Amplify Desmos Math
FLORIDA

ACC 6

Intervention and Extension Resources

Inside you'll find:

- Strategies for effective differentiation
- Mini-Lessons, including those from prior grades
- Extensions

Amplify Desmos Math FLORIDA

Accelerated 6

Intervention and Extension Resources

About Amplify

Amplify is dedicated to collaborating with educators to create learning experiences that are rigorous and riveting for all students. Amplify creates K–12 core and supplemental curriculum, assessment, and intervention programs for today's students.

A pioneer in K–12 education since 2000, Amplify is leading the way in next-generation curriculum and assessment. All of our programs provide teachers with powerful tools that help them understand and respond to the needs of every student.

Amplify gratefully acknowledges the work of distinguished program advisors from English Learners Success Forum (ELSF), who have been integral in the development of Amplify Desmos Math. ELSF is a 501(c)(3) nonprofit organization whose mission is to expand educational equity for multilingual learners by increasing the supply of high-quality instructional materials that center their cultural and linguistic assets.

55 Washington Street, Suite 800
Brooklyn, NY 11201
www.amplify.com

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Contents Mini-Lessons

Log in to the digital curriculum to review all program Mini-Lessons and Extensions.

Contents Mini-Lessons (continued)

Contents Mini-Lessons (continued)

Contents Extensions

Contents Extensions (continued)

Intervention and Extension

Amplify Desmos Math Florida provides a comprehensive suite of intervention and extension resources designed to meet the needs of all students and are intended to be used outside of the core lesson.

Assess and Respond: After the Lesson

Each lesson has a formative assessment called a ***Show What You Know***. This assessment illustrates students' progress toward key concepts in the lesson and is accompanied by a table with suggestions in three categories: students who need **support**, students who would benefit from more practice to **strengthen** their understanding, and students who are interested in a **stretch** to deepen their understanding.

Support

Provide targeted intervention for students.

Strengthen

Reinforce students' understanding of the concepts assessed.

Stretch

Challenge students and extend their learning.

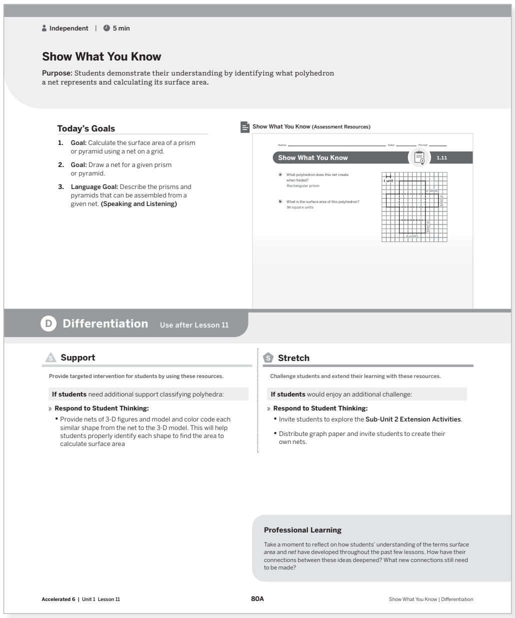
Differentiation Resources

The differentiation resources provide *beyond-the-lesson* support and challenge for all students.

Support, Strengthen, and Stretch print resources include:

- **Mini-Lessons:** Targeted intervention lessons to support students with a specific concept or skill.
- **Lesson Practices:** Practices to build and reinforce students' conceptual understanding, fluency, and application. Lesson Practices include fluency, test prep, and spiral review.
- **Extensions:** Problems aligned to the math of the sub-unit, designed for students who want to extend their thinking.

The differentiation table is available on the *Show What You Know* page of the Teacher Edition and in the *Differentiation Beyond the Lesson* tab of each lesson.


Independent | 5 min

Show What You Know

Purpose: Students demonstrate their understanding by identifying what polyhedron a net represents and calculating its surface area.

Today's Goals

1. **Goal:** Calculate the surface area of a prism or pyramid using a net on a grid.
2. **Goal:** Draw a net for a given prism or pyramid.
3. **Language Goal:** Describe the prisms and pyramids that can be assembled from a given net. **(Speaking and Listening)**

Show What You Know (Assessment Resources)

Differentiation Use after Lesson 11

Support

Provide targeted intervention for students by using these resources.

If students need additional support classifying polyhedra:

» **Respond to Student Thinking:**
- Provide nets of 3-D figures and model and color code each similar shape from the net to the 3-D model. This will help students properly identify each shape to find the area to calculate surface area

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**
- Invite students to explore the **Sub-Unit 2 Extension Activities**.
- Distribute graph paper and invite students to create their own nets.

Professional Learning

Take a moment to reflect on how students' understanding of the terms *surface area* and *net* have developed throughout the past few lessons. How have their connections between these ideas deepened? What new connections still need to be made?

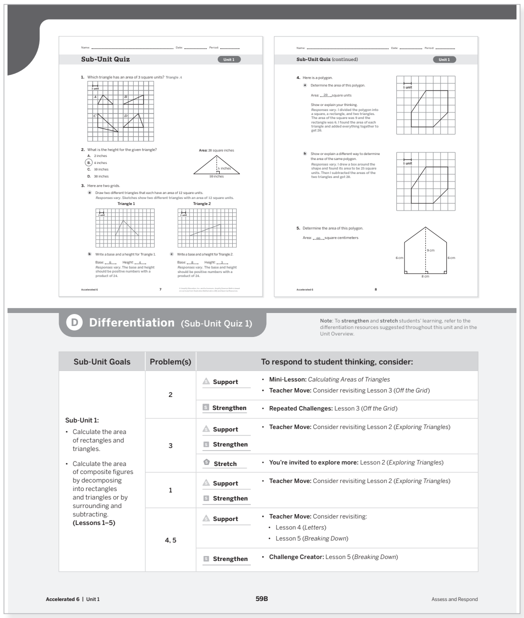
Accelerated 6 | Unit 1 Lesson 11 80A Show What You Know | Differentiation

Differentiation Beyond the Lesson table from Accelerated 6, Unit 1, Lesson 11.

Assess and Respond: After Unit Assessments

Embedded unit assessments offer key insights into students' understanding of the grade-level standards in the unit.

- Each unit includes an optional Pre-Unit Check, one or more Sub-Unit Quizzes, and an End-of-Unit Assessment.
- Each assessment is accompanied by an Assess and Respond Guide in the Teacher Edition, which includes responses to student thinking with resources that support, strengthen, and stretch learning.



D Differentiation (Sub-Unit Quiz 1)

Note: To **strengthen** and **stretch** students' learning, refer to the differentiation resources suggested throughout this unit and in the Unit Overview.

Sub-Unit Goals	Problem(s)	To respond to student thinking, consider:	
Sub-Unit 1: • Calculate the area of rectangles and triangles. • Calculate the area of composite figures by decomposing into rectangles and triangles or by surrounding and subtracting. **(Lessons 1–5)**	2	**Support**	• **Mini-Lesson:** *Calculating Areas of Triangles* • **Teacher Move:** Consider revisiting Lesson 3 (*Off the Grid*)
		Strengthen	• **Repeated Challenges:** Lesson 3 (*Off the Grid*)
	3	**Support** **Strengthen**	• **Teacher Move:** Consider revisiting Lesson 2 (*Exploring Triangles*)
		Stretch	• **You're invited to explore more:** Lesson 2 (*Exploring Triangles*)
	1	**Support** **Strengthen**	• **Teacher Move:** Consider revisiting Lesson 2 (*Exploring Triangles*)
	4, 5	**Support**	• **Teacher Move:** Consider revisiting: • Lesson 4 (*Letters*) • Lesson 5 (*Breaking Down*)
		Strengthen	• **Challenge Creator:** Lesson 5 (*Breaking Down*)

Accelerated 6 | Unit 1 59B Assess and Respond

Sub-Unit Quiz Differentiation table from Accelerated 6, Unit 1 Sub-Unit 1.

About Mini-Lessons

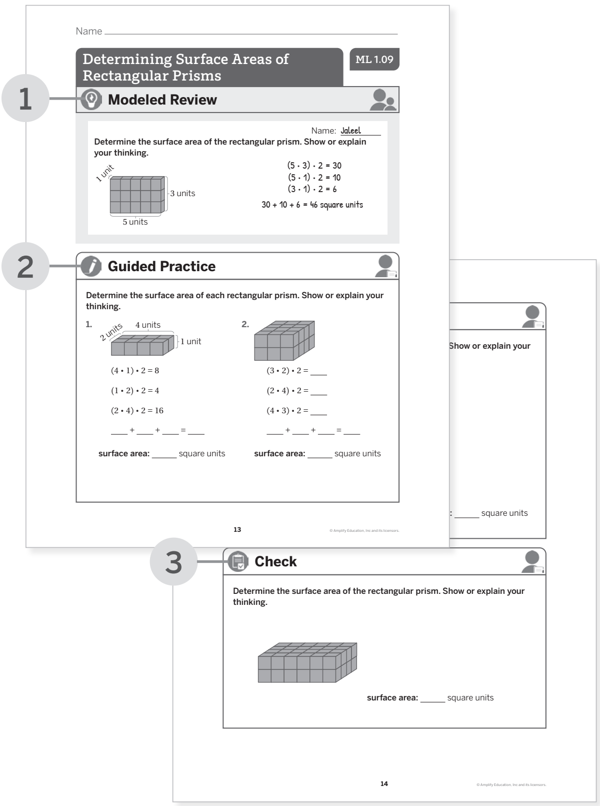
Amplify Desmos Math Florida Mini-Lessons are print activities aligned to the most critical content and skills in corresponding core lessons.

Mini-Lessons offer direct instruction and guided practice opportunities. These Mini-Lessons complement the problem-based approach, are ideal for small-group or whole-class instruction, as well as independent learning.

Student Page

Amplify Desmos Math Florida Mini-Lessons are designed based on extensive research around worked examples.[1]

The Student Page is organized to follow the flow of the Mini-Lesson: **Modeled Review**, **Guided Practice**, and **Check**.



Accelerated 6, Unit 1, Lesson 9 Mini-Lesson

1 **Modeled Review:** A worked example designed to be discussed as a group. Students make sense of a concept or process by examining the modeled student thinking.

2 **Guided Practice:** A series of problems that fades away scaffolding as students progress. Teachers can approach the Guided Practice in various ways based on their expertise and understanding of their students' needs.

3 **Check:** An opportunity for students to show what they've learned. We recommend all students complete this independently.

Considerations:

- Print or copy the Student Page in advance.
- Gather any needed materials. Any materials listed as optional on the Teacher Guide are not required to successfully implement the lesson.
- It may be helpful to have a whiteboard, but it's not required.

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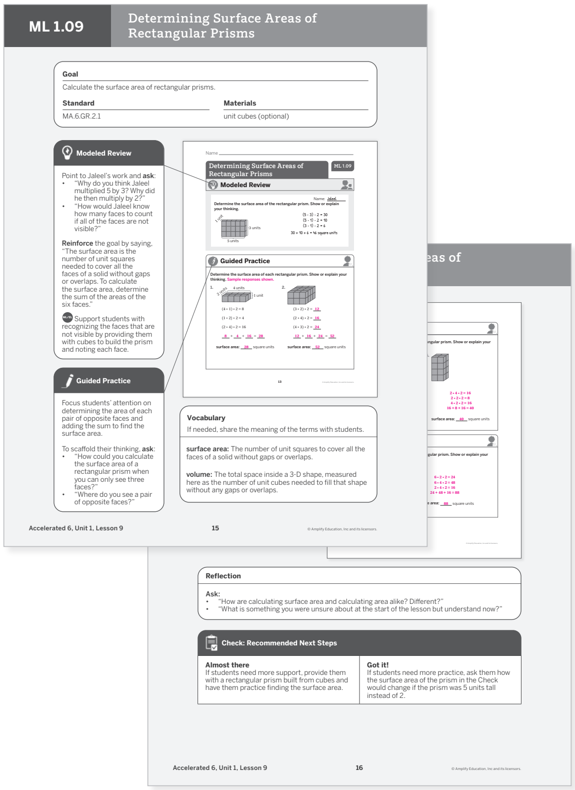
[1] Flores, R. and Inan, F. (2014). Examining the Impact of Adaptively Faded Worked Examples on Student Learning Outcomes. Journal of Interactive Learning Research, 467–485.

Teacher Guide

The Teacher Guide follows the same flow as the Student Page, with all the information needed to implement the Mini-Lesson.



Accelerated 6, Unit 1, Lesson 9 Mini-Lesson

Every Teacher Guide includes:

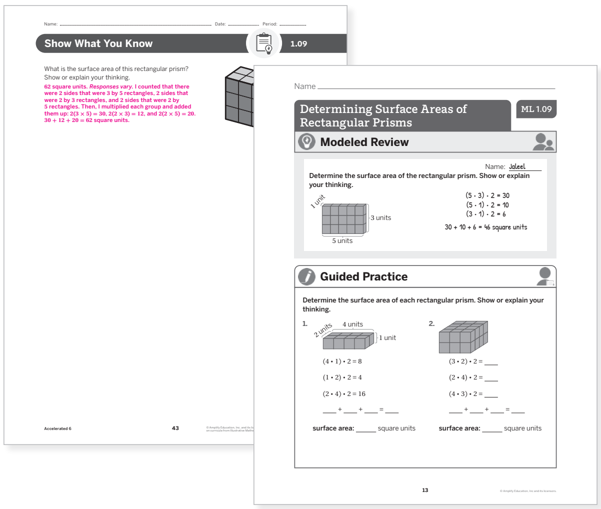
- The **lesson goal**, **materials**, and relevant **vocabulary terms**.
- Questions or statements to share with students in each part of the Mini-Lesson. **Reflection questions** are included as a way to close out the lesson.
- **Answer keys** with sample responses on the insets of the student pages.
- **Recommended next steps** for students needing more support or extra practice based on their performance on the Check problem(s).

Supporting All Learners

ELL This icon indicates suggestions for supporting **English Language Learners**.

A This icon appears at point-of-use and indicates suggestions for supporting the needs of all learners, based on the guidelines of **Universal Design for Learning (UDL)**.

Lesson and Mini-Lesson Alignment



Accelerated 6, Unit 1, Lesson 9 SWYK and Mini-Lesson

Mini-Lessons are closely aligned to the core lesson they are connected to. The **Mini-Lesson Modeled Review** is built using the *Show What You Know* (SWYK) from the lesson. Teachers can use student thinking on the *Show What You Know* to identify students who might benefit from the extra support of a Mini-Lesson.

About Extensions

Amplify Desmos Math Florida Extensions are sets of problems aligned to the math of the sub-unit. They are useful for students interested in an additional challenge or for the whole class.

Extensions build on our **student-led**, **problem-based** approach because they provide more opportunities for students to engage in creative and rigorous problems that can be approached using different strategies.

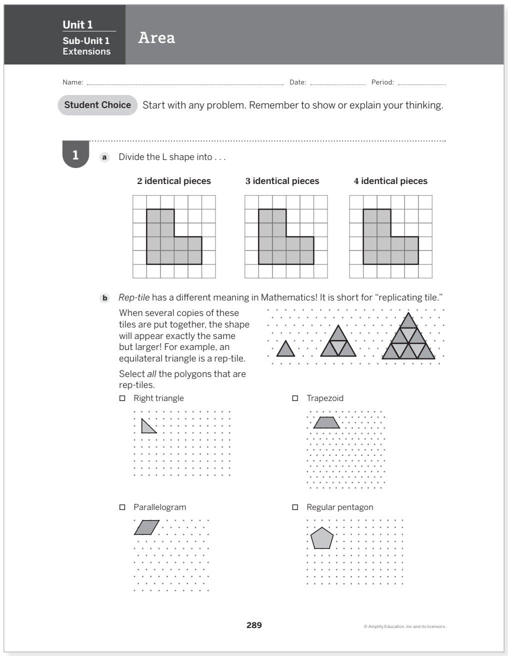
Student Page

Every sub-unit includes an Extensions problem set.

They are print-based, hands-on problems structured on the principle of student choice and designed to be student-led. The math is designed to be accessible to students at any time they are ready for more during the sub-unit.

Every sub-unit Extension includes:

- **Challenge:** Extensions focus on problem-solving and sharing thinking rather than answer-getting, with problems aligned to the math in the sub-unit.
- **Choice:** Extensions contain multiple open-ended problems, and students can start with what interests them.
- **Variety:** Some problems are designed with hands-on materials, others are discussion-based, and the rest require only a pencil and paper.



Unit 1
Sub-Unit 1
Extensions

Area

Name: ______ Date: ______ Period: ______

Student Choice Start with any problem. Remember to show or explain your thinking.

1 a Divide the L shape into . . .

2 identical pieces | 3 identical pieces | 4 identical pieces

b *Rep-tile* has a different meaning in Mathematics! It is short for "replicating tile."

When several copies of these tiles are put together, the shape will appear exactly the same but larger! For example, an equilateral triangle is a rep-tile.

Select *all* the polygons that are rep-tiles.

- ☐ Right triangle
- ☐ Trapezoid
- ☐ Parallelogram
- ☐ Regular pentagon

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Accelerated 6, Unit 1, Sub-Unit 1 Extension Student Page

Considerations:

- Invite students to choose one problem to focus on at a time.
- Prepare enough Extensions at the start of the sub-unit for all students so you can be flexible with when different students work on them.
- Think about where you will want students to store their work on the Extensions for the duration of the sub-unit.

Teacher Guide

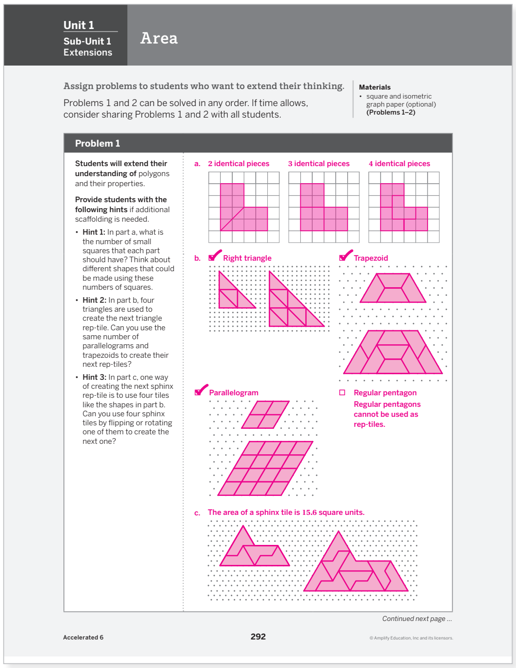
Extensions are designed to be a light lift for the teacher.

Every sub-unit Extension Teacher Guide includes:

- **Key background information** about the math in the problem.
- **Suggestions** for which problems to share with the whole class if time allows.
- **Hints** to share with students when needed.
- **Sample responses**.

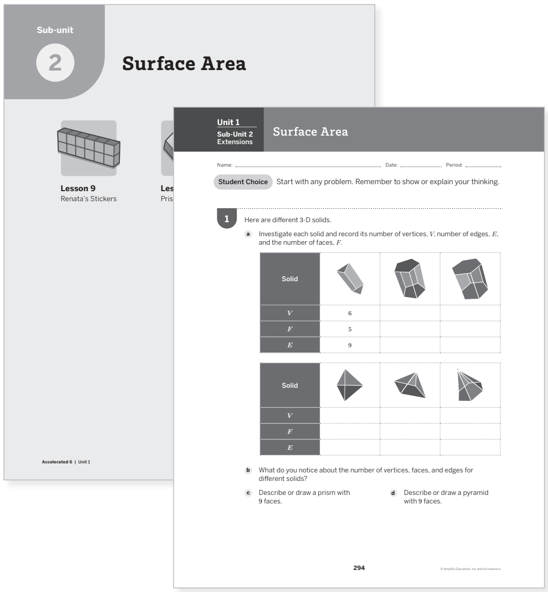
Considerations:

- Look for opportunities to introduce an Extension problem with the whole class early in the sub-unit so all students can participate.
- Help students get started on the Extension task, then let them work independently or in pairs while you work with other groups of students who may benefit from more direct support from a teacher.



Accelerated 6, Unit 1, Sub-Unit 1 Extension Teacher Guide

Lesson and Extension Alignment



Accelerated 6, Unit 1 , Sub-Unit 2 Opener and Extension

Extensions are aligned to the math of the **sub-unit**, but they also go deeper. The problems in Extensions are designed to make connections between the math of the sub-unit and other concepts. In some cases, problems will involve content from prior grades or units.

Mini-Lessons

Unit 1

Mini-Lessons

Name ______________________________

Calculating Areas of Triangles

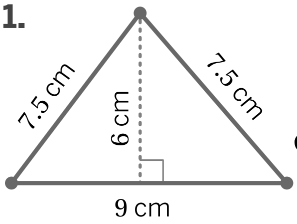
ML 1.03

Modeled Review

Name: Clare

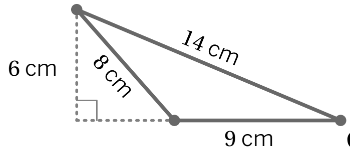
Determine the area of each triangle in square centimeters.

1.



$= \frac{1}{2}(9 \cdot 6)$
= 27 square centimeters

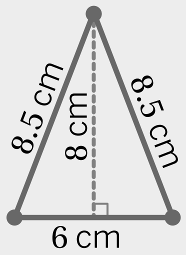
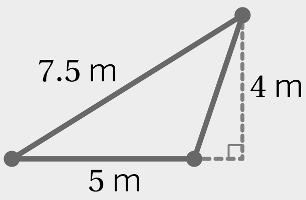
2.



$= \frac{1}{2}(9 \cdot 6)$
= 27 square centimeters

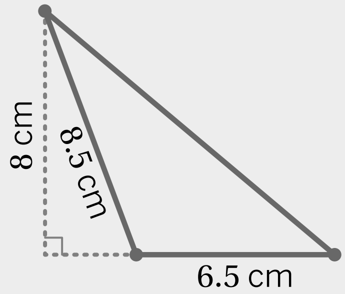
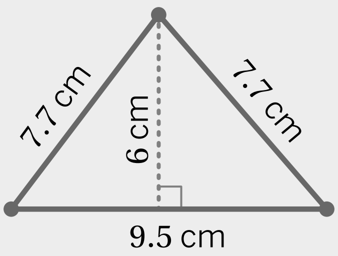
Guided Practice

1. Determine the base, height, and area of each triangle.

Triangle	Base	Height	Area
	6 cm	8 cm	
	5 m		

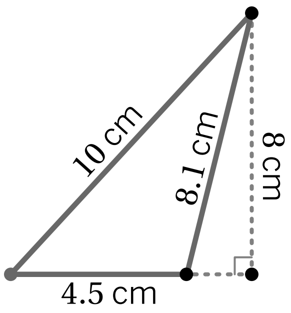
Guided Practice

2. Determine the base and height of each triangle and write an equation to represent the area of each triangle.

Triangle	Base	Height	Equation	Area
		8 cm		26 square centimeters
				28.5 square centimeters

Check

Determine the area of the triangle. Show or explain your thinking.



Calculating Areas of Triangles

Goal

Calculate the area of a triangle without a grid, given any side as its base.

Standard

MA.6.GR.2.1

Materials

coloring tools (optional), cut-out rectangles and triangles (optional)

Modeled Review

Point to Clare's work and **ask**:

- "How are the triangles alike? Different?"
- "Why did Clare use the inside height for one triangle and the outside height for the other?"

Reinforce Clare's thinking by saying, "The formula used to determine the area of any triangle is $A = \frac{1}{2} \cdot b \cdot h$."

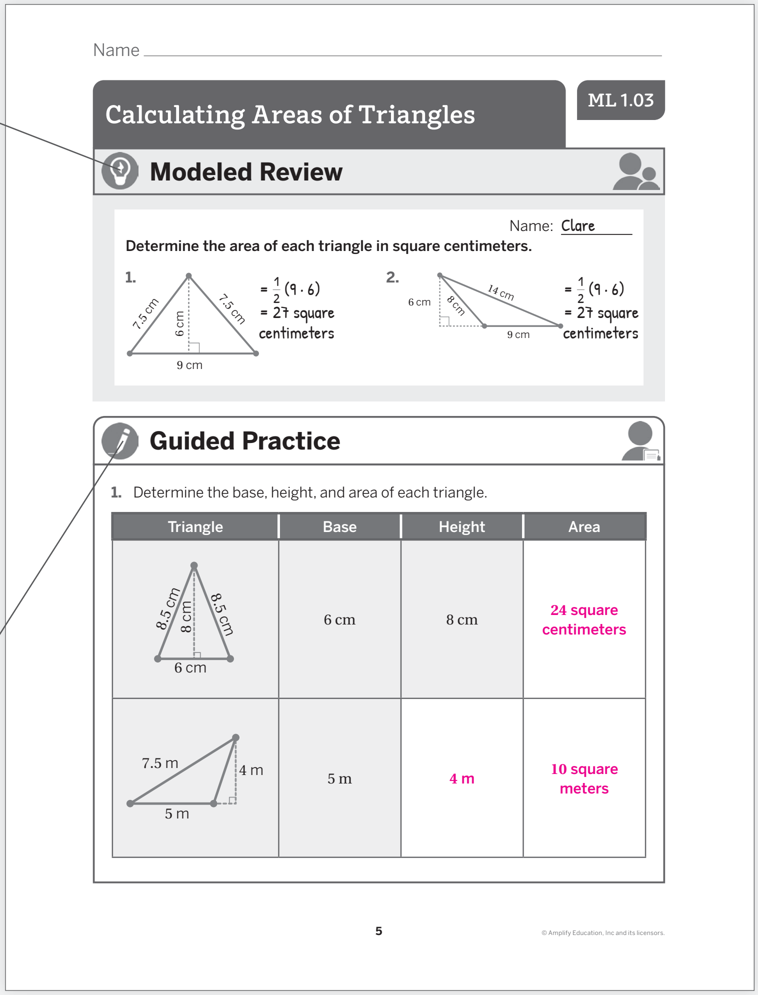
ML/EL Model by tracing and labeling the given height and/or base of each triangle.

Guided Practice

Focus students' attention on determining which measurements represent the base and height of each triangle.

To scaffold their thinking, **ask**:

- "Why is the height measurement for the first triangle 8 centimeters and not 8.5 centimeters?"
- "How do you know which measurement is the base?"
- "Why does the formula for calculating the area of a triangle use $\frac{1}{2}$?"



Name

Calculating Areas of Triangles

ML 1.03

Modeled Review

Name: Clare

Determine the area of each triangle in square centimeters.

1. 7.5 cm, 7.5 cm, 6 cm, 9 cm
$= \frac{1}{2}(9 \cdot 6)$
$= 27$ square centimeters

2. 6 cm, 8 cm, 14 cm, 9 cm
$= \frac{1}{2}(9 \cdot 6)$
$= 27$ square centimeters

Guided Practice

1. Determine the base, height, and area of each triangle.

Triangle	Base	Height	Area
8.5 cm, 8.5 cm, 8 cm, 6 cm	6 cm	8 cm	24 square centimeters
7.5 m, 4 m, 5 m	5 m	4 m	10 square meters

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Vocabulary

If needed, share the meaning of the terms with students.

area: Area measures the space inside a two-dimensional figure. It is expressed in square units.

height of a triangle: The shortest distance between a base and its opposite vertex.

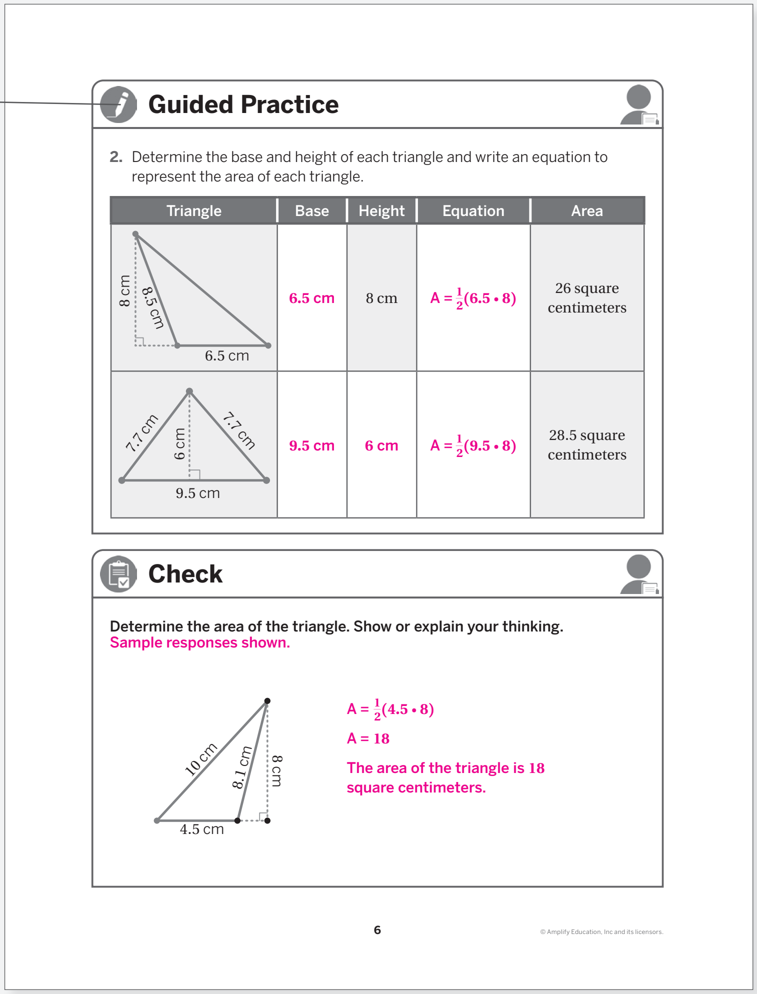
base: One side of a triangle. Any side of a triangle can be the base.

Calculating Areas of Triangles

Guided Practice

A Provide coloring tools and suggest students use color coding to annotate the base-height pairs in each triangle by labeling or circling the base in one color and its corresponding height in another color.

Key Takeaway:
Say, "A triangle can have more than one base and height pair. The height of the triangle must be perpendicular to the base of the triangle. No matter which side of a triangle is chosen as the base, its area, A, is equal to $\frac{1}{2} \cdot b \cdot h$."



Guided Practice

2. Determine the base and height of each triangle and write an equation to represent the area of each triangle.

Triangle	Base	Height	Equation	Area
8 cm, 8.5 cm, 6.5 cm	6.5 cm	8 cm	$A = \frac{1}{2}(6.5 \cdot 8)$	26 square centimeters
7.7 cm, 7.7 cm, 6 cm, 9.5 cm	9.5 cm	6 cm	$A = \frac{1}{2}(9.5 \cdot 8)$	28.5 square centimeters

Check

Determine the area of the triangle. Show or explain your thinking.
Sample responses shown.

$A = \frac{1}{2}(4.5 \cdot 8)$

$A = 18$

The area of the triangle is 18 square centimeters.

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Reflection

Ask:

- "What is always true about the base and height of a triangle when calculating the area?"
- "How did you overcome a hard problem today?"

Check: Recommended Next Steps

Almost there	Got it!
If students need more support, use cut-out triangle and rectangle shapes to show how two identical triangles can form a rectangle or parallelogram. This will help reinforce the formula $\frac{1}{2} \cdot b \cdot h$.	If students need more practice, have them solve the following problems. Then have them compare with a partner and discuss. • Determine the area of a triangle with a base of 12.5 centimeters and a height of 9 centimeters. • A triangle has an area of 30 square centimeters. If the height is 5 centimeters, what is the length of the base?

Name ______________________________

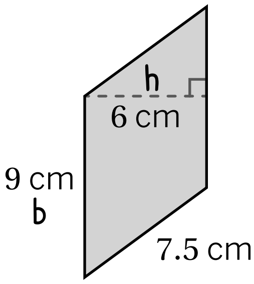
Calculating Area of Parallelograms

ML 1.06

Modeled Review

Name: Amir

Calculate the area of the parallelogram.

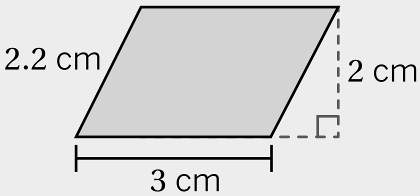
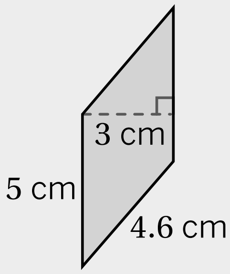


A = b · h
A = 9 · 6
A = 54

54 square centimeters

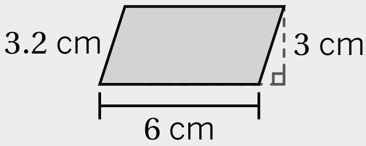
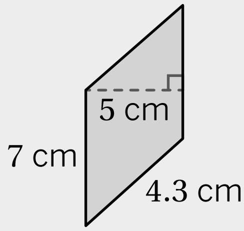
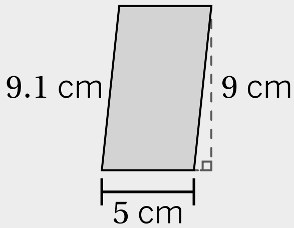
Guided Practice

1. Determine the base, height, and area of each parallelogram.

Parallelogram	Base (cm.)	Height (cm.)	Area $A = b \cdot h$
		2	6 square centimeters
	5		_____ square centimeters

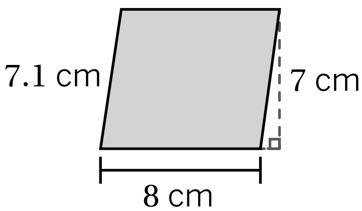
Guided Practice

2. Determine the base, height, and area of each parallelogram.

Parallelogram	Base (cm.)	Height (cm.)	Work	Area
			$A = b \cdot h$	
				
				

Check

Calculate the area of the parallelogram.



____

ML 1.06 Calculating Area of Parallelograms

Goal

Calculate the area of a parallelogram without a grid, given its base and height.

Standard

MA.7.GR.1.1

Materials

highlighter (optional)

Modeled Review

Point to Amir's work and **ask**:

- "How did Amir determine the base of the parallelogram?"
- "How can a right angle help you determine the base-height pair?"
- "How did Amir calculate the area?"

Reinforce Amir's thinking by saying, "The formula to find the area of a parallelogram is $A = b \cdot h$. A base-height pair can be identified by the right angle formed between the base and the height."

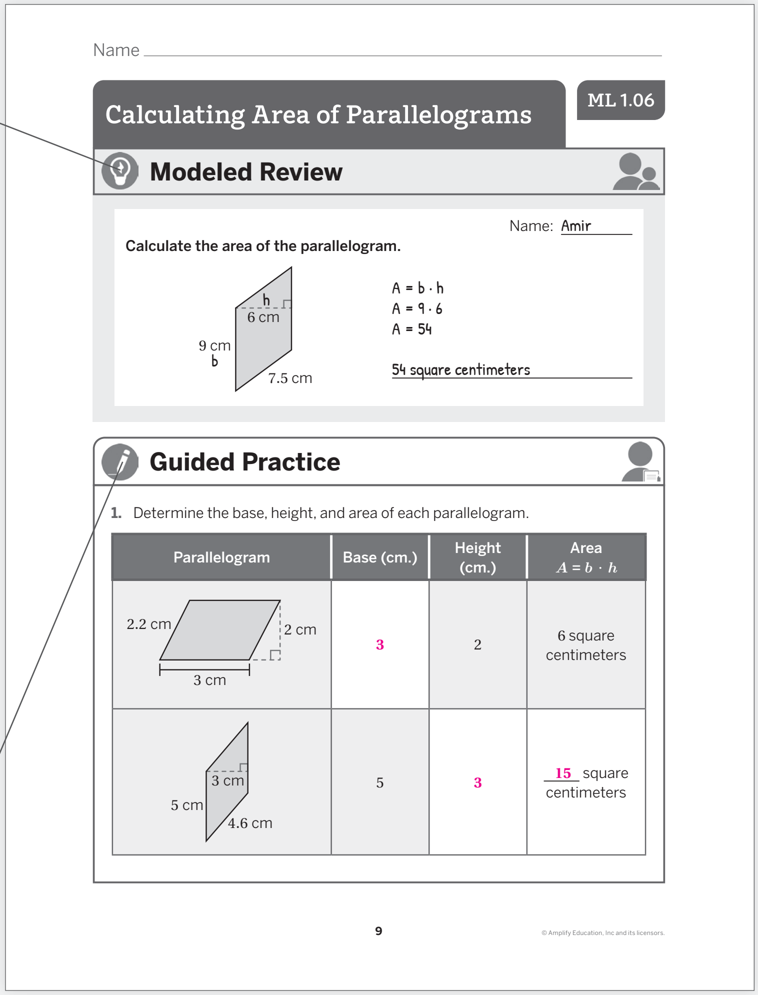
ML/EL Use a think aloud to model labeling the base and height of the parallelogram to stay organized.

Guided Practice

Focus students' attention on determining which measurements represent the base and height of each parallelogram.

To scaffold their thinking, **say**:

- "First, identify a base-height pair."
- "Next, substitute the values into the formula, $A = b \cdot h$."
- "Last, multiply the base and height to calculate the area."



Name

Calculating Area of Parallelograms ML 1.06

Modeled Review

Name: Amir

Calculate the area of the parallelogram.

A = b · h
A = 9 · 6
A = 54

54 square centimeters

Guided Practice

1. Determine the base, height, and area of each parallelogram.

Parallelogram	Base (cm.)	Height (cm.)	Area $A = b \cdot h$
2.2 cm, 2 cm, 3 cm	3	2	6 square centimeters
3 cm, 5 cm, 4.6 cm	5	3	15 square centimeters

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Vocabulary

If needed, share the meaning of the terms with students.

base of a parallelogram: The base is one side of a parallelogram. We can choose any side to be the base.

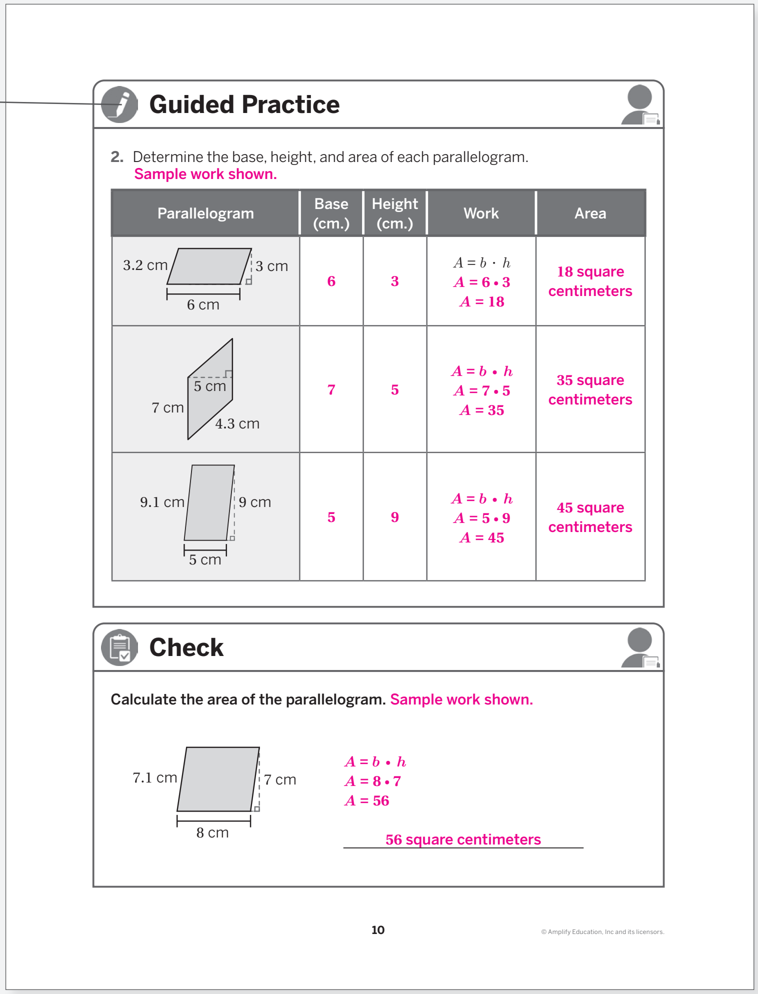
height: The height of a parallelogram is the shortest distance between a base and its opposite side.

area: Area measures the space inside a two-dimensional figure. It is expressed in square units.

Guided Practice

A Consider highlighting the right angle to show how it helps in identifying a base-height pair.

Key Takeaway:
Say, "When calculating the area of a parallelogram without a grid, you use measurements of a base and its matching height to determine the area."



Guided Practice

2. Determine the base, height, and area of each parallelogram.
Sample work shown.

Parallelogram	Base (cm.)	Height (cm.)	Work	Area
3.2 cm, 3 cm, 6 cm	6	3	$A = b \cdot h$ $A = 6 \cdot 3$ $A = 18$	18 square centimeters
5 cm, 7 cm, 4.3 cm	7	5	$A = b \cdot h$ $A = 7 \cdot 5$ $A = 35$	35 square centimeters
9.1 cm, 9 cm, 5 cm	5	9	$A = b \cdot h$ $A = 5 \cdot 9$ $A = 45$	45 square centimeters

Check

Calculate the area of the parallelogram. Sample work shown.

7.1 cm, 7 cm, 8 cm

$A = b \cdot h$
$A = 8 \cdot 7$
$A = 56$

56 square centimeters

10

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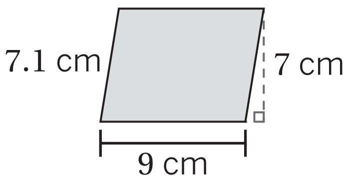
Reflection

Ask:
- "Describe how you can determine the area of any parallelogram."
- "What strategy was helpful today?"

Check: Recommended Next Steps

Almost there
If students need more support, consider having them label the base and height of the parallelogram to ensure they use the correct measurements for calculating the area.

Got it!
If students need more practice, sketch the parallelogram and ask them to calculate the area.



Name ____

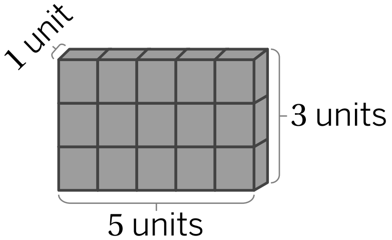
Determining Surface Areas of Rectangular Prisms

ML 1.09

Modeled Review

Name: Jaleel

Determine the surface area of the rectangular prism. Show or explain your thinking.



(5 • 3) • 2 = 30

(5 • 1) • 2 = 10

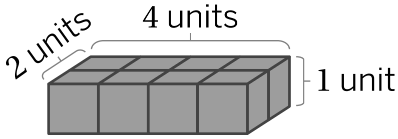
(3 • 1) • 2 = 6

30 + 10 + 6 = 46 square units

Guided Practice

Determine the surface area of each rectangular prism. Show or explain your thinking.

1.



(4 • 1) • 2 = 8

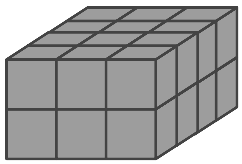
(1 • 2) • 2 = 4

(2 • 4) • 2 = 16

____ + ____ + ____ = ____

surface area: ______ square units

2.



(3 • 2) • 2 = ____

(2 • 4) • 2 = ____

(4 • 3) • 2 = ____

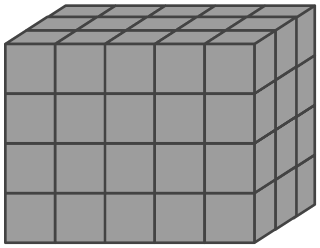
____ + ____ + ____ = ____

surface area: ______ square units

Guided Practice

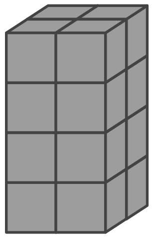
Determine the surface area of each rectangular prism. Show or explain your thinking.

3.



surface area: ______ square units

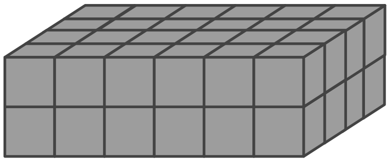
4.



surface area: ______ square units

Check

Determine the surface area of the rectangular prism. Show or explain your thinking.



surface area: ______ square units

Determining Surface Areas of Rectangular Prisms

Goal

Calculate the surface area of rectangular prisms.

Standard

MA.6.GR.2.1

Materials

unit cubes (optional)

Modeled Review

Point to Jaleel's work and **ask**:

- "Why do you think Jaleel multiplied 5 by 3? Why did he then multiply by 2?"
- "How would Jaleel know how many faces to count if all of the faces are not visible?"

Reinforce the goal by saying, "The surface area is the number of unit squares needed to cover all the faces of a solid without gaps or overlaps. To calculate the surface area, determine the sum of the areas of the six faces."

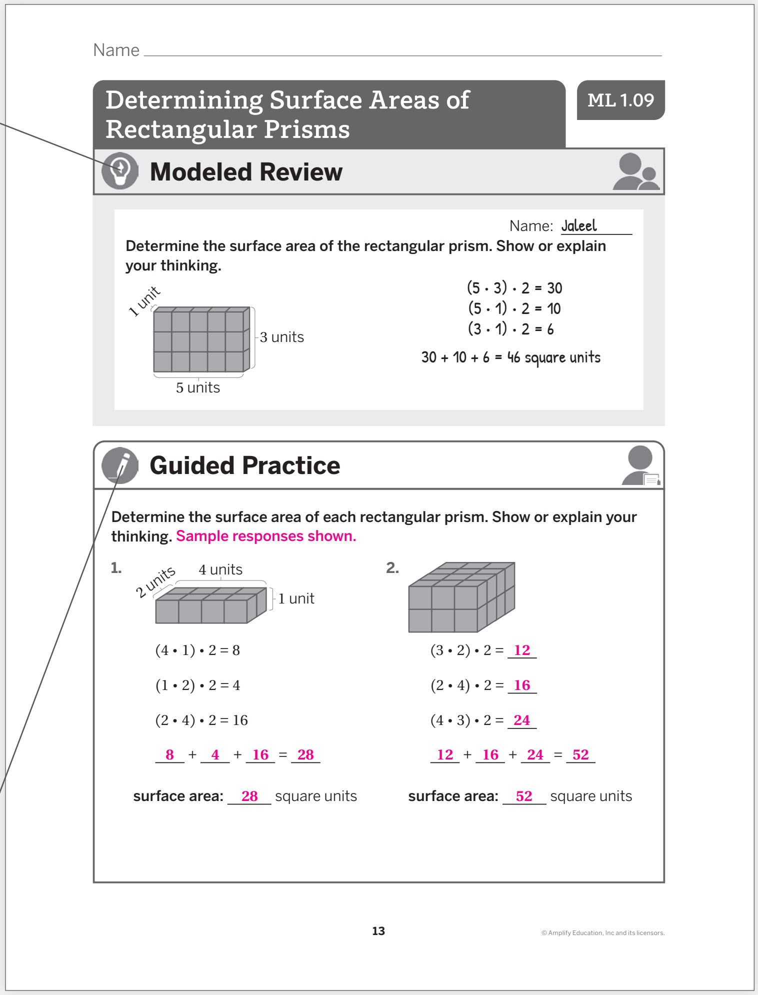
ML/EL Support students with recognizing the faces that are not visible by providing them with cubes to build the prism and noting each face.

Guided Practice

Focus students' attention on determining the area of each pair of opposite faces and adding the sum to find the surface area.

To scaffold their thinking, **ask**:

- "How could you calculate the surface area of a rectangular prism when you can only see three faces?"
- "Where do you see a pair of opposite faces?"



Name

Determining Surface Areas of Rectangular Prisms — ML 1.09

Modeled Review

Name: Jaleel

Determine the surface area of the rectangular prism. Show or explain your thinking.

(5 · 3) · 2 = 30
(5 · 1) · 2 = 10
(3 · 1) · 2 = 6
30 + 10 + 6 = 46 square units

Guided Practice

Determine the surface area of each rectangular prism. Show or explain your thinking. Sample responses shown.

1.

(4 • 1) • 2 = 8
(1 • 2) • 2 = 4
(2 • 4) • 2 = 16
8 + 4 + 16 = 28
surface area: 28 square units

2. (3 • 2) • 2 = 12
(2 • 4) • 2 = 16
(4 • 3) • 2 = 24
12 + 16 + 24 = 52
surface area: 52 square units

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Vocabulary

If needed, share the meaning of the terms with students.

surface area: The number of unit squares to cover all the faces of a solid without gaps or overlaps.

volume: The total space inside a 3-D shape, measured here as the number of unit cubes needed to fill that shape without any gaps or overlaps.

Determining Surface Areas of Rectangular Prisms

Guided Practice

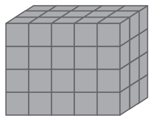
A As students interpret visual representations, provide access to unit cubes or other three-dimensional models for them to physically manipulate and hold as they make sense of the prism.

Key Takeaway:
Say, "The surface area of a rectangular prism is the sum of the areas of all its faces."

Guided Practice

Determine the surface area of each rectangular prism. Show or explain your thinking. Sample responses shown.

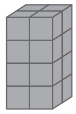
3.



$5 \cdot 4 \cdot 2 = 40$
$5 \cdot 3 \cdot 2 = 30$
$4 \cdot 3 \cdot 2 = 24$
$40 + 30 + 24 = 94$

surface area: 94 square units

4.

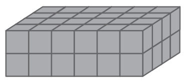


$2 \cdot 4 \cdot 2 = 16$
$2 \cdot 2 \cdot 2 = 8$
$4 \cdot 2 \cdot 2 = 16$
$16 + 8 + 16 = 40$

surface area: 40 square units

Check

Determine the surface area of the rectangular prism. Show or explain your thinking. Sample responses shown.



$6 \cdot 2 \cdot 2 = 24$
$6 \cdot 4 \cdot 2 = 48$
$2 \cdot 4 \cdot 2 = 16$
$24 + 48 + 16 = 88$

surface area: 88 square units

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Reflection

Ask:

- "How are calculating surface area and calculating area alike? Different?"
- "What is something you were unsure about at the start of the lesson but understand now?"

Check: Recommended Next Steps

Almost there
If students need more support, provide them with a rectangular prism built from cubes and have them practice finding the surface area.

Got it!
If students need more practice, ask them how the surface area of the prism in the Check would change if the prism was 5 units tall instead of 2.

Unit 2

Mini-Lessons

Name ______________________________

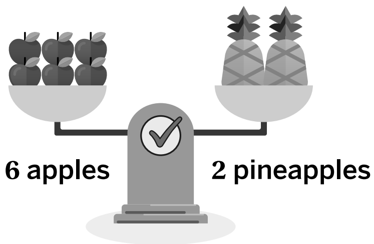
Generating Equivalent Ratios

ML 2.03

Modeled Review

Name: Tristan

The scale balances with 6 apples to 2 pineapples. Fill in the table of values that will balance the scale. Show your thinking.



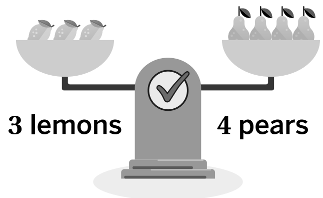
	Number of apples	Number of pineapples	
× 2	3	1	2 ×
× 2	6	2	2 ×
× 2	12	4	2 ×
	24	8	

Guided Practice

Use the tables below to record ratios that are equivalent to those shown on each balance scale.

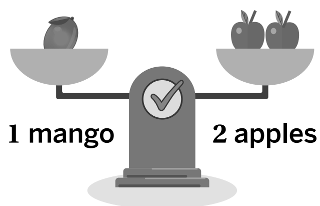
1.

	Number of lemons	Number of pears	
× 2	3	4	× 2
× 2	6		× 2
		16	



2.

Number of mangos	Number of apples
1	2
10	



Guided Practice

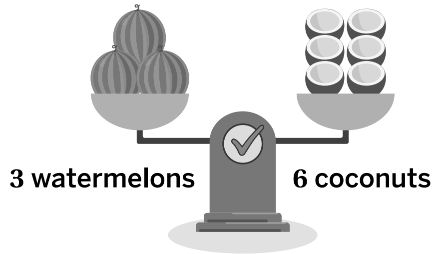
3. The scale balances with 3 watermelons to 6 coconuts. Select all of the equivalent ratios.

   A. 1 watermelon and 2 coconuts

   B. 6 watermelons and 12 coconuts

   C. 9 watermelons and 15 coconuts

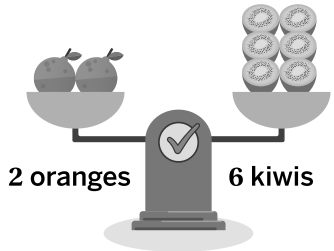
   D. 30 watermelons and 60 coconuts



For Problems 4-5, use the tables below to record ratios that are equivalent to those shown on each balance scale.

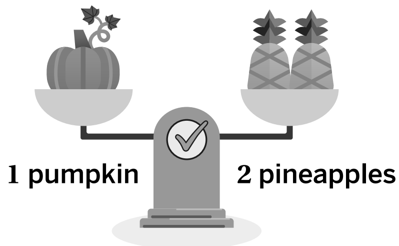
4.

Number of oranges	Number of kiwis



5.

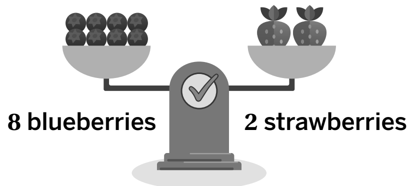
Number of pumpkins	Number of pineapples



Check

Use the table below to record ratios that are equivalent to those shown on the balance scale.

Number of blueberries	Number of strawberries
4	
	8



Generating Equivalent Ratios

Goal

Generate equivalent ratios by multiplying or dividing by the same number.

Standard

MA.6.AR.3.3

Materials

pattern blocks (optional)

Modeled Review

Point to Tristan's work and **ask**:

- "What does it mean for a scale to be balanced? Unbalanced?"
- "How does the ratio in the second row relate to the ratio in the first row? How would that relate to the scale?"

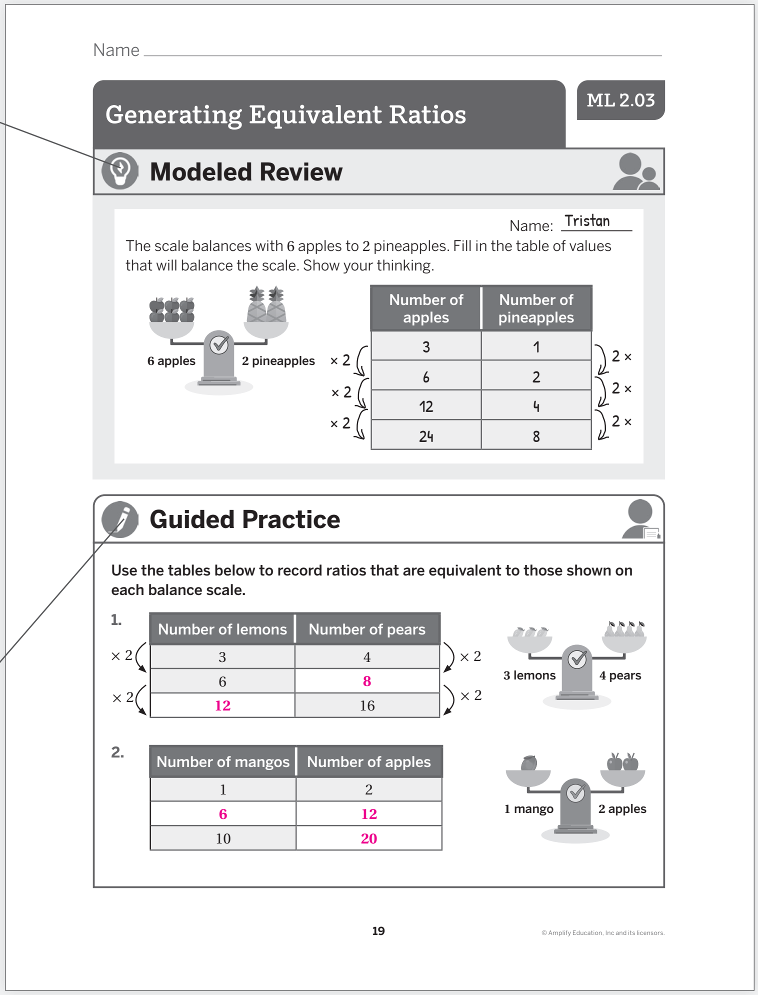
Reinforce the goal by saying, "Equivalent ratios are created by multiplying or dividing both quantities in a ratio by the same number."

Guided Practice

Focus students' attention on using multiplication and division to generate equivalent ratios.

To scaffold their thinking, **ask**:

- "How can you determine the equivalent ratios in the table?"
- "What is the 'for every' relationship between the two quantities of the original ratio?"
- "How could you use multiplication or division to determine unknown values?"



Name

Generating Equivalent Ratios — ML 2.03

Modeled Review

Name: Tristan

The scale balances with 6 apples to 2 pineapples. Fill in the table of values that will balance the scale. Show your thinking.

Number of apples	Number of pineapples
3	1
6	2
12	4
24	8

× 2, × 2, × 2 / 2 ×, 2 ×, 2 ×

Guided Practice

Use the tables below to record ratios that are equivalent to those shown on each balance scale.

1.

Number of lemons	Number of pears
3	4
6	8
12	16

× 2, × 2 / × 2, × 2

2.

Number of mangos	Number of apples
1	2
6	12
10	20

19

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Vocabulary

If needed, share the meaning of the term with students.

table: A table organizes information into horizontal rows and vertical columns. The first row or column usually tells what the numbers represent.

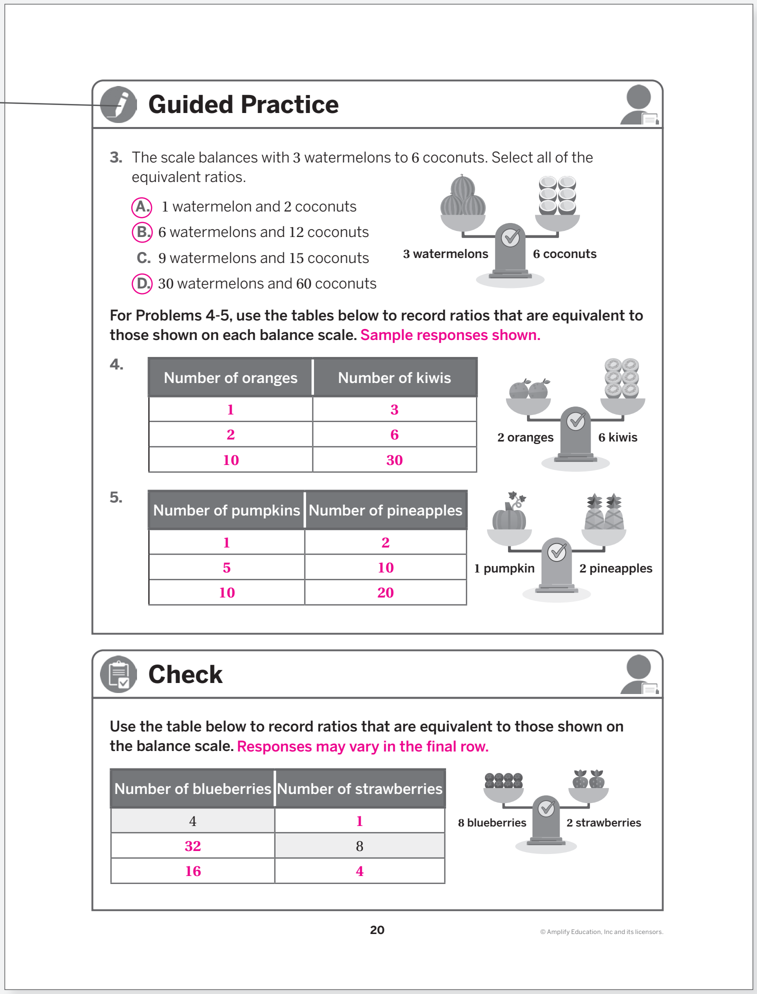
Guided Practice

A As students apply concepts to new situations, consider asking these questions:

- "What is important to think about when balancing the scale?"
- "How can you determine new quantities that keep the scale balanced? What information do you need?"

ML/EL Provide sentence frames to help students explain their thinking, e.g., "When you know a ratio balances a scale, you can create equivalent ratios by ___."

Key Takeaway:
Say, "Equivalent ratios can be used to balance a scale because when both quantities in the ratio are multiplied or divided by the same amount, the ratio relationship remains the same. You can use a table and/or multiplication and division to decide whether two ratios are equivalent."



Guided Practice

3. The scale balances with 3 watermelons to 6 coconuts. Select all of the equivalent ratios.

A. 1 watermelon and 2 coconuts
B. 6 watermelons and 12 coconuts
C. 9 watermelons and 15 coconuts
D. 30 watermelons and 60 coconuts

3 watermelons | 6 coconuts

For Problems 4-5, use the tables below to record ratios that are equivalent to those shown on each balance scale. Sample responses shown.

4.

Number of oranges	Number of kiwis
1	3
2	6
10	30

2 oranges | 6 kiwis

5.

Number of pumpkins	Number of pineapples
1	2
5	10
10	20

1 pumpkin | 2 pineapples

Check

Use the table below to record ratios that are equivalent to those shown on the balance scale. Responses may vary in the final row.

Number of blueberries	Number of strawberries
4	1
32	8
16	4

8 blueberries | 2 strawberries

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Reflection

Ask:

- "What strategy works best for you when creating equivalent ratios?"
- "How did you overcome a challenging problem today?"

Check: Recommended Next Steps

Almost there
If students need more support, provide them with pattern blocks to practice generating equivalent ratios of amounts provided. Have them record in a table similar to the ones in the lesson.

Got it!
If students need more practice, have them create a ratio table to record equivalent ratios for each ratio of fruit.

- 1 orange, 8 cherries
- 2 apples, 5 limes

Name ________________________________

Comparing Ratios

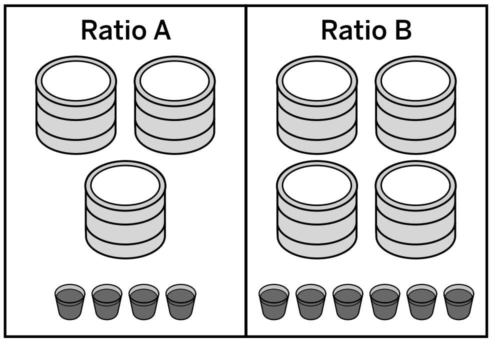
ML 2.08

Modeled Review

Name: Eva

Here are two ratios of red paint to white paint. Which ratio will make a darker red? Explain your thinking.

	Red paint
Ratio A	4 oz red: 3 gallons white
Ratio B	6 oz red: 4 gallons white



Ratio A is $\frac{4}{3}$, so it has 1.33 oz of red per gallon of white.
Ratio B is $\frac{6}{4}$, so it has 1.5 oz of red per gallon of white.

Ratio B will be darker because it has more red paint per gallon of white paint.

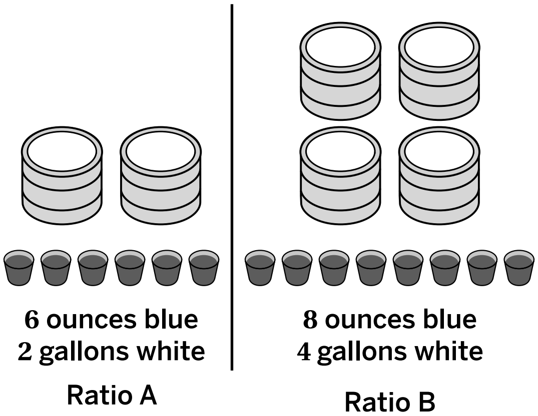
Guided Practice

Here are two ratios of blue paint to white paint. Which ratio will make a darker blue? Explain your thinking.

1. Ratio A is $\frac{6}{2}$ or _____ ounces of blue for every gallon of white.

Ratio B is _____ or _____ ounces of blue for every gallon of white.

Ratio _____ will be darker because it uses more blue paint for each gallon of white.



Guided Practice

For Problems 2-4, determine which ratio will make the darker paint color or if they would create the same color. Explain your thinking.

2.

Purple paint	
Ratio A	4 oz purple: 3 gal white
Ratio B	4 oz purple: 5 gal white

3.

Yellow Paint	
Ratio A	3 oz yellow: 6 gal white
Ratio B	14 oz yellow: 6 gal white

4.

Green Paint	
Ratio A	5 oz green: 2 gal white
Ratio B	15 oz green: 6 gal white

Check

Determine which ratio will make the darker paint color or if they would create the same color. Explain your thinking.

Red Paint	
Ratio A	2 oz red: 4 gal white
Ratio B	3 oz red: 5 gal white

ML 2.08 Comparing Ratios

Goal	Standard
Compare two ratios in context.	MA.6.AR.3.3

Modeled Review

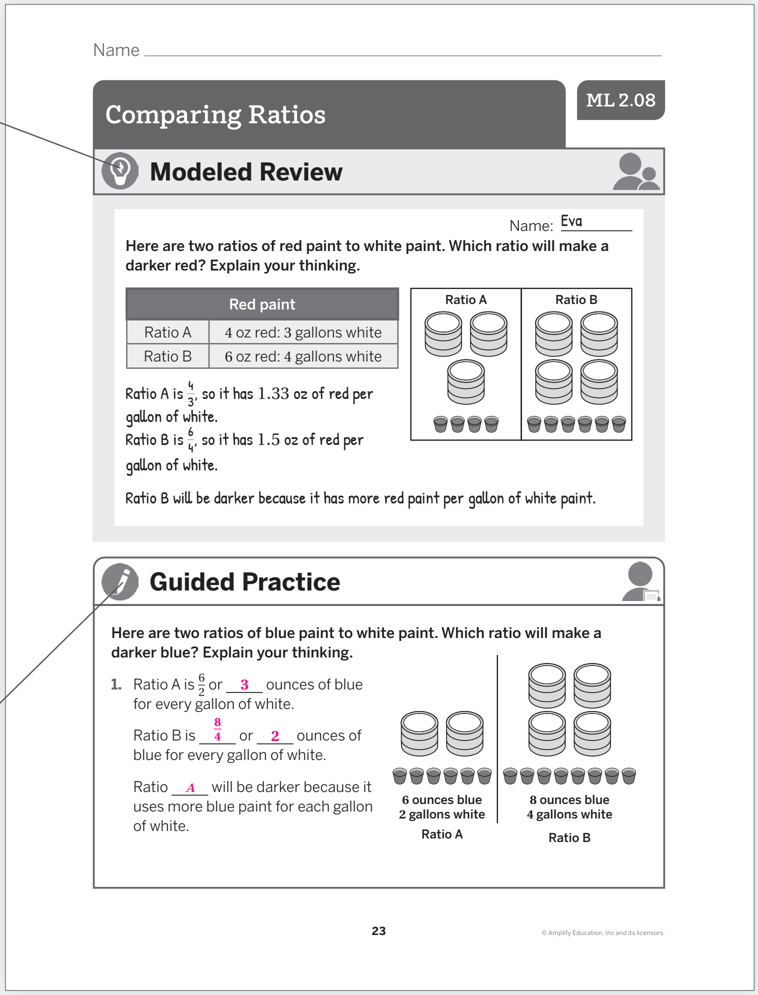
Point to Eva's work and **ask**:
- "How did Eva determine which ratio makes a darker color of red?"
- "How does changing the amount of white paint affect the color? Red paint?"

Reinforce Eva's thinking by saying, "When comparing multiple ratios, it is helpful to calculate how much each ratio is per 1 using decimals or fractions. Then the decimals or fractions can be compared."

Guided Practice

Focus students' attention on determining the ratio for 1 unit by comparing decimals or fractions.

To scaffold their thinking, **ask**:
- "How can you determine which ratio is a darker color?"
- "How will you find the ratio of blue to 1 unit of white?"


Name

Comparing Ratios

ML 2.08

Modeled Review

Name: Eva

Here are two ratios of red paint to white paint. Which ratio will make a darker red? Explain your thinking.

Red paint	
Ratio A	4 oz red: 3 gallons white
Ratio B	6 oz red: 4 gallons white

Ratio A is $\frac{4}{3}$, so it has 1.33 oz of red per gallon of white.
Ratio B is $\frac{6}{4}$, so it has 1.5 oz of red per gallon of white.

Ratio B will be darker because it has more red paint per gallon of white paint.

Guided Practice

Here are two ratios of blue paint to white paint. Which ratio will make a darker blue? Explain your thinking.

1. Ratio A is $\frac{6}{2}$ or 3 ounces of blue for every gallon of white.

   Ratio B is $\frac{8}{4}$ or 2 ounces of blue for every gallon of white.

   Ratio *A* will be darker because it uses more blue paint for each gallon of white.

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Vocabulary

If needed, share the meaning of the term with students.

ratio: A ratio $a : b$ is a relationship between two quantities. For every a of the first, there are b of the second.

Guided Practice

A To support students in carrying out multiple steps, chunk this activity into more manageable parts by inviting them to set up their fraction first to find the ratio for 1 unit. Then, compare the amounts to determine which is darker.

ML/EL Provide sentence frames for students to explain their thinking, such as "The mixture for Ratio A/B will be darker/lighter/the same color because___."

Note: For students struggling with decimal values, consider supporting their thinking with benchmark fractions.

Key Takeaway:
Say, "There are multiple strategies for comparing ratios, including calculating how much each ratio is per 1."

Guided Practice

For Problems 2-4, determine which ratio will make the darker paint color or if they would create the same color. Explain your thinking.
Sample responses shown.

2.

Purple paint	
Ratio A	4 oz purple: 3 gal white
Ratio B	4 oz purple: 5 gal white

Ratio A has $\frac{4}{3}$ or 1.33 oz of purple per gallon of white. Ratio B has $\frac{4}{5}$ or 0.80 oz of purple per gallon of white.
Ratio A will be darker.

3.

Yellow Paint	
Ratio A	3 oz yellow: 6 gal white
Ratio B	14 oz yellow: 6 gal white

Ratio A has $\frac{3}{6}$ or 0.50 oz of yellow per gallon of white.
Ratio B has $\frac{14}{6}$ or 2.3 oz of yellow per gallon of white.
Ratio B will be darker.

4.

Green Paint	
Ratio A	5 oz green: 2 gal white
Ratio B	15 oz green: 6 gal white

Ratio A has $\frac{5}{2}$ or 2.5 oz of green per gallon of white.
Ratio B has $\frac{15}{6}$ or 2.5 oz of green per gallon of white.
They will be the same color.

Check

Determine which ratio will make the darker paint color or if they would create the same color. Explain your thinking. Sample response shown.

Red Paint	
Ratio A	2 oz red: 4 gal white
Ratio B	3 oz red: 5 gal white

Ratio A has $\frac{2}{4}$ or 0.5 oz of red per gallon of white.
Ratio B has $\frac{3}{5}$ or 0.6 oz of red per gallon of white.
Ratio B will be darker.

24

Reflection

Ask:

- "How can you connect what you learned today to other real-world situations?"
- "What questions do you still have about comparing ratios?"

Check: Recommended Next Steps

Almost there
If students need more support, consider using Mini-Lesson 2.03: *Generating Equivalent Ratios*.

Got it!
If students need more practice, ask them to compare each pair of mixtures and determine which will make a darker color.

- 7 ounces of red with 5 gallons of white
  5 ounces of red with 4 gallons of white
- 2 ounces of blue with 3 gallons of white
  5 ounces of blue with 6 gallons of white

Name ______________________________

Using Ratio Tables to Solve Problems

ML 2.10

Modeled Review

Name: Wyatt

The toy factory produces 6 teddy bears for every 8 toy cars. How many toy cars are sold if 9 teddy bears are sold?

Number of Teddy Bears Sold	Number of Toy Cars Sold
6	8
1	$1\frac{1}{3}$
9	20

I created a ratio table relating the number of teddy bears sold to the number of toy cars sold. I divided 8 by 6 to get the ratio of bears to cars as 4:3 or $\frac{4}{3}$. Then, I multiplied $\frac{4}{3}$ by 9 to find the number of toy cars sold for 9 teddy bears.

Guided Practice

1. At a party store, there are 3 packs of balloons for every 12 party hats. Complete the table to determine the number of packs of balloons needed for 20 party hats. Show or explain your thinking.

Number of Packs of Balloons	Number of Party Hats
3	12
1	
	20

Guided Practice

For **Problems 2–3**, a recipe for chocolate chip cookies makes 4 dozen cookies and uses some of the ingredients below.

- $1\frac{1}{2}$ cups butter
- $1\frac{3}{4}$ cups sugar
- 2 t. vanilla
- $3\frac{1}{4}$ cups flour
- 1 t. baking soda
- $\frac{1}{4}$ t. salt
- 8 oz. chocolate chips

2. How many teaspoons of vanilla are needed to make 6 dozen cookies? Complete the table.

Number of cookies (doz)	Amount of vanilla (t)
4	2
1	

I will need ______ teaspoons of vanilla to make 6 dozen cookies.

3. How many cups of butter are needed to make 6 dozen cookies? Complete the table.

Number of cookies (doz)	Amount of butter (c)

I will need ______ cups of flour to make 6 dozen cookies.

A team builds 6 mountain bikes for every 4 road bikes. Complete the table to determine how many road bikes are built for every 9 mountain bikes. Show or explain your thinking.

Number of Mountain Bikes Built	Number of Road Bikes Built
6	4

ML 2.10 Using Ratio Tables to Solve Problems

Goal

Apply understanding of equivalent ratios using part-to-part ratio tables to solve real world applications.

Standard

MA.6.AR.3.3

Materials

None needed

Modeled Review

Point to Tristan's work and **ask**:

- Why did Wyatt divide both values by 6 first?
- What question would have required him to divide by 8 first?
- Why did Wyatt multiply both values in the second row of the table by 9 as his second step?

Reinforce the goal by saying "We can use ratio tables to create equivalent ratios to solve real world problems."

ML/EL Model the calculation of the values in the 2nd and 3rd rows of the ratio table.

Guided Practice

Focus students' attention on how to determine which value is multiplied (or divided) to calculate the values on each row.

To scaffold their thinking, **ask**:

- "How did they arrive at a 1 in the second row?"
- "What value is needed to multiply the numbers in the 2nd row to get the values in the 3rd row? "

Name

Using Ratio Tables to Solve Problems ML 2.10

Modeled Review

Name: Wyatt

The toy factory produces 6 teddy bears for every 8 toy cars. How many toy cars are sold if 9 teddy bears are sold?

Number of Teddy Bears Sold	Number of Toy Cars Sold
6	8
1	$1\frac{1}{3}$
9	20

I created a ratio table relating the number of teddy bears sold to the number of toy cars sold. I divided 8 by 6 to get the ratio of bears to cars as 4:3 or $\frac{4}{3}$. Then, I multiplied $\frac{4}{3}$ by 9 to find the number of toy cars sold for 9 teddy bears.

Guided Practice

1. At a party store, there are 3 packs of balloons for every 12 party hats. Complete the table to determine the number of packs of balloons needed for 20 party hats. Show or explain your thinking.

Number of Packs of Balloons	Number of Party Hats
3	12
1	4
5	20

5 packs of balloons. *Explanations vary*. I divided 12 by 3 to find the ratio of party hats to balloons, which is 4:1. Then, since $4 \times 5 = 20$, I multiplied 1 by 5 to get the number of packs of balloons needed.

27

Vocabulary

If needed, share the meaning of the term with students.

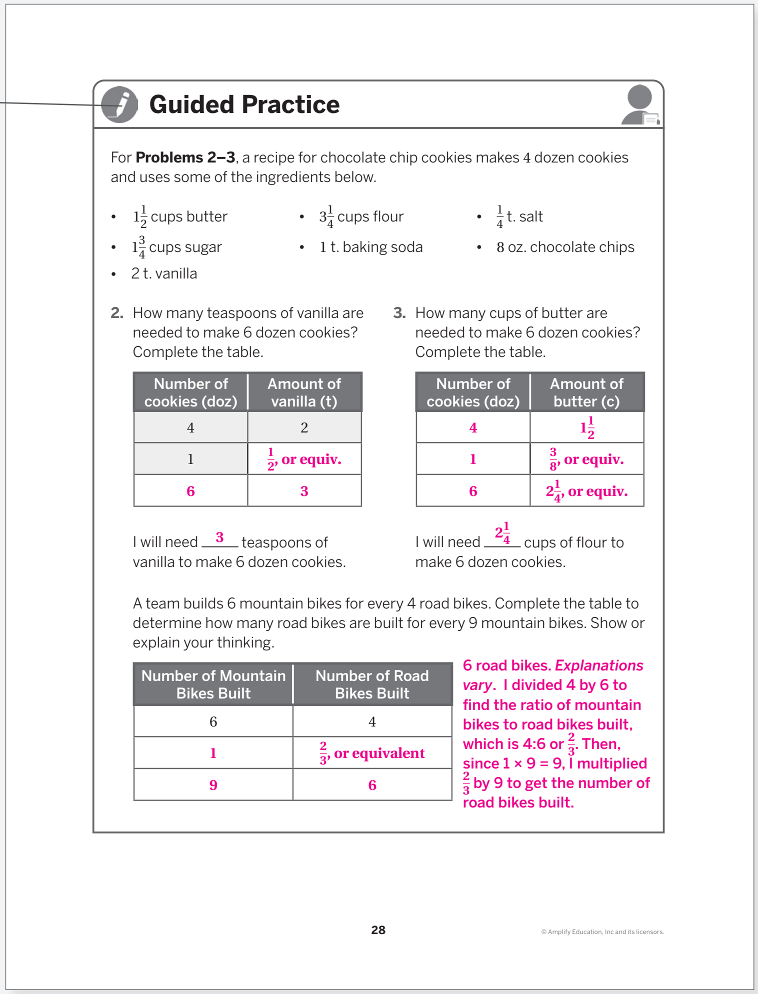
Ratio table: A table that lists ratios, where each row represents a different, but equivalent, ratio.

Part-to-part ratio: A part-to-part ratio compares different parts of a whole to each other.

Guided Practice

A As students complete the ratio tables, provide extra paper and/or area models to use when multiplying fractions. Invite students to use arrows to connect values from a row to the one below it and write the operation they are using to create equivalent ratios.

Key Takeaway:
Say, "Fill in the table with the values that you know first. Then, divide the number of (dozen) cookies by 4 to determine the amount of that ingredient needed for 1 dozen cookies. Lastly, multiply those numbers by the total number of (dozen) cookies you need for the last row."



Guided Practice

For **Problems 2–3**, a recipe for chocolate chip cookies makes 4 dozen cookies and uses some of the ingredients below.

- $1\frac{1}{2}$ cups butter
- $3\frac{1}{4}$ cups flour
- $\frac{1}{4}$ t. salt
- $1\frac{3}{4}$ cups sugar
- 1 t. baking soda
- 8 oz. chocolate chips
- 2 t. vanilla

2. How many teaspoons of vanilla are needed to make 6 dozen cookies? Complete the table.

Number of cookies (doz)	Amount of vanilla (t)
4	2
1	$\frac{1}{2}$, or equiv.
6	3

I will need 3 teaspoons of vanilla to make 6 dozen cookies.

3. How many cups of butter are needed to make 6 dozen cookies? Complete the table.

Number of cookies (doz)	Amount of butter (c)
4	$1\frac{1}{2}$
1	$\frac{3}{8}$, or equiv.
6	$2\frac{1}{4}$, or equiv.

I will need $2\frac{1}{4}$ cups of flour to make 6 dozen cookies.

A team builds 6 mountain bikes for every 4 road bikes. Complete the table to determine how many road bikes are built for every 9 mountain bikes. Show or explain your thinking.

Number of Mountain Bikes Built	Number of Road Bikes Built
6	4
1	$\frac{2}{3}$, or equivalent
9	6

6 road bikes. *Explanations vary*. I divided 4 by 6 to find the ratio of mountain bikes to road bikes built, which is 4:6 or $\frac{2}{3}$. Then, since $1 \times 9 = 9$, I multiplied $\frac{2}{3}$ by 9 to get the number of road bikes built.

28

Reflection

Ask:

- How can you use a ratio table to create equivalent ratios?
- What strategy did you use when multiplying and dividing fractions?

Check: Recommended Next Steps

Almost there	Got it!
If students need more support, consider using Mini-Lesson 2.3 *Generating Equivalent Ratios*.	If students need more practice, refer to Problem 3 and ask them to determine the amount of flour needed to make 6 dozen cookies.

Unit 3

Mini-Lessons

Name ______________________________

Converting Measurements

ML 3.02

Modeled Review

Name: Priya

A bookshelf is 6 feet wide. How many yards is that? Show your thinking.

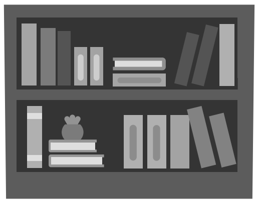
$$\frac{\textbf{1}\ \boldsymbol{yard}}{\textbf{3}\ \boldsymbol{feet}} = \frac{\boldsymbol{x}\ \boldsymbol{yards}}{\textbf{6}\ \boldsymbol{feet}}$$

$$3 \text{ ft} \times 2 = 6 \text{ ft}$$

$$\text{so } 1 \text{ yard} \times 2 = 2 \text{ yards}$$

$$2 \text{ yards}$$

6 feet



3 feet = 1 yard

Guided Practice

Use the given conversion rate to determine the approximate value of each missing quantity. Show your thinking.

1. A large measuring cup holds 48 ounces of water. Approximately how many cups is that?

$$\frac{48\ ounces}{x\ cups} = \frac{8\ ounces}{1\ cup}$$

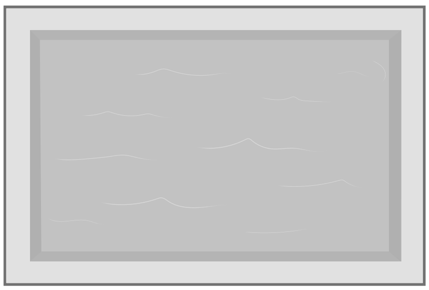
$\frac{48}{8} =$ _____   _____ cups

1 cup = 8 ounces

2. A swimming pool is 164 yards long. How many feet is that?

$$\frac{\square\ feet}{\square\ yards} = \frac{\square\ feet}{\square\ yards}$$

$3 \cdot 164 =$ _____   _____ feet



1 yard = 3 feet

Guided Practice

For Problems 3–6, use the conversion rate that makes the most sense to determine the approximate value of each missing quantity. Show your thinking.

1 yard = 3 feet	1 pound = 16 ounces
1 gallon = 16 cups	1 kilometer = 1000 meters

3. A watering can holds 5 gallons of water. Approximately how many cups is that?

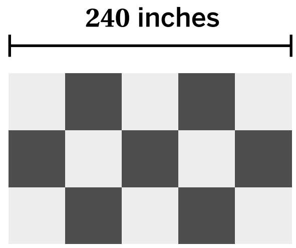
4. A bowling ball is 180 ounces. How many pounds is that?

5. A shipping container is 40 feet wide. Approximately how many yards is that?

6. A building is 0.35 kilometers tall. How many meters is that?

Check

A checkered floor is 240 inches wide. How many feet is that? Show your thinking.



1 foot = 12 inches

ML 3.02 Converting Measurements

Goal

Convert measurements from one unit to another using ratio reasoning.

Standard

MA.6.AR.3.5

Materials

highlighter (optional)

Modeled Review

Point to Priya's work and **ask**:

- "How did Priya know which conversion rate to use?"
- "How did Priya set up the ratio to convert the measurement?"
- "How did Priya know when to multiply or divide?"

Reinforce the goal by saying, "You can convert measurements from one unit to another using ratios."

ML/EL Model converting the measurements using ratios and the provided conversion rate.

Guided Practice

Focus students' attention on setting up a ratio to convert the measurement.

To scaffold their thinking, **ask**:

- "What unit are you converting to?"
- "How do we use the given conversion rate to set up a ratio?"

Name

Converting Measurements ML 3.02

Modeled Review

Name: Priya

A bookshelf is 6 feet wide. How many yards is that? Show your thinking.

$$\frac{1\ yard}{3\ feet} = \frac{x\ yards}{6\ feet}$$

3 ft × 2 = 6 ft

so 1 yard × 2 = 2 yards

2 yards

6 feet

3 feet = 1 yard

Guided Practice

Use the given conversion rate to determine the approximate value of each missing quantity. Show your thinking.

