Amplify Desmos Math FLORIDA

Teacher Edition

ACC 6

Volume 2

Amplify Desmos Math FLORIDA

Accelerated 6

Volume 2: Units 6–10

Teacher Edition

About Amplify

Amplify is dedicated to collaborating with educators to create learning experiences that are rigorous and riveting for all students. Amplify creates K–12 core and supplemental curriculum, assessment, and intervention programs for today's students.

A pioneer in K–12 education since 2000, Amplify is leading the way in next-generation curriculum and assessment. All of our programs provide teachers with powerful tools that help them understand and respond to the needs of every student.

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Amplify gratefully acknowledges the work of distinguished program advisors from English Learners Success Forum (ELSF), who have been integral in the development of Amplify Desmos Math. ELSF is a 501(c)(3) nonprofit organization whose mission is to expand educational equity for multilingual learners by increasing the supply of high-quality instructional materials that center their cultural and linguistic assets.

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Dear Teacher,

Welcome to Amplify Desmos Math Florida! We're so excited your school has chosen our program this year. Your hard work plays an essential role in your students' math experience, so we wanted to reach out to introduce ourselves.

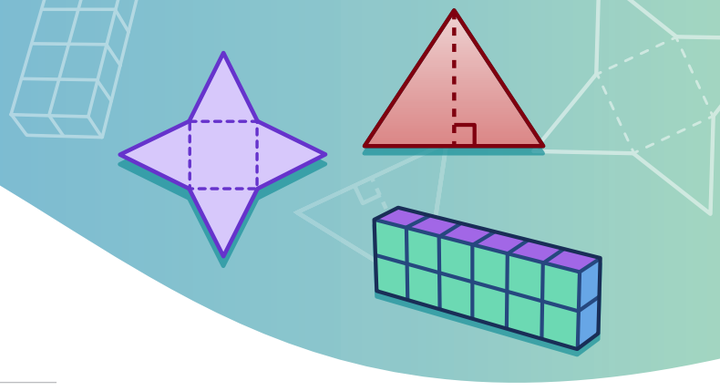
We developed the program around the idea that a structured approach to problem-based instruction builds on students' curiosity to develop lasting grade-level understanding for each and every member of your classroom. Each lesson offers opportunities for you to build on students' understanding, connect their ideas, develop their skill fluency, and empower them to ask questions, explore, and make discoveries. This year, our mission is for your students to learn math — and to love learning math!

Here's what you can expect:

- A program designed around the **Florida B.E.S.T. Standards for Mathematics**, grounded in the strands, standards, and benchmarks, and the importance of student exploration and investigation.
- **Deep and intentional alignment** to the Florida B.E.S.T. standards, Florida English Language Arts standards for K-12 Mathematics, and Florida English Language Development Standards related to the Language of Mathematics.
- **Interactive lessons** that blend paper-based and digital learning, including:
  - » Lessons that drive classroom discussions so students can work toward a shared understanding and sense of community.
  - » Manipulatives, visual models, discussions, and drawings to encourage perseverance and revision.
  - » Easy-to-follow lesson plans, with clear teaching suggestions, strategies, and Math Language Routines.
- **Lesson practice** to support fluency, help students review previous topics, and gain experience with FAST Assessment problem types.
- **Recommended differentiation moves** that meet the needs of all learners.
- **Diagnostic, formative, and summative assessments** along with lesson-level checks for understanding.
- **A caregiver resource** for each unit that includes explanations of key math concepts and problems to try with your student.

We hope your students enjoy using technology to explore math, working with classmates to solve problems, and learning new and interesting concepts. We also hope you love experiencing it with them!

—The Amplify Desmos Math Florida team



Unit 1 Area and Surface Area

Students learn to calculate areas of polygons using formulas and areas of composite figures by decomposing, rearranging, enclosing, and composing shapes. They also represent polyhedra with nets and calculate their surface areas.

Pre-Unit Check 1L

Sub-Unit 1 Area 2

		B.E.S.T. Benchmark Connection
1.01	Triangles and Rectangles | Generalizing the Area of a Triangle 3A	**1.01: MA.6.GR.2.1,** MTR.5.1
1.02	Exploring Triangles | Triangles on a Grid 10A	**1.02: MA.6.GR.2.1,** MTR.5.1
1.03	Off the Grid | Applying the Formula for the Area of a Triangle 16A	**1.03: MA.6.GR.2.1,** MTR.3.1, MTR.6.1
1.04	Letters | Area Strategies 24A	**1.04: MA.6.GR.2.1, MA.6.GR.2.2,** MTR.4.1, MTR.5.1
1.05	Breaking Down | Calculating Areas of Composite Figures Using the Area of Rectangles and Triangles 31A	**1.05: MA.6.GR.2.1, MA.6.GR.2.2,** MTR.5.1
1.06	Access to All Areas | Investigating Areas of Special Quadrilaterals 39A	**1.06: MA.7.GR.1.1,** MTR.1.1, MTR.2.1, MTR.5.1
1.07	Pile of Polygons | Investigating Polygons and Their Areas 46A	**1.07: MA.7.GR.1.2,** MTR.1.1, MTR.4.1, MTR.5.1
1.08	Shapes Inside Shapes | Area of Composite Figures 53A	**1.08: MA.7.GR.1.1, MA.7.GR.1.2,** MTR.2.1, MTR.4.1, MTR.7.1

Sub-Unit Quiz 59A

Sub-Unit 2 Surface Area 60

		B.E.S.T. Benchmark Connection
1.09	Renata's Stickers | Intro to Surface Area 61A	**1.09: MA.6.GR.2.4,** MTR.1.1, MTR.6.1
1.10	Prisms and Pyramids | Polyhedra and Their Faces 69A	**1.10: MA.6.GR.2.4,** MTR.4.1, MTR.6.1
1.11	Nothing But Nets | Nets and Surface Area on a Grid 77A	**1.11: MA.6.GR.2.4,** MTR.4.1

End-of-Unit Assessment 82B

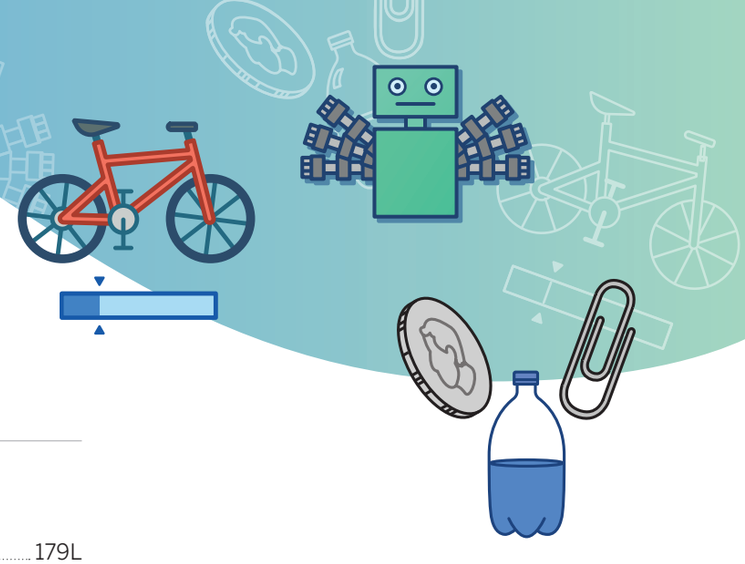
Unit 2 Introducing Ratios

Students are introduced to the concept of ratios. They represent ratios using double number lines, tables, and tape diagrams and use ratio reasoning to solve problems.

		B.E.S.T. Benchmark Connection
Pre-Unit Check	83L	
Sub-Unit 1 Ratios	84	B.E.S.T. Benchmark Connection
2.01 Ratio Relationships | Describing Ratios	85A	**2.01: MA.6.AR.3.1,** MTR.5.1
2.02 Rice Ratios | Introduction to Equivalent Ratios	93A	**2.02: MA.6.AR.3.1, MA.6.AR.3.5,** MTR.5.1
2.03 Fruit Lab | Creating Equivalent Ratios	100A	**2.03: MA.6.AR.3.3,** MTR.1.1, MTR.4.1, MTR.5.1
2.04 Balancing Act | Introduction to Double Number Lines	107A	**2.04: MA.6.AR.3.3, MA.6.AR.3.5,** MTR.4.1
2.05 Table It | Part-to-Part-to-Whole Ratio Tables	115A	**2.05: MA.6.AR.3.3,** MTR.4.1, MTR.5.1, MTR.7.1
Sub-Unit Quiz	121A	
Sub-Unit 2 Common Factors and Multiples	122	B.E.S.T. Benchmark Connection
2.06 Common Multiples | Investigating Least Common Multiple	123A	**2.06: MA.6.NSO.3.1,** MTR.1.1
2.07 Common Factors | Investigating Greatest Common Factor	131A	**2.07: MA.6.NSO.3.1,** MTR.4.1
2.08 Mixing Paint | Comparing Ratios	139A	**2.08: MA.6.AR.3.5,** MTR.4.1, MTR.6.1
Sub-Unit 3 Solving Problems With Ratios	146	B.E.S.T. Benchmark Connection
2.09 Disaster Preparation | Using Ratio Tables With Large Quantities	147A	**2.09: MA.6.AR.3.3,** MTR.2.1, MTR.7.1
2.10 Balloons | Solving Ratio Problems	155A	**2.10: MA.6.AR.3.3,** MTR.2.1, MTR.5.1
2.11 Community Life | Solving Equivalent Ratio Problems in Context	164A	**2.11: MA.6.AR.3.3,** MTR.1.1
2.12 Lunch Waste | Applying Ratio Strategies	171A	**2.12: MA.6.AR.3.5, MA.7.AR.3.2,** MTR.4.1, MTR.7.1
End-of-Unit Assessment	177A	

Unit 3 Unit Rates and Percentages

Students apply ratio reasoning from Unit 2 to unit rates and recognize that equivalent ratios have the same unit rates. They use a variety of strategies and representations of percentages to determine missing percentages, parts, and wholes.



Pre-Unit Check 179L

Sub-Unit 1 Units and Measurement 180

B.E.S.T. Benchmark Connection
3.01: Building Toward MA.6.AR.3.5, MTR.4.1
3.02: MA.6.AR.3.5, MTR.1.1, MTR.5.1

3.01 Many Measurements | Everyday Measurements 181A

3.02 Counting Classrooms | Measuring With Different Units 187A

Sub-Unit 2 Unit Rates 194

B.E.S.T. Benchmark Connection
3.03: MA.6.AR.3.2, MA.6.AR.3.5, MTR.4.1
3.04: MA.6.AR.3.2, MA.6.AR.3.5, MTR.4.1
3.05: MA.6.AR.3.2, MA.6.AR.3.3, MA.6.AR.3.5, MTR.4.1
3.06: MA.6.AR.3.2, MA.6.AR.3.3, MA.6.AR.3.5, MTR.5.1
3.07: MA.6.AR.3.2, MA.6.AR.3.3, MA.6.AR.3.5, MA.7.AR.3.2, MTR.1.1, MTR.4.1, MTR.5.1

3.03 World Records | Comparing Speeds 195A

3.04 Model Trains | Comparing Rates 202A

3.05 Soft Serve | Two Unit Rates 209A

3.06 Welcome to the Robot Factory | Using Unit Rates 216A

3.07 More Soft Serve | Solving Rate Problems 224A

Sub-Unit Quiz 230B

Sub-Unit 3 Percentages 231–232

B.E.S.T. Benchmark Connection
3.08: MA.6.NSO.3.5, MA.6.AR.3.4, MTR.1.1, MTR.3.1, MTR.4.1
3.09: MA.6.NSO.3.5, MA.6.AR.3.4, MTR.2.1, MTR.4.1
3.10: MA.6.NSO.3.5, MA.6.AR.3.4, MTR.5.1
3.11: MA.6.NSO.3.5, MA.6.AR.3.4, MTR.3.1, MTR.4.1
3.12: MA.6.NSO.3.5, MA.6.AR.3.4, MTR.3.1, MTR.4.1
3.13: MA.6.NSO.3.5, MA.6.AR.3.4, MA.7.AR.3.2, MTR.1.1, MTR.7.1

3.08 Lucky Duckies | Benchmark Percentages 233A

3.09 Bicycle Goals | Friendly Percentages 241A

3.10 What's Missing? | Solving Percentage Problems 248A

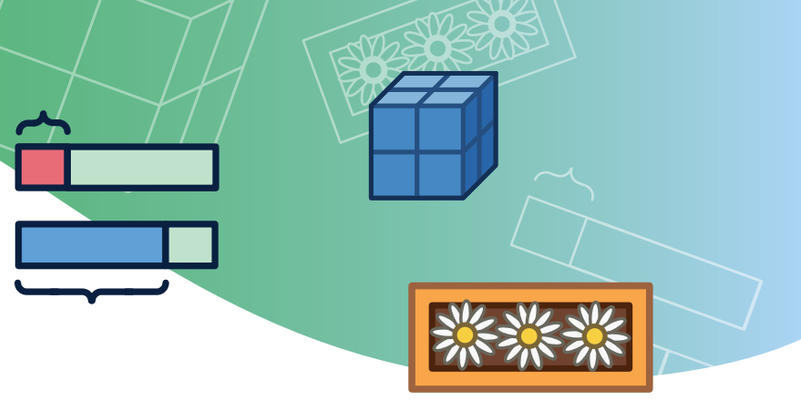
3.11 Cost Breakdown | Any Percentage of a Number 254A

3.12 More Bicycle Goals | Unknown Percentages 263A

3.13 A County as a Village | Applying Ratios, Rates, and Percentages 271A

End-of-Unit Assessment 276B

Unit 4 Multiplying and Dividing Fractions

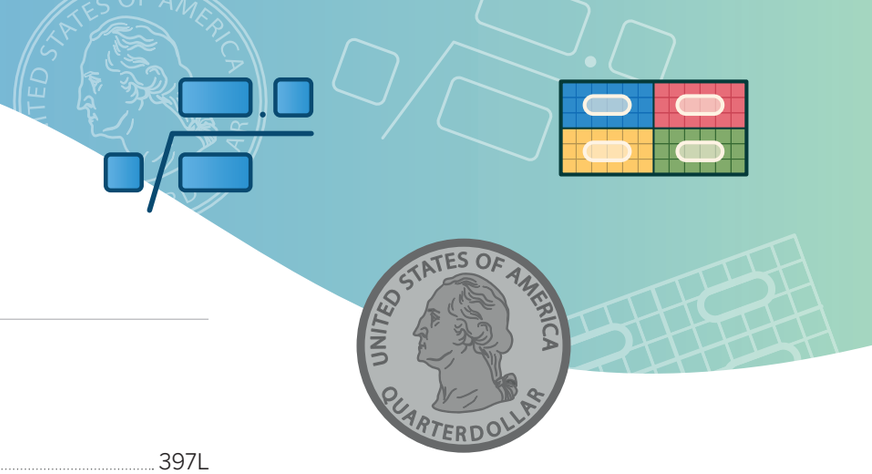


Students extend what they learned about multiplying and dividing whole numbers to multiply and divide fractions by fractions. They answer questions like "How many groups?" or "How many in 1 group?" They use a variety of strategies, such as making tape diagrams, creating common denominators, and rewriting problems using the distributive property or the reciprocal.

Pre-Unit Check	277L	
Sub-Unit 1 Multiplying Fractions	278	B.E.S.T. Benchmark Connection
4.01 Modeling Products | Multiplying Fractions	279A	**4.01: MA.6.NSO.2.2,** MTR.2.1, MTR.5.1, MTR.6.1
4.02 Mixing It Up | Multiplying Mixed Numbers	286A	**4.02: MA.6.NSO.2.2,** MTR.2.1, MTR.5.1, MTR.6.1
Sub-Unit 2 Dividing Fractions	294	B.E.S.T. Benchmark Connection
4.03 Flour Planner | How Many Groups?, Part 1	295A	**4.03: MA.6.NSO.2.2, MA.6.NSO.2.3,** MTR.2.1
4.04 Flower Planters | How Many in Each Group?	303A	**4.04: MA.6.NSO.2.3,** MTR.1.1, MTR.3.1, MTR.4.1
4.05 Garden Bricks | How Many Groups?, Part 2	311A	**4.05: MA.6.NSO.2.2, MA.6.NSO.2.3,** MTR.2.1, MTR.4.1
4.06 Fill the Gap | More or Less Than One Group?	318A	**4.06: MA.6.NSO.2.2, MA.6.NSO.2.3,** MTR.1.1, MTR.4.1, MTR.5.1
4.07 Break It Down | Using Common Denominators to Divide Fractions	326A	**4.07: MA.6.NSO.2.2, MA.6.NSO.2.3,** MTR.3.1, MTR.4.1, MTR.6.1
4.08 Potting Soil | Dividing by Unit Fractions	333A	**4.08: MA.6.NSO.2.2, MA.6.NSO.2.3,** MTR.2.1, MTR.3.1
4.09 Division Challenges | Two Strategies for Dividing Fractions	341A	**4.09: MA.6.NSO.2.2,** MTR.3.1
4.10 Action Fractions | Relating Fraction Division to Multiplication	350A	**4.10: MA.6.NSO.2.2,** MTR.1.1, MTR.3.1, MTR.4.1
4.11 Swap Meet | Division of Fractions in Context	358A	**4.11: MA.6.NSO.2.2, MA.6.NSO.2.3,** MTR.1.1, MTR.2.1, MTR.5.1
Sub-Unit Quiz	364B	
Sub-Unit 3 Area and Volume With Fractions	365–366	B.E.S.T. Benchmark Connection
4.12 Classroom Comparisons | Comparing Fractional Lengths	367A	**4.12: MA.6.NSO.2.2, MA.6.NSO.2.3,** MTR.1.1, MTR.4.1
4.13 Puzzling Areas | Areas With Fractional Dimensions	375A	**4.13: MA.6.NSO.2.2, MA.6.NSO.2.3, MA.6.GR.2.1, MA.6.GR.2.2,** MTR.1.1
4.14 Volume Challenges | Volumes With Fractional Dimensions	382A	**4.14: MA.6.NSO.2.2, MA.6.NSO.2.3, MA.6.GR.2.3,** MTR.4.1
4.15 Planter Planner | Applying Fraction Division	389A	**4.15: MA.6.NSO.2.2, MA.6.NSO.2.3, MA.6.GR.2.3,** MTR.4.1, MTR.7.1
End-of-Unit Assessment	395A	

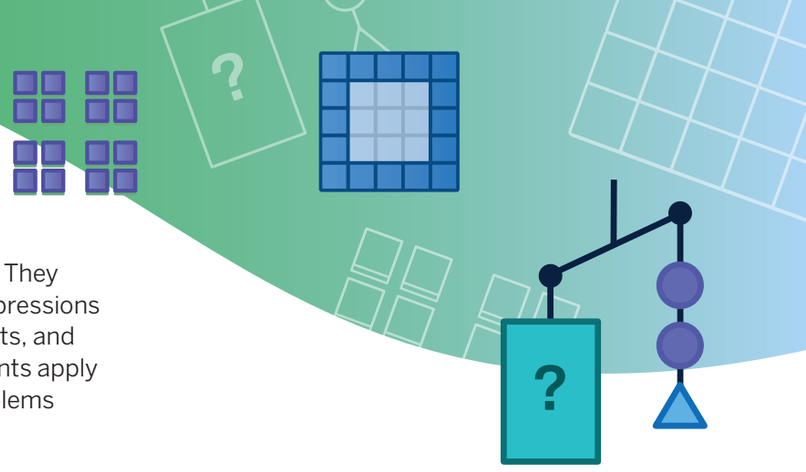
Unit 5 Decimal Arithmetic

Students develop and use a variety of strategies for multiplying and dividing decimals.



		B.E.S.T. Benchmark Connection
Pre-Unit Check	397L	
Sub-Unit 1 Multiplying Decimals	398	
5.01 Decimal Multiplication | Introduction to Multiplying Decimals	399A	**5.01: MA.6.NSO.2.1,** MTR.2.1
5.02 Garden Arrangements | Factors and Expressions	407A	**5.02: MA.6.NSO.3.2,** MTR.2.1
5.03 Multiplying With Areas | Using Area Models to Multiply Decimals	415A	**5.03: MA.6.NSO.2.1,** MTR.2.1, MTR.6.1
5.04 Multiplication Methods | Using Place Value to Multiply Decimals	423A	**5.04: MA.6.NSO.2.1,** MTR.1.1, MTR.2.1, MTR.3.1, MTR.4.1
Sub-Unit Quiz 1	429A	
Sub-Unit 2 Dividing Decimals	430	
5.05 Division Diagrams | Making Sense of Decimal Division	431A	**5.05: MA.6.NSO.2.1,** MTR.2.1
5.06 Long Division and Other Strategies | Long Division With Decimal Quotients	440A	**5.06: MA.6.NSO.2.1,** MTR.2.1
5.07 Movie Time | Dividing Decimals in Context	447A	**5.07: MA.6.NSO.2.1, MA.6.NSO.2.3,** MTR.1.1, MTR.3.1, MTR.4.1
Sub-Unit Quiz 2	452B	
Sub-Unit 3 Solving Problems With Decimals	453–454	
5.08 Budget Vehicles | Operations With Decimals in Context	455A	**5.08: MA.6.NSO.2.1, MA.6.NSO.2.3,** MTR.3.1, MTR.5.1, MTR.6.1, MTR.7.1
5.09 Coining New Representations | Percentages, Decimals, and Fractions	461A	**5.09: MA.6.NSO.2.1, MA.6.NSO.2.3, MA.6.NSO.3.5,** MTR.2.1, MTR.3.1
5.10 Grocery Prices | Percentages as Decimals	468A	**5.10: MA.6.NSO.3.5,** MTR.1.1, MTR.2.1, MTR.4.1
End-of-Unit Assessment	474B	

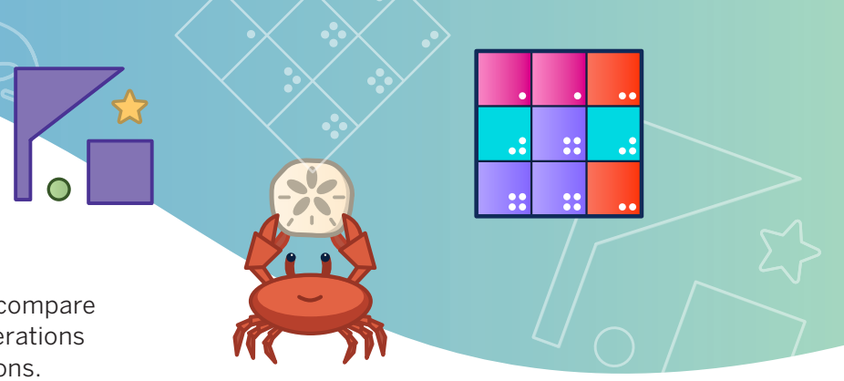
Unit 6 Expressions and Equations



Students reason about expressions and equations involving variables. They solve equations of the forms $x + p = q$ and $px = q$, write equivalent expressions using variables, make sense of variable expressions involving exponents, and create representations for relationships between two variables. Students apply what they know about ratios, proportions, and equations to solve problems involving percent change.

		B.E.S.T. Benchmark Connection
Pre-Unit Check	475L	
Sub-Unit 1 Solving Equations	476	B.E.S.T. Benchmark Connection
6.01 Weight for It | Reasoning About Unknown Values	477A	**6.01: MA.6.AR.2.2, MA.6.AR.2.3,** MTR.2.1, MTR.4.1, MTR.5.1
6.02 Five Equations | Tape Diagrams, Equations, Situations	485A	**6.02: MA.6.AR.2.1, MA.6.AR.2.2, MA.6.AR.2.3,** MTR.2.1, MTR.4.1, MTR.5.1
6.03 Hanging Around | Introduction to Balanced Hangers	491A	**6.03: MA.6.AR.2.2, MA.6.AR.2.3,** MTR.2.1, MTR.4.1, MTR.5.1
6.04 Swap and Solve | Solving Equations in Context	499A	**6.04: MA.6.AR.2.1, MA.6.AR.2.2, MA.6.AR.2.3,** MTR.2.1, MTR.4.1, MTR.5.1
6.05 Solutions of a Different Kind | Solving Equations with Decimals and Fractions	505A	**6.05: MA.6.AR.2.4,** MTR.3.1, MTR.4.1, MTR.5.1
Sub-Unit 2 Equivalent Expressions	514	B.E.S.T. Benchmark Connection
6.06 Vari-apples | Introduction to Variable Expressions	515A	**6.06: MA.6.AR.1.1, MA.6.AR.1.3,** MTR.2.1, MTR.4.1, MTR.5.1, MTR.7.1
6.07 Border Tiles | Equivalent Expressions	524A	**6.07: MA.6.AR.1.4, MA.7.AR.1.2,** MTR.2.1, MTR.4.1, MTR.5.1
6.08 Products and Sums | Distributive Property, Part 1	532A	**6.08: MA.6.AR.1.4,** MTR.4.1
6.09 Equivalent Expressions | Distributive Property, Part 2	539A	**6.09: MA.6.AR.1.4, MA.7.AR.1.2,** MTR.4.1
Sub-Unit Quiz	544B	
Sub-Unit 3 Expressions Involving Exponents	545–546	B.E.S.T. Benchmark Connection
6.10 Powers | What Are Exponents?	547A	**6.10: MA.6.NSO.3.3, MA.6.NSO.3.4,** MTR.2.1, MTR.3.1
6.11 Exponent Expressions | Exponents and Order of Operations	557A	**6.11: MA.6.NSO.3.3,** MTR.1.1, MTR.2.1, MTR.3.1, MTR.4.1
Sub-Unit 4 Applying Percentages	564	B.E.S.T. Benchmark Connection
6.12 More and Less | Percent Increase and Decrease With Tape Diagrams	565A	**6.12: MA.6.NSO.3.5, MA.7.AR.3.1,** MTR.2.1
6.13 All the Equations | Percent Increase and Decrease With Equations	573A	**6.13: MA.6.NSO.3.5, MA.7.AR.3.1,** MTR.1.1
6.14 100% | Percent Increase and Decrease With Double Number Lines	580A	**6.14: MA.6.NSO.1.2, MA.7.AR.3.1,** MTR.1.1, MTR.2.1
6.15 Tax, Tips, and Fees | Multistep Percent Problems	587A	**6.15: MA.6.NSO.3.5, MA.7.AR.3.1,** MTR.3.1, MTR.4.1
6.16 How Simple, How Interesting | Calculating Simple Interest Using a Formula	595A	**6.16: MA.6.NSO.3.5, MA.7.AR.3.1,** MTR.3.1, MTR.7.1
6.17 Bookcase Builder | Percent Error	601A	**6.17: MA.7.NSO.3.5, MA.7.AR.3.1,** MTR.3.1, MTR.4.1
End-of-Unit Assessment	609A	

Unit 7 Positive and Negative Numbers

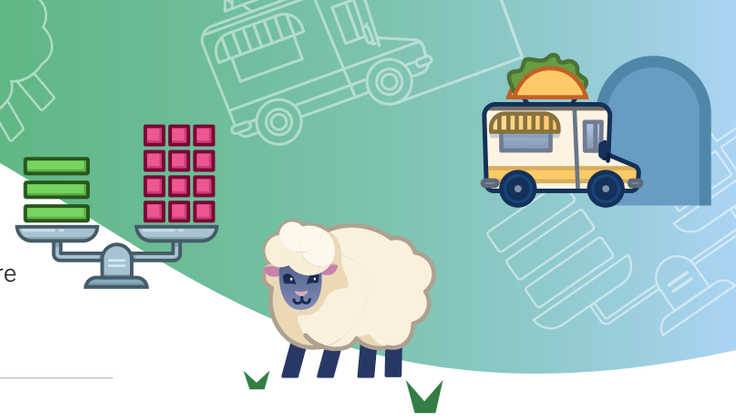


Students explore positive and negative numbers in several ways: on a number line, in expressions, and on the coordinate plane. They compare these numbers, as well as their absolute values, and use order of operations to evaluate numerical expressions and generate equivalent expressions.

		B.E.S.T. Benchmark Connection
Pre-Unit Check 611N		
Sub-Unit 1 Signed Numbers and Absolute Values 612		B.E.S.T. Benchmark Connection
7.01 Can You Dig It? | Positive and Negative Numbers 613A		**7.01: MA.6.NSO.1.1, MA.6.NSO.1.2, MA.6.NSO.4.1,** MTR.1.1, MTR.5.1
7.02 Digging Deeper | Positions on the Number Line 620A		**7.02: MA.6.NSO.1.1, MA.6.NSO.1.2,** MTR.1.1, MTR.4.1
7.03 Sub-Zero | Positive and Negative Numbers in Context 627A		**7.03: MA.6.NSO.1.1, MA.6.NSO.1.2,** MTR.2.1, MTR.3.1, MTR.6.1
7.04 Order in the Class | Comparing Numbers 634A		**7.04: MA.6.NSO.1.1, MA.6.NSO.1.2, MA.6.NSO.3.5,** MTR.1.1, MTR.4.1
7.05 Distance on the Number Line | Introduction to Absolute Value 641A		**7.05: MA.6.NSO.1.1, MA.6.NSO.1.4,** MTR.2.1, MTR.3.1
7.06 We've Got Game(s) | Comparing Numbers and Distances From Zero 649A		**7.06: MA.6.NSO.1.1, MA.6.NSO.1.2, MA.6.NSO.1.3, MA.6.NSO.1.4,** MTR.2.1, MTR.3.1
7.07 Floats and Anchors | Representing Adding and Subtracting 656A		**7.07: MA.6.NSO.4.1,** MTR.1.1, MTR.4.1, MTR.5.1
7.08 More Floats and Anchors | Adding and Subtracting Positive and Negative Numbers 664A		**7.08: MA.6.NSO.4.1,** MTR.1.1, MTR.3.1, MTR.5.1
7.09 Up, Up, and Away! | Multiplying and Dividing Integers 671A		**7.09: MA.6.NSO.4.2,** MTR.2.1
Sub-Unit Quiz 679A		
Sub-Unit 2 Operations With Rational Numbers 680		B.E.S.T. Benchmark Connection
7.10 Bumpers | Adding and Subtracting on the Number Line 681A		**7.10: MA.7.NSO.2.2,** MTR.3.1, MTR.4.1, MTR.5.1
7.11 Back in Time | Position, Rate, and Time 688A		**7.11: MA.7.NSO.2.1, MA.7.NSO.2.2, MA.7.NSO.2.3,** MTR.3.1, MTR.5.1, MTR.7.1
7.12 Speeding Turtles | Dividing Rational Numbers 696A		**7.12: MA.7.NSO.2.2, MA.7.NSO.2.3,** MTR.3.1, MTR.5.1, MTR.7.1
7.13 Order Matters | Evaluating Numerical Expressions 703A		**7.13: MA.7.NSO.2.1, MA.7.NSO.2.2, MA.7.NSO.2.3,** MTR.1.1, MTR.5.1
7.14 The Same but Different | Determining Whether Expressions Are Equivalent 711A		**7.14: MA.7.NSO.2.1, MA.7.NSO.2.2, MA.7.AR.1.2,** MTR.2.1, MTR.4.1
7.15 Collect the Squares | Adding and Subtracting Expressions 718A		**7.15: MA.7.NSO.2.1, MA.7.NSO.2.2, MA.7.AR.1.1,** MTR.3.1, MTR.4.1, MTR.5.1
Sub-Unit 3 The Coordinate Plane 726		B.E.S.T. Benchmark Connection
7.16 Sand Dollar Search | Points in the Coordinate Plane 727A		**7.16: MA.6.GR.1.1,** MTR.1.1, MTR.4.1
7.17 The A-maze-ing Coordinate Plane | Practice Plotting 735A		**7.17: MA.6.GR.1.1,** MTR.1.1, MTR.4.1
7.18 Polygon Maker | Polygons in the Plane 744A		**7.18: MA.6.GR.1.1, MA.6.GR.1.2, MA.6.GR.1.3,** MTR.1.1, MTR.4.1
7.19 Graph Telephone | The Coordinate Plane in Context 752A		**7.19: MA.6.GR.1.1, MA.6.GR.1.3,** MTR.4.1, MTR.5.1, MTR.7.1
End-of-Unit Assessment 757A		

Unit 8 Equations and Inequalities With Integers

Students solve equations with positive and negative numbers. They explore solutions to inequalities and graph the solutions on a number line.



Pre-Unit Check 759L

Sub-Unit 1 Equations 760

B.E.S.T. Benchmark Connection
8.01: MA.6.AR.2.2, MTR.2.1, MTR.5.1, MTR.7.1
8.02: MA.6.AR.2.3, MTR.2.1, MTR.7.1

8.01 Plus or Minus | Solving Equations Involving Addition and Subtraction 761A

8.02 Multiply, Divide, and Conquer | Solving Equations Involving Multiplication and Division 768A

Sub-Unit Quiz 774B

Sub-Unit 2 Inequalities 775–776

B.E.S.T. Benchmark Connection
8.03: MA.6.AR.1.2, MTR.2.1, MTR.3.1
8.04: MA.6.NSO.1.1, MA.6.AR.1.2, MA.6.AR.2.1, MTR.2.1, MTR.3.1
8.05: MA.6.AR.1.2, MA.6.AR.2.1, MTR.1.1, MTR.4.1
8.06: MA.7.NSO.2.2, MA.7.AR.2.1, MTR.2.1, MTR.3.1, MTR.4.1
8.07: MA.7.NSO.2.2, MA.7.AR.2.1, MTR.3.1, MTR.4.1, MTR.6.1

8.03 Tunnel Travels | Graphing Inequalities 777A

8.04 Shira's Solutions | Solutions to Inequalities 784A

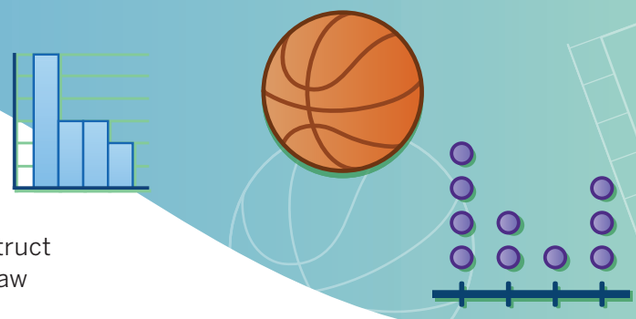
8.05 Comparing Weights | Writing Inequalities 792A

8.06 Unbalanced Hangers | Solving Inequalities 800A

8.07 Shira the Sheep | Solving Inequalities With Positive and Negative Numbers 808A

End-of-Unit Assessment 815A

Unit 9 Describing Data



Students visualize data using dot plots and histograms before calculating measures of center — mean, mode, and median — and measures of variability — range and interquartile range (IQR). They construct box plots and interpret these measures in context and use samples to draw conclusions about and compare populations.

		B.E.S.T. Benchmark Connection
Pre-Unit Check	817N	
Sub-Unit 1 Visualizing Data	818	B.E.S.T. Benchmark Connection
9.01 Screen Time | Asking Questions, Collecting Data, and Making Claims	819A	**9.01: MA.6.DP.1.1,** MTR.1.1, MTR.4.1
9.02 Survey Says . . . | Using Data to Generate Statistical Questions	827A	**9.02: MA.6.DP.1.1,** MTR.1.1, MTR.4.1
9.03 The Plot Thickens | Introduction to Histograms	834A	**9.03: MA.6.DP.1.5,** MTR.4.1, MTR.6.1
9.04 What Does It All Mean? | Describing and Interpreting Histograms and Dot Plots	841A	**9.04: MA.6.DP.1.4,** MTR.4.1, MTR.7.1
9.05 DIY Histograms | Creating Histograms	848A	**9.05: MA.6.DP.1.4, MA.6.DP.1.5,** MTR.4.1, MTR.5.1, MTR.7.1
Sub-Unit Quiz	845B	
Sub-Unit 2 Measuring Data: Mean, Median, Mode, IQR, and Range	855–856	B.E.S.T. Benchmark Connection
9.06 Snack Time | Introduction to the Mean	857A	**9.06: MA.6.DP.1.2,** MTR.2.1, MTR.3.1, MTR.6.1
9.07 Modes of Weather | Introducing the Mode	865A	**9.07: MA.6.DP.1.2,** MTR.4.1, MTR.7.1
9.08 Toy Cars | Introducing the Median	872A	**9.08: MA.6.DP.1.2,** MTR.2.1, MTR.3.1, MTR.6.1
9.09 In the News | Comparing Measures of Center	879A	**9.09: MA.6.DP.1.6,** MTR.1.1, MTR.4.1
9.10 Hoops | Calculating and Interpreting the Range	887A	**9.10: MA.6.DP.1.2, MA.6.DP.1.6,** MTR.4.1, MTR.5.1, MTR.7.1
9.11 Pumpkin Patch | Introduction to Quartiles	894A	**9.11: MA.6.DP.1.3,** MTR.1.1, MTR.4.1
9.12 Car, Plane, Bus, or Train? | Box Plots, IQR, and Range	903A	**9.12: MA.6.DP.1.3, MA.6.DP.1.5, MA.6.DP.1.6,** MTR.1.1, MTR.4.1
Sub-Unit Quiz	910B	
Sub-Unit 3 Sampling	911–912	B.E.S.T. Benchmark Connection
9.13 Car, Bike, or Train? | Using Mean, Median, and Range to Compare Groups	913A	**9.13: MA.7.NSO.2.2, MA.7.NSO.2.3, MA.7.DP.1.1, MA.7.DP.1.2,** MTR.4.1, MTR.7.1
9.14 Crab Island | Sampling From Large Populations	920A	**9.14: MA.7.DP.1.3,** MTR.1.1, MTR.4.1, MTR.7.1
9.15 Flower Power | Using Percentages to Predict Populations	928A	**9.15: MA.7.NSO.2.2, MA.7.NSO.2.3, MA.7.AR.3.2, MA.7.AR.4.5, MA.7.DP.1.3,** MTR.4.1, MTR.5.1, MTR.6.1
9.16 Plots and Samples | Using Statistical Measures to Make Predictions	935A	**9.16: MA.7.DP.1.1, MA.7.DP.1.2,** MTR.5.1, MTR.6.1
9.17 School Newspaper | Comparing Populations	943A	**9.17: MA.7.DP.1.2,** MTR.4.1, MTR.7.1
End-of-Unit Assessment	949A	

Unit 10 Probability

Students learn about probability as a way to describe the likelihood of unknown events and estimate the probability of real-world situations.

Pre-Unit Check 951L

Sub-Unit 1 Probability 952

10.01 How Likely? | Chance Experiments 953A

10.02 Prob-bear-bilities | Intro to Probability 959A

10.03 Mystery Bag | Predicting Sample Spaces 968A

10.04 Spin Class | Repeated Experiments 976A

10.05 Is It Fair? | Comparing Probabilities and Experiments 984A

End-of-Unit Assessment 991A

B.E.S.T. Benchmark Connection
10.01: MA.7.DP.2.1, MTR.2.1, MTR.3.1
10.02: MA.7.DP.2.1, MA.7.DP.2.2, MA.7.DP.2.3, MTR.2.1, MTR.3.1
10.03: MA.6.NSO.3.5, MA.7.DP.2.2, MA.7.DP.2.3, MA.7.DP.2.4, MTR.3.1, MTR.6.1
10.04: MA.6.NSO.3.5, MA.7.DP.2.2, MA.7.DP.2.3, MA.7.DP.2.4, MTR.1.1, MTR.4.1
10.05: MA.6.NSO.3.5, MA.7.DP.2.3, MA.7.DP.2.4, MTR.5.1, MTR.6.1

A curiosity-driven program that builds students' lifelong math proficiency

As we developed Amplify Desmos Math Florida, we asked ourselves: how can we support teachers in creating a collaborative classroom of learners excited about math?

With that question in mind, we built the program around four core tenets:

A structured approach to problem-based learning

The program thoughtfully combines conceptual understanding, procedural fluency, and application. Each lesson is designed to tell a story by posing problems that invite a variety of approaches before guiding students to synthesize their understanding of the learning goals.

The Teacher Edition provides guidance for teachers to anticipate and monitor strategies students may use, select and sequence students' ideas, and orchestrate productive discussions to help students make connections between their own ideas and those of their classmates.[1]

Proficiency Progression

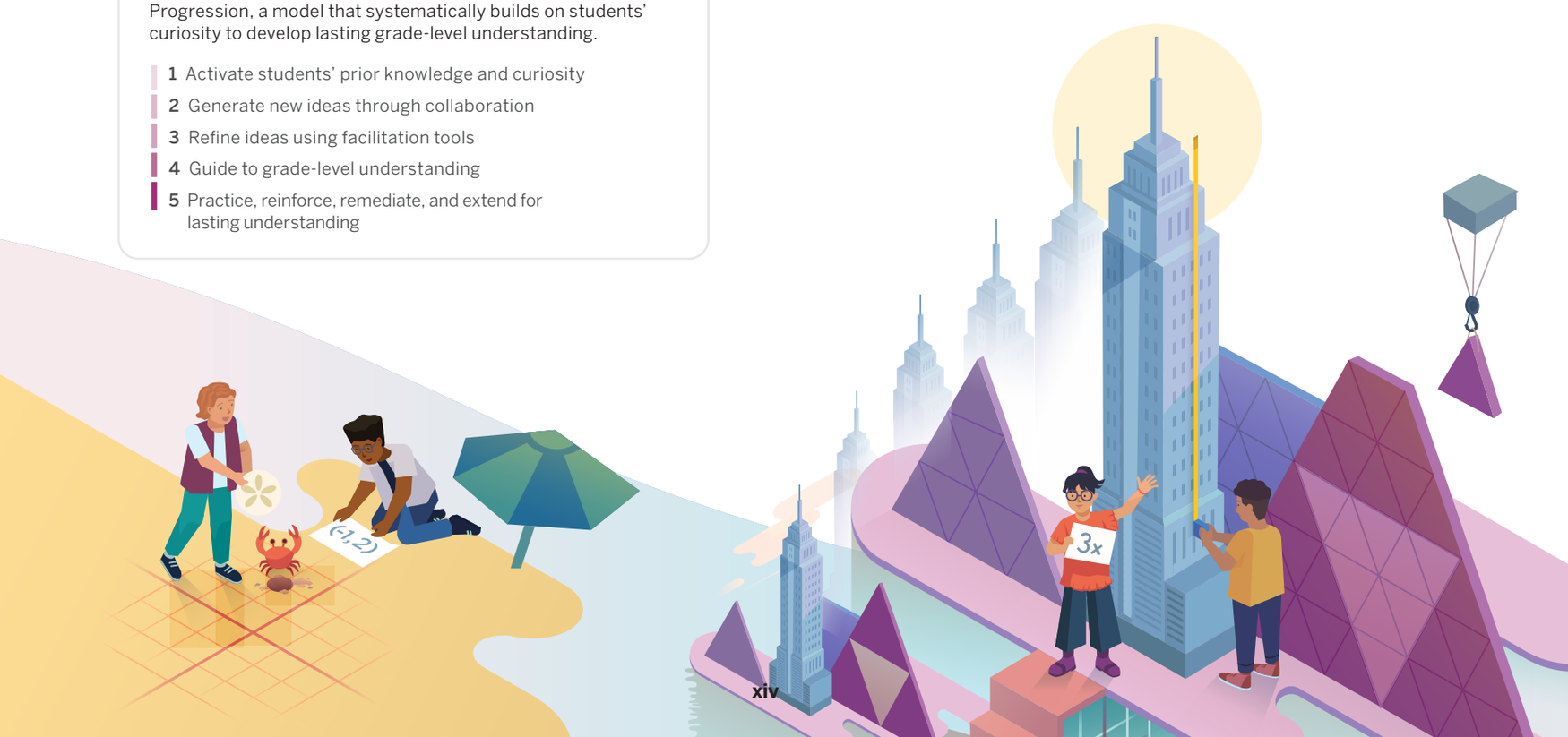
Lessons are designed around what we call the Proficiency Progression, a model that systematically builds on students' curiosity to develop lasting grade-level understanding.

1. Activate students' prior knowledge and curiosity
2. Generate new ideas through collaboration
3. Refine ideas using facilitation tools
4. Guide to grade-level understanding
5. Practice, reinforce, remediate, and extend for lasting understanding

Access to grade-level math for every student, every day

Tasks in each lesson are thoughtfully sequenced so that all students can engage with the math each day without any roadblocks. Every lesson includes suggestions for accessibility and differentiation to support, strengthen, and stretch students' understanding.

We also provide additional resources that integrate seamlessly with core instruction, including a suite of assessments, tailored practice resources that adjust to students' learning, and other intervention solutions. Cohesive differentiation and intervention resources support and challenge students on their path toward deeper understanding of the learning goals, ensuring all students a path toward a deeper understanding of the learning goals of the course.



Student thinking is valuable and can be made evident

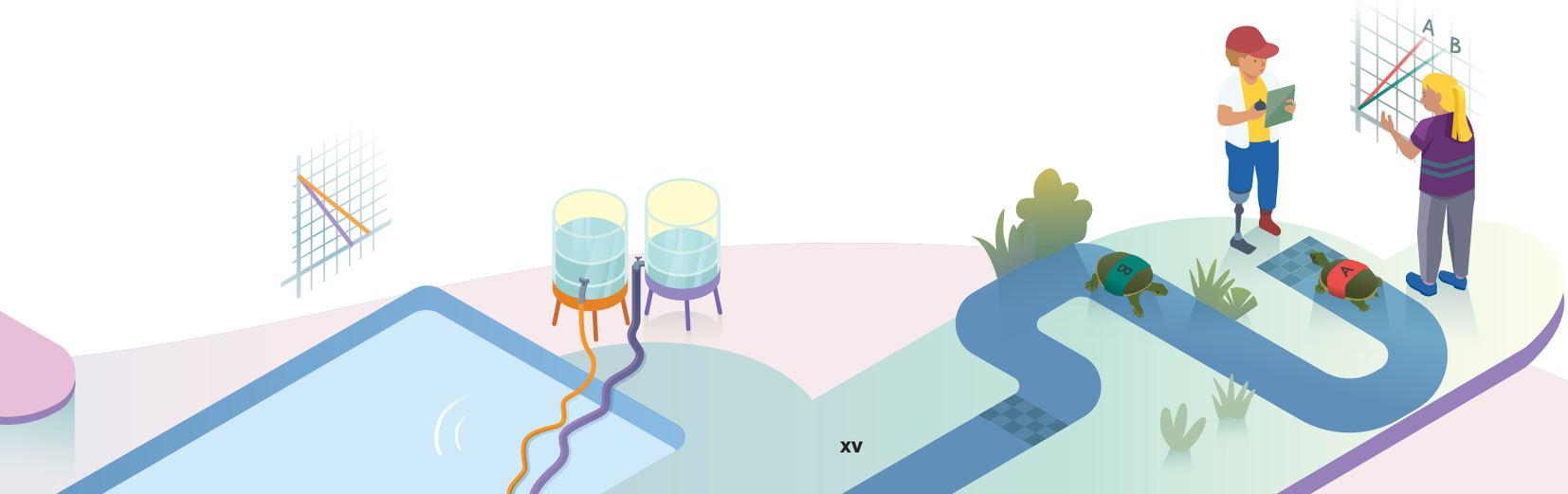
Students take an active role in developing their own ideas first and then synthesize them as a class. To guide the learning process, students see each other's thinking, engage in conversations, and connect to each other through the understanding that they can use math to make sense of the world. This fuels classroom conversations and a shared understanding of math.

Math that motivates students

Picture a classroom where students are so eagerly engaged in a lesson, they wish it would not end. It is buzzing with the sounds of natural curiosity.

Our curriculum supports collaborative classrooms, invites mathematical creativity, and evokes wonder, empowering students to see themselves and their classmates as having brilliant mathematical ideas.

[1] Smith, M.S., & Stein, M.K. (2018). *5 practices for orchestrating productive mathematics discussions* (2nd ed.). SAGE Publications.



Scope and Sequence | Grade 6–Accelerated 6

Volume 1

	Unit 1	Unit 2	Unit 3	Unit 4
Grade 6 127 days	Area and Surface Area 8 instructional days 3 assessment days	Introducing Ratios 12 instructional days 3 assessment days, Math in Action	Unit Rates and Percentages 13 instructional days 3 assessment days	Multiplying and Dividing Fractions 15 instructional days 3 assessment days, Math in Action
Grade 7 116 days	Scale Drawings 10 instructional days 3 assessment days	Introducing Proportional Relationships 13 instructional days 4 assessment days, Math in Action	Measuring Circles 6 instructional days 3 assessment days	Proportional Relationships and Percentages 12 instructional days 4 assessment days, Math in Action
Grade 8 120 days	Rigid Transformations and Congruence 10 instructional days 3 assessment days	Dilations, Similarity, and Slope 10 instructional days 4 assessment days, Math in Action	Proportional and Linear Relationships 11 instructional days 3 assessment days	Linear Equations and Linear Systems 10 instructional days 4 assessment days, Math in Action
Algebra 1 130 days	Linear Equations and Inequalities 18 instructional days 4 assessment days, Math in Action	Describing Data 17 instructional days 3 assessment days	Describing Functions 12 instructional days 4 assessment days, Math in Action	Describing Functions 10 instructional days 3 assessment days

Volume 1

	Unit 1	Unit 2	Unit 3	Unit 4	Unit 5
Accelerated 6 160 days	Area and Surface Area 11 instructional days 3 assessment days 14 days total	Introducing Ratios 12 instructional days 4 assessment days, Math in Action 16 days total	Unit Rates and Percentages 13 instructional days 3 assessment days 16 days total	Multiplying and Dividing Fractions 15 instructional days 4 assessment days, Math in Action 19 days total	Decimal Arithmetic 10 instructional days 4 assessment days 14 days total
Accelerated 7 164 days	Rigid Transformations and Congruence 10 instructional days 3 assessment Days 13 days total	Scale Factors, Dilations, Similarity, and Slope 16 instructional days 5 assessment days, Math in Action 21 days total	Introducing Proportional Relationships 11 instructional days 3 assessment Days 14 days total	Linear Relationships and Equations 8 instructional days 4 assessment days, Math in Action 12 days total	Linear Equations and Linear Systems 15 instructional days 3 assessment Days 18 days total

Volume 2

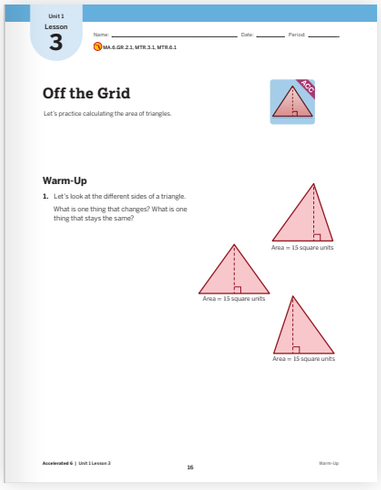
Unit 5	Unit 6	Unit 7	Unit 8	Unit 9
Decimal Arithmetic 10 instructional days 3 assessment Days	Expressions and Equations 11 instructional days 4 assessment Days, Math in Action	Positive and Negative Numbers 18 instructional days 3 assessment Days	Describing Data 12 instructional days 4 assessment Days, Math in Action	
Positive and Negative Rational Numbers 10 instructional days 3 assessment days	Equations, Expressions, and Inequalities 16 instructional days 4 assessment days, Math in Action	Polygons and Cylinders 8 instructional days 3 assessment days	Probability and Sampling 13 instructional days 4 assessment days, Math in Action	
Functions 7 instructional days 3 assessment days	Associations in Data 9 instructional days 4 assessment days, Math in Action	Exponents and Scientific Notation 13 instructional days 3 assessment days	The Pythagorean Theorem and Irrational Numbers 15 instructional days 4 assessment days, Math in Action	Probability 5 instructional days 2 assessment days
Exponential Functions 15 instructional days 4 assessment days, Math in Action	Quadratic Functions 15 instructional days 3 assessment days	Quadratic Equations 18 instructional days 4 assessment days, Math in Action		

Volume 2

Unit 6	Unit 7	Unit 8	Unit 9	Unit 10	Unit 11
Expressions and Equations 17 instructional days 4 assessment days, Math in Action 21 days total	Positive and Negative Numbers 19 instructional days 3 assessment days 22 days total	Equations and Inequalities with Integers 7 instructional days 4 assessment days, Math in Action 11 days total	Positive and Negative Numbers 17 instructional days 3 assessment days 20 days total	Describing Data 5 instructional days 2 assessment days 7 days total	
Functions 7 instructional days 4 assessment days, Math in Action 11 days total	Circles and Cylinders 9 instructional days 3 assessment Days 12 days total	Data 12 instructional days 4 assessment days, Math in Action 16 days total	The Pythagorean Theorem and Irrational Numbers 18 instructional days 3 assessment Days 21 days total	Polygons and Cylinders 16 instructional days 3 assessment days 19 days total	Probability 5 instructional days 2 assessment Days 7 days total

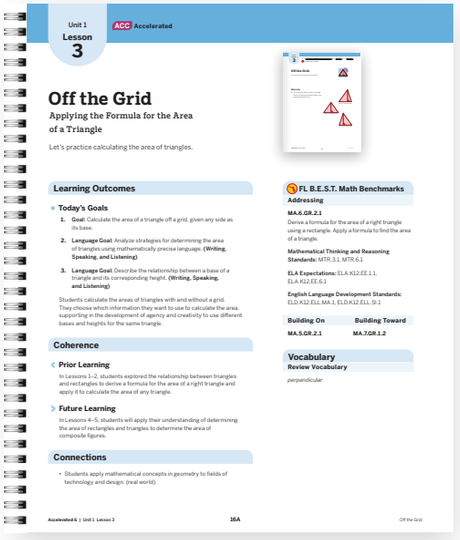
Program Resources

For Students


Unit 1
Lesson 3
Name: Date: Period:
MA.6.GR.2.1, MTR.3.1, MTR.6.1
Off the Grid
Let's practice calculating the area of triangles.
ACC
Warm-Up
1. Let's look at the different sides of a triangle.
What is one thing that changes? What is one thing that stays the same?
Area = 15 square units
Area = 15 square units
Area = 15 square units
Accelerated 6 | Unit 1 Lesson 3
16
Warm-Up

- Student Edition (two-volume)

For Teachers


Unit 1
Lesson 3
ACC Accelerated
Off the Grid
Applying the Formula for the Area of a Triangle
Let's practice calculating the area of triangles.
Learning Outcomes
Today's Goals
1. Goal: Calculate the area of a triangle off a grid, given any side as its base.
2. Language Goal: Analyze strategies for determining the area of triangles using mathematically precise language. (Writing, Speaking, and Listening)
3. Language Goal: Describe the relationship between a base of a triangle and its corresponding height. (Writing, Speaking, and Listening)
Students calculate the areas of triangles with and without a grid. They choose which information they want to use to calculate the area, supporting in the development of agency and creativity to use different bases and heights for the same triangle.
Coherence
Prior Learning
In Lessons 1–2, students explored the relationship between triangles and rectangles to derive a formula for the area of a right triangle and apply it to calculate the area of any triangle.
Future Learning
In Lessons 4–5, students will apply their understanding of determining the area of rectangles and triangles to determine the area of composite figures.
Connections
• Students apply mathematical concepts in geometry to fields of technology and design. (real world)
FL B.E.S.T. Math Benchmarks
Addressing
MA.6.GR.2.1
Derive a formula for the area of a right triangle using a rectangle. Apply a formula to find the area of a triangle.
Mathematical Thinking and Reasoning Standards: MTR.3.1, MTR.6.1
ELA Expectations: ELA.K12.EE.1.1, ELA.K12.EE.6.1
English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1
Building On
MA.5.GR.2.1
Building Toward
MA.7.GR.1.2
Vocabulary
Review Vocabulary
perpendicular
Accelerated 6 | Unit 1 Lesson 3
16A
Off the Grid

- Teacher Edition (two-volume)

Additional Resources

Assessment and Lesson Resources

- Unit Assessments
- Show What You Know Lesson Assessments
- Rubrics and Teacher Answer Keys
- Activity Sheets and Cards

Intervention and Extension Resources

- Mini-Lessons
- Prior grade Mini-Lessons
- Extensions

Optional: Manipulative Kits

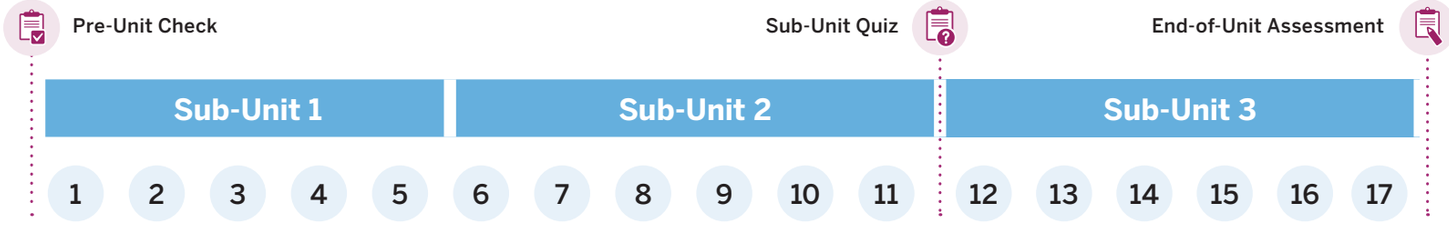
hand2mind

Program Architecture

Course

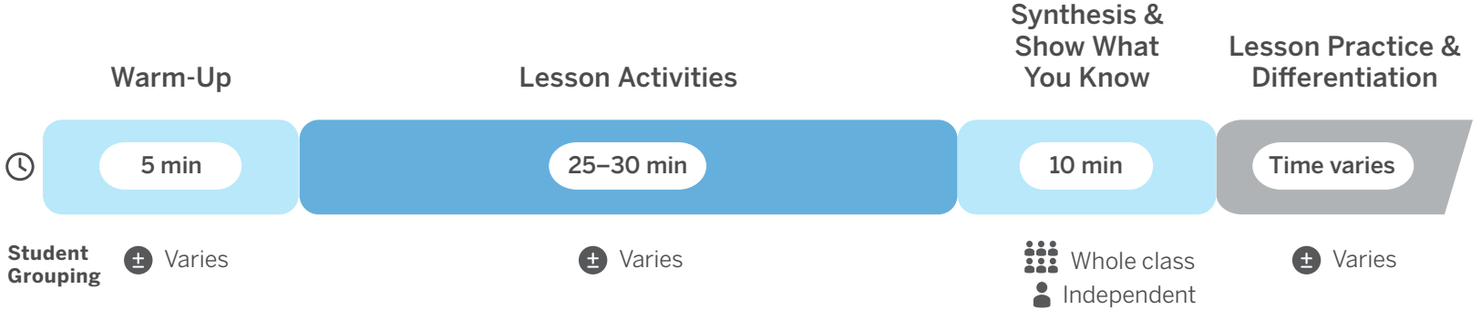
Note: The number of lessons varies from unit to unit. See Scope and Sequence pages for the full program scope.

Unit



Note: The number of sub-units, quizzes, and lessons within each unit varies. This depiction shows the general structure of a unit. See the course Table of Contents for more details.

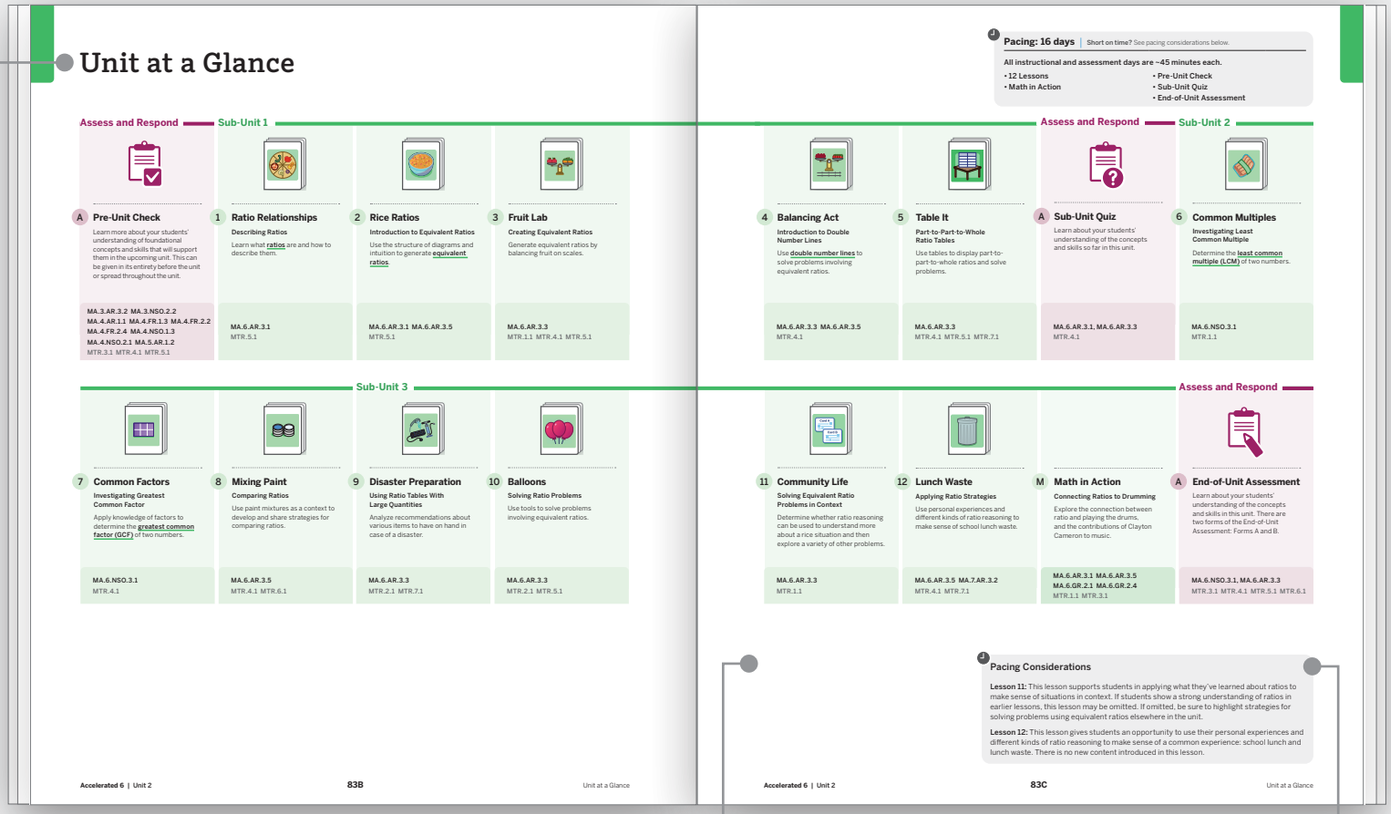
Lesson



Navigating This Program

Unit & Sub-Unit Resources

Each unit includes a range of resources designed to support teachers in thinking through the progression of mathematics that students will engage with over the course of the unit. These resources can support teachers in their unit planning, as well as choices they make in response to students' thinking, strengths, and needs that arise over the course of the unit.



Unit at a Glance

Assess and Respond — Sub-Unit 1

A Pre-Unit Check
Learn more about your students' understanding of foundational concepts and skills that will support them in the upcoming unit. This can be given in its entirety before the unit or spread throughout the unit.
MA.3.AR.3.2 MA.3.NSO.2.2 MA.4.AR.1.1 MA.4.FR.1.3 MA.4.FR.2.2 MA.4.FR.2.4 MA.4.NSO.1.3 MA.4.NSO.2.1 MA.5.AR.1.2
MTR.3.1 MTR.4.1 MTR.5.1

1 Ratio Relationships
Describing Ratios
Learn what ratios are and how to describe them.
MA.6.AR.3.1
MTR.5.1

2 Rice Ratios
Introduction to Equivalent Ratios
Use the structure of diagrams and intuition to generate equivalent ratios.
MA.6.AR.3.1 MA.6.AR.3.5
MTR.5.1

3 Fruit Lab
Creating Equivalent Ratios
Generate equivalent ratios by balancing fruit on scales.
MA.6.AR.3.3
MTR.1.1 MTR.4.1 MTR.5.1

Sub-Unit 3

7 Common Factors
Investigating Greatest Common Factor
Apply knowledge of factors to determine the greatest common factor (GCF) of two numbers.
MA.6.NSO.3.1
MTR.4.1

8 Mixing Paint
Comparing Ratios
Use paint mixtures as a context to develop and share strategies for comparing ratios.
MA.6.AR.3.5
MTR.4.1 MTR.6.1

9 Disaster Preparation
Using Ratio Tables With Large Quantities
Analyze recommendations about various items to have on hand in case of a disaster.
MA.6.AR.3.3
MTR.2.1 MTR.7.1

10 Balloons
Solving Ratio Problems
Use tools to solve problems involving equivalent ratios.
MA.6.AR.3.3
MTR.2.1 MTR.5.1

Accelerated 6 | Unit 2 83B Unit at a Glance

Pacing: 16 days | Short on time? See pacing considerations below.
All instructional and assessment days are ~45 minutes each.
• 12 Lessons
• Math in Action
• Pre-Unit Check
• Sub-Unit Quiz
• End-of-Unit Assessment

Assess and Respond — Sub-Unit 2

4 Balancing Act
Introduction to Double Number Lines
Use double number lines to solve problems involving equivalent ratios.
MA.6.AR.3.3 MA.6.AR.3.5
MTR.4.1

5 Table It
Part-to-Part-to-Whole Ratio Tables
Use tables to display part-to-part-to-whole ratios and solve problems.
MA.6.AR.3.3
MTR.4.1 MTR.5.1 MTR.7.1

A Sub-Unit Quiz
Learn about your students' understanding of the concepts and skills so far in this unit.
MA.6.AR.3.1, MA.6.AR.3.3
MTR.4.1

6 Common Multiples
Investigating Least Common Multiple
Determine the least common multiple (LCM) of two numbers.
MA.6.NSO.3.1
MTR.1.1

Assess and Respond

11 Community Life
Solving Equivalent Ratio Problems in Context
Determine whether ratio reasoning can be used to understand more about a rice situation and then explore a variety of other problems.
MA.6.AR.3.3
MTR.1.1

12 Lunch Waste
Applying Ratio Strategies
Use personal experiences and different kinds of ratio reasoning to make sense of school lunch waste.
MA.6.AR.3.5 MA.7.AR.3.2
MTR.4.1 MTR.7.1

M Math in Action
Connecting Ratios to Drumming
Explore the connection between ratio and playing the drums, and the contributions of Clayton Cameron to music.
MA.6.AR.3.1 MA.6.AR.3.5 MA.6.GR.2.1 MA.6.GR.2.4
MTR.1.1 MTR.3.1

A End-of-Unit Assessment
Learn about your students' understanding of the concepts and skills in this unit. There are two forms of the End-of-Unit Assessment: Forms A and B.
MA.6.NSO.3.1, MA.6.AR.3.3
MTR.3.1 MTR.4.1 MTR.5.1 MTR.6.1

Pacing Considerations

Lesson 11: This lesson supports students in applying what they've learned about ratios to make sense of situations in context. If students show a strong understanding of ratios in earlier lessons, this lesson may be omitted. If omitted, be sure to highlight strategies for solving problems using equivalent ratios elsewhere in the unit.

Lesson 12: This lesson gives students an opportunity to use their personal experiences and different kinds of ratio reasoning to make sense of a common experience: school lunch and lunch waste. There is no new content introduced in this lesson.

Accelerated 6 | Unit 2 83C Unit at a Glance

Every unit has a **Unit at a Glance** page that shows teachers everything they need to know to get started planning out their upcoming unit.

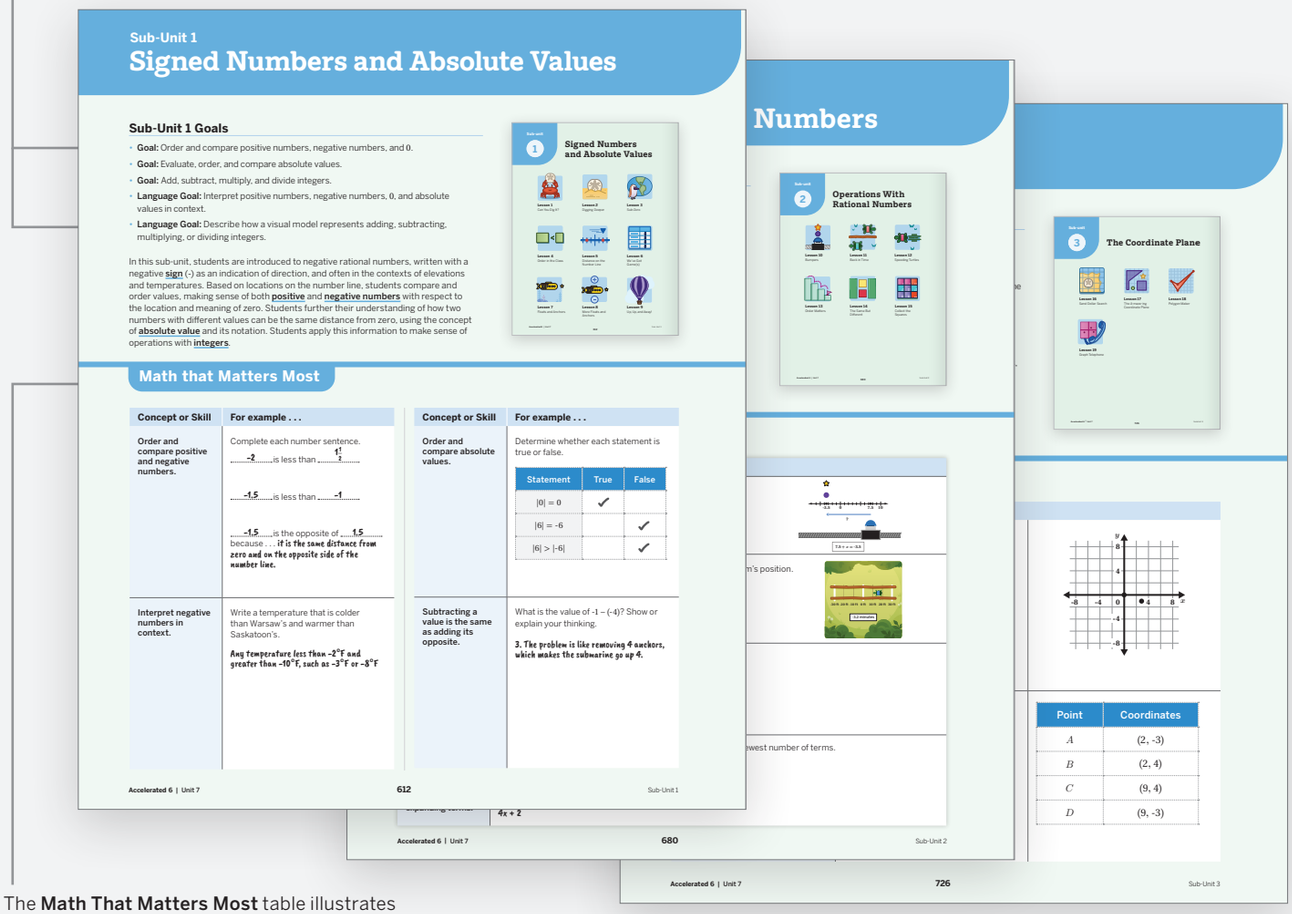
This overview of **concept development** supports teachers in making decisions about potential intervention points during the unit. It indicates at which point students have had multiple opportunities to focus on each key concept or skill. It does not necessarily mean there are no further opportunities to strengthen and develop these skills.

Teachers are provided with thoughtful **pacing considerations** for how they can adjust the pacing of the unit as needed without compromising unit learning goals.

Unit Overview pages

Teachers will find a comprehensive set of resources for each unit, including an overview of the math of the unit, a visual summary of the Unit at a Glance, a preview of each of the unit assessments, and guidance about differentiation, accessibility, language development, materials, technology, and connections to future learning. Each Unit Overview also includes a professional development activity and a formative Pre-Unit Check that teachers can use to learn more about students' understanding of foundational concepts and skills that will support them in the upcoming unit.

The **Sub-Unit Overview** page summarizes the **key goals** and **vocabulary** for this part of the unit.



The **Math That Matters Most** table illustrates for teachers the most important progressions of strategies, skills, and language that develop over the course of the sub-unit.

Sub-Unit Overview pages

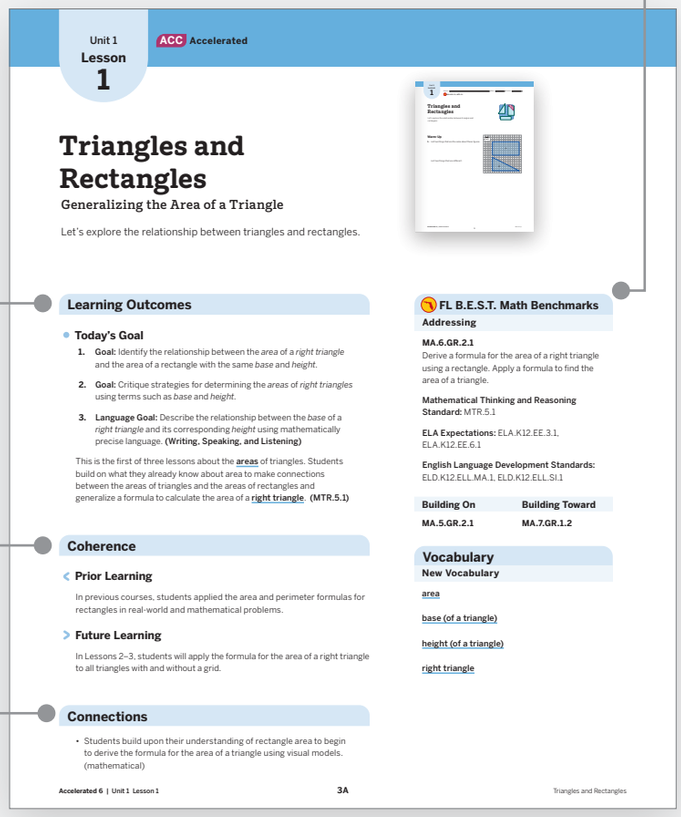
The lessons within each unit are grouped into sub-units that address a related group of concepts. All sub-units include lesson practice and Show What You Know formative assessments, and targeted sub-units include a quiz.

Navigating This Program

Lesson Supports

Throughout this Teacher Edition, lesson guidance for teachers is organized clearly and consistently so that they have all of the information they need at their fingertips.

The **Florida B.E.S.T. Math Benchmarks** section will list all standards addressed in this lesson, including the **Mathematical Thinking and Reasoning Standards,** the **ELA Expectations,** and the **English Language Development Standards.** This section shows the standards the lesson builds on and the standards the lesson builds toward.



The **Learning Outcome**s section is where teachers will find the goals of the lesson, including language goals

In the **Coherence** section, teachers will find information on prior learning that has built to the math in this lesson, as well as future learning that this lesson is helping build to.

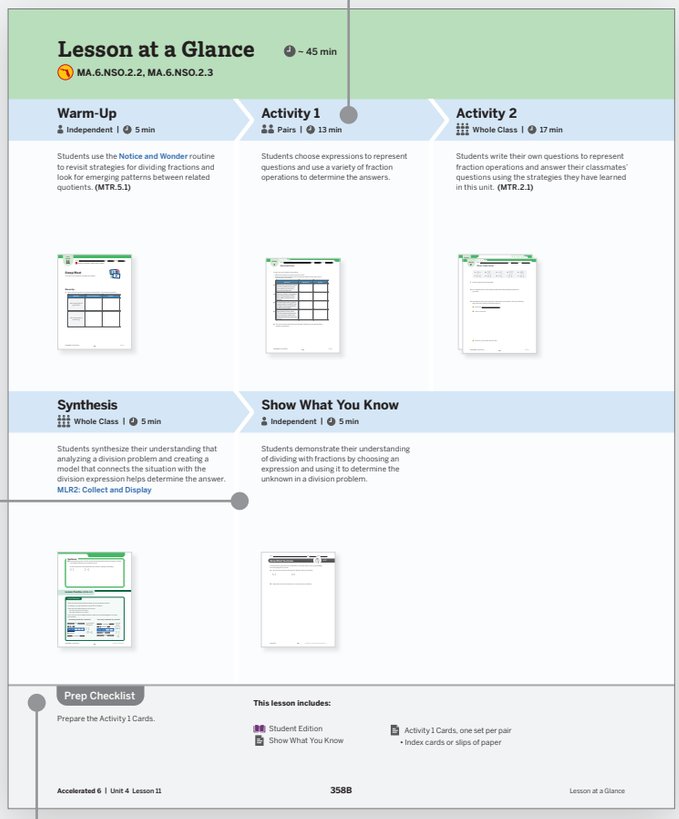
In the **Connections** section, teachers will find opportunities to help their students make connections cross-curricularly, to the real world, or to other mathematical learning.

Lesson Overview

This introductory page orients teachers to the topic, standards, and key learning goals of the lesson, including any new vocabulary terms that will be introduced.

The **Key Takeaways** and **Lesson Takeaway** summarize the most important ideas in the lesson.

The **time frame** and suggested **student grouping** is listed for each part of the lesson.



Lesson at a Glance

~ 45 min

MA.6.NSO.2.2, MA.6.NSO.2.3

Warm-Up
Independent | 5 min

Students use the Notice and Wonder routine to revisit strategies for dividing fractions and look for emerging patterns between related quotients. **(MTR.5.1)**

Activity 1
Pairs | 13 min

Students choose expressions to represent questions and use a variety of fraction operations to determine the answers.

Activity 2
Whole Class | 17 min

Students write their own questions to represent fraction operations and answer their classmates' questions using the strategies they have learned in this unit. **(MTR.2.1)**

Synthesis
Whole Class | 5 min

Students synthesize their understanding that analyzing a division problem and creating a model that connects the situation with the division expression helps determine the answer.
MLR2: Collect and Display

Show What You Know
Independent | 5 min

Students demonstrate their understanding of dividing with fractions by choosing an expression and using it to determine the unknown in a division problem.

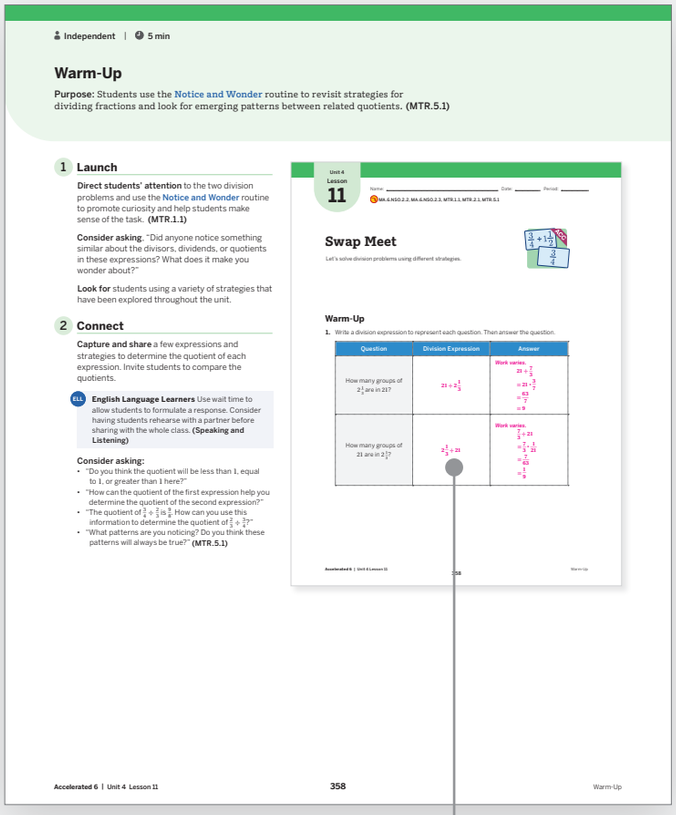
Prep Checklist

Prepare the Activity 1 Cards.

This lesson includes:

- Student Edition
- Show What You Know
- Activity 1 Cards, one set per pair
  - Index cards or slips of paper

The **Prep Checklist** displays all materials students will need for the lesson.



Independent | 5 min

Warm-Up

Purpose: Students use the Notice and Wonder routine to revisit strategies for dividing fractions and look for emerging patterns between related quotients. **(MTR.5.1)**

1 Launch

Direct students' attention to the two division problems and use the Notice and Wonder routine to promote curiosity and help students make sense of the task. **(MTR.1.1)**

Consider asking, "Did anyone notice something similar about the divisors, dividends, or quotients in these expressions? What does it make you wonder about?"

Look for students using a variety of strategies that have been explored throughout the unit.

2 Connect

Capture and share a few expressions and strategies to determine the quotient of each expression. Invite students to compare the quotients.

ELL **English Language Learners** Use wait time to allow students to formulate a response. Consider having students rehearse with a partner before sharing with the whole class. **(Speaking and Listening)**

Consider asking:

- "Do you think the quotient will be less than 1, equal to 1, or greater than 1 here?"
- "How can the quotient of the first expression help you determine the quotient of the second expression?"
- "The quotient of $\frac{3}{4} \div \frac{2}{3}$ is $\frac{9}{8}$. How can you use this information to determine the quotient of $\frac{1}{4} \div \frac{2}{3}$?"
- "What patterns are you noticing? Do you think these patterns will always be true?" **(MTR.5.1)**

Unit 4 Lesson 11

Swap Meet

Let's solve division problems using different strategies.

Warm-Up

1. Write a division expression to represent each question. Then answer the question.

Question	Division Expression	Answer
How many groups of $2\frac{1}{3}$ are in 21?	$21 \div 2\frac{1}{3}$	Work varies. $21 \div \frac{7}{3}$ $= 21 \cdot \frac{3}{7}$ $= \frac{63}{7}$ $= 9$
How many groups of 21 are in $2\frac{1}{3}$?	$2\frac{1}{3} \div 21$	Work varies. $\frac{7}{3} \div 21$ $= \frac{7}{3} \cdot \frac{1}{21}$ $= \frac{7}{63}$ $= \frac{1}{9}$

Sample student responses to the Warm-Up prompt are provided to help teachers prepare to facilitate the conversation.

Lesson at a Glance

The Lesson at a Glance page describes the purpose of the Warm-Up, Activities, Synthesis, and Show What You Know. Teachers will find suggested timing for each part of the lesson, as well as guidance on whether students should work individually, in pairs, in small groups, or with the whole class. The page also lists any hands-on materials that may be needed.

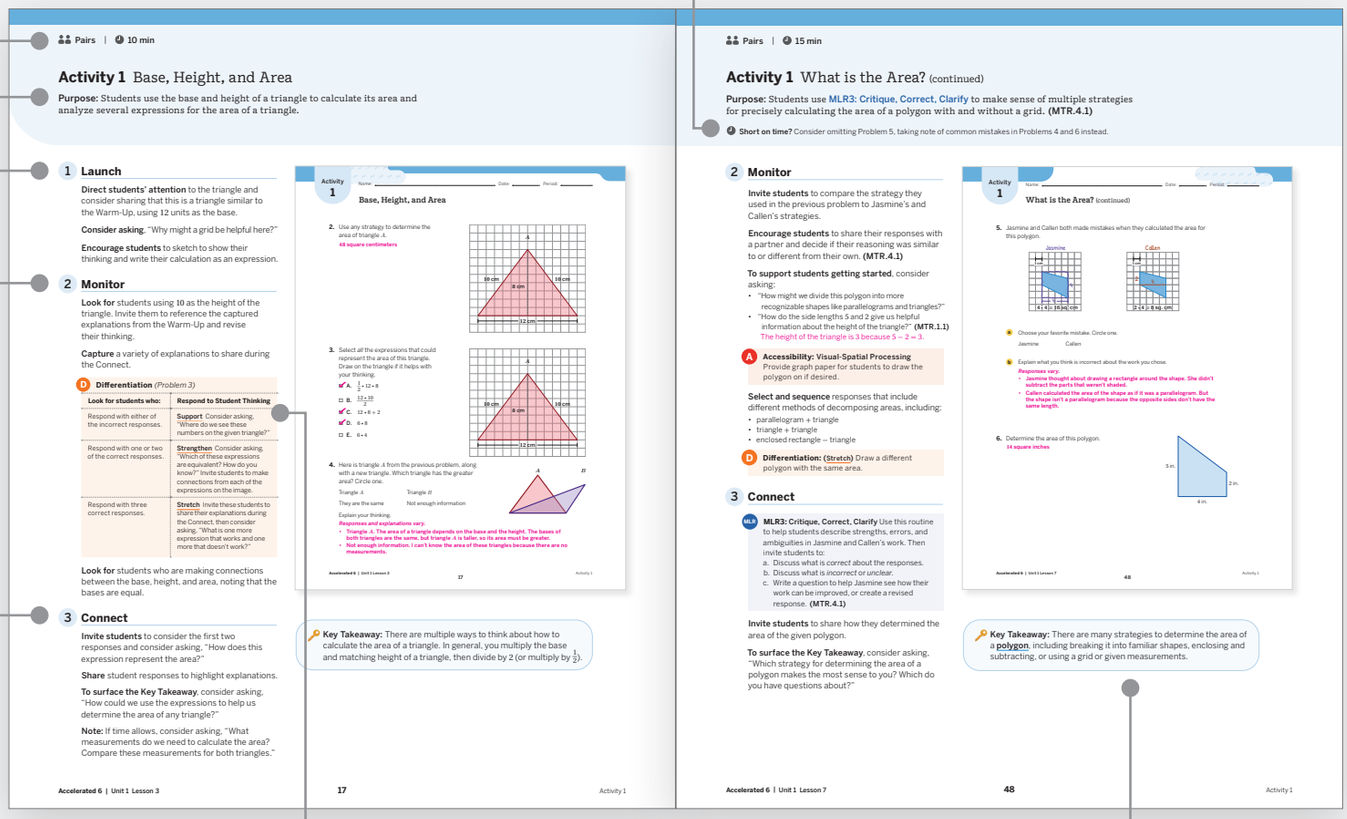
Warm-Up

Every Amplify Desmos Math Florida lesson begins with a Warm-Up to help draw students into the lesson. It might elicit information from their personal experience, remind them of a context they have seen before, invite them to think about the previous lesson, or preview a calculation that will appear in the current lesson.

Navigating This Program

The **Purpose** of each activity is highlighted here, as well as suggestions for the **student grouping** and **time frame** for the activity.

Teachers are provided with thoughtful **Short on time?** suggestions for how they can adjust their facilitation of the lesson as needed without compromising lesson goals.



Pairs | 10 min

Activity 1 Base, Height, and Area

Purpose: Students use the base and height of a triangle to calculate its area and analyze several expressions for the area of a triangle.

1 Launch

Direct students' attention to the triangle and consider sharing that this is a triangle similar to the Warm-Up, using 12 units as the base.

Consider asking. "Why might a grid be helpful here?"

Encourage students to sketch to show their thinking and write their calculation as an expression.

2 Monitor

Look for students using 10 as the height of the triangle. Invite them to reference the captured explanations from the Warm-Up and revise their thinking.

Capture a variety of explanations to share during the Connect.

D Differentiation *(Problem 3)*

Look for students who:	Respond to Student Thinking
Respond with either of the incorrect responses.	**Support** Consider asking, "Where do we see these numbers on the given triangle?"
Respond with one or two of the correct responses.	**Strengthen** Consider asking, "Which of these expressions are equivalent? How do you know?" Invite students to make connections from each of the expressions on the image.
Respond with three correct responses.	**Stretch** Invite these students to share their explanations during the Connect, then consider asking, "What is one more expression that works and one more that doesn't work?"

Look for students who are making connections between the base, height, and area, noting that the bases are equal.

3 Connect

Invite students to consider the first two responses and consider asking, "How does this expression represent the area?"

Share student responses to highlight explanations.

To surface the Key Takeaway, consider asking, "How could we use the expressions to help us determine the area of any triangle?"

Note: If time allows, consider asking, "What measurements do we need to calculate the area? Compare these measurements for both triangles."

Activity 1 — Base, Height, and Area

2. Use any strategy to determine the area of triangle *A*.
48 square centimeters

3. Select all the expressions that could represent the area of this triangle. Draw on the triangle if it helps with your thinking.
- ✓ A. $\frac{1}{2} \cdot 12 \cdot 8$
- ☐ B. $\frac{12 \cdot 10}{2}$
- ✓ C. $12 \cdot 8 \div 2$
- ✓ D. $6 \cdot 8$
- ☐ E. $6 + 6$

4. Here is triangle *A* from the previous problem, along with a new triangle. Which triangle has the greater area? Circle one.
Triangle *A* | Triangle *B* | They are the same | Not enough information
Explain your thinking.
Responses and explanations vary.
- *Triangle A. The area of a triangle depends on the base and the height. The bases of both triangles are the same, but triangle A is taller, so its area must be greater.*
- *Not enough information. I can't know the area of these triangles because there are no measurements.*

Accelerated 6 | Unit 1 Lesson 3 — 17 — Activity 1

Key Takeaway: There are multiple ways to think about how to calculate the area of a triangle. In general, you multiply the base and matching height of a triangle, then divide by 2 (or multiply by $\frac{1}{2}$).

Pairs | 15 min

Activity 1 What is the Area? (continued)

Purpose: Students use **MLR3: Critique, Correct, Clarify** to make sense of multiple strategies for precisely calculating the area of a polygon with and without a grid. **(MTR.4.1)**

Short on time? Consider omitting Problem 5, taking note of common mistakes in Problems 4 and 6 instead.

2 Monitor

Invite students to compare the strategy they used in the previous problem to Jasmine's and Callen's strategies.

Encourage students to share their responses with a partner and decide if their reasoning was similar to or different from their own. **(MTR.4.1)**

To support students getting started, consider asking:
- "How might we divide this polygon into more recognizable shapes like parallelograms and triangles?"
- "How do the side lengths 5 and 2 give us helpful information about the height of the triangle?" **(MTR.1.1)**
The height of the triangle is 3 because 5 − 2 = 3.

A Accessibility: Visual-Spatial Processing Provide graph paper for students to draw the polygon on if desired.

Select and sequence responses that include different methods of decomposing areas, including:
- parallelogram + triangle
- triangle + triangle
- enclosed rectangle − triangle

D Differentiation: (Stretch) Draw a different polygon with the same area.

3 Connect

MLR MLR3: Critique, Correct, Clarify Use this routine to help students describe strengths, errors, and ambiguities in Jasmine and Callen's work. Then invite students to:
a. Discuss what is *correct* about the responses.
b. Discuss what is *incorrect* or *unclear*.
c. Write a question to help Jasmine see how their work can be improved, or create a revised response. **(MTR.4.1)**

Invite students to share how they determined the area of the given polygon.

To surface the Key Takeaway, consider asking, "Which strategy for determining the area of a polygon makes the most sense to you? Which do you have questions about?"

Activity 1 — What is the Area? (continued)

5. Jasmine and Callen both made mistakes when they calculated the area for this polygon.
Jasmine: 4 × 4 = 16 sq. cm
Callen: 2 × 4 + 8 sq. cm
Choose your favorite mistake. Circle one. Jasmine | Callen
Explain what you think is incorrect about the work you chose.
Responses vary.
- *Jasmine thought about drawing a rectangle around the shape. She didn't subtract the parts that weren't shaded.*
- *Callen calculated the area of the shape as if it was a parallelogram. But the shape isn't a parallelogram because the opposite sides don't have the same length.*

6. Determine the area of this polygon.
14 square inches
5 in. | 2 in. | 4 in.

Accelerated 6 | Unit 1 Lesson 7 — 48 — Activity 1

Key Takeaway: There are many strategies to determine the area of a polygon, including breaking it into familiar shapes, enclosing and subtracting, or using a grid or given measurements.

In the **Launch**, **Monitor**, **Connect** guidance, teachers will find ways to help students get started, suggested facilitation moves, and discussion questions.

The guidance for every lesson includes a **D Differentiation** table to support teachers in meeting the needs of all students during the activity. This table can help teachers anticipate the ways students may approach the activity, and provides prompts that they can use during the lesson to **Support**, **Strengthen**, and **Stretch** individual students in their thinking.

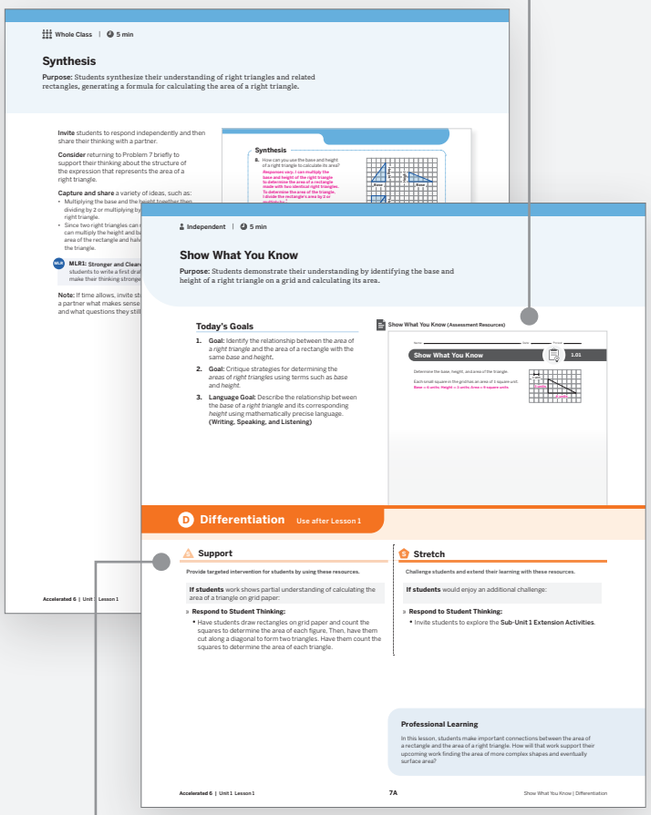
The **Key Takeaway** is called out to highlight the learning goal of the activity and provide teachers with an example of how to frame the big idea of the activity for students.

Activities

Each lesson includes one, two, or three activities. These activities are the heart of each lesson. Students notice, wonder, explore, calculate, predict, measure, explain their thinking, settle disputes, create challenges for their classmates, and more.

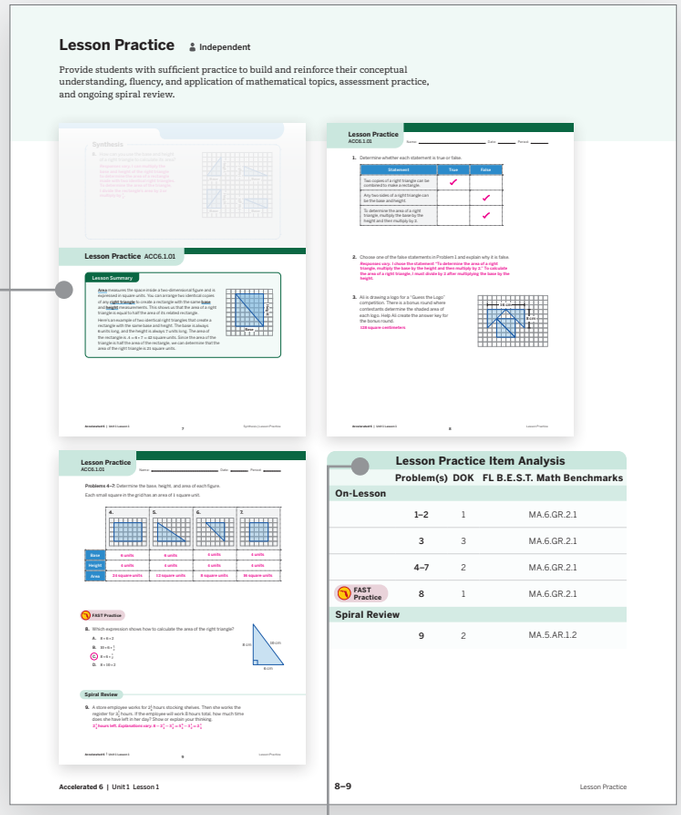
Guidance is provided to help teachers Launch, Monitor, and Connect students' thinking over the course of each activity. Teachers will also find suggestions for pacing, facilitation moves, discussion questions, examples of early student thinking, ideas for students who may enjoy a challenge, and ways to increase access to the language and content. The suggestions for facilitation provide comprehensive support for teachers as they prepare for and teach lessons.

Lessons conclude with the **Show What You Know**, which presents an opportunity for students to reflect on the main learning goals of the lesson. This is a great way for both students and teachers to access a formative check for understanding.



Each Show What You Know is accompanied by a table with suggestions in three categories: students who need **support**, would benefit from more practice to **strengthen** their understanding, or are interested in a **stretch** to deepen or extend their thinking.

Each **Lesson Practice** begins with a **Summary** of the big ideas in the lesson, often including a worked example. Students can highlight parts of the summary or share it with a caregiver or classmate.



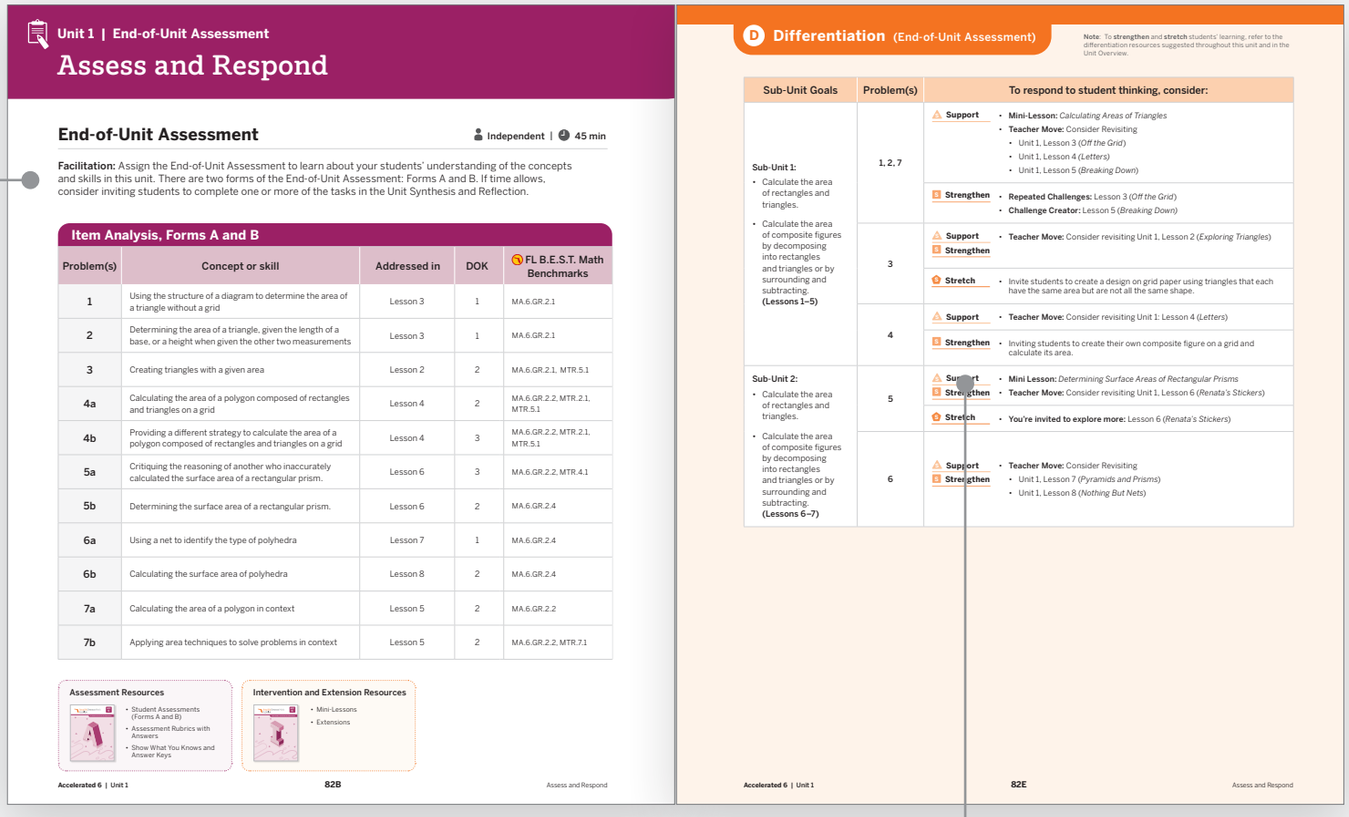
A **Practice Problem Item Analysis** table breaks down the problems by type, Depth of Knowledge (DOK), and corresponding B.E.S.T. Math Benchmarks.

Synthesis

The Synthesis is an opportunity for students to put the key ideas from the lesson into their own words. There is typically an open-ended prompt followed by a discussion for students to consolidate and refine their ideas about the learning goals. If time allows, it is also an opportunity for students to revise their responses after the discussion.

Lesson Practice

Daily practice problems for the day's lesson are included both online and in the print Student Edition, including Fluency, FAST Practice, and Spiral Review.



Unit 1 | End-of-Unit Assessment

Assess and Respond

End-of-Unit Assessment

Independent | 45 min

Facilitation: Assign the End-of-Unit Assessment to learn about your students' understanding of the concepts and skills in this unit. There are two forms of the End-of-Unit Assessment: Forms A and B. If time allows, consider inviting students to complete one or more of the tasks in the Unit Synthesis and Reflection.

Item Analysis, Forms A and B

Problem(s)	Concept or skill	Addressed in	DOK	FL B.E.S.T. Math Benchmarks
1	Using the structure of a diagram to determine the area of a triangle without a grid	Lesson 3	1	MA.6.GR.2.1
2	Determining the area of a triangle, given the length of a base, or a height when given the other two measurements	Lesson 3	1	MA.6.GR.2.1
3	Creating triangles with a given area	Lesson 2	2	MA.6.GR.2.1, MTR.5.1
4a	Calculating the area of a polygon composed of rectangles and triangles on a grid	Lesson 4	2	MA.6.GR.2.2, MTR.2.1, MTR.5.1
4b	Providing a different strategy to calculate the area of a polygon composed of rectangles and triangles on a grid	Lesson 4	3	MA.6.GR.2.2, MTR.2.1, MTR.5.1
5a	Critiquing the reasoning of another who inaccurately calculated the surface area of a rectangular prism.	Lesson 6	3	MA.6.GR.2.2, MTR.4.1
5b	Determining the surface area of a rectangular prism.	Lesson 6	2	MA.6.GR.2.4
6a	Using a net to identify the type of polyhedra	Lesson 7	1	MA.6.GR.2.4
6b	Calculating the surface area of polyhedra	Lesson 8	2	MA.6.GR.2.4
7a	Calculating the area of a polygon in context	Lesson 5	2	MA.6.GR.2.2
7b	Applying area techniques to solve problems in context	Lesson 5	2	MA.6.GR.2.2, MTR.7.1

Assessment Resources

- Student Assessments (Forms A and B)
- Assessment Rubrics with Answers
- Show What You Knows and Answer Keys

Intervention and Extension Resources

- Mini-Lessons
- Extensions

Accelerated 6 | Unit 1 82B Assess and Respond

Differentiation (End-of-Unit Assessment)

Note: To **strengthen** and **stretch** students' learning, refer to the differentiation resources suggested throughout this unit and in the Unit Overview.

Sub-Unit Goals	Problem(s)	To respond to student thinking, consider:

Sub-Unit 1:
• Calculate the area of rectangles and triangles.
• Calculate the area of composite figures by decomposing into rectangles and triangles or by surrounding and subtracting.
(Lessons 1–5)
	1, 2, 7	**Support**
		• **Mini-Lesson:** *Calculating Areas of Triangles*
		• **Teacher Move:** Consider Revisiting
		  • Unit 1, Lesson 3 (*Off the Grid*)
		  • Unit 1, Lesson 4 (*Letters*)
		  • Unit 1, Lesson 5 (*Breaking Down*)
		Strengthen
		• **Repeated Challenges:** Lesson 3 (*Off the Grid*)
		• **Challenge Creator:** Lesson 5 (*Breaking Down*)

	3	**Support** **Strengthen** • **Teacher Move:** Consider revisiting Unit 1, Lesson 2 (*Exploring Triangles*) **Stretch** • Invite students to create a design on grid paper using triangles that each have the same area but are not all the same shape.
	4	**Support** • **Teacher Move:** Consider revisiting Unit 1: Lesson 4 (*Letters*) **Strengthen** • Inviting students to create their own composite figure on a grid and calculate its area.
Sub-Unit 2: • Calculate the area of rectangles and triangles. • Calculate the area of composite figures by decomposing into rectangles and triangles or by surrounding and subtracting. **(Lessons 6–7)**	5	**Support** **Strengthen** • **Mini-Lesson:** *Determining Surface Areas of Rectangular Prisms* • **Teacher Move:** Consider revisiting Unit 1, Lesson 6 (*Renata's Stickers*) **Stretch** • **You're invited to explore more:** Lesson 6 (*Renata's Stickers*)

	6	**Support**
		Strengthen
		• **Teacher Move:** Consider Revisiting
		  • Unit 1, Lesson 7 (*Pyramids and Prisms*)
		  • Unit 1, Lesson 8 (*Nothing But Nets*)

Accelerated 6 | Unit 1 82E Assess and Respond

An **Item Analysis** is provided to show what concepts and skills are assessed in each problem.

A **Differentiation** table suggests specific materials from the unit to **Support**, **Strengthen**, or **Stretch** students' understanding based on their responses to different assessment problems.

Assess and Respond

Each unit typically includes one or two Sub-Unit Quizzes. Quizzes are designed for students to show what they know and can do based on what they have learned so far in the unit. Each unit includes Assess and Respond guidance for the Pre-Unit Check, Sub-Unit Quizzes, and End-of-Unit Assessment.

Facilitating Lesson Activities

Launch, Monitor, Connect

Amplify Desmos Math Florida is designed with a structured approach to problem-based learning that systematically builds on students' curiosity. Students are first invited to explore problems that create an intellectual need for new mathematical ideas. Then the teacher builds on strategies used by students and connects their ideas to the learning goals of the lesson. This approach is an interpretation of Smith and Stein's *5 Practices for Orchestrating Productive Mathematics Discussions*.

1 Launch

The Launch is a short, whole-class conversation that creates a need or excitement, provides clarity, or helps students connect to their prior knowledge or personal experience, which ensures that everyone has access to the upcoming work.

Considerations for Launching

- Try to keep it short. Set students up to get started with a clear and catchy invitation to the math.
- Wait to model a specific way to solve. Leave space for a variety of different student approaches.
- Pair up. Encourage small groups of students to talk through their thinking as they work.

2 Monitor

As students work individually, in pairs, or in groups, teachers explore students' thinking, ask questions, and provide support to help move the conversations closer to the intended math learning goal.

Considerations for Monitoring

- Look for students' strategies and check in with students as they work.
- Ask questions to learn what students are thinking. The suggested differentiation moves can be used to support, strengthen, and stretch their ideas.
- Use the Differentiation table to help select and sequence student ideas to highlight during the Connect discussion.

3 Connect

Teachers connect students' ideas to the key learning goals of the lesson, facilitating class discussions that help students synthesize and solidify the big ideas.

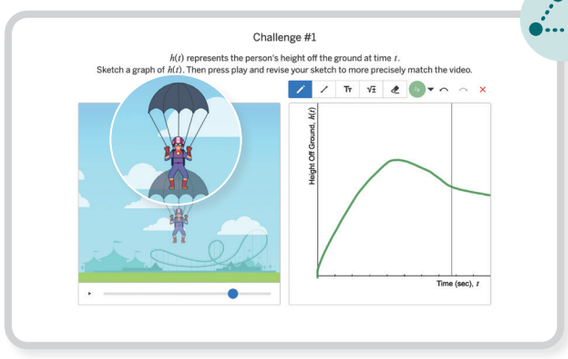
Considerations for Connecting

- Save a few minutes at the end of each lesson activity to bring students back together to discuss.
- It is okay to start the class discussion before all students have finished the activity.
- Students will be able to contribute to the discussion and learn from their classmates based on their in-progress work.
- Center discussions on students' ideas by displaying one or more of their responses and connecting the responses to the Key Takeaway of the activity.

Smith, M.S., & Stein, M.K. (2018). *5 practices for orchestrating productive mathematics discussions* (2nd ed.). SAGE Publications.

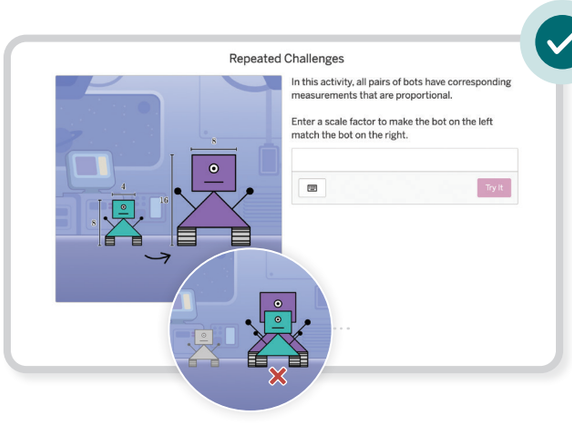
The Power of Digital

Amplify Desmos Math Florida provides the option for teachers to deliver lessons using the power of digital technology. This includes the use of visual and dynamic interactions that pique students' interest and invite all students to engage in the mathematics. These interactions and animations allow students to test predictions, get feedback, share ideas, and connect representations.



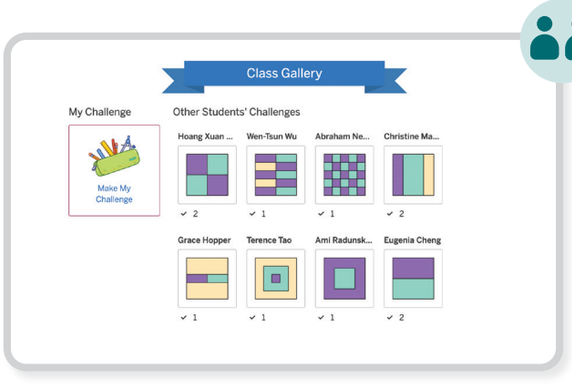
Delightful, engaging interactions

The digital interactions included in lesson activities are designed to elicit students' thinking in a way that feels fun and inviting. As students play and explore math concepts, teachers can highlight the ideas that students share, connect those ideas to other students' ideas, and build on their thinking through productive class discussion.



Responsive Feedback™

In Amplify Desmos Math Florida, students are invited to try their thinking out — even if that thinking is still in the "rough draft" phase. As students interact with the digital screens, they see visuals and simulations that respond faithfully to their inputs. This meaningful feedback allows students to experience the joy of causing the animation to react to their mathematical ideas. As a result, students may notice interesting things about both correct and incorrect answers more readily.

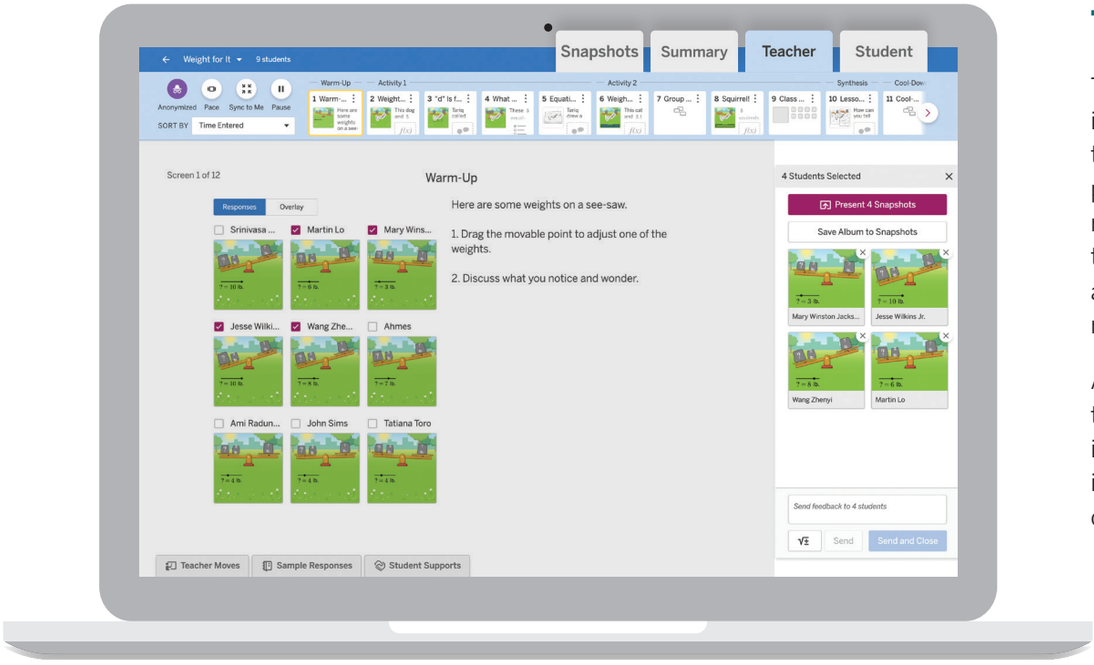


Collaborative experiences

Digital tools allow students to interact with each other's mathematical ideas in a variety of ways. Students can use the "Share With Class" button to exchange mathematical thinking with each other directly on an activity screen. Activities like Challenge Creator and Polygraph offer fun ways for students to play with the math together. Whether working solo or in tandem on devices, students are never alone as they work through activities.

Digital Facilitation Tools

Amplify Desmos Math Florida includes a suite of digital facilitation tools. These tools foster collaborative classrooms and help teachers share their students' thinking.



Teacher Dashboard

The dashboard gives the teacher insight into students' thinking in real time. Teachers can zoom in on a particular student or view all students' responses at once. This can help to identify students that may need additional support and those who are ready for extensions.

After reviewing students' thinking, teachers can select and display specific ideas or the distribution of responses to invite students into productive, student-centered discussions.

Snapshot View

Select and sequence students' work to connect mathematical ideas. Teachers can even add their own questions to prompt further thinking.

Summary View

Monitor students' progress or accuracy for a set of screens. Click into any box to see a specific student's work on that screen.

Teacher View

Answer questions such as:

- How did all students answer this question?
- What answers were most common?

Student View

Preview screens for students. For example, the teacher might work through a screen with the class or talk through upcoming screens before students work on their own.

A powerful conversation toolkit

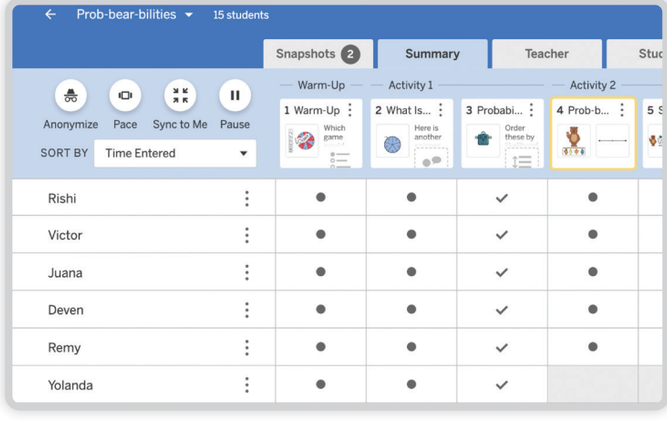
Pacing, pausing, and sync tools can support teachers as they encourage mathematical discourse and collaborative thinking in their classes.

Anonymize swaps out students' names to help them feel more comfortable sharing their ideas. Students' names are replaced by the names of famous mathematicians.

Pace allows the teacher to make a certain number of screens available for students to work on.

Sync to Me enables the teacher to bring all students to the same screen.

Pause stops students from working so that the teacher can gather everyone's attention for discussion.



Math in Action

Each Math in Action feature highlights an individual whose work in the field of mathematics, or a connected field, have made an impact on American society. These features can be used to help make connections between the mathematical content of the grade and how math shows up in a wide variety of careers.

Career Connections

Amplify Desmos Math Florida intentionally celebrates traditional mathematics fields as well as elevates work that engages mathematics in unique ways.

Representation of Mathematicians

Students can read about the contributions and work of a variety of individuals.

Unit 2 | Math in Action

Career Connection

How can ratios help drummers "keep the beat"?

Music is measured by ratios called *time signatures*, such as $\frac{4}{4}$ or $\frac{3}{4}$. The first number tells how many beats to count. The second number tells the kind of note. If the second number is 4, the notes are quarter notes. A $\frac{4}{4}$ time signature would count 1, 2, 3, 4, 1, 2, 3, 4, and so on. A $\frac{3}{4}$ measure of music would count 1, 2, 3, 1, 2, 3, and so on.

AleksKo/Shutterstock.com

Drummers keep the beat of a song by using ratios to maintain a steady rhythm, which helps other musicians playing with them. They can change the beat to give different feelings to the sound.

B.E.S.T. Mathematics Benchmark Connection

Believe it or not, musicians apply math extensively in their work. For example, the music scale and timing of different music arrangements are based on ratios (MA.6.AR.3.1, MA.6.AR.3.5) to create a certain sound, decide what speed to play certain songs, and more. Engineers and people who make musical instruments need to use their knowledge about geometry (MA.6.GR.2.1, MA.6.GR.2.4) as they cut out pieces for a musical instrument so that it makes the sound that a musician wants.

Mathematical Thinking and Reasoning Connection

People who are involved with music use thinking and reasoning skills like the ones you use for your math work! For example, they discuss their work with people they collaborate with, such as a writer, or fellow musicians in a band (MTR.1.1). When musicians study arrangements for performances such as a concert, they learn to complete the tasks of organizing and displaying the sheet music, attachments for their instruments, and other details with fluency (MTR.3.1).

Clayton Cameron

Born in Los Angeles, California, Clayton Cameron is a lecturer on Global Jazz Studies at the UCLA Herb Alpert School of Music. After receiving a degree in music from California State University at Northridge, he became a rising star in the music industry, performing as a percussionist with many award-winning acts. He led a TED Talk called "A-rhythm-etic. The Math Behind the Beats" to explain the mathematical ratios behind his drumming techniques.

Accelerated 6 | Unit 2 177

Clayton Cameron

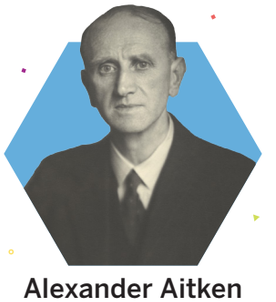
Math and music? Clayton Cameron found a way to combine his interest in both, using ratios to refine his drumming techniques.

Laura Máiz-Tomé

Math and ecology? Laura Máiz-Toméfound a way to combine her interest in both, using percentages to preserve freshwater ecosystems around the world.

Sir Jagadish Chandra Bose

Math and radio waves? Sir Jagadish Chandra Bose found a way to combine his interest in both, using mathematics and problem solving to identify patterns in radio waves.



Alexander Aitken

Math and data? Alexander Aitken found a way to combine his interest in both, using special equations to analyze data.

Facilitating Meaningful Conversations

Amplify Desmos Math Florida is designed to support interactive and collaborative classrooms, where students make sense of mathematics by sharing their thinking in paired, small-group, and whole-class conversations.

Classroom conversations are an opportunity for students to connect and share their experiences, strengths, and knowledge for the benefit of the entire class. Students are encouraged to share their early thinking, consider the ideas of others, and appreciate different approaches and changing one's mind. Lessons celebrate math as a source of joy and as a tool for analyzing and making sense of the world. Teachers will find structures and tips to support them in facilitating meaningful conversations throughout each lesson.

Centering Personal Connections

Teachers are encouraged to consider the experiences of the students in their classes and what connections they might have to various math topics and contexts. The Teacher Edition offers support for building conversations around the real-world and mathematical experiences that students bring to the classroom.

Use the Tell a Story routine to help students make sense of the turtle race animation. Invite students to write a story about the animation using their experiences, perspectives, and creativity.

In the **Turtle Time Trials** lesson, students are invited to use the Tell a Story routine to create their own narratives about what the turtles in the animation are doing and why. This is a chance for students to use their creativity while engaging deeply in the math.

1 Launch

Direct students' attention to the images of the rice dishes.

To surface connections, consider asking, "What is your favorite rice dish? When or where do you eat it?"

In the **Rice Ratios** lesson, students are introduced to the concept of equivalent ratios in the context of making rice. The Teacher Edition provides teachers with prompts they can use to help students connect the rice dishes in the activities to their own lives.

Supporting Student Collaboration

Different kinds of **groupings** are suggested for different phases of a lesson, including **whole-class** discussions and **pair** work. The conversations that these groupings encourage are opportunities for students to share their thinking and listen to each others' perspectives.

How will you support student collaboration?

- Discuss what supporting each other without giving answers could look or sound like.
- Invite a student from each group to read the task aloud.

The Teacher Edition provides suggestions for intentional groupings to support access and language development and help students support their partners without simply sharing answers. Digital activities usually suggest that pairs of students share a single device. This 2:1 structure allows students to talk through tasks together in ways that inform each other's thinking.

Celebrating Different Perspectives

Many lesson activities invite students to consider a variety of different responses, including early or unfinished thinking.

Use the Which One Doesn't Belong? routine to support students in comparing structures of two-variable equations.

Encourage students to look for more than one possibility.

In the **Subway Seats** lesson, students use the Which One Doesn't Belong? routine to begin exploring two-variable equations. Classroom conversations about activities like this are a way for teachers to elicit different kinds of student brilliance. The Teacher Edition provides suggestions for how teachers can engage students in conversation and reinforce the idea that each student has something uniquely valuable to contribute.

Accessibility

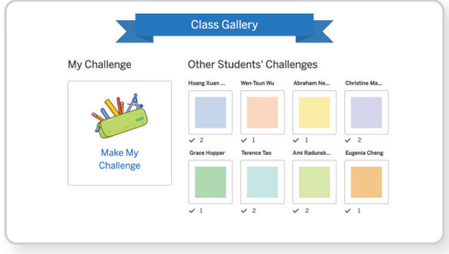
Amplify Desmos Math Florida is built to support all students in accessing and participating in meaningful and challenging learning. This support is incorporated into the curriculum structure, lesson-level guidance, and digital tools.

Universal Design for Learning

Each lesson incorporates opportunities for engagement, representation, action, and expression based on the guidelines of Universal Design for Learning (UDL). UDL is a research-based framework designed to ensure meaningful learning experiences for all students.[1]

Multiple Means of Engagement

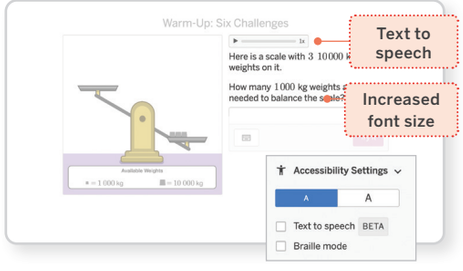
Individuals are each motivated in different ways, at different times, and in different contexts. Lessons are designed to welcome student interests, support sustaining effort and persistence, and develop an awareness of others.



Sustaining Effort and Persistence: Students are invited to build their own challenge for other students to solve, which provides opportunities for choice and autonomy, as well as joy and play.

Multiple Means of Representation

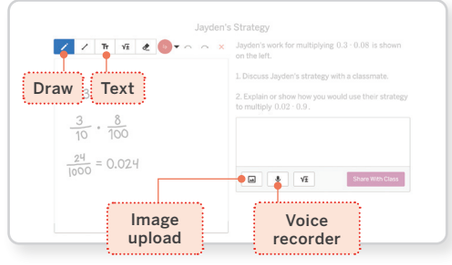
Learners make sense of information differently. Amplify Desmos Math Florida includes options for presenting information in multiple ways to support comprehension and understanding of language.



Present Information in Multiple Ways: Students and teachers can use digital tools such as text to speech, enlarged font, and dynamic visuals.

Multiple Means of Action and Expression

Learners differ in how they navigate learning environments and express what they know. Amplify Desmos Math Florida ensures that materials and interactions are accessible, support multiple means of students' expression and communication, and scaffold executive functioning.



Support Multiple Means of Student Expression and Communication: Students can communicate their ideas in multiple ways, including sketching, uploading photos, or responding with audio.

Lesson Facilitation Supports

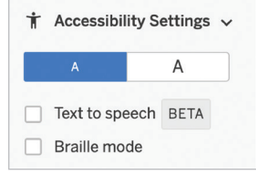
Every lesson includes at least one specific suggestion the teacher can use to increase access to the lesson without reducing the mathematical demand of the tasks. These suggestions address the following areas:

- Conceptual Processing
- Visual-Spatial Processing
- Executive Functioning
- Memory and Attention
- Fine Motor Skills
- Affective Functioning

A **Accessibility: Memory and Attention** To support keeping track of information, invite students to record the dimensions they try and whether or not they created a scaled copy.

Accessibility Tools

Students have the ability to control accessibility tools so that each learning experience is customized to their individual needs. In many instances, these tools can be turned on or off at any point of instruction.



Text to speech: Reads text instructions to students in multiple languages.

Enlarged font: Increases the size of all text on screen.

Braille mode: Includes narration of digital interactions.

Language selection: Toggles between languages.

[1] https://udlguidelines.cast.org/

Differentiation

Differentiation in Amplify Desmos Math Florida enables teachers to ensure all students have access to grade-level math content. This support is organized for teachers into three categories:

Support

Provide targeted intervention for students by using these resources.

Strengthen

Reinforce students' understanding of the concepts assessed by using these resources.

Stretch

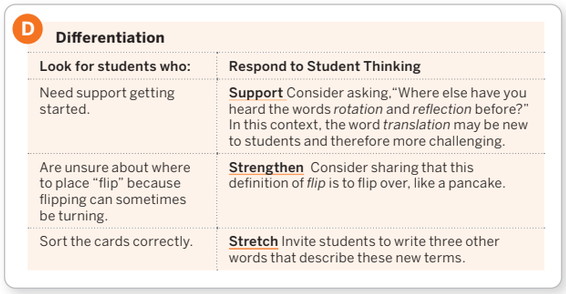
Challenge students and extend their learning with these resources.

Differentiation: In-Lesson Teacher Moves

Within every lesson activity, teachers can use the **Differentiation** suggestions to provide **in-the-moment instructional support** to learners as they engage in the work of the lesson.

Teachers are provided with clear student actions and understandings to look for, each matched with immediately usable suggestions for how to respond to the student thinking illustrated in each row of the table.

In addition to using these suggestions in the moment as teachers monitor student work, teachers can review the Differentiation table in advance to help them anticipate how students are likely to approach the activity.



Differentiation

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Consider asking, "Where else have you heard the words *rotation* and *reflection* before?" In this context, the word *translation* may be new to students and therefore more challenging.
Are unsure about where to place "flip" because flipping can sometimes be turning.	**Strengthen** Consider sharing that this definition of *flip* is to flip over, like a pancake.
Sort the cards correctly.	**Stretch** Invite students to write three other words that describe these new terms.

Differentiation: Beyond the Lesson

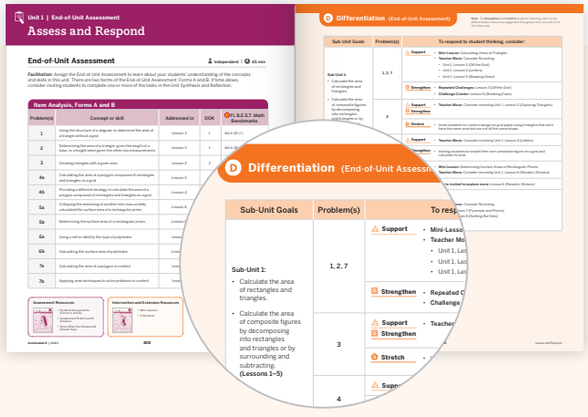
In each lesson, students' understanding of the learning goals is broken down for teachers into three categories: students who need support to understand the learning goals, students who need to continue strengthening their understanding of the learning goals, and students who are ready to stretch their learning.

Each **Support**, **Strengthen**, and **Stretch** resource is designed to take 15 minutes:

- **Mini-Lessons:** Targeted intervention for students who need additional support or need more time.
- **Extensions:** Small group or independent challenges.

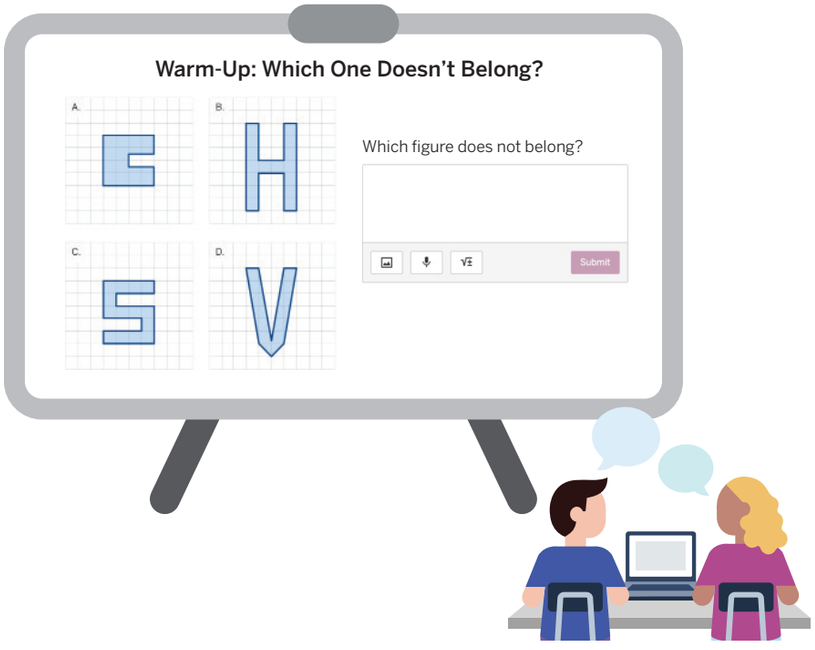
Differentiation: Assess and Respond

At each assessment point in a unit, teachers have the opportunity to respond to student understanding. For each assessment item, teachers are provided with clear suggestions for how to support, strengthen, or stretch student understanding of the corresponding concept based on their assessment responses.



Instructional Routines

Instructional routines that are used repeatedly create efficiencies for teachers so that they can attend to student thinking and communicate what is important in their classroom. Instead of focusing on the directions of an activity, students can focus on making sense of and communicating about the math.



Here are some examples:

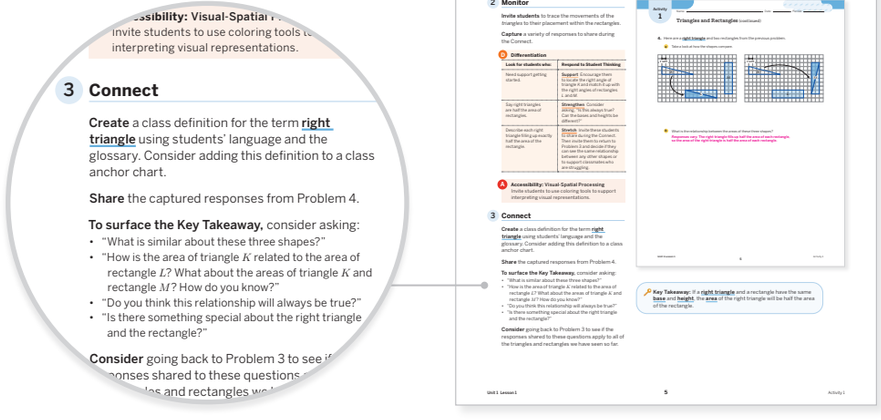
- The **Which One Doesn't Belong?** routine communicates to students that their ideas have value, that there are many ways to be correct in math, and that they can learn math by sharing their math thinking with each other.
- The **Stronger and Clearer Each Time** routine communicates the importance of feedback and creates an opportunity for students to learn from each other as they construct and refine their viable arguments.

Each instructional routine included in an Amplify Desmos Math Florida lesson creates opportunities for conversations and supports meaningful discussion. Implementing these routines can be a practical tool for establishing a classroom learning community that values students' mathematical thinking.

Instructional Routines Embedded in the Curriculum

Instructional routines can be found throughout each lesson in the Teacher Edition. Here is a list of the instructional routines used in the Amplify Desmos Math Florida curriculum:

- **MLR1:** Stronger and Clearer Each Time
- **MLR2:** Collect and Display
- **MLR3:** Critique, Correct, Clarify
- **MLR5:** Co-Craft Questions
- **MLR6:** Three Reads
- **MLR7:** Compare and Connect
- Decide and Defend
- Notice and Wonder
- Number Talk
- Tell a Story
- Think-Pair-Share
- Which One Doesn't Belong?



For more information about the Instructional Routines embedded in the Amplify Desmos Math Florida curriculum, see the Routine Facilitation Guide in the Professional Development Library.

Math Language Development

At Amplify Desmos Math Florida, we acknowledge that there is a strong interconnection between mathematical content, practices, and language. We believe that developing mathematical language is critical for all learners, while recognizing and supporting the unique needs of English Language Learners.

Our approach to math language development focuses on when, how, and why students are using language to make sense of and share their mathematical ideas. Every lesson in Amplify Desmos Math Florida includes opportunities for all students to develop mathematical language as they experience the content, while providing intentional support for English Language Learners. We purposefully progress language throughout the units by cultivating students' language and supporting students in making their arguments and explanations stronger, clearer, and more precise as they progress from lesson to lesson.

Here are four ways we support students with developing mathematical language:

Vocabulary

Units and lessons build connections between students' language and the new vocabulary for that unit. This honors the language assets that students bring into their learning.

Some **new vocabulary** is marked as *optional*; these are terms students may encounter in other settings, but are not essential for them to access the lesson. Vocabulary is *review* if has been introduced and defined in a prior unit or course. The Unit Overview also lists **contextual vocabulary**, which are context-specific non-mathematical terms critical for students to understand the lesson.

Create class definitions for the terms **pre-image** and **image**, and support students in making sense of *prime notation* ($'$). To help generate student language, consider asking:

- "How are the labels different on the two figures?"
- "What does *pre* mean? How is that connected to the term *pre-image*?"
- "Do you think the placement of A', B', C', and D' on the image matters?"

Language Goals

Language goals attend to the mathematics students are learning and are written through the lens of one or more of four language modalities: **Reading**, **Writing**, **Speaking**, and **Listening**. They describe how students will grow their ability to analyze, compare, critique, describe, discuss, explain, interpret, and justify. Language goals appear at both the lesson and sub-unit level.

- **Today's Goals**
  1. **Goal:** Calculate the area of a parallelogram on a grid using a variety of strategies.
  2. **Language Goal:** Compare and contrast strategies for determining the area of a parallelogram. **(Writing, Speaking, and Listening)**
  3. **Language Goal:** Describe parallelograms using precise mathematical language. **(Speaking and Listening)**

Math Language Routines (MLR)

MLR **Math Language Routines**[1] are used within lessons to do one or more of the following: highlight student-developed language and ideas, cultivate conversation, support mathematical sense-making, and promote meta-cognition. Tips for facilitating MLRs are included when they would be helpful within lessons.

MLR **MLR7: Compare and Connect** Invite students to analyze each strategy, compare them, and then connect to their strategy. Consider asking:

- "Who used a strategy similar to Omari's? Ivory's?"
- "Where is the constant difference in each strategy?"
- "Why did different strategies lead to the same outcome?"

English Language Learners Support

Supports for ELL **English Language Learners** are called out at intentional points within each lesson. These suggested supports are specific, targeted actions that are beneficial for English Language Learners. They often describe a modification to increase access to the task or support with contextual or mathematical language development that can often be supportive of all learners. English Language Learner supports may also be attached to MLRs.

ELL **English Language Learners** Encourage students to paraphrase each other's ideas to help them make connections and incorporate vocabulary from the unit. **(Speaking and Listening)**

[1] Zwiers, J., Dieckmann, J., Rutherford-Quach, S., Daro, V., Skarin, R., Weiss, S., & Malamut, J. (2017). Principles for the Design of Mathematics Curricula: Promoting Language and Content Development. Retrieved from Stanford University, UL/SCALE website: http://ell.stanford.edu/content/mathematics-resources-additional-resources

Developing Fluency

Procedural fluency is the ability to apply procedures efficiently, flexibly, and accurately; to transfer procedures to different problems and contexts; to build or modify procedures from other procedures; and to recognize when one strategy or procedure is more appropriate to apply than another*.

In Accelerated 6, Florida students are expected to perform all four operations with rational numbers with procedural fluency.

Expectation

Exploration

To develop understanding through the use of manipulatives, visual models, discussions, estimation and drawings.

Procedural Reliability

To utilize skills to develop an accurate, reliable method that aligns with the student's understanding and learning style.

Procedural Fluency

To utilize skills to become fluent with an efficient, generalizable and accurate procedure, including a standard algorithm.

How Amplify Desmos Math Florida Supports the Expectation

- Manipulatives and visual models and diagrams are used extensively to support student's exploration of new concepts.
- Discussions are strongly encouraged in each lesson, including 💬 Discuss prompts in the student edition as well as suggested discussion prompts in the teacher's edition.
- Students are encouraged to estimate and compare estimations to actual answers to support reasonableness of problem solving solutions.
- Students are often asked to analyze a drawing or to make a drawing to explain their reasoning and strengthen their understanding of the math concepts being studied.

- Frequently in each course, students are asked to analyze and evaluate work presented in their student edition that represents work done by other students for accuracy, preference to learning style, and for efficiency.

- Program features such as practice, spiral reviews, mini lessons, and pre-unit assessments are all designed to assist students with achieving procedural fluency.

*National Research Council [2001, 2005, 2012]; Star [2005]

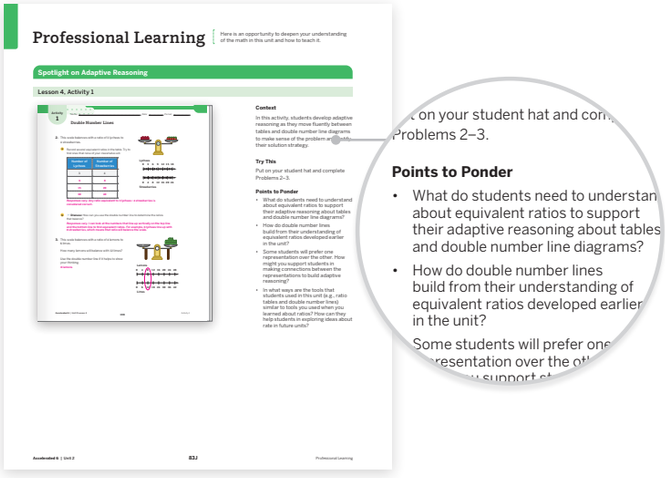
Professional Learning

The Amplify Desmos Math Florida program includes regular opportunities for teachers to deepen their understanding while they teach.

Embedded Professional Development

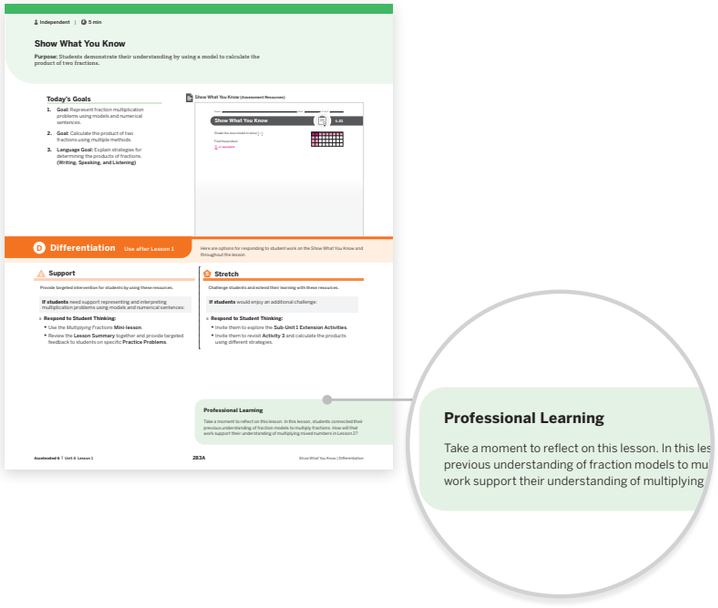
Unit-Level Professional Development

The Unit Overview for each unit includes a Professional Development page, which lists opportunities for teachers to deepen their understanding of the math in the unit and how to teach it.



Lesson-Level Professional Development

Each lesson provides a moment of professional learning as an opportunity for teachers to reflect on how well the lesson went. These professional learning moments can relate to principles, instruction, pedagogy, or program content.



Professional Development Sessions

Amplify's professional development team supports both teachers and administrators throughout their entire implementation journey. These sessions are available as on-site or virtual options.

Launch

Launch sessions introduce Amplify programs and support strong implementation. After learning about the program's foundational principles, you'll practice implementing the program in a collaborative environment.

Strengthen

Strengthen sessions deepen understanding of the program. These offerings are targeted and are meant to take your practice – and your students' learning to the next level.

Coach

These sessions are tailored to elevate instructional practices and meet the unique needs of teachers and leaders. These sessions can be customized leveraging our menu of supports.

Professional learning on your schedule

The PD Library serves as a centralized location where educators can access an ever-growing collection of professional learning resources and courses designed to support the implementation of Amplify Desmos Math Florida. Educators will find bite-sized how to videos, unit and lesson planning guides and templates, resources for leaders, and so much more!

Program Assessments

A variety of performance data in Amplify Desmos Math Florida provides evidence of student learning while helping students bolster their skills and understanding.

Throughout lessons, units, and the entire program, you will find summative and formative assessments meant to provide insights into students' conceptual understandings. Student learning is never a surprise at the end of a unit — with Amplify Desmos Math Florida, understanding is made continually visible.

Lesson-Level Assessments

Amplify Desmos Math Florida lessons are centered around sense-making and in-the-moment feedback. Daily moments of assessment provide insights into students' understandings of concepts and skills.

Show What You Know

Each lesson has a daily formative assessment focused on one of the **key concepts** in the lesson. Show What You Know moments are opportunities for students to show their teacher what they understand and what they are still learning.

Unit-Level Assessments

Our **embedded unit assessments** offer key insights into students' conceptual understanding of math. These assessments provide regular, actionable information about how students are thinking about and processing math, with both **auto-scoring** and **in-depth rubrics** that help teachers anticipate and respond to students' learning needs.

Pre-Unit Check

Each unit begins with a check to determine students' **proficiency** with **prerequisite skills** that are helpful for success in the upcoming unit. This check serves as an affirmation of the knowledge and skills with which students come into the unit.

Sub-Unit Quizzes

In Sub-Unit Quizzes, students are assessed on a subset of **conceptual understandings and procedural fluencies** from the unit, with rubrics that help illuminate students' current understanding and provides guidance for responding to student thinking.

End-of-Unit Assessment

Students engage with rigorous grade-level mathematics through a variety of formats and tasks in the End-of-Unit Assessment. A combination of auto-scored and rubric-scored items provide deep conceptual insight. Two forms are included to help facilitate assessment administration in the classroom.

Florida's B.E.S.T. Standards for Mathematics

The following correlations show the alignment of Amplify Desmos Math Florida to the FL B.E.S.T. Mathematics Standards and Benchmarks for Grade 6.

Benchmark	B.E.S.T Mathematics Benchmark	Amplify Desmos Math Lesson(s)
	Number Sense and Operations	
MA.6.NSO.1.1	Extend previous understanding of numbers to define rational numbers. Plot, order and compare rational numbers.	**Unit 7** Lessons 1, 2, 3, 4, 5, 6 **Unit 8** Lesson 4
MA.6.NSO.1.2	Given a mathematical or real-world context, represent quantities that have opposite direction using rational numbers. Compare them on a number line and explain the meaning of zero within its context.	**Unit 7** Lessons 1, 2, 3, 4, 6
MA.6.NSO.1.3	Given a mathematical or real-world context, interpret the absolute value of a number as the distance from zero on a number line. Find the absolute value of rational numbers.	**Unit 7** Lesson 6
MA.6.NSO.1.4	Solve mathematical and real-world problems involving absolute value, including the comparison of absolute value.	**Unit 7** Lessons 5, 6
MA.6.NSO.2.1	Multiply and divide positive multi-digit numbers with decimals to the thousandths, including using a standard algorithm with procedural fluency.	**Unit 5** Lessons 1, 3, 4, 5, 6, 7, 8, 9
MA.6.NSO.2.2	Extend previous understanding of multiplication and division to compute products and quotients of positive fractions by positive fractions, including mixed numbers, with procedural fluency.	**Unit 4** Lessons 1, 2, 3, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15 **Unit 8** Lesson 6
MA.6.NSO.2.3	Solve multi-step real-world problems involving any of the four operations with positive multi-digit decimals or positive fractions, including mixed numbers.	**Unit 4** Lessons 3, 4, 5, 6, 7, 8, 11, 12, 13, 14, 15 **Unit 5** Lessons 7, 8, 9
MA.6.NSO.3.1	Given a mathematical or real-world context, find the greatest common factor and least common multiple of two whole numbers.	**Unit 2** Lessons 6, 7
MA.6.NSO.3.2	Rewrite the sum of two composite whole numbers having a common factor, as a common factor multiplied by the sum of two whole numbers.	**Unit 5** Lesson 2
MA.6.NSO.3.3	Evaluate positive rational numbers and integers with natural number exponents.	**Unit 6** Lessons 10, 11
MA.6.NSO.3.4	Express composite whole numbers as a product of prime factors with natural number exponents.	**Unit 6** Lesson 10
MA.6.NSO.3.5	Rewrite positive rational numbers in different but equivalent forms including fractions, terminating decimals and percentages.	**Unit 3** Lessons 8, 9, 10, 11, 12, 13 **Unit 5** Lessons 9, 10 **Unit 6** Lessons 12, 13, 14, 15, 16, 17 **Unit 7** Lesson 4 **Unit 10** Lessons 3, 4, 5
MA.6.NSO.4.1	Apply and extend previous understandings of operations with whole numbers to add and subtract integers with procedural fluency.	**Unit 7** Lessons 1, 7, 8
MA.6.NSO.4.2	Apply and extend previous understandings of operations with whole numbers to multiply and divide integers with procedural fluency.	**Unit 7** Lesson 9

Florida's B.E.S.T. Standards for Mathematics

MA.7.NSO.2.1	Solve mathematical problems using multi-step order of operations with rational numbers including grouping symbols, whole-number exponents and absolute value.	**Unit 7** Lessons 11, 13, 14, 15
MA.7.NSO.2.2	Add, subtract, multiply and divide rational numbers with procedural fluency.	**Unit 7** Lessons 10, 11, 12, 13, 14, 15 **Unit 8** Lessons 6, 7 **Unit 9** Lesson 15
MA.7.NSO.2.3	Solve real-world problems involving any of the four operations with rational numbers.	**Unit 7** Lessons 11, 12, 13 **Unit 9** Lesson 15
	Algebraic Reasoning	
MA.6.AR.1.1	Given a mathematical or real-world context, translate written descriptions into algebraic expressions and translate algebraic expressions into written descriptions.	**Unit 6** Lesson 6
MA.6.AR.1.2	Translate a real-world written description into an algebraic inequality in the form of $x > a$, $x < a$, $x \geq a$ or $x \leq a$. Represent the inequality on a number line.	**Unit 6** Lessons 7, 9 **Unit 8** Lessons 3, 4, 5
MA.6.AR.1.3	Evaluate algebraic expressions using substitution and order of operations.	**Unit 6** Lesson 6
MA.6.AR.1.4	Apply the properties of operations to generate equivalent algebraic expressions with integer coefficients.	**Unit 6** Lessons 7, 8, 9
MA.6.AR.2.1	Given an equation or inequality and a specified set of integer values, determine which values make the equation or inequality true or false.	**Unit 6** Lessons 2, 4 **Unit 8** Lessons 4, 5
MA.6.AR.2.2	Write and solve one-step equations in one variable within a mathematical or real-world context using addition and subtraction, where all terms and solutions are integers.	**Unit 6** Lessons 1, 2, 3, 4 **Unit 8** Lesson 1
MA.6.AR.2.3	Write and solve one-step equations in one variable within a mathematical or real-world context using multiplication and division, where all terms and solutions are integers.	**Unit 6** Lessons 1, 2, 3, 4 **Unit 8** Lesson 2
MA.6.AR.2.4	Determine the unknown decimal or fraction in an equation involving any of the four operations, relating three numbers, with the unknown in any position.	**Unit 6** Lesson 5
MA.6.AR.3.1	Given a real-world context, write and interpret ratios to show the relative sizes of two quantities using appropriate notation: a/b, a to b, or $a : b$ where $b \neq 0$.	**Unit 2** Lessons 1, 2
MA.6.AR.3.2	Given a real-world context, determine a rate for a ratio of quantities with different units. Calculate and interpret the corresponding unit rate.	**Unit 3** Lessons 3, 4, 5, 6, 7
MA.6.AR.3.3	Extend previous understanding of fractions and numerical patterns to generate or complete a two- or three-column table to display equivalent part-to-part ratios and part-to-part-to-whole ratios.	**Unit 2** Lessons 3, 4, 5, 9, 10, 11 **Unit 3** Lessons 5, 6, 7
MA.6.AR.3.4	Apply ratio relationships to solve mathematical and real-world problems involving percentages using the relationship between two quantities.	**Unit 3** Lessons 8, 9, 10, 11, 12, 13

Florida's B.E.S.T. Standards for Mathematics

MA.6.AR.3.5	Solve mathematical and real-world problems involving ratios, rates and unit rates, including comparisons, mixtures, ratios of lengths and conversions within the same measurement system.	**Unit 2** Lessons 2, 4, 8, 12 **Unit 3** Lessons 1, 2, 3, 4, 5, 6, 7
MA.7.AR.1.1	Apply properties of operations to add and subtract linear expressions with rational coefficients.	**Unit 7** Lesson 15
MA.7.AR.1.2	Determine whether two linear expressions are equivalent.	**Unit 7** Lesson 14
MA.7.AR.2.1	Write and solve one-step inequalities in one variable within a mathematical context and represent solutions algebraically or graphically.	**Unit 8** Lessons 6, 7
MA.7.AR.3.1	Apply previous understanding of percentages and ratios to solve multi-step real-world percent problems.	**Unit 6** Lessons 12, 13, 14, 15, 16, 17
MA.7.AR.3.2	Apply previous understanding of ratios to solve real-world problems involving proportions.	**Unit 2** Lesson 12 **Unit 3** Lessons 7, 13
MA.7.AR.4.5	Solve real-world problems involving proportional relationships.	**Unit 9** Lesson 15
	Geometric Reasoning	
MA.6.GR.1.1	Extend previous understanding of the coordinate plane to plot rational number ordered pairs in all four quadrants and on both axes. Identify the x- or y-axis as the line of reflection when two ordered pairs have an opposite x- or y-coordinate.	**Unit 1** Lesson 2 **Unit 7** Lessons 16, 17, 18, 19
MA.6.GR.1.2	Find distances between ordered pairs, limited to the same x-coordinate or the same y-coordinate, represented on the coordinate plane.	**Unit 1** Lesson 7 **Unit 7** Lesson 18
MA.6.GR.1.3	Solve mathematical and real-world problems by plotting points on a coordinate plane, including finding the perimeter or area of a rectangle.	**Unit 7** Lessons 18, 19
MA.6.GR.2.1	Derive a formula for the area of a right triangle using a rectangle. Apply a formula to find the area of a triangle.	**Unit 1** Lessons 1, 2, 3, 4, 5 **Unit 4** Lesson 13
MA.6.GR.2.2	Solve mathematical and real-world problems involving the area of quadrilaterals and composite figures by decomposing them into triangles or rectangles.	**Unit 1** Lessons 4, 5 **Unit 4** Lesson 13
MA.6.GR.2.3	Solve mathematical and real-world problems involving the volume of right rectangular prisms with positive rational number edge lengths using a visual model and a formula.	**Unit 4** Lessons 14, 15
MA.6.GR.2.4	Given a mathematical or real-world context, find the surface area of right rectangular prisms and right rectangular pyramids using the figure's net.	**Unit 1** Lessons 9, 10, 11
MA.7.GR.1.1	Apply formulas to find the areas of trapezoids, parallelograms and rhombi.	**Unit 1** Lessons 6, 8
MA.7.GR.1.2	Solve mathematical or real-world problems involving the area of polygons or composite figures by decomposing them into triangles or quadrilaterals.	**Unit 1** Lessons 7, 8

Florida's B.E.S.T. Standards for Mathematics

Data Analysis and Probability		
MA.6.DP.1.1	Recognize and formulate a statistical question that would generate numerical data.	**Unit 9** Lessons 1, 2
MA.6.DP.1.2	Given a numerical data set within a real-world context, find and interpret mean, median, mode and range.	**Unit 9** Lessons 6, 7, 8, 10
MA.6.DP.1.3	Given a box plot within a real-world context, determine the minimum, the lower quartile, the median, the upper quartile and the maximum. Use this summary of the data to describe the spread and distribution of the data.	**Unit 9** Lessons 11, 12
MA.6.DP.1.4	Given a histogram or line plot within a real-world context, qualitatively describe and interpret the spread and distribution of the data, including any symmetry, skewness, gaps, clusters, outliers and the range.	**Unit 9** Lessons 4, 5
MA.6.DP.1.5	Create box plots and histograms to represent sets of numerical data within real-world contexts.	**Unit 9** Lessons 3, 5, 12
MA.6.DP.1.6	Given a real-world scenario, determine and describe how changes in data values impact measures of center and variation.	**Unit 9** Lessons 9, 10, 12
MA.7.DP.1.1	Determine an appropriate measure of center or measure of variation to summarize numerical data, represented numerically or graphically, taking into consideration the context and any outliers.	**Unit 9** Lessons 13, 16
MA.7.DP.1.2	Given two numerical or graphical representations of data, use the measure(s) of center and measure(s) of variability to make comparisons, interpret results and draw conclusions about the two populations.	**Unit 9** Lessons 13, 16, 17
MA.7.DP.1.3	Given categorical data from a random sample, use proportional relationships to make predictions about a population.	**Unit 9** Lessons 14, 15
MA.7.DP.2.1	Determine the sample space for a simple experiment.	**Unit 10** Lessons 1, 2
MA.7.DP.2.2	Given the probability of a chance event, interpret the likelihood of it occurring. Compare the probabilities of chance events.	**Unit 10** Lessons 2, 3, 4
MA.7.DP.2.3	Find the theoretical probability of an event related to a simple experiment.	**Unit 10** Lessons 2, 3, 4, 5
MA.7.DP.2.4	Use a simulation of a simple experiment to find experimental probabilities and compare them to theoretical probabilities.	**Unit 10** Lessons 3, 4, 5

Mathematical Thinking and Reasoning Standards

The following correlations show the alignment of Amplify Desmos Math Florida to the Mathematical Thinking and Reasoning Standards.

MA.K12.MTR.1.1 Actively participate in effortful learning both individually and collectively.

Unit 1 Lessons 6, 7, 9
Unit 2 Lessons 3, 6, 11
Unit 3 Lessons 2, 7, 8, 13
Unit 4 Lessons 4, 6, 10, 11, 12, 13
Unit 5 Lessons 4, 7, 10
Unit 6 Lessons 11, 13, 14
Unit 7 Lessons 1, 2, 4, 7, 8, 13, 16, 17, 18
Unit 8 Lesson 5
Unit 9 Lessons 1, 2, 9, 11, 12, 14
Unit 10 Lesson 4

MA.K12.MTR.2.1 Demonstrate understanding by representing problems in multiple ways.

Unit 1 Lessons 6, 8
Unit 2 Lessons 9, 10
Unit 3 Lesson 9
Unit 4 Lessons 1, 2, 3, 5, 8, 11
Unit 5 Lessons 1, 2, 3, 4, 5, 6, 9, 10
Unit 6 Lessons 1, 2, 3, 4, 6, 7, 10, 11, 12, 14
Unit 7 Lessons 3, 5, 6, 9, 14
Unit 8 Lessons 1, 2, 3, 4, 6
Unit 9 Lessons 6, 8
Unit 10 Lessons 1, 2

MA.K12.MTR.3.1 Complete tasks with mathematical fluency.

Unit 1 Lesson 3
Unit 3 Lessons 8, 11, 12
Unit 4 Lessons 4, 7, 8, 9, 10
Unit 5 Lessons 4, 7, 8, 9
Unit 6 Lessons 5, 10, 11, 15, 16, 17
Unit 7 Lessons 3, 5, 6, 8, 10, 11, 12, 15
Unit 8 Lessons 3, 4, 6, 7
Unit 9 Lessons 6, 8
Unit 10 Lessons 1, 2, 3

MA.K12.MTR.4.1 Engage in discussions that reflect on the mathematical thinking of self and others.

Unit 1 Lessons 4, 7, 8, 10, 11
Unit 2 Lessons 3, 4, 5, 7, 8, 12
Unit 3 Lessons 1, 3, 4, 5, 7, 8, 9, 11, 12
Unit 4 Lessons 4, 5, 6, 7, 10, 12, 14, 15
Unit 5 Lessons 4, 7, 10
Unit 6 Lessons 1, 2, 3, 4, 5, 6, 7, 8, 9, 11, 15, 17
Unit 7 Lessons 2, 4, 7, 10, 14, 15, 16, 17, 18, 19
Unit 8 Lessons 5, 6, 7
Unit 9 Lessons 1, 2, 3, 4, 5, 7, 9, 10, 11, 12, 13, 14, 15, 17
Unit 10 Lesson 4

MA.K12.MTR.5.1 Use patterns and structure to help understand and connect mathematical concepts.

Unit 1 Lessons 1, 2, 4, 5, 6, 7
Unit 2 Lessons 1, 2, 3, 5, 10
Unit 3 Lessons 2, 6, 7, 10
Unit 4 Lessons 1, 2, 6, 11
Unit 5 Lesson 8
Unit 6 Lessons 1, 2, 3, 4, 5, 6, 7
Unit 7 Lessons 1, 7, 8, 10, 11, 12, 13, 15, 19
Unit 8 Lesson 1
Unit 9 Lessons 5, 10, 15, 16
Unit 10 Lesson 5

MA.K12.MTR.6.1 Assess the reasonableness of solutions.

Unit 1 Lessons 3, 9, 10
Unit 2 Lesson 8
Unit 4 Lessons 1, 2, 7
Unit 5 Lessons 3, 8
Unit 7 Lesson 3
Unit 8 Lesson 7
Unit 9 Lessons 3, 6, 8, 15, 16
Unit 10 Lessons 3, 5

MA.K12.MTR.7.1 Apply mathematics to real-world contexts.

Unit 1 Lesson 8
Unit 2 Lessons 5, 9, 12
Unit 3 Lesson 13
Unit 4 Lesson 15
Unit 5 Lesson 8
Unit 6 Lessons 6, 16
Unit 7 Lessons 11, 12, 19
Unit 8 Lessons 1, 2
Unit 9 Lessons 4, 5, 7, 10, 13, 14, 17

English Language Arts B.E.S.T. Standards

ELA.K12.EE.1.1 Cite evidence to explain and justify reasoning.

Unit 1 Lessons 3, 8, 9
Unit 2 Lessons 4, 5
Unit 3 Lessons 4, 8, 12
Unit 4 Lessons 4, 6, 9, 12, 14
Unit 5 Lessons 2, 3, 7, 10
Unit 6 Lessons 1, 5, 6, 10, 11, 15, 17
Unit 7 Lessons 2, 3, 4, 5, 6, 7, 8, 9, 10, 12, 13, 15, 17, 19
Unit 8 Lessons 1, 3, 5, 6
Unit 9 Lessons 2, 3, 5, 6, 7, 8, 9, 10, 12, 14, 15, 16, 17
Unit 10 Lessons 3, 4

ELA.K12.EE.2.1 Read and comprehend grade-level complex texts proficiently.

Unit 2 Lessons 3, 6, 11
Unit 4 Lesson 1
Unit 6 Lessons 2, 4

ELA.K12.EE.3.1 Make inferences to support comprehension.

Unit 1 Lessons 1, 4
Unit 2 Lessons 1, 2, 4, 10
Unit 3 Lessons 2, 7, 8, 9, 10, 11, 12, 13
Unit 4 Lessons 2, 3, 8, 10, 11, 13
Unit 6 Lessons 1, 2, 3, 14
Unit 7 Lessons 1, 2, 19
Unit 8 Lessons 2, 3
Unit 9 Lessons 1, 15

ELA.K12.EE.4.1 Use appropriate collaborative techniques and active listening skills when engaging in discussions in a variety of situations.

Unit 1 Lessons 6, 7
Unit 2 Lesson 9
Unit 3 Lessons 1, 2, 3
Unit 4 Lesson 9
Unit 5 Lessons 2, 3, 6, 9
Unit 6 Lessons 2, 4, 5, 6, 9, 10, 11, 12, 13, 14, 15, 16
Unit 7 Lessons 5, 9, 11, 14
Unit 8 Lessons 1, 6
Unit 9 Lessons 3, 4, 13, 14

ELA.K12.EE.5.1 Use the accepted rules governing a specific format to create quality work.

Unit 1 Lessons 2, 4, 6, 7, 8, 9, 10, 11
Unit 2 Lessons 4, 5, 8, 9, 10, 11
Unit 3 Lessons 4, 6, 8, 9, 10, 11, 13
Unit 4 Lessons 1, 2, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14
Unit 5 Lessons 1, 2, 3, 4, 5, 6, 8, 9
Unit 6 Lessons 7, 8, 12, 15
Unit 7 Lessons 1, 3, 4, 5, 6, 11, 12, 14, 15, 16, 17, 18
Unit 8 Lessons 4, 7
Unit 9 Lessons 3, 4, 6, 7, 8, 11, 12, 13, 14, 16
Unit 10 Lessons 1, 2, 3, 5

ELA.K12.EE.6.1 Use appropriate voice and tone when speaking or writing.

Unit 1 Lessons 1, 3, 5, 8, 10
Unit 2 Lessons 1, 2, 3, 6, 7, 12
Unit 3 Lessons 1, 3, 4, 7, 12
Unit 4 Lessons 3, 4, 15
Unit 6 Lessons 4, 5, 6, 7, 13, 16, 17
Unit 7 Lessons 2, 9, 12, 13, 18
Unit 9 Lessons 1, 2, 7, 8, 17

English Language Development Standards

ELD.K12.ELL.MA.1 English language learners communicate information, ideas and concepts necessary for academic success in the content area of Mathematics.

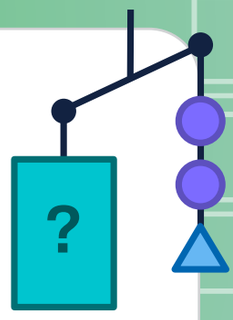
Unit 1, Lessons - All lessons
Unit 2, Lessons - All lessons
Unit 3, Lessons - All lessons
Unit 4, Lessons - All lessons
Unit 5, Lessons - All lessons
Unit 6, Lessons - All lessons
Unit 7, Lessons - All lessons
Unit 8, Lessons - All lessons
Unit 9, Lessons - All lessons
Unit 10, Lessons - All lessons

ELD.K12.ELL.SI.1 English language learners communicate for social and instructional purposes within the school setting.

Unit 1, Lessons - All lessons
Unit 2, Lessons - All lessons
Unit 3, Lessons - All lessons
Unit 4, Lessons - All lessons
Unit 5, Lessons - All lessons
Unit 6, Lessons - All lessons
Unit 7, Lessons - All lessons
Unit 8, Lessons - All lessons
Unit 9, Lessons - All lessons
Unit 10, Lessons - All lessons

UNIT 6

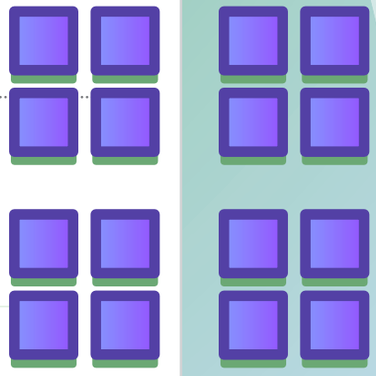
Expressions and Equations



Students build on their understanding of the equal sign and use it as a representation of equality (or balance), at which point they can perform balanced operations to solve for unknown values and relate equivalent expressions, including those involving two variables or exponents. Students apply what they know about ratios, proportions, and equations to solve problems involving percent change.

Essential Questions

- What does it mean for an equation to be true? Can an equation be false?
- What does it mean for two expressions to be equivalent?
- If multiplication is repeated addition, how do we represent repeated multiplication?
- How can percentages represent change in the real world?



Sub-Unit Goals

Sub-Unit 1

- Write and solve equations of the form $x + p = q$ and $px = q$.

Sub-Unit 2

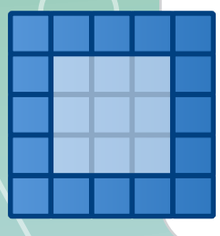
- Use properties to write equivalent expressions with variables.

Sub-Unit 3

- Evaluate numerical and variable expressions with natural number exponents and one or more other operations.

Sub-Unit 4

- Solve problems involving percent change using representations, such as tape diagrams, double number lines, and equations.
- Use ratios to solve problems in real-world situations involving percent increase, percent decrease, and percent error.
- Interpret situations involving percent change and apply appropriate representations to solve problems.



The Math of the Unit

Unit Outcomes Telling a story that narrows and deepens with the grade

Florida B.E.S.T. Standards for Mathematics

Apply properties of operations to rewrite numbers in equivalent forms. **(MA.6.NSO.3)**

Apply previous understanding of arithmetic expressions to algebraic expressions. **(MA.6.AR.1)**

Develop an understanding for solving equations and inequalities. Write and solve one-step equations in one variable. **(MA.6.AR.2)**

Rewrite algebraic expressions in equivalent forms. **(MA.7.AR.1)**

Use percentages and proportional reasoning to solve problems. **(MA.7.AR.3)**

Benchmarks Addressed in This Unit: MA.6.NSO.3.3 MA.6.NSO.3.4 MA.6.NSO.3.5 MA.6.AR.1.1 MA.6.AR.1.3 MA.6.AR.1.4 MA.6.AR.2.1 MA.6.AR.2.2 MA.6.AR.2.3 MA.6.AR.2.4 MA.7.AR.1.2 MA.7.AR.3.1

Mathematical Thinking and Reasoning Standards

- Demonstrate understanding by representing problems in multiple ways. **(MTR.2.1)**
- Complete tasks with mathematical fluency. **(MTR.3.1)**
- Engage in discussions that reflect on the mathematical thinking of self and others. **(MTR.4.1)**
- Use patterns and structure to help understand and connect mathematical concepts. **(MTR.5.1)**

Also addressed: **MTR.1.1, MTR.7.1**

Coherence Telling a story within and across grades

In this Unit

Students build on their understanding of the equal sign and use it as a representation of equality (or balance). They perform balanced operations to solve for unknown values and relate equivalent expressions. They apply ratio reasoning and proportions to solve a variety of problems involving percent change.

Prior Learning

- Determine factor pairs and distinguish whole numbers as prime, composite, or neither **(MA.4.AR.3.1)**
- Write and interpret numerical expressions and equations **(MA.5.AR.2.1, MA.5.AR.2.4)**
- **Unit 3:** Unit rate problems **(MA.6.AR.3.2, MA.6.AR.3.5)**
- **Unit 3:** Calculating percentages **(MA.6.AR.3.4)**
- **Units 4–5:** Fraction division and operations with decimals **(MA.6.NSO.2)**

Future Learning

- **Unit 8:** Solving equations with integers **(MA.6.AR.2.2, MA.6.AR.2.3)**
- Proportional relationships **(MA.7.AR.4.1, MA.7.AR.4.2, MA.7.AR.4.3, MA.7.AR.4.4)**
- Solve equations **(MA.7.AR.2.2)**
- Factor and expand expressions **(MA.7.NSO.2.2, MA.7.AR.1, MA.7.AR.2.2)**

See the Connections to Future Learning page for more information, including explanations and examples.

Rigor and Balance Integrating the aspects of rigor in ways that support one another

Here are examples of the aspects of rigor and balance in this unit. See the first page of each lesson's Teacher Edition for more information.

Conceptual Understanding

In **Lessons 1–3**, **6–8**, and **10–11**, students use visual models to represent equations and equivalent expressions. They represent relationships between variables using images, tables, equations, and graphs.

Strategic Competence

In **Lessons 1** and **9**, students choose a strategy when solving equations, identifying equivalent expressions, and evaluating expressions with exponents.

Procedural Fluency

In **Lessons 3** and **9–11**, students apply properties of operations to solve equations, generate equivalent expressions, and evaluate expressions with exponents.

Adaptive Reasoning

In **Lessons 2**, **5**, and **6**, students use multiple strategies to solve equations and justify whether expressions are equivalent. They make sense of problems and patterns using images, tables, equations, and graphs.

Application

In **Lesson 4**, students represent and solve real-world problems using equations, expressions, tables, and graphs.

Productive Disposition

In **Lessons 1** and **4**, students see themselves and their thinking as a valuable asset to the learning of others as they develop strategies for solving equations.

Unit at a Glance

Assess and Respond

A Pre-Unit Check

Learn more about your students' understanding of foundational concepts and skills that will support them in the upcoming unit. This can be given in its entirety before the unit or spread throughout the unit.

MA.4.AR.2.2 MA.5.NSO.1.1 MA.5.NSO.2.3 MA.5.AR.2.2 MA.5.AR.2.4 MA.5.GR.2.1
MTR.5.1

Sub-Unit 1

1 Weight for It

Reasoning About Unknown Values

Use reasoning, equations, and tape diagrams to determine unknown weights on a seesaw.

MA.6.AR.2.2 MA.6.AR.2.3
MTR.2.1 MTR.4.1 MTR.5.1

2 Five Equations

Tape Diagrams, Equations, Situations

Connect tape diagrams, equations, and situations, and use them as tools to answer questions in context.

MA.6.AR.2.1 MA.6.AR.2.2 MA.6.AR.2.3
MTR.2.1 MTR.4.1 MTR.5.1

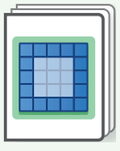
Sub-Unit 2

6 Vari-apples

Introduction to Variable Expressions

Use expressions with **variables** to represent mathematical and real-world situations (in this case, the cost of some number of pounds of fruit).

MA.6.AR.1.1 MA.6.AR.1.3
MTR.2.1 MTR.4.1 MTR.5.1 MTR.7.1



7 Border Tiles

Equivalent Expressions

Explore the concept of **equivalent expressions** (two different expressions that describe the same thing) in the context of the number of tiles that border a square.

MA.6.AR.1.4 MA.7.AR.1.2
MTR.2.1 MTR.4.1 MTR.5.1

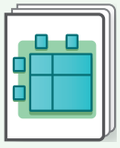
8 Products and Sums

Distributive Property, Part 1

Use variable expressions to represent areas of rectangles in two different ways: as the sum of two areas and as the product of the two side lengths.

MA.6.AR.1.4
MTR.4.1

Sub-Unit 4

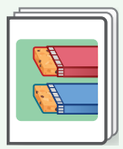


11 Exponent Expressions

Exponents and Order of Operations

Use areas to make sense of more complex expressions with exponents, particularly evaluating expressions that include an exponent and one other operation.

MA.6.NSO.3.3
MTR.1.1 MTR.2.1 MTR.3.1 MTR.4.1

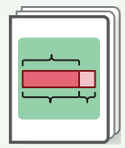


12 More and Less

Percent Increase and Decrease With Tape Diagrams

Use both tape diagrams and tables to represent **percent increase** and **percent decrease**, and then use these representations to solve problems.

MA.6.NSO.3.5 MA.7.AR.3.1
MTR.2.1



13 All the Equations

Percent Increase and Decrease With Equations

Write and use equations to describe the relationship between original and new amounts after a percent increase or decrease.

MA.6.NSO.3.5 MA.7.AR.3.1
MTR.1.1

Pacing: 21 days | **Short on time?** See pacing considerations below.

All instructional and assessment days are ~45 minutes each.

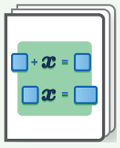
- 17 Lessons
- Math in Action
- Pre-Unit Check
- Sub-Unit Quiz
- End-of-Unit Assessment

3 Hanging Around

Introduction to Balanced Hangers

Use hangers to represent equations and then determine unknown values that balance each hanger.

MA.6.AR.2.2 MA.6.AR.2.3
MTR.2.1 MTR.4.1 MTR.5.1

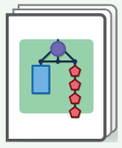


4 Swap and Solve

Solving Equations in Context

Connect equations to situations by writing situations to match equations and then trade situations with classmates.

MA.6.AR.2.1 MA.6.AR.2.2 MA.6.AR.2.3
MTR.2.1 MTR.4.1 MTR.5.1



5 Solutions of a Different Kind

Solving Equations With Decimals and Fractions

Apply reasoning to solve equations that include decimals and fractions.

MA.6.AR.2.4
MTR.3.1 MTR.4.1 MTR.5.1

Assess and Respond

9 Equivalent Expressions

Distributive Property, Part 2

Extend understanding of equivalent expressions to expressions with the distributive property and other operations.

MA.6.AR.1.4 MA.7.AR.1.2
MTR.4.1

A Sub-Unit Quiz

Learn about your students' understanding of the concepts and skills so far in this unit.

MA.6.AR.1.1 MA.6.AR.1.3 MA.6.AR.1.4
MA.6.AR. 2.2 MA.6.AR.2.3 MA.6.AR.2.4
MTR.5.1 MTR.7.1

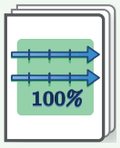
Sub-Unit 3

10 Powers

What Are Exponents?

Make sense of **exponents**, both when the **base** is a whole number and when it is a fraction.

MA.6.NSO.3.3 MA.6.NSO.3.4
MTR.2.1 MTR.3.1



14 100%

Percent Increase and Decrease With Double Number Lines

Expand on understanding of double number line diagrams and percentages to model and make sense of problems involving percent change.

MA.6.NSO.3.5 MA.7.AR.3.1
MTR.1.1 MTR.2.1

15 Tax, Tips, and Fees

Multistep Percent Problems

Solve multistep percent problems in a common context and calculate the total cost after applying sales tax, tips, and fees.

MA.6.NSO.3.5 MA.7.AR.3.1
MTR.3.1 MTR.4.1

16 How Simple, How Interesting

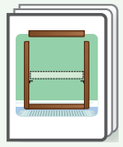
Calculating Simple Interest Using a Formula

Use the simple interest formula to determine the interest earned on deposits and **interest** paid on loans.

MA.6.NSO.3.5 MA.7.AR.3.1
MTR.3.1 MTR.7.1

Unit at a Glance

continued



17 **Bookcase Builder**

Percent Error

Consider why **percent error** is useful, determine percent error, and decide whether a measurement is within an acceptable range.

MA.6.NSO.3.5 MA.7.AR.3.1
MTR.3.1 MTR.4.1

M **Math in Action**

Connecting Equations to Telecommunications

Explore the connection between equations, telecommunication towers, and the contributions of physicist Sir Jagadish Chandra Bose.

MA.6.AR.1.4
MTR.1.1

Assess and Respond

A **End-of-Unit Assessment**

Learn about your students' understanding of the concepts and skills in this unit. There are two forms of the End-of-Unit Assessment: Forms A and B

MA.6.AR.1.1 MA.6.AR.1.3 MA.6.AR.1.4
MA.6.AR.2.1 MA.6.AR.2.4 MA.6.NSO.3.3
MA.7.AR.1.2 MA.7.AR.3.1 MA.7.AR.3.2
MA.7.AR.4.2 MA.7.AR.4.5
MTR.2.1 MTR.4.1 MTR.5.1 MTR.7.1

Pacing Considerations

Lesson 1: The purpose of this lesson is to lay the foundation for solving equations, including revisiting tape diagrams and introducing equations. If students show a strong understanding of determining unknown values and connecting tape diagrams to equations in the Pre-Unit Check, then this lesson may be omitted.

Lesson 4: This lesson invites students to use what they have learned to write their own situations based on equations, then interpret the meaning of those solutions in the situation. It may be omitted if students show a strong proficiency in moving fluidly between situations, equations, and solutions in earlier lessons.

Lesson Materials and Prep

Here are the materials and resources needed within each lesson, how to prepare them, and where to find them.

Lesson 1
- **Copy:** Show What You Know, Activity 3 Sheet (*one per student*)

Lesson 2
- **Copy:** Show What You Know
- **Cut:** Activity 2 Cards (*one set per pair*)

Lesson 3
- **Copy:** Show What You Know, Activity 3 Sheet (*one per student*)
- **Materials:** clothes on hangers (*as needed*)

Lesson 4
- **Copy:** Show What You Know
- **Materials:** index cards or slips of paper

Lesson 5
- **Copy:** Show What You Know

Lesson 6
- **Copy:** Show What You Know
- **Materials:** calculators (*as needed*)

Lesson 7
- **Copy:** Show What You Know
- **Materials:** algebra tiles (*as needed*), graph paper (*as needed*), highlighters (*as needed*)

Lesson 8
- **Copy:** Show What You Know, Activity 3 Sheet (*one per student*)
- **Materials:** algebra tiles (*as needed*)

Lesson 9
- **Copy:** Show What You Know
- **Cut:** Activity 1 Cards (*one set per pair*)

Lesson 10
- **Copy:** Show What You Know

Lesson 11
- **Copy:** Show What You Know
- **Cut:** Activity 1 Cards (*one set per pair*)

Lesson 12
- **Copy:** Show What You Know

Lesson 13
- **Copy:** Show What You Know
- **Materials:** calculators (*as needed*)

Lesson 14
- **Copy:** Show What You Know

Lesson 15
- **Copy:** Show What You Know
- **Materials:** calculators

Lesson 16
- **Copy:** Show What You Know
- **Materials:** calculators

Lesson 17
- **Copy:** Show What You Know

Resource Books

Assessment and Lesson Resources

- Pre-Unit Check
- Sub-Unit Quizzes
- End-of-Unit Assessment, Forms A & B
- Assessment Rubrics
- Show What You Knows
- Activity Sheets and Cards

Intervention and Extension Resources

- Mini-Lessons
- Prior Grade Mini-Lessons
- Extensions

Caregiver Support

These resources can be shared with students' caregivers. They provide background on the mathematics in this unit, as well as suggestions for supporting students at home.

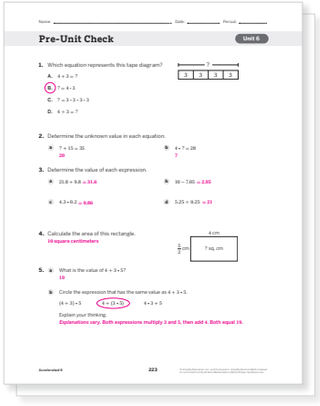
Assessments

Every unit includes a variety of formative and summative assessments, designed to highlight students' understanding of a variety of concepts and skills.

Pre-Unit Check

Assign the **Pre-Unit Check** to learn more about your students' understanding of foundational concepts and skills that will support them in the upcoming unit. This can be given in its entirety before the unit or spread throughout the unit.

Standards addressed: MA.4.AR.2.2, MA.5.NSO.1.1, MA.5.NSO.2.3, MA.5.AR.2.2, MA.5.AR.2.4, MA.5.GR.2.1


Pre-Unit Check

Unit 6

1. Which equation represents this tape diagram?

A. $4 + 3 = 7$

B. $7 = 4 + 3$

C. $7 = 3 - 3 - 3 - 3$

D. $4 + 3 = 7$

2. Determine the unknown value in each equation.

a. $7 + 13 = 35$, $n = 28$

b. $4 \cdot 7 = 28$, $n = 7$

3. Determine the value of each expression.

a. $21.8 + 9.8 = 31.6$

b. $10 - 7.65 = 2.35$

c. $4.3 \cdot 9.2 = 0.86$

d. $5.25 \div 0.25 = 21$

4. Calculate the area of this rectangle.

10 square centimeters

5. a. What is the value of $4 + 3 \cdot 5$?

19

b. Circle the expression that has the same value as $4 + 3 \cdot 5$.

$(4 + 3) \cdot 5$ $4 + (3 \cdot 5)$ $4 \cdot 3 + 5$

Explain your thinking.

Explanations vary. Both expressions multiply 3 and 5, then add 4. Both equal 19.

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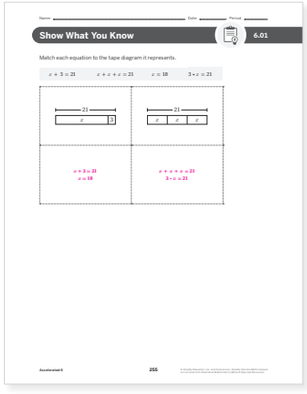
Lesson | Show What You Know

Use the **Show What You Know** for each lesson to learn more about your students' progress toward the lesson goal(s). Here is one example from this unit.

Standard addressed on Lesson 1 Show What You Know: MA.6.NSO.2.2, MA.6.NSO.2.3


Show What You Know

6.01

Match each equation to the tape diagram it represents.

$x + 3 = 21$ $x + x + x = 21$ $x = 18$ $3 \cdot x = 21$

$x = 3 \cdot 21$
$x = 18$

$x + x + x = 21$
$3 \cdot x = 21$

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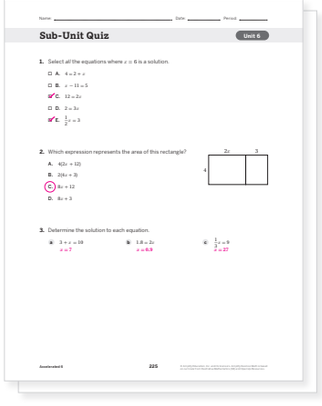
Sub-Unit Quiz

Assign the **Sub-Unit Quiz** to learn about your students' understanding of the concepts and skills so far in this unit.

Standards addressed on the Sub-Unit Quiz: MA.6.AR.1.1, MA.6.AR.1.3, MA.6.AR.1.4, MA.6.AR.2.1, MA.6.AR.2.2, MA.6.AR.2.3, MA.6.AR.2.4


Sub-Unit Quiz

Unit 6

1. Select all the equations where $x = 6$ is a solution.

A. $4 = 2 + x$

B. $x - 11 = 5$

C. $12 = 2x$

D. $2 = 3x$

E. $\frac{1}{2}x = 3$

2. Which expression represents the area of this rectangle?

A. $4(2x + 12)$

B. $2(4x + 3)$

C. $8x + 12$

D. $8x + 3$

3. Determine the solution to each equation.

a. $3 + x = 19$, $x = 7$

b. $1.8 = 2x$, $x = 0.9$

c. $\frac{1}{3}x = 9$, $x = 27$

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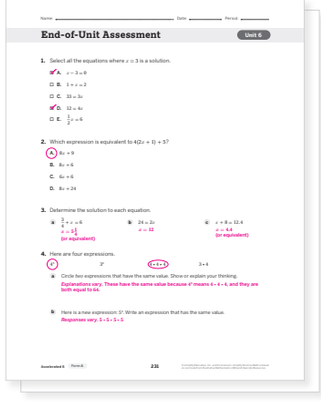
End-of-Unit Assessment

Assign the **End-of-Unit Assessment** to learn about your students' understanding of the concepts and skills in this unit. There are two forms of the End-of-Unit Assessment: Forms A and B.

Standards addressed: MA.6.AR.1.1, MA.6.AR.1.3, MA.6.AR.1.4, MA.6.AR.2.1, MA.6.AR.2.4, MA.6.NSO.3.3, MA.7.AR.1.2, MA.7.AR.3.1, MA.7.AR.3.2. MA.7.AR.4.2, MA.7.AR.4.5


End-of-Unit Assessment

Unit 6

1. Select all the equations where $x = 3$ is a solution.

A. $x - 3 = 0$

B. $1 + x = 2$

C. $10 = 3x$

D. $12 = 4x$

E. $\frac{1}{3}x = 6$

2. Which expression is equivalent to $4(2x + 1) + 5$?

A. $8x + 9$

B. $8x + 6$

C. $6x + 6$

D. $8x + 24$

3. Determine the solution to each equation.

a. $\frac{1}{2}x = 6$, $x = 3$ (or equivalent)

b. $24 = 2x$, $x = 12$

c. $x + 8 = 12.4$, $x = 4.4$ (or equivalent)

4. Here are four expressions.

A. $4 + 4$ B. 3^4 C. $4 \cdot 4$ D. $3 + 4$

a. Circle two expressions that have the same value. Show or explain your thinking.

Explanations vary. These have the same value because 4² means 4 • 4, and they are both equal to 64.

b. Here is a new expression: 5^4. Write an expression that has the same value.

Responses vary. $5 \cdot 5 \cdot 5 \cdot 5$

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Differentiation

Every unit includes additional resources to support, strengthen, and stretch student learning.

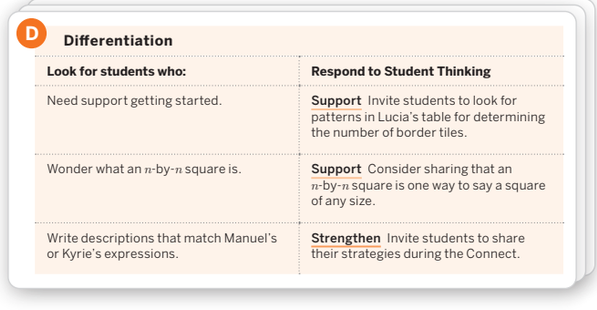
Differentiation Across the Unit

	Support Provide targeted intervention for students by using these resources.	Strengthen Reinforce students' understanding of the concepts assessed by using these resources.	Stretch Challenge students and extend their learning with these resources.
Sub-Unit 1 (Lessons 1–5) • Write and solve equations of the form $x + p = q$ and $px = q$.	**Mini-Lesson** • *Writing and Solving Equations*	**Repeated Challenges** • Lesson 3 (*Hanging Around*) • Lesson 5 (*Solutions of a Different Kind*) **Challenge Creator** • Lesson 1 (*Weight for It*) • Lesson 3 (*Hanging Around*) **Practice** • Lessons 1–5	**DOK3 Practice** • Lessons 1–2, 4
Sub-Unit 2 (Lessons 6–9) • Use properties to write equivalent expressions with variables.	**Mini-Lesson** • *Writing Equivalent Expressions Using the Area Model*	**Challenge Creator** • Lesson 8 (*Products and Sums*) **Practice** • Lessons 6–9	**You're invited to explore more.** • Lesson 6 (*Vari-apples*) **DOK3 Practice** • Lessons 6–9
Sub-Unit 3 (Lessons 10–11) • Evaluate numerical and variable expressions with natural number exponents and one or more other operations.	**Mini-Lesson** • *Evaluating Expressions With Exponents*	**Repeated Challenges** • Lesson 10 (*Powers*) **Practice** • Lessons 10–11	**You're invited to explore more.** • Lesson 11 (*Exponent Expressions*) **DOK3 Practice** • Lesson 11
Sub-Unit 4 (Lessons 12–17) • Solve problems involving percent change using representations, such as tape diagrams, double number lines, and equations. • Use ratios to solve problems in real-world situations involving percent increase, percent decrease, and percent error. • Interpret situations involving percent change and apply appropriate representations to solve problems.	**Mini-Lessons** • *Calculating Percentages* • *Finding Sales Tax and Tip*	**Repeated Challenges** • Lesson 17 (*Bookcase Builder*) **Practice** • Lessons 12–17	**You're invited to explore more.** • Lesson 12 (*More and Less*) • Lesson 14 (*100%*) • Lesson 17 (*Bookcase Builder*) **DOK3 Practice** • Lessons 14–16

Differentiation Within the Lesson

Each Teacher Edition includes point-of-use differentiation suggestions to modify or adjust instruction during the lesson to accommodate the needs of all learners.

Here is one example from **Lesson 7**, where students compare and describe various strategies for determining the number of the border tiles for an n-by-n square.



Differentiation

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Invite students to look for patterns in Lucia's table for determining the number of border tiles.
Wonder what an n-by-n square is.	**Support** Consider sharing that an n-by-n square is one way to say a square of any size.
Write descriptions that match Manuel's or Kyrie's expressions.	**Strengthen** Invite students to share their strategies during the Connect.

Accessibility

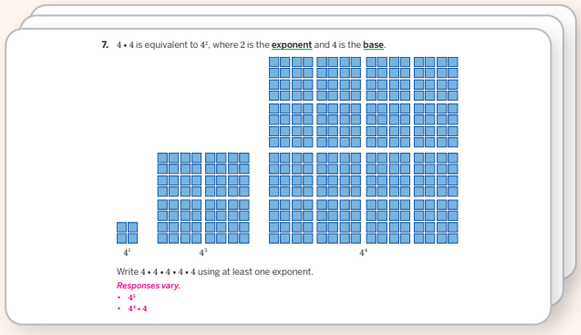
Every lesson is designed using the principles of Universal Design for Learning. Every lesson also includes support suggestions that build on students' strengths.

Universal Design for Learning

Here is one example from a lesson designed to support Representation.

Spotlight on Language & Symbols . . .

Lesson 10: In this lesson, students explore using exponents to identify and write equivalent expressions, including the prime factorization of a composite number.



Just-In-Time Accommodations and Supports

Each Teacher Edition includes suggested supports that are based on the areas of cognitive function recognized by the Education Development Center (EDC).

Spotlight on Conceptual Processing . . .

Here are two suggestions in this unit for supporting students with thinking abstractly and applying concepts to new situations.

Lesson 1, Activity 1

A **Accessibility: Conceptual Processing** Invite students to read the equation aloud with the variable's meaning instead of d (e.g., The weight of the dog plus 5 pounds is equal to 17 pounds.).

Lesson 10, Activity 3

A **Accessibility: Conceptual Processing** To support making connections between new problems and prior work, consider asking, "How are $\left(\frac{1}{2}\right)^2$ and 2^2 alike? How are they different?"

Language Development

Every lesson includes opportunities for all students to develop fluency with mathematical language.

Vocabulary

New Vocabulary	Lesson
variable	1
solution to an equation	2
term	6
equivalent expressions	7
coefficient	9
base (of a power) **exponent** **prime factorization**	10
percent decrease **percent increase**	12
markdown **markup**	14
interest **principal** **simple interest**	16
percent error	17

Review Vocabulary
distributive property
equation
expression
order of operations
product
sum

Additional Vocabulary
associative property
commutative property
like terms
substitute

Math Language Routines and Language Goals

Math Language Routine	Lesson(s)
MLR1: Stronger and Clearer Each Time	**4 and 6**
MLR2: Collect and Display	**1–7, 9–13, and 15–17**
MLR6: Three Reads	**2**
MLR7: Compare and Connect	**5 and 7**

Sub-Unit Language Goals

Students engage in the following language functions:

- Describe a situation that could be represented by an equation of the form $x + p = q$ or $px = q$, for a rational number p and unknown x.
- Interpret diagrams that represent equations of the form $p + x = q$ or $px = q$.
- Analyze expressions to determine if they are equivalent.
- Describe a pattern that could be expressed using exponents.
- Interpret expressions with exponents using language like *to the power.*
- Interpret scenarios depicting percent change and percent error.

English Language Learners Support

Every Teacher Edition includes suggestions for supporting the needs of students learning English.

Here is one example from **Lesson 10**, where students use sentence frames to help explain their strategy for determining the value of 2^4.

ELL **English Language Learners** Provide sentence frames to help students explain their strategy (e.g., First, _____. Then, _____.). **(Writing and Speaking)**

Connections to Future Learning

Here is how the content in this unit connects to where your students are headed in their math journeys.

Solving Equations With Integers

In this unit, students are introduced to solving 1-step equations with positive rational numbers. **In Unit 8**, they will solve equations with integers.

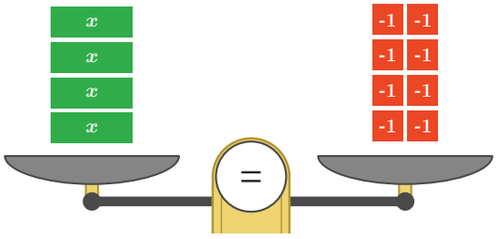
FL B.E.S.T. Math Benchmark:
MA.6.AR.2.3

Example:

The scale represents the equation $4x = -8$. We can match each x-tile with 2 negative 1-tiles, so $x = -2$.

$\frac{4x}{4} = \frac{-8}{4}$

$x = -2$



Proportional Relationships

In this unit, students represent real-world situations using equations. **In a future course**, they will explore proportional relationships.

FL B.E.S.T. Math Benchmark:
MA.7.AR.4.4

Example:

The cost of the carpet is 1.5 times the number of square feet. We can represent this relationship with the equation shown.

$y = 1.5x$

x represents the number of square feet of carpet bought.

y represents the cost of the carpet, in dollars.

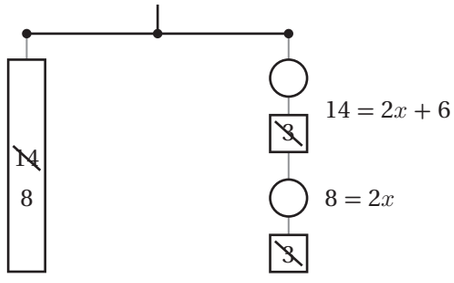
Solving More Complex Equations

In this unit, students solve one-step equations with rational numbers. **In a future course**, they will solve multi-step equations of the form $px + q = r$ and $p(x + q) = r$.

FL B.E.S.T. Math Benchmark:
MA.7.AR.2.2

Example:

We can solve the equation $14 = 2x + 6$ by removing 6 from the left and right to create the equation $8 = 2x$, which can be solved by using any method from this unit in Grade 6.



Factoring and Expanding Expressions

In this unit, students use the distributive property and area models to visualize expressions. **In a future course**, they will use the distributive property to expand expressions and work backwards to factor expressions.

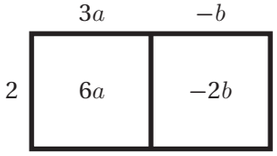
FL B.E.S.T. Math Benchmark:
MA.7.AR.2.2

Example:

The expanded expression $6a - 2b$ is represented inside the area model.

The 2 in front is the value being factored out. Dividing $6a$ and $-2b$ by 2 gives $3a$ and $-b$, respectively.

The factored form of $6a - 2b$ is $2(3a - b)$.



Factored $2(3a - b)$

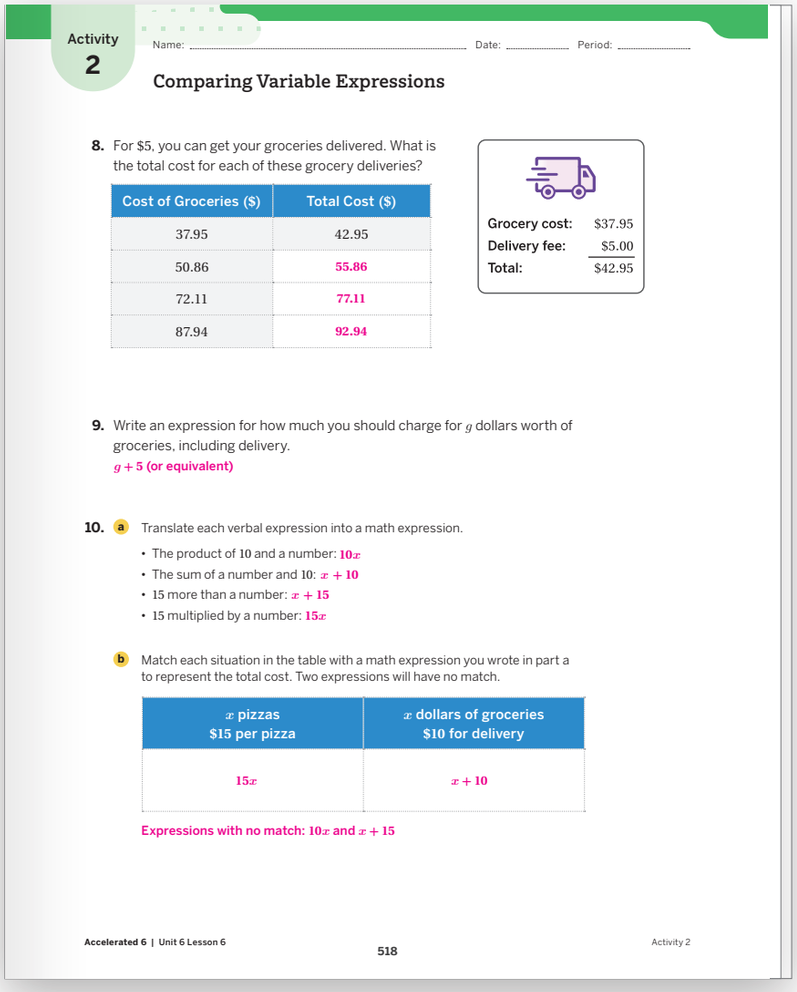
Expanded $6a - 2b$

Professional Learning

Here is an opportunity to deepen your understanding of the math in this unit and how to teach it.

Spotlight on Adaptive Reasoning

Lesson 6, Activity 2



Activity 2

Name: _____ Date: _____ Period: _____

Comparing Variable Expressions

8. For $5, you can get your groceries delivered. What is the total cost for each of these grocery deliveries?

Cost of Groceries ($)	Total Cost ($)
37.95	42.95
50.86	55.86
72.11	77.11
87.94	92.94

Grocery cost: $37.95
Delivery fee: $5.00
Total: $42.95

9. Write an expression for how much you should charge for g dollars worth of groceries, including delivery.
$g + 5$ (or equivalent)

10. a. Translate each verbal expression into a math expression.

- The product of 10 and a number: $10x$
- The sum of a number and 10: $x + 10$
- 15 more than a number: $x + 15$
- 15 multiplied by a number: $15x$

b. Match each situation in the table with a math expression you wrote in part a to represent the total cost. Two expressions will have no match.

x pizzas $15 per pizza	x dollars of groceries $10 for delivery
$15x$	$x + 10$

Expressions with no match: $10x$ and $x + 15$

Accelerated 6 | Unit 6 Lesson 6 518 Activity 2

Context

In this activity, students develop adaptive reasoning as they determine what variable expression can be used to represent each scenario.

Try This

Put on your student hat and complete Problems 8–10.

Points to Ponder

- What do students need to understand about the relationship between the values in each expression to support their adaptive reasoning about the operations that connect them?
- How does determining the variable expression in context build from their understanding of solving equations in context?
- How will writing variable expressions support their understanding of identifying and writing equivalent expressions in the future?

Notes:

Assess and Respond

Pre-Unit Check

Independent | 45 min

Facilitation: Assign the Pre-Unit Check to learn more about your students' understanding of foundational concepts and skills that will support them in the upcoming unit. This can be given in its entirety before the unit or spread throughout the unit. If time allows, consider inviting students to complete one or more of the tasks in the Unit Synthesis and Reflection.

Item Analysis

Problem(s)	Concept or skill	Preparation for	DOK	FL B.E.S.T. Math Benchmarks
1	Writing an equation to represent a tape diagram	Lesson 1	1	MA.5.AR.2.4, MTR.5.1
2a, 2b	Determining unknowns in equations	Lesson 1	1	MA.4.AR.2.2, MTR.5.1
3a–3d	Evaluating numerical expressions involving decimal operations	Lesson 5	1	MA.5.NSO.2.3
4	Calculating areas of rectangles with fractional side lengths	Lesson 5	1	MA.5.GR.2.1
5a, 5b	Evaluating numerical expressions involving grouping symbols and determining equivalence	Lesson 8	1	MA.5.AR.2.2
6	Explaining how the structure of an expression involving exponents shows its value	Lesson 10	1	MA.5.NSO.1.1

Assessment and Lesson Resources

- Student Assessments
- Show What You Know and Answer Keys

Intervention and Extension Resources

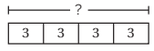
- Mini-Lessons
- Extensions

Name: ..... Date: ..... Period: .....

Pre-Unit Check

Unit 6

1. Which equation represents this tape diagram?



A. $4 + 3 = ?$

B. $? = 4 \cdot 3$

C. $? = 3 \cdot 3 \cdot 3 \cdot 3$

D. $4 \div 3 = ?$

2. Determine the unknown value in each equation.

a. $? + 15 = 35$
20

b. $4 \cdot ? = 28$
7

3. Determine the value of each expression.

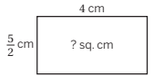
a. $21.8 + 9.8$ **$= 31.6$**

b. $10 - 7.05$ **$= 2.95$**

c. $4.3 \cdot 0.2$ **$= 0.86$**

d. $5.25 \div 0.25$ **$= 21$**

4. Calculate the area of this rectangle.
10 square centimeters



5. a. What is the value of $4 + 3 \cdot 5$?
19

b. Circle the expression that has the same value as $4 + 3 \cdot 5$.

$(4 + 3) \cdot 5$ $4 + (3 \cdot 5)$ $4 \cdot 3 + 5$

Explain your thinking.
***Explanations vary*. Both expressions multiply 3 and 5, then add 4. Both equal 19.**

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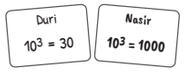
Name: ..... Date: ..... Period: .....

Pre-Unit Check (continued)

Unit 6

6. Duri says the value of 10^3 is 30.

Nasir says the value of 10^3 is $1{,}000$.



Who is correct? Circle one.

Duri Nasir Both Neither

Explain your thinking.
***Explanations vary*. Nasir is correct because 10^3 means $10 \cdot 10 \cdot 10$, which is equal to $1{,}000$.**

D Differentiation (Pre-Unit Check)

Note: To **strengthen** and **stretch** students' learning, refer to the differentiation resources suggested throughout this unit and in the Unit Overview.

Sub-Unit Goals	Problem(s)	Respond to Student Thinking
Preparation for Sub-Unit 1: Write and solve equations of the form $x + p = q$ and $px = q$. **(Lessons 1–5)**	1, 2	**Support** • **Mini-Lesson:** *Solving Comparison Problems Using Tape Diagrams* • **Teacher Move:** Prior to Lesson 1, consider creating tape diagrams using the equations in Problems 1–2 that you can reference throughout the lessons.

	3, 4	**Support**
		• **Mini-Lesson:**
		  • *Adding Decimals*
		  • *Multiplying Whole Numbers by Fractions and Mixed Numbers*
		• **Teacher Move:** Prior to Lesson 5, consider creating or referencing an anchor chart from Unit 5 with common strategies for adding, subtracting, multiplying, and dividing decimals.

	5	**Support** • **Mini-Lesson:** *Multiplying Whole Numbers by Fractions and Mixed Numbers* • **Teacher Move:** Prior to Lesson 8, consider reviewing order of operations and the purpose of parentheses by evaluating each expression in Problem 5.
	6	**Support** • **Mini-Lesson:** *Representing Powers of 10 With Exponents* • **Teacher Move:** Prior to Lesson 10, consider creating an anchor chart for students to reference throughout Lesson 10.

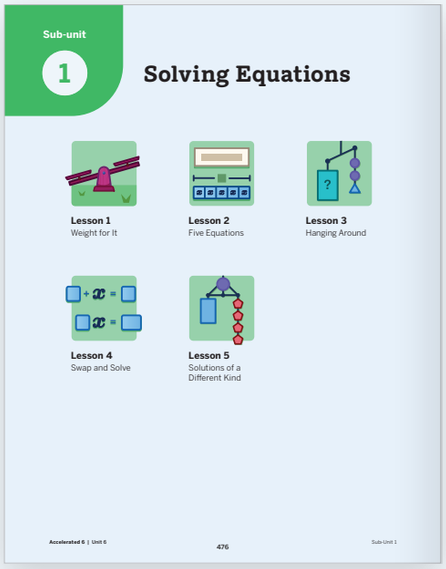
Sub-Unit 1

Solving Equations

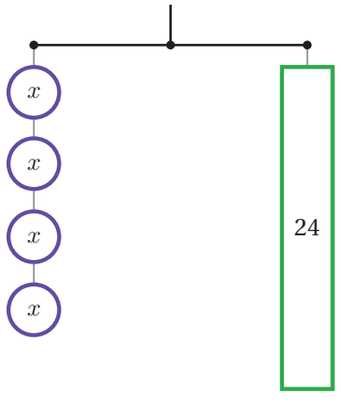
Sub-Unit 1 Goals

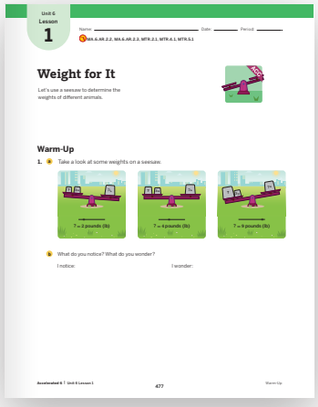
- **Goal:** Write and solve equations of the form $x + p = q$ and $px = q$.
- **Language Goal:** Describe a situation that could be represented by an equation of the form $x + p = q$ or $px = q$, for a rational number p and unknown x.
- **Language Goal:** Interpret diagrams that represent equations of the form $p + x = q$ or $px = q$.

In this sub-unit, students write and solve equations of the form $x + p = q$ and $px = q$ in and out of context.



Math That Matters Most

Concept or Skill	For example . . .
Write and solve equations of the form $x + p = q$ and $px = q$.	Yasmine is biking 5 miles to her friend's house. After she biked 2 miles, she had x miles left to go. Write an equation to match this situation. **2 + x = 5** What is the solution to the equation? **x = 3**
Interpret diagrams that represent equations of the form $p + x = q$ or $px = q$.	Here is a balanced hanger. Write an equation to represent this hanger. **4x = 24** 
Use reasoning to determine the unknown rational number in an equation of the form $x + p = q$ or $px = q$.	How can you use the relationship between the numbers in $\frac{5}{8} + x = \frac{12}{8}$ to determine the solution? **The whole has 12 eighths. The known part of the whole is 5 eighths. 5 + 7 = 12, so x must be $\frac{7}{8}$.**



Weight for It

Reasoning About Unknown Values

Let's use a seesaw to determine the weights of different animals.

Learning Outcomes

Today's Goals

1. **Goal:** Use reasoning and tape diagrams to determine unknown values.
2. **Language Goal:** Describe connections between diagrams that represent equations of the form $x + p = q$ or $px = q$. **(Speaking, Listening, and Writing)**

Students use reasoning, *equations*, and tape diagrams to determine unknown weights on a seesaw. They are introduced to **variables** as letters that represent unknown quantities. Students make connections between the visuals of the seesaw, equations, and tape diagrams. **(MTR.5.1)**

Coherence

Prior Learning

In a previous course, students used letters to represent unknown amounts. In Unit 3, students used tape diagrams to reason about percentages and determine unknown values.

Future Learning

In Lesson 2, students will use tape diagrams to write and solve equations with a variable and describe what solutions mean in context.

Connections

- Students use balance models and animal weights to build connections between variables and equations. (mathematical, real world)

FL B.E.S.T. Math Benchmarks

Addressing

MA.6.AR.2.2
Write and solve one-step equations in one variable within a mathematical or real-world context using addition and subtraction, where all terms and solutions are integers.

MA.6.AR.2.3
Write and solve one-step equations in one variable within a mathematical or real-world context using multiplication and division, where all terms and solutions are integers.

Mathematical Thinking and Reasoning Standards: MTR.2.1, MTR.4.1, MTR.5.1

ELA Expectations: ELA.K12.EE.1.1, ELA.K12.EE.3.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.5.AR.1.1	**MA.7.AR.2.2**
MA.5.AR.2.1	
MA.5.AR.2.4	

Vocabulary

New Vocabulary

variable

Review Vocabulary

equation

Lesson at a Glance

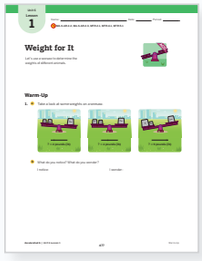
~ 45 min

MA.6.AR.2.2, MA.6.AR.2.3

Warm-Up

Whole Class | 5 min

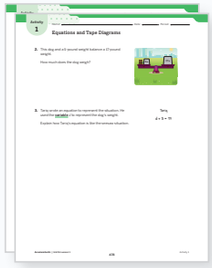
Students use the **Notice and Wonder** routine to explore how changing the weight of an object affects a seesaw.



Activity 1

Pairs | 10 min

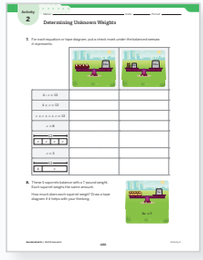
Students use *equations* and tape diagrams to represent seesaw situations and to determine unknown animal weights. **(MTR.2.1, MTR.5.1)**



Activity 2

Pairs | 10 min

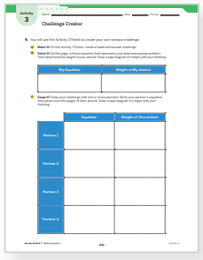
Students connect tape diagrams, equations, and seesaws and use them to determine unknown animal weights. **(MTR.2.1, MTR.5.1)**



Activity 3

Pairs | 10 min

Students use their creativity to make their own seesaw problem for other students to solve.

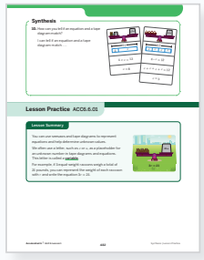


Synthesis

Whole Class | 5 min

Students synthesize their understanding of how equations and tape diagrams can represent situations with unknowns.

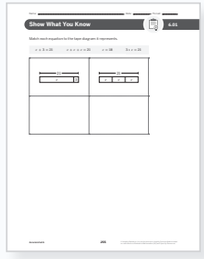
MLR2: Collect and Display



Show What You Know

Independent | 5 min

Students demonstrate their understanding by matching equations to tape diagrams.



Prep Checklist

Prepare Activity 3 Sheets

This lesson includes:

- Student Edition
- Show What You Know
- Activity 3 Sheets, one per student

Whole Class | 5 min

Warm-Up

Purpose: Students use the Notice and Wonder routine to explore how changing the weight of an object affects a seesaw.

1 Launch

Direct students' attention to the images of the seesaws.

ELL **English Language Learners** Invite students to share what they know about seesaws to increase access to the task. It may be helpful to show other images or videos of seesaws. **(Reading and Listening)**

To encourage playfulness, consider asking, "What is your favorite item on a playground?"

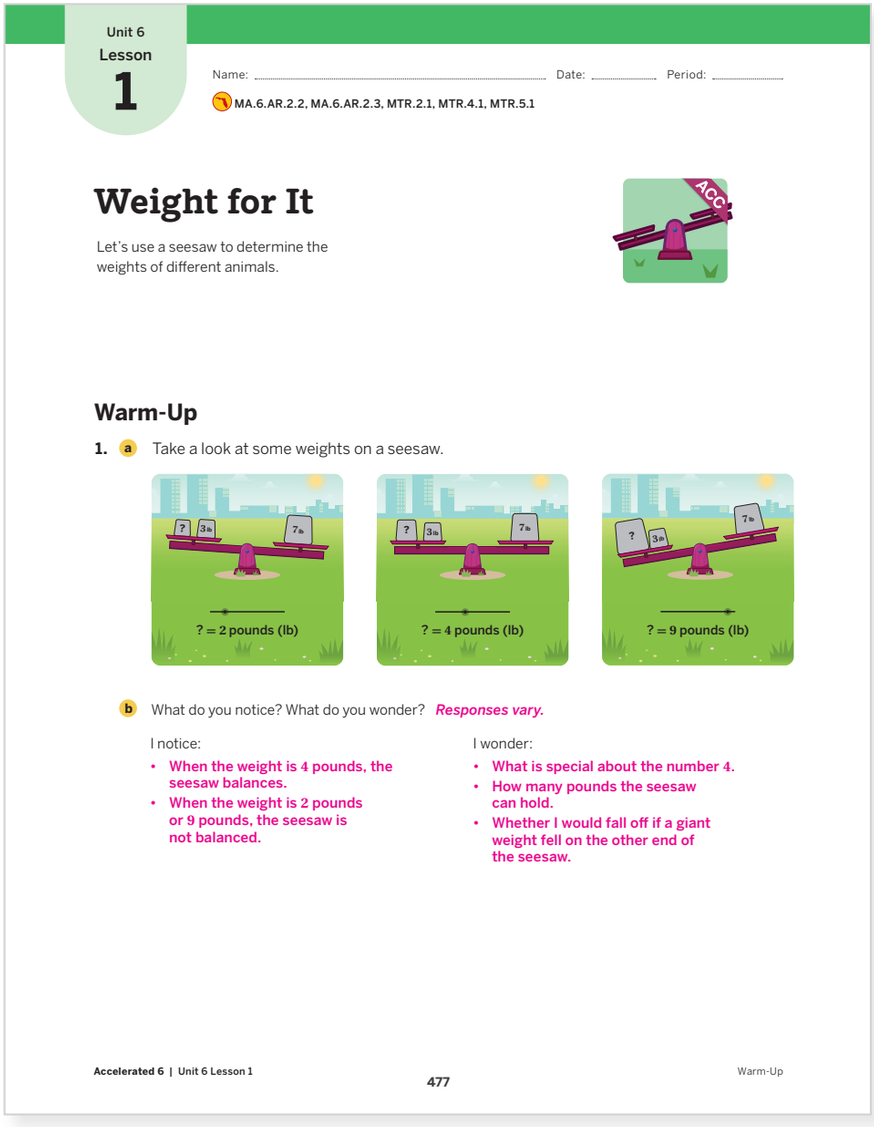
Use the Notice and Wonder routine to promote curiosity and help students make sense of balancing a seesaw. See the Routine Facilitation Guide for more information.

2 Connect

Invite students to share their noticings and wonderings.

Consider asking:

- "What happens when the weight is less than 4 pounds? Greater than 4 pounds?"
- "What happens when the weight is exactly 4 pounds? Why might that make sense?"



Unit 6
Lesson
1

Name: ..... Date: ..... Period: .....

MA.6.AR.2.2, MA.6.AR.2.3, MTR.2.1, MTR.4.1, MTR.5.1

Weight for It

Let's use a seesaw to determine the weights of different animals.

Warm-Up

1. a Take a look at some weights on a seesaw.

b What do you notice? What do you wonder? *Responses vary.*

I notice:
- When the weight is 4 pounds, the seesaw balances.
- When the weight is 2 pounds or 9 pounds, the seesaw is not balanced.

I wonder:
- What is special about the number 4.
- How many pounds the seesaw can hold.
- Whether I would fall off if a giant weight fell on the other end of the seesaw.

Accelerated 6 | Unit 6 Lesson 1 477 Warm-Up

Pairs | 10 min

Activity 1 Equations and Tape Diagrams

Purpose: Students use *equations* and tape diagrams to represent seesaw situations and to determine unknown animal weights. **(MTR.2.1, MTR.5.1)**

1 Launch

To encourage connections, consider asking, "How is this situation similar to the Warm-Up? How is it different?" **(MTR.4.1)**

Encourage students to get feedback from a partner to help them revise their responses.

2 Monitor

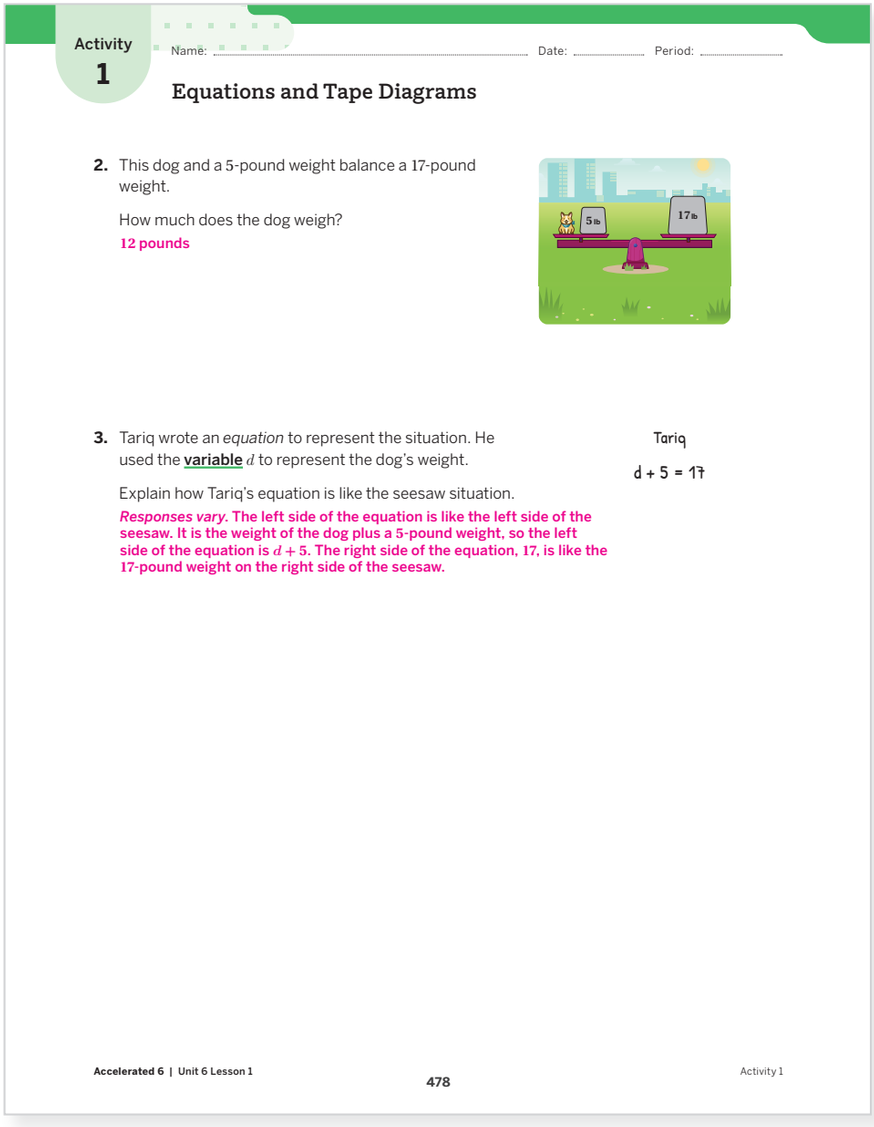
Invite students to discuss their strategy for determining the dog's weight with a partner.

A **Accessibility: Conceptual Processing** Invite students to read the equation aloud with the variable's meaning instead of d (e.g., The weight of the dog plus 5 pounds is equal to 17 pounds.).

Pause to invite students to share their ideas about how the *equation* is like the seesaw. **(MTR.2.1)**

ELL **English Language Learners** Invite students to share strategies in their primary languages. Encourage all students to use gestures and/or visuals as they share. **(Speaking and Listening)**

Activity 1 continued >



Activity 1

Name: Date: Period:

Equations and Tape Diagrams

2. This dog and a 5-pound weight balance a 17-pound weight.

How much does the dog weigh?

12 pounds

3. Tariq wrote an *equation* to represent the situation. He used the variable d to represent the dog's weight.

Explain how Tariq's equation is like the seesaw situation.

Responses vary. The left side of the equation is like the left side of the seesaw. It is the weight of the dog plus a 5-pound weight, so the left side of the equation is $d + 5$. The right side of the equation, 17, is like the 17-pound weight on the right side of the seesaw.

Tariq

$d + 5 = 17$

Pairs | 10 min

Activity 1 Equations and Tape Diagrams (continued)

Purpose: Students use *equations* and tape diagrams to represent seesaw situations and to determine unknown animal weights. **(MTR.2.1, MTR.5.1)**

2 Monitor

D Differentiation

Look for students who:	Respond to Student Thinking
Select $3 + x = 18$.	**Support** Invite them to compare this seesaw to the situation in the previous problem.
Select $3 + 18 = x$.	**Support** Consider asking, "What does the x represent for this seesaw?"
Select one of the two correct equations.	**Strengthen** Consider asking, "Is this the only equation that represents the situation? How do you know?"
Make connections between $3 \cdot x = 18$ and $x + x + x = 18$.	**Strengthen** Invite students to share their thinking during the Connect.

Listen for a variety of descriptions of how the tape diagram and the equation(s) are alike to share during the Connect.

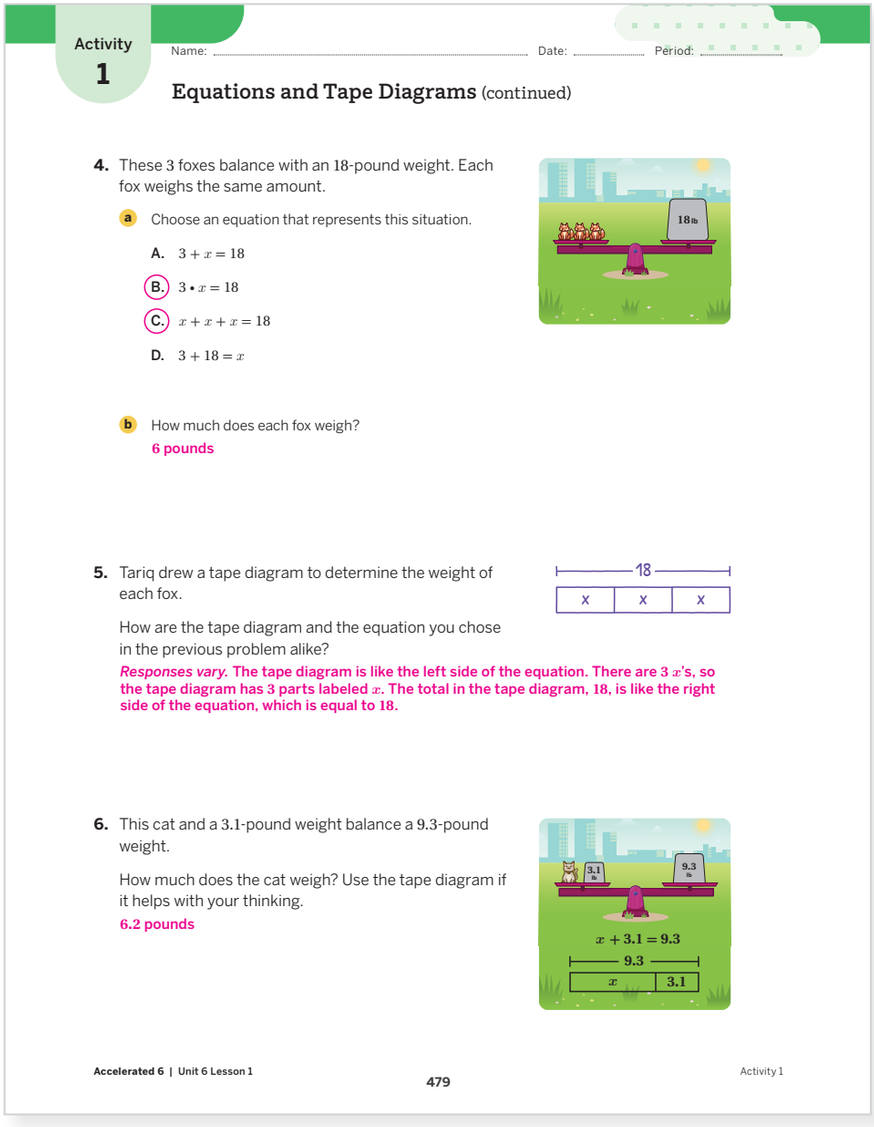
Look for a variety of strategies, such as creating a tape diagram, annotating the seesaw, or writing a subtraction expression, to highlight during the Connect. **(MTR.2.1)**

D Differentiation: (Stretch) Invite students to draw a tape diagram to represent this situation and compare it to the tape diagram in the next problem.

3 Connect

Create a class definition of **variable** using students' ideas from Problems 3–6. Consider asking, "Why might Tariq have used different variables in the first example and the second example?" *Responses vary.* To represent the weights of different animals.

To surface the Key Takeaway, consider:
- Inviting students to explain how they knew which equations matched the fox situation.
- Sharing descriptions and invite students to make connections between them. **(MTR.5.1)**



Activity 1

Name: Date: Period:

Equations and Tape Diagrams (continued)

4. These 3 foxes balance with an 18-pound weight. Each fox weighs the same amount.

a. Choose an equation that represents this situation.

A. $3 + x = 18$

B. $3 \cdot x = 18$

C. $x + x + x = 18$

D. $3 + 18 = x$

b. How much does each fox weigh?

6 pounds

5. Tariq drew a tape diagram to determine the weight of each fox.

How are the tape diagram and the equation you chose in the previous problem alike?

Responses vary. The tape diagram is like the left side of the equation. There are 3 x's, so the tape diagram has 3 parts labeled x. The total in the tape diagram, 18, is like the right side of the equation, which is equal to 18.

6. This cat and a 3.1-pound weight balance a 9.3-pound weight.

How much does the cat weigh? Use the tape diagram if it helps with your thinking.

6.2 pounds

Accelerated 6 | Unit 6 Lesson 1 479 Activity 1

Key Takeaway: Seesaws, tape diagrams, and *equations* can be used to represent balanced relationships and determine unknown weights. These unknown weights are often represented by **variables** — letters or symbols that represent a number.

Pairs | 10 min

Activity 2 Determining Unknown Weights

Purpose: Students connect tape diagrams, equations, and seesaws and use them to determine unknown animal weights. **(MTR.5.1)**

1 Launch

To surface connections, consider asking, "Which of the previous seesaws do these balanced seesaws remind you of?"

A **Accessibility: Executive Functioning** Consider chunking this activity by inviting students to work on one equation/tape diagram at a time.

2 Monitor

Encourage students to share their reasoning with their partner and work together to reach an agreement about which representations match. **(MTR.4.1)**

Note: If time allows, invite pairs to compare their choices, justify their thinking, and make revisions based on their conversation.

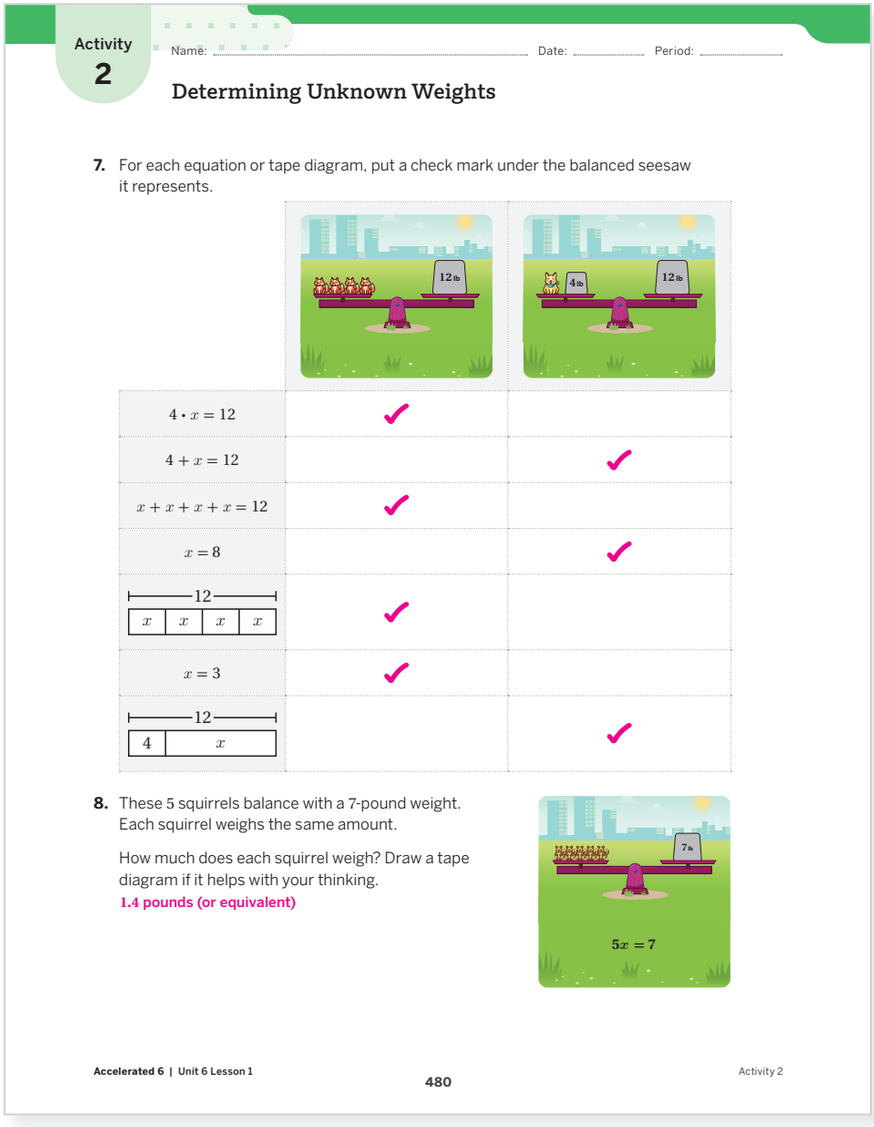
D **Differentiation**

Look for students who:	Respond to Student Thinking
Draw a tape diagram that represents $5 + x = 7$.	**Support** Consider asking, "Using your tape diagram, how much would 1 squirrel weigh? Does this make sense with the picture of the seesaw?"
Use strategies to determine the unknown weight, such as: • Creating a tape diagram with 5 equal parts. • Writing the expression $7 \div 5$. • Writing the equation $5x = 7$.	**Strengthen** Invite students to compare their strategy with a classmate's. Select and sequence these to share during the Connect.

3 Connect

Invite students to share their strategies for determining the weight of each squirrel.

Consider asking, "How was determining the weight of the cat and the weight of the squirrels alike? How was it different?" **(MTR.5.1)**



Activity 2

Name: Date: Period:

Determining Unknown Weights

7. For each equation or tape diagram, put a check mark under the balanced seesaw it represents.

$4 \cdot x = 12$	✓	
$4 + x = 12$		✓
$x + x + x + x = 12$	✓	
$x = 8$		✓
12 bar: x x x x	✓	
$x = 3$	✓	
12 bar: 4 x		✓

8. These 5 squirrels balance with a 7-pound weight. Each squirrel weighs the same amount.

How much does each squirrel weigh? Draw a tape diagram if it helps with your thinking.

1.4 pounds (or equivalent)

Accelerated 6 | Unit 6 Lesson 1 480 Activity 2

Key Takeaway: There is more than one strategy for determining the weight of the animal on a seesaw. These include drawing a tape diagram, using reasoning, and writing an equation.

Pairs | 10 min

Activity 3 Challenge Creator

Purpose: Students use their creativity to make their own seesaw problem for other students to solve.

Short on time? Consider having students only complete one challenge from a classmate.

1 Launch

Consider sharing that students will create their own challenge similar to the previous problem.

Distribute the Activity 3 Sheet to students so they can create their seesaw challenge.

To encourage creativity, invite students to make a challenge they think will be different from ones their classmates create.

A **Accessibility: Executive Functioning** Invite students to work with a partner on a challenge before trying one independently.

2 Monitor

Listen to student strategies and discussions, offering help or encouragement as needed.

Invite students to check their answers with the person who created the seesaw they're solving. Encourage them to discuss and resolve any differences. **(MTR.4.1)**

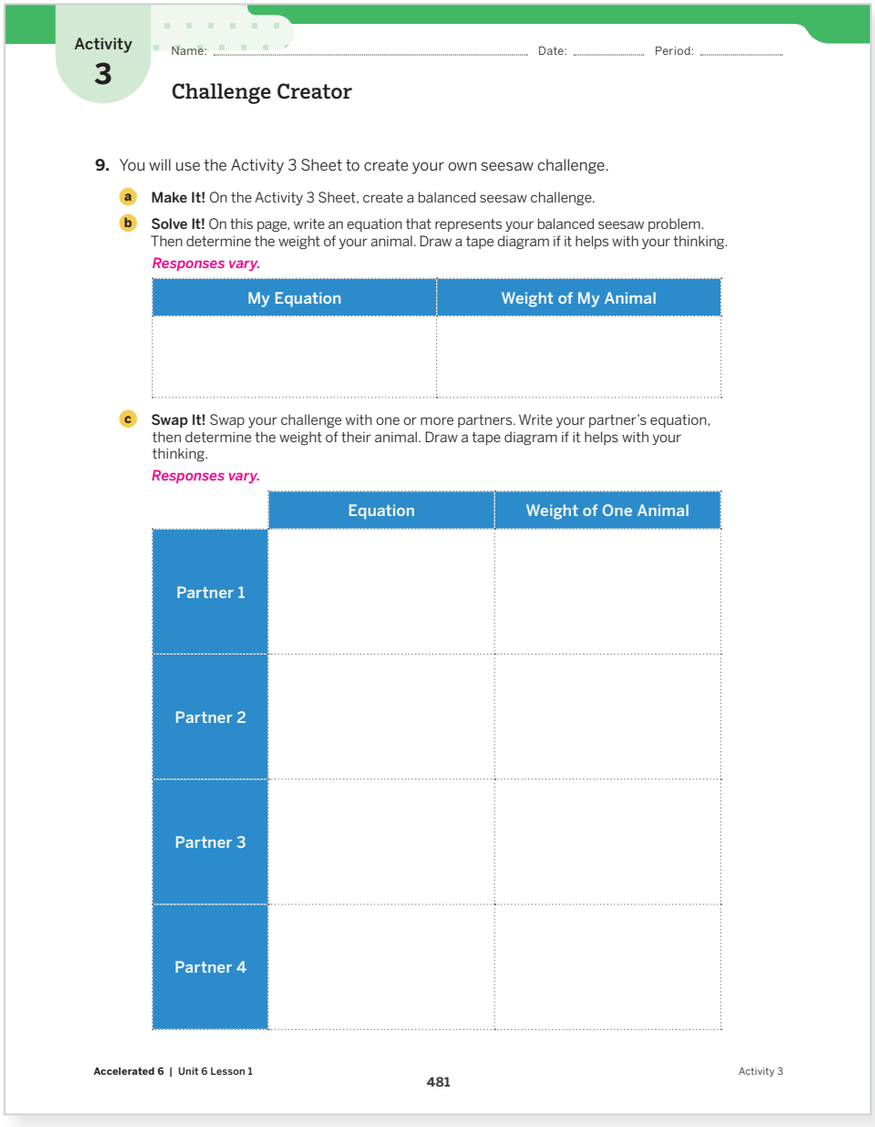
Look for several students' seesaws to share during the Connect.

3 Connect

Invite students to share how they decided how many animals and/or how much weight to put on each side of their seesaw.

Invite students to share a variety of strategies for determining the unknown weight in each seesaw problem. **(MTR.2.1)**

Consider asking, "Did you hear a strategy that surprised you?"



Activity 3

Name: _____ Date: _____ Period: _____

Challenge Creator

9. You will use the Activity 3 Sheet to create your own seesaw challenge.

a **Make It!** On the Activity 3 Sheet, create a balanced seesaw challenge.

b **Solve It!** On this page, write an equation that represents your balanced seesaw problem. Then determine the weight of your animal. Draw a tape diagram if it helps with your thinking.

Responses vary.

My Equation	Weight of My Animal

c **Swap It!** Swap your challenge with one or more partners. Write your partner's equation, then determine the weight of their animal. Draw a tape diagram if it helps with your thinking.

Responses vary.

	Equation	Weight of One Animal
Partner 1		
Partner 2		
Partner 3		
Partner 4		

Accelerated 6 | Unit 6 Lesson 1 481 Activity 3

Whole Class | 5 min

Synthesis

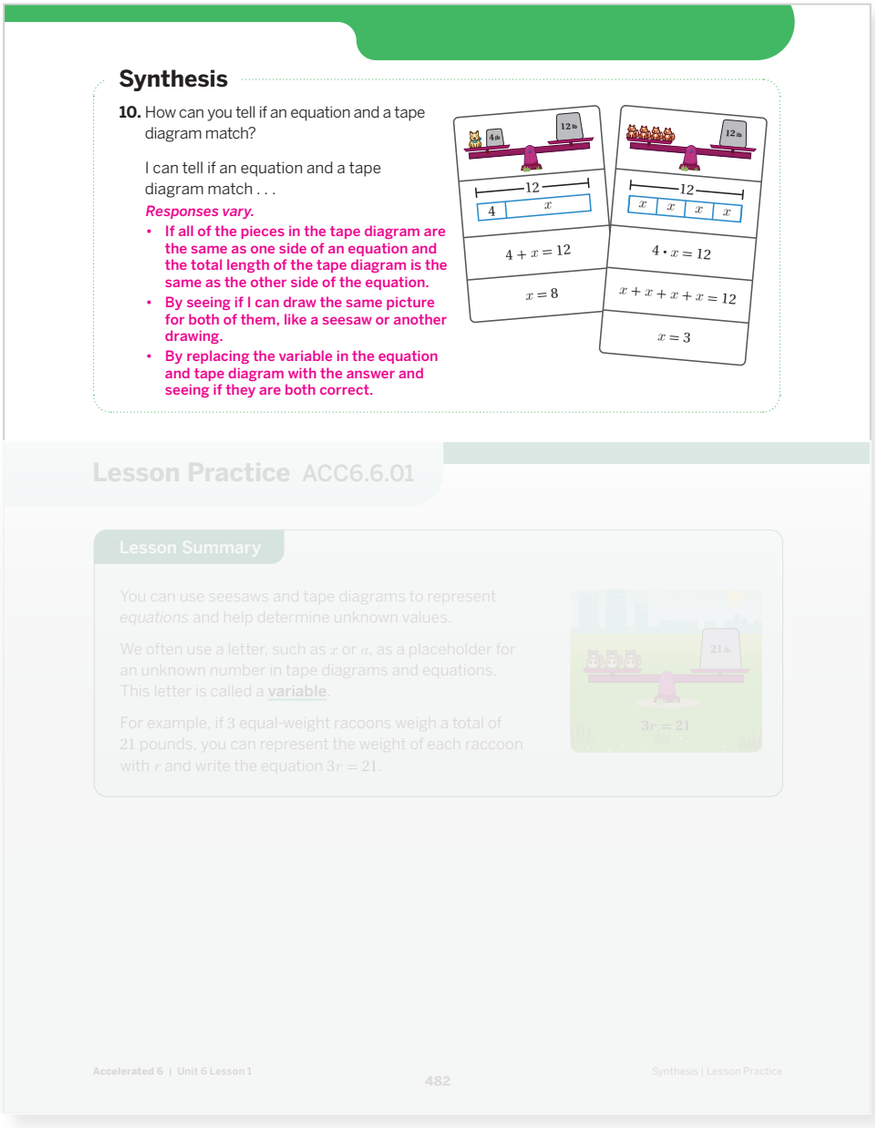
Purpose: Students synthesize their understanding of how equations and tape diagrams can represent situations with unknowns.

Invite students to respond independently and then share their thinking with a partner.

Look for and share a variety of ideas, including the ones from the sample responses.

MLR **MLR2: Collect and Display** Use this routine to formalize the definition of *variable* using students' language. Display any new language, examples, or diagrams for students to reference later.

Note: If time allows, invite students to share with a partner what makes sense about each strategy and what questions they still have.



Synthesis

10. How can you tell if an equation and a tape diagram match?

I can tell if an equation and a tape diagram match . . .

Responses vary.

- If all of the pieces in the tape diagram are the same as one side of an equation and the total length of the tape diagram is the same as the other side of the equation.
- By seeing if I can draw the same picture for both of them, like a seesaw or another drawing.
- By replacing the variable in the equation and tape diagram with the answer and seeing if they are both correct.

Lesson Practice ACC6.6.01

Lesson Summary

You can use seesaws and tape diagrams to represent *equations* and help determine unknown values.

We often use a letter, such as x or a, as a placeholder for an unknown number in tape diagrams and equations. This letter is called a **variable**.

For example, if 3 equal-weight racoons weigh a total of 21 pounds, you can represent the weight of each raccoon with r and write the equation $3r = 21$.

Accelerated 6 | Unit 6 Lesson 1 482 Synthesis | Lesson Practice

Lesson Takeaway: Seesaws, tape diagrams, and *equations* can all be used to model relationships with an unknown amount. This unknown amount is represented by a **variable**.

Independent | 5 min

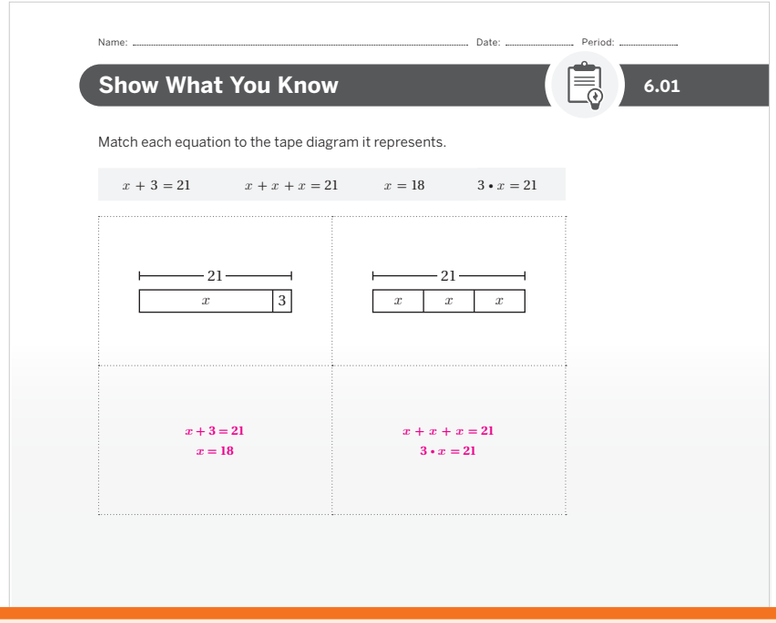
Show What You Know

Purpose: Students demonstrate their understanding by matching equations to tape diagrams.

Today's Goals

1. **Goal:** Use reasoning and tape diagrams to determine unknown values.
2. **Language Goal:** Describe connections between diagrams that represent equations of the form $x + p = q$ or $px = q$. **(Speaking, Listening, and Writing)**

Show What You Know (Assessment Resources)



Name: _____ Date: _____ Period: _____

Show What You Know 6.01

Match each equation to the tape diagram it represents.

$x + 3 = 21$ $x + x + x = 21$ $x = 18$ $3 \cdot x = 21$

$x + 3 = 21$
$x = 18$

$x + x + x = 21$
$3 \cdot x = 21$

Differentiation Use after Lesson 1

Support

Provide targeted intervention for students by using these resources.

If students need support understanding how to determine which equation represents a given situation:

» **Respond to Student Thinking:**

- Students will have more opportunities to develop their understanding of these concepts in Lesson 2. You may choose not to intervene now.
- Select a few related **Practice Problems** and give individualized feedback.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite them to explore the **Sub-Unit 1 Extension Activities**.

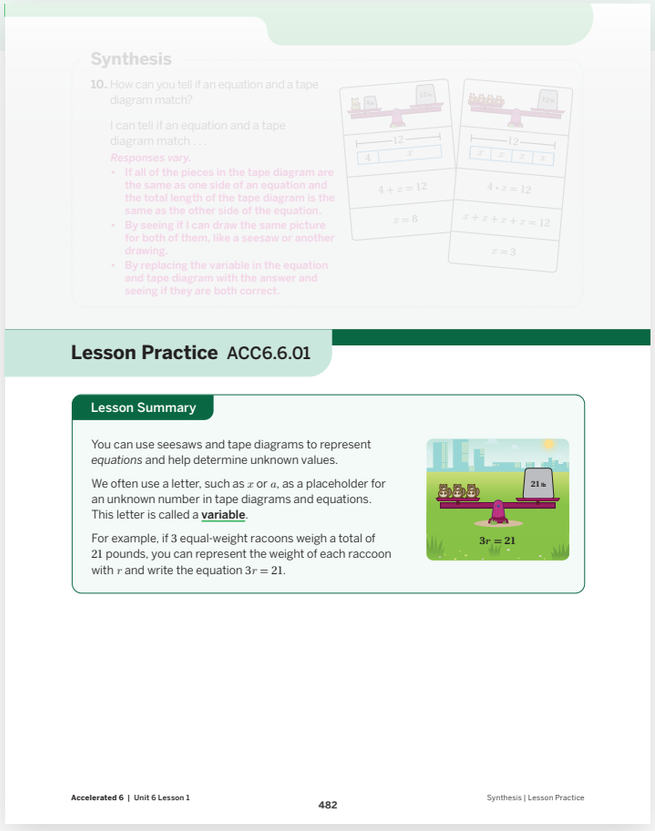
Professional Learning

In this lesson, students made connections between the visuals of the seesaw, *equations*, and tape diagrams. How will that support them in solving equations and describing what solutions mean in context?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.



Synthesis

10. How can you tell if an equation and a tape diagram match?

I can tell if an equation and a tape diagram match . . .

Responses vary.

- If all of the pieces in the tape diagram are the same as one side of an equation and the total length of the tape diagram is the same as the other side of the equation.
- By seeing if I can draw the same picture for both of them, like a seesaw or another drawing.
- By replacing the variable in the equation and tape diagram with the answer and seeing if they are both correct.

Lesson Practice ACC6.6.01

Lesson Summary

You can use seesaws and tape diagrams to represent *equations* and help determine unknown values.

We often use a letter, such as x or a, as a placeholder for an unknown number in tape diagrams and equations. This letter is called a **variable**.

For example, if 3 equal-weight racoons weigh a total of 21 pounds, you can represent the weight of each raccoon with r and write the equation $3r = 21$.

Lesson Practice
ACC6.6.01

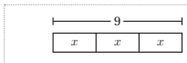
Name: _____ Date: _____ Period: _____

1. Determine the weight of 1 fox.
13 pounds

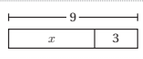
2. All 4 cats weigh the same amount. Determine the weight of 1 cat.
9 pounds

3. Match each equation to the tape diagram it represents.

$3 + x = 9$ $x + x + x = 9$ $x = 9 \div 3$ $3 \cdot x = 9$ $x = 9 - 3$

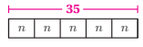


$x + x + x = 9$
$x = 9 \div 3$
$3 \cdot x = 9$



$3 + x = 9$
$x = 9 - 3$

Problems 4–5: Kwabena is trying to determine the value of n in the equation $5 \cdot n = 35$. He begins drawing a tape diagram, but isn't sure how to complete it.



4. Complete Kwabena's tape diagram so it represents the equation $5 \cdot n = 35$.

5. Determine the value of n.
$n = 7$

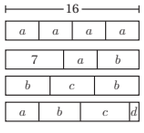
Lesson Practice
ACC6.6.01

Name: _____ Date: _____ Period: _____

FAST Practice

6. Determine the value of a, b, c, and d.

$a =$ **4**
$b =$ **5**
$c =$ **6**
$d =$ **1**



Spiral Review

Problems 7–8: Calculate the price per pound for each item.

7. $2.52 for 4.5 pounds of potatoes
$0.56 per pound of potatoes

8. $7.75 for 2.5 pounds of broccoli
$3.10 per pound of broccoli

Problems 9–12: Fill in each box to create a true equation.

9. $7 + \boxed{3} = 10$

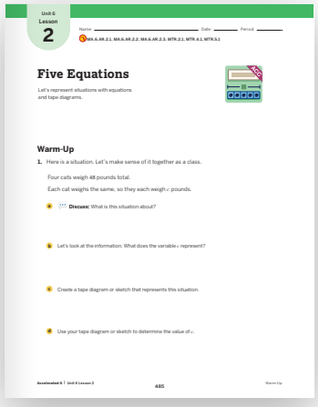
10. $\boxed{9} \cdot 5 = 45$

11. $23 - \boxed{12} = 11$

12. $\boxed{32} \div 4 = 8$

Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1	1	MA.6.AR.2.2
	2, 4–5	2	MA.6.AR.2.3
	3	2	MA.6.AR.2.2, MA.6.AR.2.3
FAST Practice	**6**	3	MA.6.AR.2.2, MA.6.AR.2.3
Spiral Review			
	7–8	2	MA.6.AR.3.2
	9, 11	1	MA.2.AR.2.2
	10, 12	1	MA.3.AR.2.3


Unit 6
Lesson 2
Five Equations
Let's represent situations with equations and tape diagrams.
Warm-Up
1. Here is a situation. Let's make sense of it together as a class.
Four cats weigh 48 pounds total.
Each cat weighs the same, so they each weigh c pounds.
Discuss: What is this situation about?
Let's look at the information. What does the variable c represent?
Create a tape diagram or sketch that represents this situation.
Use your tape diagram or sketch to determine the value of c.
Accelerated 6 | Unit 6 Lesson 2
485
Warm-Up

Five Equations

Tape Diagrams, Equations, Situations

Let's represent situations with equations and tape diagrams.

Learning Outcomes

Today's Goals

1. **Goal:** Interpret the meaning of variables and *solutions to equations* in context.
2. **Language Goal:** Interpret tape diagrams that represent equations of the form $p + x = q$ or $px = q$. **(Speaking, Listening, and Writing)**

Students connect tape diagrams and equations to make sense of situations in context. They use these tools to determine **solutions to equations** and interpret the meanings of those solutions in context. **(MTR.2.1, MTR.5.1)**

Coherence

Prior Learning

In Lesson 1, students explored equations and tape diagrams as ways to represent balancing weights on a seesaw.

Future Learning

In Lessons 3 and 5, students will explore other strategies for determining unknown values, like using balanced hangers and inverse operations. In Lesson 4, students will return to situations in context. In Unit 8, students will be introduced to the term *solutions to an inequality*.

Connections

- Students analyze models to make connections to equations. (mathematical)

FL B.E.S.T. Math Benchmarks

Addressing

MA.6.AR.2.1
Given an equation or inequality and a specified set of integer values, determine which values make the equation or inequality true or false.

MA.6.AR.2.2
Write and solve one-step equations in one variable within a mathematical or real-world context using addition and subtraction, where all terms and solutions are integers.

MA.6.AR.2.3
Write and solve one-step equations in one variable within a mathematical or real-world context using multiplication and division, where all terms and solutions are integers.

Mathematical Thinking and Reasoning Standards: MTR.2.1, MTR.4.1, MTR.5.1

ELA Expectations: ELA.K12.EE.2.1, ELA.K12.EE.3.1, ELA.K12.EE.4.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.5.AR.1.1	MA.7.AR.2.2
MA.5.AR.2.1	
MA.5.AR.2.2	
MA.5.AR.2.3	
MA.5.AR.2.4	

Vocabulary

New Vocabulary

solution to an equation

Lesson at a Glance

~ 45 min

MA.6.AR.2.1, MA.6.AR.2.2, MA.6.AR.2.3

Warm-Up

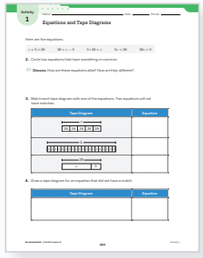
Whole Class | 10 min

Students use MLR6: Three Reads to make sense of a situation in words. **(MTR.5.1)**

Activity 1

Pairs | 10 min

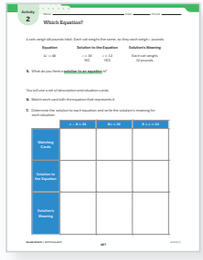
Students make connections between equations and tape diagrams. **(MTR.2.1, MTR.5.1)**



Activity 2

Pairs | 15 min

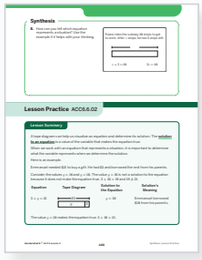
Students connect equations with descriptions and situations, then determine and interpret the meaning of each **solution to an equation**.
Routine: Think-Pair-Share **(MTR.2.1)**



Synthesis

Whole Class | 5 min

Students synthesize their understanding of the connections between equations, tape diagrams, and situations. MLR2: Collect and Display



Show What You Know

Independent | 5 min

Students demonstrate their understanding by selecting an equation to match a situation, determining its solution, and interpreting the solution in the situation.

Prep Checklist

Prepare the Activity 2 Cards.

This lesson includes:

- Student Edition
- Show What You Know
- Activity 2 Cards, 1 set per pair

Whole Class | 10 min

Warm-Up

Purpose: Students use MLR6: Three Reads to make sense of a situation in words. **(MTR.5.1)**

1 Launch

MLR **MLR6: Three Reads** Invite students to read the text three times, each with a particular focus, to support students in making sense of the scenario. **(MTR.5.1)**

Direct students' attention to the situation and invite a student to read it aloud, then discuss Problem 1a.

To surface connections, invite students to share a story they have about cats.

Direct students' attention to the situation again and invite a student to read it aloud, then discuss Problem 1b.

Invite students to share how they decided what the variable represents.

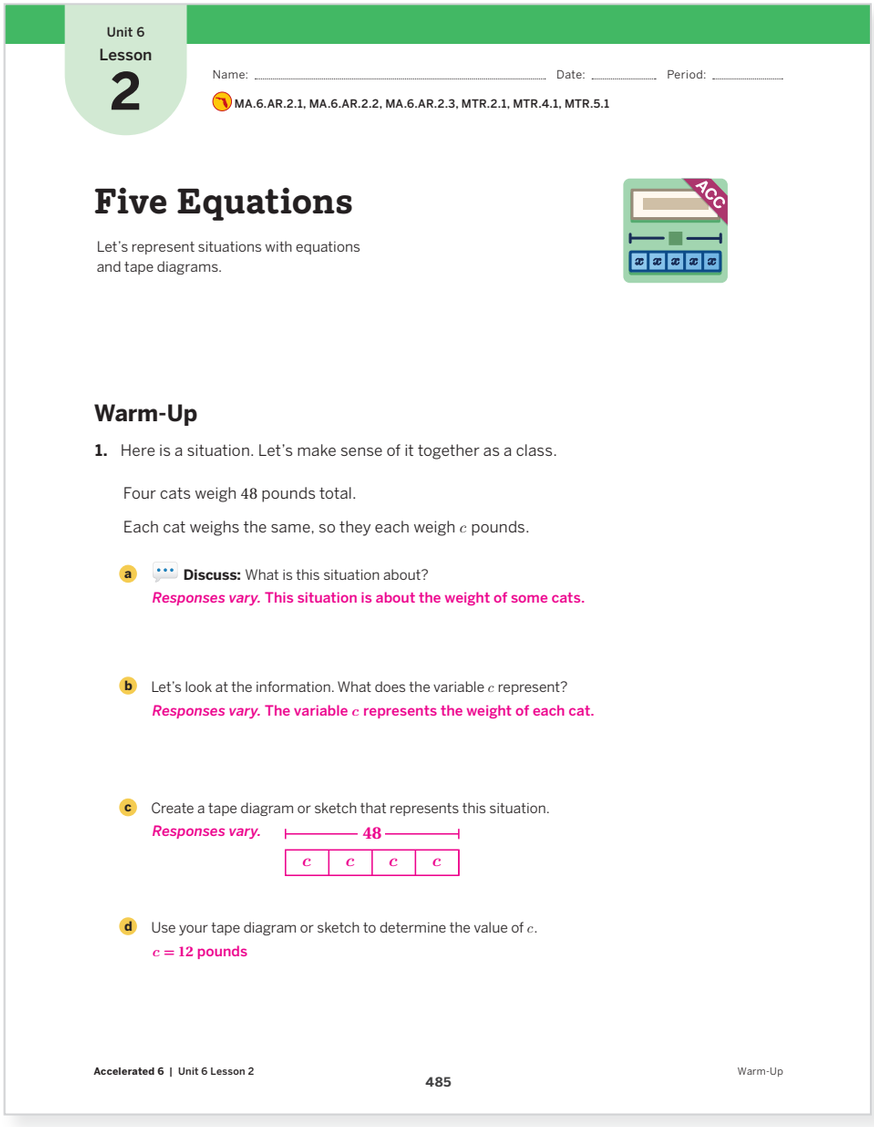
Direct students' attention to the situation a third time with a focus on the new information. After the third read, invite students to discuss Problem 1c–d with a partner, then as a class. **(MTR.4.1)**

To encourage creativity, celebrate the variety of tape diagrams and sketches students create.

2 Connect

Invite students to share their values for c by polling the class. Encourage students to justify their choice and to share which representation they used to help them choose.

Consider asking, "How much does 1 cat weigh?"



Unit 6
Lesson
2

Name: Date: Period:

MA.6.AR.2.1, MA.6.AR.2.2, MA.6.AR.2.3, MTR.2.1, MTR.4.1, MTR.5.1

Five Equations

Let's represent situations with equations and tape diagrams.

Warm-Up

1. Here is a situation. Let's make sense of it together as a class.

Four cats weigh 48 pounds total.

Each cat weighs the same, so they each weigh c pounds.

a. **Discuss:** What is this situation about?
Responses vary. **This situation is about the weight of some cats.**

b. Let's look at the information. What does the variable c represent?
Responses vary. **The variable c represents the weight of each cat.**

c. Create a tape diagram or sketch that represents this situation.
Responses vary.

d. Use your tape diagram or sketch to determine the value of c.
$c = 12$ pounds

Accelerated 6 | Unit 6 Lesson 2 485 Warm-Up

Pairs | 10 min

Activity 1 Equations and Tape Diagrams

Purpose: Students make connections between equations and tape diagrams. **(MTR.2.1, MTR.5.1)**

Short on time? Consider omitting the last problem.

1 Launch

Direct students' attention to the five equations.

Consider asking, "What do you think it means when there is a number in front of a variable, like the 5 before the x in $5x = 20$?"

Note: This may be the first time students see a variable and a number next to each other (e.g., $5x$).

2 Monitor

A Accessibility: Conceptual Processing To support abstract thinking, consider labeling $5x$ in words like "5 times x."

Encourage students to justify each match with their partner before completing the next one.

D Differentiation

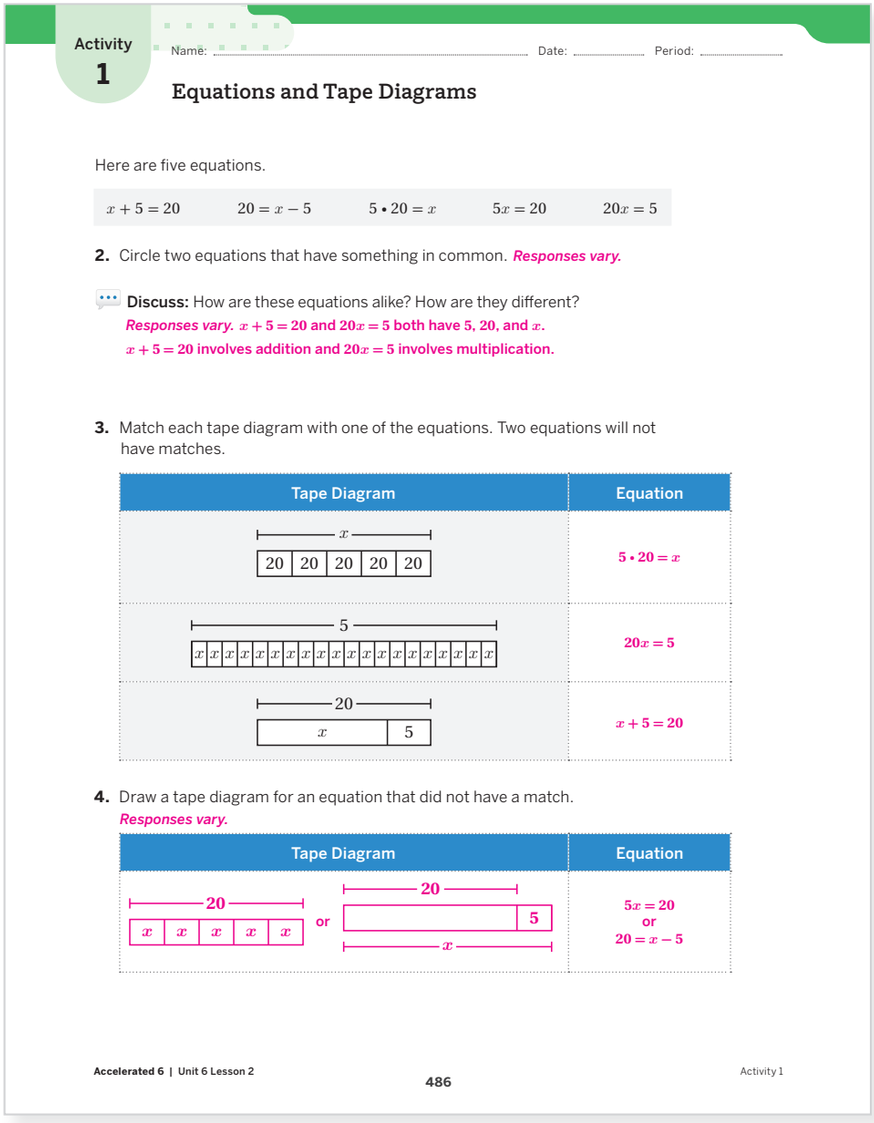
Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Invite them to describe what one equation means in words, like "an unknown number and 5 more is 20."
Use strategies like: • Testing values for x. • Looking at the number of equal groups. • Looking at the total.	**Strengthen** Invite students to share these strategies during the Connect.
Want to explore more.	**Stretch** Consider asking, "Can two different equations match the same tape diagram? Why or why not?"

3 Connect

Invite students to share the tape diagrams they created for $5x = 20$ and $x = 20 + 5$. **(MTR.2.1)**

To surface the Key Takeaway, consider asking:

- "How are these two tape diagrams alike and different? How are they the same as or different from their equations?"
- "How does each part of a tape diagram connect to each part of an equation?" **(MTR.5.1)**



Activity 1

Name: Date: Period:

Equations and Tape Diagrams

Here are five equations.

$x + 5 = 20$ $\quad 20 = x - 5$ $\quad 5 \cdot 20 = x$ $\quad 5x = 20$ $\quad 20x = 5$

2. Circle two equations that have something in common. *Responses vary.*

Discuss: How are these equations alike? How are they different?
Responses vary. $x + 5 = 20$ and $20x = 5$ both have 5, 20, and x.
$x + 5 = 20$ involves addition and $20x = 5$ involves multiplication.

3. Match each tape diagram with one of the equations. Two equations will not have matches.

Tape Diagram	Equation
x: 20 20 20 20 20	$5 \cdot 20 = x$
5: x x x x x x x x x x x x x x x x x x x x	$20x = 5$
20: x | 5	$x + 5 = 20$

4. Draw a tape diagram for an equation that did not have a match.
Responses vary.

Tape Diagram	Equation
20: x x x x x or 20: 5 / x	$5x = 20$ or $20 = x - 5$

Accelerated 6 | Unit 6 Lesson 2 486 Activity 1

Key Takeaway: Tape diagrams and equations can represent the same information. An equation like $5x = 20$ is represented on a tape diagram as 5 groups of x with a total of 20.

Pairs | 15 min

Activity 2 Which Equation?

Purpose: Students connect equations with descriptions and situations, then determine and interpret the meaning of each **solution to an equation**. **(MTR.2.1)**

Short on time? Consider working on the first problem as a whole class.

1 Launch

Direct students' attention to the situation from the Warm-Up, including the equation, solution, and solution's meaning.

Use the **Think-Pair-Share** routine to support student-to-student discourse. Invite students to think independently and then take turns sharing with a partner. **(MTR.4.1)**

Create a class definition of a **solution to an equation** using students' ideas and language.

ELL **English Language Learners** Invite students to share what *solution to an equation* reminds them of or how to say it in a different language. **(Reading and Listening)**

Distribute a set of the Activity 2 Cards to each pair.

2 Monitor

D Differentiation

Look for students who:	Respond to Student Thinking
Need support making sense of the situations.	**Support** Encourage them to read each situation three times and answer questions similar to the Warm-Up.
Match "22 is 9 more than x" with $x - 9 = 22$, or a similar mismatch.	**Support** Consider asking, "22 is 9 more than what number?" Invite them to test that value in the equation they chose.
Use imprecise language for a solution's meaning, like "13 stops."	**Support** Consider asking, "How could you describe what the 13 means more precisely?"
Use strategies like: • Making a tape diagram. • Using reasoning. • Drawing on their personal experience.	**Strengthen** Invite students to share these during the Connect.
Would like to explore this concept more.	**Stretch** Invite them to write their own situation, then trade it with a classmate who will write an equation and determine its solution.

3 Connect

Direct students' attention to the equations, situations, and descriptions. As a class, match each equation to its situation and description. **(MTR.2.1)**

To surface the Key Takeaway, consider asking:

- "What was your strategy for determining the solution to the equation $x - 9 = 22$?"
- "What does the solution mean in Mohamed's situation?"

Activity 2

Name: Date: Period:

Which Equation?

4 cats weigh 48 pounds total. Each cat weighs the same, so they each weigh c pounds.

Equation	Solution to the Equation		Solution's Meaning
$4c = 48$	$c = 10$ NO	$c = 12$ YES	Each cat weighs 12 pounds.

5. What do you think a **solution to an equation** is?

Responses vary. The solution to an equation is the number that can replace the variable to make the equation true. 4 times 10 equals 40, not 48, so $c = 10$ is not the solution to the equation. 4 times 12 equals 48, so the solution to $4c = 48$ is 12.

You will use a set of description and situation cards.

6. Match each card with the equation that represents it.

7. Determine the solution to each equation and write the solution's meaning for each situation.

	$x - 8 = 24$	$8x = 24$	$8 + x = 24$
Matching Cards	Card 3 Card 5	Card 4 Card 6	Card 1 Card 2
Solution to the Equation	$x = 32$	$x = 3$	$x = 16$
Solution's Meaning	Card 3: Mohamed made $32. Card 5: 8 less than 32 is 24.	Card 4: The product of 8 and 3 is 24. Card 6: Rebecca can afford to buy 3 day passes.	Card 1: Kwasi has 16 stops left on the bus. Card 2: 24 is 8 more than 16.

Accelerated 6 | Unit 6 Lesson 2 487 Activity 2

Key Takeaway: A **solution to an equation** is a value that makes the equation true. A solution's meaning is what that value represents in context. For example, the solution to $x - 9 = 22$ is $x = 31$. In this activity, 31 represents the $31 Mohamed earned from mowing lawns.

Whole Class | 5 min

Synthesis

Purpose: Students synthesize their understanding of the connections between equations, tape diagrams, and situations.

Invite students to respond independently, and then share their thinking with a partner.

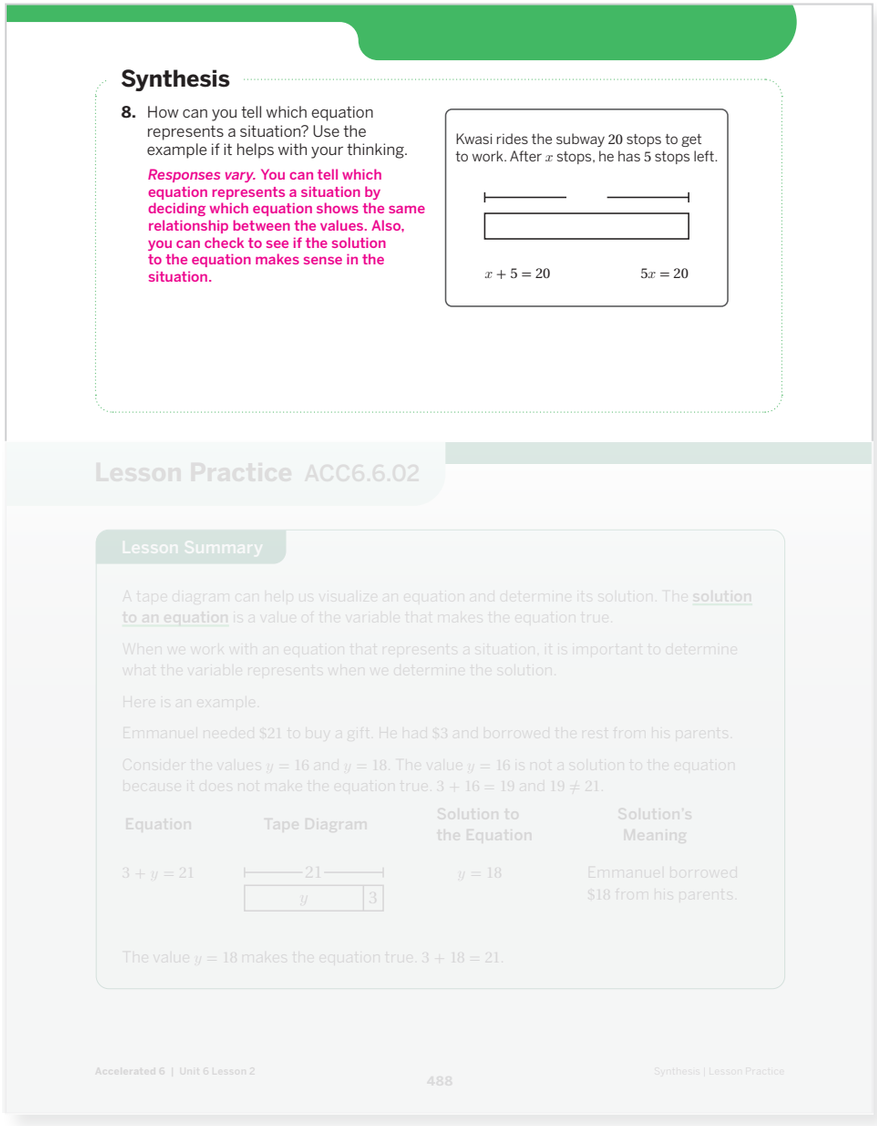
Loof for and share a variety of ideas, such as:

- Drawing a tape diagram to make sense of the situation.
- Identifying the operation that connects the values (addition, multiplication, etc.).
- Determining the solution and checking if it makes sense in the context of the problem.

Invite students to share which equation matches this situation and how they decided.

MLR **MLR2: Collect and Display** Use this routine to formalize the definition of a *solution to an equation* using students' language. Display any new language, examples, or diagrams publicly for students to reference later.

Note: If time allows, invite students to revise their first response based on the discussion.



Synthesis

8. How can you tell which equation represents a situation? Use the example if it helps with your thinking.

Responses vary. You can tell which equation represents a situation by deciding which equation shows the same relationship between the values. Also, you can check to see if the solution to the equation makes sense in the situation.

Kwasi rides the subway 20 stops to get to work. After x stops, he has 5 stops left.

$x + 5 = 20$ $5x = 20$

Lesson Practice ACC6.6.02

Lesson Summary

A tape diagram can help us visualize an equation and determine its solution. The **solution to an equation** is a value of the variable that makes the equation true.

When we work with an equation that represents a situation, it is important to determine what the variable represents when we determine the solution.

Here is an example.

Emmanuel needed \$21 to buy a gift. He had \$3 and borrowed the rest from his parents.

Consider the values $y = 16$ and $y = 18$. The value $y = 16$ is not a solution to the equation because it does not make the equation true. $3 + 16 = 19$ and $19 \neq 21$.

Equation	Tape Diagram	Solution to the Equation	Solution's Meaning
$3 + y = 21$	21 / y | 3	$y = 18$	Emmanuel borrowed \$18 from his parents.

The value $y = 18$ makes the equation true. $3 + 18 = 21$.

Accelerated 6 | Unit 6 Lesson 2 488 Synthesis | Lesson Practice

Lesson Takeaway: Tape diagrams and equations can represent situations and descriptions in words. The **solution to an equation** is the value of the *variable* that makes the equation true. Its meaning is what that value represents in the situation.

Independent | 5 min

Show What You Know

Purpose: Students demonstrate their understanding by selecting an equation to match a situation, determining its solution, and interpreting the solution in the situation.

Today's Goals

1. **Goal:** Interpret the meaning of variables and *solutions to equations* in context.
2. **Language Goal:** Interpret tape diagrams that represent equations of the form $p + x = q$ or $px = q$. **(Speaking, Listening, and Writing)**

Show What You Know (Assessment Resources)

Name: _____ Date: _____ Period: _____

Show What You Know 6.02

Yasmine is biking 5 miles to her friend's house. After she bikes 2 miles, she has x miles left to go.

a. Which equation represents this situation?

A. $2 + x = 5$ B. $2x = 5$ C. $x = 5 + 2$ D. $5 \cdot 2 = x$

b. What is the solution to the equation you chose?

$x = 3$

c. What is the solution's meaning in this situation?

Responses vary. Yasmine has 3 miles left to go before she gets to her friend's house.

D Differentiation Use after Lesson 2

Support

Provide targeted intervention for students by using these resources.

If students need support understanding how to determine which equation represents a given situation:

» **Respond to Student Thinking:**

- Select a few related **Practice Problems** and give individualized feedback.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite them to explore the **Sub-Unit 1 Extension Activities**.
- Invite students to create situations that match the tape diagrams in Activity 1.

Professional Learning

How did MLR6: Three Reads support students with interpreting the meaning of variables and **solutions to equations** in context?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

8. How can you tell which equation represents a situation? Use the example if it helps with your thinking.

Responses vary. You can tell which equation represents a situation by deciding which equation shows the same relationship between the values. Also, you can check to see if the solution to the equation makes sense in the situation.

Kwasi rides the subway 20 stops to get to work. After x stops, he has 5 stops left.

$x + 5 = 20$ $5x = 20$

Lesson Practice ACC6.6.02

Lesson Summary

A tape diagram can help us visualize an equation and determine its solution. The **solution to an equation** is a value of the variable that makes the equation true.

When we work with an equation that represents a situation, it is important to determine what the variable represents when we determine the solution.

Here is an example.

Emmanuel needed \$21 to buy a gift. He had \$3 and borrowed the rest from his parents.

Consider the values $y = 16$ and $y = 18$. The value $y = 16$ is not a solution to the equation because it does not make the equation true. $3 + 16 = 19$ and $19 \neq 21$.

Equation	Tape Diagram	Solution to the Equation	Solution's Meaning
$3 + y = 21$	21 / y | 3	$y = 18$	Emmanuel borrowed \$18 from his parents.

The value $y = 18$ makes the equation true. $3 + 18 = 21$.

Lesson Practice ACC6.6.02

Name: _____ Date: _____ Period: _____

1. Match each equation to the tape diagram that represents it.

a. 12 / m | 4 — d. $12 = 4m$

b. m / 12 | 4 — c. $m \div 4 = 12$

c. m / 12 | 12 | 12 | 12 — b. $12 + 4 = m$

d. 12 / m | m | m | m — a. $12 - 4 = m$

Problems 2–5: Aaliyah filled a water bottle with 24 ounces of water before school. She drank 15 ounces at lunch. There are x ounces of water left.

2. Draw a tape diagram to represent the situation.

24 / 15 | x

3. Select *all* the equations that could represent this situation.

☑ A. $24 - 15 = x$ ☐ B. $24 + 15 = x$ ☑ C. $x + 15 = 24$

☐ D. $15x = 24$ ☐ E. $24 \div 15 = x$

4. For each of the numbers below, determine which are solutions and which are not solutions to one of the equations you selected in Problem 3.

3 9 12 39

Solutions: 9

Not solutions: 3, 12, 39

5. Explain the solution's meaning in this situation.

There are 9 ounces of water left.

Lesson Practice ACC6.6.02

Name: _____ Date: _____ Period: _____

FAST Practice

6. Each bag of beans weighs 32 ounces. A restaurant owner needs 224 ounces of beans. The equation $32b = 224$ represents the situation, where b represents the number of bags the owner needs. Which value makes the equation true?

(A.) 7 bags B. 8 bags C. 9 bags D. 10 bags

Spiral Review

Problems 7–9: Fill in each blank to create a true equation.

7. $2.83 - 1.6 =$ 1.23 **8.** 4.9 $+ 2.1 = 7$ **9.** $\frac{3}{4} \cdot$ $\frac{32}{3}$ $= 8$

10. Fill in each blank using whole numbers from 0 to 9 so that x is the same value in each equation. Use each number only once.

$x =$ 2 $\cdot$ 4 $x =$ 3 $+$ 5 $x +$ 0 $=$ 8

11. Select *all* the true equations.

☐ A. $5 + 0 = 0$ ☑ B. $15 \cdot 0 = 0$ ☑ C. $1.4 + 2.7 = 4.1$

☐ D. $\frac{2}{3} \cdot \frac{5}{9} = \frac{7}{12}$ ☑ E. $4\frac{2}{3} = 5 - \frac{1}{3}$

12. Hailey paid \$40 for a jacket. The regular price was \$50. What percent of the regular price did Hailey pay? Use the double number line if it helps with your thinking.

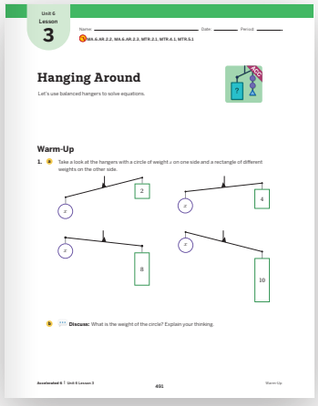
Price (\$) 0 — 50

Percent 0 — 100

80%

Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1	2	MA.6.AR.2.2, MA.6.AR.2.3
	2–5	2	MA.6.AR.2.2
FAST Practice	6	2	MA.6.AR.2.1
Spiral Review			
	7–9	2	MA.6.AR.2.4
	10	3	MA.6.AR.2.2
	11	1	MA.5.AR.2.3
	12	2	MA.6.AR.3.4


Unit 6
Lesson 3

Hanging Around

Let's use balanced hangers to solve equations.

Warm-Up

1. Take a look at the hangers with a circle of weight x on one side and a rectangle of different weights on the other side.

Discuss: What is the weight of the circle? Explain your thinking.

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Hanging Around

Introduction to Balanced Hangers

Let's use balanced hangers to solve equations.

Learning Outcomes

Today's Goals

1. **Goal:** Make connections between balanced hangers and true equations.
2. **Goal:** Use balanced hangers to explain how to solve equations of the form $x + p = q$ and $px = q$.
3. **Language Goal:** Interpret hanger diagrams that represent equations of the form $x + p = q$ and $px = q$. **(Speaking, Listening, and Writing)**

Students make connections between the structures of hanger diagrams and equations. They use hangers to represent equations and to determine unknown values. **(MTR.5.1)**

Coherence

Prior Learning

In Lesson 2, students connected tape diagrams and equations with situations. They determined the solution and the meaning of the solution in the context of the situation.

Future Learning

In Lesson 4, students will use a variety of tools to solve equations representing real-life situations. In Lesson 5, students will solve equations using reasoning and inverse operations.

Connections

- Students investigate hanger models to understand solving equations. (mathematical)

FL B.E.S.T. Math Benchmarks

Addressing

MA.6.AR.2.2
Write and solve one-step equations in one variable within a mathematical or real-world context using addition and subtraction, where all terms and solutions are integers.

MA.6.AR.2.3
Write and solve one-step equations in one variable within a mathematical or real-world context using multiplication and division, where all terms and solutions are integers.

Mathematical Thinking and Reasoning Standards: MTR.2.1, MTR.4.1, MTR.5.1

ELA Expectation: ELA.K12.EE.3.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.5.AR.1.1	MA.7.AR.2.2
MA.5.AR.2.1	
MA.5.AR.2.4	

Lesson at a Glance

~ 45 min

MA.6.AR.2.2, MA.6.AR.2.3

Warm-Up

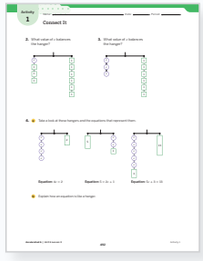
Whole Class | 5 min

Students make sense of how a hanger works and make connections to the balancing work of Lesson 1.

Activity 1

Pairs | 10 min

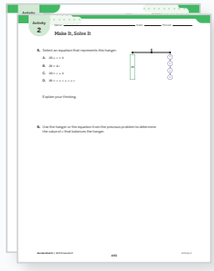
Students make connections between equations and the hangers that represent them. **(MTR.2.1, MTR.5.1)**



Activity 2

Pairs | 10 min

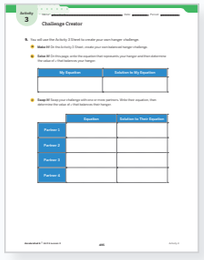
Students create hangers to represent and solve equations. **(MTR.2.1)**



Activity 3

Pairs | 10 min

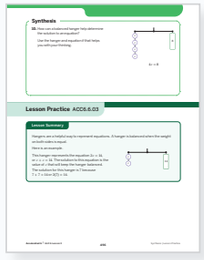
Students use their creativity to make their own hanger challenge for other students to solve.



Synthesis

Whole Class | 5 min

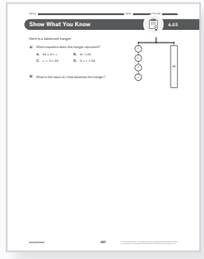
Students synthesize their understanding of using hanger diagrams to represent and solve equations. MLR2: Collect and Display



Show What You Know

Independent | 5 min

Students demonstrate their understanding by selecting the equation that represents the hanger and then determining the value of x that balances the hanger.



Prep Checklist

Prepare Activity 3 Sheets

This lesson includes:

- Student Edition
- Show What You Know
- Activity 3 sheets, one per student
- Clothes on hangers (as needed)

Whole Class | 5 min

Warm-Up

Purpose: Students make sense of how a hanger works and make connections to the balancing work of Lesson 1.

1 Launch

Direct students' attention to the hangers.

A **Accessibility: Conceptual Processing** Use objects like clothes on hangers to demonstrate how a hanger works in real life.

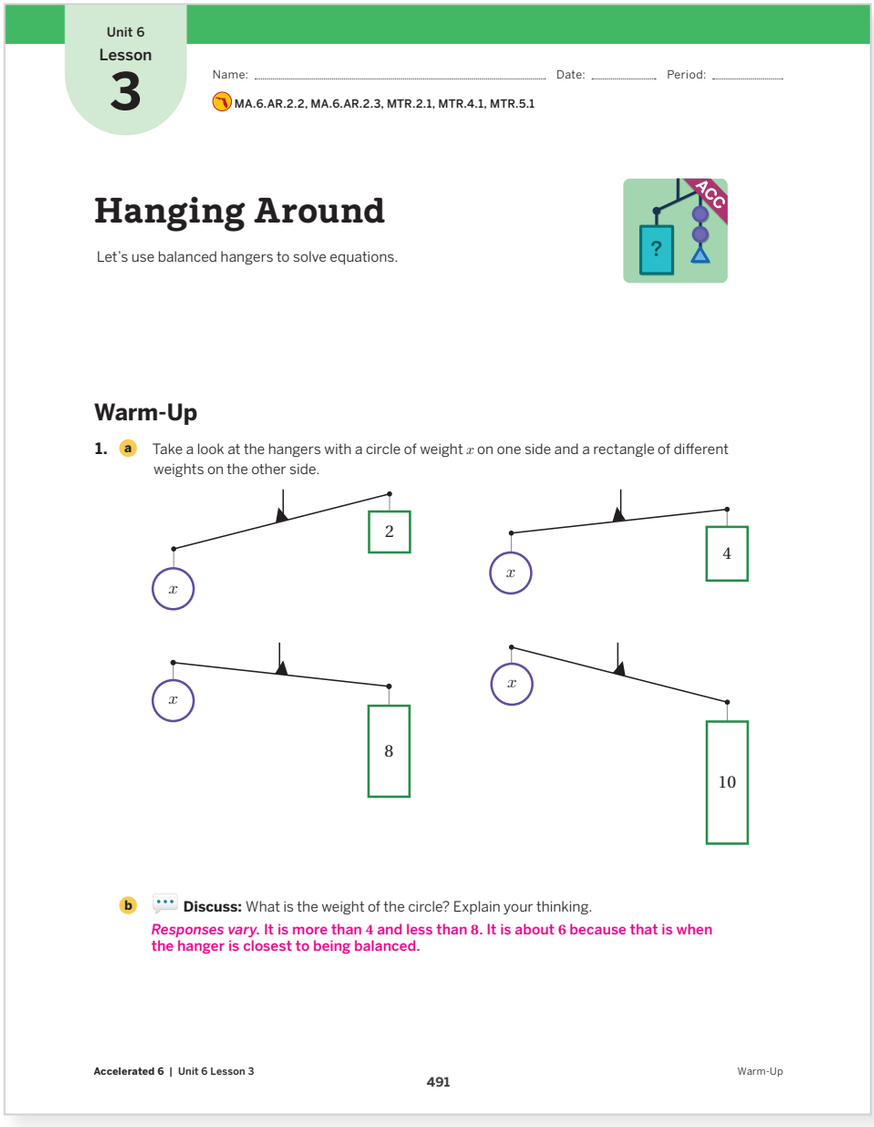
To surface connections, consider asking, "Where have you seen something like this in real life?"

2 Connect

Invite students to share how they approximated the weight of the circle.

Discuss what it means when the hanger is tilted in one direction or the other. **(MTR.4.1)**

ELL **English Language Learners** Encourage students to use gestures or hand motions to help them describe what it means for a hanger to be "balanced." **(Reading and Listening)**



Unit 6
Lesson
3

Name: Date: Period:

MA.6.AR.2.2, MA.6.AR.2.3, MTR.2.1, MTR.4.1, MTR.5.1

Hanging Around

Let's use balanced hangers to solve equations.

Warm-Up

1. a Take a look at the hangers with a circle of weight x on one side and a rectangle of different weights on the other side.

b **Discuss:** What is the weight of the circle? Explain your thinking.

Responses vary. It is more than 4 and less than 8. It is about 6 because that is when the hanger is closest to being balanced.

Accelerated 6 | Unit 6 Lesson 3 491 Warm-Up

Pairs | 10 min

Activity 1 Connect It

Purpose: Students make connections between equations and the hangers that represent them. **(MTR.2.1, MTR.5.1)**

1 Launch

Invite students to share a value of x that they know *does not* balance the hanger and justify their thinking.

2 Monitor

D Differentiation

Look for students who:	Respond to Student Thinking
Draw a tape diagram to represent each hanger.	**Support** Consider asking, "How did you use the tape diagram? What would that strategy look like on the hanger?"
Use a variety of strategies, like: • Removing the same weight from both sides or dividing the hanger into equal groups. • Writing expressions or equations.	**Strengthen** Invite these students to share their strategies during the Connect.

Invite students to share their thinking with a partner. **(MTR.4.1)**

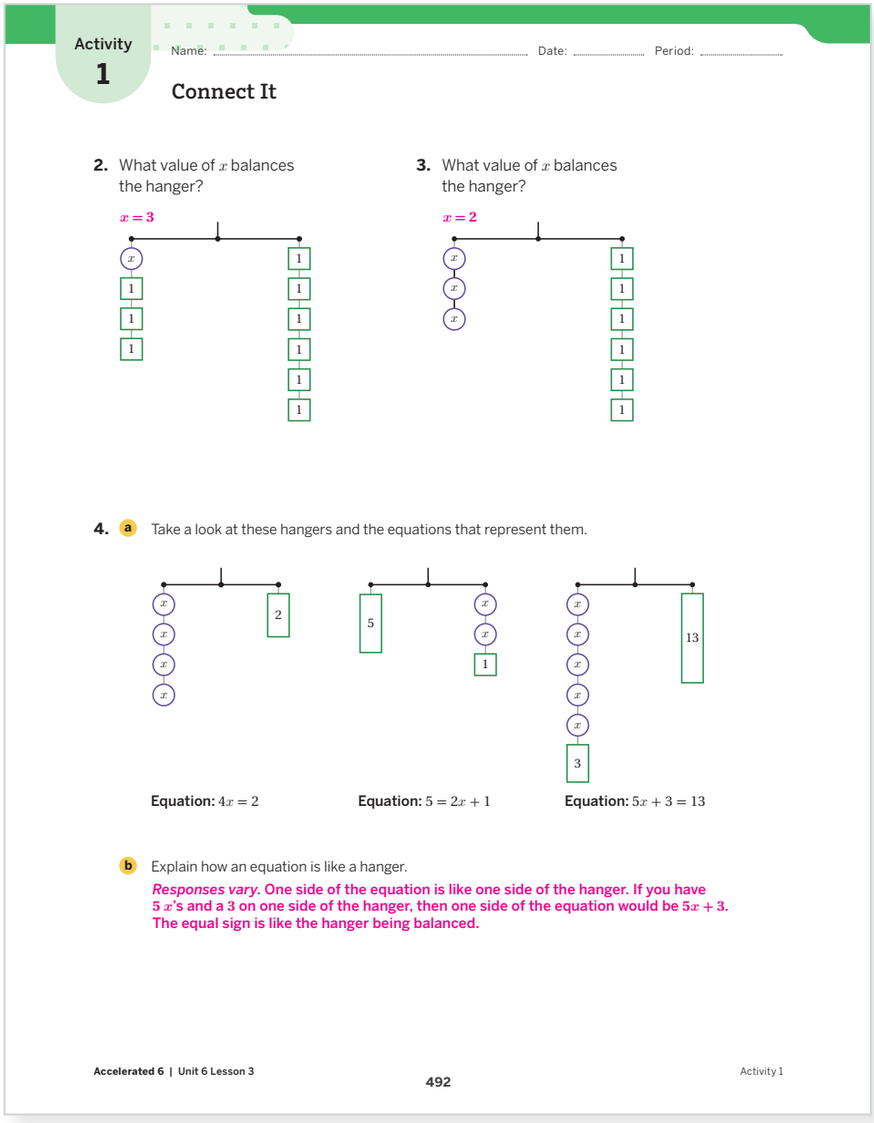
Look for a variety of hangers and equations to share during the Connect.

3 Connect

Invite students to share a variety of strategies for determining the value of x on each hanger. **(MTR.2.1)**

Consider asking, "How were your strategies for each problem alike? How were they different?"

Share student hanger diagrams and equations, and discuss how students see the same information in each matching representation. **(MTR.2.1, MTR.4.1, MTR.5.1)**



Activity 1

Name: Date: Period:

Connect It

2. What value of x balances the hanger?

$x = 3$

3. What value of x balances the hanger?

$x = 2$

4. a Take a look at these hangers and the equations that represent them.

Equation: $4x = 2$

Equation: $5 = 2x + 1$

Equation: $5x + 3 = 13$

b Explain how an equation is like a hanger.

Responses vary. One side of the equation is like one side of the hanger. If you have 5 x's and a 3 on one side of the hanger, then one side of the equation would be $5x + 3$. The equal sign is like the hanger being balanced.

Accelerated 6 | Unit 6 Lesson 3 492 Activity 1

Key Takeaway: Equations can be used to represent hanger diagrams. Both hanger diagrams and their matching equations can be used to determine the unknown value of x.

Pairs | 10 min

Activity 2 Make It, Solve It

Purpose: Students create hangers to represent and solve equations. **(MTR.2.1)**

Short on time? Consider completing Problems 5–6 as a class.

1 Launch

To surface connections, consider asking, "How are hanger diagrams and tape diagrams alike? How are they different?"

2 Monitor

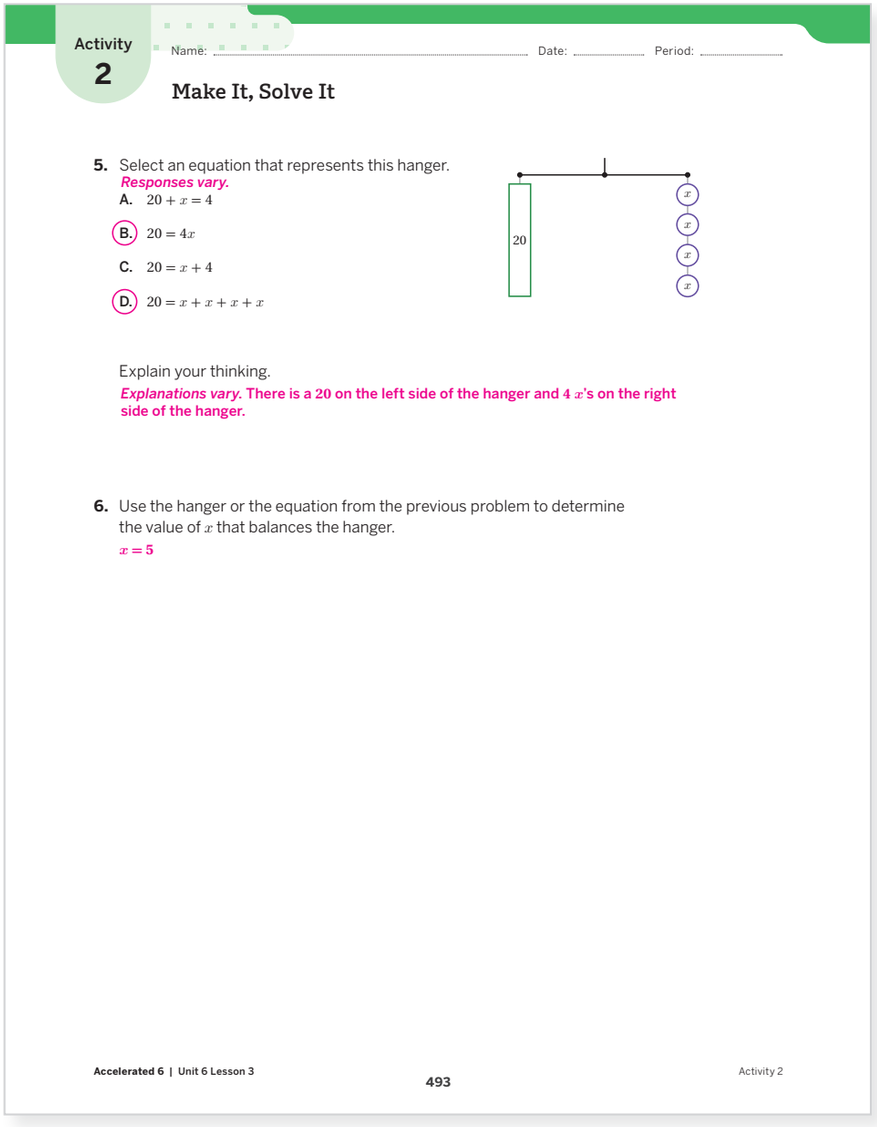
Look for students who select each of the correct responses and for students who debate between $11 = 4x$ and $11 = x + x + x + x$.

A **Accessibility: Visual-Spatial Processing** Invite students to compare and contrast each equation before responding to support recognizing the differences between each equation.

Look for students who apply or build on their strategies from the previous activity. **(MTR.5.1)**

D **Differentiation: (Support)** Look for students who use number sense to reason that the value of x is slightly less than 3. Consider asking, "How would you determine the unknown weight if you used a seesaw?"

Activity 2 continued >



Activity 2

Name: Date: Period:

Make It, Solve It

5. Select an equation that represents this hanger.

Responses vary.

A. $20 + x = 4$

B. $20 = 4x$

C. $20 = x + 4$

D. $20 = x + x + x + x$

Explain your thinking.

Explanations vary. There is a 20 on the left side of the hanger and 4 x's on the right side of the hanger.

6. Use the hanger or the equation from the previous problem to determine the value of x that balances the hanger.

$x = 5$

Accelerated 6 | Unit 6 Lesson 3 493 Activity 2

Pairs | 10 min

Activity 2 Make It, Solve It (continued)

Purpose: Students create hangers to represent and solve equations. **(MTR.2.1)**

2 Monitor

Look for both correct and incorrect hangers to share during the Connect.

A **Accessibility: Memory and Attention** Invite students with working memory challenges to look for similarities and differences between their task here and the hanger in the previous problem.

D **Differentiation**

Look for students who:	Respond to Student Thinking
Represent $6 = x + 2$ with two x's on one side and a weight of 6 on the other side.	**Support** Consider asking, "How would you represent the equation $6 = 2x$?"
Use a variety of strategies, like: • Using reasoning and number sense. • Crossing off equal weights from the hanger. • Using inverse operations in their equation. **(MTR.2.1)**	**Strengthen** Invite them to share their strategy during the Connect.

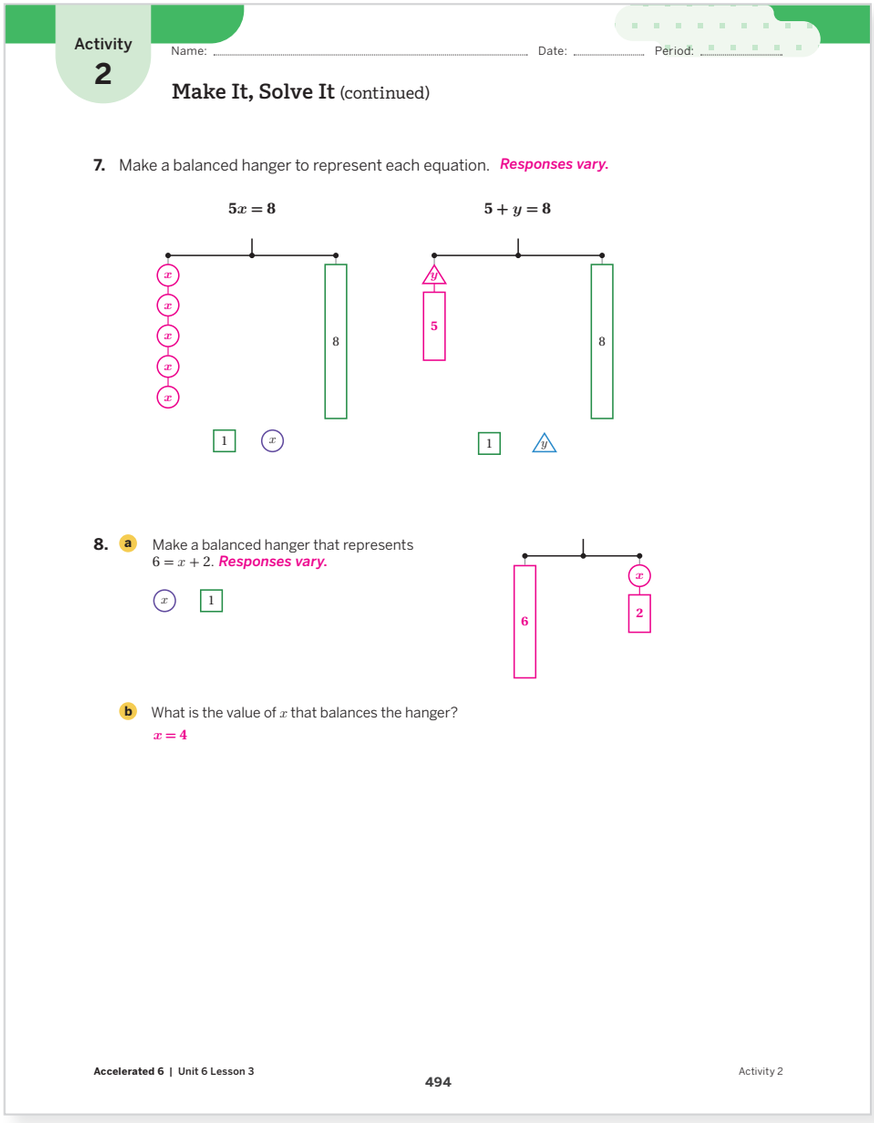
3 Connect

Share student hanger diagrams and equations. **Invite students** to decide which equation each hanger matches, or to write a new equation if it matches neither.

Consider asking, "How is 5 represented in each hanger diagram?"

Invite students to share strategies for creating a hanger and for determining the solution to the equation $6 = x + 2$.

Note: If it does not come up naturally, consider asking, "Is solving $6 = x + 2$ different from solving $x + 2 = 6$? Why or why not?" No. They are equivalent because they are both represented by a weight of 6 on one side of the hanger and weights of 2 and x on the other side. **(MTR.5.1)**



Activity 2

Name: Date: Period:

Make It, Solve It (continued)

7. Make a balanced hanger to represent each equation. ***Responses vary.***

$5x = 8$

$5 + y = 8$

8. **a** Make a balanced hanger that represents $6 = x + 2$. ***Responses vary.***

b What is the value of x that balances the hanger?

$x = 4$

Accelerated 6 | Unit 6 Lesson 3 494 Activity 2

Pairs | 10 min

Activity 3 Challenge Creator

Purpose: Students use their creativity to make their own hanger challenge for other students to solve.

1 Launch

Consider sharing that students will create their own challenge similar to the previous problem.

Distribute the Activity 3 Sheets.

To encourage creativity, invite students to make a challenge they think will be different from ones their classmates create.

A **Accessibility: Executive Functioning** Invite students to work with a partner on a challenge before trying one independently.

2 Monitor

Listen to student strategies and discussions, and offer help or encouragement as needed.

Invite students to check their answer with the person who created the hanger they are solving. Encourage them to discuss and resolve any differences. **(MTR.4.1)**

Look for several students' hanger diagrams to share during the Connect.

3 Connect

Share student hanger diagrams and invite students to share their strategies for writing and solving equations to determine the unknown weight. **(MTR.2.1, MTR.5.1)**

Consider asking:

- "How did you pick the challenges you wanted to solve?"
- "Was there a strategy you heard that surprised you?"

Activity 3

Name: _____ Date: _____ Period: _____

Challenge Creator

9. You will use the Activity 3 Sheet to create your own hanger challenge.

a **Make It!** On the Activity 3 Sheet, create your own balanced hanger challenge.

b **Solve It!** On this page, write the equation that represents your hanger and then determine the value of x that balances your hanger.

Responses vary.

My Equation	Solution to My Equation

c **Swap It!** Swap your challenge with one or more partners. Write their equation, then determine the value of x that balances their hanger.

Responses vary.

	Equation	Solution to Their Equation
Partner 1		
Partner 2		
Partner 3		
Partner 4		

Accelerated 6 | Unit 6 Lesson 3 495 Activity 3

Whole Class | 5 min

Synthesis

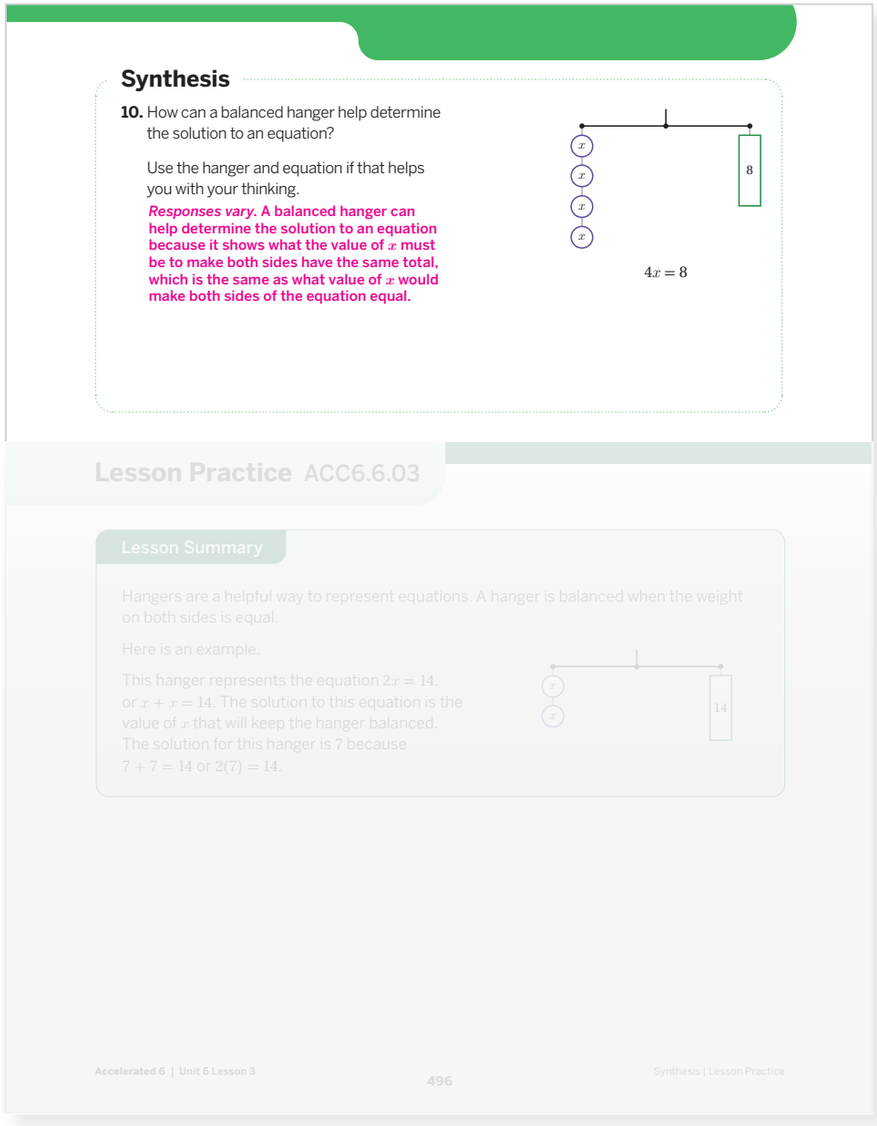
Purpose: Students synthesize their understanding of using hanger diagrams to represent and solve equations.

Invite students to respond independently, and then share their thinking with a partner.

Look for and share a variety of ideas, such as:
- Annotating diagrams.
- Using inverse operations.
- Describing the relationship between hanger diagrams and equations.

MLR **MLR2: Collect and Display** Use this routine to collect students' ideas and strategies as they discuss with a partner. Display these publicly for students to reference or add to for the rest of the unit.

Note: If time allows, invite students to revise their previous thinking based on the discussion.



Synthesis

10. How can a balanced hanger help determine the solution to an equation?

Use the hanger and equation if that helps you with your thinking.

Responses vary. A balanced hanger can help determine the solution to an equation because it shows what the value of x must be to make both sides have the same total, which is the same as what value of x would make both sides of the equation equal.

Lesson Practice ACC6.6.03

Lesson Summary

Hangers are a helpful way to represent equations. A hanger is balanced when the weight on both sides is equal.

Here is an example.

This hanger represents the equation $2x = 14$, or $x + x = 14$. The solution to this equation is the value of x that will keep the hanger balanced. The solution for this hanger is 7 because $7 + 7 = 14$ or $2(7) = 14$.

Accelerated 6 | Unit 6 Lesson 3 496 Synthesis | Lesson Practice

Lesson Takeaway: There are different strategies for determining an unknown weight on a hanger diagram. In each strategy, you must remove an equal amount of weight from each side to keep the hanger balanced.

Independent | 5 min

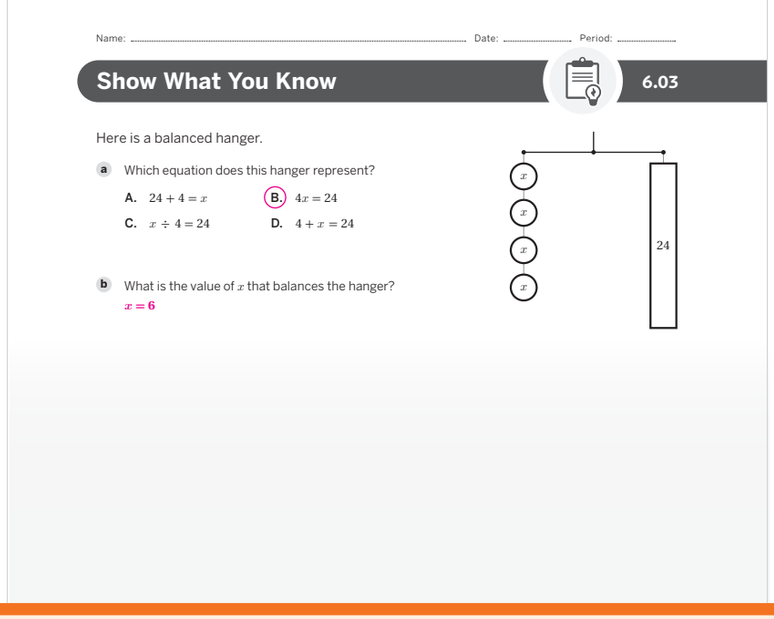
Show What You Know

Purpose: Students demonstrate their understanding by selecting the equation that represents the hanger and then determining the value of x that balances the hanger.

Today's Goals

1. **Goal:** Make connections between balanced hangers and true equations.
2. **Goal:** Use balanced hangers to explain how to solve equations of the form $x + p = q$ and $px = q$.
3. **Language Goal:** Interpret hanger diagrams that represent equations of the form $x + p = q$ and $px = q$. **(Speaking, Listening, and Writing)**

Show What You Know (Assessment Resources)



Name: _____ Date: _____ Period: _____

Show What You Know 6.03

Here is a balanced hanger.

a. Which equation does this hanger represent?

A. $24 + 4 = x$
B. $4x = 24$
C. $x \div 4 = 24$
D. $4 + x = 24$

b. What is the value of x that balances the hanger?

$x = 6$

D Differentiation Use after Lesson 3

Support

Provide targeted intervention for students by using these resources.

If students need support understanding how to make connections between balanced hangers and true equations:

» **Respond to Student Thinking:**

- Students will have more opportunities to solve equations using hangers. You may choose not to intervene now.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite them to explore the **Sub-Unit 1 Extension Activities**.
- Inviting students to create hanger problems on paper with different numbers of unknown weights on both sides. Encourage them to create a strategy for solving them.

Professional Learning

Consider the next several lessons you will teach. How will students' work in making connections between hanger diagrams and equations prepare them for Lessons 4–5, where they'll be solving equations without the use of visual models?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

10. How can a balanced hanger help determine the solution to an equation?

Use the hanger and equation if that helps you with your thinking.

Responses vary. A balanced hanger can help determine the solution to an equation because it shows what the value of x must be to make both sides have the same total, which is the same as what value of x would make both sides of the equation equal.

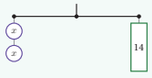
Lesson Practice ACC6.6.03

Lesson Summary

Hangers are a helpful way to represent equations. A hanger is balanced when the weight on both sides is equal.

Here is an example.

This hanger represents the equation $2x = 14$, or $x + x = 14$. The solution to this equation is the value of x that will keep the hanger balanced. The solution for this hanger is 7 because $7 + 7 = 14$ or $2(7) = 14$.

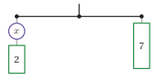


Lesson Practice ACC6.6.03

Name: _____ Date: _____ Period: _____

1. Anushka says that to balance this hanger the value of x must be 7. Is she correct? Explain your thinking.

No. ***Explanations vary.*** **The hanger would not be balanced if the value of x was 7 because the left side of the hanger would equal 9, and the right side of the hanger equals 7.**

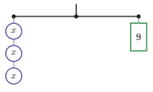


Problems 2–3: Determine the value of x that balances the hanger.

2.

$x = 8$

3.

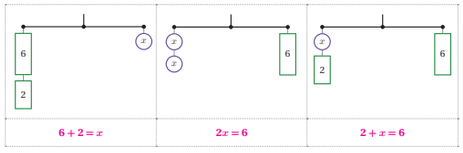


$x = 3$

FAST Practice

4. Match each equation to the hanger it represents. One equation will have no match.

$2x = 6$ $2 + x = 6$ $6 + 2 = x$ $3 \cdot 2 = x$



Equation with no match: $3 \cdot 2 = x$

Lesson Practice ACC6.6.03

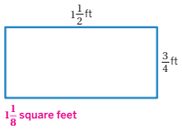
Name: _____ Date: _____ Period: _____

Spiral Review

Problems 5–8: Determine the value of each expression.

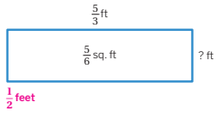
5. $12 + 2.4 = 14.4$ **6.** $12 \cdot 2.4 = 28.8$ **7.** $12 - 2.4 = 9.6$ **8.** $12 \div 2.4 = 5$

9. Calculate the area of this rectangle.



$1\frac{1}{8}$ **square feet**

10. Calculate the length of this rectangle.



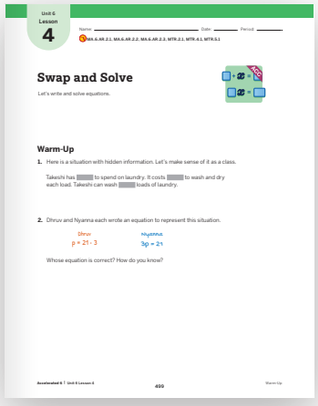
$\frac{1}{2}$ **feet**

11. Precious set a goal to save $20 for a new game. Complete the table to show how much money Precious saved at different percentages of her goal.

Percentage of Goal (%)	Money Saved ($)
25	5
75	15
125	25

Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1	2	MA.6.AR.2.2
	2–3	2	MA.6.AR.2.3
FAST Practice	**4**	2	MA.6.AR.2.2, MA.6.AR.2.3
Spiral Review			
	5, 7	1	MA.5.NSO.2.3
	6, 8	1	MA.6.NSO.2.1
	9	1	MA.5.GR.2.1
	10	2	MA.6.GR.2.2
	11	2	MA.6.AR.3.4


Unit 6
Lesson
4
Swap and Solve
Let's write and solve equations.
Warm-Up
1. Here is a situation with hidden information. Let's make sense of it as a class.
Takeshi has ▓▓▓ to spend on laundry. It costs ▓▓▓ to wash and dry each load. Takeshi can wash ▓▓▓ loads of laundry.
2. Dhruv and Nyamia each wrote an equation to represent this situation.
Dhruv: $p + 21 = 3$
Nyamia: $3p = 21$
Whose equation is correct? How do you know?
499

Swap and Solve

Solving Equations in Context

Let's write and solve equations.

Learning Outcomes

Today's Goals

1. **Goal:** Use equations of the form $x + p = q$ or $px = q$ to solve problems in context and interpret the meaning of the solution.
2. **Language Goal:** Describe a situation that can be represented by an equation of the form $x + p = q$ or $px = q$. **(Reading, Writing, Speaking, and Listening)**

Students write their own situations to match equations, and then trade situations with classmates. They solve their own equations and their classmates', check the accuracy of their solutions, and interpret the solutions in the context of the situations. **(MTR.2.1)**

Coherence

Prior Learning

In Lesson 2, students matched equations with situations and determined their solutions. In Lesson 3, students learned a variety of strategies for solving an equation out of context.

Future Learning

In a future course, students will write and solve equations of the form $px + q = r$ and $p(x + q) = r$ to represent more complex situations.

Connections

- Students demonstrate understanding of equations as they write, solve, check, and interpret the meaning of the solution. (mathematical, real world)

FL B.E.S.T. Math Benchmarks

Addressing

MA.6.AR.2.2
Write and solve one-step equations in one variable within a mathematical or real-world context using addition and subtraction, where all terms and solutions are integers.

MA.6.AR.2.3
Write and solve one-step equations in one variable within a mathematical or real-world context using multiplication and division, where all terms and solutions are integers.

Also Addressing: **MA.6.AR.2.1**

Mathematical Thinking and Reasoning Standards: MTR.2.1, MTR.4.1, MTR.5.1

ELA Expectations: ELA.K12.EE.2.1, ELA.K12.EE.4.1, ELA.K12.EE.6.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.5.AR.2.1 **MA.5.AR.2.4**	**MA.7.AR.2.2**

Vocabulary

Additional Vocabulary

substitute

Lesson at a Glance

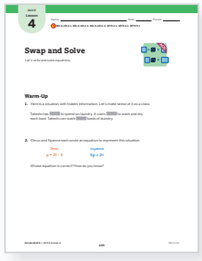
~ 45 min

MA.6.AR.2.1, MA.6.AR.2.2, MA.6.AR.2.3

Warm-Up

Independent | 10 min

Students use MLR6: Three Reads to make sense of a situation described in words. **(MTR.4.1, MTR.5.1)**



Activity 1

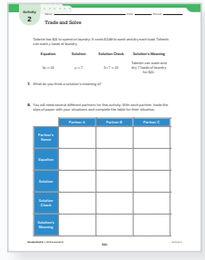
Pairs | 10 min

Students write situations that match equations, and then use MLR1: Stronger and Clearer Each Time to refine their ideas and language. **(MTR.2.1)**

Activity 2

Pairs | 15 min

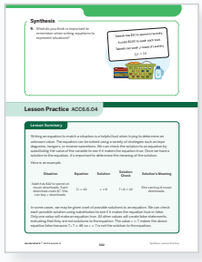
Students trade their situation with several classmates, and then write and solve an equation for each classmate's situation.
Routine: Think-Pair-Share



Synthesis

Whole Class | 5 min

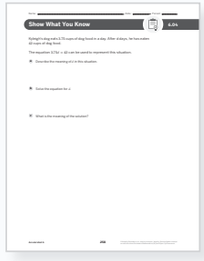
Students synthesize their understanding of writing equations to represent situations.
MLR2: Collect and Display



Show What You Know

Independent | 5 min

Students demonstrate their understanding by writing a situation for an equation, and then determining its solution and the meaning of the solution.



Prep Checklist

This lesson includes:

- Student Edition
- Show What You Know
- Index cards or slips of paper

Independent | 10 min

Warm-Up

Purpose: Students use MLR6: Three Reads to make sense of a situation described in words. **(MTR.4.1, MTR.5.1)**

1 Launch

MLR **MLR6: Three Reads** Invite students to read the text three times, each with a particular focus, to support students in making sense of the scenario. **(MTR.4.1, MTR.5.1)**

Direct students' attention to the situation with the hidden information and invite a student to read it aloud, then discuss the prompt.

To surface connections, invite students to share a time they've helped with laundry.

Share that the hidden information is Takeshi has $21 to spend, it costs $3.00 to wash and dry each load, and Takeshi can wash p loads. Encourage students to add this information in their Student Edition. Then discuss the prompt.

Invite students to share how they decided what the variable represents.

Direct students' attention to the situation again with a focus on the two equations. After the third read, invite students to discuss the question with a partner, then as a class.

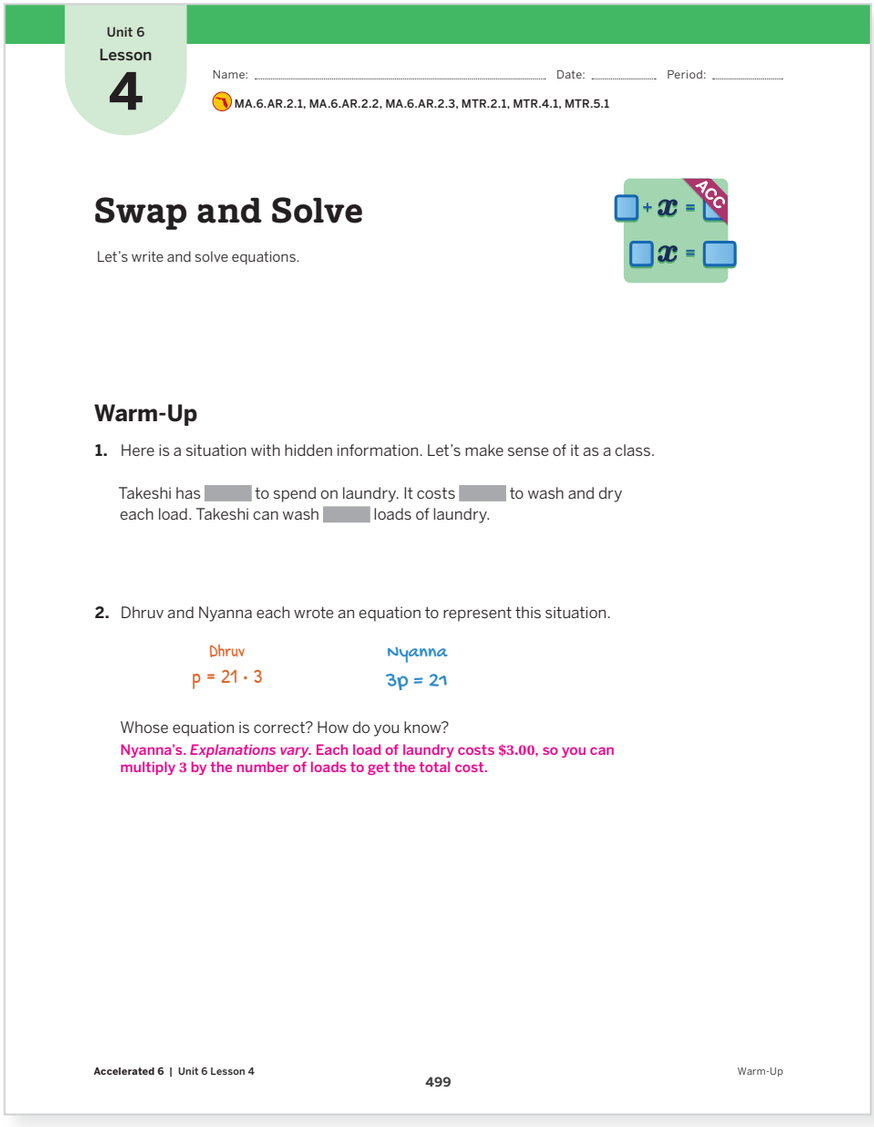
A **Accessibility: Memory and Attention** Invite students to record their work in their Student Edition or on paper to support students with keeping track of information.

2 Connect

Invite students to share which equation is correct by polling the class. Encourage students to justify their choices.

Consider asking, "What question could you ask Dhruv to help him understand his mistake?"

Note: If time allows, invite students to solve Nyanna's equation and interpret the solution in Takeshi's situation.



Unit 6
Lesson
4

Name: _____ Date: _____ Period: _____

MA.6.AR.2.1, MA.6.AR.2.2, MA.6.AR.2.3, MTR.2.1, MTR.4.1, MTR.5.1

Swap and Solve

Let's write and solve equations.

Warm-Up

1. Here is a situation with hidden information. Let's make sense of it as a class.

   Takeshi has [redacted] to spend on laundry. It costs [redacted] to wash and dry each load. Takeshi can wash [redacted] loads of laundry.

2. Dhruv and Nyanna each wrote an equation to represent this situation.

   Dhruv: $p = 21 \cdot 3$

   Nyanna: $3p = 21$

   Whose equation is correct? How do you know?

   Nyanna's. *Explanations vary.* Each load of laundry costs $3.00, so you can multiply 3 by the number of loads to get the total cost.

Accelerated 6 | Unit 6 Lesson 4 499 Warm-Up

Pairs | 10 min

Activity 1 Stronger and Clearer Each Time

Purpose: Students write situations that match equations, and then use MLR1: Stronger and Clearer Each Time to refine their ideas and language. **(MTR.2.1)**

1 Launch

Direct students' attention to the equations and the directions for the activity.

2 Monitor

D Differentiation

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Invite students to return to their work from Lessons 2–3. Consider asking, "What strategy do you want to start with?"
Use the same operation instead of the inverse operation to solve for the variable.	**Support** Invite them to substitute their solution for the variable and check to see if the equation is true.

Note: It may be helpful to review the routine before students begin.

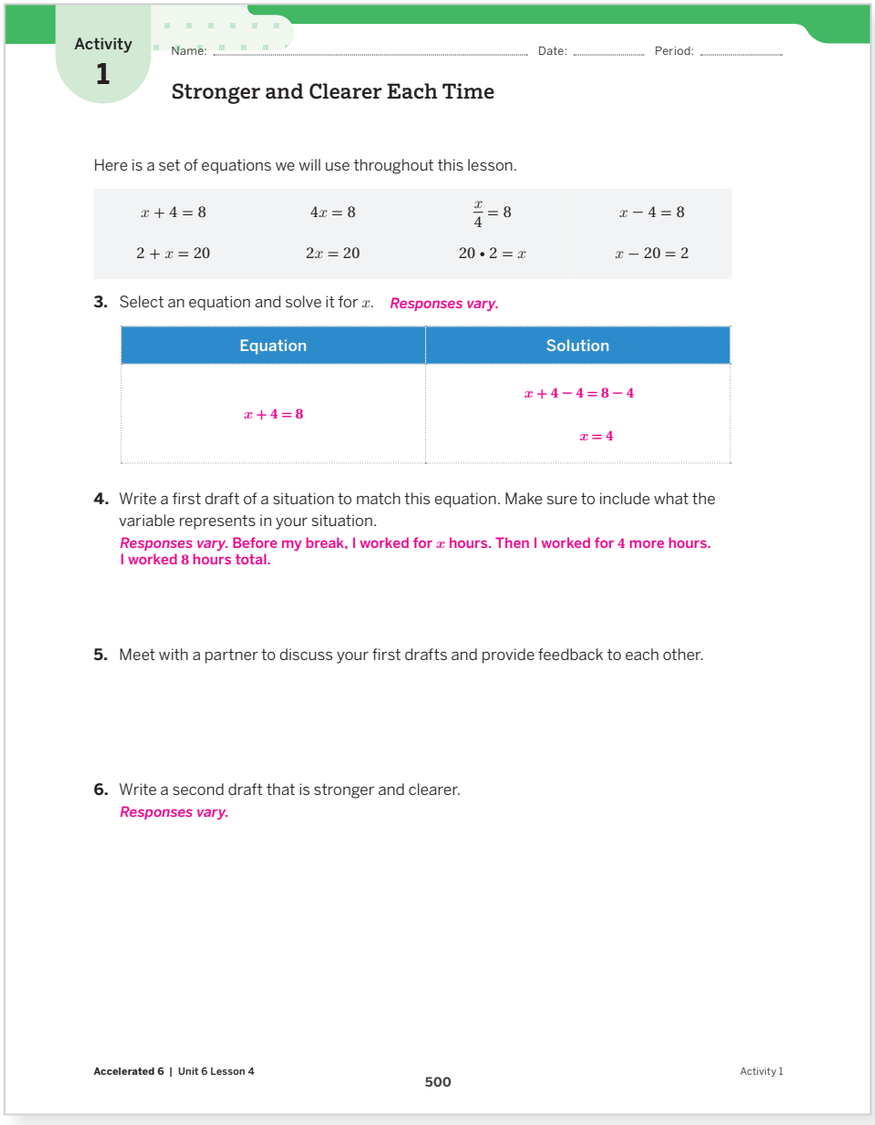
MLR **MLR1: Stronger and Clearer Each Time** Use this routine to support students in developing precise language. Invite students to meet with one or two different partners, give feedback on each other's situations, then use the feedback to write a second clearer draft. See the Routine Facilitation Guide for more information. **(MTR.4.1)**

ELL **English Language Learners** Invite students to write in their primary language and then translate their response into English during the second draft. **(Writing)**

3 Connect

Direct students' attention to the equations. Invite students to share which equation they selected and how working with their partner helped them write a situation that was stronger and clearer. **(MTR.2.1, MTR.4.1)**

Note: In the next activity, students will need to write and solve equations for the situations their classmates wrote, so they should not share their solving strategies yet.



Activity 1

Name: Date: Period:

Stronger and Clearer Each Time

Here is a set of equations we will use throughout this lesson.

$x + 4 = 8$	$4x = 8$	$\frac{x}{4} = 8$	$x - 4 = 8$
$2 + x = 20$	$2x = 20$	$20 \cdot 2 = x$	$x - 20 = 2$

3. Select an equation and solve it for x. *Responses vary.*

Equation	Solution
$x + 4 = 8$	$x + 4 - 4 = 8 - 4$ $x = 4$

4. Write a first draft of a situation to match this equation. Make sure to include what the variable represents in your situation.
Responses vary. Before my break, I worked for x hours. Then I worked for 4 more hours. I worked 8 hours total.

5. Meet with a partner to discuss your first drafts and provide feedback to each other.

6. Write a second draft that is stronger and clearer.
Responses vary.

Accelerated 6 | Unit 6 Lesson 4 500 Activity 1

Pairs | 15 min

Activity 2 Trade and Solve

Purpose: Students trade their situation with several classmates, and then write and solve an equation for each classmate's situation.

Short on time? Consider having students only trade situations with one or two classmates.

1 Launch

Direct students' attention to the situation, equation, solution, solution check, and solution's meaning from the Warm-Up.

Use the Think-Pair-Share routine to help students shape their ideas about what a solution's meaning is. See the Routine Facilitation Guide for more information. **(MTR.4.1)**

Direct students' attention to the directions in Problem 8 and review them as a class.

Distribute one index card or slip of paper to each student. Invite them to write the second draft of their situation on it.

2 Monitor

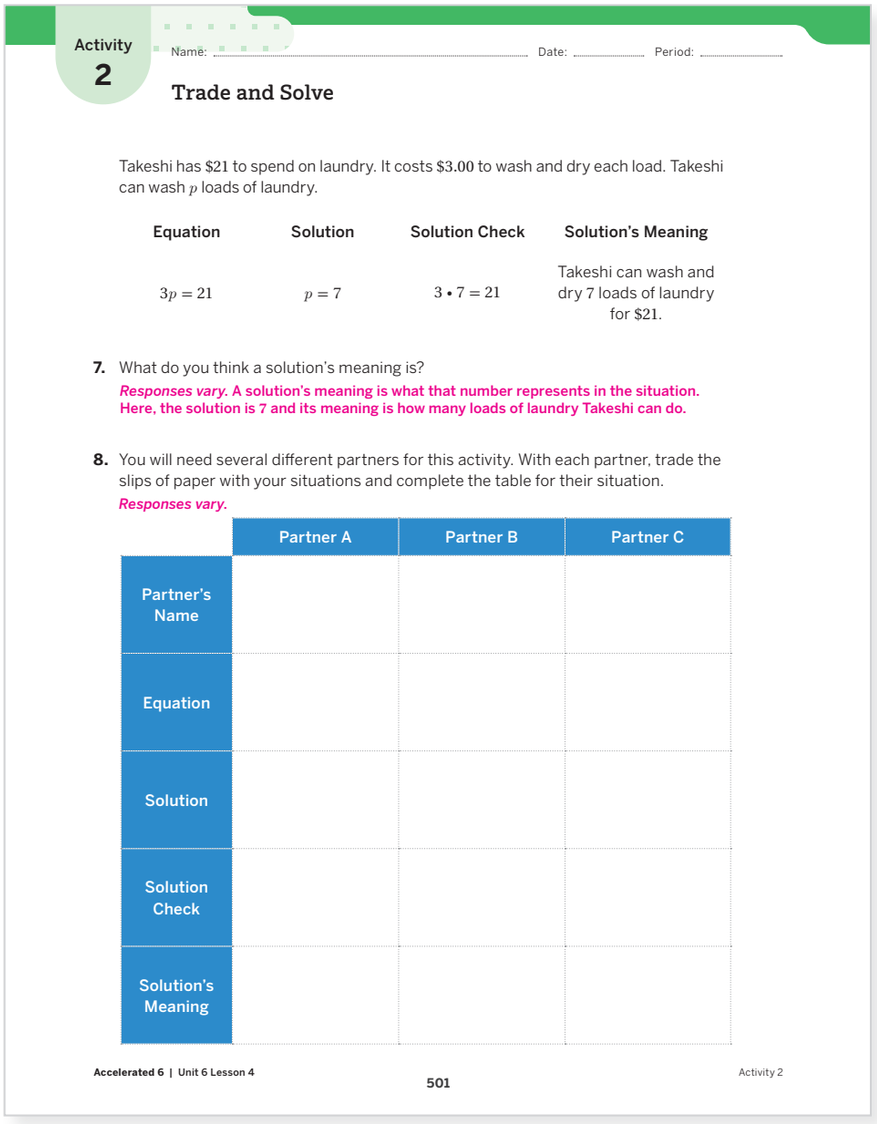
D Differentiation

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Invite them to look at the equations from Activity 1. Consider asking, "Is there anything about your situation that could help you decide which equation it matches?" **(MTR.5.1)**
Use imprecise language for a solution's meaning, like "13 stops."	**Support** Consider asking, "How could you describe what $x = 13$ means more precisely?"
Use the inverse operation to solve the equation that matches their situation.	**Strengthen** Consider asking, "How did you decide which operation to use when solving this equation?"
Would like to explore this idea more.	**Stretch** Invite them to create a different situation where the solution is different but the meaning is the same.

3 Connect

Direct students' attention to the list of equations and invite students to share their strategies for writing equations that represent their classmates' situations.

To surface the Key Takeaway, consider inviting students to share their equations, solutions, and meanings for different situations that match the same equation. **(MTR.2.1)**



Activity 2

Name: Date: Period:

Trade and Solve

Takeshi has \$21 to spend on laundry. It costs \$3.00 to wash and dry each load. Takeshi can wash p loads of laundry.

Equation	Solution	Solution Check	Solution's Meaning
$3p = 21$	$p = 7$	$3 \cdot 7 = 21$	Takeshi can wash and dry 7 loads of laundry for \$21.

7. What do you think a solution's meaning is?

Responses vary. A solution's meaning is what that number represents in the situation. Here, the solution is 7 and its meaning is how many loads of laundry Takeshi can do.

8. You will need several different partners for this activity. With each partner, trade the slips of paper with your situations and complete the table for their situation.

Responses vary.

	Partner A	Partner B	Partner C
Partner's Name			
Equation			
Solution			
Solution Check			
Solution's Meaning			

Accelerated 6 | Unit 6 Lesson 4 501 Activity 2

Key Takeaway: One equation can represent multiple situations. The solution to the equation will be the same but its meaning will depend on the situation.

Whole Class | 5 min

Synthesis

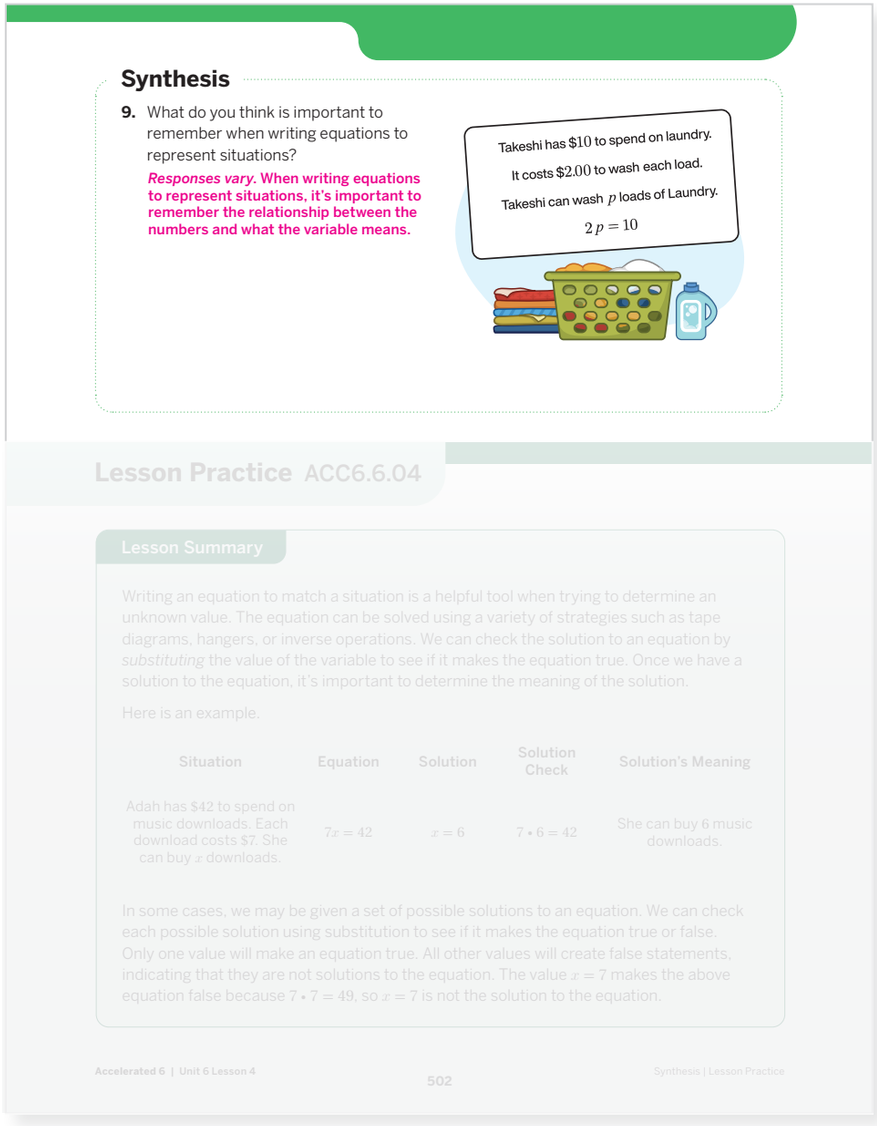
Purpose: Students synthesize their understanding of writing equations to represent situations.

Invite students to respond independently, and then share their thinking with a partner.

Listen for and share a variety of ideas, such as paying close attention to the relationship between the numbers and the variables.

MLR **MLR2: Collect and Display** Use this routine to collect students' ideas as they discuss with a partner what is important to remember when writing equations. Display for students to reference and add to for the rest of the unit.

Note: If time allows, invite students to revise their previous thinking based on the discussion.



Synthesis

9. What do you think is important to remember when writing equations to represent situations?

Responses vary. When writing equations to represent situations, it's important to remember the relationship between the numbers and what the variable means.

Takeshi has $\$10$ to spend on laundry.
It costs $\$2.00$ to wash each load.
Takeshi can wash p loads of Laundry.
$2p = 10$

Lesson Practice ACC6.6.04

Lesson Summary

Writing an equation to match a situation is a helpful tool when trying to determine an unknown value. The equation can be solved using a variety of strategies such as tape diagrams, hangers, or inverse operations. We can check the solution to an equation by *substituting* the value of the variable to see if it makes the equation true. Once we have a solution to the equation, it's important to determine the meaning of the solution.

Here is an example.

Situation	Equation	Solution	Solution Check	Solution's Meaning
Adah has $42 to spend on music downloads. Each download costs $7. She can buy x downloads.	$7x = 42$	$x = 6$	$7 \cdot 6 = 42$	She can buy 6 music downloads.

In some cases, we may be given a set of possible solutions to an equation. We can check each possible solution using substitution to see if it makes the equation true or false. Only one value will make an equation true. All other values will create false statements, indicating that they are not solutions to the equation. The value $x = 7$ makes the above equation false because $7 \cdot 7 = 49$, so $x = 7$ is not the solution to the equation.

Accelerated 6 | Unit 6 Lesson 4 502 Synthesis | Lesson Practice

Lesson Takeaway: Writing and solving an equation can help us determine unknown information in a situation. A solution's meaning depends on the situation the equation represents.

Independent | 5 min

Show What You Know

Purpose: Students demonstrate their understanding by writing a situation for an equation, and then determining its solution and the meaning of the solution.

Today's Goals

1. **Goal:** Use equations of the form $x + p = q$ or $px = q$ to solve problems in context and interpret the meaning of the solution.
2. **Language Goal:** Describe a situation that can be represented by an equation of the form $x + p = q$ or $px = q$. **(Reading, Writing, Speaking, and Listening)**

Show What You Know (Assessment Resources)

Name: _____ Date: _____ Period: _____

Show What You Know 6.04

Kyleigh's dog eats 3.75 cups of dog food in a day. After d days, he has eaten 42 cups of dog food.

The equation $3.75d = 42$ can be used to represent this situation.

a. Describe the meaning of d in this situation.
Explanations vary. d represents how many days it takes Kyleigh's dog to eat 42 cups of dog food.

b. Solve the equation for d.
$d = 11.2$

c. What is the meaning of the solution?
Explanations vary. It will take Kyleigh's dog 11.2 days to eat 42 cups of dog food.

D Differentiation Use after Lesson 4

Support

Provide targeted intervention for students by using these resources.

If students need support with writing a situation to represent an equation:

» **Respond to Student Thinking:**

- Use the ***Writing and Solving Equations*** **Mini-lesson**.
- Review the **Lesson Summary**.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite them to explore the **Sub-Unit 1 Extension Activities**.
- Invite students to create their own situations and equations that model them. Encourage them to solve each other's problems.

Professional Learning

Take a moment to reflect on the use of MLR1: Stronger and Clearer Each Time in this lesson.

- What about this routine felt successful? What are some areas for growth?
- What other opportunities might there be in the next few lessons for developing precise language for writing situations that match equations?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

9. What do you think is important to remember when writing equations to represent situations?

Responses vary. When writing equations to represent situations, it's important to remember the relationship between the numbers and what the variable means.

Lesson Practice ACC6.6.04

Lesson Summary

Writing an equation to match a situation is a helpful tool when trying to determine an unknown value. The equation can be solved using a variety of strategies such as tape diagrams, hangers, or inverse operations. We can check the solution to an equation by *substituting* the value of the variable to see if it makes the equation true. Once we have a solution to the equation, it's important to determine the meaning of the solution.

Here is an example.

Situation	Equation	Solution	Solution Check	Solution's Meaning
Adah has $42 to spend on music downloads. Each download costs $7. She can buy x downloads.	$7x = 42$	$x = 6$	$7 \cdot 6 = 42$	She can buy 6 music downloads.

In some cases, we may be given a set of possible solutions to an equation. We can check each possible solution using substitution to see if it makes the equation true or false. Only one value will make an equation true. All other values will create false statements, indicating that they are not solutions to the equation. The value $x = 7$ makes the above equation false because $7 \cdot 7 = 49$, so $x = 7$ is not the solution to the equation.

Lesson Practice
ACC6.6.04

Name: _____ Date: _____ Period: _____

1. Tiara buys a pack of paper with 200 sheets. She divides the sheets of paper equally into 5 binders. Select *all* the equations that represent the number of sheets of paper in each binder, b.

- ☑ A. $b = 200 \div 5$
- ☐ B. $200 \div b = 5$
- ☐ C. $b = 5 \cdot 200$
- ☐ D. $b \div 5 = 200$
- ☑ E. $5b = 200$

Problems 2–3: Here is an equation: $2 + x = 6$.

2. Write a situation that the equation could represent.

Responses vary. I ate 2 crackers this morning. I ate x crackers this afternoon. In total, I have eaten 6 crackers today.

3. Describe the meaning of the x in your situation.

Responses vary. x represents the number of crackers eaten this afternoon.

Problems 4–5: A plant in Zahra's garden grows 3 inches taller each month. After x months, the plant has grown 12 inches.

4. Write an equation that could represent this situation.

$3x = 12$ (or equivalent)

5. Describe the meaning of the x in the situation.

x represents the number of months it takes the plant to grow 12 inches.

6. Use the numbers 0 to 9 to complete each equation so that the value of x is the same. Use each number only once.

Responses vary.

$\boxed{3}x = \boxed{9}$ $\quad x + \boxed{4} = \boxed{7}$ $\quad x - \boxed{2} = \boxed{1}$

Lesson Practice
ACC6.6.04

Name: _____ Date: _____ Period: _____

FAST Practice

7. Anika buys 5 notebooks that contain 60 pages each. Select *all* the equations that represent the total number of pages, p.

- ☐ A. $p = 60 \div 5$
- ☑ B. $p \div 5 = 60$
- ☐ C. $5 + 60 = p$
- ☐ D. $5p = 60$
- ☑ E. $p = 5 \cdot 60$

Spiral Review

8. Select *all* the equations that have a solution of $c = 2$.

- ☐ A. $4c = 80$
- ☑ B. $c + 98 = 100$
- ☐ C. $c - 12 = 10$
- ☑ D. $6c = 12$
- ☑ E. $18c = 36$

Problems 9–11: Solve each equation.

9. $6m = 42$
$m = 7$

10. $p + 7 = 11$
$p = 4$

11. $n - 14 = 19$
$n = 33$

12. Compare the information given about triangle C and triangle D.

Triangle C	Triangle D
Base = 12 inches	Base = 15 inches
Height = 8 inches	Height = 6.5 inches

Which triangle has the greater area?

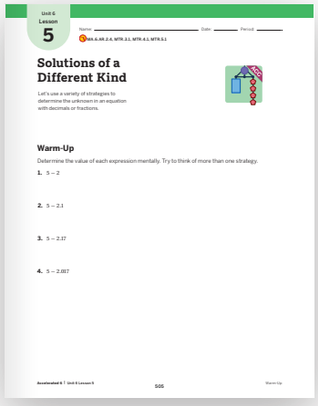
Triangle D

Explain your thinking.

Explanations vary. The area of triangle C is 48 square inches and the area of triangle D is 48.75 square inches.

Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1, 4–5	2	MA.6.AR.2.3
	2–3	3	MA.6.AR.2.2
	6	3	MA.6.AR.2.2, MA.6.AR.2.3
FAST Practice	7	2	MA.6.AR.2.2, MA.6.AR.2.3
Spiral Review			
	8–11	1	MA.6.AR.2.2, MA.6.AR.2.3
	12	1	MA.6.GR.2.1


Unit 6
Lesson 5
Name _____ Date _____ Period _____
MA.6.AR.2.4, MTR.3.1, MTR.4.1, MTR.5.1
Solutions of a Different Kind
Let's use a variety of strategies to determine the unknown in an equation with decimals or fractions.
Warm-Up
Determine the value of each expression mentally. Try to think of more than one strategy.
1. $5 - 2$
2. $5 - 2.1$
3. $5 - 2.17$
4. $5 - 2.017$
Accelerated 6 | Unit 6 Lesson 5
505
Warm-Up

Solutions of a Different Kind

Solving Equations With Decimals and Fractions

Let's use a variety of strategies to determine the unknown in an equation with decimals or fractions.

Learning Outcomes

Today's Goals

1. **Goal:** Use a variety of strategies to determine the unknown decimal or fraction in an equation of the form $x + p = q$ or $px = q$.
2. **Language Goal:** Explain the process of determining the unknown in an equation. **(Writing, Speaking, and Listening)**

Students apply reasoning to solve equations that include decimals and fractions. They revisit connections between equations and tape diagrams, then reason about what number makes an equation true. Students use a variety of tools, including strategies with and without diagrams, to determine the unknown value in an equation. **(MTR.5.1)**

Coherence

Prior Learning

In Lesson 3, students represented equations with hangers to help them determine the solutions to those equations.

Future Learning

In Unit 8, students will use a variety of strategies to solve equations with integers. In a future course, students will solve two-step equations where all terms are rational numbers.

Connections

- Students demonstrate their fluency with decimals and fractions as they solve equations. (mathematical)

FL B.E.S.T. Math Benchmarks

Addressing

MA.6.AR.2.4
Determine the unknown decimal or fraction in an equation involving any of the four operations, relating three numbers, with the unknown in any position.

Mathematical Thinking and Reasoning Standards: MTR.3.1, MTR.4.1, MTR.5.1

ELA Expectations: ELA.K12.EE.1.1, ELA.K12.EE.4.1, ELA.K12.EE.6.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.5.NSO.2.5	**MA.7.AR.2.2**
MA.5.FR.2.1	
MA.5.FR.2.3	

Lesson at a Glance

~ 45 min

MA.6.AR.2.4

Warm-Up

Whole Class | 5 min

Students engage in the **Number Talk** routine as they revisit strategies from Unit 5 for adding and subtracting with decimals.

Activity 1

Pairs | 10 min

Students use models, number sense, and reasoning to understand solutions to equations involving fractions and decimals. **(MTR.5.1)**

Activity 2

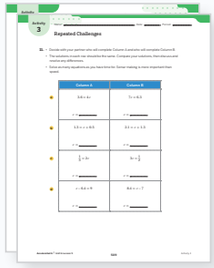
Pairs | 10 min

Students match solutions to equations and use **MLR7: Compare and Connect** to analyze two strategies for solving equations with decimals and fractions.

Activity 3

Pairs | 10 min

Students complete a variety of challenges to develop fluency in solving equations. **(MTR.3.1)**

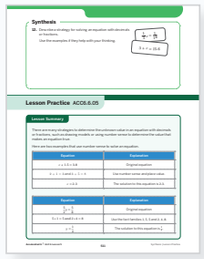


Synthesis

Whole Class | 5 min

Students synthesize their understanding of strategies for solving an equation with decimals or fractions.

MLR2: Collect and Display



Show What You Know

Independent | 5 min

Students demonstrate their understanding by solving an equation with a decimal solution.

Prep Checklist

This lesson includes:

- Student Edition
- Show What You Know

Whole Class | 5 min

Warm-Up

Purpose: Students engage in the Number Talk routine as they revisit strategies from Unit 5 for adding and subtracting with decimals.

1 Launch

Use the Number Talk routine to support students' ability to think flexibly about place value. **(MTR.4.1)**

Invite students to evaluate the expression independently and then share their strategies aloud.

Create a record of each strategy that students share, along with the name of the student who shared it.

Consider asking:
- "Who thought about it in a different way?"
- "Can someone explain ___'s strategy in their own words?"

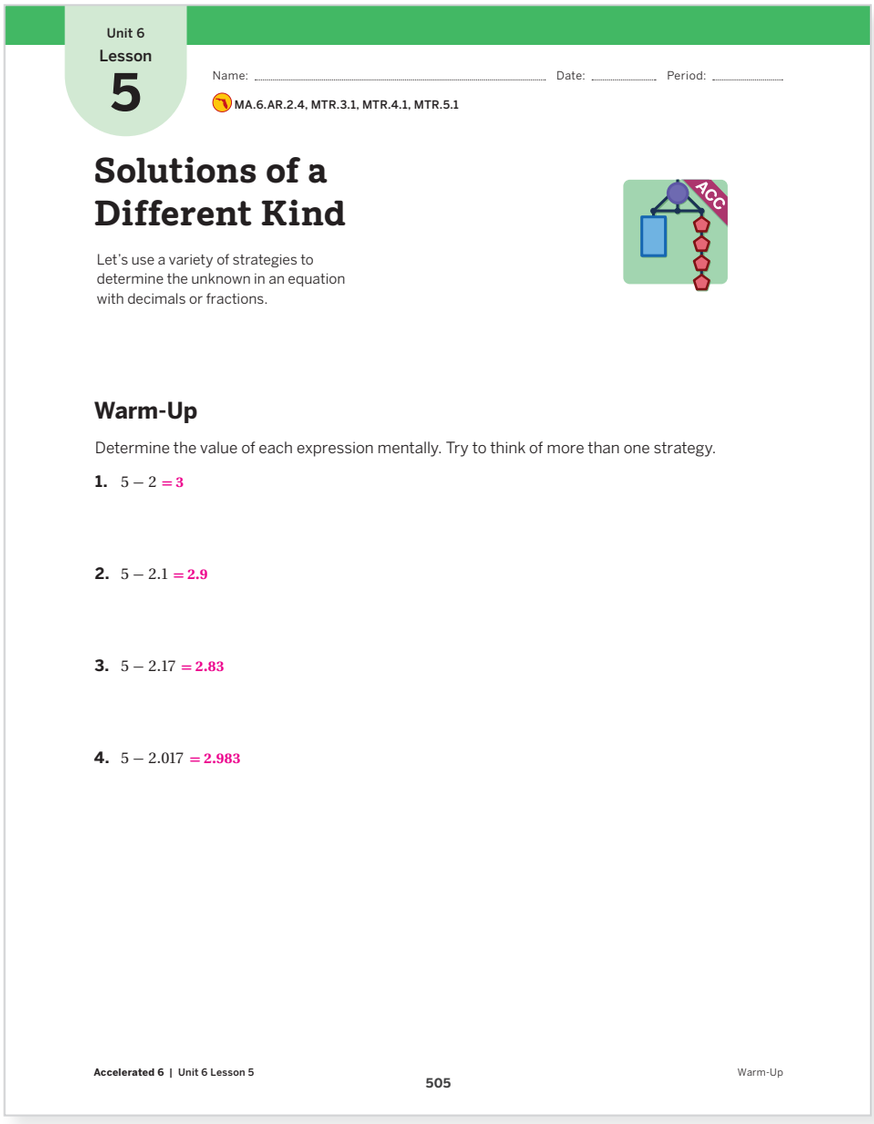
Note: Repeat this structure for each expression in the Number Talk.

A **Accessibility: Memory and Attention** Invite students to refer back to notes they took earlier on decimal operations to surface prior knowledge.

2 Connect

Invite students to compare and contrast the different expressions. **(MTR.5.1)**

Consider asking:
- "What connections do you notice among the strategies we've discussed?"
- "What connections do you notice among the different expressions?"
- "How did the place value affect the value of each expression?"



Unit 6
Lesson
5

Name: ..... Date: ..... Period: .....

MA.6.AR.2.4, MTR.3.1, MTR.4.1, MTR.5.1

Solutions of a Different Kind

Let's use a variety of strategies to determine the unknown in an equation with decimals or fractions.

Warm-Up

Determine the value of each expression mentally. Try to think of more than one strategy.

1. $5 - 2 = 3$

2. $5 - 2.1 = 2.9$

3. $5 - 2.17 = 2.83$

4. $5 - 2.017 = 2.983$

Accelerated 6 | Unit 6 Lesson 5 505 Warm-Up

Pairs | 10 min

Activity 1 What's the Relationship?

Purpose: Students use models, number sense, and reasoning to understand solutions to equations involving fractions and decimals. **(MTR.5.1)**

1 Launch

To surface prior knowledge, consider asking, "How do the numbers in an addition equation relate to one another?"

Invite students to think independently before discussing their ideas with a partner.

2 Monitor

Look for students who use a variety of descriptions of how equations and visual models represent relationships between numbers in addition and multiplication equations.

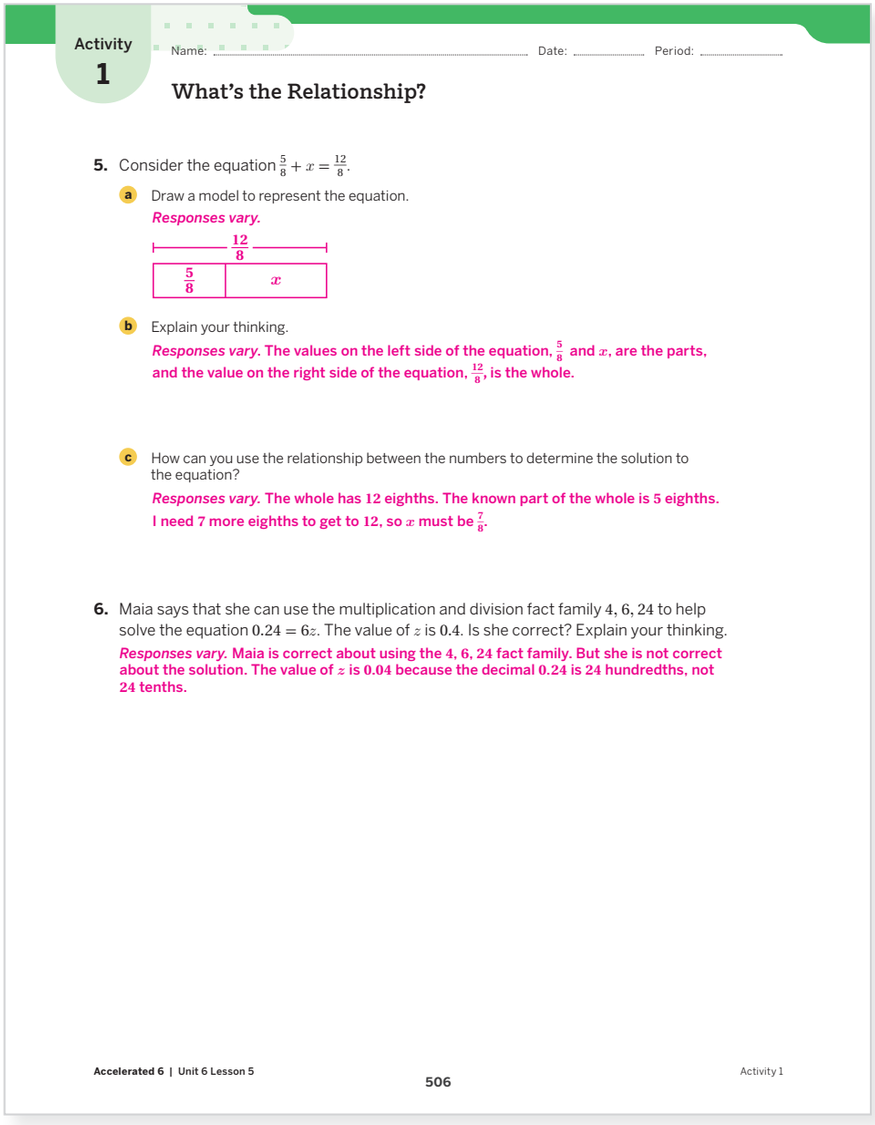
A **Accessibility: Conceptual Processing** Invite students to look at the relationship between the two equations and strategies identified to help understand them to support identifying structure. **(MTR.5.1)**

Listen for explanations to share during the Connect.