1. A large measuring cup holds 48 ounces of water. Approximately how many cups is that?

$$\frac{48\ ounces}{x\ cups} = \frac{8\ ounces}{1\ cup}$$

$\frac{48}{8}$ = 6   6 cups

1 cup = 8 ounces

2. A swimming pool is 164 yards long. How many feet is that?

$$\frac{3\ feet}{1\ yards} = \frac{x\ feet}{164\ yards}$$

3 • 164 = 492   492 feet

1 yard = 3 feet

33

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Vocabulary

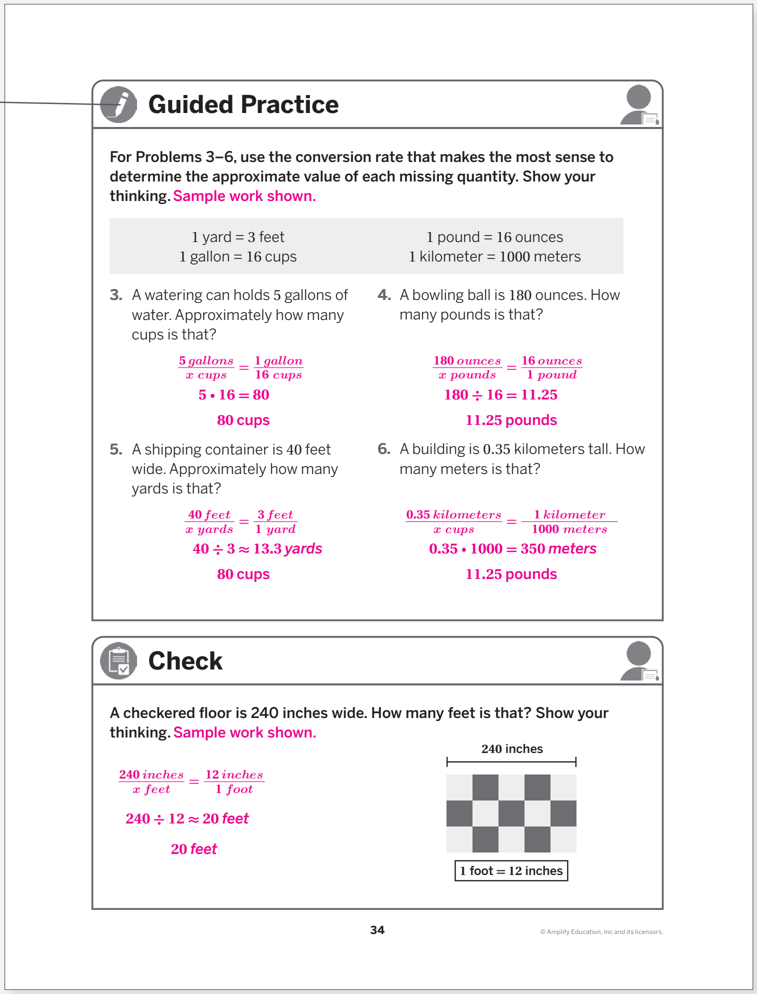
If needed, share the meaning of the term with students.

rate: A rate is a ratio that describes how two quantities change together.

Guided Practice

A Chunk this task into smaller, more manageable parts by having students first identify the conversion rate that is needed and then set up a ratio to solve the problem.

Key Takeaway:
Say, "Ratios in terms of 1 can be used to compare and convert measurements in same measurement system



Guided Practice

For Problems 3–6, use the conversion rate that makes the most sense to determine the approximate value of each missing quantity. Show your thinking. Sample work shown.

1 yard = 3 feet 1 gallon = 16 cups	1 pound = 16 ounces 1 kilometer = 1000 meters

3. A watering can holds 5 gallons of water. Approximately how many cups is that?

$\frac{5\ gallons}{x\ cups} = \frac{1\ gallon}{16\ cups}$

$5 \cdot 16 = 80$

80 cups

4. A bowling ball is 180 ounces. How many pounds is that?

$\frac{180\ ounces}{x\ pounds} = \frac{16\ ounces}{1\ pound}$

$180 \div 16 = 11.25$

11.25 pounds

5. A shipping container is 40 feet wide. Approximately how many yards is that?

$\frac{40\ feet}{x\ yards} = \frac{3\ feet}{1\ yard}$

$40 \div 3 \approx 13.3$ *yards*

80 cups

6. A building is 0.35 kilometers tall. How many meters is that?

$\frac{0.35\ kilometers}{x\ cups} = \frac{1\ kilometer}{1000\ meters}$

$0.35 \cdot 1000 = 350$ *meters*

11.25 pounds

Check

A checkered floor is 240 inches wide. How many feet is that? Show your thinking. Sample work shown.

$\frac{240\ inches}{x\ feet} = \frac{12\ inches}{1\ foot}$

$240 \div 12 \approx 20$ *feet*

20 feet

34

Reflection

Ask:

- "How does knowing 'how much per 1' help you convert between units of measurement?"
- "What makes sense? What is still confusing?"

Check: Recommended Next Steps

Almost there	Got it!
If students need more support, ask them to highlight the units in the Check to help identify the conversion rate needed. Then have them set up a ratio using the same units.	If students need more practice, refer to Problem 6 and ask them what the measurement of the building would be in decimeters, if one meter is equal to 10 decimeters.

Name ____________________

Calculating Unit Rates

ML 3.05

Modeled Review

Name: Jason

During a sale, a clothing store is selling 5 shirts for $30. What is the unit rate? What does it represent?

30 ÷ 5 = 6

The unit rate is $6/shirt. This represents the price of one shirt.

Guided Practice

1. A commuter train travels 60 miles in 30 minutes without stopping. A subway train travels 5 miles in 2 minutes without stopping. Which train is traveling faster? How do you know?

Commuter train

Ratio: 60 miles/ 30 minutes

Unit rate: 2 miles/minute

Subway train

Ratio: _____ miles/ _____ minutes

Unit rate: _____ miles/minute

Faster train: The __________ train. It has a greater unit rate.

2. A local grocery store sells three gallons of milk for $12.00. A supermarket sells two gallons of milk for $8.50. Which is the better deal? How do you know?

Grocery store

Ratio: _____ dollars/ _____ gallons

Unit rate: _____ dollars/gallon

Supermarket

Ratio: _____ dollars/ _____ gallons

Unit rate: _____ dollars/gallon

Better deal: ______________________________

Guided Practice

3. A cheetah has been observed traveling a distance of 315 meters in 10 seconds. How fast was the cheetah?

Cheetah

Ratio: _____ meters/ _____ seconds

Unit rate: _____ meters/second

A light aircraft needs to travel 30 meters per second to take off. Was the cheetah moving faster than this? How do you know?

____

4. You are setting up a lemonade stand. Your goal is to sell 25 cups of lemonade. The recipe you use says that 12 lemons make 4 cups. You purchase 100 lemons to make 25 cups. Will your lemonade have more or less lemon flavor than the recipe? How do you know?

Recipe

Ratio: ______________________

Unit rate: ___________________

Lemonade stand

Ratio: ______________________

Unit rate: ___________________

____

____

Check

A red model train moves 180 centimeters in 30 seconds. A green model train moves 320 centimeters in 40 seconds. Which train is moving faster? How do you know?

____

____

ML 3.05 Calculating Unit Rates

Goal

Determine unit rates in real-life situations and compare them in context.

Standard

MA.6.AR.3.2

Materials

coloring tools (optional)

Modeled Review

Point to Jason's work and **ask**:

- "What is a unit rate?"
- "What information is needed to calculate the unit rate?"

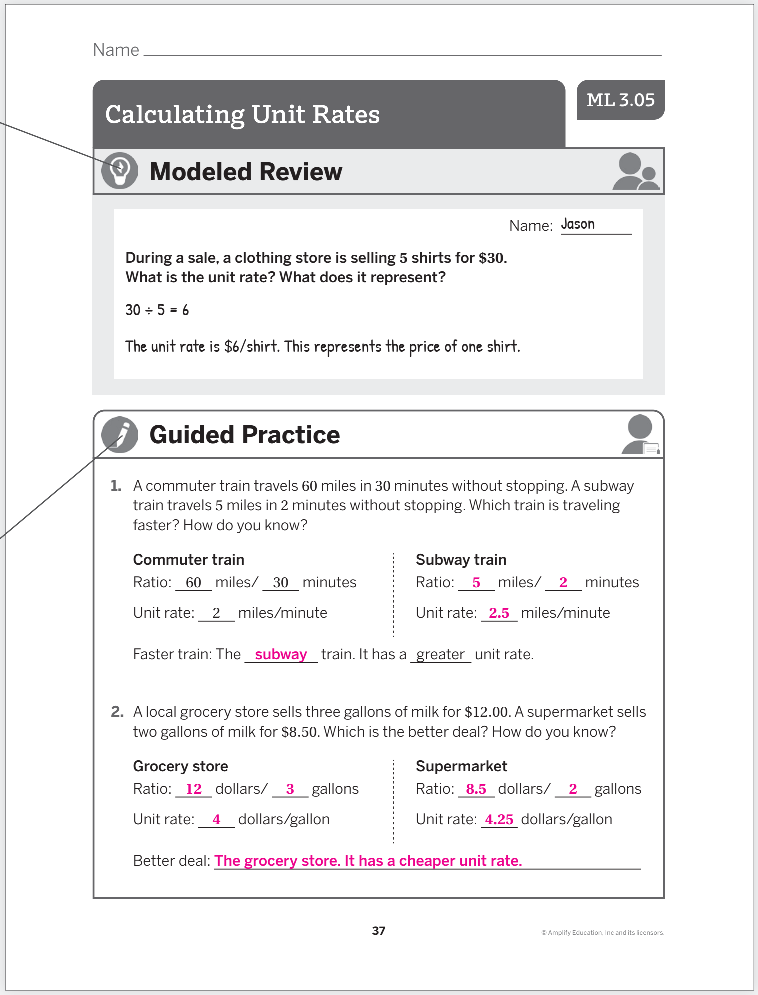
Reinforce the goal by saying, "A unit rate is a rate in which one of the quantities being compared is 1."

Guided Practice

Focus students' attention on calculating and comparing unit rates.

To scaffold their thinking, **ask**:

- "What units are represented in the problem?"
- "What are the steps you could take to find the unit rate?"
- "How do we use the determined unit rates to make comparisons?"



Name ______

Calculating Unit Rates | ML 3.05

Modeled Review

Name: Jason

During a sale, a clothing store is selling 5 shirts for $30. What is the unit rate? What does it represent?

30 ÷ 5 = 6

The unit rate is $6/shirt. This represents the price of one shirt.

Guided Practice

1. A commuter train travels 60 miles in 30 minutes without stopping. A subway train travels 5 miles in 2 minutes without stopping. Which train is traveling faster? How do you know?

Commuter train
Ratio: 60 miles/ 30 minutes
Unit rate: 2 miles/minute

Subway train
Ratio: 5 miles/ 2 minutes
Unit rate: 2.5 miles/minute

Faster train: The subway train. It has a greater unit rate.

2. A local grocery store sells three gallons of milk for $12.00. A supermarket sells two gallons of milk for $8.50. Which is the better deal? How do you know?

Grocery store
Ratio: 12 dollars/ 3 gallons
Unit rate: 4 dollars/gallon

Supermarket
Ratio: 8.5 dollars/ 2 gallons
Unit rate: 4.25 dollars/gallon

Better deal: The grocery store. It has a cheaper unit rate.

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Vocabulary

If needed, share the meaning of the terms with students.

rate: A rate is a ratio that describes how two quantities change together.

unit price: The cost for one item or the cost per item.

unit rate: A rate that describes how one quantity changes when the other quantity changes by exactly 1 unit.

Guided Practice

(A) Guide visualization by suggesting students color code the ratios in each problem.

(ML/EL) Invite students to write their explanations in their home language.

Key Takeaway:
Say, "Rate is a comparison of how two quantities change together. Typically, this is described by the unit rate, meaning how much one quantity changes when the other quantity changes by exactly 1 unit."

Guided Practice

3. A cheetah has been observed traveling a distance of 315 meters in 10 seconds. How fast was the cheetah?

Cheetah

Ratio: 315 meters/ 10 seconds

Unit rate: 31.5 meters/second

A light aircraft needs to travel 30 meters per second to take off. Was the cheetah moving faster than this? How do you know?

Yes. Its unit rate was greater than 30 meters/second.

4. You are setting up a lemonade stand. Your goal is to sell 25 cups of lemonade. The recipe you use says that 12 lemons make 4 cups. You purchase 100 lemons to make 25 cups. Will your lemonade have more or less lemon flavor than the recipe? How do you know?

Recipe	Lemonade stand
Ratio: 12 lemons/4 cups	Ratio: 100 lemons/25 cups
Unit rate: 3 lemons/cup	Unit rate: 4 lemons/cup

My lemonade will have more lemon flavor because I will use more lemons per cup.

Check

A red model train moves 180 centimeters in 30 seconds. A green model train moves 320 centimeters in 40 seconds. Which train is moving faster? How do you know?

The green train is faster. Its unit rate of 8 centimeters/second is greater than the red train's unit rate of 6 centimeters/second.

38

Reflection

Ask:

- "How does using ratios help solve unit rates in real-life situations?"
- "What questions do you still have?"

Check: Recommended Next Steps

Almost there
If students need more support, consider having them identify the unit rate for the problem.

An amusement park is selling 5 tickets for $20. What is the unit rate? What does it represent?

Got it!
If students need more practice, consider having them find and compare the unit rates to solve the problem.

A car travels 8 miles in 12 minutes at a constant speed. A motorcycle travels 6 miles in 10 minutes at a constant speed. Which is traveling faster? Explain your thinking.

Name ______________________________

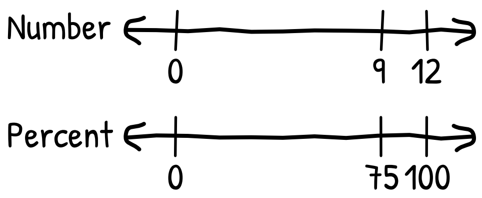
Calculating Unknown Percentages

ML 3.12

Modeled Review

Name: Tristan

What is 9 out of 12 as a percent?



75%

Name: Santiago

What is 9 out of 12 as a percent?

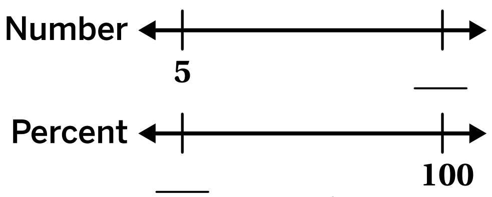
$\frac{9}{12} \cdot 100 = \underline{75}$

75%

Guided Practice

For Problems 1 and 2, calculate the unknown percentage. Show your thinking.

1. What is 5 out of 10 as a percent?



___%

2. What is 5 out of 50 as a percent?

$\frac{5}{50} \cdot 100 =$ _____

_____ %

Guided Practice

Calculate the unknown percentage. Show your thinking.

3. What is 18 out of 24 as a percent?

$\frac{18}{24} \cdot 100 =$ _____

4. What is 4 out of 16 as a percent?

_____ $\cdot 100 =$ _____

5. What is 13 out of 26 as a percent?

6. What is 3 out of 15 as a percent?

7. What is 7 out of 70 as a percent?

8. What is 19 out of 38 as a percent?

Check

What is 16 out of 64 as a percent? Show your thinking.

ML 3.12 Calculating Unknown Percentages

Goal

Calculate an unknown percentage.

Standard

MA.6.AR.3.4

Materials

double number line template (optional)

Modeled Review

Point to the problem in the Modeled Review and **ask**:

- "How are Tristan's and Santiago's work alike? How are they different?"
- "How did Tristan use a double number line diagram to calculate the percent?"
- "How did Santiago use an equation to calculate the percent?"

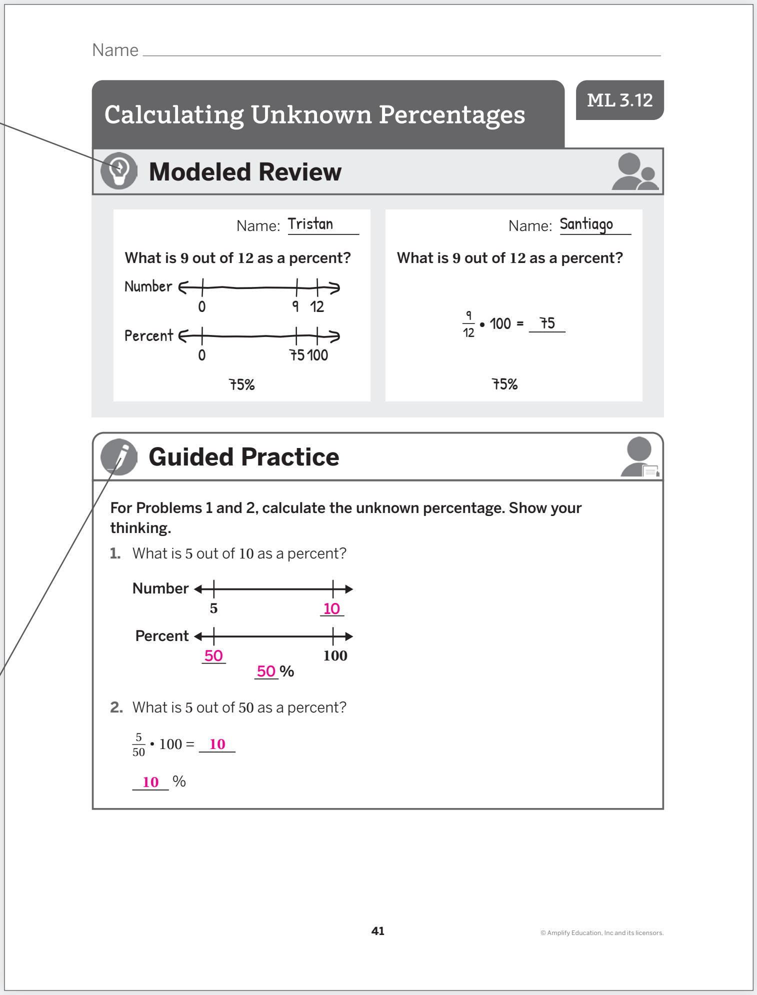
Reinforce the goal by saying, "The percent of a number can be calculated using double number line diagrams or equations."

Guided Practice

Focus students' attention on how the percent of a number can be calculated using a double number line an equation.

To scaffold their thinking, **ask**:

- "How can you use a double number line to calculate the percent?"
- "How can you use an equation to calculate the percent?"



Name

Calculating Unknown Percentages ML 3.12

Modeled Review

Name: Tristan

What is 9 out of 12 as a percent?

Number 0, 9, 12

Percent 0, 75, 100

75%

Name: Santiago

What is 9 out of 12 as a percent?

$\frac{9}{12} \cdot 100 =$ 75

75%

Guided Practice

For Problems 1 and 2, calculate the unknown percentage. Show your thinking.

1. What is 5 out of 10 as a percent?

Number 5, 10

Percent 50, 100

50 %

2. What is 5 out of 50 as a percent?

$\frac{5}{50} \cdot 100 =$ 10

10 %

41

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Vocabulary

If needed, share the meaning of the term with students.

rate: A rate is a ratio that describes how two quantities change together.

Guided Practice

A Provide students with access to double number line diagrams and support them with marking benchmark percentages (i.e., 50%, 25%, 10%) before starting the problems.

ML/EL Provide sentence frames to support students as they explain their strategies. For example, "I know that the percentage is ________ because ________."

Key Takeaway:
Say, "Percent of a number can be calculated using a variety of strategies, including double number line diagrams and equations."

Guided Practice

Calculate the unknown percentage. Show your thinking.
Sample work shown.

3. What is 18 out of 24 as a percent?

   $\frac{18}{24} \cdot 100 = \underline{75}$

   75%

4. What is 4 out of 16 as a percent?

   $\frac{4}{16} \cdot 100 = \underline{25}$

   25%

5. What is 13 out of 26 as a percent?

   $\frac{13}{26} \cdot 100 = 50$

   50%

6. What is 3 out of 15 as a percent?

   $\frac{3}{15} \cdot 100 = 20$

   20%

7. What is 7 out of 70 as a percent?

   $\frac{7}{70} \cdot 100 = 10$

   10%

8. What is 19 out of 38 as a percent?

   $\frac{19}{38} \cdot 100 = 50$

   50%

Check

What is 16 out of 64 as a percent? Show your thinking.
Sample work shown.

$\frac{16}{64} \cdot 100 = 25$

25%

42

Reflection

Ask:
- "How are equations helpful in calculating the unknown percentage?"
- "What questions do you still have?"

Check: Recommended Next Steps

Almost there
If students need more support, ask them to circle the information given to help identify what they are solving for and set up their equation with the given information.

Got it!
If students need more practice, ask them to calculate the unknown percentage and show their thinking.

- What is 6 out of 30 as a percentage?
- What is 8 out of 32 as a percentage?

Unit 4

Mini-Lessons

Name ____________________

Multiplying Fractions

ML 4.02

Modeled Review

Name: Ivy

Bella studied for $2\frac{1}{2}$ hours last night. $\frac{2}{3}$ of this time was spent working on an art project. How much time was spent on her art project?

I multiplied $2\frac{1}{2}$ and $\frac{2}{3}$ to figure out the time spent on Bella's art project.

Bella spent $1\frac{2}{3}$ hours working on her art project.

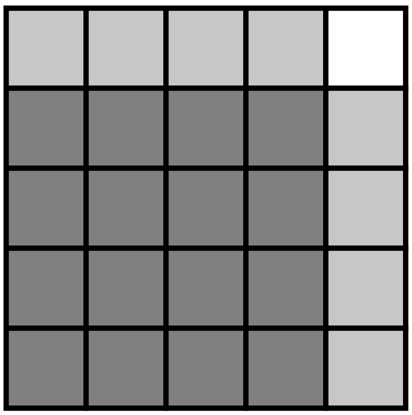
$2\frac{1}{2} \cdot \frac{2}{3}$

$\frac{5}{2} \cdot \frac{2}{3}$

$\frac{10}{6} = \frac{5}{3} = 1\frac{2}{3}$

Guided Practice

1. a. Write the expressions represented in the area model shown.



b. What is the product? Show or explain your thinking.

Guided Practice

2. What is the product of $\frac{3}{4} \cdot \frac{10}{3}$? Show your thinking.

3. What is the product of $2\frac{1}{4} \cdot 3\frac{1}{2}$? Show your thinking.

4. A bead kit has $3\frac{1}{2}$ bags of beads, and each bag contains $\frac{7}{8}$ cup of beads. How many cups of beads are there in total?

Yusuf is making a new shelf. He has a wooden board that is $5\frac{3}{4}$ feet long but he only needs $\frac{2}{3}$ of it for the shelf. How long is the new shelf?

Goal

Multiply fractions, including mixed numbers, with procedural fluency and apply this understanding to solve problems within a mathematical or real-world context.

Standard

MA.6.NSO.2.2

Materials

Colored pencils or highlighters (optional)

Modeled Review

Point to Ivy's work and **ask**:

- "How did Ivy know this problem could be solved using multiplication?"
- "Which multiplication strategy did Ivy use to multiply the fractions?"
- "What is another way Ivy could have solved this problem?"

Reinforce the goal by saying "One strategy that can be used when multiplying mixed fractions is to convert them to improper fractions first. Then, multiply the numerators and multiply the denominators."

Guided Practice

Focus students' attention on how an area model can be used to multiply fractions.

To scaffold their thinking, **ask/say**:

- "Which part of the area model represents $\frac{3}{4}$? $\frac{4}{5}$?"
- "How can we use the overlapping shaded region to determine the answer to the problem?"

Name

Multiplying Fractions

ML 4.02

Modeled Review

Name: Ivy

Bella studied for $2\frac{1}{2}$ hours last night. $\frac{2}{3}$ of this time was spent working on an art project. How much time was spent on her art project?

I multiplied $2\frac{1}{2}$ and $\frac{2}{3}$ to figure out the time spent on Bella's art project.

Bella spent $1\frac{2}{3}$ hours working on her art project.

$2\frac{1}{2} \cdot \frac{2}{3}$

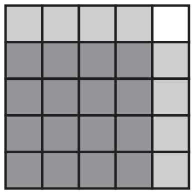
$\frac{5}{2} \cdot \frac{2}{3}$

$\frac{10}{6} = \frac{5}{3} = 1\frac{2}{3}$

Guided Practice

1. **a.** Write the expressions represented in the area model shown.

$\frac{3}{4} \cdot \frac{4}{5}$



b. What is the product? Show or explain your thinking.

$\frac{3}{5}$, or equivalent. *Methods vary.* 12 of the 20 squares are shaded the darkest so the product is $\frac{12}{20}$ or $\frac{3}{5}$.

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Vocabulary

If needed, share the meaning of the terms with students.

Area model: Arectangular diagram used to solve multiplication problems, in which the factors getting multiplied define the length and width of a rectangle.

Product: The result of a multiplication problem

Guided Practice

A If students use the area model method when multiplying fractions, invite them to use colored pencils or highlighters to shade in the parts of the rectangle represented by each fraction.

Key Takeaway:
Say, "There are multiple strategies that can be used to multiply fractions. The strategy you choose can be influenced by the form of the fractions (mixed, improper, simple), as well as the one(s) that make the most sense to you."

Note: Problems 2 and 3 show different strategies for multiplying fractions. Students can use the strategy that makes the most sense to them.

Guided Practice

2. What is the product of $\frac{3}{4} \cdot \frac{10}{3}$? Show your thinking.

$\frac{5}{2}$, or equivalent. *Methods vary.*

$$\frac{3}{4} \cdot \frac{10}{3} = \frac{30}{12} = \frac{5}{2}$$

3. What is the product of $2\frac{1}{4} \cdot 3\frac{1}{2}$? Show your thinking.

$7\frac{7}{8}$, or equivalent. *Methods vary.*

$$2\frac{1}{4} \cdot 3\frac{1}{2} = \left(2 + \frac{1}{4}\right)\frac{7}{2} = \frac{14}{2} + \frac{7}{8} = \frac{56}{8} + \frac{7}{8} = \frac{63}{8}$$

4. A bead kit has $3\frac{1}{2}$ bags of beads, and each bag contains $\frac{7}{8}$ cup of beads. How many cups of beads are there in total?

$3\frac{1}{16}$, cups of beads, or equivalent. *Methods vary.*

$$3\frac{1}{2} \cdot \frac{7}{8} = \left(3 + \frac{1}{2}\right)\frac{7}{8} = \frac{21}{8} + \frac{7}{16} = \frac{42}{16} + \frac{7}{16} = \frac{49}{16}$$

Yusuf is making a new shelf. He has a wooden board that is $5\frac{3}{4}$ feet long but he only needs $\frac{2}{3}$ of it for the shelf. How long is the new shelf?

$3\frac{5}{6}$ feet long or equivalent. *Methods vary.*

$$5\frac{3}{4} \cdot \frac{2}{3} = \frac{23}{4} \cdot \frac{2}{3} = \frac{46}{12} = \frac{23}{6}$$

48

Reflection

Ask:

- "Which strategies for multiplying fractions did you use?"
- "What strategy for multiplying fractions would you like to get better at using?"

Check: Recommended Next Steps

Almost there	Got it!
If students need more support, provide opportunities for them to practice using different strategies for the same multiplication problem. This can help them to make connections between the strategies.	If students need more practice, have them solve the problem below. You are planning to plant flowers in a rectangular garden. The length of the garden is $5\frac{1}{2}$ feet and the width is $2\frac{2}{3}$ feet. What is the area of the garden?

Name ______________________________

Determining How Many in Each Group

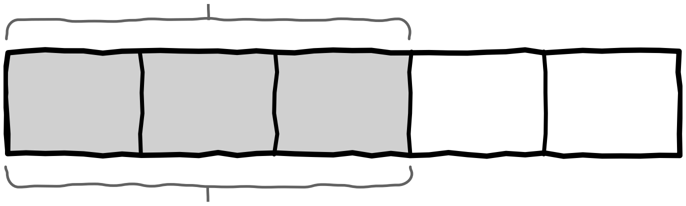
ML 4.04

Modeled Review

Name: Avery

6 flowers fill $\frac{3}{5}$ of a planter. How many flowers fill 1 planter? Draw a diagram if it is helpful.

$\frac{3}{5}$ of a planter



6 flowers

$6 \div \frac{3}{5}$

$6 \times \frac{5}{3} = \frac{30}{3} = 10$

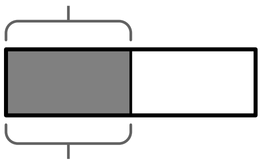
10 flowers

Guided Practice

Determine how many candles fill 1 box. Use the diagrams to model your thinking.

1. 8 candles fill $\frac{1}{2}$ of a box.

$\frac{1}{2}$ of a box



8 candles

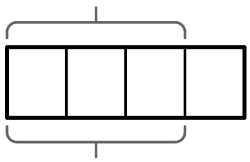
$8 \div$ ______

$8 \times$ ______ $=$ ______ $=$ ______

______ **candles**

2. 6 candles fill $\frac{3}{4}$ of a box.

______ of a box

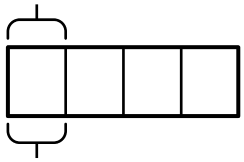


______ candles

____

Guided Practice

Micah is placing candles into different-sized boxes. Complete the table to determine the number of candles that can fill one box.

Scenarios	Diagram	Division expression	Number of candles that fill one box
6 candles fill $\frac{1}{4}$ of a box			
3 candles fill $\frac{3}{5}$ of a box			
9 candles fill $\frac{3}{4}$ of a box			

Check

3 candles fill $\frac{1}{5}$ of the box. How many candles fill 1 box? Draw a diagram if it is helpful.

ML 4.04 Determining How Many in Each Group

Goal	Standard
Solve division problems using a diagram.	MA.6.NSO.2.3

Modeled Review

Point to Avery's work and **ask**:

- "How did Avery draw a tape diagram to visualize the scenario?"
- "How did Avery calculate how many flowers fill 1 planter?"

Reinforce the goal by saying, "You can solve division problems by using tape diagrams when the number of groups is a fraction."

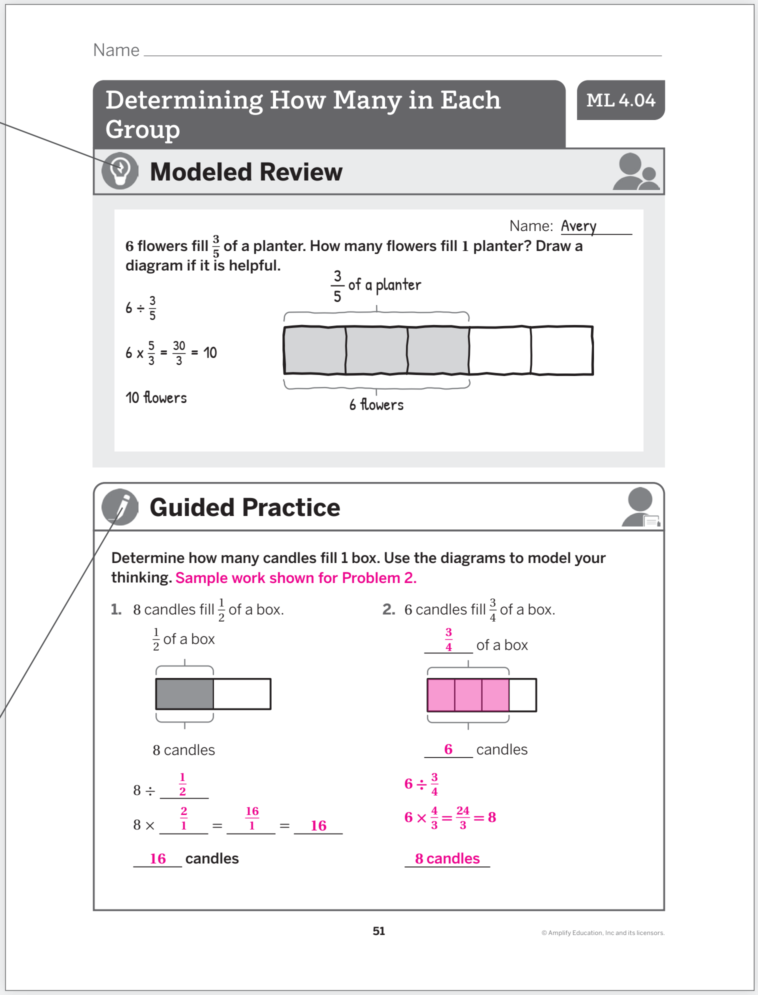
ML/EL Provide sentence frames to support students as they explain their strategies. For example, "I noticed ______. Then I ______."

Guided Practice

Focus students' attention on using the tape diagram to determine how many candles fill 1 box.

To scaffold student's thinking, **ask**:

- "How does the diagram represent the scenario?"
- "How could you represent the scenario with a division equation? Multiplication equation?"


Name

Determining How Many in Each Group — ML 4.04

Modeled Review

Name: Avery

6 flowers fill $\frac{3}{5}$ of a planter. How many flowers fill 1 planter? Draw a diagram if it is helpful.

$\frac{3}{5}$ of a planter

$6 \div \frac{3}{5}$

$6 \times \frac{5}{3} = \frac{30}{3} = 10$

10 flowers

6 flowers

Guided Practice

Determine how many candles fill 1 box. Use the diagrams to model your thinking. Sample work shown for Problem 2.

1. 8 candles fill $\frac{1}{2}$ of a box.

$\frac{1}{2}$ of a box

8 candles

$8 \div \frac{1}{2}$

$8 \times \frac{2}{1} = \frac{16}{1} = 16$

16 candles

2. 6 candles fill $\frac{3}{4}$ of a box.

$\frac{3}{4}$ of a box

6 candles

$6 \div \frac{3}{4}$

$6 \times \frac{4}{3} = \frac{24}{3} = 8$

8 candles

51

Vocabulary

If needed, share the meaning of the term with students.

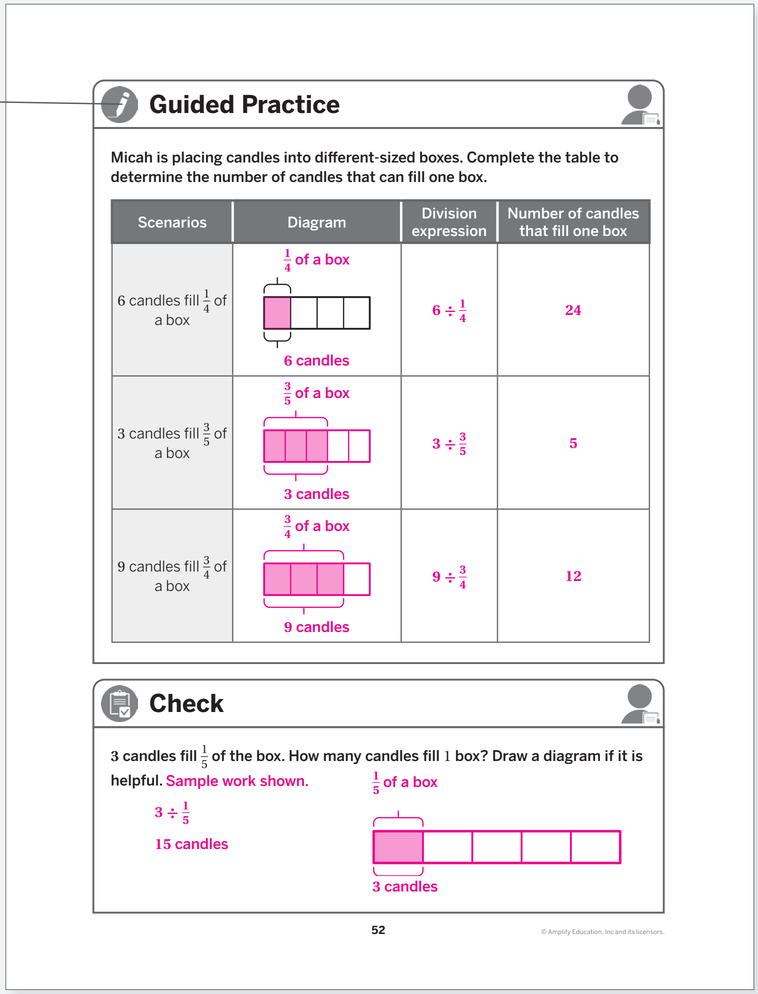
quotient: A quotient is the result of dividing two numbers.

Determining How Many in Each Group

Guided Practice

A Invite students to draw tape diagrams to help them visualize some of the more complex expressions in the table.

Key Takeaway:
Say, "Tape diagrams and division or multiplication expressions can help answer 'How many are in 1 group?' questions when the number of groups is a fraction."



Guided Practice

Micah is placing candles into different-sized boxes. Complete the table to determine the number of candles that can fill one box.

Scenarios	Diagram	Division expression	Number of candles that fill one box
6 candles fill $\frac{1}{4}$ of a box	$\frac{1}{4}$ of a box, 6 candles	$6 \div \frac{1}{4}$	24
3 candles fill $\frac{3}{5}$ of a box	$\frac{3}{5}$ of a box, 3 candles	$3 \div \frac{3}{5}$	5
9 candles fill $\frac{3}{4}$ of a box	$\frac{3}{4}$ of a box, 9 candles	$9 \div \frac{3}{4}$	12

Check

3 candles fill $\frac{1}{5}$ of the box. How many candles fill 1 box? Draw a diagram if it is helpful. Sample work shown.

$3 \div \frac{1}{5}$

15 candles

$\frac{1}{5}$ of a box

3 candles

52

Reflection

Ask:

- "How does using a diagram help solve division problems?"
- "What makes sense? What is still confusing?"

Check: Recommended Next Steps

Almost there
If students need more support, model creating a tape diagram to represent the problem in the Check. Then model using multiplication to solve the division expression.

Got it!
If students need more practice, ask them to solve the following problems:

- 2 candles fill $\frac{2}{3}$ of the box. How many candles fill 1 box?
- 4 candles fill $\frac{1}{5}$ of the box. How many candles fill 1 box?

Name ______________________________

Dividing Fractions Using Common Denominators

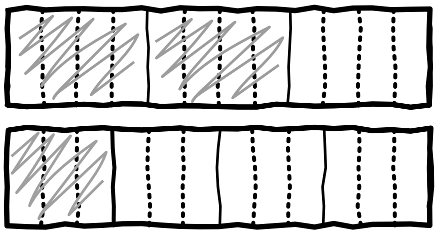
ML 4.07

Modeled Review

Name: Santiago

Calculate $\frac{2}{3} \div \frac{1}{4}$. Draw a diagram if it is helpful.

The common denominator is 12, so I cut each whole into 12 pieces.



$\frac{2}{3} \div \frac{1}{4} = \frac{8}{12} \div \frac{3}{12}$

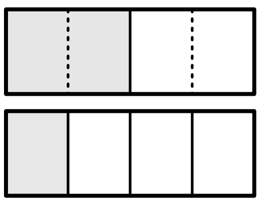
$= 8 \div 3$

$= \frac{8}{3}$

Guided Practice

Complete the given diagram to find the common denominator. Then, calculate the quotient.

1. $\frac{1}{2} \div \frac{1}{4}$

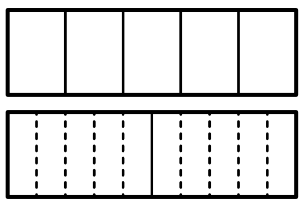


$\frac{1}{2} \div \frac{1}{4} = \frac{2}{4} \div$ ________

$= 2 \div$ ________

$=$ ________

2. $\frac{2}{5} \div \frac{1}{2}$



$\frac{2}{5} \div \frac{1}{2} = \frac{4}{10} \div$ ________

$=$ ________

$=$ ________

Guided Practice

Calculate each quotient. Draw a diagram if it is helpful.

3. $\frac{1}{8} \div \frac{3}{4}$

4. $\frac{2}{3} \div \frac{1}{2}$

5. $\frac{1}{4} \div \frac{1}{3}$

6. $\frac{3}{4} \div \frac{1}{6}$

Check

Calculate $\frac{1}{3} \div \frac{3}{4}$. Draw a diagram if it is helpful.

ML 4.07 Dividing Fractions Using Common Denominators

Goal

Divide fractions by using common denominators.

Standard

MA.6.NSO.2.2

Materials

colored pencils (optional)

Modeled Review

Point to Santiago's work and **ask**:

- "How did Santiago determine the common denominator?"
- "How did Santiago use a diagram to divide fractions?"

Reinforce Santiago's thinking by saying, "Diagrams and finding common denominators can help you divide fractions."

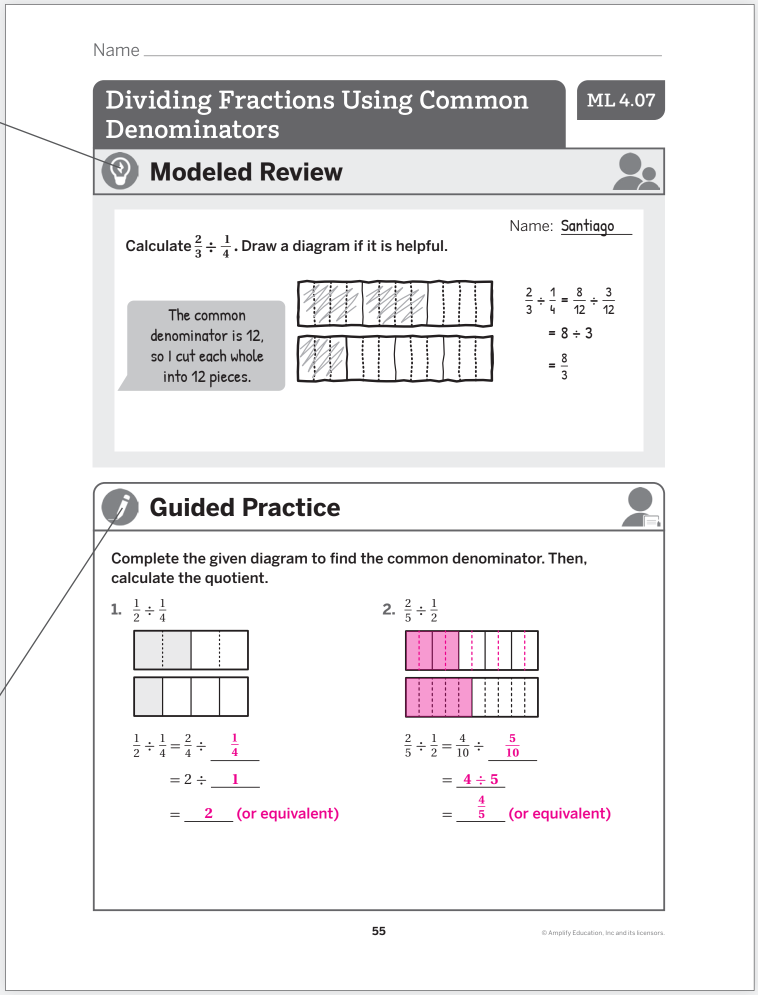
ML/EL Highlight the similarities between the tape diagrams and calculations.

Guided Practice

Focus students' attention on how to divide fractions using common denominators.

To scaffold their thinking, **ask**:

- "How can you create a diagram with an equivalent number of parts?"
- "Once you have common denominators, how do you divide?"



Name ______________________

Dividing Fractions Using Common Denominators ML 4.07

Modeled Review

Calculate $\frac{2}{3} \div \frac{1}{4}$. Draw a diagram if it is helpful. Name: Santiago

The common denominator is 12, so I cut each whole into 12 pieces.

$\frac{2}{3} \div \frac{1}{4} = \frac{8}{12} \div \frac{3}{12}$

$= 8 \div 3$

$= \frac{8}{3}$

Guided Practice

Complete the given diagram to find the common denominator. Then, calculate the quotient.

1. $\frac{1}{2} \div \frac{1}{4}$

$\frac{1}{2} \div \frac{1}{4} = \frac{2}{4} \div \frac{1}{4}$

$= 2 \div 1$

$= 2$ (or equivalent)

2. $\frac{2}{5} \div \frac{1}{2}$

$\frac{2}{5} \div \frac{1}{2} = \frac{4}{10} \div \frac{5}{10}$

$= 4 \div 5$

$= \frac{4}{5}$ (or equivalent)

55

Vocabulary

If needed, share the meaning of the term with students.

common denominator: Two fractions have a common denominator when the denominator (the bottom number in each fraction) is the same.

Dividing Fractions Using Common Denominators

Guided Practice

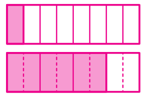
A Break this task into smaller, more manageable parts by having students first generate equivalent fractions with a common denominator, then solve.

Key Takeaway:
Say, "Finding common denominators is an efficient strategy for dividing fractions."

Guided Practice

Calculate each quotient. Draw a diagram if it is helpful.
Sample work shown.

3. $\frac{1}{8} \div \frac{3}{4}$



$\frac{1}{8} \div \frac{3}{4} = \frac{1}{8} \div \frac{6}{8}$

$= 1 \div 6$

$= \frac{1}{6}$

$\frac{1}{6}$ (or equivalent)

4. $\frac{2}{3} \div \frac{1}{2}$

$\frac{2}{3} \div \frac{1}{2} = \frac{4}{6} \div \frac{3}{6}$

$= 4 \div 3$

$= \frac{4}{3}$

$\frac{4}{3}$ (or equivalent)

5. $\frac{1}{4} \div \frac{1}{3}$

$\frac{1}{4} \div \frac{1}{3} = \frac{3}{12} \div \frac{4}{12}$

$= 3 \div 4$

$\frac{3}{4}$ (or equivalent)

6. $\frac{3}{4} \div \frac{1}{6}$

$\frac{3}{4} \div \frac{1}{6} = \frac{9}{12} \div \frac{2}{12}$

$= 9 \div 2$

$\frac{9}{2}$ (or equivalent)

Check

Calculate $\frac{1}{3} \div \frac{3}{4}$. Draw a diagram if it is helpful. Sample work shown.

$\frac{1}{3} \div \frac{3}{4} = \frac{4}{12} \div \frac{9}{12}$

$= 4 \div 9$

$\frac{4}{9}$ (or equivalent)

56

Reflection

Ask:

- "How can finding a common denominator be useful when dividing two fractions?"
- "How was the lesson helpful to you today?"

Check: Recommended Next Steps

Almost there
If students need more support, encourage them to organize their work by using different colored pencils when sketching a diagram and dividing fractions.

Got it!
If students need more practice, ask them to calculate the following quotients:

- $\frac{3}{8} \div \frac{1}{4}$
- $\frac{3}{5} \div \frac{1}{2}$
- $\frac{5}{8} \div \frac{1}{4}$

Name ______________________________

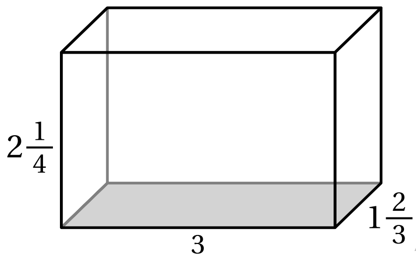
Determining the Volume of Prisms With Fractional Dimensions

ML 4.14

Modeled Review

Name: Dylan

Determine the volume of the prism. Show your thinking.



First, I need to convert the mixed numbers to equivalent fractions that are greater than 1.

Next, I need to find the volume of the prism and change it into a mixed number.

$V = l \cdot w \cdot h$

$V = 3 \cdot 1\frac{2}{3} \cdot 2\frac{1}{4}$

$1\frac{2}{3} = \frac{3}{3} + \frac{2}{3}$

$1\frac{2}{3} = \frac{5}{3}$

$2\frac{1}{4} = \frac{8}{4} + \frac{1}{4}$

$2\frac{1}{4} = \frac{9}{4}$

$V = \frac{3}{1} \cdot \frac{5}{3} \cdot \frac{9}{4} = \frac{135}{12} = \frac{45}{4}$

$V = \frac{45}{4} = 11\frac{1}{4}$

$V = 11\frac{1}{4}$ cubic units

Guided Practice

Convert the mixed number to an equivalent fraction greater than 1.

1. $3\frac{1}{5}$

$3 = \frac{15}{5}$

$\frac{15}{5} +$ _______ $=$ _______

2. $2\frac{1}{5}$

$2 = \frac{10}{5}$

_______ $+$ _______ $=$ _______

3. $2\frac{3}{4}$

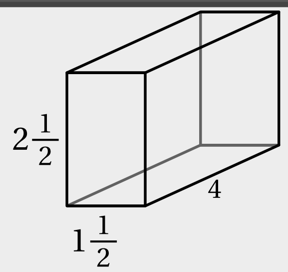
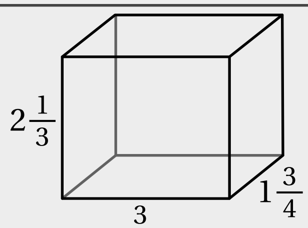
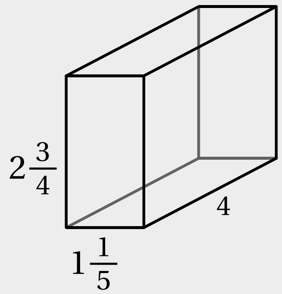
$2 =$ _______

_______ $+$ _______ $=$ _______

4. $3\frac{1}{4}$

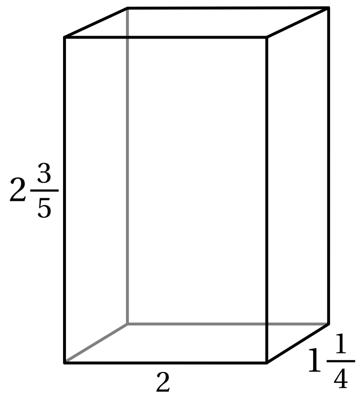
Guided Practice

5. Calculate the volume of each rectangular prism.

Prism	Dimensions $(l \cdot w \cdot h)$	Volume (cubic units)
		
		
		

Check

Determine the volume of the prism. Show your thinking.



ML 4.14 Determining the Volume of Prisms With Fractional Dimensions

Goal

Calculate the volume of rectangular prisms with fractional dimensions.

Standard

MA.6.GR.2.3

Materials

highlighter (optional), tape diagrams (optional)

Modeled Review

Point to Dylan's work and **ask**:

- "How does Dylan change the mixed numbers to fractions greater than 1?"
- "How does Dylan multiply the side lengths to calculate the volume?"

Reinforce Dylan's thinking by saying, "Mixed numbers can be changed to fractions greater than 1 to efficiently calculate the volume."

Guided Practice

Focus students' attention on changing mixed numbers to fractions greater than 1 fractions.

To scaffold their thinking, **say**:

- "Change mixed numbers to fractions greater than 1 by converting the whole number to an equivalent fraction using the same denominator and adding it to the fractional part."
- "Multiply the numerators. Multiply the denominators. Simplify."

Name

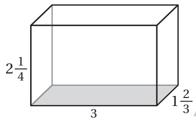
Determining the Volume of Prisms With Fractional Dimensions

ML 4.14

Modeled Review

Name: Dylan

Determine the volume of the prism. Show your thinking.



First, I need to convert the mixed numbers to equivalent fractions that are greater than 1.

Next, I need to find the volume of the prism and change it into a mixed number.

$V = l \cdot w \cdot h$

$V = 3 \cdot 1\frac{2}{3} \cdot 2\frac{1}{4}$

$1\frac{2}{3} = \frac{3}{3} + \frac{2}{3}$

$1\frac{2}{3} = \frac{5}{3}$

$2\frac{1}{4} = \frac{8}{4} + \frac{1}{4}$

$2\frac{1}{4} = \frac{9}{4}$

$V = \frac{3}{1} \cdot \frac{5}{3} \cdot \frac{9}{4} = \frac{135}{12} = \frac{45}{4}$

$V = \frac{45}{4} = 11\frac{1}{4}$

$V = 11\frac{1}{4}$ cubic units

Guided Practice

Convert the mixed number to an equivalent fraction greater than 1.

Sample work shown for Problem 4.

1. $3\frac{1}{5}$

   $3 = \frac{15}{5}$

   $\frac{15}{5} + \frac{1}{5} = \frac{16}{5}$

2. $2\frac{1}{5}$

   $2 = \frac{10}{5}$

   $\frac{10}{5} + \frac{1}{5} = \frac{11}{5}$

3. $2\frac{3}{4}$

   $2 = \frac{8}{4}$

   $\frac{8}{4} + \frac{3}{4} = \frac{11}{4}$

4. $3\frac{1}{4}$

   $3 = \frac{12}{4}$

   $\frac{12}{4} + \frac{1}{4} = \frac{13}{4}$

59

Vocabulary

If needed, share the meaning of the terms with students.

base: The face that gives the solid its name.

volume: The number of unit cubes needed to fill a three-dimensional shape without gaps or overlaps.

Determining the Volume of Prisms With Fractional Dimensions

Guided Practice

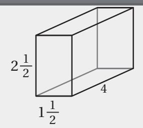
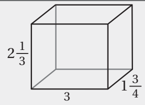
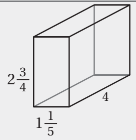
A Break this task into smaller, more manage parts by having students first change the mixed numbers into fractions greater than 1. Then, multiply.

Note: Students may multiply the fractions in any order.

Key Takeaway:
Say, "The volume of a prism is the product of its length, width, and height, regardless of whether its edge lengths are whole numbers or fractions."

Guided Practice

5. Calculate the volume of each rectangular prism. Sample work shown.

Prism	Dimensions $(l \cdot w \cdot h)$	Volume (cubic units)
	$1\frac{1}{2} \cdot 4 \cdot 2\frac{1}{2}$ or $\frac{3}{2} \cdot \frac{4}{1} \cdot \frac{5}{2}$	15
	$3 \cdot 1\frac{3}{4} \cdot 2\frac{1}{3}$ or $\frac{3}{1} \cdot \frac{7}{4} \cdot \frac{7}{3}$	$12\frac{1}{4}$
	$1\frac{1}{5} \cdot 4 \cdot 2\frac{3}{4}$ or $\frac{6}{5} \cdot \frac{4}{1} \cdot \frac{11}{4}$	$13\frac{1}{5}$

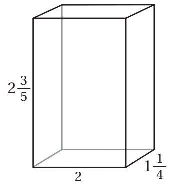
Check

Determine the volume of the prism. Show your thinking.
Sample work shown.

$2 \cdot 1\frac{1}{4} \cdot 2\frac{3}{5} = \frac{2}{1} \cdot \frac{5}{4} \cdot \frac{13}{5}$

$= \frac{13}{2}$ or $6\frac{1}{2}$

$6\frac{1}{2}$ cubic units (or equivalent)



60

Reflection

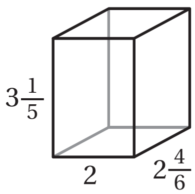
Ask:

- "Why is it helpful to change mixed numbers to fractions greater than 1 when finding the volume of prisms with fractional side lengths?"
- "What strategy was helpful today?"

Check: Recommended Next Steps

Almost there
If students need more support, consider using Grade 5 Pre-Requisite Mini-Lesson 1.6: *Determining the Volume of Solid Rectangular Prisms*.

Got it!
If students need more practice, sketch the following rectangular prism and ask them to calculate the volume.



Unit 5

Mini-Lessons

Name ______________________________

Multiplying Decimals Using Area Models

ML 5.03

Modeled Review

Name: Priya

Determine the product $4.2 \cdot 2.6$. Show your thinking.

8 + 0.4 + 2.4 + 0.12 = 10.92

10.92

	4	0.2
2	8	0.4
0.6	2.4	0.12

Guided Practice

For Problems 1–2, use the area model that represents $3 \cdot 1.5$.

1. Calculate the areas of A and B.

A	B

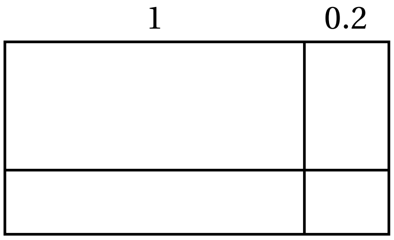
	3
1	A
0.5	B

2. What is the value of $3 \cdot 1.5$?

_____ + _____ = _____

Guided Practice

3. Calculate the product using an area model.

Problem	Area Model	Product
$1.2 \cdot 1.5$		
$2.1 \cdot 1.6$		
$3.3 \cdot 2.5$		

Check

Determine the product $3.5 \cdot 2.7$ **using an area model.**

Multiplying Decimals Using Area Models

Goal

Calculate products of decimals using an area model.

Standard

MA.6.NSO.2.1

Materials

highlighter or colored pencils (optional), base-ten blocks (optional)

Modeled Review

Point to Priya's work and **ask**:

- "How did Priya use an area model to calculate the product?"
- "How did Priya calculate the partial products? How did she use the partial products to find the total product?"

Reinforce Priya's thinking by saying, "Using area models can help you calculate the products of decimals in an organized way."

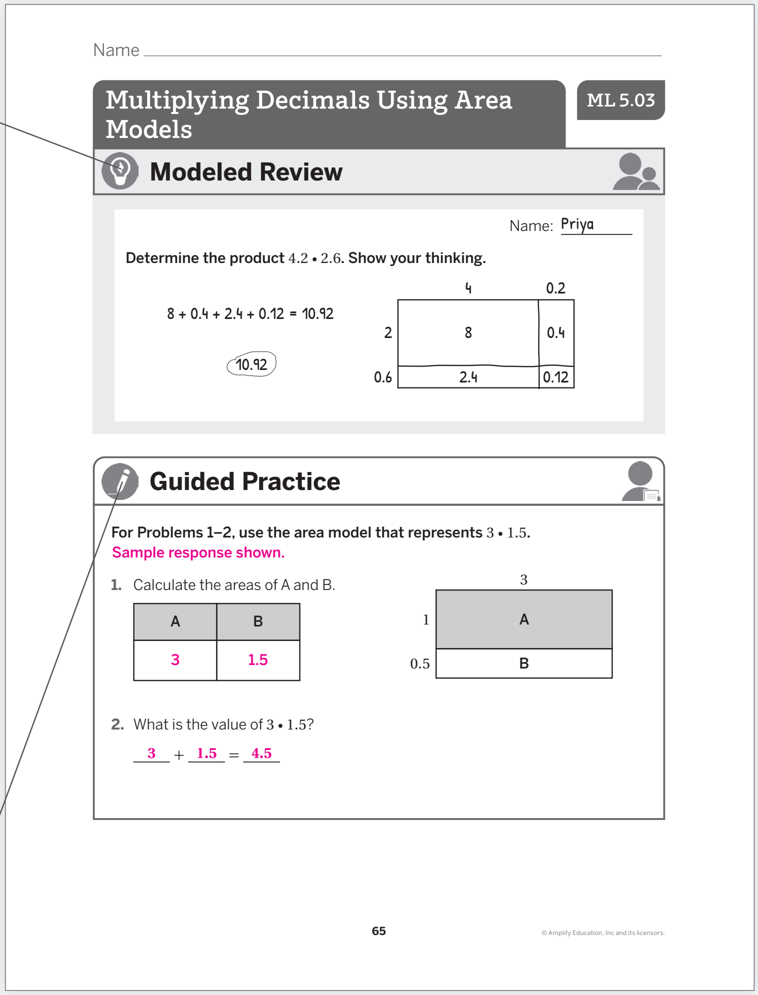
ML/EL To support students with making connections between the problem and area model, invite them to use highlighters or colored pencils to draw connections between the numbers and how they are represented in the area model.

Guided Practice

Focus students' attention on using the area model to calculate the product.

To scaffold their thinking, **say**:

- "Calculate the area of each part."
- "Add all the areas together."



Name

Multiplying Decimals Using Area Models ML 5.03

Modeled Review

Name: Priya

Determine the product $4.2 \cdot 2.6$. Show your thinking.

8 + 0.4 + 2.4 + 0.12 = 10.92

10.92

	4	0.2
2	8	0.4
0.6	2.4	0.12

Guided Practice

For Problems 1–2, use the area model that represents $3 \cdot 1.5$.
Sample response shown.

1. Calculate the areas of A and B.

A	B
3	1.5

	3
1	A
0.5	B

2. What is the value of $3 \cdot 1.5$?

3 + 1.5 = 4.5

65

ML 5.03 Multiplying Decimals Using Area Models

Guided Practice

A Model blocking out the sections of the area model that are not currently being calculated to help students concentrate on one step of the process at a time.

Note: Students may set up the rectangle with either number as the length and width.

Key Takeaway:
Say, "An area model can be used to multiply two decimals by separating by place value and using partial products."

Guided Practice

3. Calculate the product using an area model. Sample work shown.

Problem	Area Model	Product
1.2 • 1.5	columns 1, 0.2; rows 1: 1, 0.2; rows 0.5: 0.5, 0.1	1.8
2.1 • 1.6	columns 2, 0.1; rows 1: 2, 0.1; rows 0.6: 1.2, 0.06	3.36
3.3 • 2.5	columns 3, 0.3; rows 2: 6, 0.6; rows 0.5: 1.5, 0.15	8.25

Check

Determine the product 3.5 • 2.7 using an area model. Sample work shown.

9.45

	3	0.5
2	6	1
0.7	2.1	0.35

66

Reflection

Ask:

- "What challenges did you face when multiplying decimals with an area model and how did you overcome them?"
- "What makes sense? What is still confusing?"

Check: Recommended Next Steps

Almost there
If students need more support, consider having them use base-ten blocks to decompose the decimals before completing the area model.

Got it!
If students need more practice, present them with the following problems and ask them to calculate the product using an area model.

- 3.6 • 2.4
- 4.2 • 1.5

Name ______________________________

Dividing Decimals Using Long Division

ML 5.06

Modeled Review

Name: Diego

Calculate $3.18 \div 0.6$**. Show your thinking.**

$3.18 \div 0.6 = \frac{318}{100} \div \frac{60}{100}$

$$\begin{array}{r} 5.3 \\ 60 \overline{)\,318.0} \\ -\,300 \\ \hline 18 \\ -\,18 \\ \hline 0 \end{array}$$

(5.3)

I need to multiply both decimals by 100 for the numbers to both be whole numbers.

Guided Practice

Divide the decimals by completing the long division.

1. $0.8 \div 0.4 =$ ____

$0.8 \div 0.4 = \frac{8}{10} \div \frac{}{10}$

$$\begin{array}{r} 4 \overline{)\,8} \\ -\,\square \\ \hline \square \end{array}$$

2. $0.62 \div 0.2 =$ ____

$0.62 \div 0.2 = \frac{62}{100} \div \frac{}{100}$

$$\overline{)\,62.0}$$
$-$

Guided Practice

Divide the decimals using long division.

3. $1.34 \div 0.2 =$ ____

4. $2.15 \div 0.5 =$ ____

5. $2.25 \div 0.3 =$ ____

6. $3.36 \div 0.6 =$ ____

Check

Calculate $2.55 \div 0.3$. Show your thinking.

ML 5.06 Dividing Decimals Using Long Division

Goal

Use long division to divide multi-digit. decimals.

Standard

MA.6.NSO.2.1

Modeled Review

Point to Diego's work and **ask**:

- "Why did Diego multiply both the divisor and dividend by 100?"
- "How is using long division to divide decimals similar to dividing whole numbers?"

Reinforce Diego's thinking by saying, "Long division can be used to divide decimals by multiplying both the dividend and divisor by the same power of 10."

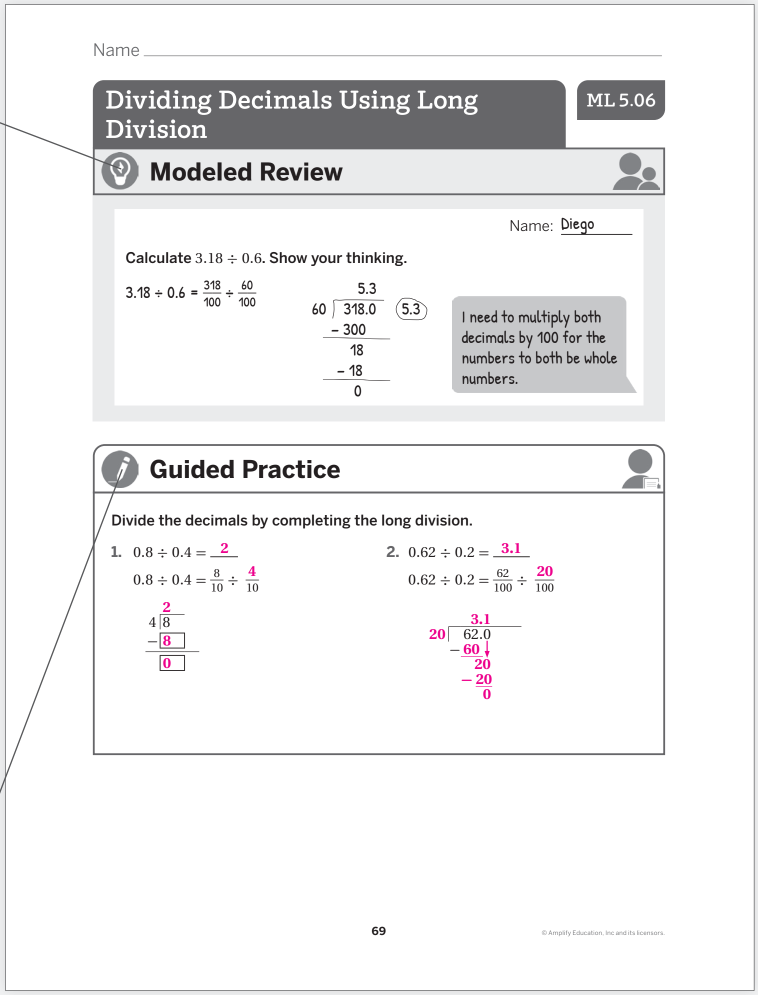
ML/EL Provide students with the following sentence frame to support them as they explain their thinking about place value, "3.18 divided by 0.6 is equivalent to 318 divided by ______."

Guided Practice

Focus students' attention on using long division to calculate the quotient.

To scaffold their thinking, **ask**:

- "How do you convert both decimals to whole numbers?"
- "How do you set up and divide using long division?"



Name ______

Dividing Decimals Using Long Division ML 5.06

Modeled Review

Name: Diego

Calculate $3.18 \div 0.6$. Show your thinking.

$3.18 \div 0.6 = \frac{318}{100} \div \frac{60}{100}$

60) 318.0 = 5.3
− 300
18
− 18
0

(5.3)

I need to multiply both decimals by 100 for the numbers to both be whole numbers.

Guided Practice

Divide the decimals by completing the long division.

1. $0.8 \div 0.4 = 2$

$0.8 \div 0.4 = \frac{8}{10} \div \frac{4}{10}$

4) 8 = 2
− 8
0

2. $0.62 \div 0.2 = 3.1$

$0.62 \div 0.2 = \frac{62}{100} \div \frac{20}{100}$

20) 62.0 = 3.1
− 60
20
− 20
0

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Vocabulary

If needed, share the meaning of the term with students.

long division: Long division is a way to divide numbers in decimal form. When we use long division, we determine the quotient one digit at a time, from left to right.

ML 5.06 Dividing Decimals Using Long Division

Guided Practice

A To support students in carrying out multiple steps, model the process for setting up the algorithm, including placeholders represented by zeros and underlining the numbers being divided.

Key Takeaway:
Say, "The dividend and divisor can be multiplied by the same power of 10 so that the divisor is a whole number."

Guided Practice

Divide the decimals using long division. Sample work shown.

3. $1.34 \div 0.2 = \underline{6.7}$

$1.34 \div 0.2 = \frac{134}{100} \div \frac{20}{100}$

$20 \overline{)134.0}$ = 6.7; -120, 140, -140, 0

4. $2.15 \div 0.5 = \underline{4.3}$

$2.15 \div 0.5 = \frac{215}{100} \div \frac{50}{100}$

$50 \overline{)215.0}$ = 4.3; -200, 150, -150, 0

5. $2.25 \div 0.3 = \underline{7.5}$

$2.25 \div 0.3 = \frac{225}{100} \div \frac{30}{100}$

$30 \overline{)225.0}$ = 7.5; -210, 150, -150, 0

6. $3.36 \div 0.6 = \underline{5.6}$

$3.36 \div 0.6 = \frac{336}{100} \div \frac{60}{100}$

$60 \overline{)336.0}$ = 5.6; -300, 360, -360, 0

Check

Calculate $2.55 \div 0.3$. Show your thinking. Sample work shown.

8.5

$2.55 \div 0.3 = \frac{225}{100} \div \frac{30}{100}$

$30 \overline{)225.0}$ = 8.5; -240, 150, -150, 0

70

Reflection

Ask:
- "What is important to remember when dividing with decimals?"
- "How did you overcome a hard problem today?"

Check: Recommended Next Steps

Almost there
If students need more support, consider modeling how to convert the decimals to equivalent fractions by focusing on their place values and multiplying by the highest power of 10 between the two decimals.

Got it!
If students need more practice, present them with the following problems and ask them to divide the decimals using long division.

- $2.44 \div 0.4$
- $3.25 \div 0.5$
- $2.22 \div 0.3$

Name ____________________

Converting Between Fractions, Percents, and Decimals

ML 5.09

Modeled Review

Name: Tristan

Write 45% as its fraction and decimal equivalent. Show your thinking.

$\frac{45}{100} = 45. = 0.45$

$\frac{45}{100} \div \frac{5}{5} = \frac{9}{20}$

A percent is always out of 100.

Guided Practice

Convert the fraction to determine its decimal and percent equivalents.

1. Convert $\frac{1}{4}$ to its decimal equivalent.

Convert $\frac{1}{4}$ to its percent equivalent.

____ • 100 = ____

2. Convert $\frac{1}{5}$ to its decimal equivalent.

Convert $\frac{1}{5}$ to its percent equivalent.

____ • ____ = ____

Guided Practice

3. Determine the missing values in each row.

Percent (%)	Decimal	Fraction
15	$\frac{}{100} =$	$\frac{15}{100} \div \frac{5}{5} =$ ____
	0.40	
5		
		$\frac{7}{20}$

Check

Write 55% as its fraction and decimal equivalents. Show your thinking.

ML 5.09 Converting Between Fractions, Percents, and Decimals

Goal	**Standard**
Convert between values represented as percentages, decimals, and fractions.	MA.6.NSO.3.5

Modeled Review

Point to Tristan's work and **ask**:

- "How is dividing by 100 equivalent to moving the decimal two places to the left?"
- "Why did Tristan divide both the numerator and denominator by 5?"

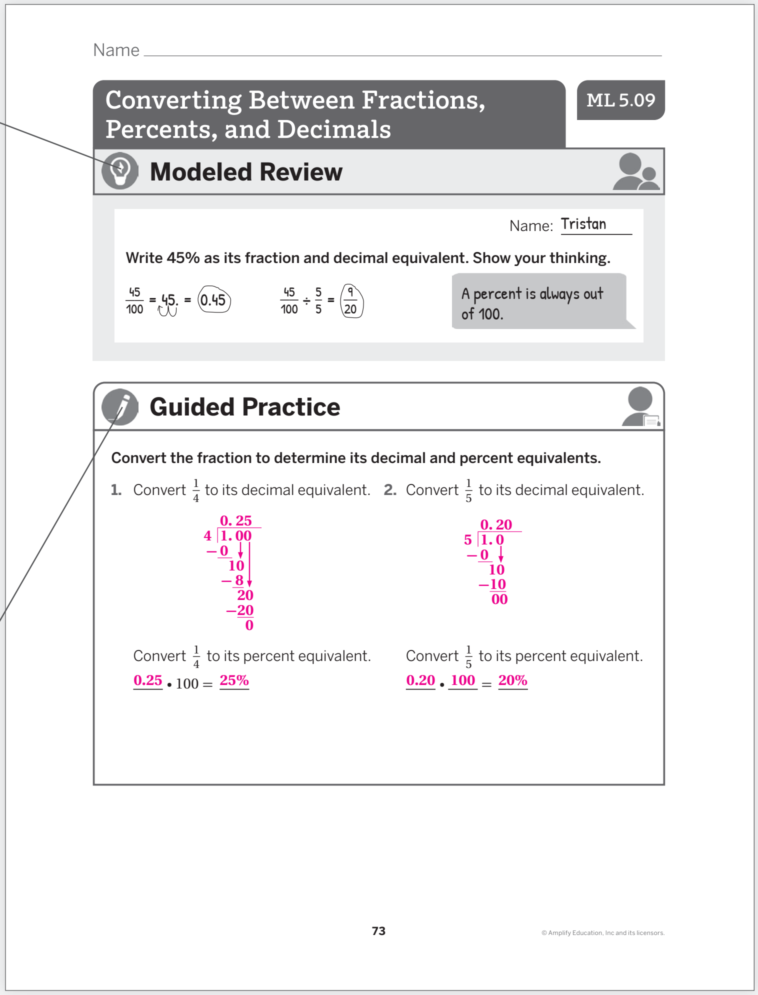
Reinforce Tristan's thinking by saying, "Use your prior knowledge of multiplying and dividing to convert between percents, decimals and fractions."

Guided Practice

Focus students' attention on using division and multiplication to convert between percents, decimals, and fractions.

To scaffold their thinking, **ask**:

- "What form are you given?"
- "What form do you need to convert to? What operation do you need to perform in order to convert?"



Name

Converting Between Fractions, Percents, and Decimals

ML 5.09

Modeled Review

Name: Tristan

Write 45% as its fraction and decimal equivalent. Show your thinking.

$\frac{45}{100} = 45. = 0.45$ $\quad \frac{45}{100} \div \frac{5}{5} = \frac{9}{20}$

A percent is always out of 100.

Guided Practice

Convert the fraction to determine its decimal and percent equivalents.

1. Convert $\frac{1}{4}$ to its decimal equivalent.

$4 \overline{)1.00} = 0.25$; -0, 10, -8, 20, -20, 0

Convert $\frac{1}{4}$ to its percent equivalent.

$0.25 \cdot 100 = 25\%$

2. Convert $\frac{1}{5}$ to its decimal equivalent.

$5 \overline{)1.0} = 0.20$; -0, 10, -10, 00

Convert $\frac{1}{5}$ to its percent equivalent.

$0.20 \cdot 100 = 20\%$

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Converting Between Fractions, Percents, and Decimals

Guided Practice

A Chunk this task into more manageable parts by presenting one question at a time.

ML/EL Encourage students to pay attention to place value when reading each decimal. This can help them convert the decimals to fractions and decide if further simplification is required.

Key Takeaway:
Say, "The relationship between place value, decimals, and fractions can be applied to convert between percents, fractions, and decimals."

Guided Practice

3. Determine the missing values in each row.

Percent (%)	Decimal	Fraction
15	$\frac{15}{100} = 0.15$	$\frac{15}{100} \div \frac{5}{5} = \frac{3}{20}$
40	0.40	$\frac{2}{5}$
5	0.05	$\frac{1}{20}$
35	0.35	$\frac{7}{20}$

Check

Write 55% as its fraction and decimal equivalents. Show your thinking.

$\frac{55}{100} = 0.55$ $\frac{55}{100} \div \frac{5}{5} = \frac{11}{20}$

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Reflection

Ask:

- "How can you convert between percents, fractions, and decimals?"
- "How does what you learned today connect to your prior learning?"

Check: Recommended Next Steps

Almost there
If students need more support, consider creating an anchor chart that outlines the process for converting percentages, decimals, and fractions using division.

Got it!
If students need more practice, present them with the following problems and ask them to show their thinking.

- Write 0.65 as its fraction and percent equivalents.
- Write $\frac{4}{5}$ as its decimal and percent equivalents.

Unit 6

Mini-Lessons

Name ____________________

Writing and Solving Equations

ML 6.04

Modeled Review

Name: Han

For Problems 1–3, use the equation $x + 2 = 10$.

1. Write a situation that represents this equation. Explain what x represents in your situation.

   Tristan spent $10 on a box of tissues and medicine. The box of tissues cost $2. The medicine cost x dollars.

2. Determine the solution to the equation.

   $x = 8$

   $$\begin{aligned} x + 2 &= 10 \\ -2 \quad & \;\; -2 \\ x &= 8 \end{aligned}$$

3. Explain what the solution means in your situation.

   The medicine cost $8.

Guided Practice

For each situation, write the equation from the bank that best represents it.

$x + 6 = 13$	$6x = 13$	$x + 2 = 7$
$2x = 7$	$x - 2 = 7$	$2x = 10$

1. Santiago spent $7 on a cup of coffee and a croissant. The cup of coffee cost $2. The croissant cost x dollars.

   ____

2. Jada spent $10 on two secondhand books. Each book cost x dollars.

   ____

3. Diego spent $13 on an herb plant and a pot. The plant cost $6. The pot cost x dollars.

   ____

Guided Practice

4. Complete the table for each equation shown.

Equation	Situation	Solution	Solution's meaning
$x + 6 = 15$	Rebecca spent $15 on a deck of cards and a phone charger. The deck of cards cost $6. The phone charger cost x dollars.	$x = 9$	
$4x = 20$	Esteban spent $20 on four greeting cards. Each greeting card cost x dollars.		
$x + 3 = 14$			

Check

For Problems 1–3, use the equation $x + 5 = 12$.

1. Write a situation that represents the equation. Explain what x represents in your situation.

2. Determine the solution to the equation.

3. Explain what the solution means in your situation.

ML 6.04 Writing and Solving Equations

Goal	Standard
Write descriptions for and solve one-step equations.	MA.6.AR.2.2

Modeled Review

Point to Han's work and **ask**:

- "How did Han write a situation to represent the equation?"
- "Why did Han subtract 2 from both sides of the equal sign?"
- "How did Han know the solution represented the cost of the medicine?"

Reinforce Han's thinking by saying, "An equation can be written as situations by focusing on the operation and its solution."

ML/EL To increase accessibility throughout the lesson, provide students with the opportunity to share their situations orally rather than in writing.

Guided Practice

Focus students' attention on representing each situation with an equation from the bank.

To scaffold their thinking, **ask**:

- "Which operation is represented by the situation?"
- "Is there anything about the situation that could help you decide which equation it matches?"
- "What does the solution mean according to the situation?"

Name ____________________

Writing and Solving Equations ML 6.04

Modeled Review

Name: Han

For Problems 1–3, use the equation $x + 2 = 10$.

1. Write a situation that represents this equation. Explain what x represents in your situation.

   Tristan spent \$10 on a box of tissues and medicine. The box of tissues cost \$2. The medicine cost x dollars.

2. Determine the solution to the equation.

   $x + 2 = 10$
   $-2 \quad -2$
   $x = 8$

   $x = 8$

3. Explain what the solution means in your situation.

   The medicine cost \$8.

Guided Practice

For each situation, write the equation from the bank that best represents it.

$x + 6 = 13$	$6x = 13$	$x + 2 = 7$
$2x = 7$	$x - 2 = 7$	$2x = 10$

1. Santiago spent \$7 on a cup of coffee and a croissant. The cup of coffee cost \$2. The croissant cost x dollars.

   $x + 2 = 7$

2. Jada spent \$10 on two secondhand books. Each book cost x dollars.

   $2x = 10$

3. Diego spent \$13 on an herb plant and a pot. The plant cost \$6. The pot cost x dollars.

   $x + 6 = 13$

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Guided Practice

A To support students with their written responses, consider providing sentence frames, such as "Jack spent ___ on ___ packs of gum. Each pack cost ___ dollars."

Key Takeaway:
Say, "Writing and solving equations can help us determine unknown information in a situation. A solution's meaning depends on the situation the equation represents."

Guided Practice

4. Complete the table for each equation shown. Sample situation shown for Row 3.

Equation	Situation	Solution	Solution's meaning
$x + 6 = 15$	Rebecca spent \$15 on a deck of cards and a phone charger. The deck of cards cost \$6. The phone charger cost x dollars.	$x = 9$	The phone charger cost \$9.
$4x = 20$	Esteban spent \$20 on four greeting cards. Each greeting card cost x dollars.	$x = 5$	Each greeting card cost \$5.
$x + 3 = 14$	Riya spent \$14 on a box of crayons and a pack of paint brushes. The box of crayons cost \$3. The pack of paint brushes cost x dollars.	$x = 11$	The pack of paint brushes cost \$11.

Check

For Problems 1–3, use the equation $x + 5 = 12$.
Sample responses shown for Problems 1 and 3.

1. Write a situation that represents the equation. Explain what x represents in your situation.

   Jack spent \$12 on a pair of sunglasses and a shirt. The sunglasses cost \$5. The shirt cost x dollars.

2. Determine the solution to the equation.

   $x = 7$

3. Explain what the solution means in your situation.

   The shirt cost \$7.

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Reflection

Ask:

- "What do you think is important to remember when writing equations to represent situations?"
- "Reflect on your learning today. What were you most proud of?"

Check: Recommended Next Steps

Almost there

If students need more support, consider providing them with a list of topics they can use when generating situations to represent the equations.

- Grocery store
- Concession stand at a game

Got it!

If students need more practice, present them with the following problems. Ask them to write a situation and solve. Then describe what the solution means in their situation.

- $x + 7 = 20$
- $6x = 42$

Name ____________________

Writing Equivalent Expressions Using the Area Model

ML 6.08

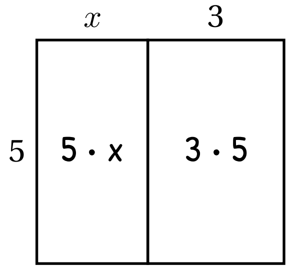
Modeled Review

Name: Jada

Write *two* equivalent expressions that represent the area of the rectangle.

Expression 1: 5(x + 3)

Expression 2: 5x + 15



5 times x + 3 or 5(x + 3)

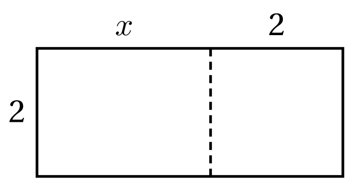
5 • x + 5 • 3 or 5 • (x + 3)

5x + 15

Guided Practice

Write *two* equivalent expressions using the area of a rectangle.

1.



$2 \cdot (x +$ ___$)$

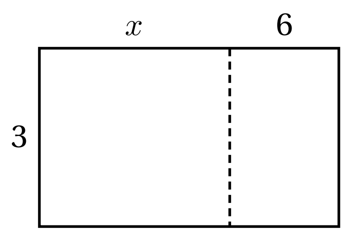
$2 \cdot x +$ ____

____ + ____

Expression 1: __________

Expression 2: __________

2.



$3 \cdot ($___ + ___$)$

____ + ____

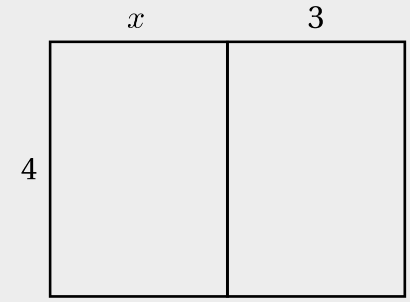
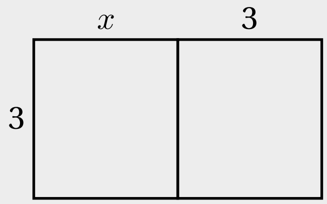
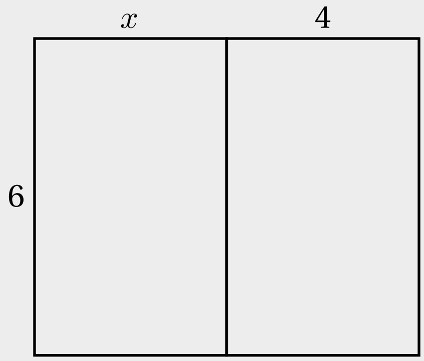
____ + ____

Expression 1: __________

Expression 2: __________

Guided Practice

3. Write *two* equivalent expressions that represent the area of each rectangle.

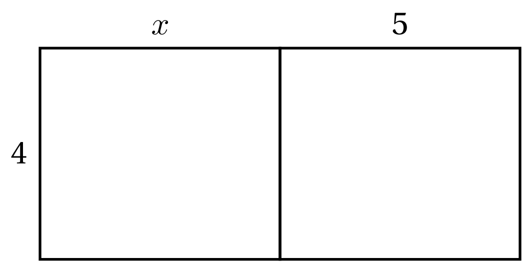
Area Model	Expression 1	Expression 2
	$4(x+3)$	
		
		

Check

Write *two* equivalent expressions that represent the area of the rectangle.

Expression 1: ____________

Expression 2: ____________



ML 6.08 Writing Equivalent Expressions Using the Area Model

Goal

Write equivalent expressions using the areas of rectangles.