3 Connect

Invite students to share their models. Consider asking, "What are the advantages of using each model to solve equations?

To surface the Key Takeaway, consider asking, "Can you use the same strategy to determine the unknown value in any equation with decimals or fractions?" **(MTR.4.1, MTR.5.1)**



Activity 1

Name: Date: Period:

What's the Relationship?

5. Consider the equation $\frac{5}{8} + x = \frac{12}{8}$.

a. Draw a model to represent the equation.

Responses vary.

$\frac{12}{8}$

$\frac{5}{8}$	x

b. Explain your thinking.

Responses vary. The values on the left side of the equation, $\frac{5}{8}$ and x, are the parts, and the value on the right side of the equation, $\frac{12}{8}$, is the whole.

c. How can you use the relationship between the numbers to determine the solution to the equation?

Responses vary. The whole has 12 eighths. The known part of the whole is 5 eighths. I need 7 more eighths to get to 12, so x must be $\frac{7}{8}$.

6. Maia says that she can use the multiplication and division fact family 4, 6, 24 to help solve the equation $0.24 = 6z$. The value of z is 0.4. Is she correct? Explain your thinking.

Responses vary. Maia is correct about using the 4, 6, 24 fact family. But she is not correct about the solution. The value of z is 0.04 because the decimal 0.24 is 24 hundredths, not 24 tenths.

Accelerated 6 | Unit 6 Lesson 5 506 Activity 1

Key Takeaway: There are many strategies for determining the unknown in an equation with decimals or fractions, including using models and fact families.

Pairs | 10 min

Activity 2 Solving and Solutions

Purpose: Students match solutions to equations and use **MLR7: Compare and Connect** to analyze two strategies for solving equations with decimals and fractions.

1 Launch

To surface prior knowledge, invite students to brainstorm the strategies they've learned for determining solutions to equations.

2 Monitor

Encourage students to share their reasoning with a partner and work together to reach an agreement about how to sort the solutions. **(MTR.4.1)**

Note: If time allows, invite pairs to compare their matches, justify their thinking, and make revisions based on their conversation.

Look for indications of student progress. Lead a discussion if students need support.

A **Accessibility: Executive Functioning** Invite students to focus on one equation at a time to support students in carrying out multiple steps.

D **Differentiation**

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Consider asking, "How can you check if $\frac{1}{9}$ is a solution? $\frac{1}{3}$?"
Select "Both."	**Support** Encourage them to substitute both solutions into the equation to see which one makes it true.

Note: If time allows, pause to invite students to justify their choice and listen to their classmates' reasoning.

Activity 2 continued >

Activity 2

Name: Date: Period:

Solving and Solutions

7. Match each solution to its equation. Two solutions will not have a match.

0.5	0.1	0.8
$\frac{1}{9}$	$\frac{7}{10}$	$\frac{1}{3}$

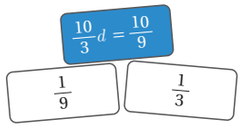
$\frac{10}{3}d = \frac{10}{9}$	$12.6 = b + 12.1$	$c + \frac{3}{10} = 1$	$4 = \frac{a}{0.2}$
$\frac{1}{3}$	0.5	$\frac{7}{10}$	0.8

Solutions with no match: 0.1 and $\frac{1}{9}$

8. Imani and Deiondre solved this equation.

Imani said the solution is $d = \frac{1}{9}$.

Deiondre said the solution is $d = \frac{1}{3}$.



Whose solution is correct? Circle one.

Imani's (Diondre's) Both Neither

Explain your thinking.

Explanations vary. I am trying to determine what number times $\frac{10}{3}$ equals $\frac{10}{9}$, and $3 \cdot 3 = 9$ I know it can't be $d = \frac{1}{9}$ because the denominator of the right side of the equation would be 27 instead of 9.

Accelerated 6 | Unit 6 Lesson 5 507 Activity 2

Pairs | 10 min

Activity 2 Solving and Solutions (continued)

Purpose: Students match solutions to equations and use **MLR7: Compare and Connect** to analyze two strategies for solving equations with decimals and fractions.

2 Monitor

D Differentiation

Look for students who:	Respond to Student Thinking
Recognize that both strategies lead to the same solution.	**Strengthen** Invite them to share their thinking during the Connect.
Describe Fabiana's strategy as using the relationships between the numbers in the equation.	

Look for students' understanding to support the conversation in the Connect.

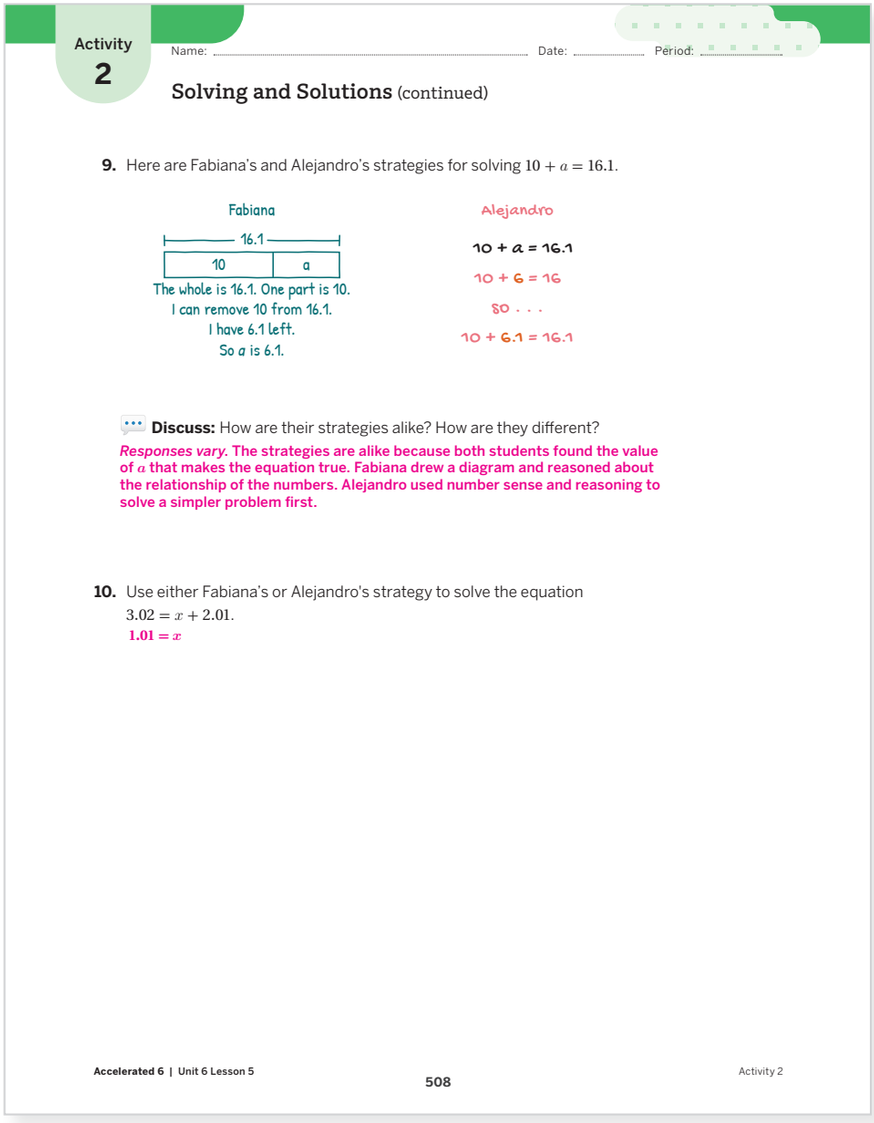
3 Connect

MLR7: Compare and Connect Invite students to analyze and compare each strategy, and then connect to their strategy. **(MTR.5.1)**

Consider asking:
- "What did each student do?"
- "Where do you see the same information in Fabiana's and Alejandro's strategies?"
- "How does each strategy compare to your own?"
- "When might we use each strategy?"

Invite students to share their work. Consider creating a record of strategies for students to refer to throughout the lesson and unit.

To surface the Key Takeaway, consider asking:
- "What would Fabiana's strategy look like if the equation included subtraction instead of addition? What if it was multiplication?"
- "When might Fabiana's strategy be useful? When might Alejandro's?"



Activity 2

Name: Date: Period:

Solving and Solutions (continued)

9. Here are Fabiana's and Alejandro's strategies for solving $10 + a = 16.1$.

Fabiana

The whole is 16.1. One part is 10.
I can remove 10 from 16.1.
I have 6.1 left.
So a is 6.1.

Alejandro

$10 + a = 16.1$
$10 + 6 = 16$
So . . .
$10 + 6.1 = 16.1$

Discuss: How are their strategies alike? How are they different?

Responses vary. The strategies are alike because both students found the value of a that makes the equation true. Fabiana drew a diagram and reasoned about the relationship of the numbers. Alejandro used number sense and reasoning to solve a simpler problem first.

10. Use either Fabiana's or Alejandro's strategy to solve the equation $3.02 = x + 2.01$.

$1.01 = x$

Accelerated 6 | Unit 6 Lesson 5 508 Activity 2

Key Takeaway: Using models to show number relationships and solving a simpler problem are two strategies for solving equations. No matter the strategy, the solution will be the same every time because the same number makes the equation true.

Pairs | 10 min

Activity 3 Repeated Challenges

Purpose: Students complete a variety of challenges to develop fluency in solving equations. **(MTR.3.1)**

Short on time? Consider having students only complete four to six problems.

1 Launch

Direct students' attention to the instructions and invite a student to read them aloud.

Invite students to set their own goal for how many problems they would like to complete, sharing that sense-making is more important than speed.

Encourage students to use their partner's feedback to help them revise their thinking. **(MTR.4.1)**

2 Monitor

Invite students to solve problems in any order or to choose any four problems to solve.

A **Accessibility: Conceptual Processing** Invite students to revisit their work from earlier in the lesson to help apply strategies to new situations.

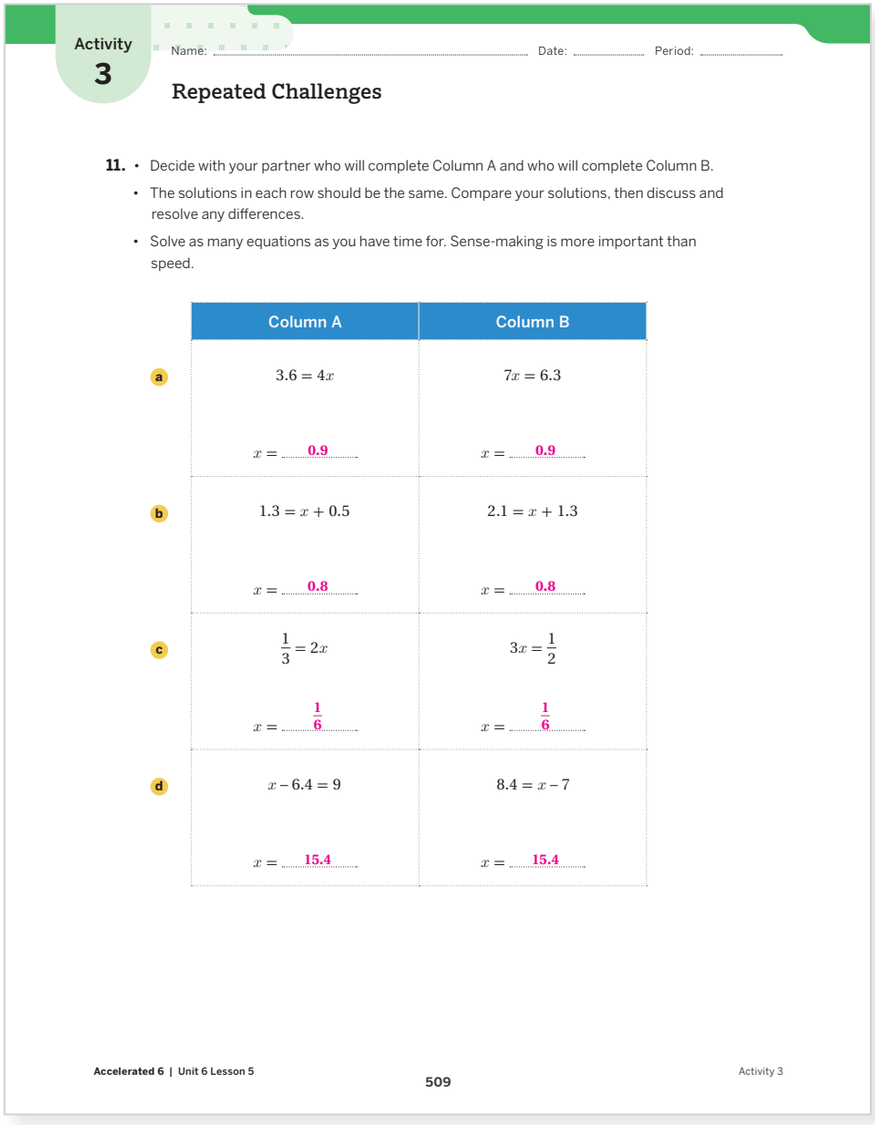
Listen for student strategies and partner discussions, offering help or encouragement as needed.

D **Differentiation**

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Consider asking, "How might drawing a tape diagram or seesaw help you?"
Subtract the coefficient in equations of the form $px = q$.	**Support** Consider asking, "What operation is being applied to the variable?"
Use a variety of strategies like: • Creating a tape diagram. • Using number sense. • Using fact families.	**Strengthen** Invite students to share during the Connect.

Note: Look for how many problems each student has completed correctly.

Activity 3 continued >



Activity 3

Name: _____ Date: _____ Period: _____

Repeated Challenges

11. • Decide with your partner who will complete Column A and who will complete Column B.
• The solutions in each row should be the same. Compare your solutions, then discuss and resolve any differences.
• Solve as many equations as you have time for. Sense-making is more important than speed.

	Column A	Column B
a	$3.6 = 4x$ $x =$ 0.9	$7x = 6.3$ $x =$ 0.9
b	$1.3 = x + 0.5$ $x =$ 0.8	$2.1 = x + 1.3$ $x =$ 0.8
c	$\frac{1}{3} = 2x$ $x = \frac{1}{6}$	$3x = \frac{1}{2}$ $x = \frac{1}{6}$
d	$x - 6.4 = 9$ $x =$ 15.4	$8.4 = x - 7$ $x =$ 15.4

Accelerated 6 | Unit 6 Lesson 5 509 Activity 3

Pairs | 10 min

Activity 3 Repeated Challenges (continued)

Purpose: Students complete a variety of challenges to develop fluency in solving equations. **(MTR.3.1)**

Short on time? Consider having students only complete four to six problems.

2 Monitor

Note: Refer to the guidance on the previous page to support students as they work on these repeated challenges.

Look for students with a variety of strategies and invite them to share during the Connect.

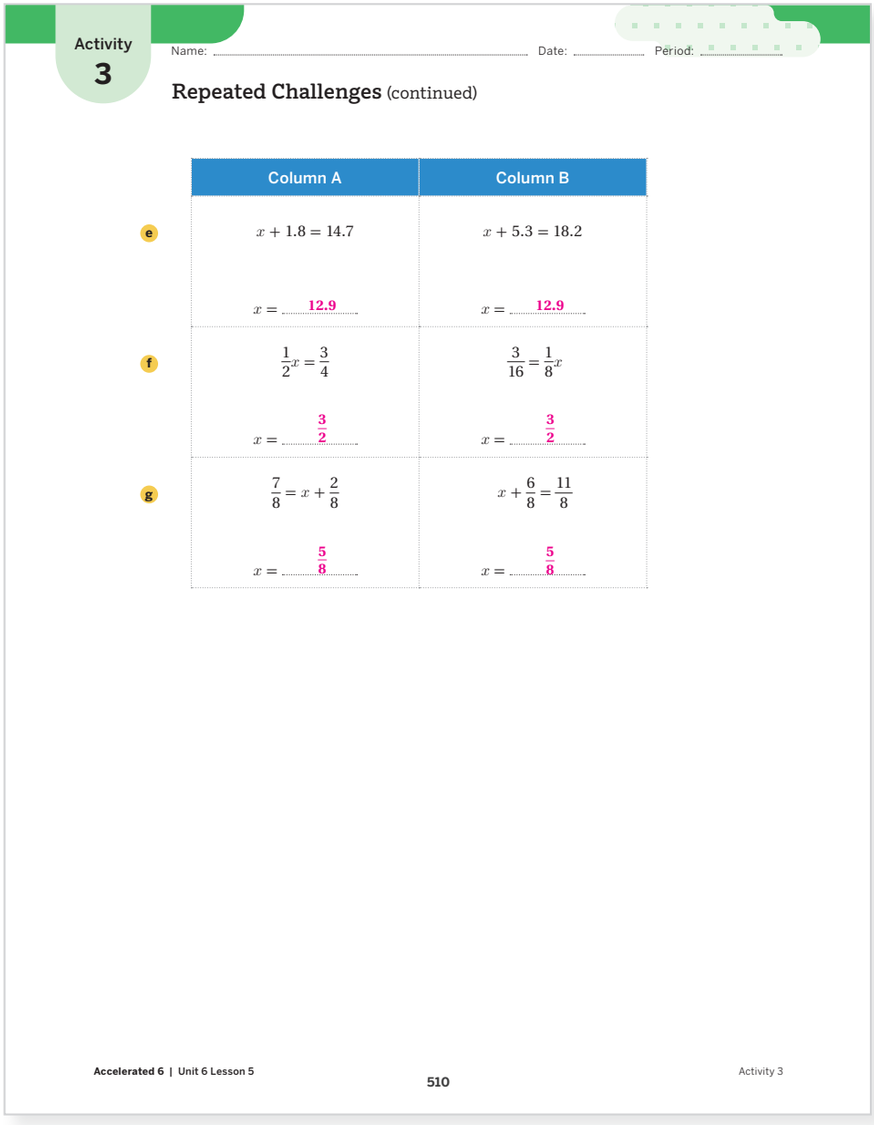
3 Connect

Invite students to share strategies that they found helpful or what they learned from the mistakes they made while solving these challenges. **(MTR.3.1)**

ELL **English Language Learners** Invite students to share strategies in their primary languages. Encourage all students to use gestures and/or visuals as they share. **(Speaking and Listening)**

Consider asking:

- "Did your strategy change when the problem included fractions? Decimals?"
- "What is your favorite strategy to use when solving these types of math problems?"



Activity 3

Name: Date: Period:

Repeated Challenges (continued)

	Column A	Column B
e	$x + 1.8 = 14.7$ $x =$ 12.9	$x + 5.3 = 18.2$ $x =$ 12.9
f	$\frac{1}{2}x = \frac{3}{4}$ $x =$ $\frac{3}{2}$	$\frac{3}{16} = \frac{1}{8}x$ $x =$ $\frac{3}{2}$
g	$\frac{7}{8} = x + \frac{2}{8}$ $x =$ $\frac{5}{8}$	$x + \frac{6}{8} = \frac{11}{8}$ $x =$ $\frac{5}{8}$

Accelerated 6 | Unit 6 Lesson 5 510 Activity 3

Whole Class | 5 min

Synthesis

Purpose: Students synthesize their understanding of strategies for solving an equation with decimals or fractions.

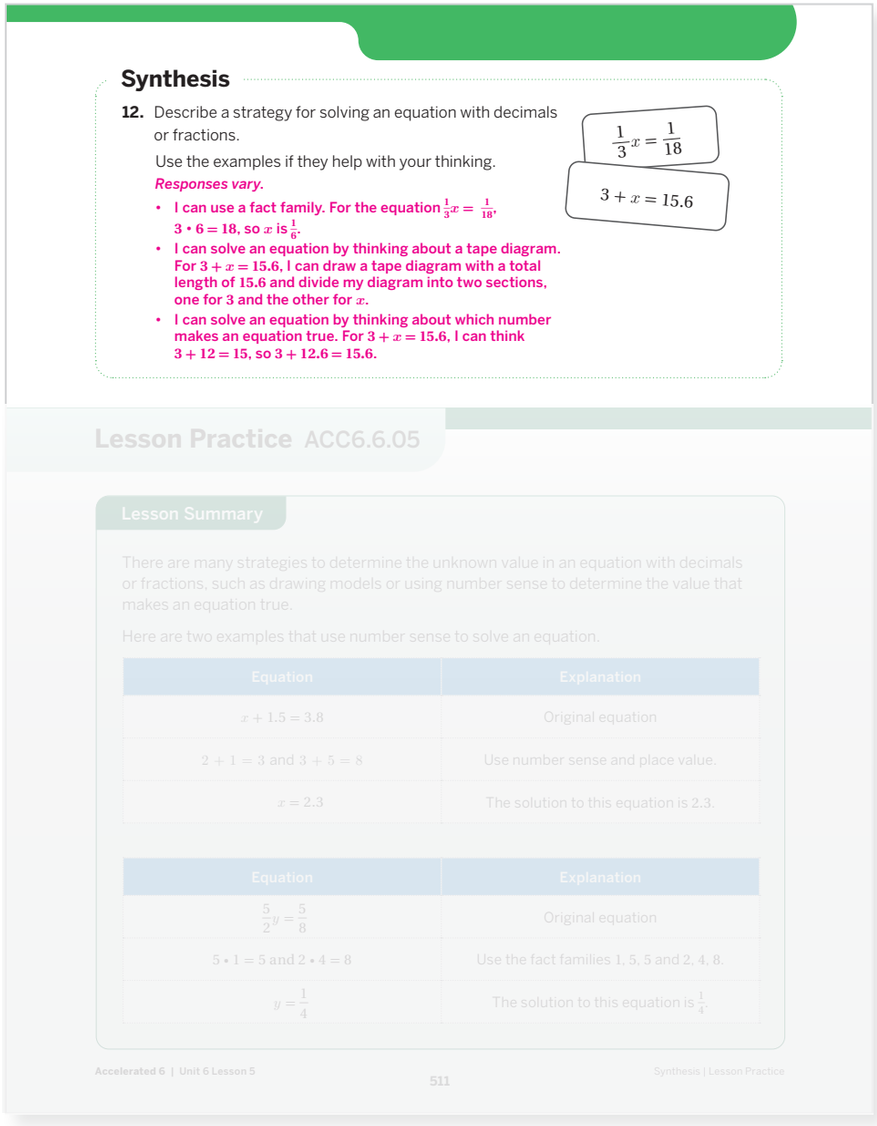
Invite students to respond independently, and then share their thinking with a partner.

Look for and share a variety of ideas, such as:

- Using a tape diagram, hanger, or seesaw.
- Using related fact families to make connections among values.
- Using number sense to reason about values that could make the equation true.

Note: If time allows, invite students to share with a partner what makes sense about each strategy and what questions they still have.

MLR **MLR2: Collect and Display** Use this routine to collect students' strategies as they share during the Connect. Display these publicly for students to reference or add to for the rest of the unit.



Synthesis

12. Describe a strategy for solving an equation with decimals or fractions.

Use the examples if they help with your thinking.

$\frac{1}{3}x = \frac{1}{18}$

$3 + x = 15.6$

Responses vary.

- I can use a fact family. For the equation $\frac{1}{3}x = \frac{1}{18}$, $3 \cdot 6 = 18$, so x is $\frac{1}{6}$.
- I can solve an equation by thinking about a tape diagram. For $3 + x = 15.6$, I can draw a tape diagram with a total length of 15.6 and divide my diagram into two sections, one for 3 and the other for x.
- I can solve an equation by thinking about which number makes an equation true. For $3 + x = 15.6$, I can think $3 + 12 = 15$, so $3 + 12.6 = 15.6$.

Lesson Practice ACC6.6.05

Lesson Summary

There are many strategies to determine the unknown value in an equation with decimals or fractions, such as drawing models or using number sense to determine the value that makes an equation true.

Here are two examples that use number sense to solve an equation.

Equation	Explanation
$x + 1.5 = 3.8$	Original equation
$2 + 1 = 3$ and $3 + 5 = 8$	Use number sense and place value.
$x = 2.3$	The solution to this equation is 2.3.

Equation	Explanation
$\frac{5}{2}y = \frac{5}{8}$	Original equation
$5 \cdot 1 = 5$ and $2 \cdot 4 = 8$	Use the fact families 1, 5, 5 and 2, 4, 8.
$y = \frac{1}{4}$	The solution to this equation is $\frac{1}{4}$.

Accelerated 6 | Unit 6 Lesson 5 511 Synthesis | Lesson Practice

Lesson Takeaway: Multiple strategies can be used to determine the unknown value in an equation with decimals or fractions, including drawing diagrams and using number sense.

Independent | 5 min

Show What You Know

Purpose: Students demonstrate their understanding by solving an equation with a decimal solution.

Today's Goals

1. **Goal:** Use a variety of strategies to determine the unknown decimal or fraction in an equation of the form $x + p = q$ or $px = q$.
2. **Language Goal:** Explain the process of determining the unknown in an equation. **(Writing, Speaking, and Listening)**

Show What You Know (Assessment Resources)

Name: _____ Date: _____ Period: _____

Show What You Know 6.05

Find the value of y for each equation. Show your thinking.

a. $\frac{2}{3}y = \frac{3}{2}$

$2\frac{1}{4}$ (or equivalent)

b. $y + 10.2 = 12.1$

1.9 (or equivalent)

D Differentiation Use after Lesson 5

Support

Provide targeted intervention for students by using these resources.

If students need support understanding how to use algebraic strategies:

» **Respond to Student Thinking:**

- Revisit **Activities 1–2** (particularly Fabiana's work) with students before providing individualized feedback on the relevant Practice Problems.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite them to explore the **Sub-Unit 1 Extension Activities**.

Professional Learning

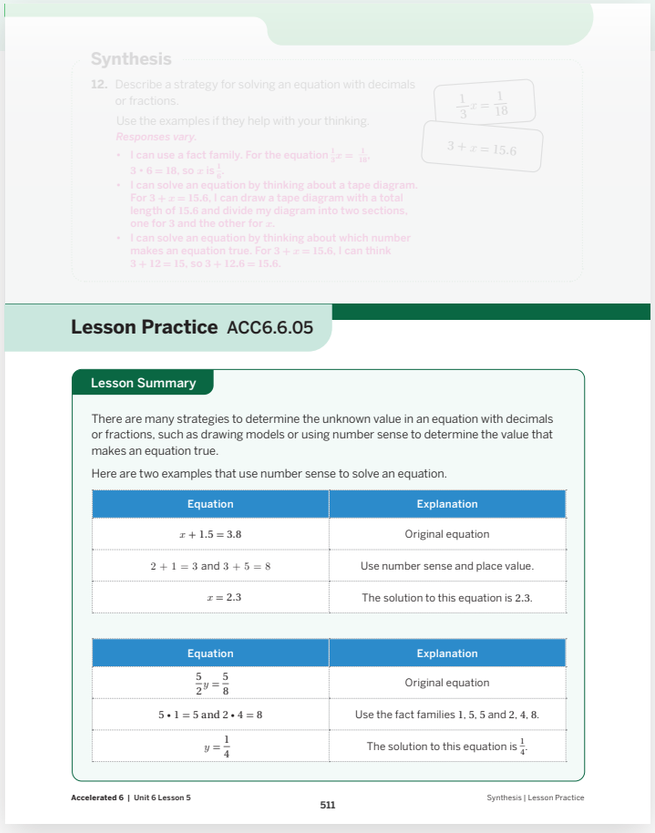
Take a moment to reflect on the strategies students used in this lesson.

- What opportunities were there to celebrate specific students' strategies for solving equations with decimals and fractions?
- What opportunities might there be in future lessons to connect back to the strategies students shared?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.



Synthesis

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Use the examples if they help with your thinking.

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$3 + x = 15.6$

Responses vary.

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Lesson Practice ACC6.6.05

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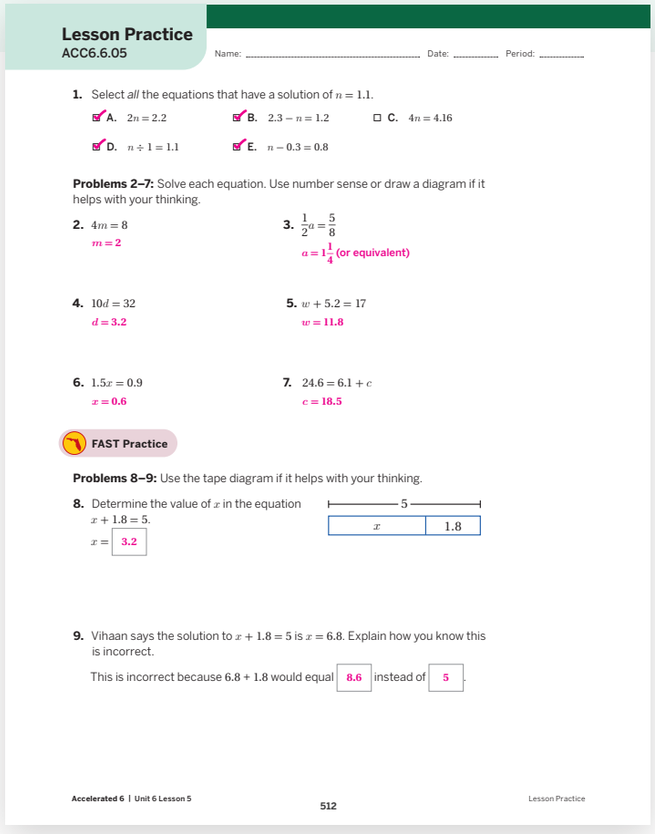
$y = \frac{1}{4}$	The solution to this equation is $\frac{1}{4}$.



Lesson Practice
ACC6.6.05

Name: _____ Date: _____ Period: _____

1. Select *all* the equations that have a solution of $n = 1.1$.

☑ A. $2n = 2.2$ ☑ B. $2.3 - n = 1.2$ ☐ C. $4n = 4.16$

☑ D. $n \div 1 = 1.1$ ☑ E. $n - 0.3 = 0.8$

Problems 2–7: Solve each equation. Use number sense or draw a diagram if it helps with your thinking.

2. $4m = 8$
   $m = 2$

3. $\frac{1}{2}a = \frac{5}{8}$
   $a = 1\frac{1}{4}$ (or equivalent)

4. $10d = 32$
   $d = 3.2$

5. $w + 5.2 = 17$
   $w = 11.8$

6. $1.5x = 0.9$
   $x = 0.6$

7. $24.6 = 6.1 + c$
   $c = 18.5$

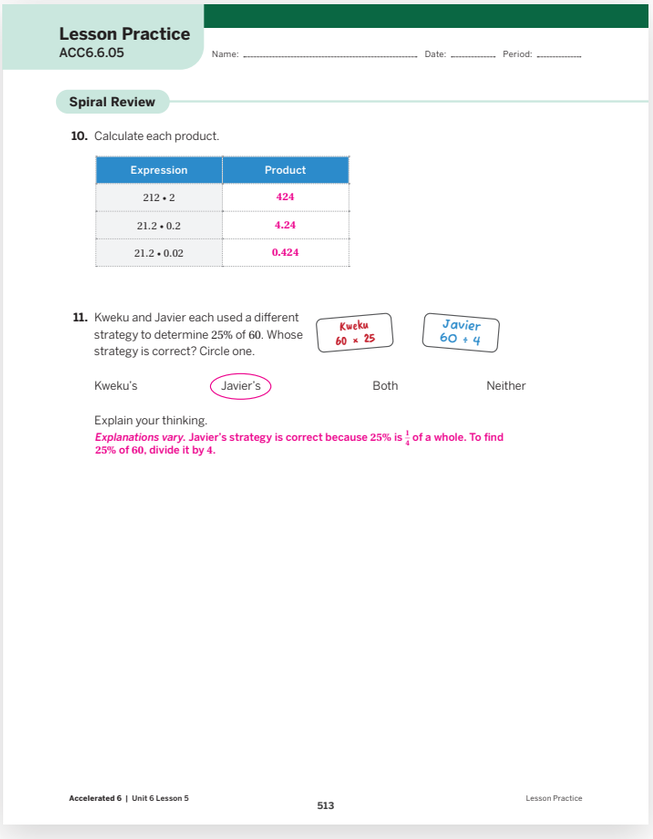
FAST Practice

Problems 8–9: Use the tape diagram if it helps with your thinking.

8. Determine the value of x in the equation $x + 1.8 = 5$.
   $x =$ 3.2

9. Vihaan says the solution to $x + 1.8 = 5$ is $x = 6.8$. Explain how you know this is incorrect.

This is incorrect because 6.8 + 1.8 would equal 8.6 instead of 5.



Lesson Practice
ACC6.6.05

Name: _____ Date: _____ Period: _____

Spiral Review

10. Calculate each product.

Expression	Product
$212 \cdot 2$	424
$21.2 \cdot 0.2$	4.24
$21.2 \cdot 0.02$	0.424

11. Kweku and Javier each used a different strategy to determine 25% of 60. Whose strategy is correct? Circle one.

Kweku: 60 × 25 Javier: 60 ÷ 4

Kweku's Javier's Both Neither

Explain your thinking.

Explanations vary. Javier's strategy is correct because 25% is $\frac{1}{4}$ of a whole. To find 25% of 60, divide it by 4.

Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1–7	1	MA.6.AR.2.4
FAST Practice	8	1	MA.6.AR.2.4
	9	2	MA.6.AR.2.4
Spiral Review			
	10	1	MA.6.NSO.2.1
	11	2	MA.6.AR.3.4

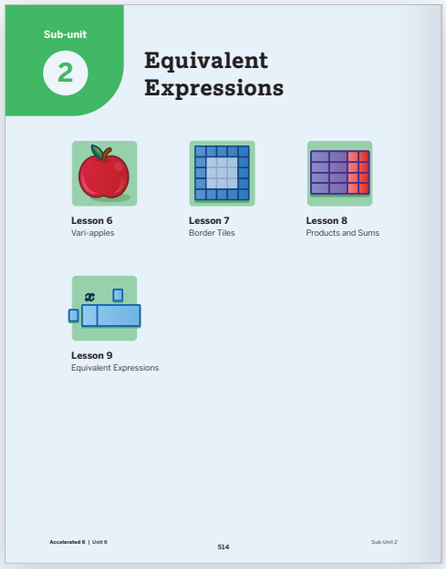
Sub-Unit 2

Equivalent Expressions

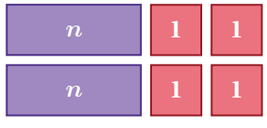
Sub-Unit 2 Goals

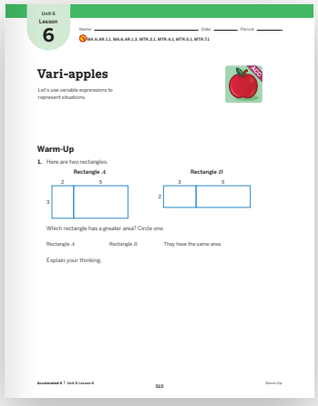
- **Goal:** Use properties to write equivalent expressions with variables.
- **Language Goal:** Analyze expressions to determine if they are equivalent.

In this sub-unit, students explore what **equivalent expressions** with variables are and use the distributive property to write equivalent expressions.



Math That Matters Most

Concept or Skill	For example . . .
Use properties to write equivalent expressions with variables.	Write an expression that is equivalent to $2x + 4(x + 3)$. ***6x + 12***
Analyze expressions to determine if they are equivalent.	Select all the expressions that are equivalent to $2n + 4$. Use the image if it helps with your thinking  ☑ $n + n + 1 + 1 + 1 + 1$ ☐ $2n + 2n + 4$ ☑ $(n + 2) + (n + 2)$ ☐ $(n + 4) + (n + 4)$ ☐ $(2n + 1) + (2n + 1)$


Unit 6
Lesson
6
Vari-apples
Let's use variable expressions to represent situations.
Warm-Up
1. Here are two rectangles.
Rectangle A
Rectangle B
Which rectangle has a greater area? Circle one.
Rectangle A
Rectangle B
They have the same area
Explain your thinking.

Vari-apples

Introduction to Variable Expressions

Let's use variable expressions to represent situations.

Learning Outcomes

Today's Goals

1. **Goal:** Write expressions involving variables to represent situations.
2. **Goal:** Evaluate algebraic expressions using substitution and order of operations.
3. **Language Goal:** Explain how variable expressions with multiplication or addition describe real-world situations. **(Writing, Speaking, and Listening)**

Students extend their understanding of multiplicative and additive relationships to represent mathematical and real-world situations with tables and *expressions*. They learn about using variable expressions as a way to express answers to questions like, "How can you describe the cost of any number of pounds of apples?" and practice writing these types of expressions. Students also evaluate algebraic expressions to solve problems. **(MTR.2.1, MTR.7.1)**

Coherence

Prior Learning

In Lessons 1–5, students used variables to write and solve equations of the form $x + p = q$ and $px = q$ in and out of context.

Future Learning

In Lesson 7, students will reason about what it means for expressions to be *equivalent*.

Connections

- Students use modeling and problems in context to develop understanding of variable expressions. (mathematical, real world)

FL B.E.S.T. Math Benchmarks

Addressing

MA.6.AR.1.1
Given a mathematical or real-world context, translate written descriptions into algebraic expressions and translate algebraic expressions into written descriptions.

MA.6.AR.1.3
Evaluate algebraic expressions using substitution and order of operations.

Mathematical Thinking and Reasoning Standards: MTR.2.1, MTR.4.1, MTR.5.1, MTR.7.1

ELA Expectations: ELA.K12.EE.1.1, ELA.K12.EE.4.1, ELA.K12.EE.6.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.5.AR.2.1	MA.7.AR.2.2
MA.5.AR.2.2	MA.7.NSO.1.1

Vocabulary

New Vocabulary

term

Review Vocabulary

expression

Lesson at a Glance

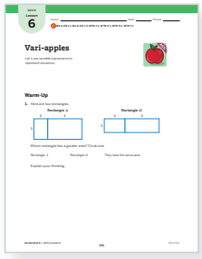
~ 45 min

MA.6.AR.1.1, MA.6.AR.1.3

Warm-Up

Pairs | 5 min

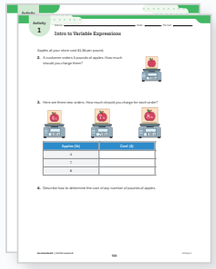
Students use the Think-Pair-Share routine to surface different strategies for calculating the area of rectangles made up of two parts.



Activity 1

Pairs | 14 min

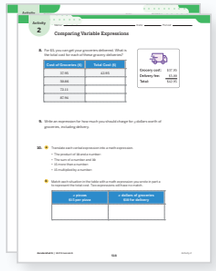
Students use variables to represent any number of some quantity (in this case, any number of pounds of various fruits).
Routine: Decide and Defend **(MTR.7.1)**



Activity 2

Pairs | 8 min

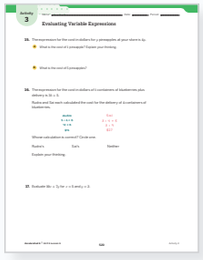
Students explore variable *expressions* that involve addition, and compare and contrast them to expressions involving multiplication. MLR1: Stronger and Clearer Each Time **(MTR.5.1)**



Activity 3

Pairs | 8 min

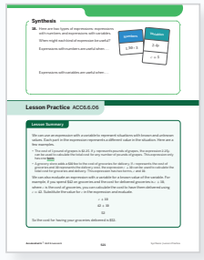
Students evaluate variable *expressions* using substitution and order of operations.
Routine: Decide and Defend



Synthesis

Whole Class | 5 min

Students synthesize their understanding of writing expressions to represent situations.
MLR2: Collect and Display



Show What You Know

Independent | 5 min

Students demonstrate their understanding by writing expressions to determine the cost of 6 pounds and x pounds of oranges.

Prep Checklist

This lesson includes:

- Student Edition
- Show What You Know
- Calculators (as needed)

Pairs | 5 min

Warm-Up

Purpose: Students use the Think-Pair-Share routine to surface different strategies for calculating the area of rectangles made up of two parts.

Short on time? Consider omitting the Warm-Up.

1 Launch

To surface prior knowledge, consider discussing what students remember about how to calculate the area of a rectangle.

Use the Think-Pair-Share routine to support student-to-student discourse. See the Routine Facilitation Guide for more information. **(MTR.4.1)**

Look for a variety of student strategies for calculating the area of each rectangle such as:

- Determining the area of each small rectangle, then calculating their sum.
- Adding the two measurements for the width and calculating the area of the whole rectangle at once.
- Comparing the rectangles on the right because the small rectangle on the left is the same size in both rectangles.

2 Connect

Invite students to share which rectangle has the greater area by polling the class.

Share student strategies. Encourage students to justify how they calculated the total area of each rectangle.

Unit 6
Lesson
6

Name: _____ Date: _____ Period: _____

MA.6.AR.1.1, MA.6.AR.1.3, MTR.2.1, MTR.4.1, MTR.5.1, MTR.7.1

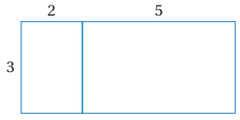
Vari-apples

Let's use variable expressions to represent situations.

Warm-Up

1. Here are two rectangles.

Rectangle A



Rectangle B

Which rectangle has a greater area? Circle one.

Rectangle A Rectangle B They have the same area

Explain your thinking.

Explanations vary.

- I added up the areas of the small rectangles. Rectangle A's area is $6 + 15 = 21$ square units. Rectangle B's area is $6 + 10 = 16$ square units.
- Rectangle A is a 3-by-7 rectangle and rectangle B is a 2-by-8 rectangle. $3 \cdot 7 = 21$ and $2 \cdot 8 = 16$.
- The small rectangle on the left is the same size in rectangles A and B, just turned. We only have to compare the rectangles on the right. A 3-by-5 rectangle is bigger than a 2-by-5 rectangle.

Accelerated 6 | Unit 6 Lesson 6 515 Warm-Up

Pairs | 14 min

Activity 1 Intro to Variable Expressions

Purpose: Students use variables to represent any number of some quantity (in this case, any number of pounds of various fruits). **(MTR.7.1)**

1 Launch

To surface connections, consider asking:

- "What is your favorite type of apple?"
- "How many apples do you think there are in 3 pounds of apples?" *Responses vary.* About 9–10 medium-sized apples.

2 Monitor

To support students getting started, consider asking, "What does the term *per* tell you?"

Select and sequence the various strategies students use and describe.

D Differentiation

Look for students who:	Respond to Student Thinking
Add 1.50 to the number of apples in each row.	**Support** Consider asking, "How can creating a double number line or drawing a picture help us make sense of the problem?"
Use a variety of strategies, such as: • Creating a double number line diagram. • Adding 1.50 until they get to the number of pounds of apples. • Multiplying the number of pounds by the unit rate. • Using the patterns in the numbers such as doubling 4 to 8 and adding the total for 3 pounds and 4 pounds to determine the cost of 7 pounds. **(MTR.5.1)** • Writing an equation or expression relating the pounds of apples to the total cost.	**Strengthen** Invite students to share and compare their strategies with their partner. Listen for and share these strategies and explanations to discuss during the Connect.

Pause to share the student explanations for determining the cost of any number of pounds of apples. Consider asking, "Why might it make sense to use multiplication or repeated addition here?" **(MTR.4.1, MTR.7.1)**

Activity 1 continued >

Activity 1

Name: Date: Period:

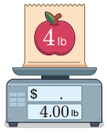
Intro to Variable Expressions

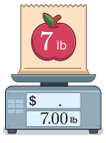
Apples at your store cost $1.50 per pound.

2. A customer orders 3 pounds of apples. How much should you charge them?

$4.50

3. Here are three new orders. How much should you charge for each order?





Apples (lb)	Cost ($)
4	6.00
7	10.50
8	12.00

4. Describe how to determine the cost of *any* number of pounds of apples.

Responses vary.

- Multiply the number of pounds you want by 1.50 to get the cost.
- Keep adding 1.50 until you get to the number of pounds you want.

Accelerated 6 | Unit 6 Lesson 6 516 Activity 1

Pairs | 14 min

Activity 1 Intro to Variable Expressions (continued)

Purpose: Students use variables to represent any number of some quantity (in this case, any number of pounds of various fruits). **(MTR.7.1)**

2 Monitor

Look for a variety of strategies for determining which *expression* represents the cost of p pounds of apples, such as:

- Substituting in values for p.
- Recognizing the multiplicative relationship between pounds and cost.
- Recognizing that adding 1.50 to p is not equivalent to multiplying p by 1.50. **(MTR.2.1)**

Invite students to write the expression in the missing blank of the table.

ELL **English Language Learners** Invite students to talk through their thinking with a partner before writing it down. **(Writing and Speaking)**

A **Accessibility: Conceptual Processing** To support abstract thinking, invite students to think of a concrete case first by asking, "How much would you pay for 1 pound of limes? 2 pounds of limes?"

Create an anchor chart with input from students of a variety of verbal expressions that each of the four operations represent.

3 Connect

Collect and share the distribution of responses.

Use the Decide and Defend routine to support students in making arguments and critiquing the reasoning of others. **(MTR.4.1)**

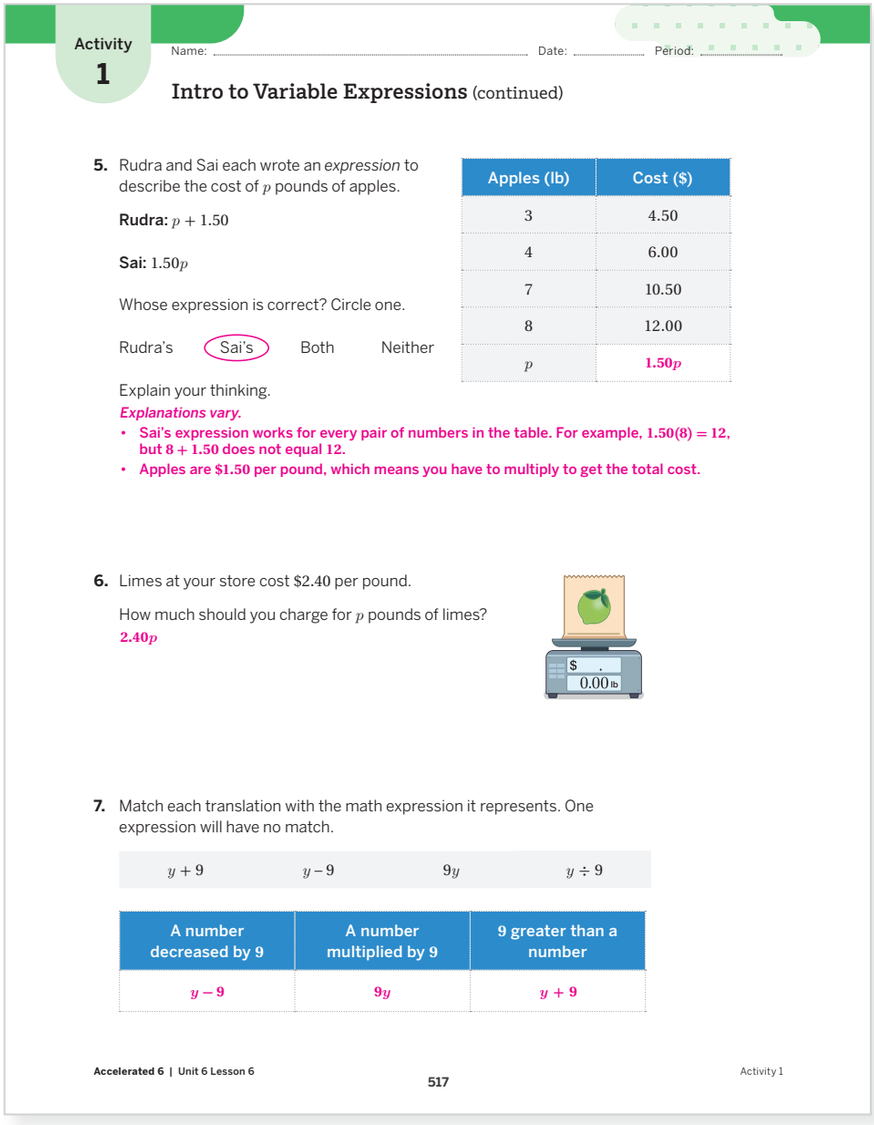
Invite students who made different choices to use reasoning to convince their classmates.

Consider asking:

- "What reasoning or evidence supports your claim?"
- "What evidence or language was most convincing to you?"

Invite students to share their expressions for the cost of limes. **(MTR.7.1)**

Consider asking, "How did you determine what operation to use in your expression?"



Activity 1

Name: Date: Period:

Intro to Variable Expressions (continued)

5. Rudra and Sai each wrote an *expression* to describe the cost of p pounds of apples.

Rudra: $p + 1.50$

Sai: $1.50p$

Whose expression is correct? Circle one.

Rudra's (Sai's) Both Neither

Explain your thinking.

Explanations vary.

- Sai's expression works for every pair of numbers in the table. For example, $1.50(8) = 12$, but $8 + 1.50$ does not equal 12.
- Apples are $1.50 per pound, which means you have to multiply to get the total cost.

Apples (lb)	Cost ($)
3	4.50
4	6.00
7	10.50
8	12.00
p	$1.50p$

6. Limes at your store cost $2.40 per pound.

How much should you charge for p pounds of limes?

$2.40p$

7. Match each translation with the math expression it represents. One expression will have no match.

$y + 9$ $y - 9$ $9y$ $y \div 9$

A number decreased by 9	A number multiplied by 9	9 greater than a number
$y - 9$	$9y$	$y + 9$

Accelerated 6 | Unit 6 Lesson 6 517 Activity 1

Pairs | 8 min

Activity 2 Comparing Variable Expressions

Purpose: Students explore variable *expressions* that involve addition, and compare and contrast them to expressions involving multiplication. **(MTR.5.1)**

1 Launch

To surface connections, consider asking, "Have you ever paid a fee for delivery?"

Consider asking, "Why would the total cost for $37.95 of groceries be $42.95?"

2 Monitor

Invite students to check their thinking for the table with a partner before moving on. **(MTR.4.1)**

A **Accessibility: Memory and Attention** Invite students to use their Student Edition, additional paper, or a calculator to support them with performing calculations.

To support students getting started, consider asking them to describe in words how they determined the total cost of each grocery delivery and how they could apply that for any original cost of groceries. **(MTR.7.1)**

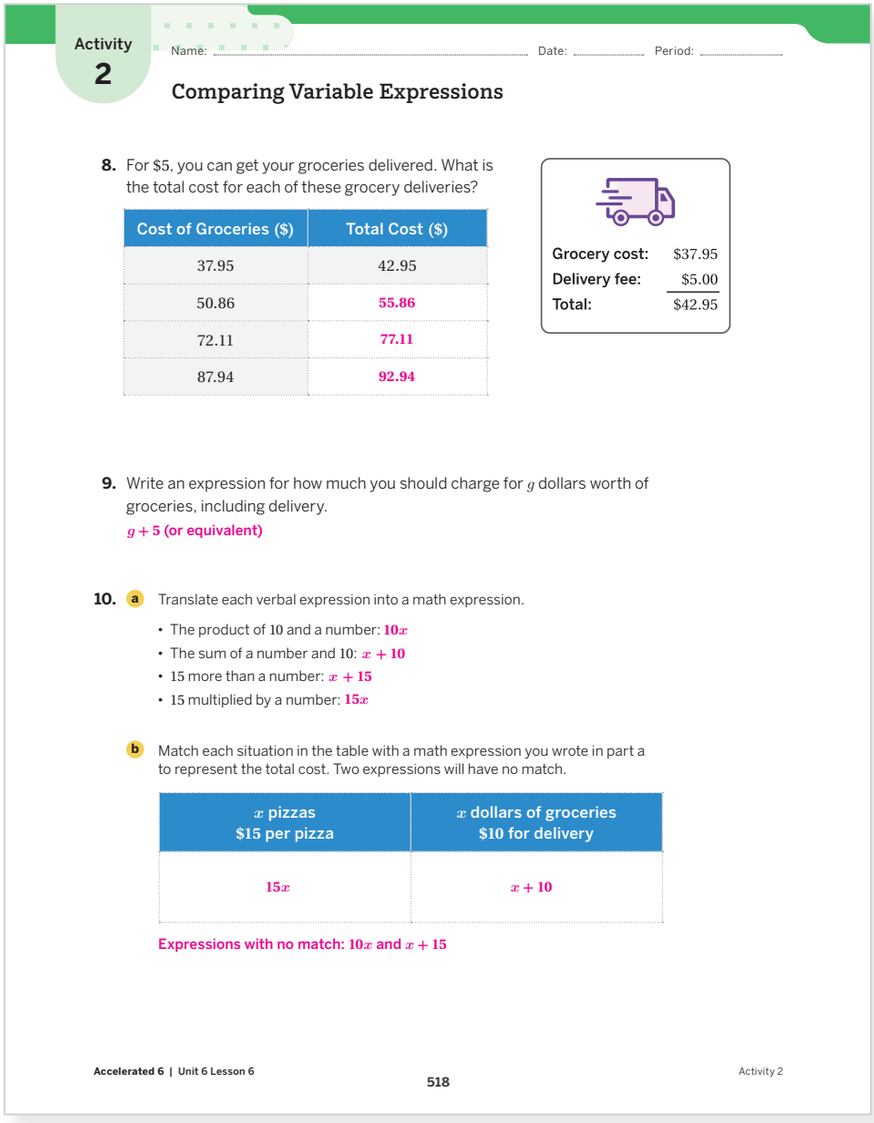
D **Differentiation:** (Support) Look for students who write an equation, like $g + 5 = c$, instead of an expression. Consider asking, "What is the difference between an expression and an equation?"

Encourage students to share their reasoning with a partner.

D **Differentiation**

Look for students who:	Respond to Student Thinking
Need support getting started.	Support Encourage them to refer to the anchor chart while reading each verbal expression, identifying key words that represent an operation. Support Encourage them to choose one scenario: pizzas or delivery. Consider asking, "Are there any expressions you can eliminate?"
Use a variety of strategies to determine the matching expression, like: • Determining the cost for specific values of x, then generalizing for any value. • Recognizing *per* as a multiplicative relationship.	Strengthen Invite students to share their strategies during the Connect.

Activity 2 continued >



Activity 2

Name: Date: Period:

Comparing Variable Expressions

8. For $5, you can get your groceries delivered. What is the total cost for each of these grocery deliveries?

Cost of Groceries ($)	Total Cost ($)
37.95	42.95
50.86	55.86
72.11	77.11
87.94	92.94

Grocery cost: $37.95
Delivery fee: $5.00
Total: $42.95

9. Write an expression for how much you should charge for g dollars worth of groceries, including delivery.

$g + 5$ (or equivalent)

10. a. Translate each verbal expression into a math expression.

- The product of 10 and a number: $10x$
- The sum of a number and 10: $x + 10$
- 15 more than a number: $x + 15$
- 15 multiplied by a number: $15x$

b. Match each situation in the table with a math expression you wrote in part a to represent the total cost. Two expressions will have no match.

x pizzas $15 per pizza	x dollars of groceries $10 for delivery
$15x$	$x + 10$

Expressions with no match: $10x$ and $x + 15$

Accelerated 6 | Unit 6 Lesson 6 518 Activity 2

Pairs | 8 min

Activity 2 Comparing Variable Expressions (continued)

Purpose: Students explore variable *expressions* that involve addition, and compare and contrast them to expressions involving multiplication. **(MTR.5.1)**

2 Monitor

MLR1: Stronger and Clearer Each Time Consider using this routine to support students in developing precise language. Invite students to meet with a partner, give feedback on each other's drafts, then use the feedback to write a second clearer draft. See the Routine Facilitation Guide for more information. **(MTR.4.1)**

English Language Learners Provide sentence frames to help students explain their thinking (e.g., I know that _____ matches this situation because _____.). **(Writing and Speaking)**

Select and sequence several student responses, including ones similar to the sample responses.

To support students getting started, consider asking:

- "What is the same about the two expressions? What is different?"
- "Why do you think $15x$ has one term while $15 + x$ has two?" **(MTR.5.1)**

Differentiation: You're invited to explore more. (Stretch) Invite students who would like to explore writing expressions further to complete this optional task. Encourage them to discuss their thinking with a partner.

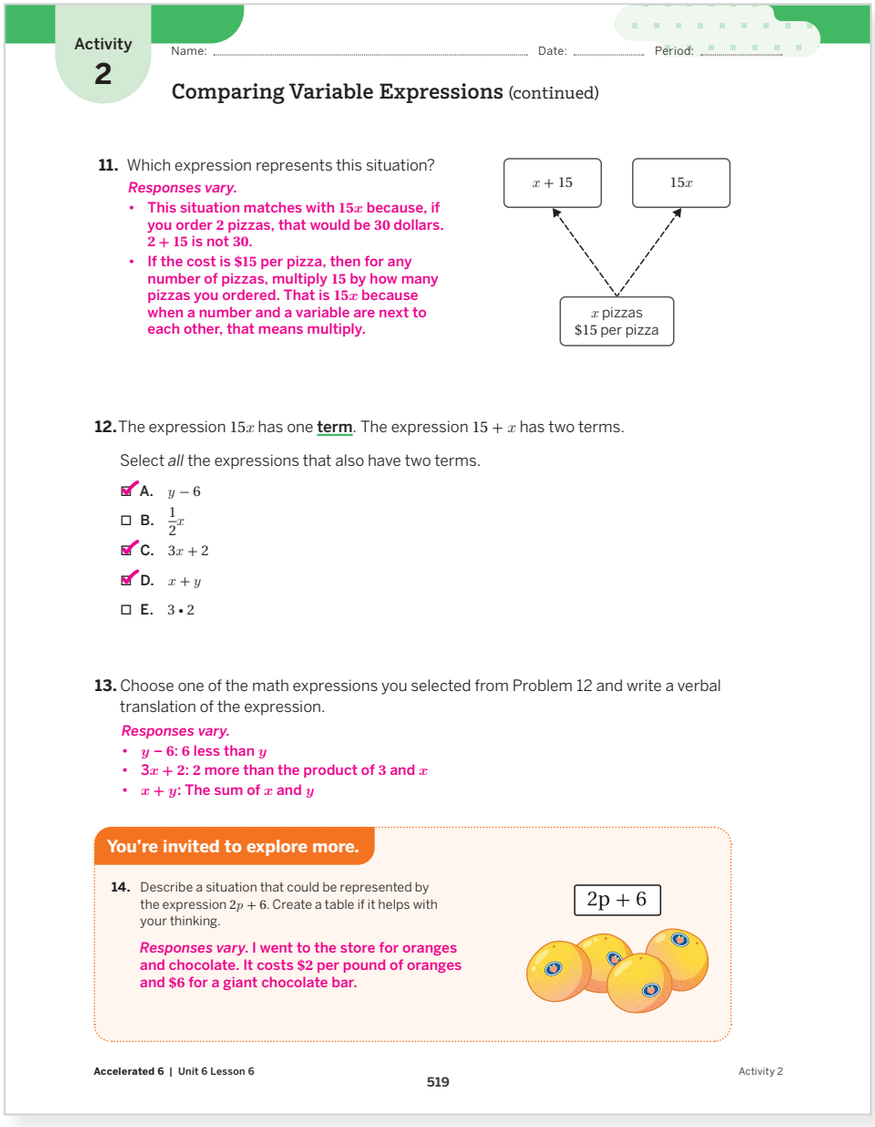
3 Connect

Invite students to share how they determined which expression matched each situation, including responses for why the total cost of pizza can be represented by $15x$.

To surface the Key Takeaway, consider asking, "How did you know which relationship used multiplication and which used addition?"

Create a class definition for **term** using students' ideas and language and the glossary. If it does not come up naturally, share that terms can be separated by addition or subtraction, but not multiplication or division.

Invite students to share their verbal translations. Consider asking, "Is there only one correct way to translate each expression?"



Activity 2

Name: Date: Period:

Comparing Variable Expressions (continued)

11. Which expression represents this situation?

Responses vary.

- This situation matches with $15x$ because, if you order 2 pizzas, that would be 30 dollars. $2 + 15$ is not 30.
- If the cost is \$15 per pizza, then for any number of pizzas, multiply 15 by how many pizzas you ordered. That is $15x$ because when a number and a variable are next to each other, that means multiply.

12. The expression $15x$ has one **term**. The expression $15 + x$ has two terms.

Select *all* the expressions that also have two terms.

- ☑ A. $y - 6$
- ☐ B. $\frac{1}{2}x$
- ☑ C. $3x + 2$
- ☑ D. $x + y$
- ☐ E. $3 \cdot 2$

13. Choose one of the math expressions you selected from Problem 12 and write a verbal translation of the expression.

Responses vary.

- $y - 6$: 6 less than y
- $3x + 2$: 2 more than the product of 3 and x
- $x + y$: The sum of x and y

You're invited to explore more.

14. Describe a situation that could be represented by the expression $2p + 6$. Create a table if it helps with your thinking.

Responses vary. I went to the store for oranges and chocolate. It costs \$2 per pound of oranges and \$6 for a giant chocolate bar.

Accelerated 6 | Unit 6 Lesson 6 519 Activity 2

Key Takeaway: *Expressions* with variables are used to model situations for any value. Whether an expression uses multiplication or addition depends on the context. One characteristic of expressions is the number of **terms** that they have.

Pairs | 8 min

Activity 3 Evaluating Variable Expressions

Purpose: Students evaluate variable *expressions* using substitution and order of operations.

1 Launch

To surface connections, consider asking, "What does the 4 represent in the expression?"

2 Monitor

Look for a variety of student thinking to share during the Connect.

Encourage students to discuss the meaning of each part of the expression in context. Consider asking, "How does the expression show the cost for blueberries as well as the delivery cost?" **(MTR.7.1)**

D **Differentiation:** (Support) Look for students who use addition before multiplication. Consider asking, "What are the two operations in this problem? Which operation should we evaluate first?"

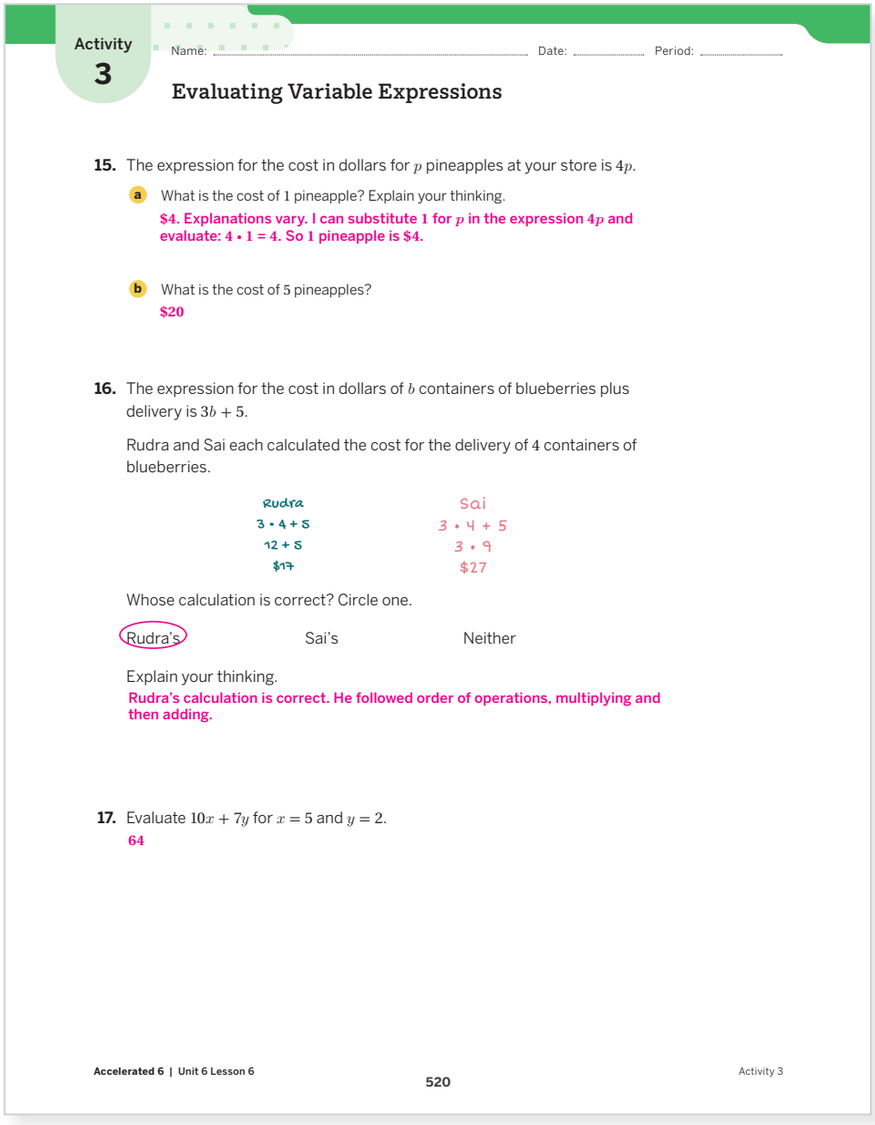
3 Connect

Invite students to share their thinking. Consider asking, "What strategy did you use to help determine the cost of 5 pineapples?"

Use the Decide and Defend routine to support students in making arguments and critiquing the reasoning of others. **(MTR.4.1)**

Invite students to share how they evaluated the expression, making connections between the expressions and the values of the variables and order of operations. **(MTR.5.1)**

If time allows, challenge students to create their own problems that involve evaluating variable expressions for their classmates to solve.



Activity 3

Name: Date: Period:

Evaluating Variable Expressions

15. The expression for the cost in dollars for p pineapples at your store is $4p$.

a. What is the cost of 1 pineapple? Explain your thinking.

$4. Explanations vary. I can substitute 1 for p in the expression $4p$ and evaluate: $4 \cdot 1 = 4$. So 1 pineapple is $4.

b. What is the cost of 5 pineapples?

$20

16. The expression for the cost in dollars of b containers of blueberries plus delivery is $3b + 5$.

Rudra and Sai each calculated the cost for the delivery of 4 containers of blueberries.

Rudra	Sai
$3 \cdot 4 + 5$	$3 \cdot 4 + 5$
$12 + 5$	$3 \cdot 9$
$17	$27

Whose calculation is correct? Circle one.

Rudra's Sai's Neither

Explain your thinking.

Rudra's calculation is correct. He followed order of operations, multiplying and then adding.

17. Evaluate $10x + 7y$ for $x = 5$ and $y = 2$.

64

Accelerated 6 | Unit 6 Lesson 6 520 Activity 3

Key Takeaway: You can evaluate a variable expression for a given value of the variable using substitution and order of operations.

Whole Class | 5 min

Synthesis

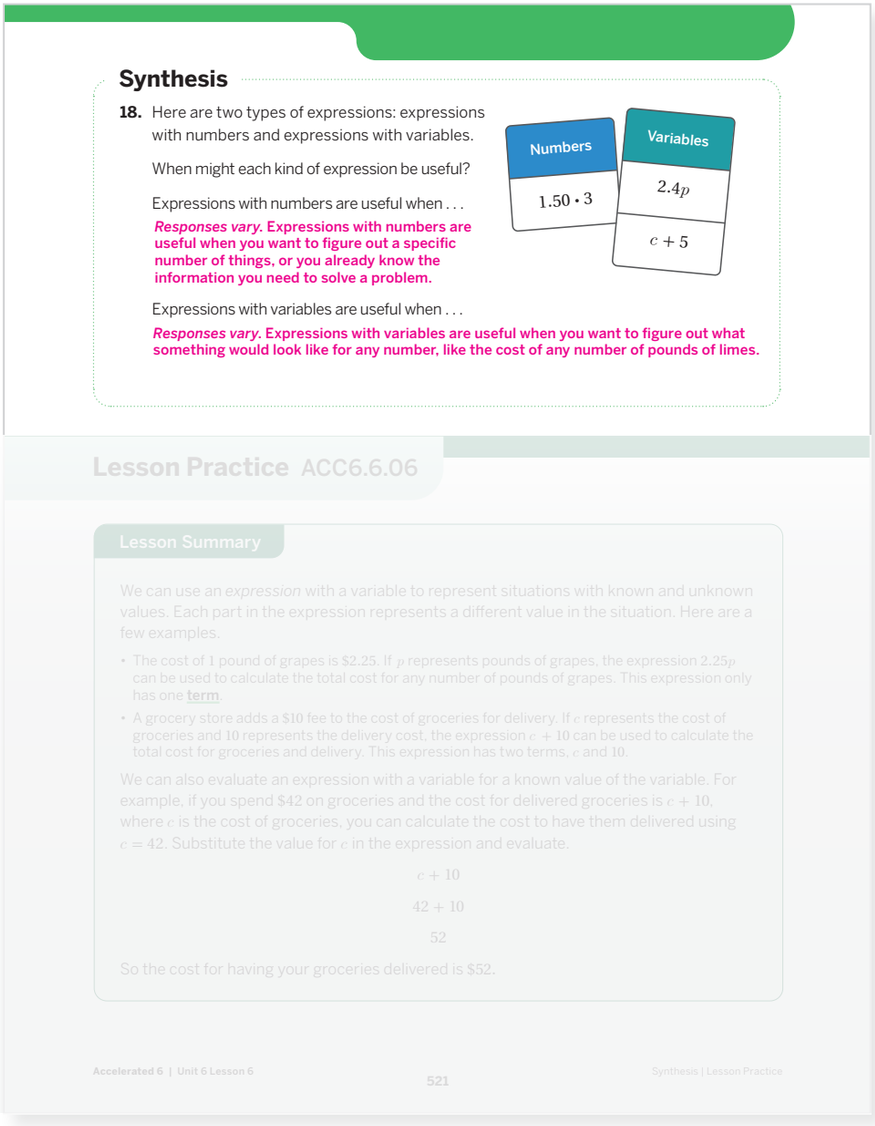
Purpose: Students synthesize their understanding of writing expressions to represent situations.

Invite students to respond independently, and then share their thinking with a partner.

Look for and share a variety of ideas, such as:
- Using expressions with numbers to determine exact values.
- Using expressions with variables to represent the scenario for any value.

MLR **MLR2: Collect and Display** Use this routine to formalize the definition of *term* using students' language. Display any new language, examples, or diagrams publicly for students to reference later.

Note: If time allows, invite students to revise their previous thinking based on the discussion.



Synthesis

18. Here are two types of expressions: expressions with numbers and expressions with variables.

When might each kind of expression be useful?

Expressions with numbers are useful when . . .

Responses vary. **Expressions with numbers are useful when you want to figure out a specific number of things, or you already know the information you need to solve a problem.**

Numbers	Variables
$1.50 \cdot 3$	$2.4p$
	$c + 5$

Expressions with variables are useful when . . .

Responses vary. **Expressions with variables are useful when you want to figure out what something would look like for any number, like the cost of any number of pounds of limes.**

Lesson Practice ACC6.6.06

Lesson Summary

We can use an *expression* with a variable to represent situations with known and unknown values. Each part in the expression represents a different value in the situation. Here are a few examples.

- The cost of 1 pound of grapes is \$2.25. If p represents pounds of grapes, the expression $2.25p$ can be used to calculate the total cost for any number of pounds of grapes. This expression only has one term.
- A grocery store adds a \$10 fee to the cost of groceries for delivery. If c represents the cost of groceries and 10 represents the delivery cost, the expression $c + 10$ can be used to calculate the total cost for groceries and delivery. This expression has two terms, c and 10.

We can also evaluate an expression with a variable for a known value of the variable. For example, if you spend \$42 on groceries and the cost for delivered groceries is $c + 10$, where c is the cost of groceries, you can calculate the cost to have them delivered using $c = 42$. Substitute the value for c in the expression and evaluate.

$$c + 10$$
$$42 + 10$$
$$52$$

So the cost for having your groceries delivered is \$52.

Accelerated 6 | Unit 6 Lesson 6 521 Synthesis | Lesson Practice

Lesson Takeaway: Variable *expressions* are useful when representing a situation with an unknown quantity. You can then use any given value for the variable to evaluate the expression using substitution and order of operations.

Independent | 5 min

Show What You Know

Purpose: Students demonstrate their understanding by writing expressions to determine the cost of 6 pounds and x pounds of oranges.

Today's Goals

1. **Goal:** Write expressions involving variables to represent situations.
2. **Goal:** Evaluate algebraic expressions using substitution and order of operations.
3. **Language Goal:** Explain how variable expressions with multiplication or addition describe real-world situations. **(Writing, Speaking, and Listening)**

Show What You Know (Assessment Resources)

Name: _____ Date: _____ Period: _____

Show What You Know 6.06

Oranges cost $2 per pound.

a. What is the cost of x pounds of oranges?

A. 2
B. $x + 2$
C. $2x$
D. $x + 4$

b. What is the cost of 6 pounds of oranges?

$12

D Differentiation Use after Lesson 6

Support

Provide targeted intervention for students by using these resources.

If students need support representing real-world scenarios with variable expressions:

» **Respond to Student Thinking:**

- Revisiting **Lesson 3, Activity 2** (*Hanging Around*) as a class and inviting students to share strategies for writing equations that model situations.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite them to explore the **Sub-Unit 2 Extension Activities**.
- Invite students to complete the **You're invited to explore more** task.

Professional Learning

Take a moment to reflect on this lesson. In this lesson, students connected their previous understanding of additive and multiplicative relationships to write and evaluate algebraic expressions. How will that work support their understanding of equivalent expressions in Lesson 7?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

18. Here are two types of expressions: expressions with numbers and expressions with variables.

When might each kind of expression be useful?

Expressions with numbers are useful when . . .

Responses vary. Expressions with numbers are useful when you want to figure out a specific number of things, or you already know the information you need to solve a problem.

Expressions with variables are useful when . . .

Responses vary. Expressions with variables are useful when you want to figure out what something would look like for any number, like the cost of any number of pounds of limes.

Lesson Practice ACC6.6.06

Lesson Summary

We can use an *expression* with a variable to represent situations with known and unknown values. Each part in the expression represents a different value in the situation. Here are a few examples.

- The cost of 1 pound of grapes is \$2.25. If p represents pounds of grapes, the expression $2.25p$ can be used to calculate the total cost for any number of pounds of grapes. This expression only has one **term**.
- A grocery store adds a \$10 fee to the cost of groceries for delivery. If c represents the cost of groceries and 10 represents the delivery cost, the expression $c + 10$ can be used to calculate the total cost for groceries and delivery. This expression has two terms, c and 10.

We can also evaluate an expression with a variable for a known value of the variable. For example, if you spend \$42 on groceries and the cost for delivered groceries is $c + 10$, where c is the cost of groceries, you can calculate the cost to have them delivered using $c = 42$. Substitute the value for c in the expression and evaluate.

$$c + 10$$
$$42 + 10$$
$$52$$

So the cost for having your groceries delivered is \$52.

Lesson Practice ACC6.6.06

Name: _____ Date: _____ Period: _____

Problems 1–3: Oranges cost \$1.25 per pound. How much would it cost to buy:

1. 2 pounds of oranges?
\$2.50

2. 5 pounds of oranges?
\$6.25

3. x pounds of oranges?
$1.25x$ dollars

4. You need red and blue ribbon for a craft project. The instructions say that the red ribbon should be 7 inches longer than the blue ribbon.

Complete the table to show how long the red ribbon should be for different lengths of blue ribbon.

Blue Ribbon (in.)	Red Ribbon (in.)
10	**17**
27	**34**
x	**$x + 7$**

Problems 5–6: The variable s represents the number of students in one class in your school.

5. What does $\frac{1}{2}s$ represent?
$\frac{1}{2}s$ represents half of the number of students in one class.

6. What does $s + 1$ represent?
$s + 1$ represents all of the students in a class, plus 1 extra student.

Problems 7–9: Evaluate the expression $3m + 5$ for each value of m.

Example
$m = 7$
$3(7) + 5 = 26$

7. $m = 8$
$3(8) + 5 = 29$

8. $m = 12$
$3(12) + 5 = 41$

9. $m = 15$
$3(15) + 5 = 50$

Lesson Practice ACC6.6.06

Name: _____ Date: _____ Period: _____

10. Choose any value for a.

$a =$ ☐

Use your value to complete the puzzle.

***Responses vary depending on the value of the variable students choose.* The expressions for each response are shown.**

$3a$	+	4	=	$3a + 4$				$10a + 4$
−				+				−
a		$4a$	+	$4a$	−	$3a$	=	$5a$
−		+		=				=
a		$3a$		$7a + 4$	−	$2a$	=	$5a + 4$
−		=				+		
a	+	$6a$	−	$3a$	=	$4a$		
=		+				=		
0		a		a	+	$6a$	=	$7a$

FAST Practice

11. 35 riders are on a bus, and n riders get off at the same stop. In this scenario, what does the expression $35 - n$ represent?

$35 - n$ represents the number of riders that [A. get off/B. are still on] the bus after n riders [A. get on/B. get off] the bus at the same stop.

Spiral Review

12. LaShawn's class raised \$500 for a fundraiser. They used 10% of the money to cover the cost of materials, saved 20% for the next fundraising project, and donated the rest. How much money did LaShawn's class donate?
\$350

Explain your thinking.

***Explanations vary.* LaShawn's class used 30% of the money on materials and savings, so they had 70% left for a donation. $500 \cdot 0.70 = 350$.**

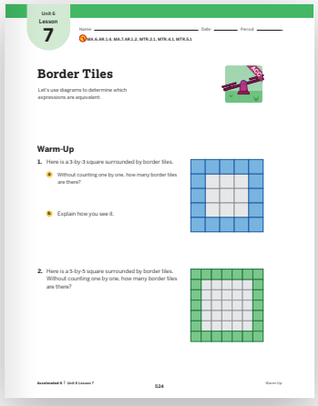
13. A garbage bin can hold 50 gallons of waste.

Complete the table to show what percent of the bin would be filled for different amounts of waste.

Waste (gal)	Percent Filled (%)
5	**10**
30	**60**
45	**90**

Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1–4	2	MA.6.AR.1.1, MA.6.AR.1.3
	5–6	2	MA.6.AR.1.1
	7–9	2	MA.6.AR.1.3
	10	3	MA.6.AR.1.3
FAST Practice	11	2	MA.6.AR.1.1
Spiral Review			
	12–13	2	MA.6.AR.3.4


Unit 6
Lesson 7

Name _____ Date _____ Period _____

MA.6.AR.1.4, MA.7.AR.1.2, MTR.2.1, MTR.4.1, MTR.5.1

Border Tiles

Let's use diagrams to determine which expressions are equivalent.

Warm-Up

1. Here is a 3-by-3 square surrounded by border tiles.
   a. Without counting one by one, how many border tiles are there?
   b. Explain how you see it.

2. Here is a 5-by-5 square surrounded by border tiles. Without counting one by one, how many border tiles are there?

Accelerated 6 | Unit 6 Lesson 7 524 Warm-Up

Border Tiles

Equivalent Expressions

Let's use diagrams to determine which expressions are equivalent.

Learning Outcomes

Today's Goals

1. **Goal:** Determine whether two expressions are equivalent.
2. **Language Goal:** Explain what it means for two expressions to be equivalent. **(Reading, Writing, Speaking, and Listening)**

Students are introduced to the concept of **equivalent expressions** in the context of the border tiles. They describe the number of border tiles for squares of different sizes, then use the structure of the border tiles to generalize to a square of any size. They analyze a variety of strategies to determine if given expressions are equivalent. **(MTR.2.1, MTR.5.1)**

Coherence

Prior Learning

In Lesson 6, students used expressions with variables to express answers to questions like, "How can you describe the cost of any number of pounds of apples?"

Future Learning

In Lessons 8–9, students will use the distributive property and order of operations to write equivalent expressions.

Connections

- Students use models to understand the characteristics of equivalent expressions. (mathematical)

FL B.E.S.T. Math Benchmarks

Addressing

MA.6.AR.1.4
Apply the properties of operations to generate equivalent algebraic expressions with integer coefficients.

MA.7.AR.1.2
Determine whether two linear expressions are equivalent.

Mathematical Thinking and Reasoning Standards: MTR.2.1, MTR.4.1, MTR.5.1

ELA Expectations: ELA.K12.EE.5.1, ELA.K12.EE.6.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.5.AR.2.2	MA.7.AR.1.1 MA.8.AR.1.1

Vocabulary

New Vocabulary

equivalent expressions

Lesson at a Glance

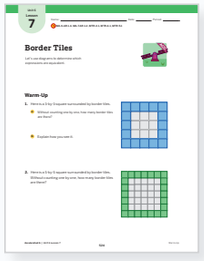
~ 45 min

MA.6.AR.1.4, MA.7.AR.1.2

Warm-Up

Pairs | 5 min

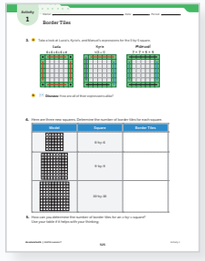
Students are introduced to the context of the lesson by determining the number of border tiles for 3-by-3 and 5-by-5 squares.



Activity 1

Pairs | 10 min

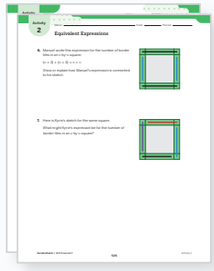
Students explore, compare, and describe various strategies for determining the number of border tiles for an n-by-n square.
MLR7: Compare and Connect



Activity 2

Pairs | 10 min

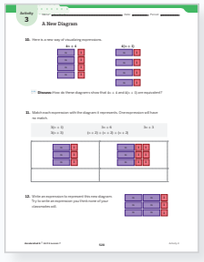
Students make sense of fictional students' expressions for the number of border tiles of an n-by-n square, and come to recognize them as **equivalent expressions**. **(MTR.5.1)**



Activity 3

Pairs | 10 min

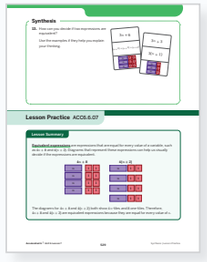
Students apply what they learned about equivalent expressions to new rectangular diagrams made of n-tiles and 1-tiles.



Synthesis

Whole Class | 5 min

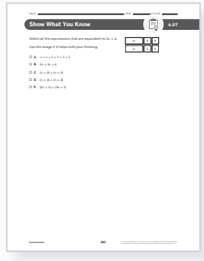
Students synthesize their understanding of equivalent expressions. MLR2: Collect and Display



Show What You Know

Independent | 5 min

Students demonstrate their understanding by selecting all expressions that are equivalent to a given expression.



Prep Checklist

This lesson includes:

- Student Edition
- Show What You Know

- Algebra tiles (as needed)
- Graph paper (as needed)
- Highlighters (as needed)

Pairs | 5 min

Warm-Up

Purpose: Students are introduced to the context of the lesson by determining the number of border tiles for 3-by-3 and 5-by-5 squares.

1 Launch

Invite students to read the prompt out loud and discuss what they think "without counting one by one" means.

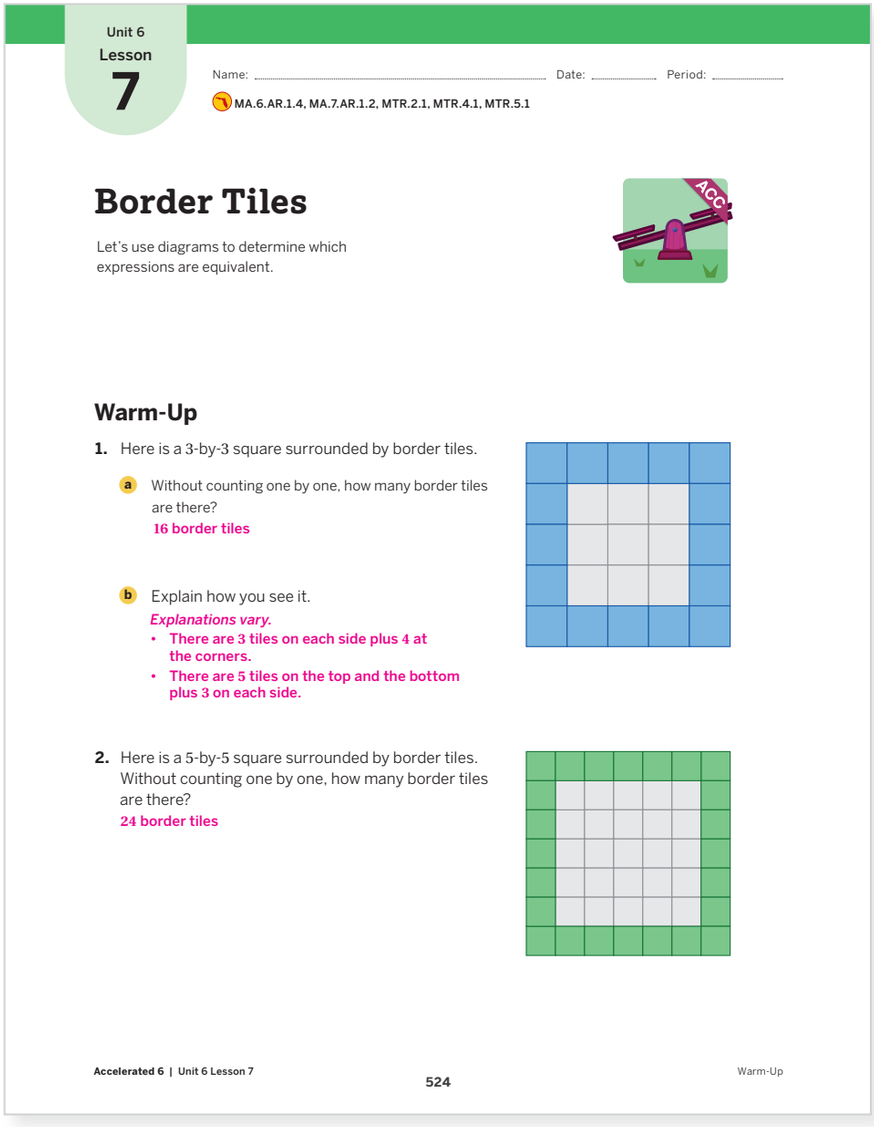
Encourage students to use their Student Edition to annotate their counting strategies.

A **Accessibility: Visual-Spatial Processing** To support students in making sense of images, invite them to use graph paper to draw their own diagrams or highlighters to highlight the given diagram in the Student Edition.

Note: If time allows, invite students to look at a classmate's sketch to see how it compares to their own sketch.

2 Connect

Invite students to share their strategies for determining the number of border tiles for the 3-by-3 and 5-by-5 squares.



Unit 6
Lesson
7

Name: ..... Date: ..... Period: .....

MA.6.AR.1.4, MA.7.AR.1.2, MTR.2.1, MTR.4.1, MTR.5.1

Border Tiles

Let's use diagrams to determine which expressions are equivalent.

Warm-Up

1. Here is a 3-by-3 square surrounded by border tiles.

   a. Without counting one by one, how many border tiles are there?
   16 border tiles

   b. Explain how you see it.
   Explanations vary.
   - There are 3 tiles on each side plus 4 at the corners.
   - There are 5 tiles on the top and the bottom plus 3 on each side.

2. Here is a 5-by-5 square surrounded by border tiles. Without counting one by one, how many border tiles are there?
   24 border tiles

Accelerated 6 | Unit 6 Lesson 7 524 Warm-Up

Pairs | 10 min

Activity 1 Border Tiles

Purpose: Students explore, compare, and describe various strategies for determining the number of border tiles for an n-by-n square.

Short on time? Consider sharing the answers for Problem 4, then have students complete Problem 5.

1 Launch

Invite students to discuss with their partner how the three strategies are alike. **(MTR.4.1)**

2 Monitor

Listen for connections students make between the expressions and the annotated diagrams. **(MTR.5.1)**

MLR **MLR7: Compare and Connect** Invite students to analyze and compare each strategy, and then connect to their strategy.

Consider asking:
- "What did each student do?"
- "Where do we see the same information in Lucia's expression? Kyrie's? Manuel's?"
- "How does each strategy compare to your own?"
- "How are these strategies like strategies we've used before?"

Invite students to check their thinking with a partner.

Look for students who write expressions to calculate the number of border tiles. These may be helpful as students describe their thinking in the next problem.

Select and sequence a variety of responses.

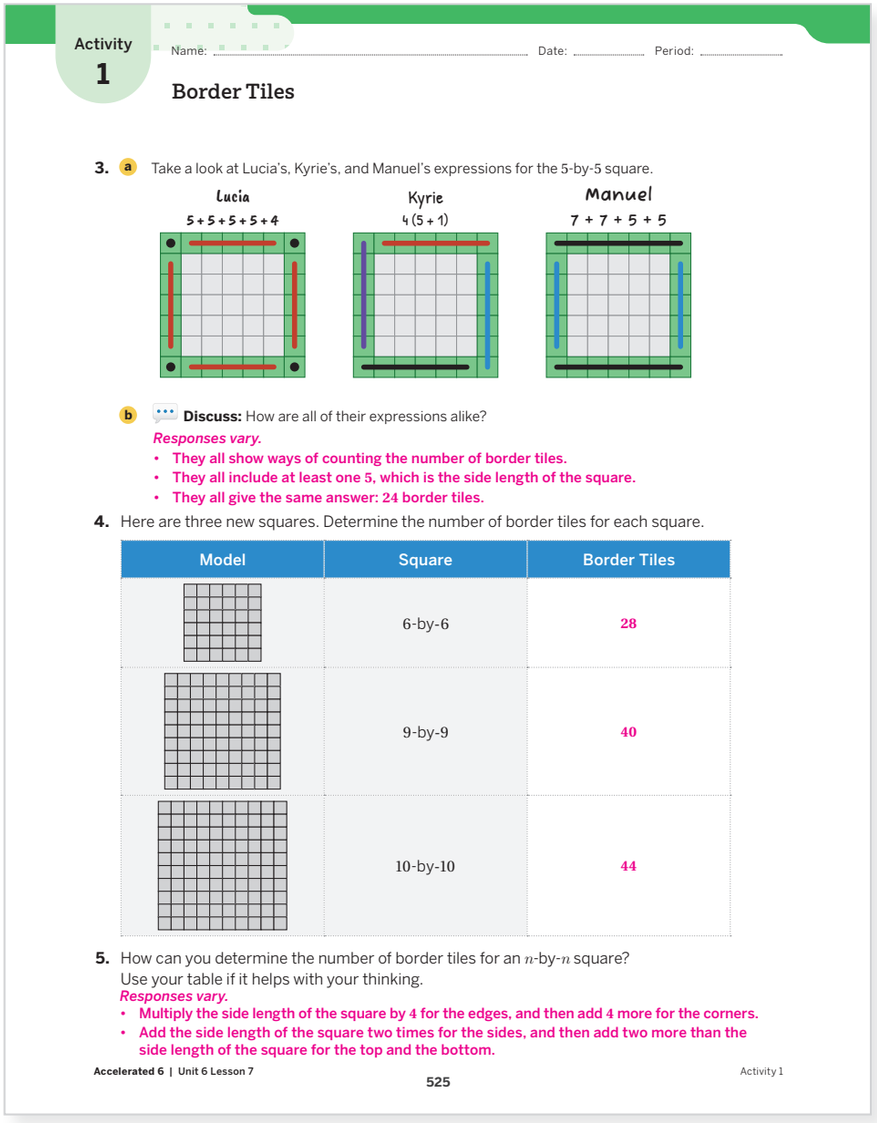
D Differentiation

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Invite students to look for patterns in Lucia's table for determining the number of border tiles.
Wonder what an n-by-n square is.	**Support** Consider sharing that an n-by-n square is one way to say a square of any size.
Write descriptions that match Manuel's or Kyrie's expressions.	**Strengthen** Invite students to share their strategies during the Connect.

3 Connect

Discuss student strategies for determining the number of border tiles for an n-by-n square. **(MTR.4.1)**

Note: Students will write expressions to represent these strategies in the next activity.



Activity 1

Name: Date: Period:

Border Tiles

3. a Take a look at Lucia's, Kyrie's, and Manuel's expressions for the 5-by-5 square.

Lucia: $5+5+5+5+4$ Kyrie: $4(5+1)$ Manuel: $7+7+5+5$

b **Discuss:** How are all of their expressions alike?

Responses vary.
- They all show ways of counting the number of border tiles.
- They all include at least one 5, which is the side length of the square.
- They all give the same answer: 24 border tiles.

4. Here are three new squares. Determine the number of border tiles for each square.

Model	Square	Border Tiles
	6-by-6	28
	9-by-9	40
	10-by-10	44

5. How can you determine the number of border tiles for an n-by-n square? Use your table if it helps with your thinking.

Responses vary.
- Multiply the side length of the square by 4 for the edges, and then add 4 more for the corners.
- Add the side length of the square two times for the sides, and then add two more than the side length of the square for the top and the bottom.

Accelerated 6 | Unit 6 Lesson 7 525 Activity 1

Pairs | 10 min

Activity 2 Equivalent Expressions

Purpose: Students make sense of fictional students' expressions for the number of border tiles of an n-by-n square, and come to recognize them as **equivalent expressions**. **(MTR.5.1)**

1 Launch

To surface connections, consider asking, "How does Manuel's expression connect to his work earlier in the lesson?" **(MTR.5.1)**

2 Monitor

To support students getting started, invite them to annotate the side lengths with the variable n. Encourage them to look back at Manuel's earlier work, and consider asking, "How are the lengths of 5 and 7 related to n and $n + 2$?" **(MTR.2.1)**

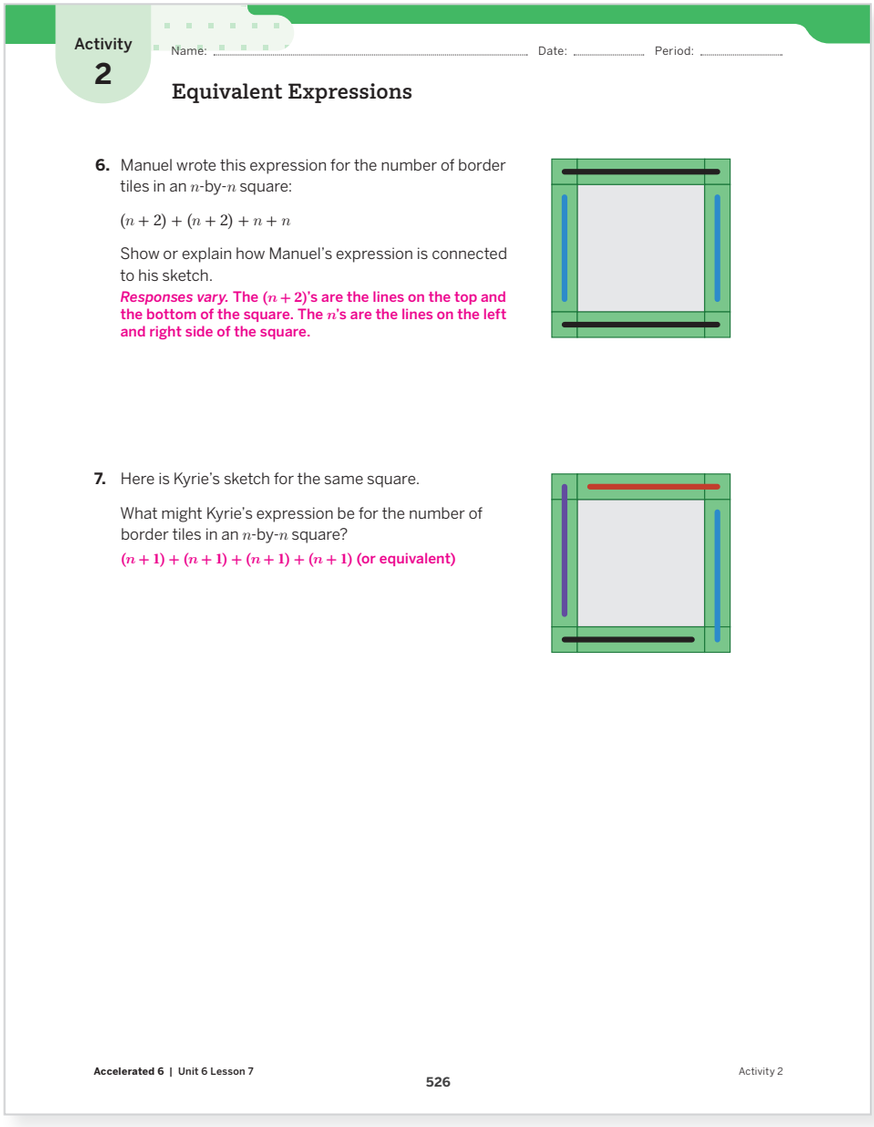
Invite students to read others' responses and decide if others' descriptions were similar to or different from their own.

Consider asking, "How is Kyrie's sketch alike and different from Manuel's?"

To support students getting started, invite them to write an expression for the line at the top of the diagram.

Note: It is okay to lack consensus about Kyrie's expression at this stage. The activity will build toward consensus in the next problem when students consider why Kyrie's and Manuel's expressions are equivalent.

Activity 2 continued >



Activity 2

Name: Date: Period:

Equivalent Expressions

6. Manuel wrote this expression for the number of border tiles in an n-by-n square:

$(n + 2) + (n + 2) + n + n$

Show or explain how Manuel's expression is connected to his sketch.

Responses vary. The $(n + 2)$'s are the lines on the top and the bottom of the square. The n's are the lines on the left and right side of the square.

7. Here is Kyrie's sketch for the same square.

What might Kyrie's expression be for the number of border tiles in an n-by-n square?

$(n + 1) + (n + 1) + (n + 1) + (n + 1)$ (or equivalent)

Accelerated 6 | Unit 6 Lesson 7 526 Activity 2

Pairs | 10 min

Activity 2 Equivalent Expressions (continued)

Purpose: Students make sense of fictional students' expressions for the number of border tiles of an n-by-n square, and come to recognize them as **equivalent expressions**. **(MTR.5.1)**

2 Monitor

A **Accessibility: Conceptual Processing**
To assist students in recognizing the connections between new problems and prior work, consider inviting students to connect Kyrie's and Manuel's expressions to their sketches before responding. **(MTR.5.1)**

Listen for students' thinking about equivalent expressions.

D **Differentiation**

Look for students who:	Respond to Student Thinking
Need support getting started evaluating each expression.	**Support** Encourage them to read the text aloud and to paraphrase it in their own words.
Get two different sums when they evaluate the expressions.	**Support** Consider asking, "Does it make sense for the number of border tiles to be different in each expression?"
Write a variety of explanations for why the expressions are equivalent, like: • Describing that the expressions have the same value when $n = 8$. • Checking additional values of n to make sure that the expressions are still equivalent. • Recognizing that both expressions have four n's and four 1's so they are equivalent. **(MTR.5.1)**	**Strengthen** Invite students to share their explanations during the Connect.

Look for students who try each option. Consider asking, "Now that you know which expression is equivalent, why does it make sense?"

3 Connect

Invite students to share why the expressions are equivalent.

Create a class definition of the term **equivalent expressions** using students' language and the glossary.

Invite students to share their responses.

Consider asking:
- "What would $4n + 4$ look like on the diagram? Where would the $4n$ be? What about the $+ 4$?"
- "How can we tell that $4n + 1$ is not equivalent to $4(n + 1)$? Why might someone think that it is?"

Activity 2

Name: Date: Period:

Equivalent Expressions (continued)

8. **Equivalent expressions** are expressions that are equal for every value of a variable.

Here are two expressions.

Kyrie: $4(n + 1)$

Manuel: $(n + 2) + (n + 2) + n + n$

a. Use each expression to calculate the number of border tiles when $n = 8$.

Kyrie	Manuel
$4(n + 1)$	$(n + 2) + (n + 2) + n + n$
$4(8 + 1) = 36$	$(8 + 2) + (8 + 2) + 8 + 8 = 36$
36 tiles	36 tiles

b. **Discuss:** How can you tell that Kyrie's and Manuel's expressions are equivalent?

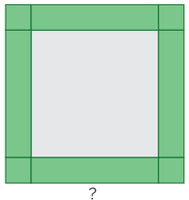
Responses vary. Kyrie's and Manuel's expressions are equivalent because when I replace n with a side length of the square I get the same number of border tiles in each expression.

9. Which expression is also equivalent to Kyrie's and Manuel's expressions?

A. $(n + 1)(n + 1)$

B. $4n + 1$

C. $4n + 4$



Accelerated 6 | Unit 6 Lesson 7 527 Activity 2

Key Takeaway: **Equivalent expressions** are equal for every value of a variable. Evaluating different values of a variable can be helpful but is not enough to show two expressions are equivalent. This comes from showing that both expressions represent the same diagram, in this case the number of border tiles on an n-by-n square.

Pairs | 10 min

Activity 3 A New Diagram

Purpose: Students apply what they learned about equivalent expressions to new rectangular diagrams made of n-tiles and 1-tiles.

Short on time? Consider discussing Problem 10 as a class.

1 Launch

To surface connections, consider asking, "How are these expressions related to the expressions we've seen so far in this lesson?"

2 Monitor

Accessibility: Visual-Spatial Processing Invite students to manipulate and rearrange algebra tiles to support interpreting visual representations.

Listen for different ways students use this new diagram to show that two expressions are equivalent.

English Language Learners Encourage students to paraphrase each other's ideas to help them make connections and incorporate vocabulary from the unit. **(Speaking and Listening)**

Encourage students to share their reasoning with a partner and work together to reach an agreement about how to match diagrams and expressions. **(MTR.2.1)**

Note: If time allows, consider asking pairs to compare and justify their matches and make revisions based on their conversation. **(MTR.4.1)**

To support students getting started, consider asking, "How can your work from the previous problem help you here?"

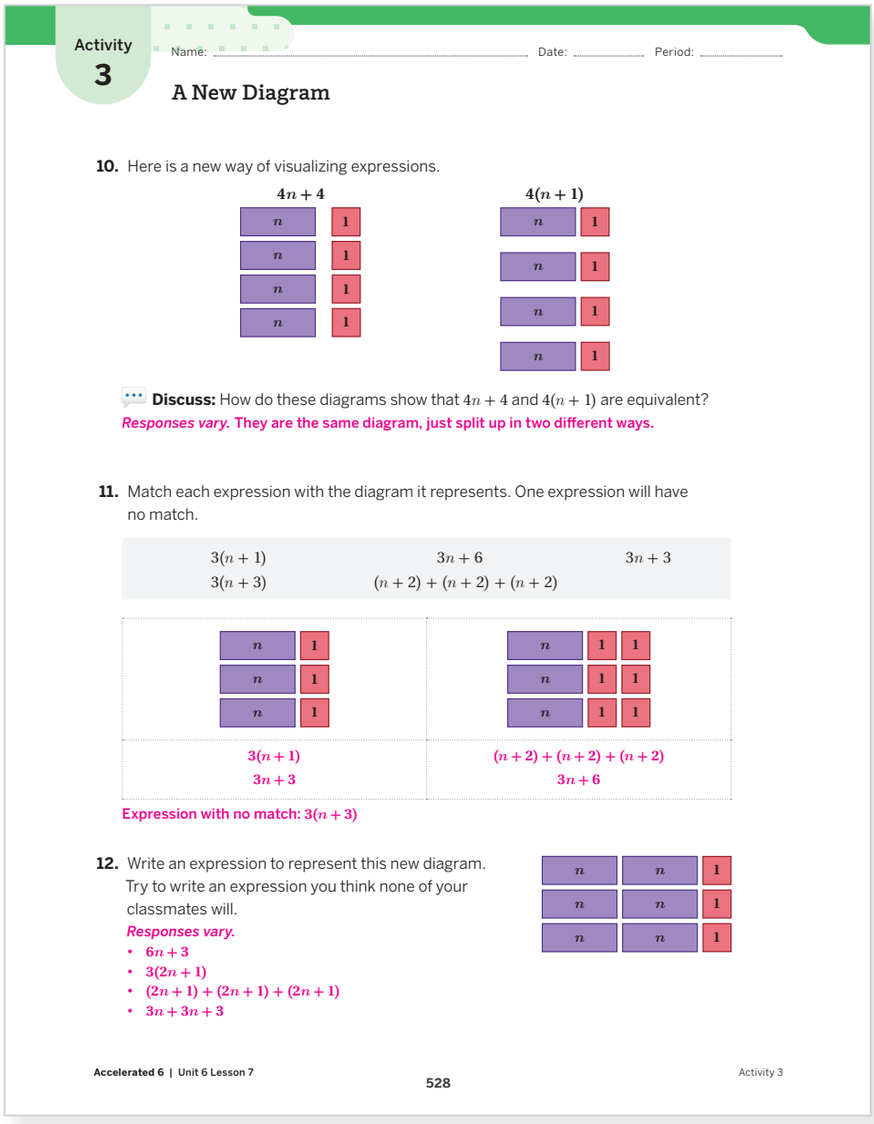
Look for a variety of expressions, including ones similar to those in the sample responses.

3 Connect

Share examples of student expressions, and invite students to justify why each expression matches the diagram. **(MTR.2.1, MTR.5.1)**

To surface connections, consider asking:

- "How do you know all of these are equivalent expressions?"
- "What is another way to determine if two expressions are equivalent without drawing diagrams?"



Activity 3

Name: Date: Period:

A New Diagram

10. Here is a new way of visualizing expressions.

$4n + 4$ $\quad$ $4(n + 1)$

Discuss: How do these diagrams show that $4n + 4$ and $4(n + 1)$ are equivalent?

Responses vary. They are the same diagram, just split up in two different ways.

11. Match each expression with the diagram it represents. One expression will have no match.

$3(n + 1)$	$3n + 6$	$3n + 3$
$3(n + 3)$	$(n + 2) + (n + 2) + (n + 2)$	

$3(n + 1)$ $3n + 3$	$(n + 2) + (n + 2) + (n + 2)$ $3n + 6$

Expression with no match: $3(n + 3)$

12. Write an expression to represent this new diagram. Try to write an expression you think none of your classmates will.

Responses vary.

- $6n + 3$
- $3(2n + 1)$
- $(2n + 1) + (2n + 1) + (2n + 1)$
- $3n + 3n + 3$

Accelerated 6 | Unit 6 Lesson 7 528 Activity 3

Whole Class | 5 min

Synthesis

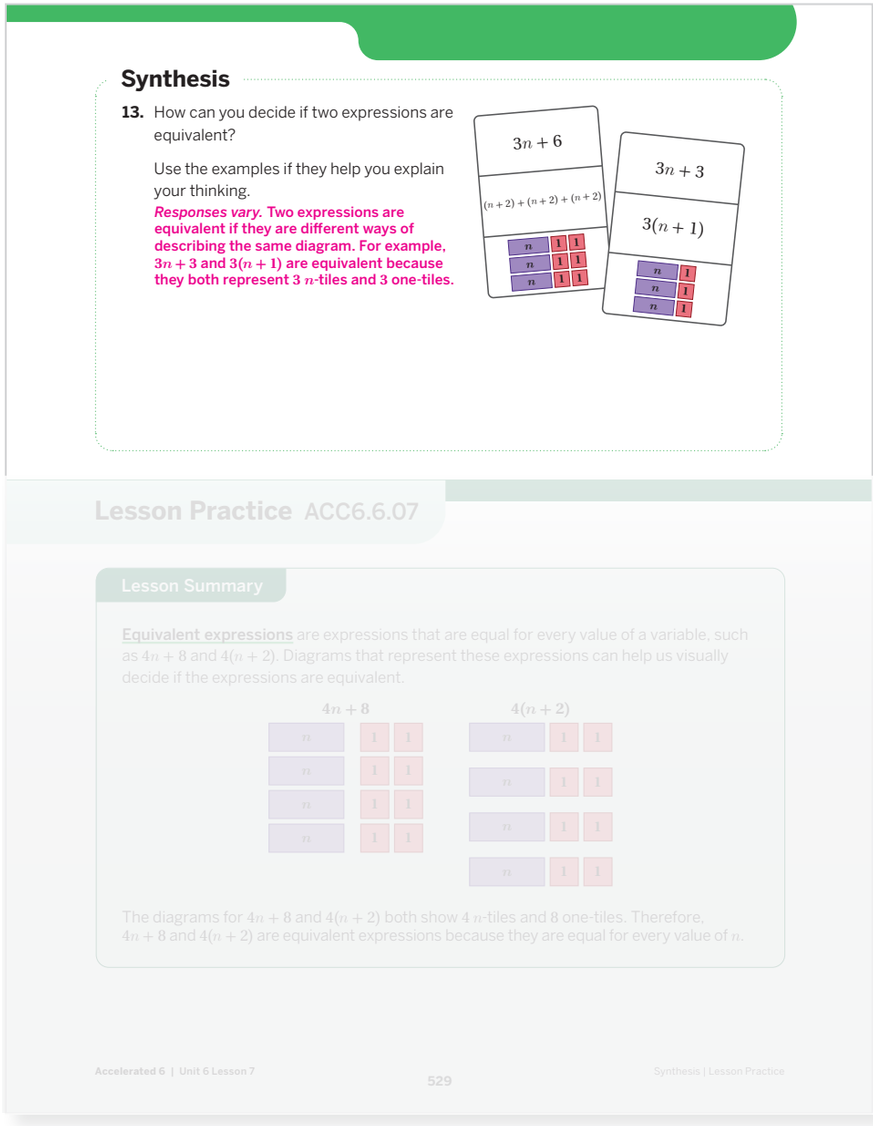
Purpose: Students synthesize their understanding of equivalent expressions.

Invite students to respond independently, and then share their thinking with a partner.

Collect and share a variety of ideas, such as justifying equivalence using the given diagrams.

MLR **MLR2: Collect and Display** Use this routine to formalize the definition of *equivalent expressions* using students' language. Display any new language, examples, or diagrams publicly for students to reference later.

Note: If time allows, invite students to revise their previous thinking based on the discussion.



Synthesis

13. How can you decide if two expressions are equivalent?

Use the examples if they help you explain your thinking.

Responses vary. Two expressions are equivalent if they are different ways of describing the same diagram. For example, $3n + 3$ and $3(n + 1)$ are equivalent because they both represent 3 n-tiles and 3 one-tiles.

Lesson Practice ACC6.6.07

Lesson Summary

Equivalent expressions are expressions that are equal for every value of a variable, such as $4n + 8$ and $4(n + 2)$. Diagrams that represent these expressions can help us visually decide if the expressions are equivalent.

$4n + 8$ $\quad$ $4(n + 2)$

The diagrams for $4n + 8$ and $4(n + 2)$ both show 4 n-tiles and 8 one-tiles. Therefore, $4n + 8$ and $4(n + 2)$ are equivalent expressions because they are equal for every value of n.

Accelerated 6 | Unit 6 Lesson 7 529 Synthesis | Lesson Practice

Lesson Takeaway: Equivalent expressions are equal for every value of a variable. One way to show that expressions are equivalent is to use the same diagram to represent both expressions. One way to show that expressions are *not* equivalent is to substitute the same value into both expressions and show that they result in different values.

Independent | 5 min

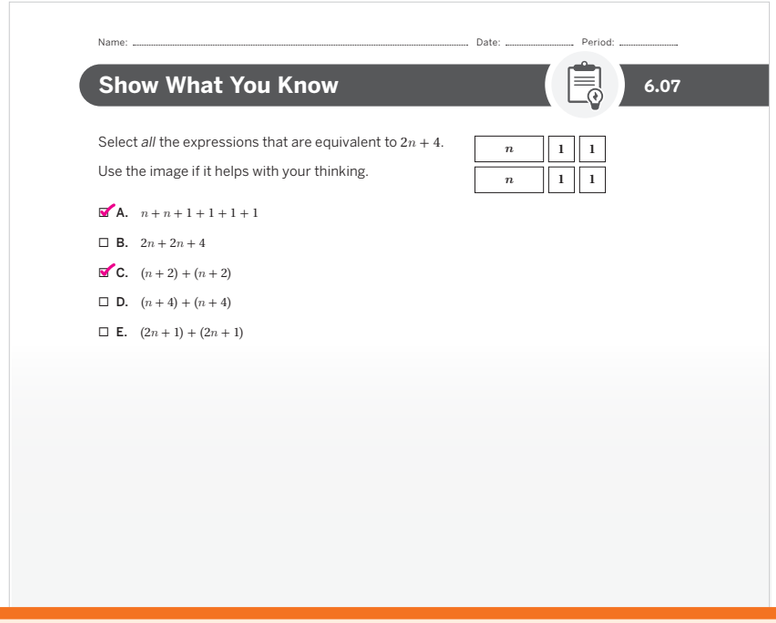
Show What You Know

Purpose: Students demonstrate their understanding by selecting all expressions that are equivalent to a given expression.

Today's Goals

1. **Goal:** Determine whether two expressions are equivalent.
2. **Language Goal:** Explain what it means for two expressions to be equivalent. **(Reading, Writing, Speaking, and Listening)**

Show What You Know (Assessment Resources)



Name: _____ Date: _____ Period: _____

Show What You Know 6.07

Select *all* the expressions that are equivalent to $2n + 4$.

Use the image if it helps with your thinking.

- ☑ A. $n + n + 1 + 1 + 1 + 1$
- ☐ B. $2n + 2n + 4$
- ☑ C. $(n + 2) + (n + 2)$
- ☐ D. $(n + 4) + (n + 4)$
- ☐ E. $(2n + 1) + (2n + 1)$

D Differentiation Use after Lesson 7

Support

Provide targeted intervention for students by using these resources.

If students need support determining whether expressions are equivalent:

» **Respond to Student Thinking:**

- Review the **Lesson Summary**.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite them to explore the **Sub-Unit 2 Extension Activities**.
- Invite students to create their own unique border designs surrounding a square and write multiple expressions that represent them. Encourage them to share with their classmates.

Professional Learning

In this lesson, students developed early ideas of **equivalent expressions**. How will their understanding of equivalent expressions build over the next several lessons?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

13. How can you decide if two expressions are equivalent?

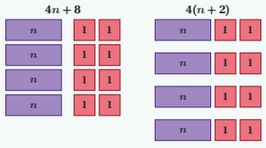
Use the examples if they help you explain your thinking.

Responses vary. Two expressions are equivalent if they are different ways of describing the same diagram. For example, $3n + 3$ and $3(n + 1)$ are equivalent because they both represent 3 n-tiles and 3 one-tiles.

Lesson Practice ACC6.6.07

Lesson Summary

Equivalent expressions are expressions that are equal for every value of a variable, such as $4n + 8$ and $4(n + 2)$. Diagrams that represent these expressions can help us visually decide if the expressions are equivalent.



The diagrams for $4n + 8$ and $4(n + 2)$ both show 4 n-tiles and 8 one-tiles. Therefore, $4n + 8$ and $4(n + 2)$ are equivalent expressions because they are equal for every value of n.

Lesson Practice ACC6.6.07

Name: _____ Date: _____ Period: _____

Problems 1–3: Here are examples of an n-by-1 rectangle.

1. How many border tiles are there in the 4-by-1 rectangle?

14 tiles

2. How many border tiles are in the 6-by-1 rectangle?

18 tiles

3. Diego says $2n + 6$ represents the number of tiles needed for the border of an n-by-1 rectangle. Explain why his strategy is correct.

Explanations vary. **Diego's strategy is correct because along the top and bottom of the n-by-1 rectangle, there are two strips of tiles that are n-units long, and there are always 3 extra tiles on the left and right side. I also know his strategy is correct because $2n + 6$ works for the 4-by-1 and 6-by-1 rectangles.**

Problems 4–5: Here are five expressions.

$2 + 6n$ $\quad 2(n + 3)$ $\quad n + 3$ $\quad n + n + 6$ $\quad (n + 3) + (n + 3)$

4. Write *all* of the expressions that are equivalent to $2n + 6$.

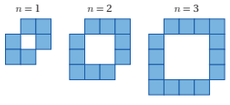
$2(n + 3)$, $n + n + 6$, $(n + 3) + (n + 3)$

5. Choose an expression that is *not* equivalent to $2n + 6$. Explain how you know it is not equivalent.

Responses vary.

- **$2 + 6n$ is not equivalent to $2n + 6$ because $6n$ is not the same as $2n$.**
- **$n + 3$ is not equivalent to $2n + 6$ because there is only a single n instead of 2.**

Problems 6–7: Here is a pattern.



6. Write an expression that describes the number of tiles for any stage, n.

Responses vary.

- **$(n + (n + 1)) \cdot 2$**
- **$4n + 2$**

7. Write an equivalent expression.

Responses vary.

Lesson Practice ACC6.6.07

Name: _____ Date: _____ Period: _____

FAST Practice

8. Which expression is equivalent to $8n + 2$?

A. $4(2n + 2)$ B. $2(4n + 1)$ C. $4n + 2 + n + 2$ D. $n + 8 + 2$

Spiral Review

Problems 9–10: Write an equation to represent each situation.

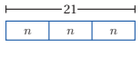
9. Aba's dog was 6 inches tall when it was a puppy but is now 14 inches tall. Aba's dog grew n inches.

$6 + n = 14$ (or equivalent)

10. Apples cost \$2 per pound. Darius bought x pounds of apples for a total cost of \$6.

$2x = 6$ (or equivalent)

Problems 11–12: Here is a tape diagram.



11. Write an equation to represent the tape diagram.

Responses vary.

- **$3n = 21$**
- **$21 \div n = 3$**
- **$21 \div 3 = n$**

12. Determine the value of n.

$n = 7$

Problems 13–16: Evaluate each expression for $b = 5$.

13. $3.5b$

$3.5(5) = 17.5$

14. $6b + 1$

$6(5) + 1 = 31$

15. $\frac{1}{4} + b$

$\frac{1}{4} + (5) = 5\frac{1}{4}$ (or equivalent)

16. $\frac{1}{2}b$

$\frac{1}{2}(5) = \frac{5}{2}$ (or equivalent)

Lesson Practice Item Analysis

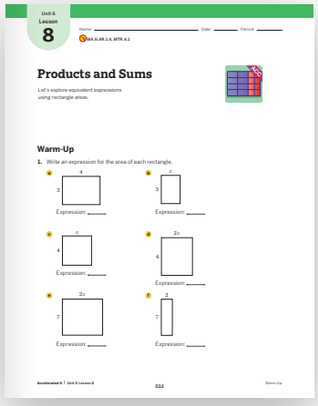
	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1–5	2	MA.6.AR.1.4
	6–7	3	MA.6.AR.1.4
FAST Practice	8	2	MA.6.AR.1.4
Spiral Review			
	9	2	MA.6.AR.2.2
	10–12	2	MA.6.AR.2.3
	13–16	1	MA.6.AR.1.3

Unit 6

Lesson

8

ACC Accelerated


Unit 6
Lesson
8
Name: Date: Period:
MA.6.AR.1.4, MTR.4.1
Products and Sums
Let's explore equivalent expressions using rectangle areas.
Warm-Up
1. Write an expression for the area of each rectangle.
Expression:
532
Warm-Up

Products and Sums

Distributive Property, Part 1

Let's explore equivalent expressions using rectangle areas.

Learning Outcomes

Today's Goals

1. **Goal:** Use areas of rectangles to write equivalent expressions.
2. **Language Goal:** Compare and contrast product and sum expressions. **(Speaking and Listening)**

Students are introduced to areas of rectangles (area models) with variable side lengths. This is the first of two lessons that focus on the *distributive property* of multiplication over addition. Students use variable expressions to represent areas of partitioned rectangles in two different ways: as the *sum* of two areas and as the *product* of the two side lengths.

Coherence

Prior Learning

In Lesson 7, students learned what it meant for two expressions to be equivalent and justified their reasoning.

Future Learning

In Lesson 9, students will generate product and sum expressions with one or more variables, and determine if expressions are equivalent.

Connections

- Students continue to make connections between models and algebraic expressions. (mathematical)

FL B.E.S.T. Math Benchmarks

Addressing

MA.6.AR.1.4

Apply the properties of operations to generate equivalent algebraic expressions with integer coefficients.

Mathematical Thinking and Reasoning Standard: MTR.4.1

ELA Expectation: ELA.K12.EE.5.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.5.AR.2.2	**MA.7.AR.1.1** **MA.7.AR.1.2**

Vocabulary

Review Vocabulary

sum

product

distributive property

Lesson at a Glance

~ 45 min

MA.6.AR.1.4

Warm-Up

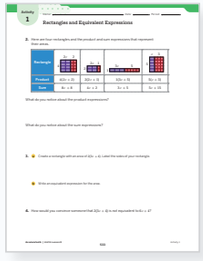
Independent | 5 min

Students revisit how to calculate the area of a rectangle, including when one side has a variable length.

Activity 1

Pairs | 10 min

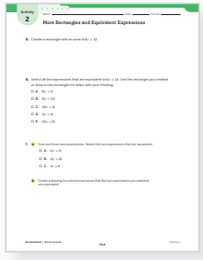
Students explore two ways of expressing the area of a partitioned rectangle with a variable length: as the *product* of the length and width, or as the *sum* of smaller areas.



Activity 2

Pairs | 10 min

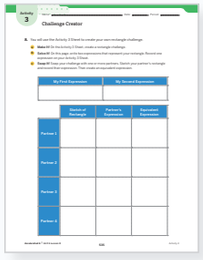
Students extend their work on using area models to reason about equivalent expressions.



Activity 3

Pairs | 10 min

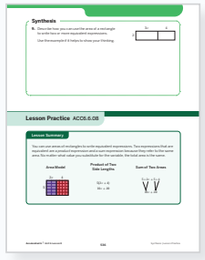
Students use their creativity to make a rectangle challenge that their classmates write equivalent expressions for.



Synthesis

Whole Class | 5 min

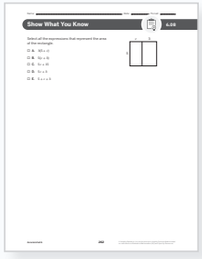
Students synthesize their understanding of using partitioned area models to write equivalent expressions.



Show What You Know

Independent | 5 min

Students demonstrate their understanding by identifying equivalent expressions that represent an area model.



Prep Checklist

Prepare Activity 3 Sheets

This lesson includes:

- Student Edition
- Show What You Know
- Activity 3 Sheets, one per student
- Algebra tiles (as needed)

Independent | 5 min

Warm-Up

Purpose: Students revisit how to calculate the area of a rectangle, including when one side has a variable length.

Short on time? Consider omitting the Warm-Up and reviewing calculating area as a class.

1 Launch

To surface prior knowledge, consider asking, "How do you determine the area of a rectangle?"

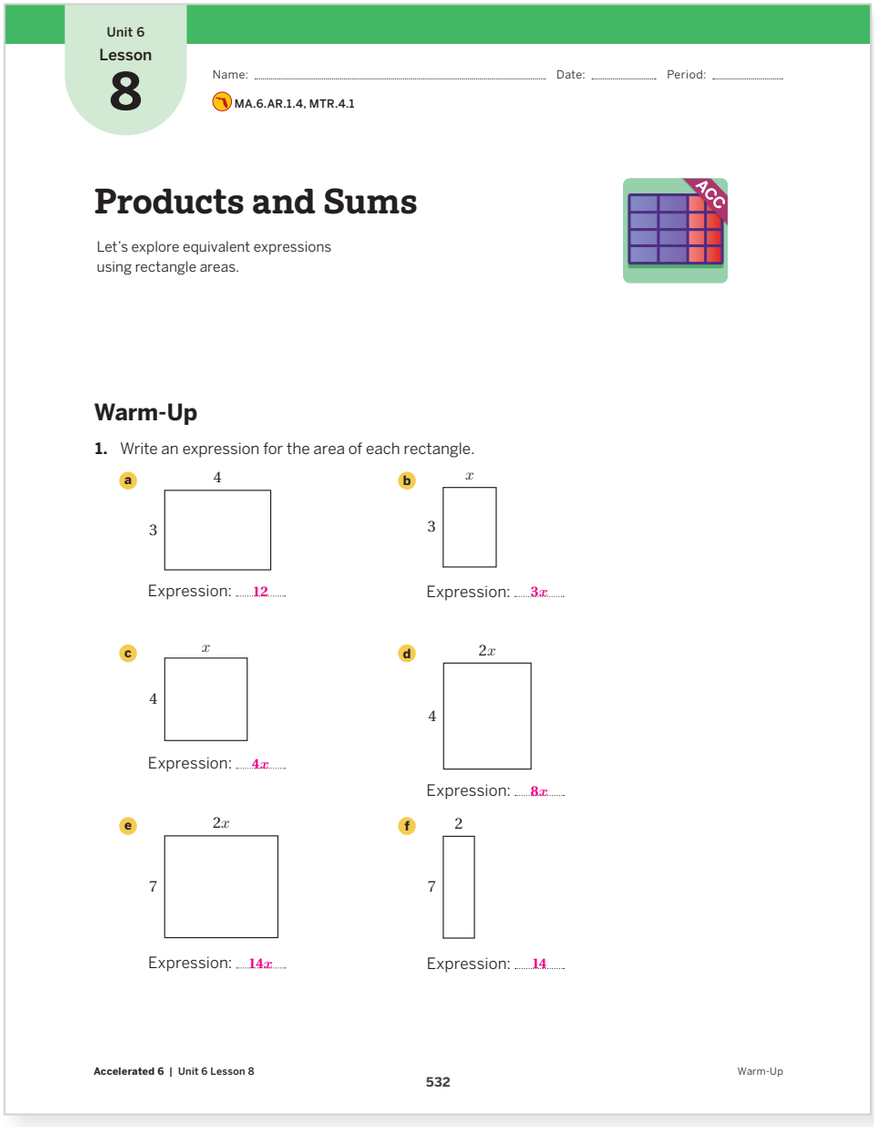
Invite students to set their own goal for how many problems they would like to complete, sharing that sense-making is more important than speed.

2 Connect

Look for and share students' area expressions, including ones with variable side lengths. Invite them to share why each expression matches each rectangle.

Consider asking:

- "How did you calculate the area when one of the side lengths was a variable?"
- "What did you notice about the two types of tiles? How are they alike? Different?"
- "Do you think multiple expressions can represent the same area? Why or why not?"



Unit 6
Lesson
8

Name: Date: Period:

MA.6.AR.1.4, MTR.4.1

Products and Sums

Let's explore equivalent expressions using rectangle areas.

Warm-Up

1. Write an expression for the area of each rectangle.

a 4 × 3
Expression: 12

b x × 3
Expression: $3x$

c x × 4
Expression: $4x$

d $2x$ × 4
Expression: $8x$

e $2x$ × 7
Expression: $14x$

f 2 × 7
Expression: 14

Accelerated 6 | Unit 6 Lesson 8 532 Warm-Up

Pairs | 10 min

Activity 1 Rectangles and Equivalent Expressions

Purpose: Students explore two ways of expressing the area of a partitioned rectangle with a variable length: as the *product* of the length and width, or as the *sum* of smaller areas.

1 Launch

Direct students' attention to the table.

To surface connections, review the terms *product* and *sum*. Consider asking, "Why do you think the expression $4(2x + 2)$ is called a *product*? Why do you think the expression $8x + 8$ is called a *sum*?"

2 Monitor

To support students getting started, consider asking:

- "Can you change the width from $2x$ to $3x$? To $1x$?"
- "Can you change the length from 4 to 5? To 2?"

Select and sequence a variety of expressions.

D Differentiation

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Encourage them to refer to the product and sum expressions from the previous problem. For each expression, consider asking, "What does each part of the expression represent?"
Use a variety of strategies to write equivalent expressions, like: • Writing a sum expression. • Counting the x tiles and 1 tiles. • Applying the distributive property.	**Strengthen** Invite students to share their strategies for writing equivalent expressions in the Connect.

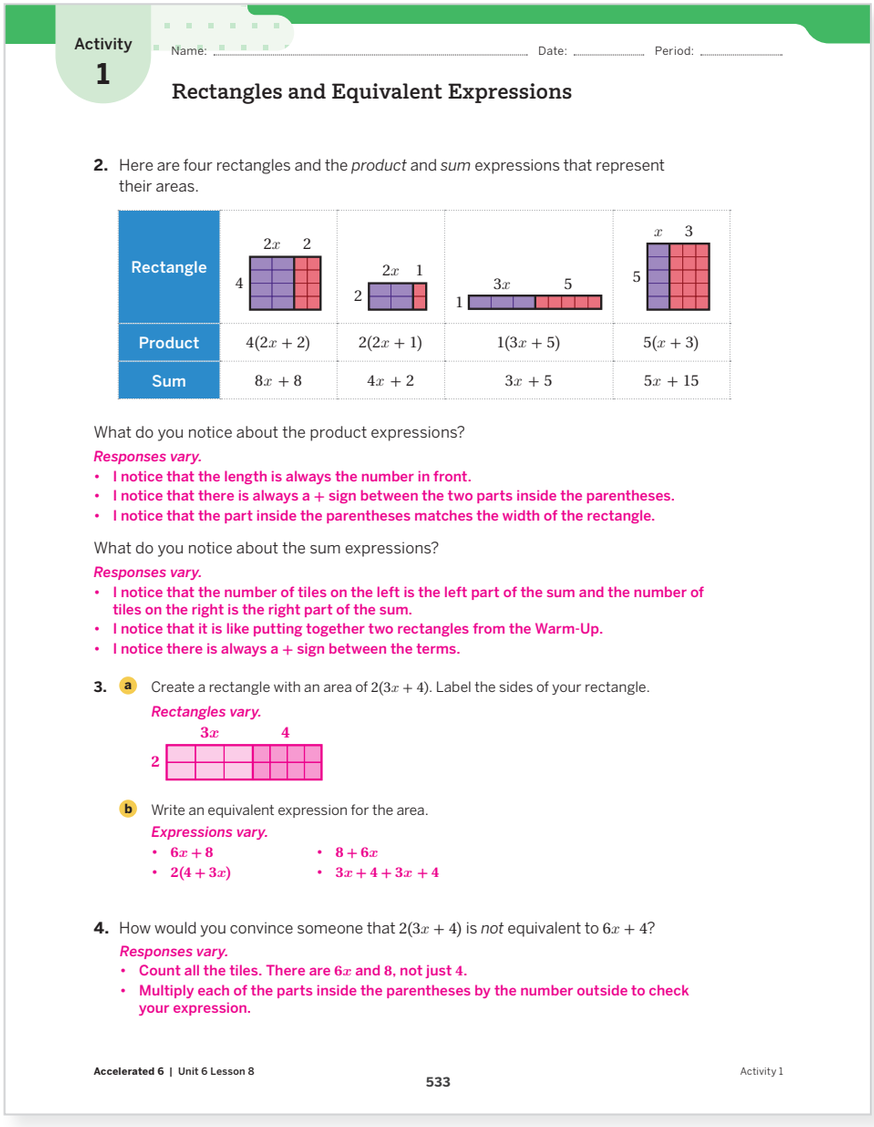
Look for a variety of explanations, including ones similar to the sample responses.

3 Connect

Share student expressions and invite students to share more. For each strategy, invite students to share how they wrote each equivalent expression.

Consider asking:

- "Why might someone think these two expressions are equivalent? Why are they not equivalent?" **(MTR.4.1)**
- "What could you change about $6x + 4$ to make it equivalent to $2(3x + 4)$?"



Activity 1

Name: Date: Period:

Rectangles and Equivalent Expressions

2. Here are four rectangles and the *product* and *sum* expressions that represent their areas.

Rectangle				
Product	$4(2x + 2)$	$2(2x + 1)$	$1(3x + 5)$	$5(x + 3)$
Sum	$8x + 8$	$4x + 2$	$3x + 5$	$5x + 15$

What do you notice about the product expressions?

Responses vary.

- I notice that the length is always the number in front.
- I notice that there is always a + sign between the two parts inside the parentheses.
- I notice that the part inside the parentheses matches the width of the rectangle.

What do you notice about the sum expressions?

Responses vary.

- I notice that the number of tiles on the left is the left part of the sum and the number of tiles on the right is the right part of the sum.
- I notice that it is like putting together two rectangles from the Warm-Up.
- I notice there is always a + sign between the terms.

3. a. Create a rectangle with an area of $2(3x + 4)$. Label the sides of your rectangle.

Rectangles vary.

b. Write an equivalent expression for the area.

Expressions vary.

- $6x + 8$
- $2(4 + 3x)$
- $8 + 6x$
- $3x + 4 + 3x + 4$

4. How would you convince someone that $2(3x + 4)$ is *not* equivalent to $6x + 4$?

Responses vary.

- Count all the tiles. There are $6x$ and 8, not just 4.
- Multiply each of the parts inside the parentheses by the number outside to check your expression.

Accelerated 6 | Unit 6 Lesson 8 533 Activity 1

Pairs | 10 min

Activity 2 More Rectangles and Equivalent Expressions

Purpose: Students extend their work on using area models to reason about equivalent expressions.

1 Launch

To encourage creativity, invite students to create a rectangle that is different from their classmates'.

2 Monitor

To support students getting started, consider asking, "What are possible lengths and widths for a rectangle with an area of 12? 6? $6x$?"

Invite students to check their thinking with a partner.

Look for the distribution of expressions students choose.

D Differentiation

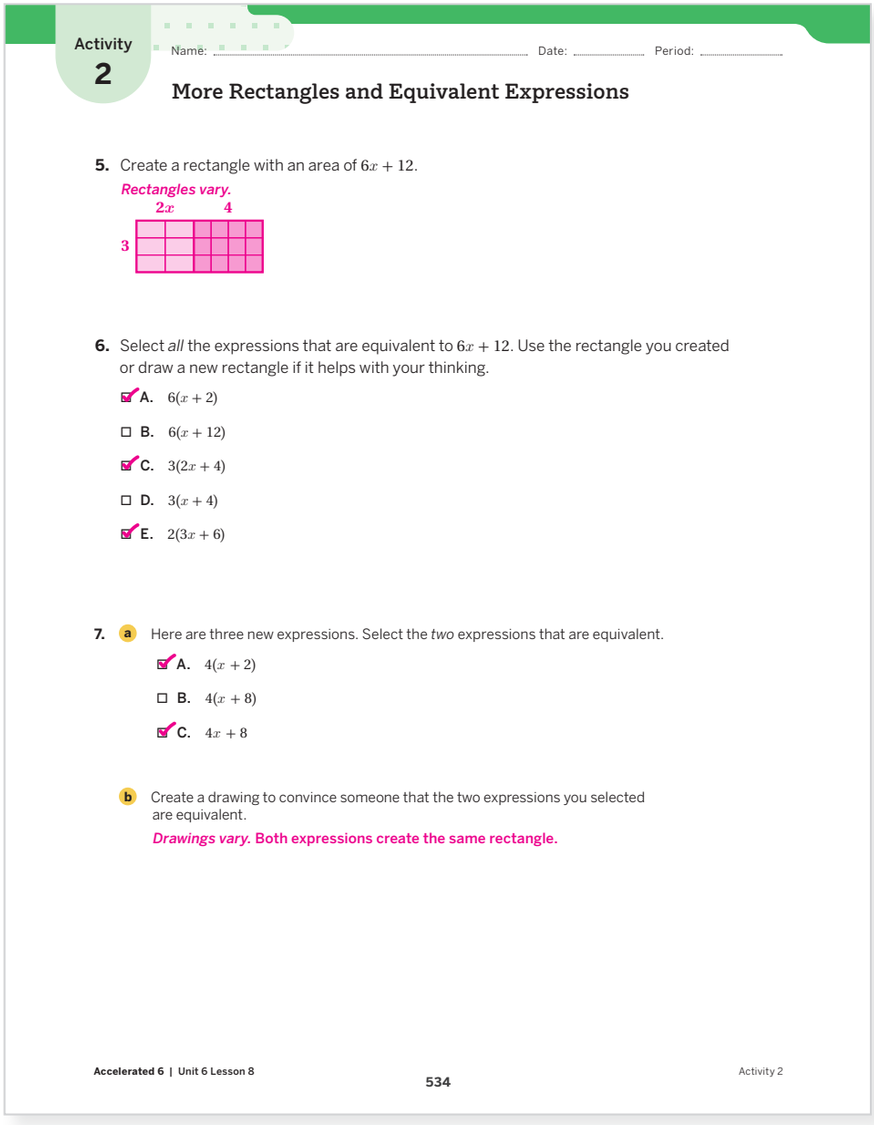
Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Consider asking, "Do any of these expressions match the rectangle you created in the previous problem?"
Use a variety of strategies to determine the equivalent expressions, like: • Drawing a diagram for each option and looking for connections. • Applying the distributive property to each product expression to create a sum.	**Strengthen** Invite them to share their strategies during the Connect. **Stretch** Invite students to create an equivalent expression for every expression they did not select.

3 Connect

Invite students to share their responses and strategies for determining which expressions were equivalent to $6x + 12$, calling attention to any conflict or consensus.

ELL **English Language Learners** Provide sentence frames to help students explain their thinking (e.g., [Expression 1] is not equivalent to [Expression 2] because _____.). **(Writing and Speaking)**

To surface the Key Takeaway, consider asking:
- "Which of these expressions are product expressions? Which are sum expressions?"
- "How can areas of rectangles, also called area models, help you identify or create equivalent expressions?"



Activity 2

Name: Date: Period:

More Rectangles and Equivalent Expressions

5. Create a rectangle with an area of $6x + 12$.

Rectangles vary.

6. Select *all* the expressions that are equivalent to $6x + 12$. Use the rectangle you created or draw a new rectangle if it helps with your thinking.

- ☑ A. $6(x + 2)$
- ☐ B. $6(x + 12)$
- ☑ C. $3(2x + 4)$
- ☐ D. $3(x + 4)$
- ☑ E. $2(3x + 6)$

7. a. Here are three new expressions. Select the *two* expressions that are equivalent.

- ☑ A. $4(x + 2)$
- ☐ B. $4(x + 8)$
- ☑ C. $4x + 8$

b. Create a drawing to convince someone that the two expressions you selected are equivalent.

Drawings vary. **Both expressions create the same rectangle.**

Accelerated 6 | Unit 6 Lesson 8 534 Activity 2

Key Takeaway: An area model is one way to generate equivalent expressions. The *product* of the length and width of a rectangle is one expression and the *sum* of the two smaller areas is another.

Pairs | 10 min

Activity 3 Challenge Creator

Purpose: Students use their creativity to make a rectangle challenge that their classmates write equivalent expressions for.

Short on time? Consider inviting students to only complete one rectangle challenge.

1 Launch

Consider sharing that students will be creating their own rectangles similar to the other ones in this lesson. Encourage creativity by inviting students to make a challenge they think will be different from ones their classmates create.

A **Accessibility: Visual-Spatial Processing** Provide access to algebra tiles to model each of the expressions.

Distribute the Activity 3 Sheet to students so they can create their rectangle challenge.

2 Monitor

Listen to student strategies and discussions, and offer help or encouragement where needed.

Invite students to check their equivalent expression with the one written by the creator of the challenge. Encourage them to discuss and resolve any differences. **(MTR.4.1)**

A **Accessibility: Executive Functioning** Invite students to work together on a challenge before trying one independently.

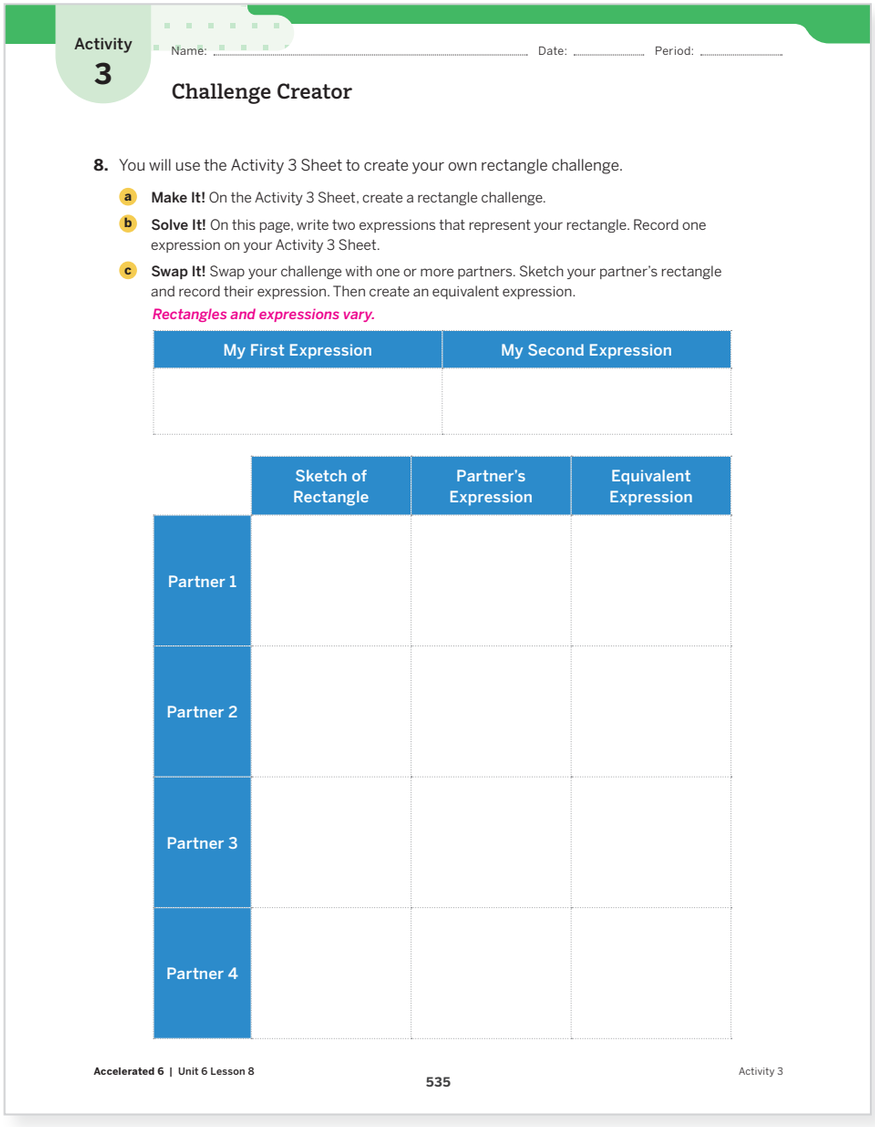
Look for several students' rectangles and expressions to share during the Connect.

3 Connect

Share student rectangles and expressions. Invite students to share their strategies for creating equivalent expressions for each rectangle.

Consider asking, "Was there a strategy you heard that surprised you?"

Note: If it does not come up naturally, consider discussing the relationship between the sum and product expressions and the *distributive property*. **(MTR.4.1)**



Activity 3

Name: Date: Period:

Challenge Creator

8. You will use the Activity 3 Sheet to create your own rectangle challenge.

a. **Make It!** On the Activity 3 Sheet, create a rectangle challenge.

b. **Solve It!** On this page, write two expressions that represent your rectangle. Record one expression on your Activity 3 Sheet.

c. **Swap It!** Swap your challenge with one or more partners. Sketch your partner's rectangle and record their expression. Then create an equivalent expression.

Rectangles and expressions vary.

My First Expression	My Second Expression

	Sketch of Rectangle	Partner's Expression	Equivalent Expression
Partner 1			
Partner 2			
Partner 3			
Partner 4			

Whole Class | 5 min

Synthesis

Purpose: Students synthesize their understanding of using partitioned area models to write equivalent expressions.

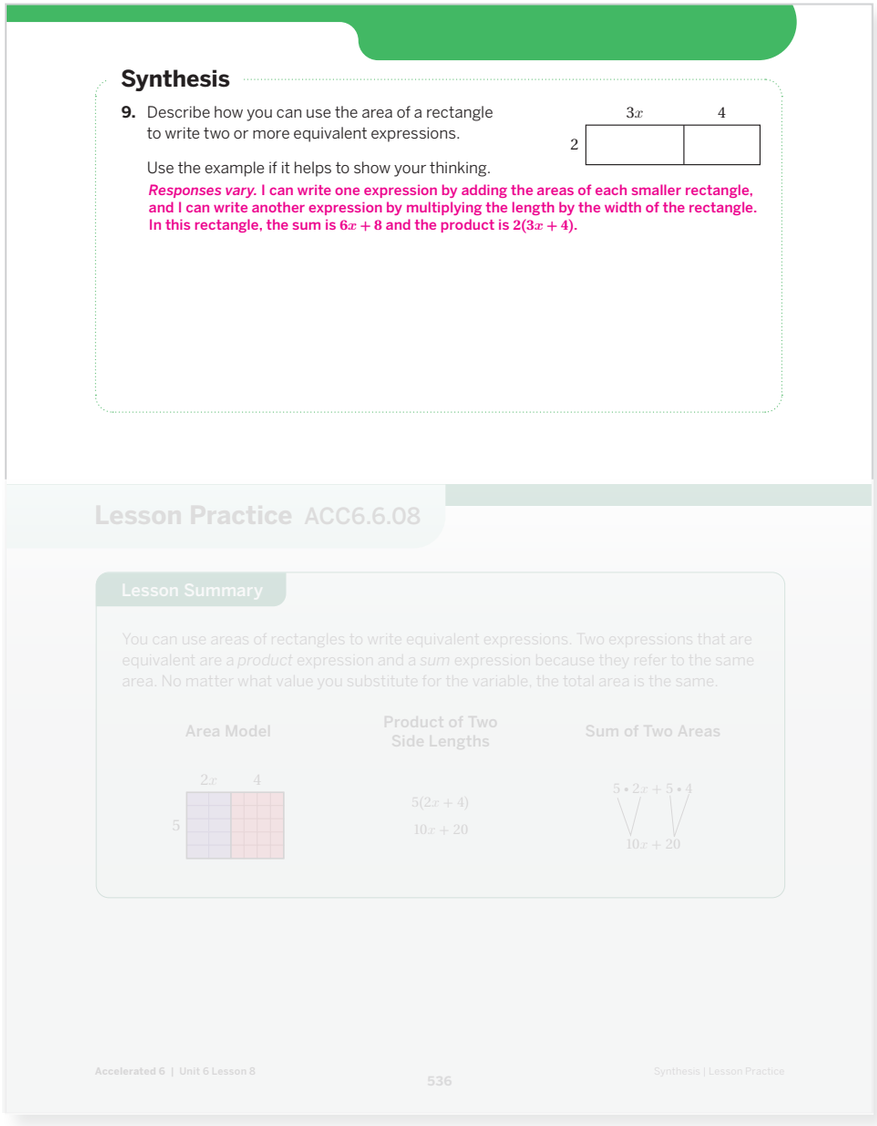
Invite students to respond independently, and then share their thinking with a partner.

Listen for and share a variety of ideas, such as:

- Two ways to find the area of a rectangle: the product of the length and width, or the sum of two smaller areas.
- How to rewrite a product as a sum or a sum as a product.
- The connection between sum and product expressions and the distributive property.

MLR **MLR1: Stronger and Clearer Each Time (optional)** Consider using this routine to support students in developing precise language. Invite students to meet with a partner, give feedback on each other's drafts, then use the feedback to write a second clearer draft. See the Routine Facilitation Guide for more information.

Note: If time allows, invite students to revise their previous thinking based on the discussion.



Synthesis

9. Describe how you can use the area of a rectangle to write two or more equivalent expressions.

Use the example if it helps to show your thinking.

Responses vary. I can write one expression by adding the areas of each smaller rectangle, and I can write another expression by multiplying the length by the width of the rectangle. In this rectangle, the sum is $6x + 8$ and the product is $2(3x + 4)$.

Lesson Practice ACC6.6.08

Lesson Summary

You can use areas of rectangles to write equivalent expressions. Two expressions that are equivalent are a *product* expression and a *sum* expression because they refer to the same area. No matter what value you substitute for the variable, the total area is the same.

Area Model	Product of Two Side Lengths	Sum of Two Areas
	$5(2x + 4)$ $10x + 20$	

Accelerated 6 | Unit 6 Lesson 8 536 Synthesis | Lesson Practice

Lesson Takeaway: You can use an area model of a partitioned rectangle to help you create two equivalent expressions: one that represents the *sum* of two areas and one that represents the *product* of the two side lengths.

Independent | 5 min

Show What You Know

Purpose: Students demonstrate their understanding by identifying equivalent expressions that represent an area model.

Today's Goals

1. **Goal:** Use areas of rectangles to write equivalent expressions.
2. **Language Goal:** Compare and contrast product and sum expressions. **(Speaking and Listening)**

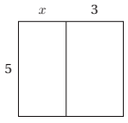
Show What You Know (Assessment Resources)

Name: _____ Date: _____ Period: _____

Show What You Know 6.08

Select *all* the expressions that represent the area of the rectangle.

- ☐ **A.** $3(5 + x)$
- ☑ **B.** $5(x + 3)$
- ☑ **C.** $5x + 15$
- ☐ **D.** $5x + 3$
- ☐ **E.** $5 + x + 3$



Differentiation Use after Lesson 8

Support

Provide targeted intervention for students by using these resources.

If students need support understanding of equivalent expressions and the distributive property:

» **Respond to Student Thinking:**

- Use the Writing ***Equivalent Expressions Using the Area* Model Mini-Lesson**.
- Revisit **Lesson 6** (*Vari-apples*). Invite students to describe areas and encourage them to use the same process with diagrams containing unknown lengths.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite them to explore the **Sub-Unit 2 Extension Activities**.

Professional Learning

In this lesson, students are introduced to area models that support their understanding of writing equivalent expressions. How does this model integrate with other representations of *product* and *sum* expressions?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

9. Describe how you can use the area of a rectangle to write two or more equivalent expressions.

Use the example if it helps to show your thinking.

Responses vary. I can write one expression by adding the areas of each smaller rectangle, and I can write another expression by multiplying the length by the width of the rectangle. In this rectangle, the sum is $6x + 8$ and the product is $2(3x + 4)$.

Lesson Practice ACC6.6.08

Lesson Summary

You can use areas of rectangles to write equivalent expressions. Two expressions that are equivalent are a *product* expression and a *sum* expression because they refer to the same area. No matter what value you substitute for the variable, the total area is the same.

Area Model	Product of Two Side Lengths	Sum of Two Areas
	$5(2x + 4)$ $10x + 20$	$5 \cdot 2x + 5 \cdot 4$ $10x + 20$

Lesson Practice ACC6.6.08

Name: _____ Date: _____ Period: _____

1. Select *all* the expressions that are equivalent to $4b$.

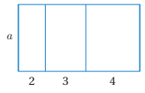
- ☑ A. $b + b + b + b$
- ☐ B. $b + 4$
- ☐ C. $b \cdot b \cdot b \cdot b$
- ☑ D. $2b + 2b$
- ☑ E. $4 \cdot b$

2. Zola wrote the total area of the rectangle as $2a + 3a + 4a$.

Amir wrote the total area as $(2 + 3 + 4)a$.

Explain why they are *both* correct.

Responses vary. Zola's expression and Amir's expression are equivalent. Zola added the areas of each small rectangle and Amir added the lengths before multiplying by the width.



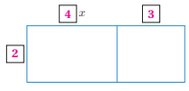
3. Write two equivalent expressions that could be used to represent the area of the rectangle.

Expression 1: $4(3a + 2)$

Expression 2: $12a + 8$

4. Use the numbers 0 to 9 to fill in each blank so that each rectangle has the same area. Use each number only once.

Responses vary.

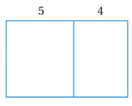


Lesson Practice ACC6.6.08

Name: _____ Date: _____ Period: _____

FAST Practice

5. Select *all* the expressions that represent the area of the rectangle.



- ☐ A. $t + 5 + 4$
- ☑ B. $5t + 4t$
- ☑ C. $9t$
- ☐ D. $4 \cdot 5 \cdot t$
- ☑ E. $(5 + 4)t$

Spiral Review

Problems 6–8: Titus's aunt is 17 years older than he is.

6. How old will his aunt be when Titus is 15 years old?

32 years old

7. How old will his aunt be when Titus is 30 years old?

47 years old

8. How old will his aunt be when Titus is x years old?

$x + 17$ years old (or equivalent)

Problems 9–11: Solve each equation. Show your thinking.

9. $10m = 30$

$m = 3$. *Strategies vary.*

$10m \div 10 = 30 \div 10$

$m = 3$

10. $13 = h + 4$

$9 = h$. *Strategies vary.*

$13 - 4 = h + 4 - 4$

$9 = h$

11. $k + 15 = 41$

$k = 26$. *Strategies vary.*

$k + 15 - 15 = 41 - 15$

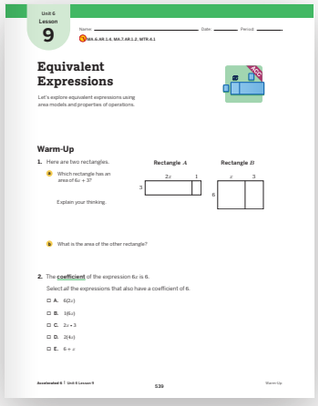
$k = 26$

Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1–3	2	MA.6.AR.1.4
	4	3	MA.6.AR.1.4
FAST Practice	5	2	MA.6.AR.1.4
Spiral Review			
	6–7	1	MA.5.AR.1.1
	8	1	MA.6.AR.1.1
	9	1	MA.6.AR.2.3
	10–11	1	MA.6.AR.2.2

Unit 6
Lesson
9

ACC Accelerated


Unit 6
Lesson
9

Equivalent Expressions

Let's explore equivalent expressions using area models and properties of operations.

Warm-Up

1. Here are two rectangles.

Which rectangle has an area of $6x + 3$?

Explain your thinking.

What is the area of the other rectangle?

2. The coefficient of the expression $6x$ is 6.

Select all the expressions that also have a coefficient of 6.

A. $6(x)$
B. $18x$
C. $2x + 3$
D. $3(4x)$
E. $6 + x$

539

Equivalent Expressions

Distributive Property, Part 2

Let's explore equivalent expressions using area models and properties of operations.

Learning Outcomes

Today's Goals

1. **Goal:** Analyze expressions to determine if they are equivalent.
2. **Language Goal:** Describe strategies for determining which expressions are equivalent. **(Writing, Speaking, and Listening)**

Students strengthen their work from the previous lesson to reason about more complicated expressions. They identify equivalent expressions, including those that involve the distributive property and one or more other operations. Students may use area models or other strategies to support their thinking.

Coherence

Prior Learning

In Lesson 8, students used variable expressions to represent areas of rectangles in two different ways: as the sum of two areas and as the product of the two side lengths.

Future Learning

In Lesson 11, students will apply their understanding to expressions involving exponents. In Unit 7, they will apply operations to generate equivalent expressions with rational coefficients.

Connections

- Students demonstrate their proficiency with models and algebraic expressions through a series of challenge problems. (mathematical)

FL B.E.S.T. Math Benchmarks

Addressing

MA.6.AR.1.4
Apply the properties of operations to generate equivalent algebraic expressions with integer coefficients.

MA.7.AR.1.2
Determine whether two linear expressions are equivalent.

Mathematical Thinking and Reasoning Standard: MTR.4.1

ELA Expectation: ELA.K12.EE.4.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.5.AR.2.2	**MA.7.AR.1.1** **MA.8.AR.1.1**

Vocabulary

New Vocabulary

coefficient

Additional Vocabulary

commutative property
associative property
like terms

Lesson at a Glance

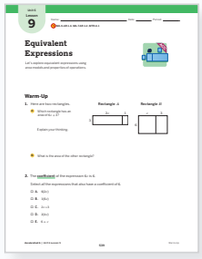
~ 45 min

MA.6.AR.1.4, MA.7.AR.1.2

Warm-Up

Pairs | 10 min

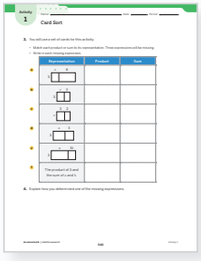
Students determine which area model matches an expression and create a class definition for the term **coefficient**.



Activity 1

Pairs | 15 min

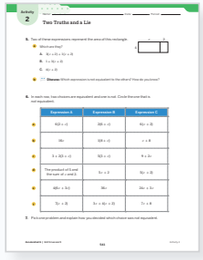
Students connect expressions written as sums and as products to area models, including expressions that have multiple variables and whose common factor is a variable.



Activity 2

Pairs | 10 min

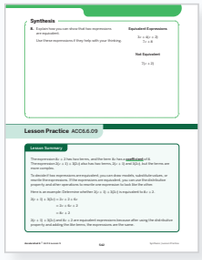
Students determine which expression, in a set of three, is not equivalent to the others and justify their reasoning.
Routine: Think-Pair-Share **(MTR.4.1)**



Synthesis

Whole Class | 5 min

Students synthesize their understanding of equivalent expressions. MLR2: Collect and Display



Show What You Know

Independent | 5 min

Students demonstrate their understanding by writing an equivalent expression.

Prep Checklist

Prepare the Activity 1 Cards.

This lesson includes:

- Student Edition
- Show What You Know
- Activity 1 Cards, one set per pair

Pairs | 10 min

Warm-Up

Purpose: Students determine which area model matches an expression and create a class definition for the term **coefficient**.

Short on time? Consider inviting students to complete Problem 1 only.

1 Launch

Direct students' attention to the rectangles and the expression.

To surface prior knowledge, consider asking, "What strategies did you use in the previous lesson to write expressions that represent the area of each rectangle?"

Look and listen for students who annotate their rectangles or use language like *sum*, *product*, or *term*.

2 Connect

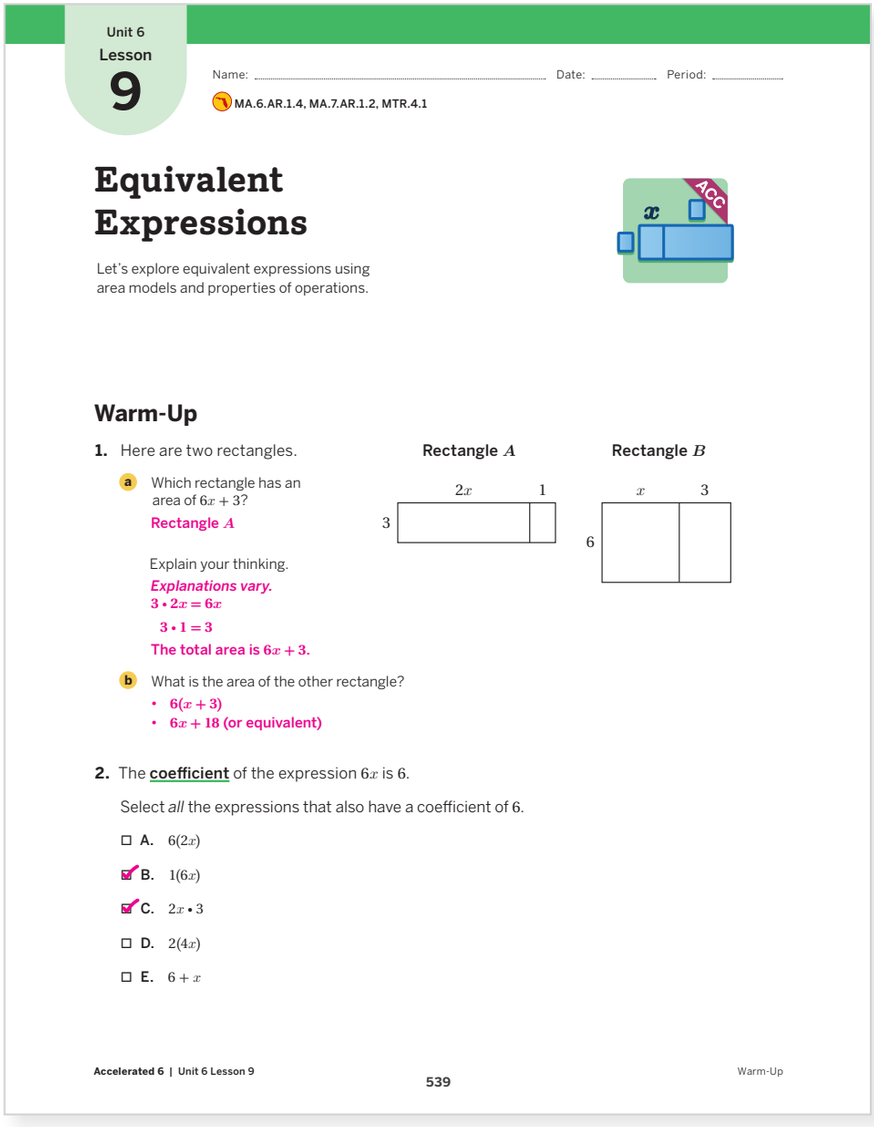
Invite students to share which rectangle they chose by polling the class. Consider asking, "Rectangle A has an area of $6x + 3$. Someone said that $6x + 3$ is equal to $9x$. Do you agree?"

Invite students to share the expressions they wrote for the area of Rectangle B. Consider asking, "Are the expressions you shared *equivalent*? How do you know?" Yes. They represent the same area, and I can use the distributive property to show they are the same.

Direct students' attention to the last Warm-Up problem.

Encourage students to justify why each expression does or does not have a coefficient of 6.

Create a class definition for the term **coefficient** using student language and the glossary.



Unit 6
Lesson
9

Name: ..... Date: ..... Period: .....

MA.6.AR.1.4, MA.7.AR.1.2, MTR.4.1

Equivalent Expressions

Let's explore equivalent expressions using area models and properties of operations.

Warm-Up

1. Here are two rectangles.

Rectangle A

	$2x$	1
3		

Rectangle B

	x	3
6		

a. Which rectangle has an area of $6x + 3$?

Rectangle A

Explain your thinking.

Explanations vary.

$3 \cdot 2x = 6x$

$3 \cdot 1 = 3$

The total area is $6x + 3$.

b. What is the area of the other rectangle?

- $6(x + 3)$
- $6x + 18$ (or equivalent)

2. The **coefficient** of the expression $6x$ is 6.

Select *all* the expressions that also have a coefficient of 6.

- ☐ A. $6(2x)$
- ☑ B. $1(6x)$
- ☑ C. $2x \cdot 3$
- ☐ D. $2(4x)$
- ☐ E. $6 + x$

Accelerated 6 | Unit 6 Lesson 9 539 Warm-Up

Pairs | 15 min

Activity 1 Card Sort

Purpose: Students connect expressions written as sums and as products to area models, including expressions that have multiple variables and whose common factor is a variable.

Short on time? Consider discussing Problem 4 as a class.

1 Launch

Direct students' attention to the instructions for the card sort.

To surface prior knowledge, consider asking, "What is an example of a product expression? A sum expression?"

Distribute one set of the Activity 1 Cards to each pair of students.

2 Monitor

Encourage students to share their reasoning with a partner and work together to reach an agreement about which representations and expressions match. **(MTR.4.1)**

A **Accessibility: Executive Functioning** Consider chunking this activity by inviting students to choose one card at a time to sort.

D **Differentiation**

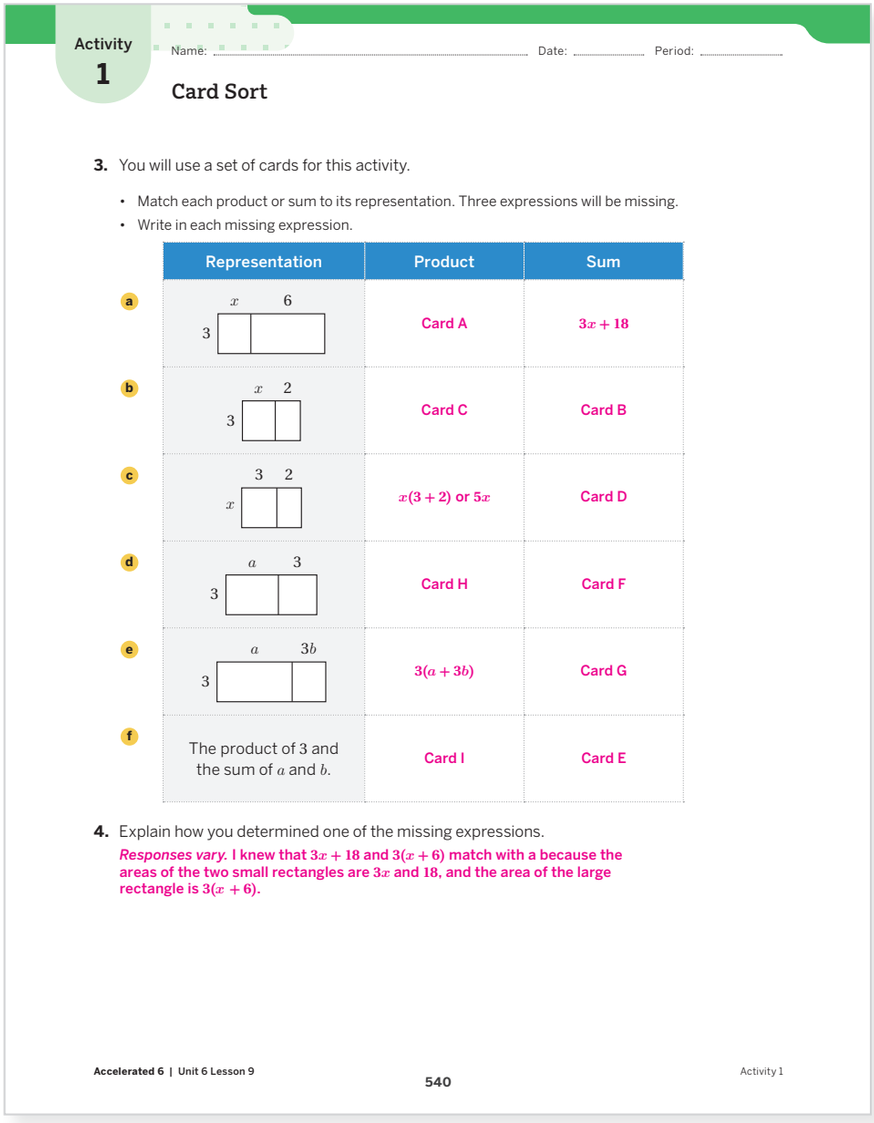
Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Encourage students to cover all but one representation. Consider asking, "How would you describe the area of this figure?"
Use a variety of strategies to complete the table, such as: • Annotating the area diagrams. • Matching cards with equivalent expressions. • Using the distributive property to determine and generate equivalent expressions.	**Strengthen** Invite students to compare their strategy with their partner's. Look for these strategies to share during the Connect.

Select and sequence a variety of explanations, including explanations similar to the sample responses and other creative explanations.

3 Connect

Direct students' attention to the cards, sort them as a class, and invite students to share their strategies for both sorting and writing the missing expressions.

Consider asking:
- "What strategies did you use to determine the expressions that matched each representation?"
- "How can you decide if two expressions are equivalent without using the area models?"



Activity 1

Name: _____ Date: _____ Period: _____

Card Sort

3. You will use a set of cards for this activity.

- Match each product or sum to its representation. Three expressions will be missing.
- Write in each missing expression.

	Representation	Product	Sum
a	3 × (x, 6)	Card A	$3x + 18$
b	3 × (x, 2)	Card C	Card B
c	x × (3, 2)	$x(3 + 2)$ or $5x$	Card D
d	3 × (a, 3)	Card H	Card F
e	3 × (a, 3b)	$3(a + 3b)$	Card G
f	The product of 3 and the sum of a and b.	Card I	Card E

4. Explain how you determined one of the missing expressions.

Responses vary. I knew that $3x + 18$ and $3(x + 6)$ match with a because the areas of the two small rectangles are $3x$ and 18, and the area of the large rectangle is $3(x + 6)$.

Accelerated 6 | Unit 6 Lesson 9 540 Activity 1

Pairs | 10 min

Activity 2 Two Truths and a Lie

Purpose: Students determine which expression, in a set of three, is not equivalent to the others and justify their reasoning. **(MTR.4.1)**

1 Launch

Direct students' attention to the area model and the expressions.

Use the Think-Pair-Share routine to support student-to-student discourse. Encourage students to justify their reasoning as they share. **(MTR.4.1)**

Direct students' attention to and review the instructions for completing the problems in the table.

2 Monitor

Select and sequence a variety of strategies for determining which expression is not equivalent.

D Differentiation

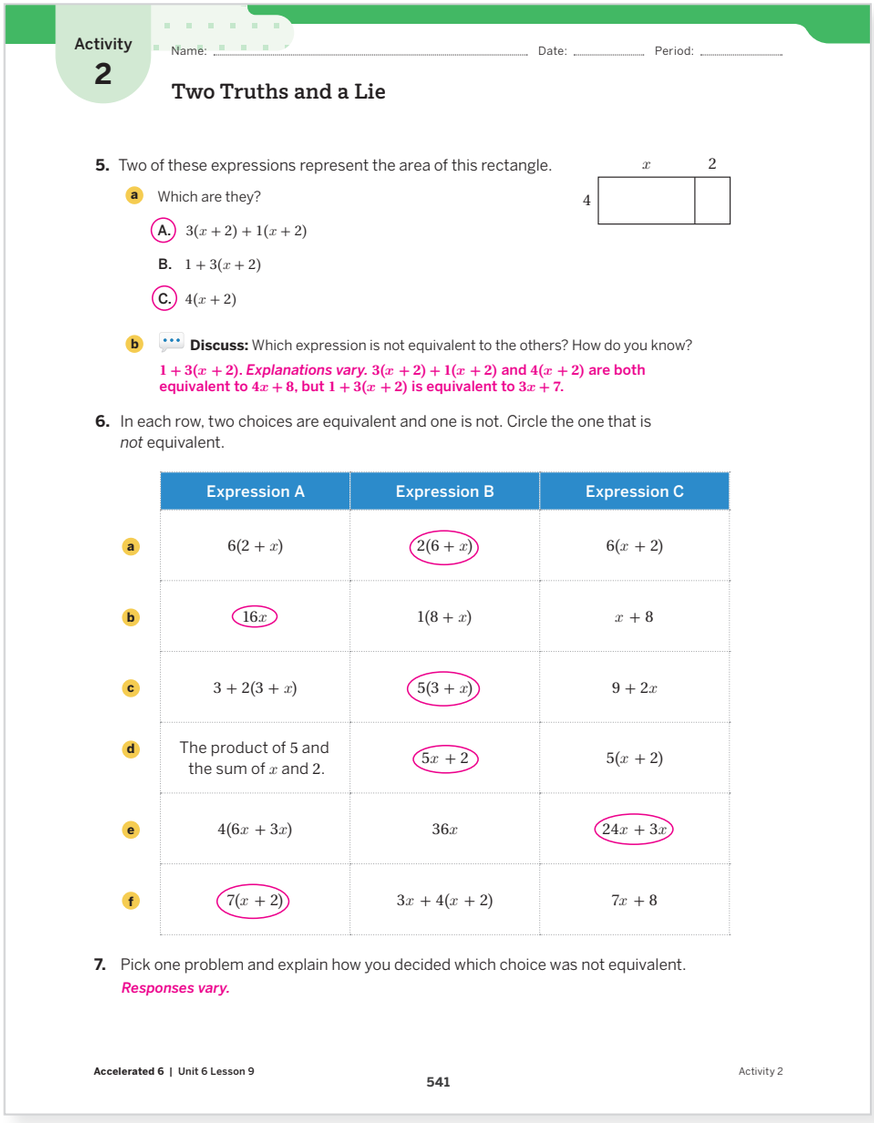
Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Encourage students to reference strategies from the beginning of the activity.
Use a variety of strategies, such as: • Drawing area diagrams. • Testing different values for x to check for equivalence. • Using the distributive property to rewrite equivalent expressions.	**Strengthen** Invite students to share their strategies during the Connect.
Would like to explore equivalent expressions further.	**Stretch** Invite students to create their own problem on a slip of paper, then trade with a classmate.

Note: Some students may make connections between these expressions and the *associative property* or *commutative property* from previous grades.

ELL **English Language Learners** Invite students to write their explanation in their primary language first and then translate it to English. **(Writing)**

3 Connect

Direct students' attention to the question and invite them to share their thinking with a partner before beginning a whole class discussion. Consider recording student ideas and inviting students to add others' ideas to their Student Edition as they are shared. **(MTR.4.1)**



Activity 2

Name: Date: Period:

Two Truths and a Lie

5. Two of these expressions represent the area of this rectangle.

a. Which are they?

A. $3(x + 2) + 1(x + 2)$

B. $1 + 3(x + 2)$

C. $4(x + 2)$

b. **Discuss:** Which expression is not equivalent to the others? How do you know?

$1 + 3(x + 2)$. *Explanations vary.* $3(x + 2) + 1(x + 2)$ and $4(x + 2)$ are both equivalent to $4x + 8$, but $1 + 3(x + 2)$ is equivalent to $3x + 7$.

6. In each row, two choices are equivalent and one is not. Circle the one that is *not* equivalent.

	Expression A	Expression B	Expression C
a	$6(2 + x)$	$2(6 + x)$	$6(x + 2)$
b	$16x$	$1(8 + x)$	$x + 8$
c	$3 + 2(3 + x)$	$5(3 + x)$	$9 + 2x$
d	The product of 5 and the sum of x and 2.	$5x + 2$	$5(x + 2)$
e	$4(6x + 3x)$	$36x$	$24x + 3x$
f	$7(x + 2)$	$3x + 4(x + 2)$	$7x + 8$

7. Pick one problem and explain how you decided which choice was not equivalent.

Responses vary.

Accelerated 6 | Unit 6 Lesson 9 541 Activity 2

Key Takeaway: Drawing models, substituting values, and rewriting expressions are strategies for deciding whether two expressions are equivalent. When rewriting expressions, make sure to distribute to both terms, multiply before adding, and only add like terms.

Whole Class | 5 min

Synthesis

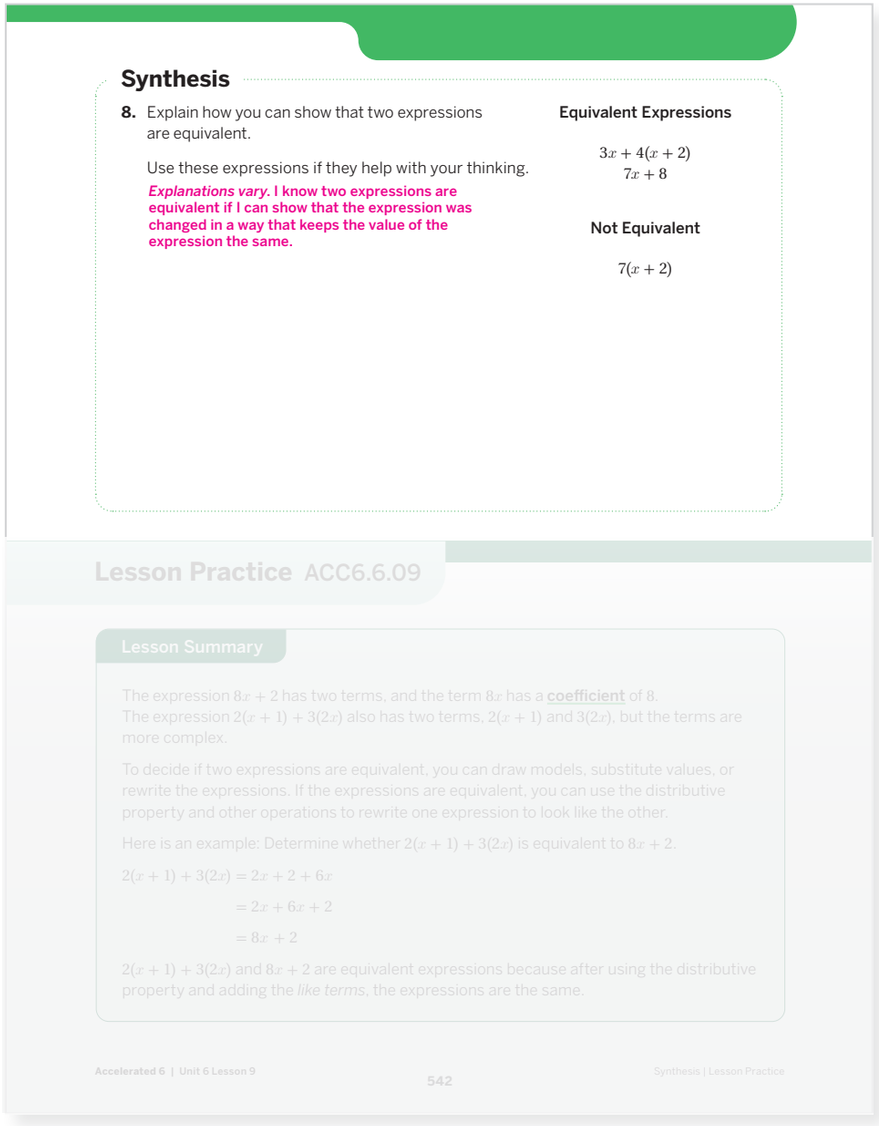
Purpose: Students synthesize their understanding of equivalent expressions.

Invite students to respond independently, and then share their thinking with a partner.

Look for and share a variety of ideas, such as:
- Drawing and annotating area diagrams.
- Substituting in values for x to check for equivalence.
- Using the distributive property and other properties to rearrange one expression so that it matches one of the other expressions.

MLR **MLR2: Collect and Display** Use this routine to formalize the definition of **coefficient** using students' language. Display any new language, examples, or diagrams publicly for students to reference later.

Note: If time allows, invite students to share with a partner what makes sense about each strategy and what questions they still have.



Synthesis

8. Explain how you can show that two expressions are equivalent.

Use these expressions if they help with your thinking.

Explanations vary. I know two expressions are equivalent if I can show that the expression was changed in a way that keeps the value of the expression the same.

Equivalent Expressions

$3x + 4(x + 2)$
$7x + 8$

Not Equivalent

$7(x + 2)$

Lesson Practice ACC6.6.09

Lesson Summary

The expression $8x + 2$ has two terms, and the term $8x$ has a coefficient of 8.
The expression $2(x + 1) + 3(2x)$ also has two terms, $2(x + 1)$ and $3(2x)$, but the terms are more complex.

To decide if two expressions are equivalent, you can draw models, substitute values, or rewrite the expressions. If the expressions are equivalent, you can use the distributive property and other operations to rewrite one expression to look like the other.

Here is an example: Determine whether $2(x + 1) + 3(2x)$ is equivalent to $8x + 2$.

$2(x + 1) + 3(2x) = 2x + 2 + 6x$
$= 2x + 6x + 2$
$= 8x + 2$

$2(x + 1) + 3(2x)$ and $8x + 2$ are equivalent expressions because after using the distributive property and adding the *like terms*, the expressions are the same.

Accelerated 6 | Unit 6 Lesson 9 542 Synthesis | Lesson Practice

Lesson Takeaway: Two expressions are equivalent to each other if one can be rewritten to look the same as the other. Strategies like using area models and the distributive property are helpful for rewriting expressions.

Independent | 5 min

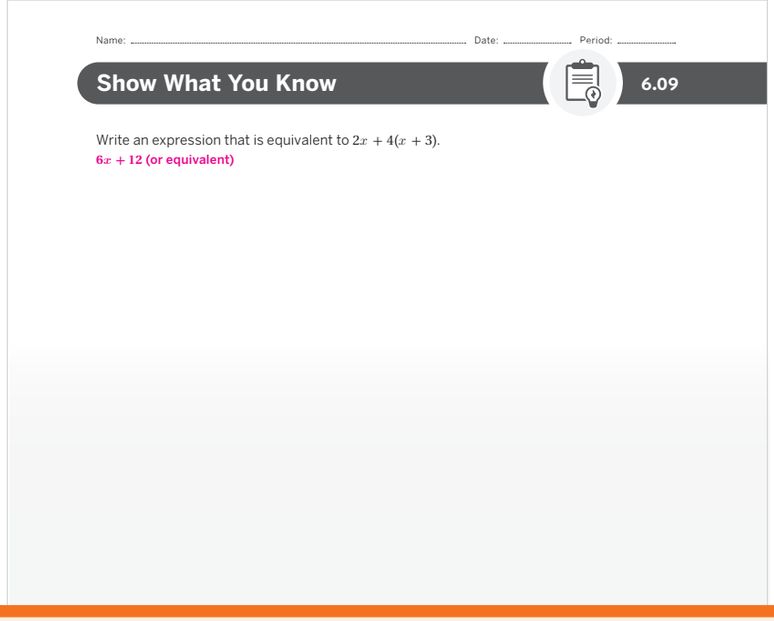
Show What You Know

Purpose: Students demonstrate their understanding by writing an equivalent expression.

Today's Goals

1. **Goal:** Analyze expressions to determine if they are equivalent.
2. **Language Goal:** Describe strategies for determining which expressions are equivalent. **(Writing, Speaking, and Listening)**

Show What You Know (Assessment Resources)



Name: _____ Date: _____ Period: _____

Show What You Know 6.09

Write an expression that is equivalent to $2x + 4(x + 3)$.

$6x + 12$ (or equivalent)

D Differentiation Use after Lesson 9

Support

Provide targeted intervention for students by using these resources.

If students need support with applying strategies to write equivalent expressions:

» **Respond to Student Thinking:**

- Revisit **Lesson 7, Activity 2**. Invite students to look for connections between visual representations and algebraic expressions, and encourage them to verify whether their two expressions are equivalent.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite them to explore the **Sub-Unit 2 Extension Activities**.
- Invite students to create other Two Truths and a Lie problems.

Professional Learning

Reflect on students' progress with expressions with variables in Sub-Units 1–2.

- In what ways have students' understanding and skills grown?
- What concepts do students need additional support with?
- How can you use students' strengths to help everyone feel prepared for their upcoming assessment?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

8. Explain how you can show that two expressions are equivalent.

Use these expressions if they help with your thinking.

Explanations vary. I know two expressions are equivalent if I can show that the expression was changed in a way that keeps the value of the expression the same.

Equivalent Expressions

$3x + 4(x + 2)$

$7x + 8$

Not Equivalent

$7(x + 2)$

Lesson Practice ACC6.6.09

Lesson Summary

The expression $8x + 2$ has two terms, and the term $8x$ has a **coefficient** of 8. The expression $2(x + 1) + 3(2x)$ also has two terms, $2(x + 1)$ and $3(2x)$, but the terms are more complex.

To decide if two expressions are equivalent, you can draw models, substitute values, or rewrite the expressions. If the expressions are equivalent, you can use the distributive property and other operations to rewrite one expression to look like the other.

Here is an example: Determine whether $2(x + 1) + 3(2x)$ is equivalent to $8x + 2$.

$$2(x + 1) + 3(2x) = 2x + 2 + 6x$$
$$= 2x + 6x + 2$$
$$= 8x + 2$$

$2(x + 1) + 3(2x)$ and $8x + 2$ are equivalent expressions because after using the distributive property and adding the *like terms*, the expressions are the same.

Lesson Practice
ACC6.6.09

Name: _____ Date: _____ Period: _____

1. Complete the table.

Rectangle	2 by 4 and x	5 by m and 3	3 by $2a$ and b
Product	$2(4 + x)$	$5(m + 3)$	$3(2a + b)$
Sum	$8 + 2x$	$5m + 15$	$6a + 3b$

2. Latifa and Joel are trying to rewrite $8y + 24$ as a product expression.

Latifa: $8(y + 3)$

Joel: $2(4y + 12)$

Are Latifa's and Joel's expressions both equivalent to $8y + 24$? Circle one.

(Yes) No

Explain your thinking.

Explanations vary. Latifa's expression is equivalent because 8 times y is $8y$, and 8 times 3 is 24. Joel's expression is equivalent because 2 times $4y$ is $8y$, and 2 times 12 is 24.

Problems 3–5: Determine whether each pair of expressions are equivalent.

	Expression A	Expression B	Equivalent?
3.	$2(x + 8) + 3$	$2x + 19$	(Yes) No
4.	$5 + 2(y + 4)$	$7y + 28$	Yes (No)
5.	$3(z + 1) + 3(z + 1)$	$6(z + 1)$	(Yes) No

Lesson Practice
ACC6.6.09

Name: _____ Date: _____ Period: _____

6. Complete the table.

Product Expression	Sum Expression
$4(x + 2)$	$4x + 8$
$(6 + 8)d$	$6d + 8d$
$(10 + 7)m$	$10m + 7m$
$3(2b + 5)$	$6b + 15$
$6(u + 2t)$	$6u + 12t$

7. The area of a rectangle is $30 + 12x$.

List three possibilities for the length and the width of the rectangle.

Responses vary.

Length	Width
3	$10 + 4x$
2	$15 + 6x$
6	$5 + 2x$

FAST Practice

8. Select *all* the expressions that are equivalent to $4x + 8$.

- ☑ A. $4(x + 2)$
- ☐ B. $(4 + 8)x$
- ☑ C. $2(2x + 4)$
- ☐ D. $2(2x + 6)$
- ☑ E. $4 + 4(x + 1)$

Spiral Review

Problems 9–11: Solve each equation. Show your thinking.

9. $x + 5 = 11$

$x = 6$. Strategies vary.

$x + 5 - 5 = 11 - 5$

$x = 6$

10. $6y = 18$

$y = 3$. Strategies vary.

$6y \div 6 = 18 \div 6$

$y = 3$

11. $\frac{w}{5} = 15$

$w = 75$. Strategies vary.

$\frac{w}{5} \cdot 5 = 15 \cdot 5$

$w = 75$

Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1–6	2	MA.6.AR.1.4
	7	3	MA.6.AR.1.4
FAST Practice	8	2	MA.6.AR.1.4
Spiral Review			
	9	1	MA.6.AR.2.2
	10–11	1	MA.6.AR.2.3

Assess and Respond

Sub-Unit Quiz 1 & 2

Independent | 45 min

Facilitation: Assign the Sub-Unit Quiz to learn about your students' understanding of the concepts and skills so far in this unit. If time allows, consider inviting students to complete one or more of the tasks in the Unit Synthesis and Reflection.

Item Analysis

Problem(s)	Concept or skill	Addressed in	DOK	FL B.E.S.T. Math Benchmarks
1	Using the structure of equations of the form $x + p = q$ and $px = q$ to determine whether given values are solutions	Lesson 4	1	MA.6.AR.2.1
2	Using the structure of expressions to represent areas of rectangles	Lesson 8	1	MA.6.AR.1.4
3a	Using the structure of equations of the form $x + p = q$ and $px = q$ to determine their solutions	Lessons 2, 3	1	MA.6.AR.2.2
3b, 3c	Using the structure of equations of the form $x + p = q$ and $px = q$ to determine their solutions	Lesson 5	1	MA.6.AR.2.4
4a, 4c	Using the structure of algebraic expressions to evaluate equivalent expressions	Lessons 7, 8	1	MA.6.AR.1.3, MTR.5.1
4b	Using the structure of algebraic expressions to write equivalent expressions	Lesson 9	2	MA.6.AR.1.4, MTR.5.1
5a	Making sense of real-world situations and modeling them with equations	Lesson 4	2	MA.6.AR.1.1, MA.6.AR.2.4, MTR.6.1, MTR.7.1
5b, 5c	Describing situations for which given equations could model, solving those equations, and interpreting the solutions within context	Lesson 4	3	MA.6.AR.2.3, MTR.6.1, MTR.7.1

Assessment Resources

- Student Assessments
- Assessment Rubrics with Answers
- Show What You Knows and Answer Keys

Intervention and Extension Resources

- Mini-Lessons
- Extensions

Name: ..... Date: ..... Period: .....

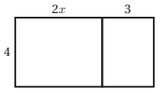
Sub-Unit Quiz

Unit 6

1. Select *all* the equations where $x = 6$ is a solution.

- ☐ **A.** $4 = 2 + x$
- ☐ **B.** $x - 11 = 5$
- ☑ **C.** $12 = 2x$
- ☐ **D.** $2 = 3x$
- ☑ **E.** $\frac{1}{2}x = 3$

2. Which expression represents the area of this rectangle?



- **A.** $4(2x + 12)$
- **B.** $2(4x + 3)$
- **C.** $8x + 12$
- **D.** $8x + 3$

3. Determine the solution to each equation.

a $3 + x = 10$
$x = 7$

b $1.8 = 2x$
$x = 0.9$

c $\frac{1}{3}x = 9$
$x = 27$

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Name: ..... Date: ..... Period: .....

Sub-Unit Quiz (continued)

Unit 6

4. Here is an expression: $3(2a + 6)$.

a What is the value of this expression when $a = 4$?
42

b Write an equivalent expression. Draw a rectangle if it helps with your thinking.
Responses vary.
- $6a + 18$
- $6(a + 3)$
- $3(6 + 2a)$

c What is the value of your expression when $a = 4$?
42. Note: Students' responses depend on their responses in Problem 4b. Students who respond 42 or who correctly evaluate their expressions are considered correct.

5. Guiying has $10 to buy tacos that cost $2.50 each. Guiying can buy x tacos in total.

a Which equation represents this situation?

A. $10x = 2.50$ **B.** $2.50x = 10$ **C.** $x + 2.50 = 10$ **D.** $x + 10 = 2.50$

b Here is a new equation: $12 + y = 20$. Describe a situation that could represent this equation.
Responses vary. Prisha has 12 eggs. She needs 20 eggs to make an egg dish for a party. How many more eggs does she need?

c Determine the value of y.
$y = 8$

Explain what it means in your situation.
Explanations vary. 8 represents how many more eggs Prisha needs to have a total of 20 eggs.

D Differentiation (Sub-Unit Quiz 1 & 2)

Note: To **strengthen** and **stretch** students' learning, refer to the differentiation resources suggested throughout this unit and in the Unit Overview.

Sub-Unit Goals	Problem(s)	To respond to student thinking, consider:

Sub-Unit 1:
• Write and solve equations of the form $x + p = q$ and $px = q$. **(Lessons 1–5)**
	3a	**Support**
		• **Teacher Move:** Consider revisiting:
		  • Lesson 2 *(Five Equations)*
		  • Lesson 3 *(Hanging Around)*

		Strengthen
		• **Challenge Creator:**
		  • Lesson 1 *(Weight for It)*
		  • Lesson 3 *(Hanging Around)*

	1, 5a–c	**Support** • **Mini-Lesson:** *Writing and Solving Equations* **Strengthen** • **Teacher Move:** Consider revisiting: • Lesson 4 *(Swap and Solve)*
	3b, 3c	**Support** • **Teacher Move:** Consider revisiting: • Lesson 5 *(Solutions of a Different Kind)*
		Strengthen • **Repeated Challenges:** • Lesson 5 *(Solutions of a Different Kind)*

Sub-Unit 2:
Use properties to write equivalent expressions with variables. **(Lessons 6–9)**
	2, 4a, 4c	**Support**
		• **Mini-Lesson:** *Writing Equivalent Expressions Using the Area Model*
		• **Teacher Move:** Consider revisiting:
		  • Lesson 7 *(Border Tiles)*
		  • Lesson 8 *(Products and Sums)*

		Strengthen • **Challenge Creator:** • Lesson 8 *(Products and Sums)*
		Stretch • **You're invited to explore more:** • Lesson 6 *(Vari-Apples)*
	4b	**Support** **Strengthen** • **Teacher Move:** Consider revisiting: • Lesson 9 *(Equivalent Expressions)*

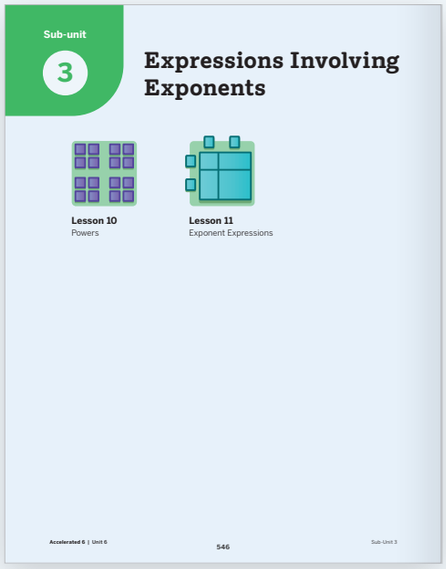
Sub-Unit 3

Expressions Involving Exponents

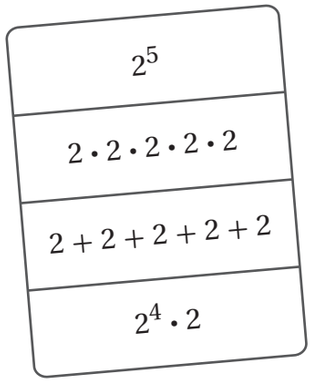
Sub-Unit 3 Goals

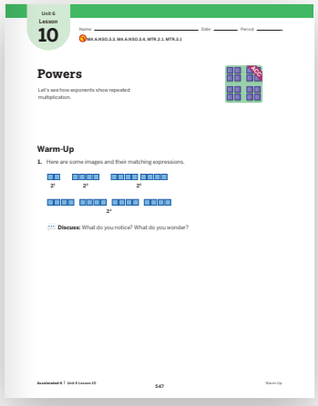
- **Goal:** Evaluate numerical and variable expressions with natural number exponents and one or more other operations.
- **Language Goal:** Describe a pattern that could be expressed using exponents.
- **Language Goal:** Interpret expressions with exponents using language like *to the power.*

In this sub-unit, students extend their work with expressions to evaluate numerical and variable expressions with natural number **exponents**.



Math That Matters Most

Concept or Skill	For example . . .
Evaluate numerical and variable expressions with whole number exponents and one or more other operations.	Determine the value of this expression. $2 \cdot 4^2$ **32**
Explain the meaning of an expression with an exponent, such as 2^5.	One expression in this group is not equivalent to the others. **2 + 2 + 2 + 2 + 2** 
Rewrite composite whole numbers as a product of prime factors using exponents.	Rewrite 45 as a product of prime factors using exponents. **$3^2 \cdot 5$**


Unit 6
Lesson
10

Powers

Let's see how exponents show repeated multiplication.

Warm-Up

1. Here are some images and their matching expressions.

Discuss: What do you notice? What do you wonder?

547

Powers

What Are Exponents?

Let's see how exponents show repeated multiplication.

Learning Outcomes

Today's Goals

1. **Goal:** Decide if two expressions involving *exponents* are equivalent and explain the reasoning.
2. **Language Goal:** Explain the meaning of an expression with an *exponent*, such as 3^5. **(Writing, Speaking, and Listening)**

Students are introduced to **exponents**, both when the **base** is a whole number and when it is a fraction. They use visuals to explore how changing an exponent changes the value of an expression and use repeated reasoning to connect these visuals to expressions involving exponents. They also use the language of equivalent expressions from earlier in the unit to write expressions with exponents as repeated multiplication and vice versa.

Coherence

Prior Learning

In Lesson 7, students used border tiles to generate equivalent expressions.

Future Learning

In Lesson 11, students will use the order of operations to evaluate expressions with two or more terms, one of which has a whole number exponent.

Connections

- Students are introduced to the concept of exponent using patterns and models. (mathematical)

FL B.E.S.T. Math Benchmarks

Addressing

MA.6.NSO.3.3
Evaluate positive rational numbers and integers with natural number exponents.

MA.6.NSO.3.4
Express composite whole numbers as a product of prime factors with natural number exponents.

Mathematical Thinking and Reasoning Standards: MTR.2.1, MTR.3.1

ELA Expectations: ELA.K12.EE.1.1, ELA.K12.EE.4.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.5.NSO.2.1	MA.7.NSO.1.1
MA.5.FR.2.2	MA.7.NSO.2.1

Vocabulary

New Vocabulary

base (of a power)
exponent
prime factorization

Lesson at a Glance

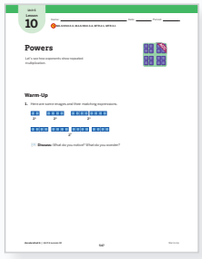
~ 45 min

MA.6.NSO.3.3, MA.6.NSO.3.4

Warm-Up

Pairs | 5 min

Students use the **Notice and Wonder** routine to visually make sense of exponents.



Activity 1

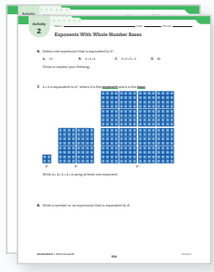
Pairs | 10 min

Students use visuals and expressions to describe exponents with whole number bases. **(MTR.2.1)**

Activity 2

Pairs | 10 min

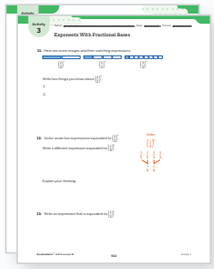
Students make connections between expressions with exponents and repeated multiplication, defining the terms **exponent**, **base (of a power)**, and **prime factorization**. **(MTR.2.1)**



Activity 3

Pairs | 10 min

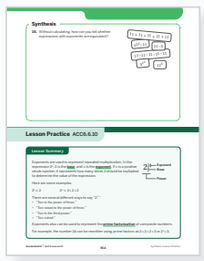
Students extend what they explored in Activities 1–2 to powers with bases that are fractions.



Synthesis

Whole Class | 5 min

Students synthesize their understanding of exponents, including making connections between exponents and repeated multiplication. **MLR2: Collect and Display**



Show What You Know

Independent | 5 min

Students demonstrate their understanding by grouping equivalent expressions involving repeated addition, multiplication, and exponents.

Prep Checklist

This lesson includes:

- Student Edition
- Show What You Know

Pairs | 5 min

Warm-Up

Purpose: Students use the Notice and Wonder routine to visually make sense of exponents.

1 Launch

Direct students' attention to how the image and the expression change.

Use the Notice and Wonder routine, and invite students to discuss with a partner prior to facilitating a whole-class discussion.

2 Connect

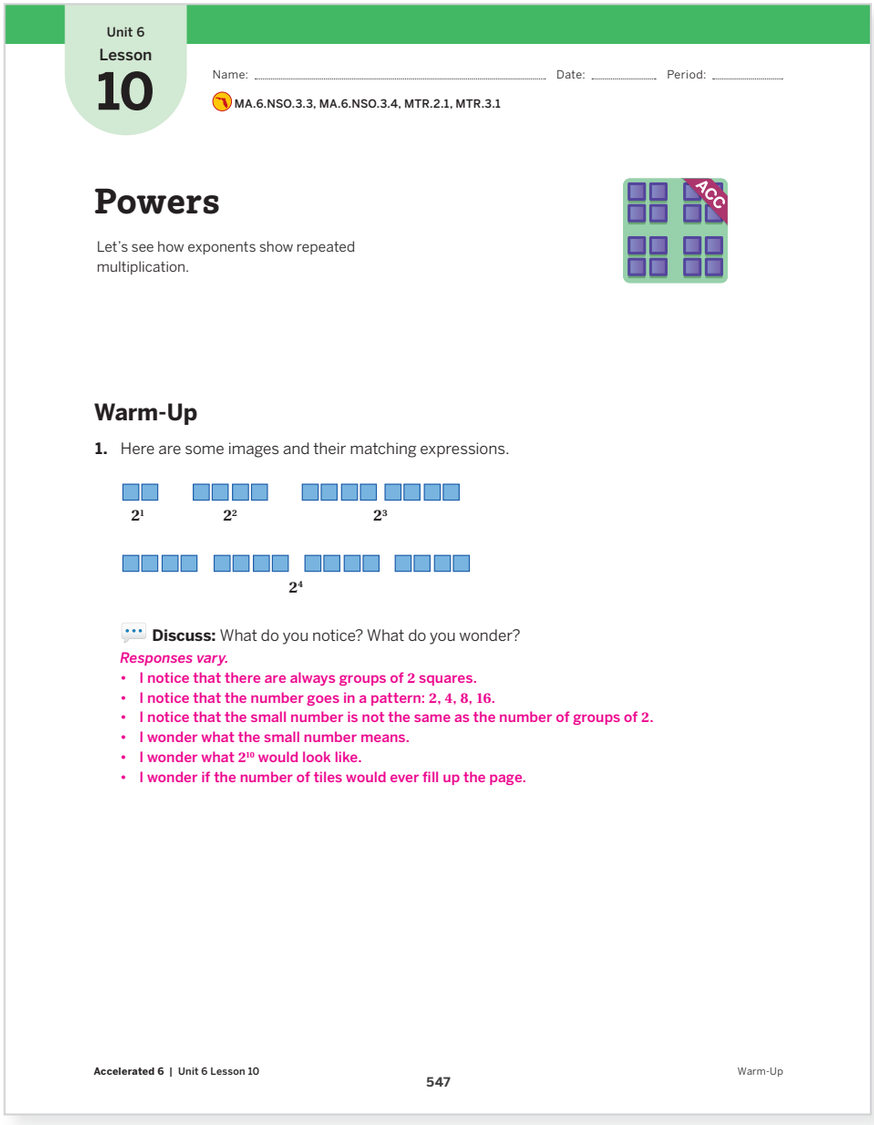
Invite students to share what they notice and wonder.

To surface connections, invite students to share if they have ever seen an expression that looks like this before.

Discuss how to read each expression aloud. For example, 2^4 can be read as:

- "Two to the fourth power."
- "Two raised to the power of four."
- "Two to the fourth."

Note: It may be helpful to introduce the term *squared* to prepare students for Lesson 11.



Unit 6
Lesson
10

Name: Date: Period:

MA.6.NSO.3.3, MA.6.NSO.3.4, MTR.2.1, MTR.3.1

Powers

Let's see how exponents show repeated multiplication.

Warm-Up

1. Here are some images and their matching expressions.

2^1 2^2 2^3 2^4

Discuss: What do you notice? What do you wonder?

Responses vary.

- I notice that there are always groups of 2 squares.
- I notice that the number goes in a pattern: 2, 4, 8, 16.
- I notice that the small number is not the same as the number of groups of 2.
- I wonder what the small number means.
- I wonder what 2^{10} would look like.
- I wonder if the number of tiles would ever fill up the page.

Accelerated 6 | Unit 6 Lesson 10 547 Warm-Up

Pairs | 10 min

Activity 1 Powers of 2

Purpose: Students use visuals and expressions to describe exponents with whole number bases. **(MTR.2.1)**

1 Launch

Consider asking, "How is 2^3 related to 2^4?"

2 Monitor

To support students getting started, invite them to return to the images from the Warm-Up.

ELL **English Language Learners** Provide sentence frames to help students explain their strategy (e.g., First, _____. Then, _____.). **(Writing and Speaking)**

Look for several expressions and values for 2^5.

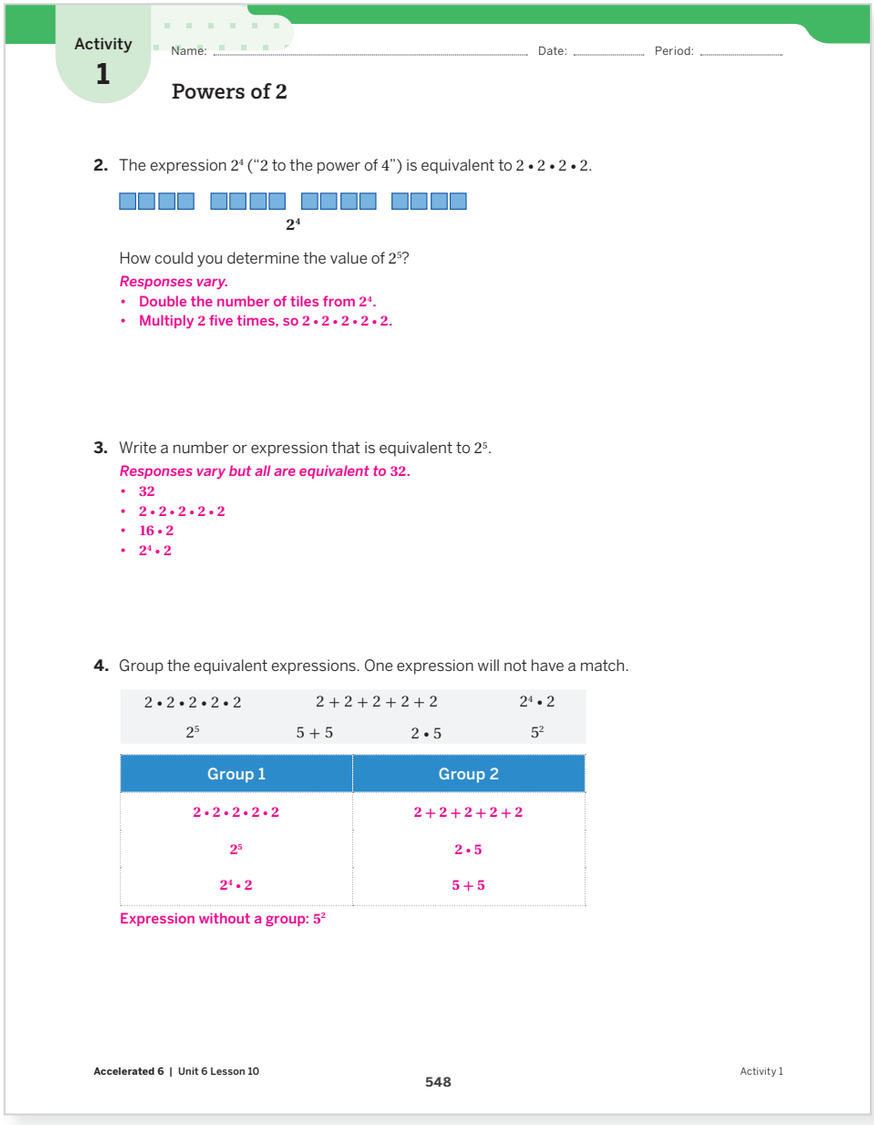
Pause to discuss students' strategies for evaluating 2^5.

Encourage students to share their reasoning and work together to reach an agreement about how to group the expressions.

A **Accessibility: Conceptual Processing** To support students in making connections between new problems and prior work, invite them to revisit writing an equivalent value or expression for 2^5 and ask, "How are the expressions you wrote for 2^5 related to what you are trying to do here?"

Look and listen for strategies and language that connect addition, multiplication, and exponents, such as using terms like *doubling* or *repeated multiplication*, and annotating diagrams. Encourage students to share their thinking during the Connect. **(MTR.2.1)**

Activity 1 continued >



Activity 1

Name: Date: Period:

Powers of 2

2. The expression 2^4 ("2 to the power of 4") is equivalent to $2 \cdot 2 \cdot 2 \cdot 2$.

2^4

How could you determine the value of 2^5?

Responses vary.

- Double the number of tiles from 2^4.
- Multiply 2 five times, so $2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$.

3. Write a number or expression that is equivalent to 2^5.

Responses vary but all are equivalent to 32.

- 32
- $2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$
- $16 \cdot 2$
- $2^4 \cdot 2$

4. Group the equivalent expressions. One expression will not have a match.

$2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$	$2+2+2+2+2$	$2^4 \cdot 2$	
2^5	$5+5$	$2 \cdot 5$	5^2

Group 1	Group 2
$2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$	$2+2+2+2+2$
2^5	$2 \cdot 5$
$2^4 \cdot 2$	$5+5$

Expression without a group: 5^2

Accelerated 6 | Unit 6 Lesson 10 548 Activity 1

Pairs | 10 min

Activity 1 Powers of 2 (continued)

Purpose: Students use visuals and expressions to describe exponents with whole number bases. **(MTR.2.1)**

2 Monitor

D Differentiation

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Invite students to select each expression one at a time and determine its value.
Think that $2^4 \cdot 2$ is not equivalent to 2^5 because it does not have five 2s.	**Support** Consider asking, "How could you write 2^4 using multiplication?"
Recognize that $2+2+2+2+2$ is the only expression with repeated addition.	**Strengthen** Consider asking, "How did you decide that this expression was not equivalent?" Invite these students to share during the Connect.
Would like to explore equivalent expressions more.	**Stretch** Encourage them to write other expressions that are equivalent to this group.

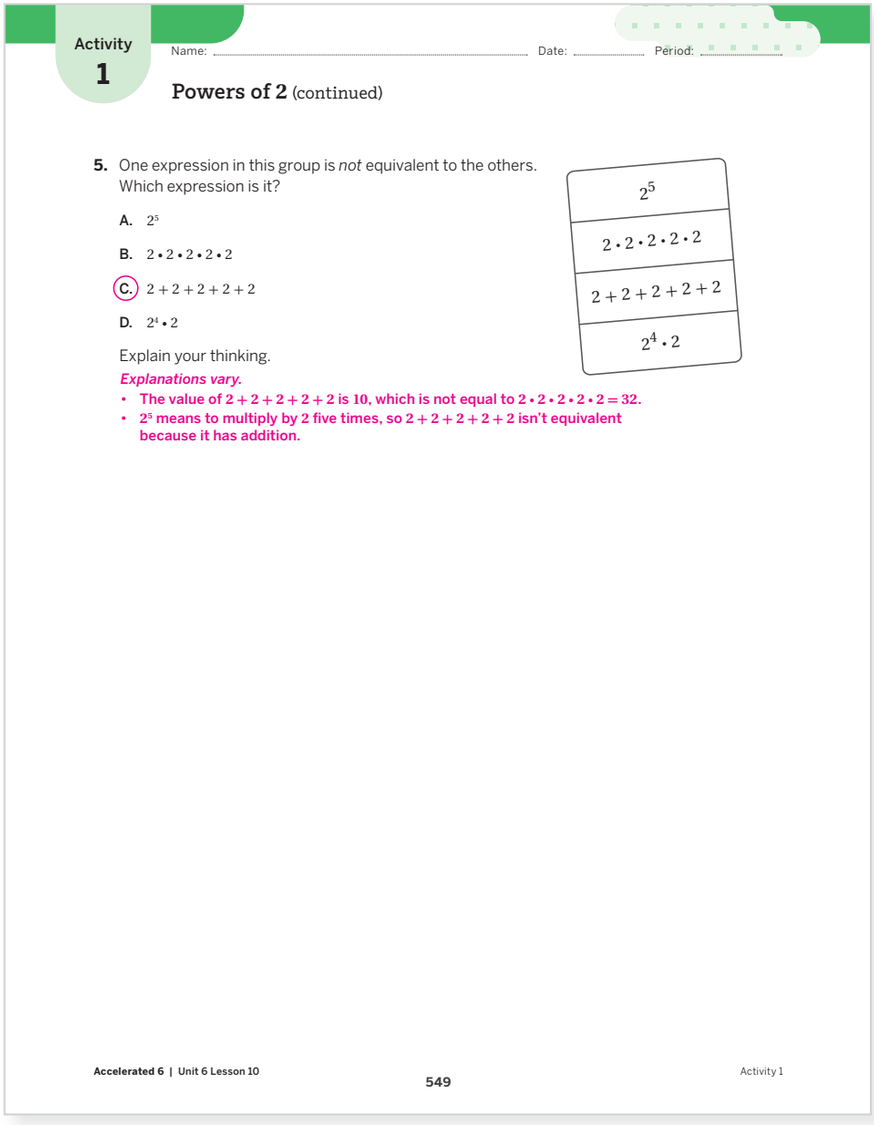
A Accessibility: Visual-Spatial Processing Invite students to read each expression aloud and come to an agreement about what it says, or to choose expressions that look similar and explain how they are different to support students in distinguishing between related expressions.

3 Connect

Invite students to share their responses, calling attention to any conflict or consensus.

Invite students to share their reasoning for how they chose which expression was not equivalent to the others. **(MTR.3.1)**

ELL English Language Learners Encourage students to rephrase their partners' reasoning. Provide sentence frames, such as, "I think you are saying _____. Is that correct?" or "First, you found _____ because _____. Is that correct?" **(Speaking and Listening)**



Activity 1

Name: Date: Period:

Powers of 2 (continued)

5. One expression in this group is *not* equivalent to the others. Which expression is it?

A. 2^5

B. $2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$

C. $2+2+2+2+2$

D. $2^4 \cdot 2$

Explain your thinking.

Explanations vary.

- The value of $2+2+2+2+2$ is 10, which is not equal to $2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 32$.
- 2^5 means to multiply by 2 five times, so $2+2+2+2+2$ isn't equivalent because it has addition.

Accelerated 6 | Unit 6 Lesson 10 549 Activity 1

Pairs | 10 min

Activity 2 Exponents With Whole Number Bases

Purpose: Students make connections between expressions with exponents and repeated multiplication, defining the terms **exponent**, **base (of a power)**, and **prime factorization**. **(MTR.2.1)**

Short on time? Consider omitting Problem 6.

1 Launch

Consider asking, "What does 3^4 represent?"

2 Monitor

Look for students who identify that both $3 \cdot 3 \cdot 3 \cdot 3$ and 81 are equivalent to 3^4. **(MTR.2.1)**

Encourage students to use the visual representations to make sense of $4 \cdot 4 \cdot 4 \cdot 4 \cdot 4$.

Look for several expressions to share during the Connect.

D Differentiation

Look for students who:	Respond to Student Thinking
Think that 4^3 represents 4.	**Support** Encourage them to return to the visual representations of the powers of 4.
Verbally describe 4^3 as "four thirds" or "three fours."	**Support** Review the different ways to read 4^3 aloud, making the connections between the term *power* and repeated multiplication.
Identify a variety of expressions equivalent to 4^3. **(MTR.2.1)**	**Stretch** Invite students to decide which is greater, 4^5 or 5^4, and then to convince a classmate.

Activity 2 continued >

Activity 2

Name: Date: Period:

Exponents With Whole Number Bases

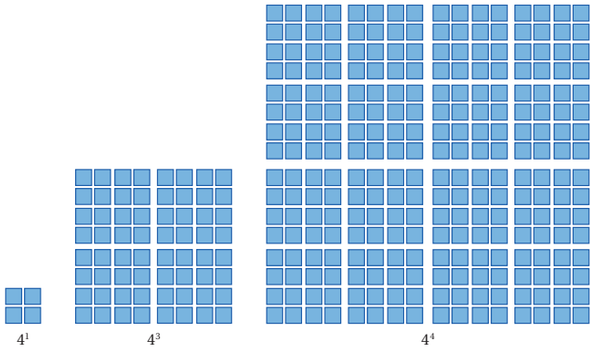
6. Select *one* expression that is equivalent to 3^4.

A. 12 B. $4 \cdot 4 \cdot 4$ C. $3 \cdot 3 \cdot 3 \cdot 3$ D. 81

Show or explain your thinking.

Explanations vary.

7. $4 \cdot 4$ is equivalent to 4^2, where 2 is the **exponent** and 4 is the **base**.



Write $4 \cdot 4 \cdot 4 \cdot 4 \cdot 4$ using at least one exponent.

Responses vary.

- 4^5
- $4^4 \cdot 4$

8. Write a number or an expression that is equivalent to 4^3.

Responses vary but all are equivalent to 64.

- 64
- $4 \cdot 4 \cdot 4$
- $16 \cdot 4$
- $4^2 \cdot 4$

Accelerated 6 | Unit 6 Lesson 10 550 Activity 2

Pairs | 10 min

Activity 2 Exponents With Whole Number Bases (continued)

Purpose: Students make connections between expressions with exponents and repeated multiplication, defining the terms **exponent**, **base (of a power)**, and **prime factorization**. **(MTR.2.1)**

Short on time? Consider omitting Problem 6.

2 Monitor

Listen for students who discuss the factor tree using the terms *factors* and *prime numbers*. Consider asking, "What are some examples of prime numbers? What are some non-examples?"

To support students getting started, consider asking, "How are the expressions alike? How are they different?"

Listen for a variety of explanations to share during the Connect.

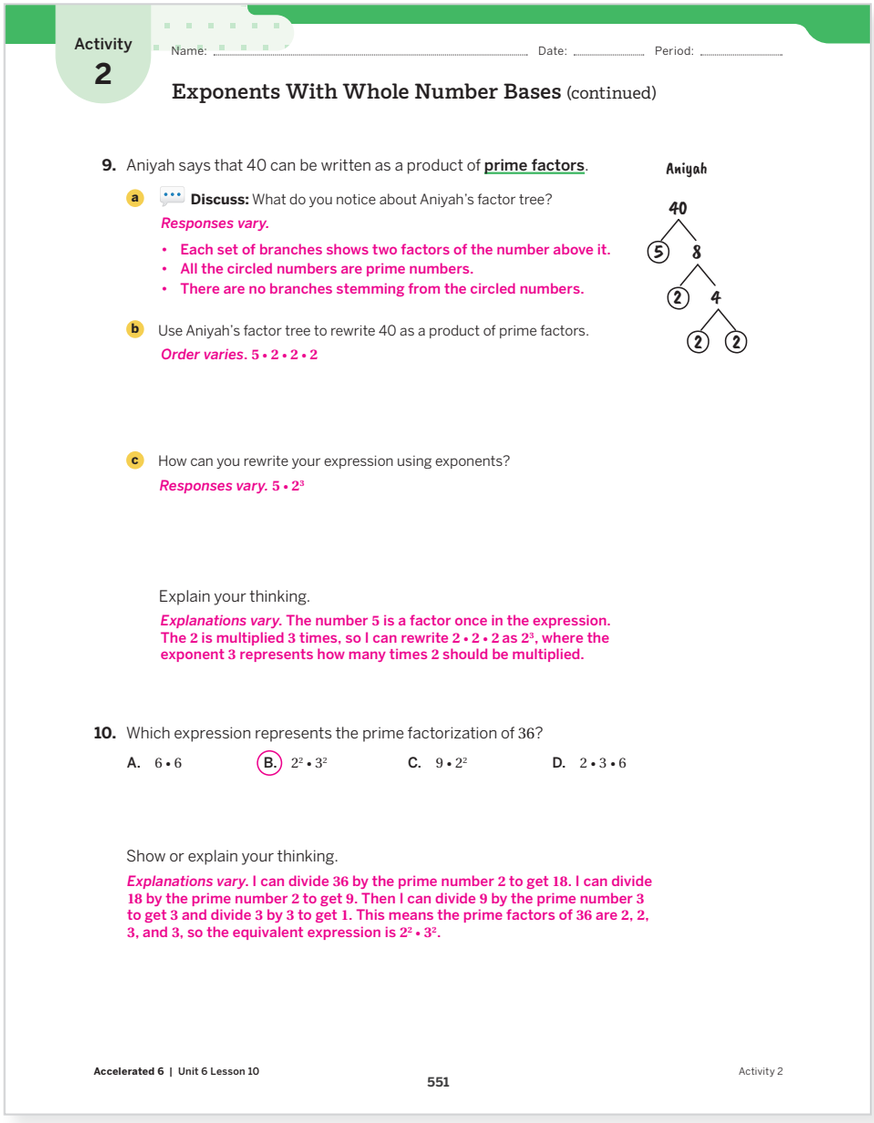
3 Connect

Create a class definition of the terms **base (of a power)**, **exponent**, and **prime factorization** using students' language and the glossary.

Consider asking, "How would you read 4^3 aloud? What is the base? What is the exponent?

Share student expressions for 4^3. Invite students to share how they know each expression is equivalent, making connections between the expressions and the visual representations. **(MTR.2.1)**

Invite students to share their explanations for representing 36 as a product of prime factors using exponents. **(MTR.2.1)**



Activity 2

Name: Date: Period:

Exponents With Whole Number Bases (continued)

9. Aniyah says that 40 can be written as a product of **prime factors**.

Aniyah

a. **Discuss:** What do you notice about Aniyah's factor tree?

Responses vary.

- Each set of branches shows two factors of the number above it.
- All the circled numbers are prime numbers.
- There are no branches stemming from the circled numbers.

b. Use Aniyah's factor tree to rewrite 40 as a product of prime factors.

Order varies. $5 \cdot 2 \cdot 2 \cdot 2$

c. How can you rewrite your expression using exponents?

Responses vary. $5 \cdot 2^3$

Explain your thinking.

Explanations vary. The number 5 is a factor once in the expression. The 2 is multiplied 3 times, so I can rewrite $2 \cdot 2 \cdot 2$ as 2^3, where the exponent 3 represents how many times 2 should be multiplied.

10. Which expression represents the prime factorization of 36?

A. $6 \cdot 6$ B. $2^2 \cdot 3^2$ C. $9 \cdot 2^2$ D. $2 \cdot 3 \cdot 6$

Show or explain your thinking.

Explanations vary. I can divide 36 by the prime number 2 to get 18. I can divide 18 by the prime number 2 to get 9. Then I can divide 9 by the prime number 3 to get 3 and divide 3 by 3 to get 1. This means the prime factors of 36 are 2, 2, 3, and 3, so the equivalent expression is $2^2 \cdot 3^2$.

Accelerated 6 | Unit 6 Lesson 10 551 Activity 2

Key Takeaway: Exponents express repeated multiplication. For example, the power $2^3 = 2 \cdot 2 \cdot 2$, where 2 is the **base** and 3 is the **exponent**. You can write the **prime factorization** of composite numbers using exponents. The prime factorization of 8 using exponents is 2^3.

Pairs | 10 min

Activity 3 Exponents With Fractional Bases

Purpose: Students extend what they explored in Activities 1–2 to powers with bases that are fractions.

1 Launch

To surface connections, consider inviting students to predict how bases that are fractions might act differently from bases that are whole numbers.

2 Monitor

A **Accessibility: Conceptual Processing** To support making connections between new problems and prior work, consider asking, "How are $\left(\frac{1}{2}\right)^2$ and 2^2 alike? How are they different?"

To support students getting started, consider asking:

- "Where do you see $\frac{1}{2}$ in the visual of $\left(\frac{1}{2}\right)^3$?"
- "What do you think the 3 represents in $\left(\frac{1}{2}\right)^3$?"

A **Accessibility: Conceptual Processing** Invite students to talk aloud about each step of Victor's work to help them verbalize what concepts make sense and which ones don't.

Look for strategies and language students used when creating equivalent expressions.

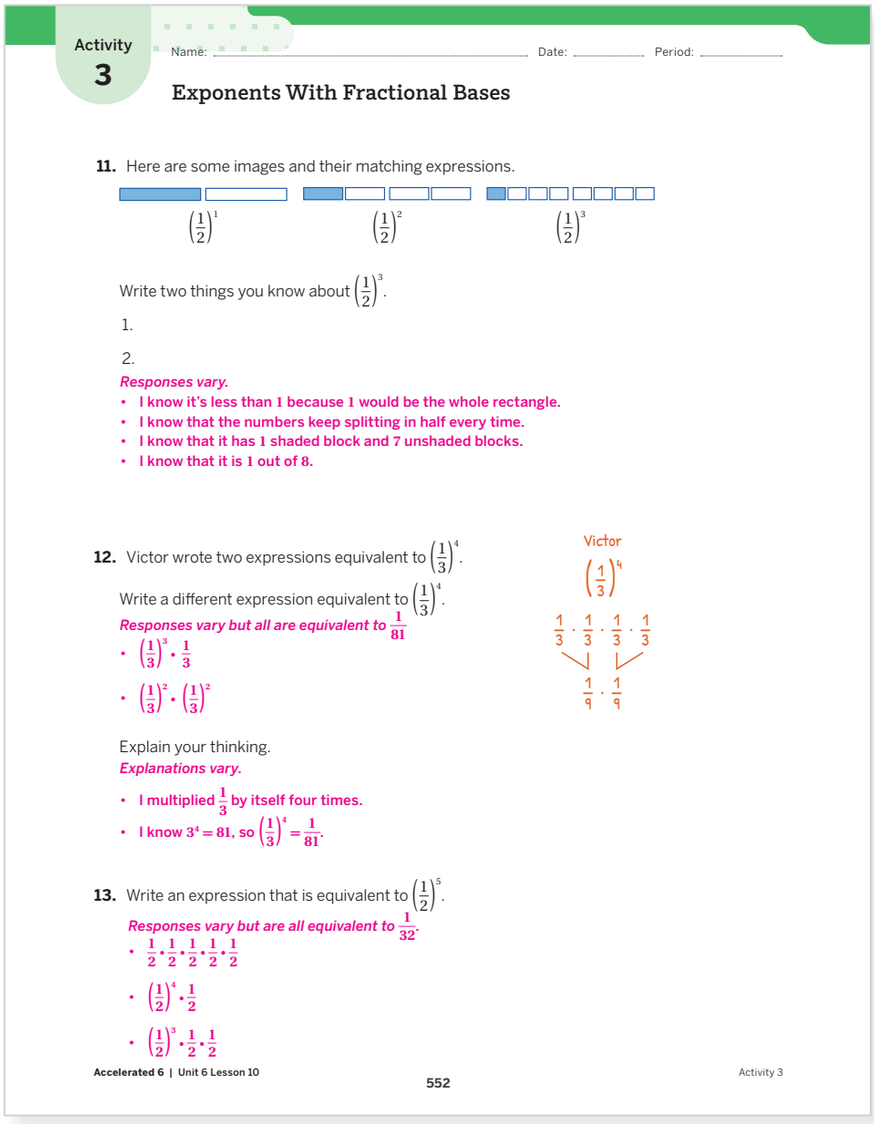
D **Differentiation**

Look for students who:	Respond to Student Thinking
Write an expression using operations other than exponents or multiplication that is equivalent to the power shown.	**Support** Encourage them to return to their previous work in this lesson. Invite them to make connections between $\left(\frac{1}{3}\right)^4$ and 3^4.
Write a variety of equivalent expressions using exponents and multiplication. **(MTR.2.1)**	**Strengthen** Invite students to compare their expressions with a partner and determine if their expressions are equivalent.

Pause to discuss students' strategies for writing equivalent expressions when the base is a fraction.

Note: Consider discussing the importance of parentheses when writing powers with fractional bases to prepare students for the next lesson on order of operations.

Activity 3 continued >



Activity 3

Name: Date: Period:

Exponents With Fractional Bases

11. Here are some images and their matching expressions.

$\left(\frac{1}{2}\right)^1$ $\left(\frac{1}{2}\right)^2$ $\left(\frac{1}{2}\right)^3$

Write two things you know about $\left(\frac{1}{2}\right)^3$.

1.

2.

Responses vary.

- I know it's less than 1 because 1 would be the whole rectangle.
- I know that the numbers keep splitting in half every time.
- I know that it has 1 shaded block and 7 unshaded blocks.
- I know that it is 1 out of 8.

12. Victor wrote two expressions equivalent to $\left(\frac{1}{3}\right)^4$.

Victor

$\left(\frac{1}{3}\right)^4$

$\frac{1}{3} \cdot \frac{1}{3} \cdot \frac{1}{3} \cdot \frac{1}{3}$

$\frac{1}{9} \cdot \frac{1}{9}$

Write a different expression equivalent to $\left(\frac{1}{3}\right)^4$.

Responses vary but all are equivalent to $\frac{1}{81}$

- $\left(\frac{1}{3}\right)^3 \cdot \frac{1}{3}$
- $\left(\frac{1}{3}\right)^2 \cdot \left(\frac{1}{3}\right)^2$

Explain your thinking.

Explanations vary.

- I multiplied $\frac{1}{3}$ by itself four times.
- I know $3^4 = 81$, so $\left(\frac{1}{3}\right)^4 = \frac{1}{81}$.

13. Write an expression that is equivalent to $\left(\frac{1}{2}\right)^5$.

Responses vary but are all equivalent to $\frac{1}{32}$.

- $\frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2}$
- $\left(\frac{1}{2}\right)^4 \cdot \frac{1}{2}$
- $\left(\frac{1}{2}\right)^3 \cdot \frac{1}{2} \cdot \frac{1}{2}$

Accelerated 6 | Unit 6 Lesson 10 552 Activity 3

Pairs | 10 min

Activity 3 Exponents With Fractional Bases (continued)

Purpose: Students extend what they explored in Activities 1–2 to powers with bases that are fractions.

2 Monitor

Listen for student strategies during partner discussions, and offer help or encouragement where needed.

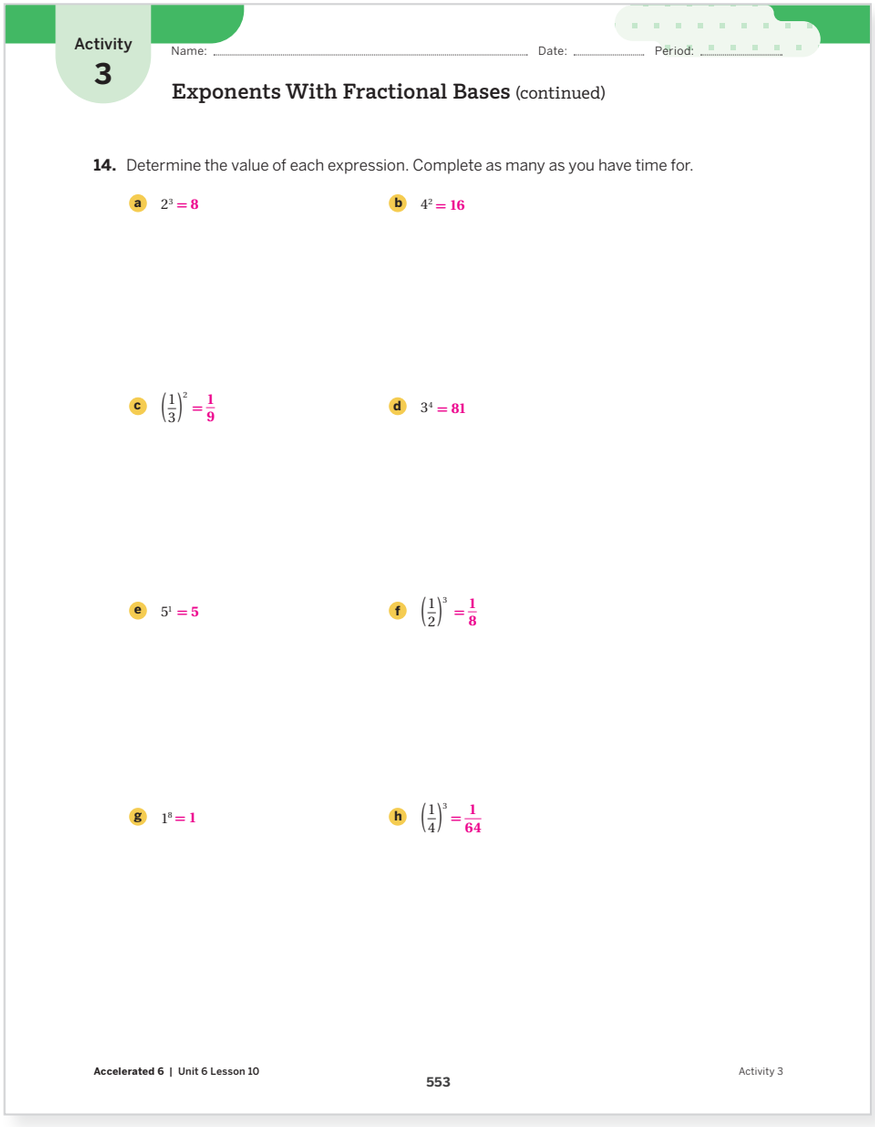
Invite students to check their answers with a partner. Encourage them to discuss and resolve any differences.

A **Accessibility: Memory and Attention** Consider inviting students with working memory challenges to record their work for each problem in their Student Edition to refer back to throughout the remainder of the lesson and unit.

3 Connect

Invite students to share their strategies for determining the value of each expression. **(MTR.3.1)**

Consider asking, "What does it mean for an expression to have an exponent of 1 like in 5^1?" if it does not come up naturally.



Activity 3

Name: Date: Period:

Exponents With Fractional Bases (continued)

14. Determine the value of each expression. Complete as many as you have time for.

a. $2^3 = 8$

b. $4^2 = 16$

c. $\left(\frac{1}{3}\right)^2 = \frac{1}{9}$

d. $3^4 = 81$

e. $5^1 = 5$

f. $\left(\frac{1}{2}\right)^3 = \frac{1}{8}$

g. $1^8 = 1$

h. $\left(\frac{1}{4}\right)^3 = \frac{1}{64}$

Accelerated 6 | Unit 6 Lesson 10 553 Activity 3

Whole Class | 5 min

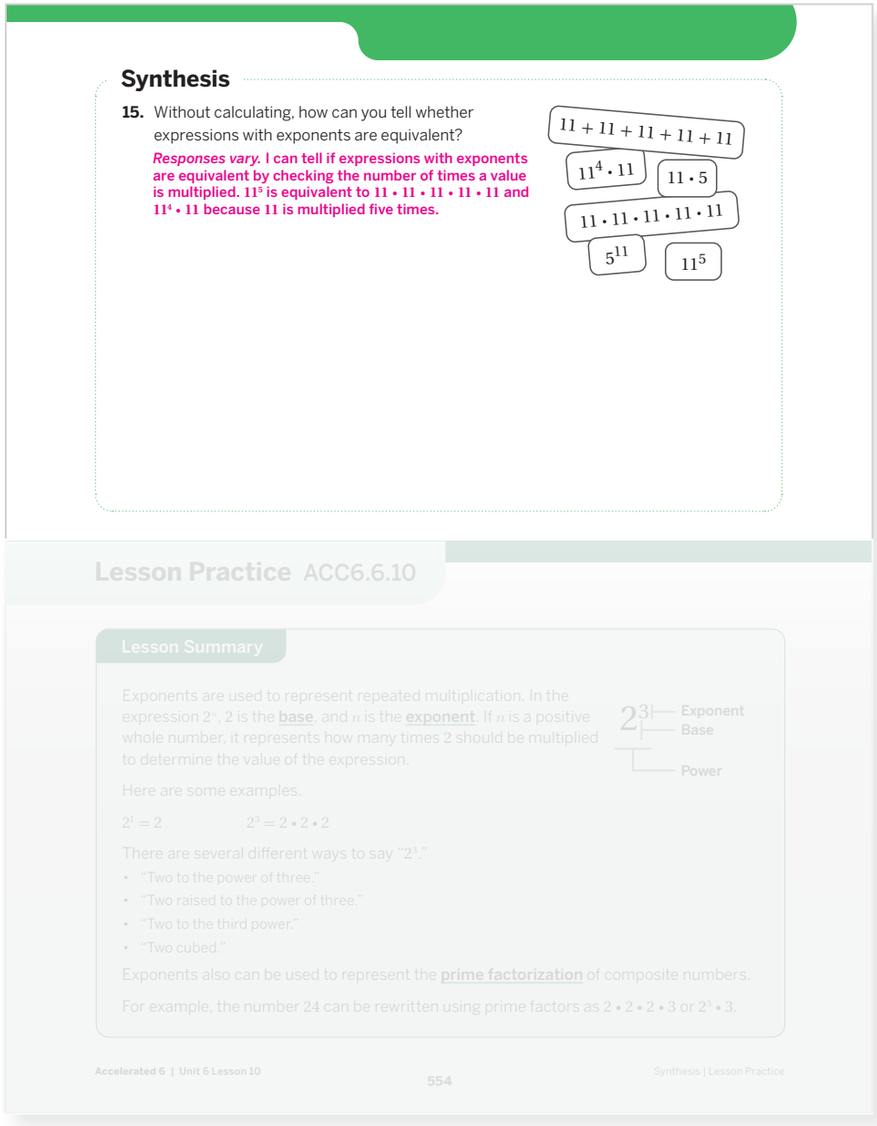
Synthesis

Purpose: Students synthesize their understanding of exponents, including making connections between exponents to repeated multiplication.

Invite students to respond independently, and then share their thinking with a partner.

Collect for and share a variety of ideas, such as:
- Rewriting powers as repeated multiplication.
- Using vocabulary such as *power*, *base*, *exponent*, and *prime factors* to justify their thinking.

MLR **MLR2: Collect and Display** Use this routine to formalize the definitions of *base*, *exponent*, and *prime factorization* using students' language. Display any new language, examples, or diagrams publicly for students to reference later.



Synthesis

15. Without calculating, how can you tell whether expressions with exponents are equivalent?

Responses vary. I can tell if expressions with exponents are equivalent by checking the number of times a value is multiplied. 11^5 is equivalent to $11 \cdot 11 \cdot 11 \cdot 11 \cdot 11$ and $11^4 \cdot 11$ because 11 is multiplied five times.

$11 + 11 + 11 + 11 + 11$

$11^4 \cdot 11$

$11 \cdot 5$

$11 \cdot 11 \cdot 11 \cdot 11 \cdot 11$

5^{11}

11^5

Lesson Practice ACC6.6.10

Lesson Summary

Exponents are used to represent repeated multiplication. In the expression 2^n, 2 is the base, and n is the exponent. If n is a positive whole number, it represents how many times 2 should be multiplied to determine the value of the expression.

2^3 — Exponent, Base, Power

Here are some examples.

$2^1 = 2$ $\quad 2^3 = 2 \cdot 2 \cdot 2$

There are several different ways to say "2^3."
- "Two to the power of three."
- "Two raised to the power of three."
- "Two to the third power."
- "Two cubed."

Exponents also can be used to represent the prime factorization of composite numbers.

For example, the number 24 can be rewritten using prime factors as $2 \cdot 2 \cdot 2 \cdot 3$ or $2^3 \cdot 3$.

Accelerated 6 | Unit 6 Lesson 10 554 Synthesis | Lesson Practice

Lesson Takeaway: **Exponents** express repeated multiplication whether the **base** is a whole number or a fraction. For example: $2^3 = 2 \cdot 2 \cdot 2$ and $\left(\frac{1}{2}\right)^3 = \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2}$.

Independent | 5 min

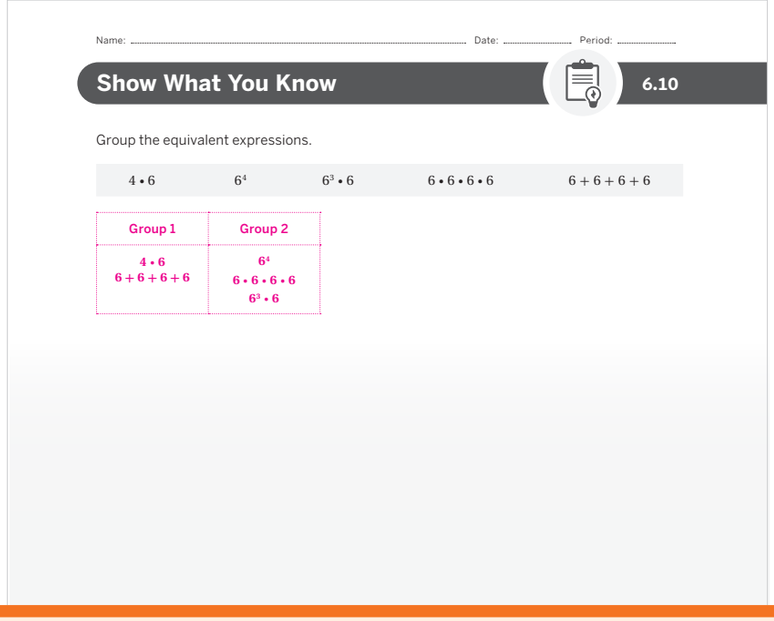
Show What You Know

Purpose: Students demonstrate their understanding by grouping equivalent expressions involving repeated addition, multiplication, and exponents.

Today's Goals

1. **Goal:** Decide if two expressions involving *exponents* are equivalent and explain the reasoning.
2. **Language Goal:** Explain the meaning of an expression with an *exponent*, such as 3^5. **(Writing, Speaking, and Listening)**

Show What You Know (Assessment Resources)



Name: _____ Date: _____ Period: _____

Show What You Know 6.10

Group the equivalent expressions.

$4 \cdot 6$ 6^4 $6^3 \cdot 6$ $6 \cdot 6 \cdot 6 \cdot 6$ $6 + 6 + 6 + 6$

Group 1	Group 2
$4 \cdot 6$ $6 + 6 + 6 + 6$	6^4 $6 \cdot 6 \cdot 6 \cdot 6$ $6^3 \cdot 6$

Differentiation Use after Lesson 10

Support

Provide targeted intervention for students by using these resources.

If students need support with determining whether two expressions that include exponents are equivalent:

» **Respond to Student Thinking:**

- Revisiting Activity 2 (particularly visualizations showing exponentiation) with students before providing individualized feedback on the relevant **Practice Problems**.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite them to explore the **Sub-Unit 3 Extension Activities**.

Professional Learning

In this lesson, students developed early ideas about **bases (of a power)** and **exponents**. How will their understanding of bases and exponents build in the next lesson?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

15. Without calculating, how can you tell whether expressions with exponents are equivalent?

Responses vary. I can tell if expressions with exponents are equivalent by checking the number of times a value is multiplied. 11^5 is equivalent to $11 \cdot 11 \cdot 11 \cdot 11 \cdot 11$ and $11^4 \cdot 11$ because 11 is multiplied five times.

Lesson Practice ACC6.6.10

Lesson Summary

Exponents are used to represent repeated multiplication. In the expression 2^n, 2 is the **base**, and n is the **exponent**. If n is a positive whole number, it represents how many times 2 should be multiplied to determine the value of the expression.

2^3 — 3: Exponent; 2: Base; Power

Here are some examples.

$2^1 = 2$ $\quad 2^3 = 2 \cdot 2 \cdot 2$

There are several different ways to say "2^3."

- "Two to the power of three."
- "Two raised to the power of three."
- "Two to the third power."
- "Two cubed."

Exponents also can be used to represent the **prime factorization** of composite numbers.

For example, the number 24 can be rewritten using prime factors as $2 \cdot 2 \cdot 2 \cdot 3$ or $2^3 \cdot 3$.

Lesson Practice ACC6.6.10

Name: _____ Date: _____ Period: _____

1. Determine the value of each expression.

Expression	Value
$3+3+3+3$	12
$3 \cdot 3 \cdot 3 \cdot 3$	81
$4(3)$	12
3^4	81

2. Complete the table. The first row has been completed for you.

Expression With Exponent	Expression Without Exponent
3^5	$3 \cdot 3 \cdot 3 \cdot 3 \cdot 3$
2^4	$2 \cdot 2 \cdot 2 \cdot 2$
4^3	$4 \cdot 4 \cdot 4$
5^1	5
$\left(\frac{1}{2}\right)^5$	$\frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2}$
$\left(\frac{1}{3}\right)^4$	$\frac{1}{3} \cdot \frac{1}{3} \cdot \frac{1}{3} \cdot \frac{1}{3}$

3. Circle the two expressions that have the same value.

(6 + 6 + 6) $\quad 6^3 \quad 3^6 \quad$ (3 • 6)

Lesson Practice ACC6.6.10

Name: _____ Date: _____ Period: _____

4. Write the expression that represents each description, then determine its value.

Description	Expression	Value
Three to the third power	3^3	27
Five to the second power	5^2	25
Two to the power of five	2^5	32
Three to the second power	3^2	9

5. Rewrite 75 as a product of prime factors using exponents.

$3 \cdot 5^2$

FAST Practice

6. Select *all* the expressions that are equal to 16.

- ☐ A. 8^2
- ☑ B. 4^2
- ☑ C. 2^4
- ☑ D. 16^1
- ☐ E. 2^8

Spiral Review

Problems 7–9: Solve each equation.

7. $a - 2 = 5$ $\quad a = 7$
8. $b + 2 = 5$ $\quad b = 3$
9. $6c = 30$ $\quad c = 5$

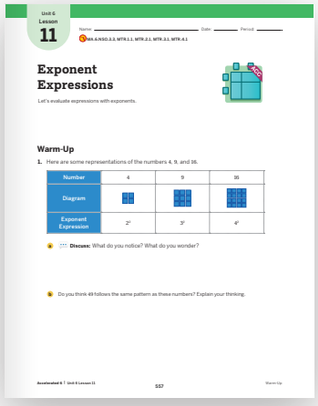
10. Write two expressions for the area of the rectangle.

Product: $4(m + 3)$

Sum: $4m + 12$

Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1–2, 4	1	MA.6.NSO.3.3
	3	2	MA.6.NSO.3.3
	5	2	MA.6.NSO.3.4
FAST Practice	6	2	MA.6.NSO.3.3
Spiral Review			
	7–8	1	MA.6.AR.2.2
	9	1	MA.6.AR.2.3
	10	2	MA.6.AR.1.4


Unit 6
Lesson
11

Exponent Expressions

Let's evaluate expressions with exponents.

Warm-Up

1. Here are some representations of the numbers 4, 9, and 16.

Number	4	9	16
Diagram			
Exponent Expression	2^2	3^2	4^2

Discuss: What do you notice? What do you wonder?

Do you think 49 follows the same pattern as these numbers? Explain your thinking.

557

Exponent Expressions

Exponents and Order of Operations

Let's evaluate expressions with exponents.

Learning Outcomes

Today's Goal

1. **Goal:** Evaluate numerical expressions that have an exponent and one or more other operations.

Students use area models to make sense of more complex expressions with exponents, particularly expressions that include an exponent and one other operation. They notice that expressions with the same numbers but with different operations or arrangements represent different visuals and have different values. **(MTR.2.1)**

Coherence

Prior Learning

In Lesson 10, students connected exponents and repeated multiplication, and they evaluated expressions with exponents. In a previous course, students used the order of operations to evaluate expressions with parentheses, brackets, and braces.

Future Learning

In a future course, students will extend what they've learned to evaluate expressions with variables that involve exponents and other operations.

Connections

- Students build their understanding of exponents as they analyze models, write expressions, and find their values. (mathematical)

FL B.E.S.T. Math Benchmarks

Addressing

MA.6.NSO.3.3
Evaluate positive rational numbers and integers with natural number exponents.

Mathematical Thinking and Reasoning Standards: MTR.1.1, MTR.2.1, MTR.3.1, MTR.4.1

ELA Expectations: ELA.K12.EE.1.1, ELA.K12.EE.4.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.5.NSO.2.1	**MA.7.NSO.1.1** **MA.7.NSO.2.1**

Vocabulary

Review Vocabulary

order of operations

Lesson at a Glance

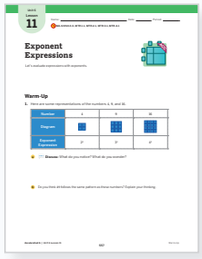
~ 45 min

MA.6.NSO.3.3

Warm-Up

Pairs | 5 min

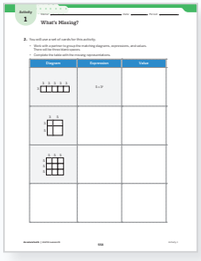
Students use the **Notice and Wonder** routine to make sense of expressions with exponents.



Activity 1

Pairs | 13 min

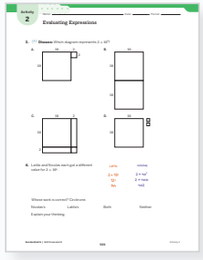
Students attend to the structure of different expressions as they match them with diagrams and calculate their values. **(MTR.2.1)**



Activity 2

Pairs | 7 min

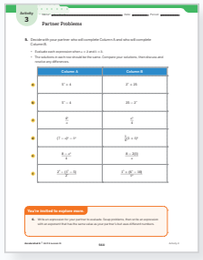
Students use the **Decide and Defend** routine to settle a dispute about the correct *order of operations* when dealing with exponents. **Routine:** Think-Pair-Share **(MTR.1.1)**



Activity 3

Pairs | 10 min

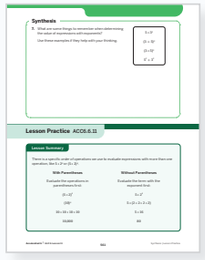
Students develop fluency in evaluating expressions involving exponents. **(MTR.3.1)**



Synthesis

Whole Class | 5 min

Students synthesize their understanding of evaluating expressions with exponents.
MLR2: Collect and Display



Show What You Know

Independent | 5 min

Students demonstrate their understanding by evaluating expressions with exponents.

Prep Checklist

Prepare the Activity 1 Cards.

This lesson includes:

- Student Edition
- Show What You Know
- Activity 1 Cards, one set per pair

Pairs | 5 min

Warm-Up

Purpose: Students use the Notice and Wonder routine to make sense of expressions with exponents.

1 Launch

Direct students' attention to the table and use the Notice and Wonder routine to support students in making connections between the representations.

Create a record of the connections students noticed and wondered, along with the names of the students who shared them.

Listen for a variety of explanations and invite students to share during the Connect.

A **Accessibility: Memory and Attention** To surface prior knowledge, invite students to refer back to the anchor chart about exponents and make connections to where they see exponents in this Warm-Up.

2 Connect

Invite students to share by polling the class about whether 49 can follow the same pattern, then invite individual students to share their reasoning.

To surface the Key Takeaway, consider asking:

- "What are some other examples of numbers that follow this pattern?"
- "What are examples of numbers that do not follow this pattern?" **(MTR.1.1)**

Unit 6
Lesson
11

Name: _____ Date: _____ Period: _____

MA.6.NSO.3.3, MTR.1.1, MTR.2.1, MTR.3.1, MTR.4.1

Exponent Expressions

Let's evaluate expressions with exponents.

Warm-Up

1. Here are some representations of the numbers 4, 9, and 16.

Number	4	9	16
Diagram			
Exponent Expression	2^2	3^2	4^2

a. **Discuss:** What do you notice? What do you wonder?

Responses vary.

- I notice that all of the exponents are 2.
- I notice that we can make an exact square out of smaller squares.
- I wonder what other numbers follow this pattern.

b. Do you think 49 follows the same pattern as these numbers? Explain your thinking.

Yes. *Explanations vary.* I could draw an exact square out of 49 smaller squares.

Accelerated 6 | Unit 6 Lesson 11 557 Warm-Up

Key Takeaway: When a number is squared (i.e., it has an exponent of two), it can be represented using an area model. 49 follows this pattern because $7 \cdot 7 = 7^2$ and $7^2 = 49$, but 55 does not follow this pattern because no number can be squared to equal 55.

Pairs | 13 min

Activity 1 What's Missing?

Purpose: Students attend to the structure of different expressions as they match them with diagrams and calculate their values. **(MTR.2.1)**

1 Launch

Direct students' attention to the instructions for the activity and review them as a class.

Distribute one set of the Activity 1 Cards to each pair of students.

2 Monitor

To support students getting started, encourage them to make sense of the diagrams in their Student Edition before sorting their cards.

A **Accessibility: Executive Functioning** Consider chunking this activity by inviting students to choose one card at a time.

D **Differentiation**

Look for students who:	Respond to Student Thinking
Think that Card A: $5^2 + 3^2$ and Card B: $(3 + 5)^2$ have the same value.	**Support** Consider asking, "For Card A, which operation would you do first? For Card B, which operation would you do first?"
Highlight connections between the diagrams, expressions, and final values. **(MTR.2.1)**	**Strengthen** Invite students to share their diagrams during the Connect.
Draw a variety of diagrams for the expression $5^2 + 3^2$.	
Finish early.	**Stretch** Invite students to create their own diagrams, expressions, and values.

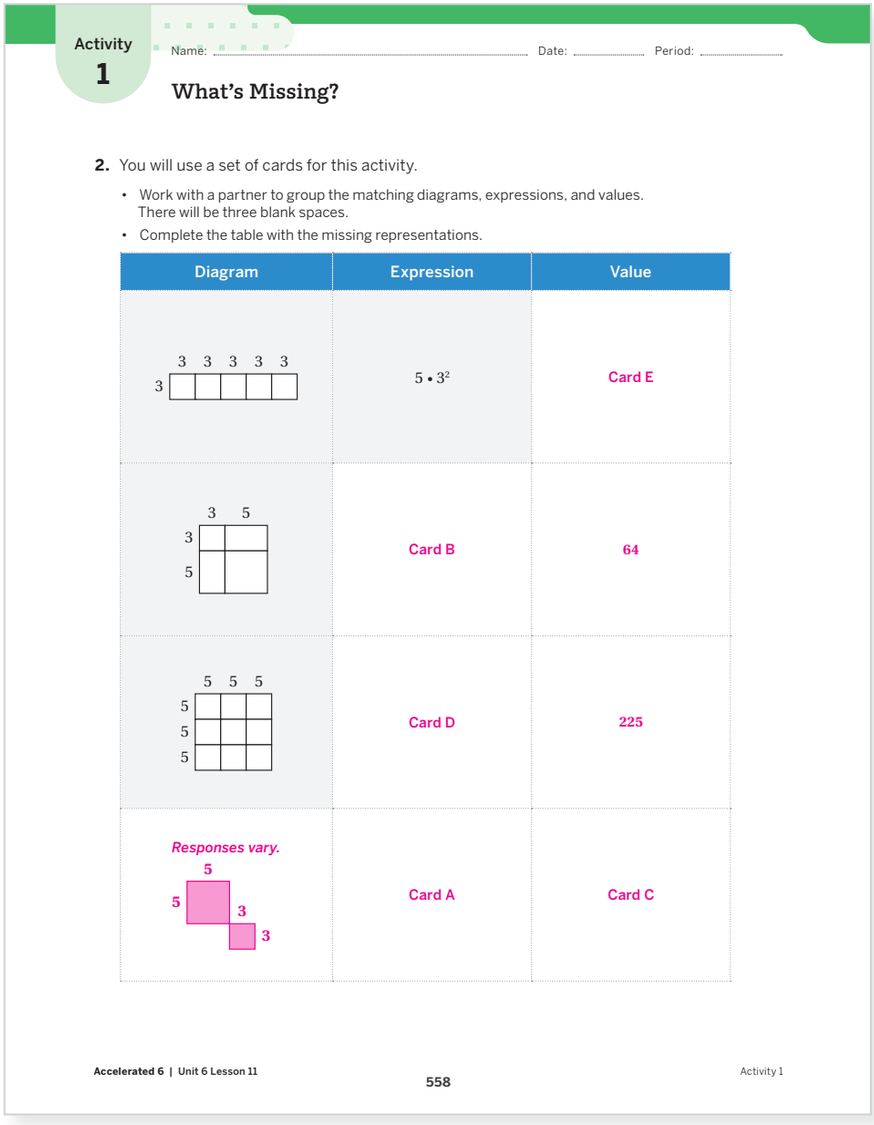
3 Connect

Direct students' attention to the set of cards. Invite students to share their strategies for matching diagrams, expressions, and values.

Consider asking:

- "What is the same about each of these expressions?"
- "What is different about each of these expressions?"
- "How did you decide which diagram matched $(3 + 5)^2$? What was its value?"

Invite students to share a variety of diagrams for the expression $5^2 + 3^2$ and then make connections between the diagrams and the expression. **(MTR.1.1, MTR.2.1)**



Activity 1

Name: Date: Period:

What's Missing?

2. You will use a set of cards for this activity.

- Work with a partner to group the matching diagrams, expressions, and values. There will be three blank spaces.
- Complete the table with the missing representations.

Diagram	Expression	Value
3 3 3 3 3 / 3	$5 \cdot 3^2$	Card E
3 5 / 3 / 5	Card B	64
5 5 5 / 5 / 5 / 5	Card D	225
Responses vary. 5 / 5 3 / 3	Card A	Card C

Accelerated 6 | Unit 6 Lesson 11 558 Activity 1

Pairs | 7 min

Activity 2 Evaluating Expressions

Purpose: Students use the Decide and Defend routine to settle a dispute about the correct *order of operations* when dealing with exponents. **(MTR.1.1)**

1 Launch

Direct students' attention to the four diagrams. Use the Think-Pair-Share routine to support student-to-student discourse about connections between diagrams and expressions. **(MTR.4.1)**

Note: Discussing which diagram matches $2 + 10^2$ may help students decide whose work is correct in the next problem.

2 Monitor

D Differentiation

Look for students who:	Respond to Student Thinking
Think both students could be correct.	**Support** Invite students to make connections between the diagrams in Problem 3 and each student's work in Problem 4.
Think Latifa's work is correct because she evaluated from left to right.	
Recognize that exponents are repeated multiplication, so they evaluate exponents before addition. **(MTR.1.1)**	**Strengthen** Invite students to share their reasoning during the Connect.

3 Connect

Direct students' attention to the two expressions.

Use the Decide and Defend routine to strengthen students' ability to make arguments and critique the reasoning of others. See the Routine Facilitation Guide for more information. **(MTR.1.1)**

ELL **English Language Learners** Provide a word bank to support students in expressing their ideas, including but not limited to: *exponent*, *order of operations*, *sum*, etc. **(Writing and Speaking)**

To surface the Key Takeaway, consider asking, "What would your first step be to evaluate $6 + 3^2$? $(6 + 3)^2$?"

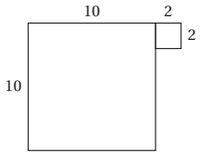
Activity 2

Name: Date: Period:

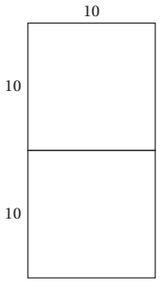
Evaluating Expressions

3. **Discuss:** Which diagram represents $2 + 10^2$?

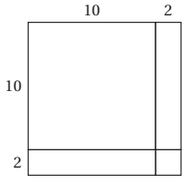
A.



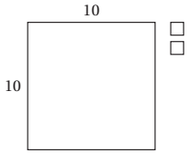
B.



C.



D.



Explanations vary. The large square represents 10^2 and the two small squares represent 2.

4. Latifa and Nicolas each got a different value for $2 + 10^2$.

Latifa
$2 + 10^2$
12^2
144

Nicolas
$2 + 10^2$
$2 + 100$
102

Whose work is correct? Circle one.

(Nicolas's) Latifa's Both Neither

Explain your thinking.

Explanations vary. Nicolas is correct because his answer matches the total area of Choice D. There is a square with an area of 100 square units and 2 additional squares, so the total area is 102 square units.

Accelerated 6 | Unit 6 Lesson 11 559 Activity 2

Key Takeaway: When an expression has an exponent and a second term, the *order of operations* says to evaluate the term with the exponent first.

Pairs | 10 min

Activity 3 Partner Problems

Purpose: Students develop fluency in evaluating expressions involving exponents. **(MTR.3.1)**

Short on time? Consider inviting students to complete half of the problems.

1 Launch

Direct students' attention to the instructions and invite students in each pair to determine who will be Partner A and Partner B.

2 Monitor

A **Accessibility: Executive Functioning** To support with organization in problem solving, consider giving students lined paper or a graphic organizer to record their thinking one step at a time.

Listen for the mathematical language students use in discussing and justifying their thinking, such as *exponent, parentheses, addition, multiplication,* and *grouping*. **(MTR.4.1)**

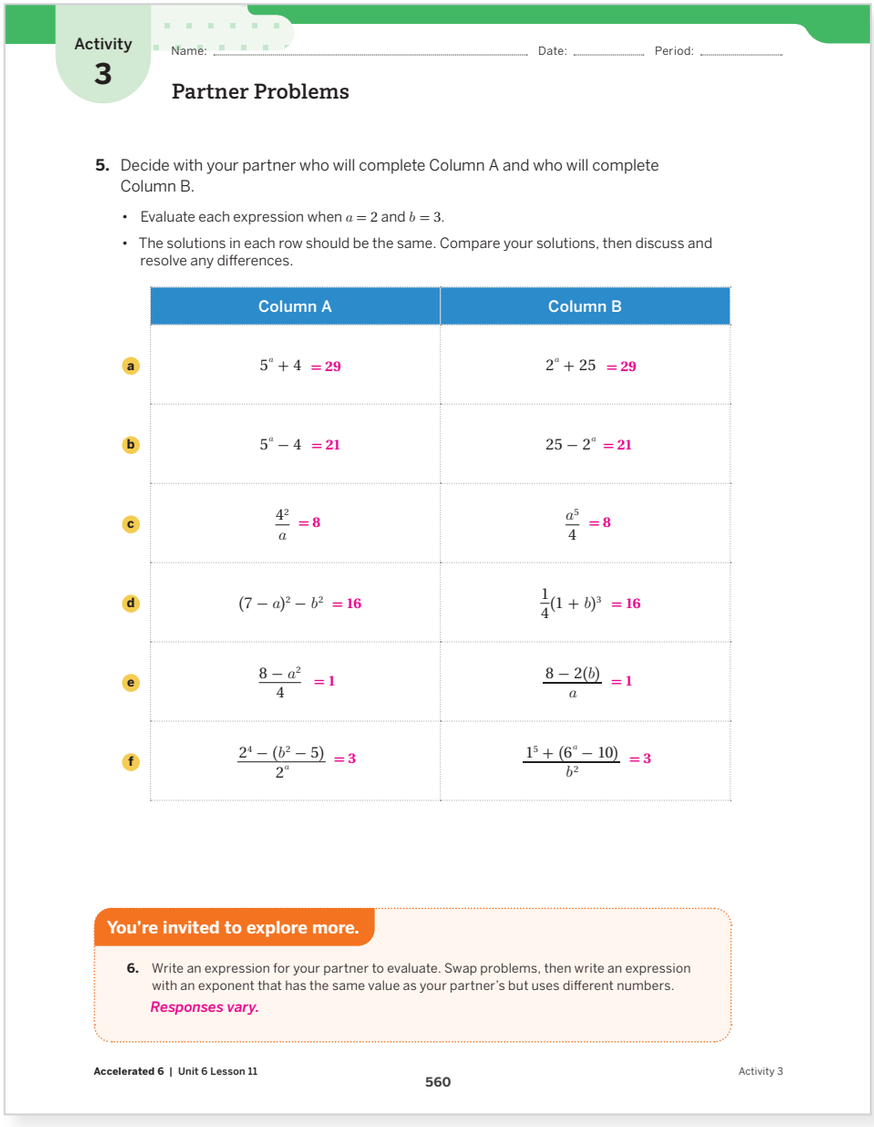
D **Differentiation**

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Invite students to return to their work from earlier in the lesson. Consider asking, "Why did Nicolas evaluate the exponent as the first step?"
Multiply and divide, or add and subtract, before evaluating the term with the exponent.	
Attend to precision by showing each step of their thinking.	**Strengthen** Invite these students to share their strategies during the Connect.
Would like to explore evaluating expressions further.	**Stretch** **You're invited to explore more.** Invite students to complete this optional task. Encourage them to discuss their thinking with a partner.

3 Connect

Direct students' attention to the set of problems and invite the class to choose ones to discuss.

Consider asking, "How did you decide which part of each expression to evaluate first?" **(MTR.3.1)**



Activity 3

Name: Date: Period:

Partner Problems

5. Decide with your partner who will complete Column A and who will complete Column B.

- Evaluate each expression when $a = 2$ and $b = 3$.
- The solutions in each row should be the same. Compare your solutions, then discuss and resolve any differences.

	Column A	Column B
a	$5^a + 4 = 29$	$2^a + 25 = 29$
b	$5^a - 4 = 21$	$25 - 2^a = 21$
c	$\frac{4^2}{a} = 8$	$\frac{a^5}{4} = 8$
d	$(7 - a)^2 - b^2 = 16$	$\frac{1}{4}(1 + b)^3 = 16$
e	$\frac{8 - a^2}{4} = 1$	$\frac{8 - 2(b)}{a} = 1$
f	$\frac{2^4 - (b^2 - 5)}{2^a} = 3$	$\frac{1^5 + (6^a - 10)}{b^2} = 3$

You're invited to explore more.

6. Write an expression for your partner to evaluate. Swap problems, then write an expression with an exponent that has the same value as your partner's but uses different numbers.

Responses vary.

Accelerated 6 | Unit 6 Lesson 11 560 Activity 3

Whole Class | 5 min

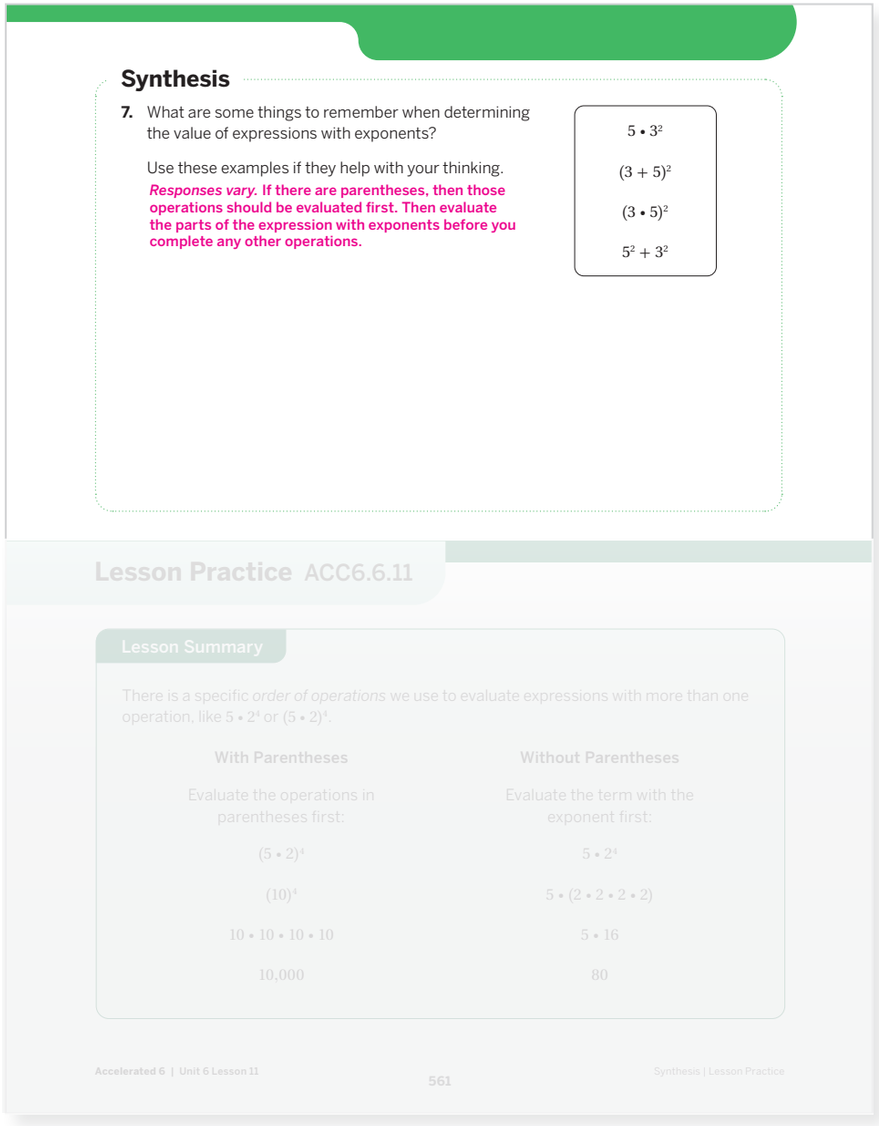
Synthesis

Purpose: Students synthesize their understanding of evaluating expressions with exponents.

Invite students to respond independently, and then share their thinking with a partner.

Collect and share a variety of ideas, such as:
- Explaining how to use the order of operations to evaluate each expression.
- Using mathematical vocabulary, such as *exponents*, *bases*, *grouping symbols*, and *parentheses* to explain their thinking.

MLR **MLR2: Collect and Display** Use this routine to collect students' strategies as they discuss with a partner. Display these publicly for students to reference or add to for the rest of the unit.



Synthesis

7. What are some things to remember when determining the value of expressions with exponents?

Use these examples if they help with your thinking.

Responses vary. If there are parentheses, then those operations should be evaluated first. Then evaluate the parts of the expression with exponents before you complete any other operations.

$5 \cdot 3^2$

$(3 + 5)^2$

$(3 \cdot 5)^2$

$5^2 + 3^2$

Lesson Practice ACC6.6.11

Lesson Summary

There is a specific *order of operations* we use to evaluate expressions with more than one operation, like $5 \cdot 2^4$ or $(5 \cdot 2)^4$.

With Parentheses	Without Parentheses
Evaluate the operations in parentheses first:	Evaluate the term with the exponent first:
$(5 \cdot 2)^4$	$5 \cdot 2^4$
$(10)^4$	$5 \cdot (2 \cdot 2 \cdot 2 \cdot 2)$
$10 \cdot 10 \cdot 10 \cdot 10$	$5 \cdot 16$
$10{,}000$	80

Accelerated 6 | Unit 6 Lesson 11 561 Synthesis | Lesson Practice

Lesson Takeaway: When evaluating expressions, evaluate exponents first, unless there are grouping symbols, like parentheses or the fraction bar. When there are grouping symbols, perform the operation(s) inside them first.

Independent | 5 min

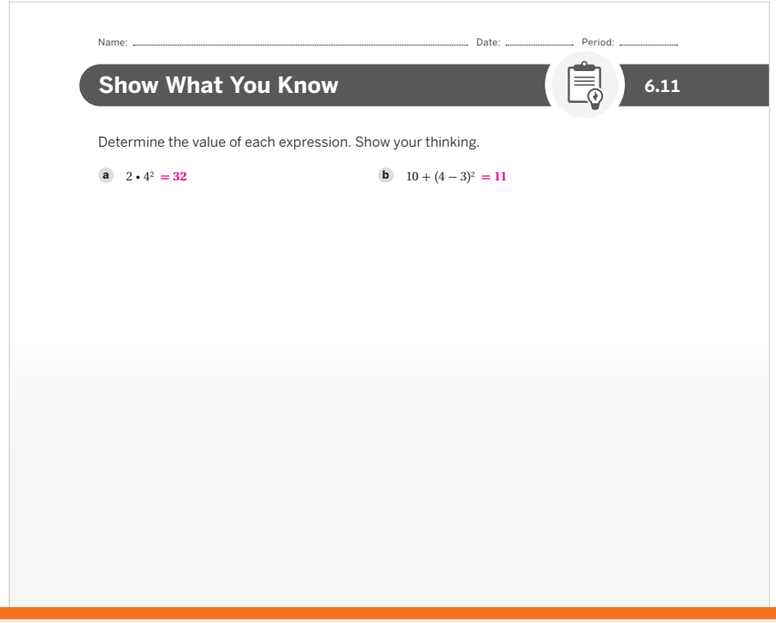
Show What You Know

Purpose: Students demonstrate their understanding by evaluating expressions with exponents.

Today's Goal

1. **Goal:** Evaluate numerical expressions that have an exponent and one or more other operations.

Show What You Know (Assessment Resources)



Name: _____ Date: _____ Period: _____

Show What You Know 6.11

Determine the value of each expression. Show your thinking.

a $2 \cdot 4^2 = 32$

b $10 + (4 - 3)^2 = 11$

Differentiation Use after Lesson 11

Support

Provide targeted intervention for students by using these resources.

If students need support determining the value of an expression that has exponents and other operations:

» **Respond to Student Thinking:**

- Using the ***Evaluating Expressions With Exponents*** **Mini-Lesson**.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite them to explore the **Sub-Unit 3 Extension Activities**.
- Invite students to complete the **Explore More** task and discuss their thinking with a partner.

Professional Learning

How did the **Decide and Defend** routine support students in evaluating expressions involving exponents?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

7. What are some things to remember when determining the value of expressions with exponents?

Use these examples if they help with your thinking.

$5 \cdot 3^2$

$(3 + 5)^2$

$(3 \cdot 5)^2$

$5^2 + 3^2$

Responses vary. If there are parentheses, then those operations should be evaluated first. Then evaluate the parts of the expression with exponents before you complete any other operations.

Lesson Practice ACC6.6.11

Lesson Summary

There is a specific *order of operations* we use to evaluate expressions with more than one operation, like $5 \cdot 2^4$ or $(5 \cdot 2)^4$.

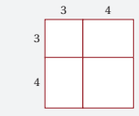
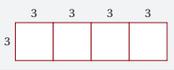
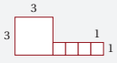
With Parentheses	Without Parentheses
Evaluate the operations in parentheses first:	Evaluate the term with the exponent first:
$(5 \cdot 2)^4$	$5 \cdot 2^4$
$(10)^4$	$5 \cdot (2 \cdot 2 \cdot 2 \cdot 2)$
$10 \cdot 10 \cdot 10 \cdot 10$	$5 \cdot 16$
$10{,}000$	80

Lesson Practice ACC6.6.11

Name: _____ Date: _____ Period: _____

1. Match each expression with the diagram it represents. Evaluate the expression to determine the area of each diagram.

$4 \cdot 3^2$ $\quad 3^2 + 4$ $\quad (3 + 4)^2$

Diagram	Expression	Area (sq. units)
	$(3 + 4)^2$	49
	$4 \cdot 3^2$	36
	$3^2 + 4$	13

Problems 2–3: Evaluate each expression for $f = 2$ and $g = 3$.

2.

Expression	Value
$5 + 4^f$	21
$(g + f)^3$	125
$f^2 \cdot 5$	20
$8 \cdot \left(\frac{1}{2}\right)^f$	2

3.

Expression	Value
$42 - 9 \cdot f^2$	6
$\frac{12 - g^2}{6}$	$\frac{1}{2}$
$\frac{g + (2 + 1)^f}{4}$	3
$\frac{1}{4}(g - 1)^3$	2

Lesson Practice ACC6.6.11

Name: _____ Date: _____ Period: _____

FAST Practice

4. What is the value of the expression $\frac{3 + 5^2}{4}$?

A. 16 B. 4 C. 7 D. $\frac{13}{4}$

Spiral Review

5. Use the numbers **0** to **9** to fill in each blank so that the values of the expressions are in order from *least* to *greatest*. Use each number only once.

Responses vary.

$4^{\boxed{1}}$	$\boxed{2}^4$	$\boxed{5}^2$	$3^{\boxed{4}}$	$\boxed{6}^3$	$2^{\boxed{8}}$

Least Greatest

Problems 6–9: Determine the value of each expression.

6. $3(5) = 15$

7. $3(5) + 2 = 17$

8. $3(5 + 2) = 21$

9. $3(2 + 5) = 21$

10. Select *all* the expressions that are equal to 3^4.

- ☑ A. $3 \cdot 3 \cdot 3 \cdot 3$
- ☐ B. 12
- ☐ C. $3 + 3 + 3 + 3$
- ☑ D. $9 \cdot 9$
- ☑ E. 81

Lesson Practice Item Analysis

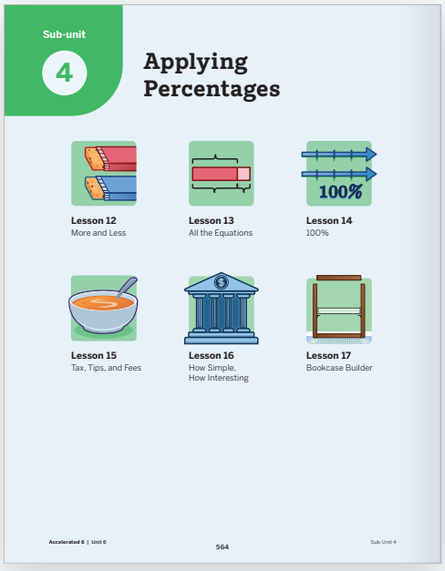
	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1–3	2	MA.6.NSO.3.3
FAST Practice	4	2	MA.6.NSO.3.3
Spiral Review			
	5	3	MA.6.NSO.3.3
	6–9	1	MA.5.AR.2.2
	10	2	MA.6.NSO.3.3

Applying Percentages

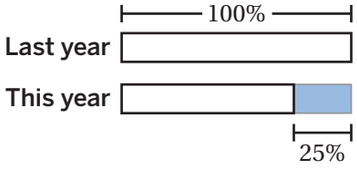
Sub-Unit 4 Goals

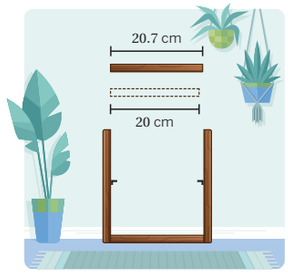
- **Goal:** Solve problems involving percent change using representations, such as tape diagrams, double number lines, and equations.
- **Goal:** Use ratios to solve problems in real-world situations involving percent increase, percent decrease, and percent error.
- **Goal:** Interpret situations involving percent change and apply appropriate representations to solve problems.
- **Language Goal:** Interpret scenarios depicting percent change and percent error.

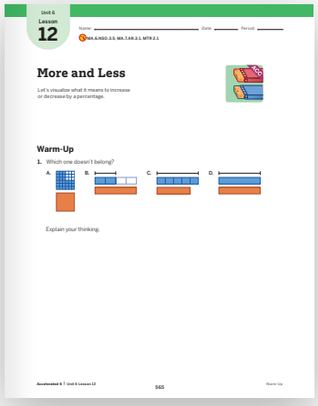
In this sub-unit, students use tape diagrams, double number lines, and equations to represent and solve problems involving **percent increase** and **percent decrease**. They also interpret and solve real-world problems involving tax and tips, **simple interest**, and **percent error**.



Math That Matters Most

Concept or Skill	Example
Use tape diagrams and equations to represent situations involving a percent increase or decrease.	If 80 pounds of peaches were harvested last year, how many pounds were harvested this year?  This year's harvest is 75% of 80, and $0.75 \cdot 80 = 60$. So 60 pounds of peaches were harvested this year. I can also use the equation $t = (1 - 0.25)l$, where l is the pounds of peaches harvested last year and t is the pounds harvested this year.
Calculate an original amount, new amount, or percent change in the context of sales tax and tip.	A store offers a 20% off coupon. The tax rate is 7.5% of the subtotal. If an item is listed at \$15, what is the total after the coupon and tax have been applied? To determine the price after the coupon, I multiplied 15 by 0.8 and got 12. Then, to determine the price after tax, I multiplied \$12 by 1.075 to get 12.9.

Concept or Skill	Example
Calculate simple interest earned on deposits or paid on loans.	You deposit \$120 into an account that pays 3% simple interest. If you leave the money in the account for 10 years, how much interest will your original deposit earn? The formula for simple interest is $I = prt$. The principal is \$120, the interest rate is 3%, and the time is 10 years: $I = 120(0.03)(10) = 36$. The account will earn \$36 in interest on the original deposit.
Decide whether a value is within an acceptable percent error.	The acceptable percent error is 5% for a shelf to fit. Will the shelf fit? Why or why not?  The shelf will fit because the error is 0.7 centimeters and the original is 20 centimeters. 0.7 is 3.5% of 20, which is less than 5%.



More and Less

Percent Increase and Decrease With Tape Diagrams

Let's visualize what it means to increase or decrease by a percentage.

Learning Outcomes

Today's Goals

1. **Goal:** Use tape diagrams and tables to represent situations involving a *percent increase* or *decrease*.
2. **Goal:** Determine a new amount given an original amount and a *percent increase* or *decrease*.
3. **Language Goal:** Interpret a verbal description to solve problems about *percent increase* or *decrease*. **(Reading and Writing)**

Students visualize what it means to increase or decrease an amount by a percentage, and use that visual to help them calculate unknown values. They see both tape diagrams and tables used to represent **percent increase** and **percent decrease**, and they use these representations to solve problems. **(MTR.2.1)**

Coherence

Prior Learning

In Units 2, 3, and 5, students solved problems involving ratios and percents.

Future Learning

In Lesson 13, students will use equations to represent percent increase and decrease.

Connections

- Students use models to solve problems involving percent increase and percent decrease in real world contexts such as comparing the lengths of granola bars and the amount of fruit harvested over two years. (real world)

FL B.E.S.T. Math Benchmarks

Addressing

MA.6.NSO.3.5
Rewrite positive rational numbers in different but equivalent forms including fractions, terminating decimals and percentages.

MA.7.AR.3.1
Apply previous understanding to percentages and ratios to solve real-world multi-step percentage problems.

Mathematical Thinking and Reasoning Standard: MTR.2.1

ELA Expectations: ELA.K12.EE.4.1, ELA.K12.EE.5.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.6.AR.3.4	MA.7.NSO.1.2

Vocabulary

New Vocabulary

percent decrease	percent increase

Lesson at a Glance

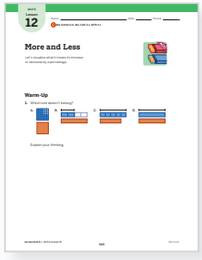
~ 45 min

MA.6.NSO.3.5, MA.7.AR.3.1

Warm-Up

Pairs | 5 min

Students make sense of the structure of tape diagrams to be ready to use them to represent a relationship between two variables. **Routine:** Which One Doesn't Belong?



Activity 1

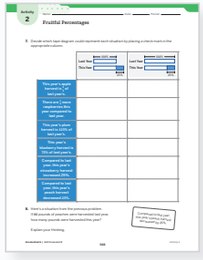
Pairs | 13 min

Students use the context of granola bars and representations, including tape diagrams and tables, to visualize **percent increase** and **percent decrease**.
Routine: Think-Pair-Share

Activity 2

Pairs | 10 min

Students apply the concepts from Activity 1 to new contexts and continue to make connections between representations.
MLR2: Collect and Display (MTR.2.1)



Activity 3

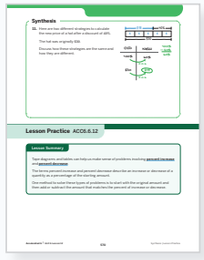
Pairs | 7 min

Students distinguish between a percent of a number and percent change.

Synthesis

Whole Class | 5 min

Students synthesize their understanding of different strategies and tools to solve for percent increase and percent decrease.



Show What You Know

Independent | 5 min

Students demonstrate their understanding by interpreting a percent decrease situation.

Prep Checklist

This lesson includes:

- Student Edition
- Show What You Know

Pairs | 5 min

Warm-Up

Purpose: Students make sense of the structure of tape diagrams to be ready to use them to represent a relationship between two variables.

1 Launch

Use the Which One Doesn't Belong? routine to support students in noticing the structure of tape diagrams.

Consider sharing that each diagram does not belong in some way. Encourage students to look for more than one possibility.

A **Accessibility: Visual-Spatial Processing** Invite students to answer a more concrete question first to support interpreting visual representations. Consider asking, "How would you describe the relationship between the blue and orange figure in diagram A? B?"

ELL **English Language Learners** Invite students to rehearse their ideas with a partner before they share their ideas with others. **(Writing and Speaking)**

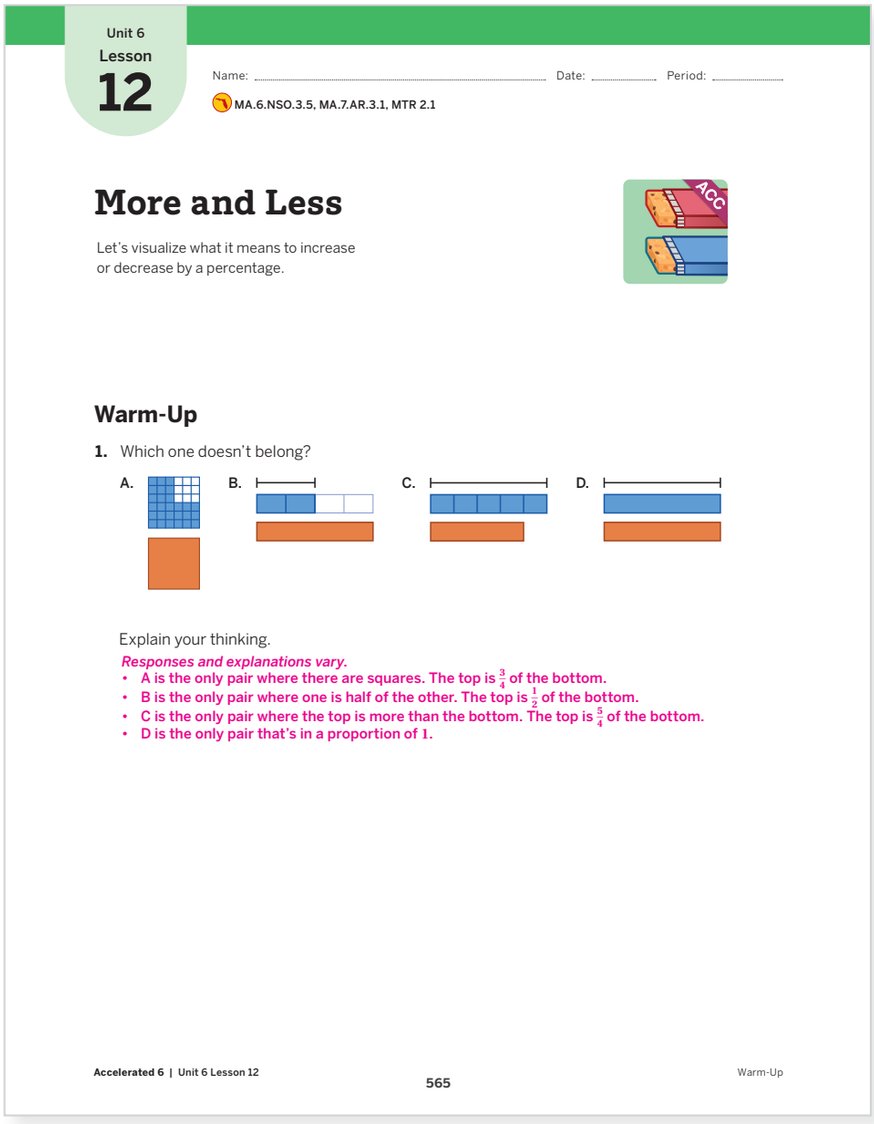
2 Connect

Invite students to share at least one reason why each diagram doesn't belong.

Consider asking:

- "What is another way to say why diagram ___ doesn't belong?"
- "Is there another reason why diagram ___ doesn't belong?"

Note: If it doesn't come up naturally, consider asking, "How could we use these diagrams to represent percentages?"



Unit 6
Lesson
12

Name: Date: Period:

MA.6.NSO.3.5, MA.7.AR.3.1, MTR 2.1

More and Less

Let's visualize what it means to increase or decrease by a percentage.

Warm-Up

1. Which one doesn't belong?

A. B. C. D.

Explain your thinking.

Responses and explanations vary.

- A is the only pair where there are squares. The top is $\frac{3}{4}$ of the bottom.
- B is the only pair where one is half of the other. The top is $\frac{1}{2}$ of the bottom.
- C is the only pair where the top is more than the bottom. The top is $\frac{5}{4}$ of the bottom.
- D is the only pair that's in a proportion of 1.

Accelerated 6 | Unit 6 Lesson 12 565 Warm-Up

Pairs | 13 min

Activity 1 Granola Bars

Purpose: Students use the context of granola bars and representations, including tape diagrams and tables, to visualize **percent increase** and **percent decrease**.

Short on time? Consider omitting Problem 6 to allocate time on the strategies for percent increase and percent decrease.

1 Launch

For Problem 2, **Direct students' attention** to the granola bar. Consider asking, "Where have you seen phrases such as 20% more or 20% longer in your lives?"

ELL **English Language Learners** Invite students to read the problem aloud and paraphrase what it means. Clarify any words they don't know. **(Reading)**

Note: This may be the first time students encounter a **percent increase**. Encourage students to use blank paper to help them revise their thinking.

2 Monitor

To support students getting started, consider asking, "How long is 10% of this granola bar? How can that help you find 20%?"

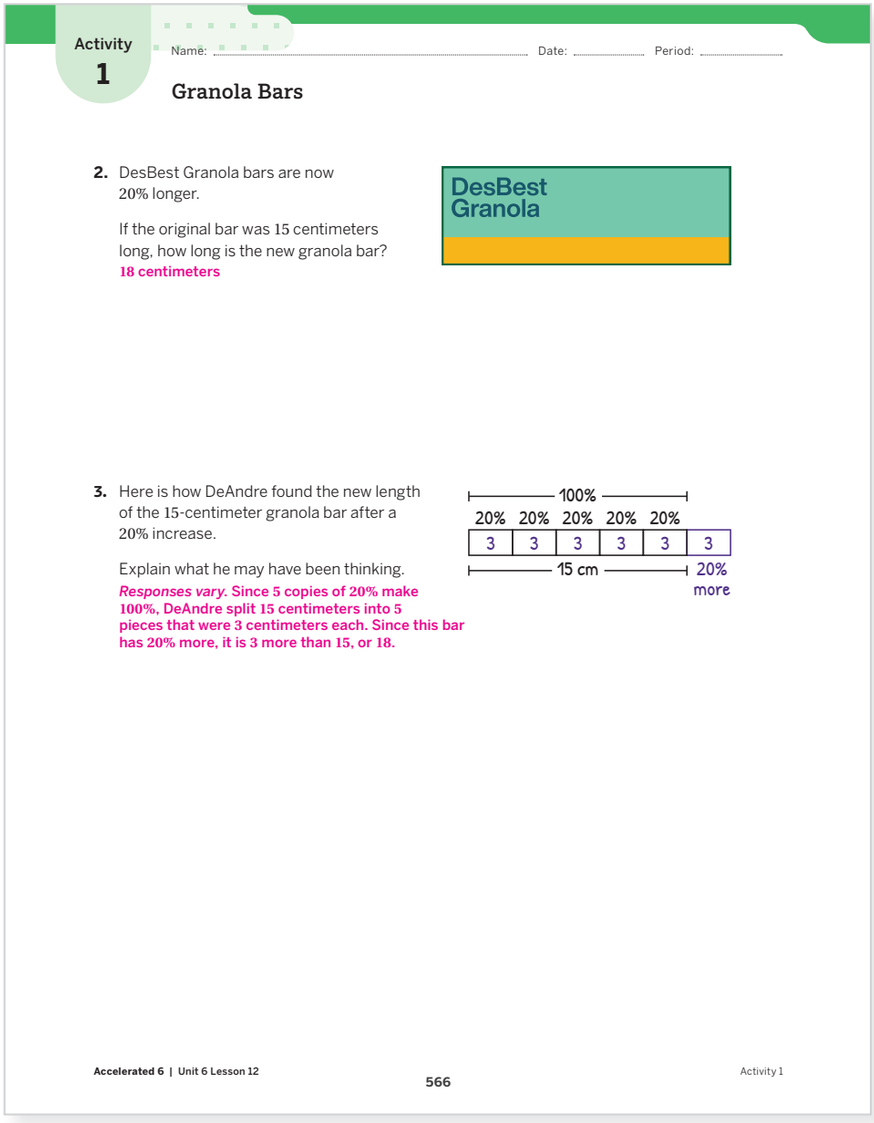
For Problem 3, **look for and share** a variety of student explanations for the steps in DeAndre's strategy, such as:

- 20% is $\frac{1}{5}$ of 100% or 5 copies of 20% make 100%.
- 3 is $\frac{1}{5}$ of 15 or 3 is 20% of 15.

Encourage students to read other student responses. Consider asking, "How are the explanations similar to or different from your own?"

Pause briefly to discuss DeAndre's strategy. Consider asking, "Why do you think DeAndre decided to divide the rectangle into five equal parts? Where did the number 3 come from?"

Activity 1 continued >



Activity 1

Name: Date: Period:

Granola Bars

2. DesBest Granola bars are now 20% longer.

If the original bar was 15 centimeters long, how long is the new granola bar?

18 centimeters

3. Here is how DeAndre found the new length of the 15-centimeter granola bar after a 20% increase.

Explain what he may have been thinking.

Responses vary. Since 5 copies of 20% make 100%, DeAndre split 15 centimeters into 5 pieces that were 3 centimeters each. Since this bar has 20% more, it is 3 more than 15, or 18.

Accelerated 6 | Unit 6 Lesson 12 566 Activity 1

Pairs | 13 min

Activity 1 Granola Bars (continued)

Purpose: Students use the context of granola bars and representations, including tape diagrams and tables, to visualize **percent increase** and **percent decrease.**

Short on time? Consider omitting Problem 6 to allocate time on the strategies for percent increase and percent decrease.

2 Monitor

For Problem 4, **Use the Think-Pair-Share routine** to help students shape their ideas about **percent decrease**. See the Routine Facilitation Guide for more information.

ELL **English Language Learners** Encourage students to paraphrase each other's ideas to help them make connections and incorporate vocabulary from the unit. **(Speaking and Listening)**

Listen for a variety of student strategies to solve for the length of the new granola bar.

D **Differentiation**

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Consider asking, "How is this problem similar to the last one? How is it different? What might that mean about how we solve for the new length?"
Multiply 0.1 by 15 for their solution.	**Support** Consider asking, "What would 50% of the granola bar be? Should this be more or less than that amount?"
Use the same strategy as the last problem and add 1.5 to 15.	**Support** Invite them to read the problem aloud and highlight key information for solving this problem. Consider asking, "How is this problem similar or different to the one you just did?"

For Problem 5, **encourage students** to read other student responses. Consider asking, "How are the explanations similar to or different from your own?"

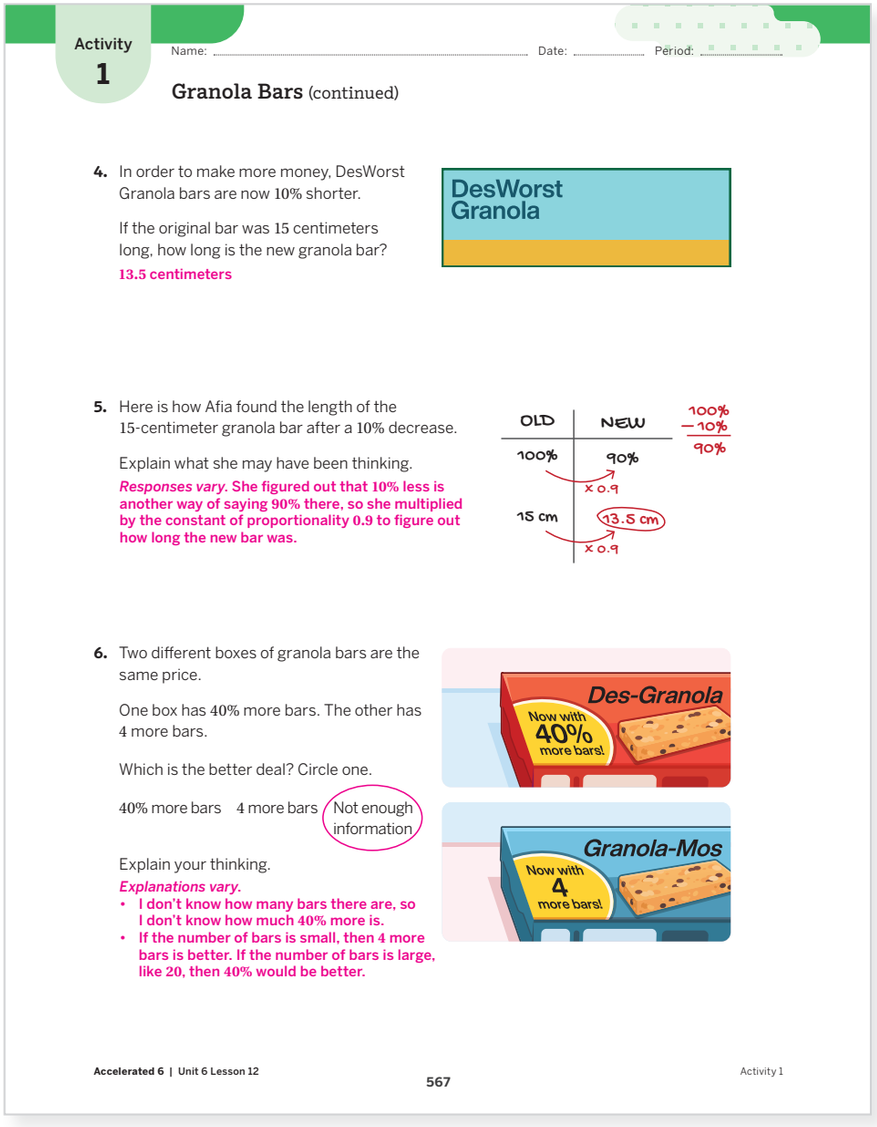
Consider asking, "How would DeAndre draw 40% more bars?"

3 Connect

For Problem 5, consider asking:

- "How do we see parts of the question in Afia's work?"
- "How is this strategy similar to DeAndre's? Different?"

Note: If time allows, consider asking, "How is this table similar to or different from tables we have used so far this year?"



Activity 1

Name: Date: Period:

Granola Bars (continued)

4. In order to make more money, DesWorst Granola bars are now 10% shorter.

If the original bar was 15 centimeters long, how long is the new granola bar?

13.5 centimeters

5. Here is how Afia found the length of the 15-centimeter granola bar after a 10% decrease.

Explain what she may have been thinking.

Responses vary. **She figured out that 10% less is another way of saying 90% there, so she multiplied by the constant of proportionality 0.9 to figure out how long the new bar was.**

6. Two different boxes of granola bars are the same price.

One box has 40% more bars. The other has 4 more bars.

Which is the better deal? Circle one.

40% more bars 4 more bars (Not enough information)

Explain your thinking.

Explanations vary.

- **I don't know how many bars there are, so I don't know how much 40% more is.**
- **If the number of bars is small, then 4 more bars is better. If the number of bars is large, like 20, then 40% would be better.**

Accelerated 6 | Unit 6 Lesson 12 567 Activity 1

Key Takeaway: One strategy for determining the total after a **percent increase** and **percent decrease** is to find a benchmark percentage to add to or subtract from 100%. Another strategy is to multiply by a decimal that is representative of the ending desired percentage.

Pairs | 10 min

Activity 2 Fruitful Percentages

Purpose: Students apply the concepts from Activity 1 to new contexts and continue to make connections between representations. **(MTR.2.1)**

1 Launch

Direct students' attention to the diagrams in the table. Consider asking, "What do you notice about the two diagrams?"

2 Monitor

Encourage students to discuss what words or phrases help them decide whether a situation represents a percent increase or decrease and work together to reach an agreement about which diagram represents each situation. **(MTR.2.1)**

A **Accessibility: Executive Function** Invite students to focus on one situation at a time to support students in carrying out multiple steps.

Note: If time allows, invite pairs to compare their tables, justify their thinking, and make revisions based on their conversation.

Listen for student responses to share during the Connect and listen for student strategies for distinguishing between situations for percent increase and decrease.

For Problem 8, **look for** a variety of student explanations for percent decrease.

D **Differentiation: (Support)** To support students getting started, consider asking, "How might you use DeAndre's or Alfi's strategies to solve for the pounds of peaches?"

3 Connect

For Problem 7, **invite students** to share their work and explain how they distinguished between situations for percent increase and decrease.

To surface the Key Takeaway, consider asking:

- "Which words in a situation connect to a percent increase diagram? A percent decrease diagram?"
- "Why might it be helpful to have more than one way to ask for the same percent increase or decrease?"

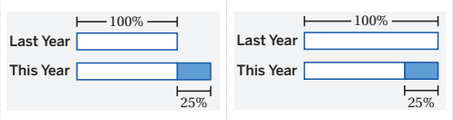
MLR **MLR2: Collect and Display** Use this routine to create a class definition of the terms *percent increase* and *percent decrease* using students' language. Display any new language, examples, or diagrams for students to reference later.

Activity 2

Name: Date: Period:

Fruitful Percentages

7. Decide which tape diagram could represent each situation by placing a check mark in the appropriate column.



This year's apple harvest is $\frac{3}{4}$ of last year's.		✓
There are $\frac{1}{4}$ more raspberries this year compared to last year.	✓	
This year's plum harvest is 125% of last year's.	✓	
This year's blueberry harvest is 75% of last year's.		✓
Compared to last year, this year's strawberry harvest increased 25%.	✓	
Compared to last year, this year's peach harvest decreased 25%.		✓

8. Here's a situation from the previous problem.

If 80 pounds of peaches were harvested last year, how many pounds were harvested this year?

60 pounds

Explain your thinking.

Explanations vary. **I know this year's harvest is 75% of 80, and $0.75 \cdot 80 = 60$.**

Compared to last year, this year's peach harvest decreased by 25%.

Accelerated 6 | Unit 6 Lesson 12 568 Activity 2

Key Takeaway: A percent increase can be expressed using words like more, percentages higher than 100, and a tape diagram with a shaded region that is more than the original length. A percent decrease can be expressed using words like less, percentages lower than 100, and a tape diagram with a shaded region that is less than the original length.

Pairs | 7 min

Activity 3 Percent Practice

Purpose: Students distinguish between a percent of a number and percent change.

1 Launch

For Problem 9, consider asking, "Where have you seen percentages? When might you want to compare percentages?"

2 Monitor

Listen for a variety of strategies students use to order the values.

D Differentiation

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Consider asking, "What words or phrases can you use to help estimate and then compute the values?"
Are unsure how to compute 25% more of 20 or 75% less than 60.	**Support** Consider asking, "How might you use DeAndre's or Alfi's strategies to solve ____?"
Use different tools to support thinking, such as a table, a tape diagram, or a calculator.	**Strengthen** Invite students to think about how they might share their strategy during the Connect.

A Accessibility: Executive Functioning Invite students to make a table on a separate piece of paper to organize their thinking.

ELL English Language Learners Encourage students to discuss how to order the values in their primary language, clarifying any unknown words. **(Speaking and Reading)**

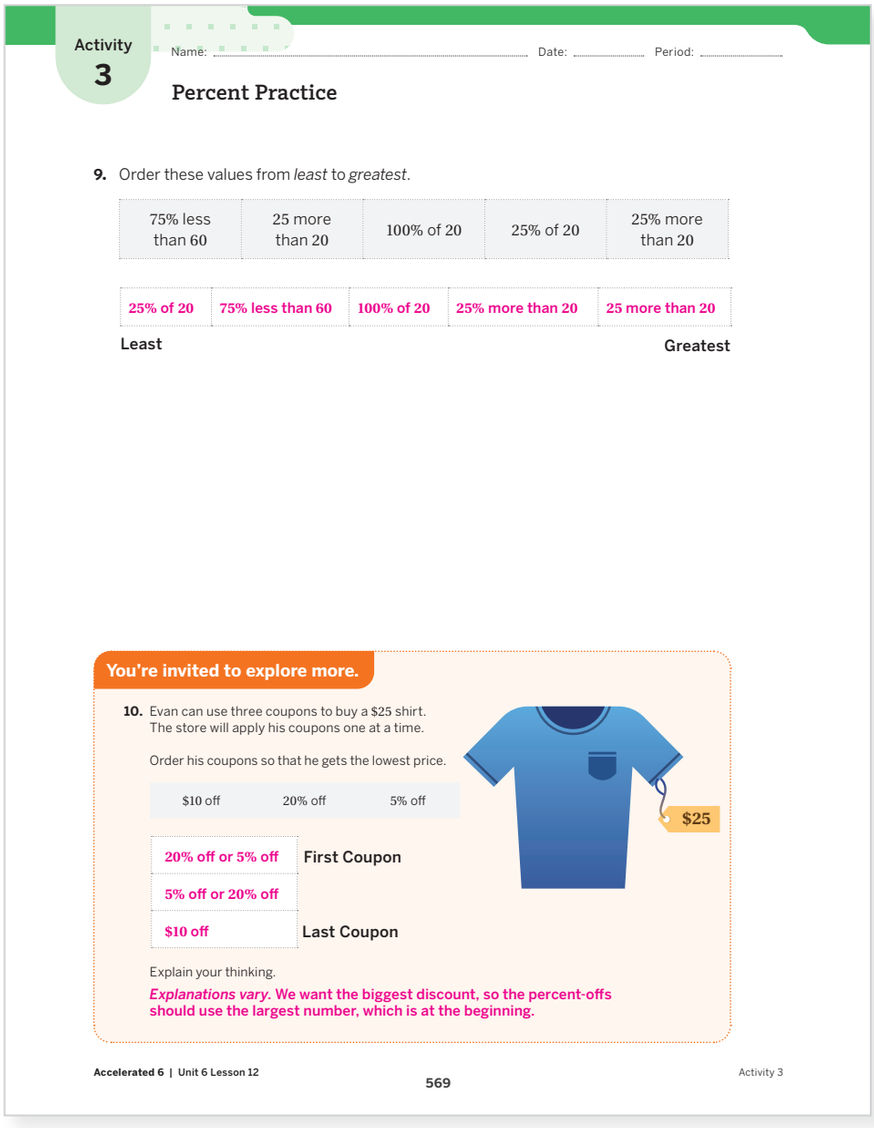
D Differentiation: You're invited to explore more. (Stretch) Invite students who want to explore percent increase and decrease further to complete this optional task. Encourage them to discuss their thinking with a partner.

3 Connect

For Problem 9, **display** the values and invite students to share strategies for ordering the values.

Consider asking, "What was easy about this ordering task? What was challenging? What do you want to remember about determining percentages?"

Note: If time allows, try to have students that used different models share how they were helpful for this problem and add them to the anchor chart.



Activity 3

Name: Date: Period:

Percent Practice

9. Order these values from *least* to *greatest*.

75% less than 60	25 more than 20	100% of 20	25% of 20	25% more than 20

25% of 20	75% less than 60	100% of 20	25% more than 20	25 more than 20

Least Greatest

You're invited to explore more.

10. Evan can use three coupons to buy a $25 shirt. The store will apply his coupons one at a time.

Order his coupons so that he gets the lowest price.

$10 off	20% off	5% off

20% off or 5% off	First Coupon
5% off or 20% off	
$10 off	Last Coupon

Explain your thinking.

Explanations vary. We want the biggest discount, so the percent-offs should use the largest number, which is at the beginning.

Accelerated 6 | Unit 6 Lesson 12 569 Activity 3

Whole Class | 5 min

Synthesis

Purpose: Students synthesize their understanding of different strategies and tools to solve for percent increase and percent decrease.

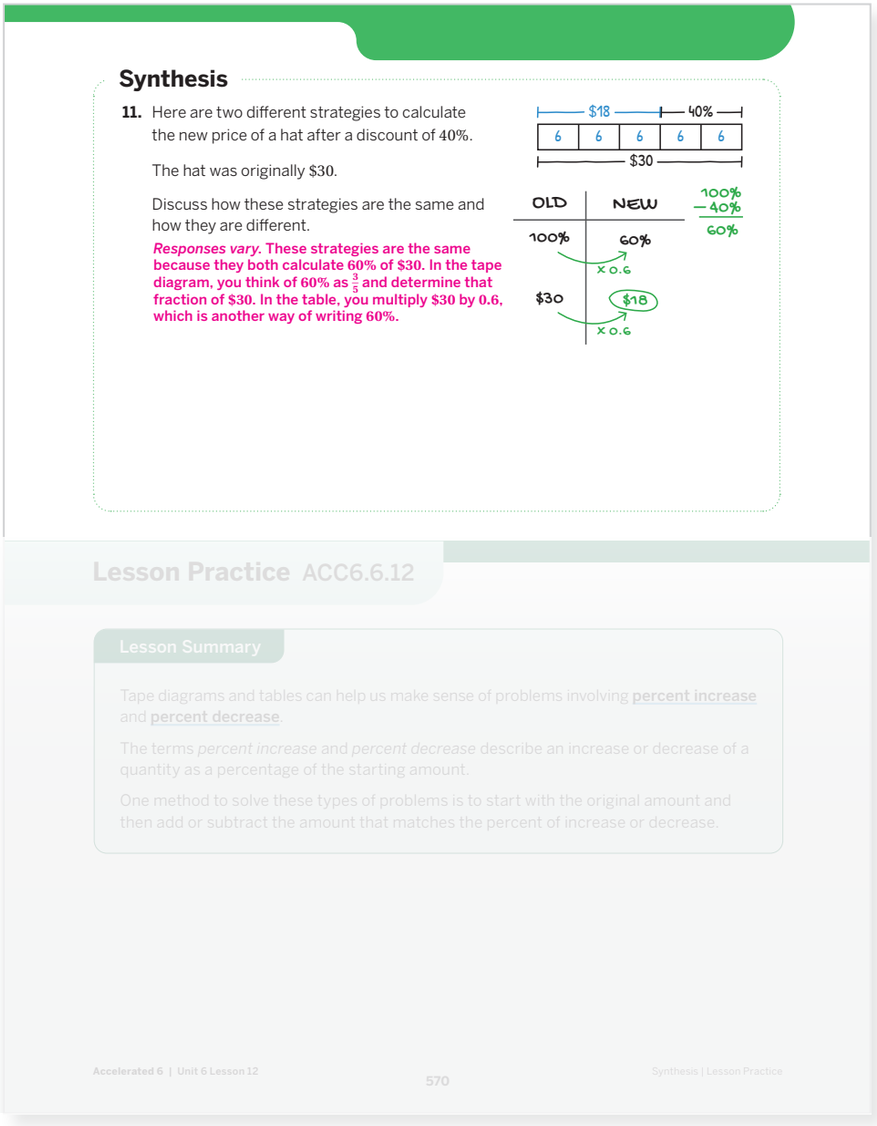
For Problem 11, **invite students** to respond independently, and then share their thinking with a partner.

Listen for and share a variety of ideas, such as:

- A tape diagram can be a useful representation in displaying a percent discount, where the segments in a tape diagram align with the number necessary to represent the percent decrease in a given problem.
- A table can be a useful representation in explaining percent discount as the same constant of proportionality can be applied to a row displaying percentages and a row displaying price.

Consider asking, "Why might it be helpful to have multiple strategies for solving for percent decrease?"

Note: If time allows, invite students to share with a partner what makes sense about each strategy and what questions they still have.



Synthesis

11. Here are two different strategies to calculate the new price of a hat after a discount of 40%.

The hat was originally $30.

Discuss how these strategies are the same and how they are different.

Responses vary. These strategies are the same because they both calculate 60% of $30. In the tape diagram, you think of 60% as $\frac{3}{5}$ and determine that fraction of $30. In the table, you multiply $30 by 0.6, which is another way of writing 60%.

Lesson Practice ACC6.6.12

Lesson Summary

Tape diagrams and tables can help us make sense of problems involving percent increase and percent decrease.

The terms *percent increase* and *percent decrease* describe an increase or decrease of a quantity as a percentage of the starting amount.

One method to solve these types of problems is to start with the original amount and then add or subtract the amount that matches the percent of increase or decrease.

Accelerated 6 | Unit 6 Lesson 12 570 Synthesis | Lesson Practice

Lesson Takeaway: Tape diagrams, tables, or other tools can help make sense of problems involving **percent increase** and **percent decrease**.

Independent | 5 min

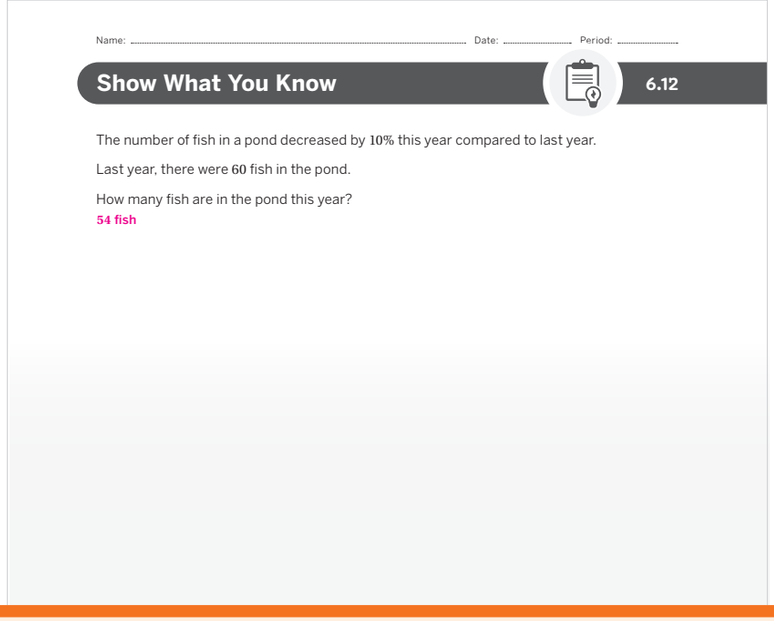
Show What You Know

Purpose: Students demonstrate their understanding by interpreting a percent decrease situation.

Today's Goals

1. **Goal:** Use tape diagrams and tables to represent situations involving a *percent increase* or *decrease*.
2. **Goal:** Determine a new amount given an original amount and a *percent increase* or *decrease*.
3. **Language Goal:** Interpret a verbal description to solve problems about *percent increase* or *decrease*. **(Reading and Writing)**

Show What You Know (Assessment Resources)



Name: _____ Date: _____ Period: _____

Show What You Know 6.12

The number of fish in a pond decreased by 10% this year compared to last year.

Last year, there were 60 fish in the pond.

How many fish are in the pond this year?

54 fish

Differentiation Use after Lesson 12

Support

Provide targeted intervention for students by using these resources.

If students need support with understanding percent increase and percent decrease:

» **Respond to Student Thinking:**

- Students will have more opportunities to develop their understanding of these concepts in Lessons 13–14. You may choose not to intervene now.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 4 Extension Activities**.
- Invite students to create another problem similar to the **Explore More** task, then challenge a partner to solve for the final price.

Professional Learning

In this lesson, students are introduced to tape diagrams as a model to support their understanding of **percent increase** and **percent decrease**. How does this model integrate with other representations (i.e., double number lines, tables, etc.) of percent increase and percent decrease?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

11. Here are two different strategies to calculate the new price of a hat after a discount of 40%.

The hat was originally $30.

Discuss how these strategies are the same and how they are different.

Responses vary. These strategies are the same because they both calculate 60% of $30. In the tape diagram, you think of 60% as $\frac{3}{5}$ and determine that fraction of $30. In the table, you multiply $30 by 0.6, which is another way of writing 60%.

Lesson Practice ACC6.6.12

Lesson Summary

Tape diagrams and tables can help us make sense of problems involving **percent increase** and **percent decrease**.

The terms *percent increase* and *percent decrease* describe an increase or decrease of a quantity as a percentage of the starting amount.

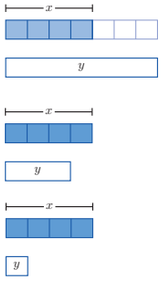
One method to solve these types of problems is to start with the original amount and then add or subtract the amount that matches the percent of increase or decrease.

Lesson Practice ACC6.6.12

Name: _____ Date: _____ Period: _____

1. Match each situation with a diagram.

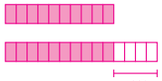
Situation	Diagram
a. Hoang drinks x ounces of juice. Nekeisha drinks $\frac{1}{4}$ less than that.	b
b. Hoang runs x miles. Nekeisha runs $\frac{3}{4}$ more than that.	a
c. Hoang buys x pounds of almonds. Nekeisha buys $\frac{1}{4}$ of that.	c



Problems 2–3: Draw a diagram to represent each situation.

2. The amount of flour that the bakery used this month increased 40% compared to last month.

Responses vary.



3. The amount of milk that the bakery used this month decreased 75% compared to last month.

Responses vary.

Lesson Practice ACC6.6.12

Name: _____ Date: _____ Period: _____

Problems 4–6: Fill in the blanks to describe each increase or decrease as a percentage of the original amount.

4. This year, there was 40% more snow than last year.

The amount of snow this year is 140 % of the amount last year.

5. This year, there were 25% fewer sunny days than last year.

The number of sunny days this year is 75 % of the number of sunny days last year.

6. A restaurant adds a 93% markup to the price of the ingredients to set the menu price.

The menu price is 193 % of the price of ingredients.

FAST Practice

7. At the beginning of the month, there were 80 ounces of peanut butter in the pantry. Since then, a family has eaten 30% of it. Which expression represents the ounces of peanut butter left in the pantry?

A. $0.7 \cdot 80$ B. $0.3 \cdot 80$ C. $8 - 0.30$ D. $(1 + 0.3) \cdot 80$

Spiral Review

Problems 8–9: A store sells strawberries for $3 per pound.

8. Write an expression for the cost of p pounds of strawberries.

$3p$

9. What is the cost of 14 pounds of strawberries?

$42

Problems 10–11: Solve each equation.

10. $x \cdot \frac{7}{3} = 1$

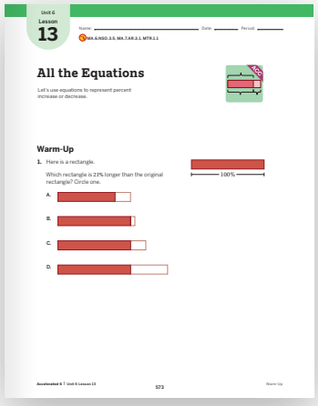
$x = \frac{3}{7}$

11. $1 \div \frac{11}{2} = x$

$x = \frac{2}{11}$

Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1	2	MA.7.AR.3.1
	2–6	2	MA.7.AR.3.1
FAST Practice	7	2	MA.7.AR.3.1
Spiral Review			
	8	2	MA.6.AR.1.1
	9	2	MA.6.AR.1.3
	10–11	1	MA.6.AR.2.3



All the Equations

Percent Increase and Decrease With Equations

Let's use equations to represent percent increase or decrease.

Learning Outcomes

Today's Goals

1. **Goal:** Use equations to represent situations involving a percent increase or decrease.
2. **Goal:** Use a tape diagram where 100% corresponds to the original value to represent adding or subtracting a percent of the original value.
3. **Language Goal:** Interpret a verbal description to write an equation involving percent increase or decrease. **(Reading)**

Students write and use equations to describe the relationship between original and new amounts after a percent increase or decrease. They also make connections between different ways to write their equations.

Coherence

Prior Learning

In Lesson 12, students used tape diagrams to visualize percent increase and decrease and to calculate unknown values.

Future Learning

In Lesson 14, students will practice calculating the original amount, new amount, or the percent change given the other two quantities using representations such as a double number line diagram, table, and equation.

Connections

- Students leverage different forms of equations to describe the relationship between the original and new amounts of an item after a percent increase or decrease in many real world situations such as the paper used in a copy shop. (real world)

FL B.E.S.T. Math Benchmarks

Addressing

MA.6.NSO.3.5
Rewrite positive rational numbers in different but equivalent forms including fractions, terminating decimals and percentages.

MA.7.AR.3.1
Apply previous understanding of percentages and ratios to solve multi-step real-world percent problems.

Mathematical Thinking and Reasoning Standard: MTR.1.1

ELA Expectations: ELA.K12.EE.4.1, ELA.K12.EE.6.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.6.AR.3.4	MA.7.NSO.1.2 MA.912.AR.1.1

Lesson at a Glance

~ 45 min

MA.6.NSO.3.5, MA.7.AR.3.1

Warm-Up

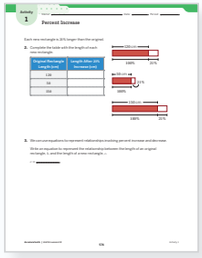
Pairs | 5 min

Students visualize increasing by the percentages they will represent with an equation in Activity 1.
Routine: Think-Pair-Share

Activity 1

Pairs | 10 min

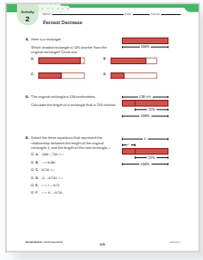
Students consider the relationship between original and new amounts after a percent increase and write an equation to represent it.
MLR2: Collect and Display **(MTR.1.1)**



Activity 2

Pairs | 12 min

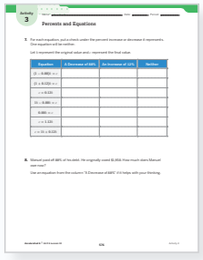
Students make sense of using equations to represent percent decrease.



Activity 3

Pairs | 8 min

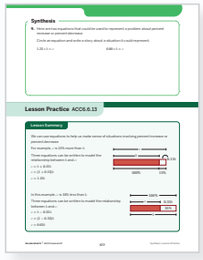
Students deepen their understanding of the connection between the equations that represent percent increase and percent decrease.



Synthesis

Whole Class | 5 min

Students synthesize their understanding of percent increase and decrease by using the Tell a Story routine to describe a given equation.



Show What You Know

Independent | 5 min

Students demonstrate their understanding by writing an equation to represent the relationship between an original and new amount after a percent increase.

Prep Checklist

This lesson includes:

- Student Edition
- Show What You Know
- Calculators (as needed)

Pairs | 5 min

Warm-Up

Purpose: Students visualize increasing by the percentages they will represent with an equation in Activity 1.

1 Launch

Use the Think-Pair-Share routine to help students shape their ideas about percent increase. Invite students to think independently about how to identify a rectangle that is 21% longer than the original rectangle and then take turns sharing with a partner.

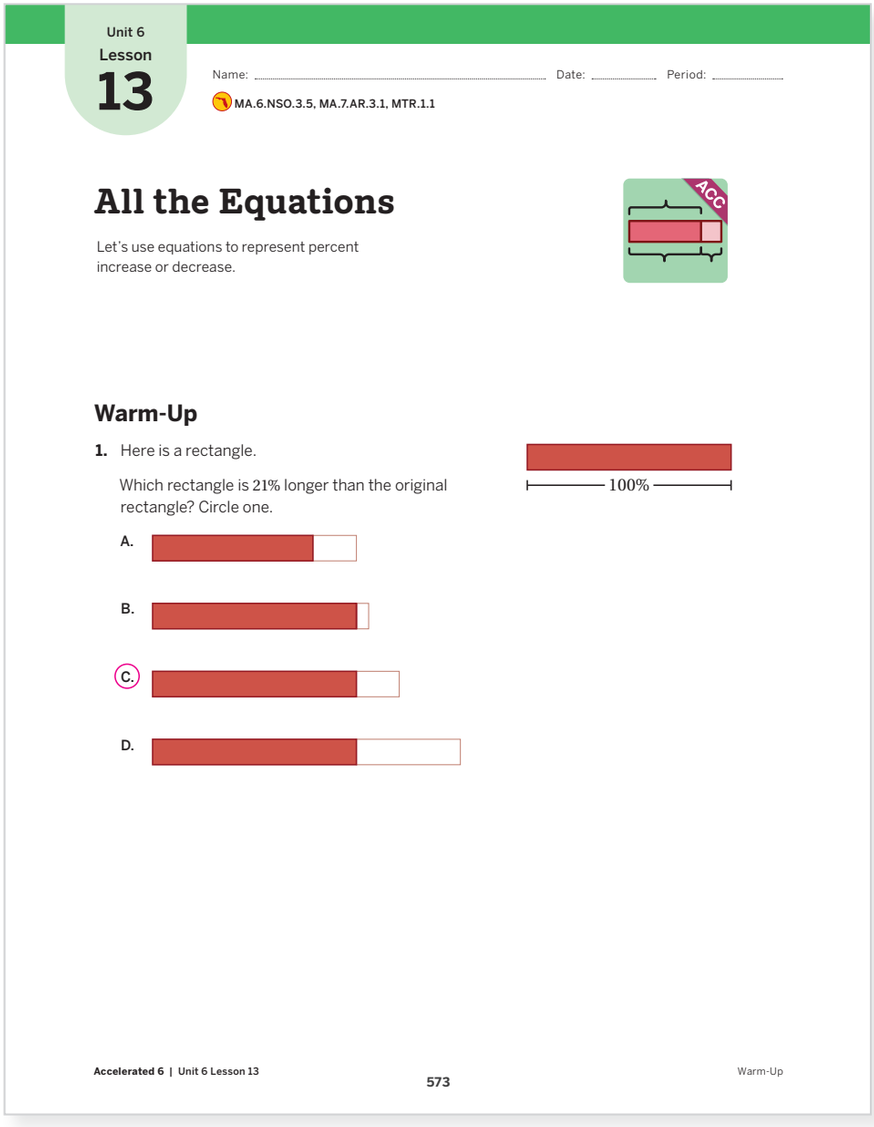
Invite students to consider the rectangle choices. Consider asking, "How are the rectangles similar to or different from the original?"

ELL **English Language Learners** Use gestures to illustrate the term longer. (**Reading and Listening**)

2 Connect

Invite students to share their or their partner's idea for identifying the rectangle that is 21% longer with the class. Encourage several students to share.

Consider asking, "How did you determine how wide the section representing 21% should be based on the model for 100%?"



Unit 6
Lesson
13

Name: Date: Period:

MA.6.NSO.3.5, MA.7.AR.3.1, MTR.1.1

All the Equations

ACC

Let's use equations to represent percent increase or decrease.

Warm-Up

1. Here is a rectangle.

Which rectangle is 21% longer than the original rectangle? Circle one.

A.

B.

C.

D.

Accelerated 6 | Unit 6 Lesson 13 573 Warm-Up

Pairs | 10 min

Activity 1 Percent Increase

Purpose: Students consider the relationship between original and new amounts after a percent increase and write an equation to represent it. **(MTR.1.1)**

Short on time? Consider having students complete two of the three rows in Problem 2.

1 Launch

To surface connections, consider asking, "Is 21% the same amount for each of these rectangles? How do you know?"

2 Monitor

Look for a variety of student strategies to compute the length of the new rectangles.

D **Differentiation:** (Support) To support students getting started, consider working together with students on the first rectangle before inviting them to try one independently.

Note: This relationship is challenging and students may need support writing an equation to represent it. Encourage them to write their equation even if they are unsure; they can edit their equation after getting feedback from their partner. **(MTR.1.1)**

Select and sequence student equations to share during the Connect.

D **Differentiation**

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Consider asking them to write any equation that fits one pair of values and use the feedback to revise their thinking.
Write the equation $c = 0.21b$.	**Support** Consider asking, "Why do you think the value decreased? How could we represent the 100%?"
Represent the 21% increase as the decimal 1.21.	**Strengthen** Invite students to think about how they might share their strategy during the Connect.
Want to explore writing percent increase equations further.	**Stretch** Invite students to think of a different correct equation for this relationship.

3 Connect

Share student equations and consider asking:

- "Where do we see 21% in this equation?"
- "Where do we see the value of the original rectangle?"
- "How do we know this equation will result in an increase?"

MLR **MLR2: Collect and Display** Use this routine to synthesize strategies for writing an equation for percent increase using students' language. Display any new language, examples, or diagrams for students to reference later.

Activity 1

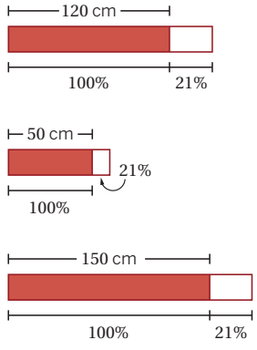
Name: Date: Period:

Percent Increase

Each new rectangle is 21% longer than the original.

2. Complete the table with the length of each new rectangle.

Original Rectangle Length (cm)	Length After 21% Increase (cm)
120	145.2
50	60.5
150	181.5



3. We can use equations to represent relationships involving percent increase and decrease.

Write an equation to represent the relationship between the length of an original rectangle, b, and the length of a new rectangle, c.

$c =$ _____

Responses vary.

- $c = 1.21b$
- $c = 1b + 0.21b$
- $c = (1 + 0.21)b$

Accelerated 6 | Unit 6 Lesson 13 574 Activity 1

Pairs | 12 min

Activity 2 Percent Decrease

Purpose: Students make sense of using equations to represent percent decrease.

1 Launch

To surface connections, consider asking, "How is this like our Warm-Up? How is it different?"

2 Monitor

To support students getting started, consider asking, "Will the amount shaded be larger or smaller than half of the original rectangle?"

Listen for a variety of student strategies to compute the length of the new rectangles.

D Differentiation

Look for students who:	Respond to Student Thinking
Need support getting started or are using only a guess and check method.	**Support** Consider asking, "How would you approach this problem if the percentage was 75%?" Encourage students to build from this strategy.
Use strategies to calculate the length of the shorter rectangle, such as: • Calculating the remaining percentage of the rectangle's length (28%). • Calculating 72% of 120 and subtracting it from 120.	**Strengthen** Invite students to think about how their strategy might be captured as an equation.

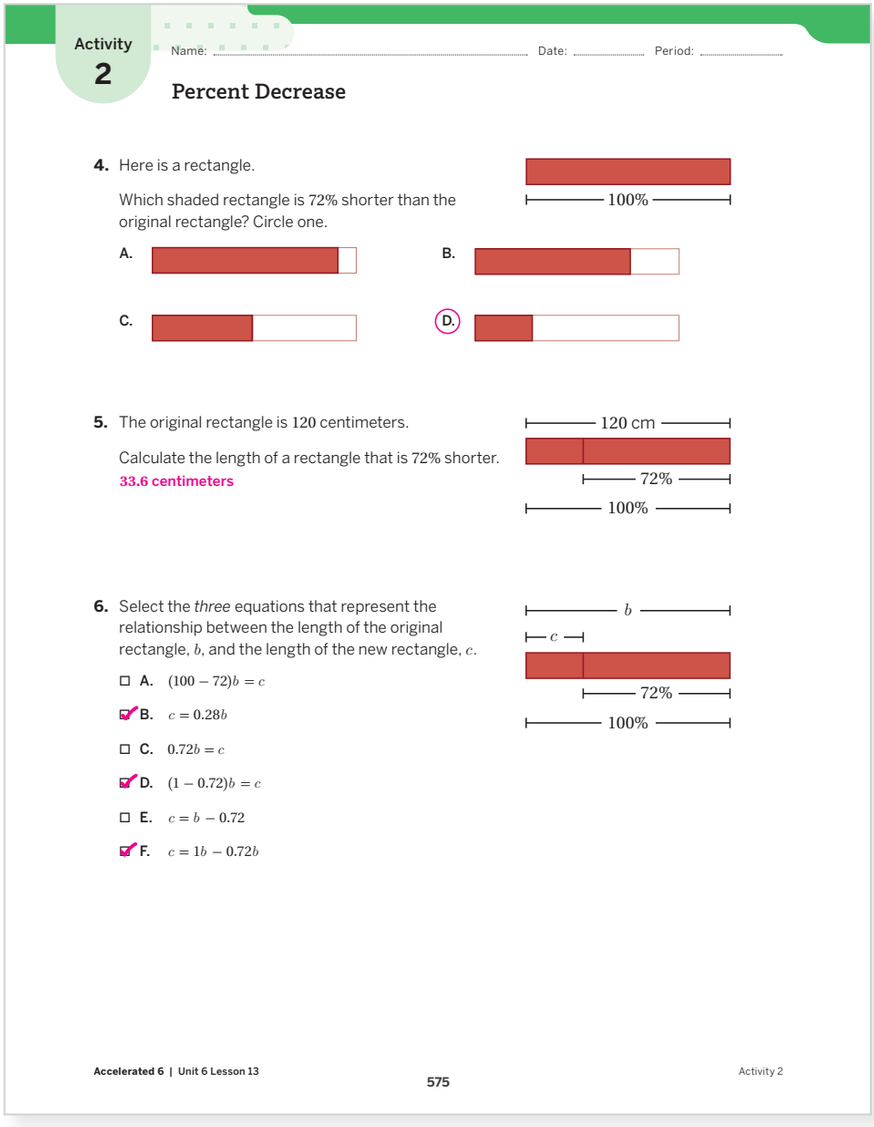
Encourage students to work together to reach an agreement about which three equations represent the relationship.

D Differentiation: (Support) For students that need support getting started, consider inviting them to test the values 120 and 33.6 from the previous problem in different equations and see what they notice.

3 Connect

Invite students to share their work. Consider adding equations to the anchor chart from Activity 1 for all students to reference throughout the unit.

To surface the Key Takeaway, consider asking, "How are each of the three correct equations related?"



Activity 2

Name: Date: Period:

Percent Decrease

4. Here is a rectangle.

Which shaded rectangle is 72% shorter than the original rectangle? Circle one.

A.

B.

C.

D.

5. The original rectangle is 120 centimeters.

Calculate the length of a rectangle that is 72% shorter.

33.6 centimeters

6. Select the *three* equations that represent the relationship between the length of the original rectangle, b, and the length of the new rectangle, c.

- ☐ A. $(100 - 72)b = c$
- ☑ B. $c = 0.28b$
- ☐ C. $0.72b = c$
- ☑ D. $(1 - 0.72)b = c$
- ☐ E. $c = b - 0.72$
- ☑ F. $c = 1b - 0.72b$

Accelerated 6 | Unit 6 Lesson 13 575 Activity 2

Key Takeaway: There are multiple equivalent ways to write an equation for percent decrease. These all are different versions of writing the percentage as a decimal, subtracting it from 1 (or 100%), and then multiplying by the original value.

Pairs | 8 min

Activity 3 Percents and Equations

Purpose: Students deepen their understanding of the connection between the equations that represent percent increase and percent decrease.

1 Launch

Encourage students to share their reasoning with a partner and work together to reach an agreement about how to identify the equations that represent percent increase and decrease.

Note: If time allows, invite pairs to compare their responses, justify their thinking, and make revisions based on their conversation.

2 Monitor

Listen for student strategies for distinguishing between equations for percent increase and decrease.

ELL **English Language Learners** Provide sentence frames to help students explain their strategy (e.g., "This equation represents ____ because ____." or "This equation does not represent an increase or decrease because ____."). **(Writing and Speaking)**

Pause briefly to invite students to share strategies for distinguishing between equations for percent increase and decrease.

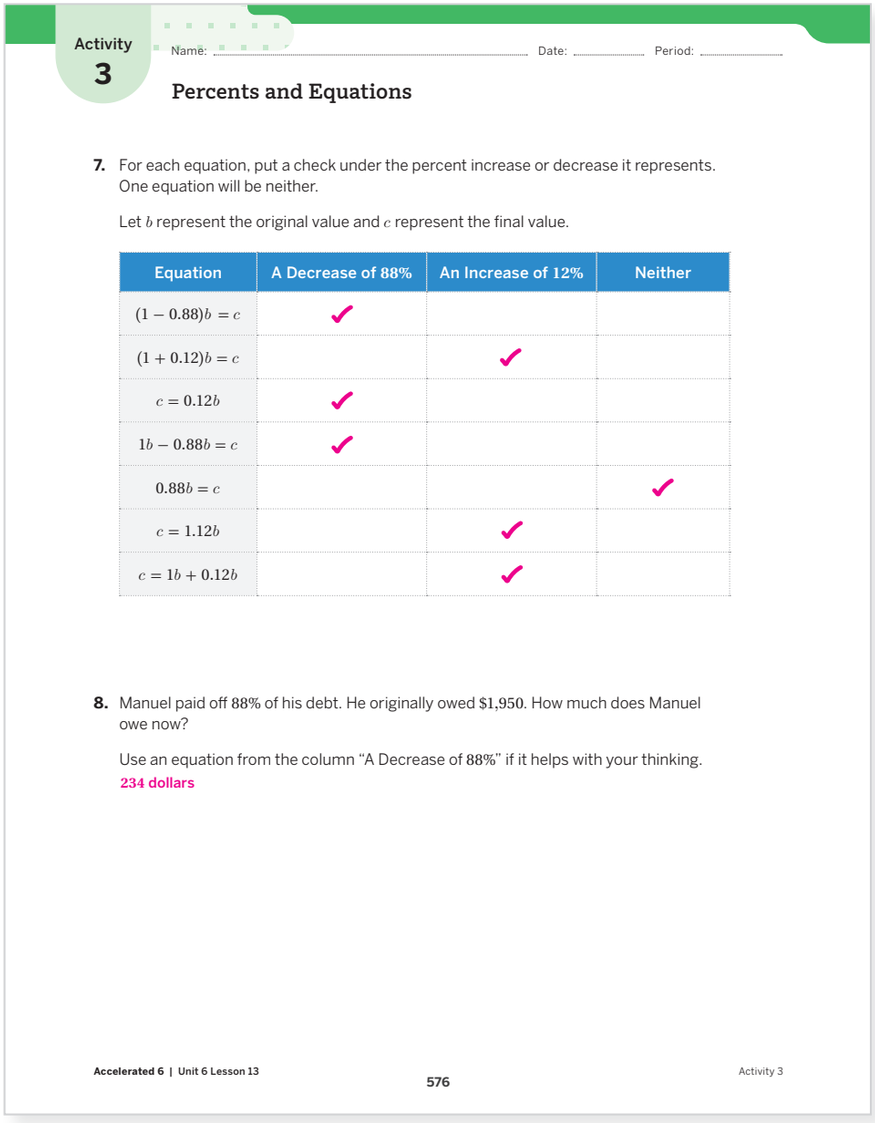
To support students getting started, consider asking, "Will 1950 represent b or c?"

Look for a variety of student strategies (e.g., tape diagrams, tables, equations, or individual strategies) to determine how much Manuel owes.

3 Connect

Discuss the three equations in the image. Consider asking:

- "Does the 1950 represent b or c in each equation?"
- "Is the amount he owes now more or less than half of what he owed originally? How do you know?"
- "What does the 0.12 represent in the equation? Where does it come from?"



Activity 3

Name: Date: Period:

Percents and Equations

7. For each equation, put a check under the percent increase or decrease it represents. One equation will be neither.

Let b represent the original value and c represent the final value.

Equation	A Decrease of 88%	An Increase of 12%	Neither
$(1-0.88)b=c$	✓		
$(1+0.12)b=c$		✓	
$c=0.12b$	✓		
$1b-0.88b=c$	✓		
$0.88b=c$			✓
$c=1.12b$		✓	
$c=1b+0.12b$		✓	

8. Manuel paid off 88% of his debt. He originally owed $1,950. How much does Manuel owe now?

Use an equation from the column "A Decrease of 88%" if it helps with your thinking.

234 dollars

Accelerated 6 | Unit 6 Lesson 13 576 Activity 3

Whole Class | 5 min

Synthesis

Purpose: Students synthesize their understanding of percent increase and decrease by using the Tell a Story routine to describe a given equation.

Invite students to spend 2 minutes responding to the prompt. Ask them to share their thinking with a partner.

Use the Tell a Story routine to help students make sense of percent increase or decrease using their unique experiences, perspectives and creativity.

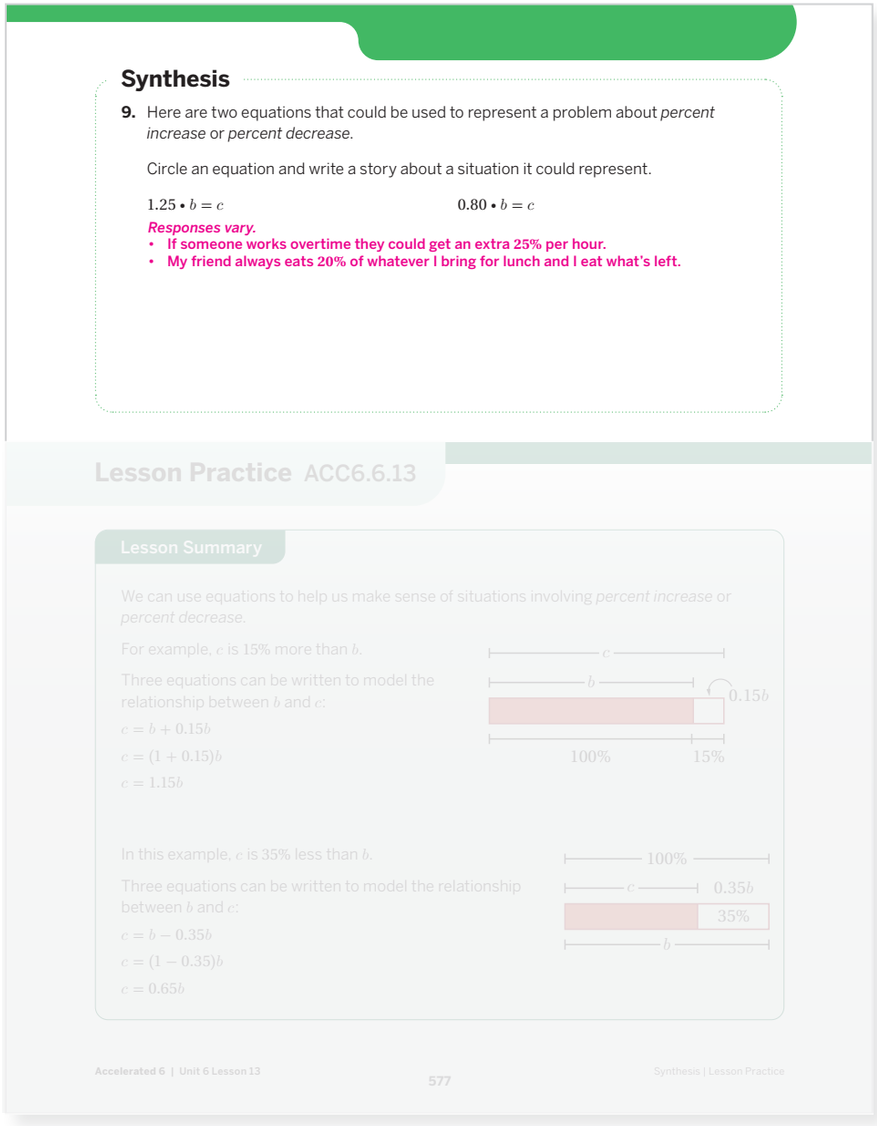
Consider sharing that stories do not have to be complete or perfect to be valuable.

ELL **English Language Learners** Invite students to write first in their primary language and then translate it to English during the second draft. **(Writing)**

Encourage students to ask questions about each other's stories. Consider asking:

- "How do you see the parts of ___'s story in the equation?"
- "What parts of each of our stories are alike? How are they different?"
- "What questions do you have about each other's stories?"

Note: If time allows, return to the anchor chart and add several of the student stories, with their names, as examples to refer back to throughout the unit.



Synthesis

9. Here are two equations that could be used to represent a problem about *percent increase* or *percent decrease*.

Circle an equation and write a story about a situation it could represent.

$1.25 \cdot b = c$ $0.80 \cdot b = c$

Responses vary.

- If someone works overtime they could get an extra 25% per hour.
- My friend always eats 20% of whatever I bring for lunch and I eat what's left.

Lesson Practice ACC6.6.13

Lesson Summary

We can use equations to help us make sense of situations involving *percent increase* or *percent decrease*.

For example, c is 15% more than b.

Three equations can be written to model the relationship between b and c:

$c = b + 0.15b$

$c = (1 + 0.15)b$

$c = 1.15b$

In this example, c is 35% less than b.

Three equations can be written to model the relationship between b and c:

$c = b - 0.35b$

$c = (1 - 0.35)b$

$c = 0.65b$

Accelerated 6 | Unit 6 Lesson 13 577 Synthesis | Lesson Practice

Lesson Takeaway: Equations are one way to represent percent increase or decrease. If you write a percentage as a decimal, then add or subtract it from 1 (or 100%), you can multiply that number by the original to get the new value.

Independent | 5 min

Show What You Know

Purpose: Students demonstrate their understanding by writing an equation to represent the relationship between an original and new amount after a percent increase.

Today's Goals

1. **Goal:** Use equations to represent situations involving a percent increase or decrease.
2. **Goal:** Use a tape diagram where 100% corresponds to the original value to represent adding or subtracting a percent of the original value.
3. **Language Goal:** Interpret a verbal description to write an equation involving percent increase or decrease. **(Reading)**

Show What You Know (Assessment Resources)

Name: _____ Date: _____ Period: _____

Show What You Know 6.13

Jayla's bank account increased by 7% this year.

Write an equation to represent the relationship between the amount that Jayla started with, b, and the amount she has now, c.

Responses vary.

- $1b + 0.07b = c$
- $1.07b = c$
- $c = (1 + 0.07)b$

D Differentiation Use after Lesson 13

Support

Provide targeted intervention for students by using these resources.

If students need support understanding how to use equations to represent percent change:

» **Respond to Student Thinking:**

- Revisit Activity 3 with students before providing individualized feedback on the relevant Practice Problems.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 4 Extension Activities**.

Professional Learning

Take a moment to reflect on this lesson. Did the order in which you sequenced student ideas support the learning goals of the lesson? What might you change for the next time you facilitate a lesson like this?

Lesson Practice

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

9. Here are two equations that could be used to represent a problem about *percent increase* or *percent decrease*.

Circle an equation and write a story about a situation it could represent.

$1.25 \cdot b = c$ $\quad$ $0.80 \cdot b = c$

Responses vary.
- If someone works overtime they could get an extra 25% per hour.
- My friend always eats 20% of whatever I bring for lunch and I eat what's left.

Lesson Practice ACC6.6.13

Lesson Summary

We can use equations to help us make sense of situations involving *percent increase* or *percent decrease*.

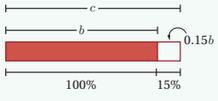
For example, c is 15% more than b.

Three equations can be written to model the relationship between b and c:

$c = b + 0.15b$

$c = (1 + 0.15)b$

$c = 1.15b$



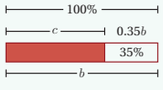
In this example, c is 35% less than b.

Three equations can be written to model the relationship between b and c:

$c = b - 0.35b$

$c = (1 - 0.35)b$

$c = 0.65b$



Lesson Practice ACC6.6.13

Name: _____ Date: _____ Period: _____

1. Draw a diagram that represents this situation:

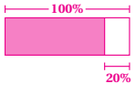
The number of people in a town has increased by 50% in the past decade.

2. Write a story that matches the diagram.

Responses vary. The number of pets in my family increased by 25%.

3. Draw a diagram that represents this situation:

The amount of paper that the copy shop used this month decreased by 20% compared to what they used last month.



4. A new video game costs $60 on the day it gets released. The price of a new game generally drops by about 12% within a few weeks. What will the price of this new game be a few weeks after release? Show or explain your thinking.

$52.80. *Explanations vary.*

$100\% - 12\% = 88\%$

Let c represent the original cost of the game and s represent the price. Then $0.88 \cdot c = s$. If $c = 60$, then $0.88 \cdot 60 = s$, so $52.80 = s$.

5. Kai and Amari both wrote an equation to represent an increase of 3%, where x represents the amount before the increase and y represents the new amount.

Kai wrote the equation $y = 1x + 0.03x$. Amari wrote the equation $y = 1.03x$.

Whose equation is correct? Circle one.

Kai Amari (Both) Neither

Explain your thinking.

Explanations vary. If I substitute a number in for either equation, I get a value that is 3% more than the value I started with.

Lesson Practice ACC6.6.13

Name: _____ Date: _____ Period: _____

Problems 6–7: A sneaker store raised its prices 15% compared to last year.

6. If x is the price before the increase and y is the price after the increase, which equations are correct? Select *all* that apply.

☑ A. $y = 1.15x$ ☐ B. $y = x + 0.15$ ☑ C. $y = x + 0.15x$

☐ D. $y = 15 + x$ ☑ E. $y = (1 + 0.15)x$

7. A pair of designer sneakers was $120 last year. What is the price this year?

$138

FAST Practice

8. Two stores each advertise a discount on the same type of sweatshirt. At both stores, the original price of the sweatshirt is $25.
- Store A discounts the price of the sweatshirt by 20%.
- Store B discounts the price of the sweatshirt by 15%.

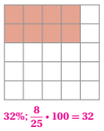
How much less is the discounted price of the sweatshirt at Store A than the discounted price of the sweatshirt at Store B?

A. $0.75 B. $1.00 (C.) $1.25 D. $1.75

Spiral Review

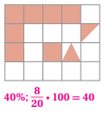
Problems 9–10: Determine the percentage of each grid that is shaded.

9.



32%; $\frac{8}{25} \cdot 100 = 32$

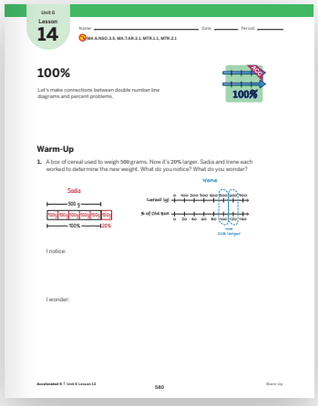
10.



40%; $\frac{8}{20} \cdot 100 = 40$

Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1–7	2	MA.7.AR.3.1
FAST Practice	8	2	MA.7.AR.3.1
Spiral Review			
	9–10	1	MA.6.AR.3.4



100%

Percent Increase and Decrease With Double Number Lines

Let's make connections between double number line diagrams and percent problems.

Learning Outcomes

Today's Goals

1. **Goal:** Use double number lines to represent situations involving a percent increase or decrease.
2. **Goal:** Determine the original amount given the new amount and a percent increase or decrease.
3. **Language Goal:** Interpret a situation to solve problems about percent increase or decrease using mathematically precise language. **(Reading and Writing)**

Students expand on their understanding of double number line diagrams and percentages to make sense of and model problems involving percent change. Later in the lesson, students make connections between tables, equations, and double number line diagrams to determine the new value when given the original value and the percent of increase or decrease. **(MTR.2.1)**

Coherence

Prior Learning

In Lesson 13, students represented situations involving percent increase or decrease using equations.

Future Learning

In Lesson 15, students will solve multistep percent problems as they calculate tax, tips, and fees in various situations.

Connections

- Students determine the new value of an item when given the original value and the percent of increase (or decrease) in real world contexts such as the decrease in the size of popcorn bags in a movie theater. (real world)

FL B.E.S.T. Math Benchmarks

Addressing

MA.6.NSO.3.5
Rewrite positive rational numbers in different but equivalent forms including fractions, terminating decimals and percentages.

MA.7.AR.3.1
Apply previous understanding of percentages and ratios to solve multi-step real-world percent problems.

Mathematical Thinking and Reasoning Standards: MTR.1.1, MTR.2.1

ELA Expectations: ELA.K12.EE.3.1, ELA.K12.EE.4.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.6.AR.3.4	MA.7.NSO.1.2

Vocabulary

New Vocabulary

markdown markup

Lesson at a Glance

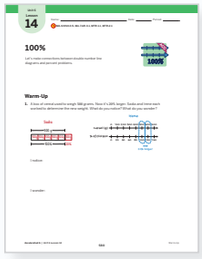
~ 45 min

MA.6.NSO.3.5, MA.7.AR.3.1

Warm-Up

Pairs | 5 min

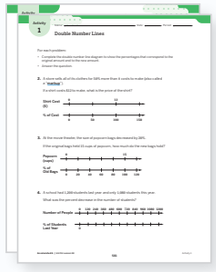
Students use the **Notice and Wonder** routine to analyze two student work samples for a percent increase problem. **(MTR.2.1)**



Activity 1

Pairs | 15 min

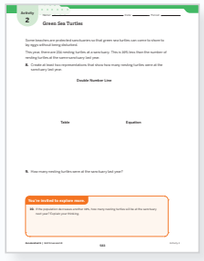
Students use double number line diagrams to calculate the original amount, new amount, or percent change, given the other two quantities.



Activity 2

Pairs | 15 min

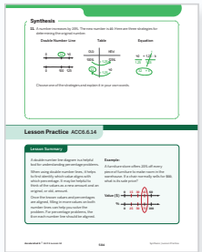
Students use multiple representations to determine how a turtle population has changed over a year. **(MTR.2.1)**



Synthesis

Whole Class | 5 min

Students synthesize their understanding of using tables, equations, and double number line diagrams to solve problems involving percent increase and decrease.



Show What You Know

Independent | 5 min

Students demonstrate their understanding by solving for the new value when given the percent increase and original value.

Prep Checklist

This lesson includes:

- Student Edition
- Show What You Know

Pairs | 5 min

Warm-Up

Purpose: Students use the Notice and Wonder routine to analyze two student work samples for a percent increase problem. **(MTR.2.1)**

1 Launch

To generate engagement, consider asking, "What is your favorite cereal? How would you feel if the box was 20% larger?"

Use the Notice and Wonder routine to promote curiosity and help students make sense of the math representations. **(MTR.2.1)**

ELL **English Language Learners** Invite students to read the prompt aloud and paraphrase what it means. Clarify any words or phrases they don't know. **(Reading)**

Encourage students to think independently and then share their thinking with a partner about the two diagrams.

2 Connect

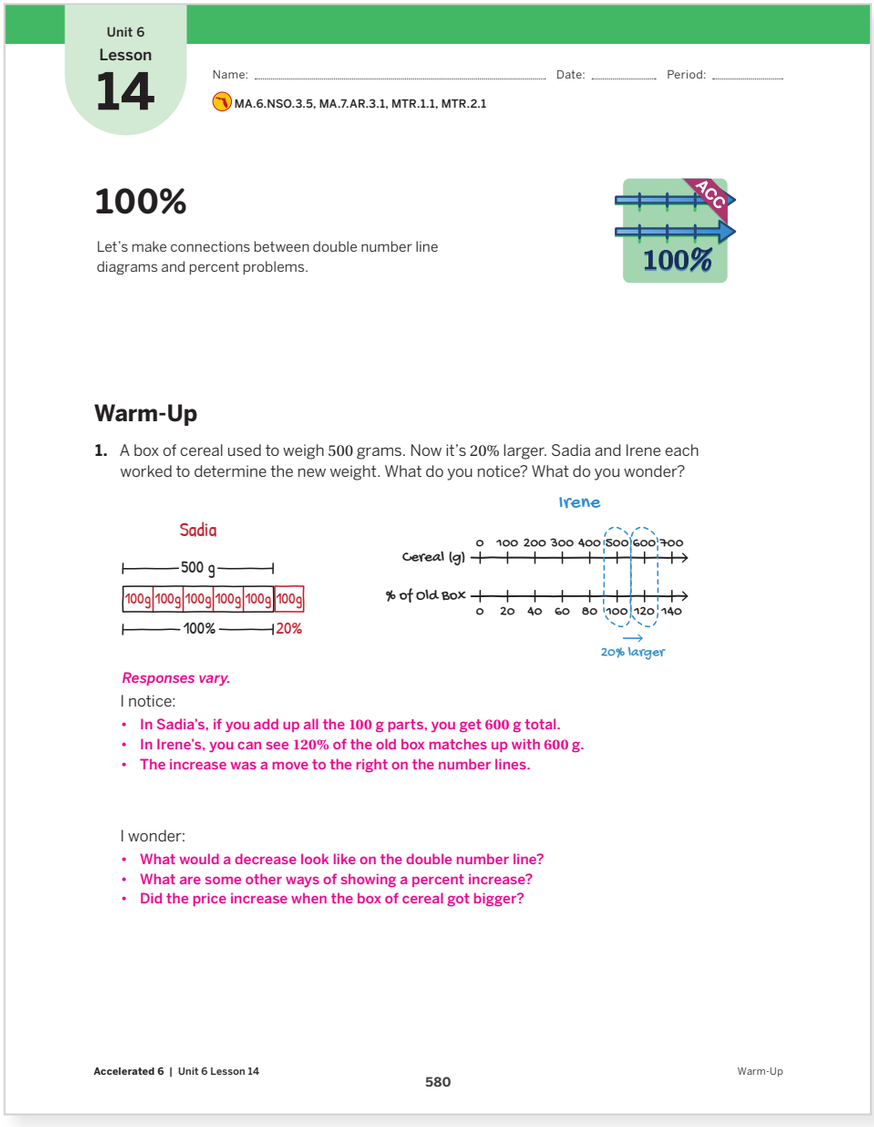
Discuss student explanations for interpreting each representation. Consider asking:

- "How are these two representations connected?"
- "Where can you see the percent increase in both representations?"
- "What would a percent decrease look like in each of these representations?"

Create a record of what students noticed and wondered, along with the name of the students who shared them.

A **Accessibility: Visual-Spatial Processing** Invite students to pick a specific point on each representation and describe what that point means to support interpreting visual representations.

Note: If it doesn't come up naturally, consider asking, "What do you recall about double number lines?"



Unit 6
Lesson
14

Name: Date: Period:

MA.6.NSO.3.5, MA.7.AR.3.1, MTR.1.1, MTR.2.1

100%

Let's make connections between double number line diagrams and percent problems.

Warm-Up

1. A box of cereal used to weigh 500 grams. Now it's 20% larger. Sadia and Irene each worked to determine the new weight. What do you notice? What do you wonder?

Responses vary.

I notice:

- In Sadia's, if you add up all the 100 g parts, you get 600 g total.
- In Irene's, you can see 120% of the old box matches up with 600 g.
- The increase was a move to the right on the number lines.

I wonder:

- What would a decrease look like on the double number line?
- What are some other ways of showing a percent increase?
- Did the price increase when the box of cereal got bigger?