Standard

MA.6.AR.1.4

Materials

algebra tiles (optional), highlighter (optional)

Modeled Review

Point to Jada's work and **ask**:

- "How did Jada use the area of the rectangle to write equivalent expressions?"
- "How are the two expressions alike? Different?"

Reinforce Jada's thinking by saying, "The area of a rectangle can be used to create equivalent expressions."

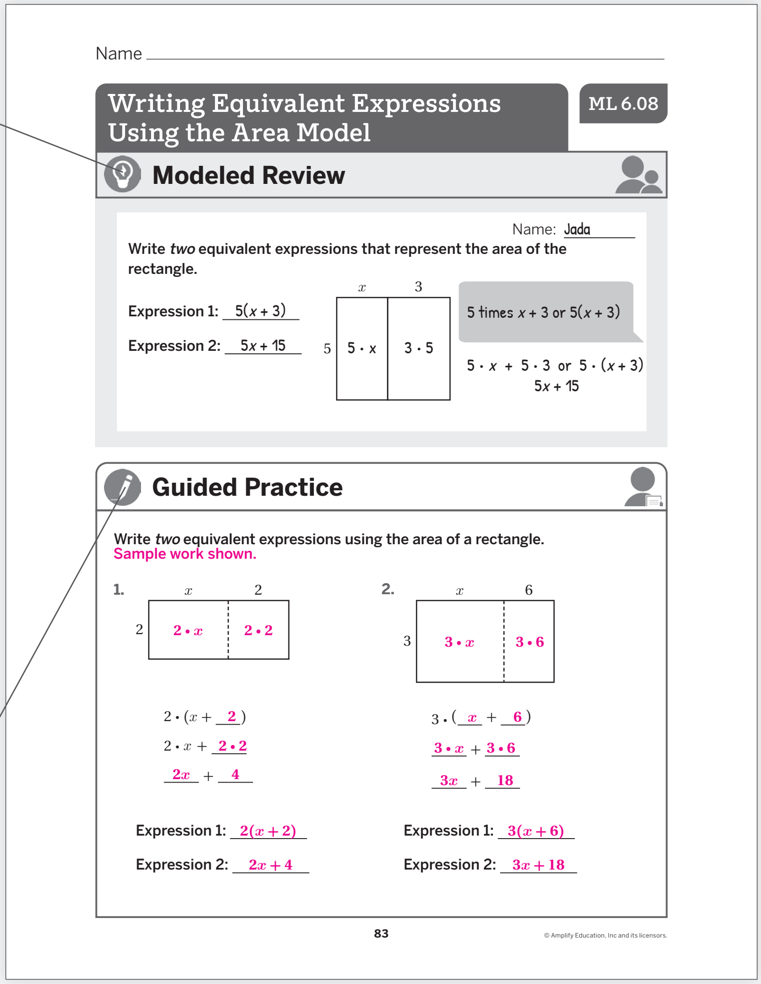
ML/EL Provide sentence frames to help students explain their thinking (e.g., Expression 1 is equivalent to Expression 2 because _______.).

Guided Practice

Focus students' attention on using the area of a rectangle to write equivalent expressions.

To scaffold their thinking, **say**:

- "Determine the product of the length and width."
- "Determine the sum of the two smaller areas within the rectangle."



Name

Writing Equivalent Expressions Using the Area Model

ML 6.08

Modeled Review

Name: Jada

Write *two* equivalent expressions that represent the area of the rectangle.

Expression 1: $5(x+3)$

Expression 2: $5x + 15$

Guided Practice

Write *two* equivalent expressions using the area of a rectangle.

Sample work shown.

1.

$2 \cdot (x + \underline{2})$

$2 \cdot x + \underline{2 \cdot 2}$

$\underline{2x} + \underline{4}$

Expression 1: $2(x+2)$

Expression 2: $2x+4$

2.

$3 \cdot (\underline{x} + \underline{6})$

$\underline{3 \cdot x} + \underline{3 \cdot 6}$

$\underline{3x} + \underline{18}$

Expression 1: $3(x+6)$

Expression 2: $3x+18$

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Vocabulary

If needed, share the meaning of the terms with students.

equivalent expressions: Expressions that are equal for every value of a variable.

product: The value of two or more quantities when multiplied.

sum: The value of two or more quantities when added together.

Writing Equivalent Expressions Using the Area Model

Guided Practice

A Invite students to use algebra tiles to represent each expression.

Key Takeaway:
Say, "An area model is one way to generate equivalent expressions. The product of the length and width of a rectangle is one expression and the sum of the two smaller areas is another."

Guided Practice

3. Write *two* equivalent expressions that represent the area of each rectangle.

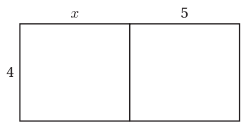
Area Model	Expression 1	Expression 2
x, 3; 4	$4(x+3)$	$4x+12$
x, 3; 3	$3(x+3)$	$3x+9$
x, 4; 6	$6(x+4)$	$6x+24$

Check

Write *two* equivalent expressions that represent the area of the rectangle.

Expression 1: $4(x+5)$

Expression 2: $4x+20$



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Reflection

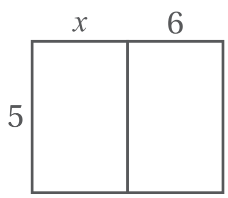
Ask:

- "How can areas of rectangles, also called area models, help you identify or create equivalent expressions?"
- "What questions do you still have?"

Check: Recommended Next Steps

Almost there
If students need more support, consider color coding the diagram in the Check to show the connection to the expressions. Highlight the length and width of the large rectangle in one color and the length and width of the two small rectangles in another color.

Got it!
If students need more practice, sketch the following area model and ask them to write two equivalent expressions that represent the area of the rectangle.



Name ______________________________

Evaluating Expressions With Exponents

ML 6.11

Modeled Review

Name: Kai

Determine the value of each expression.

1. $2 \cdot 3^2$ = 18

$2 \cdot (3 \cdot 3)$

$2 \cdot (9)$

18

I need to evaluate the part of the expression with the exponent first and then multiply.

2. $5 + (4 - 3)^2$ = 6

$5 + (1)^2$

$5 + (1 \cdot 1)$

$5 + 1$

6

I need to evaluate the grouped part of the expression first, then the exponent part. Then I can add.

Guided Practice

Determine the value of each expression.

1. 2^2

$2 \cdot$ ___

______

2. $2 + 3^2$

$2 + ($___ $\cdot$ ___$)$

___ $+$ ___

______

3. $(6 - 2)^2$

$($___$)^2$

$($______$)$

______

4. $1 + (3 - 2)^2$

$1 + ($___$)^2$

____

______

______

Guided Practice

5. Determine the value of each expression.

Expression	Value
$3^2 + 6$	
$(4 + 3)^2$	
$3 + (4 + 1)^2$	
$2 \cdot 4^2$	
$(7 - 1)^2 + 2$	
$6 + 7^2$	

Check

Determine the value of each expression.

1. $3 \cdot 5^2$ ______

2. $7 + (5 - 2)^2$ ______

Evaluating Expressions With Exponents

Goal

Evaluate expressions with exponents using the order of operations.

Standard

MA.6.AR.1.3

Materials

highlighter (optional)

Modeled Review

Point to Kai's work and **ask**:

- "How did Kai know which operation to evaluate first in each expression?"
- "Can you explain the order of operations Kai used to evaluate the expressions?"
- "What does the exponent 2 represent in each expression?"

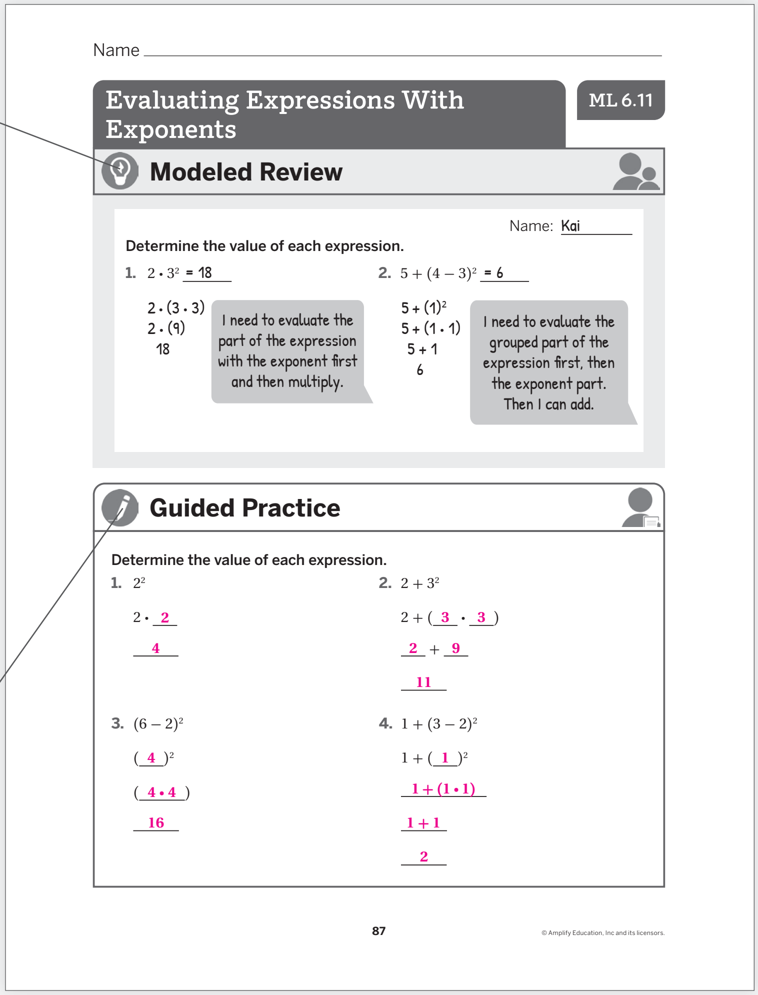
Reinforce Kai's thinking by saying, "The order of operations can be used to efficiently evaluate expressions with exponents."

Guided Practice

Focus students' attention on using the order of operations to evaluate each expression.

To scaffold their thinking, **say**:

- "First, evaluate brackets or parentheses."
- "Next, evaluate exponents."
- "Then, evaluate multiplication or division (left to right)."
- "Lastly, evaluate addition or subtraction (left to right)."



Name

Evaluating Expressions With Exponents — ML 6.11

Modeled Review

Name: Kai

Determine the value of each expression.

1. $2 \cdot 3^2$ = 18

   $2 \cdot (3 \cdot 3)$
   $2 \cdot (9)$
   18

   I need to evaluate the part of the expression with the exponent first and then multiply.

2. $5 + (4 - 3)^2$ = 6

   $5 + (1)^2$
   $5 + (1 \cdot 1)$
   $5 + 1$
   6

   I need to evaluate the grouped part of the expression first, then the exponent part. Then I can add.

Guided Practice

Determine the value of each expression.

1. 2^2

   $2 \cdot$ 2
   4

2. $2 + 3^2$

   $2 + ($ 3 $\cdot$ 3 $)$
   2 + 9
   11

3. $(6 - 2)^2$

   (4 $)^2$
   ($4 \cdot 4$)
   16

4. $1 + (3 - 2)^2$

   $1 + ($ 1 $)^2$
   $1 + (1 \cdot 1)$
   $1 + 1$
   2

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Vocabulary

If needed, share the meaning of the terms with students.

order of operations: A consistent order applied to an expression with multiple operations so that the expression is evaluated the same way by everyone.

exponent: A number used to describe repeated multiplication. Exponents are sometimes called powers.

Evaluating Expressions With Exponents

Guided Practice

A Use visual aids, such as anchor charts or diagrams, to illustrate the order of operations and guide students in understanding the correct sequence of steps to evaluate expressions with exponents.

ML/EL Demonstrate the step-by-step process of evaluating expressions with exponents, narrating your thinking aloud to show students how to apply the order of operations correctly.

Key Takeaway:
Say, "When evaluating expressions, evaluate exponents first, unless there are grouping symbols, like parentheses or a fraction bar. When there are grouping symbols, perform the operation(s) inside them first."

Guided Practice

5. Determine the value of each expression.

Expression	Value
$3^2 + 6$	15
$(4 + 3)^2$	49
$3 + (4 + 1)^2$	28
$2 \cdot 4^2$	32
$(7 - 1)^2 + 2$	38
$6 + 7^2$	55

Check

Determine the value of each expression.

1. $3 \cdot 5^2$ $= 75$

2. $7 + (5 - 2)^2$ $= 16$

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Reflection

Ask:

- "What are some things to remember when determining the value of expressions with exponents?"
- "How did you overcome a hard problem today?"

Check: Recommended Next Steps

Almost there
If students need more support, consider using a highlighter to color-code different operations and parts of the expression.

Got it!
If students need more practice, have them evaluate the following expressions:

- $9 \cdot 5^2$
- $20 - (5 - 1)^2$
- $36 - 2 \cdot 3^2$

Name ______________________________

Calculating Percentages

ML 6.14

Modeled Review

Name: Dylan

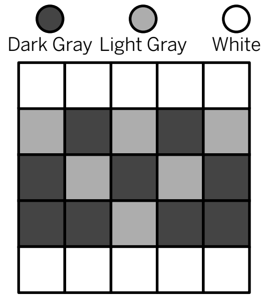
What percentage of the larger grid is white? Explain your thinking.

I counted 25 squares. Each square has a value of 4% because 100 divided by 25 is 4. There are 10 white squares. I can multiply 4 by 10 to get 40%.

$\frac{100}{25} = 4\%$

$4 \cdot 10 = 40$

40%



Guided Practice

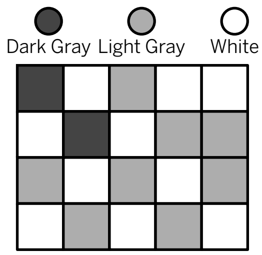
1. What percentage of the grid is light gray or white? Show your thinking.

Light Gray:

$\frac{100}{20} =$ ______

White:

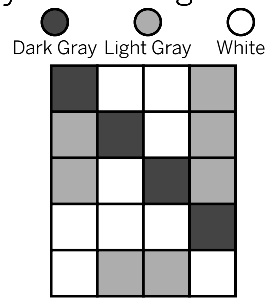
Color	Percentage
Dark Gray	10%
Light Gray	
White	



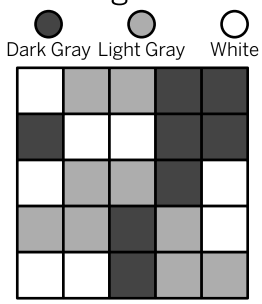
Guided Practice

Determine the percentages of the grids that are shaded each color.

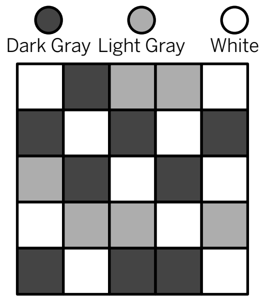
2. What percentage of the grid is dark gray? Show your thinking.



3. What percentage of the grid is white? Show your thinking.

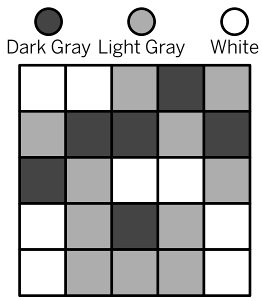


4. What percentage of the grid is light gray? Show your thinking.



Check

What percentage of the grid is white? Show your thinking.



Goal	Standard
Calculate the percentage of a quantity on a grid.	MA.7.AR.3.1

Modeled Review

Point to Dylan's work and **ask**:

- "What percentage represents 1 whole?"
- "How did Dylan use the grid to calculate the percentage of the white squares?"

Reinforce Dylan's thinking by saying, "Finding the percentage each square represents can help you determine the total percentage of each color."

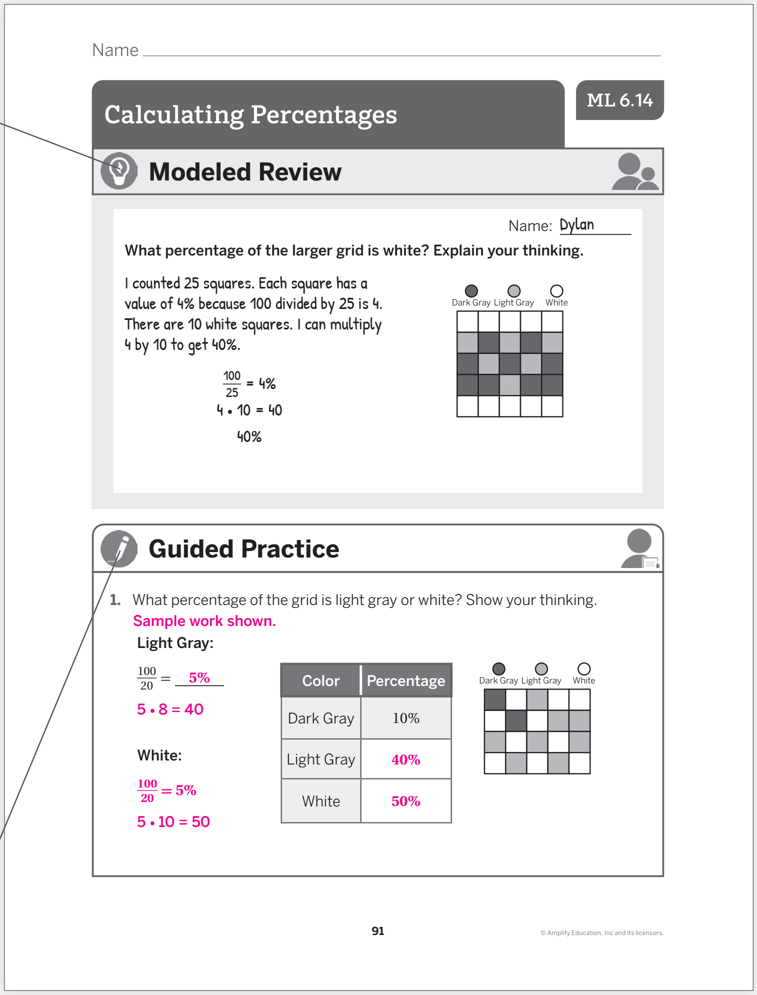
ML/EL Provide sentence frames to support students as they explain how they determined the percentage shaded for each colored section. For example, "I counted _______ sections in the larger square." or "There are _____ dark gray/light gray/white sections, so I _____."

Guided Practice

Focus students' attention on calculating the percentage of each color in the grid.

To scaffold their thinking, **ask**:

- "How many sections is the grid broken into?"
- "How many sections are there of each color?"
- "How do you calculate the percentage of each color?"



Name

Calculating Percentages ML 6.14

Modeled Review

Name: Dylan

What percentage of the larger grid is white? Explain your thinking.

I counted 25 squares. Each square has a value of 4% because 100 divided by 25 is 4. There are 10 white squares. I can multiply 4 by 10 to get 40%.

$\frac{100}{25} = 4\%$

$4 \cdot 10 = 40$

40%

Guided Practice

1. What percentage of the grid is light gray or white? Show your thinking.

Sample work shown.

Light Gray:

$\frac{100}{20} = 5\%$

$5 \cdot 8 = 40$

White:

$\frac{100}{20} = 5\%$

$5 \cdot 10 = 50$

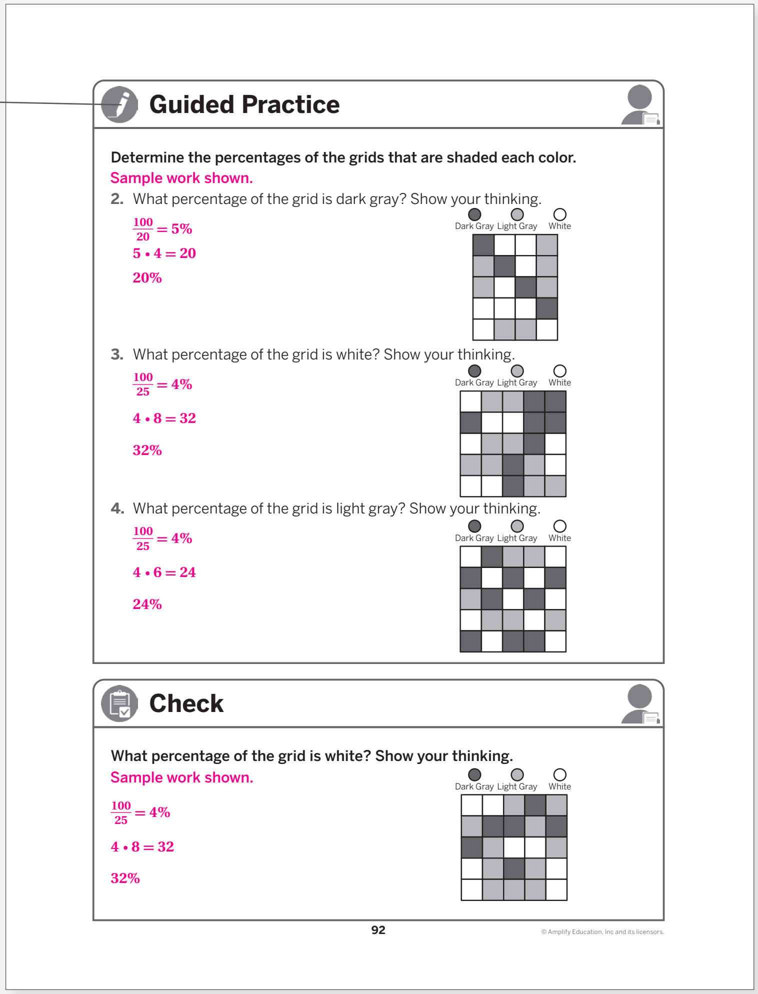
Color	Percentage
Dark Gray	10%
Light Gray	40%
White	50%

Guided Practice

A Chunk this task into smaller, more manageable parts by having students create a table that can be used to organize their data, including how many sections the grid is broken into and how many sections there are of each color.

Note: Students can check if they accurately determined the percentage for each color by ensuring all of the percentages add to 100%.

Key Takeaway:
Say, "Several strategies can be used to calculate the percent of a number such as multiplying the percentage by the whole, using benchmark fractions, or using a visual representation."



Guided Practice

Determine the percentages of the grids that are shaded each color.
Sample work shown.

2. What percentage of the grid is dark gray? Show your thinking.
$\frac{100}{20} = 5\%$
$5 \cdot 4 = 20$
20%

3. What percentage of the grid is white? Show your thinking.
$\frac{100}{25} = 4\%$
$4 \cdot 8 = 32$
32%

4. What percentage of the grid is light gray? Show your thinking.
$\frac{100}{25} = 4\%$
$4 \cdot 6 = 24$
24%

Check

What percentage of the grid is white? Show your thinking.
Sample work shown.
$\frac{100}{25} = 4\%$
$4 \cdot 8 = 32$
32%

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Reflection

Ask:
- "How is using a grid helpful in visualizing benchmark percentages?"
- "What questions do you still have?"

Check: Recommended Next Steps

Almost there	**Got it!**
If students need more support, consider using Mini-Lesson 3.12: *Calculating Unknown Percentages*.	If students need more practice, ask them to look at Problem 2 and determine the percentage of the remaining colors in the grid.

Name ______________________________

Finding Sales Tax and Tip

ML 6.15

Modeled Review

Name: Eva

Determine the missing values for the receipt.

Clare's meal costs $30.00 before tax.
There is a 7% sales tax.
What is the total after tax?

Original Cost	$30.00
7% Tax:	$2.10
Total:	$32.10

7% is $\frac{7}{100}$ or 0.07.

$30 \cdot 0.07 = 2.10$
$30 + 2.10 = 32.10$

Guided Practice

Determine the missing values for these receipts.

1.

Smoothie:	$5.00
10% Tax:	$
Total:	$

$5.00 \cdot 0.10 =$ ______

$5.00 +$ ______ $=$ ______

2.

Meal:	$20.00
20% Tip:	$
Total:	$

$20.00 \cdot 0.20 =$ ______

$20.00 +$ ______ $=$ ______

Guided Practice

Determine the missing values for these receipts.

3. Han's meal costs $20.00 before tax.
There is a 7% sales tax.
What is the total after tax?

Original Cost	$20.00
7% Tax:	$
Total:	$

4. Tristan's meal costs $30.00 before tip.
Tristan tips 20%.
What is the total after tip?

Original Cost	$30.00
20% Tip:	$
Total:	$

5. Diego's meal cost $50.00 before tax.
There is a 5% sales tax.
What is the total after tax?

Original Cost	$50.00
5% Tax:	$
Total:	$

Check

Determine the missing values for the receipt.

Priya's meal costs $40.00 before tax.
There is an 8% sales tax.
What is the total after tax?

Original Cost	$40.00
8% Tax:	$
Total:	$

ML 6.15 Finding Sales Tax and Tip

Goal	**Standard**
Calculate the sales tax or tip percentage to determine the new total.	MA.7.AR.3.1

Modeled Review

Point to Eva's work and **ask**:

- "Why does Eva change the percentages to decimals?"
- "How does Eva use the given sales tax to calculate the total?"

Reinforce Eva's thinking by saying, "Convert the given sales tax or tip to a decimal to calculate the total."

ML/EL Provide sentence frames to support students as they explain their strategies for converting percentages to decimals. For example, "First, I _______ because _______." or "I divided _________ by 100 because _______."

Guided Practice

Focus students' attention on using the sales tax or tip to calculate the total.

To scaffold their thinking, **ask**:

- "How do you convert the given sales tax or tip to a decimal?"
- "How do you determine the total?"

Name ______________________________

Finding Sales Tax and Tip

ML 6.15

Modeled Review

Name: Eva

Determine the missing values for the receipt.

Clare's meal costs $30.00 before tax.
There is a 7% sales tax.
What is the total after tax?

Original Cost	$30.00
7% Tax:	$2.10
Total:	$32.10

7% is $\frac{7}{100}$ or 0.07.

$30 \cdot 0.07 = 2.10$
$30 + 2.10 = 32.10$

Guided Practice

Determine the missing values for these receipts.

1.

Smoothie:	$5.00
10% Tax:	$0.50
Total:	$5.50

5.00 • 0.10 = 0.50

5.00 + 0.50 = 5.50

2.

Meal:	$20.00
20% Tip:	$4.00
Total:	$24.00

20.00 • 0.20 = 4.00

20.00 + 4.00 = 24.00

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Finding Sales Tax and Tip

Guided Practice

A Chunk this task into smaller, more manageable parts by having students first convert the percentages into decimals then determine the missing values for the receipts.

Key Takeaway:
Say, "To calculate sales tax or tip, change the percentage to a decimal, multiply by the cost, and add the result to the original cost."

Guided Practice

Determine the missing values for these receipts.

3. Han's meal costs $20.00 before tax.
   There is a 7% sales tax.
   What is the total after tax?
   $21.40

Original Cost	$20.00
7% Tax:	**$1.40**
Total:	**$21.40**

4. Tristan's meal costs $30.00 before tip.
   Tristan tips 20%.
   What is the total after tip?
   $36.00

Original Cost	$30.00
20% Tip:	**$6.00**
Total:	**$36.00**

5. Diego's meal cost $50.00 before tax.
   There is a 5% sales tax.
   What is the total after tax?
   $52.50

Original Cost	$50.00
5% Tax:	**$2.50**
Total:	**$52.50**

Check

Determine the missing values for the receipt.

Priya's meal costs $40.00 before tax.
There is an 8% sales tax.
What is the total after tax?
$43.20

Original Cost	$40.00
8% Tax:	$**3.20**
Total:	$**43.20**

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Reflection

Ask:
- "Why is it helpful to convert percentages to decimals when calculating the total amount including sales tax or tip?"
- "What is something you weren't sure about at the start of the lesson but understand now?"

Check: Recommended Next Steps

Almost there
If students need more support, model converting percentages to decimals. When multiplying numbers, they may need to be reminded that they don't have to line up the decimals (like in addition).

Got it!
If students need more practice, have them look at Problem 3 and ask how the total would change if the sales tax was 8%.

Unit 7

Mini-Lessons

Name ______________________________

Ordering and Comparing Positive and Negative Numbers

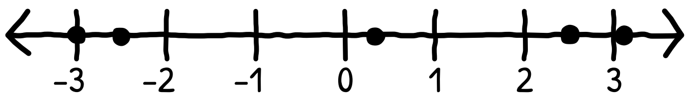
ML 7.04

Modeled Review

Name: Adam

Order the numbers from *least* to *greatest*.

−3	3.1	−2.5	2.5	0.25



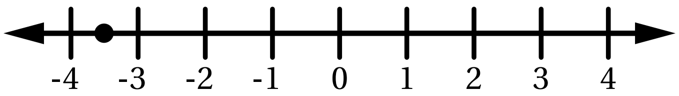
−3 (least) | −2.5 | 0.25 | 2.5 | 3.1 (greatest)

Guided Practice

Order each set of numbers from least to *greatest*. Use the number lines if they are helpful.

1.

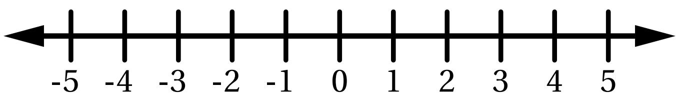
0	-3.5	3	-1



-3.5 (least) | ________ | ________ | ________ (greatest)

2.

4.5	-2	-4	0.25



________ (least) | ________ | ________ | ________ (greatest)

Guided Practice

Order each set of numbers from *least* to *greatest*.

3.

0.2	-2	3.5	3	1.25

______ least ______ ______ ______ ______ greatest

4.

$\frac{1}{2}$	3	$1\frac{1}{2}$	-2	1

______ least ______ ______ ______ ______ greatest

5.

2.5	2	-2.5	-0.5	-3

______ least ______ ______ ______ ______ greatest

6.

$-3\frac{1}{2}$	$-\frac{1}{5}$	$2\frac{1}{2}$	-5	3

______ least ______ ______ ______ ______ greatest

Check

Order the numbers from *least* to *greatest*.

$-\frac{1}{2}$	1	-3	4	$-4\frac{1}{2}$

______ least ______ ______ ______ ______ greatest

ML 7.04 Ordering and Comparing Positive and Negative Numbers

Goal	Standards
Order and compare positive and negative numbers represented as fractions and decimals.	MA.6.NSO.1.1

Modeled Review

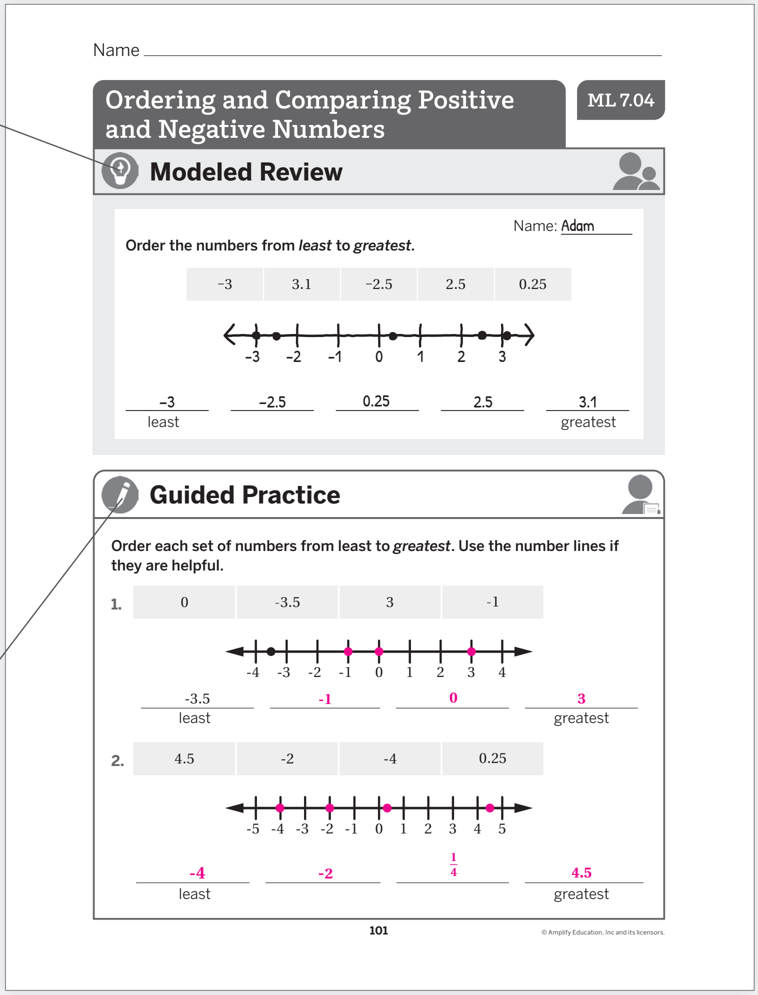
Point to Adam's work and **ask**:
- "How could Adam tell if one number was greater than or less than another number?"
- "How could Adam tell if two numbers were opposites?"
- "How was using a number line helpful?"

Reinforce Adam's thinking by saying, "Using a number line can help efficiently order and compare positive and negative numbers."

Guided Practice

Focus students' attention on ordering and comparing numbers using a number line.

To scaffold their thinking, **say**:
- "Find the smallest number and add it to the number line."
- "Then, look for the next smallest number and add it to the number line."
- "Continue this process until you have placed all of the numbers on the number line from least to the greatest."



Name

Ordering and Comparing Positive and Negative Numbers — ML 7.04

Modeled Review

Name: Adam

Order the numbers from *least* to *greatest*.

-3	3.1	-2.5	2.5	0.25

-3 (least), -2.5, 0.25, 2.5, 3.1 (greatest)

Guided Practice

Order each set of numbers from least to *greatest*. Use the number lines if they are helpful.

1.

0	-3.5	3	-1

-3.5 (least), -1, 0, 3 (greatest)

2.

4.5	-2	-4	0.25

-4 (least), -2, $\frac{1}{4}$, 4.5 (greatest)

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Vocabulary

If needed, share the meaning of the terms with students.

sign: The sign of a number (other than '0') is either positive or negative.

opposite: Two numbers that are the same distance from 0 and on different sides of the number line. For example, 4 and -4 are opposites.

rational number: All positive and negative numbers that can be written as fractions, including whole numbers, are called rational numbers.

ML 7.04 Ordering and Comparing Positive and Negative Numbers

Guided Practice

A Model crossing off the numbers in the list as they are ordered to ensure that all numbers have been included and none are overlooked.

ML/EL Guide processing by having students identify the decimal represented by each fraction before ordering the numbers from least to greatest.

Key Takeaway:
Say, "When ordering rational numbers, list them in the direction they appear on the number line: left to right for least to greatest."

Guided Practice

Order each set of numbers from *least* to *greatest*.

3.

0.2	-2	3.5	3	1.25
-2	**0.2**	**1.25**	**3**	**3.5**
least				greatest

4.

$\frac{1}{2}$	3	$1\frac{1}{2}$	-2	1
-2	**0.5**	**1**	**$1\frac{1}{2}$**	**3**
least				greatest

5.

2.5	2	-2.5	-0.5	-3
-3	**-2.5**	**$-\frac{1}{2}$**	**2**	**2.5**
least				greatest

6.

$-3\frac{1}{2}$	$-\frac{1}{5}$	$2\frac{1}{2}$	-5	3
-5	**-3.5**	**$-\frac{1}{5}$**	**2.5**	**3**
least				greatest

Check

Order the numbers from *least* to *greatest*.

$-\frac{1}{2}$	1	-3	4	$-4\frac{1}{2}$
-4.5	**-3**	**$-\frac{1}{2}$**	**1**	**4**
least				greatest

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Reflection

Ask:
- "What steps should you take when ordering positive and negative numbers from least to greatest?"
- "What strategy was helpful today?"

Check: Recommended Next Steps

Almost there
If students need more support, consider having them revisit the problem in the Check. Then model drawing a number line and adding tick marks to support students with ordering the numbers.

Got it!
If students need more practice, present them with the following numbers and ask them to order these numbers from least to greatest.

-0.75	2	-5	3	-2.5

Name ____________________

Adding and Subtracting Positive and Negative Numbers

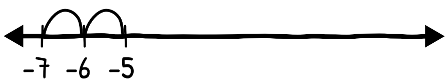
ML 7.08

Modeled Review

Name: Tristan

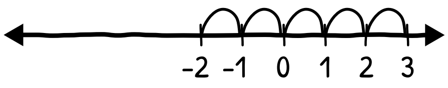
Evaluate each expression.

1. -5 + (-2) <u>-7</u>



I need to start at -5, and move left 2 times to represent adding -2.

2. -2 − (-5) <u>3</u>

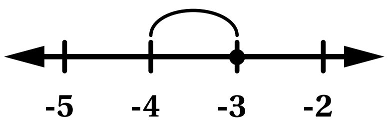


I need to start at -2, and move right 5 times.

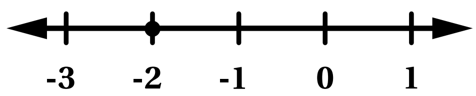
Guided Practice

Evaluate each expression using a number line.

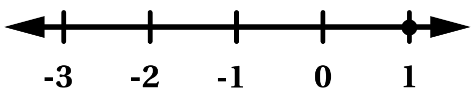
1. -3 − 2 ____



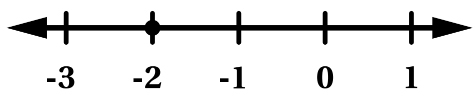
2. -2 + 3 ____



3. 1 − 3 ____



4. -2 − (-1) ____



Guided Practice

5. Evaluate each expression.

Addition Expression	Subtraction Expression
-4 + 1 ____	4 – 1 ____
2 + (-3) ____	3 – (-2) ____
-1 + 5 ____	-5 – 1 ____
-3 + -3 ____	-3 – (-3) ____
-2 + 5 ____	5 – (-2) ____

Check

Evaluate each expression.

Addition Expression	Subtraction Expression
-4 + (-2) ____	-2 – (-4) ____

ML 7.08 Adding and Subtracting Positive and Negative Numbers

Goal

Calculate sums and differences of positive and negative numbers.

Standard

MA.6.NSO.4.1

Materials

number line (optional)

Modeled Review

Point to Tristan's work and **ask**:

- "How did Tristan use a number line to add integers? Subtract?"
- "Why is subtracting negative numbers equivalent to adding positive numbers?"

Reinforce Tristan's thinking by saying, "Number lines can be used to add and subtract integers."

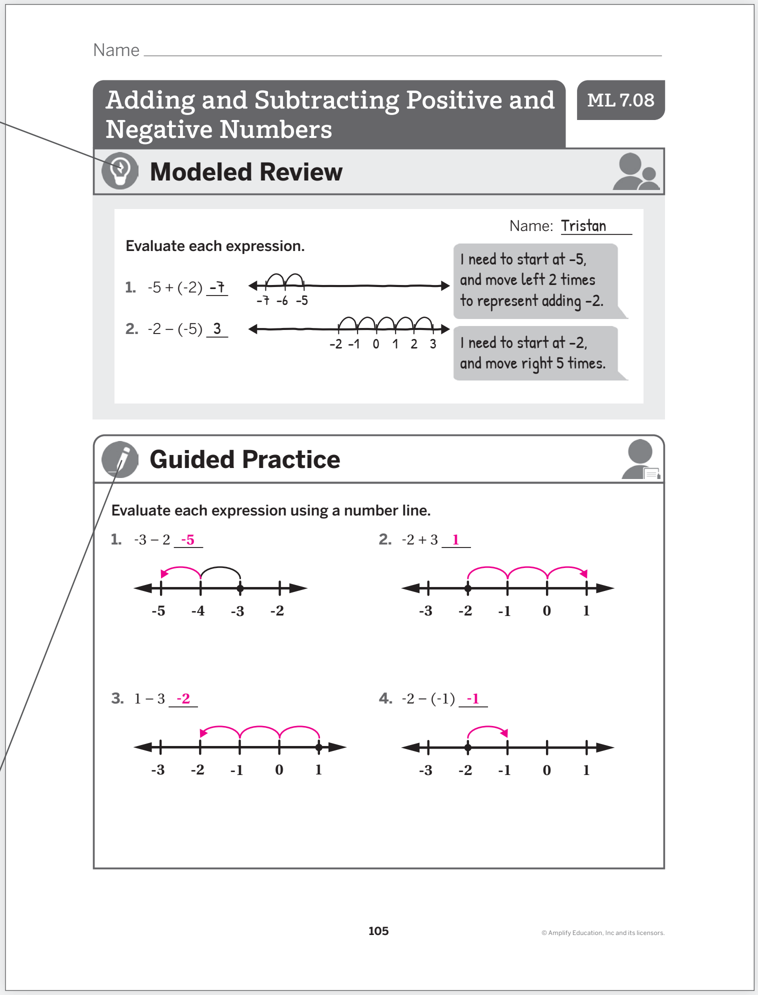
ML/EL Consider using a think aloud to reinforce that adding a negative number is equivalent to subtracting a positive number and subtracting a negative number is equivalent to adding a positive number.

Guided Practice

Focus students' attention on adding and subtracting integers.

To scaffold their thinking, **ask**:

- "What operation is in the expression?"
- "How can you use the number line to add or subtract integers?"



Name

Adding and Subtracting Positive and Negative Numbers ML 7.08

Modeled Review

Name: Tristan

Evaluate each expression.

1. -5 + (-2) -7
2. -2 − (-5) 3

Guided Practice

Evaluate each expression using a number line.

1. -3 − 2 -5
2. -2 + 3 1
3. 1 − 3 -2
4. -2 − (-1) -1

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Vocabulary

If needed, share the meaning of the term with students.

integers: Whole numbers, their opposites, and 0.

ML 7.08 Adding and Subtracting Positive and Negative Numbers

Guided Practice

A Provide students with a number line to use as they evaluate each expression.

Key Takeaway:
Say, "Subtracting a value is equivalent to adding its opposite."

Guided Practice

5. Evaluate each expression.

Addition Expression	Subtraction Expression
-4 + 1 __-3__	4 – 1 __3__
2 + (-3) __-1__	3 – (-2) __5__
-1 + 5 __4__	-5 – 1 __-6__
-3 + -3 __-6__	-3 – (-3) __0__
-2 + 5 __3__	5 – (-2) __7__

Check

Evaluate each expression.

Addition Expression	Subtraction Expression
-4 + (-2) __-6__	-2 – (-4) __2__

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Reflection

Ask:

- "How does using a number line help add and subtract integers?"
- "What makes sense? What is still confusing?"

Check: Recommended Next Steps

Almost there
If students need more support, consider using the concepts of money to help make adding and subtracting integers less abstract. Positive numbers can represent having or receiving money. Negative numbers can represent situations involving expenses or debts.

Got it!
If students need more practice, have them evaluate the following expressions:

- -6 + (-2)
- -2 – (-6)
- -7 + (-3)
- -3 – (-7)

Name ____________________

Solving Real-World Problems Involving Positive and Negative Numbers

ML 7.09

Modeled Review

Name: Santiago

Use the table to determine the change.

Date	Temperature in Kazan, Russia (°F)
December 20, 2019	-11
July 27, 2020	60

Positive change

What is the change in Kazan's temperature from December 20th to July 27th?

$60 - (-11) = 60 + 11 = 71$
71 degrees Fahrenheit

Since I am calculating the change, I need to find the difference between the final temperature and the initial temperature.

Guided Practice

Use the tables to determine if each temperature change in Fitzgerald, Canada is positive or negative.

1.

Date	Temperature (°F)
January 29, 2020	10
May 30, 2020	50

Positive Change or Negative Change

2.

Date	Temperature (°F)
June 24, 2019	60
February 9, 2020	-5

Positive Change or Negative Change

3.

Date	Temperature (°F)
December 19, 2019	-4
January 28, 2020	-8

Positive Change or Negative Change

4.

Date	Temperature (°F)
December 14, 2019	-2
July 8, 2020	70

Positive Change or Negative Change

Guided Practice

Use the tables to determine the change in temperature.

5.

Date	Temperature in Siberia, Russia (°F)
July 24, 2019	50
February 11, 2020	-10

What is the change in Siberia's temperature from July 24th to February 11th?

6.

Date	Temperature in Fairbanks, Alaska (°F)
January 2, 2020	-5
September 25, 2020	45

What is the change in Fairbanks's temperature from January 2nd to September 25th?

7.

Date	Temperature in Harbin, China (°F)
December 13, 2019	-8
August 20, 2020	68

What is the change in Harbin's temperature from December 13th to August 20th?

Check

Use the table to determine the temperature change.

Date	Temperature in Prospect Creek, Alaska (°F)
February 26, 2020	-7
August 9, 2020	63

What is the change in Prospect Creek's temperature from February 26th to August 9th?

ML 7.09 Solving Real-World Problems Involving Positive and Negative Numbers

Goal

Add and subtract positive and negative numbers to solve problems involving real-world situations.

Standard

MA.6.NSO.4.1

Materials

highlighter (optional), number line (optional)

Modeled Review

Point to Santiago's work and **ask**:

- "Why does Santiago say it is a positive change?"
- "Why does Santiago subtract the initial from the final temperature?"
- "How does knowing the table reflects a positive change help you check the calculated answer?"

Reinforce Santiago's thinking by saying, "Use your knowledge of adding and subtracting positive and negative numbers to solve real-world situations."

Guided Practice

Focus students' attention on determining if each temperature change is positive or negative.

To scaffold their thinking, **ask**:

- "What's the initial temperature?"
- "What's the final temperature?"
- "Is that change a positive (getting warmer) or negative (getting colder) change?"

Name

Solving Real-World Problems Involving Positive and Negative Numbers ML 7.09

Modeled Review

Name: Santiago

Use the table to determine the change.

Date	Temperature in Kazan, Russia (°F)
December 20, 2019	-11
July 27, 2020	60

Positive change

What is the change in Kazan's temperature from December 20th to July 27th?

$60 - (-11) = 60 + 11 = 71$

71 degrees Fahrenheit

Since I am calculating the change, I need to find the difference between the final temperature and the initial temperature.

Guided Practice

Use the tables to determine if each temperature change in Fitzgerald, Canada is positive or negative.

1.

Date	Temperature (°F)
January 29, 2020	10
May 30, 2020	50

Positive Change or Negative Change

2.

Date	Temperature (°F)
June 24, 2019	60
February 9, 2020	-5

Positive Change or Negative Change

3.

Date	Temperature (°F)
December 19, 2019	-4
January 28, 2020	-8

Positive Change or Negative Change

4.

Date	Temperature (°F)
December 14, 2019	-2
July 8, 2020	70

Positive Change or Negative Change

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Vocabulary

If needed, share the meaning of the term with students.

integers: Whole numbers and their opposites.

ML 7.09 Solving Real-World Problems Involving Positive and Negative Numbers

Guided Practice

(A) Model color coding positive and negative numbers with a highlighter for students to understand whether it will be a positive or negative change.

(ML/EL) Consider modeling, or inviting a student to model, annotating the problem to make sense of it, such as labeling the first temperature as the initial and second as the final.

Key Takeaway:
Say, "Adding and subtracting positive and negative values help to solve problems involving real-world situations."

Guided Practice

Use the tables to determine the change in temperature.

5.

Date	Temperature in Siberia, Russia (°F)
July 24, 2019	50
February 11, 2020	-10

What is the change in Siberia's temperature from July 24th to February 11th?
-60 degrees Fahrenheit

6.

Date	Temperature in Fairbanks, Alaska (°F)
January 2, 2020	-5
September 25, 2020	45

What is the change in Fairbanks's temperature from January 2nd to September 25th?
50 degrees Fahrenheit

7.

Date	Temperature in Harbin, China (°F)
December 13, 2019	-8
August 20, 2020	68

What is the change in Harbin's temperature from December 13th to August 20th?
76 degrees Fahrenheit

Check

Use the table to determine the temperature change.

Date	Temperature in Prospect Creek, Alaska (°F)
February 26, 2020	-7
August 9, 2020	63

What is the change in Prospect Creek's temperature from February 26th to August 9th? **70 degrees Fahrenheit**

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Reflection

Ask:
- "What new questions do you have about adding and subtracting positive and negative numbers?"
- "How does what you learned today connect to your prior learning?"

Check: Recommended Next Steps

Almost there
If students need more support, consider orienting a number line vertically so they can understand whether a positive or negative change is occurring in the context of temperature, and then check it against their calculated solution.

Got it!
If students need more practice, present them with the following table and ask them to determine the change in Fairbanks' weather from February 29th to March 28th.

Month	Temperature in Fairbanks, Alaska (°F)
February 29, 2020	-33
March 28, 2020	-4

Name ______________________________

Dividing Integers

ML 7.12

Modeled Review

Name: Han

Select the two expressions that have the same value.

(A.) $\frac{-20}{4}$ -5

B. $\frac{-20}{-4}$ 5

(C.) $\frac{20}{-4}$ -5

When dividing integers, like signs result in a positive quotient. Unlike signs result in a negative quotient.

Guided Practice

1. Evaluate each expression. Circle *all* the expressions with a negative quotient.

A. $\frac{-6}{2}$

B. $\frac{6}{-2}$

C. $\frac{-6}{-2}$

2. Evaluate each expression. Circle *all* the expressions with a negative quotient.

A. $\frac{-12}{3}$

B. $\frac{-12}{-3}$

C. $\frac{12}{-3}$

3. Evaluate each expression. Circle *all* the expressions with a negative quotient.

A. $\frac{-15}{5}$

B. $\frac{15}{-5}$

C. $\frac{-15}{-5}$

Guided Practice

Calculate the quotients of the expressions.

4. $\frac{10}{-2}$

5. $\frac{-12}{-6}$

6. $\frac{-20}{10}$

7. $\frac{36}{-6}$

8. $\frac{-18}{-9}$

9. $\frac{-16}{4}$

10. $\frac{-24}{-6}$

11. $\frac{-9}{-3}$

Check

Select the two expressions that have the same value.

A. $\frac{30}{-5}$

B. $\frac{-30}{-5}$

C. $\frac{-30}{5}$

Dividing Integers

Goal

Divide positive and negative numbers.

Standard

MA.7.NSO.2.2

Materials

highlighter (optional), coloring tools (optional)

Modeled Review

Point to Han's work and **ask**:

- "How did Han use the rules for dividing integers to identify the expressions that have the same quotients?"
- "What do A and C have in common? How is B different?"

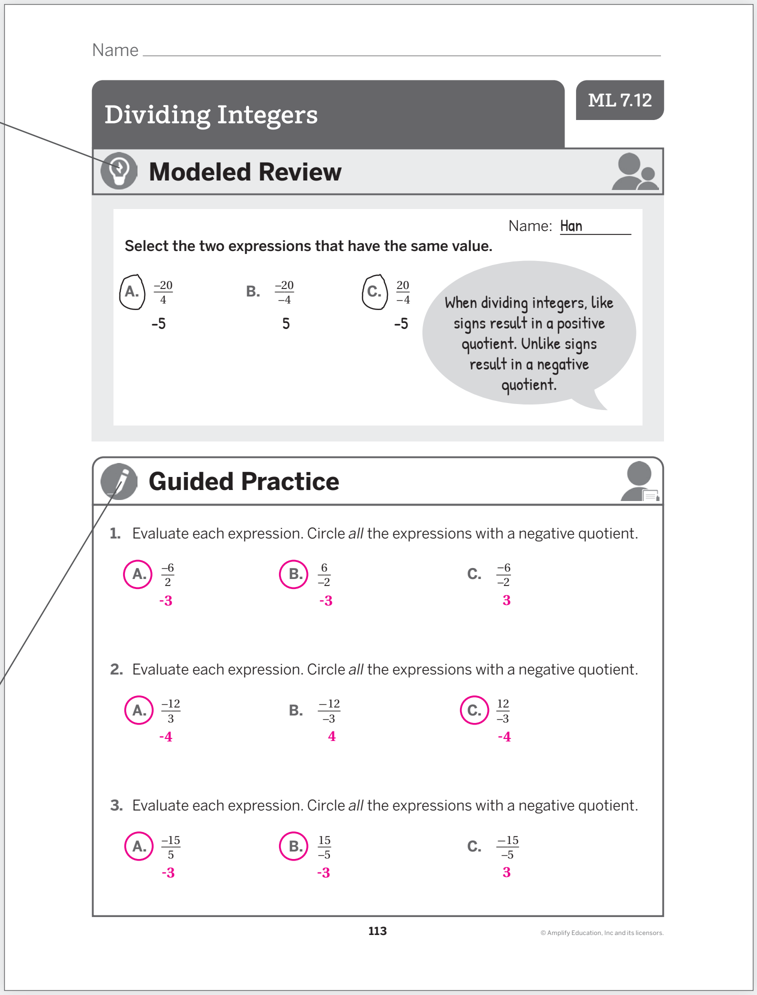
Reinforce Han's thinking by saying, "When you divide integers that have the same sign, you get a positive number. When you divide integers that have different signs, you get a negative number."

Guided Practice

Focus students' attention on using the signs of the numbers to determine the sign of the quotient.

To scaffold their thinking, **ask**:

- "Do the integers have the same or different signs?"
- "If the signs are the same, what will be the sign of the quotient? What if the signs are different?"



Name

Dividing Integers

ML 7.12

Modeled Review

Name: Han

Select the two expressions that have the same value.

(A.) $\frac{-20}{4}$ -5 B. $\frac{-20}{-4}$ 5 (C.) $\frac{20}{-4}$ -5

When dividing integers, like signs result in a positive quotient. Unlike signs result in a negative quotient.

Guided Practice

1. Evaluate each expression. Circle *all* the expressions with a negative quotient.

(A.) $\frac{-6}{2}$ -3 (B.) $\frac{6}{-2}$ -3 C. $\frac{-6}{-2}$ 3

2. Evaluate each expression. Circle *all* the expressions with a negative quotient.

(A.) $\frac{-12}{3}$ -4 B. $\frac{-12}{-3}$ 4 (C.) $\frac{12}{-3}$ -4

3. Evaluate each expression. Circle *all* the expressions with a negative quotient.

(A.) $\frac{-15}{5}$ -3 (B.) $\frac{15}{-5}$ -3 C. $\frac{-15}{-5}$ 3

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Vocabulary

If needed, share the meaning of the term with students.

integers: Whole numbers and their opposites.

Guided Practice

A Guide visualization by suggesting students color code the positive and negative numbers.

ML/EL Support students in explaining their answers by providing them with the following sentence frames:

- "A positive number divided by a negative number always results in ________."
- "A negative number divided by a negative number always results in ________."

Key Takeaway:
Say, "The sign of a quotient is dependent on the signs of the numbers being divided."

Guided Practice

Calculate the quotients of the expressions.

4. $\frac{10}{-2}$
-5

5. $\frac{-12}{-6}$
2

6. $\frac{-20}{10}$
-2

7. $\frac{36}{-6}$
-6

8. $\frac{-18}{-9}$
2

9. $\frac{-16}{4}$
-4

10. $\frac{-24}{-6}$
4

11. $\frac{-9}{-3}$
3

Check

Select the two expressions that have the same value.

A. $\frac{30}{-5}$ (circled)
-6

B. $\frac{-30}{-5}$
6

C. $\frac{-30}{5}$ (circled)
-6

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Reflection

Ask:

- "Do $\frac{-6}{3}$ and $\frac{6}{-3}$ have the same value? How do you know?"
- "How was the lesson helpful to you today?"

Check: Recommended Next Steps

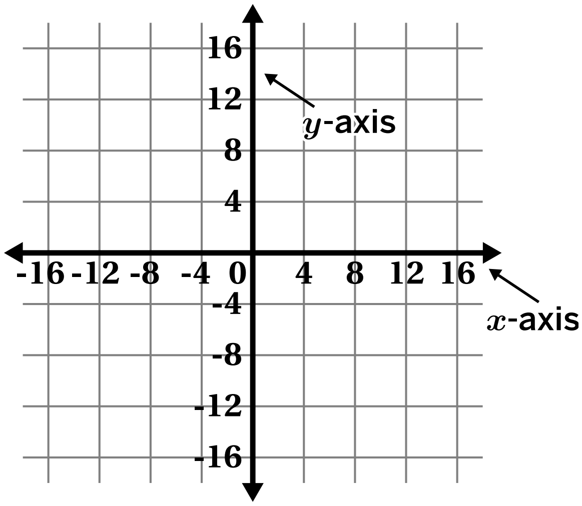
Almost there	Got it!
If students need more support, consider rewriting the division problem as a multiplication equation to help students understand how the signs of the divisor and dividend impact the sign of the quotient.	If students need more practice, present them with the following expressions and ask them to circle the two that have the same value. A. $\frac{40}{-8}$ B. $\frac{-40}{-8}$ C. $\frac{-40}{8}$

Name ______________________________

Plotting Points on the Coordinate Plane

ML 7.17

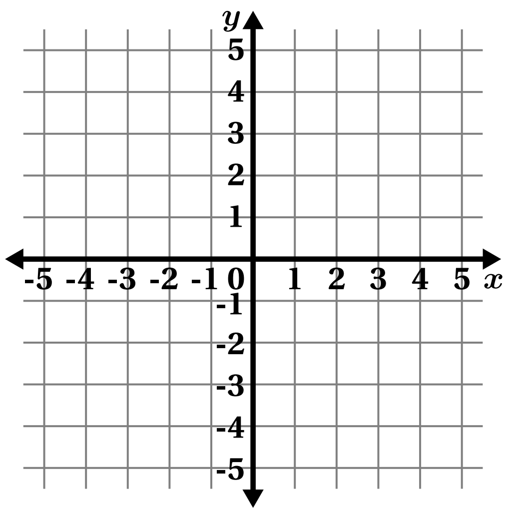
Modeled Review

Coordinate Plane	In an ordered pair (x, y):
	• The first number (x) represents how far to move **horizontally** along the ***x-axis.*** • The second number (y) represents how far to move **vertically** along the ***y-axis.***

Guided Practice

1. Plot and label each point on the graph.

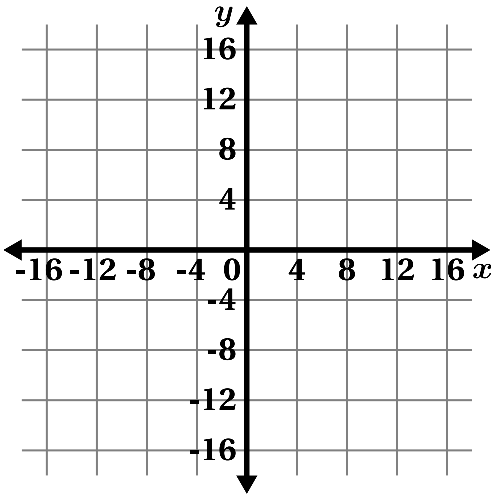
A(-4, 2)
B(4, -2)
C(-5, -1)



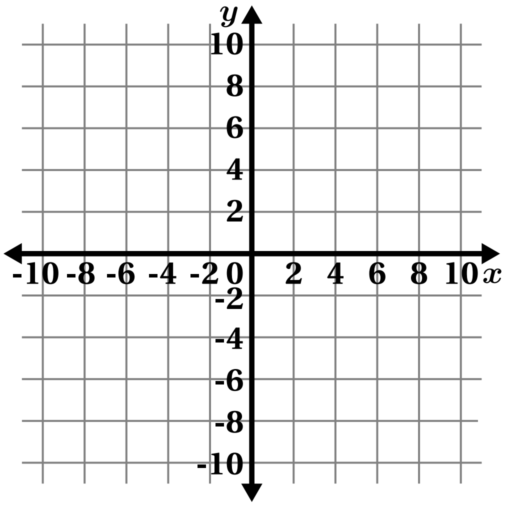
Guided Practice

Plot and label each point on the graphs.

2. $A(-8, 2)$
$B(-8, -6)$
$C(0, 8)$



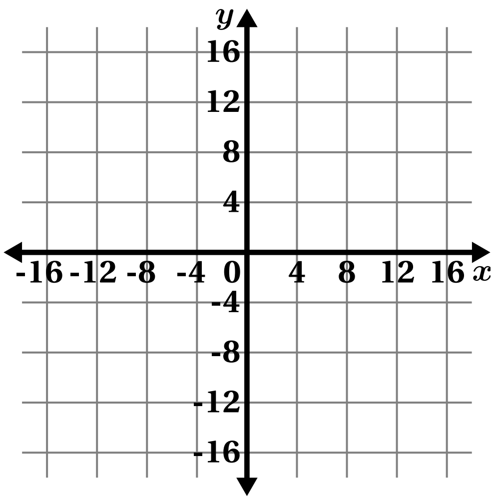
3. $A(-3, 2)$
$B(5, 5)$
$C(-4, 6)$



Check

Plot and label each point on the graph.

$A(6, -10)$
$B(-6, 4)$
$C(-6, -4)$



ML 7.17 Plotting Points on the Coordinate Plane

Goal

Plot points with positive and negative coordinates.

Standards

MA.6.GR.1.1

Materials

coloring tools (optional)

Modeled Review

Point to the problem in the Modeled Review and **ask**:

- "How are each of the quadrants different in the coordinate plane?"
- "Where are positive numbers represented on the coordinate plane? Negative numbers?"
- "What do you notice about the *x*- and *y*-axes? Why do you have to pay attention to the scale on a coordinate plane?"

Reinforce the goal by saying, "You can efficiently plot points on a coordinate plane by paying attention to the scale and labels on the *x*- and *y*-axes."

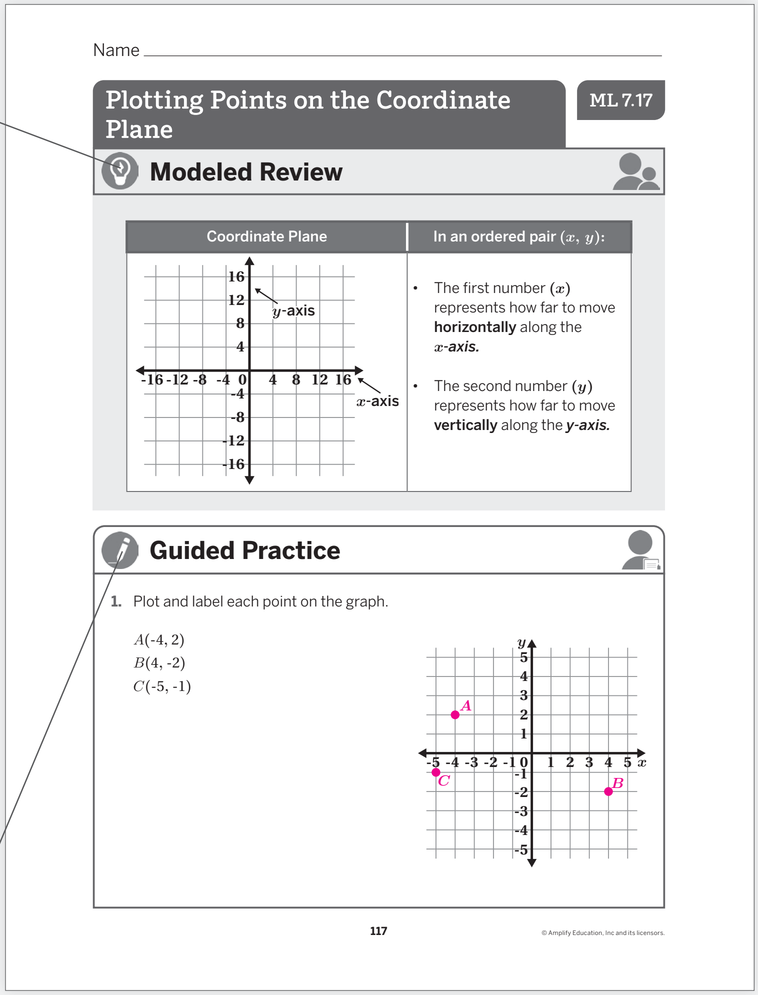
ML/EL Use a think aloud to model plotting a point on the coordinate plane.

Guided Practice

Focus students' attention on identifying the scale of the coordinate plane before plotting the points.

To scaffold their thinking, **ask**:

- "What is the scale of the coordinate plane?"
- "What axis do you move along when plotting the first point? Second?"
- "How can you verify that each point is plotted correctly?"



Name

Plotting Points on the Coordinate Plane

ML 7.17

Modeled Review

Coordinate Plane	In an ordered pair (x, y):
y-axis, *x*-axis	• The first number (x) represents how far to move **horizontally** along the *x-axis*. • The second number (y) represents how far to move **vertically** along the ***y*-axis**.

Guided Practice

1. Plot and label each point on the graph.

$A(-4, 2)$
$B(4, -2)$
$C(-5, -1)$

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Vocabulary

If needed, share the meaning of the terms with students.

origin: The point (0, 0) on the coordinate plane. This is where the *x*-axis and the *y*-axis intersect.

***x*-axis:** One of the perpendicular number lines that form the coordinate plane. The *x*-axis is the horizontal number line.

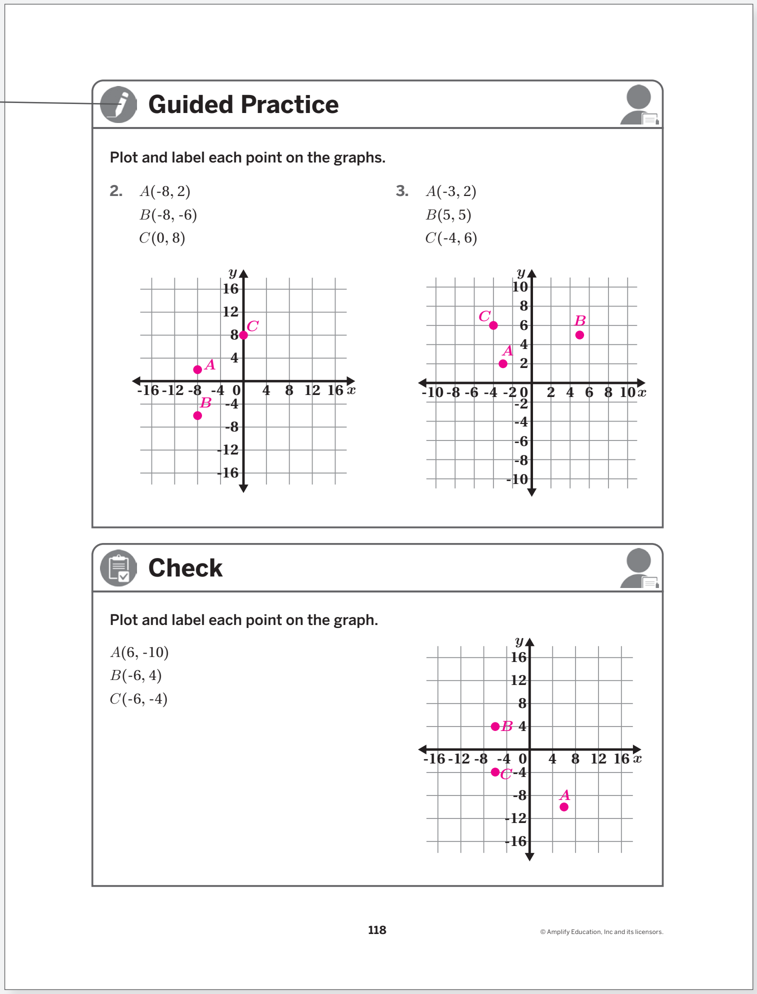
***y*-axis:** One of the perpendicular number lines that form the coordinate plane. The *y*-axis is the vertical number line.

Plotting Points on the Coordinate Plane

Guided Practice

A Model annotating the graph by marking values between intervals to help efficiently plot points on the coordinate plane when the scale is greater than 1.

Key Takeaway:
Say, "When graphing points on the coordinate plane, it can be helpful to look at the axis scale before plotting any points."



Guided Practice

Plot and label each point on the graphs.

2. $A(-8, 2)$
$B(-8, -6)$
$C(0, 8)$

3. $A(-3, 2)$
$B(5, 5)$
$C(-4, 6)$

Check

Plot and label each point on the graph.

$A(6, -10)$
$B(-6, 4)$
$C(-6, -4)$

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Reflection

Ask:

- "What advice would you give someone who is struggling with plotting points on the coordinate plane?"
- "What makes sense? What is still confusing?"

Check: Recommended Next Steps

Almost there	Got it!
If students need more support, consider having them revisit the problem in the Check. Then have them color-code the positive and negative sides of both axes.	If students need more practice, have them plot the following points on the graph in Problem 3: $D(1, -3)$ $E(-1, 3)$ $F(8, -3)$

Unit 8

Mini-Lessons

Name ______________________________

Writing and Solving One-Step Equations

ML 8.02

Modeled Review

Name: Lyn

On Wednesday, Evan spent $12 at the school bookstore, leaving him with a balance of $26 left in his account. How much money did Evan have in his account at the beginning of the day on Wednesday?

a. Write an equation that represents this situation.

$x + (-12) = 26$

b. Solve your equation.

$x + (-12) = 26$

$+12 \qquad +12$

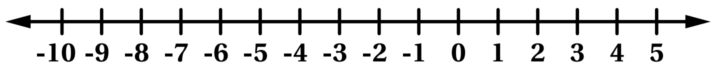
$x = 38$

Evan had $38 in his account at the beginning of the day on Wednesday.

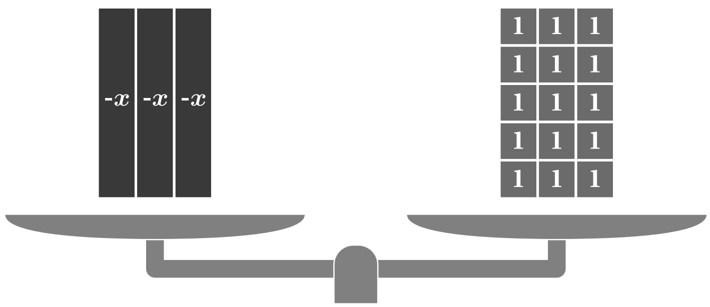
Guided Practice

Solve each equation for the value of *x*. Show or explain your thinking.

1. $x + 4 = -9$



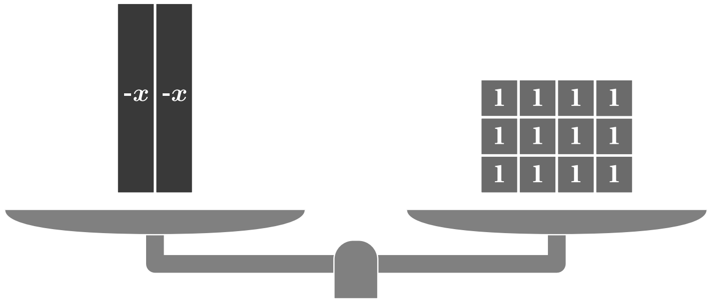
2. $-3x = 15$



Guided Practice

3. A balanced scale is given.

a. Write an equation that represents this situation.



b. Solve the equation. Show or explain your thinking.

4. Kylie was hiking at her favorite park. She started at an elevation 5 feet below sea level. She hiked to a height of 21 feet above sea level. What was Kylie's change in elevation?

a. Write an equation that represents this situation.

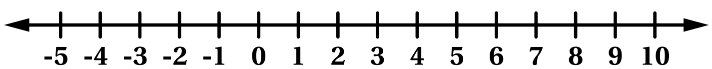
b. Solve the equation. Show your thinking.

Check

Solve each equation for *x*. Show your thinking.

1. $4x = -20$

2. $-3 = x + 8$



ML 8.02 Writing and Solving One-Step Equations

Goal

Write and solve one-step equations in one variable involving terms and solutions which are integers within a mathematical or real-world context.

Standards

MA.6.AR.2.2; MA.6.AR.2.3

Materials

Blank number lines, tape diagrams, and balanced scale models (optional)

Modeled Review

Point to Priya's work and **ask**:

- "What does the "x" represent in Lyn's equation?"
- "Why did Lyn add 12 to each side of the equation?"

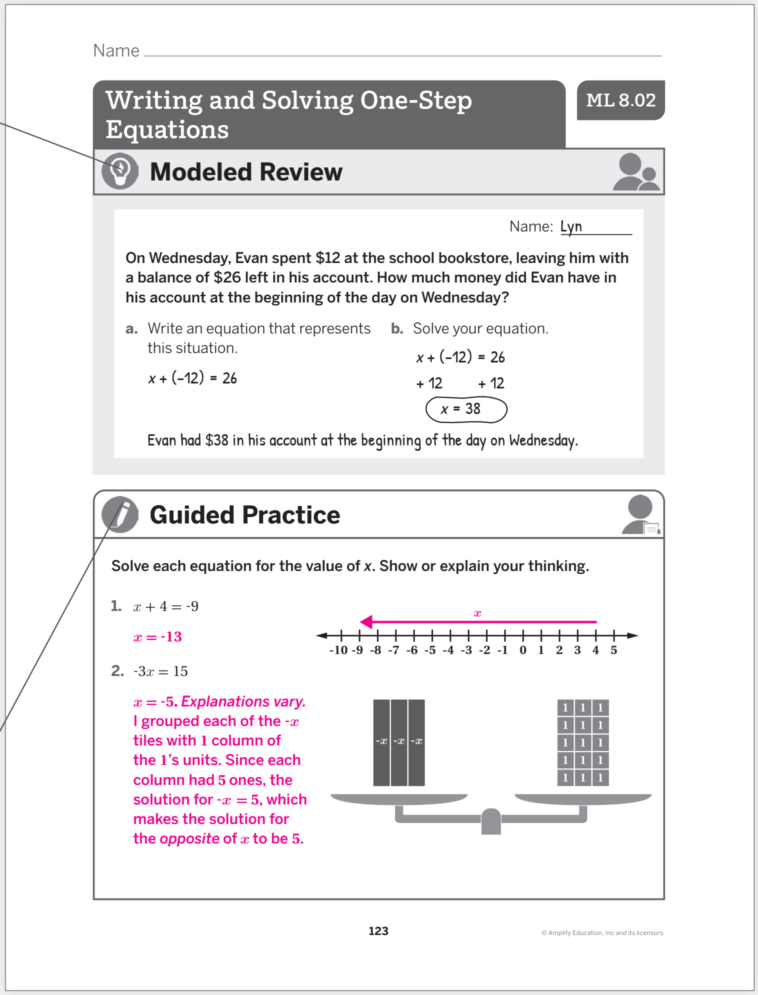
Reinforce the goal by saying "When working with equations that include negative numbers, use properties of equality and/or models, such as number lines, tape diagrams, or balanced scales, to solve the equation for the value of the variable."

ML/EL Model using a different strategy to solve this equation, such as using a number line, to check the solution.

Guided Practice

Focus students' attention on how different strategies can be used to solve an equation.

Encourage students to check their answers by substituting them into the equations or using a different strategy to solve the equation.


Name

Writing and Solving One-Step Equations ML 8.02

Modeled Review

Name: Lyn

On Wednesday, Evan spent $12 at the school bookstore, leaving him with a balance of $26 left in his account. How much money did Evan have in his account at the beginning of the day on Wednesday?

a. Write an equation that represents this situation.

$x + (-12) = 26$

b. Solve your equation.

$x + (-12) = 26$

$+12 \quad +12$

$x = 38$

Evan had $38 in his account at the beginning of the day on Wednesday.

Guided Practice

Solve each equation for the value of x. Show or explain your thinking.

1. $x + 4 = -9$

$x = -13$

2. $-3x = 15$

$x = -5$. *Explanations vary.* I grouped each of the $-x$ tiles with 1 column of the 1's units. Since each column had 5 ones, the solution for $-x = 5$, which makes the solution for the *opposite* of x to be 5.

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Vocabulary

If needed, share the meaning of the terms with students.

Equation: a statement of equality between two expressions, such *as* $-2 + x = 6$.

Integers: All positive and negative whole numbers and 0.

Writing and Solving One-Step Equations

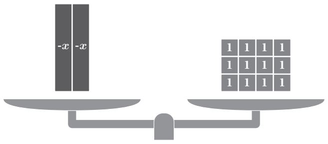
Guided Practice

A To support students in writing equations for real world contexts, provide a list of commonly used key terms, such as elevation, sea-level, deposit, withdrawal, etc., with a visual representation of these terms next to their definition.

Key Takeaway:
Say, "There are many strategies to solve one-step equations that include integers. Practice using more than one strategy to grow your fluency with solving equations for a variable."

Guided Practice

3. A balanced scale is given.



a. Write an equation that represents this situation.

$-2x = 12$

b. Solve the equation. Show or explain your thinking.

$x = -6$. *Explanations vary.* If it takes 12 positive ones to balance 2 negative x's, then it will take 6 positive ones to balance one $-x$, or 6 negative x's to balance one positive x.

4. Kylie was hiking at her favorite park. She started at an elevation 5 feet below sea level. She hiked to a height of 21 feet above sea level. What was Kylie's change in elevation?

a. Write an equation that represents this situation.

$-5 + x = 21$, or equivalent

b. Solve the equation. Show your thinking.

26 feet. *Methods vary.*

$-5 + x = 21$

$+5 \quad\quad +5$

$x = 26$

Check

Solve each equation for *x*. Show your thinking.

1. $4x = -20$

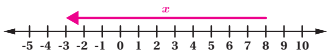
$x = -5$. *Methods vary.*

$\frac{4x}{4} = \frac{-20}{4}$

$x = -5$

2. $-3 = x + 8$

$x = -11$. *Methods vary.*



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Reflection

Ask:

- "What strategy did you find most useful when solving multiplication problems? Addition problems?"
- "How did you overcome a hard problem today?"

Check: Recommended Next Steps

Almost there	Got it!
If students need more support, consider using Mini-Lesson 7.08: *Adding and Subtracting Positive and Negative Numbers*.	If students need more practice, have them solve the following problem with a partner, each using a different strategy. Then have them compare their results and discuss. $-6 + x = -4$

Name ______________________________

Identifying Solutions to Inequalities

ML 8.04

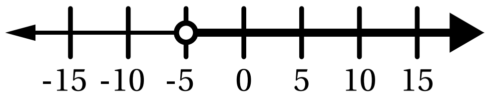
Modeled Review

Name: Emmanuel

Here is an inequality and its graph. Select *three* solutions to the inequality.

- ☐ -20
- ☐ -5
- ☑ 0
- ☑ 5
- ☑ 20

$x > -5$



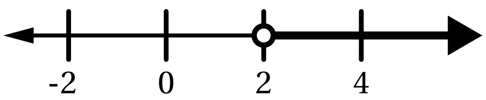
For a number to be a solution to this inequality, it must be greater than -5. The numbers 0, 5, and 20 are greater than -5.

Guided Practice

Here are inequalities and their graphs. Select *three* solutions to each inequality.

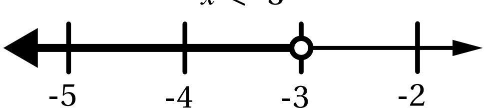
1.
- ☐ 0
- ☐ 2
- ☑ 4
- ☐ 6
- ☐ 8

$x > 2$



2.
- ☐ -6
- ☐ -5
- ☐ -4
- ☐ -3
- ☐ -2

$x < -3$

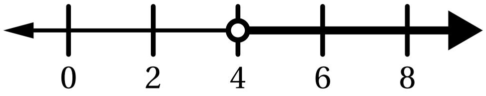


Guided Practice

Here are inequalities and their graphs. Select *three* solutions to each inequality.

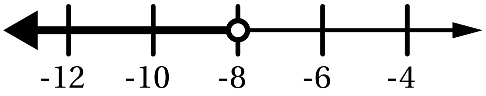
3.
- ☐ 2
- ☐ 4
- ☐ 6
- ☐ 8
- ☐ 10

$x > 4$



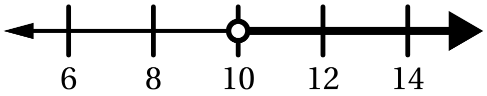
4.
- ☐ -14
- ☐ -12
- ☐ -10
- ☐ -8
- ☐ -6

$x < -8$



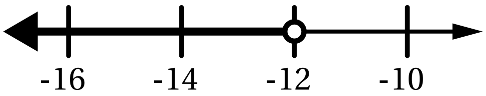
5.
- ☐ 8
- ☐ 10
- ☐ 12
- ☐ 14
- ☐ 16

$x > 10$



6.
- ☐ -18
- ☐ -16
- ☐ -14
- ☐ -12
- ☐ -10

$x < -12$

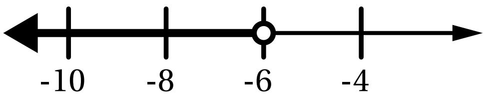


Check

Here is an inequality and its graph. Select *three* solutions to the inequality.

- ☐ -12
- ☐ -10
- ☐ -8
- ☐ -6
- ☐ -4

$x < -6$



Identifying Solutions to Inequalities

Goal

Identify solutions to inequalities by inspecting the inequalities and their graphs.

Standard

MA.6.AR.2.1

Modeled Review

Point to Emmanuel's work and **ask**:

- "How did Emmanuel determine if a specific number is a solution to the inequality $x > -5$?"
- "Why is -5 not included as a possible solution?"
- "How could Emmanuel check if his selections are correct?"

Reinforce the goal by saying, "Graphs can help identify the multiple solutions to an inequality."

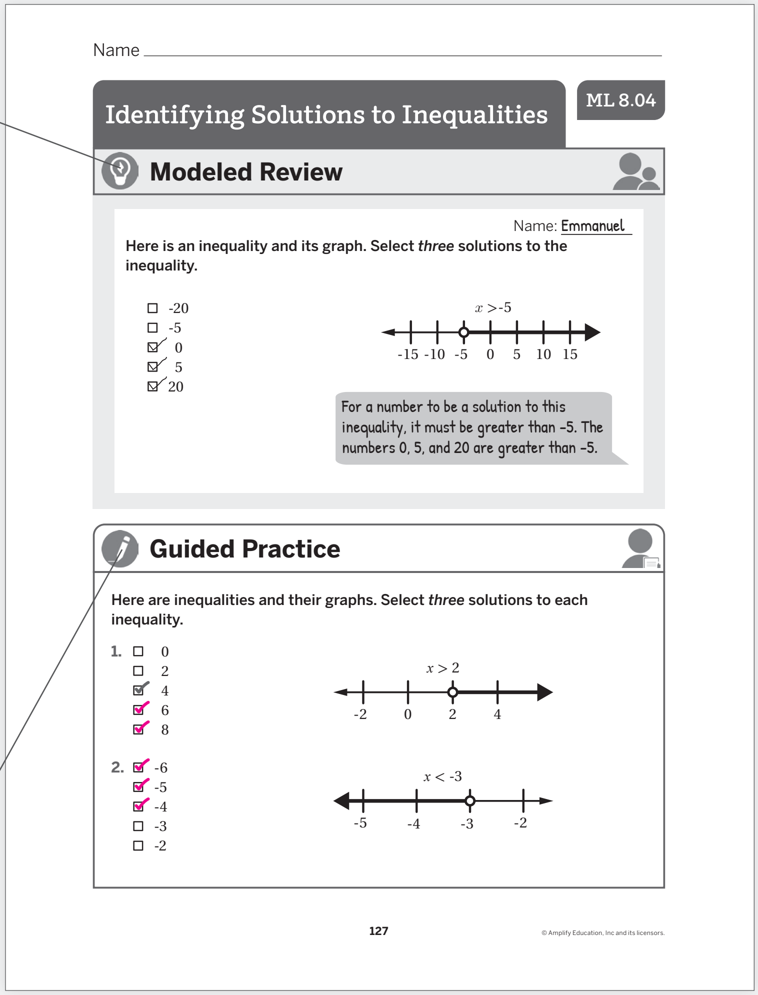
ML/EL Invite students to read each inequality aloud in their home language before identifying the solutions.

Guided Practice

Focus students' attention on using the graph to determine all the solutions for the inequality.

To scaffold their thinking, **say**:

- "First, read the inequality statement and look at the graph."
- "Next, see which side is shaded."
- "Then, select the numbers that belong to the shaded part of the number line."



Name

Identifying Solutions to Inequalities

ML 8.04

Modeled Review

Name: Emmanuel

Here is an inequality and its graph. Select *three* solutions to the inequality.

- ☐ -20
- ☐ -5
- ☑ 0
- ☑ 5
- ☑ 20

$x > -5$

-15 -10 -5 0 5 10 15

For a number to be a solution to this inequality, it must be greater than -5. The numbers 0, 5, and 20 are greater than -5.

Guided Practice

Here are inequalities and their graphs. Select *three* solutions to each inequality.