Accelerated 6 | Unit 6 Lesson 14 580 Warm-Up

Pairs | 15 min

Activity 1 Double Number Lines

Purpose: Students use double number line diagrams to calculate the original amount, new amount, or percent change, given the other two quantities.

Short on time? Consider omitting Problem 3, as students will work with double number lines with 5 partitions and percent decrease in other problems.

1 Launch

Direct students' attention to Irene's work in the Warm-Up.

- "Where do you see connections between Irene's work and the representation?" **(MTR.2.1)**
- "Why did Irene choose the number of partitions for her number line?" **(MTR.2.1)**
- "What might be a problem that would benefit from using 4 partitions? 10 partitions?" **(MTR.2.1)**

Invite students to do a first read of Problems 2–7 to initially determine whether each situation shows a percent increase or decrease and consider what number of partitions will be most helpful.

Create a class definition for the terms **markup** and **markdown** using students' language and the glossary.

Encourage students to move several times throughout the activity to share their reasoning with a partner and reach an agreement on the double number line that represents the problem.

2 Monitor

To support students getting started, invite them to determine what value represents 100% (original amount) in each problem or if that value is unknown.

ELL **English Language Learners** Invite students to use one color to highlight the original amount — even if unknown — in each problem and use the same color to locate that quantity on the double number line diagram. Have them repeat this process for the new amount, using a different color. **(Reading)**

D **Differentiation: (Support)** For students who think that the given amount always represents the original value (100%), encourage them to think of relatable examples, such as items marked on sale and how the price shown is not always the original price.

A **Accessibility: Executive Functioning** To support students in keeping track of information, consider providing or displaying a table (column suggestions: original amount, percent increase/decrease [if applicable], new amount) that students could use to record these quantities for each problem.

Activity 1 continued >

Activity 1

Name: Date: Period:

Double Number Lines

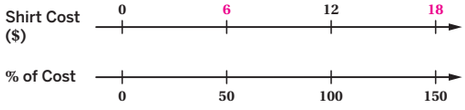
For each problem:

- Complete the double number line diagram to show the percentages that correspond to the original amount and to the new amount.
- Answer the question.

2. A store sells all of its clothes for 50% more than it costs to make (also called a "markup").

If a shirt costs $12 to make, what is the price of the shirt?

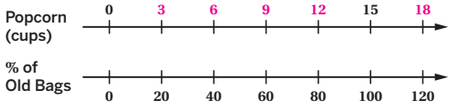
$18



3. At the movie theater, the size of popcorn bags decreased by 20%.

If the original bags held 15 cups of popcorn, how much do the new bags hold?

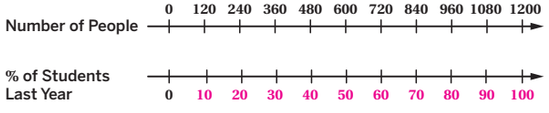
12 cups



4. A school had 1,200 students last year and only 1,080 students this year.

What was the percent decrease in the number of students?

10% decrease



Accelerated 6 | Unit 6 Lesson 14 581 Activity 1

Pairs | 15 min

Activity 1 Double Number Lines (continued)

Purpose: Students use double number line diagrams to calculate the original amount, new amount, or percent change, given the other two quantities.

Short on time? Consider omitting Problem 3, as students will work with double number lines with 5 partitions and percent decrease in other problems.

2 Monitor

Look for a variety of student strategies, such as:

- Making sense of the problems to decide whether the amount given would increase or decrease based on the description of the percentage. **(MTR.1.1)**
- Realizing that the previous year's or previous week's value corresponds to 100% on the double number line diagram. *(Problems 3–4)*
- Dividing 100% in equal intervals on the double number line diagram. *(Problems 5–6)*

D **Differentiation: (Support)** For students who are unsure how many partitions to utilize in their double number line diagrams, consider asking, "How does the percent increase or decrease for this problem help us determine how many partitions are needed?"

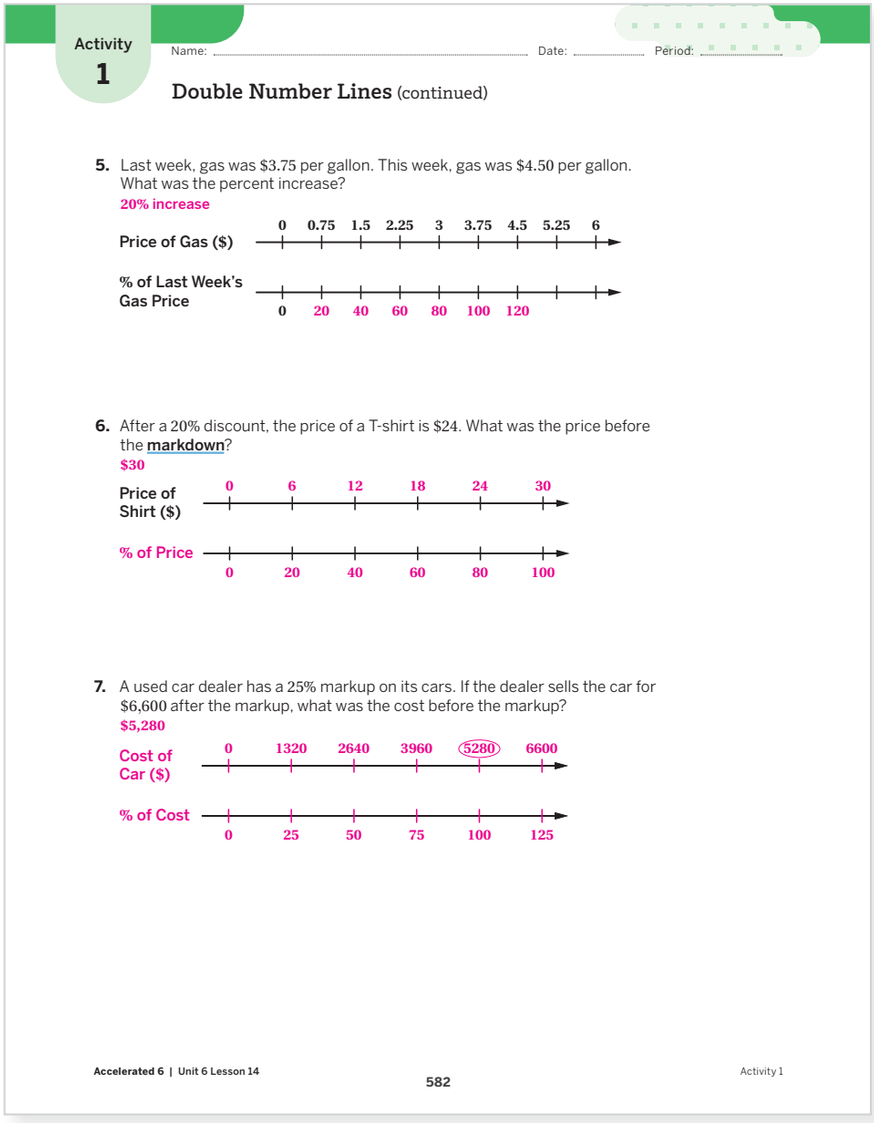
Look for good examples of students' double number lines for Problem 6 to share during the Connect.

ELL **English Language Learners** During this lesson, consider personally inviting students to share responses, asking if they would feel comfortable sharing with the class. Consider inviting students to respond in their primary languages. **(Speaking and Listening)**

3 Connect

To surface the Key Takeaway, consider asking:

- "How did you decide how many partitions were useful for this context?"
- "How does your representation help you see how much the original price of the car was before the markup?" **(MTR.2.1)**



Activity 1

Name: Date: Period:

Double Number Lines (continued)

5. Last week, gas was $3.75 per gallon. This week, gas was $4.50 per gallon. What was the percent increase?

20% increase

Price of Gas ($): 0, 0.75, 1.5, 2.25, 3, 3.75, 4.5, 5.25, 6

% of Last Week's Gas Price: 0, 20, 40, 60, 80, 100, 120

6. After a 20% discount, the price of a T-shirt is $24. What was the price before the markdown?

$30

Price of Shirt ($): 0, 6, 12, 18, 24, 30

% of Price: 0, 20, 40, 60, 80, 100

7. A used car dealer has a 25% markup on its cars. If the dealer sells the car for $6,600 after the markup, what was the cost before the markup?

$5,280

Cost of Car ($): 0, 1320, 2640, 3960, 5280, 6600

% of Cost: 0, 25, 50, 75, 100, 125

Accelerated 6 | Unit 6 Lesson 14 582 Activity 1

Key Takeaway: You can use a double number line diagram to represent a percent increase or a percent decrease. To create the diagram, fill in the information that's given and then use those clues to complete the diagram. For example, the original amount always aligns with 100%. A percent increase always aligns with that percent plus 100%. To determine how to partition the rest of the diagram, consider the difference between the percentages of quantities given. For example, for a 25% increase, make tick marks every 25%. For a decrease from 40 to 32, make tick marks every 8.

Pairs | 15 min

Activity 2 Green Sea Turtles

Purpose: Students use multiple representations to determine how a turtle population has changed over a year. **(MTR.2.1)**

1 Launch

Direct students' attention to the instructions and read them aloud, clarifying them as a class.

To surface connections, share with students that these mathematical representations can support sensemaking beyond markups and markdowns. Consider reading the prompt aloud and sharing that a sanctuary is defined as a safe place.

2 Monitor

To support students getting started, consider asking, "Does 234 represent the original amount or the new amount of nesting turtles?"

Look for a variety of student ideas, such as:

- Creating tables, double number line diagrams, or equations to showcase clear thinking.
- Realizing the equation would utilize 0.90 to represent the percent decrease for this context.
- Realizing that the number of nesting turtles would be greater than 234 in the previous year and less than 234 next year.

D **Differentiation: You're invited to explore more. (Stretch)** Invite students who would like to explore percent decrease in context further to complete this optional task. Encourage them to discuss their thinking with a partner.

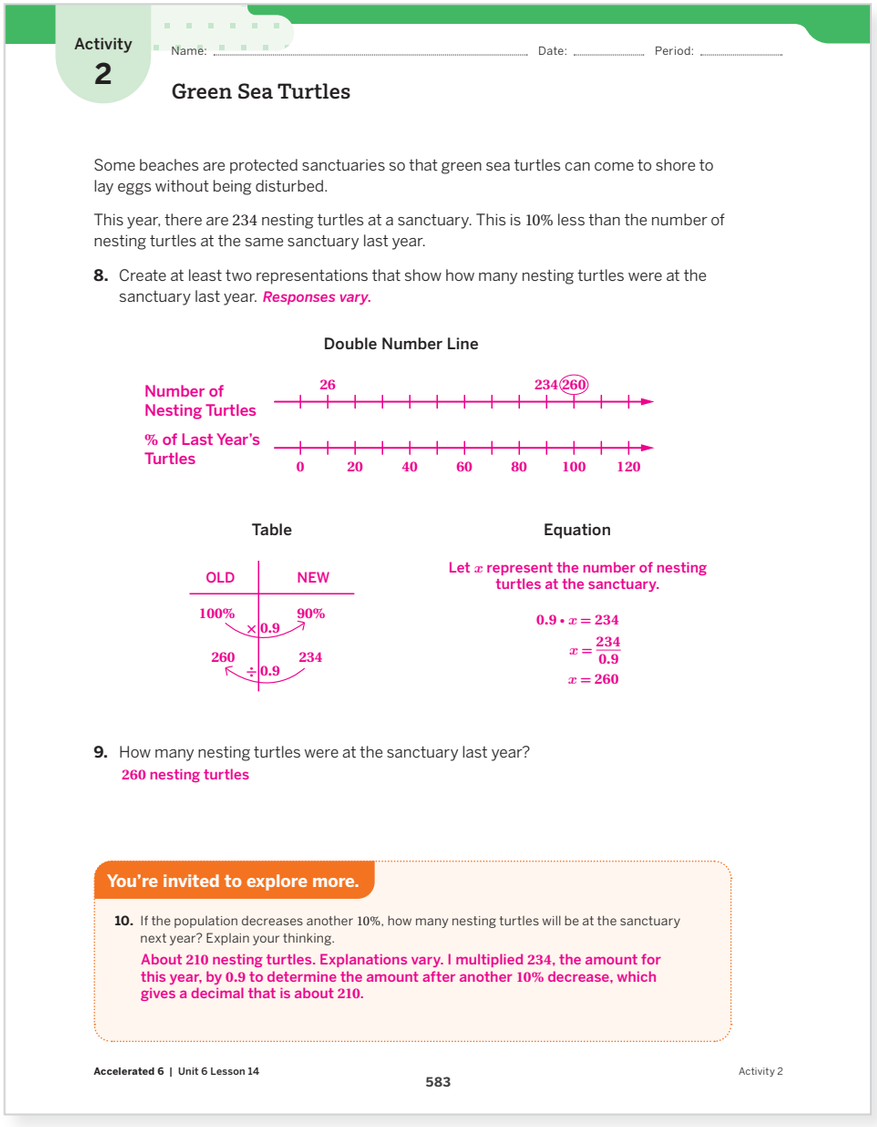
3 Connect

Invite students to share their representations for the context around sea turtles. Consider asking, "Which representation do you prefer? Why?"

ELL **English Language Learners** Provide sentence frames to support students as they share their preferred representations. For example, "I like the double number line diagram because it shows _____." or "The table helps me understand _____ because _____." **(Reading and Speaking)**

To surface the Key Takeaway, consider asking:

- "Where do you see the same information in the double number line diagram, table, and equation?"
- "How do your representations help you see how many turtles are at the sanctuary?" **(MTR.2.1)**



Activity 2

Name: _____ Date: _____ Period: _____

Green Sea Turtles

Some beaches are protected sanctuaries so that green sea turtles can come to shore to lay eggs without being disturbed.

This year, there are 234 nesting turtles at a sanctuary. This is 10% less than the number of nesting turtles at the same sanctuary last year.

8. Create at least two representations that show how many nesting turtles were at the sanctuary last year. *Responses vary.*

Double Number Line

Table

OLD	NEW
100%	90%
260	234

×0.9 ÷0.9

Equation

Let x represent the number of nesting turtles at the sanctuary.

$0.9 \cdot x = 234$

$x = \frac{234}{0.9}$

$x = 260$

9. How many nesting turtles were at the sanctuary last year?

260 nesting turtles

You're invited to explore more.

10. If the population decreases another 10%, how many nesting turtles will be at the sanctuary next year? Explain your thinking.

About 210 nesting turtles. Explanations vary. I multiplied 234, the amount for this year, by 0.9 to determine the amount after another 10% decrease, which gives a decimal that is about 210.

Key Takeaway: You can use multiple representations, including double number lines, tables, equations, to determine the original or new amount or to determine the percent change given the other two quantities.

Whole Class | 5 min

Synthesis

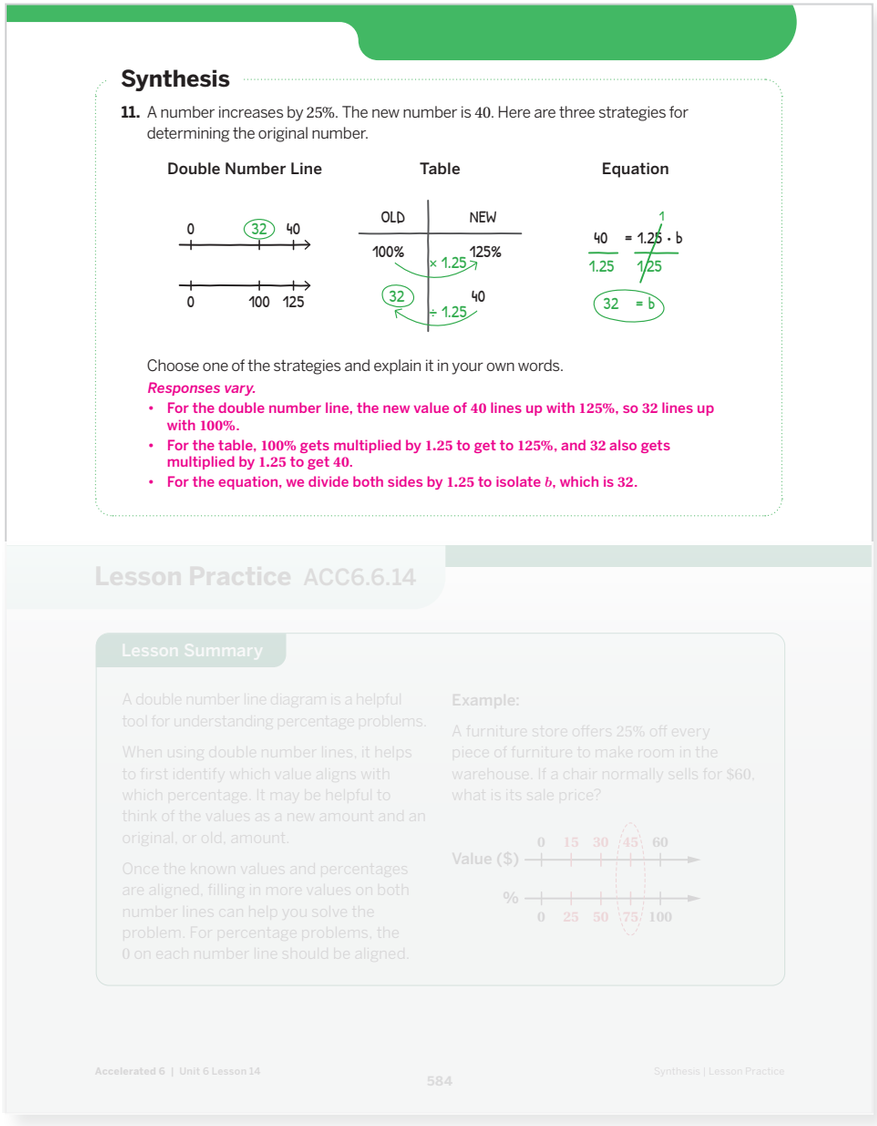
Purpose: Students synthesize their understanding of using tables, equations, and double number line diagrams to solve problems involving percent increase and decrease.

Invite students to respond independently, and then share their thinking with a partner.

Collect and share a variety of ideas, such as:

- Explaining the relationship in the double number line diagram and how the percent increase aligns with the new value.
- Using a constant of proportionality in the table to multiply by 100% in the first row and divide the new value by in the second row to find the original number.
- Converting the percent increase to a decimal, writing an equation that relates the new number to the original number, and solving the equation.

Note: If time allows, invite students to share with a partner what makes sense about each strategy and what questions they still have.



Synthesis

11. A number increases by 25%. The new number is 40. Here are three strategies for determining the original number.

Double Number Line

Table

OLD	NEW
100%	125%
32	40

Equation

$\frac{40}{1.25} = \frac{1.25 \cdot b}{1.25}$

$32 = b$

Choose one of the strategies and explain it in your own words.

Responses vary.

- For the double number line, the new value of 40 lines up with 125%, so 32 lines up with 100%.
- For the table, 100% gets multiplied by 1.25 to get to 125%, and 32 also gets multiplied by 1.25 to get 40.
- For the equation, we divide both sides by 1.25 to isolate b, which is 32.

Lesson Practice ACC6.6.14

Lesson Summary

A double number line diagram is a helpful tool for understanding percentage problems.

When using double number lines, it helps to first identify which value aligns with which percentage. It may be helpful to think of the values as a new amount and an original, or old, amount.

Once the known values and percentages are aligned, filling in more values on both number lines can help you solve the problem. For percentage problems, the 0 on each number line should be aligned.

Example:

A furniture store offers 25% off every piece of furniture to make room in the warehouse. If a chair normally sells for $60, what is its sale price?

Accelerated 6 | Unit 6 Lesson 14

584

Synthesis | Lesson Practice

Lesson Takeaway: A double number line diagram can be an appropriate tool to represent a percent increase or a percent decrease situation. The diagram can help you solve for the original amount, new amount, or percent change, when given the other two quantities. Other representations, such as a table or an equation, are more concise tools for solving the same types of problems.

Independent | 5 min

Show What You Know

Purpose: Students demonstrate their understanding by solving for the new value when given the percent increase and original value.

Today's Goals

1. **Goal:** Use double number lines to represent situations involving a percent increase or decrease.
2. **Goal:** Determine the original amount given the new amount and a percent increase or decrease.
3. **Language Goal:** Interpret a situation to solve problems about percent increase or decrease using mathematically precise language. **(Reading and Writing)**

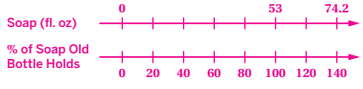
Show What You Know (Assessment Resources)

Name: _____ Date: _____ Period: _____

Show What You Know 6.14

A company's new container holds 40% more laundry soap. If the original container held 53 fluid ounces of soap, how much does the new container hold? Show or explain your thinking.

74.2 fluid ounces. *Explanations vary.* 53 fluid ounces represents the original 100%. I multiplied 53 by 1.4 to determine how much the new container holds.



D Differentiation Use after Lesson 14

Support

Provide targeted intervention for students by using these resources.

If students need support understanding how to use double number lines to solve percent increase and decrease problems:

» **Respond to Student Thinking:**

- Use the ***Calculating Percentages*** **Mini-lesson**.
- Assigning a few **Practice Problems** in class and inviting students to share their strategies for representing percent increase and percent decrease with a double number line.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 4 Extension Activities**.

Professional Learning

In this lesson, students were reintroduced to double number lines as a representation for solving percent increase and percent decrease problems. How will this representation connect to other strategies in the upcoming lessons?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

11. A number increases by 25%. The new number is 40. Here are three strategies for determining the original number.

Double Number Line

Table

Equation

Choose one of the strategies and explain it in your own words.

Responses vary.

- For the double number line, the new value of 40 lines up with 125%, so 32 lines up with 100%.
- For the table, 100% gets multiplied by 1.25 to get to 125%, and 32 also gets multiplied by 1.25 to get 40.
- For the equation, we divide both sides by 1.25 to isolate b, which is 32.

Lesson Practice ACC6.6.14

Lesson Summary

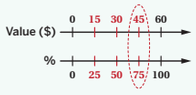
A double number line diagram is a helpful tool for understanding percentage problems.

When using double number lines, it helps to first identify which value aligns with which percentage. It may be helpful to think of the values as a new amount and an original, or old, amount.

Once the known values and percentages are aligned, filling in more values on both number lines can help you solve the problem. For percentage problems, the 0 on each number line should be aligned.

Example:

A furniture store offers 25% off every piece of furniture to make room in the warehouse. If a chair normally sells for $60, what is its sale price?



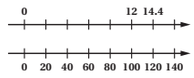
Lesson Practice ACC6.6.14

Name: _____ Date: _____ Period: _____

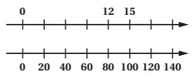
1. Abdullah and Kanna are working on this problem:

A juice box has 20% more juice in its new packaging. The original packaging held 12 fluid ounces. How many fluid ounces of juice does the new packaging hold?

Abdullah's Double Number Line



Kanna's Double Number Line



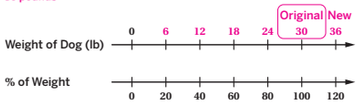
Whose double number line represents the problem? Explain your thinking.

Abdullah's. *Explanations vary.* The number 12 should be aligned with 100% because that is the original amount of fluid ounces.

Problems 2–3: Read each scenario and then answer the question. Complete the double number line diagram if it helps with your thinking.

2. A dog weighs 20% more than it did three months ago. It weighs 36 pounds now. How much did the dog weigh three months ago?

30 pounds



3. A bakery used 25% less butter this month than last month. The bakery used 240 kilograms of butter this month. How much did it use last month?

320 kilograms

Lesson Practice ACC6.6.14

Name: _____ Date: _____ Period: _____

Problems 4–5: Next week, the price of oranges at the farmer's market will decrease by 20%.

4. Select *all* the equations that represent the relationship between the price of oranges this week, x, and the price of oranges next week, y.

- ☐ A. $y = \frac{1}{5}x$
- ☑ B. $y = \frac{4}{5}x$
- ☑ C. $y = x - \frac{1}{5}x$
- ☐ D. $y = 0.2x$
- ☑ E. $y = 0.8x$

5. The price of oranges this week is $4.50. What will the price be next week?

$3.60

6. Without determining the actual percent increase, explain why it is not reasonable to say that a price increase from $4.00 to $5.00 for a bag of clementines represents a 125% increase.

***Responses vary.* A 125% increase would mean the new price is 225% of the original price, so it more than doubles. $5.00 is not more than twice $4.00.**

FAST Practice

7. The number of fish in a lake decreased by 15% between last year and this year. This year, there are 51 fish in the lake. What was the population last year?

60 fish

Spiral Review

Problems 8–9: A store drops its prices on athletic wear by 7%.

8. If the price before the decrease is x, write an equation to represent the price after the decrease, y.

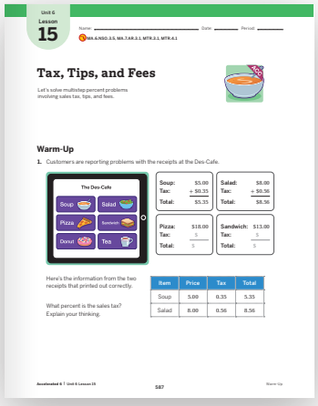
$y = 0.93x$ (or equivalent)

9. A pair of workout shorts was originally $35. What is the new cost of the shorts?

$32.55

Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1–5	2	MA.7.AR.3.1
	6	3	MA.7.AR.3.1
FAST Practice	7	2	MA.7.AR.3.1
Spiral Review			
	8–9	2	MA.7.AR.3.1



Tax, Tips, and Fees

Multistep Percent Problems

Let's solve multistep percent problems involving sales tax, tips, and fees.

Learning Outcomes

Today's Goals

1. **Goal:** Calculate an original amount, new amount, or percent change in the context of sales tax and tip, including situations that involve multiple percent increases or decreases (e.g., sales tax and a discount).
2. **Language Goal:** Interpret situations involving tax and tip using mathematically precise language. **(Speaking and Listening)**

Students apply what they've learned about percent increase and decrease to solve multistep percent problems. They determine the total cost after applying sales tax, tips, and fees and calculate the total after two percentages are applied (e.g., a discount followed by sales tax).

Coherence

Prior Learning

In Lesson 14, students made connections between double number line diagrams and percent problems. They learned that percent increase and percent decrease can be represented with double number line diagrams, tables, and equations.

Future Learning

Lessons 16–17 will continue to give students ample opportunity to work with percentages in new contexts.

Connections

- Students calculate the total price after two percentages in real world contexts, such as calculating the cost of an item in a store after a coupon and sales tax. (real world)

FL B.E.S.T. Math Benchmarks

Addressing

MA.6.NSO.3.5
Rewrite positive rational numbers in different but equivalent forms including fractions, terminating decimals and percentages.

MA.7.AR.3.1
Apply previous understanding of percentages and ratios to solve multi-step real world percent problems.

Mathematical Thinking and Reasoning Standards: MTR.3.1, MTR.4.1

ELA Expectations: ELA.K12.EE.1.1, ELA.K12.EE.4.1, ELA.K12.EE.5.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.6.AR.3.4	MA.7.NSO.1.2
	MA.8.AR.2.1
	MA.912.AR.1.1

Lesson at a Glance

~ 45 min

MA.6.NSO.3.5, MA.7.AR.3.1

Warm-Up

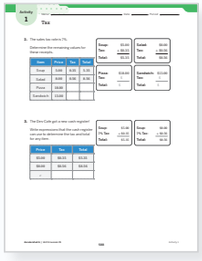
Independent | 5 min

Students explore cash register receipts as an introduction to the concept of sales tax.

Activity 1

Pairs | 10 min

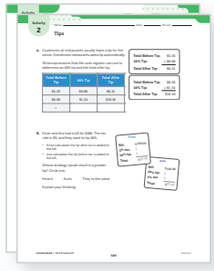
Students write expressions for the cost of a meal, including sales tax, to make sense of how tax increases the total amount.



Activity 2

Pairs | 10 min

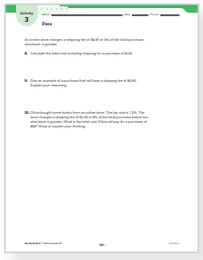
Students calculate tips to make sense of how to solve multistep percentage problems.
Routine: Decide and Defend



Activity 3

Pairs | 10 min

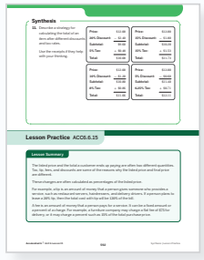
Students calculate the total cost of a purchase including shipping fees, which are calculated as the greater amount of a flat fee or a percentage of the total purchase.



Synthesis

Whole Class | 5 min

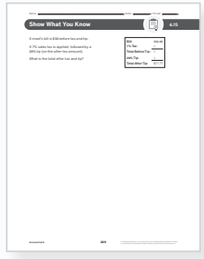
Students synthesize strategies for solving problems that involve calculating the total cost for something after multiple percentage increases or decreases. MLR2: Collect and Display



Show What You Know

Independent | 5 min

Students demonstrate their understanding by determining the cost of a meal that includes tax and tip.



Prep Checklist

This lesson includes:

- Student Edition
- Show What You Know
- Calculators

Independent | 5 min

Warm-Up

Purpose: Students explore cash register receipts as an introduction to the concept of sales tax.

1 Launch

To surface connections, consider asking, "Have you ever had something cost more than you expected because of tax?"

To support students getting started, consider asking, "How do you think the cash register is calculating the amount of the tax? Could you create an expression or equation to represent what it is doing?"

ELL **English Language Learners** Consider inviting students to share what they know about the terms *taxes* and *percentages*. Discuss how the meanings of these terms are alike and different. **(Reading and Listening)**

A **Accessibility: Memory and Attention** Invite students to use a calculator, double number line, or ratio table to support them with performing calculations.

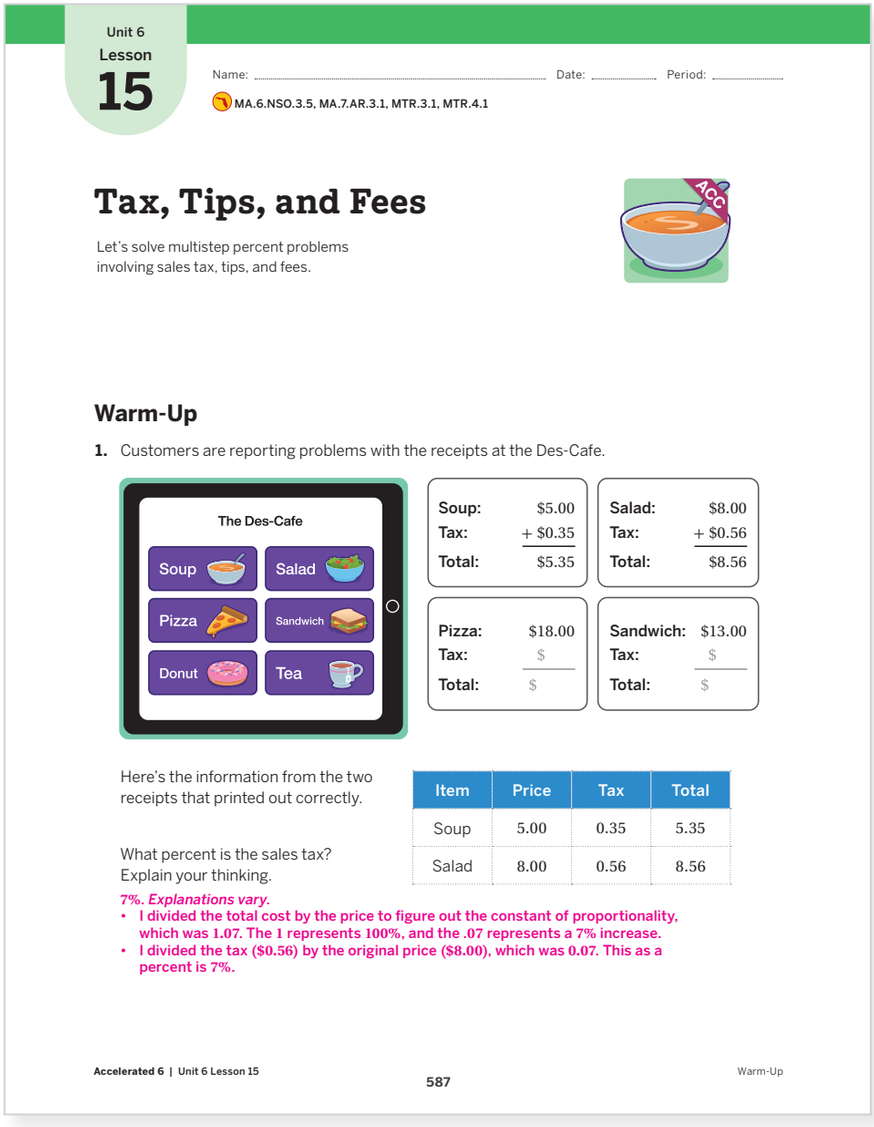
2 Connect

Encourage students to read others' responses. Consider asking, "How are others' strategies similar or different to your own?" **(MTR.4.1)**

Discuss as a whole-class what the values 0.07 and 1.07 represent in this context.

To surface the Key Takeaway, consider asking:

- "If we know the price and the tax, how can we figure out the sales tax percentage?"
- "How are the numbers 0.07 and 1.07 related to each other in this situation?"



Unit 6
Lesson
15

Name: Date: Period:

MA.6.NSO.3.5, MA.7.AR.3.1, MTR.3.1, MTR.4.1

Tax, Tips, and Fees

Let's solve multistep percent problems involving sales tax, tips, and fees.

Warm-Up

1. Customers are reporting problems with the receipts at the Des-Cafe.

The Des-Cafe: Soup, Salad, Pizza, Sandwich, Donut, Tea

Soup:	$5.00	Salad:	$8.00
Tax:	+ $0.35	Tax:	+ $0.56
Total:	$5.35	Total:	$8.56
Pizza:	$18.00	Sandwich:	$13.00
Tax:	$	Tax:	$
Total:	$	Total:	$

Here's the information from the two receipts that printed out correctly.

Item	Price	Tax	Total
Soup	5.00	0.35	5.35
Salad	8.00	0.56	8.56

What percent is the sales tax? Explain your thinking.

7%. Explanations vary.
- I divided the total cost by the price to figure out the constant of proportionality, which was 1.07. The 1 represents 100%, and the .07 represents a 7% increase.
- I divided the tax ($0.56) by the original price ($8.00), which was 0.07. This as a percent is 7%.

Accelerated 6 | Unit 6 Lesson 15 587 Warm-Up

Key Takeaway: To determine the sales tax rate, you can divide the tax by the price to get the percentage as a decimal. You can divide the total by the pre-tax price to find the constant of proportionality for the percent increase.

Pairs | 10 min

Activity 1 Tax

Purpose: Students write expressions for the cost of a meal, including sales tax, to make sense of how tax increases the total amount.

1 Launch

Invite students to use the sales tax rate they discussed in the Warm-Up to figure out the values on the missing receipts.

2 Monitor

Look for strategies students use to determine the tax and total for their items, including those that show expressions in the table.

D Differentiation

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Consider asking, "What strategies can you use to represent 7%?" Consider referring to the strategies from Lesson 13.
Use the decimal 0.7 to represent 7%.	**Support** Consider asking, "What does 7% mean? How can we represent that as a decimal?"
Determine 1% of the price and use that to determine the tax and total.	**Strengthen** Celebrate their thinking. Consider asking, "How can we determine the tax with just one calculation?"
Type expressions that involve multiplying by 0.07 for the tax or by 1.07 for the total.	**Strengthen** Consider asking, "How does this support you writing your equations in Problem 3?"
Write the expression $0.07c + c$ to represent calculating the tax, then adding the cost.	**Strengthen** Consider asking, "Can you write this as a single multiplication expression?" **(MTR.3.1)**

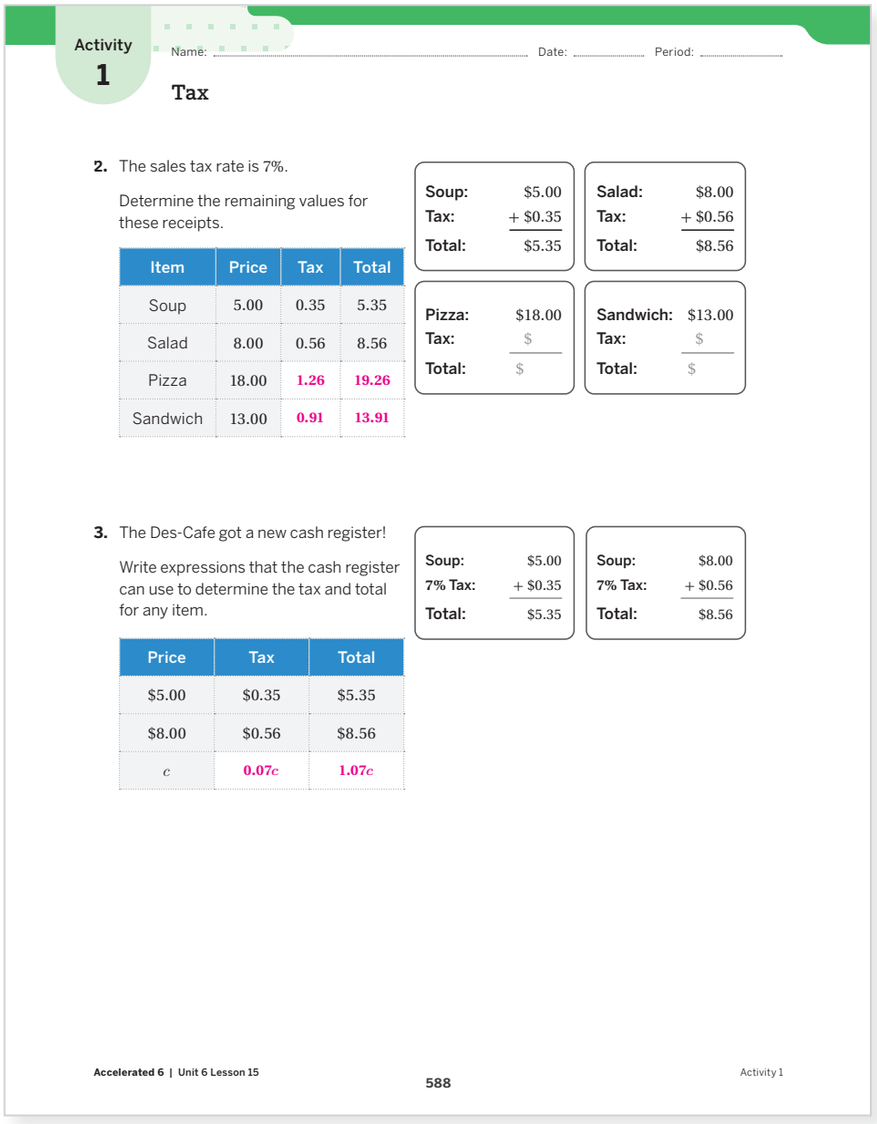
A

Accessibility: Memory and Attention Invite students to record their calculations for tax and total on a separate sheet of paper so they can reference their work when trying to determine an equation on Problem 4.

3 Connect

Invite students to share their strategies for determining tax and total. Invite students to share how the different pieces of their equations represent the tax and total.

To surface the Key Takeaway, consider asking, "How can we determine the sales tax and total for any item sold at the Des-Cafe?"



Activity 1

Name: Date: Period:

Tax

2. The sales tax rate is 7%.

Determine the remaining values for these receipts.

Item	Price	Tax	Total
Soup	5.00	0.35	5.35
Salad	8.00	0.56	8.56
Pizza	18.00	1.26	19.26
Sandwich	13.00	0.91	13.91

Soup:	$5.00	Salad:	$8.00
Tax:	+ $0.35	Tax:	+ $0.56
Total:	$5.35	Total:	$8.56
Pizza:	$18.00	Sandwich:	$13.00
Tax:	$	Tax:	$
Total:	$	Total:	$

3. The Des-Cafe got a new cash register!

Write expressions that the cash register can use to determine the tax and total for any item.

Price	Tax	Total
$5.00	$0.35	$5.35
$8.00	$0.56	$8.56
c	$0.07c$	$1.07c$

Soup:	$5.00	Soup:	$8.00
7% Tax:	+ $0.35	7% Tax:	+ $0.56
Total:	$5.35	Total:	$8.56

Accelerated 6 | Unit 6 Lesson 15 588 Activity 1

Key Takeaway: To determine a percentage of an amount, you can write the percent as a decimal or fraction, then multiply it by the amount. To increase an amount by a certain percent, you can multiply it by 100% plus the percent of the increase. For example, to determine the total after an 8% increase, you can multiply the original amount by 1.08. This is the same as finding 8% of the amount and adding it to the original.

Pairs | 10 min

Activity 2 Tips

Purpose: Students calculate tips to make sense of how to solve multistep percentage problems.

Short on time? Consider completing Problem 7 as a class.

1 Launch

To surface connections, consider asking, "What do you know about tipping at a cafe?"

Note: There are many different customs for tipping. Consider each strategy that a student offers for tipping equitably and with care, acknowledging how it can be discussed mathematically.

2 Monitor

To support students getting started, consider asking, "How can the strategies we discussed for determining tax be used here?"

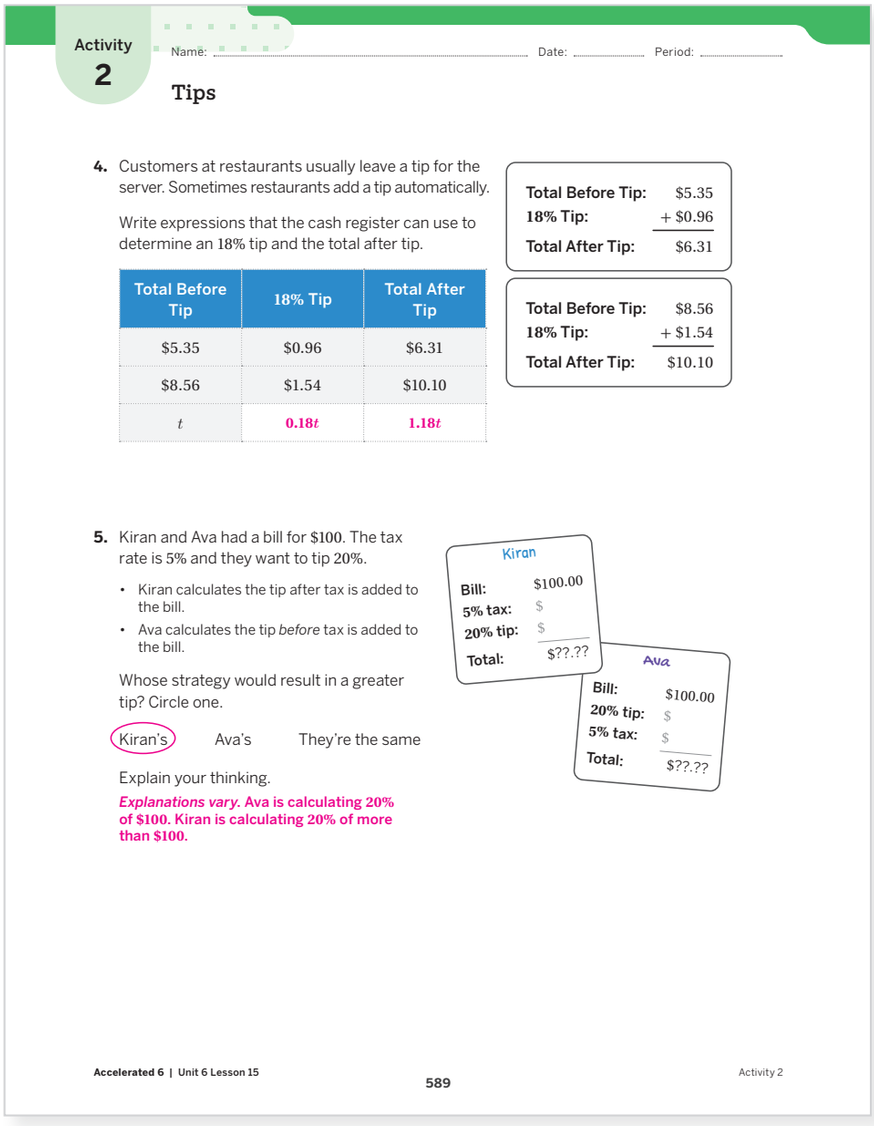
Invite students to spend 1–2 minutes thinking independently about the problem before discussing it with a partner.

A **Accessibility: Executive Functioning** Consider providing a graphic organizer to support students in calculating multiple percentages for a single problem.

Use the Decide and Defend routine to support students in making arguments and critiquing the reasoning of others. **(MTR.4.1)**

Consider asking, "Why does the order we calculate tax and tip matter for the total bill?" **(MTR.3.1)**

Activity 2 continued >



Activity 2

Name: Date: Period:

Tips

4. Customers at restaurants usually leave a tip for the server. Sometimes restaurants add a tip automatically.

Write expressions that the cash register can use to determine an 18% tip and the total after tip.

Total Before Tip	18% Tip	Total After Tip
$5.35	$0.96	$6.31
$8.56	$1.54	$10.10
t	$0.18t$	$1.18t$

Total Before Tip:	$5.35
18% Tip:	+ $0.96
Total After Tip:	$6.31

Total Before Tip:	$8.56
18% Tip:	+ $1.54
Total After Tip:	$10.10

5. Kiran and Ava had a bill for $100. The tax rate is 5% and they want to tip 20%.

- Kiran calculates the tip after tax is added to the bill.
- Ava calculates the tip *before* tax is added to the bill.

Whose strategy would result in a greater tip? Circle one.

Kiran's Ava's They're the same

Explain your thinking.

Explanations vary. Ava is calculating 20% of $100. Kiran is calculating 20% of more than $100.

Kiran
Bill: $100.00
5% tax: $
20% tip: $
Total: $??.??

Ava
Bill: $100.00
20% tip: $
5% tax: $
Total: $??.??

Accelerated 6 | Unit 6 Lesson 15 589 Activity 2

Pairs | 10 min

Activity 2 Tips (continued)

Purpose: Students calculate tips to make sense of how to solve multistep percentage problems.

Short on time? Consider completing Problem 7 as a class.

2 Monitor

To surface connections, consider asking, "How is a discount similar to or different from a tax?"

ELL **English Language Learners** Invite students to share situations in which they have used coupons or gotten discounts on things they have purchased to increase access to the task. **(Reading and Listening)**.

Look for a variety of student strategies for determining the total price.

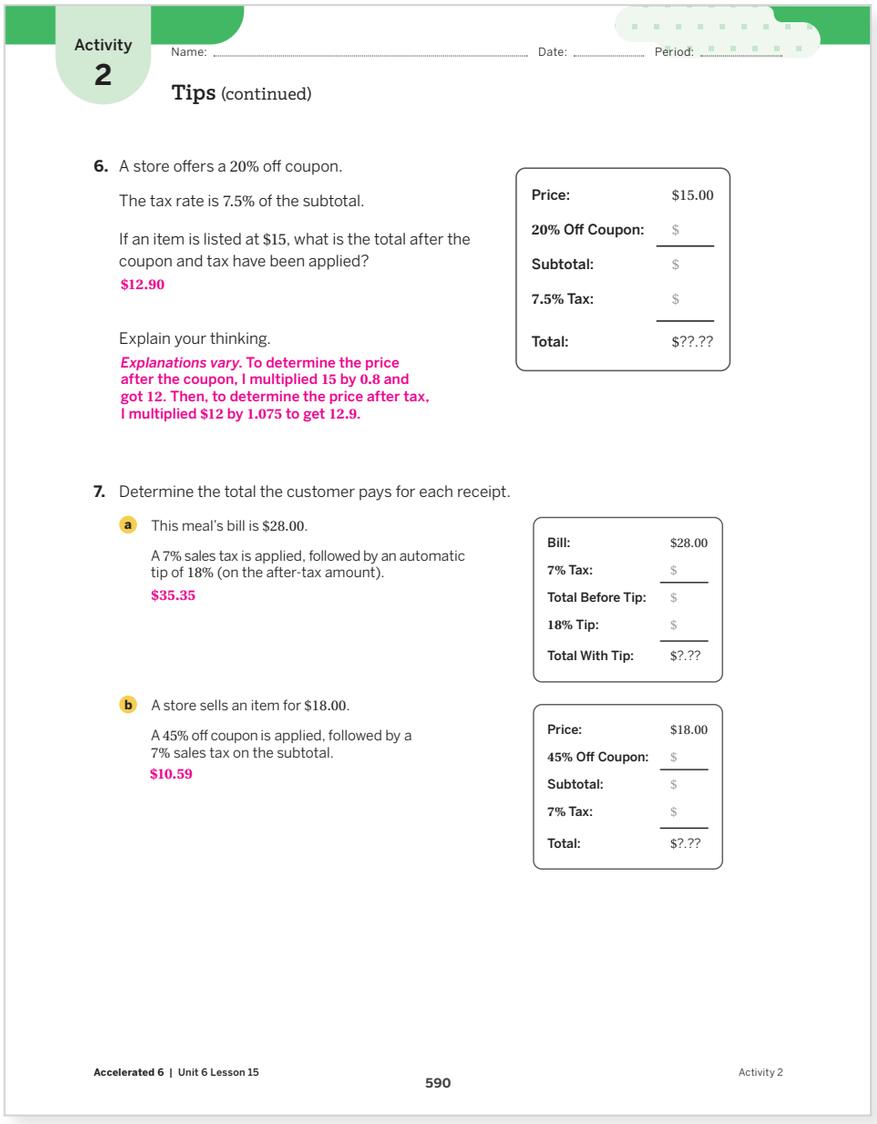
D Differentiation

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Consider asking, "Does a 20% coupon represent a percent increase or decrease?"
Calculate the tax before the coupon.	**Support** Encourage them to follow the order on the receipt. Consider asking, "Why does the order matter?"
Subtract the tax from the discounted price.	**Support** Consider asking, "Should the tax increase or decrease the total?"
Represent the total as an expression like $15(.8)(1 + 0.075)$.	**Stretch** Invite students to share during the Connect. Consider asking, "How do each of your values or operations represent something in the situation?"

3 Connect

Invite students to share how they determined the total price after the discount and tax.

To surface the Key Takeaway, consider asking, "Why is it important to consider the order in which taxes, discounts, or tips are applied to a total?"



Activity 2

Name: Date: Period:

Tips (continued)

6. A store offers a 20% off coupon.

The tax rate is 7.5% of the subtotal.

If an item is listed at $15, what is the total after the coupon and tax have been applied?

$12.90

Explain your thinking.

Explanations vary. **To determine the price after the coupon, I multiplied 15 by 0.8 and got 12. Then, to determine the price after tax, I multiplied $12 by 1.075 to get 12.9.**

Price:	$15.00
20% Off Coupon:	$
Subtotal:	$
7.5% Tax:	$
Total:	$??.??

7. Determine the total the customer pays for each receipt.

a This meal's bill is $28.00.

A 7% sales tax is applied, followed by an automatic tip of 18% (on the after-tax amount).

$35.35

Bill:	$28.00
7% Tax:	$
Total Before Tip:	$
18% Tip:	$
Total With Tip:	$?.??

b A store sells an item for $18.00.

A 45% off coupon is applied, followed by a 7% sales tax on the subtotal.

$10.59

Price:	$18.00
45% Off Coupon:	$
Subtotal:	$
7% Tax:	$
Total:	$?.??

Accelerated 6 | Unit 6 Lesson 15 590 Activity 2

Key Takeaway: It is important to consider the order in which percentages of a total are added to a tip. Taking off 20% and then adding an 8% tax is different from the opposite order because we apply the percentages to different amounts depending on the order.

Pairs | 10 min

Activity 3 Fees

Purpose: Students calculate the total cost of a purchase including shipping fees, which are calculated as the greater amount of a flat fee or a percentage of the total purchase.

1 Launch

Consider sharing that fees can be a fixed amount, a percentage of a charge, or both.

2 Monitor

To support students getting started, invite them to calculate 6% of different purchase amounts to see which ones result in a shipping fee that is less than $6.95.

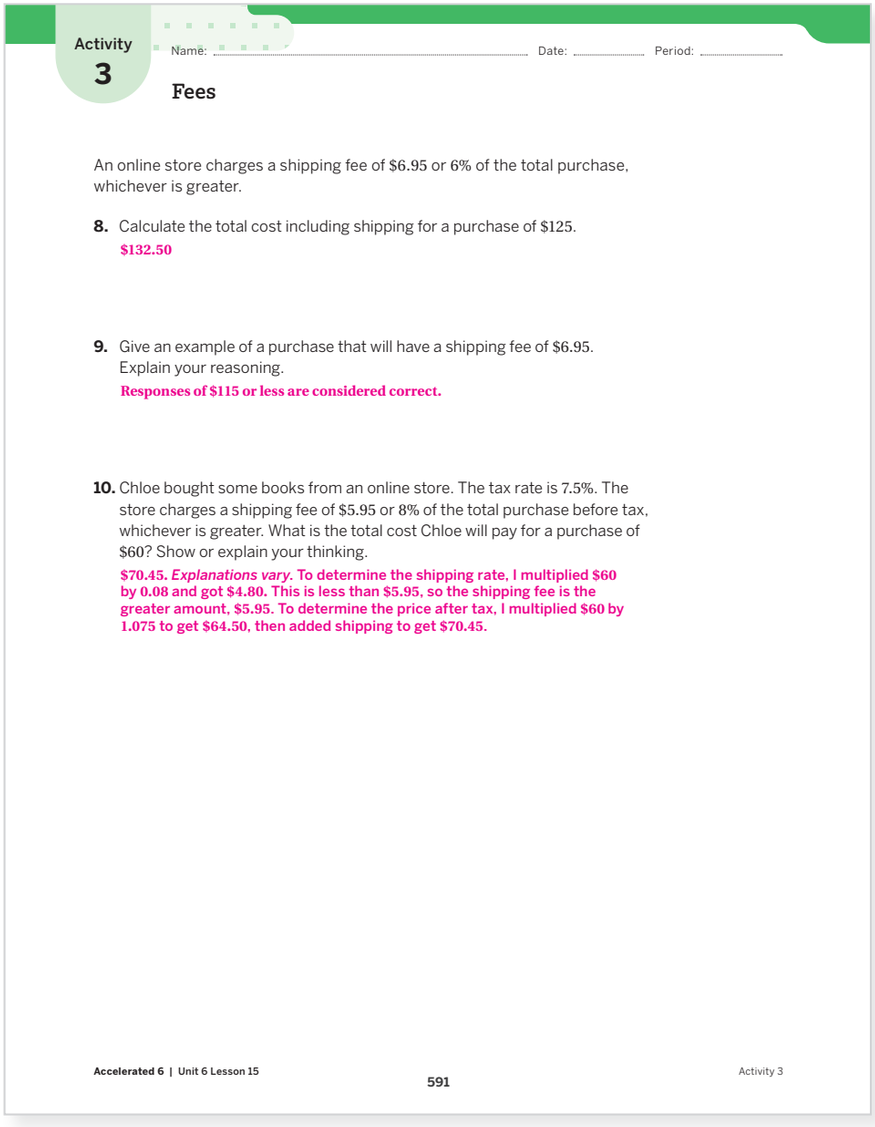
Look for students who write an equation to determine the exact purchase price that changes from a fixed shipping fee to a percentage of the purchase.

3 Connect

Invite students to share how they determined the total price after tax and shipping.

Consider asking:

- "What is the minimum shipping rate?"
- "Is there a maximum shipping rate?"
- "Why do you think some companies offer free shipping when the total purchase is above a certain amount?"



Activity 3

Name: Date: Period:

Fees

An online store charges a shipping fee of $6.95 or 6% of the total purchase, whichever is greater.

8. Calculate the total cost including shipping for a purchase of $125.

$132.50

9. Give an example of a purchase that will have a shipping fee of $6.95. Explain your reasoning.

Responses of $115 or less are considered correct.

10. Chloe bought some books from an online store. The tax rate is 7.5%. The store charges a shipping fee of $5.95 or 8% of the total purchase before tax, whichever is greater. What is the total cost Chloe will pay for a purchase of $60? Show or explain your thinking.

$70.45. *Explanations vary.* To determine the shipping rate, I multiplied $60 by 0.08 and got $4.80. This is less than $5.95, so the shipping fee is the greater amount, $5.95. To determine the price after tax, I multiplied $60 by 1.075 to get $64.50, then added shipping to get $70.45.

Accelerated 6 | Unit 6 Lesson 15 591 Activity 3

Whole Class | 5 min

Synthesis

Purpose: Students synthesize strategies for solving problems that involve calculating the total cost for something after multiple percentage increases or decreases.

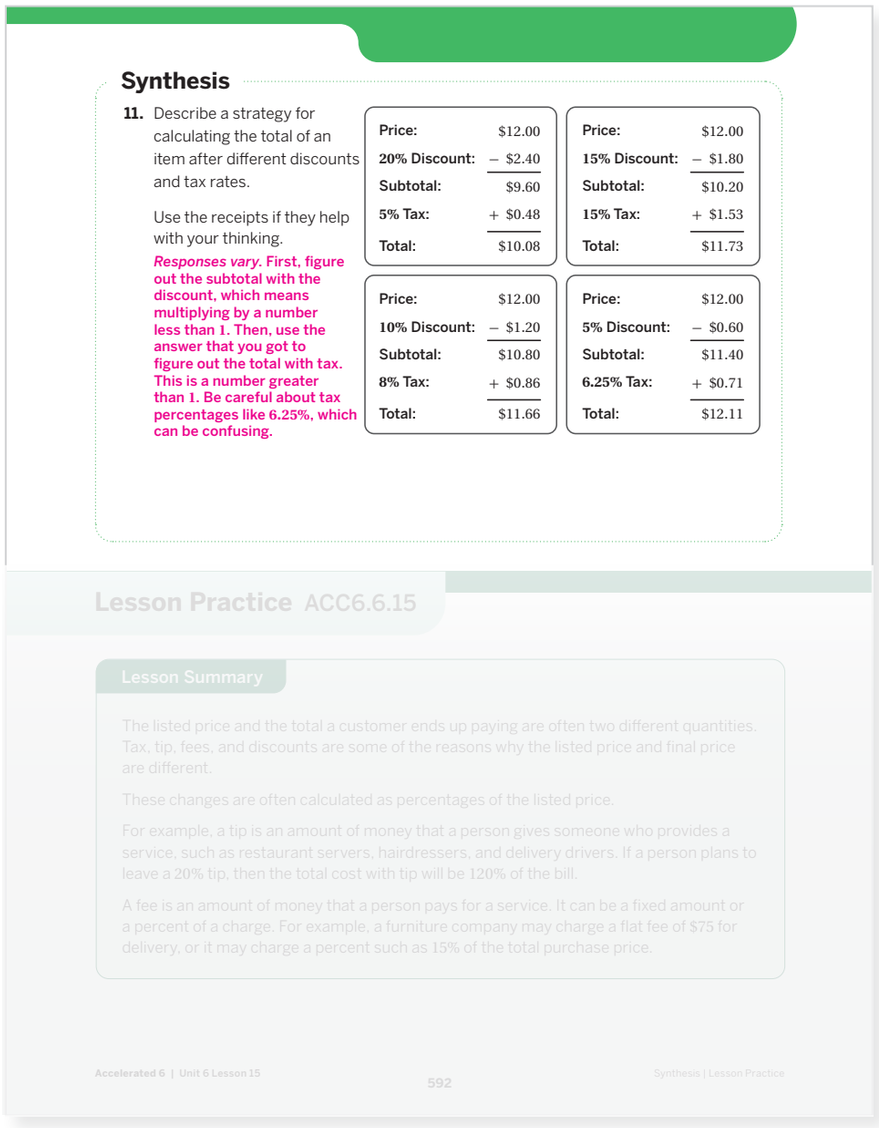
Invite students to respond independently and then share their thinking with a partner.

Collect and share a variety of ideas, such as:

- Using a representation to show the percent increase or decrease.
- Writing a single expression that includes multiplying by a constant of proportionality to represent the percent change.
- Dividing the total costs by tax rates to determine what percent the tax is of the total.

MLR **MLR2: Collect and Display** Use this routine to collect students' strategies around determining a percent change as they share during the Synthesis. Display for students to reference and add to for the rest of the unit.

Note: If time allows, invite students to share with a partner what makes sense about each strategy and what questions they still have.



Synthesis

11. Describe a strategy for calculating the total of an item after different discounts and tax rates.

Use the receipts if they help with your thinking.

Responses vary. First, figure out the subtotal with the discount, which means multiplying by a number less than 1. Then, use the answer that you got to figure out the total with tax. This is a number greater than 1. Be careful about tax percentages like 6.25%, which can be confusing.

Price:	$12.00
20% Discount:	− $2.40
Subtotal:	$9.60
5% Tax:	+ $0.48
Total:	$10.08

Price:	$12.00
15% Discount:	− $1.80
Subtotal:	$10.20
15% Tax:	+ $1.53
Total:	$11.73

Price:	$12.00
10% Discount:	− $1.20
Subtotal:	$10.80
8% Tax:	+ $0.86
Total:	$11.66

Price:	$12.00
5% Discount:	− $0.60
Subtotal:	$11.40
6.25% Tax:	+ $0.71
Total:	$12.11

Lesson Practice ACC6.6.15

Lesson Summary

The listed price and the total a customer ends up paying are often two different quantities. Tax, tip, fees, and discounts are some of the reasons why the listed price and final price are different.

These changes are often calculated as percentages of the listed price.

For example, a tip is an amount of money that a person gives someone who provides a service, such as restaurant servers, hairdressers, and delivery drivers. If a person plans to leave a 20% tip, then the total cost with tip will be 120% of the bill.

A fee is an amount of money that a person pays for a service. It can be a fixed amount or a percent of a charge. For example, a furniture company may charge a flat fee of $75 for delivery, or it may charge a percent such as 15% of the total purchase price.

Accelerated 6 | Unit 6 Lesson 15 592 Synthesis | Lesson Practice

Lesson Takeaway: Sales tax, tips, and discounts are a few examples of contexts that involve percent increases or decreases.

Independent | 5 min

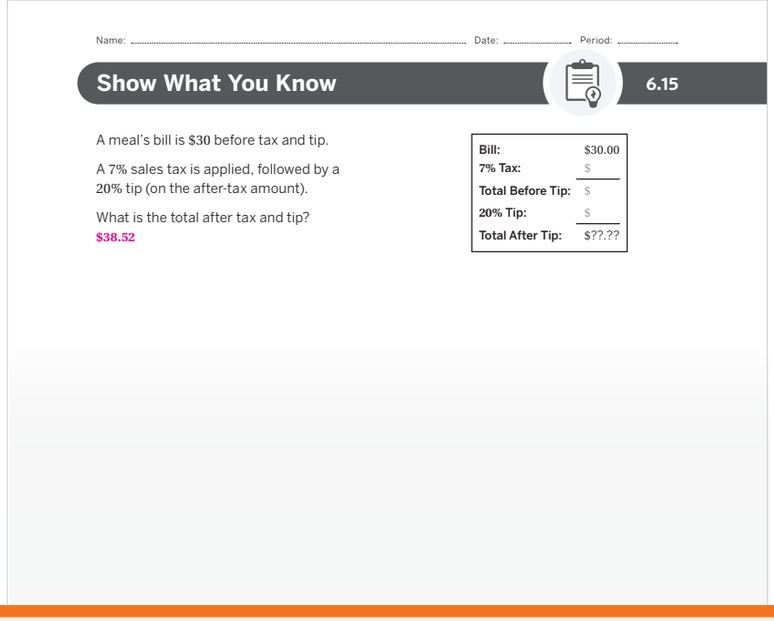
Show What You Know

Purpose: Students demonstrate their understanding by determining the cost of a meal that includes tax and tip.

Today's Goals

1. **Goal:** Calculate an original amount, new amount, or percent change in the context of sales tax and tip, including situations that involve multiple percent increases or decreases (e.g., sales tax and a discount).
2. **Language Goal:** Interpret situations involving tax and tip using mathematically precise language. **(Speaking and Listening)**

Show What You Know (Assessment Resources)



Name: _____ Date: _____ Period: _____

Show What You Know 6.15

A meal's bill is $30 before tax and tip.

A 7% sales tax is applied, followed by a 20% tip (on the after-tax amount).

What is the total after tax and tip?

$38.52

Bill:	$30.00
7% Tax:	$
Total Before Tip:	$
20% Tip:	$
Total After Tip:	$??.??

D Differentiation Use after Lesson 15

Support

Provide targeted intervention for students by using these resources.

If students need support understanding multistep percentage problems in the context of money:

» **Respond to Student Thinking:**

- Use the ***Finding Sales Tax and Tip*** **Mini-lesson**.
- Revisit the Synthesis and discuss each example as a class.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 4 Extension Activities**.

Professional Learning

Today, students explored the topics of sales tax, discounts, and tipping. Were there any moments of the conversation about the amount to tip that became difficult or ambiguous? How can you prepare to acknowledge differences that students bring to a conversation while keeping the discussion focused on mathematical topics (in this sub-unit around percentage)?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

11. Describe a strategy for calculating the total of an item after different discounts and tax rates.

Use the receipts if they help with your thinking.

Responses vary. First, figure out the subtotal with the discount, which means multiplying by a number less than 1. Then, use the answer that you got to figure out the total with tax. This is a number greater than 1. Be careful about tax percentages like 6.25%, which can be confusing.

Price:	$12.00	Price:	$12.00
20% Discount:	− $2.40	15% Discount:	− $1.80
Subtotal:	$9.60	Subtotal:	$10.20
5% Tax:	+ $0.48	15% Tax:	+ $1.53
Total:	$10.08	Total:	$11.73
Price:	$12.00	Price:	$12.00
10% Discount:	− $1.20	5% Discount:	− $0.60
Subtotal:	$10.80	Subtotal:	$11.40
8% Tax:	+ $0.86	6.25% Tax:	+ $0.71
Total:	$11.66	Total:	$12.11

Lesson Practice ACC6.6.15

Lesson Summary

The listed price and the total a customer ends up paying are often two different quantities. Tax, tip, fees, and discounts are some of the reasons why the listed price and final price are different.

These changes are often calculated as percentages of the listed price.

For example, a tip is an amount of money that a person gives someone who provides a service, such as restaurant servers, hairdressers, and delivery drivers. If a person plans to leave a 20% tip, then the total cost with tip will be 120% of the bill.

A fee is an amount of money that a person pays for a service. It can be a fixed amount or a percent of a charge. For example, a furniture company may charge a flat fee of $75 for delivery, or it may charge a percent such as 15% of the total purchase price.

Lesson Practice ACC6.6.15

Name: _____ Date: _____ Period: _____

1. Maia orders a meal that costs $15. There is no tax on the meal, and Maia decides to tip 20%. Select the expression that represents the total cost, in dollars, including the tip.

A. $15 + 0.20$ **B.** $15 + 1.20$

C. $15 \cdot 0.20$ **D.** $15 \cdot 1.20$

2. In a city in Ohio, the sales tax rate is 7.25%. Complete the table.

Item	Price Before Tax ($)	Sales Tax ($)	Price Including Tax ($)
Pillow	8.00	0.58	8.58
Blanket	24.00	1.74	25.74
Trash can	16.00	1.16	17.16

Problems 3–4: A family eats at a restaurant. The bill is $42. There is no tax on the meal, and the family leaves a tip and spends $49.77 total.

3. How much money does the family tip?
$7.77

4. How much is the tip as a percent of the bill?
18.5% of the bill

5. Some credit card companies charge a fee for cash advances. Company A charges a fee of 4%. Company B charges a flat fee of $25. What company has the lesser fee for a cash advance of $500?
Company A

6. A stock trading company charges a 0.2% fee for every stock purchase or sale. How much is the fee for a stock purchase of $4,000?
$8

Lesson Practice ACC6.6.15

Name: _____ Date: _____ Period: _____

7. A store sells an item for $75.00.

Fatima uses a 20% off coupon and then tax is applied to the subtotal.

The total after coupon and tax is $75.00.

What is the tax rate? Explain your thinking.

25%. *Explanations vary.* I wrote the equation $75 \cdot 0.8 \cdot x = 75$ and then solved it by dividing. I found that x must be 1.25, which means the tax rate is 25%.

Price:	$75.00
20% Off Coupon:	$ _____
Subtotal:	$
?% Tax:	$ _____
Total:	$75.00

FAST Practice

8. A clothing store is having a sale: 25% off shirts and 30% off jeans.

- Kyle bought a shirt with an original price of $24.
- Kyle bought a pair of jeans with an original price of $32.

Determine how much money Kyle saved.

$ **15.60**

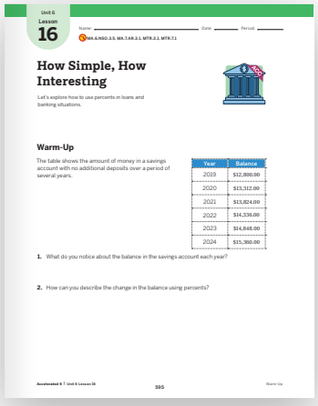
Spiral Review

9. The monthly subscription for online access to a newspaper is $8.00 per month. Next month, the newspaper is raising the price by 15%. What is the new monthly subscription cost?
$9.20

10. A skateboard that normally costs $45 is marked down by 20%. What is the sale price of the skateboard?
$36

Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1–5	2	MA.7.AR.3.1
	6	1	MA.7.AR.3.1
	7	3	MA.7.AR.3.1
FAST Practice	8	2	MA.7.AR.3.1
Spiral Review			
	9–10	2	MA.7.AR.3.1


Unit 6
Lesson
16

How Simple, How Interesting

Let's explore how to use percents in loans and banking situations.

Warm-Up

The table shows the amount of money in a savings account with no additional deposits over a period of several years.

Year	Balance
2019	$12,800.00
2020	$13,312.00
2021	$13,824.00
2022	$14,336.00
2023	$14,848.00
2024	$15,360.00

1. What do you notice about the balance in the savings account each year?
2. How can you describe the change in the balance using percents?

595

Warm-Up

How Simple, How Interesting

Calculating Simple Interest Using a Formula

Let's explore how to use percentages in loans and banking situations.

Learning Outcomes

Today's Goals

1. **Goal:** Solve real-world problems involving *simple interest* earned.
2. **Language Goal:** Describe how to use the *simple interest* formula to solve real-world problems about borrowing and investing money. **(Reading, Writing)**

Students extend their knowledge of using percents to solve real-world problems to solving problems involving **simple interest**. They use the simple interest formula to determine the **interest** earned on deposits and interest paid on loans.

Coherence

Prior Learning

Throughout Sub-Unit 4, students have solved a variety of mathematical and real-world problems using percents, such as discounts, markups, tax, tips, and fees.

Future Learning

In a future course, students will solve compound interest problems, where interest accumulates on both the principal and on the previous interest.

Connections

- Students calculate the simple interest paid on a loan over a period of time using the formula $I = prt$. (real world)

FL B.E.S.T. Math Benchmarks

Addressing

MA.6.NSO.3.5
Rewrite positive rational numbers in different but equivalent forms including fractions, terminating decimals and percentages..

MA.7.AR.3.1
Apply previous understanding of percentages and ratios to solve multi-step real-world percent problems.

Mathematical Thinking and Reasoning Standards: MTR.3.1, MTR.7.1

ELA Expectations: ELA.K12.EE.4.1, ELA.K12.EE.6.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.6.AR.3.4	MA.7.NSO.1.2 MA.912.FL.3.1 MA.912.FL.3.2

Vocabulary

New Vocabulary

interest
principal
simple interest

Lesson at a Glance

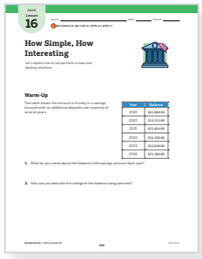
~ 45 min

MA.6.NSO.3.5, MA.7.AR.3.1

Warm-Up

Pairs | 5 min

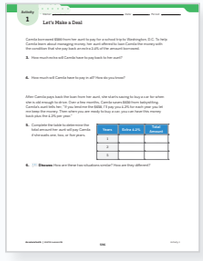
Students compare the values in a table, including using percents.



Activity 1

Pairs | 15 min

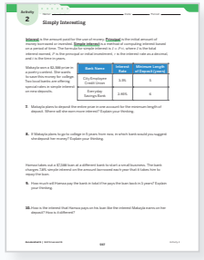
Students explore real-world situations about borrowing and lending money.
Routine: Think-Pair-Share **(MTR.7.1)**



Activity 2

Pairs | 15 min

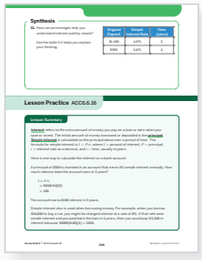
Students use the **simple interest** formula to solve real-world problems about banking.
MLR2: Collect and Display **(MTR.3.1)**



Synthesis

Whole Class | 5 min

Students synthesize their understanding of simple interest.



Show What You Know

Independent | 5 min

Students demonstrate their understanding by calculating simple interest using a formula.

Prep Checklist

This lesson includes:

- Student Edition
- Show What You Know
- Calculators

Pairs | 5 min

Warm-Up

Purpose: Students compare the values in a table, including using percents.

1 Launch

Direct students' attention to the table of values.

Invite students to select several rows from the table and compare consecutive rows.

A **Accessibility: Memory and Attention** Invite students to use a calculator to support them with performing calculations.

2 Connect

Encourage students to share their responses with a partner. Consider asking, "How are others' responses similar to or different from your own?"

Discuss students' strategies for calculating the percents they used to compare the annual balances in the table.

Look for different ways that students compare the balances:

- Constant amount of increase
- Percent of the balance in 2019
- Decreasing percent of the annual balance
- $512 per year
- About 3.5% to 4% each year

ELL **English Language Learners** Invite students to share their ideas with a partner before they share them with others. **(Writing and Speaking)**

Unit 6
Lesson
16

Name: _____ Date: _____ Period: _____

MA.6.NSO.3.5, MA.7.AR.3.1, MTR.3.1, MTR.7.1

How Simple, How Interesting

Let's explore how to use percents in loans and banking situations.

Warm-Up

The table shows the amount of money in a savings account with no additional deposits over a period of several years.

Year	Balance
2019	$12,800.00
2020	$13,312.00
2021	$13,824.00
2022	$14,336.00
2023	$14,848.00
2024	$15,360.00

1. What do you notice about the balance in the savings account each year?
Responses vary. The balance increases by a constant amount of $512 each year.

2. How can you describe the change in the balance using percents?
Responses vary. The balance increases by 4% of the 2019 balance each year.

Accelerated 6 | Unit 6 Lesson 16 595 Warm-Up

Pairs | 13 min

Activity 1 Let's Make a Deal

Purpose: Students explore real-world situations about borrowing and lending money. **(MTR.7.1)**

Short on time? Consider discussing Problem 6 as a class.

1 Launch

Use the Think-Pair-Share routine to help students shape their ideas about the deal that Camila's aunt makes with her. See the Routine Facilitation Guide for more information.

2 Monitor

Select and sequence strategies for determining the interest and total amount for different numbers of years.

Look for connections between percents and equations.

D Differentiation

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Encourage them to look for patterns in the table like in the Warm-Up.
Have difficulty calculating.	**Strengthen** Invite them to solve a simpler problem using a number that is easier to calculate with, like 2%.
Use the table to calculate the total.	**Strengthen** Consider asking, "How can you show this with an equation?"
Quickly identify the interest paid and total.	**Stretch** Challenge them to write and solve a similar problem, where a greater amount of money is involved.

3 Connect

Discuss students' responses to Problem 6. Consider asking:

- "What kinds of values are involved in both problems?"
- "How does time factor into solving these problems?"

To surface the Key Takeaway, consider asking, "How did you use percents to solve these problems?" **(MTR.7.1)**

Activity 1

Name: Date: Period:

Let's Make a Deal

Camila borrowed $500 from her aunt to pay for a school trip to Washington, D.C. To help Camila learn about managing money, her aunt offered to loan Camila the money with the condition that she pay back an extra 2.4% of the amount borrowed.

3. How much extra will Camila have to pay back to her aunt?

$12

4. How much will Camila have to pay in all? How do you know?

$512. *Responses vary.* She will have to pay back $500, plus the $12 that is 2.4% of $500.

After Camila pays back the loan from her aunt, she starts saving to buy a car for when she is old enough to drive. Over a few months, Camila saves $650 from babysitting. Camila's aunt tells her, "If you lend me the $650, I'll pay you 4.2% for each year you let me keep the money. Then when you are ready to buy a car, you can have this money back plus the 4.2% per year."

5. Complete the table to determine the total amount her aunt will pay Camila if she waits one, two, or five years.

Years	Extra 4.2%	Total Amount
1	$27.30	$677.30
2	$54.60	$704.60
5	$136.50	$786.50

6. **Discuss:** How are these two situations similar? How are they different?

Responses vary. In both situations, one person is giving money to the other and is adding a percent of the loan to the total amount to be paid back. In the first situation, Camila is borrowing money, and the percent is applied to the original amount once. In the second situation, Camila is lending money, and the percent is applied multiple times, once every year.

Accelerated 6 | Unit 6 Lesson 16 596 Activity 1

Key Takeaway: Percents are used to solve problems about the cost of borrowing money or the value of lending money.

Pairs | 15 min

Activity 2 Simply Interesting

Purpose: Students use the **simple interest** formula to solve real-world problems about banking. **(MTR.3.1)**

Short on time? Consider discussing Problem 10 as a class.

1 Launch

Direct students' attention to the *simple interest* formula.

To surface connections, consider asking, "What steps do you take to use a formula to solve problems?"

Consider creating an anchor chart of the simple interest formula emphasizing the connection between the variables (I,p,r, and t) and their meanings for students to refer to throughout the lesson.

2 Monitor

To support students getting started, consider asking, "How can you use a formula to calculate the interest earned at each bank?" **(MTR.3.1)**

Listen for students who notice that 5 years is less than the minimum length of deposit for the interest rate at Everyday Savings Bank.

Look for students who use the percent without changing the value to a decimal. Consider asking, "How do you write a percent as a decimal?"

A **Accessibility: Executive Function** Consider providing tables for students with columns labeled Principal (P), Interest Rate (r), Time (t), and Interest (I).

3 Connect

Invite students to share their strategies for determining the total that Hamza pays the bank. If students do not suggest it, note that they could write an expression like $7500(1 + 0.078)(3)$ instead of adding the interest to the amount borrowed.

MLR **MLR2: Collect and Display** Use this routine to formalize the definitions of **interest**, **simple interest**, and **principal** using students' language. Display any new language, examples, or diagrams for students to reference later.

Invite students to share their responses and justify their thinking.

ELL **English Language Learners** Provide sentence frames to help students explain their thinking (e.g., They are similar because both are _____. They are different because Hamza_____ while Makayla _____.). **(Writing and Speaking)**

Activity 2

Name: _____ Date: _____ Period: _____

Simply Interesting

Interest is the amount paid for the use of money. **Principal** is the initial amount of money borrowed or invested. **Simple interest** is a method of computing interest based on a period of time. The formula for simple interest is $I = Prt$, where I is the total interest earned, P is the principal or initial investment, r is the interest rate as a decimal, and t is the time in years.

Makayla won a $2,500 prize in a poetry contest. She wants to save this money for college. Two local banks are offering special rates in simple interest on new deposits.

Bank Name	Interest Rate	MInimum Length of Deposit (years)
City Employee Credit Union	3.5%	5
Everyday Savings Bank	2.95%	6

7. Makayla plans to deposit the entire prize in one account for the minimum length of deposit. Where will she earn more interest? Explain your thinking.

Everyday Savings Bank; *Explanations vary.* At the credit union, she will earn $2500(0.035)(5) = \$437.50$ interest over 5 years, but at the savings bank, she will earn $2500(0.0295)(6) = \$442.50$ over 6 years.

8. If Makayla plans to go to college in 5 years from now, in which bank would you suggest she deposit her money? Explain your thinking.

City Employee Credit Union; *Responses vary.* She should use the City Employee Credit Union because the minimum length of deposit at Everyday Savings Bank is 6 years to get the 2.95% interest rate and she wants to go to college before that time is up.

Hamza takes out a $7,500 loan at a different bank to start a small business. The bank charges 7.8% simple interest on the amount borrowed each year that it takes him to repay the loan.

9. How much will Hamza pay the bank in total if he pays the loan back in 3 years? Explain your thinking.

$9,255. *Explanations vary.* He will pay $7,500 plus the interest owed of $1,755.

10. How is the interest that Hamza pays on his loan like the interest Makayla earns on her deposit? How is it different?

Responses vary. Both are examples of simple interest. The interest rates are different rates, but both are determined by a percent of the original amount over a number of years. Makayla deposited money, so she will earn interest, but Hamza borrowed money, so he will pay interest.

Accelerated 6 | Unit 6 Lesson 16 597 Activity 2

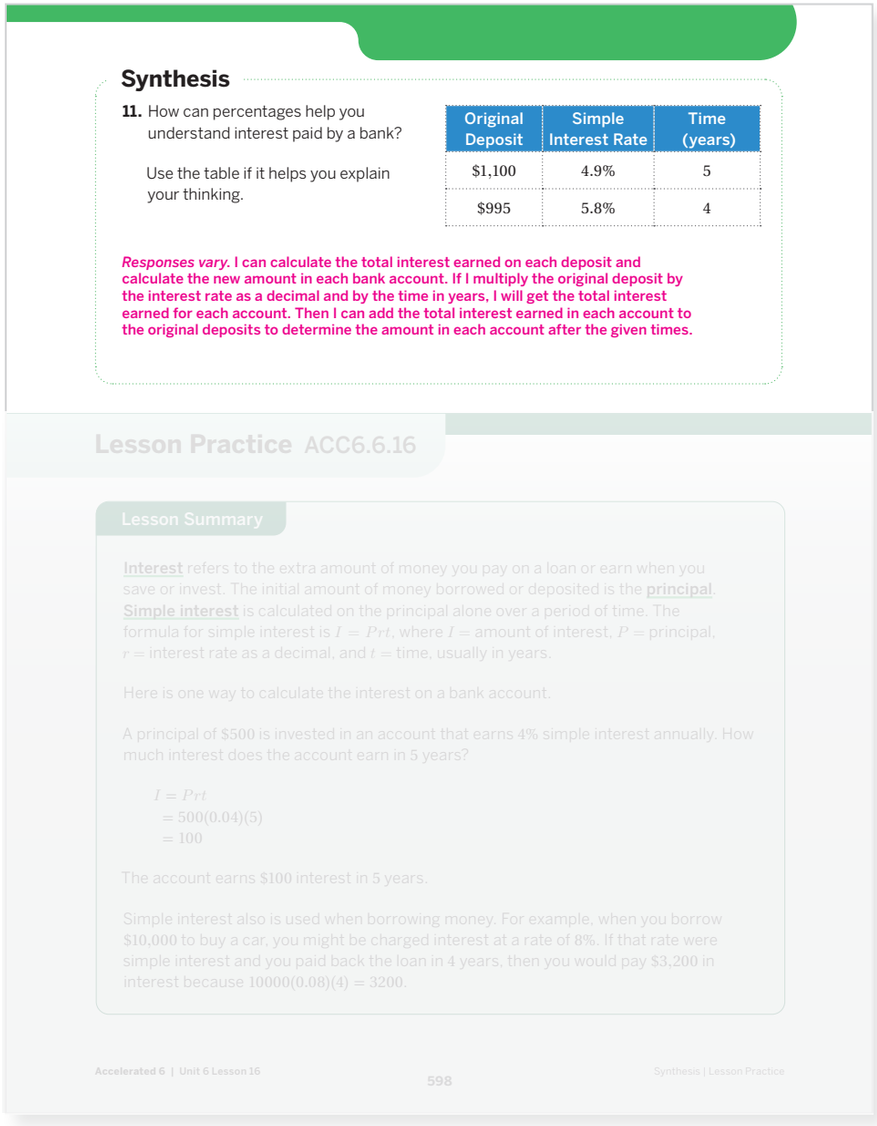
Whole Class | 5 min

Synthesis

Purpose: Students synthesize their understanding of simple interest.

Invite students to respond independently, and then share their thinking about simple interest with a partner.

Create a record of how students calculated simple interest or how they described how to calculate it.



Synthesis

11. How can percentages help you understand interest paid by a bank?

Use the table if it helps you explain your thinking.

Original Deposit	Simple Interest Rate	Time (years)
$1,100	4.9%	5
$995	5.8%	4

Responses vary. I can calculate the total interest earned on each deposit and calculate the new amount in each bank account. If I multiply the original deposit by the interest rate as a decimal and by the time in years, I will get the total interest earned for each account. Then I can add the total interest earned in each account to the original deposits to determine the amount in each account after the given times.

Lesson Practice ACC6.6.16

Lesson Summary

Interest refers to the extra amount of money you pay on a loan or earn when you save or invest. The initial amount of money borrowed or deposited is the principal. Simple interest is calculated on the principal alone over a period of time. The formula for simple interest is $I = Prt$, where I = amount of interest, P = principal, r = interest rate as a decimal, and t = time, usually in years.

Here is one way to calculate the interest on a bank account.

A principal of $500 is invested in an account that earns 4% simple interest annually. How much interest does the account earn in 5 years?

$$\begin{aligned} I &= Prt \\ &= 500(0.04)(5) \\ &= 100 \end{aligned}$$

The account earns $100 interest in 5 years.

Simple interest also is used when borrowing money. For example, when you borrow $10,000 to buy a car, you might be charged interest at a rate of 8%. If that rate were simple interest and you paid back the loan in 4 years, then you would pay $3,200 in interest because $10000(0.08)(4) = 3200$.

Accelerated 6 | Unit 6 Lesson 16 598 Synthesis | Lesson Practice

Lesson Takeaway: Simple interest may be earned on deposits or paid on loans and is calculated using the formula $I = Prt$.

Independent | 5 min

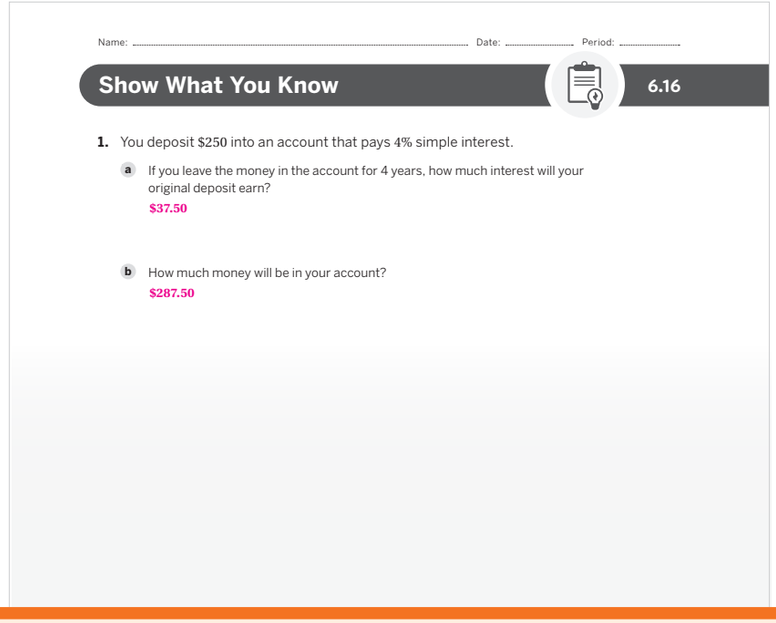
Show What You Know

Purpose: Students demonstrate their understanding by calculating simple interest using a formula.

Today's Goals

1. **Goal:** Solve real-world problems involving simple interest earned.
2. **Language Goal:** Describe how to use the simple interest formula to solve real-world problems about borrowing and investing money. **(Reading and Writing)**

Show What You Know (Assessment Resources)



Name: _____ Date: _____ Period: _____

Show What You Know 6.16

1. You deposit $250 into an account that pays 4% simple interest.
   a. If you leave the money in the account for 4 years, how much interest will your original deposit earn?
   $37.50
   b. How much money will be in your account?
   $287.50

D Differentiation Use after Lesson 16

Support

Provide targeted intervention for students by using these resources.

If students need support understanding how to calculate simple interest using a formula:

» **Respond to Student Thinking:**

- Select a few related **Practice Problems** and give individualized feedback.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 4 Extension Activities**.

Professional Learning

Take a moment to reflect on how students' understanding of the term percent has developed throughout this sub-unit. How has their definition of percent expanded over the last few lessons?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

11. How can percentages help you understand interest paid by a bank?

Use the table if it helps you explain your thinking.

Original Deposit	Simple Interest Rate	Time (years)
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Responses vary. I can calculate the total interest earned on each deposit and calculate the new amount in each bank account. If I multiply the original deposit by the interest rate as a decimal and by the time in years, I will get the total interest earned for each account. Then I can add the total interest earned in each account to the original deposits to determine the amount in each account after the given times.

Lesson Practice ACC6.6.16

Lesson Summary

Interest refers to the extra amount of money you pay on a loan or earn when you save or invest. The initial amount of money borrowed or deposited is the **principal**. **Simple interest** is calculated on the principal alone over a period of time. The formula for simple interest is $I = Prt$, where $I =$ amount of interest, $P =$ principal, $r =$ interest rate as a decimal, and $t =$ time, usually in years.

Here is one way to calculate the interest on a bank account.

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The account earns $100 interest in 5 years.

Simple interest also is used when borrowing money. For example, when you borrow $10,000 to buy a car, you might be charged interest at a rate of 8%. If that rate were simple interest and you paid back the loan in 4 years, then you would pay $3,200 in interest because $10000(0.08)(4) = 3200$.

Lesson Practice
ACC6.6.16

Name: _____ Date: _____ Period: _____

Problems 1–2: You deposit $1,200 into an account that pays 3% simple interest.

1. Complete the table to determine the interest earned and the total amount in the account for the first five years the account is open.

Years	Interest	Total Amount
1	$36	$1,236
2	$72	$1,272
3	$108	$1,308
4	$144	$1,344
5	$180	$1,380

2. If you leave the money in the account for 10 years, what will be the total amount of money in the account? Explain your thinking.

$1,560. *Explanations vary.* The account will earn $360 in interest because $(1200)(0.03)(10) = 360$. Add this to the initial deposit: $1200 + 360 = 1560$.

3. Jin purchased a brand new car. She got a loan for $35,000, which she will pay back over 6 years. The terms of Jin's loan are that she will pay 9% simple interest on the original principal each year that she is paying off the loan. How much interest will Jin pay?

$18,900

Problems 4–5: A bank is offering a special where it will charge 6.75% simple interest annually on any loan paid back within 7 years.

4. What will the total amount paid back be if the borrower takes 7 years to pay back a $1,780 loan?

$2,621.05

5. If the total paid on a $980 loan was $1,310.75 including the simple interest, how many years did it take to pay off the loan? Explain your thinking.

5 years. *Explanations vary.* If $1,310.75 was paid on a $980 loan, then $330.75 was the amount paid in interest because $1310.75 - 980 = 330.75$. I can use the simple interest formula to calculate that $66.15 is paid in interest each year $980(0.0675)(3) = 66.15$. Finally, I will divide the amount of interest paid by the interest per year to determine how many years it took to pay off the loan: $330.75 \div 66.15 = 5$.

Lesson Practice
ACC6.6.16

Name: _____ Date: _____ Period: _____

Problems 6–8: Greenville Credit Union and Parkville Bank are both offering special rates on new deposits. The simple interest rate and minimum deposit for new accounts at each bank are shown in the table.

Bank	Interest Rate	Minimum Deposit
Greenville Credit Union	5.75%	$500
Parkville Bank	6.1%	$1,500

6. How much interest will a $3,000 deposit earn at each bank in 4 years?

Greenville Credit Union: $690

Parkville Bank: $732

7. What will the value be in an account with a $850 deposit at Greenville Credit Union in 3 years? Show or explain your thinking.

About $996.63. *Explanations vary.* $850(0.0575)(3) = 146.625$, and $850 + 146.625 \approx 996.63$.

8. What will the value be in an account with a $850 deposit at Parkville Bank after 3 years? Show or explain your thinking.

***Explanations vary.* You cannot deposit $850 into a new account at Parkville Bank because there is a minimum deposit of $1,500.**

FAST Practice

9. Your bank is offering a simple interest rate of 7.5% for a loan paid back in 3 years. Determine the total amount you would pay if you borrowed $900 at this rate.

The simple interest on a loan for $900 at a rate of 7.5% over 3 years is $ 202.50 .

So I would pay back a total of $ 1,102.50 .

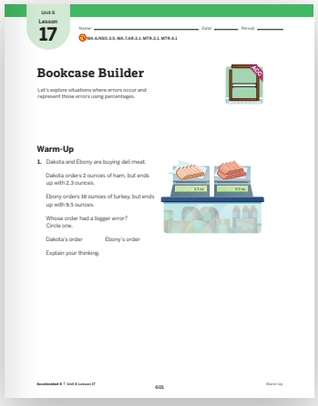
Spiral Review

10. The sales tax rate is 5.5%. Determine the price after tax for an item that sells for $40 regularly and that is 20% off for this week. Explain your reasoning.

$33.76. *Explanations vary.* The discounted price is $40(0.8) = 32$. The cost including tax is $32(1.055) = 33.76$.

Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1	1	MA.7.AR.3.1
	2–4, 6–8	2	MA.7.AR.3.1
	5	3	MA.7.AR.3.1
FAST Practice	9	2	MA.7.AR.3.1
Spiral Review			
	10	2	MA.7.AR.3.1


Unit 6
Lesson 17
Name: _____ Date: _____ Period: _____
MA.6.NSO.3.5, MA.7.AR.3.1, MTR.3.1, MTR.4.1
Bookcase Builder
Let's explore situations where errors occur and represent those errors using percentages.
Warm-Up
1. Dakota and Ebony are buying deli meat.
Dakota orders 2 ounces of ham, but ends up with 2.3 ounces.
Ebony orders 10 ounces of turkey, but ends up with 9.5 ounces.
Whose order had a bigger error? Circle one.
Dakota's order Ebony's order
Explain your thinking.
601
Warm-Up

Bookcase Builder

Percent Error

Let's explore situations where errors occur and represent those errors using percentages.

Learning Outcomes

Today's Goal

1. **Goal:** Know that *percent error* expresses error as a percentage of a desired value.
2. **Goal:** Decide whether a value is within an acceptable *percent error*.
3. **Language Goal:** Explain a strategy for solving a problem involving *percent error*. **(Writing)**

Students will be introduced to real-world contexts in which error occurs. Students will consider why **percent error** is useful, practice determining percent error, and express whether a measurement is within an acceptable range.

Coherence

Prior Learning

In Lesson 15, students worked with contexts involving percent change, such as tax, tip, and discounts.

Future Learning

In a future course, students will continue to explore proportional relationships by comparing them using equations, tables, and graphs.

Connections

- Students calculate percent error and express whether a measurement is within an acceptable range in real-world contexts, such as the size of screws when building a bookshelf. (real world)

FL B.E.S.T. Math Benchmarks

Addressing

MA.6.NSO.3.5
Rewrite positive rational numbers in different but equivalent forms including fractions, terminating decimals and percentages.

MA.7.AR.3.1
Apply previous understanding of percentages and ratios to solve multi-step real-world percent problems.

Mathematical Thinking and Reasoning Standards: MTR.3.1, MTR.4.1

ELA Expectations: ELA.K12.EE.1.1, ELA.K12.EE.6.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.6.AR.3.4	MA.7.NSO.1.2 MA.8.AR.2.1

Vocabulary

New Vocabulary

percent error

Lesson at a Glance

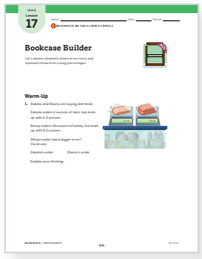
~ 45 min

MA.6.NSO.3.5, MA.7.AR.3.1

Warm-Up

Independent | 5 min

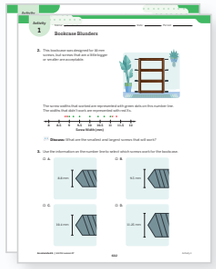
Students encounter a context involving two errors and use the Decide and Defend routine to describe which error is larger. **(MTR.4.1)**



Activity 1

Pairs | 15 min

Students come to understand that **percent error** is one way to describe how far a value is above or below a desired measurement. MLR2: Collect and Display



Activity 2

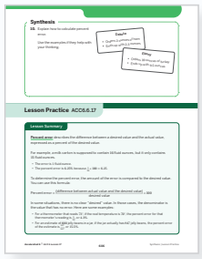
Pairs | 15 min

Students practice deciding whether a value is within a range of acceptable percent errors. **(MTR.3.1)**

Synthesis

Whole Class | 5 min

Students synthesize their understanding by explaining how to find percent error for values in context. MLR2: Collect and Display



Show What You Know

Independent | 5 min

Students demonstrate their understanding by determining the percent error for a new context.

Prep Checklist

This lesson includes:

- Student Edition
- Show What You Know

Independent | 5 min

Warm-Up

Purpose: Students encounter a context involving two errors and use the Decide and Defend routine to describe which error is larger. **(MTR.4.1)**

1 Launch

To surface connections, consider asking students whether they have ever had an experience at a store similar to Dakota's and Ebony's experiences.

Invite students to decide whose order had a bigger error and to explain their reasoning.

A **Accessibility: Affective Functioning** To support students in sharing their thinking, ask students in advance if they, you, or a classmate can share their thinking during the whole class discussion.

2 Connect

Invite students to share their work.

Use the Decide and Defend routine to support students in making arguments and critiquing the reasoning of others. **(MTR.4.1)**

ELL **English Language Learners** As students share in partner and whole-class discussions, provide sentence frames such as:

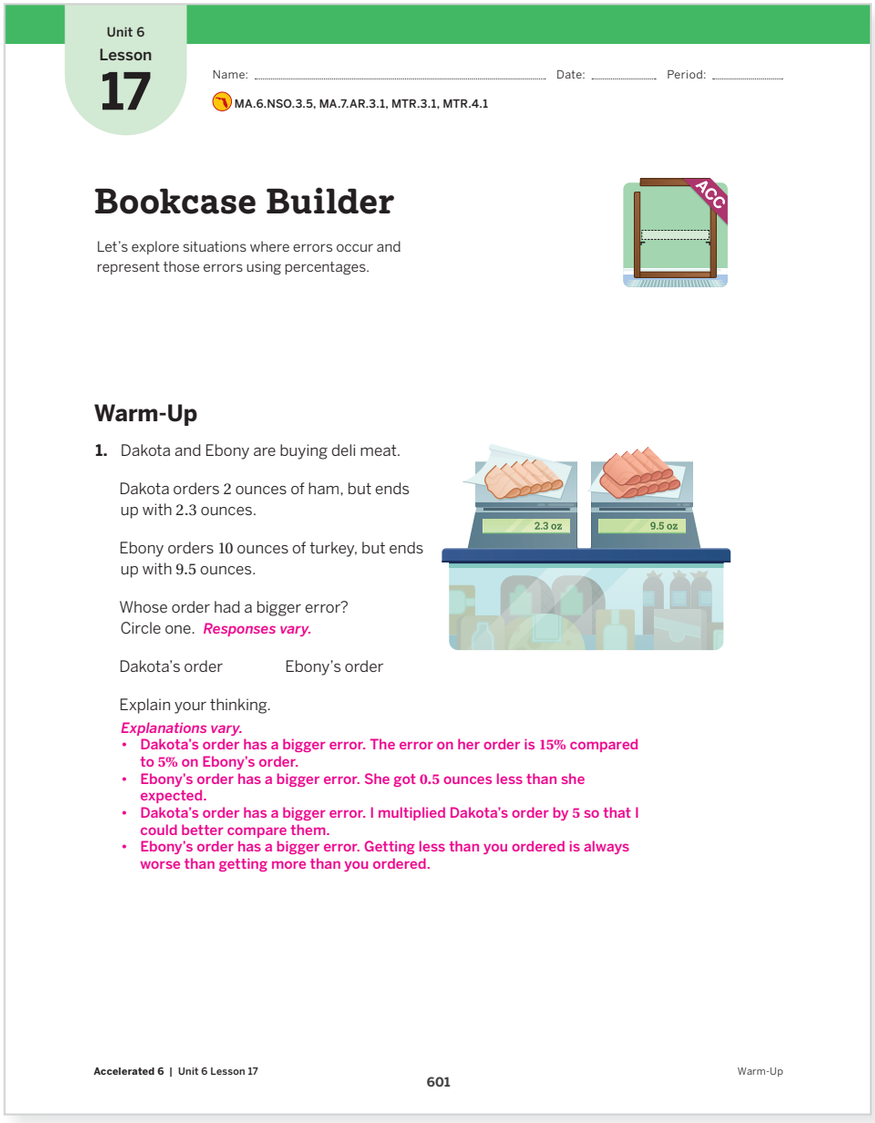
- I agree/disagree with you because . . .
- I think that makes sense because . . .
- How do you know?

(Writing and Speaking)

Note: If everyone is in agreement, consider making a class argument using different students' reasoning.

Consider asking:

- "What reasoning or evidence supports your claim?"
- "What evidence or language was most convincing to you?"
- "Where do you see mathematical thinking in selecting either order?" **(MTR.4.1)**



Unit 6
Lesson
17

Name: _____ Date: _____ Period: _____

MA.6.NSO.3.5, MA.7.AR.3.1, MTR.3.1, MTR.4.1

Bookcase Builder

Let's explore situations where errors occur and represent those errors using percentages.

Warm-Up

1. Dakota and Ebony are buying deli meat.

Dakota orders 2 ounces of ham, but ends up with 2.3 ounces.

Ebony orders 10 ounces of turkey, but ends up with 9.5 ounces.

Whose order had a bigger error? Circle one. *Responses vary.*

Dakota's order Ebony's order

Explain your thinking.

Explanations vary.

- Dakota's order has a bigger error. The error on her order is 15% compared to 5% on Ebony's order.
- Ebony's order has a bigger error. She got 0.5 ounces less than she expected.
- Dakota's order has a bigger error. I multiplied Dakota's order by 5 so that I could better compare them.
- Ebony's order has a bigger error. Getting less than you ordered is always worse than getting more than you ordered.

Accelerated 6 | Unit 6 Lesson 17 601 Warm-Up

Pairs | 15 min

Activity 1 Bookcase Blunders

Purpose: Students come to understand that **percent error** is one way to describe how far a value is above or below a desired measurement.

1 Launch

To surface connections, consider asking, "Have you ever needed to assemble furniture when given a set of instructions and parts? Does this match your experience with a building project?"

Direct students' attention to the line plot. Consider asking, "What are some measurements that lie between the tick marks on the number line?"

2 Monitor

Encourage students to determine the largest and smallest screw sizes that will work.

A **Accessibility: Visual-Spatial Processing**
Invite students to answer a more concrete question first to support interpreting visual representations. Consider asking, "What does the green dot over 9.5 mean? What about the red X between 11 and 11.5?"

Listen for a variety of student strategies to determine how they estimated the location of each measurement on the line plot. Consider inviting them to share with a partner to reach agreement before continuing.

Activity 1 continued >

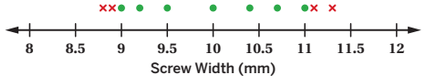
Activity 1

Name: Date: Period:

Bookcase Blunders

2. This bookcase was designed for 10 mm screws, but screws that are a little bigger or smaller are acceptable.

The screw widths that worked are represented with green dots on this number line. The widths that didn't work are represented with red Xs.

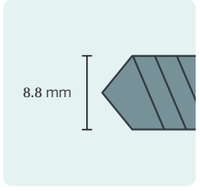


Discuss: What are the smallest and largest screws that will work?

The smallest screw would be 9 mm and the largest screw would be 11 mm.

3. Use the information on the number line to select which screws work for the bookcase.

☐ A.



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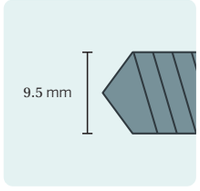
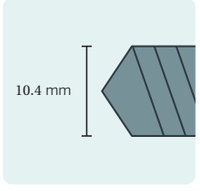
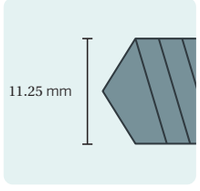


☑ C.



☐ D.



Accelerated 6 | Unit 6 Lesson 17 602 Activity 1

Pairs | 15 min

Activity 1 Bookcase Blunders (continued)

Purpose: Students come to understand that **percent error** is one way to describe how far a value is above or below a desired measurement.

2 Monitor

Encourage students to determine the largest and smallest screw sizes that will work.

Listen for explanations for both similarities and differences to share during the discussion.

D **Differentiation:** (Strengthen) For students that find the acceptable range is between $\frac{1}{10}$ (or 10%) smaller and the same amount larger than the desired value for both screws, invite these students to share their thinking during the discussion.

Consider asking, "What is similar about the range of acceptable values for both the 6-millimeter and 10-millimeter screws?"

Consider sharing that **percent error** is a way to describe error, expressed as a percentage of the desired value. Invite students to calculate the percent error of each screw.

D **Differentiation**

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Work together on the provided example to determine the percent error for the 6 mm screw. Encourage them to refer to this example as they continue.
Want to explore percent error further.	**Stretch** Encourage them to think of a rule that might be used to figure out what screws work for any desired size screw.

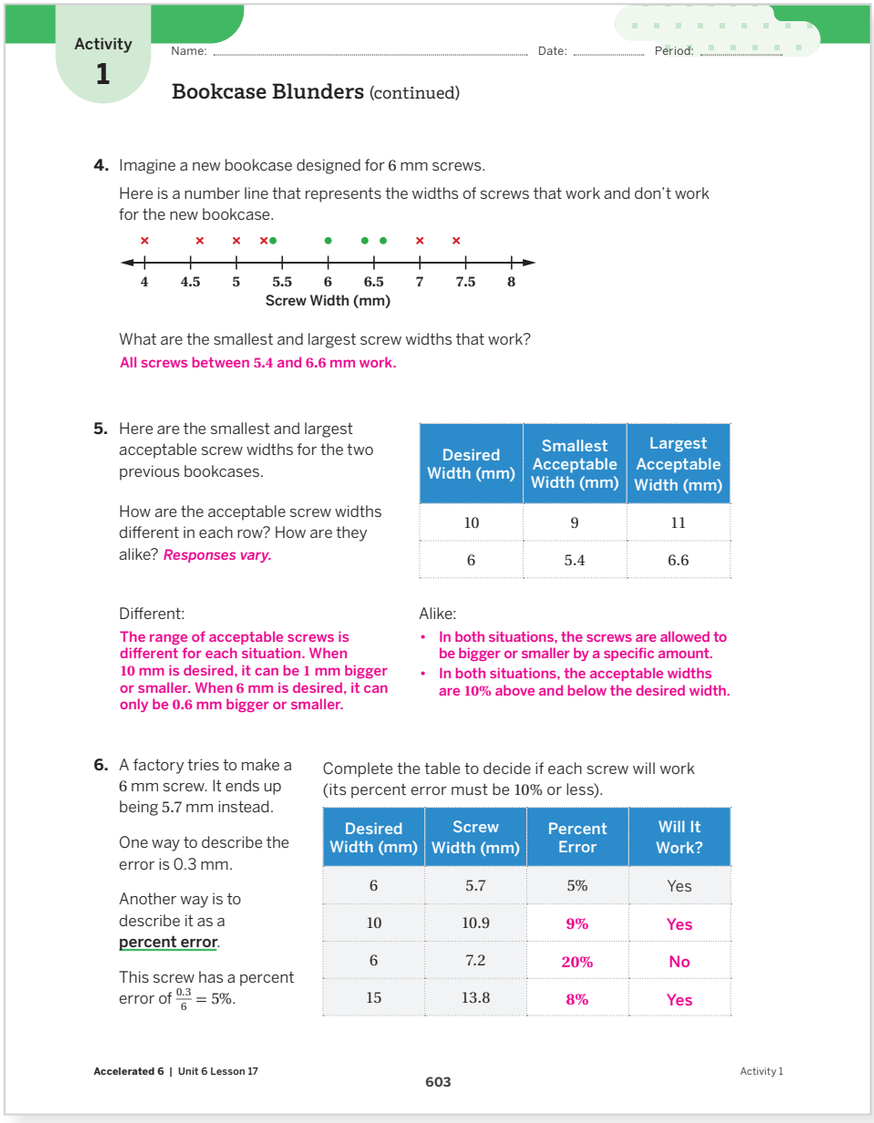
3 Connect

Share and discuss the percent error table.

MLR **MLR2: Collect and Display** Use this routine to create a class definition or anchor chart for *percent error* using students' language. Display any new language, examples, or diagrams for students to reference later.

Consider asking:

- "When do you think calculating percent error might be useful in your life or in the lives of people in your community?"
- "When would it be most necessary to have a small percent error?"
- "When would it be acceptable to have a larger percent error?"



Activity 1

Name: Date: Period:

Bookcase Blunders (continued)

4. Imagine a new bookcase designed for 6 mm screws.

Here is a number line that represents the widths of screws that work and don't work for the new bookcase.

What are the smallest and largest screw widths that work?

All screws between 5.4 and 6.6 mm work.

5. Here are the smallest and largest acceptable screw widths for the two previous bookcases.

How are the acceptable screw widths different in each row? How are they alike? *Responses vary.*

Desired Width (mm)	Smallest Acceptable Width (mm)	Largest Acceptable Width (mm)
10	9	11
6	5.4	6.6

Different:

The range of acceptable screws is different for each situation. When 10 mm is desired, it can be 1 mm bigger or smaller. When 6 mm is desired, it can only be 0.6 mm bigger or smaller.

Alike:

- In both situations, the screws are allowed to be bigger or smaller by a specific amount.
- In both situations, the acceptable widths are 10% above and below the desired width.

6. A factory tries to make a 6 mm screw. It ends up being 5.7 mm instead.

One way to describe the error is 0.3 mm.

Another way is to describe it as a **percent error**.

This screw has a percent error of $\frac{0.3}{6} = 5\%$.

Complete the table to decide if each screw will work (its percent error must be 10% or less).

Desired Width (mm)	Screw Width (mm)	Percent Error	Will It Work?
6	5.7	5%	Yes
10	10.9	9%	Yes
6	7.2	20%	No
15	13.8	8%	Yes

Accelerated 6 | Unit 6 Lesson 17 603 Activity 1

Pairs | 15 min

Activity 2 Challenge Your Shelf

Purpose: Students practice deciding whether a value is within a range of acceptable percent errors. **(MTR.3.1)**

Short on time? Consider inviting students to complete fewer repeated challenges in Problem 8.

1 Launch

Direct students' attention to the bookcase and consider asking, "What might be some sources of error when building a bookshelf?" *Responses vary.* You might drill holes at the wrong height, cut the shelves too short, or cut the shelves too long.

Invite students to use the strategies they tried in the previous problem to determine if the shelf will fit, and to explain their reasoning.

2 Monitor

Invite students to solve as many challenges as they have time for. Consider inviting students to set their own goal for a streak before beginning.

Note: Share that sense-making is more important than speed to encourage students to prioritize deepening their understanding of percent error.

A **Accessibility: Memory and Attention** Invite students who would benefit from having more time to think deeply to complete fewer challenges.

Listen for a variety of student strategies and partner discussions, offering help or encouragement where needed.

D **Differentiation**

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Encourage students to look back at the example for determining percent error for the 6 mm screw. Encourage them to refer to this example as they continue.
Repeatedly use guessing for whether the shelf will fit or not.	**Support** Consider asking, "What strategies shared from Activity 1 might help you to improve your challenge streak?"
Repeatedly calculate percent error with a reliable strategy (i.e., applying the percent error equation). **(MTR.3.1)**	**Strengthen** Invite students to think about how they would share their strategies during the Connect.

Activity 2 continued >

Activity 2

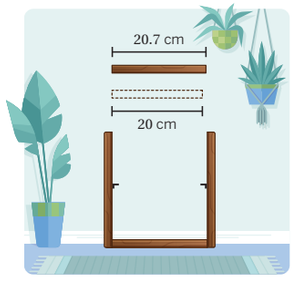
Name: Date: Period:

Challenge Your Shelf

7. Nikolai is making a shelf for his new bookcase.

He wants it to be 20 centimeters long. It ends up being 20.7 centimeters long.

The acceptable percent error is 5% for a shelf to fit.



Will the shelf fit? Circle one.

(Yes) No

Explain your thinking.

Explanations vary. **The error is 0.7 centimeters and the original is 20 centimeters. 0.7 is 3.5% of 20, which is less than 5%.**

8. Determine whether the shelf will fit given the acceptable error.

Answer as many as you have time for.

a
- Shelf length: 19.3 cm
- Desired shelf length: 20 cm
- Acceptable error: 5%

Will the shelf fit? Circle one.

(Yes) No

b
- Shelf length: 12 cm
- Desired shelf length: 15 cm
- Acceptable error: 5%

Will the shelf fit? Circle one.

Yes (No)

c
- Shelf length: 19.45 cm
- Desired shelf length: 18 cm
- Acceptable error: 5%

Will the shelf fit? Circle one.

Yes (No)

d
- Shelf length: 21 cm
- Desired shelf length: 20.5 cm
- Acceptable error: 10%

Will the shelf fit? Circle one.

(Yes) No

Accelerated 6 | Unit 6 Lesson 17 604 Activity 2

Pairs | 15 min

Activity 2 Challenge Your Shelf (continued)

Purpose: Students practice deciding whether a value is within a range of acceptable percent errors. **(MTR.3.1)**

Short on time? Consider inviting students to complete fewer repeated challenges in Problem 8.

2 Monitor

Note: Refer to the previous page for guidance.

D **Differentiation: You're invited to explore more. (Stretch)** Invite students who want to explore percent error further to complete this optional task. Encourage them to discuss their thinking with a partner.

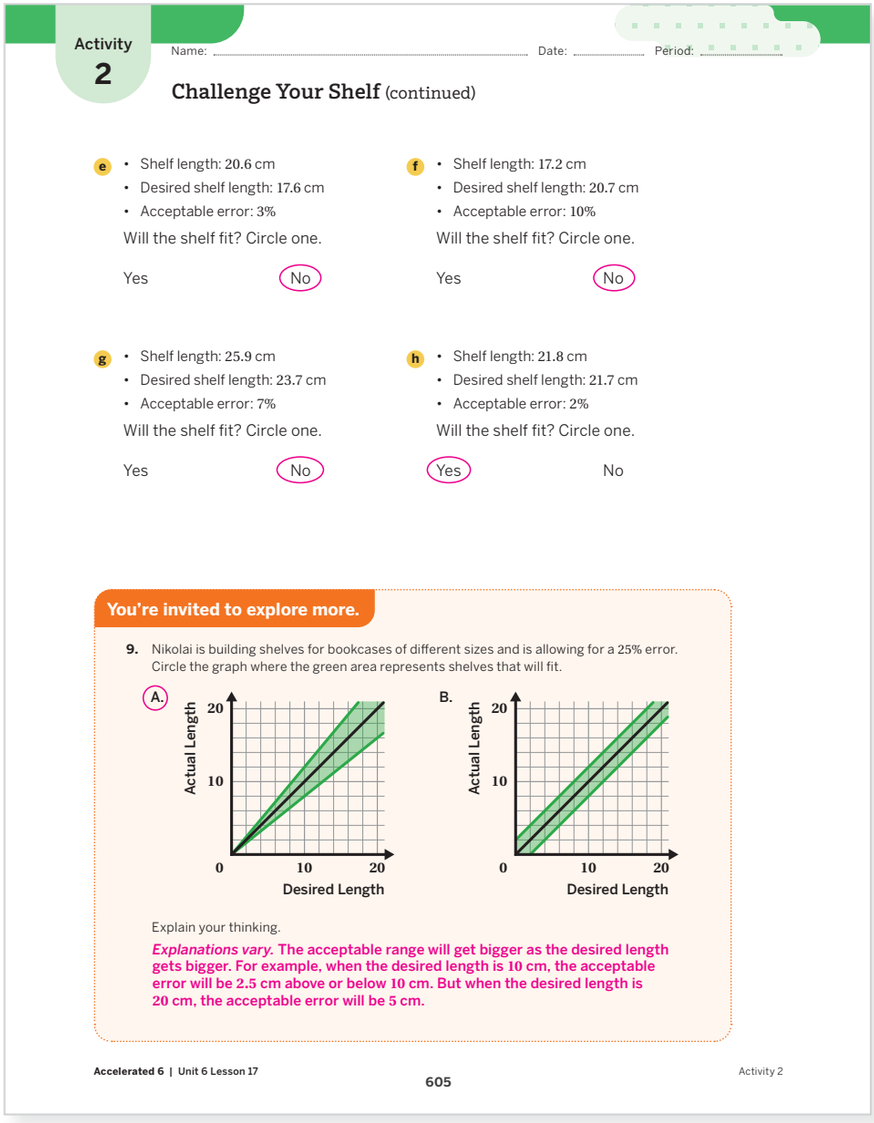
3 Connect

Invite students to share strategies they found helpful to reach their streak goal of solving multiple challenges. **(MTR.3.1)**

ELL **English Language Learners** Invite students to share strategies in their primary languages. Encourage all students to use gestures and/or visuals as they share. **(Speaking and Listening)**

To surface the Key Takeaway, consider asking:

- "Were there any mathematical representations that helped you to solve for whether a shelf would fit or not for multiple challenges?"
- "How did you adjust the representation for each new shelf scenario?"
- "What can we add to our definition of *percent error*?"



Activity 2

Name: Date: Period:

Challenge Your Shelf (continued)

e
- Shelf length: 20.6 cm
- Desired shelf length: 17.6 cm
- Acceptable error: 3%

Will the shelf fit? Circle one.

Yes (No)

f
- Shelf length: 17.2 cm
- Desired shelf length: 20.7 cm
- Acceptable error: 10%

Will the shelf fit? Circle one.

Yes (No)

g
- Shelf length: 25.9 cm
- Desired shelf length: 23.7 cm
- Acceptable error: 7%

Will the shelf fit? Circle one.

Yes (No)

h
- Shelf length: 21.8 cm
- Desired shelf length: 21.7 cm
- Acceptable error: 2%

Will the shelf fit? Circle one.

(Yes) No

You're invited to explore more.

9. Nikolai is building shelves for bookcases of different sizes and is allowing for a 25% error. Circle the graph where the green area represents shelves that will fit.

(A.)

B.

Explain your thinking.

Explanations vary. The acceptable range will get bigger as the desired length gets bigger. For example, when the desired length is 10 cm, the acceptable error will be 2.5 cm above or below 10 cm. But when the desired length is 20 cm, the acceptable error will be 5 cm.

Accelerated 6 | Unit 6 Lesson 17 605 Activity 2

Key Takeaway: Knowing the desired measurement for a situation and the **percent error** helps you calculate the highest and lowest acceptable values. For example, if the desired value is 30 and the percent error is 5%, you can calculate 5% of 30, then add that amount to or subtract it from 30 to find other values that will work.

Whole Class | 5 min

Synthesis

Purpose: Students synthesize their understanding by explaining how to find percent error for values in context.

Invite students to respond independently, and then share their thinking with a partner.

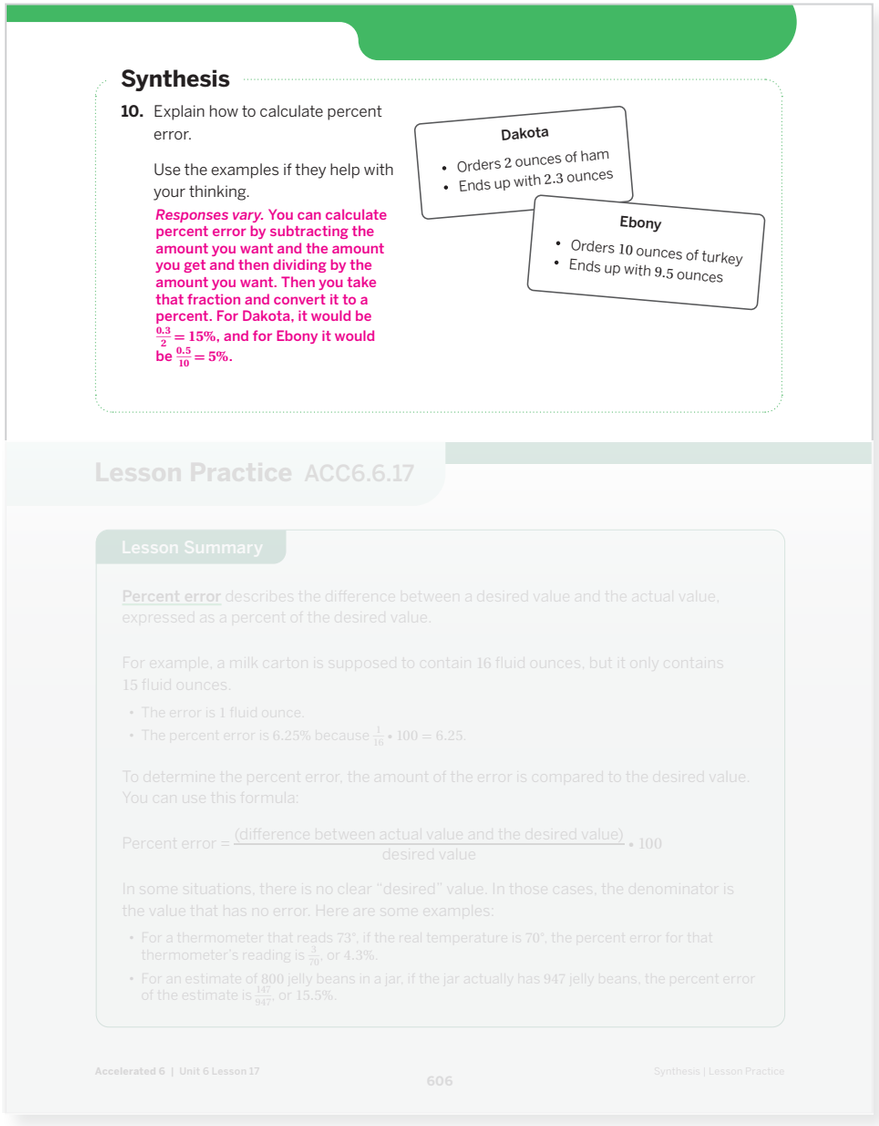
Collect and share a variety of student ideas, such as:

- Explaining calculating percent error using the relationship between actual value and desired value.
- Determining the percent error for Dakota's and Ebony's deli orders.

MLR **MLR2: Collect and Display** Use this routine to formalize the definition of *percent error* using students' language. Display any new language, examples, or diagrams for students to reference later.

Note: If time allows, consider asking students, "How can percent error be useful in situations like these deli meat orders?" *Responses vary.* Percent error allows us to compare situations with different amounts.

Note: If time allows, invite students to come up with a new situation where percent error might be helpful.



Synthesis

10. Explain how to calculate percent error.

Use the examples if they help with your thinking.

Responses vary. **You can calculate percent error by subtracting the amount you want and the amount you get and then dividing by the amount you want. Then you take that fraction and convert it to a percent. For Dakota, it would be $\frac{0.3}{2} = 15\%$, and for Ebony it would be $\frac{0.5}{10} = 5\%$.**

Dakota
- Orders 2 ounces of ham
- Ends up with 2.3 ounces

Ebony
- Orders 10 ounces of turkey
- Ends up with 9.5 ounces

Lesson Practice ACC6.6.17

Lesson Summary

Percent error describes the difference between a desired value and the actual value, expressed as a percent of the desired value.

For example, a milk carton is supposed to contain 16 fluid ounces, but it only contains 15 fluid ounces.

- The error is 1 fluid ounce.
- The percent error is 6.25% because $\frac{1}{16} \cdot 100 = 6.25$.

To determine the percent error, the amount of the error is compared to the desired value. You can use this formula:

$$\text{Percent error} = \frac{(\text{difference between actual value and the desired value})}{\text{desired value}} \cdot 100$$

In some situations, there is no clear "desired" value. In those cases, the denominator is the value that has no error. Here are some examples:

- For a thermometer that reads 73°, if the real temperature is 70°, the percent error for that thermometer's reading is $\frac{3}{70}$, or 4.3%.
- For an estimate of 800 jelly beans in a jar, if the jar actually has 947 jelly beans, the percent error of the estimate is $\frac{147}{947}$, or 15.5%.

Accelerated 6 | Unit 6 Lesson 17 606 Synthesis | Lesson Practice

Lesson Takeaway: **Percent error** is one way to describe how far above or below a desired measurement a value is. To compute percent error, find the difference between the desired value and actual value divided by the desired value (for example, $\frac{0.3}{2} = 15\%$ computes the percent error of 2.3 ounces of ham when 2 ounces of ham is desired).

Independent | 5 min

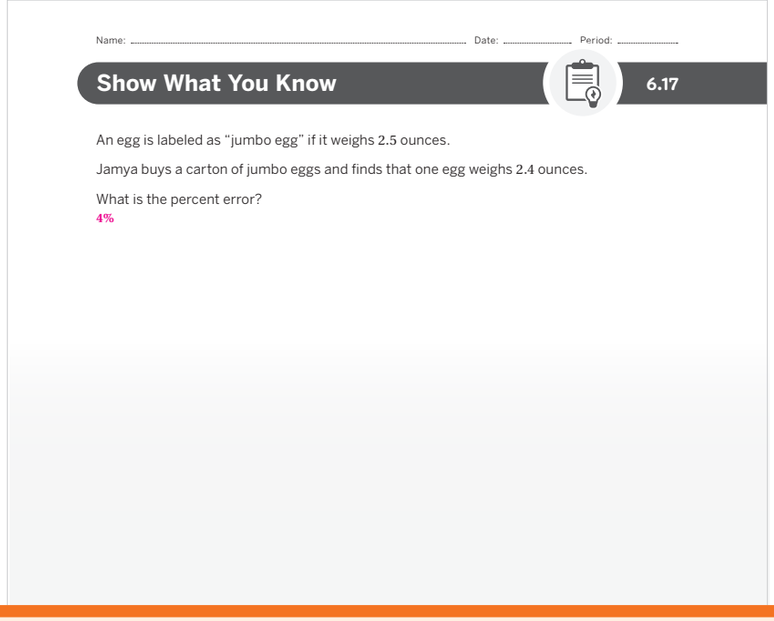
Show What You Know

Purpose: Students demonstrate their understanding by determining the percent error for a new context.

Today's Goals

1. **Goal:** Know that *percent error* expresses error as a percentage of a desired value.
2. **Goal:** Decide whether a value is within an acceptable *percent error*.
3. **Language Goal:** Explain a strategy for solving a problem involving *percent error*. **(Writing)**

Show What You Know (Assessment Resources)



Name: _____ Date: _____ Period: _____

Show What You Know 6.17

An egg is labeled as "jumbo egg" if it weighs 2.5 ounces.

Jamya buys a carton of jumbo eggs and finds that one egg weighs 2.4 ounces.

What is the percent error?

4%

D Differentiation Use after Lesson 17

Support

Provide targeted intervention for students by using these resources.

If students need support understanding how to determine whether a value is within an acceptable percent error:

» **Respond to Student Thinking:**

- Revisit the Synthesis and invite students to share strategies for calculating percent error in each example.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 4 Extension Activities**.

Professional Learning

Reflect on students' progress with percent increase, percent decrease, and percent error in Sub-Unit 4. In what ways have students' understanding and skills grown? What concepts do students need additional support with? How can you use students' strengths to help everyone feel prepared for their upcoming assessment?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

10. Explain how to calculate percent error.

Use the examples if they help with your thinking.

Responses vary. You can calculate percent error by subtracting the amount you want and the amount you get and then dividing by the amount you want. Then you take that fraction and convert it to a percent. For Dakota, it would be $\frac{0.3}{2} = 15\%$, and for Ebony it would be $\frac{0.5}{10} = 5\%$.

Dakota
- Orders 2 ounces of ham
- Ends up with 2.3 ounces

Ebony
- Orders 10 ounces of turkey
- Ends up with 9.5 ounces

Lesson Practice ACC6.6.17

Lesson Summary

Percent error describes the difference between a desired value and the actual value, expressed as a percent of the desired value.

For example, a milk carton is supposed to contain 16 fluid ounces, but it only contains 15 fluid ounces.

- The error is 1 fluid ounce.
- The percent error is 6.25% because $\frac{1}{16} \cdot 100 = 6.25$.

To determine the percent error, the amount of the error is compared to the desired value. You can use this formula:

$$\text{Percent error} = \frac{\text{(difference between actual value and the desired value)}}{\text{desired value}} \cdot 100$$

In some situations, there is no clear "desired" value. In those cases, the denominator is the value that has no error. Here are some examples:

- For a thermometer that reads 73°, if the real temperature is 70°, the percent error for that thermometer's reading is $\frac{3}{70}$, or 4.3%.
- For an estimate of 800 jelly beans in a jar, if the jar actually has 947 jelly beans, the percent error of the estimate is $\frac{147}{947}$, or 15.5%.

Lesson Practice ACC6.6.17

Name: _____ Date: _____ Period: _____

1. A baker needs 500 grams of flour for a batter, but accidentally only puts 300 grams of flour into the mixing bowl. What is the percent error?

40%

2. A speed detection device measured the speed of a baseball at 93 miles per hour. If the baseball was actually going 90 miles per hour, what was the percent error in this measurement?

About 3.3%

Problems 3–4: In Burlington, all city departments get an annual budget. They are allowed to spend 1.5% over their budget. If they spend more than that, the mayor conducts a review.

3. The Department of Health has a budget of $90,000. What is the largest acceptable amount for their spending? Show or explain your thinking.

$91,350. *Explanations vary.*
$90000 \cdot 0.015 = 1350$
$90000 + 1350 = 91350$

4. The Department of Parks and Recreation has a budget of $30,000, and they spent $31,000. Will the mayor conduct a review of their spending? Show or explain your thinking.

Yes. *Explanations vary.*
$31000 - 30000 = 1000$
$1000 \div 30000 = 0.0333\ldots$
$0.033 \cdot 100 = 3.3$
3.3% is more than 1.5%.

FAST Practice

5. Any can of seltzer should have 12 fluid ounces of liquid to be appropriately carbonated. One can on the assembly line contains 13.5 fluid ounces. What is the percent error?

12.5 %

Lesson Practice ACC6.6.17

Name: _____ Date: _____ Period: _____

Spiral Review

6. Hoang is buying 5 concert tickets that cost $95.00 each. He finds a coupon to receive a 10% discount. A sales tax of 8% is applied after the discount. How much did he pay after the discount and sales tax?

A. $92.34 B. $427.50 C. $461.70 D. $513.00

7. Kimaya invests $800 in an account that earns 3.5% simple interest annually. How much interest will Kimaya's account earn in 7 years?

$196

Problems 8–11: Evaluate each expression for $n = 3$.

8. $5^n - 2n$

119

9. $2n^4$

162

10. $\left(\frac{1}{2}\right)^n$

$\frac{1}{8}$

11. $\frac{39}{n} + 1^n$

14

Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1–4	2	MA.7.AR.3.1
FAST Practice	5	2	MA.7.AR.3.1
Spiral Review			
	6	2	MA.7.AR.3.1
	7	2	MA.7.AR.3.1
	8–11	2	MA.6.NSO.3.3

Math in Action

Purpose: Students consider the math they learned in this unit and how it applies to the workplace, their world, and communities.

In this Unit

Students built on their understanding of the equal sign and used it as a representation of equality. They performed balanced operations to solve for unknown values and related equivalent expressions, including those involving two variables or exponents.

Career Connection | Meet Sir Jagadish Chandra Bose

Invite students to read about scientist Sir Jagadish Chandra Bose and how his experiments led to the development of short radio waves.

Consider asking:

- "Communication towers are not always found in open areas like the one shown in the photo. Do you know of any towers in your town or neighborhood?" Sample response: You may see them on the roof of an office building or television station.
- "What are some other ways that math and telecommunications might be connected?" Sample response: Sometimes you see large round-shaped dishes on rooftops or even certain cable TV services. Maybe there is a relationship between the size of the round dish and the amount of information you connect to.

B.E.S.T. Benchmark Connections

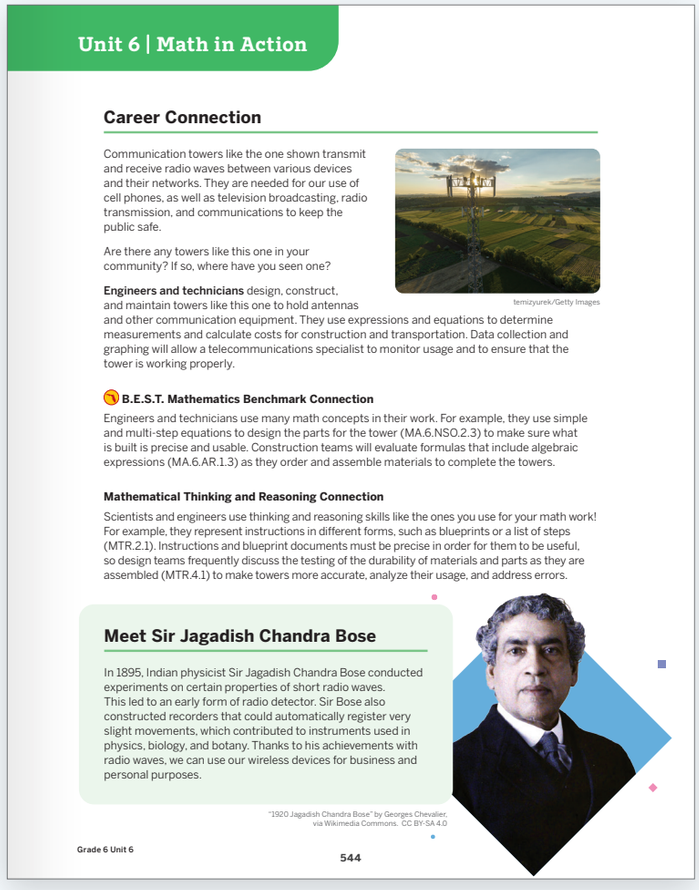
Consider asking:

- "What other math concepts have you studied in this unit that you could you apply to communication?" Sample response: You can solve equations for certain variables when you are trying to find an unknown number. (MA.6.AR.1.4)

Mathematical Thinking and Reasoning Connections

Consider asking:

- "Engineers often work in teams, and attending meetings, classes, and conferences are important learning opportunities. How are these actions similar to how you engage with math through Mathematical Thinking and Reasoning?" Sample response: Actively participate in effortful learning both individually and collectively. (MTR.1.1) Working on activities in this unit, sometimes on your own, and other times with a partner or team, can help you better understand concepts like algebraic expressions and equations.



Unit 6 | Math in Action

Career Connection

Communication towers like the one shown transmit and receive radio waves between various devices and their networks. They are needed for our use of cell phones, as well as television broadcasting, radio transmission, and communications to keep the public safe.

Are there any towers like this one in your community? If so, where have you seen one?

temizyurek/Getty Images

Engineers and technicians design, construct, and maintain towers like this one to hold antennas and other communication equipment. They use expressions and equations to determine measurements and calculate costs for construction and transportation. Data collection and graphing will allow a telecommunications specialist to monitor usage and to ensure that the tower is working properly.

B.E.S.T. Mathematics Benchmark Connection

Engineers and technicians use many math concepts in their work. For example, they use simple and multi-step equations to design the parts for the tower (MA.6.NSO.2.3) to make sure what is built is precise and usable. Construction teams will evaluate formulas that include algebraic expressions (MA.6.AR.1.3) as they order and assemble materials to complete the towers.

Mathematical Thinking and Reasoning Connection

Scientists and engineers use thinking and reasoning skills like the ones you use for your math work! For example, they represent instructions in different forms, such as blueprints or a list of steps (MTR.2.1). Instructions and blueprint documents must be precise in order for them to be useful, so design teams frequently discuss the testing of the durability of materials and parts as they are assembled (MTR.4.1) to make towers more accurate, analyze their usage, and address errors.

Meet Sir Jagadish Chandra Bose

In 1895, Indian physicist Sir Jagadish Chandra Bose conducted experiments on certain properties of short radio waves. This led to an early form of radio detector. Sir Bose also constructed recorders that could automatically register very slight movements, which contributed to instruments used in physics, biology, and botany. Thanks to his achievements with radio waves, we can use our wireless devices for business and personal purposes.

"1920 Jagadish Chandra Bose" by Georges Chevalier, via Wikimedia Commons. CC BY-SA 4.0

Grade 6 Unit 6

544

Assess and Respond

End-of-Unit Assessment

Independent | 45 min

Facilitation: Assign the End-of-Unit Assessment to learn about your students' understanding of the concepts and skills in this unit. There are two forms of the End-of-Unit Assessment: Forms A and B. If time allows, consider inviting students to complete one or more of the tasks in the Unit Synthesis and Reflection.

Item Analysis, Forms A and B

Problem(s)	Concept or skill	Addressed in	DOK	FL B.E.S.T. Math Benchmarks
1	Using the structure of equations of the form $x + p = q$ and $px = q$ to determine whether given values are solutions	Lesson 4	1	MA.6.AR.2.1
2	Using the structure determine equivalent expressions	Lesson 9	1	MA.6.AR.1.4, MA.7.AR.1.2
3a–3c	Using the structure of equations of the form $x + p = q$ and $px = q$ to determine their solutions	Lesson 5	2	MA.6.AR.2.4
4a	Determining equivalent numerical expressions, including those involving whole-number exponents	Lesson 10	2	MA.6.NSO.3.3, MA.7.AR.1.2, MTR.4.1
4b	Generating equivalent numerical expressions for an expression involving whole-number exponents	Lesson 10	2	MA.6.NSO.3.4
5a, 5b	Writing expressions to model real-world problems	Lesson 6	2	MA.6.AR.1.1, MTR.7.1
5c	Using equations to solve problems and justifying the response	Lesson 6	2	MA.6.AR.2.4, MTR.7.1
6a	Using the structure to represent the areas of given figures	Lesson 11	1	MA.6.AR.1.4, MTR.5.1
6b, 6c	Using the structure to evaluate equivalent expressions	Lesson 11	1	MA.6.AR.1.3
7	Selecting true statements associated with ratios of fractions	Grade 7, Lesson 10	2	MA.7.AR.3.2, MA.7.AR.4.2, MA.7.AR.4.5
8	Calculating the percent error given a desired measurement and an actual measurement	Lesson 17	1	MA.7.AR.3.1
9	Reasoning about multistep percent problems	Lesson 15	2	MA.7.AR.3.1
10	Calculating the interest paid over a period of time	Lesson 16	1	MA.7.AR.3.1, MTR.7.1

Assessment Resources

- Student Assessments (Forms A and B)
- Assessment Rubrics with Answers
- Show What You Knows and Answer Keys

Intervention and Extension Resources

- Mini-Lessons
- Extensions

Name: Date: Period:

End-of-Unit Assessment

Unit 6

1. Select *all* the equations where $x = 3$ is a solution.

- [x] **A.** $x - 3 = 0$
- [ ] **B.** $1 + x = 2$
- [ ] **C.** $33 = 3x$
- [x] **D.** $12 = 4x$
- [ ] **E.** $\frac{1}{2}x = 6$

2. Which expression is equivalent to $4(2x + 1) + 5$?

A. $8x + 9$

B. $8x + 6$

C. $6x + 6$

D. $8x + 24$

3. Determine the solution to each equation.

a $\frac{3}{4} + x = 6$
$x = 5\frac{1}{4}$
(or equivalent)

b $24 = 2x$
$x = 12$

c $x + 8 = 12.4$
$x = 4.4$
(or equivalent)

4. Here are four expressions.

4^3 3^4 $4 \cdot 4 \cdot 4$ $3 \cdot 4$

a Circle *two* expressions that have the same value. Show or explain your thinking.
Explanations vary. These have the same value because 4^3 means $4 \cdot 4 \cdot 4$, and they are both equal to 64.

b Here is a new expression: 5^4. Write an expression that has the same value.
Responses vary. $5 \cdot 5 \cdot 5 \cdot 5$

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Name: Date: Period:

End-of-Unit Assessment (continued)

Unit 6

5. Tyani is selling pizza to raise money for a field trip. She sells slices of pizza for $1.25 each.

a Complete the table.

Number of Pizza Slices Sold	Amount of Money Earned ($)
1	1.25
2	2.50
20	25
50	62.50

b Write an expression that represents the amount of money earned if she sells s slices of pizza.
$1.25s$ (or equivalent)

c How many slices of pizza does Tyani need to sell to earn $100?
80 slices of pizza

Explain your thinking.
Explanations vary.
- I wrote and solved the equation $100 = 1.25s$.
- I counted groups of 20 slices (each group earns $25) until I got to $100.

Name: Date: Period:

End-of-Unit Assessment (continued)

Unit 6

6. Here are three different diagrams.

Diagram A

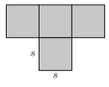
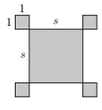


Diagram B

Diagram C



a Match each diagram with an expression that describes its area in square units.

$4 + s^2$ $4s^2$ $4s$

Diagram C Diagram A Diagram B

b Calculate the area of each diagram if $s = 3$.

$4 + s^2$
13 square units

$4s^2$
36 square units

$4s$
12 square units

c Habib drew a new diagram that has an area of $6 + 4s^2$.

What is the area of Habib's diagram when $s = \frac{1}{2}$?
7 square units (or equivalent)

Name: Date: Period:

End-of-Unit Assessment (continued)

Unit 6

7. Dalia is painting her room. After painting $\frac{1}{10}$ of her room, Dalia has used $\frac{1}{2}$ of a can of paint. Select *all* of the true statements.

- [ ] **A.** Each can of paint will cover $\frac{1}{20}$ of the Dalia's room.
- [x] **B.** 3 cans of paint will cover 60% of the Dalia's room.
- [x] **C.** Dalia's entire room requires 5 cans of paint.
- [ ] **D.** Dalia's entire room requires 20 cans of paint.
- [ ] **E.** Painting $\frac{1}{2}$ of Dalia's room requires 10 cans of paint.

8. Oliver wants to cut a piece of metal to be 30 centimeters long. The piece of metal ends up being 29.4 centimeters long. What is the percent error in this situation?

A. 1.02% **B.** 0.6% **C.** 2% **D.** 20%

9. A lamp originally costs $30. Krishna has a 5% off coupon for the lamp. After the coupon, a 5% sales tax is applied. He will pay:

A. More than $30. **B.** Less than $30. **C.** Exactly $30.

Explain your thinking.
Explanations vary. After a 5% discount, the lamp costs $28.50. Then after a 5% tax, the lamp costs about $29.93.

10. A loan is taken out to cover the cost of a home renovation. The bank offers a simple interest rate of 6.2% per year for a 4-year term. If the loan amount is $1,200, what is the total amount that must be repaid at the end of 4 years?
$1,297.60

Problem 8			Standard: MA.7.AR.3.1
4 Meeting	**3 Approaching**	**2 Developing**	**1 Beginning**
Correct choice: 2%			**Incorrect** choice. Students who select 0.6% may have calculated the actual error in centimeters and not the percent error. Students who select 20% may have correctly calculated $\frac{0.6}{30}$, but did not correctly convert this to a percentage. Students who select 1.02% may have calculated what percentage 30 is of 29.4.

Problem 9			Standard: MA.7.AR.3.1
4 Meeting	**3 Approaching**	**2 Developing**	**1 Beginning**
Correct response and **complete** explanation. Less than \$30. *Explanations vary.* After a 5% discount, the lamp costs \$28.50. Then after a 5% tax, the lamp costs about \$29.93.	**Correct** response with **minor flaws** in explanation. **Incorrect** response with logical and **complete explanation**.	**Correct** response with **incomplete** explanation. **Incorrect** response with explanation that shows **partial understanding** of multistep percentage problems.	**Incorrect** response with **no** explanation.

Problem 10			Standards: MA.7.AR.3.1, MTR.7.1
4 Meeting	**3 Approaching**	**2 Developing**	**1 Beginning**
Correct response: \$1297.60	Response shows **conceptual understanding** with minor errors. Students who write \$297.60 may have solved the problem, "How much interest will be paid at the end of 4 years?"	Response shows **incomplete understanding** with significant errors. Students who write \$30,960 may not have written the interest rate as a decimal. Students who write \$29,760 may not have written the interest rate as a decimal or added the interest to the initial amount of the loan.	Response shows **limited understanding**.

Name: _____ Date: _____ Period: _____

End-of-Unit Assessment

Unit 6

1. Select *all* the equations where $x = 4$ is a solution.

- ☐ A. $\frac{1}{2}x = 8$
- ☑ B. $x - 4 = 0$
- ☐ C. $44 = 4x$
- ☑ D. $20 = 5x$
- ☐ E. $1 + x = 3$

2. Which expression is equivalent to $5(2x + 1) + 4$?

- A. $10x + 5$
- B. $7x + 5$
- C. $10x + 25$
- (D.) $10x + 9$

3. Determine the solution to each equation.

a. $\frac{2}{3} + x = 7$
$x = 6\frac{1}{3}$
(or equivalent)

b. $36 = 4x$
$x = 9$

c. $x + 6 = 10.2$
$x = 4.2$
(or equivalent)

4. Here are four expressions.

(3⁴) 4^3 $3 \cdot 4$ (3 • 3 • 3 • 3)

a. Circle *two* expressions that have the same value. Show or explain your thinking.
Explanations vary. These have the same value because 3^4 means 3 • 3 • 3 • 3, and they are both equal to 81.

b. Here is a new expression: 6^3. Write an expression that has the same value.
Responses vary. 6 • 6 • 6

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Name: _____ Date: _____ Period: _____

End-of-Unit Assessment (continued)

Unit 6

5. Sora is buying colored pencils at a store. Each pencil costs \$1.20.

a. Complete the table.

Number of Pencils Bought	Total Cost (\$)
1	1.20
2	2.40
20	24
50	60

b. Write an expression that represents the total cost if Sora buys p pencils.
$1.20p$ (or equivalent)

c. How many pencils could Sora buy with \$150?
125 pencils

Explain your thinking.
Explanations vary.
- I wrote and solved the equation $150 = 1.20p$.
- I figured out how many groups of \$60 I need to get to \$150. Each group is 50 pencils, so I need 2.5 groups of 50, which is 125.

D Differentiation (End-of-Unit Assessment)

Note: To **strengthen** and **stretch** students' learning, refer to the differentiation resources suggested throughout this unit and in the Unit Overview.

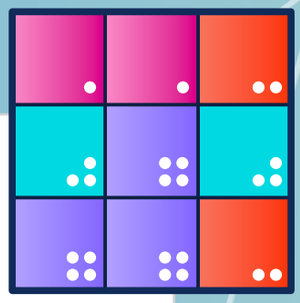
Sub-Unit Goals	Problem(s)	To respond to student thinking, consider:

Sub-Unit 1:
• Write and solve equations of the form $x + p = q$ and $px = q$. **(Lessons 1–5)**
	1, 3	**Support**
		• **Mini-Lesson:** ***Writing and Solving Equations***
		• **Teacher Moves:** Consider revisiting:
		  • Lesson 4 (*Swap and Solve*)
		  • Lesson 5 (*Solutions of a Different Kind*)

		Strengthen
		• **Repeated Challenges:**
		  • Lesson 3 (*Hanging Around*)
		  • Lesson 5 (*Solutions of a Different Kind*)

Sub-Unit 2:
• Use properties to write equivalent expressions with variables. **(Lessons 6–9)**
	2	**Support**
		• **Mini-Lesson:** ***Writing Equivalent Expressions Using the Area Model***
		• **Teacher Moves:** Consider revisiting:
		  • Lesson 8 (*Products and Sums*)
		  • Lesson 9 (*Equivalent Expressions*)

		Strengthen • **Challenge Creator:** • Lesson 8 (*Products and Sums*)
	5	**Support** **Strengthen** • **Teacher Moves:** Consider revisiting: • Lesson 6 (*Vari-Apples*)
		Stretch • **You're invited to explore more**: • Lesson 6 (*Vari-apples*)
Sub-Unit 3: • Evaluate numerical and variable expressions with whole number exponents. **(Lessons 10–11)**	4	**Support** • **Teacher Moves:** Consider revisiting: • Lesson 10 (*Powers*)
		Strengthen • **Challenge Creator:** • Lesson 10 (*Powers*)
	6	**Support** **Strengthen** • **Mini-Lesson:** ***Evaluating Expressions With Exponents*** • **Teacher Moves:** Consider revisiting: • Lesson 11 (*Exponent Expressions*)
		Stretch • **You're invited to explore more**: • Lesson 11 (*Exponent Expressions*)
Sub-Unit 4: • Solve problems involving percent change using representations, such as tape diagrams, double number lines, and equations. • Use ratios to solve problems in real-world situations involving percent increase, percent decrease, and percent error. • Interpret situations involving percent change and apply appropriate representations to solve problems.	8	**Support** • **Teacher Moves:** Consider revisiting: • Lesson 17 (*Bookcase Builder*)
		Strengthen • **Challenge Your Shelf:** • Lesson 17 (*Bookcase Builder*)
		Stretch • **You're invited to explore more**: • Lesson 17 (*Bookcase Builder*)
	9	**Support** • **Mini-Lesson:** Finding Sales Tax and Tips • **Teacher Moves:** Consider revisiting: • Lesson 15 (*Tax, Tips, and Fees*)
	10	**Support** • **Teacher Moves:** Consider revisiting: • Lesson 16 (*How Simple, How Interesting*)



UNIT 7

Positive and Negative Numbers

Students explore positive and negative numbers in several ways: on a number line, in expressions, and on the coordinate plane. They compare these numbers, as well as their absolute values, and use order of operations to evaluate numerical expressions and generate equivalent expressions.

Essential Questions

- What does it mean for a value to be less than zero?
- How is solving problems with positive and negative fractions and decimals the same as or different from solving problems with integers?
- How do you know what operation to do first when there are multiple grouping symbols?
- How do we represent points with negative numbers on a coordinate plane?

Sub-Unit Goals

Sub-Unit 1

- Order and compare positive numbers, negative numbers, and 0.
- Evaluate, order, and compare absolute values.
- Add, subtract, multiply, and divide integers.

Sub-Unit 2

- Perform all four operations with positive and negative rational numbers using a variety of strategies.
- Use order of operations to evaluate numerical expressions.

Sub-Unit 3

- Graph points with positive and negative coordinates.
- Draw polygons on the coordinate plane and determine their vertical and horizontal side lengths.

The Math of the Unit

Unit Outcomes Telling a story that narrows and deepens with the grade

Florida B.E.S.T. Standards for Mathematics

Extend knowledge of numbers to negative numbers and develop an understanding of absolute value. **(MA.6.NSO.1)**

Apply properties of operations to rewrite numbers in equivalent forms. **(MA.6.NSO.3)**

Extend understanding of operations with integers. **(MA.6.NSO.4)**

Apply previous understanding of the coordinate plane to solve problems. **(MA.6.GR.1)**

Add, subtract, multiply and divide rational numbers. **(MA.7.NSO.2)**

Rewrite algebraic expressions in equivalent forms. **(MA.7.AR.1)**

Benchmarks Addressed in This Unit: MA.6.NSO.1.1 MA.6.NSO.1.2 MA.6.NSO.1.3 MA.6.NSO.1.4 MA.6.NSO.3.5 MA.6.NSO.4.1 MA.6.NSO.4.2 MA.6.GR.1.1 MA.6.GR.1.2 MA.6.GR.1.3 MA.7.NSO.2.1 MA.7.NSO.2.2 MA.7.NSO.2.3 MA.7.AR.1.1 MA.7.AR.1.2

Mathematical Thinking and Reasoning Standards

- Actively participate in effortful learning both individually and collectively. **(MTR.1.1)**
- Demonstrate understanding by representing problems in multiple ways. **(MTR.2.1)**
- Complete tasks with mathematical fluency. **(MTR.3.1)**
- Apply mathematics to real-world contexts. **(MTR.7.1)**

Also addressed: **MTR.4.1, MTR.5.1, MTR.6.1**

Coherence Telling a story within and across grades

In this Unit

Students explore positive and negative numbers in several contexts: on a number line, in expressions, and in the coordinate plane.

Prior Learning

- Represent fractions on a number line. **(MA.3.FR.2.1)**
- Identify and plot points with non-negative coordinates. **(MA.5.GR.4)**
- **Units Grade 4–5, and Identify 5:** Operations and plot with points positive with rational non-negative numbers. **(MA.6.NSO.2)**
- **Unit 6:** Write, read, and evaluate expressions. **(MA.6.AR.1.1, MA.6.AR.1.3, MA.6.AR.1.4)**

Future Learning

- Rational and irrational numbers. **(MA.8.NSO.1.1, MA.8.NSO.1.2)**
- Transformations in the coordinate plane. **(MA.8.GR.2.3)**

See the Connections to Future Learning page for more information, including explanations and examples.

Rigor and Balance Integrating the aspects of rigor in ways that support one another

Here are examples of the aspects of rigor and balance in this unit. See the first page of each lesson's Teacher Edition for more information.

Conceptual Understanding

In **Lessons 1–3**, students develop conceptual understanding of negative numbers using number lines and the context of elevation.

Strategic Competence

In **Lessons 8 and 10,** students develop strategic competence as they use different strategies to add and subtract integers.

Procedural Fluency

In **Lesson 17**, students develop procedural fluency by plotting positive and negative coordinates on a grid.

Adaptive Reasoning

In **Lesson 18**, students develop adaptive reasoning as they move between the coordinate plane and tables to describe distances.

Application

In **Lesson 19,** students apply their understanding of plotting points in a four-quadrant coordinate plant to represent and interpret situations in context.

Productive Disposition

In **Lesson 11,** students have opportunities to develop a productive disposition as they relate multiplying negative integers to real-world scenarios.

Unit at a Glance

Assess and Respond

A Pre-Unit Check

Learn more about your students' understanding of foundational concepts and skills that will support them in the upcoming unit. This can be given in its entirety before the unit or spread throughout the unit.

MA.5.NSO.1.4 MA.5.GR.4.1 MA.5.GR.4.2
MTR.2.1 MTR.3.1 MTR.5.1

Sub-Unit 1

1 Can You Dig It?

Positive and Negative Numbers

Use **positive** and **negative numbers** to find sand dollars on a horizontal number line.

MA.6.NSO.1.1 MA.6.NSO.1.2 MA.6.NSO.4.1
MTR.1.1 MTR.5.1

2 Digging Deeper

Positions on the Number Line

Describe and use strategies for identifying and plotting negative rational numbers on the number line.

MA.6.NSO.1.1 MA.6.NSO.1.2
MTR.1.1 MTR.4.1

6 We've Got Game(s)

Comparing Numbers and Distances From Zero

Apply understanding of absolute value and ordering values to real-world scenarios.

MA.6.NSO.1.1 MA.6.NSO.1.2 MA.6.NSO.1.3 MA.6.NSO.1.4
MTR.2.1 MTR.3.1

7 Floats and Anchors

Representing Adding and Subtracting

Engage with a visual model for adding and subtracting positive and negative numbers.

MA.6.NSO.4.1
MTR.1.1 MTR.4.1 MTR.5.1

8 More Floats and Anchors

Adding and Subtracting Positive and Negative Numbers

Use floats and anchors to reason about adding and subtracting positive and negative numbers.

MA.6.NSO.4.1
MTR.1.1 MTR.3.1 MTR.5.1

Pacing: 22 days | **Short on time?** See pacing considerations below.

All instructional and assessment days are ~45 minutes each.

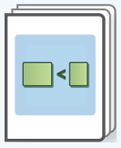
- 19 Lessons
- Pre-Unit Check
- Sub-Unit Quiz
- End-of-Unit Assessment

3 Sub-Zero

Positive and Negative Numbers in Context

Apply the understanding of positive and negative numbers in the contexts of elevations and temperatures around the world.

MA.6.NSO.1.1 MA.6.NSO.1.2
MTR.2.1 MTR.3.1 MTR.6.1



4 Order in the Class

Comparing Numbers

Practice comparing positive and negative numbers in a social way.

MA.6.NSO.1.1 MA.6.NSO.1.2 MA.6.NSO.3.5
MTR.1.1 MTR.4.1

5 Distance on the Number Line

Introduction to Absolute Value

Introduce **absolute value** and practice comparing values and absolute values through puzzles.

MA.6.NSO.1.1 MA.6.NSO.1.4
MTR.2.1 MTR.3.1

9 Up, Up, and Away!

Multiplying and Dividing Integers

Use hot-air balloons to make sense of multiplying and dividing integers.

MA.6.NSO.4.2
MTR.2.1

Assess and Respond

A Sub-Unit Quiz

Learn about your students' understanding of the concepts and skills so far in this unit.

MA.6.NSO.1.1 MA.6.NSO.1.2
MA.6.NSO.1.3 MA.6.NSO.1.4
MTR.4.1

Sub-Unit 2

10 Bumpers

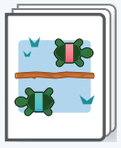
Adding and Subtracting on the Number Line

Add and subtract positive and negative decimals and fractions on a number line.

MA.7.NSO.2.2
MTR.3.1 MTR.4.1 MTR.5.1

Unit at a Glance

continued

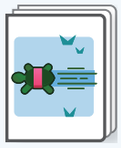


11 Back in Time

Position, Rate, and Time

Make sense of a different model for multiplying positive and negative numbers involving position, rate, and time.

MA.7.NSO.2.1 MA.7.NSO.2.2 MA.7.NSO.2.3
MTR.3.1 MTR.5.1 MTR.7.1



12 Speeding Turtles

Dividing Rational Numbers

Make sense of division of positive and negative numbers using position, rate, and time.

MA.7.NSO.2.2 MA.7.NSO.2.3
MTR.3.1 MTR.5.1 MTR.7.1

13 Order Matters

Evaluating Expressions with Exponents

Use order of operations to evaluate numerical expressions involving grouping symbols, exponents, and absolute value.

MA.7.NSO.2.1 MA.7.NSO.2.2
MTR.1.1 MTR.5.1

17 The A-maze-ing Coordinate Plane

Practice Plotting

Practice plotting points in the context of solving mazes, using different scales for the axes and patterns in coordinates that differ only by the sign.

MA.6.GR.1.1
MTR.1.1 MTR.4.1

18 Polygon Maker

Polygons in the Plane

Apply coordinates with positive and negative numbers to create polygons and determine distances in the coordinate plane.

MA.6.GR.1.1 MA.6.GR.1.2 MA.6.GR.1.3
MTR.1.1 MTR.4.1

19 Graph Telephone

The Coordinate Plane in Context

Plot and interpret coordinates in order to make sense of situations in context.

MA.6.GR.1.1 MA.6.GR.1.3
MTR.4.1 MTR.5.1 MTR.7.1

Sub-Unit 3

14 The Same but Different

Determining Whether Expressions Are Equivalent

Use algebra tiles, area models, and properties of operations to determine whether more complex expressions are equivalent.

MA.7.NSO.2.1 MA.7.NSO.2.2 MA.7.AR.1.2
MTR.2.1 MTR.4.1

15 Collect the Squares

Adding and Subtracting Expressions

Write expressions with fewer terms by adding, subtracting, and expanding terms.

MA.7.NSO.2.1 MA.7.NSO.2.2 MA.7.AR.1.1
MTR.3.1 MTR.4.1 MTR.5.1

16 Sand Dollar Search

Points in the Coordinate Plane

Develop an understanding of negative numbers in the coordinate plane.

MA.6.GR.1.1
MTR.1.1 MTR.4.1

Assess and Respond

A End-of-Unit Assessment

Learn about your students' understanding of the concepts and skills in this unit. There are two forms of the End-of-Unit Assessment: Forms A and B.

MA.6.AR.2.2 MA.6.AR.2.3 MA.6.NSO.1.1
MA.6.NSO.1.2 MA.6.NSO.1.3 MA.6.GR.1.1
MA.6.GR.1.3 MA.7.NSO.2.1 MA.7.NSO.2.2
MTR.3.1 MTR.4.1 MTR.5.1

Pacing: 22 days | **Short on time?** See pacing considerations below.

All instructional and assessment days are ~45 minutes each.

- 19 Lessons
- Pre-Unit Check
- Sub-Unit Quiz
- End-of-Unit Assessment

Pacing Considerations

Lesson 19: This lesson may be omitted because there is no new content introduced in this lesson. It gives students an opportunity to apply the concepts they learned in this unit to plot and interpret coordinates in order to make sense of situations in context.

Lesson Materials and Prep

Here are the materials and resources needed within each lesson, how to prepare them, and where to find them.

Lesson 1
- **Copy:** Show What You Know

Lesson 2
- **Copy:** Show What You Know

Lesson 3
- **Copy:** Show What You Know, Activity 1 Sheet *(one per pair)*

Lesson 4
- **Copy:** Show What You Know
- **Cut:** Activities 1 & 2 Cards *(one card per student)*

Lesson 5
- **Copy:** Show What You Know

Lesson 6
- **Copy:** Show What You Know
- **Cut:** Activity 2 Cards *(one set per group)*

Lesson 7
- **Copy:** Show What You Know

Lesson 8
- **Copy:** Show What You Know

Lesson 9
- **Copy:** Show What You Know
- **Materials:** 2-color counters *(optional)*

Lesson 10
- **Copy:** Show What You Know, Horizontal Number Lines Sheet (*one per student, optional*)

Lesson 11
- **Copy:** Show What You Know, Activity 3 You're Invited to Explore More Sheet (*one per student, as needed*)

Lesson 12
- **Copy:** Show What You Know

Lesson 13
- **Copy:** Show What You Know
- **Cut:** Activity 2 Cards *(one per group)*

Lesson 14
- **Copy:** Show What You Know

Lesson 15
- **Copy:** Show What You Know

Lesson 16
- **Copy:** Show What You Know
- **Cut:** Activity 3 Cards *(one set per pair)*
- **Materials:** rulers *(as needed)*

Lesson 17
- **Copy:** Show What You Know
- **Materials:** rulers

Lesson 18
- **Copy:** Show What You Know
- **Materials:** rulers

Lesson 19
- **Copy:** Show What You Know, Activity 2 Sheets *(one set per group)*

Resource Books

Assessment and Lesson Resources

- Pre-Unit Check
- Sub-Unit Quizzes
- End-of-Unit Assessment, Forms A & B
- Assessment Rubrics
- Show What You Knows
- Activity Sheets and Cards

Intervention and Extension Resources

- Mini-Lessons
- Prior Grade Mini-Lessons
- Extensions

Caregiver Support

These resources can be shared with students' caregivers. They provide background on the mathematics in this unit, as well as suggestions for supporting students at home.

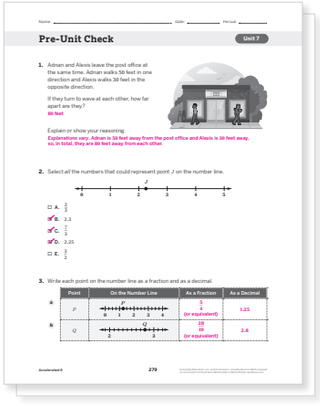
Assessments

Every unit includes a variety of formative and summative assessments, designed to highlight students' understanding of a variety of concepts and skills.

Pre-Unit Check

Assign the **Pre-Unit Check** to learn more about your students' understanding of foundational concepts and skills that will support them in the upcoming unit. This can be given in its entirety before the unit or spread throughout the unit.

FL B.E.S.T. Math Benchmarks assessed: MA.5.NSO.1.4, MA.5.GR.4.1, MA.5.GR.4.2


Pre-Unit Check

Unit 7

1. Adnan and Alexis leave the post office at the same time. Adnan walks 50 feet in one direction and Alexis walks 30 feet in the opposite direction.

If they turn to wave at each other, how far apart are they?

80 feet

Explain or show your reasoning.

Explanations vary. Adnan is 50 feet away from the post office and Alexis is 30 feet away, so, in total, they are 80 feet away from each other.

2. Select all the numbers that could represent point J on the number line.

A. $\frac{1}{3}$

B. 2.3

C. $\frac{7}{3}$

D. 2.25

E. $\frac{3}{1}$

3. Write each point on the number line as a fraction and as a decimal.

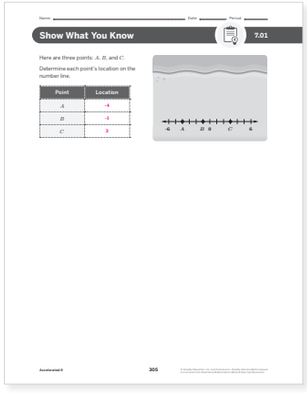
	Point	On the Number Line	As a Fraction	As a Decimal
a.	P	P	$\frac{5}{4}$ (or equivalent)	1.25
b.	Q	Q	$\frac{10}{30}$ (or equivalent)	2.8

Accelerated 6 279

Lesson | Show What You Know

Use the **Show What You Know** for each lesson to learn more about your students' progress toward the lesson goal(s). Here is one example from this unit.

FL B.E.S.T. Math Benchmarks assessed on Lesson 1 Show What You Know: MA.6.NSO.1.1


Show What You Know

7.01

Here are three points: A, B, and C.

Determine each point's location on the number line.

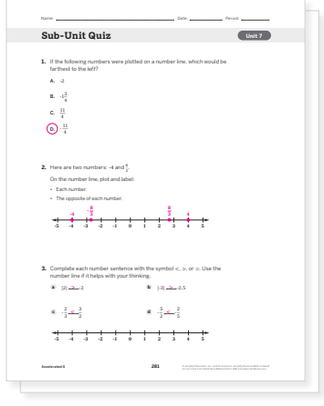
Point	Location
A	-6
B	-4
C	6

Accelerated 6 305

Sub-Unit Quiz

Assign the **Sub-Unit Quiz** to learn about your students' understanding of the concepts and skills so far in this unit.

FL B.E.S.T. Math Benchmarks assessed: on the Sub-Unit Quiz: MA.6.NSO.1.1, MA.6.NSO.1.2, MA.6.NSO.1.3, MA.6.NSO.1.4


Sub-Unit Quiz

Unit 7

1. If the following numbers were plotted on a number line, which would be farthest to the left?

A. -2

B. $-\frac{3}{4}$

C. $\frac{11}{4}$

D. $-\frac{11}{4}$

2. Here are two numbers: -4 and $\frac{3}{2}$.

On the number line, plot and label:

- Each number.
- The opposite of each number.

3. Complete each number sentence with the symbol <, >, or =. Use the number line if it helps with your thinking.

a. $|3|$ ___ 2

b. $|-3|$ ___ 0.5

c. $-\frac{3}{2}$ ___ $-\frac{3}{4}$

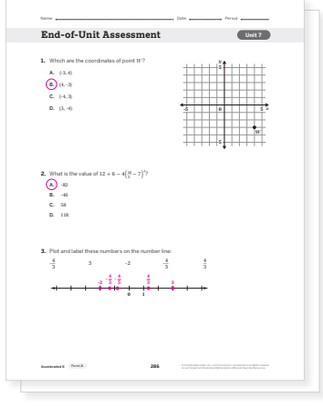
d. $\frac{3}{2}$ ___ $\frac{2}{3}$

Accelerated 6 281

End-of-Unit Assessment

Assign the **End-of-Unit Assessment** to learn about your students' understanding of the concepts and skills in this unit. There are two forms of the End-of-Unit Assessment: Forms A and B.

FL B.E.S.T. Math Benchmarks assessed: MA.6.AR.2.2, MA.6.AR.2.3, MA.6.NSO.1.1, MA.6.NSO.1.2, MA.6.NSO.1.3, MA.6.GR.1.1, MA.6.GR.1.3, MA.7.NSO.2.1, MA.7.NSO.2.2


End-of-Unit Assessment

Unit 7

1. Which are the coordinates of point W?

A. (-3, 4)

B. (4, -3)

C. (-4, 3)

D. (3, -4)

2. What is the value of $12 \div 6 - 4\left(\frac{2}{3} - 2\right)^2$?

A. 10

B. -46

C. 58

D. 118

3. Plot and label these numbers on the number line:

$-\frac{4}{3}$, 3, -2, $-\frac{4}{5}$, $\frac{4}{3}$

Accelerated 6 Form A 286

D Differentiation

Every unit includes additional resources to support, strengthen, and stretch student learning.

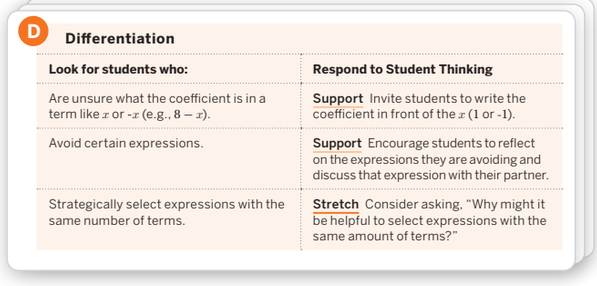
Differentiation Across the Unit

	Support	Strengthen	Stretch
	Provide targeted intervention for students by using these resources.	Reinforce students' understanding of the concepts assessed by using these resources.	Challenge students and extend their learning with these resources.
Sub-Unit 1 (Lessons 1–9) • Order and compare positive numbers, negative numbers, and 0. • Evaluate, order, and compare absolute values. • Add, subtract, multiply, and divide integers.	**Mini-Lesson** • *Ordering and Comparing Positive and Negative Numbers* • *Adding and Subtracting Positive and Negative Numbers* • *Solving Real-World Problems Involving Positive and Negative Numbers*	**Repeated Challenges** • Lesson 1 *(Can You Dig It?)* • Lesson 2 *(Digging Deeper)* • Lesson 7 (*Floats and Anchors*) **Practice** • Lessons 1–9	**You're invited to explore more.** • Lesson 3 (*Sub-Zero*) • Lesson 5 *(Distance on the Number Line)* • Lesson 6 *(We've Got Game(s))* • Lesson 8 (*More Floats and Anchors*) **DOK3 Practice** • Lessons 2, 4–5, 7–9
Sub-Unit 2 (Lessons 10–15) • Perform all four operations with positive and negative rational numbers using a variety of strategies. • Use order of operations to evaluate numerical expressions.	**Mini-Lesson** • *Dividing Integers*	**Repeated Challenges** • Lesson 10 (*Bumpers*) • Lesson 11 (*Back in Time*) • Lesson 12 (*Speeding Turtles*) **Practice** • Lessons 10–15	**You're invited to explore more.** • Lesson 11 (*Back in Time*) • Lesson 15 (*Collect the Squares*) **DOK3 Practice** • Lessons 13–15
Sub-Unit 3 (Lessons 16–19) • Graph points with positive and negative coordinates. • Draw polygons on the coordinate plane and determine their vertical and horizontal side lengths.	**Mini-Lesson** • *Plotting Points on the Coordinate Plane*	**Challenge Creator** • Lesson 18 *(Polygon Maker)* **Practice** • Lessons 16–19	**You're invited to explore more.** • Lesson 16 *(Sand Dollar Search)* • Lesson 17 *(The A-maze-ing Coordinate Plane)* • Lesson 19 *(Graph Telephone)* **DOK3 Practice** • Lessons 15–16, 18

Differentiation Within the Lesson

Each Teacher Edition includes point-of-use differentiation suggestions to modify or adjust instruction during the lesson to accommodate the needs of all learners.

Here is one example from **Lesson 15**, where students write equivalent expressions using the fewest number of terms.



D Differentiation

Look for students who:	Respond to Student Thinking
Are unsure what the coefficient is in a term like x or $-x$ (e.g., $8 - x$).	**Support** Invite students to write the coefficient in front of the x (1 or -1).
Avoid certain expressions.	**Support** Encourage students to reflect on the expressions they are avoiding and discuss that expression with their partner.
Strategically select expressions with the same number of terms.	**Stretch** Consider asking, "Why might it be helpful to select expressions with the same amount of terms?"

Accessibility

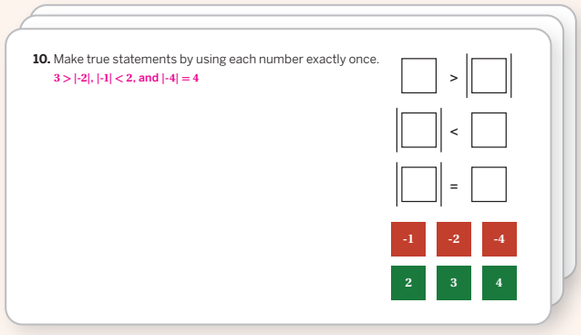
Every lesson is designed using the principles of Universal Design for Learning. Every lesson also includes support suggestions that build on students' strengths.

Universal Design for Learning

Here is one example from a lesson designed to support Action & Expression.

Spotlight on Strategy Development. . .

Lesson 5: In this lesson, students have opportunities for strategizing, planning, and monitoring progress when they are challenged to create true statements using the given numbers.



Just-In-Time Accommodations and Supports

Each Teacher Edition includes suggested supports based on the areas of cognitive function from the Education Development Center (EDC).

Spotlight on Memory and Attention. . .

Here are two suggestions in this unit for supporting students with working memory challenges.

Lesson 1, Activity 3

A **Accessibility: Memory and Attention** Invite students with working memory challenges to review the definition of *negative numbers* before responding.

Lesson 7, Activity 3

A **Accessibility: Memory and Attention** Invite students to draw the initial submarine position and the actions in each problem.

Language Development

Every lesson includes opportunities for all students to develop fluency with mathematical language.

Vocabulary

New Vocabulary	Lesson(s)
negative number **positive number**	1
integers **opposite**	2
sign	4
absolute value	5
quadrant	16

Review Vocabulary
constant
coordinate plane
exponent
inequality
ordered pair
origin
polygon
term
variable
x-axis
x-coordinate
y-axis
y-coordinate

Additional Vocabulary
additive inverse
like terms
rational number
reflection

Math Language Routines and Language Goals

Math Language Routine	Lesson(s)
MLR1: Stronger and Clearer Each Time	9, 12–13, 17
MLR2: Collect and Display	1–5, 12, 16–19
MLR3: Critique, Correct, Clarify	1, 11, 14–16
MLR7: Compare and Connect	1, 6

Unit Language Goals

Students engage in the following language functions:

- Interpret positive numbers, negative numbers, 0, and absolute values in context.
- Describe how a visual model represents adding, subtracting, multiplying, or dividing integers.
- Describe strategies for determining the sum, difference, product, or quotient of two rational numbers.
- Describe and use strategies for writing equivalent expressions that involve adding, subtracting, and expanding terms.
- Describe the relationship between ordered pairs that differ only by the signs of their coordinates, and interpret the meaning of an ordered pair in context.

English Language Learners Support

Every Teacher Edition includes suggestions for supporting the needs of students learning English.

Here is one example from **Lesson 3**, where students apply what they have learned about positive and negative numbers in the contexts of elevations and temperatures around the world.

ELL **English Language Learners** Invite students to read the two claims aloud and paraphrase what they mean. Clarify any words or symbols they don't know. **(Reading)**

Connections to Future Learning

Here is how the content in this unit connects to where your students are headed in their math journeys.

Rational and Irrational Numbers

In this unit, students develop strategies for performing operations with rational numbers. **In a future course**, students will consider what makes a number rational as they compare them to irrational numbers.

FL B.E.S.T. Math Benchmarks:
MA.8.NSO.1.1, MA.8.NSO.1.2

Example:

Rational numbers are numbers that can be written as a fraction of two integers. Some examples of rational numbers are 5, 2.4, 2, $\frac{1}{2}$, $\sqrt{16}$, and $\frac{17}{3}$.

We call numbers that cannot be written this way irrational numbers. Some examples of irrational numbers are $\sqrt{3}$, $3\sqrt{2}$, 2π, and $1.73204082\ldots$ (neither repeats nor ends).

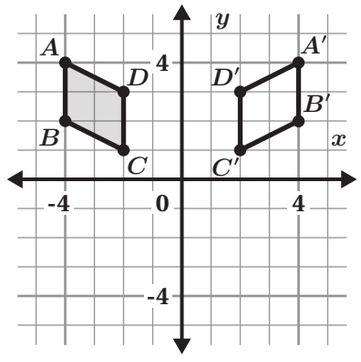
Transformations in the Coordinate Plane

In this unit, students graph points using positive and negative values in the coordinate plane. **In a future course**, students will apply this understanding to visualize, describe, and perform translations, rotations, and reflections in the plane.

FL B.E.S.T. Math Benchmark:
MA.8.GR.2.3

Example:

For example, to take the pre-image $ABCD$ to the image $A'B'C'D'$, reflect the polygon over the y-axis.

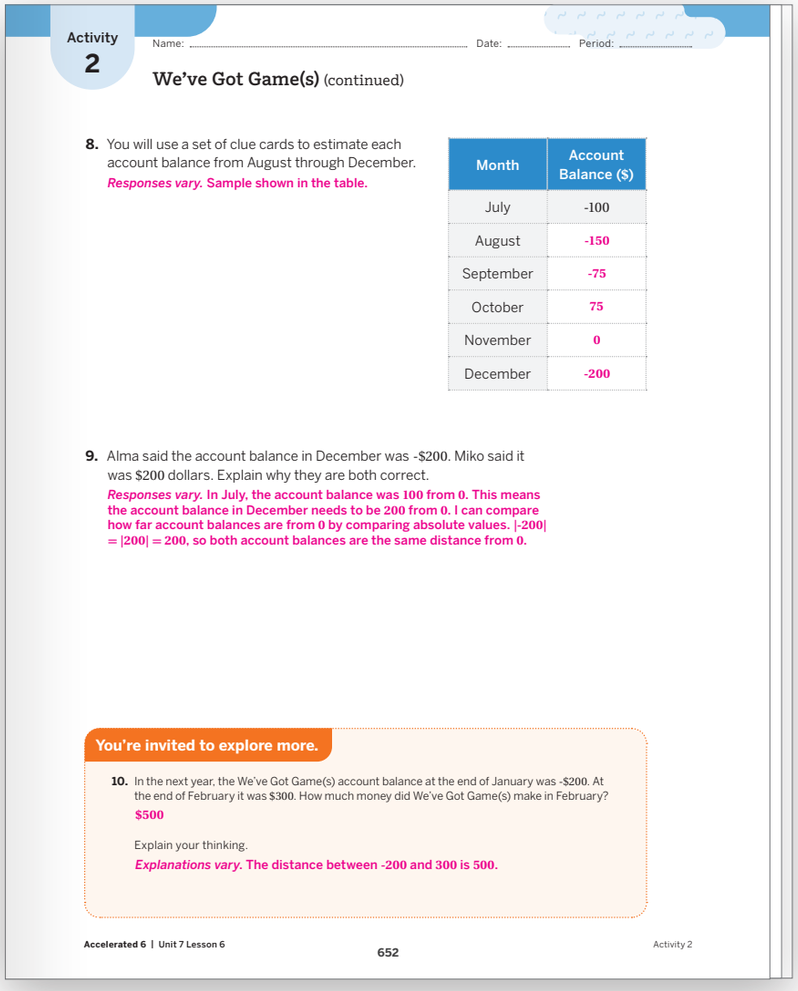


Professional Learning

Here is an opportunity to deepen your understanding of the math in this unit and how to teach it.

Spotlight on Application

Lesson 6, Activity 2



Activity 2

Name: Date: Period:

We've Got Game(s) (continued)

8. You will use a set of clue cards to estimate each account balance from August through December.

Responses vary. Sample shown in the table.

Month	Account Balance ($)
July	-100
August	-150
September	-75
October	75
November	0
December	-200

9. Alma said the account balance in December was -$200. Miko said it was $200 dollars. Explain why they are both correct.

Responses vary. In July, the account balance was 100 from 0. This means the account balance in December needs to be 200 from 0. I can compare how far account balances are from 0 by comparing absolute values. |-200| = |200| = 200, so both account balances are the same distance from 0.

You're invited to explore more.

10. In the next year, the We've Got Game(s) account balance at the end of January was -$200. At the end of February it was $300. How much money did We've Got Game(s) make in February?

$500

Explain your thinking.

Explanations vary. The distance between -200 and 300 is 500.

Accelerated 6 | Unit 7 Lesson 6 652 Activity 2

Context

In this activity, students use and interpret absolute values and negative numbers that represent and compare changes in amounts of money.

Try This

Put on your student hat and complete Problems 6 and 7.

Points to Ponder

- This is the first time students use a set of clue cards to estimate the monthly account balance of a company as they reason using positive and negative numbers. What do students need to understand about clues prior to answering questions about the account balance for each month?
- Some students may think that there is only one correct amount for each balance. How might you support your students in progressing to more efficient strategies?

Notes:

Assess and Respond

Pre-Unit Check

Independent | 45 min

Facilitation: Assign the Pre-Unit Check to learn more about your students' understanding of foundational concepts and skills that will support them in the upcoming unit. This can be given in its entirety before the unit or spread throughout the unit. If time allows, consider inviting students to complete one or more of the tasks in the Unit Synthesis and Reflection.

Item Analysis

Problem(s)	Concept or skill	Preparation for	DOK	FL B.E.S.T. Math Benchmarks
1	Solving real-world problems involving opposite directions and explaining the solution	Lesson 1	2	MA.5.NSO.1.4
2	Estimating the locations of fractions and decimals on a number line	Lesson 2	1	MA.5.NSO.1.4 MTR.2.1
3	Expressing points on the number line as fractions and decimals	Lesson 2	1	MA.5.NSO.1.4 MTR.2.1
4	Using the symbols <, >, or = to compare numbers written as decimals and fractions	Lesson 3	2	MA.5.NSO.1.4 MTR.3.1
5	Using the structure of the coordinate plane to write the coordinates of plotted points	Lesson 15	1	MA.5.GR.4.1 MTR.2.1
6	Using the structure of the coordinate plane to plot points that represent real-world situations	Lesson 18	1	MA.5.GR.4.2 MTR.5.1

Assessment and Lesson Resources

- Student Assessments
- Show What You Knows and Answer Keys

Intervention and Extension Resources

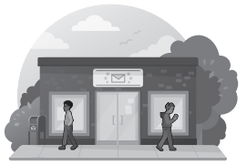
- Mini-Lessons
- Extensions

Name: Date: Period:

Pre-Unit Check

Unit 7

1. Adnan and Alexis leave the post office at the same time. Adnan walks 50 feet in one direction and Alexis walks 30 feet in the opposite direction.



If they turn to wave at each other, how far apart are they?

80 feet

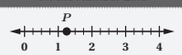
Explain or show your reasoning.

***Explanations vary*. Adnan is 50 feet away from the post office and Alexis is 30 feet away, so, in total, they are 80 feet away from each other.**

2. Select *all* the numbers that could represent point J on the number line.

- ☐ A. $\frac{2}{3}$
- ☑ B. 2.3
- ☑ C. $\frac{7}{3}$
- ☑ D. 2.25
- ☐ E. $\frac{3}{2}$

3. Write each point on the number line as a fraction and as a decimal.

	Point	On the Number Line	As a Fraction	As a Decimal
a	P		$\frac{5}{4}$ (or equivalent)	1.25
b	Q		$\frac{28}{10}$ (or equivalent)	2.8

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Name: Date: Period:

Pre-Unit Check (continued)

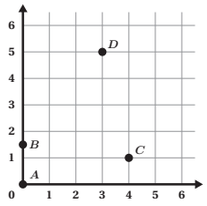
Unit 7

4. Complete each number sentence with the symbol <, >, or =.

a. $\frac{4}{3}$ > $\frac{3}{4}$ b. 1.5 = $\frac{3}{2}$ c. $\frac{4}{5}$ > $\frac{4}{7}$ d. 1.41 < 1.5

5. Write the coordinates of each point in the table.

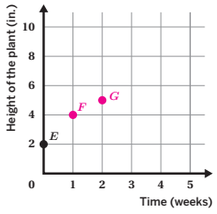
Point	Coordinates
A	$(0, 0)$
B	$(0, 1.5)$
C	$(4, 1)$
D	$(3, 5)$



6. Sneha tracked the height of a plant as it grew from week to week.

a. After one week, the plant was 4 inches tall. Plot and label this point F.

b. After two weeks, the plant was 5 inches tall. Plot and label this point G.



c. What does point E tell us about the plant?

***Responses vary*. The plant was 2 inches tall when Sneha started recording its height.**

D Differentiation (Pre-Unit Check)

Note: To **strengthen** and **stretch** students' learning, refer to the differentiation resources suggested throughout this unit and in the Unit Overview.

Prior Knowledge	Problem(s)	Respond to Student Thinking
Preparation for Sub-Unit 1: • Order and compare positive numbers, negative numbers, and 0. • Evaluate, order, and compare absolute values. • Add, subtract, multiply, and divide integers. **(Lessons 1–9)**	1	**Support** • **Teacher Move:** Before Lesson 1, consider inviting two students to act as Adnan and Liam, and discussing their distance apart as a class using a number line.
	2, 3	**Support** • **Teacher Move:** Before Lesson 2, consider inviting students to work with a partner to place each of the five choices on the number line. Then facilitate a brief whole-class discussion to surface strategies.
	4	**Support** • **Teacher Move:** During Lesson 3's Warm-Up, consider sharing examples of incorrect responses and discuss the errors.
	5	**Support** • **Mini-Lesson:** *Using the Coordinate Grid to Locate Points* • **Teacher Move:** Before Lesson 15, consider creating an anchor chart of the coordinate plane with labeled sample coordinates.
	6	**Support** • **Mini-Lesson:** *Plotting Points on the Coordinate Grid* • **Teacher Move:** Consider inviting students to share strategies they used with a partner before sharing with the class.

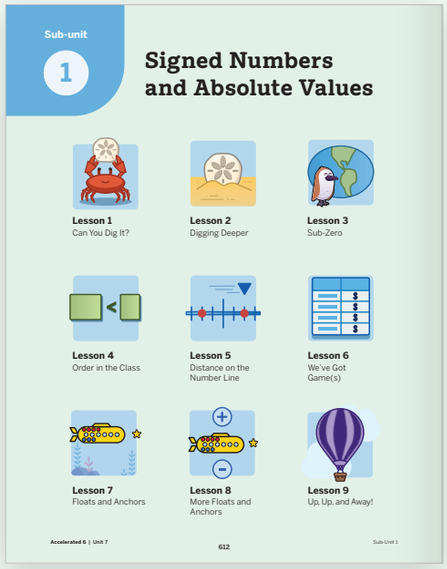
Sub-Unit 1

Signed Numbers and Absolute Values

Sub-Unit 1 Goals

- **Goal:** Order and compare positive numbers, negative numbers, and 0.
- **Goal:** Evaluate, order, and compare absolute values.
- **Goal:** Add, subtract, multiply, and divide integers.
- **Language Goal:** Interpret positive numbers, negative numbers, 0, and absolute values in context.
- **Language Goal:** Describe how a visual model represents adding, subtracting, multiplying, or dividing integers.

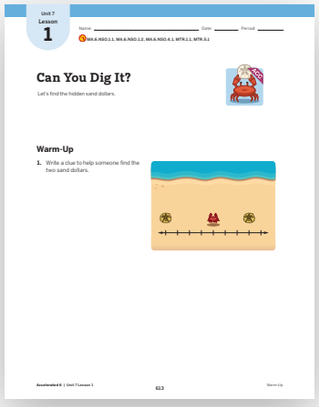
In this sub-unit, students are introduced to negative rational numbers, written with a negative **sign** (-) as an indication of direction, and often in the contexts of elevations and temperatures. Based on locations on the number line, students compare and order values, making sense of both **positive** and **negative numbers** with respect to the location and meaning of zero. Students further their understanding of how two numbers with different values can be the same distance from zero, using the concept of **absolute value** and its notation. Students apply this information to make sense of operations with **integers**.



Math that Matters Most

Concept or Skill	For example . . .
Order and compare positive and negative numbers.	Complete each number sentence. -2 is less than $1\frac{1}{2}$. -1.5 is less than -1. -1.5 is the opposite of 1.5 because . . . it is the same distance from zero and on the opposite side of the number line.
Interpret negative numbers in context.	Write a temperature that is colder than Warsaw's and warmer than Saskatoon's. Any temperature less than -2°F and greater than -10°F, such as -3°F or -8°F

Concept or Skill	For example . . .
Order and compare absolute values.	Determine whether each statement is true or false. Statement / True / False $\|0\| = 0$ / ✓ / $\|6\| = -6$ / / ✓ $\|6\| > \|-6\|$ / / ✓
Subtracting a value is the same as adding its opposite.	What is the value of -1 – (-4)? Show or explain your thinking. 3. The problem is like removing 4 anchors, which makes the submarine go up 4.


Unit 7
Lesson
1

Can You Dig It?

Let's find the hidden sand dollars.

Warm-Up

1. Write a clue to help someone find the two sand dollars.

613

Can You Dig It?

Positive and Negative Numbers

Let's find the hidden sand dollars.

Learning Outcomes

Today's Goals

1. **Goal:** Use the symbol (-) and the word *negative* to describe numbers that are less than 0.
2. **Language Goal:** Explain what *positive* and *negative numbers* are. **(Reading, Writing, Speaking, and Listening)**

Students explore numbers that are less than 0 on the number line. They learn what **negative** and **positive numbers** are and how they are represented on a number line in the context of a crab searching for sand dollars. **(MTR.5.1)**

Coherence

Prior Learning

In Units 4 and 6, students developed their understanding of and fluency operating with positive rational numbers. In previous courses, they used both horizontal and vertical number lines to represent and compare the values of positive numbers.

Future Learning

In Lesson 2, students will extend their understanding of positive rational numbers to describe, identify, and plot negative rational numbers on the number line.

Connections

- Students begin their learning about signed numbers as they describe the behavior of an object along a number line. (mathematical, real world)

FL B.E.S.T. Math Benchmarks

Addressing

MA.6.NSO.1.1
Extend previous understanding of numbers to define rational numbers. Plot, order and compare rational numbers.

MA.6.NSO.1.2
Given a mathematical or real-world context, represent quantities that have opposite direction using rational numbers. Compare them on a number line and explain the meaning of zero within its context.

Also Addressing: **MA.6.NSO.4.1**

Mathematical Thinking and Reasoning Standards: MTR.1.1, MTR.5.1

ELA Expectations: ELA.K12.EE.3.1, ELA.K12.EE.5.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.5.NSO.1.4	**MA.7.NSO.2.1**
	MA.7.NSO.2.2
	MA.8.NSO.1.1
	MA.8.NSO.1.2

Vocabulary

New Vocabulary

negative number	positive number

Lesson at a Glance

~ 45 min

MA.6.NSO.1.1, MA.6.NSO.1.2, MA.6.NSO.4.1

Warm-Up

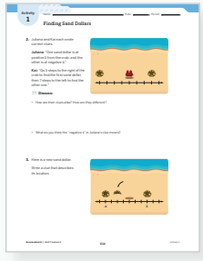
Independent | 5 min

Students are introduced to the context of a crab searching for sand dollars and make sense of precise descriptions of locations on the number line.

Activity 1

Pairs | 10 min

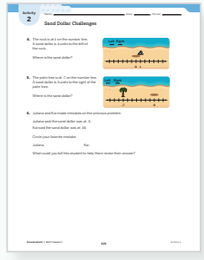
Students use **positive** and **negative numbers** to describe the locations of sand dollars on the number line. MLR7: Compare and Connect (MTR.5.1)



Activity 2

Pairs | 10 min

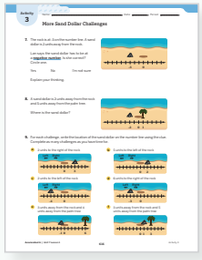
Students use negative numbers to represent the location of the sand dollars on the number line.
MLR3: Critique, Correct, Clarify



Activity 3

Pairs | 10 min

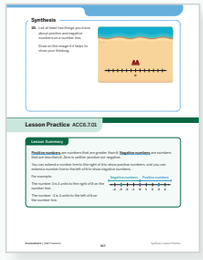
Students practice identifying hidden sand dollars on the number line. **Routine:** Think-Pair-Share



Synthesis

Whole Class | 5 min

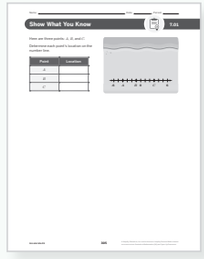
Students synthesize their understanding of how to represent positive and negative numbers on a number line. MLR2: Collect and Display



Show What You Know

Independent | 5 min

Students demonstrate their understanding by representing points on the number line with positive and negative numbers.



Prep Checklist

This lesson includes:

- Student Edition
- Show What You Know

Independent | 5 min

Warm-Up

Purpose: Students are introduced to the context of a crab searching for sand dollars and make sense of precise descriptions of locations on the number line.

1 Launch

To generate excitement, consider asking, "What is your favorite thing to do at a beach?"

Direct students' attention to the sand dollars.

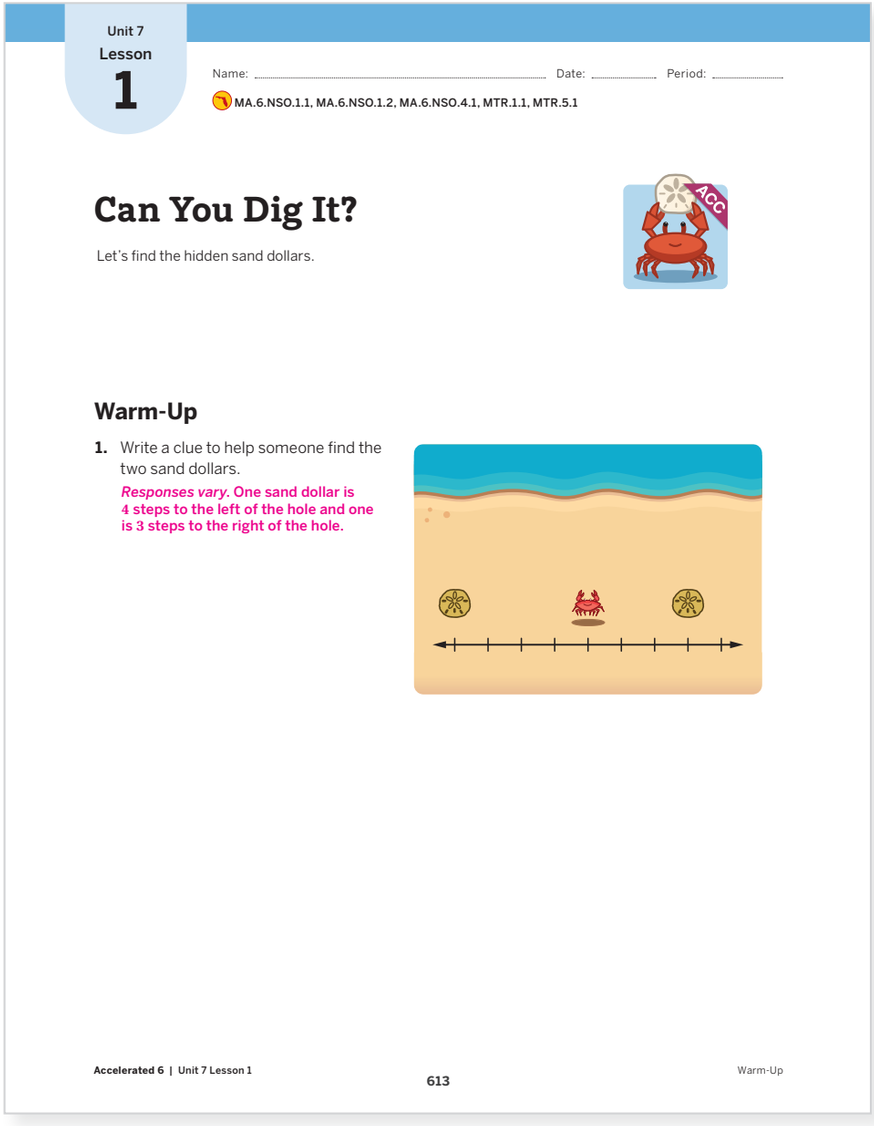
Select and sequence different clues.

2 Connect

Share examples of student clues showing unique or creative ways of describing the location of the sand dollars.

Consider asking, "What is helpful about each clue?"

Discuss the importance of using precise language when describing a location, if it doesn't come up naturally.



Unit 7
Lesson
1

Name: _____ Date: _____ Period: _____

MA.6.NSO.1.1, MA.6.NSO.1.2, MA.6.NSO.4.1, MTR.1.1, MTR.5.1

Can You Dig It?

Let's find the hidden sand dollars.

Warm-Up

1. Write a clue to help someone find the two sand dollars.

   Responses vary. One sand dollar is 4 steps to the left of the hole and one is 3 steps to the right of the hole.

Pairs | 10 min

Activity 1 Finding Sand Dollars

Purpose: Students use **positive** and **negative numbers** to describe the locations of sand dollars on the number line. **(MTR.5.1)**

1 Launch

Invite students to read both clues aloud,

MLR **MLR7: Compare and Connect** Invite students to analyze each response, then find ways in which the responses are different (compare) and alike (connect). Invite students to describe where they see the 3 and -4 in each response. Consider asking:

- "How is the clue '3 steps to the right' related to positive 3?"
- "How is the clue '7 steps to the left' related to negative 4?" **(MTR.1.1, MTR.5.1)**

Encourage students to sketch to show their thinking.

2 Monitor

D **Differentiation**

Look for students who:	Respond to Student Thinking
Count the starting mark for the hole as a move.	**Support** Use the Warm-Up to connect how the crab moves with each step.
Only state values but do not give direction or signs.	**Support** Use the Warm-Up to remind students that describing locations requires both the distance and the direction.
• Label the hole as 0 and the marks on the number line with positive and negative values. • Label the left side of the hole with negative numbers.	**Strengthen** Invite students to share their sketches during the Connect.

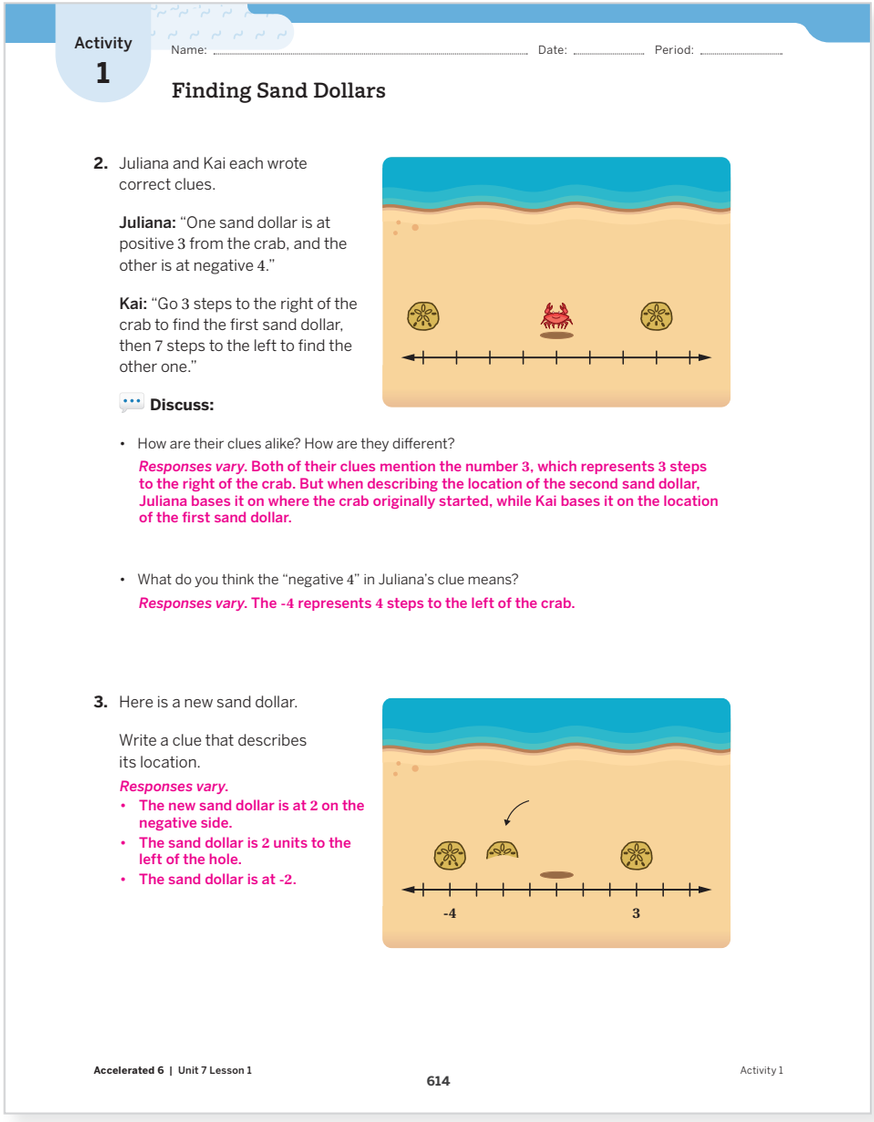
Select and sequence the clues to share during the Connect. Invite students to write multiple clues.

3 Connect

To surface the Key Takeaway, consider displaying a variety of student clues to connect "-2" and "2 to the left of the hole." **(MTR.5.1)**

Create class definitions of the terms **positive numbers** and **negative numbers**. To help generate student language, consider asking:

- "When do you need to use positive and negative numbers?"
- "What number is in the middle of the number line?"
- "How do you represent a negative number? A positive number?" **(MTR.1.1)**



Activity 1

Name: Date: Period:

Finding Sand Dollars

2. Juliana and Kai each wrote correct clues.

Juliana: "One sand dollar is at positive 3 from the crab, and the other is at negative 4."

Kai: "Go 3 steps to the right of the crab to find the first sand dollar, then 7 steps to the left to find the other one."

Discuss:

- How are their clues alike? How are they different?

Responses vary. Both of their clues mention the number 3, which represents 3 steps to the right of the crab. But when describing the location of the second sand dollar, Juliana bases it on where the crab originally started, while Kai bases it on the location of the first sand dollar.

- What do you think the "negative 4" in Juliana's clue means?

Responses vary. The -4 represents 4 steps to the left of the crab.

3. Here is a new sand dollar.

Write a clue that describes its location.

Responses vary.

- The new sand dollar is at 2 on the negative side.
- The sand dollar is 2 units to the left of the hole.
- The sand dollar is at -2.

Accelerated 6 | Unit 7 Lesson 1 614 Activity 1

Key Takeaway: The word *negative* can be used to describe locations to the left of 0 on the number line. When reading numbers written with the - symbol, we say "negative."

Positive numbers are numbers greater than zero. On a horizontal number line, positive numbers are to the right of 0.

Negative numbers are numbers less than zero. On a horizontal number line, negative numbers are to the left of 0.

Pairs | 10 min

Activity 2 Sand Dollar Challenges

Purpose: Students use negative numbers to represent the location of the sand dollars on the number line.

1 Launch

Invite students to label the numbers on the right side of the rock on the number line.

To surface connections, consider asking students how they would define these numbers.

2 Monitor

D Differentiation

Look for students who:	Respond to Student Thinking
Count tick marks instead of spaces.	**Support** Consider asking, "What number is 1 unit to the right of -7? Where do you label that number?"
Think the sand dollar is at positive 4 because the direction in Problem 5 says "to the right."	**Support** Invite them to label all the numbers on the left side of the hole.

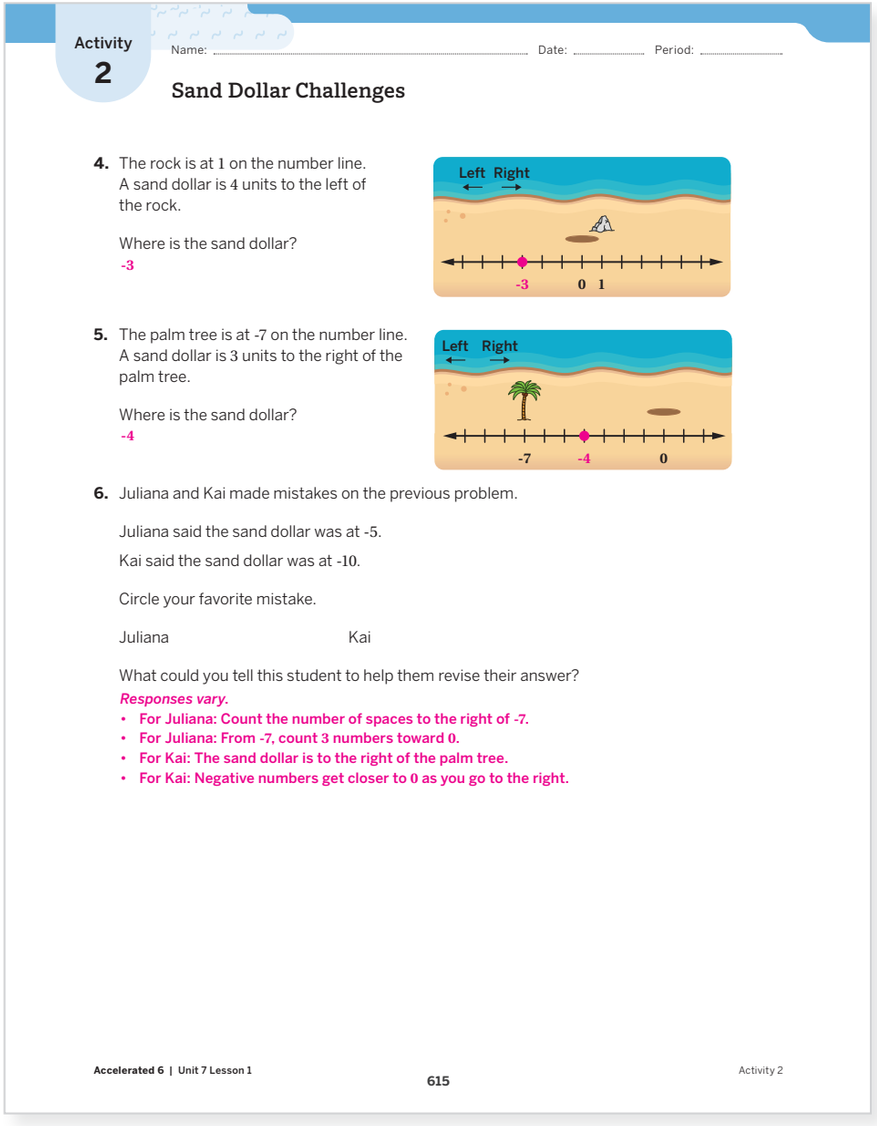
3 Connect

Invite students to share their responses.

MLR MLR3: Critique, Correct, Clarify Use this routine to help students describe strengths, errors, and ambiguities in Juliana's or Kai's work. Then invite students to:

a. Discuss what is *correct* about the response.
b. Discuss what is *incorrect* or *unclear*.
c. Write a question to help Juliana or Kai see how their work can be improved, or create a revised response. **(MTR.1.1)**

Invite students to write new clues so that each fictional student's response is correct.



Activity 2

Name: Date: Period:

Sand Dollar Challenges

4. The rock is at 1 on the number line. A sand dollar is 4 units to the left of the rock.

Where is the sand dollar?

-3

5. The palm tree is at -7 on the number line. A sand dollar is 3 units to the right of the palm tree.

Where is the sand dollar?

-4

6. Juliana and Kai made mistakes on the previous problem.

Juliana said the sand dollar was at -5.

Kai said the sand dollar was at -10.

Circle your favorite mistake.

Juliana Kai

What could you tell this student to help them revise their answer?

Responses vary.

- For Juliana: Count the number of spaces to the right of -7.
- For Juliana: From -7, count 3 numbers toward 0.
- For Kai: The sand dollar is to the right of the palm tree.
- For Kai: Negative numbers get closer to 0 as you go to the right.

Accelerated 6 | Unit 7 Lesson 1 615 Activity 2

Pairs | 10 min

Activity 3 More Sand Dollar Challenges

Purpose: Students practice identifying hidden sand dollars on the number line.

Short on time? Consider decreasing the number of challenges students are to complete in Problem 9.

1 Launch

Invite students to read the text aloud.

Use the Think-Pair-Share routine to help students shape their ideas about Lan's reasoning. See the Routine Facilitation Guide for more information. **(MTR.1.1)**

A **Accessibility: Memory and Attention** Invite students with working memory challenges to review the definition of negative numbers before responding.

2 Monitor

D **Differentiation: (Support)** Students might think that the word *away* is too ambiguous to determine the location of the sand dollar. Invite them to determine all possible locations of the sand dollar before thinking about Lan's claim.

To support students getting started, encourage them to sketch their ideas on the number line.

Listen for student strategies, partner discussions, and offer help or encouragement as needed.

A **Accessibility: Memory and Attention** For students who would benefit from having more time to think deeply on fewer problems, invite students to complete fewer challenges.

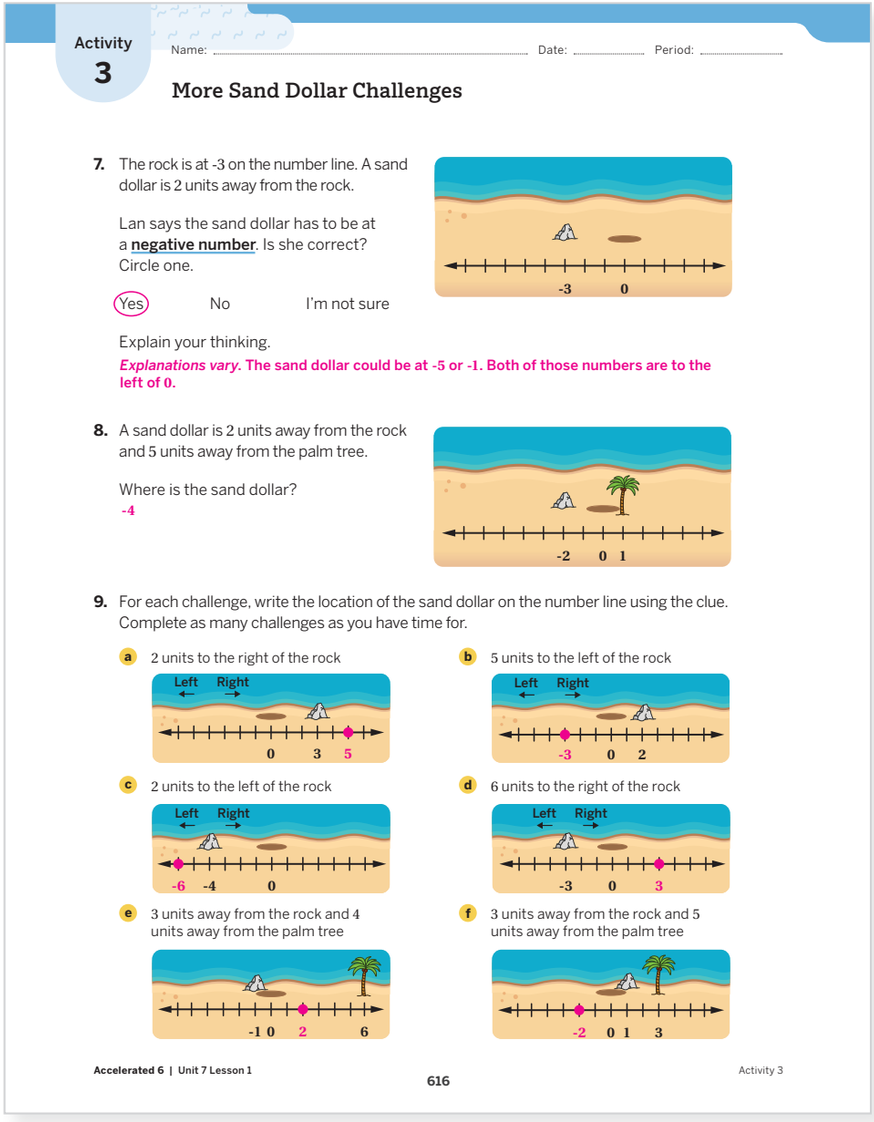
Note: Look for how many challenges each student has completed correctly.

3 Connect

Invite students to share strategies that they found helpful or mistakes they made while determining the location of each sand dollar.

Consider asking:

- "How did your strategy change when there were two clues rather than one clue?"
- "How did you use the number line to help you determine the location of each sand dollar?"
- "How did you decide if the location of the sand dollar was positive or negative?" **(MTR.5.1)**



Activity 3

Name: Date: Period:

More Sand Dollar Challenges

7. The rock is at -3 on the number line. A sand dollar is 2 units away from the rock.

Lan says the sand dollar has to be at a **negative number**. Is she correct? Circle one.

(Yes) No I'm not sure

Explain your thinking.

Explanations vary. The sand dollar could be at -5 or -1. Both of those numbers are to the left of 0.

-3 0

8. A sand dollar is 2 units away from the rock and 5 units away from the palm tree.

Where is the sand dollar?

-4

-2 0 1

9. For each challenge, write the location of the sand dollar on the number line using the clue. Complete as many challenges as you have time for.

a. 2 units to the right of the rock

Left Right

0 3 5

b. 5 units to the left of the rock

Left Right

-3 0 2

c. 2 units to the left of the rock

Left Right

-6 -4 0

d. 6 units to the right of the rock

Left Right

-3 0 3

e. 3 units away from the rock and 4 units away from the palm tree

-1 0 2 6

f. 3 units away from the rock and 5 units away from the palm tree

-2 0 1 3

Accelerated 6 | Unit 7 Lesson 1 616 Activity 3

Whole Class | 5 min

Synthesis

Purpose: Students synthesize their understanding of how to represent positive and negative numbers on a number line.

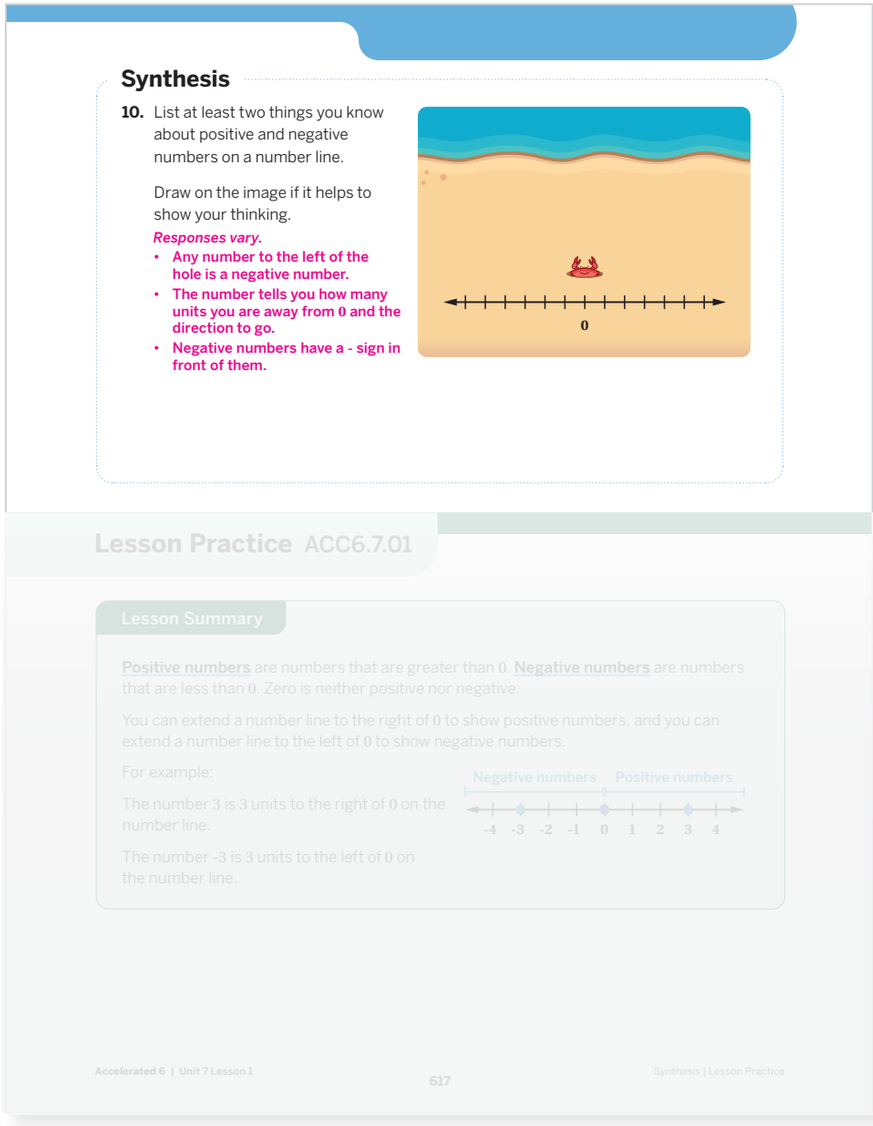
Invite students to respond independently, and then share their thinking with a partner.

Collect and share a variety of ideas, such as:

- Any number to the left of 0 is a negative number and any number to the right of 0 is a positive number.
- Negative and positive numbers tell you how many units you are away from 0 and the direction to go.
- Negative numbers have a - sign in front of them.

MLR **MLR2: Collect and Display** Use this routine to formalize the definition of *positive numbers* and *negative numbers* using students' language. Display any new language, examples, or diagrams publicly for students to reference later.

Note: If time allows, invite students to revise their previous thinking based on the discussion.



Synthesis

10. List at least two things you know about positive and negative numbers on a number line.

Draw on the image if it helps to show your thinking.

Responses vary.

- Any number to the left of the hole is a negative number.
- The number tells you how many units you are away from 0 and the direction to go.
- Negative numbers have a - sign in front of them.

Lesson Practice ACC6.7.01

Lesson Summary

Positive numbers are numbers that are greater than 0. Negative numbers are numbers that are less than 0. Zero is neither positive nor negative.

You can extend a number line to the right of 0 to show positive numbers, and you can extend a number line to the left of 0 to show negative numbers.

For example:

The number 3 is 3 units to the right of 0 on the number line.

The number -3 is 3 units to the left of 0 on the number line.

Accelerated 6 | Unit 7 Lesson 1 617 Synthesis | Lesson Practice

Lesson Takeaway: Positive and negative numbers are used together to describe quantities having opposite directions from zero on a horizontal number line. Negative numbers are written with a - sign in front.

Independent | 5 min

Show What You Know

Purpose: Students demonstrate their understanding by representing the points on the number line with positive and negative numbers.

Today's Goals

1. **Goal:** Use the symbol (-) and the word *negative* to describe numbers that are less than 0.
2. **Language Goal:** Explain what *positive* and *negative numbers* are. **(Reading, Writing, Speaking, and Listening)**

Show What You Know (Assessment Resources)

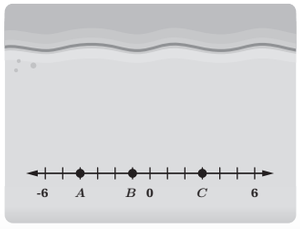
Name: _____ Date: _____ Period: _____

Show What You Know 7.01

Here are three points: A, B, and C.

Determine each point's location on the number line.

Point	Location
A	-4
B	-1
C	3



D Differentiation Use after Lesson 1

Support

Provide targeted intervention for students by using these resources.

If students need support representing points on a number line with positive and negative numbers:

» **Respond to Student Thinking:**

- Review the **Lesson Summary**.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite them to explore the **Sub-Unit 1 Extension Activities**.
- Invite them to create their own sand dollar problems like the ones in Activity 3.

Professional Learning

Reflect on the facilitation of **MLR3: Critique, Correct, Clarify** in this lesson. Students will use this routine again in Lesson 16.

- What about the routine felt successful?
- What are some areas for growth the next time your class uses this routine?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

10. List at least two things you know about positive and negative numbers on a number line.

Draw on the image if it helps to show your thinking.

Responses vary.

- Any number to the left of the hole is a negative number.
- The number tells you how many units you are away from 0 and the direction to go.
- Negative numbers have a - sign in front of them.

Lesson Practice ACC6.7.01

Lesson Summary

Positive numbers are numbers that are greater than 0. **Negative numbers** are numbers that are less than 0. Zero is neither positive nor negative.

You can extend a number line to the right of 0 to show positive numbers, and you can extend a number line to the left of 0 to show negative numbers.

For example:

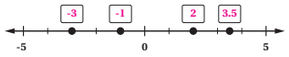
The number 3 is 3 units to the right of 0 on the number line.

The number -3 is 3 units to the left of 0 on the number line.

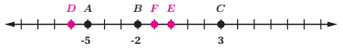
Lesson Practice ACC6.7.01

Name: _____ Date: _____ Period: _____

1. Fill in the blanks on the number line.



Problems 2–6: Here is a number line.



2. Describe where you would plot -100 on the number line.

Responses vary. The number -100 would be really far to the left of 0 on the number line.

3. Point D is 1 unit to the left of point A. Plot point D.

Response shown on the number line.

4. Point E is at 0. Plot point E.

Response shown on the number line.

5. List both locations that are 4 units away from point B.

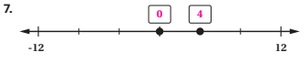
-6 and 2

6. Point F is the same distance from point A and point C. Plot point F.

Response shown on the number line.

Problems 7–9: Fill in the blanks on the number lines.

7.



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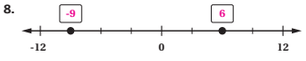

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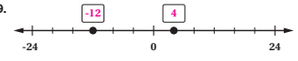


FAST Practice

10. Which pair of numbers are on opposite sides of zero on the number line?

A. 0 and 5 B. 2 and -5 C. -2 and -5 D. 2 and 5

Lesson Practice ACC6.7.01

Name: _____ Date: _____ Period: _____

Spiral Review

11. A rectangle has an area of 24 square centimeters. If one side is $2\frac{2}{5}$ centimeters long, how long is the other side?

$2\frac{2}{5}$ cm | 24 sq. cm

10 centimeters

Explain your thinking.

Work varies. $24 \div 2\frac{2}{5} = 24 \div \frac{12}{5} = 24 \cdot \frac{5}{12} = 10$

Problems 12–14: Two friends at soccer practice are drinking from their water bottles. Julian drinks $\frac{3}{5}$ of his 15-ounce bottle. De'Andre drinks 15 ounces of water, which is $\frac{3}{5}$ of his bottle.

12. How are these two situations alike? How are they different?

Responses vary.

- Both situations are about a person drinking from a water bottle.
- Both situations use the number 15 and the fraction $\frac{3}{5}$.
- In Julian's situation, the whole bottle is 15 ounces. In De'Andre's situation, the 15 ounces is how much water he drinks.
- De'Andre's water bottle is bigger than Julian's water bottle.

13. Write an expression to represent the amount of water Julian drinks.

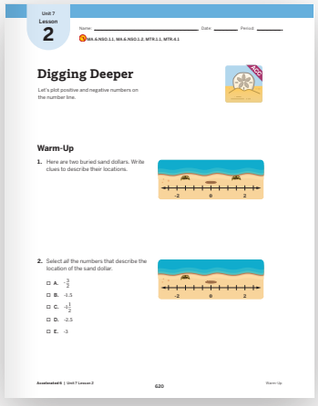
$\frac{3}{5} \cdot 15$ (or equivalent)

14. Write an expression to represent the total amount of water in De'Andre's bottle.

$15 \div \frac{3}{5}$ (or equivalent)

Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1	1	MA.6.NSO.1.1
	2, 7–9	2	MA.6.NSO.1.1
	3–4	1	MA.6.NSO.1.1, MA.6.NSO.1.2
	5–6	2	MA.6.NSO.1.1, MA.6.NSO.1.2
FAST Practice	10	1	MA.6.NSO.1.2
Spiral Review			
	11	2	MA.6.GR.2.2
	12	1	MA.5.AR.2.1
	13–14	2	MA.5.AR.2.1


Unit 7
Lesson 2
Digging Deeper
Warm-Up
1. Here are two buried sand dollars. Write clues to describe their locations.
2. Select all the numbers that describe the location of the sand dollar.
A. $-\frac{3}{2}$
B. -1.5
C. $-1\frac{1}{2}$
D. -2.5
E. -3
620

Digging Deeper

Positions on the Number Line

Let's plot positive and negative numbers on the number line.

Learning Outcomes

Today's Goals

1. **Goal:** Identify and plot positive and negative numbers on the number line.
2. **Goal:** Recognize that the *opposite* of the opposite of a number is the number itself, e.g., $-(-3) = 3$.
3. **Language Goal:** Describe numbers that are the same distance from 0 but on opposite sides of the number line using the symbol (-) and the word *opposite*. **(Reading, Writing, Speaking, and Listening)**

Students extend what they learned about negative numbers in Lesson 1 to learn about non-integers. They describe and use strategies for identifying and plotting negative rational numbers on the number line. This lesson also introduces what it means for two numbers to be **opposites**.

Coherence

Prior Learning

In Lesson 1, students learned what negative and positive numbers are and how they are represented on a number line in the context of a crab searching for sand dollars.

Future Learning

In Lesson 3, students will apply their understanding of positive and negative numbers in the contexts of elevations and temperatures.

Connections

- Students continue their learning about signed numbers as they describe the behavior of an object along a number line. (mathematical, real world)

FL B.E.S.T. Math Benchmarks

Addressing

MA.6.NSO.1.1
Extend previous understanding of numbers to define rational numbers. Plot, order and compare rational numbers.

MA.6.NSO.1.2
Given a mathematical or real-world context, represent quantities that have opposite direction using rational numbers. Compare them on a number line and explain the meaning of zero within its context.

Mathematical Thinking and Reasoning Standards: MTR.1.1, MTR.4.1

ELA Expectations: ELA.K12.EE.1.1, ELA.K12.EE.3.1, ELA.K12.EE.6.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.5.NSO.1.4	MA.8.NSO.1.1 MA.8.NSO.1.2

Vocabulary

New Vocabulary

integers
opposite

Additional Vocabulary

rational number

Lesson at a Glance

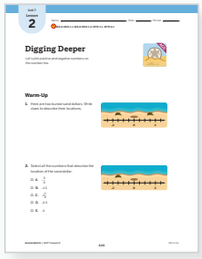
~ 45 min

MA.6.NSO.1.1, MA.6.NSO.1.2

Warm-Up

Pairs | 10 min

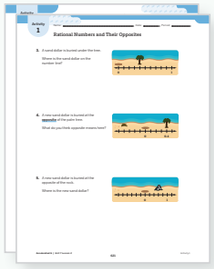
Students use fractions and decimals to describe the location of a negative value on the number line. **Routine:** Notice and Wonder



Activity 1

Pairs | 15 min

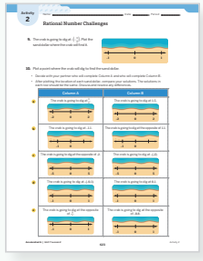
Students make sense of and apply the term **opposite** while using clues to locate new sand dollars. MLR2: Collect and Display **(MTR.1.1)**



Activity 2

Pairs | 10 min

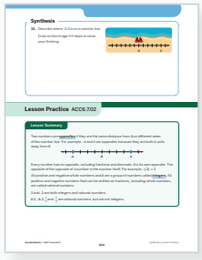
Students practice plotting positive and negative rational numbers on the number line.



Synthesis

Whole Class | 5 min

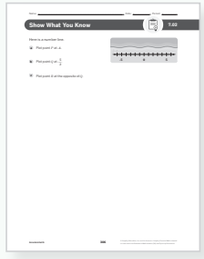
Students synthesize strategies for plotting negative rational numbers on the number line. MLR2: Collect and Display



Show What You Know

Independent | 5 min

Students demonstrate their understanding by plotting rational numbers on the number line.



Prep Checklist

This lesson includes:

- Student Edition
- Show What You Know

Pairs | 10 min

Warm-Up

Purpose: Students use fractions and decimals to describe the location of a negative value on the number line.

1 Launch

Use the Notice and Wonder routine to promote curiosity and help students make sense of the situation. **(MTR.1.1)**

Listen for students who use different ways of describing the sand dollars to the right and the left of 0 and encourage them to share their thinking with their partner. **(MTR.4.1)**

A **Accessibility: Executive Functioning** Consider chunking this activity by supporting students in writing a clue for one sand dollar at a time, while covering up the other sand dollar.

To support students getting started, invite them to label other points on the number line that they know or think they know.

2 Connect

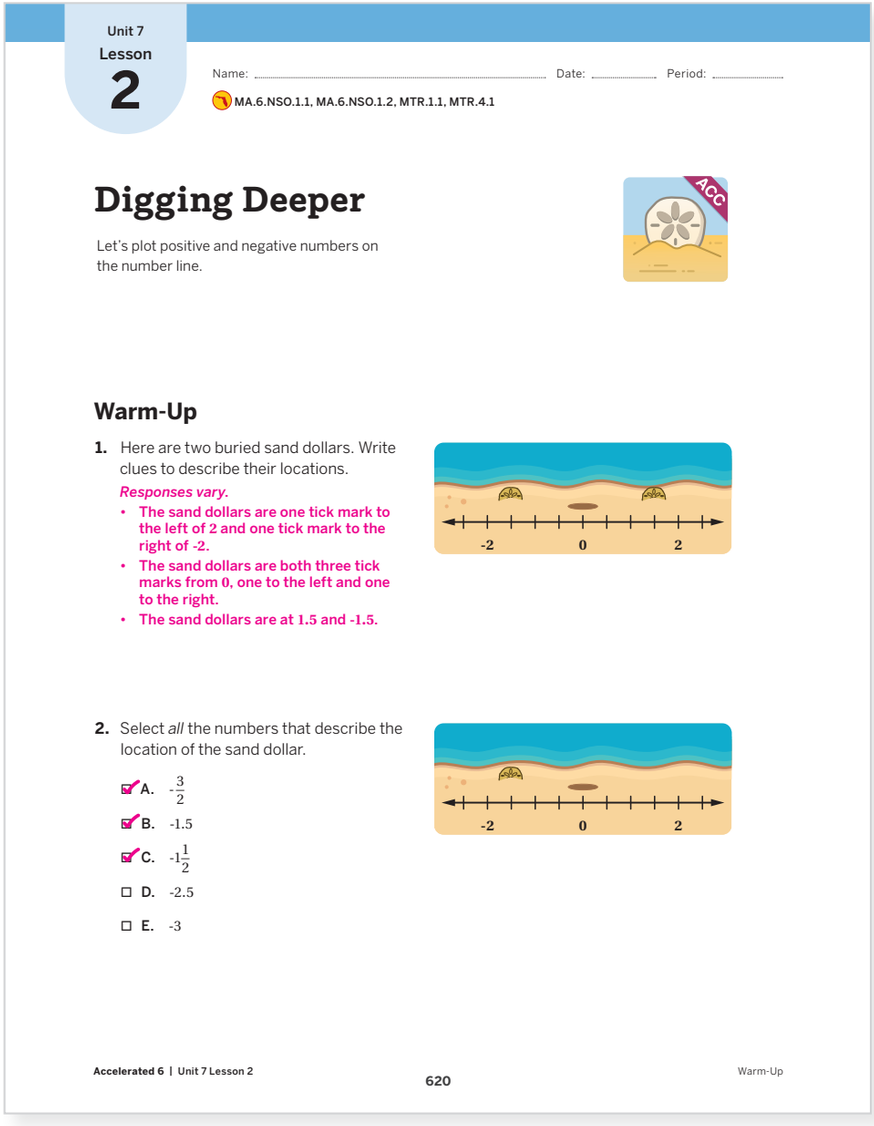
Collect the distribution of responses, calling attention to any conflict or consensus.

Invite students to defend each response to the class to reach consensus.

Encourage students to revise their responses as their classmates share.

To surface the Key Takeaway, consider asking:

- "What other values are on this number line that aren't yet labeled?"
- "What are some differences in the way positive and negative numbers are ordered on a number line?"



Unit 7
Lesson
2

Name: Date: Period:

MA.6.NSO.1.1, MA.6.NSO.1.2, MTR.1.1, MTR.4.1

Digging Deeper

ACC

Let's plot positive and negative numbers on the number line.

Warm-Up

1. Here are two buried sand dollars. Write clues to describe their locations.

Responses vary.

- The sand dollars are one tick mark to the left of 2 and one tick mark to the right of -2.
- The sand dollars are both three tick marks from 0, one to the left and one to the right.
- The sand dollars are at 1.5 and -1.5.

2. Select *all* the numbers that describe the location of the sand dollar.

- ☑ A. $-\frac{3}{2}$
- ☑ B. -1.5
- ☑ C. $-1\frac{1}{2}$
- ☐ D. -2.5
- ☐ E. -3

Accelerated 6 | Unit 7 Lesson 2 620 Warm-Up

Key Takeaway: Positive numbers are to the right of 0, increasing in value the farther they are to the right. Negative numbers are to the left of 0, decreasing in value as you move farther left.

Pairs | 15 min

Activity 1 Rational Numbers and Their Opposites

Purpose: Students make sense of and apply the term **opposite** while using clues to locate new sand dollars. **(MTR.1.1)**

Short on time? Consider completing Problem 3 as a class.

1 Launch

Encourage students to label the number line to represent their thinking.

2 Monitor

Look for students' responses such as 0.4 and $\frac{4}{10}$.

Encourage students to check their response with a partner to help them revise their thinking.

MLR **MLR2: Collect and Display** Pause briefly and use this routine to collect students' ideas and sketches as they describe **opposites**. Display these publicly for students to reference or add to for the rest of the lesson.

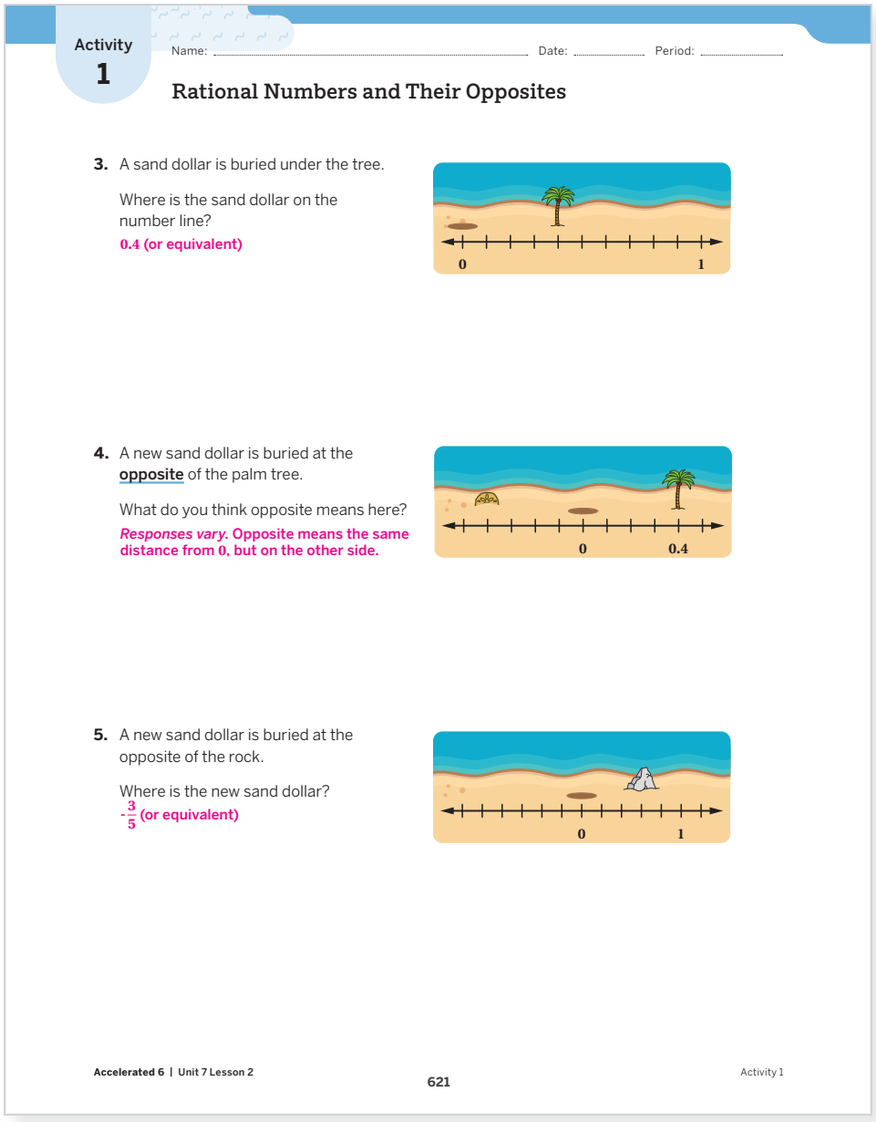
ELL **English Language Learners** Invite students to rehearse their ideas with a partner before they share their ideas with others. **(Writing and Speaking)**

D Differentiation

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Invite them to label where the rock is and each tick mark on the number line.
Write $\frac{3}{5}$ because they notice that's the location of the rock.	**Support** Consider sharing with these students that the sand dollar's location is opposite of the rock's location and ask what they think that means.
Represent the sand dollar's location as -0.6 or $-\frac{3}{5}$.	**Stretch** Consider asking them to determine the distance between the rock and the sand dollar.

Consider sharing that two points can be the same distance from 0 but on opposite sides, which means they represent different numbers — one positive and one negative.

Activity 1 continued >



Activity 1

Name: _____ Date: _____ Period: _____

Rational Numbers and Their Opposites

3. A sand dollar is buried under the tree.

Where is the sand dollar on the number line?

0.4 (or equivalent)

4. A new sand dollar is buried at the **opposite** of the palm tree.

What do you think opposite means here?

***Responses vary*. Opposite means the same distance from 0, but on the other side.**

5. A new sand dollar is buried at the opposite of the rock.

Where is the new sand dollar?

$-\frac{3}{5}$ (or equivalent)

Accelerated 6 | Unit 7 Lesson 2 621 Activity 1

Pairs | 15 min

Activity 1 Rational Numbers and Their Opposites (continued)

Purpose: Students make sense of and apply the term **opposite** while using clues to locate new sand dollars. **(MTR.1.1)**

2 Monitor

Look for students who write -(-1.25) as the location of the sand dollar and invite them to share during the Connect.

To support students getting started, consider reading the text out loud as a class. Students may refer to $-a$ as "negative a." Consider sharing with students that it can be helpful to refer to it as "the opposite of a."

Look for strategies to share.

D Differentiation

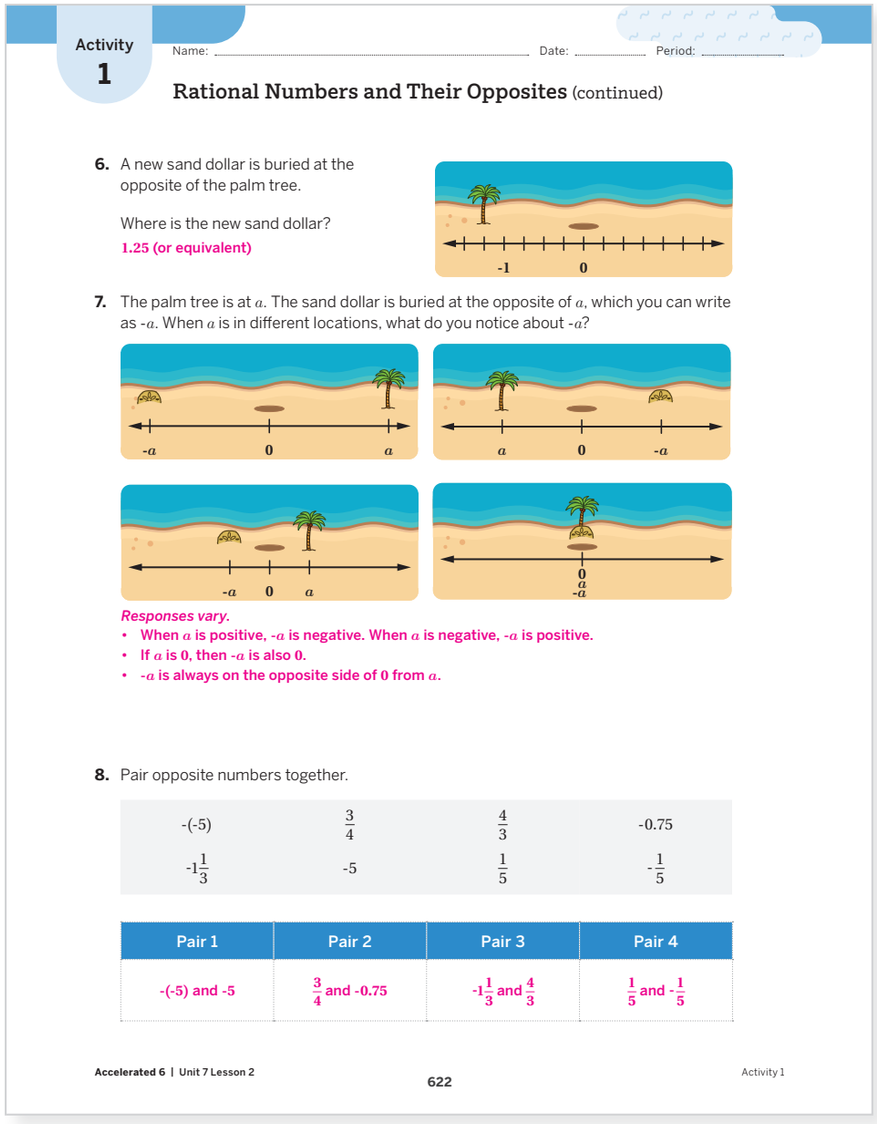
Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Consider providing number lines as support for students as they pair the numbers.
Notice that $\frac{3}{4}$ and $\frac{4}{3}$ are reciprocals and match them together.	**Support** Consider asking, "Where would each number be on the number line? What is another way of writing these numbers?"
Recognize -(-5) as a positive number to pair with -5.	**Stretch** Invite students to add another number to each pair.

3 Connect

Direct students' attention to the numbers. Invite students to share how they determined which values were opposites. Consider asking, "If -5 is read aloud as 'negative 5,' what is -(-5)?" *Responses vary.* "Negative negative five," "the opposite of negative five," or "positive five."

To surface the Key Takeaway, consider asking, "What is the opposite of a positive number? A negative number?" **(MTR.1.1)**

ELL **English Language Learners** Invite students to research how to say the new terms in other languages. Create an anchor chart with the terms in multiple languages, along with examples. **(Reading and Listening)**



Activity 1

Name: _____ Date: _____ Period: _____

Rational Numbers and Their Opposites (continued)

6. A new sand dollar is buried at the opposite of the palm tree.

Where is the new sand dollar?

1.25 (or equivalent)

7. The palm tree is at a. The sand dollar is buried at the opposite of a, which you can write as $-a$. When a is in different locations, what do you notice about $-a$?

Responses vary.

- When a is positive, $-a$ is negative. When a is negative, $-a$ is positive.
- If a is 0, then $-a$ is also 0.
- $-a$ is always on the opposite side of 0 from a.

8. Pair opposite numbers together.

-(-5)	$\frac{3}{4}$	$\frac{4}{3}$	-0.75
$-1\frac{1}{3}$	-5	$\frac{1}{5}$	$-\frac{1}{5}$

Pair 1	Pair 2	Pair 3	Pair 4
-(-5) and -5	$\frac{3}{4}$ and -0.75	$-1\frac{1}{3}$ and $\frac{4}{3}$	$\frac{1}{5}$ and $-\frac{1}{5}$

Accelerated 6 | Unit 7 Lesson 2 622 Activity 1

Key Takeaway: **Opposites** are two numbers that are the same distance from 0, but on different sides of the number line.

Pairs | 10 min

Activity 2 Rational Number Challenges

Purpose: Students practice plotting positive and negative rational numbers on the number line.

Short on time? Consider inviting students to complete fewer Repeated Challenges.

1 Launch

To surface connections, consider asking, "Do you think the sand dollar should go to the left or right of 0?"

Discuss that $-n$ can be seen as the opposite of a number, n, even if that number itself is negative, if it does not come up naturally. Consider making connections with -5 and -(-5) from the previous activity. **(MTR.4.1)**

2 Monitor

Invite students to set their own goal for how many challenges they would like to complete, sharing that sense-making is more important than speed.

Invite students to compare their solutions to pairs of problems. **(MTR.1.1, MTR.4.1)**

A **Accessibility: Memory and Attention** For students who would benefit from having more time to think deeply on fewer problems, invite them to complete fewer challenges.

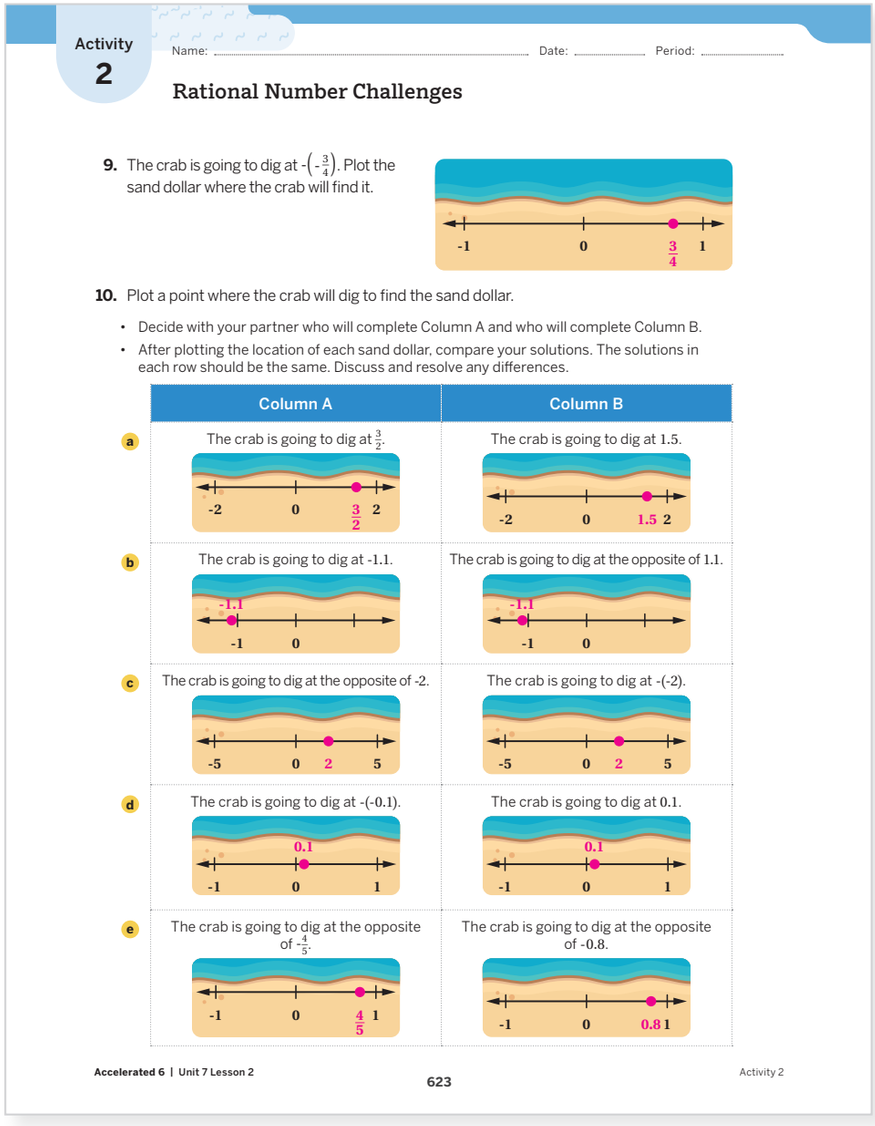
Listen for student strategies and partner discussions, offering help or encouragement as needed.

D **Differentiation**

Look for students who:	Respond to Student Thinking
Only recognize numbers that are expressed in the same form.	**Support** Consider asking, "What is another way to write your number?"
Think that all opposites are negative numbers.	**Support** Consider asking, "Where is the opposite of a point located to the left of 0?"
Rewrite -(-2) as 2 and read it as "the opposite of negative 2" by recognizing that the opposite of a negative is a positive.	**Stretch** Consider asking, "What is the value of -(-(-2))? How does the number of (-) signs affect the overall sign of the value?"

3 Connect

Invite students to share their strategies for deciding where to place the sand dollars.



Activity 2

Name: Date: Period:

Rational Number Challenges

9. The crab is going to dig at $-\left(-\frac{3}{4}\right)$. Plot the sand dollar where the crab will find it.

10. Plot a point where the crab will dig to find the sand dollar.

- Decide with your partner who will complete Column A and who will complete Column B.
- After plotting the location of each sand dollar, compare your solutions. The solutions in each row should be the same. Discuss and resolve any differences.

	Column A	Column B
a	The crab is going to dig at $\frac{3}{2}$.	The crab is going to dig at 1.5.
b	The crab is going to dig at -1.1.	The crab is going to dig at the opposite of 1.1.
c	The crab is going to dig at the opposite of -2.	The crab is going to dig at -(-2).
d	The crab is going to dig at -(-0.1).	The crab is going to dig at 0.1.
e	The crab is going to dig at the opposite of $-\frac{4}{5}$.	The crab is going to dig at the opposite of -0.8.

Accelerated 6 | Unit 7 Lesson 2 623 Activity 2

Whole Class | 5 min

Synthesis

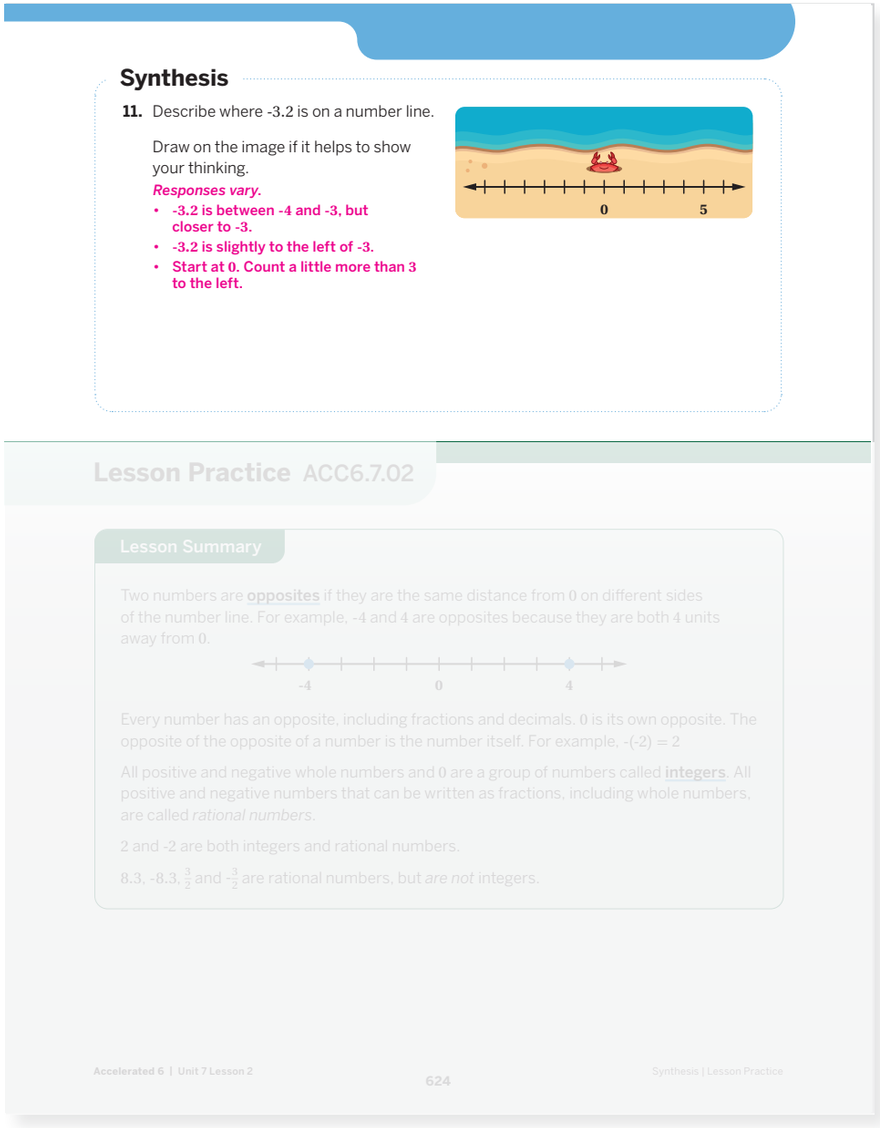
Purpose: Students synthesize strategies for plotting negative rational numbers on the number line.

Invite students to respond independently, and then share their thinking with a partner.

Collect and share a variety of ideas, such as:
- A negative number is on the left side of 0 on the number line.
- -3.2 is a number between -3 and -4 but closer to -3.
- -3.2 is the opposite of 3.2.

MLR **MLR2: Collect and Display** Use this routine to formalize the definitions of *opposite* and *integers* using students' language. Display any new language, examples, or diagrams publicly for students to reference later.

Note: If time allows, invite students to revise their previous thinking based on the discussion.



Synthesis

11. Describe where -3.2 is on a number line.

Draw on the image if it helps to show your thinking.

Responses vary.
- -3.2 is between -4 and -3, but closer to -3.
- -3.2 is slightly to the left of -3.
- Start at 0. Count a little more than 3 to the left.

Lesson Practice ACC6.7.02

Lesson Summary

Two numbers are opposites if they are the same distance from 0 on different sides of the number line. For example, -4 and 4 are opposites because they are both 4 units away from 0.

Every number has an opposite, including fractions and decimals. 0 is its own opposite. The opposite of the opposite of a number is the number itself. For example, -(-2) = 2

All positive and negative whole numbers and 0 are a group of numbers called integers. All positive and negative numbers that can be written as fractions, including whole numbers, are called *rational numbers*.

2 and -2 are both integers and rational numbers.

8.3, -8.3, $\frac{3}{2}$ and $-\frac{3}{2}$ are rational numbers, but *are not* integers.

Accelerated 6 | Unit 7 Lesson 2 624 Synthesis | Lesson Practice

Lesson Takeaway: Opposites are two numbers that are the same distance from 0, but on different sides of the number line. The opposite of the opposite of a number is the number itself (e.g., -(-3) = 3) and 0 is its own opposite.

Independent | 5 min

Show What You Know

Purpose: Students demonstrate their understanding by plotting rational numbers on the number line.

Today's Goals

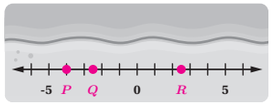
1. **Goal:** Identify and plot positive and negative numbers on the number line.
2. **Goal:** Recognize that the *opposite* of the opposite of a number is the number itself, e.g., $-(-3) = 3$.
3. **Language Goal:** Describe numbers that are the same distance from 0 but on opposite sides of the number line using the symbol (-) and the word *opposite*. **(Reading, Writing, Speaking, and Listening)**

Show What You Know (Assessment Resources)

Name: _____ Date: _____ Period: _____

Show What You Know 7.02

Here is a number line.



a. Plot point P at -4.

b. Plot point Q at $-\frac{5}{2}$.

c. Plot point R at the opposite of Q.

Responses shown on the number line.

Differentiation Use after Lesson 2

Support

Provide targeted intervention for students by using these resources.

If students need support plotting numbers and their opposites on a number line:

» **Respond to Student Thinking:**

- Revisit **Activity 1** before providing individualized feedback on the relevant **Practice Problems**.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite them to explore the **Sub-Unit 1 Extension Activities**.

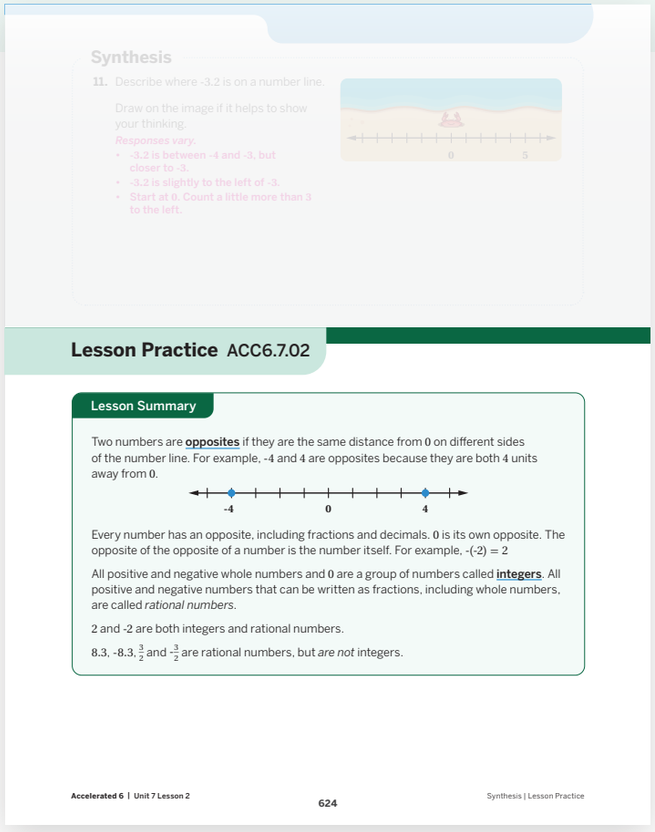
Professional Learning

Take a moment to reflect on this lesson. How did the **Collect and Display** routine support students in recognizing and describing the opposite of a number?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.



Synthesis

11. Describe where -3.2 is on a number line.

Draw on the image if it helps to show your thinking.

Responses vary.

- -3.2 is between -4 and -3, but closer to -3.
- -3.2 is slightly to the left of -3.
- Start at 0. Count a little more than 3 to the left.

Lesson Practice ACC6.7.02

Lesson Summary

Two numbers are **opposites** if they are the same distance from 0 on different sides of the number line. For example, -4 and 4 are opposites because they are both 4 units away from 0.

-4 0 4

Every number has an opposite, including fractions and decimals. 0 is its own opposite. The opposite of the opposite of a number is the number itself. For example, $-(-2) = 2$

All positive and negative whole numbers and 0 are a group of numbers called **integers**. All positive and negative numbers that can be written as fractions, including whole numbers, are called *rational numbers*.

2 and -2 are both integers and rational numbers.

8.3, -8.3, $\frac{3}{2}$ and $-\frac{3}{2}$ are rational numbers, but *are not* integers.

Lesson Practice
ACC6.7.02

Name: ..... Date: ..... Period: .....

1. Plot each number in its approximate location on the number line.

-3 $\frac{3}{2}$ $-\frac{4}{3}$ $\frac{9}{4}$

-3 $-\frac{4}{3}$ $\frac{3}{2}$ $\frac{9}{4}$

-5 0 5

2. Plot each number in its approximate location on the number line.

-2 $\frac{5}{4}$ -3.4 0

-3.4 -2 0 $\frac{5}{4}$

-4 4

Problems 3–6: Complete each statement below.

3. The opposite of -2 is 2.
4. The opposite of $\frac{5}{4}$ is $-\frac{5}{4}$.
5. The opposite of -3.4 is 3.4.
6. The opposite of 0 is 0.

Problems 7–8: Rebecca incorrectly plotted the point $-1\frac{7}{10}$ on the number line.

$-1\frac{7}{10}$ $-1\frac{7}{10}$

-2 -1 0 1 2

7. What question could you ask to help Rebecca understand her mistake?

Questions vary. Is $-1\frac{7}{10}$ closer to -1 or -2?

8. Plot $-1\frac{7}{10}$ in the correct location on the number line.

Response shown on the number line.

Lesson Practice
ACC6.7.02

Name: ..... Date: ..... Period: .....

Problems 9–11: Plot and label where zero is located on each number line.

9. 0

-5 3

10. 0

-9 6

11. 0

-10 -2

FAST Practice

12. Points K, L, M, and N are plotted on the number line.

K L M N

-4 -3 -2 -1 0 1 2 3 4

What point represents the location of -0.5?

A. Point K (B.) Point L C. Point M D. Point N

Spiral Review

Problems 13–16: Solve each equation.

13. $3x = 6$

$x = 2$

14. $\frac{1}{3}x = 6$

$x = 18$

15. $\frac{1}{3} = 6x$

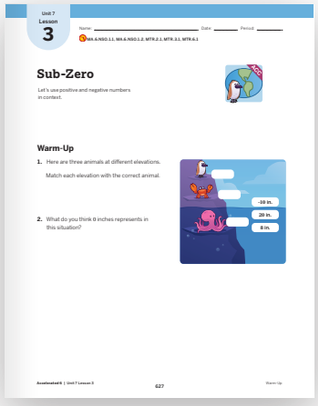
$x = \frac{1}{18}$ (or equivalent)

16. $\frac{1}{3} = \frac{1}{6}x$

$x = 2$ (or equivalent)

Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1–2, 8	1	MA.6.NSO.1.1
	3–6	1	MA.6.NSO.1.2
	7–8	2	MA.6.NSO.1.1
	9–11	3	MA.6.NSO.1.1
FAST Practice	12	1	MA.6.NSO.1.1
Spiral Review			
	13–16	1	MA.6.AR.2.4


Unit 7
Lesson 3
Sub-Zero
Let's use positive and negative numbers in context.
Warm-Up
1. Here are three animals at different elevations. Match each elevation with the correct animal.
-10 m
20 m
0 m
2. What do you think 0 inches represents in this situation?
627

Sub-Zero

Positive and Negative Numbers in Context

Let's use positive and negative numbers in context.

Learning Outcomes

Today's Goals

1. **Goal:** Compare positive and negative numbers in context using words and symbols.
2. **Language Goal:** Explain what positive numbers, negative numbers, and 0 mean in contexts. **(Reading, Writing, Speaking, and Listening)**

Students apply their understanding of positive and negative numbers in the contexts of elevations and temperatures around the world. They compare positive and negative temperatures and elevations using words and symbols. Students then abstract temperatures to positive and negative numbers on a vertical number line to compare the relative location of each temperature. **(MTR.2.1)**

Coherence

Prior Learning

In Lesson 2, students described and used strategies for identifying and plotting positive and negative rational numbers on the number line.

Future Learning

In Lesson 4, students will practice comparing positive and negative rational numbers, applying and reinforcing their understanding of the number line. In Lessons 5–6, they will build on their knowledge of opposites and explore the concept of absolute value.

Connections

- Students use signed numbers in the context of analyzing temperatures to solve problems. (mathematical, real world)

FL B.E.S.T. Math Benchmarks

Addressing

MA.6.NSO.1.1
Extend previous understanding of numbers to define rational numbers. Plot, order and compare rational numbers.

MA.6.NSO.1.2
Given a mathematical or real-world context, represent quantities that have opposite direction using rational numbers. Compare them on a number line and explain the meaning of zero within its context.

Mathematical Thinking and Reasoning Standards: MTR.2.1, MTR.3.1, MTR.6.1

ELA Expectations: ELA.K12.EE.1.1, ELA.K12.EE.5.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.5.NSO.1.4	**MA.7.NSO.2.1** **MA.8.NSO.1.1** **MA.8.NSO.1.2**

Vocabulary

Review Vocabulary

inequality

Lesson at a Glance

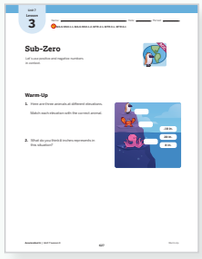
~ 45 min

MA.6.NSO.1.1, MA.6.NSO.1.2

Warm-Up

Independent | 10 min

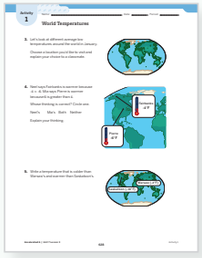
Students reason about zero elevation (sea level), positive elevations, and negative elevations.



Activity 1

Pairs | 10 min

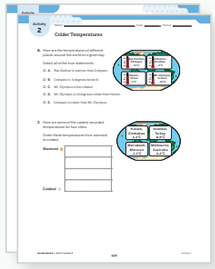
Students continue to build understanding of negative numbers using temperatures from all around the world. **Routines:** Notice and Wonder, Decide and Defend



Activity 2

Pairs | 15 min

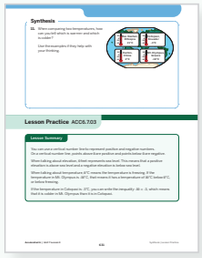
Students compare and order cold temperatures around the world using a vertical number line. **(MTR.2.1)**



Synthesis

Whole Class | 5 min

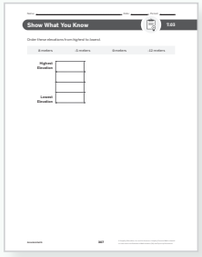
Students synthesize their understanding of strategies for comparing positive and negative temperatures. MLR2: Collect and Display



Show What You Know

Independent | 5 min

Students demonstrate their understanding by ordering rational numbers in the context of elevations.



Prep Checklist

Prepare Activity 1 Sheet

This lesson includes:

- Student Edition
- Show What You Know
- Activity 1 Sheet, one per pair

Independent | 10 min

Warm-Up

Purpose: Students reason about zero elevation (sea level), positive elevations, and negative elevations.

1 Launch

To generate excitement, invite students to share anything they know about the animals with a classmate.

Direct students' attention to the image of the animals and the water and consider sharing that elevation is a word that represents the vertical distance and direction from sea level (0) to a location.

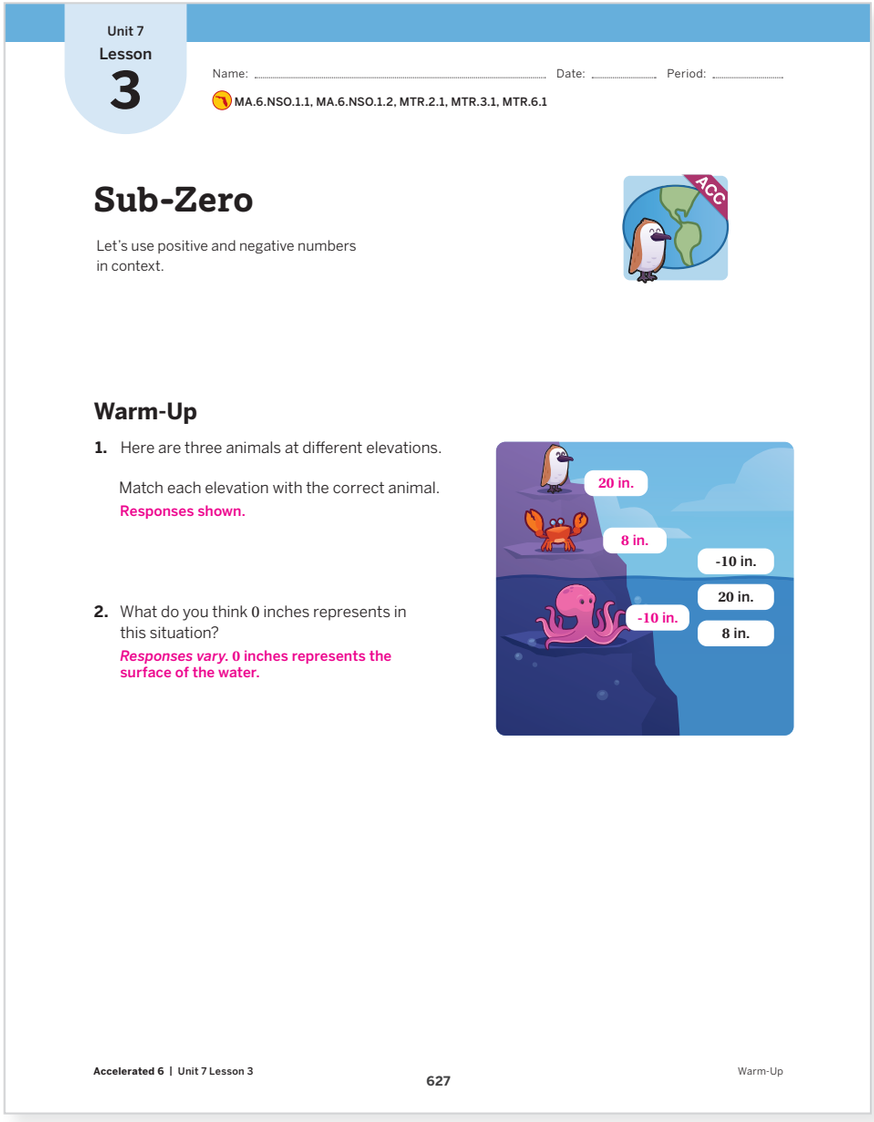
2 Connect

Invite students to share their strategies to determine the elevation of each animal focusing on those who use terms such as *above* and *below sea level*.

Consider asking:

- "If you had a map that showed elevations using positive and negative numbers, how could you quickly tell which areas are higher and which are lower?"
- "Can you think of an animal that lives at an elevation above sea level? Below sea level? What do you think their elevations might be?"
- "What do you think the elevations might be for activities such as hiking, swimming, and diving?"

To surface the Key Takeaway, consider asking, "Why is using positive and negative numbers helpful when talking about elevations?"



Unit 7
Lesson
3

Name: ..... Date: ..... Period: .....

MA.6.NSO.1.1, MA.6.NSO.1.2, MTR.2.1, MTR.3.1, MTR.6.1

Sub-Zero

Let's use positive and negative numbers in context.

Warm-Up

1. Here are three animals at different elevations.

   Match each elevation with the correct animal.

   Responses shown.

2. What do you think 0 inches represents in this situation?

   Responses vary. 0 inches represents the surface of the water.

Accelerated 6 | Unit 7 Lesson 3 627 Warm-Up

Key Takeaway: The elevation chart is like a vertical number line. 0 elevation represents sea level. Elevations above sea level are labeled by positive numbers and elevations below sea level are labeled by negative numbers. For example, an elevation of -3 meters means 3 meters below sea level and an elevation of 25 meters means 25 meters above sea level.

Pairs | 10 min

Activity 1 World Temperatures

Purpose: Students continue to build understanding of negative numbers using temperatures from all around the world.

1 Launch

Direct students' attention to the world map.

Distribute one copy of the Activity 1 Sheet to each pair of students.

To generate excitement, consider inviting students to search for the hottest temperature, the coldest temperature, and the temperature closest to 0°F.

Use the Notice and Wonder routine to promote curiosity and help students make sense of the positive and negative temperatures. See the Routine Facilitation Guide for more information.

2 Monitor

ELL **English Language Learners** Invite students to read the two claims aloud and paraphrase what they mean. Clarify any words or symbols they don't know. **(Reading)**

D **Differentiation**

Look for students who:	Respond to Student Thinking
Switch the meanings of < and >.	**Support** Display $1 < 2$ and $2 > 1$. Consider sharing that both of these inequalities are true, but are read differently.
Reason that -6 is farther away from 0 than -4, so -6 is greater than -4.	**Support** Consider asking, "What happens to the integers as you move from 0 to the left?"
Evaluate the argument by: • Translating the inequality symbol into words. • Drawing a number line and plotting -4 and -6.	**Strengthen** Invite students to share their ideas and explanations during the Connect. **(MTR.2.1)**

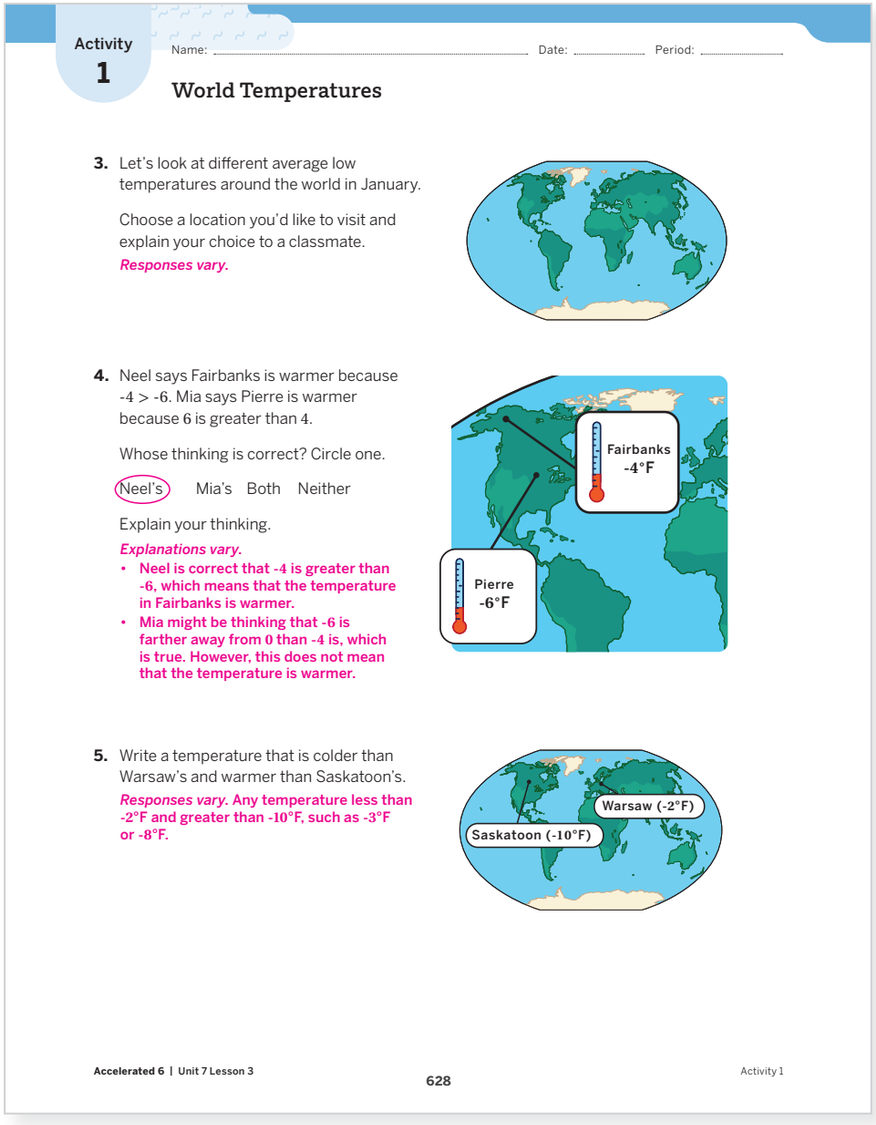
3 Connect

Invite a few students to share their thinking.

Use the Decide and Defend routine to support students in making arguments and critiquing the reasoning of others. **(MTR.3.1)**

Consider asking, "What does $-4 > -6$ say about which location is warmer? How do you know? What would need to be true for Mia's statement to be correct?" **(MTR.2.1)**

Invite students to share the temperatures they chose. Consider asking, "Why is there more than one possible answer?"



Activity 1

Name: _____ Date: _____ Period: _____

World Temperatures

3. Let's look at different average low temperatures around the world in January.

Choose a location you'd like to visit and explain your choice to a classmate.

Responses vary.

4. Neel says Fairbanks is warmer because $-4 > -6$. Mia says Pierre is warmer because 6 is greater than 4.

Whose thinking is correct? Circle one.

(Neel's) Mia's Both Neither

Explain your thinking.

Explanations vary.

- Neel is correct that -4 is greater than -6, which means that the temperature in Fairbanks is warmer.
- Mia might be thinking that -6 is farther away from 0 than -4 is, which is true. However, this does not mean that the temperature is warmer.

5. Write a temperature that is colder than Warsaw's and warmer than Saskatoon's.

Responses vary. Any temperature less than -2°F and greater than -10°F, such as -3°F or -8°F.

Accelerated 6 | Unit 7 Lesson 3 628 Activity 1

Pairs | 15 min

Activity 2 Colder Temperatures

Purpose: Students compare and order cold temperatures around the world using a vertical number line. **(MTR.2.1)**

Short on time? Complete Problem 6 as a whole class.

1 Launch

To surface connections, consider asking, "What is the coldest day you remember? Do you know what the temperature was?"

2 Monitor

D **Differentiation** *(Problem 6)*

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Invite students to evaluate each statement one at a time.
Think that "Cotopaxi is -3 degrees below 0" is a correct statement.	**Support** Invite them to represent 3 degrees below 0 with an integer, then compare both statements.
Use a variety of strategies, such as: • Ordering all temperatures using the inequality symbol to determine the coldest one. • Using a number line to check if Mt. Olympus is 12 degrees colder than Harbin.	**Strengthen** Invite students to share their strategies during the Connect. **(MTR.2.1)**

Discuss each statement one at a time as a class, inviting students to justify whether or not the statement is true.

Consider sharing that, in the next several problems, students will be investigating the coldest temperatures ever recorded in places around the world using degrees Celsius (°C). 0°C represents the freezing point of water.

Look for students who draw a number line and plot the given temperatures in order to compare them. **(MTR.2.1)**

A **Accessibility: Conceptual Processing** Invite students to revisit their work on ordering integers and help them apply the concepts to ordering rational numbers.

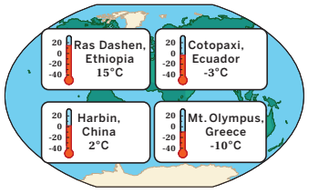
Activity 2 continued >

Activity 2

Name: Date: Period:

Colder Temperatures

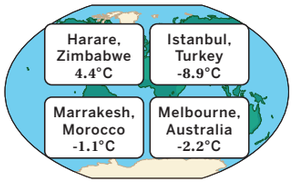
6. Here are the temperatures of different places around the world on a given day.



Select *all* of the true statements.

- ☑ A. Ras Dashen is warmer than Cotopaxi.
- ☐ B. Cotopaxi is -3 degrees below 0.
- ☑ C. Mt. Olympus is the coldest.
- ☑ D. Mt. Olympus is 12 degrees colder than Harbin.
- ☐ E. Cotopaxi is colder than Mt. Olympus.

7. Here are some of the coldest recorded temperatures for four cities.



Order these temperatures from *warmest* to *coldest*.

Warmest	4.4°C (Harare)
	-1.1°C (Marrakesh)
	-2.2°C (Melbourne)
Coldest	-8.9°C (Istanbul)

Accelerated 6 | Unit 7 Lesson 3 629 Activity 2

Pairs | 15 min

Activity 2 Colder Temperatures (continued)

Purpose: Students compare and order cold temperatures around the world using a vertical number line. **(MTR.2.1)**

2 Monitor

Listen for students expressing ideas about vertical number lines in different ways and consider using their language during the Connect. **(MTR.2.1, MTR.3.1)**

Collect student responses to take note of trends in student responses and follow up individually with students whose responses aren't between -5 and -10.

Record a variety of *inequalities* to share during the Connect. **(MTR.2.1)**

D Differentiation

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Invite students to compare Rome to another city using words like *warmer* and *colder* before translating the sentence to symbols.
Describe the inequality correctly but use incorrect symbols.	**Strengthen** Consider asking, "Which of the numbers you wrote represents a higher temperature?" Remind students that the inequality symbol opens *toward* the higher temperature.
Write one correct inequality using the structure of the number line.	**Stretch** Invite students to write a new unique response.

D Differentiation: You're invited to explore more. (Stretch) Invite students who would like to explore this concept further to complete this optional task. Encourage them to discuss their thinking with a partner.

3 Connect

Display a variety of examples of student inequalities and invite students to share their strategies for checking to make sure their response is accurate.

Consider asking:

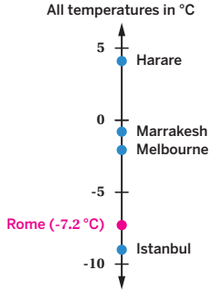
- "Does the integer farther from 0 always have the greater value?" No. For example, -20 is farther away from 0 than 5, but 5 is greater than -20.
- "Does the integer closer to 0 always have the smaller value?" No. For example, 3 is closer to 0 than -7 but -7 is smaller than 3.

Activity 2

Name: _____ Date: _____ Period: _____

Colder Temperatures (continued)

8. This number line shows some of the lowest recorded temperatures for the same four cities.



One of the lowest recorded temperatures in Rome, Italy, is -7.2°C.

Plot the point for Rome at its approximate location on the number line.

Response shown on the number line. Responses between -6 and -8 are considered correct.

9. One way to write that Harare's temperature is warmer than Marrakesh's is $4.4 > -1.1$.

Write your own *inequality* comparing Rome's temperature to another city's temperature.

Inequalities vary.

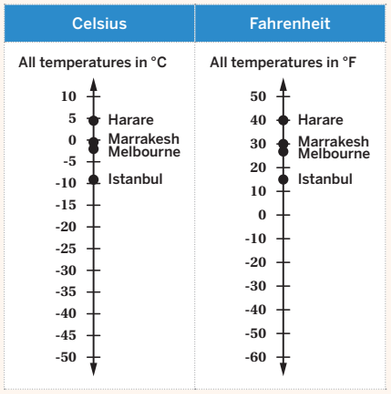
- $-7.2 > -8.9$
- $-7.2 < -2.2$
- $-1.1 > -7.2$

You're invited to explore more.

10. Look up the lowest recorded temperature in a city near you.

Plot and label a point to represent this temperature on one of the number lines.

Responses vary.



Accelerated 6 | Unit 7 Lesson 3 630 Activity 2

Whole Class | 5 min

Synthesis

Purpose: Students synthesize their understanding of strategies for comparing positive and negative temperatures.

Invite students to respond independently, and then share their thinking with a partner.

Collect and share a variety of ideas, such as:
- When both temperatures are negative, the one closer to 0 is warmer because its value is larger.
- A positive number is always larger than a negative number, so positive temperatures are warmer than negative ones.

MLR **MLR2: Collect and Display** Use this routine to formalize the strategy for comparing rational numbers using students' language. Display any new language, examples, or diagrams publicly for students to reference later.

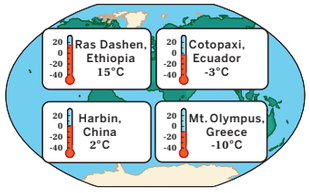
Note: If time allows, invite students to revise their previous thinking based on the discussion.

Synthesis

11. When comparing two temperatures, how can you tell which is warmer and which is colder?

Use the examples if they help with your thinking.

Responses vary.
- A warmer temperature is another way to say a greater temperature, so whatever number is greater will be warmer. For example, -3 > -10, so -3°C is warmer than -10°C.
- Any positive number is greater than any negative number, so any positive temperature is warmer than any negative temperature.



Lesson Practice ACC6.7.03

Lesson Summary

You can use a vertical number line to represent positive and negative numbers. On a vertical number line, points above 0 are positive and points below 0 are negative.

When talking about elevation, 0 feet represents sea level. This means that a positive elevation is above sea level and a negative elevation is below sea level.

When talking about temperature, 0°C means the temperature is freezing. If the temperature in Mt. Olympus is -10°C, that means it has a temperature of 10°C below 0°C, or below freezing.

If the temperature in Cotopaxi is -3°C, you can write the *inequality* -10 < -3, which means that it is colder in Mt. Olympus than it is in Cotopaxi.

Accelerated 6 | Unit 7 Lesson 3 631 Synthesis | Lesson Practice

Lesson Takeaways:
- When using negative numbers in context, we either say 3 meters below zero or -3 meters of elevation, and 10 degrees Celsius below zero or -10°C.
- Vertical and horizontal number lines help to compare numbers both in and out of context. Values to the right/above are greater than values to the left/lower.
- The *inequality* symbols < and > are used to write statements comparing positive and negative values and 0. For example, you can write -3 > -10 or -10 < -3.

Independent | 5 min

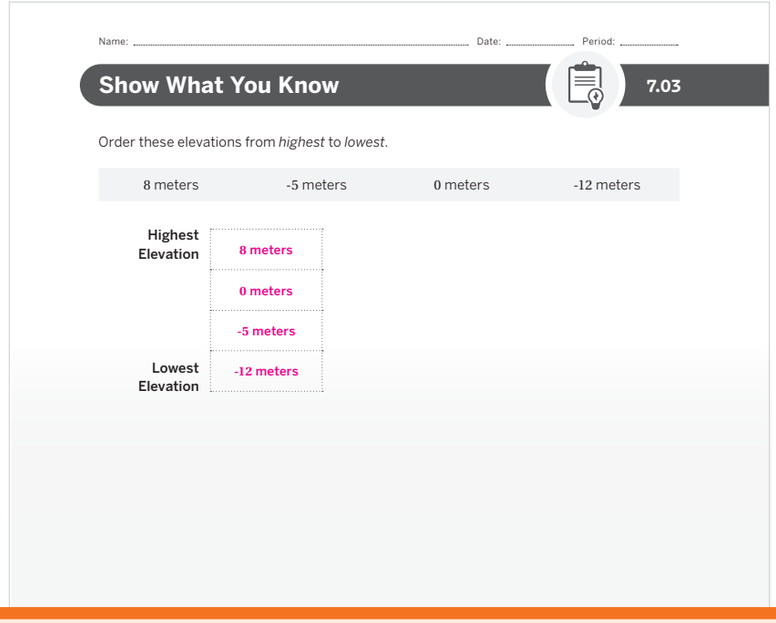
Show What You Know

Purpose: Students demonstrate their understanding by ordering rational numbers in the context of elevations.

Today's Goals

1. **Goal:** Compare positive and negative numbers in context using words and symbols.
2. **Language Goal:** Explain what positive numbers, negative numbers, and 0 mean in contexts. **(Reading, Writing, Speaking, and Listening)**

Show What You Know (Assessment Resources)



Name: _____ Date: _____ Period: _____

Show What You Know 7.03

Order these elevations from *highest* to *lowest*.

8 meters	-5 meters	0 meters	-12 meters

Highest Elevation	8 meters
	0 meters
	-5 meters
Lowest Elevation	-12 meters

Differentiation Use after Lesson 3

Support

Provide targeted intervention for students by using these resources.

If students need support comparing and ordering rational numbers in context:

» **Respond to Student Thinking:**

- Students will have more opportunities to compare and order rational numbers in Lesson 4. You may choose not to intervene now.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite them to explore the **Sub-Unit 1 Extension Activities**.
- Inviting them to complete the **Explore More** task and discuss their thinking with a partner.

Professional Learning

Consider the important issues raised in this lesson.

- What aspects of world temperatures are you more knowledgeable about? Less knowledgeable about?
- How might you support students in making connections between the math they learn and their world?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

11. When comparing two temperatures, how can you tell which is warmer and which is colder?

Use the examples if they help with your thinking.

Responses vary.

- A warmer temperature is another way to say a greater temperature, so whatever number is greater will be warmer. For example, -3 > -10, so -3°C is warmer than -10°C.
- Any positive number is greater than any negative number, so any positive temperature is warmer than any negative temperature.

Lesson Practice ACC6.7.03

Lesson Summary

You can use a vertical number line to represent positive and negative numbers. On a vertical number line, points above **0** are positive and points below **0** are negative.

When talking about elevation, **0** feet represents sea level. This means that a positive elevation is above sea level and a negative elevation is below sea level.

When talking about temperature, **0**°C means the temperature is freezing. If the temperature in Mt. Olympus is **-10**°C, that means it has a temperature of **10**°C below **0**°C, or below freezing.

If the temperature in Cotopaxi is -3°C, you can write the *inequality* **-10 < -3**, which means that it is colder in Mt. Olympus than it is in Cotopaxi.

Lesson Practice ACC6.7.03

Name: _____ Date: _____ Period: _____

Problems 1–3: Determine if each statement is true or false.

Statement	True	False
1. An elevation of 35 feet is the same as an elevation of -35 feet.		✓
2. A city that has an elevation of -17 meters is closer to sea level than a city that has an elevation of -40 meters.	✓	
3. A temperature of -4°F is the same as a temperature 4°F below zero.	✓	

Problems 4–7: Here is a table that shows some elevations in several cities around the world.

Location	Elevation (ft)
San Juan, Puerto Rico	-4
New Orleans, Louisiana	-7
Amsterdam, Netherlands	-2
Jakarta, Indonesia	3
Taipei City, Taiwan	5
Tunis, Tunisia	0

4. A city, not listed in the table, has a higher elevation than San Juan, Puerto Rico.

Select *all* the numbers that could represent the city's elevation in feet.

- ☑ **A.** 0
- ☑ **B.** 4
- ☑ **C.** -2
- ☐ **D.** -10
- ☐ **E.** -7

5. Plot all of the elevations on the number line.

Responses shown on the number line.

6. Mio says: *I know that 2 is less than 4, so -2 must be less than -4. This means Amsterdam has a lower elevation than San Juan.* Is Mio correct? Explain your thinking.

No. *Explanations vary.* Mio is incorrect because -4 is located below -2 on the number line.

7. Nasir says: *3 is less than -7 because 3 is closer to 0 on the number line.* Is Nasir correct? Explain your thinking.

No. *Explanations vary.* Nasir is incorrect because 3 is positive and -7 is negative. Positive numbers are always greater than negative numbers, no matter how close they are to 0.

Lesson Practice ACC6.7.03

Name: _____ Date: _____ Period: _____

FAST Practice

Problems 8–10: Here is a table that shows some of the lowest temperatures recorded in five U.S. locations.

Location	Temperature (°F)
Boca, CA	-45
Coventry, CT	-32
Tallahassee, FL	-2
Mt. Carroll, IL	-38
CCC Fire Camp F-16, GA	-17

8. Which of these locations had the lowest record temperature?

- (A.) Boca, CA
- **B.** Tallahassee, FL
- **C.** Coventry, CT
- **D.** CCC Fire Camp F-16, GA

9. Which location had a lower record temperature: Tallahassee, FL, or CCC Fire Camp F-16, GA? Write a statement using < or > to compare the recorded temperatures.

CCC Fire Camp F-16, GA. -17 < -2 or -2 > -17

10. How does the temperature in Coventry, CT compare to the temperature in Mt. Carroll, IL? Complete the statement using < or > to compare the recorded temperatures.

Temperature in Coventry, CT [>] Temperature in Mt. Carroll, IL

Spiral Review

Problems 11–14: Determine the value of each expression.

11. $2^3 \cdot 4 = 32$

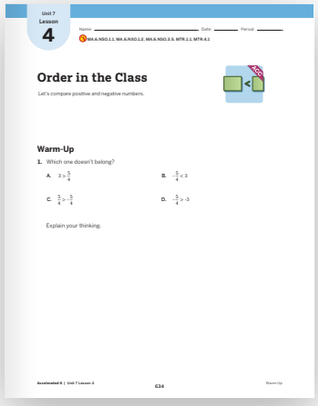
12. $\frac{2^3}{4} = 2$

13. $2^4 - 4 = 12$

14. $2^3 + 4^3 = 72$

Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1–3	1	MA.6.NSO.1.2
	4	1	MA.6.NSO.1.1
	5	2	MA.6.NSO.1.1
	6–7	2	MA.6.NSO.1.2
FAST Practice	**8–10**	2	MA.6.NSO.1.1
Spiral Review			
	11–14	2	MA.6.NSO.3.3


Unit 7
Lesson
4
Name: _____ Date: _____ Period: _____
MA.6.NSO.1.1, MA.6.NSO.1.2, MA.6.NSO.3.5, MTR.1.1, MTR.4.1
Order in the Class
Let's compare positive and negative numbers.
Warm-Up
1. Which one doesn't belong?
A. $3 > \frac{3}{4}$
B. $-\frac{3}{4} < 3$
C. $\frac{3}{4} > -\frac{3}{4}$
D. $-\frac{3}{4} > -3$
Explain your thinking.
Accelerated 6 | Unit 7 Lesson 4
634
Warm-Up

Order in the Class

Comparing Numbers

Let's compare positive and negative numbers.

Learning Outcomes

Today's Goals

1. **Goal:** Use a number line to compare and order positive and negative numbers.
2. **Language Goal:** Compare positive and negative numbers using the terms *greater than*, *less than*, and *opposite* and the symbols > and <. **(Reading, Writing, Speaking, and Listening)**

Students practice comparing rational numbers in a social way. They are given number cards and arrange themselves into small groups based on rules related to their number card. Students then complete comparison and ordering tasks. They also connect and compare positive and negative numbers represented as fractions and decimals. **(MTR.1.1)**

Coherence

Prior Learning

In Lesson 3, students applied their understanding of positive and negative numbers in the contexts of elevations and temperatures to make comparisons using words and symbols.

Future Learning

In Lessons 5–6, students will explore the concept of absolute value and continue to practice comparing and ordering rational numbers.

Connections

- Students apply their understanding of comparing and ordering numbers to signed numbers. (mathematical)

FL B.E.S.T. Math Benchmarks

Addressing

MA.6.NSO.1.1
Extend previous understanding of numbers to define rational numbers. Plot, order and compare rational numbers.

MA.6.NSO.1.2
Given a mathematical or real-world context, represent quantities that have opposite direction using rational numbers. Compare them on a number line and explain the meaning of zero within its context.

Also Addressing: **MA.6.NSO.3.5**

Mathematical Thinking and Reasoning Standards: MTR.1.1, MTR.4.1

ELA Expectations: ELA.K12.EE.1.1, ELA.K12.EE.5.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.5.NSO.1.4	MA.8.NSO.1.1 MA.8.NSO.1.2

Vocabulary

New Vocabulary

sign

Lesson at a Glance

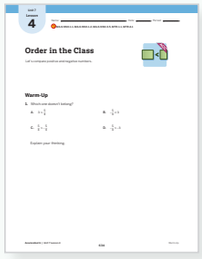
~ 45 min

MA.6.NSO.1.1, MA.6.NSO.1.2, MA.6.NSO.3.5

Warm-Up

Independent | 5 min

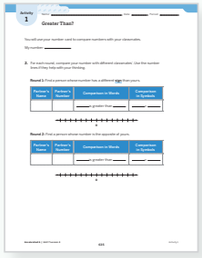
Students compare and contrast inequalities with positive and negative numbers.
Routine: Which One Doesn't Belong?


Unit 7 Lesson 4
Order in the Class
Let's compare positive and negative numbers.
Warm-Up
1. Which one doesn't belong?
A. $\frac{1}{2}$
B. $\frac{1}{4}$
C. $\frac{3}{4}$
D. $\frac{1}{3}$
Explain your thinking.
634

Activity 1

Pairs | 10 min

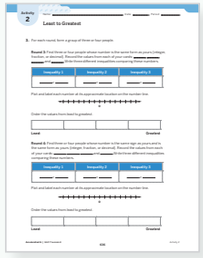
Students use words and symbols to make comparisons between pairs of rational numbers.


Activity 1
Greater Than?
You will use your number card to compare numbers with your classmates.
My number: _____
3. For each round, compare your number with different classmates'. Use the number lines if they help with your thinking.
Round 1: Find a person whose number has a different sign than yours.
Partner's Name | Partner's Number | Comparison in Words | Comparison in Symbols
_____ is greater than _____
Round 2: Find a person whose number is the opposite of yours.
Partner's Name | Partner's Number | Comparison in Words | Comparison in Symbols
_____ is greater than _____
633

Activity 2

Small Groups | 10 min

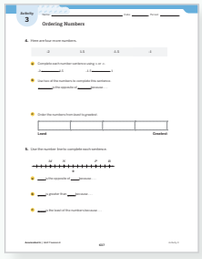
Students compare and order groups of rational numbers.


Activity 2
Least to Greatest
3. For each round, form a group of three or four people.
Round 1: Find three or four people whose number is the same form as yours (integer, fraction, or decimal). Record the values from each of your cards.
Inequality 1 | Inequality 2 | Inequality 3
Plot and label each number at the approximate location on the number line.
Order the values from least to greatest.
Least | Greatest
Round 2: Find three or four people whose number is the same sign as yours and is the same form as yours (integer, fraction, or decimal). Record the values from each of your cards.
Inequality 1 | Inequality 2 | Inequality 3
Plot and label each number at the approximate location on the number line.
Order the values from least to greatest.
Least | Greatest
634

Activity 3

Pairs | 10 min

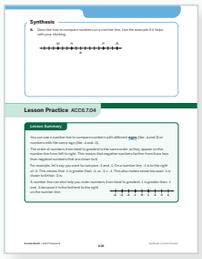
Students surface strategies for comparing two negative numbers and ordering rational numbers from least to greatest. **(MTR.1.1)**


Activity 3
Ordering Numbers
A. Here are four more numbers.
Choose a card number and keep it secret.
Use two of the numbers to complete this sentence.
is greater than
Order the numbers from least to greatest.
Least | Greatest
B. Use the number line to complete each sentence.
is greater than
is to the left of
is the same as the number represented by
633

Synthesis

Whole Class | 5 min

Students synthesize their understanding of strategies to compare rational numbers using a number line. MLR2: Collect and Display


Synthesis
A. Discuss how they compare numbers on a number line. Use the example if it helps with your thinking.
Lesson Practice ACC6.7.04
Lesson Summary
634

Show What You Know

Independent | 5 min

Students demonstrate their understanding by ordering rational numbers from least to greatest.

Show What You Know
5 pts
Order these numbers from least to greatest.
Least | Greatest
635

Prep Checklist

Prepare the Activities 1 & 2 Cards.

This lesson includes:

- Student Edition
- Show What You Know
- Activity 1 & 2 Cards, one card per student, pre-cut

Independent | 5 min

Warm-Up

Purpose: Students compare and contrast inequalities with positive and negative numbers.

Short on time? Consider completing the Warm-Up as a whole class.

1 Launch

Direct students' attention to the inequalities.

Use the Which One Doesn't Belong routine to support students in noticing and precisely describing properties of inequalities.

Encourage students to look for more than one possibility.

2 Connect

Discuss why *each* representation doesn't belong, asking students to justify their choice. **(MTR.4.1)**

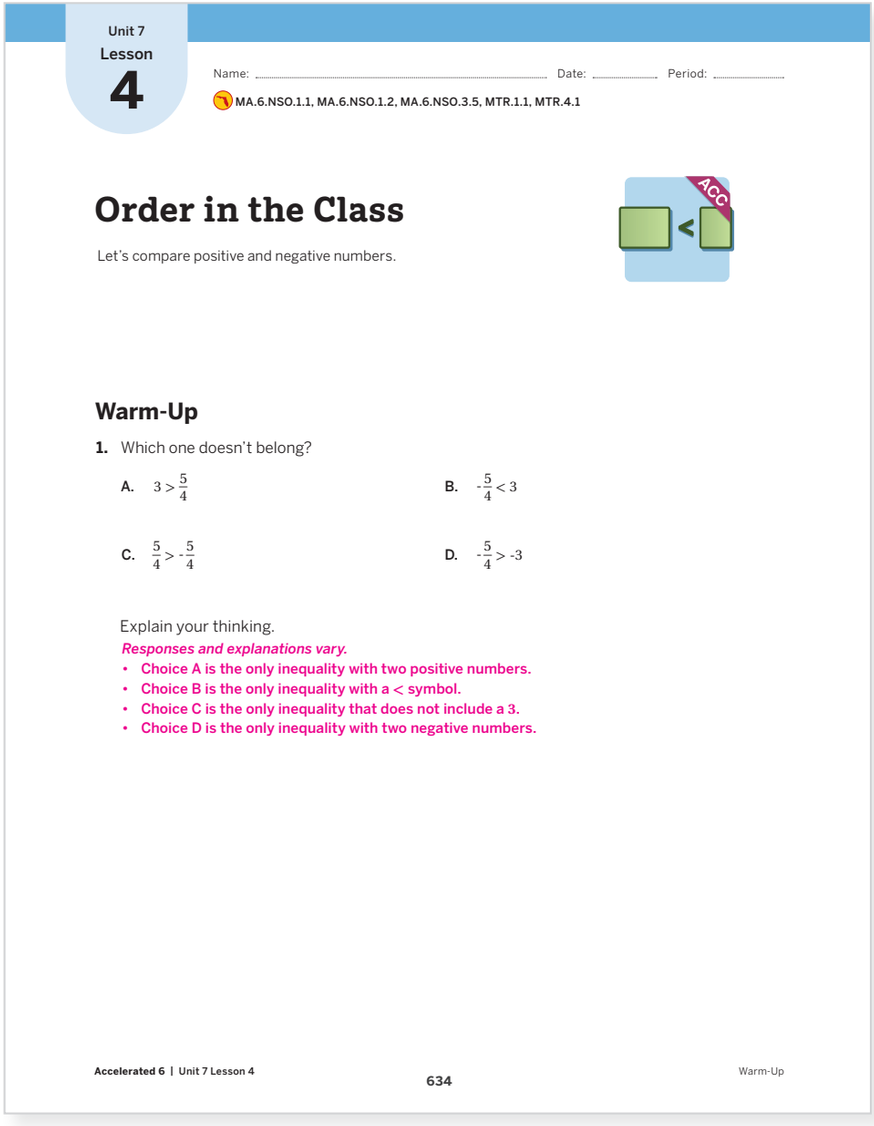
ELL **English Language Learners** Provide sentence frames (e.g., "_____ doesn't belong because _____." or " _____ is the only one that has/ doesn't have_____."). **(Writing and Speaking)**

Note: If it does not come up naturally, consider asking, "What kind of numbers are being compared in each inequality?"

Create a class definition of the term **sign**. To help generate student language, consider asking:

- "Do you think $+3$ and 3 have the same sign? What about -3 and $-\frac{5}{4}$?"
- "What do you think is the sign of 0?" **(MTR.1.1)**

ELL **English Language Learners** Invite students to research how to say the new term in other languages. Create an anchor chart with the term in multiple languages, along with examples. **(Reading and Listening)**



Unit 7
Lesson
4

Name: _____ Date: _____ Period: _____

MA.6.NSO.1.1, MA.6.NSO.1.2, MA.6.NSO.3.5, MTR.1.1, MTR.4.1

Order in the Class

Let's compare positive and negative numbers.

ACC

Warm-Up

1. Which one doesn't belong?

A. $3 > \frac{5}{4}$

B. $-\frac{5}{4} < 3$

C. $\frac{5}{4} > -\frac{5}{4}$

D. $-\frac{5}{4} > -3$

Explain your thinking.

Responses and explanations vary.

- Choice A is the only inequality with two positive numbers.
- Choice B is the only inequality with a $<$ symbol.
- Choice C is the only inequality that does not include a 3.
- Choice D is the only inequality with two negative numbers.

Accelerated 6 | Unit 7 Lesson 4 634 Warm-Up

Pairs | 10 min

Activity 1 Greater Than?

Purpose: Students use words and symbols to make comparisons between pairs of rational numbers.

1 Launch

Direct students' attention to the instructions for the activity and explain to students that they will complete this activity in rounds, locating a new partner for each round.

Distribute one number card per student from the Activities 1 & 2 Cards.

Invite students to record and get to know their number.

Consider asking:
- "What is the sign of your number?"
- "What could your number describe in the real world?"
- "Where is your number on the number line?"

2 Monitor

Direct students' attention to prompts for each round, one at a time, reminding students that they should record their work.

Note: If students are unable to find a partner in a given round (e.g., there is an odd number of students, etc.), invite them to join a pair of students and have each student compare two different numbers.

Select and sequence student strategies to share during the Connect for each round.

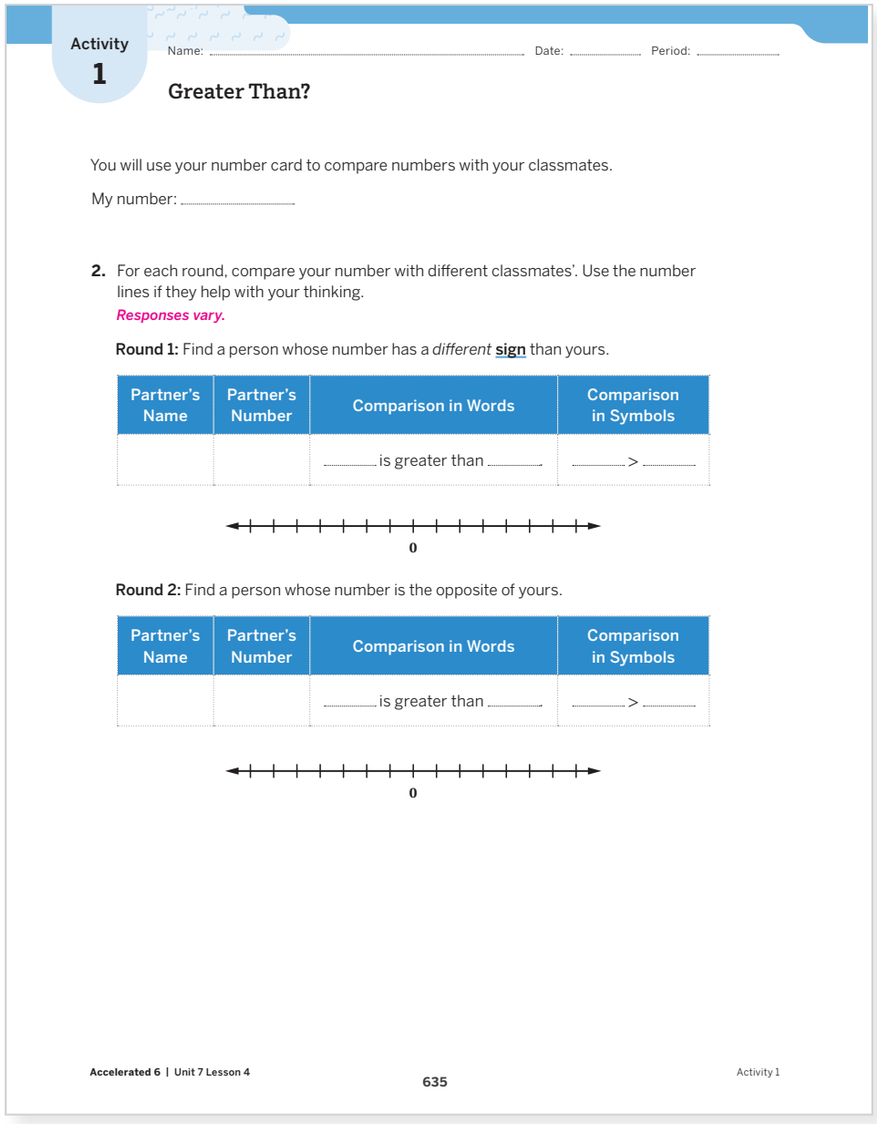
D Differentiation

Look for students who:	Respond to Student Thinking
Think $+3$ and 3 have different signs.	**Support** Encourage them to plot both numbers on the number line to help them recognize they are the same number.
Think $\frac{2}{5}$ has no opposite.	**Support** Consider asking, "How else can you think about the number $\frac{2}{5}$? Is there another way to write it?"
Complete both comparisons using words and inequality symbols.	**Stretch** Consider inviting them to think of a number that is less than both partners' numbers or between their numbers.

3 Connect

Invite students to share their comparisons and opposite pairs.

To surface the Key Takeaway, consider asking:
- "Can a negative number ever be greater than a positive number? Greater than 0?"
- "What is the sign of 0? What is its opposite?"



Activity 1

Name: _____ Date: _____ Period: _____

Greater Than?

You will use your number card to compare numbers with your classmates.

My number: _____

2. For each round, compare your number with different classmates'. Use the number lines if they help with your thinking.

Responses vary.

Round 1: Find a person whose number has a *different* **sign** than yours.

Partner's Name	Partner's Number	Comparison in Words	Comparison in Symbols
		_____ is greater than _____	_____ > _____

0

Round 2: Find a person whose number is the opposite of yours.

Partner's Name	Partner's Number	Comparison in Words	Comparison in Symbols
		_____ is greater than _____	_____ > _____

0

Accelerated 6 | Unit 7 Lesson 4 635 Activity 1

Key Takeaway: Any number that is located to the left of a number n is less than n. And any number that is located to the right of a number n is greater than n.
This means that any positive number is greater than any negative number, and any negative number is less than any positive number.

Small Groups | 10 min

Activity 2 Least to Greatest

Purpose: Students compare and order groups of rational numbers.

Short on time? Consider completing only Round 3.

1 Launch

For Problem 3, Direct students' attention to instructions for the activity. Explain to students they will also complete this activity in rounds, locating new partners for each round, and they will be using the same number card from Activity 1.

2 Monitor

Direct students' attention to prompts for each round, one at a time, reminding students that they should record their work with a new group each time.

To support students getting started, encourage them to compare one pair of values at a time.

ELL **English Language Learners** Invite students to use the sentence frames from Activity 1 to support their explanations.

Listen for students who use information from Activity 1 to compare and order positive and negative numbers. **(MTR.1.1)**

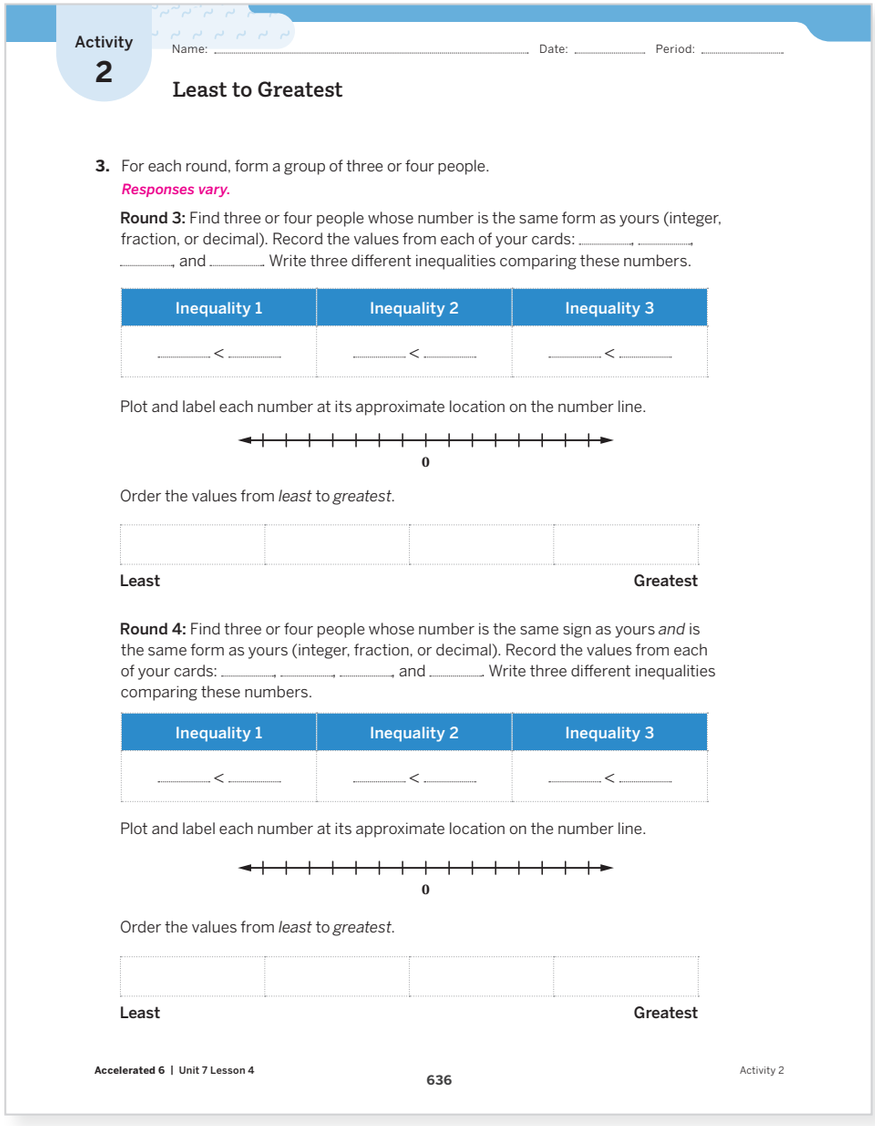
Note: If time allows, extend the activity by inviting students to form a line with everyone in the class so that all the numbers are in order from least to greatest.

3 Connect

Invite students to share their methods for comparing and ordering rational numbers, focusing on productive and generalizable strategies. **(MTR.1.1)**

Consider asking:

- "What is the sign of any number to the right of/above 0? To the left of/below 0?"
- "If you plot two numbers on a number line, how can you tell which one has a greater value?" The number to the right has a greater value than the number to the left.
- "How can you compare rational numbers in different forms?" It is helpful to compare rational numbers when they are in the same form and written to the same place value.



Activity 2

Name: _____ Date: _____ Period: _____

Least to Greatest

3. For each round, form a group of three or four people.

Responses vary.

Round 3: Find three or four people whose number is the same form as yours (integer, fraction, or decimal). Record the values from each of your cards: _____, _____, _____, and _____. Write three different inequalities comparing these numbers.

Inequality 1	Inequality 2	Inequality 3
_____ < _____	_____ < _____	_____ < _____

Plot and label each number at its approximate location on the number line.

0

Order the values from *least* to *greatest*.

Least Greatest

Round 4: Find three or four people whose number is the same sign as yours *and* is the same form as yours (integer, fraction, or decimal). Record the values from each of your cards: _____, _____, _____, and _____. Write three different inequalities comparing these numbers.

Inequality 1	Inequality 2	Inequality 3
_____ < _____	_____ < _____	_____ < _____

Plot and label each number at its approximate location on the number line.

0

Order the values from *least* to *greatest*.

Least Greatest

Accelerated 6 | Unit 7 Lesson 4 636 Activity 2

Pairs | 10 min

Activity 3 Ordering Numbers

Purpose: Students surface strategies for comparing two negative numbers and ordering rational numbers from least to greatest. **(MTR.1.1)**

Short on time? Consider completing only Problem 5.

1 Launch

Direct students' attention to the new set of numbers.

Encourage students to try each problem on their own first before comparing and discussing their answers with their partner. **(MTR.4.1)**

2 Monitor

Listen for students' explanations for why $-1.5 < -1$ and why -2 is the least of the numbers in the group.

ELL **English Language Learners** Invite students to talk through their thinking with a partner before writing it down. **(Writing and Speaking)**

A **Accessibility: Conceptual Processing** Invite students to solve a similar question with possible number sets to replace M, N, P, and R to support thinking abstractly.

D **Differentiation**

Look for students who:	Respond to Student Thinking
Need support getting started. (*Problem 4*)	**Support** Encourage them to create a number line and plot the given numbers.
List the numbers in the order -2, -1.5, -1, 1.5. (*Problem 4c*)	**Strengthen** Consider asking, "How would you explain to a peer why -2 is less than -1.5? Where would 0 be in this collection of numbers?"
Use the structure of the number line (i.e., which numbers are to the left and right of zero, and which numbers are further from zero) to justify responses. (*Problems 5a–5c*) **(MTR.1.1)**	**Stretch** Invite students to share during the Connect. Consider asking, "Are there other possible responses for Problems 5a, 5b, and 5c? How do you know?"

Look for students who use the structure of the number line to write inequalities, determine opposites, and evaluate and critique arguments about the connection between the distance from 0 and direction on the number line. **(MTR.1.1)**

3 Connect

Direct students' attention to the number line showing M, N, P, and R.

Consider asking:

- "Is ___ less than or greater than ___? How do you know?"
- "How would you explain why M is less than the other numbers? Why R is greater than the other numbers?"
- "How do you know if two numbers are opposites?"
- "How do you know which of two negative numbers is greater?"

Activity 3

Name: _____ Date: _____ Period: _____

Ordering Numbers

4. Here are four more numbers.

-2	1.5	-1.5	-1

a. Complete each number sentence using > or <.

-2 < 1.5 -1.5 < -1

b. Use two of the numbers to complete this sentence.

-1.5 is the opposite of 1.5 because . . . it is the same distance from 0 and on the opposite side of the number line.

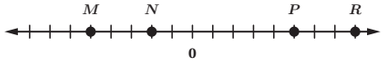
c. Order the numbers from *least* to *greatest*.

-2	-1.5	-1	1.5

Least Greatest

5. Use the number line to complete each sentence.

Responses and explanations vary.



a. M is the opposite of P because . . . it is the same distance from 0 and on the opposite side of the number line.

b. R is greater than N because . . . it is farther to the right on the number line.

c. M is the least of the numbers because . . . it is the farthest to the left on the number line.

Accelerated 6 | Unit 7 Lesson 4 637 Activity 3

Key Takeaway: The order of numbers from least to greatest is the same as they appear on a number line from left to right.

Whole Class | 5 min

Synthesis

Purpose: Students synthesize their understanding of strategies to compare rational numbers using a number line.

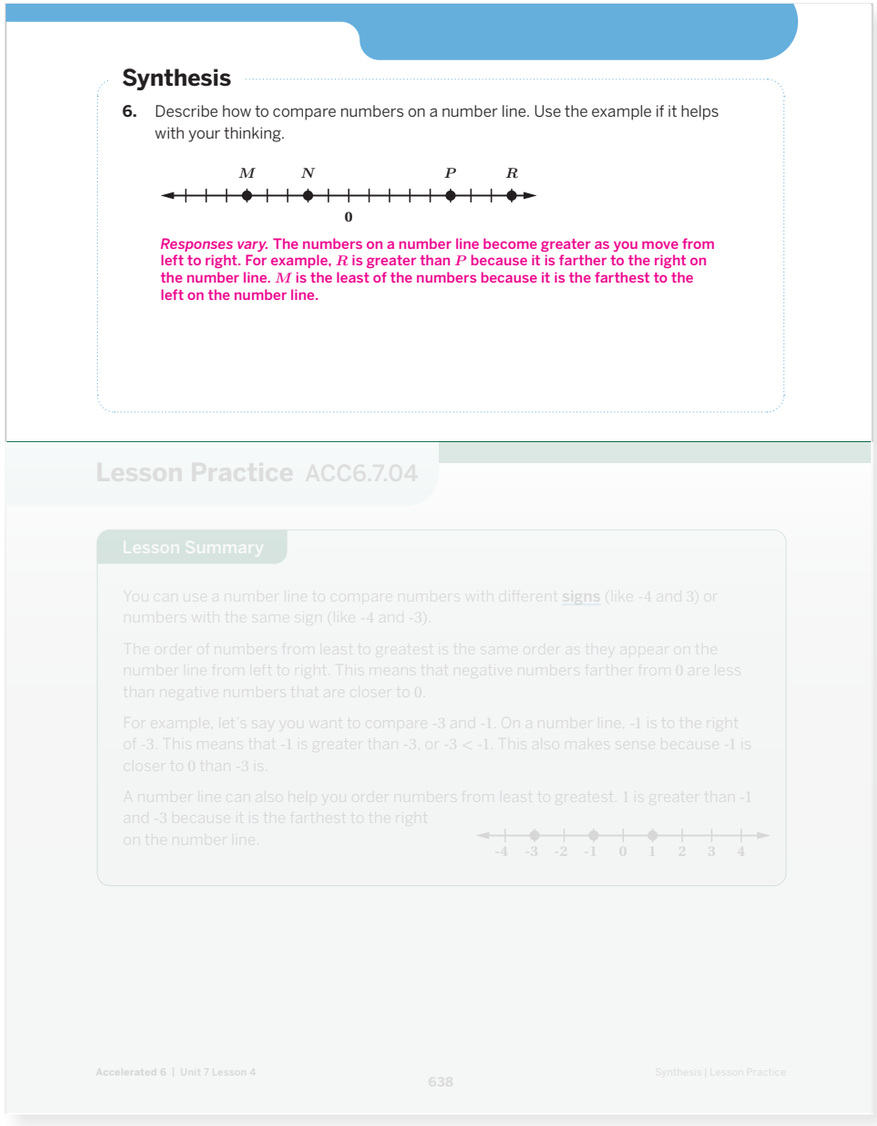
Invite students to respond independently, and then share their thinking with a partner.

Collect and share a variety of ideas, such as:

- On a horizontal number line, the number to the right has a greater value than the number to the left.
- All positive numbers and 0 are larger than all negative numbers.
- When comparing two negative numbers, the one closer to 0 is larger.

MLR **MLR2: Collect and Display** Use this routine to formalize the definition of *sign* and the strategies to compare and order rational numbers using students' language. Display any new language, examples, or diagrams publicly for students to reference later.

Note: If time allows, invite students to share with a partner what makes sense about each strategy and what questions they still have.



Synthesis

6. Describe how to compare numbers on a number line. Use the example if it helps with your thinking.

Responses vary. The numbers on a number line become greater as you move from left to right. For example, R is greater than P because it is farther to the right on the number line. M is the least of the numbers because it is the farthest to the left on the number line.

Lesson Practice ACC6.7.04

Lesson Summary

You can use a number line to compare numbers with different signs (like -4 and 3) or numbers with the same sign (like -4 and -3).

The order of numbers from least to greatest is the same order as they appear on the number line from left to right. This means that negative numbers farther from 0 are less than negative numbers that are closer to 0.

For example, let's say you want to compare -3 and -1. On a number line, -1 is to the right of -3. This means that -1 is greater than -3, or $-3 < -1$. This also makes sense because -1 is closer to 0 than -3 is.

A number line can also help you order numbers from least to greatest. 1 is greater than -1 and -3 because it is the farthest to the right on the number line.

Accelerated 6 | Unit 7 Lesson 4 638 Synthesis | Lesson Practice

Lesson Takeaway: When ordering three or more rational numbers from least to greatest, list them in the order they appear on the number line, moving from left to right. When ordering three or more rational numbers from greatest to least, list them in the order they appear on the number line, moving from right to left.

Independent | 5 min

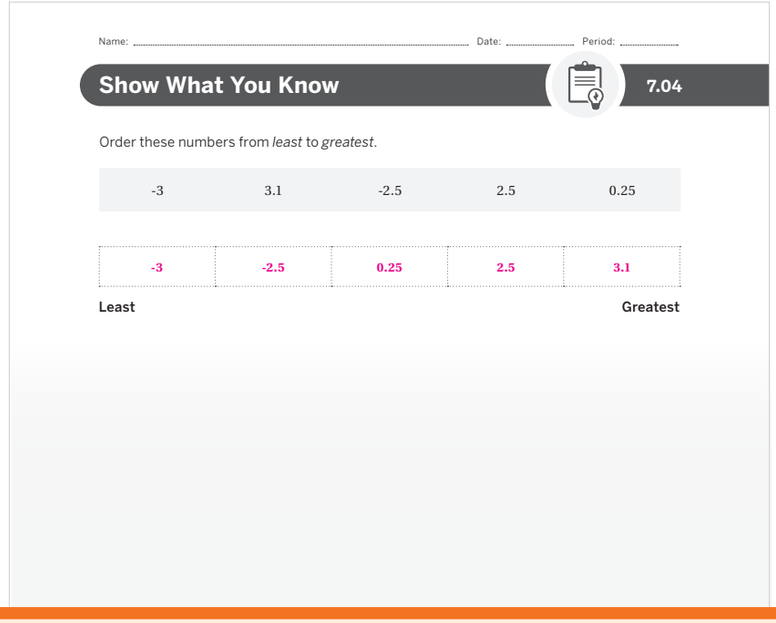
Show What You Know

Purpose: Students demonstrate their understanding by ordering rational numbers from least to greatest.

Today's Goals

1. **Goal:** Use a number line to compare and order positive and negative numbers.
2. **Language Goal:** Compare positive and negative numbers using the terms *greater than*, *less than*, and *opposite* and the symbols > and <. **(Reading, Writing, Speaking, and Listening)**

Show What You Know (Assessment Resources)



Name: _____ Date: _____ Period: _____

Show What You Know 7.04

Order these numbers from *least* to *greatest*.

-3	3.1	-2.5	2.5	0.25

-3	-2.5	0.25	2.5	3.1

Least Greatest

D Differentiation Use after Lesson 4

Support

Provide targeted intervention for students by using these resources.

If students need support comparing and ordering rational numbers:

» **Respond to Student Thinking:**

- Use the ***Ordering and Comparing Positive and Negative Numbers*** **Mini-lesson**.
- Review the **Lesson Summary**.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite them to explore the **Sub-Unit 1 Extension Activities**.

Professional Learning

In this lesson, students used number lines to compare and order rational numbers. How will that support them with comparing rational numbers that involve absolute values?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

6. Describe how to compare numbers on a number line. Use the example if it helps with your thinking.

Responses vary. The numbers on a number line become greater as you move from left to right. For example, R is greater than P because it is farther to the right on the number line. M is the least of the numbers because it is the farthest to the left on the number line.

Lesson Practice ACC6.7.04

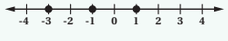
Lesson Summary

You can use a number line to compare numbers with different **signs** (like -4 and 3) or numbers with the same sign (like -4 and -3).

The order of numbers from least to greatest is the same order as they appear on the number line from left to right. This means that negative numbers farther from 0 are less than negative numbers that are closer to 0.

For example, let's say you want to compare -3 and -1. On a number line, -1 is to the right of -3. This means that -1 is greater than -3, or $-3 < -1$. This also makes sense because -1 is closer to 0 than -3 is.

A number line can also help you order numbers from least to greatest. 1 is greater than -1 and -3 because it is the farthest to the right on the number line.

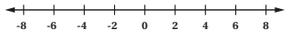


Lesson Practice
ACC6.7.04

Name: _____ Date: _____ Period: _____

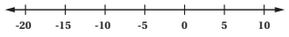
1. Complete each number sentence with a number that makes it true. Use the number line if it helps with your thinking. *Responses vary.*

a. $\square < 5$ b. $\square < -5$ c. $-5 < \square$ d. $-5 > \square$



2. Complete each number sentence with the symbol <, >, or =. Use the number line if it helps with your thinking.

a. $-5 < 2$ b. $5 > -5$ c. $-12 > -15$ d. $-12.5 < -12$



3. Circle whether each statement is *true* or *false*.

Statement A: -8.4 is to the right of -8.7 on the number line. (True) False

Statement B: -2.4 is greater than -2.3. True (False)

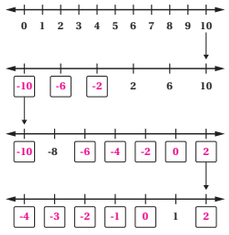
Statement C: $-\frac{11}{12} < -\frac{7}{12}$ (True) False

Choose one statement and explain your thinking.

Responses and explanations vary. **Statement B is false because -2.4 is farther left on the number line, so $-2.4 < -2.3$.**

4. Here is a number line maze.

Each number line has a different scale but at least one matching number, which is labeled with an arrow. Determine the missing values in this number line maze.



Lesson Practice
ACC6.7.04

Name: _____ Date: _____ Period: _____

FAST Practice

5. Here are five numbers: $-\frac{2}{5}$, -1, $\frac{4}{3}$, 1, $-\frac{3}{2}$.

These numbers are plotted on a horizontal number line. Which statement about the locations of the numbers is true?

A. $-\frac{2}{5}$ is the farthest to the left, and 1 is farthest to the right.

B. -1 is the farthest to the left, and 1 is farthest to the right.

C. $\frac{4}{3}$ is the farthest to the left, and $-\frac{3}{2}$ is farthest to the right.

(D.) $-\frac{3}{2}$ is the farthest to the left, and $\frac{4}{3}$ is farthest to the right.

Spiral Review

6. Solve each equation. Write each solution as a fraction and as a decimal.

Equation	Solution (Fraction)	Solution (Decimal)
$2x = 3$	$x = \frac{3}{2}$ (or equivalent)	$x = 1.5$
$5y = 3$	$y = \frac{3}{5}$ (or equivalent)	$y = 0.6$
$0.3z = 0.09$	$z = \frac{3}{10}$ (or equivalent)	$z = 0.3$

Problems 7–10: Each lap around a track is 400 meters. How many meters will someone run in:

7. 2 laps? **800 meters**

8. 5 laps? **2,000 meters**

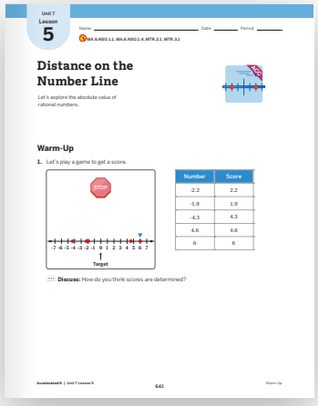
9. x laps? **$400x$ meters**

10. If Sol ran 7,600 meters, how many laps did he run? Explain your thinking.

19 laps. *Explanations vary.* If Sol ran 8,000 meters, that would be 20 laps. Because he ran 400 meters less than 8,000 meters, he ran $20 - 1 = 19$ laps.

Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1–3	2	MA.6.NSO.1.1, MA.6.NSO.1.2
	4	3	MA.6.NSO.1.1
FAST Practice	5	2	MA.6.NSO.1.1, MA.6.NSO.1.2
Spiral Review			
	6	2	MA.6.AR.2.4
	7–8	1	MA.6.AR.3.5
	9	2	MA.6.AR.1.1
	10	2	MA.6.AR.3.5


Unit 7
Lesson 5
Distance on the Number Line
Let's explore the absolute value of rational numbers.
Warm-Up
1. Let's play a game to get a score.
Number | Score
-2.2 | 2.2
-1.9 | 1.9
4.3 | 4.3
4.6 | 4.6
6 | 6
Target
Discuss: How do you think scores are determined?

Distance on the Number Line

Introduction to Absolute Value

Let's explore the absolute value of rational numbers.

Learning Outcomes

Today's Goals

1. **Goal:** Use inequality symbols to compare rational numbers and *absolute values*.
2. **Language Goal:** Understand the absolute value of a number as its distance from 0 on the number line. **(Reading, Writing, Speaking, and Listening)**

Students learn and apply the term **absolute value** to describe the distance a value is from 0. They recognize that opposite numbers have the same absolute value, and compare and contrast integers and their absolute values using inequalities.

Coherence

Prior Learning

In Lessons 2–3, students learned about locating rational numbers on the number line. In Lesson 4, they practiced comparing and ordering rational numbers.

Future Learning

In Lesson 6, students will continue to explore the concept of absolute value using the contexts of elevation and account balance in real-world situations.

Connections

- Students are introduced to the concept of absolute value using number lines. (mathematical)

FL B.E.S.T. Math Benchmarks

Addressing

MA.6.NSO.1.1
Extend previous understanding of numbers to define rational numbers. Plot, order and compare rational numbers.

MA.6.NSO.1.4
Solve mathematical and real-world problems involving absolute value, including comparison of absolute value.

Mathematical Thinking and Reasoning Standards: MTR.2.1, MTR.3.1

ELA Expectations: ELA.K12.EE.1.1, ELA.K12.EE.4.1, ELA.K12.EE.5.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.4.NSO.1.3	MA.7.NSO.2.1 MA.8.NSO.1.2

Vocabulary

New Vocabulary

absolute value

Lesson at a Glance

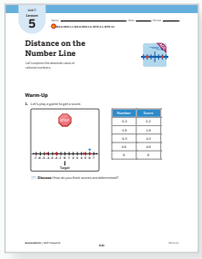
~ 45 min

MA.6.NSO.1.1, MA.6.NSO.1.4

Warm-Up

Independent | 5 min

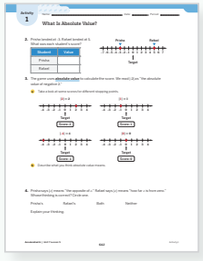
Students play a game where their score is their distance from a target number.



Activity 1

Pairs | 10 min

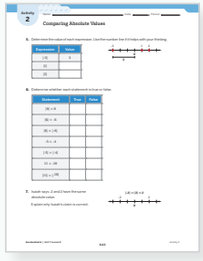
Students recognize that the **absolute value** of a number is its distance from 0. **Routines:** Think-Pair-Share, Decide and Defend



Activity 2

Pairs | 10 min

Students extend their understanding of absolute value to make comparisons that include absolute value.



Activity 3

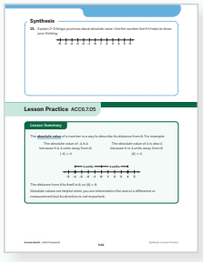
Pairs | 10 min

Students complete number puzzles by using inequalities to compare rational numbers and absolute values. **Routine:** Think-Pair-Share

Synthesis

Whole Class | 5 min

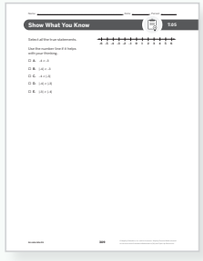
Students synthesize their understanding of absolute value. MLR2: Collect and Display



Show What You Know

Independent | 5 min

Students demonstrate their understanding by comparing rational numbers and absolute values.



Prep Checklist

This lesson includes:

- Student Edition
- Show What You Know

Independent | 5 min

Warm-Up

Purpose: Students play a game where their score is their distance from a target number.

Short on time? Consider completing the Warm-Up as a whole class.

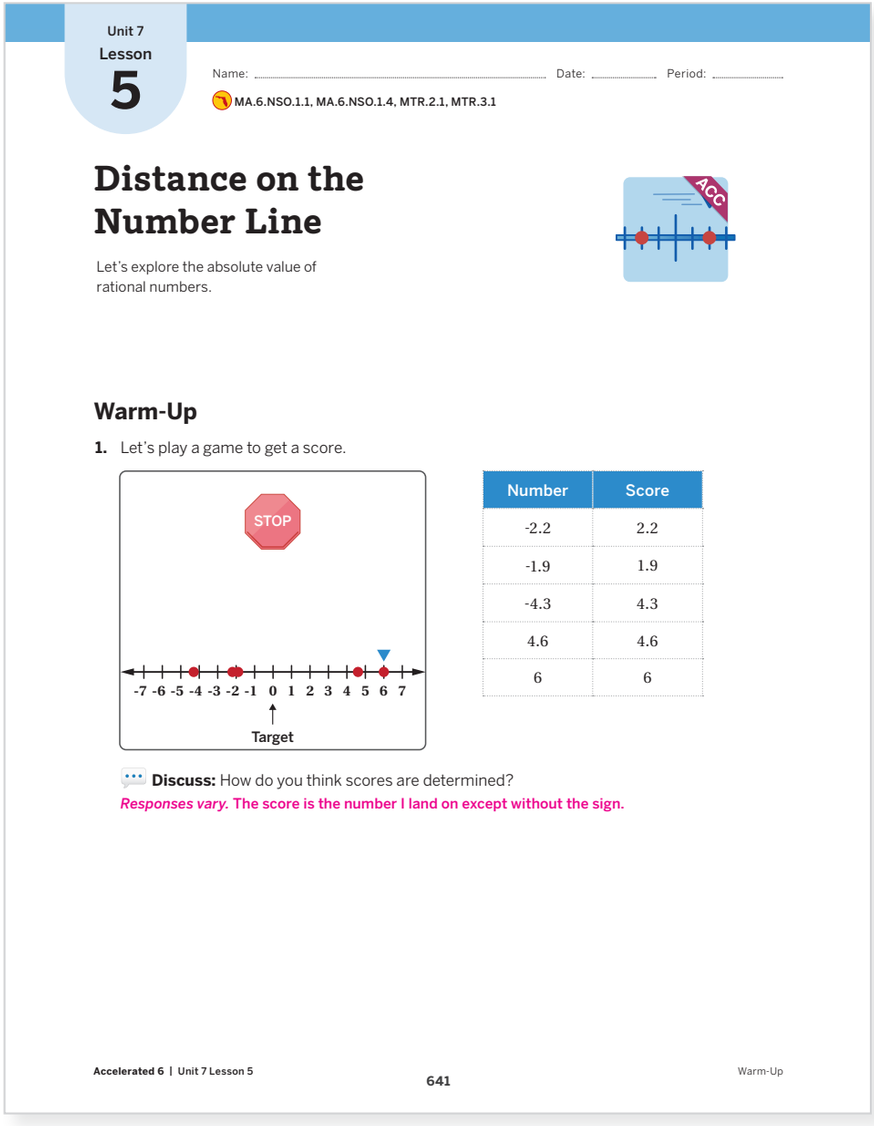
1 Launch

Direct students' attention to the game images. Share that, in the game, the arrow continues to move back and forth along the number line until you press the stop button. The points shown along the number line are places where the arrow has stopped. The table shows the score for each time the arrow has stopped.

A **Accessibility: Conceptual Processing**
As students identify patterns and relationships, invite them to create a hypothesis about how the score is determined. Consider asking:
- "What patterns do you see in the table?"
- "How could we check to see if this is true?"

2 Connect

Consider asking:
- "What patterns do you notice? What were the scores for a negative number? Positive number?"
- "Could a player stop at 0?"
- "How is the score related to the target number? What would my score be if I stopped at -3? -5.6? 2.3?"



Unit 7
Lesson 5

Name: ..... Date: ..... Period: .....

MA.6.NSO.1.1, MA.6.NSO.1.4, MTR.2.1, MTR.3.1

Distance on the Number Line

Let's explore the absolute value of rational numbers.

ACC

Warm-Up

1. Let's play a game to get a score.

Number	Score
-2.2	2.2
-1.9	1.9
-4.3	4.3
4.6	4.6
6	6

Discuss: How do you think scores are determined?

Responses vary. The score is the number I land on except without the sign.

Accelerated 6 | Unit 7 Lesson 5 641 Warm-Up

Pairs | 10 min

Activity 1 What Is Absolute Value?

Purpose: Students recognize that the **absolute value** of a number is its distance from 0.

1 Launch

Invite students to determine Prisha's and Rafael's scores and invite a few of them to share their thinking with the class.

Use the Think-Pair-Share routine to help students shape their ideas about absolute value. Invite students to think independently and then take turns sharing with a partner.

ELL **English Language Learners** Consider grouping students with different English language proficiency levels, so they can listen to and learn from their peers. **(Speaking and Listening)**

2 Monitor

Select and sequence a variety of responses.

Look for students who connect the value's distance from the 0 (the target) to the absolute value. **(MTR.3.1)**

D **Differentiation: (Support)** Students might think Prisha is correct. Consider asking, "What would the score be if she landed on 2? -3?"

3 Connect

Display the distribution of responses.

Use the Decide and Defend routine to support students in making arguments and critiquing the reasoning of others. Consider asking, "What reasoning or evidence supports your claim?"

Create a class definition for **absolute value** using students' language and the glossary. To help generate student language, consider asking:

- "How do you connect the scores from the Target Number game with the concept of absolute value?"
- "How would you write 'the absolute value of -4'?" **(MTR.2.1)**
- "How would you read |-3|? |7|?" **(MTR.3.1)**
- "If a number is 7 units away from zero on the number line, what is its absolute value?"

Activity 1

Name: _____ Date: _____ Period: _____

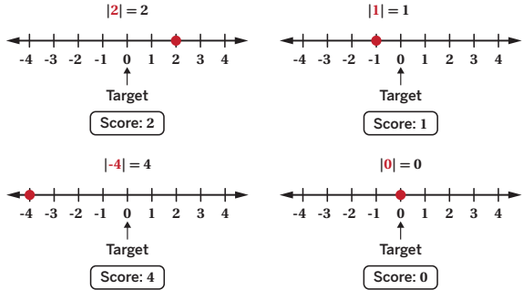
What Is Absolute Value?

2. Prisha landed at -3. Rafael landed at 5. What was each student's score?

Student	Value
Prisha	3
Rafael	5

3. The game uses **absolute value** to calculate the score. We read |-2| as "the absolute value of negative 2."

a. Take a look at some scores for different stopping points.



b. Describe what you think absolute value means.

Responses vary.

- Absolute value means the number without the sign.
- Absolute value means how far away the number is from 0.
- Absolute value means take all the negative numbers and make them positive.

4. Prisha says $|x|$ means "the opposite of x." Rafael says $|x|$ means "how far x is from zero." Whose thinking is correct? Circle one.

Prisha's (Rafael's) Both Neither

Explain your thinking.

Explanations vary.

- Sometimes $|x|$ has the same value as the opposite of x, and sometimes it does not. For example, the opposite of 3 is -3, but |3| is 3.
- The absolute value is always a positive number or 0. "How far x is from zero" is also always positive or 0 because you can't be a negative distance away from something. The opposite of a number can be positive or negative.

Accelerated 6 | Unit 7 Lesson 5 642 Activity 1

Pairs | 10 min

Activity 2 Comparing Absolute Values

Purpose: Students extend their understanding of absolute value to make comparisons that include absolute value.

1 Launch

Encourage students to get feedback from a partner to help them revise their thinking.

2 Monitor

To support students getting started, consider inviting them to explain what they think the measurement line on the left has to do with the first row of the table. **(MTR.2.1)**

Encourage students to share their reasoning with a partner and work together to reach an agreement about how to determine if each statement is true or false.

Note: The statements intentionally use similar numbers in order to support students in attending to precision. **(MTR.3.1)**

D Differentiation *(Problem 6)*

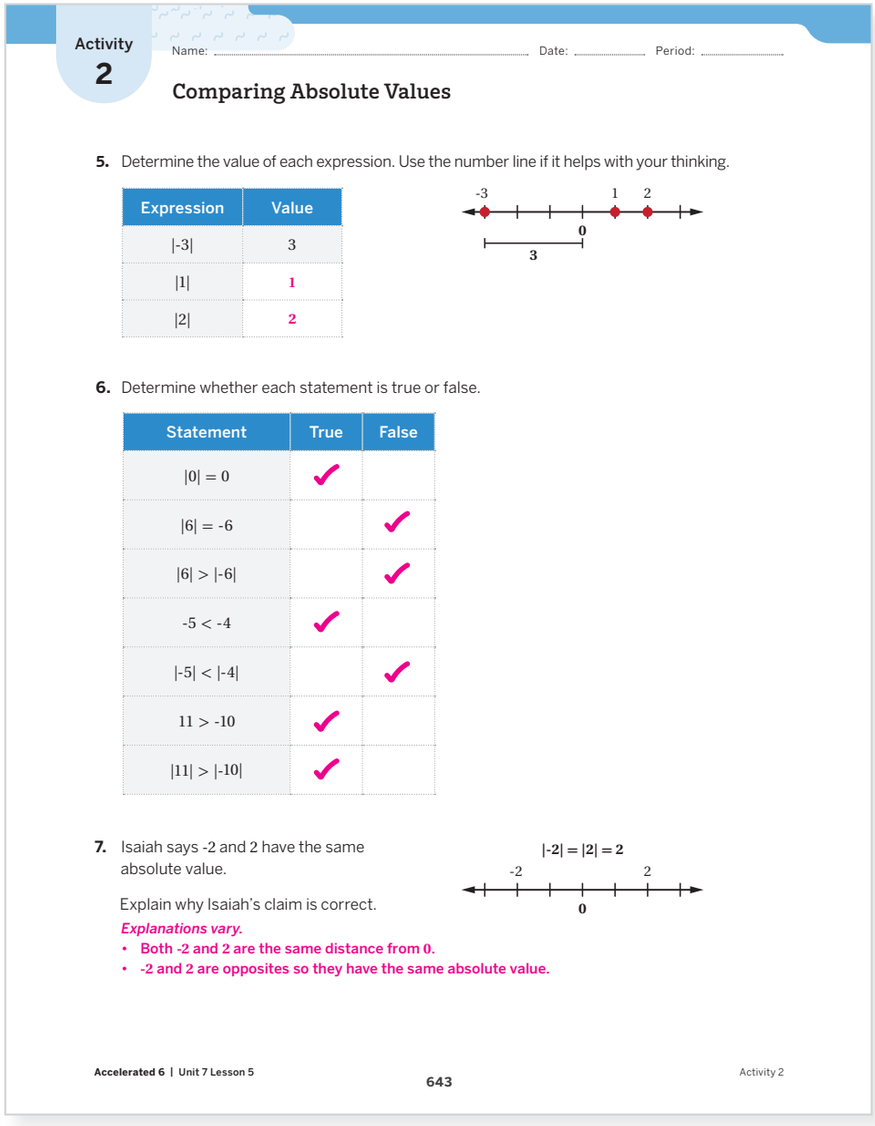
Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Encourage them to choose a statement without an absolute value comparison to start with.
May not be sure if it is possible to have $\|0\|$.	**Support** Consider asking these students how far away they are from where they are currently sitting or standing.
Recognize: • The absolute value of any non-zero number is always positive. • Opposite pairs have the same absolute value.	**Stretch** Encourage them to create their own set of statements that they think might be challenging, then to trade statements with a classmate.

Look for explanations and strategies to compare rational numbers to share during the Connect.

3 Connect

Invite students to share their strategies and explanations to compare the distances from 0.

To surface the Key Takeaway, consider asking:
- "Does finding a number's absolute value always mean changing the sign?"
- "What is the absolute value of 0? Why is it different from other numbers?"
- "Can two numbers have the same absolute value?"



Activity 2

Name: Date: Period:

Comparing Absolute Values

5. Determine the value of each expression. Use the number line if it helps with your thinking.

Expression	Value
$\|-3\|$	3
$\|1\|$	1
$\|2\|$	2

6. Determine whether each statement is true or false.

Statement	True	False
$\|0\| = 0$	✓	
$\|6\| = -6$		✓
$\|6\| > \|-6\|$		✓
$-5 < -4$	✓	
$\|-5\| < \|-4\|$		✓
$11 > -10$	✓	
$\|11\| > \|-10\|$	✓	

7. Isaiah says -2 and 2 have the same absolute value.

Explain why Isaiah's claim is correct.

Explanations vary.
- Both -2 and 2 are the same distance from 0.
- -2 and 2 are opposites so they have the same absolute value.

Accelerated 6 | Unit 7 Lesson 5 643 Activity 2

Key Takeaway: The **absolute value** of a number is its distance from 0 on the number line. Absolute value of any non-zero number is always positive, whereas $|0| = 0$. Opposites have the same absolute value because they are the same distance from zero.

Pairs | 10 min

Activity 3 Absolute Value Puzzles

Purpose: Students complete number puzzles by using inequalities to compare rational numbers and absolute values.

1 Launch

Support students getting started by displaying a false inequality, like $|-1| > 1$.

Use the Think-Pair-Share routine to help students shape their ideas about how they know this inequality is false and then to create a true inequality. Invite students to think independently and then take turns sharing with a partner. Invite several students to share their or their partner's reasoning with the class.

ELL **English Language Learners** Provide sentence frames to help students explain their thinking (e.g., I know my inequality is true because _____.).

2 Monitor

Invite students to check their statements with a partner.

D Differentiation

Look for students who:	Respond to Student Thinking
Think the absolute value of a number is greater than the number itself.	**Support** Consider inviting students to evaluate $\|-5\|$ and $\|5\|$. Then compare the values with the original numbers.
Think that absolute value means the opposite.	**Support** Remind students that absolute value represents the distance of the number from 0, so it is always positive regardless of sign or direction.
Interpret the inequality symbols and signs appropriately.	**Stretch** Invite them to think of all the integers that have an absolute value of less than 3.

Activity 3 continued >

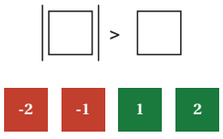
Activity 3

Name: _____ Date: _____ Period: _____

Absolute Value Puzzles

8. a Make a true inequality by using each number at most once.

Responses vary.
- $|-2| > 1$
- $|1| > -1$



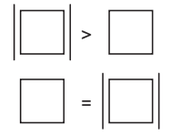
-2 -1 1 2

b Explain how you know your inequality is true.

Explanations vary.
- $|-2| > 1$ is true because $|-2| = 2$ and $2 > 1$.
- $|1| > -1$ is true because $|1|$ is positive and all positive numbers are greater than all negative numbers.

9. Make true statements by using each number exactly once.

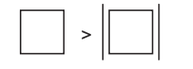
Responses vary.
- $|-5| > 3$ and $4 = |-4|$
- $|3| > -5$ and $4 = |-4|$

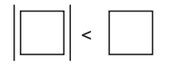


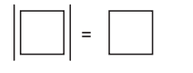
3 4 -4 -5

10. Make true statements by using each number exactly once.

$3 > |-2|$, $|-1| < 2$, and $|-4| = 4$







-1 -2 -4

2 3 4

Accelerated 6 | Unit 7 Lesson 5 644 Activity 3

Pairs | 10 min

Activity 3 Absolute Value Puzzles (continued)

Purpose: Students complete number puzzles by using inequalities to compare rational numbers and absolute values.

2 Monitor

Select and sequence one to two responses for each of the most popular choices to display, and invite those students to justify why their inequalities are true during the Connect.

D **Differentiation: You're invited to explore more. (Stretch)** Invite students who would like to explore absolute value further to complete this optional task. Encourage them to discuss their thinking with a partner.

3 Connect

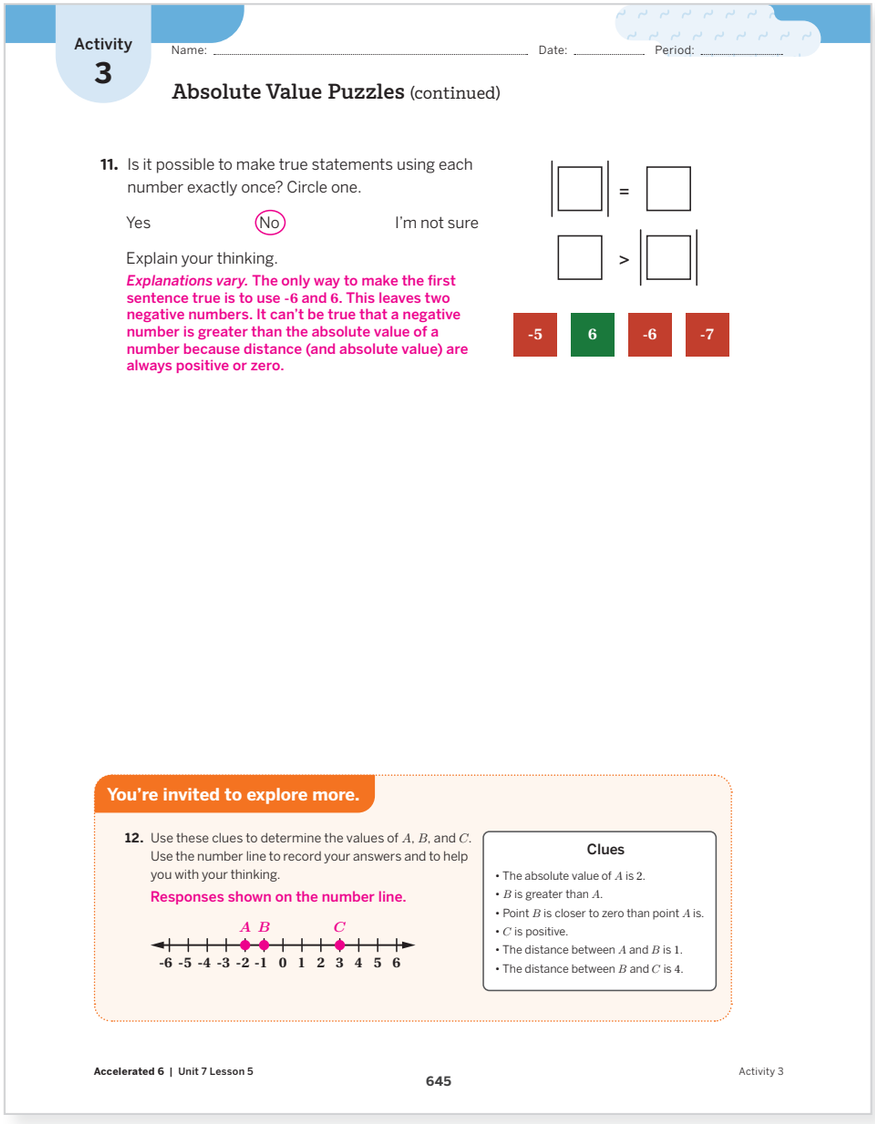
Invite students to share their responses, calling attention to any conflict or consensus you see.

Consider asking:

- "What might someone who chose a different answer have been thinking?"
- "How could you change the puzzle to make it possible?"

To surface the Key Takeaway, consider asking:

- "What are some integers less than -3? Less than |-3|?"
- "If $|x| = 2$, what are the possible values that x can be?"
- "If $|x| < 2$, what are the possible values that x can be?"



Activity 3

Name: _____ Date: _____ Period: _____

Absolute Value Puzzles (continued)

11. Is it possible to make true statements using each number exactly once? Circle one.

Yes (No) I'm not sure

Explain your thinking.

Explanations vary. The only way to make the first sentence true is to use -6 and 6. This leaves two negative numbers. It can't be true that a negative number is greater than the absolute value of a number because distance (and absolute value) are always positive or zero.

$|\square| = \square$

$\square > |\square|$

-5 6 -6 -7

You're invited to explore more.

12. Use these clues to determine the values of A, B, and C. Use the number line to record your answers and to help you with your thinking.

Responses shown on the number line.

Clues

- The absolute value of A is 2.
- B is greater than A.
- Point B is closer to zero than point A is.
- C is positive.
- The distance between A and B is 1.
- The distance between B and C is 4.

Accelerated 6 | Unit 7 Lesson 5 645 Activity 3

Key Takeaway: The absolute value of a number can have a different value than the number, so inequalities for these comparisons can use different symbols. For example, $-5 < 2$ but $|-5| > 2$.

Whole Class | 5 min

Synthesis

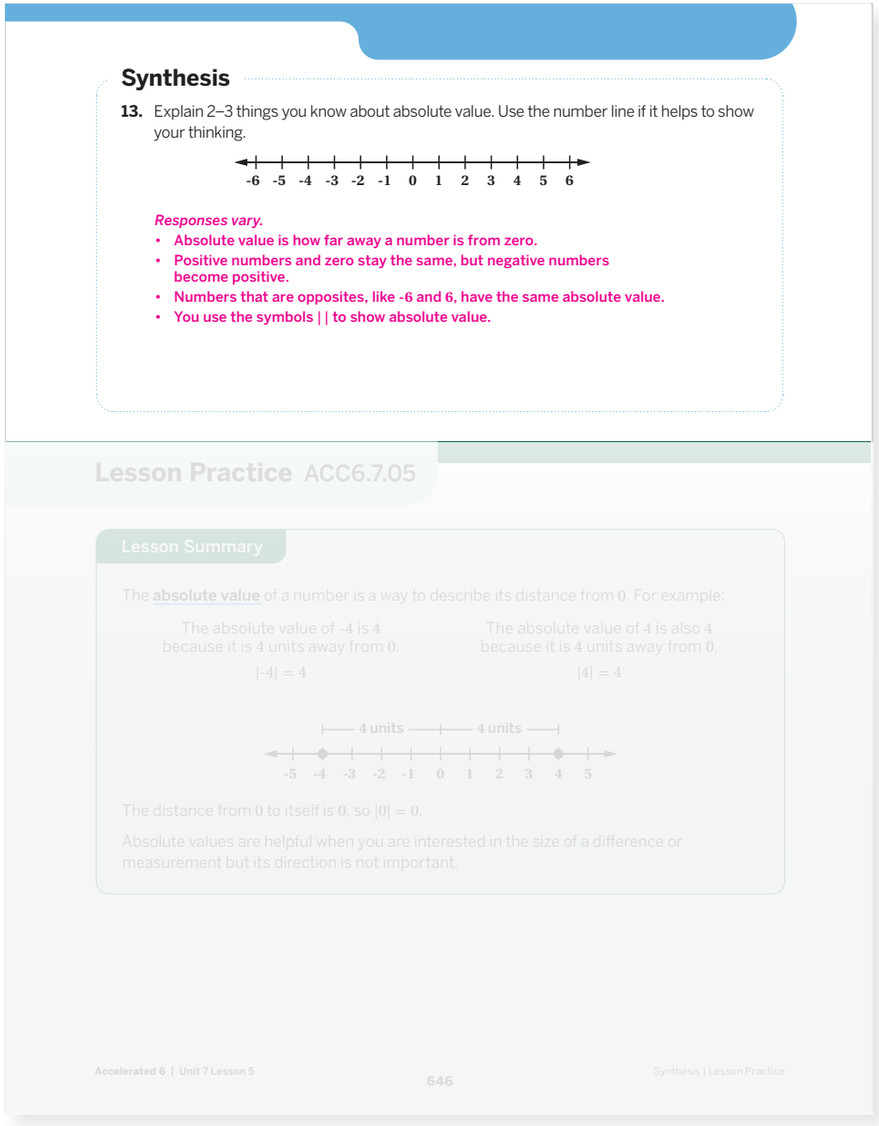
Purpose: Students synthesize their understanding of absolute value.