1. ☐ 0, ☐ 2, ☑ 4, ☑ 6, ☑ 8

$x > 2$

-2 0 2 4

2. ☑ -6, ☑ -5, ☑ -4, ☐ -3, ☐ -2

$x < -3$

-5 -4 -3 -2

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Vocabulary

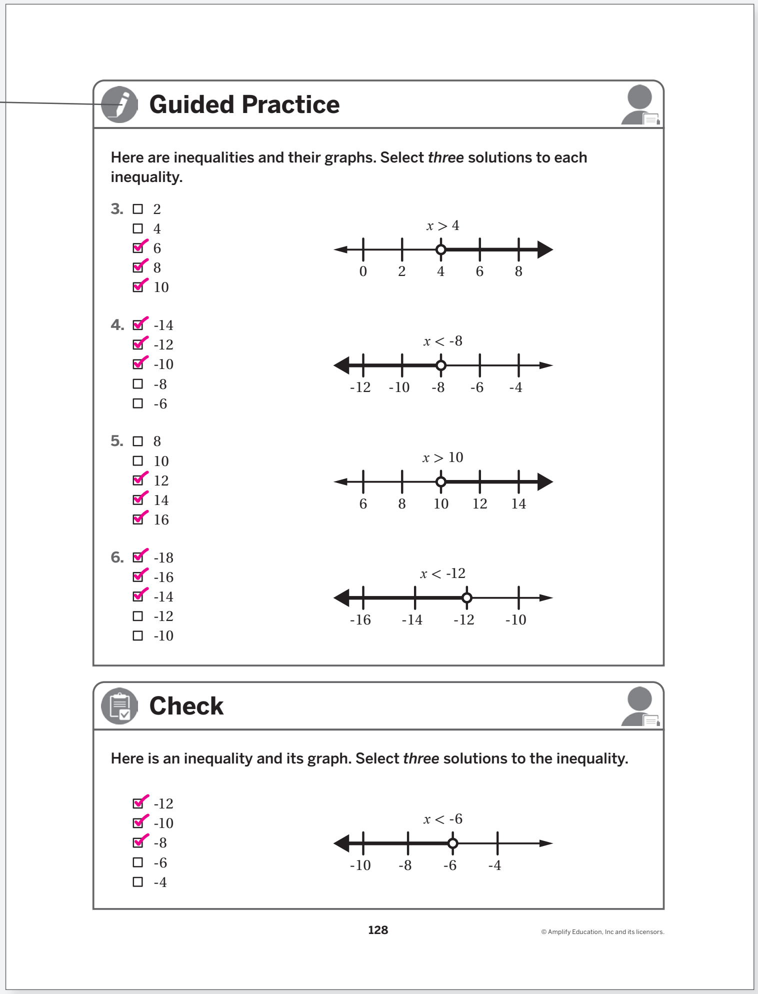
If needed, share the meaning of the term with students.

solution to an inequality: Any value of a variable that makes the inequality true.

Guided Practice

A Invite students to focus on one value at a time and answer the question, "Is this value greater than or less than ___?"

Key Takeaway:
Say, "A number line can be used to show inequality solutions. A value is a solution if it makes the inequality true."



Guided Practice

Here are inequalities and their graphs. Select *three* solutions to each inequality.

3. ☐ 2 ☐ 4 ☑ 6 ☑ 8 ☑ 10 — $x > 4$ (number line: 0, 2, 4, 6, 8)

4. ☑ -14 ☑ -12 ☑ -10 ☐ -8 ☐ -6 — $x < -8$ (number line: -12, -10, -8, -6, -4)

5. ☐ 8 ☐ 10 ☑ 12 ☑ 14 ☑ 16 — $x > 10$ (number line: 6, 8, 10, 12, 14)

6. ☑ -18 ☑ -16 ☑ -14 ☐ -12 ☐ -10 — $x < -12$ (number line: -16, -14, -12, -10)

Check

Here is an inequality and its graph. Select *three* solutions to the inequality.

☑ -12 ☑ -10 ☑ -8 ☐ -6 ☐ -4 — $x < -6$ (number line: -10, -8, -6, -4)

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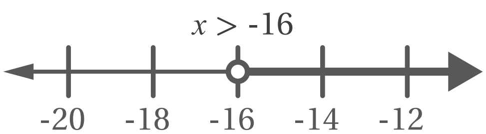
Reflection

Ask:
- "What does it mean for a number to be a solution to an inequality?"
- "What strategy did someone else share today that was helpful?"

Check: Recommended Next Steps

Almost there
If students need more support, consider having them revisit the problem in the Check. Then model how to determine if the solutions are true by substituting each value into the inequality.

Got it!
If students need more practice, present them with the following inequality and graph. Then have them list *two* possible solutions to the inequality.



Name ______________________________

Solving Inequalities

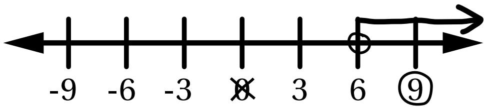
ML 8.07

Modeled Review

Name: Priya

Solve and graph the solution to $x - 1 > 5$.

x > 6



x - 1 = 5
+1 +1
x = 6

x < 6	x > 6
0 - 1 > 5	(9) - 1 > 5
-1 > 5	8 > 5
False	True

Guided Practice

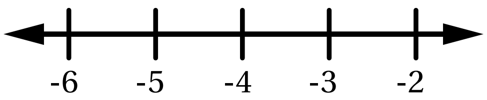
1. Graph the solution to the inequality $3x \geq 9$ by finding the boundary point and testing values on both sides of the boundary point.

Moves	Work
Move 1: Find the boundary point.	$3x$ = 9 x = ____
Move 2: Test the values on both sides of the point and determine whether the statement is true or false.	**Less than or equal to** $3(\_\_\_\_) \geq 9$ $\_\_\_\_ \geq 9$ ________ **Greater than or equal to** $3(\_\_\_\_) \geq 9$ $\_\_\_\_ \geq 9$ ________
Move 3: Graph the solution.	x ____ 3 

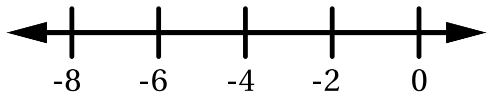
Guided Practice

Solve and graph the solution to each of the inequalities.

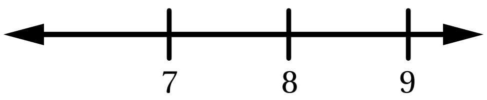
2. $2x \leq -8$ __________



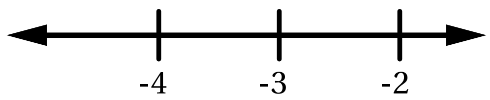
3. $x - 1 > -7$ __________



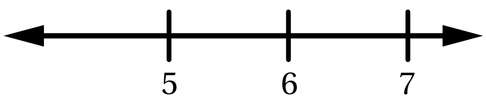
4. $x + 11 \geq 18$ __________



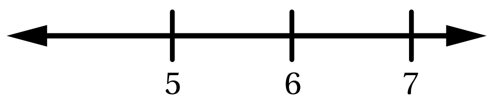
5. $-6x > 18$ __________



6. $x - 12 \geq -7$ __________



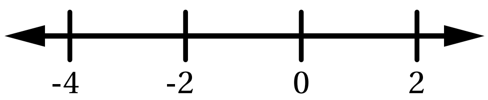
7. $-5x > -30$ __________



Check

Solve and graph the solution to $-4x < 8$**.**

____



Goal	Standard
Solve linear equations that involve positive and negative numbers.	MA.7.AR.2.1

Modeled Review

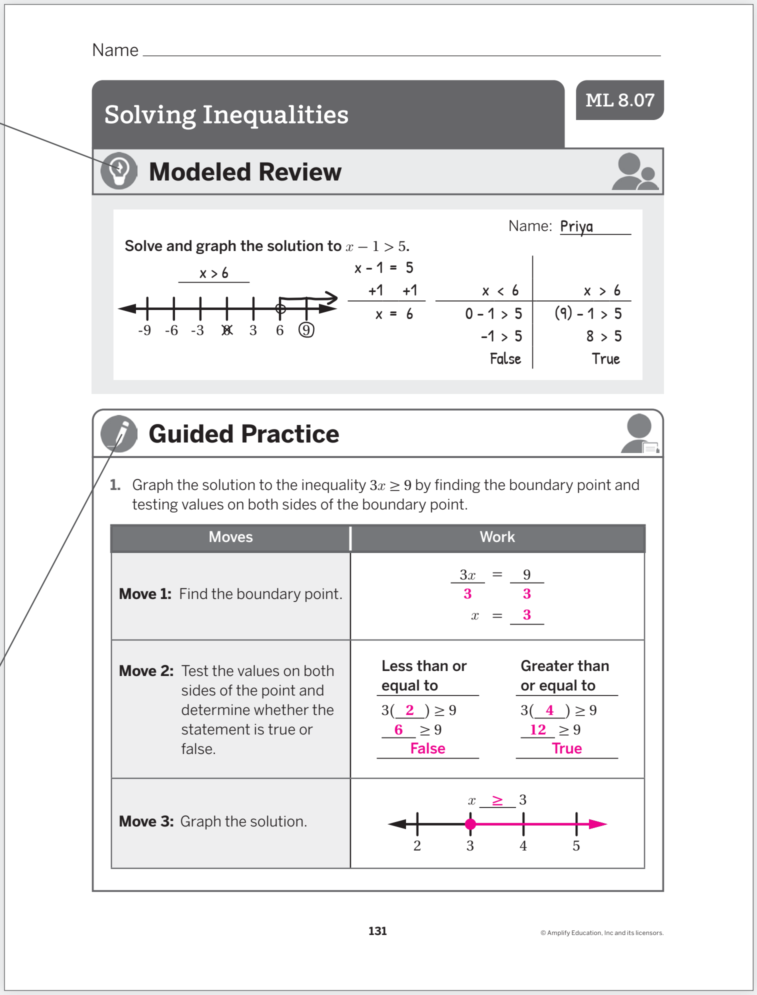
Point to Avery's work and **ask**:
- "How did Avery determine the boundary point?"
- "Why did Avery substitute 0 and 9 into the inequality?"
- "How does testing numbers help to determine which inequality symbol to use?"

Reinforce Avery's thinking by saying, "After finding the boundary point, test values to determine which inequality symbol makes the solution statement true."

Guided Practice

Focus students' attention on finding the boundary point and testing values on both sides of the point.

To scaffold their thinking, **say**:
- "Determine the boundary point."
- "Choose numbers to the left and right of the boundary point and substitute them into the inequality."
- "Decide which number satisfies the inequality. Based on this, determine the correct inequality sign to use."



Name

Solving Inequalities

ML 8.07

Modeled Review

Name: Priya

Solve and graph the solution to $x - 1 > 5$.

x > 6

x - 1 = 5
+1 +1
x = 6

-9 -6 -3 0 3 6 9

x < 6	x > 6
0 - 1 > 5	(9) - 1 > 5
-1 > 5	8 > 5
False	True

Guided Practice

1. Graph the solution to the inequality $3x \geq 9$ by finding the boundary point and testing values on both sides of the boundary point.

Moves	Work
Move 1: Find the boundary point.	$\frac{3x}{3} = \frac{9}{3}$ $x = 3$
Move 2: Test the values on both sides of the point and determine whether the statement is true or false.	Less than or equal to: $3(2) \geq 9$, $6 \geq 9$, False Greater than or equal to: $3(4) \geq 9$, $12 \geq 9$, True
Move 3: Graph the solution.	$x \geq 3$ 2 3 4 5

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Vocabulary

If needed, share the meaning of the terms with students.

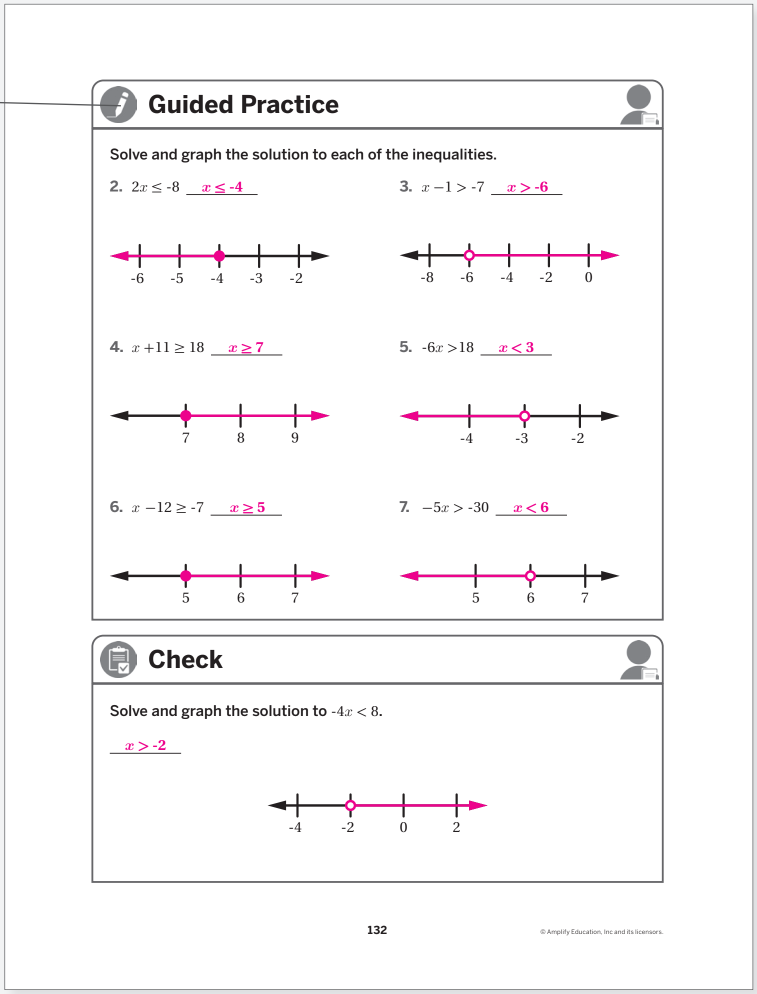
inequality: A comparison statement that uses the symbols < or >. Inequalities are used to represent the relationship between numbers, variables, or expressions that are not always equal.

solution to an inequality: All of the values of a variable that make that inequality true.

Guided Practice

(A) Emphasize that inequality signs that include 'equal to' should be represented with a shaded circle on the number line.

Key Takeaway:
Say, "Solving inequalities is similar to solving equations. The solutions to an inequality can be determined by first solving an equation, then testing whether the values on the left or right side of the boundary point make the inequality true."



Guided Practice

Solve and graph the solution to each of the inequalities.

2. $2x \le -8$ $x \le -4$
-6 -5 -4 -3 -2

3. $x - 1 > -7$ $x > -6$
-8 -6 -4 -2 0

4. $x + 11 \ge 18$ $x \ge 7$
7 8 9

5. $-6x > 18$ $x < 3$
-4 -3 -2

6. $x - 12 \ge -7$ $x \ge 5$
5 6 7

7. $-5x > -30$ $x < 6$
5 6 7

Check

Solve and graph the solution to $-4x < 8$.

$x > -2$
-4 -2 0 2

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Reflection

Ask:

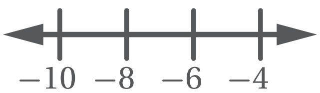
- "What steps can you take to graph the solutions to an inequality?"
- "How are solutions to an inequality different from an equation?"

Check: Recommended Next Steps

Almost there
If students need more support, consider having them revisit the problem in the Check. Then model graphing the solution by marking off the value on the number line that does not make statement true and circling the one that does. Invite them to use that information to determine how they should draw the arrow.

Got it!
If students need more practice, present students with the following problem and ask them to graph the solution:

$-7x \le 42$



Unit 9

Mini-Lessons

Name ______________________________

Creating Dot Plots

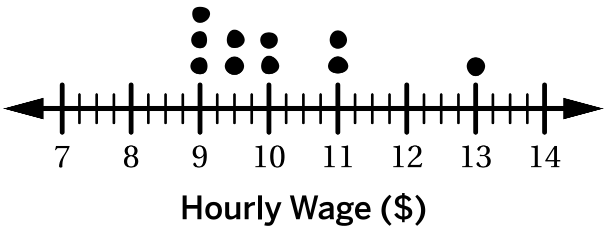
ML 9.03

Modeled Review

Name: Clare

10 fast food companies in Nebraska reported their hourly wages for new employees. Complete the line to display this data.

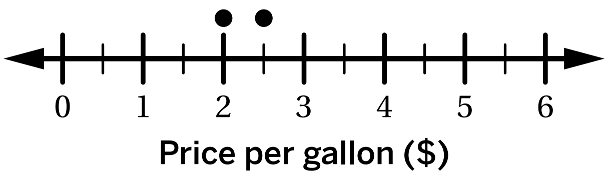
Hourly Wage				
$11.00	$9.00	$9.00	$10.00	$9.50
$10.00	$9.00	$11.00	$9.50	$13.00



Guided Practice

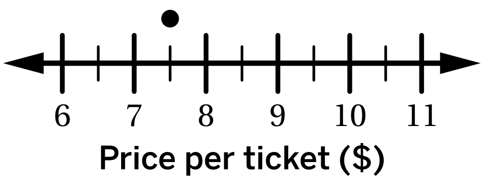
1. Here are the prices per gallon of gasoline at 6 gas stations in Georgia. Complete the line plot that shows this data.

Price per gallon		
$2.00	$3.00	$2.50
$2.00	$3.50	$4.00



2. Here are the ticket prices for movies at 6 theaters in Texas. Complete the dot plot to display this data.

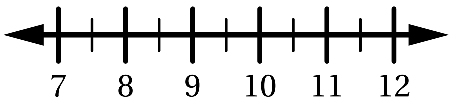
Price per ticket		
$10.00	$8.00	$7.50
$8.50	$7.00	$8.00



Guided Practice

3. 8 smoothie shops in Arizona reported the price of their large smoothie. Complete the dot plot to display this data.

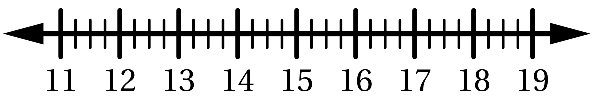
Price per smoothie			
$10.00	$9.50	$9.00	$10.50
$11.00	$10.00	$9.50	$11.00



Price per smoothie ($)

4. 10 parking lots in Chicago reported their daily parking fees. Complete the dot plot to display this data.

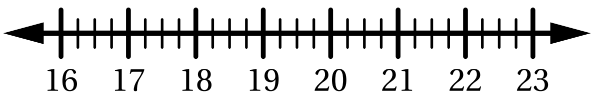
Daily parking fee				
$12.50	$14.00	$18.00	$18.00	$12.00
$18.00	$15.00	$12.00	$15.50	$15.00



Daily parking fee ($)

5. 10 deli shops in New York reported the price of a large sandwich. Complete the dot plot to display this data.

Price per sandwich				
$18.00	$21.00	$21.00	$23.00	$19.50
$20.50	$19.00	$20.00	$22.00	$20.00

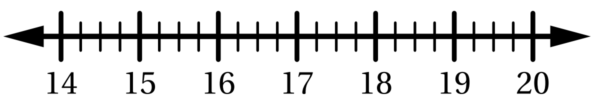


Price per sandwich ($)

Check

10 bookstores in Illinois reported the price of their best-selling book. Complete the dot plot to display this data.

Price per book				
$15.00	$20.00	$16.00	$15.00	$16.00
$18.00	$18.50	$15.00	$17.00	$18.50



Price per book ($)

ML 9.03 Creating Dot Plots

Goal	Standard
Create a dot plot to visualize a data set.	MA.6.DP.1.4

Modeled Review

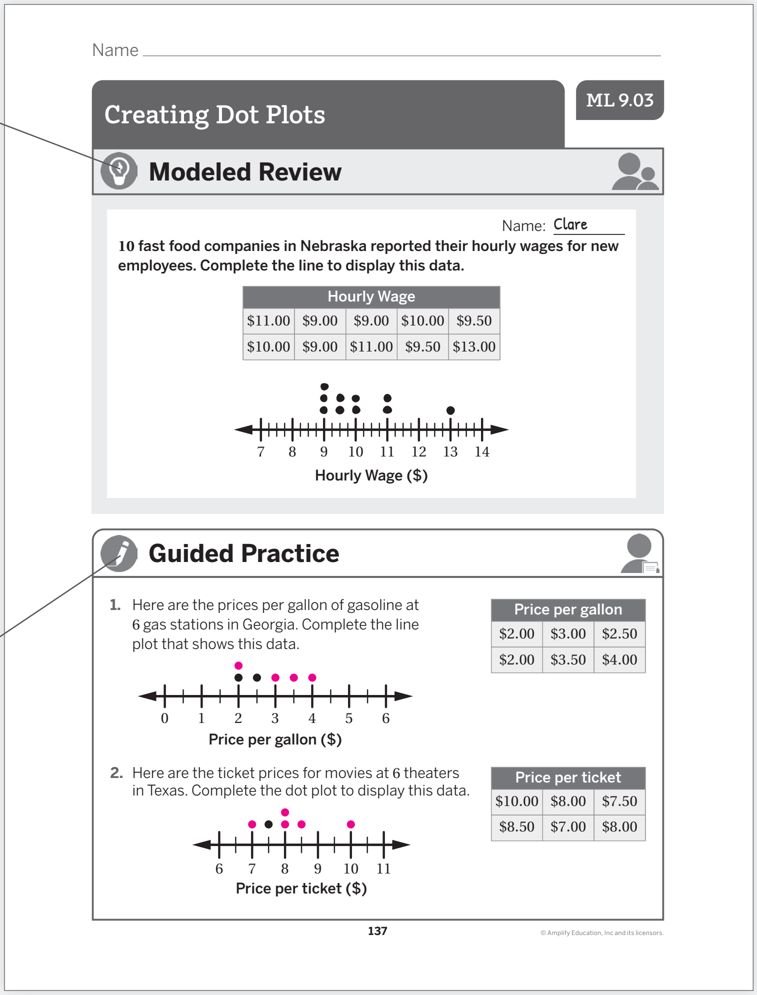
Point to Clare's work and **ask**:
- "How did Clare know to place three dots above the number 9?"
- "Why is it important to stack the dots vertically on a dot plot?"
- "How can Clare check that her dot plot matches the data table?"

Reinforce the goal by saying, "A data set can be visualized by creating a dot plot."

Guided Practice

Focus students' attention on creating a dot plot that represents the data set provided.

To scaffold their thinking, **say**:
- "First, place a dot above the corresponding number on the *x*-axis for each data point."
- "Then, if a value repeats, stack the dots vertically above that number."
- "Last, check your line plot to make sure all values are plotted correctly and the number of dots matches your data."



Name

Creating Dot Plots

ML 9.03

Modeled Review

Name: Clare

10 fast food companies in Nebraska reported their hourly wages for new employees. Complete the line to display this data.

Hourly Wage				
$11.00	$9.00	$9.00	$10.00	$9.50
$10.00	$9.00	$11.00	$9.50	$13.00

Guided Practice

1. Here are the prices per gallon of gasoline at 6 gas stations in Georgia. Complete the line plot that shows this data.

Price per gallon		
$2.00	$3.00	$2.50
$2.00	$3.50	$4.00

2. Here are the ticket prices for movies at 6 theaters in Texas. Complete the dot plot to display this data.

Price per ticket		
$10.00	$8.00	$7.50
$8.50	$7.00	$8.00

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Vocabulary

If needed, share the meaning of the term with students.

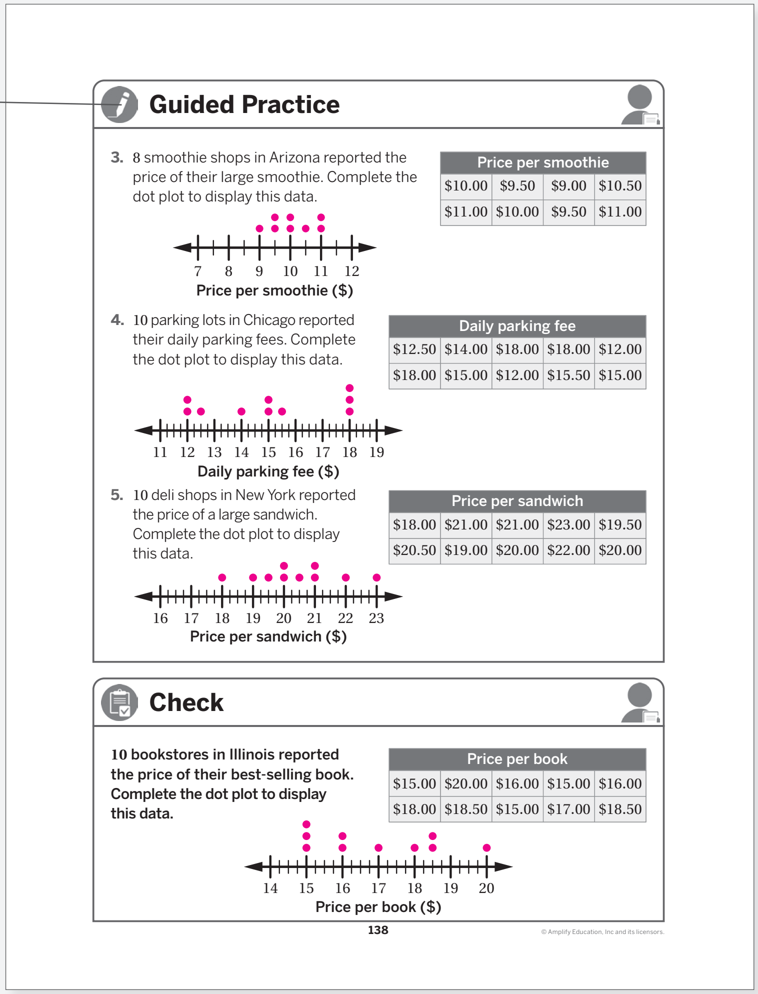
dot plot: A way to visualize numerical data sets, where each data point is represented by a dot on a number line. Data points with the same value are stacked on top of each other.

Guided Practice

A Model crossing off values in the table so that students don't miss or double count any data.

ML/EL Use a think aloud as you model plotting the data points on the dot plot.

Key Takeaway:
Say, "Dot plots can be used to display and visualize numerical data."



Guided Practice

3. 8 smoothie shops in Arizona reported the price of their large smoothie. Complete the dot plot to display this data.

Price per smoothie			
$10.00	$9.50	$9.00	$10.50
$11.00	$10.00	$9.50	$11.00

4. 10 parking lots in Chicago reported their daily parking fees. Complete the dot plot to display this data.

Daily parking fee				
$12.50	$14.00	$18.00	$18.00	$12.00
$18.00	$15.00	$12.00	$15.50	$15.00

5. 10 deli shops in New York reported the price of a large sandwich. Complete the dot plot to display this data.

Price per sandwich				
$18.00	$21.00	$21.00	$23.00	$19.50
$20.50	$19.00	$20.00	$22.00	$20.00

Check

10 bookstores in Illinois reported the price of their best-selling book. Complete the dot plot to display this data.

Price per book				
$15.00	$20.00	$16.00	$15.00	$16.00
$18.00	$18.50	$15.00	$17.00	$18.50

Reflection

Ask:
- "What do you think is important to remember when creating a dot plot?"
- "How does what you learned today connect to your prior learning?"

Check: Recommended Next Steps

Almost there
If students need more support, consider modeling how to label fractional values on the axis to help them with plotting points.

Got it!
If students need more practice, ask them to add the following sandwich prices to the line plot in Problem 5.

Price per sandwich				
$22.50	$21.00	$21.50	$16.50	$17.00

Name ______________________________

Calculating Measures of Center and Spread

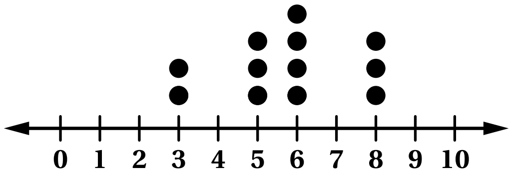
ML 9.10

Modeled Review

Name: Tristan

The line plot shows the number of text messages Deven sent every day for 12 days. Calculate the mean, median, and range of this data.

Number of text messages Deven sent



To find the mean, I divide the total number of texts (69) by the number of days (12).
To find the median, I counted in from each side to find the number in the middle of the data.
To find the range, I subtract the smallest value (3) from the largest value (8).

Mean = $\frac{2(3) + 5(3) + 6(4) + 8(3)}{12} = \frac{69}{12} = 5.75$

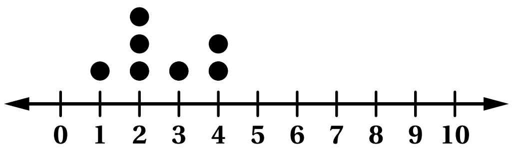
Range = 8-3=5 texts

Median = The middle number of 12 numbers is halfway between the 6th and 7th number, which is 6 texts.

Guided Practice

1. The line plot shows the number of books Jin read each month for 7 months. Calculate the mean, median, and range of this data.

Number of books Jin read



Mean = $\frac{1(\ \ )+2(\ \ )+3(\ \ )+4(\ \ )}{7} = \frac{\square}{7} \approx$ _____

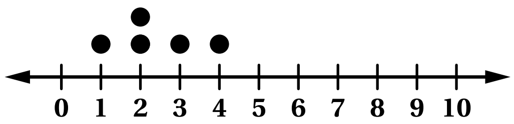
Median = The middle value is _____.

Range = _____ − _____ = _____

Guided Practice

2. The line plot shows the number of homework assignments Neel completed each day for 5 days. Calculate the mean, median, and range of this data.

Number of assignments Neel completed



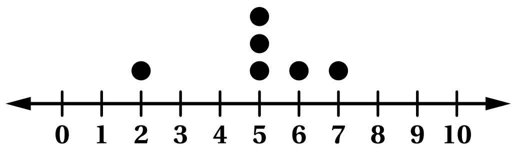
Mean = ____________________________

Median = _____

Range = _____

3. The line plot shows the number of sales Peter made each day for 6 days. Calculate the mean, median, and range of this data.

Number of sales Peter made



Mean = ____________________________

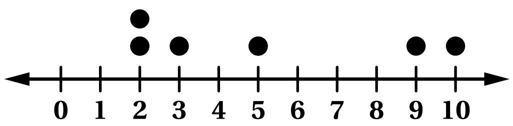
Median = _____

Range = _____

Check

The line plot shows the number of birds Tyler saw each day for 6 days. Calculate the mean, median, and range of this data.

Number of birds Tyler saw



Mean = ________________________________

Median = _____

Range = _____

ML 9.10 Calculating Measures of Center and Spread

Goal

Calculate the mean, median, and range of a data set.

Standard

MA.7.DP.1.1

Materials

calculator (optional)

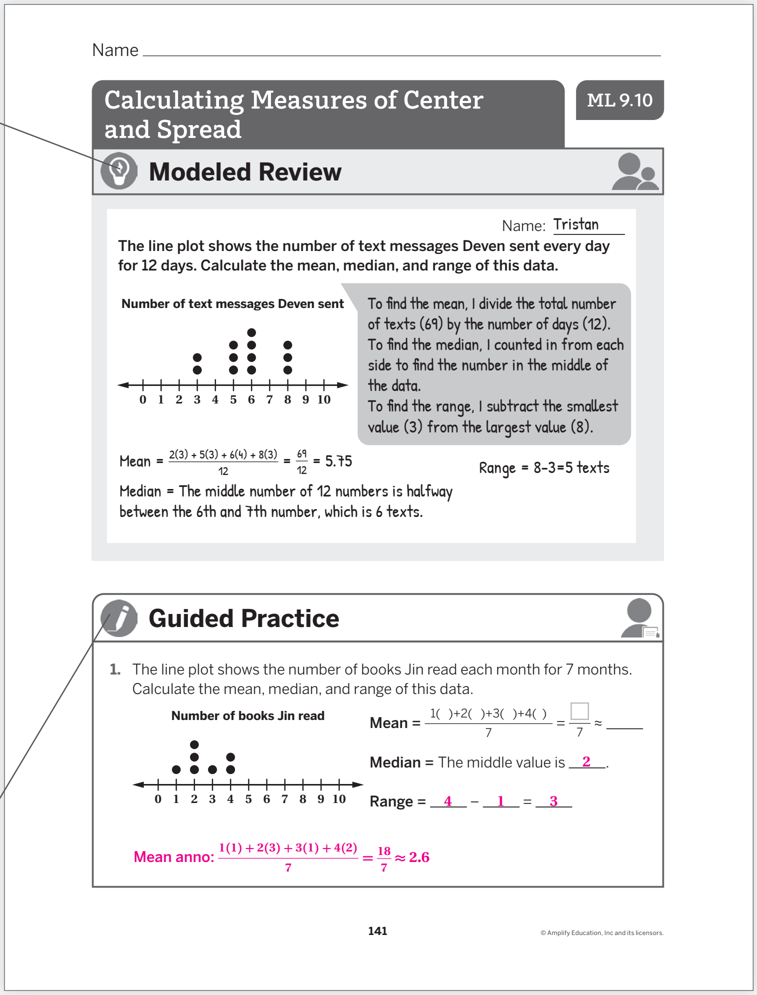
Modeled Review

Point to Tristan's work and **ask**:

- "Where did Tristan find the numbers he added?"
- "How did Tristan determine the median of the data?"
- "What does the range of 5 represent?"

Reinforce Tristan's thinking by saying, "The mean can be calculated by adding all of the number of text messages together and dividing by the total number of text messages."

ML/EL Discuss the vocabulary mean, median, range.


Name

Calculating Measures of Center and Spread — ML 9.10

Modeled Review

Name: Tristan

The line plot shows the number of text messages Deven sent every day for 12 days. Calculate the mean, median, and range of this data.

Number of text messages Deven sent

To find the mean, I divide the total number of texts (69) by the number of days (12).
To find the median, I counted in from each side to find the number in the middle of the data.
To find the range, I subtract the smallest value (3) from the largest value (8).

Mean = $\frac{2(3) + 5(3) + 6(4) + 8(3)}{12} = \frac{69}{12} = 5.75$

Range = 8−3=5 texts

Median = The middle number of 12 numbers is halfway between the 6th and 7th number, which is 6 texts.

Guided Practice

1. The line plot shows the number of books Jin read each month for 7 months. Calculate the mean, median, and range of this data.

Number of books Jin read

Mean = $\frac{1(\ )+2(\ )+3(\ )+4(\ )}{7} = \frac{\square}{7} \approx$ ____

Median = The middle value is __2__.

Range = __4__ − __1__ = __3__

Mean anno: $\frac{1(1)+2(3)+3(1)+4(2)}{7} = \frac{18}{7} \approx 2.6$

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Guided Practice

Focus students' attention on calculating the mean for the data on the line plot.

To scaffold their thinking, **say**:

- "First, count the number of books over each dot on the line plot and multiply them by the value of the dot."
- "Then, divide this total number of books read by the total number of months, 7."

Vocabulary

If needed, share the meaning of the terms with students.

mean: The arithmetic average of a set of numbers found by dividing the sum of all values by the number of values. It is a measure of central tendency.

median: The middle of an ordered list of values. It is a measure of central tendency.

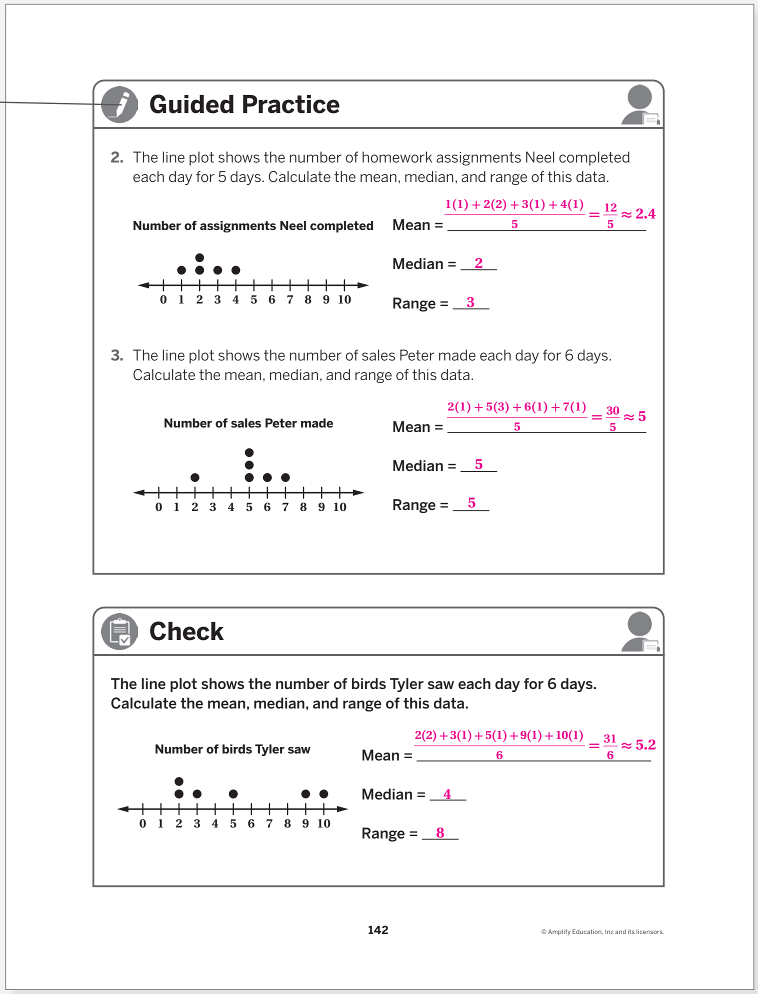
range: The difference between the highest value and lowest value of a data set.

Calculating Measures of Center and Spread

Guided Practice

A Consider providing students with calculators to efficiently calculate the mean.

Key Takeaway:
Say, "The mean is the value found by finding the sum of all of the data values and dividing it by the total number of data points."



Guided Practice

2. The line plot shows the number of homework assignments Neel completed each day for 5 days. Calculate the mean, median, and range of this data.

Number of assignments Neel completed

0 1 2 3 4 5 6 7 8 9 10

Mean = $\frac{1(1)+2(2)+3(1)+4(1)}{5} = \frac{12}{5} \approx 2.4$

Median = 2

Range = 3

3. The line plot shows the number of sales Peter made each day for 6 days. Calculate the mean, median, and range of this data.

Number of sales Peter made

0 1 2 3 4 5 6 7 8 9 10

Mean = $\frac{2(1)+5(3)+6(1)+7(1)}{5} = \frac{30}{5} \approx 5$

Median = 5

Range = 5

Check

The line plot shows the number of birds Tyler saw each day for 6 days. Calculate the mean, median, and range of this data.

Number of birds Tyler saw

0 1 2 3 4 5 6 7 8 9 10

Mean = $\frac{2(2)+3(1)+5(1)+9(1)+10(1)}{6} = \frac{31}{6} \approx 5.2$

Median = 4

Range = 8

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Reflection

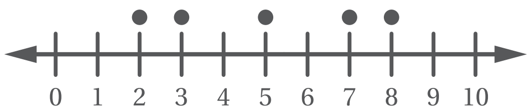
Ask:
- "What step did you find most challenging when calculating the mean, median, and range?"
- "Reflect on your learning today. What were you most proud of?"

Check: Recommended Next Steps

Almost there
If students need more support, consider revisiting the problem in the Check and using a think-aloud to finding the mean from the dot plot.

Got it!
If students need more practice, sketch the dot plot and ask them to calculate the mean.

Number of miles employees travel to work



Name ______________________________

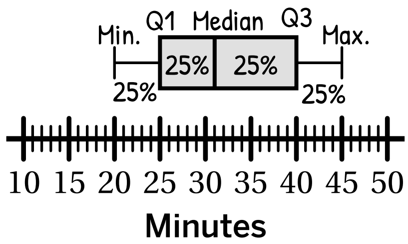
Interpreting Box Plots

ML 9.12

Modeled Review

Name: Santiago

Inola took the bus to school most days in January. She wrote down how many minutes her journey took each day and made this box plot.



1. Determine the median, IQR, and range for this data.

median: 30　　**IQR:** 15　　**range:** 25

40 – 25 = 15　　45 – 20 = 25

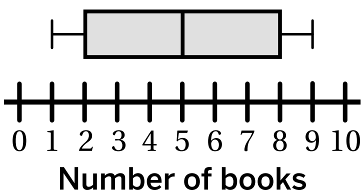
2. What percent of Inola's journey to school took 40 minutes or less?

A. 25%　　**B.** 50%　　**C.** 75% (circled)　　**D.** 100%

Guided Practice

For Problems 1–2, refer to the box plots to identify the statistics of the data set.

1. Mia measured the number of books she read each day for a week in June.

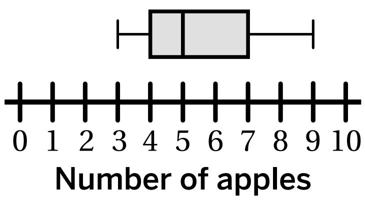


Min.	Q1	Median	Q3	Max.
1	2			

What percent of the data was more than **5** books?

A. 25%　　**B.** 50%
C. 75%　　**D.** 100%

2. Zoe tracked the number of apples she ate each day for a week in January.



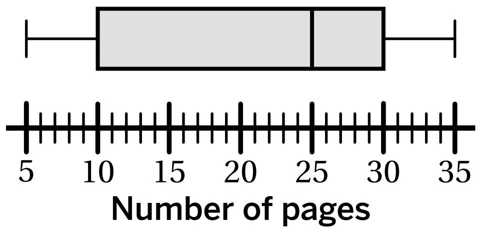
Min.	Q1	Median	Q3	Max.

What percent of the data was between **4** and **5** apples?

A. 25%　　**B.** 50%
C. 75%　　**D.** 100%

Guided Practice

Eliza tracked the number of pages she read each day for a week. She wrote down the amounts and created this box plot.



3. Determine the median, IQR, and range for this data.

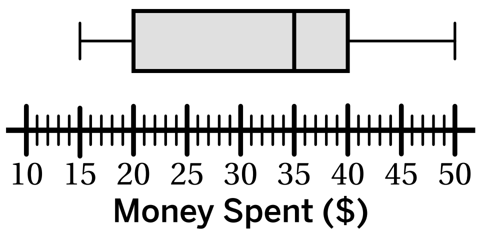
median: ______ **IQR:** ______ **range:** ______

4. On what percent of days did Eliza read less than 25 pages?

A. 25% **B.** 50% **C.** 75% **D.** 100%

Check

Evan tracked the amount of money he spent on clothes in January. He wrote down the amounts and created this box plot.



1. Determine the median, IQR, and range for this data.

median: ______ **IQR:** ______ **range:** ______

2. What percent of Evan's clothes cost at least $15?

A. 25% **B.** 50% **C.** 75% **D.** 100%

ML 9.12 Interpreting Box Plots

Goal

Determine and interpret the median, interquartile range (IQR), and range of a given data set in a box plot.

Standard

MA.6.DP.1.3

Materials

colored pencils (optional), highlighter (optional)

Modeled Review

Point to Santiago's work and **ask**:

- "How did Santiago determine the median? IQR?"
- "How did Santiago identify the minimum and maximum on the box plot to calculate the range?"
- "How did Santiago know that 75% of Inola's journeys took 40 minutes or less?"

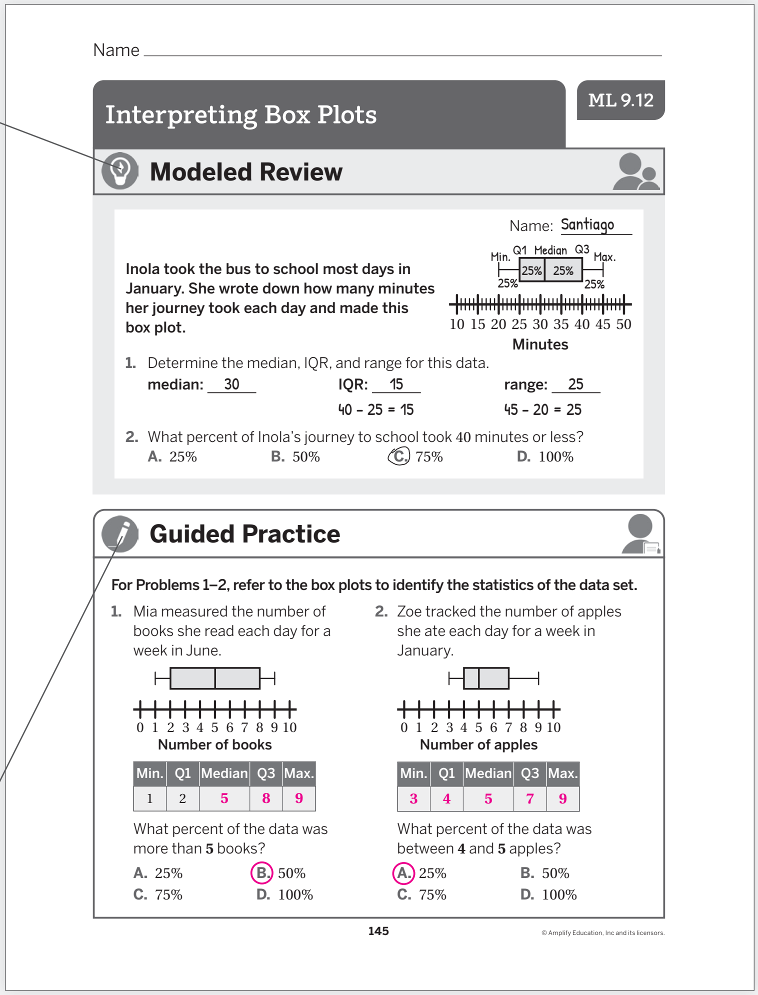
Reinforce Santiago's thinking by saying, "The median, IQR, and range can be determined by identifying and measuring the distances between key points on a box plot, with each section representing 25% of the data."

Guided Practice

Focus students' attention on identifying key points on each box plot to find the statistics for each data set.

To scaffold their thinking, **say**:

- "Identify the median of the box."
- "To calculate the IQR, determine the distance from Q1 to Q3."
- "To calculate the range, determine the distance from minimum to the maximum."
- "Each section of the box plot represents 25%."



Name

Interpreting Box Plots — ML 9.12

Modeled Review

Name: Santiago

Inola took the bus to school most days in January. She wrote down how many minutes her journey took each day and made this box plot.

1. Determine the median, IQR, and range for this data.
   median: 30 IQR: 15 range: 25
   40 − 25 = 15 45 − 20 = 25
2. What percent of Inola's journey to school took **40** minutes or less?
   A. 25% B. 50% (C.) 75% D. 100%

Guided Practice

For Problems 1–2, refer to the box plots to identify the statistics of the data set.

1. Mia measured the number of books she read each day for a week in June.

Min.	Q1	Median	Q3	Max.
1	2	5	8	9

What percent of the data was more than **5** books?
A. 25% (B.) 50% C. 75% D. 100%

2. Zoe tracked the number of apples she ate each day for a week in January.

Min.	Q1	Median	Q3	Max.
3	4	5	7	9

What percent of the data was between **4** and **5** apples?
(A.) 25% B. 50% C. 75% D. 100%

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Vocabulary

If needed, share the meaning of the terms with students.

box plot: A way to visualize quantitative data. The data is divided into four sections using five values: the minimum, Q1, Q2 (the median), Q3, and the maximum. A box is drawn between Q1 and Q3, and the line inside the box represents the median.

range: A measure of spread. It is the difference between the maximum and minimum values in a data set.

ML 9.12 Interpreting Box Plots

Guided Practice

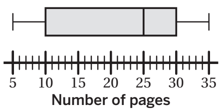
(A) Invite students to use colored pencils or arrows to annotate the box plot and make connections between the median, IQR, and range.

(ML/EL) Consider providing key vocabulary in students' home language to scaffold their understanding of box plots.

Key Takeaway:
Say, "Identifying key points on a box plot and measuring the distances between them can help you determine the median, interquartile range (IQR), and range."

Guided Practice

Eliza tracked the number of pages she read each day for a week. She wrote down the amounts and created this box plot.



3. Determine the median, IQR, and range for this data.

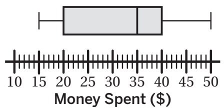
median: 25 IQR: 20 range: 30

4. On what percent of days did Eliza read less than 25 pages?

A. 25% **B.** 50% C. 75% D. 100%

Check

Evan tracked the amount of money he spent on clothes in January. He wrote down the amounts and created this box plot.



1. Determine the median, IQR, and range for this data.

median: 35 IQR: 20 range: 35

2. What percent of Evan's clothes cost at least $15?

A. 25% B. 50% C. 75% **D.** 100%

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Reflection

Ask:
- "What is important to remember when determining the median, IQR, and range in a box plot?"
- "What questions do you still have?"

Check: Recommended Next Steps

Almost there
If students need more support, consider annotating or highlighting the percentage each section of the box plot represents. For example, highlight the area from the minimum to Q1 and write "25%" next to it. Repeat the same for Q1 to median, median to Q3, and Q3 to maximum by labeling each section as "25%."

Got it!
If students need more practice, ask them to revisit the box plot in Problems 3–4 and answer the following questions:

- What percent of days did Eliza read at least 5 pages?
- What percent of days did Eliza read between 10 and 35 pages?

Name ____________________

Predicting Population Using Sample Data

ML 9.15

Modeled Review

Name: Caleb

20 random students from Median Middle School were asked what superpower they wanted. The results are shown in the table. Median Middle School has 500 students. Estimate the total number of students who prefer teleportation.

$\frac{4}{20} = \frac{x}{500}$

$500 \cdot 0.2 = \frac{x}{500} \cdot 500$

$100 = x$

100 students

Superpower	Number of students
Teleportation	4
Flight	3
Super strength	3
Time travel	5
Invisibility	5

Guided Practice

1. 10 random 7th grade students from Birchwood Middle School were asked about their favorite type of pet. The results are shown in the table. Birchwood Middle School has 50 students in 7th grade. Estimate the total number of students who prefer fish.

$\frac{\_\_}{10} = \frac{x}{\_\_}$

$\_\_ \cdot \_\_ = \frac{x}{\_\_} \cdot \_\_$

$\_\_ = x$

____ students

Pet	Number of students
Dogs	6
Cats	3
Fish	1

Guided Practice

2. 15 random students from Pinecrest Academy were asked what their favorite type of music was. The results are shown in the table. Pinecrest Academy has 100 students. Estimate the total number of students who prefer country music.

_____ students

$\frac{\_\_\_}{\_\_\_} = \frac{x}{\_\_\_}$

$\_\_\_ \cdot \frac{\_\_\_}{\_\_\_} = \frac{x}{\_\_\_} \cdot \_\_\_$

$\frac{\_\_\_}{\_\_\_} = x$

Music	Number of students
Pop	1
Rock	3
Hip-Hop	2
Country	9

3. 20 random students from Oakwood Elementary were asked what their favorite fruit was. The results are shown in the table. Oakwood Elementary has 400 students. Estimate the total number of students who prefer pineapple.

_____ students

Fruit	Number of students
Apples	5
Bananas	3
Oranges	3
Strawberries	5
Pineapple	4

Check

25 random students from Valley High School were asked what their favorite sport is. The results are shown in the table. Valley High School has 400 students. Estimate the total number of students who prefer basketball.

_____ students

Sport	Number of students
Soccer	8
Basketball	10
Baseball	3
Tennis	4

ML 9.15 Predicting Population Using Sample Data

Goal

Estimate unknown information about a population using equivalent ratios.

Standard

MA.7.DP.1.3

Materials

graph paper (optional), highlighter (optional)

Modeled Review

Point to Caleb's work and **ask**:

- "How did Caleb use equivalent ratios to estimate the number of students who would prefer teleportation as a superpower?"
- "What steps did Caleb take to solve for the unknown value?"
- "Is Caleb's prediction reasonable? How do you know?"

Reinforce Caleb's thinking by saying, "Equivalent ratios can be used to make accurate predictions from the given sample."

Guided Practice

Focus students' attention on making a prediction using proportions.

To scaffold their thinking, **say**:

- "First, identify the given ratio in the sample."
- "Next, set up a proportion using the known ratio and the population you're predicting."
- "Last, solve the proportion and check if the prediction makes sense."

Name ______________________

Predicting Population Using Sample Data ML 9.15

Modeled Review

Name: Caleb

20 random students from Median Middle School were asked what superpower they wanted. The results are shown in the table. Median Middle School has 500 students. Estimate the total number of students who prefer teleportation.

$\frac{4}{20} = \frac{x}{500}$

$500 \cdot 0.2 = \frac{x}{500} \cdot 500$

$100 = x$

100 students

Superpower	Number of students
Teleportation	4
Flight	3
Super strength	3
Time travel	5
Invisibility	5

Guided Practice

1. 10 random 7th grade students from Birchwood Middle School were asked about their favorite type of pet. The results are shown in the table. Birchwood Middle School has 50 students in 7th grade. Estimate the total number of students who prefer fish.

$\frac{1}{10} = \frac{x}{50}$

$50 \cdot 0.1 = \frac{x}{50} \cdot 50$

$5 = x$

5 students

Pet	Number of students
Dogs	6
Cats	3
Fish	1

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Vocabulary

If needed, share the meaning of the terms with students.

equivalent ratios: Two ratios are equivalent if you can multiply each of the values in the first ratio by the same number to get the values in the second ratio.

proportional: Quantities are proportional if they form equivalent ratios.

sample: A part of a population.

Predicting Population Using Sample Data

Guided Practice

Ⓐ Provide students with graph paper to organize their work.

ML/EL Consider using a highlighter to color-code and represent differing parts of the proportion (e.g., numerator and denominator) for clarity.

Key Takeaway:
Say, "When information is known about a sample, equivalent ratios help us make accurate predictions about the entire population."

Guided Practice

2. 15 random students from Pinecrest Academy were asked what their favorite type of music was. The results are shown in the table. Pinecrest Academy has 100 students. Estimate the total number of students who prefer country music.

60 students

$\frac{9}{15} = \frac{x}{100}$

$100 \cdot 0.6 = \frac{x}{100} \cdot 100$

$60 = x$

Music	Number of students
Pop	1
Rock	3
Hip-Hop	2
Country	9

3. 20 random students from Oakwood Elementary were asked what their favorite fruit was. The results are shown in the table. Oakwood Elementary has 400 students. Estimate the total number of students who prefer pineapple.

80 students

Fruit	Number of students
Apples	5
Bananas	3
Oranges	3
Strawberries	5
Pineapple	4

Check

25 random students from Valley High School were asked what their favorite sport is. The results are shown in the table. Valley High School has 400 students. Estimate the total number of students who prefer basketball.

160 students

Sport	Number of students
Soccer	8
Basketball	10
Baseball	3
Tennis	4

150

Reflection

Ask:

- "Describe how you could use a sample and equivalent ratios to estimate information about a population."
- "Reflect on your learning today. What were you most proud of?"

Check: Recommended Next Steps

Almost there
If students need more support, consider having them revisit the Check. Then, model identifying the given ratio, setting up the proportion using the known ratio and population, and solving the proportion and checking to make sure the prediction makes sense.

Got it!
If students need more practice, have them revisit Problem 3 and predict the total populations that favored the other fruits.

Unit 10

Mini-Lessons

Name ______________________________

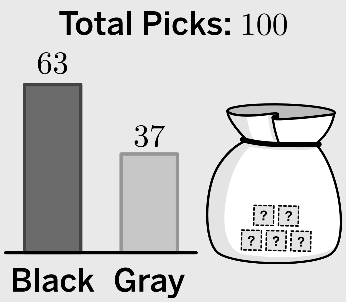
Predicting Sample Spaces Using Ratios and Percentages

ML 10.03

Modeled Review

100 blocks were picked one at a time, and the results are shown in the bar graph.

A new bag has 5 blocks. Some blocks are black and the others are gray. Based on the results of the bar graph, how many blocks do you think are black?



Amir's work

$\frac{63}{100} = \frac{x}{5}$

$5 \cdot 0.63 = \frac{x}{5} \cdot 5$

$3.15 = x$

$x \cong 3$

3 blocks

Axel's work

$\frac{63}{100} \cong 60\%$

$x \cong 0.60 \cdot 5$

$x \cong 3$

3 blocks

Guided Practice

10 blocks were picked one at a time, and the results are shown in the bar graph.

1. A new bag has 4 blocks. Some are black and some are gray. Based on the results of the bar graph, how many blocks do you think are gray?

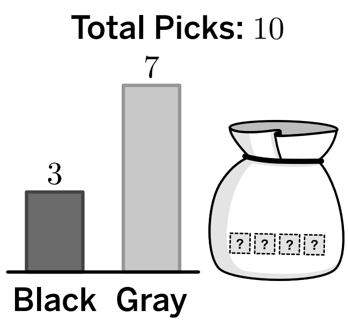
$\frac{\_\_\_\_}{10} = \_\_\_\_ \%$

$x = \_\_\_\_ \cdot 4$

$x = \_\_\_\_$

$x \cong \_\_\_\_$

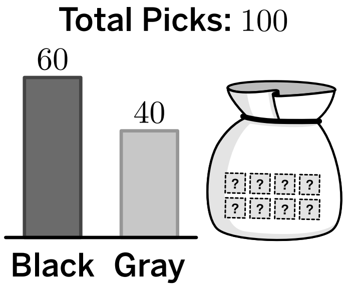
______ blocks



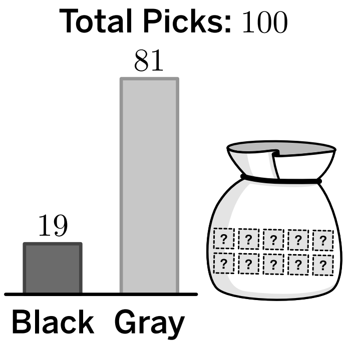
Guided Practice

100 blocks were picked one at a time, and the results are shown in the bar graph.

2. A new bag has 8 blocks. Some are black and some are gray. Based on the results of the bar graph, how many blocks do you think are black?



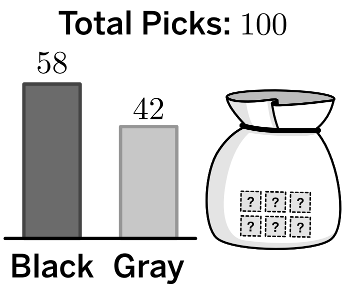
3. The bag has 10 blocks. Some are black and some are gray. Based on the results of the bar graph, how many blocks do you think are gray?



Check

100 blocks were picked one at a time, and the results are shown in the bar graph.

A new bag has 6 blocks. Some blocks are black and the others are gray. Based on the results of the bar graph, how many blocks do you think are black?



ML 10.03 Predicting Sample Spaces Using Ratios and Percentages

Goal

Use ratios and repeated experiments to calculate probability.

Standard

MA.7.DP.2.3

Materials

cubes (optional)

Modeled Review

Point to the Modeled Review and **ask**:

- "How are Amir's and Axel's strategies alike? Different?"
- "How did both Amir and Axel determine that the number of black blocks was probably 3?"
- "What do you think is helpful about each strategy?"

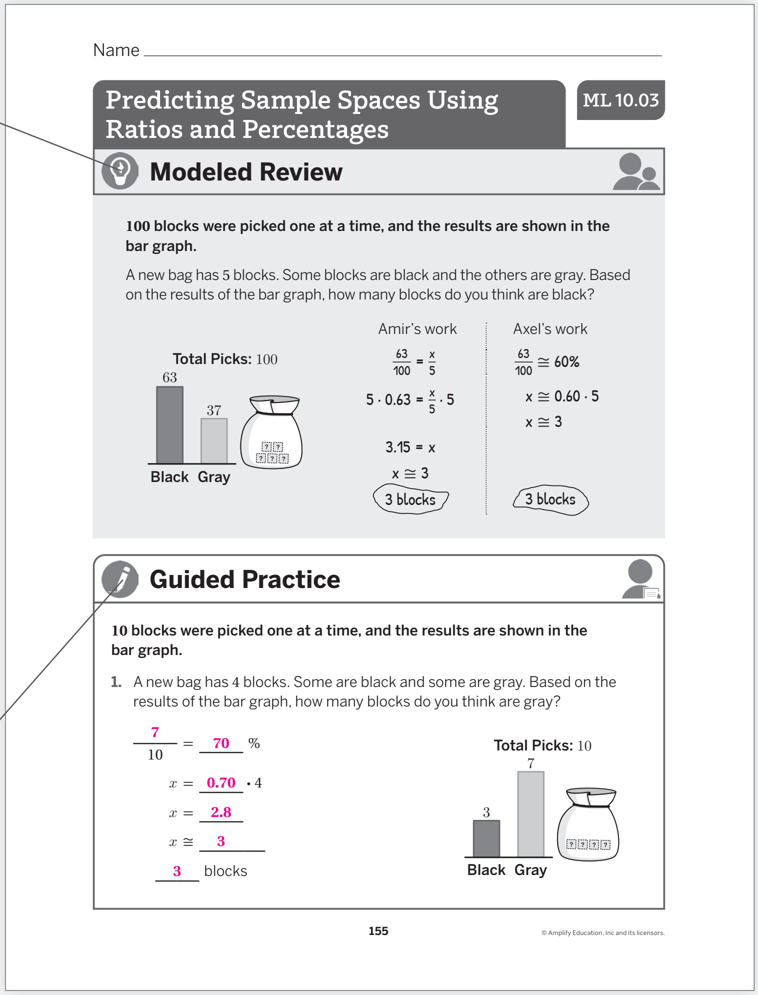
Reinforce the goal by saying, "Ratios and percentages can be used to make predictions based on data from experiments."

Guided Practice

Focus students' attention on using ratios and percentages to make predictions.

To scaffold their thinking, **say**:

- "First, identify what needs to be predicted."
- "Next, divide the values of the ratio to determine the percentage."
- "Then, multiply the percentage by the total number of blocks."
- "Finally, round your answer to the nearest whole number if the prediction does not make sense."



Name ______

Predicting Sample Spaces Using Ratios and Percentages — ML 10.03

Modeled Review

100 blocks were picked one at a time, and the results are shown in the bar graph.

A new bag has 5 blocks. Some blocks are black and the others are gray. Based on the results of the bar graph, how many blocks do you think are black?

Amir's work

$\frac{63}{100} = \frac{x}{5}$

$5 \cdot 0.63 = \frac{x}{5} \cdot 5$

$3.15 = x$

$x \cong 3$

3 blocks

Axel's work

$\frac{63}{100} \cong 60\%$

$x \cong 0.60 \cdot 5$

$x \cong 3$

3 blocks

Guided Practice

10 blocks were picked one at a time, and the results are shown in the bar graph.

1. A new bag has 4 blocks. Some are black and some are gray. Based on the results of the bar graph, how many blocks do you think are gray?

$\frac{7}{10} = $ 70 %

$x = $ 0.70 $\cdot 4$

$x = $ 2.8

$x \cong $ 3

3 blocks

Vocabulary

If needed, share the meaning of the term with students.

ratio: A ratio $a : b$ is a relationship between two quantities. For every a of the first, there are b of the second.

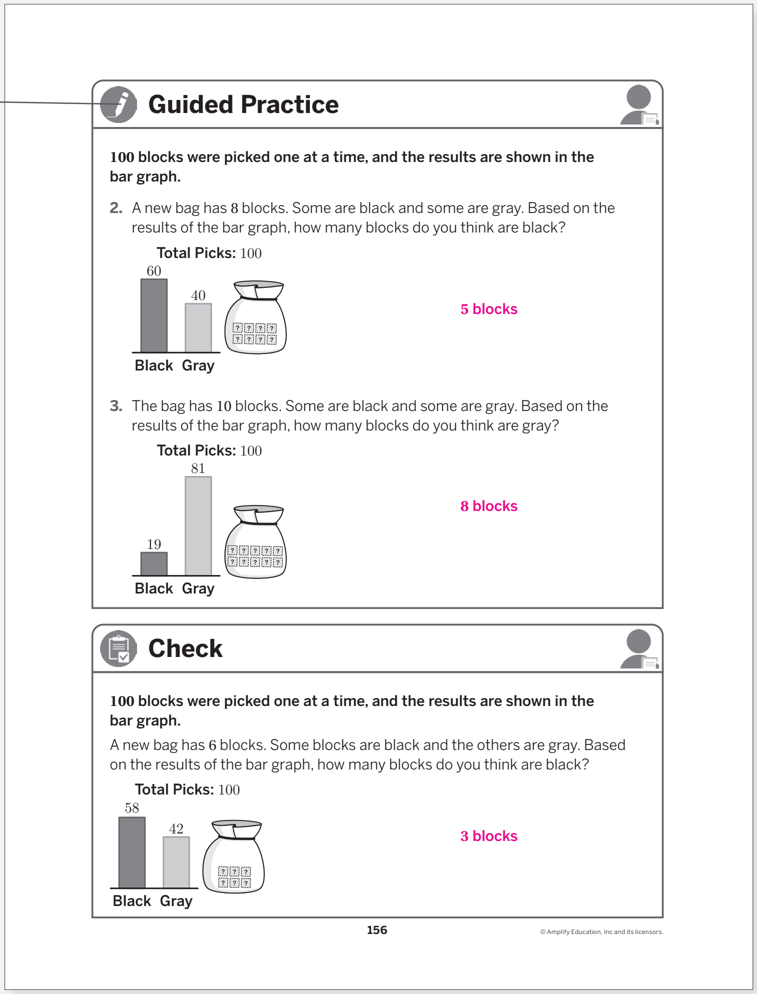
ML 10.03 Predicting Sample Spaces Using Ratios and Percentages

Guided Practice

(A) Provide students with cubes that can be used to replicate the mystery blocks in each problem. The cubes will serve as a hands-on tool to help clarify the solution students are working toward.

(ML/EL) Use a think aloud as you round to the nearest whole number by checking the digit after the decimal point. If it's 5 or greater, round up; otherwise, round down.

Key Takeaway:
Say, "Using ratios and percentages from the results of experiments can help make predictions."



Guided Practice

100 blocks were picked one at a time, and the results are shown in the bar graph.

2. A new bag has 8 blocks. Some are black and some are gray. Based on the results of the bar graph, how many blocks do you think are black?

5 blocks

3. The bag has 10 blocks. Some are black and some are gray. Based on the results of the bar graph, how many blocks do you think are gray?

8 blocks

Check

100 blocks were picked one at a time, and the results are shown in the bar graph.

A new bag has 6 blocks. Some blocks are black and the others are gray. Based on the results of the bar graph, how many blocks do you think are black?

3 blocks

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Reflection

Ask:

- "Describe your favorite strategy when predicting sample spaces using ratios."
- "How did you overcome a hard problem today?"

Check: Recommended Next Steps

Almost there	Got it!
If students need more support, consider creating an anchor chart that outlines the steps for determining ratios and percentages to aid in making predictions.	If students need more practice, have them use the bar graph from Problem 3 to determine how many blocks would be black if there are 5 blocks in the bag.

Prerequisite Skills and Concepts

Mini-Lessons

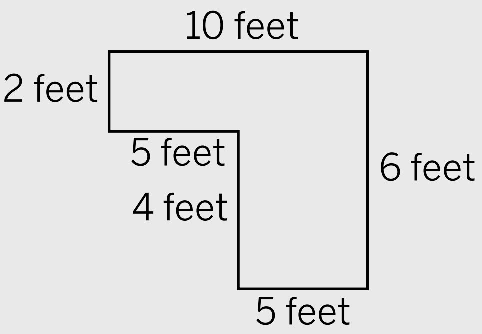
Name ____________________

Finding Area of Composite Figures Without Grids

ML 2.11

Modeled Review

Students were asked to determine the area of the figure.

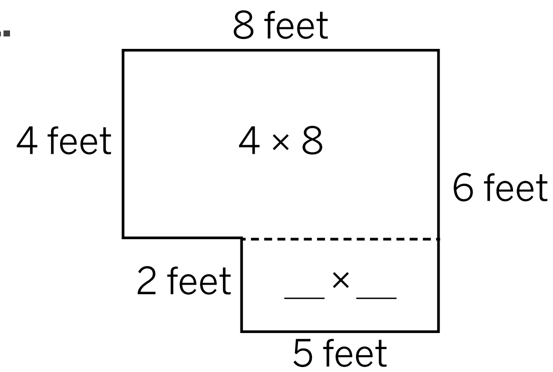


- Maya says you can determine the area by adding 2 × 10 and 4 × 5.
- Jack says you can determine the area by adding 2 × 5 and 6 × 5.

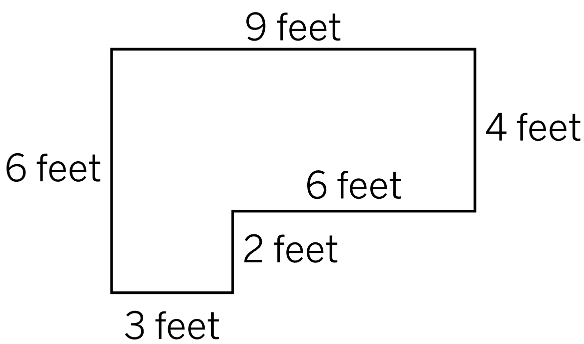
Guided Practice

Draw a line to decompose each figure into rectangles. Then write expressions that represent the area of each rectangle.

1.



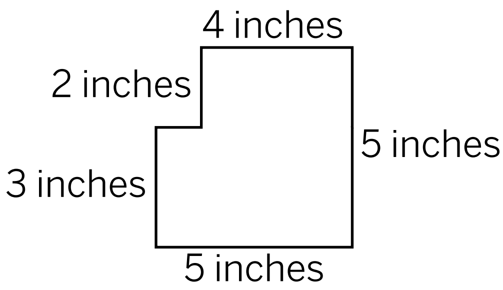
2.



Guided Practice

Determine the area of each figure. Show or explain your thinking.

3.



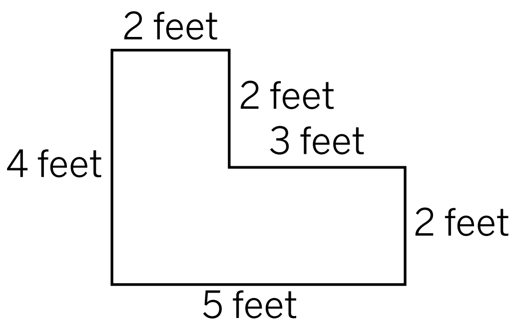
____ × ____ = ____

____ × ____ = ____

total area: ____ + ____ = ____

answer: ____ square inches

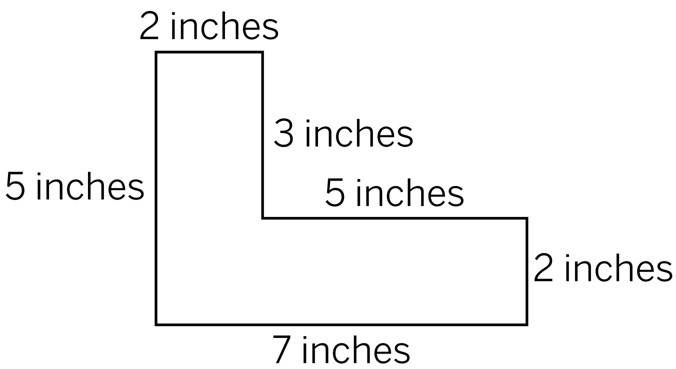
4.



answer: ________________

Check

Determine the area of the figure. Show or explain your thinking.



answer: ________________

ML 2.11 Finding Area of Composite Figures Without Grids

Goal

Determine the areas of composite figures composed of rectangles.

Standard

MA.3.GR.2.4

Modeled Review

Point to the problem in the Modeled Review and **ask**:

- "Where in the figure do you see the expressions that Maya used? Jack used?"
- "How could Maya show her thinking by annotating the figure?"

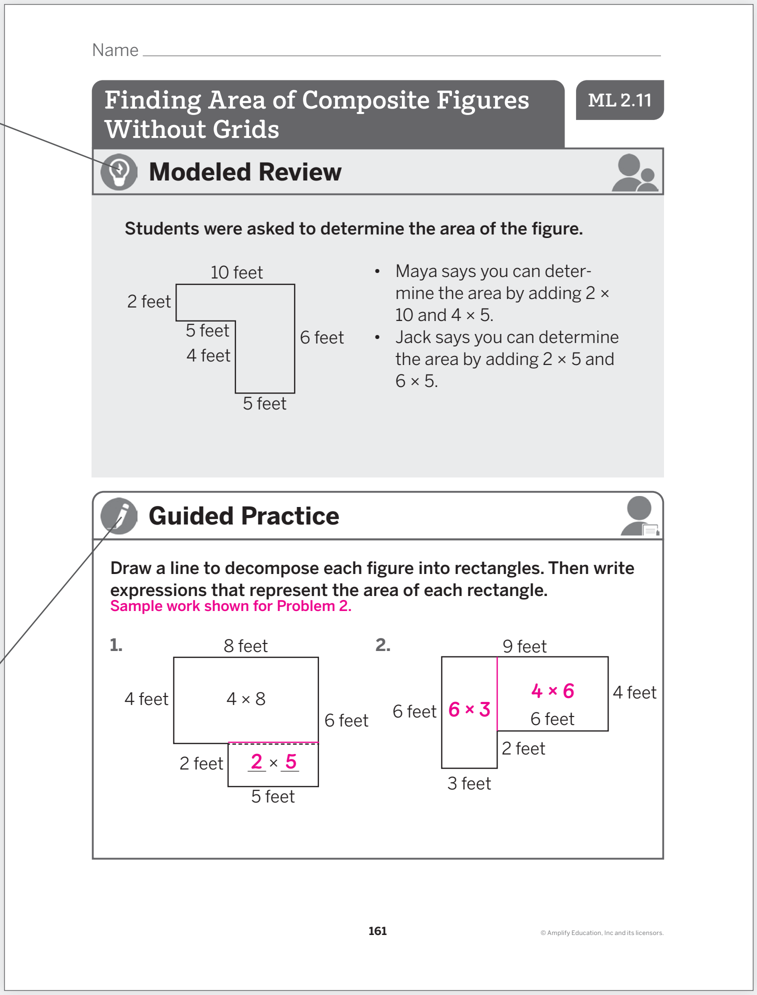
Reinforce the goal by saying, "You can break the figure into rectangles, multiply the side lengths, and then add those areas together to find the total area."

Guided Practice

Focus students' attention on decomposing the figure into two rectangles to help determine the area.

To scaffold their thinking, **ask**:

- "Where do you see rectangles in the figure?"
- "What expressions show the areas of the rectangles?"
- "Which given side lengths will you not use to calculate the area?"


Name

Finding Area of Composite Figures Without Grids

ML 2.11

Modeled Review

Students were asked to determine the area of the figure.

- Maya says you can determine the area by adding 2 × 10 and 4 × 5.
- Jack says you can determine the area by adding 2 × 5 and 6 × 5.

Guided Practice

Draw a line to decompose each figure into rectangles. Then write expressions that represent the area of each rectangle.

Sample work shown for Problem 2.

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Vocabulary

If needed, share the meaning of the term with students.

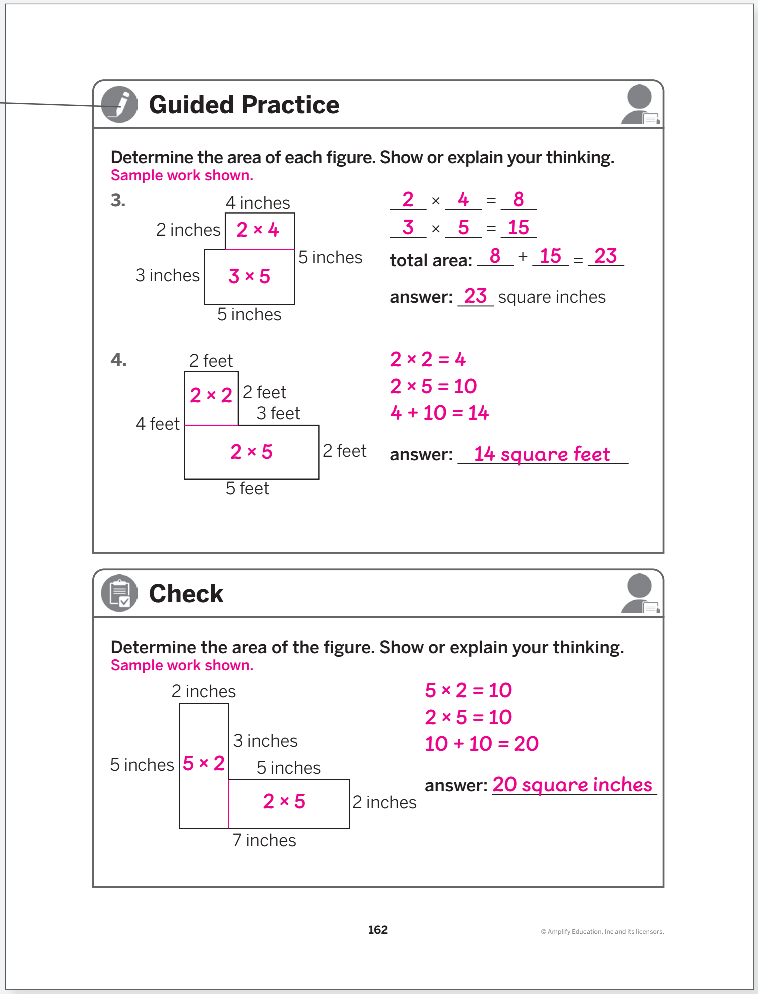
decompose: To break apart.

Finding Area of Composite Figures Without Grids

Guided Practice

A Invite students to share how they decomposed each figure and the expressions they determined for the area of each rectangle.

Key Takeaway:
Say, "There are different ways to decompose a figure made of rectangles. No matter how you decompose the figure, you can use the side lengths of the rectangles to determine their areas. Then you add the areas of the rectangles together to calculate the total area of the figure."



Guided Practice

Determine the area of each figure. Show or explain your thinking.
Sample work shown.

3.

2 × 4 = 8
3 × 5 = 15
total area: 8 + 15 = 23
answer: 23 square inches

4.

2 × 2 = 4
2 × 5 = 10
4 + 10 = 14
answer: 14 square feet

Check

Determine the area of the figure. Show or explain your thinking.
Sample work shown.

5 × 2 = 10
2 × 5 = 10
10 + 10 = 20
answer: 20 square inches

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Reflection

Ask:
- "What are some different ways to determine the area of a figure made of rectangles?"
- "What is something you were proud of from today's lesson?"

Check: Recommended Next Steps

Almost there	Got it!
If students need more support, consider using having students model two different ways they decomposed a composite figure into two rectangles.	If students need more practice, have them determine the total area of the figures in Problems 1 and 2.

Name ______________________________

Identifying Multiples

ML 1.05

Modeled Review

Name: Avery

Place a check mark next to the numbers that are multiples of 5.

Number	Multiple of 5?
20	✓
38	
50	✓
72	

I can skip count by 5 to find multiples of 5.
5, 10, 15, 20, 25, 30, 35, 40, 45, 50...

Guided Practice

1. Skip count by 4 up to 40. Use a hundreds chart if it is helpful.

4, _____, _____, _____, _____, _____, _____, _____, _____, _____

2. Place a check mark next to the numbers that are multiples of 4. Use your skip counting pattern from Problem 1 if it is helpful.

Number	Multiple of 4?
16	
20	
22	
36	

Guided Practice

Place a check mark next to the numbers that are multiples of the given number. Use a hundreds chart if it is helpful.

3.

Number	Multiple of 5?
20	✓
35	
56	
75	

4.

Number	Multiple of 10?
40	
68	
80	
96	

5.

Number	Multiple of 6?
18	
32	
36	
40	

6.

Number	Multiple of 9?
19	
25	
50	
54	

Check

Place a check mark next to the numbers that are multiples of 3.

Number	Multiple of 3?
14	
18	
23	
33	

ML 1.05 Identifying Multiples

Goal

Determine if one number is a multiple of another.

Standard

MA.4.AR.3.1

Materials

multiplication chart (optional)

Modeled Review

Point to Avery's work and **ask**:

- "What strategy did Avery use to determine if the numbers are multiples of 5?"
- "How could skip counting be used to determine multiples of other numbers?"
- "What other strategies could be used to determine multiples of numbers?"

Reinforce Avery's thinking by saying, "A multiple of a number is the result of multiplying the number by any whole number."

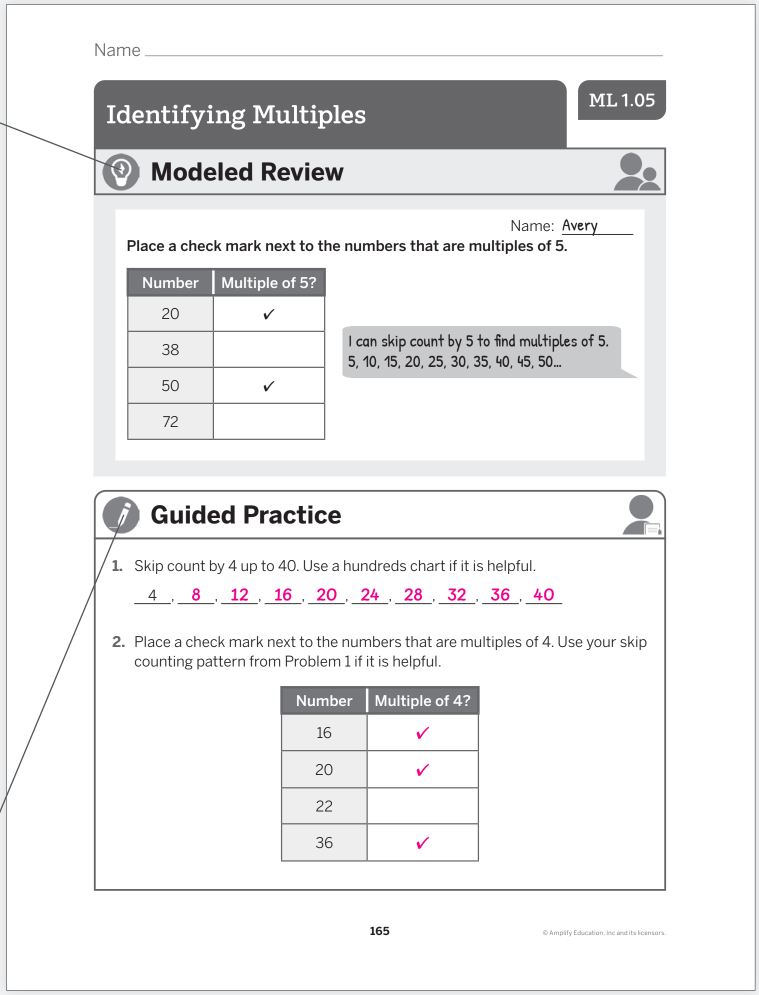
ML/EL Throughout the lesson, invite students to skip count in their home language if it is helpful.

Guided Practice

Focus students' attention on identifying multiples of a given number.

To scaffold their thinking, **ask**:

- "How could number patterns help you determine multiples?"
- "What strategies could you use to determine multiples?"


Name

Identifying Multiples

ML 1.05

Modeled Review

Name: Avery

Place a check mark next to the numbers that are multiples of 5.

Number	Multiple of 5?
20	✓
38	
50	✓
72	

I can skip count by 5 to find multiples of 5.
5, 10, 15, 20, 25, 30, 35, 40, 45, 50...

Guided Practice

1. Skip count by 4 up to 40. Use a hundreds chart if it is helpful.

4, 8, 12, 16, 20, 24, 28, 32, 36, 40

2. Place a check mark next to the numbers that are multiples of 4. Use your skip counting pattern from Problem 1 if it is helpful.

Number	Multiple of 4?
16	✓
20	✓
22	
36	✓

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Vocabulary

If needed, share the meaning of the term with students.

multiple: A number you get by multiplying another number (for example, some multiples of 4 are 4, 8, 12, and 16).

Guided Practice

A Provide students with a multiplication chart that can be used to identify multiples of a given number.

Key Takeaway:
Say, "You could use what you know about multiplication to determine whether a number is a multiple of another number."

Guided Practice

Place a check mark next to the numbers that are multiples of the given number. Use a hundreds chart if it is helpful.

3.

Number	Multiple of 5?
20	✓
35	✓
56	
75	✓

4.

Number	Multiple of 10?
40	✓
68	
80	✓
96	

5.

Number	Multiple of 6?
18	✓
32	
36	✓
40	

6.

Number	Multiple of 9?
19	
25	
50	
54	✓

Check

Place a check mark next to the numbers that are multiples of 3.

Number	Multiple of 3?
14	
18	✓
23	
33	✓

166

Reflection

Ask:
- "How could you determine if a number is a multiple of another number?"
- "How did today's learning connect to your previous learning?"

Check: Recommended Next Steps

Almost there
If students need more support, consider using modeling how to use a multiplication chart to identify the patterns of multiples of numbers.