Invite students to respond independently, and then share their thinking with a partner.

Collect and share a variety of ideas, such as:
- Absolute value is the distance of a number from 0 on the number line.
- Opposite numbers have the same absolute value.

MLR **MLR2: Collect and Display** Use this routine to formalize the definition of *absolute value* using students' language. Display any new language, examples, or diagrams publicly for students to reference later.

Note: If time allows, invite students to revise their previous thinking based on the discussion.



Synthesis

13. Explain 2–3 things you know about absolute value. Use the number line if it helps to show your thinking.

-6 -5 -4 -3 -2 -1 0 1 2 3 4 5 6

Responses vary.
- Absolute value is how far away a number is from zero.
- Positive numbers and zero stay the same, but negative numbers become positive.
- Numbers that are opposites, like -6 and 6, have the same absolute value.
- You use the symbols | | to show absolute value.

Lesson Practice ACC6.7.05

Lesson Summary

The absolute value of a number is a way to describe its distance from 0. For example:

The absolute value of -4 is 4 because it is 4 units away from 0. $|-4| = 4$

The absolute value of 4 is also 4 because it is 4 units away from 0. $|4| = 4$

4 units 4 units

-5 -4 -3 -2 -1 0 1 2 3 4 5

The distance from 0 to itself is 0, so $|0| = 0$.

Absolute values are helpful when you are interested in the size of a difference or measurement but its direction is not important.

Accelerated 6 | Unit 7 Lesson 5 646 Synthesis | Lesson Practice

Lesson Takeaway: The distance of a number, n, from 0 on a number line is called its **absolute value**.

It is written as $|n|$ and read as "the absolute value of n." Opposites have the same absolute value because they are the same distance from zero. The absolute value of any non-zero number is always positive and the absolute value of 0 is 0.

Independent | 5 min

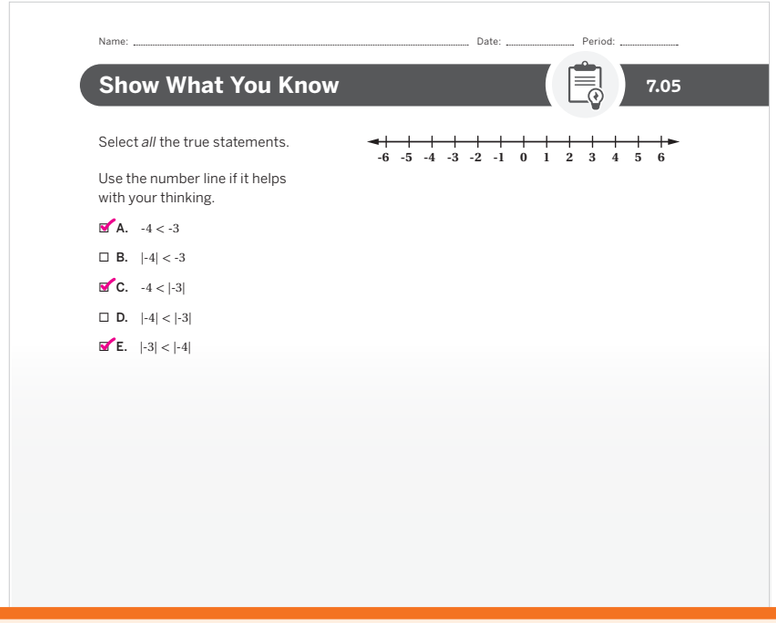
Show What You Know

Purpose: Students demonstrate their understanding by comparing rational numbers and absolute values.

Today's Goals

1. **Goal:** Use inequality symbols to compare rational numbers and *absolute values*.
2. **Language Goal:** Understand the absolute value of a number as its distance from 0 on the number line. **(Reading, Writing, Speaking, and Listening)**

Show What You Know (Assessment Resources)



Name: _____ Date: _____ Period: _____

Show What You Know 7.05

Select *all* the true statements.

Use the number line if it helps with your thinking.

-6 -5 -4 -3 -2 -1 0 1 2 3 4 5 6

- ☑ A. $-4 < -3$
- ☐ B. $|-4| < -3$
- ☑ C. $-4 < |-3|$
- ☐ D. $|-4| < |-3|$
- ☑ E. $|-3| < |-4|$

D Differentiation Use after Lesson 5

Support

Provide targeted intervention for students by using these resources.

If students need support comparing and ordering numbers and absolute values:

» **Respond to Student Thinking:**

- Revisit the **Warm-Up** as a class and invite students to share strategies for calculating the scores.
- Review the **Lesson Summary**.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite them to explore the **Sub-Unit 1 Extension Activities**.
- Invite them to create their own card puzzles like those in Activity 3. Encourage them to share and solve each other's problems.
- Invite them to complete the **Explore More** task.

Professional Learning

In this lesson, students developed early ideas of **absolute value**. How will their understanding of absolute value build over the next lesson?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

13. Explain 2–3 things you know about absolute value. Use the number line if it helps to show your thinking.

Responses vary.

- Absolute value is how far away a number is from zero.
- Positive numbers and zero stay the same, but negative numbers become positive.
- Numbers that are opposites, like -6 and 6, have the same absolute value.
- You use the symbols | | to show absolute value.

Lesson Practice ACC6.7.05

Lesson Summary

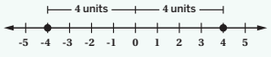
The **absolute value** of a number is a way to describe its distance from 0. For example:

The absolute value of -4 is 4 because it is 4 units away from 0.

$|-4| = 4$

The absolute value of 4 is also 4 because it is 4 units away from 0.

$|4| = 4$



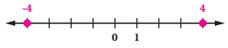
The distance from 0 to itself is 0, so $|0| = 0$.

Absolute values are helpful when you are interested in the size of a difference or measurement but its direction is not important.

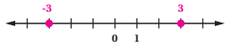
Lesson Practice
ACC6.7.05

Name: _____ Date: _____ Period: _____

1. Plot and label all numbers that are 4 units away from 0.



2. Plot and label all numbers with an absolute value of 3.



3. Ivory claims that a number and its opposite will always have the same absolute value. Is Ivory correct? Explain your thinking.

Yes. *Explanations vary.* Opposite numbers are the same distance away from 0. Because absolute value is another way of saying how far away a number is from 0, opposites have the same absolute value.

Problems 4–11: Complete each number sentence with the symbol <, >, or =.

4. -2 ___<___ 1

5. |-2| ___>___ |1|

6. 2 ___>___ -1

7. |2| ___>___ |-1|

8. 3 ___>___ -3

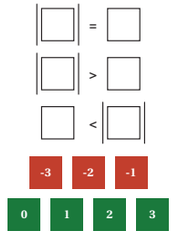
9. |3| ___=___ |-3|

10. |-2| ___<___ |-4|

11. |-2| ___>___ -4

12. Make each statement true by using each number at most once.

Responses vary. $|-3| = 3$, $|-2| > 1$, and $-1 < |2|$



-3 -2 -1

0 1 2 3

Lesson Practice
ACC6.7.05

Name: _____ Date: _____ Period: _____

FAST Practice

13. Which list of absolute value expressions are ordered from *least* to *greatest* value?

A. |0|, |1|, |-5|, |-7|

B. |-7|, |-5|, |1|, |0|

C. |-7|, |-5|, |0|, |1|

D. |1|, |0|, |-5|, |-7|

Spiral Review

Problems 14–17: Determine the value of each quotient.

14. $24 \div 15 = 1.6$ (or equivalent)

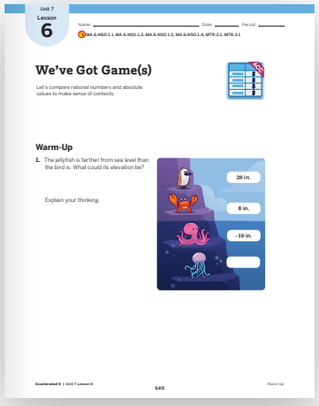
15. $0.24 \div 0.15 = 1.6$ (or equivalent)

16. $0.24 \div 0.015 = 16$

17. $0.024 \div 0.015 = 1.6$ (or equivalent)

Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1, 4, 6, 8	1	MA.6.NSO.1.1
	2, 5, 7, 9–11	1	MA.6.NSO.1.4
	3	2	MA.6.NSO.1.4
	12	3	MA.6.NSO.1.1, MA.6.NSO.1.4
FAST Practice	13	2	MA.6.NSO.1.1, MA.6.NSO.1.4
Spiral Review			
	14–17	1	MA.6.NSO.2.1



We've Got Game(s)

Comparing Numbers and Distances From Zero

Let's compare rational numbers and absolute values to make sense of contexts.

Learning Outcomes

Today's Goals

1. **Goal:** Use absolute values to compare rational numbers in context.
2. **Language Goal:** Justify whether it makes sense to use numbers or their absolute values when making a comparison in context. **(Reading, Writing, Speaking, and Listening)**

Students use precise language to distinguish between the order of rational numbers and their absolute values. They apply their understanding to visualize the possible elevations of several animals and compare absolute locations with relative distances. Students then reason with rational numbers and absolute values to write and explain mathematical comparison statements about account balances. **(MTR.2.1, MTR.3.1)**

Coherence

Prior Learning

In Lesson 5, students explored absolute values and reasoning with distances from 0 with positive and negative numbers. They practiced ordering and comparing rational numbers and absolute values.

Future Learning

In Unit 8, students will be introduced to inequalities with variables and connect verbal descriptions, symbols, and number line representations of inequalities.

Connections

- Students apply their understanding of absolute value to solve problems. (mathematical, real world)

FL B.E.S.T. Math Benchmarks

Addressing

MA.6.NSO.1.3
Given a mathematical or real-world context, interpret the absolute value of a number as the distance from zero on a number line. Find the absolute value of rational numbers.

MA.6.NSO.1.4
Solve mathematical and real-world problems involving absolute value, including the comparison of absolute value.

Also Addressing: **MA.6.NSO.1.1, MA.6.NSO.1.2**

Mathematical Thinking and Reasoning Standards: MTR.2.1, MTR.3.1

ELA Expectations: ELA.K12.EE.1.1, ELA.K12.EE.5.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.5.NSO.1.4	**MA.7.NSO.2.1**
	MA.8.NSO.1.1
	MA.8.NSO.1.2

Lesson at a Glance

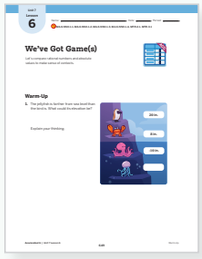
~ 45 min

MA.6.NSO.1.1, MA.6.NSO.1.2, MA.6.NSO.1.3, MA.6.NSO.1.4

Warm-Up

Pairs | 5 min

Students reason about the order of negative elevations using a vertical number line.



Activity 1

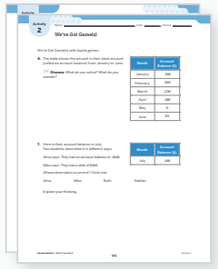
Pairs | 10 min

Students distinguish between absolute value and order in the context of elevation and then make sense of inequality statements involving distance and location. **Routines:** Notice and Wonder, MLR7: Compare and Connect

Activity 2

Small Groups | 20 min

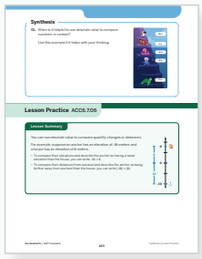
Students use and interpret absolute values and negative numbers that represent and compare changes in amounts of money. **Routines:** Notice and Wonder, Decide and Defend (MTR.2.1)



Synthesis

Whole Class | 5 min

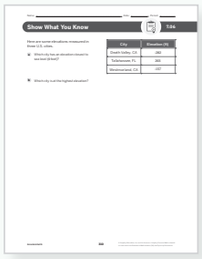
Students synthesize their understanding of comparing absolute values versus rational numbers to make sense of contexts.



Show What You Know

Independent | 5 min

Students demonstrate their understanding by comparing elevations and distances from sea level.



Prep Checklist

Prepare the Activity 2 Cards.

This lesson includes:

- Student Edition
- Show What You Know
- Activity 2 cards, one set per group, pre-cut

Pairs | 5 min

Warm-Up

Purpose: Students reason about the order of negative elevations using a vertical number line.

1 Launch

Direct students' attention to the elevation chart of animals.

ELL **English Language Learners** Invite students to highlight key words, such as *farther from sea level* and *elevation*, to help them decide what information is known and what needs to be determined. **(Reading)**

To surface connections, consider asking:

- "How can positive or negative numbers be used to describe the elevations for the animals?"
- "How far away is the bird from the sea level? How can you represent its distance?"
- "If the jellyfish were at the same distance from sea level as the bird, what elevation would it have?"

Invite students to think independently before discussing their ideas with a partner.

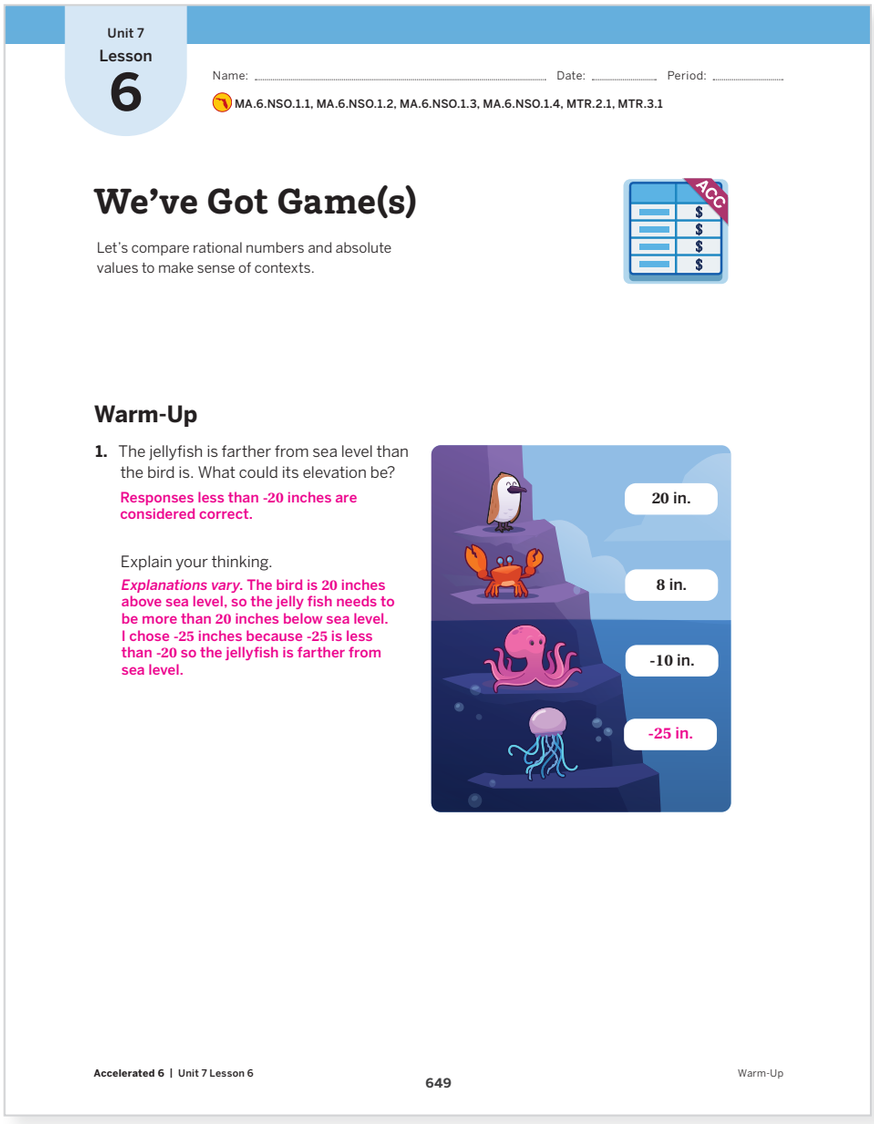
A **Accessibility: Visual-Spatial Processing** Invite students to pick a specific point on the negative side of the number line and describe whether the point is farther from sea level than the bird.

2 Connect

Invite students to share their strategies to determine the possible elevations for the jellyfish, focusing on finding the opposite of the bird's location on the number line.

To surface Key Takeaway, consider asking, "How would you describe all the possible elevations of the jellyfish? How do you know?"

Consider sharing that all possible elevations for the jellyfish can be represented with a value less than -20 inches if it does not come up naturally.



Unit 7
Lesson
6

Name: ..... Date: ..... Period: .....

MA.6.NSO.1.1, MA.6.NSO.1.2, MA.6.NSO.1.3, MA.6.NSO.1.4, MTR.2.1, MTR.3.1

We've Got Game(s)

Let's compare rational numbers and absolute values to make sense of contexts.

ACC

Warm-Up

1. The jellyfish is farther from sea level than the bird is. What could its elevation be?

   Responses less than -20 inches are considered correct.

   Explain your thinking.

   Explanations vary. The bird is 20 inches above sea level, so the jelly fish needs to be more than 20 inches below sea level. I chose -25 inches because -25 is less than -20 so the jellyfish is farther from sea level.

Accelerated 6 | Unit 7 Lesson 6 649 Warm-Up

Key Takeaway: Comparing elevations differs from comparing distances from sea level. Two different elevations can be the same distance from sea level, where one is below (a negative) and one is above (a positive).

Pairs | 10 min

Activity 1 Comparing Distances From Sea Level

Purpose: Students distinguish between absolute value and order in the context of elevation and then make sense of inequality statements involving distance and location.

Short on time? Consider completing Problems 2–3 as a whole class.

1 Launch

Direct students' attention to the elevation chart of animals including the jellyfish and the turtle.

Use the Notice and Wonder routine to promote curiosity and help students make sense of the chart. See the Routine Facilitation Guide for more information.

2 Monitor

To support students getting started, consider asking, "What is the elevation of the turtle? How far away is the turtle from sea level?"

Encourage students to sketch to show their thinking.

D Differentiation

Look for students who:	Respond to Student Thinking
Think "farther away from sea level" always means greater than 0.	**Support** Consider reminding them that this is true for positive numbers, but is not true for negative numbers.
Think that the elevation of the sea turtle is less than the octopus, therefore it is closer to sea level.	**Support** Consider asking them to represent the distances using absolute values.
Reason using absolute values and inequality symbols. **(MTR.3.1)**	**Strengthen** Consider asking them to think of the elevation of another creature that is higher than the octopus and closer to sea level than the crab.

3 Connect

MLR7: Compare and Connect Invite students to analyze each statement comparing elevation (greater or less elevation) versus comparing distance from sea level, then find ways in which the statements are different (compare) and alike (connect). Consider asking, "How are these two representations connected? Where can you see the distance from sea level in both representations?"

To surface the Key Takeaway, consider asking:

- "What is the distance of the octopus from sea level? How do you represent it? Is the octopus or the jellyfish closer to sea level?"
- "What would an inequality that compares distances from sea level look like? How is it different from comparing elevations?" **(MTR.2.1)**

Activity 1

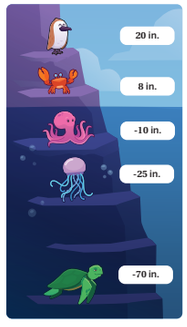
Name: _____ Date: _____ Period: _____

Comparing Distances From Sea Level

2. Which animal has the lowest elevation? The highest elevation?

Lowest elevation: The sea turtle

Highest elevation: The bird



3. Which animal is farthest from sea level? Closest to sea level?

Farthest from sea level: The sea turtle

Closest to sea level: The crab

4. Order the elevations of the animals from *closest* to sea level to *farthest* from sea level.

Crab (|8| inches)	Octopus (|-10| inches)	Bird (|20| inches)	Jellyfish (|-25| inches)	Sea turtle (|-70| inches)

Closest to sea level — Farthest from sea level

5. Here are two ways to compare the locations of the crab and the octopus. Match each inequality to the statement it represents.

a. $-10 < 8$ ___b___ The crab is closer to sea level than the octopus is.

b. $|8| < |-10|$ ___a___ The crab has a higher elevation than the octopus.

Accelerated 6 | Unit 7 Lesson 6 650 Activity 1

Key Takeaway: The distance of an elevation from sea level is represented by its absolute value. Therefore when comparing distances, you need to compare the absolute values of these elevations. For example, you can write:

- $-25 > -70$ to represent that the jellyfish has a higher elevation than the turtle.
- $|-70| > |-25|$ to represent that the turtle is farther away from sea level than the jellyfish.

Small Groups | 20 min

Activity 2 We've Got Game(s)

Purpose: Students use and interpret absolute values and negative numbers that represent and compare changes in amounts of money. **(MTR.2.1)**

1 Launch

Use the Notice and Wonder routine to promote curiosity and help students make sense of the account balance table. If it doesn't come up naturally, consider asking, "What do you think the negative amounts represent?" Clarify that account balances represent the balances at the end of each month.

Look for students who connect receiving money to positive amounts and spending or owing money to negative amounts. **(MTR.2.1)**

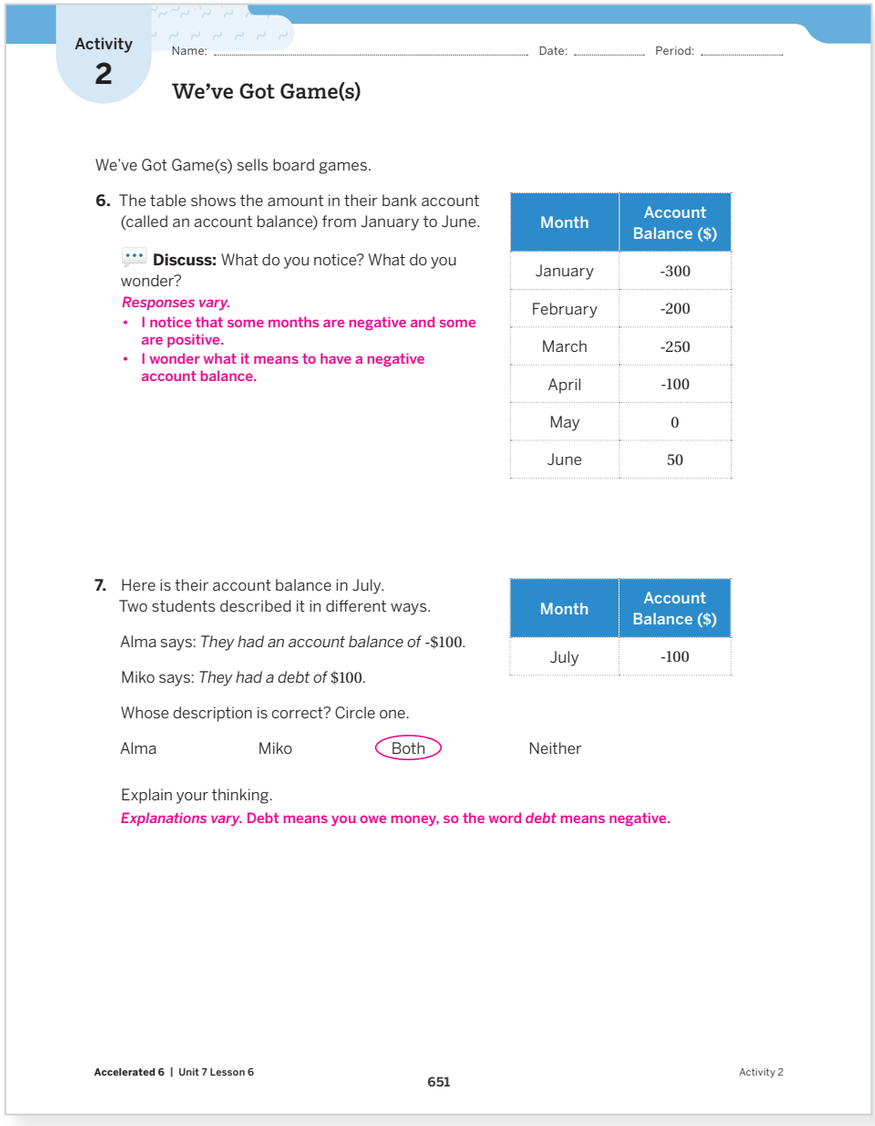
ELL **English Language Learners** Create a list of words about the context of money that students might not know, along with visuals or translations of each word into languages used by your students. **(Reading and Listening)**

2 Monitor

Pause briefly and direct students' attention to the account balance for July. Discuss how to read negative balances aloud such as -100 means $100 of debt, not -$100 of debt.

Use the Decide and Defend routine to support students in making arguments about the account balance showing -100 and critiquing the reasoning of others. **(MTR.3.1)**

Activity 2 continued >



Activity 2

Name: _____ Date: _____ Period: _____

We've Got Game(s)

We've Got Game(s) sells board games.

6. The table shows the amount in their bank account (called an account balance) from January to June.

Month	Account Balance ($)
January	-300
February	-200
March	-250
April	-100
May	0
June	50

Discuss: What do you notice? What do you wonder?

Responses vary.

- I notice that some months are negative and some are positive.
- I wonder what it means to have a negative account balance.

7. Here is their account balance in July. Two students described it in different ways.

Month	Account Balance ($)
July	-100

Alma says: *They had an account balance of* -$100.

Miko says: *They had a debt of* $100.

Whose description is correct? Circle one.

Alma Miko (Both) Neither

Explain your thinking.

Explanations vary. Debt means you owe money, so the word *debt* means negative.

Accelerated 6 | Unit 7 Lesson 6 651 Activity 2

Activity 2 We've Got Game(s) (continued)

Purpose: Students use and interpret absolute values and negative numbers that represent and compare changes in amounts of money. **(MTR.2.1)**

2 Monitor

Distribute one set of Activity 2 Cards per group of students, pre-cut.

Encourage students to share their reasoning with a partner and reach an agreement about how to evaluate each clue before completing the table.

A Accessibility: Executive Functioning

To support students:

- In interpreting each clue, invite them to highlight key information on each card.
- In initiating complex tasks, identify possible places to start.

D Differentiation

Look for students who:	Respond to Student Thinking
Think there is only one correct value for each month.	**Support** Consider reminding them to estimate the values rather than determine the exact values.
Interpret having a lower account balance as having a value with a lower magnitude (e.g., thinking that -50 is less than -100 because $50 < 100$).	**Support** Consider asking, "Is this correct for both negative and positive balances?"
Recognize that all values represent change and compare change using the distance from 0. **(MTR.2.1)**	**Stretch You're invited to explore more.** Invite students who would like to explore comparing rational numbers using distances further to complete this optional task. Encourage them to discuss their thinking with a partner.

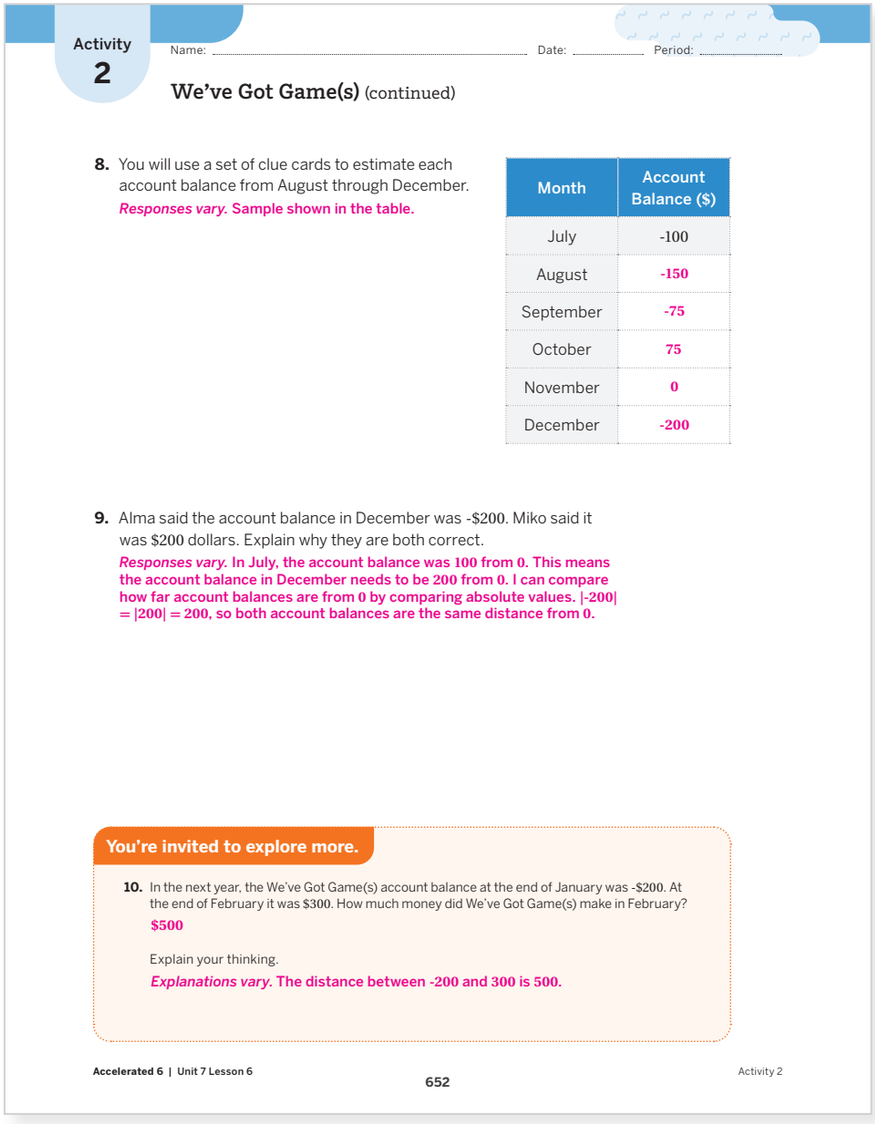
Listen for strategies and explanations to share during the Connect.

3 Connect

Invite students to share the possible values and their strategies to determine the values in the account balance table from July to December, focusing on how the positive and negative numbers are relative to the type of change in the context of money. **(MTR.2.1, MTR.3.1)**

Consider asking:

- "What is the one month that must be the same in everyone's table? Why?"
- "Why are both Alma and Miko correct? When might we use each statement?"



Activity 2

Name: _____ Date: _____ Period: _____

We've Got Game(s) (continued)

8. You will use a set of clue cards to estimate each account balance from August through December.

Responses vary. Sample shown in the table.

Month	Account Balance ($)
July	-100
August	-150
September	-75
October	75
November	0
December	-200

9. Alma said the account balance in December was -$200. Miko said it was $200 dollars. Explain why they are both correct.

Responses vary. In July, the account balance was 100 from 0. This means the account balance in December needs to be 200 from 0. I can compare how far account balances are from 0 by comparing absolute values. $|-200| = |200| = 200$, so both account balances are the same distance from 0.

You're invited to explore more.

10. In the next year, the We've Got Game(s) account balance at the end of January was -$200. At the end of February it was $300. How much money did We've Got Game(s) make in February?

$500

Explain your thinking.

Explanations vary. The distance between -200 and 300 is 500.

Accelerated 6 | Unit 7 Lesson 6 652 Activity 2

Whole Class | 5 min

Synthesis

Purpose: Students synthesize their understanding of comparing absolute values versus rational numbers to make sense of contexts.

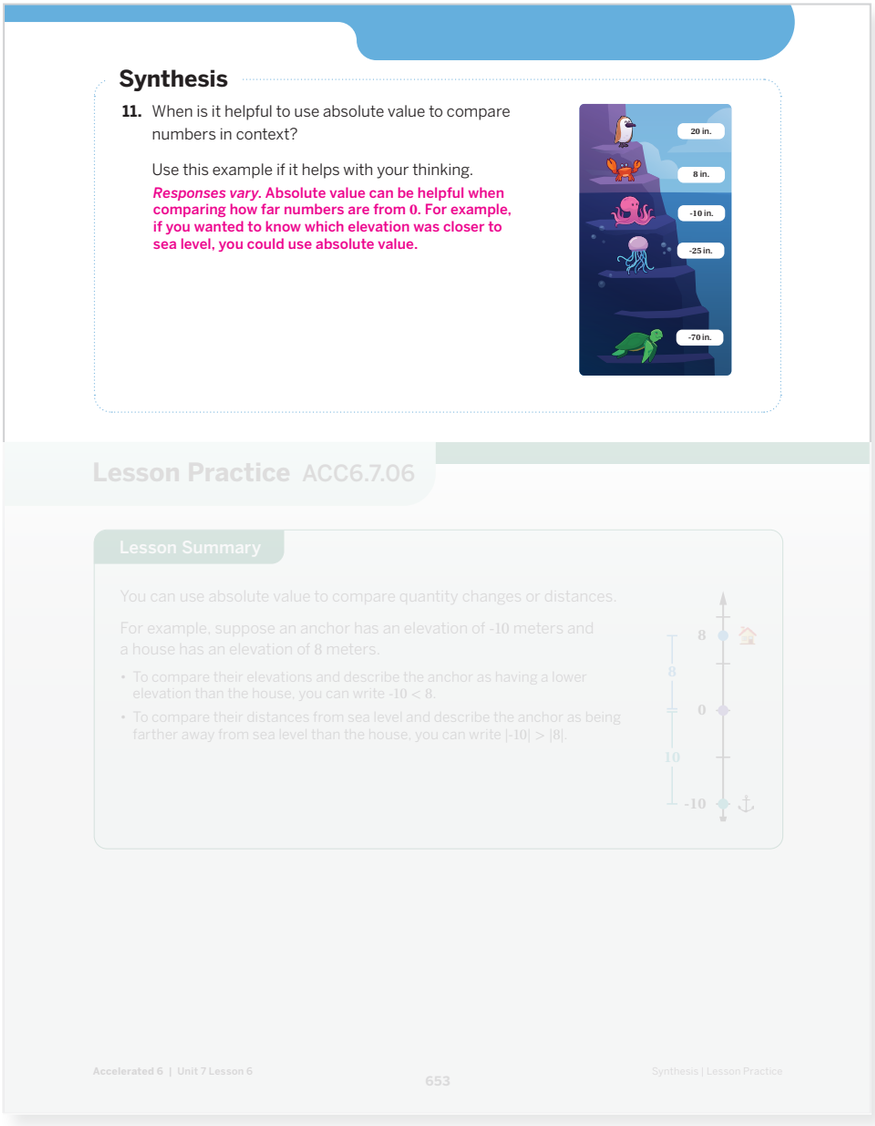
Invite students to respond independently, and then share their thinking with a partner.

Collect and share a variety of ideas, such as:

- Two different numbers can be the same distance from zero, where one is negative and one is positive.
- Distance from 0 and absolute value are always positive for non-zero numbers.

MLR **MLR1: Stronger and Clearer Each Time (optional)** Consider using this routine to support students in developing precise language. Invite students to meet with a partner, give feedback on each other's drafts, then use the feedback to write a second clearer draft. See the Routine Facilitation Guide for more information.

Note: If time allows, invite students to revise their previous thinking based on the discussion.



Synthesis

11. When is it helpful to use absolute value to compare numbers in context?

Use this example if it helps with your thinking.

Responses vary. **Absolute value can be helpful when comparing how far numbers are from 0. For example, if you wanted to know which elevation was closer to sea level, you could use absolute value.**

Lesson Practice ACC6.7.06

Lesson Summary

You can use absolute value to compare quantity changes or distances.

For example, suppose an anchor has an elevation of -10 meters and a house has an elevation of 8 meters.

- To compare their elevations and describe the anchor as having a lower elevation than the house, you can write $-10 < 8$.
- To compare their distances from sea level and describe the anchor as being farther away from sea level than the house, you can write $|-10| > |8|$.

Accelerated 6 | Unit 7 Lesson 6 653 Synthesis | Lesson Practice

Lesson Takeaway: Using numbers or their absolute values when making a comparison depends on the context. For example, when you're determining which value is closer to a target point such as sea level or $0 account balance, using absolute values helps you focus on how much the values differ in distance, without worrying about whether the value is positive or negative.

Independent | 5 min

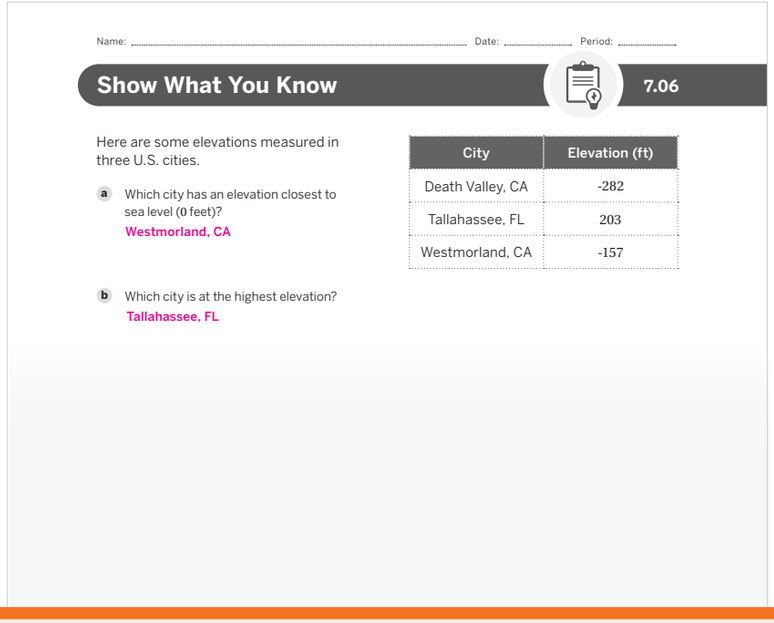
Show What You Know

Purpose: Students demonstrate their understanding by comparing elevations and distances from sea level.

Today's Goals

1. **Goal:** Use absolute values to compare rational numbers in context.
2. **Language Goal:** Justify whether it makes sense to use numbers or their absolute values when making a comparison in context. **(Reading, Writing, Speaking, and Listening)**

Show What You Know (Assessment Resources)



Name: _____ Date: _____ Period: _____

Show What You Know 7.06

Here are some elevations measured in three U.S. cities.

City	Elevation (ft)
Death Valley, CA	-282
Tallahassee, FL	203
Westmorland, CA	-157

a. Which city has an elevation closest to sea level (0 feet)?
Westmorland, CA

b. Which city is at the highest elevation?
Tallahassee, FL

Differentiation Use after Lesson 6

Support

Provide targeted intervention for students by using these resources.

If students need support comparing quantities and absolute values in context:

» **Respond to Student Thinking:**

- Revisit **Activity 1** and invite students to share their thinking for comparing quantities and absolute values.
- Select a few related **Practice Problems** and give individualized feedback.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite them to explore the **Sub-Unit 1 Extension Activities**.
- Invite them to complete the **Explore More** task and discuss their thinking with a partner.

Professional Learning

Take a moment to reflect on the **Decide and Defend** routine in this lesson and Lesson 5. Students will use this routine again in future lessons.

- How can you support students in making a clear and concise claim, asking clarifying questions, and posing counterarguments?
- What about this routine felt successful? What are some areas for future growth?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

11. When is it helpful to use absolute value to compare numbers in context?

Use this example if it helps with your thinking.

Responses vary. Absolute value can be helpful when comparing how far numbers are from 0. For example, if you wanted to know which elevation was closer to sea level, you could use absolute value.

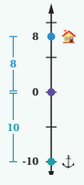
Lesson Practice ACC6.7.06

Lesson Summary

You can use absolute value to compare quantity changes or distances.

For example, suppose an anchor has an elevation of -10 meters and a house has an elevation of 8 meters.

- To compare their elevations and describe the anchor as having a lower elevation than the house, you can write $-10 < 8$.
- To compare their distances from sea level and describe the anchor as being farther away from sea level than the house, you can write $|-10| > |8|$.

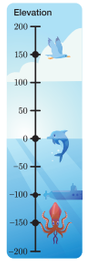


Lesson Practice ACC6.7.06

Name: _____ Date: _____ Period: _____

1. Use the number line to compare each statement. Determine which value is *greater*.

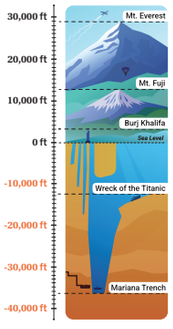
Statement A	Statement B	A	B	Same
The elevation of the submarine	The elevation of the giant squid	✓		
The distance the submarine is from sea level	The distance the giant squid is from sea level		✓	
The distance the seagull is from sea level	The distance the giant squid is from sea level			✓
Sea level	The elevation of the dolphin			✓



Problems 2–3: Here is a number line.

2. What is the approximate elevation of each location and its distance from sea level?

Location	Elevation (ft)	Distance from Sea Level (ft)
Mt. Everest	29,000	29,000
Mt. Fuji	12,500	12,500
Burj Khalifa	3,000	3,000
Wreck of the Titanic	-12,500	12,500
Trench in Mariana Trench	-36,500	36,500



3. Select the two locations that are the same distance from sea level.

Mt. Fuji and the wreck of the *Titanic*

Lesson Practice ACC6.7.06

Name: _____ Date: _____ Period: _____

Problems 4–5: This table shows a bakery owner's bank account transactions.

Date	Item	Amount
May 3	Electricity	-$294
May 3	Catering order	$240
May 5	Baking supplies	-$147
May 6	Catering order	$428

4. What does the number -147 represent in this context?

The owner spent $147 on baking supplies.

5. Did the bakery owner receive more money or spend more money on May 3? Explain your thinking.

The owner spent more money. *Explanations vary.* $|-294|$ is greater than $|240|$.

FAST Practice

6. Anushka thinks her freezer is broken, so she tracks the temperature every 2 hours. The table shows the temperature at different times of the day. At what time was her freezer's temperature closest to freezing level (0°C)?

Time	Temperature (°C)
9 AM	0.4
11 AM	-0.3
1 PM	0.1
3 PM	-0.7

A. 9 AM
B. 11 AM
C. 1 PM
D. 3 PM

Spiral Review

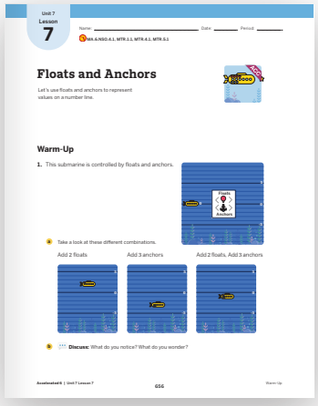
Problems 7–12: Use the symbols <, >, or = to make true comparisons.

7. -32 ___<___ $|15|$ **8.** $|-32|$ ___>___ $|15|$ **9.** 32 ___>___ $|-15|$

10. 5 ___>___ -5 **11.** 5 ___=___ $|-5|$ **12.** -5 ___<___ $|-5|$

Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1	1	MA.6.NSO.1.1, MA.6.NSO.1.2
	2	1	MA.6.NSO.1.1, MA.6.NSO.1.2
	3	1	MA.6.NSO.1.2
	4	1	MA.6.NSO.1.2
	5	2	MA.6.NSO.1.1, MA.6.NSO.1.2
FAST Practice	6	2	MA.6.NSO.1.2
Spiral Review			
	7–12	1	MA.6.NSO.1.3, MA.6.NSO.1.4


Unit 7
Lesson 7

Floats and Anchors

Let's use floats and anchors to represent values on a number line.

Warm-Up

1. This submarine is controlled by floats and anchors.

Take a look at these different combinations.

Add 2 floats | Add 3 anchors | Add 2 floats, Add 3 anchors

Discuss: What do you notice? What do you wonder?

Floats and Anchors

Representing Adding and Subtracting

Let's use floats and anchors to represent values on a number line.

Learning Outcomes

Today's Goals

1. **Goal:** Use a visual model to represent adding and subtracting positive and negative numbers.
2. **Goal:** Use a visual model to represent that pairs of numbers can be combined to make 0.
3. **Language Goal:** Identify different ways to represent the same change (e.g., removing 2 floats or adding 2 anchors). **(Writing, Speaking, and Listening)**

Students engage with a visual model for adding and subtracting positive and negative numbers. They investigate the results of adding or removing floats and anchors and connect these actions to numeric expressions. **(MTR.5.1)**

Coherence

Prior Learning

In Lesson 6, students compared and ordered rational numbers and used absolute value to represent their distance from 0.

Future Learning

In Lesson 8, students will use floats and anchors to reason about adding and subtracting positive and negative numbers. Students will identify different addition and subtraction expressions that have the same value.

Connections

- Students begin their exploration of integer addition and subtraction using context models. (mathematical, real world)

FL B.E.S.T. Math Benchmarks

Addressing

MA.6.NSO.4.1
Apply and extend previous understandings of operations with whole numbers to add and subtract integers with procedural fluency.

Mathematical Thinking and Reasoning Standards: MTR.1.1, MTR.4.1, MTR.5.1

ELA Expectation: ELA.K12.EE.1.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.3.NSO.2.1	MA.7.NSO.2.2

Lesson at a Glance

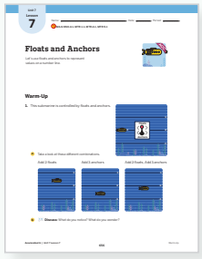
~ 45 min

MA.6.NSO.4.1

Warm-Up

Pairs | 5 min

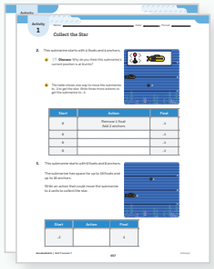
Students are introduced to the context of controlling a submarine's position on a vertical number line by adding or removing floats and anchors. **Routine:** Notice and Wonder



Activity 1

Pairs | 12 min

Students explore the impact of adding or removing floats and anchors, including situations where there is an equal number of floats and anchors.



Activity 2

Pairs | 10 min

Students determine the final position of the submarine based on adding or removing floats and anchors and connect those actions to numeric expressions. **Routine:** Decide and Defend **(MTR.5.1)**

Activity 3

Pairs | 8 min

Students complete repeated challenges to determine the final position of a submarine with different starting positions and actions.

Synthesis

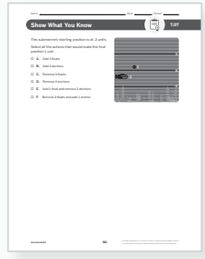
Whole Class | 5 min

Students synthesize strategies for adding and subtracting positive and negative numbers by describing a series of actions to move a submarine.

Show What You Know

Independent | 5 min

Students demonstrate their understanding by using floats and anchors to represent adding and subtracting positive and negative numbers.



Prep Checklist

This lesson includes:

- Student Edition
- Show What You Know

Pairs | 5 min

Warm-Up

Purpose: Students are introduced to the context of controlling a submarine's position on a vertical number line by adding or removing floats and anchors.

1 Launch

Use the Notice and Wonder routine to promote curiosity and help students make sense of how floats and anchors impact the submarine's position. **(MTR.1.1)**

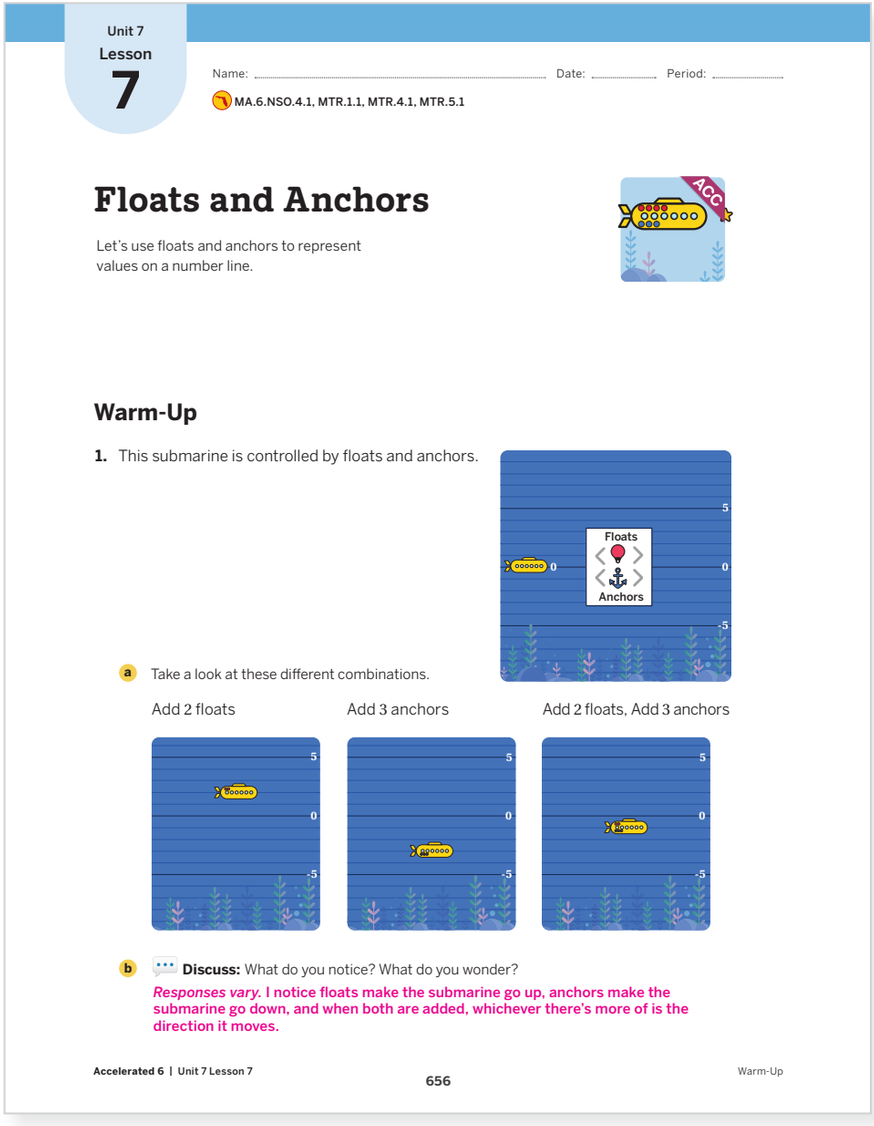
To surface connections, invite students to share what floats and anchors remind them of (e.g., "When I hear [float/anchor], I think . . ."). Consider sharing that floats help raise a submarine from its starting position and anchors lower a submarine from its starting position.

2 Connect

Create a record of the observations students noticed and wondered, along with the name of the students who shared them.

Consider asking:

- "Did anyone notice what happened when a float was added? An anchor?"
- "Why might we wonder about combinations of floats and anchors?"



Unit 7
Lesson 7

Name: ..... Date: ..... Period: .....

MA.6.NSO.4.1, MTR.1.1, MTR.4.1, MTR.5.1

Floats and Anchors

Let's use floats and anchors to represent values on a number line.

ACC

Warm-Up

1. This submarine is controlled by floats and anchors.

a. Take a look at these different combinations.

Add 2 floats | Add 3 anchors | Add 2 floats, Add 3 anchors

b. **Discuss:** What do you notice? What do you wonder?

Responses vary. I notice floats make the submarine go up, anchors make the submarine go down, and when both are added, whichever there's more of is the direction it moves.

Accelerated 6 | Unit 7 Lesson 7 656 Warm-Up

Pairs | 12 min

Activity 1 Collect the Star

Purpose: Students explore the impact of adding or removing floats and anchors, including situations where there is an equal number of floats and anchors.

1 Launch

To generate excitement, consider sharing with students they will engage with a variety of challenges to collect stars.

Note: If it doesn't come up naturally, consider asking:

- "If there are 7 anchors on a submarine, what can be added to the submarine to move it to a position of 0 units?" 7 floats
- "How are these related to each other?" *Responses vary.* The two values are opposites.

2 Monitor

To support students getting started, consider asking, "Which direction does the submarine need to move to collect the star at -3? By how many spaces?"

Listen for student strategies combining floats and anchors.

D Differentiation

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Invite students to guess and check, marking the image, and then think of a strategy for their next action.
Name that adding a float or removing an anchor results in the submarine moving up one unit.	**Strengthen** Consider asking, "How might that noticing help to solve any challenge?" Invite students to think about how they might share their thinking during the Connect.
Name that removing a float or adding an anchor results in the submarine moving down one unit.	
Want to explore submarine challenges further.	**Stretch** Encourage students to try a combination of floats and anchors they think will be different from others and prepare to share during the Connect.

Pause briefly to discuss multiple strategies for moving the submarine, including unique or creative approaches students tried. **(MTR.1.1)**

Activity 1 continued >

Activity 1

Name: _____ Date: _____ Period: _____

Collect the Star

2. This submarine starts with 4 floats and 4 anchors.

a. **Discuss:** Why do you think this submarine's current position is at 0 units?

Responses vary. The anchors and floats are balanced out so it doesn't go up or down.

b. The table shows one way to move the submarine to -3 to get the star. Write three more actions to get the submarine to -3.

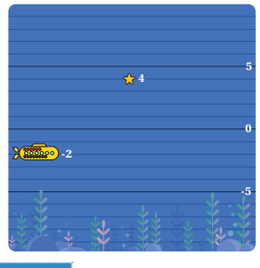
Responses vary.

Start	Action	Final
0	Remove 1 float Add 2 anchors	-3
0	Remove 3 floats	-3
0	Remove 2 floats Add 1 anchor	-3
0	Remove 4 floats Remove 1 anchor	-3

3. This submarine starts with 6 floats and 8 anchors.

The submarine has space for up to 10 floats and up to 10 anchors.

Write an action that could move the submarine to 4 units to collect the star.



Start	Action	Final
-2	*Responses vary.* • Remove 6 anchors • Add 4 floats and remove 2 anchors	4

Accelerated 6 | Unit 7 Lesson 7 657 Activity 1

Pairs | 12 min

Activity 1 Collect the Star (continued)

Purpose: Students explore the impact of adding or removing floats and anchors, including situations where there is an equal number of floats and anchors.

2 Monitor

To surface connections, consider sharing that given the multiple solutions for the previous challenge, there are often a variety of approaches to move the submarine to a specific position.

D **Differentiation: (Support)** To support students getting started, invite students to sketch the actions to determine which ones move the sub to -5 units.

Encourage students to share their reasoning with a partner and work together to reach an agreement about how to sort the actions describing sums and differences for the position of the submarine. **(MTR.1.1, MTR.4.1)**

A **Accessibility: Executive Functioning** Invite students to focus on sorting one set of actions at a time to support students in carrying out multiple steps.

3 Connect

Invite students to share their responses, calling attention to any conflict or consensus.

To surface the Key Takeaway, consider asking:

- "Why does adding 3 floats and adding 4 anchors result in the submarine moving down?"
- "Why does removing 5 floats and removing 5 anchors result in the submarine staying in the same position?"
- "Why does adding 8 floats and removing 8 anchors not keep the submarine in the same position?"

Note: If time allows, invite pairs of students to share their unique responses from the earlier problems in the activity.

Activity 1

Name: _____ Date: _____ Period: _____

Collect the Star (continued)

4. This submarine starts at -3 units.

Select *all* the actions that would move it to -5 units.

- ☐ **A.** Add 2 floats
- ☑ **B.** Add 2 anchors
- ☑ **C.** Remove 1 float and add 1 anchor
- ☑ **D.** Add 3 floats and add 5 anchors
- ☐ **E.** Remove 2 floats and add 4 anchors

5. Imagine a new submarine. For each action, put a check for whether the submarine would go up, go down, or stay in the same position.

Action	Up	Same Position	Down
Add 3 floats Add 4 Anchors			✓
Remove 10 anchors	✓		
Remove 5 floats Remove 5 anchors		✓	
Add 8 floats Remove 8 anchors	✓		
Add 6 floats Add 2 anchors	✓		
Remove 7 floats Add 3 anchors			✓

Accelerated 6 | Unit 7 Lesson 7 658 Activity 1

Key Takeaway: Visual models, such as submarines with floats and anchors, can be used to represent sums and differences and can be shown on a number line. Adding 1 float and adding 1 anchor are opposite actions and do not move the submarine.

Pairs | 10 min

Activity 2 Sea-king Stars

Purpose: Students determine the final position of the submarine based on adding or removing floats and anchors and connect those actions to numeric expressions. **(MTR.5.1)**

1 Launch

To surface connections, consider asking, "How is this problem different from the previous ones?"

2 Monitor

Listen for student explanations for determining the submarine's final position that may be similar to Leo's and Mai's expressions.

D Differentiation

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Consider asking, "Which direction does the submarine move when adding 3 floats?" Have students mark this on the vertical number line. Repeat this step for the second part of the action.
Think that removing 5 anchors would move the submarine down 5 units.	**Support** Remind students that removing signals subtraction but anchors also represent a negative number.
Determine the submarine's final position to be -4 units.	**Support** Consider asking, "What would happen to the submarine if you only removed 5 anchors?"

Listen for explanations of what each expression would mean in terms of floats and anchors and why each may be correct to share during the Connect. **(MTR.5.1)**

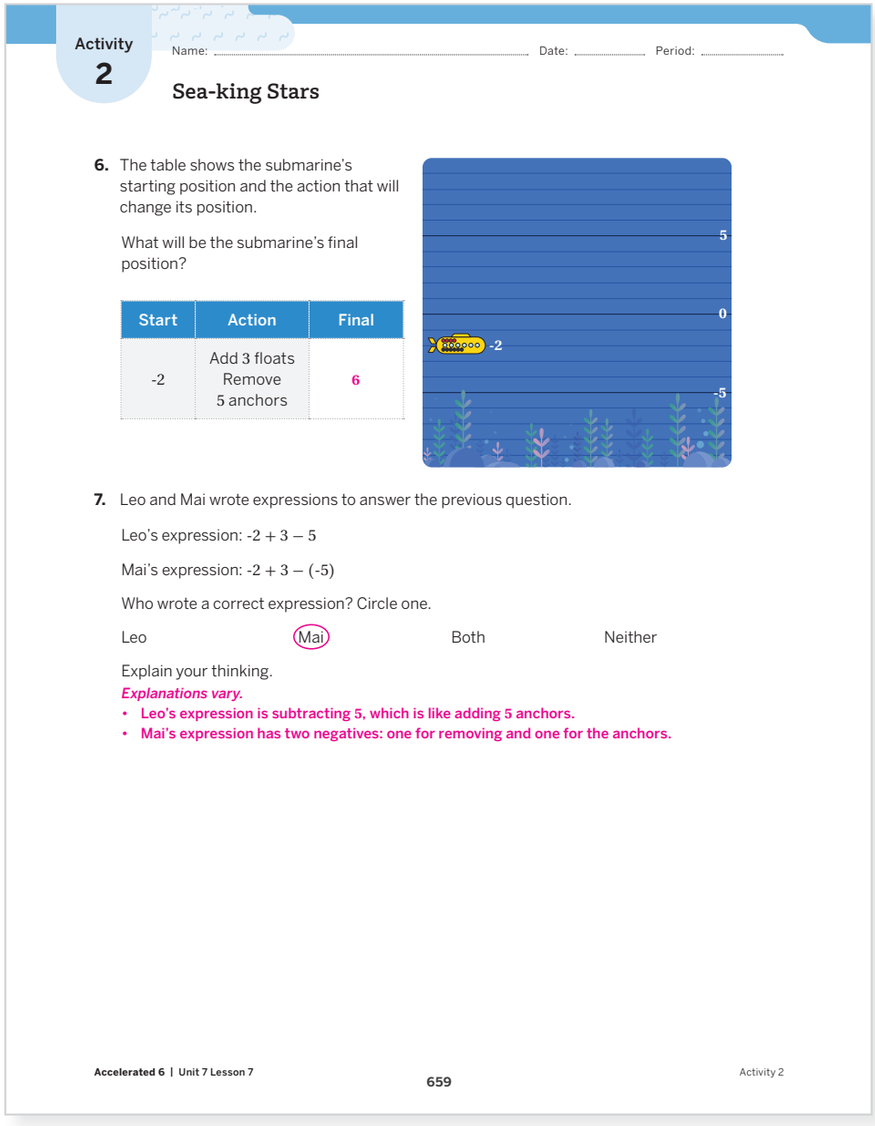
3 Connect

Invite students to share their responses, calling attention to any conflict or consensus.

Use the Decide and Defend routine to support students in making arguments and critiquing the reasoning of others. **(MTR.1.1, MTR.4.1)**

Invite students who made different choices to use reasoning to convince their classmates. Consider asking, "What reasoning or evidence supports your claim?"

Note: If everyone is in agreement, consider making a class argument using different students' reasoning.



Activity 2

Name: Date: Period:

Sea-king Stars

6. The table shows the submarine's starting position and the action that will change its position.

What will be the submarine's final position?

Start	Action	Final
-2	Add 3 floats Remove 5 anchors	6

7. Leo and Mai wrote expressions to answer the previous question.

Leo's expression: $-2 + 3 - 5$

Mai's expression: $-2 + 3 - (-5)$

Who wrote a correct expression? Circle one.

Leo (Mai) Both Neither

Explain your thinking.

Explanations vary.

- Leo's expression is subtracting 5, which is like adding 5 anchors.
- Mai's expression has two negatives: one for removing and one for the anchors.

Accelerated 6 | Unit 7 Lesson 7 659 Activity 2

Pairs | 8 min

Activity 3 Captain's Challenge

Purpose: Students complete repeated challenges to determine the final position of a submarine with different starting positions and actions.

Short on time? Consider inviting students to complete fewer challenges.

1 Launch

Invite students to set a goal for how many stars they hope to collect as they work on the repeated challenges. Share that it's more important to make sense of the challenges than it is to work quickly. Students will solve and compare their solutions to problems with a partner.

A **Accessibility: Memory and Attention** Invite students to draw the initial submarine position and the actions in each problem.

2 Monitor

Listen for strategies students use while discussing how to solve the problems, such as generalizing how different actions with floats and anchors impact the final position. **(MTR.1.1, MTR.4.1, MTR.5.1)**

D **Differentiation**

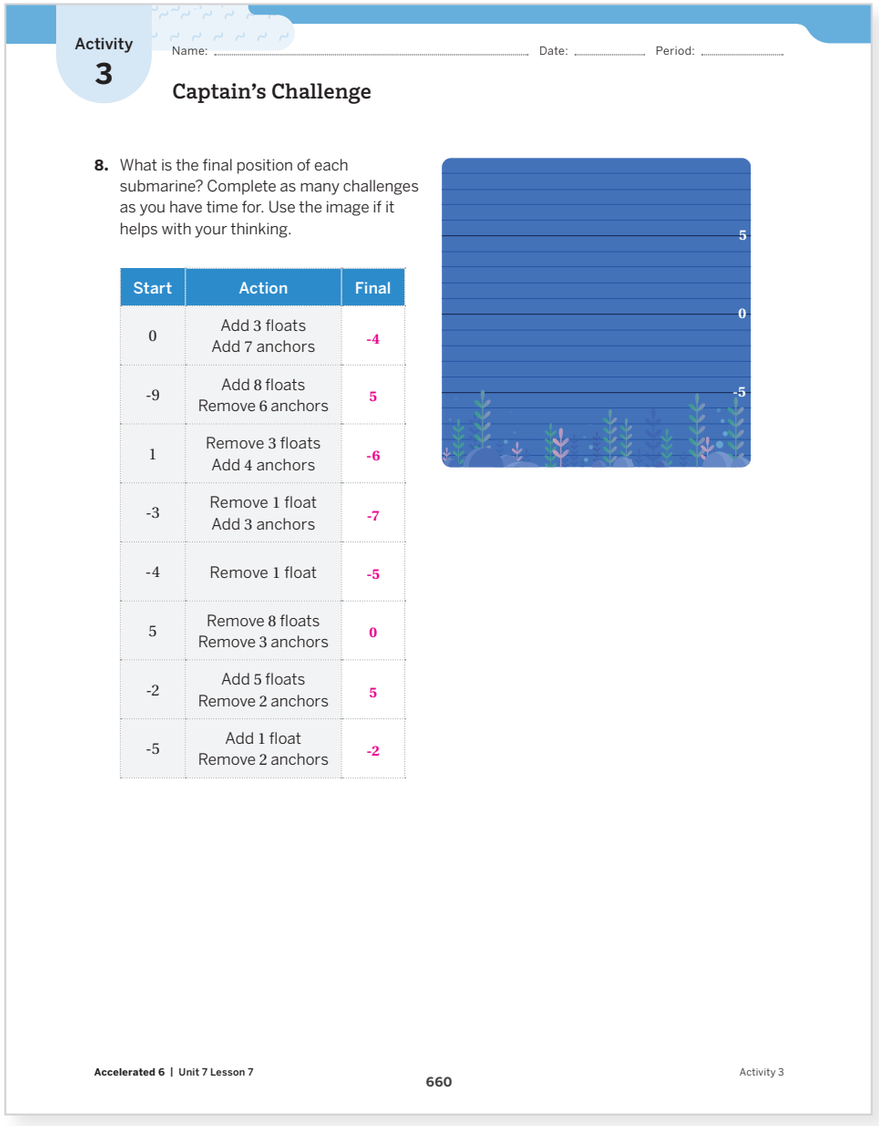
Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Consider asking, "Which direction does the submarine move when adding __ floats?" Have students mark this on the vertical number line. Repeat this step for the second part of the action, if appropriate.
Repeatedly guess the final position of their submarine.	**Support** Consider asking, "What strategy shared from Activities 1 and 2 could you use to try to get the next challenge star?"

3 Connect

Invite students to share strategies that they found helpful or what they learned from the mistakes they made while solving these challenges.

To surface the Key Takeaway, consider asking:

- "How can we use floats and anchors to represent subtracting a positive value? How would you write that as an expression?" **(MTR.5.1)**
- "How does the submarine display subtracting a negative value? How would you write that as an expression?"



Activity 3

Name: _____ Date: _____ Period: _____

Captain's Challenge

8. What is the final position of each submarine? Complete as many challenges as you have time for. Use the image if it helps with your thinking.

Start	Action	Final
0	Add 3 floats Add 7 anchors	-4
-9	Add 8 floats Remove 6 anchors	5
1	Remove 3 floats Add 4 anchors	-6
-3	Remove 1 float Add 3 anchors	-7
-4	Remove 1 float	-5
5	Remove 8 floats Remove 3 anchors	0
-2	Add 5 floats Remove 2 anchors	5
-5	Add 1 float Remove 2 anchors	-2

Accelerated 6 | Unit 7 Lesson 7 660 Activity 3

Key Takeaway: A model can be a useful tool for visualizing adding and subtracting positive and negative numbers. In particular, removing 5 anchors in our submarine model helps us to visualize the value of –(-5) as an increase in 5 on the number line.

Whole Class | 5 min

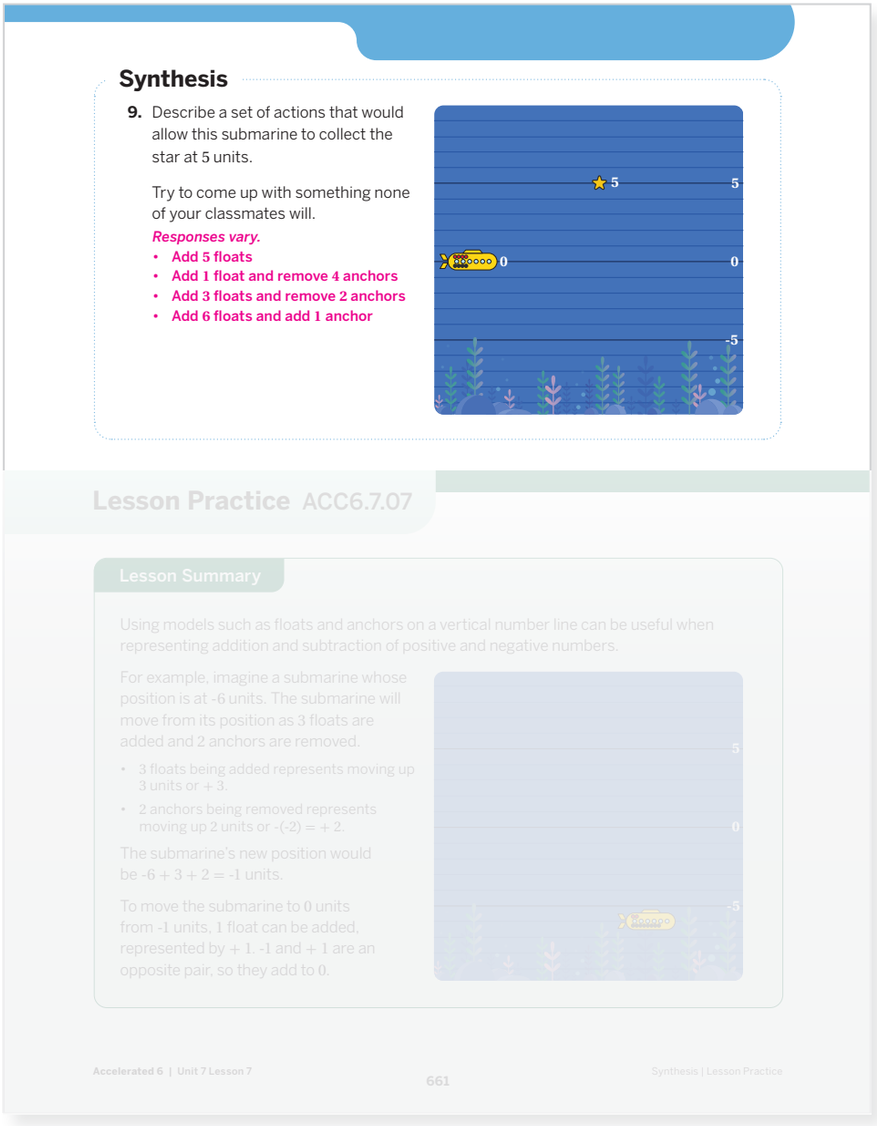
Synthesis

Purpose: Students synthesize strategies for adding and subtracting positive and negative numbers by describing a series of actions to move a submarine.

Invite students to respond independently, and then share their thinking with a partner.

Collect and share a variety of ideas, such as:
- Adding floats to raise the submarine 5 units.
- Adding and/or removing a combination of floats and anchors that moves the submarine to 5 units.

Note: If time allows, invite students to revise their previous thinking based on the discussion.



Synthesis

9. Describe a set of actions that would allow this submarine to collect the star at 5 units.

Try to come up with something none of your classmates will.

Responses vary.
- Add 5 floats
- Add 1 float and remove 4 anchors
- Add 3 floats and remove 2 anchors
- Add 6 floats and add 1 anchor

Lesson Practice ACC6.7.07

Lesson Summary

Using models such as floats and anchors on a vertical number line can be useful when representing addition and subtraction of positive and negative numbers.

For example, imagine a submarine whose position is at -6 units. The submarine will move from its position as 3 floats are added and 2 anchors are removed.

- 3 floats being added represents moving up 3 units or $+3$.
- 2 anchors being removed represents moving up 2 units or $-(-2) = +2$.

The submarine's new position would be $-6 + 3 + 2 = -1$ units.

To move the submarine to 0 units from -1 units, 1 float can be added, represented by $+1$. -1 and $+1$ are an opposite pair, so they add to 0.

Accelerated 6 | Unit 7 Lesson 7 661 Synthesis | Lesson Practice

Key Takeaway: Picturing floats and anchors on a submarine is one way to make sense of adding and subtracting positive and negative numbers. Adding positive numbers (floats) and subtracting negative numbers (anchors) both make a greater value. Adding negative numbers and subtracting positive numbers both make a lesser value.

Independent | 5 min

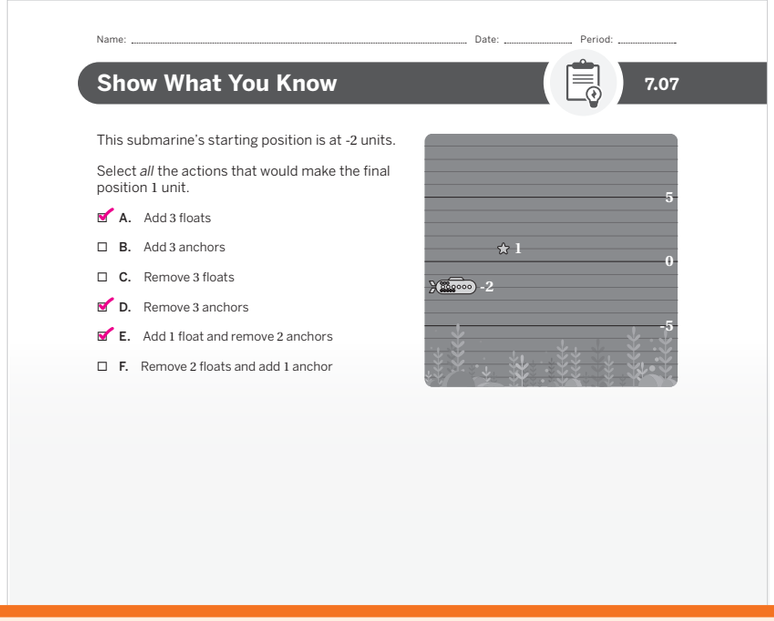
Show What You Know

Purpose: Students demonstrate their understanding by using floats and anchors to represent adding and subtracting positive and negative numbers.

Today's Goals

1. **Goal:** Use a visual model to represent adding and subtracting positive and negative numbers.
2. **Goal:** Use a visual model to represent that pairs of numbers can be combined to make 0.
3. **Language Goal:** Identify different ways to represent the same change (e.g., removing 2 floats or adding 2 anchors). **(Writing, Speaking, and Listening)**

Show What You Know (Assessment Resources)



Name: _____ Date: _____ Period: _____

Show What You Know 7.07

This submarine's starting position is at -2 units.

Select *all* the actions that would make the final position 1 unit.

- [x] A. Add 3 floats
- [ ] B. Add 3 anchors
- [ ] C. Remove 3 floats
- [x] D. Remove 3 anchors
- [x] E. Add 1 float and remove 2 anchors
- [ ] F. Remove 2 floats and add 1 anchor

Differentiation Use after Lesson 7

Support

Provide targeted intervention for students by using these resources.

If students need support adding and subtracting positive numbers:

» **Respond to Student Thinking:**

- Students will have more opportunities to develop their understanding of these concepts in Lesson 8 and throughout the unit. You may choose not to intervene now.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite them to explore the **Sub-Unit 1 Extension Activities**.

Professional Learning

Reflect on students' use of the submarine model throughout the lesson. How did this model support students with communicating their ideas or participating in the lesson? What support might students need to apply the concepts from this model effectively in the future?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

9. Describe a set of actions that would allow this submarine to collect the star at 5 units.

Try to come up with something none of your classmates will.

Responses vary.

- Add 5 floats
- Add 1 float and remove 4 anchors
- Add 3 floats and remove 2 anchors
- Add 6 floats and add 1 anchor

Lesson Practice ACC6.7.07

Lesson Summary

Using models such as floats and anchors on a vertical number line can be useful when representing addition and subtraction of positive and negative numbers.

For example, imagine a submarine whose position is at -6 units. The submarine will move from its position as 3 floats are added and 2 anchors are removed.

- 3 floats being added represents moving up 3 units or $+3$.
- 2 anchors being removed represents moving up 2 units or $-(-2) = +2$.

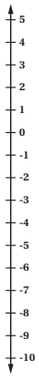
The submarine's new position would be $-6 + 3 + 2 = -1$ units.

To move the submarine to 0 units from -1 units, 1 float can be added, represented by $+1$. -1 and $+1$ are an opposite pair, so they add to 0.

Lesson Practice
ACC6.7.07

Name: ..... Date: ..... Period: .....

Problems 1–5: One moment in December, it was -8°C in Harbin, China, and -2°C in Beijing, China. Use the number line if it helps with your thinking.



1. Which city was colder?
   Harbin

2. At the same moment, it was 7 degrees warmer in Shanghai than it was in Beijing. What was the temperature in Shanghai?
   5°C

3. How many degrees warmer was it in Shanghai than in Harbin?
   13 degrees

4. Later in the day, Beijing got 5 degrees colder. What was Beijing's new temperature?
   -7°C

5. Later in the day, Harbin's temperature was 1°C. By how much did Harbin's temperature change? Explain your thinking.
   9 degrees. *Explanations vary.* The distance from -8 to 1 on a number line is 9 units.

6. This submarine has 0 floats and 0 anchors. The submarine can hold up to 10 floats and 10 anchors.

   a. List *all* the combinations of floats and anchors that could collect the star at -9 units.
      - **9 anchors**
      - **10 anchors and 1 float**

   b. How do you know there are no other combinations?
      ***Responses vary.* It takes 9 anchors to get down to -9. Every other anchor needs to be balanced with a float. Since the submarine only has space for 10 anchors, you can only add 1 float.**

Lesson Practice
ACC6.7.07

Name: ..... Date: ..... Period: .....

FAST Practice

7. A submarine is at the position -5 units. The expression $-5 + (-2)$ represents the submarine's new position after adding 2 anchors. What is the submarine's new position?

   A. -7 units **B.** -3 units **C.** 3 units **D.** 7 units

Spiral Review

8. Fill in each blank using the symbols >, <, or =.

3	>	-3
12	<	24
-12	>	-24
7	<	7.2
-7	>	-7.2

Problems 9–10: A color of green paint is made by mixing 2 cups of yellow paint with 3.5 cups of blue paint.

9. Complete the table to show how many cups of yellow and blue paint will make the same color of green but in a smaller amount.
   ***Responses vary.* Sample shown in table.**

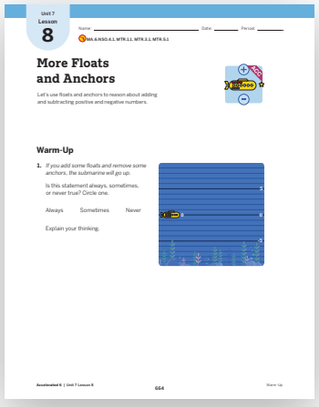
Yellow Paint (cups)	Blue Paint (cups)
1	1.75

10. Complete the table to show how many cups of yellow and blue paint will make the same color of green but in a larger amount.
    ***Responses vary.* Sample shown in table.**

Yellow Paint (cups)	Blue Paint (cups)
4	7

Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1–5	2	MA.6.NSO.4.1
	6	3	MA.6.NSO.4.1
FAST Practice	7	2	MA.6.NSO.4.1
Spiral Review			
	8	1	MA.6.NSO.1.1
	9–10	2	MA.6.AR.3.5


Unit 7
Lesson
8

More Floats and Anchors

Let's use floats and anchors to reason about adding and subtracting positive and negative numbers.

Warm-Up

1. If you add some floats and remove some anchors, the submarine will go up.

Is this statement always, sometimes, or never true? Circle one.

Always Sometimes Never

Explain your thinking.

664

More Floats and Anchors

Adding and Subtracting Positive and Negative Numbers

Let's use floats and anchors to reason about adding and subtracting positive and negative numbers.

Learning Outcomes

Today's Goals

1. **Goal:** Connect different visual models that represent addition and subtraction.
2. **Goal:** Add and subtract positive and negative numbers.
3. **Language Goal:** Describe how a visual model represents adding or subtracting positive and negative numbers. **(Reading, Writing, Speaking, and Listening)**

Students transition from manipulating floats and anchors to using floats and anchors to reason about adding and subtracting positive and negative numbers. Students connect specific actions with mathematical expressions and discuss expressions that have the same value in terms of floats and anchors (e.g., $5 + 3$ and $5 - (-3)$). **(MTR.5.1)**

Coherence

Prior Learning

In Lesson 7, students were introduced to floats and anchors and used them to represent adding and subtracting positive and negative numbers.

Future Learning

In Lesson 9, students will extend their understanding of operations with whole numbers to multiply and divide integers. In Unit 8, students will apply what they know about adding and subtracting integers to solve equations.

Connections

- Students continue to build their understanding of integer addition and subtraction using context models. (mathematical, real world)

FL B.E.S.T. Math Benchmarks

Addressing

MA.6.NSO.4.1
Apply and extend previous understandings of operations with whole numbers to add and subtract integers with procedural fluency.

Mathematical Thinking and Reasoning Standards: MTR.1.1, MTR.3.1, MTR.5.1

ELA Expectations: ELA.K12.EE.1.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.3.NSO.2.1	MA.7.NSO.2.2

Vocabulary

Additional Vocabulary

additive inverses

Lesson at a Glance

~ 45 min

MA.6.NSO.4.1

Warm-Up

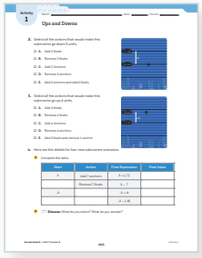
Pairs | 5 min

Students reason about how adding floats and removing anchors affects the position of the submarine to decide whether a statement is always, sometimes, or never true. **(MTR.1.1)**

Activity 1

Pairs | 15 min

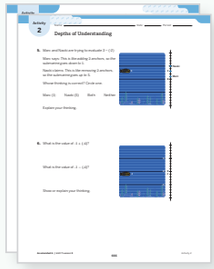
Students connect adding and removing floats and anchors to adding and subtracting positive and negative numbers and encounter pairs of expressions with the same value (e.g., $5 + (-3)$ and $5 - 3$). **Routine:** Notice and Wonder **(MTR.5.1)**



Activity 2

Pairs | 15 min

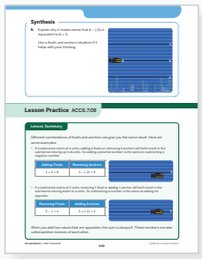
Students use floats and anchors as a model and begin to recognize patterns that help them identify different expressions with the same value. **Routine:** Decide and Defend **(MTR.5.1)**



Synthesis

Whole Class | 5 min

Students synthesize their understanding of pairs of equivalent expressions for computing sums and differences of positive and negative numbers.



Show What You Know

Independent | 5 min

Students demonstrate their understanding by evaluating expressions involving adding and subtracting positive and negative numbers.

Prep Checklist

This lesson includes:

- Student Edition
- Show What You Know

Pairs | 5 min

Warm-Up

Purpose: Students reason about how adding floats and removing anchors affects the position of the submarine to decide whether a statement is always, sometimes, or never true. **(MTR.1.1)**

1 Launch

To surface connections, consider asking, "What do the terms floats and anchors mean in this context?"

Invite students to read the statement aloud and then think independently before discussing their ideas with a partner.

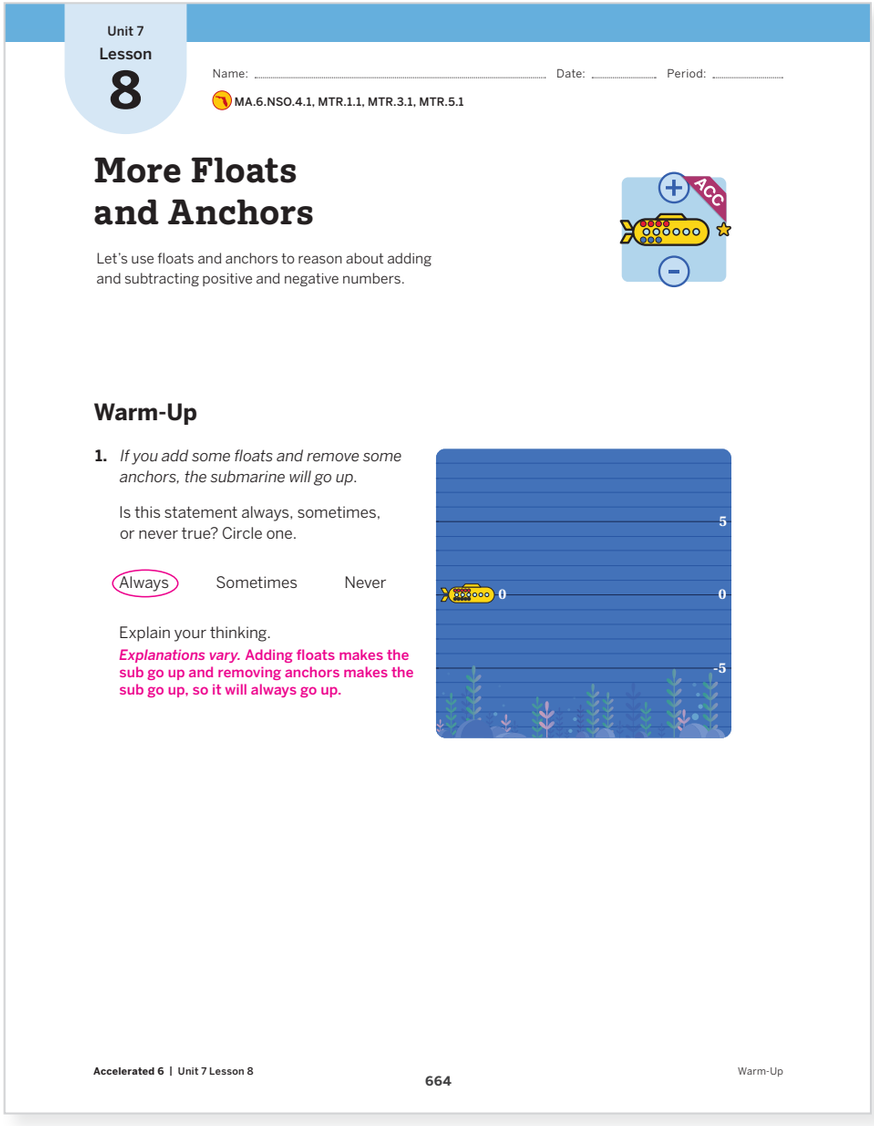
ELL **English Language Learners** Invite students to read the claim aloud and paraphrase what it means. Clarify any words they don't know. **(Reading)**

2 Connect

Invite students to share their responses, calling attention to any conflict or consensus.

Invite students who made different choices to share their reasoning with their classmates. Consider asking, "How did you decide whether this statement is always, sometimes, or never true?" **(MTR.1.1)**

Note: If time allows, consider inviting students to write a statement about adding or removing floats and anchors that is sometimes true or never true.



Unit 7
Lesson
8

Name: ..... Date: ..... Period: .....

MA.6.NSO.4.1, MTR.1.1, MTR.3.1, MTR.5.1

More Floats and Anchors

Let's use floats and anchors to reason about adding and subtracting positive and negative numbers.

ACC

Warm-Up

1. *If you add some floats and remove some anchors, the submarine will go up.*

   Is this statement always, sometimes, or never true? Circle one.

   Always Sometimes Never

   Explain your thinking.

   Explanations vary. **Adding floats makes the sub go up and removing anchors makes the sub go up, so it will always go up.**

Accelerated 6 | Unit 7 Lesson 8 664 Warm-Up

Pairs | 15 min

Activity 1 Ups and Downs

Purpose: Students connect adding and removing floats and anchors to adding and subtracting positive and negative numbers and encounter pairs of expressions with the same value (e.g., $5 + (-3)$ and $5 - 3$). **(MTR.5.1)**

Short on time? Consider omitting Problem 2 or Problem 3 to allow more time for discussion around Problems 4–5.

1 Launch

To surface prior knowledge, consider asking, "What are actions that cause a submarine to move up? Down?"

A **Accessibility: Memory and Attention** Invite students to draw what is happening for each action.

2 Monitor

Listen for student strategies for connecting expressions with the submarine model, including references to the starting position and actions with floats and anchors. **(MTR.5.1)**

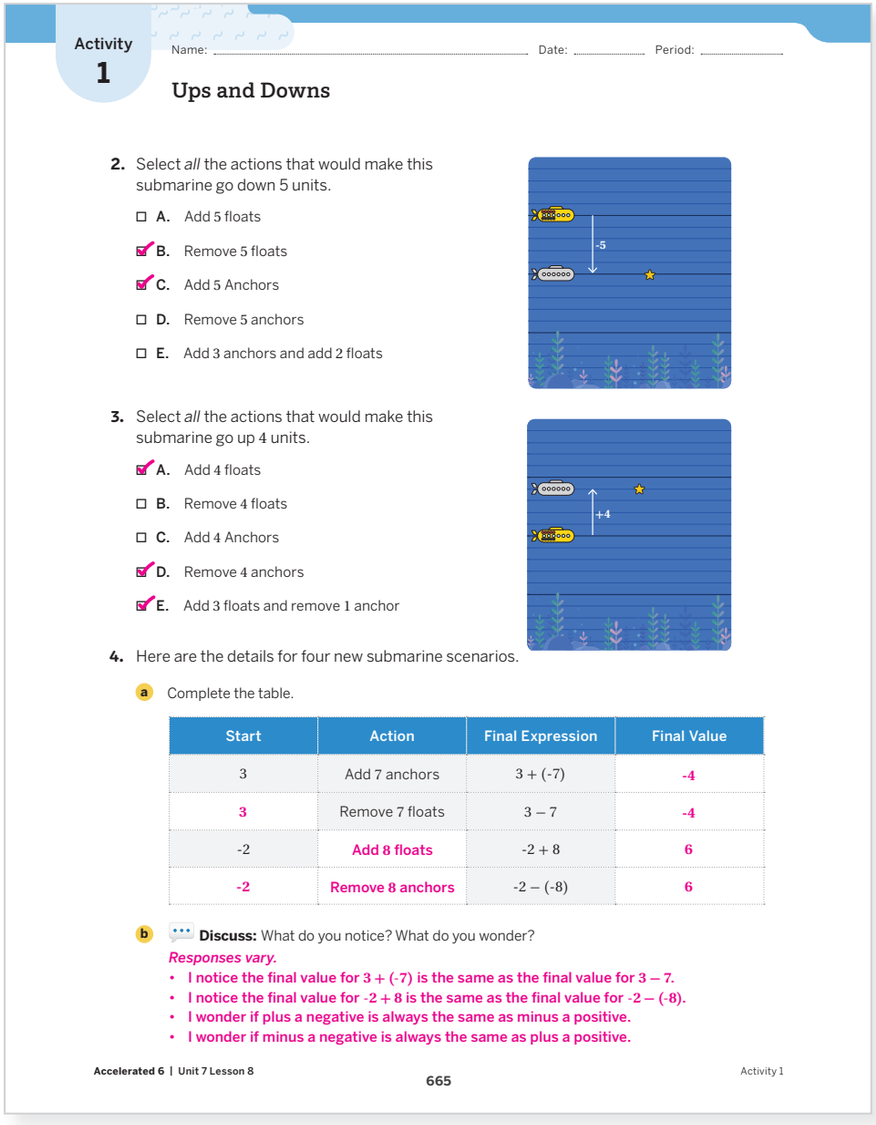
D **Differentiation** *(Problem 4)*

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Consider asking, "Can you describe two different ways floats and anchors can be used to move a submarine down?"
Think that $3 + (-7)$ has a different value than $3 - 7$.	**Support** Consider asking, "If the submarine starts at a position of 3 units, what does adding 7 anchors do? Removing 7 floats?"
Recognize that adding a negative number is equivalent to subtracting a positive number.	**Strengthen** Consider asking, "How might that noticing help you write an expression for any set of actions?" Invite students to think about how they might share their thinking during the Connect.
Recognize that adding a positive number is equivalent to subtracting a negative number.	
Want to explore addition and subtraction of positive and negative numbers further.	**Stretch** Encourage students to write an expression with positive and negative numbers that has the same final value as one of the final values in this problem.

3 Connect

Use the Notice and Wonder routine to promote curiosity and help students make sense of patterns in the structure of the expressions. See the Routine Facilitation Guide for more information. **(MTR.1.1, MTR.5.1)**

Consider asking, "Why do the first two rows have the same final value? The last two rows?" **(MTR.5.1)**



Activity 1

Name: _____ Date: _____ Period: _____

Ups and Downs

2. Select *all* the actions that would make this submarine go down 5 units.

- ☐ **A.** Add 5 floats
- ☑ **B.** Remove 5 floats
- ☑ **C.** Add 5 Anchors
- ☐ **D.** Remove 5 anchors
- ☐ **E.** Add 3 anchors and add 2 floats

3. Select *all* the actions that would make this submarine go up 4 units.

- ☑ **A.** Add 4 floats
- ☐ **B.** Remove 4 floats
- ☐ **C.** Add 4 Anchors
- ☑ **D.** Remove 4 anchors
- ☑ **E.** Add 3 floats and remove 1 anchor

4. Here are the details for four new submarine scenarios.

a Complete the table.

Start	Action	Final Expression	Final Value
3	Add 7 anchors	$3 + (-7)$	-4
3	Remove 7 floats	$3 - 7$	-4
-2	Add 8 floats	$-2 + 8$	6
-2	Remove 8 anchors	$-2 - (-8)$	6

b **Discuss:** What do you notice? What do you wonder?

Responses vary.

- I notice the final value for $3 + (-7)$ is the same as the final value for $3 - 7$.
- I notice the final value for $-2 + 8$ is the same as the final value for $-2 - (-8)$.
- I wonder if plus a negative is always the same as minus a positive.
- I wonder if minus a negative is always the same as plus a positive.

Accelerated 6 | Unit 7 Lesson 8 665 Activity 1

Pairs | 15 min

Activity 2 Depths of Understanding

Purpose: Students use floats and anchors as a model and begin to recognize patterns that help them identify different expressions with the same value. **(MTR.5.1)**

1 Launch

To surface connections, consider reading the prompt aloud and asking, "How does the number line display Marc's and Naoki's thinking?"

Use the Decide and Defend routine to support students in making arguments and critiquing the reasoning of others. **(MTR.1.1)**

A **Accessibility: Visual-Spatial Processing** Invite pairs to think about key features of the figure and discuss them aloud before responding to support connecting the prompt and visual representation.

2 Monitor

Invite students to read others' responses and decide if others' thinking was similar to or different from their own.

Pause to discuss student responses and invite students who made different choices to use reasoning to convince their classmates.

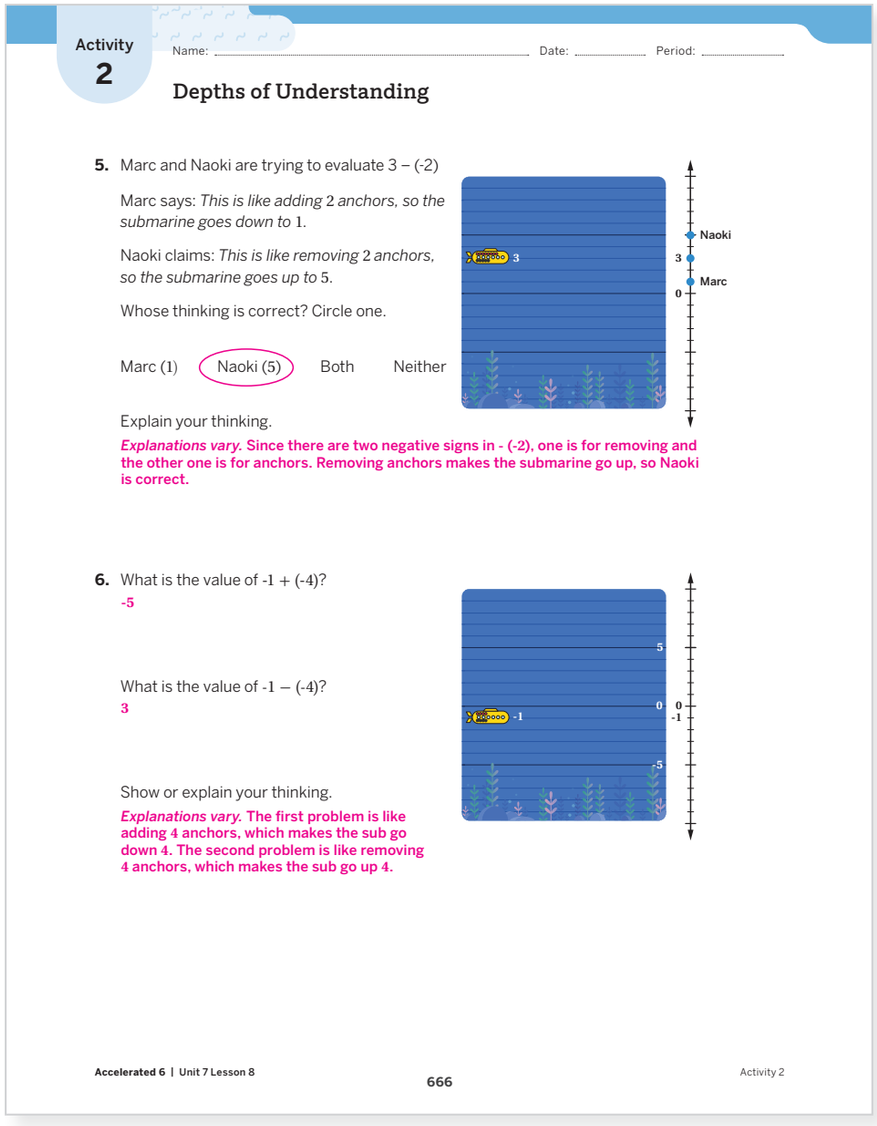
Consider asking, "Based on this discussion, what advice might you give Marc in reconsidering this problem?"

Invite students to continue to Problems 7 and 8. Listen for student explanations on computing the sums and differences.

D **Differentiation** *(Problem 6)*

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Encourage them to model each expression with floats and anchors to make sense of the differences.
Think that both expressions have the same value.	**Support** Consider asking, "What is the difference between these two expressions?"
Recognize one expression represents going up by 4 and the other represents going down by 4, but may not be sure which yet.	**Strengthen** Consider inviting these students to use the language of floats and anchors to support their thinking, e.g., "Start at -1 and add 4 anchors."

Activity 2 continued >



Activity 2

Name: _____ Date: _____ Period: _____

Depths of Understanding

5. Marc and Naoki are trying to evaluate 3 – (-2)

Marc says: *This is like adding 2 anchors, so the submarine goes down to 1.*

Naoki claims: *This is like removing 2 anchors, so the submarine goes up to 5.*

Whose thinking is correct? Circle one.

Marc (1) Naoki (5) Both Neither

Explain your thinking.

Explanations vary. Since there are two negative signs in - (-2), one is for removing and the other one is for anchors. Removing anchors makes the submarine go up, so Naoki is correct.

6. What is the value of -1 + (-4)?

-5

What is the value of -1 – (-4)?

3

Show or explain your thinking.

Explanations vary. The first problem is like adding 4 anchors, which makes the sub go down 4. The second problem is like removing 4 anchors, which makes the sub go up 4.

Accelerated 6 | Unit 7 Lesson 8 666 Activity 2

Pairs | 15 min

Activity 2 Depths of Understanding (continued)

Purpose: Students use floats and anchors as a model and begin to recognize patterns that help them identify different expressions with the same value. **(MTR.5.1)**

2 Monitor

Encourage students to share their reasoning with a partner and work together to reach an agreement about how to sort the pairs of expressions. **(MTR.1.1)**

D **Differentiation: (Stretch)** Listen for students who notice that adding a value is the same thing as subtracting its opposite and consider asking, "Will this always be true?" Invite these students to share during the Connect. **(MTR.3.1, MTR.5.1)**

Note: If time allows, invite pairs to compare their groups, justify their thinking, and make revisions based on their conversation.

A **Accessibility: Executive Function** Invite students to focus on one expression at a time to support students in carrying out multiple steps.

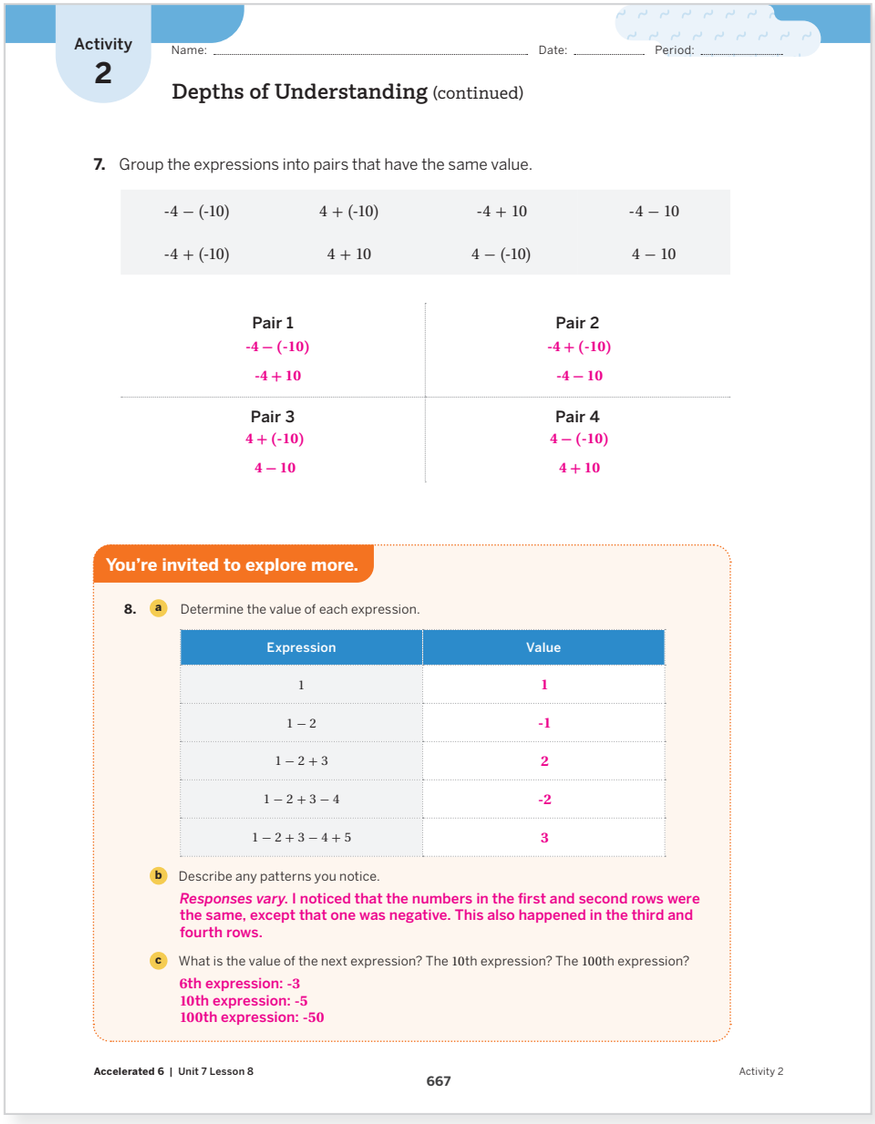
D **Differentiation: You're invited to explore more. (Stretch)** Invite students who would like to explore positive and negative numbers further to complete this optional task. Encourage them to discuss their thinking with a partner.

3 Connect

Invite students to share their responses, calling attention to any conflict or consensus.

To surface the Key Takeaway, consider asking:

- "Why are adding -10 and subtracting 10 a pair?"
  The value -10 can be represented by 10 anchors and adding 10 anchors lowers the submarine 10 units just like subtracting 10 floats.
- "Why are subtracting -10 and adding 10 a pair?"
  The value -10 can be represented by 10 anchors and subtracting 10 anchors raises the submarine 10 units just like adding 10 floats.
- "What are two different expressions you could write to represent subtracting 3 from 7?" **(MTR.3.1)**
  7 − 3 or 7 + (-3)



Activity 2

Name: _____ Date: _____ Period: _____

Depths of Understanding (continued)

7. Group the expressions into pairs that have the same value.

-4 − (-10)	4 + (-10)	-4 + 10	-4 − 10
-4 + (-10)	4 + 10	4 − (-10)	4 − 10

Pair 1	Pair 2
-4 − (-10) -4 + 10	-4 + (-10) -4 − 10
Pair 3	**Pair 4**
4 + (-10) 4 − 10	4 − (-10) 4 + 10

You're invited to explore more.

8. a. Determine the value of each expression.

Expression	Value
1	1
1 − 2	-1
1 − 2 + 3	2
1 − 2 + 3 − 4	-2
1 − 2 + 3 − 4 + 5	3

b. Describe any patterns you notice.
Responses vary. I noticed that the numbers in the first and second rows were the same, except that one was negative. This also happened in the third and fourth rows.

c. What is the value of the next expression? The 10th expression? The 100th expression?
6th expression: -3
10th expression: -5
100th expression: -50

Accelerated 6 | Unit 7 Lesson 8 667 Activity 2

Key Takeaway: Subtracting a value is equivalent to adding its opposite. So when you are adding or subtracting positive and negative numbers, a pair of expressions might have the same value. For example, 3 + (-7) is equivalent to 3 − 7.

Whole Class | 5 min

Synthesis

Purpose: Students synthesize their understanding of pairs of equivalent expressions for computing sums and differences of positive and negative numbers.

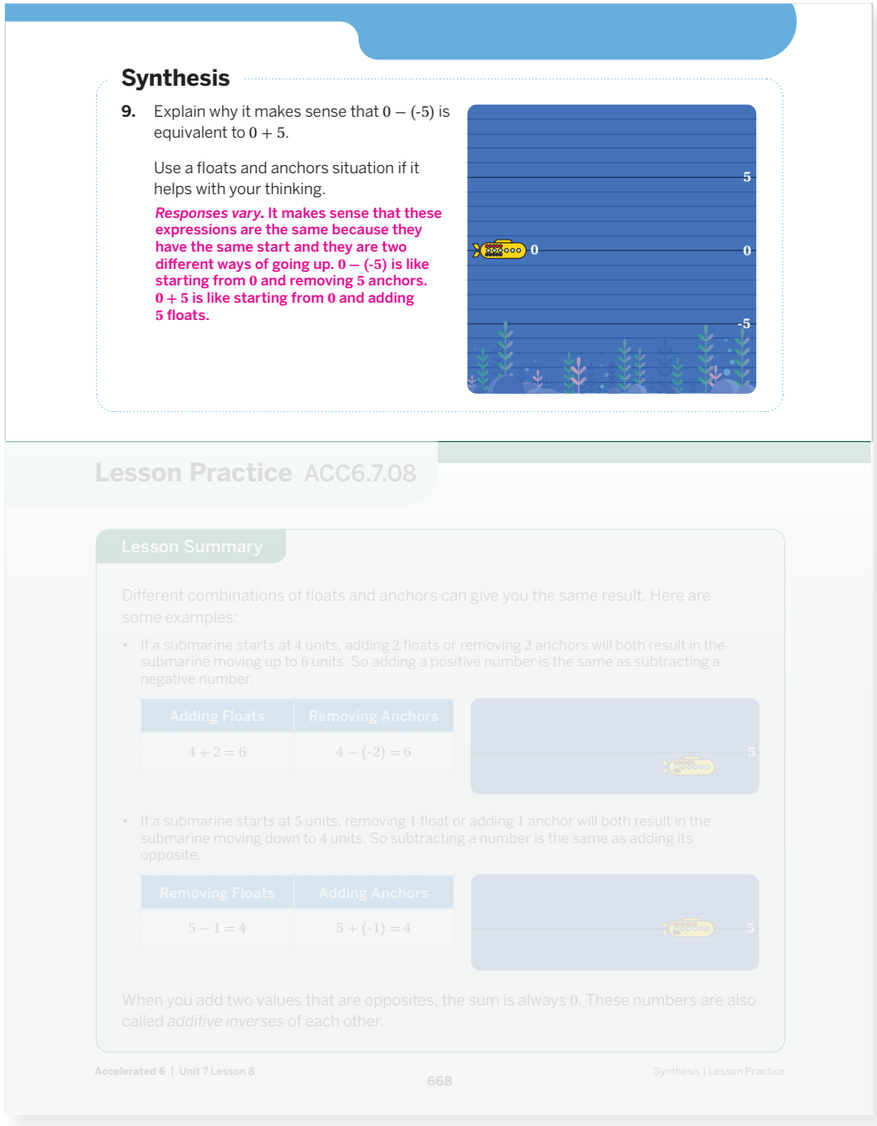
Invite students to respond independently, and then share their thinking with a partner.

Collect and share a variety of ideas, such as:

- The value -5 can be represented by 5 anchors and taking away 5 anchors raises the submarine 5 units just like adding 5 floats.
- The expressions $0 - (-5)$ and $0 + 5$ are equivalent because 5 and -5 are opposites and subtracting a value is equivalent to adding its opposite.

Consider asking, "Why is subtracting a value the same as adding its opposite? Why is this helpful?"

Note: If time allows, invite students to revise their previous thinking based on the discussion.



Synthesis

9. Explain why it makes sense that $0 - (-5)$ is equivalent to $0 + 5$.

Use a floats and anchors situation if it helps with your thinking.

Responses vary. **It makes sense that these expressions are the same because they have the same start and they are two different ways of going up. $0 - (-5)$ is like starting from 0 and removing 5 anchors. $0 + 5$ is like starting from 0 and adding 5 floats.**

Lesson Practice ACC6.7.08

Lesson Summary

Different combinations of floats and anchors can give you the same result. Here are some examples:

- If a submarine starts at 4 units, adding 2 floats or removing 2 anchors will both result in the submarine moving up to 6 units. So adding a positive number is the same as subtracting a negative number.

Adding Floats	Removing Anchors
$4 + 2 = 6$	$4 - (-2) = 6$

- If a submarine starts at 5 units, removing 1 float or adding 1 anchor will both result in the submarine moving down to 4 units. So subtracting a number is the same as adding its opposite.

Removing Floats	Adding Anchors
$5 - 1 = 4$	$5 + (-1) = 4$

When you add two values that are opposites, the sum is always 0. These numbers are also called *additive inverses* of each other.

Accelerated 6 | Unit 7 Lesson 8 668 Synthesis | Lesson Practice

Lesson Takeaway: A visual model can be a helpful tool to write expressions when adding and subtracting positive and negative numbers. You can write equivalent pairs of expressions for sums and differences because subtracting a value is the same as adding its opposite.

Independent | 5 min

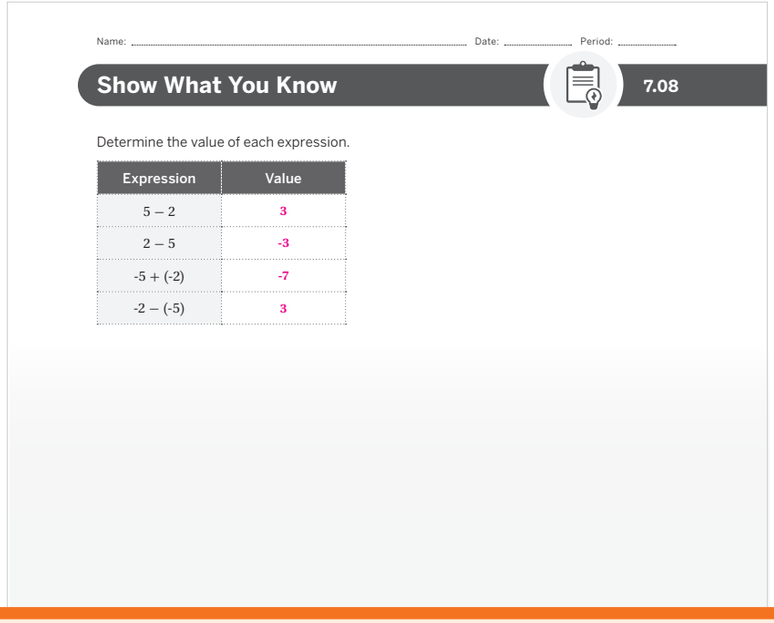
Show What You Know

Purpose: Students demonstrate their understanding by evaluating expressions involving adding and subtracting positive and negative numbers.

Today's Goals

1. **Goal:** Connect different visual models that represent addition and subtraction.
2. **Goal:** Add and subtract positive and negative numbers.
3. **Language Goal:** Describe how a visual model represents adding or subtracting positive and negative numbers. **(Reading, Writing, Speaking, and Listening)**

Show What You Know (Assessment Resources)



Name: _____ Date: _____ Period: _____

Show What You Know 7.08

Determine the value of each expression.

Expression	Value
$5 - 2$	3
$2 - 5$	-3
$-5 + (-2)$	-7
$-2 - (-5)$	3

Differentiation Use after Lesson 8

Support

Provide targeted intervention for students by using these resources.

If students need support adding and subtracting positive and negative numbers:

» **Respond to Student Thinking:**

- Use the ***Adding and Subtracting Positive and Negative Numbers*** **Mini- Lesson**.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite them to explore the **Sub-Unit 1 Extension Activities**.
- Invite them to complete the **You're Invited to Explore More** task and discuss their thinking with a partner.

Professional Learning

Take a moment to reflect on this lesson. How did the **Decide and Defend** routine support students in determining sums and differences of positive and negative numbers?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

9. Explain why it makes sense that $0 - (-5)$ is equivalent to $0 + 5$.

Use a floats and anchors situation if it helps with your thinking.

Responses vary. It makes sense that these expressions are the same because they have the same start and they are two different ways of going up. $0 - (-5)$ is like starting from 0 and removing 5 anchors. $0 + 5$ is like starting from 0 and adding 5 floats.

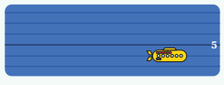
Lesson Practice ACC6.7.08

Lesson Summary

Different combinations of floats and anchors can give you the same result. Here are some examples:

- If a submarine starts at 4 units, adding 2 floats or removing 2 anchors will both result in the submarine moving up to 6 units. So adding a positive number is the same as subtracting a negative number.

Adding Floats	Removing Anchors
$4 + 2 = 6$	$4 - (-2) = 6$



- If a submarine starts at 5 units, removing 1 float or adding 1 anchor will both result in the submarine moving down to 4 units. So subtracting a number is the same as adding its opposite.

Removing Floats	Adding Anchors
$5 - 1 = 4$	$5 + (-1) = 4$

When you add two values that are opposites, the sum is always 0. These numbers are also called *additive inverses* of each other.

Lesson Practice ACC6.7.08

Name: ..... Date: ..... Period: .....

Problems 1–4: Determine the value of each expression.

1. $5 + (-3) = 2$

2. $-5 + 3 = -2$

3. $-5 - 3 = -8$

4. $-5 - (-3) = -2$

Problems 5–6: The table shows eight expressions.

5. Determine the value of each expression.

	Expression	Value
Expression 1	$1 + 2 - 3$	0
Expression 2	$1 + 2 - 3 + 4$	4
Expression 3	$1 + 2 - 3 + 4 - 5$	-1
Expression 4	$1 + 2 - 3 + 4 - 5 + 6$	5
Expression 5	$1 + 2 - 3 + 4 - 5 + 6 - 7$	-2
Expression 6	$1 + 2 - 3 + 4 - 5 + 6 - 7 + 8$	6
Expression 7	$1 + 2 - 3 + 4 - 5 + 6 - 7 + 8 - 9$	-3
Expression 8	$1 + 2 - 3 + 4 - 5 + 6 - 7 + 8 - 9 + 10$	7

6. What is the value of the next expression? The 10th expression? The 20th expression?

9th expression: -4
10th expression: 8
20th expression: 13

7. A swimmer was 8 feet underwater. Then he swam 3 feet deeper. Riku wrote the expression $-8 - 3$. Charlie wrote the expression $-8 + (-3)$. Explain why both Riku and Charlie are correct.

Responses vary. 8 feet underwater can be represented by -8. Swimming 3 feet deeper can be represented by -3. Whether I start at -8 and subtract 3 or add -3, I will still move down 3 feet to a position of -11 feet.

Lesson Practice ACC6.7.08

Name: ..... Date: ..... Period: .....

8. The temperature was -13°F and then rose to 5°F. What was the *change* in temperature? Explain your thinking.

18 degrees. *Explanations vary.* If I use a vertical number line and plot a point at -13 and go up to 5, I first count 13 steps up to reach 0 and then have to go up 5 more steps to reach 5.

9. The temperature was 13°F and then dropped 5 degrees. What was the final temperature?

8°F

FAST Practice

10. The temperature was -13°F and then dropped 5 degrees. What was the final temperature?

-18 °F

Spiral Review

Problems 11–13: Complete each statement with a value that makes the statement true.

11. ..... < 13

Responses less than 13 are considered correct.

12. ..... < -0.1

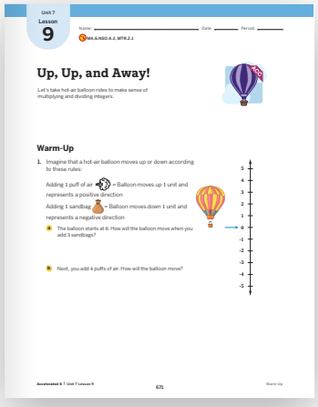
Responses less than -0.1 are considered correct.

13. ..... > -2

Responses greater than -2 are considered correct.

Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1–4	1	MA.6.NSO.4.1
	5–6	3	MA.6.NSO.4.1
	7–9	2	MA.6.NSO.4.1
FAST Practice	10	2	MA.6.NSO.4.1
Spiral Review			
	11–13	1	MA.6.NSO.1.1


Unit 7
Lesson 9
Name: Date: Period:
MA.6.NSO.4.2, MTR.2.1
Up, Up, and Away!
Let's take hot-air balloon rides to make sense of multiplying and dividing integers.
Warm-Up
1. Imagine that a hot-air balloon moves up or down according to these rules:
Adding 1 puff of air = Balloon moves up 1 unit and represents a positive direction
Adding 1 sandbag = Balloon moves down 1 unit and represents a negative direction
a. The balloon starts at 0. How will the balloon move when you add 3 sandbags?
b. Next, you add 4 puffs of air. How will the balloon move?
671
Warm-Up

Up, Up, and Away!

Multiplying and Dividing Integers

Let's take hot-air balloon rides to make sense of multiplying and dividing integers.

Learning Outcomes

Today's Goals

1. **Goal:** Multiply and divide integers with procedural fluency.
2. **Goal:** Use models, number lines, and the properties of operations to generalize rules about multiplication and division of signed numbers.
3. **Language Goal:** Explain why the product or quotient of two integers is negative or positive. **(Reading, Writing, Speaking, and Listening)**

Students use hot-air balloon models and manipulatives to make sense of multiplying and dividing integers. They investigate the movement of a hot-air balloon using positive and negative numbers. Students use counters to generalize patterns about products and quotients of positive and negative integers.

Coherence

Prior Learning

In Lesson 8, students used floats and anchors to add and subtract positive and negative numbers.

Future Learning

In Unit 8, students will solve equations involving multiplication and division with integers.

Connections

- Students begin their exploration of integer multiplication and division using context models. (mathematical, real world)

FL B.E.S.T. Math Benchmarks

Addressing

MA.6.NSO.4.2
Apply and extend previous understandings of operations with whole numbers to multiply and divide integers with procedural fluency.

Mathematical Thinking and Reasoning Standard: MTR.2.1

ELA Expectations: ELA.K12.EE.1.1, ELA.K12.EE.4.1, ELA.K12.EE.6.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.5.NSO.2.1	MA.7.AR.2.1
MA.5.NSO.2.2	MA.7.AR.2.2

Lesson at a Glance

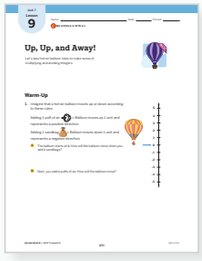
~ 45 min

MA.6.NSO.4.2

Warm-Up

Whole Class | 5 min

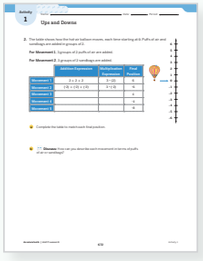
Students are introduced to the context of adding puffs of air or sandbags to control the position of a hot-air balloon.



Activity 1

Pairs | 5 min

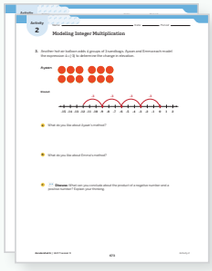
Students use the Think-Pair-Share routine to explore representing changes in the hot-air balloon's position using multiplication.



Activity 2

Pairs | 13 min

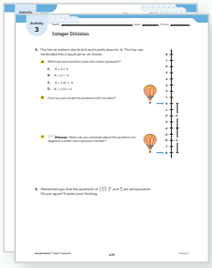
Students use models to investigate products of integers. MLR1: Stronger and Clearer Each Time (MTR.2.1)



Activity 3

Pairs | 12 min

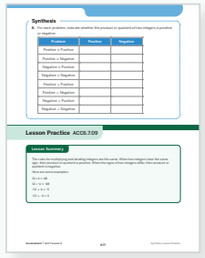
Students use the hot-air balloon context and counters to investigate quotients of integers.
Routine: Decide and Defend (MTR.2.1)



Synthesis

Whole Class | 5 min

Students synthesize their understanding of multiplying and dividing integers and generalize rules.



Show What You Know

Independent | 5 min

Students demonstrate their understanding by multiplying and dividing positive and negative numbers in context.

Prep Checklist

This lesson includes:

- Student Edition
- Show What You Know
- 2-color counters (optional)

Whole Class | 5 min

Warm-Up

Purpose: Students are introduced to the context of adding puffs of air or sandbags to control the position of a hot-air balloon.

1 Launch

To generate excitement, consider inviting students to share whether or not they'd like to go on a hot-air balloon ride someday. Consider displaying images of colorful hot-air balloons or showing a video of a hot-air balloon ride.

ELL **English Language Learners** Create an anchor chart that displays the results of adding puffs of air and sandbags to aid in reasoning throughout this lesson. Consider coming back to the chart and adding more information in subsequent activities. **(Reading and Listening)**

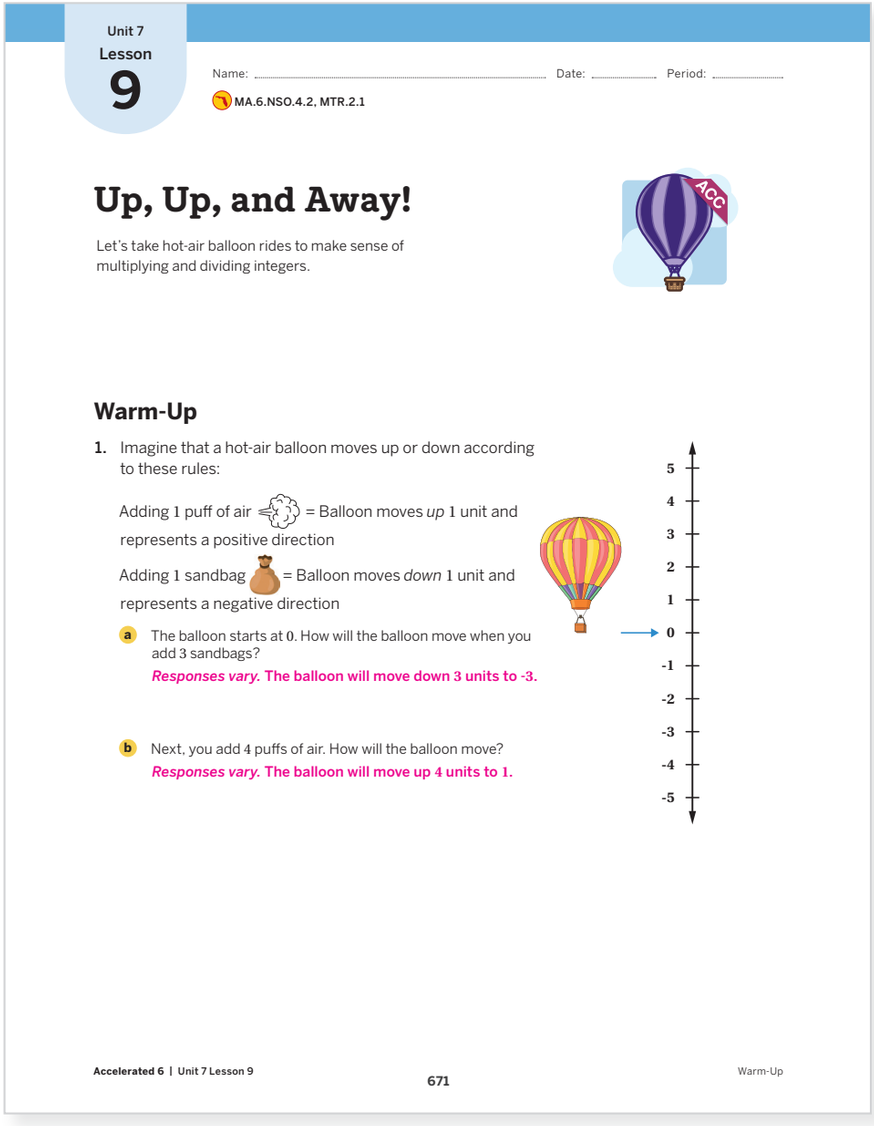
Look for students who might need help understanding how a hot-air balloon moves up or down.

2 Connect

Invite students to share their responses with the class.

Consider asking:

- "When you add puffs of air, does the balloon go up or down?"
- "When you add sandbags, does the balloon go up or down?"
- "When the balloon goes up, does this represent a positive or negative number?"
- "When the balloon goes down, does this represent a positive or negative number?"



Unit 7
Lesson
9

Name: ..... Date: ..... Period: .....

MA.6.NSO.4.2, MTR.2.1

Up, Up, and Away!

Let's take hot-air balloon rides to make sense of multiplying and dividing integers.

Warm-Up

1. Imagine that a hot-air balloon moves up or down according to these rules:

   Adding 1 puff of air = Balloon moves *up* 1 unit and represents a positive direction

   Adding 1 sandbag = Balloon moves *down* 1 unit and represents a negative direction

   a. The balloon starts at 0. How will the balloon move when you add 3 sandbags?

   Responses vary. The balloon will move down 3 units to -3.

   b. Next, you add 4 puffs of air. How will the balloon move?

   Responses vary. The balloon will move up 4 units to 1.

5 4 3 2 1 0 -1 -2 -3 -4 -5

Accelerated 6 | Unit 7 Lesson 9 671 Warm-Up

Pairs | 5 min

Activity 1 Ups and Downs

Purpose: Students use the Think-Pair-Share routine to explore representing changes in the hot-air balloon's position using multiplication.

1 Launch

Use the Think-Pair-Share routine to help students make sense of the groups of puffs of air and sandbags. See the Routine Facilitation Guide for more information.

2 Monitor

Encourage students to first think independently when completing the table, then consult with their partner.

D Differentiation

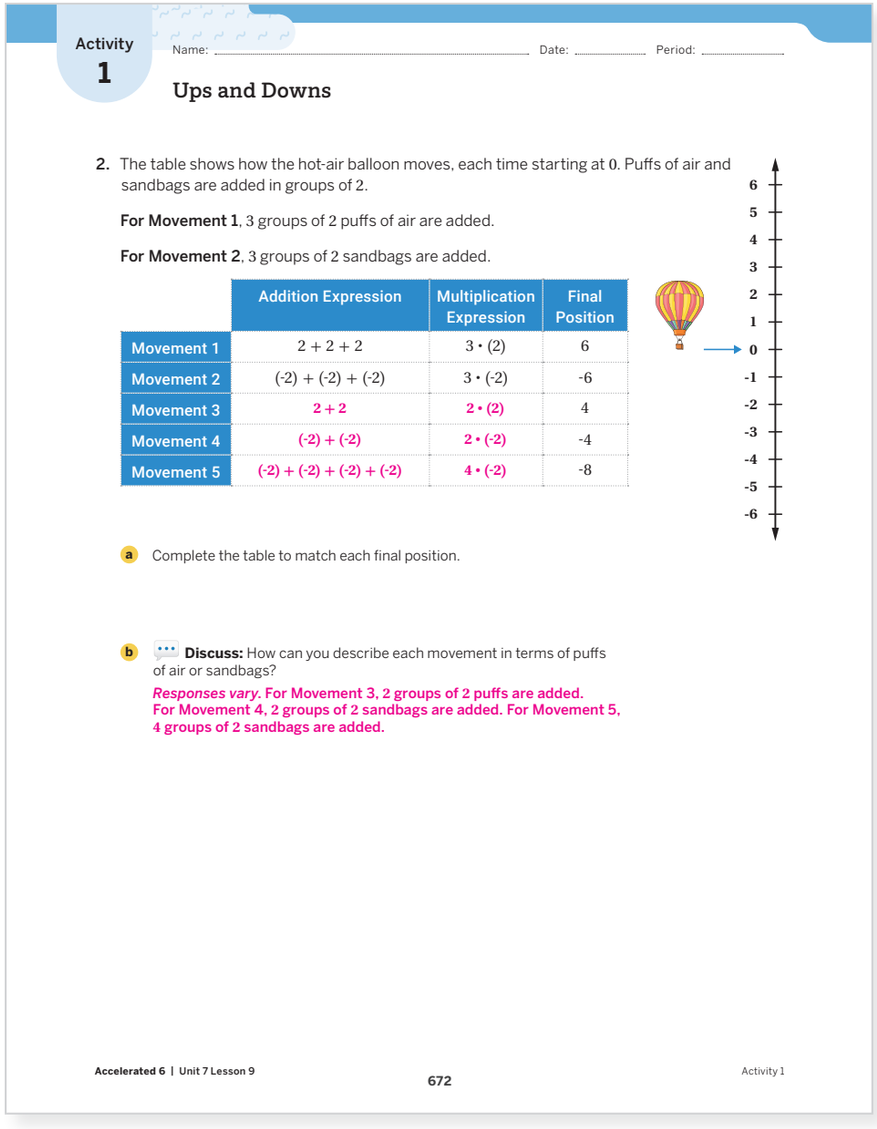
Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Encourage them to think about what is happening in the first two rows. Consider asking, "How are the addition and multiplication expressions related?"
Do not correctly write the addition expressions.	**Support** Encourage them to consider how many 2s add up to 4 and how many -2s add up to -4.
Correctly fill in the missing parts in the table.	**Strengthen** Celebrate their thinking and invite them to share with the class how they determined their solutions.
Want to explore the multiplication of integers further.	**Stretch** Consider asking, "Does it matter if the negative number is first or second?"

ELL **English Language Learners** Provide sentence frames to help students explain their reasoning. For example, "I would add _____ groups of _____ because the final position is _____." **(Reading and Speaking)**

3 Connect

Invite students to share their descriptions for each movement.

Listen for students using key words such as *opposite* or *backward* when they are discussing the hot-air balloon's movement.



Activity 1

Name: _____ Date: _____ Period: _____

Ups and Downs

2. The table shows how the hot-air balloon moves, each time starting at 0. Puffs of air and sandbags are added in groups of 2.

For Movement 1, 3 groups of 2 puffs of air are added.

For Movement 2, 3 groups of 2 sandbags are added.

	Addition Expression	Multiplication Expression	Final Position
Movement 1	2 + 2 + 2	3 • (2)	6
Movement 2	(-2) + (-2) + (-2)	3 • (-2)	-6
Movement 3	2 + 2	2 • (2)	4
Movement 4	(-2) + (-2)	2 • (-2)	-4
Movement 5	(-2) + (-2) + (-2) + (-2)	4 • (-2)	-8

a Complete the table to match each final position.

b **Discuss:** How can you describe each movement in terms of puffs of air or sandbags?

Responses vary. For Movement 3, 2 groups of 2 puffs are added. For Movement 4, 2 groups of 2 sandbags are added. For Movement 5, 4 groups of 2 sandbags are added.

Accelerated 6 | Unit 7 Lesson 9 672 Activity 1

Pairs | 13 min

Activity 2 Modeling Integer Multiplication

Purpose: Students use models to investigate products of integers. **(MTR.2.1)**

1 Launch

Consider asking, "How does the multiplication expression model the movement of the hot-air balloon?"

Look for students who can verbally explain why they like each method and have them share their thinking with the class. **(MTR.2.1)**

ELL **English Language Learners** Consider pairing students with partners who speak the same primary language and allow them to share their thinking using their primary language. **(Writing, Speaking, and Listening)**

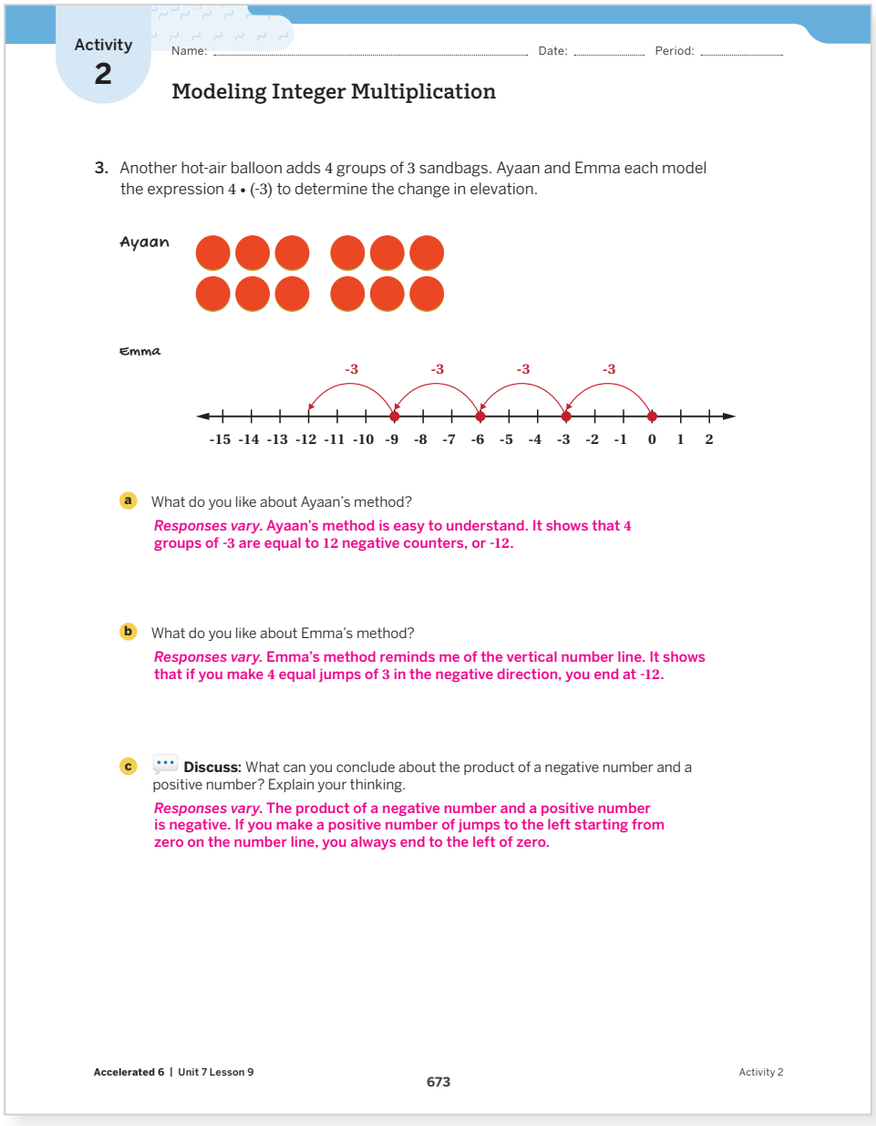
2 Monitor

Encourage students to share their conclusions with a partner.

MLR **MLR1: Stronger and Clearer Each Time** Use this routine to support students in developing and explaining their conclusions. Invite students to meet with one or two different partners, give feedback on each other's drafts, then use the feedback to write a second clearer draft. See the Routine Facilitation Guide for more information.

A **Accessibility: Conceptual Processing** Provide students with a set of 2-color counters to model the expressions in this Lesson.

Activity 2 continued >



Activity 2

Name: _____ Date: _____ Period: _____

Modeling Integer Multiplication

3. Another hot-air balloon adds 4 groups of 3 sandbags. Ayaan and Emma each model the expression 4 • (-3) to determine the change in elevation.

Ayaan

Emma

-3 -3 -3 -3

-15 -14 -13 -12 -11 -10 -9 -8 -7 -6 -5 -4 -3 -2 -1 0 1 2

a. What do you like about Ayaan's method?

Responses vary. Ayaan's method is easy to understand. It shows that 4 groups of -3 are equal to 12 negative counters, or -12.

b. What do you like about Emma's method?

Responses vary. Emma's method reminds me of the vertical number line. It shows that if you make 4 equal jumps of 3 in the negative direction, you end at -12.

c. **Discuss:** What can you conclude about the product of a negative number and a positive number? Explain your thinking.

Responses vary. The product of a negative number and a positive number is negative. If you make a positive number of jumps to the left starting from zero on the number line, you always end to the left of zero.

Accelerated 6 | Unit 7 Lesson 9 673 Activity 2

Key Takeaway: The product of a negative and a positive integer is negative, and the product of two negative integers is positive.

Pairs | 13 min

Activity 2 Modeling Integer Multiplication (continued)

Purpose: Students use models to investigate products of integers. **(MTR.2.1)**

2 Monitor

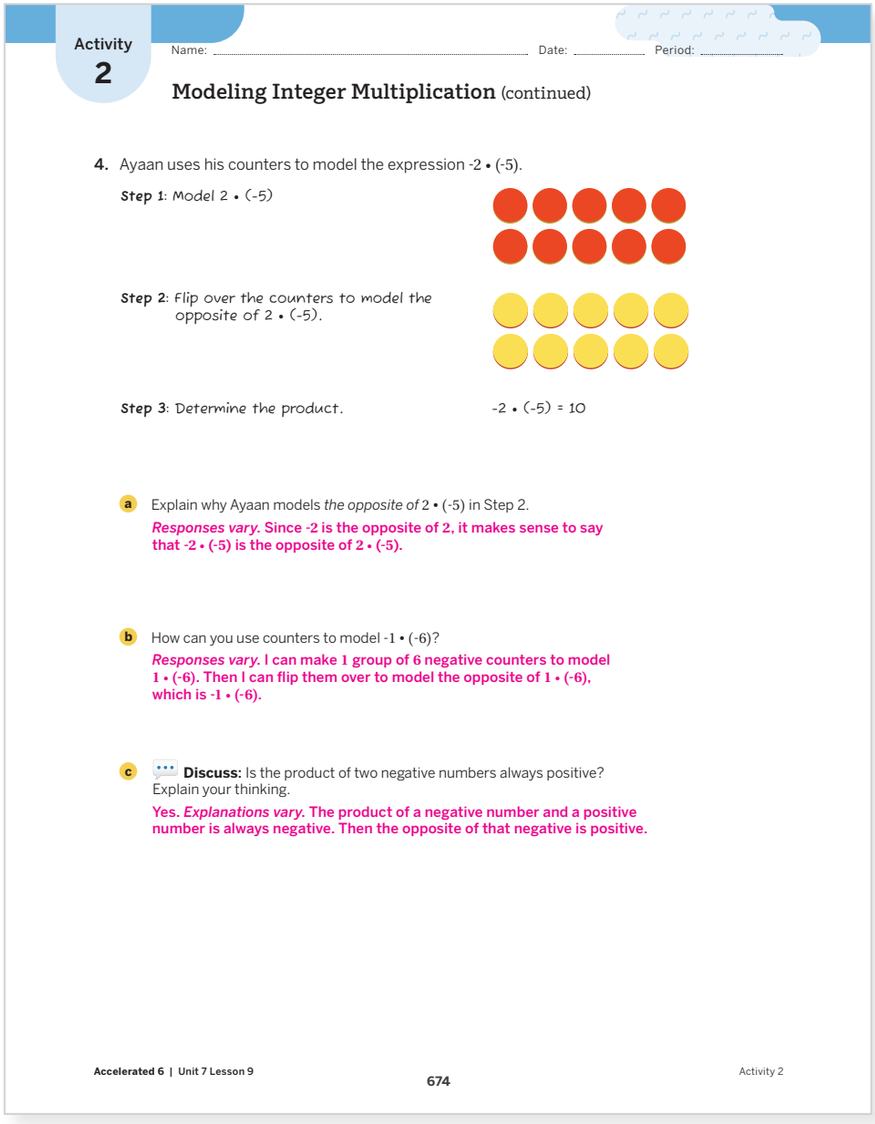
Look and listen for how students make sense of modeling integer multiplication and generalizations about integer multiplication. Invite students to share their thinking during the Connect.

3 Connect

Invite students to share their explanations for Problem 4a and their models for Problem 4b.

To surface the Key Takeaway, consider asking:

- "What happens when you multiply a positive integer and a negative integer?"
- "What happens when you multiply two negative integers together?"



Activity 2

Name: _____ Date: _____ Period: _____

Modeling Integer Multiplication (continued)

4. Ayaan uses his counters to model the expression -2 • (-5).

Step 1: Model 2 • (-5)

Step 2: Flip over the counters to model the opposite of 2 • (-5).

Step 3: Determine the product.

-2 • (-5) = 10

a. Explain why Ayaan models *the opposite of* 2 • (-5) in Step 2.

Responses vary. Since -2 is the opposite of 2, it makes sense to say that -2 • (-5) is the opposite of 2 • (-5).

b. How can you use counters to model -1 • (-6)?

Responses vary. I can make 1 group of 6 negative counters to model 1 • (-6). Then I can flip them over to model the opposite of 1 • (-6), which is -1 • (-6).

c. **Discuss:** Is the product of two negative numbers always positive? Explain your thinking.

Yes. *Explanations vary.* The product of a negative number and a positive number is always negative. Then the opposite of that negative is positive.

Accelerated 6 | Unit 7 Lesson 9 674 Activity 2

Key Takeaway: The product of a negative and a positive integer is negative, and the product of two negative integers is positive.

Pairs | 12 min

Activity 3 Integer Division

Purpose: Students use the hot-air balloon context and counters to investigate quotients of integers. **(MTR.2.1)**

Short on time? Consider skipping Problem 5b.

1 Launch

Consider asking, "How is this situation the same as the previous problems with the hot-air balloon? How is it different?"

Invite students to share their ideas about how to model the problem with counters. **(MTR.2.1)**

2 Monitor

Use the Decide and Defend routine to support students in making arguments and critiquing the reasoning of others. See the Routine Facilitation Guide for more information.

Look for students who use properties of operations to justify Mohamed's reasoning.

Look for explanations and ideas to share during the Connect.

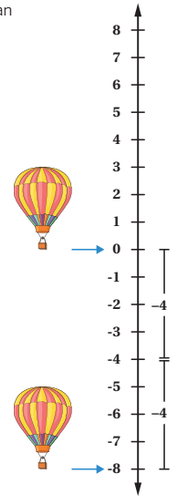
Activity 3 continued >

Activity 3

Name: _____ Date: _____ Period: _____

Integer Division

5. The hot-air balloon starts at 0 and travels down to -8. The trip can be divided into 2 equal parts, as shown.



a. Which division problem does the model represent?

A. $8 \div 2 = 4$

B. $-8 \div 2 = -4$

C. $8 \div (-2) = -4$

D. $-8 \div (-2) = 4$

b. How can you model the problem with counters?

I can take 8 negative counters and divide them into 2 equal groups, each with 4 negative counters.

c. **Discuss:** What can you conclude about the quotient of a negative number and a positive number?

Responses vary. The quotient of a negative number and a positive number is negative.

6. Mohamed says that the quotients of $-\left(\frac{12}{6}\right)$, $\frac{-12}{6}$, and $\frac{12}{-6}$ are all equivalent. Do you agree? Explain your thinking.

Responses vary. I agree because each expression shows the opposite of $12 \div 6$.

Accelerated 6 | Unit 7 Lesson 9 675 Activity 3

Pairs | 12 min

Activity 3 Integer Division (continued)

Purpose: Students use the hot-air balloon context and counters to investigate quotients of integers. **(MTR.2.1)**

Short on time? Consider skipping Problem 5b.

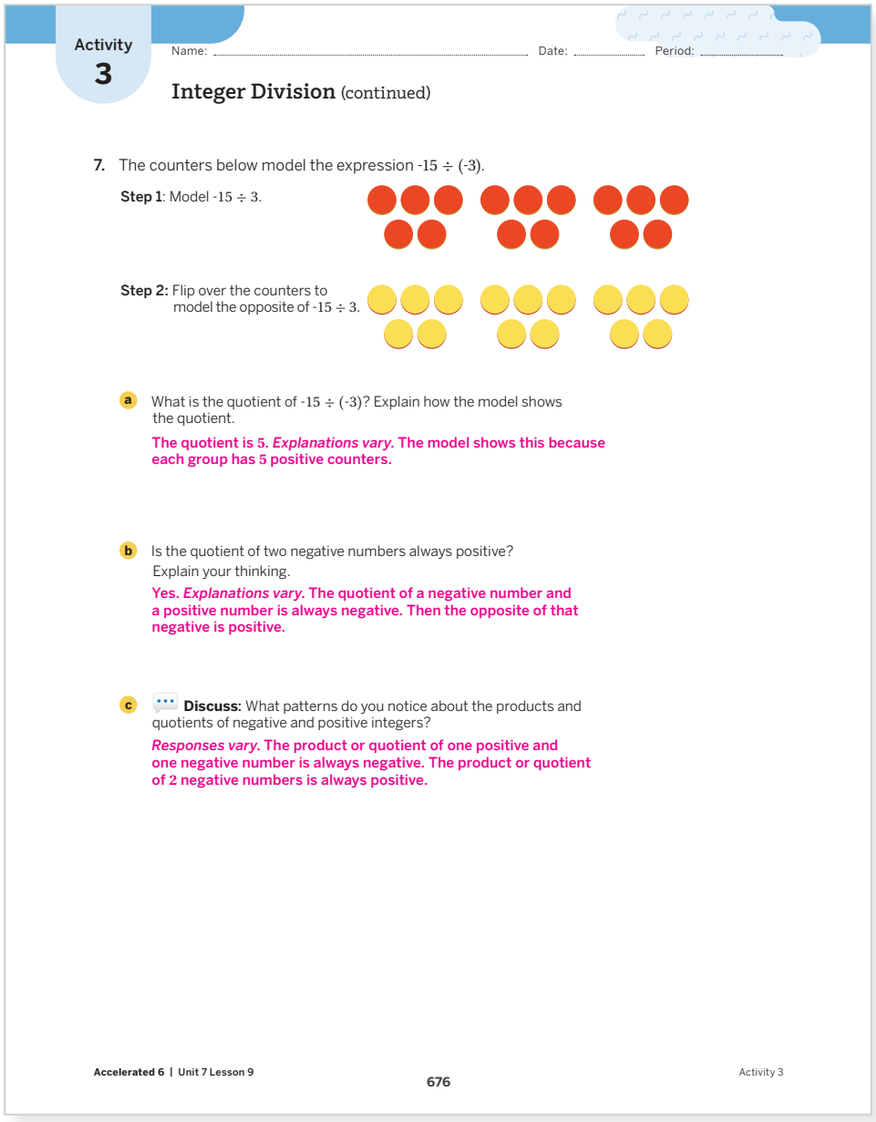
2 Monitor

D **Differentiation: Support** Consider providing 2-color counters so students can perform the steps themselves.

3 Connect

To surface connections, consider discussing how the counters were used to model a negative times a negative in the previous activity.

To surface the Key Takeaway, invite students to share their observations about the patterns of integer signs and their quotients.



Activity 3

Name: _____ Date: _____ Period: _____

Integer Division (continued)

7. The counters below model the expression $-15 \div (-3)$.

Step 1: Model $-15 \div 3$.

Step 2: Flip over the counters to model the opposite of $-15 \div 3$.

a. What is the quotient of $-15 \div (-3)$? Explain how the model shows the quotient.

The quotient is 5. *Explanations vary.* The model shows this because each group has 5 positive counters.

b. Is the quotient of two negative numbers always positive? Explain your thinking.

Yes. *Explanations vary.* The quotient of a negative number and a positive number is always negative. Then the opposite of that negative is positive.

c. **Discuss:** What patterns do you notice about the products and quotients of negative and positive integers?

Responses vary. The product or quotient of one positive and one negative number is always negative. The product or quotient of 2 negative numbers is always positive.

Accelerated 6 | Unit 7 Lesson 9 676 Activity 3

Key Takeaway: The quotient of a negative and a positive integer is negative, and the quotient of two negative integers is positive.

Whole Class | 5 min

Synthesis

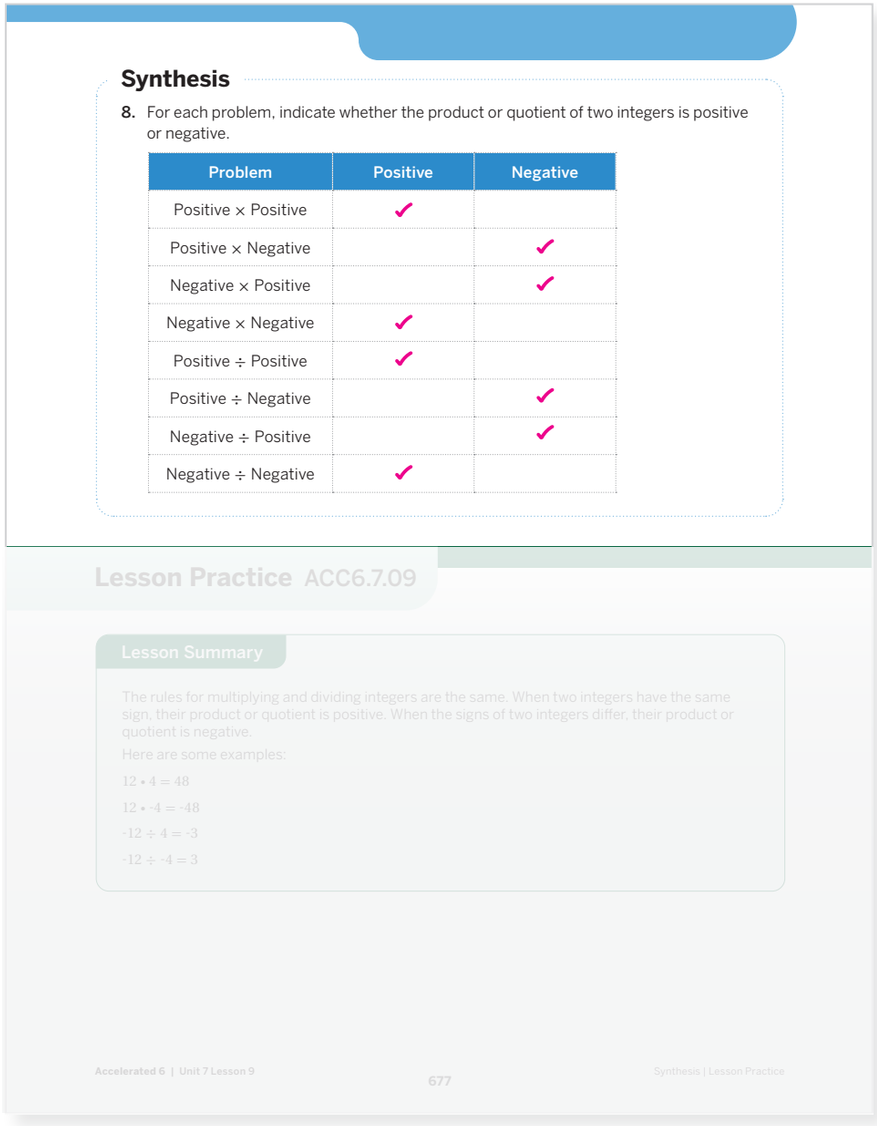
Purpose: Students synthesize their understanding of multiplying and dividing integers and generalize rules.

Invite students to respond independently, and then share their thinking with a partner.

Collect and share a variety of ideas, such as:
- Different expressions that have the same value.
- Patterns between multiplication and division.
- Words or phrases that represent negative quantities.
- Words or phrases that represent positive quantities.

Create an anchor chart of rules that students were able to generalize during the lesson when multiplying or dividing integers. Allow them to use this anchor chart in future lessons as needed.

Note: If time allows, invite students to revise their previous thinking based on the discussion.



Synthesis

8. For each problem, indicate whether the product or quotient of two integers is positive or negative.

Problem	Positive	Negative
Positive × Positive	✓	
Positive × Negative		✓
Negative × Positive		✓
Negative × Negative	✓	
Positive ÷ Positive	✓	
Positive ÷ Negative		✓
Negative ÷ Positive		✓
Negative ÷ Negative	✓	

Lesson Practice ACC6.7.09

Lesson Summary

The rules for multiplying and dividing integers are the same. When two integers have the same sign, their product or quotient is positive. When the signs of two integers differ, their product or quotient is negative.

Here are some examples:

$12 \cdot 4 = 48$

$12 \cdot -4 = -48$

$-12 \div 4 = -3$

$-12 \div -4 = 3$

Accelerated 6 | Unit 7 Lesson 9 677 Synthesis | Lesson Practice

Lesson Takeaway: When multiplying and dividing two numbers whose signs are the same, the answer is positive. If they are different, the answer is negative.

Independent | 5 min

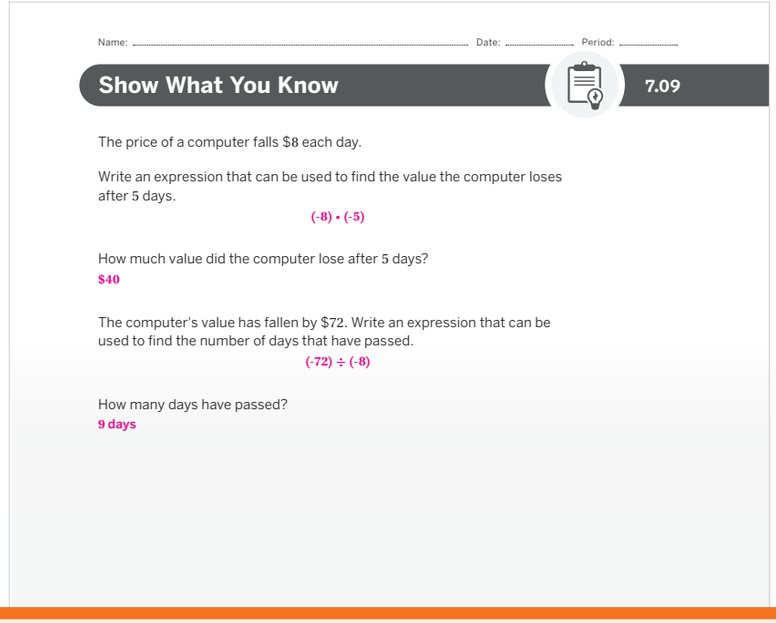
Show What You Know

Purpose: Students demonstrate their understanding by multiplying and dividing positive and negative numbers in context.

Today's Goals

1. **Goal:** Multiply and divide integers with procedural fluency.
2. **Goal:** Use models, number lines, and the properties of operations to generalize rules about multiplication and division of signed numbers.
3. **Language Goal:** Explain why the product or quotient of two integers is negative or positive. **(Reading, Writing, Speaking, and Listening)**

Show What You Know (Assessment Resources)



Name: _____ Date: _____ Period: _____

Show What You Know 7.09

The price of a computer falls $8 each day.

Write an expression that can be used to find the value the computer loses after 5 days.

$(-8) \cdot (-5)$

How much value did the computer lose after 5 days?

$40

The computer's value has fallen by $72. Write an expression that can be used to find the number of days that have passed.

$(-72) \div (-8)$

How many days have passed?

9 days

D Differentiation Use after Lesson 9

Support

Provide targeted intervention for students by using these resources.

If students need support multiplying and dividing positive and negative numbers in context:

» **Respond to Student Thinking:**

- Use the ***Solving Real-World Problems Involving Positive and Negative Numbers*** **Mini-Lesson**.
- Revisit the **Lesson Summary**.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite them to explore the **Sub-Unit 1 Extension Activities**.

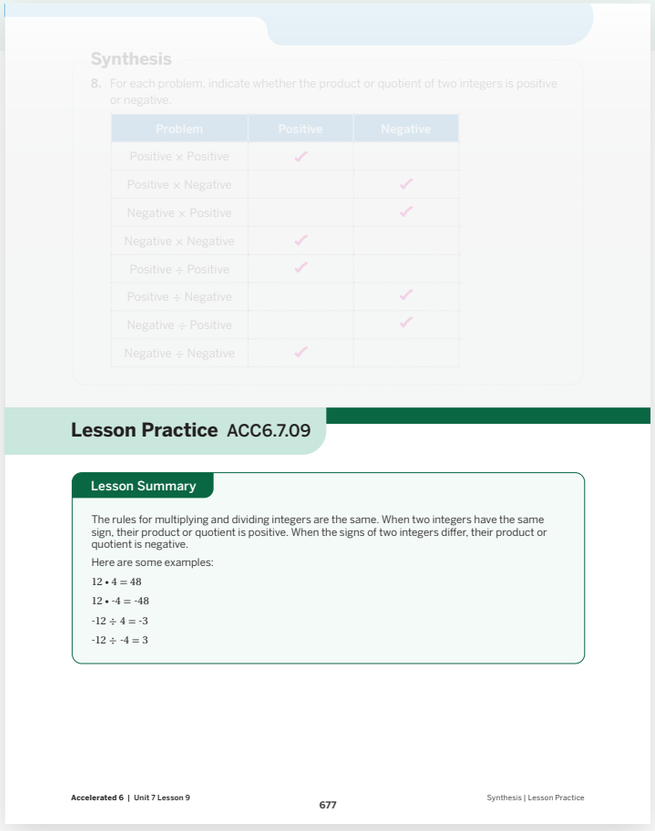
Professional Learning

How can you use the work students did in today's lesson to prepare them for working with rational numbers in future lessons?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.



Synthesis

8. For each problem, indicate whether the product or quotient of two integers is positive or negative.

Problem	Positive	Negative
Positive × Positive	✓	
Positive × Negative		✓
Negative × Positive		✓
Negative × Negative	✓	
Positive ÷ Positive	✓	
Positive ÷ Negative		✓
Negative ÷ Positive		✓
Negative ÷ Negative	✓	

Lesson Practice ACC6.7.09

Lesson Summary

The rules for multiplying and dividing integers are the same. When two integers have the same sign, their product or quotient is positive. When the signs of two integers differ, their product or quotient is negative.

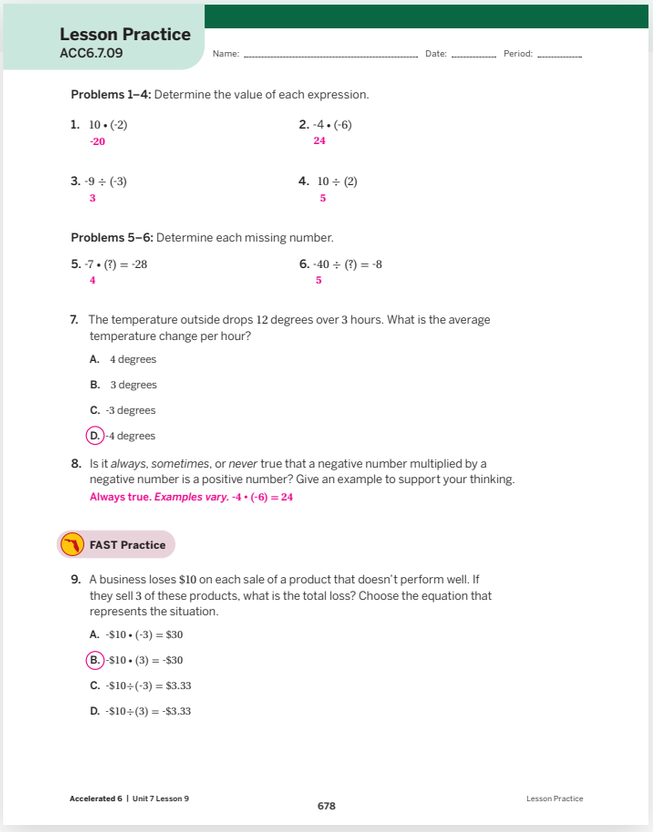
Here are some examples:

$12 \cdot 4 = 48$

$12 \cdot -4 = -48$

$-12 \div 4 = -3$

$-12 \div -4 = 3$



Lesson Practice
ACC6.7.09

Name: _____ Date: _____ Period: _____

Problems 1–4: Determine the value of each expression.

1. $10 \cdot (-2)$
   -20
2. $-4 \cdot (-6)$
   24
3. $-9 \div (-3)$
   3
4. $10 \div (2)$
   5

Problems 5–6: Determine each missing number.

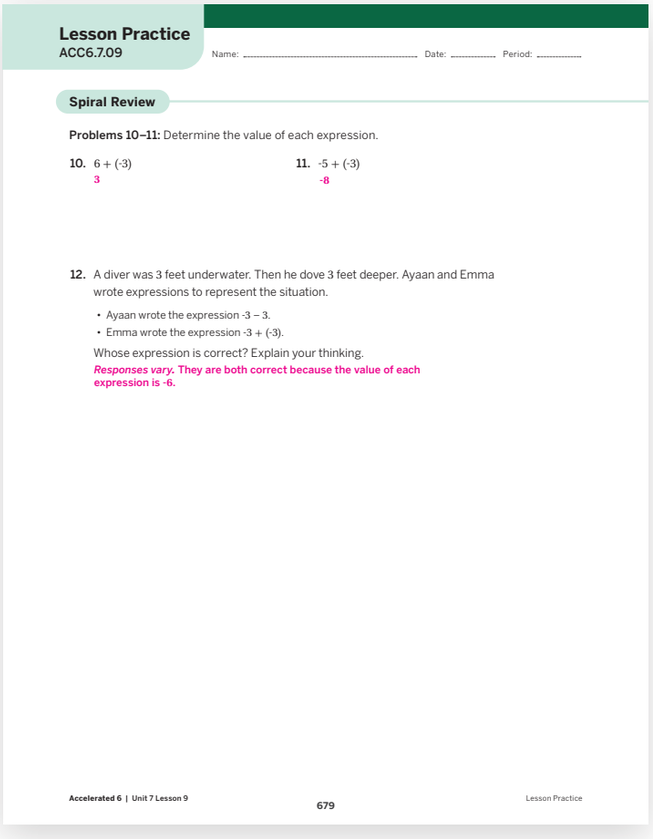
5. $-7 \cdot (?) = -28$
   4
6. $-40 \div (?) = -8$
   5

7. The temperature outside drops 12 degrees over 3 hours. What is the average temperature change per hour?
   - A. 4 degrees
   - B. 3 degrees
   - C. -3 degrees
   - D. -4 degrees

8. Is it *always*, *sometimes*, or *never* true that a negative number multiplied by a negative number is a positive number? Give an example to support your thinking.
   Always true. *Examples vary.* $-4 \cdot (-6) = 24$

FAST Practice

9. A business loses $10 on each sale of a product that doesn't perform well. If they sell 3 of these products, what is the total loss? Choose the equation that represents the situation.
   - A. $-\$10 \cdot (-3) = \$30$
   - B. $-\$10 \cdot (3) = -\$30$
   - C. $-\$10 \div (-3) = \$3.33$
   - D. $-\$10 \div (3) = -\$3.33$



Lesson Practice
ACC6.7.09

Name: _____ Date: _____ Period: _____

Spiral Review

Problems 10–11: Determine the value of each expression.

10. $6 + (-3)$
    3
11. $-5 + (-3)$
    -8

12. A diver was 3 feet underwater. Then he dove 3 feet deeper. Ayaan and Emma wrote expressions to represent the situation.
    - Ayaan wrote the expression $-3 - 3$.
    - Emma wrote the expression $-3 + (-3)$.

    Whose expression is correct? Explain your thinking.
    Responses vary. They are both correct because the value of each expression is -6.

Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1–4	1	MA.6.NSO.4.2
	5–7	2	MA.6.NSO.4.2
	8	3	MA.6.NSO.4.2
FAST Practice	9	2	MA.6.NSO.4.2
Spiral Review			
	10–11	1	MA.6.NSO.4.1
	12	3	MA.6.NSO.4.1

Assess and Respond

Sub-Unit Quiz 1

Independent | 45 min

Facilitation: Assign the Sub-Unit Quiz to learn about your students' understanding of the concepts and skills so far in this unit. If time allows, consider inviting students to complete one or more of the tasks in the Unit Synthesis and Reflection.

Item Analysis

Problem(s)	Concept or skill	Addressed in	DOK	FL B.E.S.T. Math Benchmarks
1	Comparing and ordering positive and negative numbers by reasoning about the structure of number lines	Lesson 2	1	MA.6.NSO.1.1
2	Using the structure of a number line to plot and label positive and negative numbers and their opposites	Lesson 4	1	MA.6.NSO.1.2
3	Using the symbols $<$, $>$, or $=$ to compare numbers and expressions involving absolute value	Lessons 4, 5	1	MA.6.NSO.1.1, MA.6.NSO.1.4
4a	Using negative numbers to represent quantities in context	Lessons 3, 6	2	MA.6.NSO.1.2
4b	Distinguishing comparisons of absolute value from comparisons about order and justifying the response	Lesson 5	2	MA.6.NSO.1.3, MA.6.NSO.1.4, MTR.4.1
4c	Comparing and ordering positive and negative numbers within the context of real-world situations	Lessons 3, 6	1	MA.6.NSO.1.1
5a, 5b	Determining if statements involving absolute value are are always, sometimes, or never true and explaining the reasoning used	Lesson 5	3	MA.6.NSO.1.3, MA.6.NSO.1.4, MTR.4.1

Assessment Resources

- Student Assessments
- Assessment Rubrics with Answers
- Show What You Knows and Answer Keys

Intervention and Extension Resources

- Mini-Lessons
- Extensions

Name: ..... Date: ..... Period: .....

Sub-Unit Quiz

Unit 7

1. If the following numbers were plotted on a number line, which would be farthest to the *left*?

A. -2

B. $-1\frac{3}{4}$

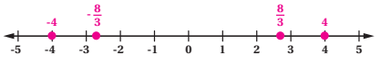
C. $\frac{11}{4}$

D. $-\frac{11}{4}$

2. Here are two numbers: -4 and $\frac{8}{3}$.

On the number line, plot and label:

- Each number.
- The opposite of each number.



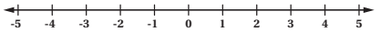
3. Complete each number sentence with the symbol $<$, $>$, or $=$. Use the number line if it helps with your thinking.

a. $|2| __>__ -2$

b. $|-3| __>__ -2.5$

c. $-\frac{2}{3} __<__ \frac{3}{2}$

d. $-\frac{5}{2} __<__ -\frac{2}{5}$



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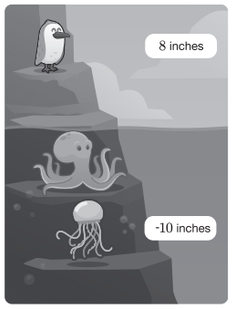
Name: ..... Date: ..... Period: .....

Sub-Unit Quiz (continued)

Unit 7

4. Here is a snapshot of some creatures on the coast. The surface of the water has an elevation of 0 inches.



a. The octopus is under the water above the jellyfish. What could be the elevation of the octopus?

Responses vary. **Any elevation less than 0 inches and greater than -10 inches.**

b. Which creature's elevation has the greatest absolute value?

Jellyfish

Explain how you know.

Explanations vary. **The absolute value is a number's distance from 0. In this situation, 0 inches represents the surface of the water and the jellyfish is the farthest away from the surface of the water.**

c. Here are elevations of three other creatures.

Andean condor: 800 feet | Anglerfish: -2,000 feet | Giant squid: -1,600 feet

Order their elevations from *highest* to *lowest*.

Andean condor: 800 feet	Giant squid: -1,600 feet	Anglerfish: -2,000 feet
Highest Elevation		**Lowest Elevation**

5. Decide if each statement is always, sometimes, or never true. Circle one.

a. The absolute value of a number is negative.

Always Sometimes (Never)

Explain your reasoning.

Explanations vary. **The absolute value is a number's distance from 0. Distances aren't negative, so absolute values can't be negative.**

b. A number and its opposite have the same absolute value. Circle one.

(Always) Sometimes Never

Explain your reasoning.

Explanations vary. **The absolute value is a number's distance from 0. Opposites are always the same distance away from 0, so their absolute values are the same.**

D Differentiation (Sub-Unit Quiz 1)

Note: To **strengthen** and **stretch** students' learning, refer to the differentiation resources suggested throughout this unit and in the Unit Overview.

Sub-Unit Goal	Problem(s)	To respond to student thinking, consider:

Sub-Unit 1:
• Order and compare positive numbers, negative numbers, and 0. **(Lessons 1–9)**
	1, 2	**Support**
		• **Mini-Lesson:** *Ordering and Comparing Positive and Negative Numbers*
		• **Teacher Move:** Consider revisiting:
		  • Lesson 2 *(Digging Deeper)*
		  • Lesson 4 *(Order in the Class)*

		Strengthen
		• **Repeated Challenges:**
		  • Lesson 1 *(Can You Dig It?)*
		  • Lesson 2 *(Digging Deeper)*

	4a, 4c	**Support**
		Strengthen
		• **Teacher Move:** Consider revisiting:
		  • Lesson 3 *(Sub-Zero)*
		  • Lesson 6 *(We've Got Game(s))*

		Stretch
		• **You're invited to explore more:**
		  • Lesson 3 *(Sub-Zero)*
		  • Lesson 6 *(We've Got Game(s))*

	3, 4b, 5	**Support**
		Strengthen
		• **Teacher Move:** Consider revisiting:
		  • Lesson 4 *(Order in the Class)*
		  • Lesson 5 *(Distance on the Number Line)*

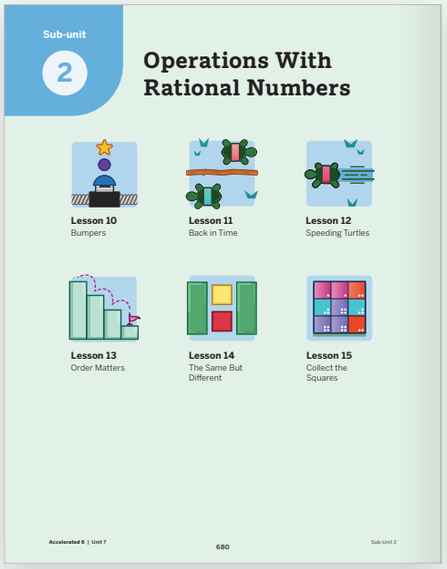
		Stretch • **You're invited to explore more:** • Lesson 5 *(Distance on the Number Line)*

Operations With Rational Numbers

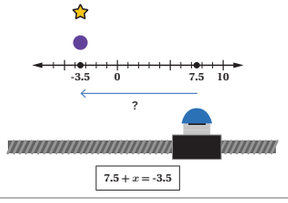
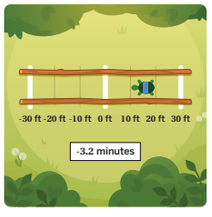
Sub-Unit 2 Goals

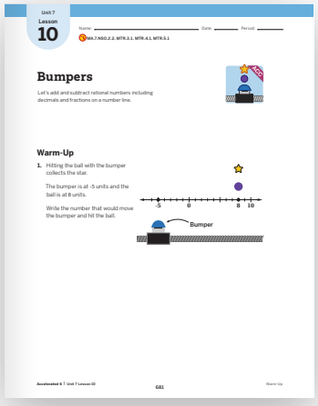
- **Goal:** Perform all four operations with positive and negative rational numbers using a variety of strategies.
- **Goal:** Use order of operations to evaluate numerical expressions.
- **Language Goal:** Describe strategies for determining the sum, difference, product, or quotient of two rational numbers.
- **Language Goal:** Describe and use strategies for writing equivalent expressions that involve adding, subtracting, and expanding terms.

In this sub-unit, students develop models to add, subtract, multiply, and divide rational numbers. They extend their learning by using order of operations to evaluate numerical expressions that include grouping symbols, absolute value, and exponents. They also write equivalent expressions using fewer terms by adding, subtracting, and expanding.



Math That Matters Most

Concept or Skill	For example . . .
A number line can help to identify the start, change, and end to determine an unknown value in equations.	The bumper is at 7.5 units and the ball is at -3.5 units. Write the number that would move the bumper and hit the ball. **-11 units** 
We can determine the position of an object by multiplying the rate it is traveling by the time.	One student wrote the expression $(-5)(-3.2) = 16$ to determine Tam's position. Explain what each number represents in the situation. **-5 represents that Tam is walking left 5 feet every minute.** **-3.2 represents 3.2 minutes back in time.** **16 represents where Tam was at that time.** 
Use order of operations to evaluate numerical expressions.	What is the value of the expression $18 - 3^2 + 10 \times 2$? **First, evaluate the term with the exponent.** **18 – 9 + 10 × 2** **Next, multiply 10 times 2.** **18 – 9 + 20** **Finally, subtract and add from left to right.** **9 + 20** **The value of the expression is 29.**
Write equivalent expressions using a variety of strategies including adding, subtracting, and expanding terms.	Here is an expression. Write an equivalent expression using the fewest number of terms. $6(x + 1) + 8 - x + (-x - 12)$ **6x + 6 + 8 – x + (-x – 12)** **6x + 14 – x – x – 12** **4x + 2**



Bumpers

Adding and Subtracting on the Number Line

Let's add and subtract rational numbers including decimals and fractions on a number line.

Learning Outcomes

Today's Goals

1. **Goal:** Add and subtract positive and negative numbers, including decimals and fractions, on a number line.
2. **Language Goal:** Describe a strategy for adding and subtracting positive and negative decimals and fractions. **(Reading, Writing, Speaking, and Listening)**

Students use number lines as a model for adding and subtracting rational numbers (e.g., positive and negative integers, decimals, and fractions). The lesson begins with the context of a bumper moving to hit a ball and progresses toward performing operations without a number line to develop fluency in adding and subtracting rational numbers. **(MTR.3.1)**

Coherence

Prior Learning

In Lesson 8, students added and subtracted integers.

Future Learning

In Lesson 11, students will extend their knowledge of multiplication of integers to multiply positive and negative rational numbers.

Connections

- Students explore how to represent real world scenarios in the context of a bumper and ball on a number line. (mathematical)

FL B.E.S.T. Math Benchmarks

Addressing

MA.7.NSO.2.2
Add, subtract, multiply, and divide rational numbers with procedural fluency.

Mathematical Thinking and Reasoning Standards: MTR.3.1, MTR.4.1, MTR.5.1

ELA Expectation: ELA.K12.EE.1.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.6.NSO.4.1	**MA.8.NSO.1.5** **MA.8.NSO.1.7**

Lesson at a Glance

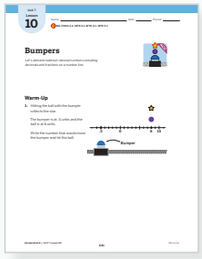
~ 45 min

MA.7.NSO.2.2

Warm-Up

Pairs | 5 min

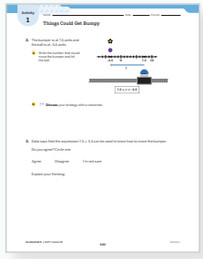
Students are introduced to the context of moving a bumper along a number line to model addition of positive and negative numbers.



Activity 1

Pairs | 10 min

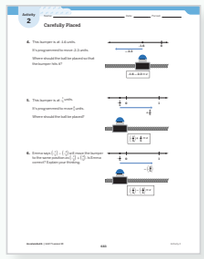
Students determine an unknown value in a bumper challenge involving positive and negative decimals.



Activity 2

Pairs | 10 min

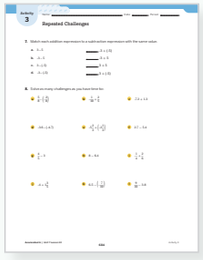
Students use number lines to determine an unknown value in an equation by adding or subtracting positive and negative decimals and fractions. **Routine:** Decide and Defend



Activity 3

Pairs | 10 min

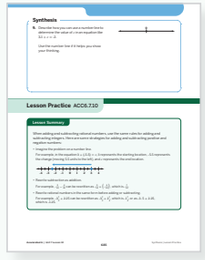
Students complete repeated challenges to practice adding and subtracting rational numbers. **(MTR.3.1)**



Synthesis

Whole Class | 5 min

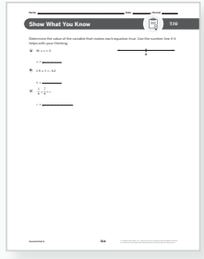
Students synthesize their understanding of how to model adding positive and negative decimals using a number line.



Show What You Know

Independent | 5 min

Students demonstrate their understanding by determining an unknown amount in a one-step addition equation with positive and negative fractions and decimals.



Prep Checklist

Prepare Horizontal Number Sheets

This lesson includes:

- Student Edition
- Show What You Know
- Horizontal Number Lines Sheet, one per student (optional)

Pairs | 5 min

Warm-Up

Purpose: Students are introduced to the context of moving a bumper along a number line to model addition of positive and negative numbers.

1 Launch

Consider sharing that the bumper can move to the left and to the right along the number line.

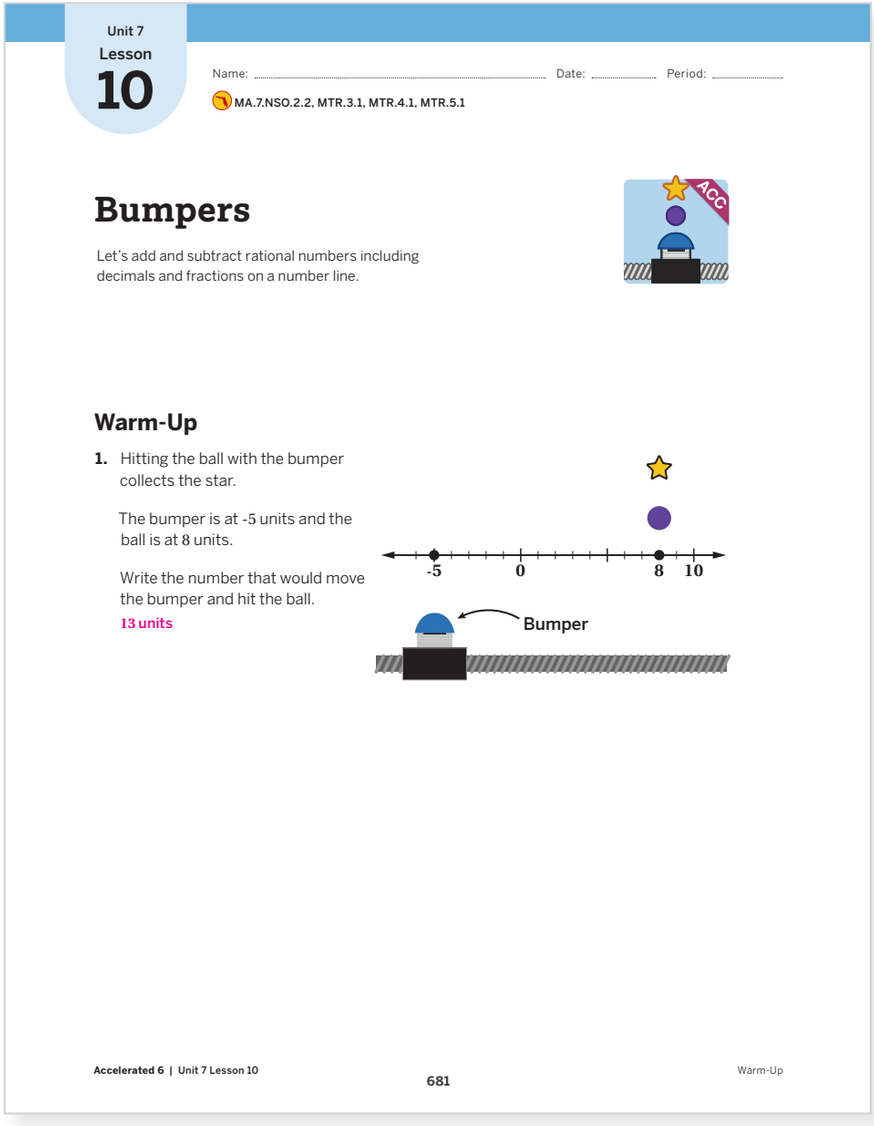
A **Accessibility: Visual-Spatial Processing**
Consider inviting students to use their fingers or pencils to count the spaces on the number line.

2 Connect

Invite students to share their responses.

Consider asking:

- "Why does it make sense that the change from -5 to 8 is 13?"
- "How would you explain this to someone else?"
- "What equation could you write to represent the number line?"



Unit 7
Lesson
10

Name: Date: Period:

MA.7.NSO.2.2, MTR.3.1, MTR.4.1, MTR.5.1

Bumpers

Let's add and subtract rational numbers including decimals and fractions on a number line.

Warm-Up

1. Hitting the ball with the bumper collects the star.

The bumper is at -5 units and the ball is at 8 units.

Write the number that would move the bumper and hit the ball.

13 units

Accelerated 6 | Unit 7 Lesson 10 681 Warm-Up

Pairs | 10 min

Activity 1 Things Could Get Bumpy

Purpose: Students determine an unknown value in a bumper challenge involving positive and negative decimals.

1 Launch

To surface connections, consider discussing how it is possible to move the bumper to the left by adding a number. **(MTR.5.1)**

Consider distributing the Horizontal Number Lines Sheet to each student as a space to write their thinking throughout the lesson.

2 Monitor

Look for a variety of ways students make sense of the challenge.

D Differentiation

Look for students who:	Respond to Student Thinking
Estimate the distance and use the equation to revise or check their work.	**Strengthen** Consider asking, "How is the answer you determined related to the values -3.5 and 7.5?"
Determine the distances on each side of 0 and add them.	**Strengthen** Celebrate this strategy. Consider asking, "Which direction does the bumper need to move? How will you communicate that?"

A Accessibility: Visual-Spatial Processing Invite pairs to think about key features of the visual, such as the start, change, and end points. Encourage pairs to discuss them aloud and make connections with the equation representation.

Look for a variety of reasoning students use to evaluate Dalia's claim.

ELL English Language Learners Invite students to talk through their thinking with a partner before writing it down. **(Writing and Speaking)**

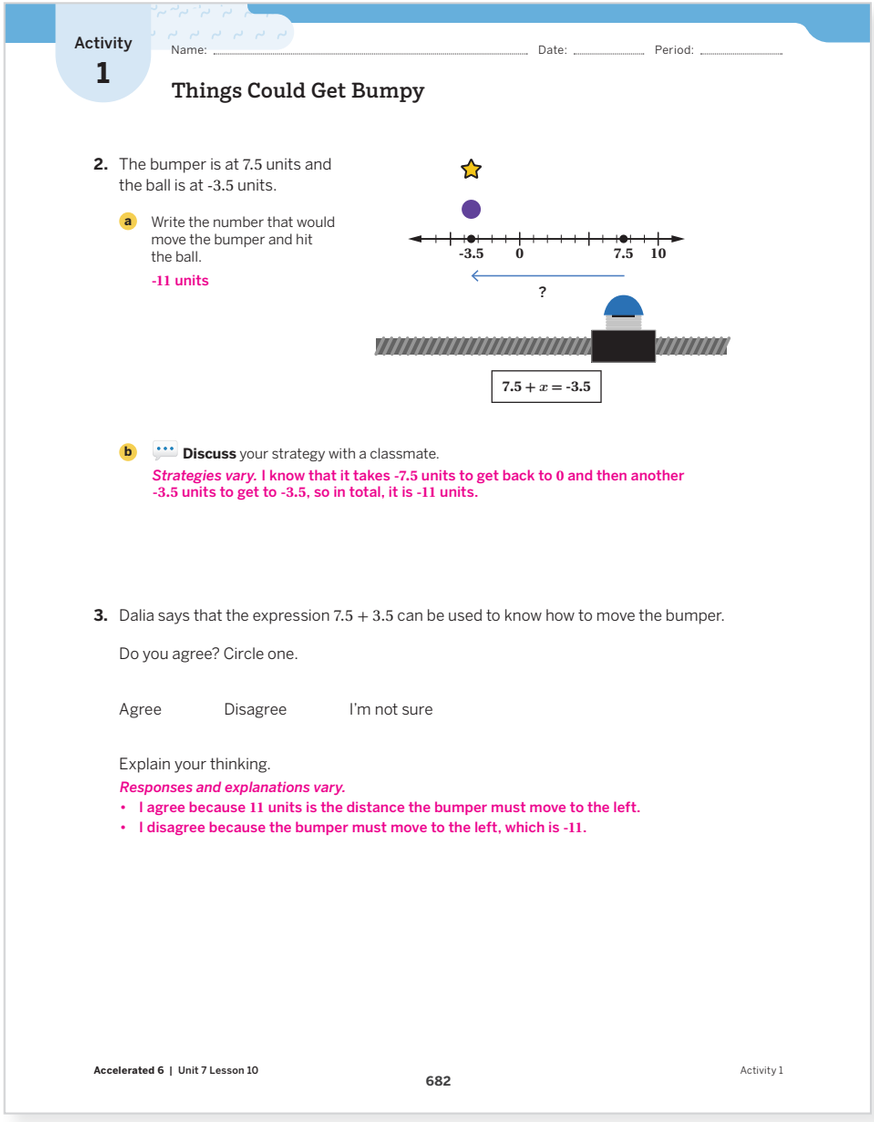
3 Connect

Record the distribution of responses by polling the class, calling attention to any conflict or consensus.

Invite students who made different choices to share their reasoning.

To surface the Key Takeaway, consider asking:

- "How does what you see on the number line connect with what you see in Dalia's work?"
- "What is correct about Dalia's work?"
- "What could you add or change to make Dalia's claim stronger and clearer?"



Activity 1

Name: Date: Period:

Things Could Get Bumpy

2. The bumper is at 7.5 units and the ball is at -3.5 units.

a. Write the number that would move the bumper and hit the ball.

-11 units

b. **Discuss** your strategy with a classmate.

Strategies vary. I know that it takes -7.5 units to get back to 0 and then another -3.5 units to get to -3.5, so in total, it is -11 units.

3. Dalia says that the expression $7.5 + 3.5$ can be used to know how to move the bumper.

Do you agree? Circle one.

Agree Disagree I'm not sure

Explain your thinking.

Responses and explanations vary.

- I agree because 11 units is the distance the bumper must move to the left.
- I disagree because the bumper must move to the left, which is -11.

Accelerated 6 | Unit 7 Lesson 10 682 Activity 1

Key Takeaway: When solving a bumper challenge, it is important to consider both the distance and the direction the bumper needs to travel. If the bumper is crossing zero, you can determine the distance on either side of zero and add them together. Adding a negative number moves the bumper to the left, and adding a positive number moves the bumper to the right.

Pairs | 10 min

Activity 2 Carefully Placed

Purpose: Students use number lines to determine an unknown value in an equation by adding or subtracting positive and negative decimals and fractions.

1 Launch

Direct students' attention to the bumper and equation in Problem 4. Consider asking, "How is this like the previous challenge? How is it different?" They both have a bumper on a number line, but this time we are deciding where to put the ball instead of how to move the bumper. The previous challenge involved addition, but this challenge involves subtraction.

2 Monitor

To support students getting started, consider inviting them to model $-1 - 2 = x$ on the Horizontal Number Lines Sheet. Then ask, "How is this challenge similar to and different from that problem?"

Look for a variety of strategies students use to decide where the ball should be placed.

D Differentiation

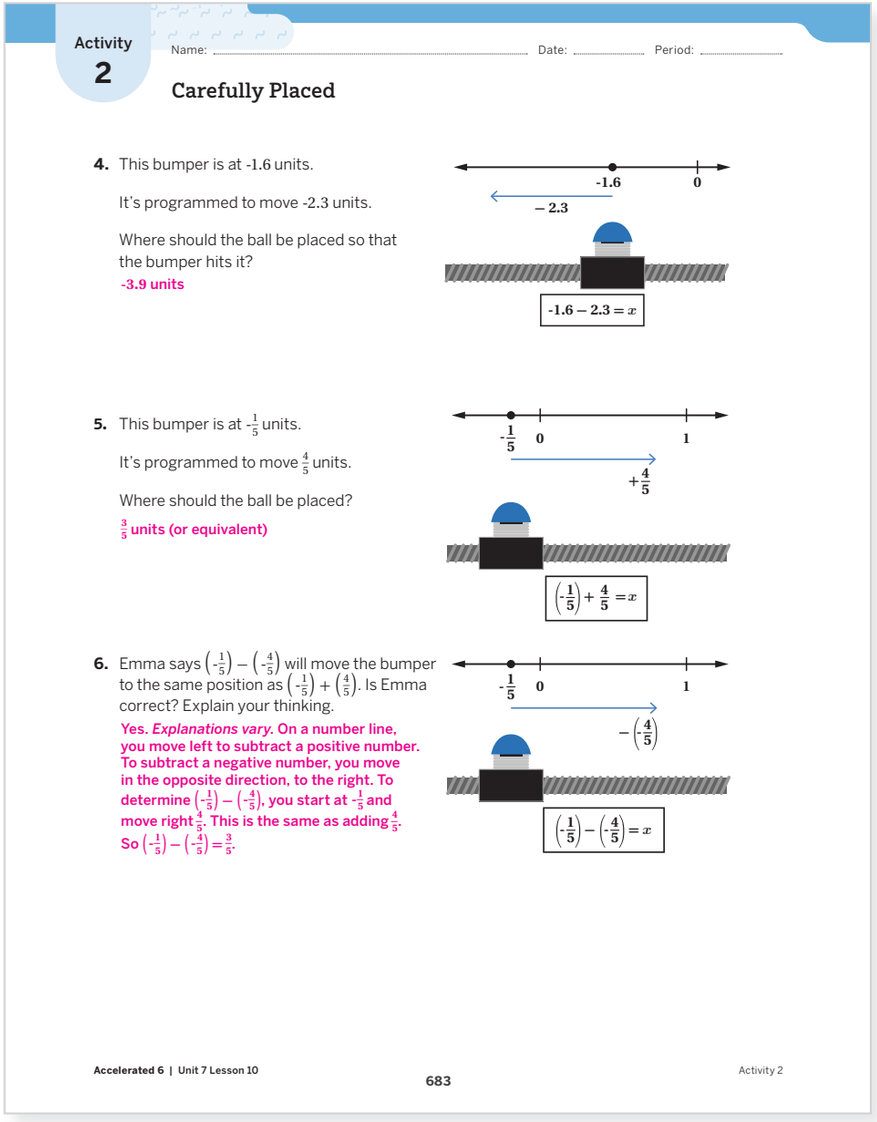
Look for students who:	Respond to Student Thinking
Think the ball should be placed at $\frac{5}{5}$ because $\frac{1}{5}+\frac{4}{5}=\frac{5}{5}$.	**Support** Consider asking, "How many fifths do you need to move the ball to 0? How many more fifths past 0?"
Add the denominators of the fractions, e.g., write $\frac{3}{10}$ or $\frac{5}{10}$.	**Support** Consider asking, "How big is $\frac{1}{10}$ compared to $\frac{1}{5}$? Where would your answer be on the number line?"
Use a strategy such as: • Counting 4 fifths forward from $-\frac{1}{5}$. • Subtracting $\frac{1}{5}$ from $\frac{4}{5}$.	**Strengthen** Invite students to compare their strategy with a classmate.

3 Connect

Use the Decide and Defend routine to support students in making arguments and critiquing the reasoning of others. **(MTR.4.1)**

Consider asking:
- "What reasoning or evidence supports your claim?"
- "What evidence or language was most convincing to you?"

To surface the Key Takeaway, consider asking:
- "Why is subtracting a negative value like adding the opposite of the value?"
- "What direction would we move on the number line if we subtracted -2.3 from a number?"



Activity 2

Name: Date: Period:

Carefully Placed

4. This bumper is at -1.6 units.

It's programmed to move -2.3 units.

Where should the ball be placed so that the bumper hits it?

-3.9 units

5. This bumper is at $-\frac{1}{5}$ units.

It's programmed to move $\frac{4}{5}$ units.

Where should the ball be placed?

$\frac{3}{5}$ units (or equivalent)

6. Emma says $\left(-\frac{1}{5}\right)-\left(-\frac{4}{5}\right)$ will move the bumper to the same position as $\left(-\frac{1}{5}\right)+\left(\frac{4}{5}\right)$. Is Emma correct? Explain your thinking.

Yes. *Explanations vary.* On a number line, you move left to subtract a positive number. To subtract a negative number, you move in the opposite direction, to the right. To determine $\left(-\frac{1}{5}\right)-\left(-\frac{4}{5}\right)$, you start at $-\frac{1}{5}$ and move right $\frac{4}{5}$. This is the same as adding $\frac{4}{5}$. So $\left(-\frac{1}{5}\right)-\left(-\frac{4}{5}\right)=\frac{3}{5}$.

Accelerated 6 | Unit 7 Lesson 10 683 Activity 2

Key Takeaway: Subtraction is the opposite of addition. To subtract a negative number, move to the right on a number line because subtracting a negative value has the same change as adding its positive value.

Pairs | 10 min

Activity 3 Bumper Challenge

Purpose: Students complete repeated challenges to practice adding and subtracting rational numbers. **(MTR.3.1)**

Short on time? Consider inviting students to complete fewer challenges.

1 Launch

Consider asking, "Will you always be able to represent a subtraction problem as addition?"

Share that it's more important to make sense of the challenges than it is to work quickly.

A **Accessibility: Memory and Attention** Invite students who would benefit from having more time to think deeply to complete fewer challenges.

2 Monitor

Listen for strategies students use in discussing how to solve the problems, such as:

- Drawing each challenge on a number line.
- Determining the start, change, and end.
- Using 0 as a benchmark, e.g., determining the movement necessary to get to 0 and adding the movement past 0.
- Generalizing about when to use subtraction or addition to calculate the difference. **(MTR.3.1)**

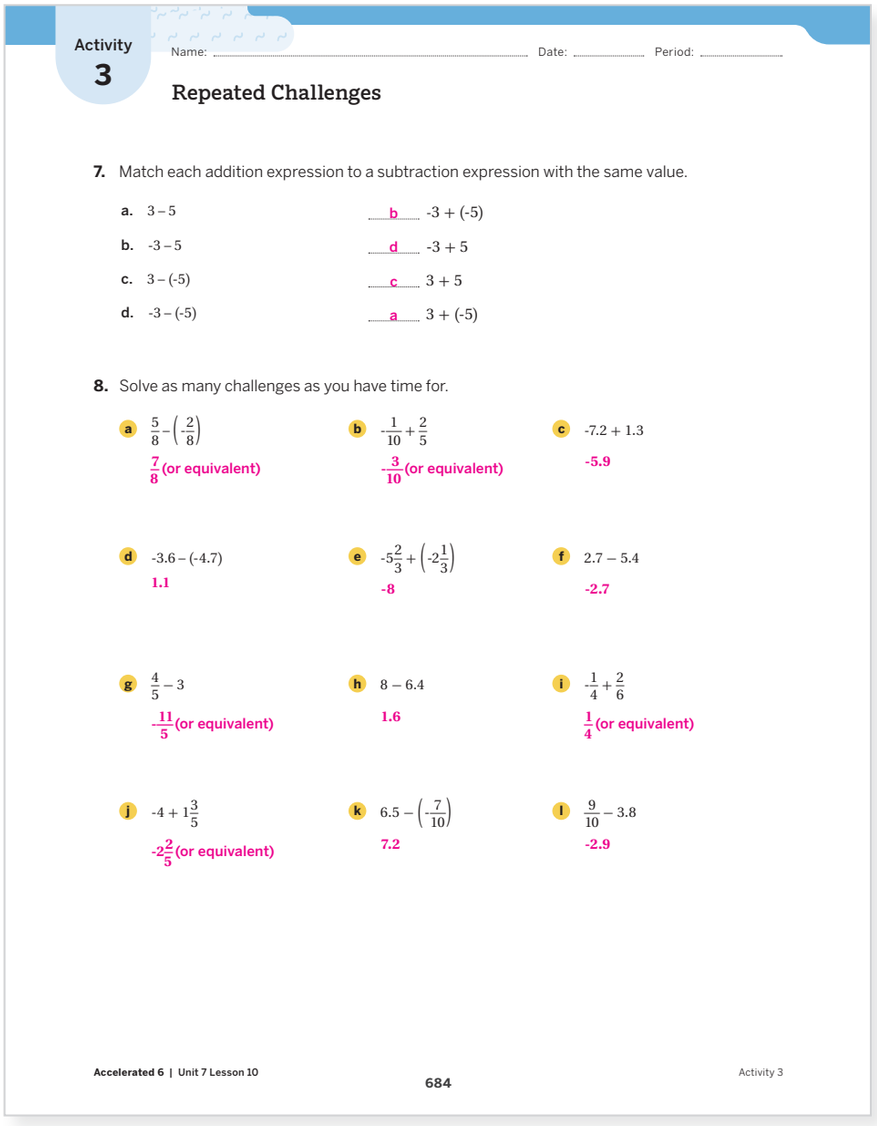
D **Differentiation: (Support)** Students may need support performing calculations involving fractions. Encourage them to choose a unit fraction to scale their number line before labeling the values from the problem.

3 Connect

Discuss the strategies students used to approach the repeated challenges.

Consider asking:

- "What strategies were useful for more than one challenge?"
- "What patterns did you notice?" **(MTR.5.1)**



Activity 3

Name: Date: Period:

Repeated Challenges

7. Match each addition expression to a subtraction expression with the same value.

a. $3-5$	b $-3+(-5)$
b. $-3-5$	d $-3+5$
c. $3-(-5)$	c $3+5$
d. $-3-(-5)$	a $3+(-5)$

8. Solve as many challenges as you have time for.

a. $\frac{5}{8}-\left(\frac{2}{8}\right)$ — $\frac{7}{8}$ (or equivalent)

b. $-\frac{1}{10}+\frac{2}{5}$ — $\frac{3}{10}$ (or equivalent)

c. $-7.2+1.3$ — -5.9

d. $-3.6-(-4.7)$ — 1.1

e. $-5\frac{2}{3}+\left(-2\frac{1}{3}\right)$ — -8

f. $2.7-5.4$ — -2.7

g. $\frac{4}{5}-3$ — $-\frac{11}{5}$ (or equivalent)

h. $8-6.4$ — 1.6

i. $-\frac{1}{4}+\frac{2}{6}$ — $\frac{1}{4}$ (or equivalent)

j. $-4+1\frac{3}{5}$ — $-2\frac{2}{5}$ (or equivalent)

k. $6.5-\left(-\frac{7}{10}\right)$ — 7.2

l. $\frac{9}{10}-3.8$ — -2.9

Accelerated 6 | Unit 7 Lesson 10 684 Activity 3

Whole Class | 5 min

Synthesis

Purpose: Students synthesize their understanding of how to model adding positive and negative decimals using a number line.

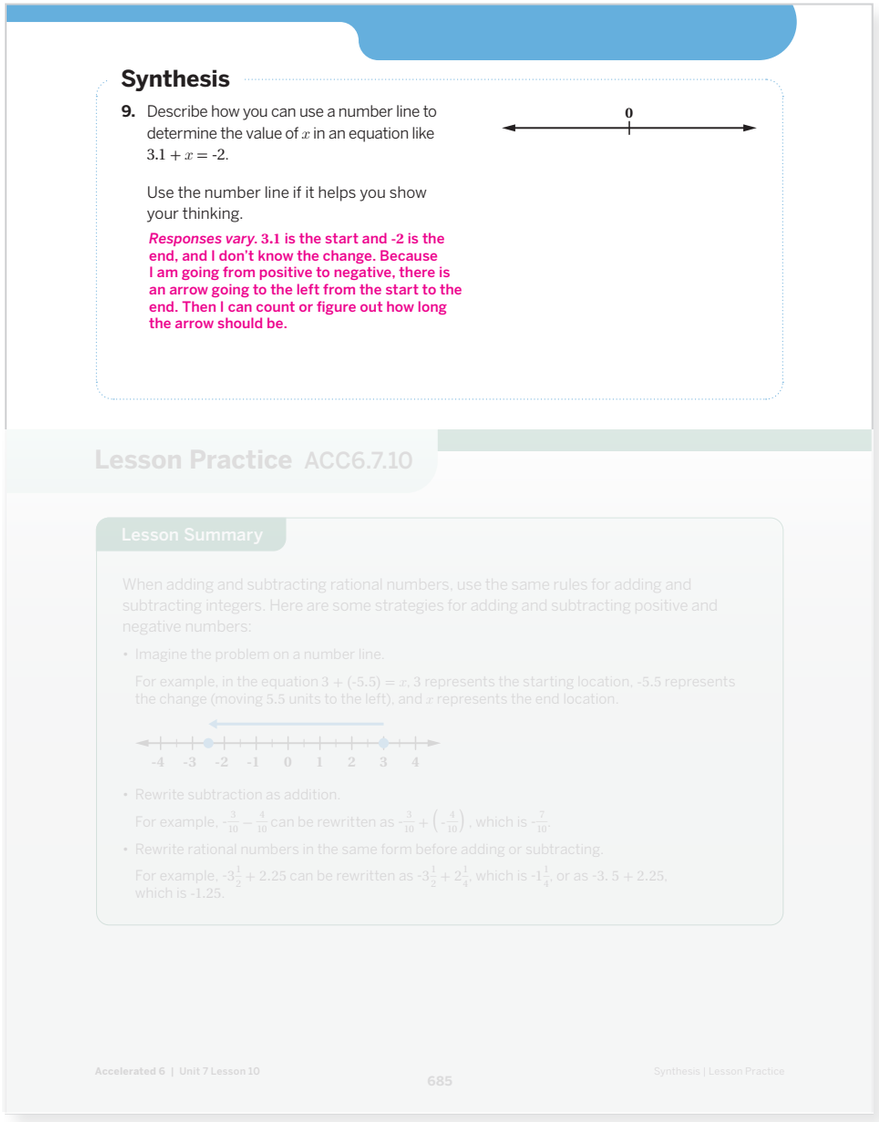
Invite students to respond independently, and then share their thinking with a partner.

Collect and share a variety of ideas, such as:

- Using 3.1 as a starting point and -2 as the end, then determining the distance between them.
- Noticing that the value of x must be negative because the number line shows moving from a positive to a negative number.
- Determining the distance from 3.1 to 0 and then adding the distance from 0 to -2.

Consider asking, "How could the number line visually show each part of the equation? Why does that make the number line an effective tool for these problems?"

Note: If time allows, invite students to revise their previous thinking based on the discussion.



Synthesis

9. Describe how you can use a number line to determine the value of x in an equation like $3.1 + x = -2$.

Use the number line if it helps you show your thinking.

Responses vary. **3.1 is the start and -2 is the end, and I don't know the change. Because I am going from positive to negative, there is an arrow going to the left from the start to the end. Then I can count or figure out how long the arrow should be.**

Lesson Practice ACC6.7.10

Lesson Summary

When adding and subtracting rational numbers, use the same rules for adding and subtracting integers. Here are some strategies for adding and subtracting positive and negative numbers:

- Imagine the problem on a number line.

  For example, in the equation $3 + (-5.5) = x$, 3 represents the starting location, -5.5 represents the change (moving 5.5 units to the left), and x represents the end location.

- Rewrite subtraction as addition.

  For example, $-\frac{3}{10} - \frac{4}{10}$ can be rewritten as $-\frac{3}{10} + \left(-\frac{4}{10}\right)$, which is $-\frac{7}{10}$.

- Rewrite rational numbers in the same form before adding or subtracting.

  For example, $-3\frac{1}{2} + 2.25$ can be rewritten as $-3\frac{1}{2} + 2\frac{1}{4}$, which is $-1\frac{1}{4}$, or as $-3.5 + 2.25$, which is -1.25.

Accelerated 6 | Unit 7 Lesson 10 685 Synthesis | Lesson Practice

Lesson Takeaway: Using a number line to identify the start, change, and end can help you determine an unknown value in equations involving adding and subtracting positive and negative numbers, including equations with fractions and decimals.

Independent | 5 min

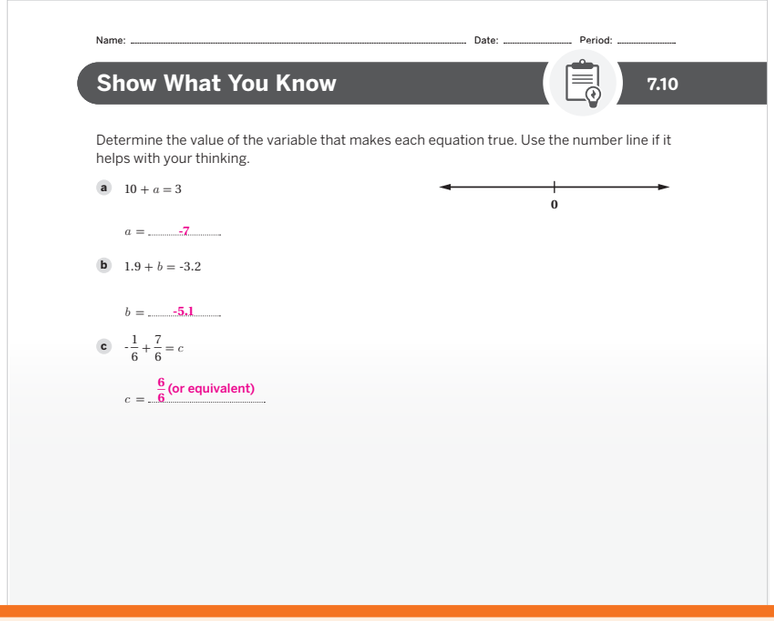
Show What You Know

Purpose: Students demonstrate their understanding by determining an unknown amount in a one-step addition equation with positive and negative fractions and decimals.

Today's Goals

1. **Goal:** Add and subtract positive and negative numbers, including decimals and fractions, on a number line.
2. **Language Goal:** Describe a strategy for adding and subtracting positive and negative decimals and fractions. **(Reading, Writing, Speaking, and Listening)**

Show What You Know (Assessment Resources)



Name: _____ Date: _____ Period: _____

Show What You Know 7.10

Determine the value of the variable that makes each equation true. Use the number line if it helps with your thinking.

a. $10 + a = 3$

$a =$ -7

b. $1.9 + b = -3.2$

$b =$ -5.1

c. $-\frac{1}{6} + \frac{7}{6} = c$

$c =$ $\frac{6}{6}$ (or equivalent)

D Differentiation Use after Lesson 10

Support

Provide targeted intervention for students by using these resources.

If students need support adding and subtracting rational numbers using a number line:

» **Respond to Student Thinking:**

- Revisit Activity 2 and invite students to share strategies for adding and subtracting fractions and decimals.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 2 Extension Activities**.
- Invite students to create their own number line challenges, trade them with a partner, and discuss strategies for solving.

Professional Learning

In this lesson, students are introduced to a model that supports their understanding of adding and subtracting rational numbers. In Lesson 11, they will continue to use number line models as they solve multiplication problems. How does this model integrate with other representations of adding and subtracting positive and negative numbers?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

9. Describe how you can use a number line to determine the value of x in an equation like $3.1 + x = -2$.

Use the number line if it helps you show your thinking.

Responses vary. 3.1 is the start and -2 is the end, and I don't know the change. Because I am going from positive to negative, there is an arrow going to the left from the start to the end. Then I can count or figure out how long the arrow should be.

Lesson Practice ACC6.7.10

Lesson Summary

When adding and subtracting rational numbers, use the same rules for adding and subtracting integers. Here are some strategies for adding and subtracting positive and negative numbers:

- Imagine the problem on a number line.

  For example, in the equation $3 + (-5.5) = x$, 3 represents the starting location, -5.5 represents the change (moving 5.5 units to the left), and x represents the end location.

- Rewrite subtraction as addition.

  For example, $-\frac{3}{10} - \frac{4}{10}$ can be rewritten as $-\frac{3}{10} + \left(-\frac{4}{10}\right)$, which is $-\frac{7}{10}$.

- Rewrite rational numbers in the same form before adding or subtracting.

  For example, $-3\frac{1}{2} + 2.25$ can be rewritten as $-3\frac{1}{2} + 2\frac{1}{4}$, which is $-1\frac{1}{4}$, or as -3. 5 + 2.25, which is -1.25.

Lesson Practice ACC6.7.10

Name: _____ Date: _____ Period: _____

Problems 1–3: Determine the value of the variable that makes each equation true.

1. $40 + a = 30$
$a = -10$

2. $-3.5 + c = 4.5$
$c = 8$

3. $d + 2.4 = 0.9$
$d = -1.5$

Problems 4–9: Evaluate each expression.

4. $-\frac{1}{10} + \frac{2}{5}$
$\frac{3}{10}$ (or equivalent)

5. $-7.2 + 1.3$
-5.9

6. $-5\frac{7}{12} + \left(-2\frac{5}{12}\right)$
-8

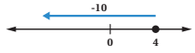
7. $3.7 - 5.4$
-1.7

8. $8 - 6.09$
1.91

9. $\frac{1}{4} + \frac{2}{6}$
$\frac{1}{4}$ (or equivalent)

FAST Practice

10. Select *all* the equations that represent the number line.



- ☑ A. $4 + (-10) = x$
- ☐ B. $4 - (-10) = x$
- ☐ C. $4 + 10 = x$
- ☑ D. $4 - 10 = x$
- ☐ E. $-4 + (-10) = x$

Lesson Practice ACC6.7.10

Name: _____ Date: _____ Period: _____

Spiral Review

11. Last week, it rained g inches. This week, the amount of rain decreased by 5%. Which expressions represent the amount of rain that fell this week? Select *all* that apply.

- ☐ A. $g - 0.05$
- ☑ B. $g - 0.05g$
- ☑ C. $0.95g$
- ☐ D. $0.05g$
- ☑ E. $(1 - 0.05)g$

Problems 12–13: Determine the value of each expression.

12. $7 \cdot (-5)$
-35

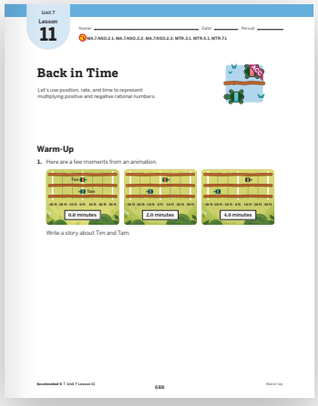
13. $-40 \div (-20)$
2

14. Tiam bought a train ticket online. The original price of the train ticket was $83.00. He used a coupon to receive a 20% discount. A sales tax of 9% was applied after the discount. How much did Tiam end up paying?

A. $18.09 B. $56.44 C. $72.38 D. $75.00

Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1–9	1	MA.7.NSO.2.2
FAST Practice	10	2	MA.7.NSO.2.2
Spiral Review			
	11	2	MA.6.AR.1.1
	12–13	1	MA.6.NSO.4.2
	14	2	MA.7.AR.3.1



Back in Time

Position, Rate, and Time

Let's use position, rate, and time to represent multiplying positive and negative rational numbers.

Learning Outcomes

Today's Goals

1. **Goal:** Represent rate, time, and position using the product of positive and negative rational numbers.
2. **Language Goal:** Explain why the product of two negative numbers is positive in the context of rate, time, and position. **(Writing)**

Students make sense of a different model for multiplying positive and negative rational numbers (rate • time = position).

Coherence

Prior Learning

In Lesson 9, students multiplied with integers. In Lesson 10, students made sense of adding and subtracting positive and negative rational numbers.

Future Learning

In Lesson 12, students will extend the model in this lesson to make sense of division with positive and negative rational numbers.

Connections

- Students explore annimation to represent multiplying postive and negative rational numbers. (real-world)

FL B.E.S.T. Math Benchmarks

Addressing

MA.7.NSO.2.2

Add, subtract, **multiply** and divide **rational numbers with procedural fluency.**

MA.7.NSO.2.3

Solve real-world problems involving any of the four operations with rational numbers.

Also Addressing: **MA.7.NSO.2.1**

Mathematical Thinking and Reasoning Standards: MTR.3.1, MTR.5.1, MTR.7.1

ELA Expectations: ELA.K12.EE.4.1, ELA.K12. EE.5.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.6.NSO.2.1	MA.8.NSO.1.5
MA.6.NSO.2.2	MA.8.NSO.1.6
MA.6.NSO.2.3	MA.8.NSO.1.7
MA.6.NSO.4.2	

Lesson at a Glance

~ 45 min

MA.7.NSO.2.1, MA.7.NSO.2.2, MA.7.NSO.2.3

Warm-Up

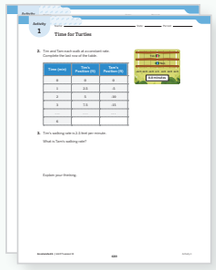
Pairs | 5 min

Students are introduced to the context of turtles walking at different rates in opposite directions.
Routine: Tell a Story

Activity 1

Pairs | 15 min

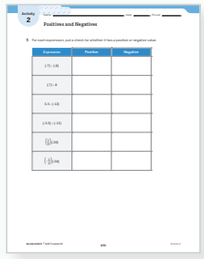
Students use rate, time, and position to make sense of multiplying rational numbers.
MLR3: Critique, Correct, Clarify **(MTR.5.1)**



Activity 2

Pairs | 5 min

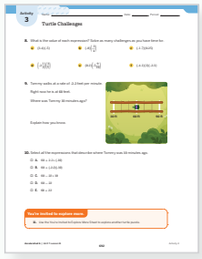
Students practice determining when multiplication expressions are positive or negative.



Activity 3

Pairs | 10 min

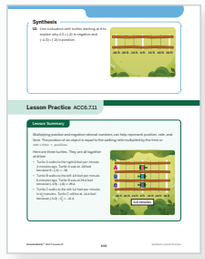
Students extend what they have learned to consider expressions that involve both addition and multiplication and gain fluency with multiplying positive and negative rational numbers. **(MTR.3.1)**



Synthesis

Whole Class | 5 min

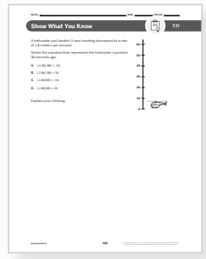
Students synthesize their understanding of why the product of two numbers is positive or negative using the turtle context.



Show What You Know

Independent | 5 min

Students demonstrate their understanding by determining which equation represents a situation in context using distance, time, and rate.



Prep Checklist

Prepare Activity 3 Sheets

This lesson includes:

- Student Edition
- Show What You Know
- Activity 3 You're Invited to Explore More Sheet, one per student (as needed)

Pairs | 5 min

Warm-Up

Purpose: Students are introduced to the context of turtles walking at different rates in opposite directions.

1 Launch

Use the Tell a Story routine to help students make sense of the images of the two turtles. Invite students to write a story about the images using their unique experiences, perspectives, and creativity. **(MTR.7.1)**

Consider sharing that stories do not have to be complete or perfect to be valuable.

Select several student graphs and stories. Consider capturing stories that include:

- A mention that both turtles start from the same location.
- Tim and Tam traveling in opposite directions.
- Tam traveling a further distance or faster than Tim in the same amount of time.
- A creative way to explain Tim's and Tam's journeys.

A **Accessibility: Conceptual Processing** As students connect visual representations to the context of different rates in different directions, consider asking:

- "What do you notice about the rate at which Tim and Tam are walking?"
- "What do you notice about the direction that Tim and Tam are traveling?"

ELL **English Language Learners** Give students time to rehearse and formulate their stories before sharing them with the class. **(Speaking and Listening)**

2 Connect

Invite students to share their stories.

Invite students to share about their stories and what inspired them in their story writing.

Encourage students to ask questions about each other's stories. Consider asking:

- "How do you see the parts of ___'s story in the images?"
- "What parts of each of our stories are alike? How are they different?"

Unit 7
Lesson
11

Name: _____ Date: _____ Period: _____

MA.7.NSO.2.1, MA.7.NSO.2.2, MA.7.NSO.2.3, MTR.3.1, MTR.5.1, MTR.7.1

Back in Time

Let's use position, rate, and time to represent multiplying positive and negative rational numbers.

Warm-Up

1. Here are a few moments from an animation.

Write a story about Tim and Tam.

Responses vary. Tim and Tam are taking some time to process an argument they just had, so they are walking away from each other toward their houses. Tam is walking faster than Tim is.

Accelerated 6 | Unit 7 Lesson 11 688 Warm-Up

Pairs | 15 min

Activity 1 Time for Turtles

Purpose: Students use rate, time, and position to make sense of multiplying rational numbers. **(MTR.5.1)**

1 Launch

To surface connections, consider asking, "Is the information in the table consistent with the stories you told in the Warm-Up?"

Invite students to predict both positions after 6 seconds.

A **Accessibility: Conceptual Processing** Invite students to look at the relationship between the turtle's starting position, the values in the table, and the turtle's ending position to support identifying patterns. **(MTR.5.1)**

Note: The position of each turtle is measured from the shell stripe.

2 Monitor

Encourage students to make connections between the walking rate and their work with proportional relationships. **(MTR.5.1)**

Listen for different ways students explain Tam's walking rate, including those that use a positive value for the rate.

D **Differentiation**

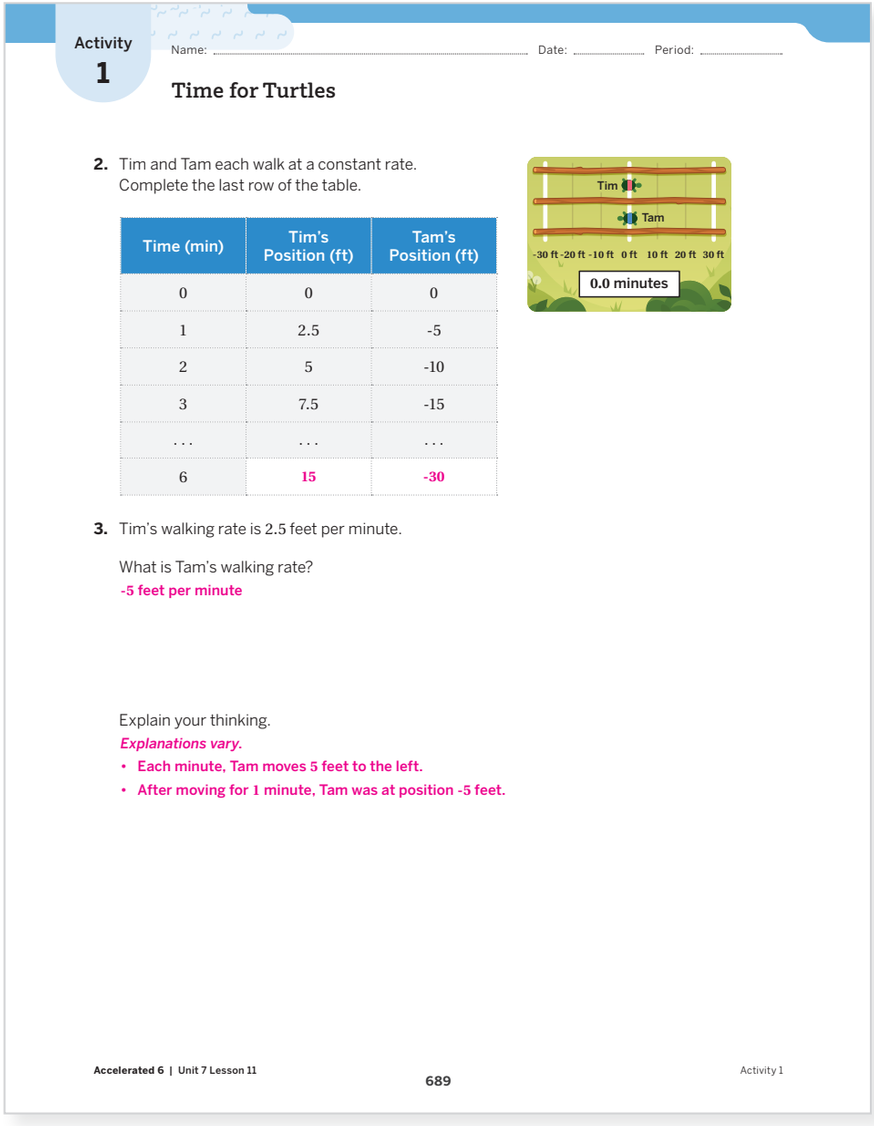
Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Encourage them to focus on one row to determine the number of feet per second each turtle walks.
Use a positive value to represent Tam's walking rate.	**Support** Consider asking, "What direction on the number line would a positive walking rate represent? What direction did Tam go in the image?"
Determine the walking rate of -5 feet per second quickly.	**Stretch** Consider asking, "When will Tim and Tam be 150 feet apart?"

Encourage students to read others' responses and decide if others' strategies were similar to or different from their own.

MLR **MLR3: Critique, Correct, Clarify** Use this routine to help students describe strengths, errors, and ambiguities in each other's responses. As a class:

- Discuss what is *correct* about each other's responses.
- Discuss what is *incorrect* or *unclear*.
- Create a *correct and clear* solution.

Activity 1 continued >



Activity 1

Name: Date: Period:

Time for Turtles

2. Tim and Tam each walk at a constant rate. Complete the last row of the table.

Time (min)	Tim's Position (ft)	Tam's Position (ft)
0	0	0
1	2.5	-5
2	5	-10
3	7.5	-15
...	...	...
6	15	-30

3. Tim's walking rate is 2.5 feet per minute.

What is Tam's walking rate?

-5 feet per minute

Explain your thinking.

Explanations vary.

- Each minute, Tam moves 5 feet to the left.
- After moving for 1 minute, Tam was at position -5 feet.

Accelerated 6 | Unit 7 Lesson 11 689 Activity 1

Pairs | 15 min

Activity 1 Time for Turtles (continued)

Purpose: Students use rate, time, and position to make sense of multiplying rational numbers. **(MTR.5.1)**

2 Monitor

Consider sharing they are going to use Tam's rate of change to make predictions about positions at various points in time.

Encourage students to look for connections between Problems 4 and 5. Consider asking, "How are the prompts similar? Different?"

Select and sequence student responses that explain the connection between the equations and the image of Tam from Problems 4 and 5. **(MTR.5.1)**

D Differentiation

Look for students who:	Respond to Student Thinking
Use Tam's position in the image to describe his position.	**Strengthen** Consider asking, "What about the equation shows that Tam will be facing to the left?"
Say that Tam moves to the right because of the positive 16, but don't mention the rate or going back in time.	**Strengthen** Invite students to write about Tam's final position using the -5 and the -3.2, and consider asking what each number represents.
Use language such as "to the right", "to the left", or "back in time" to describe Tam's movements.	**Stretch** Consider asking, "What are other combinations of positive and negative rates and times that would result in Tam moving to the right? To the left?"

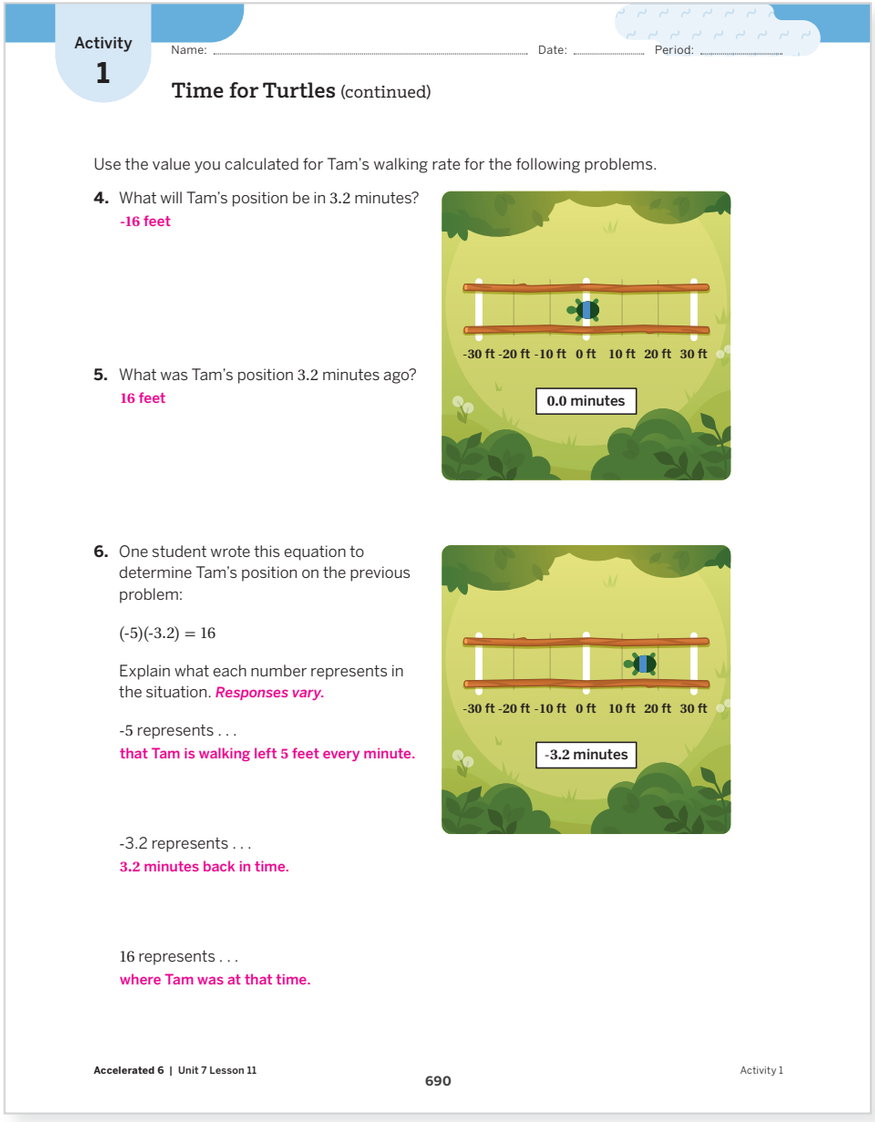
ELL **English Language Learners** Create an anchor chart that publicly displays words describing the turtle scenario to aid explanations and reasoning. **(Reading and Listening)**

3 Connect

Invite students to explain what each value in the expression means and why it makes sense that the position is positive. **(MTR.7.1)**

To surface the Key Takeaway, consider asking:

- "Why does it make sense that Tam's position is positive in this situation?"
- "Will Tam's position always be positive if both the rate and time are negative? Why or why not?"



Activity 1

Name: Date: Period:

Time for Turtles (continued)

Use the value you calculated for Tam's walking rate for the following problems.

4. What will Tam's position be in 3.2 minutes?
-16 feet

5. What was Tam's position 3.2 minutes ago?
16 feet

6. One student wrote this equation to determine Tam's position on the previous problem:

$(-5)(-3.2) = 16$

Explain what each number represents in the situation. *Responses vary.*

-5 represents . . .
that Tam is walking left 5 feet every minute.

-3.2 represents . . .
3.2 minutes back in time.

16 represents . . .
where Tam was at that time.

Accelerated 6 | Unit 7 Lesson 11 690 Activity 1

Key Takeaway: The direction and time an object travels helps us to determine the final position of the object and whether it will be positive or negative. On a number line, moving to the right represents a positive change while moving to the left represents a negative change. The rate multiplied by the time represents the position.

Pairs | 5 min

Activity 2 Positives and Negatives

Purpose: Students practice determining when multiplication expressions are positive or negative.

1 Launch

Encourage students to share their reasoning with a partner and to work together to reach an agreement about how to sort the expressions.

A **Accessibility: Visual-Spatial Processing**
To support distinguishing between different expressions with positive and negative numbers, invite pairs to read each equation aloud and discuss how they represent Tam's position from the previous problems. **(MTR.5.1)**

2 Monitor

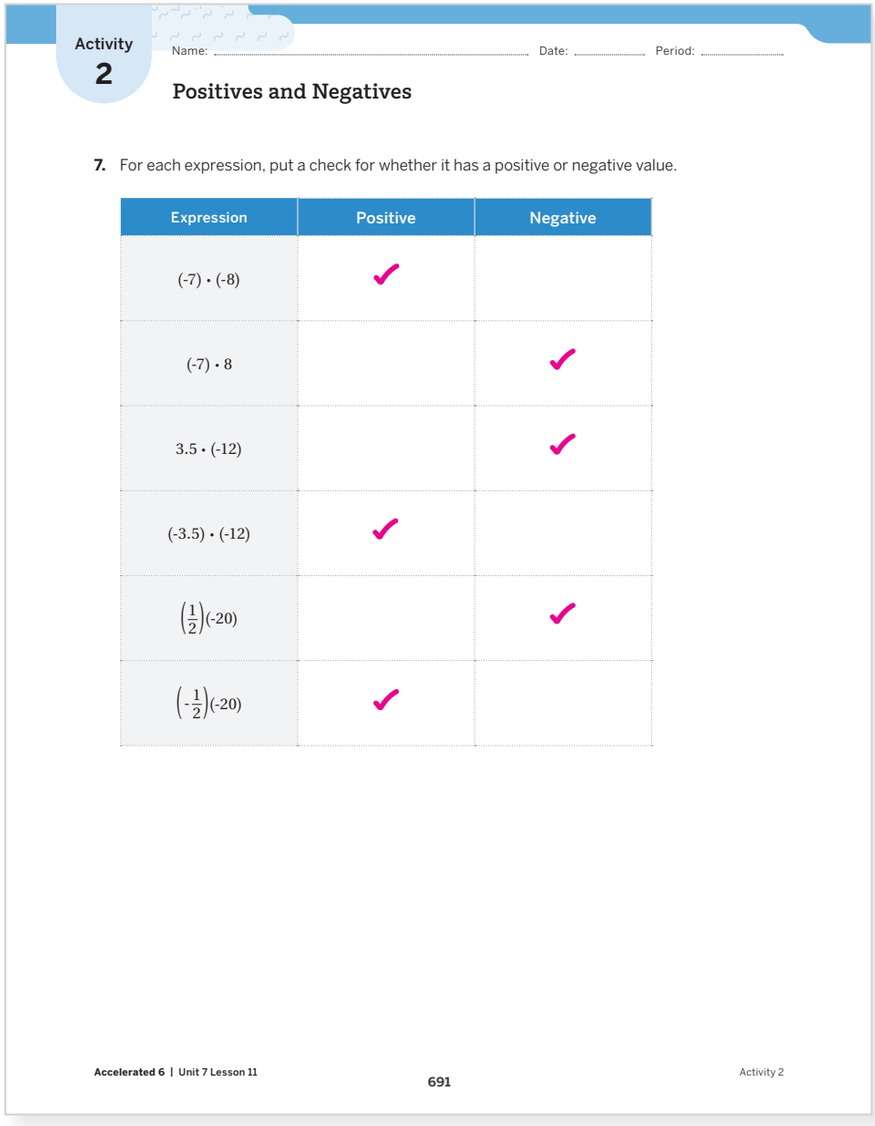
Invite students to compare their answers with another pair of students, justify their decisions, and make revisions based on their conversations.

D **Differentiation: (Stretch)** Invite students who finish early to write about how one of the expressions could represent Tam's position from the previous problems.

3 Connect

Invite students to share strategies they used to group the expressions.

Consider asking, "If we think about how these expressions represent our turtle's position, how do $3.5 \cdot (-12)$ and $(-7) \cdot 8$ both represent a turtle at a negative final position?" *Responses vary.* $3.5 \cdot (-12)$ represents a turtle that traveled 3.5 feet per minute starting 12 minutes ago so they will start in a negative position. $(-7) \cdot 8$ represents a turtle that travels 7 feet to the left every minute for 8 minutes so they will end up in a negative position.



Activity 2

Name: Date: Period:

Positives and Negatives

7. For each expression, put a check for whether it has a positive or negative value.

Expression	Positive	Negative
$(-7) \cdot (-8)$	✓	
$(-7) \cdot 8$		✓
$3.5 \cdot (-12)$		✓
$(-3.5) \cdot (-12)$	✓	
$\left(\frac{1}{2}\right)(-20)$		✓
$\left(-\frac{1}{2}\right)(-20)$	✓	

Accelerated 6 | Unit 7 Lesson 11 691 Activity 2

Pairs | 10 min

Activity 3 Turtle Challenges

Purpose: Students extend what they have learned to consider expressions that involve both addition and multiplication and gain fluency with multiplying positive and negative rational numbers. **(MTR.3.1)**

Short on time? Consider omitting Problems 8–9.

1 Launch

Invite students to discuss strategies for multiplying decimals and fractions. Consider asking, "How does multiplying positive and negative rational numbers compare to multiplying integers?" **(MTR.3.1)**

A **Accessibility: Conceptual Processing** Invite students to make a table of values or write out where Tommy was at shorter intervals to make sense of going back in time with a negative rate. (For example, 1 minute ago Tommy was at 62.2 feet.)

2 Monitor

Look for how many challenges each student has completed.

Look for students using negative numbers to make sense of moving to the left and going back in time.

D **Differentiation**

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Consider asking, "Since Tommy was moving -2.2 feet per minute, would he be further to the left or right 10 minutes ago?"
Write expressions to represent Tommy's position, like $60 + (-2.2)(-10)$.	**Stretch** Encourage them to write out in words what each number in their expression represents to share during the Connect.

Listen for students who notice that 60 is in each expression because it is the starting position.

D **Differentiation: You're invited to explore more. (Stretch)** Invite students who would like to explore the relationship between position, rate, and time further to complete this optional task. Encourage them to discuss their thinking with a partner. **Note**: Students will need copies of the You're Invited to Explore More Sheet.

3 Connect

Display the distribution of responses, calling attention to any conflicts or consensus you see.

To surface the Key Takeaway, consider asking, "How does each expression connect to a turtle scenario? How does that help determine its value?"

Activity 3

Name: Date: Period:

Turtle Challenges

8. What is the value of each expression? Solve as many challenges as you have time for.

a. $(3.4)(-5)$ — **-17**

b. $(-8)\left(-\frac{3}{4}\right)$ — **6**

c. $(-1.7)(0.25)$ — **-0.425**

d. $\left(-2\frac{1}{2}\right)\left(\frac{3}{5}\right)$ — **$-1\frac{1}{2}$ (or equivalent)**

e. $(0.2)\left(-5\frac{9}{10}\right)$ — **-1.18 (or equivalent)**

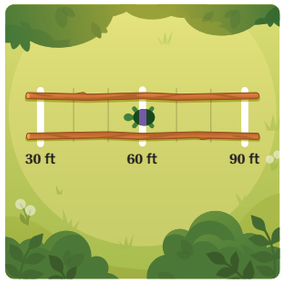
f. $(-4.1)(3)(-2.3)$ — **28.29**

9. Tommy walks at a rate of -2.2 feet per minute.

Right now he is at 60 feet.

Where was Tommy 10 minutes ago?

82 feet



Explain how you know.

Explanations vary. **Tommy was at 82 feet because he started 22 feet to the right of where he is now.**

10. Select *all* the expressions that describe where Tommy was 10 minutes ago.

- ☐ A. $60 + 2.2 \cdot (-10)$
- ☑ B. $60 + (-2.2)(-10)$
- ☐ C. $60 - 22 \cdot 10$
- ☐ D. $60 - 22$
- ☑ E. $60 + 22$

You're invited to explore more.

11. Use the You're Invited to Explore More Sheet to explore another turtle puzzle.

Accelerated 6 | Unit 7 Lesson 11 692 Activity 3

Key Takeaway: When an object doesn't start at 0, the starting position is added to, or subtracted from, the value of the rate multiplied by the time to get the final position.

Whole Class | 5 min

Synthesis

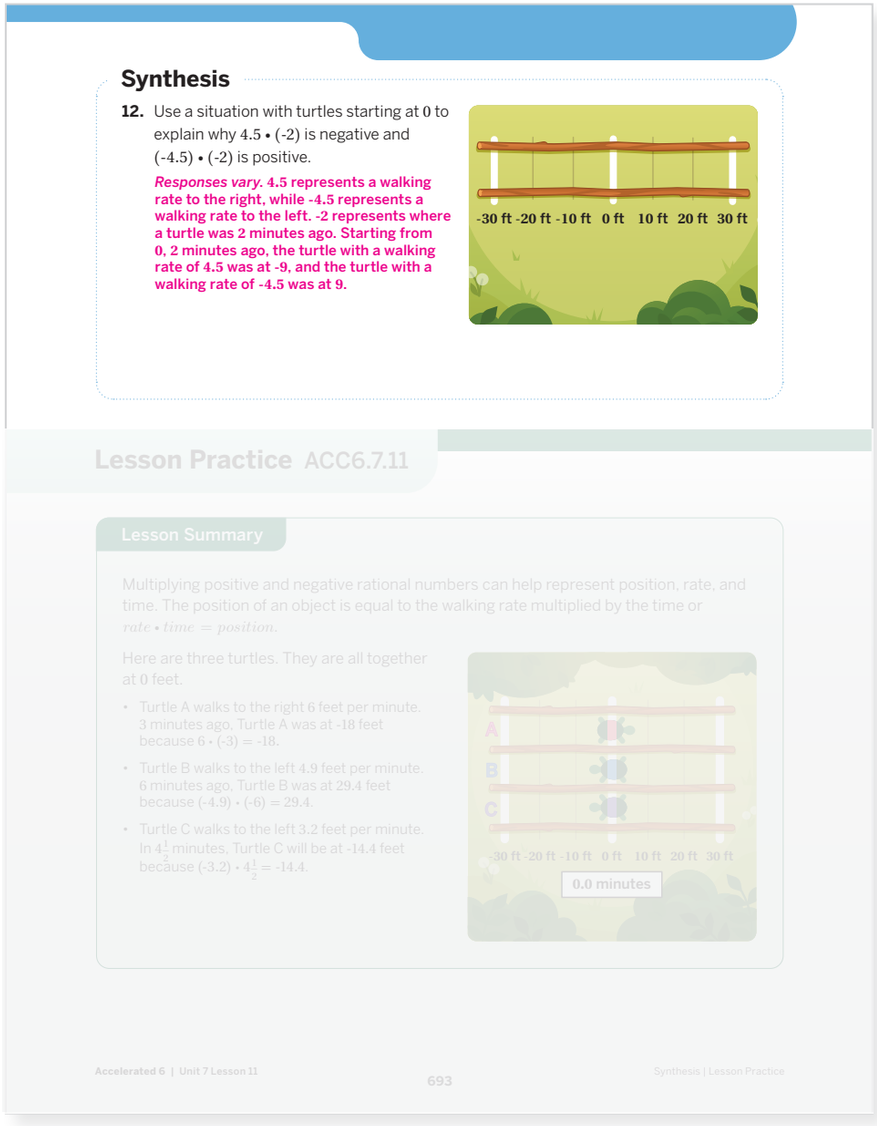
Purpose: Students synthesize their understanding of why the product of two numbers is positive or negative using the turtle context.

Invite students to respond independently, and then share their thinking with a partner.

Collect and share a variety of ideas, such as:

- A turtle that traveled 4.5 feet per minute two minutes ago would have been at -9 feet to begin with.
- A turtle that traveled -4.5 feet per minute two minutes ago would have been at 9 feet to begin with.
- If we go back in time two minutes with a turtle that is traveling to the left, it would have been in the positive position to start.

Note: If time allows, invite students to revise their previous thinking based on the discussion.



Synthesis

12. Use a situation with turtles starting at 0 to explain why $4.5 \cdot (-2)$ is negative and $(-4.5) \cdot (-2)$ is positive.

Responses vary. 4.5 represents a walking rate to the right, while -4.5 represents a walking rate to the left. -2 represents where a turtle was 2 minutes ago. Starting from 0, 2 minutes ago, the turtle with a walking rate of 4.5 was at -9, and the turtle with a walking rate of -4.5 was at 9.

Lesson Practice ACC6.7.11

Lesson Summary

Multiplying positive and negative rational numbers can help represent position, rate, and time. The position of an object is equal to the walking rate multiplied by the time or $rate \cdot time = position$.

Here are three turtles. They are all together at 0 feet.

- Turtle A walks to the right 6 feet per minute. 3 minutes ago, Turtle A was at -18 feet because $6 \cdot (-3) = -18$.
- Turtle B walks to the left 4.9 feet per minute. 6 minutes ago, Turtle B was at 29.4 feet because $(-4.9) \cdot (-6) = 29.4$.
- Turtle C walks to the left 3.2 feet per minute. In $4\frac{1}{2}$ minutes, Turtle C will be at -14.4 feet because $(-3.2) \cdot 4\frac{1}{2} = -14.4$.

Accelerated 6 | Unit 7 Lesson 11 693 Synthesis | Lesson Practice

Lesson Takeaway: We can determine the position of an object by multiplying the rate it is traveling by the time. A positive number represents a walking rate to the right, and a negative number represents a walking rate to the left. Traveling forward in time is represented by a positive number, and going back in time is represented by a negative number.

Independent | 5 min

Show What You Know

Purpose: Students demonstrate their understanding by determining which equation represents a situation in context using distance, time, and rate.

Today's Goals

1. **Goal:** Represent rate, time, and position using the product of positive and negative rational numbers.
2. **Language Goal:** Explain why the product of two negative numbers is positive in the context of rate, time, and position. **(Writing)**

Show What You Know (Assessment Resources)

Name: _____ Date: _____ Period: _____

Show What You Know 7.11

A helicopter just landed. It was traveling downward at a rate of 1.8 meters per second.

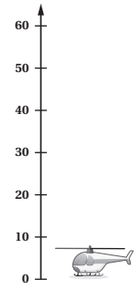
Select the equation that represents the helicopter's position 30 seconds ago.

A. $(-1.8)(-30) = -54$

B. $(-1.8)(-30) = 54$

C. $(-1.8)(30) = -54$

D. $(-1.8)(30) = 54$



Explain your thinking.

Explanations vary. The helicopter's rate can be represented using a negative number since it was moving downward to land. The time is also represented using a negative number since it refers to a time in the past. Multiplying two negative numbers has a positive value. In this case, this makes sense because 30 seconds ago, the helicopter would have still been in the air, so its position would be represented by a positive number.

D Differentiation Use after Lesson 11

Support

Provide targeted intervention for students by using these resources.

If students need support understanding how to model multiplication using time, rate, and position:

» **Respond to Student Thinking:**

- Select a few related **Practice Problems** and give individualized feedback.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 2 Extension Activities**.

Professional Learning

Consider the next several lessons you will teach. How will students' work today in Activity 1 prepare them for dividing rational numbers in Lesson 12? What are ways that you might call back to students' strategies from this lesson?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

12. Use a situation with turtles starting at 0 to explain why $4.5 \cdot (-2)$ is negative and $(-4.5) \cdot (-2)$ is positive.

Responses vary. 4.5 represents a walking rate to the right, while -4.5 represents a walking rate to the left. -2 represents where a turtle was 2 minutes ago. Starting from 0, 2 minutes ago, the turtle with a walking rate of 4.5 was at -9, and the turtle with a walking rate of -4.5 was at 9.

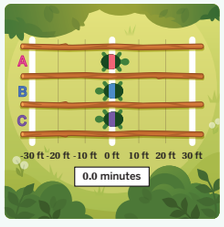
Lesson Practice ACC6.7.11

Lesson Summary

Multiplying positive and negative rational numbers can help represent position, rate, and time. The position of an object is equal to the walking rate multiplied by the time or $rate \cdot time = position$.

Here are three turtles. They are all together at 0 feet.

- Turtle A walks to the right 6 feet per minute. 3 minutes ago, Turtle A was at -18 feet because $6 \cdot (-3) = -18$.
- Turtle B walks to the left 4.9 feet per minute. 6 minutes ago, Turtle B was at 29.4 feet because $(-4.9) \cdot (-6) = 29.4$.
- Turtle C walks to the left 3.2 feet per minute. In $4\frac{1}{2}$ minutes, Turtle C will be at -14.4 feet because $(-3.2) \cdot 4\frac{1}{2} = -14.4$.



Lesson Practice ACC6.7.11

Name: _____ Date: _____ Period: _____

Problems 1–4: Determine the value of each expression.

1. $5(-3) = -15$

2. $-5 \cdot 3 = -15$

3. $(-5)(-3) = 15$

4. $-5 \cdot (-0.3) = 1.5$

Problems 5–8: Anton the Ant can travel up and down an anthill. Currently, his crawling rate is -6.7 centimeters per second, which means he's crawling down the anthill. Anton passes ground level at position 0.

5. Which equation represents Anton's position 8 seconds *after* he passes ground level?

A. $-6.7 \cdot 8 = 53.6$ (B.) $-6.7 \cdot 8 = -53.6$ C. $-6.7 \cdot -8 = 53.6$ D. $-6.7 \cdot -8 = -53.6$

6. Which equation represents Anton's position 8 seconds *before* he passes ground level?

A. $-6.7 \cdot 8 = 53.6$ B. $-6.7 \cdot 8 = -53.6$ (C.) $-6.7 \cdot -8 = 53.6$ D. $-6.7 \cdot -8 = -53.6$

7. Write an ant story that could be represented by the equation $5 \cdot \left(-6\frac{1}{2}\right) = -32\frac{1}{2}$.

Responses vary. An ant is crawling up an anthill at a rate of 5 centimeters per second. The ant passes ground level. The expression shows the position of the ant $6\frac{1}{2}$ seconds before it passes ground level.

8. Anton has friends in nearby hills named Anto and Anty. For each description, write an equation in the table that represents it. Assume each ant starts at position 0.

Responses vary.

Description	Equation
Anto crawls for 28 seconds at -2.4 centimeters per second.	$-2.4 \cdot 28 = -67.2$
Anty crawls for $3\frac{1}{2}$ seconds at $-8\frac{1}{4}$ centimeters per second.	$-8\frac{1}{4} \cdot 3\frac{1}{2} = -28\frac{7}{8}$

Lesson Practice ACC6.7.11

Name: _____ Date: _____ Period: _____

FAST Practice

9. What is the value of the expression $(0.8)\left(-\frac{2}{9}\right)$?

$-\frac{8}{45}$ (or equivalent)

Spiral Review

10. A tank of water breaks and begins draining at a constant rate. Complete the table.

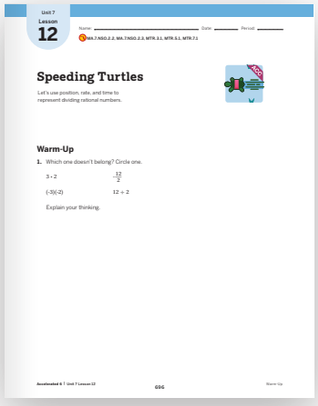
Time (min)	Water in the Tank (L)
0	770
1	756
2	742
. . .	. . .
5	700

11. Angela buys a pair of shoes. The shoes cost $40. The store is having a 20% off sale on everything in the store. How much money does Angela save from the sale?

$8.00

Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1–4	1	MA.7.NSO.2.2
	5–8	2	MA.7.NSO.2.3
FAST Practice	9	2	MA.7.NSO.2.2
Spiral Review			
	10	2	MA.7.AR.3.2
	11	2	MA.7.AR.3.1


Unit 7
Lesson 12
Name: Date: Period:
MA.7.NSO.2.2, MA.7.NSO.2.3, MTR.3.1, MTR.5.1, MTR.7.1
Speeding Turtles
Let's use position, rate, and time to represent dividing rational numbers.
Warm-Up
1. Which one doesn't belong? Circle one.
3 • 2
$-\frac{12}{2}$
(-3)(-2)
12 ÷ 2
Explain your thinking.
696
Warm-Up

Speeding Turtles

Dividing Rational Numbers

Let's use position, rate, and time to represent dividing rational numbers.

Learning Outcomes

Today's Goals

1. **Goal:** Divide positive and negative rational numbers.
2. **Language Goal:** Describe the operations used to determine position, time, and rate and when values will be positive or negative. **(Writing)**

Students make sense of division of positive and negative rational numbers using the context of position, rate, and time as a model. Students use the turtle context to determine unknown rates and times, reason about their signs, and build fluency with dividing rational numbers.

Coherence

Prior Learning

In Lesson 11, students made sense of multiplying positive and negative rational numbers using the context of position, rate, and time as a model.

Future Learning

In Lesson 13, students will use order of operations to evaluate expressions with rational numbers.

Connections

- Students use rate, time, and position to make sense of multiplying rational numbers through the context of walking turtles. (real-world)

FL B.E.S.T. Math Benchmarks

Addressing

MA.7.NSO.2.2
Add, subtract, multiply and divide rational numbers with procedural fluency.

MA.7.NSO.2.3
Solve real-world problems involving any of the four operations with rational numbers.

Mathematical Thinking and Reasoning Standards: MTR.3.1, MTR.5.1, MTR.7.1

ELA Expectations: ELA.K12.EE.1.1, ELA.K12. EE.5.1, ELA.K12.EE.6.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.6.NSO.2.1	MA.8.NSO.1.5
MA.6.NSO.2.2	MA.8.NSO.1.6
MA.6.NSO.2.3	MA.8.NSO.1.7
MA.6.NSO.4.2	

Lesson at a Glance

~ 45 min

MA.7.NSO.2.2, MA.7.NSO.2.3

Warm-Up

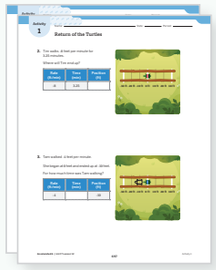
Independent | 5 min

Students reason about numerical expressions involving multiplication and division. **Routine:** Which One Doesn't Belong?

Activity 1

Pairs | 15 min

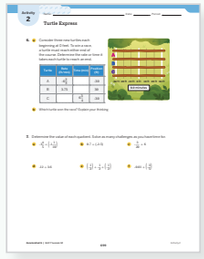
Students multiply and divide rational numbers in context to determine unknown positions, rates, and times. MLR1: Stronger and Clearer Each Time **(MTR.7.1)**



Activity 2

Pairs | 15 min

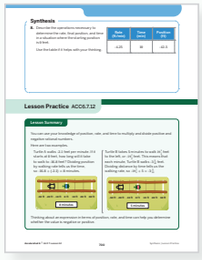
Students build fluency in determining quotients of rational numbers. **(MTR.3.1)**



Synthesis

Whole Class | 5 min

Students synthesize their understanding of dividing positive and negative rational numbers. MLR2: Collect and Display



Show What You Know

Independent | 5 min

Students demonstrate their understanding by determining which two expressions have the same value.

Prep Checklist

This lesson includes:

- Student Edition
- Show What You Know

Independent | 5 min

Warm-Up

Purpose: Students reason about numerical expressions involving multiplication and division.

1 Launch

Use the Which One Doesn't Belong? routine to support students in describing properties of expressions with positive and negative numbers. **(MTR.5.1)**

Consider sharing that each expression does not belong in some way. Encourage students to look for more than one possibility.

ELL **English Language Learners** Provide sentence frames to help students explain their reasoning (e.g., "_____ doesn't belong because _____."). **(Writing and Speaking)**

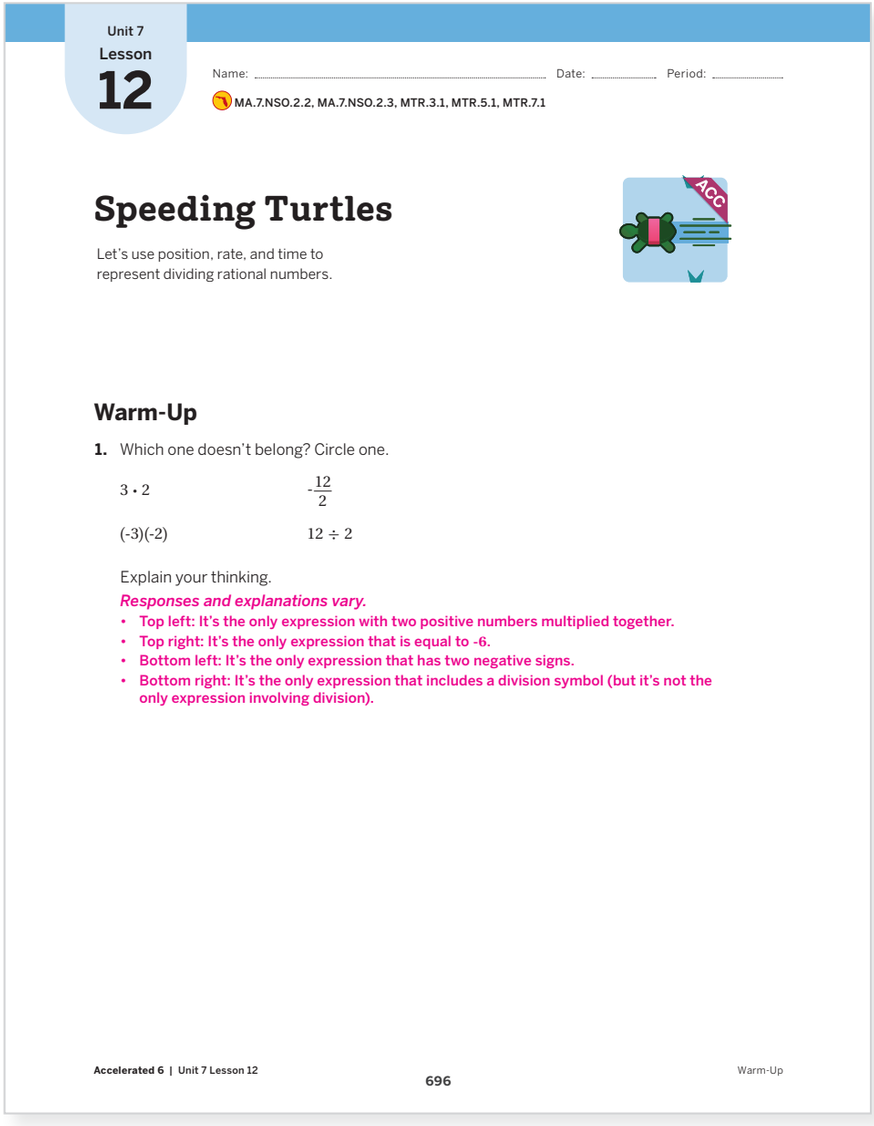
2 Connect

Invite students to share at least one reason why *each* expression doesn't belong.

Create a record of why each expression doesn't belong, along with the name of the student who shared the reasoning.

Consider asking:

- "Does the expression $-\frac{12}{2}$ represent division?"
- "Does $3 \cdot 2$ have the same value as $-3 \cdot -2$?"



Unit 7
Lesson
12

Name: _____ Date: _____ Period: _____

MA.7.NSO.2.2, MA.7.NSO.2.3, MTR.3.1, MTR.5.1, MTR.7.1

Speeding Turtles

Let's use position, rate, and time to represent dividing rational numbers.

Warm-Up

1. Which one doesn't belong? Circle one.

$3 \cdot 2$ $\quad -\frac{12}{2}$

$(-3)(-2)$ $\quad 12 \div 2$

Explain your thinking.

Responses and explanations vary.

- Top left: It's the only expression with two positive numbers multiplied together.
- Top right: It's the only expression that is equal to -6.
- Bottom left: It's the only expression that has two negative signs.
- Bottom right: It's the only expression that includes a division symbol (but it's not the only expression involving division).

Accelerated 6 | Unit 7 Lesson 12 696 Warm-Up

Pairs | 15 min

Activity 1 Return of the Turtles

Purpose: Students multiply and divide rational numbers in context to determine unknown positions, rates, and times. **(MTR.7.1)**

1 Launch

To surface prior knowledge, consider asking, "What do you remember about negative rates and times from the turtles in the last lesson?" **(MTR.7.1)**

Note: The position of each turtle is measured from the shell stripe.

A **Accessibility: Memory and Attention** Provide students with a graphic organizer to support them with keeping track of the relationship between the position, rate, and time.

2 Monitor

Look for a variety of student strategies for determining the time including writing expressions such as $\frac{-10}{-4}$ or setting up equations.

D **Differentiation**

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Consider asking, "How much time did it take Tam to reach -8 feet? -12 feet?"
Multiply the rate and the position to get the time.	**Support** Consider asking, "What would the units be for the value you calculated? Does that match the units for time?"
Write an equation like $-10 = -4 \cdot t$.	**Strengthen** Celebrate the way they used the relationship to make sense of the situation. Encourage them to solve the equation to determine the time. **(MTR.5.1)**
Divide the position by the rate to determine the time.	**Stretch** Consider asking, "How do the units for each of your values help you to make sense of the division?"

Pause briefly to discuss students' strategies for determining the unknown time. Consider asking:

- "How do the units of position and rate help to determine the unknown time?" When I divide the units of feet by feet/minute, the result is minutes. Minutes is what time is measured in so I know that I need to divide the two values.
- "Based on the starting and ending positions, how do we know that the time was positive?" Since the rate is negative, and the turtle ends to the left of where it started, I know that time had to be positive.

Note: Consider creating an anchor chart of rate · time = position and displaying it with examples to support students' reasoning.

Activity 1 continued >

Activity 1

Name: Date: Period:

Return of the Turtles

2. Tim walks -8 feet per minute for 3.25 minutes.

Where will Tim end up?

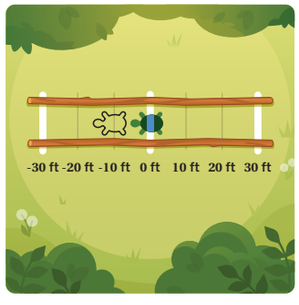
Rate (ft/min)	Time (min)	Position (ft)
-8	3.25	-26

3. Tam walked -4 feet per minute.

She began at 0 feet and ended up at -10 feet.

For how much time was Tam walking?

Rate (ft/min)	Time (min)	Position (ft)
-4	2.5	-10



Accelerated 6 | Unit 7 Lesson 12 697 Activity 1

Pairs | 15 min

Activity 1 Return of the Turtles (continued)

Purpose: Students multiply and divide rational numbers in context to determine unknown positions, rates, and times. **(MTR.7.1)**

2 Monitor

Invite students to continue to Problem 4, writing a first draft about why Fabiana's equation makes sense.

MLR **MLR1: Stronger and Clearer Each Time** Use this routine to support students in developing precise language. Invite students to meet with a partner, give feedback on each other's drafts, then use the feedback to write a second clearer draft. See the Routine Facilitation Guide for more information.

Encourage students to read others' responses and decide if others' strategies were similar to or different from their own. Consider asking, "Is there anything in someone else's explanation that you can use to make your writing stronger and clearer?"
Select and sequence student responses that calculate the correct unit rate.

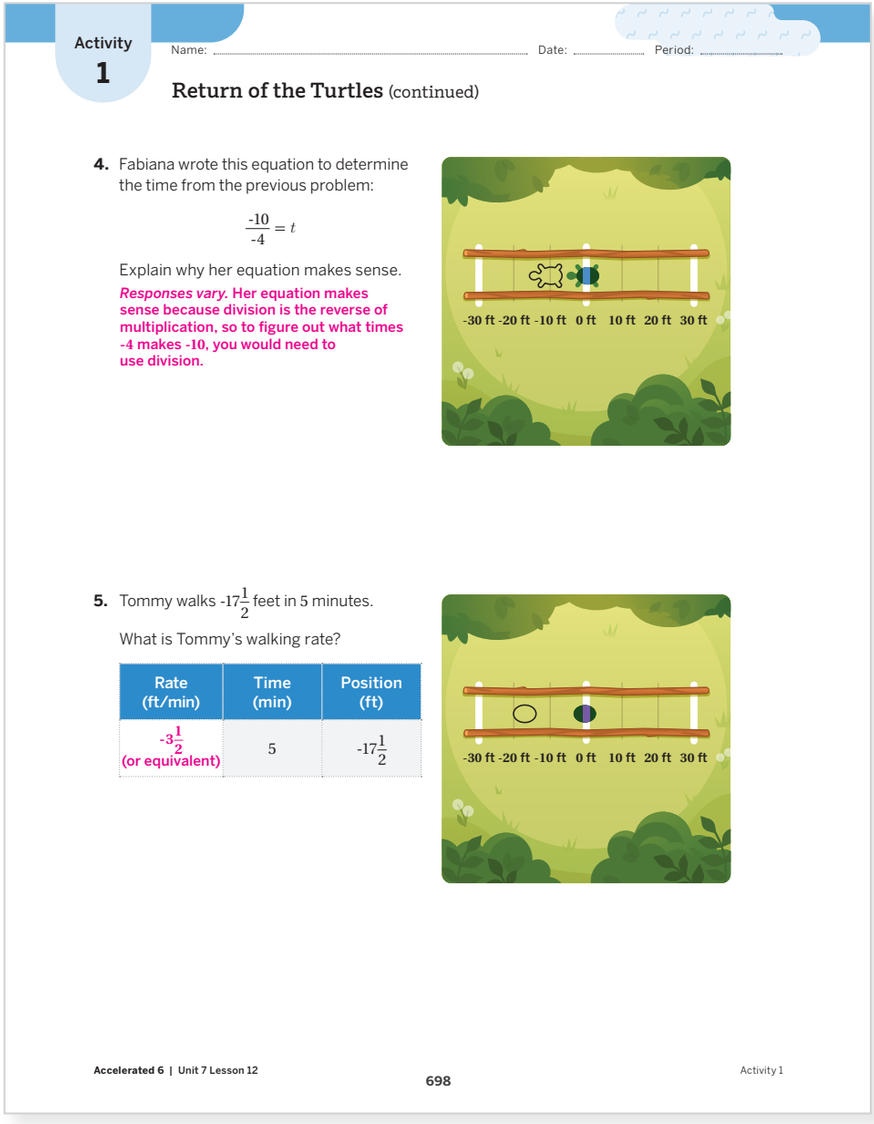
D Differentiation

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Consider asking, "How can you use Fabiana's strategy to help make sense of this problem?" **(MTR.5.1)**
Multiply the time and position to try to determine the rate.	**Support** Consider asking, "What are the units if we multiply position by time?"
Enter a division expression $\left(\frac{-17}{5}\right)$ for the rate.	**Strengthen** Consider asking, "Why is the walking rate negative here? How does your expression help to show the negative position?"

3 Connect

Invite students to share their strategies for determining the unknown rate.

To surface the Key Takeaway, consider asking, "Why do we use division to determine an unknown rate when we know the position and time?"



Activity 1

Name: Date: Period:

Return of the Turtles (continued)

4. Fabiana wrote this equation to determine the time from the previous problem:

$$\frac{-10}{-4} = t$$

Explain why her equation makes sense.

Responses vary. Her equation makes sense because division is the reverse of multiplication, so to figure out what times -4 makes -10, you would need to use division.

5. Tommy walks $-17\frac{1}{2}$ feet in 5 minutes.

What is Tommy's walking rate?

Rate (ft/min)	Time (min)	Position (ft)
$-3\frac{1}{2}$ (or equivalent)	5	$-17\frac{1}{2}$

Accelerated 6 | Unit 7 Lesson 12 698 Activity 1

Key Takeaway: When using the relationship position = rate • time, we can use division to figure out an unknown rate or time.

Pairs | 15 min

Activity 2 Turtle Express

Purpose: Students build fluency in determining quotients of rational numbers. **(MTR.3.1)**

Short on time? Consider completing Problem 6 as a class.

1 Launch

Invite a student to read the problem aloud, then discuss the table.

To surface connections, invite students to share experiences they've had with foot races. Consider asking, "Have you ever run a race where people run in different directions?"

ELL **English Language Learners** Provide sentence frames to help students explain their thinking (e.g., Turtle _____ had the _____ rate, so this turtle _____.). **(Writing and Speaking)**

2 Monitor

Look for students who make sense of rate and time using the relationship between position, time, and rate.

Encourage students to share their reasoning with a partner and revise their solutions based on feedback and collective decisions.

3 Connect

Invite students to share their responses, calling attention to any conflicts or consensus you see.

Consider asking:

- "Can you follow the same process as with dividing integers to determine the sign of the answer?" **(MTR.3.1)**
- "How are multiplying and dividing positive and negative rational numbers alike? How are they different?"

Activity 2

Name: _____ Date: _____ Period: _____

Turtle Express

6. a Consider three new turtles each beginning at 0 feet. To win a race, a turtle must reach either end of the course. Determine the rate or time it takes each turtle to reach an end.

Turtle	Rate (ft/min)	Time (min)	Position (ft)
A	$-6\frac{2}{3}$	$-4\frac{1}{2}$	-30
B	3.75	8	30
C	$-4\frac{4}{9}$	$6\frac{3}{4}$	-30

b Which turtle won the race? Explain your thinking

Turtle A won the race. *Explanations vary.* Turtle A had the fastest rate, so it went the distance in the shortest amount of time.

7. Determine the value of each quotient. Solve as many challenges as you have time for.

a $-3\frac{4}{5} \div \left(-1\frac{1}{10}\right)$

$\frac{38}{11}$ (or equivalent)

b $0.7 \div (-2.5)$

-0.28

c $-\frac{3}{20} \div 6$

$-\frac{1}{40}$ (or equivalent)

d $-12 \div 3.6$

$-3.\overline{3}$

e $\left(-\frac{1}{2}\right) \div -\frac{1}{2} \div \left(-\frac{1}{2}\right)$

-2

f $-0.01 \div \left(-\frac{3}{5}\right)$

60

Accelerated 6 | Unit 7 Lesson 12 699 Activity 2

Whole Class | 5 min

Synthesis

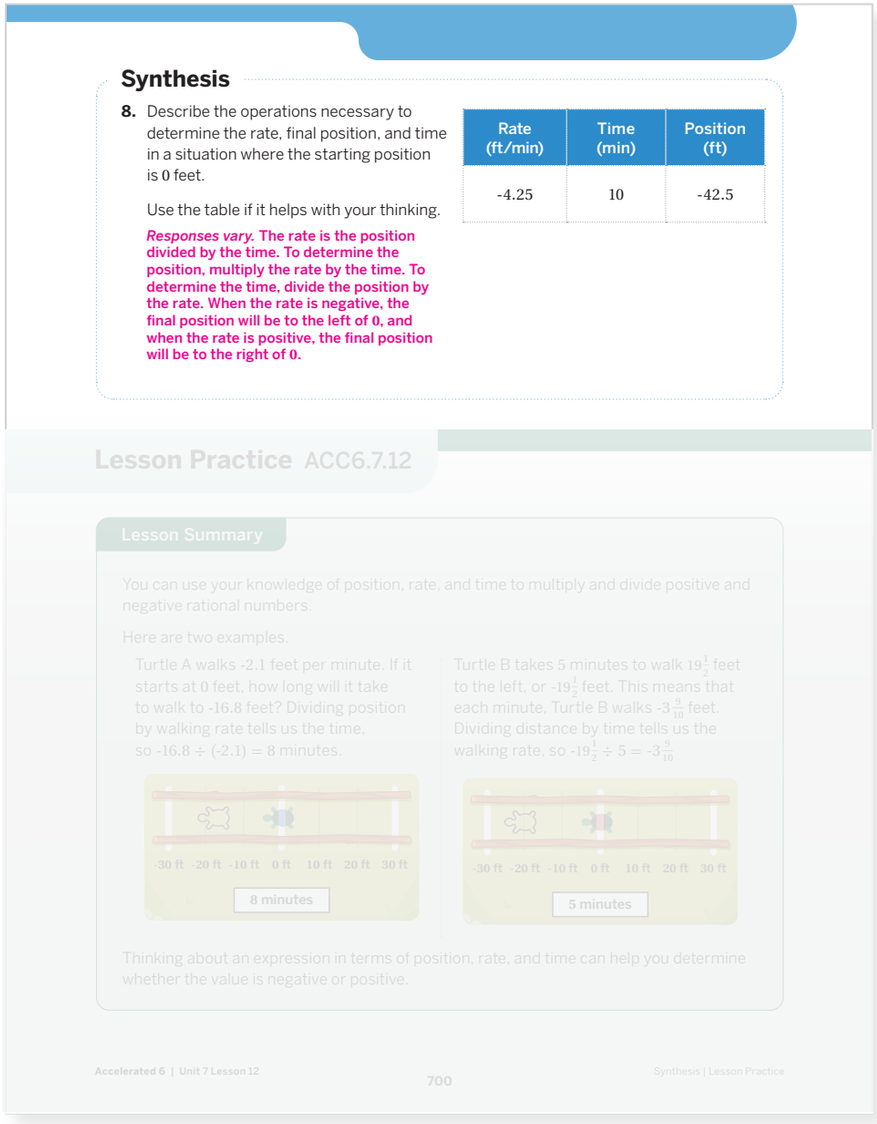
Purpose: Students synthesize their understanding of dividing positive and negative rational numbers.

Invite students to respond independently, and then share their thinking with a partner.

Collect and share a variety of ideas, such as:

- We can use the equation position = rate • time to determine the unknown quantity.
- The unit of measure for time is minutes. When you divide the position (in feet) by the rate (in feet/minute), the unit of measure that remains is minutes.
- Negative numbers are to the left of 0, and positive numbers are to the right of 0.

MLR **MLR2: Collect and Display** Use this routine to collect students' strategies for dividing positive and negative rational numbers as they discuss with a partner. Consider asking, "When dividing two rational numbers, how do you know whether a quotient is positive or negative?" Display for students to reference and add to for the rest of the unit.



Synthesis

8. Describe the operations necessary to determine the rate, final position, and time in a situation where the starting position is 0 feet.

Use the table if it helps with your thinking.

Rate (ft/min)	Time (min)	Position (ft)
-4.25	10	-42.5

Responses vary. The rate is the position divided by the time. To determine the position, multiply the rate by the time. To determine the time, divide the position by the rate. When the rate is negative, the final position will be to the left of 0, and when the rate is positive, the final position will be to the right of 0.

Lesson Practice ACC6.7.12

Lesson Summary

You can use your knowledge of position, rate, and time to multiply and divide positive and negative rational numbers.

Here are two examples.

Turtle A walks -2.1 feet per minute. If it starts at 0 feet, how long will it take to walk to -16.8 feet? Dividing position by walking rate tells us the time, so $-16.8 \div (-2.1) = 8$ minutes.

Turtle B takes 5 minutes to walk $19\frac{1}{2}$ feet to the left, or $-19\frac{1}{2}$ feet. This means that each minute, Turtle B walks $-3\frac{9}{10}$ feet. Dividing distance by time tells us the walking rate, so $-19\frac{1}{2} \div 5 = -3\frac{9}{10}$

Thinking about an expression in terms of position, rate, and time can help you determine whether the value is negative or positive.

Accelerated 6 | Unit 7 Lesson 12 700 Synthesis | Lesson Practice

Lesson Takeaway: The sign of the quotient of a positive number divided by a negative number (or a negative number divided by a positive number) is negative. The sign of the quotient of a negative number divided by a negative number is positive. We can use models, such as the relationship between position, rate, and time to help us make sense of why the signs of these division problems are positive or negative.

Independent | 5 min

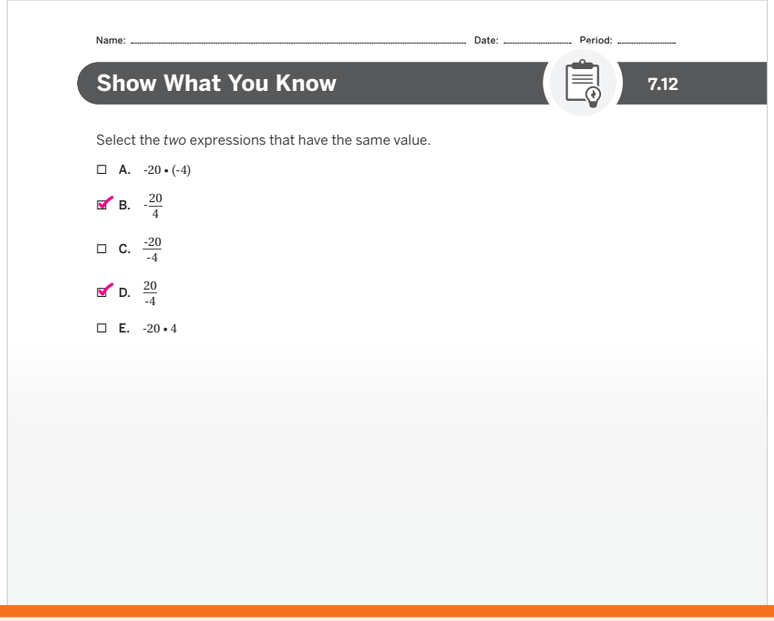
Show What You Know

Purpose: Students demonstrate their understanding by determining which two expressions have the same value.

Today's Goals

1. **Goal:** Divide positive and negative rational numbers.
2. **Language Goal:** Describe the operations used to determine position, time, and rate and when values will be positive or negative. **(Writing)**

Show What You Know (Assessment Resources)



Name: _____ Date: _____ Period: _____

Show What You Know 7.12

Select the *two* expressions that have the same value.

- ☐ A. $-20 \cdot (-4)$
- ☑ B. $-\frac{20}{4}$
- ☐ C. $\frac{-20}{-4}$
- ☑ D. $\frac{20}{-4}$
- ☐ E. $-20 \cdot 4$

D Differentiation Use after Lesson 12

Support

Provide targeted intervention for students by using these resources.

If students need support understanding how to of multiplying and dividing integers:

» **Respond to Student Thinking:**

- ***Mini-Lesson: Dividing Integers***
- Select a few related **Practice Problems** and give individualized feedback.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 2 Extension Activities**.

Professional Learning

Reflect on your facilitation of the **MLR1: Stronger and Clearer Each Time** routine in this lesson. How did students engage with making their response stronger about why the equation made sense? What about how you facilitated the routine resulted in students revising their responses to make them stronger? How might you facilitate it differently in the future?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

8. Describe the operations necessary to determine the rate, final position, and time in a situation where the starting position is 0 feet.

Use the table if it helps with your thinking.

Rate (ft/min)	Time (min)	Position (ft)
-4.25	10	-42.5

Responses vary. The rate is the position divided by the time. To determine the position, multiply the rate by the time. To determine the time, divide the position by the rate. When the rate is negative, the final position will be to the left of 0, and when the rate is positive, the final position will be to the right of 0.

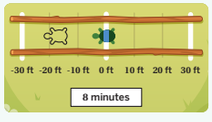
Lesson Practice ACC6.7.12

Lesson Summary

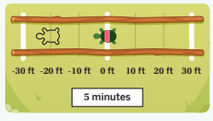
You can use your knowledge of position, rate, and time to multiply and divide positive and negative rational numbers.

Here are two examples.

Turtle A walks -2.1 feet per minute. If it starts at 0 feet, how long will it take to walk to -16.8 feet? Dividing position by walking rate tells us the time, so $-16.8 \div (-2.1) = 8$ minutes.



Turtle B takes 5 minutes to walk $19\frac{1}{2}$ feet to the left, or $-19\frac{1}{2}$ feet. This means that each minute, Turtle B walks $-3\frac{9}{10}$ feet. Dividing distance by time tells us the walking rate, so $-19\frac{1}{2} \div 5 = -3\frac{9}{10}$



Thinking about an expression in terms of position, rate, and time can help you determine whether the value is negative or positive.

Lesson Practice ACC6.7.12

Name: _____ Date: _____ Period: _____

1. Select *all* the expressions that have a negative value.

- ☑ A. $\frac{-15}{12}$
- ☑ B. $\frac{15}{-12}$
- ☐ C. $\frac{-15}{-12}$
- ☑ D. $-\frac{15}{12}$
- ☐ E. $\frac{15}{12}$

Problems 2–3: Determine each quotient.

2. $6.3 \div (-3)$

-2.1

3. $0.36 \div \left(-1\frac{1}{5}\right)$

-0.3

Problems 4–5: A machine that drills holes for wells drilled at a constant rate to a depth of $-76\frac{4}{5}$ feet in one day (24 hours).

4. How much did the depth change each hour? Make sure to show whether the change is positive or negative.

$-3\frac{1}{5}$ feet per hour (or equivalent)

5. What was the depth after 15 hours?

-48 feet

FAST Practice

6. Which expression has the same value as (-0.75)(-0.6)?

- (A.) $0.135 \div 0.3$
- B. $-0.135 \div 0.3$
- C. $11\frac{1}{4} \div \frac{1}{4}$
- D. $11\frac{1}{4} \div -\frac{1}{4}$

Lesson Practice ACC6.7.12

Name: _____ Date: _____ Period: _____

Spiral Review

7. The equation $30 + (-30) = 0$ is an example of two numbers whose sum is 0. Write a different equation with two numbers whose sum is 0.

Responses vary. $44 + (-44) = 0$

8. Write an equation with three numbers whose sum is 0.

Responses vary. $33 + (-22) + (-11) = 0$

9. Write an equation with *four* numbers, none of which are opposites, whose sum is 0. One example of opposites is -30 and 30.

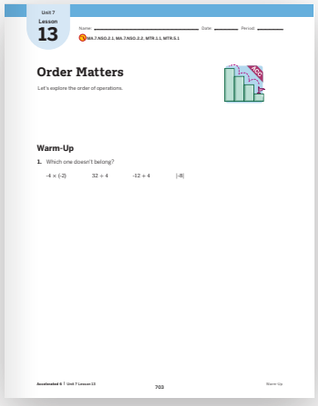
Responses vary. $44 + (-33) + (-22) + 11 = 0$

10. Write an equation using *four* numbers that multiply to 0.

Responses vary. $33 \cdot 22 \cdot 11 \cdot 0 = 0$

Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1	1	MA.7.NSO.2.2
	2–3	2	MA.7.NSO.2.2
	4–5	2	MA.7.NSO.2.3
FAST Practice	6	2	MA.7.NSO.2.2
Spiral Review			
	7–10	2	MA.7.NSO.2.2


Unit 7
Lesson 13
Order Matters
Let's explore the order of operations.
Warm-Up
1. Which one doesn't belong?
-4 × (-2)
32 ÷ 4
-12 ÷ 4
|-8|
703

Order Matters

Evaluating Numerical Expressions

Let's explore how to evaluate numerical expressions using order of operations.

Learning Outcomes

Today's Goals

1. **Goal:** Use order of operations to evaluate numerical expressions.
2. **Language Goal:** Describe how order of operations is used to evaluate a numerical expression. **(Reading, Writing, Speaking, and Listening)**

Students use structure when using the order of operations to evaluate numerical expressions. They notice that they should perform operations within grouping symbols first, followed by *exponents*, then multiplication and division in order from left to right, and finally addition and subtraction in order from left to right. **(MTR.5.1)**

Coherence

Prior Learning

In Sub-Unit 1, students used order of operations to evaluate algebraic expressions involving integers.

Future Learning

In Lesson 14, students will determine whether two expressions are equivalent using models and properties of operations.

Connections

- Students explore evaluating expressions through analyzing student work. (mathematical)

FL B.E.S.T. Math Benchmarks

Addressing

MA.7.NSO.2.1
Solve mathematical problems using multi-step order of operations with rational numbers including grouping symbols, whole-number exponents and absolute value.

MA.7.NSO.2.2
Add, subtract, multiply, and divide rational numbers with procedural fluency.

Mathematical Thinking and Reasoning Standards: MTR.1.1, MTR.5.1

ELA Expectations: ELA.K12.EE.1.1, ELA.K12. EE.6.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.6.NSO.2 **MA.6.NSO.4** **MA.6.AR.1.3**	**MA.8.NSO.1.7**

Vocabulary

Review Vocabulary

exponent

Lesson at a Glance

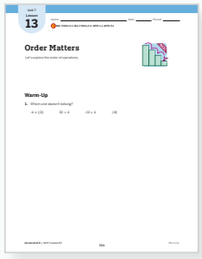
~ 45 min

MA.7.NSO.2.1, MA.7.NSO.2.2

Warm-Up

Pairs | 5 min

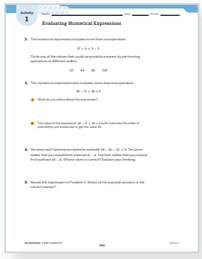
Students use the **Which One Doesn't Belong?** routine to surface their understanding of expressions and operations.



Activity 1

Pairs | 15 min

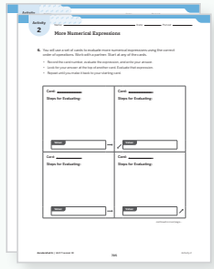
Students use order of operations to evaluate numerical expressions.



Activity 2

Small Groups | 10 min

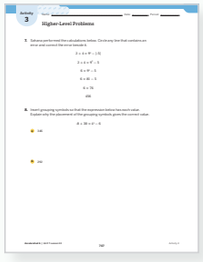
Students apply what they have learned to evaluate more numerical expressions. **(MTR.5.1)**



Activity 3

Pairs | 10 min

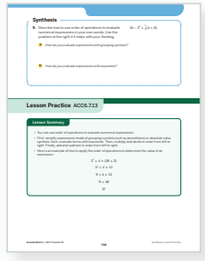
Students analyze student work for potential errors and place grouping symbols in an expression to create different values.
MLR1: Stronger and Clearer Each Time **(MTR.5.1)**



Synthesis

Whole Class | 5 min

Students synthesize their understanding about using order of operations to evaluate numerical expressions.
Routine: **Collect and Display**



Show What You Know

Independent | 5 min

Students demonstrate their understanding by critiquing the process of another student when evaluating a numerical expression and explaining their thinking.

Prep Checklist

Prepare Activity 2 Sheet

This lesson includes:

- Student Edition
- Show What You Know
- Activity 2 Sheet, one per group, pre-cut

Pairs | 5 min

Warm-Up

Purpose: Students use the Which One Doesn't Belong? routine to surface their understanding of expressions and operations.

1 Launch

Use the Which One Doesn't Belong routine to support students in noticing signs of numbers, absolute value, and operations.

Direct students' attention to the four expressions. Invite students to choose one they believe doesn't belong and justify their choice.

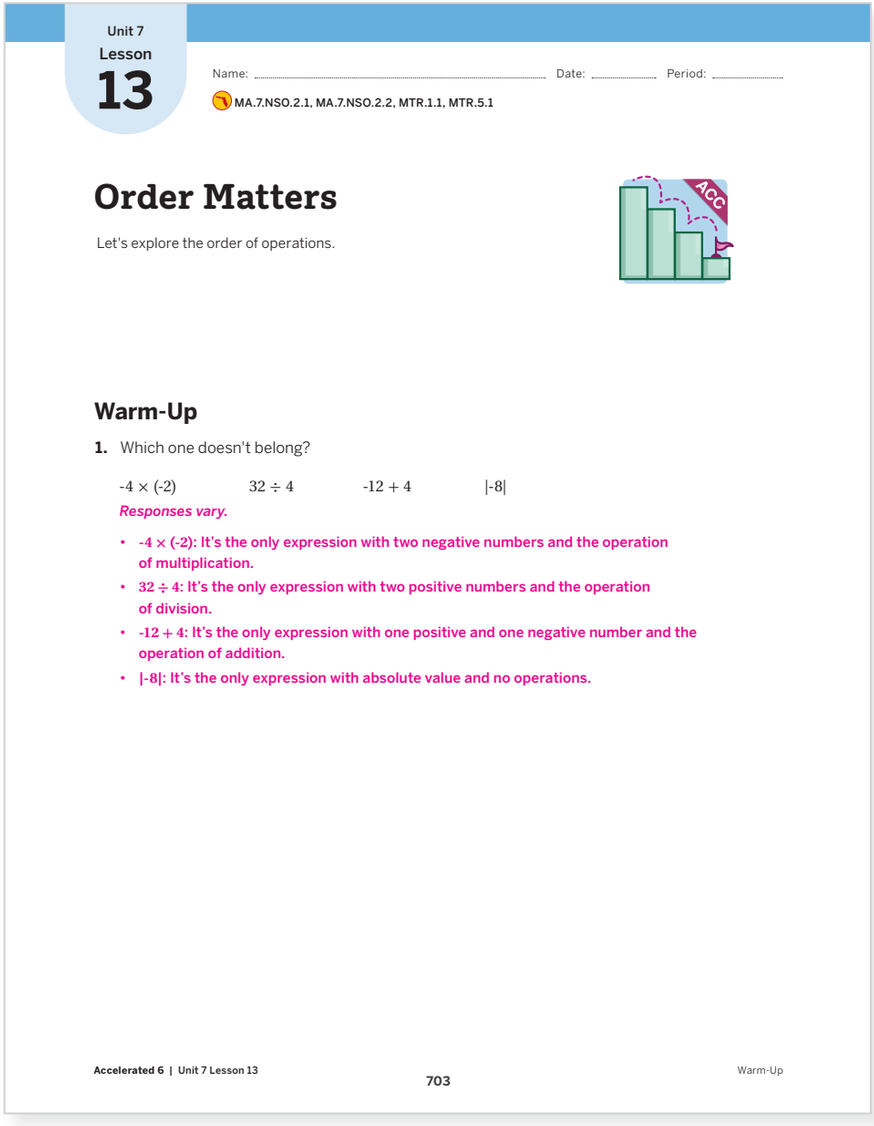
Consider sharing that each expression does not belong in some way. Encourage students to look for more than one possibility.

A **Accessibility: Conceptual Processing** Encourage students who struggle with determining which one doesn't belong to think about what could be different about the one that doesn't belong. Ask students to discuss what they notice about each given expression.

2 Connect

Invite students to share at least one reason why each expression doesn't belong. Consider asking, "What were you thinking while you were determining which expression doesn't belong?"

ELL **English Language Learners** Provide sentence frames (e.g., "_____ doesn't belong because _____." or " _____ is the only one that has/doesn't have _____."). **(Writing and Speaking)**



Unit 7
Lesson
13

Name: Date: Period:

MA.7.NSO.2.1, MA.7.NSO.2.2, MTR.1.1, MTR.5.1

Order Matters

Let's explore the order of operations.

Warm-Up

1. Which one doesn't belong?

$-4 \times (-2)$ $32 \div 4$ $-12 + 4$ $|-8|$

Responses vary.

- $-4 \times (-2)$: It's the only expression with two negative numbers and the operation of multiplication.
- $32 \div 4$: It's the only expression with two positive numbers and the operation of division.
- $-12 + 4$: It's the only expression with one positive and one negative number and the operation of addition.
- $|-8|$: It's the only expression with absolute value and no operations.

Accelerated 6 | Unit 7 Lesson 13 703 Warm-Up

Pairs | 10 min

Activity 1 Evaluating Numerical Expressions

Purpose: Students use order of operations to evaluate numerical expressions.

1 Launch

Direct students' attention to the numerical expression. Invite students to discuss how this numerical expression is similar to or different from the expressions in the Warm-up.

Consider asking:

- "Do you think you will evaluate this expression the same as your classmates?"
- "How can you come up with different possible answers?"
- "Is there only one correct answer?"

Pause briefly to have students share their descriptions.

2 Monitor

Encourage students to share their descriptions with a partner and work together to reach an agreement about the order of operations used to get the value of 29.

D Differentiation

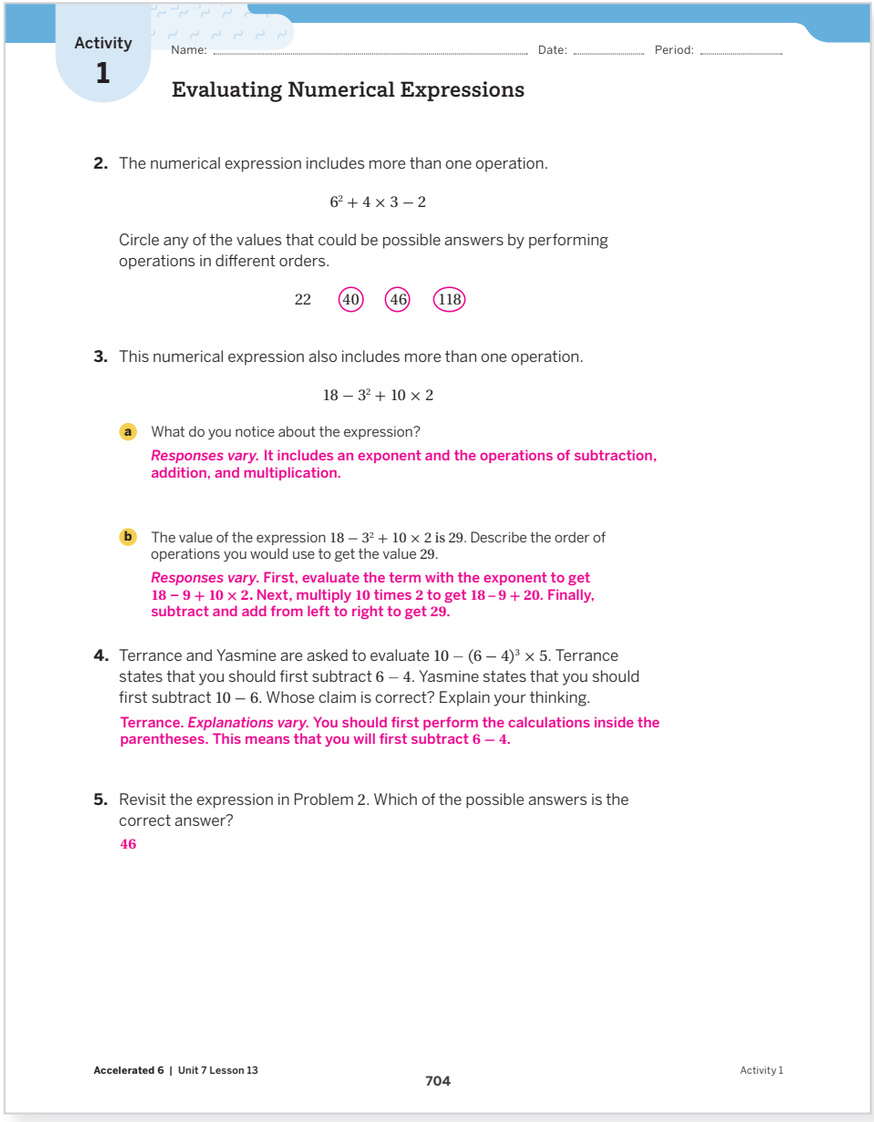
Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Encourage students to state aloud which operation should be completed first. Consider asking, "Can you just perform the operations from left to right as they appear? Why or why not?"
Add $-9 + 10$.	**Support** Consider asking, "Can you perform addition before multiplication in the order of operations?"
Show two different ways to calculate 29.	**Stretch** Challenge students to write a statement that says that addition and subtraction are on the same level.

3 Connect

Create an anchor chart that explains what operations to perform first when they see other numerical expressions in this form. Record it for students to refer to later in the unit.

Capture and share students' answers. Consider creating a bar graph to show the students' answers using the possible answers they got for Problem 2 as the values along the horizontal axis.

Invite students to share how they know their steps and value are correct.



Activity 1

Name: Date: Period:

Evaluating Numerical Expressions

2. The numerical expression includes more than one operation.

$$6^2 + 4 \times 3 - 2$$

Circle any of the values that could be possible answers by performing operations in different orders.

22 (40) (46) (118)

3. This numerical expression also includes more than one operation.

$$18 - 3^2 + 10 \times 2$$

a. What do you notice about the expression?

Responses vary. It includes an exponent and the operations of subtraction, addition, and multiplication.

b. The value of the expression $18 - 3^2 + 10 \times 2$ is 29. Describe the order of operations you would use to get the value 29.

Responses vary. First, evaluate the term with the exponent to get $18 - 9 + 10 \times 2$. Next, multiply 10 times 2 to get $18 - 9 + 20$. Finally, subtract and add from left to right to get 29.

4. Terrance and Yasmine are asked to evaluate $10 - (6 - 4)^3 \times 5$. Terrance states that you should first subtract $6 - 4$. Yasmine states that you should first subtract $10 - 6$. Whose claim is correct? Explain your thinking.

Terrance. *Explanations vary.* You should first perform the calculations inside the parentheses. This means that you will first subtract $6 - 4$.

5. Revisit the expression in Problem 2. Which of the possible answers is the correct answer?

46

Accelerated 6 | Unit 7 Lesson 13 704 Activity 1

Small Groups | 10 min

Activity 2 More Numerical Expressions

Purpose: Students apply what they have learned to evaluate more numerical expressions. **(MTR5.1)**

1 Launch

Distribute the Activity 2 Sheet, one per group.

Direct students' attention to the numerical expressions on the cards. Ask students to describe what they notice about these expressions. **(MTR.5.1)**

Consider asking:

- "What operations are involved in these expressions?"
- "Which expressions require you to work with a grouping symbol inside a grouping symbol?"
- "At what point should you evaluate absolute value in an expression?"

Pause briefly to have students share their thoughts.

Demonstrate how to determine the next card. Consider sharing that the values for the expressions can be found at the top of a card in the set. If students do not see their answer at the top of any card, they should check their work.

2 Monitor

Listen for students' strategies for evaluating operations, particularly when there are operations on the same level when performing the order of operations.

Consider asking:

- "In this expression, is there a place where you have a choice in which operation you do next?"
- "What are some examples of operations that are on the same level?"

Encourage students to select a numerical expression to discuss with the class and share the steps they used to evaluate it.

Collect and share students' discussions.

ELL **English Language Learners** Invite students to restate another group's steps for evaluating to help solidify understanding and use appropriate mathematical language and terms.

Activity 2 continued >

Activity 2

Name: _____ Date: _____ Period: _____

More Numerical Expressions

6. You will use a set of cards to evaluate more numerical expressions using the correct order of operations. Work with a partner. Start at any of the cards.

- Record the card number, evaluate the expression, and write your answer.
- Look for your answer at the top of another card. Evaluate that expression.
- Repeat until you make it back to your starting card.

The card students start with varies.

Card: 1

Steps for Evaluating:

$|-8|\left(\frac{1}{4}\times 8\right)\times 3+1$

$8\left(\frac{1}{4}\times 8\right)\times 3+1$

$8(2)\times 3+1$

$16\times 3+1$

$48+1$

Value: 49 →

Card: 5

Steps for Evaluating:

$20\div(3+|2|)-1\times 4$

$20\div(3+2)-1\times 4$

$20\div 5-1\times 4$

$4-4$

↙ **Value: 0**

Card: 3

Steps for Evaluating:

$-6+\left(-\frac{1}{4}\times(-48)\right)^2-|-7|$

$-6+(12)^2-|-7|$

$-6+(12)^2-7$

$-6+144-7$

$138-7$

Value: 131 →

Card: 2

Steps for Evaluating:

$(13-|2|)+2\div 0.4$

$(13-2)+2\div 0.4$

$11+2\div 0.4$

$11+5$

Value: 16 ↗

continued on next page...

Activity 2 More Numerical Expressions (continued)

Purpose: Students apply what they have learned to evaluate more numerical expressions. **(MTR5.1)**

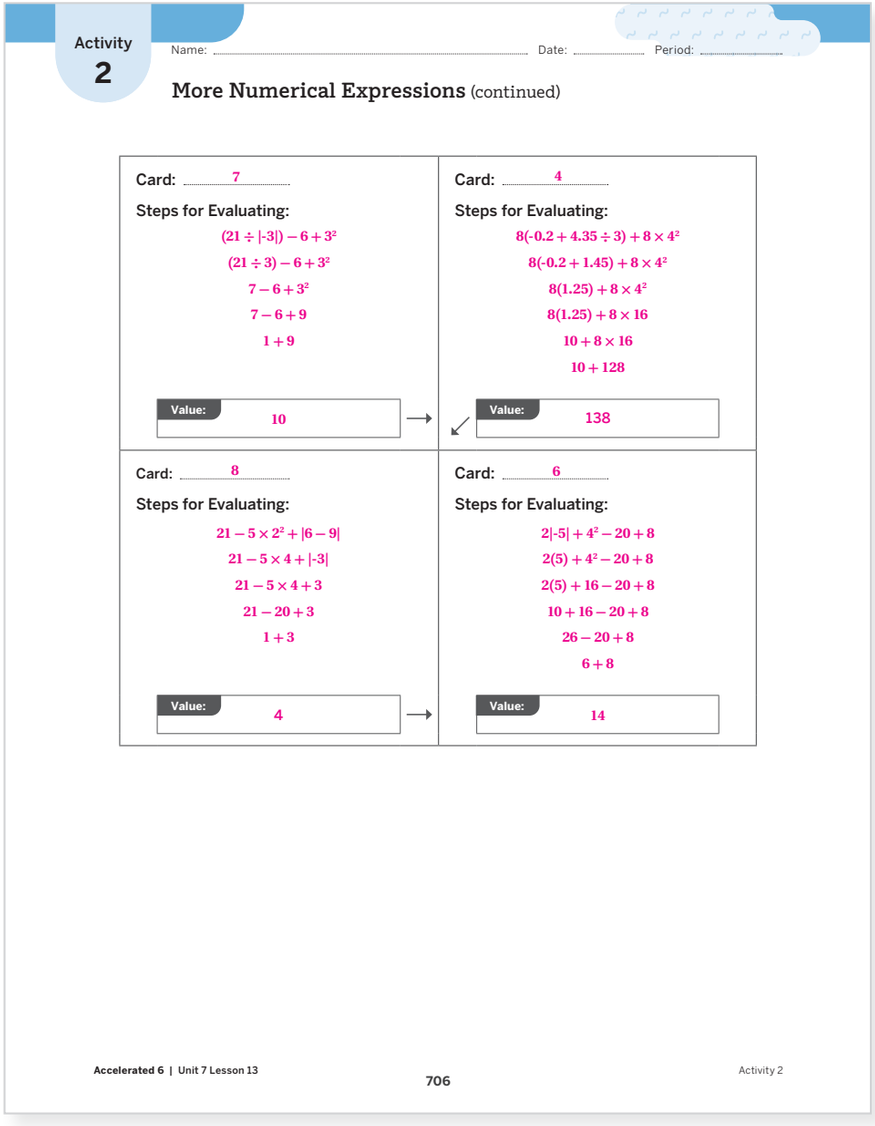
2 Monitor

Note: Refer to the guidance on the previous page to support students as they work through the remainder of the cards.

3 Connect

Invite students to share any difficulties they encountered when working through the activity. Encourage them to ask questions about anything that is still not clear. **(MTR.1.1)**

To surface the Key Takeaway, consider asking, "From what you have seen, how should you approach any numerical expression when working to determine its value? What is an example of a mistake you could make?"



Activity 2

Name: Date: Period:

More Numerical Expressions (continued)

Card: 7

Steps for Evaluating:

$(21 \div |-3|) - 6 + 3^2$

$(21 \div 3) - 6 + 3^2$

$7 - 6 + 3^2$

$7 - 6 + 9$

$1 + 9$

Value: 10

Card: 4

Steps for Evaluating:

$8(-0.2 + 4.35 \div 3) + 8 \times 4^2$

$8(-0.2 + 1.45) + 8 \times 4^2$

$8(1.25) + 8 \times 4^2$

$8(1.25) + 8 \times 16$

$10 + 8 \times 16$

$10 + 128$

Value: 138

Card: 8

Steps for Evaluating:

$21 - 5 \times 2^2 + |6 - 9|$

$21 - 5 \times 4 + |-3|$

$21 - 5 \times 4 + 3$

$21 - 20 + 3$

$1 + 3$

Value: 4

Card: 6

Steps for Evaluating:

$2|-5| + 4^2 - 20 + 8$

$2(5) + 4^2 - 20 + 8$

$2(5) + 16 - 20 + 8$

$10 + 16 - 20 + 8$

$26 - 20 + 8$

$6 + 8$

Value: 14

Accelerated 6 | Unit 7 Lesson 13 706 Activity 2

Key Takeaway: The order of operations must be followed when evaluating a numerical expression to get the correct value.

Pairs | 10 min

Activity 3 Higher-Level Problems

Purpose: Students analyze student work for potential errors and place grouping symbols in an expression to create different values. **(MTR5.1)**

1 Launch

To support students getting started, encourage them to say the numerical expression in words. Then ask them to identify what they first notice about the student's work.

2 Monitor

Pause after most students have finished Problem 7 to discuss the errors in Sahana's work.

Consider asking:

- "What should you be thinking when deciding on where to place the grouping symbols?"
- "How can you check your work?"

3 Connect

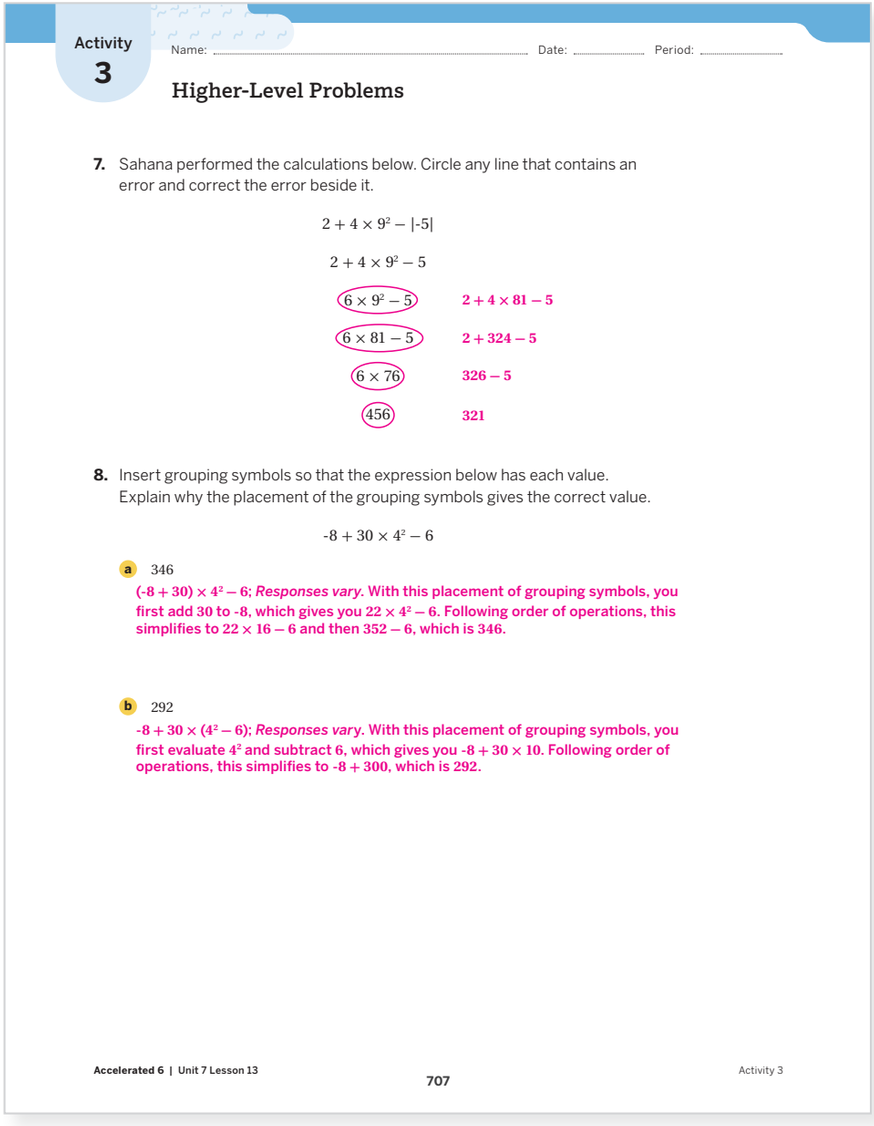
Invite students to use structure when providing the explanation for why the numerical expression has the correct value. **(MTR.5.1)**

Consider asking:

- "What was difficult about completing this problem?"
- "What strategy can be used to determine where to include grouping symbols in an expression to show a certain value?"
- "Prior to working through Problem 8, did you think about the importance of grouping symbols and how they impact the value of a numerical expression?" **(MTR.1.1)**

Invite students to share their strategies and ideas.

MLR **MLR1: Stronger and Clearer Each Time (optional)** Consider using this routine to support students in developing precise language when explaining why their expressions have the given values. Invite students to meet with other groups, give feedback on each other's explanations, and then use the feedback to write a second clearer explanation. See the Routine Facilitation Guide for more information.



Activity 3

Name: Date: Period:

Higher-Level Problems

7. Sahana performed the calculations below. Circle any line that contains an error and correct the error beside it.

$2 + 4 \times 9^2 - |-5|$

$2 + 4 \times 9^2 - 5$

$6 \times 9^2 - 5$ $\quad 2 + 4 \times 81 - 5$

$6 \times 81 - 5$ $\quad 2 + 324 - 5$

6×76 $\quad 326 - 5$

456 $\quad 321$

8. Insert grouping symbols so that the expression below has each value. Explain why the placement of the grouping symbols gives the correct value.

$-8 + 30 \times 4^2 - 6$

a. 346

$(-8 + 30) \times 4^2 - 6$; *Responses vary.* With this placement of grouping symbols, you first add 30 to -8, which gives you $22 \times 4^2 - 6$. Following order of operations, this simplifies to $22 \times 16 - 6$ and then $352 - 6$, which is 346.

b. 292

$-8 + 30 \times (4^2 - 6)$; *Responses vary.* With this placement of grouping symbols, you first evaluate 4^2 and subtract 6, which gives you $-8 + 30 \times 10$. Following order of operations, this simplifies to $-8 + 300$, which is 292.

Accelerated 6 | Unit 7 Lesson 13 707 Activity 3

Whole Class | 5 min

Synthesis

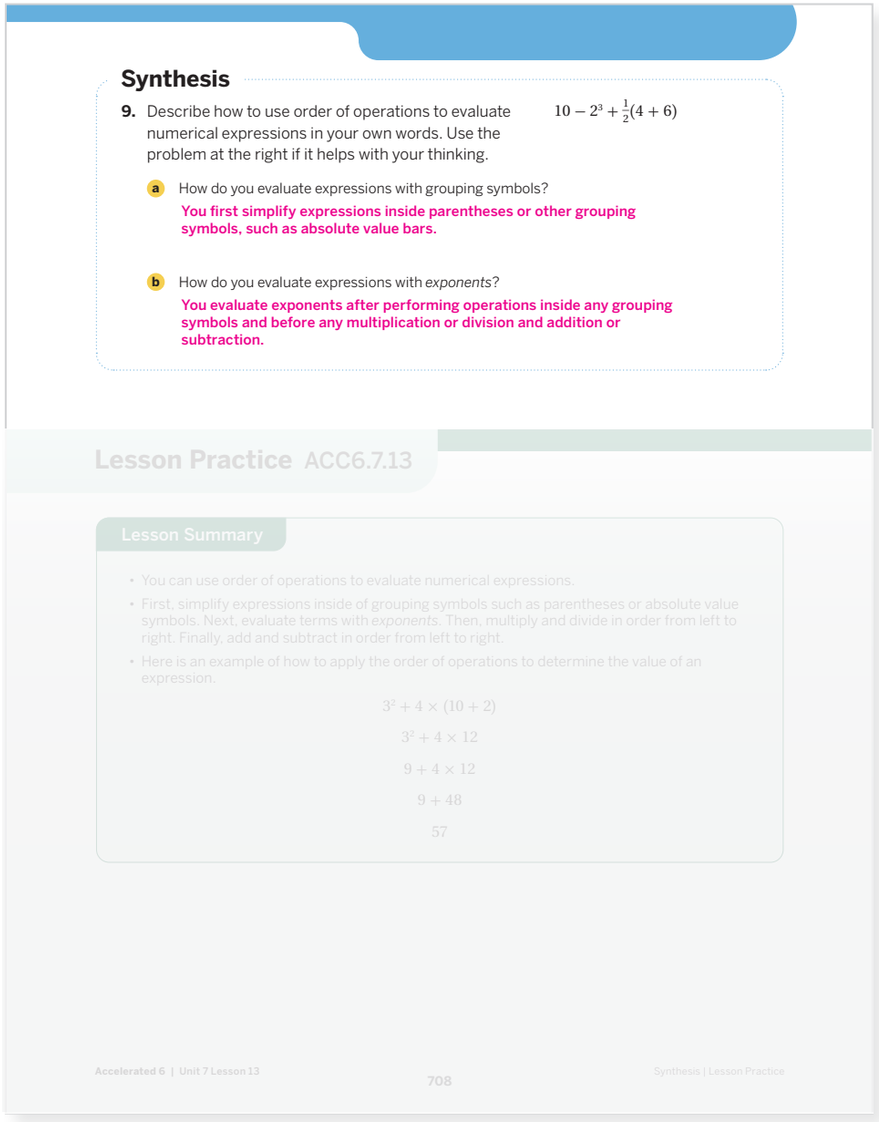
Purpose: Students synthesize their understanding about using order of operations to evaluate numerical expressions.

Invite students to respond independently, and then share their thinking with a partner.

Collect and share students' ideas about evaluating numerical expressions using the order of operations.

MLR **MLR2: Collect and Display** Use this routine to collect students' ideas about using the order of operations to evaluate numerical expressions. Display for students to reference and add to for the rest of the unit.

Note: If time allows, invite students to discuss examples of numerical expressions that will require even more care in order to evaluate.



Synthesis

9. Describe how to use order of operations to evaluate numerical expressions in your own words. Use the problem at the right if it helps with your thinking. $10 - 2^3 + \frac{1}{2}(4 + 6)$

a. How do you evaluate expressions with grouping symbols?

You first simplify expressions inside parentheses or other grouping symbols, such as absolute value bars.

b. How do you evaluate expressions with *exponents*?

You evaluate exponents after performing operations inside any grouping symbols and before any multiplication or division and addition or subtraction.

Lesson Practice ACC6.7.13

Lesson Summary

- You can use order of operations to evaluate numerical expressions.
- First, simplify expressions inside of grouping symbols such as parentheses or absolute value symbols. Next, evaluate terms with *exponents*. Then, multiply and divide in order from left to right. Finally, add and subtract in order from left to right.
- Here is an example of how to apply the order of operations to determine the value of an expression.

$3^2 + 4 \times (10 + 2)$

$3^2 + 4 \times 12$

$9 + 4 \times 12$

$9 + 48$

57

Accelerated 6 | Unit 7 Lesson 13 708 Synthesis | Lesson Practice

Lesson Takeaway: When using the order of operations, first simplify expressions inside grouping symbols, next evaluate *exponents*, then perform multiplication and division in order from left to right, and finally perform addition and subtraction from left to right.

Independent | 5 min

Show What You Know

Purpose: Students demonstrate their understanding by critiquing the process of another student when evaluating a numerical expression and explaining their thinking.

Today's Goals

1. **Goal:** Use order of operations to evaluate numerical expressions.
2. **Language Goal:** Describe how order of operations is used to evaluate a numerical expression. **(Reading, Writing, Speaking, and Listening)**

Show What You Know (Assessment Resources)

Name: _____ Date: _____ Period: _____

Show What You Know 7.13

Seth was asked to evaluate $(6 - 9)^3 + 3 \div 2$. His work is shown below. Do you agree with his work? Explain your thinking.

$$(-3)^3 + 3 \div 2$$
$$-27 + 3 \div 2$$
$$-24 \div 2$$
$$-12$$

Responses vary. No, Seth should have divided 3 by 2 and added the quotient to -27 to get a value of -25.5.

D Differentiation Use after Lesson 13

Support

Provide targeted intervention for students by using these resources.

If students need support understanding how to evaluate numerical expressions:

» **Respond to Student Thinking:**

- Select a few related **Practice Problems** and give individualized feedback.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 2 Extension Activities**.

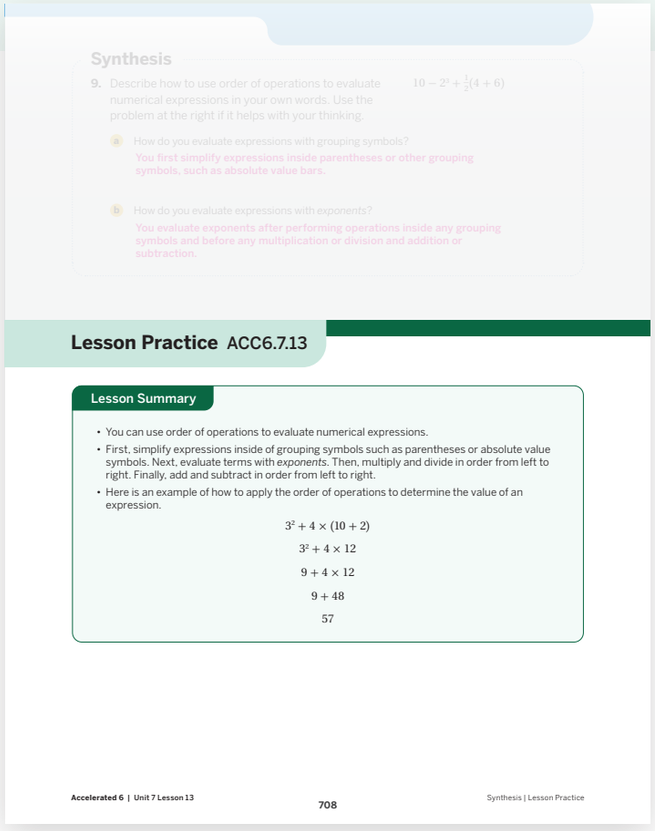
Professional Learning

Take a moment to reflect on this lesson. How will students' work today prepare them for determining whether expressions are equivalent in Lesson 14?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.



Synthesis

9. Describe how to use order of operations to evaluate numerical expressions in your own words. Use the problem at the right if it helps with your thinking. $10 - 2^3 + \frac{1}{2}(4 + 6)$

a. How do you evaluate expressions with grouping symbols?

You first simplify expressions inside parentheses or other grouping symbols, such as absolute value bars.

b. How do you evaluate expressions with *exponents*?

You evaluate exponents after performing operations inside any grouping symbols and before any multiplication or division and addition or subtraction.

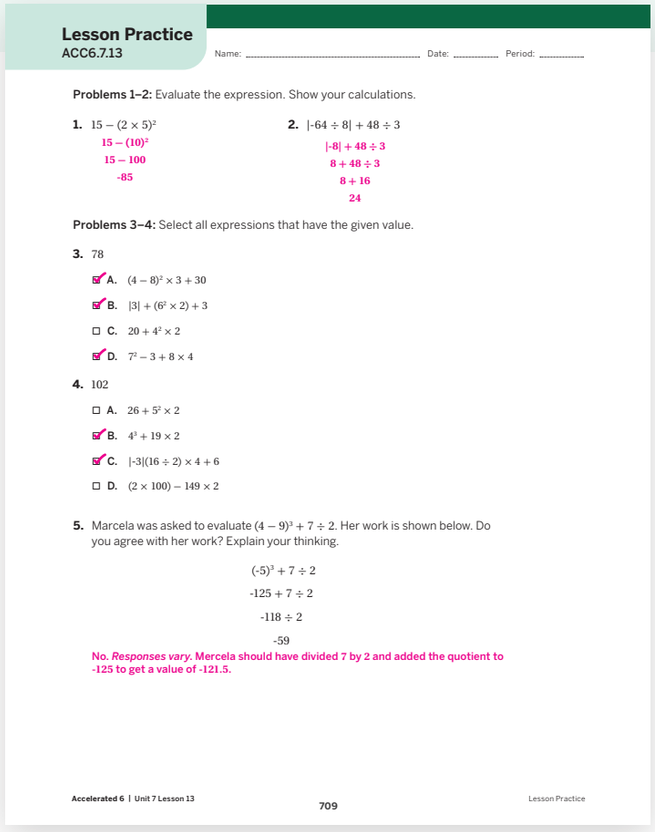
Lesson Practice ACC6.7.13

Lesson Summary

- You can use order of operations to evaluate numerical expressions.
- First, simplify expressions inside of grouping symbols such as parentheses or absolute value symbols. Next, evaluate terms with *exponents*. Then, multiply and divide in order from left to right. Finally, add and subtract in order from left to right.
- Here is an example of how to apply the order of operations to determine the value of an expression.

$$3^2 + 4 \times (10 + 2)$$
$$3^2 + 4 \times 12$$
$$9 + 4 \times 12$$
$$9 + 48$$
$$57$$

Accelerated 6 | Unit 7 Lesson 13 708 Synthesis | Lesson Practice



Lesson Practice
ACC6.7.13

Name: _____ Date: _____ Period: _____

Problems 1–2: Evaluate the expression. Show your calculations.

1. $15 - (2 \times 5)^2$

$15 - (10)^2$
$15 - 100$
-85

2. $|-64 \div 8| + 48 \div 3$

$|-8| + 48 \div 3$
$8 + 48 \div 3$
$8 + 16$
24

Problems 3–4: Select all expressions that have the given value.

3. 78

- [x] A. $(4 - 8)^2 \times 3 + 30$
- [x] B. $|3| + (6^2 \times 2) + 3$
- [ ] C. $20 + 4^2 \times 2$
- [x] D. $7^2 - 3 + 8 \times 4$

4. 102

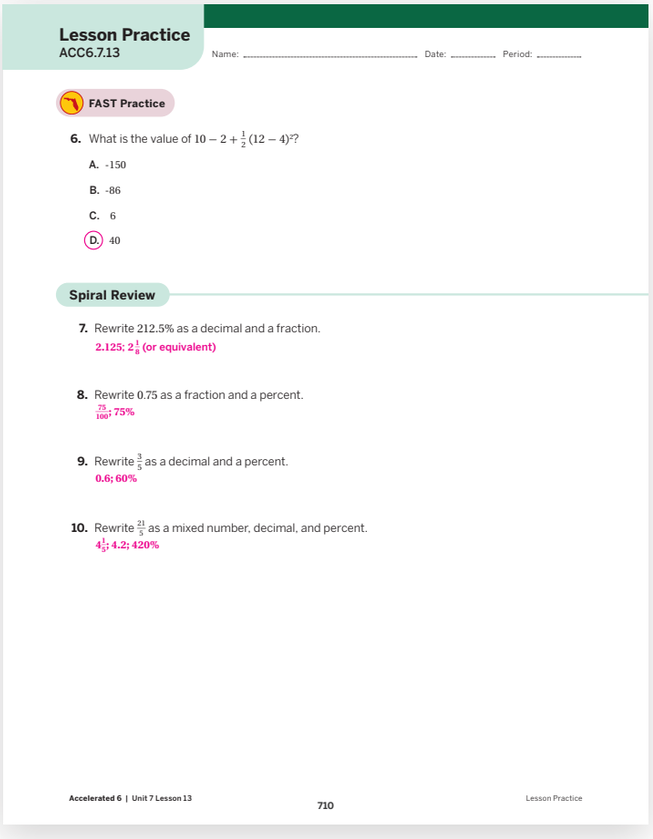
- [ ] A. $26 + 5^2 \times 2$
- [x] B. $4^3 + 19 \times 2$
- [x] C. $|-3|(16 \div 2) \times 4 + 6$
- [ ] D. $(2 \times 100) - 149 \times 2$

5. Marcela was asked to evaluate $(4 - 9)^3 + 7 \div 2$. Her work is shown below. Do you agree with her work? Explain your thinking.

$(-5)^3 + 7 \div 2$
$-125 + 7 \div 2$
$-118 \div 2$
-59

No. *Responses vary.* Mercela should have divided 7 by 2 and added the quotient to -125 to get a value of -121.5.

Accelerated 6 | Unit 7 Lesson 13 709 Lesson Practice



Lesson Practice
ACC6.7.13

Name: _____ Date: _____ Period: _____

FAST Practice

6. What is the value of $10 - 2 + \frac{1}{2}(12 - 4)^2$?

A. -150
B. -86
C. 6
D. 40

Spiral Review

7. Rewrite 212.5% as a decimal and a fraction.

2.125; $2\frac{1}{8}$ (or equivalent)

8. Rewrite 0.75 as a fraction and a percent.

$\frac{75}{100}$; 75%

9. Rewrite $\frac{3}{5}$ as a decimal and a percent.

0.6; 60%

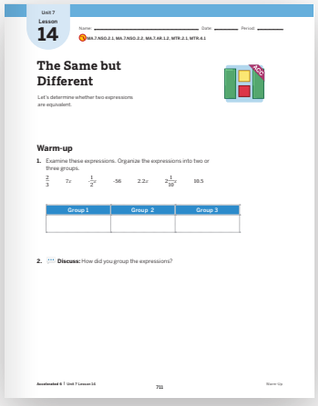
10. Rewrite $\frac{21}{5}$ as a mixed number, decimal, and percent.

$4\frac{1}{5}$; 4.2; 420%

Accelerated 6 | Unit 7 Lesson 13 710 Lesson Practice

Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1–4	2	MA.7.NSO.2.1
	5	3	MA.7.NSO.2.1
FAST Practice	6	2	MA.7.NSO.2.1
Spiral Review			
	7–10	1	MA.6.NSO.3.5


Unit 7
Lesson
14
Name: Date: Period:
MA.7.NSO.2.1, MA.7.NSO.2.2, MA.7.AR.1.2, MTR.2.1, MTR.4.1
The Same but Different
Let's determine whether two expressions are equivalent.
Warm-up
1. Examine these expressions. Organize the expressions into two or three groups.
$\frac{2}{3}$ $7r$ $\frac{1}{2^r}$ -56 $2.2r$ $2\frac{1}{2}r$ 10.5
Group 1 Group 2 Group 3
2. Discuss: How did you group the expressions?
711
Warm-up

The Same but Different

Determining Whether Expressions Are Equivalent

Let's determine whether two expressions are equivalent.

Learning Outcomes

Today's Goals

1. **Goal:** Determine whether two linear expressions in any form with rational coefficients are equivalent.
2. **Language Goal:** Describe the steps used to simplify expressions. **(Reading, Writing)**

Students extend their knowledge of simplifying expressions to determine whether two linear expressions are equivalent. They use manipulatives such as algebra tiles and area models to visualize whether expressions are the same. They also compare more complex expressions by using the properties of operations to simplify them.

Coherence

Prior Learning

In Unit 6, students were introduced to equivalent expressions.

Future Learning

In Lesson 15, students will write equivalent expressions with fewer terms.

Connections

- Students determine if two expressions are equivalent using algebra tiles. (mathematical)

FL B.E.S.T. Math Benchmarks

Addressing

MA.7.NSO.2.1
Solve mathematical problems using multi-step order of operations with rational numbers including grouping symbols, whole-number exponents and absolute value.

MA.7.AR.1.2
Determine whether two linear expressions are equivalent.

Mathematical Thinking and Reasoning Standards: MTR.2.1, MTR.4.1

Also Addressing: **MA.7.NSO.2.2**

ELA Expectations: ELA.K12.EE.4.1, ELA.K12.EE.5.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.6.AR.1.4	**MA.8.AR.1.1** **MA.8.AR.1.3**

Vocabulary

Review Vocabulary

constant *variable*

Additional Vocabulary

like terms

Lesson at a Glance

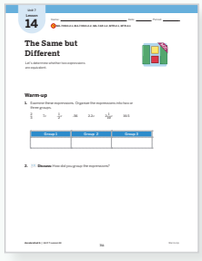
~ 45 min

MA.7.NSO.2.1, MA.7.NSO.2.2, MA.7.AR.1.2

Warm-Up

Pairs | 5 min

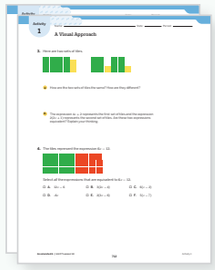
Students sort expressions into groups using any criteria they choose.



Activity 1

Pairs | 15 min

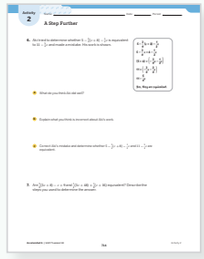
Students use algebra tiles and area models to explore equivalent expressions.
Routine: Think-Pair-Share **(MTR.2.1)**



Activity 2

Pairs | 15 min

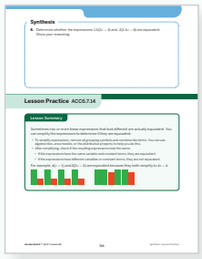
Students use the properties of operations to simplify expressions and determine equivalence. MLR3: Critique, Correct, Clarify **(MTR.4.1)**



Synthesis

Whole Class | 5 min

Students synthesize their understanding of equivalent expressions.



Show What You Know

Independent | 5 min

Students demonstrate their understanding by determining whether two linear expressions are equivalent.

Prep Checklist

This lesson includes:

- Student Edition
- Show What You Know

Pairs | 5 min

Warm-Up

Purpose: Students sort expressions into groups using any criteria they choose.

1 Launch

Direct students' attention to the expressions. Consider sharing that there are many different ways to sort the expressions and that the purpose of this sort is to think about differences among the expressions and to notice what they have in common.

2 Connect

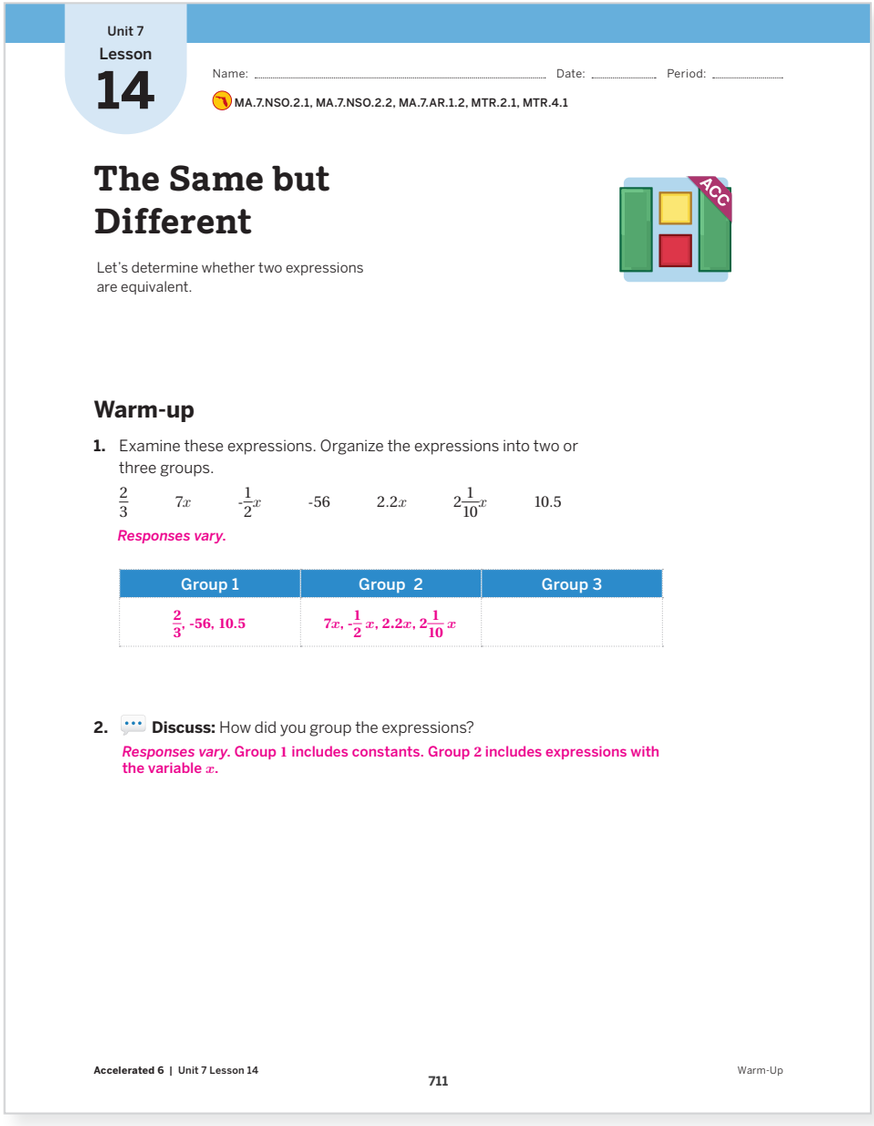
Pause and invite students to share the groups they made.

Create a record of the way students grouped the terms, along with the name of the students who shared them.

Look for and celebrate the different ways that students group the expressions, such as:

- *Constants* and expressions with *variables*.
- Expressions with whole numbers, fractions, and decimals.
- Positive and negative numbers/coefficients.
- Numbers/coefficients that are negative, between 0 and 1, and greater than 1.

ELL **English Language Learners** Invite students to rehearse their ideas with a partner before they share them with others. (Writing and Speaking)



Unit 7
Lesson
14

Name: ..... Date: ..... Period: .....

MA.7.NSO.2.1, MA.7.NSO.2.2, MA.7.AR.1.2, MTR.2.1, MTR.4.1

The Same but Different

Let's determine whether two expressions are equivalent.

ACC

Warm-up

1. Examine these expressions. Organize the expressions into two or three groups.

$\frac{2}{3}$ $7x$ $-\frac{1}{2}x$ -56 $2.2x$ $2\frac{1}{10}x$ 10.5

Responses vary.

Group 1	Group 2	Group 3
$\frac{2}{3}$, -56, 10.5	$7x$, $-\frac{1}{2}x$, $2.2x$, $2\frac{1}{10}x$	

2. **Discuss:** How did you group the expressions?

Responses vary. Group 1 includes constants. Group 2 includes expressions with the variable x.

Accelerated 6 | Unit 7 Lesson 14 711 Warm-Up

Pairs | 15 min

Activity 1 A Visual Approach

Purpose: Students use algebra tiles and area models to explore equivalent expressions. **(MTR.2.1)**

1 Launch

Use the Think-Pair-Share routine to help students shape their ideas about equivalent expressions. Encourage students to think independently and then take turns sharing with a partner.

To encourage movement, invite students to find a partner in a different location.

2 Monitor

Select and sequence responses to share during the Connect, such as students who:

- Use the algebra tiles to justify their answer.
- Use the properties of operations to justify their answer.

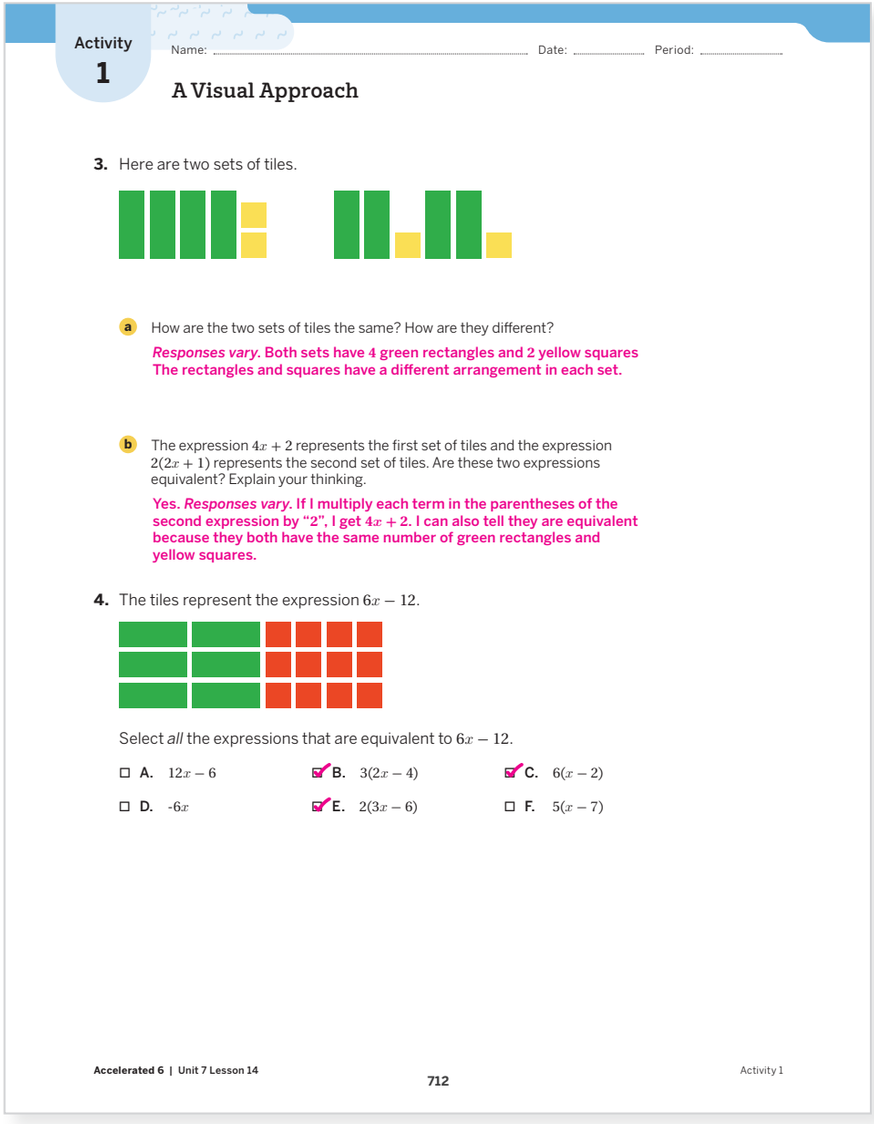
D Differentiation *(Problems 3–4)*

Look for students who:	Respond to Student Thinking
Need support getting started. *(Problem 4)*	**Support** Encourage them to draw their own algebra tiles to model the options.
Use the properties of equality. *(Problem 3)*	**Strengthen** Invite them to name the properties of equality they are using.
Use Algebra tiles. *(Problem 3)*	**Strengthen** Consider asking, "How can you show this algebraically?"
Quickly identify the equivalent expressions. *(Problem 4)*	**Stretch** Challenge them to write a different equivalent expression. Hint: They can use fractions or decimals.

Listen for connections between visual manipulatives and the properties of operations.

A Accessibility: Visual-Spatial Processing Provide students with algebra tiles and encourage them to separate them into equal groups to identify equivalent expressions.

Activity 1 continued >



Activity 1

Name: Date: Period:

A Visual Approach

3. Here are two sets of tiles.

a. How are the two sets of tiles the same? How are they different?

Responses vary. Both sets have 4 green rectangles and 2 yellow squares The rectangles and squares have a different arrangement in each set.

b. The expression $4x + 2$ represents the first set of tiles and the expression $2(2x + 1)$ represents the second set of tiles. Are these two expressions equivalent? Explain your thinking.

Yes. *Responses vary.* If I multiply each term in the parentheses of the second expression by "2", I get $4x + 2$. I can also tell they are equivalent because they both have the same number of green rectangles and yellow squares.

4. The tiles represent the expression $6x - 12$.

Select *all* the expressions that are equivalent to $6x - 12$.

- ☐ A. $12x - 6$
- ☑ B. $3(2x - 4)$
- ☑ C. $6(x - 2)$
- ☐ D. $-6x$
- ☑ E. $2(3x - 6)$
- ☐ F. $5(x - 7)$

Accelerated 6 | Unit 7 Lesson 14 712 Activity 1

Activity 1 A Visual Approach (continued)

Purpose: Students use algebra tiles and area models to explore equivalent expressions. **(MTR.2.1)**

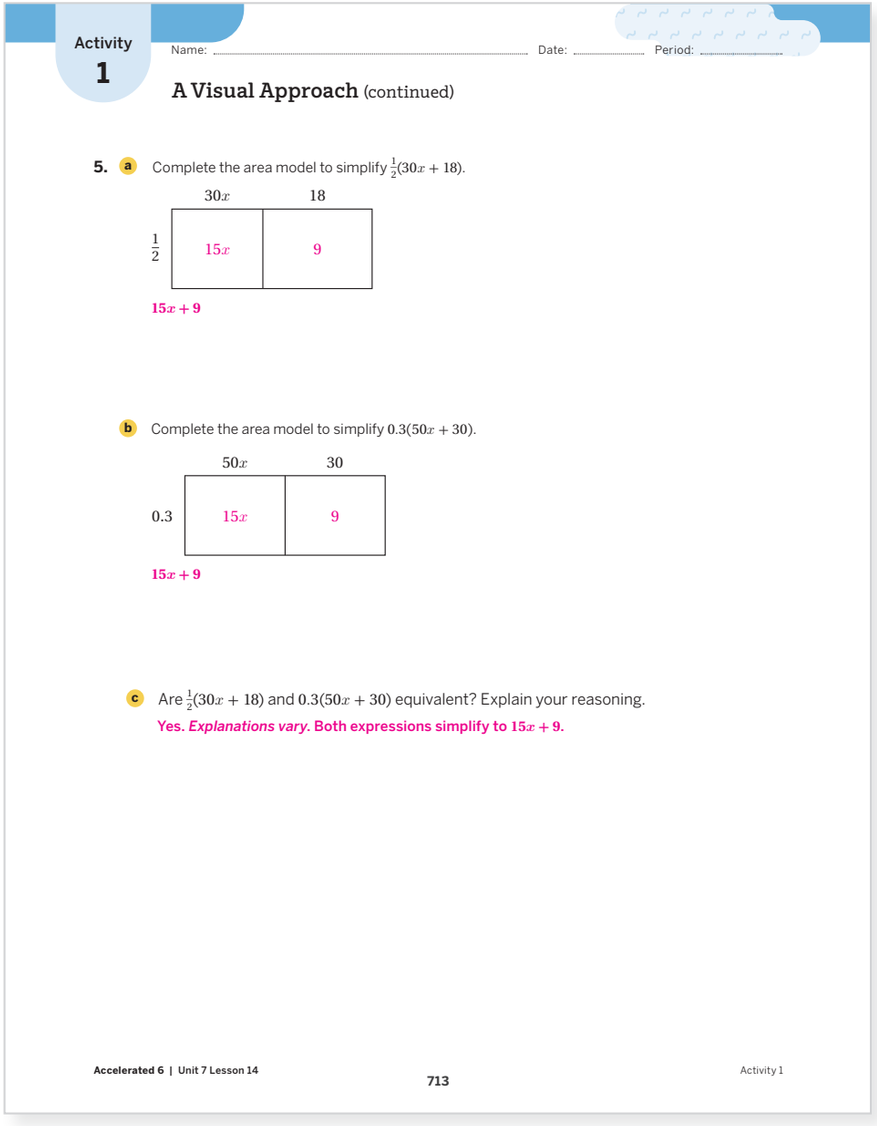
2 Monitor

To support students getting started, consider asking:

- "How do you calculate the area of a rectangle? How does that relate here?"
- "Does the fraction or decimal change the process?"

3 Connect

To surface the Key Takeaway, consider asking, "What can you say about all of the equivalent expressions in this activity?" **(MTR.2.1)**



Activity 1

Name: Date: Period:

A Visual Approach (continued)

5. **a** Complete the area model to simplify $\frac{1}{2}(30x + 18)$.

	$30x$	18
$\frac{1}{2}$	$15x$	9

$15x + 9$

b Complete the area model to simplify $0.3(50x + 30)$.

	$50x$	30
0.3	$15x$	9

$15x + 9$

c Are $\frac{1}{2}(30x + 18)$ and $0.3(50x + 30)$ equivalent? Explain your reasoning.

Yes. *Explanations vary.* Both expressions simplify to $15x + 9$.

Accelerated 6 | Unit 7 Lesson 14 713 Activity 1

Key Takeaway: Two expressions are equivalent if they simplify or expand to the same expression.

Pairs | 15 min

Activity 2 A Step Further

Purpose: Students use the properties of operations to simplify expressions and determine equivalence. **(MTR.4.1)**

Short on time? Consider discussing Problems 6a and 6b as a class.

1 Launch

To support students getting started, consider asking, "Why might Aki have used this method instead of using algebra tiles?"

MLR **MLR3: Critique, Correct, Clarify** Use this routine to help students describe strengths, errors, and ambiguities in Aki's work. Invite students to share their responses from the activity. See the Routine Facilitation Guide for more information. **(MTR.4.1)**

ELL **English Language Learners** Provide sentence frames to help students explain their thinking (e.g., _____ is correct/incorrect. _____ did _____ correctly/incorrectly.). (Writing and Speaking)

2 Monitor

Look for students who list different things that Aki did well to have them share during the Connect.

A **Accessibility: Executive Functioning** Consider chunking this activity by inviting students to analyze one step of Aki's work at a time. To promote student focus, consider covering up later steps until students are ready to address them.

3 Connect

Invite students to discuss what they observed about Aki's work and how they corrected it.

To surface connections, consider asking, "What steps do you take to simplify an expression? Do fractions change how you do that?"

Invite several students to share their responses and justify their thinking.

D **Differentiation: (Support)** Provide students who need support with different-colored highlighters and encourage them to highlight constants in one color and x-terms in another color.

Note: If it does not come up naturally, consider highlighting that, for every expression in this activity, students applied the distributive property and then combined *like terms*.

Activity 2

Name: Date: Period:

A Step Further

6. Aki tried to determine whether $5-\frac{3}{4}(x+8)-\frac{1}{2}x$ is equivalent to $11-\frac{5}{4}x$ and made a mistake. His work is shown.

$5-\frac{3}{4}(x+8)-\frac{1}{2}x$

$5-\frac{3}{4}x+6-\frac{1}{2}x$

$(5+6)+\left(-\frac{3}{4}x-\frac{1}{2}x\right)$

$11+\left(-\frac{3}{4}x-\frac{2}{4}x\right)$

$11-\frac{5}{4}x$

Yes, they are equivalent.

a. What do you think Aki did well?

Responses vary. Aki correctly distributed the $-\frac{3}{4}$ to the x in the parentheses and then combined the variable terms correctly.

b. Explain what you think is incorrect about Aki's work.

Responses vary. Aki only distributed a negative to the first term in the parentheses, not the second, which made the 6 positive instead of negative. He should subtract 6 from 5 instead of adding.

c. Correct Aki's mistake and determine whether $5-\frac{3}{4}(x+8)-\frac{1}{2}x$ and $11-\frac{5}{4}x$ are equivalent.

$5-\frac{3}{4}x-6-\frac{1}{2}x$

$(5-6)+\left(-\frac{3}{4}x-\frac{1}{2}x\right)$

$-1-\frac{5}{4}x$

No, they are not equivalent.

7. Are $\frac{5}{8}(3x+8)-x+9$ and $\frac{1}{4}(3x+48)+\frac{1}{8}(x+16)$ equivalent? Describe the steps you used to determine the answer.

Yes. *Responses vary.* I used the distributive property and combined like terms to find that they both simplify to $\frac{7}{8}x+14$.

$\frac{5}{8}(3x+8)-x+9$

$\frac{15}{8}x+5-x+9$

$\frac{15}{8}x-\frac{8}{8}x+5+9$

$\frac{7}{8}x+14$

$\frac{1}{4}(3x+48)+\frac{1}{8}(x+16)$

$\frac{3}{4}x+12+\frac{1}{8}x+2$

$\frac{6}{8}x+\frac{1}{8}x+12+2$

$\frac{7}{8}x+14$

Accelerated 6 | Unit 7 Lesson 14 714 Activity 2

Whole Class | 5 min

Synthesis

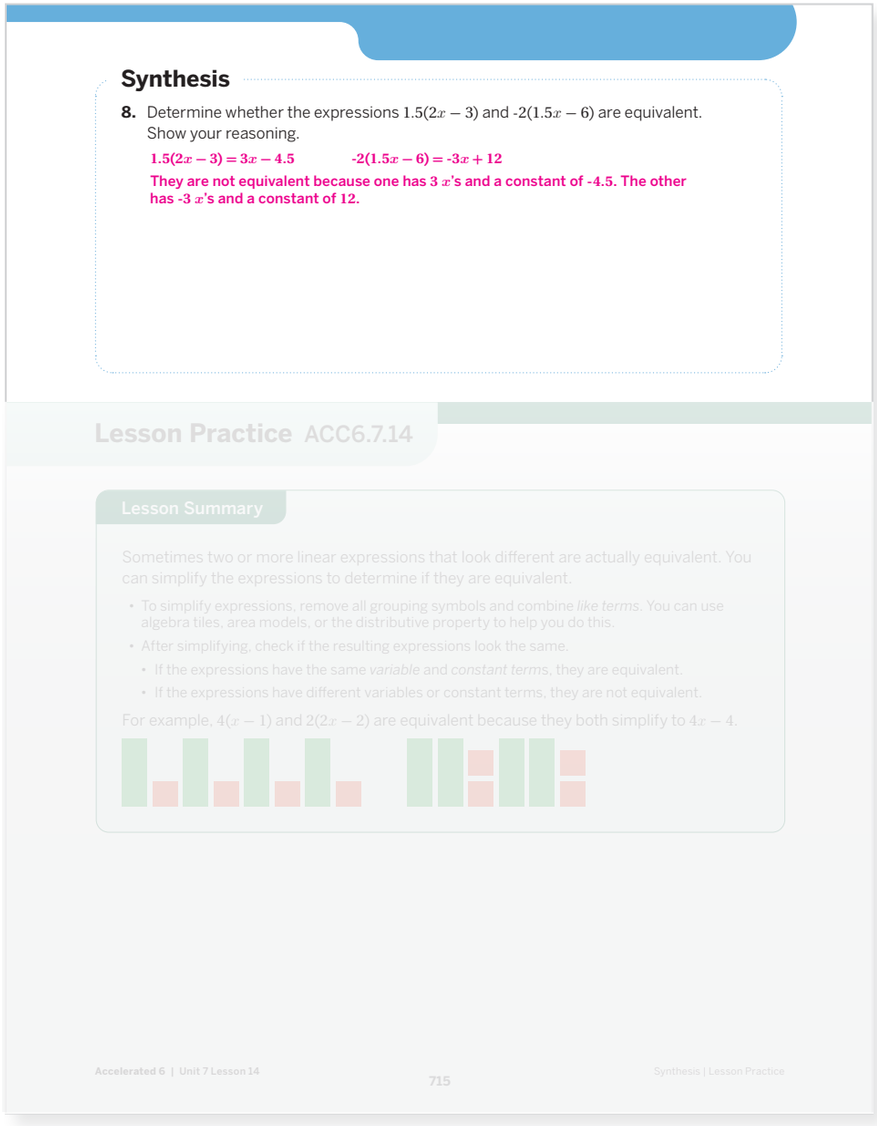
Purpose: Students synthesize their understanding of equivalent expressions.

Invite students to respond independently, and then share their thinking with a partner

ELL **English Language Learners** As students share in partner and whole-class discussions, provide sentence frames such as:

- I agree/disagree with you because _____.
- I think that makes sense because _____.
- How did you know _____. (Writing and Speaking)

A **Accessibility: Memory and Attention** Invite students to refer back to notes they took earlier on determining whether expressions are equivalent to activate prior knowledge.



Synthesis

8. Determine whether the expressions $1.5(2x - 3)$ and $-2(1.5x - 6)$ are equivalent. Show your reasoning.

$1.5(2x - 3) = 3x - 4.5$ $\quad -2(1.5x - 6) = -3x + 12$

They are not equivalent because one has 3 x's and a constant of -4.5. The other has -3 x's and a constant of 12.

Lesson Practice ACC6.7.14

Lesson Summary

Sometimes two or more linear expressions that look different are actually equivalent. You can simplify the expressions to determine if they are equivalent.

- To simplify expressions, remove all grouping symbols and combine *like terms*. You can use algebra tiles, area models, or the distributive property to help you do this.
- After simplifying, check if the resulting expressions look the same.
  - If the expressions have the same *variable* and *constant terms*, they are equivalent.
  - If the expressions have different variables or constant terms, they are not equivalent.

For example, $4(x - 1)$ and $2(2x - 2)$ are equivalent because they both simplify to $4x - 4$.

Accelerated 6 | Unit 7 Lesson 14 715 Synthesis | Lesson Practice

Lesson Takeaway: Expressions with rational numbers can be simplified using properties of operations to determine if they are equivalent.

Independent | 5 min

Show What You Know

Purpose: Students demonstrate their understanding by determining whether two linear expressions are equivalent.

Today's Goals

1. **Goal:** Determine whether two linear expressions in any form with rational coefficients are equivalent.
2. **Language Goal:** Describe the steps used to simplify expressions. (Reading, Writing)

Show What You Know (Assessment Resources)

Name: _____ Date: _____ Period: _____

Show What You Know 7.14

Determine whether these expressions are equivalent. Explain your reasoning.

$\frac{2}{3}(3x - 6) + 4$ and $\frac{1}{2}(12x + 6) - 4x$

Solution: *Explanations vary.* No, these are not equivalent because the first expression simplifies to $2x$ and the second expression simplifies to $2x + 3$.

Differentiation Use after Lesson 14

Support

Provide targeted intervention for students by using these resources.

If students need support understanding how to determine if two expressions are equivalent:

» **Respond to Student Thinking:**

- Select a few related **Practice Problems** and give individualized feedback.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 2 Extension Activities**.

Professional Learning

Take a moment to reflect on this lesson. In this lesson, students used properties of operations to create and identify equivalent expressions. How will that work support solving equations?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

8. Determine whether the expressions $1.5(2x - 3)$ and $-2(1.5x - 6)$ are equivalent. Show your reasoning.

$1.5(2x - 3) = 3x - 4.5$ $\quad -2(1.5x - 6) = -3x + 12$

They are not equivalent because one has 3 x's and a constant of -4.5. The other has -3 x's and a constant of 12.

Lesson Practice ACC6.7.14

Lesson Summary

Sometimes two or more linear expressions that look different are actually equivalent. You can simplify the expressions to determine if they are equivalent.

- To simplify expressions, remove all grouping symbols and combine *like terms*. You can use algebra tiles, area models, or the distributive property to help you do this.
- After simplifying, check if the resulting expressions look the same.
  - If the expressions have the same *variable* and *constant terms*, they are equivalent.
  - If the expressions have different variables or constant terms, they are not equivalent.

For example, $4(x - 1)$ and $2(2x - 2)$ are equivalent because they both simplify to $4x - 4$.

Lesson Practice
ACC6.7.14

Name: _____ Date: _____ Period: _____

1. Which expression is equivalent to $13 + \frac{3}{5}(x - 10) + 4$?

A. $\frac{3}{5}x + 11$ $\quad$ B. $\frac{3}{5}x + 7$

C. $\frac{3}{5}x + 11$ $\quad$ D. $\frac{3}{5}x - 3$

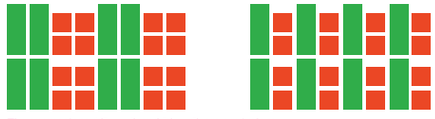
2. Use area models to determine whether $15(x - 2)$ and $-3(10 - 5x)$ are equivalent. Explain your thinking.

	x	-2
15	$15x$	-30

	10	$-5x$
-3	-30	$15x$

Yes, they are equivalent. *Explanations vary.* Both expressions simplify to $15x - 30$.

3. Use the two sets of algebra tiles to write a pair of equivalent expressions.



The expressions $4(2x - 4)$ and $8(x - 2)$ are equivalent.

Lesson Practice
ACC6.7.14

Name: _____ Date: _____ Period: _____

Problems 4–7: Determine whether each pair of expressions is equivalent. Explain your reasoning.

4. $2(1.4x - 3) + 3(2x + 1)$ and $4(0.7x - 1.5) + 6(x + 0.5)$

Yes, they are equivalent. They both simplify to $8.8x - 3$.

5. $2(3x + 5)$ and $6\left(x + \frac{5}{3}\right)$

Yes, they are equivalent. They both simplify to $6x + 10$.

6. $\frac{2}{3}(3x + 9) - 7$ and $\frac{3}{4}(8 + 12x) - 7x$

No, they are not equivalent. $\frac{2}{3}(3x + 9) - 7 = 2x - 1$ and $\frac{3}{4}(8 + 12x) - 7x = 6 + 2x$, which are not equal.

7. $\frac{1}{2}(6x + 4) + \frac{1}{3}(9x - 3)$ and $6x + 1$

Yes, they are equivalent. The first expression simplifies to $6x + 1$.

FAST Practice

8. Select all the expressions equivalent to $-7x + 4$.

- ☑ A. $-2(-2 + 3.5x)$
- ☐ B. $2(-3.5x - 2)$
- ☑ C. $7(1 - x) - 3$
- ☑ D. $-(2x + 5) - (5x - 9)$
- ☐ E. $-(6 - 7x) + 10$

Spiral Review

Problems 9–10: Evaluate each expression.

9. $5\frac{1}{4} \div \left(\frac{3}{8}\right)$

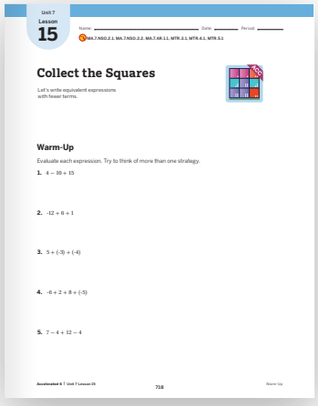
-14

10. $-6.75 + 4(1.8) - 8.2$

-7.75

Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1–2	1	MA.7.AR.1.2
	3	3	MA.7.AR.1.2
	4–7	2	MA.7.AR.1.2
FAST Practice	8	2	MA.7.AR.1.2
Spiral Review			
	9–10	2	MA.7.NSO.2.2


Unit 7
Lesson 15
Collect the Squares
Let's write equivalent expressions with fewer terms.
Warm-Up
Evaluate each expression. Try to think of more than one strategy.
1. 4 − 39 + 15
2. -12 + 6 + 1
3. 5 + (-3) + (-4)
4. -6 + 2 + 8 + (-5)
5. 7 − 4 + 12 − 4

Collect the Squares

Adding Expressions

Let's write equivalent expressions with fewer terms.

Learning Outcomes

Today's Goals

1. **Goal:** Write equivalent expressions with fewer terms by expanding, adding, and subtracting terms.
2. **Language Goal:** Explain how to write equivalent expressions using fewer terms. **(Speaking and Writing)**

Students explore the concept of a *term* and why writing an expression with fewer terms may be helpful for solving problems. This lesson will support students with "combining like terms" as they make use of structure to rewrite expressions with fewer terms by adding, subtracting, and expanding. **(MTR.5.1)**

Coherence

Prior Learning

In Unit 6, students used the distributive property to write equivalent expressions and described what it meant for two expressions to be equal.

Future Learning

In a future course, students will begin solving more complex equations including adding and expanding expressions.

Connections

- Students use visual representaions to simplifying expressions by grouping together terms that have the same squared variable. (mathematical)

FL B.E.S.T. Math Benchmarks

Addressing

MA.7.NSO.2.1
Solve mathematical problems using multi-step order of operations with rational numbers including grouping symbols, whole-number exponents and absolute value.

MA.7.AR.1.1
Apply properties of operations to add and subtract linear expressions with rational coefficients.

Also Addressing: **MA.7.NSO.2.2**

Mathematical Thinking and Reasoning Standards: MTR.3.1, MTR.4.1, MTR.5.1

ELA Expectations: ELA.K12.EE.1.1, ELA.K12.EE.5.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.6.AR.1.4	**MA.8.AR.1.2** **MA.8.AR.1.3**

Vocabulary

Review Vocabulary

term

Lesson at a Glance

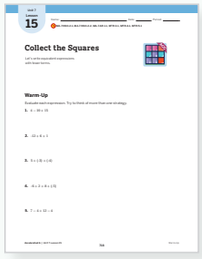
~ 45 min

MA.7.NSO.2.1, MA.7.NSO.2.2, MA.7.AR.1.1

Warm-Up

Whole Class | 10 min

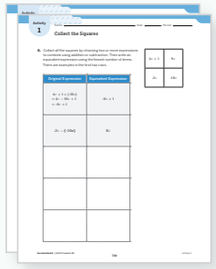
Students use the **Number Talk** routine to think flexibly about evaluating addition and subtraction expressions with integers. **(MTR.5.1)**



Activity 1

Pairs | 10 min

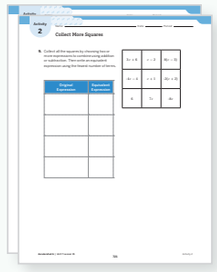
Students practice writing equivalent expressions with fewer *terms* by expanding, adding, and subtracting terms, addressing common errors throughout. **MLR3: Critique, Correct, Clarify**



Activity 2

Pairs | 15 min

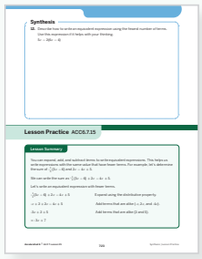
Students develop fluency by selecting expressions to combine and rewriting them with fewer terms. **(MTR.3.1)**



Synthesis

Whole Class | 5 min

Students synthesize their understanding of how to write an expression using fewer terms.



Show What You Know

Independent | 5 min

Students demonstrate their understanding by writing expressions with the fewest number of terms.

Prep Checklist

This lesson includes:

- Student Edition
- Show What You Know

Whole Class | 10 min

Warm-Up

Purpose: Students use the Number Talk routine to think flexibly about evaluating addition and subtraction expressions with integers. **(MTR.5.1)**

1 Launch

Use the Number Talk routine to support students in making use of structure when evaluating expressions with integers. See the Routine Facilitation Guide for more information. **(MTR.5.1)**

Invite students to evaluate the first expression independently. Then record a variety of strategies, along with the name of the students who shared them.

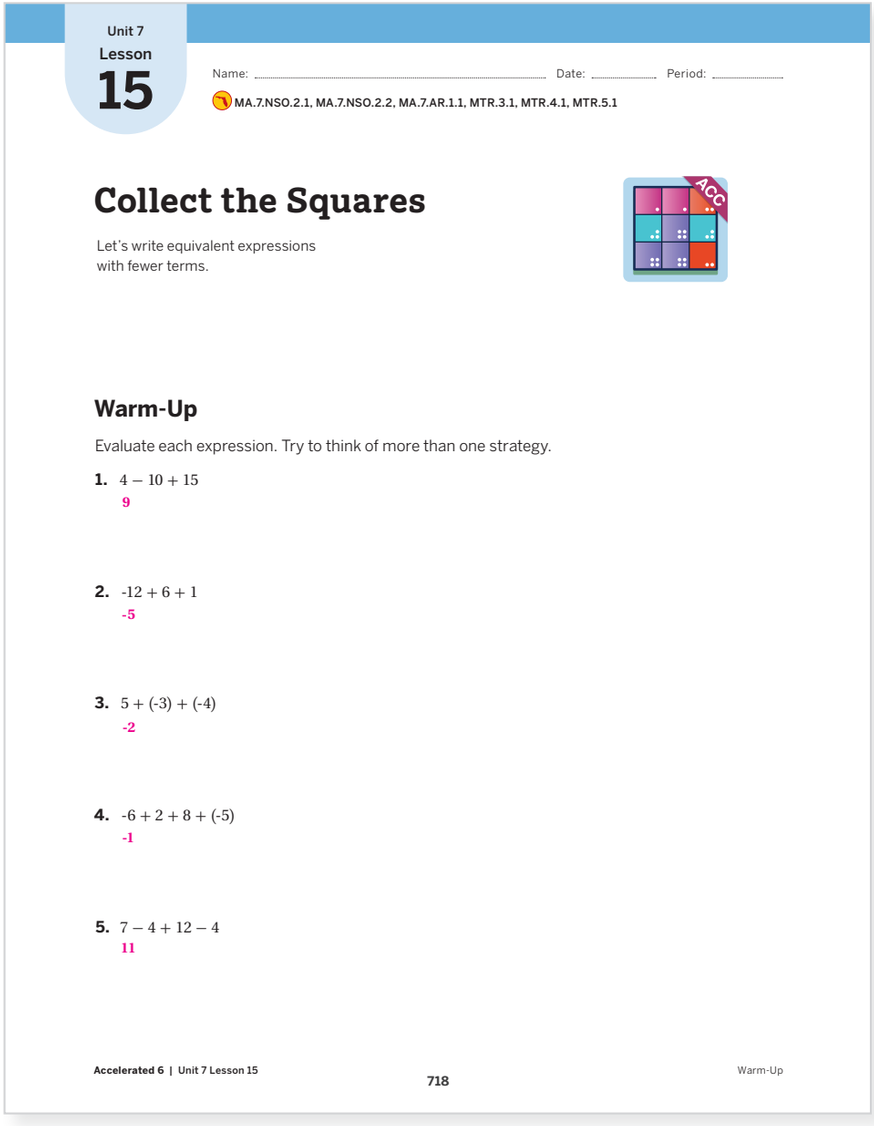
Consider asking, "Who thought about it in a different way?"

Note: This structure is repeated for each problem in the Number Talk.

2 Connect

Consider asking:

- "How are the expressions in Problems 2–4 alike? How do they differ from those in Problems 1 and 5?"
- "Is there more than one way to evaluate each expression?"



Unit 7
Lesson
15

Name: ..... Date: ..... Period: .....

MA.7.NSO.2.1, MA.7.NSO.2.2, MA.7.AR.1.1, MTR.3.1, MTR.4.1, MTR.5.1

Collect the Squares

Let's write equivalent expressions with fewer terms.

ACC

Warm-Up

Evaluate each expression. Try to think of more than one strategy.

1. $4 - 10 + 15$
   9
2. $-12 + 6 + 1$
   -5
3. $5 + (-3) + (-4)$
   -2
4. $-6 + 2 + 8 + (-5)$
   -1
5. $7 - 4 + 12 - 4$
   11

Accelerated 6 | Unit 7 Lesson 15 718 Warm-Up

Pairs | 10 min

Activity 1 Collect the Squares

Purpose: Students practice writing equivalent expressions with fewer *terms* by expanding, adding, and subtracting terms, addressing common errors throughout.

1 Launch

Direct students' attention to the examples in the table. Consider asking, "How do you combine expressions with addition and subtraction?"

Discuss the example $4x + 1 + (-10x)$, emphasizing why 1 cannot be combined with the other *terms*.

Consider sharing that the first expression has three *terms* if it does not come up naturally. Invite students to share what they think *term* means, celebrating multiple interpretations.

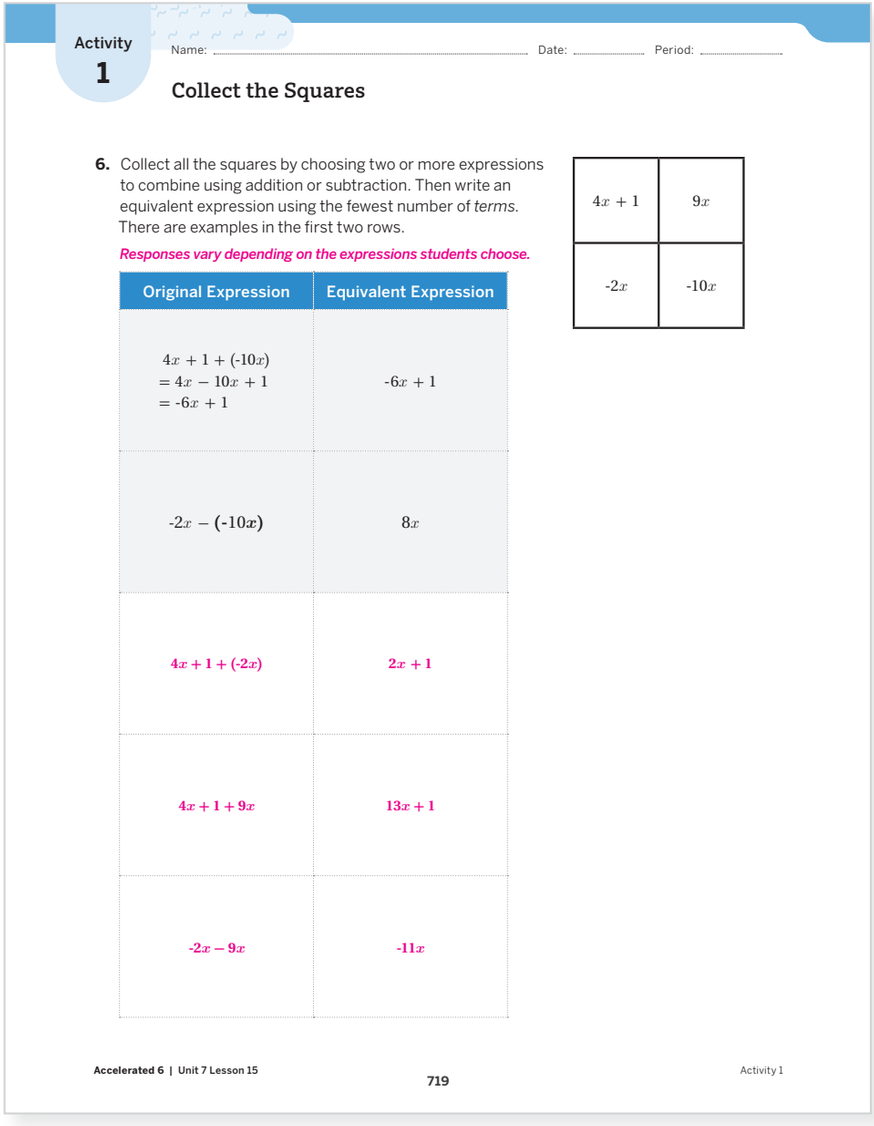
Encourage students to record the expressions they picked and make their thinking visible as they rewrite them.

2 Monitor

Invite students to share their thinking with a partner and give feedback on each other's expressions.

Listen for several combinations and equivalent expressions to share during the Connect.

Activity 1 continued >



Activity 1

Name: Date: Period:

Collect the Squares

6. Collect all the squares by choosing two or more expressions to combine using addition or subtraction. Then write an equivalent expression using the fewest number of *terms*. There are examples in the first two rows.

Responses vary depending on the expressions students choose.

Original Expression	Equivalent Expression
$4x + 1 + (-10x)$ $= 4x - 10x + 1$ $= -6x + 1$	$-6x + 1$
$-2x - (-10x)$	$8x$
$4x + 1 + (-2x)$	$2x + 1$
$4x + 1 + 9x$	$13x + 1$
$-2x - 9x$	$-11x$

$4x + 1$	$9x$
$-2x$	$-10x$

Accelerated 6 | Unit 7 Lesson 15 719 Activity 1

Pairs | 10 min

Activity 1 Collect the Squares (continued)

Purpose: Students practice writing equivalent expressions with fewer *terms* by expanding, adding, and subtracting terms, addressing common errors throughout.

2 Monitor

Select and sequence a variety of correct and incorrect ideas to highlight during the Connect.

D Differentiation

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Encourage them to share what they recognize from the expression and what feels confusing.
Add before distributing (e.g., $-3 + 3(x + 1) = 0(x + 1)$).	**Support** Consider asking, "What does it mean for a term to be on the outside of parentheses? Why might it be important to expand first?"
Add terms with a variable and terms without one (e.g., $2x + 8 = 10x$).	**Support** Invite them to share what $2x$ means and how that differs from 8 and that they cannot be combined because we don't yet know what x is. Note: This error is similar to Caleb's error. Consider capturing.

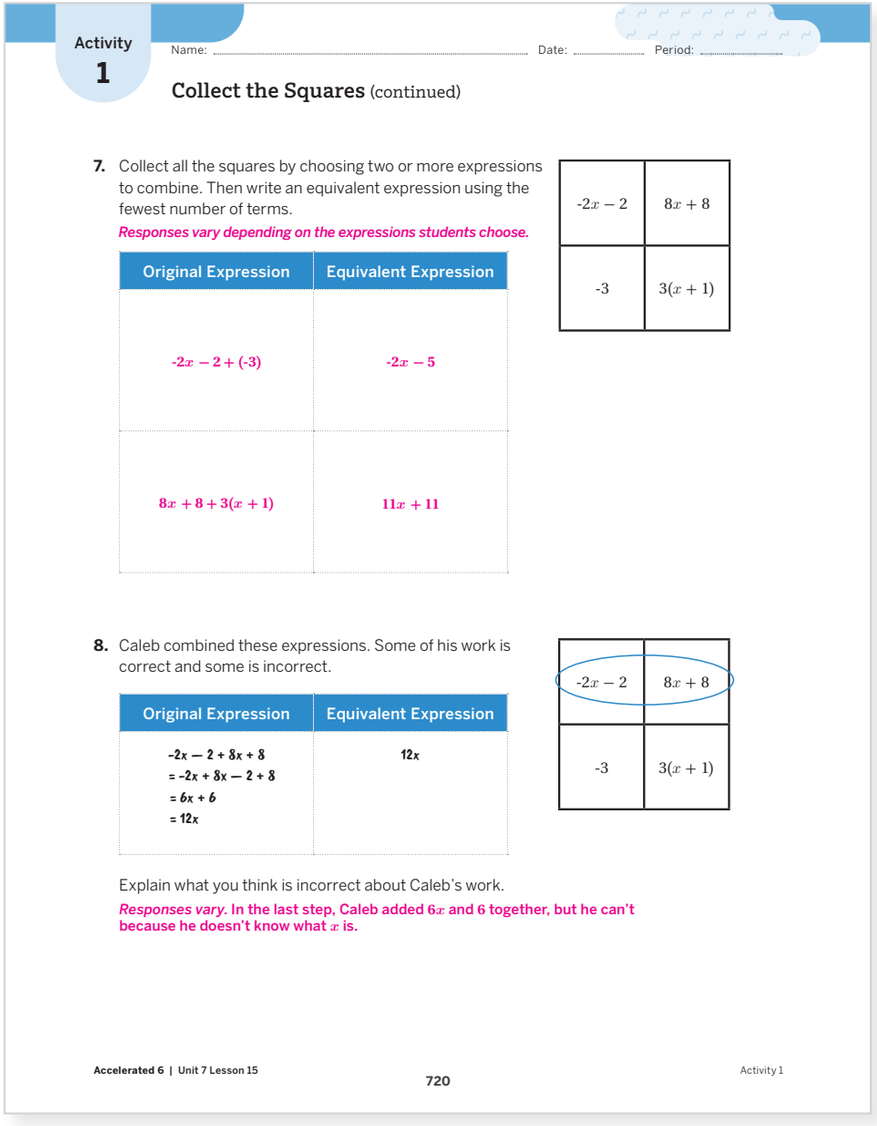
Listen for two or more explanations of Caleb's error that amplify why the sum of $6x$ and 6 is not $12x$ (e.g., without knowing the value of x, the structure of the terms is not the same).

D Differentiation: (Stretch) Invite students to add all of the squares at once. Consider asking, "What is a fifth expression that we could add to make the total sum equal to zero?"

3 Connect

MLR MLR3: Critique, Correct, Clarify Use this routine to help students describe strengths, errors, and ambiguities in Caleb's work. Invite students to share their responses from the activity. See the Routine Facilitation Guide for more information. **(MTR.4.1)**

To surface the Key Takeaway, consider inviting students to share how they would describe terms that can be combined.



Activity 1

Name: Date: Period:

Collect the Squares (continued)

7. Collect all the squares by choosing two or more expressions to combine. Then write an equivalent expression using the fewest number of terms.

Responses vary depending on the expressions students choose.

Original Expression	Equivalent Expression
$-2x - 2 + (-3)$	$-2x - 5$
$8x + 8 + 3(x + 1)$	$11x + 11$

$-2x - 2$	$8x + 8$
-3	$3(x + 1)$

8. Caleb combined these expressions. Some of his work is correct and some is incorrect.

Original Expression	Equivalent Expression
$-2x - 2 + 8x + 8$ $= -2x + 8x - 2 + 8$ $= 6x + 6$ $= 12x$	$12x$

$-2x - 2$	$8x + 8$
-3	$3(x + 1)$

Explain what you think is incorrect about Caleb's work.

Responses vary. In the last step, Caleb added $6x$ and 6 together, but he can't because he doesn't know what x is.

Accelerated 6 | Unit 7 Lesson 15 720 Activity 1

Key Takeaway: It can be helpful to think about which *terms* can be combined and which cannot when writing equivalent expressions.

Pairs | 15 min

Activity 2 Collect More Squares

Purpose: Students develop fluency by selecting expressions to combine and rewriting them with fewer terms. **(MTR.3.1)**

Short on time? Consider inviting students to choose either Problem 9 or 10 to complete.

1 Launch

To surface connections to Activity 1, ask:

- "Which expression stands out to you? Which other expression do you think would be least difficult to add to that one? The most difficult? Why?"
- "Which strategies from earlier might help us combine some of these expressions?" **(MTR.3.1)**

A **Accessibility: Executive Functioning** Chunk this activity into more manageable parts (e.g., asking students to collect one row or two squares).

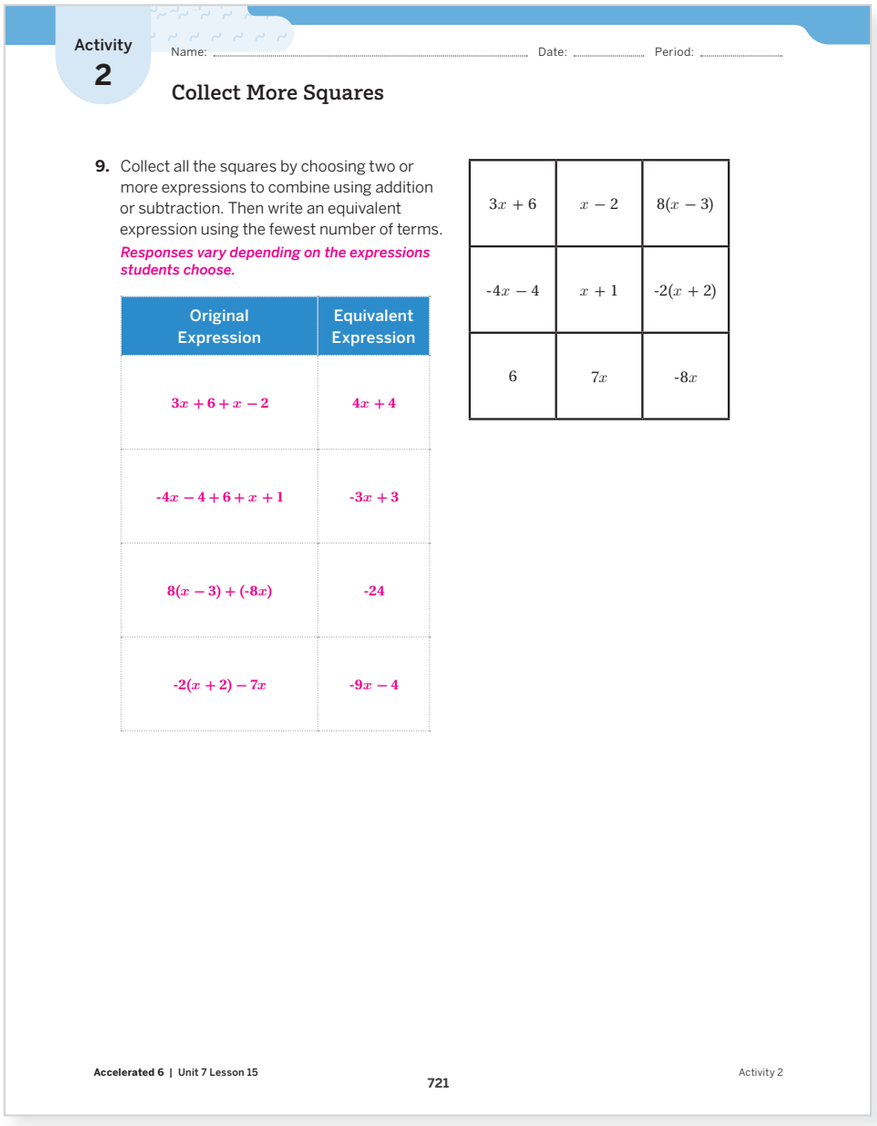
2 Monitor

Look for trends in the squares that students select first. Consider asking, "Why did you choose to combine those two expressions first?"

D **Differentiation**

Look for students who:	Respond to Student Thinking
Are unsure what the coefficient is in a term like x or $-x$ (e.g., $8 - x$).	**Support** Invite students to write the coefficient in front of the x (1 or -1).
Avoid certain expressions.	**Support** Encourage students to reflect on the expressions they are avoiding and discuss that expression with their partner.
Strategically select expressions with the same number of terms.	**Strengthen** Consider asking: "Why might it be helpful to select expressions with the same amount of terms? What should we do with expressions with parentheses?" Consider capturing a few of these responses for the Connect.
Use addition or subtraction to collect terms (e.g., $7 - 8x$ or $7 + (-8x)$).	**Strengthen** Consider asking, "Do both of these produce equivalent expressions? Why?" Consider capturing one of these responses for the Connect.

Activity 2 continued >



Activity 2

Name: Date: Period:

Collect More Squares

9. Collect all the squares by choosing two or more expressions to combine using addition or subtraction. Then write an equivalent expression using the fewest number of terms.

Responses vary depending on the expressions students choose.

Original Expression	Equivalent Expression
$3x + 6 + x - 2$	$4x + 4$
$-4x - 4 + 6 + x + 1$	$-3x + 3$
$8(x - 3) + (-8x)$	-24
$-2(x + 2) - 7x$	$-9x - 4$

$3x + 6$	$x - 2$	$8(x - 3)$
$-4x - 4$	$x + 1$	$-2(x + 2)$
6	$7x$	$-8x$

Accelerated 6 | Unit 7 Lesson 15 721 Activity 2

Pairs | 15 min

Activity 2 Collect More Squares (continued)

Purpose: Students develop fluency by selecting expressions to combine and rewriting them with fewer terms. **(MTR.3.1)**

Short on time? Consider inviting students to choose either Problem 9 or 10 to complete.

2 Monitor

D **Differentiation: You're invited to explore more. (Stretch)** Invite students who would like to explore expressions further to complete this optional task. Encourage them to discuss their thinking with a partner.

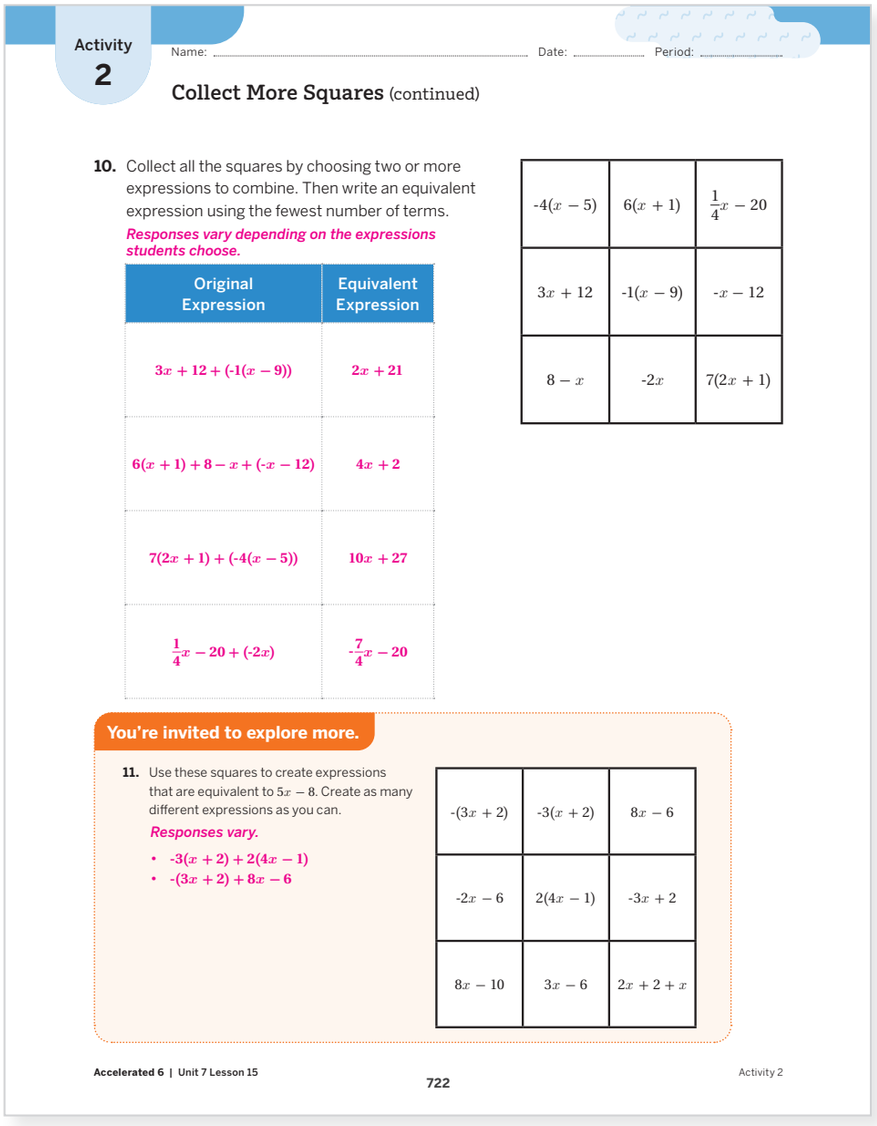
3 Connect

Display a collection of responses.

Consider asking:

- "Which expressions did you start with? Why?"
- "Which expressions did you leave until the end? Why?"
- "What happens when we select an expression with a negative sign in front?"
- "What are the strategies that you found helpful?"
- "How would we know if we have written an expression with the fewest possible terms?"

ELL **English Language Learners** Encourage students to paraphrase each other's ideas to help make connections and incorporate vocabulary from the unit.



Activity 2

Name: Date: Period:

Collect More Squares (continued)

10. Collect all the squares by choosing two or more expressions to combine. Then write an equivalent expression using the fewest number of terms.

Responses vary depending on the expressions students choose.

Original Expression	Equivalent Expression
$3x + 12 + (-1(x - 9))$	$2x + 21$
$6(x + 1) + 8 - x + (-x - 12)$	$4x + 2$
$7(2x + 1) + (-4(x - 5))$	$10x + 27$
$\frac{1}{4}x - 20 + (-2x)$	$-\frac{7}{4}x - 20$

$-4(x - 5)$	$6(x + 1)$	$\frac{1}{4}x - 20$
$3x + 12$	$-1(x - 9)$	$-x - 12$
$8 - x$	$-2x$	$7(2x + 1)$

You're invited to explore more.

11. Use these squares to create expressions that are equivalent to $5x - 8$. Create as many different expressions as you can.

Responses vary.

- $-3(x + 2) + 2(4x - 1)$
- $-(3x + 2) + 8x - 6$

$-(3x + 2)$	$-3(x + 2)$	$8x - 6$
$-2x - 6$	$2(4x - 1)$	$-3x + 2$
$8x - 10$	$3x - 6$	$2x + 2 + x$

Accelerated 6 | Unit 7 Lesson 15 722 Activity 2

Whole Class | 5 min

Synthesis

Purpose: Students synthesize their understanding of how to write an expression using fewer terms.

Synthesis

Invite students to respond independently, and then share their thinking with a partner.

ELL **English Language Learners** Provide sentence frames to help students explain their strategy (e.g., First, _____. Then, _____.). **(Speaking and Listening)**

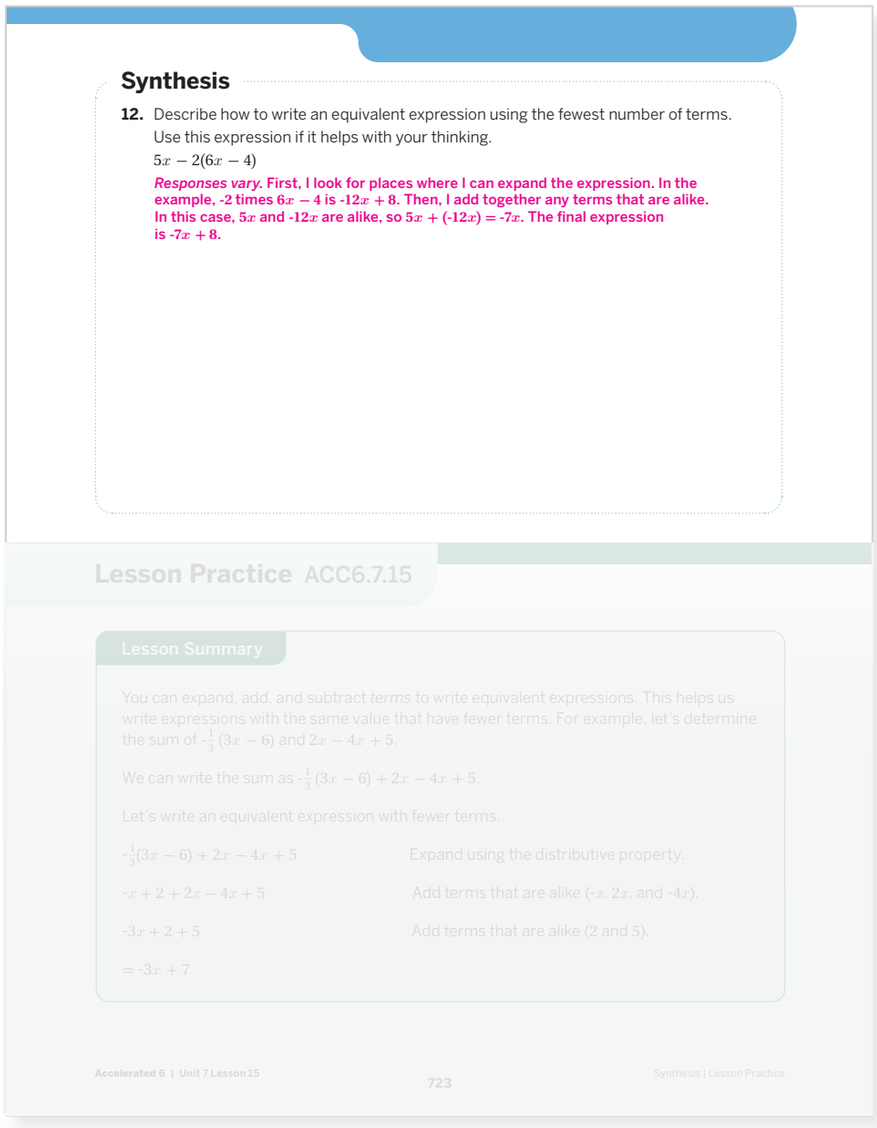
Collect and share a variety of ideas, such as:

- Expanding the expression first.
- Analyzing the structure of each of the terms to decide if they can be combined.

Consider asking:

- "What are some ways we could rewrite this expression using expanding, adding, or subtracting?"
- "What are some common mistakes someone might make on this problem?"

Note: If time allows, invite students to revise their previous thinking based on the discussion.



Synthesis

12. Describe how to write an equivalent expression using the fewest number of terms. Use this expression if it helps with your thinking.
$5x - 2(6x - 4)$

Responses vary. First, I look for places where I can expand the expression. In the example, -2 times $6x - 4$ is $-12x + 8$. Then, I add together any terms that are alike. In this case, $5x$ and $-12x$ are alike, so $5x + (-12x) = -7x$. The final expression is $-7x + 8$.

Lesson Practice ACC6.7.15

Lesson Summary

You can expand, add, and subtract *terms* to write equivalent expressions. This helps us write expressions with the same value that have fewer terms. For example, let's determine the sum of $-\frac{1}{3}(3x - 6)$ and $2x - 4x + 5$.

We can write the sum as $-\frac{1}{3}(3x - 6) + 2x - 4x + 5$.

Let's write an equivalent expression with fewer terms.

$-\frac{1}{3}(3x - 6) + 2x - 4x + 5$ Expand using the distributive property.

$-x + 2 + 2x - 4x + 5$ Add terms that are alike ($-x$, $2x$, and $-4x$).

$-3x + 2 + 5$ Add terms that are alike (2 and 5).

$= -3x + 7$

Accelerated 6 | Unit 7 Lesson 15 723 Synthesis | Lesson Practice

Lesson Takeaway: Equivalent expressions can be written with fewer *terms* by expanding, adding, and subtracting terms.

Independent | 5 min

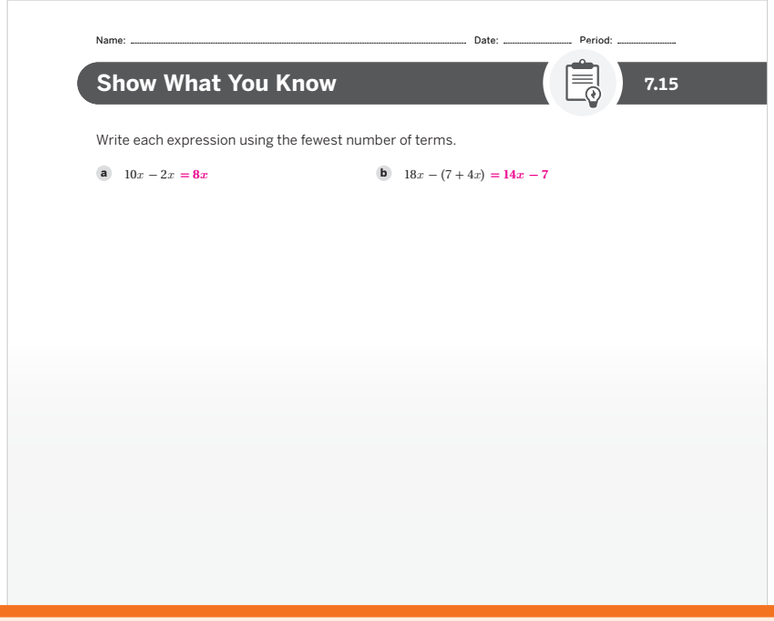
Show What You Know

Purpose: Students demonstrate their understanding by writing expressions with the fewest number of terms.

Today's Goals

1. **Goal:** Write equivalent expressions with fewer terms by expanding, adding, and subtracting terms.
2. **Language Goal:** Explain how to write equivalent expressions using fewer terms. **(Speaking and Writing)**

Show What You Know (Assessment Resources)



Name: _____ Date: _____ Period: _____

Show What You Know 7.15

Write each expression using the fewest number of terms.

a $10x - 2x = 8x$

b $18x - (7 + 4x) = 14x - 7$

Differentiation Use after Lesson 15

Support

Provide targeted intervention for students by using these resources.

If students need support with rewriting an expression with fewer terms by adding, subtracting, and expanding:

» **Respond to Student Thinking:**

- Revisit the **Synthesis** before giving individualized feedback on the relevant **Practice Problems**.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 2 Extension Activities**.
- Invite students to complete the **You're invited to Explore More task**.

Professional Learning

Which students' ideas were you able to highlight during Activity 1? Did students prefer to combine expressions using addition rather than subtraction?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

12. Describe how to write an equivalent expression using the fewest number of terms. Use this expression if it helps with your thinking.

$5x - 2(6x - 4)$

Responses vary. First, I look for places where I can expand the expression. In the example, -2 times $6x - 4$ is $-12x + 8$. Then, I add together any terms that are alike. In this case, $5x$ and $-12x$ are alike, so $5x + (-12x) = -7x$. The final expression is $-7x + 8$.

Lesson Practice ACC6.7.15

Lesson Summary

You can expand, add, and subtract *terms* to write equivalent expressions. This helps us write expressions with the same value that have fewer terms. For example, let's determine the sum of $-\frac{1}{3}(3x - 6)$ and $2x - 4x + 5$.

We can write the sum as $-\frac{1}{3}(3x - 6) + 2x - 4x + 5$.

Let's write an equivalent expression with fewer terms.

$-\frac{1}{3}(3x - 6) + 2x - 4x + 5$	Expand using the distributive property.
$-x + 2 + 2x - 4x + 5$	Add terms that are alike ($-x$, $2x$, and $-4x$).
$-3x + 2 + 5$	Add terms that are alike (2 and 5).
$= -3x + 7$	

Lesson Practice ACC6.7.15

Name: _____ Date: _____ Period: _____

1. Fill in the blanks to make each equation true.

Responses vary.

$6x +$ ___4x___ $= 10x$

$6x +$ ___-4x___ $= 2x$

$6x +$ ___-16x___ $= -10x$

$6x +$ ___4x + 5___ $= 10x + 5$

$6x -$ ___4x___ $= 2x$

$6x -$ ___5x___ $= x$

$6x +$ ___-6x + 10___ $= 10$

$6x - ($ ___2x + 10___ $) = 4x - 10$

2. Fill in each blank with a number or expression such that each row and column has the same sum.

$2x$	$x + 2$	$2 - x$
$5 - x$	x	$2x - 1$
$x - 1$	2	$x + 3$

FAST Practice

3. Select *all* of the expressions that are equivalent to $4x - 5 + 6$.

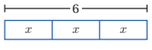
- ☑ **A.** $4x + (-5) + 6$
- ☐ **B.** $4x - 6 + 5$
- ☑ **C.** $4x + 1$
- ☐ **D.** $5x$
- ☐ **E.** $5 + 6 - 4x$

Lesson Practice ACC6.7.15

Name: _____ Date: _____ Period: _____

Spiral Review

Problems 4–6: This diagram can be represented by the equation $6 = 3x$.



4. Explain where you can see the 3 in the diagram.

Explanations vary. There are 3 parts labeled x.

5. Determine the value of x.

$x = 2$

6. Select *all* the situations that could be represented by this equation.

- ☑ **A.** Aaliyah is studying 6 hours this week for end-of-year exams. She spends an equal number of hours each on math, science, and history.
- ☐ **B.** Lan spends $3 on 6 markers.
- ☑ **C.** Sneha gives 6 grapes to 3 friends. She gives each friend the same number of grapes.
- ☑ **D.** Kiri has a goal to read 6 total books within the next 3 weeks. She will read the same number of books each week.
- ☐ **E.** Adriana runs 6 miles every day for 3 days.

Problems 7–10: Solve each equation.

7. $5n = 60$

$n = 12$

8. $t + 3 = 25$

$t = 22$

9. $p - 8 = 22$

$p = 30$

10. $\frac{j}{5} = 4$

$j = 20$

11. A small town had a population of 960 people last year. The population grew to 1,200 people this year. By what percent did the population grow?

25%

Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1	2	MA.7.AR.1.1
	2	3	MA.7.AR.1.1
FAST Practice	3	1	MA.7.AR.1.1
Spiral Review			
	4–7, 10	2	MA.6.AR.2.3
	8–9	2	MA.6.AR.2.2
	11	2	MA.7.AR.3.1

Sub-Unit 3

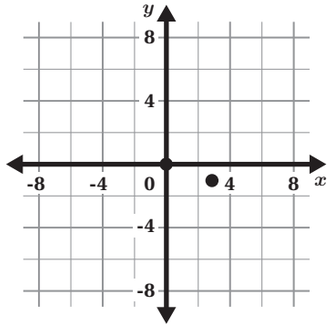
The Coordinate Plane

Sub-Unit 3 Goals

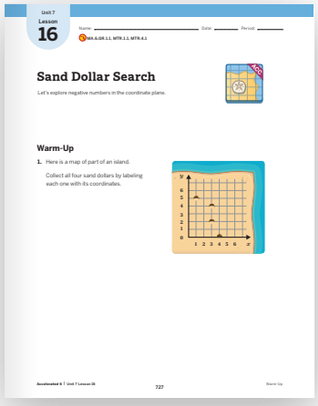
- **Goal:** Graph points with positive and negative coordinates.
- **Goal:** Draw polygons on the coordinate plane and determine their vertical and horizontal side lengths.
- **Language Goal:** Describe the relationship between ordered pairs that differ only by the signs of their coordinates.
- **Language Goal:** Interpret the meaning of an ordered pair in context.

In this sub-unit, students extend what they learned about the coordinate plane in a previous course to include points with positive and negative coordinates in all four **quadrants**. Students solve real-world and mathematical problems by graphing points, and draw polygons given coordinates for the vertices.

Math That Matters Most

Concept or Skill	For example . . .	
Graph points with positive and negative coordinates.	Plot a point at the coordinates (3, -1). Explain how you chose where to plot the point. *I know the point should be 3 to the right and 1 down. Because the graph is going by 2s, that means that it is really 1.5 squares to the right and 0.5 squares down.*	
Draw polygons on the coordinate plane.	Here are Vicente's coordinates. Did he make a square? *Yes* Show or explain your thinking. *Vicente's polygon is made up of straight horizontal and vertical lines and every side is 7 units long so he did build a square.*	

Point	Coordinates
A	(2, -3)
B	(2, 4)
C	(9, 4)
D	(9, -3)


Unit 7
Lesson 16
Sand Dollar Search
Let's explore negative numbers in the coordinate plane.
Warm-Up
1. Here is a map of part of an island.
Collect all four sand dollars by labeling each one with its coordinates.
727

Sand Dollar Search

Points in the Coordinate Plane

Let's explore negative numbers in the coordinate plane.

Learning Outcomes

Today's Goals

1. **Goal:** Write coordinates for points on the coordinate plane.
2. **Goal:** Estimate the location of a point on the coordinate plane using the signs of its coordinates.
3. **Language Goal:** Explain what the coordinate plane looks like with positive and negative numbers. **(Writing)**

Students develop an understanding of negative numbers on the *coordinate plane* and use the context of searching for sand dollars to move between locations and identify the coordinates. Students also use structure to construct an argument for the location of a point on the coordinate plane using the signs of its coordinates. **(MTR.1.1)**

Coherence

Prior Learning

In Lessons 1–5, students explored locations on the number line, beginning to describe them using positive and negative numbers. In a previous course, students began their work with the coordinate system, expanding their previous understanding of the number line to include a vertical and horizontal number line and naming the two axes the x-axis and y-axis.

Future Learning

In Lessons 17–19, students will deepen their understanding of the coordinate plane to understand distance and make sense of new contexts.

Connections

- Students draw on their prior experience of graphing in the coordinate plane in quadrant 1 to expand their understanding of the other quadrants in context. (mathematical, real world)

FL B.E.S.T. Math Benchmarks

Addressing

MA.6.GR.1.1
Extend previous understanding of the coordinate plane to plot rational number ordered pairs in all four quadrants and on both axes. Identify the x -or y-axis as the line of reflection when two ordered pairs have an opposite x- or y-coordinate.

Mathematical Thinking and Reasoning Standards: MTR.1.1, MTR.4.1

ELA Expectation: ELA.K12.EE.5.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.5.GR.4.1	MA.7.AR.4.3
	MA.8.GR.2.3

Vocabulary

New Vocabulary

quadrant

Review Vocabulary

origin
ordered pair
coordinate plane
x-coordinate
x-axis
y-coordinate
y-axis

Lesson at a Glance

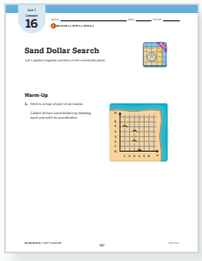
~ 45 min

MA.6.GR.1.1

Warm-Up

Pairs | 5 min

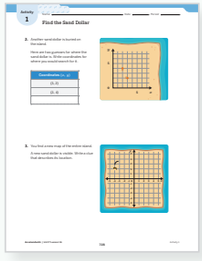
Students graph points in the first quadrant of a *coordinate plane* and are introduced to the context of sand dollars buried at locations described by coordinates.



Activity 1

Pairs | 5 min

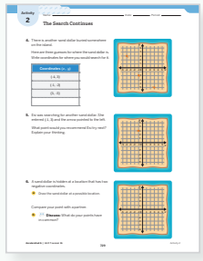
Students extend their understanding of the coordinate plane from one **quadrant** to four.



Activity 2

Pairs | 10 min

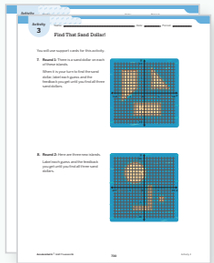
Students determine possible locations of sand dollars on the coordinate plane, sometimes only using the signs of their coordinates. MLR3: Critique, Correct, Clarify



Activity 3

Pairs | 15 min

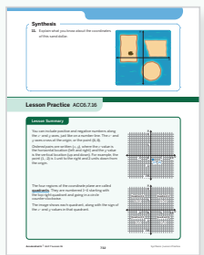
Students use knowledge of the signed coordinates in each quadrant to search for sand dollars on various islands, including planes without gridlines.



Synthesis

Whole Class | 5 min

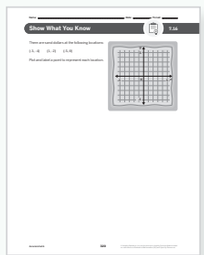
Students synthesize their understanding of points with negative and positive coordinates on the coordinate plane. MLR2: Collect and Display



Show What You Know

Independent | 5 min

Students demonstrate their understanding by plotting and labeling points.



Prep Checklist

Prepare Activity 3 Sheets.

This lesson includes:

- Student Edition
- Show What You Know
- Activity 3 Sheets, one set per pair, pre-cut.
- Rulers (as needed)

Pairs | 5 min

Warm-Up

Purpose: Students graph points in the first quadrant of a *coordinate plane* and are introduced to the context of sand dollars buried at locations described by coordinates.

1 Launch

To generate engagement, ask students if they have ever been to the beach or seen a sand dollar. Invite students to share these experiences.

Invite students to find all of the sand dollars.

2 Connect

Invite students to share their strategy for locating each sand dollar.

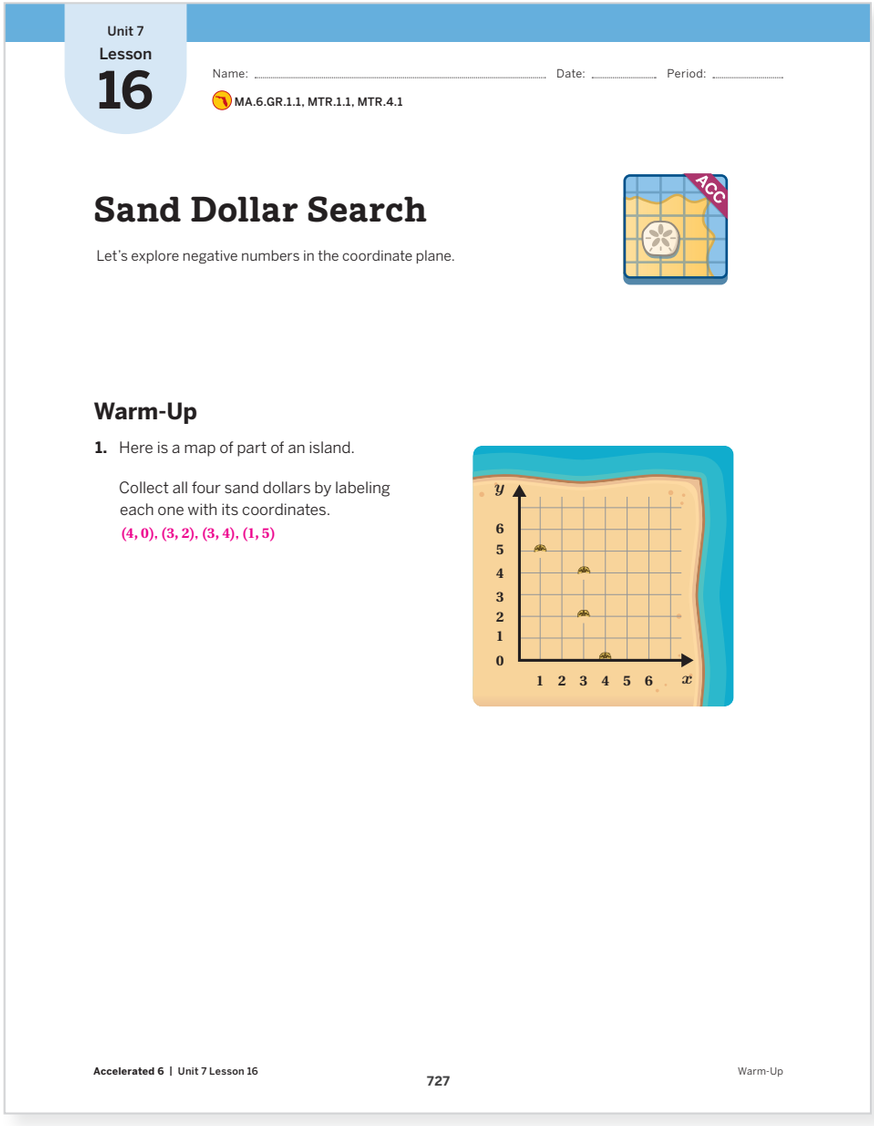
Consider asking:

- "How would you describe where the point $(4, 0)$ is?"
- "Why is the origin an important consideration when plotting points?"

Invite students to show how they would move with their hands, starting from the origin to reach a given point.

A **Accessibility: Executive Functioning**
To support students in keeping track of information, create a class anchor chart that describes how we locate a coordinate on the plane.

To surface prior knowledge, consider asking, "What is meant by *ordered pair*?" The ordered pair is the name of a point. It is written in the form (x, y) and tells you the location of a point in the coordinate plane. The first number is the x-coordinate and the second number is the y-coordinate.



Unit 7
Lesson
16

Name: ..... Date: ..... Period: .....

MA.6.GR.1.1, MTR.1.1, MTR.4.1

Sand Dollar Search

Let's explore negative numbers in the coordinate plane.

Warm-Up

1. Here is a map of part of an island.

Collect all four sand dollars by labeling each one with its coordinates.
$(4, 0), (3, 2), (3, 4), (1, 5)$

Pairs | 5 min

Activity 1 Find the Sand Dollar

Purpose: Students extend their understanding of the coordinate plane from one **quadrant** to four.

1 Launch

To surface connections, consider asking students how the guesses in the table connect to the arrows on the image.

2 Monitor

Look for students who say the location of the sand dollar is $(3, 2)$ instead of $(2, 3)$. Invite them to reference the anchor chart for the order of the coordinates.

Invite students to look at the map of the island. Pause briefly to have students share what they notice about the new map.

Record a variety of student clues describing the location of the sand dollar. Look for students who use the signs of the coordinates, making connections to their previous work with extending number lines. **(MTR.1.1)**

3 Connect

Display the recorded clues and consider asking:

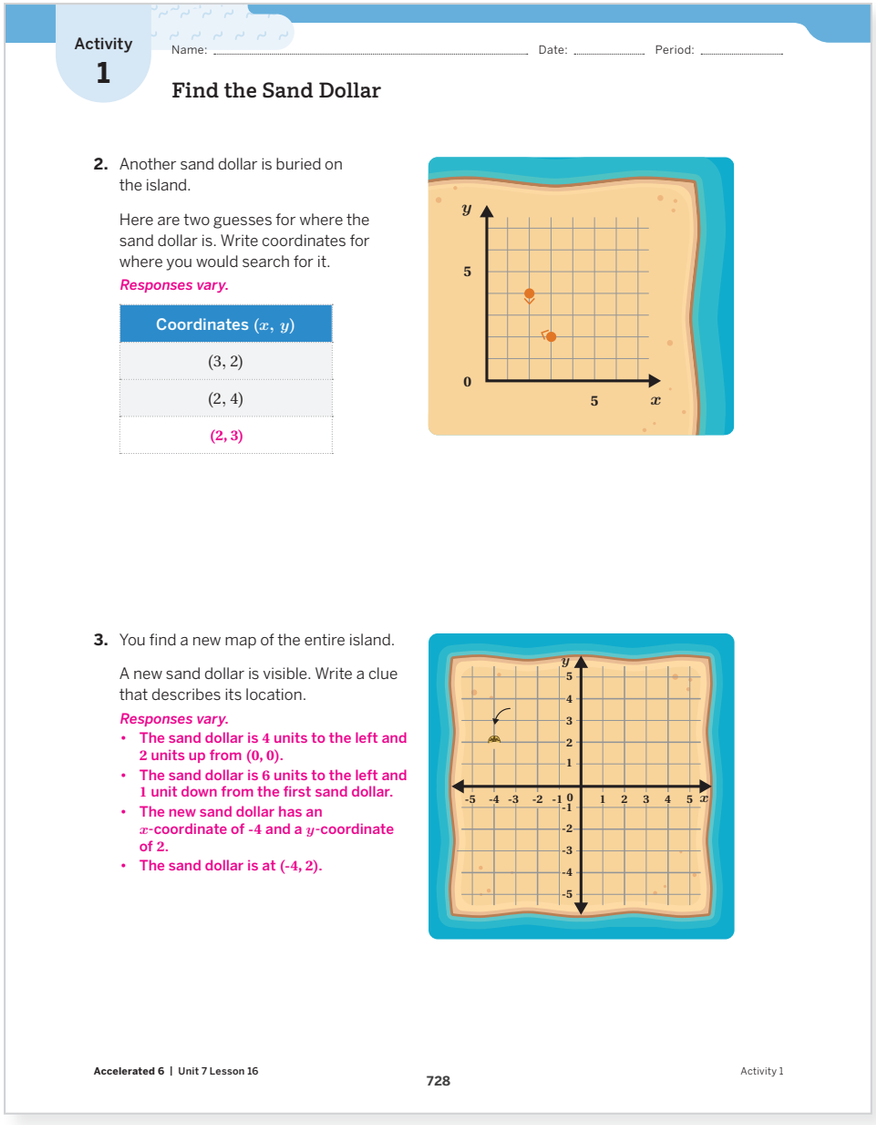
- "What is helpful about this clue?"
- "Does this clue describe one location? More than one?"
- "How might you use an ordered pair to describe the location of the new sand dollar?" You can use the point $(-4, 2)$.

To surface the Key Takeaway, consider asking, "How would you label the extended parts of the x- and y- axis?"

Share that the *x-axis* and *y-axis* separate the *coordinate plane* into four quadrants.

Create a class definition of the word **quadrant** using students' language and the glossary.

Note: You may choose to share the convention that mathematicians have chosen to name each quadrant (I, II, III, and IV).



Activity 1

Name: _____ Date: _____ Period: _____

Find the Sand Dollar

2. Another sand dollar is buried on the island.

Here are two guesses for where the sand dollar is. Write coordinates for where you would search for it.

Responses vary.

Coordinates (x, y)
$(3, 2)$
$(2, 4)$
$(2, 3)$

3. You find a new map of the entire island.

A new sand dollar is visible. Write a clue that describes its location.

Responses vary.

- The sand dollar is 4 units to the left and 2 units up from (0, 0).
- The sand dollar is 6 units to the left and 1 unit down from the first sand dollar.
- The new sand dollar has an x-coordinate of -4 and a y-coordinate of 2.
- The sand dollar is at (-4, 2).

Accelerated 6 | Unit 7 Lesson 16 728 Activity 1

Key Takeaway: The value of the *x-coordinate* indicates the direction and distance left and right of the *origin*. The *y-coordinate* indicates the direction and distance up and down from the origin. The *x-axis* and *y-axis* separate the *coordinate plane* into four sections called **quadrants**.

Pairs | 10 min

Activity 2 The Search Continues

Purpose: Students determine possible locations of sand dollars on the coordinate plane, sometimes only using the signs of their coordinates.

1 Launch

Encourage students to consider many points as they search for more sand dollars.

2 Monitor

Look for students using the clues and consider asking them to voice their strategies out loud for students who are struggling to get started.

A **Accessibility: Visual-Spatial Processing** Provide rulers to support processing visual information.

Encourage students to check their answers with a partner. **(MTR.1.1)**

Listen for explanations to share in the Connect.

D **Differentiation** *(Problem 5)*

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Invite students to describe Esi's current point and use the structure to name another point.
Name a point with reversed x- and y-coordinates.	**Support** Encourage them to reference the anchor chart.
Write (-2, 3), (-4, 3), or $(x, 3)$ where $x < -1$.	**Strengthen** Consider asking, "What are all of the points that are possible guesses? What do they all have in common?"

Listen for students who make connections between the signs of the values in the ordered pair and the quadrants.

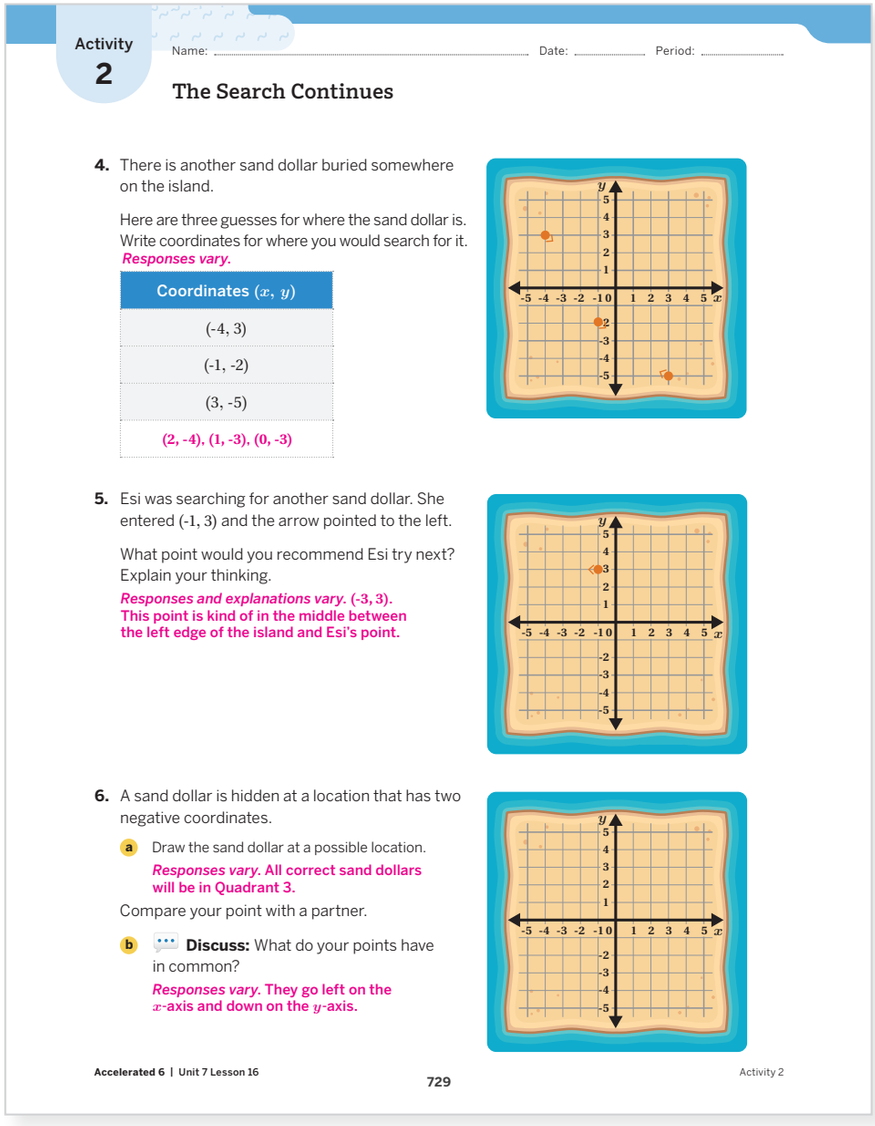
3 Connect

Display student responses.

MLR **MLR3: Critique, Correct, Clarify** Use this routine to help students describe strengths, errors, and ambiguities in the class' responses for Esi. Display a few student responses then invite students to:

a. Discuss what is *correct* about the response.
b. Discuss what is *incorrect* or *unclear*.
c. Write a question to help the student see how their work can be improved, or encourage students to create a revised response. **(MTR.1.1, MTR.4.1)**

Consider asking, "Could (-3, 0) be a possible location of the sand dollar? Why or why not?" No. *Responses vary.* 0 is not a negative number and it would be a point on the x-axis, not in the bottom left quadrant.



Activity 2

Name: Date: Period:

The Search Continues

4. There is another sand dollar buried somewhere on the island.

Here are three guesses for where the sand dollar is. Write coordinates for where you would search for it.
Responses vary.

Coordinates (x, y)
(-4, 3)
(-1, -2)
(3, -5)
(2, -4), (1, -3), (0, -3)

5. Esi was searching for another sand dollar. She entered (-1, 3) and the arrow pointed to the left.

What point would you recommend Esi try next? Explain your thinking.
Responses and explanations vary. (-3, 3). This point is kind of in the middle between the left edge of the island and Esi's point.

6. A sand dollar is hidden at a location that has two negative coordinates.

a. Draw the sand dollar at a possible location.
Responses vary. All correct sand dollars will be in Quadrant 3.

Compare your point with a partner.

b. **Discuss:** What do your points have in common?
Responses vary. They go left on the x-axis and down on the y-axis.

Accelerated 6 | Unit 7 Lesson 16 729 Activity 2

Pairs | 15 min

Activity 3 Find That Sand Dollar!

Purpose: Students use knowledge of the signed coordinates in each quadrant to search for sand dollars on various islands, including planes without gridlines.

Short on time? Consider omitting Problem 8.

1 Launch

Encourage students to use the feedback from their partner to help them find all three sand dollars. **(MTR.1.1)**

Distribute one set of the Activity 3 Cards to each pair of students.

2 Monitor

Look for students who are focusing on one island and using the location of the island on the plane to predict what the coordinates would be.

A **Accessibility: Visual-Spatial Processing** Invite students to count the number of squares along the horizontal axis and then move vertically to support interpreting the points visually.

Activity 3 continued >

Activity 3

Name: _____ Date: _____ Period: _____

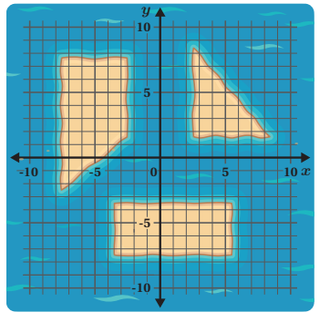
Find That Sand Dollar!

You will use support cards for this activity.

7. **Round 1:** There is a sand dollar on each of these islands.

   When it is your turn to find the sand dollar, label each guess and the feedback you get until you find all three sand dollars.

   Partner A: Sand dollars are at (-7, -1), (5, 4), and (0, -7).
   Partner B: Sand dollars are at (-7, -2), (5, 2), and (0, -5).

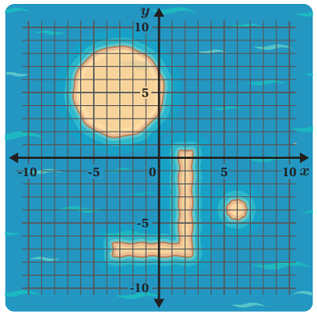


8. **Round 2:** Here are three new islands.

   Label each guess and the feedback you get until you find all three sand dollars.

   Partner A: Sand dollars are at (6, -4), (-2, -7), (-2, 6).

   Partner B: Sand dollars are at (6, -4), (-3, -7), (-5, 5).



Accelerated 6 | Unit 7 Lesson 16 730 Activity 3

Pairs | 15 min

Activity 3 Find That Sand Dollar! (continued)

Purpose: Students use knowledge of the signed coordinates in each quadrant to search for sand dollars on various islands, including planes without gridlines.

2 Monitor

D Differentiation

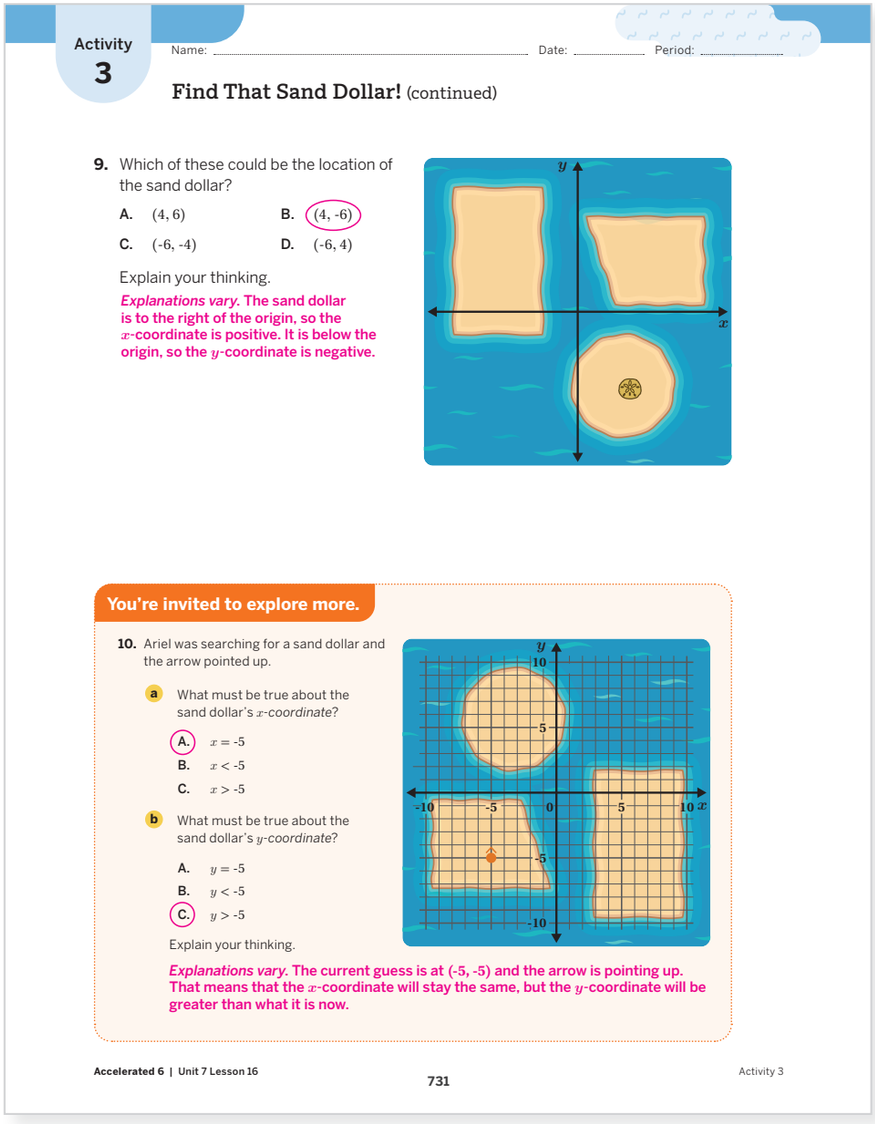
Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Consider asking, "Is there a coordinate you know that cannot be the location of the sand dollar?"
Make connections between the quadrant's location and the sign of values on the x-and y-axis bordering that quadrant.	**Strengthen** Celebrate and invite students to share during the Connect.
Would like to explore the coordinate plane further.	**Stretch** **You're invited to explore more.** Invite students who would like to explore coordinates on a grid further to complete this optional task. Encourage them to discuss their thinking with a partner. **(MTR.4.1)**

3 Connect

Invite students to share what is alike and different about each of the ordered pairs.

Invite students to share their responses. Consider asking, "Based on where we see the sand dollar, what do you know about the sign of the x-coordinate? The y-coordinate?"

To surface the Key Takeaway, consider asking, "How does the structure of the ordered pair help us determine which quadrant the point is in?"



Activity 3

Name: _____ Date: _____ Period: _____

Find That Sand Dollar! (continued)

9. Which of these could be the location of the sand dollar?

A. $(4, 6)$ B. $(4, -6)$

C. $(-6, -4)$ D. $(-6, 4)$

Explain your thinking.

Explanations vary. The sand dollar is to the right of the origin, so the x-coordinate is positive. It is below the origin, so the y-coordinate is negative.

You're invited to explore more.

10. Ariel was searching for a sand dollar and the arrow pointed up.

a What must be true about the sand dollar's x-*coordinate*?

A. $x = -5$

B. $x < -5$

C. $x > -5$

b What must be true about the sand dollar's y-*coordinate*?

A. $y = -5$

B. $y < -5$

C. $y > -5$

Explain your thinking.

Explanations vary. The current guess is at (-5, -5) and the arrow is pointing up. That means that the x-coordinate will stay the same, but the y-coordinate will be greater than what it is now.

Accelerated 6 | Unit 7 Lesson 16 731 Activity 3

Key Takeaway: The signs of the coordinates indicate what quadrant the point is in. For example, points in the upper left quadrant (Quadrant 2) all have a negative x-value and a positive y-value.

Whole Class | 5 min

Synthesis

Purpose: Students synthesize their understanding of points with negative and positive coordinates on the coordinate plane.

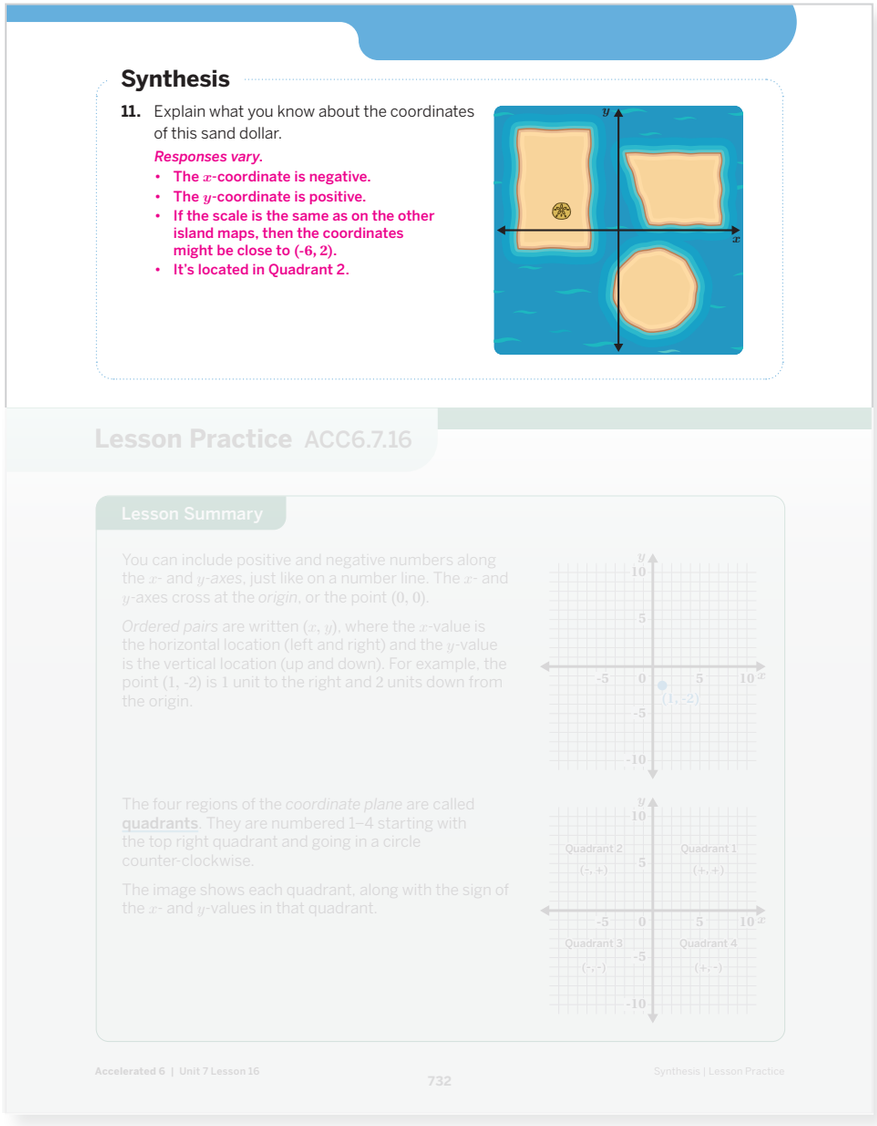
Invite students to respond independently, and then share their thinking with a partner.

Collect and share a variety of ideas, such as:

- The x-coordinate is negative.
- The y-coordinate is positive.
- If the axes have the same scale, then the absolute value of the x-coordinate is greater than the absolute value of the y-coordinate.
- If the scale is the same as on the other island maps, then the coordinates might be close to (-6, 2).

Note: If time allows, invite students to share with a partner what makes sense about the points in each quadrant and what they still have questions about.

MLR **MLR2: Collect and Display** Use this routine to formalize the definition of *quadrant* using students' language. Display any new language, examples, or diagrams publicly for students to reference later.



Synthesis

11. Explain what you know about the coordinates of this sand dollar.

Responses vary.

- The x-coordinate is negative.
- The y-coordinate is positive.
- If the scale is the same as on the other island maps, then the coordinates might be close to (-6, 2).
- It's located in Quadrant 2.

Lesson Practice ACC6.7.16

Lesson Summary

You can include positive and negative numbers along the x- and y-*axes*, just like on a number line. The x- and y-axes cross at the *origin*, or the point (0, 0).

Ordered pairs are written (x, y), where the x-value is the horizontal location (left and right) and the y-value is the vertical location (up and down). For example, the point (1, -2) is 1 unit to the right and 2 units down from the origin.

The four regions of the *coordinate plane* are called **quadrants**. They are numbered 1–4 starting with the top right quadrant and going in a circle counter-clockwise.

The image shows each quadrant, along with the sign of the x- and y-values in that quadrant.

Accelerated 6 | Unit 7 Lesson 16

732

Synthesis | Lesson Practice

Lesson Takeaway: The *coordinate plane* can be extended to include positive and negative coordinates. The x-axis and y-axis separate the coordinate plane into four sections called **quadrants**. The signs of the x- and y-*coordinates* indicate which quadrant the point is in.

Independent | 5 min

Show What You Know

Purpose: Students demonstrate their understanding by plotting and labeling points.

Today's Goals

1. **Goal:** Write coordinates for points on the coordinate plane.
2. **Goal:** Estimate the location of a point on the coordinate plane using the signs of its coordinates.
3. **Language Goal:** Explain what the coordinate plane looks like with positive and negative numbers. **(Writing)**

Show What You Know (Assessment Resources)

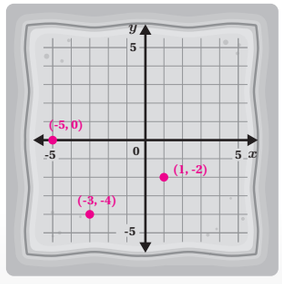
Name: _____ Date: _____ Period: _____

Show What You Know 7.16

There are sand dollars at the following locations:

(-3, -4) (1, -2) (-5, 0)

Plot and label a point to represent each location.



D Differentiation Use after Lesson 16

Support

Provide targeted intervention for students by using these resources.

If students need support writing coordinates for points on the coordinate plane:

» **Respond to Student Thinking:**

- Revisit **Activity 3** and invite students to share strategies for graphing points.
- Select a few related **Practice Problems** and give individualized feedback.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite them to explore the **Sub-Unit 3 Extension Activities**.
- Invite students to complete the **You're Invited to Explore More** task and discuss their thinking with a partner.

Professional Learning

Why was the context of sand dollar search in this lesson particularly useful in building student understanding of negative numbers on the coordinate plane?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

11. Explain what you know about the coordinates of this sand dollar.

Responses vary.

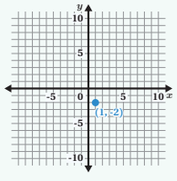
- The x-coordinate is negative.
- The y-coordinate is positive.
- If the scale is the same as on the other island maps, then the coordinates might be close to (-6, 2).
- It's located in Quadrant 2.

Lesson Practice ACC6.7.16

Lesson Summary

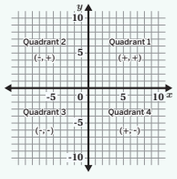
You can include positive and negative numbers along the x- and y-*axes*, just like on a number line. The x- and y-axes cross at the *origin*, or the point **(0, 0)**.

Ordered pairs are written (x, y), where the x-value is the horizontal location (left and right) and the y-value is the vertical location (up and down). For example, the point **(1, -2)** is 1 unit to the right and 2 units down from the origin.



The four regions of the *coordinate plane* are called **quadrants**. They are numbered 1–4 starting with the top right quadrant and going in a circle counter-clockwise.

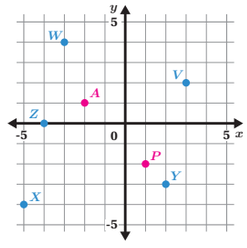
The image shows each quadrant, along with the sign of the x- and y-values in that quadrant.



Lesson Practice
ACC6.7.16

Name: _____ Date: _____ Period: _____

Problems 1–5: Here is a coordinate plane.



1. Write the coordinates of each point in the table.

Point	Coordinates
V	(3, 2)
W	(-3, 4)
X	(-5, -4)
Y	(2, -3)
Z	(-4, 0)

2. Plot and label point A at (-2, 1).

3. Plot and label point P at (1, -2).

4. Plot two points that are each 2 units away from point V. Label each point with its coordinates.

Responses vary.

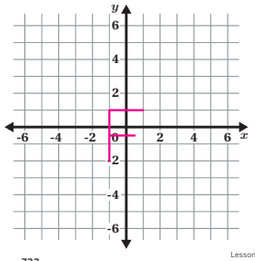
5. Point Q is more than 3 units directly to the left of point Y. Write at least one thing you know about the coordinates of point Q.

Responses vary.

- Both coordinates are negative.
- The y-coordinate will be -3.

6. Draw the first letter of your name on the coordinate plane so that at least part of your drawing is in each quadrant.

Responses vary. Sample shown on the graph.



Lesson Practice
ACC6.7.16

Name: _____ Date: _____ Period: _____

FAST Practice

7. Select *all* the possible locations on the coordinate plane of the point $(x, -2)$, where $x \leq 0$.

- ☐ A. Quadrant 2
- ☑ B. Quadrant 3
- ☐ C. Quadrant 4
- ☐ D. x-axis
- ☑ E. y-axis

Spiral Review

8. The height requirement for an amusement park ride is written as $h > 42$, where h represents a rider's height in inches. Write a sentence or sketch a sign that describes these rules as clearly as possible.

Responses vary. Anyone over 42 inches tall can ride!

9. Select *all* the values of x that are solutions to the inequality $x > -2$.

- ☑ A. -1
- ☐ B. -2
- ☐ C. -3
- ☐ D. -2.1
- ☑ E. -1.8

Problems 10–13: Solve each equation.

10. $\frac{3}{5}a = \frac{12}{5}$

$a = 4$

11. $b + 8.3 = 8.9$

$b = 0.6$

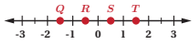
12. $1 = \frac{1}{4}c$

$c = 4$

13. $2d = 6.4$

$d = 3.2$

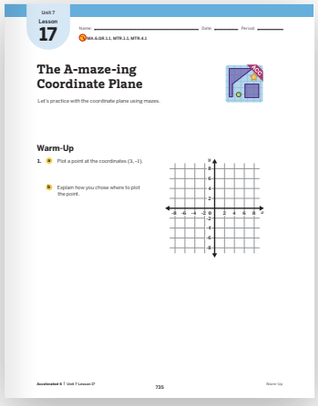
14. Here is a number line with points Q, R, S, and T. Which point represents $-1\frac{1}{2}$?



Point Q

Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1–3	1	MA.6.GR.1.1
	4, 6	2	MA.6.GR.1.1
	5	2	MA.6.GR.1.1
FAST Practice	7	2	MA.6.GR.1.1
Spiral Review			
	8	2	MA.6.AR.1.2
	9	1	MA.6.AR.2.1
	10–13	1	MA.6.AR.2.4
	14	1	MA.6.NSO.1.1



The A-maze-ing Coordinate Plane

Practice Plotting

Let's practice with the coordinate plane using mazes.

Learning Outcomes

Today's Goals

1. **Goal:** Plot points with positive and negative coordinates on grids with different scales.
2. **Language Goal:** Describe the relationship between ordered pairs that differ only by the signs of the coordinates. **(Writing, Speaking, and Listening)**

Students practice plotting points in all four quadrants of the coordinate plane within the context of solving mazes. They are introduced to grids that use different scales for the axes and use structure to identify patterns in coordinates that differ only by the sign. Students should gain confidence in identifying coordinates with positive and negative numbers and be able to critique the reasoning of others' claims about the location of points when reflected across one or both axes. **(MTR.1.1)**

Coherence

Prior Learning

In Lesson 16, students extended their previous understanding of the coordinate plane to include negative numbers. They wrote coordinates for points in all four quadrants and estimated the location of a point based on the signs of its coordinates.

Future Learning

In Lessons 18–19, students will use all four quadrants in the coordinate plane to make sense of situations in context. In a future course, students will engage more deeply with rigid transformations, including what happens when figures are reflected on a grid.

Connections

- Students build on their understanding of the coordinate plane using context situations. (mathematical, real world)

FL B.E.S.T. Math Benchmarks

Addressing

MA.6.GR.1.1

Extend previous understanding of the coordinate plane to plot rational number ordered pairs in all four quadrants and on both axes. Identify the x-or y-axis as the line of reflection when two ordered pairs have an opposite x-or y-coordinate.

Mathematical Thinking and Reasoning Standards: MTR.1.1, MTR.4.1

ELA Expectations: ELA.K12.EE.1.1, ELA.K12.EE.5.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.5.GR.4.1	MA.7.AR.4.3
	MA.8.GR.2.3

Vocabulary

Additional Vocabulary

reflection

Lesson at a Glance

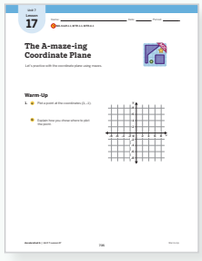
~ 45 min

MA.6.GR.1.1

Warm-Up

Pairs | 5 min

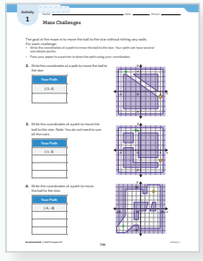
Students share what they know about plotting points with positive and negative numbers and are introduced to a grid not scaled by 1.



Activity 1

Pairs | 10 min

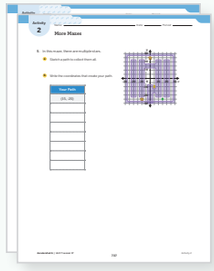
Students familiarize themselves with the maze interface and use structure to identify how to determine coordinates in axes with different scales.



Activity 2

Pairs | 10 min

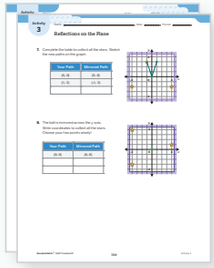
Students use the **Decide and Defend** routine to deepen their understanding of points on coordinate planes with different scaled axes. **(MTR.1.1)**



Activity 3

Pairs | 10 min

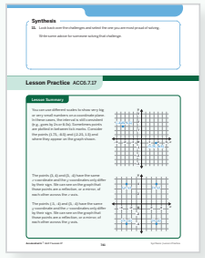
Students use reflections to solve more challenges and construct an argument about the structure of points that are reflections over an axis. **(MTR.1.1)**



Synthesis

Whole Class | 5 min

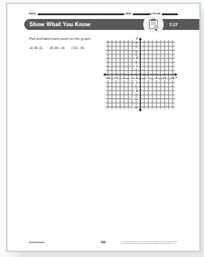
Students synthesize their understanding of strategies for plotting points on the coordinate plane with different scales and points that are reflected. **MLR2: Collect and Display**



Show What You Know

Independent | 5 min

Students demonstrate their understanding by plotting and labeling points at given locations on a grid.



Prep Checklist

This lesson includes:

- Student Edition
- Show What You Know
- Rulers

Pairs | 5 min

Warm-Up

Purpose: Students share what they know about plotting points with positive and negative numbers and are introduced to a grid not scaled by 1.

1 Launch

Direct students' attention to the coordinate plane and invite students to share what they notice. Invite students to look specifically at the x-axis and y-axis.

To surface prior knowledge, consider asking, "What about this coordinate plane looks similar to others you have looked at previously? What looks different?"

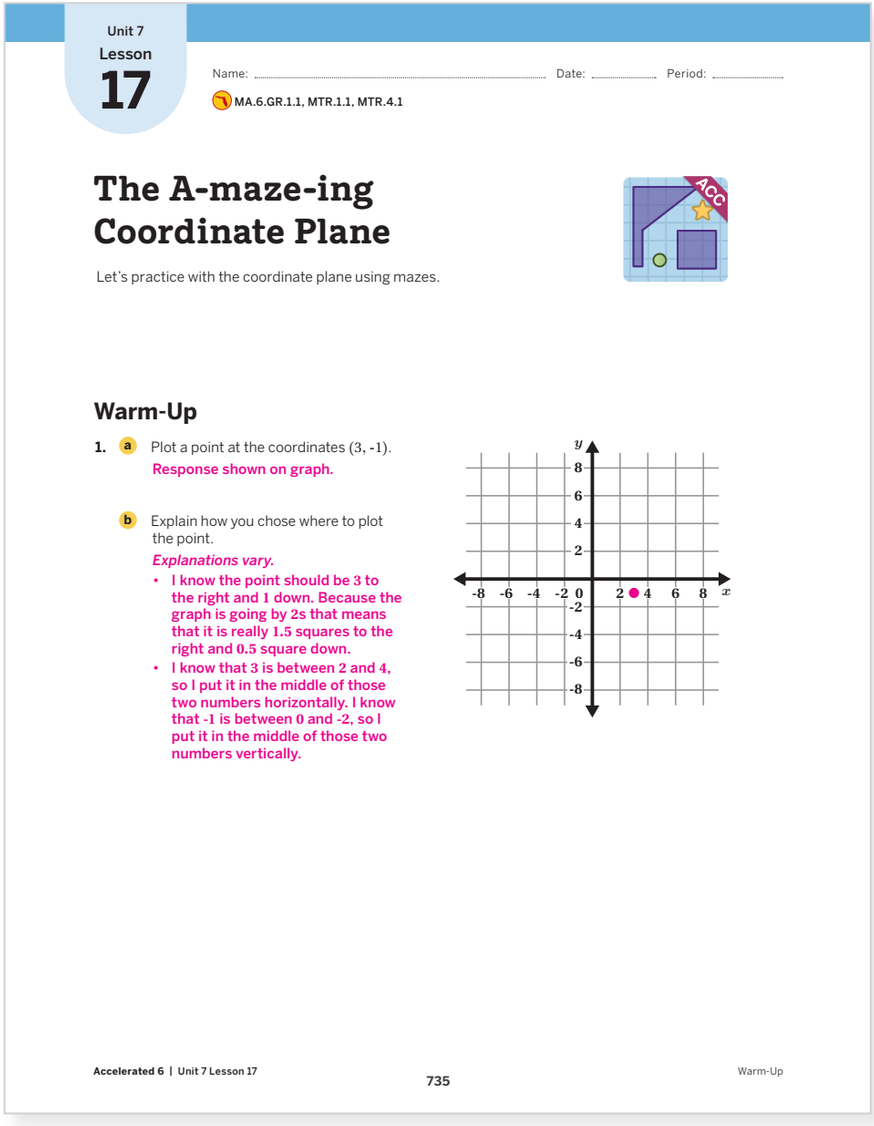
Invite students to plot a point at $(3, -1)$ and compare their responses with a few of their classmates.

2 Connect

Listen for and share a few student explanations, highlighting students who reference the origin and discuss how it is possible to locate 3 and -1 when they aren't labeled on x-axis and y-axis. **(MTR.4.1)**

Consider asking:

- "Is it ok that the point is not located on the grid lines? Why or why not?" Yes. *Responses vary.* Points that are not on grid lines have x - and y- values that are between the values marked on the scale of the axes.
- "Are there other points that exist that would not be on grid lines? How many and where?"



Unit 7
Lesson
17

Name: ..... Date: ..... Period: .....

MA.6.GR.1.1, MTR.1.1, MTR.4.1

The A-maze-ing Coordinate Plane

Let's practice with the coordinate plane using mazes.

Warm-Up

1. a Plot a point at the coordinates $(3, -1)$.
   Response shown on graph.

   b Explain how you chose where to plot the point.
   Explanations vary.
   - **I know the point should be 3 to the right and 1 down. Because the graph is going by 2s that means that it is really 1.5 squares to the right and 0.5 square down.**
   - **I know that 3 is between 2 and 4, so I put it in the middle of those two numbers horizontally. I know that -1 is between 0 and -2, so I put it in the middle of those two numbers vertically.**

Accelerated 6 | Unit 7 Lesson 17 735 Warm-Up

Pairs | 10 min

Activity 1 Maze Challenges

Purpose: Students familiarize themselves with the maze interface and use structure to identify how to determine coordinates in axes with different scales.

Short on time? Consider omitting Problem 3 or Problem 4.

1 Launch

To encourage playfulness, invite students to share any experiences they have had with mazes.

Invite students to use a ruler to draw the path of the ball on their maze.

A **Accessibility: Memory and Attention** Invite students to refer back to the anchor chart from Lesson 16 and make connections between the points on the table and the image.

2 Monitor

Invite students to use strategies they heard from their peers about the previous challenge to begin.

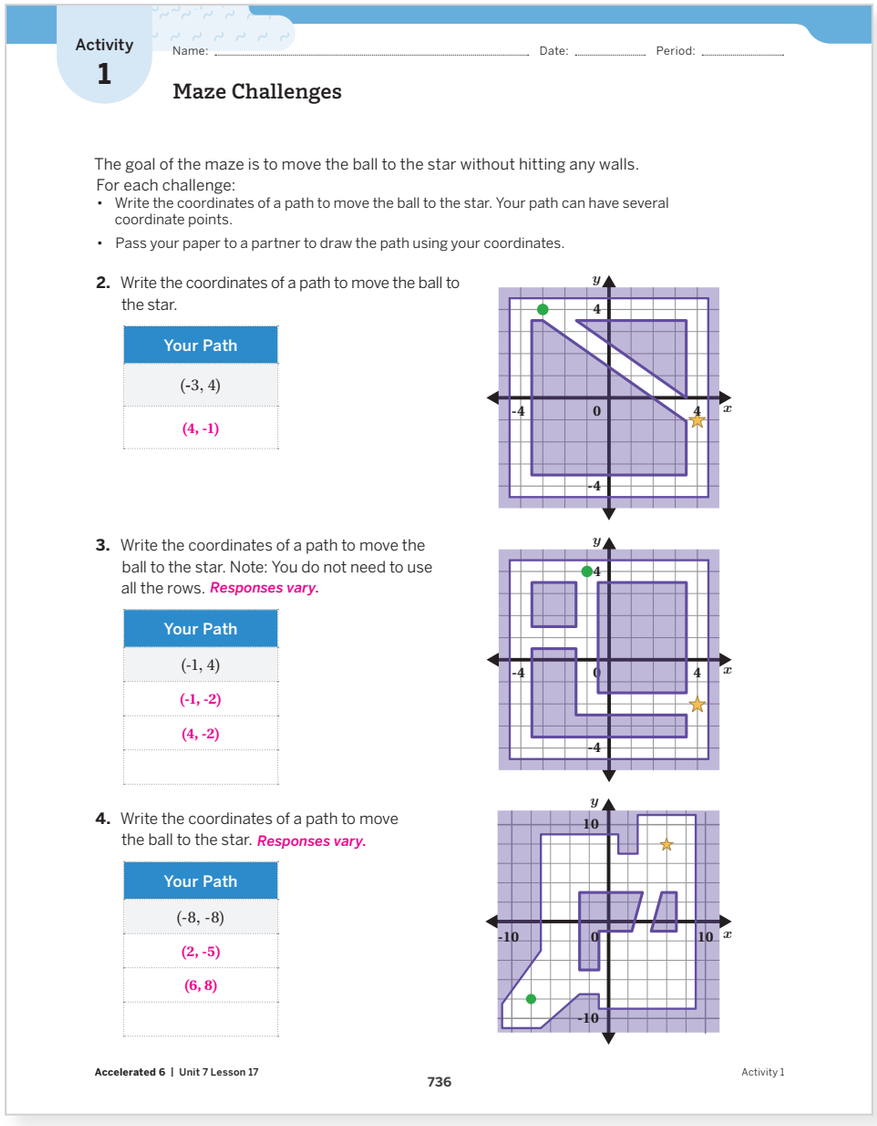
D **Differentiation**

Look for students who:	Respond to Student Thinking
Count by 1s between grid lines.	**Support** Demonstrate with a number line that uses increments of 2. Consider asking, "If each tick mark represents 2 units, what would be the first coordinate of a point located halfway between the horizontal tick marks representing 6 and 8?"
Use the numbered grid lines correctly but aren't yet estimating the numbers in between the markings.	**Strengthen** Consider asking, "What is similar and different between the two grids?" Invite students to annotate the numbers on each line.
Are precise with their coordinates and succeed in moving the ball to the star. **(MTR.1.1)**	**Stretch** Consider asking, "What would it mean if a point is placed on a plane where two gridlines don't meet? What would it represent? Could a point be placed anywhere?"

3 Connect

Invite students to share their responses and predict if it will make it to the star. Highlight correct responses and support students with revisions if needed.

Consider asking, "What strategy was most helpful to you?"



Activity 1

Name: _____ Date: _____ Period: _____

Maze Challenges

The goal of the maze is to move the ball to the star without hitting any walls.
For each challenge:

- Write the coordinates of a path to move the ball to the star. Your path can have several coordinate points.
- Pass your paper to a partner to draw the path using your coordinates.

2. Write the coordinates of a path to move the ball to the star.

Your Path
(-3, 4)
(4, -1)

3. Write the coordinates of a path to move the ball to the star. Note: You do not need to use all the rows. *Responses vary.*

Your Path
(-1, 4)
(-1, -2)
(4, -2)

4. Write the coordinates of a path to move the ball to the star. *Responses vary.*

Your Path
(-8, -8)
(2, -5)
(6, 8)

Accelerated 6 | Unit 7 Lesson 17 736 Activity 1

Pairs | 10 min

Activity 2 More Mazes

Purpose: Students use the **Decide and Defend** routine to deepen their understanding of points on coordinate planes with different scaled axes. **(MTR.1.1)**

1 Launch

Direct students' attention to the maze and invite students to share what they notice about the x- and y-axis.

Consider asking, "What is the distance between each grid line? How would you plot a point like $(1, 1)$?"

2 Monitor

To support students getting started, consider asking them to point at the places they want the ball to go with one finger while tracing the path with another finger.

Look for students who use the coordinates of one point to determine the coordinates of another point (e.g., the ball starts at $(15, -25)$ and we need to get to $(15, 25)$). Consider highlighting this while students are working. **(MTR.1.1)**

Activity 2 continued >

Activity 2

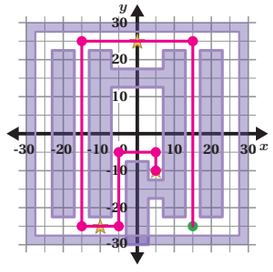
Name: Date: Period:

More Mazes

5. In this maze, there are multiple stars.

   a. Sketch a path to collect them all.
   Sketches vary.

   b. Write the coordinates that create your path.
   Responses vary.



Your Path
(15, -25)
(15, 25)
(-15, 25)
(-15, -25)
(-5, -25)
(-5, -5)
(5, -5)
(5, -10)

Accelerated 6 | Unit 7 Lesson 17 737 Activity 2

Pairs | 10 min

Activity 2 More Mazes (continued)

Purpose: Students use the Decide and Defend routine to deepen their understanding of points on coordinate planes with different scaled axes. **(MTR.1.1)**

2 Monitor

Listen for arguments for each response to share during the Connect.

D Differentiation

Look for students who:	Respond to Student Thinking
Select "Felipe" and reference the lines.	**Support** Invite them to label the missing increments on the grid.
Select "Annika" and reference the -10 on the y-axis.	**Support** Consider asking, "If this is -10, what are the values on the x-axis?"
Select "Neither."	**Stretch** Consider asking, "What was their misconception? Is there any way they could be correct?"

3 Connect

Invite students to share and discuss their responses. **(MTR.4.1)**

Use the Decide and Defend routine to support students in making arguments and critiquing the reasoning of others. **(MTR.1.1)**

Invite students who made different choices to use reasoning to convince their classmates.

Consider asking:
- "What reasoning or evidence supports your claim?"
- "What evidence or language was most convincing to you?"

Activity 2

Name: _____ Date: _____ Period: _____

More Mazes (continued)

6. Felipe and Annika were working on the previous maze.

Felipe says: *The star's coordinates are* (1, -2).

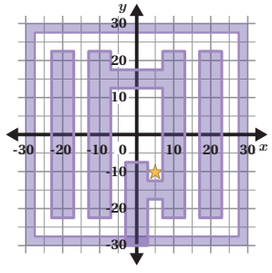
Annika says: *The coordinates are* (1, -10).

Whose claim is correct? Circle one.

Felipe's Annika's (Neither)

Explain your thinking.

Explanations vary. **They are both wrong because the graph goes by 5s. This means that 1 over and 2 down from (0, 0) is (5, -10).**



Accelerated 6 | Unit 7 Lesson 17 738 Activity 2

Key Takeaway: Sometimes axes have a scale that is not equal to 1. In these cases, the interval is still consistent (e.g., goes by 2s). It can be helpful to look at the axis scale before plotting any points.

Pairs | 10 min

Activity 3 Reflections on the Plane

Purpose: Students use reflections to solve more challenges and construct an argument about the structure of points that are reflections over an axis. **(MTR.1.1)**

1 Launch

Direct students' attention to the graph and invite students to share what they notice.

Listen for students who are surprised about there being two paths. Consider asking, "What is similar about the two paths? Can we describe what is happening using one word or phrase?" The path to (1, 3) also goes to (-1, 3). Those points are mirrors or reflections.

To surface prior knowledge, invite students to share what they think of when they hear the word "mirrored path."

2 Monitor

Consider asking, "Is there more than one solution possible here?"

D Differentiation

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Invite students to draw a line to each star and make note of how the lines might "mirror" across the axis.
Think the goal is to collect any two stars rather than all four stars with any two moves.	**Strengthen** Celebrate the stars that are collected and then invite students to read the prompt to make sure the question was answered completely.
Recognize that the mirrored line drawn from the origin to the star at (-4, -3) passes through the star at (2, -1.5).	**Stretch** Consider asking, "Are there other ways to capture all of the stars?" See sample responses for both solutions.

A Accessibility: Executive Functioning To support students initiating complex tasks, identify one of the stars to start with and invite them to write that in the "Your Path" column.

Activity 3 continued >

Activity 3

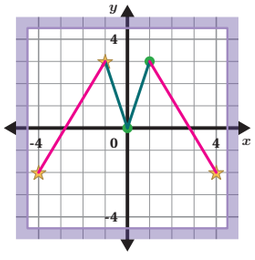
Name: _____ Date: _____ Period: _____

Reflections on the Plane

7. Complete the table to collect all the stars. Sketch the new paths on the graph.

Responses vary.

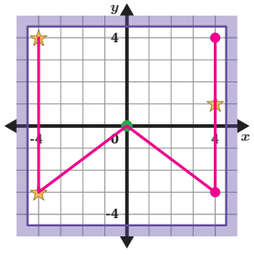
Your Path	Mirrored Path
(0, 0)	(0, 0)
(1, 3)	(-1, 3)
(4, -2)	(-4, -2)



8. The ball is mirrored across the y-axis. Write coordinates to collect all the stars. Choose your two points wisely!

Responses vary.

Your Path	Mirrored Path
(0, 0)	(0, 0)
(4, -3)	(-4, -3)
(4, 4)	(-4, 4)



Accelerated 6 | Unit 7 Lesson 17 739 Activity 3

Pairs | 10 min

Activity 3 Reflections on the Plane (continued)

Purpose: Students use reflections to solve more challenges and construct an argument about the structure of points that are reflections over an axis. **(MTR.1.1)**

2 Monitor

Listen for students who notice patterns between the reflections over each axis. Consider asking, "What is similar about the reflection across each axis? What is different?"

Note: This problem identifies a "ball and its reflection" whereas previous problems identified this as the "mirrored path." Consider making the connection between these two terms explicit.

ELL **English Language Learners** Invite students to talk through their thinking before writing it down. **(Writing and Speaking)**

D **Differentiation: You're invited to explore more. (Stretch)** Invite students who would like to explore mazes further to complete this optional task. Encourage them to discuss their thinking with a partner. **(MTR.4.1)**

3 Connect

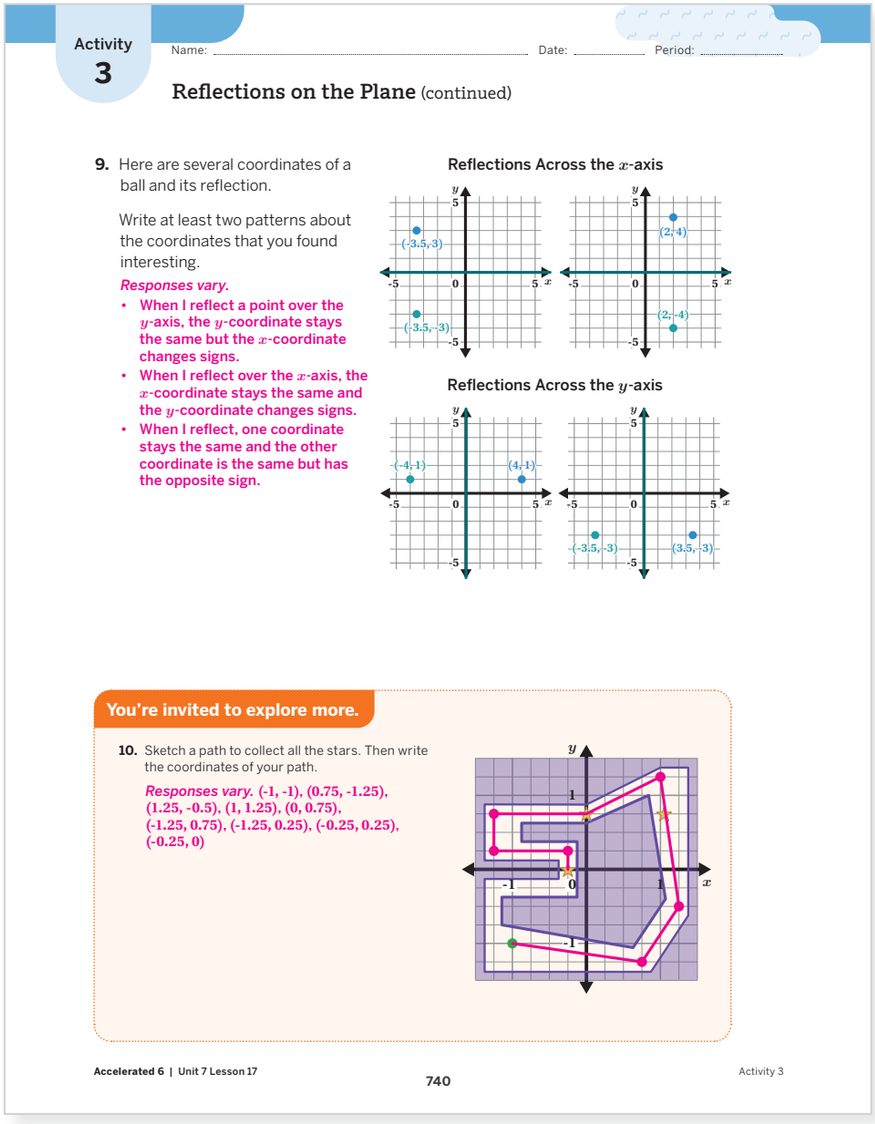
Invite students to test patterns other students found and then use reasoning to justify whether or not the pattern would always make sense. **(MTR.1.1)**

Consider asking:

- "What did you notice about the coordinates when you reflected over one of the axes? Why does this pattern make sense?"
- "How are these patterns related to what we know about opposites?"

To surface the Key Takeaway, invite students to work together with a partner to express the pattern as concisely as possible. **(MTR.1.1, MTR.4.1)**

Note: Reflection will be formally defined in a future course.



Activity 3

Name: _____ Date: _____ Period: _____

Reflections on the Plane (continued)

9. Here are several coordinates of a ball and its reflection.

Write at least two patterns about the coordinates that you found interesting.

Responses vary.

- When I reflect a point over the y-axis, the y-coordinate stays the same but the x-coordinate changes signs.
- When I reflect over the x-axis, the x-coordinate stays the same and the y-coordinate changes signs.
- When I reflect, one coordinate stays the same and the other coordinate is the same but has the opposite sign.

Reflections Across the x-axis

Reflections Across the y-axis

You're invited to explore more.

10. Sketch a path to collect all the stars. Then write the coordinates of your path.

Responses vary. (-1, -1), (0.75, -1.25), (1.25, -0.5), (1, 1.25), (0, 0.75), (-1.25, 0.75), (-1.25, 0.25), (-0.25, 0.25), (-0.25, 0)

Accelerated 6 | Unit 7 Lesson 17 740 Activity 3

Key Takeaway: When a point is reflected over the y-axis, the y-coordinate stays the same, but the x-coordinate has the opposite sign. When a point is reflected over the x-axis, the x-coordinate stays the same, but the y-coordinate has the opposite sign.

Whole Class | 5 min

Synthesis

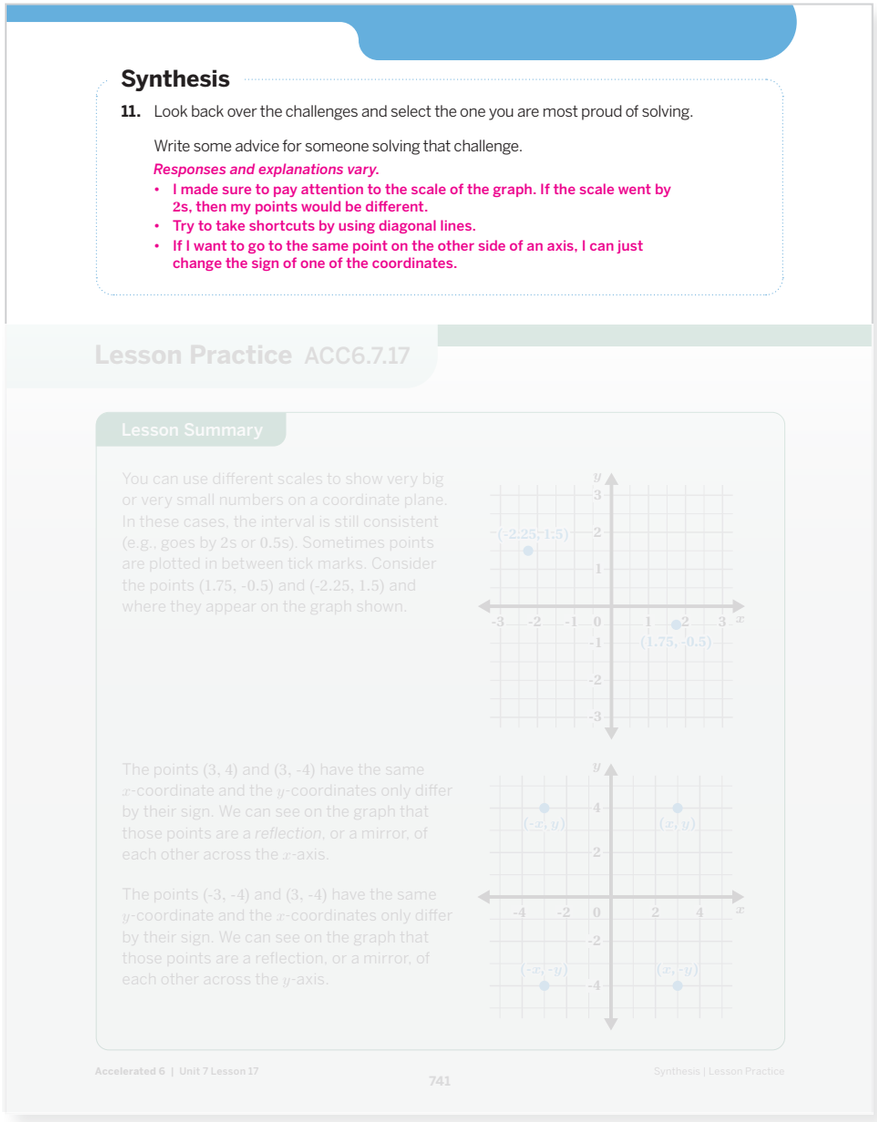
Purpose: Students synthesize their understanding of strategies for plotting points on the coordinate plane with different scales and points that are reflected.

Invite students to review the challenges from this lesson and then respond independently before sharing their thinking with a partner.

Collect and share a variety of ideas, such as:

- Make sure to pay attention to the scale of the graph. If the scale went by 2s, then the points would be different.
- Try to take shortcuts by using diagonal lines.
- To go to the same point on the other side of the axis, you can just change the sign of one of the coordinates.

MLR **MLR2: Collect and Display** Use this routine to formalize strategies for plotting points on the coordinate plane with different scales using students' language. Display any new language, examples, or diagrams publicly for students to reference later.



Synthesis

11. Look back over the challenges and select the one you are most proud of solving.

Write some advice for someone solving that challenge.

Responses and explanations vary.

- I made sure to pay attention to the scale of the graph. If the scale went by 2s, then my points would be different.
- Try to take shortcuts by using diagonal lines.
- If I want to go to the same point on the other side of an axis, I can just change the sign of one of the coordinates.

Lesson Practice ACC6.7.17

Lesson Summary

You can use different scales to show very big or very small numbers on a coordinate plane. In these cases, the interval is still consistent (e.g., goes by 2s or 0.5s). Sometimes points are plotted in between tick marks. Consider the points (1.75, -0.5) and (-2.25, 1.5) and where they appear on the graph shown.

The points (3, 4) and (3, -4) have the same x-coordinate and the y-coordinates only differ by their sign. We can see on the graph that those points are a *reflection*, or a mirror, of each other across the x-axis.

The points (-3, -4) and (3, -4) have the same y-coordinate and the x-coordinates only differ by their sign. We can see on the graph that those points are a reflection, or a mirror, of each other across the y-axis.

Accelerated 6 | Unit 7 Lesson 17 741 Synthesis | Lesson Practice

Lesson Takeaway: Here are two things to pay attention to when graphing points on the coordinate plane.

- The scale of the axes may not be 1.
- Points reflected over the x-axis will have the same x-coordinate and the y- coordinate changes signs. Points reflected over the y-axis will have the same y-coordinate but the x-coordinate changes signs.

Independent | 5 min

Show What You Know

Purpose: Students demonstrate their understanding by plotting and labeling points at given locations on a grid.

Today's Goals

1. **Goal:** Plot points with positive and negative coordinates on grids with different scales.
2. **Language Goal:** Describe the relationship between ordered pairs that differ only by the signs of the coordinates. **(Writing, Speaking, and Listening)**

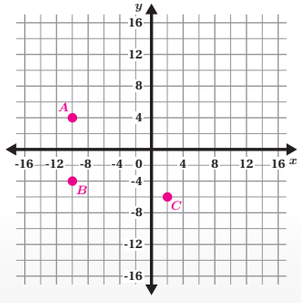
Show What You Know (Assessment Resources)

Name: _____ Date: _____ Period: _____

Show What You Know 7.17

Plot and label each point on the graph.

$A(-10, 4)$ $B(-10, -4)$ $C(2, -6)$



D Differentiation Use after Lesson 17

Support

Provide targeted intervention for students by using these resources.

If students need support with plotting points:

» **Respond to Student Thinking:**

- Use the ***Plotting Points on the Coordinate Plane*** **Mini-Lesson**.
- Review the **Lesson Summary**.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite them to explore the **Sub-Unit 3 Extension Activities**.
- Invite students to complete the **You're Invited to Explore More** task and discuss their thinking with a partner.

Professional Learning

In what ways did students' work in Activity 3 surprise you today?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

11. Look back over the challenges and select the one you are most proud of solving.

Write some advice for someone solving that challenge.

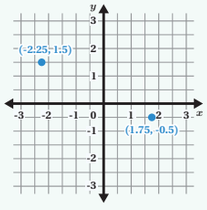
Responses and explanations vary.

- I made sure to pay attention to the scale of the graph. If the scale went by 2s, then my points would be different.
- Try to take shortcuts by using diagonal lines.
- If I want to go to the same point on the other side of an axis, I can just change the sign of one of the coordinates.

Lesson Practice ACC6.7.17

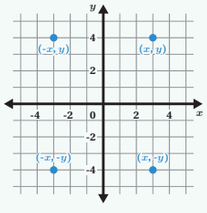
Lesson Summary

You can use different scales to show very big or very small numbers on a coordinate plane. In these cases, the interval is still consistent (e.g., goes by 2s or 0.5s). Sometimes points are plotted in between tick marks. Consider the points (1.75, -0.5) and (-2.25, 1.5) and where they appear on the graph shown.



The points (3, 4) and (3, -4) have the same x-coordinate and the y-coordinates only differ by their sign. We can see on the graph that those points are a *reflection*, or a mirror, of each other across the x-axis.

The points (-3, -4) and (3, -4) have the same y-coordinate and the x-coordinates only differ by their sign. We can see on the graph that those points are a reflection, or a mirror, of each other across the y-axis.



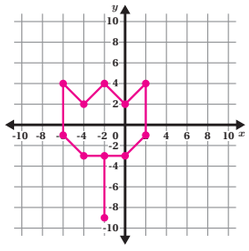
Lesson Practice ACC6.7.17

Name: _____ Date: _____ Period: _____

Problems 1–2: Here is a graph.

1. Plot and connect each point in order. Plot all of Column 1 first, then Column 2.

Column 1	Column 2
(-2, -9)	(-2, 4)
(-2, -3)	(0, 2)
(-4, -3)	(2, 4)
(-6, -1)	(2, -1)
(-6, 4)	(0, -3)
(-4, 2)	(-2, -3)



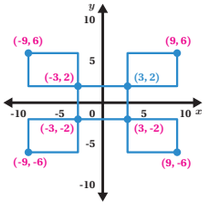
2. Describe your strategy for plotting the point (-2, -9) in Problem 1.

Responses vary. For the x-coordinate, I counted 2 units to the left of 0. For the y-coordinate, I knew that -9 is halfway between -10 and -8.

Problems 3–4: Here are identical rectangles on a coordinate plane. The origin is in the center of the middle rectangle.

3. Label the coordinates of the remaining points.

Response shown on graph.



4. What patterns do you notice?

Responses vary. The points that are mirror images of each other have similar numbers. For example, (3, 2) and (3, -2) are on opposite sides of the x-axis.

Lesson Practice ACC6.7.17

Name: _____ Date: _____ Period: _____

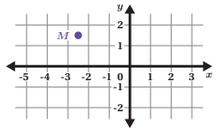
5. Point A is located at $(3, n)$ in the coordinate plane. Point B is located at $(-3, n)$. What do you know about points A and B?

Responses vary. The points are mirror images of each other. One point is on the left side of the coordinate plane and one is on the right side of the coordinate plane.

FAST Practice

6. What is the value of the x-coordinate of point M? Your answer should be a decimal rounded to the nearest 0.5.

-2.5



Spiral Review

Problems 7–10: Complete each number sentence with the symbol <, >, or =. Use a number line if it helps with your thinking.

7. $\left|-\frac{3}{2}\right| > -\frac{2}{3}$

8. $-\frac{3}{2} < -\frac{2}{3}$

9. $-\frac{3}{2} < \frac{2}{3}$

10. $\frac{3}{2} > \left|\frac{2}{3}\right|$

Problems 11–12: DeShawn's dog weighs 34 pounds. Jacy's dog weighs 12 pounds more than DeShawn's dog.

11. Select *all* the equations that show the weight of Jacy's dog, j.

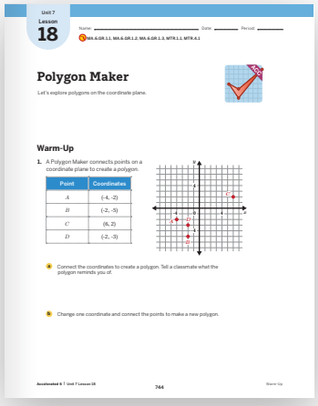
- ☑ A. $j = 34 + 12$
- ☐ B. $j = 34 - 12$
- ☐ C. $j + 12 = 34$
- ☑ D. $j - 12 = 34$
- ☐ E. $j = 34 \cdot 12$

12. Determine how much Jacy's dog weighs.

46 pounds

Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1, 3	1	MA.6.GR.1.1
	2, 5	2	MA.6.GR.1.1
	4	3	MA.6.GR.1.1
FAST Practice	6	1	MA.6.GR.1.1
Spiral Review			
	7–10	1	MA.6.NSO.1.1, MA.6.NSO.1.4
	11	2	MA.6.AR.2.2
	12	1	MA.6.AR.2.2


Unit 7
Lesson
18

Polygon Maker

Let's explore polygons on the coordinate plane.

Warm-Up

1. A Polygon Maker connects points on a coordinate plane to create a polygon.

Point	Coordinates
A	$(-4, -2)$
B	$(-2, -5)$
C	$(6, 2)$
D	$(-2, -3)$

Connect the coordinates to create a polygon. Tell a classmate what the polygon reminds you of.

Change one coordinate and connect the points to make a new polygon.

744

Polygon Maker

Polygons in the Plane

Let's explore polygons on the coordinate plane.

Learning Outcomes

Today's Goals

1. **Goal:** Draw a polygon on the coordinate plane given the coordinates for its vertices.
2. **Goal:** Determine lengths and distances between points on the coordinate plane that are vertically or horizontally aligned.
3. **Language Goal:** Justify whether a given set of coordinates create a square or rectangle on the coordinate plane. **(Writing, Speaking, and Listening)**

Students apply their understanding of positive and negative coordinates to explore *polygons* on the coordinate plane. They make use of structure on a graph and in tables to calculate the distance between two points and determine if a given set of coordinates make a described polygon. Students create their own polygon and challenge their classmates to recreate it. **(MTR.1.1)**

Coherence

Prior Learning

In Lesson 16, students plotted points on the coordinate plane, extending their understanding to include positive and negative coordinates.

Future Learning

In Lesson 19, students will use all four quadrants of the coordinate plane to make sense of changes in temperature at different elevations.

Connections

- Students draw on their understanding of geometric figures as they plot various figures in the four-quadrant coordinate plane. (mathematical)

FL B.E.S.T. Math Benchmarks

Addressing

MA.6.GR.1.2

Find distances between ordered pairs, limited to the same x-coordinate or the same y-coordinate, represented on the coordinate plane.

MA.6.GR.1.3

Solve mathematical and real-world problems by plotting points on a coordinate plane, including finding the perimeter or area of a rectangle.

Also Addressing: **MA.6.GR.1.1**

Mathematical Thinking and Reasoning Standards: MTR.1.1, MTR.4.1

ELA Expectations: ELA.K12.EE.5.1, ELA.K12.EE.6.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.3.GR.2.3	**MA.7.GR.1.1**
MA.5.GR.4.1	**MA.8.GR.1.2**
MA.5.GR.4.2	

Vocabulary

Review Vocabulary

polygon

Lesson at a Glance

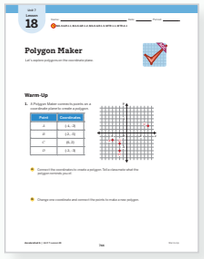
~ 45 min

MA.6.GR.1.1, MA.6.GR.1.2, MA.6.GR.1.3

Warm-Up

Pairs | 5 min

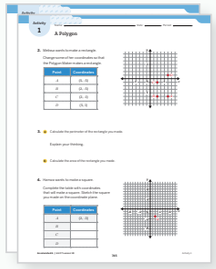
Students are introduced to a Polygon Maker which creates *polygons* from a set of coordinates. They use their creativity to create a unique quadrilateral by changing coordinates.



Activity 1

Pairs | 15 min

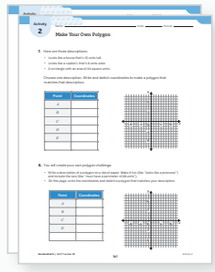
Students use coordinates to create polygons with specific requirements and surface strategies to calculate the side lengths of a polygon.
MLR1: Stronger and Clearer Each Time



Activity 2

Pairs | 15 min

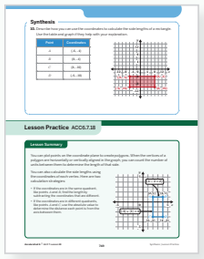
Students create their own polygon to practice using coordinates that match specific descriptions before creating their own challenge and solving classmates' challenges.



Synthesis

Whole Class | 5 min

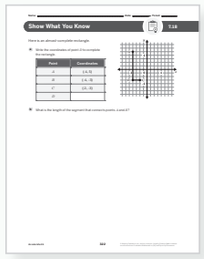
Students synthesize their understanding of strategies for calculating vertical and horizontal side lengths of polygons on the coordinate plane.
MLR2: Collect and Display



Show What You Know

Independent | 5 min

Students demonstrate their understanding by identifying coordinates that complete a rectangle and finding the length of a given segment using a table or graph.



Prep Checklist

This lesson includes:

- Student Edition
- Show What You Know
- Rulers

Pairs | 5 min

Warm-Up

Purpose: Students are introduced to a Polygon Maker which creates *polygons* from a set of coordinates. They use their creativity to create a unique quadrilateral by changing coordinates.

1 Launch

To surface connections to Unit 1, consider asking, "What do you think of when you hear the word *polygon*?"

Invite students to share examples of polygons and the definition in their own words. Connect to the shared definition from Unit 1, a polygon is a closed 2-D shape with straight sides that do not cross each other.

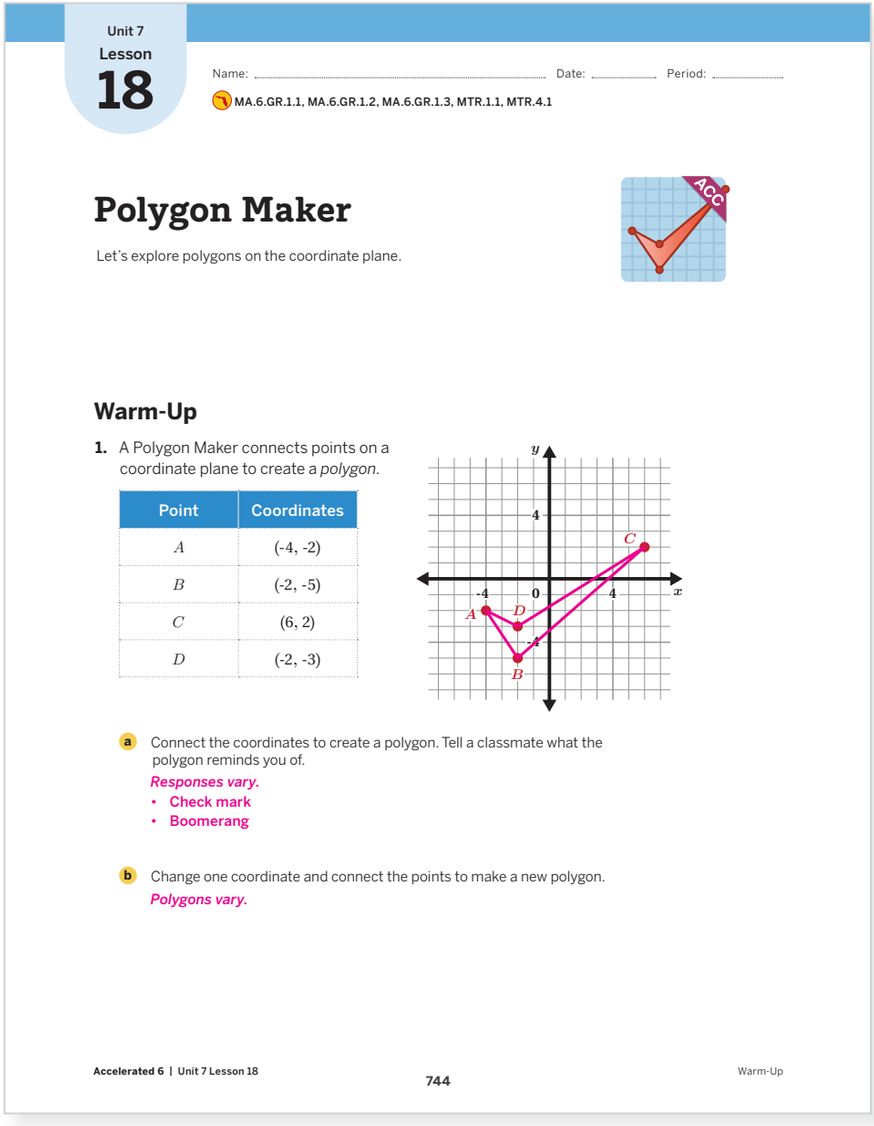
Note: Provide access to rulers to support students in creating polygons on the coordinate plane throughout this lesson.

D **Differentiation:** (Stretch) Encourage students to make a quadrilateral and then challenge a classmate to determine the coordinates that make the quadrilateral they sketched.

2 Connect

Invite two to three student to share their modifications to the polygon. Consider asking, "What does this polygon remind you of? What coordinates did they use to create it?"

Consider asking, "How would you describe what a Polygon Maker does?" It connects coordinates in the order they are written to create polygons in the coordinate plane.



Unit 7
Lesson
18

Name: ..... Date: ..... Period: .....

MA.6.GR.1.1, MA.6.GR.1.2, MA.6.GR.1.3, MTR.1.1, MTR.4.1

Polygon Maker

Let's explore polygons on the coordinate plane.

Warm-Up

1. A Polygon Maker connects points on a coordinate plane to create a *polygon*.

Point	Coordinates
A	(-4, -2)
B	(-2, -5)
C	(6, 2)
D	(-2, -3)

a. Connect the coordinates to create a polygon. Tell a classmate what the polygon reminds you of.

Responses vary.
- Check mark
- Boomerang

b. Change one coordinate and connect the points to make a new polygon.

Polygons vary.

Accelerated 6 | Unit 7 Lesson 18 744 Warm-Up

Pairs | 15 min

Activity 1 A Polygon

Purpose: Students use coordinates to create polygons with specific requirements and surface strategies to calculate the side lengths of a polygon.

1 Launch

Consider asking:

- "How do you know this is not a rectangle?"
- "How would you know if you had created a rectangle?"

ELL **English Language Learners** Invite students to share what they picture when they hear the word *rectangle* (e.g., "When I hear rectangle, I picture . . ."). This may help surface multiple pictures and encourage students to connect to their prior knowledge. **(Reading and Listening)**

2 Monitor

To support students getting started, invite them to identify the point they want to move and where they would like to move the point to. Support them in sense-making by connecting the words they use to the numeric values of the point. **(MTR.1.1)**

Look for students with different responses and invite them to share their thoughts with one another. Consider asking, "Could you make it a rectangle by changing one point? What if you changed two points? Three points?"

Invite students to remind one another what is meant by the terms *perimeter* and *area* and encourage them to support one another in strategies for calculating the perimeter and area of their rectangle. **(MTR.1.1)**

Look for a variety of strategies for determining the perimeter and area including:

- Annotating their image.
- For perimeter, calculating the length of two sides and doubling them.
- Counting the length of each side.

To support getting started, consider asking, "What are some qualities of a square?"

A **Accessibility: Visual-Spatial Processing** Encourage students to draw their square before determining the unknown coordinates.

Listen for students using different side lengths from the person next to them and invite them to share their coordinates and look for what is alike and what is different. **(MTR.1.1)**

Activity 1 continued >

Activity 1

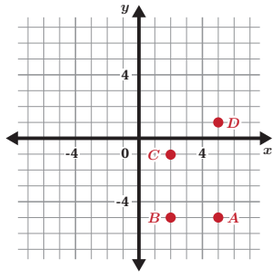
Name: _____ Date: _____ Period: _____

A Polygon

2. Melissa wants to make a rectangle.

Change some of her coordinates so that the Polygon Maker makes a rectangle.

Point	Coordinates
A	(5, -5)
B	(2, -5)
C	(2, -1)
D	(5, 1)



Responses vary.

- Change C from (2, -1) to (2, 1).
- Change D from (5, 1) to (5, -1).

3. a Calculate the perimeter of the rectangle you made.

Responses vary. 18 units or 14 units.

Explain your thinking.

Explanations vary. For the horizontal sides, going from 2 to 5 is 3 units. For the vertical sides, it is 4 units from -5 to -1. This makes the perimeter 14 units.

b Calculate the area of the rectangle you made.

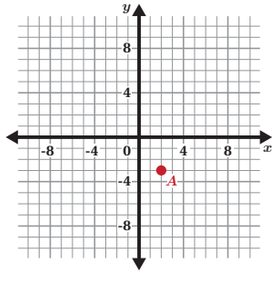
Responses vary. 18 square units or 12 square units

4. Hamza wants to make a square.

Complete the table with coordinates that will make a square. Sketch the square you made on the coordinate plane.

Responses vary.

Point	Coordinates
A	(2, -3)
B	(-2, -9)
C	(-4, -9)
D	(-4, -3)



Accelerated 6 | Unit 7 Lesson 18 745 Activity 1

Pairs | 15 min

Activity 1 A Polygon (continued)

Purpose: Students use coordinates to create polygons with specific requirements and surface strategies to calculate the side lengths of a polygon.

2 Monitor

Select and sequence descriptions and sketches of Vicente's square to share during the Connect.

D Differentiation

Look for students who:	Respond to Student Thinking
Need support getting started or think that Vicente did not make a square.	**Support** Encourage them to plot coordinates on the graph first, and then consider if what they drew looks like a square.
Reference the graph and count to find the length of the sides.	**Strengthen** Consider asking, "How does the graph connect with what is in the table? What initial conjectures can we make? Could you determine the lengths *without* the graph?"
Explain that the distance between the x-coordinates and the distance between the y-coordinates are the same when their corresponding y- and x- coordinates are equivalent.	**Stretch** Consider asking, "How could you use this strategy to change Vicente's coordinates to create a 10-by-10 square?"

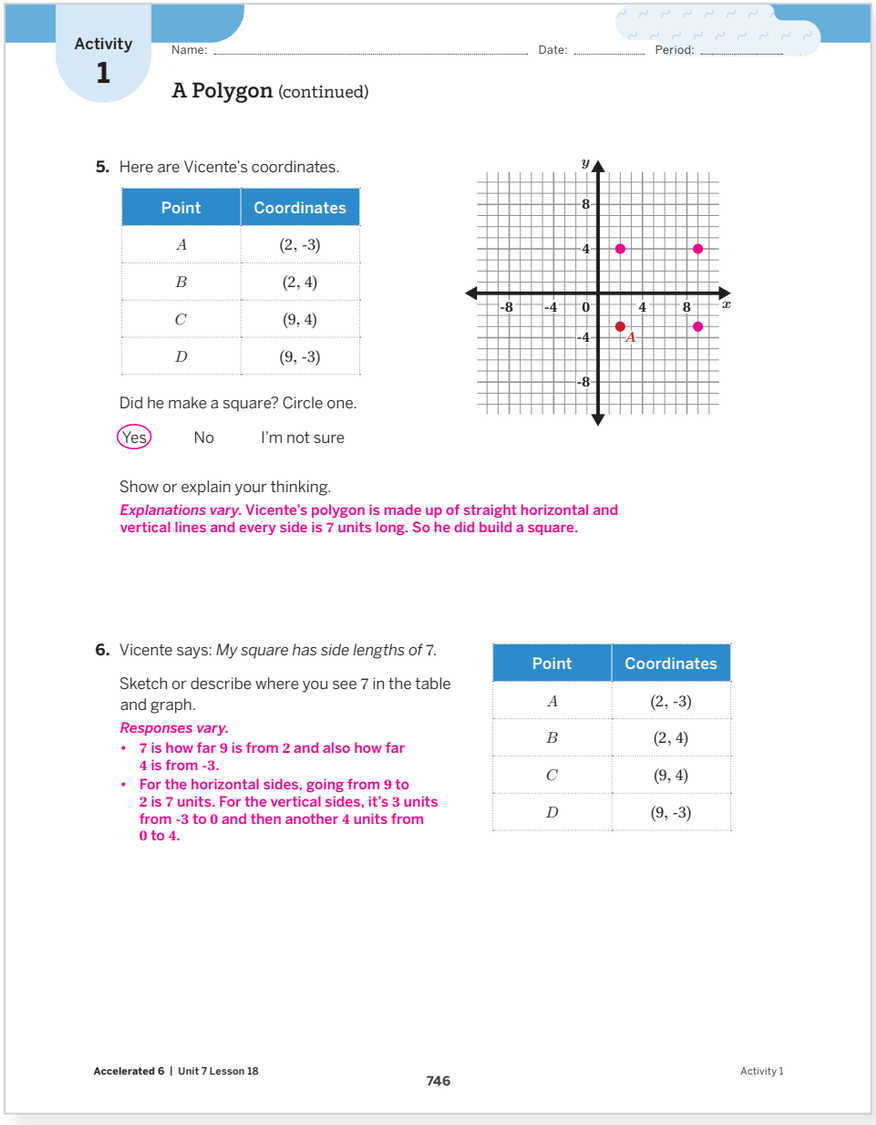
Look for student work to share during the Connect.

3 Connect

Display the distribution of student responses. Invite students to share their explanation, inviting other students to restate their thinking, and ask if they agree or disagree. **(MTR.1.1, MTR.4.1)**

MLR **MLR1: Stronger and Clearer Each Time**
Consider using this routine to support students in developing precise language. Invite students to meet with a partner, give feedback on each other's drafts, then use the feedback to write a second clearer draft. See the Routine Facilitation Guide for more information. **(MTR.1.1)**

To surface the Key Takeaway, consider asking, "What operation did we use to describe the distance from 0? How could you use absolute value to help determine the distance between two points?"



Activity 1

Name: _____ Date: _____ Period: _____

A Polygon (continued)

5. Here are Vicente's coordinates.

Point	Coordinates
A	(2, -3)
B	(2, 4)
C	(9, 4)
D	(9, -3)

Did he make a square? Circle one.

(Yes) No I'm not sure

Show or explain your thinking.

Explanations vary. Vicente's polygon is made up of straight horizontal and vertical lines and every side is 7 units long. So he did build a square.

6. Vicente says: *My square has side lengths of 7.*

Sketch or describe where you see 7 in the table and graph.

Responses vary.

- 7 is how far 9 is from 2 and also how far 4 is from -3.
- For the horizontal sides, going from 9 to 2 is 7 units. For the vertical sides, it's 3 units from -3 to 0 and then another 4 units from 0 to 4.

Point	Coordinates
A	(2, -3)
B	(2, 4)
C	(9, 4)
D	(9, -3)

Accelerated 6 | Unit 7 Lesson 18 746 Activity 1

Key Takeaway: There are many strategies for determining the distance between two points that share a coordinate: totaling absolute values, subtracting coordinates (in specific cases), and counting units.

Pairs | 15 min

Activity 2 Make Your Own Polygon

Purpose: Students create their own polygon to practice using coordinates that match specific descriptions before creating their own challenge and solving classmates' challenges.

Short on time? Consider omitting either Problem 7 or the Challenge Creator.

1 Launch

Invite students to read through the options and select the one that sounds fun to create.

2 Monitor

Encourage students to add points that are outside of the first quadrant to make their shapes more complex.

D Differentiation

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Encourage them to place one starting point and count using the constraint that they chose to place the next point.
Have the right shape but the wrong size.	**Support** Consider asking, "How could we determine the length of a side if the vertices are in different quadrants?"
Start by sketching the figure, then placing the points.	**Strengthen** Consider asking, "Is there a way you could make this same figure but in a different location on the grid? What would change?"

A **Accessibility: Executive Functioning** Consider chunking this activity into manageable parts (e.g., sketching a polygon, then identifying the coordinates, then entering the coordinates).

Consider sharing that students will create their own challenge similar to the previous problem. Demonstrate how students can create their polygon.

Note: Provide a piece of paper for students to record their clue.

Encourage students to make their description something silly (like "looks like a pinecone") and include size requirements (like "must have a perimeter of 20 units").

Activity 2 continued >

Activity 2

Name: _____ Date: _____ Period: _____

Make Your Own Polygon

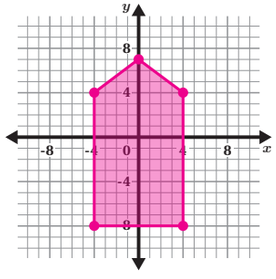
7. Here are three descriptions:

- Looks like a house that's 15 units tall.
- Looks like a capital L that's 6 units wide.
- A rectangle with an area of 24 square units.

Choose one description. Write and sketch coordinates to make a polygon that matches that description.

Points and polygons vary.

Point	Coordinates
A	(0, 7)
B	(4, 4)
C	(4, -8)
D	(-4, -8)
E	(-4, 4)

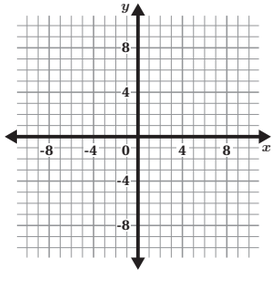


8. You will create your own polygon challenge.

- Write a description of a polygon on a slip of paper. Make it fun (like "looks like a pinecone") and include the size (like "must have a perimeter of 20 units").
- On this page, write the coordinates and sketch a polygon that matches your description.

Points and polygons vary.

Point	Coordinates
A	
B	
C	
D	



Accelerated 6 | Unit 7 Lesson 18 747 Activity 2

Pairs | 15 min

Activity 2 Make Your Own Polygon (continued)

Purpose: Students create their own polygon to practice using coordinates that match specific descriptions before creating their own challenge and solving classmate's challenges.

Short on time? Consider omitting either Problem 7 or the Challenge Creator.

2 Monitor

Look for students who are in need of support choosing a challenge and invite them to select a classmate's polygon that they can visualize easily. If they have time, invite them to pick a polygon that sounds very challenging.

Invite students to check their answers with the person who created the polygon they are drawing. Encourage them to discuss and resolve any differences. **(MTR.4.1)**

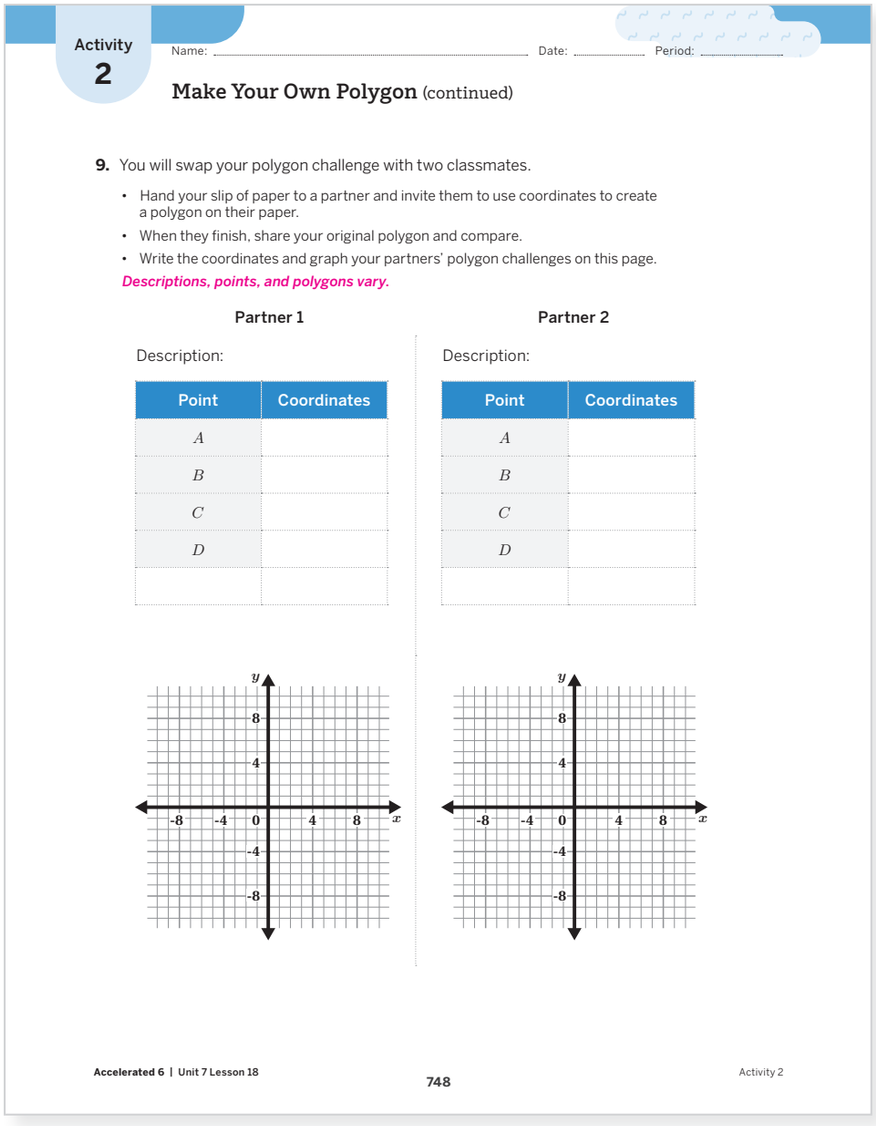
Look for several students' polygons to share during the Connect.

3 Connect

Invite students to share the inspiration for their shapes.

Invite students to share how they decided how to recreate their partner's polygon.

Consider asking, "Was there a strategy you heard that surprised you?"



Activity 2

Name: _____ Date: _____ Period: _____

Make Your Own Polygon (continued)

9. You will swap your polygon challenge with two classmates.

- Hand your slip of paper to a partner and invite them to use coordinates to create a polygon on their paper.
- When they finish, share your original polygon and compare.
- Write the coordinates and graph your partners' polygon challenges on this page.

Descriptions, points, and polygons vary.

Partner 1

Description:

Point	Coordinates
A	
B	
C	
D	

Partner 2

Description:

Point	Coordinates
A	
B	
C	
D	

Accelerated 6 | Unit 7 Lesson 18 748 Activity 2

Whole Class | 5 min

Synthesis

Purpose: Students synthesize their understanding of strategies for calculating vertical and horizontal side lengths of polygons on the coordinate plane.

Synthesis

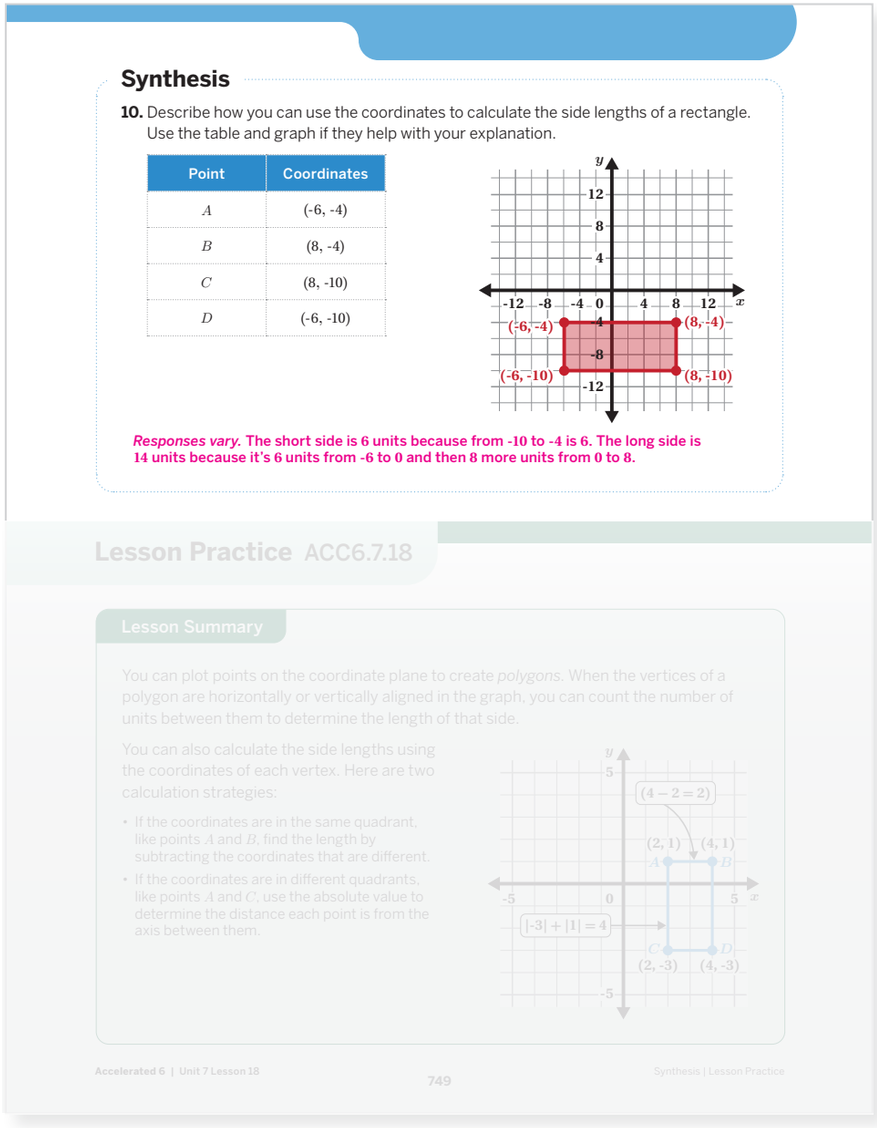
Invite students to respond independently, and then share their thinking with a partner.

Collect and share a variety of ideas, such as:

- Draw a quick sketch of the rectangle using the points and then use the image to count the length of the sides.
- The short side is 6 units because from -10 to -4 is 6.
- The long side is 14 units because it is 6 units from -6 to 0 and then 8 more units from 0 to 8.

MLR **MLR2: Collect and Display** Use this routine to formalize the strategy for calculating distances on the coordinate plane using student language. Display any new language, examples, or diagrams publicly for students to reference later.

Note: If time allows, invite students to share with a partner what makes sense about each strategy and what questions they still have.



Synthesis

10. Describe how you can use the coordinates to calculate the side lengths of a rectangle. Use the table and graph if they help with your explanation.

Point	Coordinates
A	(-6, -4)
B	(8, -4)
C	(8, -10)
D	(-6, -10)

Responses vary. The short side is 6 units because from -10 to -4 is 6. The long side is 14 units because it's 6 units from -6 to 0 and then 8 more units from 0 to 8.

Lesson Practice ACC6.7.18

Lesson Summary

You can plot points on the coordinate plane to create *polygons*. When the vertices of a polygon are horizontally or vertically aligned in the graph, you can count the number of units between them to determine the length of that side.

You can also calculate the side lengths using the coordinates of each vertex. Here are two calculation strategies:

- If the coordinates are in the same quadrant, like points A and B, find the length by subtracting the coordinates that are different.
- If the coordinates are in different quadrants, like points A and C, use the absolute value to determine the distance each point is from the axis between them.

Lesson Takeaway: You can draw *polygons* on the coordinate plane using points for their vertices. Distances or absolute value are useful for calculating any side lengths whose vertices are vertically or horizontally aligned.

Independent | 5 min

Show What You Know

Purpose: Students demonstrate their understanding by identifying coordinates that complete a rectangle and finding the length of a given segment using a table or graph.

Today's Goals

1. **Goal:** Draw a polygon on the coordinate plane given the coordinates for its vertices.
2. **Goal:** Determine lengths and distances between points on the coordinate plane that are vertically or horizontally aligned.
3. **Language Goal:** Justify whether a given set of coordinates create a square or rectangle on the coordinate plane. **(Writing, Speaking, and Listening)**

Show What You Know (Assessment Resources)

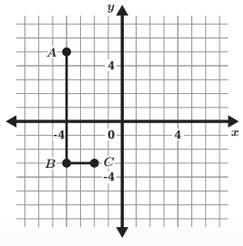
Name: _____ Date: _____ Period: _____

Show What You Know 7.18

Here is an almost-complete rectangle.

a. Write the coordinates of point D to complete the rectangle.

Point	Coordinates
A	(-4, 5)
B	(-4, -3)
C	(-2, -3)
D	(-2, 5)



b. What is the length of the segment that connects points A and B?
8 units

Differentiation Use after Lesson 18

Support

Provide targeted intervention for students by using these resources.

If students need support with calculating distances on the coordinate plane:

» **Respond to Student Thinking:**

- Revisit the **Lesson Summary**.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite them to explore the **Sub-Unit 3 Extension Activities**.

Professional Learning

How did MLR1: Stronger and Clearer Each Time support students in calculating the side lengths of a polygon on the coordinate plane?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

10. Describe how you can use the coordinates to calculate the side lengths of a rectangle. Use the table and graph if they help with your explanation.

Point	Coordinates
A	(-6, -4)
B	(8, -4)
C	(8, -10)
D	(-6, -10)

Responses vary. The short side is 6 units because from -10 to -4 is 6. The long side is 14 units because it's 6 units from -6 to 0 and then 8 more units from 0 to 8.

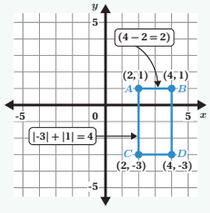
Lesson Practice ACC6.7.18

Lesson Summary

You can plot points on the coordinate plane to create *polygons*. When the vertices of a polygon are horizontally or vertically aligned in the graph, you can count the number of units between them to determine the length of that side.

You can also calculate the side lengths using the coordinates of each vertex. Here are two calculation strategies:

- If the coordinates are in the same quadrant, like points A and B, find the length by subtracting the coordinates that are different.
- If the coordinates are in different quadrants, like points A and C, use the absolute value to determine the distance each point is from the axis between them.



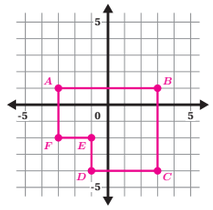
Lesson Practice ACC6.7.18

Name: _____ Date: _____ Period: _____

Problems 1–4: Here is a set of coordinates.

Point	Coordinates
A	(-3, 1)
B	(3, 1)
C	(3, -4)
D	(-1, -4)
E	(-1, -2)
F	(-3, -2)

1. Plot each point.



2. Connect the points to create polygon $ABCDEF$.

3. Determine the length of the segment between point A and point B.
6 units

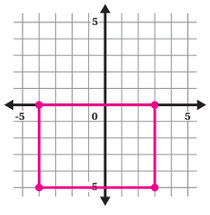
4. Determine the perimeter of polygon $ABCDEF$.
22 units

Problems 5–7: Three points of a rectangle are (3, 0), (3, -5), and (-4, -5).

5. What are the coordinates of the missing point?
(-4, 0)

6. Sketch the rectangle and calculate its perimeter.
24 units

7. Calculate the area of the rectangle.
35 square units



Lesson Practice ACC6.7.18

Name: _____ Date: _____ Period: _____

FAST Practice

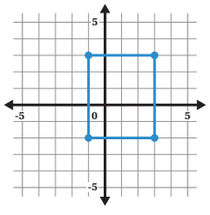
8. Gabriela drew this rectangle. Which statement is incorrect about Gabriela's drawing?

A. One of the coordinates of the rectangle is (-1, -2).

B. The area of the rectangle is 20 square units.

C. The perimeter of the rectangle is 9 units.

D. The side lengths of the rectangle are 5 units and 4 units.



Spiral Review

9. Select *all* the values of x that are solutions to the inequality $-0.5 > x$.

☐ A. 0 ☑ B. -1 ☐ C. -0.40

☑ D. -0.6 ☑ E. -0.55

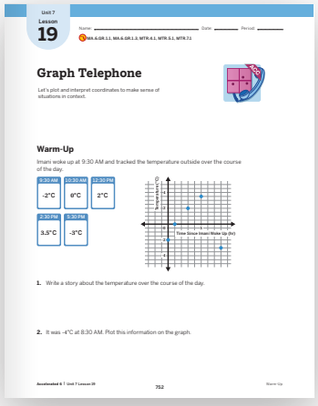
10. Is $\frac{12}{5} \div \frac{3}{5}$ greater than, less than, or equal to 1?
Greater than 1

11. What is the value of $\frac{9}{5} \div \frac{3}{5}$?
3

12. At top speed, an elephant can run 25 miles per hour and a giraffe can run 16 miles in $\frac{1}{2}$ hour. Which animal runs faster?
Giraffe

Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1	1	MA.6.GR.1.1
	2	1	MA.6.GR.1.3
	3	1	MA.6.GR.1.2
	4	2	MA.6.GR.1.2, MA.6.GR.1.3
	5, 7	2	MA.6.GR.1.3
	6	2	MA.6.GR.1.1, MA.6.GR.1.2, MA.6.GR.1.3
FAST Practice	8	2	MA.6.GR.1.2, MA.6.GR.1.3
Spiral Review			
	9	1	MA.6.AR.2.1
	10	2	MA.6.NSO.2.2
	11	1	MA.6.NSO.2.2
	12	1	MA.6.AR.3.5


Unit 7
Lesson 19
Name: Date: Period:
MA.6.GR.1.1, MA.6.GR.1.3, MTR.4.1, MTR.5.1, MTR.7.1
Graph Telephone
Let's plot and interpret coordinates to make sense of situations in context.
Warm-Up
Imani woke up at 9:30 AM and tracked the temperature outside over the course of the day.
9:00 AM -2°C
10:00 AM 6°C
12:00 PM 2°C
2:00 PM 3.5°C
3:00 PM -3°C
Temperature (°C)
Time Since Imani Woke Up (hr)
1. Write a story about the temperature over the course of the day.
2. It was -4°C at 8:30 AM. Plot this information on the graph.
Accelerated 6 | Unit 7 Lesson 19
752
Warm-Up

Graph Telephone

The Coordinate Plane in Context

Let's plot and interpret coordinates to make sense of situations in context.

Learning Outcomes

Today's Goals

1. **Goal:** Use points with positive and negative coordinates on a graph to make sense of situations in context.
2. **Language Goal:** Describe the meaning of points on a coordinate plane from a situation. **(Speaking and Listening)**

Students plot and interpret coordinates in order to make sense of situations in context. They look at a situation collectively to make sense of an elevation over time. Then each student receives a separate story in the form of images or words and plays the game Graph Telephone, taking turns reasoning quantitatively by creating a graph of the story and then retelling it abstractly in words. **(MTR.7.1)**

Coherence

Prior Learning

In Lessons 16–17, students explored coordinate planes with positive and negative coordinates out of context. In Lesson 18, they extended their understanding to coordinate planes with axes that have different scales.

Future Learning

In a future course, students will explore data plotted on the coordinate plane and begin to use trend lines to model and make predictions.

Connections

- Students continue to build their understanding of graphing in the coordinate plane to solve problems in context. (mathematical, cross-curricular, real world)

FL B.E.S.T. Math Benchmarks

Addressing

MA.6.GR.1.1

Extend previous understanding of the coordinate plane to plot rational number ordered pairs in all four quadrants and on both axes. Identify the x-or y-axis as the line of reflection when two ordered pairs have an opposite x-or y-coordinate.

MA.6.GR.1.3

Solve mathematical and real-world problems by plotting points on a coordinate plane, including finding the perimeter or area of a rectangle.

Mathematical Thinking and Reasoning Standards: MTR.4.1, MTR.5.1, MTR.7.1

ELA Expectations: ELA.K12.EE.1.1, ELA.K12.EE.3.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.5.GR.4.1	MA.7.AR.4.1
	MA.7.AR.4.3
	MA.8.GR.2.3

Lesson at a Glance

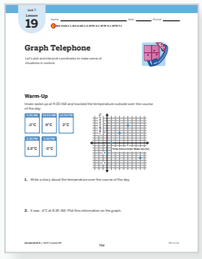
~ 45 min

MA.6.GR.1.1, MA.6.GR.1.3

Warm-Up

Pairs | 5 min

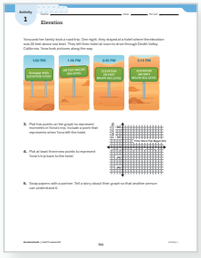
Students use the **Tell a Story** routine to give meaning to negative x-coordinates and y-coordinates in context.



Activity 1

Pairs | 10 min

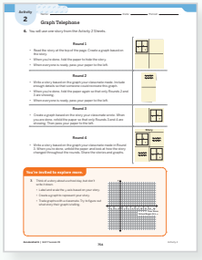
Students create a graph to represent time-stamped pictures from a family's trip to Death Valley. **(MTR.7.1)**



Activity 2

Small Groups | 20 min

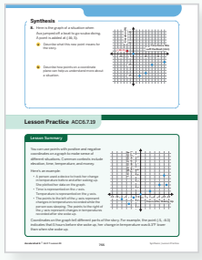
Students practice moving between stories in context and graphs on the coordinate plane using a social structure similar to the game Telephone. **MLR2: Collect and Display** **(MTR.5.1)**



Synthesis

Whole Class | 5 min

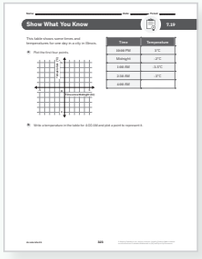
Students synthesize their understanding of how to interpret points with positive and negative coordinates on the plane in context.



Show What You Know

Independent | 5 min

Students demonstrate their understanding by plotting and interpreting coordinates to make sense of a situation involving temperature.



Prep Checklist

Prepare the Activity 2 Sheets.

This lesson includes:

- Student Edition
- Show What You Know
- Activity 2 Sheets, one set per group

Pairs | 5 min

Warm-Up

Purpose: Students use the Tell a Story routine to give meaning to negative x-coordinates and y-coordinates in context.

1 Launch

Direct students' attention to the temperatures Imani collected and the accompanying graph.

Use the Tell a Story routine to help students make sense of the context. Invite students to write a story about the graph and temperatures using their unique experiences, perspectives, and creativity. **(MTR.7.1)**

ELL **English Language Learners** Invite students to write stories in their primary language. **(Writing)**

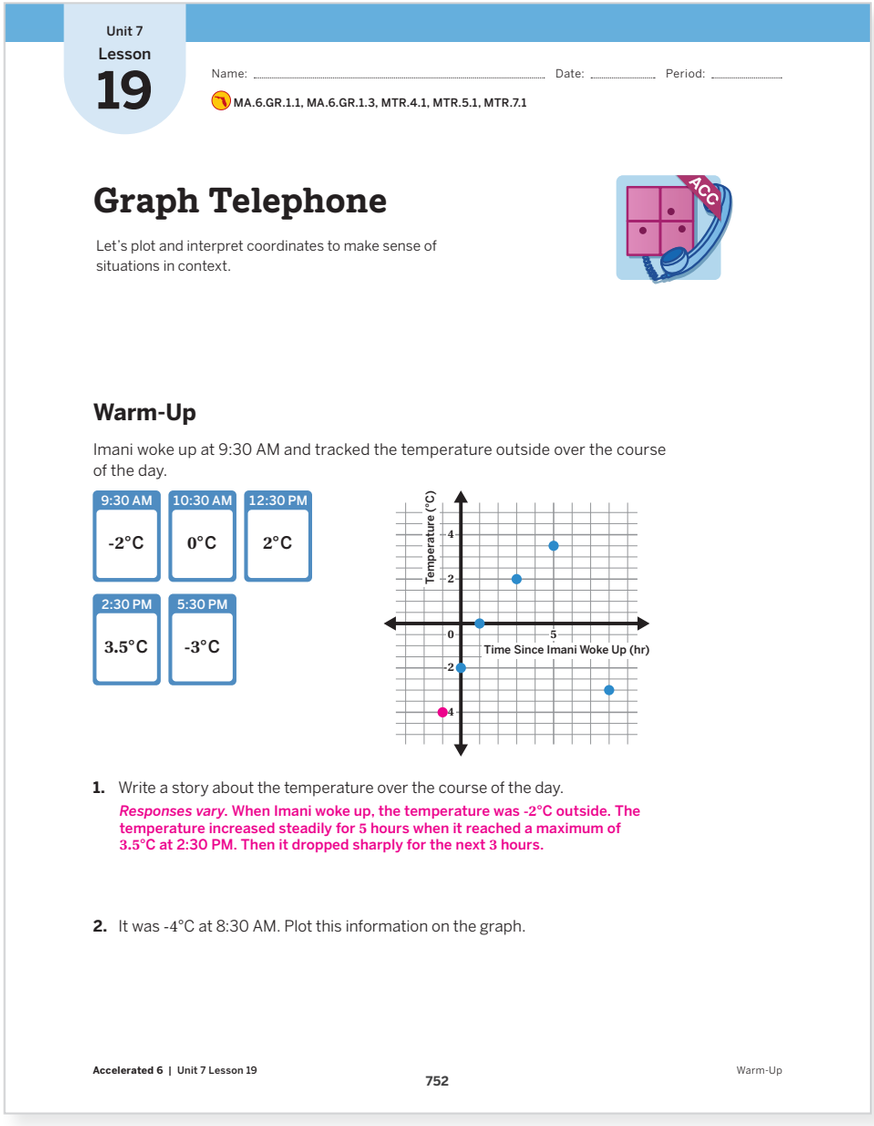
2 Connect

Invite several students to share their stories, encouraging them to make connections to the graph as they share.

Encourage students to ask questions about each other's stories. **(MTR.4.1)**

Consider asking:

- "How does your story connect to the graph?"
- "What value on the x-axis represents 8:30 AM?"
- "Does it make sense that there are time values that are negative?"
- "What other times can be represented by negative x-values on the graph?"



Unit 7
Lesson
19

Name: _____ Date: _____ Period: _____

MA.6.GR.1.1, MA.6.GR.1.3, MTR.4.1, MTR.5.1, MTR.7.1

Graph Telephone

Let's plot and interpret coordinates to make sense of situations in context.

Warm-Up

Imani woke up at 9:30 AM and tracked the temperature outside over the course of the day.

9:30 AM	10:30 AM	12:30 PM
-2°C	0°C	2°C

2:30 PM	5:30 PM
3.5°C	-3°C

1. Write a story about the temperature over the course of the day.

   Responses vary. When Imani woke up, the temperature was -2°C outside. The temperature increased steadily for 5 hours when it reached a maximum of 3.5°C at 2:30 PM. Then it dropped sharply for the next 3 hours.

2. It was -4°C at 8:30 AM. Plot this information on the graph.

Accelerated 6 | Unit 7 Lesson 19 752 Warm-Up

Pairs | 10 min

Activity 1 Elevation

Purpose: Students create a graph to represent time-stamped pictures from a family's trip to Death Valley. **(MTR.7.1)**

1 Launch

Direct students' attention to the image and invite students to share what they recall about the phrase *below sea level*. Clarify that below sea level does not necessarily mean under water.

A **Accessibility: Conceptual Processing** Invite students to make connections to familiar contexts to support thinking about Yona's trip.

2 Monitor

Look for students who plot different points to represent different options for Yona's return trip. Use these graphs during the discussion.

D **Differentiation**

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Encourage them to label the times on the x-axis (e.g. 1:00 PM at $x = 0$, 2:00 PM at $x = 1$, etc.).
Ignore "above sea level" and "below sea level" and plot all points as positive.	**Support** Invite them to reference the graph about the temperature and use it to determine if the point should be positive or negative.
Place points for her return trip in different places and justify the placement with a story.	**Strengthen** Celebrate variety in student explanations, reminding students that math is about being creative and making lots of connections.

Listen for variety and creativity in storytelling as students swap papers and discuss one another's thinking. **(MTR.4.1)**

3 Connect

Direct students' attention to the graph from Activity 1 and invite a few students to discuss how they connected Yona's photos to each point. **(MTR.4.1, MTR.5.1)**

Consider asking:

- "What does $(5, -50)$ mean in the context of Yona's trip?" **(MTR.7.1)**
- "What story would you tell about Yona's trip to Death Valley and back to the hotel from looking at this graph?"

Activity 1

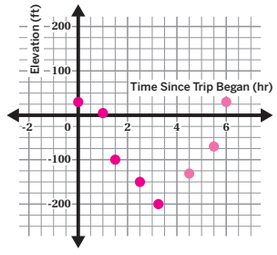
Name: Date: Period:

Elevation

Yona and her family took a road trip. One night, they stayed at a hotel where the elevation was 35 feet above sea level. They left their hotel at noon to drive through Death Valley, California. Yona took pictures along the way.

3. Plot five points on the graph to represent moments in Yona's trip. Include a point that represents when Yona left the hotel.
   Sample shown on graph.

4. Plot at least three new points to represent Yona's trip back to the hotel.
   Responses vary. Sample shown on graph.



5. Swap papers with a partner. Tell a story about their graph so that another person can understand it.
   Responses vary. Yona's family drove through Death Valley National Park and stopped at many spots that were below sea level. At 3:15 PM, they reached the lowest point at 200 feet below sea level. They followed a similar path back, arriving at the hotel at 6:00 PM.

Accelerated 6 | Unit 7 Lesson 19 753 Activity 1

Key Takeaway: You can use graphs to represent situations in context and add new points to tell the story with information that was previously missing.

Small Groups | 20 min

Activity 2 Graph Telephone

Purpose: Students practice moving between stories in context and graphs on the coordinate plane using a social structure similar to the game Telephone. **(MTR.5.1)**

Short on time? Consider having students complete just two rounds of the Graph Telephone game.

1 Launch

Consider asking if students have played the game telephone. Invite students to describe how the game works. Share that they will be playing a similar game today where they will take turns either creating graphs or writing stories.

Distribute one set of Activity 2 Sheets to each group so that each student receives a different story. Each student in the group should take one story card and not share the information with the others in their group. For groups of three, there will be one story card not used.

2 Monitor

Direct students' attention to the directions to the activity and review them as a class.

Note: It may be helpful to set a two-minute timer for each round.

D Differentiation

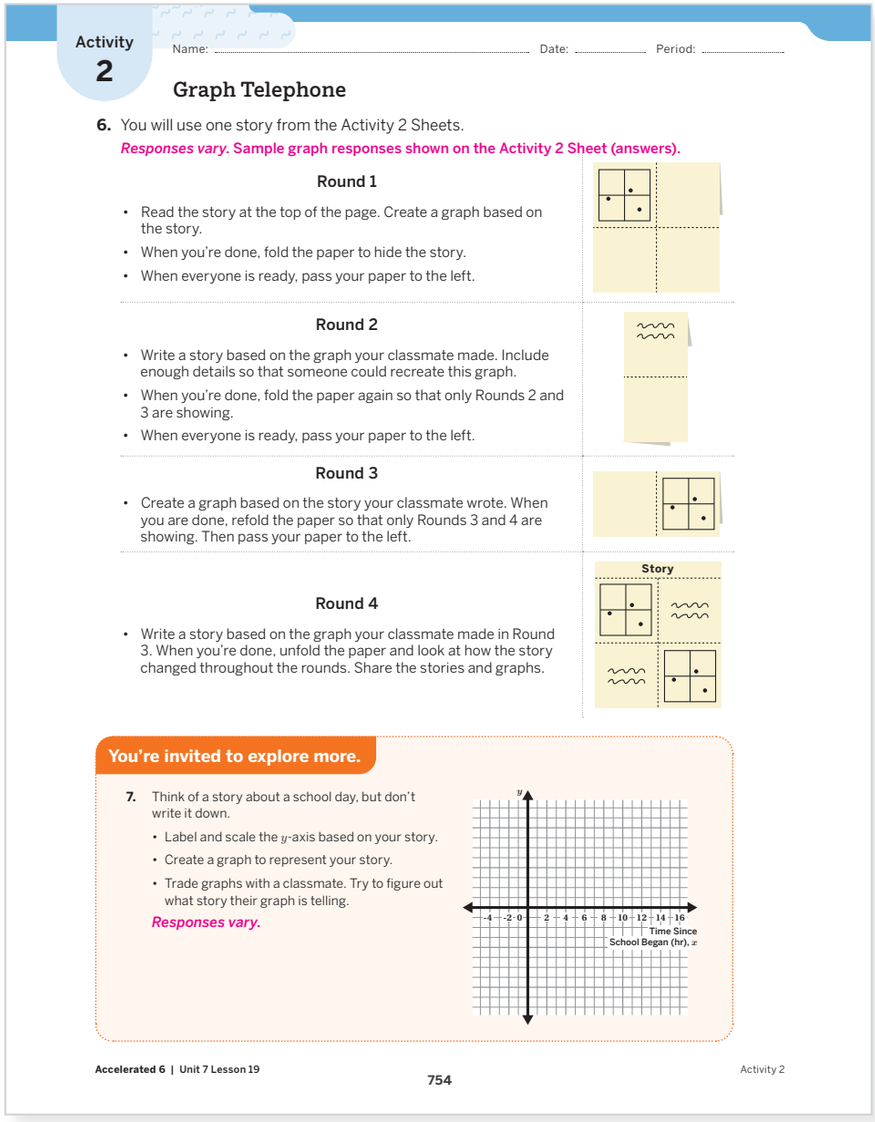
Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Invite students to select one point on the graph and describe what each coordinate means.
Write stories that describe specific points of a given situation.	**Support** Invite them to take note of broader trends in the information and include words like *increasing, decreasing*, or discuss the highest values or lowest values of the situation.
Attend to precision in their graphs or stories.	**Stretch** Consider asking, "What other points could we plot on this graph that would make sense for this story?"

Listen for students making connections between the stories and the graphs. **(MTR.5.1)**

3 Connect

Invite students to share their stories, encouraging connections to the graph.

MLR2: Collect and Display Use this routine to collect students' language as they formalize the connections between stories and graphs. Display these publicly for students to reference or add to for the rest of the unit.



Activity 2

Name: Date: Period:

Graph Telephone

6. You will use one story from the Activity 2 Sheets.

Responses vary. Sample graph responses shown on the Activity 2 Sheet (answers).

Round 1

- Read the story at the top of the page. Create a graph based on the story.
- When you're done, fold the paper to hide the story.
- When everyone is ready, pass your paper to the left.

Round 2

- Write a story based on the graph your classmate made. Include enough details so that someone could recreate this graph.
- When you're done, fold the paper again so that only Rounds 2 and 3 are showing.
- When everyone is ready, pass your paper to the left.

Round 3

- Create a graph based on the story your classmate wrote. When you are done, refold the paper so that only Rounds 3 and 4 are showing. Then pass your paper to the left.

Round 4

- Write a story based on the graph your classmate made in Round 3. When you're done, unfold the paper and look at how the story changed throughout the rounds. Share the stories and graphs.

Story

You're invited to explore more.

7. Think of a story about a school day, but don't write it down.

- Label and scale the y-axis based on your story.
- Create a graph to represent your story.
- Trade graphs with a classmate. Try to figure out what story their graph is telling.

Responses vary.

y

-4 -2 0 2 4 6 8 10 12 14 16

Time Since School Began (hr), x

Accelerated 6 | Unit 7 Lesson 19 754 Activity 2

Key Takeaway: The same information can be described using positive and negative coordinates on a graph and in a story.

Whole Class | 5 min

Synthesis

Purpose: Students synthesize their understanding of how to interpret points with positive and negative coordinates on the plane in context.

Invite students to respond independently, and then share their thinking with a partner.

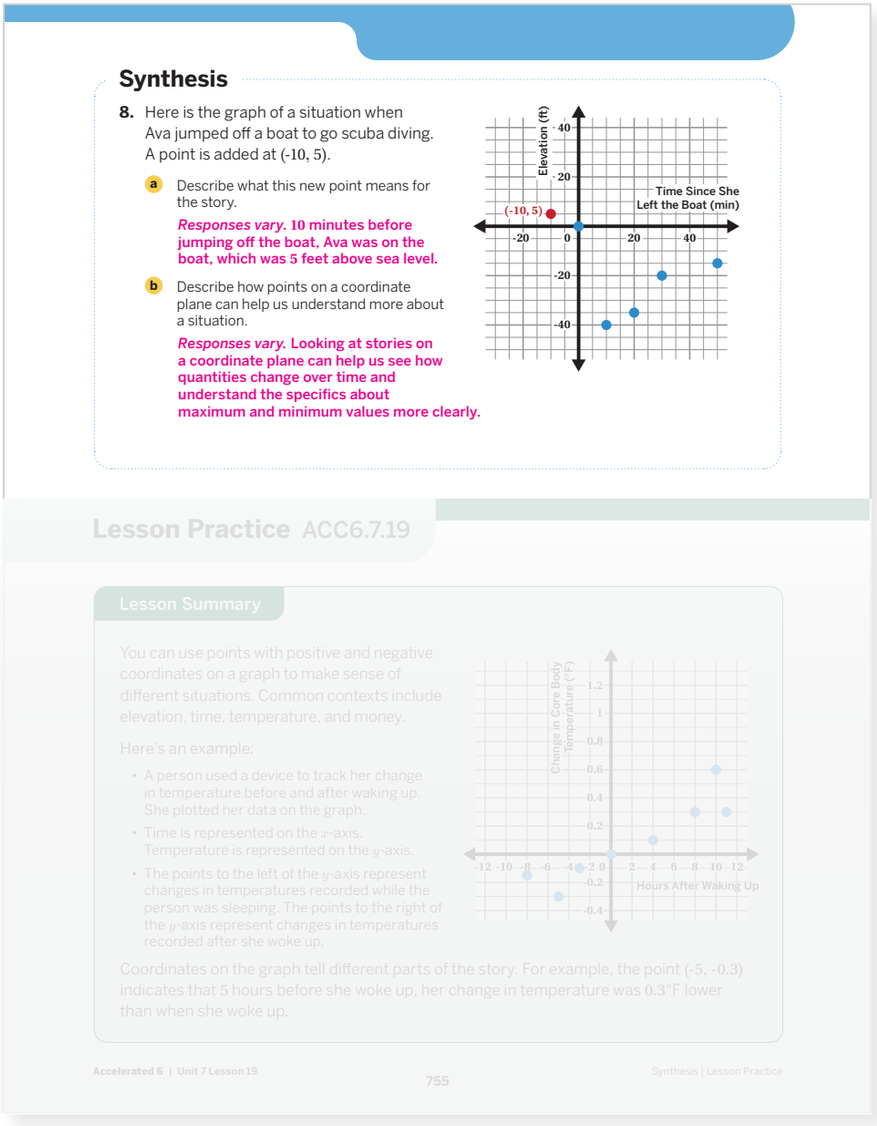
Collect and share a variety of ideas, such as:

- Points that are negative for the x-axis connect to the time before she left the boat.
- Points that are negative for the y-axis connect to her being underwater.
- Looking at stories on graphs can help us see how quantities change over time.

Consider asking:

- "What does this point mean about Ava's scuba trip?"
- "How could you show that Ava came back to the surface an hour after jumping off the boat?"
- "Which do you like more, the graphical representation or the written story? Why?"

Note: If time allows, invite students to share with a partner what makes sense about each strategy and what questions they still have.



Synthesis

8. Here is the graph of a situation when Ava jumped off a boat to go scuba diving. A point is added at (-10, 5).

a. Describe what this new point means for the story.

Responses vary. 10 minutes before jumping off the boat, Ava was on the boat, which was 5 feet above sea level.

b. Describe how points on a coordinate plane can help us understand more about a situation.

Responses vary. Looking at stories on a coordinate plane can help us see how quantities change over time and understand the specifics about maximum and minimum values more clearly.

Lesson Practice ACC6.7.19

Lesson Summary

You can use points with positive and negative coordinates on a graph to make sense of different situations. Common contexts include elevation, time, temperature, and money.

Here's an example:

- A person used a device to track her change in temperature before and after waking up. She plotted her data on the graph.
- Time is represented on the x-axis. Temperature is represented on the y-axis.
- The points to the left of the y-axis represent changes in temperatures recorded while the person was sleeping. The points to the right of the y-axis represent changes in temperatures recorded after she woke up.

Coordinates on the graph tell different parts of the story. For example, the point (-5, -0.3) indicates that 5 hours before she woke up, her change in temperature was 0.3°F lower than when she woke up.

Accelerated 6 | Unit 7 Lesson 19 755 Synthesis | Lesson Practice

Lesson Takeaway: When a real-world scenario compares two things (e.g., time and elevation) it can be represented on a coordinate plane with positive and negative coordinates.

Independent | 5 min

Show What You Know

Purpose: Students demonstrate their understanding by plotting and interpreting coordinates to make sense of a situation involving temperature.

Today's Goals

1. **Goal:** Use points with positive and negative coordinates on a graph to make sense of situations in context.
2. **Language Goal:** Describe the meaning of points on a coordinate plane from a situation. **(Speaking and Listening)**

Show What You Know (Assessment Resources)

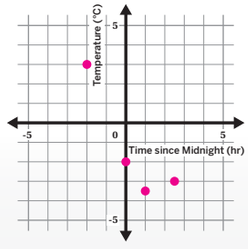
Name: _____ Date: _____ Period: _____

Show What You Know 7.19

This table shows some times and temperatures for one day in a city in Illinois.

Time	Temperature
10:00 PM	3°C
Midnight	-2°C
1:00 AM	-3.5°C
2:30 AM	-3°C
4:00 AM	*Responses vary.*

a. Plot the first four points.



b. Write a temperature in the table for 4:00 AM and plot a point to represent it.
Responses vary.

Differentiation Use after Lesson 19

Support

Provide targeted intervention for students by using these resources.

If students need support with calculating distances on the coordinate plane:

» **Respond to Student Thinking:**

- Revisit the **Lesson Summary**.
- Revisit the **Warm-Up** and **Activity 1** before providing individualized feedback on the relevant **Practice Problems**.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite them to explore the **Sub-Unit 4 Extension Activities**.
- Invite students to complete the **You're Invited to Explore More** task and discuss their thinking with a partner.

Professional Learning

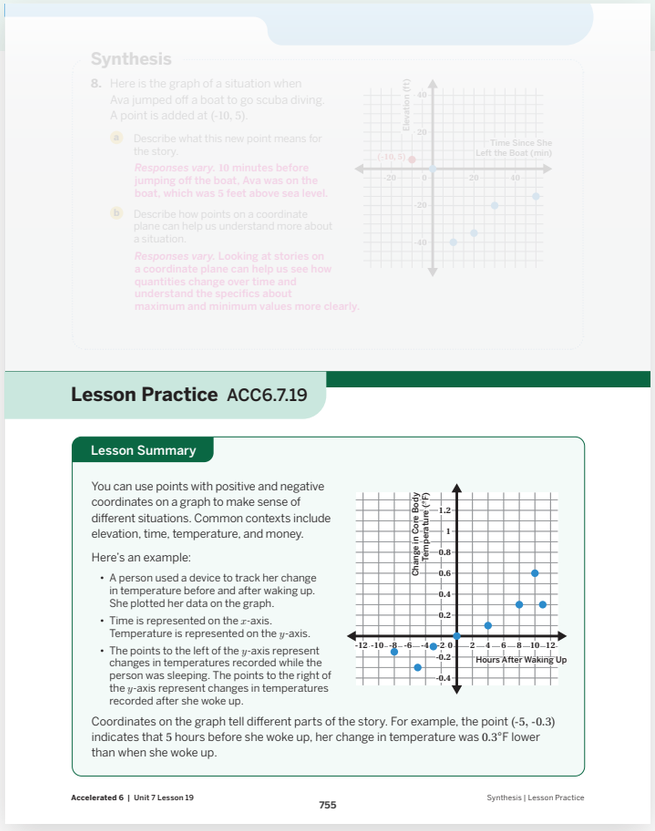
Reflect on students' progress with positive and negative numbers throughout the unit.

- In what ways have students' understanding and skills grown?
- What concepts do students need additional support with?
- How can you use students' strengths to help everyone feel prepared for their upcoming assessment?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.



Synthesis

8. Here is the graph of a situation when Ava jumped off a boat to go scuba diving. A point is added at (-10, 5).

a. Describe what this new point means for the story.

Responses vary. 10 minutes before jumping off the boat, Ava was on the boat, which was 5 feet above sea level.

b. Describe how points on a coordinate plane can help us understand more about a situation.

Responses vary. Looking at stories on a coordinate plane can help us see how quantities change over time and understand the specifics about maximum and minimum values more clearly.

Elevation (ft)

Time Since She Left the Boat (min)

(-10, 5)

Lesson Practice ACC6.7.19

Lesson Summary

You can use points with positive and negative coordinates on a graph to make sense of different situations. Common contexts include elevation, time, temperature, and money.

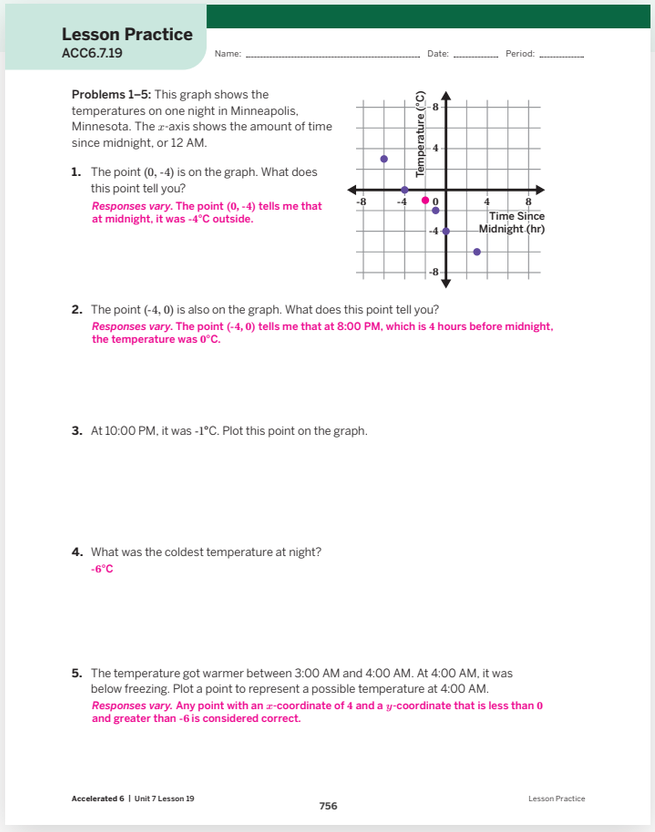
Here's an example:

- A person used a device to track her change in temperature before and after waking up. She plotted her data on the graph.
- Time is represented on the x-axis. Temperature is represented on the y-axis.
- The points to the left of the y-axis represent changes in temperatures recorded while the person was sleeping. The points to the right of the y-axis represent changes in temperatures recorded after she woke up.

Change in Core Body Temperature (°F)

Hours After Waking Up

Coordinates on the graph tell different parts of the story. For example, the point (-5, -0.3) indicates that 5 hours before she woke up, her change in temperature was 0.3°F lower than when she woke up.



Lesson Practice
ACC6.7.19

Name: _____ Date: _____ Period: _____

Problems 1–5: This graph shows the temperatures on one night in Minneapolis, Minnesota. The x-axis shows the amount of time since midnight, or 12 AM.

Temperature (°C)

Time Since Midnight (hr)

1. The point (0, -4) is on the graph. What does this point tell you?

Responses vary. The point (0, -4) tells me that at midnight, it was -4°C outside.

2. The point (-4, 0) is also on the graph. What does this point tell you?

Responses vary. The point (-4, 0) tells me that at 8:00 PM, which is 4 hours before midnight, the temperature was 0°C.

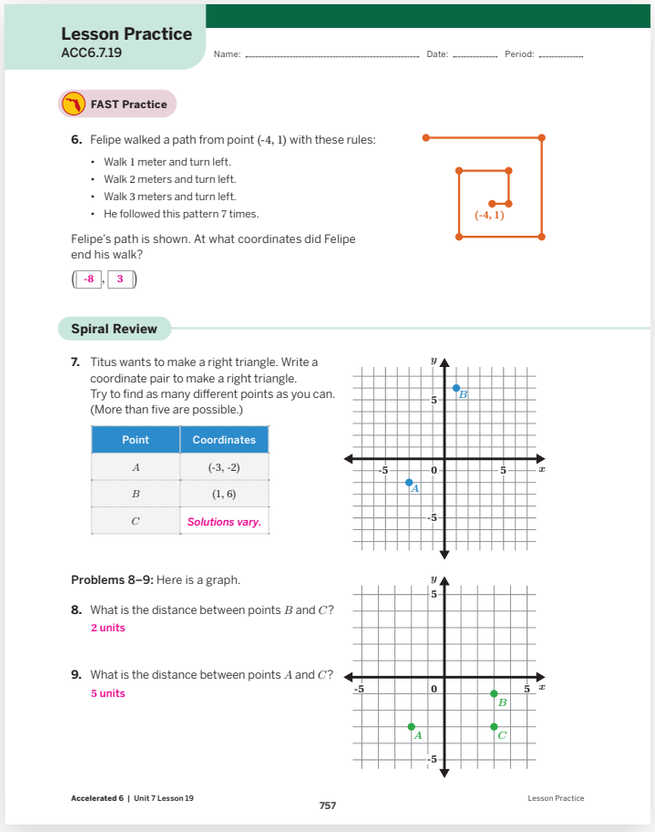
3. At 10:00 PM, it was -1°C. Plot this point on the graph.

4. What was the coldest temperature at night?

-6°C

5. The temperature got warmer between 3:00 AM and 4:00 AM. At 4:00 AM, it was below freezing. Plot a point to represent a possible temperature at 4:00 AM.

Responses vary. Any point with an x-coordinate of 4 and a y-coordinate that is less than 0 and greater than -6 is considered correct.



Lesson Practice
ACC6.7.19

Name: _____ Date: _____ Period: _____

FAST Practice

6. Felipe walked a path from point (-4, 1) with these rules:

- Walk 1 meter and turn left.
- Walk 2 meters and turn left.
- Walk 3 meters and turn left.
- He followed this pattern 7 times.

(-4, 1)

Felipe's path is shown. At what coordinates did Felipe end his walk?

(-8 , 3)

Spiral Review

7. Titus wants to make a right triangle. Write a coordinate pair to make a right triangle. Try to find as many different points as you can. (More than five are possible.)

Point	Coordinates
A	(-3, -2)
B	(1, 6)
C	*Solutions vary.*

Problems 8–9: Here is a graph.

8. What is the distance between points B and C?

2 units

9. What is the distance between points A and C?

5 units

Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1–2, 4	1	MA.6.GR.1.3
	3	1	MA.6.GR.1.1
	5	2	MA.6.GR.1.1
FAST Practice	6	3	MA.6.GR.1.3
Spiral Review			
	7	3	MA.6.GR.1.3
	8–9	1	MA.6.GR.1.2

Assess and Respond

End-of-Unit Assessment

Independent | 45 min

Facilitation: Assign the End-of-Unit Assessment to learn about your students' understanding of the concepts and skills in this unit. There are two forms of the End-of-Unit Assessment: Forms A and B. If time allows, consider inviting students to complete one or more of the tasks in the Unit Synthesis and Reflection.

Item Analysis, Forms A and B

Problem(s)	Concept or skill	Addressed in	DOK	FL B.E.S.T. Math Benchmarks
1	Identifying the coordinates of plotted points	Lesson 16	1	MA.6.NSO.1.1
2	Evaluating a numerical expression that includes, exponents, integers, and fractions	Lessons 11–12	1	MA.7.NSO.2.1
3	Plotting positive and negative numbers on a number line	Lesson 2	1	MA.6.NSO.1.1
4	Using the symbols <, >, or = to compare numbers and expressions involving absolute values	Lessons 4, 5	1	MA.6.NSO.1.3, MTR.3.1
5a, 5d	Solving one-step equations using multiplication and division, where all terms and solutions are integers	Lesson 9	2	MA.6.NSO.1.2, MTR.4.1
5c, 5c	Solving one-step equations using addition and subtraction, where all terms and solutions are integers	Lesson 8	2	MA.7.NSO.2.2, MA.6.AR.2.2
6a	Plotting and labeling points in all four quadrants of the coordinate plane	Lesson 17	1	MA.6.NSO.1.1, MA.6.GR.1.1, MTR.5.1
6b	Determining distances between points with the same first coordinate	Lesson 18	1	MA.6.GR.1.1 MA.6.GR.1.3
7a, 7b	Interpreting plotted points within real-world contexts	Lesson 19	2	MA.6.NSO.1.1, MA.6.NSO.1.2
7c	Plotting points to represent real-world contexts	Lesson 19	2	MA.6.NSO.1.1, MA.6.NSO.1.2
7d	Writing coordinates that do not make sense within real-world contexts and explaining the reason	Lesson 19	3	MA.6.NSO.1.1, MA.6.NSO.1.2 MA.6.GR.1.1, MTR.4.1

Assessment Resources

- Student Assessments (Forms A and B)
- Assessment Rubrics with Answers
- Show What You Knows and Answer Keys

Intervention and Extension Resources

- Mini-Lessons
- Extensions

Name: Date: Period:

End-of-Unit Assessment

Unit 7

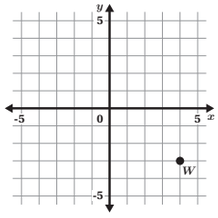
1. Which are the coordinates of point W?

A. (-3, 4)

B. (4, -3)

C. (-4, 3)

D. (3, -4)



2. What is the value of $12 + 6 - 4\left(\frac{10}{5} - 7\right)^2$?

A. -82

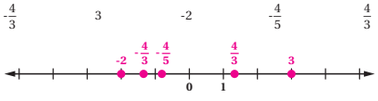
B. -46

C. 58

D. 118

3. Plot and label these numbers on the number line:

$-\frac{4}{3}$ 3 -2 $-\frac{4}{5}$ $\frac{4}{3}$



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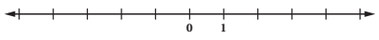
Name: Date: Period:

End-of-Unit Assessment (continued)

Unit 7

4. Complete each number sentence with the symbol <, >, or =.

Use the number line if it helps with your thinking.



a. $\frac{3}{2}$ > $-\frac{3}{2}$

b. |-5| > |-4.5|

c. $-\frac{5}{3}$ < $-\frac{3}{5}$

d. |-5| = 5

5. Solve each equation.

a. $8n = 72$
$n = 9$

b. $t - 11 = -17$
$t = -6$

c. $p + 5 = 12$
$p = 7$

d. $\frac{a}{4} = -3$
$a = -12$

Name: Date: Period:

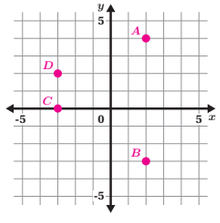
End-of-Unit Assessment (continued)

Unit 7

6. Here are four points.

- A (2, 4)
- B (2, -3)
- C (-3, 0)
- D (-3, 2)

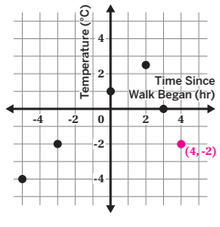
a. Plot and label each point.



b. What is the length of the segment between A and B? Connect the points if it helps with your thinking.
7 units

7. On Saturday, Aba went for a walk and made a graph of the temperature outside at different times. She started her walk at time 0.

a. What was the temperature outside when Aba started her walk?
1°C



b. The point (-5, -4) is on Aba's graph. What does this point tell us?
Responses vary. 5 hours before Aba's walk began, it was -4°C outside.

c. Aba walked for 4 hours. When Aba ended her walk, it was -2°C outside.
Add this point to her graph.

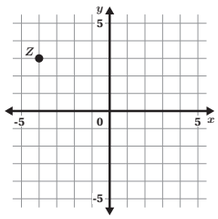
d. Write coordinates for a new point that would *not* make sense in this situation. Explain how you know it doesn't make sense.
Responses vary.
- (2, 1) because there cannot be two different temperatures at the same time.
- (1, 100) because 100°C doesn't make sense on a winter day.

Name: Date: Period:

End-of-Unit Assessment

Unit 7

1. Which are the coordinates of point Z?



A. (-3, 4)

B. (4, -3)

C. (-4, 3)

D. (3, -4)

2. What is the value of $15 + |3 - 8| - 4^2$?

A. -6

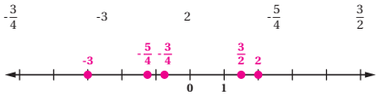
B. -3

C. 4

D. 10

3. Plot and label these numbers on the number line:

$-\frac{3}{4}$ -3 2 $-\frac{5}{4}$ $\frac{3}{2}$



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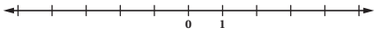
Name: Date: Period:

End-of-Unit Assessment (continued)

Unit 7

4. Complete each number sentence with the symbol <, >, or =.

Use the number line if it helps with your thinking.



a $-\frac{1}{4} < \frac{1}{4}$

b $|3| > |-3.5|$

c $-\frac{2}{3} > -\frac{3}{2}$

d $4 = |4|$

5. Solve each equation.

a $7n = 49$

$n = 7$

b $t - 13 = -20$

$t = -7$

c $p + 4 = 9$

$p = 5$

c $\frac{a}{3} = -6$

$a = -18$

Name: Date: Period:

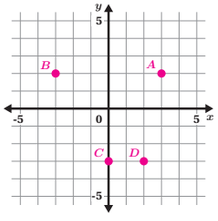
End-of-Unit Assessment (continued)

Unit 7

6. Here are four points.

- A (3, 2)
- B (-3, 2)
- C (0, -3)
- D (2, -3)

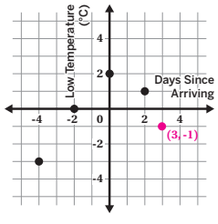
a Plot and label each point.



b What is the length of the segment between A and B? Connect the points if it helps with your thinking.

6 units

7. A scientist took a trip to Montreal, Canada. He made a graph of the low temperatures in Montreal.



a What was the low temperature the day he arrived?

2°C

b The point (-4, -3) is on his graph. What does this point tell us?

Responses vary. 4 days before the scientist arrived, the low temperature was -3°C.

c 3 days after he arrived, the low temperature was -1°C. Add this point to his graph.

d Write coordinates for a new point that would *not* make sense in this situation. Explain how you know it doesn't make sense.

Responses vary.

- (0, 3) because there cannot be two different lowest temperatures on the same day.
- (1, 100) because 100°C isn't a reasonable temperature.

D Differentiation (End-of-Unit Assessment)

Note: To **strengthen** and **stretch** students' learning, refer to the differentiation resources suggested throughout this unit and in the Unit Overview.

Sub-Unit Goals	Problem(s)	To respond to student thinking, consider:	

Sub-Unit 1:
• Order and compare positive numbers, negative numbers, and 0
• Evaluate, order, and compare absolute values
• Add, subtract, multiply, and divide integers **(Lessons 1–9)**
	3–4	**Support**	• **Mini-Lessons:**
			  • *Ordering and Comparing Positive and Negative Numbers*
			• **Teacher Move:** Consider revisiting:
			  • Lesson 2 *(Digging Deeper)*
			  • Lesson 4 *(Order in the Class)*
			  • Lesson 5 *(Distance on the Number Line)*

		Strengthen	• **Repeated Challenges:** • Lesson 2 *(Digging Deeper)*

		Stretch	• **You're invited to explore more:**
			  • Lesson 3 *(Sub-Zero)*
			  • Lesson 5 *(Distance on the Number Line)*

	5	**Support**	• **Mini-Lesson:**
			  • *Adding and Subtracting Positive and Negative Numbers*
			  • *Solving Real-World Problems Involving Positive and Negative Numbers*
			• **Teacher Move:** Consider revisiting:
			  • Lesson 8 *(More Floats and Anchors)*
			  • Lesson 9 *(Up, Up, and Away!)*

		Stretch	• **You're invited to explore more:** • Lesson 8 *(More Floats and Anchors)*

Sub-Unit 2:
• Perform all four operations with positive and negative rational numbers using a variety of strategies.
• Use order of operations to evaluate numerical expressions. **(Lessons 11–15)**
	2	**Support**	• **Mini-Lesson:** *Writing and Solving One-Step Equations*
			• **Teacher Move:** Consider revisiting:
			  • Lesson 11 *(Back in Time)*
			  • Lesson 12 *(Speeding Turtles)*

		Strengthen	• **Repeated Challenges:**
			  • Lesson 11 *(Back in Time)*
			  • Lesson 12 *(Speeding Turtles)*

		Stretch	• **You're invited to explore more:** • Lesson 11 *(Back in Time)*

Sub-Unit 3:
• Graph points with positive and negative coordinates.
• Draw polygons on the coordinate plane and determine their vertical and horizontal side lengths. **(Lessons 16–19)**
	1, 6a, 6b	**Support**	• **Mini-Lesson:** *Plotting Points on the Coordinate Plane*
			• **Teacher Move:** Consider revisiting:
			  • Lesson 17 *(The A-maze-ing Coordinate Plane)*
			  • Lesson 18 *(Polygon Maker)*

		Strengthen	• **Repeated Challenges:** • Lesson 18 *(Polygon Maker*
		Stretch	• **You're invited to explore more:** Lesson 17 *(The A-maze-ing Coordinate Plane)*
	7	**Support**	• **Mini-Lesson:** Consider revisiting Lesson 19 *(Graph Telephone)*
		Stretch	• **You're invited to explore more:** Lesson 19 *(Graph Telephone)*

UNIT 8

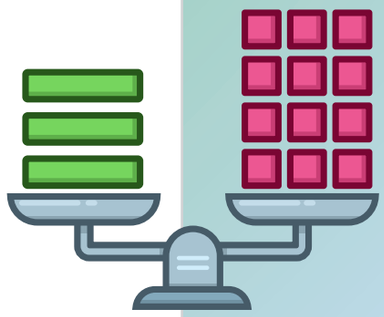
Equations and Inequalities With Integers

Students solve equations with positive and negative numbers. They explore solutions to inequalities and graph the solutions on a number line.

Essential Questions

- How are solving equations with positive numbers and with negative numbers alike and different?
- How do we represent all the numbers that are less than or greater than a value?
- Which strategies for solving simple equations or inequalities can help you solve more complex ones?

Sub-Unit Goals

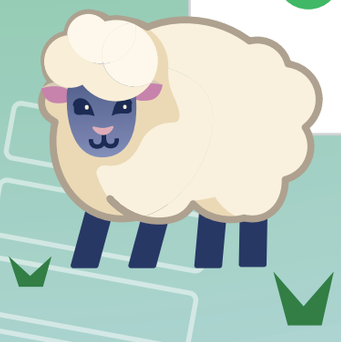


Sub-Unit 1

- Write and solve equations of the form $p + x = q$, $x + p = q$, $px = q$, and $\frac{x}{p} = q$, where p and q are integers.

Sub-Unit 2

- Write inequalities using symbols and words.
- Graph solutions to inequalities on a number line.
- Solve one-step inequalities with positive and negative coefficients.



The Math of the Unit

Unit Outcomes Telling a story that narrows and deepens with the grade

Florida B.E.S.T. Standards for Mathematics

Extend knowledge of numbers to negative numbers and develop an understanding of absolute value. **(MA.6.NSO.1)**

Apply previous understanding of arithmetic expressions to algebraic expressions. **(MA.6.AR.1)**

Develop an understanding for solving equations and inequalities. Write and solve one-step equations in one variable. **(MA.6.AR.2)**

Add, subtract, multiply and divide rational numbers. **(MA7.NSO.2)**

Write and solve equations and inequalities in one variable. **(MA.7.AR.2)**

Benchmarks Addressed in This Unit: MA.6.NSO.1.1 MA.6.AR.1.2 MA.6.AR.2.1 MA.6.AR.2.2 MA.6.AR.2.3 MA.7.NSO.2.2 MA.7.AR.2.1

Mathematical Thinking and Reasoning Standards

- Actively participate in effortful learning both individually and collectively. **(MTR.1.1)**
- Demonstrate understanding by representing problems in multiple ways. **(MTR.2.1)**
- Complete tasks with mathematical fluency. **(MTR.3.1)**
- Apply mathematics to real-world contexts. **(MTR.7.1)**

Also addressed: **MTR.4.1, MTR.5.1, MTR.6.1**

Coherence Telling a story within and across grades

In this Unit

Students solve equations and inequalities with positive and negative numbers. They graph solutions to inequalities on a number line.

Prior Learning

- Represent fractions on a number line. **(MA.3.FR.2.1)**
- Compare non-negative values using inequalities. **(MA.5.NSO.1.4)**
- **Unit 6:** Write and solve equations with positive rational numbers. **(MA.6.AR.2)**

Future Learning

- Solve linear equations with variables on both sides. **(MA.8.AR.2.1)**
- Solve two-step linear inequalities. **(MA.8.AR.2.2)**
- Solve systems of linear equations. **(MA.8.AR.4.3)**

See the Connections to Future Learning page for more information, including explanations and examples.

Rigor and Balance Integrating the aspects of rigor in ways that support one another

Here are examples of the aspects of rigor and balance in this unit. See the first page of each lesson's Teacher Edition for more information.

Strategic Competence

In **Lesson 4**, students develop strategic competence as they use different strategies to justify their solutions to inequalities.

Procedural Fluency

In **Lessons 1–2**, students develop procedural fluency by solving equations involving the four operations with integers.

Productive Disposition

In **Lesson 3**, students have opportunities to develop a productive disposition as they relate inequalities to real-world scenarios.

Unit at a Glance

Assess and Respond

A Pre-Unit Check

Learn more about your students' understanding of foundational concepts and skills that will support them in the upcoming unit. This can be given in its entirety before the unit or spread throughout the unit.

MA.5.NSO.1 MA.5.NSO.1.4 MA.5.GR.4.1 MA.5.GR.4.2 MTR.2.1 MTR.3.1 MTR.5.1

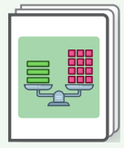
Sub-Unit 1

1 Plus or Minus

Solving Equations Involving Addition and Subtraction

Develop and solve equations for real-world and mathematical situations that require addition or subtraction of integers.

MA.6.AR.2.2
MTR.2.1 MTR.5.1 MTR.7.1



2 Multiply, Divide, and Conquer

Solving Equations Involving Multiplication and Division

Use models to understand and solve multiplication and division equations with integers.

MA.6.AR.2.3
MTR.2.1 MTR.7.1

Assess and Respond

A Sub-Unit Quiz

Learn about your students' understanding of the concepts and skills so far in this unit.

MA.6.AR.2.2 MA.6.AR.2.3
MTR.3.1 MTR.7.1

7 Shira the Sheep

Solving Inequalities With Positive and Negative Numbers

Solve inequalities with both positive and negative coefficients and connect the solutions of inequalities to their graphs.

MA.7.NSO.2.2 MA.7.AR.2.1
MTR.3.1 MTR.4.1 MTR.6.1

M Math in Action

Connecting Equations to Data Collection

Explore the connection between equations and data collection, and the contributions of mathematician Alexander Aitken.

MA.6.AR.2.3 MA.6.DP.1.2
MTR.4.1 MTR.5.1 MTR.7.1

Assess and Respond

A End-of-Unit Assessment

Learn about your students' understanding of the concepts and skills in this unit. There are two forms of the End-of-Unit Assessment: Forms A and B.

MA.6.NSO.1.2 MA.6.AR.2.1 MA.6.AR.2.2 MA.7.AR.2.1 MA.7.AR.2.2
MTR.2.1 MTR.3.1 MTR.7.1

Pacing: 11 days | **Short on time?** See pacing considerations below.

All instructional and assessment days are ~45 minutes each.

- 7 Lessons
- Math in Action
- Pre-Unit Check
- Sub-Unit Quiz
- End-of-Unit Assessment

Sub-Unit 2

3 Tunnel Travels

Graphing Inequalities

Introduce inequalities with variables and connect verbal descriptions, symbols, and number line representations of inequalities.

MA.6.AR.1.2
MTR.2.1 MTR.3.1

4 Shira's Solutions

Solutions to Inequalities

Consolidate and apply the understanding of inequalities on the number line and learn the term **solution to an inequality**.

MA.6.NSO.1.1 MA.6.AR.1.2 MA.6.AR.2.1
MTR.2.1 MTR.3.1

5 Comparing Weights

Writing Inequalities

Use the hanger representation from earlier in Grade 6 to interpret and write inequalities that involve one variable and more than one variable.

MA.6.AR.1.2 MA.6.AR.2.1
MTR.1.1 MTR.4.1

6 Unbalanced Hangers

Solving Inequalities

Make sense of how to solve inequalities using hanger diagrams, a familiar representation.

MA.7.NSO.2.2 MA.7.AR.2.1
MTR.2.1 MTR.3.1 MTR.4.1

Pacing Considerations

Lesson 5: This lesson can be omitted if most students show a strong understanding of inequalities on the Pre-Unit Check.

Lesson Materials and Prep

Here are the materials and resources needed within each lesson, how to prepare them, and where to find them.

Lesson 1

- **Copy:** Show What You Know

Lesson 2

- **Copy:** Show What You Know
- **Cut:** Activity 3 Cards *(one set per pair)*

Lesson 3

- **Copy:** Show What You Know, Activity 2 Sheet *(one per group)*
- **Materials:** measuring tapes or meter sticks *(optional)*

Lesson 4

- **Copy:** Show What You Know

Lesson 5

- **Copy:** Show What You Know, Activity 3 Sheet *(one per student)*

Lesson 6

- **Copy:** Show What You Know

Lesson 7

- **Copy:** Show What You Know

Resource Books

Assessment and Lesson Resources

- Pre-Unit Check
- Sub-Unit Quizzes
- End-of-Unit Assessment, Forms A & B
- Assessment Rubrics
- Show What You Knows
- Activity Sheets and Cards

Intervention and Extension Resources

- Mini-Lessons
- Prior Grade Mini-Lessons
- Extensions

Caregiver Support

These resources can be shared with students' caregivers. They provide background on the mathematics in this unit, as well as suggestions for supporting students at home.

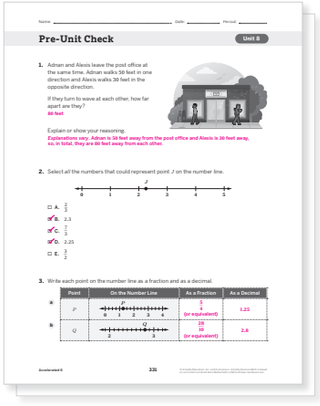
Assessments

Every unit includes a variety of formative and summative assessments, designed to highlight students' understanding of a variety of concepts and skills.

Pre-Unit Check

Assign the **Pre-Unit Check** to learn more about your students' understanding of foundational concepts and skills that will support them in the upcoming unit. This can be given in its entirety before the unit or spread throughout the unit.

FL B.E.S.T. Math Benchmarks assessed: MA.5.NSO.1, MA.5.NSO.1.4, MA.5.GR.4.1, MA.5.GR.4.2



Pre-Unit Check

Unit 8

1. Adnan and Alexis leave the post office at the same time. Adnan walks 50 feet in one direction and Alexis walks 30 feet in the opposite direction.

If they turn to wave at each other, how far apart are they?

80 feet

Explain or show your reasoning.

Explanations vary. Adnan is 50 feet away from the post office and Alexis is 30 feet away, so, in total, they are 80 feet away from each other.

2. Select all the numbers that could represent point J on the number line.

J

0 1 2 3 4 5

- ☐ A. $\frac{5}{3}$
- ☑ B. 2.3
- ☑ C. $\frac{7}{3}$
- ☑ D. 2.25
- ☐ E. $\frac{3}{2}$

3. Write each point on the number line as a fraction and as a decimal.

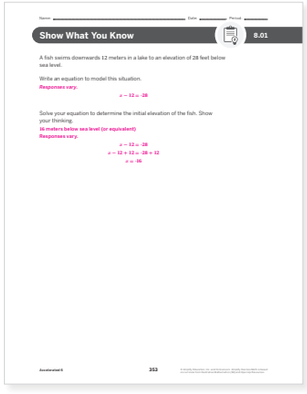
	Point	On the Number Line	As a Fraction	As a Decimal
a.	P	P 0 1 2 3 4	$\frac{5}{4}$ (or equivalent)	1.25
b.	Q	Q 2 3	$\frac{18}{10}$ (or equivalent)	1.8

Accelerated 6 331

Lesson | Show What You Know

Use the **Show What You Know** for each lesson to learn more about your students' progress toward the lesson goal(s). Here is one example from this unit.

FL B.E.S.T. Math Benchmarks assessed: on Lesson 1 Show What You Know: MA.6.AR.2.2



Show What You Know

8.01

A fish swims downwards 12 meters in a lake to an elevation of 28 feet below sea level.

Write an equation to model this situation.

Responses vary.

$x - 12 = -28$

Solve your equation to determine the initial elevation of the fish. Show your thinking.

16 meters below sea level (or equivalent)

Responses vary.

$x - 12 = -28$

$x - 12 + 12 = -28 + 12$

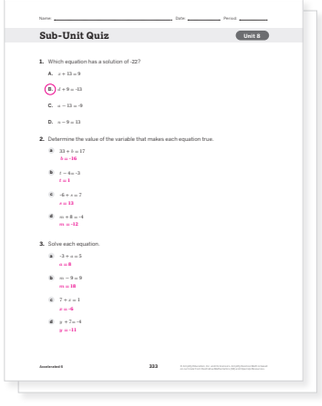
$x = -16$

Accelerated 6 353

Sub-Unit Quiz

Assign the **Sub-Unit Quiz** to learn about your students' understanding of the concepts and skills so far in this unit.

FL B.E.S.T. Math Benchmarks assessed: MA.6.AR.2.2, MA.6.AR.2.3



Sub-Unit Quiz

Unit 8

1. Which equation has a solution of -22?

A. $x + 13 = 9$

B. $x + 9 = -13$

C. $x - 13 = -9$

D. $x - 9 = 13$

2. Determine the value of the variable that makes each equation true.

a. $33 + h = 17$
$h = -16$

b. $t - 4 = -3$
$t = 1$

c. $-6 + n = 7$
$n = 13$

d. $m + 8 = -4$
$m = -12$

3. Solve each equation.

a. $-3 = a + 5$
$a = 8$

b. $m - 9 = 9$
$m = 18$

c. $7 = n + 1$
$n = -6$

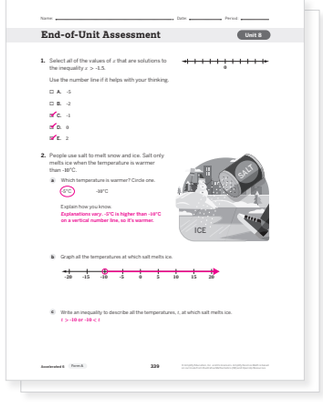
d. $y + 7 = -4$
$y = -11$

Accelerated 6 333

End-of-Unit Assessment

Assign the **End-of-Unit Assessment** to learn about your students' understanding of the concepts and skills in this unit. There are two forms of the End-of-Unit Assessment: Forms A and B.

FL B.E.S.T. Math Benchmarks assessed: MA.6.NSO.1.2, MA.6.AR.2.1, MA.6.AR.2.2, MA.7.AR.2.1, MA.7.AR.2.2



End-of-Unit Assessment

Unit 8

1. Select all of the values of x that are solutions to the inequality $x > -1.5$.

Use the number line if it helps with your thinking.

- ☐ A. -5
- ☐ B. -2
- ☑ C. -1
- ☑ D. 0
- ☑ E. 2

2. People use salt to melt snow and ice. Salt only melts ice when the temperature is warmer than -18 °C.

a. Which temperature is warmer? Circle one.

(-10 °C) -18 °C

Explain how you know.

Explanations vary. -10 °C is higher than -18 °C on a vertical number line, so it's warmer.

b. Graph all the temperatures at which salt melts ice.

-20 -15 -10 -5 0 5 10 15 20

c. Write an inequality to describe all the temperatures, t, at which salt melts ice.

$t > -10$ or $-10 < t$

Accelerated 6 Form A 339

Differentiation

Every unit includes additional resources to support, strengthen, and stretch student learning.

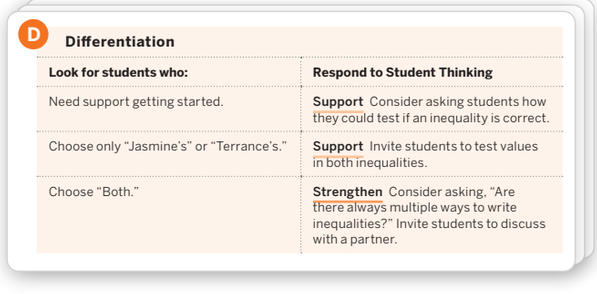
Differentiation Across the Unit

	Support	Strengthen	Stretch
	Provide targeted intervention for students by using these resources.	Reinforce students' understanding of the concepts assessed by using these resources.	Challenge students and extend their learning with these resources.
Sub-Unit 1 (Lessons 1–2) • Write and solve equations of the form $p + x = q$, $x + p = q$, $px = q$, and $\frac{x}{p} = q$, where p and q are integers.	**Mini-Lesson** • *Writing and Solving One-Step Equations*	**Practice** • Lessons 1–2	**DOK3 Practice** • Lessons 1–2
Sub-Unit 2 (Lessons 3–7) • Write inequalities using symbols and words. • Graph solutions to inequalities on a number line.	**Mini-Lessons** • *Identifying Solutions to Inequalities* • *Solving Inequalities*	**Repeated Challenges** • Lesson 6 *(Unbalanced Hangers)* • Lesson 7 *(Shira the Sheep)* **Challenge Creator** • Lesson 5 *(Comparing Weights)* **Practice** • Lessons 3–7	**You're invited to explore more.** • Lesson 3 *(Tunnel Travels)* • Lesson 4 *(Shira's Solutions)* **DOK3 Practice** • Lessons 4–6

Differentiation Within the Lesson

Each Teacher Edition includes point-of-use differentiation suggestions to modify or adjust instruction during the lesson to accommodate the needs of all learners.

Here is one example from **Lesson 5**, where students make connections between symbols that represent inequalities and unknown weights on hangers, and surface why it is possible to write more than one inequality to represent a hanger.



Differentiation

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Consider asking students how they could test if an inequality is correct.
Choose only "Jasmine's" or "Terrance's."	**Support** Invite students to test values in both inequalities.
Choose "Both."	**Strengthen** Consider asking, "Are there always multiple ways to write inequalities?" Invite students to discuss with a partner.

Accessibility

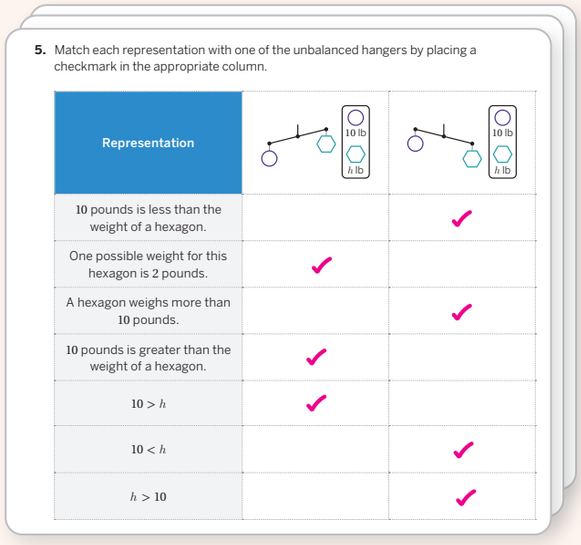
Every lesson is designed using the principles of Universal Design for Learning. Every lesson also includes support suggestions that build on students' strengths.

Universal Design for Learning

Here is one example from a lesson designed to support Representation.

Spotlight on Building Knowledge. . .

Lesson 5: In this lesson, students have opportunities to explore variations of unbalanced hangers and identify critical features to become more efficient in their learning.



5. Match each representation with one of the unbalanced hangers by placing a checkmark in the appropriate column.

Representation		
10 pounds is less than the weight of a hexagon.		✓
One possible weight for this hexagon is 2 pounds.	✓	
A hexagon weighs more than 10 pounds.		✓
10 pounds is greater than the weight of a hexagon.	✓	
$10 > h$	✓	
$10 < h$		✓
$h > 10$		✓

Just-In-Time Accommodations and Supports

Each Teacher Edition includes suggested supports based on the areas of cognitive function from the Education Development Center (EDC).

Spotlight on Conceptual Processing. . .

Here are two suggestions in this unit for supporting students with integrating ideas to understand and represent concepts.

Lesson 3, Activity 2

Accessibility: Conceptual Processing Ask students what they could do to test if an inequality is correct. If it does not come up naturally, consider inviting students to choose a number that is shaded on the graph and test it in each inequality.

Lesson 7, Activity 1

Accessibility: Conceptual Processing Invite students to talk aloud about their thinking to support self-monitoring of understanding.

Language Development

Every lesson includes opportunities for all students to develop fluency with mathematical language.

Vocabulary

New Vocabulary	Lesson(s)	Review Vocabulary	Additional Vocabulary
≥ (greater-than-or-equal-to) **≤ (less-than-or-equal-to)**	3	solution to an equation variable	boundary point
solution to an inequality	4		

Math Language Routines and Language Goals

Math Language Routine	Lesson(s)
MLR2: Collect and Display	1–4
MLR3: Critique, Correct, Clarify	7

Unit Language Goals

Students engage in the following language functions:

- Interpret diagrams and real-world situations that represent one-step equations.
- Interpret the meaning of an inequality in context, and justify whether a number is a solution to an inequality.

English Language Learners Support

Every Teacher Edition includes suggestions for supporting the needs of students learning English.

Here is one example from **Lesson 6**, where students use hanger diagrams to help visualize and determine solutions to inequalities.

ELL **English Language Learners** Encourage students to use gestures or hand motions to help them predict what the hanger should look like. **(Reading and Listening)**

Connections to Future Learning

Here is how the content in this unit connects to where your students are headed in their math journeys.

Solving Linear Equations with Variables on Both Sides

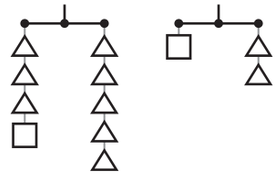
In this unit, students focus on solving equations with a variable only on one side. **In a future course**, students will learn to solve equations with variables on both sides.

FL B.E.S.T. Math Benchmark:
MA.8.AR.2.1

Example:

We can also represent equations with two variables using hanger diagrams. One way to solve the equation $3x + 5 = 5x$ is by subtracting $3x$ from both sides (or removing three triangles from both sides).

This creates the equivalent equation $5 = 2x$.
The solution to both equations is $x = \frac{5}{2}$.



$3x + 5 = 5x$ $\quad$ $5 = 2x$

Solve Two-Step Linear Inequalities

In this unit, students solve one-step inequalities and represent solutions algebraically and graphically. **In a future course**, students will solve two-step inequalities in the forms $px \pm q > r$ and $p(x \pm q) > r$ where p, q, and r are rational numbers.

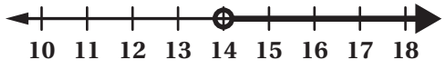
FL B.E.S.T. Math Benchmark:
MA.8.AR.2.2

Example:

You can use properties of inequalities to solve a two-step inequality. Apply the same operations to both sides of the inequality so that the inequality remains true.

The solution is $x > 14$.

$5x - 24 > 46$	Given inequality
$5x - 24 + 24 > 46 + 24$	Add 24 to both sides.
$5x > 70$	Simplify.
$\frac{5x}{5} > \frac{70}{5}$	Divide by 5 on both sides.
$x > 14$	Simplify.



Solving Systems of Linear Equations

In this unit, students solve one-step equations in and out of context. **In a future course**, students will use these solving strategies to help them solve two-step equations and systems of linear equations.

FL B.E.S.T. Math Benchmark:
MA.8.AR.4.3

Example:

Yona and Haruto's school is 600 meters from their home. Yona is running home from school at 4 meters per second and, at the same time, Haruto starts walking from home to school at 2 meters per second. When will they pass each other?

First, write an equation for each child's distance from home. When Yona and Haruto pass each other, their distances from home will be equal. You can set the expressions equal to each other, then solve for the time.

Yona: $d = 600 - 4t$

Haruto: $d = 2t$

$600 - 4t = 2t$

$600 = 6t$

$100 = t$

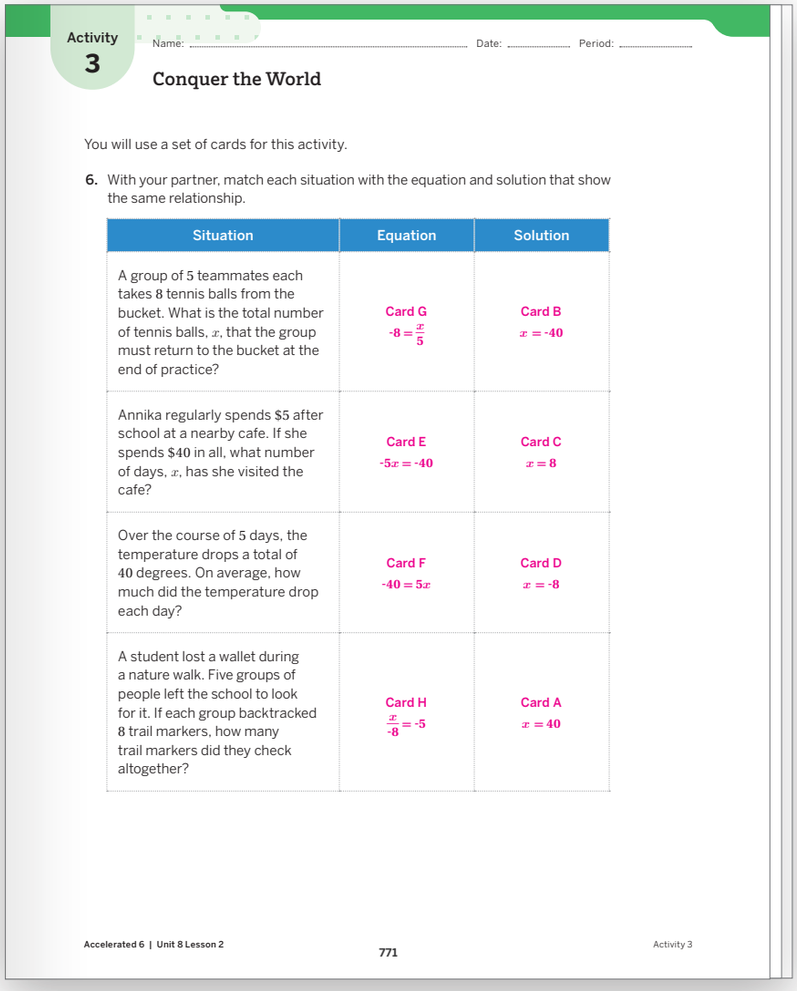
Yona and Haruto pass each other after 100 seconds.

Professional Learning

Here is an opportunity to deepen your understanding of the math in this unit and how to teach it.

Spotlight on Conceptual Understanding

Lesson 2, Activity 3



Activity 3

Name: Date: Period:

Conquer the World

You will use a set of cards for this activity.

6. With your partner, match each situation with the equation and solution that show the same relationship.

Situation	Equation	Solution
A group of 5 teammates each takes 8 tennis balls from the bucket. What is the total number of tennis balls, x, that the group must return to the bucket at the end of practice?	Card G $-8 = \frac{x}{5}$	Card B $x = -40$
Annika regularly spends \$5 after school at a nearby cafe. If she spends \$40 in all, what number of days, x, has she visited the cafe?	Card E $-5x = -40$	Card C $x = 8$
Over the course of 5 days, the temperature drops a total of 40 degrees. On average, how much did the temperature drop each day?	Card F $-40 = 5x$	Card D $x = -8$
A student lost a wallet during a nature walk. Five groups of people left the school to look for it. If each group backtracked 8 trail markers, how many trail markers did they check altogether?	Card H $\frac{x}{-8} = -5$	Card A $x = 40$

Accelerated 6 | Unit 8 Lesson 2 771 Activity 3

Context

In this activity, students develop procedural fluency as they write and solve multiplication and division equations with integers for real-word situations.

Try This

Put on your student hat and complete Problem 6.

Points to Ponder

- What do students need to understand about properties of equality and evaluating expressions with integers to support writing and solving one-step multiplication or division equations?
- How does writing and solving equations build from their understanding of writing and evaluating expressions with integers?

Notes:

Assess and Respond

Pre-Unit Check

Independent | 45 min

Facilitation: Assign the Pre-Unit Check to learn more about your students' understanding of foundational concepts and skills that will support them in the upcoming unit. This can be given in its entirety before the unit or spread throughout the unit. If time allows, consider inviting students to complete one or more of the tasks in the Unit Synthesis and Reflection.

Item Analysis

Problem	Concept or Skill	Preparation for	DOK	FL B.E.S.T. Math Benchmarks
1	Solving real-world problems involving opposite directions and explaining the solution	Lesson 1	2	MA.5.NSO.1.4
2	Estimating the locations of fractions and decimals on a number line	Lesson 3	1	MA.5.NSO.1. MTR.2.1
3a–3b	Expressing points on the number line as fractions and decimals	Lesson 3	1	MA.5.NSO.1.4 MTR. 2.1
4a–4b	Using the symbols <, >, or = to compare numbers written as decimals and fractions	Lesson 3	2	MA.5.NSO.1. MTR.3.1
5	Using the structure of the coordinate plane to write the coordinates of plotted points	Lesson 3	1	MA.5.GR.4.1 MTR. 2.1
6a–6b	Using the structure of the coordinate plane to plot points that represent real-world situations	Lesson 3	1	MA.5.GR.4.2 MTR.5.1

Assessment and Lesson Resources

- Student Assessments
- Show What You Knows and Answer Keys

Intervention and Extension Resources

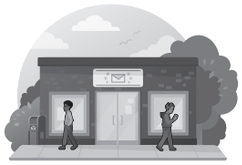
- Mini-Lessons
- Extensions

Name: Date: Period:

Pre-Unit Check

Unit 8

1. Adnan and Alexis leave the post office at the same time. Adnan walks 50 feet in one direction and Alexis walks 30 feet in the opposite direction.



If they turn to wave at each other, how far apart are they?

80 feet

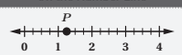
Explain or show your reasoning.

Explanations vary. Adnan is 50 feet away from the post office and Alexis is 30 feet away, so, in total, they are 80 feet away from each other.

2. Select *all* the numbers that could represent point J on the number line.

- ☐ A. $\frac{2}{3}$
- ☑ B. 2.3
- ☑ C. $\frac{7}{3}$
- ☑ D. 2.25
- ☐ E. $\frac{3}{2}$

3. Write each point on the number line as a fraction and as a decimal.

	Point	On the Number Line	As a Fraction	As a Decimal
a	P		$\frac{5}{4}$ (or equivalent)	1.25
b	Q		$\frac{28}{10}$ (or equivalent)	2.8

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Name: Date: Period:

Pre-Unit Check (continued)

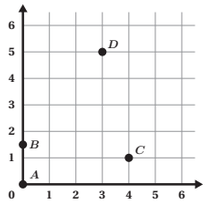
Unit 8

4. Complete each number sentence with the symbol <, >, or =.

a $\frac{4}{3}$ > $\frac{3}{4}$ b 1.5 = $\frac{3}{2}$ c $\frac{4}{5}$ > $\frac{4}{7}$ d 1.41 < 1.5

5. Write the coordinates of each point in the table.

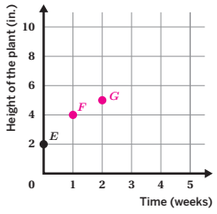
Point	Coordinates
A	$(0, 0)$
B	$(0, 1.5)$
C	$(4, 1)$
D	$(3, 5)$



6. Sneha tracked the height of a plant as it grew from week to week.

a After one week, the plant was 4 inches tall. Plot and label this point F.

b After two weeks, the plant was 5 inches tall. Plot and label this point G.



c What does point E tell us about the plant?

Responses vary. The plant was 2 inches tall when Sneha started recording its height.

D Differentiation (Pre-Unit Check)

Note: To **strengthen** and **stretch** students' learning, refer to the differentiation resources suggested throughout this unit and in the Unit Overview.

Sub-Unit Goals	Problem(s)	Respond to Student Thinking
Preparation for Sub-Unit 1: Write and solve equations of the form $p + x = q$, $x + p = q$, $px = q$, and $x/p = q$, where p and q are integers. **(Lessons 1–2)**	1	**Support** • **Teacher Move:** Before Lesson 1, consider inviting two students to act as Adnan and Alexis, and discussing their distance apart as a class using a number line.
	2, 3	**Support** • **Teacher Move:** Before Lesson 3, consider inviting students to work with a partner to place each of the five choices in Problem 2 on the number line. Then facilitate a brief whole-class discussion to surface strategies.
	4	**Support** • **Teacher Move:** Before Lesson 3, consider creating an anchor chart of the symbols <, >, and = along with expressions that use these symbols to compare numbers and a brief description in words of what the expressions mean.
	5	**Support** • **Mini-Lesson:** *Using the Coordinate Grid to Locate Points* • **Teacher Move:** Consider creating an anchor chart of the coordinate plane with labeled sample coordinates.
	6	**Support** • **Mini-Lesson:** *Plotting Points on the Coordinate Grid* • **Teacher Move:** Consider inviting students to share strategies they used with a partner before sharing with the class.

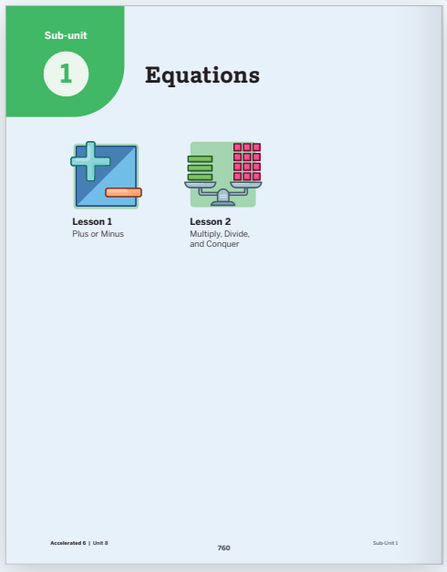
Sub-Unit 1

Equations

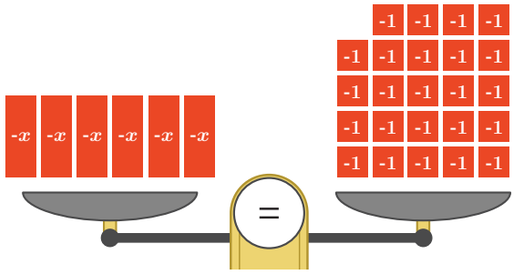
Sub-Unit 1 Goals

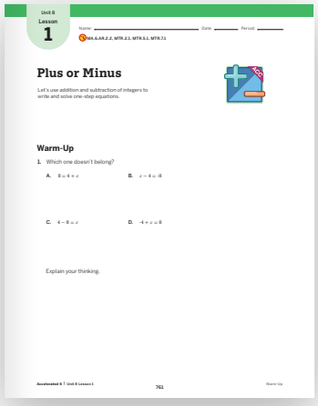
- **Goal:** Write and solve equations of the form $p + x = q$, $x + p = q$, $px = q$, and $\frac{x}{p} = q$, where p and q are integers.
- **Language Goal:** Interpret diagrams and real-world situations that represent one-step equations.

In this sub-unit, students build on their work of writing expressions and equations with variables from Unit 6 and their work with integers from Unit 7 to develop strategies for writing and solving one-step equations with integers.



Math That Matters Most

Concept or Skill	For example . . .
Write and solve addition or subtraction equations in context.	A rock drops 6 feet in a well and reaches an elevation of -15 feet. What was the rock's initial elevation? Write and solve an equation that represents the situation. $x + (-6) = -15$ $x + (-6) - (-6) = -15 - (-6)$ $x = -15 + 6$ $x = -9$
Write and solve multiplication or division equations.	Write and solve an equation that the scale represents. $-6x = -24$ $x = 4$ 


Unit 8
Lesson 1
Plus or Minus
Let's use addition and subtraction of integers to write and solve one-step equations.
Warm-Up
1. Which one doesn't belong?
A. $8 = 4 + x$
B. $x = 4 + (-8)$
C. $4 - 8 = x$
D. $-4 \neq x + 8$
Explain your thinking.
761

Plus or Minus

Solving Equations Involving Addition and Subtraction

Let's use addition and subtraction of integers to write and solve one-step equations.

Learning Outcomes

Today's Goals

1. **Goal:** Write an equation to represent a given situation.
2. **Goal:** Write and solve equations of the form $p + x = q$ and $x + p = q$, where p and q are integers.
3. **Language Goal:** Interpret real-world and mathematical situations that represent equations of the form of $p + x = q$ and $x + p = q$. **(Reading, Writing, Speaking, and Listening)**

Students use models and prior knowledge about equations and expressions to develop equations to represent real-world and mathematical situations that require addition or subtraction of integers. They apply properties of operations and equality to determine *solutions to equations*. **(MTR.2.1, MTR.5.1, MTR.7.1)**

Coherence

Prior Learning

In Unit 7, students evaluated addition and subtraction expressions with integers. In Unit 6, students wrote and solved one-step addition and subtraction equations with whole numbers.

Future Learning

In Lessons 6–7, students will solve one-step inequalities. In a future course, students will solve two-step equations involving rational numbers.

Connections

- Students analyze information given in models or real-world context to develop and solve one-step equations involving addition and subtraction of integers. (mathematical, real world)

FL B.E.S.T. Math Benchmarks

Addressing

MA.6.AR.2.2
Write and solve one-step equations in one variable within a mathematical or real-world context using addition and subtraction, where all terms and solutions are integers.

Mathematical Thinking and Reasoning Standards: MTR.2.1, MTR.5.1, MTR.7.1

ELA Expectations: ELA.K12.EE.1.1, ELA.K12.EE.4.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.5.AR.1.1	**MA.7.AR.2.1**
MA.5.AR.2.1	**MA.7.AR.2.2**
MA.5.AR.2.2	
MA.5.AR.2.3	
MA.5.AR.2.4	
MA.6.NSO.4.1	
MA.6.AR.1.1	

Vocabulary

Review Vocabulary

solution to an equation *variable*

Lesson at a Glance

~ 45 min

MA.6.AR.2.2

Warm-Up

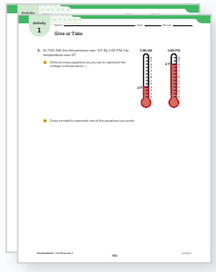
Pairs | 5 min

Students look for patterns and structure in one-step addition and subtraction equations with integers. **Routine:** Which One Doesn't Belong? **(MTR.5.1)**

Activity 1

Pairs | 15 min

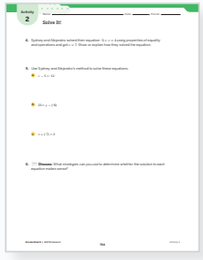
Students represent a real-world situation involving integers using multiple equations and models. **(MTR.2.1, MTR.7.1)**



Activity 2

Pairs | 15 min

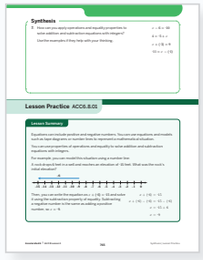
Students solve one-step addition and subtraction equations involving integers. **Routine:** Think-Pair-Share



Synthesis

Whole Class | 5 min

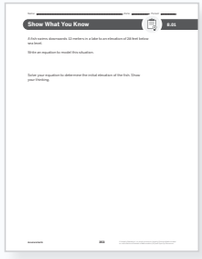
Students synthesize their understanding of solving one-step addition and subtraction equations with integers. MLR2: Collect and Display



Show What You Know

Independent | 5 min

Students demonstrate their understanding by writing and solving a one-step addition and subtraction equations involving integers in a real-world context.



Prep Checklist

This lesson includes:

- Student Edition
- Show What You Know

Pairs | 5 min

Warm-Up

Purpose: Students look for patterns and structure in one-step addition and subtraction equations with integers. **(MTR.5.1)**

1 Launch

Direct students' attention to the four equations and invite them to reflect on which equation does not belong.

Use the Which One Doesn't Belong? routine to support students in looking for patterns and structure in the equations and justifying their thinking precisely. **(MTR.5.1)**

Consider sharing that each equation does not belong in some way. Encourage students to look for more than one possibility.

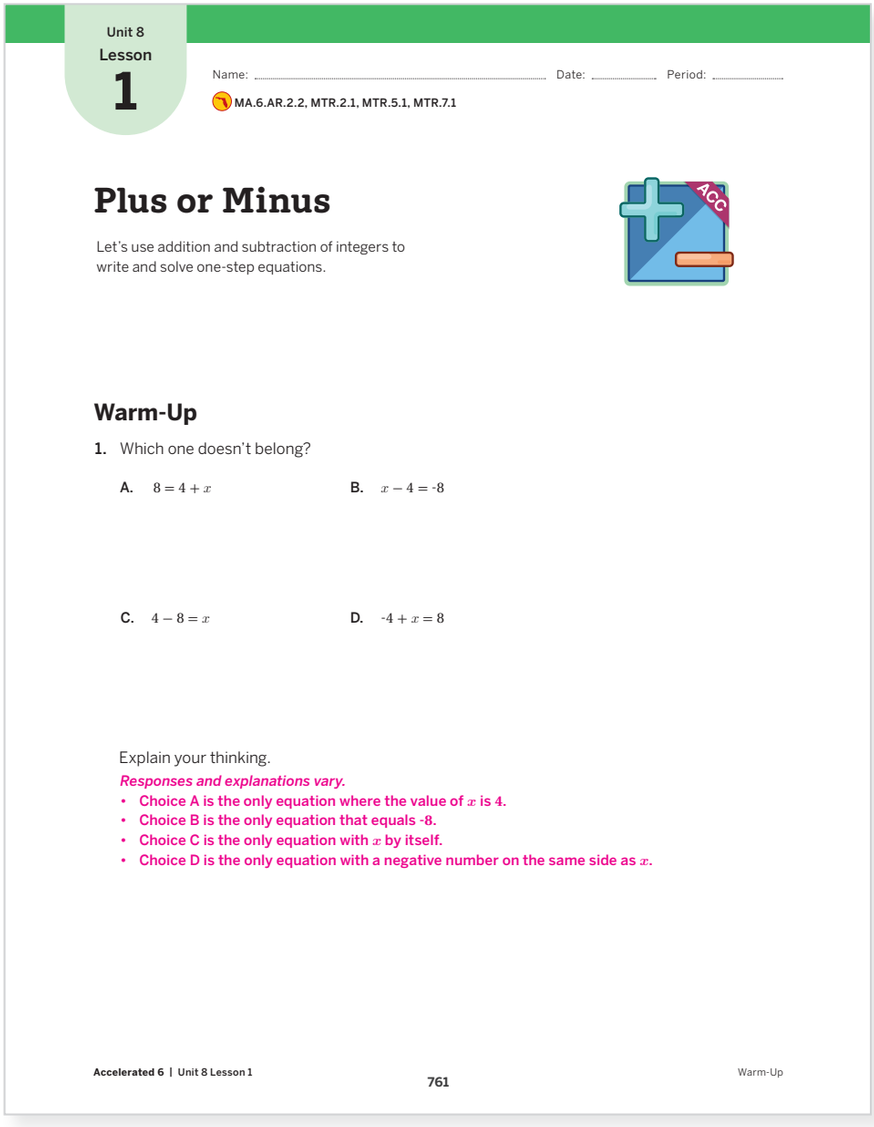
ELL **English Language Learners** Provide sentence frames. (e.g., "_____doesn't belong because _____." or " _____is the only one that has/doesn't have _____."). **(Writing and Speaking)**

2 Connect

Invite students to share at least one reason why *each* equation doesn't belong.

Consider asking:

- "Is there another reason why each equation doesn't belong?"
- "Do you see any reasons why you might group pairs of the equations?"



Unit 8
Lesson
1

Name: ..... Date: ..... Period: .....

MA.6.AR.2.2, MTR.2.1, MTR.5.1, MTR.7.1

Plus or Minus

Let's use addition and subtraction of integers to write and solve one-step equations.

Warm-Up

1. Which one doesn't belong?

A. $8 = 4 + x$

B. $x - 4 = -8$

C. $4 - 8 = x$

D. $-4 + x = 8$

Explain your thinking.

Responses and explanations vary.

- Choice A is the only equation where the value of x is 4.
- Choice B is the only equation that equals -8.
- Choice C is the only equation with x by itself.
- Choice D is the only equation with a negative number on the same side as x.

Accelerated 6 | Unit 8 Lesson 1 761 Warm-Up

Pairs | 15 min

Activity 1 Give or Take

Purpose: Students represent a real-world situation involving integers using multiple equations and models. **(MTR.2.1, MTR.7.1)**

1 Launch

Direct students' attention to the thermometers.

To surface prior knowledge, consider asking, "What is an example of an equation with a variable that involves addition? That involves subtraction?"

2 Monitor

A Accessibility: Visual-Spatial Processing
To support students in interpreting visual representations, invite students to highlight key information.

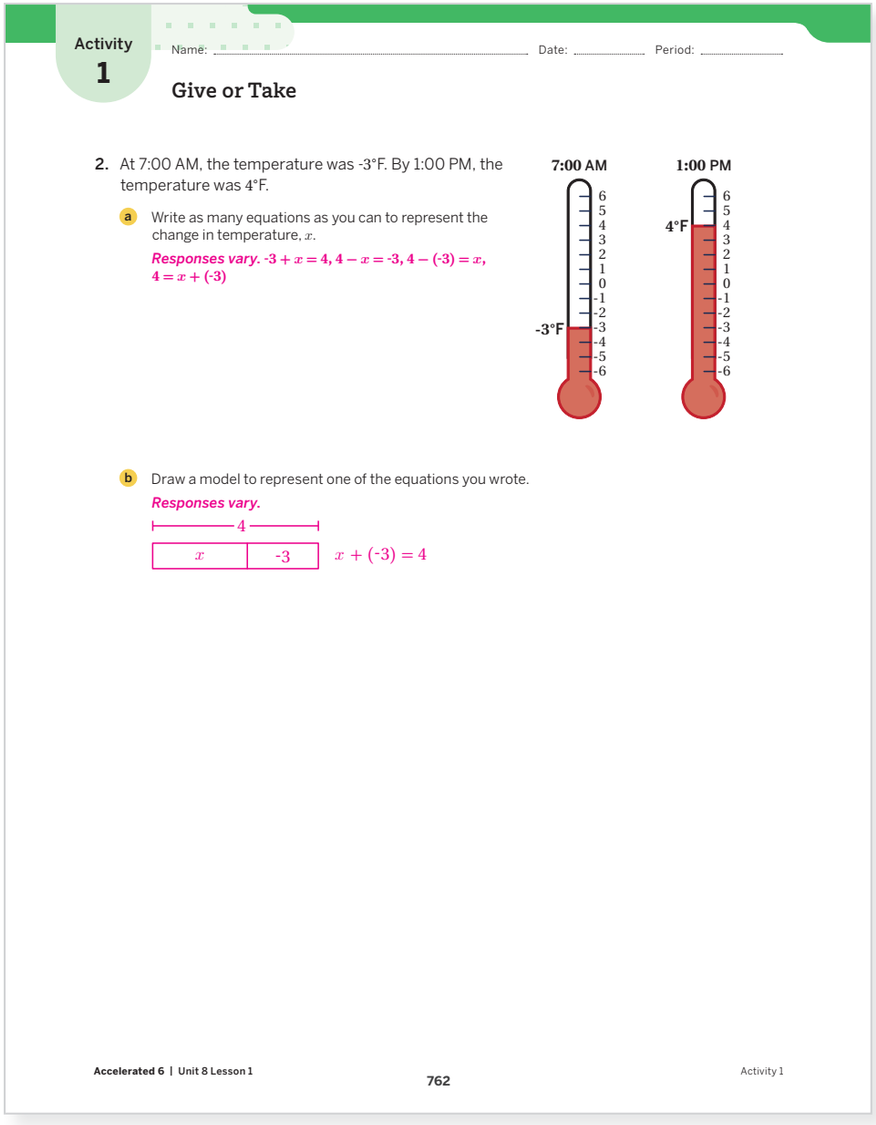
Encourage students to generate as many equations as possible to represent the situation. **(MTR.7.1)**

Look for a variety of student equations and models to share during the Connect.

D Differentiation

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Encourage students to write one equation to start and then use properties of operations to modify the equation.
Write both addition and subtraction equations. **(MTR.2.1)**	**Strengthen** Consider asking, "How are these equations alike? How are they different?"
Model an equation that does not feature an isolated variable.	**Stretch** Invite these students to share their strategy and reasoning during the Connect.

Activity 1 continued >



Activity 1

Name: _____ Date: _____ Period: _____

Give or Take

2. At 7:00 AM, the temperature was -3°F. By 1:00 PM, the temperature was 4°F.

a. Write as many equations as you can to represent the change in temperature, x.

Responses vary. $-3 + x = 4$, $4 - x = -3$, $4 - (-3) = x$, $4 = x + (-3)$

b. Draw a model to represent one of the equations you wrote.

Responses vary.

$x + (-3) = 4$

Accelerated 6 | Unit 8 Lesson 1 762 Activity 1

Pairs | 15 min

Activity 1 Give or Take (continued)

Purpose: Students represent a real-world situation involving integers using multiple equations and models. **(MTR.2.1, MTR.7.1)**

2 Monitor

Listen for explanations of student thinking and invite students to share more.

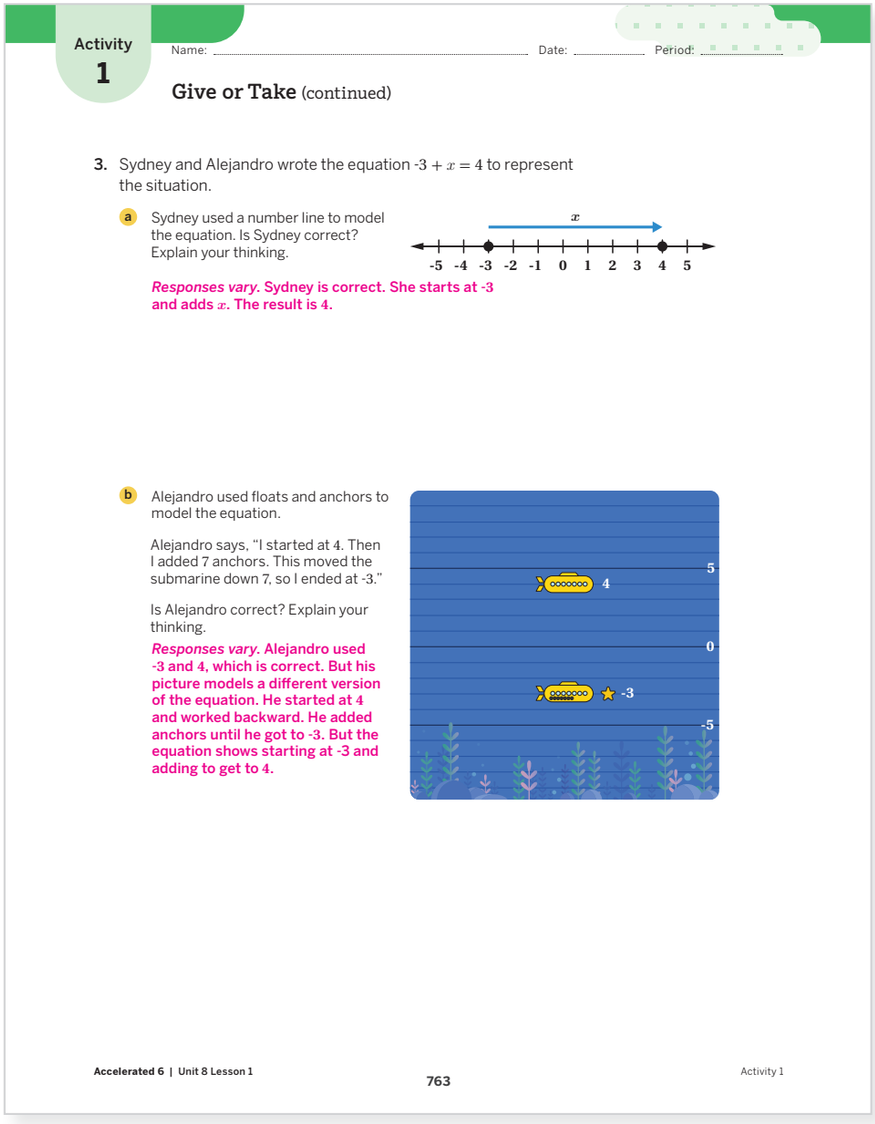
Note: Students can refer back to Unit 7, Lessons 7 and 8 to review models with floats and anchors.

3 Connect

Share student responses. Encourage students to justify their representations.

Consider asking, "How can you determine whether the model correctly represents the equation?"

To surface the Key Takeaway, consider asking, "Which equations and models did you find most helpful for understanding the situation?" **(MTR.2.1)**



Activity 1

Name: _____ Date: _____ Period: _____

Give or Take (continued)

3. Sydney and Alejandro wrote the equation $-3 + x = 4$ to represent the situation.

a. Sydney used a number line to model the equation. Is Sydney correct? Explain your thinking.

Responses vary. Sydney is correct. She starts at -3 and adds x. The result is 4.

b. Alejandro used floats and anchors to model the equation.

Alejandro says, "I started at 4. Then I added 7 anchors. This moved the submarine down 7, so I ended at -3."

Is Alejandro correct? Explain your thinking.

Responses vary. Alejandro used -3 and 4, which is correct. But his picture models a different version of the equation. He started at 4 and worked backward. He added anchors until he got to -3. But the equation shows starting at -3 and adding to get to 4.

Accelerated 6 | Unit 8 Lesson 1 763 Activity 1

Key Takeaway: You can represent a situation involving integers using an equation with a *variable*.

Pairs | 15 min

Activity 2 Solve It!

Purpose: Students solve one-step addition and subtraction equations involving integers.

Short on time? Consider inviting students to solve fewer equations in Problem 5.

1 Launch

Invite students to share what they know about properties of equality and operations.

Consider asking:
- "What is the opposite of addition?"
- "When you have an addition equation, what must you do to both sides of the equation to keep it balanced?"

2 Monitor

Look for several student explanations to share during the Connect.

Look for students who correctly use the addition and subtraction properties of equality.

Encourage students to use a visual representation if it helps their thinking. **(MTR.2.1)**

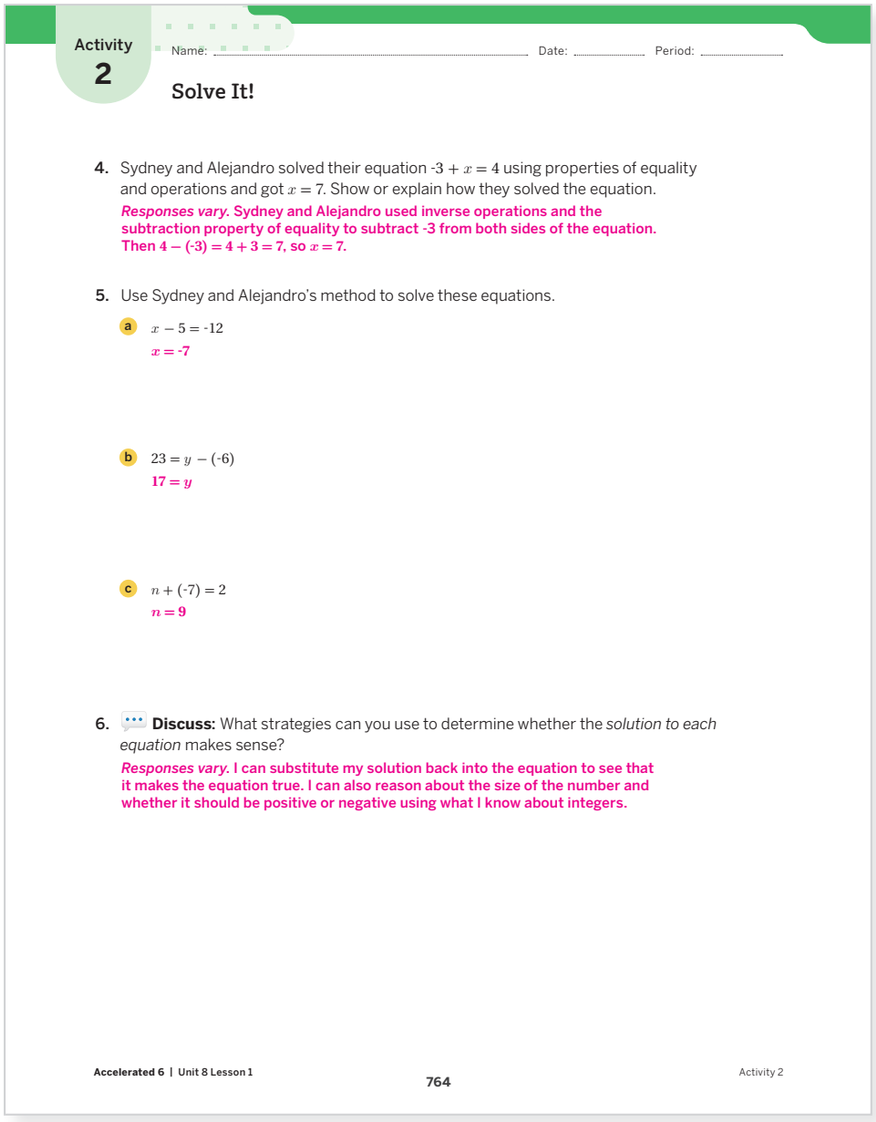
A **Accessibility: Conceptual Processing** Use objects like algebra tiles, chips, or small blocks to support students in manipulating variables in equations.

Direct students' attention to the discuss question. Invite a student to read the question aloud. Use the Think-Pair-Share routine to support student-to-student discourse about connections between equations with integers and their solutions. **(MTR.5.1)**

3 Connect

Share the student explanations.

To surface the Key Takeaway, consider asking:
- "What strategies did you use to solve the equations?"
- "How can you check if the answer is correct?"



Activity 2

Name: _____ Date: _____ Period: _____

Solve It!

4. Sydney and Alejandro solved their equation $-3 + x = 4$ using properties of equality and operations and got $x = 7$. Show or explain how they solved the equation.

Responses vary. Sydney and Alejandro used inverse operations and the subtraction property of equality to subtract -3 from both sides of the equation. Then $4 - (-3) = 4 + 3 = 7$, so $x = 7$.

5. Use Sydney and Alejandro's method to solve these equations.

a. $x - 5 = -12$

$x = -7$

b. $23 = y - (-6)$

$17 = y$

c. $n + (-7) = 2$

$n = 9$

6. **Discuss:** What strategies can you use to determine whether the *solution to each equation* makes sense?

Responses vary. I can substitute my solution back into the equation to see that it makes the equation true. I can also reason about the size of the number and whether it should be positive or negative using what I know about integers.

Accelerated 6 | Unit 8 Lesson 1 764 Activity 2

Key Takeaway: You can use what you know about properties of equality and evaluating integer expressions to determine *solutions to equations* and check that the solutions are reasonable.

Whole Class | 5 min

Synthesis

Purpose: Students synthesize their understanding of solving one-step addition and subtraction equations with integers.

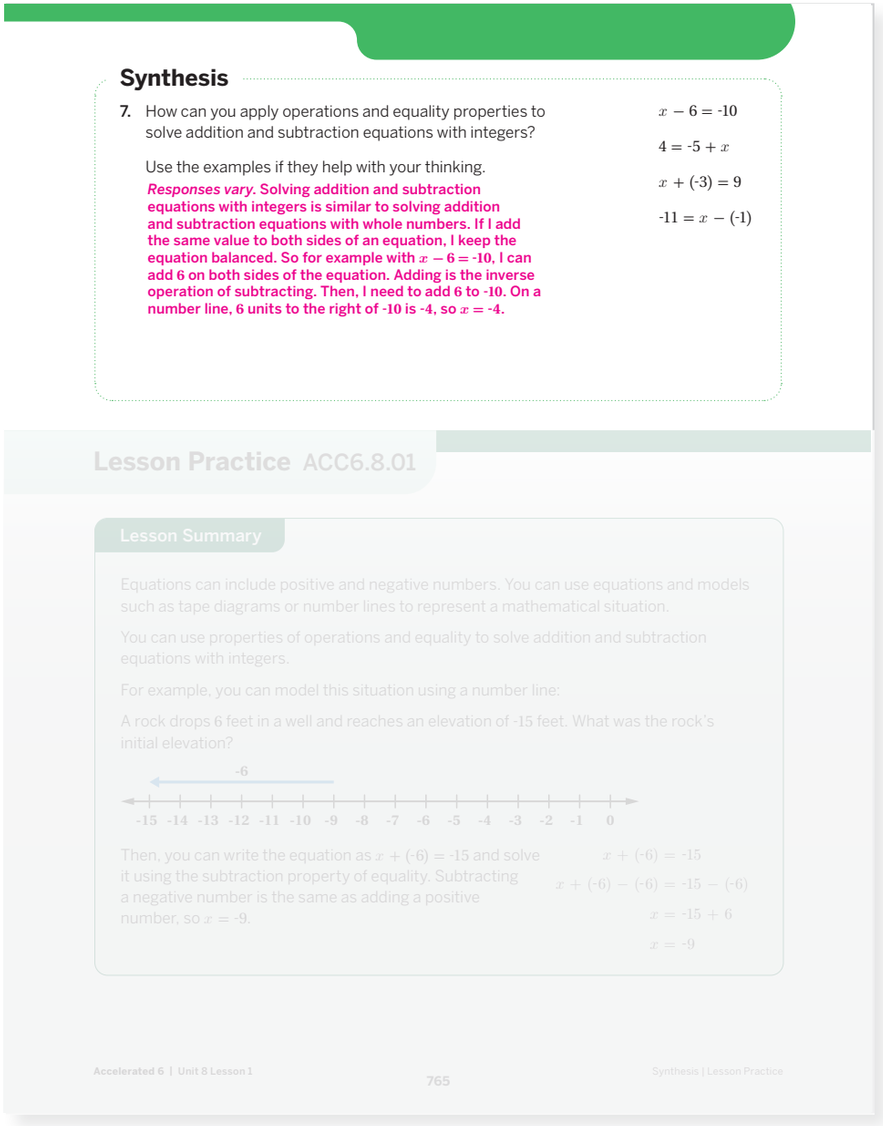
Invite students to respond independently, and then share their thinking with a partner.

Collect and share a variety of ideas, such as:

- Use inverse operations to isolate the variable.
- The opposite of addition is subtraction.
- What you do to one side of the equation you must do to the other side of the equation.
- Subtracting a negative number is the same as adding a positive number.

MLR **MLR2: Collect and Display** Use this routine to collect students' examples and strategies as they share during the Connect. Display for students to reference and add to for the rest of the unit.

Note: If time allows, invite students to share with a partner what makes sense about each strategy and what questions they still have.



Synthesis

7. How can you apply operations and equality properties to solve addition and subtraction equations with integers?

   Use the examples if they help with your thinking.

   Responses vary. Solving addition and subtraction equations with integers is similar to solving addition and subtraction equations with whole numbers. If I add the same value to both sides of an equation, I keep the equation balanced. So for example with $x - 6 = -10$, I can add 6 on both sides of the equation. Adding is the inverse operation of subtracting. Then, I need to add 6 to -10. On a number line, 6 units to the right of -10 is -4, so $x = -4$.

$x - 6 = -10$

$4 = -5 + x$

$x + (-3) = 9$

$-11 = x - (-1)$

Lesson Practice ACC6.8.01

Lesson Summary

Equations can include positive and negative numbers. You can use equations and models such as tape diagrams or number lines to represent a mathematical situation.

You can use properties of operations and equality to solve addition and subtraction equations with integers.

For example, you can model this situation using a number line:

A rock drops 6 feet in a well and reaches an elevation of -15 feet. What was the rock's initial elevation?

-6

-15 -14 -13 -12 -11 -10 -9 -8 -7 -6 -5 -4 -3 -2 -1 0

Then, you can write the equation as $x + (-6) = -15$ and solve it using the subtraction property of equality. Subtracting a negative number is the same as adding a positive number, so $x = -9$.

$x + (-6) = -15$

$x + (-6) - (-6) = -15 - (-6)$

$x = -15 + 6$

$x = -9$

Accelerated 6 | Unit 8 Lesson 1 765 Synthesis | Lesson Practice

Lesson Takeaway: Solving equations with integers involves similar strategies as solving equations with whole numbers. When solving equations with integers, apply what you know about adding and subtracting integers and properties of equality.

Independent | 5 min

Show What You Know

Purpose: Students demonstrate their understanding by writing and solving a one-step addition and subtraction equations involving integers in a real-world context.

Today's Goals

1. **Goal:** Write an equation to represent a given situation.
2. **Goal:** Write and solve equations of the form $p + x = q$ and $x + p = q$, where p and q are integers.
3. **Language Goal:** Interpret real-world and mathematical situations that represent equations of the form of $p + x = q$ and $x + p = q$. **(Reading, Writing, Speaking, and Listening)**

Show What You Know (Assessment Resources)

Name: _____ Date: _____ Period: _____

Show What You Know 8.01

A fish swims downwards 12 meters in a lake to an elevation of 28 feet below sea level.

Write an equation to model this situation.

Responses vary.

$x - 12 = -28$

Solve your equation to determine the initial elevation of the fish. Show your thinking.

16 meters below sea level (or equivalent)

Responses vary.

$x - 12 = -28$

$x - 12 + 12 = -28 + 12$

$x = -16$

Differentiation Use after Lesson 1

Support

Provide targeted intervention for students by using these resources.

If students need support writing and solving one-step addition and subtraction equations involving integers in a real-world context:

» **Respond to Student Thinking:**

- Revisit **Activity 1** (particularly Sydney and Alejandro's methods) with students before providing individualized feedback on the relevant **Practice Problems**.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 1 Extension Activities**.

Professional Learning

In this lesson, students extended their understanding of equations to include integers. How will that work support their understanding of equations that include multiplication, division, fractions, and decimals?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

7. How can you apply operations and equality properties to solve addition and subtraction equations with integers?

   Use the examples if they help with your thinking.

   Responses vary. Solving addition and subtraction equations with integers is similar to solving addition and subtraction equations with whole numbers. If I add the same value to both sides of an equation, I keep the equation balanced. So for example with $x - 6 = -10$, I can add 6 on both sides of the equation. Adding is the inverse operation of subtracting. Then, I need to add 6 to -10. On a number line, 6 units to the right of -10 is -4, so $x = -4$.

$x - 6 = -10$

$4 = -5 + x$

$x + (-3) = 9$

$-11 = x - (-1)$

Lesson Practice ACC6.8.01

Lesson Summary

Equations can include positive and negative numbers. You can use equations and models such as tape diagrams or number lines to represent a mathematical situation.

You can use properties of operations and equality to solve addition and subtraction equations with integers.

For example, you can model this situation using a number line:

A rock drops 6 feet in a well and reaches an elevation of -15 feet. What was the rock's initial elevation?

Then, you can write the equation as $x + (-6) = -15$ and solve it using the subtraction property of equality. Subtracting a negative number is the same as adding a positive number, so $x = -9$.

$x + (-6) = -15$

$x + (-6) - (-6) = -15 - (-6)$

$x = -15 + 6$

$x = -9$

Lesson Practice ACC6.8.01

Name: _____ Date: _____ Period: _____

Problems 1–4: Seawater freezes at about -2°C. The temperature of a certain sea drops 13 degrees, and the water starts freezing. What was the original temperature of the sea?

1. Draw a model to represent the situation.

   Responses vary.

2. Select all equations that can represent the situation.

   - ☑ A. $t = 13 + (-2)$
   - ☐ B. $-2 + t = 13$
   - ☐ C. $13 - (-2) = t$
   - ☑ D. $-2 = t - 13$
   - ☑ E. $t + (-13) = -2$

3. Choose one of the equations that can represent the situation and determine the solution.

   Responses vary.

   $t = 13 + (-2)$

   $t = 11$

4. Explain why your solution makes sense.

   Explanations vary. The original temperature of the sea is 11°C. Because the temperature dropped to reach freezing, the original temperature must be greater than the freezing temperature. 11 is greater than -2, so the solution makes sense.

5. Solve the equation $-8 = x - (-12)$. Show or explain your thinking.

   $-20 = x$. *Explanations vary.*

   $-8 = x - (-12)$

   $-8 + (-12) = x - (-12) + (-12)$

   $-20 = x$

Lesson Practice ACC6.8.01

Name: _____ Date: _____ Period: _____

Problems 6–7: Write an equation for each situation.

6. The hiker's starting elevation in a river valley is 6 feet below sea level. The hiker reaches a height of 14 feet below sea level. What was the hiker's change in elevation?

   Responses vary. $-6 + x = -14$

7. The number -2 is 3 less than what number?

   Responses vary. $-2 = x - 3$

8. Solve the equation you wrote in Problem 7.

   $1 = x$

FAST Practice

9. Solve the equation $-12 + z = 13$.

   $z =$ 25

Spiral Review

10. Select *all* the expressions that are equivalent to -20.

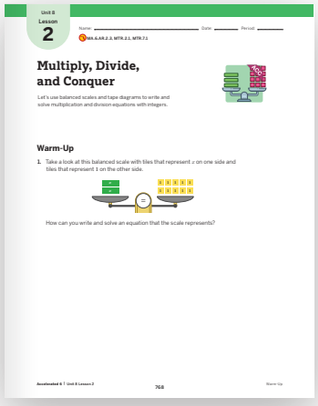
    - ☐ A. $-10 \cdot (-2)$
    - ☑ B. $-4 \cdot 5$
    - ☑ C. $40 \div (-2)$
    - ☐ D. $-100 \div (-50)$
    - ☑ E. $1 \cdot (-20)$

11. Determine the missing number. $-3 \cdot (?) = (-15)$

    $? = 5$

Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1–3	2	MA.6.AR.2.2
	4	3	MA.6.AR.2.2
	5–8	2	MA.6.AR.2.2
FAST Practice	9	2	MA.6.AR.2.2
Spiral Review			
	10	1	MA.6.NSO.4.2
	11	2	MA.6.NSO.4.2


Unit 8
Lesson
2
Name: Date: Period:
MA.6.AR.2.3, MTR.2.1, MTR.7.1
Multiply, Divide, and Conquer
Let's use balanced scales and tape diagrams to write and solve multiplication and division equations with integers.
Warm-Up
1. Take a look at this balanced scale with tiles that represent x on one side and tiles that represent 3 on the other side.
How can you write and solve an equation that the scale represents?
Accelerated 6 | Unit 8 Lesson 2
768
Warm-Up

Multiply, Divide, and Conquer

Solving Equations Involving Multiplication and Division

Let's use balanced scales and tape diagrams to write and solve multiplication and division equations with integers.

Learning Outcomes

Today's Goals

1. **Goal:** Make connections between diagrams and equations.
2. **Goal:** Write and solve equations of the form $px = q$ and $\frac{x}{p} = q$, where p and q are integers.
3. **Language Goal:** Interpret diagrams and real-world situations that represent equations of the form $px = q$ and $\frac{x}{p} = q$. **(Reading, Writing, Speaking, and Listening)**

Students connect balanced scales and tape diagrams with one-step multiplication and division equations with integers to represent problems in context. They use these tools to determine solutions to equations and interpret the meanings of those solutions. **(MTR.2.1)**

Coherence

Prior Learning

In Unit 7, students evaluated multiplication and division expressions with integers. In Unit 6, students wrote and solved one-step multiplication and division equations with whole numbers.

Future Learning

In Lessons 6–7, students will solve one-step inequalities. In a future course, students will solve two-step equations involving rational numbers.

Connections

- Students analyze information given in models or context to develop and solve one-step equations involving multiplication and division of integers. (mathematical, cross-curricular, real world)

FL B.E.S.T. Math Benchmarks

Addressing

MA.6.AR.2.3
Write and solve one-step equations in one variable within a mathematical or real-world context using multiplication and division, where all terms and solutions are integers.

Mathematical Thinking and Reasoning Standards: MTR.2.1, MTR.7.1

ELA Expectation: ELA.K12.EE.3.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.5.AR.2.4	MA.6.AR.2.4
MA.6.NSO.4.2	MA.7.AR.2.1
MA.6.AR.2.2	MA.7.AR.2.2

Lesson at a Glance

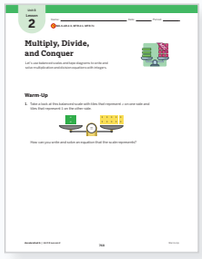
~ 45 min

MA.6.AR.2.3

Warm-Up

Pairs | 5 min

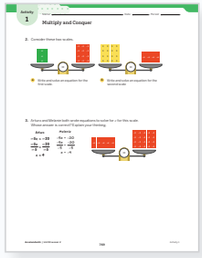
Students relate a balanced scale diagram to an equation and its solution.



Activity 1

Pairs | 10 min

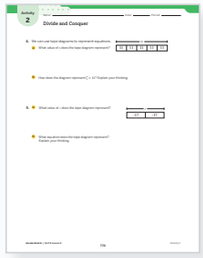
Students use balanced scales to write and solve one-step multiplication equations with integers. **(MTR.2.1)**



Activity 2

Pairs | 10 min

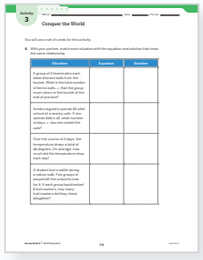
Students use tape diagrams to write and solve one-step division equations with integers. **(MTR.2.1)**



Activity 3

Pairs | 10 min

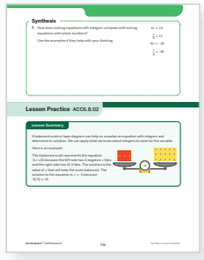
Students match multiplication and division equations and their solutions with real-world situations. **(MTR.2.1, MTR.7.1)**



Synthesis

Whole Class | 5 min

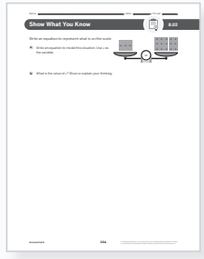
Students synthesize their understanding of solving multiplication and division equations with integers. MLR2: Collect and Display



Show What You Know

Independent | 5 min

Students demonstrate their understanding by writing a one-step multiplication equation involving integers to model a balanced scale.



Prep Checklist

Prepare Activity 3 cards.

This lesson includes:

- Student Edition
- Show What You Know
- Activity 3 cards, one set per pair, pre-cut.

Pairs | 5 min

Warm-Up

Purpose: Students relate a balanced scale diagram to an equation and its solution.

1 Launch

Encourage students to discuss the diagram with a partner.

To surface prior knowledge, consider discussing how this problem relates to problems students saw in Unit 6.

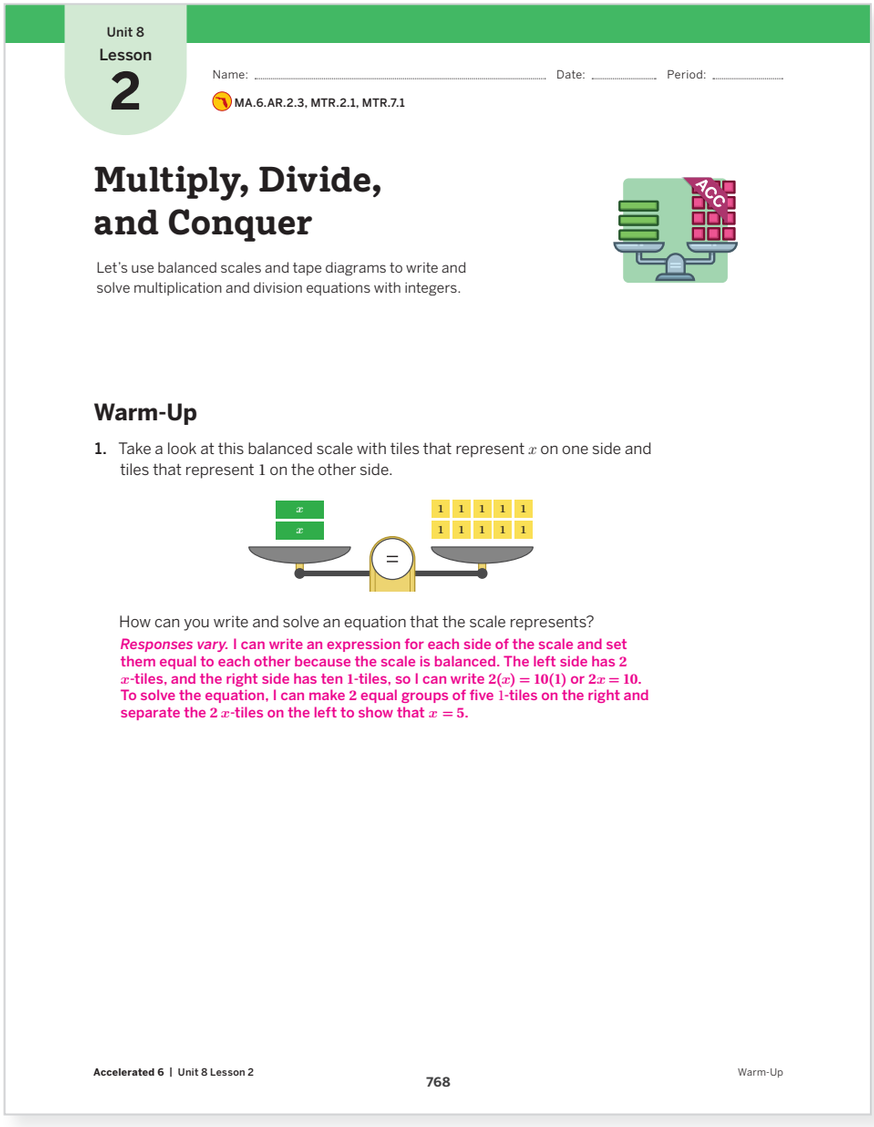
2 Connect

Invite students to share their equations and solutions to the equation.

Consider asking:

- "How is this diagram like others you have used to represent equations?"
- "Can different equations be used to represent this diagram?"
- "What methods can you use to determine the solution to the equation?"

D **Differentiation:** (Stretch) Encourage students to create a real-world problem that this diagram could represent.



Unit 8
Lesson
2

Name: ..... Date: ..... Period: .....

MA.6.AR.2.3, MTR.2.1, MTR.7.1

Multiply, Divide, and Conquer

Let's use balanced scales and tape diagrams to write and solve multiplication and division equations with integers.

Warm-Up

1. Take a look at this balanced scale with tiles that represent x on one side and tiles that represent 1 on the other side.

How can you write and solve an equation that the scale represents?

Responses vary. I can write an expression for each side of the scale and set them equal to each other because the scale is balanced. The left side has 2 x-tiles, and the right side has ten 1-tiles, so I can write $2(x) = 10(1)$ or $2x = 10$. To solve the equation, I can make 2 equal groups of five 1-tiles on the right and separate the 2 x-tiles on the left to show that $x = 5$.

Accelerated 6 | Unit 8 Lesson 2 768 Warm-Up

Pairs | 10 min

Activity 1 Multiply and Conquer

Purpose: Students use balanced scales to write and solve one-step multiplication equations with integers. **(MTR.2.1)**

1 Launch

Direct students' attention to the balanced scales. Invite students to share what they notice.

2 Monitor

Encourage students to justify each solution with their partner.

D Differentiation

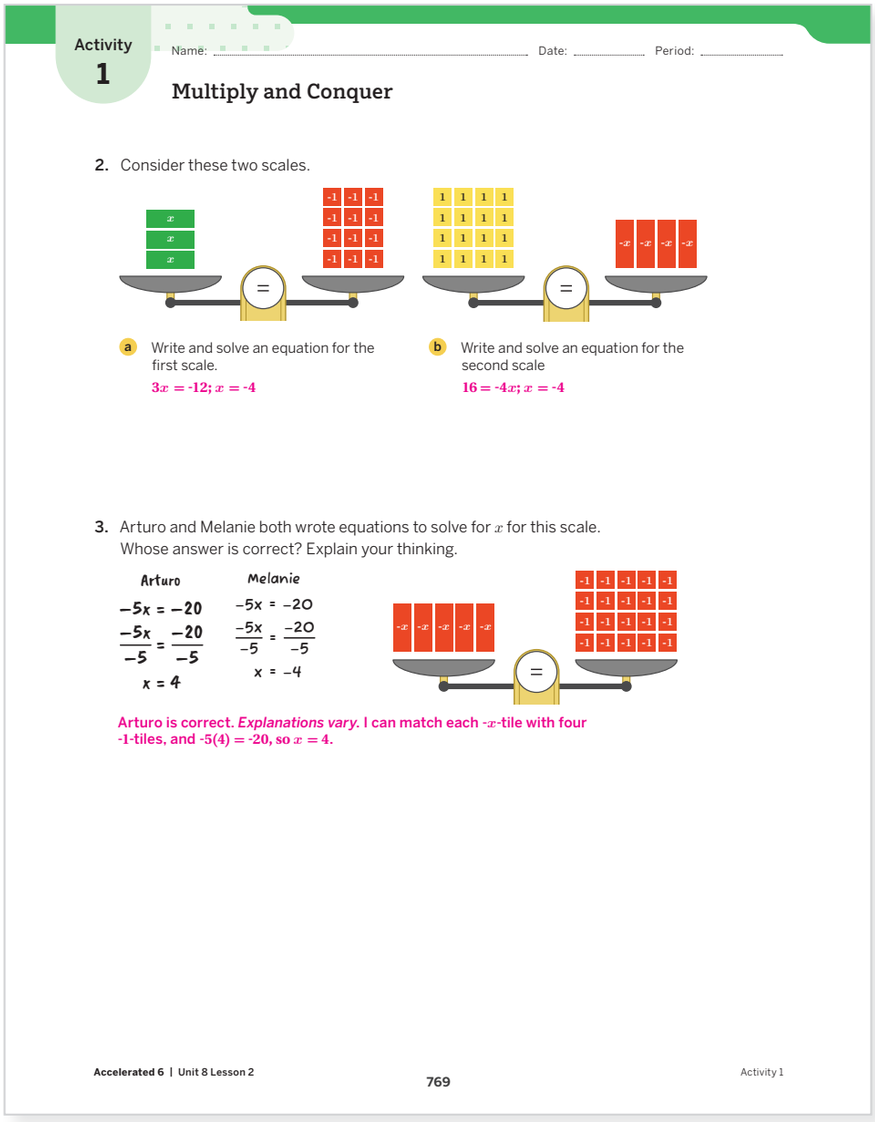
Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Consider asking, "How many x tiles do you count? How many negative 1 tiles do you count?"
Use a variety of strategies like: • Removing the same weight from both sides. • Dividing the elements on the scale into equal groups. • Writing expressions or equations. **(MTR.2.1)**	**Strengthen** Invite students to compare their strategy with a classmate's.
Substitute the solution into the equation to check.	**Stretch** Invite these students to share their strategy and reasoning during the Connect.

3 Connect

Invite students to share a variety of strategies for determining the value of x for each balanced scale.

Note: If it does not come up naturally, consider asking, "Does it matter whether the variables are on the left side or on the right side of a scale or an equation? Why or why not?" No. As long as the scale is balanced or the equation is true, you can represent the variables on the left side or the right side.

To surface the Key Takeaway, consider asking, "How does solving these equations relate to evaluating expressions with integers?"



Activity 1

Name: _____ Date: _____ Period: _____

Multiply and Conquer

2. Consider these two scales.

a. Write and solve an equation for the first scale.

$3x = -12; x = -4$

b. Write and solve an equation for the second scale

$16 = -4x; x = -4$

3. Arturo and Melanie both wrote equations to solve for x for this scale. Whose answer is correct? Explain your thinking.

Arturo

$-5x = -20$

$\frac{-5x}{-5} = \frac{-20}{-5}$

$x = 4$

Melanie

$-5x = -20$

$\frac{-5x}{-5} = \frac{-20}{-5}$

$x = -4$

Arturo is correct. *Explanations vary.* I can match each $-x$-tile with four -1-tiles, and $-5(4) = -20$, so $x = 4$.

Accelerated 6 | Unit 8 Lesson 2 769 Activity 1

Key Takeaway: Use what you know about evaluating expressions with integers to determine the solution to an equation with integers.

Pairs | 10 min

Activity 2 Divide and Conquer

Purpose: Students use tape diagrams to write and solve one-step division equations with integers. **(MTR.2.1)**

1 Launch

Direct students' attention to the tape diagram. Invite students to describe the diagram in their own words.

Note: This may be the first time students see a variable and a constant together in a fraction (e.g., $\frac{n}{5}$). Discuss with students how to interpret this notation.

2 Monitor

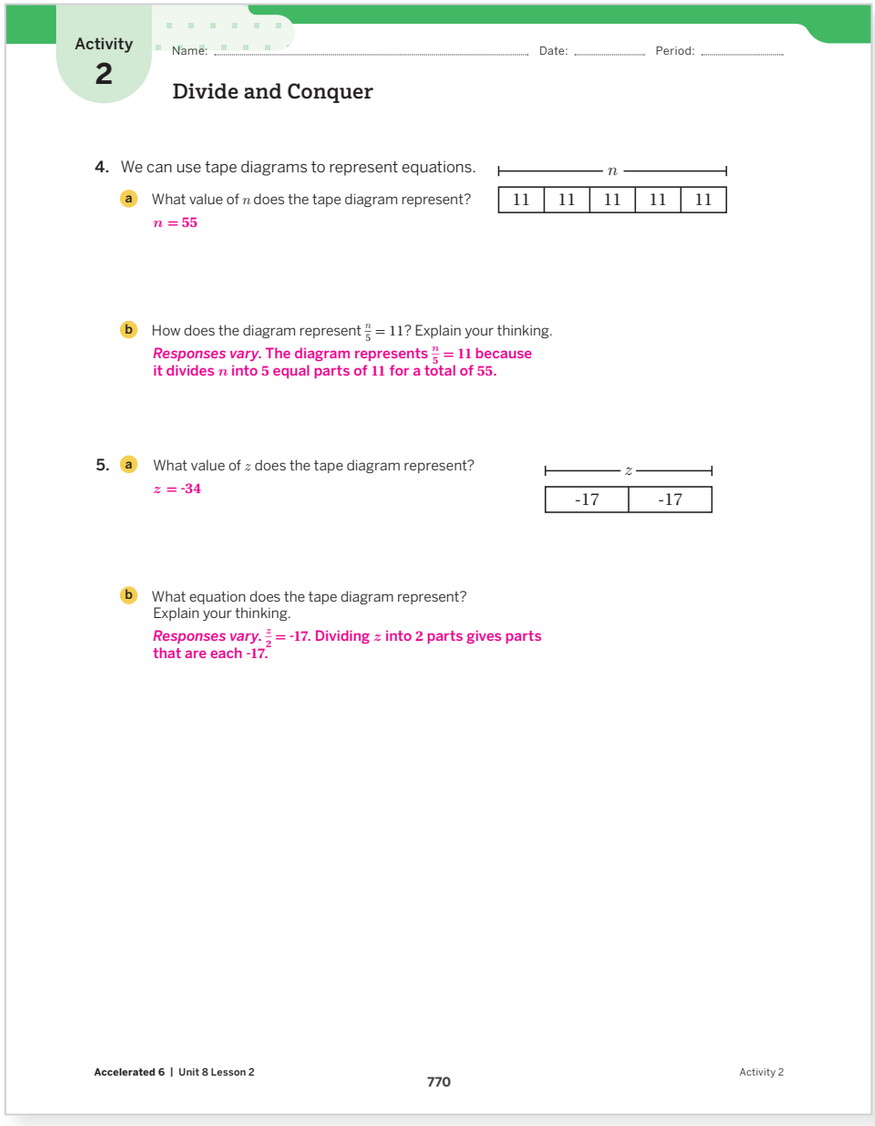
Invite students to share the equations they wrote. Discuss whether there are other equations that they could write to represent the tape diagram. **(MTR.2.1)**

Look for students who use terms like *division*, *equal parts*, and *whole* or *total* in their reasoning.

3 Connect

Invite students to share their reasoning for how each tape diagram represents each equation and how they determined the value of the variable in each problem.

Note: If this has not been discussed yet, this is a good time to mention the concept of using inverse operations. "These tape diagrams represent division equations, which can be solved using multiplication, while the balanced scales show multiplication equations, which can be solved using division."



Activity 2

Name: Date: Period:

Divide and Conquer

4. We can use tape diagrams to represent equations.

a. What value of n does the tape diagram represent?

$n = 55$

b. How does the diagram represent $\frac{n}{5} = 11$? Explain your thinking.

Responses vary. The diagram represents $\frac{n}{5} = 11$ because it divides n into 5 equal parts of 11 for a total of 55.

5. a. What value of z does the tape diagram represent?

$z = -34$

b. What equation does the tape diagram represent? Explain your thinking.

Responses vary. $\frac{z}{2} = -17$. Dividing z into 2 parts gives parts that are each -17.

Accelerated 6 | Unit 8 Lesson 2 770 Activity 2

Pairs | 10 min

Activity 3 Conquer the World

Purpose: Students match multiplication and division equations and their solutions with real-world situations. **(MTR.2.1, MTR.7.1)**

Short on time? Leave discussion for how students matched equations and solutions for another day.

1 Launch

Direct students' attention to the instructions for the activity and review them as a class.

Distribute one set of the Activity 3 Cards to each pair of students.

ELL **English Language Learners** Invite students to read each situation aloud and paraphrase what it means. Clarify any words they don't know. **(Reading)**

2 Monitor

A **Accessibility: Visual-Spatial Processing** Invite students to compare and contrast each equation before responding to support recognizing the differences among the equations.

Look for students who select an incorrect equation to represent a situation. Ask them how they think the equation represents that situation.

Invite students to share how they determined the solution for each equation.

Listen for explanations that students have for how they chose an equation to represent each real-world situation. **(MTR.2.1, MTR.7.1)**

3 Connect

Invite students to share their explanations.

To surface the Key Takeaway, consider asking, "What words in each situation provide clues about whether each number is positive or negative?"

Activity 3

Name: _____ Date: _____ Period: _____

Conquer the World

You will use a set of cards for this activity.

6. With your partner, match each situation with the equation and solution that show the same relationship.

Situation	Equation	Solution
A group of 5 teammates each takes 8 tennis balls from the bucket. What is the total number of tennis balls, x, that the group must return to the bucket at the end of practice?	Card G $-8 = \frac{x}{5}$	Card B $x = -40$
Annika regularly spends \$5 after school at a nearby cafe. If she spends \$40 in all, what number of days, x, has she visited the cafe?	Card E $-5x = -40$	Card C $x = 8$
Over the course of 5 days, the temperature drops a total of 40 degrees. On average, how much did the temperature drop each day?	Card F $-40 = 5x$	Card D $x = -8$
A student lost a wallet during a nature walk. Five groups of people left the school to look for it. If each group backtracked 8 trail markers, how many trail markers did they check altogether?	Card H $\frac{x}{-8} = -5$	Card A $x = 40$

Accelerated 6 | Unit 8 Lesson 2 771 Activity 3

Key Takeaway: You can represent real-world situations with multiplication and division equations using integers.

Whole Class | 5 min

Synthesis

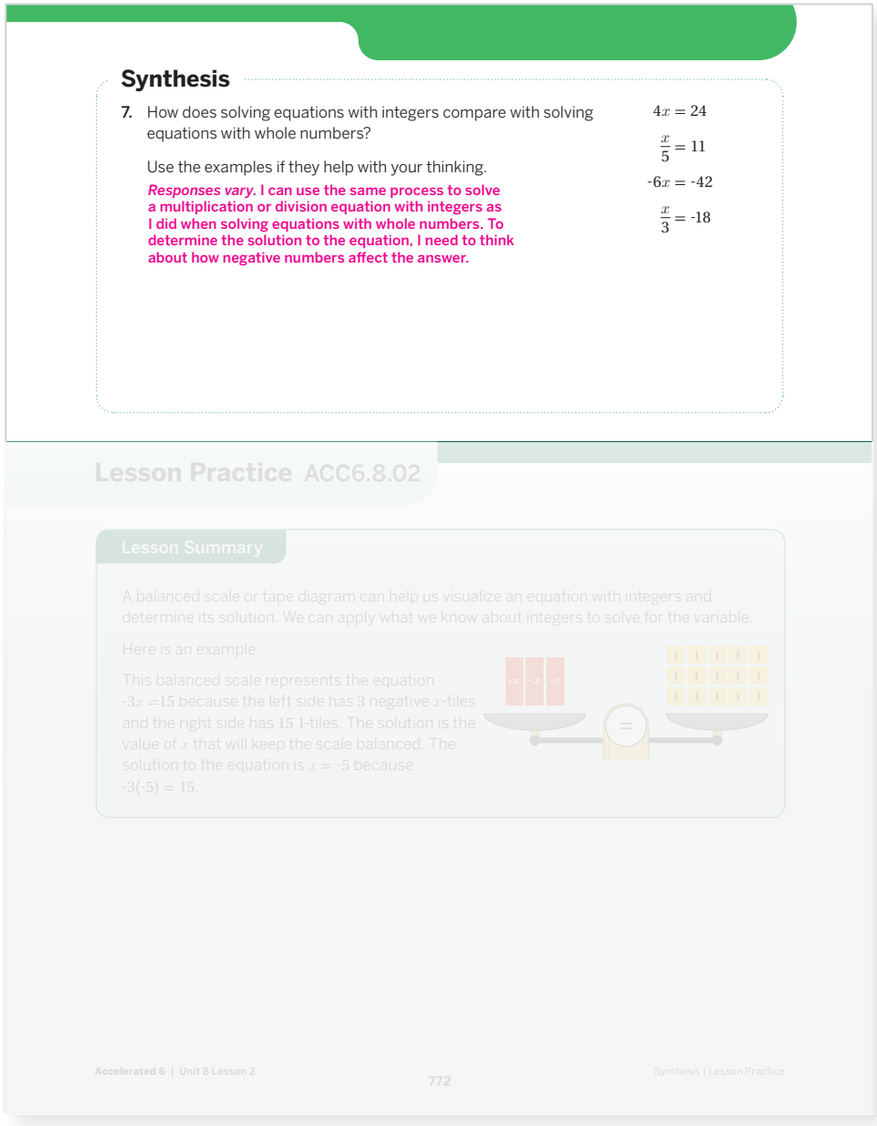
Purpose: Students synthesize their understanding of solving multiplication and division equations with integers.

Invite students to respond independently, and then share their thinking with a partner.

Collect and share a variety of ideas, such as:
- Using inverse operations.
- Relating equations with algebra tiles.
- Applying strategies for evaluating expressions with integers.

MLR **MLR2: Collect and Display** Use this routine to formalize the strategy for solving one-step multiplication and division problems with integers using students' language. Display any new language, examples, or diagrams publicly for students to reference later.

Note: If time allows, invite students to share how they would use a tape diagram or balanced scale to solve problems in real-world situations involving multiplication and division of integers.



Synthesis

7. How does solving equations with integers compare with solving equations with whole numbers?

   Use the examples if they help with your thinking.

   Responses vary. I can use the same process to solve a multiplication or division equation with integers as I did when solving equations with whole numbers. To determine the solution to the equation, I need to think about how negative numbers affect the answer.

$4x = 24$

$\frac{x}{5} = 11$

$-6x = -42$

$\frac{x}{3} = -18$

Lesson Practice ACC6.8.02

Lesson Summary

A balanced scale or tape diagram can help us visualize an equation with integers and determine its solution. We can apply what we know about integers to solve for the variable.

Here is an example.

This balanced scale represents the equation $-3x = 15$ because the left side has 3 negative x-tiles and the right side has 15 1-tiles. The solution is the value of x that will keep the scale balanced. The solution to the equation is $x = -5$ because $-3(-5) = 15$.

Accelerated 6 | Unit 8 Lesson 2 772 Synthesis | Lesson Practice

Lesson Takeaway: You can apply the same process for solving equations with whole numbers to solving multiplication and division equations with integers. To determine the solution to an equation, apply what you know about integer operations.

Independent | 5 min

Show What You Know

Purpose: Students demonstrate their understanding by writing a one-step multiplication equation involving integers to model a balanced scale.

Today's Goals

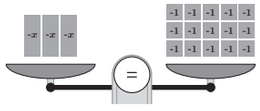
1. **Goal:** Make connections between diagrams and equations.
2. **Goal:** Write and solve equations of the form $px = q$ and $\frac{x}{p} = q$, where p and q are integers.
3. **Language Goal:** Interpret diagrams and real-world situations that represent equations of the form $px = q$, and $\frac{x}{p} = q$. **(Reading, Writing, Speaking, and Listening)**

Show What You Know (Assessment Resources)

Name: _____ Date: _____ Period: _____

Show What You Know 8.02

Write an equation to represent what is on the scale.



a. Write an equation to model this situation. Use x as the variable.

$-3x = -15$ (or equivalent)

b. What is the value of x? Show or explain your thinking.

Explanations vary.

For every $-x$ on the left side of the scale, there are 5(-1)'s on the right side of the scale. So, 1 group of 5(-1)'s matches each $-x$.

The solution for x is 3 because $-3(5) = -15$.

Differentiation Use after Lesson 2

Support

Provide targeted intervention for students by using these resources.

If students need support writing a one-step multiplication equation involving integers to model a balanced scale.

» **Respond to Student Thinking:**

- Using the ***Writing and Solving One-Step Equations*** **Mini-Lesson**.
- Revisit the **Lesson Summary**.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 1 Extension Activities**.

Professional Learning

Consider how you will teach the next several lessons. How will students' work relating diagrams with solving multiplication and division equations with integers prepare them for solving inequalities?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

7. How does solving equations with integers compare with solving equations with whole numbers?

Use the examples if they help with your thinking.

Responses vary. I can use the same process to solve a multiplication or division equation with integers as I did when solving equations with whole numbers. To determine the solution to the equation, I need to think about how negative numbers affect the answer.

$4x = 24$

$\frac{x}{5} = 11$

$-6x = -42$

$\frac{x}{3} = -18$

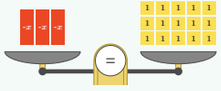
Lesson Practice ACC6.8.02

Lesson Summary

A balanced scale or tape diagram can help us visualize an equation with integers and determine its solution. We can apply what we know about integers to solve for the variable.

Here is an example.

This balanced scale represents the equation $-3x = 15$ because the left side has 3 negative x-tiles and the right side has 15 1-tiles. The solution is the value of x that will keep the scale balanced. The solution to the equation is $x = -5$ because $-3(-5) = 15$.

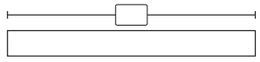


Lesson Practice ACC6.8.02

Name: _____ Date: _____ Period: _____

Problems 1–4: A scuba diver goes from sea level to the ocean floor in 7 minutes. If he dives down 4 meters each minute, what is the elevation of the ocean floor?

1. Draw a tape diagram to represent the situation.



Responses vary. The total is unknown, so it can be represented by a variable. The tape diagram has 7 equal groups of -4.

2. Select *all* the equations that can represent this situation.

- ☐ A. $7m = -4$
- ☐ B. $4m = -7$
- ☑ C. $\frac{m}{7} = -4$
- ☑ D. $\frac{m}{-4} = 7$
- ☑ E. $m = 7(-4)$

3. Determine the solution to one of the selected equations from Problem 2.

$m = -28$

4. Explain the solution's meaning in this situation.

Responses vary. The ocean floor is 28 meters below sea level.

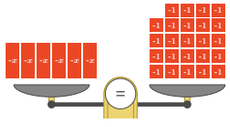
Problems 5–6: Consider the balanced scale.

5. Write an equation that the scale represents.

$-6x = -24$

6. Solve the equation for the value of x.

$x = 4$



Lesson Practice ACC6.8.02

Name: _____ Date: _____ Period: _____

FAST Practice

7. Solve the equation $\frac{y}{-8} = 6$.

$y =$ -48

Spiral Review

Problems 8–11: Determine the value of each expression.

8. $-5 + 7$

2

9. $-5 - 7$

-12

10. $-3 + 9 + (-7) + 4 + (-6) + (-5)$

-8

11. $-8 - (-7) + (-6) - 5 + 9$

-3

Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1–3, 5–6	2	MA.6.AR.2.3
	4	3	MA.6.AR.2.3
FAST Practice	7	2	MA.6.AR.2.3
Spiral Review			
	8–11	1	MA.6.NSO.4.1

Assess and Respond

Sub-Unit Quiz 1

Independent | 45 min

Facilitation: Assign the Sub-Unit Quiz to learn about your students' understanding of the concepts and skills so far in this unit. If time allows, consider inviting students to complete one or more of the tasks in the Unit Synthesis and Reflection.

Item Analysis

Problem(s)	Concept or skill	Addressed in	DOK	FL B.E.S.T. Math Benchmarks
1	Identifying solutions to one-step equations involving addition and subtraction of integers	Lesson 1	1	MA.6.AR.2.2 MTR.3.1
2a–2d	Determining the value of the variable that makes one-step equations true	Lesson 1	1	MA.6.AR.2.2 MTR.3.1
3a–3d	Solving one-step equations involving addition and subtraction of integers	Lesson 1	1	MA.6.AR.2.2 MTR.3.1
4a–4d	Solving one-step equations involving multiplication and division of integers	Lesson 2	1	MA.6.AR.2.3 MTR.3.1
5	Solving a real-world problem involving multiplication and division of integers	Lesson 2	2	MA.6.AR.2.3 MTR.7.1

Assessment Resources

- Student Assessments
- Assessment Rubrics with Answers
- Show What You Knows and Answer Keys

Intervention and Extension Resources

- Mini-Lessons
- Extensions

Name: ..... Date: ..... Period: .....

Sub-Unit Quiz

Unit 8

1. Which equation has a solution of -22?

A. $x + 13 = 9$

B. $d + 9 = -13$

C. $a - 13 = -9$

D. $n - 9 = 13$

2. Determine the value of the variable that makes each equation true.

a. $33 + b = 17$
$b = -16$

b. $t - 4 = -3$
$t = 1$

c. $-6 + s = 7$
$s = 13$

d. $m + 8 = -4$
$m = -12$

3. Solve each equation.

a. $-3 + a = 5$
$a = 8$

b. $m - 9 = 9$
$m = 18$

c. $7 + x = 1$
$x = -6$

d. $y + 7 = -4$
$y = -11$

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Name: ..... Date: ..... Period: .....

Sub-Unit Quiz (continued)

Unit 8

4. Solve each equation.

a. $3k = 21$
$k = 7$

b. $-4g = 20$
$g = -5$

c. $\frac{d}{3} = -2$
$d = -6$

d. $\frac{p}{-5} = -7$
$p = 35$

5. A mole starts digging at ground level. Ground level is represented as 0. The mole is digging at a rate of -2 inches per minute, which indicates digging deeper underground. The equation $-2m = -28$ represents how many minutes, m, it will take the mole to reach 28 inches below ground level. How long will it take the mole to reach 28 inches below ground level?
14 minutes

D Differentiation (Sub-Unit Quiz 1)

Note: To **strengthen** and **stretch** students' learning, refer to the differentiation resources suggested throughout this unit and in the Unit Overview.

Sub-Unit Goals	Problem(s)	To respond to student thinking, consider:	

Sub-Unit 1:
• Write and solve equations of the form $p + x = q$, $x + p = q$, $px = q$, and $x/p = q$, where p and q are integers. **(Lessons 1–2)**
	1, 2a–2d, 3a–3d	**Support**	• **Mini-Lesson:** *Writing and Solving One-Step Equations*
			• **Teacher Move:** Consider revisiting:
			  • Unit 7, Lesson 7 (*Floats and Anchors*)
			  • Unit 7, Lesson 8 (*More Floats and Anchors*)

		Strengthen	• **You're Invited to Explore More** • Unit 7, Lesson 8 (*More Floats and Anchors*)

	4a–4d, 5	**Support**
		Strengthen
			• **Mini-Lesson:** Writing and Solving One-Step Equations
			  • Unit 7, Lesson 3 (*Sub-Zero*)
			  • Unit 7, Lesson 9 (*Up, Up, and Away!*)

		Stretch	• **You're invited to explore more:** • Unit 7, Lesson 3 (*Sub-Zero*)

Sub-Unit 2

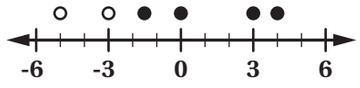
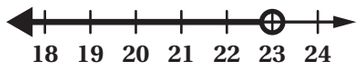
Inequalities

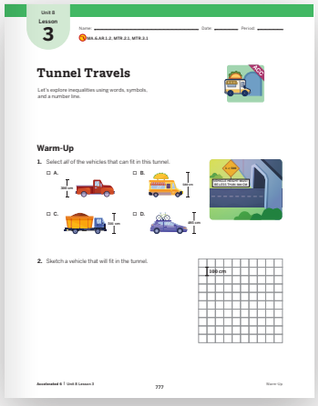
Sub-Unit 2 Goals

- **Goal:** Write inequalities using symbols and words.
- **Goal:** Graph solutions to inequalities on a number line.
- **Goal:** Solve one-step inequalities with positive and negative coefficients.
- **Language Goal:** Interpret the meaning of an inequality in context.
- **Language Goal:** Justify whether a number is a solution to an inequality.

In this sub-unit, students write verbal and symbolic inequality statements with variables representing upper or lower bounds. Combined with the development of negative rational numbers in Unit 7, students identify and graph **solutions to an inequality** on a number line that can contain negative values. They also use what they have learned about solving equations to solve inequalities that represent mathematical and real-world problems.

Math That Matters Most

Concept or Skill	For example . . .
Write inequalities using words and symbols.	Write an inequality so that all of the solid points are solutions and none of the open points are.  $x \geq -1.5$ $-2.5 < x$
Graph solutions to inequalities on a number line.	Rewrite the inequality $x > 23$ so that it matches the graph.  $x < 23$ or $23 > x$
Write and interpret inequalities in context.	At a book sale, all books cost less than \$5. Write an inequality to represent the cost of a book at the book sale, b. $b < 5$ or $5 > b$
Solve inequalities with positive and negative coefficients.	Solve this inequality. $-6x < 60$ $x > -10$



Tunnel Travels

Graphing Inequalities

Let's explore inequalities using words, symbols, and a number line.

Learning Outcomes

Today's Goals

1. **Goal:** Connect verbal, symbolic, and number line representations of inequalities in context.
2. **Language Goal:** Describe and interpret real-world inequalities. **(Reading, Writing, Speaking, and Listening)**

Students are introduced to inequalities with variables using the contexts of vehicles fitting in a tunnel and children riding an amusement park ride. They connect verbal descriptions, symbols, and number line representations of inequalities.

Coherence

Prior Learning

In Unit 7, students compared numbers using inequality symbols and plotted rational numbers on number line diagrams.

Future Learning

In Lesson 4, students will draw and label a number line diagram to represent the solutions to an inequality. They will also justify whether a value is a solution to a given inequality.

Connections

- Students explore inequality representations using context and number line models. (mathematical, real world)

FL B.E.S.T. Math Benchmarks

Addressing

MA.6.AR.1.2
Translate a real-world written description into an algebraic inequality in the form of $x > a$, $x < a$, $x \geq a$ or $x \leq a$. Represent the inequality on a number line.

Mathematical Thinking and Reasoning Standards: MTR.2.1, MTR.3.1

ELA Expectations: ELA.K12.EE.1.1, ELA.K12.EE.3.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.5.NSO.1.4	MA.7.AR.2.1
MA.5.AR.2.1	
MA.5.AR.2.4	

Vocabulary

New Vocabulary

≥ (greater-than-or-equal-to)
≤ (less-than-or-equal-to)

Additional Vocabulary

boundary point

Lesson at a Glance

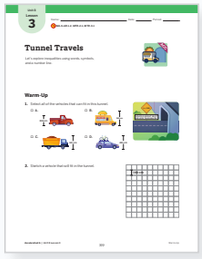
~ 45 min

MA.6.AR.1.2

Warm-Up

Independent | 5 min

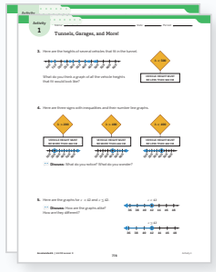
Students explore inequalities with variables in context.



Activity 1

Pairs | 20 min

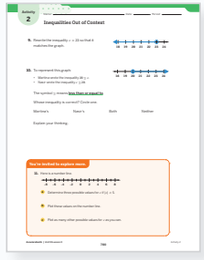
Students connect verbal descriptions, symbols, and number line representations of inequalities.
Routines: Notice and Wonder; MLR2: Collect and Display **(MTR.2.1)**



Activity 2

Pairs | 10 min

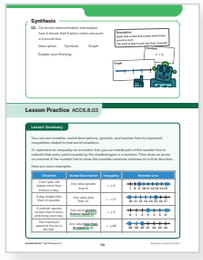
Students make sense of inequality symbols and graphs out of context and explore inequalities with the variable on the right side, such as $19 < x$.
Routine: Decide and Defend



Synthesis

Whole Class | 5 min

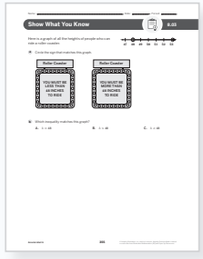
Students synthesize their understanding of different representations of inequalities.



Show What You Know

Independent | 5 min

Students demonstrate their understanding by matching words and inequalities to a graph.



Prep Checklist

This lesson includes:

- Student Edition
- Show What You Know
- Measuring tapes or meter sticks (optional)

Independent | 5 min

Warm-Up

Purpose: Students explore inequalities with variables in context.

Short on time? Consider omitting Problem 2.

1 Launch

Direct students' attention to the vehicles and the tunnel and invite a student to read the signs aloud.

To surface connections, invite students to share any experiences they have with height limits or similar signs. Consider using a meter stick or measuring tape and asking questions like:

- "Would you fit in the tunnel? How do you know?"
- "Would a vehicle as tall as the classroom fit in the tunnel?"

Invite students to share their responses, calling attention to any conflict or consensus. Invite students to share how they determined which vehicle fit in the tunnel.

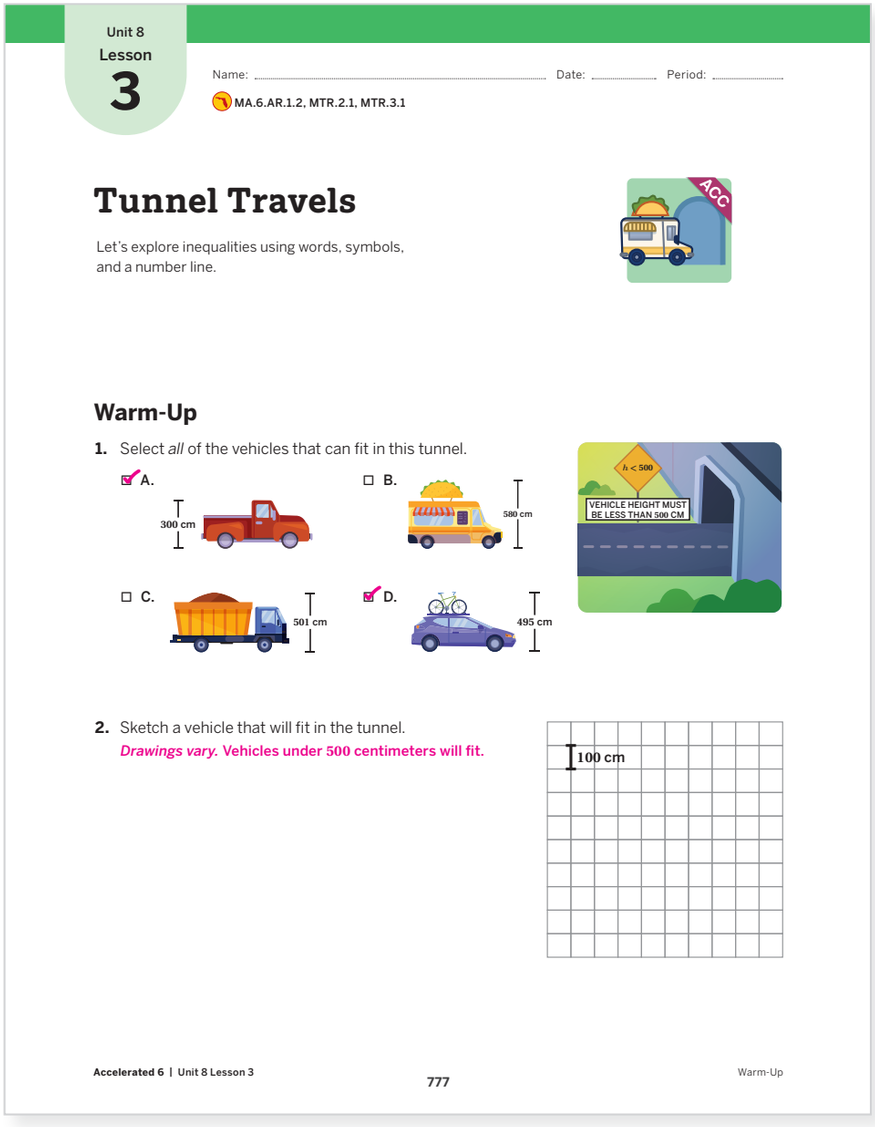
To encourage playfulness, invite students to create their own vehicle to travel through the tunnel.

To support students getting started, consider drawing their attention to the scale on the grid and asking, "What does the height of each square represent? How many squares would you need to represent a height of 100 centimeters? 200 centimeters? 500 centimeters?"

2 Connect

Invite students to share their sketches and consider asking:

- "Will this vehicle fit? How do you know?"
- "How can you use the scale on the grid to help you decide if your vehicle fits?"
- "How could you change the sign so that this vehicle fits (or doesn't fit)? How would the words change? How would the symbols change?" **(MTR.2.1)**



Unit 8
Lesson
3

Name: ..... Date: ..... Period: .....

MA.6.AR.1.2, MTR.2.1, MTR.3.1

Tunnel Travels

Let's explore inequalities using words, symbols, and a number line.

ACC

Warm-Up

1. Select *all* of the vehicles that can fit in this tunnel.

☑ A. 300 cm

☐ B. 580 cm

☐ C. 501 cm

☑ D. 495 cm

2. Sketch a vehicle that will fit in the tunnel.

Drawings vary. **Vehicles under 500 centimeters will fit.**

Pairs | 20 min

Activity 1 Tunnels, Garages, and More!

Purpose: Students connect verbal descriptions, symbols, and number line representations of inequalities. **(MTR.2.1)**

Short on time? Consider inviting students to choose either Problem 6 or Problem 7 to complete.

1 Launch

ELL **English Language Learners** Read the prompt aloud for students who benefit from extra processing time. **(Reading)**

Consider asking:

- "Why do you think there are no dots to the right of 500?"
- "Is there another way to show all the heights than making dots?"
- "Where do you see *less than* in the symbols, words, and number line?" **(MTR.2.1)**

Look for students who use creative or concise text responses and sketches. Invite these students to share their thinking during the Connect.

2 Monitor

Direct students' attention to the signs and number lines. Use the **Notice and Wonder** routine to promote curiosity and help students make sense of inequalities.

Listen for what students noticed and wondered, including patterns across the number lines and inequalities, and consider using their language during the Connect.

D **Differentiation: (Support)** Provide students a T-chart to record what they notice along with the inequality they used before they are expected to share their ideas with others.

Listen for student responses that include:

- The less than inequality has an open circle while the inequality with a line under it has a solid (closed) circle.
- Both inequalities have many solutions.

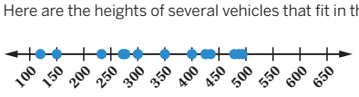
Activity 1 continued >

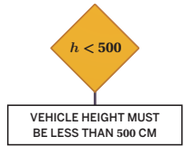
Activity 1

Name: _____ Date: _____ Period: _____

Tunnels, Garages, and More!

3. Here are the heights of several vehicles that fit in the tunnel.



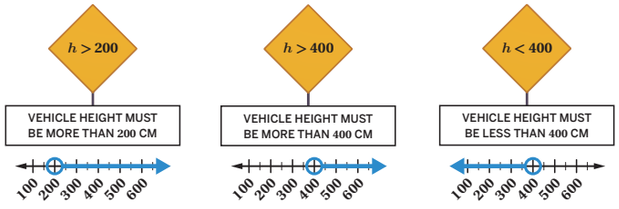


What do you think a graph of *all* the vehicle heights that fit would look like?

Responses vary.

- It would look like a lot of dots squished together, all to the left of 500.
- All the vehicles' heights would make a thick line from the right of 0 (because you can't have a vehicle with a negative height or no height) to the left of 500.

4. Here are three signs with inequalities and their number line graphs.

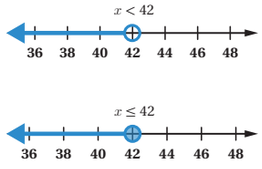


Discuss: What do you notice? What do you wonder?

Responses vary.

- I notice that when the shaded part of the number line is on the left side of the circle, the vehicle height is less than the number in the inequality.
- I notice the circle is at 200 for the inequality $h > 200$. I wonder if 200 is a solution.
- I wonder how I can change the inequality $h > 200$ to include a vehicle height of 200 centimeters.

5. Here are the graphs for $x < 42$ and $x \leq 42$.



Discuss: How are the graphs alike? How are they different?

Responses vary.

- Both graphs have a circle at 42 and a line with an arrow going to the left.
- One graph has a circle that is filled in and the other has a circle that is not filled in.

Accelerated 6 | Unit 8 Lesson 3 778 Activity 1

Pairs | 20 min

Activity 1 Tunnels, Garages, and More! (continued)

Purpose: Students connect verbal descriptions, symbols, and number line representations of inequalities. **(MTR.2.1)**

Short on time? Consider inviting students to choose either Problem 6 or Problem 7 to complete.

2 Monitor

Look for student responses to share during the Connect.

D Differentiation *(Problem 6)*

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Demonstrate an incorrect solution like $h > 200$ to provide access to students who benefit from clear and explicit instructions.
Draw the graph with the arrow pointed in the incorrect direction.	**Support** Consider inviting students to pick a solution from the number line and ask, "Would this vehicle fit in the parking garage?"
Correctly identify the *boundary point* and the direction of the inequality. **(MTR.3.1)**	**Strengthen** Celebrate their thinking and invite them to share during the Connect.

ELL English Language Learners Invite students to read the scenario aloud and paraphrase what it means. Clarify any words they don't know. **(Reading)**

Encourage students to share their reasoning with a partner and work together to reach an agreement about how to sort the number lines, scenarios, and inequalities. **(MTR.2.1)**

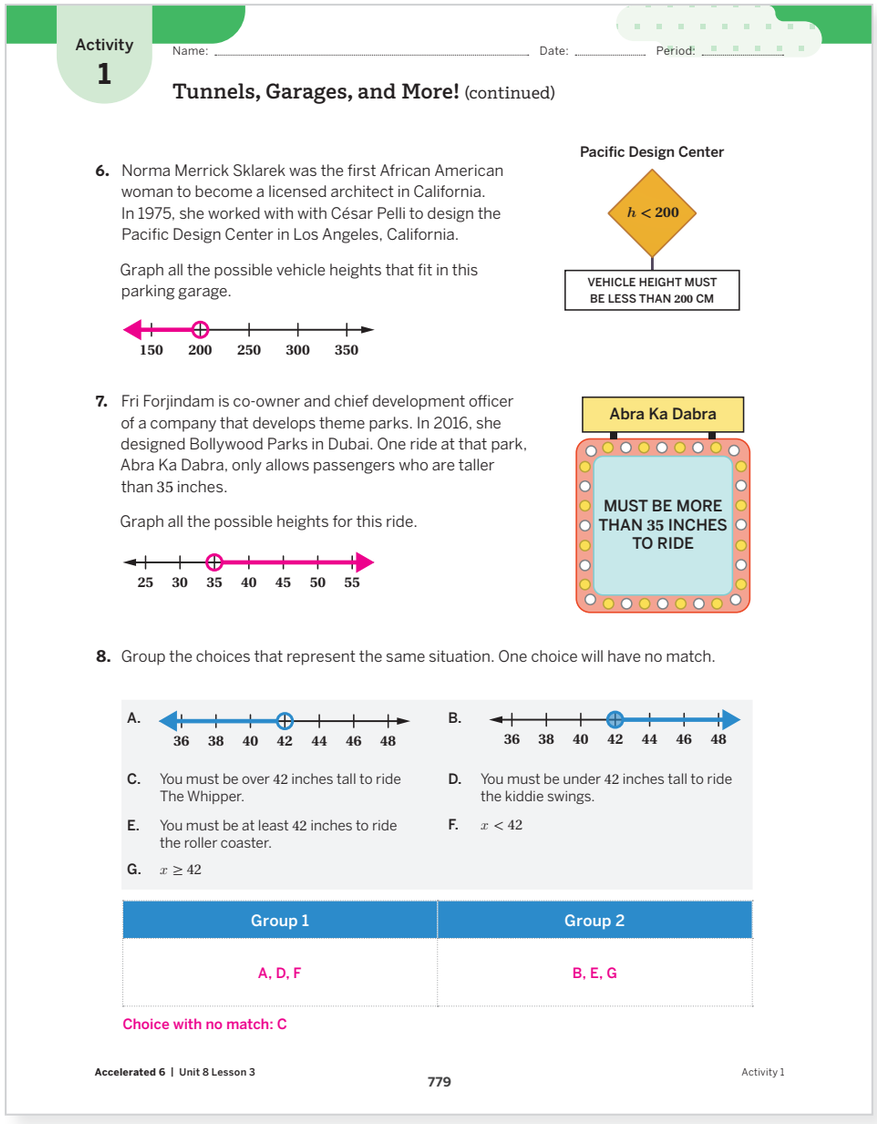
Listen for student discussions that make connections between the structure of the inequality and its graph.

3 Connect

Share student responses showing the correct number lines representing each inequality.

To surface the Key Takeaway, consider asking, "How can we use inequalities to represent contextual situations?"

MLR MLR2: Collect and Display Use this routine to collect students' ideas as they share with the class. Display these publicly for students to reference or add to for the rest of the unit/lesson.



Activity 1

Name: _____ Date: _____ Period: _____

Tunnels, Garages, and More! (continued)

6. Norma Merrick Sklarek was the first African American woman to become a licensed architect in California. In 1975, she worked with with César Pelli to design the Pacific Design Center in Los Angeles, California.

Graph all the possible vehicle heights that fit in this parking garage.

150 200 250 300 350

Pacific Design Center

$h < 200$

VEHICLE HEIGHT MUST BE LESS THAN 200 CM

7. Fri Forjindam is co-owner and chief development officer of a company that develops theme parks. In 2016, she designed Bollywood Parks in Dubai. One ride at that park, Abra Ka Dabra, only allows passengers who are taller than 35 inches.

Graph all the possible heights for this ride.

25 30 35 40 45 50 55

Abra Ka Dabra

MUST BE MORE THAN 35 INCHES TO RIDE

8. Group the choices that represent the same situation. One choice will have no match.

A. 36 38 40 42 44 46 48

B. 36 38 40 42 44 46 48

C. You must be over 42 inches tall to ride The Whipper.

D. You must be under 42 inches tall to ride the kiddie swings.

E. You must be at least 42 inches to ride the roller coaster.

F. $x < 42$

G. $x \geq 42$

Group 1	Group 2
A, D, F	B, E, G

Choice with no match: C

Accelerated 6 | Unit 8 Lesson 3 779 Activity 1

Key Takeaway: Inequalities can represent real-world contexts on a number line diagram. A *boundary point* is marked with an open or closed circle, and an arrow can be used to indicate multiple solutions.

Pairs | 10 min

Activity 2 Inequalities Out of Context

Purpose: Students make sense of inequality symbols and graphs out of context and explore inequalities with the variable on the right side, such as $19 < x$.

1 Launch

Direct students' attention to the graph and invite students to share with a partner what is incorrect about the inequality before fixing it.

2 Monitor

ELL **English Language Learners** Invite students to read the prompt in pairs so they can practice speaking and listening to phrases like "20 is less than or equal to x" and "x is less than or equal to 20." **(Speaking and Listening)**

Create a class definition for the term **less-than-or-equal-to** using students' language and the glossary.

Select one to two explanations for each of the choices.

A **Accessibility: Conceptual Processing** Ask students what they could do to test if an inequality is correct. If it does not come up naturally, consider inviting students to choose a number that is shaded on the graph and test it in each inequality.

D **Differentiation: You're invited to explore more. (Stretch)** Invite students who would like to explore inequalities further to complete this optional task. Encourage them to discuss their thinking with a partner.

3 Connect

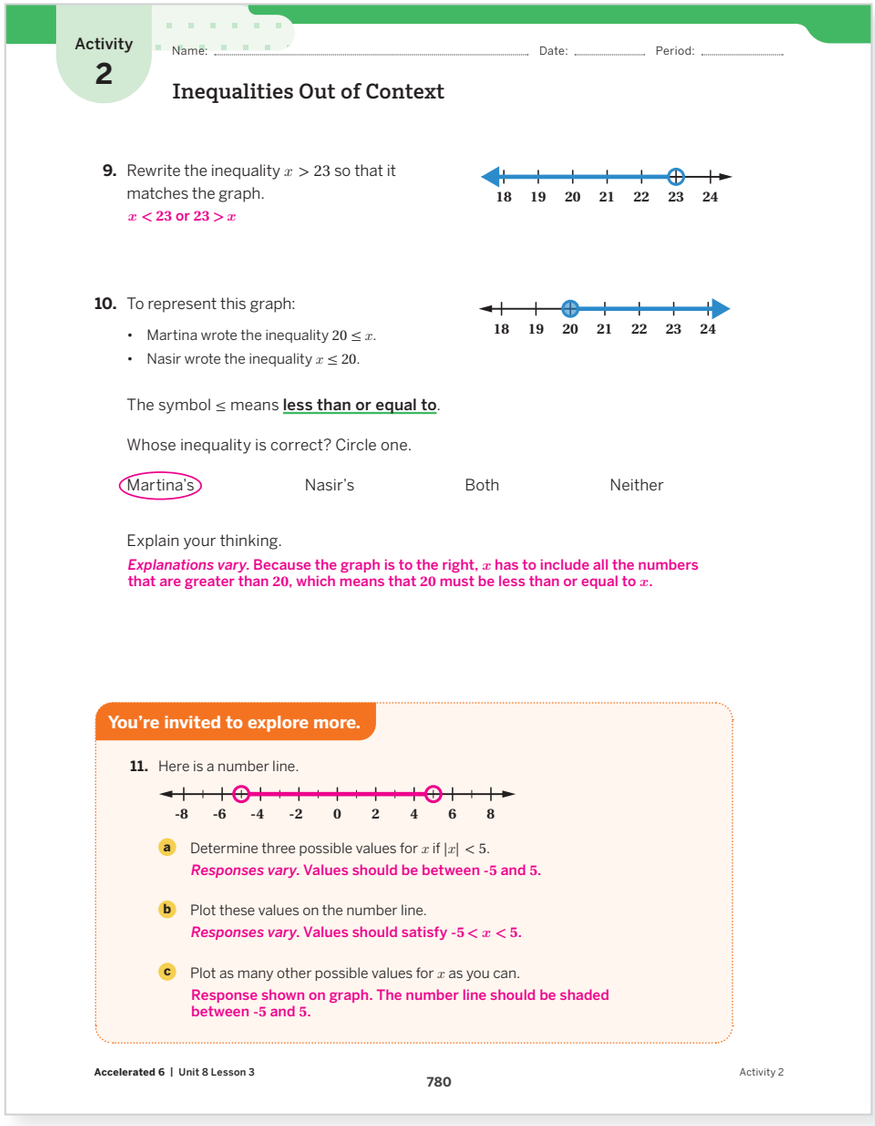
Invite students to share their responses.

Use the Decide and Defend routine to support students in making arguments and critiquing the reasoning of others. **(MTR.3.1)**

Consider asking:

- "How did you decide whose inequality was correct?"
- "Is there another correct way to describe this inequality?"
- "If ≤ represents less than or equal to, what do you think ≥ represents?"

ELL **English Language Learners** Provide sentence frames to help students explain their reasoning (e.g., _____ is correct because _____.). **(Writing and Speaking)**



Activity 2

Name: Date: Period:

Inequalities Out of Context

9. Rewrite the inequality $x > 23$ so that it matches the graph.

$x < 23$ or $23 > x$

10. To represent this graph:

- Martina wrote the inequality $20 \leq x$.
- Nasir wrote the inequality $x \leq 20$.

The symbol ≤ means **less than or equal to**.

Whose inequality is correct? Circle one.

Martina's Nasir's Both Neither

Explain your thinking.

Explanations vary. Because the graph is to the right, x has to include all the numbers that are greater than 20, which means that 20 must be less than or equal to x.

You're invited to explore more.

11. Here is a number line.

a. Determine three possible values for x if $|x| < 5$.

Responses vary. Values should be between -5 and 5.

b. Plot these values on the number line.

Responses vary. Values should satisfy $-5 < x < 5$.

c. Plot as many other possible values for x as you can.

Response shown on graph. The number line should be shaded between -5 and 5.

Accelerated 6 | Unit 8 Lesson 3 780 Activity 2

Whole Class | 5 min

Synthesis

Purpose: Students synthesize their understanding of different representations of inequalities.

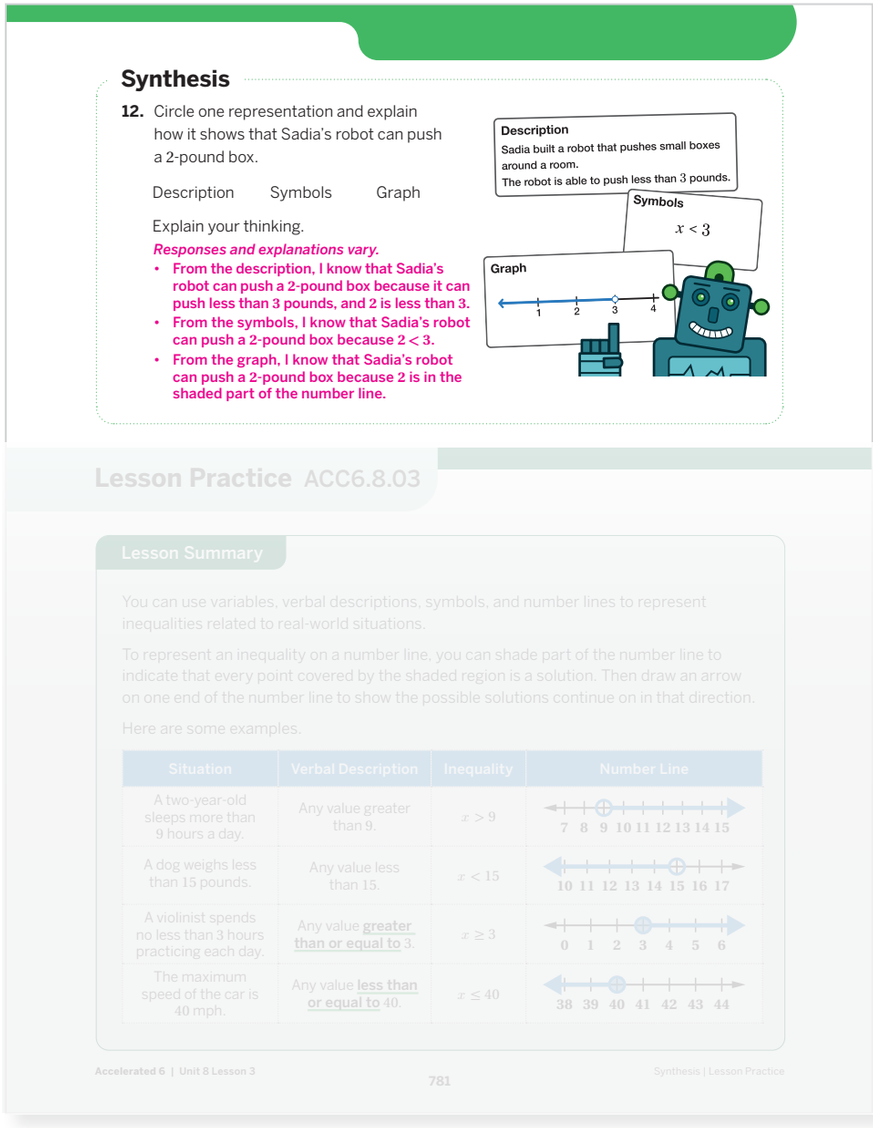
Invite students to respond independently, and then share their thinking with a partner.

Collect and share a variety of ideas, such as:
- Descriptions give contextual meaning.
- Symbols represent the inequality clearly out of context using variables, numbers, and either $>$ or $<$.
- Graphs visually show all of the solutions to the inequality.

Consider asking:
- "What are the advantages of the words? The symbols? The graph?"
- "What are the disadvantages of the words? The symbols? The graph?"
- "Which representation (words, symbols, graphs) feels most helpful to you? Why?"

MLR **MLR2: Collect and Display** Use this routine to formalize the definitions of *less-than-or-equal-to* and *greater-than-or-equal-to* using students' language. Display any new language, examples, or diagrams for students to reference later.

Note: If time allows, invite students to revise their previous thinking based on the discussion.



Synthesis

12. Circle one representation and explain how it shows that Sadia's robot can push a 2-pound box.

Description Symbols Graph

Explain your thinking.

Responses and explanations vary.
- From the description, I know that Sadia's robot can push a 2-pound box because it can push less than 3 pounds, and 2 is less than 3.
- From the symbols, I know that Sadia's robot can push a 2-pound box because $2 < 3$.
- From the graph, I know that Sadia's robot can push a 2-pound box because 2 is in the shaded part of the number line.

Lesson Practice ACC6.8.03

Lesson Summary

You can use variables, verbal descriptions, symbols, and number lines to represent inequalities related to real-world situations.

To represent an inequality on a number line, you can shade part of the number line to indicate that every point covered by the shaded region is a solution. Then draw an arrow on one end of the number line to show the possible solutions continue on in that direction.

Here are some examples.

Situation	Verbal Description	Inequality	Number Line
A two-year-old sleeps more than 9 hours a day.	Any value greater than 9.	$x > 9$	7 8 9 10 11 12 13 14 15
A dog weighs less than 15 pounds.	Any value less than 15.	$x < 15$	10 11 12 13 14 15 16 17
A violinist spends no less than 3 hours practicing each day.	Any value greater than or equal to 3.	$x \geq 3$	0 1 2 3 4 5 6
The maximum speed of the car is 40 mph.	Any value less than or equal to 40.	$x \leq 40$	38 39 40 41 42 43 44

Accelerated 6 | Unit 8 Lesson 3 781 Synthesis | Lesson Practice

Lesson Takeaway: Inequalities can be represented through verbal descriptions, symbols, and number lines.

Independent | 5 min

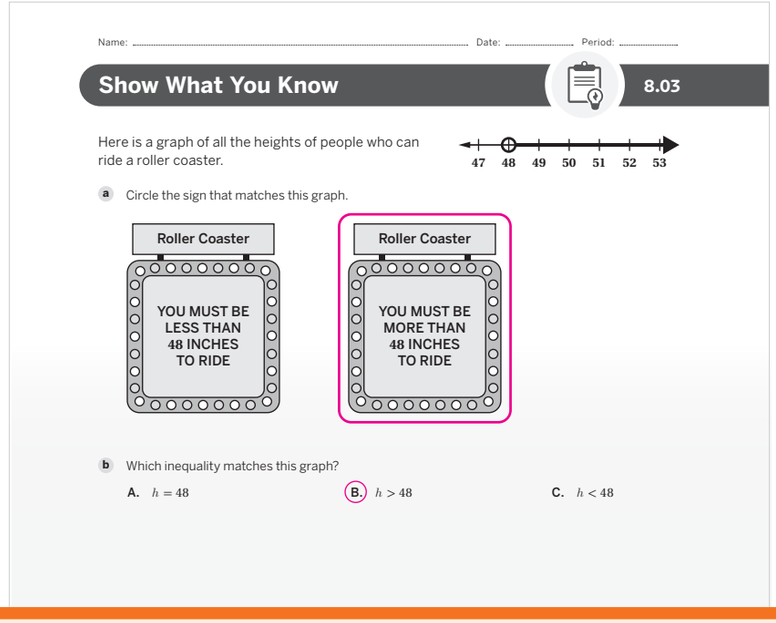
Show What You Know

Purpose: Students demonstrate their understanding by matching words and inequalities to a graph.

Today's Goals

1. **Goal:** Connect verbal, symbolic, and number line representations of inequalities in context.
2. **Language Goal:** Describe and interpret real-world inequalities. **(Reading, Writing, Speaking, and Listening)**

Show What You Know (Assessment Resources)



Name: _____ Date: _____ Period: _____

Show What You Know 8.03

Here is a graph of all the heights of people who can ride a roller coaster.

47 48 49 50 51 52 53

a. Circle the sign that matches this graph.

Roller Coaster
YOU MUST BE LESS THAN 48 INCHES TO RIDE

Roller Coaster
YOU MUST BE MORE THAN 48 INCHES TO RIDE

b. Which inequality matches this graph?

A. $h = 48$ B. $h > 48$ C. $h < 48$

D Differentiation Use after Lesson 3

Support

Provide targeted intervention for students by using these resources.

If students need support matching a real world context to the graph of an inequality:

» **Respond to Student Thinking:**

- Review the **Lesson Summary**.
- Select a few related **Practice Problems** and provide individualized feedback.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 2 Extension Activities**.
- Invite students to complete the **Explore More** task.

Professional Learning

Consider the next several lessons you will teach. How will students' work in connecting verbal, symbolic, and number line representations of inequalities prepare students for representing the solutions to inequalities in Lessons 4–7?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

12. Circle one representation and explain how it shows that Sadia's robot can push a 2-pound box.

Description | Symbols | Graph

Explain your thinking.

Responses and explanations vary.

- From the description, I know that Sadia's robot can push a 2-pound box because it can push less than 3 pounds, and 2 is less than 3.
- From the symbols, I know that Sadia's robot can push a 2-pound box because $2 < 3$.
- From the graph, I know that Sadia's robot can push a 2-pound box because 2 is in the shaded part of the number line.

Lesson Practice ACC6.8.03

Lesson Summary

You can use variables, verbal descriptions, symbols, and number lines to represent inequalities related to real-world situations.

To represent an inequality on a number line, you can shade part of the number line to indicate that every point covered by the shaded region is a solution. Then draw an arrow on one end of the number line to show the possible solutions continue on in that direction.

Here are some examples.

Situation	Verbal Description	Inequality	Number Line
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A violinist spends no less than 3 hours practicing each day.	Any value greater than or equal to 3.	$x \geq 3$	0 1 2 3 4 5 6
The maximum speed of the car is 40 mph.	Any value less than or equal to 40.	$x \leq 40$	38 39 40 41 42 43 44

Lesson Practice ACC6.8.03

Name: _____ Date: _____ Period: _____

Problems 1–3: At a book sale, all books cost less than $5.

1. List *three* possible prices for a book at this book sale.

Responses vary. $1.49, $3.00, $4.99

2. Write an inequality to represent the cost of a book at the book sale, b.

$b < 5$ or $5 > b$

3. Graph all the possible prices of books at the sale.

Problems 4–7: n represents the number of eggs a sea turtle lays.

4. What does $50 < n$ mean in this situation?

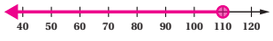
A sea turtle laid more than 50 eggs.

5. Graph this inequality.

6. What does $n \leq 110$ mean in this situation?

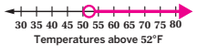
A sea turtle laid no more than 110 eggs.

7. Graph this inequality.

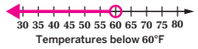


Problems 8–9: One day in Boston, the temperature was above 52°F and below 60°F.

8. Make two graphs, one to represent temperatures above 52°F and another to represent temperatures below 60°F.



Temperatures above 52°F



Temperatures below 60°F

9. Write two inequalities to represent the possible temperatures, T, on that day.

$T > 52$ (or equivalent) and $T < 60$ (or equivalent)

Lesson Practice ACC6.8.03

Name: _____ Date: _____ Period: _____

FAST Practice

10. The minimum amount of electricity used to run a computer is 30 watts. Write an inequality to represent the electricity, e, the computer needs.

$e \geq 30$

Spiral Review

11. Select *all* of the temperatures that are warmer than -10°F.

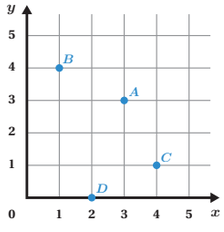
☑ A. 10°F ☑ B. 0°F ☑ C. -5°F ☐ D. -11°F ☐ E. -20°F

12. Anika wrote the number sentence $|-5| = |5|$. Explain what it means in your own words.

Responses vary. -5 and 5 are both 5 units away from 0 on the number line.

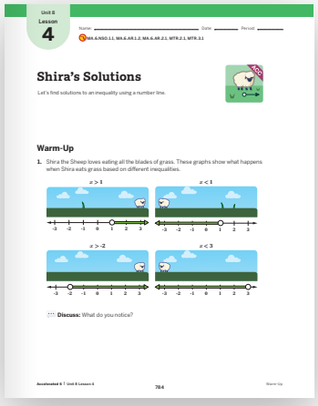
13. Write the coordinates of each point shown on the graph.

Point	Coordinates
A	(3, 3)
B	(1, 4)
C	(4, 1)
D	(2, 0)



Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1–3, 5, 7–9	1	MA.6.AR.1.2
	4, 6	2	MA.6.AR.1.2
FAST Practice	10	1	MA.6.AR.1.2
Spiral Review			
	11	1	MA.6.NSO.1.1
	12	2	MA.6.NSO.1.4
	13	1	MA.5.GR.4.1



Shira's Solutions

Solutions to Inequalities

Let's find solutions to an inequality using a number line.

Learning Outcomes

Today's Goals

1. **Goal:** Draw and label a number line diagram to represent the *solutions to an inequality*.
2. **Language Goal:** Justify whether a value is a solution to a given inequality. **(Writing)**
3. **Language Goal:** Explain that an inequality has infinitely many solutions. **(Speaking, Listening, and Writing)**

Students continue to connect graphs, verbal descriptions, and symbolic representations in the contexts of helping a fictional sheep eat grass according to an inequality. They draw and label a number line diagram to represent the **solutions to an inequality**. They justify whether a value is a solution to a given inequality, and explain why an inequality may have infinitely many solutions. **(MTR.2.1)**

Coherence

Prior Learning

In Lesson 3, students connected verbal descriptions, symbols, and number line representations of inequalities in context.

Future Learning

In Lesson 5, students will write and interpret inequalities that represent relationships between weights on an unbalanced hanger.

Connections

- Students develop an understanding of solution as it relates to inequalities through analyzing and solving context problems. (mathematical, real world)

FL B.E.S.T. Math Benchmarks

Addressing

MA.6.NSO.1.1
Extend previous understanding of numbers to define rational numbers. Plot, order and compare rational numbers.

MA.6.AR.1.2
Translate a real-world written description into an algebraic inequality in the form of $x > a$, $x < a$, $x \geq a$ or $x \leq a$. Represent the inequality on a number line.

Also Addressing: **MA.6.AR.2.1**

Mathematical Thinking and Reasoning Standards: MTR.2.1, MTR.3.1

ELA Expectation: ELA.K12.EE.5.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.5.NSO.1.4	MA.7.AR.2.1
MA.5.AR.2.1	MA.8.NSO.1.2
MA.5.AR.2.3	

Vocabulary

New Vocabulary

solution to an inequality

Lesson at a Glance

~ 45 min

MA.6.NSO.1.1, MA.6.AR.1.2, MA.6.AR.2.1

Warm-Up

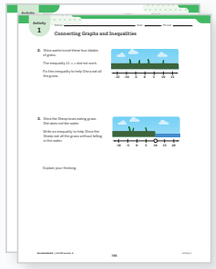
Independent | 5 min

Students get familiarized with the context they will explore throughout the lesson: a fictional sheep who eats grass according to an inequality.

Activity 1

Pairs | 15 min

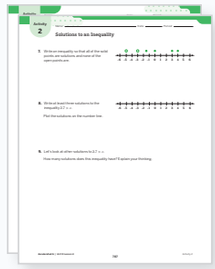
Students write inequalities that include a boundary point and specific solutions to control how Shira the Sheep moves.



Activity 2

Pairs | 15 min

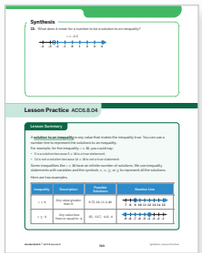
Students make connections between graphs, numeric solutions, and symbolic representations of inequalities and investigate the number of **solutions to an inequality**. **(MTR.2.1)**



Synthesis

Whole Class | 5 min

Students synthesize their understanding of what it means for a number to be a solution to an inequality. MLR2: Collect and Display



Show What You Know

Independent | 5 min

Students demonstrate their understanding by selecting solutions to an inequality.

Prep Checklist

This lesson includes:

- Student Edition
- Show What You Know

Independent | 5 min

Warm-Up

Purpose: Students get familiarized with the context they will explore throughout the lesson: a fictional sheep who eats grass according to an inequality.

1 Launch

Direct students' attention to the four images to introduce students to Shira the Sheep.

To generate excitement, invite students to share any experiences they've had with sheep.

2 Connect

ELL **English Language Learners** Give students time to think independently before sharing their ideas verbally. **(Speaking)**

Invite students to share inequalities that they thought had interesting or surprising results, and any inequalities that made Shira eat all the grass.

Unit 8
Lesson
4

Name: ..... Date: ..... Period: .....

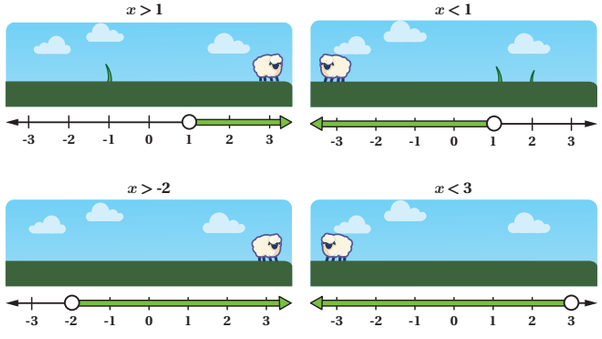
MA.6.NSO.1.1, MA.6.AR.1.2, MA.6.AR.2.1, MTR.2.1, MTR.3.1

Shira's Solutions

Let's find solutions to an inequality using a number line.

Warm-Up

1. Shira the Sheep loves eating all the blades of grass. These graphs show what happens when Shira eats grass based on different inequalities.



Discuss: What do you notice?

Responses vary.

- I notice there are multiple inequalities where Shira eats all the grass.
- I notice for the inequality $x > 1$ Shira only eats some of the grass.
- I notice that sometimes Shira eats grass one way and sometimes she eats grass the other way.

Accelerated 6 | Unit 8 Lesson 4 784 Warm-Up

Pairs | 15 min

Activity 1 Connecting Graphs and Inequalities

Purpose: Students write inequalities that include a boundary point and specific solutions to control how Shira the Sheep moves.

Short on time? Consider inviting students to choose either Problem 5 or Problem 6 to complete.

1 Launch

Share with students that there is often more than one inequality that will help Shira eat all the grass.

2 Monitor

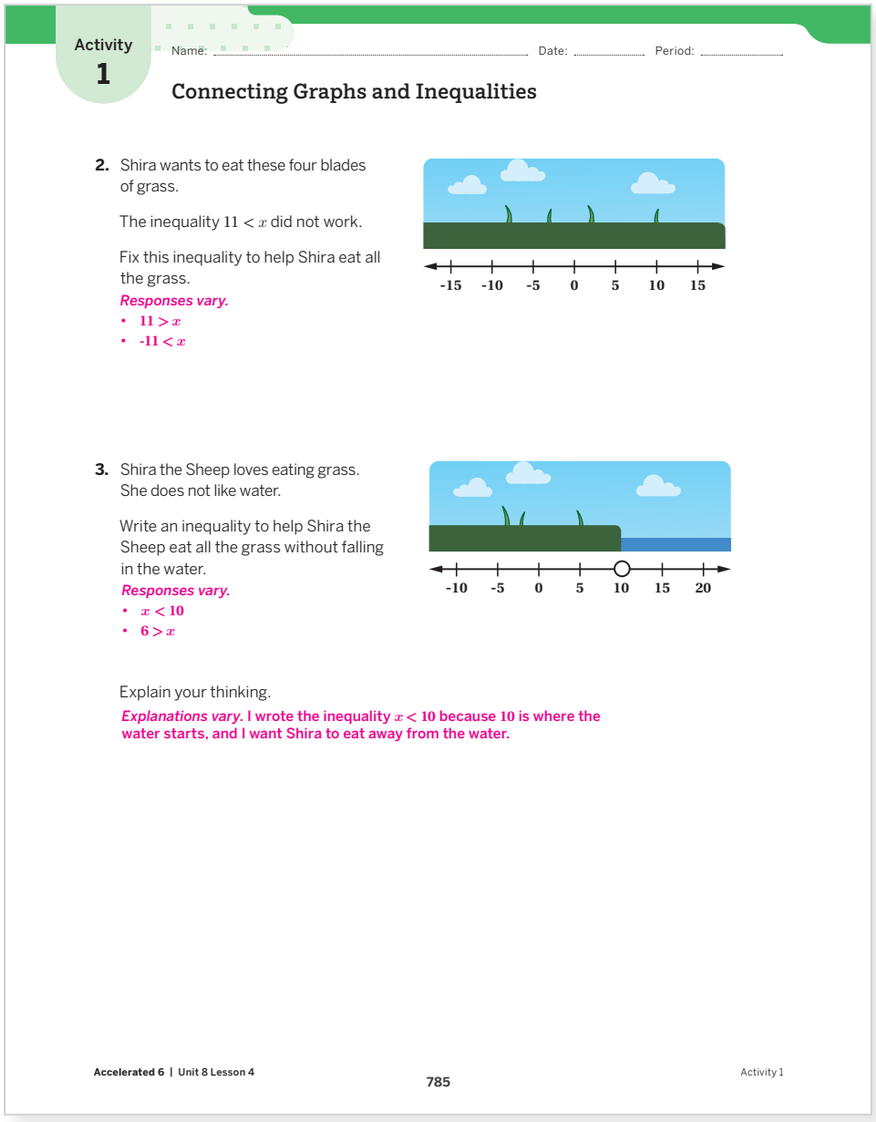
Invite students to try several different ways to fix the inequality and then check their inequalities with a partner.

Encourage students to revise their solutions based on the feedback from their partner.

To support students getting started, consider asking, "Why does the inequality $x > -5$ not work?"

Invite students to share with a partner what they learned from inequalities that did not work.

Activity 1 continued >



Activity 1

Name: _____ Date: _____ Period: _____

Connecting Graphs and Inequalities

2. Shira wants to eat these four blades of grass.

The inequality $11 < x$ did not work.

Fix this inequality to help Shira eat all the grass.

Responses vary.

- $11 > x$
- $-11 < x$

3. Shira the Sheep loves eating grass. She does not like water.

Write an inequality to help Shira the Sheep eat all the grass without falling in the water.

Responses vary.

- $x < 10$
- $6 > x$

Explain your thinking.

Explanations vary. I wrote the inequality $x < 10$ because 10 is where the water starts, and I want Shira to eat away from the water.

Accelerated 6 | Unit 8 Lesson 4 785 Activity 1

Pairs | 15 min

Activity 1 Connecting Graphs and Inequalities (continued)

Purpose: Students write inequalities that include a boundary point and specific solutions to control how Shira the Sheep moves.

Short on time? Consider inviting students to choose either Problem 5 or Problem 6 to complete.

2 Monitor

Invite students to discuss their thinking with a partner.

Look for student responses to share during the Connect.

D Differentiation

Look for students who:	Respond to Student Thinking
Guess and check random inequalities.	**Support** Consider asking, "How do you determine the starting point (also known as the boundary point)?"
Use a correct boundary value, but incorrect inequality symbol. Shira the Sheep moves in the wrong direction and goes into the water.	**Support** Invite students to consider which direction the grass is from the boundary value. Consider asking, "Is the blade of grass at ___ greater than or less than your boundary point?"
Use an inequality that captures all blades of grass, but Shira goes into the water.	**Support** Invite students to try a new boundary value or inequality symbol.

Invite students to share their inequalities with a partner.

3 Connect

Invite students to share what happens at the boundary point of an inequality focusing on the importance of being precise on what value is chosen. **(MTR.3.1)**

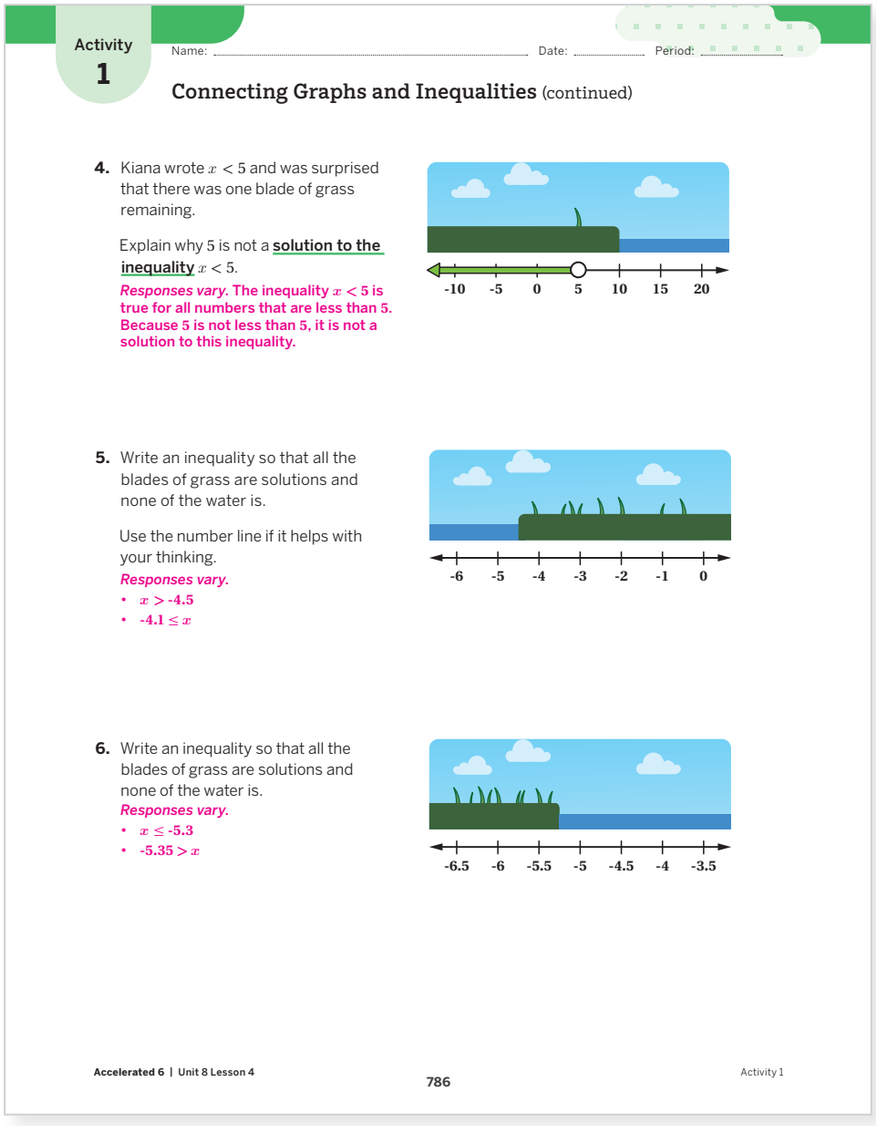
Consider asking:

- "What do you think *solution* means here?"
- "What are solutions to $x < 5$?"
- "How could we change Kiana's inequality so 5 is a solution?"

ELL English Language Learners Give students time to rehearse their ideas with a partner before they are expected to share their ideas with others. **(Speaking)**

Create a class definition for the term **solution to an inequality** using students' language and the glossary.

To surface the Key Takeaway, consider asking students to justify which inequalities will help Shira and which will not, and to critique each other's reasoning. **(MTR.3.1)**



Activity 1

Name: _____ Date: _____ Period: _____

Connecting Graphs and Inequalities (continued)

4. Kiana wrote $x < 5$ and was surprised that there was one blade of grass remaining.

Explain why 5 is not a **solution to the inequality** $x < 5$.

Responses vary. The inequality $x < 5$ is true for all numbers that are less than 5. Because 5 is not less than 5, it is not a solution to this inequality.

5. Write an inequality so that all the blades of grass are solutions and none of the water is.

Use the number line if it helps with your thinking.

Responses vary.

- $x > -4.5$
- $-4.1 \leq x$

6. Write an inequality so that all the blades of grass are solutions and none of the water is.

Responses vary.

- $x \leq -5.3$
- $-5.35 > x$

Accelerated 6 | Unit 8 Lesson 4 786 Activity 1

Key Takeaway: To determine the direction of an inequality, you can test values on either side of the boundary point. The direction of the inequality is determined by which values make the inequality true.

Pairs | 15 min

Activity 2 Solutions to an Inequality

Purpose: Students make connections between graphs, numeric solutions, and symbolic representations of inequalities and investigate the number of **solutions to an inequality**. **(MTR.2.1)**

1 Launch

Demonstrate an incorrect solution like $x > 2$ and invite students to share with a partner what is incorrect about the inequality before fixing it.

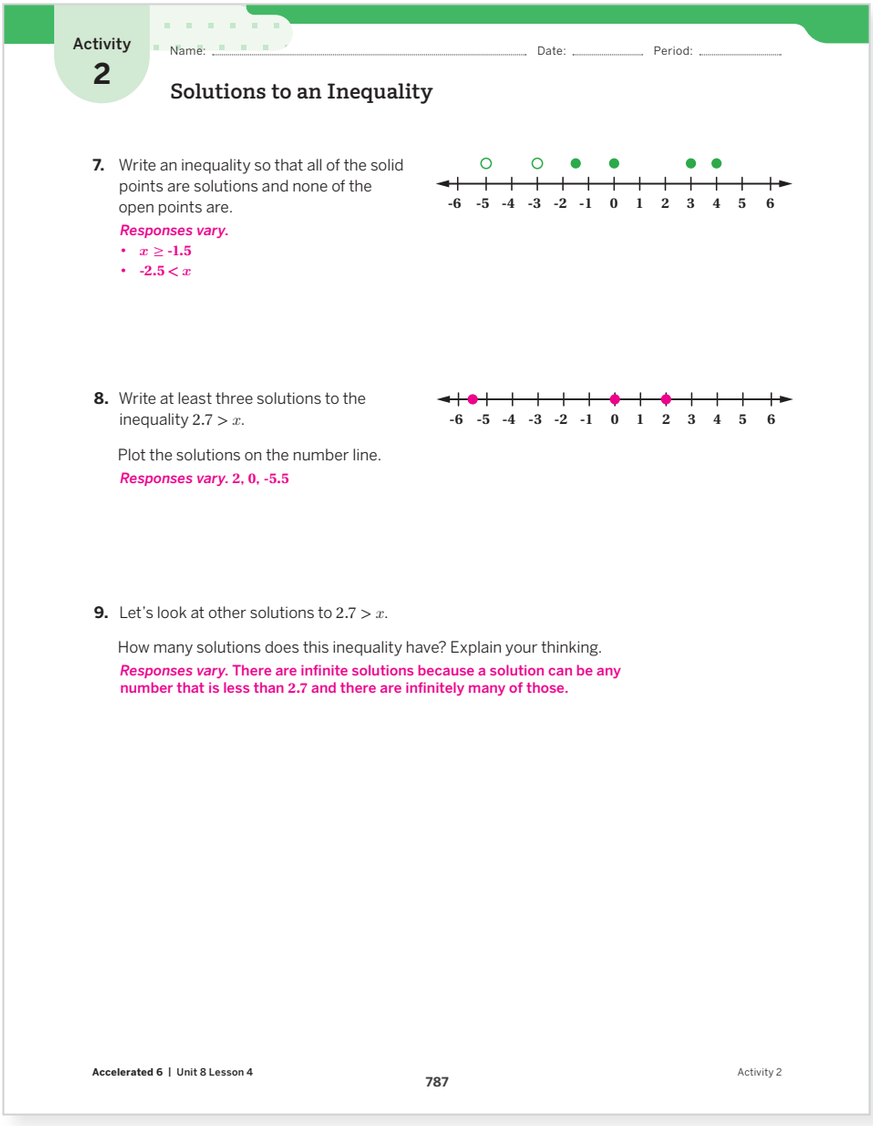
2 Monitor

Invite students to check their inequalities with a partner.

Invite students to replace x with a number they think is a solution and to decide if that statement is true or false.

Encourage students to sketch the graph of all of the solutions.

Activity 2 continued >



Activity 2

Name: Date: Period:

Solutions to an Inequality

7. Write an inequality so that all of the solid points are solutions and none of the open points are.

Responses vary.

- $x \geq -1.5$
- $-2.5 < x$

8. Write at least three solutions to the inequality $2.7 > x$.

Plot the solutions on the number line.

Responses vary. 2, 0, -5.5

9. Let's look at other solutions to $2.7 > x$.

How many solutions does this inequality have? Explain your thinking.

Responses vary. There are infinite solutions because a solution can be any number that is less than 2.7 and there are infinitely many of those.