Got it!
If students need more practice, have them determine if the following numbers are multiples of the given number:

- 8: 16, 20, 34, 64
- 10: 12, 25, 30, 90
- 7: 19, 21, 49, 58

Name ______________________________

Identifying Factor Pairs

ML 1.06

Modeled Review

Name: Jada

Use the factor pairs of 15 to write multiplication expressions.

Number	Factor pairs
15	1 × 15, 3 × 5

I can find factor pairs by thinking about which two numbers I can multiply to equal the number.

Guided Practice

Determine *all* the factor pairs of the given number.

Number	Factor pairs
12	1 × 12, 2 × 6, 3 × ______
9	1 × 9, 3 × ______
10	1 × 10, ______ × ______
16	1 × 16, 2 × 8, ______ × ______
21	1 × 21, ______ × ______
19	1 × ______

Guided Practice

Use *all* the factor pairs of the given number to write multiplication expressions.

Number	Factor pairs
30	1 × 30, __________, __________, __________
7	1 × ______
35	
20	
36	

Check

Use *all* the factor pairs of the given number to write multiplication expressions.

Number	Factor pairs
18	
11	

ML 1.06 Identifying Factor Pairs

Goal

Use multiplication facts to identify all factor pairs of a given number.

Standard

MA.4.NSO.2.1

Materials

inch tiles (optional), graph paper (optional)

Modeled Review

Point to Jada's work and **ask**:
- "What strategy did Jada use to find the factor pairs of 15?"
- "Why do you think 2 is *not* listed in one of the factor pairs?"

Reinforce Jada's thinking by saying, "Factor pairs for a given number are two factors that result in that number when multiplied together."

ML/EL Invite students to discuss the problem in the Modeled Review with a partner by providing the following sentence frame: "2 is not a factor of 15 because ________."

Guided Practice

Focus students' attention on determining all of the factor pairs for a given number.

To scaffold their thinking, **say**:
- "You could use multiplication facts to help you determine factor pairs."
- "Think of the factors in numerical order, so you do not miss any factor pairs."
- "After 1 and the given number, consider if 2 can be multiplied by a number to result in the given number, and then 3, 4, and so on."

Name ________

Identifying Factor Pairs

ML 1.06

Modeled Review

Name: Jada

Use the factor pairs of 15 to write multiplication expressions.

Number	Factor pairs
15	1 × 15, 3 × 5

I can find factor pairs by thinking about which two numbers I can multiply to equal the number.

Guided Practice

Determine *all* the factor pairs of the given number.

Number	Factor pairs
12	1 × 12, 2 × 6, 3 × 4
9	1 × 9, 3 × 3
10	1 × 10, 2 × 5
16	1 × 16, 2 × 8, 4 × 4
21	1 × 21, 3 × 7
19	1 × 19

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Vocabulary

If needed, share the meaning of the term with students.

factor pair: A set of two numbers that arrive at a specific number when multiplied (ex: 4 and 6 is a factor pair of 24).

ML 1.06 Identifying Factor Pairs

Guided Practice

A Consider having tiles or graph paper available for students to determine factor pairs using side lengths of rectangles for additional support.

Key Takeaway:
Say, "A factor pair of a whole number is a set of two whole numbers that multiply to result in that number. Factor pairs could be written as multiplication expressions."

Guided Practice

Use *all* the factor pairs of the given number to write multiplication expressions.

Number	Factor pairs
30	1 × 30, 2 × 15, 3 × 10, 5 × 6
7	1 × 7
35	1 × 35, 5 × 7
20	1 × 20, 2 × 10, 4 × 5
36	1 × 36, 2 × 18, 3 × 12, 4 × 9, 6 × 6

Check

Use *all* the factor pairs of the given number to write multiplication expressions.

Number	Factor pairs
18	1 × 18, 2 × 9, 3 × 6
11	1 × 11

170

Reflection

Ask:
- "What strategies could you use to determine factor pairs of any whole number?"
- "What is something you weren't sure about at the start of the lesson but understand now?"

Check: Recommended Next Steps

Almost there
If students need more support, consider building strategies and conceptual understanding to recall factor pairs by building off their understanding of known multiplication facts.

Got it!
If students need more practice, have them list *all* the factor pairs of 40.

Name ____________________

Comparing Decimals and Fractions

ML 4.07

Modeled Review

Name: Tristan

Complete the comparison statement using < , >, or =.

1. $\frac{1}{10}$ ___<___ 1.10

$1.10 = \frac{11}{10}$

2. 0.5 ___=___ $\frac{50}{100}$

3. $\frac{45}{100}$ ___>___ 0.2

$0.2 = \frac{2}{10} = \frac{20}{100}$

Guided Practice

Complete the comparison statement using <, >, or =.

1. 0.3 ______ $\frac{4}{10}$

$0.3 = \frac{3}{10}$

2. $\frac{43}{100}$ ______ 0.65

$0.65 = \frac{65}{100}$

3. $\frac{56}{100}$ ______ 0.4

4. 0.60 ______ $\frac{6}{10}$

5. $\frac{2}{10}$ ______ 0.22

Guided Practice

Complete the comparison statement using <, >, or =.

6. 0.5 ______ $\frac{15}{100}$

7. $\frac{72}{100}$ ______ 0.75

8. 0.3 ______ 0.40

9. 0.28 ______ $\frac{28}{100}$

10. $\frac{4}{10}$ ______ 0.32

Check

Complete the comparison statement using <, >, or =.

1. 0.1 ______ $\frac{1}{10}$

2. $\frac{12}{100}$ ______ 0.15

3. $\frac{6}{10}$ ______ 0.72

4. $\frac{54}{100}$ ______ 0.54

ML 4.07 Comparing Decimals and Fractions

Goal

Compare decimals and fractions to the hundredths by reasoning about their values.

Standard

MA.4.FR.1.4

Modeled Review

Point to Tristan's work and **ask**:

- "How are fractions and decimals related?"
- "How does Tristan use equivalent fractions to help him make the comparisons?"

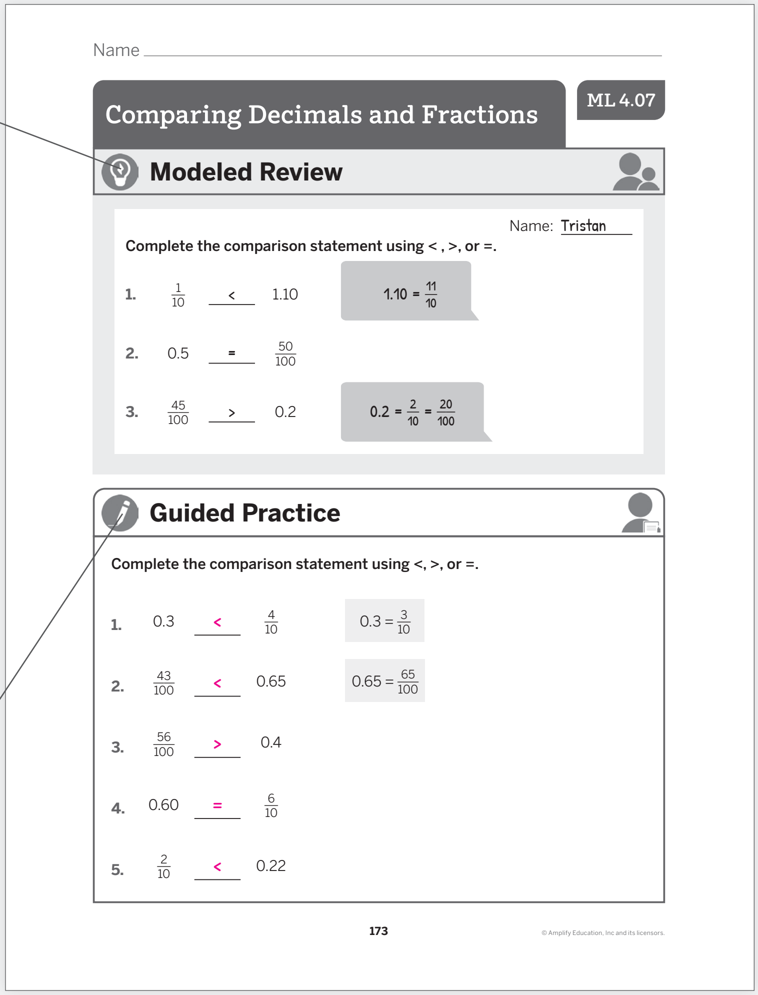
Reinforce the goal by saying, "There are a variety of strategies you can use to compare fractions and decimals, such as using benchmark numbers or rewriting them as fractions with the same denominator and comparing their values."

Guided Practice

Focus students' attention on completing the comparison statements.

To scaffold their thinking, **ask**:

- "How could you rewrite the fraction or decimal to help you compare the numbers?"
- "What do you notice about the value of the fraction or decimal?"



Name ______

Comparing Decimals and Fractions — ML 4.07

Modeled Review

Name: Tristan

Complete the comparison statement using <, >, or =.

1. $\frac{1}{10}$ < 1.10 ($1.10 = \frac{11}{10}$)
2. 0.5 = $\frac{50}{100}$
3. $\frac{45}{100}$ > 0.2 ($0.2 = \frac{2}{10} = \frac{20}{100}$)

Guided Practice

Complete the comparison statement using <, >, or =.

1. 0.3 < $\frac{4}{10}$ ($0.3 = \frac{3}{10}$)
2. $\frac{43}{100}$ < 0.65 ($0.65 = \frac{65}{100}$)
3. $\frac{56}{100}$ > 0.4
4. 0.60 = $\frac{6}{10}$
5. $\frac{2}{10}$ < 0.22

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Vocabulary

If needed, share the meaning of the terms with students.

- least
- greatest

ML 4.07 Comparing Decimals and Fractions

Guided Practice

A Encourage students to write the decimals as fractions with the same denominator as the fraction from the problem.

ML/EL Consider using a think-aloud as you review and annotate how to determine an equivalent decimal to complete the comparison statement.

Key Takeaway:
Say, "Fractions and decimals with tenths or hundredths can be compared by reasoning about the values of the decimals as fractions."

Guided Practice

Complete the comparison statement using <, >, or =.

6. 0.5 __>__ $\frac{15}{100}$

7. $\frac{72}{100}$ __<__ 0.75

8. 0.3 __<__ 0.40

9. 0.28 __=__ $\frac{28}{100}$

10. $\frac{4}{10}$ __>__ 0.32

Check

Complete the comparison statement using <, >, or =.

1. 0.1 __=__ $\frac{1}{10}$

2. $\frac{12}{100}$ __<__ 0.15

3. $\frac{6}{10}$ __<__ 0.72

4. $\frac{54}{100}$ __=__ 0.54

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Reflection

Ask:

- "What strategy was the most helpful when comparing decimals and fractions?"
- "What questions do you still have?"

Check: Recommended Next Steps

Almost there
If students need more support, consider helping students extend understanding by generating equivalent fractions with common numerators or common denominators to compare and order fractions.

Got it!
If students need more practice, have them complete the following comparison statements using <, >, or =:

- 0.28 ____ $\frac{4}{10}$
- $\frac{65}{100}$ ____ 0.72

Name ______________________________

Solving Multiplicative Comparison Problems

ML 5.05

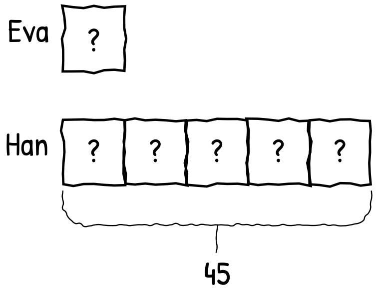
Modeled Review

Name: Tristan

Eva has some stickers. Han has 45, which is 5 times as many stickers as Eva has. How many stickers does Eva have?

$5 \times ? = 45$

$45 \div 5 = ?$



answer: 9 stickers

Guided Practice

Solve each problem to determine the unknown value. Show your thinking.

1. Avery has some trading cards. Jada has 24, which is 6 times as many trading cards as Avery has. How many trading cards does Avery have?

answer: ______ trading cards

2. Maya has 7 beads and Diego has 28 beads. How many times as many beads does Diego have compared to the number of beads that Maya has?

answer: ______ times as many

Guided Practice

Solve each problem to determine the unknown value. Show your thinking.

3. Dylan has some baseballs. Han has 27, which is 3 times as many baseballs as Dylan has. How many baseballs does Dylan have?

answer: ____________________

4. Shawn has 6 tennis balls and Clare has 42. How many times as many tennis balls does Clare have compared to the number of tennis balls that Shawn has?

answer: ____________________

5. Tristan scored some points in basketball. Priya scored 18, which is 6 times as many points as Tristan scored. How many points did Tristan score?

answer: ____________________

Check

Solve the problem to determine the unknown value. Show your thinking.

Jack scored some points in football. Santiago has 21, which is 3 times as many points as Jack has. How many points does Jack have?

answer: ____________________

ML 5.05 Solving Multiplicative Comparison Problems

Goal	Standard
Solve multiplicative situations using multiplicative and division equations.	MA.4.AR.1.1

Modeled Review

Point to Tristan's work and **ask**:

- "What information is known? What is unknown?"
- "How did Tristan use the information given to create a multiplication equation? Division equation?"
- "How did the equations help him determine the answer?"

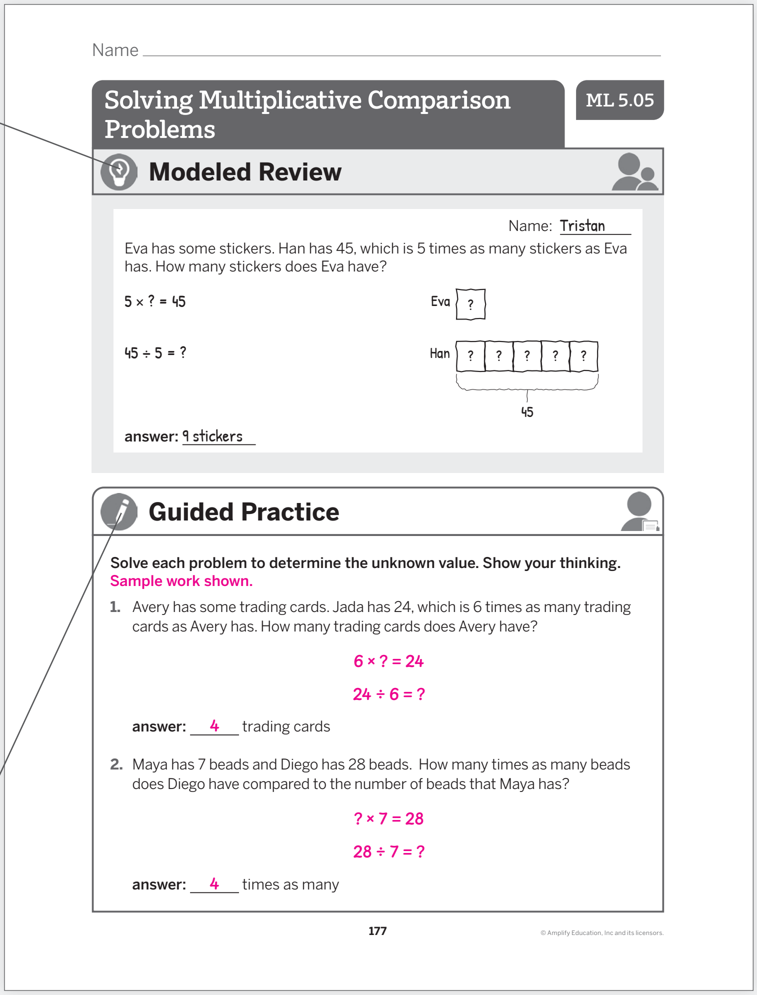
Reinforce the goal by saying, "When you represent a comparison situation using a multiplication or division equation, you can represent the unknown value with a symbol, such as a question mark or with a letter."

Guided Practice

Focus students' attention on using the situations to determine the answer.

To scaffold their thinking, **ask**:

- "What information is known? Unknown?"
- "How can you use a multiplication and division equation to find the answer?"



Name

Solving Multiplicative Comparison Problems

ML 5.05

Modeled Review

Name: Tristan

Eva has some stickers. Han has 45, which is 5 times as many stickers as Eva has. How many stickers does Eva have?

$5 \times ? = 45$

$45 \div 5 = ?$

answer: 9 stickers

Guided Practice

Solve each problem to determine the unknown value. Show your thinking.
Sample work shown.

1. Avery has some trading cards. Jada has 24, which is 6 times as many trading cards as Avery has. How many trading cards does Avery have?

   $6 \times ? = 24$

   $24 \div 6 = ?$

   answer: 4 trading cards

2. Maya has 7 beads and Diego has 28 beads. How many times as many beads does Diego have compared to the number of beads that Maya has?

   $? \times 7 = 28$

   $28 \div 7 = ?$

   answer: 4 times as many

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Guided Practice

A Annotate the situation while reinforcing how the known information could be used to write a multiplication equation and division equation.

ML/EL Encourage students to discuss with a partner what is known and unknown before having them write their equations.

Key Takeaway:
Say, "Multiplication and division are related operations that can be used to represent and solve multiplicative comparison problems where the unknown could be the number of groups or the number in each group."

Guided Practice

Solve each problem to determine the unknown value. Show your thinking.
Sample work shown.

3. Dylan has some baseballs. Han has 27, which is 3 times as many baseballs as Dylan has. How many baseballs does Dylan have?

$3 \times ? = 27$

$27 \div 3 = ?$

answer: 9 baseballs

4. Shawn has 6 tennis balls and Clare has 42. How many times as many tennis balls does Clare have compared to the number of tennis balls that Shawn has?

$? \times 6 = 42$

$42 \div 6 = ?$

answer: 7 times as many

5. Tristan scored some points in basketball. Priya scored 18, which is 6 times as many points as Tristan scored. How many points did Tristan score?

$6 \times ? = 18$

$18 \div 6 = ?$

answer: 3 points

Check

Solve the problem to determine the unknown value. Show your thinking.
Sample work shown.

Jack scored some points in football. Santiago has 21, which is 3 times as many points as Jack has. How many points does Jack have?

$3 \times ? = 21$

$21 \div 3 = ?$

answer: 7 points

178

Reflection

Ask:
- "Can you write a multiplication and division equation for every comparison problem involving 'times as many'? Why or why not?"
- "What strategy was helpful today?"

Check: Recommended Next Steps

Almost there
If students need more support, consider including opportunities to engage in guided practice completing real-world problems involving remainders.

Got it!
If students need more practice, have them solve the following problem.

Maya has some marbles. Kai has 42, which is 6 times as many marbles as Maya has. How many marbles does Maya have?

Name ______________________

Dividing Multi-Digit Numbers Using Partial Quotients

ML 6.14

Modeled Review

Name: Diego

Calculate the quotient.

672 ÷ 3 = 224

```
3)672
 -600     3 × 200     200
   72                  20
  -60     3 × 20     +  4
   12                 224
  -12     3 × 4
    0
```

Guided Practice

Calculate the quotient.

1. 245 ÷ 5 = ______

```
5)245
 -200     5 × 40      40
   45               +  9
  -45     5 × 9     ____
    0
```

2. 1,233 ÷ 3 = ______

```
3)1,233
 -1,200     3 × 400      400
     33                + ___
  - ___     3 × ___    _____
    ___
```

Guided Practice

Calculate the quotient.

3. 920 ÷ 2 = __________

$2\overline{)920}$

4. 2,416 ÷ 4 = __________

$4\overline{)2,416}$

5. 292 ÷ 4 = __________

6. 1,120 ÷ 5 = __________

Check

Calculate the quotient.

856 ÷ 4 = __________

ML 6.14 Dividing Multi-Digit Numbers Using Partial Quotients

Goal

Divide three-digit and four-digit dividends by one-digit divisors.

Standard

MA.4.NSO.2.4

Materials

multiplication chart (optional)

Modeled Review

Point to Diego's work and **ask**:

- "What did Diego do as his first step? Why did he decide to multiply 3 by 200?"
- "Why did he subtract 600?"
- "Why did he subtract until he got to zero?"
- "Why did he add 200, 20, and 4?"

Reinforce Diego's thinking by saying, "To divide multi-digit numbers, one strategy is to use partial quotients. You find how many times the divisor fits into parts of the dividend, and then add those parts together to get the final quotient."

Guided Practice

Focus students' attention on using partial quotients to divide.

To scaffold their thinking, **ask**:

- "How can using multiples of 10 or 100 as possible factors help you solve?"
- "What does each multiplication expression represent?"
- "What do you need to do to find the final quotient?"

Name

Dividing Multi-Digit Numbers Using Partial Quotients ML 6.14

Modeled Review

Name: Diego

Calculate the quotient.

672 ÷ 3 = 224

```
3)672
 -600    3 × 200    200
   72                20
  -60    3 × 20    +  4
   12               224
  -12    3 × 4
    0
```

Guided Practice

Calculate the quotient.

1. 245 ÷ 5 = 49

```
5)245
 -200    5 × 40     40
   45              + 9
  -45    5 × 9      49
    0
```

2. 1,233 ÷ 3 = 411

```
3)1,233
 -1,200    3 × 400    400
     33              + 11
   - 33    3 × 11     411
      0
```

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Vocabulary

If needed, share the meaning of the terms with students.

quotient: The result obtained by dividing two numbers representing either the number of equal-sized groups or the size of each group.

partial quotient: The value obtained when multiplying the divisor in a division equation by another number, then subtracting that number from the dividend (which is then added to other partial quotients to arrive at the total quotient).

Dividing Multi-Digit Numbers Using Partial Quotients

Guided Practice

(A) Provide students with a multiplication chart to help them strategically choose partial quotients.

(ML/EL) Invite students to discuss how they determined their partial quotients and provide them with sentence frames to support their discussion. For example, "First, I thought about _____ because______" or "I chose _____ as my first partial quotient because _____."

Key Takeaway:
Say, "When dividing multi-digit numbers, you can try to determine the greatest possible multiple of each place value for the partial quotients. This may help you solve the problem with fewer steps."

Guided Practice

Calculate the quotient. Sample work shown.

3. 920 ÷ 2 = 460

2)920
− 800 (2 × 400)
120
− 120 (2 × 60)
0

400 + 60 = 460

4. 2,416 ÷ 4 = 604

4)2,416
− 2,400 (4 × 600)
16
− 16 (4 × 4)
0

600 + 4 = 604

5. 292 ÷ 4 = 73

4)292
− 200 (4 × 50)
92
− 80 (4 × 20)
12
− 12 (4 × 3)
0

50 + 20 + 3 = 73

6. 1,120 ÷ 5 = 224

5)1,120
− 1,000 (5 × 200)
120
− 100 (5 × 20)
20
− 20 (5 × 4)
0

200 + 20 + 4 = 224

Check

Calculate the quotient. Sample work shown.

856 ÷ 4 = 214

4)856
− 800 (4 × 200)
56
− 40 (4 × 10)
16
− 16 (4 × 4)
0

200 + 10 + 4 = 214

182

Reflection

Ask:
- "How can you use place value to choose partial quotients when dividing multi-digit numbers?"
- "What makes sense? What is still confusing?"

Check: Recommended Next Steps

Almost there
If students need more support, consider connecting place value with the partial products model.

Got it!
If students need more practice, invite them to calculate the quotient for the following expressions:

- 864 ÷ 3
- 1,272 ÷ 4
- 1,080 ÷ 5

Name ____________________

Solving Problems Involving Area and Perimeter

ML 6.21

Modeled Review

Name: Tristan

A square has a perimeter of 252 inches. What is the length of each side?

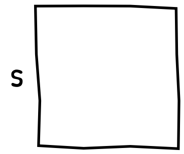
252 ÷ 4 = s

240 + 12

240 ÷ 4 = (60)

12 ÷ 4 = (3)

60 + 3 = 63



answer: 63 inches

Guided Practice

1. A rectangle has an area of 446 square inches. The length is 2 inches. What is the width?

answer: ________________

2. A square has a perimeter of 136 inches. What is the length of each side?

answer: ________________

Guided Practice

3. A rectangle measures 3 feet wide and 257 feet long. What is the area of the rectangle?

answer: ____________________

4. A rectangle measures 32 feet wide and 26 feet long. What is the area of the rectangle?

answer: ____________________

Check

A rectangle measures 17 inches wide and 48 inches long. What is the area of the rectangle?

answer: ____________________

ML 6.21 Solving Problems Involving Area and Perimeter

Goal	Standard
Solve problems involving perimeter and area.	MA.4.GR.2.1

Modeled Review

Point to Tristan's work and **ask**:

- "What information was known? Unknown?"
- "What steps did Tristan take to find the side lengths?"
- "Why did Tristan divide 252 by 4 to find the side lengths of the square?"

Reinforce Tristan's thinking by saying, "You can use what you know about squares and rectangles to find missing side lengths if you know the perimeter or area."

ML/EL Display an anchor chart with area and perimeter examples and formulas. Explain the information to students, emphasizing the differences between area and perimeter.

Guided Practice

Focus students' attention on determining the unknown side lengths.

To scaffold their thinking, **ask**:

- "What do you know about the side lengths of a rectangle? Square?"
- "How can you use the known information to determine the unknown side lengths?"

Name ____________

Solving Problems Involving Area and Perimeter — ML 6.21

Modeled Review

Name: Tristan

A square has a perimeter of 252 inches. What is the length of each side?

252 ÷ 4 = s
240 + 12

240 ÷ 4 = (60)
12 ÷ 4 = (3)

60 + 3 = 63

s

answer: 63 inches

Guided Practice

1. A rectangle has an area of 446 square inches. The length is 2 inches. What is the width? **Sample work shown.**

400 ÷ 2 = 200
40 ÷ 2 = 20
6 ÷ 2 = 3

200
20
+ 3
223

answer: 223 inches

2. A square has a perimeter of 136 inches. What is the length of each side? **Sample work shown.**

136 ÷ 4 = s
120 + 16

120 ÷ 4 = 30
16 ÷ 4 = 4

30 + 4 = 34

answer: 34 inches

185

Vocabulary

If needed, share the meaning of the terms with students.

area: A measurement of the space inside a two-dimensional shape.

perimeter: The total length of the boundary of a two-dimensional shape.

Solving Problems Involving Area and Perimeter

Guided Practice

(A) Invite students to draw a rectangle labeling the known side lengths to help them visualize the problem and determine the area.

Key Takeaway:
Say, "Knowing the formulas for area and perimeter can help you figure out missing side lengths. If you are given the side lengths, you can use them to calculate the area or perimeter."

Guided Practice

3. A rectangle measures 3 feet wide and 257 feet long. What is the area of the rectangle? Sample work shown.

	200	50	7
3	200 × 3 = 600	50 × 3 = 150	7 × 3 = 21

600
150
+ 21
771

answer: 771 square feet

4. A rectangle measures 32 feet wide and 26 feet long. What is the area of the rectangle? Sample work shown.

	30	2
20	30 × 20 = 600	2 × 20 = 40
6	30 × 6 = 180	2 × 6 = 12

600
40
180
+ 12
832

answer: 832 square inches

Check

A rectangle measures 17 inches wide and 48 inches long. What is the area of the rectangle? Sample work shown.

	40	8
10	40 × 10 = 400	8 × 10 = 80
7	40 × 7 = 280	8 × 7 = 56

400
80
280
+ 56
816

answer: 816 square inches

186

Reflection

Ask:
- "How can you determine the area of a figure when some side lengths are missing?"
- "How did today's learning connect to your previous learning?"

Check: Recommended Next Steps

Almost there
If students need more support, consider using a think-aloud to model the step-by-step process for finding the area in Problem 4. Then invite students to revisit the Check.

Got it!
If students need more practice, invite them to determine the missing measurement.

- A square has a perimeter of 732 feet. What is the length of each side?
- A rectangle has a length of 58 inches and a width of 23 inches. What is the area of the rectangle?

Name ______________________________

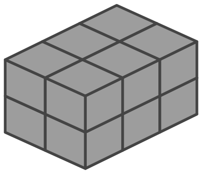
Using the Structure of Rectangular Prisms to Determine Volume

ML 1.04

Modeled Review

Name: Gabriela

Determine the volume of the prism. Show or explain your thinking.

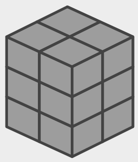
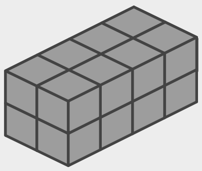
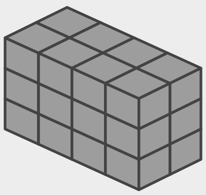


The bottom layer has 6 cubes.
There are 2 layers. 2 layers of
6 cubes is 12 cubes.

answer: 12 cubic units

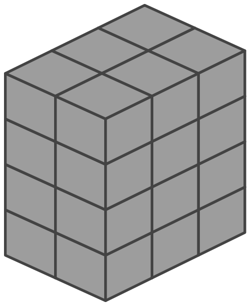
Guided Practice

1. Complete the table.

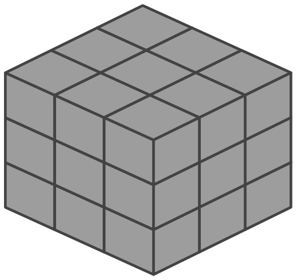
Prism	Number of cubes in the bottom layer	Number of layers	Volume (cubic units)
	4	3	
	8		
			

Guided Practice

Determine the volume of each prism. Show or explain your thinking.

2. 

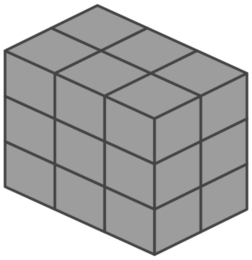
answer: ____________________

3. 

answer: ____________________

Check

Determine the volume of the prism. Show or explain your thinking.



answer: ____________________

Using the Structure of Rectangular Prisms to Determine Volume

Goal

Use the layered structure of a rectangular prism to explain how to determine volume.

Standard

MA.5.GR.3.1

Materials

unit cubes (optional)

Modeled Review

Point to Gabriela's work and **ask**:

- "How did Gabriela know the bottom layer has 6 cubes?"
- "How did Gabriela know there are 2 layers?"
- "How would Gabriela's thinking change if the prism had 4 more layers?"
- "What steps did Gabriela take to determine the volume?"

Reinforce the goal by saying, "You can determine the volume of a rectangular prism shown in a two-dimensional image by using your understanding of layers composed of equal groups of unit cubes."

Guided Practice

Focus students' attention on the number of cubes in a layer and how they can use multiplication to find the volume.

To scaffold their thinking, **ask**:

- "How does the top layer help you to know the amount of cubes in the bottom layer?"
- "How can you use multiplication to find the volume?"

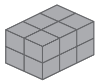
Name ______________________

Using the Structure of Rectangular Prisms to Determine Volume — ML 1.04

Modeled Review

Name: Gabriela

Determine the volume of the prism. Show or explain your thinking.

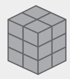
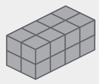
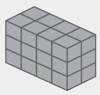


The bottom layer has 6 cubes.
There are 2 layers. 2 layers of 6 cubes is 12 cubes.

answer: 12 cubic units

Guided Practice

1. Complete the table.

Prism	Number of cubes in the bottom layer	Number of layers	Volume (cubic units)
	4	3	12 cubic units
	8	2	16 cubic units
	6	3	18 cubic units

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Vocabulary

If needed, share the meaning of the terms with students.

rectangular prism: A solid figure with 6 faces that are all rectangles.

unit cube: A cube, whose sides are 1 unit long, used to measure volume.

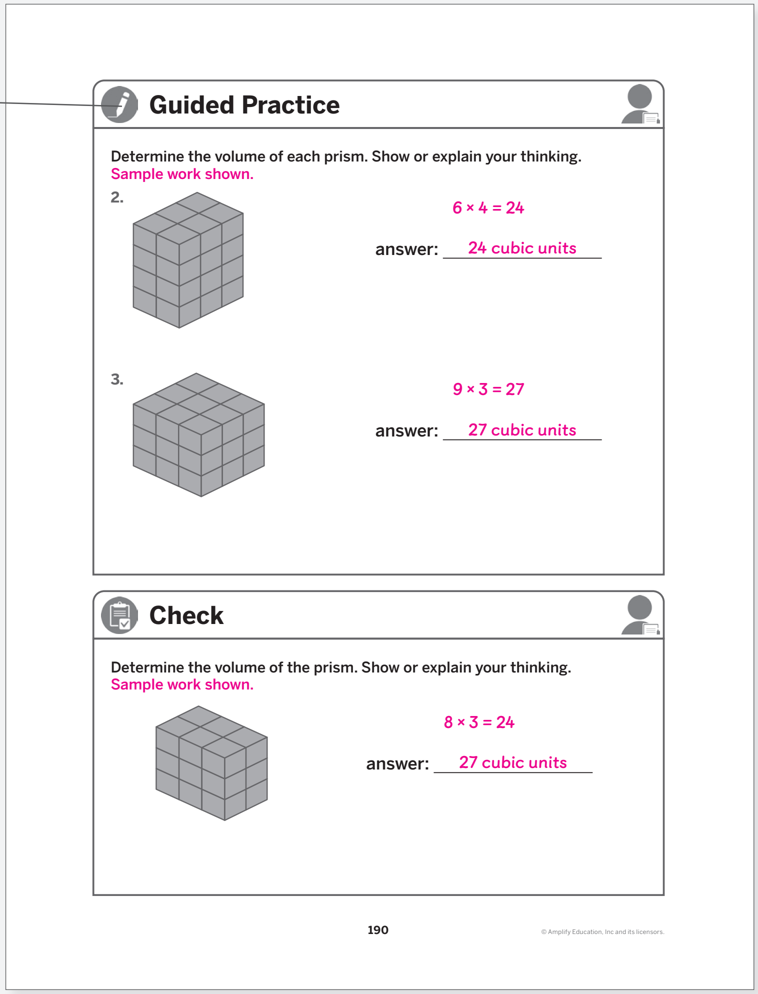
volume: The amount of space a three-dimensional figure takes up.

ML 1.04 Using the Structure of Rectangular Prisms to Determine Volume

Guided Practice

A Provide access to unit cubes and have students build the prisms they see in the images.

Key Takeaway:
Say, "The volume of a rectangular prism can be determined by counting equal groups of any one layer. No matter which layer you use, the volume will be the same."


Guided Practice

Determine the volume of each prism. Show or explain your thinking.
Sample work shown.

2. 6 × 4 = 24

answer: 24 cubic units

3. 9 × 3 = 27

answer: 27 cubic units

Check

Determine the volume of the prism. Show or explain your thinking.
Sample work shown.

8 × 3 = 24

answer: 27 cubic units

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Reflection

Ask:

- "Does the volume of a prism change depending on which side of the prism you are viewing? Why or why not?"
- "How was the lesson helpful to you today?"

Check: Recommended Next Steps

Almost there	**Got it!**
If students need more support, consider using unit cubes and have students build different rectangular prisms with specific dimensions and then calculating the volume.	If students need more practice, consider having them determine the volume of each prism. • A prism with 5 layers and 10 cubes on the bottom layer. • A prism with 8 layers and 5 cubes on the bottom layer.

Name ______________________________

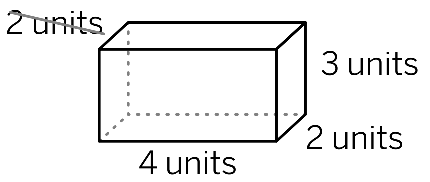
Determining the Volume of Solid Rectangular Prisms

ML 1.06

Modeled Review

Name: Han

Determine the volume of the prism in cubic units.

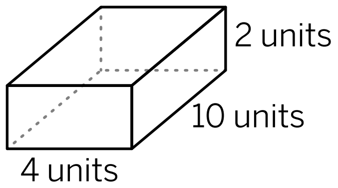


V = length × width × height
V = l × w × h
V = 4 × 2 × 3
answer: 24 cubic units

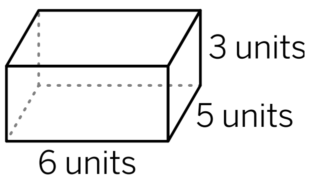
Guided Practice

1. Use the prisms to complete the table.

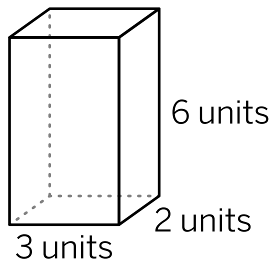
Prism A



Prism B



Prism C

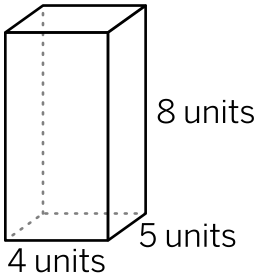


	length × width × height	volume (cubic units) $V = l \times w \times h$
Prism A	4 × 10 × 2	80 cubic units
Prism B	6 × ___ × ___	
Prism C	___ × ___ × ___	

Guided Practice

Determine the volume of each prism. Show or explain your thinking.

2.



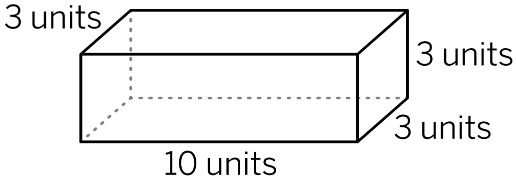
$V = l \times w \times h$

$V = 4 \times 5 \times 8$

$V =$ ______

answer: ______ cubic units

3.



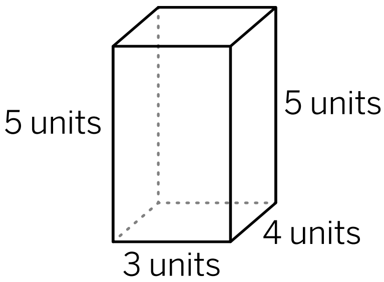
$V = l \times w \times h$

$V =$ ______ × ______ × ______

$V =$ ______

answer: ________________

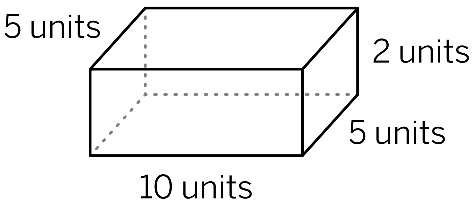
4.



answer: ________________

Check

Determine the volume of the prism. Show or explain your thinking.



answer: ________________

ML 1.06 Determining the Volume of Solid Rectangular Prisms

Goal

Calculate the volume of a rectangular prism using the formula $V = \ell \times w \times h$.

Standard

MA.5.GR.3.2

Materials

unit cubes (optional)

Modeled Review

Point to Han's work and **ask**:
- "Why did Han cross out one of the edge lengths?"
- "How does the expression Han wrote represent the volume of the prism?"

Reinforce the goal by saying, "The formula, length times width times height, can be used to determine the volume of a rectangular prism."

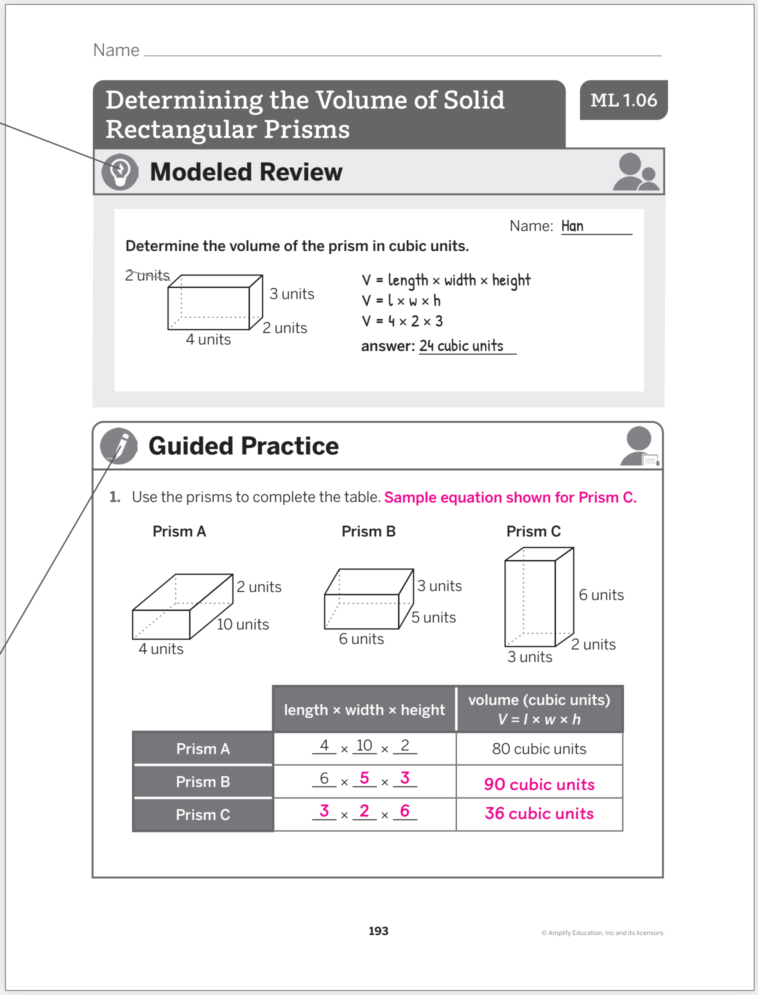
ML/EL If needed, model identifying the needed edge lengths to write an expression.

Guided Practice

Focus students' attention on using the dimensions of a rectangular prism to determine the volume.

To scaffold their thinking, **say**:
- "A rectangular prism has three dimensions–length, width, and height."
- "The product of these dimensions is the volume of the prism."

Note: Students may record the dimensions in any order.



Name

Determining the Volume of Solid Rectangular Prisms ML 1.06

Modeled Review

Name: Han

Determine the volume of the prism in cubic units.

~~2 units~~ 3 units 2 units 4 units

V = length × width × height
V = l × w × h
V = 4 × 2 × 3
answer: 24 cubic units

Guided Practice

1. Use the prisms to complete the table. Sample equation shown for Prism C.

Prism A: 2 units, 10 units, 4 units
Prism B: 3 units, 5 units, 6 units
Prism C: 6 units, 2 units, 3 units

	length × width × height	volume (cubic units) $V = l \times w \times h$
Prism A	4 × 10 × 2	80 cubic units
Prism B	6 × 5 × 3	90 cubic units
Prism C	3 × 2 × 6	36 cubic units

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Vocabulary

If needed, share the meaning of the terms with students.

Associative Property of Multiplication: The product of three or more numbers remains the same regardless of how the numbers are grouped.

rectangular prism: A solid figure with 6 faces that are all rectangles.

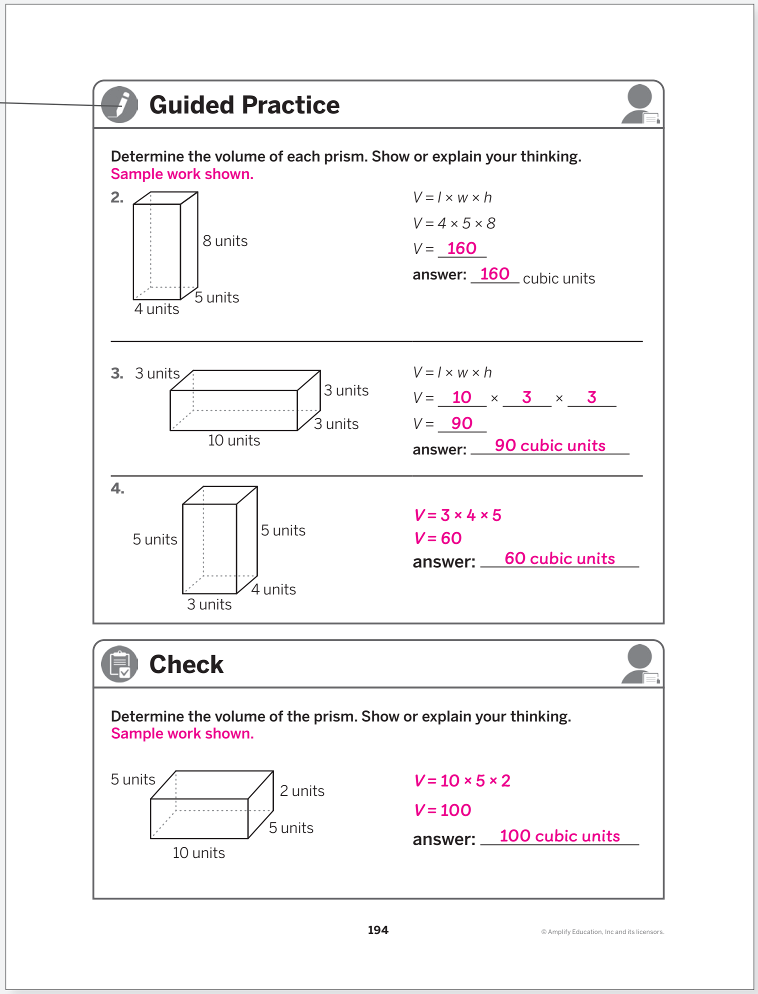
volume: The amount of space a three-dimensional figure takes up.

Determining the Volume of Solid Rectangular Prisms

Guided Practice

Ⓐ Remind students that sometimes more information may be given than is needed to calculate the volume. Invite students to circle or highlight the length, width, and height of each prism and cross out any edge lengths that are not needed.

Key Takeaway:
Say, "One way you can determine the volume of any rectangular prism is by using the formula $V = \ell \times w \times h$. The volume is measured in cubic units, such as cubic inches."



Guided Practice

Determine the volume of each prism. Show or explain your thinking.
Sample work shown.

2.

$V = l \times w \times h$
$V = 4 \times 5 \times 8$
$V =$ 160
answer: 160 cubic units

3.

$V = l \times w \times h$
$V =$ 10 × 3 × 3
$V =$ 90
answer: 90 cubic units

4.

$V = 3 \times 4 \times 5$
$V = 60$
answer: 60 cubic units

Check

Determine the volume of the prism. Show or explain your thinking.
Sample work shown.

$V = 10 \times 5 \times 2$
$V = 100$
answer: 100 cubic units

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Reflection

Ask:

- "How can you determine whether an expression represents the volume of a rectangular prism?"
- "What strategy was helpful today?"

Check: Recommended Next Steps

Almost there	Got it!
If students need more support, have them build a prism with cubes and connect the length, width, and height of the model with the formula.	If students need more practice, refer to the prism in the Modeled Review and ask them what the volume would be if the height was 6 units.

Name ______________________________

Multiplying Whole Numbers by Fractions and Mixed Numbers

ML 2.14

Modeled Review

Name: Diego

Evaluate the expressions. Explain your thinking.

1. $\frac{7}{4} \times 12$

$\frac{(7 \times 12)}{4}$

$84 \div 4 = 21$

2. $3\frac{2}{5} \times 8$

$(8 \times 3) + (8 \times \frac{2}{5})$

$24 + \frac{16}{5}$

$\frac{16}{5} = 3\frac{1}{5}$

$24 + 3\frac{1}{5} = 27\frac{1}{5}$

Guided Practice

Evaluate the equations.

1. $\frac{1}{4} \times 16 =$ ____________

2. $\frac{5}{9} \times 3 =$ ____________

3. $\frac{1}{12} \times 24 =$ ____________

Guided Practice

4. Evaluate the expressions in the table.

Expression	Work	Solution
$7\frac{2}{3} \times 3$	$(3 \times 7) + (3 \times \frac{2}{3})$	
$2\frac{3}{4} \times 3$	(3 × ____) + (3 × ____)	
$\frac{7}{3} \times 4$		
$6\frac{3}{4} \times 5$		
$\frac{15}{2} \times 5$		

Check

Evaluate the equations. Show your thinking.

1. $\frac{14}{3} \times 6 =$ ________

2. $3\frac{4}{5} \times 5 =$ ________

ML 2.14 Multiplying Whole Numbers by Fractions and Mixed Numbers

Goal

Solve problems involving multiplying whole numbers by fractions and mixed numbers.

Standard

MA.5.FR.2.2

Materials

fraction tiles (optional)

Modeled Review

Point to Diego's work and **ask**:

- "What strategy did Diego use to solve Problem 1? Problem 2?"
- "Which strategy do you think is more efficient? Why?"

Reinforce the goal by saying, "When multiplying whole numbers by mixed numbers, it can be more efficient to use the distributive property to solve rather than converting the mixed number into a fraction greater than 1 first."

Guided Practice

Focus students' attention on evaluating the equations.

To scaffold their thinking, **ask**:

- "What are you trying to determine in each equation?"
- "Would you evaluate each equation the same way? Why or why not?"
- "Would decomposing a mixed number be helpful in determining the solution? Why or why not?"

Name ______

Multiplying Whole Numbers by Fractions and Mixed Numbers ML 2.14

Modeled Review

Name: Diego

Evaluate the expressions. Explain your thinking.

1. $\frac{7}{4} \times 12$

   $\frac{(7 \times 12)}{4}$

   $84 \div 4 = 21$

2. $3\frac{2}{5} \times 8$

   $(8 \times 3) + (8 \times \frac{2}{5})$

   $24 + \frac{16}{5}$

   $\frac{16}{5} = 3\frac{1}{5}$

   $24 + 3\frac{1}{5} = 27\frac{1}{5}$

Guided Practice

Evaluate the equations. Sample responses shown.

1. $\frac{1}{4} \times 16 =$ $\frac{16}{4}$ or 4
2. $\frac{5}{9} \times 3 =$ $\frac{15}{9}$ or $1\frac{2}{3}$
3. $\frac{1}{12} \times 24 =$ $\frac{24}{12}$ or 2

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Vocabulary

If needed, share the meaning of the terms with students.

mixed number: A number expressed as a whole number and a fraction less than 1.

Distributive Property: Multiplying the sum of two or more addends by a factor will give the same result as multiplying each addend individually by the factor and then adding the products together.

Multiplying Whole Numbers by Fractions and Mixed Numbers

Guided Practice

(A) Model thinking aloud as you use arrows to annotate the connection between the expressions in the first two columns of the table.

(ML/EL) Provide students with fraction tiles to solve the problems to support making connections with decomposing a mixed number to determine a solution.

Key Takeaway:
Say, "When multiplying whole numbers and fractions, you can multiply by a fraction greater than 1 or by a mixed number. Consider the numbers to determine which way would be more helpful."

Guided Practice

4. Evaluate the expressions in the table. Sample work shown for Rows 3–5.

Expression	Work	Solution
$7\frac{2}{3} \times 3$	$(3 \times 7) + (3 \times \frac{2}{3})$	23
$2\frac{3}{4} \times 3$	$(3 \times \underline{2}) + (3 \times \underline{\frac{3}{4}})$	$8\frac{1}{4}$
$\frac{7}{3} \times 4$	$\frac{(7 \times 4)}{3} = \frac{28}{3}$	$9\frac{1}{3}$
$6\frac{3}{4} \times 5$	$(5 \times 6) + (5 \times \frac{3}{4})$	$33\frac{3}{4}$
$\frac{15}{2} \times 5$	$\frac{(15 \times 5)}{2} = \frac{75}{2}$	$37\frac{1}{2}$

Check

Evaluate the equations. Show your thinking. Sample work shown.

1. $\frac{14}{3} \times 6 = \underline{28}$

$\frac{(14 \times 6)}{3} = \frac{84}{3}$

$\frac{84}{3} = 28$

2. $3\frac{4}{5} \times 5 = \underline{19}$

$(5 \times 3) + (5 \times \frac{4}{5})$

$15 + \frac{20}{5}$

$15 + 4 = 19$

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Reflection

Ask:

- "Was decomposing a mixed number helpful in determining the solution to the equation? Why or why not?"
- "What is something you weren't sure about at the start of the lesson but understand now?"

Check: Recommended Next Steps

Almost there
If students need more support, consider using visual fraction models (area models, tape diagrams, number lines) created by students.

Got it!
If students need more practice, have them evaluate the following expressions and show their thinking.

- $6 \times 4\frac{3}{5}$
- $5 \times \frac{13}{4}$

Name ______________________________

Multiplying With Fractions and Mixed Numbers

ML 3.07

Modeled Review

Name: Priya

Determine the product of each expression. Show your thinking.

1. $\frac{2}{4} \times \frac{2}{3}$

$\frac{2}{4} \times \frac{2}{3} = \frac{2 \times 2}{4 \times 3} = \frac{4}{12}$

answer: $\frac{4}{12}$

2. $2\frac{2}{5} \times \frac{3}{4}$

$2\frac{2}{5} = \frac{5}{5} + \frac{5}{5} + \frac{2}{5} = \frac{12}{5}$ $\quad \frac{12 \times 3}{5 \times 4} = \frac{36}{20}$

answer: $\frac{36}{20}$

Guided Practice

Determine the product of each expression. Show your thinking.

1. $\frac{2}{3} \times \frac{3}{4}$

$\frac{2}{3} \times \frac{3}{4} = \frac{2 \times 3}{3 \times 4} = \frac{6}{\square}$

2. $\frac{4}{5} \times \frac{1}{2}$

$\frac{4}{5} \times \frac{1}{2} = \frac{4 \times 1}{5 \times 2} = \frac{\square}{10}$

3. $\frac{5}{6} \times \frac{1}{4}$

4. $\frac{4}{5} \times \frac{2}{3}$

5. $2\frac{1}{4} \times \frac{2}{5}$

$2\frac{1}{4} = \frac{4}{4} + \frac{4}{4} + \frac{1}{4} = \frac{\square}{4}$

$\frac{9}{4} \times \frac{2}{5} = \frac{\square \times 2}{4 \times 5} = \frac{\square}{\square}$

6. $2\frac{1}{3} \times \frac{4}{6}$

$2\frac{1}{3} = \frac{\square}{3} + \frac{\square}{3} + \frac{1}{3} = \frac{\square}{\square}$

$\frac{\square}{3} \times \frac{4}{6} = \frac{\square \times \square}{\square \times \square} = \frac{\square}{\square}$

Guided Practice

Determine the product of each expression. Show your thinking.

7. $1\frac{2}{4} \times \frac{2}{3}$

$1\frac{2}{4} =$

answer: ______________

8. $1\frac{3}{4} \times \frac{2}{3}$

answer: ______________

Check

Determine the product of each expression. Show your thinking.

1. $\frac{3}{4} \times \frac{4}{5}$

answer: ______________

2. $2\frac{2}{3} \times \frac{2}{5}$

answer: ______________

Multiplying With Fractions and Mixed Numbers

Goal	Standard
Determine the product of two fractions or a fraction and a mixed number.	MA.5.FR.2.2

Modeled Review

Point to Priya's work and **ask**:

- "What numbers does Priya multiply to calculate the product of the numerators? Denominators?"
- "What does Priya do to the mixed number before multiplying it by the other factor in the expression?"
- "What other strategies could be used to convert a mixed number to a fraction greater than 1?"

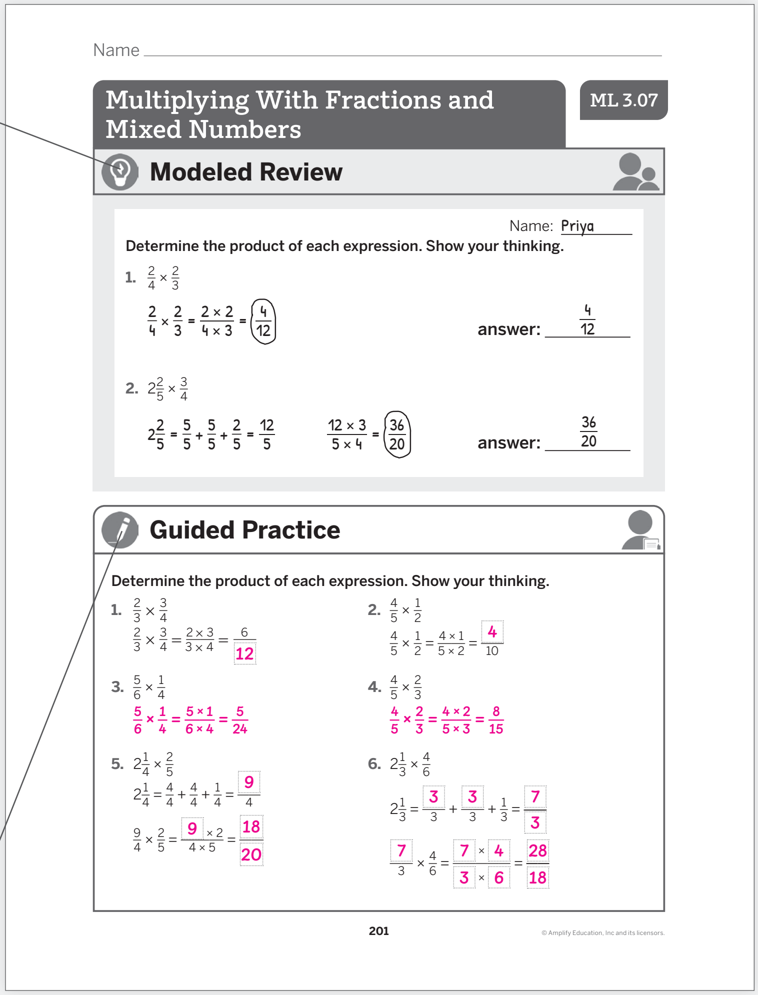
Reinforce the goal by saying, "You can determine the product of two fractions by multiplying the numerators of the factors and the denominators of the factors. When multiplying with mixed numbers, convert the whole number into a fraction and add the numerators. The denominator stays the same."

Guided Practice

Focus students' attention on following the steps to determine the product of each expression.

To scaffold their thinking, **ask**:

- "What is the first step you could take to determine the product of two fractions? Second step?"
- "What additional steps are needed when a mixed number is represented in the expression?"



Name

Multiplying With Fractions and Mixed Numbers — ML 3.07

Modeled Review

Name: Priya

Determine the product of each expression. Show your thinking.

1. $\frac{2}{4} \times \frac{2}{3}$

$\frac{2}{4} \times \frac{2}{3} = \frac{2 \times 2}{4 \times 3} = \frac{4}{12}$ answer: $\frac{4}{12}$

2. $2\frac{2}{5} \times \frac{3}{4}$

$2\frac{2}{5} = \frac{5}{5} + \frac{5}{5} + \frac{2}{5} = \frac{12}{5}$ $\frac{12 \times 3}{5 \times 4} = \frac{36}{20}$ answer: $\frac{36}{20}$

Guided Practice

Determine the product of each expression. Show your thinking.

1. $\frac{2}{3} \times \frac{3}{4}$

$\frac{2}{3} \times \frac{3}{4} = \frac{2 \times 3}{3 \times 4} = \frac{6}{12}$

2. $\frac{4}{5} \times \frac{1}{2}$

$\frac{4}{5} \times \frac{1}{2} = \frac{4 \times 1}{5 \times 2} = \frac{4}{10}$

3. $\frac{5}{6} \times \frac{1}{4}$

$\frac{5}{6} \times \frac{1}{4} = \frac{5 \times 1}{6 \times 4} = \frac{5}{24}$

4. $\frac{4}{5} \times \frac{2}{3}$

$\frac{4}{5} \times \frac{2}{3} = \frac{4 \times 2}{5 \times 3} = \frac{8}{15}$

5. $2\frac{1}{4} \times \frac{2}{5}$

$2\frac{1}{4} = \frac{4}{4} + \frac{4}{4} + \frac{1}{4} = \frac{9}{4}$

$\frac{9}{4} \times \frac{2}{5} = \frac{9 \times 2}{4 \times 5} = \frac{18}{20}$

6. $2\frac{1}{3} \times \frac{4}{6}$

$2\frac{1}{3} = \frac{3}{3} + \frac{3}{3} + \frac{1}{3} = \frac{7}{3}$

$\frac{7}{3} \times \frac{4}{6} = \frac{7 \times 4}{3 \times 6} = \frac{28}{18}$

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Vocabulary

If needed, share the meaning of the terms with students.

mixed number: A number expressed as a whole number and a fraction less than 1.

non-unit fraction: A number that describes more than one part of a whole that has been partitioned into equal parts.

product: The value when two or more numbers are multiplied.

Multiplying With Fractions and Mixed Numbers

Guided Practice

A Chunk this task into smaller, more manageable parts by having students first convert the mixed numbers to fractions greater than 1. Then, multiply.

ML/EL Model converting a mixed number to create a fraction greater than 1 by connecting it to the procedural representation in Problem 5.

Key Takeaway:
Say, "To determine the product of two fractions, multiply the numerators and then the denominators of the factors. Mixed numbers can be rewritten as fractions greater than 1."

Guided Practice

Determine the product of each expression. Show your thinking.
Sample work shown.

7. $1\frac{2}{4} \times \frac{2}{3}$

$1\frac{2}{4} = \frac{4}{4} + \frac{2}{4} = \frac{6}{4}$

$\frac{6}{4} \times \frac{2}{3} = \frac{6 \times 2}{4 \times 3} = \frac{12}{12}$

answer: $\frac{12}{12}$ or 1

8. $1\frac{3}{4} \times \frac{2}{3}$

$\frac{7}{4} \times \frac{2}{3} = \frac{7 \times 2}{4 \times 3} = \frac{14}{12}$

answer: $\frac{14}{12}$ or $\frac{7}{6}$ or $1\frac{1}{6}$

Check

Determine the product of each expression. Show your thinking.
Sample work shown.

1. $\frac{3}{4} \times \frac{4}{5}$

$\frac{3}{4} \times \frac{4}{5} = \frac{3 \times 4}{4 \times 5} = \frac{12}{20}$

answer: $\frac{12}{20}$ or $\frac{6}{10}$ or $\frac{3}{5}$

2. $2\frac{2}{3} \times \frac{2}{5}$

$\frac{8}{3} \times \frac{2}{5} = \frac{8 \times 2}{3 \times 5} = \frac{16}{15}$

answer: $\frac{16}{15}$ or $1\frac{1}{15}$

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Reflection

Ask:
- "What could you do to find the product of two fractions if one or both are mixed numbers?"
- "How was the lesson helpful to you today?"

Check: Recommended Next Steps

Almost there
If students need more support, consider allowing opportunities for students to use models to check their work.

Got it!
If students need more practice, invite them to determine the product for the following expressions:

- $1\frac{1}{3} \times \frac{2}{5}$
- $\frac{3}{4} \times 3\frac{2}{6}$

Name ______________________

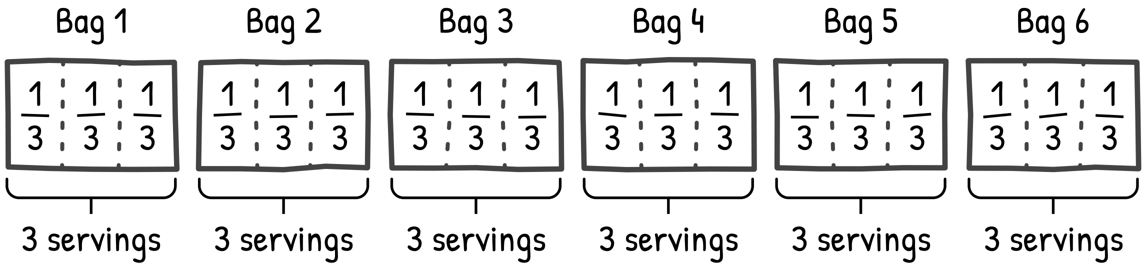
Representing Dividing Whole Numbers by Unit Fractions

ML 3.12

Modeled Review

Name: Jada

One serving of popcorn is $\frac{1}{3}$ of a bag. How many servings are there in 6 bags?



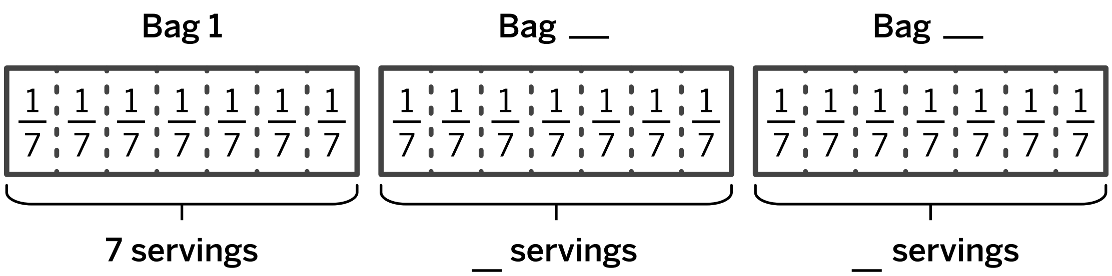
6 × 3 = 18 servings

equation: $6 \div \frac{1}{3} = 18$ **answer:** 18 servings

Guided Practice

Fill in the missing information in the diagram to show your thinking.

1. One serving of pretzels is $\frac{1}{7}$ of a bag. How many servings are there in 3 bags?

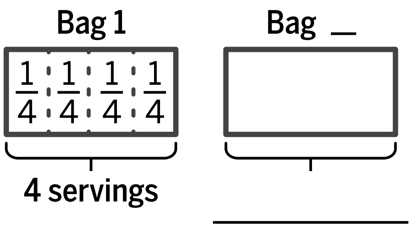


equation: $3 \div \frac{1}{7} =$ __________ **answer:** __________________

Guided Practice

Write a division equation to represent the situation. Then solve the problem. Show or explain your thinking.

2. One serving of granola is $\frac{1}{4}$ of a bag. How many servings are in 2 bags?



equation: ________________ **answer:** ________________

3. One serving of hamburger is $\frac{1}{3}$ pounds. How many hamburgers can be made with 5 pounds of hamburger meat?

equation: ________________ **answer:** ________________

Check

Write a division equation to represent the situation. Then solve the problem. Show or explain your thinking.

One serving of dog food is $\frac{1}{6}$ of a can. How many servings are there in 4 cans?

equation: ________________ **answer:** ________________

ML 3.12 Representing Dividing Whole Numbers by Unit Fractions

Goal

Represent a situation in which a whole number is divided by a unit fraction using diagrams and equations.

Standard

MA.5.FR.2.4

Materials

connecting cubes (optional)

Modeled Review

Point to Jada's work and **ask**:

- "What does $\frac{1}{3}$ represent? 6?"
- "How does Jada use the diagram to help her create a division equation?"
- "How does the diagram help Jada find the total number of servings?"

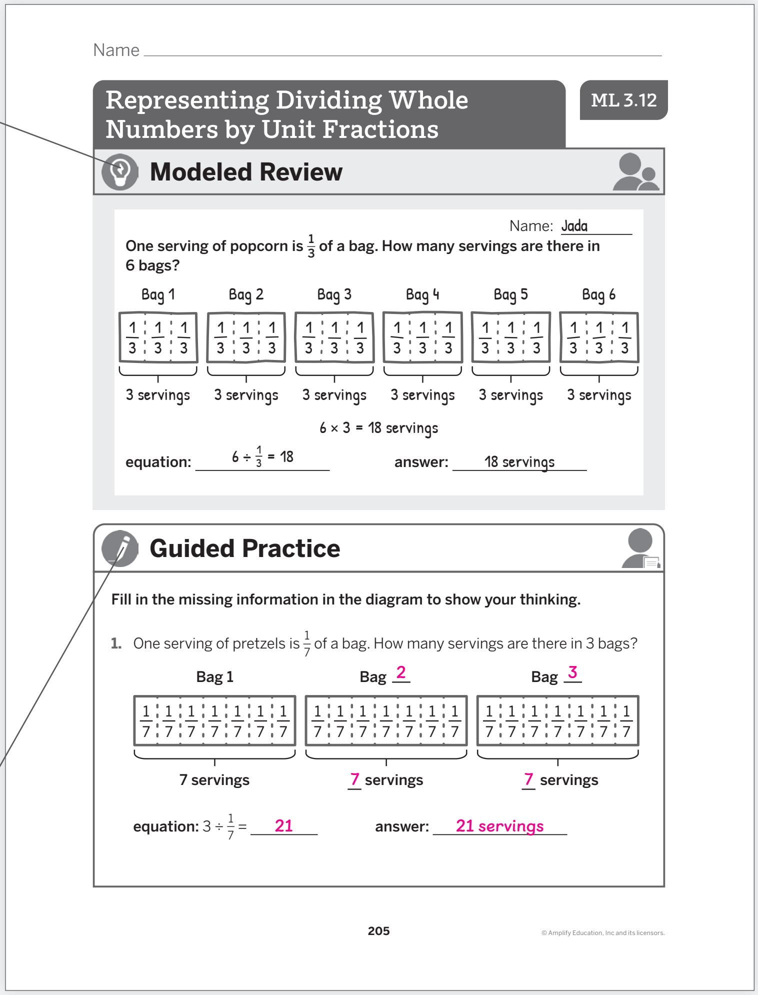
Reinforce Jada's thinking by saying, "You can use division equations and diagrams to represent equal-sharing situations. In the equation, the dividend is the total amount, the divisor is the size of the parts, and the quotient is the number of parts in the total amount."

Guided Practice

Focus students' attention on filling in the missing information in the diagram and using that to solve the division equation.

To scaffold their thinking, **ask**:

- "What do you know from the problem? How is that represented on the diagram?"
- "How can you find the quotient without counting each part in the diagram?"
- "What connection do you see between division and multiplication in each story problem?"



Name

Representing Dividing Whole Numbers by Unit Fractions — ML 3.12

Modeled Review

Name: Jada

One serving of popcorn is $\frac{1}{3}$ of a bag. How many servings are there in 6 bags?

Bag 1, Bag 2, Bag 3, Bag 4, Bag 5, Bag 6 — each divided into $\frac{1}{3}$ | $\frac{1}{3}$ | $\frac{1}{3}$; 3 servings each

$6 \times 3 = 18$ servings

equation: $6 \div \frac{1}{3} = 18$ answer: 18 servings

Guided Practice

Fill in the missing information in the diagram to show your thinking.

1. One serving of pretzels is $\frac{1}{7}$ of a bag. How many servings are there in 3 bags?

Bag 1, Bag 2, Bag 3 — each divided into sevenths ($\frac{1}{7}$); 7 servings, 7 servings, 7 servings

equation: $3 \div \frac{1}{7} =$ 21 answer: 21 servings

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Vocabulary

If needed, share the meaning of the terms with students.

dividend: The number being divided, representing the total number being equally distributed or divided.

divisor: The number that follows the division symbol that represents either the number of equal-sized groups or the size of each group.

quotient: The result obtained by dividing two numbers representing either the number of equal-sized groups or the size of each group.

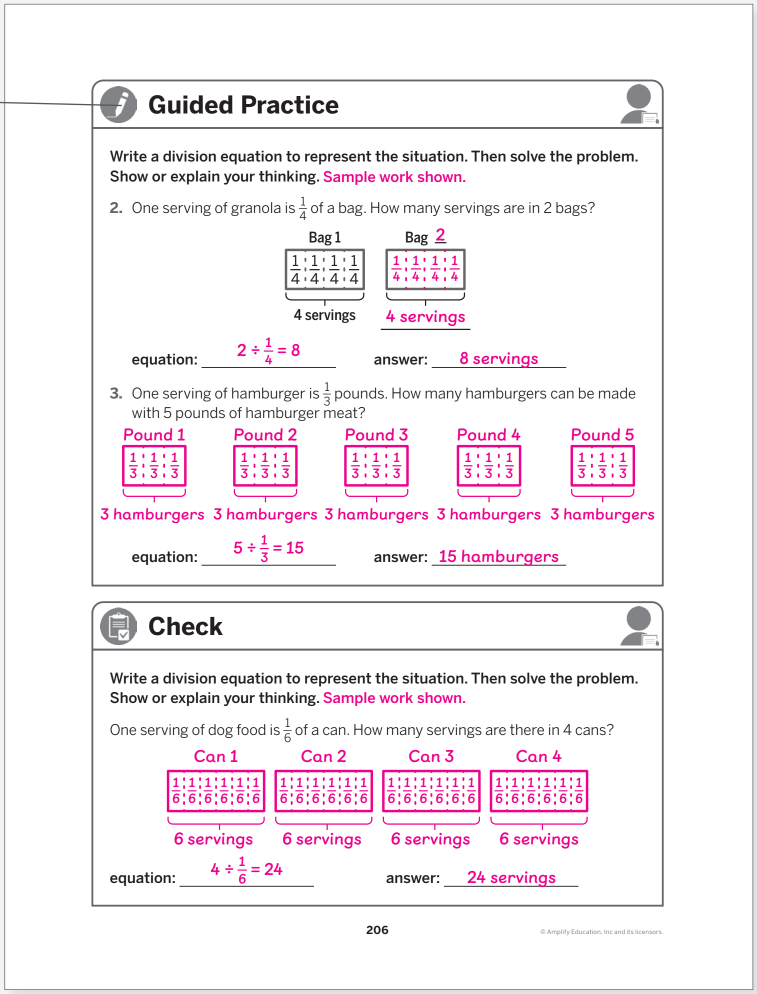
Representing Dividing Whole Numbers by Unit Fractions

Guided Practice

A Chunk this task into smaller, more manageable parts by having students first annotate the equation with what each number represents. Then, solve.

ML/EL Invite students to share how they created their diagram using the story problem.

Key Takeaway:
Say, "When determining the number of servings, you are determining how many fractional-sized parts fit into the dividend. The quotient represents the number of parts in all of the wholes."



Guided Practice

Write a division equation to represent the situation. Then solve the problem. Show or explain your thinking. Sample work shown.

2. One serving of granola is $\frac{1}{4}$ of a bag. How many servings are in 2 bags?

Bag 1: $\frac{1}{4}$ | $\frac{1}{4}$ | $\frac{1}{4}$ | $\frac{1}{4}$ — 4 servings
Bag 2: $\frac{1}{4}$ | $\frac{1}{4}$ | $\frac{1}{4}$ | $\frac{1}{4}$ — 4 servings

equation: $2 \div \frac{1}{4} = 8$ answer: 8 servings

3. One serving of hamburger is $\frac{1}{3}$ pounds. How many hamburgers can be made with 5 pounds of hamburger meat?

Pound 1: $\frac{1}{3}$ | $\frac{1}{3}$ | $\frac{1}{3}$ — 3 hamburgers
Pound 2: $\frac{1}{3}$ | $\frac{1}{3}$ | $\frac{1}{3}$ — 3 hamburgers
Pound 3: $\frac{1}{3}$ | $\frac{1}{3}$ | $\frac{1}{3}$ — 3 hamburgers
Pound 4: $\frac{1}{3}$ | $\frac{1}{3}$ | $\frac{1}{3}$ — 3 hamburgers
Pound 5: $\frac{1}{3}$ | $\frac{1}{3}$ | $\frac{1}{3}$ — 3 hamburgers

equation: $5 \div \frac{1}{3} = 15$ answer: 15 hamburgers

Check

Write a division equation to represent the situation. Then solve the problem. Show or explain your thinking. Sample work shown.

One serving of dog food is $\frac{1}{6}$ of a can. How many servings are there in 4 cans?

Can 1: $\frac{1}{6}$ | $\frac{1}{6}$ | $\frac{1}{6}$ | $\frac{1}{6}$ | $\frac{1}{6}$ | $\frac{1}{6}$ — 6 servings
Can 2: $\frac{1}{6}$ | $\frac{1}{6}$ | $\frac{1}{6}$ | $\frac{1}{6}$ | $\frac{1}{6}$ | $\frac{1}{6}$ — 6 servings
Can 3: $\frac{1}{6}$ | $\frac{1}{6}$ | $\frac{1}{6}$ | $\frac{1}{6}$ | $\frac{1}{6}$ | $\frac{1}{6}$ — 6 servings
Can 4: $\frac{1}{6}$ | $\frac{1}{6}$ | $\frac{1}{6}$ | $\frac{1}{6}$ | $\frac{1}{6}$ | $\frac{1}{6}$ — 6 servings

equation: $4 \div \frac{1}{6} = 24$ answer: 24 servings

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Reflection

Ask:
- "How does creating a diagram help you solve division problems?"
- "What strategy did someone else share today that was helpful?"

Check: Recommended Next Steps

Almost there
If students need more support, provide them with connecting cubes that can be used to represent each situation.

Got it!
If students need more practice, invite them to refer to the Modeled Review and ask them to find the amount of servings in 10 bags of popcorn.

Name ____________________

Connecting Division Expressions and Story Problems

ML 3.13

Modeled Review

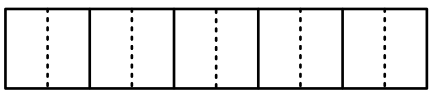
Story problem	Jada has 2 pounds of fish. She wants to serve each person $\frac{1}{4}$ pound of fish. How many people can she serve?	Jada and her friend split $\frac{1}{4}$ pound of fish evenly between them. How much fish does each person get?
Problem type	“how many parts”	equal sharing
Expression	$2 \div \frac{1}{4}$ (2: starting amount; $\frac{1}{4}$: size of each part)	$\frac{1}{4} \div 2$ ($\frac{1}{4}$: starting amount; 2: number of shares/groups)
Size of quotient	larger than 2	less than $\frac{1}{4}$

Guided Practice

Represent each story problem with the matching expression from the bank.

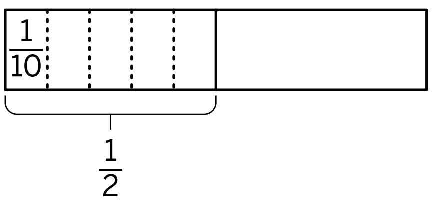
$\frac{1}{2} \div 5$	$5 \div \frac{1}{2}$

1. Jada has 5 cups of popcorn she wants to share with her friends. She gives each friend $\frac{1}{2}$ cup. How many friends were given popcorn?



____

2. Jada has $\frac{1}{2}$ pound of granola to split with friends. If she shares the granola between her and 4 friends, how much does each person get?



____

Guided Practice

Represent each story problem with the matching expression from the bank.

$4 \div \frac{1}{3}$	$\frac{1}{3} \div 4$

3. Jada has $\frac{1}{3}$ of a bag of dog food for her dog over the next 4 days. How much of the bag of food can her dog eat each day?

____

4. Jada bought 4 pounds of fruit at the market. A serving size of fruit is $\frac{1}{3}$ pound. How many single servings will Jada be able to make?

____

5. Jada brings 4 sub sandwiches to a picnic. She wants to cut each sandwich into thirds. How many people will she be able to serve?

____

6. Jada picked $\frac{1}{3}$ pound of blueberries. She wants to share the blueberries equally among her four friends. How many pounds of blueberries will each friend get?

____

Check

Represent each story problem with the matching expression from the bank.

$\frac{1}{4} \div 3$	$3 \div \frac{1}{4}$

1. Jada has 3 cups of popcorn. She gives each friend $\frac{1}{4}$ cup. How many friends were given popcorn?

____

2. Jada buys $\frac{1}{4}$ pound of granola to share between her and 2 friends. How much granola does each person get?

____

ML 3.13 Connecting Division Expressions and Story Problems

Goal

Relate story problems with division expressions that include whole numbers and unit fractions.

Standard

MA.5.AR.1.3

Modeled Review

Point to the chart in the Modeled Review and **ask**:

- "The starting amount is the dividend. What is the dividend in each story problem?"
- "In the first type of story problem, what does the fraction represent? In the second type?"

Reinforce the goal by saying, "You can relate story problems and division expressions by considering the meaning of the dividend and divisor, determining whether the problem is an equal-sharing or 'how many parts?' problem. You can reason about whether the quotient for the expression should be greater or less than the dividend."

ML/EL Model creating a diagram for each story problem in the Modeled Review.

Guided Practice

Focus students' attention on representing each story problem with an expression.

To scaffold their thinking, **ask**:

- "What is being shared? How many groups are sharing it?"
- "Is the quotient greater or less than the dividend? How do you know?"

Name ______________________

Connecting Division Expressions and Story Problems ML 3.13

Modeled Review

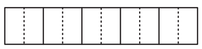
Story problem	Jada has 2 pounds of fish. She wants to serve each person $\frac{1}{4}$ pound of fish. How many people can she serve?	Jada and her friend split $\frac{1}{4}$ pound of fish evenly between them. How much fish does each person get?
Problem type	"how many parts"	equal sharing
Expression	$2 \div \frac{1}{4}$ (starting amount; size of each part)	$\frac{1}{4} \div 2$ (starting amount; number of shares/groups)
Size of quotient	larger than 2	less than $\frac{1}{4}$

Guided Practice

Represent each story problem with the matching expression from the bank.

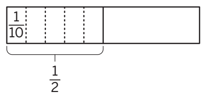
$\frac{1}{2} \div 5$ $5 \div \frac{1}{2}$

1. Jada has 5 cups of popcorn she wants to share with her friends. She gives each friend $\frac{1}{2}$ cup. How many friends were given popcorn?



$5 \div \frac{1}{2}$

2. Jada has $\frac{1}{2}$ pound of granola to split with friends. If she shares the granola between her and 4 friends, how much does each person get?



$\frac{1}{2} \div 5$

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Vocabulary

If needed, share the meaning of the terms with students.

dividend: The number being divided, representing the total number being equally distributed or divided.

divisor: The number that follows the division symbol that represents either the number of equal-sized groups or the size of each group.

quotient: The result obtained by dividing two numbers representing either the number of equal-sized groups or the size of each group.

ML 3.13 Connecting Division Expressions and Story Problems

Guided Practice

A Chunk this task into smaller, more manageable parts by reviewing Problem 3 with students before having them complete the remaining problems.

Key Takeaway:
Say, "You can connect story problems and division expressions by figuring out the dividend and divisor's roles, deciding if it's about equal sharing or 'how many parts?'"

Guided Practice

Represent each story problem with the matching expression from the bank.

$4 \div \frac{1}{3}$	$\frac{1}{3} \div 4$

3. Jada has $\frac{1}{3}$ of a bag of dog food for her dog over the next 4 days. How much of the bag of food can her dog eat each day?
   $\frac{1}{3} \div 4$

4. Jada bought 4 pounds of fruit at the market. A serving size of fruit is $\frac{1}{3}$ pound. How many single servings will Jada be able to make?
   $4 \div \frac{1}{3}$

5. Jada brings 4 sub sandwiches to a picnic. She wants to cut each sandwich into thirds. How many people will she be able to serve?
   $4 \div \frac{1}{3}$

6. Jada picked $\frac{1}{3}$ pound of blueberries. She wants to share the blueberries equally among her four friends. How many pounds of blueberries will each friend get?
   $\frac{1}{3} \div 4$

Check

Represent each story problem with the matching expression from the bank.

$\frac{1}{4} \div 3$	$3 \div \frac{1}{4}$

1. Jada has 3 cups of popcorn. She gives each friend $\frac{1}{4}$ cup. How many friends were given popcorn?
   $3 \div \frac{1}{4}$

2. Jada buys $\frac{1}{4}$ pound of granola to share between her and 2 friends. How much granola does each person get?
   $\frac{1}{4} \div 3$

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Reflection

Ask:

- "What strategies did you use to help you find the matching expression?"
- "How was the lesson helpful to you today?"

Check: Recommended Next Steps

Almost there
If students need more support, consider using Grade 5 Mini-Lesson 3.12: *Representing Dividing Whole Numbers by Unit Fractions*.

Got it!
If students need more practice, invite them to determine which expression from the Check matches the following story problem:

Jada bought 3 pounds of fruit. A serving size of fruit is $\frac{1}{4}$ pound. How many single servings will Jada be able to make?

Name ________________________________

Finding Products Using the Standard Algorithm With Composing in More Than One Place

ML 4.08

Modeled Review

Name: Jack

Determine the product using the standard algorithm.

583 × 67 = 39,061

			4	1	
			5	2	
			5	8	3
×				6	7
		4,	0	8	1
+	3	4,	9	8	0
	3	9,	0	6	1

Guided Practice

Determine the product using the standard algorithm.

1. 749 × 58 = ________

			2	4	
			3	7	
			7	4	9
×				5	8
		5,	9	9	2
+	3	7,	4	5	0

2. 326 × 89 = ________

			2	5	
			3	2	6
×				8	9
		2,	9	3	4
+					0

Guided Practice

Determine the product using the standard algorithm.

3. 248 × 75 = ________

$$\begin{array}{r} {\scriptstyle 2\ 4\ \ } \\ 2\ 4\ 8 \\ \times \quad 7\ 5 \\ \hline 1{,}\ 2\ 4\ 0 \\ +\ \square\ \square\ \square\ \square\ \square \\ \hline \square\ \square\ \square\ \square\ \square \end{array}$$

4. 395 × 62 = ________

$$\begin{array}{r} 3\ 9\ 5 \\ \times \quad 6\ 2 \\ \hline \square\ \square\ \square \\ +\ \square\ \square\ \square\ \square\ \square \\ \hline \square\ \square\ \square\ \square\ \square \end{array}$$

5. 417 × 86 = ________

6. 539 × 94 = ________

Check

Determine the product using the standard algorithm.