Accelerated 6 | Unit 8 Lesson 4 787 Activity 2

Pairs | 15 min

Activity 2 Solutions to an Inequality (continued)

Purpose: Students make connections between graphs, numeric solutions, and symbolic representations of inequalities and investigate the number of **solutions to an inequality**. **(MTR.2.1)**

2 Monitor

Look for student responses to share during the Connect.

D Differentiation

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Invite students to think about several values that would make the inequality true.
Have the arrow pointing towards the left.	**Support** Invite students to test their solutions to the inequality.
Place the boundary value at -1.5 rather than -0.5.	**Strengthen** Celebrate student thinking and invite students to compare their strategy with a classmate's.

A Accessibility: Executive Functioning Chunk this activity into more manageable parts by inviting students to choose one graph at a time and find all the choices that match it.

Encourage students to share their reasoning with a partner and work together to reach an agreement about how to sort the inequalities or solutions. If time allows, consider asking pairs to compare their choices, justify their placements, and make revisions based on their observations. **(MTR.2.1)**

D Differentiation: You're invited to explore more. (Stretch) Invite students who would like to explore inequalities further to complete this optional task. Encourage them to discuss their thinking with a partner.

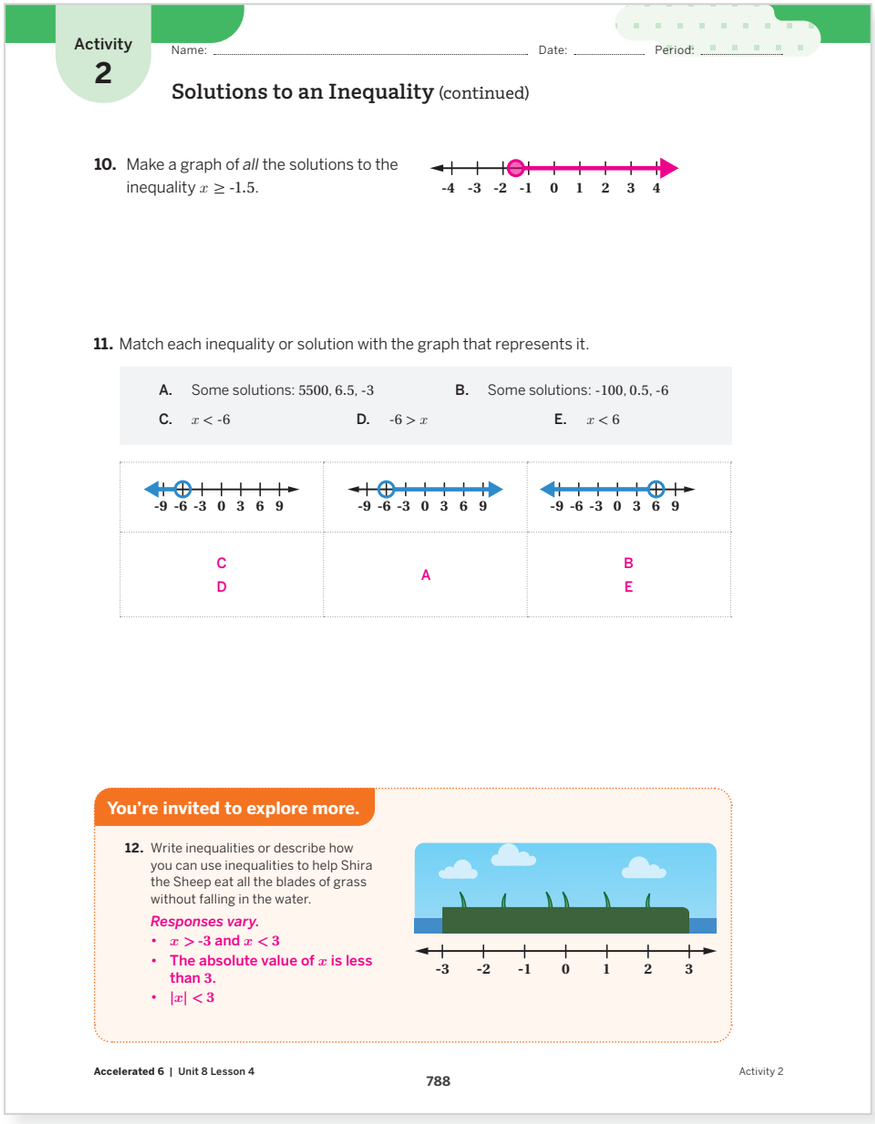
3 Connect

Consider asking:

- "How do you know -1.5 and 2 are solutions to this inequality, but -3 is not?"
- "What does it mean to have infinite solutions? Is this special to this inequality?"
- "How do we show infinite solutions on the graph?"

Invite students to share their responses and strategies on graphing solutions to inequalities. If it doesn't come up naturally, mention using an arrow indicating infinite solutions.

To surface the Key Takeaway, consider asking, "How can we determine if a value is a solution to an inequality?"



Activity 2

Name: _____ Date: _____ Period: _____

Solutions to an Inequality (continued)

10. Make a graph of *all* the solutions to the inequality $x \geq -1.5$.

11. Match each inequality or solution with the graph that represents it.

A. Some solutions: 5500, 6.5, -3
B. Some solutions: -100, 0.5, -6
C. $x < -6$
D. $-6 > x$
E. $x < 6$

C
D

A

B
E

You're invited to explore more.

12. Write inequalities or describe how you can use inequalities to help Shira the Sheep eat all the blades of grass without falling in the water.

Responses vary.

- $x > -3$ and $x < 3$
- The absolute value of x is less than 3.
- $|x| < 3$

Accelerated 6 | Unit 8 Lesson 4 788 Activity 2

Key Takeaway: You can determine whether a value is a **solution to an inequality** by substituting that value into the inequality. An inequality may have infinitely many solutions.

Whole Class | 5 min

Synthesis

Purpose: Students synthesize their understanding of what it means for a number to be a solution to an inequality.

Invite students to respond independently, and then share their thinking with a partner.

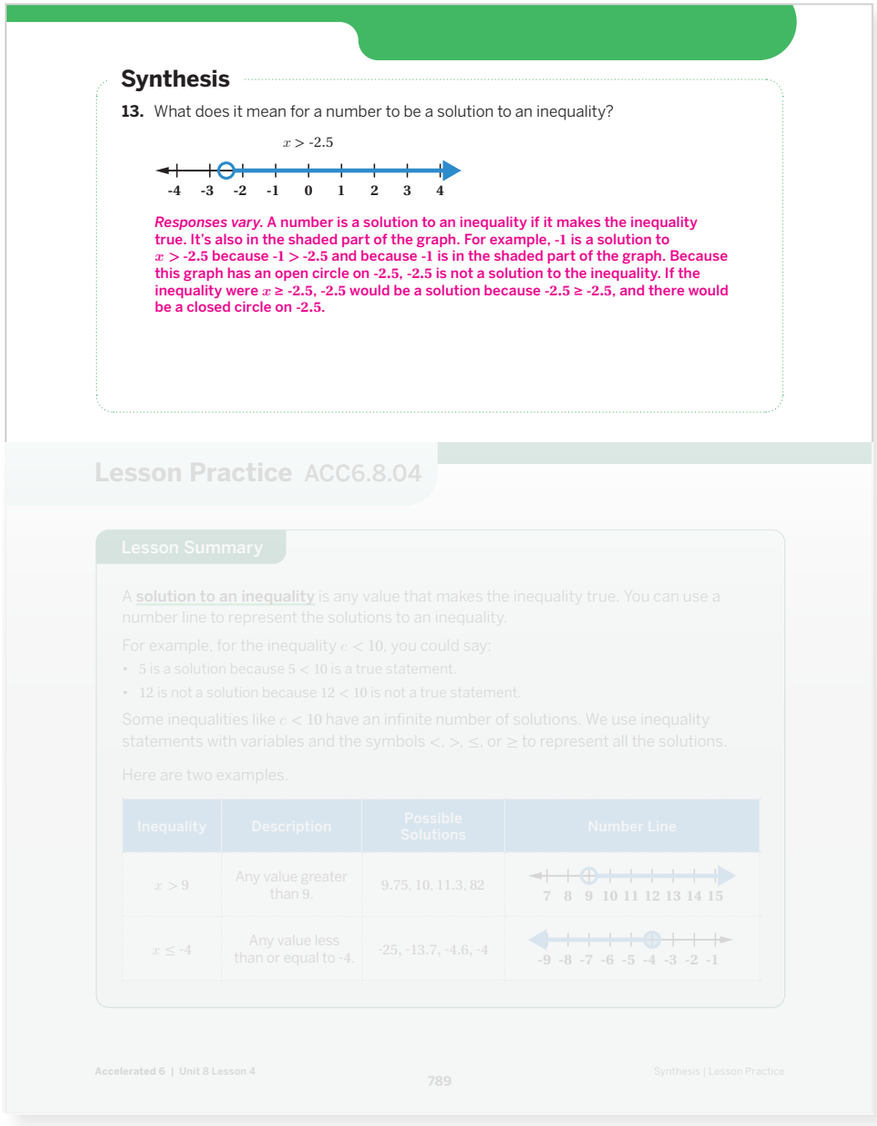
Collect and share a variety of ideas, such as:

- A number is a solution to an inequality if that number makes the inequality true.
- The solution is located in the shaded part of the graph.

ELL **English Language Learners** Provide sentence frames to help students explain their thinking (e.g., For a number to be a solution to an inequality, _____.). **(Writing and Speaking)**

MLR **MLR2: Collect and Display** Use this routine to formalize the definition of *solution to an inequality* using students' language. Display any new language, examples, or diagrams publicly for students to reference later.

Note: If time allows, invite students to revise their previous thinking based on the discussion.



Synthesis

13. What does it mean for a number to be a solution to an inequality?

Responses vary. A number is a solution to an inequality if it makes the inequality true. It's also in the shaded part of the graph. For example, -1 is a solution to $x > -2.5$ because $-1 > -2.5$ and because -1 is in the shaded part of the graph. Because this graph has an open circle on -2.5, -2.5 is not a solution to the inequality. If the inequality were $x \geq -2.5$, -2.5 would be a solution because $-2.5 \geq -2.5$, and there would be a closed circle on -2.5.

Lesson Practice ACC6.8.04

Lesson Summary

A **solution to an inequality** is any value that makes the inequality true. You can use a number line to represent the solutions to an inequality.

For example, for the inequality $c < 10$, you could say:

- 5 is a solution because $5 < 10$ is a true statement.
- 12 is not a solution because $12 < 10$ is not a true statement.

Some inequalities like $c < 10$ have an infinite number of solutions. We use inequality statements with variables and the symbols $<$, $>$, $\leq$, or $\geq$ to represent all the solutions.

Here are two examples.

Inequality	Description	Possible Solutions	Number Line
$x > 9$	Any value greater than 9.	9.75, 10, 11.3, 82	7 8 9 10 11 12 13 14 15
$x \leq -4$	Any value less than or equal to -4.	-25, -13.7, -4.6, -4	-9 -8 -7 -6 -5 -4 -3 -2 -1

Accelerated 6 | Unit 8 Lesson 4 789 Synthesis | Lesson Practice

Lesson Takeaway: A number line diagram can be used to represent the solutions to an inequality. A value is a **solution to an inequality** if that value makes the inequality true. An inequality can have infinite solutions, and this can be indicated on a number line with an arrow.

Independent | 5 min

Show What You Know

Purpose: Students demonstrate their understanding by selecting solutions to an inequality.

Today's Goals

1. **Goal:** Draw and label a number line diagram to represent the *solutions to an inequality*.
2. **Language Goal:** Justify whether a value is a solution to a given inequality. **(Writing)**
3. **Language Goal:** Explain that an inequality has infinitely many solutions. **(Speaking, Listening, and Writing)**

Show What You Know (Assessment Resources)

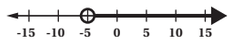
Name: ..... Date: ..... Period: .....

Show What You Know 8.04

Here is an inequality and its graph.

Select *all* the solutions to the inequality.

$x > -5$



- ☐ A. -20
- ☐ B. -5
- ☑ C. 0
- ☑ D. 5
- ☑ E. 20

Differentiation Use after Lesson 4

Support

Provide targeted intervention for students by using these resources.

If students need support using hangers to write inequalities:

» **Respond to Student Thinking:**

- Using the ***Identifying Solutions to Inequalities*** **Mini-Lesson**.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to complete the **Explore More** task and discuss their thinking with a partner.
- Invite students to explore the **Sub-Unit 2 Extension Activities**.

Professional Learning

How did partner feedback in this lesson support student learning?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

13. What does it mean for a number to be a solution to an inequality?

Responses vary. A number is a solution to an inequality if it makes the inequality true. It's also in the shaded part of the graph. For example, -1 is a solution to $x > -2.5$ because $-1 > -2.5$ and because -1 is in the shaded part of the graph. Because this graph has an open circle on -2.5, -2.5 is not a solution to the inequality. If the inequality were $x \geq -2.5$, -2.5 would be a solution because $-2.5 \geq -2.5$, and there would be a closed circle on -2.5.

Lesson Practice ACC6.8.04

Lesson Summary

A **solution to an inequality** is any value that makes the inequality true. You can use a number line to represent the solutions to an inequality.

For example, for the inequality $c < 10$, you could say:

- 5 is a solution because $5 < 10$ is a true statement.
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$x \leq -4$	Any value less than or equal to -4.	-25, -13.7, -4.6, -4	-9 -8 -7 -6 -5 -4 -3 -2 -1

Lesson Practice ACC6.8.04

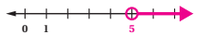
Name: _____ Date: _____ Period: _____

Problems 1–2: Here is the inequality $k > 5$.

1. Select *all* the values of k that are solutions to the inequality.

☐ A. 4.9 ☐ B. 5 ☑ C. 6 ☑ D. 5.2 ☐ E. -5.01

2. Make a graph of all the solutions to the inequality.



3. List three numbers that are solutions and three numbers that are not solutions to the inequality $x \leq -2.25$. Use the number line if it helps with your thinking.

Responses vary.

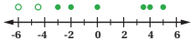
Solutions	Not Solutions
-3	-2
-5.225	0
-2.25	2.3

4. Write an inequality so that -0.75, -1.5, and -5 *are* solutions and 2, 0, and 100 *are not* solutions. Use the number line if it helps with your thinking.

Responses vary. $x < -0.5$

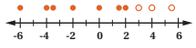
Problems 5–6: Write an inequality so that all of the solid points and none of the open points are solutions.

5.



Responses vary. $x > -4$

6.



Responses vary. $x < 2.5$

Lesson Practice ACC6.8.04

Name: _____ Date: _____ Period: _____

7. Use each number exactly once so that all of the statements are true.

-3 -2 -1 0 1 2 3

Responses vary.

- $x < 3$, $x > -1$. Solutions: 0, 1, 2. Not solutions: -2, -3
- $x < 2$, $x > -2$. Solutions: -1, 0, 1. Not solutions: -3, 3
- $x < 1$, $x > -3$. Solutions: 0, -1, -2. Not solutions: 2, 3

$x <$ ☐ and $x >$ ☐

Solutions to both inequalities: ☐ ☐ ☐

Not solutions to both inequalities: ☐ ☐

FAST Practice

8. Which value is a solution to both $-2.4 \geq x$ and $x < -3$?

(A.) -4.9 B. -3 C. -2.4 D. -1

Spiral Review

Problems 9–12: Complete each number sentence with the symbol $<$, $>$, or $=$.

9. $|-45|$ ___>___ -50

10. $|-50|$ ___>___ 45

11. $|-45|$ ___<___ 50

12. $|-45|$ ___<___ $|-50|$

13. Different numbers of hot sauce bottles and their costs are given in the table.

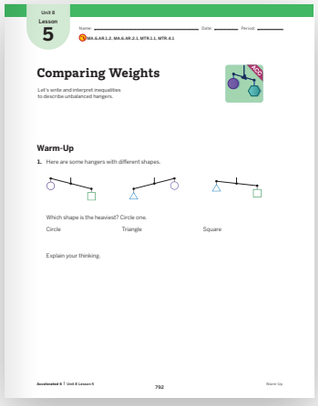
What is the unit rate in dollars per bottle?

$3.54 per bottle

Number of Bottles	Cost
3	$10.62
5	$17.70
7	$24.78

Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1, 3	1	MA.6.NSO.1.1, MA.6.AR.2.1
	2, 5–6	1	MA.6.AR.1.2
	4	2	MA.6.NSO.1.1, MA.6.AR.1.2, MA.6.AR.2.1
	7	3	MA.6.AR.1.2, MA.6.AR.2.1
FAST Practice	8	2	MA.6.NSO.1.1, MA.6.AR.2.1
Spiral Review			
	9–12	1	MA.6.NSO.1.4
	13	1	MA.6.AR.3.2


Unit 8
Lesson 5

Comparing Weights

Let's write and interpret inequalities to describe unbalanced hangers.

Warm-Up

1. Here are some hangers with different shapes.

Which shape is the heaviest? Circle one.
Circle Triangle Square

Explain your thinking.

792

Comparing Weights

Writing Inequalities

Let's write and interpret inequalities to describe unbalanced hangers.

Learning Outcomes

Today's Goal

1. **Goal:** Write and interpret inequalities that represent relationships between weights on an unbalanced hanger.

Students develop a deeper understanding of using inequality symbols to describe unknown values. They use hanger diagrams to interpret and write inequalities with one *variable* and then more than one variable. They dig deeper into the different ways of writing an inequality to describe the same relationship (e.g., $s < 8$ and $8 > s$).

Coherence

Prior Learning

In Lesson 4, students drew and labeled a number line diagram to represent solutions to an inequality. They also justified whether a value is a solution to a given inequality. In Unit 6, students used balanced hanger diagrams to represent and solve equations.

Future Learning

In Lessons 6–7, students will solve inequalities that represent situations in and out of context. They will use hanger diagrams to determine solutions to inequalities.

Connections

- Students revisit hanger models to analyze inequality relationships. (mathematical)

FL B.E.S.T. Math Benchmarks

Addressing

MA.6.AR.1.2
Translate a real-world written description into an algebraic inequality in the form of $x > a$, $x < a$, $x \geq a$ or $x \leq a$. Represent the inequality on a number line.

MA.6.AR.2.1
Given an equation or inequality and a specified set of integer values, determine which values make the equation or inequality true or false.

Mathematical Thinking and Reasoning Standards: MTR.1.1, MTR.4.1

ELA Expectation: ELA.K12.EE.1.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.5.AR.2.1	MA.7.AR.2.1
MA.5.AR.2.3	MA.7.AR.2.2
MA.5.AR.2.4	

Lesson at a Glance

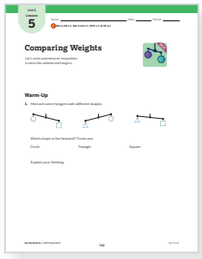
~ 45 min

MA.6.AR.1.2, MA.6.AR.2.1

Warm-Up

Independent | 5 min

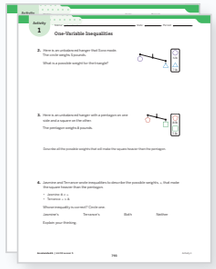
Students explore unbalanced hangers and compare the relative weights of objects. **Routine:** Decide and Defend



Activity 1

Pairs | 13 min

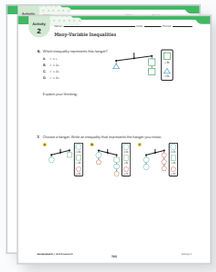
Students interpret inequalities that represent relationships between weights on unbalanced hangers. **(MTR.1.1)**



Activity 2

Pairs | 7 min

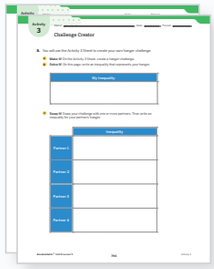
Students create unbalanced hangers and write inequalities using only variables to compare unknown weights.



Activity 3

Pairs | 10 min

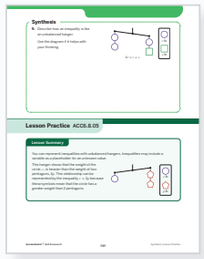
Students use their creativity to build their own unbalanced hanger for other students to represent using inequalities. **(MTR.1.1)**



Synthesis

Whole Class | 5 min

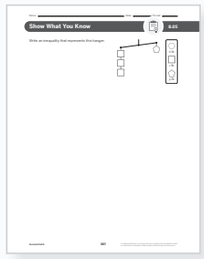
Students synthesize their understanding of the connection between an unbalanced hanger with unknown weights and an inequality written in symbols with variables.



Show What You Know

Independent | 5 min

Students demonstrate their understanding by writing an inequality to represent an unbalanced hanger.



Prep Checklist

Prepare Activity 3 Sheets.

This lesson includes:

- Student Edition
- Show What You Know
- Activity 3 Sheets, one per student

Independent | 5 min

Warm-Up

Purpose: Students explore unbalanced hangers and compare the relative weights of objects.

1 Launch

To surface prior knowledge, consider asking, "What did it mean when a hanger diagram was *balanced* previously this year? What does it mean for a hanger diagram to be unbalanced?"

2 Connect

Display the distribution of responses by polling the class.

Use the Decide and Defend routine to support students in making arguments and critiquing the reasoning of others. Encourage students to justify how they know which side of the hanger is heavier. **(MTR.1.1)**

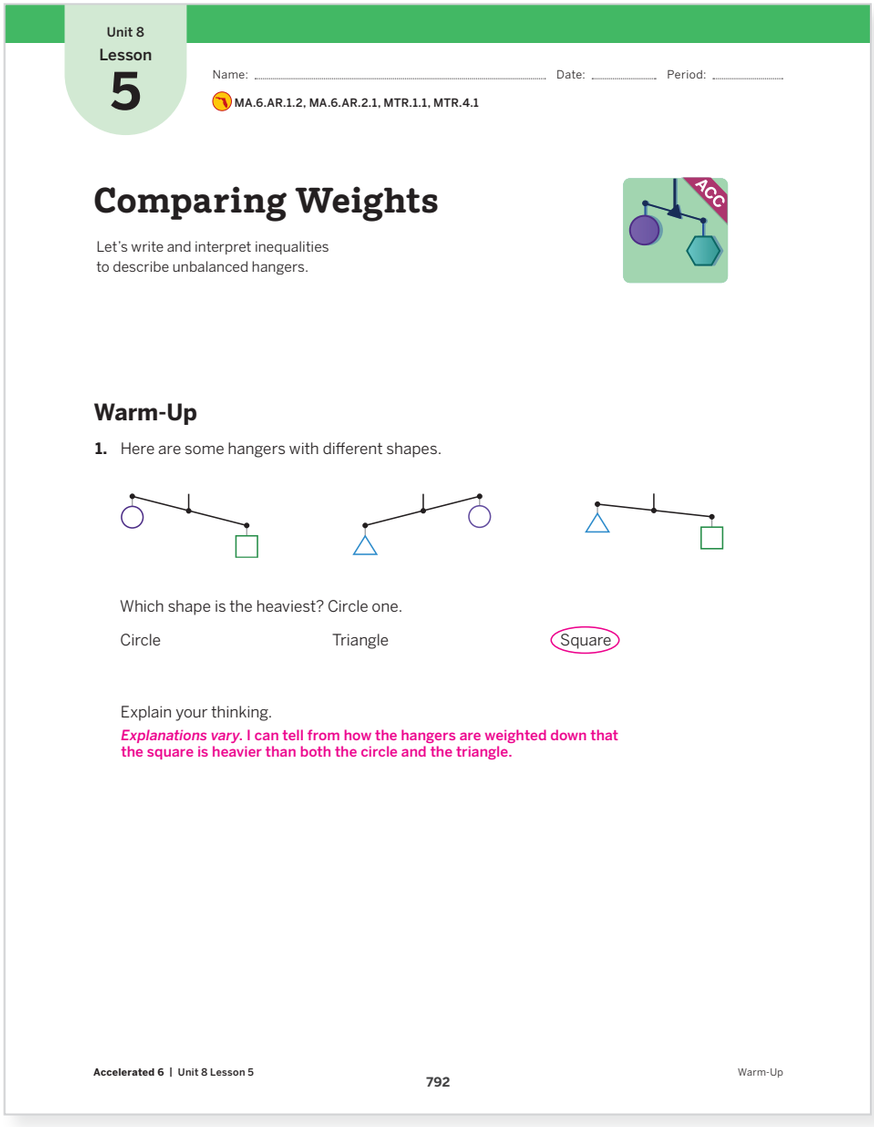
Note: As students explain their reasoning, it may be helpful to write inequalities that students describe on the board for future reference. For example, $c < s$ would represent the circle on the left and the square on the right, where the square is heavier than the circle.

ELL **English Language Learners** As students share in partner and whole-class discussions, provide sentence frames such as:

- I agree/disagree with you because . . .
- I think that makes sense because . . .
- How did you know . . .

(Writing and Speaking)

Encourage students to use a different strategy to defend their argument, or try to create a balanced hanger.



Unit 8
Lesson
5

Name: ..... Date: ..... Period: .....

MA.6.AR.1.2, MA.6.AR.2.1, MTR.1.1, MTR.4.1

Comparing Weights

ACC

Let's write and interpret inequalities to describe unbalanced hangers.

Warm-Up

1. Here are some hangers with different shapes.

Which shape is the heaviest? Circle one.

Circle Triangle Square

Explain your thinking.

Explanations vary. I can tell from how the hangers are weighted down that the square is heavier than both the circle and the triangle.

Accelerated 6 | Unit 8 Lesson 5 792 Warm-Up

Pairs | 13 min

Activity 1 One-Variable Inequalities

Purpose: Students interpret inequalities that represent relationships between weights on unbalanced hangers. **(MTR.1.1)**

1 Launch

Direct students' attention to the hanger and consider asking, "Which object is heavier? How do you know?"

2 Monitor

Invite students to share their weights with a partner and decide if their partner's weight makes sense for this hanger.

Listen to partner discussions for student language and descriptions to highlight in the Connect. **(MTR.4.1)**

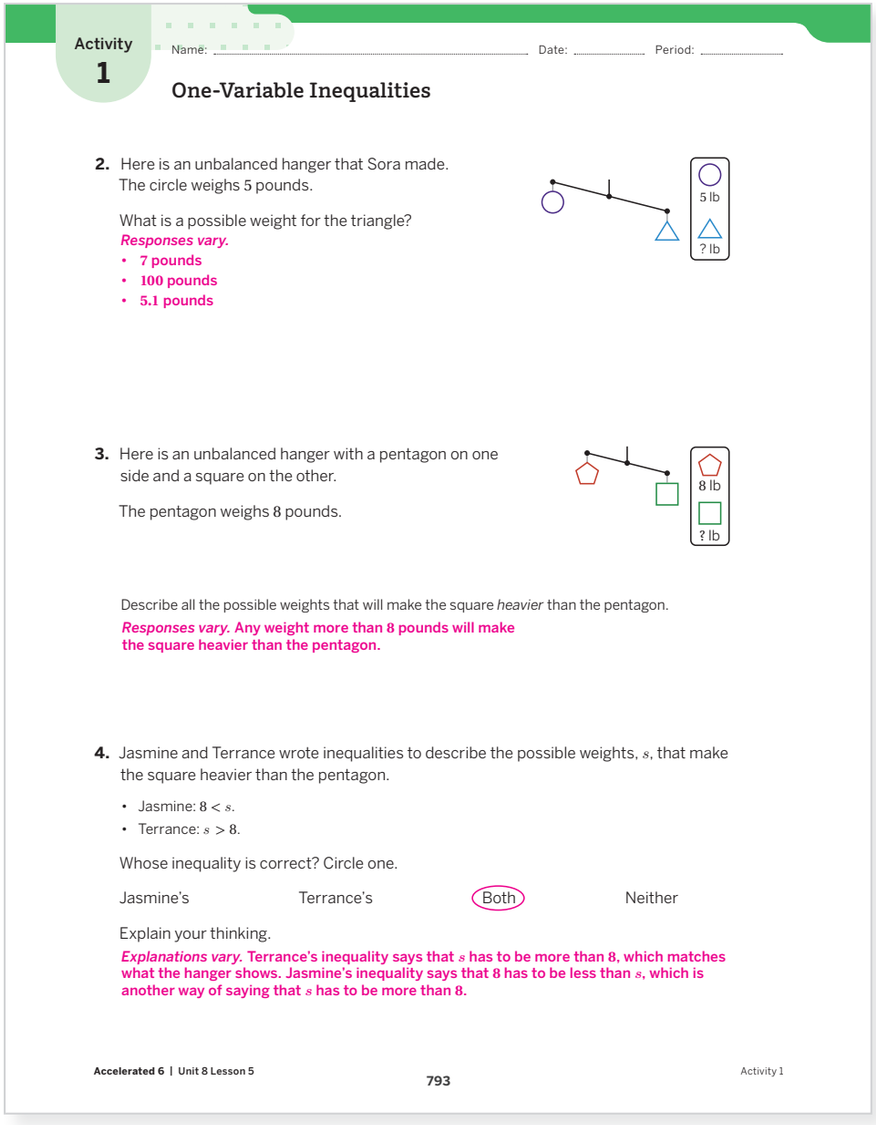
Invite students to read the prompt aloud in pairs so they can practice speaking and listening to phrases like "8 is less than s" and "s is greater than 8."

Listen for student responses to share during the Connect.

D Differentiation

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Consider asking students how they could test if an inequality is correct.
Choose only "Jasmine's" or "Terrance's."	**Support** Invite students to test values in both inequalities.
Choose "Both."	**Strengthen** Consider asking, "Are there always multiple ways to write inequalities?" Invite students to discuss with a partner. **(MTR.4.1)**
Would like to explore using inequalities further.	**Stretch** Invite students to create their own inequality that represents the hanger Sora made.

Activity 1 continued >



Activity 1

Name: _____ Date: _____ Period: _____

One-Variable Inequalities

2. Here is an unbalanced hanger that Sora made. The circle weighs 5 pounds.

What is a possible weight for the triangle?

Responses vary.

- 7 pounds
- 100 pounds
- 5.1 pounds

5 lb
? lb

3. Here is an unbalanced hanger with a pentagon on one side and a square on the other.

The pentagon weighs 8 pounds.

8 lb
? lb

Describe all the possible weights that will make the square *heavier* than the pentagon.

Responses vary. Any weight more than 8 pounds will make the square heavier than the pentagon.

4. Jasmine and Terrance wrote inequalities to describe the possible weights, s, that make the square heavier than the pentagon.

- Jasmine: $8 < s$.
- Terrance: $s > 8$.

Whose inequality is correct? Circle one.

Jasmine's Terrance's (Both) Neither

Explain your thinking.

Explanations vary. Terrance's inequality says that s has to be more than 8, which matches what the hanger shows. Jasmine's inequality says that 8 has to be less than s, which is another way of saying that s has to be more than 8.

Accelerated 6 | Unit 8 Lesson 5 793 Activity 1

Pairs | 13 min

Activity 1 One-Variable Inequalities (continued)

Purpose: Students interpret inequalities that represent relationships between weights on unbalanced hangers. **(MTR.1.1)**

2 Monitor

Encourage students to share their reasoning with a partner and work together to reach an agreement about how to match the representations. **(MTR.1.1)**

Note: If time allows, invite pairs to compare their matches, justify their thinking, and make revisions based on their conversation.

D Differentiation

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Invite students to choose one hanger at a time and find all of the representations that match it.
Match $10 > h$ and $h > 10$ in the same group.	**Support** Consider inviting these students to choose a specific value for h or to say out loud what each inequality means.
Match each representation correctly.	**Stretch** Encourage students who finish early to create their own set of representations that they think might be challenging.

3 Connect

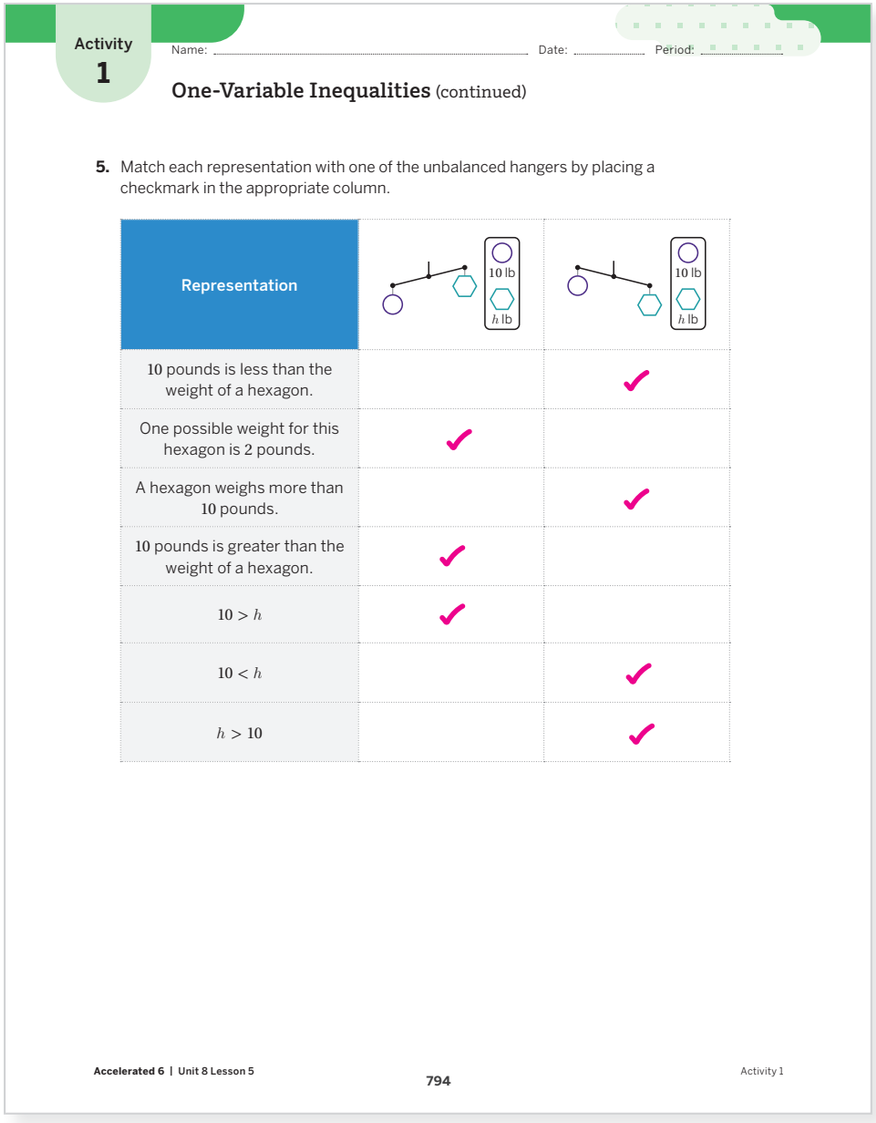
Invite students to share their responses and their reasoning about which inequality represents the hanger, calling attention to any conflict or consensus.

ELL **English Language Learners** Provide sentence frames to help students explain their reasoning (e.g., _____ is correct because _____.). **(Writing and Speaking)**

Consider asking:

- "How would you say each students' inequality in words?"
- "How can both students be correct?"
- "What would a hanger for the inequality $s < 8$ look like?"

To surface the Key Takeaway, consider asking, "How can we use inequalities to compare unknown weights?"



Activity 1

Name: _____ Date: _____ Period: _____

One-Variable Inequalities (continued)

5. Match each representation with one of the unbalanced hangers by placing a checkmark in the appropriate column.

Representation		
10 pounds is less than the weight of a hexagon.		✓
One possible weight for this hexagon is 2 pounds.	✓	
A hexagon weighs more than 10 pounds.		✓
10 pounds is greater than the weight of a hexagon.	✓	
$10 > h$	✓	
$10 < h$		✓
$h > 10$		✓

Accelerated 6 | Unit 8 Lesson 5 794 Activity 1

Key Takeaway: You can use *variables* to represent unknown weights on a hanger diagram, and symbols such as greater than or less than to compare those weights. There are multiple ways to represent the same inequality. For example, $8 < s$ and $s > 8$ both indicate that the value of s is greater than 8.

Pairs | 7 min

Activity 2 Many-Variable Inequalities

Purpose: Students create unbalanced hangers and write inequalities using only variables to compare unknown weights.

1 Launch

To surface connections, consider asking, "How are these inequalities similar to or different from the ones in the last activity?"

2 Monitor

To support students getting started, consider asking:

- "Is this hanger balanced?"
- "Which side of the hanger is heavier? How is this seen in an inequality?"

Invite students to think independently before discussing their answers with a partner.

Invite students to share with a partner or a student using digital.

A **Accessibility: Conceptual Processing** Invite students to describe what they have tried to support self-monitoring of understanding.

Note: Remind students that they do not need to include all three shapes in their hanger.

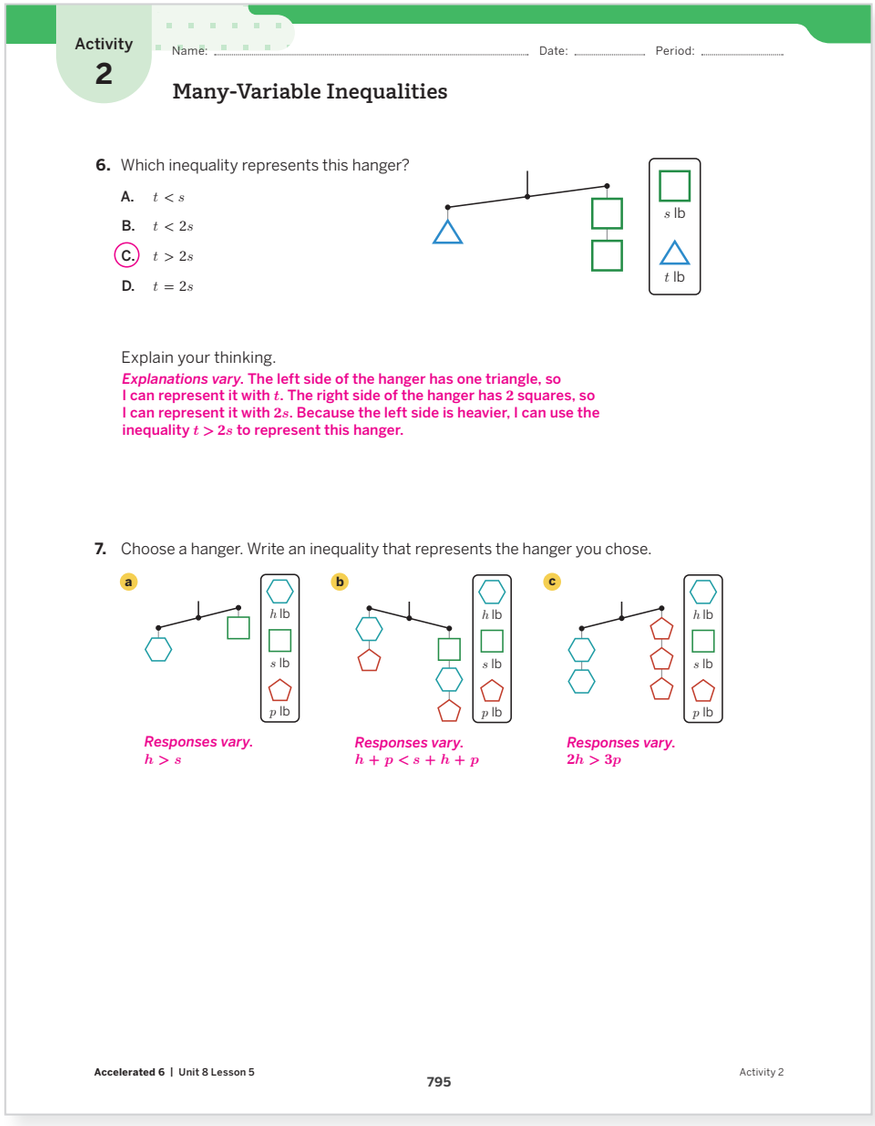
D **Differentiation: (Support)** Look for students who write the total of two weights as a product (e.g. hp) rather than a sum ($h + p$). Consider asking them to substitute different values for the weights and then check to see if the product is equal to the total they expect from the hanger.

Look for unique or creative hangers to share in the Connect.

3 Connect

Invite students to share how they determined which inequality represented the hanger diagram.

Consider asking, "How can you be certain that an inequality and hanger match?"



Activity 2

Name: Date: Period:

Many-Variable Inequalities

6. Which inequality represents this hanger?

A. $t < s$
B. $t < 2s$
C. $t > 2s$
D. $t = 2s$

Explain your thinking.

Explanations vary. The left side of the hanger has one triangle, so I can represent it with t. The right side of the hanger has 2 squares, so I can represent it with $2s$. Because the left side is heavier, I can use the inequality $t > 2s$ to represent this hanger.

7. Choose a hanger. Write an inequality that represents the hanger you chose.

a

Responses vary.
$h > s$

b

Responses vary.
$h + p < s + h + p$

c

Responses vary.
$2h > 3p$

Accelerated 6 | Unit 8 Lesson 5 795 Activity 2

Pairs | 10 min

Activity 3 Challenge Creator

Purpose: Students use their creativity to build their own unbalanced hanger for other students to represent using inequalities. **(MTR.1.1)**

Short on time? Consider omitting this activity.

1 Launch

Consider sharing that students will create their own challenge similar to the previous problem. Demonstrate how students can create their hanger diagram.

Distribute the Activity 3 Sheet to students so they can create their hanger diagram challenge.

To encourage creativity, invite students to create their own hanger challenge.

2 Monitor

Listen to student strategies and discussions, offering help or encouragement as needed.

Invite students to check their answers with the person who created the hanger they're representing. Encourage them to discuss and resolve any differences. **(MTR.4.1)**

Note: This Challenge Creator may take 10 or more minutes.

D Differentiation

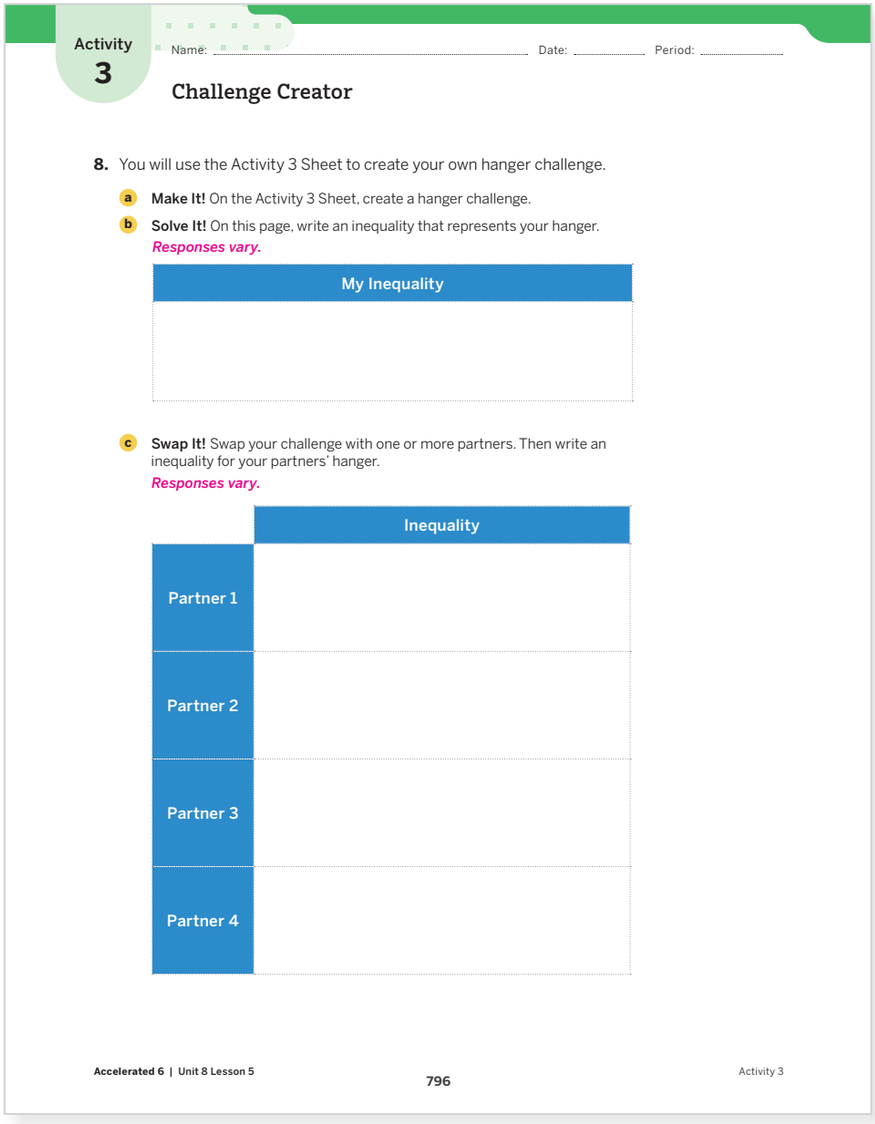
Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Invite students to start with a simpler hanger using two shapes instead of all five.
Write an incorrect inequality.	**Support** Consider asking, "Do the variables and symbols in the inequality match your hanger diagram?"
Correctly write an inequality for their hanger.	**Strengthen** Celebrate students thinking and invite them to share during the Connect. **Stretch** Consider asking, "What other inequalities can we write to represent your hanger?"

Invite students to share their hanger diagram responses during the Connect.

3 Connect

Share hangers and invite students to write an inequality for their classmates' hangers.

Invite students to share a variety of strategies for creating their hanger diagrams. Consider asking, "Was there a strategy you heard that surprised you?" **(MTR.1.1)**



Activity 3

Name: _____ Date: _____ Period: _____

Challenge Creator

8. You will use the Activity 3 Sheet to create your own hanger challenge.

a. **Make It!** On the Activity 3 Sheet, create a hanger challenge.

b. **Solve It!** On this page, write an inequality that represents your hanger.

Responses vary.

My Inequality

c. **Swap It!** Swap your challenge with one or more partners. Then write an inequality for your partners' hanger.

Responses vary.

	Inequality
Partner 1	
Partner 2	
Partner 3	
Partner 4	

Accelerated 6 | Unit 8 Lesson 5 796 Activity 3

Whole Class | 5 min

Synthesis

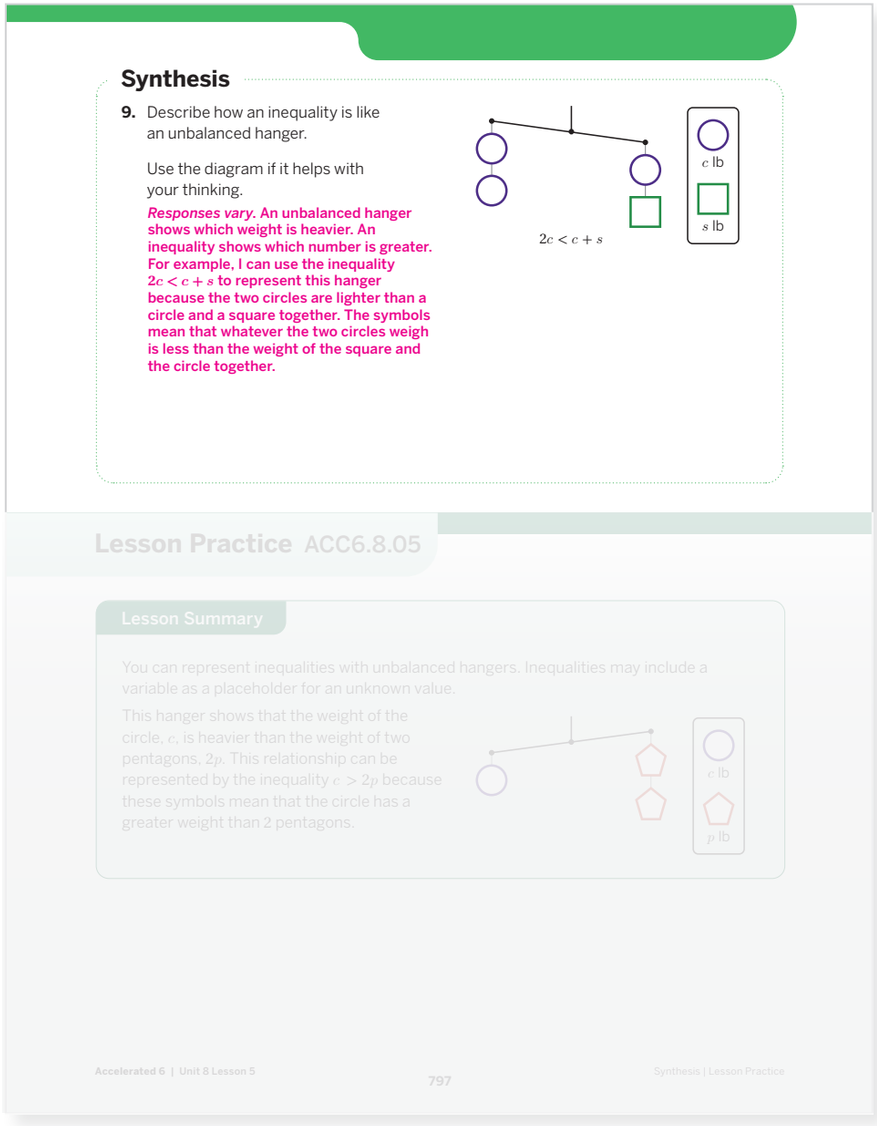
Purpose: Students synthesize their understanding of the connection between an unbalanced hanger with unknown weights and an inequality written in symbols with variables.

Invite students to respond independently, and then share their thinking with a partner.

Collect and share a variety of ideas, such as:

- An unbalanced hanger can show which weight is heavier.
- An inequality includes symbols and variables that show which side is greater.

Note: If time allows, invite students to revise their previous thinking based on the discussion.



Synthesis

9. Describe how an inequality is like an unbalanced hanger.

Use the diagram if it helps with your thinking.

Responses vary. An unbalanced hanger shows which weight is heavier. An inequality shows which number is greater. For example, I can use the inequality $2c < c + s$ to represent this hanger because the two circles are lighter than a circle and a square together. The symbols mean that whatever the two circles weigh is less than the weight of the square and the circle together.

Lesson Practice ACC6.8.05

Lesson Summary

You can represent inequalities with unbalanced hangers. Inequalities may include a variable as a placeholder for an unknown value.

This hanger shows that the weight of the circle, c, is heavier than the weight of two pentagons, $2p$. This relationship can be represented by the inequality $c > 2p$ because these symbols mean that the circle has a greater weight than 2 pentagons.

Accelerated 6 | Unit 8 Lesson 5 797 Synthesis | Lesson Practice

Lesson Takeaway: Inequalities can describe unbalanced hangers. Variables represent the unknown weights, and symbols compare the different weights. Every inequality can be written in two ways. For example, $c > s$ is the same relationship as $s < c$.

Independent | 5 min

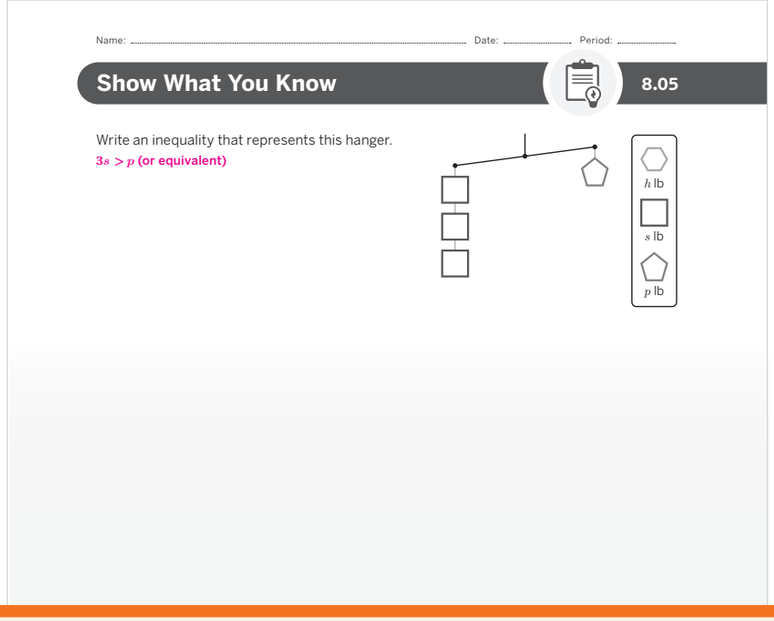
Show What You Know

Purpose: Students demonstrate their understanding by writing an inequality to represent an unbalanced hanger.

Today's Goal

1. **Goal:** Write and interpret inequalities that represent relationships between weights on an unbalanced hanger.

Show What You Know (Assessment Resources)


Name: Date: Period:

Show What You Know 8.05

Write an inequality that represents this hanger.

$3s > p$ (or equivalent)

Differentiation Use after Lesson 5

Support

Provide targeted intervention for students by using these resources.

If students need support with using hangers to solve inequalities:

» **Respond to Student Thinking:**

- Revisit **Activity 2, Problem 7** and invite students to explain how they determined the inequality for each hanger.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 2 Extension Activities**.

Professional Learning

Take a moment to reflect on the strategies students used in this lesson.

- What opportunities were there to celebrate specific students' strategies for interpreting and writing inequalities?
- What opportunities might there be in future lessons to connect back to the strategies students shared?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

9. Describe how an inequality is like an unbalanced hanger.

Use the diagram if it helps with your thinking.

Responses vary. An unbalanced hanger shows which weight is heavier. An inequality shows which number is greater. For example, I can use the inequality $2c < c + s$ to represent this hanger because the two circles are lighter than a circle and a square together. The symbols mean that whatever the two circles weigh is less than the weight of the square and the circle together.

Lesson Practice ACC6.8.05

Lesson Summary

You can represent inequalities with unbalanced hangers. Inequalities may include a variable as a placeholder for an unknown value.

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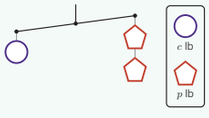
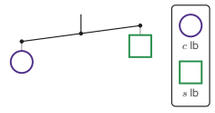


Lesson Practice
ACC6.8.05

Name: _____ Date: _____ Period: _____

Problems 1–4: Here is an unbalanced hanger.



1. Write an inequality that compares the weights of the two shapes. Explain your inequality in words.

Responses vary. $c > s$ or $s < c$. *Explanations vary.* The circle c is weighted down, which means that it's heavier than the square s.

2. The square weighs 5 pounds. Rewrite your inequality to represent this.

$c > 5$ (or equivalent)

3. List *three* possible weights for the circle. Use your inequality if it helps your thinking.

Responses vary. 5.49, 6, 7.2

4. Graph all of the possible solutions for your inequality that represents the hanger.

Problems 5–7: There is leftover food that has been in Jin's refrigerator for d days.

5. What does the inequality $d < 7$ tell you about Jin's food?

Responses vary. It means that all the leftover food is less than 7 days old.

6. What does the inequality $d > 0$ tell you about Jin's food?

Responses vary. It means that all the leftover food is more than 0 days old.

7. List *three* possible values of d that make both $d < 7$ and $d > 0$ true.

Responses vary. 0.8, 5, 4.5

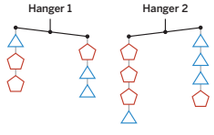
8. Gabriel is 12 years old. He has an older brother named Alejandro. Select *all* the inequalities that show the relationship between Gabriel's age, g, and Alejandro's age, a.

☐ **A.** $a < g$ ☑ **B.** $g < a$ ☑ **C.** $a > g$ ☐ **D.** $g > a$ ☑ **E.** $a > 12$

Lesson Practice
ACC6.8.05

Name: _____ Date: _____ Period: _____

9. If Hanger 1 correctly shows how the pentagons and triangles balance, is it possible to create a hanger that looks like Hanger 2? Explain your thinking.



No. Explanations vary. If Hanger 1 is correct, then the triangle has to be heavier than the pentagon. This means that the right side of Hanger 2 should also be heavier than the left side.

FAST Practice

10. Which of the following is a solution to the inequality $3c > 12$?

A. $c = 1$ **B.** $c = 3$ **C.** $c = 4$ **D.** $c = 9$

Spiral Review

Problems 11–12: Angel's family is driving to their grandmother's house, which is 325 miles away.

11. What percent of the distance have they traveled after driving 26 miles?

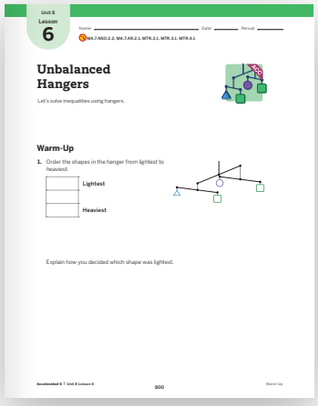
8%

12. How far have they driven if they are 72% of the way to grandmother's house?

234 miles

Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1–2, 4	1	MA.6.AR.1.2
	3	1	MA.6.AR.2.1
	5–6, 8	2	MA.6.AR.1.2
	7	3	MA.6.AR.2.1
	9	3	MA.6.AR.1.2
FAST Practice	10	2	MA.6.AR.2.1
Spiral Review			
	11–12	1	MA.6.AR.3.4


Unit 8
Lesson 6
Name: Date: Period:
MA.7.NSO.2.2, MA.7.AR.2.1, MTR.2.1, MTR.3.1, MTR.4.1
Unbalanced Hangers
Let's solve inequalities using hangers.
Warm-Up
1. Order the shapes in the hanger from lightest to heaviest.
Lightest
Heaviest
Explain how you decided which shape was lightest.
Accelerated 6 | Unit 8 Lesson 6
800
Warm-Up

Unbalanced Hangers

Solving Inequalities

Let's solve inequalities using hangers.

Learning Outcomes

Today's Goals

1. **Goal:** Determine the solutions to an inequality with only positive numbers.
2. **Language Goal:** Compare and contrast solutions to equations and solutions to inequalities. **(Speaking and Listening)**

Students determine solutions to inequalities using a familiar representation: hanger diagrams. Students connect solving an inequality with determining the values of x that will keep one side of a hanger heavier than the other, and they recognize that the solutions to inequalities are themselves inequalities.

Coherence

Prior Learning

In Lesson 5, students wrote inequalities to describe unbalanced hangers.

Future Learning

In Lesson 7, students will practice solving inequalities with both positive and negative coefficients and connect the solutions of inequalities to their graphs.

Connections

- Students use hanger diagrams as a visual way to solve inequalities by representing the relationship between variables and constants. (mathematical)

FL B.E.S.T. Math Benchmarks

Addressing

MA.7.AR.2.1
Write and solve one-step inequalities in one variable within a mathematical context and represent solutions algebraically or graphically.

Also Addressing: **MA.7.NSO.2.2**

Mathematical Thinking and Reasoning Standards: MTR.2.1, MTR.3.1, MTR.4.1

ELA Expectations: ELA.K12.EE.1.1, ELA.K12.EE.4.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.6.AR.1.2	**MA.8.AR.2.2**
MA.6.AR.2.1	

Lesson at a Glance

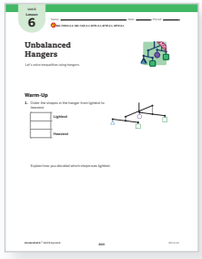
~ 45 min

MA.7.NSO.2.2, MA.7.AR.2.1

Warm-Up

Pairs | 5 min

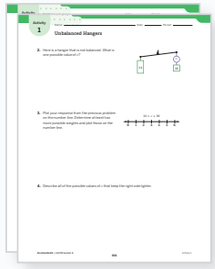
Students revisit hangers and reason about relative weights of objects on an unbalanced hanger. **Routine:** Think-Pair-Share



Activity 1

Pairs | 15 min

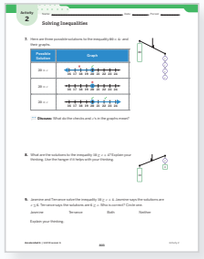
Students connect unbalanced hangers, inequalities, and their solutions. They explore why the solutions to an inequality are also inequalities. **Routine:** Decide and Defend **(MTR.2.1)**



Activity 2

Pairs | 10 min

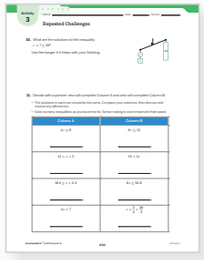
Students develop strategies for determining solutions to inequalities, making connections to equations, and using unbalanced hangers to help them revise their thinking.



Activity 3

Independent | 5 min

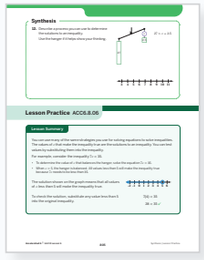
Students practice solving inequalities with positive values using a set of partner problems. **(MTR.3.1)**



Synthesis

Whole Class | 5 min

Students synthesize their understanding of how to solve an inequality.



Show What You Know

Independent | 5 min

Students demonstrate their understanding by solving an inequality and explaining their thinking.

Prep Checklist

This lesson includes:

- Student Edition
- Show What You Know

Pairs | 5 min

Warm-Up

Purpose: Students revisit hangers and reason about relative weights of objects on an unbalanced hanger.

1 Launch

Invite students to share what they notice about the hanger.

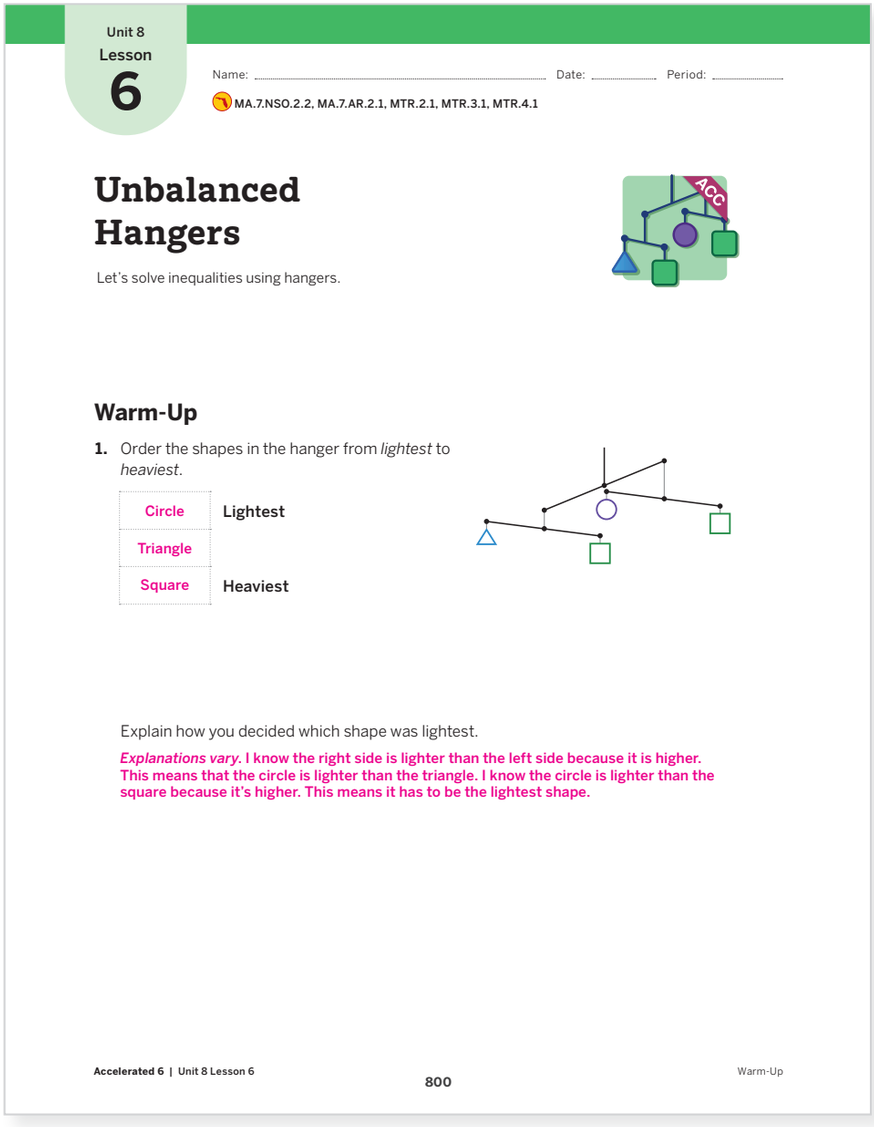
Use the Think-Pair-Share routine to support student-to-student discourse. Invite students to think independently and then take turns sharing with a partner. Invite several students to share their or their partner's strategy with the class. **(MTR.4.1)**

ELL **English Language Learners** Provide sentence frames to help students explain their strategy (e.g., The ____ is lighter/heavier than the ____ because ____.). **(Writing and Speaking)**

2 Connect

Display a few student explanations. Invite several students to share their reasoning.

To surface the Key Takeaway, consider asking, "Which side of an unbalanced hanger is heavier? How do you know?"



Unit 8
Lesson
6

Name: ..... Date: ..... Period: .....

MA.7.NSO.2.2, MA.7.AR.2.1, MTR.2.1, MTR.3.1, MTR.4.1

Unbalanced Hangers

Let's solve inequalities using hangers.

ACC

Warm-Up

1. Order the shapes in the hanger from *lightest* to *heaviest*.

Circle	Lightest
Triangle	
Square	Heaviest

Explain how you decided which shape was lightest.

Explanations vary. I know the right side is lighter than the left side because it is higher. This means that the circle is lighter than the triangle. I know the circle is lighter than the square because it's higher. This means it has to be the lightest shape.

Accelerated 6 | Unit 8 Lesson 6 800 Warm-Up

Key Takeaway: Unbalanced hangers are a visual representation for inequalities.

Pairs | 15 min

Activity 1 Unbalanced Hangers

Purpose: Students connect unbalanced hangers, inequalities, and their solutions. They explore why the solutions to an inequality are also inequalities. **(MTR.2.1)**

1 Launch

Invite students to share what they notice and wonder about the hanger. Listen for and amplify words such as *heavier*, *lighter*, *tilted*, and *unbalanced*.

2 Monitor

Look for students' progress, checking in with students as needed.

Encourage students to record several values that do and do not make the left side of the hanger heavier.

Invite students to check their values with a partner.

Look for several responses to share during the Connect.

D Differentiation

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Encourage students to make connections between the dots on the number line and the hanger.
Discuss 4 as a point that makes it equal or balanced.	**Support** Consider asking, "Does that keep the right side lighter? What is important about this point?" Consider capturing this to show during the Connect.
List specific numbers (1, 2, 5, etc.).	**Strengthen** Consider asking, "Are there other solutions that aren't listed here yet? How could we incorporate those?"
Write "any number less than 4."	**Strengthen** Consider asking, "How might we represent that using symbols?"
Write $x < 4$.	**Stretch** Challenge students to describe the largest and the smallest solutions.

ELL **English Language Learners** Provide sentence frames to help students explain their reasoning (e.g., All of the possible weights are _____ because _____.). **(Writing and Speaking)**

Activity 1 continued >

Activity 1

Name: Date: Period:

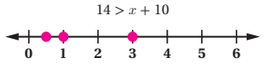
Unbalanced Hangers

2. Here is a hanger that is not balanced. What is one possible value of x?

Responses less than 4 are considered correct.

3. Plot your response from the previous problem on the number line. Determine at least two more possible weights and plot those on the number line.

Sample shown on number line.



4. Describe *all* of the possible values of x that keep the right side lighter.

Responses vary. **The weight could be any number less than 4. 4 is not included because there is a $<$ sign, not a $\leq$ sign.**

Pairs | 15 min

Activity 1 Unbalanced Hangers (continued)

Purpose: Students connect unbalanced hangers, inequalities, and their solutions. They explore why the solutions to an inequality are also inequalities. **(MTR.2.1)**

2 Monitor

Display the distribution of responses by polling the class.

Use the Decide and Defend routine to support students in making arguments and critiquing the reasoning of others. **(MTR.4.1)**

Note: If it doesn't come up naturally, consider asking:

- "What would the hanger look like for each of the responses?"
- "What strategies can you use to solve inequalities such as $3x \leq 24$?"

Listen for students who reference the graph of $3x = 24$ as they create the graph of $3x \leq 24$.

Invite students to share their thoughts aloud during the whole-class discussion.

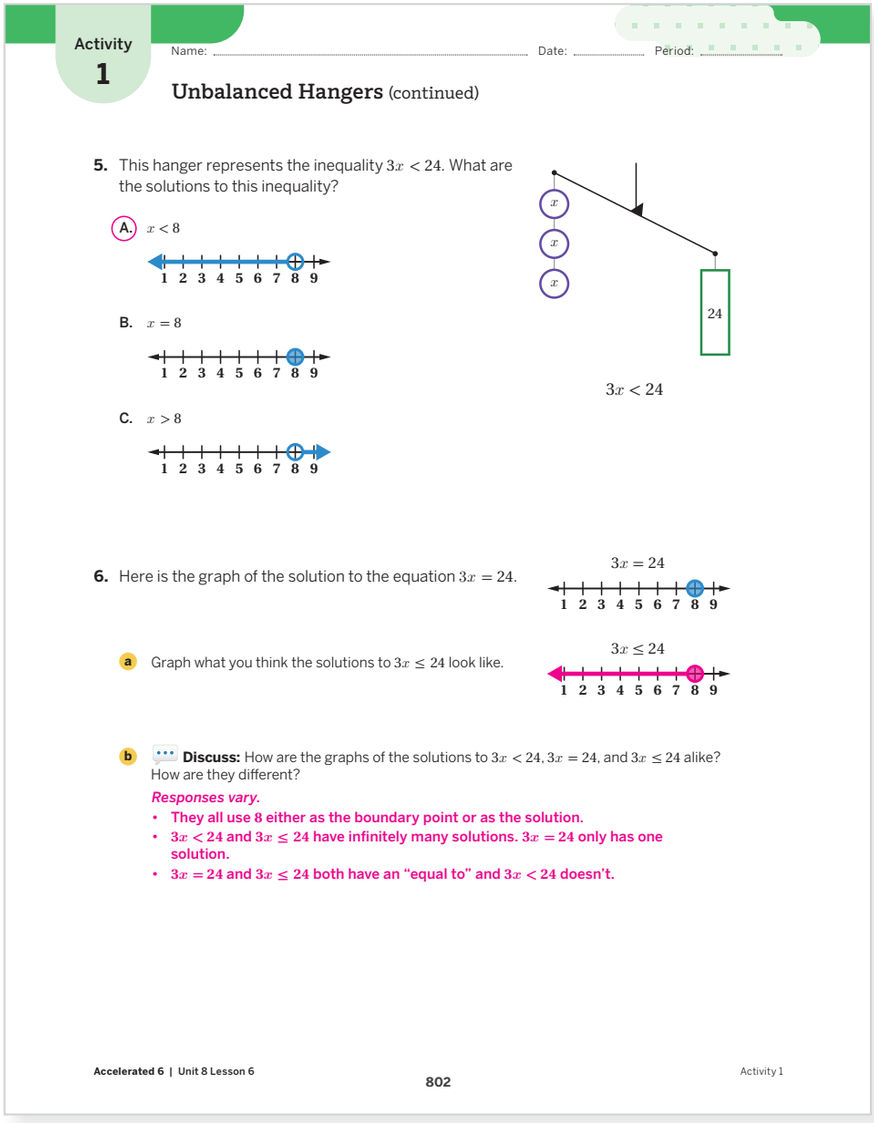
3 Connect

Share the strategies to connect the unbalanced hanger, the inequality, and the graph. Consider opening with a strategy where the student identified 4 as the balancing point. **(MTR.2.1)**

Consider asking, "How can you tell from each representation that the 4 isn't included in the inequality?"

To surface connections with Problem 4, consider asking, "How can you tell that the 8 is included in the inequality?"

To surface the Key Takeaway, consider asking, "What do solutions to inequalities look like graphically? Symbolically?"



Activity 1

Name: Date: Period:

Unbalanced Hangers (continued)

5. This hanger represents the inequality $3x < 24$. What are the solutions to this inequality?

$3x < 24$

A. $x < 8$

B. $x = 8$

C. $x > 8$

6. Here is the graph of the solution to the equation $3x = 24$.

$3x = 24$

a. Graph what you think the solutions to $3x \leq 24$ look like.

$3x \leq 24$

b. **Discuss:** How are the graphs of the solutions to $3x < 24$, $3x = 24$, and $3x \leq 24$ alike? How are they different?

Responses vary.

- They all use 8 either as the boundary point or as the solution.
- $3x < 24$ and $3x \leq 24$ have infinitely many solutions. $3x = 24$ only has one solution.
- $3x = 24$ and $3x \leq 24$ both have an "equal to" and $3x < 24$ doesn't.

Accelerated 6 | Unit 8 Lesson 6 802 Activity 1

Key Takeaway: Solutions to inequalities are written as inequalities because there are a range of values that make them true.

Pairs | 10 min

Activity 2 Solving Inequalities

Purpose: Students develop strategies for determining solutions to inequalities, making connections to equations, and using unbalanced hangers to help them revise their thinking.

1 Launch

To surface connections, consider asking, "How could you solve this problem if it were an equation?"

A **Accessibility: Visual-Spatial Processing** Invite students to answer a more concrete question first to support interpreting visual representations. Consider asking, "What does the green check above 20 mean? What about the red x above 20?"

2 Monitor

ELL **English Language Learners** Encourage students to use gestures or hand motions to help them predict what the hanger should look like. **(Reading and Listening)**

Look for students who justify the response "both" in creative ways, and look for several responses to share during the Connect.

D **Differentiation**

Look for students who:	Respond to Student Thinking
Have minimal or incomplete explanations.	**Support** Encourage students to justify their thinking because their ideas are valuable for the class discussion that is coming up.
Choose Terrance because the inequality sign is facing the same direction as the original.	**Support** Consider asking, "What is the difference between Jasmine's and Terrance's solutions?" Encourage students to test values.
Choose both.	**Strengthen** Consider asking, "Are there always multiple ways to write inequalities? How do you know?"

3 Connect

Invite students to share their responses, calling attention to any conflict or consensus.

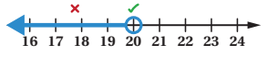
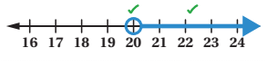
To surface the Key Takeaway, consider asking, "How is solving an inequality like solving an equation? How is it different?"

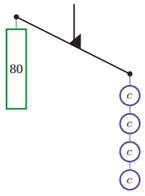
Activity 2

Name: Date: Period:

Solving Inequalities

7. Here are three possible solutions to the inequality $80 < 4c$ and their graphs.

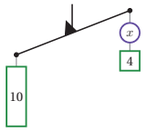
Possible Solution	Graph
$20 > c$	
$20 = c$	
$20 < c$	



Discuss: What do the checks and x's in the graphs mean?

Responses vary. In the last row of the table, the check on the open circle on 20 means 20 is the boundary. The x in the middle row means that 20 is not included. The check on the arrow facing right means this is correct because the solution for c includes all of the values that are greater than 20.

8. What are the solutions to the inequality $10 \geq x + 4$? Explain your thinking. Use the hanger if it helps with your thinking.



$x \leq 6$ or $6 \geq x$. *Explanations vary.* First, I determined the boundary point of 6, which keeps the hanger balanced because of the "equal to" in the inequality. A value greater than 6 will make the right side of the hanger heavier. A value less than 6 will keep the hanger lighter on the right as shown in the hanger.

9. Jasmine and Terrance solve the inequality $10 \geq x + 4$. Jasmine says the solutions are $x \leq 6$. Terrance says the solutions are $6 \geq x$. Who is correct? Circle one.

Jasmine Terrance (Both) Neither

Explain your thinking.

Explanations vary. These both mean the same thing because $x \leq 6$ and $6 \geq x$ are both true when x is any number less than or equal to 6.

Accelerated 6 | Unit 8 Lesson 6 803 Activity 2

Key Takeaway: One strategy for solving inequalities is to solve a related equation to determine the boundary point, then test values to determine the direction of the solutions.

Independent | 5 min

Activity 3 Repeated Challenges

Purpose: Students practice solving inequalities with positive values using a set of partner problems. **(MTR.3.1)**

Short on time? Consider omitting this activity.

1 Launch

Invite students to make a prediction about the solution before solving.

2 Monitor

Look for a variety of strategies and consider creating a record of them to support students with the repeated challenges.

Invite students to set their own goal for how many repeated challenges they would like to complete and share that sense-making is more important than speed.

Encourage students to solve and compare their solutions with a partner.

3 Connect

Invite students to share strategies that they found helpful or mistakes they made while solving these challenges. **(MTR.3.1)**

Activity 3

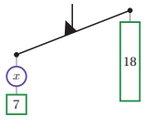
Name: Date: Period:

Repeated Challenges

10. What are the solutions to the inequality $x + 7 \geq 18$?

Use the hanger if it helps with your thinking.

$x \geq 11$ or $11 \leq x$



11. Decide with a partner who will complete Column A and who will complete Column B.

- The solutions in each row should be the same. Compare your solutions, then discuss and resolve any differences.
- Solve as many inequalities as you have time for. Sense-making is more important than speed.

Column A	Column B
$4x \leq 8$ $x \leq 2$	$6x \leq 12$ $x \leq 2$
$12 > x + 5$ $7 > x$	$15 > 3x$ $5 > x$
$10.4 \leq x + 2.2$ $8.2 \leq x$	$8x \geq 32.8$ $x \geq 4.1$
$2x > 7$ $x > \frac{7}{2}$	$x + \frac{2}{3} > \frac{40}{3}$ $x > \frac{38}{3}$

Accelerated 6 | Unit 8 Lesson 6 804 Activity 3

Whole Class | 5 min

Synthesis

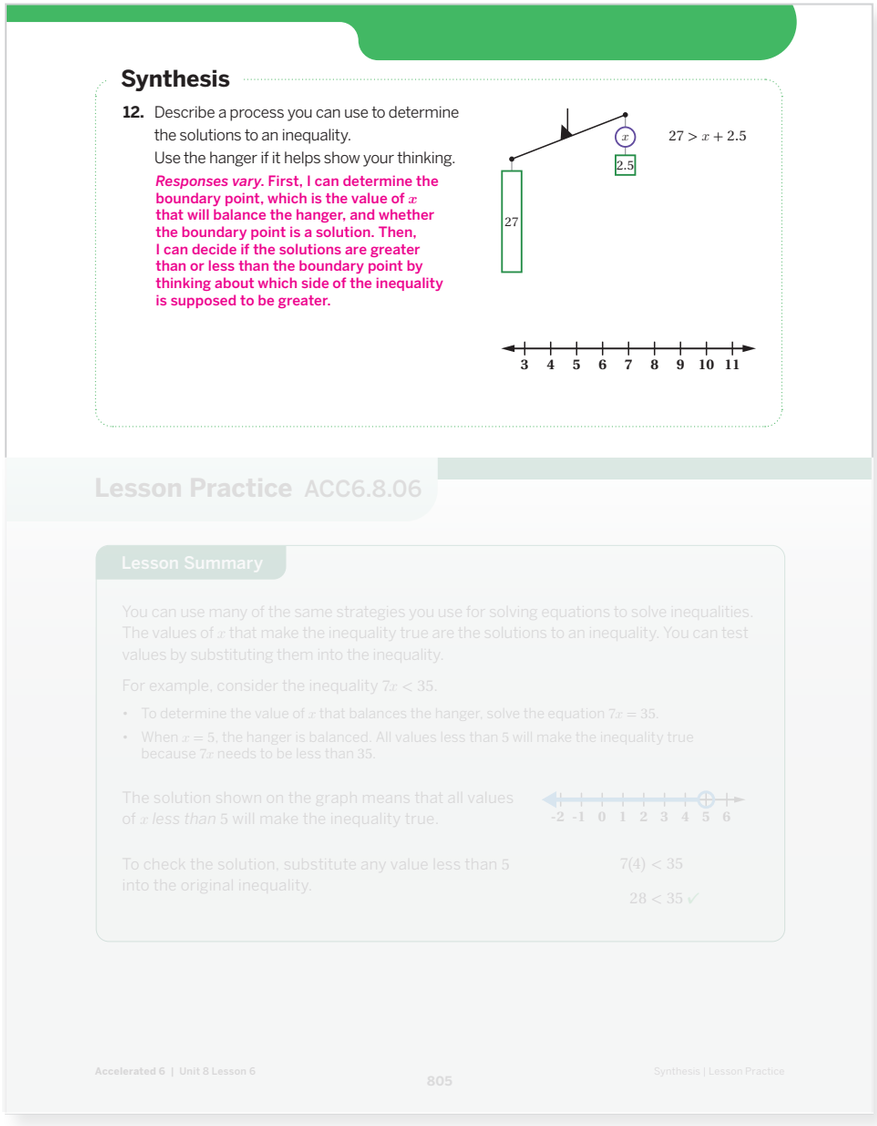
Purpose: Students synthesize their understanding of how to solve an inequality.

Invite students to respond independently, and then share their thinking with a partner.

Listen for and share a variety of ideas, such as:

- Connecting the inequality to the hanger.
- Solving as if this were an equation to determine the boundary point.
- Substituting values for x and testing if they make the inequality true.

Note: If time allows, invite students to share with a partner what makes sense about each strategy and what questions they still have.



Synthesis

12. Describe a process you can use to determine the solutions to an inequality.
Use the hanger if it helps show your thinking.

Responses vary. First, I can determine the boundary point, which is the value of x that will balance the hanger, and whether the boundary point is a solution. Then, I can decide if the solutions are greater than or less than the boundary point by thinking about which side of the inequality is supposed to be greater.

Lesson Practice ACC6.8.06

Lesson Summary

You can use many of the same strategies you use for solving equations to solve inequalities. The values of x that make the inequality true are the solutions to an inequality. You can test values by substituting them into the inequality.

For example, consider the inequality $7x < 35$.

- To determine the value of x that balances the hanger, solve the equation $7x = 35$.
- When $x = 5$, the hanger is balanced. All values less than 5 will make the inequality true because $7x$ needs to be less than 35.

The solution shown on the graph means that all values of x *less than* 5 will make the inequality true.

-2 -1 0 1 2 3 4 5 6

To check the solution, substitute any value less than 5 into the original inequality.

$7(4) < 35$

$28 < 35$ ✓

Accelerated 6 | Unit 8 Lesson 6 805 Synthesis | Lesson Practice

Lesson Takeaway: An inequality can be solved by determining the boundary point and then testing a value to determine the direction for the graph.

Independent | 5 min

Show What You Know

Purpose: Students demonstrate their understanding by solving an inequality and explaining their thinking.

Today's Goals

1. **Goal:** Determine the solutions to an inequality with only positive numbers.
2. **Language Goal:** Compare and contrast solutions to equations and solutions to inequalities. **(Speaking and Listening)**

Show What You Know (Assessment Resources)

Name: _____ Date: _____ Period: _____

Show What You Know 8.06

Solve the inequality $19 \geq 2x$. Explain your thinking.

$9.5 \geq x$ (or equivalent). *Explanations vary.* First, I found the boundary point, which is $x = 9.5$ because that is the solution to the equation $19 = 2x$. The solution to the inequality is $9.5 \geq x$ because 9.5 is the greatest value that makes the inequality true.

Differentiation Use after Lesson 6

Support

Provide targeted intervention for students by using these resources.

If students need support understanding how to solve an inequality:

» **Respond to Student Thinking:**

- Students will have more opportunities to develop understanding of inequalities in Lessons 7. You may choose not to intervene now.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 2 Extension Activities**.

Professional Learning

In this lesson, students continued to develop ideas of solutions to an inequality. How will their understanding of *solutions to an inequality* build during the next lesson?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

12. Describe a process you can use to determine the solutions to an inequality.
Use the hanger if it helps show your thinking.

Responses vary. First, I can determine the boundary point, which is the value of x that will balance the hanger, and whether the boundary point is a solution. Then, I can decide if the solutions are greater than or less than the boundary point by thinking about which side of the inequality is supposed to be greater.

Lesson Practice ACC6.8.06

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The solution shown on the graph means that all values of x *less than* 5 will make the inequality true.

To check the solution, substitute any value less than 5 into the original inequality.

$7(4) < 35$

$28 < 35$ ✓

Lesson Practice ACC6.8.06

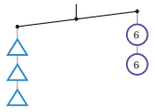
Name: _____ Date: _____ Period: _____

Problems 1–2: Here are two unbalanced hangers. Write an inequality to represent the relationship between the weights on each hanger. Use t to represent the weight of the triangle in grams.

1.

$t > 1$ (or equivalent)

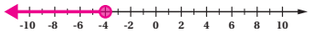
2.



$t > 4$ (or equivalent)

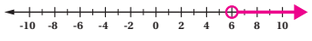
Problems 3–5: Use the number line diagrams to graph possible solutions for each inequality.

3. $5x \leq -20$



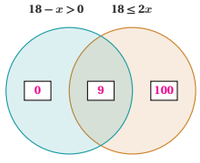
4. $30 > 6x$

5. $x + 3 > 9$



6. List three values for x that would make $5x \leq -20$ true.
Responses less than or equal to -4 are considered correct.

7. Write a value in each region that makes the inequality or inequalities true.
Responses vary. Sample shown on diagram.



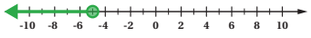
Lesson Practice ACC6.8.06

Name: _____ Date: _____ Period: _____

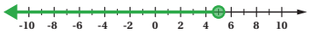
FAST Practice

8. Which number line represents the solutions to the inequality $x + 8 \leq 13$?

A.

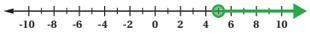


B.



C.

D.



Spiral Review

9. Select *all* the values that are solutions to $x \leq -4$.

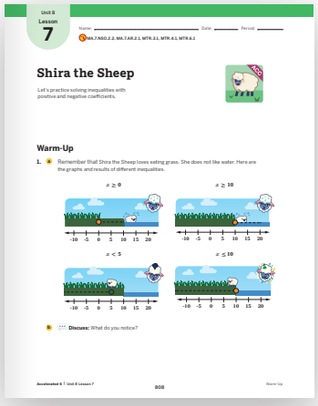
☐ A. 4 ☑ B. -4 ☐ C. -3.99
☑ D. -4.01 ☐ E. 0

10. Finish writing $\frac{5}{8}$ as a decimal.

$$
\begin{array}{r}
0.625 \\
8\overline{)5.000} \\
-48\downarrow \\
\hline
20 \\
-16\downarrow \\
\hline
40 \\
-40 \\
\hline
0
\end{array}
$$

Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1–5	2	MA.7.AR.2.1
	6	1	MA.7.AR.2.1
	7	3	MA.7.AR.2.1
FAST Practice	8	2	MA.7.AR.2.1
Spiral Review			
	9	1	MA.6.AR.2.1
	10	1	MA.7.NSO.1.2



Shira the Sheep

Solving Inequalities With Positive and Negative Numbers

Let's practice solving inequalities with positive and negative coefficients.

Learning Outcomes

Today's Goals

1. **Goal:** Solve inequalities with positive and negative coefficients, and graph their solutions.
2. **Goal:** Test values to determine which inequality symbol represents the solutions to an inequality.
3. **Language Goal:** Explain how to determine the graph of the solutions to any inequality. **(Writing, Speaking, and Listening)**

Students practice solving inequalities with positive and negative coefficients, and connect the solutions of inequalities to their graphs. They reason about the direction of an inequality when solving by asking, "What happens if I make x larger or smaller than the boundary?"

Coherence

Prior Learning

In Lesson 6, students made sense of solving inequalities using hanger diagrams.

Future Learning

In a future course, students will extend their thinking to solve two-step inequalities in one variable with rational numbers.

Connections

- Students determine the range of values for the variable that a sheep can safely graze on. (real world)

FL B.E.S.T. Math Benchmarks

Addressing

MA.7.AR.2.1
Write and solve one-step inequalities in one variable within a mathematical context and represent solutions algebraically or graphically.

Also addressing: **MA.7.NSO.2.2**

Mathematical Thinking and Reasoning Standards: MTR.3.1, MTR.4.1, MTR.6.1

ELA Expectation: ELA.K12.EE.5.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.6.AR.1.2	**MA.8.AR.2.2**
MA.6.AR.2.1	

Lesson at a Glance

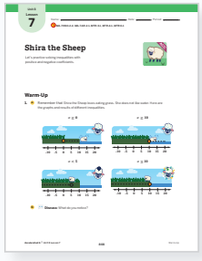
~ 45 min

MA.7.NSO.2.2, MA.7.AR.2.1

Warm-Up

Independent | 5 min

Students revisit the context of a fictional sheep who eats grass according to an inequality.



Activity 1

Pairs | 15 min

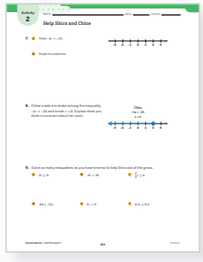
Students solve inequalities with positive and negative coefficients to solve a variety of Shira the Sheep challenges.

Activity 2

Pairs | 15 min

Students practice solving inequalities and critique other students' solutions.

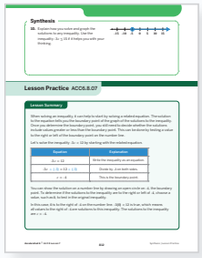
MLR3: Critique, Correct, Clarify (MTR.4.1)



Synthesis

Whole Class | 5 min

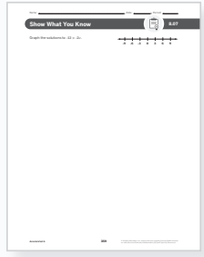
Students synthesize their understanding of solving and graphing inequalities with a negative coefficient.



Show What You Know

Independent | 5 min

Students demonstrate their understanding by graphing the solutions to an inequality.



Prep Checklist

This lesson includes:

- Student Edition
- Show What You Know

Independent | 5 min

Warm-Up

Purpose: Students revisit the context of a fictional sheep who eats grass according to an inequality.

1 Launch

Direct students' attention to the inequalities and their graphs.

Consider sharing that Shira the Sheep starts at the circle and moves along the dotted line.

To support students getting started, consider asking, "How is the dotted line related to the inequality in each situation?"

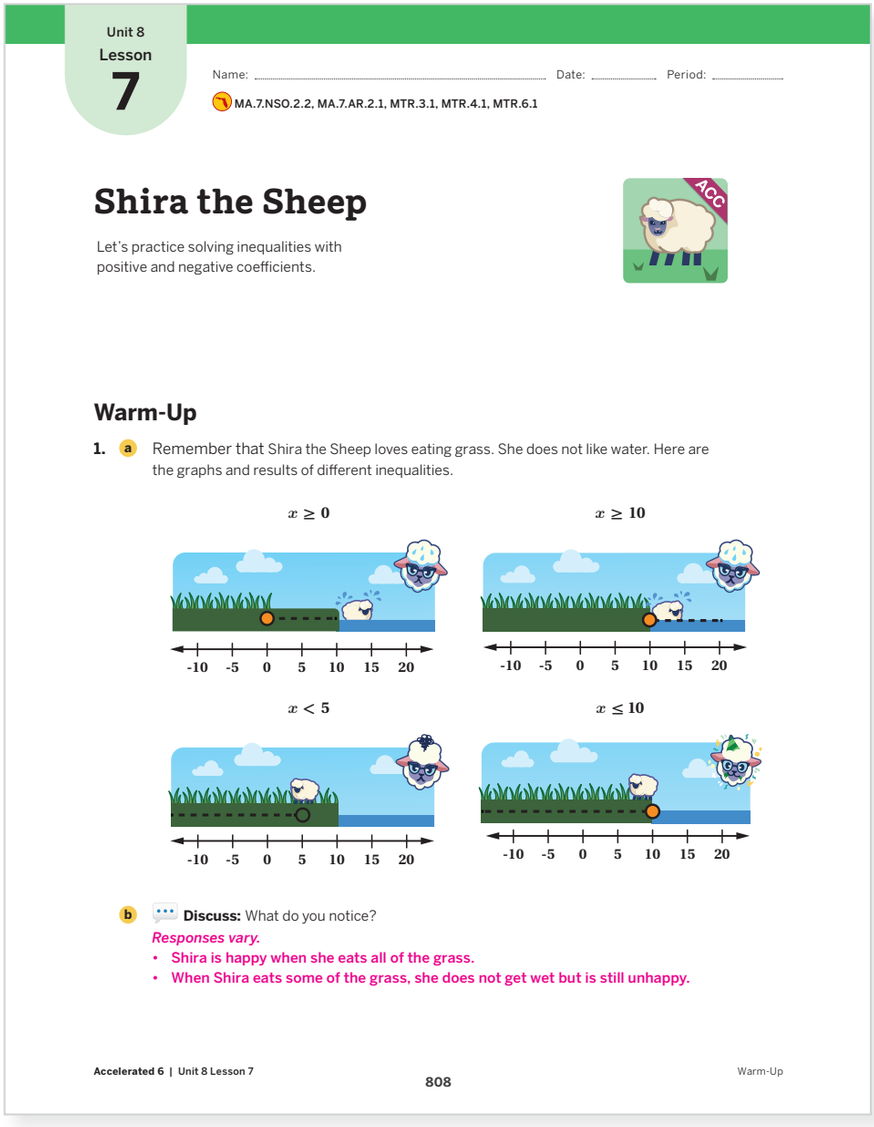
Invite students to share what they notice about Shira's reaction in each case.

2 Connect

Invite students to share what they noticed.

Note: If it does not come up naturally, consider asking:

- "What does the grass represent?"
- "Which inequalities cause Shira to get wet?"
- "Which inequality allows Shira to eat some but not all of the grass without getting wet?"
- "Which inequality allows Shira to eat all of the grass without getting wet?".



Unit 8
Lesson
7

Name: _____ Date: _____ Period: _____

MA.7.NSO.2.2, MA.7.AR.2.1, MTR.3.1, MTR.4.1, MTR.6.1

Shira the Sheep

Let's practice solving inequalities with positive and negative coefficients.

Warm-Up

1. a Remember that Shira the Sheep loves eating grass. She does not like water. Here are the graphs and results of different inequalities.

$x \geq 0$

$x \geq 10$

$x < 5$

$x \leq 10$

b **Discuss:** What do you notice?

Responses vary.

- Shira is happy when she eats all of the grass.
- When Shira eats some of the grass, she does not get wet but is still unhappy.

Accelerated 6 | Unit 8 Lesson 7 808 Warm-Up

Pairs | 15 min

Activity 1 Shira the Sheep

Purpose: Students solve inequalities with positive and negative coefficients to solve a variety of Shira the Sheep challenges.

1 Launch

Consider asking, "What is the same about this image and the one from the Warm-Up? What is different?"

Invite students to check their inequalities with a partner.

2 Monitor

Encourage students to revise their solutions based on the feedback from their partner.

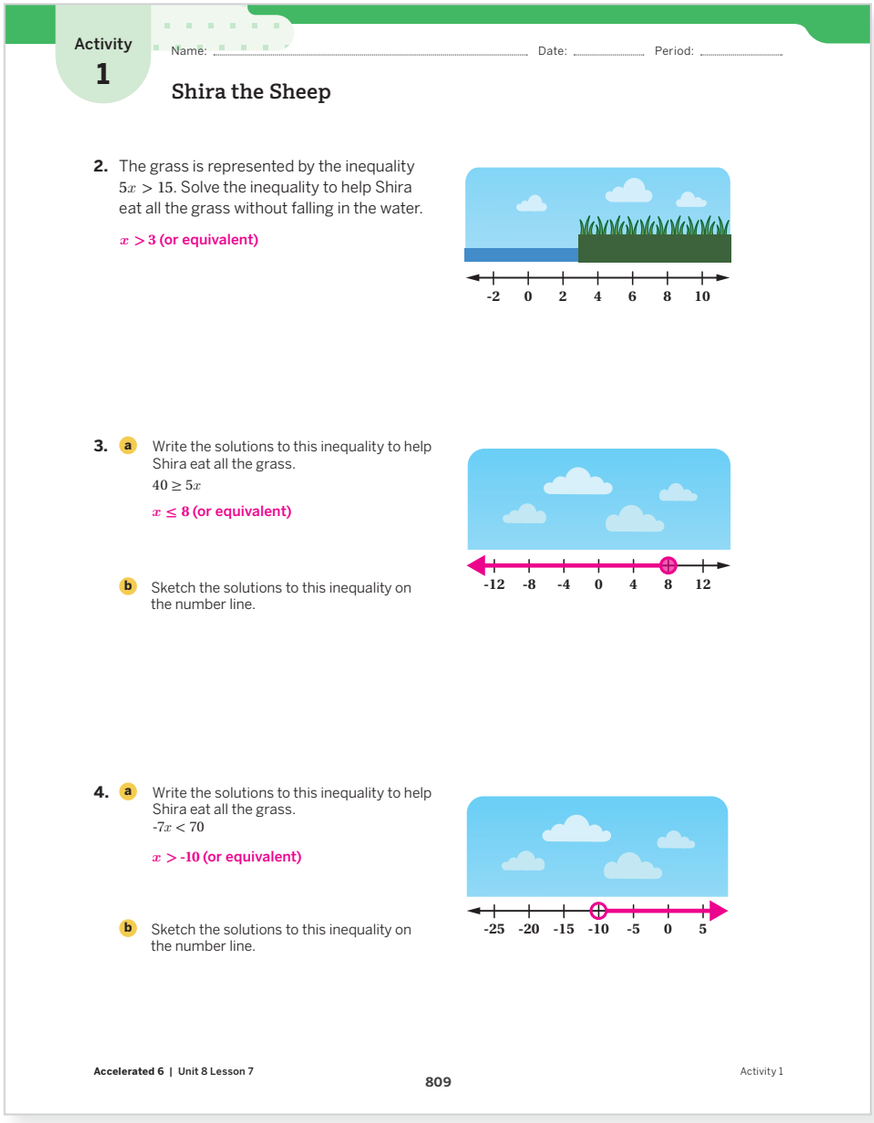
Look for students who use a variety of strategies to solve each inequality, including students who make connections to solving equations. **(MTR.3.1)**

Note: If time allows, pause to celebrate the different ways students solve the inequalities.

D **Differentiation: (Support)** Look for students who wrote $x < -10$ as the solution. Encourage them to test the value then ask, "How does the inequality need to change to prevent Shira from falling in the water?" **(MTR.6.1)**

A **Accessibility: Conceptual Processing** Invite students to talk aloud about their thinking to support self-monitoring of understanding.

Activity 1 continued >



Activity 1

Name: Date: Period:

Shira the Sheep

2. The grass is represented by the inequality $5x > 15$. Solve the inequality to help Shira eat all the grass without falling in the water.

$x > 3$ (or equivalent)

-2 0 2 4 6 8 10

3. a Write the solutions to this inequality to help Shira eat all the grass.

$40 \geq 5x$

$x \leq 8$ (or equivalent)

b Sketch the solutions to this inequality on the number line.

-12 -8 -4 0 4 8 12

4. a Write the solutions to this inequality to help Shira eat all the grass.

$-7x < 70$

$x > -10$ (or equivalent)

b Sketch the solutions to this inequality on the number line.

-25 -20 -15 -10 -5 0 5

Accelerated 6 | Unit 8 Lesson 7 809 Activity 1

Pairs | 15 min

Activity 1 Shira the Sheep (continued)

Purpose: Students solve inequalities with positive and negative coefficients to solve a variety of Shira the Sheep challenges.

2 Monitor

Look for several student responses to discuss during the Connect, including ones similar to the sample responses or other creative ideas.

Select and sequence strategies for solving inequalities with negative coefficients.

D Differentiation

Look for students who:	Respond to Student Thinking
Guess and check values.	**Support** Consider asking, "How could solving an equation help determine the boundary value?"
Write the equation $-2x = 4$ before solving.	**Strengthen** Invite studnets to share their strategy during the Connect.
Test a value on either side of the boundary point to determine the direction of the inequality.	
Recognize that the negative coefficient is changing the inequality sign.	**Stretch** Invite these students to test their theory and explain why this might be.

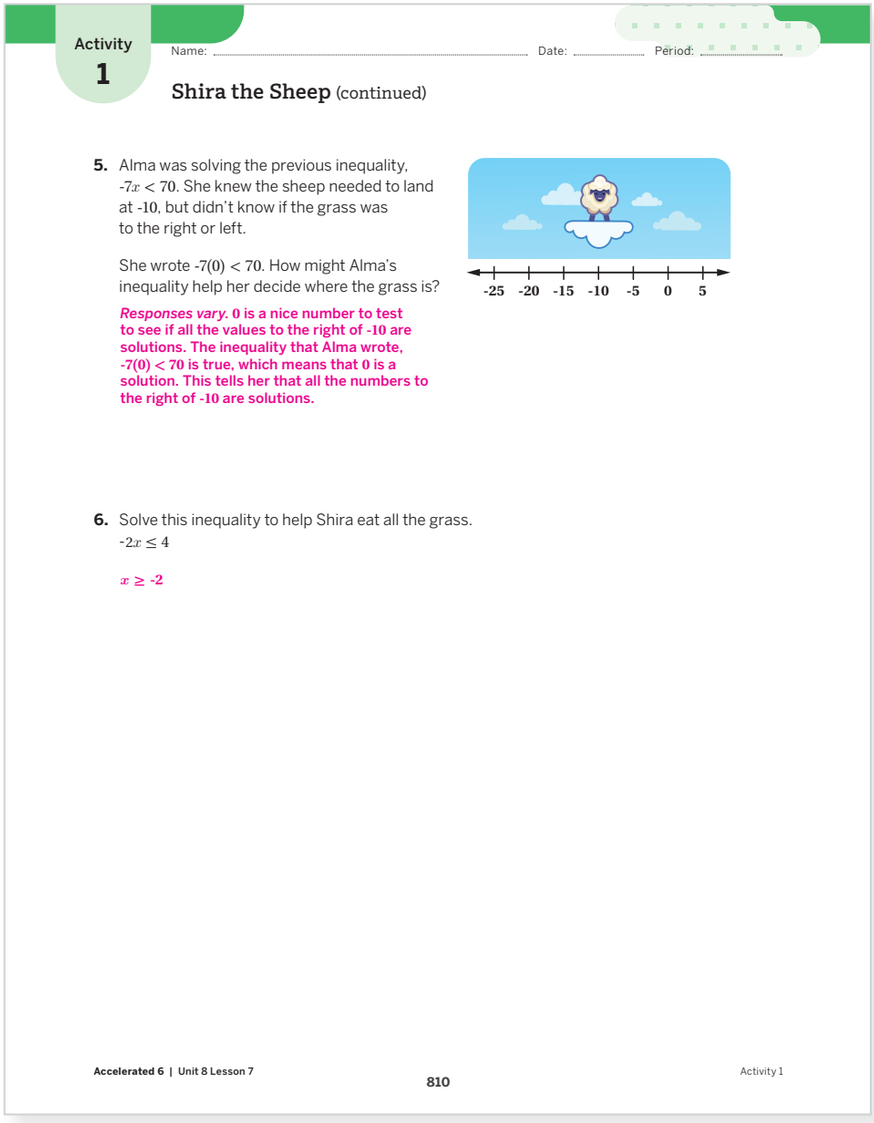
3 Connect

Display several student responses and invite students to critique Alma's reasoning. **(MTR.4.1)**

Consider asking:

- "Is 0 a solution to the original inequality? Why or why not?"
- "What do you think would happen if you tried a number less than -10, like -100 or -1,000?"
- "How does testing numbers help to determine which inequality symbol to use?"

To surface the Key Takeaway, share the strategies and discuss the connection between solving equations and solving inequalities.



Activity 1

Name: Date: Period:

Shira the Sheep (continued)

5. Alma was solving the previous inequality, $-7x < 70$. She knew the sheep needed to land at -10, but didn't know if the grass was to the right or left.

She wrote $-7(0) < 70$. How might Alma's inequality help her decide where the grass is?

Responses vary. 0 is a nice number to test to see if all the values to the right of -10 are solutions. The inequality that Alma wrote, $-7(0) < 70$ is true, which means that 0 is a solution. This tells her that all the numbers to the right of -10 are solutions.

6. Solve this inequality to help Shira eat all the grass.

$-2x \leq 4$

$x \geq -2$

Accelerated 6 | Unit 8 Lesson 7 810 Activity 1

Key Takeaway: Solving inequalities is similar to solving equations. To determine the direction of the inequality, you can test values on either side of the boundary point. The direction of the inequality is determined by which values make the inequality true.

Pairs | 15 min

Activity 2 Help Shira and Chloe

Purpose: Students practice solving inequalities and critique other students' solutions. **(MTR.4.1)**

Short on time? Consider setting a timer for students to attempt as many challenges as they can in Problem 9.

1 Launch

To encourage connections, consider asking, "Is 0 a solution to this inequality? Why or why not?"

Invite students to check their inequalities with a partner.

Note: It is okay if students are feeling stuck here. They will revisit this inequality in the next problem.

2 Monitor

Select and sequence students' solving strategies.

D Differentiation

Look for students who:	Respond to Student Thinking
Think Chloe is correct.	**Support** Consider asking, "If you substitute 1 for x, is your inequality true?"
Test values on either side of the boundary point to determine the direction of the inequality.	**Strengthen** Invite them to share during the Connect.
Recognize that the negative coefficient is why the inequality sign is > and not <.	**Stretch** Invite them to test their theory and explain why this might be.

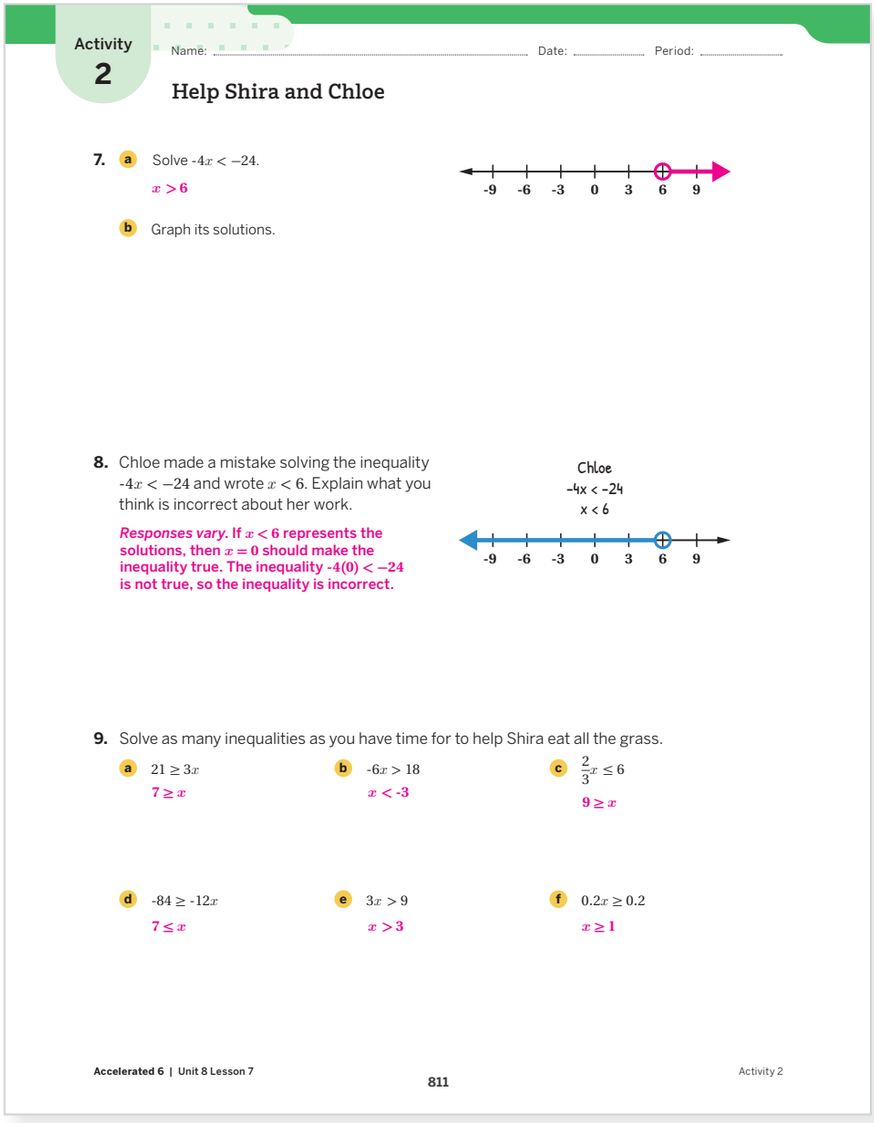
Invite students to set their own goal for how many problems they would like to complete.

To support students getting started, ask, "How might you determine the boundary point?"

3 Connect

MLR MLR3: Critique, Correct, Clarify Use this routine to help students describe strengths, errors, and ambiguities in Chloe's work. **(MTR.4.1)**
As a class:
- Discuss what is *correct* about her response.
- Discuss what is *incorrect* or *unclear*.
- Create a *correct* and *clear* solution.

Invite students to share strategies that they found helpful or what they learned from the mistakes they made while solving these problems. **(MTR.3.1)**



Activity 2

Name: Date: Period:

Help Shira and Chloe

7. **a** Solve $-4x < -24$.

$x > 6$

b Graph its solutions.

8. Chloe made a mistake solving the inequality $-4x < -24$ and wrote $x < 6$. Explain what you think is incorrect about her work.

Responses vary. If $x < 6$ represents the solutions, then $x = 0$ should make the inequality true. The inequality $-4(0) < -24$ is not true, so the inequality is incorrect.

9. Solve as many inequalities as you have time for to help Shira eat all the grass.

a $21 \geq 3x$

$7 \geq x$

b $-6x > 18$

$x < -3$

c $\frac{2}{3}x \leq 6$

$9 \geq x$

d $-84 \geq -12x$

$7 \leq x$

e $3x > 9$

$x > 3$

f $0.2x \geq 0.2$

$x \geq 1$

Accelerated 6 | Unit 8 Lesson 7 811 Activity 2

Whole Class | 5 min

Synthesis

Purpose: Students synthesize their understanding of solving and graphing inequalities with a negative coefficient.

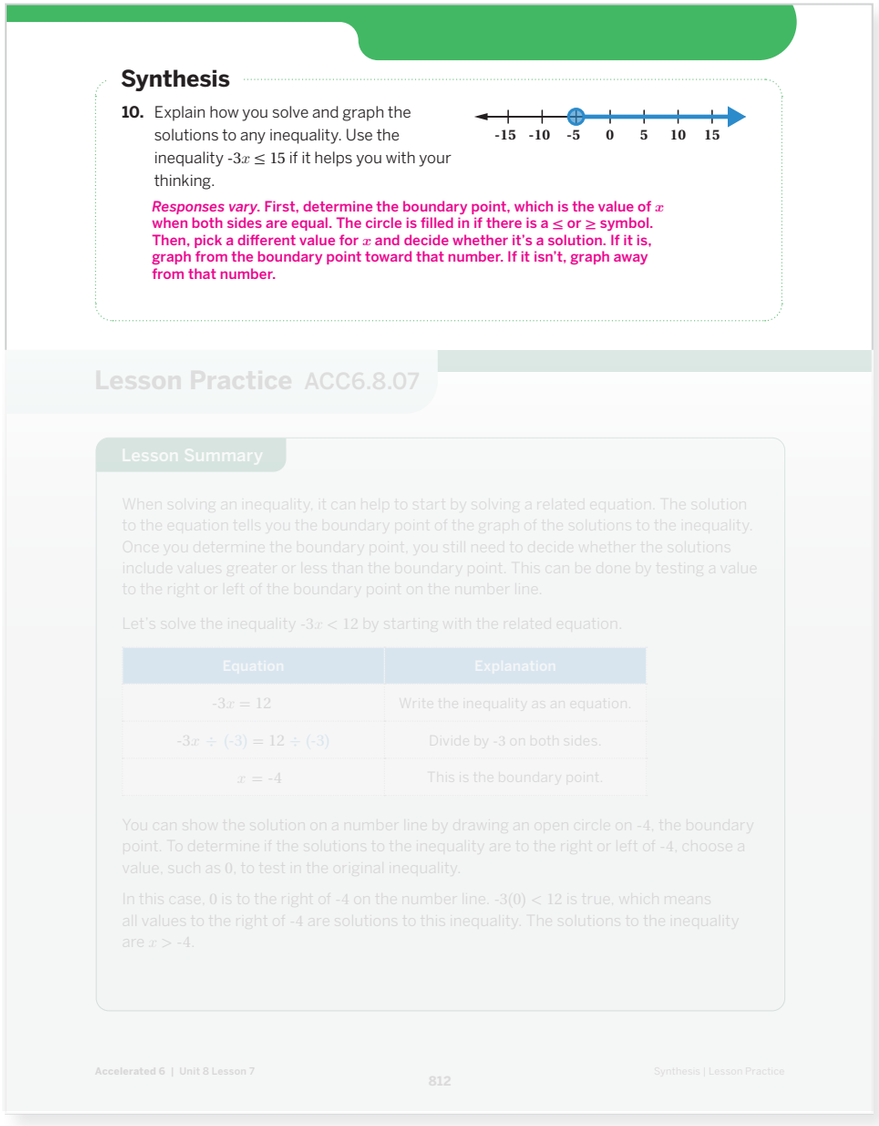
Invite students to respond independently, and then share their thinking with a partner.

Look for and share a variety of student responses to display, such as:

- Rewriting the inequality as an equation before solving.
- Testing values on both sides of the boundary point.
- Substituting 0 to test the solution.
- Referencing the inequality sign changing.

Discuss the connections students see between responses.

Note: If time allows, invite students to revise their previous thinking based on the discussion.



Synthesis

10. Explain how you solve and graph the solutions to any inequality. Use the inequality $-3x \leq 15$ if it helps you with your thinking.

Responses vary. First, determine the boundary point, which is the value of x when both sides are equal. The circle is filled in if there is a $\leq$ or $\geq$ symbol. Then, pick a different value for x and decide whether it's a solution. If it is, graph from the boundary point toward that number. If it isn't, graph away from that number.

Lesson Practice ACC6.8.07

Lesson Summary

When solving an inequality, it can help to start by solving a related equation. The solution to the equation tells you the boundary point of the graph of the solutions to the inequality. Once you determine the boundary point, you still need to decide whether the solutions include values greater or less than the boundary point. This can be done by testing a value to the right or left of the boundary point on the number line.

Let's solve the inequality $-3x < 12$ by starting with the related equation.

Equation	Explanation
$-3x = 12$	Write the inequality as an equation.
$-3x \div (-3) = 12 \div (-3)$	Divide by -3 on both sides.
$x = -4$	This is the boundary point.

You can show the solution on a number line by drawing an open circle on -4, the boundary point. To determine if the solutions to the inequality are to the right or left of -4, choose a value, such as 0, to test in the original inequality.

In this case, 0 is to the right of -4 on the number line. $-3(0) < 12$ is true, which means all values to the right of -4 are solutions to this inequality. The solutions to the inequality are $x > -4$.

Accelerated 6 | Unit 8 Lesson 7 812 Synthesis | Lesson Practice

Lesson Takeaway: Solving inequalities is similar to solving equations. The solutions to an inequality can be determined by first solving an equation, then testing whether the values on the left or right side of the boundary point make the inequality true. Testing a value is especially important when there are negative signs in the inequality.

Independent | 5 min

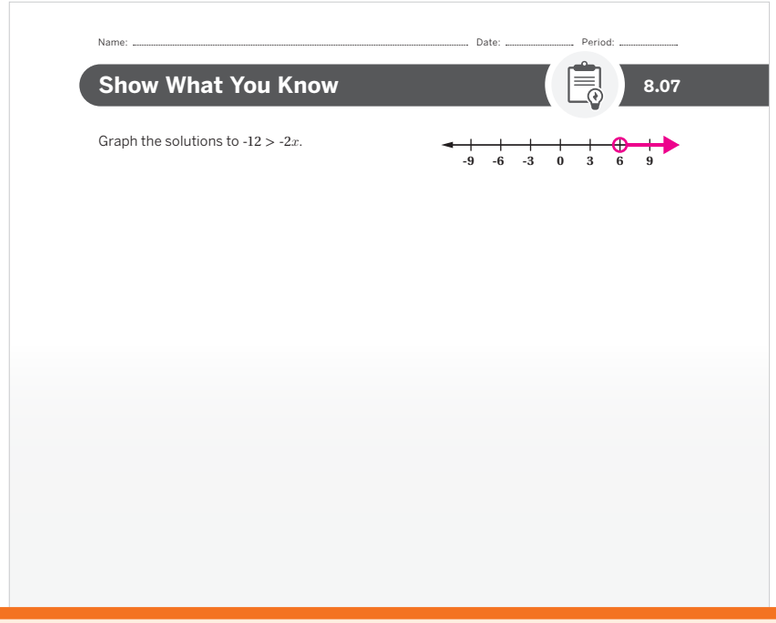
Show What You Know

Purpose: Students demonstrate their understanding by graphing the solutions to an inequality.

Today's Goals

1. **Goal:** Solve inequalities with positive and negative coefficients, and graph their solutions.
2. **Goal:** Test values to determine which inequality symbol represents the solutions to an inequality.
3. **Language Goal:** Explain how to determine the graph of the solutions to any inequality. **(Writing, Speaking, and Listening)**

Show What You Know (Assessment Resources)


Name: Date: Period:

Show What You Know 8.07

Graph the solutions to $-12 > -2x$.

-9 -6 -3 0 3 6 9

D Differentiation Use after Lesson 7

Support

Provide targeted intervention for students by using these resources.

If students need support understanding how to of solve inequalities with negative coefficients:

» **Respond to Student Thinking:**

- Use the ***Solving Inequalities*** **Mini-lesson**.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 2 Extension Activities**.

Professional Learning

Reflect on how students solved inequalities with positive and negative coefficients.

- How did they talk about mistakes?
- Were they able to notice the correct and incorrect parts of Alma's and Chloe's work?
- How can you support students in valuing learning from both correct and incorrect thinking?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

10. Explain how you solve and graph the solutions to any inequality. Use the inequality $-3x \leq 15$ if it helps you with your thinking.

Responses vary. First, determine the boundary point, which is the value of x when both sides are equal. The circle is filled in if there is a $\leq$ or $\geq$ symbol. Then, pick a different value for x and decide whether it's a solution. If it is, graph from the boundary point toward that number. If it isn't, graph away from that number.

Lesson Practice ACC6.8.07

Lesson Summary

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Let's solve the inequality $-3x < 12$ by starting with the related equation.

Equation	Explanation
$-3x = 12$	Write the inequality as an equation.
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$x = -4$	This is the boundary point.

You can show the solution on a number line by drawing an open circle on -4, the boundary point. To determine if the solutions to the inequality are to the right or left of -4, choose a value, such as 0, to test in the original inequality.

In this case, 0 is to the right of -4 on the number line. $-3(0) < 12$ is true, which means all values to the right of -4 are solutions to this inequality. The solutions to the inequality are $x > -4$.

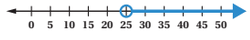
Lesson Practice ACC6.8.07

Name: _____ Date: _____ Period: _____

1. Select *all* the values of x that make the inequality $-x + 6 \geq 10$ true.

- ☐ A. -3.9
- ☐ B. 4
- ☑ C. -4.01
- ☑ D. -4
- ☐ E. 4.01
- ☐ F. 3.9
- ☐ G. 0
- ☑ H. -7

2. A library is having a party for any student who read at least 25 books over the summer. Match each situation or number line graph with an inequality.

Situation/Graph	Inequality
a. Ricardo read x books and was invited to the party.	b. $x < 25$
b. Prisha read x books but was not invited to the party.	d. $x > 25$
c.	c. $x \leq 25$
d. 	a. $x \geq 25$

Problems 3–4: Use the inequality $-3x \geq -150$.

3. Select *all* the values of x that make the inequality $-3x \geq -150$ true.

☑ A. 0 ☑ B. 50 ☑ C. -50 ☑ D. 49.9 ☐ E. 50.1

4. In order to solve the inequality $-3x \geq -150$, Makalya solved the equation $-3x = -150$ and got $x = 50$. What is the solution to the inequality?

$x \leq 50$

5. Diego is solving the inequality $-3x \geq 45$. He solves the equation $-3x = 45$ to determine $x = -15$. What is the solution to the inequality?

A. $x < -15$
B. $x > -15$
C. $x \leq -15$
D. $x \geq -15$

Lesson Practice ACC6.8.07

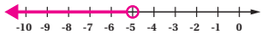
Name: _____ Date: _____ Period: _____

Problems 6–7: Complete the table to determine the solutions to each inequality. Write the solutions as an inequality and graph them on the number line.

6. $-6x > 30$

$x < -5$

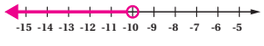
x	-7	-6	-5	-4	-3	-2	-1	0
$-6x$	42	36	30	24	18	12	6	0



7. $\frac{x}{2} < -5$

$x < -10$

x	-16	-14	-12	-10	-8	-6	-4	-2
$\frac{x}{2}$	-8	-7	-6	-5	-4	-3	-2	-1



FAST Practice

8. Solve the inequality $x + 2 < 5$.

$x < 3$

Spiral Review

9. Alma makes 5 cups of her favorite color of purple paint by mixing 3 cups of blue paint, $1\frac{1}{2}$ cups of red paint, and $\frac{1}{2}$ a cup of white paint. Alma has 2 cups of white paint. How much blue paint and red paint will Alma need to use with the 2 cups of white paint?

12 cups of blue paint and 6 cups of red paint

10. A backpack normally costs $25, but is on sale for $21. What percent is the discount? Show your thinking.

16%. *Work varies.* $25 - 21 = 4$, $\frac{4}{25} \cdot 100 = 16$

Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1	1	MA.7.AR.2.1
	2–7	2	MA.7.AR.2.1
FAST Practice	8	1	MA.7.AR.2.1
Spiral Review			
	9	2	MA.7.AR.3.2
	10	2	MA.7.AR.3.1

Math in Action

Purpose: Students think about the math learned in this unit and how it applies to the workplace and real-world situations.

In this Unit

Students summarized data using dot plots, histograms, and box plots and used measures of center and variability to describe data.

Career Connection | Meet Alexander Aitken

Invite students to read about mathematician Alexander Aitken.

Consider asking:

- "What kinds of data have we collected during our study of this unit?" Sample response: screen time data, snacks, information in the news, and transportation.
- "Is there a particular type of data plot from the ones we studied in this unit that you found to be a helpful way to summarize the data?" Sample response: Line plots let you see the data collected and you can use them to find the mean, median, mode, and range.

B.E.S.T. Benchmark Connections

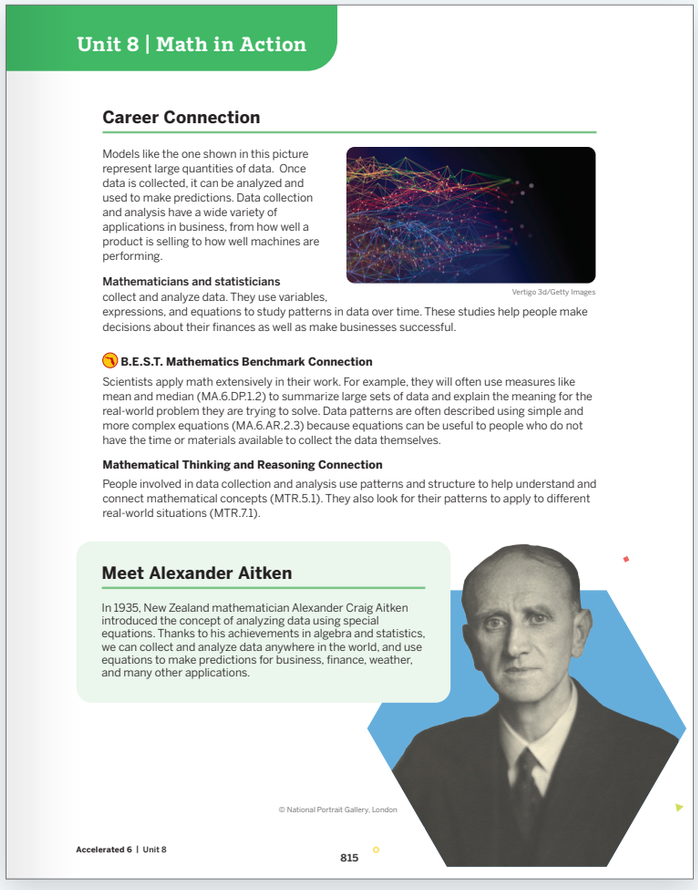
Consider asking:

- "What topic of data collection from this unit surprised you when you represented it using a plot or graph?" Answers will vary.

Mathematical Thinking and Reasoning Connections

Consider asking:

- "Why do you think that scientists and mathematicians who lived after Alexander Aitken apply Mathematical Thinking and Reasoning standard MTR.4.1 in their work?" Sample response: Mathematicians, inventors, and many others analyze the thinking of others so that they can apply their ideas in new and exciting ways.



Unit 8 | Math in Action

Career Connection

Models like the one shown in this picture represent large quantities of data. Once data is collected, it can be analyzed and used to make predictions. Data collection and analysis have a wide variety of applications in business, from how well a product is selling to how well machines are performing.

Vertigo 3d/Getty Images

Mathematicians and statisticians collect and analyze data. They use variables, expressions, and equations to study patterns in data over time. These studies help people make decisions about their finances as well as make businesses successful.

B.E.S.T. Mathematics Benchmark Connection

Scientists apply math extensively in their work. For example, they will often use measures like mean and median (MA.6.DP.1.2) to summarize large sets of data and explain the meaning for the real-world problem they are trying to solve. Data patterns are often described using simple and more complex equations (MA.6.AR.2.3) because equations can be useful to people who do not have the time or materials available to collect the data themselves.

Mathematical Thinking and Reasoning Connection

People involved in data collection and analysis use patterns and structure to help understand and connect mathematical concepts (MTR.5.1). They also look for their patterns to apply to different real-world situations (MTR.7.1).

Meet Alexander Aitken

In 1935, New Zealand mathematician Alexander Craig Aitken introduced the concept of analyzing data using special equations. Thanks to his achievements in algebra and statistics, we can collect and analyze data anywhere in the world, and use equations to make predictions for business, finance, weather, and many other applications.

© National Portrait Gallery, London

Accelerated 6 | Unit 8

815

Assess and Respond

End-of-Unit Assessment

Independent | 45 min

Facilitation: Assign the End-of-Unit Assessment to learn about your students' understanding of the concepts and skills in this unit. There are two forms of the End-of-Unit Assessment: Forms A and B. If time allows, consider inviting students to complete one or more of the tasks in the Unit Synthesis and Reflection.

Item Analysis, Forms A and B

Problem(s)	Concept or skill	Addressed in	DOK	FL B.E.S.T. Math Benchmarks
1	Identifying solutions to inequalities	Lesson 4	1	MA.6.AR.2.1
2a	Comparing rational numbers within a real-world context	Lesson 3	2	MA.6.NSO.1.2 MTR.4.1
2b	Representing the graph of a real-world situation involving an inequality on the number line	Lesson 3	1	MA.6.AR.1.2
2c	Writing an inequality to represent a real-world situation	Lesson 5	1	MA.6.AR.2.2
3	Graphing inequalities on the number line	Lesson 3	1	MA.7.AR.2.1
4	Writing an inequality to represent a real-world situation	Lesson 5	2	MA.7.AR.2.1 MTR.6.1
5a–5b	Solving two-step equations involving integers	Lesson 1	2	MA.7.AR.2.2
6a–6b	Writing an inequality to represent a real-world situation	Lesson 5	2	MA.7.AR.2.1 MTR.6.1
6c	Solving and interpreting inequalities within context	Lesson 7	2	MA.7.AR.2.1, MA.7.AR.2.2 MTR.7.1

Assessment Resources

- Student Assessments (Forms A and B)
- Assessment Rubrics with Answers
- Show What You Knows and Answer Keys

Intervention and Extension Resources

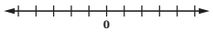
- Mini-Lessons
- Extensions

Name: Date: Period:

End-of-Unit Assessment

Unit 8

1. Select *all* of the values of x that are solutions to the inequality $x > -1.5$.



Use the number line if it helps with your thinking.

- ☐ **A.** -5
- ☐ **B.** -2
- ☑ **C.** -1
- ☑ **D.** 0
- ☑ **E.** 2

2. People use salt to melt snow and ice. Salt only melts ice when the temperature is warmer than -10°C.

a Which temperature is warmer? Circle one.

(-5°C) -10°C

Explain how you know.

Explanations vary. -5°C is higher than -10°C on a vertical number line, so it's warmer.

b Graph all the temperatures at which salt melts ice.

c Write an inequality to describe all the temperatures, t, at which salt melts ice.

$t > -10$ or $-10 < t$

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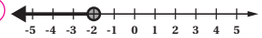
Name: Date: Period:

End-of-Unit Assessment (continued)

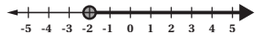
Unit 8

3. Which number line shows all of the values of x that make the inequality $3x \leq -6$ true?

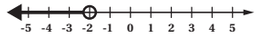
(A.)



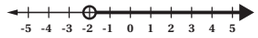
B.



C.



D.



4. Parv has a $50 gift card. He wants to buy n movies. Each movie costs $3.99.

Which inequality describes how many movies Parv can buy?

- **A.** $3.99n > 50$
- (**B.**) $3.99n \leq 50$
- **C.** $3.99n \geq 50$
- **D.** $3.99n < 50$

5. Solve each equation.

a $4(x + 2) = 40$

$x = 8$

b $-2x - 10 = -6$

$x = -2$

Name: Date: Period:

End-of-Unit Assessment (continued)

Unit 8

6. Joel's family car has a 14-gallon gas tank. The car uses about 0.5 gallons of gas each day.

a If Joel's family starts with a full tank, can they drive the car for 15 days? Circle one.

(Yes) No

Explain how you know.

Explanations vary. Joel's family would use 7.5 gallons of gas in 15 days, so there would still be $14 - 7.5 = 6.5$ gallons of gas left in the tank.

b Which expression describes the gallons of gas in the tank after d days?

(**A.**) $14 - 0.5d$ **B.** $14d - 0.5$ **C.** $0.5 - 14d$ **D.** $0.5d - 14$

Write an inequality that represents the number of days Joel's family can drive.

$0.5d < 14$ (or equivalent)

c Solve the inequality you wrote.

$d < 28$

Explain what the solutions mean in this situation.

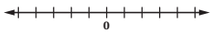
Explanations vary. The solutions to this inequality represent the number of days that Joel's family can drive.

Name: ..... Date: ..... Period: .....

End-of-Unit Assessment

Unit 8

1. Select *all* of the values of x that are solutions to the inequality $x < -2.5$.



Use the number line if it helps with your thinking.

- ☑ **A.** -5
- ☑ **B.** -4
- ☐ **C.** -2
- ☐ **D.** 0
- ☐ **E.** 1

2. Frostbite can occur quickly when skin is exposed to temperatures colder than -5°C.

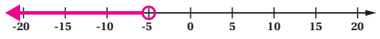
a Which temperature is colder? Circle one.

-5°C (-15°C)

Explain how you know.

Explanations vary. **-15°C is lower than -5°C on a vertical number line, so it's colder.**

b Graph all the temperatures at which frostbite can occur.



c Write an inequality to describe all the temperatures, t, at which frostbite can occur.

$t < -5$ **or** $-5 > t$

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Name: ..... Date: ..... Period: .....

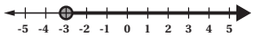
End-of-Unit Assessment (continued)

Unit 8

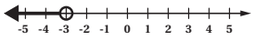
3. Which number line shows all of the values of x that make the inequality $2x < -4$ true?

(A)

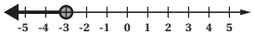
B.



C.



D.



4. Kayla has a goal to hike at least 75 miles. She plans to hike n miles each week for the next 6 weeks.

Which inequality describes the number of miles Kayla should hike each week to meet her goal?

- **A.** $6n > 75$
- **B.** $6n \leq 75$
- (**C.**) $6n \geq 75$
- **D.** $6n < 75$

5. Solve each equation.

a $30 = 2(x + 3)$

$x = 12$

b $-4x - 17 = -5$

$x = -3$

Name: ..... Date: ..... Period: .....

End-of-Unit Assessment (continued)

Unit 8

6. When fully charged, Jaylin's computer works for 20 hours. She uses her computer for about 1.5 hours each day.

a If Jaylin's computer starts with a full charge, can she use it for 13 days? Circle one.

(Yes) No

Explain how you know.

Explanations vary. **Jaylin would use the computer for 19.5 hours in 13 days, so there would only be 0.5 hours remaining in the battery.**

b Which expression describes the hours of computer charge remaining after d days?

A. $20 - 2d$ **B.** $1.5 - 20d$ (**C.**) $20 - 1.5d$ **D.** $1.5d - 20$

Write an inequality that represents the number of days Jaylin can use her computer.

$1.5d < 20$ **(or equivalent)**

c Solve the inequality you wrote.

$d < 13\frac{1}{3}$

Explain what the solutions mean in this situation.

Explanations vary. **The solutions to this inequality represent the number of days that Jaylin can use the computer.**

D Differentiation (End-of-Unit Assessment)

Note: To **strengthen** and **stretch** students' learning, refer to the differentiation resources suggested throughout this unit and in the Unit Overview.

Sub-Unit Goals	Problem(s)	Respond to Student Thinking
Sub-Unit 1: Write and solve equations of the form $p + x = q$, $x + p = q$, $px = q$, and $x/p = q$, where p and q are integers. **(Lessons 1–2)**	5a–5b	**Support** • **Mini-Lesson:** *Writing and Solving One-Step Equations* • **Teacher Move:** Consider revisiting Lesson 1 (*Plus or Minus*)
Sub-Unit 2: • Write inequalities using symbols and words. • Graph solutions to inequalities on a number line. • Solve one-step inequalities with positive and negative coefficients. **(Lessons 3–7)**	1	**Support** • **Mini-Lesson:** *Identifying Solutions to Inequalities* • **Teacher Move:** Consider revisiting Lesson 4 (*Shira's Solutions*)
		Stretch • **You're invited to explore more.** • Lesson 4 (*Shira's Solutions*)

	2a–2c	**Support**
		• **Teacher Move:** Consider revisiting:
		  • Lesson 3 (*Tunnel Travels*)
		  • Lesson 5 (*Comparing Weights*)

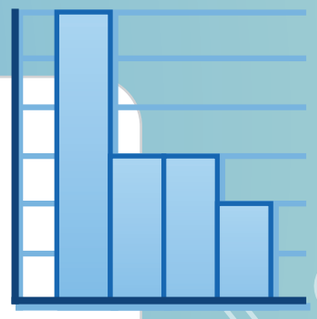
		Strengthen • **Repeated Challenges:** • Lesson 5 (*Comparing Weights*)
	3	**Support** • **Teacher Move:** Consider revisiting Lesson 3 (*Tunnel Travels*)
	4	**Support** • **Teacher Move:** Consider revisiting Lesson 5 (*Comparing Weights*)
		Strengthen • **Repeated Challenges:** • Lesson 5 (*Comparing Weights*)

	6a–6c	**Support**
		• **Mini-Lesson:** *Solving Inequalities*
		• **Teacher Move:** Consider revisiting:
		  • Lesson 5 (*Comparing Weights*)
		  • Lesson 7 (*Shira the Sheep*)

		Strengthen • **Repeated Challenges:** • Lesson 5 (*Comparing Weights*)

UNIT 9

Describing Data



Students visualize data using dot plots and histograms before calculating measures of center — mean, mode, and median — and measures of variability — range and interquartile range (IQR). They construct box plots and interpret these measures in context and use samples to draw conclusions about and compare populations.

Essential Questions

- What are statistical questions and how are they used?
- What are different ways to represent numerical data?
- How do we measure the center and the spread of a data set?
- How do samples help us make predictions about a larger population?

Sub-Unit Goals

Sub-Unit 1

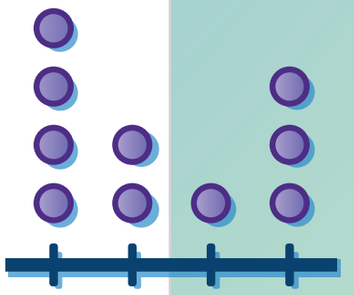
- Use data sets to make claims about data.
- Create histograms to visualize data.

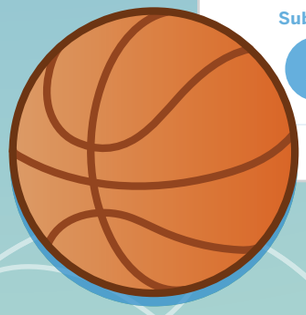
Sub-Unit 2

- Calculate the mean, median, and mode of a data set.
- Calculate the quartiles, interquartile range (IQR), and range of a data set.
- Create and use box plots to make claims about data.

Sub-Unit 3

- Use measures of center and measures of spread from random samples to draw conclusions about and compare populations.





The Math of the Unit

Unit Outcomes Telling a story that narrows and deepens with the grade

Florida B.E.S.T. Standards for Mathematics

Develop an understanding of statistics and determine measures of center and measures of variability. Summarize statistical distributions graphically and numerically. **(MA.6.DP.1)**

Add, subtract, multiply and divide rational numbers. **(MA.7.NSO.2)**

Use percentages and proportional reasoning to solve problems. **(MA.7.AR.3)**

Represent and interpret numerical and categorical data. **(MA.7.DP.1)**

Benchmarks Addressed in This Unit: MA.6.DP.1.1 MA.6.DP.1.2 MA.6.DP.1.3 MA.6.DP.1.4 MA.6.DP.1.5 MA.6.DP.1.6 MA.7.NSO.2.2 MA.7.NSO.2.3 MA.7.AR.3.2 MA.7.DP.1.1 MA.7.DP.1.2 MA.7.DP.1.3

Mathematical Thinking and Reasoning Standards

- Actively participate in effortful learning both individually and collectively. **(MTR.1.1)**
- Engage in discussions that reflect on the mathematical thinking of self and others. **(MTR.4.1)**
- Assess the reasonableness of solutions. **(MTR.6.1)**
- Apply mathematics to real-world contexts. **(MTR.7.1)**

Also Addressed: **MTR.2.1, MTR.3.1, MTR.5.1**

Coherence Telling a story within and across grades

In this Unit

Students use dot plots, histograms, and box plots to visualize data and make statements about its center and spread. They explore the mean, median, and mode as measures of center, range and interquartile range as measures of spread, and apply their understanding of these statistics to describe the effects of adding or removing a data value. Students use samples to draw conclusions about and compare populations.

Prior Learning

- Represent fractions on a number line **(MA.3.FR.2.1, MA.4.FR.1.4)**
- Calculate distances on a number line **(MA.4.M.2.1)**
- Visualize and interpret data using line plots **(MA.4.DP.1.1, MA.4.DP.1.2, MA.5.DP.1)**
- **Unit 3:** Calculate percentages **(MA.6.AR.3.4)**

Future Learning

- Associations in bivariate data **(MA.8.DP.1.2)**
- Standard deviation and outliers **(MA.912.DP.2.1)**

Rigor and Balance Integrating the aspects of rigor in ways that support one another

Here are examples of the aspects of rigor and balance in this unit. See the first page of each lesson's Teacher Edition for more information.

Conceptual Understanding

In **Lessons 1–4**, students develop conceptual understanding of representing data in different ways and making sense of how to summarize a set of numerical data with a single number.

Strategic Competence

In **Lesson 9**, students develop strategic competence as they justify their reasoning with statistics they calculated.

Procedural Fluency

In **Lessons 8–11**, students develop procedural fluency with calculating the median and IQR of a data set.

Adaptive Reasoning

In **Lesson 1**, students develop adaptive reasoning as they determine which type of data a question is collecting, categorical or numerical.

Application

In **Lesson 12**, students apply their understanding of measures of center and spread to make claims about large data sets.

Productive Disposition

In **Lesson 5**, students have opportunities to develop a productive disposition as they make sense of histograms.

Unit at a Glance

Assess and Respond

A Pre-Unit Check

Learn more about your students' understanding of foundational concepts and skills that will support them in the upcoming unit. This can be given in its entirety before the unit or spread throughout the unit.

MA.3.FR.3.1 MA.4.NSO.1.5 MA.5.DP.1.1 MA.5.AR.2.2 MA.6.AR.3.4

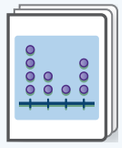
Sub-Unit 1

1 Screen Time

Asking Questions, Collecting Data, and Making Claims

Begin the unit by making connections between questions, claims, and two types of data.

MA.6.DP.1.1
MTR.1.1 MTR.4.1



2 Survey Says . . .

Using Data to Generate Statistical Questions

Interpret data and understand what is meant by the term **statistical question**.

MA.6.DP.1.1
MTR.1.1 MTR.4.1

Sub-Unit 2

6 Snack Time

Introduction to the Mean

Explore **mean** as an equal share and make sense of mean as a measure of center.

MA.6.DP.1.2
MTR.2.1 MTR.3.1 MTR.6.1

7 Modes of Weather

Introducing the Mode

Determine the **mode** of a data set and explore when a data display may not be helpful in identifying the mode.

MA.6.DP.1.2
MTR.4.1 MTR.7.1

8 Toy Cars

Introducing the Median

Explore a new measure of center: the **median**.

MA.6.DP.1.2
MTR.2.1 MTR.3.1 MTR.6.1

Pacing: 20 days | **Short on time?** See pacing considerations below.

All instructional and assessment days are ~45 minutes each.

- 17 Lessons
- Pre-Unit Check
- Sub-Unit Quizzes
- End-of-Unit Assessment

3 The Plot Thickens

Introduction to Histograms

Explore a new way to visualize data: in a **histogram**.

MA.6.DP.1.5
MTR.4.1 MTR.6.1

4 What Does It All Mean?

Describing and Interpreting Histograms and Dot Plots

Describe and interpret real-world data by its **spread** and **shape**.

MA.6.DP.1.4
MTR.4.1 MTR.7.1

5 DIY Histograms

Creating Histograms

Create histograms by hand and use histograms to compare data sets.

MA.6.DP.1.4 MA.6.DP.1.5
MTR.4.1 MTR.5.1 MTR.7.1

9 In the News

Comparing Measures of Center

Investigate how the shape of a data set affects the mean and median, and decide which measure of center best describes the data.

MA.6.DP.1.6
MTR.1.1 MTR.4.1

10 Hoops

Calculating and Interpreting the Range

Calculate the range of a data set and analyze how adding or removing a data value affects the range.

MA.6.DP.1.2 MA.6.DP.1.6
MTR.4.1 MTR.5.1 MTR.7.1

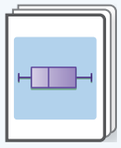
11 Pumpkin Patch

Introduction to Quartiles

Understand what is meant by **quartiles** and why they are useful when describing data.

MA.6.DP.1.3
MTR.1.1 MTR.4.1

Unit at a Glance



12 **Car, Plane, Bus, or Train?**

Box Plots, IQR, and Range

Explore a new way to visualize data: a **box plot**, and a new measure of spread: **interquartile range (IQR)**.

MA.6.DP.1.3 MA.6.DP.1.5 MA.6.DP.1.6
MTR.1.1 MTR.4.1

Assess and Respond

A **Sub-Unit Quiz**

Learn about your students' understanding of the concepts and skills so far in this unit.

MA.6.DP.1.1 MA.6.DP.1.2 MA.6.DP.1.4
MA.6.DP.1.5

Sub-Unit 3

13 **Car, Bike, or Train?**

Using Mean, Median, and Range to Compare Groups

Analyze and compare sets of data by calculating the mean, median, and range to make a recommendation for a real-world situation.

MA.7.NSO.2.2 MA.7.NSO.2.3 MA.7.DP.1.1
MA.7.DP.1.2
MTR.4.1 MTR.7.1

17 **School Newspaper**

Comparing Populations

Students compare two populations using the difference between the centers of the samples, then compare the difference to the measures of spread, or variability.

MA.7.DP.1.2
MTR.4.1 MTR.7.1

Assess and Respond

A **End-of-Unit Assessment**

Learn about your students' understanding of the concepts and skills in this unit. There are two forms of the End-of-Unit Assessment: Forms A and B.

MA.6.DP.1.2 MA.6.DP.1.3 MA.6.DP.1.4
MA.7.DP.1.1 MA.7.DP.1.2
MTR.4.1 MTR.6.1

14 Crab Island

Sampling From Large Populations

Explore the advantages and disadvantages of using samples to answer a question about a population.

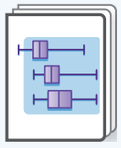
MA.7.DP.1.3
MTR.1.1 MTR.4.1 MTR.7.1

15 Flower Power

Using Percentages to Predict Populations

Use data from a sample to estimate the number of marigolds in a large flower garden using ratios and percentages.

MA.7.NSO.2.2 MA.7.NSO.2.3 MA.7.AR.3.2
MA.7.AR.4.5 MA.7.DP.1.3
MTR.41 MTR.5.1 MTR.6.1



16 Plots and Samples

Using Statistical Measures to Make Predictions

Draw conclusions about a population using quantitative data and estimate a measure of center of a population based on one or more samples of the population.

MA.7.DP.1.1 MA.7.DP.1.2
MTR.5.1 MTR.6.1

Pacing Considerations

Lessons 1–2: The first lesson shows students how to differentiate between categorical and numerical data, and the second lesson introduces statistical questions. If time runs short, consider combining these lessons into one by having students complete the Warm-Up and Activity 2 from Lesson 1, then Activity 2 from Lesson 2.

Lesson Materials and Prep

Here are the materials and resources needed within each lesson, how to prepare them, and where to find them.

Lesson 1
- **Copy:** Show What You Know

Lesson 2
- **Copy:** Show What You Know

Lesson 3
- **Copy:** Show What You Know
- **Materials:** highlighters *(optional)*, straightedges

Lesson 4
- **Copy:** Show What You Know, Warm-Up Sheet *(one per student)*

Lesson 5
- **Copy:** Show What You Know, Activity 2 Sheets *(one per student)*, Activity 2 Sheets *(answers)*
- **Materials:** straightedges

Lesson 6
- **Copy:** Show What You Know
- **Materials:** calculators, chips/counters/paper clips *(as needed)*

Lesson 7
- **Copy:** Show What You Know

Lesson 8
- **Copy:** Show What You Know

Lesson 9
- **Copy:** Show What You Know
- **Materials:** calculators

Lesson 10
- **Copy:** Show What You Know, Warm-Up Sheet *(one per pair)*
- **Materials:** counters

Lesson 11
- **Copy:** Show What You Know
- **Materials:** chips or blocks *(as needed)*

Lesson 12
- **Copy:** Show What You Know
- **Cut:** Activity 3 Cards *(one set per pair)*
- **Materials:** coloring tools *(as needed)*, straightedges

Lesson 13
- **Copy:** Show What You Know

Lesson 14
- **Copy:** Show What You Know

Lesson 15
- **Copy:** Show What You Know

Lesson 16
- **Copy:** Show What You Know

Lesson 17
- **Copy:** Show What You Know

Resource Books

Assessment and Lesson Resources

- Pre-Unit Check
- Sub-Unit Quizzes
- End-of-Unit Assessment, Forms A & B
- Assessment Rubrics
- Show What You Knows
- Activity Sheets and Cards

Intervention and Extension Resources

- Mini-Lessons
- Prior Grade Mini-Lessons
- Extensions

Caregiver Support

These resources can be shared with students' caregivers. They provide background on the mathematics in this unit, as well as suggestions for supporting students at home.

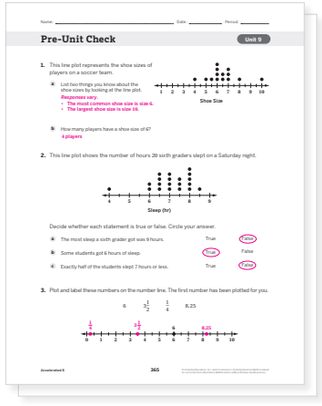
Assessments

Every unit includes a variety of formative and summative assessments, designed to highlight students' understanding of a variety of concepts and skills.

Pre-Unit Check

Assign the **Pre-Unit Check** to learn more about your students' understanding of foundational concepts and skills that will support them in the upcoming unit. This can be given in its entirety before the unit or spread throughout the unit.

Standards addressed: MA.3.FR.3.1, MA.4.NSO.1.5, MA.5.DP.1.1, MA.5.AR.2.2, MA.6.AR.3.4


Pre-Unit Check

Unit 9

1. This line plot represents the shoe sizes of players on a soccer team.

Shoe Size

Responses vary.
- The most common shoe size is size 6.
- The largest shoe size is size 10.

How many players have a shoe size of 6?
4 players

2. This line plot shows the number of hours 28 sixth graders slept on a Saturday night.

Sleep (hr)

Decide whether each statement is true or false. Circle your answer.

- The most sleep a sixth grader got was 9 hours. True False
- Some students got 6 hours of sleep. True False
- Exactly half of the students slept 7 hours or less. True False

3. Plot and label these numbers on the number line. The first number has been plotted for you.

6 $\frac{1}{2}$ $\frac{1}{4}$ 8.25

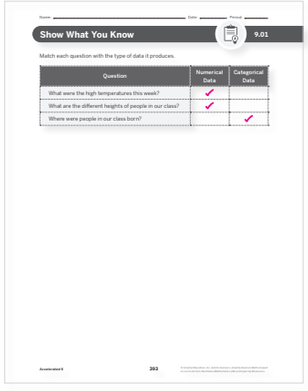
$\frac{3}{4}$ $\frac{1}{2}$ 8.25

365

Lesson | Show What You Know

Use the **Show What You Know** for each lesson to learn more about your students' progress toward the lesson goal(s). Here is one example from this unit.

Standard addressed on Lesson 1 Show What You Know: MA.6.DP.1.1


Show What You Know

9.01

Match each question with the type of data it produces.

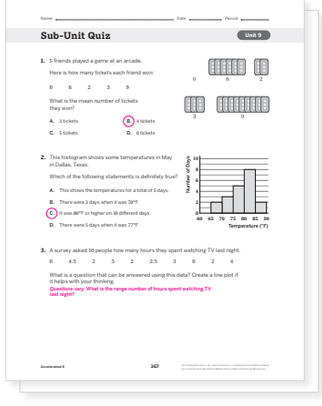
Question	Numerical Data	Categorical Data
What were the high temperatures this week?	✓	
What are the different heights of people in our class?	✓	
Where were people in our class born?		✓

393

Sub-Unit Quiz

Assign the **Sub-Unit Quiz** to learn about your students' understanding of the concepts and skills so far in this unit.

Standards addressed on the Sub-Unit Quiz: MA.6.DP.1.1, MA.6.DP.1.2, MA.6.DP.1.4, MA.6.DP.1.5


Sub-Unit Quiz

Unit 9

1. 5 friends played a game at an arcade.

Here is how many tickets each friend won:

0 6 2 3 9

What is the mean number of tickets they won?

A. 3 tickets B. 4 tickets

C. 5 tickets D. 6 tickets

2. This histogram shows some temperatures in May in Dallas, Texas.

Which of the following statements is definitely true?

A. This shows the temperatures for a total of 5 days.

B. There were 3 days when it was 78°F.

C. It was 86°F or higher on 18 different days.

D. There were 5 days when it was 77°F.

Number of Days

Temperature (°F)

3. A survey asked 18 people how many hours they spent watching TV last night.

0 4.5 2 3 2 2.5 3 0 2 4

What is a question that can be answered using this data? Create a line plot if it helps with your thinking.

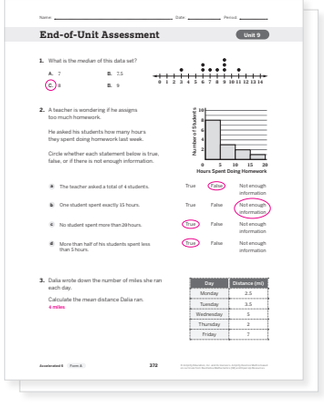
Questions vary. What is the range number of hours spent watching TV last night?

347

End-of-Unit Assessment

Assign the **End-of-Unit Assessment** to learn about your students' understanding of the concepts and skills in this unit. There are two forms of the End-of-Unit Assessment: Forms A and B.

Standards addressed: MA.6.DP.1.2, MA.6.DP.1.3, MA.6.DP.1.4, MA.7.DP.1.1, MA.7.DP.1.2


End-of-Unit Assessment

Unit 9

1. What is the median of this data set?

A. 7 B. 7.5

C. 8 D. 9

2. A teacher is wondering if he assigns too much homework.

He asked his students how many hours they spent doing homework last week.

Circle whether each statement below is true, false, or if there is not enough information.

Number of Students

Hours Spent Doing Homework

- The teacher asked a total of 4 students. True False Not enough information
- One student spent exactly 15 hours. True False Not enough information
- No student spent more than 28 hours. True False Not enough information
- More than half of his students spent less than 5 hours. True False Not enough information

3. Dalia wrote down the number of miles she ran each day.

Calculate the mean distance Dalia ran.

4 miles

Day	Distance (mi)
Monday	2.5
Tuesday	3.5
Wednesday	5
Thursday	2
Friday	7

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Differentiation

Every unit includes additional resources to support, strengthen, and stretch student learning.

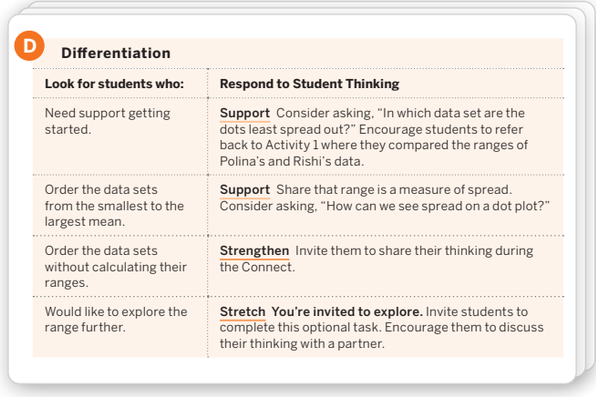
Differentiation Across the Unit

	Support	Strengthen	Stretch
	Provide targeted intervention for students by using these resources.	Reinforce students' understanding of the concepts assessed by using these resources.	Challenge students and extend their learning with these resources.
Sub-Unit 1 (Lessons 1–5) • Use data sets to make claims about data. • Create histograms to visualize data.	**Mini-Lesson** • *Creating Dot Plots*	**Challenge Creator** • Lesson 1 (*Screen Time*) **Practice** • Lessons 1–5	**DOK3 Practice** • Lessons 1–2, 4–5
Sub-Unit 2 (Lessons 6–12) • Calculate the mean, median, and mode of a data set. • Calculate the quartiles, interquartile range (IQR), and range of a data set. • Create and use box plots to make claims about data.	**Mini-Lesson** • *Calculating Measures of Center and Spread* • *Interpreting Box Plots*	**Repeated Challenges** • Lesson 8 (*Toy Cars*) • Lesson 11 (*Pumpkin Patch*) **Practice** • Lessons 6–12	**You're invited to explore more.** • Lesson 6 (*Snack Time*) • Lesson 9 (*In The News*) • Lesson 10 (*Hoops*) **DOK3 Practice** • Lessons 6–8, 11–12
Sub-Unit 3 (Lessons 13–17) • Use measures of center and measures of spread from random samples to draw conclusions about and compare populations.	**Mini-Lesson** • *Predicting Population Using Sample Data*	**Practice** • Lessons 13–17	**You're invited to explore more.** • Lesson 14 (*Crab Island*) **DOK3 Practice** • Lessons 14–16

Differentiation Within the Lesson

Each Teacher Edition includes point-of-use differentiation suggestions to modify or adjust instruction during the lesson to accommodate the needs of all learners.

Here is one example from **Lesson 10**, where students make sense of range as a measure of spread by examining three dot plots and ordering them according to their ranges.



Differentiation

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Consider asking, "In which data set are the dots least spread out?" Encourage students to refer back to Activity 1 where they compared the ranges of Polina's and Rishi's data.
Order the data sets from the smallest to the largest mean.	**Support** Share that range is a measure of spread. Consider asking, "How can we see spread on a dot plot?"
Order the data sets without calculating their ranges.	**Strengthen** Invite them to share their thinking during the Connect.
Would like to explore the range further.	**Stretch** **You're invited to explore.** Invite students to complete this optional task. Encourage them to discuss their thinking with a partner.

Accessibility

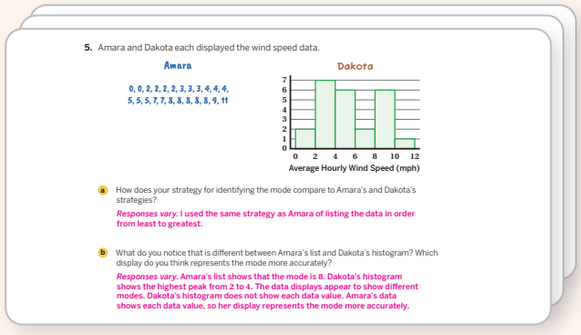
Every lesson is designed using the principles of Universal Design for Learning. Every lesson also includes support suggestions that build on students' strengths.

Universal Design for Learning

Here is one example from a lesson designed to support Engagement.

Spotlight on Collaborative Thinking . . .

Lesson 7: In this lesson, students discuss the ways in which representations of data are different and alike and reflect on their thinking and the thinking of their classmates.



5. Amara and Dakota each displayed the wind speed data.

Amara

0, 0, 2, 2, 2, 2, 3, 3, 3, 4, 4, 4, 5, 5, 5, 7, 7, 8, 8, 8, 8, 8, 9, 11

Dakota

a. How does your strategy for identifying the mode compare to Amara's and Dakota's strategies?

Responses vary. I used the same strategy as Amara of listing the data in order from least to greatest.

b. What do you notice that is different between Amara's list and Dakota's histogram? Which display do you think represents the mode more accurately?

Responses vary. Amara's list shows that the mode is 8. Dakota's histogram shows the highest peak from 2 to 4. The data displays appear to show different modes. Dakota's histogram does not show each data value. Amara's data shows each data value, so her display represents the mode more accurately.

Just-In-Time Accommodations and Supports

Each Teacher Edition includes suggested supports based on the areas of cognitive function from the Education Development Center (EDC).

Spotlight on Conceptual Processing . . .

Here are two suggestions in this unit for supporting students with interpreting histograms and visualizing the mean of a data set as an equal share.

Lesson 3, Activity 3

A **Accessibility: Conceptual Processing** Invite students to talk aloud about their thinking to support self-monitoring of understanding.

Lesson 6, Activity 1

A **Accessibility: Conceptual Processing** Use objects like chips, paper clips, or counters to support students' understanding of the mean as an equal share. Encourage them to use these objects to represent the number of tickets each friend has and to physically redistribute them to help them visualize the process.

Language Development

Every lesson includes opportunities for all students to develop fluency with mathematical language.

Vocabulary

New Vocabulary	Lesson
categorical data **numerical data**	1
statistical question	2
bin (of a histogram) **histogram**	3
cluster **outlier** **range** **shape** **skewed** **spread** **symmetry (of a data distribution)**	4
center	5
mean (average) statistic	6
mode	7
median	8
quartile	11
box plot **interquartile range (IQR)**	12
population **sample**	14

Review Vocabulary
dot plot
frequency table
maximum
minimum
proportional

Additional Vocabulary
measure of center
measure of spread

Math Language Routines and Language Goals

Math Language Routine	Lesson(s)
MLR1: Stronger and Clearer Each Time	17
MLR2: Collect and Display	1–4, 6–8, 11–14, 16
MLR3: Critique, Correct, Clarify	6, 8
MLR7: Compare and Connect	7, 15

Sub-Unit Language Goals

Students engage in the following language functions:

- Describe data sets using terms such as *shape*, *center*, and *spread*.
- Compare and contrast the mean and median as measures of center.
- Interpret the median and IQR in the context of a data set.
- Explain the purpose of sampling and which methods of obtaining a sample tend to produce representative samples.

English Language Learners Support

Every Teacher Edition includes suggestions for supporting the needs of students learning English.

Here is one example from **Lesson 11**, where students are reminded of the definition and examples of *median* to support them in their explanations and reasoning of a new concept: quartiles.

ELL **English Language Learners** If possible, review an anchor chart that publicly displays the definition of *median* along with examples in order to aid students in explanations and reasoning. It may be helpful to add Quartile 1 and Quartile 3 definitions to this chart throughout the lesson.

Connections to Future Learning

Here is how the content in this unit connects to where your students are headed in their math journeys.

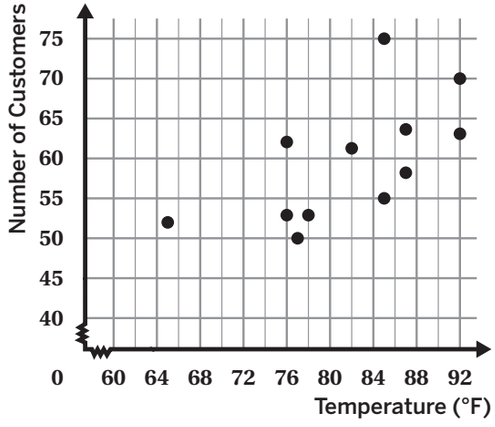
Associations in Bivariate Data

In this unit, students create and analyze histograms, and are introduced to shape, center, spread, and outliers of data in one variable. **In a future course,** students will analyze numerical data in two variables using scatter plots and fitted lines.

FL B.E.S.T. Math Benchmark:
MA.8.DP.1.2

Example:

An ice cream store collected data on the outside temperature and number of customers over time. The scatter plot shows how these variables are related and can be used to answer several questions, including: *Is there an association between the outside temperature and the number of customers at the ice cream store? What is the predicted number of customers if the temperature is 78°F?*



Standard Deviation and Outliers

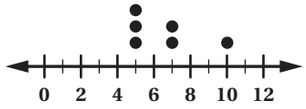
In this unit, students calculate summary statistics. **In a future course,** they will summarize one-variable data with mean, median, and standard deviation. Students also more formally identify outliers in data sets.

FL B.E.S.T. Math Benchmark:
MA.912.DP.2.1

Example:

Here are the masses (in grams) of fish in two tanks. Tank A's fish have a more consistent mass than Tank's B fish because Tank A has a smaller standard deviation.

Tank A: 5, 5, 5, 7, 7, 10

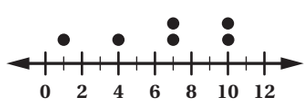


Standard Deviation

$A = [5, 5, 5, 7, 7, 10]$

$\text{stdevp}(A)$ ≈ 1.8

Tank B: 1, 4, 7, 7, 10, 10



Standard Deviation

$B = [1, 4, 7, 7, 10, 10]$

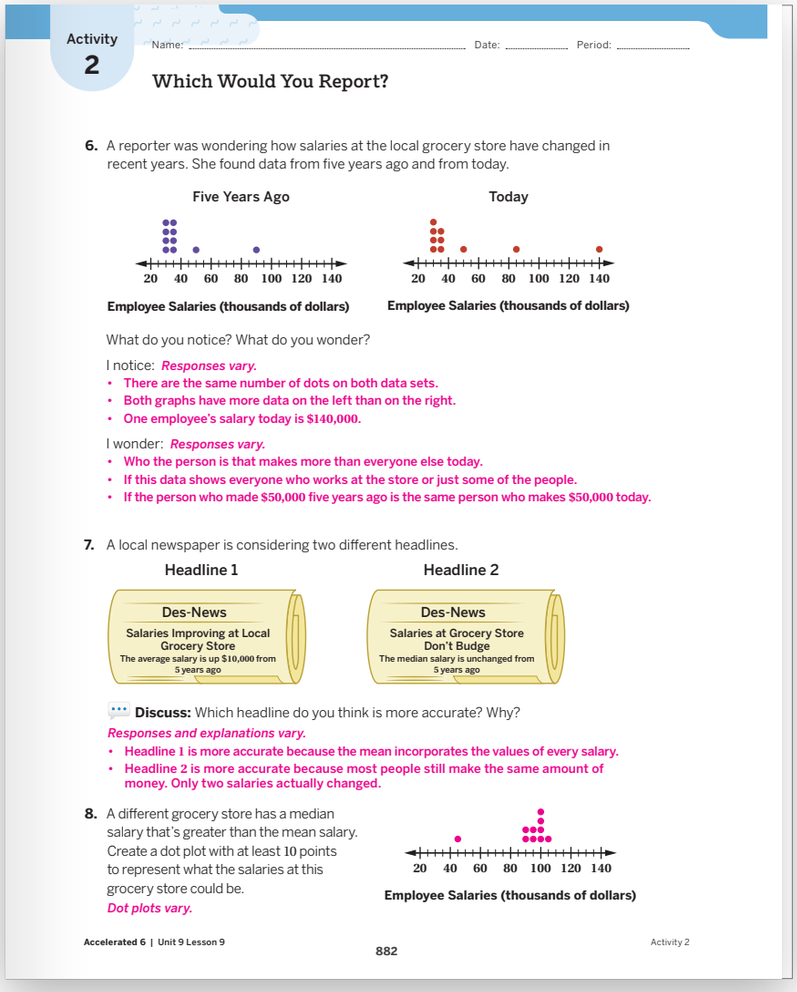
$\text{stdevp}(B)$ ≈ 3.2

Professional Learning

Here is an opportunity to deepen your understanding of the math in this unit and how to teach it.

Spotlight on Strategic Competence

Lesson 9, Activity 2



Activity 2

Name: _____ Date: _____ Period: _____

Which Would You Report?

6. A reporter was wondering how salaries at the local grocery store have changed in recent years. She found data from five years ago and from today.

Five Years Ago

Employee Salaries (thousands of dollars)

Today

Employee Salaries (thousands of dollars)

What do you notice? What do you wonder?

I notice: *Responses vary.*
- There are the same number of dots on both data sets.
- Both graphs have more data on the left than on the right.
- One employee's salary today is $140,000.

I wonder: *Responses vary.*
- Who the person is that makes more than everyone else today.
- If this data shows everyone who works at the store or just some of the people.
- If the person who made $50,000 five years ago is the same person who makes $50,000 today.

7. A local newspaper is considering two different headlines.

Headline 1

Headline 2

Discuss: Which headline do you think is more accurate? Why?

Responses and explanations vary.
- Headline 1 is more accurate because the mean incorporates the values of every salary.
- Headline 2 is more accurate because most people still make the same amount of money. Only two salaries actually changed.

8. A different grocery store has a median salary that's greater than the mean salary. Create a dot plot with at least 10 points to represent what the salaries at this grocery store could be.

Dot plots vary.

Employee Salaries (thousands of dollars)

Accelerated 6 | Unit 9 Lesson 9 882 Activity 2

Context

In this activity, students develop strategic competence as they consider the shape of a data set, any extreme data values, and the context, then decide which measure of center more accurately describes it.

Try This

Put on your student hat and complete Problems 6–8.

Points to Ponder

- What do students need to understand about how the shape of a data set affects the mean and median to support their decision about which measure of center best describes the data?
- Some students may choose a measure of center based solely on the shape of the dot plot. How might you support students in also using the context of the data to make a decision?
- How will students' understanding of the median as a measure of center support their understanding of box plots in the future?

Notes:

Assess and Respond

Pre-Unit Check

Independent | 45 min

Facilitation: Assign the Pre-Unit Check to learn more about your students' understanding of foundational concepts and skills that will support them in the upcoming unit. This can be given in its entirety before the unit or spread throughout the unit. If time allows, consider inviting students to complete one or more of the tasks in the Unit Synthesis and Reflection.

Item Analysis

Problem	Concept or skill	Preparation for	DOK	FL B.E.S.T. Math Benchmarks
1, 2	Interpreting line plots.	Lesson 2	1	MA.5.DP.1.1, MTR.7.1
3	Plotting fractions and decimals on a number line	Lesson 3	1	MA.5.DP.1.1
4	Evaluating expressions with different operations and grouping symbols	Lesson 6	1	MA.5.AR.2.2
5	Calculating percentages that are multiples of 25%	Lesson 11	1	MA.6.DP.1.3, MTR.2.1

Assessment and Lesson Resources

- Student Assessments
- Show What You Knows and Answer Keys

Intervention and Extension Resources

- Mini-Lessons
- Extensions

Name: ..... Date: ..... Period: .....

Pre-Unit Check

Unit 9

1. This line plot represents the shoe sizes of players on a soccer team.

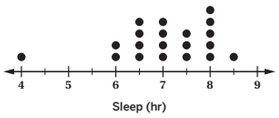
a. List *two* things you know about the shoe sizes by looking at the line plot.

Responses vary.
- The most common shoe size is size 6.
- The largest shoe size is size 10.

b. How many players have a shoe size of 6?

4 players

2. This line plot shows the number of hours 20 sixth graders slept on a Saturday night.

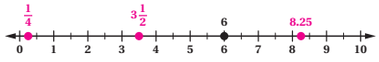


Decide whether each statement is true or false. Circle your answer.

a.	The most sleep a sixth grader got was 9 hours.	True	(False)
b.	Some students got 6 hours of sleep.	(True)	False
c.	Exactly half of the students slept 7 hours or less.	True	(False)

3. Plot and label these numbers on the number line. The first number has been plotted for you.

6 $3\frac{1}{2}$ $\frac{1}{4}$ 8.25



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Name: ..... Date: ..... Period: .....

Pre-Unit Check (continued)

Unit 9

4. Evaluate each expression.

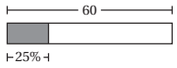
a. $\frac{5+11+9+15}{4} = 10$

b. $(4+2+9) \div 3 = 5$

5. Calculate each percent.

a. 25% of 60

Use the tape diagram if it helps you with your thinking.



15

b. 75% of 40

30

c. 25% of 30

7.5

D Differentiation (Pre-Unit Check)

Note: To **strengthen** and **stretch** students' learning, refer to the differentiation resources suggested throughout this unit and in the Unit Overview.

Sub-Unit Goals	Problem(s)	Respond to Student Thinking
Preparation for Sub-Unit 1: • Use data sets to make claims about data. • Create histograms to visualize data. **(Lessons 1–5)**	1, 2	**Support** • **Teacher Move:** During Lesson 2, consider inviting students to share what they know about each dot plot.
	3	**Support** • **Mini-Lesson:** Creating Dot Plots • **Teacher Move:** Invite students to place each value in the appropriate place on the number line and justify their placement with a partner.
	4	**Support** • **Teacher Move:** Before Lesson 6, consider sharing incorrect work for this problem and inviting students in groups to explain what the student did well and what error(s) they made.
	5	**Support** • **Teacher Move:** After Lesson 11, consider revisiting this problem and inviting students to make connections between quartiles and percentages.

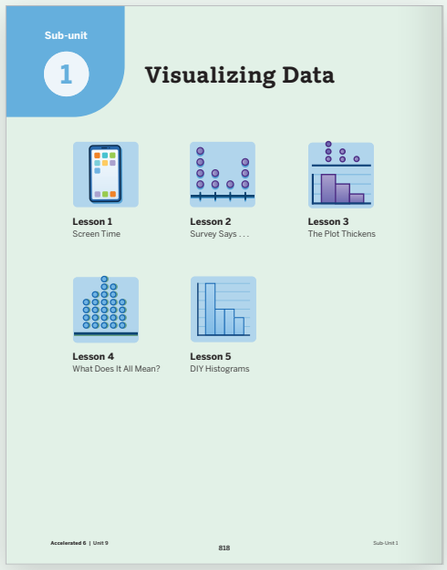
Sub-Unit 1

Visualizing Data

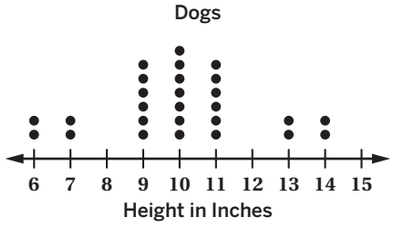
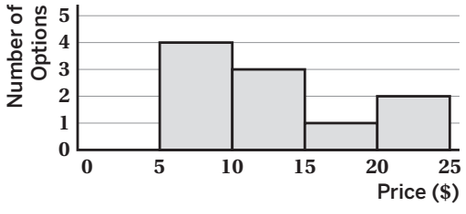
Sub-Unit 1 Goals

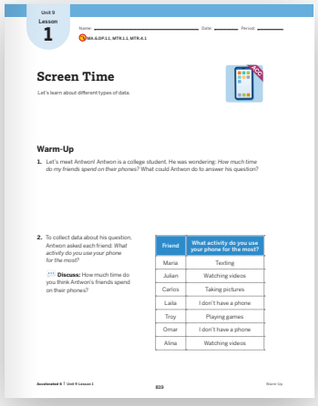
- **Goal:** Use data sets to make claims about data.
- **Goal:** Create histograms to visualize data.
- **Language Goal:** Describe data sets using terms such as *shape*, *center*, and *spread*.

In this sub-unit, students answer **statistical questions** and analyze data sets, dot plots, and **histograms** by describing their **shape**, **center**, and **spread**.



Math That Matters Most

Concept or Skill	For example . . .
Explain the difference between categorical and numerical data.	Describe the difference between numerical and categorical data and give an example of a question that would produce each type. *Numerical data is data that are numbers, resulting from questions like, "How many pets do you own?"* *Categorical data is data that are words, resulting from questions like, "What's your favorite type of music?"*
Describe and interpret the center, spread, and shape of a data set.	Describe the spread and center of this dog height data. *There is a wider range of heights in the data set, but the data is symmetrical. This means more dogs have a medium height and fewer dogs are shorter or taller.* 
Create a histogram to visualize data.	The prices of 10 options at a restaurant are \$8.00, \$7.25, \$15.50, \$13.50, \$14.00, \$9.75, \$24.60, \$7.25, \$13.00, and \$20.00. Create a histogram of the data. 


Unit 9
Lesson 1
Name: _____ Date: _____ Period: _____
MA.6.DP.1.1, MTR.1.1, MTR.4.1
Screen Time
Let's learn about different types of data.
Warm-Up
1. Let's meet Antwoni Antwon is a college student. He was wondering: How much time do my friends spend on their phones? What could Antwon do to answer his question?
2. To collect data about his question, Antwon asked each friend: What activity do you use your phone for the most?
??? Discuss: How much time do you think Antwoni's friends spend on their phones?

Friend	What activity do you use your phone for the most?
Maria	Texting
Julian	Watching videos
Carlos	Taking pictures
Laila	I don't have a phone
Troy	Playing games
Omar	I don't have a phone
Alina	Watching videos

Accelerated 6 | Unit 9 Lesson 1
819
Warm-Up

Screen Time

Asking Questions, Collecting Data, and Making Claims

Let's learn about different types of data.

Learning Outcomes

Today's Goals

1. **Goal:** Make connections between questions, claims, and data.
2. **Language Goal:** Explain the difference between *categorical* and *numerical data*. **(Writing and Speaking)**

Students make connections between questions, claims, and data. They consider how the type of question you ask can be more or less helpful for gathering data needed to make a claim. Students define the terms and learn how to distinguish between two types of data: **categorical data** and **numerical data**. **(MTR.1.1)**

Coherence

Prior Learning

In a previous course, students represented and interpreted data on line plots.

Future Learning

In Lesson 2, students will learn about statistical questions as they explore data represented in tables and lists.

Connections

- Students are introduced to new vocabulary to categorize data by analyzing and describing context situations. (mathematical, cross-curricular, real world)

FL B.E.S.T. Math Benchmarks

Addressing

MA.6.DP.1.1
Recognize and formulate a statistical question that would generate numerical data.

Mathematical Thinking and Reasoning Standards: MTR.1.1, MTR.4.1

ELA Expectations: ELA.K12.EE.3.1, ELA.K12.EE.6.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.5.DP.1.1	MA.7.DP.1.5

Vocabulary

New Vocabulary

categorical data
numerical data

Lesson at a Glance

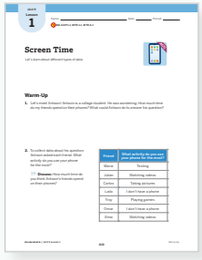
~ 45 min

MA.6.DP.1.1

Warm-Up

Whole Class | 5 min

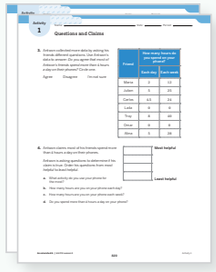
Students explore collecting data to investigate a question. **Routine:** Think-Pair-Share



Activity 1

Pairs | 12 min

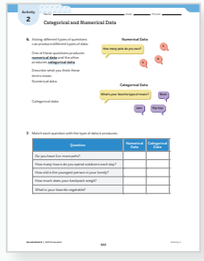
Students consider how questions can be helpful in gathering data needed to make a claim about a group of people.



Activity 2

Pairs | 8 min

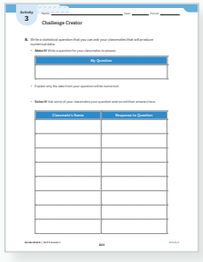
Students organize the types of questions and the data produced from them into two categories: **numerical data** and **categorical data**.



Activity 3

Pairs | 10 min

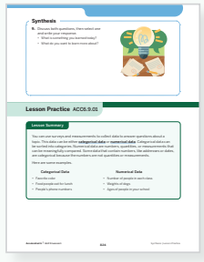
Students use their creativity to write their own question and gather numerical data. **(MTR.1.1)**



Synthesis

Whole Class | 5 min

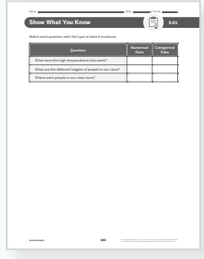
Students synthesize their understanding of categorical and numerical data and the types of questions that can be used to gather data. MLR2: Collect and Display



Show What You Know

Independent | 5 min

Students demonstrate their understanding by categorizing the data a question produces as either categorical or numerical.



Prep Checklist

This lesson includes:

Student Edition
Show What You Know

Whole Class | 5 min

Warm-Up

Purpose: Students explore collecting data to investigate a question.

1 Launch

Direct students' attention to the information about Antwon.

Use the Think-Pair-Share routine to support student-to-student discourse about what Antwon could do to answer his question. See the Routine Facilitation Guide for more information. **(MTR.4.1)**

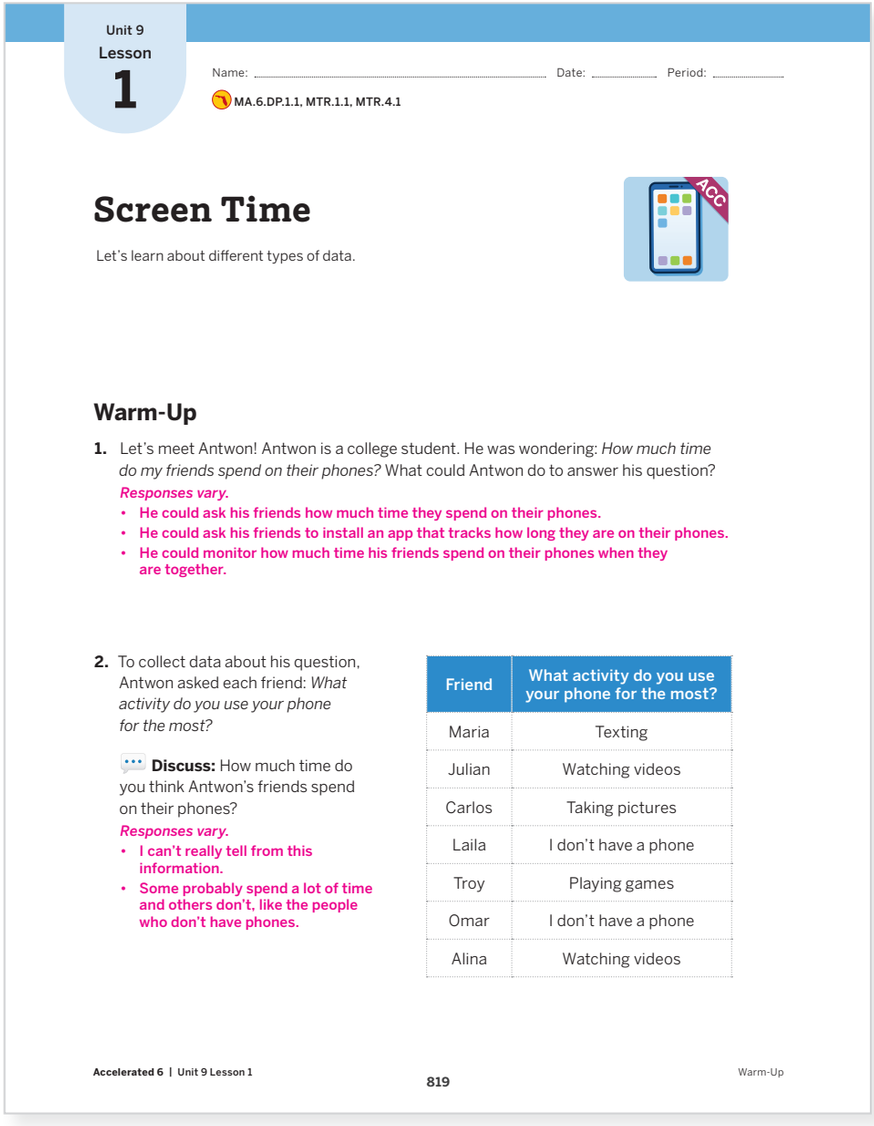
Invite students to share their advice for Antwon. Discuss each possible response.

Invite students to think independently before discussing their ideas with a partner. **(MTR.4.1)**

2 Connect

To surface connections, consider asking:

- "Do you think the activity each student uses the phone for plays a role in how much time they spend on it?"
- "Who do you think spends the most time on their phone? The least?"



Unit 9
Lesson
1

Name: _____ Date: _____ Period: _____

MA.6.DP.1.1, MTR.1.1, MTR.4.1

Screen Time

Let's learn about different types of data.

ACC

Warm-Up

1. Let's meet Antwon! Antwon is a college student. He was wondering: *How much time do my friends spend on their phones?* What could Antwon do to answer his question?

Responses vary.

- He could ask his friends how much time they spend on their phones.
- He could ask his friends to install an app that tracks how long they are on their phones.
- He could monitor how much time his friends spend on their phones when they are together.

2. To collect data about his question, Antwon asked each friend: *What activity do you use your phone for the most?*

Discuss: How much time do you think Antwon's friends spend on their phones?

Responses vary.

- I can't really tell from this information.
- Some probably spend a lot of time and others don't, like the people who don't have phones.

Friend	What activity do you use your phone for the most?
Maria	Texting
Julian	Watching videos
Carlos	Taking pictures
Laila	I don't have a phone
Troy	Playing games
Omar	I don't have a phone
Alina	Watching videos

Accelerated 6 | Unit 9 Lesson 1 819 Warm-Up

Pairs | 12 min

Activity 1 Questions and Claims

Purpose: Students consider how questions can be helpful in gathering data needed to make a claim about a group of people.

1 Launch

Consider sharing that in this activity students will think about what claims they can make about Antwon's friends based on the types of questions asked and the data gathered.

Invite students to determine whether they agree or disagree with Antwon and justify their reasoning.

2 Monitor

Encourage students to explore how the data changes depending on which question was asked.

Look for students who change their mind about whether they agree or disagree with Antwon's claim. Invite them to share their thinking with a partner and compare it with their original answer. **(MTR.4.1)**

Pause briefly and invite students to share their results and then compare them.

Invite students to justify their reasoning for their responses and respond to others' reasoning. **(MTR.1.1)**

D Differentiation

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Invite students to determine which questions were most and least helpful then discuss the remaining questions with their partner.
Think that Question b is more helpful than Question d because it has exact values.	**Support** Consider asking, "Do we need to know exactly how long each student spent on the phone to decide if Antwon's claim is true?"
Order all the questions correctly.	**Strengthen** Consider asking, "How did you determine how to rank each question?" Invite students to share their thinking during the Connect.

Listen for the language students use to justify their ordering and invite them to share that reasoning with one another, reminding them that a variety of responses might be correct for this question. **(MTR.1.1)**

Activity 1 continued >

Activity 1

Name: Date: Period:

Questions and Claims

3. Antwon collected more data by asking his friends different questions. Use Antwon's data to answer: *Do you agree that most of Antwon's friends spend more than 4 hours a day on their phones?* Circle one.

Agree Disagree I'm not sure

Responses vary.

- Agree. 4 out of Antwon's 7 friends are on their phones for more than 4 hours a day.
- Disagree. If you look at the number of hours per week, only Alina and Troy are on their phones 4 or more hours a day.
- I'm not sure. People aren't very good at knowing how long they spend on their phone, so just asking might not be very accurate.

Friend	How many hours do you spend on your phone?	
	Each day	Each week
Maria	2	12
Julian	5	25
Carlos	4.5	24
Laila	0	0
Troy	8	40
Omar	0	0
Alina	5	28

4. Antwon claims most of his friends spend more than 4 hours a day on their phones.

Antwon is asking questions to determine if his claim is true. Order his questions from *most helpful* to *least helpful*. *Orders vary.*

d	Most helpful
b	
c	
a	Least helpful

a. What activity do you use your phone for the most?

b. How many hours are you on your phone each day?

c. How many hours are you on your phone each week?

d. Do you spend more than 4 hours a day on your phone?

Accelerated 6 | Unit 9 Lesson 1 820 Activity 1

Pairs | 12 min

Activity 1 Questions and Claims (continued)

Purpose: Students consider how questions can be helpful in gathering data needed to make a claim about a group of people.

2 Monitor

Collect examples of student claims to share during the Connect.

MLR **MLR1: Stronger and Clearer Each Time (optional)** Consider using this routine to support students in developing precise language. Invite students to meet with a partner, give feedback on each other's drafts, then use the feedback to write a second clearer draft. See the Routine Facilitation Guide for more information. **(MTR.1.1)**

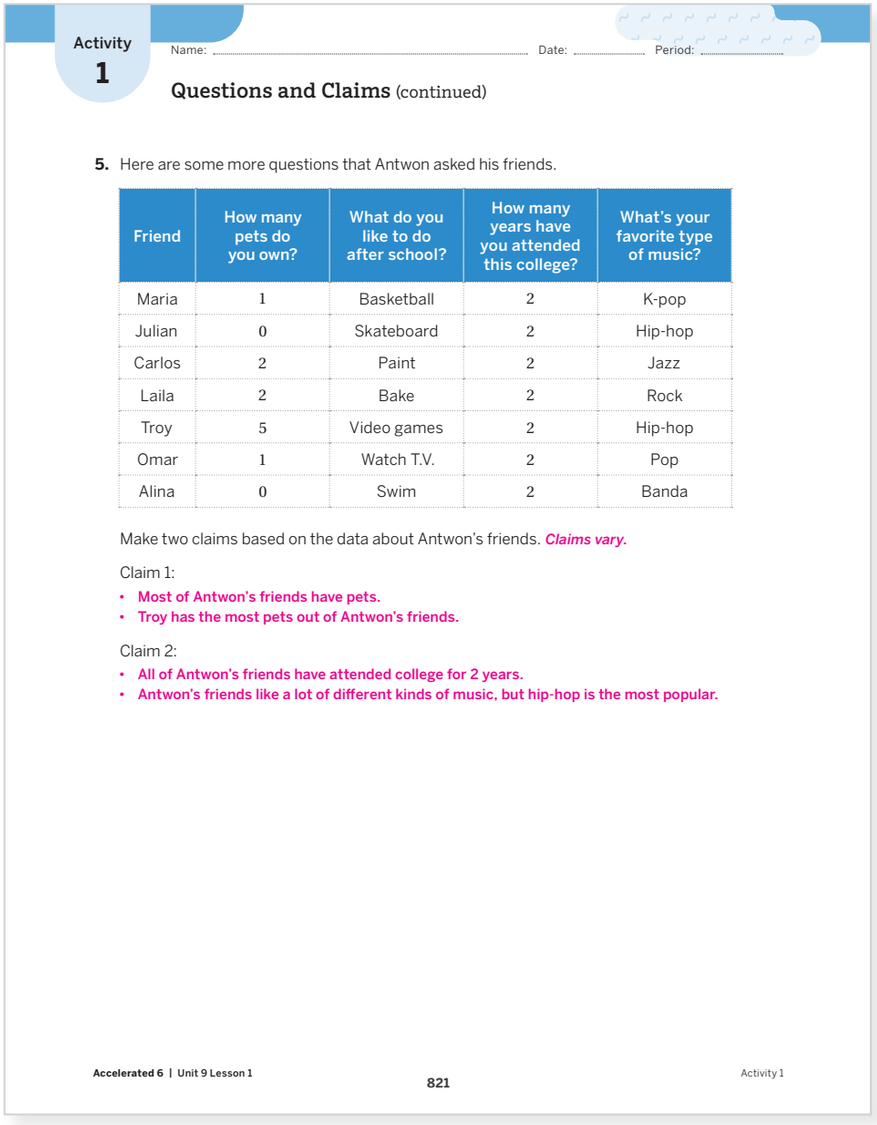
3 Connect

Display the distribution of responses for ranking the questions by helpfulness.

Share collected claims showing unique or creative ways of interpreting the data.

To surface the Key Takeaway, consider asking:

- "What do you notice about the questions and the data that was collected?"
- "What is one more question you might ask Antwon's friends to learn about how they spend time on their phones?"
- "What is a claim you could make using the data you gather from asking that question?"



Activity 1

Name: Date: Period:

Questions and Claims (continued)

5. Here are some more questions that Antwon asked his friends.

Friend	How many pets do you own?	What do you like to do after school?	How many years have you attended this college?	What's your favorite type of music?
Maria	1	Basketball	2	K-pop
Julian	0	Skateboard	2	Hip-hop
Carlos	2	Paint	2	Jazz
Laila	2	Bake	2	Rock
Troy	5	Video games	2	Hip-hop
Omar	1	Watch T.V.	2	Pop
Alina	0	Swim	2	Banda

Make two claims based on the data about Antwon's friends. *Claims vary.*

Claim 1:

- Most of Antwon's friends have pets.
- Troy has the most pets out of Antwon's friends.

Claim 2:

- All of Antwon's friends have attended college for 2 years.
- Antwon's friends like a lot of different kinds of music, but hip-hop is the most popular.

Accelerated 6 | Unit 9 Lesson 1 821 Activity 1

Key Takeaway: The types of questions that are asked help determine the kind of data that is collected.

Pairs | 8 min

Activity 2 Categorical and Numerical Data

Purpose: Students organize the types of questions and the data produced from them into two categories: **numerical data** and **categorical data**.

1 Launch

Invite students to compare the two questions with their partner before describing the difference between numerical and categorical data. **(MTR.4.1)**

Create a class definition of the terms **numerical data** and **categorical data** using students' language and the glossary. Record it for students to refer to later in the unit. **(MTR.1.1)**

Consider asking, "What is another question we might ask that produces numerical data?"

ELL **English Language Learners** Invite students to research how to say the new terms in other languages. Create an anchor chart with the terms in multiple languages, along with examples. **(Reading and Listening)**

2 Monitor

D **Differentiation**

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Pause the class and invite groups to share their thoughts on the first two questions.
Select "Numerical Data" for "Do you have 2 or more pets?"	**Support** Invite them to create sample data and then determine if "Numerical Data" is still the correct answer.

3 Connect

Invite students to share how they determined whether a question would result in numerical or categorical data.

To surface the Key Takeaway, consider asking:

- "How would you explain the differences between each type of data to someone who is not in our class?"
- "Can you think of a question that would result in numerical data? Categorical data?"

Activity 2

Name: ..... Date: ..... Period: .....

Categorical and Numerical Data

6. Asking different types of questions can produce different types of data.

One of these questions produces **numerical data** and the other produces **categorical data**.

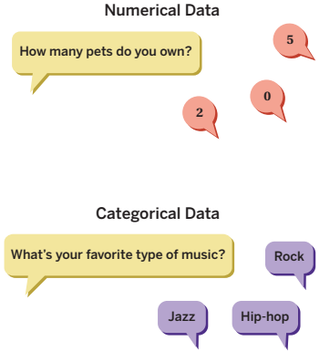
Describe what you think these terms mean.

Numerical data:

Responses vary. Data that are numbers, like 1 or 13.

Categorical data:

Responses vary. Data that are words, like *beach* or *park*.



7. Match each question with the type of data it produces.

Question	Numerical Data	Categorical Data
Do you have 2 or more pets?		✓
How many hours do you spend outdoors each day?	✓	
How old is the youngest person in your family?	✓	
How much does your backpack weigh?	✓	
What is your favorite vegetable?		✓

Accelerated 6 | Unit 9 Lesson 1 822 Activity 2

Key Takeaway: The type of question asked can determine whether the responses will produce **numerical data** or **categorical data**.

Pairs | 10 min

Activity 3 Challenge Creator

Purpose: Students use their creativity to write their own question and gather numerical data. **(MTR.1.1)**

Short on time? Consider having students only answer one question from a classmate.

1 Launch

Consider sharing that pairs of students will write their own question similar to the numerical questions in the previous activity. Students will gather data by walking around the classroom to ask their questions to their peers, documenting their responses in the table in their Student Edition.

To encourage creativity, invite pairs of students to write a question they think will be different from ones their classmates create. **(MTR.1.1)**

A **Accessibility: Executive Functioning** Invite students to work with a partner on a challenge before trying one independently.

2 Monitor

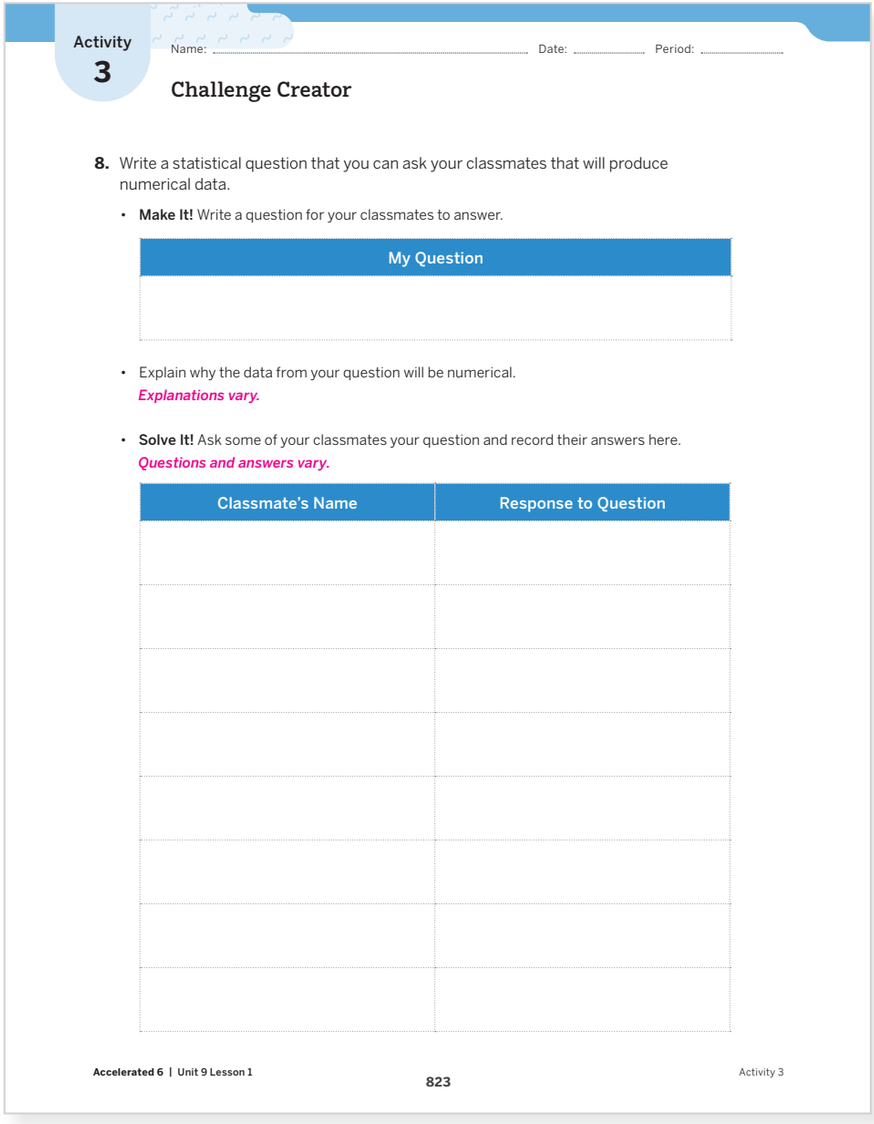
Listen to student strategies, discussions, and offer help or encouragement as needed. **(MTR.4.1)**

Select several pairs of students' questions to share during the Connect.

3 Connect

Share selected questions. Invite students to share how they decided that the data they would be gathering is numerical data.

Consider asking, "Was there a question you heard that surprised you?"



Activity 3

Name: ..... Date: ..... Period: .....

Challenge Creator

8. Write a statistical question that you can ask your classmates that will produce numerical data.

- **Make It!** Write a question for your classmates to answer.

My Question

- Explain why the data from your question will be numerical.
  Explanations vary.

- **Solve It!** Ask some of your classmates your question and record their answers here.
  Questions and answers vary.

Classmate's Name	Response to Question

Accelerated 6 | Unit 9 Lesson 1 823 Activity 3

Whole Class | 5 min

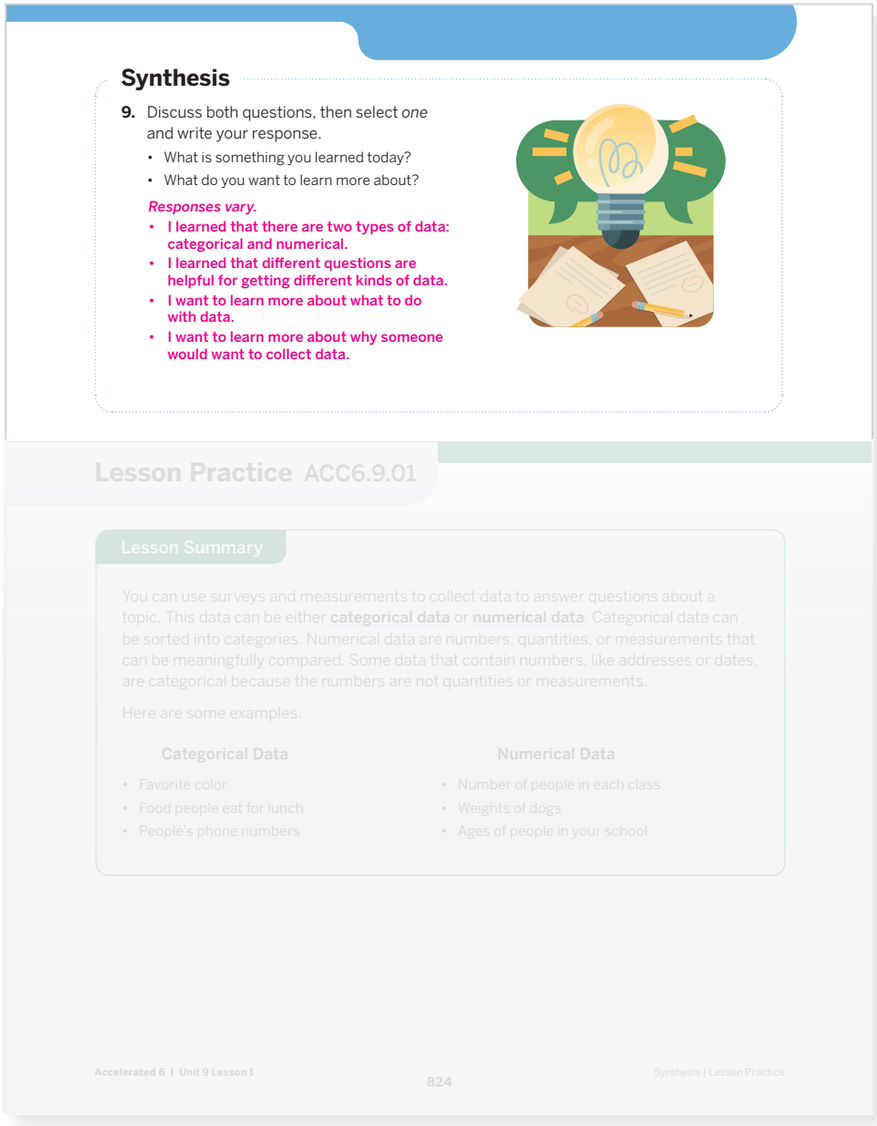
Synthesis

Purpose: Students synthesize their understanding of categorical and numerical data and the types of questions that can be used to gather data.

Invite students to respond independently, and then share their thinking with a partner.

Collect and share a variety of ideas, such as:
- Descriptions of the difference between categorical data and numerical data.
- Strategies for identifying the types of questions that can be used to gather numerical data.

MLR **MLR2: Collect and Display** Use this routine to formalize the definitions of *categorical data* and *numerical data* using students' language. Display any new language, examples, or diagrams for students to reference later.



Synthesis

9. Discuss both questions, then select *one* and write your response.
- What is something you learned today?
- What do you want to learn more about?

Responses vary.
- I learned that there are two types of data: categorical and numerical.
- I learned that different questions are helpful for getting different kinds of data.
- I want to learn more about what to do with data.
- I want to learn more about why someone would want to collect data.

Lesson Practice ACC6.9.01

Lesson Summary

You can use surveys and measurements to collect data to answer questions about a topic. This data can be either **categorical data** or **numerical data**. Categorical data can be sorted into categories. Numerical data are numbers, quantities, or measurements that can be meaningfully compared. Some data that contain numbers, like addresses or dates, are categorical because the numbers are not quantities or measurements.

Here are some examples.

Categorical Data	Numerical Data
• Favorite color	• Number of people in each class
• Food people eat for lunch	• Weights of dogs
• People's phone numbers	• Ages of people in your school

Accelerated 6 | Unit 9 Lesson 1 824 Synthesis | Lesson Practice

Lesson Takeaway: Data can be categorized by the type of responses a question produces. When a question produces values that are numbers that can be measured and compared, it is called **numerical data**. When a question produces data that is in words or values that can be sorted into categories, it is called **categorical data**.

Independent | 5 min

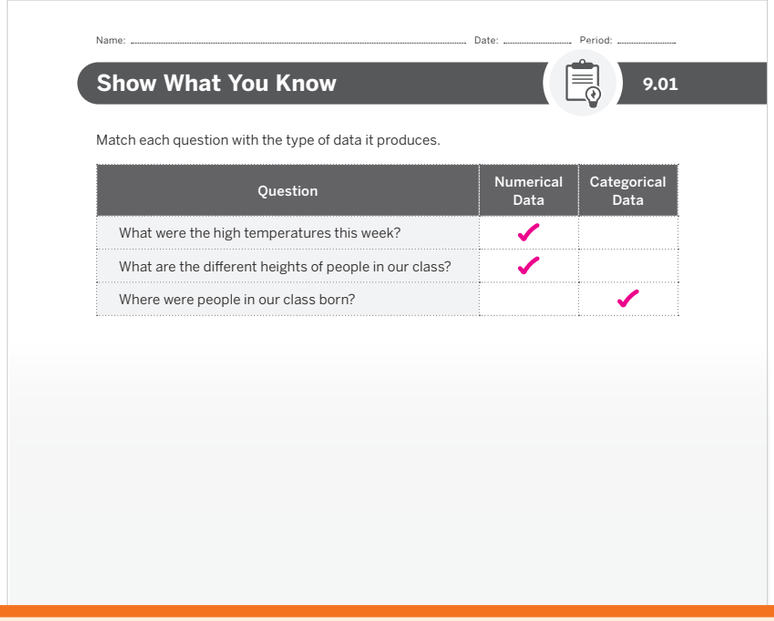
Show What You Know

Purpose: Students demonstrate their understanding by categorizing the data a question produces as either categorical or numerical.

Today's Goals

1. **Goal:** Make connections between questions, claims, and data.
2. **Language Goal:** Explain the difference between *categorical* and *numerical data*. **(Writing and Speaking)**

Show What You Know (Assessment Resources)



Name: _____ Date: _____ Period: _____

Show What You Know 9.01

Match each question with the type of data it produces.

Question	Numerical Data	Categorical Data
What were the high temperatures this week?	✓	
What are the different heights of people in our class?	✓	
Where were people in our class born?		✓

Differentiation Use after Lesson 1

Support

Provide targeted intervention for students by using these resources.

If students need support determining the difference between numerical and categorical data:

» **Respond to Student Thinking:**

- Review the **Lesson Summary**.
- Select a few related **Practice Problems** and give individualized feedback.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 1 Extension Activities**.

Professional Learning

In this lesson, students considered the distinction between different types of questions that can be used to gather data. How will that work support their understanding of creating and analyzing data displays throughout the rest of the unit?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

9. Discuss both questions, then select *one* and write your response.

- What is something you learned today?
- What do you want to learn more about?

Responses vary.

- I learned that there are two types of data: categorical and numerical.
- I learned that different questions are helpful for getting different kinds of data.
- I want to learn more about what to do with data.
- I want to learn more about why someone would want to collect data.

Lesson Practice ACC6.9.01

Lesson Summary

You can use surveys and measurements to collect data to answer questions about a topic. This data can be either **categorical data** or **numerical data**. Categorical data can be sorted into categories. Numerical data are numbers, quantities, or measurements that can be meaningfully compared. Some data that contain numbers, like addresses or dates, are categorical because the numbers are not quantities or measurements.

Here are some examples.

Categorical Data	Numerical Data
• Favorite color	• Number of people in each class
• Food people eat for lunch	• Weights of dogs
• People's phone numbers	• Ages of people in your school

Lesson Practice
ACC6.9.01

Name: _____ Date: _____ Period: _____

1. Determine whether each question would produce numerical or categorical data.

Question	Numerical Data	Categorical Data
What is your favorite breakfast food?		✓
How did you get to school this morning?		✓
How many people live in your home?	✓	
What is the last thing you ate or drank?		✓
How many minutes did it take you to get ready this morning, from waking up to leaving for school?	✓	

2. Choose one question from the table that would produce numerical data. Explain your thinking.

Responses vary.

3. Choose one question from the table that would produce categorical data. Explain your thinking.

Responses vary.

Problems 4–6: A pizza shop owner wants to improve her dinner service, so she collects data from her sales. Here is a copy of a receipt she is examining.

Booking type:	Online
Booking time:	5:30 PM
Server: Amir	Party Size: 4
1 × Pepperoni	12.00
2 × Vegetarian	21.00
1 × Chef's Special	13.50
Subtotal	46.50
Tax	4.65
Tip	10.00
Total	$61.15
Payment Method	Credit Card
Time	6:47 PM

4. Where do you see categorical data in this receipt?

Responses vary. The types of pizza people ordered is categorical data.

5. Where do you see numerical data?

Responses vary. The party size is numerical data.

6. What questions could the pizza shop owner answer using the information from this receipt and others like it?

Responses vary. She could use receipts like this to answer the question: *Do more people pay with cash or credit card?*

Lesson Practice
ACC6.9.01

Name: _____ Date: _____ Period: _____

7. Lan claims that more students in her class walk to school than take the bus. Write a question that Lan could ask each of her classmates to investigate the claim.

Responses vary.

- How do you get to school most days?
- Do you walk to school or take the bus?

FAST Practice

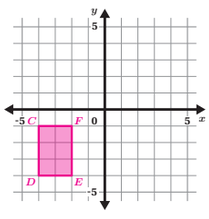
8. Select *all* the questions that can be answered using numerical data from receipts from the pizza shop.

- ☐ **A.** What types of pizzas did each party order?
- ☑ **B.** What was the tip for each order?
- ☑ **C.** How many pizzas did parties order?
- ☐ **D.** Who was the server for each order?
- ☐ **E.** What method of payment did each party use?

Spiral Review

Problems 9–10: Rectangle $CDEF$ has vertices $C = (-4, -1)$, $D = (-4, -4)$, $E = (-2, -4)$, and $F = (-2, -1)$.

9. Plot the rectangle on the coordinate plane and label the vertices.

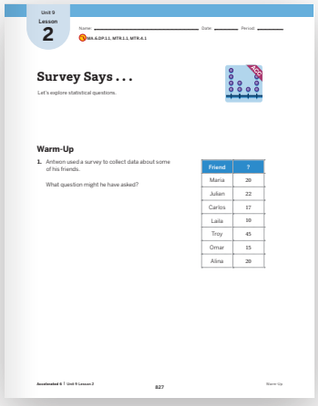


10. What is the area of the rectangle?

6 square units

Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1	1	MA.6.DP.1.1
	2–5, 7	2	MA.6.DP.1.1
	6	3	MA.6.DP.1.1
FAST Practice	8	1	MA.6.DP.1.1
Spiral Review			
	9–10	1	MA.6.GR.1.3


Unit 9
Lesson 2
Survey Says . . .
Let's explore statistical questions.
Warm-Up
1. Antwon used a survey to collect data about some of his friends.
What question might he have asked?
827

Survey Says . . .

Using Data to Generate Statistical Questions

Let's explore statistical questions.

Learning Outcomes

Today's Goals

1. **Language Goal:** Describe and interpret a data set to help answer a *statistical question*. **(Reading, Writing, and Speaking)**
2. **Language Goal:** Describe how to use a data set to create a *statistical question*. **(Writing, Speaking, and Listening)**

Students make sense of data and explore what it means for a question to be a **statistical question**.

Coherence

Prior Learning

In a previous course, students created and interpreted data presented in tables and dot plots.

Future Learning

In Lessons 3–4, students will use dot plots and histograms to represent and interpret real-world data.

Connections

- Students are introduced to the characteristics of a statistical question by exploring situations. (mathematical, cross-curricular, real world)

FL B.E.S.T. Math Benchmarks

Addressing

MA.6.DP.1.1
Recognize and formulate a statistical question that would generate numerical data.

Mathematical Thinking and Reasoning Standards: MTR.1.1, MTR.4.1

ELA Expectations: ELA.K12.EE.1.1, ELA.K12.EE.6.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.5.DP.1.1	MA.7.DP.1.5

Vocabulary

New Vocabulary

statistical question

Lesson at a Glance

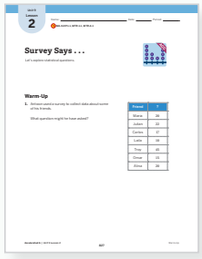
~ 45 min

MA.6.DP.1.1

Warm-Up

Whole Class | 5 min

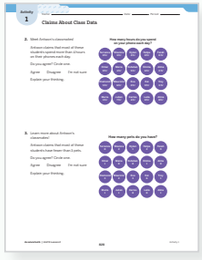
Students generate their own questions about a set of data.



Activity 1

Pairs | 10 min

Students explore claims about sets of data.
Routine: Notice and Wonder



Activity 2

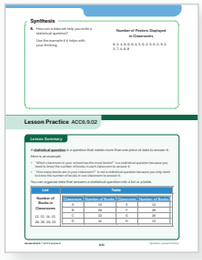
Pairs | 20 min

Students consider what type of question a set of data can help answer and define the term **statistical question**. **(MTR.1.1)**

Synthesis

Whole Class | 5 min

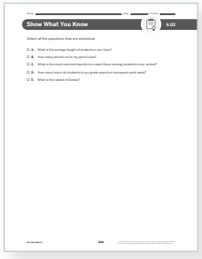
Students synthesize their understanding of using data to create a statistical question.
MLR2: Collect and Display



Show What You Know

Independent | 5 min

Students demonstrate their understanding by identifying which questions are statistical from a provided list.



Prep Checklist

This lesson includes:

Student Edition
Show What You Know

Whole Class | 5 min

Warm-Up

Purpose: Students generate their own questions about a set of data.

1 Launch

To surface connections, consider discussing how this set of data is similar to the data from the previous lesson.

To support students getting started, consider asking, "Is this categorical or numerical data? How do you know?"

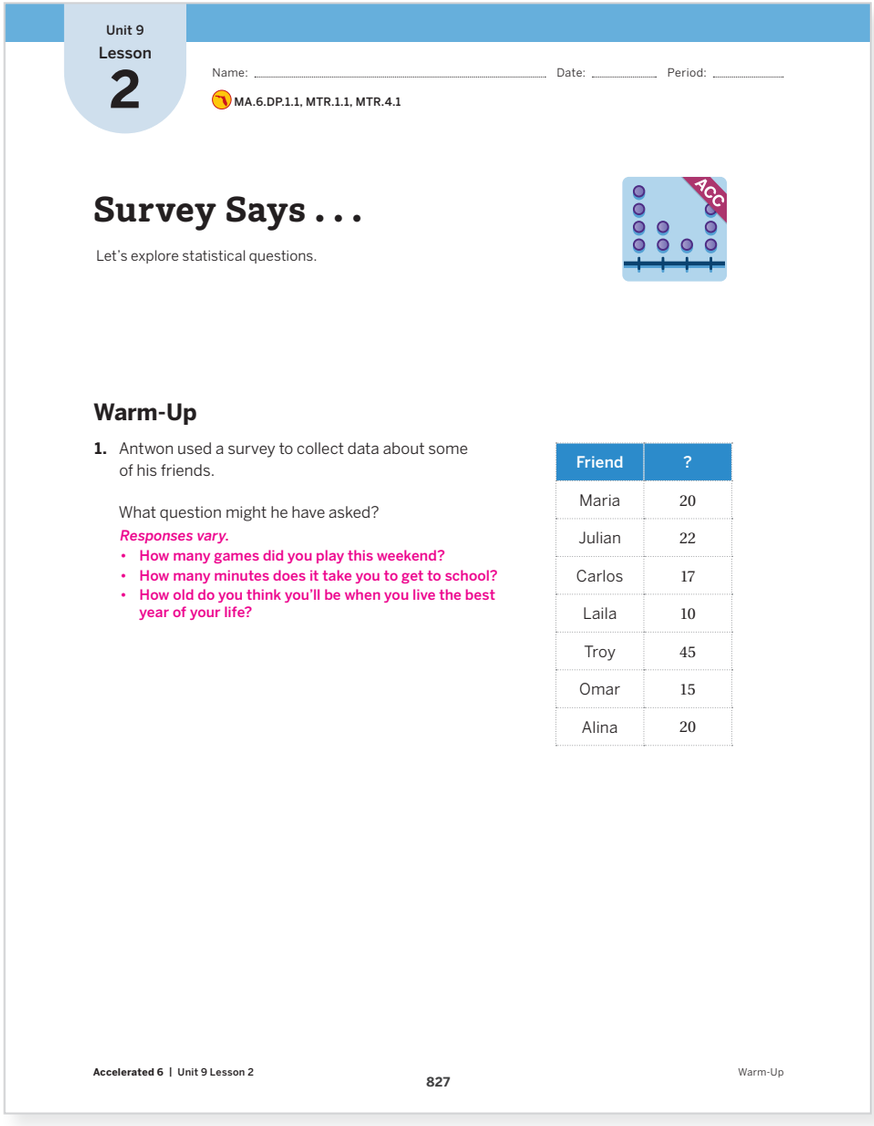
Invite students to write their own questions before discussing their idea with a partner. **(MTR.4.1)**

ELL **English Language Learners** Invite students to write questions in their primary language. **(Writing)**

2 Connect

Collect and share several student responses.

Consider asking, "Do you think Antwon could have asked this question? Why or why not?" **(MTR.1.1)**



Unit 9
Lesson
2

Name: _____ Date: _____ Period: _____

MA.6.DP.1.1, MTR.1.1, MTR.4.1

Survey Says . . .

Let's explore statistical questions.

Warm-Up

1. Antwon used a survey to collect data about some of his friends.

What question might he have asked?

Responses vary.

- How many games did you play this weekend?
- How many minutes does it take you to get to school?
- How old do you think you'll be when you live the best year of your life?

Friend	?
Maria	20
Julian	22
Carlos	17
Laila	10
Troy	45
Omar	15
Alina	20

Accelerated 6 | Unit 9 Lesson 2 827 Warm-Up

Pairs | 10 min

Activity 1 Claims About Class Data

Purpose: Students explore claims about sets of data.

1 Launch

Consider asking, "Why might Antwon be interested in how many hours students in his class spend on their phones?"

Listen to students' strategies for deciding if they agree with Antwon's claim.

Invite students to share their responses, calling attention to any conflict or consensus. It's okay—even desirable—to lack consensus at this stage.

Use the Notice and Wonder routine to promote curiosity and help students make sense of the data sets. See the Routine Facilitation Guide for more information. **(MTR.1.1)**

2 Monitor

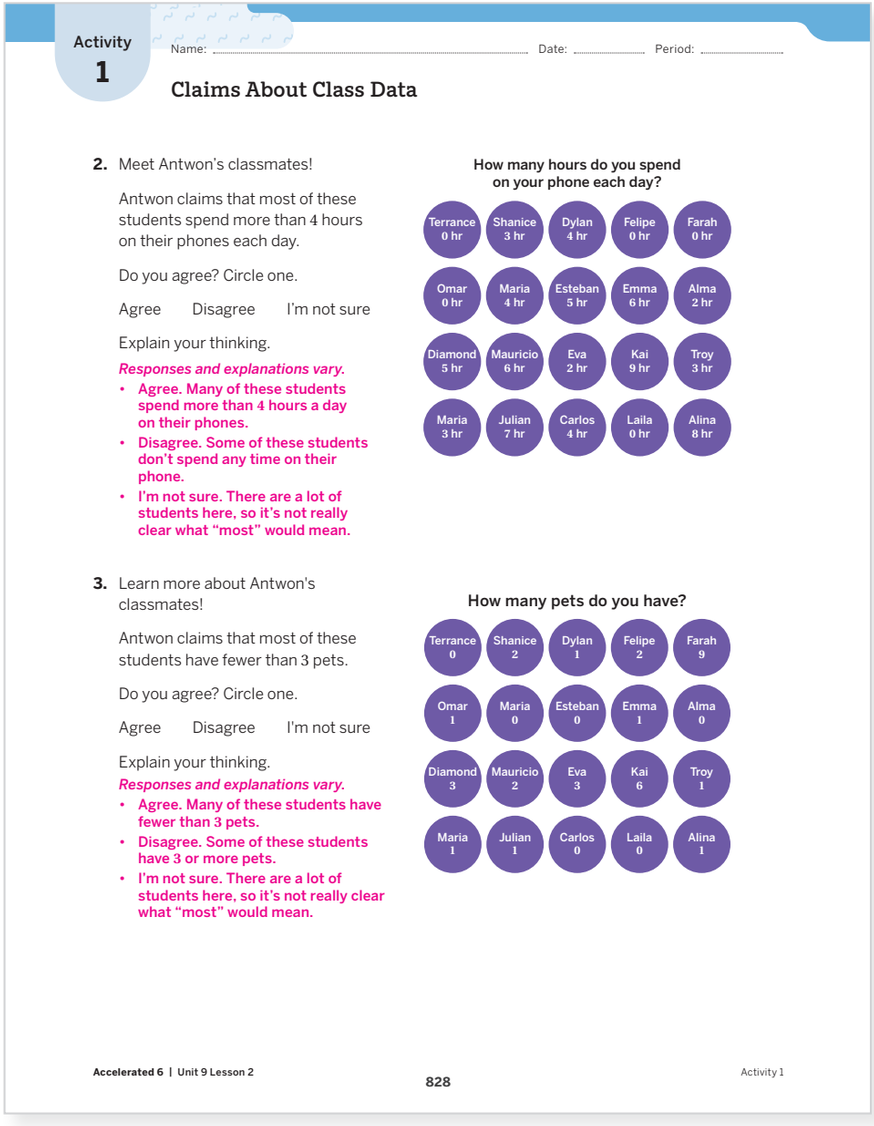
Encourage students to discuss their thinking with a partner.

Listen for students' ideas to highlight during the Connect.

ELL **English Language Learners** Create a list of words about the context that students might not know, along with visuals or translations of each word in languages used by your students. **(Reading and Listening)**

3 Connect

Invite students to share their thinking. Consider asking, "How can you organize the data to help make decisions about the claims?"



Activity 1

Name: Date: Period:

Claims About Class Data

2. Meet Antwon's classmates!

Antwon claims that most of these students spend more than 4 hours on their phones each day.

Do you agree? Circle one.

Agree Disagree I'm not sure

Explain your thinking.

Responses and explanations vary.

- Agree. Many of these students spend more than 4 hours a day on their phones.
- Disagree. Some of these students don't spend any time on their phone.
- I'm not sure. There are a lot of students here, so it's not really clear what "most" would mean.

How many hours do you spend on your phone each day?

Terrance 0 hr	Shanice 3 hr	Dylan 4 hr	Felipe 0 hr	Farah 0 hr
Omar 0 hr	Maria 4 hr	Esteban 5 hr	Emma 6 hr	Alma 2 hr
Diamond 5 hr	Mauricio 6 hr	Eva 2 hr	Kai 9 hr	Troy 3 hr
Maria 3 hr	Julian 7 hr	Carlos 4 hr	Laila 0 hr	Alina 8 hr

3. Learn more about Antwon's classmates!

Antwon claims that most of these students have fewer than 3 pets.

Do you agree? Circle one.

Agree Disagree I'm not sure

Explain your thinking.

Responses and explanations vary.

- Agree. Many of these students have fewer than 3 pets.
- Disagree. Some of these students have 3 or more pets.
- I'm not sure. There are a lot of students here, so it's not really clear what "most" would mean.

How many pets do you have?

Terrance 0	Shanice 2	Dylan 1	Felipe 2	Farah 9
Omar 1	Maria 0	Esteban 0	Emma 1	Alma 0
Diamond 3	Mauricio 2	Eva 3	Kai 6	Troy 1
Maria 1	Julian 1	Carlos 0	Laila 0	Alina 1

Accelerated 6 | Unit 9 Lesson 2 828 Activity 1

Pairs | 20 min

Activity 2 Questionable Data

Purpose: Students consider what type of question a set of data can help answer and define the term **statistical question**. **(MTR.1.1)**

Short on time? Consider limiting the time for discussion in Problem 4.

1 Launch

To generate excitement, consider asking:

- "What questions would you be interested in asking to our class?"
- "What do you imagine the data would look like?"

A **Accessibility: Visual-Spatial Processing**
Invite students to list the numbers from least to greatest to increase access to the task.

2 Monitor

Listen for students who use evidence from the tables to support their claims about Antwon's classmates such as:

- Ordering the data in a list and then counting the number of occurrences for each data value.
- Using ratio language, such as "The ratio of classmates who have pets to those who don't have pets is $14 : 6$."
- Using percentages in their noticings, such as "25% of Antwon's classmates don't have siblings." **(MTR.1.1)**

Pause briefly and invite students to share what they learned about Antwon's classmates.

Activity 2 continued >

Activity 2

Name: Date: Period:

Questionable Data

4. Antwon surveyed his classmates about the number of siblings they each have and made a table of their responses.

Name	Number of Siblings	Name	Number of Siblings
Terance	0	Diamond	2
Shanice	1	Mauricio	2
Dylan	1	Eva	0
Felipe	3	Kai	0
Farah	2	Troy	1
Omar	2	Maria	3
Maria	3	Julian	1
Esteban	1	Carlos	5
Emma	6	Laila	2
Alma	1	Alina	0

Discuss: What are two things you learned about Antwon's classmates from this data?

Responses vary.

- There are 4 students who are the "only child" in Antwon's class.
- Most students have 2 or fewer siblings.
- Only 2 students have more than 4 siblings.
- None of Antwon's classmates have 4 siblings.

5. Antwon surveyed his classmates about the number of books each student read this month and made a table of their responses.

Name	Number of Books Read	Name	Number of Books Read
Terance	7	Diamond	7
Shanice	5	Mauricio	0
Dylan	8	Eva	1
Felipe	7	Kai	6
Farah	5	Troy	7
Omar	6	Maria	1
Maria	7	Julian	5
Esteban	12	Carlos	4
Emma	4	Laila	3
Alma	5	Alina	8

Discuss: What are two things you learned about Antwon's classmates from this data?

Responses vary.

- There are 5 students who read 7 books this month.
- Most students read 6 or fewer books.
- Only 1 student read more than 8 books.
- None of Antwon's classmates read more than 12 books.

Accelerated 6 | Unit 9 Lesson 2 829 Activity 2

Pairs | 20 min

Activity 2 Questionable Data (continued)

Purpose: Students consider what type of question a set of data can help answer and define the term **statistical question**. **(MTR.1.1)**

2 Monitor

D Differentiation

Look for students who:	Respond to Student Thinking
Write a question like: "How many books did you read this month?"	**Support** Consider asking, "How would the data help you answer this question?" If it is helpful, consider revisiting the discussion of the previous problem.
Write questions that are examples of statistical questions.	**Strengthen** Celebrate student creativity and collect these to share during the Connect.

Encourage students to discuss each question one at a time with their partner before determining if it is a statistical question or not. **(MTR.1.1, MTR.4.1)**

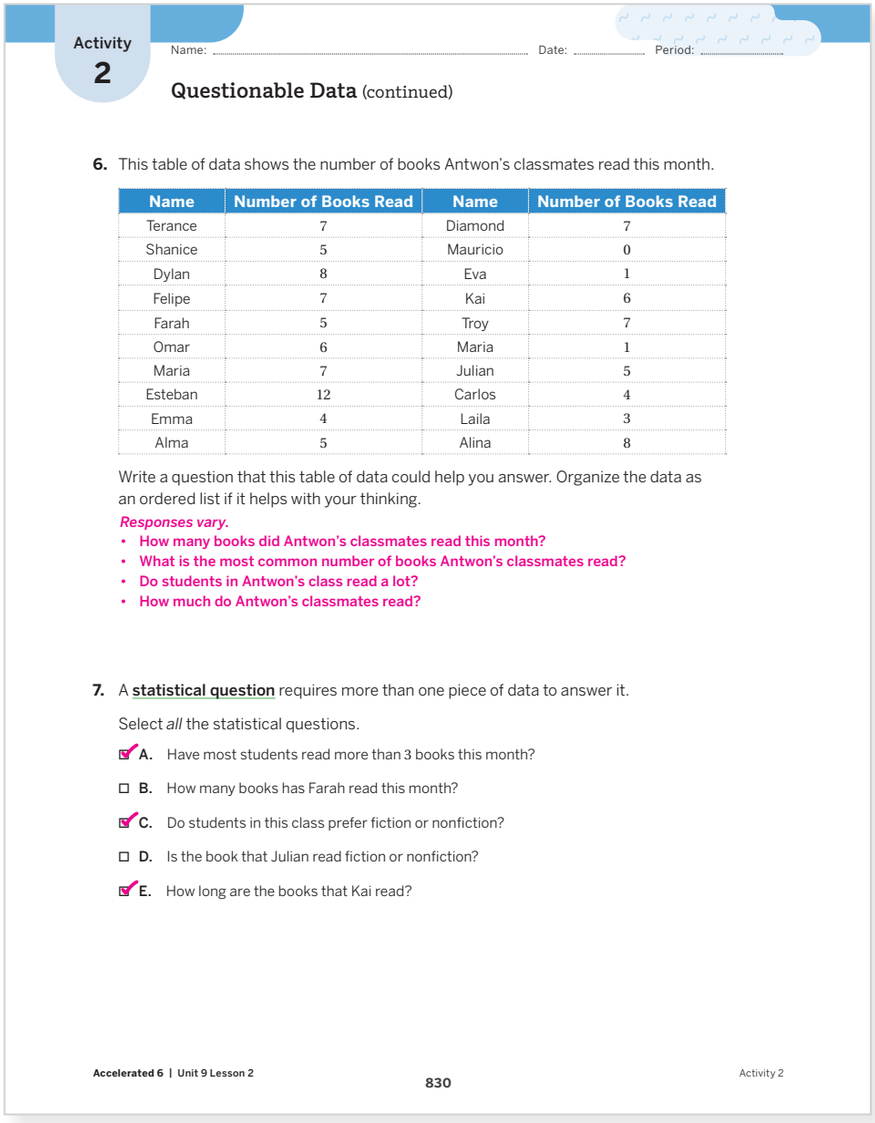
3 Connect

Invite students to share their responses, calling attention to any conflict or consensus.

Discuss each possible response. Invite students to share their reasoning in order to surface features of statistical questions.

Create a class definition for the term **statistical question** using students' language and the glossary. **(MTR.1.1)**

To surface the Key Takeaway, share the questions for the table. Consider asking, "Which of these questions are statistical questions?"



Activity 2

Name: Date: Period:

Questionable Data (continued)

6. This table of data shows the number of books Antwon's classmates read this month.

Name	Number of Books Read	Name	Number of Books Read
Terance	7	Diamond	7
Shanice	5	Mauricio	0
Dylan	8	Eva	1
Felipe	7	Kai	6
Farah	5	Troy	7
Omar	6	Maria	1
Maria	7	Julian	5
Esteban	12	Carlos	4
Emma	4	Laila	3
Alma	5	Alina	8

Write a question that this table of data could help you answer. Organize the data as an ordered list if it helps with your thinking.

Responses vary.

- How many books did Antwon's classmates read this month?
- What is the most common number of books Antwon's classmates read?
- Do students in Antwon's class read a lot?
- How much do Antwon's classmates read?

7. A **statistical question** requires more than one piece of data to answer it.

Select *all* the statistical questions.

- ☑ **A.** Have most students read more than 3 books this month?
- ☐ **B.** How many books has Farah read this month?
- ☑ **C.** Do students in this class prefer fiction or nonfiction?
- ☐ **D.** Is the book that Julian read fiction or nonfiction?
- ☑ **E.** How long are the books that Kai read?

Accelerated 6 | Unit 9 Lesson 2 830 Activity 2

Key Takeaway: A **statistical question** is one that requires collecting more than one piece of data to answer it.

Whole Class | 5 min

Synthesis

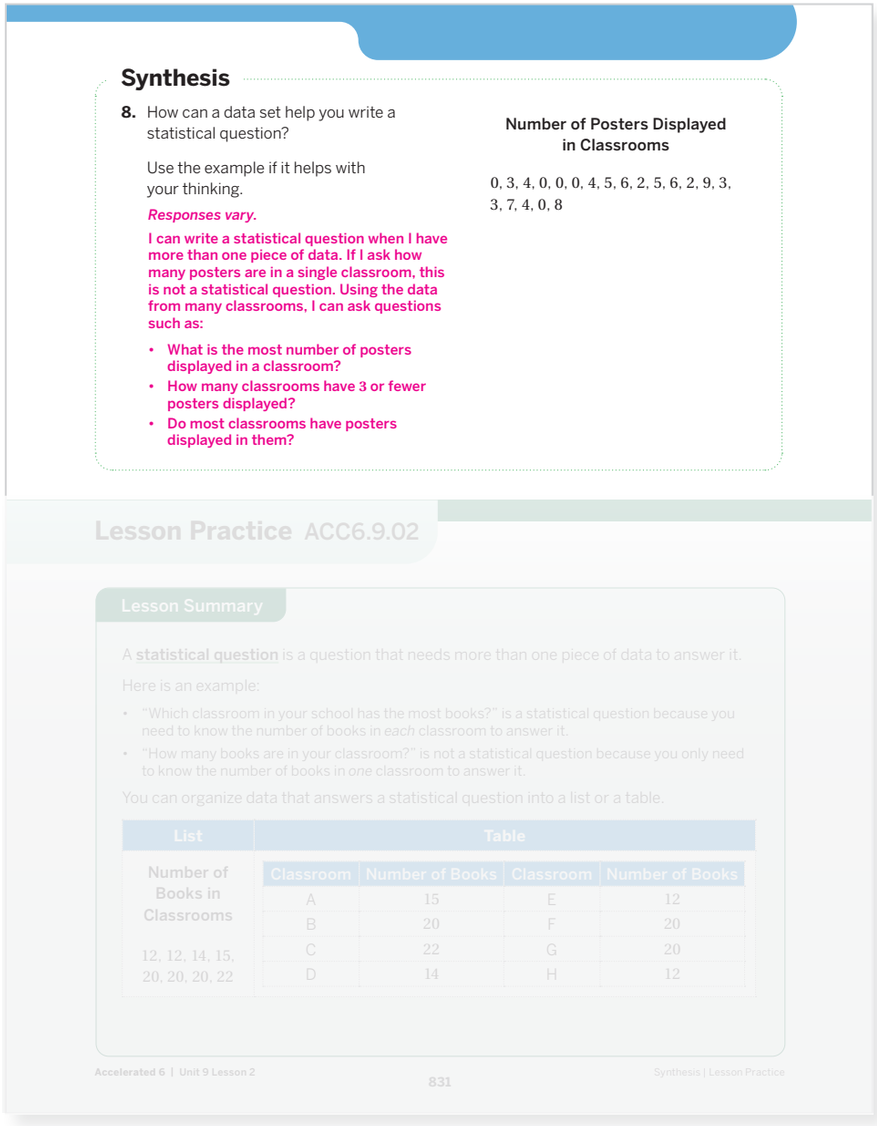
Purpose: Students synthesize their understanding of using data to create a statistical question.

Invite students to respond independently, and then share their thinking with a partner.

Collect and share a variety of ideas, such as:
- A data set provides more than one piece of information.
- Values in a data set can be analyzed and compared to create statistical questions.

MLR **MLR2: Collect and Display** Use this routine to formalize the definition of *statistical question* using students' language. Display any new language, examples, or diagrams for students to reference later.

Note: If time allows, invite students to revise their previous thinking based on the discussion.



Synthesis

8. How can a data set help you write a statistical question?

Use the example if it helps with your thinking.

Responses vary.

I can write a statistical question when I have more than one piece of data. If I ask how many posters are in a single classroom, this is not a statistical question. Using the data from many classrooms, I can ask questions such as:

- What is the most number of posters displayed in a classroom?
- How many classrooms have 3 or fewer posters displayed?
- Do most classrooms have posters displayed in them?

Number of Posters Displayed in Classrooms

0, 3, 4, 0, 0, 0, 4, 5, 6, 2, 5, 6, 2, 9, 3, 3, 7, 4, 0, 8

Lesson Practice ACC6.9.02

Lesson Summary

A statistical question is a question that needs more than one piece of data to answer it.

Here is an example:

- "Which classroom in your school has the most books?" is a statistical question because you need to know the number of books in *each* classroom to answer it.
- "How many books are in your classroom?" is not a statistical question because you only need to know the number of books in *one* classroom to answer it.

You can organize data that answers a statistical question into a list or a table.

List	Table			
Number of Books in Classrooms	Classroom	Number of Books	Classroom	Number of Books
12, 12, 14, 15, 20, 20, 20, 22	A	15	E	12
	B	20	F	20
	C	22	G	20
	D	14	H	12

Accelerated 6 | Unit 9 Lesson 2 831 Synthesis | Lesson Practice

Key Takeaway: Statistical questions require collecting more than one piece of data to answer them. You can organize data from a survey in a table or a list.

Independent | 5 min

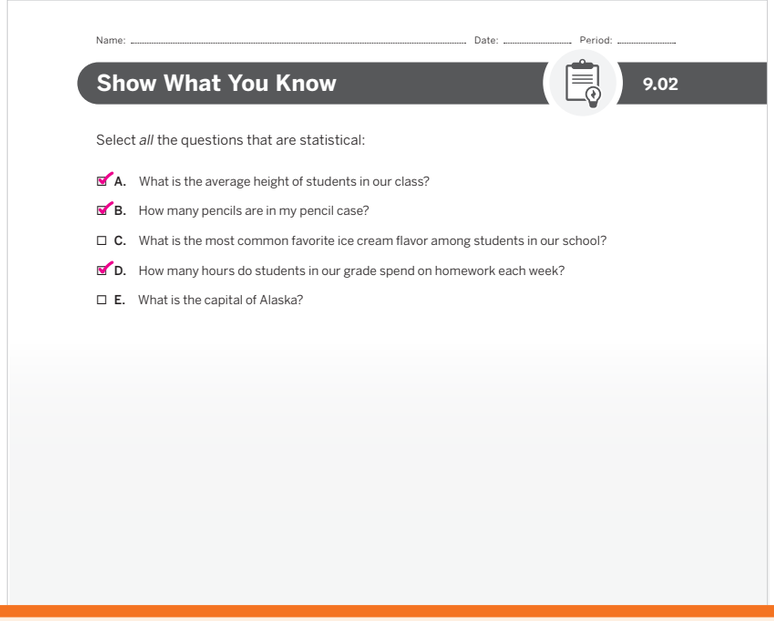
Show What You Know

Purpose: Students demonstrate their understanding by identifying which questions are statistical from a provided list.

Today's Goals

1. **Language Goal:** Describe and interpret a data set to help answer a *statistical question.* **(Reading, Writing, and Speaking)**
2. **Language Goal:** Describe how to use a data set to create a *statistical question*. **(Writing, Speaking, and Listening)**

Show What You Know (Assessment Resources)



Name: _____ Date: _____ Period: _____

Show What You Know 9.02

Select *all* the questions that are statistical:

- ☑ **A.** What is the average height of students in our class?
- ☑ **B.** How many pencils are in my pencil case?
- ☐ **C.** What is the most common favorite ice cream flavor among students in our school?
- ☑ **D.** How many hours do students in our grade spend on homework each week?
- ☐ **E.** What is the capital of Alaska?

D Differentiation Use after Lesson 2

Support

Provide targeted intervention for students by using these resources.

If students need support identifying a statistical question:

» **Respond to Student Thinking:**

- Review **Activity 2, Problem 7** and provide targeted feedback to students on specific **Practice Problems**.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite them to explore the **Sub-Unit 1 Extension Activities**.

Professional Learning

Take a moment to reflect on the Notice and Wonder routine.

- How did this routine support students in making sense of the data sets?
- What about the routine felt successful?
- What are some areas for growth the next time your class uses this routine?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

8. How can a data set help you write a statistical question?

Use the example if it helps with your thinking.

Responses vary.

I can write a statistical question when I have more than one piece of data. If I ask how many posters are in a single classroom, this is not a statistical question. Using the data from many classrooms, I can ask questions such as:

- What is the most number of posters displayed in a classroom?
- How many classrooms have 3 or fewer posters displayed?
- Do most classrooms have posters displayed in them?

Number of Posters Displayed in Classrooms

0, 3, 4, 0, 0, 0, 4, 5, 6, 2, 5, 6, 2, 9, 3, 3, 7, 4, 0, 8

Lesson Practice ACC6.9.02

Lesson Summary

A **statistical question** is a question that needs more than one piece of data to answer it.

Here is an example:

- "Which classroom in your school has the most books?" is a statistical question because you need to know the number of books in *each* classroom to answer it.
- "How many books are in your classroom?" is not a statistical question because you only need to know the number of books in *one* classroom to answer it.

You can organize data that answers a statistical question into a list or a table.

List	Table			
Number of Books in Classrooms 12, 12, 14, 15, 20, 20, 20, 22	Classroom	Number of Books	Classroom	Number of Books
	A	15	E	12
	B	20	F	20
	C	22	G	20
	D	14	H	12

Lesson Practice ACC6.9.02

Name: _____ Date: _____ Period: _____

Problems 1–5: Five sixth-grade students at a school were each asked the following survey questions:

Question A: What grade are you in?

Question B: How many books did you read in the last year?

Question C: How many inches are in 1 foot?

Question D: How many dogs and cats do you have?

Their answers are shown in the table. Write the letter of the question that could have produced each line of data.

	Question	Diya	Peter	Tiana	Marc	Callen
1.	D	0	1	1	3	0
2.	C	12	12	12	12	12
3.	A	6	6	6	6	6
4.	B	11	5	18	20	9

5. How are Questions A and C different from the other questions?

Responses vary. Questions A and C have only one correct answer. The other questions have a lot of different possible answers.

Problems 6–7: Anya asked 10 students how many minutes it takes them to get to school each morning.

6. Which list could represent the data that Anya collected?

B

A. -5, 0, 2, 3, 3, 3, 4, 5, 5, 7

B. 8, 8, 10, 10, 10, 12, 13, 16, 18, 20

C. 120, 140, 140, 160, 180, 240, 260, 260, 280, 300

7. Which list could not represent Anya's data? Explain your thinking.

Responses vary.

- List A cannot represent Anya's data because it would not make sense for someone to take -5 minutes to get to school.
- List C would not make sense because 300 minutes is 5 hours, which is probably too long for someone to get to school.

Lesson Practice ACC6.9.02

Name: _____ Date: _____ Period: _____

FAST Practice

8. Which question is a statistical question?

A. How old are you?

B. How old are the students in our school?

C. How old is your teacher?

D. What is the difference between your age and your teacher's age?

9. Choose an answer from Problem 8 that you did not select. Why do you think this is not a statistical question?

Responses vary.

A. Asking a single person for their age is not enough data for a statistical question.

C. This question refers to a single teacher, so there is only one answer instead of a variety of answers.

D. There is only one calculation with one answer for the difference between my age and my teacher's age, so it is not a statistical question.

Spiral Review

10. Order these values from *least* to *greatest*.

|-17|	|-18|	-18	|19|	20
-18	**|-17|**	**|-18|**	**|19|**	**20**

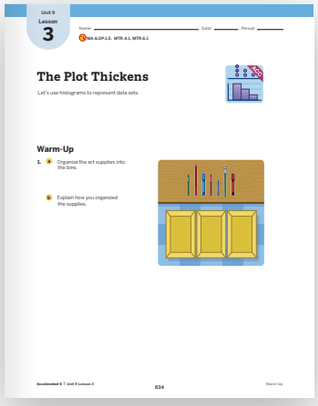
Least **Greatest**

Problems 11–13: Determine each quotient.

11. $45052 \div 28 = 1609$ **12.** $6052 \div 17 = 356$ **13.** $60.52 \div 1.7 = 35.6$

Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1–4, 6	2	MA.6.DP.1.1
	5, 7, 9	3	MA.6.DP.1.1
FAST Practice	8	1	MA.6.DP.1.1
Spiral Review			
	10	1	MA.6.NSO.1.4
	11–13	1	MA.6.NSO.2.1



The Plot Thickens

Introduction to Histograms

Let's use histograms to represent data sets.

Learning Outcomes

Today's Goals

1. **Goal:** Describe and interpret a *histogram* that represents a data set.
2. **Language Goal:** Compare and contrast dot plots and *histograms* to visualize data. **(Writing, Speaking, and Listening)**

Students are introduced to another way to visualize data: in a **histogram**. They compare and contrast histograms and *dot plots*. They also spend time considering the important features of a histogram.

Coherence

Prior Learning

In Lesson 2, students used lists and tables of data to answer statistical questions.

Future Learning

In Lessons 5–6, students will analyze data using histograms and dot plots and work together to create their own histograms.

Connections

- Students draw on prior knowledge to explore data displays and their characteristics for given situations. (mathematical, real world)

FL B.E.S.T. Math Benchmarks

Addressing

MA.6.DP.1.5
Create box plots and histograms to represent sets of numerical data within real-world contexts.

Mathematical Thinking and Reasoning Standards: MTR.4.1, MTR.6.1

ELA Expectations: ELA.K12.EE.1.1, ELA.K12.EE.4.1, ELA.K12.EE.5.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.5.DP.1.1	MA.7.DP.1.5

Vocabulary

New Vocabulary

bin (of a histogram)
histogram

Review Vocabulary

dot plot

Lesson at a Glance

~ 45 min

MA.6.DP.1.5

Warm-Up

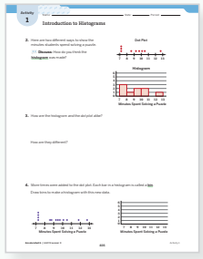
Independent | 5 min

Students consider different ways of grouping items to introduce the concept of grouping data.

Activity 1

Pairs | 10 min

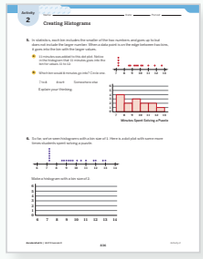
Students are introduced to **histograms** as another way to visualize numerical data.



Activity 2

Pairs | 10 min

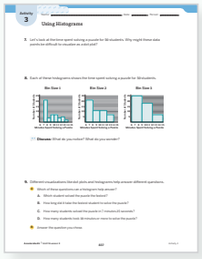
Students use the **Think-Pair-Share** routine to shape their ideas about creating histograms from *dot plots* representing data on time spent solving a puzzle. **(MTR.4.1)**



Activity 3

Pairs | 10 min

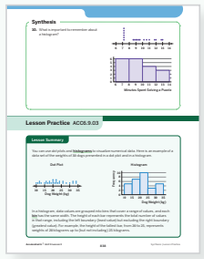
Students use the **Notice and Wonder** routine to explore when a histogram is the most useful visualization of a data set and explore what questions a histogram can and cannot answer.



Synthesis

Whole Class | 5 min

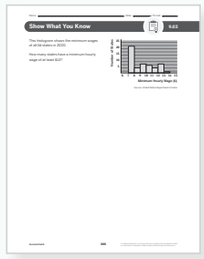
Students synthesize their understanding of the relationship between histograms and dot plots. **MLR2: Collect and Display**



Show What You Know

Independent | 5 min

Students demonstrate their understanding by analyzing a histogram to determine the number of states with a minimum wage or less than $12.



Prep Checklist

This lesson includes:

- Student Edition
- Show What You Know
- Highlighters (optional)
- Straightedges

Independent | 5 min

Warm-Up

Purpose: Students consider different ways of grouping items to introduce the concept of grouping data.

1 Launch

To surface connections, consider asking, "Who has a messy drawer at home or at school that could use some organization?"

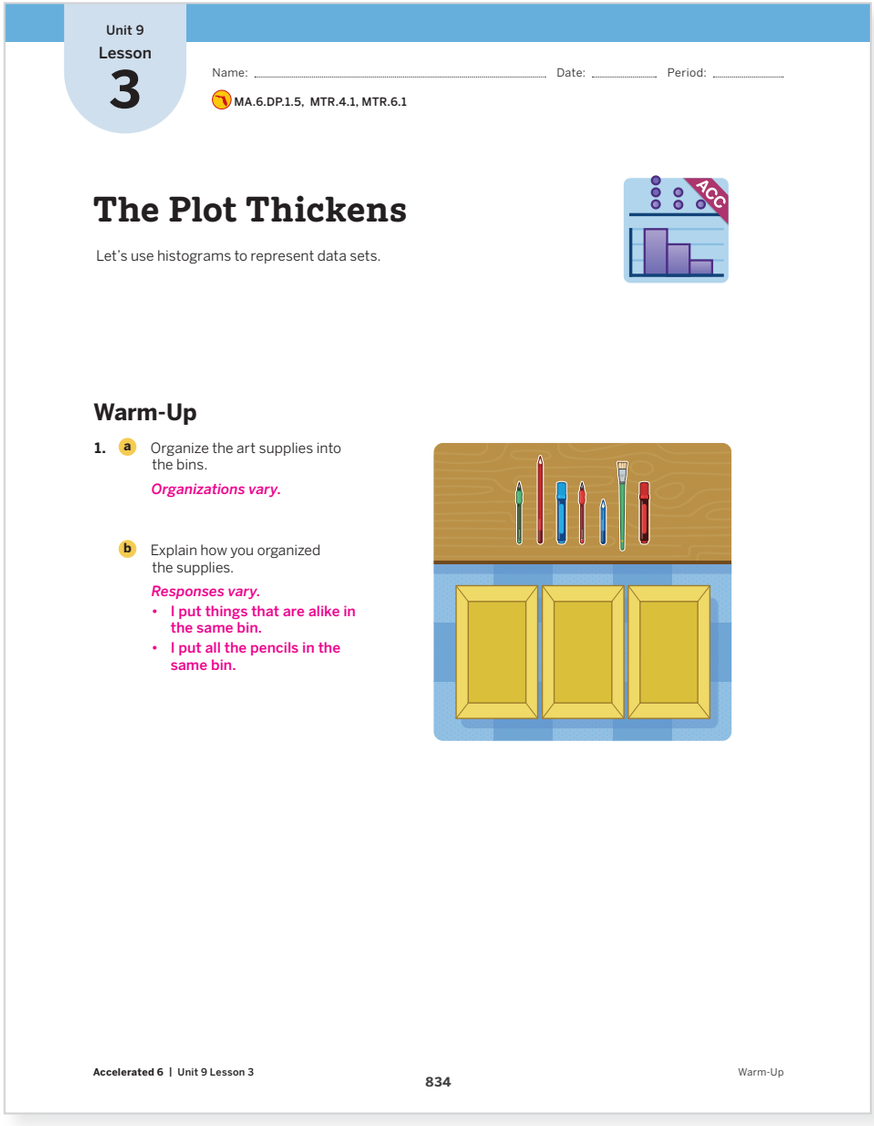
Watch for several different strategies that students use to organize their supplies during the Connect.

2 Connect

Share different strategies and have students discuss the variation in groupings.

Consider asking, "What if you were trying to sort a group of values instead of objects? What are different ways you could create groups?"

Invite students to construct a viable argument for how they organized their bins and allow other students to critique their reasoning using their own language. **(MTR.4.1)**



Unit 9
Lesson
3

Name: _____ Date: _____ Period: _____

MA.6.DP.1.5, MTR.4.1, MTR.6.1

The Plot Thickens

Let's use histograms to represent data sets.

ACC

Warm-Up

1. a Organize the art supplies into the bins.

Organizations vary.

b Explain how you organized the supplies.

Responses vary.

- I put things that are alike in the same bin.
- I put all the pencils in the same bin.

Accelerated 6 | Unit 9 Lesson 3 834 Warm-Up

Pairs | 10 min

Activity 1 Introduction to Histograms

Purpose: Students are introduced to **histograms** as another way to visualize numerical data.

1 Launch

Consider asking, "How can we tell that there are 9 students represented here?"

Encourage students to use highlighters to make connections between the *dot plot* and the histogram.

2 Monitor

Select and sequence student responses to use during the Connect discussion.

Look for students who recognize that the graphs show the same data. Encourage students to justify their thinking to a partner. **(MTR.4.1)**

Note: Students need access to a straightedge for the remainder of this lesson.

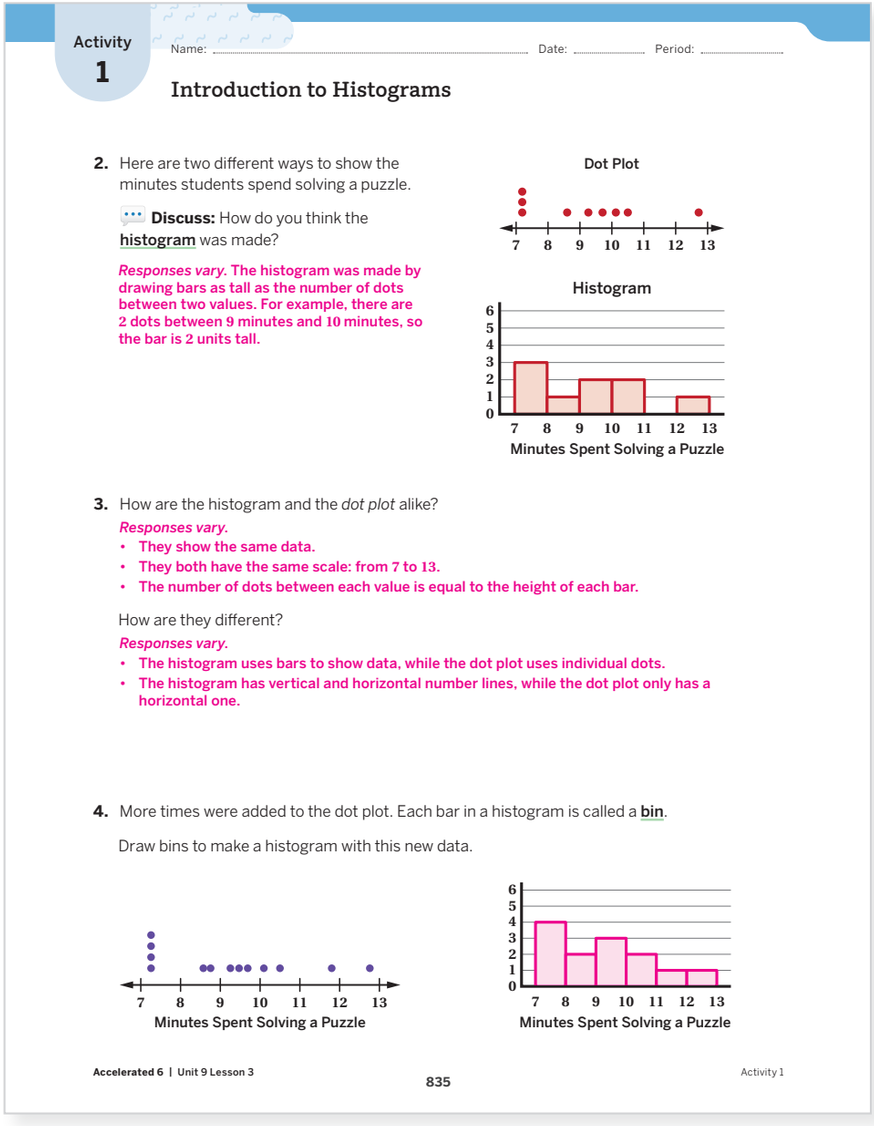
3 Connect

Consider asking:

- "If we added a time of 12 minutes 50 seconds, where would it go in the dot plot? The histogram?"
- "What do you think are some important features of histograms?"

To surface the Key Takeaway, consider inviting students to share how they determined the height of each bin.

Create a class definition for the terms **histogram** and **bin** using students' language and the glossary.



Activity 1

Name: Date: Period:

Introduction to Histograms

2. Here are two different ways to show the minutes students spend solving a puzzle.

Discuss: How do you think the **histogram** was made?

Responses vary. The histogram was made by drawing bars as tall as the number of dots between two values. For example, there are 2 dots between 9 minutes and 10 minutes, so the bar is 2 units tall.

Dot Plot

Histogram

Minutes Spent Solving a Puzzle

3. How are the histogram and the *dot plot* alike?

Responses vary.

- They show the same data.
- They both have the same scale: from 7 to 13.
- The number of dots between each value is equal to the height of each bar.

How are they different?

Responses vary.

- The histogram uses bars to show data, while the dot plot uses individual dots.
- The histogram has vertical and horizontal number lines, while the dot plot only has a horizontal one.

4. More times were added to the dot plot. Each bar in a histogram is called a **bin**.

Draw bins to make a histogram with this new data.

Minutes Spent Solving a Puzzle

Minutes Spent Solving a Puzzle

Accelerated 6 | Unit 9 Lesson 3 835 Activity 1

Key Takeaway: **Histograms** are another way to model numerical data. They can be created from a *dot plot* by sorting data into groups or **bins**.

Pairs | 10 min

Activity 2 Creating Histograms

Purpose: Students use the Think-Pair-Share routine to shape their ideas about creating histograms from *dot plots* representing data on time spent solving a puzzle. **(MTR.4.1)**

1 Launch

Use the Think-Pair-Share routine to help students shape their ideas about creating histograms from dot plots. See the Routine Facilitation Guide for more information. **(MTR.4.1)**

2 Monitor

Pause briefly to display the distribution of responses, calling attention to any conflict or consensus. Invite students to make arguments and justify their reasoning for each response. **(MTR.4.1)**

D Differentiation

Look for students who:	Respond to Student Thinking
Include the dot for 12 in the bin for 10 to 12.	**Support** Invite students to go back to the previous problem and observe which bin 11 minutes goes into.
Create a precise histogram and describe how they made the decisions that they did.	**Stretch** Encourage students to create a different dot plot that would result in the same histogram.

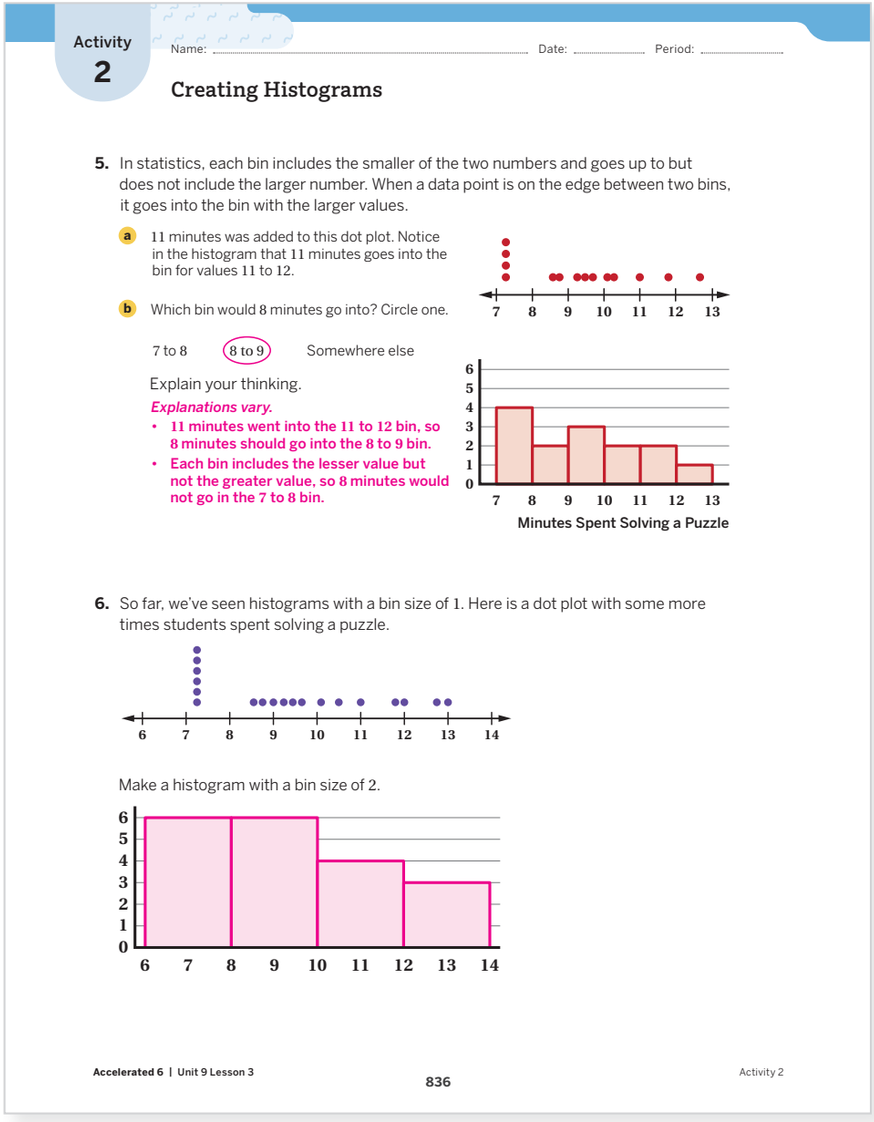
Look for a correct and incorrect histogram to share during the Connect.

3 Connect

Display the correct and incorrect histograms.

Consider asking:

- "Which histogram represents the information in the dot plot?"
- "What feedback would you give to the person who created the other histogram?"
- "How did you determine how tall to make each bin?"
- "If we added another time of 10 minutes, which bin would that value go in?"
- "How would this histogram change if the bin size were 3?"



Activity 2

Name: ..... Date: ..... Period: .....

Creating Histograms

5. In statistics, each bin includes the smaller of the two numbers and goes up to but does not include the larger number. When a data point is on the edge between two bins, it goes into the bin with the larger values.

a. 11 minutes was added to this dot plot. Notice in the histogram that 11 minutes goes into the bin for values 11 to 12.

b. Which bin would 8 minutes go into? Circle one.

7 to 8 (8 to 9) Somewhere else

Explain your thinking.

Explanations vary.

- 11 minutes went into the 11 to 12 bin, so 8 minutes should go into the 8 to 9 bin.
- Each bin includes the lesser value but not the greater value, so 8 minutes would not go in the 7 to 8 bin.

Minutes Spent Solving a Puzzle

6. So far, we've seen histograms with a bin size of 1. Here is a dot plot with some more times students spent solving a puzzle.

Make a histogram with a bin size of 2.

Accelerated 6 | Unit 9 Lesson 3 836 Activity 2

Pairs | 10 min

Activity 3 Using Histograms

Purpose: Students use the Notice and Wonder routine to explore when a histogram is the most useful visualization of a data set and explore what questions a histogram can and cannot answer.

Short on time? Consider omitting Problem 7.

1 Launch

Direct student's attention to the data for time spent solving a puzzle for 50 students.

Encourage students to read their classmates' responses and decide if their reasoning is alike or different.

Consider asking, "Why would someone want to visualize this data?"

Use the Notice and Wonder routine to promote curiosity and help students make sense of the bin size used in each histogram.

2 Monitor

D **Differentiation: (Support)** Look for students who select any of the first three questions in Problem 9. Consider asking, "Where do you see the answer to that question on the histogram?"

A **Accessibility: Conceptual Processing** Invite students to talk aloud about their thinking to support self-monitoring of understanding.

3 Connect

Discuss each possible response. Invite students to share their reasoning for why a histogram can or cannot answer each question. **(MTR.4.1, MTR.6.1)**

To surface the Key Takeaway, consider asking:

- "How does the bin size change the shape of the graph?"
- "What do you think are the advantages or disadvantages of using a small bin size? Large bin size?"
- "What do you recognize about the relationship between the heights of the bars when the width of the bin changes?"

Activity 3

Name: ..... Date: ..... Period: .....

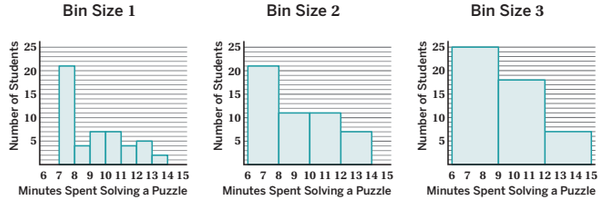
Using Histograms

7. Let's look at the time spent solving a puzzle for 50 students. Why might these data points be difficult to visualize as a dot plot?

Responses vary.

- There are a lot of data points. The number of dots on a dot plot could be overwhelming.
- Some of the values might be hard to put on a dot plot, like 8 minutes 56 seconds or 10 minutes 10 seconds.
- There might be dots that are too close together, like 8 minutes 56 seconds and 8 minutes 59 seconds.

8. Each of these histograms shows the time spent solving a puzzle for 50 students.



Discuss: What do you notice? What do you wonder?

Responses vary.

- I notice that the scale of both axes stays the same even when the bin size changes. The horizontal axis goes from 6 to 15, and the vertical axis goes from 0 to 25.
- I notice that when the bin size increases, the bin gets wider.
- I notice that if you add the heights of the smaller bins, you get the height of the larger bin.
- I wonder how you pick the best bin size.
- I wonder why someone would want a bin size of 3.
- I wonder if the histograms show 50 students no matter the bin size.

9. Different visualizations like dot plots and histograms help answer different questions.

a Which of these questions can a histogram help answer?

A. Which student solved the puzzle the fastest?

B. How long did it take the fastest student to solve the puzzle?

C. How many students solved the puzzle in 7 minutes 25 seconds?

D. How many students took 10 minutes or more to solve the puzzle?

b Answer the question you chose.

18 students

Accelerated 6 | Unit 9 Lesson 3 837 Activity 3

Key Takeaway: As the width of the size of the bin in a histogram is increased, the height of the bar may change.

Whole Class | 5 min

Synthesis

Purpose: Students synthesize their understanding of the relationship between histograms and dot plots.

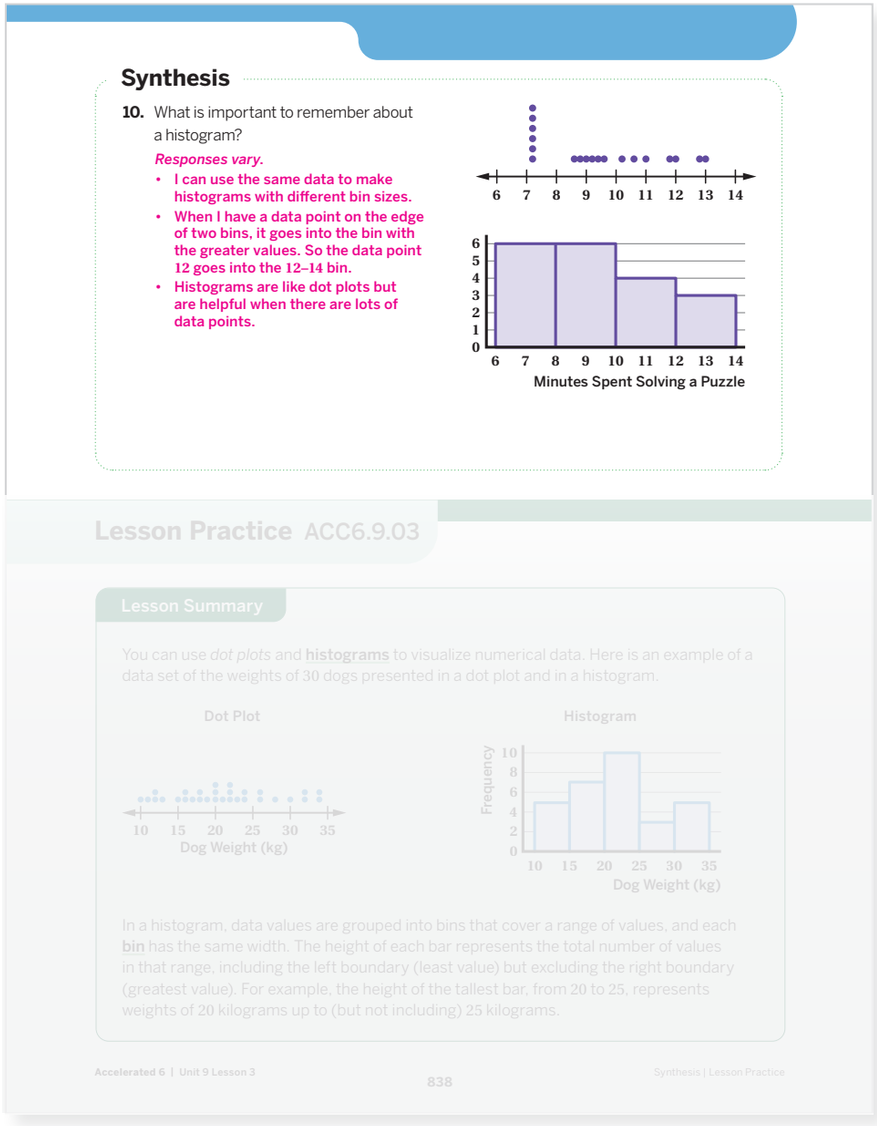
Invite students to respond independently, and then share their thinking with a partner.

Collect and share a variety of ideas, such as:

- Histograms are helpful to understand trends when there are lots of data points, but the individual data points are not shown.
- Key features of a histogram include bars touching, bin widths, etc.
- When a data point is on the edge of two bins, it goes into the bin with the greater values.

MLR **MLR2: Collect and Display** Use this routine to formalize the definitions of *histogram* and *bin* using students' language. Display any new language, examples, or diagrams for students to reference later.

Note: If time allows, invite students to revise their previous thinking based on the discussion.



Synthesis

10. What is important to remember about a histogram?

Responses vary.

- I can use the same data to make histograms with different bin sizes.
- When I have a data point on the edge of two bins, it goes into the bin with the greater values. So the data point 12 goes into the 12–14 bin.
- Histograms are like dot plots but are helpful when there are lots of data points.

Lesson Practice ACC6.9.03

Lesson Summary

You can use *dot plots* and histograms to visualize numerical data. Here is an example of a data set of the weights of 30 dogs presented in a dot plot and in a histogram.

Dot Plot

Histogram

In a histogram, data values are grouped into bins that cover a range of values, and each bin has the same width. The height of each bar represents the total number of values in that range, including the left boundary (least value) but excluding the right boundary (greatest value). For example, the height of the tallest bar, from 20 to 25, represents weights of 20 kilograms up to (but not including) 25 kilograms.

Accelerated 6 | Unit 9 Lesson 3 838 Synthesis | Lesson Practice

Lesson Takeaway: Histograms are another way of displaying numerical data by grouping values into **bins** of equal size. Different visualizations like *dot plots* and histograms help answer different questions.

Independent | 5 min

Show What You Know

Purpose: Students demonstrate their understanding by analyzing a histogram to determine the number of states with a minimum wage of less than $12.

Today's Goals

1. **Goal:** Describe and interpret a *histogram* that represents a data set.
2. **Language Goal:** Compare and contrast dot plots and *histograms* to visualize data. **(Writing, Speaking, and Listening)**

Show What You Know (Assessment Resources)

Name: ..... Date: ..... Period: .....

Show What You Know 9.03

This histogram shows the minimum wages of all 50 states in 2020.

How many states have a minimum hourly wage of at least $12?

8 states

Source: *United States Department of Labor*

D Differentiation Use after Lesson 3

Support

Provide targeted intervention for students by using these resources.

If students need support interpreting a histogram:

» **Respond to Student Thinking:**

- Revisit the **Lesson Summary** and ask questions that can be answered by interpreting the histogram.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite them to explore the **Sub-Unit 1 Extension Activities**.

Professional Learning

In the next lesson, students will describe and interpret histograms and dot plots. How might you prepare to invite students' experiences into this lesson?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

10. What is important to remember about a histogram?

Responses vary.

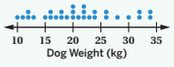
- I can use the same data to make histograms with different bin sizes.
- When I have a data point on the edge of two bins, it goes into the bin with the greater values. So the data point 12 goes into the 12–14 bin.
- Histograms are like dot plots but are helpful when there are lots of data points.

Lesson Practice ACC6.9.03

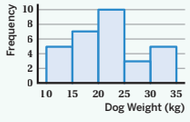
Lesson Summary

You can use *dot plots* and **histograms** to visualize numerical data. Here is an example of a data set of the weights of 30 dogs presented in a dot plot and in a histogram.

Dot Plot



Histogram

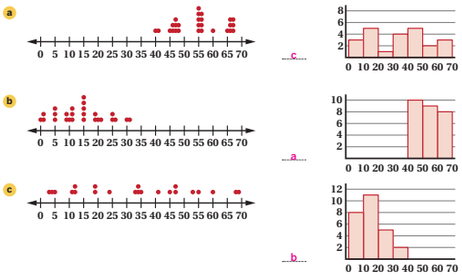


In a histogram, data values are grouped into bins that cover a range of values, and each **bin** has the same width. The height of each bar represents the total number of values in that range, including the left boundary (least value) but excluding the right boundary (greatest value). For example, the height of the tallest bar, from 20 to 25, represents weights of 20 kilograms up to (but not including) 25 kilograms.

Lesson Practice ACC6.9.03

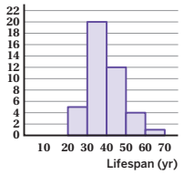
Name: ..... Date: ..... Period: .....

1. Match each histogram with the dot plot that represents the same data set.



2. Here is a histogram showing the lifespans of 42 chimpanzees that lived in the wild. How many chimpanzees lived at least 50 years and less than 70 years?

5 chimpanzees

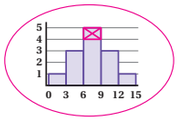


Problems 3–4: Here are three representations of the same data set.

List

2, 3, 3, 4, 6, 6, 6, 7, 9, 9, 11, 12

Histogram



Dot Plot

3. Circle the representation that doesn't match the other two.

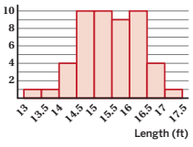
4. Revise your circled representation to make it match the other two representations.

Lesson Practice ACC6.9.03

Name: ..... Date: ..... Period: .....

FAST Practice

5. A marine biologist is studying a group of sharks. She made a histogram of the lengths of a group of adult sharks.



Select *all* of the false statements.

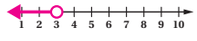
- ☑ **A.** A total of 9 sharks were measured.
- ☐ **B.** Two of the sharks are less than 14 feet long.
- ☐ **C.** A typical shark is about 14.5 to 16.5 feet long.
- ☑ **D.** The longest shark measured was 10 feet long.
- ☑ **E.** The smallest shark measured was 11 feet long.

Spiral Review

Problems 6–7: Graph each inequality on a number line.

6. $m > 6$

7. $3 > n$



Problems 8–11: Determine the value of each expression.

8. $3.727 + 1.384 = 5.111$

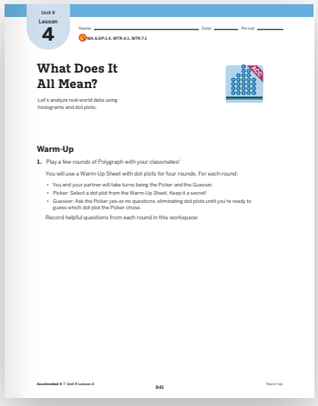
9. $3.727 - 1.384 = 2.343$

10. $5.01 \cdot 4.8 = 24.048$

11. $5.01 \div 4.8 = 1.04375$

Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1–2, 4	2	MA.6.DP.1.5
	3	1	MA.6.DP.1.5
FAST Practice	5	2	MA.6.DP.1.5
Spiral Review			
	6–7	2	MA.6.AR.1.2
	8–9	2	MA.5.NSO.2.3
	10–11	2	MA.6.NSO.2.1


Unit 9
Lesson
4
What Does It All Mean?
Let's analyze real-world data using histograms and dot plots.
Warm-Up
1. Play a few rounds of Polygraph with your classmates!
You will use a Warm-Up Sheet with dot plots for four rounds. For each round:
• You and your partner will take turns being the Picker and the Guesser.
• Picker: Select a dot plot from the Warm-Up Sheet. Keep it a secret!
• Guesser: Ask the Picker yes-or-no questions, eliminating dot plots until you're ready to guess which dot plot the Picker chose.
Record helpful questions from each round in this workspace:
841

What Does It All Mean?

Describing and Interpreting Histograms and Dot Plots

Let's analyze real-world data using histograms and dot plots.

Learning Outcomes

Today's Goals

1. **Goal:** Describe the *spread* and *shape* of a histogram or dot plot that represents a data set.
2. **Language Goal:** Interpret the *spread* and *shape* of a histogram or dot plot in a real-world context. **(Reading, Writing, Speaking, and Listening)**

Students use Polygraph to explore different ways of describing dot plots. They also describe and interpret sets of real-world data displayed as dot plots and histograms. They discuss **clusters** and **outliers** and analyze the **spreads** and **shapes** of the displays.

Coherence

Prior Learning

In Lessons 2–3, students used dot plots and histograms to visualize data sets.

Future Learning

In Lesson 5, students will work together to create their own histograms to represent and compare real-world data.

Connections

- Students are introduced to new vocabulary while describing characteristics of different data displays for problem situations. (mathematical, real world)

FL B.E.S.T. Math Benchmarks

Addressing

MA.6.DP.1.4
Given a histogram or line plot within a real-world context, qualitatively describe and interpret the spread and distribution of the data, including any symmetry, skewness, gaps, clusters, outliers and the range.

Mathematical Thinking and Reasoning Standards: MTR.4.1, MTR.7.1

ELA Expectations: ELA.K12.EE.4.1, ELA.K12.EE.5.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.5.DP.1.1	MA.7.DP.1.5

Vocabulary

New Vocabulary

cluster	outlier
range	shape
skewed	spread
symmetry (of a data distribution)	

Lesson at a Glance

~ 45 min

MA.6.DP.1.4

Warm-Up

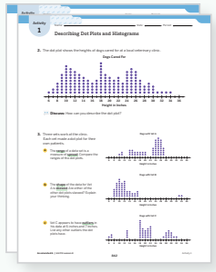
Pairs | 5 min

Students play the Polygraph game to practice describing dot plots using informal language.
MLR2: Collect and Display

Activity 1

Pairs | 20 min

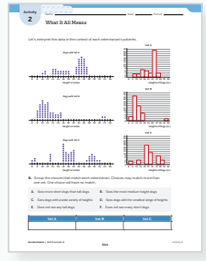
Students use the Think-Pair-Share routine to describe characteristics of dot plots and histograms, including **range**, **symmetry** and **skewness**, **clusters**, gaps, and **outliers**. **(MTR.4.1)**



Activity 2

Pairs | 10 min

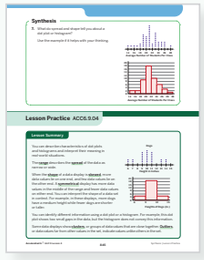
Students interpret the **shape** and **spread** of dot plots and histograms that represent heights of dogs. **(MTR.7.1)**



Synthesis

Whole Class | 5 min

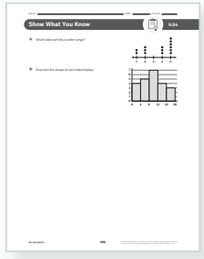
Students synthesize their understanding of the shape and spread of a dot plot and a histogram.
MLR2: Collect and Display



Show What You Know

Independent | 5 min

Students demonstrate their understanding by describing the spread and shape of a histogram and dot plot that represent a data set.



Prep Checklist

Prepare Warm-Up Sheets.

This lesson includes:

- Student Edition
- Show What You Know
- Warm-Up Sheets, two per pair.

Pairs | 5 min

Warm-Up

Purpose: Students play the Polygraph game to practice describing dot plots using informal language.

1 Launch

Encourage students to try to guess the card using as few questions as possible.

Distribute two copies of the Warm-Up Sheet to each pair. Invite one partner to be the Picker and the other to be the Guesser in the first round, then have them switch roles for the next round. For Rounds 3–4, invite them to trade partners.

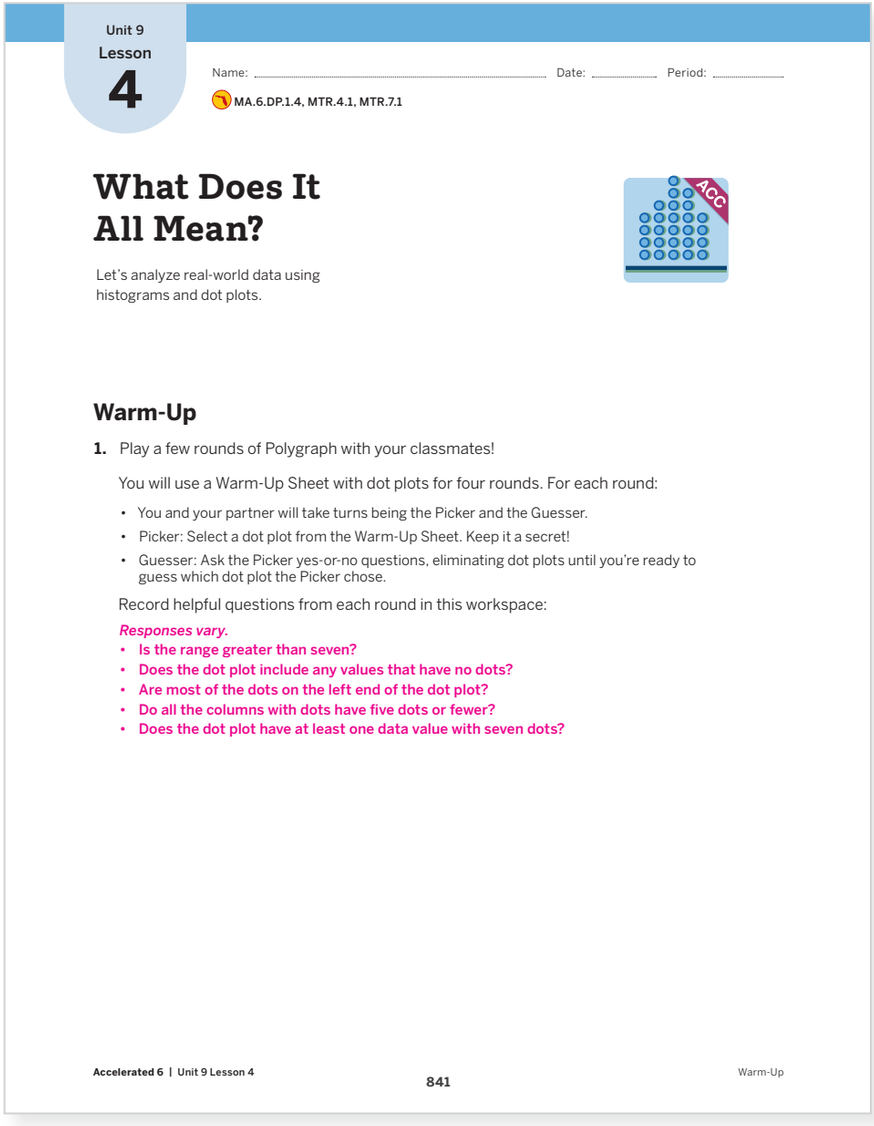
Look for interesting or effective questions, including ones about shape or spread.

2 Connect

Invite students to share key phrases that they found helpful when describing dot plots.

MLR **MLR2: Collect and Display** Use this routine to collect students' language as they discuss with a partner. Display for students to reference or add to for the rest of the lesson.

Note: Students will learn more precise language to describe features of dot plots in the next activity.



Unit 9
Lesson
4

Name: _____ Date: _____ Period: _____

MA.6.DP.1.4, MTR.4.1, MTR.7.1

What Does It All Mean?

Let's analyze real-world data using histograms and dot plots.

Warm-Up

1. Play a few rounds of Polygraph with your classmates!

   You will use a Warm-Up Sheet with dot plots for four rounds. For each round:

   - You and your partner will take turns being the Picker and the Guesser.
   - Picker: Select a dot plot from the Warm-Up Sheet. Keep it a secret!
   - Guesser: Ask the Picker yes-or-no questions, eliminating dot plots until you're ready to guess which dot plot the Picker chose.

   Record helpful questions from each round in this workspace:

   Responses vary.
   - Is the range greater than seven?
   - Does the dot plot include any values that have no dots?
   - Are most of the dots on the left end of the dot plot?
   - Do all the columns with dots have five dots or fewer?
   - Does the dot plot have at least one data value with seven dots?

Accelerated 6 | Unit 9 Lesson 4 841 Warm-Up

Pairs | 20 min

Activity 1 Describing Dot Plots and Histograms

Purpose: Students use the Think-Pair-Share routine to describe characteristics of dot plots and histograms, including range, symmetry and skewness, clusters, gaps, and outliers. **(MTR.4.1)**

1 Launch

Direct students' attention to the dot plots.

Use the Think-Pair-Share routine to support student-to-student discourse about describing the shape and spread of dot plots. Encourage students to think independently and then take turns sharing with a partner. Invite several students to share their or their partner's thinking with the class. **(MTR.4.1)**

Note: Students may not realize that three smaller dot plots include the same information as the larger dot plot, only separated by vet. Consider sharing this information with students.

2 Monitor

To surface connections, consider discussing common meanings of the terms *range*, *spread*, *shape*, and *skewed* and having students discover that the term *outlier* is made up of the words *out* and *lie*.

ELL **English Language Learners** Invite students to research how to say the new terms in other languages. Create an anchor chart with the terms in multiple languages, along with examples. **(Reading and Listening)**

Listen to partner discussions for student ideas to highlight during the Connect.

A **Accessibility: Visual-Spatial Processing** Throughout this activity, invite students to pick a specific point on a dot plot or a specific bin and bar on a histogram and describe what that item means to support them in describing visual representations.

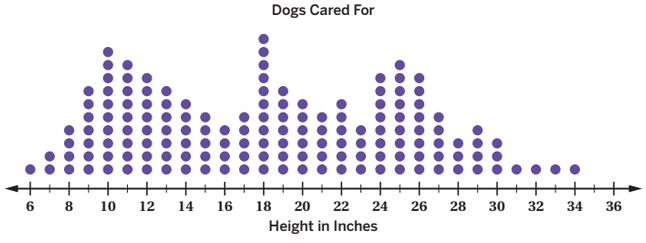
Activity 1 continued >

Activity 1

Name: Date: Period:

Describing Dot Plots and Histograms

2. The dot plot shows the heights of dogs cared for at a local veterinary clinic.



Discuss: How can you describe the dot plot?

Responses vary. There are very few dots at the right end. There are lots of dots near 10 and 11, near 18, and from 24 to 26. There are no dots before 6 or after 34.

3. Three vets work at the clinic. Each vet made a dot plot for their own patients.

a. The range of a data set is a measure of spread. Compare the ranges of the dot plots.

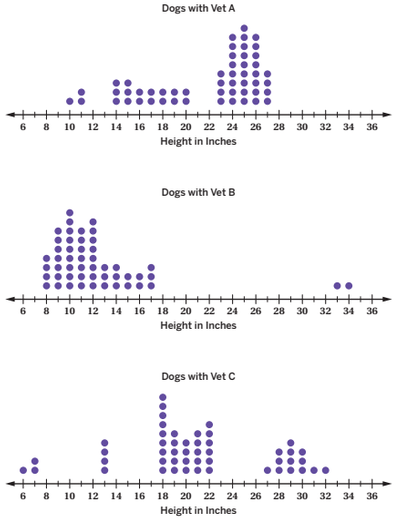
Responses vary. The data for Vet A is less spread out so it has a narrower range than the ranges for Vets B and C.

b. The shape of the data for Vet A is skewed. Are either of the other dot plots skewed? Explain your thinking.

The data for Vet B is skewed. *Explanations vary.* There are fewer data points on the right side than on the left side of the dot plot.

c. Vet C appears to have outliers in his data at 6 inches and 7 inches. List any other outliers the dot plots have.

Responses vary. The data for Vet B has outliers at 33 inches and 34 inches.



Accelerated 6 | Unit 9 Lesson 4 842 Activity 1

Pairs | 20 min

Activity 1 Describing Dot Plots and Histograms (continued)

Purpose: Students use the Think-Pair-Share routine to describe characteristics of dot plots and histograms, including **range**, **symmetry** and **skewness**, **clusters**, gaps, and **outliers**. **(MTR.4.1)**

2 Monitor

Invite students to think independently before discussing their ideas with a partner. **(MTR.4.1)**

To surface prior knowledge, consider asking, "What do you recall about symmetry in shapes and drawings?"

Listen for students who use the new vocabulary while comparing the histograms and connect these terms to the context. Invite these students to share their thinking during the Connect.

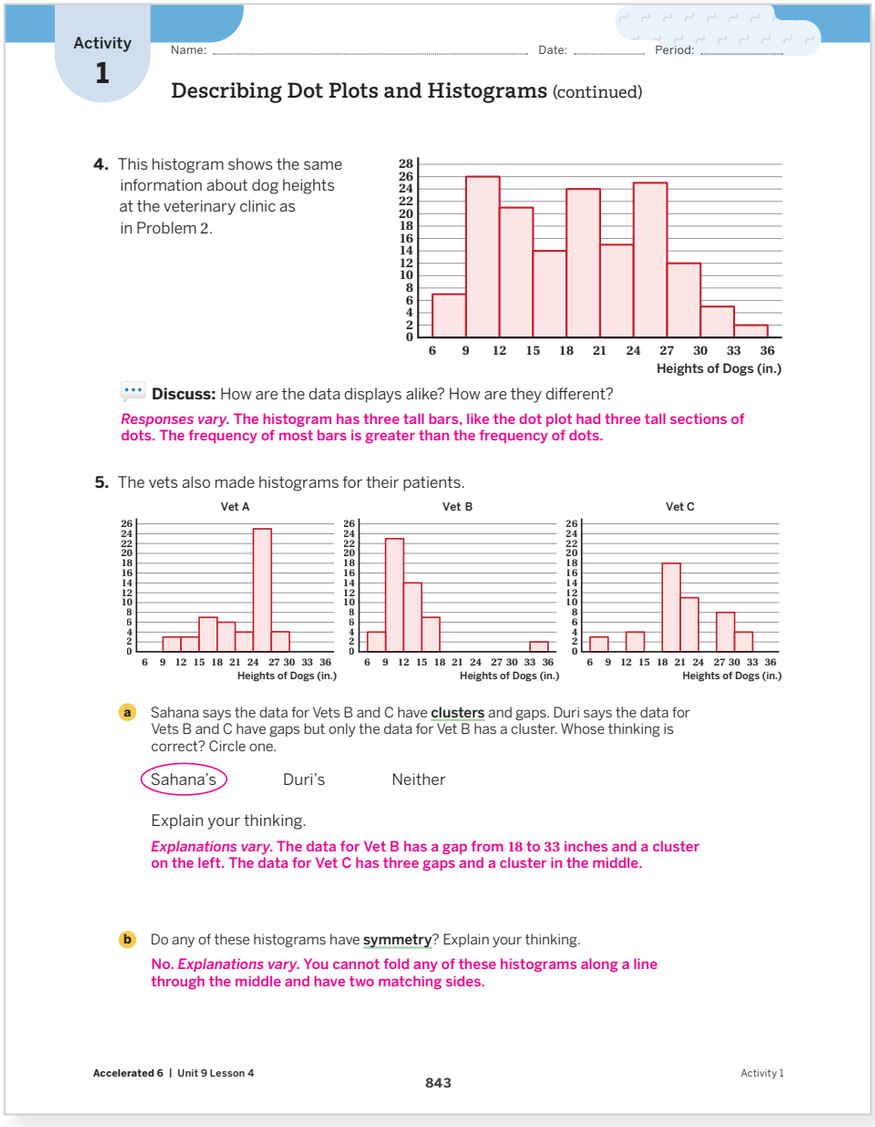
3 Connect

Invite students to share their thinking for analyzing and comparing the dot plots and histograms.

Create a class definition for the terms **range**, **spread**, **shape**, **skewed**, **outlier**, **cluster**, and **symmetry** using students' language and the glossary.

To surface the Key Takeaway, consider asking:

- "How does a graph with symmetry differ from a skewed graph?"
- "How can you identify a cluster, gap, or outlier in a dot plot or histogram?



Activity 1

Name: Date: Period:

Describing Dot Plots and Histograms (continued)

4. This histogram shows the same information about dog heights at the veterinary clinic as in Problem 2.

Discuss: How are the data displays alike? How are they different?

Responses vary. The histogram has three tall bars, like the dot plot had three tall sections of dots. The frequency of most bars is greater than the frequency of dots.

5. The vets also made histograms for their patients.

a. Sahana says the data for Vets B and C have **clusters** and gaps. Duri says the data for Vets B and C have gaps but only the data for Vet B has a cluster. Whose thinking is correct? Circle one.

(Sahana's) Duri's Neither

Explain your thinking.

Explanations vary. The data for Vet B has a gap from 18 to 33 inches and a cluster on the left. The data for Vet C has three gaps and a cluster in the middle.

b. Do any of these histograms have **symmetry**? Explain your thinking.

No. *Explanations vary.* You cannot fold any of these histograms along a line through the middle and have two matching sides.

Accelerated 6 | Unit 9 Lesson 4 843 Activity 1

Key Takeaway: You can use the **range**, **skewness** or **symmetry**, **clusters**, gaps, and **outliers** to describe dot plots and histograms.

Pairs | 10 min

Activity 2 What It All Means

Purpose: Students interpret the **shape** and **spread** of dot plots and histograms that represent heights of dogs. **(MTR.7.1)**

Short on time? Consider completing Problem 6 together as a class.

1 Launch

Direct students' attention to the dot plots and histograms. Invite a student to read aloud the problem text for the sort.

To generate excitement, invite students to share types of dogs they know and estimations of their heights.

To surface connections, invite students to create statistical questions about the data with a partner. **(MTR.7.1)**

2 Monitor

Encourage students to justify their thinking for each match.

D Differentiation

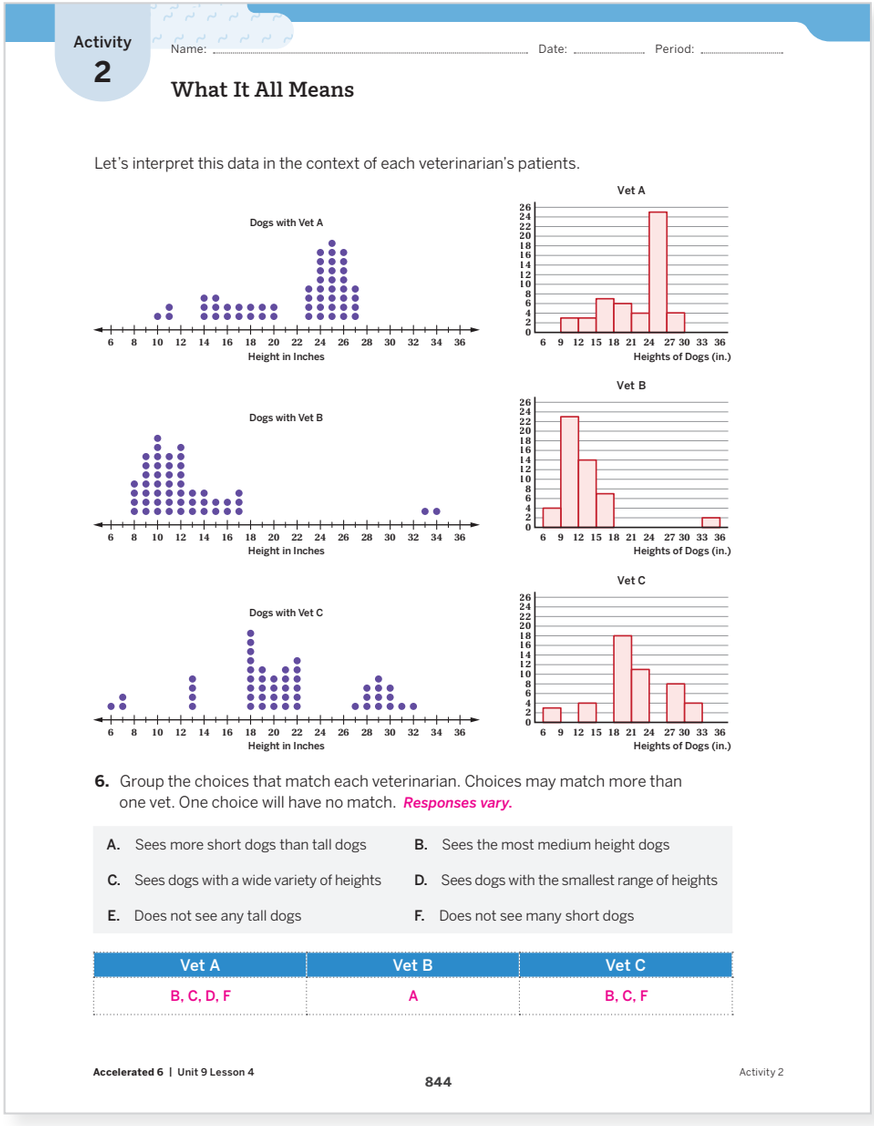
Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Encourage students to focus on one vet's data set at a time and choose the data display they are most comfortable with to analyze.
Think that Vet B sees the most medium height dogs or sees dogs with the smallest range of heights.	**Support** Consider asking: • "Where is the middle of the horizontal axis?" • "Where is the smallest data value and where is the greatest data value for each vet?"
Do not include Choice E in their matches.	**Strengthen** Invite students to explain their thinking during the Connect.
Use a variety of strategies to determine each match.	**Stretch** Invite these students to write a new interpretation of the data in context and invite them to share their interpretations during the Connect. **(MTR.7.1)**

Listen to partner discussions for student ideas to highlight during the Connect.

3 Connect

Invite students to share their responses to the problems, highlighting unique and creative thinking.

To surface the Key Takeaway, consider asking, "How can you use a dot plot or a histogram to explain what data means in a real-world situation?" **(MTR.7.1)**



Activity 2

Name: Date: Period:

What It All Means

Let's interpret this data in the context of each veterinarian's patients.

6. Group the choices that match each veterinarian. Choices may match more than one vet. One choice will have no match. ***Responses vary.***

A. Sees more short dogs than tall dogs
B. Sees the most medium height dogs
C. Sees dogs with a wide variety of heights
D. Sees dogs with the smallest range of heights
E. Does not see any tall dogs
F. Does not see many short dogs

Vet A	Vet B	Vet C
B, C, D, F	A	B, C, F

Accelerated 6 | Unit 9 Lesson 4 844 Activity 2

Key Takeaway: You can use **shape** and **spread** to interpret real-world situations represented in a dot plot or a histogram.

Whole Class | 5 min

Synthesis

Purpose: Students synthesize their understanding of the shape and spread of a dot plot and a histogram.

Invite students to respond independently, and then share their thinking with a partner.

Collect and share a variety of ideas, such as:

- Range measures the spread of the data.
- A data display can be skewed or symmetrical.
- Some data may have gaps, clusters, or outliers.
- The spread and shape can help you understand what the data means.

MLR **MLR2: Collect and Display** Use this routine to formalize the definitions of *cluster*, *outlier*, *range*, *shape*, *skewed*, *spread*, and *symmetry* (*of a data distribution*) using students' language. Display any new language, examples, or diagrams publicly for students to reference later.

Note: If time allows, invite students to share with a partner what makes sense about the answer and what questions they still have.

Synthesis

7. What do spread and shape tell you about a dot plot or histogram?

Use the example if it helps with your thinking.

Responses vary. The spread tells how wide the range of data is. This data shows a wide range of students per class. The shape tells where most of the data lies. This data is fairly symmetrical, showing more students per class in the middle of the range and fewer students per class on either end of the range.

Lesson Practice ACC6.9.04

Lesson Summary

You can describe characteristics of dot plots and histograms and interpret their meaning in real-world situations.

The **range** describes the **spread** of the data as narrow or wide.

When the **shape** of a data display is **skewed**, more data values lie on one end, and few data values lie on the other end. A **symmetrical** display has more data values in the middle of the range and fewer data values on either end. You can interpret the shape of a data set in context. For example, in these displays, more dogs have a medium height while fewer dogs are shorter or taller.

You can identify different information using a dot plot or a histogram. For example, this dot plot shows two small gaps in the data, but the histogram does not convey this information.

Some data displays show **clusters**, or groups of data values that are close together. **Outliers**, or data values far from other values in the set, indicate values unlike others in the set.

Accelerated 6 | Unit 9 Lesson 4 845 Synthesis | Lesson Practice

Lesson Takeaway: A dot plot and a histogram are useful for helping to visualize a data set. You can describe the **shape** and **spread** and interpret their meanings in context. A dot plot and a histogram can reveal different information about the same data set.

Independent | 5 min

Show What You Know

Purpose: Students demonstrate their understanding by describing the spread and shape of a histogram and dot plot that represent a data set.

Today's Goals

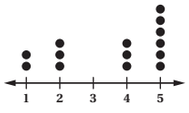
1. **Goal:** Describe the *spread* and *shape* of a histogram or dot plot that represents a data set.
2. **Language Goal:** Interpret the *spread* and *shape* of a histogram or dot plot in a real-world context. **(Reading, Writing, Speaking, and Listening)**

Show What You Know (Assessment Resources)

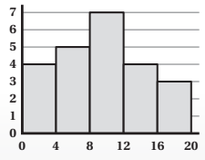
Name: _____ Date: _____ Period: _____

Show What You Know 9.04

a. Which data set has a wider range?
Data Set B



b. Describe the shape of each data display.
Responses vary.
Data set A is skewed left.
Data set B has a roughly symmetrical shape.



D Differentiation Use after Lesson 4

Support

Provide targeted intervention for students by using these resources.

If students need support with creating or describing dot plots:

» **Respond to Student Thinking:**

- Use the ***Representing Data on a Line Plot and Solving Problems*** **Mini-Lesson**.
- Use the ***Creating Dot Plots*** **Mini-lesson**.
- Review the **Lesson Summary**.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite them to explore the **Sub-Unit 1 Extension Activities**.

Professional Learning

Consider the next lessons you will teach. How will students' work in Activity 2 today prepare them for analyzing measures of center and spread in Lessons 6–11? How might you call upon students' work from this lesson?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

7. What do spread and shape tell you about a dot plot or histogram?

Use the example if it helps with your thinking.

Responses vary. The spread tells how wide the range of data is. This data shows a wide range of students per class. The shape tells where most of the data lies. This data is fairly symmetrical, showing more students per class in the middle of the range and fewer students per class on either end of the range.

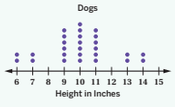
Lesson Practice ACC6.9.04

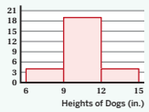
Lesson Summary

You can describe characteristics of dot plots and histograms and interpret their meaning in real-world situations.

The **range** describes the **spread** of the data as narrow or wide.

When the **shape** of a data display is **skewed**, more data values lie on one end, and few data values lie on the other end. A **symmetrical** display has more data values in the middle of the range and fewer data values on either end. You can interpret the shape of a data set in context. For example, in these displays, more dogs have a medium height while fewer dogs are shorter or taller.





You can identify different information using a dot plot or a histogram. For example, this dot plot shows two small gaps in the data, but the histogram does not convey this information.

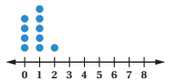
Some data displays show **clusters**, or groups of data values that are close together. **Outliers**, or data values far from other values in the set, indicate values unlike others in the set.

Lesson Practice ACC6.9.04

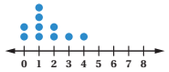
Name: ..... Date: ..... Period: .....

1. Which dot plot has the widest range?

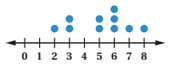
A.



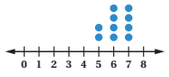
B.



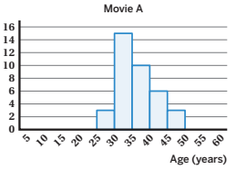
C.

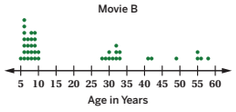


D.



Problems 2–5: These data displays represent ages of people who watched two different movies at a movie theater.





2. How are the data displays alike?

Responses vary.
- Both displays compare the ages of people at a movie.
- They both show more data on the left than on the right of each range.

3. How are they different?

Responses vary.
- Movie A has a medium spread. Movie B has a large spread.
- Movie B has many gaps while Movie A does not appear to have gaps.

4. How does the typical age for Movie A compare to people watching Movie B?

Responses vary. The typical age for Movie A is a much narrower range than for Movie B. More older people watched Movie A than Movie B.

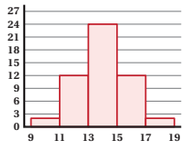
5. What kind of movie do you think Movie B is? Explain your thinking.

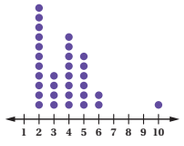
Responses vary. Movie B could be a family movie because there are a lot of kids and some older family members that probably took them to the movies.

Lesson Practice ACC6.9.04

Name: ..... Date: ..... Period: .....

Problems 6–8: Here are two displays of different data sets.





6. Describe the shape of each data display. Explain your thinking.

Responses vary. The histogram is symmetrical because it has a peak in the middle and the left and right sides have fewer data values. The dot plot is skewed because the distribution decreases from left to right.

7. Which data display could represent the age in months that babies take their first steps? Explain your thinking.

Responses vary. The histogram could represent the ages in months that babies take their first steps. The dot plot has many data values on the left, beginning at 2, but babies are probably too young to start walking that early. The histogram shows that most babies take their first steps later.

FAST Practice

8. Describe the data display that shows an outlier. Select **ONE** correct answer in each box.

The [**A.** histogram / (**B.**) dot plot] includes an outlier. The data value is [(**A.**) far from / **B.** grouped with] other data values in the display.

Spiral Review

9. Select *all* of the numbers that are greater than 5.

☐ **A.** -9 ☐ **B.** 3 ☐ **C.** -6 ☑ **D.** |-9| ☑ **E.** 7

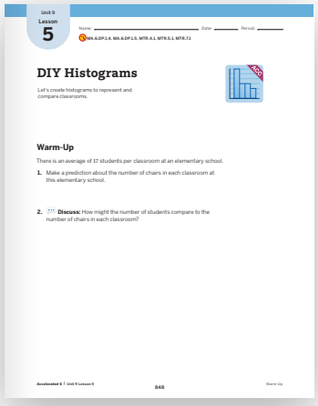
Problems 10–11: Determine the value of each expression.

10. $6.05 \cdot 2.5 = 15.125$

11. $6.05 \div 2.5 = 2.42$

Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1	1	MA.6.DP.1.4
	2–6	2	MA.6.DP.1.4
	7	3	MA.6.DP.1.4
FAST Practice	8	1	MA.6.DP.1.4
Spiral Review			
	9	1	MA.6.NSO.1.1, MA.6.NSO.1.3
	10–11	2	MA.6.NSO.2.1


Unit 9
Lesson 5
DIY Histograms
Let's create histograms to represent and compare classrooms.
Warm-Up
There is an average of 17 students per classroom at an elementary school.
1. Make a prediction about the number of chairs in each classroom at this elementary school.
2. Discuss: How might the number of students compare to the number of chairs in each classroom?

DIY Histograms

Creating Histograms

Let's create histograms to represent and compare classrooms.

Learning Outcomes

Today's Goals

1. **Goal:** Create a histogram to visualize data.
2. **Language Goal:** Compare and contrast data sets using histograms, including making statements about the shape, center, and spread. **(Writing, Speaking, and Listening)**

Students practice creating histograms by hand and use histograms to compare data sets. Students compare the number of students with the number of chairs in elementary and high school classrooms. **(MTR.5.1, MTR.7.1)**

Coherence

Prior Learning

In Lesson 3, students were introduced to histograms. They compared data sets using dot plots and histograms.

Future Learning

In Lesson 6, students will be introduced to the mean of a data set as the amount each member of a set would get if everything is distributed equally.

Connections

- Students construct displays to represent data from various situations. (mathematical, real world)

FL B.E.S.T. Math Benchmarks

Addressing

MA.6.DP.1.4
Given a histogram or line plot within a real-world context, qualitatively describe and interpret the spread and distribution of the data, including any symmetry, skewness, gaps, clusters, outliers and the range.

MA.6.DP.1.5
Create box plots and histograms to represent sets of numerical data within real-world contexts.

Mathematical Thinking and Reasoning Standards: MTR.4.1, MTR.5.1, MTR.7.1

ELA Expectation: ELA.K12.EE.1.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.5.DP.1.1	MA.7.DP.1.5

Vocabulary

New Vocabulary

center

Review Vocabulary

frequency table

Lesson at a Glance

~ 45 min

MA.6.DP.1.5

Warm-Up

Whole Class | 5 min

Students are introduced to the context of comparing the number of students to the number of chairs in classrooms. **(MTR.7.1)**

Activity 1

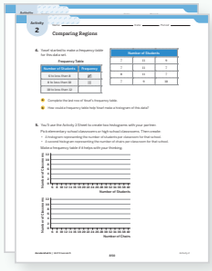
Pairs | 10 min

Students interpret histograms displaying the number of students and the number of chairs in classrooms in a school district.
Routine: Notice and Wonder **(MTR.7.1)**

Activity 2

Pairs | 20 min

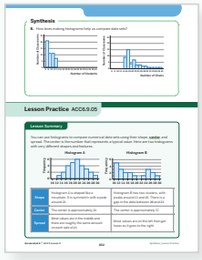
Students create histograms to compare the number of students and the number of chairs in elementary and high school classrooms.
(MTR.5.1, MTR.7.1)



Synthesis

Whole Class | 5 min

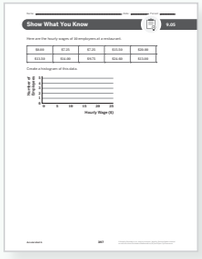
Students synthesize their understanding of how histograms are helpful when comparing data sets MLR2: Collect and Display.



Show What You Know

Independent | 5 min

Students demonstrate their understanding by creating a histogram when given 10 hourly wages for employees at a restaurant.



Prep Checklist

Prepare Activity 2 Sheets.

This lesson includes:

- Student Edition
- Show What You Know
- Straightedges
- Activity 2 Sheets, one per student

Whole Class | 5 min

Warm-Up

Purpose: Students are introduced to the context of comparing the number of students to the number of chairs in classrooms. **(MTR.7.1)**

1 Launch

Direct students' attention to the problem text and consider inviting a student to read the prompt and first question out loud to the class.

To generate excitement, consider having students count the number of students and the number of chairs in the classroom.

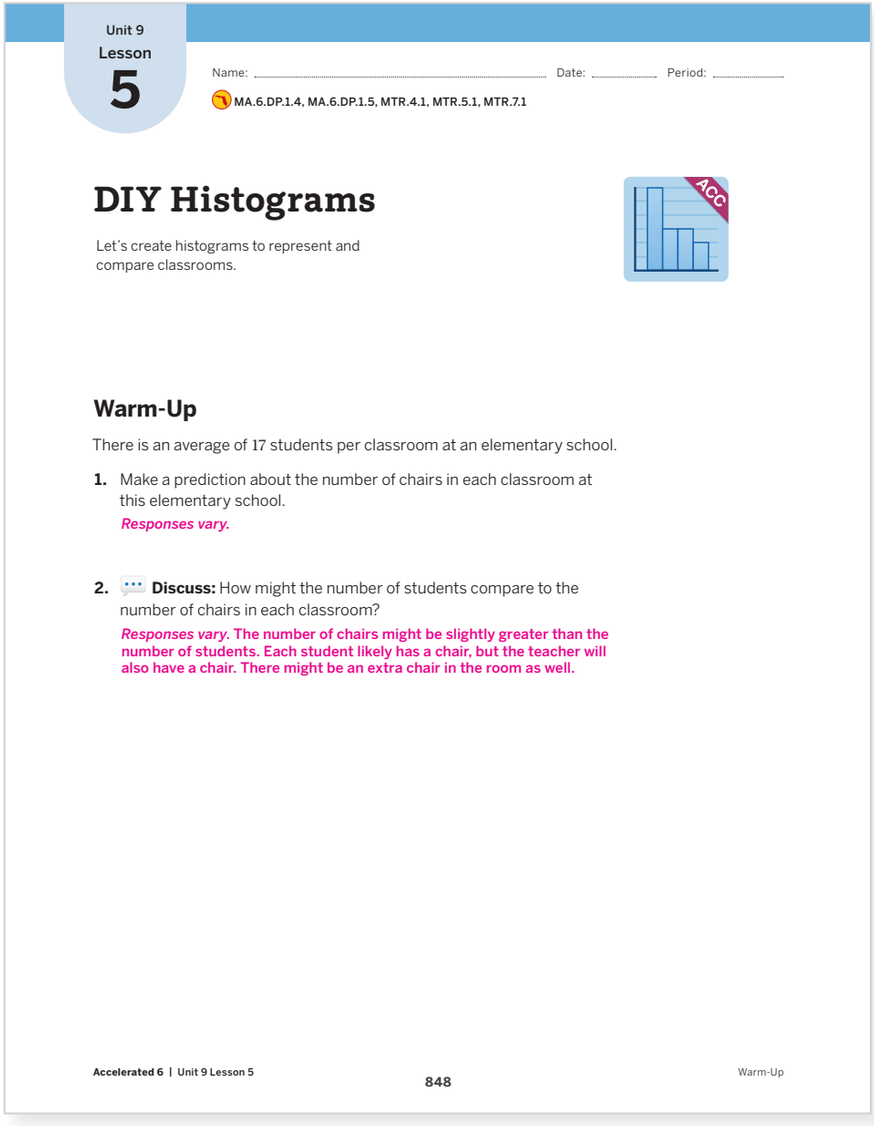
Invite students to discuss their thinking with a partner. **(MTR.4.1)**

2 Connect

Invite students to share their thinking about how the number of students and number of chairs in a classroom compare, calling attention to any conflict or consensus. **(MTR.7.1)**

Consider asking:

- "Why do you think some classrooms may need more chairs?"
- "Do you think these values would change if these were middle school classrooms? High school classrooms?"



Unit 9
Lesson
5

Name: _____ Date: _____ Period: _____

MA.6.DP.1.4, MA.6.DP.1.5, MTR.4.1, MTR.5.1, MTR.7.1

DIY Histograms

Let's create histograms to represent and compare classrooms.

ACC

Warm-Up

There is an average of 17 students per classroom at an elementary school.

1. Make a prediction about the number of chairs in each classroom at this elementary school.

   Responses vary.

2. **Discuss:** How might the number of students compare to the number of chairs in each classroom?

   Responses vary. The number of chairs might be slightly greater than the number of students. Each student likely has a chair, but the teacher will also have a chair. There might be an extra chair in the room as well.

Accelerated 6 | Unit 9 Lesson 5 848 Warm-Up

Pairs | 10 min

Activity 1 Out of Reach

Purpose: Students interpret histograms displaying the number of students and the number of chairs in classrooms in a school district. **(MTR.7.1)**

1 Launch

Direct students' attention to the histograms of classroom data.

Use the Notice and Wonder routine to promote curiosity and help students make sense of the histograms. **(MTR.4.1)**

Responses vary.
- I notice that the number of students data has a smaller spread than the other data.
- I notice that 21 classrooms have between 6 and 8 students.
- I wonder why the data for chairs is more spread out than the data for students.
- I wonder why the bins for number of students and number of chairs do not overlap.

Consider asking, "What does the first bin on the histogram tell us about the number of students in classrooms?" **(MTR.7.1)**

2 Monitor

Listen for students who describe **center** as the middle of a data set.

D Differentiation

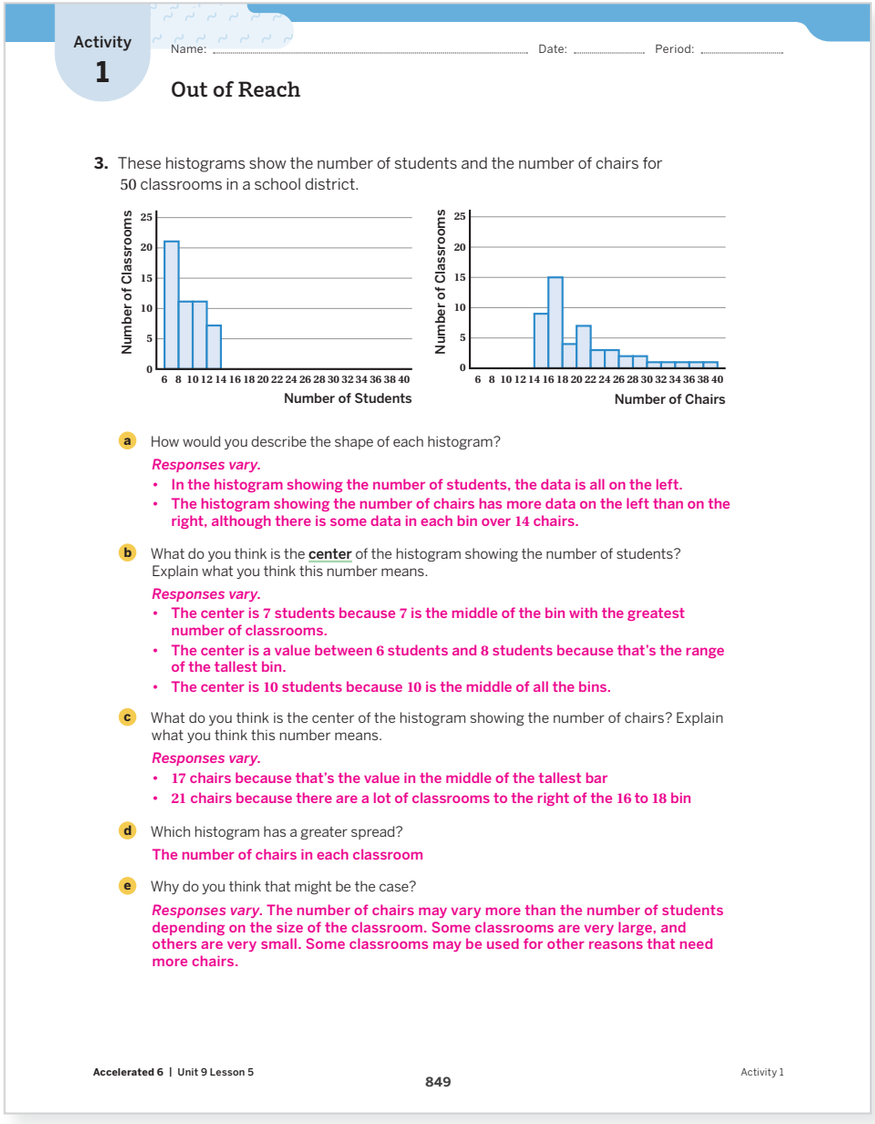
Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Invite them to read the class definitions for *shape* and *spread* that were created in Lesson 4.
Use mathematical language to describe the center, shape, and spread of the histograms.	**Strengthen** Invite these students to share their thinking during the discussion in the Connect.
Use the features of each histogram to justify their reasoning. **(MTR.5.1)**	
Respond to each prompt with precision, including thorough explanations.	**Stretch** Consider asking, "How would the histogram be different if we were looking at the percentage of classrooms instead of the number of classrooms?"

A Accessibility: Affective Functioning To support students in sharing their thinking, ask students in advance if they, or a classmate can share their thinking during the whole-class discussion.

3 Connect

Create a class definition for the term *center* using students' language and the glossary.

To surface the Key Takeaway, invite students who used mathematical language to share their descriptions and explain their thinking.



Activity 1

Name: Date: Period:

Out of Reach

3. These histograms show the number of students and the number of chairs for 50 classrooms in a school district.

a. How would you describe the shape of each histogram?

Responses vary.
- In the histogram showing the number of students, the data is all on the left.
- The histogram showing the number of chairs has more data on the left than on the right, although there is some data in each bin over 14 chairs.

b. What do you think is the **center** of the histogram showing the number of students? Explain what you think this number means.

Responses vary.
- The center is 7 students because 7 is the middle of the bin with the greatest number of classrooms.
- The center is a value between 6 students and 8 students because that's the range of the tallest bin.
- The center is 10 students because 10 is the middle of all the bins.

c. What do you think is the center of the histogram showing the number of chairs? Explain what you think this number means.

Responses vary.
- 17 chairs because that's the value in the middle of the tallest bar
- 21 chairs because there are a lot of classrooms to the right of the 16 to 18 bin

d. Which histogram has a greater spread?

The number of chairs in each classroom

e. Why do you think that might be the case?

Responses vary. The number of chairs may vary more than the number of students depending on the size of the classroom. Some classrooms are very large, and others are very small. Some classrooms may be used for other reasons that need more chairs.

Accelerated 6 | Unit 9 Lesson 5 849 Activity 1

Key Takeaway: Histograms can be described by their center, shape, and spread — just like dot plots.

Pairs | 20 min

Activity 2 Comparing Regions

Purpose: Students create histograms to compare the number of students and the number of chairs in elementary and high school classrooms. **(MTR.5.1, MTR.7.1)**

Short on time? Consider providing the solutions to the last row of Yosef's frequency table.

1 Launch

Direct students' attention to Yosef's *frequency table*.

To surface prior knowledge, invite students to share their definition of a frequency table.

Note: Students used frequency tables in a previous course but might need support getting started. Consider asking, "What do the tick marks next to each column represent?"

Distribute the Activity 2 Sheets and straightedges. Encourage students to use their straightedge to create precise bins for their histograms.

2 Monitor

Invite pairs of students to decide whether they will each select one set of data and individually create a histogram to compare, or work together to create the histograms.

D Differentiation

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Encourage them to refer to the histograms in Activity 1 and Yosef's strategy for using a frequency table.
Create a frequency table that skips values (e.g. 6–8, 9–11 instead of 6–8, 8–10, 10–12).	**Support** Select a value that is not currently represented in their table and ask, "Where would you place this value?"
Place values that are on the edge of two bins in the correct bin.	**Strengthen** Invite these students to share their strategy during the Connect and how they know which bin size to use.
	Stretch Invite them to compare their histogram with other students in the class who analyzed the same school.

Look for strategies for deciding the bin size to share during the Connect.

Activity 2 continued >

Activity 2

Name: Date: Period:

Comparing Regions

4. Yosef started to make a *frequency table* for this data set.

Frequency Table

Number of Students	Frequency
6 to less than 8	𝍸
8 to less than 10	|||
10 to less than 12	||||

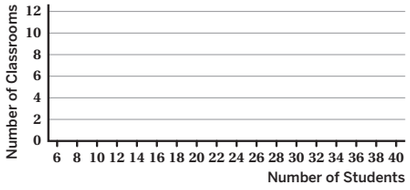
Number of Students		
7	11	9
7	11	7
8	11	7
7	9	10

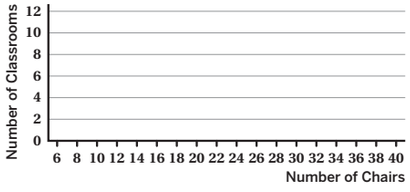
a. Complete the last row of Yosef's frequency table. **Response shown in table.**

b. How could a frequency table help Yosef make a histogram of this data?
Responses vary. **A frequency table can help Yosef organize how many data points belong in each bin.**

5. You'll use the Activity 2 Sheet to create two histograms with your partner.
Pick elementary school classrooms or high school classrooms. Then create:
- A histogram representing the number of students per classroom for that school.
- A second histogram representing the number of chairs per classroom for that school.

Make a frequency table if it helps with your thinking.





Responses vary. **Sample histograms shown on the Activity 2 Sheet (answers).**

Accelerated 6 | Unit 9 Lesson 5 850 Activity 2

Pairs | 20 min

Activity 2 Comparing Regions (continued)

Purpose: Students create histograms to compare the number of students and the number of chairs in elementary and high school classrooms. **(MTR.5.1, MTR.7.1)**

2 Monitor

Listen for students who compare their histograms using features such as center, shape, and spread and connect these features to the context. Invite these students to share their thinking during the Connect. **(MTR.5.1)**

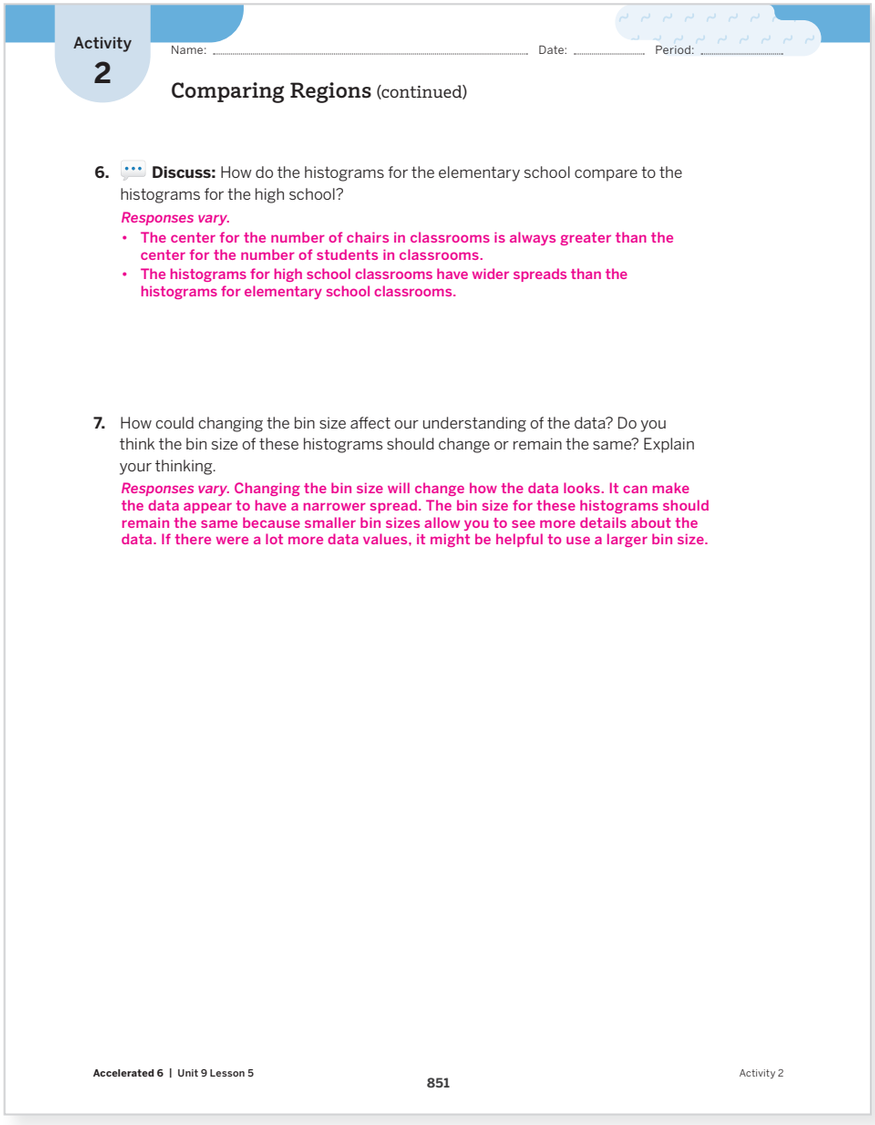
ELL **English Language Learners** Invite students to share responses to the discussion questions in their primary languages. Encourage all students to use gestures and/or visuals as they share. **(Speaking and Listening)**

3 Connect

Invite students to share their strategies for creating their histograms as well their noticings about how their histograms on classroom data compared. Highlight students who used different bin sizes to aid in the discussion. **(MTR.4.1, MTR.7.1)**

To surface the Key Takeaway, consider asking:

- "What was challenging in creating a histogram with this data?"
- "How did you decide which bin size to use in your histogram?"
- "How was looking at the histogram more helpful than looking at the data in a list? How was it less helpful?"



Activity 2

Name: Date: Period:

Comparing Regions (continued)

6. **Discuss:** How do the histograms for the elementary school compare to the histograms for the high school?

Responses vary.

- The center for the number of chairs in classrooms is always greater than the center for the number of students in classrooms.
- The histograms for high school classrooms have wider spreads than the histograms for elementary school classrooms.

7. How could changing the bin size affect our understanding of the data? Do you think the bin size of these histograms should change or remain the same? Explain your thinking.

Responses vary. Changing the bin size will change how the data looks. It can make the data appear to have a narrower spread. The bin size for these histograms should remain the same because smaller bin sizes allow you to see more details about the data. If there were a lot more data values, it might be helpful to use a larger bin size.

Accelerated 6 | Unit 9 Lesson 5 851 Activity 2

Key Takeaway: When creating a histogram, it is helpful to organize the information beforehand. *Frequency tables* are helpful in organizing the data and determining the height of each bin.

Whole Class | 5 min

Synthesis

Purpose: Students synthesize their understanding of how histograms are helpful when comparing data sets.

Invite students to respond independently, and then share their thinking with a partner.

Collect and share a variety of ideas, such as:
- How histograms help us see the shape of the data.
- Histograms are a useful tool in comparing multiple data sets, as long as the bin size is the same.

MLR **MLR2: Collect and Display** Use this routine to formalize the definition of *center* using students' language. Display any new language, examples, or diagrams for students to reference later.

Note: If time allows, invite students to revise their previous thinking based on the discussion.

Synthesis

8. How does making histograms help us compare data sets?

Responses vary. Histograms help us see the shape of the data more clearly. Visualizing the data helps us compare the data because we can tell where similar data is grouped together.

Lesson Practice ACC6.9.05

Lesson Summary

You can use histograms to compare numerical data sets using their shape, center, and spread. The center is the number that represents a typical value. Here are two histograms with very different shapes and features.

	Histogram A	Histogram B
Shape	Histogram A is shaped like a mountain. It is symmetric with a peak around 21.	Histogram B has two clusters, with peaks around 11 and 25. There is a gap in the data between 20 and 22.
Center	The center is approximately 21.	The center is approximately 17.
Spread	Most values are in the middle and there are roughly the same amount on each side of 21.	Most values are on the left then get lower as it goes to the right.

Accelerated 6 | Unit 9 Lesson 5 852 Synthesis | Lesson Practice

Lesson Takeaway: Histograms are a useful model for analyzing and comparing larger numerical sets of data.

Independent | 5 min

Show What You Know

Purpose: Students demonstrate their understanding by creating a histogram when given 10 hourly wages for employees at a restaurant.

Today's Goals

1. **Goal:** Create a histogram to visualize data.
2. **Language Goal:** Compare and contrast data sets using histograms, including making statements about the shape, center, and spread. **(Writing, Speaking, and Listening)**

Show What You Know (Assessment Resources)

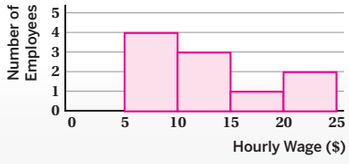
Name: _____ Date: _____ Period: _____

Show What You Know 9.05

Here are the hourly wages of 10 employees at a restaurant.

$8.00	$7.25	$7.25	$15.50	$20.00
$13.50	$14.00	$9.75	$24.60	$13.00

Create a histogram of this data.



Differentiation Use after Lesson 5

Support

Provide targeted intervention for students by using these resources.

If students need support comparing data sets represented by histograms:

» **Respond to Student Thinking:**

- Review the **Lesson Summary**.
- Select a few related **Practice Problems** and give individualized feedback.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite them to explore the **Sub-Unit 1 Extension Activities**.

Professional Learning

In Lesson 4, students developed their first definitions of shape and spread. How have their definitions of these two terms changed or expanded over the last two lessons?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

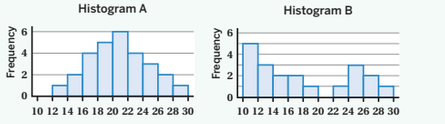
8. How does making histograms help us compare data sets?

Responses vary. Histograms help us see the shape of the data more clearly. Visualizing the data helps us compare the data because we can tell where similar data is grouped together.

Lesson Practice ACC6.9.05

Lesson Summary

You can use histograms to compare numerical data sets using their shape, **center**, and spread. The center is the number that represents a typical value. Here are two histograms with very different shapes and features.



	Histogram A	Histogram B
Shape	Histogram A is shaped like a mountain. It is symmetric with a peak around 21.	Histogram B has two clusters, with peaks around 11 and 25. There is a gap in the data between 20 and 22.
Center	The center is approximately 21.	The center is approximately 17.
Spread	Most values are in the middle and there are roughly the same amount on each side of 21.	Most values are on the left then get lower as it goes to the right.

Lesson Practice
ACC6.9.05

Name: _____ Date: _____ Period: _____

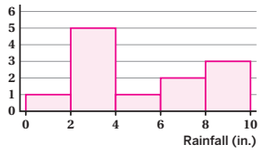
Problems 1–2: Here is the average amount of rainfall, in inches, for each month last year in Vicente's hometown.

Average Rainfall (in.)					
1.67	2	2.99	6.1	8.9	9.84
3.14	5.35	9.69	6	3.27	2.05

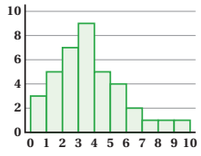
1. Complete the frequency table.

Rainfall (in.)	0 to less than 2	2 to less than 4	4 to less than 6	6 to less than 8	8 to less than 10
Frequency	1	5	1	2	3

2. Create a histogram using the data about Vicente's hometown.



Problems 3–6: Here is a histogram.



3. Which of these situations could this histogram represent?

A. The height of a group of sixth-grade students (in inches).
B. The weight of students' backpacks (in pounds).
C. The temperature for 38 days in summer (in degrees).
D. The time it takes a group of students to walk to school (in hours).

Explain your thinking.

Explanations vary. The histogram shows the numbers 1 to 10. B is the only situation where those numbers would be reasonable because backpacks can weigh between 1 to 10 pounds.

4. Describe the center of the histogram.

Descriptions vary. The center would be between 2 and 5.

5. Describe the shape of the histogram.

Descriptions vary. The shape has one tall bar in the middle and only a little data on the right.

6. Describe the spread of the histogram.

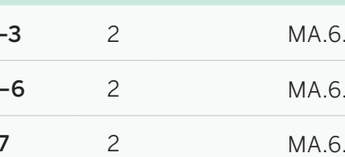
Descriptions vary. The data is spread out with lots of data between 3 and 4.

Lesson Practice
ACC6.9.05

Name: _____ Date: _____ Period: _____

FAST Practice

7. Which of these situations could this histogram represent?



A. The weight of cars (in pounds)
B. The height of trees in a forest (in feet)
C. The time it takes to drive from a city in Florida to a city in Maine (in hours)
D. The amount of water needed each week for a houseplant (in ounces)

Spiral Review

8. Order these numbers from *least* to *greatest*.

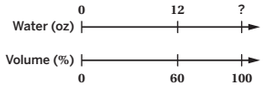
-6 0.6 0.16 6 -1.6

-6	-1.6	0.16	0.6	6

Least Greatest

9. Rebecca drank 12 ounces of water from her water bottle, which is 60% of the volume of the bottle. How much water can the bottle hold when full? Use the double number line if it helps with your thinking.

20 ounces



10. The point (-2, 3) is one vertex of a square on a coordinate plane. Name three points that could be the other vertices.

Responses vary. (2, 3), (2, -1), (-2, -1)

Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1–3	2	MA.6.DP.1.5
	4–6	2	MA.6.DP.1.4
FAST Practice	7	2	MA.6.DP.1.5
Spiral Review			
	8	2	MA.6.NSO.1.1
	9	2	MA.6.AR.3.4
	10	3	MA.6.GR.1.3

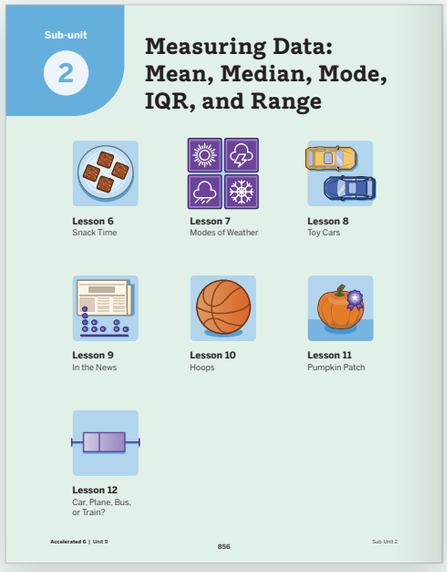
Sub-Unit 2

Measuring Data: Mean, Median, Mode, IQR, and Range

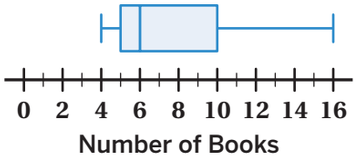
Sub-Unit 2 Goals

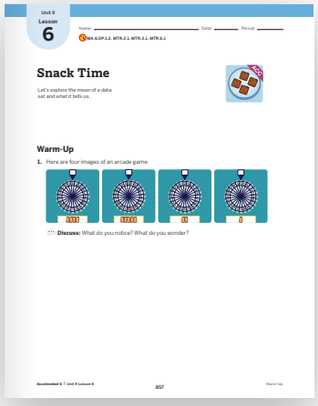
- **Goal:** Calculate the mean, median, and mode of a data set.
- **Goal:** Calculate the quartiles, interquartile range (IQR), and range of a data set.
- **Goal:** Create and use box plots to make claims about data.
- **Language Goal:** Compare and contrast the mean and median as measures of center.
- **Language Goal:** Interpret the median and IQR in the context of a data set.

In this sub-unit, students are introduced to three ways to measure center (**mean**, **median**, and **mode**) and explore two measures of spread (**interquartile range (IQR)** and **range**). They create **box plots** to visualize data and identify how changes in data values affect measures of center and spread.



Math That Matters Most

Concept or Skill	For example . . .
Describe measures of center of a data set.	For 12 days, Mai recorded how many minutes long her bus rides to school were. Here are the times she recorded. 9, 12, 6, 9, 10, 7, 6, 12, 9, 8, 10, 10 Determine the mean, median, and mode for the data and describe what each measure represents. $\frac{108}{12}$, or 9, is the mean. Mai's bus rides were 9 minutes long on average. The median length of time is 9 minutes because 9 is the middle value of the data set in order from least to greatest. The data set has two modes, 9 and 10. Most often, it took Mai 9 or 10 minutes to get to school.
Justify whether the mean or median best describes a given data set.	Ricardo got the following scores on 5 class assignments: 87, 90, 0, 95, 100. Which measure of center do you think is better for describing Ricardo's scores? Median. Ricardo did really well on 4 of his 5 assignments, so reporting the median makes more sense. Mean. Otherwise the assignment that Ricardo did badly on won't really be included.
Analyze the effects on the spread of a data set when removing a data value.	What is the range of this data set? How would removing a data value affect the range? The range of the data is 16 – 4, or 12, books. If the data value is the least or greatest of the data set, 4 or 16 books, it may decrease the range. If it is another value within the data set, the range will stay the same. 



Snack Time

Introduction to the Mean

Let's explore the mean of a data set and what it tells us.

Learning Outcomes

Today's Goals

1. **Goal:** Calculate the *mean* of a data set.
2. **Language Goal:** Describe the *mean* for a data set as a measure of center that summarizes the data using a single number. **(Reading and Writing)**

Students are introduced to the **mean** of a data set as the amount each member of the set gets if everything is distributed equally. Students determine the mean by calculating the (arithmetic) average of the data set and explore the mean as a **statistic** that summarizes the data set using a single number. Students also make a connection between the mean and the idea of "typical," or the center of a distribution, and make sense of why it is called a measure of center for a distribution.

Coherence

Prior Learning

In Lesson 5, students created and analyzed histograms and learned about center as a way to describe a data set.

Future Learning

In Lessons 7–8, students will be introduced to two more measures of center, the mode and the median.

Connections

- Students are introduced to the mean as it applies to problems in context. (mathematical, real world)

FL B.E.S.T. Math Benchmarks

Addressing

MA.6.DP.1.2
Given a numerical data set within a real-world context, find and interpret mean, median, mode and range.

Mathematical Thinking and Reasoning Standards: MTR.2.1, MTR.3.1, MTR.6.1

ELA Expectations: ELA.K12.EE.1.1, ELA.K12.EE.5.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.5.DP.1.2	**MA.7.DP.1.1**
	MA.7.DP.1.2

Vocabulary

New Vocabulary

mean (average) statistic

Additional Vocabulary

measure of center

Lesson at a Glance

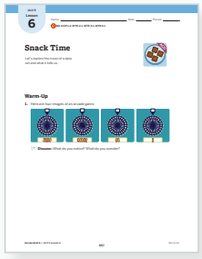
~ 45 min

MA.6.DP.1.2

Warm-Up

Pairs | 5 min

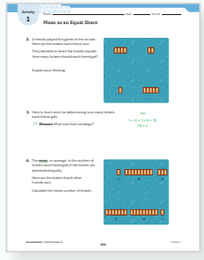
Students use the **Notice and Wonder** routine to observe how an arcade game works to introduce them to the context they will explore in the next activity.



Activity 1

Pairs | 8 min

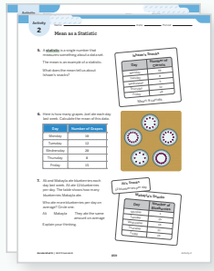
Students are introduced to the concept of **mean** as an average, in terms of the equal distribution of tickets among friends.



Activity 2

Pairs | 15 min

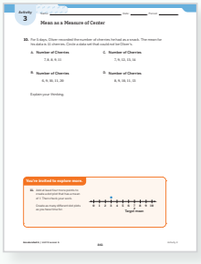
Students consider what the mean, as a **statistic**, says about the data set and review strategies for calculating the mean. **MLR3: Critique, Correct, Clarify**



Activity 3

Pairs | 7 min

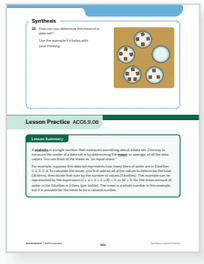
Students determine which data set could not have a mean of 11 to develop an understanding of the mean as a measure of center.



Synthesis

Whole Class | 5 min

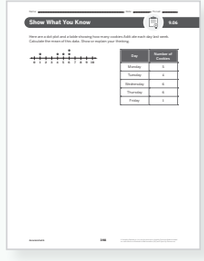
Students synthesize their understanding of mean as a statistic and a measure of center and how it is used to summarize a data set. **MLR2: Collect and Display**



Show What You Know

Independent | 5 min

Students demonstrate their understanding by calculating the mean of a data set.



Prep Checklist

This lesson includes:

- Student Edition
- Show What You Know

- Calculators
- Chips, counters, or paper clips (as needed)

Pairs | 5 min

Warm-Up

Purpose: Students use the Notice and Wonder routine to observe how an arcade game works to introduce them to the context they will explore in the next activity.

1 Launch

Direct students' attention to the four images of an arcade game.

Use the Notice and Wonder routine to promote curiosity and help students make sense of the arcade game.

Consider asking, "Have you seen or played games like this before? If so, where?"

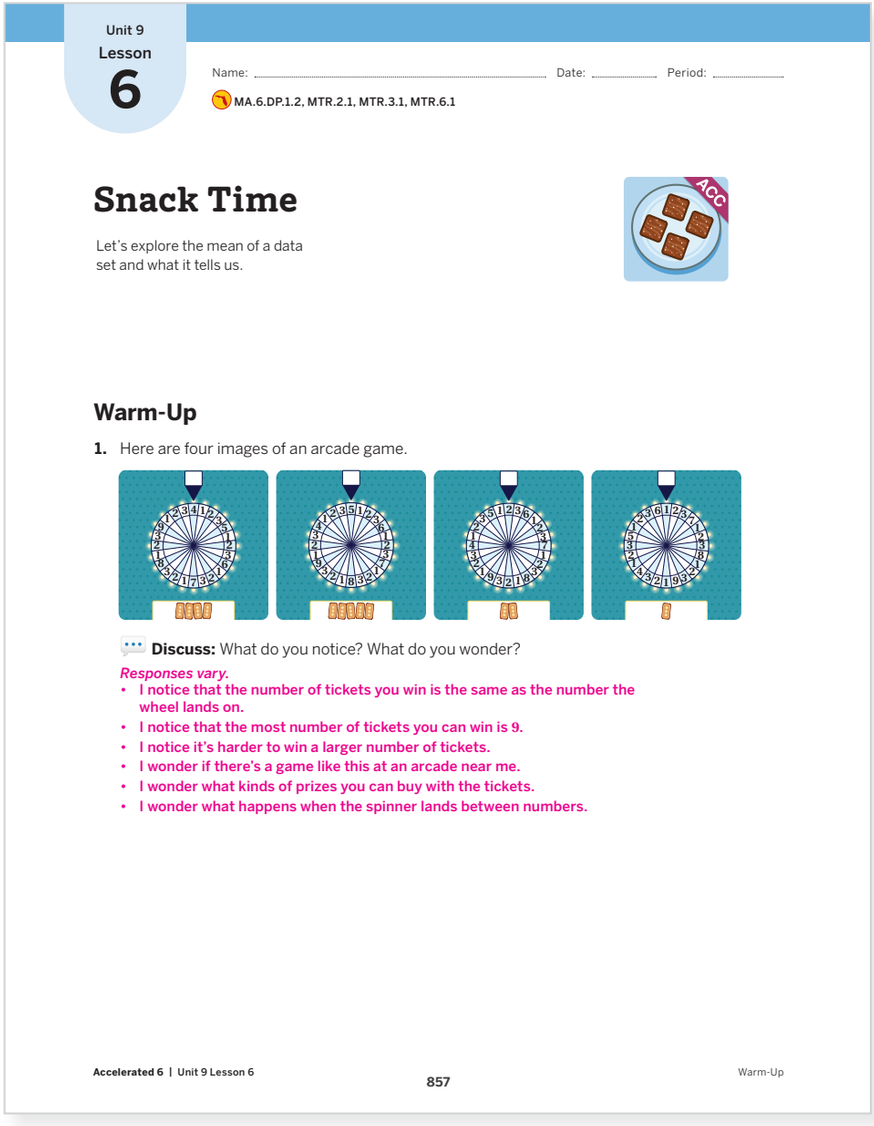
ELL **English Language Learners** Invite students to share what they know about arcade games and tickets to increase access to the task. **(Reading and Listening)**

2 Connect

Invite students to share what they noticed and wondered about the arcade game.

Consider asking, "Are you more likely to win 1 ticket or 5 tickets? Why?"

To generate excitement, consider sharing that students will consider the tickets that several friends won from the arcade game in the next activity.



Unit 9
Lesson
6

Name: _____ Date: _____ Period: _____

MA.6.DP.1.2, MTR.2.1, MTR.3.1, MTR.6.1

Snack Time

Let's explore the mean of a data set and what it tells us.

ACC

Warm-Up

1. Here are four images of an arcade game.

Discuss: What do you notice? What do you wonder?

Responses vary.

- I notice that the number of tickets you win is the same as the number the wheel lands on.
- I notice that the most number of tickets you can win is 9.
- I notice it's harder to win a larger number of tickets.
- I wonder if there's a game like this at an arcade near me.
- I wonder what kinds of prizes you can buy with the tickets.
- I wonder what happens when the spinner lands between numbers.

Accelerated 6 | Unit 9 Lesson 6 857 Warm-Up

Pairs | 8 min

Activity 1 Mean as an Equal Share

Purpose: Students are introduced to the concept of **mean** as an average, in terms of the equal distribution of tickets among friends.

1 Launch

Direct students' attention to the story about four friends and consider reading it aloud as class.

Consider asking, "Why might the friends have decided to share the tickets equally?"

2 Monitor

Look for students who show their thinking in different ways, including those who use arithmetic and those who sketch to show how they would share the tickets equally. **(MTR.2.1)**

A **Accessibility: Conceptual Processing** Use objects like chips, paper clips, or counters to support students' understanding of the mean as an equal share. Encourage them to use these objects to represent the number of tickets each friend has and to physically redistribute them to help them visualize the process.

Listen for students who explain why Ava added the tickets then divided by 4.

Look for students' strategies for calculating the mean number of tickets to share during the Connect.

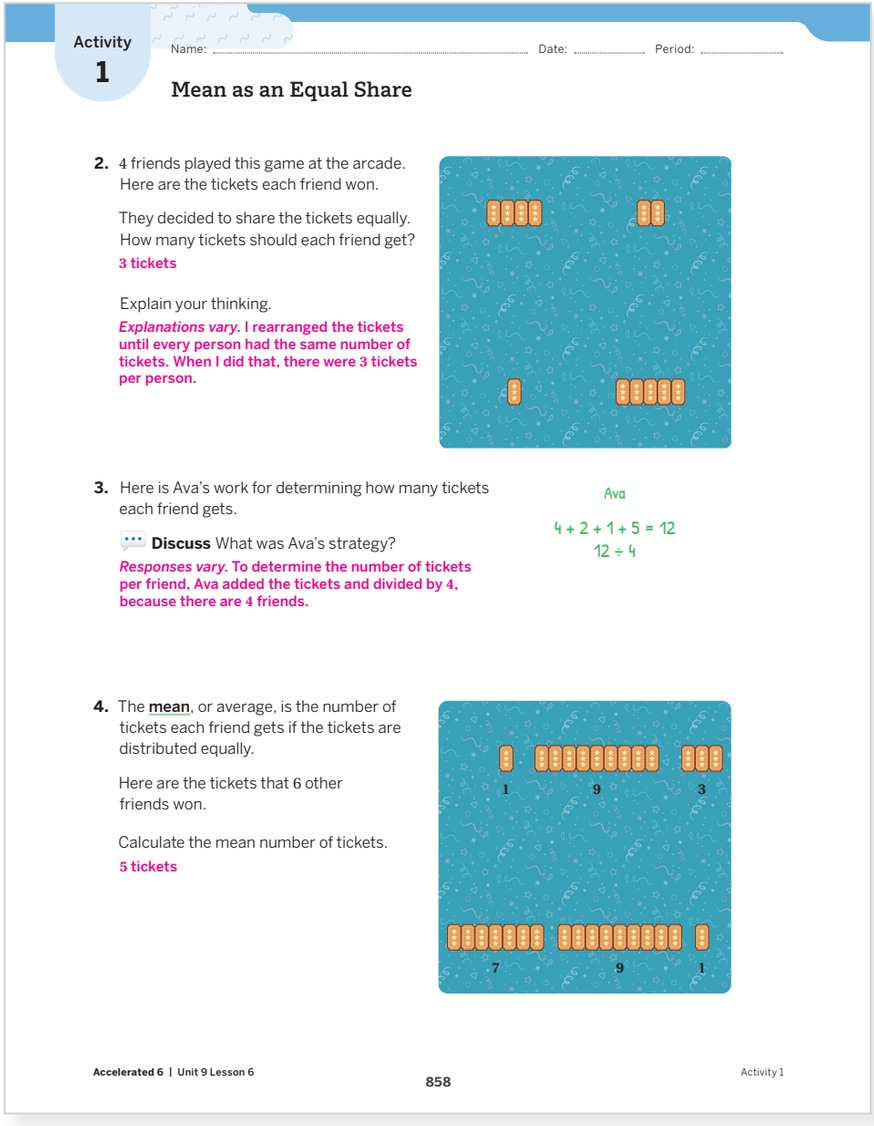
3 Connect

Discuss Ava's strategy.

Consider asking, "Why did Ava add all the tickets? Why did Ava divide by 4?"

Share student strategies for calculating the mean. Invite students to describe a general strategy for determining the mean no matter how many tickets or players there are.

Create a class definition for the term **mean (average)** using students' language and the glossary. Consider creating a visual display with a definition of mean (average) and at least one example for students to refer to throughout this lesson and unit.



Activity 1

Name: Date: Period:

Mean as an Equal Share

2. 4 friends played this game at the arcade. Here are the tickets each friend won.

They decided to share the tickets equally. How many tickets should each friend get?

3 tickets

Explain your thinking.

Explanations vary. I rearranged the tickets until every person had the same number of tickets. When I did that, there were 3 tickets per person.

3. Here is Ava's work for determining how many tickets each friend gets.

Ava

4 + 2 + 1 + 5 = 12
12 ÷ 4

Discuss What was Ava's strategy?

Responses vary. To determine the number of tickets per friend, Ava added the tickets and divided by 4, because there are 4 friends.

4. The **mean**, or average, is the number of tickets each friend gets if the tickets are distributed equally.

Here are the tickets that 6 other friends won.

Calculate the mean number of tickets.

5 tickets

Accelerated 6 | Unit 9 Lesson 6 858 Activity 1

Pairs | 15 min

Activity 2 Mean as a Statistic

Purpose: Students consider what the mean, as a **statistic**, says about the data set and review strategies for calculating the mean.

Short on time? Consider omitting Problem 6.

1 Launch

To surface prior knowledge, consider inviting students to share where they have heard the word *statistic* or *statistics* before.

Direct students' attention to the definition of **statistic** and read aloud as a class.

Consider asking, "How is the data in this problem similar to and different from the data we looked at before?"

2 Monitor

Consider asking, "In Activity 1, what did the mean tell us?"

To support students getting started, consider asking, "Which plate do you think represents Monday?"

Encourage students to use a calculator.

To support students getting started, consider asking, "How could we determine how many blueberries Makayla ate per day on average?"

ELL **English Language Learners** Consider asking, "What do you think *on average* means? How might we use what we know about the mean to help us here?" Clarify that the average and the mean are the same thing.

Activity 2 continued >

Activity 2

Name: Date: Period:

Mean as a Statistic

5. A **statistic** is a single number that measures something about a data set.

The mean is an example of a statistic.

What does the mean tell us about Ishaan's snacks?

Responses vary.

- The mean tells us that if Ishaan evened out the number of carrots he ate per day, he would have 9 carrots per day.
- The mean tells us that Ishaan ate an average of 9 carrots per day during the week he recorded his data.

Ishaan's Snacks

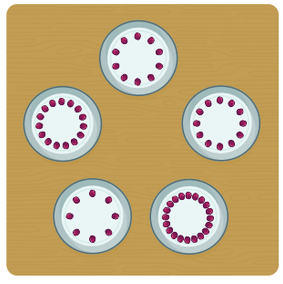
Day	Number of Carrots
Monday	10
Tuesday	8
Wednesday	5
Thursday	12
Friday	10

Mean: 9 carrots

6. Here is how many grapes Joel ate each day last week. Calculate the mean of this data.

Day	Number of Grapes
Monday	10
Tuesday	12
Wednesday	20
Thursday	8
Friday	15

13 grapes



7. Ali and Makayla ate blueberries each day last week. Ali ate 12 blueberries per day. The table shows how many blueberries Makayla ate.

Who ate more blueberries per day on average? Circle one.

(Ali) Makayla They ate the same amount on average

Explain your thinking.

Explanations vary. The mean of Makayla's data is $\frac{1+15+13+11+15}{5} = 11$. That means Makayla ate 11 blueberries per day on average. Because Ali ate 12 blueberries per day, she ate more.

Ali's Snacks
12 blueberries per day

Makayla's Snacks

Day	Number of Blueberries
Monday	1
Tuesday	15
Wednesday	13
Thursday	11
Friday	15

Accelerated 6 | Unit 9 Lesson 6 859 Activity 2

Pairs | 15 min

Activity 2 Mean as a Statistic (continued)

Purpose: Students consider what the mean, as a **statistic**, says about the data set and review strategies for calculating the mean.

2 Monitor

Select and sequence students' strategies for calculating the mean to share during the Connect.

D Differentiation

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Encourage students to use the image to help their thinking. Consider asking, "Why are there 5 plates?"
Think that the mean is 4 because they did not include 0 as a data value when calculating the mean.	**Support** Consider asking, "If the mean is 4 brownies, then how many total brownies would Omari eat in a week?"
Use strategies like: • Sketching a redistribution of brownies on each plate. • Using arithmetic to calculate the mean. **(MTR.2.1)**	**Strengthen** Celebrate and invite students to share during the Connect.

Listen for students who mention that Omari did not include the data for Tuesday in his calculation and needs to divide his sum by 5 instead of 4. **(MTR.3.1)**

3 Connect

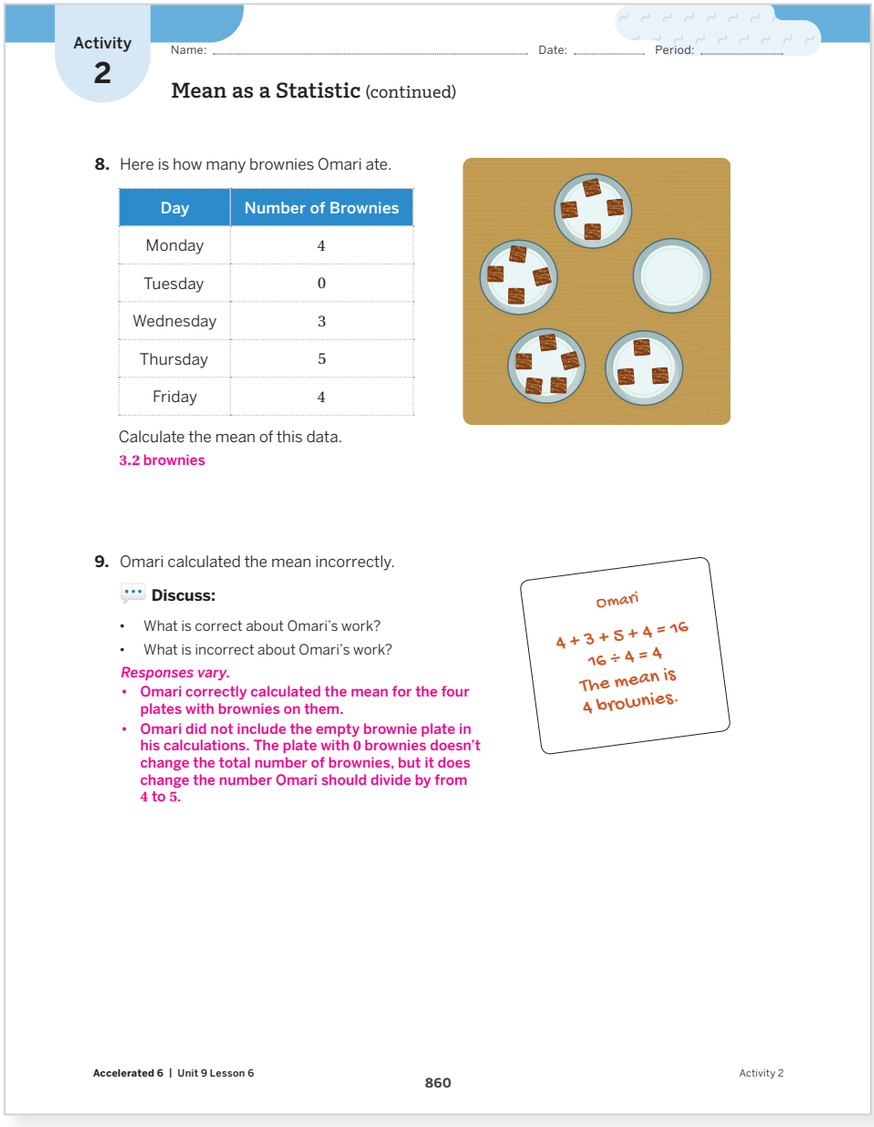
Invite students to share who ate more blueberries per day on average by polling the class. Discuss their strategies for comparing Ali's and Makayla's data.

Share that it is possible for some values in a data set to be equal to the mean, highlighting the number of blueberries Makayla ate on Thursday.

MLR MLR3: Critique, Correct, Clarify Use this routine to help students communicate about strengths, errors, and ambiguities in Omari's work. Invite students to share their responses from the activity. See the Routine Facilitation Guide for more information. **(MTR.6.1)**

Consider asking:

- "Why do you think Omari divided by 4?"
- "Is it possible for the mean to be a decimal value even when all the values are whole numbers?" **(MTR.3.1)**



Activity 2

Name: Date: Period:

Mean as a Statistic (continued)

8. Here is how many brownies Omari ate.

Day	Number of Brownies
Monday	4
Tuesday	0
Wednesday	3
Thursday	5
Friday	4

Calculate the mean of this data.

3.2 brownies

9. Omari calculated the mean incorrectly.

Discuss:

- What is correct about Omari's work?
- What is incorrect about Omari's work?

Responses vary.

- Omari correctly calculated the mean for the four plates with brownies on them.
- Omari did not include the empty brownie plate in his calculations. The plate with 0 brownies doesn't change the total number of brownies, but it does change the number Omari should divide by from 4 to 5.

Accelerated 6 | Unit 9 Lesson 6 860 Activity 2

Key Takeaway: The **mean** can be thought of as an equal redistribution, so that each value in the data set has the same frequency.

Pairs | 7 min

Activity 3 Mean as a Measure of Center

Purpose: Students determine which data set could not have a mean of 11 to develop an understanding of the mean as a measure of center.

1 Launch

Direct students' attention to the story about Oliver and consider reading it aloud as class.

2 Monitor

A **Accessibility: Executive Functioning** Consider chunking this activity by inviting students to choose one data set at a time and answering the question, "Could the mean be 11 cherries?"

Select and sequence students' strategies for selecting a data set to share during the Connect.

D **Differentiation**

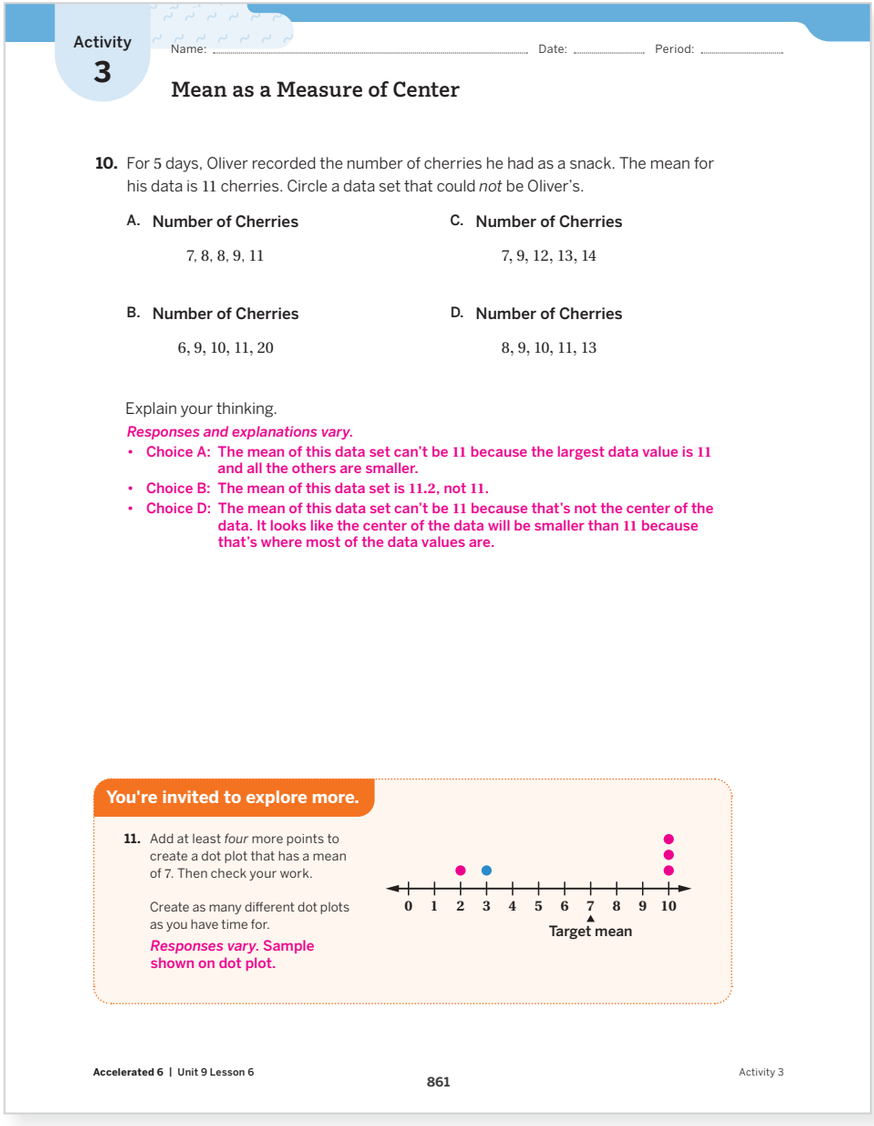
Look for students who:	Respond to Student Thinking
Calculate the mean of each data set.	**Strengthen** Consider asking, "Can you tell which data sets do not have a mean of 11 without calculating?"
Think the mean of Choice C is less than 11 because most of the data values are less than 11.	**Strengthen** Invite them to share their thinking during the Connect.
Recognize the mean of Choice A cannot be 11 because the largest data value is 11.	
Would like to explore the mean as a measure of center further.	**Stretch** **You're invited to explore more.** Invite students to complete this optional task.

3 Connect

Invite students to share their responses, calling attention to any conflicts or consensus.

Discuss each possible response one at a time and invite students to justify their reasoning for why 11 could or could not be the mean. Consider marking 11 in each data set to show the data values are not balanced on either side of 11, noting that the mean is the balance point of a data set. **(MTR.3.1)**

Consider sharing that a *measure of center* is a single number that summarizes all of the values in a data set. The mean is a measure of center because it is located at the center of the values and represents a typical value for the data set.



Activity 3

Name: Date: Period:

Mean as a Measure of Center

10. For 5 days, Oliver recorded the number of cherries he had as a snack. The mean for his data is 11 cherries. Circle a data set that could *not* be Oliver's.

A. **Number of Cherries**
7, 8, 8, 9, 11

C. **Number of Cherries**
7, 9, 12, 13, 14

B. **Number of Cherries**
6, 9, 10, 11, 20

D. **Number of Cherries**
8, 9, 10, 11, 13

Explain your thinking.

Responses and explanations vary.
- Choice A: The mean of this data set can't be 11 because the largest data value is 11 and all the others are smaller.
- Choice B: The mean of this data set is 11.2, not 11.
- Choice D: The mean of this data set can't be 11 because that's not the center of the data. It looks like the center of the data will be smaller than 11 because that's where most of the data values are.

You're invited to explore more.

11. Add at least *four* more points to create a dot plot that has a mean of 7. Then check your work.

Create as many different dot plots as you have time for.

Responses vary. Sample shown on dot plot.

Accelerated 6 | Unit 9 Lesson 6 861 Activity 3

Whole Class | 5 min

Synthesis

Purpose: Students synthesize their understanding of mean as a statistic and a measure of center and how it is used to summarize a data set.

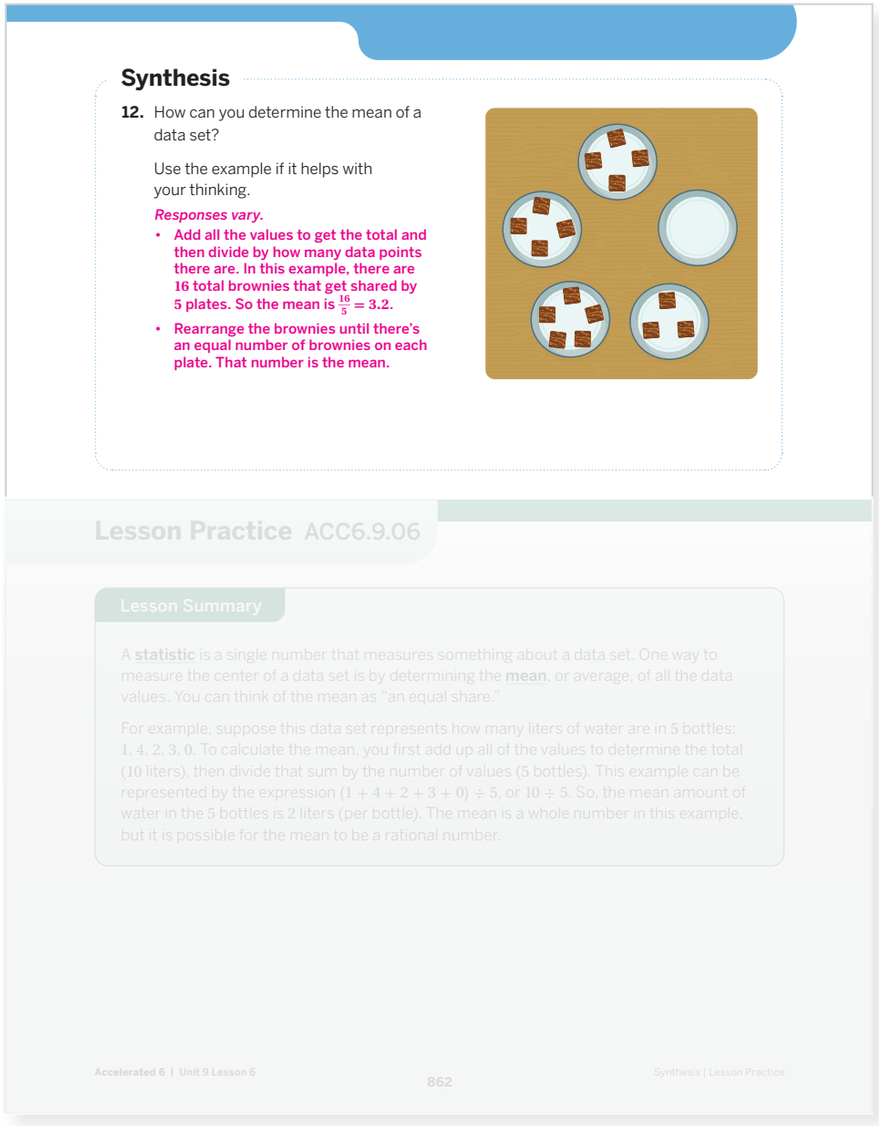
Invite students to respond independently, and then share their thinking with a partner.

Collect and share a variety of ideas, such as:

- How *mean* is an equal redistribution, or "equal share," calculated by adding together all of the data points and dividing by the number of values.
- How *mean* is a statistic and measure of center that summarizes a data set with a typical value.

MLR **MLR2: Collect and Display** Use this routine to formalize the definitions of *mean (average)* and *statistic* using students' language. Display any new language, examples, or diagrams publicly for students to reference later.

Note: If time allows, invite students to share with a partner what makes sense about the strategy for calculating the mean and what questions they still have.



Synthesis

12. How can you determine the mean of a data set?

Use the example if it helps with your thinking.

Responses vary.

- Add all the values to get the total and then divide by how many data points there are. In this example, there are 16 total brownies that get shared by 5 plates. So the mean is $\frac{16}{5} = 3.2$.
- Rearrange the brownies until there's an equal number of brownies on each plate. That number is the mean.

Lesson Practice ACC6.9.06

Lesson Summary

A **statistic** is a single number that measures something about a data set. One way to measure the center of a data set is by determining the **mean**, or average, of all the data values. You can think of the mean as "an equal share."

For example, suppose this data set represents how many liters of water are in 5 bottles: $1, 4, 2, 3, 0$. To calculate the mean, you first add up all of the values to determine the total (10 liters), then divide that sum by the number of values (5 bottles). This example can be represented by the expression $(1 + 4 + 2 + 3 + 0) \div 5$, or $10 \div 5$. So, the mean amount of water in the 5 bottles is 2 liters (per bottle). The mean is a whole number in this example, but it is possible for the mean to be a rational number.

Accelerated 6 | Unit 9 Lesson 6 862 Synthesis | Lesson Practice

Lesson Takeaway: The **mean** is a **statistic** that describes the number of items in each group if the items are distributed equally or the balance point of a data set. To calculate the mean, you can add up all the data values, and divide by the number of data points.

Independent | 5 min

Show What You Know

Purpose: Students demonstrate their understanding by calculating the mean of a data set.

Today's Goals

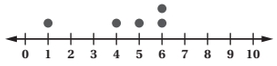
1. **Goal:** Calculate the *mean* of a data set.
2. **Language Goal:** Describe the *mean* for a data set as a measure of center that summarizes the data using a single number. **(Reading and Writing)**

Show What You Know (Assessment Resources)

Name: _____ Date: _____ Period: _____

Show What You Know 9.06

Here are a dot plot and a table showing how many cookies Aditi ate each day last week. Calculate the mean of this data. Show or explain your thinking.



Day	Number of Cookies
Monday	5
Tuesday	4
Wednesday	6
Thursday	6
Friday	1

4.4 cookies. *Responses vary.* I added all of the cookies Aditi ate over the week, which totaled 22 cookies. Then, I divided 22 by the number of days she ate cookies, which was 5. $\frac{22}{5} = 4.4$.

Differentiation Use after Lesson 6

Support

Provide targeted intervention for students by using these resources.

If students need support calculating the mean from a dot plot or a table:

» **Respond to Student Thinking:**

- Revisit **Activity 1** with students before providing individualized feedback on the relevant **Practice Problems**.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite them to explore the **Sub-Unit 2 Extension Activities**.
- Invite them to collect numerical data or use data from earlier in the unit to **calculate the mean** and describe what it represents in context.
- Invite them to complete the **Explore More** task and discuss their thinking with a partner.

Professional Learning

Reflect on the facilitation of **MLR3: Critique, Correct, Clarify** in this lesson.

- Who shared in the whole-class discussion? Who did not?
- How can you support the same or increased involvement the next time you use this routine?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

12. How can you determine the mean of a data set?

Use the example if it helps with your thinking.

Responses vary.

- Add all the values to get the total and then divide by how many data points there are. In this example, there are 16 total brownies that get shared by 5 plates. So the mean is $\frac{16}{5} = 3.2$.
- Rearrange the brownies until there's an equal number of brownies on each plate. That number is the mean.

Lesson Practice ACC6.9.06

Lesson Summary

A **statistic** is a single number that measures something about a data set. One way to measure the center of a data set is by determining the **mean**, or average, of all the data values. You can think of the mean as "an equal share."

For example, suppose this data set represents how many liters of water are in 5 bottles: 1, 4, 2, 3, 0. To calculate the mean, you first add up all of the values to determine the total (10 liters), then divide that sum by the number of values (5 bottles). This example can be represented by the expression $(1 + 4 + 2 + 3 + 0) \div 5$, or $10 \div 5$. So, the mean amount of water in the 5 bottles is 2 liters (per bottle). The mean is a whole number in this example, but it is possible for the mean to be a rational number.

Lesson Practice ACC6.9.06

Name: _____ Date: _____ Period: _____

1. A preschool teacher plans to reorganize these 4 boxes of playing blocks so that each box contains an equal number of blocks. How could the teacher determine the number of blocks to put in each box?

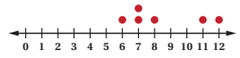
Box 1	Box 2	Box 3	Box 4
32 blocks	18 blocks	41 blocks	9 blocks

Responses vary. The teacher could add all of the blocks and then divide that number by 4.

2. 3 classes worked together to raise money for their classroom libraries. They agreed to share the money equally. The first class raised $25.50, the second class raised $49.75, and the third class raised $37.25. What is each class's equal share? Explain or show your thinking.

$37.50. *Explanations vary.* $\frac{25.50 + 49.75 + 37.25}{3} = \frac{112.50}{3}$ or $37.50

3. Kimaya guesses that 11 is the mean of this data set. Without calculating, determine if Kimaya's guess is correct. Explain your thinking.



No. *Explanations vary.* Most of the numbers are less than 11 and the only number greater than 11 is 12. So the mean is probably less than 11.

Problems 4–5: For 12 days, Mai recorded how many minutes long her bus rides to school were. Here are the times she recorded.

Time on the Bus (min)											
9	12	6	9	10	7	6	12	9	8	10	10

4. Determine the mean for Mai's data. Explain or show your thinking.

9 minutes. *Explanations vary.* $\frac{9 + 12 + 6 + 9 + 10 + 7 + 6 + 12 + 9 + 8 + 10 + 10}{12} = \frac{108}{12}$ or 9

5. What does the mean tell us about Mai's trip to school?

Responses vary. The mean tells us that it takes Mai 9 minutes to get to school on most days.

6. In her English class, Lan's teacher gives 4 quizzes, each worth 5 points. After 3 quizzes, Lan has scores of 4, 3, and 4. How many points does Lan need to get on the last quiz to have an average score of 4? Explain or show your thinking. 5 points. *Explanations vary.*

- Because 3 is one less than 4, Lan has to score $4 + 1 = 5$ points on the final quiz.
- $\frac{4 + 3 + 4 + x}{4} = 4$

$4 \cdot \left(\frac{4 + 3 + 4 + x}{4}\right) = 4 \cdot 4$

$4 + 3 + 4 + x = 16$

$11 + x = 16$

$x = 5$

Lesson Practice ACC6.9.06

Name: _____ Date: _____ Period: _____

7. Select a data set that matches each statement. You can only select each data set once.

a. The mean is 2.	b	1, 1, 2, 2, 2, 4
b. Only one value is greater than the mean.	c	1, 1, 1, 1, 2, 3
c. Two values are greater than the mean.	a	0, 1, 2, 2, 3, 4

FAST Practice

8. In a round of mini golf, Angel records his number of putts on each hole.

Number of Putts								
2	3	1	4	5	2	3	4	3

What is his mean number of putts per hole?

3 putts per hole

Spiral Review

Problems 9–11: Evaluate the expression $4x^3$ for each value of x.

9. 1
4

10. 2
32

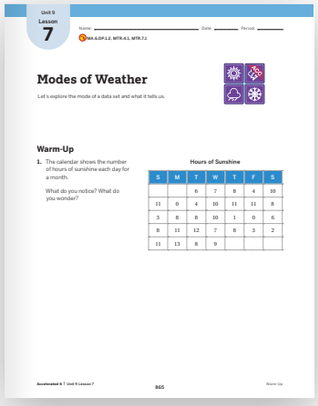
11. -3
-108

12. Select *all* of the values of x that are solutions to the inequality $x > -3$.

- ☑ A. -2
- ☑ B. |-4|
- ☐ C. -4
- ☑ D. 3.5
- ☐ E. -3.5

Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1–3, 5–6	2	MA.6.DP.1.2
	4	1	MA.6.DP.1.2
	7	3	MA.6.DP.1.2
FAST Practice	8	2	MA.6.DP.1.2
Spiral Review			
	9–11	1	MA.6.AR.1.3
	12	1	MA.6.AR.2.1


Unit 9
Lesson 7

Modes of Weather

Let's explore the mode of a data set and what it tells us.

Warm-Up

1. The calendar shows the number of hours of sunshine each day for a month.

What do you notice? What do you wonder?

Hours of Sunshine

S	M	T	W	T	F	S
		6	7	8	4	10
11	0	4	10	11	11	8
2	8	8	10	1	0	6
6	11	12	7	6	3	2
11	13	8	9			

Modes of Weather

Introducing the Mode

Let's explore the mode of a data set and what it tells us.

Learning Outcomes

Today's Goals

1. **Goal:** Identify the *mode* of a data set.
2. **Language Goal:** Interpret the *mode* of a data set in the context of real-world data. **(Reading, Writing, Speaking, and Listening)**

Students are introduced to the **mode** of a data set. Students determine the mode using a variety of data displays and explore when a data display may not be helpful in identifying the mode. Students also compare the mode to the mean and interpret the mode of weather data. **(MTR.7.1)**

Coherence

Prior Learning

In Lesson 6, students were introduced to the mean as a statistic that measures the center of a data set.

Future Learning

In Lesson 8, students are introduced to the median as the third measure of center to describe and interpret data.

Connections

- Students are introduced to the mode as it applies to problems in context. (mathematical, real world)

FL B.E.S.T. Math Benchmarks

Addressing

MA.6.DP.1.2
Given a numerical data set within a real-world context, find and interpret mean, median, mode and range.

Mathematical Thinking and Reasoning Standards: MTR.4.1, MTR.7.1

ELA Expectations: ELA.K12.EE.1.1, ELA.K12.EE.5.1, ELA.K12.EE.6.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.5.DP.1.2	**MA.7.DP.1.1** **MA.7.DP.1.2**

Vocabulary

New Vocabulary

mode

Lesson at a Glance

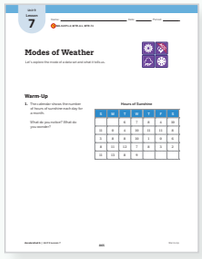
~ 45 min

MA.6.DP.1.2

Warm-Up

Whole Class | 5 min

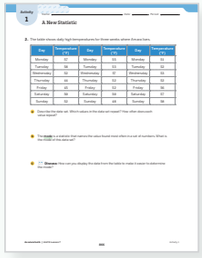
Students use the **Notice and Wonder** routine to explore a weather-related data set.



Activity 1

Pairs | 10 min

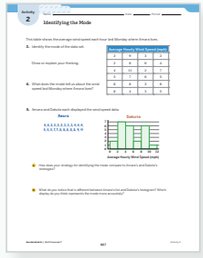
Students are introduced to the **mode** as a statistic that describes daily high temperature data. **(MTR.7.1)**



Activity 2

Pairs | 10 min

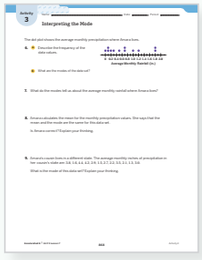
Students analyze a variety of strategies to determine the **mode**. **MLR7: Compare and Connect**



Activity 3

Pairs | 10 min

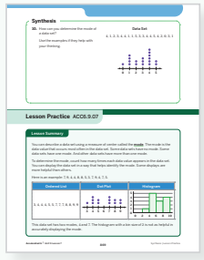
Students analyze data sets with multiple **modes** and with no mode. They interpret the mode in the context of monthly rainfall data and compare the mode to the mean. **(MTR.7.1)**



Synthesis

Whole Class | 5 min

Students synthesize their understanding of the **mode** as a statistic. **MLR2: Collect and Display**



Show What You Know

Independent | 5 min

Students demonstrate their understanding by identifying the mode of a given data set.

Prep Checklist

This lesson includes:

- Student Edition
- Show What You Know

Whole Class | 5 min

Warm-Up

Purpose: Students use the Notice and Wonder routine to explore a weather-related data set.

1 Launch

To generate excitement, consider asking:

- "What are some types of information you like to know about the weather in this area?"
- "How does the weather in this area change from day to day?"
- "How might you collect data about the weather in this area?"
- "What statistical questions could you ask to collect the data?"

ELL **English Language Learners** During this lesson, consider personally inviting students to share responses, asking if they would feel comfortable sharing with the class. Consider inviting students to respond in their primary languages. **(Speaking and Listening)**

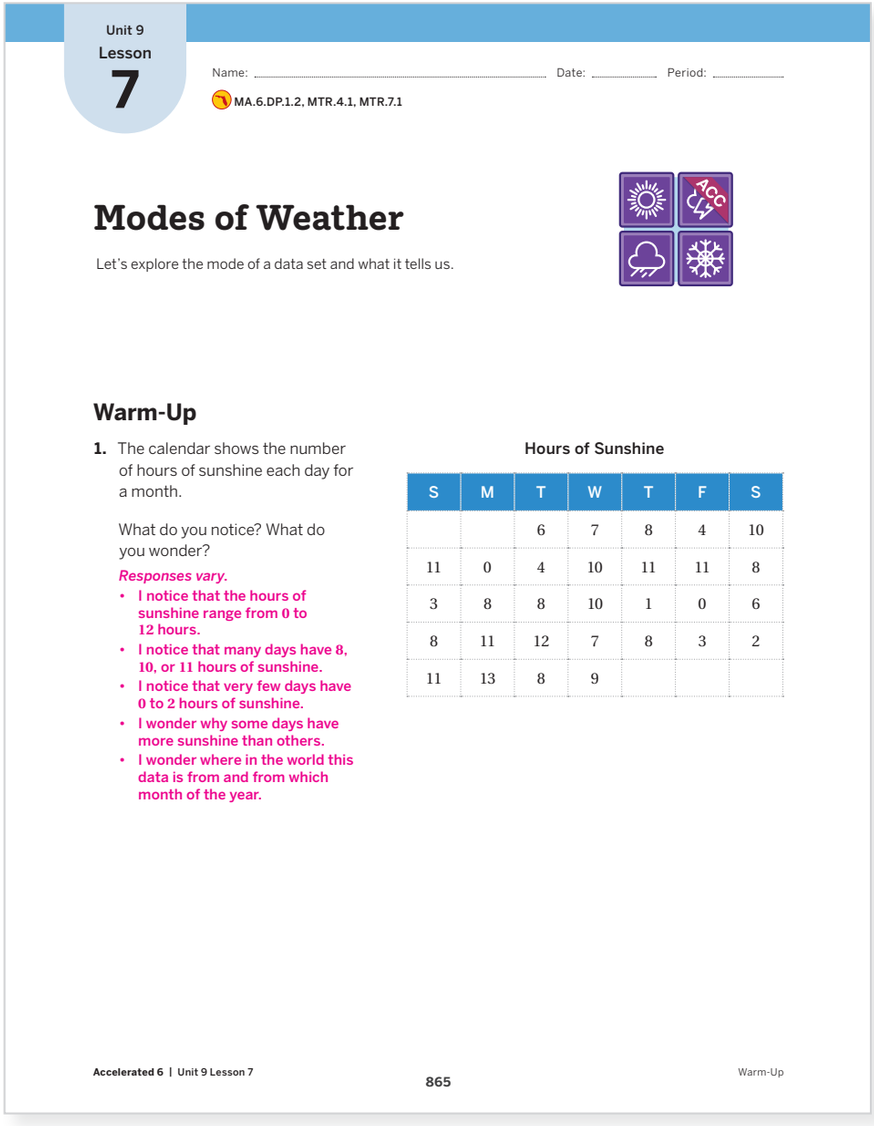
Use the Notice and Wonder routine to promote curiosity and help students make sense of the sunshine data. **(MTR.7.1)**

2 Connect

Invite students to share what they noticed and wondered.

Consider asking:

- "How can you use this data to understand the weather this month?"
- "What might this data tell us about the weather this month?"



Unit 9
Lesson
7

Name: _____ Date: _____ Period: _____

MA.6.DP.1.2, MTR.4.1, MTR.7.1

Modes of Weather

Let's explore the mode of a data set and what it tells us.

Warm-Up

1. The calendar shows the number of hours of sunshine each day for a month.

What do you notice? What do you wonder?

Responses vary.

- I notice that the hours of sunshine range from 0 to 12 hours.
- I notice that many days have 8, 10, or 11 hours of sunshine.
- I notice that very few days have 0 to 2 hours of sunshine.
- I wonder why some days have more sunshine than others.
- I wonder where in the world this data is from and from which month of the year.

Hours of Sunshine

S	M	T	W	T	F	S
		6	7	8	4	10
11	0	4	10	11	11	8
3	8	8	10	1	0	6
8	11	12	7	8	3	2
11	13	8	9			

Accelerated 6 | Unit 9 Lesson 7 865 Warm-Up

Pairs | 10 min

Activity 1 A New Statistic

Purpose: Students are introduced to the **mode** as a statistic that describes daily high temperature data. **(MTR.7.1)**

1 Launch

Direct students' attention to the data table.

To generate excitement, consider asking, "What time of year might this data be from?"

2 Monitor

Invite students to discuss the data set with a partner. **(MTR.4.1)**

Select ideas to share during the Connect.

D Differentiation

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Encourage them to order the temperatures from least to greatest.
Identify the number of times each value occurs in the data set.	**Strengthen** Consider asking, "How did you keep track of the number of times each value occurred in the data set?"
Used a data display to distinguish the mode among other values that repeat.	**Stretch** Invite them to share their data displays during the Connect.

3 Connect

Share the selected ideas, calling attention to any conflict or consensus, and invite students to explain their thinking.

Create a class definition for the term *mode* using students' language and the glossary. Consider creating a visual display indicating the definition of mode with at least one example for students to refer to throughout this lesson and unit.

Activity 1

Name: Date: Period:

A New Statistic

2. The table shows daily high temperatures for three weeks where Amara lives.

Day	Temperature (°F)	Day	Temperature (°F)	Day	Temperature (°F)
Monday	57	Monday	55	Monday	51
Tuesday	58	Tuesday	53	Tuesday	52
Wednesday	52	Wednesday	57	Wednesday	53
Thursday	44	Thursday	52	Thursday	52
Friday	45	Friday	52	Friday	56
Saturday	50	Saturday	50	Saturday	57
Sunday	52	Sunday	48	Sunday	58

a. Describe the data set. Which values in the data set repeat? How often does each value repeat?

Responses vary. Three values occur twice: 50, 53, and 58. The value 57 occurs three times. The value 52 occurs six times.

b. The **mode** is a statistic that names the value found most often in a set of numbers. What is the mode of this data set?

52

c. **Discuss:** How can you display the data from the table to make it easier to determine the mode?

Responses vary. I can rewrite the data as an ordered list or create a dot plot of the data. Then I can more easily identify the value that occurs most often.

Accelerated 6 | Unit 9 Lesson 7 866 Activity 1

Key Takeaway: The **mode** is any value that appears most often in a data set.

Pairs | 10 min

Activity 2 Identifying the Mode

Purpose: Students analyze a variety of strategies to determine the **mode**.

1 Launch

Direct students' attention to the wind speed data table. **(MTR.7.1)**

Consider asking, "What data in this table stands out to you?"

2 Monitor

Listen for students using phrases such as *frequency* and *most often* when identifying and interpreting the mode.

Look for students who identify a different mode based on the data displays.

Encourage students to think about how the data displays are alike and different.

Note: Students may think that Dakota's data display is incorrect because it indicates a different mode. Remind students that some data displays are better than others at revealing details about the data.

A **Accessibility: Visual-Spatial Processing** Invite pairs to think about key features of the data displays and discuss the features aloud before responding to support distinguishing between representations.

2 Connect

Invite students to share their strategies and explanations for the mode.

MLR **MLR7: Compare and Connect** Invite students to analyze each representation, then find ways in which the representations are different (compare) and alike (connect). Invite students to discuss why the two displays indicate different modes and consider other ways to display the data using a histogram or another method. **(MTR.4.1)**

Activity 2

Name: Date: Period:

Identifying the Mode

This table shows the average wind speed each hour last Monday where Amara lives.

Average Hourly Wind Speed (mph)			
2	9	3	2
2	8	0	4
4	11	2	7
5	7	0	5
8	8	3	8
8	4	3	5

3. Identify the mode of the data set.

8

Show or explain your thinking.

Explanations vary. 8 is the value that occurs most often. It occurs 5 times.

4. What does the mode tell us about the wind speed last Monday where Amara lives?

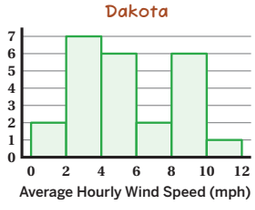
Responses vary. Last Monday where Amara lives, most often the wind speed was 8 mph.

5. Amara and Dakota each displayed the wind speed data.

Amara

0, 0, 2, 2, 2, 2, 3, 3, 3, 4, 4, 4, 5, 5, 5, 7, 7, 8, 8, 8, 8, 8, 9, 11

Dakota



a. How does your strategy for identifying the mode compare to Amara's and Dakota's strategies?

Responses vary. I used the same strategy as Amara of listing the data in order from least to greatest.

b. What do you notice that is different between Amara's list and Dakota's histogram? Which display do you think represents the mode more accurately?

Responses vary. Amara's list shows that the mode is 8. Dakota's histogram shows the highest peak from 2 to 4. The data displays appear to show different modes. Dakota's histogram does not show each data value. Amara's data shows each data value, so her display represents the mode more accurately.

Accelerated 6 | Unit 9 Lesson 7 867 Activity 2

Pairs | 10 min

Activity 3 Interpreting the Mode

Purpose: Students analyze data sets with multiple **modes** and with no mode. They interpret the mode in the context of monthly rainfall data and compare the mode to the mean. **(MTR.7.1)**

Short on time? Consider completing this activity as a whole class.

1 Launch

Direct students' attention to the dot plot displaying average monthly rainfall.

Consider asking, "What does each dot in the dot plot represent?"

2 Monitor

Encourage students to revisit the class definition of *mode* to help make sense of multiple modes in a data set.

A **Accessibility: Visual-Spatial Processing** Invite students to use coloring tools and annotate the dot plot to support describing and interpreting visual representations.

D **Differentiation: (Support)** Students may think that they need to do something different to determine the mode of data values containing decimals. Consider asking, "How can you identify the mode in a data set regardless of the numbers given?"

Invite students to think about how the modes compare to one another. Consider asking, "Do the modes represent the data well?"

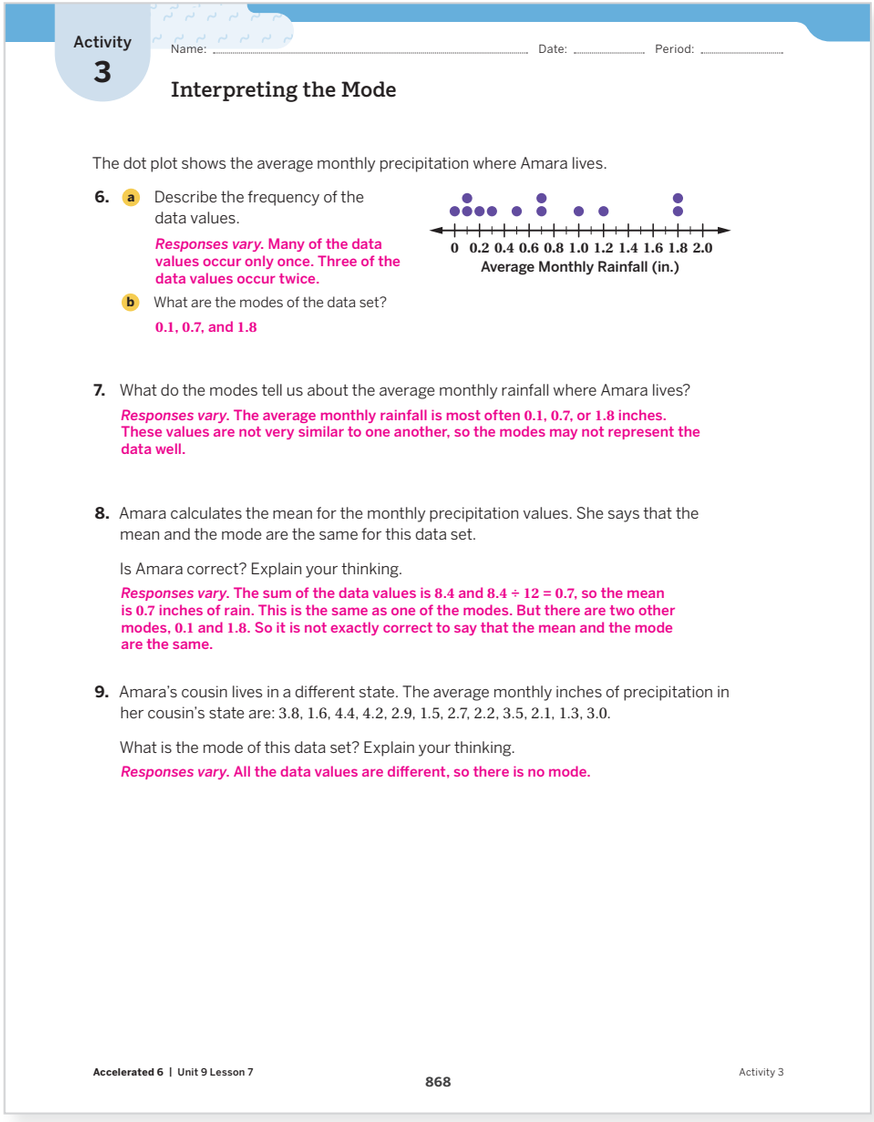
To surface prior knowledge, consider asking:
- "What does the mean measure?"
- "How can we calculate the mean of this data set?"

Look for and celebrate the different ways that students explain the lack of a mode in the data from Amara's cousin.

3 Connect

Invite students to share their thinking about the rainfall data. Discuss whether the mode or the mean is a better measure of center for the data set. **(MTR.7.1)**

To surface the Key Takeaway, consider asking:
- "How many modes does Amara's rainfall data have?"
- "How many modes does the data from Amara's cousin have?"



Activity 3

Name: Date: Period:

Interpreting the Mode

The dot plot shows the average monthly precipitation where Amara lives.

6. a. Describe the frequency of the data values.

Responses vary. Many of the data values occur only once. Three of the data values occur twice.

b. What are the modes of the data set?

0.1, 0.7, and 1.8

0 0.2 0.4 0.6 0.8 1.0 1.2 1.4 1.6 1.8 2.0
Average Monthly Rainfall (in.)

7. What do the modes tell us about the average monthly rainfall where Amara lives?

Responses vary. The average monthly rainfall is most often 0.1, 0.7, or 1.8 inches. These values are not very similar to one another, so the modes may not represent the data well.

8. Amara calculates the mean for the monthly precipitation values. She says that the mean and the mode are the same for this data set.

Is Amara correct? Explain your thinking.

Responses vary. The sum of the data values is 8.4 and 8.4 ÷ 12 = 0.7, so the mean is 0.7 inches of rain. This is the same as one of the modes. But there are two other modes, 0.1 and 1.8. So it is not exactly correct to say that the mean and the mode are the same.

9. Amara's cousin lives in a different state. The average monthly inches of precipitation in her cousin's state are: 3.8, 1.6, 4.4, 4.2, 2.9, 1.5, 2.7, 2.2, 3.5, 2.1, 1.3, 3.0.

What is the mode of this data set? Explain your thinking.

Responses vary. All the data values are different, so there is no mode.

Accelerated 6 | Unit 9 Lesson 7 868 Activity 3

Key Takeaway: The **mode** is a statistic that identifies the most common data value in a data set. Data sets may have zero modes, one mode, or more than one mode.

Whole Class | 5 min

Synthesis

Purpose: Students synthesize their understanding of the **mode** as a statistic.

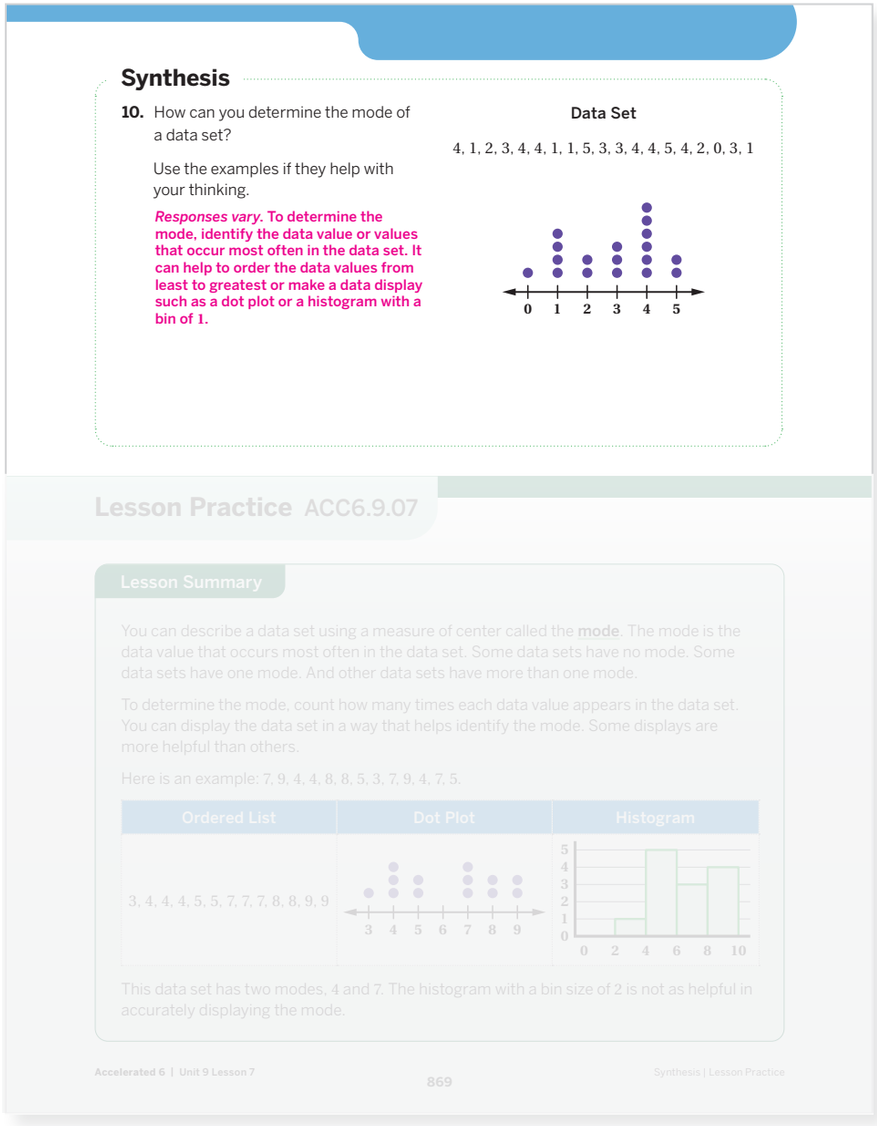
Invite students to respond independently, and then share their thinking with a partner.

Collect and share a variety of ideas, such as:

- It can be helpful to list the data values from least to greatest or from greatest to least.
- Data sets may have no mode, one mode, or more than one mode.
- The highest peak of a data display indicates the mode.
- A histogram may or may not accurately display the mode.

MLR **MLR2: Collect and Display** Use this routine to formalize the definition of *mode* using students' language. Display any new language, examples, or diagrams publicly for students to reference later.

Note: If time allows, invite students to revise their previous thinking based on the discussion.



Synthesis

10. How can you determine the mode of a data set?

Use the examples if they help with your thinking.

Responses vary. To determine the mode, identify the data value or values that occur most often in the data set. It can help to order the data values from least to greatest or make a data display such as a dot plot or a histogram with a bin of 1.

Data Set

4, 1, 2, 3, 4, 4, 1, 1, 5, 3, 3, 4, 4, 5, 4, 2, 0, 3, 1

Lesson Practice ACC6.9.07

Lesson Summary

You can describe a data set using a measure of center called the **mode**. The mode is the data value that occurs most often in the data set. Some data sets have no mode. Some data sets have one mode. And other data sets have more than one mode.

To determine the mode, count how many times each data value appears in the data set. You can display the data set in a way that helps identify the mode. Some displays are more helpful than others.

Here is an example: 7, 9, 4, 4, 8, 8, 5, 3, 7, 9, 4, 7, 5.

Ordered List	Dot Plot	Histogram
3, 4, 4, 4, 5, 5, 7, 7, 7, 8, 8, 9, 9		

This data set has two modes, 4 and 7. The histogram with a bin size of 2 is not as helpful in accurately displaying the mode.

Accelerated 6 | Unit 9 Lesson 7 869 Synthesis | Lesson Practice

Lesson Takeaway: The **mode** is a statistic that identifies the value(s) that occur most often in a data set. You can use an ordered list, a dot plot, or a histogram with a bin of 1 to determine the mode.

Independent | 5 min

Show What You Know

Purpose: Students demonstrate their understanding by identifying the mode of a given data set.

Today's Goals

1. **Goal:** Identify the *mode* of a data set.
2. **Language Goal:** Interpret the *mode* of a data set in the context of real-world data. **(Reading, Writing, Speaking, and Listening)**

Show What You Know (Assessment Resources)

Name: _____ Date: _____ Period: _____

Show What You Know 9.07

Ben kept track of how many hours he slept each night for 10 days.

8, 7, 8, 7, 8, 6, 10, 9, 8, 8

What is the mode of his data set?

8 hours

What does the mode tell us about how many hours Ben slept over the 10 days?

Responses vary. The most common number of hours per night that Ben slept over the past 10 days is 8 hours.

D Differentiation Use after Lesson 7

Support

Provide targeted intervention for students by using these resources.

If students need support identifying the mode of a data set:

» **Respond to Student Thinking:**

- Review the **Lesson Summary**.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite them to explore the **Sub-Unit 2 Extension Activities**.

Professional Learning

Take a moment to reflect on this lesson. In this lesson, students identified and interpreted the *mode*. How will that work support them in determining the best measure of center to interpret data?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

10. How can you determine the mode of a data set?

Use the examples if they help with your thinking.

Responses vary. To determine the mode, identify the data value or values that occur most often in the data set. It can help to order the data values from least to greatest or make a data display such as a dot plot or a histogram with a bin of 1.

Data Set

4, 1, 2, 3, 4, 4, 1, 1, 5, 3, 3, 4, 4, 5, 4, 2, 0, 3, 1

Lesson Practice ACC6.9.07

Lesson Summary

You can describe a data set using a measure of center called the **mode**. The mode is the data value that occurs most often in the data set. Some data sets have no mode. Some data sets have one mode. And other data sets have more than one mode.

To determine the mode, count how many times each data value appears in the data set. You can display the data set in a way that helps identify the mode. Some displays are more helpful than others.

Here is an example: 7, 9, 4, 4, 8, 8, 5, 3, 7, 9, 4, 7, 5.

Ordered List	Dot Plot	Histogram
3, 4, 4, 4, 5, 5, 7, 7, 7, 8, 8, 9, 9		

This data set has two modes, 4 and 7. The histogram with a bin size of 2 is not as helpful in accurately displaying the mode.

Lesson Practice
ACC6.9.07

Name: _____ Date: _____ Period: _____

Problems 1–4: Tyani kept track of how many miles she ran each day for 10 days. Here is the data.

Miles Run Each Day									
2.7	3.2	2.7	4.2	3.2	3.5	2.5	3.2	4.1	3.7

1. Determine the mode for Tyani's data. Show or explain your thinking.

Responses vary. The mode is 3.2. I counted how many times each data value appears in the table. The value 3.2 appears most often.

2. What does the mode tell us about Tyani's running practice?

Responses vary. Tyani ran 3.2 miles more often than she ran any other distance.

3. Basheera said that the mode of the data is 3. What could you say to help Basheera understand why her answer is incorrect?

Responses vary. Three is the number of times the value 3.2 occurs. The mode identifies the value that occurs most often, not how many times it occurs.

4. How does the mode for this data set compare to the mean?

A. The mode is greater than the mean.

B. The mode is less than the mean.

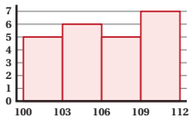
C. The mode is equal to the mean.

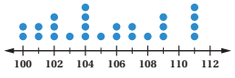
D. There is not enough information to compare.

Lesson Practice
ACC6.9.07

Name: _____ Date: _____ Period: _____

Problems 5–6: Two students made data displays to understand the same data set.





5. Which data display can you use to determine the mode? Explain your thinking.

Responses vary. The dot plot more accurately shows the values that occur most often. The histogram only shows how often each group of values in a bin occurs.

6. Determine the mode(s) for the data set.

104 and 111

7. Which data values must you remove from the data set so that the mode is $4\frac{1}{2}$?

$2\frac{3}{4}, 6\frac{1}{4}, 4\frac{1}{2}, 1, 2\frac{3}{4}, 5\frac{1}{2}, 4\frac{1}{2}, 1, \frac{1}{4}$

1 and $2\frac{3}{4}$

FAST Practice

8. What is the mode of the data set?

5.2

Pounds of Seafood									
4.8	5.4	5.2	4.4	5.4	3.9	5.2	6.0	4.5	5.2

Spiral Review

Problems 9–10: Determine the mean for each data set.

9. 0.4, 0.7, 0.5, 0.5, 0.1, 0.4, 0.2

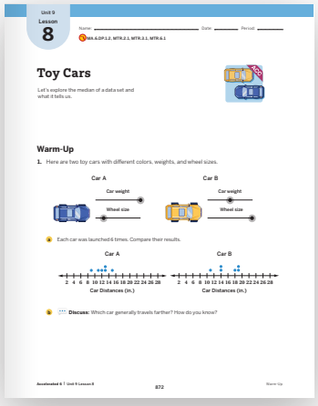
0.4

10. $2\frac{1}{2}, 2\frac{1}{2}, 2\frac{1}{2}, 3, 4, 4, 4\frac{3}{4}, 4\frac{3}{4}$

$3\frac{1}{2}$ (or equivalent)

Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1, 4–7	2	MA.6.DP.1.2
	2–3	3	MA.6.DP.1.2
FAST Practice	8	1	MA.6.DP.1.2
Spiral Review			
	9–10	1	MA.6.DP.1.2



Toy Cars

Introducing the Median

Let's explore the median of a data set and what it tells us.

Learning Outcomes

Today's Goals

1. **Goal:** Describe the *median* as another measure of center that can be used to summarize a data set.
2. **Language Goal:** Determine and interpret the *median* of a data set from a list or a dot plot. **(Reading, Writing, Speaking, and Listening)**

Students are introduced to a new measure of center: the **median**. They analyze and compare the data of toy cars and determine the median from lists and dot plots. Students reason abstractly and quantitatively as they critique the claims of others and decide whether the mean or the median represents a typical value for a data set. **(MTR.2.1)**

Coherence

Prior Learning

In Lesson 6, students explored the mean as an equal share and as a number that can be used to describe the center of a data set.

Future Learning

In Lesson 9, students will continue comparing the mean and median of a data set and consider how they are affected by the shape of the data.

Connections

- Students are introduced to the median as it applies to problems in context. (mathematical, real world)

FL B.E.S.T. Math Benchmarks

Addressing

MA.6.DP.1.2
Given a numerical data set within a real-world context, find and interpret mean, median, mode and range.

Mathematical Thinking and Reasoning Standards: MTR.2.1, MTR.3.1, MTR.6.1

ELA Expectations: ELA.K12.EE.1.1, ELA.K12.EE.5.1, ELA.K12.EE.6.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.5.DP.1.2	MA.7.DP.1.1
	MA.7.DP.1.2

Vocabulary

New Vocabulary

median

Lesson at a Glance

~ 45 min

MA.6.DP.1.2

Warm-Up

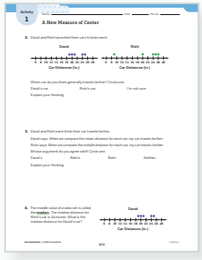
Independent | 5 min

Students compare data sets from the launches of two toy cars that each have multiple data points.

Activity 1

Pairs | 12 min

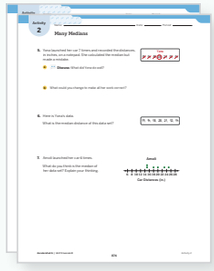
Students use the **Decide and Defend** routine to critique arguments and make sense of the **median** as a measure of center and practice identifying the median of a data set.



Activity 2

Pairs | 18 min

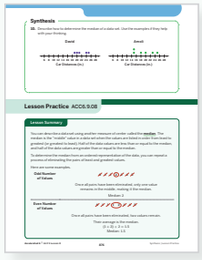
Students use **MLR3: Critique, Correct, Clarify** to analyze student work and practice identifying the median of a data set.



Synthesis

Whole Class | 5 min

Students synthesize their understanding of what the median of a data set is and how it is useful.
MLR2: Collect and Display



Show What You Know

Independent | 5 min

Students demonstrate their understanding by determining the median of the given data set.

Prep Checklist

This lesson includes:

- Student Edition
- Show What You Know

Independent | 5 min

Warm-Up

Purpose: Students compare data sets from the launches of two toy cars that each have multiple data points.

1 Launch

Consider asking, "Have you ever played with a toy car? Which elements of a toy car design would most impact how far it goes when you launch it?"

Invite students to compare the data given in their Student Edition.

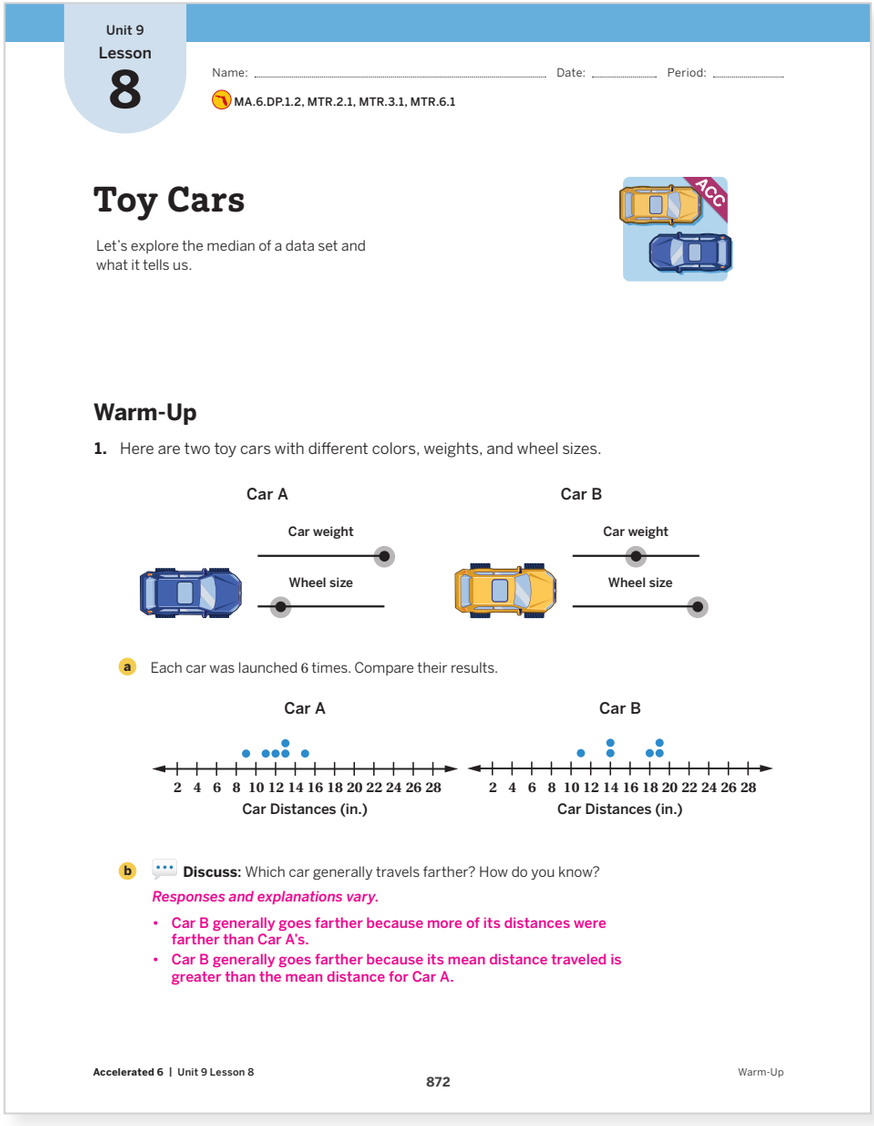
2 Connect

Invite students to share their strategies for comparing their cars' distances.

To surface prior knowledge, encourage students to reference the center, spread, or shape of each dot plot as they share their thinking.

Consider asking:

- "What is challenging about comparing cars when each car has many data points?"
- "In general, how might we decide whose car goes farther?"



Unit 9
Lesson
8

Name: _____ Date: _____ Period: _____

MA.6.DP.1.2, MTR.2.1, MTR.3.1, MTR.6.1

Toy Cars

Let's explore the median of a data set and what it tells us.

Warm-Up

1. Here are two toy cars with different colors, weights, and wheel sizes.

Car A: Car weight, Wheel size

Car B: Car weight, Wheel size

a. Each car was launched 6 times. Compare their results.

Car A: 2 4 6 8 10 12 14 16 18 20 22 24 26 28 — Car Distances (in.)

Car B: 2 4 6 8 10 12 14 16 18 20 22 24 26 28 — Car Distances (in.)

b. **Discuss:** Which car generally travels farther? How do you know?

Responses and explanations vary.

- Car B generally goes farther because more of its distances were farther than Car A's.
- Car B generally goes farther because its mean distance traveled is greater than the mean distance for Car A.

Accelerated 6 | Unit 9 Lesson 8 872 Warm-Up

Pairs | 12 min

Activity 1 A New Measure of Center

Purpose: Students use the Decide and Defend routine to critique arguments and make sense of the median as a measure of center and practice identifying the median of a data set.

1 Launch

Direct students' attention to the dot plots. Select a point on David's dot plot and consider asking, "What does this point tell us about David's car?"

Invite students to share how they chose which car generally travels farther. Encourage students to share with a partner before responding.

2 Monitor

To surface connections, annotate David's and Rishi's dot plots with their means, encouraging students to annotate in their Student Edition.

D Differentiation

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Invite students to guess which value on David's dot plot is the middle value.
Recognize that the smallest value in Rishi's data set is not a typical value.	**Strengthen** Consider asking, "How does this value affect the mean? Do you think the mean summarizes the data well?"
Interpret the middle distance as a typical distance for David's or Rishi's car. **(MTR.3.1)**	**Strengthen** Celebrate their thinking and invite them to share during the Connect.

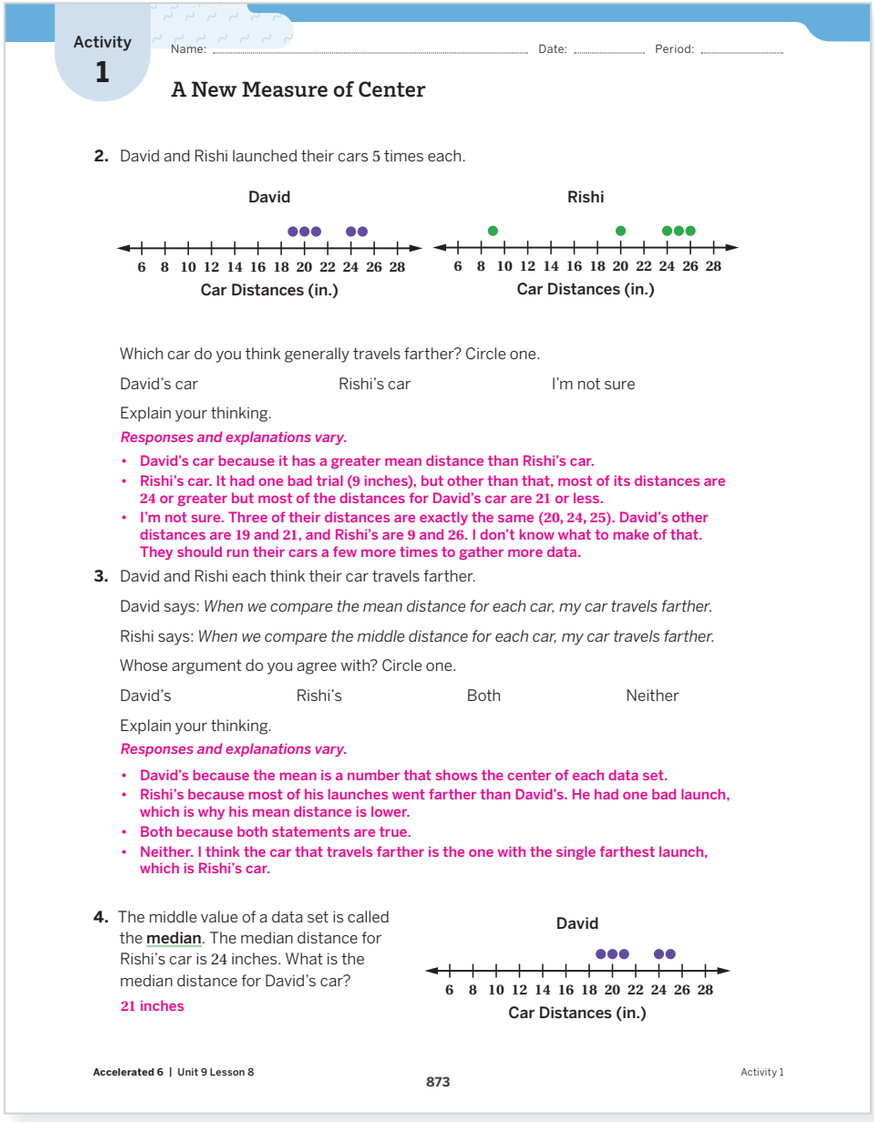
Encourage students to label the median for David's car on the dot plot. Consider asking, "How does the median compare to the mean?"

3 Connect

Use the Decide and Defend routine to support students in making arguments and critiquing the reasoning of others. Consider asking, "Would you compare the average distance or the middle distance to determine which car travels farther? Why?" **(MTR.3.1, MTR.6.1)**

Consider sharing that because the mean is sensitive to extreme values, it may not always represent a typical value for a data set. The middle value might better describe the typical value. **(MTR.2.1)**

Create a class definition for the term median using students' language and the glossary.



Activity 1

Name: Date: Period:

A New Measure of Center

2. David and Rishi launched their cars 5 times each.

Which car do you think generally travels farther? Circle one.

David's car Rishi's car I'm not sure

Explain your thinking.

Responses and explanations vary.

- David's car because it has a greater mean distance than Rishi's car.
- Rishi's car. It had one bad trial (9 inches), but other than that, most of its distances are 24 or greater but most of the distances for David's car are 21 or less.
- I'm not sure. Three of their distances are exactly the same (20, 24, 25). David's other distances are 19 and 21, and Rishi's are 9 and 26. I don't know what to make of that. They should run their cars a few more times to gather more data.

3. David and Rishi each think their car travels farther.

David says: *When we compare the mean distance for each car, my car travels farther.*

Rishi says: *When we compare the middle distance for each car, my car travels farther.*

Whose argument do you agree with? Circle one.

David's Rishi's Both Neither

Explain your thinking.

Responses and explanations vary.

- David's because the mean is a number that shows the center of each data set.
- Rishi's because most of his launches went farther than David's. He had one bad launch, which is why his mean distance is lower.
- Both because both statements are true.
- Neither. I think the car that travels farther is the one with the single farthest launch, which is Rishi's car.

4. The middle value of a data set is called the median. The median distance for Rishi's car is 24 inches. What is the median distance for David's car?

21 inches

Accelerated 6 | Unit 9 Lesson 8 873 Activity 1

Key Takeaway: The median is the middle value of the data set when the values are written in numerical order. Like mean, it is a measure of center that summarizes the data set with a typical value.

Pairs | 18 min

Activity 2 Many Medians

Purpose: Students use **MLR3: Critique, Correct, Clarify** to analyze student work and practice identifying the median of a data set.

Short on time? Consider assigning the Repeated Challenges in Problem 9 as additional practice.

1 Launch

Direct students' attention to Yona's work. Consider asking, "How do you know 28 is not the median?"

MLR **MLR3: Critique, Correct, Clarify** Use this routine to help students describe strengths, errors, and ambiguities in Yona's work. Invite students to share their responses from the activity. See the Routine Facilitation Guide for more information. **(MTR.3.1, MTR.6.1)**

2 Monitor

Look for students who arrange the data values in numerical order using a variety of strategies. **(MTR.2.1)**

Pause to elicit students' strategies for determining the median of Amoli's data set.

Invite students to share what they think the median is Problem 7.

Consider asking:

- "How did you decide what the median was?"
- "Why might someone else choose a different median?"

Activity 2 continued >

Activity 2

Name: Date: Period:

Many Medians

5. Yona launched her car 7 times and recorded the distances, in inches, on a notepad. She calculated the median but made a mistake.

Yona
~~19~~, ~~14~~, ~~18~~, 28, ~~21~~, ~~12~~, ~~14~~

a. **Discuss:** What did Yona do well?

Yona found the middle value of the list.

b. What could you change to make all her work correct?

Responses vary. I would put the numbers in order first, then determine the middle number.

6. Here is Yona's data.

19, 14, 18, 28, 21, 12, 14

What is the median distance of this data set?

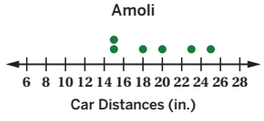
18 inches

7. Amoli launched her car 6 times.

What do you think is the median of her data set? Explain your thinking.

Responses and explanations vary.

- 19 inches because it's halfway in between the two numbers in the middle of the data set.
- 18 inches and 20 inches because those are the two numbers in the middle.
- There is no median because there is no one value in the middle.



Accelerated 6 | Unit 9 Lesson 8 874 Activity 2

Pairs | 18 min

Activity 2 Many Medians (continued)

Purpose: Students use **MLR3: Critique, Correct, Clarify** to analyze student work and practice identifying the median of a data set.

Short on time? Consider assigning the Repeated Challenges in Problem 9 as additional practice.

2 Monitor

Consider sharing that the data given is from the Warm-Up for Car A.

D Differentiation

Look for students who:	Respond to Student Thinking
Think the median could be two values.	**Support** Consider asking, "If the median is the middle, how could you determine the middle of two values?"
List the values to determine the median, but list repeated values on the dot plot only once.	**Support** Remind students that each point is a separate piece of data and needs to be included in the list.
Use a variety of strategies, such as: • Counting all the data points and dividing by 2. • Crossing off values from the ends in pairs until only the values in the middle remain.	**Strengthen** Celebrate their thinking and invite them to share during the Connect.

Listen for student strategies and discussions, offering help or encouragement as needed.

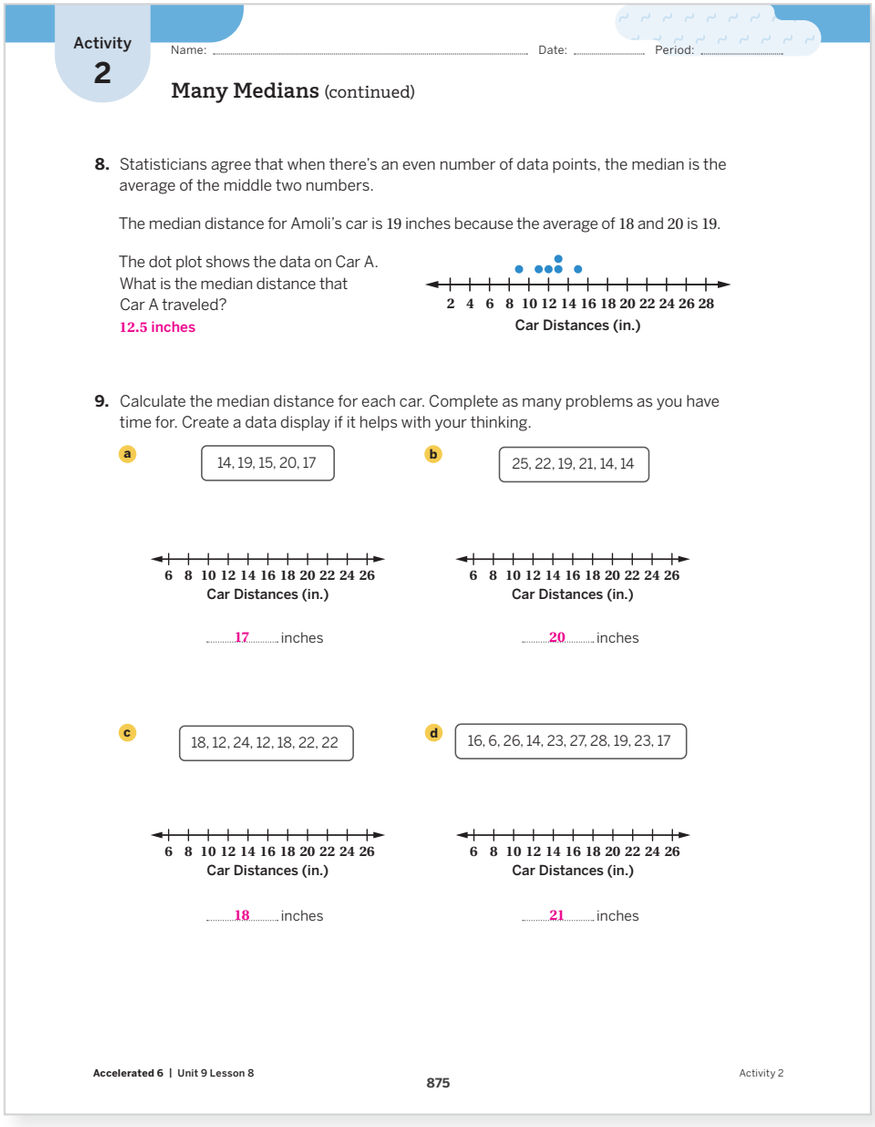
Invite students to compare their solutions with a partner.

A Accessibility: Memory and Attention Invite students who would benefit from having more time to think deeply to complete fewer challenges.

3 Connect

Invite students to share their strategies.

Consider sharing that the median may or may not be a value in the data set. Consider asking, "Can you predict when the median will not be a value in the data set?"



Activity 2

Name: Date: Period:

Many Medians (continued)

8. Statisticians agree that when there's an even number of data points, the median is the average of the middle two numbers.

The median distance for Amoli's car is 19 inches because the average of 18 and 20 is 19.

The dot plot shows the data on Car A. What is the median distance that Car A traveled?

12.5 inches

9. Calculate the median distance for each car. Complete as many problems as you have time for. Create a data display if it helps with your thinking.

a. 14, 19, 15, 20, 17

17 inches

b. 25, 22, 19, 21, 14, 14

20 inches

c. 18, 12, 24, 12, 18, 22, 22

18 inches

d. 16, 6, 26, 14, 23, 27, 28, 19, 23, 17

21 inches

Accelerated 6 | Unit 9 Lesson 8 875 Activity 2

Key Takeaway: When there's an even number of data points, the **median** is the average of the middle two numbers.

Whole Class | 5 min

Synthesis

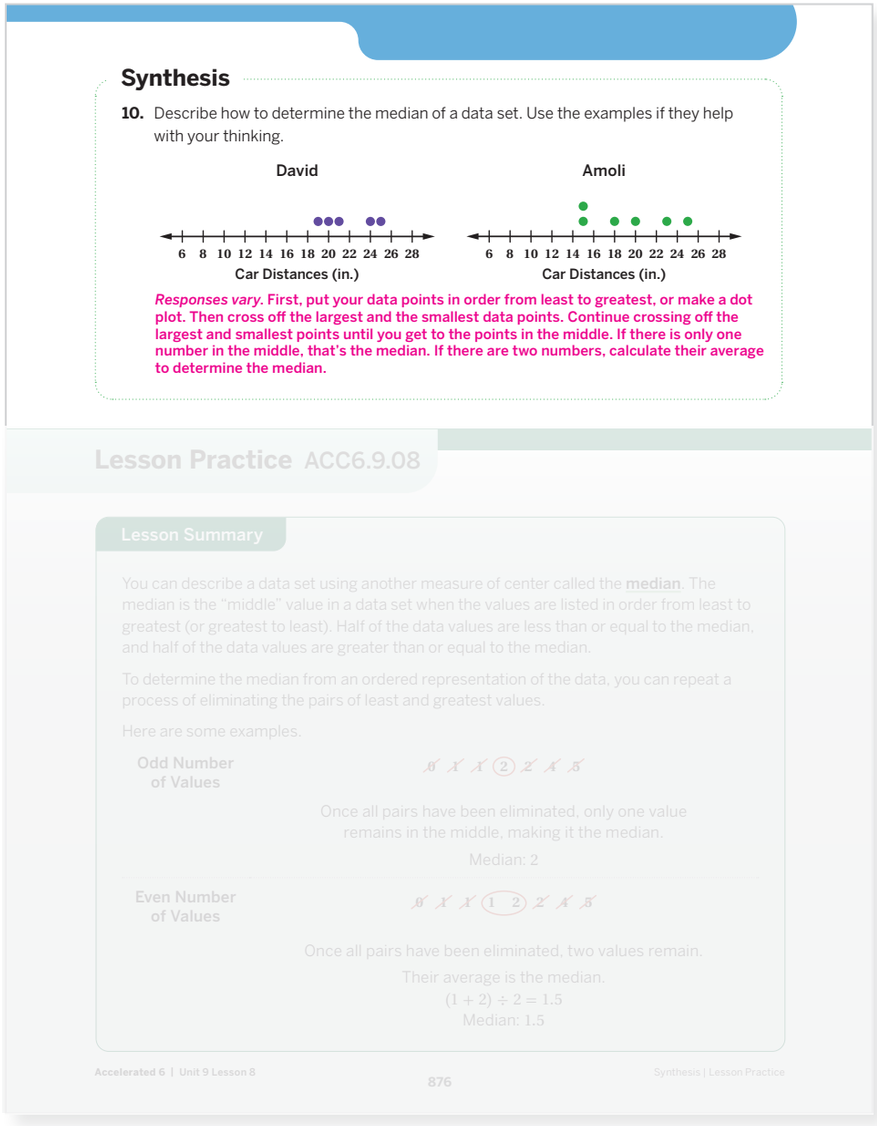
Purpose: Students synthesize their understanding of what the median of a data set is and how it is useful.

Invite students to respond independently, and then share their thinking with a partner.

Collect and share a variety of ideas, such as:
- The median is another measure of center that can be used to summarize the typical value of a data set.
- When there's an even number of data points, the median is the average of the middle two numbers.

MLR **MLR2: Collect and Display** Use this routine to formalize the definition of *median* using students' language. Display any new language, examples, or diagrams for students to reference later.

Note: If time allows, invite students to share with a partner what makes sense about using a dot plot to determine the median and what questions they still have.



Synthesis

10. Describe how to determine the median of a data set. Use the examples if they help with your thinking.

Responses vary. First, put your data points in order from least to greatest, or make a dot plot. Then cross off the largest and the smallest data points. Continue crossing off the largest and smallest points until you get to the points in the middle. If there is only one number in the middle, that's the median. If there are two numbers, calculate their average to determine the median.

Lesson Practice ACC6.9.08

Lesson Summary

You can describe a data set using another measure of center called the median. The median is the "middle" value in a data set when the values are listed in order from least to greatest (or greatest to least). Half of the data values are less than or equal to the median, and half of the data values are greater than or equal to the median.

To determine the median from an ordered representation of the data, you can repeat a process of eliminating the pairs of least and greatest values.

Here are some examples.

Odd Number of Values

6 7 7 (2) 7 7 8

Once all pairs have been eliminated, only one value remains in the middle, making it the median.

Median: 2

Even Number of Values

6 7 7 (1 2) 7 7 8

Once all pairs have been eliminated, two values remain.

Their average is the median.

$(1 + 2) \div 2 = 1.5$

Median: 1.5

Accelerated 6 | Unit 9 Lesson 8 876 Synthesis | Lesson Practice

Lesson Takeaway: The **median** is the middle value in a data set when the values are listed in order from least to greatest (or greatest to least). If there are two values in the middle, the median is their average. When the mean is not a good indication of what is typical for the data set, the median may be a better measure of center.

Independent | 5 min

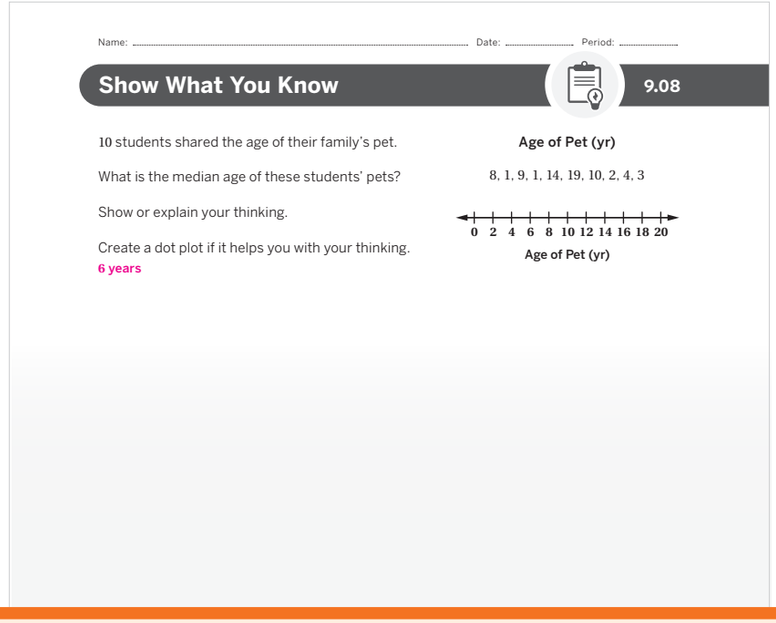
Show What You Know

Purpose: Students demonstrate their understanding by determining the median of the given data set.

Today's Goals

1. **Goal:** Describe the *median* as another measure of center that can be used to summarize a data set.
2. **Language Goal:** Determine and interpret the *median* of a data set from a list or a dot plot. **(Reading, Writing, Speaking, and Listening)**

Show What You Know (Assessment Resources)



Name: _____ Date: _____ Period: _____

Show What You Know 9.08

10 students shared the age of their family's pet.

What is the median age of these students' pets?

Show or explain your thinking.

Create a dot plot if it helps you with your thinking.

6 years

Age of Pet (yr)

8, 1, 9, 1, 14, 19, 10, 2, 4, 3

Differentiation Use after Lesson 8

Support

Provide targeted intervention for students by using these resources.

If students need support calculating the median, like crossing out values before sorting:

» **Respond to Student Thinking:**

- Revisit **Activity 2** (particularly Yona's thinking) as a class and invite students to share strategies for calculating the median.
- Review the **Lesson Summary**.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite them to explore the **Sub-Unit 2 Extension Activities**.
- Invite them to collect numerical data or use data from earlier in the unit to **calculate the median** and describe what it means in context.

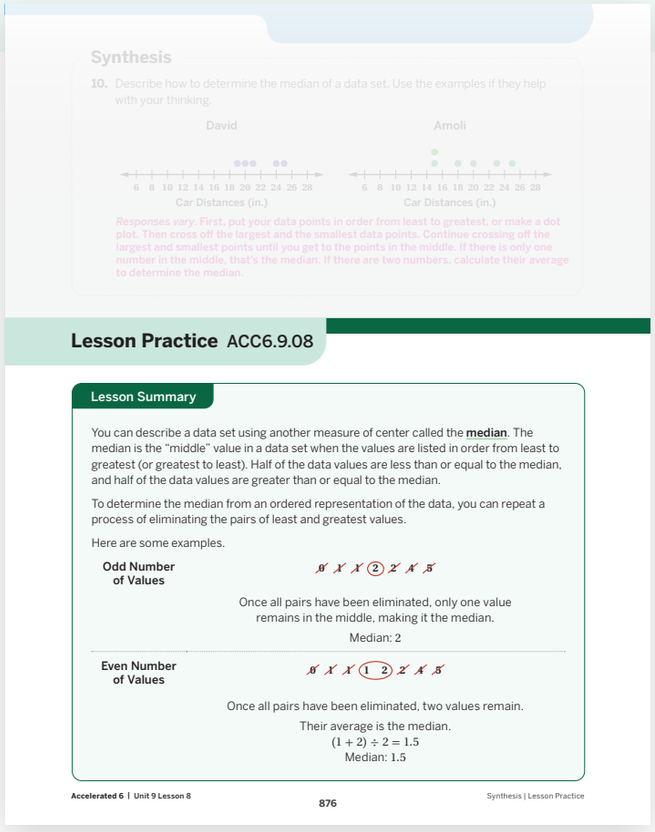
Professional Learning

Take a moment to reflect on the lesson you just taught. What about the facilitation felt successful? What are some areas for growth next time?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.



Synthesis

10. Describe how to determine the median of a data set. Use the examples if they help with your thinking.

David

Car Distances (in.)

Amoli

Car Distances (in.)

Responses vary. First, put your data points in order from least to greatest, or make a dot plot. Then cross off the largest and the smallest data points. Continue crossing off the largest and smallest points until you get to the points in the middle. If there is only one number in the middle, that's the median. If there are two numbers, calculate their average to determine the median.

Lesson Practice ACC6.9.08

Lesson Summary

You can describe a data set using another measure of center called the **median**. The median is the "middle" value in a data set when the values are listed in order from least to greatest (or greatest to least). Half of the data values are less than or equal to the median, and half of the data values are greater than or equal to the median.

To determine the median from an ordered representation of the data, you can repeat a process of eliminating the pairs of least and greatest values.

Here are some examples.

Odd Number of Values

~~6~~ ~~7~~ ~~7~~ (2) ~~2~~ ~~5~~

Once all pairs have been eliminated, only one value remains in the middle, making it the median.

Median: 2

Even Number of Values

~~6~~ ~~7~~ ~~7~~ (1 2) ~~2~~ ~~5~~

Once all pairs have been eliminated, two values remain.

Their average is the median.

$(1 + 2) \div 2 = 1.5$

Median: 1.5

Lesson Practice ACC6.9.08

Name: _____ Date: _____ Period: _____

Problems 1–2: Here is a data set.

1	6	7	6	2	9	3

1. Create a dot plot for this data.

2. Determine the median of this data.

6

Problems 3–4: Kayla wants to determine the median of the data in this dot plot. She starts solving the problem but isn't sure what to do next.

3. What could you say to help Kayla determine the median?

Responses vary. Because there are an even number of points, I would tell Kayla to calculate the average of the two points in the middle: 8 and 10.

4. What is the median of this data?

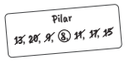
9

5. The table shows Prisha's scores after attempting the first level of a video game 10 times. What is her median score?

130	150	120	170	130
120	160	160	190	140

145

6. Pilar recorded the number of points she scored in her last 7 basketball games. She says that her median score was 8 points. Is she correct?



No

Explain your thinking.

Explanations vary. 8 is not the middle value of Pilar's data set. It's just the middle of the list. To determine the middle value of these numbers, Pilar should start by rearranging them from smallest to largest.

Lesson Practice ACC6.9.08

Name: _____ Date: _____ Period: _____

7. *Add* points to this dot plot to make the median 4.

Responses vary.

FAST Practice

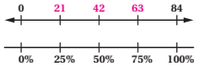
8. Cross out points to remove from this dot plot to make the median 4.

Responses vary.

Spiral Review

9. Determine 25%, 50%, and 75% of 84.

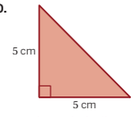
Use the double number line if it helps with your thinking.



Sample shown on diagram.

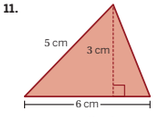
Problems 10–12: Calculate the area of each triangle.

10.



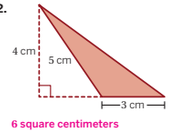
12.5 (or equivalent) square centimeters

11.



9 square centimeters

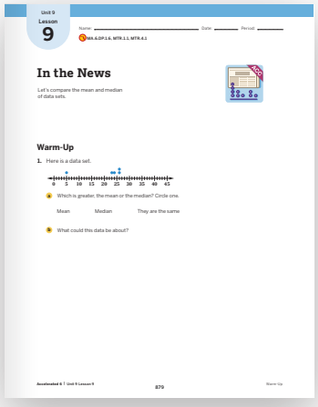
12.



6 square centimeters

Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1–2, 4–5	1	MA.6.DP.1.2
	3, 6	2	MA.6.DP.1.2
	7	3	MA.6.DP.1.2
FAST Practice	8	3	MA.6.DP.1.2
Spiral Review			
	9	1	MA.6.AR.3.4
	10–12	1	MA.6.GR.2.1



In the News

Comparing Measures of Center

Let's compare the mean and median of data sets.

Learning Outcomes

Today's Goals

1. **Language Goal:** Justify whether the mean or median best describes a given data set. **(Reading, Writing, Speaking, and Listening)**
2. **Language Goal:** Explain how the shape of a data set affects its mean and median. **(Speaking and Listening)**

Students consider how the shape of a data set affects the mean and median and then decide which measure of center more accurately describes the data. They examine data sets where the mean and median differ, and consider why the mean is more affected by extreme values than the median. Students recognize that the choice of measure of center can be subjective, and the choice should always consider the context of the problem and the desired insights or questions being asked. **(MTR.1.1)**

Coherence

Prior Learning

In Lessons 6 and 8, students used the mean and the median, respectively, as measures of center to summarize data sets.

Future Learning

In Lesson 10, students will determine the ranges of data sets and use them to describe data.

Connections

- Students build on their understanding of mean, median, and mode as the compare these values to analyze problems in context. (mathematical, real world)

FL B.E.S.T. Math Benchmarks

Addressing

MA.6.DP.1.6
Given a real-world scenario, determine and describe how changes in data values impact measures of center and variation.

Mathematical Thinking and Reasoning Standards: MTR.1.1, MTR.4.1

ELA Expectation: ELA.K12.EE.1.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.5.DP.1.2	**MA.7.DP.1.1**
	MA.7.DP.1.2

Lesson at a Glance

~ 45 min

MA.6.DP.1.6

Warm-Up

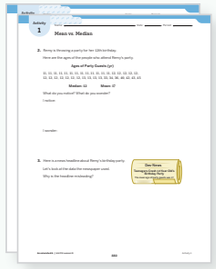
Pairs | 5 min

Students examine a data set where the mean and median differ to prompt thinking about when and why a data set may have two different measures of center.

Activity 1

Pairs | 15 min

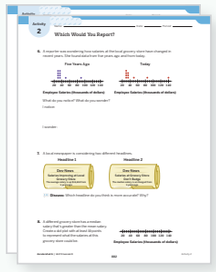
Students compare the mean and median of two different data sets, including one with extreme values, to conclude that extreme values impact the mean more than the median. **Routine:** Notice and Wonder



Activity 2

Pairs | 15 min

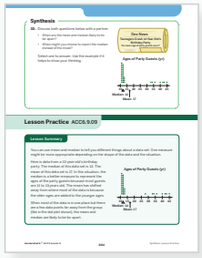
Students examine data about salaries in fictional companies and make decisions about which measure of center more accurately describes the data. **Routine:** Notice and Wonder **(MTR.1.1)**



Synthesis

Whole Class | 5 min

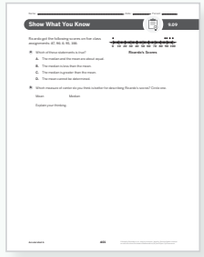
Students synthesize their understanding of how the shape of a data set affects its mean and median and how to choose the better measure of center for summarizing the data.



Show What You Know

Independent | 5 min

Students demonstrate their understanding by comparing measures of center and deciding which one more accurately describes the data set.



Prep Checklist

This lesson includes:

- Student Edition
- Show What You Know
- Calculators

Pairs | 5 min

Warm-Up

Purpose: Students examine a data set where the mean and median differ to prompt thinking about when and why a data set may have two different measures of center.

Short on time? Consider omitting Problem 1b.

1 Launch

Encourage students to list the values in the dot plot to help their thinking.

Invite students to share their ideas about what the data could be about. **(MTR.1.1)**

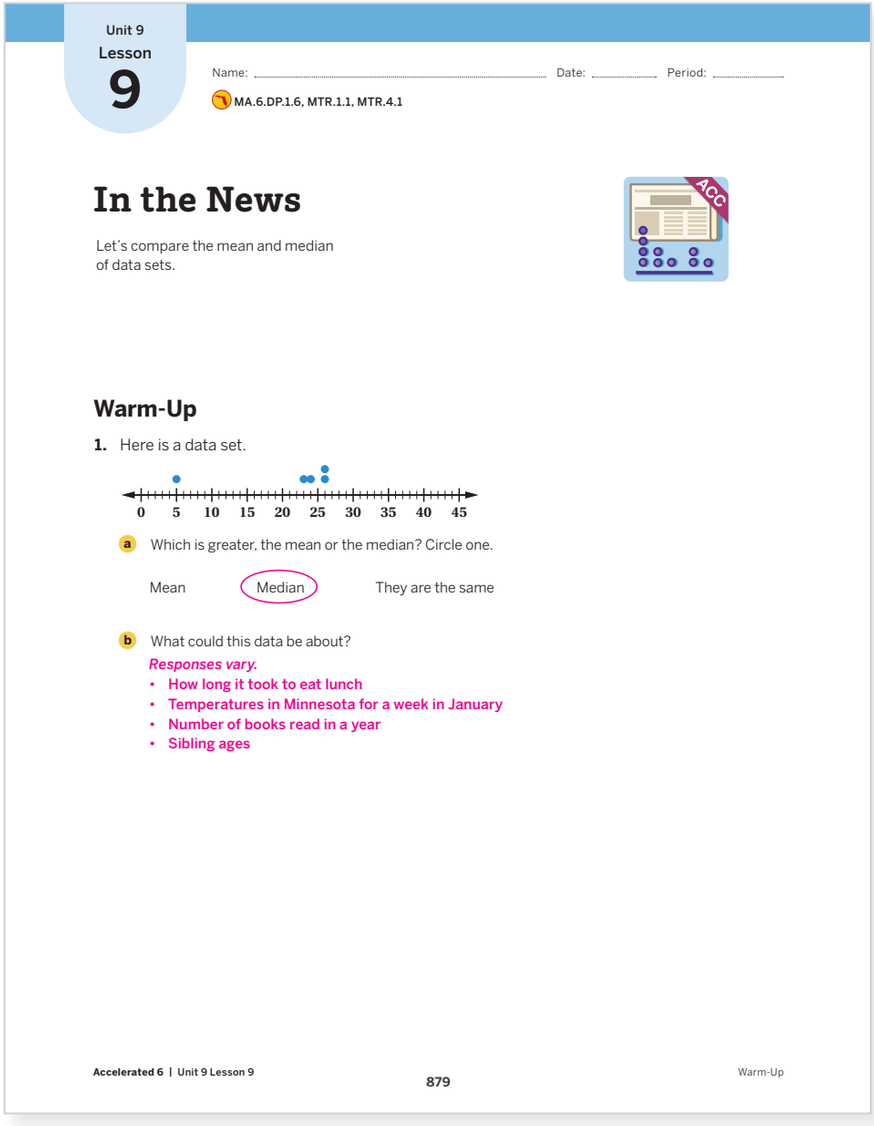
D **Differentiation: (Stretch)** Consider asking students who would like to explore the mean and median further, "Do you think it is possible to make the mean equal to 30? Do you think it is possible to make the median equal to 30? Why or why not?"

2 Connect

Invite students to share their responses.

Consider asking:

- "How did you decide which was greater: the mean or the median?"
- "If you change the location of a point, do you think the mean or the median would be more affected?"



Unit 9
Lesson
9

Name: _____ Date: _____ Period: _____

MA.6.DP.1.6, MTR.1.1, MTR.4.1

In the News

Let's compare the mean and median of data sets.

ACC

Warm-Up

1. Here is a data set.

a. Which is greater, the mean or the median? Circle one.

Mean (Median) They are the same

b. What could this data be about?

Responses vary.

- How long it took to eat lunch
- Temperatures in Minnesota for a week in January
- Number of books read in a year
- Sibling ages

Accelerated 6 | Unit 9 Lesson 9 879 Warm-Up

Pairs | 15 min

Activity 1 Mean vs. Median

Purpose: Students compare the mean and median of two different data sets, including one with extreme values, to conclude that extreme values impact the mean more than the median.

Short on time? Consider discussing Problem 5 as a whole class.

1 Launch

Consider asking, "Have you ever attended a birthday party? If so, what were the ages of the people who attended the party?"

Use the Notice and Wonder routine to promote curiosity and help students make sense of the data values and the measures of center. **(MTR.1.1)**

2 Monitor

To support students getting started, consider asking, "In what way is this headline true? In what way is it false?"

Pause and invite students to share their thinking about the headline.

Activity 1 continued >

Activity 1

Name: Date: Period:

Mean vs. Median

2. Remy is throwing a party for her 12th birthday.

Here are the ages of the people who attend Remy's party.

Ages of Party Guests (yr)

11, 11, 11, 11, 11, 11, 11, 11, 11, 11, 11, 11, 12, 12, 12, 12, 12, 12, 12, 12, 12, 12, 12, 13, 13, 13, 13, 33, 34, 36, 40, 42, 43, 45

Median: 12 **Mean: 17**

What do you notice? What do you wonder?

I notice: *Responses vary.*
- I notice that more 11-year-olds are attending the party than any other age.
- I notice that there are fewer adults than kids.
- I notice that the median is the same as Remy's age.

I wonder: *Responses vary.*
- I wonder if any of the guests are siblings.
- I wonder how the adults at the party are related to Remy.
- I wonder why the mean is greater than the median.

3. Here is a news headline about Remy's birthday party.

Let's look at the data the newspaper used.

Why is the headline misleading?

Responses vary. The headline makes it seem like 17-year-olds crashed a 12-year-old's party. In truth, all the kids at the party were between 11 and 13 years old. There were also 7 adults, probably parents, which made the mean age 17.

Accelerated 6 | Unit 9 Lesson 9 880 Activity 1

Pairs | 15 min

Activity 1 Mean vs. Median (continued)

Purpose: Students compare the mean and median of two different data sets, including one with extreme values, to conclude that extreme values impact the mean more than the median.

Short on time? Consider discussing Problem 5 as a whole class.

2 Monitor

Direct students' attention to the list of Remy's party guests.

Select and sequence students' responses to share during the Connect.

D Differentiation

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Consider asking, "How does calculating the mean differ from calculating the median?"
Recognize that: • The median is a typical value for the data set. • The mean is not a typical value for the data set.	**Strengthen** Consider asking, "Why does it make sense that the typical age of guests at Remy's party is 12? Why does 17 not make sense as the typical age of a guest at Remy's party?"
Recognize that the mean is pulled in the direction of the few extreme values (older ages) in the data set.	**Strengthen** Consider asking, "Why might the data set include these few extreme values?"

Encourage students to think about how the location of the data values with respect to the range, center, and spread of Remy's aunt's data compares to Remy's data.

3 Connect

Invite students to share why they think the mean is greater than the median in Problem 4.

Consider asking, "Why do you think the median was less affected by the ages of the older people than the mean? Which measure of center feels more accurate to you in this situation?" **(MTR.1.1)**

Discuss how the data of Remy's aunt's party compares to the data of Remy's party, especially how the data values fall within the range of each data set. Encourage students to think about how the shape of the data affects the median and mean. **(MTR.4.1)**

To surface the Key Takeaway, consider asking, "How do you think the mean and median would compare if the 7 largest data values in Remy's data set were removed?"

Activity 1

Name: Date: Period:

Mean vs. Median (continued)

4. Why do you think the mean is higher than the median?

Responses vary.

- The mean is higher than the median because the older ages, like 45, are added with the younger ages to make the mean larger. The median is just the middle of the numbers, so it doesn't matter that a few of the ages are really high.
- I see a lot of data listed for the younger guests. The older ages, mixed with younger ages, made the mean larger. The median is just the middle of the numbers, so it doesn't matter that a few of the ages are really high.

5. Remy's aunt also threw a party.

Ages of Remy's Aunt's Party Guests (yr)

36, 36, 36, 37, 37, 37, 37, 37, 37, 38, 38, 39, 39, 39, 39, 40, 40, 40, 40, 40, 40, 41, 41, 41, 41, 41, 42, 42, 42, 43, 43, 44, 44, 44, 44, 45

Median: 40 **Mean: 40**

a. Compare the data for Remy's party guests with the data for her aunt's party guests.

Responses vary.

- The ages at Remy's party are more spread out than the ages of her aunt's party.
- The center of Remy's data is much lower than the center for her aunt's data.
- The data for Remy's party has most of the values less than the mean, between 11 and 13, and few values that are much greater than the mean. The data for her aunt's party shows values all close to the mean.

b. **Discuss:** Why do you think the mean and median ages are closer together for Remy's aunt's party than Remy's party?

Responses vary. The data is closer together so that means the mean and median are also closer together.

Accelerated 6 | Unit 9 Lesson 9 881 Activity 1

Key Takeaway: Because extreme values influence the mean more than the median, they can be used to reason about the value of the mean. The mean is calculated using every data value, so it shifts away from typical values when there are extreme values in the data set.

Pairs | 15 min

Activity 2 Which Would You Report?

Purpose: Students examine data about salaries in fictional companies and make decisions about which measure of center more accurately describes the data. **(MTR.1.1)**

Short on time? Consider facilitating the **Notice and Wonder** routine as a discussion only.

1 Launch

Direct students' attention to the dot plots showing employee salaries from five years ago and today and read the text aloud. Highlight that these values are measured in thousands of dollars. Consider selecting a point on one of the dot plots and asking, "What does this dot represent? What does the 50 mean?"

Use the Notice and Wonder routine to promote curiosity and help students make sense of how the data has changed from five years ago to today. **(MTR.1.1)**

Distribute calculators to support students in calculating the mean and median of the dot plots.

2 Monitor

To support students getting started, share that many newspapers use the term *average* to report the mean.

ELL **English Language Learners** Invite students to read each headline aloud and paraphrase what it means. Clarify any words they don't know. **(Reading)**

Pause the class and poll students to see which headline they think is more accurate. It is okay — even desirable — for there to be a lack of consensus. If there is not a consensus, invite students to make arguments and justify their reasoning for each response. **(MTR.4.1)**

Encourage students to look at the dot plots showing employee salaries, noting that the median salary is less than the mean. Consider asking, "How would the shape of the dot plot change if the median salary is *greater* than the mean?"

Invite students to check their measure of center with a partner. **(MTR.4.1)**

Look for examples of dot plots students created where most of the data values are on the right with a few extreme values on the left. Invite these students to share their thinking during the Connect.

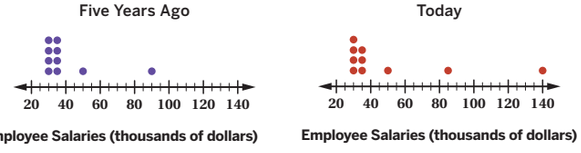
Activity 2 continued >

Activity 2

Name: Date: Period:

Which Would You Report?

6. A reporter was wondering how salaries at the local grocery store have changed in recent years. She found data from five years ago and from today.



What do you notice? What do you wonder?

I notice: *Responses vary.*
- There are the same number of dots on both data sets.
- Both graphs have more data on the left than on the right.
- One employee's salary today is $140,000.

I wonder: *Responses vary.*
- Who the person is that makes more than everyone else today.
- If this data shows everyone who works at the store or just some of the people.
- If the person who made $50,000 five years ago is the same person who makes $50,000 today.

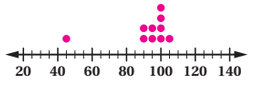
7. A local newspaper is considering two different headlines.

Discuss: Which headline do you think is more accurate? Why?

Responses and explanations vary.
- Headline 1 is more accurate because the mean incorporates the values of every salary.
- Headline 2 is more accurate because most people still make the same amount of money. Only two salaries actually changed.

8. A different grocery store has a median salary that's greater than the mean salary. Create a dot plot with at least 10 points to represent what the salaries at this grocery store could be.

Dot plots vary.



Accelerated 6 | Unit 9 Lesson 9 882 Activity 2

Pairs | 15 min

Activity 2 Which Would You Report? (continued)

Purpose: Students examine data about salaries in fictional companies and make decisions about which measure of center more accurately describes the data. **(MTR.1.1)**

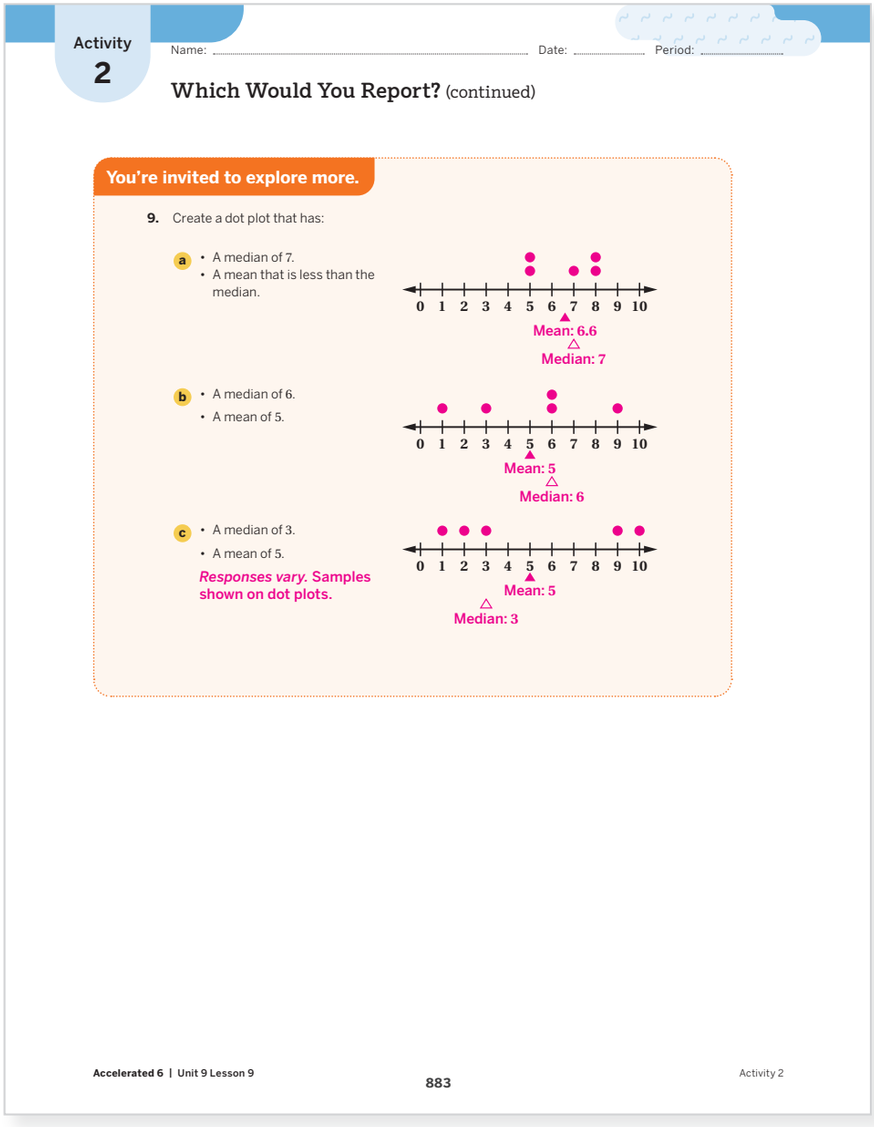
2 Monitor

D **Differentiation: You're invited to explore more.** **(Stretch)** Invite students who would like to explore dot plots with different medians and means further to complete this optional task. Encourage them to discuss their thinking with a partner. **(MTR.4.1)**

3 Connect

Share student dot plots. Consider asking, "What do all of these dot plots have in common? How are their shapes alike?"

Consider asking, "What will the dot plot look like if the mean and median are far apart? How about if they are close together?" *Responses vary.* If the mean and median are far apart, the dot plot will have most of the data on one side and only a few data points on the other side. If they are close together, the data points will be more balanced on either side of the mean.



Activity 2

Name: Date: Period:

Which Would You Report? (continued)

You're invited to explore more.

9. Create a dot plot that has:

a • A median of 7.
• A mean that is less than the median.

b • A median of 6.
• A mean of 5.

c • A median of 3.
• A mean of 5.

Responses vary. **Samples shown on dot plots.**

Accelerated 6 | Unit 9 Lesson 9 883 Activity 2

Whole Class | 5 min

Synthesis

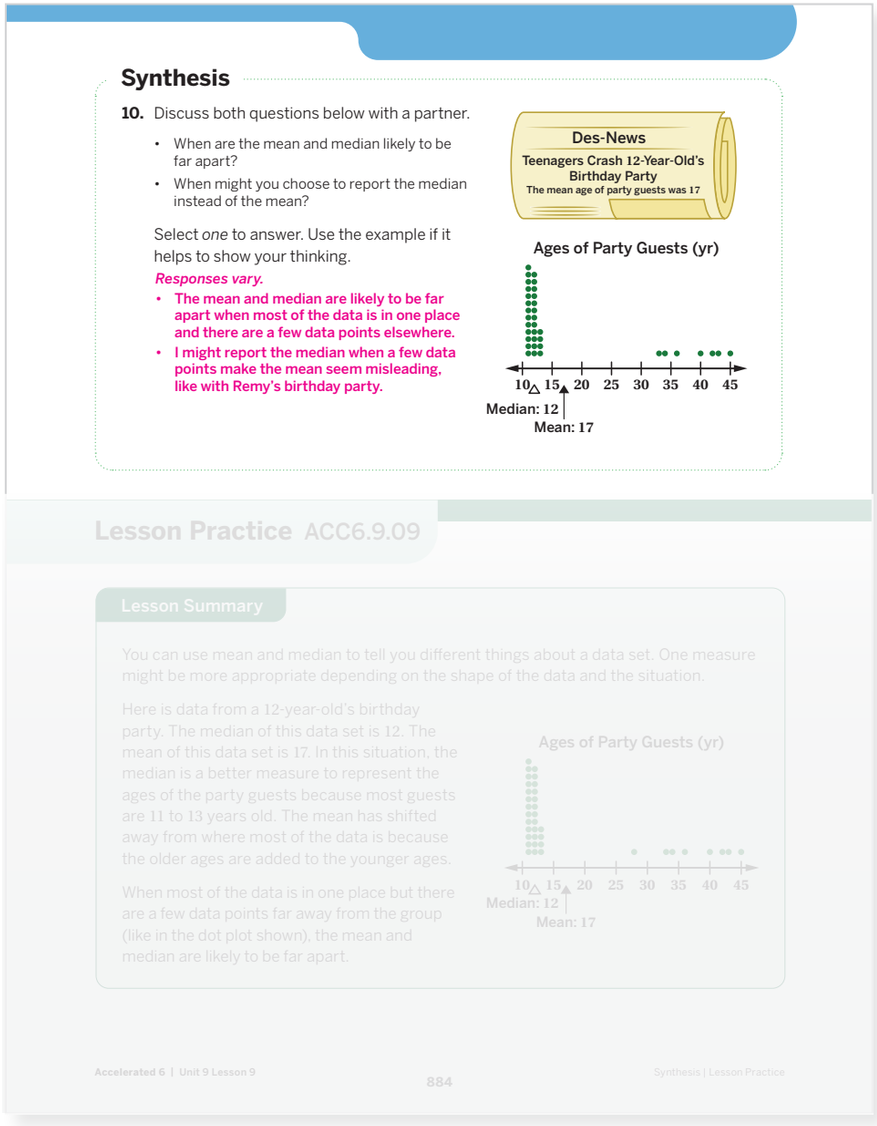
Purpose: Students synthesize their understanding of how the shape of a data set affects its mean and median and how to choose the better measure of center for summarizing the data.

Invite students to respond independently, and then share their thinking with a partner.

Collect and share a variety of ideas, such as:
- The mean is more affected by values that are far from the majority of the data values.
- The shape of the data can help to determine which measure of center, the mean or median, more accurately describes the data set.

MLR **MLR1: Stronger and Clearer Each Time (optional)** Consider using this routine to support students in developing precise language. Invite students to meet with a partner, give feedback on each other's drafts, then use the feedback to write a second clearer draft. See the Routine Facilitation Guide for more information.

Note: If time allows, invite students to share with a partner what makes sense about the strategies they used to choose the best measure of center to summarize a data set and what questions they still have.



Synthesis

10. Discuss both questions below with a partner.

- When are the mean and median likely to be far apart?
- When might you choose to report the median instead of the mean?

Select *one* to answer. Use the example if it helps to show your thinking.

Responses vary.

- The mean and median are likely to be far apart when most of the data is in one place and there are a few data points elsewhere.
- I might report the median when a few data points make the mean seem misleading, like with Remy's birthday party.

Des-News
Teenagers Crash 12-Year-Old's Birthday Party
The mean age of party guests was 17

Ages of Party Guests (yr)

Lesson Practice ACC6.9.09

Lesson Summary

You can use mean and median to tell you different things about a data set. One measure might be more appropriate depending on the shape of the data and the situation.

Here is data from a 12-year-old's birthday party. The median of this data set is 12. The mean of this data set is 17. In this situation, the median is a better measure to represent the ages of the party guests because most guests are 11 to 13 years old. The mean has shifted away from where most of the data is because the older ages are added to the younger ages.

When most of the data is in one place but there are a few data points far away from the group (like in the dot plot shown), the mean and median are likely to be far apart.

Ages of Party Guests (yr)

Accelerated 6 | Unit 9 Lesson 9 884 Synthesis | Lesson Practice

Lesson Takeaway: Sometimes one measure of center summarizes a data set better than the other. It is important to consider the shape of the data, the question being asked, and the context when making a decision about which measure of center to use.

Independent | 5 min

Show What You Know

Purpose: Students demonstrate their understanding by comparing measures of center and deciding which one more accurately describes the data set.

Today's Goals

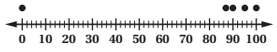
1. **Language Goal:** Justify whether the mean or median best describes a given data set. **(Reading, Writing, Speaking, and Listening)**
2. **Language Goal:** Explain how the shape of a data set affects its mean and median. **(Speaking and Listening)**

Show What You Know (Assessment Resources)

Name: _____ Date: _____ Period: _____

Show What You Know 9.09

Ricardo got the following scores on five class assignments: 87, 90, 0, 95, 100.



Ricardo's Scores

a. Which of these statements is *true*?

A. The median and the mean are about equal.
B. The median is less than the mean.
C. The median is greater than the mean.
D. The mean cannot be determined.

b. Which measure of center do you think is better for describing Ricardo's scores? Circle one.

Mean Median

Explain your thinking.

Responses and explanations vary.
- Median. Ricardo did really well on 4 of his 5 assignments, so reporting the median makes more sense.
- Mean. Otherwise, the assignment that Ricardo did badly on won't really be included.

Differentiation Use after Lesson 9

Support

Provide targeted intervention for students by using these resources.

If students need support determining how the mean or median describe data:

» **Respond to Student Thinking:**

- Revisit **Activity 2** (particularly "Headline News") as a class and invite students to share strategies for using mean and median to describe data.
- Review the **Lesson Summary**.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite them to explore the **Sub-Unit 2 Extension Activities**.
- Invite them to complete the **Explore More** task and discuss their thinking with a partner.

Professional Learning

Which students' ideas and dot plots were you able to highlight during Activity 2? How did working with a partner affect student engagement and task completion?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

10. Discuss both questions below with a partner.

- When are the mean and median likely to be far apart?
- When might you choose to report the median instead of the mean?

Select *one* to answer. Use the example if it helps to show your thinking.

Responses vary.

- The mean and median are likely to be far apart when most of the data is in one place and there are a few data points elsewhere.
- I might report the median when a few data points make the mean seem misleading, like with Remy's birthday party.

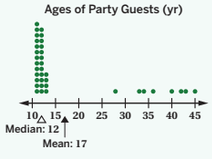
Lesson Practice ACC6.9.09

Lesson Summary

You can use mean and median to tell you different things about a data set. One measure might be more appropriate depending on the shape of the data and the situation.

Here is data from a 12-year-old's birthday party. The median of this data set is 12. The mean of this data set is 17. In this situation, the median is a better measure to represent the ages of the party guests because most guests are 11 to 13 years old. The mean has shifted away from where most of the data is because the older ages are added to the younger ages.

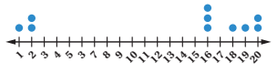
When most of the data is in one place but there are a few data points far away from the group (like in the dot plot shown), the mean and median are likely to be far apart.



Lesson Practice ACC6.9.09

Name: _____ Date: _____ Period: _____

1. Here are two statistics related to this dot plot: 13 and 16.



Without calculating, determine which number is the median and which number is the mean.

Median: 16 Mean: 13

2. Here are the scores for Sahana's history homework.

100	100	100	100	95	100	90	100	0

The history teacher uses the mean to calculate an overall grade for homework. How might Sahana argue that the median is a better way to measure her work?

Responses vary. Sahana could argue that the median more accurately reflects the center of her data. The one score of 0 decreases the mean by a lot when the rest of her grades are 90 to 100.

3. Ayaan competed in a bowling tournament. Here are his scores for each game.

Game 1	Game 2	Game 3	Game 4	Game 5	Game 6
300	250	275	120	243	290

Do you think the tournament judges should use the median or the mean to calculate Ayaan's final score? Justify your choice.

Responses vary.

- The judges should use the median because it's closer to most of Ayaan's other scores in the tournament.
- The judges should use the mean because the median would ignore Ayaan's worst game and that wouldn't be fair to the other players.

4. Create a dot plot with 5 data points that has:

- A median of 14.
- A mean less than 14.

Responses vary. Sample shown on dot plot.

Lesson Practice ACC6.9.09

Name: _____ Date: _____ Period: _____

5. Add one point to this dot plot to make the median *greater* than the mean.

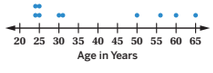
Responses vary. Sample shown on dot plot.

6. Add one point to this dot plot to make the median equal to the mean.

Responses vary. Sample shown on dot plot.

FAST Practice

7. Kweku surveyed 10 teachers at his school. The dot plot displays the ages of those teachers. Which of these statements is *true*?



A. The median is greater than the mean.

B. The median and the mean are the same.

C. The median is less than the mean.

D. The mean cannot be determined.

Spiral Review

Problems 8–11: Calculate each percentage.

8. 25% of 30
7.5

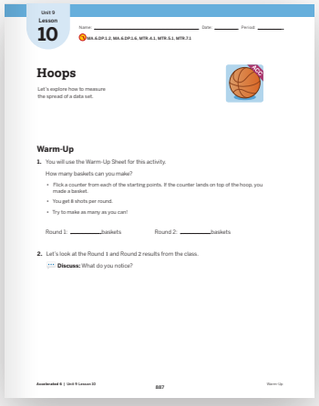
9. 75% of 30
22.5

10. 75% of 150
112.5

11. 58% of 18
10.44

Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1–6	2	MA.6.DP.1.6
FAST Practice	7	2	MA.6.DP.1.6
Spiral Review			
	8–11	1	MA.6.AR.3.4


Unit 9
Lesson
10

Name: _____ Date: _____ Period: _____

MA.6.DP.1.2, MA.6.DP.1.6, MTR.4.1, MTR.5.1, MTR.7.1

Hoops

Let's explore how to measure the spread of a data set.

Warm-Up

1. You will use the Warm-Up Sheet for this activity.

   How many baskets can you make?

   - Flick a counter from each of the starting points. If the counter lands on top of the hoop, you made a basket.
   - You get 4 shots per round.
   - Try to make as many as you can!

   Round 1: _____ baskets Round 2: _____ baskets

2. Let's look at the Round 1 and Round 2 results from the class.

   Discuss: What do you notice?

Accelerated 6 | Unit 9 Lesson 10 887 Warm-Up

Hoops

Calculating and Interpreting the Range

Let's explore how to measure the spread of a data set.

Learning Outcomes

Today's Goals

1. **Goal:** Calculate the range of a data set.
2. **Goal:** Analyze the effects on the range of a data set when adding or removing a data value.
3. **Language Goal:** Describe the range as a measure of spread. **(Speaking and Listening)**

Students make connections between consistency and the statistic of range in the context of basketball. Students learn how to calculate the range of a data set and analyze how adding or removing a data value affects the range. They also compare dot plots with different ranges, which helps them recognize that the greater the range, the greater the spread in a data set. **(MTR.5.1, MTR.7.1)**

Coherence

Prior Learning

In Lesson 4, students qualitatively described the range of a data set as a measure of spread.

Future Learning

In Lesson 12, students will compare the range and interquartile range (IQR) of real-world data sets.

Connections

- Students are introduced to range as it applies to problems in context. (mathematical, real world)

FL B.E.S.T. Math Benchmarks

Addressing

MA.6.DP.1.2

Given a numerical data set within a real-world context, find and interpret mean, median, mode and range.

MA.6.DP.1.6

Given a real-world scenario, determine and describe how changes in data values impact measures of center and variation.

Mathematical Thinking and Reasoning Standards: MTR.4.1, MTR.5.1, MTR.7.1

ELA Expectation: ELA.K12.EE.1.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.5.DP.1.2	MA.7.DP.1.1
	MA.7.DP.1.2

Vocabulary

Additional Vocabulary

measure of spread

Lesson at a Glance

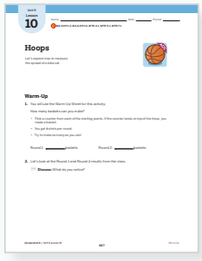
~ 45 min

MA.6.DP.1.2, MA.6.DP.1.6

Warm-Up

Independent | 5 min

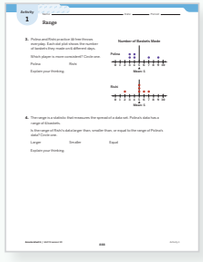
Students simulate two rounds of basketball to become familiar with the context they will use to generate data.
Routine: Notice and Wonder



Activity 1

Pairs | 12 min

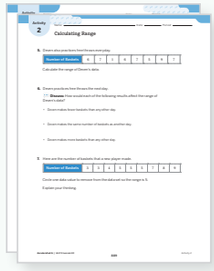
Students compare the ranges of two data sets to draw connections between consistency and range. **(MTR.5.1)**



Activity 2

Pairs | 18 min

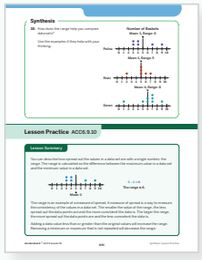
Students practice calculating range, analyze how changes in data affect the range, and order several data sets according to their ranges to make sense of range as a measure of spread. **(MTR.5.1)**



Synthesis

Whole Class | 5 min

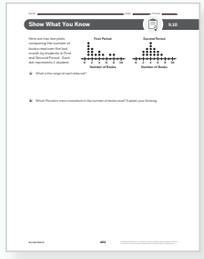
Students synthesize their understanding of how to calculate and interpret the range of a data set.



Show What You Know

Independent | 5 min

Students demonstrate their understanding by calculating the range of data sets represented by dot plots.



Prep Checklist

Prepare Warm-Up Sheet

This lesson includes:

- Student Edition
- Show What You Know
- Counters
- Warm-Up Sheet, one per pair

Independent | 5 min

Warm-Up

Purpose: Students simulate two rounds of basketball to become familiar with the context they will use to generate data.

1 Launch

Demonstrate how to throw the basketball using a counter.

Consider asking, "How is this game scored?"
By the number of baskets made.

Distribute a Warm-Up Sheet and at least one counter to each pair of students. Note the 8 starting points from which students will take each shot.

2 Connect

To encourage playfulness, invite students to play a second round of the game.

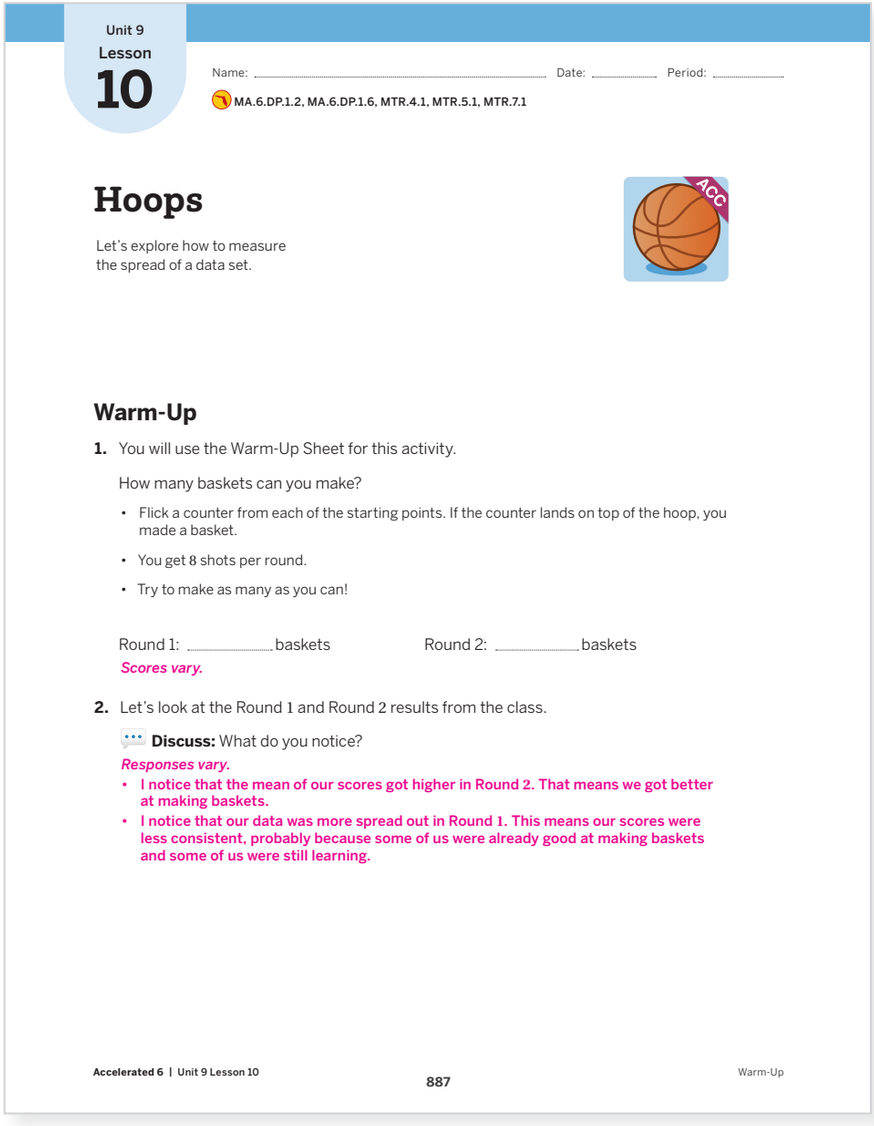
Display the class results from Round 1 and Round 2 by polling the class. Encourage students to write the lists of class data in their Student Edition.

Use the Notice and Wonder routine to promote curiosity and help students make sense of the class results from Rounds 1 and 2.

Consider asking:

- "What do these data sets tell us about how our rounds compare?"
- "Which data set is more spread out? Why do you think that is? What does this tell us about our class's free throws?" **(MTR.7.1)**

Note: If it does not come up naturally, consider asking, "As a class, did we achieve more consistent scores during Round 1 or Round 2? How can you tell?" Ensure that students understand that *consistent* means "similar each time."



Unit 9
Lesson
10

Name: _____ Date: _____ Period: _____

MA.6.DP.1.2, MA.6.DP.1.6, MTR.4.1, MTR.5.1, MTR.7.1

Hoops

Let's explore how to measure the spread of a data set.

Warm-Up

1. You will use the Warm-Up Sheet for this activity.

How many baskets can you make?

- Flick a counter from each of the starting points. If the counter lands on top of the hoop, you made a basket.
- You get 8 shots per round.
- Try to make as many as you can!

Round 1: _____ baskets Round 2: _____ baskets

Scores vary.

2. Let's look at the Round 1 and Round 2 results from the class.

Discuss: What do you notice?

Responses vary.

- I notice that the mean of our scores got higher in Round 2. That means we got better at making baskets.
- I notice that our data was more spread out in Round 1. This means our scores were less consistent, probably because some of us were already good at making baskets and some of us were still learning.

Accelerated 6 | Unit 9 Lesson 10 887 Warm-Up

Pairs | 12 min

Activity 1 Range

Purpose: Students compare the ranges of two data sets to draw connections between consistency and range. **(MTR.5.1)**

Short on time? Consider completing Problem 3 together as a class.

1 Launch

Invite students to read the prompt aloud then restate it in their own words. Consider asking, "What does each dot on Polina's dot plot represent? How does this differ from the data sets of our class results in the Warm-Up?"

2 Monitor

To support students getting started, encourage them to discuss which player they would want on their team and why.

Look for students who think the range of Rishi's data is smaller and invite them to share during the Connect.

3 Connect

Invite students to share how they know the range of Rishi's data is smaller.

To surface the key takeaway, consider asking, "How does range relate to consistency?" **(MTR.5.1)**

Activity 1

Name: Date: Period:

Range

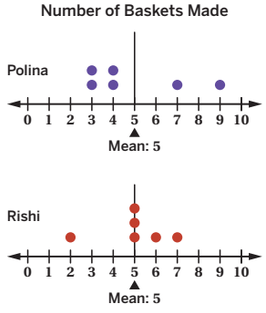
3. Polina and Rishi practice 10 free throws everyday. Each dot plot shows the number of baskets they made on 6 different days.

Which player is more consistent? Circle one.

Polina (Rishi)

Explain your thinking.

Explanations vary. Rishi is more consistent. I can tell because his dots are less spread out and most of his scores are close to the mean.



4. The range is a statistic that measures the spread of a data set. Polina's data has a range of 6 baskets.

Is the range of Rishi's data larger than, smaller than, or equal to the range of Polina's data? Circle one.

Larger (Smaller) Equal

Explain your thinking.

Responses vary. The minimum data value of Rishi's data is 2, and the maximum data value is 7. The difference between the maximum value and the minimum value is 5, so the range of Rishi's data is smaller than the range of Polina's data.

Accelerated 6 | Unit 9 Lesson 10 888 Activity 1

Key Takeaway: The range is a statistic that measures the consistency of a data set. For example, because Rishi's data is less spread out, the range of his data is smaller. The smaller the range, the more consistent the data. This indicates that Rishi is more consistent in the number of baskets he makes than Polina is.

Pairs | 18 min

Activity 2 Calculating Range

Purpose: Students practice calculating range, analyze how changes in data affect the range, and order several data sets according to their ranges to make sense of range as a measure of spread. **(MTR.5.1)**

Short on time? Consider discussing Problem 6 together as a class.

1 Launch

Direct students' attention to the table of data.

2 Monitor

To support students getting started, consider asking, "How can we organize the data so it is easier to understand?"

Listen to student discussions, offering help or encouragement as needed. Consider asking, "How can you prove that your responses are correct?" **(MTR.4.1)**

A **Accessibility: Conceptual Processing** Invite students to use trial and error, removing a data value and then calculating the range. When they determine the correct value to remove, encourage students to consider why that data value worked when others did not. **(MTR.5.1)**

Activity 2 continued >

Activity 2

Name: Date: Period:

Calculating Range

5. Deven also practices free throws everyday.

Number of Baskets	6	7	1	6	7	5	9	7

Calculate the range of Deven's data.

8 baskets

6. Deven practices free throws the next day.

Discuss: How would each of the following results affect the range of Deven's data?

- Deven makes fewer baskets than any other day.
  Responses vary. The range will be greater.
- Deven makes the same number of baskets as another day.
  Responses vary. The range will be the same.
- Deven makes more baskets than any other day.
  Responses vary. The range will be greater.

7. Here are the number of baskets that a new player made.

Number of Baskets	3	3	4	5	5	5	7	8	(9)

Circle one data value to remove from the data set so the range is 5.

Explain your thinking.

Responses vary. The range of the data set is $9 - 3 = 6$. To decrease the range, I must remove the maximum or minimum data value. However, the minimum data value repeats, so I must remove the maximum data value to change the range.

Accelerated 6 | Unit 9 Lesson 10 889 Activity 2

Pairs | 18 min

Activity 2 Calculating Range (continued)

Purpose: Students practice calculating range, analyze how changes in data affect the range, and order several data sets according to their ranges to make sense of range as a measure of spread. **(MTR.5.1)**

2 Monitor

D Differentiation

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Consider asking, "In which data set are the dots least spread out?" Encourage students to refer back to Activity 1 where they compared the ranges of Polina's and Rishi's data.
Order the data sets from the smallest to the largest mean.	**Support** Share that range is a measure of spread. Consider asking, "How can we see spread on a dot plot?"
Order the data sets without calculating their ranges.	**Strengthen** Invite them to share their thinking during the Connect.
Would like to explore the range further.	**Stretch** **You're invited to explore.** Invite students to complete this optional task. Encourage them to discuss their thinking with a partner.

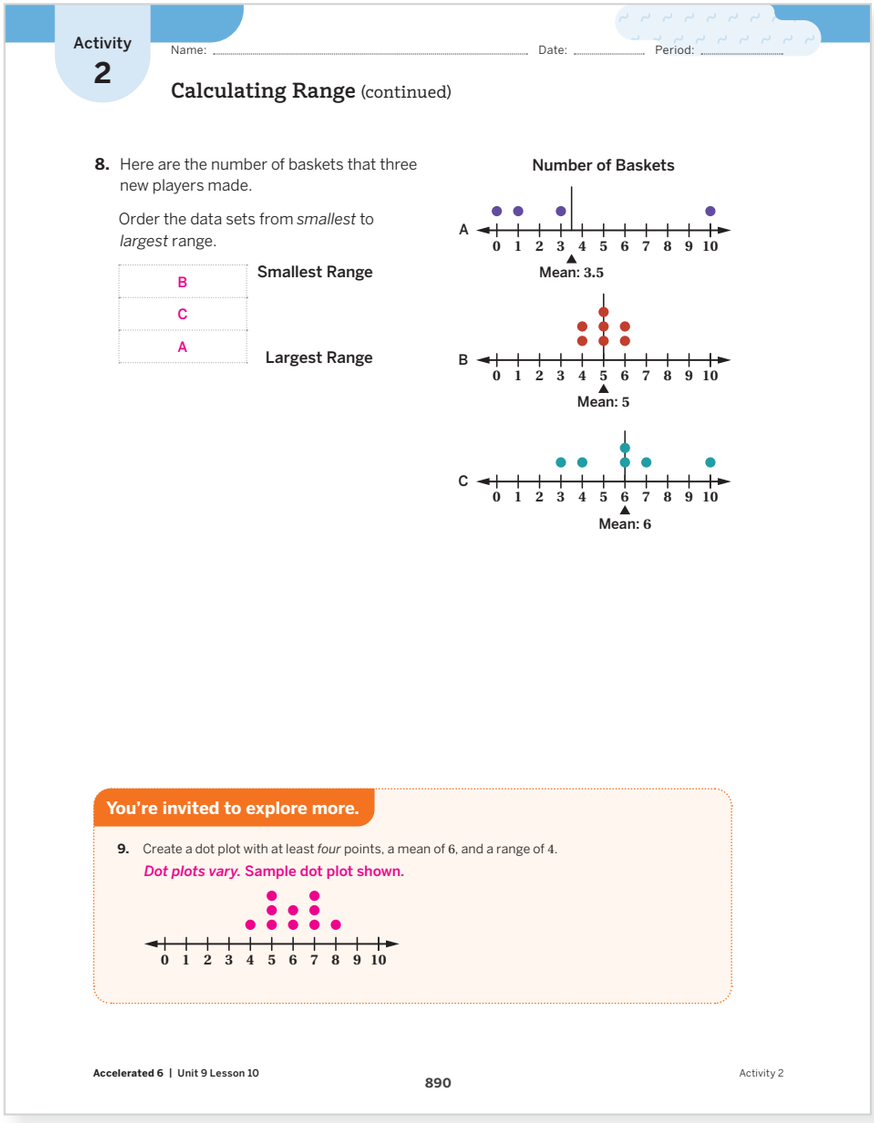
3 Connect

Invite students to share how changes to the values in a data set can affect the range. Encourage students to provide explanations or examples to support their thinking. **(MTR.5.1)**

Invite students to refine their definition of the term *measure of spread* using students' language and the glossary, noting that range is a measure of spread (just as mean, median, and mode are measures of center).

Consider asking, "How did you decide which had the smallest range? The largest range?"

Responses vary. Choice B has the smallest range because its points are the least spread out. Choice A, on the other hand, has the largest range because its points are the most spread out.



Activity 2

Name: Date: Period:

Calculating Range (continued)

8. Here are the number of baskets that three new players made.

Order the data sets from *smallest* to *largest* range.

B

C

A

Smallest Range

Largest Range

Number of Baskets

A
0 1 2 3 4 5 6 7 8 9 10
Mean: 3.5

B
0 1 2 3 4 5 6 7 8 9 10
Mean: 5

C
0 1 2 3 4 5 6 7 8 9 10
Mean: 6

You're invited to explore more.

9. Create a dot plot with at least *four* points, a mean of 6, and a range of 4.

Dot plots vary. Sample dot plot shown.

0 1 2 3 4 5 6 7 8 9 10

Accelerated 6 | Unit 9 Lesson 10 890 Activity 2

Key Takeaway: The range is a measure of spread. To calculate range, subtract the minimum value from the maximum value in a data set.

Whole Class | 5 min

Synthesis

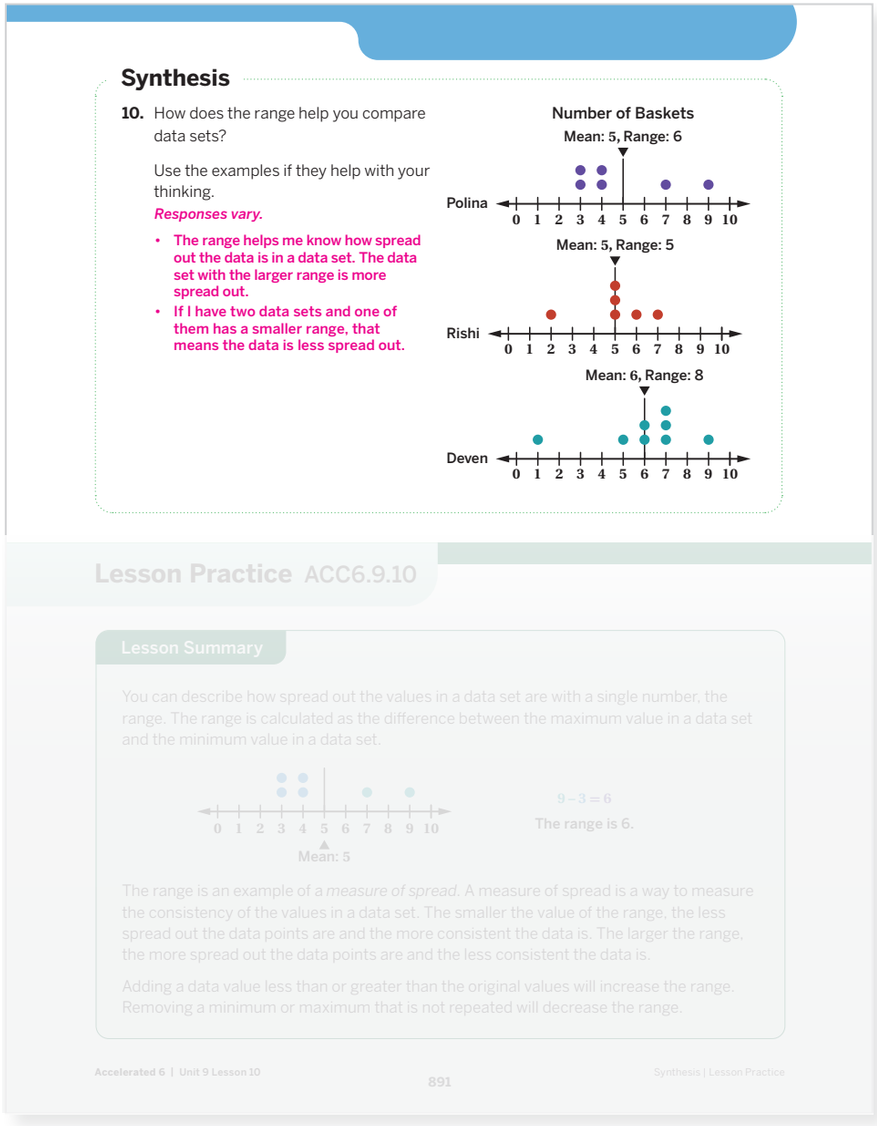
Purpose: Students synthesize their understanding of how to calculate and interpret the range of a data set.

Invite students to respond independently, and then share their thinking with a partner.

Collect and share a variety of ideas, such as:

- The range is the difference between the maximum and minimum values in a data set.
- Range is a measure of spread used to measure the consistency of a data set. The smaller the range, the more consistent the data are.

Note: If time allows, invite students to revise their first response based on the discussion.



Synthesis

10. How does the range help you compare data sets?

Use the examples if they help with your thinking.

Responses vary.

- The range helps me know how spread out the data is in a data set. The data set with the larger range is more spread out.
- If I have two data sets and one of them has a smaller range, that means the data is less spread out.

Lesson Practice ACC6.9.10

Lesson Summary

You can describe how spread out the values in a data set are with a single number, the range. The range is calculated as the difference between the maximum value in a data set and the minimum value in a data set.

$9 - 3 = 6$

The range is 6.

The range is an example of a *measure of spread*. A measure of spread is a way to measure the consistency of the values in a data set. The smaller the value of the range, the less spread out the data points are and the more consistent the data is. The larger the range, the more spread out the data points are and the less consistent the data is.

Adding a data value less than or greater than the original values will increase the range. Removing a minimum or maximum that is not repeated will decrease the range.

Accelerated 6 | Unit 9 Lesson 10 891 Synthesis | Lesson Practice

Lesson Takeaway: The range is a measure of spread that is calculated by subtracting the minimum from the maximum value in a data set. Range can be used to compare the spreads of various data sets. The lower the value of the range the less spread there is in the data.

Independent | 5 min

Show What You Know

Purpose: Students demonstrate their understanding by calculating the range of data sets represented by dot plots.

Today's Goals

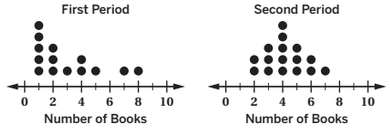
1. **Goal:** Calculate the range of a data set.
2. **Goal:** Analyze the effects on the range of a data set when adding or removing a data value.
3. **Language Goal:** Describe the range as a measure of spread. **(Speaking and Listening)**

Show What You Know (Assessment Resources)

Name: _____ Date: _____ Period: _____

Show What You Know 9.10

Here are two line plots comparing the number of books read over the last month by students in First and Second Period. Each dot represents 1 student.



a. What is the range of each data set?
First period range is 8 − 1 = 7 books.
Second period range is 7 − 2 = 5 books.

b. Which Period is more consistent in the number of books read? Explain your thinking.
Explanations vary. Second period is more consistent because the class has a smaller range of students who read books than First period.

Differentiation Use after Lesson 10

Support

Provide targeted intervention for students by using these resources.

If students need support calculating the range of data sets represented by dot plots:

» **Respond to Student Thinking:**

- Review the **Lesson Summary**.
- Selecting a few related **Practice Problems** and giving individualized feedback.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite them to explore the **Sub-Unit 2 Extension Activities**.
- Invite them to complete the **Explore More** task and discuss their thinking with a partner.

Professional Learning

In what ways did students' work in Activity 2 surprise you today?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

10. How does the range help you compare data sets?

Use the examples if they help with your thinking.

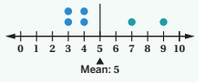
Responses vary.

- The range helps me know how spread out the data is in a data set. The data set with the larger range is more spread out.
- If I have two data sets and one of them has a smaller range, that means the data is less spread out.

Lesson Practice ACC6.9.10

Lesson Summary

You can describe how spread out the values in a data set are with a single number, the range. The range is calculated as the difference between the maximum value in a data set and the minimum value in a data set.



$9 - 3 = 6$

The range is 6.

The range is an example of a *measure of spread*. A measure of spread is a way to measure the consistency of the values in a data set. The smaller the value of the range, the less spread out the data points are and the more consistent the data is. The larger the range, the more spread out the data points are and the less consistent the data is.

Adding a data value less than or greater than the original values will increase the range. Removing a minimum or maximum that is not repeated will decrease the range.

Lesson Practice ACC6.9.10

Name: _____ Date: _____ Period: _____

1. This table shows the amount of time it takes 6 students to get to school.

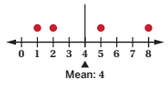
Time (minutes)	10	10	18	20	30	44

Calculate the range of this data set.

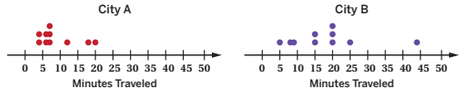
34 minutes

2. Calculate the range of this data set.

7



Problems 3–4: These dot plots show the travel times for 10 students from two cities.



3. The ranges have been calculated for you. Match each range to the correct city.

Range	City
16	A
39	B

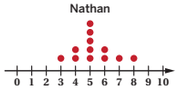
4. What data value can you add to City A so that the cities have the same range?

43 minutes

Lesson Practice ACC6.9.10

Name: _____ Date: _____ Period: _____

Problems 5–6: Nathan and Esi recorded the number of baskets they each made out of 10 attempts. They each collected 12 data points.



Esi	7	8	9	0	1	2	1	3	2	10	9	8

5. Calculate the range of each player's data.

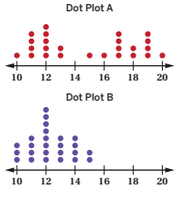
Nathan: 5; Esi: 10

6. Which player is more consistent?

Nathan

FAST Practice

7. Here are two dot plots showing the recorded speeds of two manatees.



Which of these statements is true?

A. The data for Dot Plot B is more spread out than the data for Dot Plot A.

B. The manatee recorded in Dot Plot A swims at a more consistent speed.

C. The range of Dot Plot A is greater than the range of Dot Plot B.

D. Both dot plots have approximately the same mean.

Spiral Review

8. Select *all* the expressions that are equivalent to $2(6 + 3x)$.

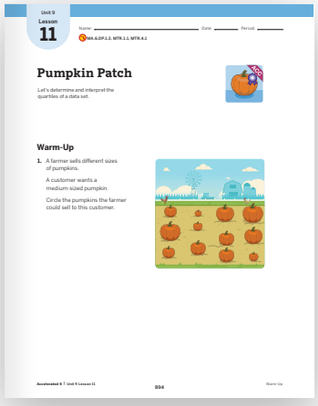
- ☐ A. $8 + 5x$
- ☐ B. $12x + 6$
- ☑ C. $12 + 6x$
- ☑ D. $6(2 + x)$
- ☐ E. $2(9x)$

9. Select *all* the expressions that represent the total area of the rectangle.

- ☑ A. $5(a + b)$
- ☐ B. $5 + ab$
- ☑ C. $5a + 5b$
- ☐ D. $2(5 + a + b)$
- ☐ E. $5ab$

Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1–3, 5	1	MA.6.DP.1.2
	4	2	MA.6.DP.1.6
	6	2	MA.6.DP.1.2
FAST Practice	7	2	MA.6.DP.1.2
Spiral Review			
	8–9	2	MA.6.AR.1.4


Unit 9
Lesson
11

Pumpkin Patch

Let's determine and interpret the quartiles of a data set.

Warm-Up

1. A farmer sells different sizes of pumpkins.

   A customer wants a medium-sized pumpkin.

   Circle the pumpkins the farmer could sell to this customer.

Pumpkin Patch

Introduction to Quartiles

Let's determine and interpret the quartiles of a data set.

Learning Outcomes

Today's Goals

1. **Goal:** Identify the *quartiles* of a data set (i.e., the first quartile (Q1), median (Q2), and third quartile (Q3)).
2. **Language Goal:** Interpret the meaning of quartiles of a data set in context. **(Writing)**

Students are introduced to **quartiles** and why they might be useful when describing data, using the context of picking pumpkins. They make connections between quartiles and the concept of the middle half of a data set.

Coherence

Prior Learning

In Lessons 6–9, students explored measures of center. In Lesson 10, students determined and interpreted the range, a measure of spread, of a data set from a table or dot plot.

Future Learning

In Lesson 12, students will create a box plot to visualize a data set. They will also calculate the range and interquartile range (IQR) of a data set to describe measures of spread.

Connections

- Students are introduced to quartiles as they explore situations represented by tables and graphs. (mathematical, real world)

FL B.E.S.T. Math Benchmarks

Addressing

MA.6.DP.1.3
Given a box plot within a real-world context, determine the minimum, the lower quartile, the median, the upper quartile and the maximum. Use this summary of the data to describe the spread and distribution of the data.

Mathematical Thinking and Reasoning Standards: MTR.1.1, MTR.4.1

ELA Expectation: ELA.K12.EE.5.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.5.DP.1.1	MA.7.DP.1.1
MA.5.DP.1.2	MA.7.DP.1.2

Vocabulary

New Vocabulary

quartile

Lesson at a Glance

~ 45 min

MA.6.DP.1.3

Warm-Up

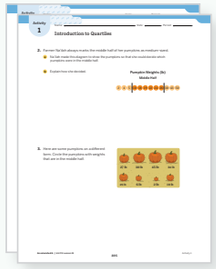
Independent | 5 min

Students identify medium-sized pumpkins to become familiar with the context of the lesson.

Activity 1

Pairs | 10 min

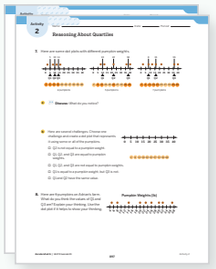
Students explore **quartiles** and how they are related to the middle half of a data set.



Activity 2

Pairs | 10 min

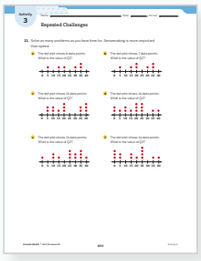
Students determine quartiles with different numbers of data points and explore how it affects the location of each quartile. **(MTR.1.1)**



Activity 3

Pairs | 10 min

Students determine quartiles of different data sets to develop strategies and generalizations. **(MTR.1.1)**

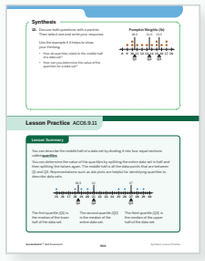


Synthesis

Whole Class | 5 min

Students synthesize their understanding of determining the value of quartiles and how they relate to the middle half of a data set.

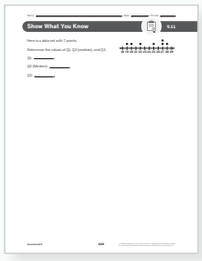
MLR2: Collect and Display



Show What You Know

Independent | 5 min

Students demonstrate their understanding by determining the values of Q1, Q2 (median), and Q3 of a data set.



Prep Checklist

Prepare Warm-Up Sheet

This lesson includes:

- Student Edition
- Show What You Know
- Chips or blocks (as needed)

Independent | 5 min

Warm-Up

Purpose: Students identify medium-sized pumpkins to become familiar with the context of the lesson.

1 Launch

Direct students' attention to the pumpkin image and consider asking, "How would you decide what a medium-sized pumpkin is?"

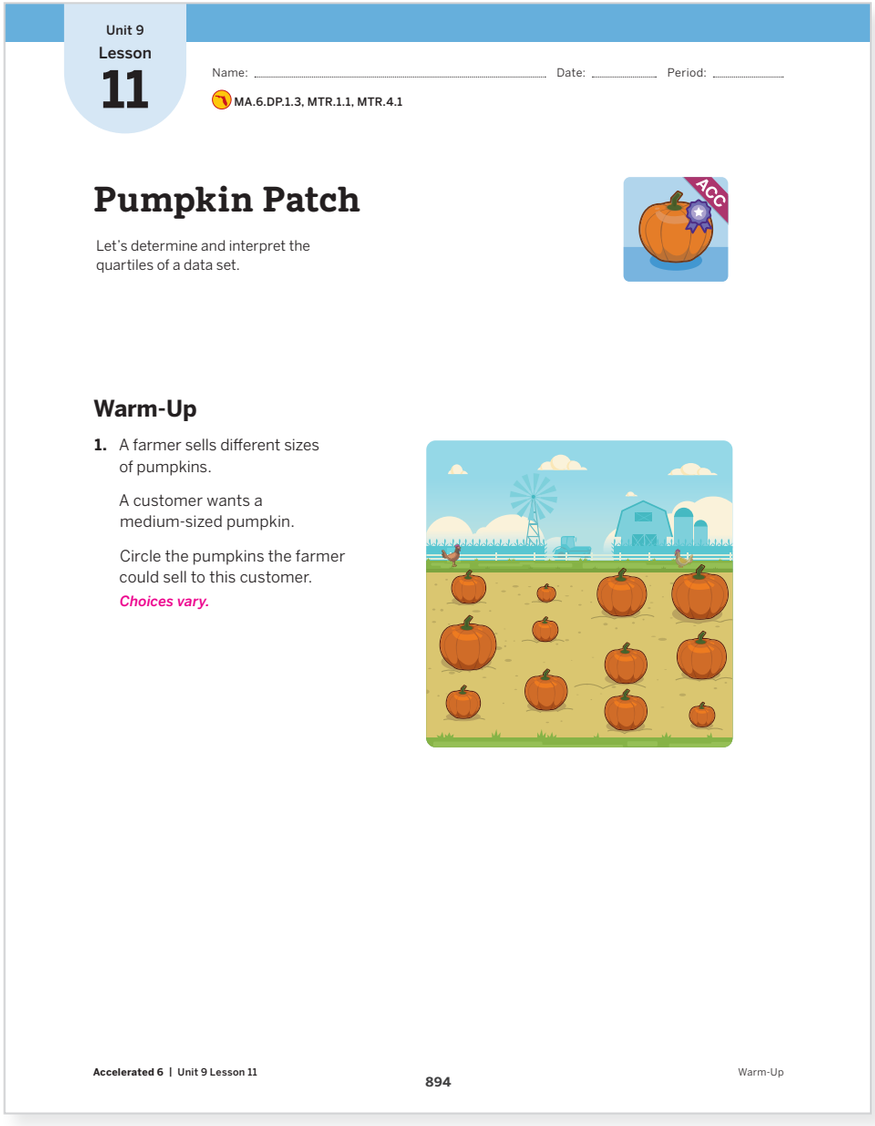
To surface connections, invite students to share any experiences they have with picking pumpkins. Consider asking questions like:

- "Why might someone want a medium-sized pumpkin?"
- "What is the largest pumpkin you've ever seen?"

2 Connect

Invite students to share which pumpkins they chose and how they decided which pumpkins were medium-sized.

Encourage students to create a rule for how to tell if any pumpkin in a group is medium-sized. **(MTR.1.1)**



Unit 9
Lesson
11

Name: _____ Date: _____ Period: _____

MA.6.DP.1.3, MTR.1.1, MTR.4.1

Pumpkin Patch

Let's determine and interpret the quartiles of a data set.

ACC

Warm-Up

1. A farmer sells different sizes of pumpkins.

   A customer wants a medium-sized pumpkin.

   Circle the pumpkins the farmer could sell to this customer.

   Choices vary.

Accelerated 6 | Unit 9 Lesson 11 894 Warm-Up

Pairs | 10 min

Activity 1 Introduction to Quartiles

Purpose: Students explore **quartiles** and how they are related to the middle half of a data set.

1 Launch

Direct students' attention to the diagram.

Consider asking, "Why do you think Na'ilah weighed her pumpkins before deciding which are medium-sized?"

2 Monitor

Look for student ideas that are similar to quartiles (e.g., split the data into four parts).

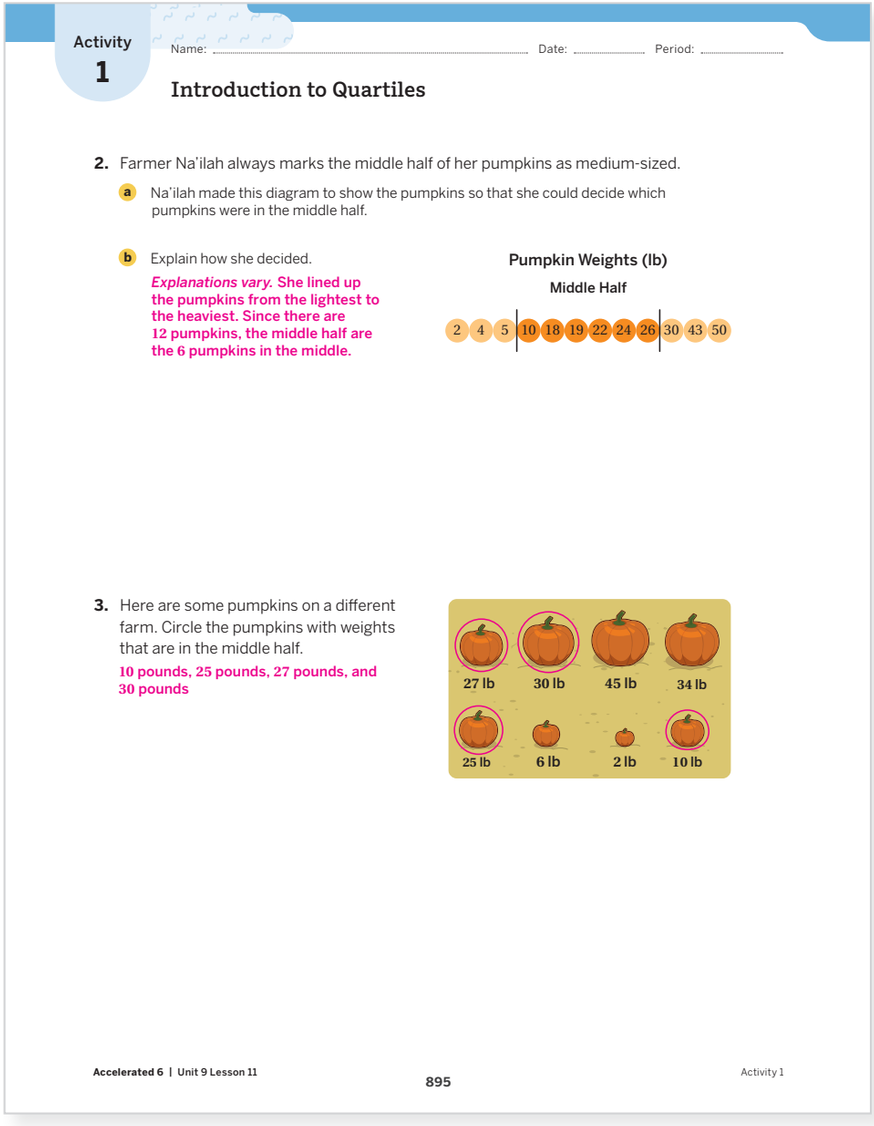
A **Accessibility: Conceptual Processing** Use objects like chips or blocks to show what Na'ilah means by "middle half." It may be helpful for students to use these objects throughout the lesson as they make sense of quartiles (lining up the data values and splitting them into four groups, etc.)

Encourage students to read others' responses and decide if others' descriptions were alike or different from their own. **(MTR.1.1)**

D **Differentiation: (Support)** Look for students who select the four pumpkins in the middle of the patch without reordering. Consider asking, "Is a 2-pound pumpkin in the middle half of pumpkins if we order them by weight?"

Consider inviting students to revisit this problem after learning about quartiles in Problem 4 and check the reasonableness of their responses.

Activity 1 continued >



Activity 1

Name: Date: Period:

Introduction to Quartiles

2. Farmer Na'ilah always marks the middle half of her pumpkins as medium-sized.

a. Na'ilah made this diagram to show the pumpkins so that she could decide which pumpkins were in the middle half.

b. Explain how she decided.

Explanations vary. She lined up the pumpkins from the lightest to the heaviest. Since there are 12 pumpkins, the middle half are the 6 pumpkins in the middle.

Pumpkin Weights (lb)

Middle Half

2 4 5 10 18 19 22 24 26 30 43 50

3. Here are some pumpkins on a different farm. Circle the pumpkins with weights that are in the middle half.

10 pounds, 25 pounds, 27 pounds, and 30 pounds

27 lb 30 lb 45 lb 34 lb

25 lb 6 lb 2 lb 10 lb

Accelerated 6 | Unit 9 Lesson 11 895 Activity 1

Pairs | 10 min

Activity 1 Introduction to Quartiles (continued)

Purpose: Students explore **quartiles** and how they are related to the middle half of a data set.

2 Monitor

Pause to read the definition of **quartiles** and ensure students know that Q2 is the median of a data set.

Select and sequence student explanations of how to determine Q3 to share during the Connect.

D Differentiation

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Encourage students to look at both the list and the dot plot views of the data.
Choose a value between 30 and 34 for Quartile 3 other than 32.	**Support** Invite students to think about how Q1 and Q2 were determined. Consider asking, "Why is 8 the value of Q1?"
Determine the correct value for Quartile 3.	**Strengthen** Celebrate student thinking and invite them to share during the Connect. **Stretch** Encourage students to describe how they would change this data set so the value of Q3 is 35.

Listen for ideas that are similar to the definitions students will see in Problems 5–6 (e.g., median of the lower half of the data).

To support students getting started, consider asking them to circle where they think the lower half of the data is and ask, "How could we find the median of these points?"

ELL **English Language Learners** If possible, review an anchor chart that publicly displays the definition of median along with examples in order to aid in explanations and reasoning. It may be helpful to add Quartile 1 and Quartile 3 definitions to this chart throughout the lesson.

Invite students to check their values with a partner.

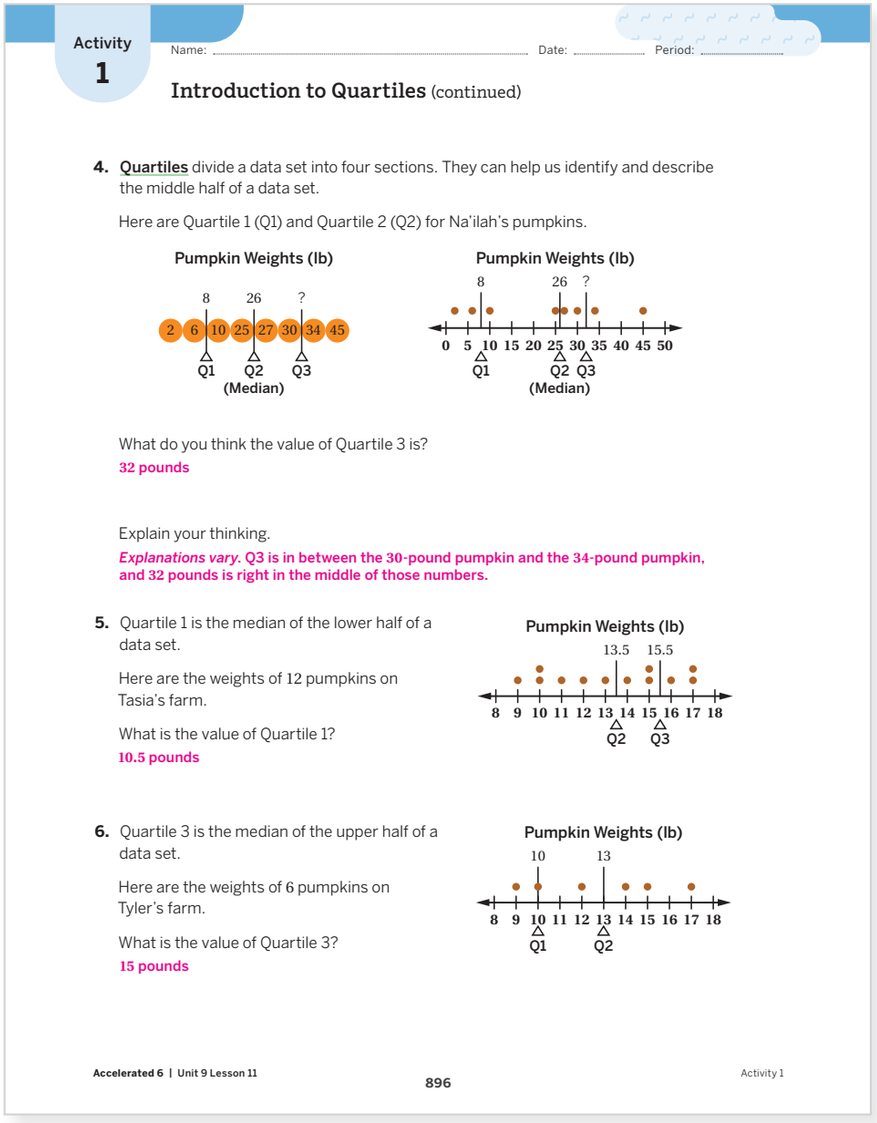
Listen to students' different strategies for determining Q3. Invite those students to share.

3 Connect

Share student explanations of how they determined the value of Quartile 3.

To surface the Key Takeaway, consider asking:

- "Why do you think these numbers are called quartiles?"
- "Someone used the phrase *upper half of the data* in their explanation. What do you think they were talking about?"
- "Are the distances between the quartiles on the number line equal? How do you know?"



Activity 1

Name: Date: Period:

Introduction to Quartiles (continued)

4. **Quartiles** divide a data set into four sections. They can help us identify and describe the middle half of a data set.

Here are Quartile 1 (Q1) and Quartile 2 (Q2) for Na'ilah's pumpkins.

Pumpkin Weights (lb)

Pumpkin Weights (lb)

What do you think the value of Quartile 3 is?

32 pounds

Explain your thinking.

Explanations vary. Q3 is in between the 30-pound pumpkin and the 34-pound pumpkin, and 32 pounds is right in the middle of those numbers.

5. Quartile 1 is the median of the lower half of a data set.

Here are the weights of 12 pumpkins on Tasia's farm.

What is the value of Quartile 1?

10.5 pounds

Pumpkin Weights (lb)

6. Quartile 3 is the median of the upper half of a data set.

Here are the weights of 6 pumpkins on Tyler's farm.

What is the value of Quartile 3?

15 pounds

Pumpkin Weights (lb)

Accelerated 6 | Unit 9 Lesson 11 896 Activity 1

Key Takeaway: **Quartiles** divide a data set into four sections. They can help identify and describe the middle half of the data set.

Pairs | 10 min

Activity 2 Reasoning About Quartiles

Purpose: Students determine quartiles with different numbers of data points and explore how it affects the location of each quartile. **(MTR.1.1)**

1 Launch

Direct students' attention to the dot plots. Consider sharing that the darker shaded circles indicate the pumpkins used in each dot plot.

Invite students to share what they notice about the quartiles.

2 Monitor

A **Accessibility: Executive Functioning** To support initiating complex tasks, identify possible places to start. Consider suggesting that students begin by choosing pumpkins to use to create a dot plot and then check whether the dot plot created represents one of the challenges.

Encourage students to share their reasoning with a partner. **(MTR.1.1, MTR.4.1)**

Pause to discuss the challenges and consider asking:

- "What happens to the quartiles when there is an odd number of pumpkins? An even number of pumpkins?"
- "When do the quartiles all equal the pumpkin weights?"
- "When are all of the quartiles between pumpkin weights?" **(MTR.4.1)**

Invite students to read others' responses and decide if others' strategies were similar to or different from their own. **(MTR.1.1)**

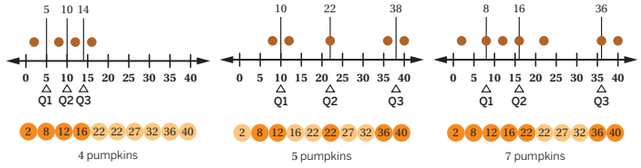
Activity 2 continued >

Activity 2

Name: Date: Period:

Reasoning About Quartiles

7. Here are some dot plots with different pumpkin weights.

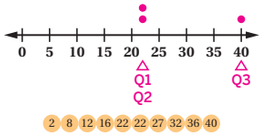


a. **Discuss:** What do you notice?

Responses vary.

- I notice that when a pumpkin is added, all of the quartiles tend to change.
- I notice that a quartile can be on a pumpkin or between two pumpkins.
- I notice that the quartiles can be close together or farther apart.

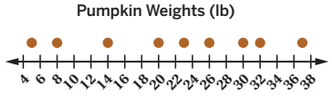
b. Here are several challenges. Choose one challenge and create a dot plot that represents it using some or all of the pumpkins.



- ☐ Q2 is not equal to a pumpkin weight.
- ☐ Q1, Q2, and Q3 are equal to pumpkin weights.
- ☐ Q1, Q2, and Q3 are not equal to pumpkin weights.
- ☐ Q1 is equal to a pumpkin weight, but Q3 is not.
- ☐ Q1 and Q2 have the same value.

Choices and graphs vary. Sample graph shown.

8. Here are 9 pumpkins on Adrian's farm. What do you think the values of Q1 and Q3 are? Explain your thinking. Use the dot plot if it helps to show your thinking.



Responses vary.

- I think Q1 is 11 and Q3 is 31. There are 9 data points, so I can't really split them in half. If I ignore the point in the middle, then there are 4 on each side. 11 is halfway between 8 and 14. 31 is halfway between 30 and 32.
- 23 is the median, so 14 is the middle number in the lower half and 30 is the middle number in the upper half.

Note: Students are introduced to the convention of excluding the median when determining the quartiles of a data set with an odd number of points in the next problem.

Accelerated 6 | Unit 9 Lesson 11 897 Activity 2

Pairs | 10 min

Activity 2 Reasoning About Quartiles (continued)

Purpose: Students determine quartiles with different numbers of data points and explore how it affects the location of each quartile. **(MTR.1.1)**

2 Monitor

Invite students to share their values for Q3 with a partner.

ELL **English Language Learners** Consider inviting pairs to explain what happened to the median of Adrian's pumpkins in the previous problem before reading the prompt and to then make connections between the prompt and the diagram. It may be helpful to use objects to demonstrate this situation.

Encourage students to reflect on their choices before trying a new strategy if you notice them guessing and checking.

To support students getting started, consider asking, "How many pumpkins do you think are in each quarter of the data set? How do you know?" Then invite students to label each quarter with 20 pumpkins.

Listen for several different responses to share in Connect.

D **Differentiation: (Support)** Look for students who may write 75 because they realize 75% of the pumpkins weigh less than 15.5. Consider validating these students' reasoning and asking, "How many pumpkins would 75% of all the pumpkins be?"

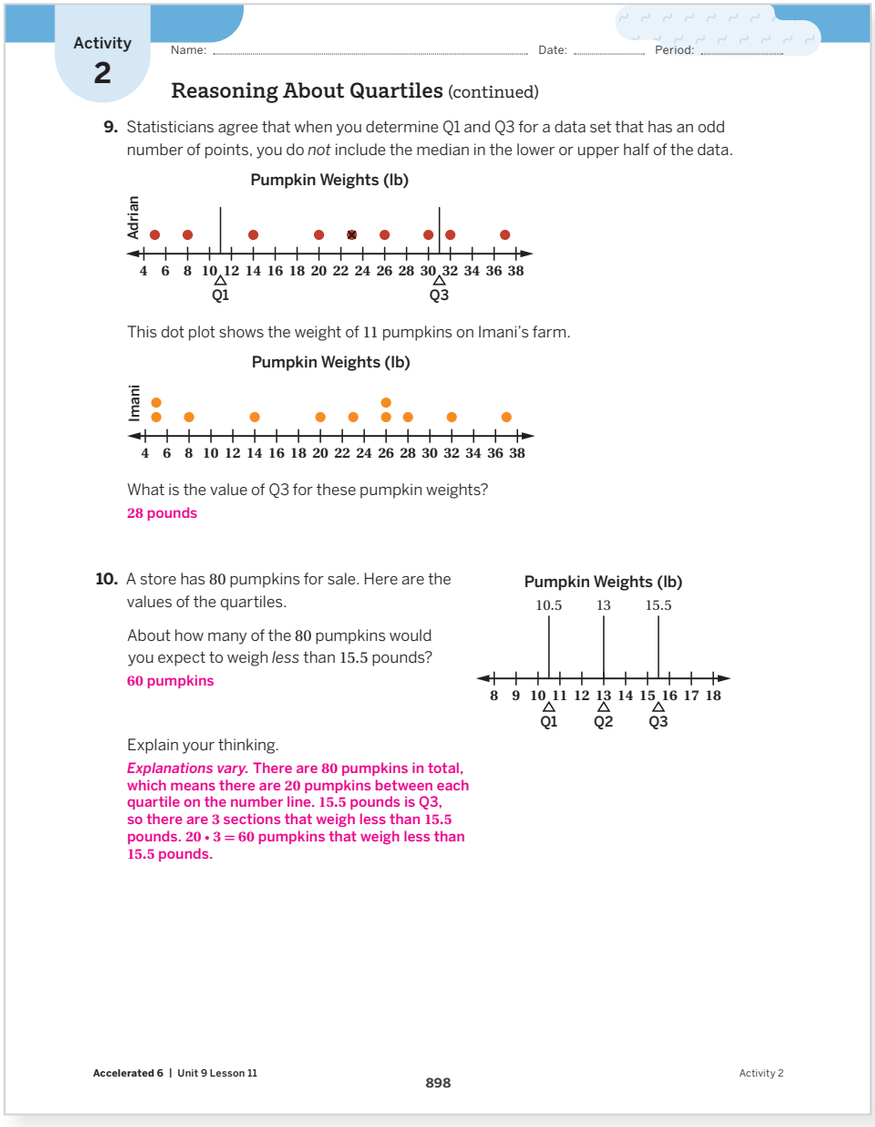
3 Connect

Consider asking:

- "Why is Q3 of Adrian's data 31?"
- "How did you decide what Q3 of Imani's data is? What did you do with the median?"

Discuss how to attend to precision in determining quartiles in situations with odd numbers of data points. **(MTR.4.1)**

Invite students to share their responses and ask students to justify their thinking. Then come to a consensus as a class. **(MTR.1.1)**



Activity 2

Name: Date: Period:

Reasoning About Quartiles (continued)

9. Statisticians agree that when you determine Q1 and Q3 for a data set that has an odd number of points, you do *not* include the median in the lower or upper half of the data.

Pumpkin Weights (lb)

Adrian

4 6 8 10 12 14 16 18 20 22 24 26 28 30 32 34 36 38

Q1 Q3

This dot plot shows the weight of 11 pumpkins on Imani's farm.

Pumpkin Weights (lb)

Imani

4 6 8 10 12 14 16 18 20 22 24 26 28 30 32 34 36 38

What is the value of Q3 for these pumpkin weights?

28 pounds

10. A store has 80 pumpkins for sale. Here are the values of the quartiles.

Pumpkin Weights (lb)

10.5 13 15.5

8 9 10 11 12 13 14 15 16 17 18

Q1 Q2 Q3

About how many of the 80 pumpkins would you expect to weigh *less* than 15.5 pounds?

60 pumpkins

Explain your thinking.

Explanations vary. There are 80 pumpkins in total, which means there are 20 pumpkins between each quartile on the number line. 15.5 pounds is Q3, so there are 3 sections that weigh less than 15.5 pounds. $20 \cdot 3 = 60$ pumpkins that weigh less than 15.5 pounds.

Accelerated 6 | Unit 9 Lesson 11 898 Activity 2

Pairs | 10 min

Activity 3 Repeated Challenges

Purpose: Students determine quartiles of different data sets to develop strategies and generalizations. **(MTR.1.1)**

Short on time? Consider decreasing the number of challenges students are to complete in Problem 11.

1 Launch

Direct students' attention to the first dot plot and invite them to share their strategies on how to start finding the quartiles.

Invite students to set their own goal for how many problems they would like to complete, sharing that sense-making is more important than speed. Encourage students to check their values with a partner to help them revise their thinking. **(MTR.1.1)**

2 Monitor

Encourage students to develop strategies and generalizations as they progress through the challenges. **(MTR.1.1)**

D Differentiation

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Encourage students to put the values on the dot plot into a list.
Confuse Q2 for Q1 or Q3.	**Support** Invite students to label the values for Q1, Q2, and Q3.
Use creative strategies for determining quartiles.	**Strengthen** Celebrate their strategies. Select and sequence these strategies to share during the Connect.

Listen to partner discussions for student ideas and strategies to highlight in the Connect. **(MTR.4.1)**

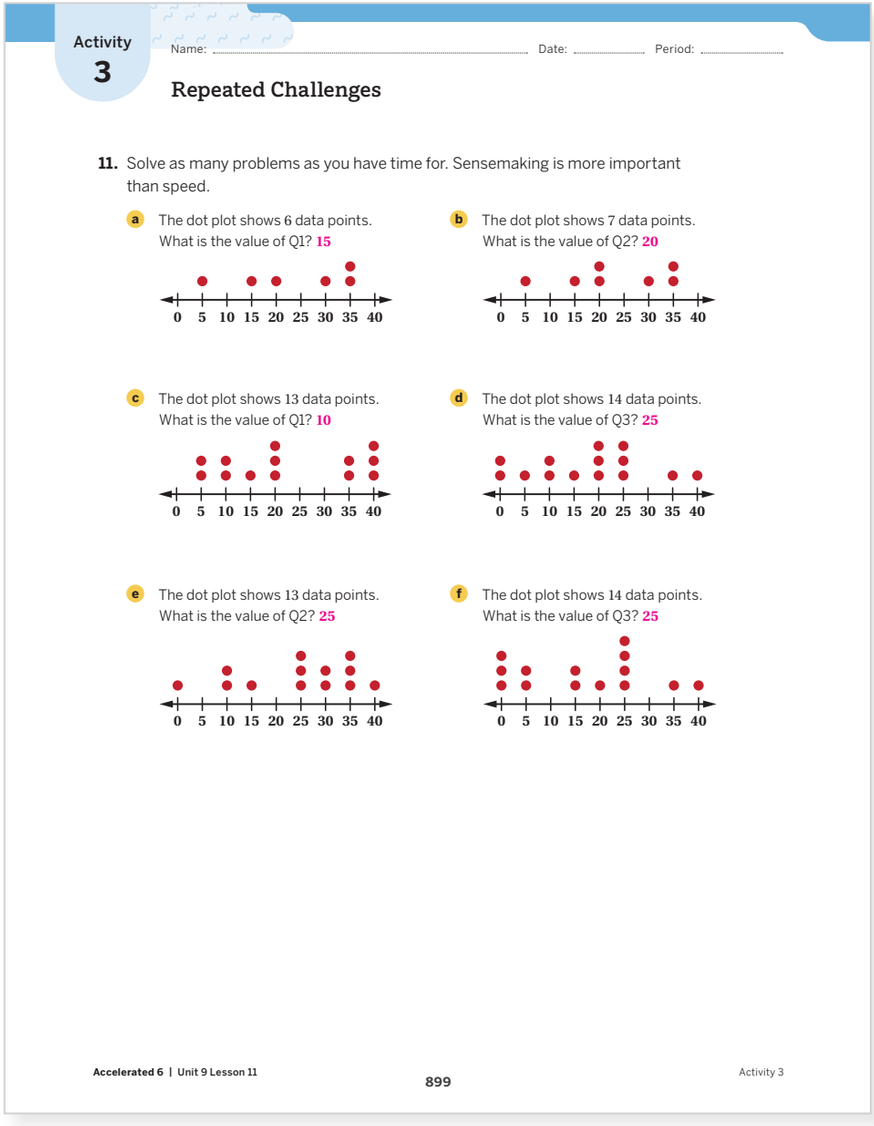
A Accessibility: Memory and Attention Consider inviting students with working memory challenges to record any strategies they come up with on a piece of blank paper to refer back to.

3 Connect

Invite students to share their ideas and strategies for finding values of Q1, Q2, and Q3.

To surface the Key Takeaway, consider asking:

- "Did your strategy change when the problem had an odd number of data points? Even number of data points? Repeated values?"
- "What is your favorite strategy to use when solving these types of problems?"



Activity 3

Name: Date: Period:

Repeated Challenges

11. Solve as many problems as you have time for. Sensemaking is more important than speed.

a. The dot plot shows 6 data points. What is the value of Q1? 15

b. The dot plot shows 7 data points. What is the value of Q2? 20

c. The dot plot shows 13 data points. What is the value of Q1? 10

d. The dot plot shows 14 data points. What is the value of Q3? 25

e. The dot plot shows 13 data points. What is the value of Q2? 25

f. The dot plot shows 14 data points. What is the value of Q3? 25

Accelerated 6 | Unit 9 Lesson 11 899 Activity 3

Key Takeaway: The value of quartiles are determined by determining the median, then determining the medians of the upper and lower half of the data. If there is a data point in the middle, then that point is not included in either half.

Whole Class | 5 min

Synthesis

Purpose: Students synthesize their understanding of determining the value of quartiles and how they relate to the middle half of a data set.

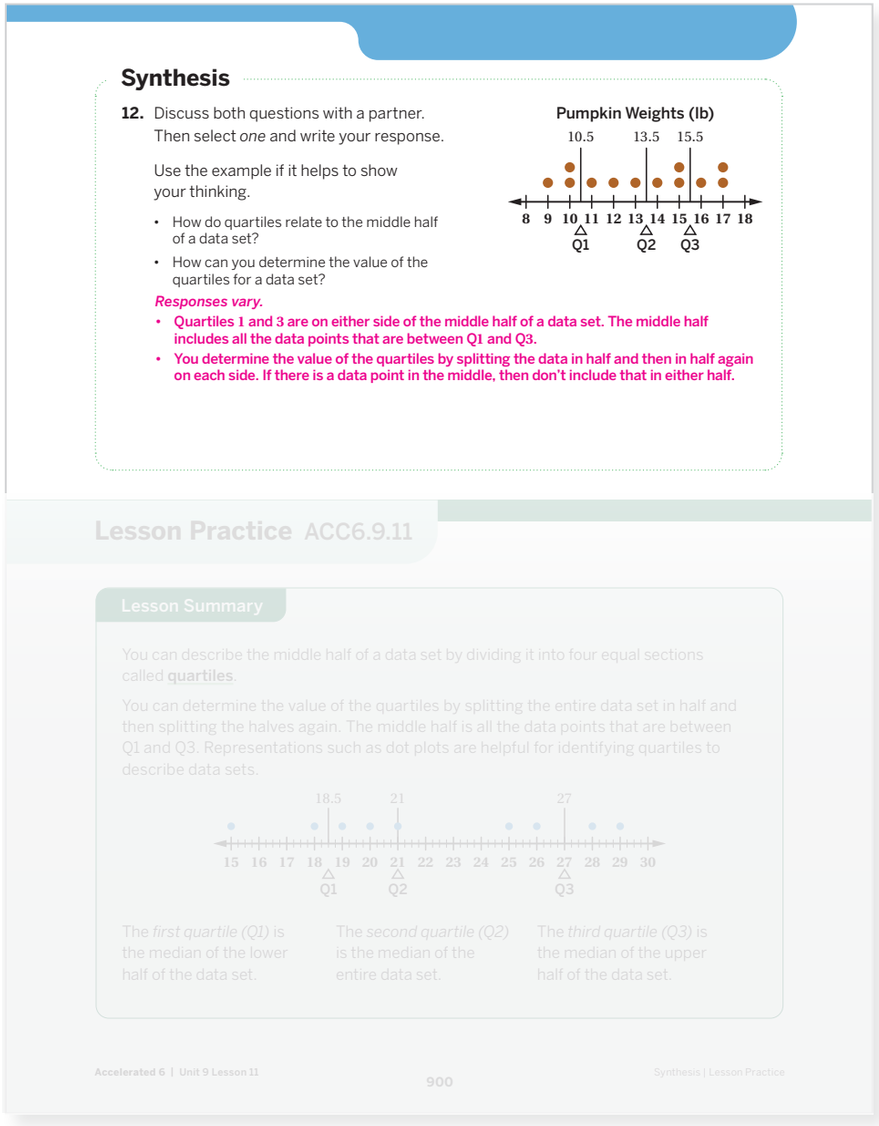
Invite students to respond independently, and then share their thinking with a partner.

Collect and share a variety of ideas, such as:

- The middle half is all the data points that are between Q1 and Q3.
- Quartiles 1 and 3 are on either side of the middle half.
- The values of quartiles are determined by splitting the data in half and then in half again on each side. If there is a data point in the middle, then that point is not included in either half.

MLR **MLR2: Collect and Display** Use this routine to formalize the definition of *quartiles* and strategies for determining the values of quartiles using students' language. Display any new language, examples, or diagrams for students to reference later.

Note: If time allows, invite students to share with a partner what makes sense about each strategy and what questions they still have.



Synthesis

12. Discuss both questions with a partner. Then select *one* and write your response.

Use the example if it helps to show your thinking.

- How do quartiles relate to the middle half of a data set?
- How can you determine the value of the quartiles for a data set?

Pumpkin Weights (lb)

Responses vary.

- Quartiles 1 and 3 are on either side of the middle half of a data set. The middle half includes all the data points that are between Q1 and Q3.
- You determine the value of the quartiles by splitting the data in half and then in half again on each side. If there is a data point in the middle, then don't include that in either half.

Lesson Practice ACC6.9.11

Lesson Summary

You can describe the middle half of a data set by dividing it into four equal sections called quartiles.

You can determine the value of the quartiles by splitting the entire data set in half and then splitting the halves again. The middle half is all the data points that are between Q1 and Q3. Representations such as dot plots are helpful for identifying quartiles to describe data sets.

The *first quartile (Q1)* is the median of the lower half of the data set.

The *second quartile (Q2)* is the median of the entire data set.

The *third quartile (Q3)* is the median of the upper half of the data set.

Accelerated 6 | Unit 9 Lesson 11 900 Synthesis | Lesson Practice

Lesson Takeaway: Quartiles divide a data set into 4 sections. Quartile 1 is the median of the lower half. Quartile 2 is the median of the entire data set. Quartile 3 is the median of the upper half. The values between Quartiles 1 and 3 are the middle half of a data set.

Independent | 5 min

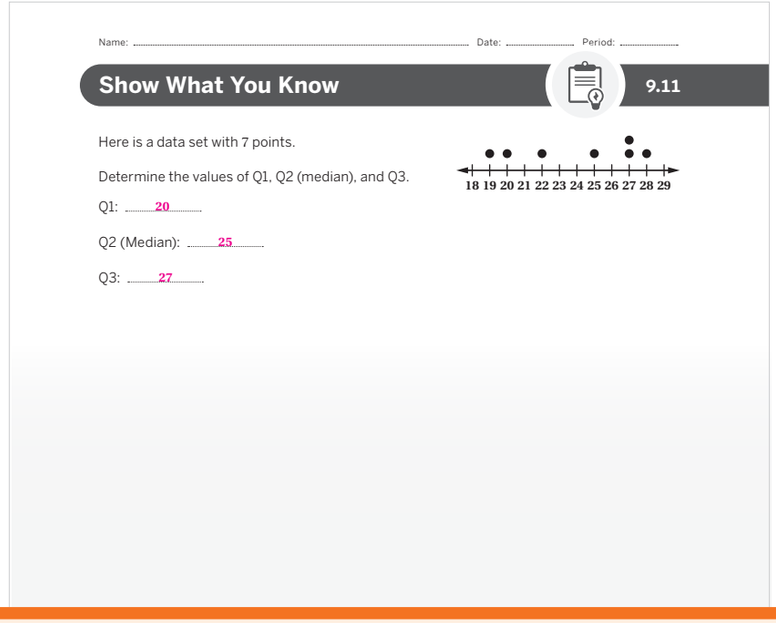
Show What You Know

Purpose: Students demonstrate their understanding by determining the values of Q1, Q2 (median), and Q3 of a data set.

Today's Goals

1. **Goal:** Identify the *quartiles* of a data set (i.e., the first quartile (Q1), median (Q2), and third quartile (Q3)).
2. **Language Goal:** Interpret the meaning of quartiles of a data set in context. **(Writing)**

Show What You Know (Assessment Resources)



Name: ..... Date: ..... Period: .....

Show What You Know 9.11

Here is a data set with 7 points.

Determine the values of Q1, Q2 (median), and Q3.

Q1: 20

Q2 (Median): 25

Q3: 27

Differentiation Use after Lesson 11

Support

Provide targeted intervention for students by using these resources.

If students need support calculating values of the quartiles from a dot plot:

» **Respond to Student Thinking:**

- Revisit **Activity 2** and give students individualized feedback.
- Review the **Lesson Summary**.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite them to explore the **Sub-Unit 2 Extension Activities**.

Professional Learning

How did the way in which you learned about **quartiles** and measures of center inform how you planned for the lesson? How did it impact how you taught the lesson?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

12. Discuss both questions with a partner. Then select *one* and write your response.

Use the example if it helps to show your thinking.

- How do quartiles relate to the middle half of a data set?
- How can you determine the value of the quartiles for a data set?

Pumpkin Weights (lb)

Responses vary.

- Quartiles 1 and 3 are on either side of the middle half of a data set. The middle half includes all the data points that are between Q1 and Q3.
- You determine the value of the quartiles by splitting the data in half and then in half again on each side. If there is a data point in the middle, then don't include that in either half.

Lesson Practice ACC6.9.11

Lesson Summary

You can describe the middle half of a data set by dividing it into four equal sections called **quartiles**.

You can determine the value of the quartiles by splitting the entire data set in half and then splitting the halves again. The middle half is all the data points that are between Q1 and Q3. Representations such as dot plots are helpful for identifying quartiles to describe data sets.

The *first quartile (Q1)* is the median of the lower half of the data set.

The *second quartile (Q2)* is the median of the entire data set.

The *third quartile (Q3)* is the median of the upper half of the data set.

Lesson Practice
ACC6.9.11

Name: ..... Date: ..... Period: .....

Problems 1–2: Here are the ages of 20 people at a family reunion, ordered from youngest to oldest.

3, 8, 9, 10, 11, 11, 12, 18, 18, 28, 30, 35, 37, 40, 53, 54, 58, 65, 70, 72

1. The value of Quartile 2 (Q2) is 29. What does that tell us about the people at the family reunion?

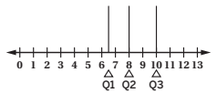
Responses vary. It means that about half of the people at the family reunion are younger than 29 years old and about half are older than 29 years old.

2. Determine the value of Q1 and Q3.

Quartile 1 (Q1): 11 Quartile 3 (Q3): 53.5

Problems 3–5: Haru recorded how long his bus ride to school took for 16 days. Here are the values of the quartiles of his data.

Bus Ride Length (min)



3. About how many rides would you expect to be less than 6.5 minutes long?

About 4 bus rides

4. About how many rides would you expect to be less than 10 minutes long?

About 12 bus rides

5. About what percent of the rides would you expect to be between 6.5 minutes and 10 minutes long?

About 50% of the bus rides

6. The heights, in inches, of Javier's classmates are 60, 68, 56, 60, 62, 58, 55, 67, 59, 61, 62, 64, 63, 63, 59, 62, 66, and 61. Determine the values of Q1, Q2 (median), and Q3.

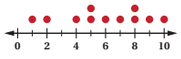
	Height (in.)
Q1	59
Q2 (Median)	61.5
Q3	63

Lesson Practice
ACC6.9.11

Name: ..... Date: ..... Period: .....

FAST Practice

7. Makayla and Axel both try to determine the quartiles for this dot plot.



Makayla says:

Q1: 4.5 Q2: 6 Q3: 8

Axel says:

Q1: 4 Q2: 6 Q3: 8

Who is correct? Explain your thinking.

A. Makayla **B.** Axel is correct. If the data set has an odd number of points, the median **A.** should **B.** should not be included when determining Q1 and Q2.

Spiral Review

Problems 8–10: Calculate each percentage.

8. 25% of 40
10

9. 25% of 120
30

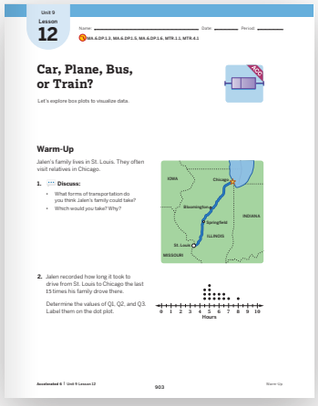
10. 25% of 90
22.5

11. Add 5 points to this dot plot without changing the median.

Sample response shown on graph.

Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1, 3–5	2	MA.6.DP.1.3
	2, 6	1	MA.6.DP.1.3
FAST Practice	7	2	MA.6.DP.1.3
Spiral Review			
	8–10	1	MA.6.AR.3.4
	11	3	MA.6.DP.1.6



Car, Plane, Bus, or Train?

Box Plots, IQR, and Range

Let's explore box plots to visualize data.

Learning Outcomes

Today's Goals

1. **Goal:** Create a *box plot* to visualize a data set.
2. **Goal:** Calculate the range and *interquartile range (IQR)* of a data set.
3. **Language Goal:** Describe the center and spread of data using terms like *median*, *interquartile range*, and *range*. **(Writing, Speaking, and Listening)**

Students are introduced to a new way to visualize data (a **box plot**) and a new way to measure spread (**IQR**). They make sense of a box plot for a data set, then interpret information on a different box plot. Students also use box plots to visualize and analyze data and explore how changing a data set affects measures of center and spread. They make a claim about which form of transportation is best to travel between St. Louis and Chicago. **(MTR.1.1)**

Coherence

Prior Learning

In Lessons 4 and 10, students explored the range of data displayed in histograms and dot plots. In Lesson 11, students explored quartiles and how they are useful when describing data. They made connections between quartiles and the concept of the middle half of a data set.

Future Learning

In Lesson 16, students will continue modeling with data by analyzing parallel box plots to draw inferences about populations.

Connections

- Students build on their understanding of data displays as they explore box plots. (mathematical, real world)

FL B.E.S.T. Math Benchmarks

Addressing

MA.6.DP.1.3
Given a box plot within a real-world context, determine the minimum, the lower quartile, the median, the upper quartile and the maximum. Use this summary of the data to describe the spread and distribution of the data.

MA.6.DP.1.5
Create box plots and histograms to represent sets of numerical data within real-world contexts.

MA.6.DP.1.6
Given a real-world scenario, determine and describe how changes in data values impact measures of center and variation.

Mathematical Thinking and Reasoning Standards: MTR.1.1, MTR.4.1

ELA Expectations: ELA.K12.EE.1.1, ELA.K12.EE.5.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.5.DP.1.1	**MA.7.DP.1.1**
MA.5.DP.1.2	**MA.7.DP.1.2**
	MA.7.DP.1.5

Vocabulary

New Vocabulary

box plot

interquartile range (IQR)

Review Vocabulary

maximum

minimum

Lesson at a Glance

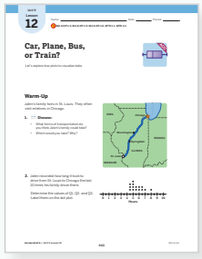
~ 45 min

MA.6.DP.1.3, MA.6.DP.1.5, MA.6.DP.1.6

Warm-Up

Pairs | 5 min

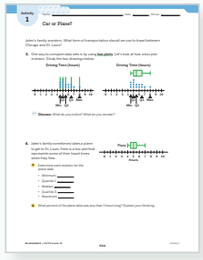
Students make sense of the context of a family trip from St. Louis to Chicago. They explore travel time data on a dot plot and identify quartiles. **(MTR.1.1)**



Activity 1

Pairs | 8 min

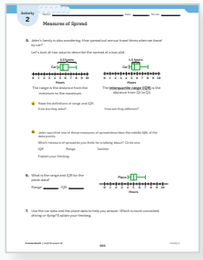
Students define **box plot** and interpret information shown on a box plot. **Routine:** Notice and Wonder



Activity 2

Pairs | 8 min

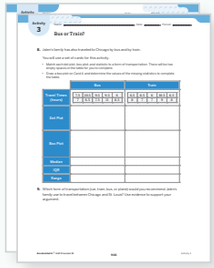
Students explore two ways to measure spread: range and **interquartile range (IQR)**, and use these measures to make claims about the consistency of data.



Activity 3

Pairs | 14 min

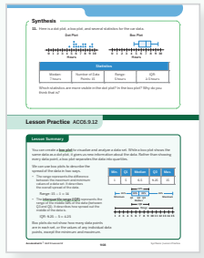
Students make connections between data, dot plots, box plots, and measures of center and spread. **(MTR.1.1)**



Synthesis

Whole Class | 5 min

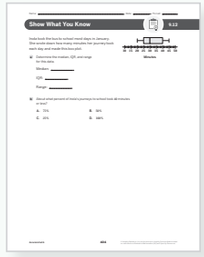
Students synthesize their understanding of key features of a box plot. MLR2: Collect and Display



Show What You Know

Independent | 5 min

Students demonstrate their understanding by calculating the median, IQR, and range of a data set represented by a box plot and interpreting what one of the quartiles means in context.



Prep Checklist

Prepare the Activity 3 Cards.

This lesson includes:

- Student Edition
- Show What You Know
- Coloring tools (as needed)
- Straightedges
- Activity 3 Cards, one set per pair, pre-cut

Pairs | 5 min

Warm-Up

Purpose: Students make sense of the context of a family trip from St. Louis to Chicago. They explore travel time data on a dot plot and identify quartiles. **(MTR.1.1)**

1 Launch

Direct students' attention to the map showing the path from St. Louis to Chicago.

To surface connections, invite students to discuss the two questions with a partner and then share out with the class. **(MTR.4.1)**

Consider asking, "How do you travel when you visit family? Is there more than one way to get there?"

Direct students' attention to the dot plot and ask a student to read the prompt for Problem 2 aloud. Invite students to work in pairs to determine the value of each quartile.

To surface prior knowledge, consider asking, "What is meant by Q1, Q2, and Q3? How is each value related to the median?"

A **Accessibility: Conceptual Processing** Invite students to select a data point on the dot plot and explain what it means to a partner before calculating each statistic.

Look for students who annotate the dot plot by marking the median, circling values, or using strategies like crossing off dots to determine each quartile. **(MTR.1.1)**

2 Connect

Invite students to share their strategies for determining the value of each quartile.

Consider asking:

- "What does this dot (point to one dot) represent in the dot plot?" Each dot represents the length of time it took Jalen's family to make one trip to Chicago.
- "How did you know where to label each quartile?" *Responses vary.* I counted the total dots and then figured out which one was in the middle. Then I found the middle of each half of the data.
- "What does each quartile tell you about the data?" *Responses vary.*
  - They tell us the median of the upper and lower half of the data.
  - They tell us how/where to divide the data into 4 parts (boundaries for where 25% of the data is).

Unit 9
Lesson
12

Name: _____ Date: _____ Period: _____

MA.6.DP.1.3, MA.6.DP.1.5, MA.6.DP.1.6, MTR.1.1, MTR.4.1

Car, Plane, Bus, or Train?

ACC

Let's explore box plots to visualize data.

Warm-Up

Jalen's family lives in St. Louis. They often visit relatives in Chicago.

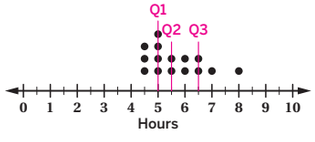
1. **Discuss:**
   - What forms of transportation do you think Jalen's family could take?
   - Which would *you* take? Why?

   ***Responses vary.* Trains, buses, cars, or planes. I would take a car because then I can be the most flexible.**

2. Jalen recorded how long it took to drive from St. Louis to Chicago the last 15 times his family drove there.

   Determine the values of Q1, Q2, and Q3. Label them on the dot plot.

   Q1: 5 hours
   Q2: 5.5 hours
   Q3: 6.5 hours



Accelerated 6 | Unit 9 Lesson 12 903 Warm-Up

Pairs | 8 min

Activity 1 Car or Plane?

Purpose: Students define **box plot** and interpret information shown on a box plot.

1 Launch

Invite students to read the statistical question aloud then make a prediction.

Direct students' attention to the data displays. Consider reviewing the terms *maximum* and *minimum*. Consider sharing that the dot plot and box plot represent car data for travel between Chicago and St. Louis.

Use the Notice and Wonder routine to promote curiosity and help students make sense of how a box plot is drawn. **(MTR.1.1)**

2 Monitor

Listen for students who make connections between the four sections on the box plot and the percent of drives that are between 5 and 5.5 hours.

D Differentiation *(Problem 4)*

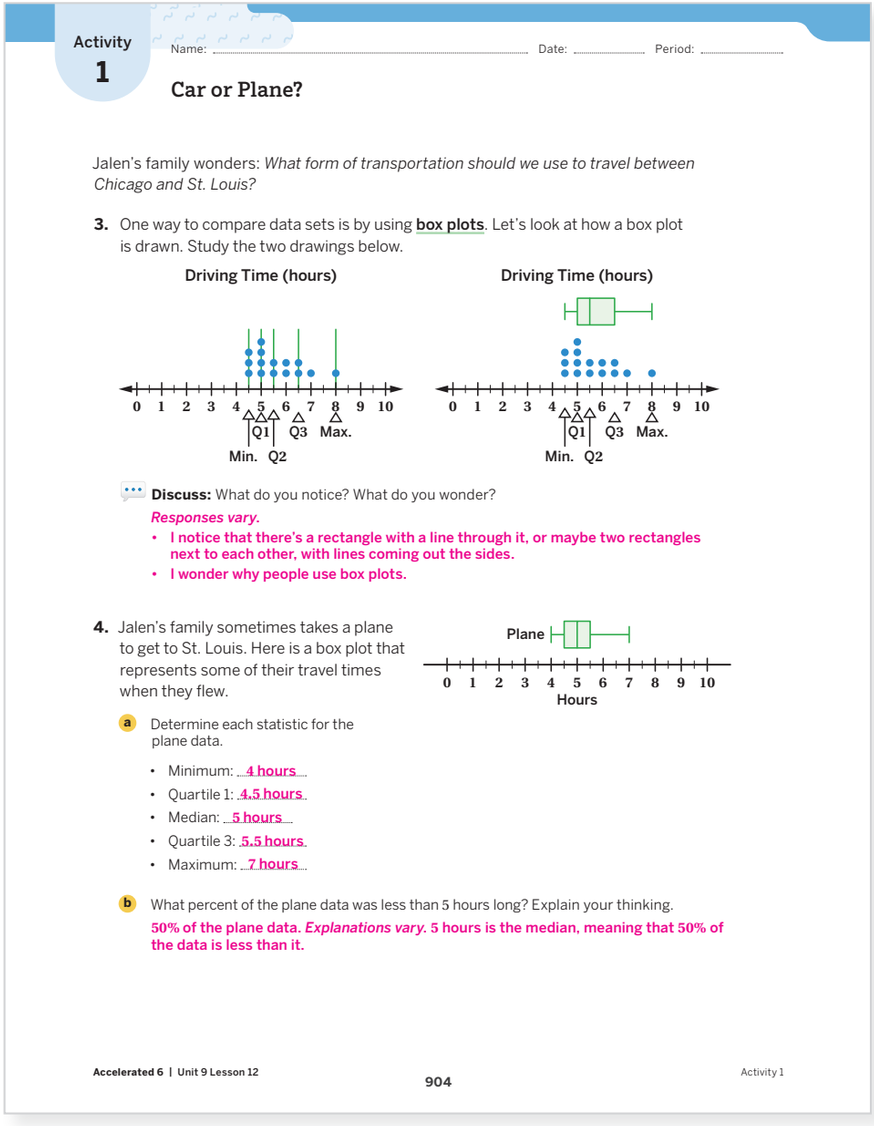
Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Invite students to compare this new box plot with the labels in the previous problem and encourage them to add labels to this graph.
Determine 33% because between 4 and 5 is $\frac{1}{3}$ of the length of the entire box plot.	**Support** Invite students to review the definition of *median*.
Determine 5 is the median (i.e., 50% of the data is less than it).	**Strengthen** Invite students to share their thinking during the Connect.
Write clear explanations for determining the percentage.	**Stretch** Consider asking, "How does this compare to the lower 50% of car trips?"

3 Connect

Invite students to share how they determined each statistic and the percent of flights that were less than 5 hours.

To surface the Key Takeaway, consider asking:

- "What does each section of a box plot represent?"
- "Do we know how many plane rides Jalen has data about?"
- "Based on the data so far, how should Jalen's family travel?"



Activity 1

Name: Date: Period:

Car or Plane?

Jalen's family wonders: *What form of transportation should we use to travel between Chicago and St. Louis?*

3. One way to compare data sets is by using **box plots**. Let's look at how a box plot is drawn. Study the two drawings below.

Driving Time (hours)

Driving Time (hours)

Discuss: What do you notice? What do you wonder?

Responses vary.

- I notice that there's a rectangle with a line through it, or maybe two rectangles next to each other, with lines coming out the sides.
- I wonder why people use box plots.

4. Jalen's family sometimes takes a plane to get to St. Louis. Here is a box plot that represents some of their travel times when they flew.

a. Determine each statistic for the plane data.

- Minimum: 4 hours
- Quartile 1: 4.5 hours
- Median: 5 hours
- Quartile 3: 5.5 hours
- Maximum: 7 hours

b. What percent of the plane data was less than 5 hours long? Explain your thinking.

50% of the plane data. *Explanations vary.* 5 hours is the median, meaning that 50% of the data is less than it.

Accelerated 6 | Unit 9 Lesson 12 904 Activity 1

Key Takeaway: **Box plots** give us a clear way to see data in quartiles. Each section of the box plot represents 25% of the data set.

Pairs | 8 min

Activity 2 Measures of Spread

Purpose: Students explore two ways to measure spread: range and interquartile range (IQR), and use these measures to make claims about the consistency of data.

1 Launch

Invite a student to read the prompt and then have each student restate the question to a partner.

Consider asking students what they notice before inviting students to read and discuss the definitions of range and IQR.

ELL **English Language Learners** Consider discussing other contexts inside and outside of math class where the word *range* is used (e.g., a range of interests) in order to make connections and to distinguish those from the use of range in this context.

2 Monitor

Listen for students who give the IQR or range as a span of numbers rather than one value. Encourage students to revisit the definition.

D **Differentiation**

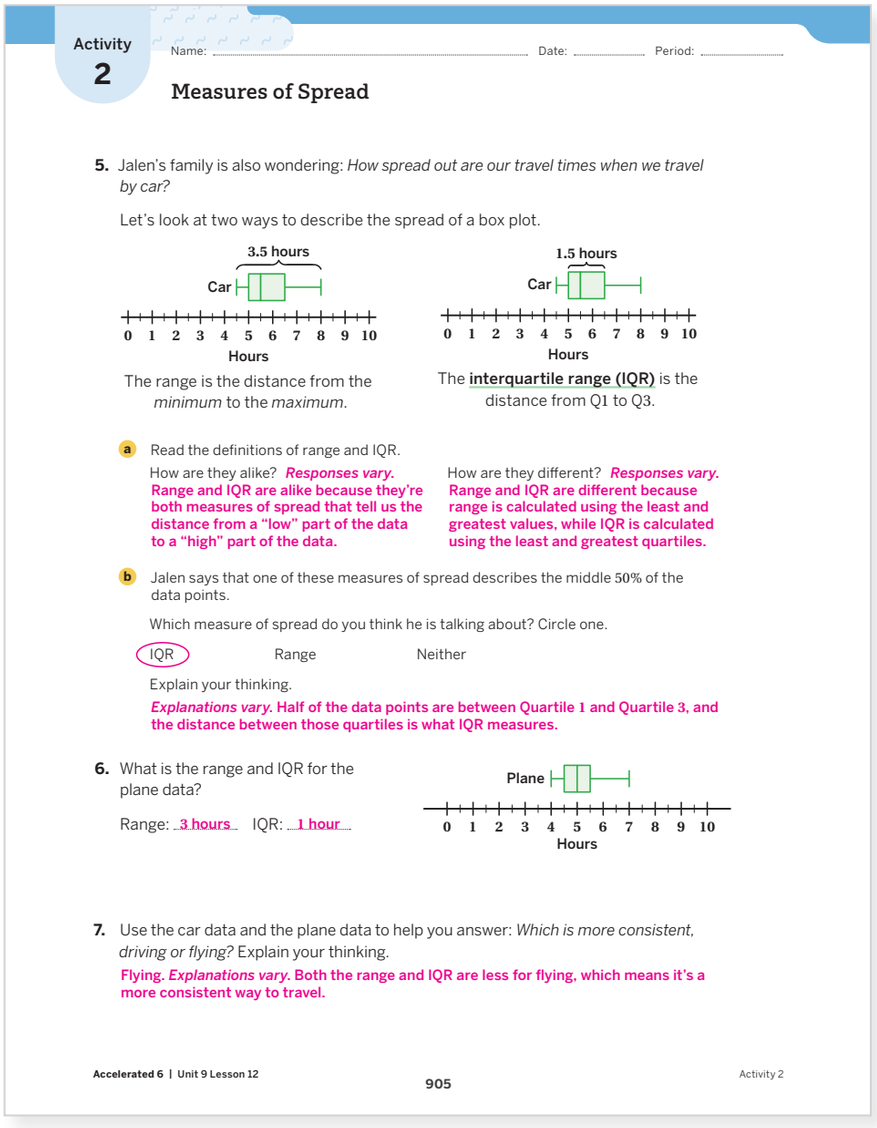
Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Consider asking, "What would data that was really consistent look like?"
Say that driving is more consistent because they compare the range of flying with the IQR of driving.	**Support** Invite students to write the IQR for driving and flying side by side and the range for driving and flying side by side to compare.
Respond that flying is more consistent because both the range and IQR are smaller.	**Stretch** Encourage students to design an experiment to calculate travel time on their way to school and consider how consistent it is. Invite students to make a plan to gather that data and share their findings with the class in the future.

3 Connect

Direct students' attention to the plane and car box plots and invite students to share how the range and IQR helped them to describe whether cars or planes were more consistent.

To surface the Key Takeaway, consider asking:

- "How do we measure the spread using a box plot?"
- "How would you describe the range in your own words? The interquartile range?"
- "How would you determine the IQR or range from a set of data not displayed on a box plot?"



Activity 2

Name: ..... Date: ..... Period: .....

Measures of Spread

5. Jalen's family is also wondering: *How spread out are our travel times when we travel by car?*

Let's look at two ways to describe the spread of a box plot.

The range is the distance from the *minimum* to the *maximum*.

The **interquartile range (IQR)** is the distance from Q1 to Q3.

a. Read the definitions of range and IQR.

How are they alike? ***Responses vary.*** **Range and IQR are alike because they're both measures of spread that tell us the distance from a "low" part of the data to a "high" part of the data.**

How are they different? ***Responses vary.*** **Range and IQR are different because range is calculated using the least and greatest values, while IQR is calculated using the least and greatest quartiles.**

b. Jalen says that one of these measures of spread describes the middle 50% of the data points.

Which measure of spread do you think he is talking about? Circle one.

(IQR) Range Neither

Explain your thinking.

Explanations vary. **Half of the data points are between Quartile 1 and Quartile 3, and the distance between those quartiles is what IQR measures.**

6. What is the range and IQR for the plane data?

Range: 3 hours IQR: 1 hour

7. Use the car data and the plane data to help you answer: *Which is more consistent, driving or flying?* Explain your thinking.

Flying. *Explanations vary.* Both the range and IQR are less for flying, which means it's a more consistent way to travel.

Accelerated 6 | Unit 9 Lesson 12 905 Activity 2

Key Takeaway: IQR and range are both measures of spread, meaning that they measure how consistent data is. Range measures the spread of the entire data set, while IQR measures the spread of the middle 50% of the data set.

Pairs | 14 min

Activity 3 Bus or Train?

Purpose: Students make connections between data, dot plots, box plots, and measures of center and spread. **(MTR.1.1)**

1 Launch

Direct students' attention to the instructions for the card sort and review them as a class.

Distribute one set of the Activity 3 Cards and straightedges to each pair of students. Encourage students to use their straightedges to create the missing box plot.

2 Monitor

Listen for students using the vocabulary from this lesson and the unit in their descriptions (e.g., *IQR*, *range*, *median*) and invite them to share their thinking during the Connect.

D Differentiation

Look for students who:	Respond to Student Thinking
Need support getting started. *(Problem 8)*	**Support** Invite students to choose one card at a time and decide if it represents the train or the bus.
First sort the cards by type of representation and then decide which column they go in by discussing individual data points that stand out to them. *(Problem 8)*	**Strengthen** Highlight this strategy, celebrating students for making use of structure to make a complex problem more solvable.
Sort the cards with precision and use the information in them to justify their argument for Problem 9. **(MTR.1.1)**	**Stretch** Consider asking what kind of data would change their answer to Problem 9.

A Accessibility: Executive Functioning

Invite students to use coloring tools or arrows to annotate the box plot and make connections between the box plot, median, IQR, and range.

Encourage students to share their reasoning with a partner and reach consensus before completing the blank cards. **(MTR.1.1)**

Activity 3 continued >

Activity 3

Name: Date: Period:

Bus or Train?

8. Jalen's family has also traveled to Chicago by bus and by train.

You will use a set of cards for this activity.

- Match each dot plot, box plot, and statistic to a form of transportation. There will be two empty spaces in the table for you to complete.
- Draw a box plot on Card A and determine the values of the missing statistics to complete the table.

	Bus	Train
Travel Times (hours)	7.5, 10.5, 9.5, 9.5, 6 7, 6.5, 7.5, 11, 8.5	6.5, 8.5, 6, 10.5, 6.5 8, 7, 7, 9, 8
Dot Plot	Card B	Card D
Box Plot	Card A	Card C
Median	8 hours	Card G 7.5 hours
IQR	Card H 2.5 hours	2 hours
Range	Card F 5 hours	Card E 4.5 hours

9. Which form of transportation (car, train, bus, or plane) would you recommend Jalen's family use to travel between Chicago and St. Louis? Use evidence to support your argument.

Responses vary.

- I would recommend a plane because its median is the lowest and its spread is low, too.
- I would recommend that Jalen's family go by plane if they weren't worried about cost. The median travel time is 5 hours compared to 7 hours, but flying by plane might be more expensive.
- I would recommend a train because the median is not that different from the others. Plus, trains avoid traffic (unlike cars and buses) and train tickets are probably cheaper than plane tickets.

Accelerated 6 | Unit 9 Lesson 12 906 Activity 3

Pairs | 14 min

Activity 3 Bus or Train? (continued)

Purpose: Students make connections between data, dot plots, box plots, and measures of center and spread. **(MTR.1.1)**

2 Monitor

Pause briefly after Problem 10a to discuss what students noticed about the travel times removed and added. **(MTR.4.1)**

Note: If it does not come up naturally, consider asking:

- "How does the removed/added data value relate to the median of the original data set?"
- "Is the removed/added data value within or outside of the original data set?"

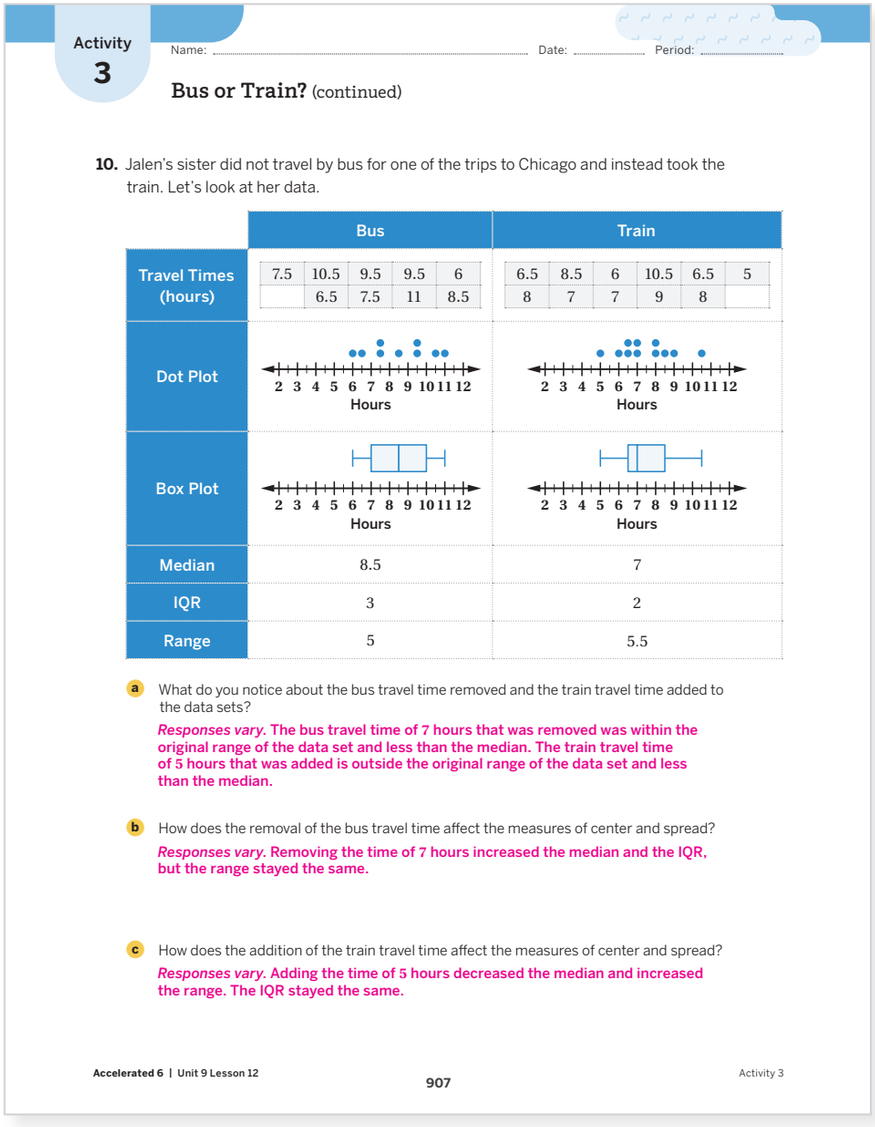
3 Connect

Invite students to share their recommended form of transportation by polling the class.

Consider asking:

- "Which graph or statistics did you use to justify the form of transportation Jalen's family should use?"
- "How did that representation in particular help you make this decision?"
- "What other graphs or statistics would you look at to continue deepening your understanding of this question?"

Invite students to share their responses about the removal and addition of travel times, encouraging them to make connections between how the data value related to the original data set and how the change affects measures of center and spread. **(MTR.1.1)**



Activity 3

Name: Date: Period:

Bus or Train? (continued)

10. Jalen's sister did not travel by bus for one of the trips to Chicago and instead took the train. Let's look at her data.

	Bus	Train
Travel Times (hours)	7.5, 10.5, 9.5, 9.5, 6 6.5, 7.5, 11, 8.5	6.5, 8.5, 6, 10.5, 6.5, 5 8, 7, 7, 9, 8
Dot Plot		
Box Plot		
Median	8.5	7
IQR	3	2
Range	5	5.5

a. What do you notice about the bus travel time removed and the train travel time added to the data sets?

Responses vary. The bus travel time of 7 hours that was removed was within the original range of the data set and less than the median. The train travel time of 5 hours that was added is outside the original range of the data set and less than the median.

b. How does the removal of the bus travel time affect the measures of center and spread?

Responses vary. Removing the time of 7 hours increased the median and the IQR, but the range stayed the same.

c. How does the addition of the train travel time affect the measures of center and spread?

Responses vary. Adding the time of 5 hours decreased the median and increased the range. The IQR stayed the same.

Accelerated 6 | Unit 9 Lesson 12 907 Activity 3

Key Takeaway: When a data value is removed from or added to a data set, it may affect measures of center and spread.

Whole Class | 5 min

Synthesis

Purpose: Students synthesize their understanding of key features of a box plot.

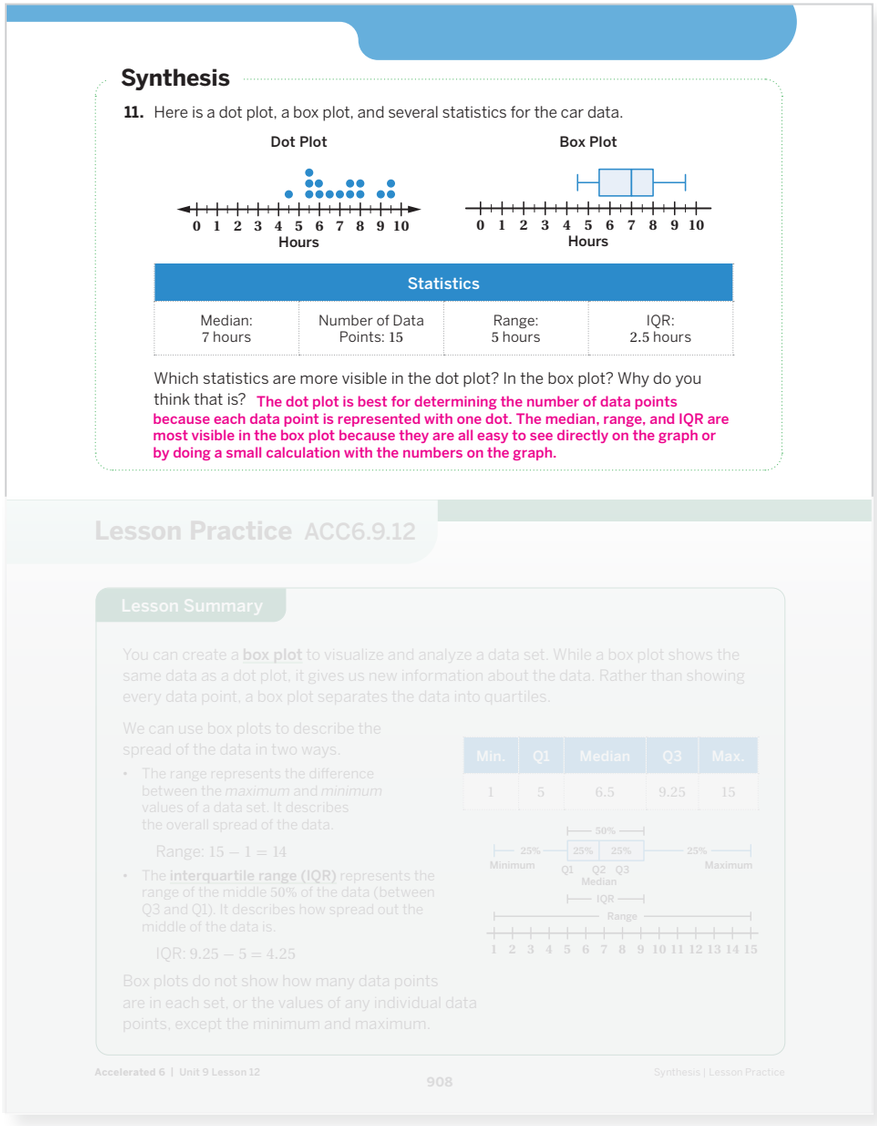
Invite students to respond independently, and then share their thinking with a partner.

Collect and share a variety of ideas, such as:
- The median is most visible in a box plot.
- The number of data points is most visible in a dot plot.
- The range is easily calculated in both representations.
- The IQR is more easily calculated in the box plot.

Consider asking:
- "What are the advantages of displaying data in a dot plot? In a box plot?"
- "When might someone want to make a box plot from data?"

MLR **MLR2: Collect and Display** Use this routine to formalize the definition of *interquartile range (IQR)* using students' language. Display any new language, examples, or diagrams publicly for students to reference later.

Note: If time allows, invite students to revise their previous thinking based on the discussion.



Synthesis

11. Here is a dot plot, a box plot, and several statistics for the car data.

Dot Plot

Box Plot

Statistics			
Median: 7 hours	Number of Data Points: 15	Range: 5 hours	IQR: 2.5 hours

Which statistics are more visible in the dot plot? In the box plot? Why do you think that is? **The dot plot is best for determining the number of data points because each data point is represented with one dot. The median, range, and IQR are most visible in the box plot because they are all easy to see directly on the graph or by doing a small calculation with the numbers on the graph.**

Lesson Practice ACC6.9.12

Lesson Summary

You can create a **box plot** to visualize and analyze a data set. While a box plot shows the same data as a dot plot, it gives us new information about the data. Rather than showing every data point, a box plot separates the data into quartiles.

We can use box plots to describe the spread of the data in two ways.

- The range represents the difference between the *maximum* and *minimum* values of a data set. It describes the overall spread of the data.

  Range: $15 - 1 = 14$

- The **interquartile range (IQR)** represents the range of the middle 50% of the data (between Q3 and Q1). It describes how spread out the middle of the data is.

  IQR: $9.25 - 5 = 4.25$

Min.	Q1	Median	Q3	Max.
1	5	6.5	9.25	15

Box plots do not show how many data points are in each set, or the values of any individual data points, except the minimum and maximum.

Accelerated 6 | Unit 9 Lesson 12 908 Synthesis | Lesson Practice

Lesson Takeaway: A **box plot** is a way to visualize data by dividing it into quartiles. A box plot shows a measure of center (the median) which divides the data set in half. It also shows two measures of variation (range and IQR). The range is a statistic that represents the difference between the *maximum* and *minimum* and the **IQR** is a statistic that measures the spread of the middle 50% of the data.

Independent | 5 min

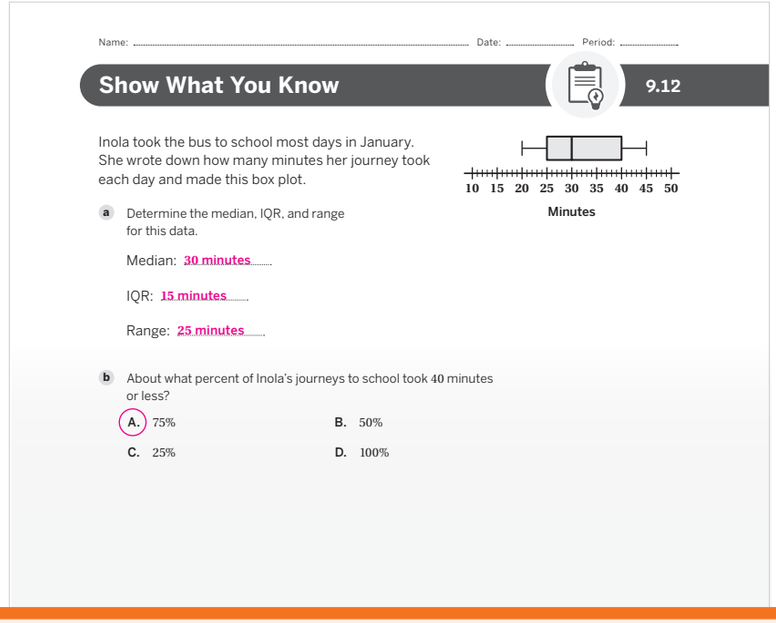
Show What You Know

Purpose: Students demonstrate their understanding by calculating the median, IQR, and range of a data set represented by a box plot and interpreting what one of the quartiles means in context.

Today's Goals

1. **Goal:** Create a *box plot* to visualize a data set.
2. **Goal:** Calculate the range and *interquartile range (IQR)* of a data set.
3. **Language Goal:** Describe the center and spread of data using terms like *median*, *interquartile range*, and *range*. **(Writing, Speaking, and Listening)**

Show What You Know (Assessment Resources)



Name: _____ Date: _____ Period: _____

Show What You Know 9.12

Inola took the bus to school most days in January. She wrote down how many minutes her journey took each day and made this box plot.

a. Determine the median, IQR, and range for this data.

Median: 30 minutes

IQR: 15 minutes

Range: 25 minutes

b. About what percent of Inola's journeys to school took 40 minutes or less?

A. 75%
B. 50%
C. 25%
D. 100%

Differentiation Use after Lesson 12

Support

Provide targeted intervention for students by using these resources.

If students need support interpreting a box plot:

» **Respond to Student Thinking:**

- Use the ***Interpreting Box Plots*** **Mini-Lesson**.
- Review the **Lesson Summary**.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite them to explore the **Sub-Unit 2 Extension Activities**.
- Invite them to collect numerical data or use data from earlier in the unit to **create a box plot and use it to draw conclusions** about the data.

Professional Learning

How did students learn from correct and incorrect thinking during this lesson? In what ways did they see the value of collaboration and changing one's mind?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

11. Here is a dot plot, a box plot, and several statistics for the car data.

Dot Plot

Box Plot

Statistics			
Median: 7 hours	Number of Data Points: 15	Range: 5 hours	IQR: 2.5 hours

Which statistics are more visible in the dot plot? In the box plot? Why do you think that is? The dot plot is best for determining the number of data points because each data point is represented with one dot. The median, range, and IQR are most visible in the box plot because they are all easy to see directly on the graph or by doing a small calculation with the numbers on the graph.

Lesson Practice ACC6.9.12

Lesson Summary

You can create a **box plot** to visualize and analyze a data set. While a box plot shows the same data as a dot plot, it gives us new information about the data. Rather than showing every data point, a box plot separates the data into quartiles.

We can use box plots to describe the spread of the data in two ways.

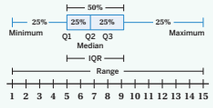
- The range represents the difference between the *maximum* and *minimum* values of a data set. It describes the overall spread of the data.

  Range: $15 - 1 = 14$

- The **interquartile range (IQR)** represents the range of the middle 50% of the data (between Q3 and Q1). It describes how spread out the middle of the data is.

  IQR: $9.25 - 5 = 4.25$

Min.	Q1	Median	Q3	Max.
1	5	6.5	9.25	15

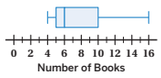


Box plots do not show how many data points are in each set, or the values of any individual data points, except the minimum and maximum.

Lesson Practice ACC6.9.12

Name: ..... Date: ..... Period: .....

Problems 1–3: Each student in a class recorded how many books they read in a school year.

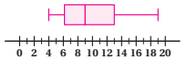


1. What is the greatest number of books that a student read this year?
   16 books
2. What is the median of this data?
   6 books
3. How would removing a data value affect the range of this data?
   Responses vary. If the data value is the least or greatest of the data set, it may decrease the range. If it is another value within the data set, the range will stay the same.

Problems 4–5: Here are five statistics about a data set.

Minimum	Q1	Median	Q3	Maximum
4	6	9	13	19

4. Create a box plot that represents this data set.



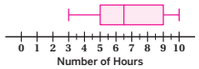
5. What is the IQR of this data set?
   7

Problems 6–9: A group of seventh-grade students recorded the number of hours they spent doing homework in one week. Here is their data: 3, 4, 5, 5, 6, 7, 7, 9, 9, 10.

6. Determine each of the statistics.

Minimum	Q1	Median	Q3	Maximum
3	5	6.5	9	10

7. Create a box plot of this data.



8. Most seventh-graders do less than 5 hours of homework in a week. Circle one. True (False)
9. 25% of seventh-grade students spent between 9 and 10 hours doing homework in a week. Circle one. (True) False

Lesson Practice ACC6.9.12

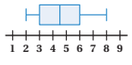
Name: ..... Date: ..... Period: .....

FAST Practice

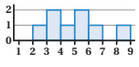
10. Here are three representations of the same data set. One representation has a mistake.

Dot Plot

Box Plot



Histogram



Select **ONE** correct answer in each box to complete the sentence to describe the mistake.

In the [**A.** dot plot (**B.** box plot) **C.** histogram], the mistake is that

[(**A.**) Q3 should be 5.5 **B.** the maximum is 8 **C.** 6 should have two data values].

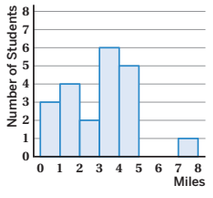
Spiral Review

11. Which of these has the largest value?

A. 25% of 80
B. 100% of 60
(**C.**) 75% of 100
D. 50% of 120

12. Nasir surveyed his classmates to determine how many miles they travel to school each day. This histogram shows the results. Select *all* the true statements.

Distance Traveled to School



☐ **A.** Most of the data is between 4 and 7 miles.
☑ **B.** The median of the data values is between 3 and 4 miles.
☐ **C.** The interquartile range of the data values is greater than 5.
☐ **D.** No students travel between 2 and 3 miles.
☑ **E.** Most of the data is between 3 and 5 miles.

Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1–2, 5	1	MA.6.DP.1.3
	3	3	MA.6.DP.1.6
	4, 7	2	MA.6.DP.1.5
	6, 8–9	2	MA.6.DP.1.3
FAST Practice	10	3	MA.6.DP.1.3
Spiral Review			
	11	2	MA.6.AR.3.4
	12	2	MA.6.DP.1.4

Assess and Respond

Sub-Unit Quiz 1

Independent | 45 min

Facilitation: Assign the Sub-Unit Quiz to learn about your students' understanding of the concepts and skills so far in this unit. If time allows, consider inviting students to complete one or more of the tasks in the Unit Synthesis and Reflection.

Item Analysis

Problem(s)	Concept or skill	Addressed in	DOK	FL B.E.S.T. Math Benchmarks
1	Determining the mean of a set of values	Lesson 6	1	MA.6.DP.1.2
2	Interpreting a histogram that represents a data set	Lessons 3, 4	1	MA.6.DP.1.4
3	Writing a question that can be answered from a given data set	Lesson 2	3	MA.6.DP.1.1
4a	Identifying and justifying which data set matches a given mean	Lesson 6	2	MA.6.DP.1.2
4b	Determining the mode and the range represented by a dot plot	Lesson 4, 7	1	MA.6.DP.1.2
5	Completing a frequency table and creating a histogram for a given data set	Lesson 5	3	MA.6.DP.1.5

Assessment Resources

- Student Assessments
- Assessment Rubrics with Answers
- Show What You Knows and Answer Keys

Intervention and Extension Resources

- Mini-Lessons
- Extensions

Name: ..... Date: ..... Period: .....

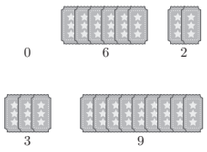
Sub-Unit Quiz

Unit 9

1. 5 friends played a game at an arcade.

Here is how many tickets each friend won:

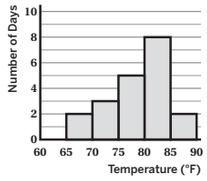
0 6 2 3 9



What is the mean number of tickets they won?

A. 3 tickets
B. 4 tickets
C. 5 tickets
D. 6 tickets

2. This histogram shows some temperatures in May in Dallas, Texas.



Which of the following statements is definitely true?

A. This shows the temperatures for a total of 5 days.
B. There were 3 days when it was 70°F.
C. It was 80°F or higher on 10 different days.
D. There were 5 days when it was 77°F.

3. A survey asked 10 people how many hours they spent watching TV last night.

0 4.5 2 3 2 2.5 3 0 2 4

What is a question that can be answered using this data? Create a line plot if it helps with your thinking.

Questions vary. What is the range number of hours spent watching TV last night?

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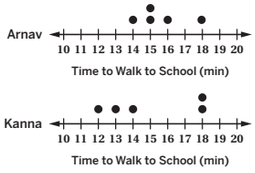
Name: ..... Date: ..... Period: .....

Sub-Unit Quiz (continued)

Unit 9

4. These line plots show the number of minutes it took Arnav and Kanna to walk to school last week.



a. Whose data has a mean of 15 minutes? Circle one.

Arnav Kanna Both Neither

Show or explain your thinking.

Explanations vary.
- The distances to the left of 15 and the distances to the right of 15 on Arnav's line plot are not equal, so his mean can't be 15.
- The total time it took Kanna to walk to school was $12 + 13 + 14 + 18 + 18 = 75$, and $75 \div 5 = 15$.

b. What are the mode and the range of Kanna's times?

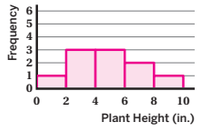
18 minutes; 6 minutes

5. Here are the heights, in inches, of plants in a garden.

Plant Height (in.)				
2.5	3.2	1.9	4.3	5.4
7.8	5.2	6.5	8.1	3.4

Complete the frequency table and make a histogram using the plant height data.

Plant Height (in.)	0 to less than 2	2 to less than 4	4 to less than 6	6 to less than 8	8 to less than 10
Frequency	1	3	3	2	1



D Differentiation (Sub-Unit Quiz 1)

Note: To **strengthen** and **stretch** students' learning, refer to the differentiation resources suggested throughout this unit and in the Unit Overview.

Sub-Unit Goals	Problem(s)	To respond to student thinking, consider:
Sub-Unit 1: • Use data sets to make claims about data. • Create histograms to visualize data. **(Lessons 1–5)**	3	**Support** **Strengthen** • **Teacher Move:** Consider revisiting: • Lesson 2 *(Survey Says...)*

	2, 4b	**Support** **Strengthen**
		• **Teacher Move:** Consider revisiting:
		  • Lesson 3 *(The Plot Thickens)*
		  • Lesson 4 *(What Does it All Mean?)*

	5	**Support** **Strengthen** • **Mini-Lesson:** *Creating Dot Plots* • **Teacher Move:** Consider revisiting: • Lesson 5 *(DIY Histograms)*
Sub-Unit 2: • Calculate the mean, median, and mode of a data set. **(Lessons 6–7)**	1, 4a	**Support** **Strengthen** • **Teacher Move:** Consider revisiting: • Lesson 6 *(Snack Time)*
	4b	**Support** **Strengthen** • **Teacher Move:** Consider revisiting: • Lesson 7 *(Toy Cars)*

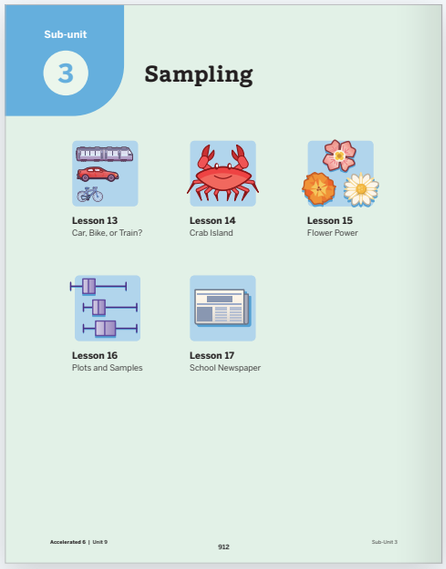
Sub-Unit 3

Sampling

Sub-Unit 3 Goals

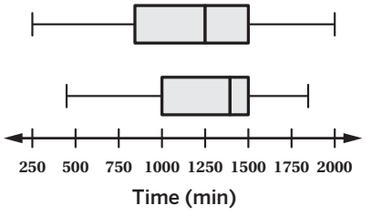
- **Goal:** Use measures of center and measures of spread from random samples to draw conclusions about and compare populations.
- **Language Goal:** Explain the purpose of sampling and which methods of obtaining a sample tend to produce representative samples.

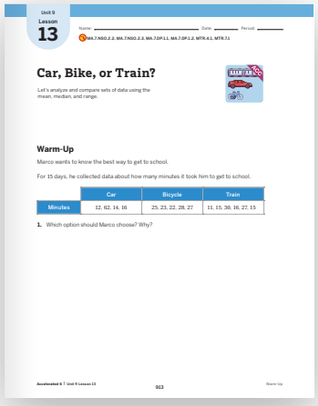
In this sub-unit, students calculate mean and median to compare and contrast the measures of center for sets of data. They conceptualize the difference between a **sample** and a **population**, analyze whether sampling strategies would likely produce a **representative** sample, and explain how that affects their understanding of a population.



Math That Matters Most

Concept or Skill	For example . . .
Calculate the mean and median for a data set.	Here is a data set: $4, 5, 5, 6, 8, 8$. **a.** Calculate the mean. *6* **b.** Determine the median. *5.5*
Use ratios and percentages to make predictions about a population based on a sample.	In a school of 580 students, one class was asked which hand they write with. "L" means they use their left hand, and "R" means they use their right hand. Here are the results: L, R, R, R, R, R, R, R, R, L, R, R, R, R, R Estimate the number of students at the school who write with their left hand. *About 77 students. In the sample, 2 out of 15 students write with their left hand, which is about 13.3%. 13.3% of 580 is 77 students.*

Concept or Skill	For example . . .
Understand why a sample is often used to study a population and determine whether a sampling method will generate a representative sample.	Jada wants to learn about the percentage of students who like the food in the cafeteria. Jada asks the first 25 students who purchase lunch at the cafeteria if they like the food. Is Jada's method likely to produce a representative sample? *No, because people who like school lunch might be more likely to line up first. Jada is also not asking any students who didn't want to buy lunch from the cafeteria.*
Use statistics from random samples to understand a population or compare two populations.	A video game developer wants to know how long it takes people to finish playing a new game. She surveyed two random samples of 20 players and asked how long it took each of them. Estimate the median time it takes a player to finish the game.  *It takes about 1300 minutes.*


Unit 9
Lesson
13
Name: Date: Period:
MA.7.NSO.2.2, MA.7.NSO.2.3, MA.7.DP.1.1, MA.7.DP.1.2, MTR.4.1, MTR.7.1
Car, Bike, or Train?
Let's analyze and compare sets of data using the mean, median, and range.
Warm-Up
Marco wants to know the best way to get to school.
For 15 days, he collected data about how many minutes it took him to get to school.

	Car	Bicycle	Train
Minutes	12, 62, 14, 16	25, 23, 22, 28, 27	11, 15, 30, 16, 27, 15

1. Which option should Marco choose? Why?

913

Car, Bike, or Train?

Using Mean, Median, and Range to Compare Groups

Let's analyze and compare sets of data using the mean, median, and range.

Learning Outcomes

Today's Goals

1. **Goal:** Calculate the mean, median, and range for a data set.
2. **Goal:** Use measures of center and spread to compare data sets.
3. **Language Goal:** Justify a choice using evidence from the data set. **(Reading, Writing, Speaking, and Listening)**

Students analyze and compare sets of data. They calculate the mean, median, and range, and use these measures to compare data and make a recommendation. They also visually determine if a data point is an outlier and determine the effect the outlier has on the mean and median.

Coherence

Prior Learning

In Sub-Unit 2, students calculated the mean, median, and mode of a data set and described the spread and distribution of data including outliers and range.

Future Learning

In Lesson 17, students will use measures of center and spread to determine if two populations are very different.

Connections

- Students effectively analyze and compare data sets for the best way to get to school by determining the mean, median, and range for a car, bicycle and train. (real world)

FL B.E.S.T. Math Benchmarks

Addressing

MA.7.DP.1.1
Determine an appropriate measure of center or measure of variation to summarize numerical data, represented numerically or graphically, taking into consideration the context and any outliers.

MA.7.DP.1.2
Given two numerical or graphical representations of data, use the measure(s) of center and measure(s) of variability to make comparisons, interpret results and draw conclusions about the two populations.

Also Addressing: **MA.7.NSO.2.2, MA.7.NSO.2.3**

Mathematical Thinking and Reasoning Standards: MTR.4.1, MTR.7.1

ELA Expectations: ELA.K12.EE.4.1, ELA.K12.EE.5.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.6.DP.1.2	**MA.912.DP.2.1**
MA.6.DP.1.6	

Lesson at a Glance

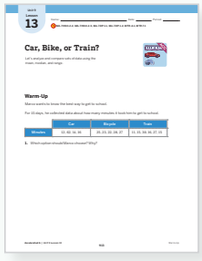
~ 45 min

MA.NSO.2.2, MA.NSO.2.3, MA.7.DP.1.1, MA..DP.1.2

Warm-Up

Pairs | 5 min

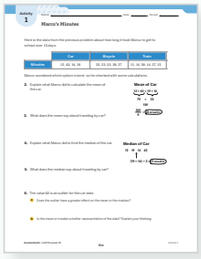
Students surface features of data that might be useful for making a decision about which transportation option to choose.
Routine: Think-Pair-Share



Activity 1

Pairs | 10 min

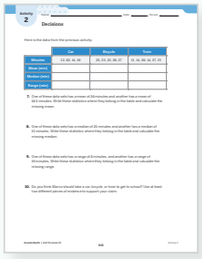
Students revisit how to measure the center of a data set using mean and median.



Activity 2

Pairs | 15 min

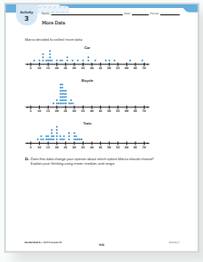
Students calculate the mean, median, and range and use these measures to make a recommendation. **Routine:** Think-Pair-Share **(MTR.4.1)**



Activity 3

Pairs | 5 min

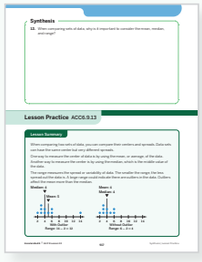
Students use their understanding of center and spread to compare and contrast data represented in dot plots.



Synthesis

Whole Class | 5 min

Students synthesize their understanding of using the mean, median, and range when analyzing a data set. MLR2: Collect and Display



Show What You Know

Independent | 5 min

Students demonstrate their understanding by calculating the mean and the mean absolute deviation of a data set.

Prep Checklist

This lesson includes:

- Student Edition
- Show What You Know

Pairs | 5 min

Warm-Up

Purpose: Students surface features of data that might be useful for making a decision about which transportation option to choose.

1 Launch

Direct students' attention to the data.

Invite students to discuss the three options with their groups before sharing as a whole class.

Use the Think-Pair-Share routine to support student-to-student discourse about data. Invite students to think independently and then take turns sharing with a partner. Invite several students to share their or their partner's thinking with the class.

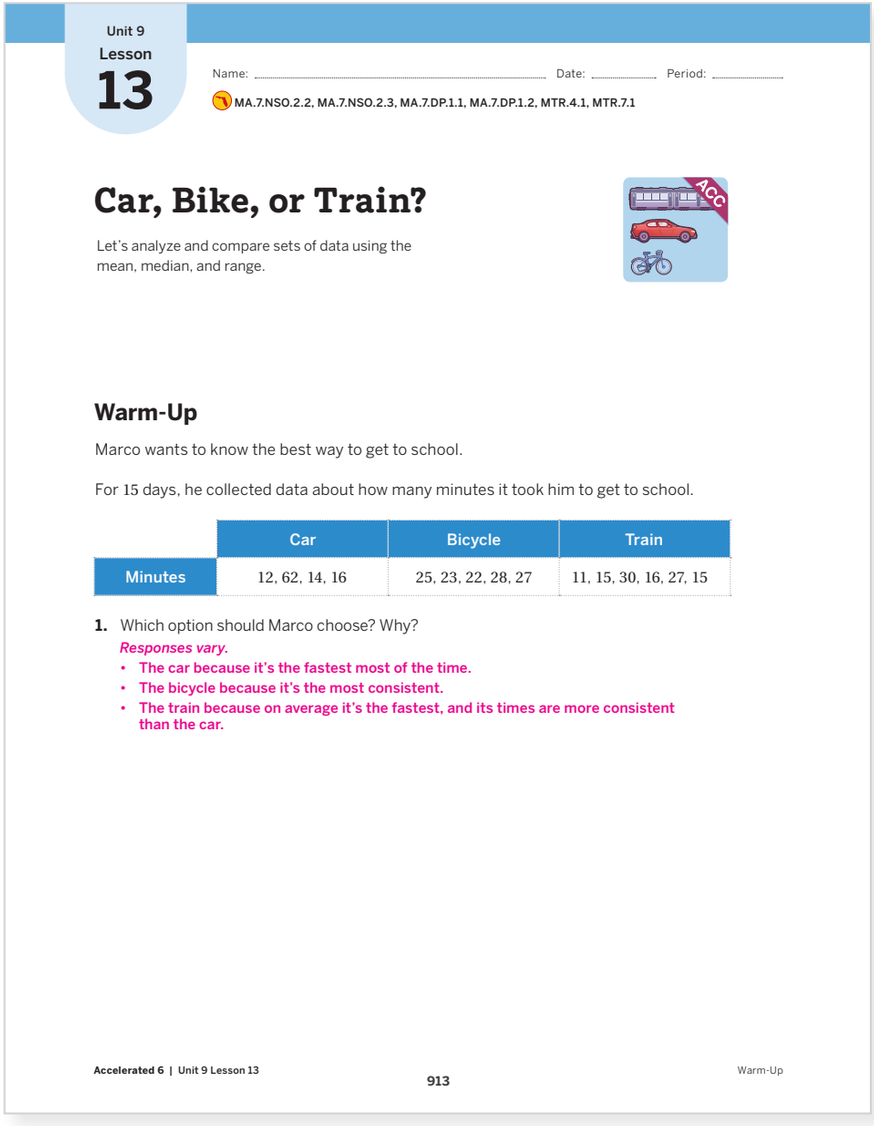
2 Connect

ELL **English Language Learners** Encourage students to paraphrase each other's ideas to help them make connections and incorporate vocabulary from the unit. **(Speaking and Listening)**

Discuss the features of the data that students paid attention to when making their choices.

A **Accessibility: Conceptual Processing** Create an anchor chart that describes features of a data set to support learning new concepts.

Consider asking, "What would make you more confident in your choice?"



Unit 9
Lesson
13

Name: _____ Date: _____ Period: _____

MA.7.NSO.2.2, MA.7.NSO.2.3, MA.7.DP.1.1, MA.7.DP.1.2, MTR.4.1, MTR.7.1

Car, Bike, or Train?

Let's analyze and compare sets of data using the mean, median, and range.

Warm-Up

Marco wants to know the best way to get to school.

For 15 days, he collected data about how many minutes it took him to get to school.

	Car	Bicycle	Train
Minutes	12, 62, 14, 16	25, 23, 22, 28, 27	11, 15, 30, 16, 27, 15

1. Which option should Marco choose? Why?

Responses vary.

- The car because it's the fastest most of the time.
- The bicycle because it's the most consistent.
- The train because on average it's the fastest, and its times are more consistent than the car.

Accelerated 6 | Unit 9 Lesson 13 913 Warm-Up

Pairs | 10 min

Activity 1 Marco's Minutes

Purpose: Students revisit how to measure the center of a data set using mean and median.

1 Launch

Consider asking, "What makes a transportation option the best?"

Note: If it doesn't come up naturally, consider asking, "How can you measure which transportation option is best?

To surface prior knowledge, invite students to turn to their partner and discuss what they remember about mean and median.

Create a class anchor chart for mean and median including definitions, how to calculate the measures with examples, when they are useful, and the effect that outliers have on them.

Consider asking, "Why might Marco be interested in calculating the mean? How will this help him decide?" **(MTR.7.1)**

2 Monitor

To support students getting started, consider asking, "What operations did Marco use?"

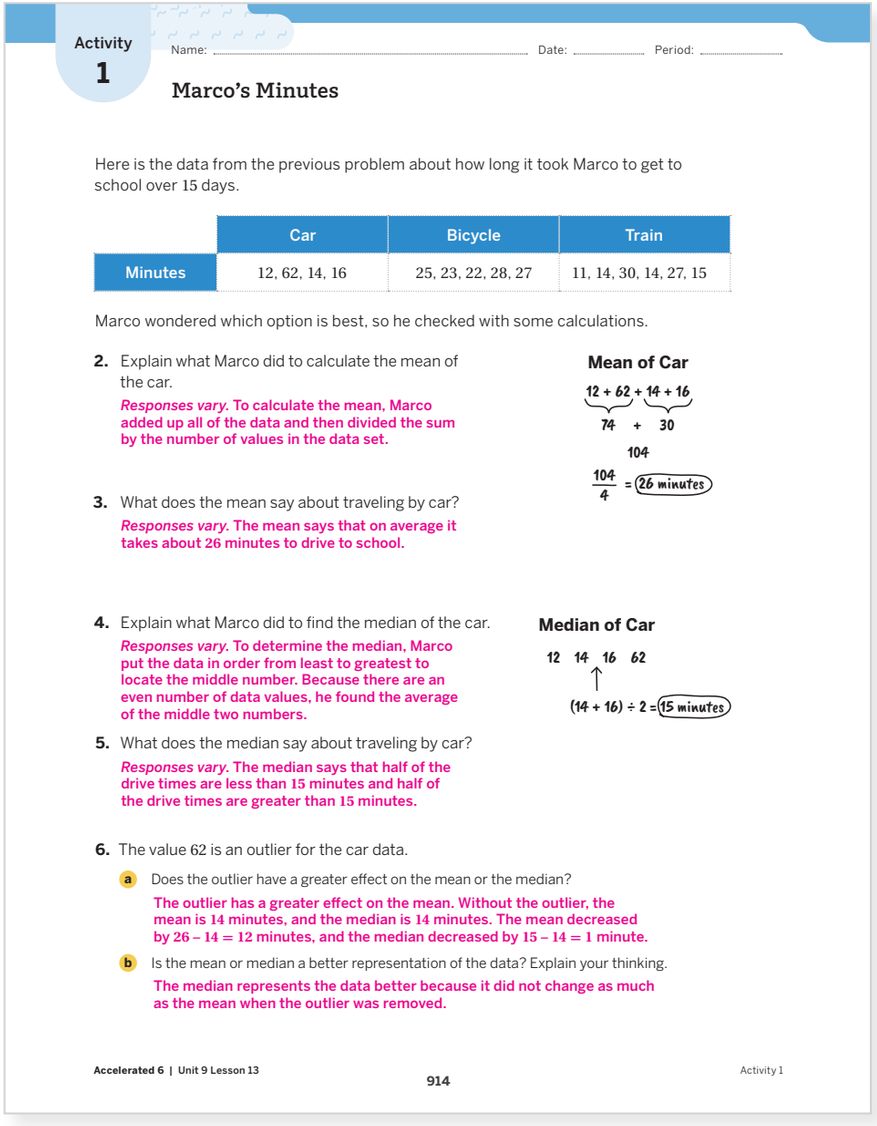
D Differentiation *(Problems 2–5)*

Look for students who:	Respond to Student Thinking
Need support getting started. *(Problem 2)*	**Support** Refer students to Marco's car calculations and consider asking, "What do each of these numbers being added represent? What does the 4 in the denominator of the fraction represent?"
Would benefit from reviewing how to determine the median. *(Problem 4)*.	**Support** Consider asking, "Why is it important to order the data before identifying the median? How is determining the median different for an odd number of data values versus an even number of data values?"
Would like to practice calculating the mean and median.	**Stretch** Encourage students to calculate the mean and median of the other two data sets.

3 Connect

To surface the Key Takeaway, consider asking:

- "Does an outlier have a greater effect on the mean or the median? Why?"
- "How do you think an outlier that was much less than the other data would affect the mean and median?"



Activity 1

Name: Date: Period:

Marco's Minutes

Here is the data from the previous problem about how long it took Marco to get to school over 15 days.

	Car	Bicycle	Train
Minutes	12, 62, 14, 16	25, 23, 22, 28, 27	11, 14, 30, 14, 27, 15

Marco wondered which option is best, so he checked with some calculations.

2. Explain what Marco did to calculate the mean of the car.

Responses vary. To calculate the mean, Marco added up all of the data and then divided the sum by the number of values in the data set.

Mean of Car

$12 + 62 + 14 + 16$

$74 + 30$

104

$\frac{104}{4} = 26$ minutes

3. What does the mean say about traveling by car?

Responses vary. The mean says that on average it takes about 26 minutes to drive to school.

4. Explain what Marco did to find the median of the car.

Responses vary. To determine the median, Marco put the data in order from least to greatest to locate the middle number. Because there are an even number of data values, he found the average of the middle two numbers.

Median of Car

12 14 16 62

$(14 + 16) \div 2 = 15$ minutes

5. What does the median say about traveling by car?

Responses vary. The median says that half of the drive times are less than 15 minutes and half of the drive times are greater than 15 minutes.

6. The value 62 is an outlier for the car data.

a. Does the outlier have a greater effect on the mean or the median?

The outlier has a greater effect on the mean. Without the outlier, the mean is 14 minutes, and the median is 14 minutes. The mean decreased by $26 - 14 = 12$ minutes, and the median decreased by $15 - 14 = 1$ minute.

b. Is the mean or median a better representation of the data? Explain your thinking.

The median represents the data better because it did not change as much as the mean when the outlier was removed.

Accelerated 6 | Unit 9 Lesson 13 914 Activity 1

Key Takeaway: When analyzing a data set, it is important to look at the center (mean and median) and consider whether there are outliers that could have an effect on the center.

Pairs | 15 min

Activity 2 Decisions

Purpose: Students calculate the mean, median, and range and use these measures to make a recommendation. **(MTR.4.1)**

Short on time? Consider having students discuss Problem 8 instead of writing their response.

1 Launch

To surface connections, consider asking:

- "How could you use the mean and median to compare the three options?"
- "What does the range tell you about the data?"

Use the Think-Pair-Share routine to support student-to-student discourse. See the Routine Facilitation Guide for more information.

2 Monitor

To support students getting started, invite them to review Marco's work from the previous activity.

Look for student strategies for estimating and calculating the missing values.

A **Accessibility: Memory and Attention** Invite students to review the anchor chart created previously on calculating the mean to activate prior knowledge.

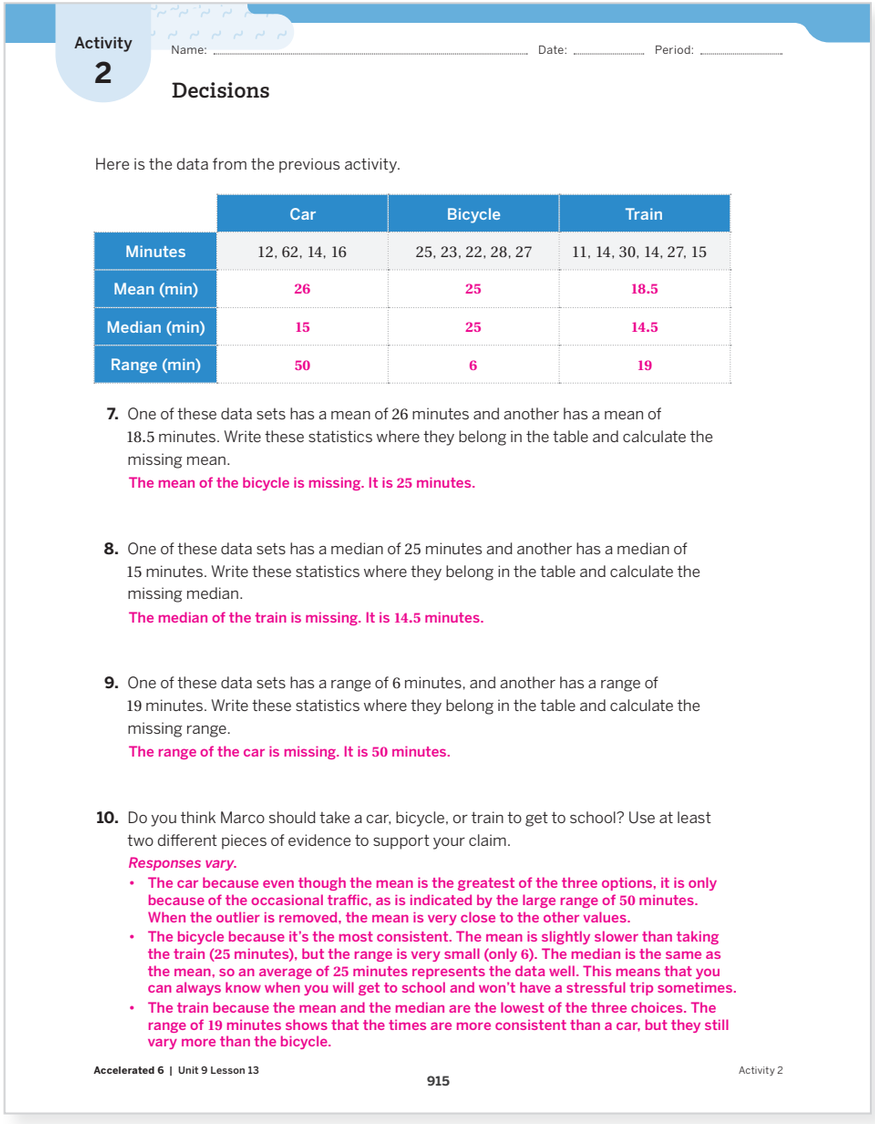
ELL **English Language Learners** Invite students to talk through their thinking with a partner before writing it down.

3 Connect

Direct students' attention to the table. Poll the class to determine which mode of transportation Marco should use. Record the results to revisit later in the lesson.

Invite students to explain their reasoning and encourage them to use evidence from their calculations. **(MTR.4.1)**

Note: It is okay — even desirable — to lack consensus at this stage. The next activity will build toward consensus when students use more data to support their claim.



Activity 2

Name: Date: Period:

Decisions

Here is the data from the previous activity.

	Car	Bicycle	Train
Minutes	12, 62, 14, 16	25, 23, 22, 28, 27	11, 14, 30, 14, 27, 15
Mean (min)	26	25	18.5
Median (min)	15	25	14.5
Range (min)	50	6	19

7. One of these data sets has a mean of 26 minutes and another has a mean of 18.5 minutes. Write these statistics where they belong in the table and calculate the missing mean.

The mean of the bicycle is missing. It is 25 minutes.

8. One of these data sets has a median of 25 minutes and another has a median of 15 minutes. Write these statistics where they belong in the table and calculate the missing median.

The median of the train is missing. It is 14.5 minutes.

9. One of these data sets has a range of 6 minutes, and another has a range of 19 minutes. Write these statistics where they belong in the table and calculate the missing range.

The range of the car is missing. It is 50 minutes.

10. Do you think Marco should take a car, bicycle, or train to get to school? Use at least two different pieces of evidence to support your claim.

Responses vary.

- The car because even though the mean is the greatest of the three options, it is only because of the occasional traffic, as is indicated by the large range of 50 minutes. When the outlier is removed, the mean is very close to the other values.
- The bicycle because it's the most consistent. The mean is slightly slower than taking the train (25 minutes), but the range is very small (only 6). The median is the same as the mean, so an average of 25 minutes represents the data well. This means that you can always know when you will get to school and won't have a stressful trip sometimes.
- The train because the mean and the median are the lowest of the three choices. The range of 19 minutes shows that the times are more consistent than a car, but they still vary more than the bicycle.

Accelerated 6 | Unit 9 Lesson 13 915 Activity 2

Pairs | 5 min

Activity 3 More Data

Purpose: Students use their understanding of center and spread to compare and contrast data represented in dot plots.

1 Launch

Direct students' attention to the dot plots and invite students to share what they notice.

2 Monitor

To support students getting started, consider asking, "Looking at these dot plots, which mode of transportation do you predict to have the greatest and least range?"

Look for students who change their opinion or use mathematically precise language such as mean, median, range, spread, or variability and invite them to share during the Connect.

A **Accessibility: Affective Functioning** To support students in sharing their thinking, ask students in advance if they, you, or a classmate can share their thinking during the whole class discussion.

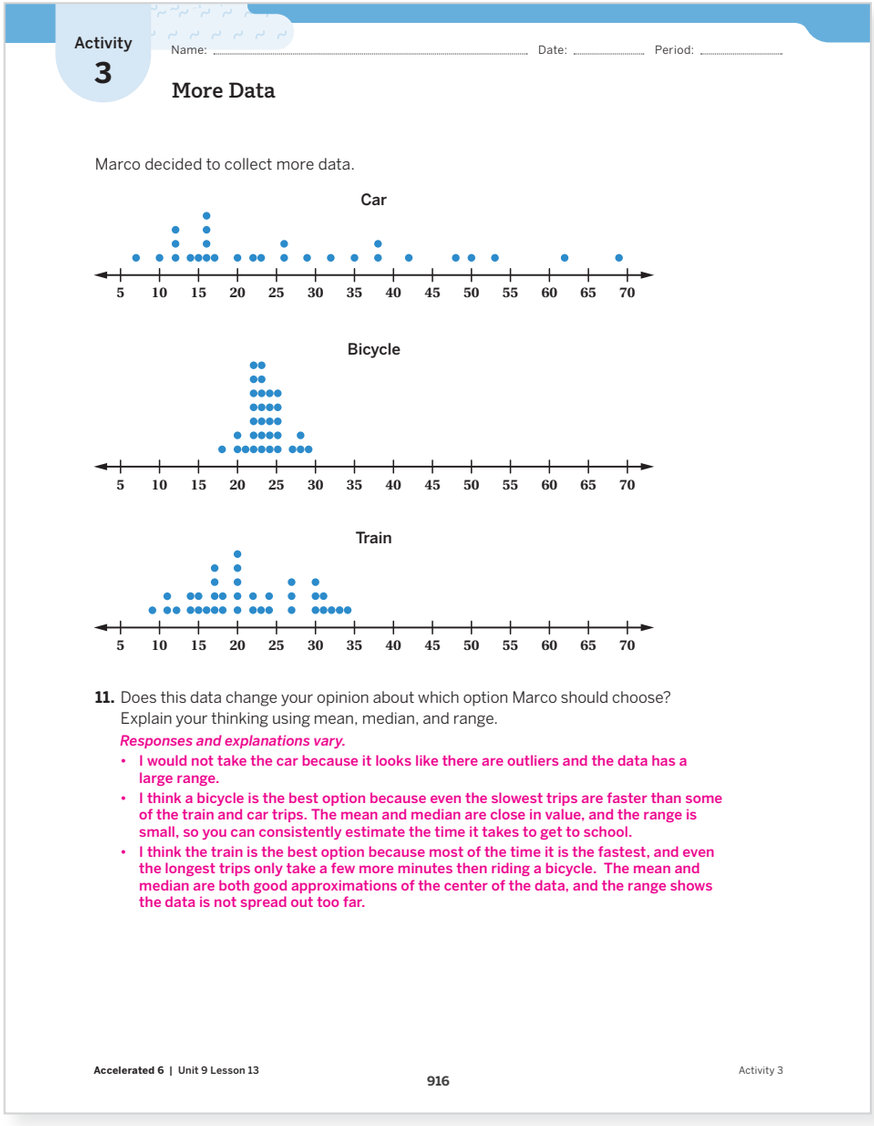
3 Connect

Invite students to share their thinking. Poll the class again to determine which mode of transportation Marco should use and compare the results to the previous activity.

Consider asking, "Did anyone's recommendation change? Why?" **(MTR.4.1)**

ELL **English Language Learners** As students share in partner and whole-class discussions, provide sentence frames such as:

- I agree/disagree with you because . . .
- I think that makes sense because . . .
- How do you know?



Activity 3

Name: Date: Period:

More Data

Marco decided to collect more data.

Car

Bicycle

Train

11. Does this data change your opinion about which option Marco should choose? Explain your thinking using mean, median, and range.

Responses and explanations vary.

- I would not take the car because it looks like there are outliers and the data has a large range.
- I think a bicycle is the best option because even the slowest trips are faster than some of the train and car trips. The mean and median are close in value, and the range is small, so you can consistently estimate the time it takes to get to school.
- I think the train is the best option because most of the time it is the fastest, and even the longest trips only take a few more minutes then riding a bicycle. The mean and median are both good approximations of the center of the data, and the range shows the data is not spread out too far.

Accelerated 6 | Unit 9 Lesson 13 916 Activity 3

Whole Class | 5 min

Synthesis

Purpose: Students synthesize their understanding of using the mean, median, and range when analyzing a data set.

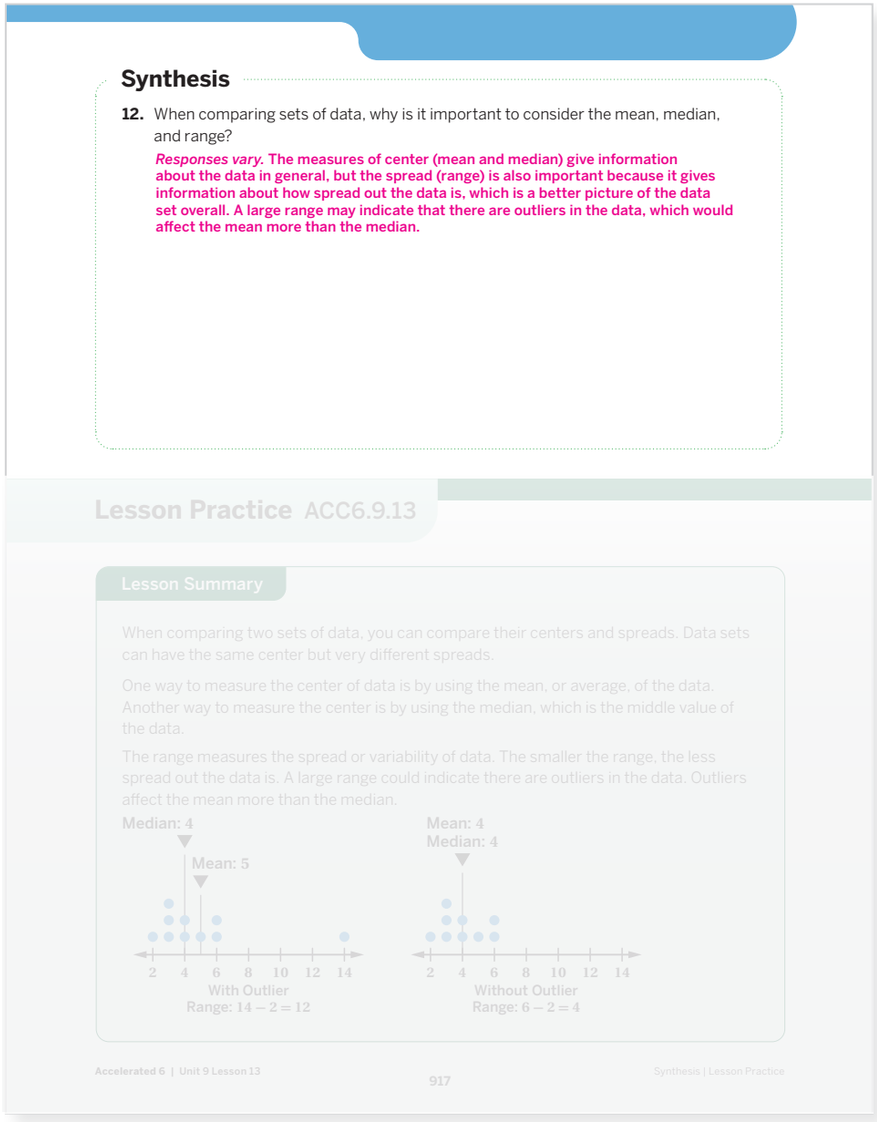
Invite students to respond independently, and then share their thinking with a partner.

Collect and share a variety of ideas, such as:

- The mean and median give information about the center of the data.
- The mean can be skewed one way or another by extremely high or low values (outliers) in the data set.
- The range gives information about the spread (variability) of the data.

Note: If time allows, invite students to revise their explanation based on the discussion.

MLR **MLR2: Collect and Display** Use this routine to collect students' ideas as they discuss with a partner the importance of considering mean, median, and range when comparing data sets. Display for students to reference and add to for the rest of the unit.



Synthesis

12. When comparing sets of data, why is it important to consider the mean, median, and range?

Responses vary. The measures of center (mean and median) give information about the data in general, but the spread (range) is also important because it gives information about how spread out the data is, which is a better picture of the data set overall. A large range may indicate that there are outliers in the data, which would affect the mean more than the median.

Lesson Practice ACC6.9.13

Lesson Summary

When comparing two sets of data, you can compare their centers and spreads. Data sets can have the same center but very different spreads.

One way to measure the center of data is by using the mean, or average, of the data. Another way to measure the center is by using the median, which is the middle value of the data.

The range measures the spread or variability of data. The smaller the range, the less spread out the data is. A large range could indicate there are outliers in the data. Outliers affect the mean more than the median.

Median: 4
Mean: 5
2 4 6 8 10 12 14
With Outlier
Range: $14 - 2 = 12$

Mean: 4
Median: 4
2 4 6 8 10 12 14
Without Outlier
Range: $6 - 2 = 4$

Accelerated 6 | Unit 9 Lesson 13 917 Synthesis | Lesson Practice

Lesson Takeaway: When comparing two sets of data, it is important to compare their centers and spread. Data sets can have a similar center but very different distributions. A larger range represents greater spread, or variability, in the data set. A smaller range represents less spread, or variability. Outliers have a greater effect on the mean than on the median.

Independent | 5 min

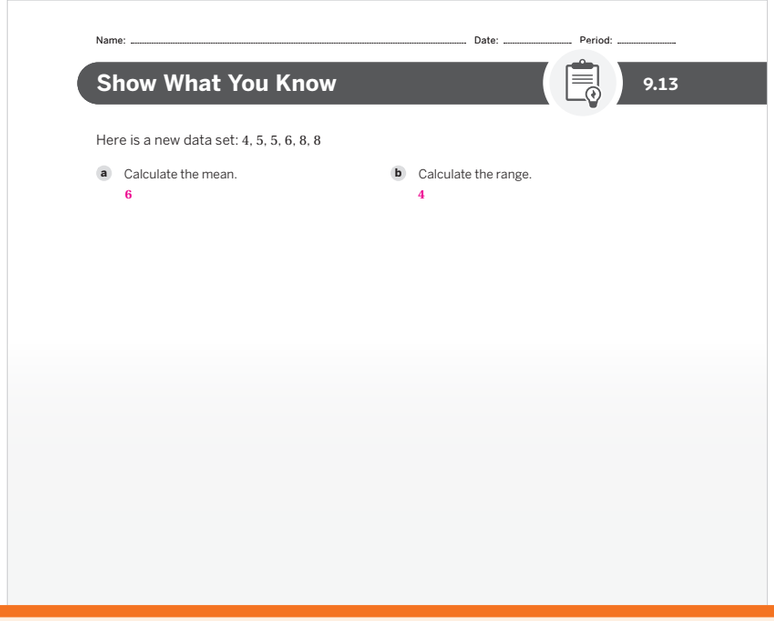
Show What You Know

Purpose: Students demonstrate their understanding by calculating the mean and mean absolute deviation of a data set.

Today's Goals

1. **Goal:** Calculate the mean and mean, median, and range for a data set.
2. **Goal:** Use measures of center and spread to compare data sets.
3. **Language Goal:** Justify a choice using evidence from the data set. **(Reading, Writing, Speaking, and Listening)**

Show What You Know (Assessment Resources)



Name: _____ Date: _____ Period: _____

Show What You Know 9.13

Here is a new data set: 4, 5, 5, 6, 8, 8

a. Calculate the mean.
6

b. Calculate the range.
4

D Differentiation Use after Lesson 13

Support

Provide targeted intervention for students by using these resources.

If students need support understanding how to calculate the mean and median from a data set:

» **Respond to Student Thinking:**

- Use the ***Calculating Measures of Center and Spread*** **Mini-lesson**.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 3 Extension Activities**.

Professional Learning

What strengths did your students exhibit today? How can you build from those strengths as students continue learning about mean, median, range, and outliers?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

12. When comparing sets of data, why is it important to consider the mean, median, and range?

Responses vary. The measures of center (mean and median) give information about the data in general, but the spread (range) is also important because it gives information about how spread out the data is, which is a better picture of the data set overall. A large range may indicate that there are outliers in the data, which would affect the mean more than the median.

Lesson Practice ACC6.9.13

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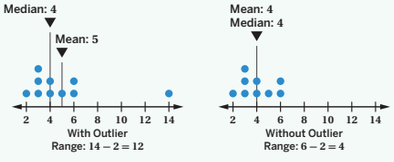


Lesson Practice ACC6.9.13

Name: _____ Date: _____ Period: _____

Problems 1–3: A school's art club holds a bake sale on Fridays to raise money for art supplies. This table shows the number of cookies they sold each week in the fall and in the spring.

Fall		Spring
8 9	1	9
0 2 6 6 7	2	0 0 2 7 7
0 1 1	3	0 0 2 3
	4	

Key: 1 | 8 = 18

1. Calculate the mean and median number of cookies sold in the fall.

Mean: 25 cookies; median: 26 cookies

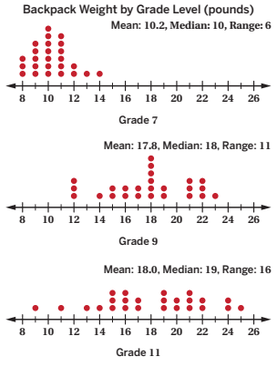
2. Calculate the mean and median number of cookies sold in the spring.

Mean: 26 cookies; median: 27 cookies

3. Based on this data, were sales generally higher in the spring than in the fall? Explain your thinking.

Yes. *Explanations vary.* Sales were slightly higher in the spring, but both the mean and median were only 1 more than in the fall, so they are not very different.

4. Compare the weights of the backpacks for students in these three grades. Include the mean and median weights and the range of weights in your response.



Responses vary.

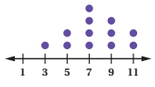
- **The backpacks tend to weigh less in grade 7 than in grades 9 and 11. The mean weight is 10.2 lb in grade 7 compared to 17.8 lb in grade 9 and 18.0 lb in grade 11. The median weight is 10 lb in grade 7 compared to 18 lb in grade 9 and 19 lb in grade 11. than for the 9th graders.**
- **The distribution of weights in grade 7 is less spread out than in grades 9 and 11. The range in grade 7 was 6 lb compared to a range of 11 lb in grade 9 and a range of 16 lb in grade 11. The backpack weights vary the most in grade 11.**

Lesson Practice ACC6.9.13

Name: _____ Date: _____ Period: _____

FAST Practice

5. Select *all* the true statements about the data shown in the dot plot.



- ☐ **A.** The mean is 7.
- ☑ **B.** The median is 7.
- ☐ **C.** The mean and median are the same.
- ☑ **D.** The mean is greater than the median.
- ☐ **E.** The range is 10.

Spiral Review

Problems 6–7: Evaluate each expression.

6. $\frac{(6-3)+(6-6)+(9-6)}{3}$ **= 2**

7. $\frac{(5-1)+(5-3)+(7-5)+(9-5)}{4}$ **= 3**

Problems 8–11: Rewrite each expression with fewer terms.

8. $4(5x-3y)-10x+6y$ **$= 10x-6y$ (or equivalent)**

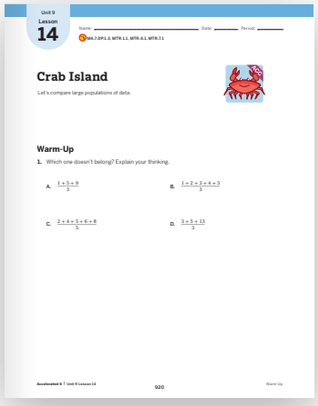
9. $2.5(2x+4y)-5(4y-x)$ **$= 10(x-y)$ (or equivalent)**

10. $7.9(5x+3y)-4.2(5x+3y)-1.7(5x+3y)$ **$= 10x+6y$ (or equivalent)**

11. $6(x+2y)-2(y-2x)$ **$= 10(x+y)$ (or equivalent)**

Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1–4	2	MA.7.DP.1.2
FAST Practice	5	2	MA.7.DP.1.2
Spiral Review			
	6–7	1	MA.7.NSO.2.1
	8–11	2	MA.7.AR.1.1


Unit 9
Lesson
14

Crab Island

Let's compare large populations of data.

Warm-Up

1. Which one doesn't belong? Explain your thinking.

A. $\frac{1+3+9}{3}$

B. $\frac{1+2+3+4+5}{5}$

C. $\frac{2+4+5+6+8}{5}$

D. $\frac{3+5+13}{3}$

920

Crab Island

Sampling from Large Populations

Let's compare large populations of data.

Learning Outcomes

Today's Goals

1. **Goal:** Compare the mean of multiple *samples* from the same *population* to the mean of the population.
2. **Language Goal:** Describe how a *sample* may be used when it is unreasonable to gather data about a *population*. **(Reading, Writing, Speaking, and Listening)**

In this lesson, students use **samples** to draw conclusions about a **population**. Students compare sample means to population means and explore the advantages and disadvantages of using samples to answer a question about a population.

Coherence

Prior Learning

In Lesson 13, students calculated and interpreted the mean, median, and range of data sets in order to compare and contrast the center and spread of data.

Future Learning

In Lesson 15, students will use data from smaller samples to make estimates about a larger population.

Connections

- Students compare large populations of data for particular species of crabs using graphs. (mathematical, cross curricular)

FL B.E.S.T. Math Benchmarks

Addressing

MA.7.DP.1.3
Given categorical data from a random sample, use proportional relationships to make predictions about a population.

Mathematical Thinking and Reasoning Standards: MTR.1.1, MTR.4.1, MTR.7.1

ELA Expectations: ELA.K12.EE.1.1, ELA.K12.EE.4.1, ELA.K12.EE.5.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.6.DP.1.1	MA.912.DP.1.4
MA.6.DP.1.2	
MA.6.DP.1.6	

Vocabulary

New Vocabulary

population	sample

Lesson at a Glance

~ 45 min

MA.7.DP.1.3

Warm-Up

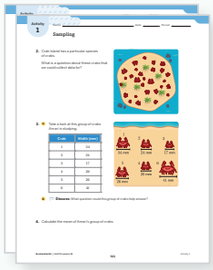
Pairs | 5 min

Students use the Which One Doesn't Belong? routine to analyze expressions.

Activity 1

Pairs | 10 min

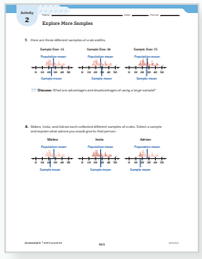
Students investigate the impact of **sample** size and randomness on how close a sample mean is to the **population** mean.
Routine: Notice and Wonder



Activity 2

Pairs | 10 min

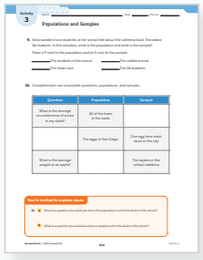
Students analyze how sample size can affect the quality of the data we collect. **(MTR.7.1)**



Activity 3

Pairs | 10 min

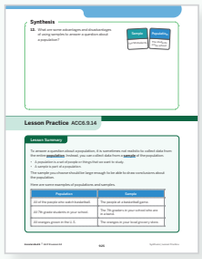
Students apply the terms population and sample to new contexts. **Routine:** Think-Pair-Share



Synthesis

Whole Class | 5 min

Students synthesize their understanding of the advantages and disadvantages of using samples to answer a question about a population. MLR2: Collect and Display



Show What You Know

Independent | 5 min

Students demonstrate their understanding by determining the population and sample in a situation.

Prep Checklist

This lesson includes:

- Student Edition
- Show What You Know

Pairs | 5 min

Warm-Up

Purpose: Students use the **Which One Doesn't Belong?** routine to analyze expressions.

1 Launch

Use the Which One Doesn't Belong? routine to support students in looking for structure in mean calculations and justifying their thinking precisely.

Consider sharing that each expression does not belong in some way. Encourage students to look for more than one possibility.

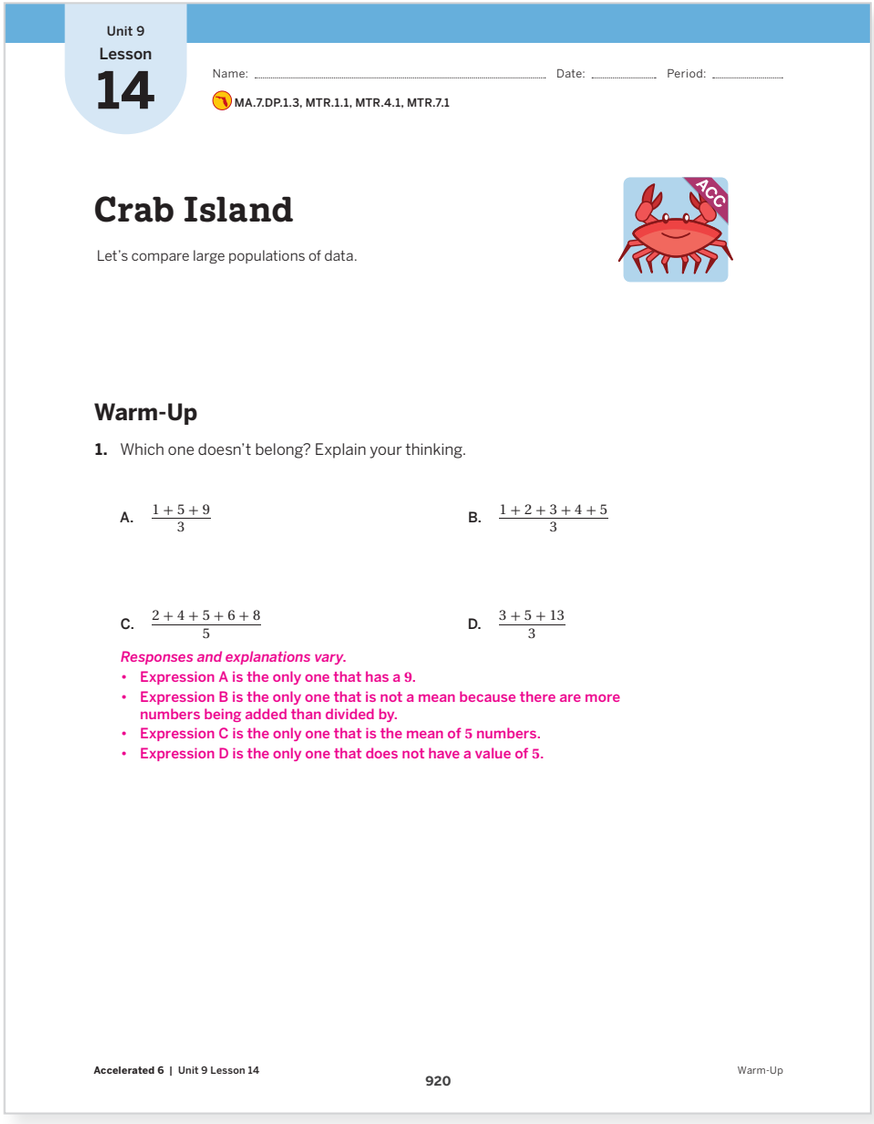
2 Connect

Discuss why each expression doesn't belong, asking students to justify their choice.

Collect and share responses and encourage students to share their thinking.

To surface connections, focus on expressions A, C, and D, and consider asking:

- "Is there a relationship between the denominator and the numerator? When have you seen this before?"
- "If you evaluate these expressions, what will the calculation represent?"



Unit 9
Lesson
14

Name: Date: Period:

MA.7.DP.1.3, MTR.1.1, MTR.4.1, MTR.7.1

Crab Island

Let's compare large populations of data.

Warm-Up

1. Which one doesn't belong? Explain your thinking.

A. $\frac{1+5+9}{3}$

B. $\frac{1+2+3+4+5}{3}$

C. $\frac{2+4+5+6+8}{5}$

D. $\frac{3+5+13}{3}$

Responses and explanations vary.

- Expression A is the only one that has a 9.
- Expression B is the only one that is not a mean because there are more numbers being added than divided by.
- Expression C is the only one that is the mean of 5 numbers.
- Expression D is the only one that does not have a value of 5.

Accelerated 6 | Unit 9 Lesson 14 920 Warm-Up

Pairs | 10 min

Activity 1 Sampling

Purpose: Students investigate the impact of **sample** size and randomness on how close a sample mean is to the **population** mean.

1 Launch

To generate excitement, direct students' attention to crab island and invite students to think about the prompt.

Invite students to share their questions.

2 Monitor

Direct students' attention to the group of crabs Amari is studying.

To support students getting started, invite them to review the expressions from the Warm-Up and describe a strategy for calculating the mean of a data set.

A **Accessibility: Memory and Attention** Invite students to use additional paper or a calculator to support them with performing calculations.

Activity 1 continued >

Activity 1

Name: Date: Period:

Sampling

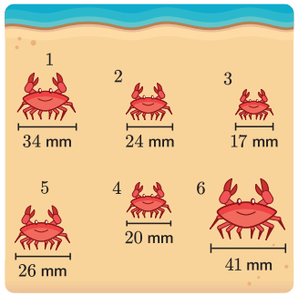
2. Crab Island has a particular species of crabs.

What is a question about these crabs that we could collect data for?

Responses vary.
- How big are the crabs' claws?
- What proportion of the crabs are babies?
- Are there more babies than adults?

3. a Take a look at this group of crabs Amari is studying.

Crab	Width (mm)
1	34
2	24
3	17
4	20
5	26
6	41



b **Discuss:** What question could this group of crabs help answer?

Responses vary. How wide are the crabs on this island?

4. Calculate the mean of Amari's group of crabs.

27 millimeters

Accelerated 6 | Unit 9 Lesson 14 921 Activity 1

Pairs | 10 min

Activity 1 Sampling (continued)

Purpose: Students investigate the impact of **sample** size and randomness on how close a sample mean is to the **population** mean.

2 Monitor

Pause and use the **Notice and Wonder** routine to promote curiosity and help students make sense of using samples that reflect the population. **(MTR.1.1)**

Create a class definition for the terms **sample** and **population** using students' language and the glossary.

Consider asking:

- "What is the population that you are collecting data from?"
- "What is the sample that you are collecting data from?"
- "Why might we use samples when conducting experiments?" **(MTR.7.1)**

ELL **English Language Learners** Consider creating an anchor chart of examples of populations and samples that students can refer to throughout the rest of the unit.

Listen for student ideas to highlight during the Connect.

D Differentiation

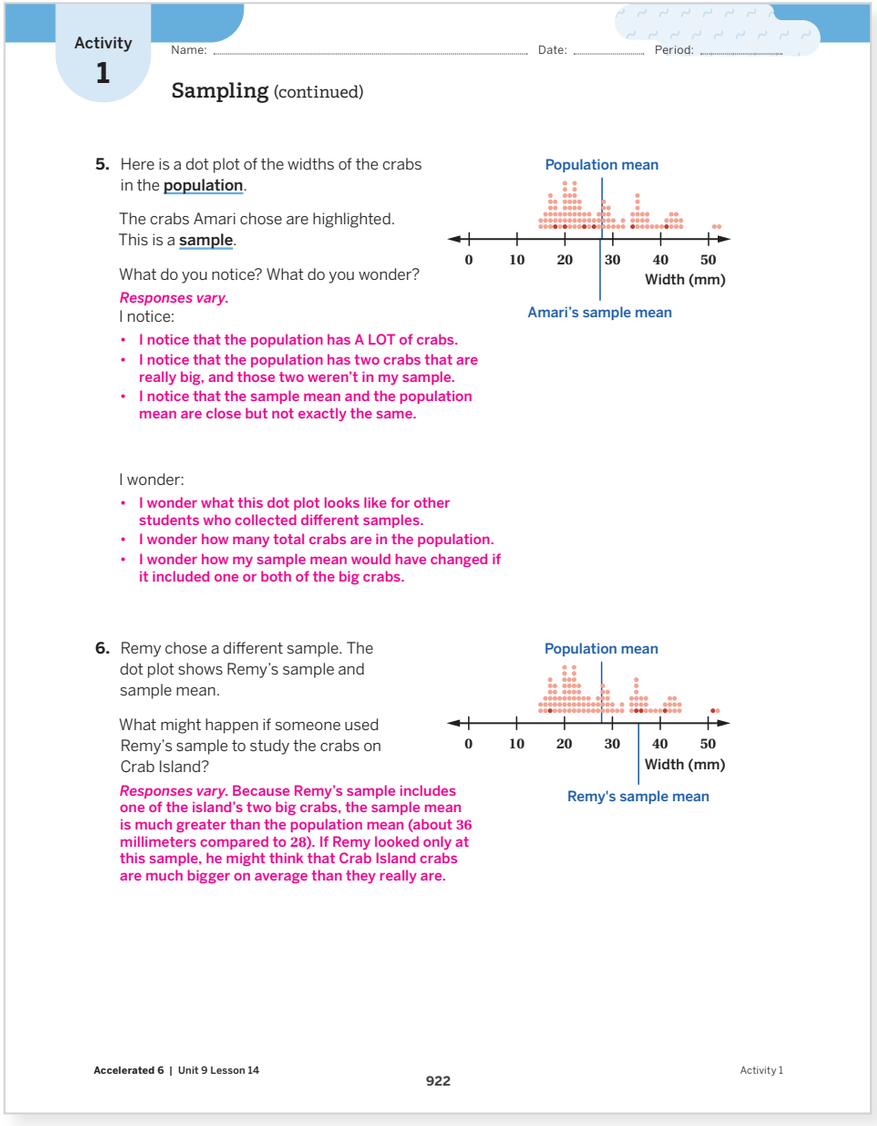
Look for students who:	Respond to Student Thinking
Only make observations about the population.	**Support** Consider asking, "What do you notice about the spread of Remy's sample?"
Would like to explore sample data more.	**Support** Encourage students to return to Problem 3 and try to select a sample that has a mean as close to the population mean as possible.
Notice that Remy's sample has more crabs that are larger.	**Stretch** Invite students to share why this may or may not reflect the population.

3 Connect

Share student responses, calling attention to any conflict or consensus.

To surface the Key Takeaway, consider asking:

- "Why might we care how close a sample mean is to the population mean?"
- "Why do you think Amari's sample mean and Remy's sample mean are different?"
- "What could someone do to be more likely to get a sample mean close to the population mean?"



Activity 1

Name: Date: Period:

Sampling (continued)

5. Here is a dot plot of the widths of the crabs in the **population**.

The crabs Amari chose are highlighted. This is a **sample**.

What do you notice? What do you wonder?

Responses vary.

I notice:

- I notice that the population has A LOT of crabs.
- I notice that the population has two crabs that are really big, and those two weren't in my sample.
- I notice that the sample mean and the population mean are close but not exactly the same.

I wonder:

- I wonder what this dot plot looks like for other students who collected different samples.
- I wonder how many total crabs are in the population.
- I wonder how my sample mean would have changed if it included one or both of the big crabs.

6. Remy chose a different sample. The dot plot shows Remy's sample and sample mean.

What might happen if someone used Remy's sample to study the crabs on Crab Island?

Responses vary. Because Remy's sample includes one of the island's two big crabs, the sample mean is much greater than the population mean (about 36 millimeters compared to 28). If Remy looked only at this sample, he might think that Crab Island crabs are much bigger on average than they really are.

Accelerated 6 | Unit 9 Lesson 14 922 Activity 1

Key Takeaway: When selecting a **sample** to gather data about a **population**, we should make sure that our sample has a variety of characteristics so that it reflects the larger population.

Pairs | 10 min

Activity 2 Explore More Samples

Purpose: Students analyze how sample size can affect the quality of the data we collect. **(MTR.7.1)**

1 Launch

Direct students' attention to the three different samples.

Share that the purpose of this activity is to explore how different samples and sample sizes affect the sample mean when compared to the population mean.

2 Monitor

Invite students to share their advice with another pair of students. Encourage them to support each other to make their advice stronger and clearer. **(MTR.4.1)**

A **Accessibility: Visual-Spatial Processing** Invite students to answer a more concrete question first to support interpreting visual representations. Consider asking, "How does each student's sample differ?"

D **Differentiation: (Stretch)** Invite students to return to Problem 3 and try to create a small sample with a mean as close to the population mean as possible.

3 Connect

Consider asking:

- "What happened to the sample mean as the sample size increased?"
- "Why might it be challenging in real life to use a large sample?"
- "What other questions might you be interested in asking where surveying the whole population would be challenging?"

A **Accessibility: Memory and Attention** Invite students to refer to the anchor chart created previously on samples and population to activate prior knowledge. Add to the anchor chart as students share.

Invite students to share their advice for one of the characters, or share the advice of another student that was helpful.

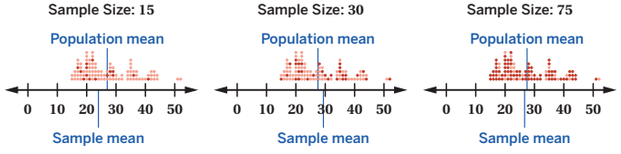
To surface the Key Takeaway, consider asking, "Whose sample mean was closest to the actual population mean? Why do you think this was the case?" **(MTR.7.1)**

Activity 2

Name: Date: Period:

Explore More Samples

7. Here are three different samples of crab widths.

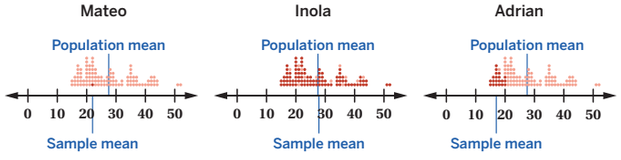


Discuss: What are advantages and disadvantages of using a large sample?

Responses vary.

- An advantage of using a large sample is that you are more likely to get a sample mean that is closer to the population mean because you are including more of the data.
- A disadvantage of using a large sample is that it might be hard to do. Tagging and measuring a lot of crabs is a lot of work!

8. Mateo, Inola, and Adrian each collected different samples of crabs. Select a sample and explain what advice you would give to that person.



Responses vary.

Advice for Mateo: Choose more crabs in your sample.

Advice for Inola: Take a smaller sample. Even with a smaller sample, your approximate sample mean can be close to the population mean.

Advice for Adrian: It looks like your sample includes a lot of small crabs. If you choose randomly, then your sample might include crabs that are more likely to represent the population.

Accelerated 6 | Unit 9 Lesson 14 923 Activity 2

Key Takeaway: There is a relationship between the **sample** mean and the **population** mean. Larger samples are more likely to have means that are closer to the population mean. It may be difficult to collect a large sample, and when it is, selecting a random sample is the most efficient way to collect data that represents the entire population.

Pairs | 10 min

Activity 3 Populations and Samples

Purpose: Students apply the terms population and sample to new contexts.

Short on time? Consider completing Problems 9 and 10 as a whole class.

1 Launch

To surface connections, consider asking students, "Who would be the best group of people to ask if you wanted to know how students feel about something?"

Note: Students may think they need to place either P or S in all blanks. Consider sharing that there will be two unused options.

Use the Think-Pair-Share routine to support student-to-student discourse. See the Routine Facilitation Guide for more information.

ELL **English Language Learners** Consider grouping students with different English language proficiency levels, so students can listen to and learn from their peers. **(Speaking and Listening)**

2 Monitor

To support students getting started, encourage them to think of any sample of trees that could be taken from the population.

Collect several responses to highlight the variety of possible answers.

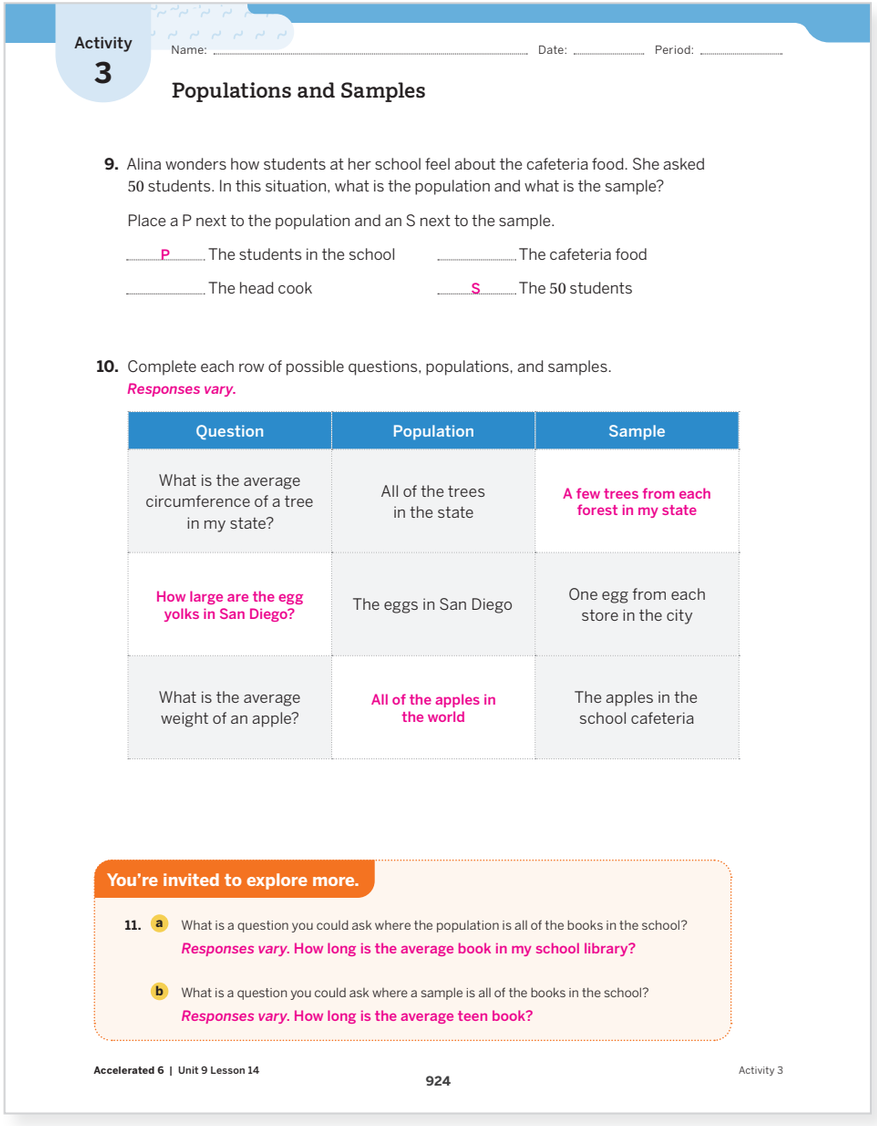
D **Differentiation: You're invited to explore more. (Stretch)** Invite students who would like to explore samples and populations further to complete this optional task. Encourage them to discuss their thinking with a partner.

3 Connect

Share the collected responses and have students discuss whether the responses coincide with the rest of the question, population, or sample.

Consider asking:

- "How do you know that is a sample?"
- "How do you know that is a population?"
- "What other samples could we use to answer this question?" **(MTR.7.1)**



Activity 3

Name: _____ Date: _____ Period: _____

Populations and Samples

9. Alina wonders how students at her school feel about the cafeteria food. She asked 50 students. In this situation, what is the population and what is the sample?

Place a P next to the population and an S next to the sample.

P The students in the school _____ The cafeteria food

_____ The head cook S The 50 students

10. Complete each row of possible questions, populations, and samples.

Responses vary.

Question	Population	Sample
What is the average circumference of a tree in my state?	All of the trees in the state	A few trees from each forest in my state
How large are the egg yolks in San Diego?	The eggs in San Diego	One egg from each store in the city
What is the average weight of an apple?	All of the apples in the world	The apples in the school cafeteria

You're invited to explore more.

11. a. What is a question you could ask where the population is all of the books in the school?

Responses vary. How long is the average book in my school library?

b. What is a question you could ask where a sample is all of the books in the school?

Responses vary. How long is the average teen book?

Accelerated 6 | Unit 9 Lesson 14 924 Activity 3

Whole Class | 5 min

Synthesis

Purpose: Students synthesize their understanding of the advantages and disadvantages of using samples to answer a question about a population.

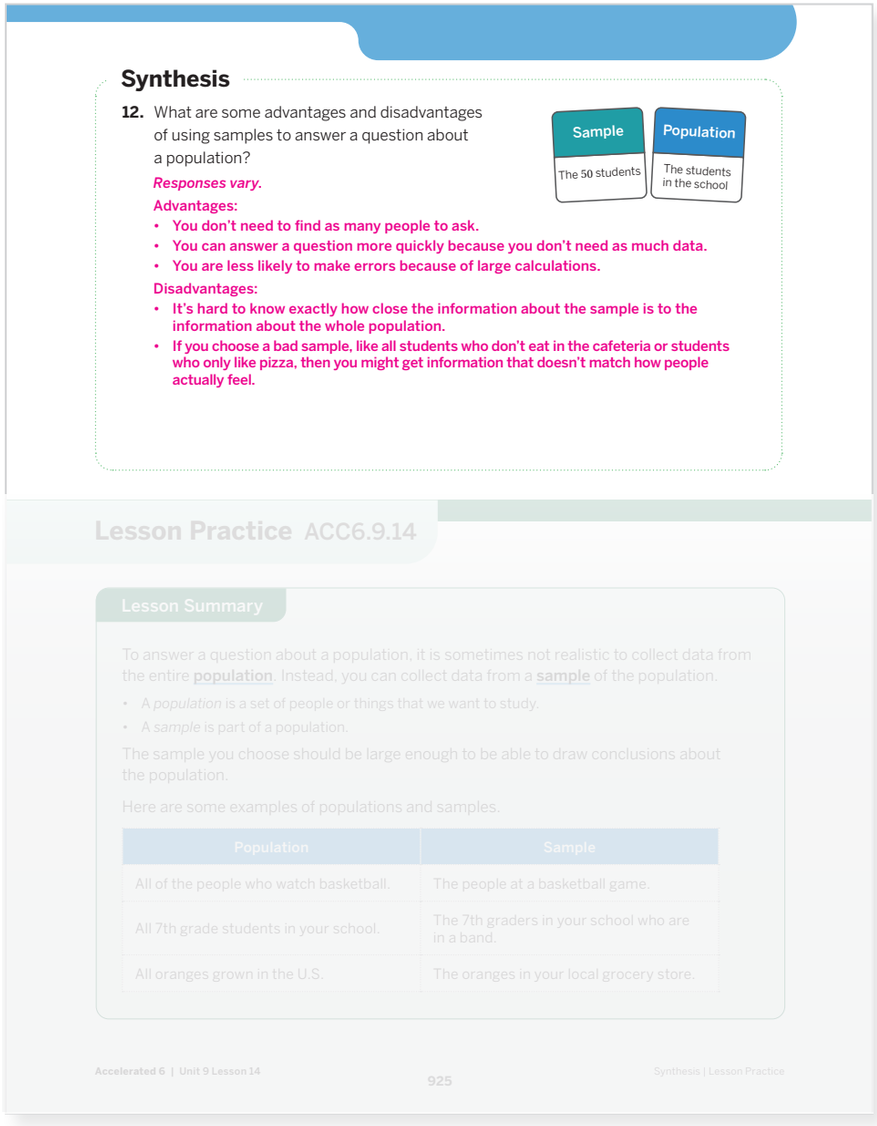
Invite students to respond independently, and then share their thinking with a partner.

Collect and share a variety of ideas, such as:

- Samples are smaller and it may be very difficult to ask an entire population a question.
- Samples may not always reflect an entire population.
- No two samples will produce the same results; depending on the sample the spread of data may be large and/or inaccurate.

MLR **MLR2: Collect and Display** Use this routine to formalize the definitions of *population* and *sample* using students' language. Display any new language, examples, or diagrams for students to reference later.

Note: If time allows, invite students to revise their previous thinking based on the discussion.



Synthesis

12. What are some advantages and disadvantages of using samples to answer a question about a population?

Responses vary.

Advantages:

- You don't need to find as many people to ask.
- You can answer a question more quickly because you don't need as much data.
- You are less likely to make errors because of large calculations.

Disadvantages:

- It's hard to know exactly how close the information about the sample is to the information about the whole population.
- If you choose a bad sample, like all students who don't eat in the cafeteria or students who only like pizza, then you might get information that doesn't match how people actually feel.

Lesson Practice ACC6.9.14

Lesson Summary

To answer a question about a population, it is sometimes not realistic to collect data from the entire **population**. Instead, you can collect data from a **sample** of the population.

- A *population* is a set of people or things that we want to study.
- A *sample* is part of a population.

The sample you choose should be large enough to be able to draw conclusions about the population.

Here are some examples of populations and samples.

Population	Sample
All of the people who watch basketball.	The people at a basketball game.
All 7th grade students in your school.	The 7th graders in your school who are in a band.
All oranges grown in the U.S.	The oranges in your local grocery store.

Accelerated 6 | Unit 9 Lesson 14 925 Synthesis | Lesson Practice

Lesson Takeaway: It is not always possible or reasonable to collect data from an entire **population**. In this case, data is collected from a **sample** of the population and used to answer any necessary questions. Samples can vary in size, but larger samples tend to better represent the population.

Independent | 5 min

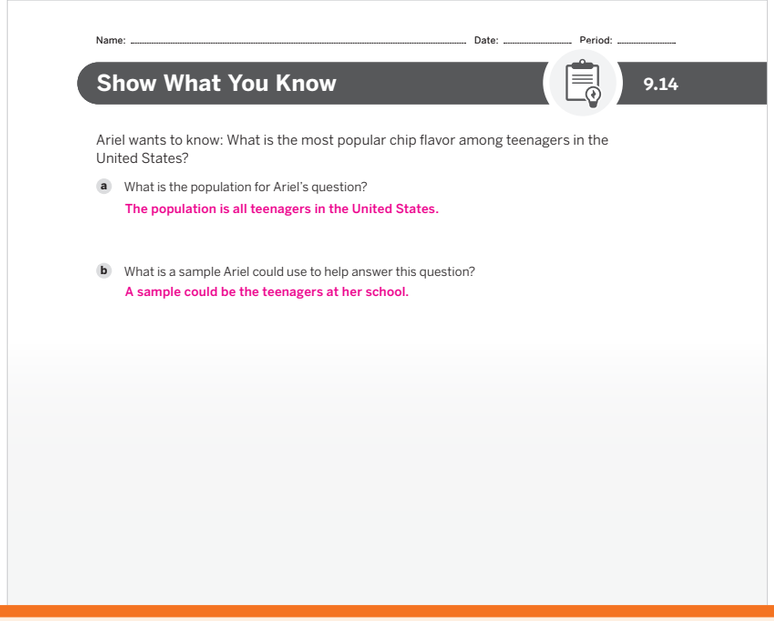
Show What You Know

Purpose: Students demonstrate their understanding by determining the population and sample in a situation.

Today's Goals

1. **Goal:** Compare the mean of multiple *samples* from the same *population* to the mean of the population.
2. **Language Goal:** Describe how a *sample* may be used when it is unreasonable to gather data about a *population*. **(Reading, Writing, Speaking, and Listening)**

Show What You Know (Assessment Resources)



Name: _____ Date: _____ Period: _____

Show What You Know 9.14

Ariel wants to know: What is the most popular chip flavor among teenagers in the United States?

a. What is the population for Ariel's question?
The population is all teenagers in the United States.

b. What is a sample Ariel could use to help answer this question?
A sample could be the teenagers at her school.

D Differentiation Use after Lesson 14

Support

Provide targeted intervention for students by using these resources.

If students need support understanding how to identify the population and sample:

» **Respond to Student Thinking:**

- Revisit **Activity 3** as a class and invite students to discuss which of the phrases describe the sample and which describes the population.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 3 Extension Activities**.
- Invite students to complete the **Explore More** task.

Professional Learning

Reflect on the facilitation of the **Which One Doesn't Belong?** routine in this lesson. Students will use this routine again in Lesson 17.

- What about the routine felt successful?
- What are some areas for growth for the next time your class uses this routine?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

12. What are some advantages and disadvantages of using samples to answer a question about a population?

Responses vary.

Advantages:

- You don't need to find as many people to ask.
- You can answer a question more quickly because you don't need as much data.
- You are less likely to make errors because of large calculations.

Disadvantages:

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Lesson Practice ACC6.9.14

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- A *sample* is part of a population.

The sample you choose should be large enough to be able to draw conclusions about the population.

Here are some examples of populations and samples.

Population	Sample
All of the people who watch basketball.	The people at a basketball game.
All 7th grade students in your school.	The 7th graders in your school who are in a band.
All oranges grown in the U.S.	The oranges in your local grocery store.

Lesson Practice ACC6.9.14

Name: _____ Date: _____ Period: _____

Problems 1–3: Aniyah wonders: How much time do 7th graders at my school spend outdoors on a typical day?

1. What is the population for Aniyah's question?

7th graders at her school.

2. Select *all* possible samples for Aniyah's question.

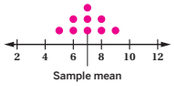
- ☑ **A.** The 20 students in a 7th grade math class.
- ☐ **B.** The first 20 people to arrive at her middle school on a particular day.
- ☐ **C.** The 7th graders participating in a science fair with students from four middle schools.
- ☑ **D.** The 10 7th graders on the school soccer team.
- ☐ **E.** The students on the high school debate team.

3. Select two samples and list a different possible population each sample could belong to. *Responses vary.*

Sample	Possible Population
The 20 students in a 7th grade math class.	All of the students in all math classes.
The first 20 people to arrive at her middle school on a particular day.	All of the people who visit the middle school.
The 7th graders participating in a science fair with students from four middle schools.	All of the students participating in the science fair.
The 10 7th graders on the school soccer team.	All of the students on the school soccer team.
The students on the high school debate team.	All of the students at the high school.

4. Here is a dot plot of a population. Create a dot plot of a sample with a mean that's the same as the population mean. Your sample should have more than 6, but fewer than 20 data points.

Population mean: 7



Sample mean

Lesson Practice ACC6.9.14

Name: _____ Date: _____ Period: _____

FAST Practice

5. The average math test score for the students in Grade 7 is 82. A teacher randomly selects 3 groups of 15 students each and calculates their average scores. The average score of Group A is 78, the average score of Group B is 84, and the average score of Group C is 82. What conclusions can you draw from these sample means?

- (A.) The sample means vary but are close to the population mean of 82.
- **B.** The sample means show that the population mean is incorrect.
- **C.** The samples cannot be used to estimate the population mean, as they differ.
- **D.** A new population mean should be calculated by averaging the sample means.

Spiral Review

Problems 6–8: Calculate the mean of each data set.

6. 8, 9, 9, 9, 10
9

7. 2, 6, 12, 16
9

8. 5, 6, 12, 13
9

Problems 9–10: A bookstore has a 15% discount on all books.

9. How much money is the 15% discount worth on a book that normally costs $18?

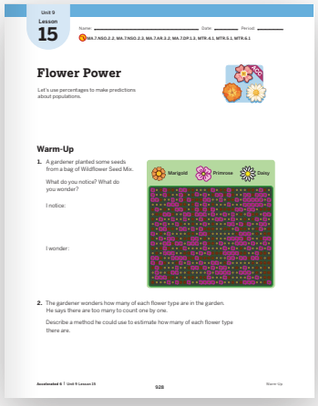
$2.70

10. After the discount, how much would the book cost?

$15.30

Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1	1	MA.7.DP.1.3
	2–3	2	MA.7.DP.1.3
	4	3	MA.7.DP.1.3
FAST Practice	5	2	MA.7.DP.1.3
Spiral Review			
	6–8	1	MA.6.DP.1.2
	9–10	1	MA.6.AR.3.4


Unit 9
Lesson 15
Flower Power
Let's use percentages to make predictions about populations.
Warm-Up
1. A gardener planted some seeds from a bag of Wildflower Seed Mix.
What do you notice? What do you wonder?
I notice:
I wonder:
Marigold
Primrose
Daisy
2. The gardener wonders how many of each flower type are in the garden. He says there are too many to count one by one.
Describe a method he could use to estimate how many of each flower type there are.
928

Flower Power

Using Percentages to Predict Populations

Let's use percentages to make predictions about populations.

Learning Outcomes

Today's Goals

1. **Goal:** Estimate unknown information about a population using proportional relationships.
2. **Language Goal:** Describe how a sample and ratios can be used to estimate information about a population. **(Reading, Writing, Speaking, and Listening)**

Students build on their knowledge of *proportional* relationships and percentages by using data from smaller samples to make estimates about a larger population. They test claims made by others by creating a new sample and comparing the results to form conclusions. **(MTR.6.1)**

Coherence

Prior Learning

In Lesson 14, students used samples to draw conclusions about a population. They explored advantages and disadvantages of using samples to answer questions about a population.

Future Learning

In Lessons 16 and 17, students will analyze how measures of center and spread can be used to compare sets of data. They will draw inferences by examining the spread of the data.

Connections

- Students understand the proportion or distribution of characteristics within a population to make predictions about wildflower seed mix. (cross curricular, real world)

FL B.E.S.T. Math Benchmarks

Addressing

MA.7.AR.3.2
Apply previous understanding of ratios to solve real-world problems involving proportions.

MA.7.DP.1.3
Given categorical data from a random sample, use proportional relationships to make predictions about a population.

Also Addressing: **MA.7.NSO.2.2, MA.7.NSO.2.3**

Mathematical Thinking and Reasoning Standards: MTR.4.1, MTR.5.1, MTR.6.1

ELA Expectations: ELA.K12.EE.1.1, ELA.K12.EE.3.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.6.AR.3.4 **MA.6.AR.3.5**	**MA.912.DP.1.4**

Vocabulary

Review Vocabulary

proportional

Lesson at a Glance

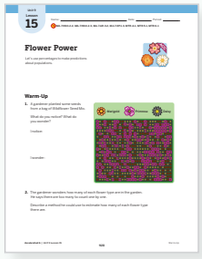
~ 45 min

MA.7.NSO.2.2, MA.7.NSO.2.3, MA.7.AR.3.2, MA.7.DP.1.3

Warm-Up

Pairs | 5 min

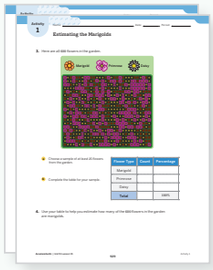
Students use the **Notice and Wonder** routine to explore flowers in a garden and consider methods for estimating how many flowers of each type there are.



Activity 1

Pairs | 15 min

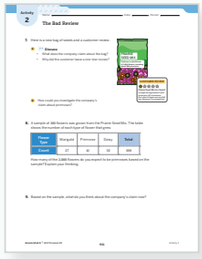
Students choose a sample of flowers from the garden to make an estimate about the total number of marigolds then analyze the estimation strategies of two other students.
MLR7: Compare and Connect



Activity 2

Pairs | 15 min

Students determine if a company's claim is true by first analyzing a customer review based on a small sample size, then using a second sample to test the claim.



Synthesis

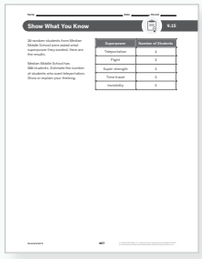
Whole Class | 5 min

Students synthesize their understanding of using a sample and ratios to estimate information about a population.

Show What You Know

Independent | 5 min

Students demonstrate their understanding by using data from a sample to make an estimate about a larger population.



Prep Checklist

This lesson includes:

- Student Edition
- Show What You Know

Pairs | 5 min

Warm-Up

Purpose: Students use the **Notice and Wonder** routine to explore flowers in a garden and consider methods for estimating how many flowers of each type there are.

1 Launch

Direct students' attention to the illustration of the garden.

Use the Notice and Wonder routine to promote curiosity about the mixture of flowers in the garden and help students make sense of ways to analyze a population.

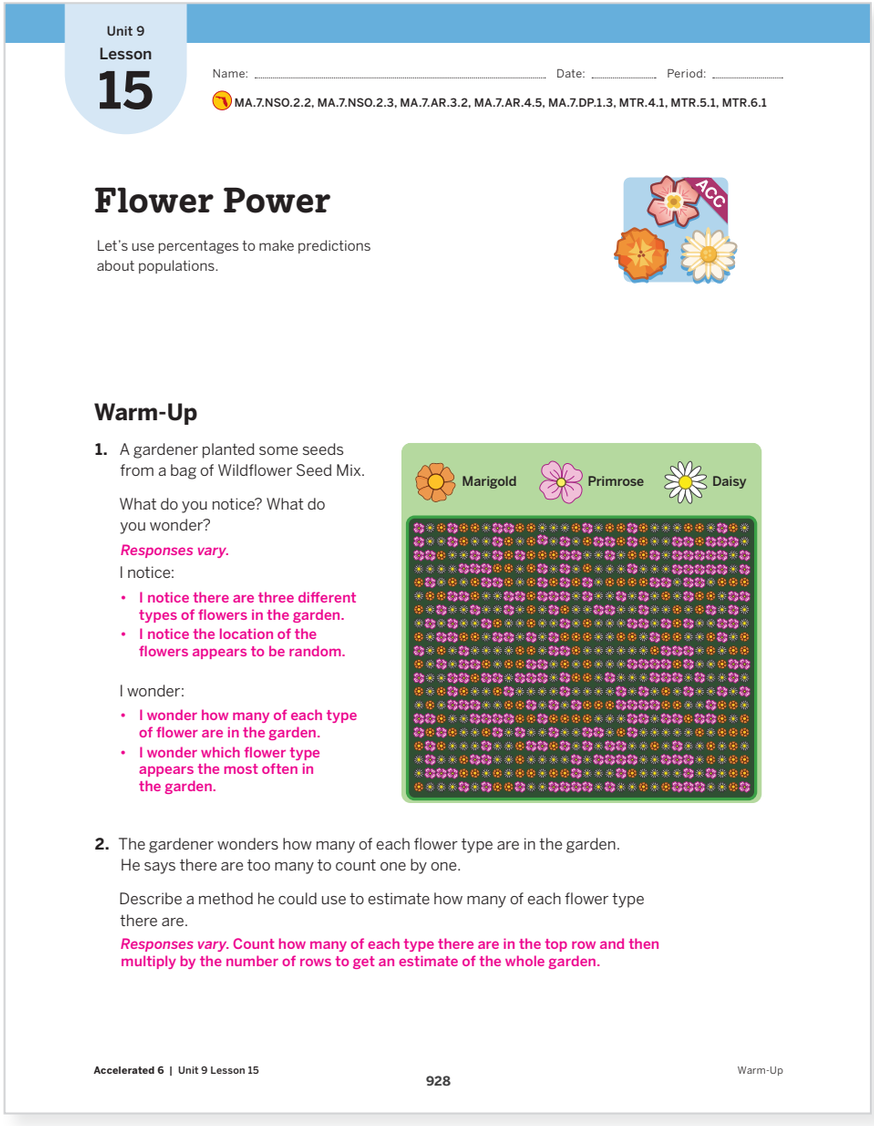
Invite students to think independently before discussing their ideas with a partner.

2 Connect

Invite students to share their methods with the class.

Consider asking:

- "What do you like about each method?"
- "Do you think this method would get a representative sample? Why or why not?"
- "What would you do if you took a random sample and one of the flower types wasn't included in your sample?"



Unit 9
Lesson
15

Name: _____ Date: _____ Period: _____

MA.7.NSO.2.2, MA.7.NSO.2.3, MA.7.AR.3.2, MA.7.AR.4.5, MA.7.DP.1.3, MTR.4.1, MTR.5.1, MTR.6.1

Flower Power

Let's use percentages to make predictions about populations.

Warm-Up

1. A gardener planted some seeds from a bag of Wildflower Seed Mix.

What do you notice? What do you wonder?

Responses vary.

I notice:

- I notice there are three different types of flowers in the garden.
- I notice the location of the flowers appears to be random.

I wonder:

- I wonder how many of each type of flower are in the garden.
- I wonder which flower type appears the most often in the garden.

2. The gardener wonders how many of each flower type are in the garden. He says there are too many to count one by one.

Describe a method he could use to estimate how many of each flower type there are.

Responses vary. Count how many of each type there are in the top row and then multiply by the number of rows to get an estimate of the whole garden.

Pairs | 15 min

Activity 1 Estimating the Marigolds

Purpose: Students choose a sample of flowers from the garden to make an estimate about the total number of marigolds then analyze the estimation strategies of two other students.

Short on time? Consider completing the discussion for Problem 6 as a whole class.

1 Launch

To surface prior knowledge, consider asking, "How can you calculate what percentage of a whole is made up of a certain item?"

2 Monitor

Support students getting started by inviting them to select a different sample size than their partner and discuss their results.

Encourage students to use a calculator as needed.

Listen for student strategies for calculating the estimate to share during the Connect.

D **Differentiation: (Support)** Students may have difficulty using their sample to create an estimate. Consider asking:

- "What percent of the whole garden do you think is marigolds? Why?"
- "How can you use equivalent ratios to estimate the number of marigolds in the garden?" **(MTR.5.1)**

Invite students that finish early to choose a different sample and refine their estimate. **(MTR.6.1)**

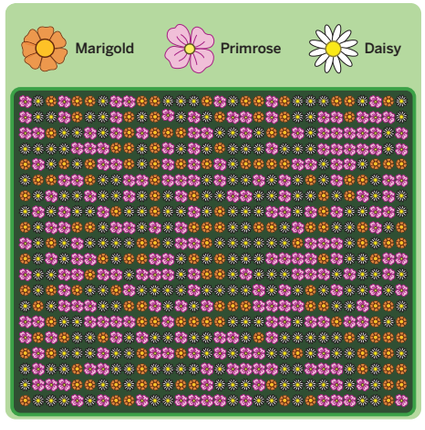
Activity 1 continued >

Activity 1

Name: Date: Period:

Estimating the Marigolds

3. Here are all 600 flowers in the garden.



a. Choose a sample of at least 25 flowers from the garden.
Samples vary.

b. Complete the table for your sample.
Responses vary based on students' sample.

Flower Type	Count	Percentage
Marigold	9	14%
Primrose	25	39%
Daisy	30	47%
Total	64	100%

4. Use your table to help you estimate how many of the 600 flowers in the garden are marigolds.
Responses vary based on students' sample.

Accelerated 6 | Unit 9 Lesson 15 929 Activity 1

Pairs | 15 min

Activity 1 Estimating the Marigolds (continued)

Purpose: Students choose a sample of flowers from the garden to make an estimate about the total number of marigolds then analyze the estimation strategies of two other students.

Short on time? Consider completing the discussion for Problem 6 as a whole class.

2 Monitor

Listen to partner discussions for student ideas about *proportional* relationships to highlight during the Connect.

A **Accessibility: Memory and Attention** To activate prior knowledge, consider asking, "What do we know about percentages and ratios that can help us to analyze Eliza's and Javier's strategies?"

MLR **MLR7: Compare and Connect** Invite students to analyze and compare each student's strategy, and then connect to their strategy for estimating the number of marigolds in the garden. Consider asking:

- "How are Eliza's and Javier's strategies alike? How are they different?"
- "What do you think was helpful about each strategy?" **(MTR.4.1)**

Listen for a variety of student responses to share during the Connect.

D **Differentiation**

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Encourage them to think about how the sizes of samples could impact an estimate of the population.
Made an estimate that was much less or much greater than the actual number of marigolds in the garden.	**Support** Consider asking "How could you make adjustments to your sample size to improve your estimate?"
Made an estimate that was close to the actual number of marigolds in the garden.	**Strengthen** Encourage them to practice with different sample sizes and create new strategies for making estimates.

3 Connect

Invite students to share ideas about how Eliza and Javier used proportional reasoning in their estimation strategies. **(MTR.5.1)**

Invite students to share what they think they could do to improve their estimates. **(MTR.6.1)**

To surface the Key Takeaway, consider asking, "Why do you think your estimate might have been different from your classmates' estimates?"

Activity 1

Name: Date: Period:

Estimating the Marigolds (continued)

5. Eliza and Javier chose a 5-by-5 sample by Eliza closing her eyes and placing her finger on the image.

Here are their strategies for estimating the total number of marigolds.

Eliza

Marigolds are 28% of the sample.

Marigolds are probably 28% of the garden.

28% of 600 is 168 marigolds.

Javier

In the sample, 7 of 25 are marigolds.

Marigolds	Total
7	25
168	600

·24 ·24

Explain each student's strategy.

Explanations vary.

- Eliza's strategy: Figure out the percent of marigolds in the sample. Then calculate that same percent of flowers in the whole garden.
- Javier's strategy: Figure out how many times larger the population is than the sample. Multiply the number of marigolds in the sample by the same scale factor.

6. The actual number of marigolds in the entire garden is 151.

Discuss:

- How did the actual number of marigolds in the garden compare to the earlier estimates?
- What do you think could have improved the estimates?

Responses vary.

- There are 151 marigolds in the garden, which is less than my estimate of 172 marigolds. This is also 17 marigolds less than Eliza and Javier's estimate.
- Using a larger sample size or looking at multiple samples might have provided a more accurate representation of the population.

Accelerated 6 | Unit 9 Lesson 15 930 Activity 1

Key Takeaway: Using a larger sample or multiple samples can provide a more accurate representation when making estimates about a population.

Pairs | 15 min

Activity 2 The Bad Review

Purpose: Students determine if a company's claim is true by first analyzing a customer review based on a small sample size, then using a second sample to test the claim.

1 Launch

To surface connections, consider asking, "What do you know about product reviews?"

Invite students to read the company's claim and the customer's review aloud then restate them in their own words.

2 Monitor

To support students getting started, encourage them to first compare the customer's review to the company's claim.

ELL **English Language Learners** Provide sentence frames to help students explain their reasoning (e.g., "The customer left a one-star review because _____." or "To investigate the company's claim, I would _____."). **(Writing and Speaking)**

Direct students' attention to the table with the results of the 100-flower sample.

Listen for student strategies using ratios to share during the Connect.

D Differentiation

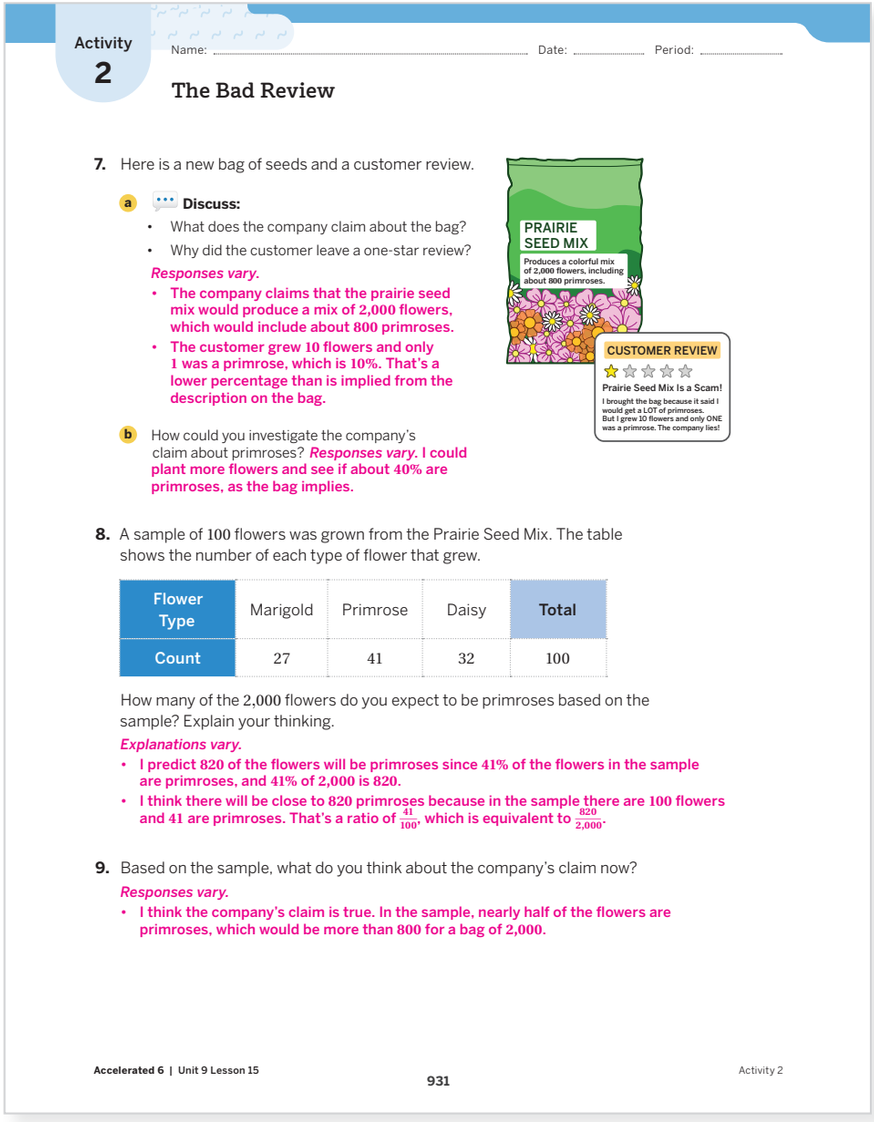
Look for students who:	Respond to Student Thinking
Make predictions that include a really high or low number of primroses.	**Support** Consider asking: • "Was there a big difference in the number of primroses in the sample compared to other flower types?" • "How did you determine how many of the 2,000 flowers were primroses?"
Make a prediction by applying ratios to the number of primroses from the sample.	**Strengthen** Consider asking, "How does your prediction compare to the customer's review about primroses?"

Look for a variety of student responses to share during the Connect.

3 Connect

Share strategies that used ratios and invite students to share how they used calculations to test the company's claim. **(MTR.6.1)**

To surface the Key Takeaway, consider asking:
- "How did using a random sample help you test the claim made by the company?"
- "How can ratios be used to make predictions about a larger population?"



Activity 2

Name: _____ Date: _____ Period: _____

The Bad Review

7. Here is a new bag of seeds and a customer review.

a. **Discuss:**
- What does the company claim about the bag?
- Why did the customer leave a one-star review?

Responses vary.
- The company claims that the prairie seed mix would produce a mix of 2,000 flowers, which would include about 800 primroses.
- The customer grew 10 flowers and only 1 was a primrose, which is 10%. That's a lower percentage than is implied from the description on the bag.

b. How could you investigate the company's claim about primroses? *Responses vary.* I could plant more flowers and see if about 40% are primroses, as the bag implies.

8. A sample of 100 flowers was grown from the Prairie Seed Mix. The table shows the number of each type of flower that grew.

Flower Type	Marigold	Primrose	Daisy	Total
Count	27	41	32	100

How many of the 2,000 flowers do you expect to be primroses based on the sample? Explain your thinking.

Explanations vary.
- I predict 820 of the flowers will be primroses since 41% of the flowers in the sample are primroses, and 41% of 2,000 is 820.
- I think there will be close to 820 primroses because in the sample there are 100 flowers and 41 are primroses. That's a ratio of $\frac{41}{100}$, which is equivalent to $\frac{820}{2,000}$.

9. Based on the sample, what do you think about the company's claim now?

Responses vary.
- I think the company's claim is true. In the sample, nearly half of the flowers are primroses, which would be more than 800 for a bag of 2,000.

Accelerated 6 | Unit 9 Lesson 15 931 Activity 2

Key Takeaway: A random sample can be used to test a claim by applying equivalent ratios to relate the sample data to a larger population to make predictions about the larger population.

Whole Class | 5 min

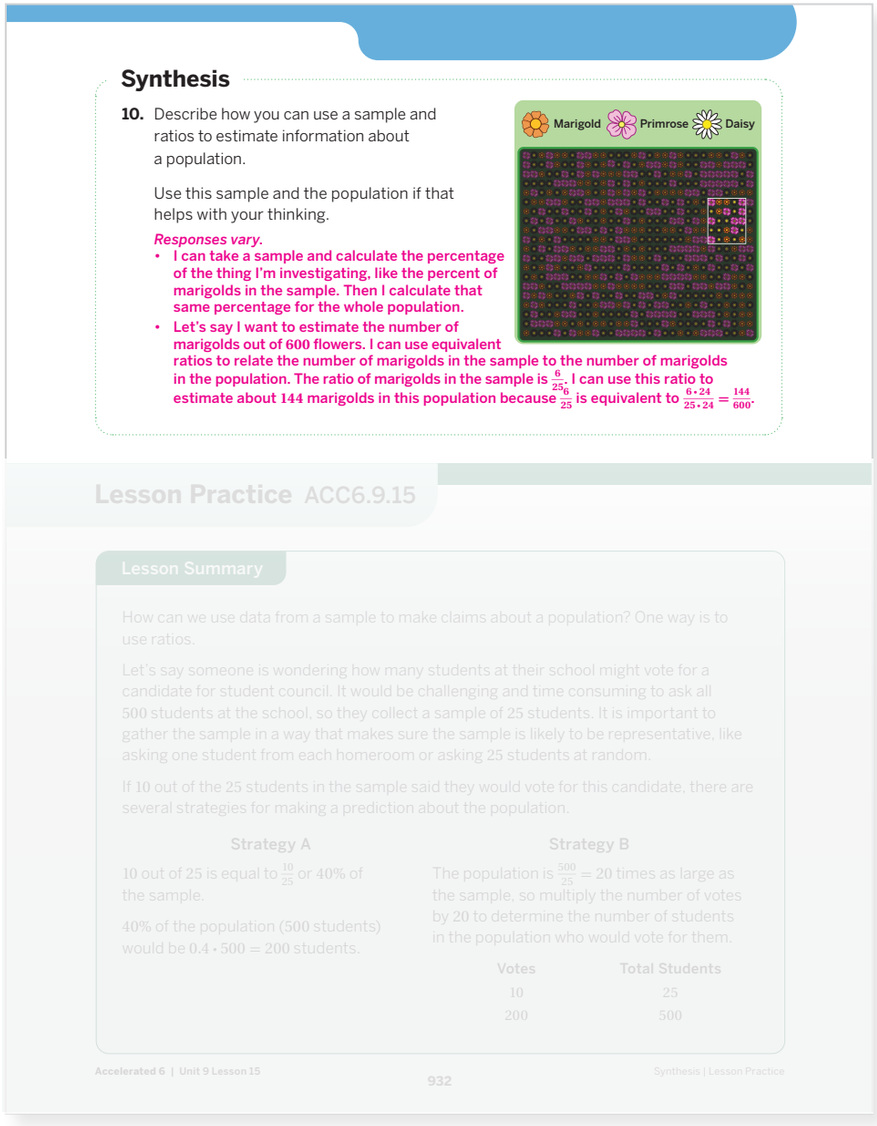
Synthesis

Purpose: Students synthesize their understanding of using a sample and ratios to estimate information about a population.

Invite students to respond independently, and then share their thinking with a partner.

Collect and share a variety of ideas, such as:
- Using ratios to calculate a percentage in a sample then calculate that percentage for the large population.
- Using equivalent ratios to estimate a value for the larger population based on the number being proportional for that object in the sample.

Note: If time allows, invite students to revise their previous thinking based on the discussion.



Synthesis

10. Describe how you can use a sample and ratios to estimate information about a population.

Use this sample and the population if that helps with your thinking.

Responses vary.
- I can take a sample and calculate the percentage of the thing I'm investigating, like the percent of marigolds in the sample. Then I calculate that same percentage for the whole population.
- Let's say I want to estimate the number of marigolds out of 600 flowers. I can use equivalent ratios to relate the number of marigolds in the sample to the number of marigolds in the population. The ratio of marigolds in the sample is $\frac{6}{25}$. I can use this ratio to estimate about 144 marigolds in this population because $\frac{6}{25}$ is equivalent to $\frac{6 \cdot 24}{25 \cdot 24} = \frac{144}{600}$.

Lesson Practice ACC6.9.15

Lesson Summary

How can we use data from a sample to make claims about a population? One way is to use ratios.

Let's say someone is wondering how many students at their school might vote for a candidate for student council. It would be challenging and time consuming to ask all 500 students at the school, so they collect a sample of 25 students. It is important to gather the sample in a way that makes sure the sample is likely to be representative, like asking one student from each homeroom or asking 25 students at random.

If 10 out of the 25 students in the sample said they would vote for this candidate, there are several strategies for making a prediction about the population.

Strategy A

10 out of 25 is equal to $\frac{10}{25}$ or 40% of the sample.

40% of the population (500 students) would be $0.4 \cdot 500 = 200$ students.

Strategy B

The population is $\frac{500}{25} = 20$ times as large as the sample, so multiply the number of votes by 20 to determine the number of students in the population who would vote for them.

Votes	Total Students
10	25
200	500

Accelerated 6 | Unit 9 Lesson 15 932 Synthesis | Lesson Practice

Lesson Takeaway: Ratios can be used to make connections between the sample data and the population data. Using larger or multiple samples is more likely to provide a more accurate representation of the population.

Independent | 5 min

Show What You Know

Purpose: Students demonstrate their understanding by using data from a sample to make an estimate about a larger population.

Today's Goals

1. **Goal:** Estimate unknown information about a population using proportional relationships.
2. **Language Goal:** Describe how a sample and ratios can be used to estimate information about a population. **(Reading, Writing, Speaking, and Listening)**

Show What You Know (Assessment Resources)

Name: _____ Date: _____ Period: _____

Show What You Know 9.15

20 random students from Median Middle School were asked what superpower they wanted. Here are the results.

Superpower	Number of Students
Teleportation	4
Flight	3
Super strength	3
Time travel	5
Invisibility	5

Median Middle School has 500 students. Estimate the number of students who want teleportation. Show or explain your thinking.

100 students. Responses vary. First, I found the percentage of students from the random group who want teleportation as their superpower, which was $\frac{4}{20}$ or 0.20. Then, I multiplied 0.20 and 500 to estimate the number of students at the school who want the same superpower, which was $0.20 \times 500 = 100$ student.

Differentiation Use after Lesson 15

Support

Provide targeted intervention for students by using these resources.

If students need support understanding how to of make an estimate about population using ratios:

» **Respond to Student Thinking:**

- Use the ***Predicting Population Using Sample Data*** **Mini-lesson**.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 3 Extension Activities**.
- Invite students to pose a question about their school's student population, then gather a sample from students in their class and use it to draw a conclusion using ratios.

Professional Learning

How did the **MLR7: Compare and Connect** routine support students in using a sample to make an estimate about a population?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

10. Describe how you can use a sample and ratios to estimate information about a population.

Use this sample and the population if that helps with your thinking.

Responses vary.

- I can take a sample and calculate the percentage of the thing I'm investigating, like the percent of marigolds in the sample. Then I calculate that same percentage for the whole population.
- Let's say I want to estimate the number of marigolds out of 600 flowers. I can use equivalent ratios to relate the number of marigolds in the sample to the number of marigolds in the population. The ratio of marigolds in the sample is $\frac{6}{25}$. I can use this ratio to estimate about 144 marigolds in this population because $\frac{6}{25}$ is equivalent to $\frac{6 \cdot 24}{25 \cdot 24} = \frac{144}{600}$.

Lesson Practice ACC6.9.15

Lesson Summary

How can we use data from a sample to make claims about a population? One way is to use ratios.

Let's say someone is wondering how many students at their school might vote for a candidate for student council. It would be challenging and time consuming to ask all 500 students at the school, so they collect a sample of 25 students. It is important to gather the sample in a way that makes sure the sample is likely to be representative, like asking one student from each homeroom or asking 25 students at random.

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Votes	Total Students
10	25
200	500

Lesson Practice ACC6.9.15

Name: ..... Date: ..... Period: .....

Problems 1–3: Faaria and Ariel wondered how many students at school would dye their hair blue. They each surveyed a different random sample of the students.

1. Faaria asked 10 students, and 2 of them said they would. Based on Faaria's sample, what percent of the students would dye their hair blue?

20%

2. Ariel asked 100 students, and 17 of them said they would. Based on Ariel's sample, what percent of the students would dye their hair blue?

17%

3. Whose percentage is likely to be closest to the percentage of all the students? Explain how you know.

Ariel. *Explanations vary.* Ariel's sample is much larger than Faaria's. The larger the random sample, the closer it usually is to representing the actual population.

Problems 4–6: In a school of 580 students, one class was asked which hand they write with. "L" means they use their left hand, and "R" means they use their right hand.

Here are the results: L, R, R, R, R, R, R, R, L, R, R, R, R, R

4. Based on this sample, estimate the percentage of students at the school who write with their left hand.

About 13.3%

5. Estimate the number of students at the school who write with their left hand.

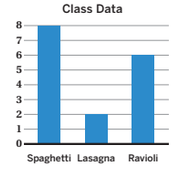
77 students

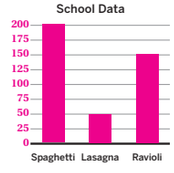
6. A different class of 18 students is surveyed. Estimate how many write with their left hand. Explain your thinking.

Responses and explanations vary. I estimate that 2 students write with their left hand because 13.3% of 18 is about 2.4 students and that number rounds down to 2.

7. 16 students at a school were asked about their favorite pasta dish. The results are shown in the bar graph. Create a new bar graph showing the possible results for all 400 students in the school. Be sure to scale and label the vertical axis.

Sample shown on graph.





Lesson Practice ACC6.9.15

Name: ..... Date: ..... Period: .....

FAST Practice

8. A gardener randomly selected 25 flowers from a garden with a total of 600 flowers. In the sample, 7 flowers were marigolds. Based on this sample, how many marigolds are likely in the entire garden?

A. 150
B. 168
C. 175
D. 200

Spiral Review

9. Match each expression in the first list with an equivalent expression from the second list.

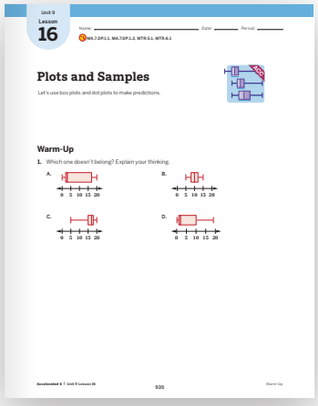
Expression	Equivalent Expression
a. $(8x + 6y) - (2x + 4y)$	c. $10x - 10y$
b. $(8x + 6y) - (2x - 4y)$	d. $10x - 2y$
c. $8x - 6y - (-2x + 4y)$	a. $6x + 2y$
d. $8x - 6y - (-2x - 4y)$	b. $6x + 10y$

10. Complete the table so that each column has an equivalent fraction, decimal, and percent.

Fraction	$\frac{2}{5}$	$\frac{3}{4}$ (or equivalent)	$\frac{3}{10}$	$\frac{1}{8}$ (or equivalent)
Decimal	0.4	0.75	0.3	0.125
Percent	40%	75%	30%	12.5%

Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1–6	2	MA.7.DP.1.3
	7	3	MA.7.DP.1.3
FAST Practice	8	2	MA.7.DP.1.3
Spiral Review			
	9	1	MA.7.AR.1.1
	10	2	MA.7.NSO.1.2


Unit 9
Lesson
16
Name: Date: Period:
MA.7.DP.1.1, MA.7.DP.1.2, MTR.5.1, MTR.6.1
Plots and Samples
Let's use box plots and dot plots to make predictions.
Warm-Up
1. Which one doesn't belong? Explain your thinking.
A.
B.
C.
D.
0 5 10 15 20
Accelerated 6 | Unit 9 Lesson 16
935
Warm-Up

Plots and Samples

Using Statistical Measures to Make Predictions

Let's use box plots and dot plots to make predictions.

Learning Outcomes

Today's Goals

1. **Goal:** Estimate a measure of center of a population based on one or more samples.
2. **Language Goal:** Interpret box plots, dot plots, and lists of data that represent real-world situations. **(Reading, Writing, Speaking, and Listening)**

Students consider the variability between samples when drawing inferences about a population. They explore medians and interquartile ranges (IQRs) of box plots to make predictions about a population. Students decide whether multiple samples have measures of center that are alike or different and use this information to form claims about a population. They analyze data sets to determine the most appropriate measures of center and spread. **(MTR.5.1, MTR.6.1)**

Coherence

Prior Learning

In Lesson 15, students revisited ratios and percentages to analyze samples and make predictions about a larger population.

Future Learning

In Lesson 17, students will compare two populations based on samples to determine whether there is a difference between the data sets.

Connections

- Students analyze box plots and dot plots to make perdictions on how long it takes students to get to school by car. (mathematical, real world)

FL B.E.S.T. Math Benchmarks

Addressing

MA.7.DP.1.1
Determine an appropriate measure of center or measure of variation to summarize numerical data, represented numerically or graphically, taking into consideration the context and any outliers.

MA.7.DP.1.2
Given two numerical or graphical representations of data, use the measure(s) of center and measure(s) of variability to make comparisons, interpret results and draw conclusions about the two populations.

Mathematical Thinking and Reasoning Standards: MTR.5.1, MTR.6.1

ELA Expectations: ELA.K12.EE.1.1, ELA.K12.EE.5.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.6.DP.1.2	**MA.912.DP.2.1**
MA.6.DP.1.3	

Lesson at a Glance

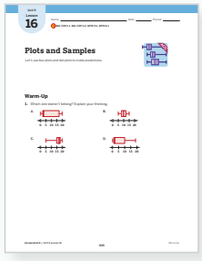
~ 45 min

MA.7.DP.1.1, MA.7.DP.1.2

Warm-Up

Pairs | 5 min

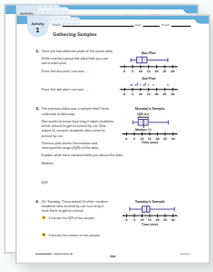
Students use the **Which One Doesn't Belong?** routine to compare the properties of four box plots and justify their thinking using precise language. **(MTR.5.1)**



Activity 1

Pairs | 14 min

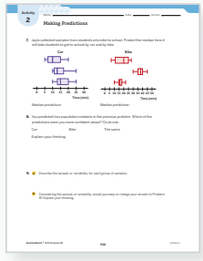
Students expand their prior knowledge of medians and interquartile ranges (IQR) by analyzing different samples represented by box plots and dot plots.



Activity 2

Pairs | 8 min

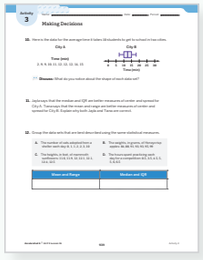
Students use data from multiple samples to draw inferences about a population then use the **Decide and Defend** routine to justify their confidence about the predictions. **(MTR.6.1)**



Activity 3

Pairs | 8 min

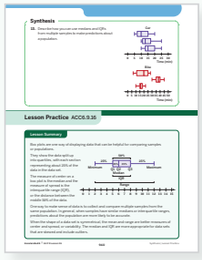
Students analyze data sets to determine the most appropriate measure of center and measure of spread. **(MTR.5.1)**



Synthesis

Whole Class | 5 min

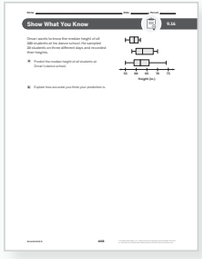
Students synthesize their understanding of using medians and interquartile ranges from multiple samples to make predictions about a population. **MLR2: Collect and Display**



Show What You Know

Independent | 5 min

Students demonstrate their understanding by predicting the median from multiple samples and determining the accuracy of their prediction.



Prep Checklist

This lesson includes:

- Student Edition
- Show What You Know

Pairs | 5 min

Warm-Up

Purpose: Students use the **Which One Doesn't Belong?** routine to compare the properties of four box plots and justify their thinking using precise language. **(MTR.5.1)**

1 Launch

Use the Which One Doesn't Belong? routine to support students in analyzing properties of box plots and justifying their thinking precisely. **(MTR.5.1)**

Encourage students to look for more than one possibility.

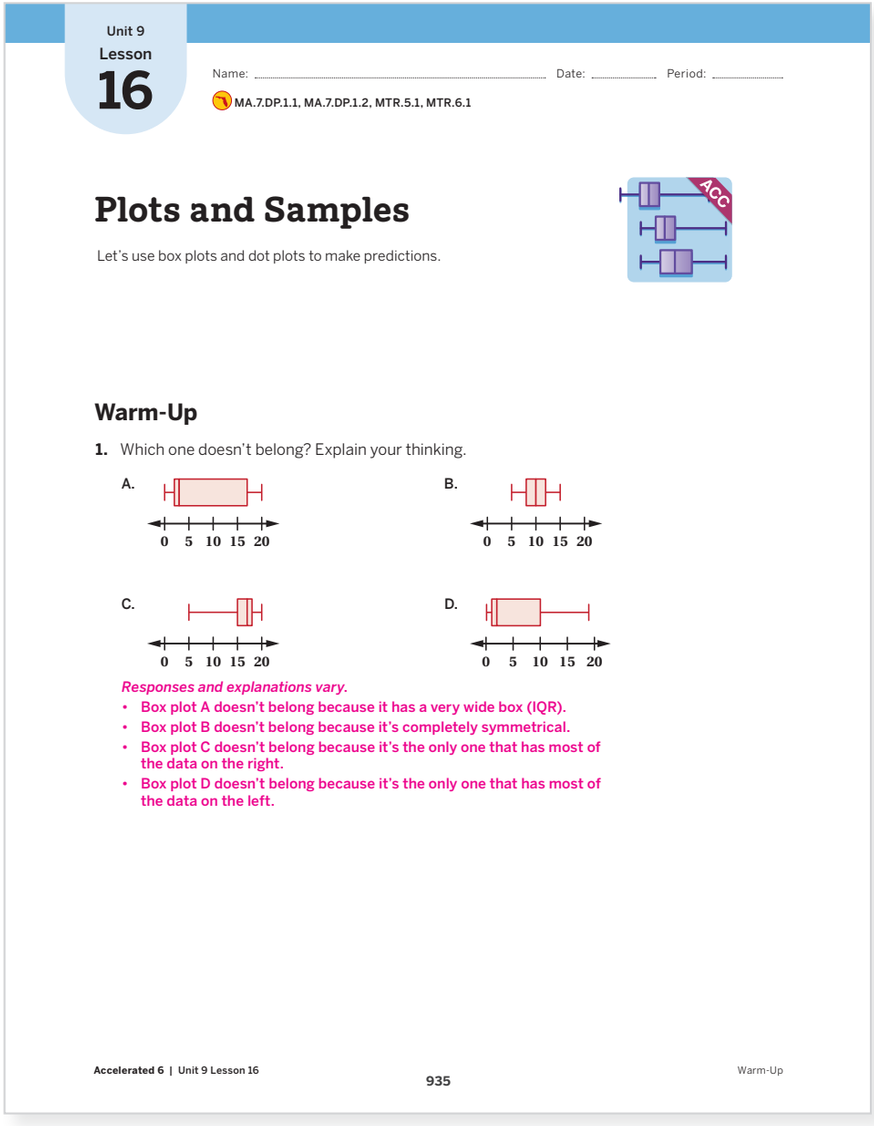
ELL **English Language Learners** Provide sentence frames (e.g., "_____ doesn't belong because _____." or " _____ is the only one that has/doesn't have _____."). **(Writing and Speaking)**

2 Connect

Discuss why each representation doesn't belong, asking students to justify their choice.

Consider asking:

- "How is box plot A different from the other box plots?"
- "What is another way to say why box plot D doesn't belong?"



Unit 9
Lesson
16

Name: _____ Date: _____ Period: _____

MA.7.DP.1.1, MA.7.DP.1.2, MTR.5.1, MTR.6.1

Plots and Samples

Let's use box plots and dot plots to make predictions.

Warm-Up

1. Which one doesn't belong? Explain your thinking.

A.

B.

C.

D.

Responses and explanations vary.

- Box plot A doesn't belong because it has a very wide box (IQR).
- Box plot B doesn't belong because it's completely symmetrical.
- Box plot C doesn't belong because it's the only one that has most of the data on the right.
- Box plot D doesn't belong because it's the only one that has most of the data on the left.

Accelerated 6 | Unit 9 Lesson 16 935 Warm-Up

Pairs | 18 min

Activity 1 Gathering Samples

Purpose: Students expand their prior knowledge of medians and interquartile ranges (IQR) by analyzing different samples represented by box plots and dot plots.

Short on time? Consider completing the questions from Problem 2 as a whole class.

1 Launch

To surface prior knowledge, consider asking, "How can you compare samples when the data is displayed using two different types of representations?"

Direct students' attention to the box plot and dot plot to promote curiosity.

2 Monitor

To support students getting started, consider asking, "How is data on a box plot represented differently than data on a dot plot?"

Invite students to think independently about each measure before discussing their ideas with a partner.

Look for student responses that discuss the median of the box plot.

Consider asking, "What do you recall about medians and interquartile ranges (IQRs)?"

Create a class definition for the term interquartile range (IQR) using students' language and the glossary.

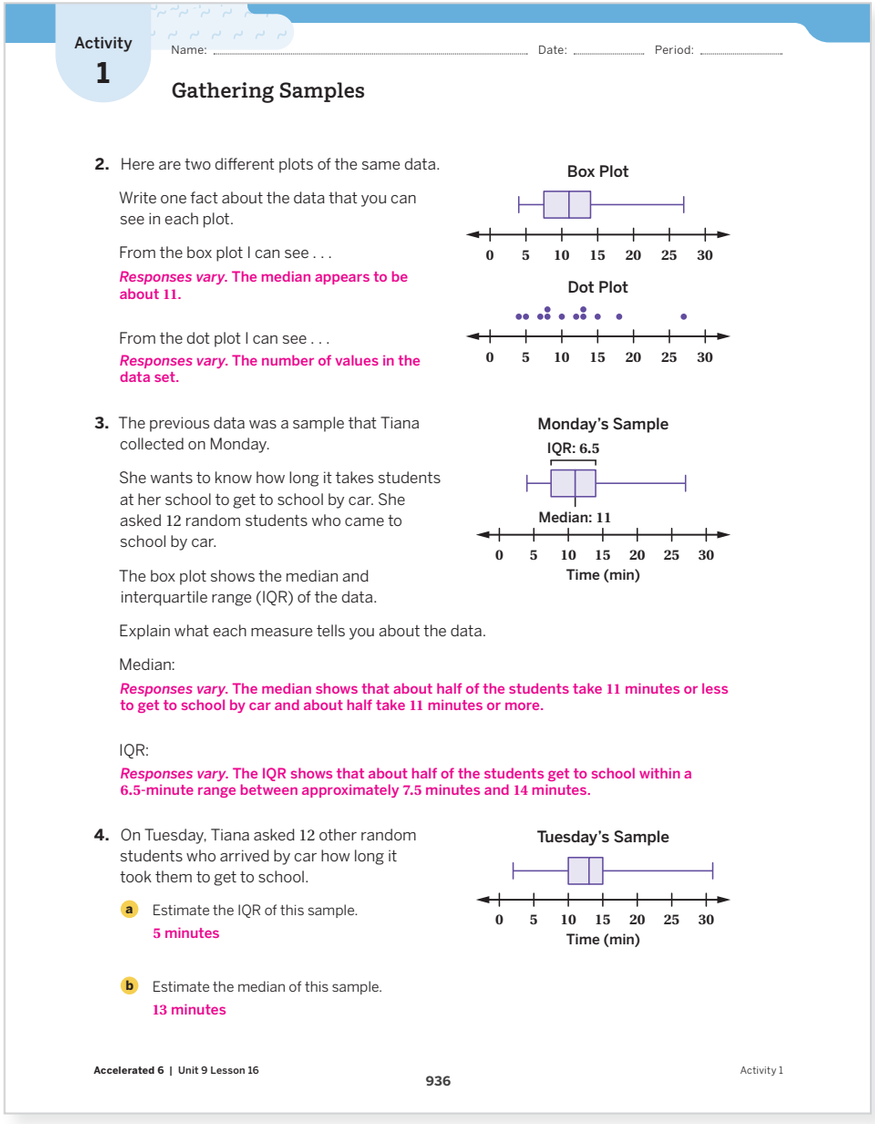
A **Accessibility: Visual-Spatial Processing** Add labels to the quartiles of the box plot to support processing visual information.

Encourage students to sketch to show their thinking.

Listen for opportunities to recognize the brilliance from students who have not been heard from recently.

D **Differentiation: (Support)** Students may think the measure of the IQR is determined by both the width of the box and the brackets. Consider inviting them to review the image in the previous problem for an example of the IQR.

Activity 1 continued >



Activity 1

Name: Date: Period:

Gathering Samples

2. Here are two different plots of the same data.

Write one fact about the data that you can see in each plot.

From the box plot I can see . . .

Responses vary. The median appears to be about 11.

From the dot plot I can see . . .

Responses vary. The number of values in the data set.

Box Plot

0 5 10 15 20 25 30

Dot Plot

0 5 10 15 20 25 30

3. The previous data was a sample that Tiana collected on Monday.

She wants to know how long it takes students at her school to get to school by car. She asked 12 random students who came to school by car.

The box plot shows the median and interquartile range (IQR) of the data.

Monday's Sample

IQR: 6.5

Median: 11

0 5 10 15 20 25 30

Time (min)

Explain what each measure tells you about the data.