462 × 74 = ________

ML 4.08 Finding Products Using the Standard Algorithm With Composing in More Than One Place

Goal

Multiply two multi-digit whole numbers, in which units need to be composed in more than one place, using the standard algorithm.

Standard

MA.5.NSO.2.1

Materials

lined paper (optional)

Modeled Review

Point to Jack's work and **ask**:

- "What was Jack's first step?"
- "Where did the digits above 583 represent?"

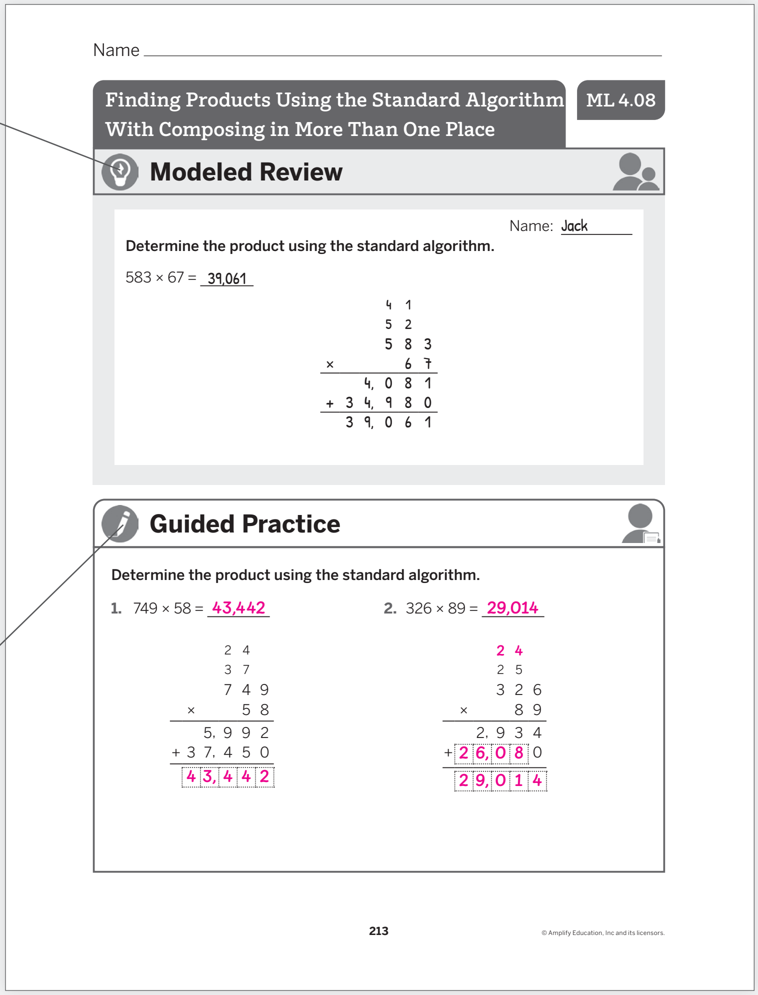
Reinforce Jack's thinking by saying, "Sometimes when using the standard algorithm, you may need to record more than one composed unit above the same place value. This occurs when you compose units for both digits in the second factor."

Guided Practice

Focus students' attention on determining the product using the standard algorithm.

To scaffold their thinking, **ask**:

- "Which two digits will you multiply first? Why?"
- "Which two digits, when multiplied, will require you to compose a unit? How do you know?"
- "In the standard algorithm, will there always be a 0 in the ones place in the second partial product? Why?"



Name

Finding Products Using the Standard Algorithm With Composing in More Than One Place — ML 4.08

Modeled Review

Name: Jack

Determine the product using the standard algorithm.

583 × 67 = 39,061

```
      4 1
      5 2
      5 8 3
×       6 7
   4, 0 8 1
+ 3 4, 9 8 0
  3 9, 0 6 1
```

Guided Practice

Determine the product using the standard algorithm.

1. 749 × 58 = 43,442

```
     2 4
     3 7
     7 4 9
×      5 8
  5, 9 9 2
+ 3 7, 4 5 0
  4 3, 4 4 2
```

2. 326 × 89 = 29,014

```
     2 4
     2 5
     3 2 6
×      8 9
  2, 9 3 4
+ 2 6, 0 8 0
  2 9, 0 1 4
```

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Vocabulary

If needed, share the meaning of the terms with students.

factor: A number that is multiplied by another number to determine the product.

product: The value when two or more numbers are multiplied.

ML 4.08 Finding Products Using the Standard Algorithm With Composing in More Than One Place

Guided Practice

(A) Provide students with lined paper oriented vertically to help organize the digits and products based on their value. Ensure that students align the products in their correct columns to help them keep track of the values as they add up the partial products to find the final product.

Key Takeaway:
Say, "Recording composed units above factors is a shorthand way to multiply single digits and add any composed units before recording the digits of the partial product."

Guided Practice

Determine the product using the standard algorithm.

3. 248 × 75 = 18,600

```
   3 5
   2 4
   2 4 8
×    7 5
  1, 2 4 0
+1 7, 3 6 0
 1 8, 6 0 0
```

4. 395 × 62 = 24,490

```
   5 3
   1 1
   3 9 5
×    6 2
     7 9 0
+2 3, 7 0 0
 2 4, 4 9 0
```

5. 417 × 86 = 35,862

```
   1 5
   1 4
   4 1 7
×    8 6
  2, 5 0 2
+3 3, 3 6 0
 3 5, 8 6 2
```

6. 539 × 94 = 50,666

```
   3 8
   1 3
   5 3 9
×    9 4
  2, 1 5 6
+4 8, 5 1 0
 5 0, 6 6 6
```

Check

Determine the product using the standard algorithm.

462 × 74 = 34,188

```
   4 1
   2
   4 6 2
×    7 4
  1, 8 4 8
+3 2, 3 4 0
 3 4, 1 8 8
```

214

Reflection

Ask:

- "How is composing units above the factors similar to how you composed units in previous lessons? How is it different?"
- "What is something you weren't sure about at the start of the lesson but understand now?"

Check: Recommended Next Steps

Almost there
If students need more support, consider creating a process/strategy anchor chart with students so they can use as a reference guide.

Got it!
If students need more practice, invite them to use the standard algorithm to determine the product for the following expressions:

- 429 × 26
- 639 × 48
- 794 × 53

Name ______________________________

Dividing Four-Digit Dividends by Two-Digit Divisors Without Remainders

ML 4.12

Modeled Review

Name: Jack

Determine the quotient.

5,676 ÷ 12 = 473

```
12 | 5, 6 7 6      (5→4, 6→16 regrouped)
   − 4, 8 0 0       12 × 400
        8 7 6
      − 8 4 0       12 × 70      473
          3 6
        − 3 6       12 × 3
            0
```

Guided Practice

Determine the quotient.

1. 8,640 ÷ 18 = ________

```
18 | 8, 6 4 0
   − 7, 2 0 0       18 × 400
     1, 4 4 0                    [ ]
   − 1, 4 4 0       18 × 80
             0
```

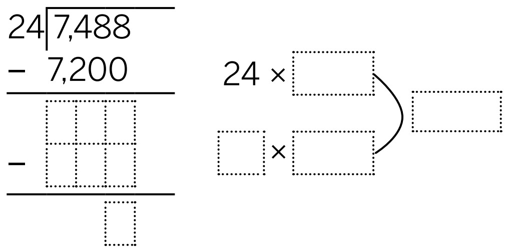
2. 9,801 ÷ 27 = ________

```
27 | 9, 8 0 1
   − 8, 1 0 0       27 × 300
     1, 7 0 1
   − 1, 6 2 0       27 × 60      [ ]
           [ ][ ]
         − [ ][ ]   27 × [ ]
              [ ]
```

Guided Practice

Determine the quotient.

3. 7,488 ÷ 24 = ________



4. 7,920 ÷ 12 = ________

12 | 7, 9 2 0

5. 9,408 ÷ 16 = ________

16 | 9,408

6. 8,670 ÷ 15 = ________

Check

Determine the quotient.

6,480 ÷ 15 = ________

ML 4.12 Dividing Four-Digit Dividends by Two-Digit Divisors Without Remainders

Goal

Divide a four-digit dividend by a two-digit divisor without remainders.

Standard

MA.5.NSO.2.2

Materials

multiplication chart (optional)

Modeled Review

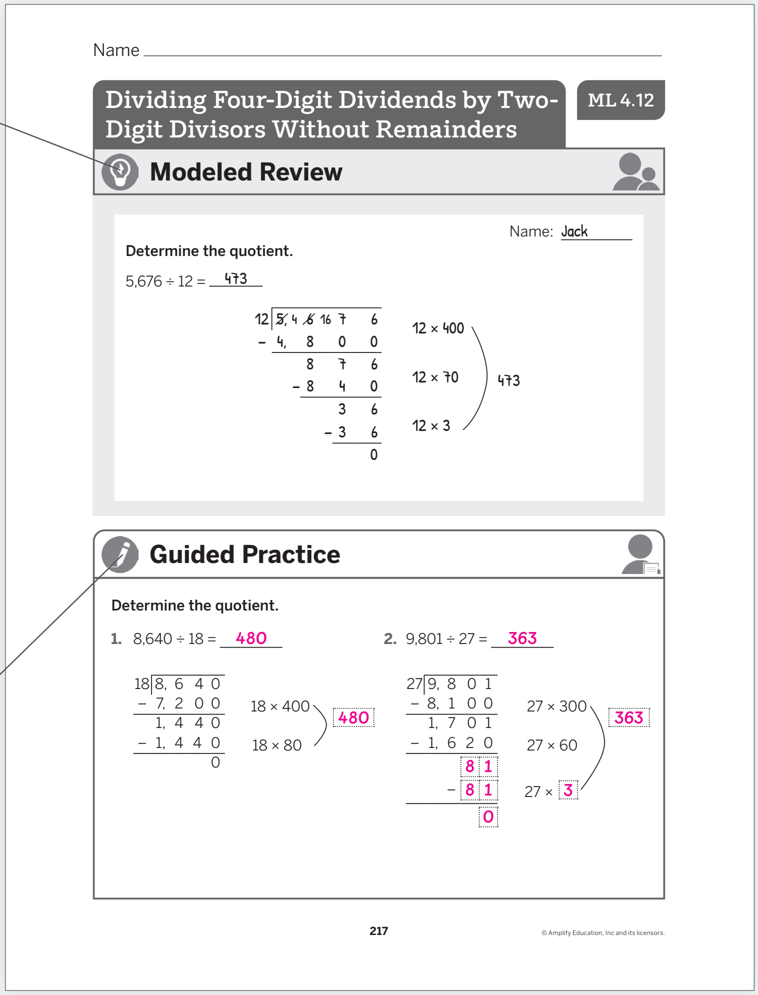
Point to Jack's work and **ask**:
- "What did Jack do as his first step?"
- "What did Jack do to get 4,800, 840, and 36?"
- "Why did Jack combine 400, 70, and 3?"

Reinforce Jack's thinking by saying, "No matter the size of the quotient, you can strategically choose your partial quotients by thinking about place value, multiplication facts, and doubles."

Guided Practice

Focus students' attention on determining the quotient.

To scaffold their thinking, **ask**:
- "When might you start with one hundred or more than one hundred for your first partial quotient?"
- "What will be the place value of the quotient? How do you know?"



Name __________

Dividing Four-Digit Dividends by Two-Digit Divisors Without Remainders

ML 4.12

Modeled Review

Name: Jack

Determine the quotient.

5,676 ÷ 12 = 473

12 | 5,676
− 4,800 (12 × 400)
876
− 840 (12 × 70)
36
− 36 (12 × 3)
0

473

Guided Practice

Determine the quotient.

1. 8,640 ÷ 18 = 480

18 | 8,640
− 7,200 (18 × 400)
1,440
− 1,440 (18 × 80)
0

480

2. 9,801 ÷ 27 = 363

27 | 9,801
− 8,100 (27 × 300)
1,701
− 1,620 (27 × 60)
81
− 81 (27 × 3)
0

363

217

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Vocabulary

If needed, share the meaning of the terms with students.

dividend: The number being divided, representing the total number being equally distributed or divided.

divisor: Either the number of equal-sized groups or the size of each group in a division expression.

partial quotients: The value obtained when multiplying the divisor in a division equation by another number, then subtracting that number from the dividend (which is then added to other partial quotients to arrive at the total quotient).

Dividing Four-Digit Dividends by Two-Digit Divisors Without Remainders

Guided Practice

(A) Provide students with a multiplication chart to help them strategically choose partial quotients by place.

(ML/EL) As students discuss how they determined their partial quotients, provide them with sentence frames to support their discussion. For example, "First, I thought about _____ because ____" or "I chose _____ as my first partial quotient because _____."

Key Takeaway:
Say, "Partial quotient strategies include using place value understanding, estimation, known facts, doubling, and halving."

Guided Practice

Determine the quotient. Sample work shown in Problems 4–6.

3. 7,488 ÷ 24 = 312

24)7,488
− 7,200
288
− 288
0

24 × 300
24 × 12
312

4. 7,920 ÷ 12 = 660

12)7, 9 2 0
− 7, 2 0 0
7 2 0
− 7 2 0
0

12 × 600
12 × 60
660

5. 9,408 ÷ 16 = 588

16)9,408
− 8,000
1,408
− 1,280
128
− 128
0

16 × 500
16 × 80
16 × 8
588

6. 8,670 ÷ 15 = 578

15)8,670
− 7,500
1,170
− 1,050
120
− 120
0

15 × 500
15 × 70
15 × 8
578

Check

Determine the quotient. Sample work shown.

6,480 ÷ 15 = 432

15)6,480
− 6,000
480
− 450
30
− 30
0

15 × 400
15 × 30
15 × 2
432

218

Reflection

Ask:

- "How can you use place value to help you determine which partial quotients to use when dividing with multi-digit numbers?"
- "How does what you learned today connect to your prior learning?"

Check: Recommended Next Steps

Almost there
If students need more support, consider modeling with estimations with reasonable values for quotients when dividing by two digit divisors.

Got it!
If students need more practice, invite them to determine the quotient for the following expressions:

- 8,064 ÷ 16
- 3,762 ÷ 18
- 4,032 ÷ 21
- 7,920 ÷ 24

Name ______________________________

Determining Missing Side Lengths in Area and Volume Problems

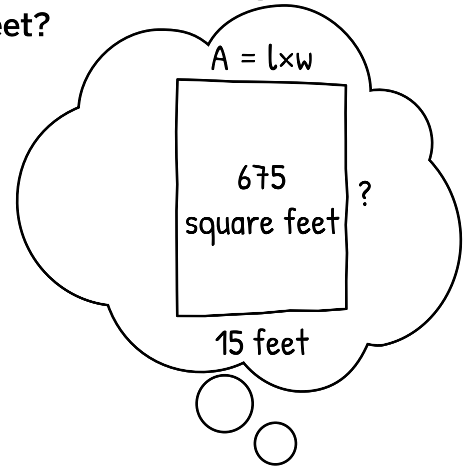
ML 4.13

Modeled Review

Name: Santiago

A rectangular garden has an area of 675 square feet and a length of 15 feet. What is the width of the garden, in feet?

```
15 ) 6 7 5
   - 6 0 0    15 × 40
       7 5                40
   -   7 5    15 × 5    +  5
         0                45
```

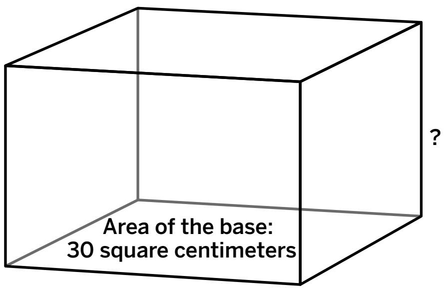


answer: 45 feet

Guided Practice

Determine the missing height of the prism in feet, using $V = B \times h$.

1. A rectangular prism has a volume of 1,050 cubic centimeters. The area of the base is 30 square centimeters. What is the height of the prism, in centimeters?



V = 1,050 cubic centimeters

```
30 ) 1, 0 5 0
   -    9 0 0    30 × 30
        _ _ _                30
      - _ _ _    _____     +  _
            _                _ _
```

answer: _____ **centimeters**

Guided Practice

Determine the missing dimension in each problem, using *A* = *b* × *h* or *V* = *B* × *h*.

2. A rectangular garden has an area of 1,272 square feet and a width of 53 feet. What is the length of the garden, in feet?

answer: ______________

3. A rectangular prism has a volume of 854 cubic feet. The area of the base is 61 square feet. What is the height of the prism in feet?

answer: ________________

Check

A rectangular garden has an area of 725 square feet and a length of 25 feet. What is the width of the garden, in feet?

answer: ______________

ML 4.13 Determining Missing Side Lengths in Area and Volume Problems

Goal

Calculate missing side or edge lengths when given information related to the area of rectangles and the volume of rectangular prisms by using the relationship between multiplication and division.

Standard	Materials
MA.5.GR.3.3	multiplication chart (optional)

Modeled Review

Point to Santiago's work and **ask**:

- "Why did Santiago divide?"
- "What multiplication strategy did Santiago use?"
- "How can Santiago check his answer?"

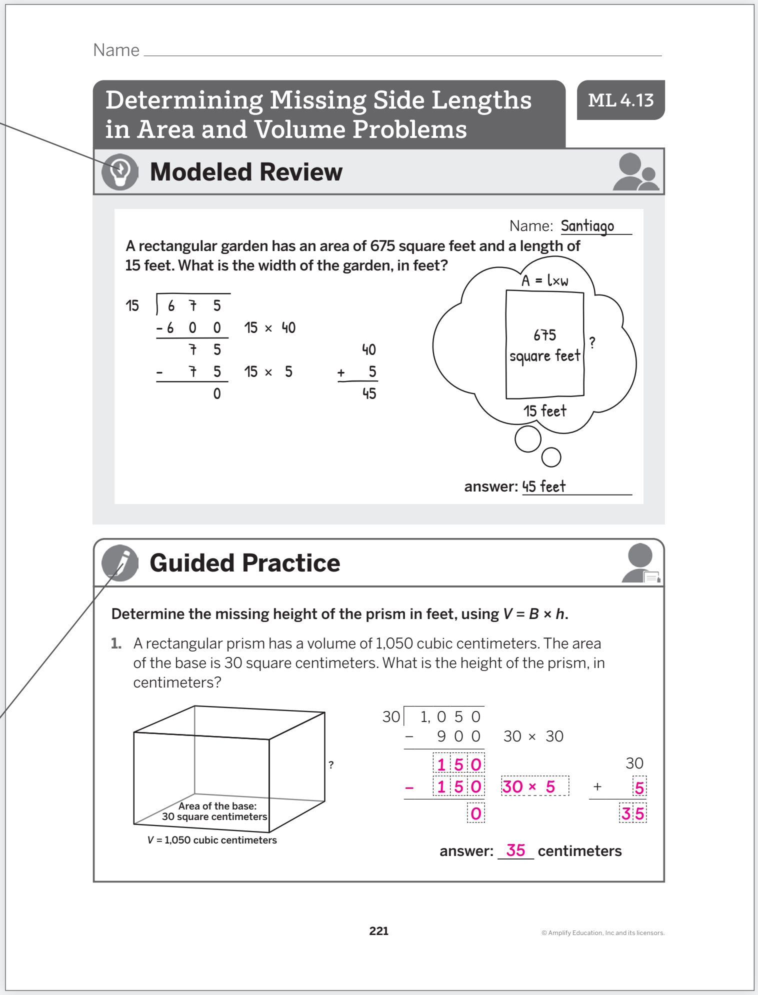
Reinforce Santiago's thinking by saying, "The missing side can be calculated using the given information and the relationship between multiplication and division, then applying the partial quotients method."

Guided Practice

Focus students' attention on calculating the height of the rectangular prism using place value to choose partial quotients.

To scaffold their thinking, **say**:

- "First, identify the known values and the missing dimension."
- "Next, set up the division problem."
- "Last, use partial quotients to solve and check your solution."



Name

Determining Missing Side Lengths in Area and Volume Problems — ML 4.13

Modeled Review

Name: Santiago

A rectangular garden has an area of 675 square feet and a length of 15 feet. What is the width of the garden, in feet?

15) 675
− 600 (15 × 40)
75
− 75 (15 × 5)
0

40 + 5 = 45

A = l×w; 675 square feet; ?; 15 feet

answer: 45 feet

Guided Practice

Determine the missing height of the prism in feet, using $V = B \times h$.

1. A rectangular prism has a volume of 1,050 cubic centimeters. The area of the base is 30 square centimeters. What is the height of the prism, in centimeters?

Area of the base: 30 square centimeters; ?; V = 1,050 cubic centimeters

30) 1,050
− 900 (30 × 30)
150
− 150 (30 × 5)
0

30 + 5 = 35

answer: 35 centimeters

221

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Vocabulary

If needed, share the meaning of the terms with students.

area: A measurement of the amount of space inside a two-dimensional shape (shown here as the number of square units that cover the shape without gaps or overlaps.)

volume: The amount of space a three-dimensional figure takes up.

rectangular prism: A solid shape with six rectangular faces.

ML 4.13 Determining Missing Side Lengths in Area and Volume Problems

Guided Practice

(A) Provide students with a multiplication chart to help them strategically choose partial quotients by place.

(ML/EL) To increase accessibility, provide students with questions they can ask themselves as they determine missing side lengths in area and volume problems. For example, "What information is given?" and "Which values do I need to divide?"

Key Takeaway:
Say, "You can use the relationship between multiplication and division to reason about the missing side lengths in area and volume problems involving rectangular prisms."

Guided Practice

Determine the missing dimension in each problem, using $A = b \times h$ or $V = B \times h$.
Sample work shown.

2. A rectangular garden has an area of 1,272 square feet and a width of 53 feet. What is the length of the garden, in feet?

```
53 | 1, 2 7 2
   - 1, 0 6 0    53 × 20      20
         2 1 2              +  4
   -     2 1 2   53 × 4       24
             0
```

answer: 24 feet

3. A rectangular prism has a volume of 854 cubic feet. The area of the base is 61 square feet. What is the height of the prism in feet?

```
61 | 8 5 4
   - 6 1 0   61 × 10      10
     2 4 4              +  4
   - 2 4 4   61 × 4       14
         0
```

answer: 14 feet

Check

A rectangular garden has an area of 725 square feet and a length of 25 feet. What is the width of the garden, in feet? Sample work shown.

```
25 | 7 2 5
   - 5 0 0   25 × 20      20
     2 2 5              +  9
   - 2 2 5   25 × 9       29
         0
```

answer: 29 feet

222

Reflection

Ask:

- "Describe a strategy for identifying the given measurements in a word problem."
- "How did you overcome a hard problem today?"

Check: Recommended Next Steps

Almost there
If students need more support, consider drawing a sketch of the problem to help them visualize and understand what is provided and what needs to be solved for.

Got it!
If students need more practice, ask them to determine the missing side length using V = B x h.

A rectangular box has a volume of 432 cubic centimeters. The area of its base is 36 square centimeters. What is the height of the box in centimeters?

Name ____________________

Selecting Equations With Parentheses to Represent Multi-Step Story Problems

ML 4.16

Modeled Review

Name: Maya

Which equation represents the story problem?

A school orders 156 whiteboard markers each month to be split evenly between 12 teachers. How many markers does each teacher receive after 10 months?

A. (156 × 12) ÷ 10 = ?

B. (156 ÷ 12) ÷ 10 = ?

C. (156 ÷ 10) × 12 = ?

D. (156 ÷ 12) × 10 = ?

Guided Practice

For Problems 1 and 2, select an expression that matches the problem.

1. A school purchases 24 packs of paper each month to split evenly between 3 printers. How many packs of paper are purchased for each printer after 4 months?

A. (24 ÷ 3) × 4

B. (24 ÷ 4) × 3

2. Twice a year, a school purchases 80 packs of pencils for 8 classrooms. How many packs of pencils does each classroom get per year?

A. (80 × 2) ÷ 8

B. (8 × 2) × 80

Guided Practice

For Problems 3–5, select the equation that represents the problem.

3. A school purchases 20 new books for the class library that are split between 5 bookshelves. If the school purchases books 4 times a year, how many books are on each bookshelf at the end of the year?

A. $(20 \times 5) \div 4 = ?$ **B.** $(20 \div 5) \times 4 = ?$

C. $(20 \div 5) \div 4 = ?$ **D.** $(20 \div 4) \times 5 = ?$

4. Three times a year, a school purchases 200 erasers for 12 classrooms. How many erasers does each classroom receive for the entire year?

A. $(200 \div 12) \div 3 = ?$ **B.** $(200 \div 3) \times 12 = ?$

C. $(200 \times 12) \div 3 = ?$ **D.** $(200 \times 3) \div 12 = ?$

5. A teacher purchases 125 highlighters for 25 students. If she purchases that same amount twice during the year, how many highlighters does each student receive?

A. $(125 \times 25) \div 2 = ?$ **B.** $(125 \div 2) \times 25 = ?$

C. $(125 \div 25) \times 2 = ?$ **D.** $(125 \div 2) \div 25 = ?$

Check

Which equation represents the story problem?

A principal purchases 75 stickers to split evenly between 5 teachers. If she purchases stickers three times a year, how many stickers does each teacher receive by the end of the year?

A. $(75 \div 5) \times 3 = ?$ **B.** $(75 \times 5) \div 3 = ?$

C. $(75 \div 3) \times 5 = ?$ **D.** $(75 \div 5) \div 3 = ?$

Selecting Equations With Parentheses to Represent Multi-Step Story Problems

Goal

Select equations using parentheses to represent multi-step story problems.

Standard

MA.5.AR.1.1

Modeled Review

Point to Maya's work and **ask**:

- "What do the parentheses represent in each equation?"
- "How did Maya determine that the equation should start with 156 ÷ 12?"
- "Why did Maya decide to select an equation that ends with multiplying by 10?"

Reinforce Maya's thinking by saying, "Parentheses are grouping symbols that can be used in expressions and equations. They indicate what is evaluated first."

ML/EL Use gestures by cupping hands in the shape of parentheses or pointing to parentheses in displayed work to provide visual cues on what is evaluated first.

Guided Practice

Focus students' attention on finding information needed to select a two-step equation using parentheses.

To scaffold their thinking, **ask**:

- "What do you know? What information do you need to determine?"
- "What is the first step? How can you use parentheses to show that step?"

Name ______________________

Selecting Equations With Parentheses to Represent Multi-Step Story Problems ML 4.16

Modeled Review

Name: Maya

Which equation represents the story problem?

A school orders 156 whiteboard markers each month to be split evenly between 12 teachers. How many markers does each teacher receive after 10 months?

A. (156 × 12) ÷ 10 = ?   **B.** (156 ÷ 12) ÷ 10 = ?

C. (156 ÷ 10) × 12 = ?   **D.** (156 ÷ 12) × 10 = ? (circled)

Guided Practice

For Problems 1 and 2, select an expression that matches the problem.

1. A school purchases 24 packs of paper each month to split evenly between 3 printers. How many packs of paper are purchased for each printer after 4 months?

   A. (24 ÷ 3) × 4 (circled)   **B.** (24 ÷ 4) × 3

2. Twice a year, a school purchases 80 packs of pencils for 8 classrooms. How many packs of pencils does each classroom get per year?

   A. (80 × 2) ÷ 8 (circled)   **B.** (8 × 2) × 80

225

Vocabulary

If needed, share the meaning of the term with students.

parenthesis: Grouping symbols that can be used in expressions or equations, such as (3 x 5) + (2 x 10).

ML 4.16 Selecting Equations With Parentheses to Represent Multi-Step Story Problems

Guided Practice

A Check for understanding by inviting students to rephrase the problem in their own words. Listen for and clarify any questions or misconceptions students have as they make sense of the problem.

Key Takeaway:
Say, "Sometimes multi-step problems can be solved using a different order of steps. Because of this, the corresponding equations will show different operations grouped using parentheses."

Guided Practice

For Problems 3–5, select the equation that represents the problem.

3. A school purchases 20 new books for the class library that are split between 5 bookshelves. If the school purchases books 4 times a year, how many books are on each bookshelf at the end of the year?

A. (20 × 5) ÷ 4 = ? **B.** (20 ÷ 5) × 4 = ?
C. (20 ÷ 5) ÷ 4 = ? **D.** (20 ÷ 4) × 5 = ?

4. Three times a year, a school purchases 200 erasers for 12 classrooms. How many erasers does each classroom receive for the entire year?

A. (200 ÷ 12) ÷ 3 = ? **B.** (200 ÷ 3) × 12 = ?
C. (200 × 12) ÷ 3 = ? **D.** (200 × 3) ÷ 12 = ?

5. A teacher purchases 125 highlighters for 25 students. If she purchases that same amount twice during the year, how many highlighters does each student receive?

A. (125 × 25) ÷ 2 = ? **B.** (125 ÷ 2) × 25 = ?
C. (125 ÷ 25) × 2 = ? **D.** (125 ÷ 2) ÷ 25 = ?

Check

Which equation represents the story problem?

A principal purchases 75 stickers to split evenly between 5 teachers. If she purchases stickers three times a year, how many stickers does each teacher receive by the end of the year?

A. (75 ÷ 5) × 3 = ? **B.** (75 × 5) ÷ 3 = ?
C. (75 ÷ 3) × 5 = ? **D.** (75 ÷ 5) ÷ 3 = ?

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Reflection

Ask:

- "What is the purpose of parentheses in an equation?"
- "What makes sense? What is still confusing?"

Check: Recommended Next Steps

Almost there
If students need more support, allowing students an opportunity to practice with word problems that require multiplication or division which can be solved by using drawings and equations.

Got it!
If students need more practice, read the problem aloud and have them choose the equation that matches the story problem.

A school buys 240 boxes of tissues each month to split evenly between 20 teachers. How many boxes does each teacher receive after 10 months?

A. (240 ÷ 10) x 20 B. (240 ÷ 20) x 10
C. (240 ÷ 20) ÷ 10 D. (240 x 20) x 10

Name ______________________________

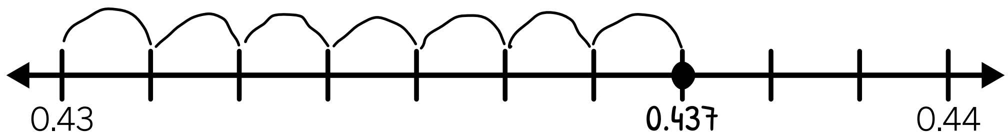
Locating Decimals on Number Lines

ML 5.05

Modeled Review

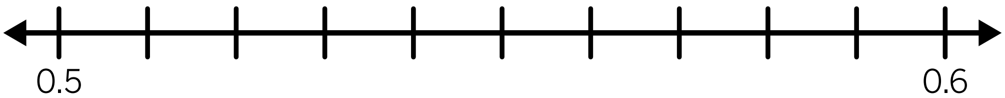
Name: Diego

1. Locate and label 0.437 on the number line.

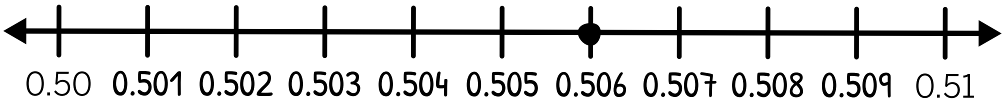


2. Which number line could you use to precisely locate and label 0.506 on a tick mark?

A.



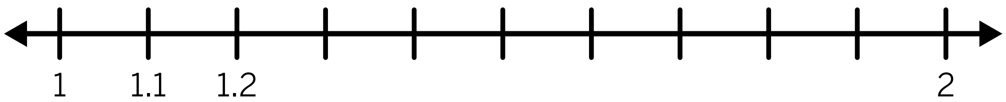
B. (circled)



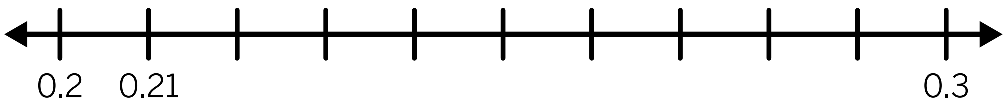
Guided Practice

Locate and label each value on the number line.

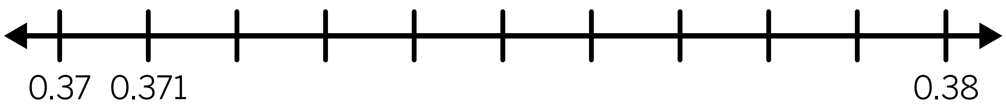
1. 1.6



2. 0.25

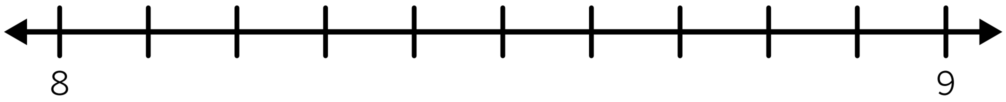


3. 0.376

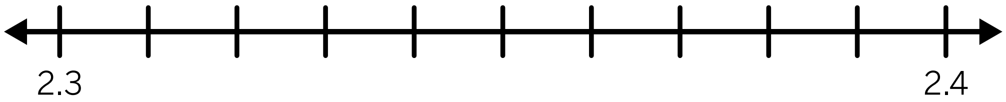


Guided Practice

4. Locate and label 8.7 on the number line.

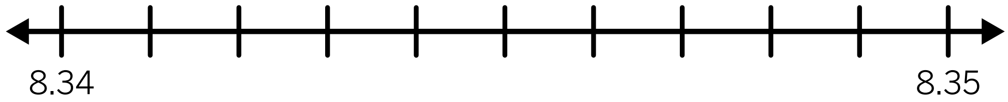


5. Locate and label 2.34 on the number line.

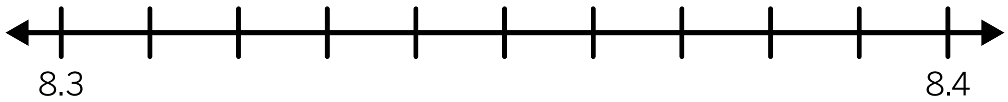


6. Which number line could you use to precisely locate and label 8.346 on a tick mark?

A.

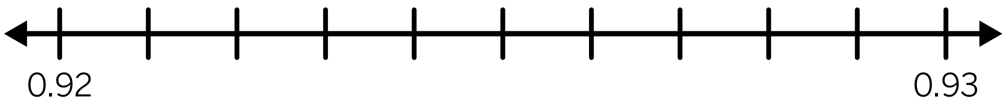


B.



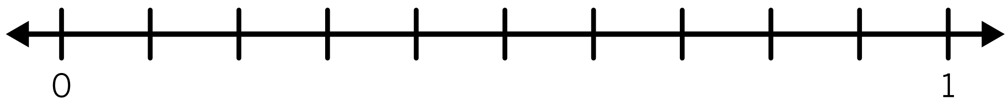
Check

1. Locate and label 0.924 on the number line.

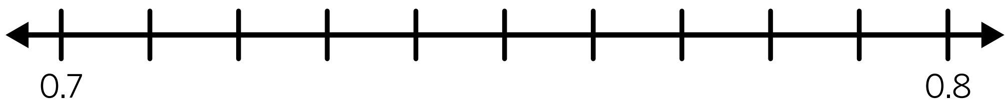


2. Which number line could you use to precisely locate and label 0.703 on a tick mark?

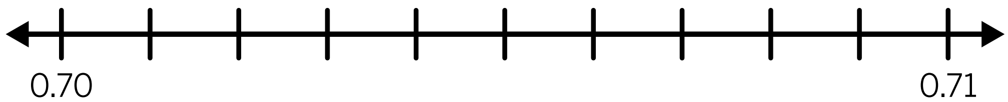
A.



B.



C.



Locating Decimals on Number Lines

Goal	**Standard**
Locate and label decimals to the thousandths on a number line.	MA.5.NSO.1.4

Modeled Review

Point to Diego's work and **ask**:

- "In Problem 1, what is the number line counting by? How do you know?"
- "How did Diego locate and label 0.437 on the number line?"
- "In Problem 2, could you locate 0.506 on the first number line? Why is it not as precise as the second number line?"

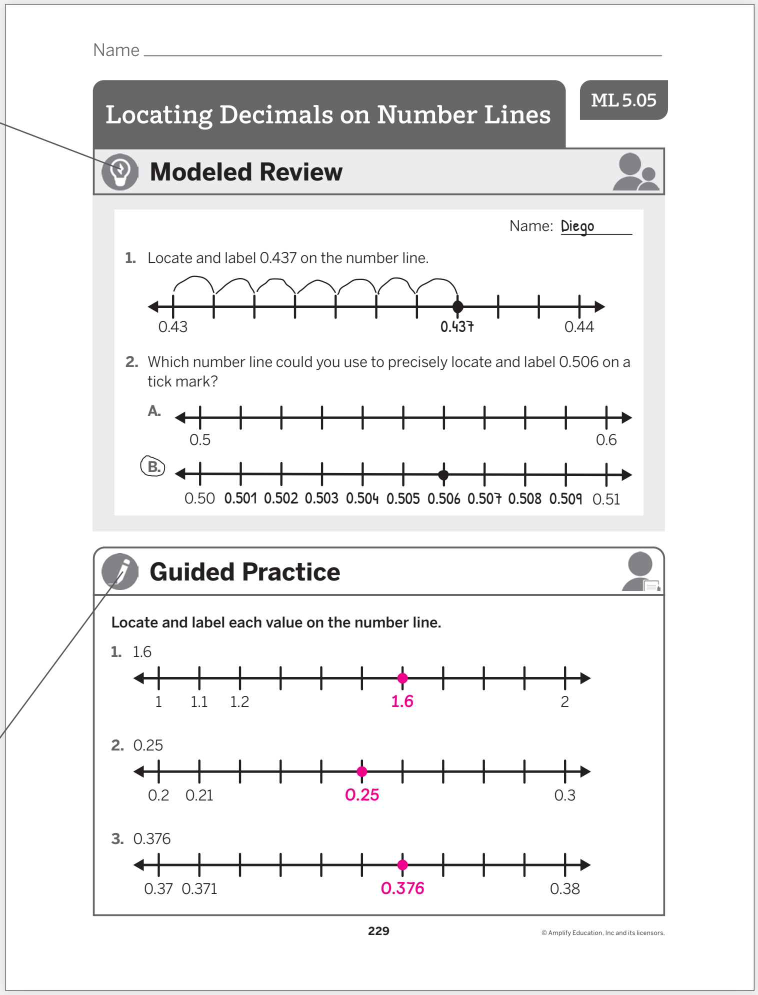
Reinforce Diego's thinking by saying, "To precisely locate and label a decimal on a number line, it is helpful if the scale of the number line is the place value of the decimal."

Guided Practice

Focus students' attention on locating and labeling each value on the number line.

To scaffold their thinking, **ask**:

- "What strategy can you use to make sure you label the correct tick mark?"
- "The first tick marks are labeled for you. What is the value of the next tick mark? How does that help you locate the given value?"



Name

Locating Decimals on Number Lines

ML 5.05

Modeled Review

Name: Diego

1. Locate and label 0.437 on the number line.

0.43 0.437 0.44

2. Which number line could you use to precisely locate and label 0.506 on a tick mark?

A. 0.5 0.6

(B.) 0.50 0.501 0.502 0.503 0.504 0.505 0.506 0.507 0.508 0.509 0.51

Guided Practice

Locate and label each value on the number line.

1. 1.6

1 1.1 1.2 1.6 2

2. 0.25

0.2 0.21 0.25 0.3

3. 0.376

0.37 0.371 0.376 0.38

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Vocabulary

If needed, share the meaning of the terms with students.

tenths: The base-ten place value unit equal to $\frac{1}{10}$ (one tenth in standard form is .1).

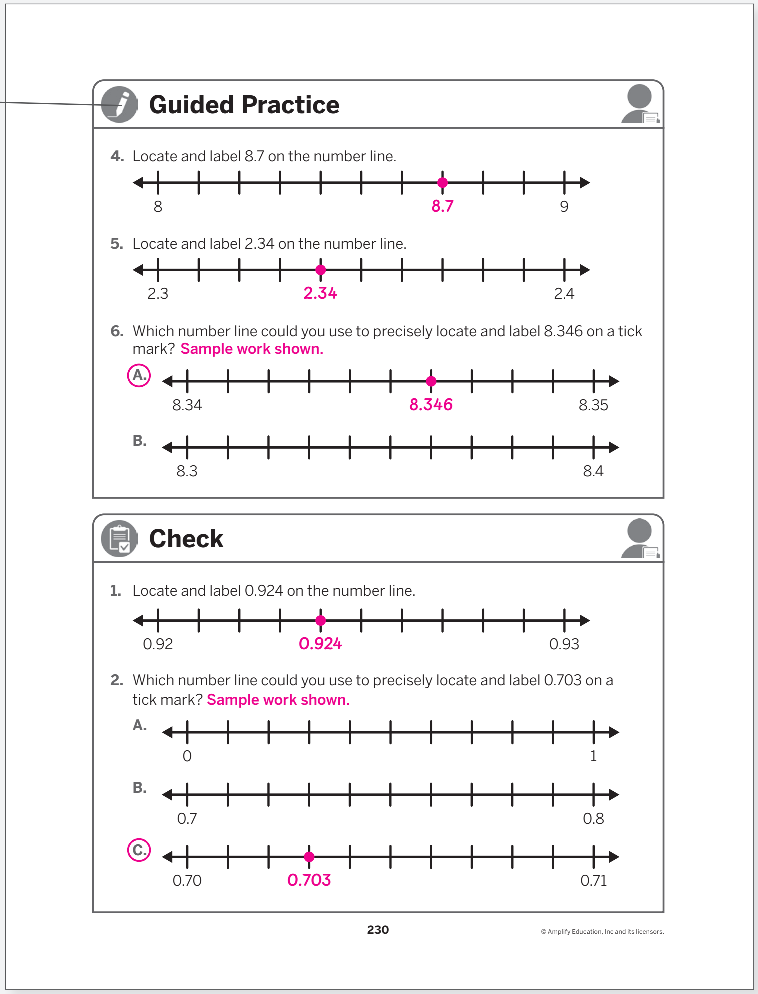
hundredths: The base-ten place value unit equal to $\frac{1}{100}$ (one hundredth in standard form is .01).

thousandths: The base-ten place value unit equal to $\frac{1}{1,000}$ (one thousandth in standard form is .001).

Guided Practice

A Model annotating the number line by writing the value of each tick mark while reinforcing that students can use the same strategy as they locate and label decimals on number lines.

Key Takeaway:
Say, “You can use your understanding of place value to locate and label decimals to the thousandths on number lines.”


Guided Practice

4. Locate and label 8.7 on the number line.

8 8.7 9

5. Locate and label 2.34 on the number line.

2.3 2.34 2.4

6. Which number line could you use to precisely locate and label 8.346 on a tick mark? Sample work shown.

A. 8.34 8.346 8.35

B. 8.3 8.4

Check

1. Locate and label 0.924 on the number line.

0.92 0.924 0.93

2. Which number line could you use to precisely locate and label 0.703 on a tick mark? Sample work shown.

A. 0 1

B. 0.7 0.8

C. 0.70 0.703 0.71

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Reflection

Ask:

- “What strategy did you learn today to help you accurately locate and label a given value on a number line?”
- “Reflect on your learning today. What were you most proud of?”

Check: Recommended Next Steps

Almost there	**Got it!**
If students need more support, consider labeling some of the tick marks on the number lines in Problems 4–6. Then ask students to annotate the remaining tick marks.	If students need more practice, have them revisit Problem 1 in the Check. Then ask them to locate and label the following decimals: • 0.921 • 0.927

Name ______________________________

Adding and Subtracting Decimals

ML 5.12

Modeled Review

Name: Han

Determine the sum or difference. Show your thinking.

1. 2.4 + 4.76 = 7.16

$$\begin{array}{r} {}^{1} \\ 2.40 \\ +\ 4.76 \\ \hline 7.16 \end{array}$$

2. 9.7 − 6.45 = 3.25

$$\begin{array}{r} {}^{6\ 10} \\ 9.\not{7}\not{0} \\ -\ 6.45 \\ \hline 3.25 \end{array}$$

Guided Practice

Determine the sum. Show your thinking.

1. 8.21 + 0.5 = ________

$$\begin{array}{r} 8.21 \\ +\ 0.50 \\ \hline \square.\square 1 \end{array}$$

2. 41.7 + 11.32 = ________

$$\begin{array}{r} {}^{1} \\ 41.70 \\ +\ 11.32 \\ \hline \square\square.\square 2 \end{array}$$

3. 9.3 + 0.74 = ________

4. 35.86 + 19.7 = ________

Guided Practice

Determine the difference. Show your thinking.

5. 7.8 − 0.45 = ________

$$\begin{array}{r} 7.80 \\ -\ 0.45 \\ \hline \end{array}$$

6. 31.6 − 14.08 = ________

$$\begin{array}{r} 31.60 \\ -\ 14.08 \\ \hline \end{array}$$

7. 3.49 − 1.7 = ________

8. 13.2 − 6.52 = ________

Check

Determine the sum or difference. Show your thinking.

1. 17.5 + 3.41 = ________

2. 21.4 − 8.75 = ________

ML 5.12 Adding and Subtracting Decimals

Goal

Add and subtract decimals to the hundredths place using the standard algorithm.

Standard

MA.5.NSO.2.3

Materials

graph or lined paper (optional)

Modeled Review

Point to Han's work and **ask**:

- "What did Han do to create decimal equivalence in both problems?"
- "How does adding zeros to the end of decimals affect the sum? Difference?"
- "How do you know when a zero needs to be added?"

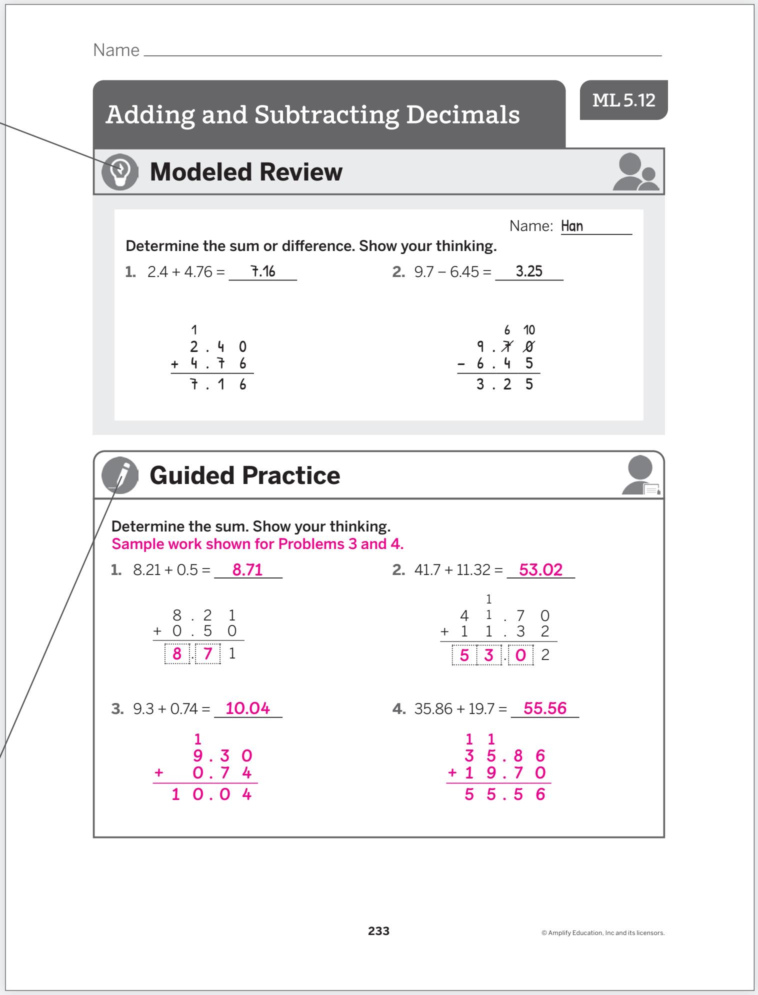
Reinforce Han's thinking by saying, "It is important to line up digits by place when adding and subtracting decimals using the standard algorithm. Zeros can be recorded to represent an equivalent place, but are not always necessary."

Guided Practice

Focus students' attention on using place value understanding to determine the sum.

To scaffold their thinking, **ask**:

- "How would you use the standard algorithm to add if these were whole numbers?"
- "Will you need to compose? If so, where? How do you know?"
- "Will you need to extend both decimals to the same place value?"



Name

Adding and Subtracting Decimals ML 5.12

Modeled Review

Name: Han

Determine the sum or difference. Show your thinking.

1. 2.4 + 4.76 = 7.16

2. 9.7 − 6.45 = 3.25

Guided Practice

Determine the sum. Show your thinking.

Sample work shown for Problems 3 and 4.

1. 8.21 + 0.5 = 8.71

2. 41.7 + 11.32 = 53.02

3. 9.3 + 0.74 = 10.04

4. 35.86 + 19.7 = 55.56

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Vocabulary

If needed, share the meaning of the terms with students.

sum: The value when two or more numbers are added.

difference: The amount you get when you subtract one number from another.

decompose: To break apart.

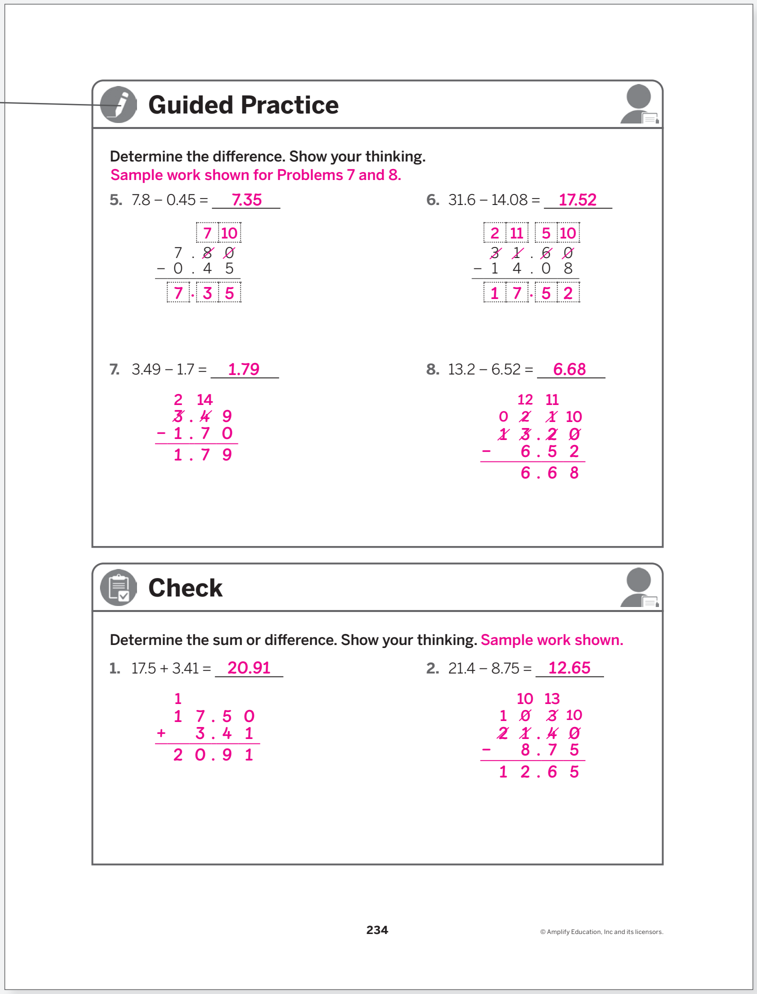
compose: To put together.

Guided Practice

A Provide students with graph paper or lined paper oriented vertically that can be used to organize their decimals by place value.

ML/EL Model thinking aloud while solving each step of the algorithm and showing students where to record decomposed units.

Key Takeaway:
Say, "The only time you must extend both decimals to the same place value is when you are subtracting and the first number has fewer digits than the second number. In all other decimal addition and subtraction cases, extending to the same place value, by adding one or more zeros, is helpful but not necessary."



Guided Practice

Determine the difference. Show your thinking.
Sample work shown for Problems 7 and 8.

5. 7.8 − 0.45 = 7.35

6. 31.6 − 14.08 = 17.52

7. 3.49 − 1.7 = 1.79

8. 13.2 − 6.52 = 6.68

Check

Determine the sum or difference. Show your thinking. Sample work shown.

1. 17.5 + 3.41 = 20.91

2. 21.4 − 8.75 = 12.65

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Reflection

Ask:

- "When is it helpful to extend both decimals to the same place value when using the standard algorithm to add or subtract decimals?"
- "What is something you weren't sure about at the start of the lesson but understand now?"

Check: Recommended Next Steps

Almost there
If students need more support, consider highlighting each place value with a different color.

Got it!
If students need more practice, invite them to determine the sum or difference for the following expressions:

- 12.47 − 7.9
- 9.5 + 3.58
- 23.48 + 6.3

Name ______________________________

Connecting Whole Number and Decimal Multiplication

ML 5.14

Modeled Review

Name: Avery

Determine each product. Show your thinking.

1. 4 × 3 = 12

4 × 3 ones = 12 ones = 12

2. 4 × 0.3 = 1.2

4 × 3 tenths = 12 tenths = 1.2

3. 4 × 0.31 = 1.24

4 × 31 hundredths = 124 hundredths = 1.24

$$\begin{array}{r} 31 \\ \times \quad 4 \\ \hline 124 \end{array}$$

Guided Practice

Determine each product.

1. 5 × 0.3

5 × 3 tenths = ____ tenths

5 × 0.3 = 1.5

2. 4 × 0.12

___ × ___ hundredths = 48 hundredths

4 × 0.12 = ______

3. 2 × 0.32

___ × ___ hundredths = 64 hundredths

___ × ______ = ______

4. 6 × 0.40

___ × ___ hundredths = 240 hundredths

___ × ______ = ______

Guided Practice

Determine each product. Show your thinking.

5. 7 × 0.2 = __________

6. 8 × 0.3 = __________

7. 6 × 0.5 = __________

8. 5 × 0.9 = __________

9. 4 × 0.15 = __________

10. 2 × 0.51 = __________

Check

Determine each product. Show your thinking.

1. 6 × 0.3 = __________

2. 3 × 0.11 = __________

Connecting Whole Number and Decimal Multiplication

Goal

Multiply decimals less than one by one-digit whole numbers.

Standard

MA.5.NSO.2.4

Materials

base-ten blocks (optional), decimal grid (optional)

Modeled Review

Point to Avery's work and **ask**:

- "What similarities do you see in the expressions? Differences?"
- "In Problem 2, why does the product, 12 tenths, have a one in the ones place?"
- "How does the product's place value help you write the product in standard form?"

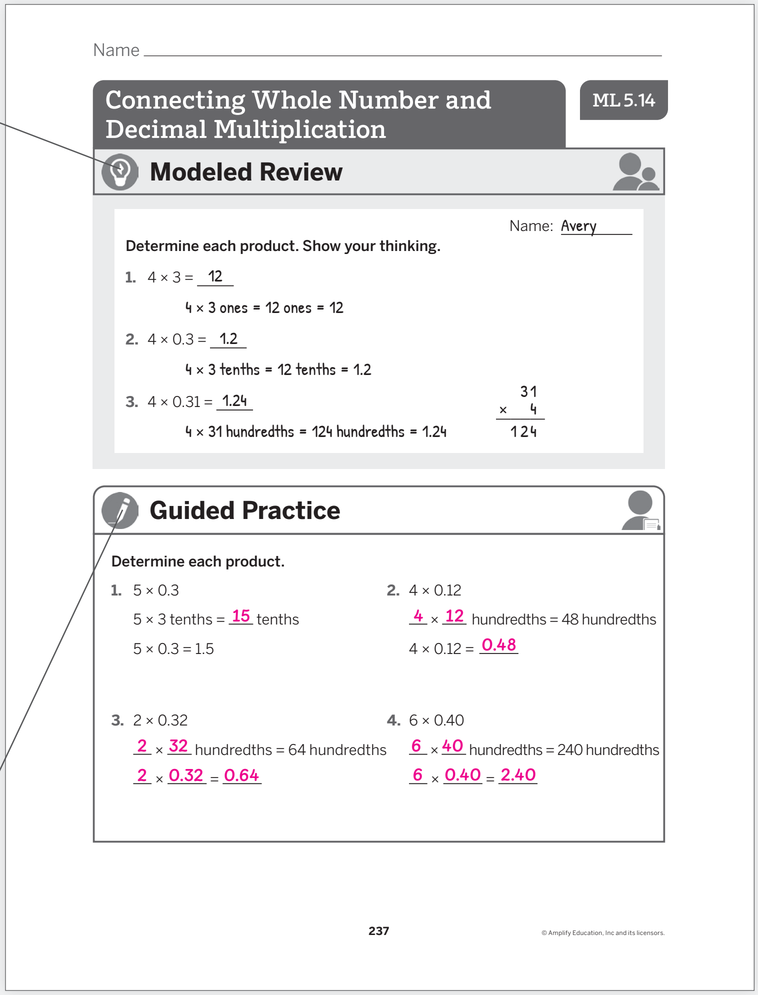
Reinforce Avery's thinking by saying, "The same representations and strategies you use to multiply whole numbers can be used to multiply whole numbers and decimals. It is important to pay attention to place value when writing your answer as a decimal."

Guided Practice

Focus students' attention on determining the product of each expression.

To scaffold their thinking, **ask**:

- "How many tenths or hundredths are there in one of the factors?"
- "How can you use what you know about multiplying whole numbers to help you determine the product?"
- "How could you write the number of tenths or hundredths in decimal notation?"



Name

Connecting Whole Number and Decimal Multiplication ML 5.14

Modeled Review

Name: Avery

Determine each product. Show your thinking.

1. 4 × 3 = 12
   4 × 3 ones = 12 ones = 12
2. 4 × 0.3 = 1.2
   4 × 3 tenths = 12 tenths = 1.2
3. 4 × 0.31 = 1.24
   4 × 31 hundredths = 124 hundredths = 1.24
   31 × 4 = 124

Guided Practice

Determine each product.

1. 5 × 0.3
   5 × 3 tenths = 15 tenths
   5 × 0.3 = 1.5
2. 4 × 0.12
   4 × 12 hundredths = 48 hundredths
   4 × 0.12 = 0.48
3. 2 × 0.32
   2 × 32 hundredths = 64 hundredths
   2 × 0.32 = 0.64
4. 6 × 0.40
   6 × 40 hundredths = 240 hundredths
   6 × 0.40 = 2.40

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Vocabulary

If needed, share the meaning of the term with students.

decimal: A fractional value written using single digits for each place value.

ML 5.14 Connecting Whole Number and Decimal Multiplication

Guided Practice

A Model writing numbers in decimal notation to tenths and hundredths. Have students practice with guidance before trying independently.

ML/EL Provide students with base-ten blocks they can use when multiplying whole numbers by decimals. For example, have them build 7 groups of 0.2 with base-ten blocks to help them understand Problem 5.

Key Takeaway:
Say, "Just like when multiplying a whole number and a fraction less than 1, when multiplying a whole number and a decimal less than 1, the product will be less than the whole number and greater than the decimal factor."

Guided Practice

Determine each product. Show your thinking. Sample work shown.

5. 7 × 0.2 = 1.4
   7 x 2 tenths = 14 tenths
   7 x 0.2 = 1.4

6. 8 × 0.3 = 2.4
   8 x 3 tenths = 24 tenths
   8 x 0.3 = 2.4

7. 6 × 0.5 = 3.0 or 3
   6 x 5 tenths = 30 tenths
   6 x 0.5 = 3.0 or 3

8. 5 × 0.9 = 4.5
   5 x 9 tenths = 45 tenths
   5 x 0.9 = 4.5

9. 4 × 0.15 = 0.60
   4 x 15 hundredths = 60 hundredths
   4 x 0.15 = 0.60

10. 2 × 0.51 = 1.02
   2 x 51 hundredths = 102 hundredths
   2 x 0.51 = 1.02

Check

Determine each product. Show your thinking. Sample work shown.

1. 6 × 0.3 = 1.8
   6 x 3 tenths = 18 tenths
   6 x 0.3 = 1.8

2. 3 × 0.11 = 0.33
   3 x 11 hundredths = 33 hundredths
   3 x 0.11 = 0.33

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Reflection

Ask:

- "Why is place value important when multiplying whole numbers and decimals?"
- "What strategy was helpful today?"

Check: Recommended Next Steps

Almost there
If students need more support, consider having them use decimal grids to model solving each problem in the Modeled Review. Ask them to compare the products they determined using a decimal grid with the products Avery found.

Got it!
If students need more practice, invite them to determine the product for the following expressions:

- 3 × 0.9
- 4 × 0.21
- 2 × 0.55

Name ____________________

Multiplying Decimals Less Than 1

ML 5.16

Modeled Review

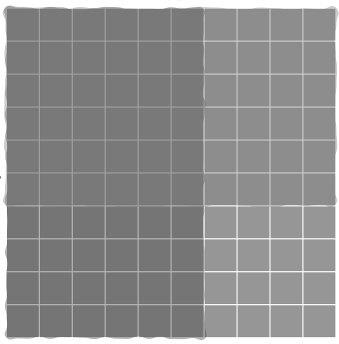
Name: Jack

Determine the product.

0.6 × 0.6 = 0.36

$\frac{6}{10}$

$\frac{6}{10}$



$\frac{6}{10} \times \frac{6}{10} = \frac{36}{100} = 0.36$

Guided Practice

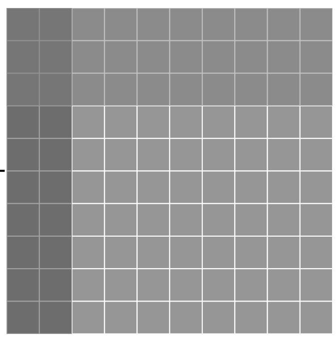
Determine the product.

1. 0.3 × 0.2 = ______

$\frac{\square}{10} \times \frac{\square}{10} = \frac{\square}{100} =$ ______

$\frac{2}{10}$

$\frac{3}{10}$

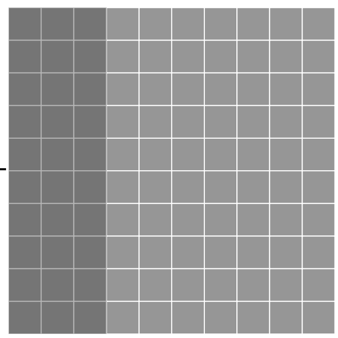


2. 0.4 × 0.3 = ______

$\frac{\square}{10} \times \frac{\square}{10} =$ ___

$\frac{3}{10}$

$\frac{4}{10}$

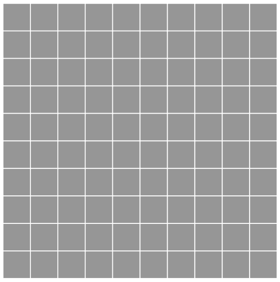


Guided Practice

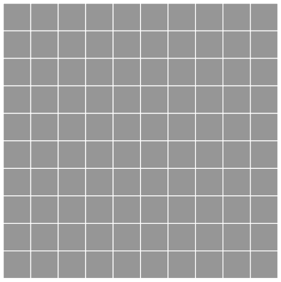
Determine the product.

3. 0.2 × 0.8 = _______

— × — = — = _______



4. 0.3 × 0.6 = _______



5. 0.5 × 0.4 = _______

6. 0.6 × 0.4 = _______

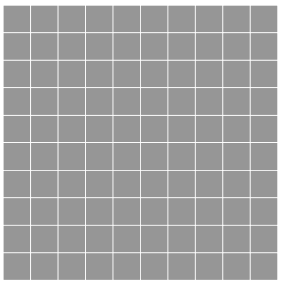
7. 0.7 × 0.3 = _______

8. 0.8 × 0.4 = _______

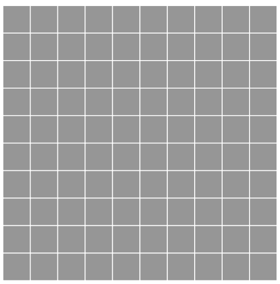
Check

Determine the product. Use a decimal grid if it is helpful.

1. 0.5 × 0.8 = _______



2. 0.3 × 0.9 = _______



Multiplying Decimals Less Than 1

Goal

Multiply two decimals (tenths only) less than 1.

Standard

MA.5.NSO.2.5

Materials

coloring tools (optional)

Modeled Review

Point to Jack's work and **ask**:

- "How is multiplying fractions similar to multiplying decimals?"
- "Where do you see the numerators being multiplied in the decimal grid?"
- "Why is the product in the hundredths place?"
- "Why is the product less than the factors in the expression?"

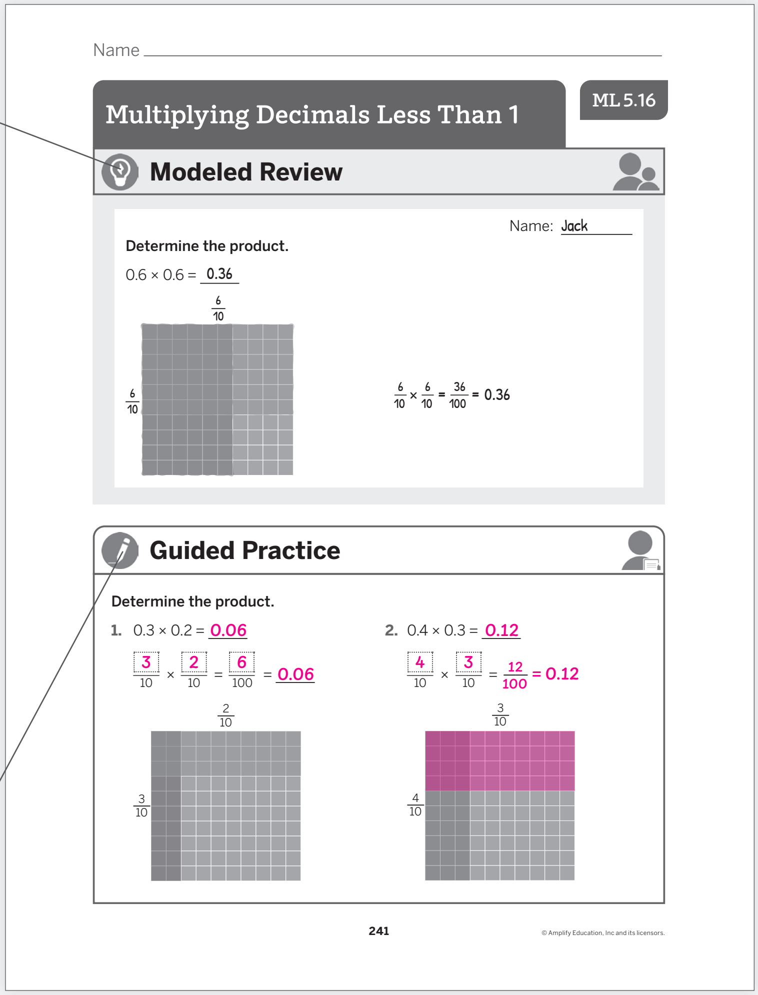
Reinforce Jack's thinking by saying, "Just like when multiplying two fractions, when you multiply two decimals less than 1, you are determining part of a part. This means the product is less than both factors."

Guided Practice

Focus students' attention on converting the decimals to fractions then determining each product.

To scaffold their thinking, **ask**:

- "What are the factors in the expression? How can you write them as fractions?"
- "How could you represent each factor on the grid to help determine the product?"
- "What is the relationship between the numerators in the factors and the numerator in the product? Denominators?"



Name

Multiplying Decimals Less Than 1

ML 5.16

Modeled Review

Name: Jack

Determine the product.

$0.6 \times 0.6 = 0.36$

$\frac{6}{10}$, $\frac{6}{10}$

$\frac{6}{10} \times \frac{6}{10} = \frac{36}{100} = 0.36$

Guided Practice

Determine the product.

1. $0.3 \times 0.2 = 0.06$

$\frac{3}{10} \times \frac{2}{10} = \frac{6}{100} = 0.06$

$\frac{2}{10}$, $\frac{3}{10}$

2. $0.4 \times 0.3 = 0.12$

$\frac{4}{10} \times \frac{3}{10} = \frac{12}{100} = 0.12$

$\frac{3}{10}$, $\frac{4}{10}$

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Vocabulary

If needed, share the meaning of the terms with students.

factor: A number that is multiplied by another number to determine the product.

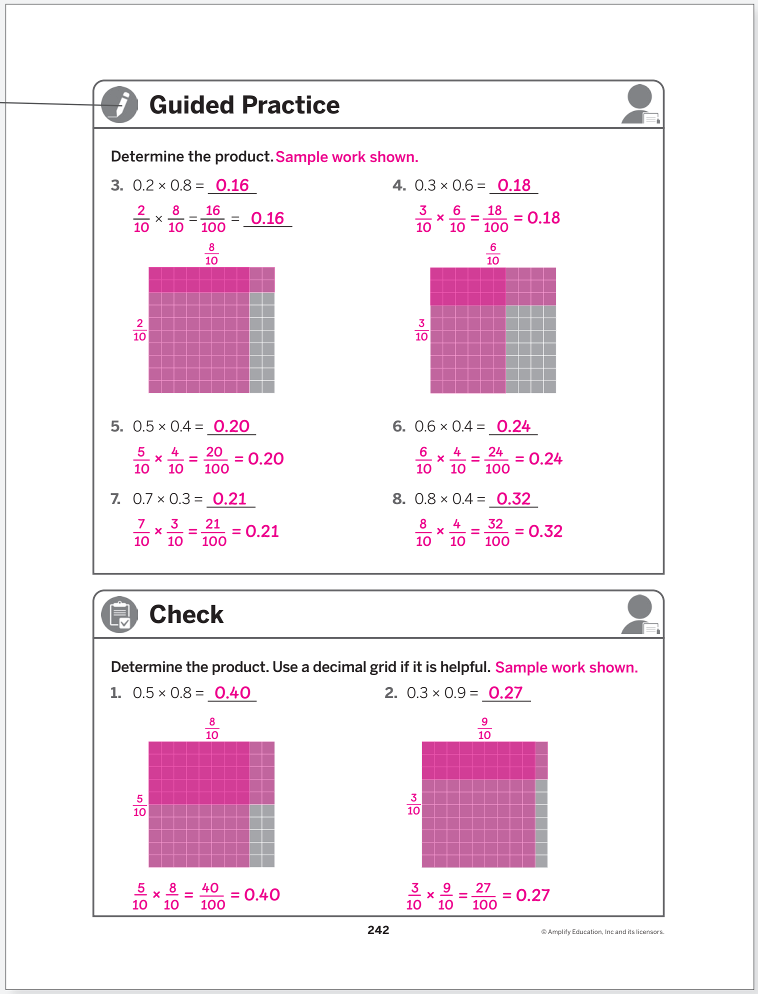
product: The value when two or more numbers are multiplied.

expression: A mathematical statement with a minimum of two numbers and at least one math operation.

Guided Practice

A Provide students with coloring tools to represent the two decimals on the grid.

Key Takeaway:
Say, "You can use what you know about fraction multiplication, whole number facts, and place value to multiply two decimals less than 1."



Guided Practice

Determine the product. Sample work shown.

3. 0.2 × 0.8 = 0.16
$\frac{2}{10} \times \frac{8}{10} = \frac{16}{100} = 0.16$

4. 0.3 × 0.6 = 0.18
$\frac{3}{10} \times \frac{6}{10} = \frac{18}{100} = 0.18$

5. 0.5 × 0.4 = 0.20
$\frac{5}{10} \times \frac{4}{10} = \frac{20}{100} = 0.20$

6. 0.6 × 0.4 = 0.24
$\frac{6}{10} \times \frac{4}{10} = \frac{24}{100} = 0.24$

7. 0.7 × 0.3 = 0.21
$\frac{7}{10} \times \frac{3}{10} = \frac{21}{100} = 0.21$

8. 0.8 × 0.4 = 0.32
$\frac{8}{10} \times \frac{4}{10} = \frac{32}{100} = 0.32$

Check

Determine the product. Use a decimal grid if it is helpful. Sample work shown.

1. 0.5 × 0.8 = 0.40
$\frac{5}{10} \times \frac{8}{10} = \frac{40}{100} = 0.40$

2. 0.3 × 0.9 = 0.27
$\frac{3}{10} \times \frac{9}{10} = \frac{27}{100} = 0.27$

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Reflection

Ask:
- "What did you learn about the product of two decimals less than 1?"
- "Reflect on your learning today. What were you most proud of?"

Check: Recommended Next Steps

Almost there	Got it!
If students need more support, consider using a think-aloud as you model converting the factors in the Check to fractions. Then represent the fractions on a grid as you reinforce how to determine the product using the grid.	If students need more practice, invite them to determine the product for the following expressions: • 0.9 × 0.4 • 0.2 × 0.5 • 0.5 × 0.7 • 0.8 × 0.3

Name ______________________________

Multiplying Two Decimals Greater than 1

ML 5.18

Modeled Review

Name: Priya

Determine the product. Show your thinking.

14.6 × 8.2 = 119.72

	14	0.6
8	112	4.8
0.2	2.8	0.12

```
  ¹
 112.00
   4.80
   2.80
+  0.12
 ------
 119.72
```

Guided Practice

Determine the product. Show your thinking.

1. 10.5 × 2.1 = ____________

	10	0.5
2	20	1
0.1	1	

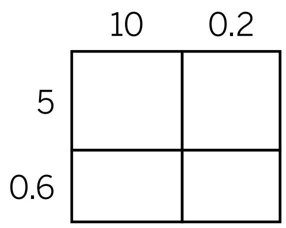
2. 10.8 × 4.3 = ____________

	10	0.8
4	40	3.2
0.3	3	

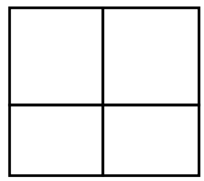
Guided Practice

Determine the product. Show your thinking.

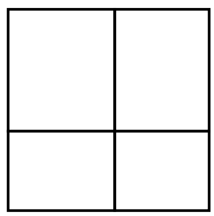
3. 10.2 × 5.6 = ___________



4. 11.5 × 7.5 = ___________



5. 12.7 × 9.1 = ___________



6. 14.3 × 8.6 = ___________

Check

Determine the product. Show your thinking.

13.4 × 8.5 = ___________

ML 5.18 Multiplying Two Decimals Greater Than 1

Goal

Multiply two decimals using area diagrams when both decimals are greater than 1.

Standard

MA.5.NSO.2.5

Materials

highlighter or colored pencils (optional)

Modeled Review

Point to Priya's work and **ask**:

- "How did Priya decompose 14.6 and 8.2 to help her multiply?"
- "How did Priya use multiplication to find the area of each rectangle?"
- "Why was Priya's last step addition?"

Reinforce Priya's thinking by saying, "When multiplying two decimals greater than 1, you can decompose the factors by place value and represent your partial products with an area diagram."

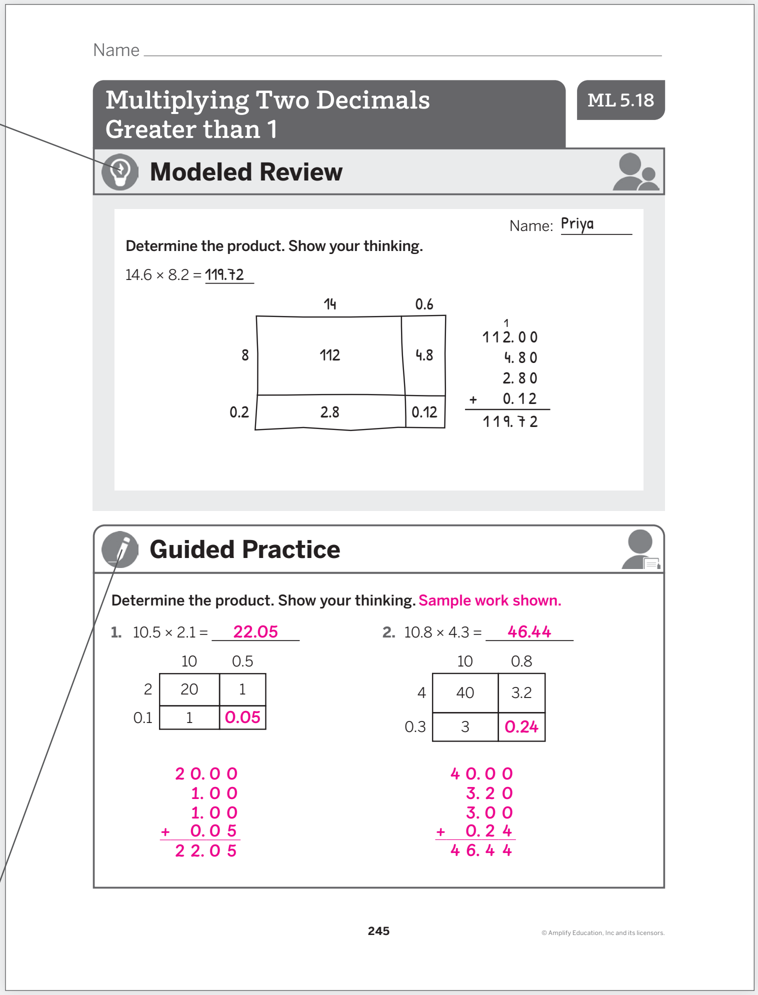
ML/EL To support students with making connections between the problem and area model, invite them to use highlighters or colored pencils to draw connections between the numbers and how they are represented in the area model.

Guided Practice

Focus students' attention on using the area model to calculate the product.

To scaffold their thinking, **say**:

- "Calculate the area of each missing part."
- "Add all the areas together."



Name

Multiplying Two Decimals Greater than 1 — ML 5.18

Modeled Review

Name: Priya

Determine the product. Show your thinking.

14.6 × 8.2 = 119.72

	14	0.6
8	112	4.8
0.2	2.8	0.12

112.00
4.80
2.80
+ 0.12
119.72

Guided Practice

Determine the product. Show your thinking. Sample work shown.

1. 10.5 × 2.1 = 22.05

	10	0.5
2	20	1
0.1	1	0.05

20.00
1.00
1.00
+ 0.05
22.05

2. 10.8 × 4.3 = 46.44

	10	0.8
4	40	3.2
0.3	3	0.24

40.00
3.20
3.00
+ 0.24
46.44

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Vocabulary

If needed, share the meaning of the terms with students.

product: The value when two or more numbers are multiplied.

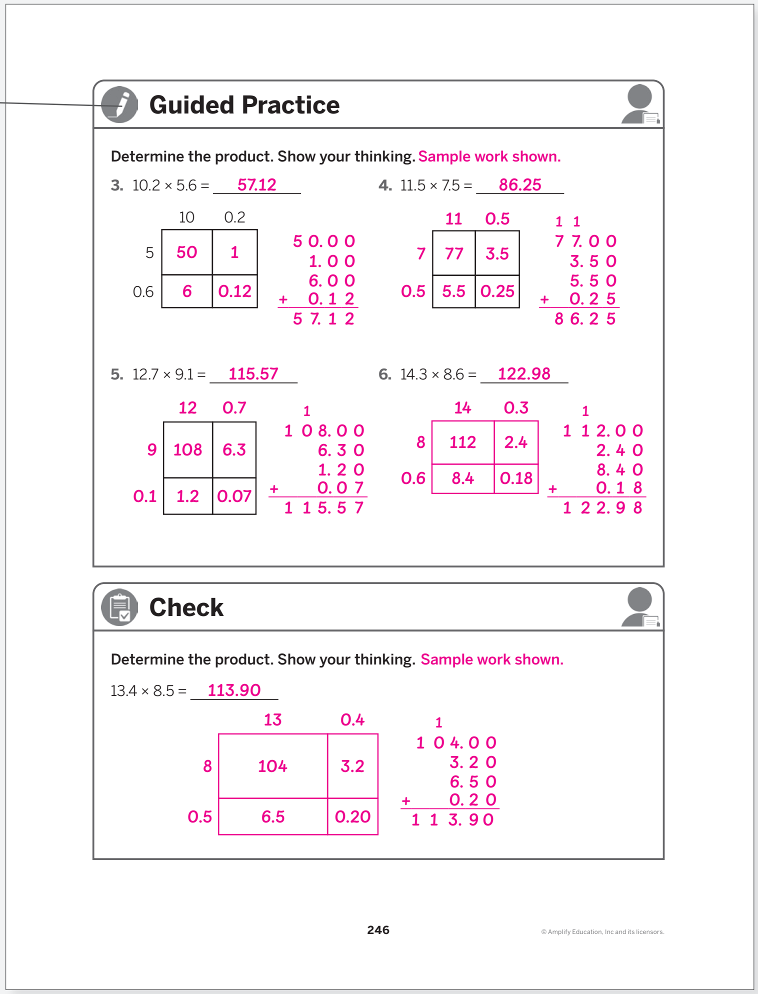
partial products: The value obtained when multiplying the parts of two numbers separately (which is then added to other partial products to arrive at the total product).

Multiplying Two Decimals Greater Than 1

Guided Practice

(A) Model covering up the sections of the area model that are not being calculated to help students concentrate on one step of the process at a time.

Key Takeaway:
Say, "When multiplying two decimals greater than 1, you can decompose the factors and draw an area diagram to help you calculate the product."



Guided Practice

Determine the product. Show your thinking. Sample work shown.

3. 10.2 × 5.6 = 57.12

	10	0.2
5	50	1
0.6	6	0.12

50.00 + 1.00 + 6.00 + 0.12 = 57.12

4. 11.5 × 7.5 = 86.25

	11	0.5
7	77	3.5
0.5	5.5	0.25

77.00 + 3.50 + 5.50 + 0.25 = 86.25

5. 12.7 × 9.1 = 115.57

	12	0.7
9	108	6.3
0.1	1.2	0.07

108.00 + 6.30 + 1.20 + 0.07 = 115.57

6. 14.3 × 8.6 = 122.98

	14	0.3
8	112	2.4
0.6	8.4	0.18

112.00 + 2.40 + 8.40 + 0.18 = 122.98

Check

Determine the product. Show your thinking. Sample work shown.

13.4 × 8.5 = 113.90

	13	0.4
8	104	3.2
0.5	6.5	0.20

104.00 + 3.20 + 6.50 + 0.20 = 113.90

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Reflection

Ask:
- "What challenges did you run into when multiplying decimals with an area model, and how did you overcome them?"
- "How does what you learned today connect to your prior learning?"

Check: Recommended Next Steps

Almost there
If students need more support, consider modeling how to align the decimals and use zeros as placeholders when adding the four partial products to find the final product.

Got it!
If students need more practice, present them with the following problems and ask them to calculate the product using an area model.

- 13.8 × 3.2
- 14.5 × 4.4

Name ______________________________

Converting Millimeters, Centimeters, Meters, and Kilometers

ML 6.06

Modeled Review

Name: Diego

Clare ran 14,500 meters last week. How many kilometers did Clare run?

14,500 ÷ 1,000 = 14.5

1 kilometer = 1,000 meters
1 meter = 100 centimeters
1 centimeter = 10 millimeters

answer: 14.5 kilometers

Guided Practice

Complete the conversions. Show your thinking.

1. 1 kilometer = 1,000 meters

4 kilometers = __________ meters

1,000 × 4 = __________

2. 1 meter = 100 centimeters

6.5 meters = _______ centimeters

3. 1 meter = 1,000 millimeters

_____ meters = 5,000 millimeters

4. 1 centimeter = 10 millimeters

_____ centimeters = 43 millimeters

Guided Practice

Use the information about the caterpillar to complete each conversion. Show your thinking.

5. A caterpillar moves 220 centimeters in 4 minutes. How many meters does the caterpillar move?

   answer: ______________________

6. A caterpillar moves 60 centimeters in 30 seconds. How many millimeters does the caterpillar move?

   answer: ______________________

7. A caterpillar moves 864 centimeters in a day. How many millimeters does the caterpillar move in a day?

   answer: ______________________

8. A caterpillar moves 15 millimeters. How many centimeters did the caterpillar move?

   answer: ______________________

Check

Clare ran a 5,500 meter race. How many kilometers did Clare run? Show your thinking.

answer: ______________________

ML 6.06 Converting Millimeters, Centimeters, Meters, and Kilometers

Goal

Convert metric lengths from a larger unit to a smaller unit and from a smaller unit to a larger unit.

Standard

MA.5.M.1.1

Modeled Review

Point to Diego's work and **ask**:

- "What is the relationship between one meter and one kilometer?"
- "How did Diego use powers of 10 to convert from meters to kilometers?"
- "Why does Diego divide by 1,000 to find the number of kilometers?"

Reinforce Diego's thinking by saying, "In the metric system, you can convert a smaller unit to a larger unit, like meters to kilometers, by dividing by a power of 10."

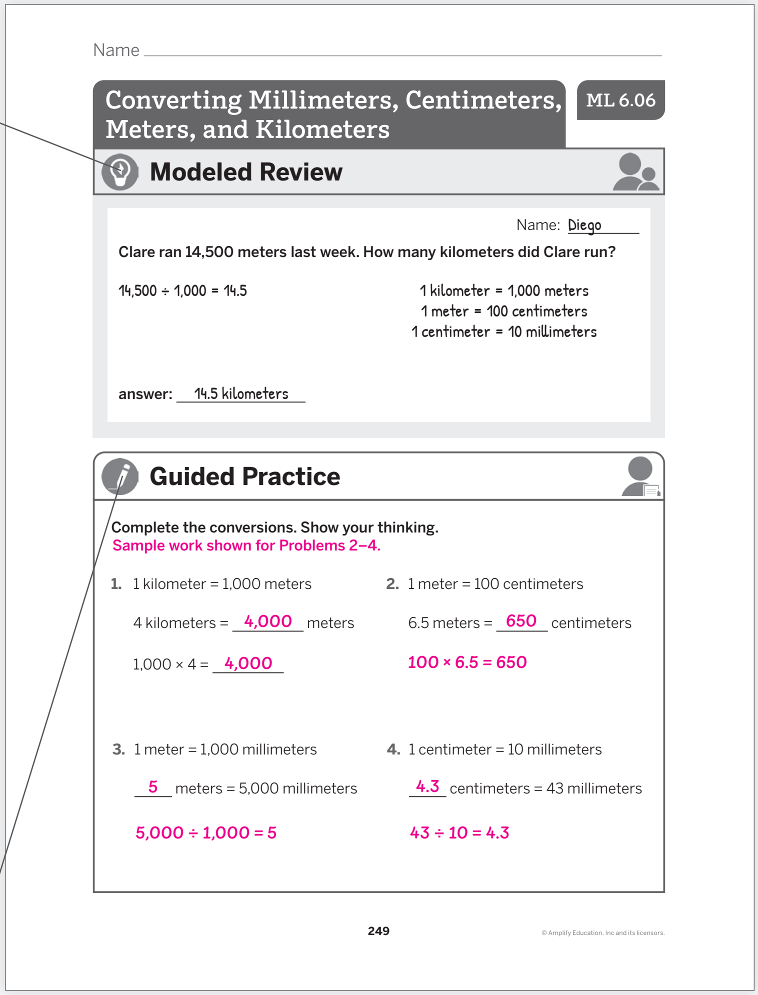
ML/EL Invite students to share what they know about kilometers, meters, centimeters, and millimeters. Connect each unit of measurement to real-world scenarios (running, walking, height, size of paper clip, etc).

Guided Practice

Focus students' attention on completing the conversions.

To scaffold their thinking, **ask**:

- "Which is a larger unit of measurement?"
- "Do you need to multiply or divide to complete the conversion?"



Name ______

Converting Millimeters, Centimeters, Meters, and Kilometers ML 6.06

Modeled Review

Name: Diego

Clare ran 14,500 meters last week. How many kilometers did Clare run?

14,500 ÷ 1,000 = 14.5

1 kilometer = 1,000 meters
1 meter = 100 centimeters
1 centimeter = 10 millimeters

answer: 14.5 kilometers

Guided Practice

Complete the conversions. Show your thinking.
Sample work shown for Problems 2–4.

1. 1 kilometer = 1,000 meters
   4 kilometers = 4,000 meters
   1,000 × 4 = 4,000
2. 1 meter = 100 centimeters
   6.5 meters = 650 centimeters
   100 × 6.5 = 650
3. 1 meter = 1,000 millimeters
   5 meters = 5,000 millimeters
   5,000 ÷ 1,000 = 5
4. 1 centimeter = 10 millimeters
   4.3 centimeters = 43 millimeters
   43 ÷ 10 = 4.3

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Vocabulary

If needed, share the meaning of the terms with students.

kilometer: A unit of length in the metric measurement system (1,000 meters is equal to 1 kilometer).

centimeter: A unit of length measurement in the metric system (100 centimeters is equal to 1 meter).

millimeter: A unit of length in the metric measurement system (1,000 millimeters is equal to 1 meter).

Converting Millimeters, Centimeters, Meters, and Kilometers

Guided Practice

(A) Encourage students to refer back to the conversions listed in the Modeled Review to support them when solving the problems.

Key Takeaway:
Say, "Metric length units are related by powers of 10. To convert from larger to smaller units, you can multiply by powers of 10. To convert from smaller to larger units, you can divide by powers of 10."

Guided Practice

Use the information about the caterpillar to complete each conversion. Show your thinking. Sample work shown.

5. A caterpillar moves 220 centimeters in 4 minutes. How many meters does the caterpillar move?

220 ÷ 100 = 2.2

answer: 2.2 meters

6. A caterpillar moves 60 centimeters in 30 seconds. How many millimeters does the caterpillar move?

60 × 10 = 600

answer: 600 millimeters

7. A caterpillar moves 864 centimeters in a day. How many millimeters does the caterpillar move in a day?

864 × 100 = 86,400

answer: 86,400 millimeters

8. A caterpillar moves 15 millimeters. How many centimeters did the caterpillar move?

15 ÷ 10 = 1.5

answer: 1.5 centimeters

Check

Clare ran a 5,500 meter race. How many kilometers did Clare run? Show your thinking. Sample work shown.

1,000 meters = 1 kilometer

5,500 ÷ 1,000 = 5.5

answer: 5.5 kilometers

250

Reflection

Ask:

- "When would you multiply by a power of 10 when converting units of measurement? Divide?"
- "How was the lesson helpful to you today?"

Check: Recommended Next Steps

Almost there
If students need more support, consider creating an anchor chart with students that includes deciding which operation to use when converting from smaller units to larger units (e.g., ounces to pounds) and when converting from larger units to smaller units (e.g., pounds to ounces).

Got it!
If students need more practice, invite them to complete the following conversions:

- 50 centimeters = ____ millimeters
- 7,000 meters = _______ kilometers
- 15 meters = _________ centimeters

Name ______________________________

Converting Inches, Feet, Yards, and Miles

ML 6.09

Modeled Review

Name: Han

A walking path is 2,340 yards long. How long is the walking path in feet?

2,340 × 3 = 7,020

$$\begin{array}{r} {}^{1}\,{}^{1} \\ 2,340 \\ \times 3 \\ \hline 7,020 \end{array}$$

1 yard = 3 feet

2,340 yards = ? feet

answer: 7,020 feet

Guided Practice

Complete the conversions. Show your thinking.

1. 1 foot = 12 inches

3 feet = ________ inches

12 × 3 = ________

2. 1 yard = 3 feet

5 yards = ________ feet

3 × 5 = ________

3. 1 foot = ________ inches

$\frac{1}{2}$ foot = ________ inches

4. 1 mile = 5,280 feet

10 miles = ________ feet

Guided Practice

Solve each story problem. Show your thinking.

5. A football field is 100 yards. How long is a football field in feet?

answer: ______________

6. The soccer team ran half a mile before practice. How many yards did they run?
1 mile = 1,760 yards

answer: ______________

7. A soccer ball was kicked 120 inches. How far is that in feet?

answer: ______________

8. A football goal post has a height of 30 feet. How tall is the goal post in inches?

answer: ______________

Check

Solve the story problem. Show your thinking.

A football player kicked a 66 yard field goal. How far is that in feet?

answer: ____________________

Converting Inches, Feet, Yards, and Miles

Goal

Solve problems involving converting inches, feet, yards, and miles.

Standard

MA.5.M.1.1

Materials

ruler (optional), yardstick (optional)

Modeled Review

Point to Han's work and **ask**:

- "What is Han trying to find?"
- "Which measurement is larger - one yard or one foot? How do you know?"
- "How does Han determine the amount of feet in 2,340 yards?"

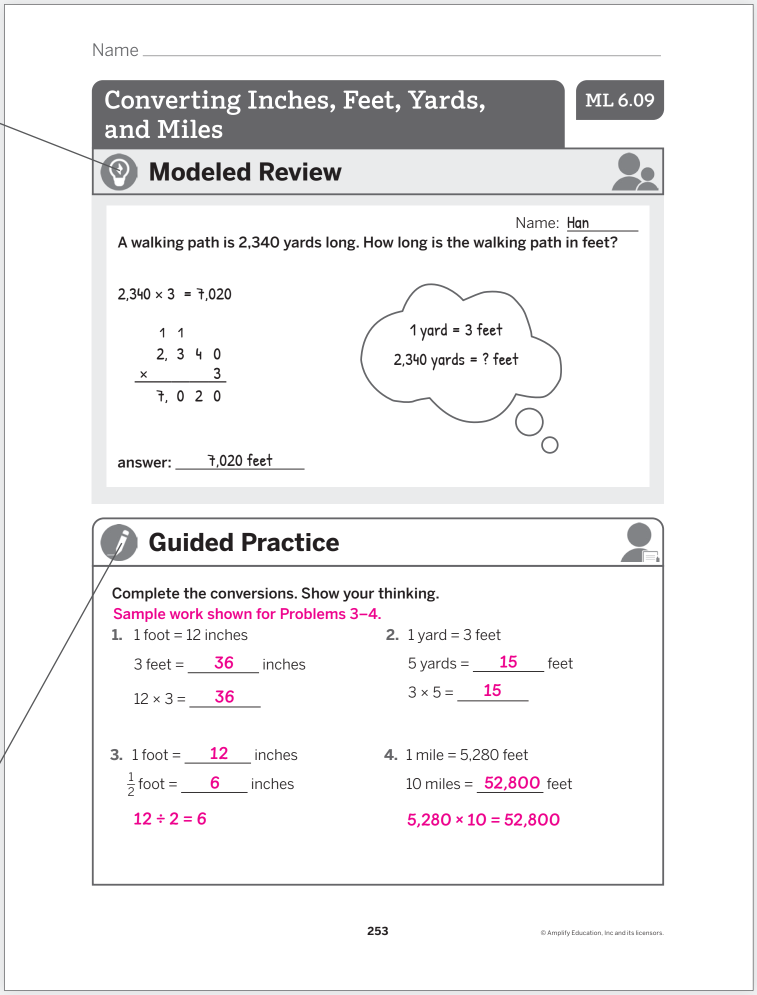
Reinforce Han's thinking by saying, "Inches, feet, yards, and miles are customary measurements. You can use multiplication and division to convert between them, but the measurements involve relationships that are not powers of 10. For example, 1 yard equals 3 feet and 1 foot equals 12 inches."

Guided Practice

Focus students' attention on completing each conversion.

To scaffold their thinking, **ask**:

- "What is the relationship between feet and inches? Yards and feet? Miles and feet?"
- "How can you use the relationship to help determine the conversion?"
- "If you need to find half of the original unit, what operation could you use?"



Name

Converting Inches, Feet, Yards, and Miles — ML 6.09

Modeled Review

Name: Han

A walking path is 2,340 yards long. How long is the walking path in feet?

2,340 × 3 = 7,020

$$\begin{array}{r} {}^{1}\,{}^{1}\quad \\ 2,340 \\ \times \quad 3 \\ \hline 7,020 \end{array}$$

1 yard = 3 feet
2,340 yards = ? feet

answer: 7,020 feet

Guided Practice

Complete the conversions. Show your thinking.

Sample work shown for Problems 3–4.

1. 1 foot = 12 inches
   3 feet = 36 inches
   12 × 3 = 36

2. 1 yard = 3 feet
   5 yards = 15 feet
   3 × 5 = 15

3. 1 foot = 12 inches
   $\frac{1}{2}$ foot = 6 inches
   12 ÷ 2 = 6

4. 1 mile = 5,280 feet
   10 miles = 52,800 feet
   5,280 × 10 = 52,800

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Vocabulary

If needed, share the meaning of the terms with students.

foot: A unit of length in the U.S. customary measurement system that is 12 inches long (plural: feet).

yard: A unit of length in the U.S. customary measurement system (1 yard is equal to three feet).

mile: A unit of length in the U.S. customary measurement system (1 mile is equal to 5,280 feet or 1,760 yards).

Converting Inches, Feet, Yards, and Miles

Guided Practice

A Provide students with access to classroom materials like a ruler and yardstick as a visual aid for inches, feet and yards. Then ask them where they have seen each measurement used in the real world.

Key Takeaway:
Say, "For both customary and metric length measurements, you can use multiplication and division to convert units. Converting metric length measurements, like centimeters, meters, and kilometers, involves relationships between units that are all powers of 10. Converting customary length measurements, like inches, feet, yards, and miles, involves all different relationships that are not powers of 10."

Guided Practice

Solve each story problem. Show your thinking. Sample work shown.

5. A football field is 100 yards. How long is a football field in feet?

   $100 \times 3 = 300$ feet

   answer: 300 feet

6. The soccer team ran half a mile before practice. How many yards did they run?
   1 mile = 1,760 yards

   $1{,}760 \div 2 = 880$ yards

   answer: 880 yards

7. A soccer ball was kicked 120 inches. How far is that in feet?

   $120 \div 12 = 10$ feet

   answer: 10 feet

8. A football goal post has a height of 30 feet. How tall is the goal post in inches?

   $30 \times 12 = 360$ inches

   answer: 360 inches

Check

Solve the story problem. Show your thinking. Sample work shown.
A football player kicked a 66 yard field goal. How far is that in feet?

$$\begin{array}{r} {}^{1} \\ 66 \\ \times\ \ 3 \\ \hline 198 \end{array}$$

answer: 198 feet

254

Reflection

Ask:

- "How do you know when to multiply or divide to change from one unit to another?"
- "How does what you learned today connect to your prior learning?"

Check: Recommended Next Steps

Almost there
If students need more support, consider identifying in each problem which unit is the smaller unit and which unit is the larger unit.

Got it!
If students need more practice, invite them to solve the story problem.

The distance from Jada's house to the beach is 2 miles. How many yards is Jada's house from the beach?

Name ____________________

Representing Data on a Line Plot and Solving Problems

ML 6.18

Modeled Review

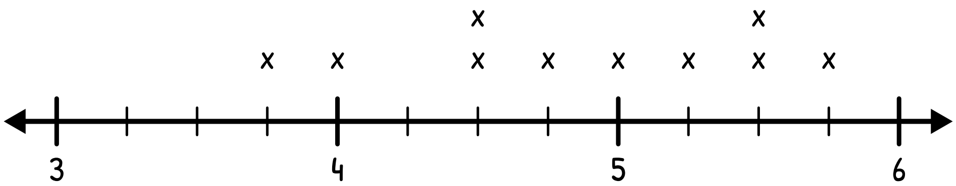
Name: Avery

Students were surveyed about the number of miles they biked in a week. The results are shown in the table.

$5\frac{1}{4}$	$4\frac{3}{4}$	$5\frac{1}{2}$	$4\frac{1}{2}$	5	$5\frac{3}{4}$	$4\frac{1}{2}$	$5\frac{1}{2}$	4	$3\frac{3}{4}$

1. Represent the data on a line plot. Include a title and label.

Miles Biked By Students in a Week



miles

2. What is the difference between the greatest number of miles biked and least number of miles biked?

$5\frac{3}{4} - 3\frac{3}{4} = 2$

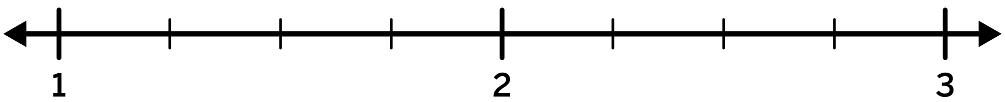
answer: 2 miles

Guided Practice

Represent the data on a line plot. Include a title and label.

1. Han surveyed students about how many hours they practiced a musical instrument each day.

$1\frac{1}{2}$	$2\frac{1}{4}$	$1\frac{3}{4}$	$1\frac{1}{4}$	2	$2\frac{1}{2}$	$1\frac{1}{2}$	$2\frac{3}{4}$	$1\frac{1}{2}$	$2\frac{3}{4}$

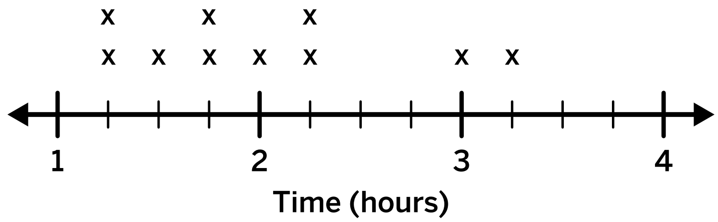


Guided Practice

Several students were surveyed about how many hours they spend on homework in a week. Use the line plot data to answer the questions.

$1\frac{1}{4}$	2	$1\frac{3}{4}$	$2\frac{1}{4}$	$1\frac{1}{2}$
$1\frac{1}{4}$	3	$2\frac{1}{4}$	$1\frac{3}{4}$	$3\frac{1}{4}$

Weekly Homework



2. How many students completed the survey? **answer:** ____________

3. What fraction of the students spent fewer than 3 hours on homework? **answer:** ____________

4. What fraction of students spent at least $2\frac{1}{4}$ hours on homework? **answer:** ____________

5. What is the difference between the greatest number of hours spent on homework and the least number of hours? **answer:** ____________

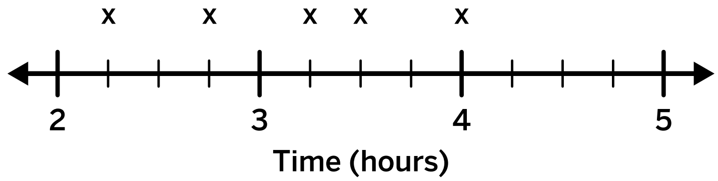
Check

Complete the line plot and answer the question.

1. Diego keeps track of the time he spends at different soccer practices each month.

$2\frac{3}{4}$	4	$3\frac{1}{2}$	$3\frac{1}{4}$	$2\frac{1}{4}$
$2\frac{3}{4}$	$3\frac{1}{2}$	$2\frac{1}{2}$	$4\frac{1}{4}$	2

Monthly Soccer Practice



2. What is the difference between the greatest number of hours Diego spent practicing and the least number of hours he spent practicing? Show or explain your thinking.

answer: ____________

Representing Data on a Line Plot and Solving Problems

Goal

Create a line plot to represent data and use the information to solve problems.

Standard

MA.5.DP.1.1

Modeled Review

Point to Avery's work and **ask**:

- "What does each X on the line plot represent?"
- "How many total students were surveyed? How do you know?"
- "How did Avery know the greatest number of miles biked? Least? What steps did she take to find the difference?"

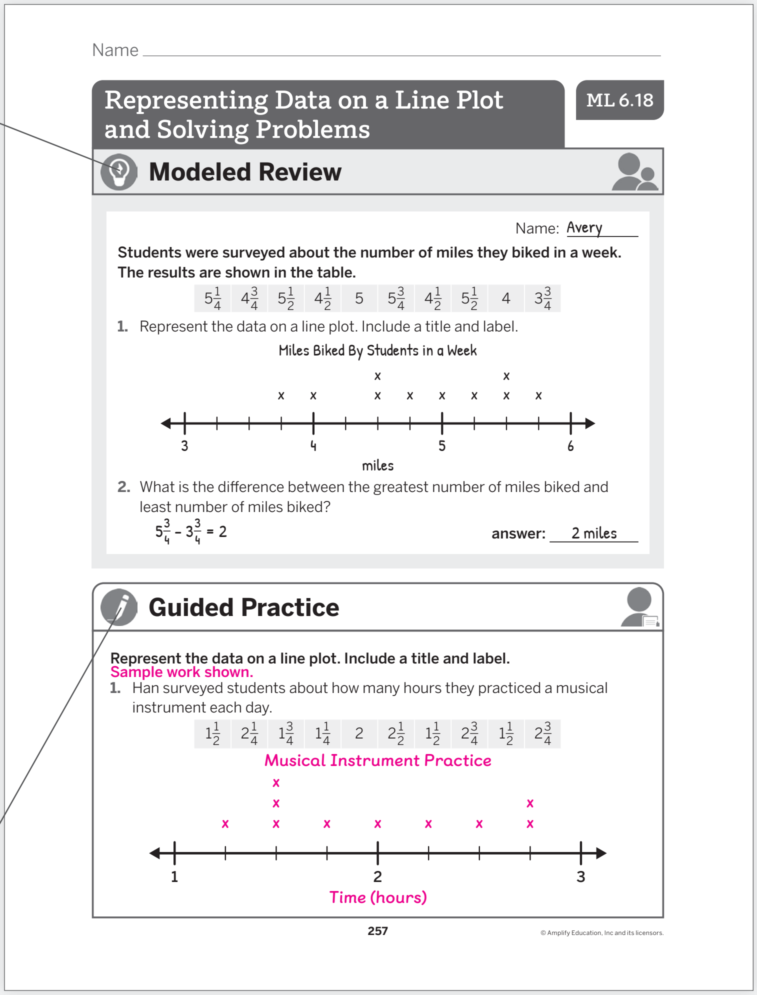
Reinforce Avery's thinking by saying, "Line plots include a number line that is labeled to show the measurement units. A symbol, such as an X, is used to mark each data value above its measurement. If a measurement is recorded multiple times, the Xs are stacked on top of one another."

Guided Practice

Focus students' attention on representing the data on a line plot.

To scaffold their thinking, **ask**:

- "What title could represent the data on the line plot?"
- "How could you label your line plot to help you know where to place the measurements?"
- "How can you show that the same measurement occurs more than once on the line plot?"



Name

Representing Data on a Line Plot and Solving Problems ML 6.18

Modeled Review

Name: Avery

Students were surveyed about the number of miles they biked in a week. The results are shown in the table.

$5\frac{1}{4}$	$4\frac{3}{4}$	$5\frac{1}{2}$	$4\frac{1}{2}$	5	$5\frac{3}{4}$	$4\frac{1}{2}$	$5\frac{1}{2}$	4	$3\frac{3}{4}$

1. Represent the data on a line plot. Include a title and label.

2. What is the difference between the greatest number of miles biked and least number of miles biked?

$5\frac{3}{4} - 3\frac{3}{4} = 2$ answer: 2 miles

Guided Practice

Represent the data on a line plot. Include a title and label.
Sample work shown.

1. Han surveyed students about how many hours they practiced a musical instrument each day.

$1\frac{1}{2}$	$2\frac{1}{4}$	$1\frac{3}{4}$	$1\frac{1}{4}$	2	$2\frac{1}{2}$	$1\frac{1}{2}$	$2\frac{3}{4}$	$1\frac{1}{2}$	$2\frac{3}{4}$

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Vocabulary

If needed, share the meaning of the term with students.

line plot: A display of measurement data in which every data value is shown with a symbol above a number line.

ML 6.18 Representing Data on a Line Plot and Solving Problems

Guided Practice

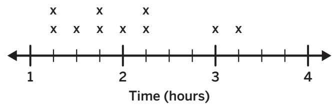
A Consider modeling or inviting a student to model strategies for annotating the line plot to make sense of it.

Key Takeaway:
Say, "Using data from a line plot to solve problems sometimes means needing to add, subtract, multiply, or divide. For some problems, you can interpret the data without needing to compute."

Guided Practice

Several students were surveyed about how many hours they spend on homework in a week. Use the line plot data to answer the questions.

$1\frac{1}{4}$	2	$1\frac{3}{4}$	$2\frac{1}{4}$	$1\frac{1}{2}$
$1\frac{1}{4}$	3	$2\frac{1}{4}$	$1\frac{3}{4}$	$3\frac{1}{4}$



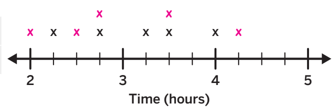
2. How many students completed the survey? answer: 10
3. What fraction of the students spent fewer than 3 hours on homework? answer: $\frac{8}{10}$ or $\frac{4}{5}$
4. What fraction of students spent at least $2\frac{1}{4}$ hours on homework? answer: $\frac{4}{10}$ or $\frac{2}{5}$
5. What is the difference between the greatest number of hours spent on homework and the least number of hours? Sample work shown. answer: 2 hours
   $3\frac{1}{4} - 1\frac{1}{4} = 2$

Check

Complete the line plot and answer the question. Sample work shown.

1. Diego keeps track of the time he spends at different soccer practices each month.

$2\frac{3}{4}$	4	$3\frac{1}{2}$	$3\frac{1}{4}$	$2\frac{1}{4}$
$2\frac{3}{4}$	$3\frac{1}{2}$	$2\frac{1}{2}$	$4\frac{1}{4}$	2



2. What is the difference between the greatest number of hours Diego spent practicing and the least number of hours he spent practicing? Show or explain your thinking.
   $4\frac{1}{4} - 2 = 2\frac{1}{4}$ answer: $2\frac{1}{4}$ hours

Reflection

Ask:
- "What strategies were most useful for creating and interpreting a line plot. Why?"
- "What questions do you still have?"

Check: Recommended Next Steps

Almost there
If students need more support, consider reviewing key vocabulary terms line plots and line graphs with students.

Got it!
If students need more practice, invite them to answer the following questions about the line plot in Problem 1:

- What fraction of students practiced less than $2\frac{1}{4}$ hours?
- What is the difference between the greatest number of hours practiced and the least number of hours practiced?

Name ______________________________

Using Line Plots to Solve Problems

ML 6.19

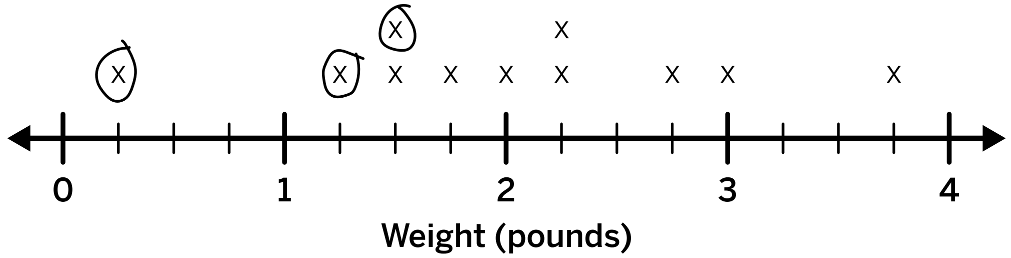
Modeled Review

Name: Jada

Use the line plot to answer the question.

What is the total combined weight of the three smallest bags of fruit?

Bags of Fruit



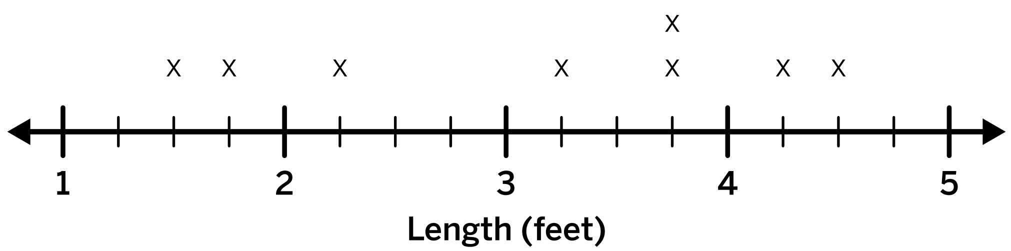
$\frac{1}{4} + 1\frac{1}{4} + 1\frac{2}{4} = 2\frac{4}{4} = 3$

answer: 3 pounds

Guided Practice

Use the line plot data to answer the questions.

Ribbon Measurements



1. How many ribbons are shown in the line plot? **answer:** ______________

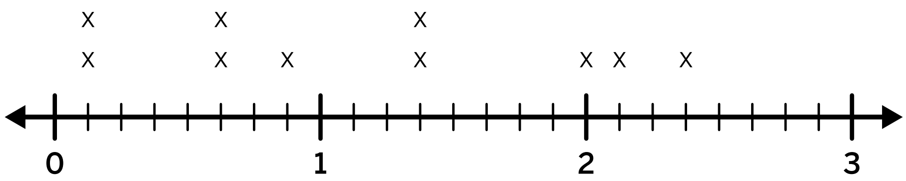
2. What is the length of the longest ribbon? **answer:** ______________

3. How many feet longer is the longest ribbon than the shortest ribbon? **answer:** ______________

Guided Practice

Several students were surveyed about how many hours they practice playing an instrument each week. Use the line plot data to answer the questions.

Hours Practiced



Time (hours)

4. What is the combined time for students who practiced 2 or more hours?

answer: ________________

5. How much more time did the student with the greatest number of hours practice than the two students with the least number of hours combined?

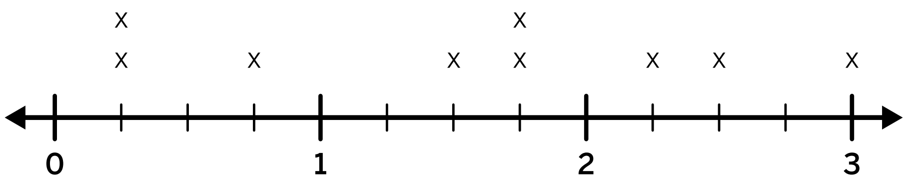
answer: ________________

Check

Use the line plot data to answer the question.

What is the combined length of fabric pieces that were at least 2 feet?

Fabric Pieces



Length (feet)

answer: ________________

ML 6.19 Using Line Plots to Solve Problems

Goal

Use data from line plots to solve problems involving operations of fractions, mixed numbers, and whole numbers.

Standard

MA.5.DP.1.1

Modeled Review

Point to Jada's work and **ask**:

- "How many total bags of fruit were recorded? How do you know?"
- "How did Jada use the line plot to help her solve the problem?"

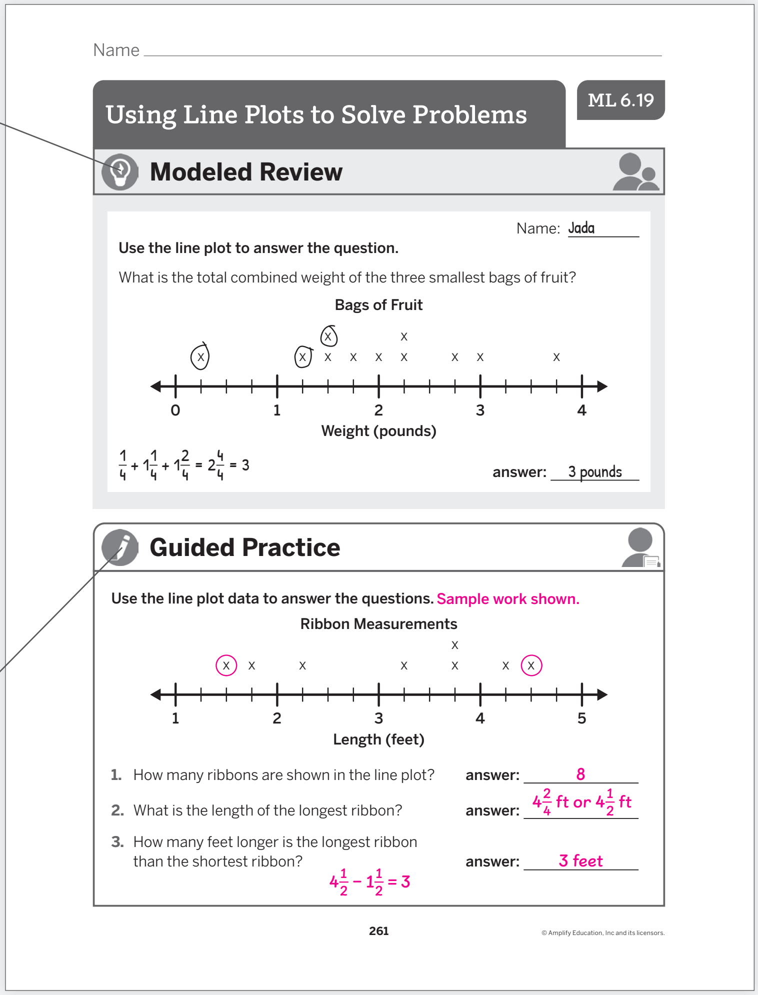
Reinforce Jada's thinking by saying, "When solving problems with line plots, make sure to carefully look at all the data points. You can use addition, subtraction, and other operations to solve problems."

Guided Practice

Focus students' attention on using the line plot data to answer the questions.

To scaffold their thinking, **ask**:

- "What does each X on the line plot represent?"
- "Are there any repeated lengths?"
- "What is the length of the shortest ribbon? Longest ribbon?"
- "What steps do you need to take to find the difference between the longest and shortest ribbon lengths?"



Name

Using Line Plots to Solve Problems ML 6.19

Modeled Review

Name: Jada

Use the line plot to answer the question.

What is the total combined weight of the three smallest bags of fruit?

Bags of Fruit

Weight (pounds)

$\frac{1}{4} + 1\frac{1}{4} + 1\frac{2}{4} = 2\frac{4}{4} = 3$

answer: 3 pounds

Guided Practice

Use the line plot data to answer the questions. Sample work shown.

Ribbon Measurements

Length (feet)

1. How many ribbons are shown in the line plot? answer: 8
2. What is the length of the longest ribbon? answer: $4\frac{2}{4}$ ft or $4\frac{1}{2}$ ft
3. How many feet longer is the longest ribbon than the shortest ribbon? answer: 3 feet

$4\frac{1}{2} - 1\frac{1}{2} = 3$

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Vocabulary

If needed, share the meaning of the term with students.

line plot: A display of measurement data in which every data value is shown with a symbol above a number line.

Using Line Plots to Solve Problems

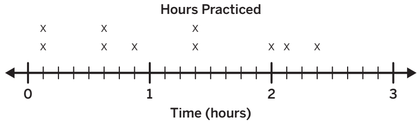
Guided Practice

A Use a think aloud as you model finding the data points on the line plot that are needed and writing an equation to solve the problem.

Key Takeaway:
Say, "There are many different questions you can ask about the data represented on a line plot. Depending on the question, you may be able to solve it using one or more of the four operations."

Guided Practice

Several students were surveyed about how many hours they practice playing an instrument each week. Use the line plot data to answer the questions. Sample work shown.



4. What is the combined time for students who practiced 2 or more hours?

$2 + 2\frac{1}{8} + 2\frac{3}{8} = 6\frac{4}{8}$

$6\frac{4}{8} = 6\frac{1}{2}$

answer: $6\frac{4}{8}$ hr or $6\frac{1}{2}$ hr

5. How much more time did the student with the greatest number of hours practice than the two students with the least number of hours combined?

$2\frac{3}{8} - (\frac{1}{8} \times 2) = 2\frac{3}{8} - \frac{2}{8}$

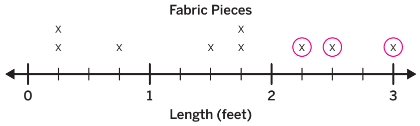
$2\frac{3}{8} - \frac{2}{8} = 2\frac{1}{8}$

answer: $2\frac{1}{8}$ hr

Check

Use the line plot data to answer the question. Sample work shown.

What is the combined length of fabric pieces that were at least 2 feet?



$2\frac{1}{4} + 2\frac{2}{4} + 3 = 7\frac{3}{4}$

answer: $7\frac{3}{4}$ ft

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Reflection

Ask:
- "What strategy was helpful when answering questions about data on a line plot?"
- Reflect on your learning today. What were you most proud of?"

Check: Recommended Next Steps

Almost there
If students need more support, consider using Grade 5 Mini-Lesson 6.18: *Representing Data on a Line Plot and Solving Problems*.

Got it!
If students need more practice, invite them to answer the following questions about the line plot in Problem 1:

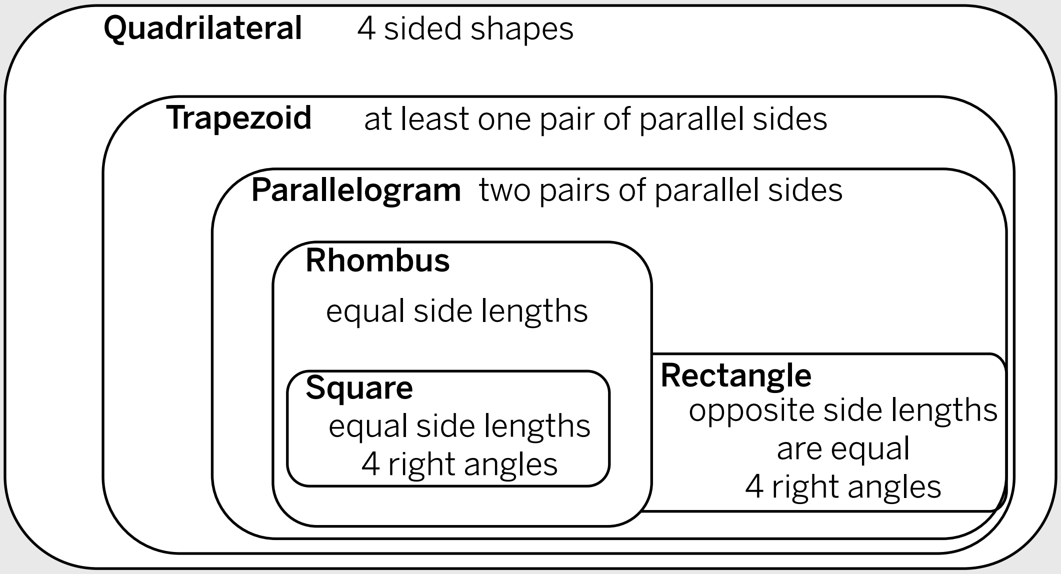
- What is the combined length of the ribbons less than 2 feet?
- What is the difference in length between the two longest ribbons?

Name ______________________________

Using a Hierarchy to Classify Quadrilaterals

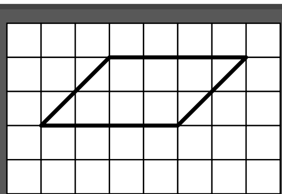
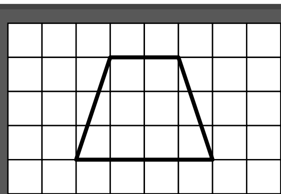
ML 7.04

Modeled Review



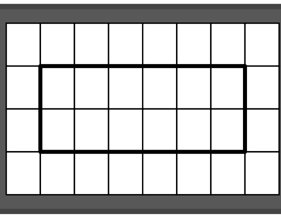
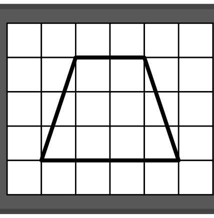
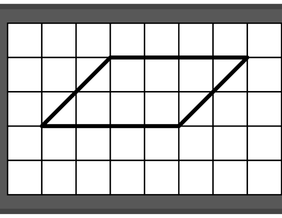
Guided Practice

1. For each shape, determine *all* the possible names that describe it. Place a check mark in the correct column.

		
Quadrilateral	✓	✓
Trapezoid		
Parallelogram		
Rhombus		
Rectangle		
Square		

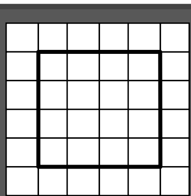
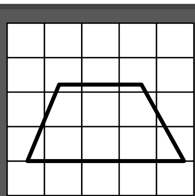
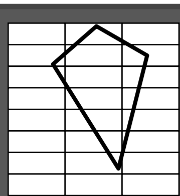
Guided Practice

2. For each shape, determine *all* the possible names that describe it. Place a check mark in the correct column.

			
Quadrilateral			
Trapezoid			
Parallelogram			
Rhombus			
Rectangle			
Square			

Check

For each shape, determine *all* the possible names that describe it. Place a check mark in the correct column.

			
Quadrilateral			
Trapezoid			
Parallelogram			
Rhombus			
Rectangle			
Square			

ML 7.04 Using a Hierarchy to Classify Quadrilaterals

Goal

Classify quadrilaterals in a hierarchy using defining attributes.

Standard

MA.5.GR.1.1

Modeled Review

Point to the Modeled Review and **ask**:

- "What attributes do quadrilaterals share? What attributes make them different?"
- "If a shape is a quadrilateral, does that mean it is always a parallelogram? How do you know?"
- "What does the placement of the rectangle in the chart tell you about its relationship with squares and rhombuses?"

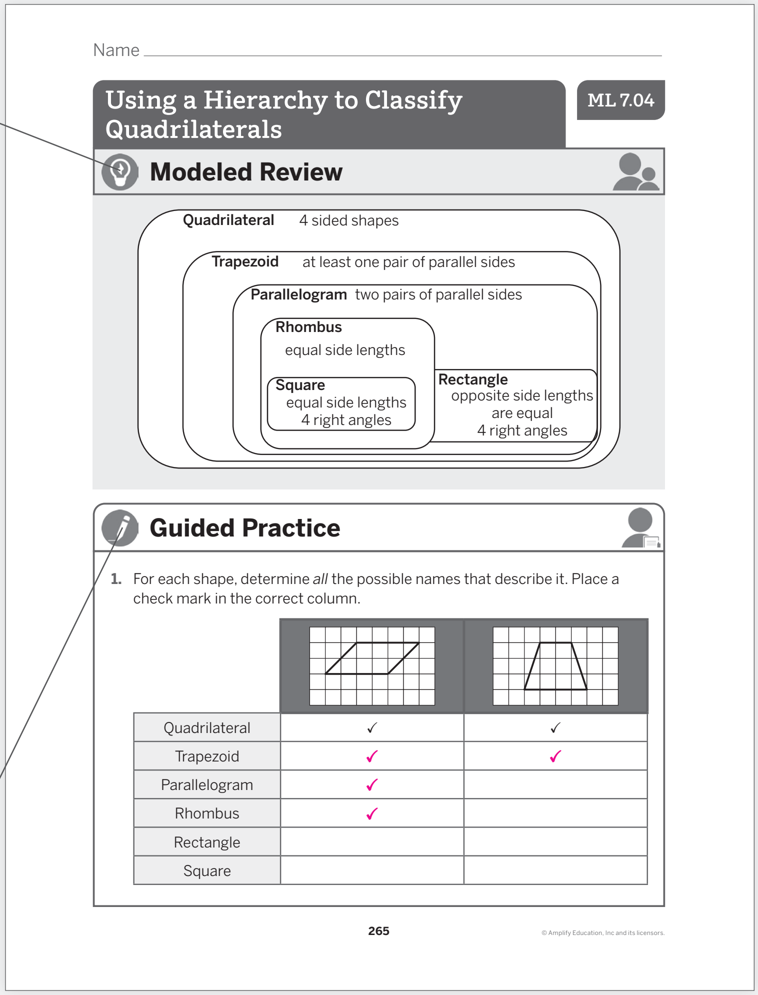
Reinforce the goal by saying, "The quadrilateral hierarchy shows how all quadrilaterals are related."

Guided Practice

Focus students' attention on determining the attributes of each shape and naming them correctly.

To scaffold their thinking, **ask**:

- "What are the attributes of a trapezoid (or any shape)? Does this shape have those attributes?"
- "What attribute do the shapes have that is different? How can you use that defining attribute to help determine the names?"



Name

Using a Hierarchy to Classify Quadrilaterals ML 7.04

Modeled Review

Quadrilateral 4 sided shapes
Trapezoid at least one pair of parallel sides
Parallelogram two pairs of parallel sides
Rhombus equal side lengths
Square equal side lengths 4 right angles
Rectangle opposite side lengths are equal 4 right angles

Guided Practice

1. For each shape, determine *all* the possible names that describe it. Place a check mark in the correct column.

Quadrilateral	✓	✓
Trapezoid	✓	✓
Parallelogram	✓	
Rhombus	✓	
Rectangle		
Square		

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Vocabulary

If needed, share the meaning of the terms with students.

parallelogram: A quadrilateral with two pairs of opposite sides that are parallel and have equal length.

rhombus: A quadrilateral with four equal-length sides.

trapezoid: A quadrilateral with at least one pair of opposite sides that are parallel.

Using a Hierarchy to Classify Quadrilaterals

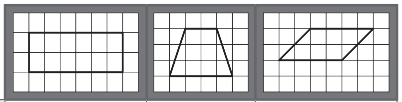
Guided Practice

A As students select possible names for the shapes in Problem 2, encourage them to explain their reasoning to classmates, which can help them solidify their understanding.

Key Takeaway:
Say, "Quadrilaterals can have many names, based on their attributes. In the quadrilateral hierarchy, every shape in a subgroup is *always* a member of each larger group around it. A shape in a larger group can *sometimes* be a member of a subgroup."

Guided Practice

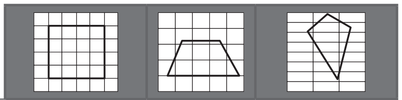
2. For each shape, determine *all* the possible names that describe it. Place a check mark in the correct column.



Quadrilateral	✓	✓	✓
Trapezoid	✓	✓	✓
Parallelogram	✓		✓
Rhombus			
Rectangle	✓		
Square			

Check

For each shape, determine *all* the possible names that describe it. Place a check mark in the correct column.



Quadrilateral	✓	✓	✓
Trapezoid	✓	✓	
Parallelogram	✓		
Rhombus	✓		
Rectangle	✓		
Square	✓		

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Reflection

Ask:
- "What are defining attributes for a trapezoid? Parallelogram? Rhombus?"
- "How does what you learned today connect to your prior learning?"

Check: Recommended Next Steps

Almost there
If students need more support, consider reviewing the chart in the Modeled Review and clearly explaining the defining attributes for each type of quadrilateral. Then have students revisit Problem 2.

Got it!
If students need more practice, invite them to determine whether each statement is true or false.

- A rectangle is never a rhombus.
- A quadrilateral is always a parallelogram.
- A parallelogram is always a trapezoid.
- A rhombus is always a square.

Name ____________________

Using the Coordinate Grid to Locate Points

ML 7.06

Modeled Review

Name: Jack

Describe the location of each point.

1. point *A*

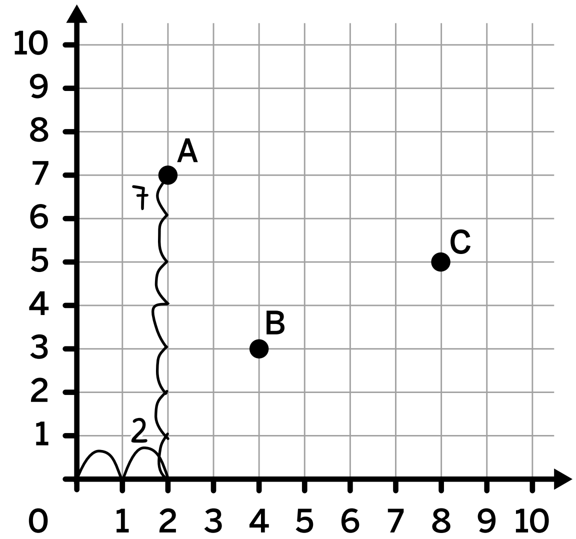
2 units right, then up 7 units

2. point *B*

(4, 3)

3. point *C*

(8, 5)



Guided Practice

Identify the point that is shown by each ordered pair.

1. (3, 5)

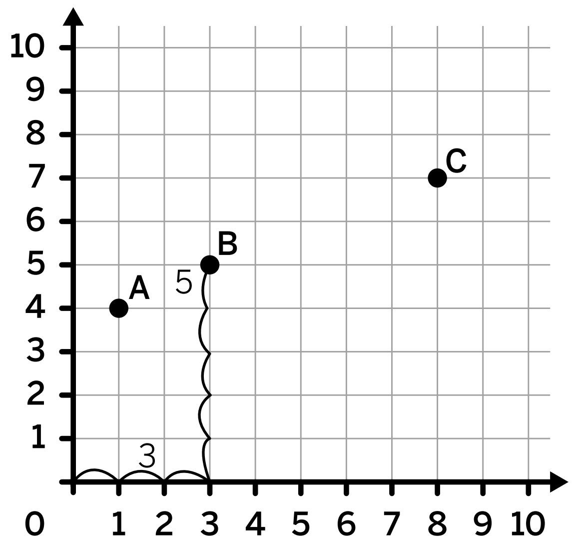
____

2. (1, 4)

____

3. (8, 7)

____



Guided Practice

Use only numbers to describe the location of each given point.

4. point *A*

1 unit to the right, then up 5 units

(1, ____)

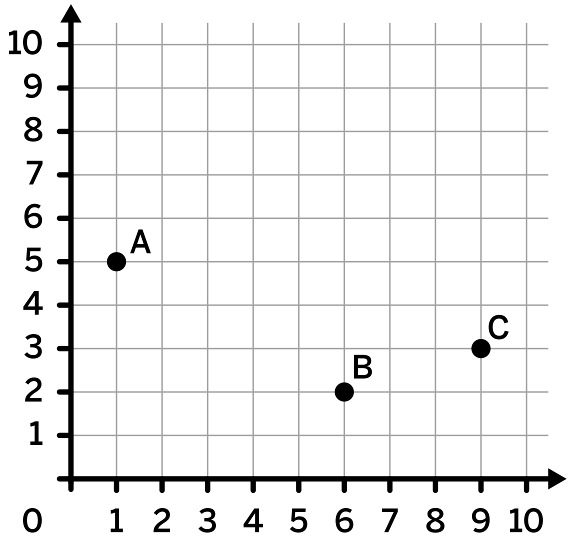
5. point *B*

6 units right, then up 2 units

(____, ____)

6. point *C*

____



Check

Use only numbers to describe the location of each given point.

1. point *A*

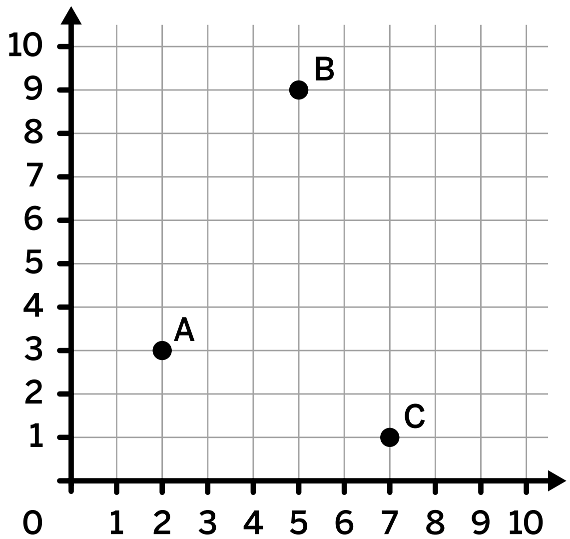
____

2. point *B*

____

3. point *C*

____



Using the Coordinate Grid to Locate Points

Goal

Use the coordinate grid to describe the location of a point.

Standard

MA.5.GR.4.1

Materials

coloring tools (optional)

Modeled Review

Point to Jack's work and **ask**:
- "Where do you see number lines on the coordinate grid?"
- "Where is Jack's starting point before moving two units right to find the location of point *A*?"
- "How could Jack have described point *A* using only two numbers and without any words?"

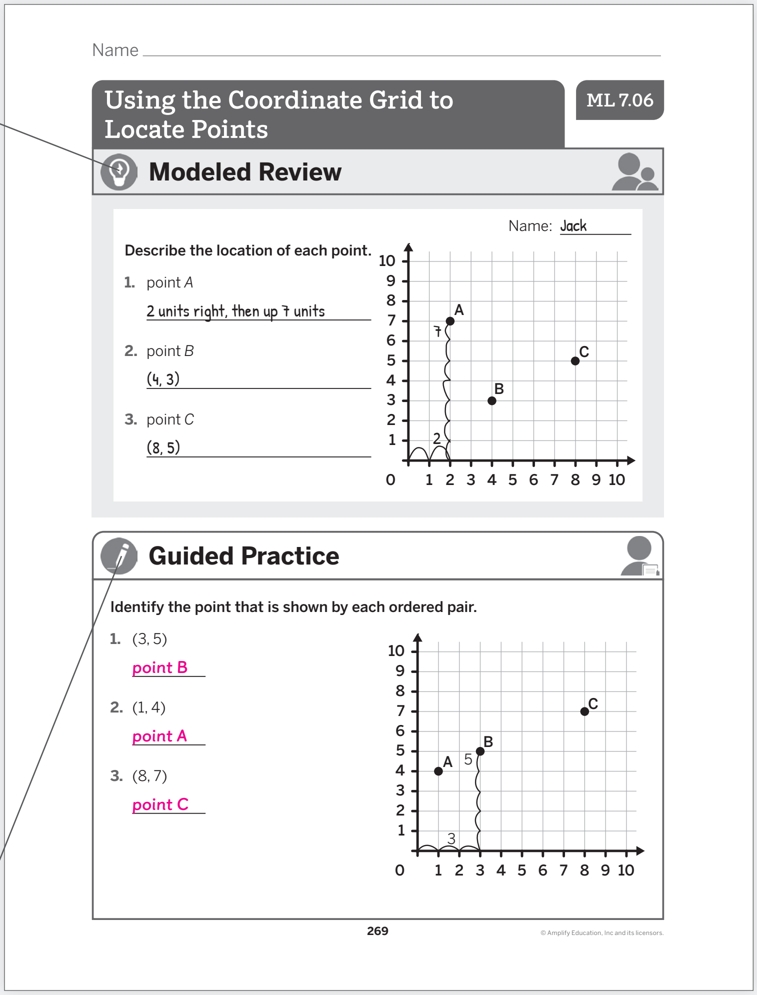
Reinforce the goal by saying, "A coordinate grid helps you precisely describe the location of points. Ordered pairs show the distance the point is from the origin in two directions."

ML/EL Use hand gestures or body movements to model the direction in which you identify points on the coordinate grid (i.e., slide right, then jump up).

Guided Practice

Focus students' attention on identifying each point on the coordinate grid.

To scaffold their thinking, **ask**:
- "When finding the location of a point, what is your starting point?"
- "How do you read the ordered pair? What direction do you move in first from the origin? Second?"



Name

Using the Coordinate Grid to Locate Points — ML 7.06

Modeled Review

Name: Jack

Describe the location of each point.

1. point *A*
   2 units right, then up 7 units
2. point *B*
   (4, 3)
3. point *C*
   (8, 5)

Guided Practice

Identify the point that is shown by each ordered pair.

1. (3, 5)
   point B
2. (1, 4)
   point A
3. (8, 7)
   point C

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Vocabulary

If needed, share the meaning of the terms with students.

coordinate grid: A two-dimensional plane formed by two perpendicular number lines.

ordered pair: A pair of two numbers that correspond to points on the x and y axes, expressed as (x, y).

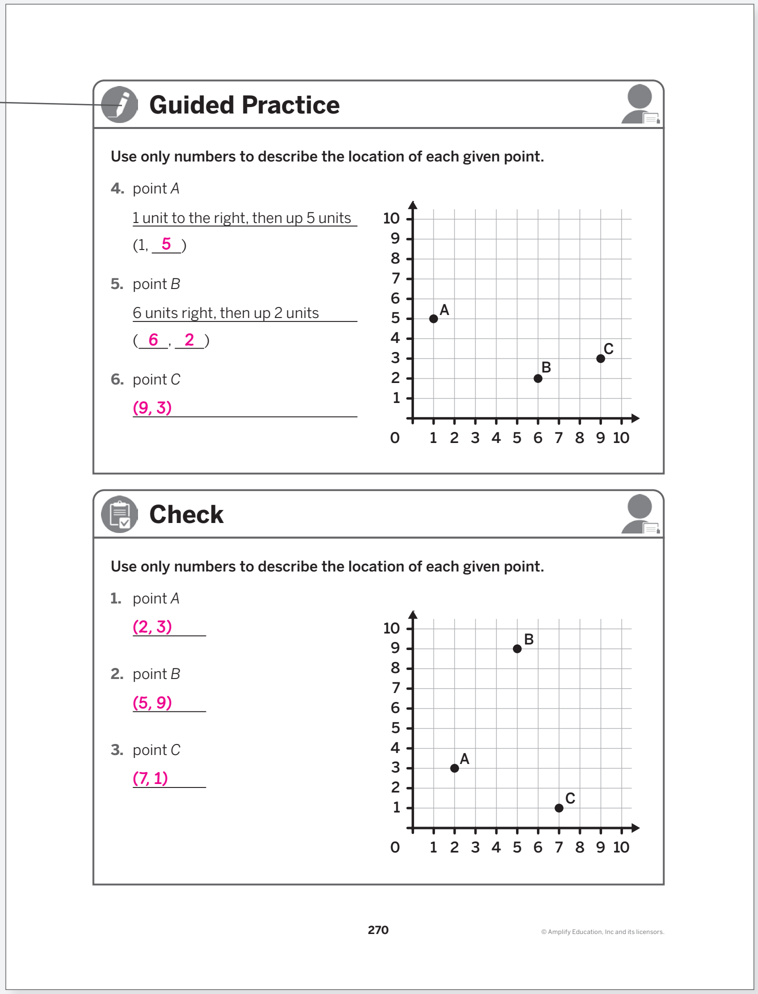
origin: The point where the two axes intersect on the coordinate plane, which is located at (0, 0).

ML 7.06 Using the Coordinate Grid to Locate Points

Guided Practice

A Guide visualization of coordinates by coloring the x-axis one color and the y-axis another. Then invite students to record the coordinates with the color matching its corresponding axis.

Key Takeaway:
Say, "The coordinate grid can be used to locate points with precision by using coordinates to describe the horizontal and vertical distances from the origin."



Guided Practice

Use only numbers to describe the location of each given point.

4. point *A*

1 unit to the right, then up 5 units

(1, 5)

5. point *B*

6 units right, then up 2 units

(6, 2)

6. point *C*

(9, 3)

Check

Use only numbers to describe the location of each given point.

1. point *A*

(2, 3)

2. point *B*

(5, 9)

3. point *C*

(7, 1)

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Reflection

Ask:

- "When identifying points on the coordinate grid, the ordered pair describes the distance from what? Where is this located?"
- "What is something you weren't sure about at the start of the lesson but understand now?"

Check: Recommended Next Steps

Almost there	Got it!
If students need more support, consider having them say the steps aloud as they identify each point in the Check to increase their understanding of the coordinate grid. For example, "Start at (0, 0). Then move ___ units right and ___ units up."	If students need more practice, mark and label a point *D* and point *E* on the coordinate grid in the Check. Then invite students to identify the ordered pair for each point.

Name ______________________________

Plotting Points on the Coordinate Grid

ML 7.07

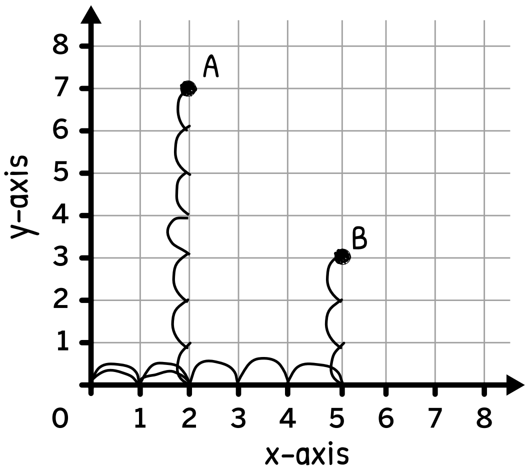
Modeled Review

Name: Eva

Plot and label each point on the coordinate grid.

Point	Ordered pair
A	(2, 7)
B	(5, 3)

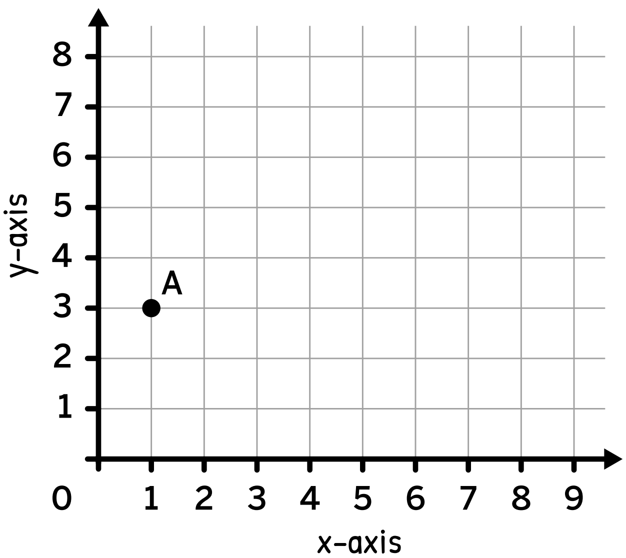
(x, y)
→, ↑



Guided Practice

1. Use the information in the table to plot and label each point on the coordinate grid.

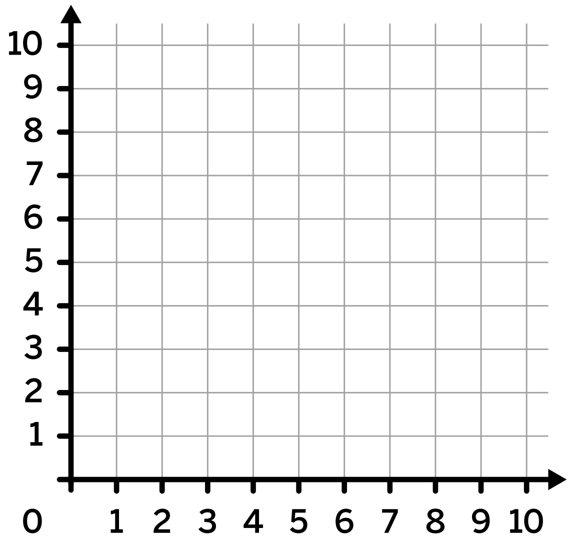
Point	Ordered pair (x, y)	Directions from origin →, ↑
A	(1, 3)	1 unit right, up 3 units
B	(3, 7)	3 units right, up 7 units
C	(5, 6)	5 units right, up 6 units
D	(8, 2)	8 units right, up 2 units



Guided Practice

2. Plot and label each point on the coordinate grid.

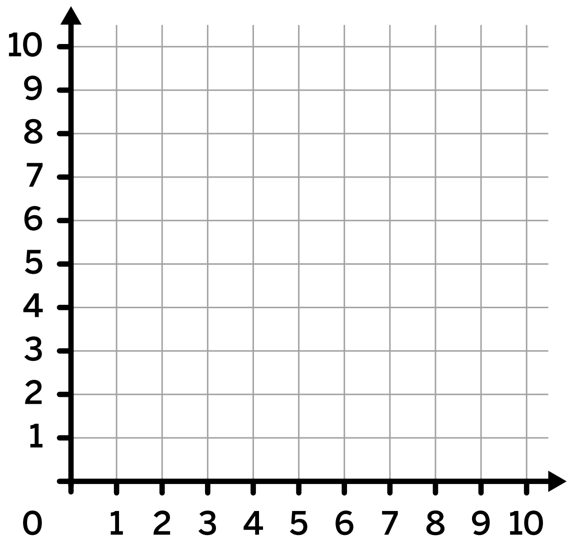
Point	Ordered pair
A	(1, 5)
B	(2, 3)
C	(7, 1)
D	(6, 4)



Check

Plot and label each point on the coordinate grid.

Point	Ordered pair
A	(4, 8)
B	(5, 2)



ML 7.07 Plotting Points on the Coordinate Grid

Goal

Plot points on the coordinate grid using ordered pairs.

Standard

MA.5.GR.4.1

Modeled Review

Point to Eva's work and **ask**:

- "Eva knew that an ordered pair is expressed as (x, y). Why did she draw 2 jumps along the x-axis? Why did she draw 7 jumps up?"
- "What was Eva's starting point when plotting both ordered pairs?"
- "What would happen to point *A* if Eva would have moved up 2, then right 7? Would it still be correct?"

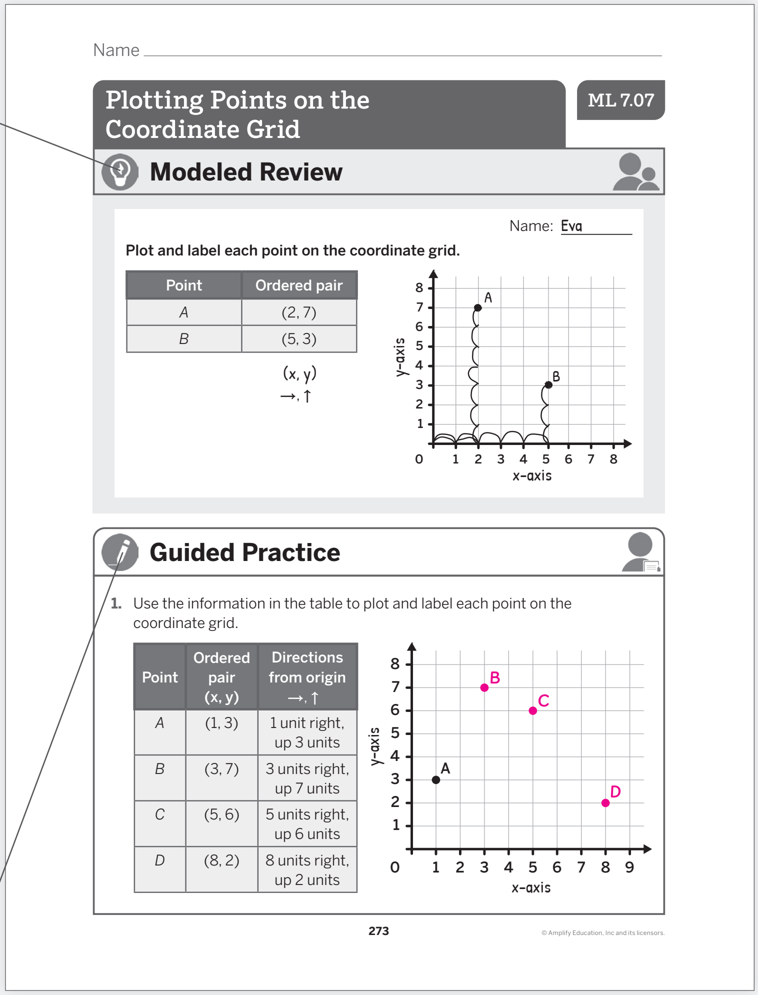
Reinforce Eva's thinking by saying, "Order matters when listing numbers in an ordered pair. In an ordered pair, the first number is the x-coordinate, which shows the distance from the origin on the horizontal axis. The second number is the y-coordinate which shows the distance from the origin on the vertical axis."

Guided Practice

Focus students' attention on using the table to plot each ordered pair.

To scaffold their thinking, **ask**:

- "What do the instructions say about where to start?"
- "Explain the directions you used to plot point *B*. Does that match the instructions given?"



Name

Plotting Points on the Coordinate Grid ML 7.07

Modeled Review

Name: Eva

Plot and label each point on the coordinate grid.

Point	Ordered pair
A	(2, 7)
B	(5, 3)

(x, y)
→, ↑

Guided Practice

1. Use the information in the table to plot and label each point on the coordinate grid.

Point	Ordered pair (x, y)	Directions from origin →, ↑
A	(1, 3)	1 unit right, up 3 units
B	(3, 7)	3 units right, up 7 units
C	(5, 6)	5 units right, up 6 units
D	(8, 2)	8 units right, up 2 units

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Vocabulary

If needed, share the meaning of the terms with students.

x-coordinate: The first number in an ordered pair, indicating how far left or right the point is from the origin on the horizontal axis.

y-coordinate: The second number in an ordered pair, indicating how far above or below the point is from the origin on the vertical axis.

Plotting Points on the Coordinate Grid

Guided Practice

A Encourage students to label the x-axis and the y-axis on the coordinate grid and annotate the table with (x, y) and arrows to remind them of the steps to plot the points.

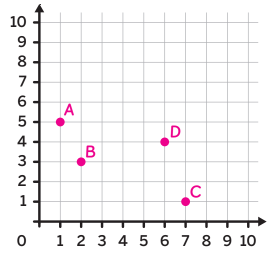
ML/EL Use gestures to highlight the difference between the x-axis and y-axis.

Key Takeaway:
Say, "In an ordered pair, the x-coordinate describes the distance from the origin on the horizontal axis. The y-coordinate describes the distance along the y-axis."

Guided Practice

2. Plot and label each point on the coordinate grid.

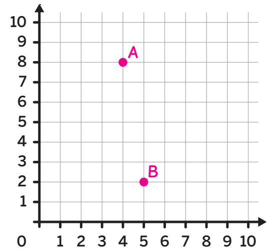
Point	Ordered pair
A	(1, 5)
B	(2, 3)
C	(7, 1)
D	(6, 4)



Check

Plot and label each point on the coordinate grid.

Point	Ordered pair
A	(4, 8)
B	(5, 2)



Reflection

Ask:
- "What directions do you use to determine the distance from the origin when plotting an ordered pair on the coordinate grid?"
- "How was the lesson helpful to you today?"

Check: Recommended Next Steps

Almost there
If students need more support, consider using Grade 5 Mini-Lesson 7.06: *Using the Coordinate Grid to Locate Points*.

Got it!
If students need more practice, invite them to label the points on the coordinate grid in the Check.

- point *C* (4, 6)
- point *D* (7, 4)
- point *E* (1, 9)

Name ____________________

Determining Relationships Between Rules

ML 7.09

Modeled Review

Name: Jada

1. Use the rules to complete the table.

   Rule 1: Start with 0 and keep adding 9.

   Rule 2: Start with 0 and keep adding 3.

Rule 1	0	9 ~~×3 = 27~~	18	27	36	45
Rule 2	0	÷ 3 3	÷ 3 6	÷ 3 9	÷ 3 12	÷ 3 15

2. Describe the relationship between the numbers in Rule 1 and Rule 2.

   Divide the number in Rule 1 by 3.

Guided Practice

1. Use the rules to complete each table.

   Rule 1: Start with 0 and keep adding 5.

   Rule 2: Start with 0 and keep adding 10.

Rule 1	0	5	10				
Rule 2							

2. **Discuss:** How could you describe the relationship between the numbers in Rule 1 and Rule 2?

Guided Practice

3. Use the rules to complete the table.

Rule 1: Start with 0 and keep adding 8.

Rule 2: Start with 0 and keep adding 2.

Rule 1						
Rule 2						

4. Describe the relationship between the numbers in Rule 1 and Rule 2.

____

____

Check

1. Use the rules to complete the table.

Rule 1: Start with 0 and keep adding 6.

Rule 2: Start with 0 and keep adding 12.

Rule 1						
Rule 2						

2. Describe the relationship between the numbers in Rule 1 and Rule 2.

____

____

ML 7.09 Determining Relationships Between Rules

Goal	Standard
Generate patterns from two given rules and relate their corresponding terms.	MA.5.AR.3.2

Modeled Review

Point to Jada's work and **ask**:
- "How did Jada use the rules to complete the table?"
- "What relationship did Jada notice between 9 and 3? The other numbers in Rule 1 and Rule 2?"
- "How did Jada describe that relationship?"

Reinforce the goal by saying, "You can use two rules to generate a pattern that has a relationship between corresponding terms. You can describe that relationship in different ways."

Guided Practice

Focus students' attention on using the rules to describe the relationship between the numbers.

To scaffold their thinking, **ask**:
- "How can you use the rules to find the next number?"
- "Look at the second number in Rule 1 and the second number in Rule 2. What do you notice? How could you describe that relationship?"
- "When the number in Rule 1 is 50, what will be the corresponding term in Rule 2? How do you know?"

Name ______________________

Determining Relationships Between Rules — ML 7.09

Modeled Review

Name: Jada

1. Use the rules to complete the table.

Rule 1: Start with 0 and keep adding 9.

Rule 2: Start with 0 and keep adding 3.

Rule 1	0	9 ~~×3 = 27~~	18	27	36	45
Rule 2	0	÷3 3	÷3 6	÷3 9	÷3 12	÷3 15

2. Describe the relationship between the numbers in Rule 1 and Rule 2.

Divide the number in Rule 1 by 3.

Guided Practice

1. Use the rules to complete each table.

Rule 1: Start with 0 and keep adding 5.

Rule 2: Start with 0 and keep adding 10.

Rule 1	0	5	10	15	20	25	30
Rule 2	0	10	20	30	40	50	60

2. **Discuss:** How could you describe the relationship between the numbers in Rule 1 and Rule 2? Oral activity: No writing expected.

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Vocabulary

If needed, share the meaning of the term with students.

pattern: A specific, replicable sequence of shapes or numbers; some patterns repeat and some patterns grow.

ML 7.09 Determining Relationships Between Rules

Guided Practice

A Encourage students to annotate the tables to support their understanding of the relationship between each rule.

Key Takeaway:
Say, "Two rules can be used to generate a pattern that has a relationship between corresponding terms. The relationship between corresponding terms can be described in different ways, including using multiplication or division."

Guided Practice

3. Use the rules to complete the table.

Rule 1: Start with 0 and keep adding 8.

Rule 2: Start with 0 and keep adding 2.

Rule 1	0	8	16	24	32	40
Rule 2	0	2	4	6	8	10

4. Describe the relationship between the numbers in Rule 1 and Rule 2.
Sample response shown.
Divide the number in Rule 1 by 4.

Check

1. Use the rules to complete the table.

Rule 1: Start with 0 and keep adding 6.

Rule 2: Start with 0 and keep adding 12.

Rule 1	0	6	12	18	24	30
Rule 2	0	12	24	36	48	60

2. Describe the relationship between the numbers in Rule 1 and Rule 2.
Sample response shown.
Multiply the number in Rule 1 by 2.

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Reflection

Ask:

- "What is one way you can describe the relationship between two rules?"
- "How was the lesson helpful to you today?"

Check: Recommended Next Steps

Almost there
If students need more support, consider using a think-aloud to complete the table in Problem 3 and describe the relationship between the numbers. Then have students revisit the Check.

Got it!
If students need more practice, invite them to describe the relationship between the two rules in the Check in a different way.

Name ______________________________

Representing Data on the Coordinate Grid

ML 7.11

Modeled Review

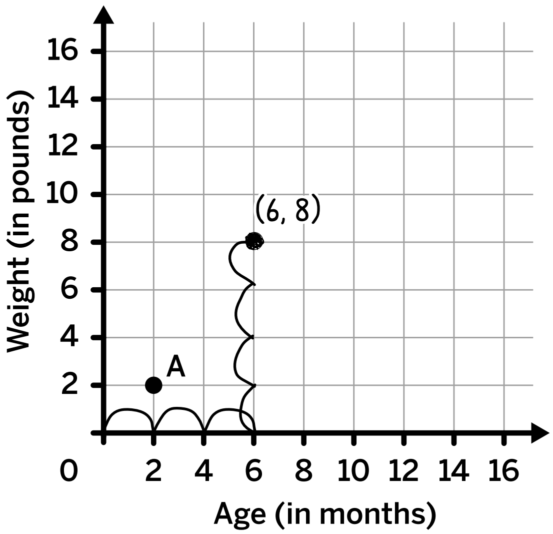
Name: Han

The graph shows the weight of a small dog as it grew. Use the graph for Problems 1–2.

1. What does point *A* represent?

When the dog was two months old, it weighed 2 pounds.

2. Plot a point on the coordinate grid to show that the dog weighed 8 pounds at 6 months. Label the point with its ordered pair.



Guided Practice

The graph shows the number of books Maya read over a 10 week period.

1. What does point *A* represent?

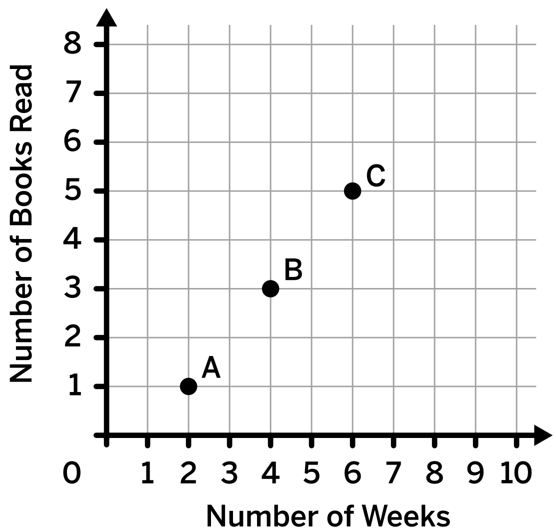
A. It takes Maya 1 week to read 2 books.

B. It takes Maya 2 weeks to read 1 book.

2. What does point *C* represent?

A. It takes Maya 6 weeks to read 5 books.

B. It takes Maya 5 weeks to read 6 books.



Guided Practice

The graph shows the amount of money earned for every hour a student worked after school. Use the graph for Problems 3–4.

3. What does point *A* represent?

____

____

4. Plot points on the coordinate grid to show that a student makes \$60 in 5 hours and \$96 after working 8 hours. Label each point with its ordered pair.

Check

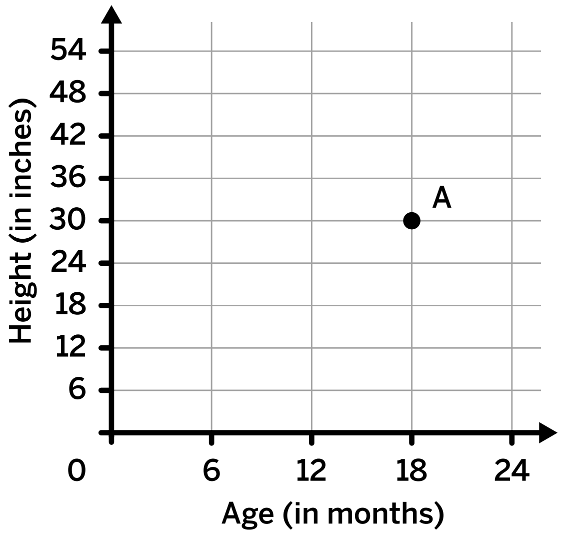
The graph shows the height of a child from birth to two years. Use the graph for Problems 1–2.

1. What does point *A* represent?

____

____

2. Plot a point on the coordinate grid to show that the child is 24 inches at 6 months. Label the point with its ordered pair.



ML 7.11 Representing Data on the Coordinate Grid

Goal	**Standard**
Represent and interpret contextual problems on a coordinate grid.	MA.5.GR.4.2

Modeled Review

Point to Han's work and **ask**:

- "What does each axis on the coordinate grid represent?"
- "How do the labels of each axis help Han answer each question?"
- "What would the point (4, 6) represent given the context of the graph?"

Reinforce Han's thinking by saying, "When a coordinate grid represents a context, you can determine what each point represents by thinking about what each axis represents."

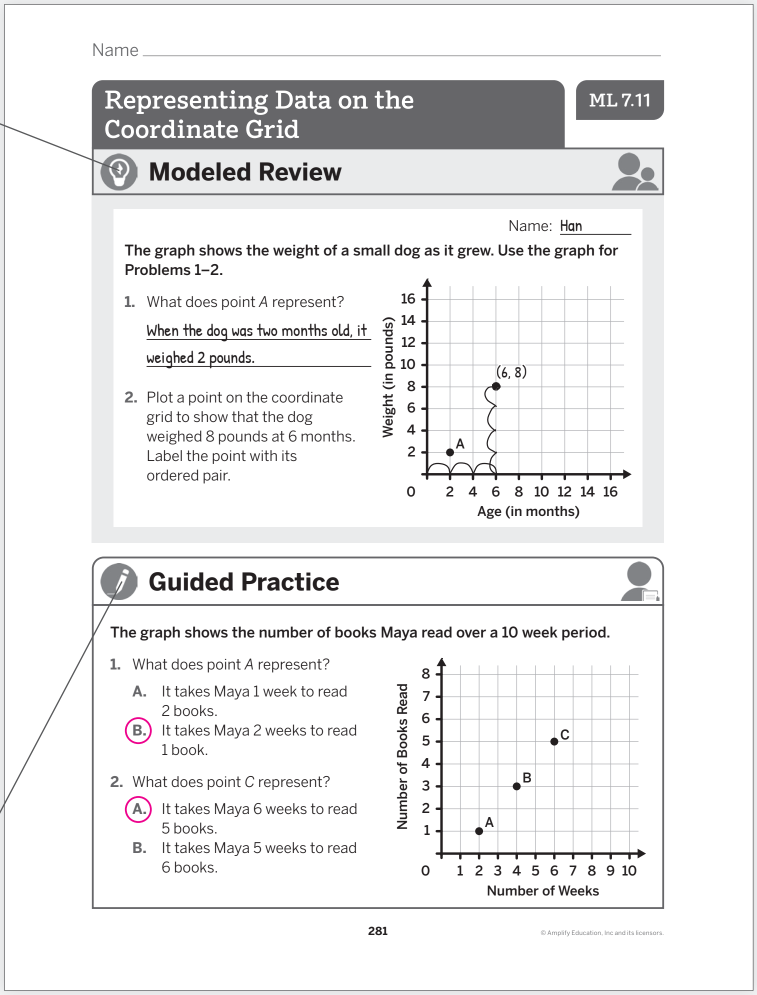
ML/EL Model using a think-aloud while plotting and labeling another point on the coordinate grid.

Guided Practice

Focus students' attention on using the graph to interpret the context of the situation.

To scaffold their thinking, **ask**:

- "What does the information on the x-axis represent? The y-axis?"
- "How many weeks does point *A* represent? How many books?"
- "How many weeks does point *C* represent? How many books?"



Name

Representing Data on the Coordinate Grid

ML 7.11

Modeled Review

Name: Han

The graph shows the weight of a small dog as it grew. Use the graph for Problems 1–2.

1. What does point *A* represent?
   When the dog was two months old, it weighed 2 pounds.
2. Plot a point on the coordinate grid to show that the dog weighed 8 pounds at 6 months. Label the point with its ordered pair.

Guided Practice

The graph shows the number of books Maya read over a 10 week period.

1. What does point *A* represent?
   A. It takes Maya 1 week to read 2 books.
   (B.) It takes Maya 2 weeks to read 1 book.
2. What does point *C* represent?
   (A.) It takes Maya 6 weeks to read 5 books.
   B. It takes Maya 5 weeks to read 6 books.

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Vocabulary

If needed, share the meaning of the terms with students.

plot: To place a point (often representing *x* and *y* coordinates) on a grid.

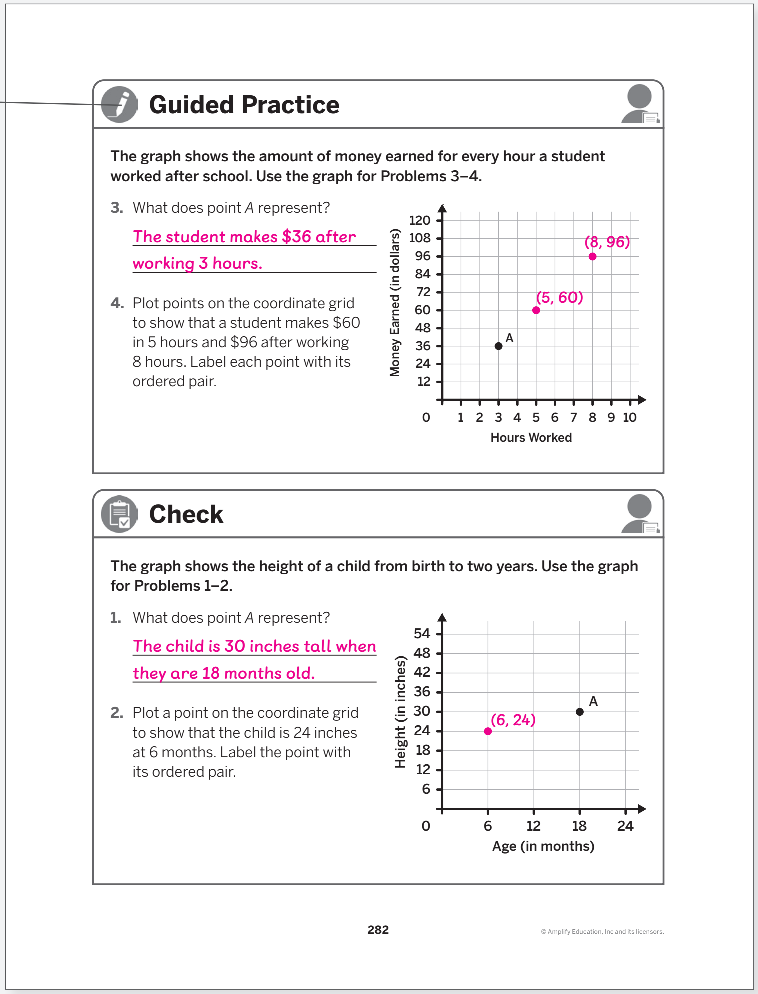
axis: The perpendicular lines that form the coordinate plane, in which the vertical axis is the line going up and down/on the left and the horizontal axis is the line running side to side/on the bottom (plural: axes).