Median:

Responses vary. The median shows that about half of the students take 11 minutes or less to get to school by car and about half take 11 minutes or more.

IQR:

Responses vary. The IQR shows that about half of the students get to school within a 6.5-minute range between approximately 7.5 minutes and 14 minutes.

4. On Tuesday, Tiana asked 12 other random students who arrived by car how long it took them to get to school.

Tuesday's Sample

0 5 10 15 20 25 30

Time (min)

a. Estimate the IQR of this sample.

5 minutes

b. Estimate the median of this sample.

13 minutes

Accelerated 6 | Unit 9 Lesson 16 936 Activity 1

Pairs | 18 min

Activity 1 Gathering Samples (continued)

Purpose: Students expand their prior knowledge of medians and interquartile ranges (IQR) by analyzing different samples represented by box plots and dot plots.

Short on time? Consider completing the questions from Problem 2 as a whole class.

2 Monitor

Encourage students to share their reasoning with a partner and work together to reach an agreement about how to sort the samples.

Note: If time allows, invite pairs to compare their groups, justify their thinking, and make revisions based on their conversation.

Listen for students who share strategies on how to match box plots and dot plots and how to determine the median and IQR from either representation. **(MTR.5.1)**

Select and sequence unique and creative student responses to share during the Connect.

D Differentiation

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Encourage them to analyze the median and IQR of each box plot before comparing all three visually.
Identify the IQRs of the three box plots as the same size.	**Support** Encourage students to draw lines to connect the edges of the IQR boxes to the axes.
Notice that the IQRs of the three box plots are similar, but that each box plot has a different median.	**Strengthen** Invite students to compare their responses with a classmate.

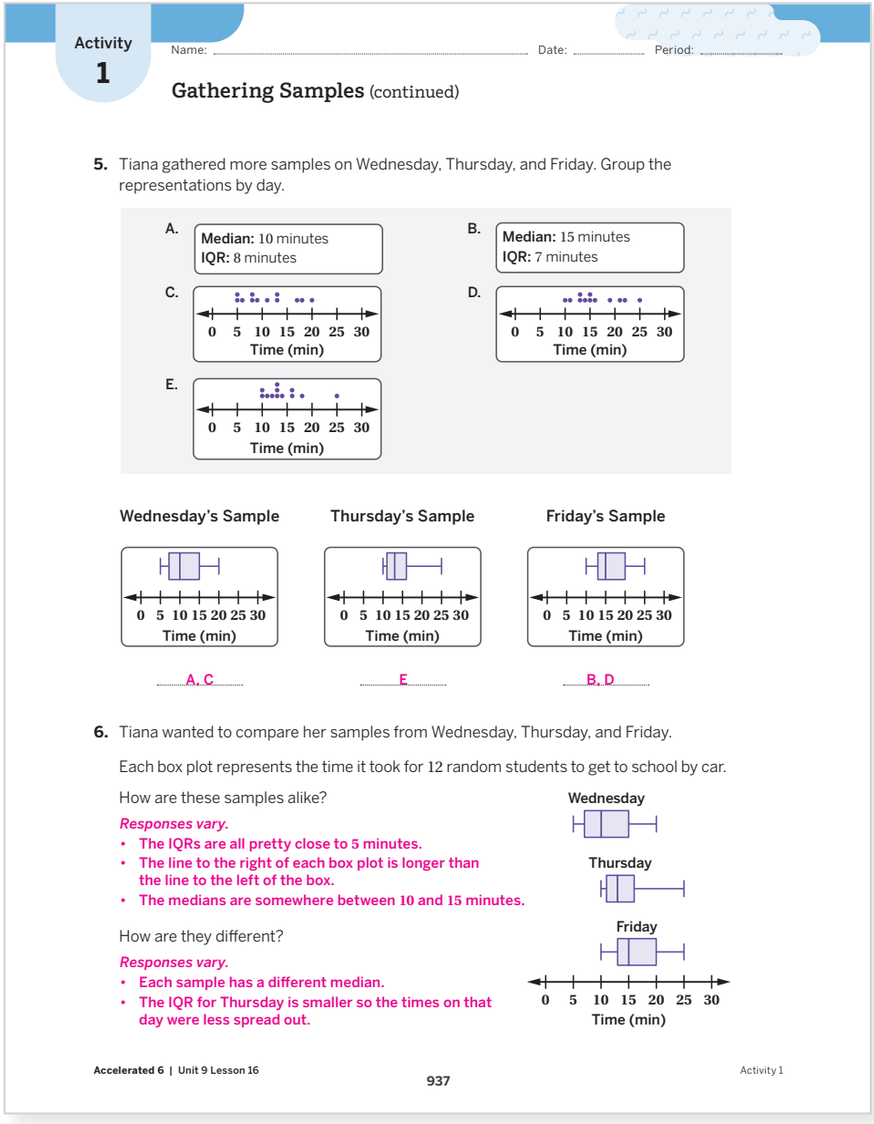
3 Connect

Invite students to share their strategies for grouping the different representations of Tiana's samples.

Consider asking, "How did you determine which box plots and dot plots represented data for the same day?"

Share collected student responses that show unique or creative ways of thinking.

To surface the Key Takeaway, consider asking, "How can you assess the accuracy of predictions made using the measures of center from multiple samples of a population?"



Activity 1

Name: Date: Period:

Gathering Samples (continued)

5. Tiana gathered more samples on Wednesday, Thursday, and Friday. Group the representations by day.

A. **Median:** 10 minutes **IQR:** 8 minutes

B. **Median:** 15 minutes **IQR:** 7 minutes

C.

D.

E.

Wednesday's Sample	Thursday's Sample	Friday's Sample
A, C	E	B, D

6. Tiana wanted to compare her samples from Wednesday, Thursday, and Friday.

Each box plot represents the time it took for 12 random students to get to school by car.

How are these samples alike?

Responses vary.

- The IQRs are all pretty close to 5 minutes.
- The line to the right of each box plot is longer than the line to the left of the box.
- The medians are somewhere between 10 and 15 minutes.

How are they different?

Responses vary.

- Each sample has a different median.
- The IQR for Thursday is smaller so the times on that day were less spread out.

Accelerated 6 | Unit 9 Lesson 16 937 Activity 1

Key Takeaway: Predictions can be made about a population by comparing the visual overlap between the measures of center and spread of multiple samples. When medians and IQRs from multiple samples are closely aligned, similarities and differences in the data are easier to identify.

Pairs | 12 min

Activity 2 Making Predictions

Purpose: Students use data from multiple samples to draw inferences about a population then use the **Decide and Defend** routine to justify their confidence about the predictions. **(MTR.6.1)**

1 Launch

To surface connections, consider asking, "How can you use samples to make predictions about a population?"

Consider sharing an inaccurate location for the population median and invite students to explain why they believe the location would not be an accurate prediction.

2 Monitor

Invite students to continue to the next problem after they've discussed their median predictions with a partner.

Collect student responses to share during the Connect.

D Differentiation

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Consider asking, "In general, how can you tell when your predictions are more likely to be accurate?"
Are unsure about their confidence in their predictions.	**Support** Consider asking, "Which median prediction do you feel is a more accurate representation of the population?"

ELL English Language Learners Invite students to rehearse their ideas with a partner before they share them with others. **(Writing and Speaking)**

Consider asking, "How consistent is the IQR in each set of samples?"

3 Connect

Invite students to share their responses.

Use the Decide and Defend routine to support students in making arguments and critiquing the reasoning of others. **(MTR.6.1)**

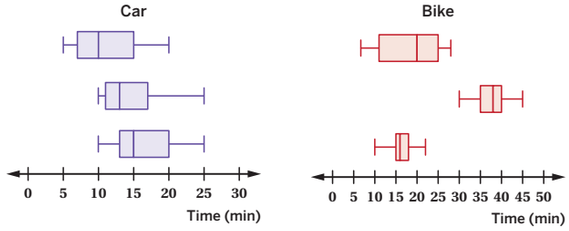
Consider asking, "How can the medians and IQRs from multiple samples help you to feel more confident that your prediction is accurate?"

Activity 2

Name: Date: Period:

Making Predictions

7. Jayla collected samples from students who bike to school. Predict the median time it will take students to get to school by car and by bike.



Median prediction:
Responses vary.

Median prediction:
Responses vary.

8. You predicted two population medians in the previous problem. Which of the predictions were you more confident about? Circle one.

Car Bike The same

Explain your thinking.

Responses and explanations vary.

- **The same. In both cases, I had three samples and I used the same approach. I used each sample's median and took the average.**
- **Car. The three car samples have medians and IQRs that are pretty consistent. The three bike samples vary quite a bit. That made me think that I have a better understanding of the car population than the bike population.**

9. a Describe the spread, or variability, for each group of samples.

***Responses vary.* The IQR for the car data is pretty consistent. The IQR for the bike data varies.**

b Considering the spread, or variability, would you keep or change your answer to Problem 8? Explain your thinking.

Responses vary.

Accelerated 6 | Unit 9 Lesson 16 938 Activity 2

Pairs | 8 min

Activity 3 Making Decisions

Purpose: Students analyze data sets to determine the most appropriate measure of center and measure of spread. **(MTR.5.1)**

1 Launch

Direct students' attention to the data sets.

To generate excitement, consider asking students how long it takes them to get to school each day, on average.

2 Monitor

Invite students to explore what is alike and different about the average times.

Listen for students using the terms *skewed*, *symmetrical*, and *outlier* in their responses to share during the Connect.

A **Accessibility: Visual-Spatial Processing** Invite students to create a data display for City A to help process the shape of the data.

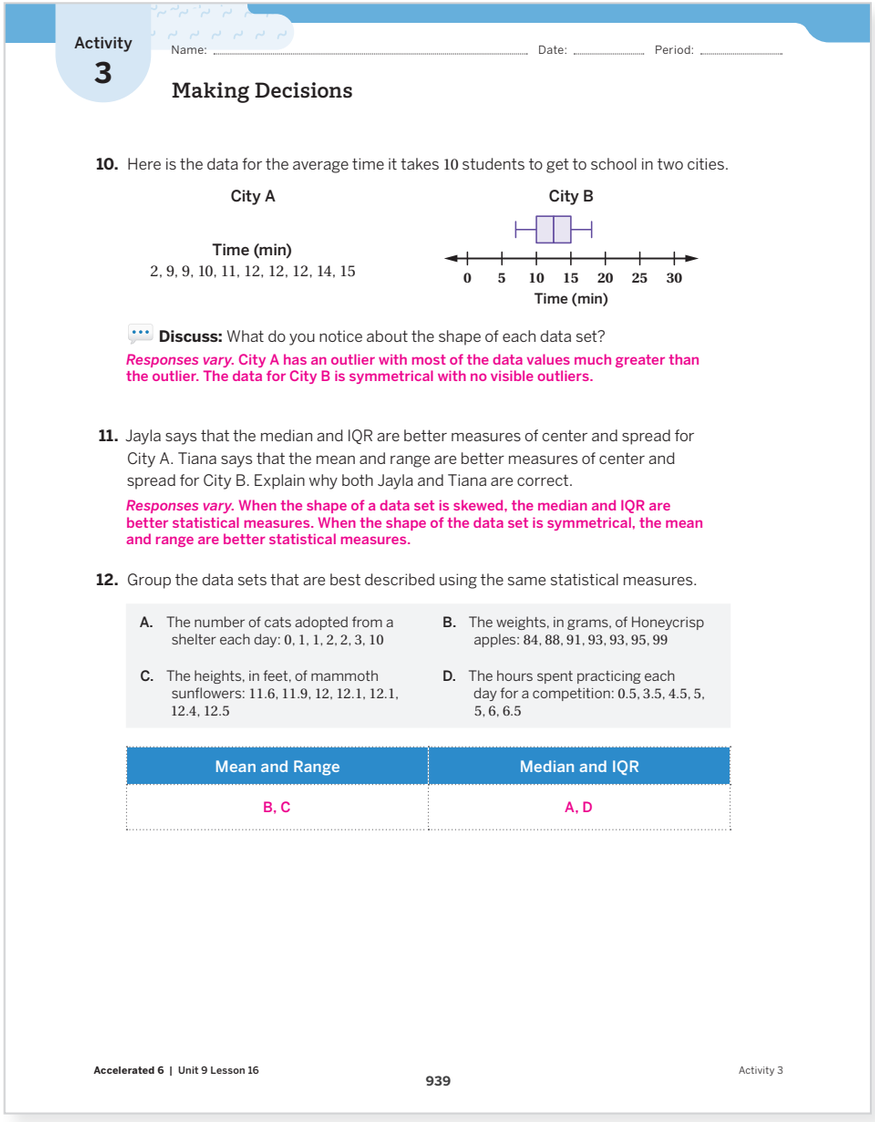
Consider asking, "How can the shape of the data help determine the most appropriate statistical measures to describe the data?" **(MTR.5.1)**

Encourage students to share their reasoning with a partner and work together to reach an agreement about how to sort the data sets.

3 Connect

Invite students to share their thinking about choosing the most appropriate measures of center and spread, or variability.

To surface the Key Takeaway, consider asking, "When is it appropriate to use the mean and range to describe a set of data? When is it more appropriate to use the median and IQR?"



Activity 3

Name: Date: Period:

Making Decisions

10. Here is the data for the average time it takes 10 students to get to school in two cities.

City A

Time (min)

2, 9, 9, 10, 11, 12, 12, 12, 14, 15

City B

Discuss: What do you notice about the shape of each data set?

Responses vary. City A has an outlier with most of the data values much greater than the outlier. The data for City B is symmetrical with no visible outliers.

11. Jayla says that the median and IQR are better measures of center and spread for City A. Tiana says that the mean and range are better measures of center and spread for City B. Explain why both Jayla and Tiana are correct.

Responses vary. When the shape of a data set is skewed, the median and IQR are better statistical measures. When the shape of the data set is symmetrical, the mean and range are better statistical measures.

12. Group the data sets that are best described using the same statistical measures.

A. The number of cats adopted from a shelter each day: 0, 1, 1, 2, 2, 3, 10

B. The weights, in grams, of Honeycrisp apples: 84, 88, 91, 93, 93, 95, 99

C. The heights, in feet, of mammoth sunflowers: 11.6, 11.9, 12, 12.1, 12.1, 12.4, 12.5

D. The hours spent practicing each day for a competition: 0.5, 3.5, 4.5, 5, 5, 6, 6.5

Mean and Range	Median and IQR
B, C	A, D

Accelerated 6 | Unit 9 Lesson 16 939 Activity 3

Key Takeaway: The shape of a data set, along with the context of a problem, can help determine the most appropriate measure of center and measure of spread for the data.

Whole Class | 5 min

Synthesis

Purpose: Students synthesize their understanding of using medians and interquartile ranges from multiple samples to make predictions about a population.

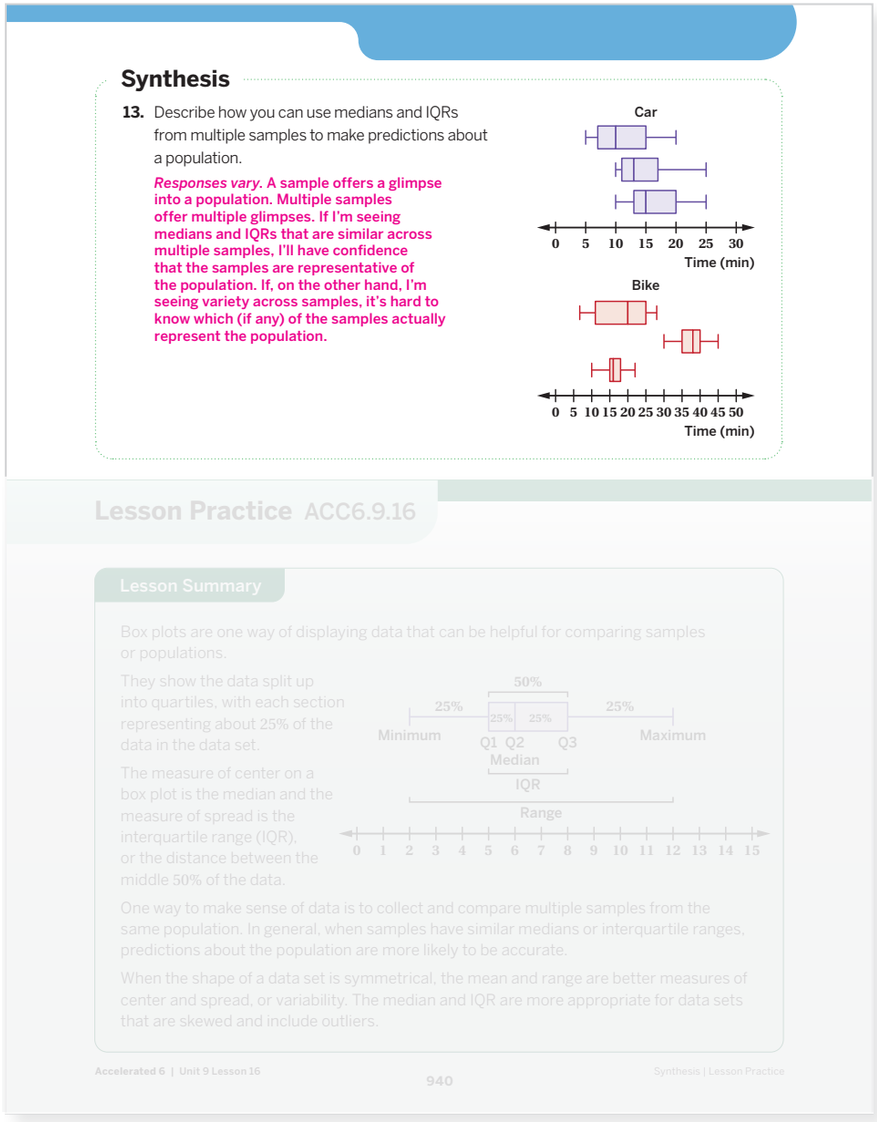
Invite students to respond independently, and then share their thinking with a partner.

Collect and share a variety of ideas, such as:

- Using multiple samples with similar medians and IQRs to make predictions provides a higher level of confidence in the prediction.
- Predictions made about a population when the medians and IQRs of multiple samples are very different are less likely to be accurate.
- The shape of a data set can help determine whether the median and IQR are appropriate measures of center and spread, or variability.

MLR **MLR2: Collect and Display** Use this routine to collect students' strategies as they discuss with a partner. Display these publicly for students to reference or add to for the rest of the unit.

Note: If time allows, invite students to revise their previous thinking based on the discussion.



Synthesis

13. Describe how you can use medians and IQRs from multiple samples to make predictions about a population.

Responses vary. A sample offers a glimpse into a population. Multiple samples offer multiple glimpses. If I'm seeing medians and IQRs that are similar across multiple samples, I'll have confidence that the samples are representative of the population. If, on the other hand, I'm seeing variety across samples, it's hard to know which (if any) of the samples actually represent the population.

Lesson Practice ACC6.9.16

Lesson Summary

Box plots are one way of displaying data that can be helpful for comparing samples or populations.

They show the data split up into quartiles, with each section representing about 25% of the data in the data set.

The measure of center on a box plot is the median and the measure of spread is the interquartile range (IQR), or the distance between the middle 50% of the data.

One way to make sense of data is to collect and compare multiple samples from the same population. In general, when samples have similar medians or interquartile ranges, predictions about the population are more likely to be accurate.

When the shape of a data set is symmetrical, the mean and range are better measures of center and spread, or variability. The median and IQR are more appropriate for data sets that are skewed and include outliers.

Accelerated 6 | Unit 9 Lesson 16 940 Synthesis | Lesson Practice

Lesson Takeaway: Comparing the medians and *interquartile ranges* from multiple samples is useful for making estimates about a population. When the *shape* of a data set is *skewed* or includes an outlier, the median and IQR may be the best measures of center and spread. The accuracy of a prediction made about a population often depends on whether the medians and IQRs of the samples are alike or different.

Independent | 5 min

Show What You Know

Purpose: Students demonstrate their understanding by predicting the median from multiple samples and determining the accuracy of their prediction.

Today's Goals

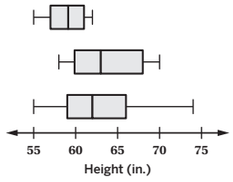
1. **Goal:** Estimate a measure of center of a population based on one or more samples.
2. **Language Goal:** Interpret box plots, dot plots, and lists of data that represent real-world situations. **(Reading, Writing, Speaking, and Listening)**

Show What You Know (Assessment Resources)

Name: _____ Date: _____ Period: _____

Show What You Know 9.16

Omari wants to know the median height of all 200 students at his dance school. He sampled 20 students on three different days and recorded their heights.



a. Predict the median height of *all* students at Omari's dance school.

Responses between 58 and 63 are considered correct.

b. Explain how accurate you think your prediction is.

Explanations vary. **Because the sample medians and IQRs are all similar, I think my prediction is pretty accurate.**

D Differentiation Use after Lesson 16

Support

Provide targeted intervention for students by using these resources.

If students need support understanding how to of estimate the median from a box plot:

» **Respond to Student Thinking:**

- Revisit the launch of **Activity 2** as a class and invite students to share strategies for estimating a population's measure of center based on samples.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 3 Extension Activities**.

Professional Learning

Consider the next lesson you will teach. How will students' work today in "Gathering Samples" prepare them for Lesson 17, where they will make comparisons between samples based on visual and quantitative differences in the measures of center?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

13. Describe how you can use medians and IQRs from multiple samples to make predictions about a population.

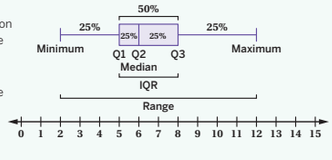
Responses vary. A sample offers a glimpse into a population. Multiple samples offer multiple glimpses. If I'm seeing medians and IQRs that are similar across multiple samples, I'll have confidence that the samples are representative of the population. If, on the other hand, I'm seeing variety across samples, it's hard to know which (if any) of the samples actually represent the population.

Lesson Practice ACC6.9.16

Lesson Summary

Box plots are one way of displaying data that can be helpful for comparing samples or populations.

They show the data split up into quartiles, with each section representing about 25% of the data in the data set.



The measure of center on a box plot is the median and the measure of spread is the interquartile range (IQR), or the distance between the middle 50% of the data.

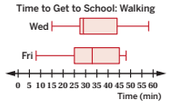
One way to make sense of data is to collect and compare multiple samples from the same population. In general, when samples have similar medians or interquartile ranges, predictions about the population are more likely to be accurate.

When the shape of a data set is symmetrical, the mean and range are better measures of center and spread, or variability. The median and IQR are more appropriate for data sets that are skewed and include outliers.

Lesson Practice ACC6.9.16

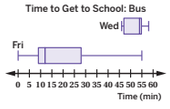
Name: _____ Date: _____ Period: _____

Problems 1–2: Kai wants to understand how long it takes for students to get to school using different methods. On Wednesday, Kai surveyed random samples of students who walked and students who took the bus. He repeated this on Friday.



1. Estimate the median time it takes to get to school by each method.

Walking: **Responses between 30 and 35 minutes are considered correct.**



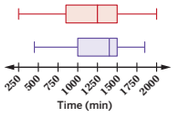
Bus: **Responses between 20 and 50 minutes are considered correct.**

2. Which estimate is more likely to be accurate? **Walking**

Explain your thinking.

Explanations vary. **There is more variability between the samples of the time it took students to get to school by bus.**

Problems 3–5: A video game developer wants to know how long it takes people to finish playing a new game. She surveyed two random samples of 20 players and asked how long it took each of them.



3. Estimate the median time it takes a player to finish the game.

Responses between 1,250 and 1,400 minutes are considered correct.

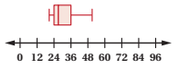
4. How close do you think your estimate is to the actual median time?

Responses vary. **I expect my estimate to be close to the actual median time because the median and IQRs for each sample are similar.**

5. How could the developer change her sampling strategy to make you more confident that the data is representative of the population?

Responses vary. **To get more representative samples, the developer could increase the size of each sample.**

6. Determine the best measure of center and measure of spread for the data set. Explain your thinking.



Responses vary. **The box plot is skewed and appears to include an outlier, so the best measure of center is the median and the best measure of spread is the IQR.**

Lesson Practice ACC6.9.16

Name: _____ Date: _____ Period: _____

FAST Practice

7. The data shows the heights, in inches, of a team of volleyball players.

Heights of Volleyball Teammates (in.)

70, 70.5, 71, 71.5, 72, 72, 73

If the team adds a new player with a height of 71 inches, will the most appropriate statistical measures change? Explain your thinking.

The original data set is **A.** skewed **B.** (fairly symmetrical), so the best measure of center is the **A.** (mean) **B.** median and the best measure of spread, or variability, is the **A.** (range) **B.** IQR. The new player's height **A.** is **B.** (is not) an outlier, so the most appropriate statistical measures **A.** will **B.** (will not) change.

Spiral Review

8. Complete the table so that each column has an equivalent fraction, decimal, and percent.

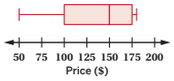
Fraction	$\frac{4}{5}$	$\frac{33}{50}$ (or equivalent)	$\frac{3}{8}$	$\frac{1}{100}$ (or equivalent)
Decimal	**0.8**	0.66	**0.375**	0.01
Percent	80%	66%	**37.5%**	**1%**

9. The recipe for a signature pizza uses 3 cups of cheese, $\frac{3}{4}$ cup of olives, and $1\frac{1}{2}$ cups of onions. How many cups of onions are needed to make 10 signature pizzas?

A. 10 cups **B.** 30 cups **C.** $7\frac{1}{2}$ cups **D.** 15 cups

Problems 10–11: Determine the median and interquartile range (IQR) for each box plot.

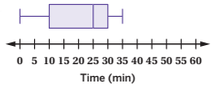
10.



Median: **150 dollars**

IQR: **75 dollars**

11.

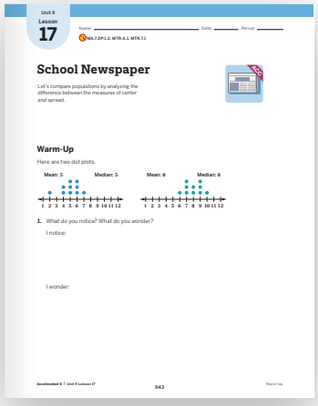


Median: **25 minutes**

IQR: **20 minutes**

Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1–5	2	MA.7.DP.1.2
	6	2	MA.7.DP.1.1
FAST Practice	7	3	MA.7.DP.1.1
Spiral Review			
	8	1	MA.6.NSO.3.5
	9	2	MA.7.NSO.2.2
	10–11	2	MA.6.DP.1.3



School Newspaper

Comparing Populations

Let's compare populations by analyzing the difference between the measures of center and spread.

Learning Outcomes

Today's Goals

1. **Goal:** Decide whether two populations are very different using measures of center and spread.
2. **Language Goal:** Justify whether a claim is supported based on visual and quantitative differences in the measures of center and spread. **(Reading, Writing, Speaking, and Listening)**

Students compare two populations based on samples. They are introduced to the idea of deciding if there is a big difference between two populations by looking at the difference between the centers of the samples and comparing the difference to the measures of spread.

Coherence

Prior Learning

In Lesson 16, students used data from samples to draw conclusions about a population by estimating a measure of center.

Future Learning

In a future course, students will analyze numerical data in scatter plots and describe patterns of associations.

Connections

- Students use a school survey to compare populations, analyze the difference between measures of center and spread. (real world)

FL B.E.S.T. Math Benchmarks

Addressing

MA.7.DP.1.2
Given two numerical or graphical representations of data, use the measure(s) of center and measure(s) of variability to make comparisons, interpret results and draw conclusions about the two populations.

Mathematical Thinking and Reasoning Standards: MTR.4.1, MTR.7.1

ELA Expectations: ELA.K12.EE.1.1, ELA.K12.EE.6.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.6.DP.1.2	MA.8.DP.1.2 MA.912.DP.2.1

Lesson at a Glance

~ 45 min

MA.7.DP.1.2

Warm-Up

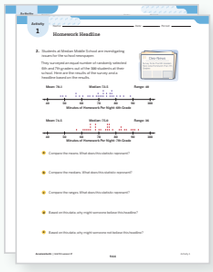
Small Groups | 5 min

Students use the **Notice and Wonder** routine to make observations of two data sets represented on dot plots.

Activity 1

Small Groups | 20 min

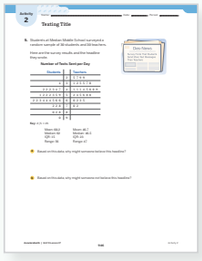
Students analyze data to determine the difference between two data sets by comparing their measures of center and spread.



Activity 2

Small Groups | 10 min

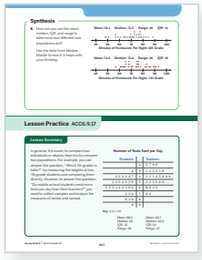
Students decide whether a headline is believable by comparing the difference between the means, medians, IQRs, and ranges of two data sets.
MLR1: Stronger and Clearer Each Time (MTR.7.1)



Synthesis

Whole Class | 5 min

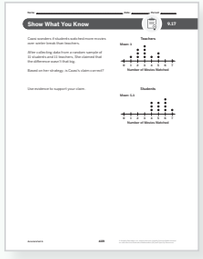
Students synthesize their understanding of using statistical measures to decide how different two data sets are.



Show What You Know

Independent | 5 min

Students demonstrate their understanding by analyzing two different data sets from samples to decide whether they support a claim.



Prep Checklist

This lesson includes:

- Student Edition
- Show What You Know

Small Groups | 5 min

Warm-Up

Purpose: Students use the **Notice and Wonder** routine to make observations of two data sets represented on dot plots.

1 Launch

Display the dot plots.

Use the Notice and Wonder routine to promote curiosity and help students make sense of the data.

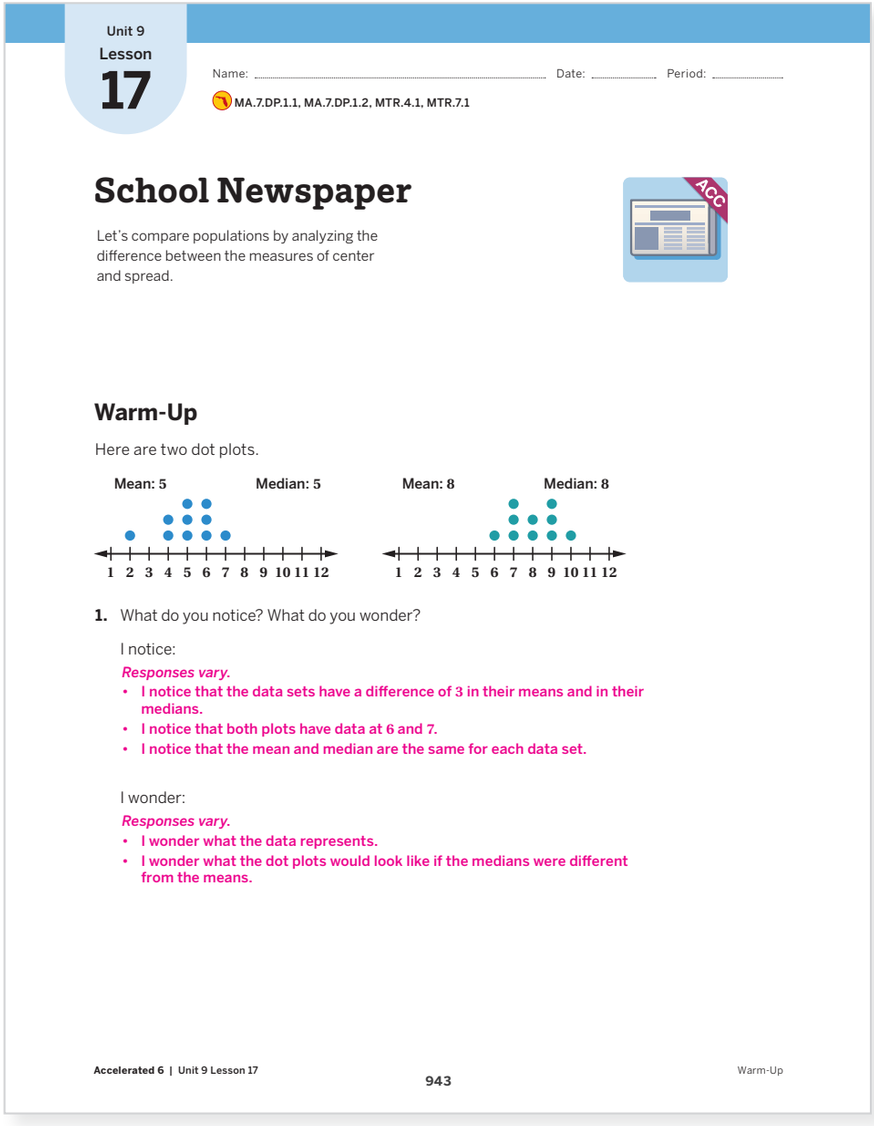
2 Connect

Invite students to share their observations.

Create a record of the things students noticed and wondered, along with the name of the students who shared them.

Note: If it does not come up naturally during the discussion, consider asking:

- "What could these dot plots represent?"
- "What do you notice about the difference in the means of both data sets? The medians?"
- "How much overlap is there between the points in the data sets?"
- "What do you notice about the spread of the data?"



Unit 9
Lesson
17

Name: ..... Date: ..... Period: .....

MA.7.DP.1.1, MA.7.DP.1.2, MTR.4.1, MTR.7.1

School Newspaper

Let's compare populations by analyzing the difference between the measures of center and spread.

ACC

Warm-Up

Here are two dot plots.

Mean: 5 Median: 5

Mean: 8 Median: 8

1. What do you notice? What do you wonder?

I notice:

Responses vary.

- I notice that the data sets have a difference of 3 in their means and in their medians.
- I notice that both plots have data at 6 and 7.
- I notice that the mean and median are the same for each data set.

I wonder:

Responses vary.

- I wonder what the data represents.
- I wonder what the dot plots would look like if the medians were different from the means.

Accelerated 6 | Unit 9 Lesson 17 943 Warm-Up

Small Groups | 20 min

Activity 1 Homework Headline

Purpose: Students analyze data to determine the difference between two data sets by comparing their measures of center and spread.

Short on time? Consider completing Problems 2a and 2b as a whole class.

1 Launch

Direct students' attention to the headline and the dot plots. Consider asking, "Do you think 6th graders or 7th graders have more homework?"

Invite students to share their thinking.

2 Monitor

Select and sequence student ideas to share during the Connect.

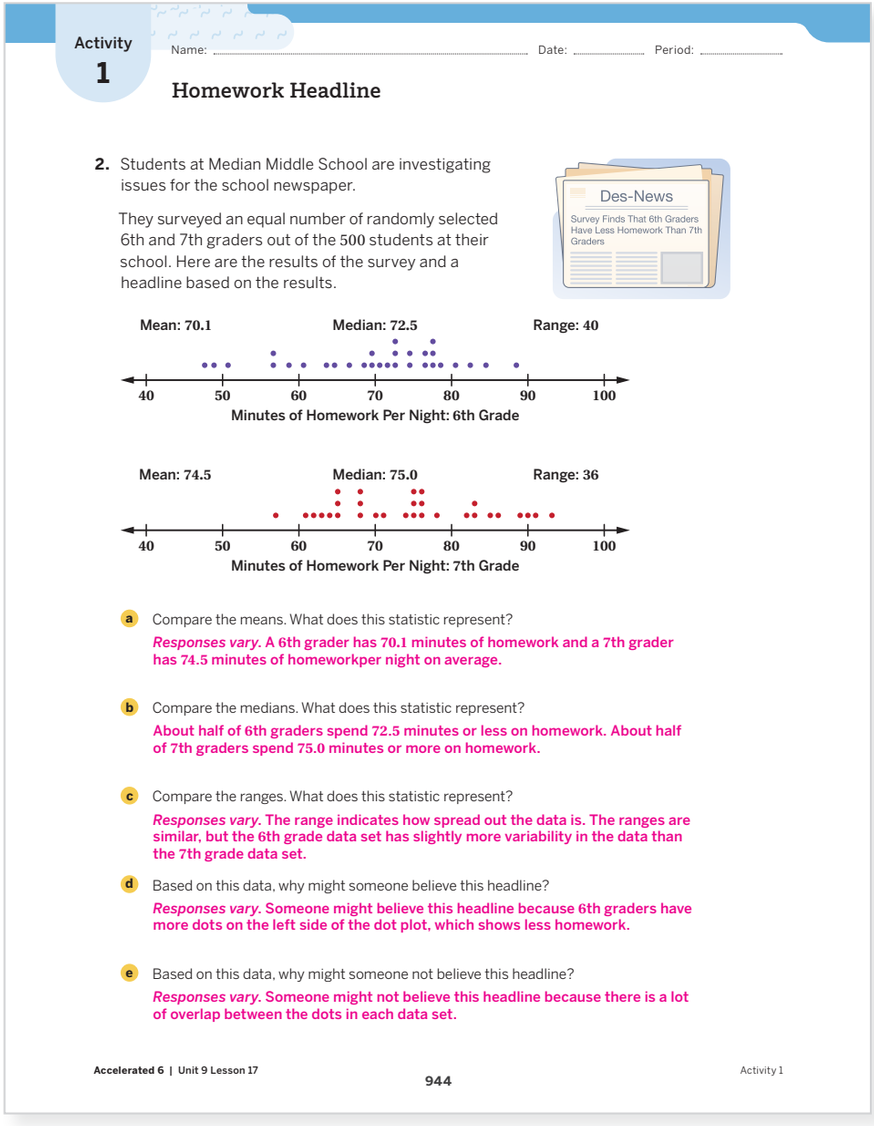
D Differentiation

Look for students who:	Respond to Student Thinking
Need support getting started. *(Problems 2a–2c)*	**Support** Consider asking, "How do you calculate the mean, median, and range?"
Describe the spread of data to justify their reasoning. *(Problems 2d–2e)*	**Strengthen** Invite them to share how the graphs can help someone decide whether they should believe the headline during the Connect.
Describe the overlap between data to justify their reasoning. *(Problems 2d–2e)*	

A Accessibility: Memory and Attention Provide students with an anchor chart that describes the meaning of mean, median, IQR, and range, and how to calculate them for students to refer to throughout the lesson.

Listen for explanations about why someone might and might not believe the headline in Problem 2 to share during the Connect.

Activity 1 continued >



Activity 1

Name: Date: Period:

Homework Headline

2. Students at Median Middle School are investigating issues for the school newspaper.

They surveyed an equal number of randomly selected 6th and 7th graders out of the 500 students at their school. Here are the results of the survey and a headline based on the results.

Des-News

Survey Finds That 6th Graders Have Less Homework Than 7th Graders

Mean: 70.1 Median: 72.5 Range: 40

40 50 60 70 80 90 100

Minutes of Homework Per Night: 6th Grade

Mean: 74.5 Median: 75.0 Range: 36

40 50 60 70 80 90 100

Minutes of Homework Per Night: 7th Grade

a. Compare the means. What does this statistic represent?

Responses vary. A 6th grader has 70.1 minutes of homework and a 7th grader has 74.5 minutes of homeworkper night on average.

b. Compare the medians. What does this statistic represent?

About half of 6th graders spend 72.5 minutes or less on homework. About half of 7th graders spend 75.0 minutes or more on homework.

c. Compare the ranges. What does this statistic represent?

Responses vary. The range indicates how spread out the data is. The ranges are similar, but the 6th grade data set has slightly more variability in the data than the 7th grade data set.

d. Based on this data, why might someone believe this headline?

Responses vary. Someone might believe this headline because 6th graders have more dots on the left side of the dot plot, which shows less homework.

e. Based on this data, why might someone not believe this headline?

Responses vary. Someone might not believe this headline because there is a lot of overlap between the dots in each data set.

Accelerated 6 | Unit 9 Lesson 17 944 Activity 1

Activity 1 Homework Headline (continued)

Purpose: Students analyze data to determine the difference between two data sets by comparing their measures of center and spread.

2 Monitor

Select and sequence explanations to share during the Connect.

D Differentiation *(Problems 3–4)*

Look for students who:	Respond to Student Thinking
Use the ranges to support their claim. (*Problem 3*)	**Strengthen** Invite them to share how the ranges helped them make their decision.
Reason that there is a big difference between the data sets. (*Problem 4*)	
Would like to further explore the difference in populations.	**Stretch** Consider asking, "Do you think people who write news articles use strategies similar to Ama's before writing headlines?"

3 Connect

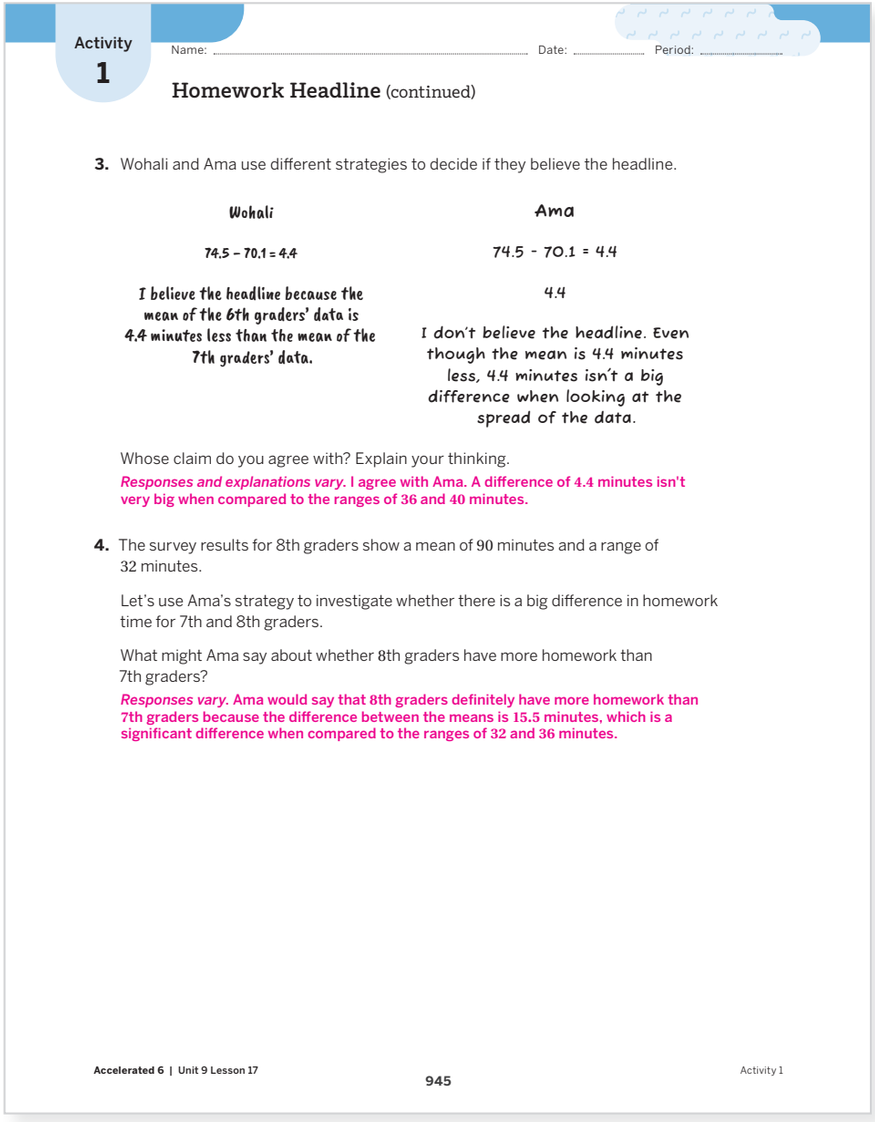
Discuss Ama's and Wohali's strategies.

Consider asking:
- "Whose strategy do you think makes more sense for this situation? Why?"
- "After analyzing each strategy, do you believe the headline in Problem 2? Why?"

Direct students' attention to the headline and data in Problem 2. Revisit their answers from the Launch to surface a connection between Ama's and Wohali's strategies and students' original answer choice. **(MTR.4.1)**

Invite students to share their responses for Problem 4 and their thinking behind it.

To surface the Key Takeaway, consider asking:
- "How does the range help you determine whether the difference in means is significant?"
- "Based on your calculations, what conclusions can you draw? How does this support Ama's claim?"



Activity 1

Name: Date: Period:

Homework Headline (continued)

3. Wohali and Ama use different strategies to decide if they believe the headline.

Wohali

74.5 − 70.1 = 4.4

I believe the headline because the mean of the 6th graders' data is 4.4 minutes less than the mean of the 7th graders' data.

Ama

74.5 - 70.1 = 4.4

4.4

I don't believe the headline. Even though the mean is 4.4 minutes less, 4.4 minutes isn't a big difference when looking at the spread of the data.

Whose claim do you agree with? Explain your thinking.

Responses and explanations vary. I agree with Ama. A difference of 4.4 minutes isn't very big when compared to the ranges of 36 and 40 minutes.

4. The survey results for 8th graders show a mean of 90 minutes and a range of 32 minutes.

Let's use Ama's strategy to investigate whether there is a big difference in homework time for 7th and 8th graders.

What might Ama say about whether 8th graders have more homework than 7th graders?

Responses vary. Ama would say that 8th graders definitely have more homework than 7th graders because the difference between the means is 15.5 minutes, which is a significant difference when compared to the ranges of 32 and 36 minutes.

Accelerated 6 | Unit 9 Lesson 17 945 Activity 1

Key Takeaway: One way to compare two samples is by subtracting the means of the populations and comparing the result to the ranges of the samples. A difference in means of 15.5 might be much more significant than a difference in means of 4.4 depending on the range of data values.

Small Groups | 10 min

Activity 2 Texting Title

Purpose: Students decide whether a headline is believable by comparing the difference between the means, medians, IQRs, and ranges of two data sets. **(MTR.7.1)**

1 Launch

To generate excitement, share with students that they will use calculations to compare the number of texts students and teachers send each day. **(MTR.7.1)**

Direct students' attention to the headline and the stem-and-leaf plot. Consider asking, "Who do you think sends more text messages?"

2 Monitor

Select and sequence student reasoning to share during the Connect.

D Differentiation

Look for students who:	Respond to Student Thinking
Use a variety of strategies, such as: • Compare the means and medians. • Compare the spread of the data. • Mention sample size.	**Strengthen** Invite them to share how these strategies and ideas help in deciding whether this headline might or might not be believable.
Would like to further explore the difference in populations.	**Stretch** Consider asking, "When might you use median and IQR instead of mean and range?"

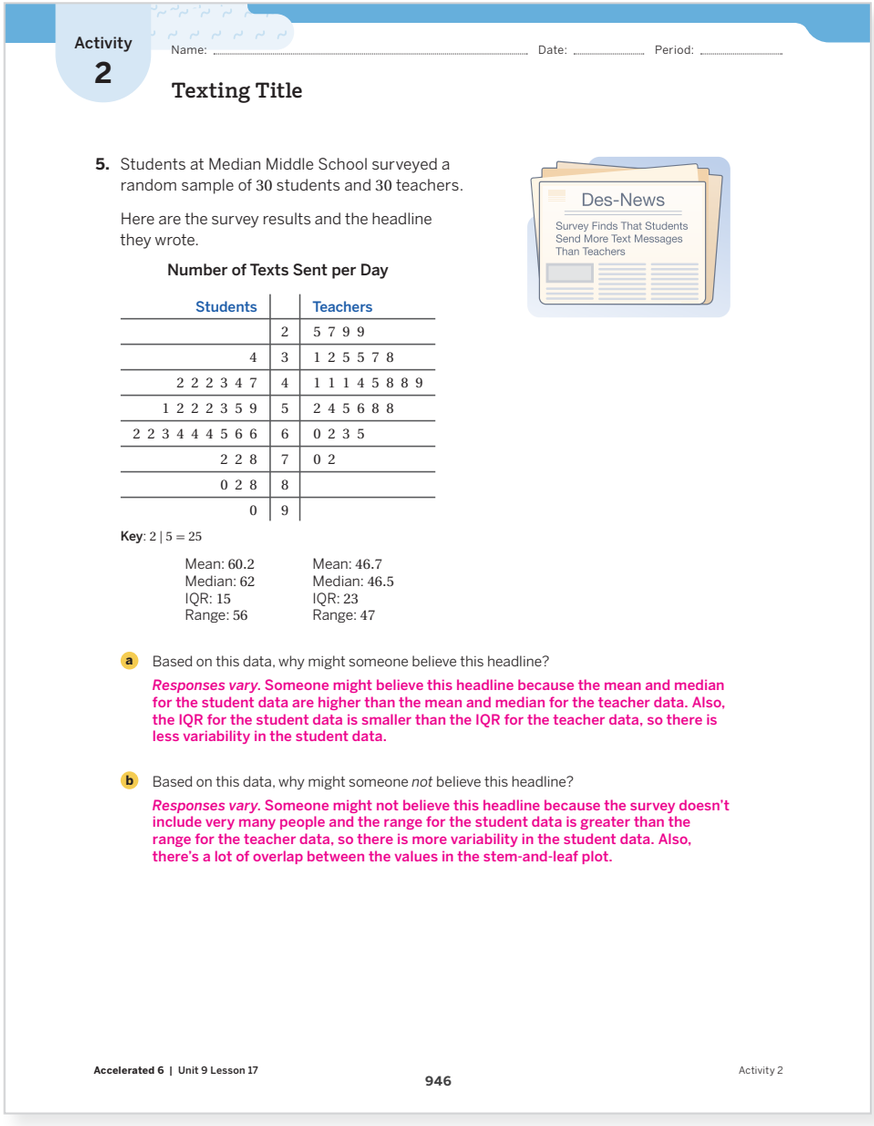
3 Connect

Invite students to share their responses from Problem 5 and if their original prediction about who sends more text messages was accurate.

MLR MLR1: Stronger and Clearer Each Time

Consider using this routine to support students in developing precise language. Invite students to meet with a partner, give feedback on each other's drafts, then use the feedback to write a second clearer draft. See the Routine Facilitation Guide for more information.

Consider asking, "How might sample size affect the results of a survey?"



Activity 2

Name: Date: Period:

Texting Title

5. Students at Median Middle School surveyed a random sample of 30 students and 30 teachers.

Here are the survey results and the headline they wrote.

Number of Texts Sent per Day

Students		Teachers
	2	5 7 9 9
4	3	1 2 5 5 7 8
2 2 2 3 4 7	4	1 1 1 4 5 8 8 9
1 2 2 2 3 5 9	5	2 4 5 6 8 8
2 2 3 4 4 4 5 6 6	6	0 2 3 5
2 2 8	7	0 2
0 2 8	8	
0	9	

Key: 2 \| 5 = 25

Mean: 60.2
Median: 62
IQR: 15
Range: 56

Mean: 46.7
Median: 46.5
IQR: 23
Range: 47

a. Based on this data, why might someone believe this headline?

Responses vary. Someone might believe this headline because the mean and median for the student data are higher than the mean and median for the teacher data. Also, the IQR for the student data is smaller than the IQR for the teacher data, so there is less variability in the student data.

b. Based on this data, why might someone *not* believe this headline?

Responses vary. Someone might not believe this headline because the survey doesn't include very many people and the range for the student data is greater than the range for the teacher data, so there is more variability in the student data. Also, there's a lot of overlap between the values in the stem-and-leaf plot.

Accelerated 6 | Unit 9 Lesson 17 946 Activity 2

Whole Class | 5 min

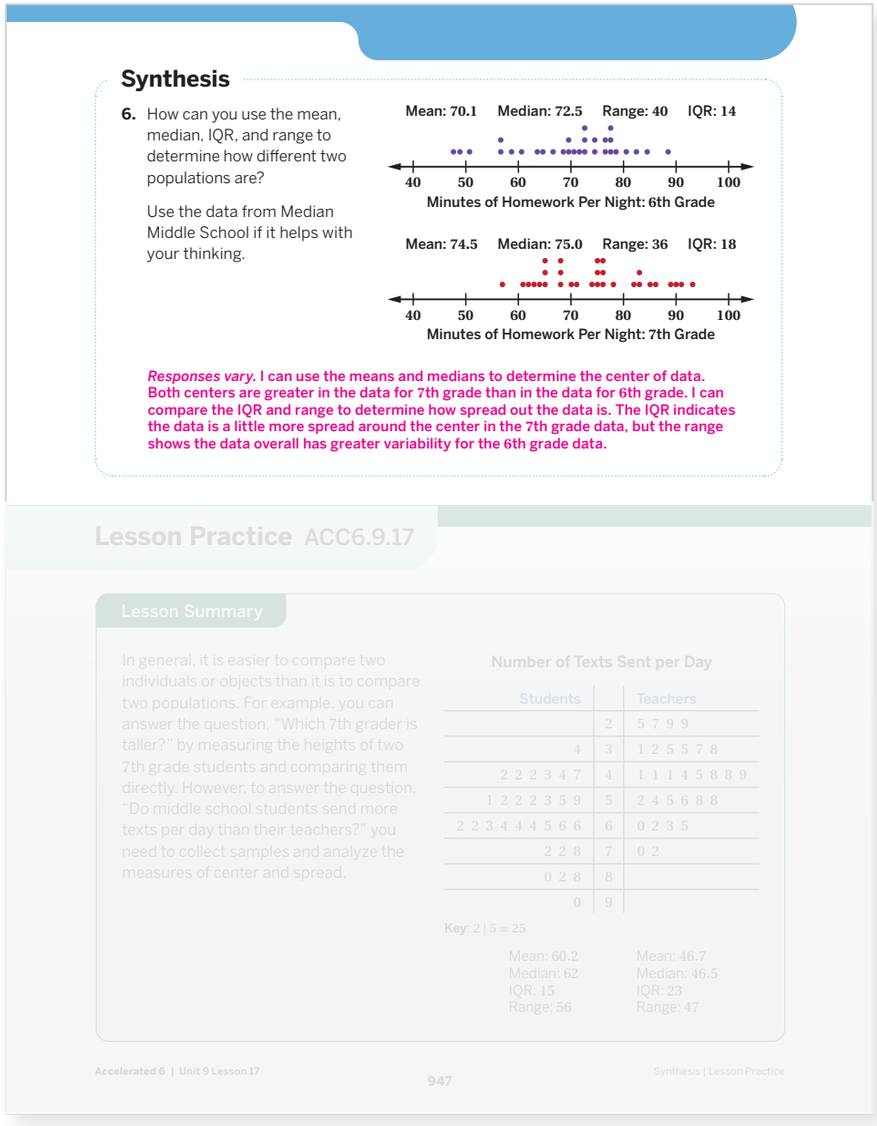
Synthesis

Purpose: Students synthesize their understanding of using statistical measures to decide how different two data sets are.

Invite students to respond independently, and then share their thinking with a partner.

Collect and share a variety of ideas, such as:
- Comparing the means and medians.
- Analyzing the spread of the data.

Note: If time allows, invite students to share with a partner what makes sense about each strategy and what questions they still have.



Synthesis

6. How can you use the mean, median, IQR, and range to determine how different two populations are?

Use the data from Median Middle School if it helps with your thinking.

Responses vary. I can use the means and medians to determine the center of data. Both centers are greater in the data for 7th grade than in the data for 6th grade. I can compare the IQR and range to determine how spread out the data is. The IQR indicates the data is a little more spread around the center in the 7th grade data, but the range shows the data overall has greater variability for the 6th grade data.

Lesson Practice ACC6.9.17

Lesson Summary

In general, it is easier to compare two individuals or objects than it is to compare two populations. For example, you can answer the question, "Which 7th grader is taller?" by measuring the heights of two 7th grade students and comparing them directly. However, to answer the question, "Do middle school students send more texts per day than their teachers?" you need to collect samples and analyze the measures of center and spread.

Number of Texts Sent per Day

Students		Teachers
	2	5 7 9 9
4	3	1 2 5 5 7 8
2 2 2 3 4 7	4	1 1 1 4 5 8 8 9
1 2 2 2 3 5 9	5	2 4 5 6 8 8
2 2 3 4 4 4 5 6 6	6	0 2 3 5
2 2 8	7	0 2
0 2 8	8	
0	9	

Key: 2 | 5 = 25

Mean: 60.2
Median: 62
IQR: 15
Range: 56

Mean: 46.7
Median: 46.5
IQR: 23
Range: 47

Accelerated 6 | Unit 9 Lesson 17 947 Synthesis | Lesson Practice

Lesson Takeaway: To calculate the differences between two data sets, first calculate the differences in the measures of center. Then compare the spread, or variability, and consider the sample size of the data sets.

Independent | 5 min

Show What You Know

Purpose: Students demonstrate their understanding by analyzing two different data sets from samples to decide whether they support a claim.

Today's Goals

1. **Goal:** Decide whether two populations are very different using measures of center and spread.
2. **Language Goal:** Justify whether a claim is supported based on visual and quantitative differences in the measures of center and spread. **(Reading, Writing, Speaking, and Listening)**

Show What You Know (Assessment Resources)

Name: _____ Date: _____ Period: _____

Show What You Know 9.17

Caasi wonders if students watched more movies over winter break than teachers.

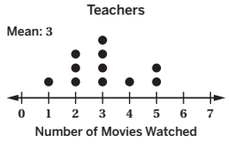
After collecting data from a random sample of 11 students and 11 teachers, She claimed that the difference wasn't that big.

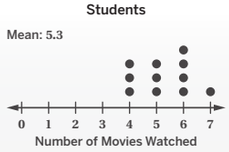
Based on her strategy, is Caasi's claim correct?

No

Use evidence to support your claim.

Explanations vary. Caasi's claim is incorrect because the difference between the means is $5.3 - 3 = 2.3$. There is a small difference between the number of movies watched by each population.





D Differentiation Use after Lesson 17

Support

Provide targeted intervention for students by using these resources.

If students need support understanding how to compare populations using mean and median:

» **Respond to Student Thinking:**

- Revisit the launch of **Activity 2** as a class and invite students to share strategies for estimating a population's measure of center based on samples.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 3 Extension Activities**.

Professional Learning

How did you create learning opportunities when students needed support interpreting the measures of center and spread to compare two data sets and draw conclusions?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

6. How can you use the mean, median, IQR, and range to determine how different two populations are?

Use the data from Median Middle School if it helps with your thinking.

Responses vary. I can use the means and medians to determine the center of data. Both centers are greater in the data for 7th grade than in the data for 6th grade. I can compare the IQR and range to determine how spread out the data is. The IQR indicates the data is a little more spread around the center in the 7th grade data, but the range shows the data overall has greater variability for the 6th grade data.

Lesson Practice ACC6.9.17

Lesson Summary

In general, it is easier to compare two individuals or objects than it is to compare two populations. For example, you can answer the question, "Which 7th grader is taller?" by measuring the heights of two 7th grade students and comparing them directly. However, to answer the question, "Do middle school students send more texts per day than their teachers?" you need to collect samples and analyze the measures of center and spread.

Number of Texts Sent per Day

Students		Teachers
	2	5 7 9 9
4	3	1 2 5 5 7 8
2 2 2 3 4 7	4	1 1 1 4 5 8 8 9
1 2 2 2 3 5 9	5	2 4 5 6 8 8
2 2 3 4 4 4 5 6 6	6	0 2 3 5
2 2 8	7	0 2
0 2 8	8	
0	9	

Key: 2 | 5 = 25

Mean: 60.2
Median: 62
IQR: 15
Range: 56

Mean: 46.7
Median: 46.5
IQR: 23
Range: 47

Lesson Practice ACC6.9.17

Name: _____ Date: _____ Period: _____

Problems 1–3: A school's art club held a fundraiser to raise money for art supplies. This data shows the number of T-shirts sold each week during each season.

Fall

Stem	Leaf
1	9 9
2	0 0 4 4 4 5 6 9

Key: 1 | 9 = 19

Spring

Stem	Leaf
1	9
2	1 1 2 5 5 5 6 7 9

Key: 1 | 9 = 19

1. Calculate the mean number of T-shirts sold in the fall and in the spring.

Fall: 23 T-shirts; Spring: 24 T-shirts

2. Calculate the range of the number of T-shirts sold in the fall and in the spring.

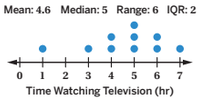
Fall: 10 T-shirts; Spring: 10 T-shirts

3. Based on this data, are sales generally higher in the spring than in the fall? Explain your thinking.

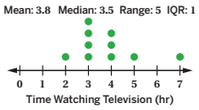
No. *Explanations vary.* There isn't much of a difference between the spring and fall sales because the ranges are the same and the means are very close together.

Problems 4–5: Abdullah compared the time that students and teachers spent watching television over the weekend. He took a random sample of 10 students and 10 teachers and made a dot plot of their responses.

Students



Teachers



4. Is there a big difference between the students' data and the teachers' data?

No

Explain your thinking.

***Explanations vary.* Teachers do watch less television than students, but the difference in the means (0.8 hours) and the difference in the medians (1.5 hours) are not very large. The IQRs and ranges are small, so there the data is pretty consistent for both groups.**

5. Abdullah then took a random sample of 10 parents. The mean was 2.6 hours, the median was 2.5 hours, the IQR was 1 hour, and the range was 5 hours.

Is there a big difference between how much television parents and students watch? Explain your thinking.

Yes. *Explanations vary.* The difference in the means is 2 hours, so parents watch 2 fewer hours of television each night on average. The median time for parents is half the median time for students. The IQR (1) and range (5) are small, so the data is consistent.

Lesson Practice ACC6.9.17

Name: _____ Date: _____ Period: _____

FAST Practice

6. These dot plots represent the time that it took new runners to run a mile.

Which statement is *best* supported by the information in the dot plots?

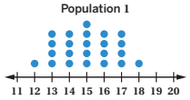
A. The two populations have an equal number of runners.

B. The two populations have the same range.

C. The two populations have different medians.

D. The two populations have the same mean.

Population 1

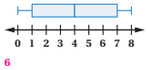


Population 2

Spiral Review

Problems 7–9: Determine the interquartile range (IQR) for each box plot.

7.



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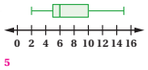

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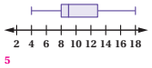


5

Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1–5	2	MA.7.DP.1.2
FAST Practice	6	2	MA.7.DP.1.2
Spiral Review			
	7–9	1	MA.6.DP.1.3

Assess and Respond

End-of-Unit Assessment

Independent | 45 min

Facilitation: Assign the End-of-Unit Assessment to learn about your students' understanding of the concepts and skills in this unit. There are two forms of the End-of-Unit Assessment: Forms A and B. If time allows, consider inviting students to complete one or more of the tasks in the Unit Synthesis and Reflection.

Item Analysis, Forms A and B

Problem(s)	Concept or skill	Addressed in	DOK	FL B.E.S.T. Math Benchmarks
1	Identifying the median of data sets	Lesson 8	1	MA.6.DP.1.2
2	Interpreting histograms as related to a context	Lessons 3, 4	2	MA.6.DP.1.4
3	Calculating the mean of data sets	Lesson 6	1	MA.6.DP.1.2
4a	Creating dot plots to represent data sets	Lessons 3, 4	2	MA.6.DP.1.4 MTR.6.1
4b	Determining quantitative measures of data sets	Lesson 11	1	MA.6.DP.1.3 MTR.6.1
5a	Comparing the range of data sets	Lesson 10	1	MA.6.DP.1.2
5b	Comparing the spread of data sets	Lesson 10	2	MA.6.DP.1.2
6a	Calculating the median of data sets	Lesson 8	1	MA.6.DP.1.3
6b - 6c	Interpreting box plots using percentages	Lesson 12	2	MA.6.DP.1.3
6d	Comparing IQRs of data sets and justifying a claim	Lesson 12	2	MA.6.DP.1.3 MTR.4.1
7	Creating dot plots that meet criteria about measures of center	Lesson 9	3	MA.6.DP.1.4
8a	Inferring about populations based on samples represented in dot plot diagrams	Lesson 17	2	MA.7.DP.1.1
8b	Comparing two populations using measures of center and variability for numerical data from random samples	Lesson 17	3	MA.7.DP.1.2

Assessment Resources

- Student Assessments (Forms A and B)
- Assessment Rubrics with Answers
- Show What You Knows and Answer Keys

Intervention and Extension Resources

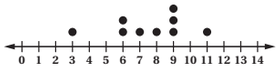
- Mini-Lessons
- Extensions

Name: Date: Period:

End-of-Unit Assessment

Unit 9

1. What is the *median* of this data set?

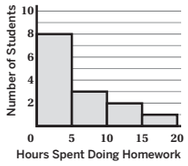


A. 7
B. 7.5
C. 8
B. 9

2. A teacher is wondering if he assigns too much homework.

He asked his students how many hours they spent doing homework last week.

Circle whether each statement below is true, false, or if there is not enough information.



		True	False	Not enough information
a	The teacher asked a total of 4 students.		(False)	
b	One student spent exactly 15 hours.			(Not enough information)
c	No student spent more than 20 hours.	(True)		
d	More than half of his students spent less than 5 hours.	(True)		

3. Dalia wrote down the number of miles she ran each day.

Calculate the *mean* distance Dalia ran.

4 miles

Day	Distance (mi)
Monday	2.5
Tuesday	3.5
Wednesday	5
Thursday	2
Friday	7

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Name: Date: Period:

End-of-Unit Assessment (continued)

Unit 9

4. Manuel has data from basketball practice about the number of free throws each teammate made.

8 5 6 5 9 10 2 5 8 10

a. Create a line plot for Manuel's team.

b. Determine each statistic for his team.

Quartile 1: 5

Median: 7

Quartile 3: 9

IQR: 4

5. Here are temperatures for five days in January for two cities.

Minneapolis

Temperatures (°F)				
32	25	35	38	20

Mean = 30°F, Range = 18°F

Ottawa

Temperatures (°F)				
31	29	27	31	32

Mean = 30°F, Range = ?°F

a. Calculate the range of the temperatures in Ottawa.

5°F

b. Which city has a wider spread of temperatures? Circle one.

(Minneapolis) Ottawa They have the same spread

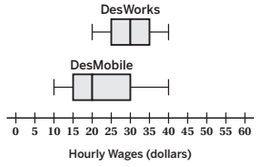
Name: Date: Period:

End-of-Unit Assessment (continued)

Unit 9

6. Two companies analyzed the hourly wages of their employees.

Here are the results.



a. What is the median hourly wage for DesMobile?

$20

b. What percentage of workers at *DesWorks* have an hourly wage of $30 or more?

50%

c. What percentage of workers at *DesMobile* have an hourly wage of $30 or more?

25%

d. Which company's wages have a greater IQR? Circle one.

DesWorks (DesMobile) They are the same

Explain how you know.

***Explanations vary*. The IQR for DesMobile employees is about $15, which is greater than the IQR for DesWorks of about $10.**

7. Create a line plot with:

- At least five data points.
- A median of 6.
- A mean that is less than the median.

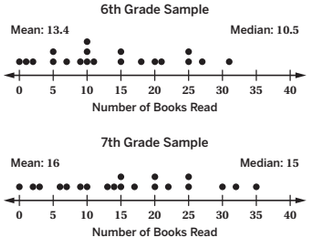
***Line plots vary*.**

Name: Date: Period:

End-of-Unit Assessment (continued)

Unit 9

8. Maneli is wondering who reads more books at her school: the 6th graders or the 7th graders. She selects a random sample of 20 students from 6th grade and 20 students from 7th grade. Then she asks the students how many books they read in the past year.



a. Dylan looks at the dot plots and says that the school only has 20 6th graders and 20 7th graders.

Do you agree?

No

Explain your thinking.

***Explanations vary*. These are only samples of the 6th graders and 7th graders at Maneli's school, not the full populations.**

b. Explain whether the data shows a big difference between the number of books read by 6th graders and 7th graders. Use evidence to justify your claim.

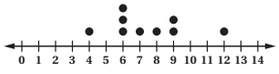
***Responses vary*. The median is a measure of center for the data. The measure of center changes from 10.5 to 15, which is a large change. This shows that there is a big difference in the number of books read by the two grades.**

Name: Date: Period:

End-of-Unit Assessment

Unit 9

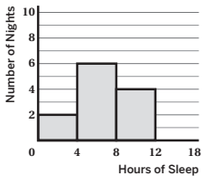
1. What is the *median* of this data set?



A. 6
B. 7
C. 7.5
B. 8

2. Anika wants to learn about how much she sleeps.

She made this histogram of the number of hours she slept each night.



Circle whether each statement below is true, false, or if there is not enough information.

a. Anika kept track of her sleep for 3 nights.	True	(False)	Not enough information
b. Anika slept less than 12 hours every night.	(True)	False	Not enough information
c. One night, Anika slept exactly 6 hours.	True	False	(Not enough information)
d. Half of the nights, Anika slept fewer than 8 hours.	True	(False)	Not enough information

3. Five friends shared a bag of cookies.

Calculate the *mean* number of cookies eaten.

3 cookies

Person	Cookies Eaten
Troy	2.5
Latifa	1.5
Sai	5
Matias	2
Katie	4

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Name: Date: Period:

End-of-Unit Assessment (continued)

Unit 9

4. Eliza has data from her soccer team about the number of goals each teammate scored.

4 7 4 5 9 8 4 1 9 7

a. Create a line plot for Eliza's team.

b. Determine each statistic for her team.

Quartile 1: **4**

Median: **6**

Quartile 3: **8**

IQR: **4**

5. Here are temperatures for five days in February for two cities.

Montreal

Temperatures (°F)				
30	29	32	35	34

Mean = 32°F, Range = 6°F

Chicago

Temperatures (°F)				
35	37	34	28	26

Mean = 32°F, Range = ?°F

a. Calculate the range of the temperatures in Chicago.

11°F

b. Which city has a wider spread of temperatures? Circle one.

Montreal (Chicago) They have the same spread

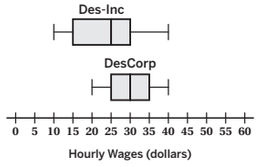
Name: Date: Period:

End-of-Unit Assessment (continued)

Unit 9

6. Two companies analyzed the hourly wages of their employees.

Here are the results.



a. What is the median hourly wage for DesCorp?

$30

b. What percentage of employees at *Des-Inc* have an hourly wage of $25 or more?

About 50%

c. What percentage of employees at *DesCorp* have an hourly wage of $25 or more?

About 75%

d. Which company's wages have a greater IQR? Circle one.

(Des-Inc) DesCorp They are the same

Explain how you know.

***Explanations vary*. The IQR for Des-Inc employees is about $15, which is greater than the IQR for DesCorp of about $10.**

7. Create a line plot with:

- At least five data points.
- A median of 7.
- A mean that is greater than the median.

***Line plots vary*.**

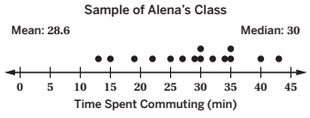
Name: Date: Period:

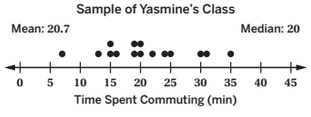
End-of-Unit Assessment (continued)

Unit 9

8. Alena is wondering who spends more time commuting: her class or her friend Yasmine's class. Alena selects a random sample of 15 students from her class. Yasmine does the same from her class.

They ask those students how long they spent commuting to school yesterday.





a. Dylan looks at the data and says that each class has 15 students. Do you agree?

No

Explain your thinking.

***Explanations vary*. They sampled 15 students from each class. This is not the whole population.**

b. Explain whether the data shows a big difference between the time Alena's classmates and Yasmine's classmates spend commuting. Use evidence to justify your claim.

***Responses vary*. The median is a measure of center for the data. The measures of center is 30 for one class but 20 for another class, which is a large difference. This shows that there is a big difference in the the time that Alena's classmates and Yasmine's classmates spend commuting.**

D Differentiation (End-of-Unit Assessment)

Note: To **strengthen** and **stretch** students' learning, refer to the differentiation resources suggested throughout this unit and in the Unit Overview.

Sub-Unit Goals	Problem(s)	To respond to student thinking, consider:

Sub-Unit 1:
• Use data sets to make claims about data.
• Create histograms to visualize data.
(Lessons 1–5)
	2, 4a	**Support**
		• **Mini-Lesson:** *Creating Dot Plots*
		• **Teacher Move:** Consider revisiting:
		  • Lesson 3 *(The Plot Thickens)*
		  • Lesson 4 *(What Does It All Mean?)*
		Strengthen
		• **Teacher Move:** Invite students to list the data from least to greatest and cross off values as they are plotted.

Sub-Unit 2:
• Calculate the mean, median, and mode of a data set.
• Calculate the quartiles, interquartile range (IQR), and range of a data set.
• Create and use box plots to make claims about data.
(Lessons 6–12)
	3	**Support**
		Strengthen
		• **Teacher Move:** Consider revisiting:
		  • Lesson 6 *(Snack Time)*
		Stretch:
		• **You're Invited to Explore More:**
		  • Lesson 6 *(Snack Time)*

	5	**Support**
		Strengthen
		• **Teacher Move:** Consider revisiting:
		  • Lesson 10 *(Hoops)*
		Stretch:
		• **You're Invited to Explore More:**
		  • Lesson 10 *(Hoops)*

	1, 6a	**Support**
		• **Teacher Move:** Consider revisiting:
		  • Lesson 8 *(Toy Cars)*
		Strengthen
		• **Repeated Challenges:**
		  • Lesson 8 *(Toy Cars)*

	4b, 7	**Support**
		• **Teacher Move:** Consider revisiting:
		  • Lesson 9 *(In the News)*
		  • Lesson 11 *(Pumpkin Patch)*
		Strengthen
		• **Repeated Challenges:**
		  • Lesson 11 *(Pumpkin Patch)*

	6b–6d	**Support** **Strengthen** • **Mini-Lesson:** *Interpreting Box Plots* • **Teacher Move:** Consider revisiting: • Lesson 12 *(Car, Plane, Bus, or Train?)* • **Teacher Move:** Invite students to label the locations of the first quartile, median, and third quartile on the box plots.
Sub-Unit 3: Use measures of center and measures of spread from random samples to draw conclusions about and compare populations. **(Lessons 13–17)**	8	**Support** • **Teacher Move:** Consider revisiting, Lesson 17 *(School Newspaper)*

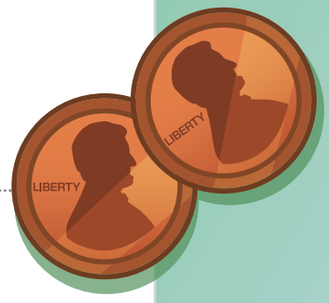
UNIT 10

Probability

Students learn about probability as a way to describe the likelihood of unknown events and estimate the probability of real-world situations.

Essential Questions

- How can we determine how likely an event is to happen?
- How can we use ratios and proportions to make predictions?
- How can we decide whether an object is fair?



Sub-Unit Goals

Sub-Unit 1

- Compare probabilities to the results of repeated experiments.
- Determine the probability of events by identifying the sample space.

The Math of the Unit

Unit Outcomes Telling a story that narrows and deepens with the grade

Florida B.E.S.T. Standards for Mathematics

Apply properties of operations to rewrite numbers in equivalent forms. **(MA.6.NSO.3)**

Develop an understanding of probability. Find and compare experimental and theoretical probabilities. **(MA.7.DP.2)**

Benchmarks Addressed in This Unit: MA.6.NSO.3.5 MA.7.DP.2.1 MA.7.DP.2.2 MA.7.DP.2.3 MA.7.DP.2.4

Mathematical Thinking and Reasoning Standards

- Demonstrate understanding by representing problems in multiple ways. **(MTR.2.1)**
- Engage in discussions that reflect on the mathematical thinking of self and others. **(MTR.4.1)**
- Use patterns and structure to help understand and connect mathematical concepts. **(MTR.5.1)**
- Assess the reasonableness of solutions. **(MTR.6.1)**

Also Addressed: **MTR.1.1, MTR.3.1**

Coherence Telling a story within and across grades

In this Unit

Students describe the likelihood of unknown events and estimate the probabilities of real-world situations.

Prior Learning

- **Unit 3:** Solve rate problems **(MA.6.AR.3.2, MA.6.AR.3.5)**
- **Unit 9:** Calculate and interpret statistical measures **(MA.6.DP.1.2)**

Future Learning

- Determine probabilities of repeated experiments **(MA.8.DP.2)**

See the Connections to Future Learning page for more information, including explanations and examples.

Connections Integrating the aspects of rigor in ways that support one another

Here are examples of the aspects of rigor and balance in this unit. See the first page of each lesson's Teacher Edition for more information.

Conceptual Understanding

In **Lessons 1–4**, students describe the likelihood and probability of events and connect the results of repeated experiments to the accuracy of their own predictions.

Adaptive Reasoning

In **Lessons 4–5**, students develop adaptive reasoning as they analyze results from experiments to approximate the probability of events.

Unit at a Glance

Assess and Respond

A Pre-Unit Check

Learn more about your students' understanding of foundational concepts and skills that will support them in the upcoming unit. This can be given in its entirety before the unit or spread throughout the unit.

MA.6.DP.1.2 MA.6.DP.1.3 MA.6.DP.1.4 MA.6.DP.1.5 MA.6.DP.1.6 MA.6.AR.3.4 MA.6.NSO.1.1 MA.6.NSO.3.5 MA.7.AR.3.1
MTR.4.1 MTR.5.1

Sub-Unit 1

1 How Likely?

Chance Experiments

Use phrases such as "likely" and "equally likely as not" to describe the likelihood of different events.

MA.7.DP.2.2
MTR.2.1 MTR.3.1

2 Prob-bear-bilities

Intro to Probability

Connect likelihood with probability written in different forms and use sample spaces to help determine probabilities.

MA.7.DP.2.1 MA.7.DP.2.2 MA.7.DP.2.3
MTR.2.1 MTR.3.1

3 Mystery Bag

Predicting Sample Spaces

Use repeated experiments and ratios to make predictions and recognize that the number of repeated experiments affects the accuracy of a prediction.

MA.6.NSO.3.5 MA.7.DP.2.2 MA.7.DP.2.3 MA.7.DP.2.4
MTR.3.1 MTR.6.1

Pacing: 7 days | **Short on time?** See pacing considerations below.

All instructional and assessment days are ~45 minutes each.

- 5 Lessons
- Pre-Unit Check
- End-of-Unit Assessment

Assess and Respond

4 Spin Class

Repeated Experiments

Recognize that the results of a repeated experiment will approach, but may not exactly match, the probability of the event.

MA.6.NSO.3.5 MA.7.DP.2.2 MA.7.DP.2.3 MA.7.DP.2.4
MTR.1.1 MTR.4.1

5 Is It Fair?

Comparing Probabilities and Experiments

Decide whether an object is fair by comparing probabilities from a model to the results of a repeated experiment.

MA.6.NSO.3.5 MA.7.DP.2.3 MA.7.DP.2.4
MTR.5.1 MTR.6.1

A End-of-Unit Assessment

Learn about your students' understanding of the concepts and skills in this unit. There are two forms of the End-of-Unit Assessment: Forms A and B.

MA.7.DP.1.2 MA.7.DP.1.3 MA.7.DP.2.2 MA.7.DP.2.3 MA.7.DP.2.4
MTR.4.1 MTR.6.1

Pacing Considerations

Lesson 1: This lesson surfaces what students know about likelihood, which will be addressed further in upcoming lessons. If omitted, the vocabulary from this lesson should be defined during Lesson 2.

Lesson Materials and Prep

Here are the materials and resources needed within each lesson, how to prepare them, and where to find them.

Lesson 1

- **Copy:** Show What You Know
- **Cut:** Activity 1 Sheet *(one per two groups)*
- **Materials:** coins, two different types *(e.g., a penny and a nickel)*, containers for paper letters *(e.g., bag, bowl, or cup)*, number cubes, paper clips for spinners

Lesson 2

- **Copy:** Show What You Know, Activity 2 and 3 Sheets *(one per pair)*, You're Invited to Explore More Sheet *(optional)*
- **Materials:** coloring tools *(optional)*, paper clips for spinners, coins, number cubes *(as needed)*

Lesson 3

- **Copy:** Show What You Know
- **Materials:** coloring tools (green and purple) *(optional)*

Lesson 4

- **Copy:** Show What You Know, Activity 3 Sheet *(one per student)*
- **Materials:** coloring tools (blue and red), paper clips *(optional)*

Lesson 5

- **Copy:** Show What You Know
- **Materials:** coins *(one per pair)*, number cubes *(one per pair)*

Resource Books

Assessment and Lesson Resources

- Pre-Unit Check
- End-of-Unit Assessment, Forms A & B
- Assessment Rubrics
- Show What You Knows
- Activity Sheets and Cards

Intervention and Extension Resources

- Mini-Lessons
- Prior Grade Mini-Lessons
- Extensions

Caregiver Support

These resources can be shared with students' caregivers. They provide background on the mathematics in this unit, as well as suggestions for supporting students at home.

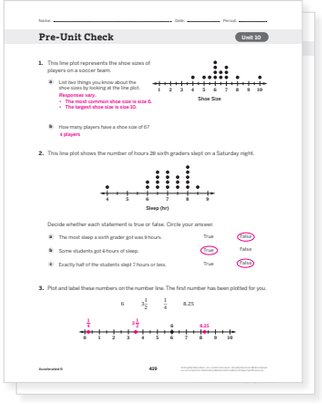
Assessments

Every unit includes a variety of formative and summative assessments designed to highlight students' understanding of a variety of concepts and skills.

Pre-Unit Check

Assign the **Pre-Unit Check** to learn more about your students' understanding of foundational concepts and skills that will support them in the upcoming unit. This can be given in its entirety before the unit or spread throughout the unit.

FL B.E.S.T. Math Benchmarks assessed: MA.6.DP.1.2, MA.6.DP.1.3, MA.6.DP.1.4, MA.6.DP.1.5, MA.6.DP.1.6, MA.6.AR.3.4, MA.6.NSO.1.1, MA.6.NSO.3.5, MA.7.AR.3.1


Pre-Unit Check

Unit 10

1. This line plot represents the shoe sizes of players on a soccer team.

Shoe Size

2. This line plot shows the number of hours 28 sixth graders slept on a Saturday night.

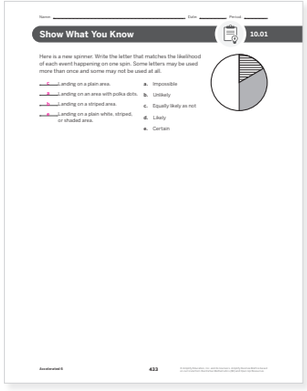
Sleep (hr)

3. Plot and label these numbers on the number line. The first number has been plotted for you.

Lesson | Show What You Know

Use the **Show What You Know** for each lesson to learn more about your students' progress toward the lesson goal(s). Here is one example from this unit.

FL B.E.S.T. Math Benchmarks assessed on Lesson 1 Show What You Know: MA.7.DP.2.1


Show What You Know

10.01

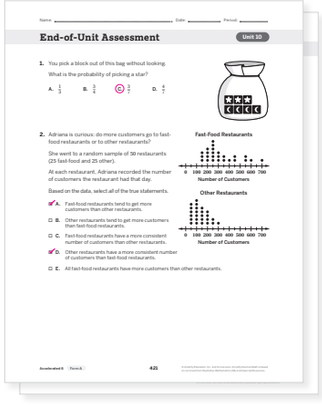
Here is a new spinner. Write the letter that matches the likelihood of each event happening on one spin. Some letters may be used more than once and some may not be used at all.

a. Impossible
b. Unlikely
c. Equally likely as not
d. Likely
e. Certain

End-of-Unit Assessment

Assign the **End-of-Unit Assessment** to learn about your students' understanding of the concepts and skills in this unit. There are two forms of the End-of-Unit Assessment: Forms A and B.

FL B.E.S.T. Math Benchmarks assessed: MA.7.DP.1.2, MA.7.DP.1.3, MA.7.DP.2.2, MA.7.DP.2.3, MA.7.DP.2.4


End-of-Unit Assessment

Unit 10

1. You pick a block out of this bag without looking.
What is the probability of picking a star?

2. Adriana is curious: do more customers go to fast-food restaurants or to other restaurants?

Fast-Food Restaurants

Number of Customers

Other Restaurants

Number of Customers

Based on the data, select all of the true statements.

Differentiation

Every unit includes additional resources to support, strengthen, and stretch student learning.

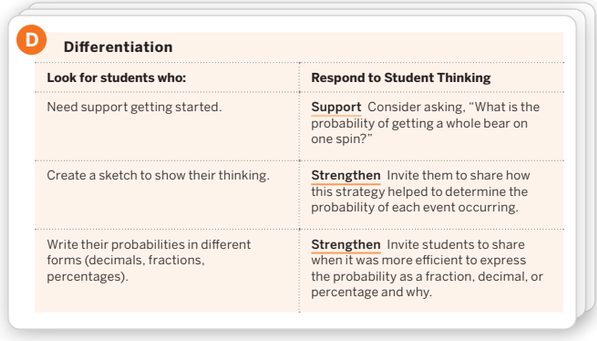
Differentiation Across the Unit

	Support	Strengthen	Stretch
	Provide targeted intervention for students by using these resources.	Reinforce students' understanding of the concepts assessed by using these resources.	Challenge students and extend their learning with these resources.
Sub-Unit 1 (Lessons 1–5) • Compare probabilities to the results of repeated experiments. • Determine the probability of events by identifying the sample space.	**Mini-Lesson** • *Predicting Sample Spaces Using Ratios and Percentages*	**Repeated Challenges** • Lesson 2 (*Prob-bear-bilities*) **Challenge Creator** • Lesson 4 (*Spin Class*) **Practice** • Lessons 1–5	**You're invited to explore more.** • Lesson 2 (*Prob-bear-bilities*) • Lesson 5 (*Is It Fair?*) **DOK3 Practice** • Lessons 1, 4, and 5

Differentiation Within the Lesson

Each Teacher Edition includes point-of-use differentiation suggestions to modify or adjust instruction during the lesson to accommodate the needs of all learners.

Here is one example from **Lesson 2**, where students determine the probability of an event.



Differentiation

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Consider asking, "What is the probability of getting a whole bear on one spin?"
Create a sketch to show their thinking.	**Strengthen** Invite them to share how this strategy helped to determine the probability of each event occurring.
Write their probabilities in different forms (decimals, fractions, percentages).	**Strengthen** Invite students to share when it was more efficient to express the probability as a fraction, decimal, or percentage and why.

Accessibility

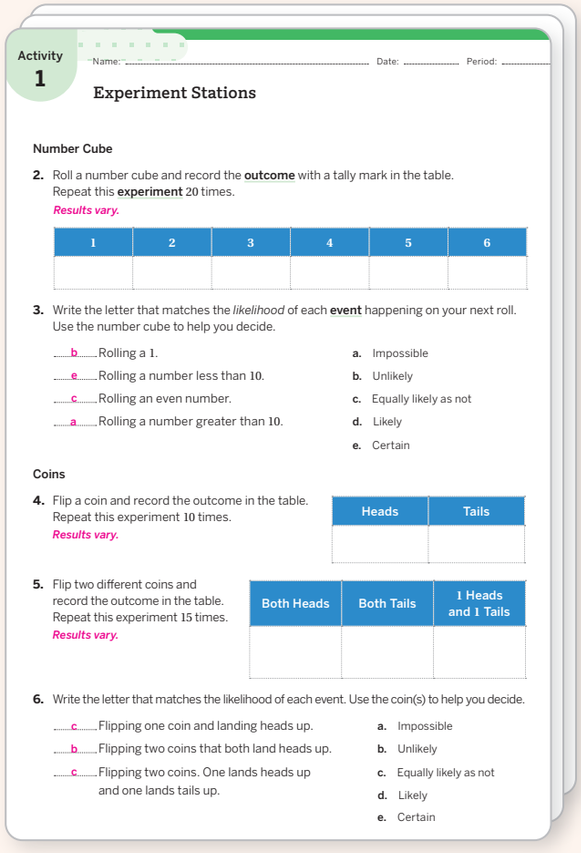
Every lesson is designed using the principles of Universal Design for Learning. Every lesson also includes support suggestions that build on students' strengths.

Universal Design for Learning

Here is one example from a lesson designed to support Action & Expression.

Spotlight on Interaction . . .

Lesson 1: In this lesson, students physically interact with manipulatives to perform experiments and move from station to station.



Activity 1

Name: Date: Period:

Experiment Stations

Number Cube

2. Roll a number cube and record the **outcome** with a tally mark in the table. Repeat this **experiment** 20 times.

Results vary.

1	2	3	4	5	6

3. Write the letter that matches the *likelihood* of each **event** happening on your next roll. Use the number cube to help you decide.

b Rolling a 1.
e Rolling a number less than 10.
c Rolling an even number.
a Rolling a number greater than 10.

a. Impossible
b. Unlikely
c. Equally likely as not
d. Likely
e. Certain

Coins

4. Flip a coin and record the outcome in the table. Repeat this experiment 10 times.

Results vary.

Heads	Tails

5. Flip two different coins and record the outcome in the table. Repeat this experiment 15 times.

Results vary.

Both Heads	Both Tails	1 Heads and 1 Tails

6. Write the letter that matches the likelihood of each event. Use the coin(s) to help you decide.

c Flipping one coin and landing heads up.
b Flipping two coins that both land heads up.
c Flipping two coins. One lands heads up and one lands tails up.

a. Impossible
b. Unlikely
c. Equally likely as not
d. Likely
e. Certain

Just-In-Time Accommodations and Supports

Each Teacher Edition includes suggested supports based on the areas of cognitive function from the Education Development Center (EDC).

Spotlight on Conceptual Processing . . .

Here are two suggestions in this unit for supporting students with thinking abstractly and applying prior knowledge to new situations.

Lesson 2, Activity 1

Accessibility: Conceptual Processing To support abstract thinking, provide probability tools such as coins or number cubes.

Lesson 3, Activity 1

Accessibility: Conceptual Processing Invite students to solve a similar question with friendlier numbers to support recognizing connections between new problems and prior work with ratios.

Language Development

Every lesson includes opportunities for all students to develop fluency with mathematical language.

Vocabulary

New Vocabulary	Lesson	Review Vocabulary
event **experiment** **outcome** **random**	1	percentage
probability **sample space**	2	
relative frequency	4	

Math Language Routines and Language Goals

Math Language Routine	Lesson(s)
MLR2: Collect and Display	1–2

Sub-Unit Language Goal

- Interpret the results from repeated experiments to determine fairness, probably, and the likelihood of events.

English Language Learners Support

Every Teacher Edition includes suggestions for supporting the needs of students learning English.

Here is one example from **Lesson 5**, where students are analyzing the fairness of a game using probability.

ELL **English Language Learners** Provide sentence frames to help students explain their reasoning (e.g., I think the probability that Player 2 wins is _____ because _____.).

Connections to Future Learning

Here is how the content in this unit connects to where your students are headed on their math journeys.

Probabilities of Repeated Experiments

In this unit, students determine probabilities for simple experiments. **In a future course**, students will represent and determine probabilities of repeated experiments.

FL B.E.S.T. Math Benchmarks:
MA.8.DP.1.1, MA.8.DP.1.2

Example:

Consider the repeated experiment of flipping a coin 2 times, where H is heads and T is tails.

The sample space is HH, HT, TH, TT. So this repeated experiment has 4 events.

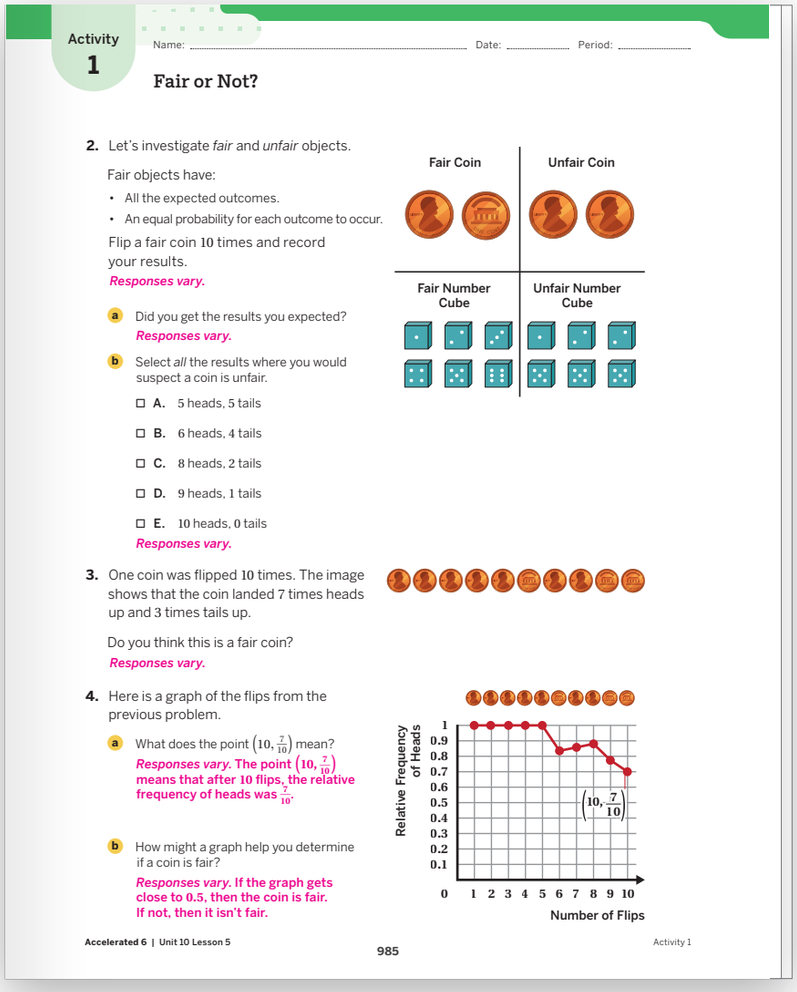
Theoretical probability is the number of outcomes in the event out of the number of outcomes in the sample space. The theoretical probability of getting at least one heads, P(at least 1 heads), is $\frac{3}{4}$, or 75%.

Professional Learning

Here is an opportunity to deepen your understanding of the math in this unit and how to teach it.

Spotlight on Adaptive Reasoning

Lesson 5, Activity 1



Activity 1

Name: _____ Date: _____ Period: _____

Fair or Not?

2. Let's investigate *fair* and *unfair* objects.

Fair objects have:

- All the expected outcomes.
- An equal probability for each outcome to occur.

Flip a fair coin 10 times and record your results.

Responses vary.

a. Did you get the results you expected?

Responses vary.

b. Select *all* the results where you would suspect a coin is unfair.

- ☐ A. 5 heads, 5 tails
- ☐ B. 6 heads, 4 tails
- ☐ C. 8 heads, 2 tails
- ☐ D. 9 heads, 1 tails
- ☐ E. 10 heads, 0 tails

Responses vary.

3. One coin was flipped 10 times. The image shows that the coin landed 7 times heads up and 3 times tails up.

Do you think this is a fair coin?

Responses vary.

4. Here is a graph of the flips from the previous problem.

a. What does the point $\left(10, \frac{7}{10}\right)$ mean?

Responses vary. The point $\left(10, \frac{7}{10}\right)$ means that after 10 flips, the relative frequency of heads was $\frac{7}{10}$.

b. How might a graph help you determine if a coin is fair?

Responses vary. If the graph gets close to 0.5, then the coin is fair. If not, then it isn't fair.

Accelerated 6 | Unit 10 Lesson 5 985 Activity 1

Context

In this activity, students are developing adaptive reasoning as they think logically to determine the fairness of objects.

Try This

Put on your student hat and complete Problems 2–4.

Points to Ponder

- What do students need to understand about repeated experiments and relative frequency to support their thinking about the fairness of objects?
- How does determining the fairness of an object build from their understanding of using repeated experiments to make predictions?

Notes:

Assess and Respond

Pre-Unit Check

Independent | 45 min

Facilitation: Assign the Pre-Unit Check to learn more about your students' understanding of foundational concepts and skills that will support them in the upcoming unit. This can be given in its entirety before the unit or spread throughout the unit. If time allows, consider inviting students to complete one or more of the tasks in the Unit Synthesis and Reflection.

Item Analysis

Problem	Concept or skill	Preparation for	DOK	FL B.E.S.T. Math Benchmarks
1, 2	Interpreting line plots	Lesson 2	1	MA.5.DP.1.1 MTR.7.1
3	Plotting fractions and decimals on a number line	Lesson 3	1	MA.5.DP.1.1
4	Evaluating expressions with different operations and grouping symbols	Lesson 6	1	MA.5.AR.2.2
5	Calculating percentages that are multiples of 25%	Lesson 11	1	MA.6.DP.1.3 MTR.2.1

Assessment and Lesson Resources

- Student Assessments
- Show What You Knows and Answer Keys

Intervention and Extension Resources

- Mini-Lessons
- Extensions

Name: ..... Date: ..... Period: .....

Pre-Unit Check

Unit 10

1. This line plot represents the shoe sizes of players on a soccer team.

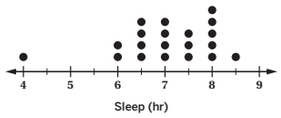
a List *two* things you know about the shoe sizes by looking at the line plot.

Responses vary.
- The most common shoe size is size 6.
- The largest shoe size is size 10.

b How many players have a shoe size of 6?

4 players

2. This line plot shows the number of hours 20 sixth graders slept on a Saturday night.

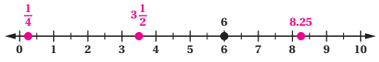


Decide whether each statement is true or false. Circle your answer.

a	The most sleep a sixth grader got was 9 hours.	True	(False)
b	Some students got 6 hours of sleep.	(True)	False
c	Exactly half of the students slept 7 hours or less.	True	(False)

3. Plot and label these numbers on the number line. The first number has been plotted for you.

$6 \quad 3\frac{1}{2} \quad \frac{1}{4} \quad 8.25$



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Name: ..... Date: ..... Period: .....

Pre-Unit Check (continued)

Unit 10

4. Evaluate each expression.

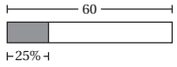
a $\frac{5+11+9+15}{4} = 10$

b $(4+2+9) \div 3 = 5$

5. Calculate each percent.

a 25% of 60

Use the tape diagram if it helps you with your thinking.



15

b 75% of 40

30

c 25% of 30

7.5

D Differentiation (Pre-Unit Check)

Note: To **strengthen** and **stretch** students' learning, refer to the differentiation resources suggested throughout this unit and in the Unit Overview.

Sub-Unit Goals	Problem(s)	Respond to Student Thinking
Preparation for Sub-Unit 1: • Use data sets to make claims about data. • Create histograms to visualize data. **(Lessons 1–5)**	1, 2	**Support** • **Teacher Move:** During Lesson 2, consider inviting students to share what they know about each dot plot.
	3	**Support** • **Mini-Lesson:** Creating Dot Plots • **Teacher Move:** Invite students to place each value in the appropriate place on the number line and justify their placement with a partner.
	4	**Support** • **Teacher Move:** Before Lesson 6, consider sharing incorrect work for this problem and inviting students in groups to explain what the student did well and what error(s) they made.
	5	**Support** • **Teacher Move:** After Lesson 11, consider revisiting this problem and inviting students to make connections between quartiles and percentages.

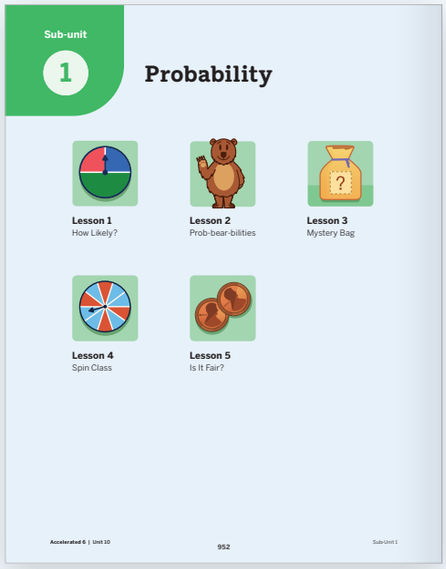
Sub-Unit 1

Probability

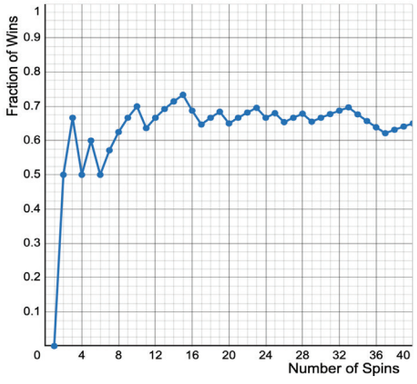
Sub-Unit 1 Goals

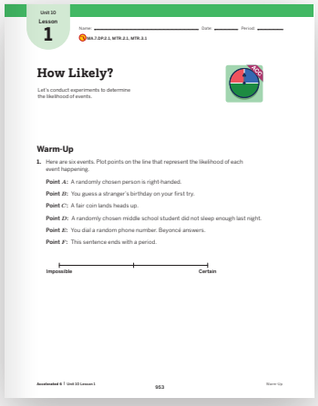
- **Goal:** Compare probabilities to the results of repeated experiments.
- **Goal:** Determine the probability of events by identifying the sample space.
- **Language Goal:** Interpret the results from repeated experiments to determine fairness, probability, and likelihood of events.

In this sub-unit, students determine the **probability** of **events** and compare the results of repeated **experiments** to the probability.



Math That Matters Most

Concept or Skill	For example . . .
Express the probability of events as a fraction, decimal, or percentage.	Calculate the probability that a spinner with equal parts red, yellow, and green lands on green on the next spin. $\frac{1}{3}$
Use data from a repeated experiment to approximate the probability of an event.	A spinner is spun 40 times for a game. This graph shows the fraction of games that are wins. Based on the graph, what is the probability of a spin winning this game?  *The probability is around 65% because the line on the graph approaches 0.65 as the number of spins increases.*


Unit 10
Lesson
1

How Likely?

Let's conduct experiments to determine the likelihood of events.

Warm-Up

1. Here are six events. Plot points on the line that represent the likelihood of each event happening.

Point A: A randomly chosen person is right-handed.
Point B: You guess a stranger's birthday on your first try.
Point C: A fair coin lands heads up.
Point D: A randomly chosen middle school student did not sleep enough last night.
Point E: You dial a random phone number. Beyoncé answers.
Point F: This sentence ends with a period.

Impossible — Certain

953

How Likely?

Chance Experiments

Let's conduct experiments to determine the likelihood of events.

Learning Outcomes

Today's Goals

1. **Language Goal:** Describe the likelihood of *events* using the words *impossible, unlikely, equally likely as not, likely,* or *certain*. **(Reading and Writing)**
2. **Language Goal:** Explain how *experiments* can help determine the likelihood of future *events*. **(Writing)**

Students experience chance as they conduct their own **experiments** and use phrases such as "likely" and "equally likely as not" to describe the likelihood of different **events**. They also encounter many of the tools they will use to simulate events later in the unit: coins, number cubes, spinners, and items in bags.

Coherence

Prior Learning

In Unit 9, students visualized data and calculated measures of center to describe and compare data sets.

Future Learning

In Lessons 2–5, students will connect likelihoods to the word probability and to methods for calculating probability. They will use repeated experiments and ratios to make predictions.

Connections

- Students explore experimental probability of flipping a coin or rolling a number cube and recording their outcomes on a table. (mathematical, real world)

FL B.E.S.T. Math Benchmarks

Addressing

MA.7.DP.2.1

Given the probability of a chance event, interpret the likelihood of it occurring. Compare the probabilities of chance events.

Mathematical Thinking and Reasoning Standards: MTR.2.1, MTR.3.1

ELA Expectation: ELA.K12.EE.5.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.6.DP.1.2	MA.7.DP.2.3 MA.7.DP.2.4

Vocabulary

New Vocabulary

event	outcome
experiment	random

Lesson at a Glance

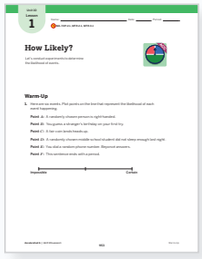
~ 45 min

MA.7.DP.2.2

Warm-Up

Pairs | 5 min

Students use their knowledge and intuition to rank the likelihoods of different **events**.



Activity 1

Small Groups | 30 min

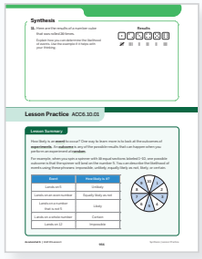
Students describe the likelihood of events using the words impossible, unlikely, equally likely as not, likely, or certain.

Synthesis

Whole Class | 5 min

Students synthesize their understanding of likelihoods by explaining how **experiments** can help determine the likelihood of events.

MLR2: Collect and Display



Show What You Know

Independent | 5 min

Students demonstrate their understanding by determining the likelihood of spinning a named section on a spinner.

Prep Checklist

This lesson includes:

Student Edition
Show What You Know

- Coins, two different types (e.g., a penny and a nickel), containers for paper letters (e.g., bag, bowl, or cup), number cubes, paper clips for spinners

Pairs | 5 min

Warm-Up

Purpose: Students use their knowledge and intuition to rank the likelihoods of different **events**.

1 Launch

Direct students' attention to the line and six events to read aloud as a class.

Consider asking, "What do you think *random* means in this context?"

Create a class definition for the terms **events** and **random** using students' language and the glossary.

To surface prior knowledge, consider asking:

- "What percentage is associated with impossible? Certain? The marker in between impossible and certain?"
- "Where on the line would you label as being unlikely? Likely?"

2 Connect

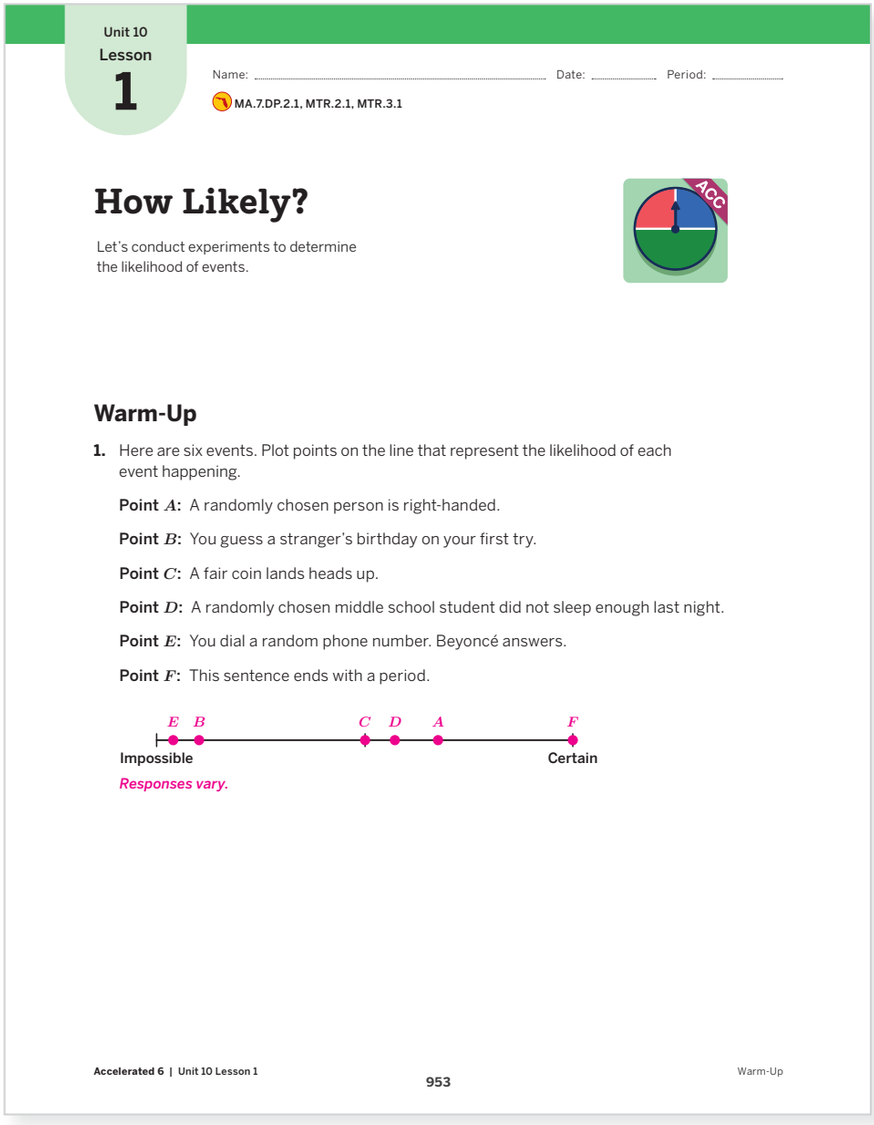
Invite students to share their responses and justify the placement of each point on the line. **(MTR.3.1)**

Encourage students to critique each other's reasoning.

ELL **English Language Learners** Provide sentence frames to help students explain their reasoning:

- I would place the statement _____ because _____.
- I think it should be placed farther left because _____.
- I think it should be placed farther right because _____.
- I think it should be placed right in the middle because _____.

Consider asking, "Which events did you think were likely to occur? Did you place them in the same position on the line? Why?"



Unit 10
Lesson
1

Name: _____ Date: _____ Period: _____

MA.7.DP.2.1, MTR.2.1, MTR.3.1

How Likely?

Let's conduct experiments to determine the likelihood of events.

Warm-Up

1. Here are six events. Plot points on the line that represent the likelihood of each event happening.

   Point A: A randomly chosen person is right-handed.

   Point B: You guess a stranger's birthday on your first try.

   Point C: A fair coin lands heads up.

   Point D: A randomly chosen middle school student did not sleep enough last night.

   Point E: You dial a random phone number. Beyoncé answers.

   Point F: This sentence ends with a period.

Responses vary.

Accelerated 6 | Unit 10 Lesson 1 953 Warm-Up

Small Groups | 30 min

Activity 1 Experiment Stations

Purpose: Students describe the likelihood of events using the words impossible, unlikely, equally likely as not, likely, or certain.

Short on time? Consider allowing students to choose two of the four stations to complete.

1 Launch

To generate excitement, consider sharing that in this activity, students will be conducting experiments at different stations.

Note: You may set up stations with physical manipulatives including a number cube, coins, deck of letter cards, and a spinner.

Demonstrate how to use the physical tools for each experiment and discuss how to record the outcomes.

Create a class definition of the terms **outcome** and **experiment** using students' language and the glossary.

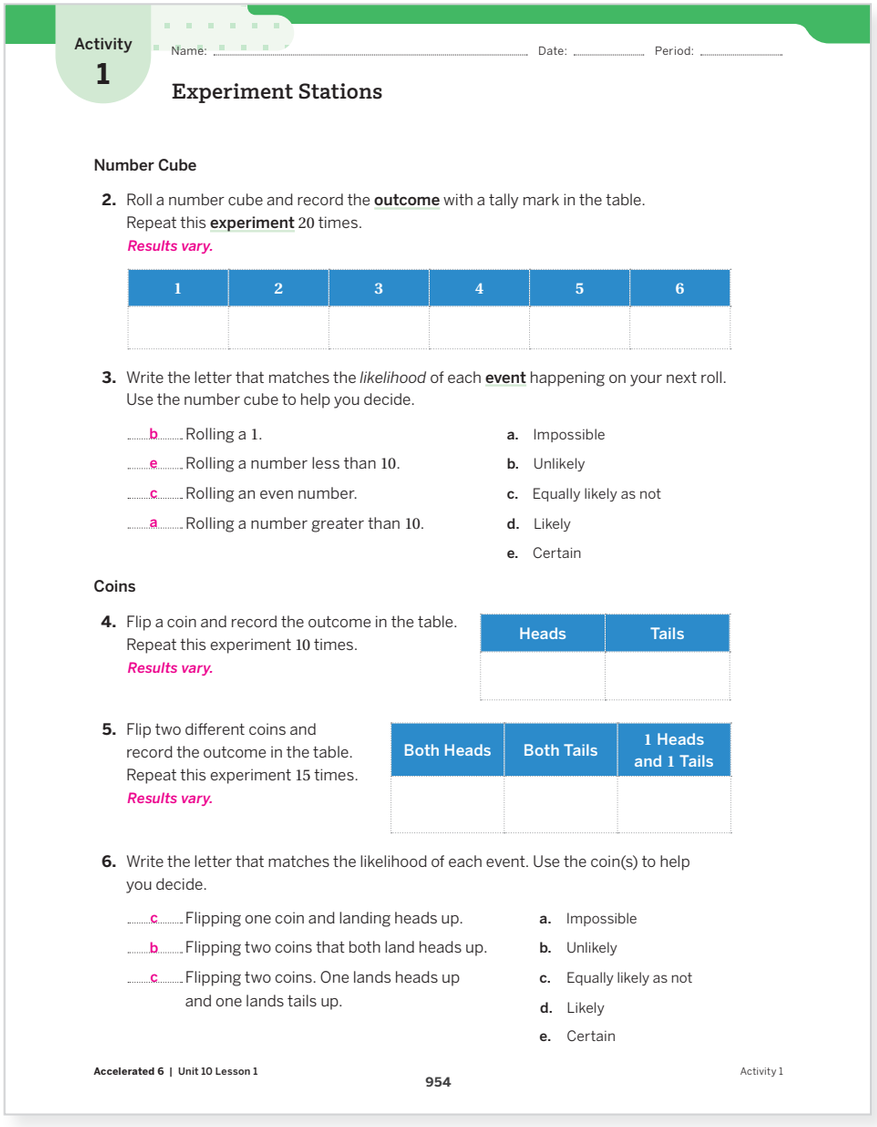
Note: Consider these options for student movement:

- As students finish a station, instruct them to move from station to station.
- After a set amount of time, instruct students to move as a group from station to station.
- Invite students to complete each station individually or in pairs.

2 Monitor

Note: Refer to the guidance on the next page to support students as they work through the experiments.

Activity 1 continued >



Activity 1

Name: Date: Period:

Experiment Stations

Number Cube

2. Roll a number cube and record the **outcome** with a tally mark in the table. Repeat this **experiment** 20 times.
Results vary.

1	2	3	4	5	6

3. Write the letter that matches the *likelihood* of each **event** happening on your next roll. Use the number cube to help you decide.

b Rolling a 1.
e Rolling a number less than 10.
c Rolling an even number.
a Rolling a number greater than 10.

a. Impossible
b. Unlikely
c. Equally likely as not
d. Likely
e. Certain

Coins

4. Flip a coin and record the outcome in the table. Repeat this experiment 10 times.
Results vary.

Heads	Tails

5. Flip two different coins and record the outcome in the table. Repeat this experiment 15 times.
Results vary.

Both Heads	Both Tails	1 Heads and 1 Tails

6. Write the letter that matches the likelihood of each event. Use the coin(s) to help you decide.

c Flipping one coin and landing heads up.
b Flipping two coins that both land heads up.
c Flipping two coins. One lands heads up and one lands tails up.

a. Impossible
b. Unlikely
c. Equally likely as not
d. Likely
e. Certain

Accelerated 6 | Unit 10 Lesson 1 954 Activity 1

Small Groups | 30 min

Activity 1 Experiment Stations (continued)

Purpose: Students describe the likelihood of events using the words impossible, unlikely, equally likely as not, likely, or certain.

Short on time? Consider allowing students to choose to complete two of the four stations.

2 Monitor

A **Accessibility: Executive Functioning** Invite students to consider one event at a time. (*Problems 3, 6, 8, and 10*)

D **Differentiation**

Look for students who:	Respond to Student Thinking
Reason that an event is impossible because it did not happen in their experiment.	**Support** Consider asking, "Is the outcome possible based on the actual tool? Do you think that event happened in any group's experiment?"
Reason that a letter in TENNESSEE cannot have an equally likely as not chance of occurring. *(Problem 8)*	**Strengthen** Consider asking, "Which letter can you remove from TENNESSEE so that one of the letters has an equally likely as not chance of happening on your next pick?"
Would like to further explore the likelihood of events.	**Stretch** Invite students to use the tools at a station to create their own experiment where the outcome is unlikely but not impossible. Consider challenging students to create an experiment with the most unlikely outcome of anyone in their group.

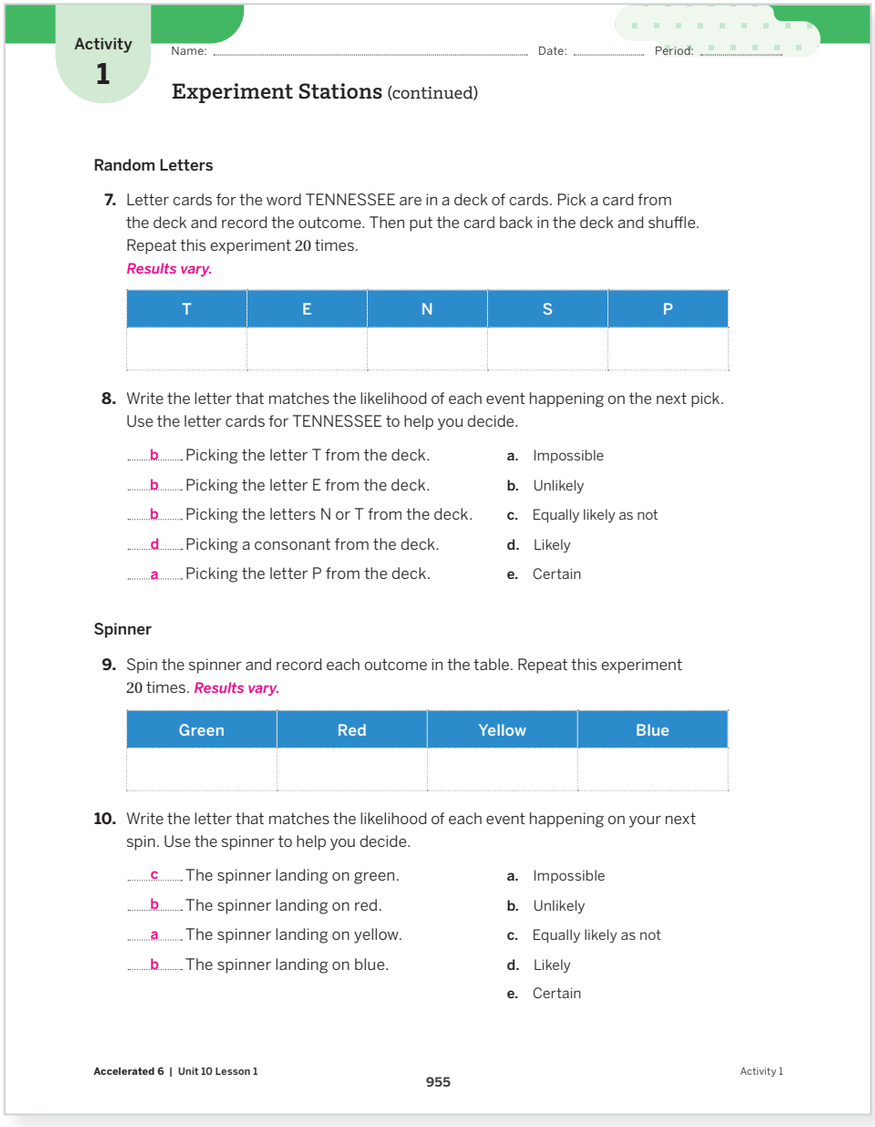
Listen for strategies students use at each station to determine the likelihood of each event to share during the Connect. **(MTR.2.1)**

3 Connect

Display a line from impossible to certain like the one from the Warm-Up on the board.

Invite students to revisit their responses from Problems 3, 6, 8, and 10, and then share where they would place the likelihood of each event on the line, one at a time.

To surface the Key Takeaway, consider asking, "Can you give an example of an event that is unlikely to occur? Where would you place its likelihood on the line?"



Activity 1

Name: Date: Period:

Experiment Stations (continued)

Random Letters

7. Letter cards for the word TENNESSEE are in a deck of cards. Pick a card from the deck and record the outcome. Then put the card back in the deck and shuffle. Repeat this experiment 20 times.

Results vary.

T	E	N	S	P

8. Write the letter that matches the likelihood of each event happening on the next pick. Use the letter cards for TENNESSEE to help you decide.

b Picking the letter T from the deck.
b Picking the letter E from the deck.
b Picking the letters N or T from the deck.
d Picking a consonant from the deck.
a Picking the letter P from the deck.

a. Impossible
b. Unlikely
c. Equally likely as not
d. Likely
e. Certain

Spinner

9. Spin the spinner and record each outcome in the table. Repeat this experiment 20 times. *Results vary.*

Green	Red	Yellow	Blue

10. Write the letter that matches the likelihood of each event happening on your next spin. Use the spinner to help you decide.

c The spinner landing on green.
b The spinner landing on red.
a The spinner landing on yellow.
b The spinner landing on blue.

a. Impossible
b. Unlikely
c. Equally likely as not
d. Likely
e. Certain

Accelerated 6 | Unit 10 Lesson 1 955 Activity 1

Key Takeaway: Events can be described on a scale from impossible (0%) to certain (100%). While some events might have an unlikely chance of occurring, how unlikely they are can differ according to the specific event.

Whole Class | 5 min

Synthesis

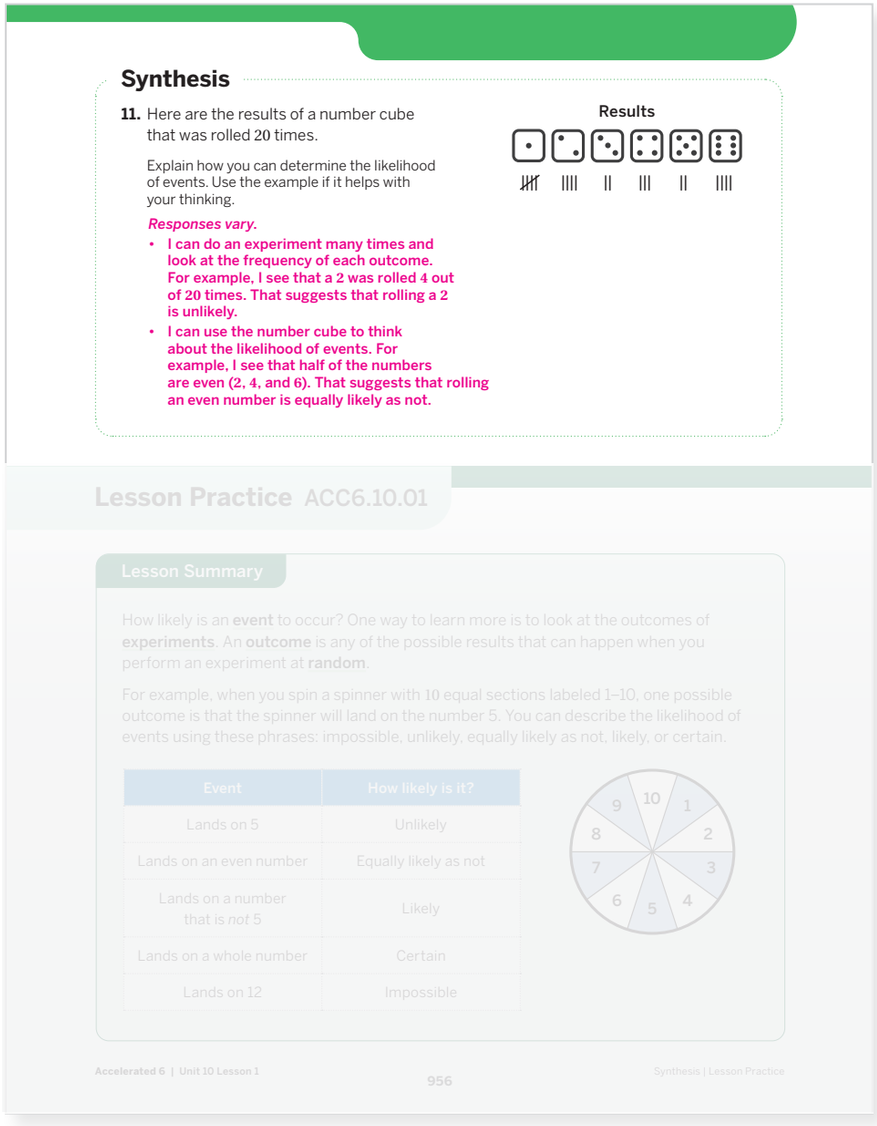
Purpose: Students synthesize their understanding of likelihoods by explaining how **experiments** can help determine the likelihood of events.

Invite students to respond independently, and then share their thinking with a partner.

Collect and share a variety of ideas, such as:
- Conducting an experiment numerous times to gather data.
- Describing the likelihood using a range of impossible to certain.
- Analyzing the object used in an experiment.

MLR **MLR2: Collect and Display** Use this routine to formalize the definitions of *event, experiment, outcome,* and *random* using students' language. Display any new language, examples, or diagrams for students to reference later.

Note: If time allows, invite students to make their response stronger and clearer based on the conversation.



Synthesis

11. Here are the results of a number cube that was rolled 20 times.

Explain how you can determine the likelihood of events. Use the example if it helps with your thinking.

Results

Responses vary.
- I can do an experiment many times and look at the frequency of each outcome. For example, I see that a 2 was rolled 4 out of 20 times. That suggests that rolling a 2 is unlikely.
- I can use the number cube to think about the likelihood of events. For example, I see that half of the numbers are even (2, 4, and 6). That suggests that rolling an even number is equally likely as not.

Lesson Practice ACC6.10.01

Lesson Summary

How likely is an **event** to occur? One way to learn more is to look at the outcomes of **experiments**. An **outcome** is any of the possible results that can happen when you perform an experiment at **random**.

For example, when you spin a spinner with 10 equal sections labeled 1–10, one possible outcome is that the spinner will land on the number 5. You can describe the likelihood of events using these phrases: impossible, unlikely, equally likely as not, likely, or certain.

Event	How likely is it?
Lands on 5	Unlikely
Lands on an even number	Equally likely as not
Lands on a number that is *not* 5	Likely
Lands on a whole number	Certain
Lands on 12	Impossible

Accelerated 6 | Unit 10 Lesson 1 956 Synthesis | Lesson Practice

Lesson Takeaway: The likelihood of **events** can be described using the words impossible, unlikely, equally likely as not, likely, or certain. The results of an **experiment** can be used to help determine the likelihood of events. Analyzing the tools used to conduct an experiment can also be helpful to more precisely describe the likelihood.

Independent | 5 min

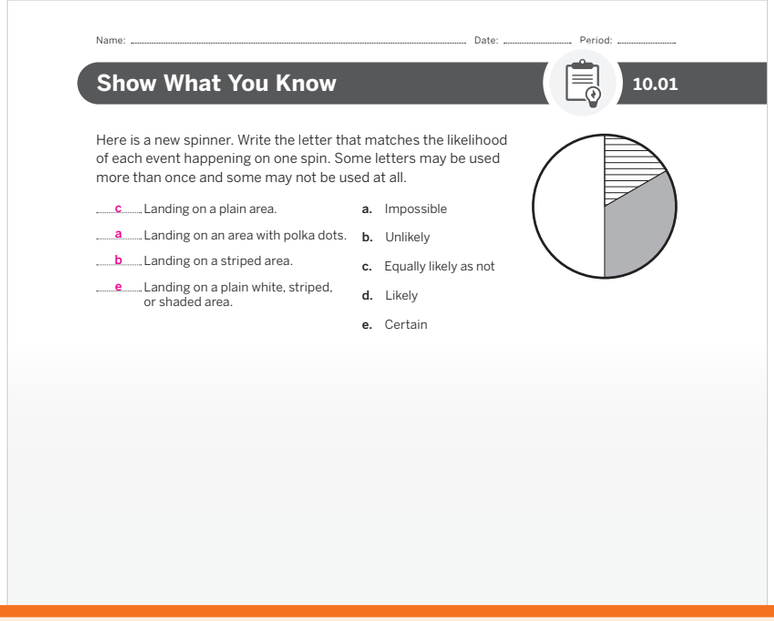
Show What You Know

Purpose: Students demonstrate their understanding by determining the likelihood of spinning a named section on a spinner.

Today's Goals

1. **Language Goal:** Describe the likelihood of *events* using the words *impossible, unlikely, equally likely as not, likely,* or *certain*. **(Reading and Writing)**
2. **Language Goal:** Explain how *experiments* can help determine the likelihood of future *events*. **(Writing)**

Show What You Know (Assessment Resources)



Name: _____ Date: _____ Period: _____

Show What You Know 10.01

Here is a new spinner. Write the letter that matches the likelihood of each event happening on one spin. Some letters may be used more than once and some may not be used at all.

_c_ Landing on a plain area.
_a_ Landing on an area with polka dots.
_b_ Landing on a striped area.
_e_ Landing on a plain white, striped, or shaded area.

a. Impossible
b. Unlikely
c. Equally likely as not
d. Likely
e. Certain

Differentiation Use after Lesson 1

Support

Provide targeted intervention for students by using these resources.

If students need support understanding how to determine the likelihood of an event:

» **Respond to Student Thinking:**

- Students will have more opportunities to develop fluency with these concepts in Lesson 2. You may choose not to intervene now.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 1 Extension Activities**.

Professional Learning

In this lesson, students described the likelihood of **events** occurring. How did students' work in the stations surprise you today?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

11. Here are the results of a number cube that was rolled 20 times.

Results

Explain how you can determine the likelihood of events. Use the example if it helps with your thinking.

Responses vary.

- I can do an experiment many times and look at the frequency of each outcome. For example, I see that a 2 was rolled 4 out of 20 times. That suggests that rolling a 2 is unlikely.
- I can use the number cube to think about the likelihood of events. For example, I see that half of the numbers are even (2, 4, and 6). That suggests that rolling an even number is equally likely as not.

Lesson Practice ACC6.10.01

Lesson Summary

How likely is an **event** to occur? One way to learn more is to look at the outcomes of **experiments**. An **outcome** is any of the possible results that can happen when you perform an experiment at **random**.

For example, when you spin a spinner with 10 equal sections labeled 1–10, one possible outcome is that the spinner will land on the number 5. You can describe the likelihood of events using these phrases: impossible, unlikely, equally likely as not, likely, or certain.

Event	How likely is it?
Lands on 5	Unlikely
Lands on an even number	Equally likely as not
Lands on a number that is *not* 5	Likely
Lands on a whole number	Certain
Lands on 12	Impossible

Lesson Practice ACC6.10.01

Name: _____ Date: _____ Period: _____

Problems 1–5: Write the letter that matches the likelihood of each event.

Use the image if it helps with your thinking.

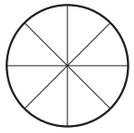
1. a Rolling a 7.
2. d Rolling a number less than 5.
3. b Rolling a 3.
4. c Rolling an even number.
5. e Rolling a number greater than 0.

a. Impossible
b. Unlikely
c. Equally likely as not
d. Likely
e. Certain

6. Label the sections on the spinner so that it has all of these likelihoods in one spin.

- Equally likely as not to spin green (G).
- Impossible to spin blue (B).
- Very unlikely to spin yellow (Y).
- Unlikely to spin red (R).

The spinner should contain four green sections, three red sections, one yellow section, and zero blue sections.



7. Sahana is curious how many of her classmates watch her favorite T.V. show, so she starts asking around at lunch. She gets the following responses:

yes yes yes no no no no no no yes no no no yes

If she asks one more person randomly in the cafeteria, do you think they will say "yes" or "no"? Explain your thinking.

***Responses vary.* I think they will say "no" because most people so far have said "no."**

Lesson Practice ACC6.10.01

Name: _____ Date: _____ Period: _____

FAST Practice

8. A bag contains 14 marbles. There are 6 blue, 7 yellow, and 1 red marble. One marble is selected at random. Determine whether each statement is true or false.

Statement	True	False
It's impossible that a green marble will be selected.	☑	☐
It's unlikely that a yellow marble will be selected.	☐	☑
It's certain that a blue marble will be selected.	☐	☑
It's unlikely that a red marble will be selected.	☑	☐

Spiral Review

Problems 9–12: Evaluate each expression.

9. $\left(\frac{1}{2}\right)^2 \cdot 8$
2

10. $3^2 + 2^3$
17

11. $(2 \cdot 3)^2$
36

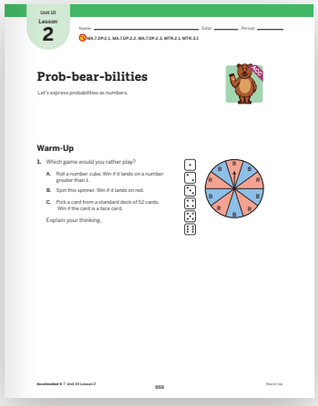
12. $\left(\frac{1}{3}\right)^2 \cdot 3^2$
1

Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1–5	1	MA.7.DP.2.2
	6	3	MA.7.DP.2.2
	7	2	MA.7.DP.2.2
FAST Practice	8	1	MA.7.DP.2.2
Spiral Review			
	9–12	1	MA.7.NSO.2.1

Unit 10
Lesson
2

ACC Accelerated


Unit 10
Lesson
2
Name: _____ Date: _____ Period: _____
MA.7.DP.2.1, MA.7.DP.2.2, MA.7.DP.2.3, MTR.2.1, MTR.3.1
Prob-bear-bilities
Let's express probabilities as numbers.
Warm-Up
1. Which game would you rather play?
A. Roll a number cube. Win if it lands on a number greater than 1.
B. Spin this spinner. Win if it lands on red.
C. Pick a card from a standard deck of 52 cards. Win if the card is a face card.
Explain your thinking.
Accelerated 6 | Unit 10 Lesson 2
959
Warm-Up

Prob-bear-bilities

Intro to Probability

Let's express probabilities as numbers.

Learning Outcomes

Today's Goals

1. **Goal:** Understand *probability* as a number between 0 and 1 that represents the likelihood of an event occurring.
2. **Goal:** Compare the *probability* of events using fractions, decimals, and percentages.
3. **Language Goal:** Explain how to use a *sample space* to determine the *probability* of an event. **(Reading and Writing)**

Students are introduced to **probability** as a measure of the likelihood of an event. They make sense of probabilities using decimals, fractions, and *percentages*, and use **sample spaces** to help determine these probabilities.

Coherence

Prior Learning

In Lesson 1, students described the likelihood of events in words.

Future Learning

In Lesson 3, students will use repeated experiments and ratios to make predictions and recognize that the number of repeated experiments affects the accuracy of their predictions.

Connections

- Students explore the likelihood of events happening, and understanding sample spaces to determine those probabilities such as spinning a spinner with five equal sections. (real world)

FL B.E.S.T. Math Benchmarks

Addressing

MA.7.DP.2.1
Determine the sample space for a simple experiment.

MA.7.DP.2.2
Given the probability of a chance event, interpret the likelihood of it occurring. Compare the probabilities of chance events.

MA.7.DP.2.3
Find the theoretical probability of an event related to a simple experiment.

Mathematical Thinking and Reasoning Standards: MTR.2.1, MTR.3.1

ELA Expectation: ELA.K12.EE.5.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.6.NSO.3.5	MA.8.DP.2.1 MA.8.DP.2.2 MA.8.DP.2.3

Vocabulary

New Vocabulary

probability, sample space

Review Vocabulary

percentage

Lesson at a Glance

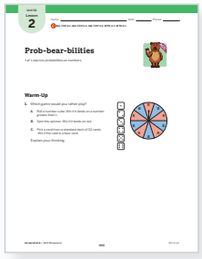
~ 45 min

MA.7.DP.2.1, MA.7.DP.2.2, MA.7.DP.2.3

Warm-Up

Independent | 5 min

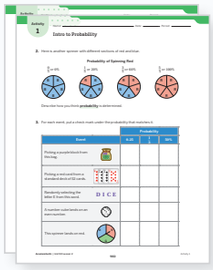
Students surface ideas about likelihood to decide which game they have a better chance of winning.



Activity 1

Pairs | 10 min

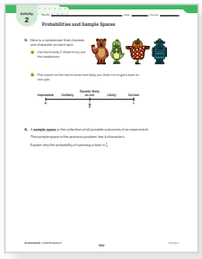
Students calculate the **probability** of an event and compare the probability of events expressed as fractions, decimals, and *percentages*. **(MTR.2.1)**



Activity 2

Pairs | 5 min

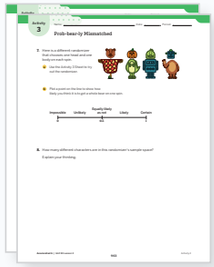
Students are introduced to the term **sample space** and use a sample space to determine the probability of an event.



Activity 3

Pairs | 15 min

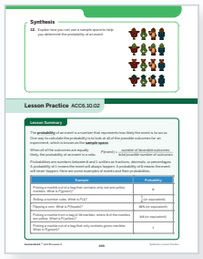
Students use a sample space to determine the probability of an event.



Synthesis

Whole Class | 5 min

Students synthesize their understanding of using a sample space to determine the probability of an event. MLR2: Collect and Display



Show What You Know

Independent | 5 min

Students demonstrate their understanding by determining the probability of choosing a particular item when choosing at random from a given menu.

Prep Checklist

This lesson includes:

- Student Edition
- Show What You Know

- Coloring tools (optional)
- Paper clips for spinners, coins, number cubes (as needed)

Independent | 5 min

Warm-Up

Purpose: Students surface ideas about likelihood to decide which game they have a better chance of winning.

1 Launch

To surface connections, consider asking if students have ever played a game that involved number cubes, spinners, or cards.

Consider sharing that a standard deck of 52 playing cards has the following characteristics:

- There are 4 suits: hearts, diamonds, clubs, and spades
- Hearts and diamonds are red, clubs and spades are black.
- There are 13 cards of each suit: ace, 2, 3, 4, 5, 6, 7, 8, 9, 10, jack, queen, king.
- The jack, queen, and king are called face cards.

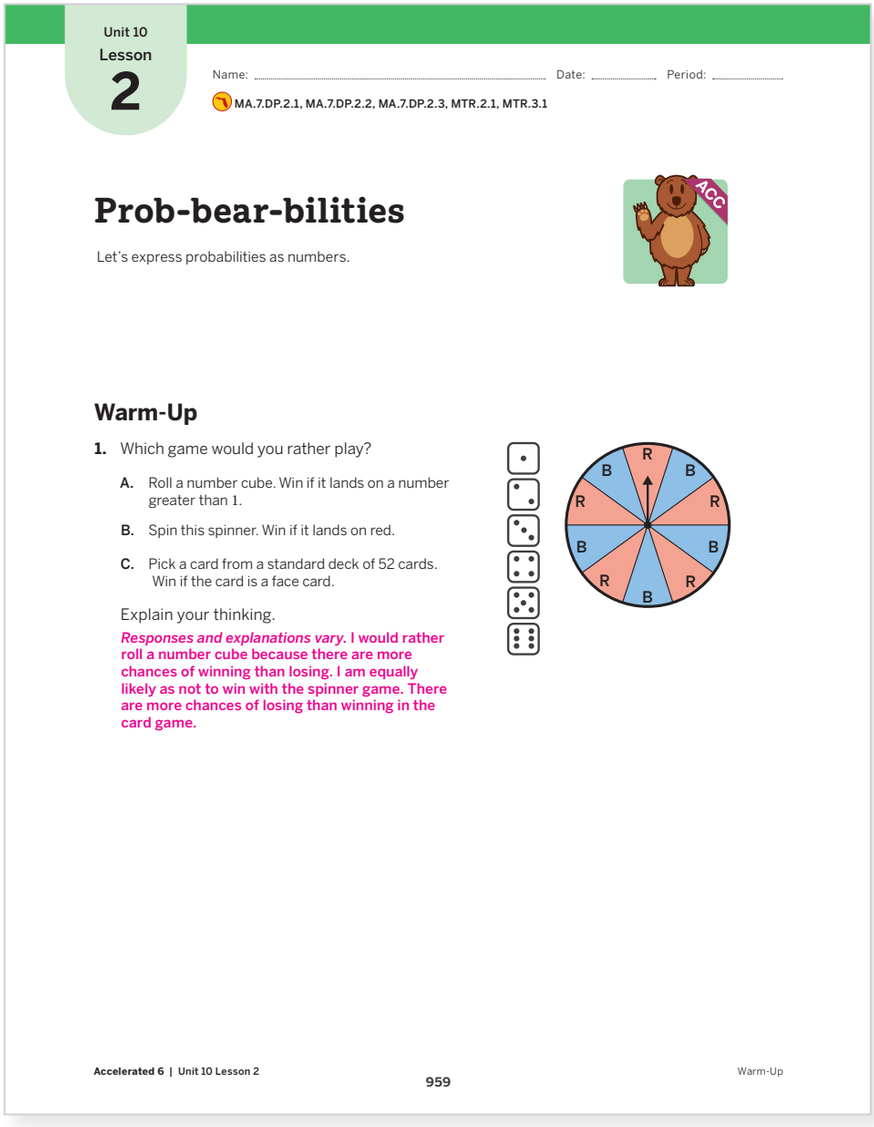
To generate engagement, invite students to imagine themselves at a fair where they get to choose which game in the Warm-Up to play to win something they've wanted.

2 Connect

Invite students to share their reasoning for which game they chose.

ELL **English Language Learners** Provide sentence frames to help students explain their reasoning (e.g., I would rather play _____ because _____.).

Consider asking, "What is the likelihood you would win the number cube game? The spinner game? The card game?"



Unit 10
Lesson
2

Name: _____ Date: _____ Period: _____

MA.7.DP.2.1, MA.7.DP.2.2, MA.7.DP.2.3, MTR.2.1, MTR.3.1

Prob-bear-bilities

Let's express probabilities as numbers.

Warm-Up

1. Which game would you rather play?
   - A. Roll a number cube. Win if it lands on a number greater than 1.
   - B. Spin this spinner. Win if it lands on red.
   - C. Pick a card from a standard deck of 52 cards. Win if the card is a face card.

Explain your thinking.

Responses and explanations vary. I would rather roll a number cube because there are more chances of winning than losing. I am equally likely as not to win with the spinner game. There are more chances of losing than winning in the card game.

Accelerated 6 | Unit 10 Lesson 2 959 Warm-Up

Pairs | 10 min

Activity 1 Intro to Probability

Purpose: Students calculate the **probability** of an event and compare the probability of events expressed as fractions, decimals, and *percentages*. **(MTR.2.1)**

Short on time? Consider having students complete only 1–2 challenges in Problem 4.

1 Launch

Invite students to study each spinner and ask what they notice about how the fraction and *percentage* were determined. **(MTR.2.1)**

ELL **English Language Learners** Provide sentence frames to help students explain their reasoning (e.g., I think probability is determined by _____.).

Create a class definition for the term **probability** using students' language and glossary. Consider sharing that probability is another way to describe the chance of an event happening.

2 Monitor

To support students getting started, invite them to write a list of the possible outcomes for each experiment.

Look for students who convert the probabilities to one form (decimal, fraction, or percentage).

A **Accessibility: Conceptual Processing** To support abstract thinking, provide probability tools such as coins or number cubes.

Activity 1 continued >

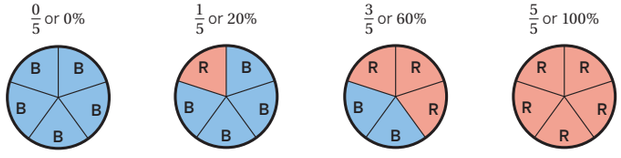
Activity 1

Name: _____ Date: _____ Period: _____

Intro to Probability

2. Here is another spinner with different sections of red and blue.

Probability of Spinning Red



Describe how you think **probability** is determined.

Responses vary. Probability is the number of red sections divided by the total number of sections, which is always 5 for this spinner.

3. For each event, put a check mark under the probability that matches it.

Event	Probability: 0.25	Probability: 1/3	Probability: 50%
Picking a purple block from this bag.	✓		
Picking a red card from a standard deck of 52 cards.			✓
Randomly selecting the letter E from this word. DICE	✓		
A number cube lands on an even number.			✓
This spinner lands on red.		✓	

Pairs | 10 min

Activity 1 Intro to Probability (continued)

Purpose: Students calculate the **probability** of an *event* and compare the probability of events expressed as fractions, decimals, and *percentages*. **(MTR.2.1)**

Short on time? Consider having students complete only 1–2 challenges in Problem 4.

2 Monitor

Select and sequence student strategies to share during the Connect.

D Differentiation

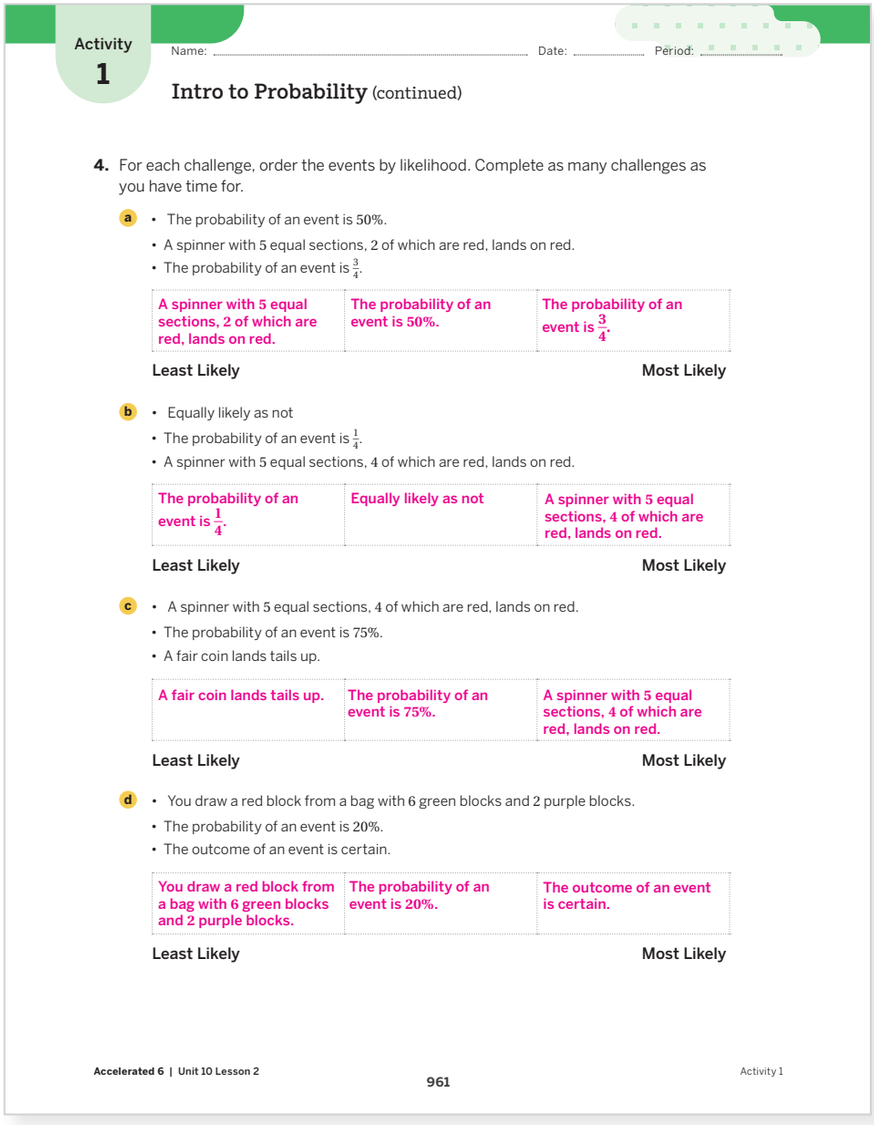
Look for students who:	Respond to Student Thinking
Use estimation or use words such as *unlikely*, *equally likely as not*, or *likely* to describe events.	**Strengthen** Consider asking, "How did your strategy help you compare the probabilities?"
Convert between forms (fractions, decimals, percentages). **(MTR.2.1)**	
Would like to explore ordering events by likelihood further.	**Stretch** Invite students to create their own challenge similar to the ones in Problem 4, and trade with a partner to complete each other's challenge.

3 Connect

Invite students to share their responses and the strategies they used to justify their placements and make revisions based on the discussion. **(MTR.3.1)**

Consider asking:

- "When is it helpful to convert between fractions, decimals, and percentages?"
- "Do you prefer using probabilities expressed as fractions, decimals, or percents to compare the likelihood of events? Why?"
- "How can you compare the likelihoods of events?"



Activity 1

Name: _____ Date: _____ Period: _____

Intro to Probability (continued)

4. For each challenge, order the events by likelihood. Complete as many challenges as you have time for.

a
- The probability of an event is 50%.
- A spinner with 5 equal sections, 2 of which are red, lands on red.
- The probability of an event is $\frac{3}{4}$.

A spinner with 5 equal sections, 2 of which are red, lands on red.	The probability of an event is 50%.	The probability of an event is $\frac{3}{4}$.

Least Likely — Most Likely

b
- Equally likely as not
- The probability of an event is $\frac{1}{4}$.
- A spinner with 5 equal sections, 4 of which are red, lands on red.

The probability of an event is $\frac{1}{4}$.	Equally likely as not	A spinner with 5 equal sections, 4 of which are red, lands on red.

Least Likely — Most Likely

c
- A spinner with 5 equal sections, 4 of which are red, lands on red.
- The probability of an event is 75%.
- A fair coin lands tails up.

A fair coin lands tails up.	The probability of an event is 75%.	A spinner with 5 equal sections, 4 of which are red, lands on red.

Least Likely — Most Likely

d
- You draw a red block from a bag with 6 green blocks and 2 purple blocks.
- The probability of an event is 20%.
- The outcome of an event is certain.

You draw a red block from a bag with 6 green blocks and 2 purple blocks.	The probability of an event is 20%.	The outcome of an event is certain.

Least Likely — Most Likely

Accelerated 6 | Unit 10 Lesson 2 961 Activity 1

Pairs | 5 min

Activity 2 Probabilities and Sample Spaces

Purpose: Students are introduced to the term **sample space** and use a sample space to determine the probability of an event.

1 Launch

Invite students to make a prediction about how likely it is to get a bear on one spin before trying the randomizer.

Distribute the Activity 2 sheet and a paper clip to each pair of students.

2 Monitor

Listen for detailed or creative explanations.

D **Differentiation: (Support)** Students may think the probability will change if there is more than one spin. Consider asking, "If you flip a coin with heads and tails more than once, does the probability change?"

ELL **English Language Learners** Provide sentence frames to help students explain their reasoning (e.g., The probability of spinning a bear is $\frac{1}{4}$ or 25% because _____.).

3 Connect

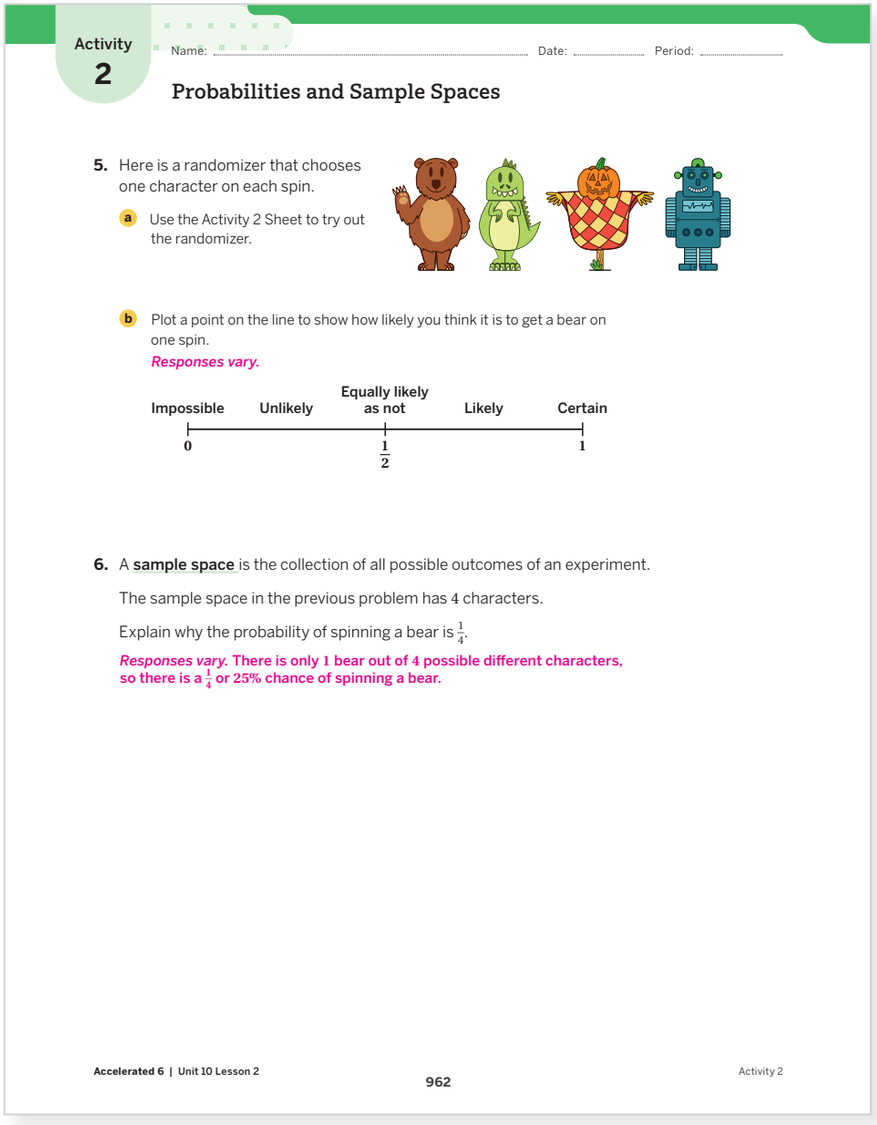
Share student explanations.

MLR **MLR1: Stronger and Clearer Each Time (optional)** Consider using this routine to support students in developing precise language. Invite students to meet with a partner, give feedback on each other's explanations, then use the feedback to write a second clearer draft. See the Routine Facilitation Guide for more information.

Consider asking:

- "What is the probability of spinning a character that is not a bear?"
- "What is the probability of spinning a character that has two legs? Two arms?"
- "How would you express these probabilities as a decimal? As a percentage?"

To surface the Key Takeaway, consider asking, "How does the sample space relate to the probability?"



Activity 2

Name: _____ Date: _____ Period: _____

Probabilities and Sample Spaces

5. Here is a randomizer that chooses one character on each spin.

a. Use the Activity 2 Sheet to try out the randomizer.

b. Plot a point on the line to show how likely you think it is to get a bear on one spin.

Responses vary.

Impossible — Unlikely — Equally likely as not — Likely — Certain

0 — $\frac{1}{2}$ — 1

6. A **sample space** is the collection of all possible outcomes of an experiment.

The sample space in the previous problem has 4 characters.

Explain why the probability of spinning a bear is $\frac{1}{4}$.

Responses vary. There is only 1 bear out of 4 possible different characters, so there is a $\frac{1}{4}$ or 25% chance of spinning a bear.

Accelerated 6 | Unit 10 Lesson 2 962 Activity 2

Key Takeaway: The **sample space** of an experiment is the collection of all possible outcomes. The **probability** can be determined using the sample space and can be represented by a number between 0 and 1.

Pairs | 15 min

Activity 3 Prob-bear-ly Mismatched

Purpose: Students use the sample space to determine the probability of an event.

1 Launch

To surface connections, consider discussing how this problem and the one from Problem 5 are alike.

Distribute the Activity 3 sheet, a paper clip, and optional coloring tools to each pair of students.

Consider asking, "Will there be more or less than four characters in the sample space? Why?"

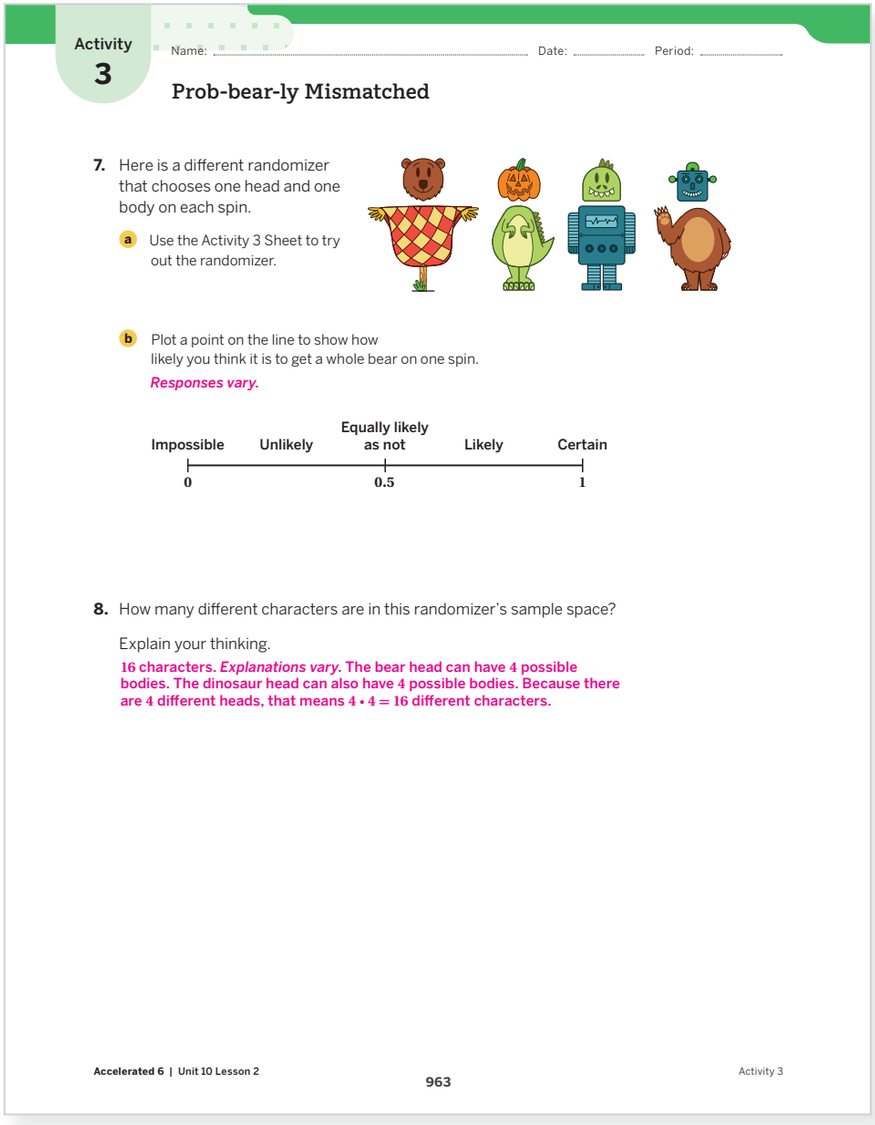
2 Monitor

Look for students who use drawings, organized lists, or other tools to help them determine the size of the sample space. **(MTR.2.1)**

ELL **English Language Learners** Provide sentence frames to help students explain their strategy (e.g., I counted the different creatures by _____.).

Pause briefly to allow students to share their strategies and how they are alike or different from other students' strategies.

Activity 3 continued >



Activity 3

Name: _____ Date: _____ Period: _____

Prob-bear-ly Mismatched

7. Here is a different randomizer that chooses one head and one body on each spin.

a. Use the Activity 3 Sheet to try out the randomizer.

b. Plot a point on the line to show how likely you think it is to get a whole bear on one spin.

Responses vary.

Impossible — Unlikely — Equally likely as not — Likely — Certain

0 — 0.5 — 1

8. How many different characters are in this randomizer's sample space?

Explain your thinking.

16 characters. *Explanations vary.* The bear head can have 4 possible bodies. The dinosaur head can also have 4 possible bodies. Because there are 4 different heads, that means $4 \cdot 4 = 16$ different characters.

Accelerated 6 | Unit 10 Lesson 2 963 Activity 3

Pairs | 15 min

Activity 3 Prob-bear-ly Mismatched (continued)

Purpose: Students use the sample space to determine the probability of an event.

2 Monitor

Listen for students making use of the structure of the sample space to organize their thinking about the missing character.

Select and sequence students' ideas to share during the Connect.

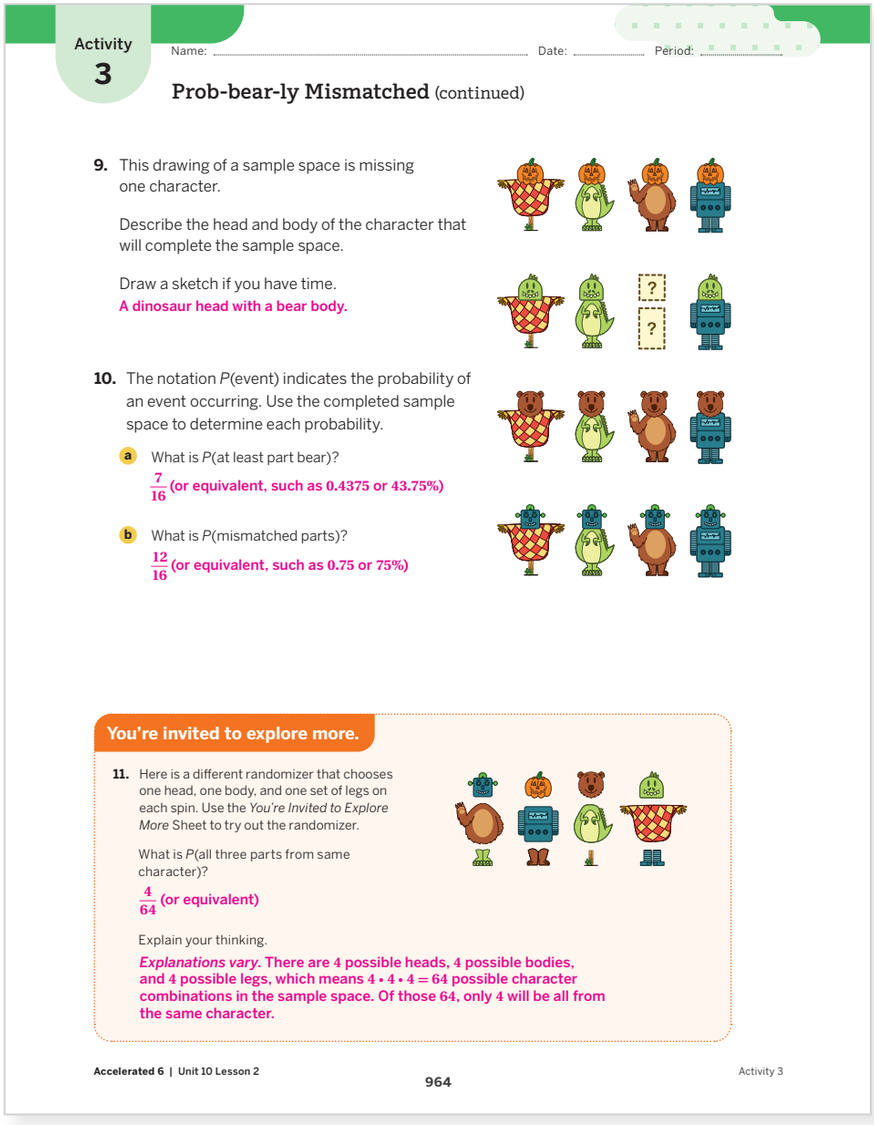
D Differentiation

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Consider asking, "What is the probability of getting a whole bear on one spin?"
Need support reading probability notation.	**Support** Encourage students to write out the event in words. For example, *P*(at least part bear) means "the probability of spinning a character that is at least part bear."
Create a sketch to show their thinking.	**Strengthen** Invite them to share how this strategy helped to determine the probability of each event occurring.
Write their probabilities in different forms (decimals, fractions, percentages). **(MTR.2.1)**	**Strengthen** Invite students to share when it was more efficient to express the probability as a fraction, decimal, or percentage and why.

D Differentiation: You're invited to explore more. (Stretch) Invite students who would like to explore probability using sample space further to complete this optional task. Encourage them to discuss their thinking with a partner.
Note: Students will need copies of the You're Invited to Explore More Sheet.

3 Connect

Invite students to share their sketches from Problem 9, and discuss strategies they used to determine the probability. Invite students to share their responses and reasoning.



Activity 3

Name: Date: Period:

Prob-bear-ly Mismatched (continued)

9. This drawing of a sample space is missing one character.

Describe the head and body of the character that will complete the sample space.

Draw a sketch if you have time.

A dinosaur head with a bear body.

10. The notation *P*(event) indicates the probability of an event occurring. Use the completed sample space to determine each probability.

a. What is *P*(at least part bear)?

$\frac{7}{16}$ **(or equivalent, such as 0.4375 or 43.75%)**

b. What is *P*(mismatched parts)?

$\frac{12}{16}$ **(or equivalent, such as 0.75 or 75%)**

You're invited to explore more.

11. Here is a different randomizer that chooses one head, one body, and one set of legs on each spin. Use the *You're Invited to Explore More* Sheet to try out the randomizer.

What is *P*(all three parts from same character)?

$\frac{4}{64}$ **(or equivalent)**

Explain your thinking.

Explanations vary. **There are 4 possible heads, 4 possible bodies, and 4 possible legs, which means $4 \cdot 4 \cdot 4 = 64$ possible character combinations in the sample space. Of those 64, only 4 will be all from the same character.**

Accelerated 6 | Unit 10 Lesson 2 964 Activity 3

Whole Class | 5 min

Synthesis

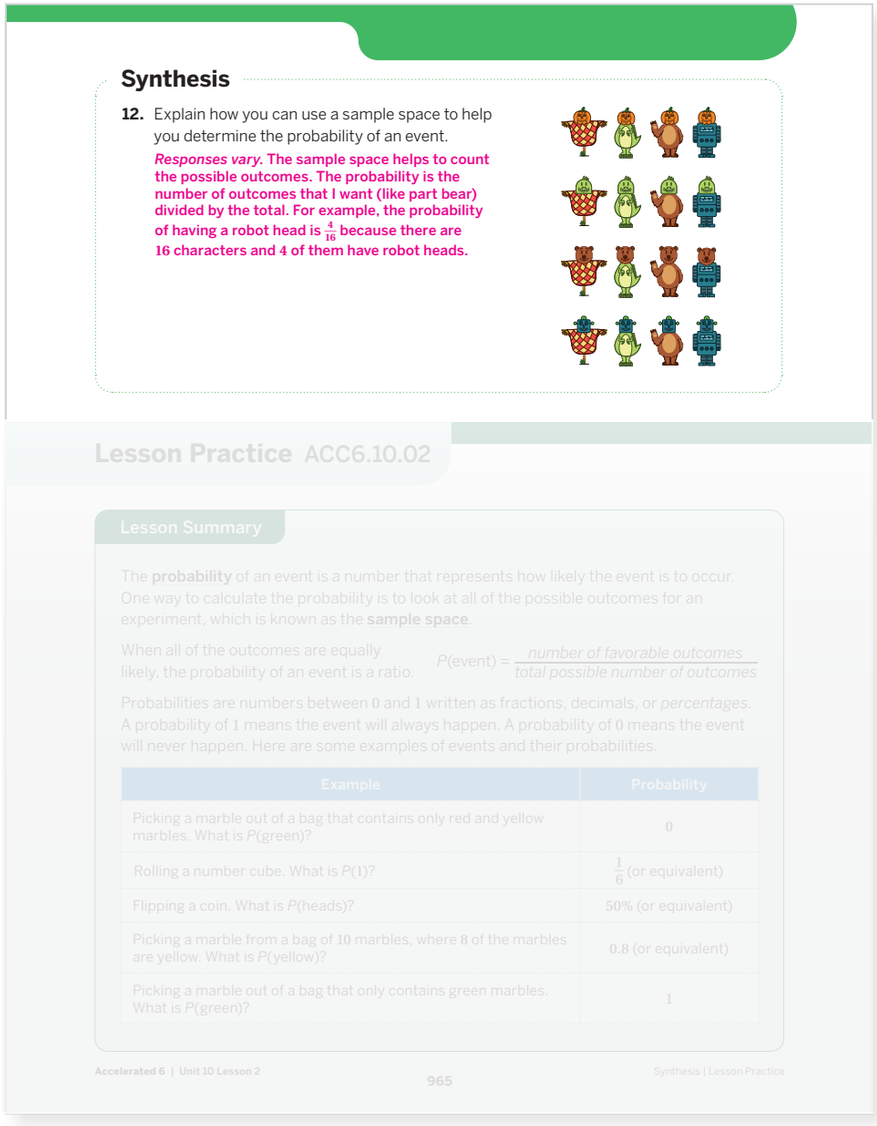
Purpose: Students synthesize their understanding of using a sample space to determine the probability of an event.

Invite students to respond independently, and then share their thinking with a partner.

Collect and share a variety of ideas, such as:
- Counting the total possible outcomes.
- Using a ratio of outcomes desired over the total possible outcomes.
- Using benchmark percentages.

MLR **MLR2: Collect and Display** Use this routine to formalize the definitions of *probability* and *sample space* using students' language. Display any new language, examples, or diagrams for students to reference later.

Note: If time allows, invite students to make their response stronger and clearer based on the conversation.



Synthesis

12. Explain how you can use a sample space to help you determine the probability of an event.

Responses vary. The sample space helps to count the possible outcomes. The probability is the number of outcomes that I want (like part bear) divided by the total. For example, the probability of having a robot head is $\frac{4}{16}$ because there are 16 characters and 4 of them have robot heads.

Lesson Practice ACC6.10.02

Lesson Summary

The **probability** of an event is a number that represents how likely the event is to occur. One way to calculate the probability is to look at all of the possible outcomes for an experiment, which is known as the **sample space**.

When all of the outcomes are equally likely, the probability of an event is a ratio. $P(\text{event}) = \frac{\text{number of favorable outcomes}}{\text{total possible number of outcomes}}$

Probabilities are numbers between 0 and 1 written as fractions, decimals, or *percentages*. A probability of 1 means the event will always happen. A probability of 0 means the event will never happen. Here are some examples of events and their probabilities.

Example	Probability
Picking a marble out of a bag that contains only red and yellow marbles. What is $P(\text{green})$?	0
Rolling a number cube. What is $P(1)$?	$\frac{1}{6}$ (or equivalent)
Flipping a coin. What is $P(\text{heads})$?	50% (or equivalent)
Picking a marble from a bag of 10 marbles, where 8 of the marbles are yellow. What is $P(\text{yellow})$?	0.8 (or equivalent)
Picking a marble out of a bag that only contains green marbles. What is $P(\text{green})$?	1

Accelerated 6 | Unit 10 Lesson 2 965 Synthesis | Lesson Practice

Lesson Takeaway: The **sample space** can be used to determine the **probability** of an event. The probability is the number of options desired divided by the total outcomes in the sample space and can be written as a fraction, decimal, or *percentage*.

Independent | 5 min

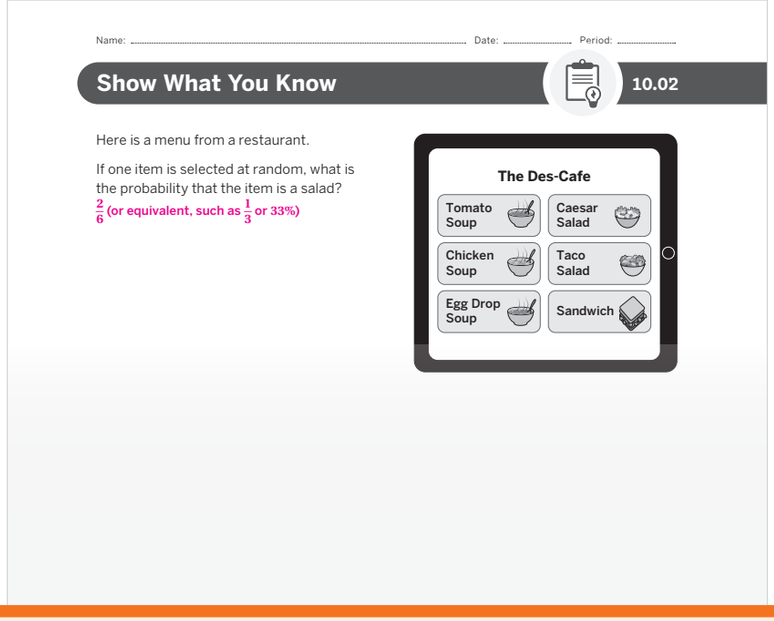
Show What You Know

Purpose: Students demonstrate their understanding by determining the probability of choosing a particular item when choosing at random from a given menu.

Today's Goals

1. **Goal:** Understand *probability* as a number between 0 and 1 that represents the likelihood of an event occurring.
2. **Goal:** Compare the *probability* of events using fractions, decimals, and percentages.
3. **Language Goal:** Explain how to use a *sample space* to determine the *probability* of an event. **(Reading and Writing)**

Show What You Know (Assessment Resources)



Name: _____ Date: _____ Period: _____

Show What You Know 10.02

Here is a menu from a restaurant.

If one item is selected at random, what is the probability that the item is a salad?

$\frac{2}{6}$ (or equivalent, such as $\frac{1}{3}$ or 33%)

Differentiation Use after Lesson 2

Support

Provide targeted intervention for students by using these resources.

If students need support understanding how to determine probabilities using sample space:

» **Respond to Student Thinking:**

- Revisit the definition of sample space in **Activity 2**. Invite students to share strategies for determining the sample space of an event and probability of an outcome.
- Select a few related **Practice Problems** and give students individualized feedback.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 1 Extension Activities**.
- Invite students to complete the **You're invited to explore more** task.

Professional Learning

Take a moment to reflect on the strategies students used in this lesson.

- What opportunities were there to celebrate specific students' strategies for using **sample space** to determine the **probability**?
- What opportunities might there be in future lessons to connect back to the strategies students shared?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

12. Explain how you can use a sample space to help you determine the probability of an event.

Responses vary. The sample space helps to count the possible outcomes. The probability is the number of outcomes that I want (like part bear) divided by the total. For example, the probability of having a robot head is $\frac{4}{16}$ because there are 16 characters and 4 of them have robot heads.

Lesson Practice ACC6.10.02

Lesson Summary

The **probability** of an event is a number that represents how likely the event is to occur. One way to calculate the probability is to look at all of the possible outcomes for an experiment, which is known as the **sample space**.

When all of the outcomes are equally likely, the probability of an event is a ratio. $P(\text{event}) = \frac{\textit{number of favorable outcomes}}{\textit{total possible number of outcomes}}$

Probabilities are numbers between 0 and 1 written as fractions, decimals, or *percentages*. A probability of 1 means the event will always happen. A probability of 0 means the event will never happen. Here are some examples of events and their probabilities.

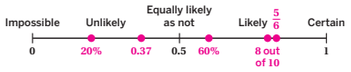
Example	Probability
Picking a marble out of a bag that contains only red and yellow marbles. What is *P*(green)?	0
Rolling a number cube. What is *P*(1)?	$\frac{1}{6}$ (or equivalent)
Flipping a coin. What is *P*(heads)?	50% (or equivalent)
Picking a marble from a bag of 10 marbles, where 8 of the marbles are yellow. What is *P*(yellow)?	0.8 (or equivalent)
Picking a marble out of a bag that only contains green marbles. What is *P*(green)?	1

Lesson Practice
ACC6.10.02

Name: _____ Date: _____ Period: _____

1. Place a point on the line to show each probability's likelihood from impossible to certain.

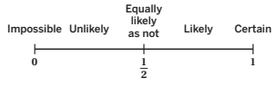
60% 8 out of 10 0.37 20% $\frac{5}{6}$



Problems 2–4: List the sample space for each experiment. For example, the sample space of flipping one fair coin is heads or tails.

2. Selecting a random season of the year.
   winter, spring, summer, fall
3. Selecting a random day of the week.
   Sunday, Monday, Tuesday, Wednesday, Thursday, Friday, Saturday
4. Rolling a number cube.
   one, two, three, four, five, six

Problems 5–10: Determine the probability of each event. Use the line if it helps with your thinking.



5. You read this sentence.
   1
6. A fair coin lands heads up when flipped.
   $\frac{1}{2}$ (or equivalent)
7. A weekend day is selected randomly from the days of the week.
   $\frac{2}{7}$ (or equivalent)
8. You see a live unicorn outside today.
   0
9. A spinner with equal parts red, yellow, and green lands on green on the next spin.
   $\frac{1}{3}$ (or equivalent)
10. You pick a green block when you randomly pick one block from a bag with 7 green blocks and 3 blue blocks.
    $\frac{7}{10}$ (or equivalent)

Lesson Practice
ACC6.10.02

Name: _____ Date: _____ Period: _____

Problems 11–13: A computer randomly selects a letter from the English alphabet.

11. How many different outcomes are possible?
    26 outcomes
12. What is *P*(A, E, I, O, or U)?
    $\frac{5}{26}$
13. What is *P*(first letter of your first name)?
    $\frac{1}{26}$

FAST Practice

14. This table shows the number of bags of different flavors of crackers in a large box. A bag will be selected at random from the box.

Flavor of Cracker	Number of Bags
Plain	12
Barbeque	16
Ranch	8
Cheddar Cheese	20

Which statement about the flavor of the crackers chosen is best supported by the information in the table?

A. Plain is the least likely flavor.
B. Choosing barbecue is twice as likely as choosing ranch.
C. There is an equal chance of choosing plain, barbeque, ranch, or cheddar cheese.
D. Choosing cheddar cheese is twice as likely as choosing ranch.

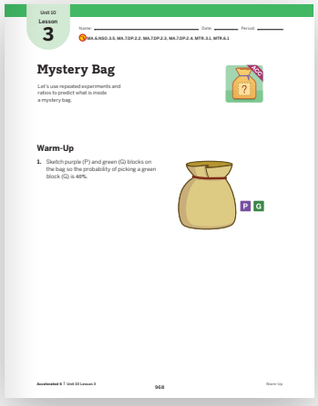
Spiral Review

Problems 15–16: Letter cards for the word COOKBOOK are in a deck of cards.

15. What is the likelihood of picking the letter B from the deck? Explain your thinking.
    Unlikely. *Explanations vary.* The letter B occurs only once in the deck of 8 cards.
16. What is the likelihood of picking the letter O from the deck? Explain your thinking.
    Equally likely as not. *Explanations vary.* There are 4 cards with the letter O out of a total of 8 cards. 4 out of 8 is half of the cards.

Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1	1	MA.7.DP.2.2
	2–4, 11	2	MA.7.DP.2.1
	5–10, 12–13	2	MA.7.DP.2.2
FAST Practice	14	2	MA.7.DP.2.2
Spiral Review			
	15–16	2	MA.7.DP.2.1



Mystery Bag

Predicting Sample Spaces

Let's use repeated experiments and ratios to predict what is inside a mystery bag.

Learning Outcomes

Today's Goals

1. **Goal:** Use ratios and data from a repeated experiment to make predictions about the sample space.
2. **Language Goal:** Describe how to use the results from a repeated experiment to make predictions. **(Reading, Writing, Speaking, and Listening)**

Students use repeated experiments and ratios to predict the contents of a mystery bag, recognizing that the number of repeated experiments affects the accuracy of their prediction. Students encounter situations in which each outcome is not equally likely. This lesson also surfaces important ideas about the difference between probabilities and the results of repeated experiments.

Coherence

Prior Learning

In Lessons 1 and 2, students described the likelihood of an event occurring and used the sample space to determine the probability of an event.

Future Learning

In Lesson 4, students will recognize that the results from repeated experiments approach the probability of the event.

Connections

- Using graphs students predict the contents of a mystery bag using repeated experiments and ratios. (mathematical, real world)

FL B.E.S.T. Math Benchmarks

Addressing

MA.7.DP.2.3
Find the theoretical probability of an event related to a simple experiment.

MA.7.DP.2.4
Use a simulation of a simple experiment to find experimental probabilities and compare them to theoretical probabilities.

Also Addressing: **MA.6.NSO.3.5, MA.7.DP.2.2**

Mathematical Thinking and Reasoning Standards: MTR.3.1, MTR.6.1

ELA Expectations: ELA.K12.EE.1.1, ELA.K12.EE.5.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.6.AR.3.2	**MA.8.DP.2.1**
MA.6.AR.3.5	**MA.8.DP.2.2**
	MA.8.DP.2.3

Lesson at a Glance

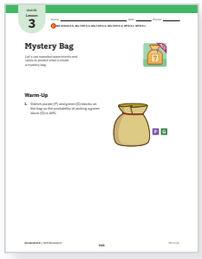
~ 45 min

MA.6.NSO.3.5, MA.7.DP.2.2, MA.7.DP.2.3, MA.7.DP.2.4

Warm-Up

Pairs | 5 min

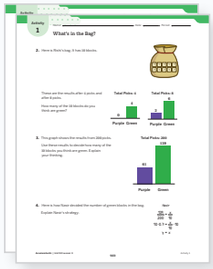
Students connect probability and the number of different colored blocks in a bag.



Activity 1

Pairs | 15 min

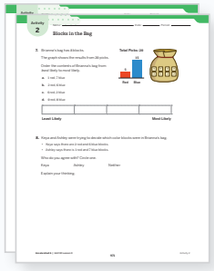
Students use probability and ratios to predict the sample space of a mystery bag. They recognize that repeated experiments lead to better predictions using the Notice and Wonder routine. **(MTR.6.1)**



Activity 2

Pairs | 15 min

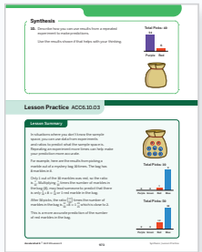
Students use ratios and the results from repeated experiments to make predictions about the sample space of a mystery bag.



Synthesis

Whole Class | 5 min

Students synthesize their understanding of using repeated experiments to make predictions.



Show What You Know

Independent | 5 min

Students demonstrate their understanding by predicting how many colored blocks are in a bag using the results from a repeated experiment.

Prep Checklist

This lesson includes:

- Student Edition
- Show What You Know
- Coloring tools, green and purple (optional)

Pairs | 5 min

Warm-Up

Purpose: Students connect probability and the number of different colored blocks in a bag.

1 Launch

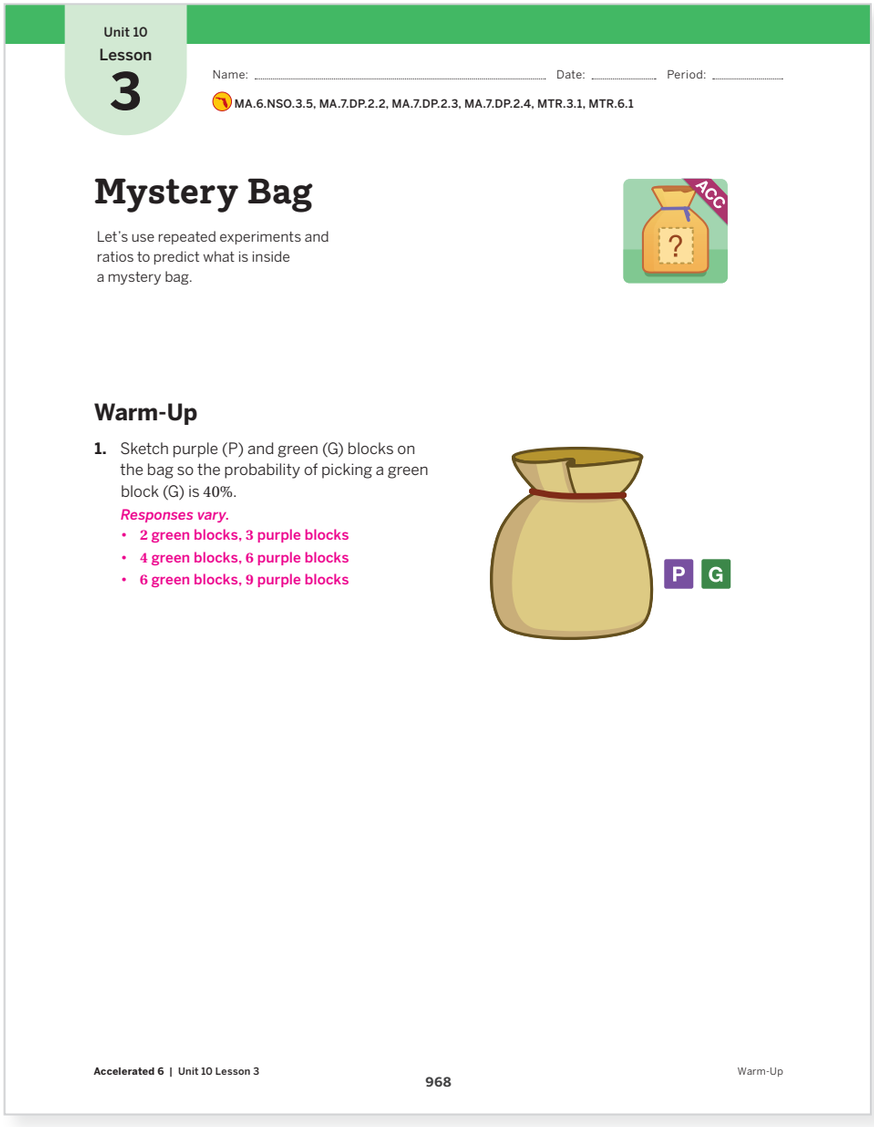
Invite students to use coloring tools, if desired.

Consider asking, "If one purple block and one green block are added to the bag, what would be the probability of picking a green block?"

2 Connect

Collect and share student responses of correct and incorrect bags. For each bag, consider asking, "What is the probability of picking a green block? How do you know?"

Invite students to share their strategies for determining how many purple and green blocks to add to the bag.



Unit 10
Lesson
3

Name: _____ Date: _____ Period: _____

MA.6.NSO.3.5, MA.7.DP.2.2, MA.7.DP.2.3, MA.7.DP.2.4, MTR.3.1, MTR.6.1

Mystery Bag

Let's use repeated experiments and ratios to predict what is inside a mystery bag.

Warm-Up

1. Sketch purple (P) and green (G) blocks on the bag so the probability of picking a green block (G) is 40%.

Responses vary.

- 2 green blocks, 3 purple blocks
- 4 green blocks, 6 purple blocks
- 6 green blocks, 9 purple blocks

Accelerated 6 | Unit 10 Lesson 3 968 Warm-Up

Pairs | 15 min

Activity 1 What's in the Bag?

Purpose: Students use probability and ratios to predict the sample space of a mystery bag. They recognize that repeated experiments lead to better predictions using the Notice and Wonder routine. **(MTR.6.1)**

1 Launch

Invite students to make a prediction about how many blocks are green.

Note: It is okay if students are unsure at this stage. Students will be shown a strategy for approaching problems like this in Problem 4.

2 Monitor

To support students getting started, consider asking, "How many of the 200 blocks picked were green? How does this relate to how many of the 10 blocks might be green?"

A **Accessibility: Conceptual Processing** Invite students to solve a similar question with friendlier numbers to support recognizing connections between new problems and prior work with ratios.

ELL **English Language Learners** Provide sentence frames to help students explain their reasoning (e.g., I think ___ of the 10 blocks are green because _____.).

Pause to discuss Nasir's strategy. Identify any students who used a strategy similar to Nasir's. **(MTR.3.1)**

To surface prior knowledge, consider asking, "What does 0.7 represent in Nasir's strategy?"

Activity 1 continued >

Activity 1

Name: Date: Period:

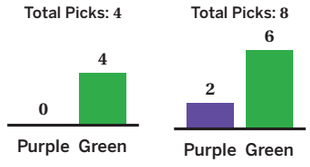
What's in the Bag?

2. Here is Rishi's bag. It has 10 blocks.

These are the results after 4 picks and after 8 picks.

How many of the 10 blocks do you think are green?

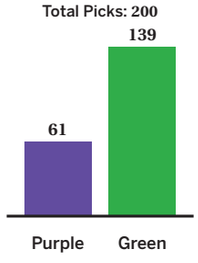
Responses vary.



3. This graph shows the results from 200 picks.

Use these results to decide how many of the 10 blocks you think are green. Explain your thinking.

Responses and explanations vary. 7 blocks. There are 139 green blocks out of 200 total, which is about 70% and 70% of 10 is 7.



4. Here is how Nasir decided the number of green blocks in the bag.

Explain Nasir's strategy.

Responses vary. Nasir set up a proportion. He divided 139 by 200 and got 0.7 after rounding. Then he multiplied the number of blocks in the bag (10) by 0.7 to determine the number of green blocks.

Nasir

$$\frac{139}{200} = \frac{x}{10}$$

$$10 \cdot 0.7 = \frac{x}{10} \cdot 10$$

$$7 = x$$

Accelerated 6 | Unit 10 Lesson 3 969 Activity 1

Pairs | 15 min

Activity 1 What's in the Bag? (continued)

Purpose: Students use probability and ratios to predict the sample space of a mystery bag. They recognize that repeated experiments lead to better predictions using the Notice and Wonder routine. **(MTR.6.1)**

2 Monitor

Use the Notice and Wonder routine to promote curiosity and help students make sense of the two dot plots.

Pause to create a record of the things students noticed and wondered, along with the name of the student who shared each one.

Consider asking, "Did anyone notice a more similar response to Nasir's answer after 200 picks? Does this make you wonder about what happens to the data after more picks?" **(MTR.6.1)**

Direct students' attention to the contents of the bag.

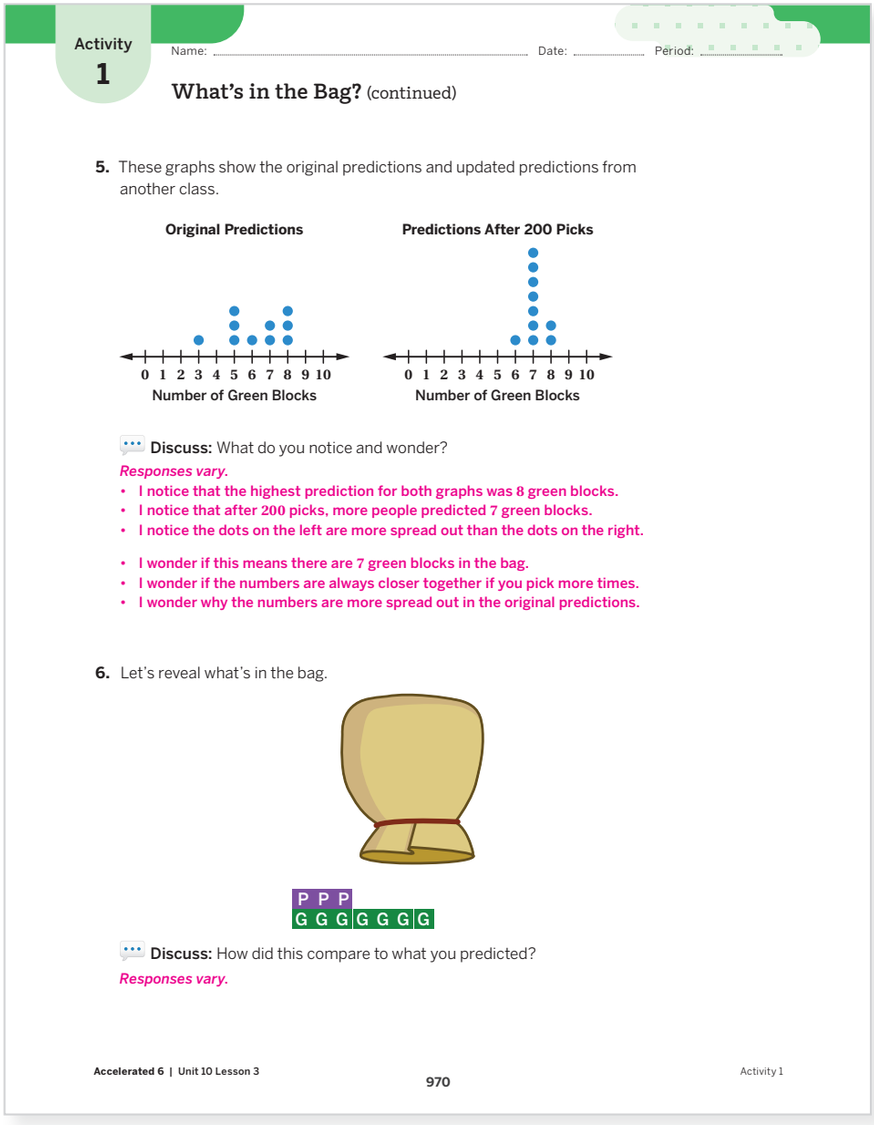
Encourage students to record their thinking.

3 Connect

Consider asking:

- "How did the predictions change after 200 picks? Why?"
- "What do these graphs tell you about the number of green blocks in the bag?"
- "Which graph would you rather use to decide? Why?"

Invite students to share if the number of green blocks matches their predictions. **(MTR.6.1)**



Activity 1

Name: Date: Period:

What's in the Bag? (continued)

5. These graphs show the original predictions and updated predictions from another class.

Original Predictions

Number of Green Blocks

Predictions After 200 Picks

Number of Green Blocks

Discuss: What do you notice and wonder?

Responses vary.

- I notice that the highest prediction for both graphs was 8 green blocks.
- I notice that after 200 picks, more people predicted 7 green blocks.
- I notice the dots on the left are more spread out than the dots on the right.
- I wonder if this means there are 7 green blocks in the bag.
- I wonder if the numbers are always closer together if you pick more times.
- I wonder why the numbers are more spread out in the original predictions.

6. Let's reveal what's in the bag.

P P P
G G G G G G

Discuss: How did this compare to what you predicted?

Responses vary.

Accelerated 6 | Unit 10 Lesson 3 970 Activity 1

Pairs | 15 min

Activity 2 Blocks in the Bag

Purpose: Students use ratios and the results from repeated experiments to make predictions about the sample space of a mystery bag.

Short on time? Consider completing only 1–2 challenges in Problem 9.

1 Launch

To surface prior knowledge, consider asking, "When making comparisons, what does it mean for something to be *more likely* to occur?"

2 Monitor

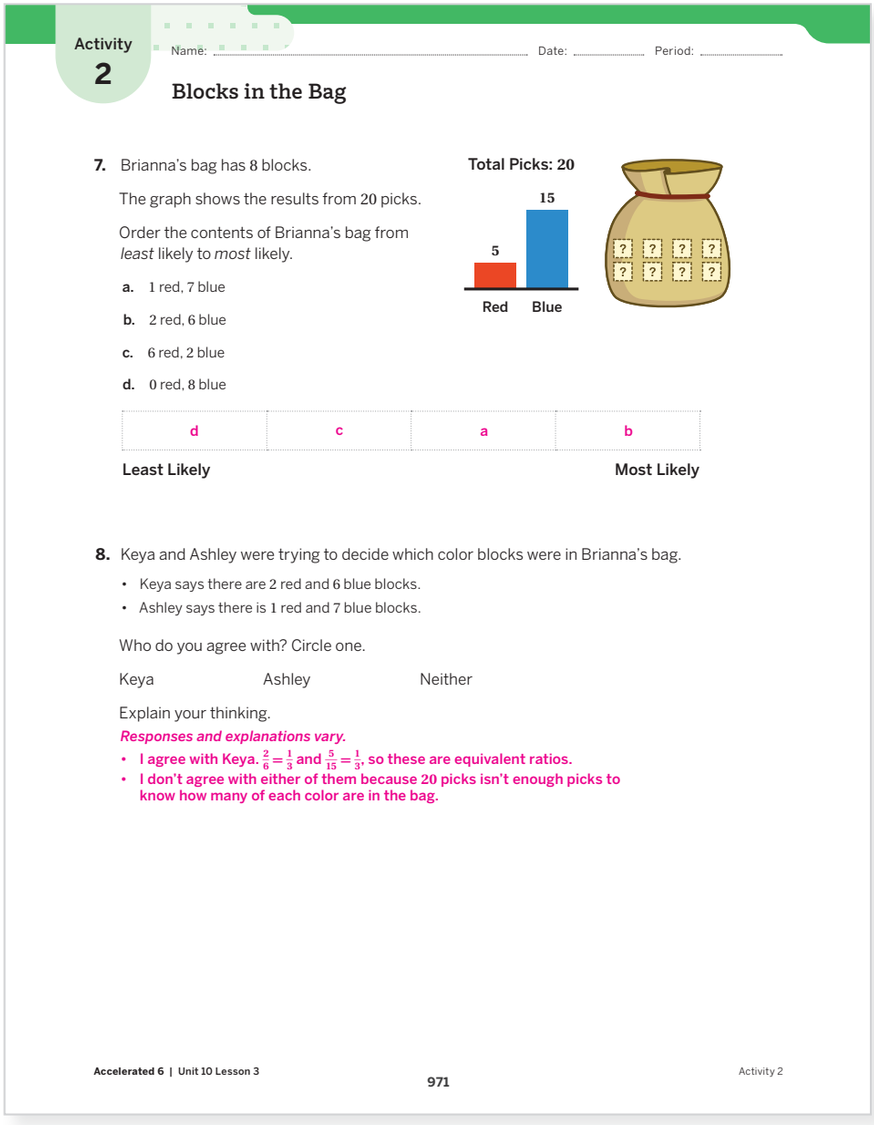
D Differentiation

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Encourage students to use ratios to order the contents of the bag. Consider asking, "If 5 of the 20 picks are red, what percentage is that?"
Compare ratios.	**Strengthen** Invite them to share how this helped them decide which options are more likely than others during the Connect.

Collect a variety of student responses to share during the Connect.

ELL **English Language Learners** Provide sentence frames to help students explain their reasoning (e.g., _____ is correct because _____.).

Activity 2 continued >



Activity 2

Name: Date: Period:

Blocks in the Bag

7. Brianna's bag has 8 blocks.

The graph shows the results from 20 picks.

Order the contents of Brianna's bag from *least* likely to *most* likely.

a. 1 red, 7 blue

b. 2 red, 6 blue

c. 6 red, 2 blue

d. 0 red, 8 blue

d	c	a	b
Least Likely			Most Likely

8. Keya and Ashley were trying to decide which color blocks were in Brianna's bag.

- Keya says there are 2 red and 6 blue blocks.
- Ashley says there is 1 red and 7 blue blocks.

Who do you agree with? Circle one.

Keya Ashley Neither

Explain your thinking.

Responses and explanations vary.

- I agree with Keya. $\frac{2}{6} = \frac{1}{3}$ and $\frac{5}{15} = \frac{1}{3}$, so these are equivalent ratios.
- I don't agree with either of them because 20 picks isn't enough picks to know how many of each color are in the bag.

Accelerated 6 | Unit 10 Lesson 3 971 Activity 2

Pairs | 15 min

Activity 2 Blocks in the Bag (continued)

Purpose: Students use ratios and the results from repeated experiments to make predictions about the sample space of a mystery bag.

Short on time? Consider completing only 1–2 challenges in Problem 9.

2 Monitor

Listen for student strategies and partner discussions, offering help or encouragement where needed.

3 Connect

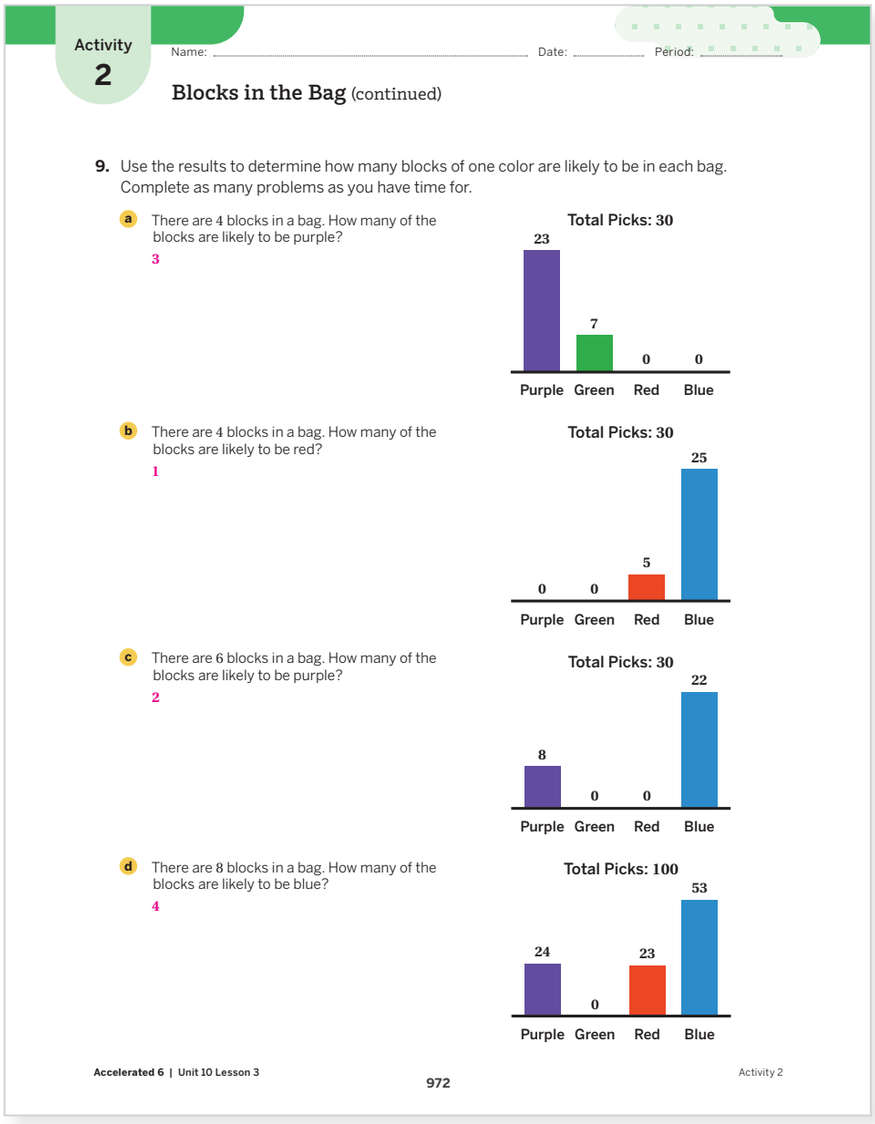
Invite students to compare their ordering with another pair of students and justify their reasoning for which descriptions were most and least likely. **(MTR.3.1)**

Consider asking, "Are this many picks enough to make you confident in your ordering?"

Share student responses, calling attention to any conflict or consensus. Invite students to share their reasoning for their choice.

Note: If it does not come up naturally, consider asking, "Are this many picks enough to know who is correct? Why or why not?"

To surface the Key Takeaway, consider asking, "How does using ratios help you to make a prediction about the contents of the bag?"



Activity 2

Name: Date: Period:

Blocks in the Bag (continued)

9. Use the results to determine how many blocks of one color are likely to be in each bag. Complete as many problems as you have time for.

a. There are 4 blocks in a bag. How many of the blocks are likely to be purple?

3

b. There are 4 blocks in a bag. How many of the blocks are likely to be red?

1

c. There are 6 blocks in a bag. How many of the blocks are likely to be purple?

2

d. There are 8 blocks in a bag. How many of the blocks are likely to be blue?

4

Accelerated 6 | Unit 10 Lesson 3 972 Activity 2

Key Takeaway: You can use ratios to make predictions from an experiment. These predictions are likely to be more accurate when the experiment is repeated many times.

Whole Class | 5 min

Synthesis

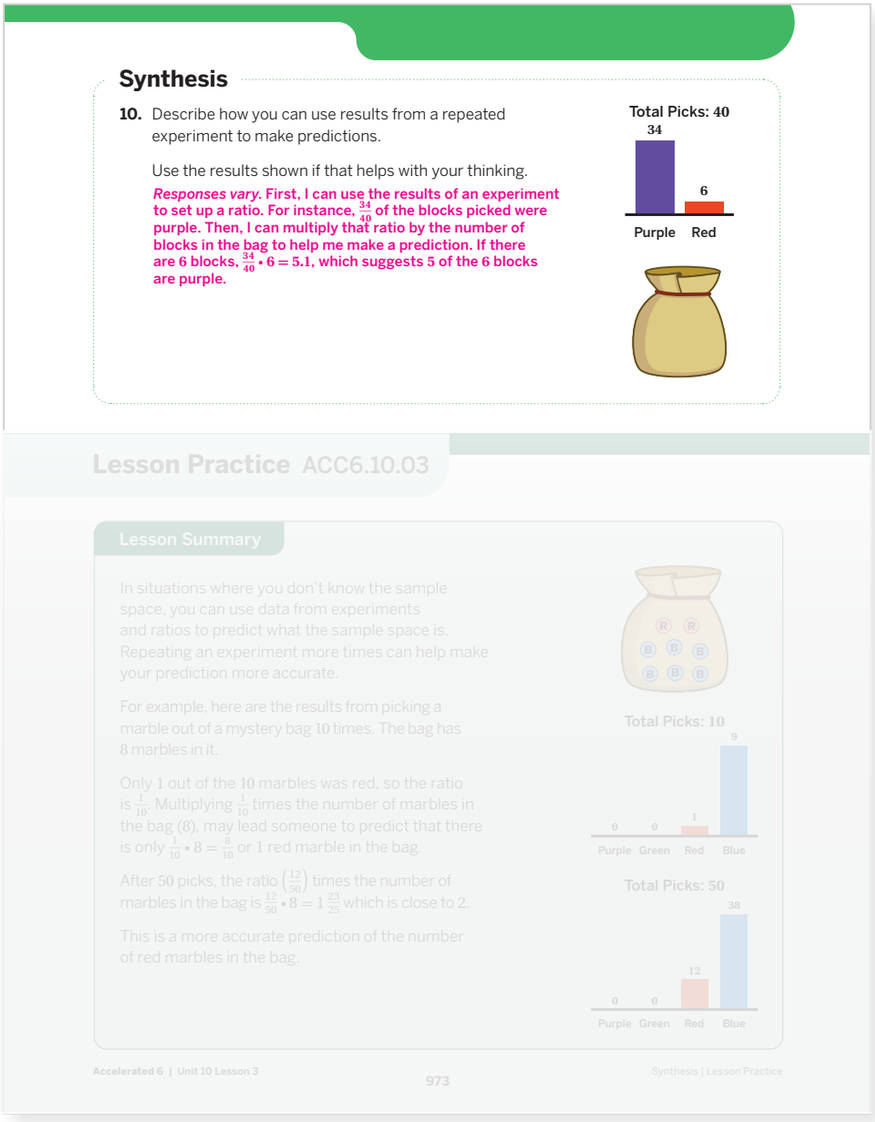
Purpose: Students synthesize their understanding of using repeated experiments to make predictions.

Invite students to respond independently, and then share their thinking with a partner.

Collect and share a variety of ideas, such as:

- Using ratios and multiplying or dividing to make a prediction.
- Rounding to the nearest whole number value if the prediction does not make sense.

Note: If time allows, invite students to revise their previous thinking based on the discussion.



Synthesis

10. Describe how you can use results from a repeated experiment to make predictions.

Use the results shown if that helps with your thinking.

Responses vary. **First, I can use the results of an experiment to set up a ratio. For instance, $\frac{34}{40}$ of the blocks picked were purple. Then, I can multiply that ratio by the number of blocks in the bag to help me make a prediction. If there are 6 blocks, $\frac{34}{40} \cdot 6 = 5.1$, which suggests 5 of the 6 blocks are purple.**

Lesson Practice ACC6.10.03

Lesson Summary

In situations where you don't know the sample space, you can use data from experiments and ratios to predict what the sample space is. Repeating an experiment more times can help make your prediction more accurate.

For example, here are the results from picking a marble out of a mystery bag 10 times. The bag has 8 marbles in it.

Only 1 out of the 10 marbles was red, so the ratio is $\frac{1}{10}$. Multiplying $\frac{1}{10}$ times the number of marbles in the bag (8), may lead someone to predict that there is only $\frac{1}{10} \cdot 8 = \frac{8}{10}$ or 1 red marble in the bag.

After 50 picks, the ratio $\left(\frac{12}{50}\right)$ times the number of marbles in the bag is $\frac{12}{50} \cdot 8 = 1\frac{23}{25}$ which is close to 2.

This is a more accurate prediction of the number of red marbles in the bag.

Accelerated 6 | Unit 10 Lesson 3 973 Synthesis | Lesson Practice

Lesson Takeaway: Results from repeated experiments can be helpful when making predictions.

Independent | 5 min

Show What You Know

Purpose: Students demonstrate their understanding by predicting how many colored blocks are in a bag using the results from a repeated experiment.

Today's Goals

1. **Goal:** Use ratios and data from a repeated experiment to make predictions about the sample space.
2. **Language Goal:** Describe how to use the results from a repeated experiment to make predictions. **(Reading, Writing, Speaking, and Listening)**

Show What You Know (Assessment Resources)

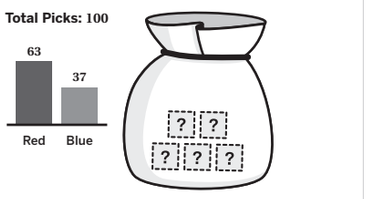
Name: _____ Date: _____ Period: _____

Show What You Know 10.03

A new bag has 5 blocks. Some are red and some are blue.

Based on these results, how many blocks are likely to be red? Show or explain your thinking.

3 blocks. *Responses vary.* I multiplied the ratio of $\frac{63}{100}$ and 5 to determine the the number of blocks out of 5 that would be red. I got 3.15 and, since there are only whole blocks, I rounded down to 3 blocks.



Differentiation Use after Lesson 3

Support

Provide targeted intervention for students by using these resources.

If students need support understanding how to make predictions about the sample space using results from a repeated experiment:

» **Respond to Student Thinking:**

- Use the ***Predicting Sample Spaces Using Proportional Relationships*** **Mini-lesson**.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 1 Extension Activities**.
- Invite students to set up their own mystery bag or other random experiment, then trade with a partner and conduct experiments to predict the sample space.

Professional Learning

Consider the next lessons you will teach. How will students' work today in "What's in the Bag?" prepare them for understanding the relationship between the probability of an outcome and its relative frequency in repeated experiments?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

10. Describe how you can use results from a repeated experiment to make predictions.

Use the results shown if that helps with your thinking.

Responses vary. First, I can use the results of an experiment to set up a ratio. For instance, $\frac{34}{40}$ of the blocks picked were purple. Then, I can multiply that ratio by the number of blocks in the bag to help me make a prediction. If there are 6 blocks, $\frac{34}{40} \cdot 6 = 5.1$, which suggests 5 of the 6 blocks are purple.

Lesson Practice ACC6.10.03

Lesson Summary

In situations where you don't know the sample space, you can use data from experiments and ratios to predict what the sample space is. Repeating an experiment more times can help make your prediction more accurate.

For example, here are the results from picking a marble out of a mystery bag 10 times. The bag has 8 marbles in it.

Only 1 out of the 10 marbles was red, so the ratio is $\frac{1}{10}$. Multiplying $\frac{1}{10}$ times the number of marbles in the bag (8), may lead someone to predict that there is only $\frac{1}{10} \cdot 8 = \frac{8}{10}$ or 1 red marble in the bag.

After 50 picks, the ratio $\left(\frac{12}{50}\right)$ times the number of marbles in the bag is $\frac{12}{50} \cdot 8 = 1\frac{23}{25}$ which is close to 2.

This is a more accurate prediction of the number of red marbles in the bag.

Lesson Practice
ACC6.10.03

Name: _____ Date: _____ Period: _____

Problems 1–2: A month is chosen at random from the months of the year.

1. What is the probability of getting a month that starts with the letter "J"? Consider listing the sample space to show your thinking.

$\frac{1}{4}$ (or equivalent)

2. If you repeat this experiment 600 times, how many times do you expect to get a month that starts with "J"?

About 150 times

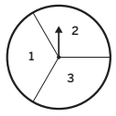
3. A bag has 6 blocks in it. Joel picks a block out of the bag 60 times. He gets a green block 43 times. Based on these results, how many blocks do you expect to be green? Explain your thinking.

4 blocks. *Explanations vary.* Joel picked a green block about 72% of the time because $\frac{43}{60} \approx 0.7167$. If the proportion of green blocks in the bag matched this, there would be 4.3 green blocks. Because 4.3 is closer to 4 than 5, it's likely that there are 4 green blocks in the bag.

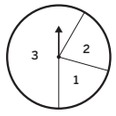
4. Miko spun a spinner with numbered sections 15 times. Here are the results. Which spinner is most likely the one Miko used?

Miko's results:
3, 3, 3, 1, 3, 2, 2, 3, 3, 1, 3, 1, 3, 3, 2

A.



B.



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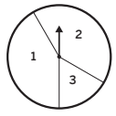


Explain your thinking.

Explanations vary. Miko spun a 3 nine times, which is three times as many times that he spun a 1 or a 2. Because Spinner B has a large section for 3 and smaller equal sections for 1 and 2, this spinner seems to match Miko's results more than the other two spinners.

Problems 5–7: A textbook has 428 numbered pages, starting with 1. You are equally likely to stop on any of the pages if you flip through the book randomly.

5. What is the probability that you turn to page 45?

$\frac{1}{428}$ (or equivalent)

6. What is the probability that you turn to an even-numbered page?

$\frac{1}{2}$ (or equivalent)

7. If you repeat this experiment 50 times, about how many times do you expect to turn to an even-numbered page?

About 25 times

Lesson Practice
ACC6.10.03

Name: _____ Date: _____ Period: _____

FAST Practice

8. The table shows the number of different colors of crayons in a case. A 7th grader will randomly choose one crayon from the case.

Color	Number of Crayons
Blue	2
Red	8
Orange	4
Yellow	5

Based on the information in the table, which statement is true?

A. The crayon is least likely to be orange.

B. The crayon is 4 times as likely to be red as it is to be blue.

C. The crayon is equally likely to be orange or yellow.

D. The crayon is more likely to be red than all other colors combined.

Spiral Review

Problems 9–11: For each word, determine the probability of selecting the letter "A" at random.

9. LAMB

$\frac{1}{4}$ (or equivalent)

10. SAFETY

$\frac{1}{6}$ (or equivalent)

11. ALABAMA

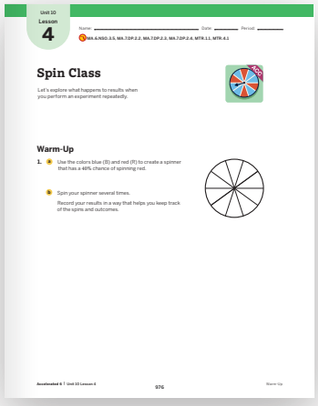
$\frac{4}{7}$ (or equivalent)

12. Select *all* the situations in which the surface area would be useful to know.

- ☑ A. Ordering tiles to replace the roof of a house.
- ☑ B. Estimating how long it will take to clean the windows of a greenhouse.
- ☐ C. Deciding whether leftover soup will fit in a container.
- ☐ D. Estimating how long it will take to fill a swimming pool with a garden hose.
- ☑ E. Buying fabric to sew a couch cover.

Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1, 5–6	1	MA.7.DP.2.3
	2–4, 7	2	MA.7.DP.2.4
FAST Practice	8	2	MA.7.DP.2.2 MA.7.DP.2.3
Spiral Review			
	9–11	1	MA.7.DP.2.3
	12	2	MA.6.GR.2.4, MA.7.GR.2.1


Unit 10
Lesson
4

Spin Class

Let's explore what happens to results when you perform an experiment repeatedly.

Warm-Up

1. Use the colors blue (B) and red (R) to create a spinner that has a 40% chance of spinning red.

Spin your spinner several times.
Record your results in a way that helps you keep track of the spins and outcomes.

976

Spin Class

Repeated Experiments

Let's explore what happens to results when you perform an experiment repeatedly.

Learning Outcomes

Today's Goals

1. **Goal:** Recognize that the results of a repeated experiment approach the probability of the event.
2. **Language Goal:** Make connections between the probability of an event and a graph representing data from a repeated experiment. **(Writing, Speaking, and Listening)**

Students recognize that if an experiment is performed repeatedly, the relative frequency will approach, but may not exactly match, the probability of each outcome. Students analyze the results of repeated experiments and examine the structure of the results. **(MTR.1.1)**

Coherence

Prior Learning

In Lesson 3, students used a repeated experiment to predict the sample space.

Future Learning

In Lesson 5, students will decide whether an object is fair by comparing the probabilities of a model to the results of a repeated experiment.

Connections

- Students analyze visuals such as line graphs, scatter plots, or histograms of the results of repeating an experiment multiple times. (mathematical)

FL B.E.S.T. Math Benchmarks

Addressing

MA.7.DP.2.3
Find the theoretical probability of an event related to a simple experiment.

MA.7.DP.2.4
Use a simulation of a simple experiment to find experimental probabilities and compare them to theoretical probabilities.

Also Addressing: **MA.6.NSO.3.5, MA.7.DP.2.2**

Mathematical Thinking and Reasoning Standards: MTR.1.1, MTR.4.1

ELA Expectation: ELA.K12.EE.1.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.6.AR.3.2	**MA.8.DP.2.1**
MA.6.AR.3.5	**MA.8.DP.2.2**
	MA.8.DP.2.3

Vocabulary

New Vocabulary

relative frequency

Lesson at a Glance

~ 45 min

MA.6.NSO.3.5, MA.7.DP.2.2, MA.7.DP.2.3, MA.7.DP.2.4

Warm-Up

Independent | 5 min

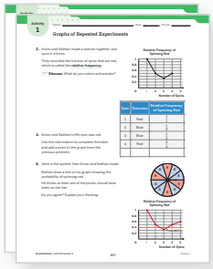
Students connect spinners with probabilities and share different ways to keep track of spins. They create a spinner similar to the one they will analyze in Activity 1.

Activity 1

Pairs | 15 min

Students interpret a graph of a repeated experiment's results to surface the idea that the **relative frequency** approaches, but may not exactly match, the probability of an outcome.

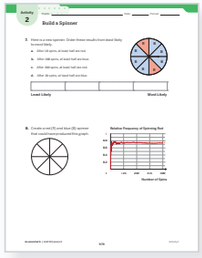
Routine: Notice and Wonder



Activity 2

Pairs | 7 min

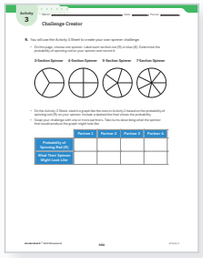
Students order the results of a repeated experiment and create a spinner to match the results of a repeated experiment on a graph.



Activity 3

Independent | 8 min

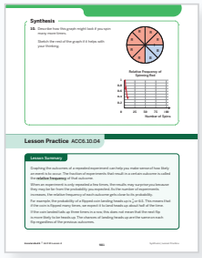
Students express their creativity by creating a spinner that could produce the results of a repeated experiment.



Synthesis

Whole Class | 5 min

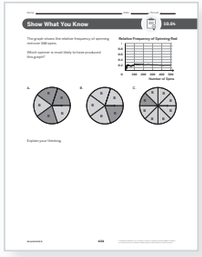
Students synthesize their understanding of how the results of a repeated experiment are related to the probability of an event.



Show What You Know

Independent | 5 min

Students demonstrate their understanding by connecting the results of a graph to a probability spinner.



Prep Checklist

This lesson includes:

- Student Edition
- Show What You Know

- Coloring tools, blue and red
- Paper clips (optional)

Independent | 5 min

Warm-Up

Purpose: Students connect spinners with probabilities and share different ways to keep track of spins. They create a spinner similar to the one they will analyze in Activity 1.

1 Launch

Demonstrate how to create the spinner.

Distribute coloring tools and paper clips to students.

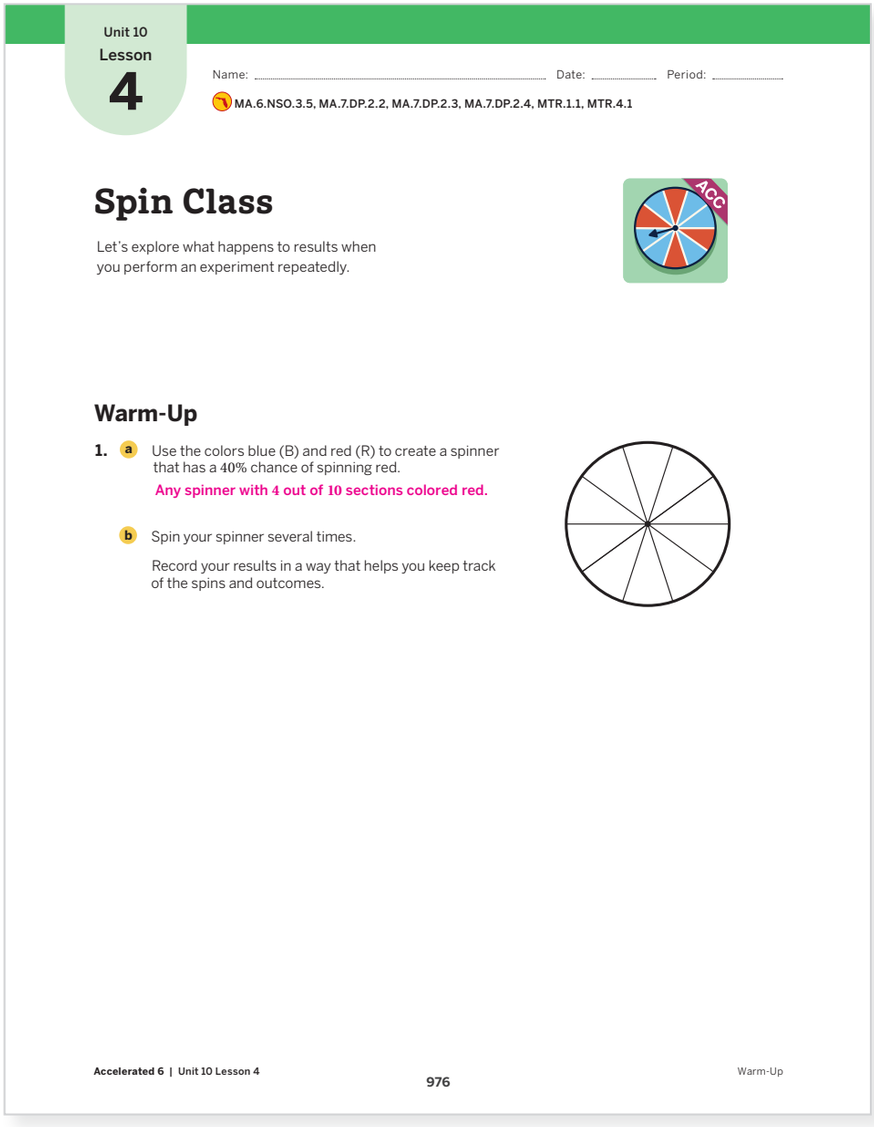
To surface prior knowledge, consider asking, "How can we change a percentage to a fraction?"

A **Accessibility: Memory and Attention** Create or review an anchor chart that displays the definition and examples of probability to aid in explanations and reasoning.

Invite students to use a paper clip as a pointer on their own spinner.

2 Connect

Invite students to share their spinners and their strategies for recording their results.



Unit 10
Lesson
4

Name: Date: Period:

MA.6.NSO.3.5, MA.7.DP.2.2, MA.7.DP.2.3, MA.7.DP.2.4, MTR.1.1, MTR.4.1

Spin Class

Let's explore what happens to results when you perform an experiment repeatedly.

Warm-Up

1. a Use the colors blue (B) and red (R) to create a spinner that has a 40% chance of spinning red.

   Any spinner with 4 out of 10 sections colored red.

   b Spin your spinner several times.

   Record your results in a way that helps you keep track of the spins and outcomes.

Accelerated 6 | Unit 10 Lesson 4 976 Warm-Up

Pairs | 15 min

Activity 1 Graphs of Repeated Experiments

Purpose: Students interpret a graph of a repeated experiment's results to surface the idea that the **relative frequency** approaches, but may not exactly match, the probability of an outcome.

1 Launch

Use the Notice and Wonder routine to promote curiosity and help students make sense of the table and graph representations. **(MTR.1.1)**

Create a class definition of the term **relative frequency** using students' language and the glossary. To help generate student language, consider asking:

- "After 4 spins, how many were red? Where do you see that information on the graph and in the table?"
- "How is the relative frequency of red related to the number of red spins?"
- "Does the relative frequency stay the same as Amari and Nathan spin more and more times?"

2 Monitor

To support students getting started, encourage them to analyze the table. Consider asking, "After 5 spins, how many were red? How does that help you determine the relative frequency?"

Collect a variety of student responses. Consider selecting responses for each of the most popular choices to discuss in the Connect.

ELL **English Language Learners** Provide sentence frames to help students explain their reasoning (e.g., I agree or disagree because _____.).

Activity 1 continued >

Activity 1

Name: Date: Period:

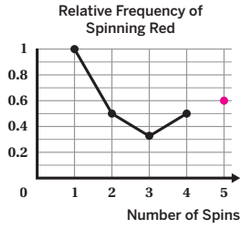
Graphs of Repeated Experiments

2. Amari and Nathan made a spinner together and spun it 4 times.

They recorded the fraction of spins that are red, which is called the **relative frequency**.

Discuss: What do you notice and wonder?

Responses vary.
- I notice that there are 4 spins in both the table and the graph.
- I notice that when the relative frequency is $\frac{1}{2}$, the graph is at 0.5.
- I notice that when they spun red on the fourth spin, the graph went back up.
- I wonder how the graph and the table are connected.
- I wonder where the fractions in the table came from.
- I wonder what the table and graph would look like if the spinner kept landing on red.



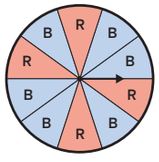
Spin	Outcome	Relative Frequency of Spinning Red
1	Red	$\frac{1}{1}$
2	Blue	$\frac{1}{2}$
3	Blue	$\frac{1}{3}$
4	Red	$\frac{2}{4}$
5	Red	$\frac{3}{5}$

3. Amari and Nathan's fifth spin was red.

Use this information to complete the table and add a point to the graph from the previous problem.

Responses shown in the table and on the graph.

4. Here is the spinner that Amari and Nathan made.

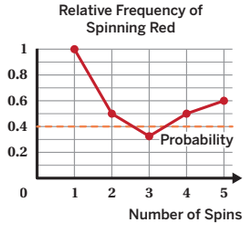


Nathan drew a line on his graph showing the probability of spinning red.

He thinks at least one of the points should have been on the line.

Do you agree? Explain your thinking.

Responses and explanations vary.
- Yes. If I perform an experiment many times, the relative frequency should match the probability at least some of the time.
- No. They only spun the spinner 5 times. That isn't enough to get close to the line. Also, because the spinner is random, we can't predict the exact fraction of times it will spin red.
- I'm not sure. The spinner is random, so it seems like the points could go anywhere on the graph.



Accelerated 6 | Unit 10 Lesson 4 977 Activity 1

Pairs | 15 min

Activity 1 Graphs of Repeated Experiments (continued)

Purpose: Students interpret a graph of a repeated experiment's results to surface the idea that the **relative frequency** approaches, but may not exactly match, the probability of an outcome.

2 Monitor

Listen for observations that may surface connections to the discussion in the next problem.

Select and sequence student observations to share during the Connect.

D Differentiation

Look for students who:	Respond to Student Thinking
Notice that the results of a repeated experiment do not exactly match the probability of the event.	**Strengthen** Consider asking, "Why do you think some graphs are closer to the probability line than others?"
Notice that the graph gets smoother toward 0.4, which is the probability.	**Strengthen** Invite students to share the relationship between the relative frequency and the probability of the outcome.
Notice that the graph varies during each of the 200 spins but gets closer as the number of spins increases.	

3 Connect

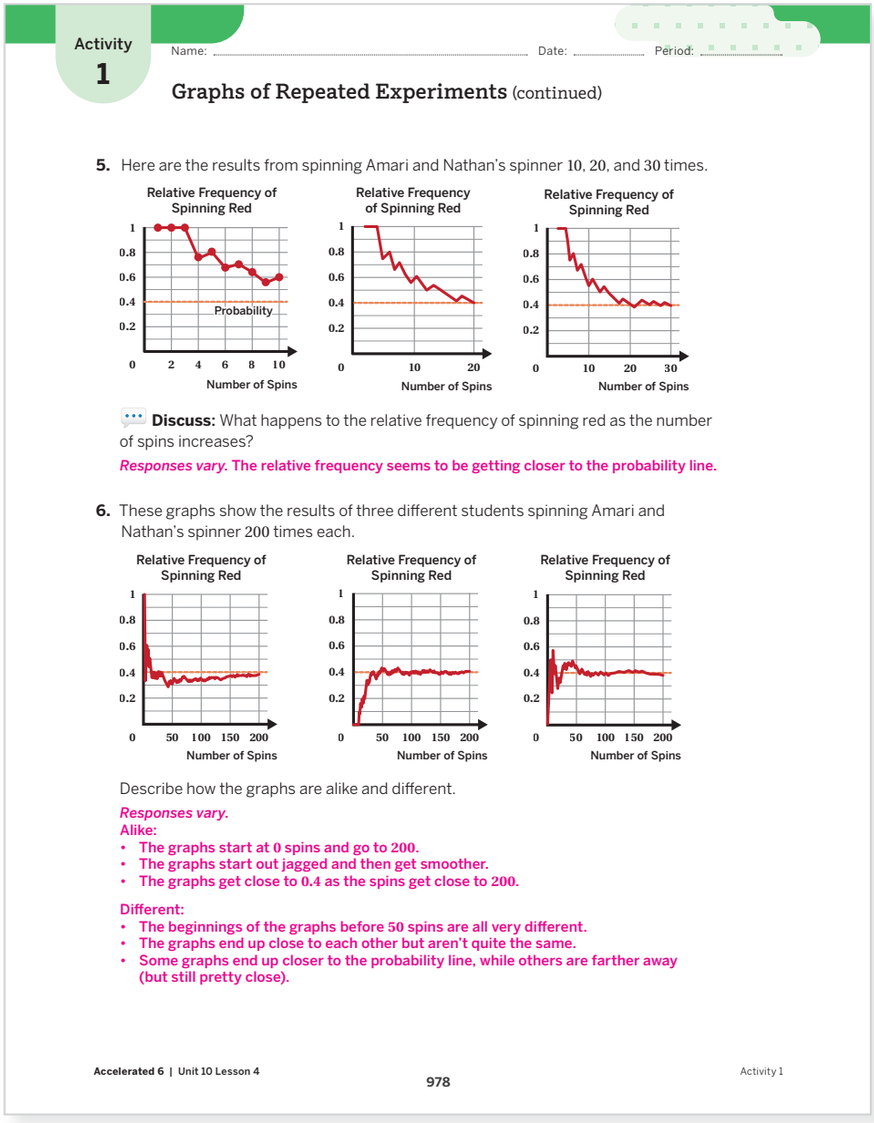
Invite students to share responses, calling attention to any conflict or consensus.

Invite students to share why they believe the points should or should not be exactly on the line to surface that the relative frequency may not exactly match the probability, especially after only 5 spins. **(MTR.4.1)**

Invite students to share their descriptions.

Consider asking, "Why do you think parts of the graphs are more alike while other parts are different?"

To surface the Key Takeaway, consider asking, "What happens to the graph as the number of spins gets closer to 200?"



Activity 1

Name: Date: Period:

Graphs of Repeated Experiments (continued)

5. Here are the results from spinning Amari and Nathan's spinner 10, 20, and 30 times.

Discuss: What happens to the relative frequency of spinning red as the number of spins increases?

Responses vary. The relative frequency seems to be getting closer to the probability line.

6. These graphs show the results of three different students spinning Amari and Nathan's spinner 200 times each.

Describe how the graphs are alike and different.

Responses vary.
Alike:
- The graphs start at 0 spins and go to 200.
- The graphs start out jagged and then get smoother.
- The graphs get close to 0.4 as the spins get close to 200.

Different:
- The beginnings of the graphs before 50 spins are all very different.
- The graphs end up close to each other but aren't quite the same.
- Some graphs end up closer to the probability line, while others are farther away (but still pretty close).

Accelerated 6 | Unit 10 Lesson 4 978 Activity 1

Key Takeaway: If you divide the number of times an outcome happened by the total number of experiments, you get the **relative frequency** of that outcome. As experiments are repeated (hundreds or thousands of times), the relative frequencies get closer to the probabilities of each outcome.

Pairs | 7 min

Activity 2 Build a Spinner

Purpose: Students order the results of a repeated experiment and create a spinner to match the results of a repeated experiment on a graph.

1 Launch

To surface connections, consider asking, "What is the probability of spinning red on this spinner? Blue?"

2 Monitor

To support students getting started, consider asking:

- "After 10 spins, what results would you expect to see? What would surprise you?"
- "Would more spins make it more or less likely to have mostly blue?"

ELL **English Language Learners** Invite one partner to read each prompt out loud before the pair discusses it. **(Reading and Listening)**

Look for students who have six colored sections of red and invite them to share their strategies during the Connect.

3 Connect

Invite students to share their strategies for how they ordered the results. Work together to reach a consensus about which results are least and most likely. **(MTR.4.1)**

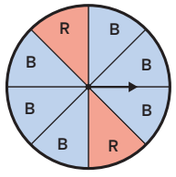
Consider asking, "How many sections of the spinner did you color red? How did you choose that number of sections?"

Activity 2

Name: _____ Date: _____ Period: _____

Build a Spinner

7. Here is a new spinner. Order these results from *least* likely to *most* likely.



a. After 10 spins, at least half are red.

b. After 100 spins, at least half are blue.

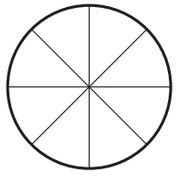
c. After 100 spins, at least half are red.

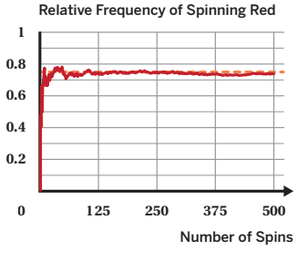
d. After 10 spins, at least half are blue.

c	a	d	b

Least Likely — Most Likely

8. Create a red (R) and blue (B) spinner that could have produced this graph.





Responses vary. Any spinner with 6 out of 8 sections colored red.

Accelerated 6 | Unit 10 Lesson 4 979 Activity 2

Independent | 8 min

Activity 3 Challenge Creator

Purpose: Students express their creativity by creating a spinner that could produce the results of a repeated experiment.

Short on time? Consider omitting this Activity.

1 Launch

To generate engagement, share with students that they will create their own "secret" spinner today.

Distribute the Activity 3 Sheet and coloring tools.

2 Monitor

Look for creative challenges.

Note: Spinners in an equivalent ratio of red sections to total sections should be accepted.

3 Connect

Invite students to share how they determined what their classmate's spinner(s) look like.

Discuss how ratios are helpful when analyzing a graph to determine the sections of a spinner.

Activity 3

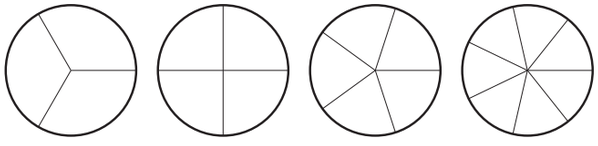
Name: Date: Period:

Challenge Creator

9. You will use the Activity 3 Sheet to create your own spinner challenge.

- On this page, choose *one* spinner. Label each section red (R) or blue (B). Determine the probability of spinning red on your spinner and record it.

3-Section Spinner | 4-Section Spinner | 5-Section Spinner | 7-Section Spinner



- On the Activity 3 Sheet, sketch a graph like the ones in Activity 2 based on the probability of spinning red (R) on your spinner. Include a dashed line that shows the probability.
- Swap your challenge with one or more partners. Take turns describing what the spinner that would produce the graph might look like. ***Responses vary.***

	Partner 1	Partner 2	Partner 3	Partner 4
Probability of Spinning Red (R)				
What Their Spinner Might Look Like				

Accelerated 6 | Unit 10 Lesson 4 980 Activity 3

Whole Class | 5 min

Synthesis

Purpose: Students synthesize their understanding of how the results of a repeated experiment are related to the probability of an event.

Invite students to respond independently, and then share their thinking with a partner.

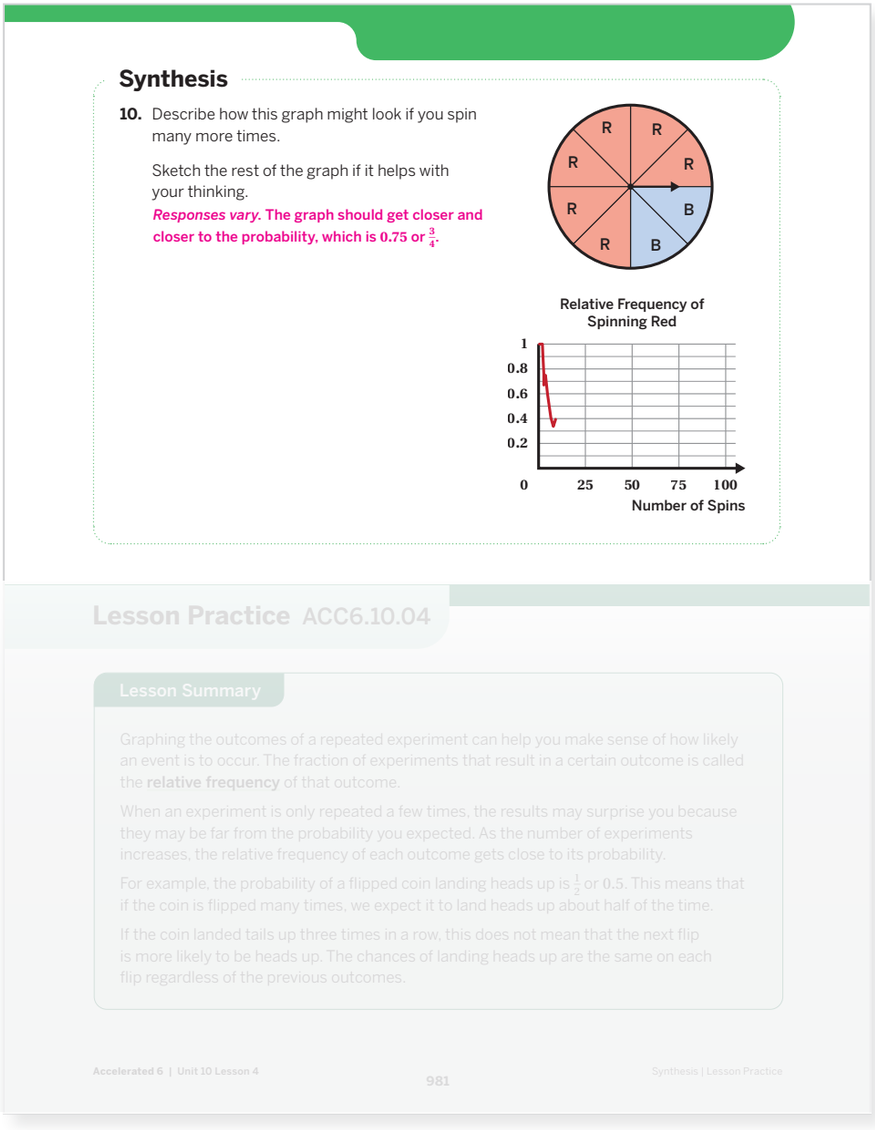
ELL **English Language Learners**

- Provide sentence frames to help students explain their reasoning (e.g., If you spin many more times, the graph will look like _____.).
- Give students time to rehearse their ideas with a partner before they are expected to share their ideas with others.

Look for and share a variety of ideas, such as:

- Approaching the probability.
- The probability being 0.75 (or equivalent).

Note: If time allows, invite students to revise their previous thinking based on the discussion.



Synthesis

10. Describe how this graph might look if you spin many more times.

Sketch the rest of the graph if it helps with your thinking.

Responses vary. The graph should get closer and closer to the probability, which is 0.75 or $\frac{3}{4}$.

Lesson Practice ACC6.10.04

Lesson Summary

Graphing the outcomes of a repeated experiment can help you make sense of how likely an event is to occur. The fraction of experiments that result in a certain outcome is called the **relative frequency** of that outcome.

When an experiment is only repeated a few times, the results may surprise you because they may be far from the probability you expected. As the number of experiments increases, the relative frequency of each outcome gets close to its probability.

For example, the probability of a flipped coin landing heads up is $\frac{1}{2}$ or 0.5. This means that if the coin is flipped many times, we expect it to land heads up about half of the time.

If the coin landed tails up three times in a row, this does not mean that the next flip is more likely to be heads up. The chances of landing heads up are the same on each flip regardless of the previous outcomes.

Accelerated 6 | Unit 10 Lesson 4 981 Synthesis | Lesson Practice

Lesson Takeaway: When you repeat an experiment many times, the relative **frequency** of each outcome approaches its probability.

Independent | 5 min

Show What You Know

Purpose: Students demonstrate their understanding by connecting the results of a graph to a probability spinner.

Today's Goals

1. **Goal:** Recognize that the results of a repeated experiment approach the probability of the event.
2. **Language Goal:** Make connections between the probability of an event and a graph representing data from a repeated experiment. **(Writing, Speaking, and Listening)**

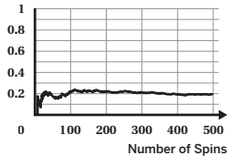
Show What You Know (Assessment Resources)

Name: _____ Date: _____ Period: _____

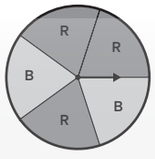
Show What You Know 10.04

The graph shows the relative frequency of spinning red over 500 spins.

Which spinner is most likely to have produced this graph?



A.



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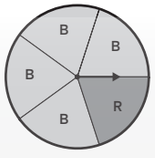
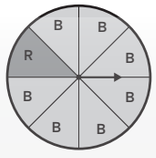


C.



Explain your thinking.

Explanations vary. The graph shows that after many spins, the relative frequency gets close to 0.2. This means the probability of red is likely $\frac{1}{5}$. A spinner with 1 red section out of 5 has a $\frac{1}{5}$ probability of landing on red.

D Differentiation Use after Lesson 4

Support

Provide targeted intervention for students by using these resources.

If students need support understanding how to recognize which spinner would have produced the results shown in the graph:

» **Respond to Student Thinking:**

- Revisit the spinners students built in **Activity 2**. Invite students to make connections between the observed relative frequencies and the characteristics of the spinner.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 1 Extension Activities**.

Professional Learning

How did the Notice and Wonder routine support students in recognizing the idea that in repeated experiments, the relative frequency of each outcome approaches, but may not exactly match, its probability?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

10. Describe how this graph might look if you spin many more times.

Sketch the rest of the graph if it helps with your thinking.

Responses vary. The graph should get closer and closer to the probability, which is 0.75 or $\frac{3}{4}$.

Lesson Practice ACC6.10.04

Lesson Summary

Graphing the outcomes of a repeated experiment can help you make sense of how likely an event is to occur. The fraction of experiments that result in a certain outcome is called the **relative frequency** of that outcome.

When an experiment is only repeated a few times, the results may surprise you because they may be far from the probability you expected. As the number of experiments increases, the relative frequency of each outcome gets close to its probability.

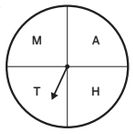
For example, the probability of a flipped coin landing heads up is $\frac{1}{2}$ or 0.5. This means that if the coin is flipped many times, we expect it to land heads up about half of the time.

If the coin landed tails up three times in a row, this does not mean that the next flip is more likely to be heads up. The chances of landing heads up are the same on each flip regardless of the previous outcomes.

Lesson Practice ACC6.10.04

Name: _____ Date: _____ Period: _____

Problems 1–2: A spinner has four equal sections, with one letter from the word MATH in each section.



1. You spin the spinner 20 times. About how many times do you expect it will land on "A"?

About 5 times

Explain your thinking.

Explanations vary. Because the spinner is split into four equal sections, there is a 25% chance of it landing on "A." 25% of 20 spins is a total of 5 times.

2. You spin the spinner 80 times. About how many times do you expect it will land on something other than "A"?

About 60 times

Explain your thinking.

Explanations vary. Because the spinner is split into four equal sections, there is a 75% chance of it landing on something other than "A." 75% of 80 is a total of 60 times.

Problems 3–5: A fair coin is flipped 100 times.

3. About how many times would you expect the coin to land heads up?

About 50 times.

4. What is another number of times that would be likely for it to land heads up?

Whole numbers between 40 and 60 times are considered correct.

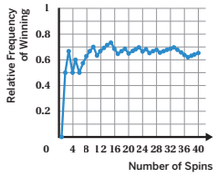
5. Adah says it's impossible for the coin to land heads up all 100 times. Basheera says it's possible, but unlikely. Who is correct?

Basheera

Explain your thinking.

Explanations vary. Because the probability of the coin landing heads up is 50%, it's very unlikely that every flip out of 100 lands heads up, but it's not impossible.

6. A spinner is spun 40 times for a game. This graph shows the relative frequency of winning the game.



Based on the graph, estimate the probability of winning this game.

Responses between 0.6 and 0.7 (or equivalent) are considered correct.

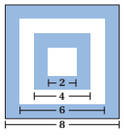
Explain your thinking.

Explanations vary. The relative frequency of winning approaches 0.65 the more times the spinner is spun.

Lesson Practice ACC6.10.04

Name: _____ Date: _____ Period: _____

Problems 7–8: Here is a square dartboard.



7. Is a dart thrown at random more likely to land in the shaded or the unshaded area? Show or explain your thinking.

Shaded. *Explanations vary.* The unshaded area is 24 square units because $2^2 + (6^2 - 4^2) = 24$. The shaded area is 40 square units because $(4^2 - 2^2) + (8^2 - 6^2) = 40$. The total area of the board is 64 square units. There is a 62.5% chance that a randomly thrown dart lands in the shaded area.

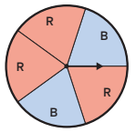
8. A dart is thrown at the board 100 times at random. About how many times is the dart likely to land in the shaded area? Explain your thinking.

Whole numbers between 60 and 65 times are considered correct. *Explanations vary.* There is a 62.5% chance that a randomly thrown dart lands in the shaded area, and 62.5% of 100 is 62.5 which rounds to 63.

FAST Practice

9. Janna spun the spinner shown 4 times. If it landed on R twice, what is the relative frequency of landing on R?

Write your answer as a decimal. 0.5



Spiral Review

10. Write each decimal as an equivalent fraction.

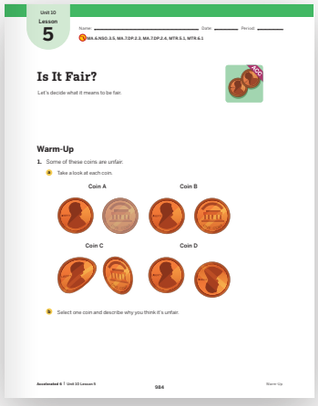
0.5 $\frac{1}{2}$ 0.4 $\frac{2}{5}$ 0.75 $\frac{3}{4}$ 0.25 $\frac{1}{4}$

11. Solve the inequality $12 < -3x$.

$x < -4$

Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1–2	2	MA.7.DP.2.4
	3	1	MA.7.DP.2.4
	4–6, 8	2	MA.7.DP.2.4
	7	3	MA.7.DP.2.4
FAST Practice	9	2	MA.7.DP.2.4
Spiral Review			
	10	1	MA.6.NSO.3.5
	11	1	MA.7.AR.2.1



Is It Fair?

Comparing Probabilities and Experiments

Let's decide what it means to be fair.

Learning Outcomes

Today's Goals

1. **Goal:** Use the results from a repeated experiment to approximate the probability of an event.
2. **Language Goal:** Explain whether an object is fair based on the results of a repeated experiment. **(Reading and Writing)**

Students are introduced to unfair objects. They decide whether an object is fair by comparing the probabilities of a model to the results of a repeated experiment. They also examine unfair coins and number cubes and explain possible sources of discrepancy between what they expect and the relative frequencies they observe.

Coherence

Prior Learning

In Lessons 3 and 4, students used the results of a repeated experiment to make predictions.

Future Learning

In a future course, students will solve real-world problems involving probabilities and will make predictions based on theoretical probability.

Connections

- Students investigate how the makeup of objects suchs a coin or die can affect the likelihood of certian outcomes. (real world)

FL B.E.S.T. Math Benchmarks

Addressing

MA.7.DP.2.3
Find the theoretical probability of an event related to a simple experiment.

MA.7.DP.2.4
Use a simulation of a simple experiment to find experimental probabilities and compare them to theoretical probabilities.

Also Addressing: **MA.6.NSO.3.5**

Mathematical Thinking and Reasoning Standards: MTR.5.1, MTR.6.1

ELA Expectation: ELA.K12.EE.5.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.6.AR.3.2	**MA.8.DP.2.1**
MA.6.AR.3.5	**MA.8.DP.2.2**
	MA.8.DP.2.3

Lesson at a Glance

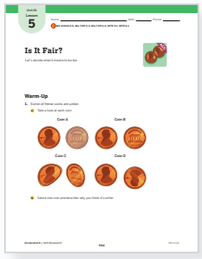
~ 45 min

MA.6.NSO.3.5, MA.7.DP.2.3, MA.7.DP.2.4

Warm-Up

Pairs | 5 min

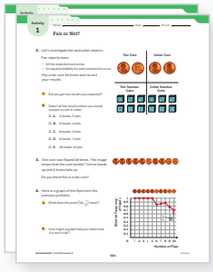
Students are introduced to fair and unfair objects.



Activity 1

Pairs | 15 min

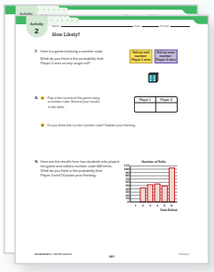
Students use the expected probabilities of fair objects and the results of a repeated experiment to decide if a coin is fair. They also approximate the probability of an unfair coin landing heads up. **(MTR.6.1)**



Activity 2

Pairs | 15 min

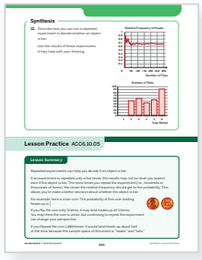
Students use the results of a repeated experiment (rolling an unfair number cube) to determine probabilities of events. Students also use these probabilities to design a game that is fair. **(MTR.5.1)**



Synthesis

Whole Class | 5 min

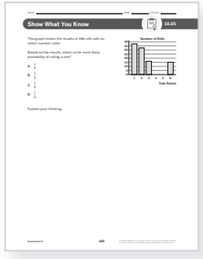
Students synthesize their understanding of using repeated experiments to decide whether an object is fair.



Show What You Know

Independent | 5 min

Students demonstrate their understanding by determining the probability of an event occurring based on the results of a repeated experiment.



Prep Checklist

This lesson includes:

- Student Edition
- Show What You Know

- Coins, one per pair
- Number cubes, one per pair

Pairs | 5 min

Warm-Up

Purpose: Students are introduced to fair and unfair objects.

1 Launch

Consider asking, "Have you ever played a game you thought was unfair or set up as a trick? What made it unfair?"

To generate engagement, distribute a fair coin to each pair of students and invite students to examine it.

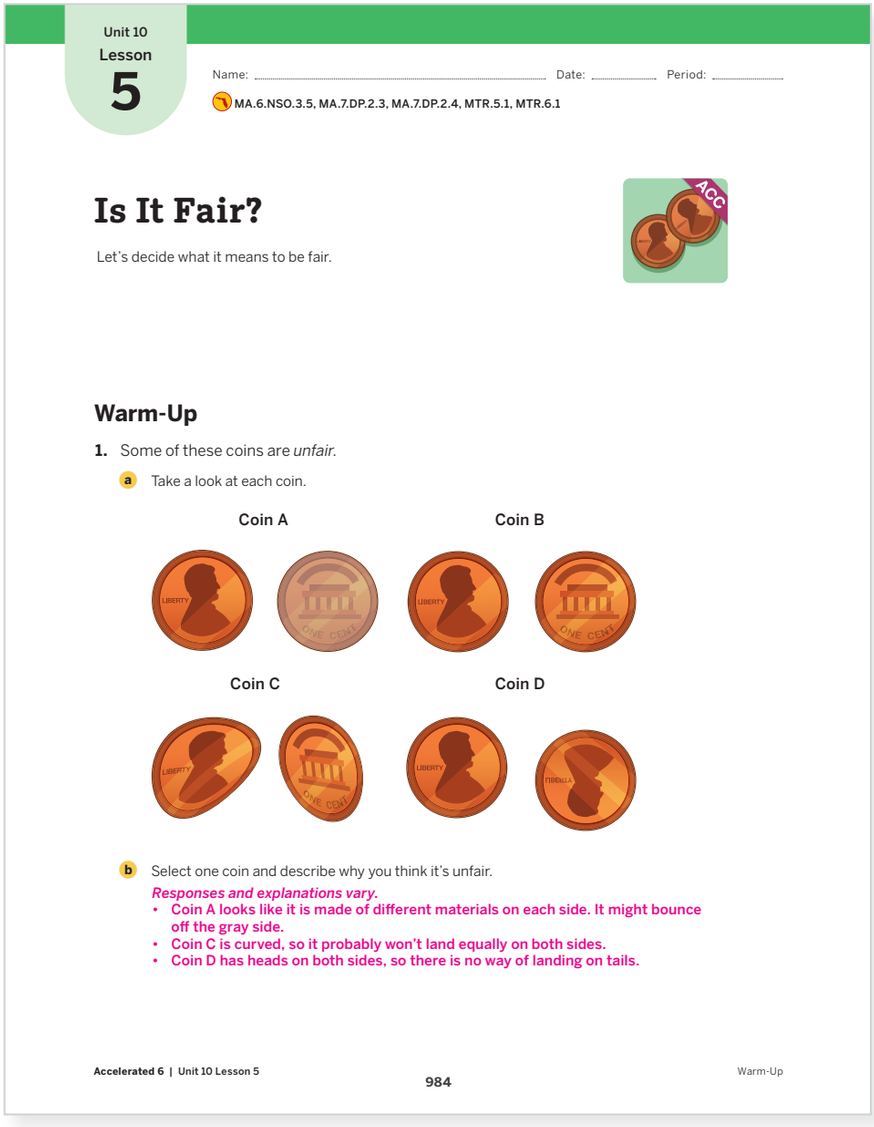
2 Connect

Invite students to share their responses.

Select and sequence 1–2 responses for each of the most popular choices to share and invite those students to share what they think makes the coin unfair.

Note: If it does not come up naturally when examining the coin, share with students that on a standard coin, the side with tails faces a different direction than the side with heads. Coin B is an example of a fair coin.

ELL **English Language Learners** Provide sentence frames to help students explain their reasoning (e.g., I think Coin ___ is unfair because _____.).



Unit 10
Lesson
5

Name: _____ Date: _____ Period: _____

MA.6.NSO.3.5, MA.7.DP.2.3, MA.7.DP.2.4, MTR.5.1, MTR.6.1

Is It Fair?

Let's decide what it means to be fair.

ACC

Warm-Up

1. Some of these coins are *unfair*.

a Take a look at each coin.

Coin A

Coin B

Coin C

Coin D

b Select one coin and describe why you think it's unfair.

Responses and explanations vary.

- Coin A looks like it is made of different materials on each side. It might bounce off the gray side.
- Coin C is curved, so it probably won't land equally on both sides.
- Coin D has heads on both sides, so there is no way of landing on tails.

Accelerated 6 | Unit 10 Lesson 5 984 Warm-Up

Pairs | 15 min

Activity 1 Fair or Not?

Purpose: Students use the expected probabilities of fair objects and the results of a repeated experiment to decide if a coin is fair. They also approximate the probability of an unfair coin landing heads up. **(MTR.6.1)**

1 Launch

Consider asking, "If you flip a fair coin 100 times, how many times do you expect it to land heads up? Why?" **(MTR.6.1)**

To encourage student engagement, distribute a coin and invite students to take turns flipping it 10 or more times.

A **Accessibility: Conceptual Processing** Invite students to read all statements or problems aloud to support extra processing time.

2 Monitor

Listen for a variety of student responses, including students who are not sure because the coin is not flipped enough times to make a prediction.

Pause to discuss student explanations, such as:

- You should compare the relative frequencies you see on the graph to 0.5, because that indicates the coin is fair.
- As the number of flips increases, you can look for the line to stay close to 0.5.
- You can pick a point on the graph with a high number of flips and see if the relative frequency is close to 0.5.

Consider asking, "How does that relate to the probability of a fair coin?" **(MTR.6.1)**

ELL **English Language Learners** Provide sentence frames to help students explain their reasoning (e.g., A graph might help you determine if a coin is fair by _____.).

Activity 1 continued >

Activity 1

Name: _____ Date: _____ Period: _____

Fair or Not?

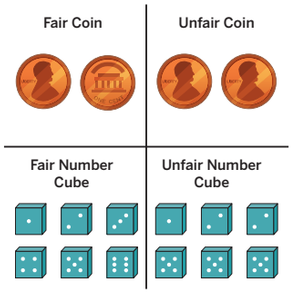
2. Let's investigate *fair* and *unfair* objects.

Fair objects have:

- All the expected outcomes.
- An equal probability for each outcome to occur.

Flip a fair coin 10 times and record your results.

Responses vary.



a. Did you get the results you expected?

Responses vary.

b. Select *all* the results where you would suspect a coin is unfair.

- ☐ A. 5 heads, 5 tails
- ☐ B. 6 heads, 4 tails
- ☐ C. 8 heads, 2 tails
- ☐ D. 9 heads, 1 tails
- ☐ E. 10 heads, 0 tails

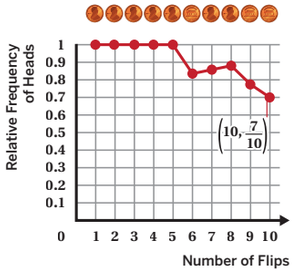
Responses vary.

3. One coin was flipped 10 times. The image shows that the coin landed 7 times heads up and 3 times tails up.

Do you think this is a fair coin?

Responses vary.

4. Here is a graph of the flips from the previous problem.



a. What does the point $\left(10, \frac{7}{10}\right)$ mean?

Responses vary. The point $\left(10, \frac{7}{10}\right)$ means that after 10 flips, the relative frequency of heads was $\frac{7}{10}$.

b. How might a graph help you determine if a coin is fair?

Responses vary. If the graph gets close to 0.5, then the coin is fair. If not, then it isn't fair.

Accelerated 6 | Unit 10 Lesson 5

985

Activity 1

Pairs | 15 min

Activity 1 Fair or Not? (continued)

Purpose: Students use the expected probabilities of fair objects and the results of a repeated experiment to decide if a coin is fair. They also approximate the probability of an unfair coin landing heads up. **(MTR.6.1)**

2 Monitor

Select and sequence a variety of explanations to share during the Connect.

D Differentiation

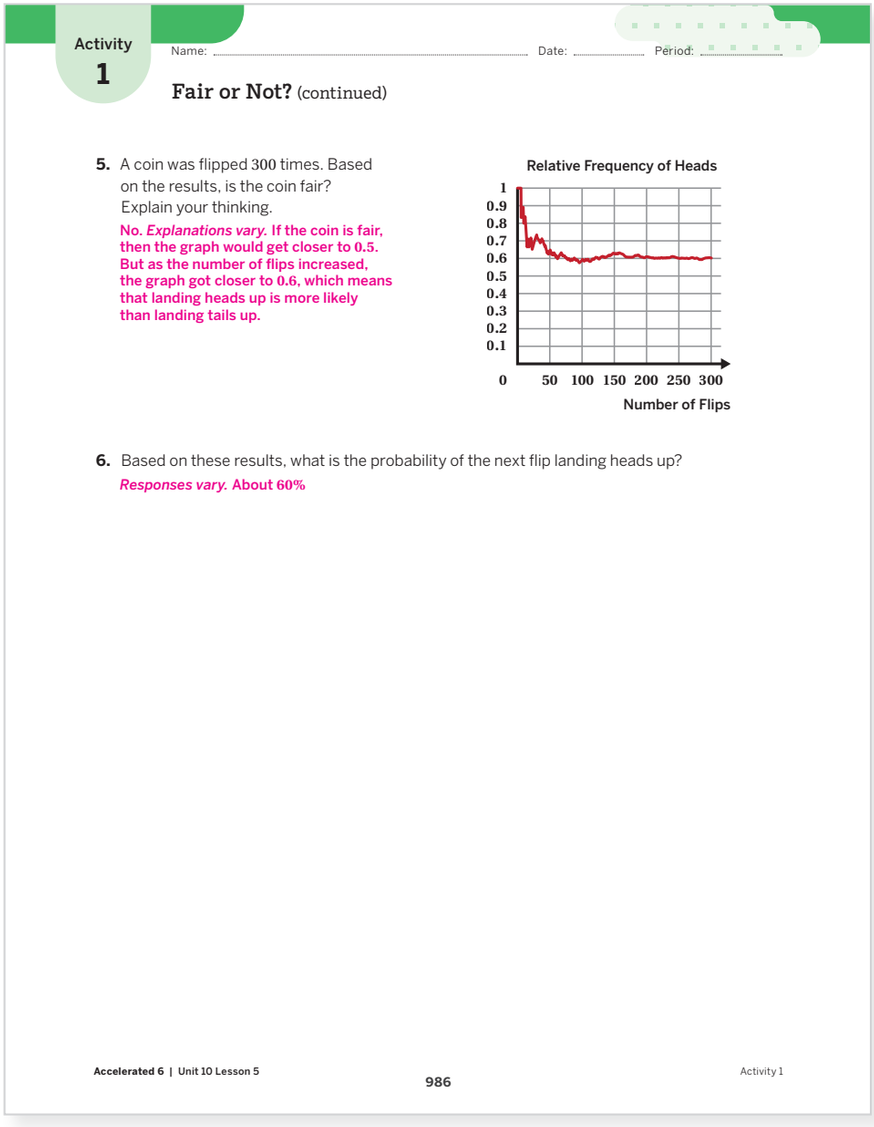
Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Consider asking, "What does the graph represent? The change in the graph over time?"
Think the coin is fair because 0.6 is close to 0.5.	**Strengthen** Encourage students to think about an example in a different context such as comparing \$0.50 per pound versus \$0.60 per pound when making a purchase at the grocery store.
Reason that heads facing up is more likely than tails.	**Strengthen** Invite them to share how they determined this during the Connect.

3 Connect

Invite students to share their responses, calling attention to any conflict or consensus.

To surface the Key Takeaway, share the selected explanations. Discuss what the graph of a fair coin might look like and how that differs from the graph of this coin.

MLR1: Stronger and Clearer Each Time (optional) Consider using this routine to support students in developing precise language. Invite students to meet with a partner, give feedback on each other's drafts, then use the feedback to write a second clearer draft. See the Routine Facilitation Guide for more information.



Activity 1

Name: Date: Period:

Fair or Not? (continued)

5. A coin was flipped 300 times. Based on the results, is the coin fair? Explain your thinking.

No. *Explanations vary.* If the coin is fair, then the graph would get closer to 0.5. But as the number of flips increased, the graph got closer to 0.6, which means that landing heads up is more likely than landing tails up.

6. Based on these results, what is the probability of the next flip landing heads up?

Responses vary. About 60%

Accelerated 6 | Unit 10 Lesson 5 986 Activity 1

Key Takeaway: Fair objects have an equal probability for each outcome to occur. When an experiment is repeated hundreds of times, the relative frequency of each outcome should be very close to the probability you expect. If it is not, the object may not be fair.

Pairs | 15 min

Activity 2 How Likely?

Purpose: Students use the results of a repeated experiment (rolling an unfair number cube) to determine probabilities of events. Students also use these probabilities to design a game that is fair. **(MTR.5.1)**

Short on time? Consider omitting Problem 8.

1 Launch

To generate excitement, invite students to choose who is Player 1 and Player 2.

Distribute a number cube to each pair of students.

2 Monitor

Look for a variety of explanations on whether the game is fair.

To support students getting started, consider asking, "What numbers result in Player 2 winning?"

ELL **English Language Learners** Provide sentence frames to help students explain their reasoning (e.g., I think the probability that Player 2 wins is ___ because _____.).

Activity 2 continued >

Activity 2

Name: Date: Period:

How Likely?

7. Here is a game involving a number cube.

What do you think is the probability that Player 2 wins on any single roll?

Responses vary.

Roll an odd number: Player 1 wins	Roll an even number: Player 2 wins

8. a Play a few rounds of the game using a number cube. Record your results in the table.

Responses vary.

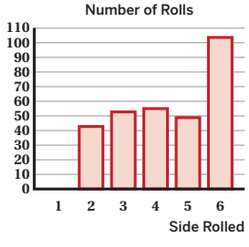
Player 1	Player 2

b Do you think this is a fair number cube? Explain your thinking.

Responses and explanations vary.

- Yes. I think this is a fair number cube because my partner and I won about an equal number of times.
- No. I don't think it's a fair number cube because my partner won more games than I did.

9. Here are the results from two students who played the game and rolled a number cube 300 times. What do you think is the probability that Player 2 wins? Explain your thinking.



Responses and explanations vary.

- It looks like the one is never rolled and the six is rolled twice as often as the others, so my guess is that there is 1 two, 1 four, and 2 sixes on this number cube. This would mean the probability of Player 2 winning is $\frac{4}{6}$.
- About 40 of the rolls were two, about 50 of them were four, and about 100 were six. So that would be $\frac{(40+50+100)}{300}$, or about a 63% chance of Player 2 winning.

Accelerated 6 | Unit 10 Lesson 5 987 Activity 2

Pairs | 15 min

Activity 2 How Likely? (continued)

Purpose: Students use the results of a repeated experiment (rolling an unfair number cube) to determine probabilities of events. Students also use these probabilities to design a game that is fair. **(MTR.5.1)**

2 Monitor

Select and sequence games that are fair and unfair to share during the Connect.

D Differentiation

Look for students who:	Respond to Student Thinking
Explain fairness in terms of probability.	Strengthen Invite students to use their game as an example and justify why it is fair.
Explain fairness in terms of the number of winning outcomes.	

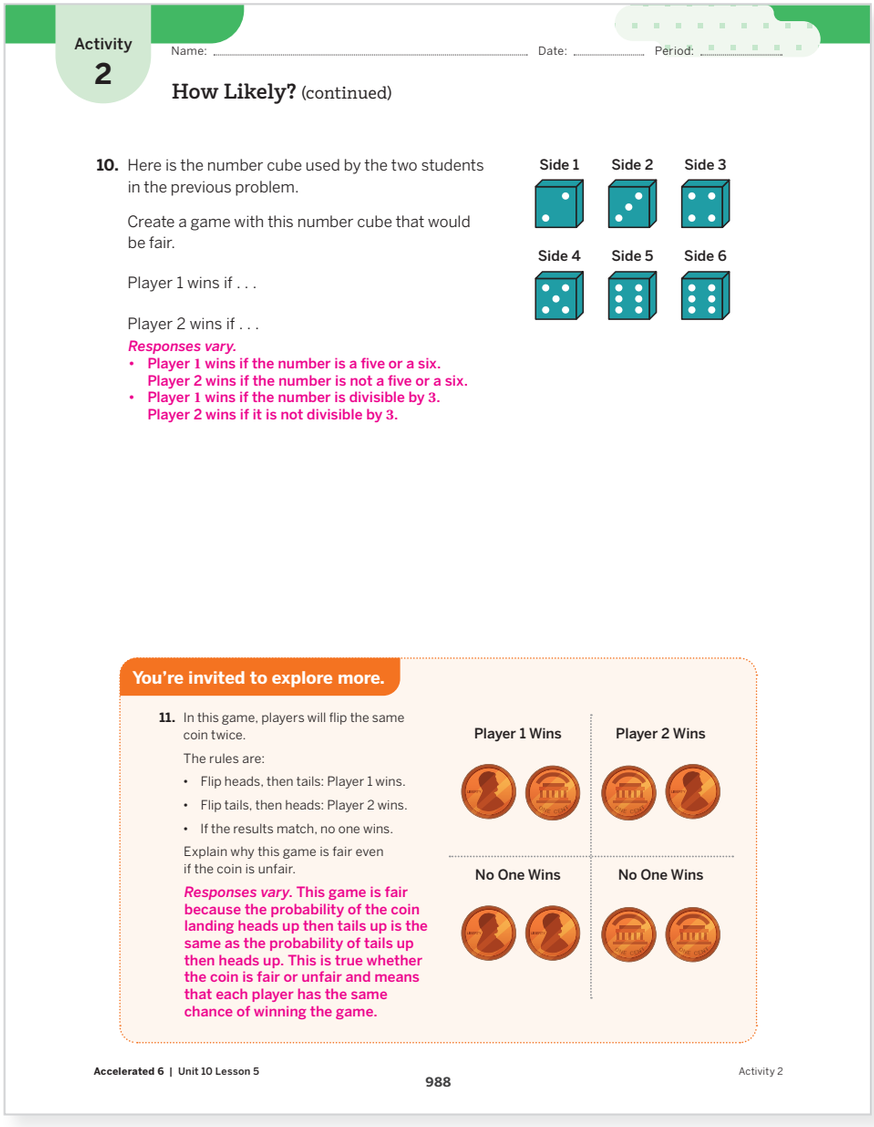
D Differentiation: You're invited to explore more. (Stretch) Invite students who would like to explore fair games further to complete this optional task. Encourage them to discuss their thinking with a partner.

3 Connect

Invite students to share the rules of their game and justify why the game is fair.

Consider asking:

- "If you roll a number cube 200 times and get a 6 every time, what can you reason about the number cube?" **(MTR.5.1)**
- "Would it be fair for two players to play your game with this number cube? Why? What is the probability each player would win?"



Activity 2

Name: Date: Period:

How Likely? (continued)

10. Here is the number cube used by the two students in the previous problem.

Create a game with this number cube that would be fair.

Player 1 wins if . . .

Player 2 wins if . . .

Side 1 Side 2 Side 3 Side 4 Side 5 Side 6

Responses vary.

- Player 1 wins if the number is a five or a six. Player 2 wins if the number is not a five or a six.
- Player 1 wins if the number is divisible by 3. Player 2 wins if it is not divisible by 3.

You're invited to explore more.

11. In this game, players will flip the same coin twice.

The rules are:

- Flip heads, then tails: Player 1 wins.
- Flip tails, then heads: Player 2 wins.
- If the results match, no one wins.

Explain why this game is fair even if the coin is unfair.

Responses vary. This game is fair because the probability of the coin landing heads up then tails up is the same as the probability of tails up then heads up. This is true whether the coin is fair or unfair and means that each player has the same chance of winning the game.

Player 1 Wins | Player 2 Wins | No One Wins | No One Wins

Accelerated 6 | Unit 10 Lesson 5 988 Activity 2

Whole Class | 5 min

Synthesis

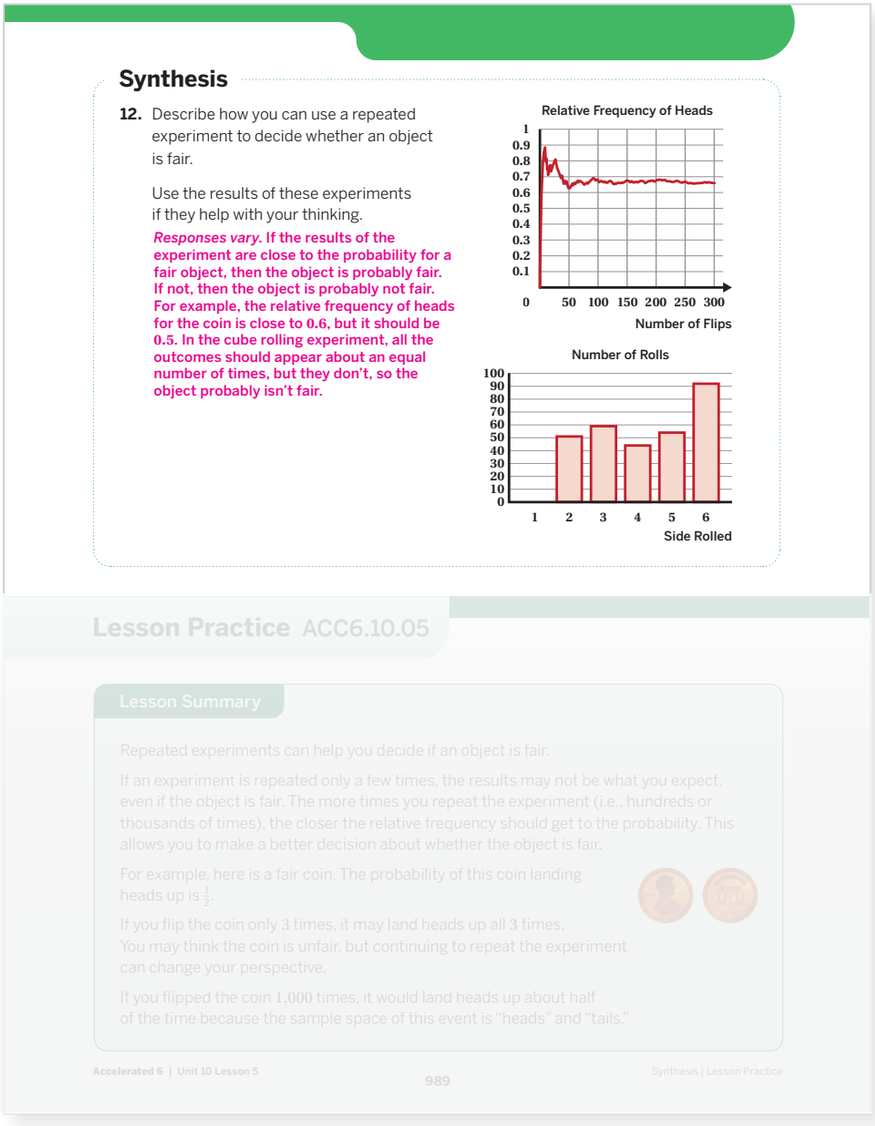
Purpose: Students synthesize their understanding of using repeated experiments to decide whether an object is fair.

Invite students to respond independently, and then share their thinking with a partner.

Collect and share a variety of ideas, such as:

- Comparing the relative frequency from the experiment to the probability you expect from a fair object.
- The importance of repeating an experiment hundreds of times.

Note: If time allows, invite students to revise their previous thinking based on the discussion.



Synthesis

12. Describe how you can use a repeated experiment to decide whether an object is fair.

Use the results of these experiments if they help with your thinking.

Responses vary. **If the results of the experiment are close to the probability for a fair object, then the object is probably fair. If not, then the object is probably not fair. For example, the relative frequency of heads for the coin is close to 0.6, but it should be 0.5. In the cube rolling experiment, all the outcomes should appear about an equal number of times, but they don't, so the object probably isn't fair.**

Lesson Practice ACC6.10.05

Lesson Summary

Repeated experiments can help you decide if an object is fair.

If an experiment is repeated only a few times, the results may not be what you expect, even if the object is fair. The more times you repeat the experiment (i.e., hundreds or thousands of times), the closer the relative frequency should get to the probability. This allows you to make a better decision about whether the object is fair.

For example, here is a fair coin. The probability of this coin landing heads up is $\frac{1}{2}$.

If you flip the coin only 3 times, it may land heads up all 3 times. You may think the coin is unfair, but continuing to repeat the experiment can change your perspective.

If you flipped the coin 1,000 times, it would land heads up about half of the time because the sample space of this event is "heads" and "tails."

Accelerated 6 | Unit 10 Lesson 5 989 Synthesis | Lesson Practice

Lesson Takeaway: The results from a repeated experiment can be used to approximate the probability of an event and help decide if an object is fair.

Independent | 5 min

Show What You Know

Purpose: Students demonstrate their understanding by determining the probability of an event occurring based on the results of a repeated experiment.

Today's Goals

1. **Goal:** Use the results from a repeated experiment to approximate the probability of an event.
2. **Language Goal:** Explain whether an object is fair based on the results of a repeated experiment. **(Reading and Writing)**

Show What You Know (Assessment Resources)

Name: _____ Date: _____ Period: _____

Show What You Know 10.05

This graph shows the results of 100 rolls with an unfair number cube.

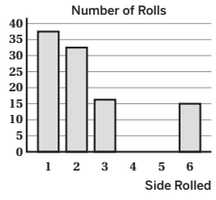
Based on the results, which is the most likely probability of rolling a one?

A. $\frac{1}{2}$

B. $\frac{1}{3}$

C. $\frac{1}{4}$

D. $\frac{1}{6}$



Explain your thinking.

Explanations vary. The results suggest that the number cube has one dot on 2 of its 6 sides. That means the probability of rolling a one is $\frac{2}{6}$ or $\frac{1}{3}$.

D Differentiation Use after Lesson 5

Support

Provide targeted intervention for students by using these resources.

If students need support understanding how to use the results from a repeated experiment to approximate the probability of an event:

» **Respond to Student Thinking:**

- Review the **Lesson 4 Summary**.
- Select a few related **Practice Problems** and give students individualized feedback.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 1 Extension Activities**.
- Invite students to complete the **Explore More** task.

Professional Learning

Which students' ideas were you able to highlight when deciding if the coin is fair? Did student-to-student discourse help engage students and aid in their understanding?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

12. Describe how you can use a repeated experiment to decide whether an object is fair.

Use the results of these experiments if they help with your thinking.

Responses vary. If the results of the experiment are close to the probability for a fair object, then the object is probably fair. If not, then the object is probably not fair. For example, the relative frequency of heads for the coin is close to 0.6, but it should be 0.5. In the cube rolling experiment, all the outcomes should appear about an equal number of times, but they don't, so the object probably isn't fair.

Lesson Practice ACC6.10.05

Lesson Summary

Repeated experiments can help you decide if an object is fair.

If an experiment is repeated only a few times, the results may not be what you expect, even if the object is fair. The more times you repeat the experiment (i.e., hundreds or thousands of times), the closer the relative frequency should get to the probability. This allows you to make a better decision about whether the object is fair.

For example, here is a fair coin. The probability of this coin landing heads up is $\frac{1}{2}$.

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If you flipped the coin 1,000 times, it would land heads up about half of the time because the sample space of this event is "heads" and "tails."

Lesson Practice ACC6.10.05

Name: _____ Date: _____ Period: _____

Problems 1–3: Deja has a six-sided number cube.

1. If this were a standard number cube, what would be the probability that the cube lands on a five?

$\frac{1}{6}$ (or equivalent)

2. Deja suspects the six-sided number cube is not standard.

- Deja rolled a five 40 times out of 100.
- Manuel rolled a five 21 times out 50.
- Santino rolled a five 11 times out of 30.

Based on these results, estimate the probability of rolling a five.

About 40% (or equivalent)

3. Is it likely this is a standard number cube? Explain your thinking.

No. *Explanations vary.* A standard number cube should land on five $\frac{1}{6}$ of the time. After 100 rolls, it should land on five about 16 or 17 times. All three people had it land on five more than twice as often. With this many rolls, there is strong evidence that this cube is not standard.

Problems 4–5: Santino wants to know if his coin is fair, so he flips it 10 times. It lands heads up 3 times and tails up 7 times.

4. Are these results enough to determine if the coin is fair? Explain your thinking.

No. *Explanations vary.* The relative frequency from experiments is usually a good approximation for the probability of an event, but only when the experiment is done many, many times. 10 flips may not be enough for the relative frequency to be close to the $\frac{1}{2}$ probability.

5. What could Santino do to be more sure of his results?

Santino should flip the coin many more times.

Problems 6–9: A game is played with two tetrahedral (4-sided) dice. The dice are in the shape of a pyramid with the numbers 1, 2, 3, and 4 written on each triangular side. After the dice are rolled, the numbers on the sides facing down are added together.

6. What are all of the possible sums for rolling two tetrahedral dice?

2, 3, 4, 5, 6, 7, 8

7. What sum do you think you will be most likely to get when rolling these dice? Least likely? Explain your thinking.

A sum of 5 is most likely and a sum of 2 or 8 are least likely. *Explanations vary.* I think rolling a sum of 5 will be the most likely because there are lot of ways to get 5: 1 and 4, 2 and 3, 3 and 2, and 4 and 1. I think rolling a sum of 2 or 8 will be the least likely because there is only one way to roll each of these: 1 and 1 and 4 and 4.

8. Create rules for a fair two-player game that uses these dice. *Responses vary.*

Player A wins if... the sum of the dice is 4. Player B wins if... the sum of the dice is 6.

Lesson Practice ACC6.10.05

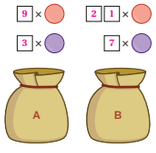
Name: _____ Date: _____ Period: _____

9. Explain why you think your game in Problem 8 is fair.

Responses vary. I think my game is fair because rolling a sum of 4 or a sum of 6 have the same likelihood of occurring. There are three ways to roll a sum of 4: $1+3$, $2+2$, and $3+1$. There are three ways to roll a sum of 6: $2+4$, $3+3$, and $4+2$. So, I assigned them the same number of points.

10. Fill in each blank using the digits 0 to 9 only once so that the probability of drawing a purple marble is the same for each bag.

Responses vary.



FAST Practice

11. A random number generator selects a digit from 1 to 5. Troy uses the generator 1,500 times. Which statement best predicts how many times the digit 3 will appear?

A. It will appear exactly 300 times.

B. It will appear exactly 340 times.

C. It will appear close to 300 times but probably not exactly 300 times.

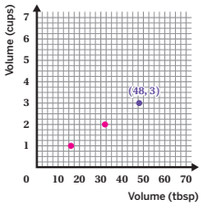
D. It will appear close to 340 times but probably not exactly 340 times.

Spiral Review

Problems 12–14: Sugar can be measured in cups and measured in tablespoons. For example, 48 tablespoons of sugar is equivalent to 3 cups of sugar.

12. Plot and list the coordinates of two more points that represent the relationship.

Points where the y-coordinate is $\frac{1}{16}$ of the x-coordinate are considered correct.



13. What is the conversion rate of tablespoons to 1 cup?

16 tablespoons = 1 cup

14. Use your conversion rate to determine the number of tablespoons in 10 cups of sugar.

160 tablespoons of sugar

Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1	1	MA.7.DP.2.3
	2–7	2	MA.7.DP.2.4
	8–10	3	MA.7.DP.2.4
FAST Practice	11	2	MA.7.DP.2.4
Spiral Review			
	12	1	MA.6.NSO.1.1, MA.6.AR.3.5
	13–14	2	MA.6.AR.3.5

Assess and Respond

End-of-Unit Assessment

Independent | 45 min

Facilitation: Assign the End-of-Unit Assessment to learn about your students' understanding of the concepts and skills in this unit. There are two forms of the End-of-Unit Assessment: Forms A and B. If time allows, consider inviting students to complete one or more of the tasks in the Unit Synthesis and Reflection.

Item Analysis, Forms A and B

Problem(s)	Concept or skill	Addressed in	DOK	FL B.E.S.T. Math Benchmarks
1	Determining the probability of events	Lesson 2	1	MA.7.DP.2.2
2	Comparing and contrasting data of boxplots in context	Lesson 3	2	MA.7.DP.1.2, MTR.6.1
3	Estimating and explaining predicted outcomes of probability events	Lesson 4	2	MA.7.DP.2.4, MTR.4.1
4a, 4b	Calculating the probability of events	Lesson 2	1	MA.7.DP.2.3
5a	Identifying the population of a random sample	Lesson 4	2	MA.7.DP.1.3, MTR.6.1
5b	Making predictions about a population	Lesson 4	2	MA.7.DP.1.3, MTR.6.1
5c	Explaining how results from multiple samples affect their confidence in predictions	Lesson 5	2	MA.7.DP.1.3, MTR.6.1

Assessment Resources

- Student Assessments (Forms A and B)
- Assessment Rubrics with Answers
- Show What You Knows and Answer Keys

Intervention and Extension Resources

- Mini-Lessons
- Extensions

Name: Date: Period:

End-of-Unit Assessment

Unit 10

1. You pick a block out of this bag without looking.

What is the probability of picking a star?

A. $\frac{1}{3}$ B. $\frac{3}{4}$ C. $\frac{3}{7}$ D. $\frac{4}{7}$

2. Adriana is curious: do more customers go to fast-food restaurants or to other restaurants?

She went to a random sample of 50 restaurants (25 fast-food and 25 other).

At each restaurant, Adriana recorded the number of customers the restaurant had that day.

Based on the data, select *all* of the true statements.

- ☑ A. Fast-food restaurants tend to get more customers than other restaurants.
- ☐ B. Other restaurants tend to get more customers than fast-food restaurants.
- ☐ C. Fast-food restaurants have a more consistent number of customers than other restaurants.
- ☑ D. Other restaurants have a more consistent number of customers than fast-food restaurants.
- ☐ E. All fast-food restaurants have more customers than other restaurants.

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Name: Date: Period:

End-of-Unit Assessment (continued)

Unit 10

3. Five friends share an apartment. Each week, they use this spinner with 5 equal sections to determine who will mop the floor.

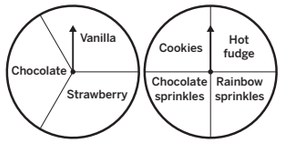
Estimate the number of times Abdel will be chosen to mop in a year (about 52 weeks).

10 or 11 are considered correct

Explain your thinking.

Explanations vary. The probability of the spinner landing on Abdel is $\frac{1}{5}$. That means the best estimate for how many spins out of 52 will land on Abdel is $\frac{1}{5}$ of 52, or 10.4, which rounds to 10.

4. The Spin N' Dine Restaurant sells ice cream for dessert. You can spin to either select a flavor of ice cream at random OR one topping at random.



a. What is the probability of getting chocolate ice cream on your spin?

$\frac{1}{3}$

b. What is the probability of NOT getting Hot fudge as a topping on your spin?

$\frac{3}{4}$

5. Rudra thinks school should have a longer lunch period and wants to know if others agree. He surveys a random sample of 20 students and find that 12 of them agree.

a. What is the population for Rudra question?

All the students at Rudra school

b. If the school has 250 students, about how many do you predict would agree?

150 students

c. The next day, Rudra surveys another 20 random students and finds that 8 of them are in favor. Does this make you more or less confident in your prediction?

Less confident

Explain your thinking.

Explanations vary. In Rudra first sample, over 50% of students agreed. In his second sample, less than 50% agreed. This means that the variation is large, which makes me less confident in my prediction.

Name: ..... Date: ..... Period: .....

End-of-Unit Assessment

Unit 10

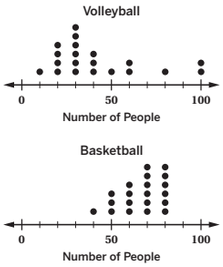
1. You pick a block out of this bag without looking.

What is the probability of picking a block that is *not* a star?

A. $\frac{1}{4}$ B. $\frac{4}{5}$ C. $\frac{4}{9}$ D. $\frac{5}{9}$

2. Ebony is curious: do more people attend volleyball or basketball games?

She went to a random sample of 40 games (20 volleyball and 20 basketball) At each game, Ebony recorded the number of people.



Based on the data in the dot plots, select *all* of the true statements.

- ☑ A. Basketball games tend to get more people to attend than volleyball games.
- ☐ B. Volleyball games tend to get more people to attend than basketball games.
- ☑ C. Basketball games have a more consistent number of people than volleyball games.
- ☐ D. Volleyball games have a more consistent number of people than basketball games.
- ☐ E. All basketball games have more people attending than volleyball games.

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Name: ..... Date: ..... Period: .....

End-of-Unit Assessment (continued)

Unit 10

3. Six friends share an apartment. Each day, they use this spinner with 6 equal sections to determine who will wash the dishes.

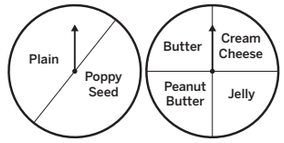
Estimate the number of times Binta will be chosen to do the dishes in a month (about 30 days).

5 times

Explain your thinking.

Explanations vary. A month has about 30 days. The probability of the spinner landing on Binta is $\frac{1}{6}$. That means the best estimate for how many spins in a month will land on Binta is about $\frac{1}{6}$ of 30, or 5.

4. The Spin N' Dine Restaurant sells bagels. You can spin to either select one random type of bagel OR one random spread.



a. What is the probability of getting a plain bagel on your spin?

$\frac{1}{2}$

b. What is the probability of NOT getting cream cheese as a topping on your spin?

$\frac{3}{4}$

5. Darius thinks school should start later and wants to know if others agree. He surveys a random sample of 40 students and finds that 26 of them agree.

a. What is the population for Darius's question?

All the students at Darius's school

b. If the school has 240 students, about how many do you predict would agree?

156 students

c. The next day, Darius surveys another 10 random students and finds that 6 of them agree. Does this make you more or less confident in your prediction?

More confident

Explain your thinking.

Explanations vary. In Darius's first sample, over 60% of the students agreed. In his second sample, 60% also agreed. This means that the variation is small, which makes me more confident in my prediction.

D Differentiation (End-of-Unit Assessment)

Note: To **strengthen** and **stretch** students' learning, refer to the differentiation resources suggested throughout this unit and in the Unit Overview.

Sub-Unit Goals	Problem(s)	To respond to student thinking, consider:	
Sub-Unit 1: Compare probabilities to the results of repeated experiments. Determine the probability of events by identifying the sample space. **(Lessons 1–5)**	1, 4a, 4b	Support	• **Teacher Move:** Consider revisiting: • Lesson 1 (*How Likely?*) • Lesson 2 (*Prob-bear-bilities*)
		Stretch	• **You're Invited to Explore More:** Lesson 2 (*Prob-bear-bilities*)
	3	Support	• **Teacher Move:** Consider revisiting: • Lesson 4 (*Spin Class*) • Lesson 5 (*Is It Fair?*)
		Strengthen	• **Challenge Creator:** Lesson 4 (*Spin Class*)
		Stretch	• **You're invited to explore more:** Lesson 5 (*Is It Fair?*)
	2	Support	• **Teacher Move:** Consider using a class project involving collecting data on the number of student's pets and creating a box plot to visualize the distribution of these results.
		Strengthen	• **Teacher Move:** Invite students to write questions from the data collection on student's pets to share with a partner.
	5	Support	• **Mini-Lesson: Predicting Population Using Sample Data**
		Strengthen	• **Teacher Move:** Invite students to share with a partner how results from multiple samples could affect predictions.

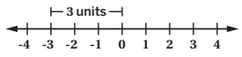
Glossary/Glosario

English

Español

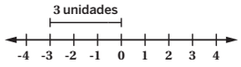
A

absolute value The distance from 0 to a number on a number line is its absolute value.



For example, the absolute value of -3 is 3 because -3 is 3 units away from 0. This is written as $|-3| = 3$.

valor absoluto La distancia del 0 a un número en una recta numérica es su valor absoluto.



Por ejemplo, el valor absoluto de -3 es 3 porque -3 está a 3 unidades del 0. Esto se escribe $|-3| = 3$.

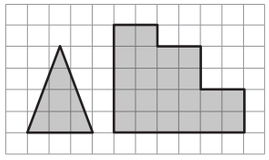
additive inverses The additive inverse of a is $-a$. The sum of two additive inverses is 0.

For example, 5 and -5 are additive inverses because $5 + (-5) = 0$.

inversos aditivos El inverso aditivo de a es $-a$. La suma de dos inversos aditivos es 0.

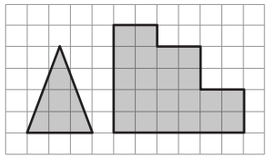
Por ejemplo, 5 y -5 son inversos aditivos porque $5 + (-5) = 0$.

area Area measures the space inside a two-dimensional figure. It is expressed in square units.



The area of the triangle is 6 square units. The area of the other shape is 22 square units.

área El área mide el espacio dentro de una figura bidimensional. Se expresa en unidades cuadradas.



El área del triángulo mide 6 unidades cuadradas. El área de la otra figura mide 22 unidades cuadradas.

associative property The property says $a + (b + c) = (a + b) + c$ and $a \cdot (b \cdot c) = (a \cdot b) \cdot c$. This means that expressions with addition or multiplication have the same sum or product no matter how the numbers in the expression are grouped.

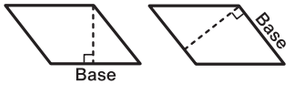
For example, $(2 + 1) + 3 = 2 + (1 + 3)$ and $(3 \cdot 4) \cdot 5 = 3 \cdot (4 \cdot 5)$.

propiedad asociativa La propiedad indica que $a + (b + c) = (a + b) + c$ y $a \cdot (b \cdot c) = (a \cdot b) \cdot c$. Esto significa que las expresiones en las que se suma o se multiplica tienen la misma suma o el mismo producto independientemente de cómo se agrupen los números en la expresión.

Por ejemplo, $(2 + 1) + 3 = 2 + (1 + 3)$ y $(3 \cdot 4) \cdot 5 = 3 \cdot (4 \cdot 5)$.

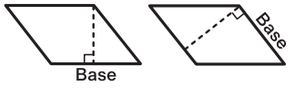
B

base (of a parallelogram) One side of a parallelogram. We can choose any side to be the base.



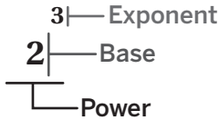
The base can also refer to the length of this side. The height of a shape is perpendicular to the base.

base (de un paralelogramo) Un lado de un paralelogramo. Podemos elegir cualquier lado como base.



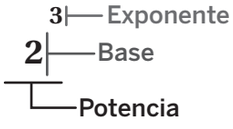
La base también puede referirse a la longitud de este lado. La altura de una figura es perpendicular a la base.

base (of a power) The number that is raised to an exponent. When determining the value of a power, the exponent tells you how many times the base should be multiplied.



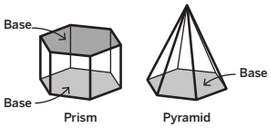
In the expression 2^3, 2 is the base.

base (de una potencia) El número elevado a un exponente. Al determinar el valor de una potencia, el exponente indica cuántas veces debe multiplicarse la base.

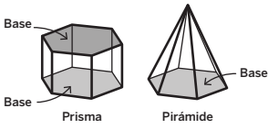


En la expresión 2^3, 2 es la base.

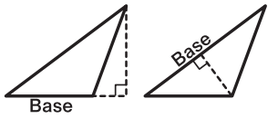
base (of a pyramid or prism) The face that gives the solid its name. A prism has two identical bases that are parallel. A pyramid has one base.



base (de una pirámide o un prisma) La cara que da el nombre al cuerpo geométrico. Un prisma tiene dos bases idénticas que son paralelas. Una pirámide tiene una base.

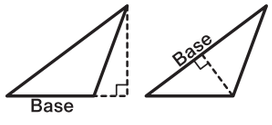


base (of a triangle) The base is one side of a triangle. We can choose any side to be the base.



The base can also refer to the length of this side. The height of a shape is perpendicular to the base.

base (de un triángulo) La base es un lado de un triángulo. Podemos elegir cualquier lado como base.

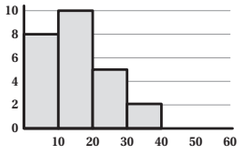


La base también puede referirse a la longitud de este lado. La altura de una figura es perpendicular a la base.

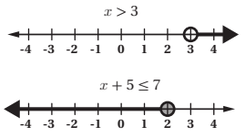
English

bin (of a histogram) The intervals used to group data values in a histogram.

For example, this histogram shows bins of 10 units each.

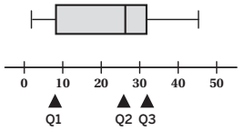


boundary point The point on a number line or graph that separates solutions of an inequality from non-solutions.
If the boundary point is a solution to the inequality (i.e., ≥ or ≤), it's represented with a closed circle on the graph. If it's not a solution (i.e., > or <), it's represented with an open circle on the graph.



The boundary point of $x > 3$ is 3. The boundary point of $x \leq 2$ is 2.

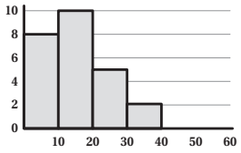
box plot A way to visualize numerical data sets. The data is divided into four sections using five values: the minimum, Q1, Q2 (or the median), Q3, and the maximum. A box is drawn between Q1 and Q3. The line inside the box represents the median.



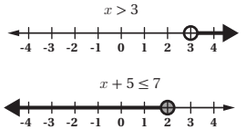
Español

intervalo (de un histograma) Los intervalos que se usan para agrupar los valores de datos en un histograma.

Por ejemplo, este histograma muestra intervalos de 10 unidades cada uno.

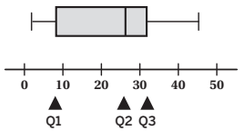


punto límite Punto en una recta numérica o una gráfica que separa las soluciones de una desigualdad de los valores que no son soluciones.
Si el punto límite es una solución de la desigualdad (es decir, ≥ o ≤), se representa con un círculo cerrado en la gráfica. Si no es una solución (es decir, > o <), se representa con un círculo abierto en la gráfica.



El punto límite de $x > 3$ es 3. El punto límite de $x \leq 2$ es 2.

diagrama de caja Una forma de visualizar conjuntos de datos numéricos. Los datos se dividen en cuatro secciones utilizando cinco valores: el mínimo, Q1, Q2 (o la mediana), Q3 y el máximo.
Se dibuja una caja entre Q1 y Q3. La línea dentro de la caja representa la mediana.



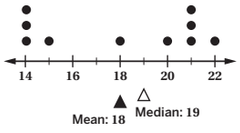
C

categorical data Data that can be sorted into categories instead of counted. Categorical data usually have values that are represented by words instead of numbers.

"What kind of pet do you have?" is a question that would result in categorical data.

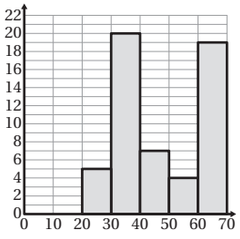
center A single value that can be used to summarize the typical value in a data set.

Mean and median are measures of center.



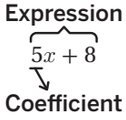
cluster A group of data values that are close together.

For example, this histogram shows a cluster between 30 and 40 and between 60 and 70.



coefficient A number that is multiplied by a variable. Usually, there is no symbol between the coefficient and the variable.

In the expression $5x + 8$, 5 is the coefficient of x.

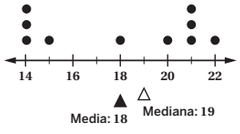


datos categóricos Datos que pueden clasificarse en categorías en lugar de contarse. Los datos categóricos suelen tener valores que se representan mediante palabras en lugar de números.

"¿Qué tipo de mascota tienes?" es una pregunta que produciría datos categóricos.

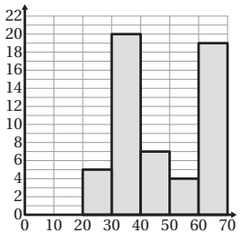
centro Un solo valor que puede utilizarse para resumir el valor típico de un conjunto de datos.

La media y la mediana son medidas de tendencia central, o sea, medidas de centro.



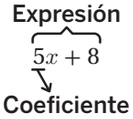
agrupación Un grupo de valores de datos que están próximos entre sí.

Por ejemplo, este histograma muestra una agrupación entre 30 y 40, y entre 60 y 70.



coeficiente Un número que se multiplica por una variable. Por lo general, no hay ningún símbolo entre el coeficiente y la variable.

En la expresión $5x + 8$, el coeficiente de x es 5.



Glossary/Glosario

English

common denominator The denominator is the bottom number in a fraction. Two fractions have a common denominator when their denominators are the same.

For example, $\frac{3}{4}$ and $\frac{5}{4}$ have a common denominator because they each split the whole into fourths.

common factor When two numbers have the same factor, we call that a common factor.

For example, the factors of 8 are 1, 2, 4, and 8 and the factors of 12 are 1, 2, 3, 4, 6, and 12.

Since 2 is a factor of 8 and also of 12, 2 is a common factor of 8 and 12.

common multiple When two numbers have the same multiple, we call that a common multiple.

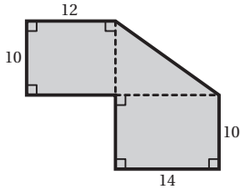
For example, 12 is a multiple of 2 and also of 3, so 12 is a common multiple of 2 and 3.

commutative property The property says $a + b = b + a$ and $a \cdot b = b \cdot a$. This means that expressions with addition or multiplication have the same sum or product no matter what order the numbers are in.

For example, $2 + 1 = 1 + 2$ or $3 \cdot 4 = 4 \cdot 3$.

composite figure A two- or three-dimensional figure that can be decomposed into smaller figures.

The composite figure can be decomposed into 2 rectangles and a triangle.



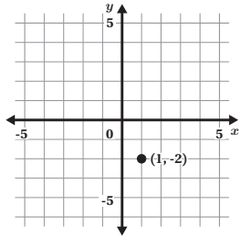
constant A value that does not change. In an algebraic expression, the constant term does not contain a variable.

In the expression $7x + 9$, 9 is the constant term.

In the expression $5x - 8$, -8 is the constant term.

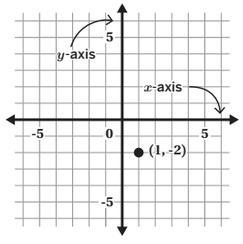
In the expression $12 - 4x$, 12 is the constant term.

coordinate plane The coordinate plane consists of two axes that intersect at 0: one horizontal (often called the x-axis) and one vertical (often called the y-axis).



coordinates A pair of numbers that shows an exact position on the coordinate plane. The first number represents a position on the x-axis and is called the x-coordinate. The second number represents a position on the y-axis and is called the y-coordinate.

The coordinates of the point on the graph are $(1, -2)$.



Español

denominador común El denominador es el número de abajo en una fracción. Dos fracciones tienen un denominador común cuando los denominadores son iguales.

Por ejemplo, $\frac{3}{4}$ y $\frac{5}{4}$ tienen un denominador común porque cada uno divide el todo en cuartos.

factor común Cuando dos números tienen el mismo factor, lo llamamos factor común.

Por ejemplo, los factores de 8 son 1, 2, 4 y 8 y los factores de 12 son 1, 2, 3, 4, 6 y 12.

Ya que 2 es un factor de 8 y también de 12, 2 es un factor común de 8 y 12.

múltiplo común Cuando dos números tienen el mismo múltiplo, lo llamamos múltiplo común.

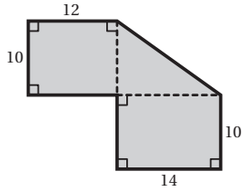
Por ejemplo, 12 es un múltiplo de 2 y también de 3, por lo tanto, 12 es un múltiplo común de 2 y 3.

propiedad conmutativa La propiedad indica que $a + b = b + a$ y $a \cdot b = b \cdot a$. Esto significa que las expresiones en las que se suma o se multiplica tienen la misma suma o el mismo producto, independientemente del orden en el que estén los números.

Por ejemplo, $2 + 1 = 1 + 2$ o $3 \cdot 4 = 4 \cdot 3$.

figura compuesta Figura bidimensional o tridimensional que puede descomponerse en figuras más pequeñas.

La figura compuesta se puede descomponer en 2 rectángulos y un triángulo.



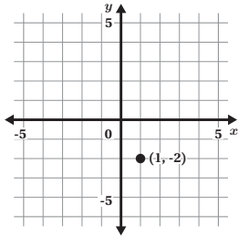
constante Un valor que no cambia. En una expresión algebraica, el término constante no contiene una variable.

En la expresión $7x + 9$, 9 es el término constante.

En la expresión $5x - 8$, -8 es el término constante.

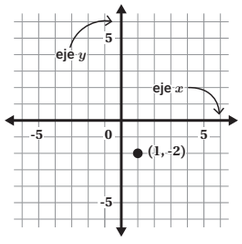
En la expresión $12 - 4x$, 12 es el término constante.

plano de coordenadas El plano de coordenadas consta de dos ejes que se intersecan en 0: uno horizontal (a menudo llamado el eje x) y uno vertical (a menudo llamado el eje y).



coordenadas Un par de números que muestran una posición exacta en el plano de coordenadas. El primer número representa una posición en el eje x y se denomina coordenada x. El segundo número representa una posición en el eje y y se denomina coordenada y.

Las coordenadas del punto en la gráfica son $(1, -2)$.

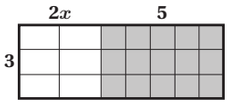


English

D

decompose Decompose means "take apart." We use the word *decompose* to describe taking a figure apart to make more than one new shape.

distributive property The property that says $a(b + c) = ab + ac$. This means that multiplying a number by the sum of two or more terms is equal to multiplying the number by each term separately before adding the products together.



Area as a Product
$3(2x + 5)$

For example, $3(2x + 5) = 3 \cdot 2x + 3 \cdot 5 = 6x + 15$.

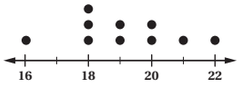
dividend The number in a division statement that is being divided.

For example, in the equation $12 \div 3 = 4$, the dividend is 12.

divisor In a division statement, the divisor describes the number of equal-sized groups or the size of each group being created.

For example, in the equation $12 \div 3 = 4$, the divisor is 3.

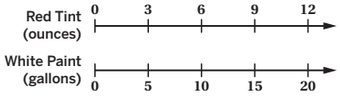
dot plot A way to visualize numerical data sets, where each data point is represented by a dot on a number line. Data points with the same value are stacked on top of each other. A dot plot is sometimes called a line plot.



Number of Jelly Beans

For example, this dot plot shows that 3 students guessed that there were 18 jelly beans in a jar.

double number line A pair of parallel number lines showing equivalent ratios. The tick marks are labeled so that the marks that line up vertically are equivalent ratios.



This double number line shows a ratio of 3 ounces of red tint : 5 gallons of white paint.

E

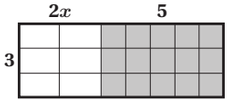
edge Each straight side of a polygon is called an edge. An edge is also a line segment where two faces of a 3-D figure meet.

This rectangle has four edges.

Español

descomponer Descomponer significa "desmontar." Usamos la palabra *descomponer* para describir que una figura se desmonta para formar más de una figura nueva.

propiedad distributiva La propiedad que indica que $a(b + c) = ab + ac$. Significa que multiplicar un número por la suma de dos o más términos equivale a multiplicar el número por cada término individualmente y luego sumar los productos.



El área como un producto
$3(2x + 5)$

Por ejemplo, $3(2x + 5) = 3 \cdot 2x + 3 \cdot 5 = 6x + 15$.

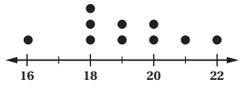
dividendo El número que se está dividiendo en un enunciado de división.

Por ejemplo, en la ecuación $12 \div 3 = 4$, el dividendo es 12.

divisor En un enunciado de división, el divisor describe la cantidad de grupos de igual tamaño o el tamaño de cada grupo que se produce.

Por ejemplo, esta ecuación $12 \div 3 = 4$, el divisor es 3.

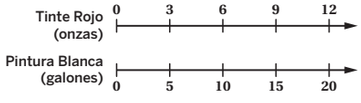
diagrama de puntos Una forma de visualizar conjuntos de datos numéricos, en la que cada punto de datos se representa mediante un punto en una recta numérica. Los puntos de datos con el mismo valor se apilan unos sobre otros. Un diagrama de puntos a veces se conoce como gráfica de puntos.



Cantidad de frijolitos

Por ejemplo, este diagrama de puntos muestra que 3 estudiantes estimaron que había 18 frijolitos de jalea en un tarro.

recta numérica doble Un par de rectas numéricas paralelas que muestran razones equivalentes. Las marcas indicadoras se denominan de forma tal que las marcas alineadas verticalmente sean razones equivalentes.



Esta recta numérica doble muestra una razón de 3 onzas de tinte rojo : 5 galones de pintura blanca.

lado, arista Cada borde recto de un polígono se llama arista o lado. Una arista también es un segmento de recta donde se unen dos caras de una figura tridimensional.

Este rectángulo tiene cuatro aristas.

Glossary/Glosario

English

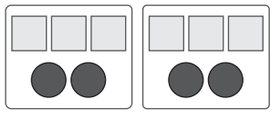
equation A mathematical statement made up of two expressions with an equal sign between them.

For example, $6m + 5 = 17$ and $12 - 15 = -3$ are equations, but $2n$ and $x > 5$ are not equations.

equivalent expressions Expressions that are equal for every value of a variable.

$x + x + x$ is equivalent to $3x$ because they both describe three copies of an unknown number, x.

equivalent ratio Two ratios are equivalent if you can multiply each of the values in the first ratio by the same number to get the values in the second ratio.



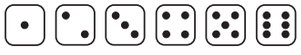
$3 : 2$ is equivalent to $6 : 4$ because $3 \cdot 2 = 6$ and $2 \cdot 2 = 4$.

One lemonade recipe uses 3 cups of water and 2 lemons. Another uses 6 cups of water and 4 lemons. The second recipe will make twice as much lemonade but both recipes will taste the same.

evaluate To evaluate is to determine a single number that represents an expression's value.

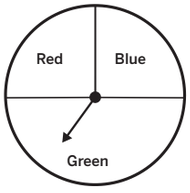
For example, to evaluate $5x + 2$ when $x = 3$, we substitute 3 for x and then calculate $5(3) + 2 = 17$.

event A set of one or more outcomes in a chance experiment.



For example, if we roll a number cube, there are six possible outcomes.

experiment An action or activity that you can do over and over again without knowing what the outcome will be each time.



For example, each time you spin the spinner, it could land on red, blue, or green.

exponent A number used to describe repeated multiplication. Exponents are sometimes called powers.

$2^4 = 16$

For example, $2 \cdot 2 \cdot 2 \cdot 2 = 16$ can be represented by the equation $2^4 = 16$, where 4 is the exponent. We can read this equation as "2 to the power of 4 equals 16" or "2 to the fourth equals 16."

expression A set of numbers, variables, operations, and grouping symbols that represent a quantity.

For example, $2n - 8$ and $21 + 37$ are expressions.

Español

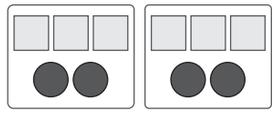
ecuación Un enunciado matemático formado por dos expresiones con un signo igual entre ellas.

Por ejemplo, $6m + 5 = 17$ y $12 - 15 = -3$ son ecuaciones, pero $2n$ y $x > 5$ no son ecuaciones.

expresiones equivalentes Expresiones que son iguales para cualquier valor de una variable.

$x + x + x$ equivale a $3x$ porque ambas describen tres copias de un número desconocido, x.

razón equivalente Dos razones son equivalentes si cada uno de los valores de la primera razón se puede multiplicar por el mismo número para obtener los valores de la segunda razón.



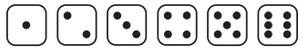
$3 : 2$ es equivalente a $6 : 4$ porque $3 \cdot 2 = 6$ y $2 \cdot 2 = 4$.

Una receta de limonada lleva 3 tazas de agua y 2 limones. Otra lleva 6 tazas de agua y 4 limones. La segunda receta rendirá el doble de limonada, pero ambas tendrán el mismo sabor.

evaluar Evaluar significa determinar el número individual que representa el valor de una expresión.

Por ejemplo, para evaluar $5x + 2$ cuando $x = 3$, sustituimos x por 3 y luego calculamos $5(3) + 2 = 17$.

evento Un conjunto de uno o más resultados en un experimento de azar.

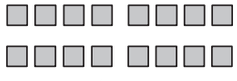


Por ejemplo, si tiramos un dado, hay seis resultados posibles.

experimento Una acción o actividad que se puede llevar a cabo una y otra vez sin saber cuál será el resultado en cada ocasión.

Por ejemplo, cada vez que se gira la ruleta, podría caer en rojo, azul o verde.

exponente Un número que se usa para describir multiplicaciones repetidas. A los exponentes a veces se les conoce como potencias.



$2^4 = 16$

Por ejemplo, $2 \cdot 2 \cdot 2 \cdot 2 = 16$ puede representarse con la ecuación $2^4 = 16$, donde 4 es el exponente. Podemos leer esta ecuación como "2 a la potencia de 4 es igual a 16" o "2 a la cuarta potencia es igual a 16".

expresión Un conjunto de números, variables, operaciones y símbolos de agrupación que representan una cantidad.

Por ejemplo, $2n - 8$ y $21 + 37$ son expresiones.

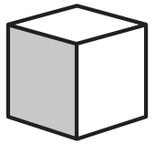
English

Español

F

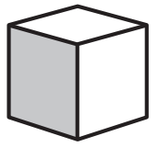
face Each flat side of a polyhedron is called a face.

A cube has six faces and they are all squares.



cara Cada lado plano de un poliedro se llama cara.

Un cubo tiene seis caras y todas son cuadradas.



factor (of a number) A whole number that divides evenly into the given number (with no remainder).

For example, 1, 2, 4, and 8 are all factors of the number 8.

factor (de un número) Un número natural por el que se puede dividir en partes iguales el número dado (sin resto).

Por ejemplo, 1, 2, 4 y 8 son factores del número 8.

frequency table A table that shows the number of times each value or category occurs in a data set.

For example, this frequency table shows that 5 students selected red as their favorite color.

Favorite Color	Frequency
Red	5
Blue	3
Pink	4

tabla de frecuencia Una tabla que muestra el número de veces que aparece cada valor o categoría en un conjunto de datos.

Por ejemplo, esta tabla de frecuencia muestra que 5 estudiantes seleccionaron el rojo como su color favorito.

Color favorito	Frecuencia
Rojo	5
Azul	3
Rosado	4

G

≥ (greater-than-or-equal-to) This symbol means "greater than or equal to."

For example, the inequality $x \geq 3$ says the value of x is either exactly 3 or any value greater than 3.

greatest common factor (GCF) The largest number that is a common factor of two numbers.

The common factors of 8 and 12 are 1, 2, and 4. The greatest common factor is 4.

≥ (mayor que o igual a) Este símbolo significa "mayor que o igual a".

Por ejemplo, la desigualdad $x \geq 3$ indica que el valor de x es exactamente 3 o cualquier valor mayor que 3.

máximo común divisor (MCD) El número mayor que es factor común de dos números.

Los factores comunes de 8 y 12 son 1, 2 y 4. El máximo común divisor es 4.

H

height (of a parallelogram) The shortest distance between a base and its opposite side. The height is always perpendicular to the base.

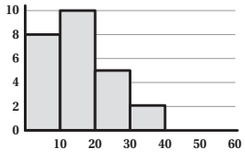
altura (de un paralelogramo) La distancia más corta entre una base y su lado opuesto. La altura siempre es perpendicular a la base.

height (of a triangle) The shortest distance between a base and its opposite vertex. Sometimes the height falls outside the shape. The height is always perpendicular to the base.

altura (de un triángulo) La distancia más corta entre una base y su vértice opuesto. A veces, la altura cae fuera de la figura. La altura siempre es perpendicular a la base.

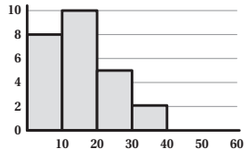
histogram A way to visualize numerical data where the data is grouped into bins represented by rectangles. The height of each rectangle shows how many values are in that bin.

For example, this histogram shows that there are 8 data values from 0 up to (but not including) 10.



histograma Una forma de visualizar datos numéricos en la que los datos se agrupan en intervalos que se representan con rectángulos. La altura de cada rectángulo muestra cuántos valores hay en ese intervalo.

Por ejemplo, este histograma muestra que hay 8 valores de datos del 0 al 10 (sin incluirlo).



Glossary/Glosario

English

Español

I

inequality A comparison statement that uses the symbols < or >. Inequalities are used to represent the relationship between numbers, variables, or expressions that are not always equal.

For example, the inequality $y > 30$ means that the value of the expression y is any number greater than 30.

desigualdad Un enunciado de comparación que utiliza los símbolos < o >. Las desigualdades se usan para representar la relación entre números, variables o expresiones que no siempre son iguales.

Por ejemplo, la desigualdad $y > 30$ significa que el valor de la expresión y es cualquier número mayor que 30.

integers All whole numbers and their opposites.

For example, 35, -15, and 1 are integers. 0.3 and $\frac{1}{3}$ are not.

enteros Todos los números enteros y sus opuestos.

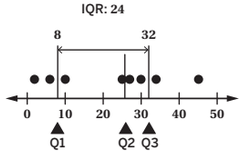
Por ejemplo, 35, -15 y 1 son números enteros. 0.3 y $\frac{1}{3}$ no lo son.

interest The extra money that you pay on a loan or the money that you earn when you save or invest.

interés El dinero extra que pagas por un préstamo o el dinero que ganas cuando ahorras o inviertes.

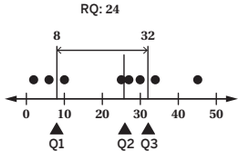
interquartile range (IQR) A measure of spread. The IQR is calculated as the distance from Q1 to Q3, or the width of the box in a box plot.

For example, the IQR of this data set is $32 - 8 = 24$.



rango intercuartílico (RQ) Una medida de dispersión. El RQ se calcula como la distancia de Q1 a Q3, o el ancho de la caja en un diagrama de caja.

Por ejemplo, el RQ de este conjunto de datos es $32 - 8 = 24$.



L

least common multiple (LCM) The smallest number that is a common multiple of two numbers.

The common multiples of 2 and 3 are 6, 12, 18, . . .
The least common multiple is 6.

mínimo común múltiplo (MCM) El número menor que es múltiplo común de dos números.

Los múltiplos comunes de 2 y 3 son 6, 12, 18. . .
El mínimo común múltiplo es 6.

≤ (less-than-or-equal-to) This symbol means "less than or equal to."

For example, the inequality $x \leq 9$ says the value of x is either exactly 9 or any value less than 9.

≤ (menor que o igual a) Este símbolo significa "menor que o igual a".

Por ejemplo, la desigualdad $x \leq 9$ indica que el valor de x es exactamente 9 o cualquier valor menor que 9.

like terms Terms with variables and exponents that are the same.

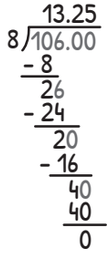
For example, $8x$ and $12x$ are like terms because both terms have a variable of x. $3x$ and $3x^2$ are not like terms because they have different exponents.

términos semejantes Términos con variables y exponentes iguales.

Por ejemplo, $8x$ y $12x$ son términos semejantes porque ambos tienen una variable que incluye x. $3x$ y $3x^2$ no son términos semejantes porque tienen exponentes diferentes.

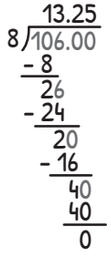
long division A way to divide numbers. When we use long division, we determine the quotient one digit at a time, from left to right.

For example, here is the long division for $106 \div 8$.



división larga Una forma de dividir números. Al usar la división larga, determinamos el cociente de izquierda a derecha, un dígito a la vez.

Por ejemplo, esta es la división larga de $106 \div 8$.



M

markdown The amount of decrease between the original price of an item and its sale price.

rebaja El monto de descuento entre el precio original de un artículo y su precio de venta.

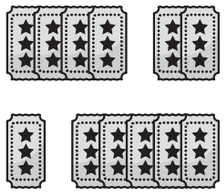
markup The amount of increase between the store's cost of an item and the item's selling price.

sobreprecio La cantidad de aumento entre el costo de un artículo en la tienda y el precio de venta del artículo.

English

maximum The greatest value in a data set.

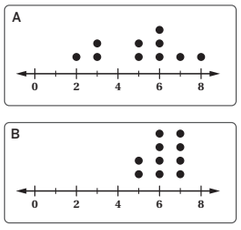
mean (average) A measure of center. If you equally distribute a set of items into different groups, the mean is the number of items in each group. It is also the balance point of a dot plot. To calculate the mean, you can add the values of all the data points, then divide by the number of data points.



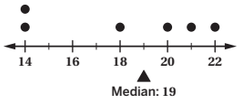
The mean in this example is 3 tickets because that is the number each person would get if the tickets were distributed equally among four people.

measure of center A single number that summarizes all of the values in a data set. It is usually a typical value for a data set. Mean and median are measures of center.

measure of spread A single number that describes the spread of a data set. Range and interquartile range are measures of spread.



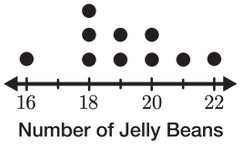
median A measure of center. It is the middle value of a data set when the values are in numerical order. When there is an even number of data points, the median is the average of the two middle values.



The median of this data set is 19 because it is the average of the two middle values: 18 and 20.

minimum The least value in a data set.

mode A measure of center. It is the value that occurs most often in a data set. There may be no mode, one mode, or more than one mode.



For example, the mode of this data set is 18 jelly beans because the value 18 occurs more times than any other data value.

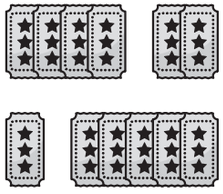
multiple The result of multiplying a number by a whole number.

For example, 10, 15, and 20 are all multiples of the number 5.

Español

máximo El mayor valor en un conjunto de datos.

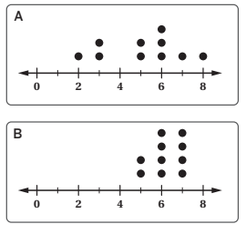
media (promedio) Una medida de tendencia central. Si se distribuye equitativamente un conjunto de elementos en diferentes grupos, la media es la cantidad de elementos en cada grupo. También es el punto de equilibrio de un diagrama de puntos. Para calcular la media, se pueden sumar los valores de todos los puntos de datos y luego dividir por la cantidad de puntos de datos.



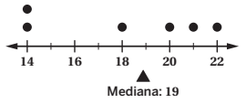
La media en este ejemplo es 3 boletos porque es la cantidad que obtendría cada persona si los boletos se distribuyeran equitativamente entre cuatro personas.

medida de tendencia central, medida de centro Un solo número que resume todos los valores de un conjunto de datos. Suele ser un valor típico de un conjunto de datos. La media y la mediana son medidas de tendencia central.

medida de dispersión Un solo número que describe la dispersión de un conjunto de datos. El rango y el rango intercuartílico son medidas de dispersión.



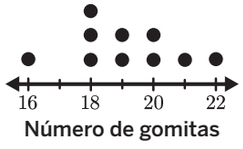
mediana Una medida de tendencia central. Es el valor del medio de un conjunto de datos cuando los valores se clasifican en orden numérico. Cuando hay un número par de puntos de datos, la mediana es el promedio de los dos números centrales.



La mediana de este conjunto de datos es 19 porque es el promedio de los dos valores centrales: 18 y 20.

mínimo El menor valor en un conjunto de datos.

moda Una medida de centro. Es el valor que aparece con mayor frecuencia en un conjunto de datos. Puede no haber ninguna moda, haber una moda o más de una moda.



Por ejemplo, la moda de este conjunto de datos es 18 gomitas, porque el valor 18 aparece más veces que cualquier otro valor de datos.

múltiplo El resultado de multiplicar un número por un número natural.

Por ejemplo, 10, 15 y 20 son múltiplos del número 5.

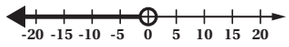
Glossary/Glosario

English

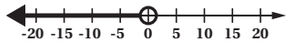
Español

N

negative number A number that is less than 0. On a horizontal number line, negative numbers are to the left of 0.

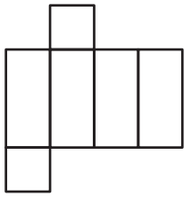


número negativo Un número que es menor que 0. En una recta numérica horizontal, los números negativos están a la izquierda del 0.



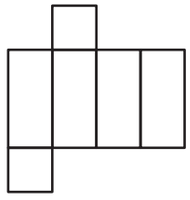
net A two-dimensional representation of a three-dimensional shape. It can be folded to make a polyhedron.

Here is a net for a rectangular prism.



red Una representación bidimensional de una figura tridimensional. Puede plegarse para formar un poliedro.

Esta es una red de un prisma rectangular.



numerical data Data that includes values that are numbers and can be measured and meaningfully compared.

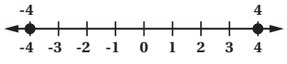
"How many pets do you have?" is a question that would result in numerical data.

datos numéricos Datos que incluyen valores numéricos que pueden medirse y compararse de forma significativa.

"¿Cuántas mascotas tienes?" es una pregunta que produciría datos numéricos.

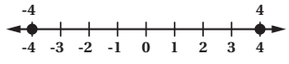
O

opposite Two numbers that are the same distance from 0 and on different sides of 0 on the number line are opposites.



For example, 4 and -4 are opposites.

opuesto Dos números son opuestos si están a la misma distancia del 0 y en diferentes lados del 0 en la recta numérica.



Por ejemplo, 4 y -4 son opuestos.

order of operations A consistent order applied to an expression with multiple operations so that the expression is evaluated the same way by everyone. The standard order of operations is parentheses/grouping symbols, exponents/roots, multiplication/division, and then addition/subtraction.

orden de las operaciones Un orden coherente aplicado a una expresión con múltiples operaciones para que cualquiera pueda evaluar la expresión de la misma manera. El orden estándar de las operaciones es paréntesis/símbolos de agrupación, exponentes/raíces, multiplicación/división y, luego, suma/resta.

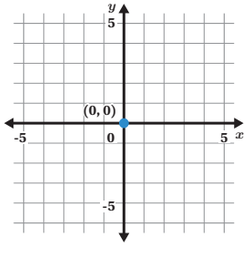
ordered pair Two values of x and y, written as (x, y), that represent a point on the coordinate plane.

For example, $(3, 5)$ represents the point where $x = 3$ and $y = 5$.

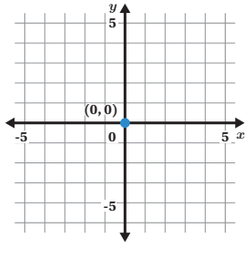
par ordenado Dos valores de x y y, escritos como (x, y), que representan un punto en el plano de coordenadas.

Por ejemplo, $(3, 5)$ representa el punto donde $x = 3$ y $y = 5$.

origin The point $(0, 0)$ on the coordinate plane. This is where the x-axis and the y-axis intersect.



origen El punto $(0, 0)$ en el plano de coordenadas. El punto en el que se intersecan los ejes x y y.



outcome One of the possible results of an experiment.

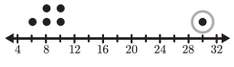
For example, the possible outcomes of tossing a coin are heads and tails.

resultado Uno de los posibles resultados de un experimento.

Por ejemplo, los posibles resultados de lanzar una moneda son cara y sello.

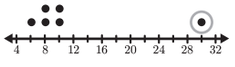
outlier A data value that is far from the other values in a data set.

The circled point is an outlier.



valor atípico Un valor de datos que está lejos de los demás valores del conjunto de datos.

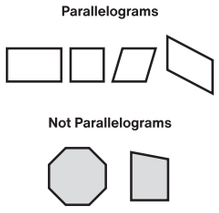
El punto encerrado en un círculo es un valor atípico.



English

P

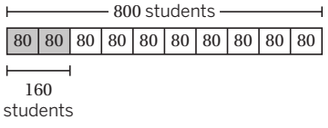
parallelogram A quadrilateral that has two pairs of parallel sides. Opposite, or parallel, sides of a parallelogram are the same length.



per The word *per* means "for each."

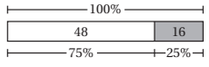
For example, if the price is $5 per ticket, that means that each ticket costs $5. Buying 4 tickets would cost $5 \cdot 4 = \$20$.

percent (percentage) Percent means "for every 100." It is represented by the percent symbol, %. We use percentages to represent ratios and fractions.



For example, 20% means 20 for every 100. 20% of a number means $\frac{20}{100}$ or $\frac{1}{5}$ of that number. Let's say there are 800 students in a school. If 20% of them are on a field trip, that means 160 students because 20 students are on the trip for every 100 students total.

percent decrease How much a quantity goes down, expressed as a percentage of the starting amount.

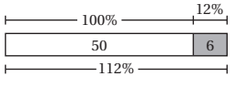


For example, a store had 64 hats in stock on Friday.
They had 48 hats left on Saturday. The amount of hats went down by 16.
This was a 25% decrease because 16 is 25% of 64.

percent error A way to describe the difference between a desired value and the actual value, expressed as a percent of the desired value.

For example, a box is supposed to have 150 folders in it. Clare counts only 147 folders in the box. This is an error of 3 folders. The percent error is 2% because 3 is 2% of 150. $\frac{3}{150} = 0.02 = 2\%$

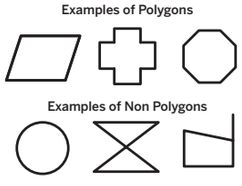
percent increase How much a quantity goes up, expressed as a percentage of the starting amount.



For example, Elena had $50 on Monday. She helped a neighbor, so she had $56 on Tuesday. The amount went up by $6.
This was a 12% increase because 6 is 12% of 50. $\frac{6}{50} = 0.12 = 12\%$

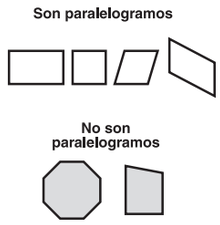
perpendicular Describes a line that crosses or meets another line at a 90° angle.

polygon A closed two-dimensional shape with straight sides that do not cross each other.



Español

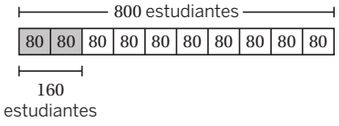
paralelogramo Un cuadrilátero que tiene dos pares de lados paralelos. Los lados opuestos, o paralelos, de un paralelogramo tienen la misma longitud.



por, por cada La palabra *por* puede significar "por cada".

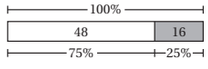
Por ejemplo, si el precio es $5 por boleto, esto significa que cada boleto cuesta $5. Comprar 4 boletos costaría $5 \cdot 4 = \$20$.

por ciento (porcentaje) Por ciento significa "por cada 100". Se representa con el símbolo de porcentaje, %. Usamos porcentajes para representar razones y fracciones.



Por ejemplo, 20% significa 20 por cada 100. 20% de un número significa $\frac{20}{100}$ o $\frac{1}{5}$ de dicho número. Supongamos que hay 800 estudiantes en una escuela. Si el 20% de ellos está en una excursión, eso significa 160 estudiantes porque 20 están de viaje por cada 100 estudiantes.

disminución porcentual Describe cuánto disminuye una cantidad, expresada como un porcentaje de la cantidad inicial.

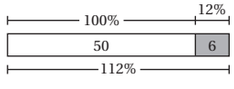


Por ejemplo, una tienda tenía 64 sombreros en inventario el viernes. El sábado le quedaban 48 sombreros.
La cantidad de sombreros se redujo en 16.
Fue una disminución del 25% porque 16 es el 25% de 64.

porcentaje de error, error porcentual Una forma de describir la diferencia entre un valor deseado y el valor real, expresada como un porcentaje del valor deseado.

Por ejemplo, se supone que una caja contiene 150 carpetas. Clare cuenta solo 147 carpetas en la caja. Ese es un error de 3 carpetas. El porcentaje de error es del 2% porque 3 es el 2% de 150. $\frac{3}{150} = 0.02 = 2\%$

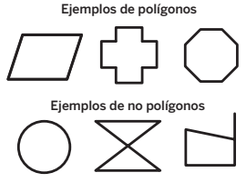
aumento porcentual Cuánto aumenta una cantidad, expresada como un porcentaje de la cantidad inicial.



Por ejemplo, Elena tenía $50 el lunes. Ayudó a un vecino, así que el martes tenía $56. La cantidad aumentó en $6.
Fue un aumento del 12% porque 6 es el 12% de 50. $\frac{6}{50} = 0.12 = 12\%$

perpendicular Describe una línea que cruza o se une con otra línea formando un ángulo de 90°.

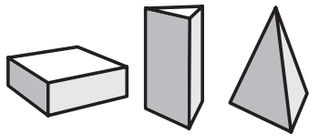
polígono Una figura bidimensional cerrada con lados rectos que no se cruzan.



Glossary/Glosario

English

polyhedron A polyhedron is a closed three-dimensional shape with flat sides. The plural of polyhedron is polyhedra. Prisms and pyramids are types of polyhedra.

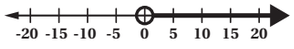


Here are some drawings of polyhedra.

population A set of people or things that are being studied.

For example, if we want to study the heights of people on different sports teams, the population would be all the people on the teams.

positive number A number that is greater than 0. On a horizontal number line, positive numbers are to the right of 0.

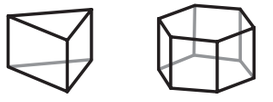


prime factorization The expression of a number as the product of prime factors.

The prime factorization of 28 is $2 \cdot 2 \cdot 7$.

principal The total amount (initial value) of money borrowed or invested, not including any interest.

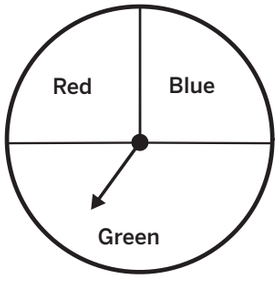
prism A three-dimensional shape, or polyhedron, with two bases that are identical copies.



Triangular Prism Hexagonal Prism

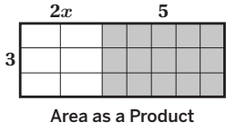
probability A number that tells us howlikely an event is to happen. A probability of 1 means the event is certain. A probability of 0 means the event is impossible.

For example, the probability of spinning red is $\frac{1}{4}$.



product The value of two or more quantities when multiplied.

For example, the area of this rectangle is the product of 3 and $2x + 5$ or $3(2x + 5)$.



Area as a Product
$3(2x + 5)$

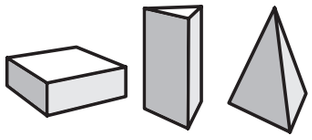
profit The money left after paying for business expenses.

proportional Quantities are proportional if they form equivalent ratios.

For example, the heights and bases of these triangles are proportional because the ratio $6 : 12$ is equivalent to $4 : 8$.

Español

poliedro Un poliedro es una figura tridimensional cerrada con caras planas. En inglés, usamos las palabras polyhedron (singular) y polyhedra (plural). Los prismas y las pirámides son tipos de poliedros.



Estos son algunos dibujos de poliedros.

población Conjunto de personas o cosas que se estudian.

Por ejemplo, si queremos estudiar las alturas de las personas de diferentes equipos deportivos, la población sería todas las personas de los equipos.

número positivo Un número que es mayor que 0. En una recta numérica horizontal, los números positivos están a la derecha del 0.

factorización prima La expresión de un número como producto de factores primos.

La factorización prima de 28 es $2 \cdot 2 \cdot 7$.

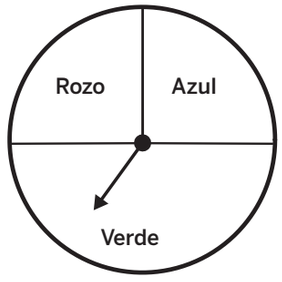
principal La cantidad total (valor inicial) de un préstamo o una inversión de dinero, excluyendo los intereses.

prisma Una figura tridimensional, o poliedro, con dos bases que son copias idénticas.

Prisma triangular Prisma hexagonal

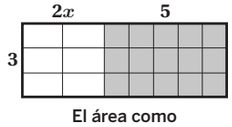
probabilidad Número que indica qué tan probable es que ocurra un evento.Una probabilidad de 1 significa que el evento es seguro. Una probabilidad de 0 significa que el evento es imposible.

Por ejemplo, la probabilidad de girar y que caiga rojo es $\frac{1}{4}$.



producto El valor de dos o más cantidades cuando se multiplican.

Por ejemplo, el área de este rectángulo es el producto de 3 y $2x + 5$ o $3(2x + 5)$.



El área como un producto
$3(2x + 5)$

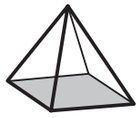
ganancia El dinero que sobra después de pagar los gastos del negocio.

proporcional Diferentes cantidades son proporcionales si forman razones equivalentes.

Por ejemplo, las alturas y las bases de estos triángulos son proporcionales porque la relación $6 : 12$ es equivalente a $4 : 8$.

English

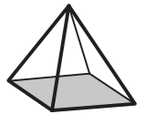
pyramid A three-dimensional shape, or polyhedron, that has one base. All of the other faces are triangles that meet at a single vertex.



Pyramid

Español

pirámide Una figura tridimensional, o poliedro, que tiene solo una base. Todas las demás caras son triángulos que se unen en un único vértice.

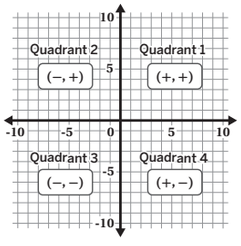


Pirámide

Q

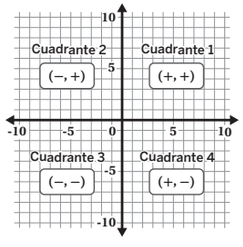
quadrant The coordinate plane is divided into 4 regions called quadrants.

The image shows each quadrant, along with the sign of the x- and y-values in that quadrant.



cuadrante El plano de coordenadas se divide en 4 regiones llamadas cuadrantes.

La imagen muestra cada cuadrante junto con el signo de los valores x y y del cuadrante correspondiente.



quadrilateral A polygon that has four sides.

Squares, rectangles, and parallelograms are examples of quadrilaterals.

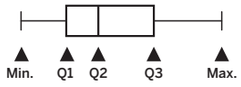
cuadrilátero Un polígono que tiene cuatro lados.

Los cuadrados, rectángulos y paralelogramos son ejemplos de cuadriláteros.

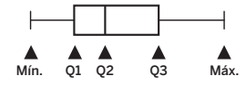
quantity The amount or the number of a thing.

cantidad Cierta cantidad o número de algo.

quartile Quartiles divide an ordered data set into four equal sections.
Quartile 1 (Q1) is the median of the lower half of the data.
Quartile 2 (Q2) is the median.
Quartile 3 (Q3) is the median of the upper half of the data.



cuartil Los cuartiles dividen un conjunto de datos ordenado en cuatro secciones iguales.
El cuartil 1 (Q1) es la mediana de la mitad inferior de los datos.
El cuartil 2 (Q2) es la mediana.
El cuartil 3 (Q3) es la mediana de la mitad superior de los datos.



quotient The result of dividing two numbers is called the quotient.

For example, in the equation $12 \div 3 = 4$, the quotient is 4.

cociente Se denomina cociente al resultado de dividir dos números.

Por ejemplo, en la ecuación $12 \div 3 = 4$, el cociente es 4.

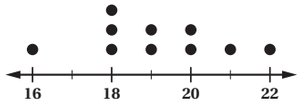
R

random Outcomes of a chance experiment are random if they are all equally likely to happen.

aleatorio Los resultados de un experimento de azar son aleatorios si todos tienen la misma probabilidad de suceder.

range A measure of spread. It is the difference between the maximum and minimum values in a data set.

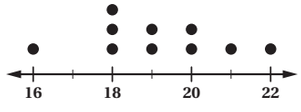
For example, the range of this data set is 6 jelly beans because $22 - 16 = 6$.



Number of Jelly Beans

rango Una medida de dispersión. Es la diferencia entre los valores máximo y mínimo de un conjunto de datos.

Por ejemplo, el rango de este conjunto de datos es 6 frijolitos de jalea porque $22 - 16 = 6$.



Cantidad de frijolitos

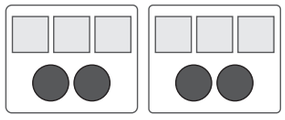
rate A comparison, or ratio, that describes how two quantities change together.

tasa Una comparación, o razón, que describe cómo cambian juntas dos cantidades.

Glossary/Glosario

English

ratio A ratio $a : b$ is a relationship between two quantities. For every a of the first, there are b of the second.



There are several ways to describe ratios.
- For every 3 squares, there are 2 circles.
- The ratio of squares to circles is 3 to 2.
- The ratio of squares to circles is $3 : 2$.

If the ratio of apples to oranges in a fruit bowl is $2 : 3$, then for every 2 apples, there are 3 oranges.

rational number Any number that can be written as a fraction, including whole numbers and negative numbers.

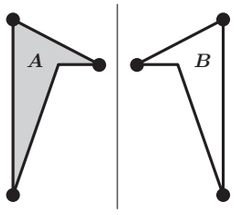
For example, -10, 2.5, $\frac{3}{7}$, and 82 are all rational numbers.

rearrange Change the position of something. We use the word *rearrange* to describe moving pieces of a figure to make a new shape.

reciprocal The reciprocal of a fraction $\frac{a}{b}$ is $\frac{b}{a}$. The product of two fractions that are reciprocals of one another is 1.

For example, $\frac{3}{2}$ and $\frac{2}{3}$ are reciprocals because $\frac{3}{2} \cdot \frac{2}{3} = 1$.

reflection A reflection moves each point on a figure across a line of reflection to a point on the opposite side of the line. The new point is the same distance from the line as it was in the original figure.



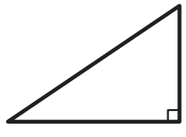
This diagram shows a reflection of A over a line that makes the mirror image B.

relative frequency The number of times an outcome happens divided by the number of repeated experiments. After several experiments, the relative frequency of an outcome gets close to its probability.

For example, if you spin a spinner 12 times and it lands on red 5 times, that means the relative frequency of landing on red is $\frac{5}{12}$.

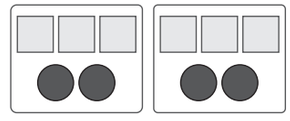
rhombus A quadrilateral containing four equal-length sides.

right triangle A triangle containing an interior right angle, or angle measuring exactly $90°$.



Español

razón Una razón $a : b$ es una relación entre dos cantidades. Por cada a del primero, hay b del segundo.



Hay varias formas de describir razones.
- Por cada 3 cuadrados hay 2 círculos.
- La razón de cuadrados a círculos es de 3 a 2.
- La razón de cuadrados a círculos es $3 : 2$.

Si la razón de manzanas a naranjas en un frutero es $2 : 3$, entonces por cada 2 manzanas hay 3 naranjas.

número racional Cualquier número que pueda escribirse como una fracción, incluidos los números enteros y los números negativos.

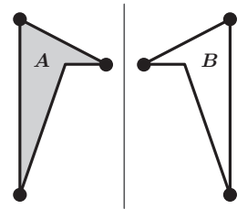
Por ejemplo, -10, 2.5, $\frac{3}{7}$ y 82 son números racionales.

reordenar Cambiar la posición de algo. Usamos la palabra *reordenar* para describir el movimiento de las partes de una figura para formar una nueva figura.

recíproco El recíproco de una fracción $\frac{a}{b}$ es $\frac{b}{a}$. El producto de dos fracciones que son recíprocas entre sí es 1.

Por ejemplo, $\frac{3}{2}$ y $\frac{2}{3}$ son recíprocos porque $\frac{3}{2} \cdot \frac{2}{3} = 1$.

reflexión Una reflexión mueve cada punto de una figura sobre una línea de reflexión a un punto en el lado opuesto de la línea. El nuevo punto está a la misma distancia de la línea que estaba en la figura original.

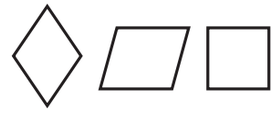


Este diagrama muestra una reflexión de A sobre una línea que produce la imagen espejo B.

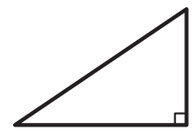
frecuencia relativa El número de veces que se produce un resultado dividido por el número de experimentos repetidos. Al cabo de varios experimentos, la frecuencia relativa de un resultado se aproxima a su probabilidad.

Por ejemplo, si se hace girar una ruleta 12 veces y cae en rojo 5 veces, la frecuencia relativa de que caiga en rojo es $\frac{5}{12}$.

rombo Un cuadrilátero que contiene cuatro lados de igual longitud.



triángulo recto Un triángulo que contiene un ángulo recto interior o un ángulo que mide exactamente $90°$.



S

sample A part of a population.

For example, a population could be all the seventh grade students at one school. One sample of that population is all the seventh grade students who are in band.

muestra Una parte de una población.

Por ejemplo, una población podría ser todos los estudiantes de séptimo grado de una escuela. Una muestra de esa población son todos los estudiantes de séptimo grado que están en la banda.

English

sample space The list of every possible outcome for a chance experiment.

For example, the sample space for tossing two coins is: heads-heads, tails-heads, heads-tails, tails-tails.

shape A description of the distribution (or pattern) of the data within a data set. Descriptions of the shape of a distribution can include: symmetry, skew, clusters, and outliers.

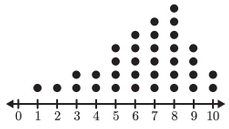
sign The sign of a number (other than 0) is either positive or negative.

For example, the sign of 4 or $+4$ is positive. The sign of -4 is negative.

simple interest Interest that is calculated solely based on the initial amount.

$I = prt$, where I = interest, p = principal, r = rate, and t = time

skewed A data distribution where there are more values concentrated on one end of the data and few values on the other end.



solution to an equation A value of a variable that makes the equation true. "Solving an equation" is any work you do to answer the question "Which values make the equation true?"

$$3x = 15$$
$$x = 15$$
$$3(5) = 15$$

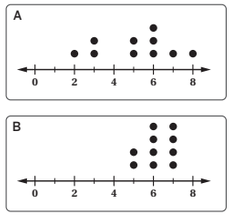
For example, 5 is a solution to the equation $3x = 15$ because $3(5) = 15$ is true. 6 is not a solution to the equation $3x = 15$ because $3(6) = 15$ is not true.

solution to an inequality Any value of a variable that makes the inequality true.

For example, 5 is a solution to the inequality $x < 10$ because $5 < 10$. Some other solutions to $x < 10$ are 9.99, 0, and -4.

spread A description of how alike or different the values in a distribution are, often in relation to the center. The spread also describes the variability of a distribution.

For example, Dot Plot A has a larger spread than Dot Plot B.



statistic A single number that measures something about a data set.

Examples of statistics include: mean, median and IQR.

Español

espacio muestral La lista de todos los posibles resultados de un experimento de azar.

Por ejemplo, el espacio muestral del lanzamiento de dos monedas es: cara-cara, sello-cara, cara-sello, sello-sello.

figura Una descripción de la distribución (o patrón) de los datos en un conjunto de datos. Las descripciones de la forma de una distribución pueden incluir: simetría, asimetría, agrupaciones y valores atípicos.

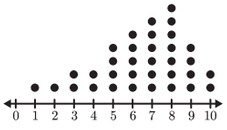
signo El signo de un número (que no sea 0) es positivo o negativo.

Por ejemplo, el signo de 4 o $+4$ es positivo. El signo de -4 es negativo.

interés simple Interés que se calcula solo según la cantidad inicial.

$I = prt$, donde I = interés, p = principal, r = tasa y t = tiempo

asimétrico Una distribución de datos en la que hay valores concentrados en un extremo de los datos y pocos valores en el otro extremo.



solución de una ecuación Un valor de una variable que hace que la ecuación sea verdadera. "Resolver una ecuación" es cualquier trabajo que se hace para responder la pregunta: "¿Qué valores hacen que la ecuación sea verdadera?".

$$3x = 15$$
$$x = 15$$
$$3(5) = 15$$

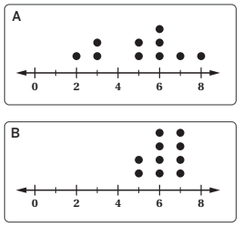
Por ejemplo, 5 es una solución de la ecuación $3x = 15$ porque $3(5) = 15$ es verdadero. 6 no es una solución de la ecuación $3x = 15$ porque $3(6) = 15$ no es verdadero.

solución de una desigualdad Cualquier valor de una variable que hace que la desigualdad sea verdadera.

Por ejemplo, 5 es una solución de la desigualdad $x < 10$ porque $5 < 10$. Algunas otras soluciones de $x < 10$ son 9.99, 0 y -4.

dispersión Una descripción de las semejanzas o diferencias de los valores en una distribución, a menudo en relación con el centro. La dispersión también describe la variabilidad de una distribución.

Por ejemplo, el diagrama de puntos A tiene una dispersión mayor que el diagrama de puntos B.



dato estadístico, estadística Un número único que mide algo de un conjunto de datos.

Ejemplos de datos estadísticos: media, mediana y RQ.

Glossary/Glosario

English

statistical question A question that requires more than one piece of data to answer.

Here are some examples of statistical questions:
- What is the most popular band at your school?
- When do students in your class typically eat dinner?

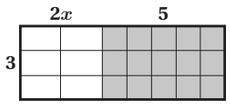
substitute To replace a variable with a value or other expression.

$$4x = 4(5)$$
$$= 20$$

In this example, 5 is substituted for x in the expression $4x$.

sum The value of two or more quantities when added together.

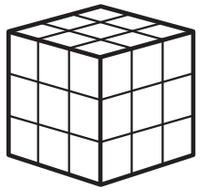
For example, the area of this rectangle is the sum of $6x$ and 15, or $6x + 15$.



Area as a Sum
$6x + 15$

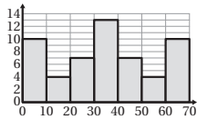
surface area The sum of the areas of a polyhedron's faces.

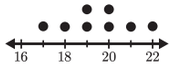
If the six faces of a cube each have an area of 9 square centimeters, then the surface area of the cube is $6 \cdot 9$, or 54 square centimeters.



symmetry (of a data distribution) A property of a data display where the left side of the distribution is a mirror image of the right side.

Here are some examples of data distributions that have symmetry.





Español

pregunta estadística Una pregunta que requiere más de un dato para responder.

Estos son algunos ejemplos de preguntas estadísticas:
- ¿Cuál es la banda más popular en tu escuela?
- ¿Cuándo suelen cenar los estudiantes de tu clase?

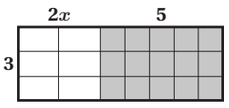
sustituir Reemplazar una variable por un valor u otra expresión.

$$4x = 4(5)$$
$$= 20$$

En este ejemplo, el 5 sustituye a la x en la expresión $4x$.

suma El valor de dos o más cantidades que se suman.

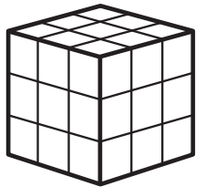
Por ejemplo, el área de este rectángulo es la suma de $6x$ y 15, o $6x + 15$.



El área como una suma
$6x + 15$

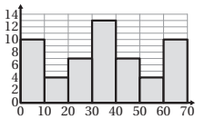
área de superficie La suma de las áreas de sus caras.

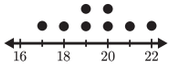
Si cada una de las seis caras de un cubo tiene un área de 9 centímetros cuadrados, el área de superficie del cubo mide $6 \cdot 9$, o 54 centímetros cuadrados.



simetría (de una distribución de datos) Una propiedad de una representación de datos donde el lado izquierdo de la distribución es una imagen reflejada del lado derecho.

Estos son algunos ejemplos de distribuciones de datos que tienen simetría.





T

table A table organizes information into horizontal rows and vertical columns. The first row or column usually tells what the numbers represent.

Here is a table showing the tail lengths of three different pets. This table has four rows and two columns.

Pet	Tail Length (in.)
Dog	22
Cat	12
Mouse	2

term A part of an expression. A term can be a single number, a variable, or a number and variable multiplied together.

For example, the expression $5x + 8$ has two terms. The first term is $5x$ and the second term is 8.

Expression
$5x + 8$
Terms

trapezoid A quadrilateral with *at least* one pair of parallel sides.

tabla Una tabla organiza la información en filas horizontales y columnas verticales. La primera fila o columna suele indicar lo que representan los números.

Esta es una tabla que muestra la longitud de la cola de tres mascotas diferentes. Esta tabla tiene cuatro filas y dos columnas.

Mascota	Longitud de cola (pulg.)
Perro	22
Gato	12
Ratón	2

término Una parte de una expresión. Un término puede ser un número individual, una variable, o una variable y un número multiplicados.

Por ejemplo, la expresión $5x + 8$ tiene dos términos. El primer término es $5x$ y el segundo término es 8.

Expresión
$5x + 8$
Términos

trapezoide Un cuadrilátero con al menos un par de lados paralelos.

English

Español

U

unit price The price per unit of an item. You can count how many units by counting the items or the weight.

For example, if 4 avocados cost $12, then the unit price is $\frac{12}{4} = \$3$ per avocado.

precio unitario El precio por unidad de un artículo. Puedes contar la cantidad de unidades contando los artículos o el peso.

Por ejemplo, si 4 aguacates cuestan $12, el precio unitario es $\frac{12}{4} = \$3$ por aguacate.

unit rate A rate that describes how one quantity changes when the other quantity changes by exactly 1 unit.

For example, if 12 people share 3 pizzas equally, then one unit rate is 4 people per pizza. Another unit rate in this situation is $\frac{1}{4}$ pizza per person.

tasa unitaria Una tasa que describe cómo cambia una cantidad cuando la otra cantidad cambia en exactamente 1 unidad.

Por ejemplo, si 12 personas se reparten 3 pizzas en partes iguales, entonces una tasa unitaria es 4 personas por pizza. Otra tasa unitaria en esta situación es $\frac{1}{4}$ de pizza por persona.

V

variable A letter or symbol that represents a value or set of values.

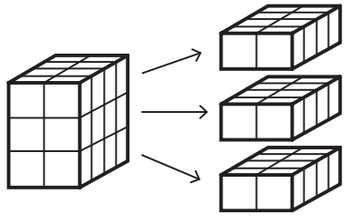
In the expression $10 - x$, the variable is x.

variable Una letra o un símbolo que representa un valor o un conjunto de valores.

En la expresión $10 - x$, la variable es x.

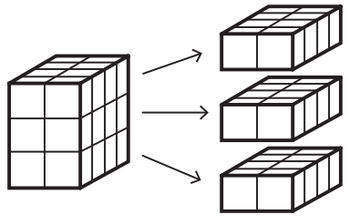
volume The number of unit cubes needed to fill a three-dimensional shape without gaps or overlaps.

The volume of this rectangular prism is 24 cubic units because it is composed of 3 layers that are each 8 cubic units.



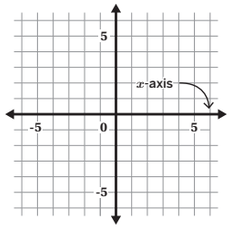
volumen La cantidad de cubos unitarios que se necesitan para llenar una figura tridimensional sin vacíos ni superposiciones.

El volumen de este prisma rectangular es de 24 unidades cúbicas porque se compone de 3 capas de 8 unidades cúbicas cada una.

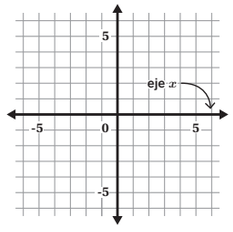


X

x-axis One of the perpendicular number lines that form the coordinate plane. The x-axis is the horizontal number line.



eje x Una de las rectas numéricas perpendiculares que forman el plano de coordenadas. El eje x es la recta numérica horizontal.

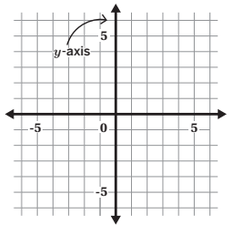


x-coordinate See *coordinates*.

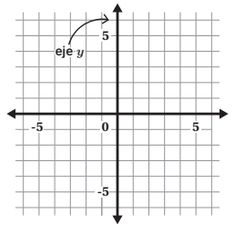
coordenada x Ver *coordenadas*.

Y

y-axis One of the perpendicular number lines that form the coordinate plane. The y-axis is the vertical number line.



eje y Una de las rectas numéricas perpendiculares que forman el plano de coordenadas. El eje y es la recta numérica vertical.



y-coordinate See *coordinates*.

coordenada y Ver *coordenadas*.

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