Representing Data on the Coordinate Grid

Guided Practice

A Provide sentence frames to support students' understanding of x- coordinates and y- coordinates. For example, "After __ hours worked, a student makes __ dollars."

Key Takeaway:
Say, "Points on a coordinate grid can represent data about the context of a given situation. The ordered pairs of each point represent the relationship between the information represented on each axis."



Guided Practice

The graph shows the amount of money earned for every hour a student worked after school. Use the graph for Problems 3–4.

3. What does point *A* represent?

The student makes $36 after working 3 hours.

4. Plot points on the coordinate grid to show that a student makes $60 in 5 hours and $96 after working 8 hours. Label each point with its ordered pair.

Check

The graph shows the height of a child from birth to two years. Use the graph for Problems 1–2.

1. What does point *A* represent?

The child is 30 inches tall when they are 18 months old.

2. Plot a point on the coordinate grid to show that the child is 24 inches at 6 months. Label the point with its ordered pair.

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Reflection

Ask:
- "How are the labels on each axis helpful when interpreting data on a coordinate grid?"
- "What makes sense? What is still confusing?"

Check: Recommended Next Steps

Almost there	Got it!
If students need more support, consider providing coordinate points to graph in quadrant 1 of the coordinate plane along with two small objects. Have students explain how they move the object along the *x*-axis and then up the *y*-axis.	If students need more practice, invite them to plot and explain the ordered pairs in the context of the situation in the graph for Problems 3 and 4. • (1, 12) • (10, 120)

Extensions

Unit 1

Extensions

Area

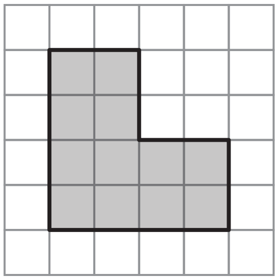
Name: .... Date: .... Period: ....

Student Choice Start with any problem. Remember to show or explain your thinking.

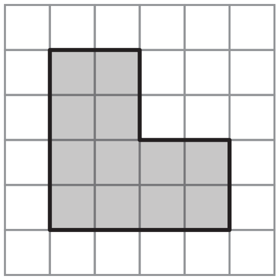
1

a Divide the L shape into . . .

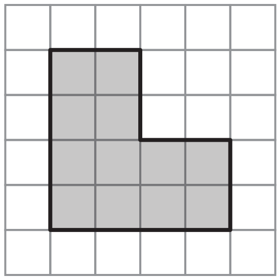
2 identical pieces



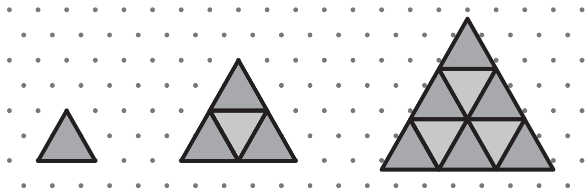
3 identical pieces



4 identical pieces

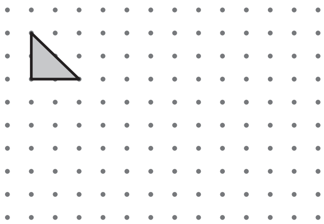


b *Rep-tile* has a different meaning in Mathematics! It is short for "replicating tile." When several copies of these tiles are put together, the shape will appear exactly the same but larger! For example, an equilateral triangle is a rep-tile.

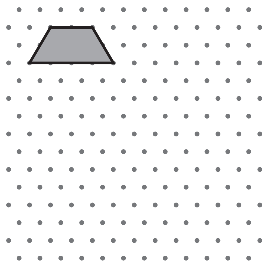


Select *all* the polygons that are rep-tiles.

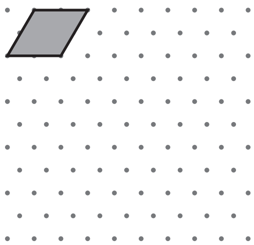
☐ Right triangle



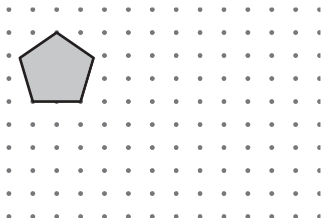
☐ Trapezoid



☐ Parallelogram



☐ Regular pentagon

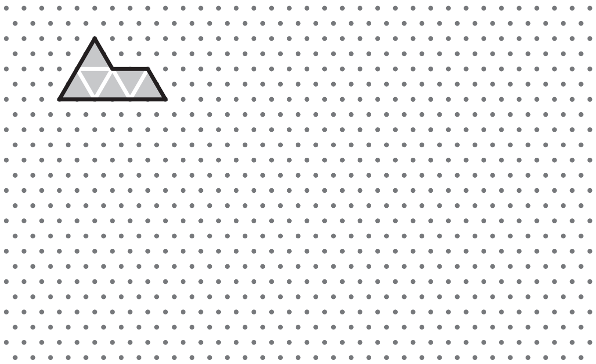


Area (continued)

Name: .... Date: .... Period: ....

c The figure shows a *sphinx tile*, which is composed of six equilateral triangles.

If the side length of each equilateral triangle is 6 units and the height is 5.2 units, draw the sphinx rep-tile and determine its area.



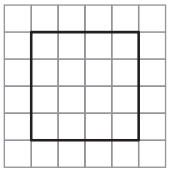
2

a Here are several shapes. For each shape:

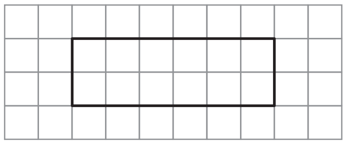
- Mark the midpoint of each side using a ruler or the square grids.
- Connect the four midpoints to form another shape.

What do you notice about the shapes you drew?

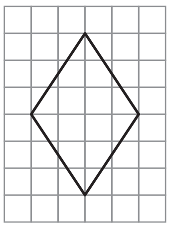
Square



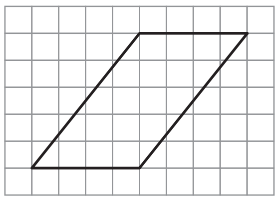
Rectangle



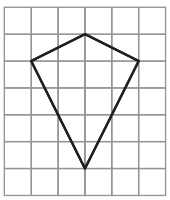
Rhombus



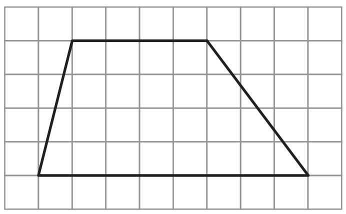
Parallelogram



Kite



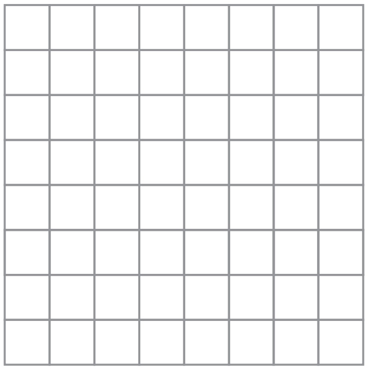
Trapezoid



Area (continued)

Name: ________________ Date: ________ Period: ________

b Draw any four sided shape and repeat the same steps. What is the new shape?



c Find the area of the original shape and compare it to the area of shape you drew inside.

Area

Assign problems to students who want to extend their thinking.

Problems 1 and 2 can be solved in any order. If time allows, consider sharing Problems 1 and 2 with all students.

Materials
- square and isometric graph paper (optional) **(Problems 1–2)**

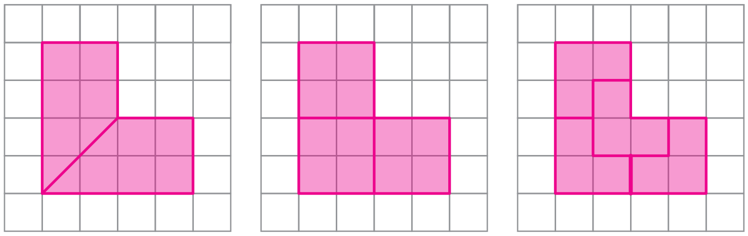
Problem 1

Students will extend their understanding of polygons and their properties.

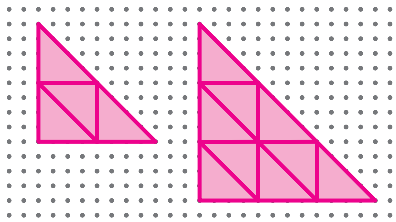
Provide students with the following hints if additional scaffolding is needed.

- **Hint 1:** In part a, what is the number of small squares that each part should have? Think about different shapes that could be made using these numbers of squares.
- **Hint 2:** In part b, four triangles are used to create the next triangle rep-tile. Can you use the same number of parallelograms and trapezoids to create their next rep-tiles?
- **Hint 3:** In part c, one way of creating the next sphinx rep-tile is to use four tiles like the shapes in part b. Can you use four sphinx tiles by flipping or rotating one of them to create the next one?

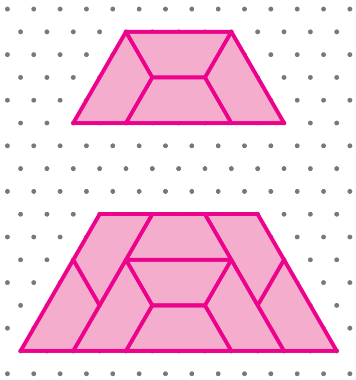
a. 2 identical pieces | 3 identical pieces | 4 identical pieces



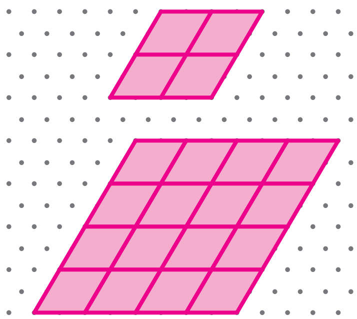
b. ☑ Right triangle



☑ Trapezoid



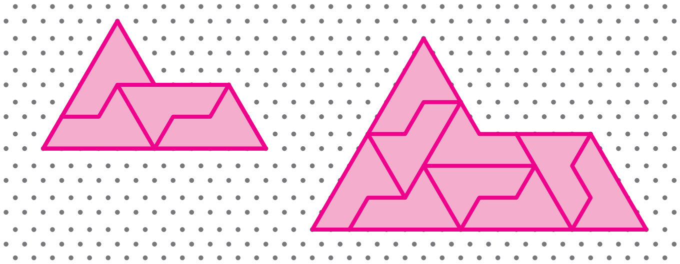
☑ Parallelogram



☐ Regular pentagon

Regular pentagons cannot be used as rep-tiles.

c. The area of a sphinx tile is 15.6 square units.



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Problem 2

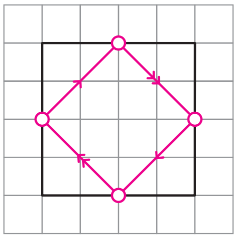
Students will extend their understanding of the properties of quadrilaterals and area by forming parallelograms connecting the midpoints of different quadrilaterals and comparing their areas.

Provide students with the following hint if additional scaffolding is needed.

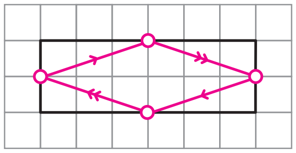
- **Hint:** In parts a and b, you can measure the side lengths and the interior angles of the new shape to identify it.

a. *Responses vary*. All the new shapes are parallelograms.

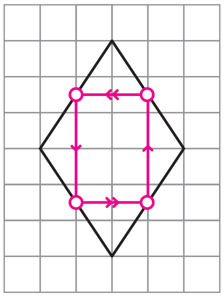
Square



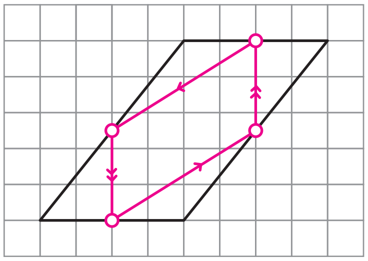
Rectangle



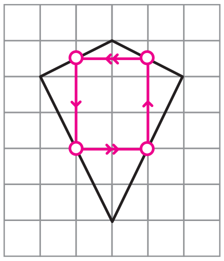
Rhombus



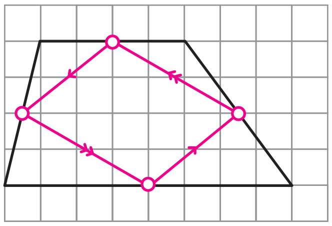
Parallelogram



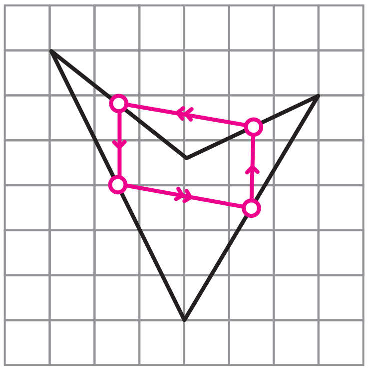
Kite



Trapezoid



b. *Initial shapes vary*. The new shape is again a parallelogram.



c. *Responses vary*. The area of the new shape is always half the area of the original shape.

Surface Area

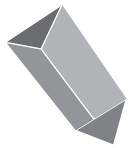
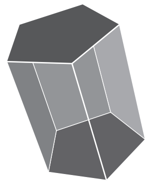
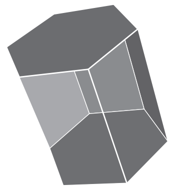
Name: .... Date: .... Period: ....

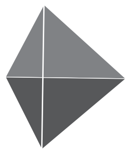
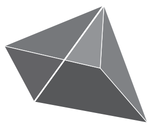
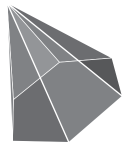
Student Choice Start with any problem. Remember to show or explain your thinking.

1

Here are different 3-D solids.

a Investigate each solid and record its number of vertices, V, number of edges, E, and the number of faces, F.

Solid			
V	6		
F	5		
E	9		

Solid			
V			
F			
E			

b What do you notice about the number of vertices, faces, and edges for different solids?

c Describe or draw a prism with 9 faces.

d Describe or draw a pyramid with 9 faces.

Surface Area (continued)

Name: .... Date: .... Period: ....

e Describe or draw a prism with 12 vertices.

f Describe or draw a pyramid with 9 vertices.

g If there a solid with 24 vertices and 14 faces, how many edges does it have?

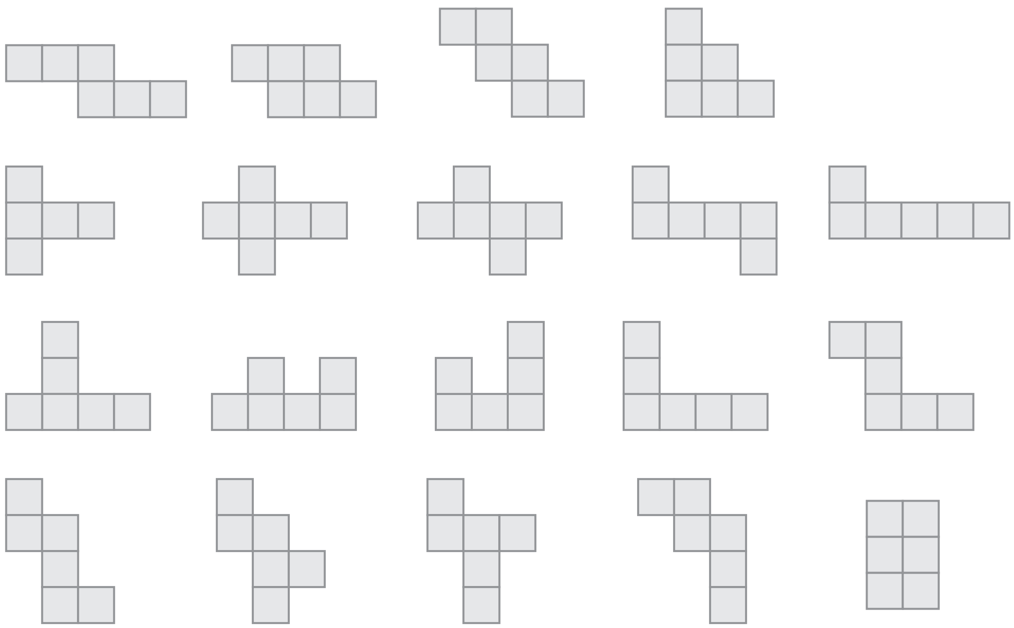
2

Create four identical triangles using only 6 identical toothpicks.

3

Here are nets of six squares.

a Circle *all* the nets that can fold into a cube.



b Create a different net of six squares that can fold into a cube.

Surface Area

Assign problems to students who want to extend their thinking.

Problems 1–3 can be solved in any order. If time allows, consider sharing Problems 1 and 3 with all students.

Materials
- 3-D solids (optional) **(Problem 1)**
- 6 sticks or toothpicks (optional) **(Problem 2)**
- scrap paper (optional) **(Problem 3)**

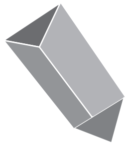
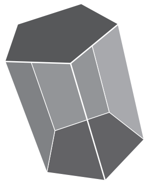
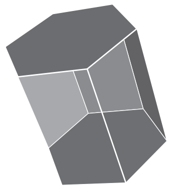
Problem 1

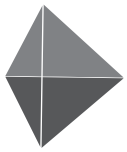
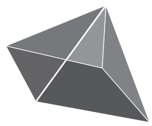
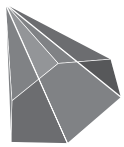
Students will extend their understanding of three-dimensional polyhedra and their properties by investigating the number of edges, faces, and vertices.

Provide students with the following hints if additional scaffolding is needed.

- **Hint 1:** In part a, how does the base shape affect the number of faces, vertices, and edges of a prism? A pyramid?
- **Hint 2:** In part g, do you see a relationship between the sum of the number of vertices and faces of a solid and its number of edges?

a.

Solid			
V	6	10	12
F	5	7	8
E	9	15	18

Solid			
V	4	5	7
F	4	5	7
E	6	8	12

b. *Responses vary.*

- For a prism, if the base has n sides, $V = 2n$, $F = n + 2$, and $E = 3n$.
- For a pyramid, if the base has n sides, $V = n + 1$, $F = n + 1$, and $E = 2n$.
- The number of vertices and faces are always the same for a pyramid.
- For any solid, $V + F = E + 2$.

c. Heptagonal prism

d. Octagonal pyramid

e. Hexagonal prism

f. Octagonal pyramid

g. 36 edges

Continued next page ...

Surface Area (continued)

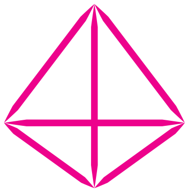
Problem 2

Students will extend their understanding of three dimensions using a famous puzzle.

Provide students with the following hint if additional scaffolding is needed.

- **Hint:** If you have 6 sticks and need to create 4 triangles, how many triangles should every toothpick be used in?

Responses vary. 6 toothpicks can be used to create a tetrahedron.



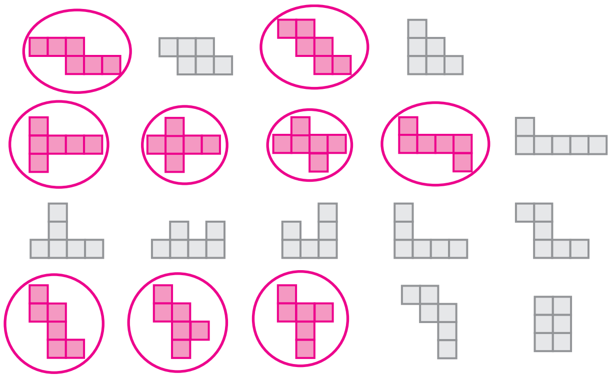
Problem 3

Students will extend their understanding of nets of a cube by selecting which figures can fold into a cube.

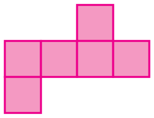
Provide students with the following hints if additional scaffolding is needed.

- **Hint 1:** In part a, you may use scrap paper to draw the figures and fold to see if they fold into a cube.
- **Hint 2:** In part b, once you create your net of six squares, you may want to check if this is a new net or one that is already shown in part a.

a.



b. *Responses vary.* Sample net shown.



Unit 2

Extensions

Ratios

Name: .... Date: .... Period: ....

Student Choice Start with any problem. Remember to show or explain your thinking.

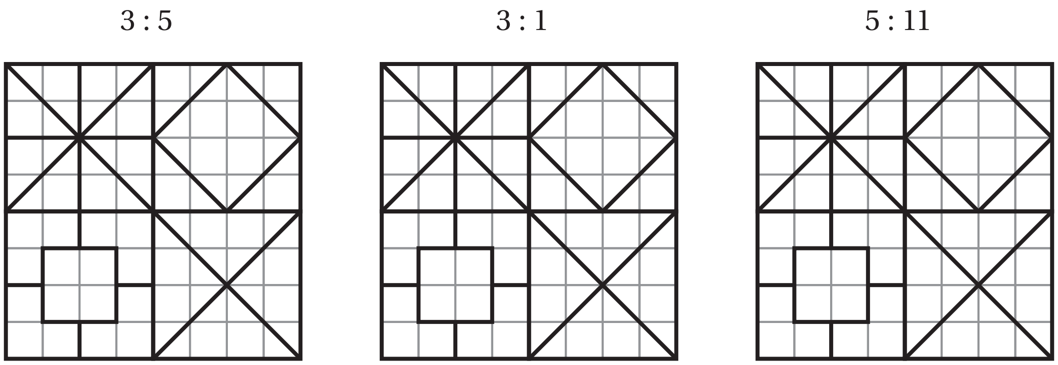
1 Determine two numbers such that . . .

a Their ratio is $4 : 5$ and their sum is 27.

b Their ratio is $2 : 5$ and their difference is 27.

c Both numbers are less than 20, and the ratio of their sum to their difference is $7 : 5$.

2 Shade the figure so that the ratio of the shaded area to the unshaded area is . . .



Unit 2

Sub-Unit 1
Extensions

Ratios

Assign problems to students who want to extend their thinking.

Problems 1 and 2 can be solved in any order. If time allows, consider sharing the problems with all students.

Materials

- coloring tools **(Problem 2)**

Problem 1

Students will extend their understanding of how ratios are generated by determining pairs numbers with given properties and ratios.

Provide students with the following hint if additional scaffolding is needed.

- **Hint:** In part a, 4 and 5 are the numbers that have a 4 : 5 ratio, and their sum is 9. What other numbers have the 4 : 5 ratio? Iterate this thinking to help you with parts b and c.

a. 12 and 15. *Explanations vary.*

b. 18 and 45. *Explanations vary.*

c. *Responses and explanations vary.*
   - 1 and 6
   - 2 and 12
   - 3 and 18

Problem 2

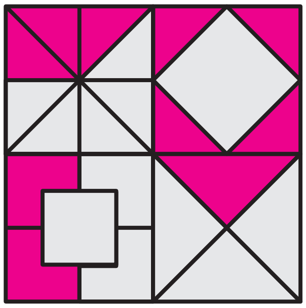
Students will extend their understanding of connections between area calculations, fractions, and ratios by shading the given figures.

Provide students with the following hint if additional scaffolding is needed.

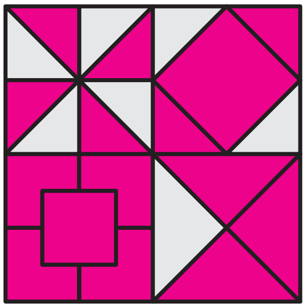
- **Hint:** Can you represent each part of the figure with a fraction? How can fractions help you to shade the given ratio?

Responses vary. Sample responses shown.

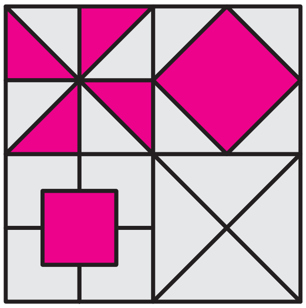
3 : 5



3 : 1



5 : 11



Common Factors and Multiples

Name: .... Date: .... Period: ....

Student Choice Start with any problem. Remember to show or explain your thinking.

1

The number 660 is divisible by 3, 4, and 11. What is the next largest number that is also divisible by 3, 4, and 11?

2

A pirate chest contains gold coins such that:

- If the coins are equally divided among 12 pirates, 10 coins are left over.
- If the coins are equally divided among 10 pirates, 8 coins are left over.
- The number of gold coins is known to be greater than 200.

What is the smallest number of coins that meets these conditions?

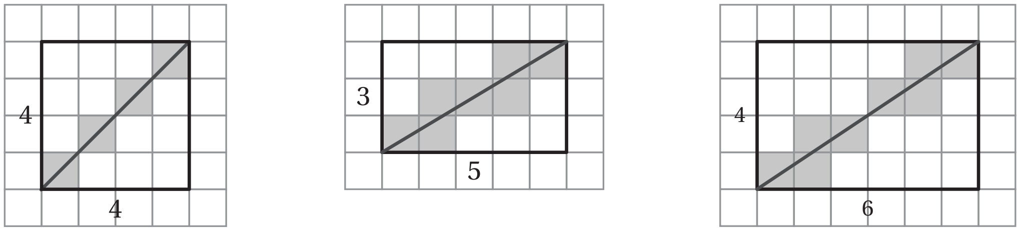
Common Factors and Multiples (continued)

Name: .... Date: .... Period: ....

3 Computer screens are large square grids of individual lights, where each light is called a *pixel*.

To see how computers draw lines on the screen, you can first draw the line using a straightedge, then shade any pixels that the line touches.

For example, to draw each of these diagonals:



4 pixels should be shaded. 7 pixels should be shaded. 8 pixels should be shaded.

Determine the number of pixels that need to be shaded to draw the diagonal of a 15×20 rectangle.

Common Factors and Multiples

Assign problems to students who want to extend their thinking.

Problems 1–3 can be solved in any order. If time allows, consider sharing Problem 3 with all students.

Materials
- coloring tools (optional) **(Problem 3)**
- graph paper **(Problem 3)**

Problem 1

Students will extend their understanding of divisibility rules, factors, and multiples.

Provide students with the following hints if additional scaffolding is needed.

- **Hint 1:** How do you know 660 is divisible by 3, 4, and 11? How can this information help you find the next largest number divisible by 3, 4, and 11?
- **Hint 2:** Is 660 the smallest number that is divisible by 3, 4, and 11?

792. *Explanations vary.*

The LCM of 3, 4, and 11 is 132, and $660 = 5 \cdot 132$. The next largest number that is divisible by 3, 4, and 11 is $6 \cdot 132 = 792$.

Problem 2

Students will extend their understanding of divisibility, factors, multiples, GCF, and LCM by solving the pirate puzzle.

Provide students with the following hint if additional scaffolding is needed.

- **Hint:** How many coins seem to be missing in each case? What would happen if the pirates had those coins?

238 coins. *Explanations vary.*

If there were 2 more coins, the total number of coins would be divisible by both 10 and 12. The LCM of 10 and 12 is 60. Because the total number of coins is more than 200, the smallest number of coins must be $60 \cdot 4 - 2 = 240 - 2 = 238$.

Continued next page ...

Problem 3

Students will extend their understanding of GCF by determining the number of pixels that need to be shaded to draw a line on a computer screen.

Provide students with the following hints if additional scaffolding is needed.

- **Hint 1:** Can you draw a smaller rectangle that helps you make sense of the larger rectangle?
- **Hint 2:** Ada thinks that the number of pixels needed to draw the diagonal is related to the GCF of the length and width. Is Ada correct? How can you help Ada organize her thinking?

30 pixels. *Explanations vary.*

- I noticed that the diagonal of the 4×6 rectangle is made from two copies of a 2×3 diagonal.

  Because 15 and 20 have a GCF of 5, I know that the diagonal will be made from 5 copies of a 3×4 diagonal.

  The 3×4 rectangle has a diagonal of 6 pixels, so the 15×20 rectangle will have a diagonal of $6 \cdot 5 = 30$ pixels.
- I organized the numbers in a table to help figure out the pattern.

Length (pixels)	Width (pixels)	GCF of Length and Width	Diagonal (pixels)
4	4	4	4
5	3	1	7
6	4	2	8
9	4	1	12
4	2	2	4

The table shows that the number of pixels needed to draw the diagonal is the sum of the length and width minus their greatest common factor.

Using this relationship for a 15×20 rectangle, I got:

$15 + 20 - 5 = 30$.

Solving Problems With Ratios

Name: .... Date: .... Period: ....

Student Choice Start with Problem 1 or Problem 2. Complete Problem 3 after Problem 2. Remember to show or explain your thinking.

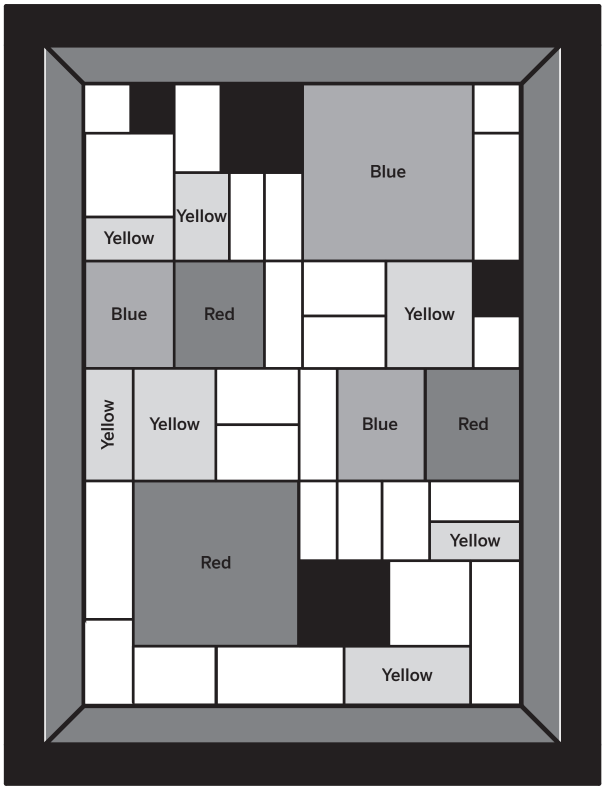
1

Dutch artist Piet Mondrian (1872–1944) is one of the best-known abstract painters.

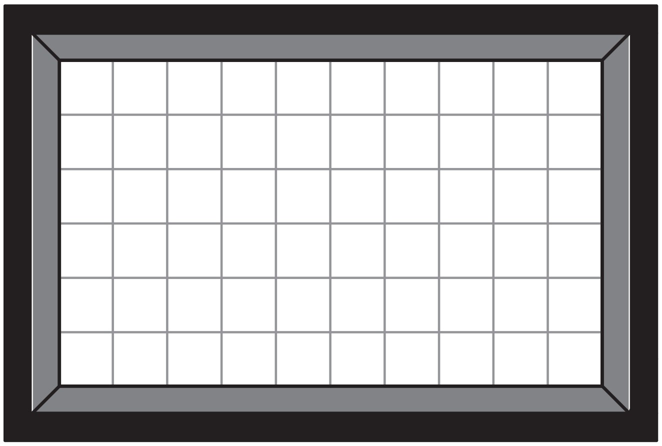
His abstract style is mostly known for his use of geometric elements colored with primary colors (red, blue, and yellow), black, and white.

Create a Mondrian-like painting using a 6×10 canvas such that:

- The ratio of red to blue to yellow is $5 : 10 : 8$.
- The ratio of black to white is $3 : 4$.
- The ratio of yellow to white is $2 : 1$.



Piet Mondrian



Solving Problems With Ratios (continued)

Name: .... Date: .... Period: ....

2

A collection of dimes and nickels is worth $5.60.

The ratio of the number of dimes to the number of nickels is 3 : 2.

Determine the number of each type of coin.

dime
10¢

nickel
5¢

3

A collection of dimes, nickels, and quarters is worth $6.90.

The ratio of the number of dimes to the number of nickels is 8 : 3.

The ratio of the number of dimes to the number of quarters is 8 : 10.

Determine the number of each type of coin.

quarter
25¢

dime
10¢

nickel
5¢

Solving Problems With Ratios

Assign problems to students who want to extend their thinking.

Problems 1 and 2 can be solved in any order. Assign Problem 3 to students who have solved Problem 2. If time allows, consider sharing Problem 1 with all students.

Materials

- coloring tools **(Problem 1)**
- coins (optional) **(Problems 2–3)**

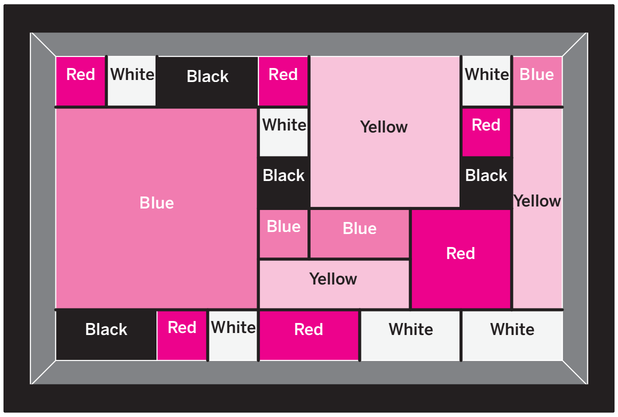
Problem 1

Students will extend their understanding of equivalent ratios by creating a Mondrian-like painting.

Provide students with the following hints if additional scaffolding is needed.

- **Hint 1:** When there is a single white square, how many yellow squares must there be? How many black? Blue? Red?
- **Hint 2:** How can you use your knowledge of GCFs and equivalent ratios to simplify the process?

Responses vary.



Problem 2

Students will extend their understanding of solving problems using ratios.

Provide students with the following hint if additional scaffolding is needed.

- **Hint:** How many cents is a nickel worth? A dime? How can you use their values to calculate the total amount of money?

42 dimes and 28 nickels. *Explanations vary*.

The ratio of the number of dimes to the number of nickels is $3:2$, meaning for every 3 dimes worth 10 cents each, there are 2 nickels worth 5 cents each: $30 + 10 = 40$ cents.

Because there are 14 groups of 40 cents in 560 cents, there are 14 groups of 3 dimes and 2 nickels:

- $14 \cdot 3 = 42$ dimes, and
- $14 \cdot 2 = 28$ nickels.

Problem 3

Students will extend their understanding of solving problems using ratios.

Provide students with the following hint if additional scaffolding is needed.

- **Hint:** How many cents is a nickel worth? A dime? A quarter? How can you use their values to calculate the total amount of money?

16 dimes, 6 nickels, and 20 quarters.
Explanations vary. The ratio of the number of dimes to the number of nickels to the number of quarters is $8:3:10$, meaning for every 8 dimes worth 10 cents each, there are 3 nickels worth 5 cents each and 10 quarters worth 25 cents each: $80 + 15 + 250 = 345$.

Because there are 2 groups of 345 cents in 690 cents, there are two groups 8 dimes, 3 nickels, and 10 quarters:

- $2 \cdot 8 = 16$ dimes
- $2 \cdot 3 = 6$ nickels
- $2 \cdot 10 = 20$ quarters

Unit 3

Extensions

Units and Measurement

Name: .... Date: .... Period: ....

Student Choice Remember to show or explain your thinking.

1 Our solar system is so big, it is almost impossible to imagine its size using units like kilometers or miles. To solve this problem, astronomers use the distance between the Earth and the Sun, approximately 149 million kilometers, as a new unit of measure called the Astronomical Unit (1 AU).

Complete the table to determine the distances of some of the planets from the Sun in Astronomical Units. Round answers to the nearest hundredth.

Planet	Approximate distance to the Sun (millions of kilometers)	Approximate distance to the Sun (AU)
Mercury	57	
Earth	149	
Mars	228	
Neptune	4,530	30.40

2 Proxima Centauri is the closest star to our planet other than the sun. It is about 268,770 AU away. Astronomers also use the light-year, the distance light travels in one year, as a unit of distance in space, especially for the distances outside of our solar system.

a Light travels at a velocity of about 300,000 kilometers per second. Determine the distance that light travels in one year.

b One light-year is about 63,241 Astronomical Units. How many light-years away is Proxima Centauri?

Name: .... Date: .... Period: ....

3 A common estimate for the time it takes Earth to complete one revolution around the Sun is 365 days and 6 hours.

Thus, we add an extra day to February every four years ($6 \cdot 4 = 24$ hours, or one day) for a leap year.

Actually, a more precise estimate is about 365 days, 5 hours, 48 minutes, and 46 seconds.

a How many extra minutes and seconds do we add to the calendar every four years?

b Determine the extra time that would be added to the calendar every 400 years.

c This extra time is accounted for by skipping leap years that are divisible by 100, but not by 400. Can you determine the last time and the next time we skip a leap year?

Units and Measurement

Assign problems to students who want to extend their thinking.

Problems 1 and 3 can be solved in any order. Assign Problem 2 to students who have solved Problem 1. If time allows, consider sharing Problems 3 with all students.

Problem 1

Students will extend their understanding of conversion between units of measurement across kilometers and Astronomical Units by using ratio reasoning.

Provide students with the following hints if additional scaffolding is needed.

- **Hint 1:** What is the unit rate of kilometers to AU conversion?

Mercury	0.38
Earth	1
Mars	1.53
Neptune	30.40

Problem 2

Students will extend their understanding of conversion between units of measurement across kilometers, Astronomical Units and light-years by using ratio reasoning.

Provide students with the following hints if additional scaffolding is needed.

- **Hint:** In part a, how many seconds are there in an hour?

a. $300000 \cdot 60 = 18000000$ kilometers in a minute.
$18000000 \cdot 60 = 1.08$ billion km in an hour.
$1.08 \cdot 24 = 25.92$ billion km in a day
$25.92 \cdot 365 = 9.4608$ trillion km in a year.

b. 4.25 light-years.

Continued next page ...

Problem 3

Students will extend their understanding of conversion between units of time measurement to explore the leap year concept.

Provide students with the following hints if additional scaffolding is needed.

- **Hint 1:** How many minutes are in an hour? How many seconds are in a minute?

a. 44 minutes, 56 seconds

In every year, the extra added time is

5 hours	59 minutes	60 seconds.
-5 hours	48 minutes	46 seconds.
0 hours	11 minutes	14 seconds.

Then in four years, the extra time is
$4 \cdot (11 \text{ minutes}, 14 \text{ seconds}) = 44 \text{ minutes}, 56 \text{ seconds}$

b. 3 days, 2 hours, 53 minutes, 20 seconds
In 400 years, the extra time is
$100 \cdot (44 \text{ minutes}, 56 \text{ seconds}) = 4400 \text{ minutes}, 5600 \text{ seconds}$
5600 seconds is 1 hour, 33 minutes, and 20 seconds, and 4400 minutes is 73 hours and 20 minutes. In total it is 3 days, 2 hours, 53 minutes, 20 seconds.

c. 1900 and 2100

Unit Rates

Name: .... Date: .... Period: ....

Student Choice Remember to show or explain your thinking.

1 The bridge will collapse in 17 minutes. Hikers A, B, C, and D must cross the bridge in pitch darkness using a flashlight.

- A maximum of two people can cross the bridge at one time, carrying the flashlight which must be returned by hand after each crossing.
- Each hiker walks at a different speed:
  - Hiker A takes 1 minute to cross the bridge
  - Hiker B takes 2 minutes to cross
  - Hiker C takes takes 5 minutes to cross
  - Hiker D takes 10 minutes to cross
- Each pair crosses the bridge at the rate of the slower hiker's pace.

Help them cross the bridge in 17 minutes or less by determining the order in which the hikers should cross.

Unit Rates (continued)

Name: .... Date: .... Period: ....

2 A snail climbed up a window 90 inches high. Every day, it climbed 11 inches up, and every night dropped back 7 inches. How many days did it take for the snail to reach the top?

3 Tiam traveled to work at her usual speed of about 60 miles per hour. Her return journey during the rush hour was 30 miles per hour. What was her average speed for the round trip?

Unit Rates

Assign problems to students who want to extend their thinking.

Problems 1 and 2 can be solved in any order. Assign Problem 3 to students who have solved Problems 1 and 2. If time allows, consider sharing Problem 1 with all students.

Problem 1

Students will extend their understanding of unit rates by working on a famous crossing puzzle.

Provide students with the following hints if additional scaffolding is needed.

- **Hint 1:** To carry the flashlight back, some hikers will need to make a back crossing. How would you choose those people?

17 mins. Responses vary.

Crossing 1	Back crossing 2	Crossing 3	Back crossing 4	Crossing 5
Let A and B cross	A goes back	C and D cross	B goes back	A and B cross
$+2$	$+1$	$+10$	$+2$	$+2$
Total time: 2 mins	Total time: 3 mins	Total time: 13 mins	Total time: 15 mins	Total time: 17 mins

Problem 2

Students will extend their understanding of calculating the speed of a snail as a unit rate for the distance the snail travels over time.

Provide students with the following hints if additional scaffolding is needed.

- **Hint 1:** How high will the snail be in the morning of the third day? Fourth day? Fifth?
- **Hint 2:** What is the speed of the snail? How can its speed help you determine its height on the 20th day? 21st day?

In 21 days. *Explanations vary*.

Day 1: the snail climbs to 11 inches and then drops back 7 inches.

Day 2 morning: 4 inches

Day 3 morning: 8 inches

Day 4 morning: 12 inches

Day 5 morning: 16 inches

. . .

Day 20 morning: 76 inches

Day 21 morning: 80 inches, so on day 21, the snail reached the window of 90 inches high because he climbed up $80 + 11 = 91$ inches before he dropped back 7 inches.

Continued next page ...

Unit Rates (continued)

Problem 3

Students will extend their understanding of calculating the average speed as a unit rate for the distance the object travels over time.

Provide students with the following hints if additional scaffolding is needed.

- **Hint 1:** How long does it take for her to travel back home since her speed was halved during the rush hour?
- **Hint 2:** In what fraction of her round trip her speed was 60 mph? In what fraction it was 30 mph?

40 miles per hour. *Explanations vary*.

During the rush hour, Tiam's speed was 30 miles per hour, which is half of her usual speed of 60 miles per hour when traveling to work. Since her speed was halved, the time it took for her to travel back home was doubled compared to the time it took to go to work.

During the $\frac{1}{3}$ of the round trip, her speed was 60 mph, and in $\frac{2}{3}$ of the trip, her speed was 30 mph, so her average speed is $\frac{60 \cdot 1 + 30 \cdot 2}{3} = 40$ miles per hour.

Percentages

Name: .... Date: .... Period: ....

Student Choice Start with any problem. Remember to show or explain your thinking.

1

a $A\% + B\% + C\% + D\% + E\% + F\% = 100\%$. If A, B, C, D, E, F are known to be odd numbers, determine their values.

b Can you find five different percentages that add up to 100%? Seven different percentages?

2

If you increase one side of a rectangle by 30% and decrease the other side by 30%, what happens to its area?

Percentages (continued)

Name: .... Date: .... Period: ....

3 In a collection of marbles, 8 are red, 1 is black, and 55% of them are blue. What is the smallest possible size of this collection?

4 After eating at a restaurant, you know that the bill before tax is $\$50$ and that the sales tax rate is 8%. You decide to leave a 20% tip for the waiter based on the pre-tax amount.

a How much is the total bill including tax and tip?

b How much is the total bill including tax and tip if the bill before tax was $\$75$?

c Determine an easy way to calculate the total bill including tax and tip for any amount of pre-tax and tip bill.

Percentages

Assign problems to students who want to extend their thinking.

Problems can be solved in any order. If time allows, consider sharing Problems 1 and 2 with all students.

Problem 1

Students will extend their understanding of percentages by solving a number sense puzzle.

Provide students with the following hints if additional scaffolding is needed.

- **Hint:** Determine two odd number percentages to add up 100%. Now determine two odd number percentages to add up to those numbers.

a. *Responses vary.*
$9 + 11 + 15 + 17 + 19 + 29 = 100$

b. No, it is not possible to determine 5 or 7 odd numbers that add up to 100 because an odd number of odd addends gives an odd sum, and 100 is an even number.

Problem 2

Students will extend their understanding of using ratio reasoning to determine unknown percentages.

Provide students with the following hints if additional scaffolding is needed.

- **Hint:** You may choose a 10 by 10 square to calculate the new side lengths and then compare their areas.

It decreases by 9%. *Explanations vary.*

If one side of a rectangle is increased by 30% and the other side is decreased by 30%, the new area of the rectangle is $1.3 \cdot 0.7 = 0.91$ which is 91% of the original area. This means the area decreases by 9%.

Continued next page ...

Problem 3

Students will extend their understanding of using ratio reasoning to determine unknown percentages, parts, and whole.

Provide students with the following hints if additional scaffolding is needed.

- **Hint 1:** How many colors are there in the collection? How do you know?
- **Hint 2:** What is the assumption you need to make to determine the total?

20. *Responses vary*.

We know there are 8 red marbles and 1 black marble, making a total of 9 marbles that are not blue. To determine the smallest possible size, I assume there are no more colors in the collection.
So, 45% of the collection is these 9 marbles. If 45% of the collection is 9 marbles, we can find the total by $9 \div 0.45 = 20$.

Problem 4

Students will extend their understanding of using ratio reasoning to determine unknown parts, wholes, and percentages.

Provide students with the following hints if additional scaffolding is needed.

- **Hint 1:** What is the percentage of the sales tax? What is its amount?
- **Hint 2:** What is the percentage of the tip? What is its amount?

a. \$64. *Explanations vary*.
To find the sales tax, multiply \$50 by 8%.
Sales tax $= 50 \cdot 0.08 = 4$
So, the sales tax is \$4.
To determine the 20% tip based on the pre-tax amount, I multiplied \$50 by 20%.
Tip $= 50 \cdot 0.20 = 10$
I then added the original bill, the sales tax, and the tip together.
Total bill $= 50 + 4 + 10 = \$64$

b. \$96. *Explanations vary*.
To find the sales tax, multiply \$75 by 8%.
Sales tax $= 75 \cdot 0.08 = 6$
So, the sales tax is \$6.
To determine the 20% tip based on the pre-tax amount, I multiplied \$75 by 20%.
Tip $= 75 \cdot 0.20 = 15$
I then added the original bill, the sales tax, and the tip together.
Total bill $= 75 + 6 + 15 = \$96$

c. *Responses vary*.
The total bill is the 128% of the pre-tax and tip bill.

Pre-tax and tip bill (\$)	Total bill (\$)
50	64
75	96
100	128

Unit 4

Extensions

Multiplying Fractions

Name: .... Date: .... Period: ....

Student Choice Start with any problem. Remember to show or explain your thinking.

1

Unit fraction $\frac{1}{2}$ can be written as $\frac{3-(2-1)}{4}$

Unit fraction $\frac{1}{3}$ can be written as $\frac{1}{4+2-3}$

a Using $1, 2, 3, 4$ write a different fraction equivalent to $\frac{1}{3}$.

b Using $1, 2, 3, 4$ write as many fractions equivalent to other unit fractions as you can.

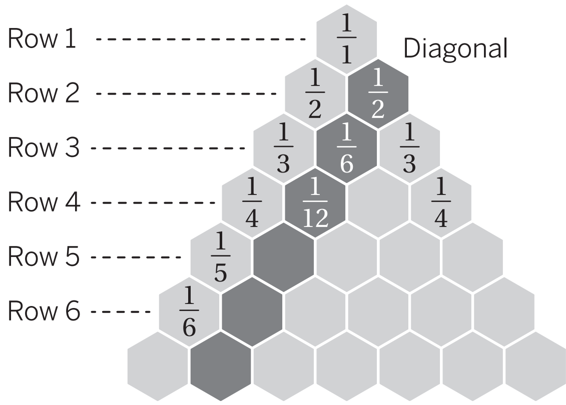
2

Suppose you have two jugs: one containing tea and the other containing milk. You pour $\frac{1}{3}$ of the tea into the milk and mix it. Then you pour $\frac{1}{3}$ of this mixture back into the tea. After these operations, is there more tea in the jug of milk or more milk in the jug of tea?

Multiplying Fractions (continued)

Name: .... Date: .... Period: ....

3 The *Leibniz harmonic triangle* is a triangular arrangement of unit fractions.



a What do you notice about each fraction and the two fractions beneath it?

b Anna thinks that the missing fraction in Row 4 is $\frac{1}{12}$. Explain how she might be able to check if she is correct?

c What else do you notice about the triangle?

Multiplying Fractions

Assign problems to students who want to extend their thinking.

Problems 1, 2 and 3 can be solved in any order. If time allows, consider sharing Problems 1, 2, and 3 with all students.

Problem 1

Students will extend their understanding of fractions, equivalent fractions, and simplifying fractions by solving part b.

Provide students with the following hint(s) if additional scaffolding is needed.

- **Hint 1:** You can use equivalent fractions such as $\frac{2}{8}$ to create the unit fraction $\frac{1}{4}$.
- **Hint 2:** How can using parentheses and exponents help you to create different unit fractions? How can using the order of operations help you to create different unit fractions?

a. *Responses vary.*
- $\frac{2}{4+3-1}$
- $\frac{3-2}{4-1}$

b. *Responses vary.*
- $\frac{1}{4}=\frac{2}{4+3+1}$
- $\frac{1}{5}=\frac{1}{4+3-2}, \frac{3-2}{4+1}$
- $\frac{1}{6}=\frac{2 \cdot 1}{4 \cdot 3}$
- $\frac{1}{7}=\frac{2-1}{4+3}$
- $\frac{1}{8}=\frac{2}{4 \cdot (1+3)}$
- $\frac{1}{9}=\frac{1}{2+3+4}$
- $\frac{1}{10}=\frac{1}{3 \cdot 4-2}\ldots$

Problem 2

Students will extend their understanding of reasoning about fractional amounts by comparing different mixtures.

Provide students with the following hint(s) if additional scaffolding is needed.

- **Hint 1:** Start by considering the initial amounts: a jug of tea and a jug of milk. After pouring $\frac{1}{3}$ of the tea into the milk, figure out how much tea is now in the milk.
- **Hint 2:** When you pour one-third of the new milk-tea mixture back into the tea, calculate how much tea and milk are in this mixture.

The proportions of tea in the milk and milk in the tea are equal.

Explanations vary. The volumes of liquids did not change after the experiment. If I remove even a drop of milk from the container with milk, we must compensate for this with an equal drop of tea, so that the amount of liquid remains at the same level. Therefore, each drop of milk in the tea must correspond to an equal drop of tea in the milk, meaning the proportions of tea in the milk and milk in the tea are equal.

Continued next page ...

Problem 3

Students will extend their understanding of fractions by exploring the Leibniz triangle.

Provide students with the following hint(s) if additional scaffolding is needed.

- **Hint:** In row 4, what fraction can be added to $\frac{1}{12}$, so that the answer is $\frac{1}{6}$?

a. *Responses vary*. Each fraction is equal to the sum of the two fractions below it.

b. She is correct. *Explanations vary*.
$\frac{1}{3} - \frac{1}{4} = \frac{1}{12}$

c. *Responses vary.*
- Each row starts with the reciprocal of the row number.
- The entries of Leibniz's triangle are symmetric with respect to a vertical line through the center.
- The sum of the first diagonal is 1.

$$= \frac{1}{2} + \frac{1}{6} + \frac{1}{12} + \dots$$
$$= 1 - \frac{1}{2} + \frac{1}{2} - \frac{1}{3} + \frac{1}{3} - \frac{1}{4} + \dots = 1$$

Unit 4
Sub-Unit 2
Extensions

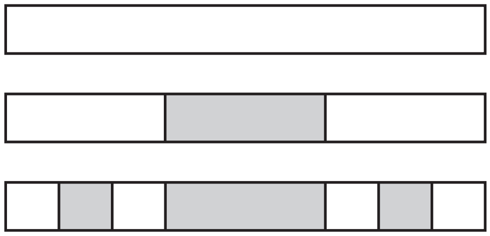
Dividing Fractions

Name: .... Date: .... Period: ....

Student Choice Start with any problem. Remember to show or explain your thinking.

1

Follow this process repeatedly to create a Cantor ternary set.



Step 1 Start with a tape diagram of length 1 unit

Step 2 Color in the middle third of the tape diagram.

Step 3 Do the same to each remaining segment that is not colored in.

a What fraction of the diagram is colored in after Step 2? Step 3? Step 5?

b If you continue this process, how much of the tape diagram will you color?

c Will the result be different if you color the first fifth instead of the middle third of each strip?

2

Use identical rectangular blocks such as dominoes, place two dominoes to have a step.

a How far can you move the top domino? What fraction of the domino's length is the overhang?

b Add one more domino to the ladder to create the max length. What fraction of the domino's length is the second overhang?

Name: .... Date: .... Period: ....

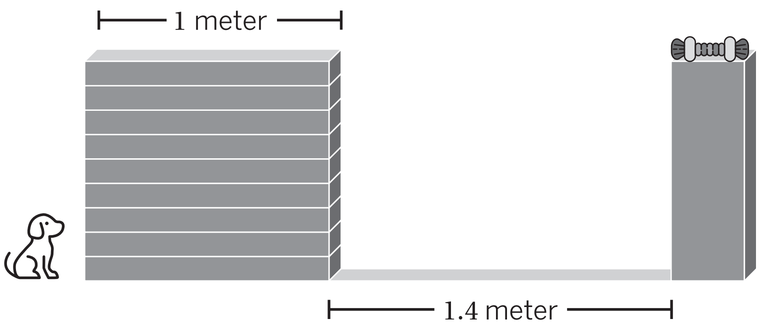
2

c Add a fourth domino to the ladder to keep creating a new max length. What fraction of the domino's length is the third overhang?

d What do you notice about the length of the maximum overhang in each step? How is it related to the length of the dominoes?

e Use the *Block-Stacking Sheet* to determine the length of the maximum overhang to build a 6-blocks tall ladder. Then, determine the length of the maximum overhang to build a 7-blocks tall ladder.

f Nine identical blocks, each 1 meter in length are stacked, with the bottom block nailed to the floor. Move the eighth block to maximize the overhang for the top block. Will the overhang be sufficient for the puppy to reach the toy that is 140 centimeters away?



Dividing Fractions

Assign problems to students who want to extend their thinking.

Problems 1 and 2 can be solved in any order. If time allows, consider sharing Problem 2 with all students.

Materials

- Extension Sheet, one per student, **(Problem 2)**
- Identical rectangular blocks, dominoes **(Problem 2)**

Problem 1

Students will extend their understanding of division problems by repeating a division pattern using tape diagrams.

Provide students with the following hints if additional scaffolding is needed.

- **Hint 1 :** How many parts will there be on the tape diagram once you divide it by 3 two times? Three times? Five times?
- **Hint 2 :** You can use the width of a paper instead of a tape diagram to help you with your thinking.

a. Step 2: $\frac{1}{3}$

Step 3: $\frac{5}{9}$

Step 5: $\frac{65}{81}$

Explanations vary. In Step 3, the first shaded part $\frac{1}{3}$ will stay as it is, and I also shade the $\frac{1}{3}$ of the remaining $\frac{2}{3}$.
$\frac{1}{3}+\frac{1}{3}\cdot\frac{2}{3}=\frac{5}{9}$

In Step 4, the shaded region will be
$\frac{5}{9}+\frac{1}{3}\cdot\frac{4}{9}=\frac{19}{27}$.

In Step 5, the shaded region will be
$\frac{19}{27}+\frac{1}{3}\cdot\frac{8}{27}=\frac{65}{81}$.

b. Almost the entire tape diagram.

c. No. *Explanations vary*. When I continue to keep coloring the first fifth, almost the entire tape diagram will be colored in.

Continued next page ...

Problem 2

Students will extend their understanding of division problems by exploring the block-stacking problem.

Provide students with the following hints if additional scaffolding is needed.

- **Hint 1:** In the two block tall ladder, what fraction of the domino's length is the overhang? You can round your answer.
- **Hint 2:** When you added the third block/domino, was the overhang shorter or longer with respect to the first one? What about the fourth domino?

a. *Responses vary*. It needs to be approximately half of the length of the block.

b. *Responses vary*. It needs to be approximately a quarter of the length of the block.

c. *Responses vary*. It needs to be approximately $\frac{1}{6}$ of the length of the block.

d. *Responses vary*.

- The length of the maximum overhang in each step decreases.
- The length of overhangs follow a pattern of $\frac{1}{2}, \frac{1}{4}, \frac{1}{6}, \frac{1}{8} \ldots$

e. For 6-blocks tall ladder, it is $\frac{1}{2} + \frac{1}{4} + \frac{1}{6} + \frac{1}{8} + \frac{1}{10} \approx 1.142.$

For 7-blocks tall ladder, it is $\frac{1}{2} + \frac{1}{4} + \frac{1}{6} + \frac{1}{8} + \frac{1}{10} + \frac{1}{12} = 1.225$

f. No. *Explanations vary*. Counting from the top, the nth block can have an overhang relative to the block below it equals to $\frac{1}{2n}$ meters. This leads to the sequence of $\frac{1}{2}, \frac{1}{4}, \frac{1}{6}, \frac{1}{8} \ldots$ which is $0.5, 0.25, 0.167, 0.125, 0.1, 0.83, 0.71$ and 0.62 meters. The total overhang is approximately 1.358 meters, a little short of 140 centimeters.

Area and Volume With Fractions

Name: .... Date: .... Period: ....

Student Choice Start with any problem. Remember to show or explain your thinking.

1

a Multiply $\frac{1}{2}$ and $\frac{2}{3}$.

b Multiply $\frac{1}{2}$, $\frac{2}{3}$, and $\frac{3}{4}$.

c What do you think happens if you keep multiplying fractions $\frac{1}{2} \cdot \frac{2}{3} \cdot \frac{3}{4} \cdot \frac{4}{5} \ldots$? Explain your thinking.

d Multiply $\frac{1}{1}$, $\frac{2}{1}$, and $\frac{1}{3}$.

e What do you think happens if you keep multiplying fractions $\frac{1}{1} \cdot \frac{2}{1} \cdot \frac{1}{3} \cdot \frac{4}{1} \cdot \frac{1}{5} \ldots$? Explain your thinking.

Area and Volume With Fractions (continued)

Name: .... Date: .... Period: ....

2

The ancient Egyptians represent fractions as the sums of distinct unit fractions. For example:

$\frac{2}{3} = \frac{1}{2} + \frac{1}{6}$ $\quad$ $\frac{2}{3} = \frac{1}{3} + \frac{1}{4} + \frac{1}{12}$ $\quad$ $\frac{2}{5} = \frac{1}{3} + \frac{1}{15}$

a Express $\frac{5}{6}$ as the sum of distinct unit fractions.

b Is there another way to to express $\frac{5}{6}$ as the sum of distinct unit fractions?

c Do you think it is possible to represent any unit fraction as a sum of two unit fractions?

d Jon thinks that he can use $\frac{1}{n} = \frac{1}{n+1} + \frac{1}{n(n+1)}$ $(n \neq 0)$ to write any unit fraction in terms of two distinct unit fractions. Is he correct?

e Can every fraction be represented as a sum of two unit fractions?

f Can every fraction be written as the sum of distinct unit fractions?

Area and Volume With Fractions

Assign problems to students who want to extend their thinking.

Problems 1 and 2 can be solved in any order. If time allows, consider sharing Problem 2 with all students.

Problem 1

Students will extend their understanding of fractional lengths by exploring patterns when multiplying fractions.

Provide students with the following hint if additional scaffolding is needed.

- **Hint:** How can simplifying help you determine the products in each part?

a. $\frac{1}{3}$

b. $\frac{1}{4}$

c. I would get close to the value of 0.

Explanations vary. Because the values of the denominator and the next numerator will continue to simplify, the result will be 1 over a very large number, which will be close to 0.

d. $\frac{2}{3}$

e. I would get close to the value of 1.

Explanations vary. With each product, the result is a little less than 1, then a little more than 1, and it keeps repeating and will continue to get closer and closer to 1 on both sides of 1.

Continued next page ...

Problem 2

Students will extend their understanding of analyzing strategies for solving problems involving unit fractions by exploring Egyptian fractions.

Provide students with the following hint(s) if additional scaffolding is needed.

- **Hint 1:** In part b, use the unit fractions you wrote in part a to create other unit fractions. For example if you wrote $\frac{5}{6} = \frac{1}{2} + \frac{1}{3}$, how can you write $\frac{1}{2}$ as the sum of two unit fractions? $\frac{1}{3}$?
- **Hint 2:** In part d, start with writing many examples. Did the expression work every time? How can writing the sum using the common denominator help you simplify the expression?
- **Hint 3:** In part f, Jon thinks that he needs to determine the largest possible unit fraction in any fraction and then subtract this unit fraction from it to get a new fraction to start finding the unit fractions. Is he correct? How would you continue this process? Give as many examples as you can.

a. *Responses vary*.
$\frac{5}{6} = \frac{1}{2} + \frac{1}{3}$

b. *Responses vary*.
- $\frac{5}{6} = \frac{1}{2} + \frac{1}{5} + \frac{1}{10} + \frac{1}{30}$
- $\frac{5}{6} = \frac{1}{3} + \frac{1}{4} + \frac{1}{6} + \frac{1}{12}$

c. Yes. *Explanations vary*. All unit fractions can be written as the sum of two unit fractions.

d. Yes, he is correct. *Explanations vary.*
- $\frac{1}{3} = \frac{1}{4} + \frac{1}{12}$
- $\frac{1}{6} = \frac{1}{7} + \frac{1}{42}$
- $\frac{1}{10} = \frac{1}{11} + \frac{1}{110}$
- When we rewrite $\frac{1}{n+1} + \frac{1}{n(n+1)}$ using common denominators, it will be $\frac{n}{n(n+1)} + \frac{1}{n(n+1)}$ which is $\frac{n+1}{n(n+1)}$. This expression can be simplified to $\frac{1}{n}$.

e. No. Not all fractions can be written as the sum of two unit fractions. *Explanations vary.*
- For example, $\frac{4}{5}$ is written as $\frac{1}{2} + \frac{1}{5} + \frac{1}{10}$, so it can only be written as the sum of at least three different unit fractions.
- Using two unit fractions will lead to either a greater or smaller value than $\frac{4}{5}$. For example, $\frac{1}{2} + \frac{1}{3} = \frac{5}{6}$ is larger than $\frac{4}{5}$, and $\frac{1}{2} + \frac{1}{4} = \frac{3}{4}$ is smaller than $\frac{4}{5}$, so any combination of two unit fractions will be smaller than $\frac{4}{5}$.

f. Yes. Every fraction can be written as the sum of distinct unit fractions. *Explanations vary*.
- Start with a fraction.
- Find the largest possible unit fraction in it.
- Subtract this unit fraction from the original fraction to get a new fraction.
- Repeat the process with the new fraction until you reach zero.

Unit 5

Extensions

Unit 5
Sub-Unit 1
Extensions

Multiplying Decimals

Name: .... Date: .... Period: ....

Student Choice Start with any problem. Remember to show or explain your thinking.

1 Jon knows he is able to buy 10 pens and 4 pencils using all of his money, but his shopping list also includes paper and an eraser. Find a way Jon can spend $25.25 and have at least one of each item on his list

- Paper at 2 sheets for $1.01
- Pens at $1.01 each
- Pencils at 2 for $5.05
- Erasers at $5.05 each

How many of each can he buy?

2 Using the digits 0 to 9 at most one time each, fill in the boxes to make a true equation.

$$\square.\square\left(\square+\square\right)=\square.\square+\square.\square$$

Multiplying Decimals (continued)

Name: .... Date: .... Period: ....

3

If you are only allowed to use the steps A, B, and C, and your starting number is 3.5, how can you get to

A: $\times 2$
B: $\times 5$
C: $+ 0.2$

a. 36 in three steps

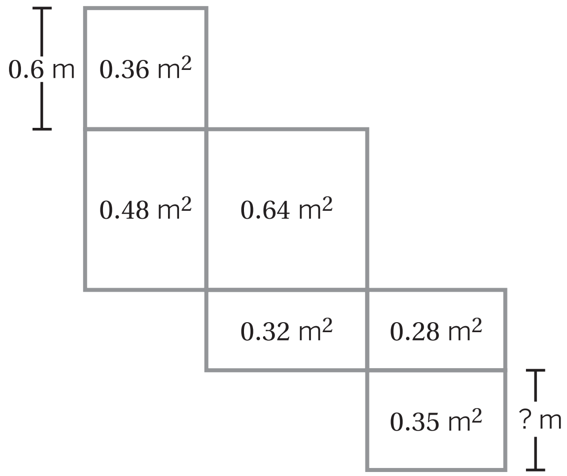
b. 37 in three steps

c. 38 in four steps

d. 39 in four steps

4

A figure made out of rectangles is given. Determine the unknown length.



Unit 5
Sub-Unit 1
Extensions

Multiplying Decimals

Assign problems to students who want to extend their thinking.

Problems can be solved in any order. If time allows, consider sharing Problems 3 and 4 with all students.

Problem 1

Students will extend their understanding of multiplying multi-digit decimals by solving a word problem.

Provide students with the following hint if additional scaffolding is needed.

- **Hint:** What are the most expensive items? How many of those items can Jon purchase at most?

Responses vary.

- **14 sheets of paper: $7.07**
- **8 pens: $8.08**
- **2 pencils: $5.05**
- **1 eraser: $5.05**

Problem 2

Students will extend their understanding of multiplying multi-digit decimals by solving an Open-Middle like problem.

Provide students with the following hint if additional scaffolding is needed.

- **Hint:** If the factor at the beginning is 0.9, how can you arrange the rest of the digits to make the equation true?

Responses vary.

$0.9(8 + 6) = 7.2 + 5.4$

Continued next page ...

Multiplying Decimals (continued)

Problem 3

Students will extend their understanding of multiplying multi-digit decimals by determining the correct order of operations to reach to the target number.

Provide students with the following hint if additional scaffolding is needed.

- **Hint:** List all possible orders for operations and determine the results.

Responses vary.

a. A, C, B

b. C, A, B or C, B, A

c. C, A, C, B

d. C, C, A, B or C, C, B, A

Problem 4

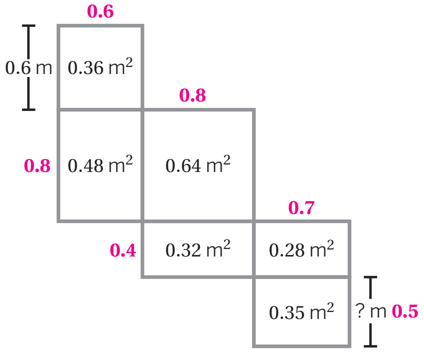
Students will extend their understanding of multiplying multi-digit decimals by determining the unknown length of a rectangle in a complex figure.

Provide students with the following hint if additional scaffolding is needed.

- **Hint:** What is the other side length of the rectangle with the area of 0.36 square meters? How can this help you find the unknown side lengths of the rectangle with the area of 0.48 square meters?

0.5 meters.

Explanations vary.



Unit 5
Sub-Unit 2
Extensions

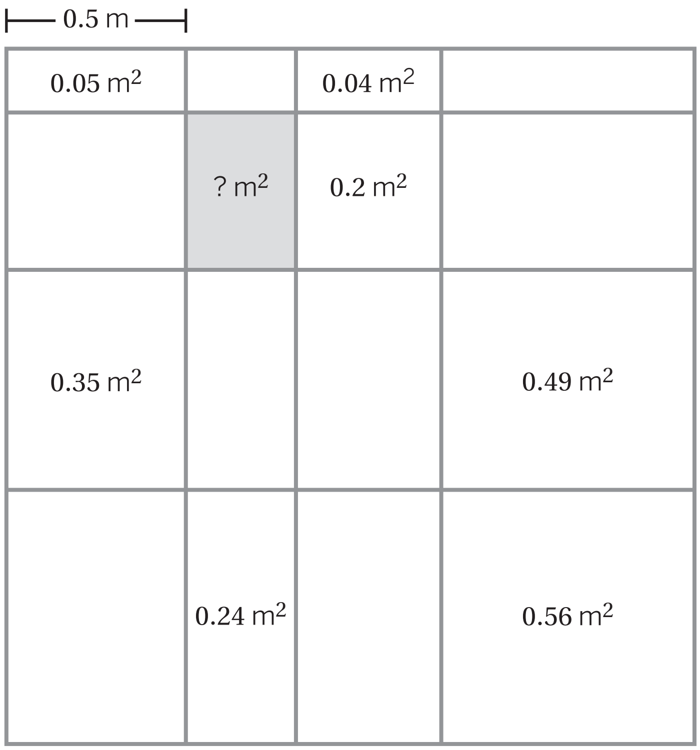
Dividing Decimals

Name: .... Date: .... Period: ....

Student Choice Remember to show or explain your thinking.

1 Use three 7s to write an expression that equals to 20.

2 A figure made out of rectangles is given. Determine the unknown area.

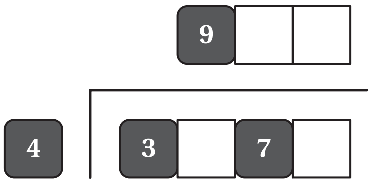


Dividing Decimals (continued)

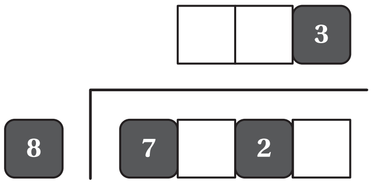
Name: .... Date: .... Period: ....

3 Use the remaining digits 1–9 without repeating any digits to make the divisions correct.

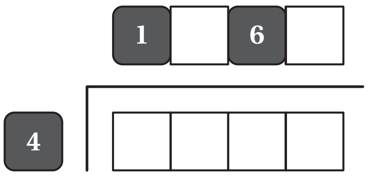
a



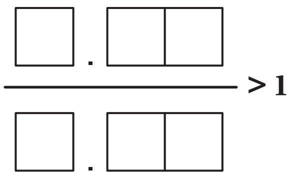
b



4 Fill in the blanks using digits 1–9, except the numbers 1, 4 and 6 to make the division correct.



5 Using the digits 0-9, without repeating any digits, find the quotient closest to 1 and also bigger than 1.



Dividing Decimals

Assign problems to students who want to extend their thinking.

Assign Problems 4 and 5 to students who have solved Problem 3. If time allows, consider sharing Problems 1–3 with all students.

Problem 1

Students will extend their understanding of dividing multi-digit numbers and decimals by solving 7s puzzle.

Provide students with the following hint if additional scaffolding is needed.

- **Hint:** One way to write the decimal 0.7 is .7, which can help you with the restriction of using only 7s.

Responses vary.

$\frac{7+7}{.7} = 20$

Problem 2

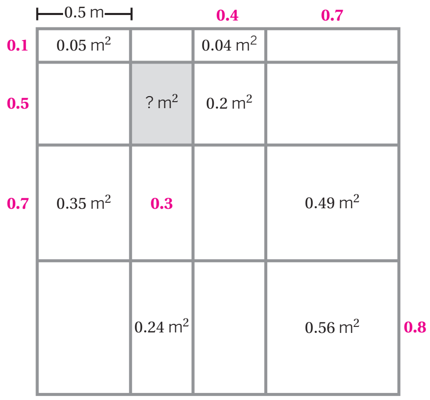
Students will extend their understanding of dividing decimals by determining the area of a rectangle.

Provide students with the following hint if additional scaffolding is needed.

- **Hint:** Use the side lengths of the rectangles with the given areas to help you determine as many side lengths as possible till you reach the ones that are needed for you to calculate the missing area.

0.15 square meters.

Explanations vary.



$0.3 \cdot 0.5 = 0.15$

Problem 3

Students will extend their understanding of dividing multi-digit numbers by determining the unknown digits in a division.

Provide students with the following hint if additional scaffolding is needed.

- **Hint:** You can start by listing the numbers 1–9 and canceling out the numbers that are already used in the division.

a. $3672 \div 4 = 918$

b. $7624 \div 8 = 953$

Continued next page ...

Dividing Decimals (continued)

Problem 4

Students will extend their understanding of dividing multi-digit numbers and decimals by determining the missing digits of the division.

Provide students with the following hint if additional scaffolding is needed.

- **Hint:** Think about the largest possible value of the thousands place of the dividend to start with.

The quotient is 1,963.

$7852 \div 4 = 1963$

Problem 5

Students will extend their understanding of dividing and decimals by solving an Open Middle-like problem to determine the closest quotient to 1.

Provide students with the following hint if additional scaffolding is needed.

- **Hint:** How do you choose the dividend and the divisor so that the quotient is very close to 1 but bigger than 1?

Responses vary.

$\frac{7.01}{6.98} = 1.0042\ldots$

Solving Problems With Decimals

Name: .... Date: .... Period: ....

Student Choice Remember to show or explain your thinking.

1 If you were offered a job for 30 days at the following rates, would you take it?

$0.01 for the first day

$0.02 for the second day

$0.04 for the third day

$0.08 for the fourth day, and so on until the last day, with each day's pay being double that of the previous day.

2 On Arnav's calculator he divided one whole number by another whole number and got the answer 3.125. He can't remember what the numbers were, but remembers both were less than 50. Determine the numbers Arnav used.

Solving Problems With Decimals (continued)

Name: .... Date: .... Period: ....

3 Alexis has a total of $1.15 in coins, which include half dollars, quarters, dimes, nickels, and pennies. With the coins she has, she cannot make exact change for the following amounts:

- $1.00 (one dollar)
- $0.50 (one half dollar)
- $0.25 (one quarter)
- $0.10 (one dime)
- $0.05 (one nickel)

What possible combination of half dollars, quarters, dimes, nickel and pennies could Alexis have?

Solving Problems With Decimals

Assign problems to students who want to extend their thinking.

Assign Problem 3 to students who have solved Problem 1 and 2. If time allows, consider sharing Problems 1 and 2 with all students.

Problem 1

Students will extend their understanding of decimal operations to solve problems in context by exploring the growing pattern in a salary.

Provide students with the following hint if additional scaffolding is needed.

- **Hint:** Organizing your calculations using a table can help you decide without needing to calculate the paycheck for the entire month.

Responses and explanations vary.

Yes. Because the paycheck gets doubled everyday. In 30 days the total amount will reach more than 10 million dollars.

Day	Pay ($)	Day	Pay ($)	Day	Pay ($)
1	0.01	11	10.24	21	10,485.76
2	0.02	12	20.48	22	20,971.52
3	0.04	13	40.96	23	41,943.04
4	0.08	14	81.92	24	83,886.08
5	0.16	15	163.84	25	167,772.16
6	0.32	16	327.68	26	335,544.32
7	0.64	17	655.36	27	671,088.64
8	1.28	18	1,310.72	28	1,342,177.28
9	2.56	19	2,621.44	29	2,684,354.56
10	5.12	20	5,242.88	30	5,368,709.12

Problem 2

Students will extend their understanding of decimal operations to solve problems in context by solving a broken calculator kind problem.

Provide students with the following hint if additional scaffolding is needed.

- **Hint:** What number needs to be multiplied by the decimal part of the quotient to result in a whole number?

25 and 8

Explanations vary. Both numbers are whole numbers. The decimal part of the quotient is .125 which can only be multiplied by 8 or multiples of 8 to give a whole number.

Because both numbers are less than 50, I started with 8. Then $8 \cdot 3.125 = 25$. Next, I tried 16; $16 \cdot 3.125 = 50$. So the numbers must be 25 and 8.

Continued next page ...

Problem 3

Students will extend their understanding of decimal operations to solve problems in context by working on a coin puzzle.
 	
Provide students with the following hints if additional scaffolding is needed.
• **Hint 1:** If Alexis couldn't change a dollar, then can she have 2 half dollars?
• **Hint 2:** If Alexis couldn't change half a dollar, then can she have 2 quarters? Five dimes?
• **Hint 3:** Continue the same logic to determine the maximum number of coins for each type.
	A half dollar, a quarter, and four dimes. *Explanations vary*.
	• **If Alexis can't make exact change for a dollar, then she cannot have 2 half dollars.**
	• **If she can't make exact change for a half dollar, then she cannot have 2 quarters, or five dimes.**
	• **If she can't make exact change for a dime, then she can have no more than one nickel.**
	• **If she can't make exact change for a nickel, then she can have no more than four pennies.**
	• **So she cannot have more than:**
	  ◦ **1 half dollar: \$0.50.**
	  ◦ **1 quarter: \$0.25.**
	  ◦ **4 dimes: \$0.40.**
	  ◦ **1 nickel: \$0.05.**
	  ◦ **4 pennies: \$0.04.**
	The total is \$1.24 which is 9 cents more than the value of her coins. The only way to make 9 cents is with a nickel and four pennies, so those are the coins that must be removed. The remaining coins—a half dollar, a quarter, and four dimes—will not provide change for a dollar or any smaller coin, and they equal \$1.15.

Unit 6

Extensions

Solving Equations

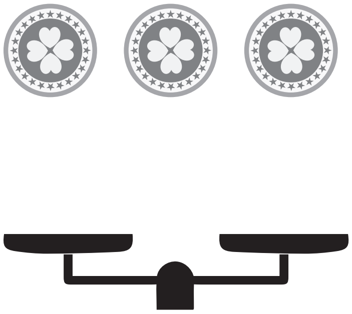
Name: .... Date: .... Period: ....

Student Choice Remember to explain your thinking.

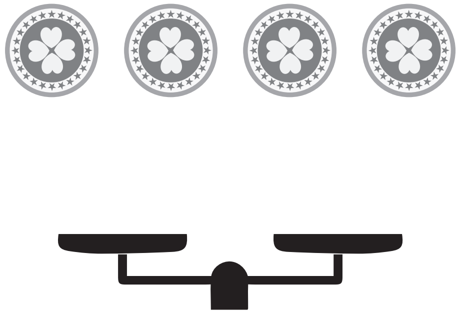
1

Here are two balance scales and some identical-looking coins. One of the coins is counterfeit and has a different weight than all the others. What is the fewest number of weighings needed to determine the counterfeit coin?

a When there are three coins?



b When there are four coins?



2

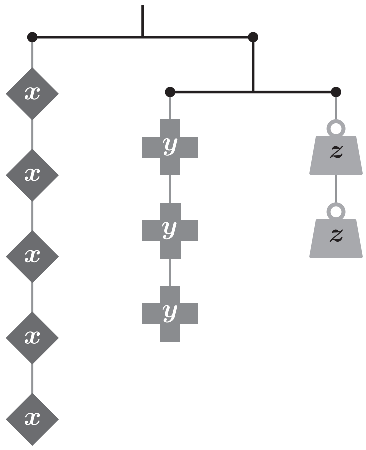
Hanger A is balanced.

a If the value of x is 4, what are the values of y and z?

b Determine three whole number values for x, y, and z to keep the hanger balanced.

c Describe all the whole number values for x, y, and z that keep the hanger balanced.

Hanger A



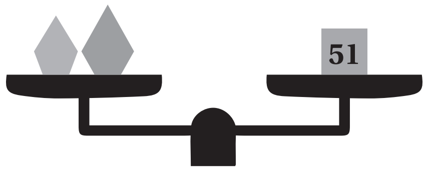
Solving Equations (continued)

Name: .... Date: .... Period: ....

3 *Consecutive numbers* are whole numbers that follow each other from smallest to biggest without skipping any number in between.

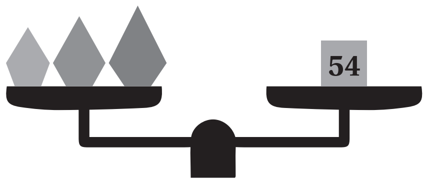
Here are three balanced scales with consecutive weights:

a In Scale A, two consecutive weights balance with a weight of 51 grams. How many grams is each consecutive weight?



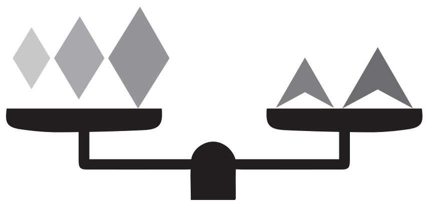
Scale A

b In Scale B, three consecutive weights balance with a weight of 54 grams. How many grams is each consecutive weight?



Scale B

c In Scale C, three consecutive weights balance with two different consecutive weights. Determine a possible solution.


Scale C

Solving Equations

Assign problems to students who want to extend their thinking.

Problems 1 and 2 can be solved in any order. Assign Problem 3 to students who have solved Problem 2. If time allows, consider sharing Problem 1 with all students.

Materials

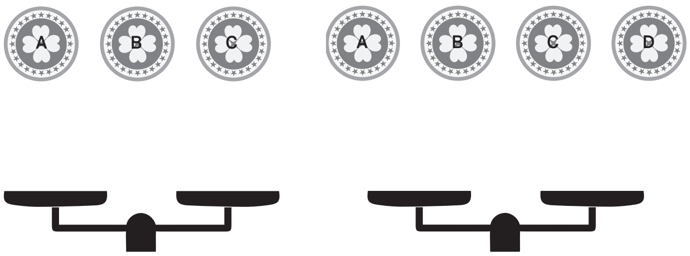
- coins, counters, or concrete objects **(optional for Problem 1)**

Problem 1

Students will extend their understanding of balance while determining the counterfeit coin with the fewest number of weighings.

Provide students with the following hints if additional scaffolding is needed.

- **Hint 1:** Think about how not knowing whether the counterfeit coin is lighter or heavier than the real coins can affect the number of weighings.
- **Hint 2:** Consider labeling the coins to help you organize your thoughts.



a. Two weighings. *Explanations vary.*

Compare Coins A and B.

If the scale is balanced, Coin C is the counterfeit. If the scale is not balanced, compare Coins A and C. If the scale is balanced, Coin B is the counterfeit. If the scale is not balanced, Coin A is counterfeit.

b. Two weighings. *Explanations vary.*

Compare Coins A and B.

If the scale is balanced with Coins A and B, the counterfeit is either Coin C or D. Compare Coins C and A. If the scale is balanced, Coin D is the counterfeit. If the scale is not balanced, Coin C is the counterfeit.

If the scale is not balanced with Coins A and B, the counterfeit is either Coin A or B. Compare Coins A and C. If the scale is balanced, Coin B is counterfeit. If the scale is not balanced, Coin A is counterfeit.

Continued next page ...

Problem 2

Students will extend their understanding of writing and solving equations with variables by representing a multi-level hanger with three variables.

Provide students with the following hints if additional scaffolding is needed.

- **Hint 1:** In part a, write equation(s) that represents Hanger A.
- **Hint 2:** In part a, determine the value of the left side of the hanger when $x = 4$. Then determine the value of each side on the right side of the hanger.

a. $y = \frac{10}{3}$ (or equivalent), $z = 5$

Explanations vary. The hanger represents $3y = 2z$ and $5x = 3y + 2z$

The left side of the hanger is equal to 20, so the right side of the hanger is also equal to 20. Because the right side is balanced, each side of that hanger is equal to 10. Because $2z = 10$ and $3y = 10$, $z = 5$ and $y = \frac{10}{3}$.

b. *Responses vary*.

- $x = 12$, $y = 10$, and $z = 15$
- $x = 24$, $y = 20$, and $z = 30$
- $x = 36$, $y = 30$, and $z = 45$

b. *Responses vary*.

Any multiple of the set of values $x = 12$, $y = 10$, and $z = 15$ is a solution for Hanger A.

Problem 3

Students will extend their understanding of writing and solving equations by representing consecutive numbers with variables.

Provide students with the following hint if additional scaffolding is needed.

- **Hint:** How can you represent the bigger number(s) if the smaller number is x? For example, 3, 4, and 5 are three consecutive numbers. If you represent 3 with x, how can you represent 4 using x? 5?

a. 25 and 26 grams. *Explanations vary*.

Let x represent the smaller number. Then the consecutive number is $x + 1$. Because the sum is 51, I can write $2x + 1 = 51$ and solve to find $x = 25$.

b. 17, 18, and 19 grams. *Explanations vary*.

Let x represent the first number. Then the consecutive numbers are $x + 1$ and $x + 2$. Because the sum is 54, I can write $3x + 3 = 54$ and solve to find $x = 17$.

c. *Responses vary*.

Sample responses:

$2 + 3 + 4 = 4 + 5$

$4 + 5 + 6 = 7 + 8$

$6 + 7 + 8 = 10 + 11$

$8 + 9 + 10 = 13 + 14 \ldots$

Equivalent Expressions

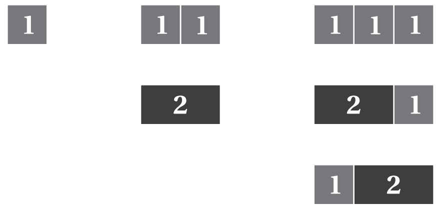
Name: .... Date: .... Period: ....

Student Choice Start with any problem. Remember to show or explain your thinking.

1

Antwon is building rods of different lengths by combining smaller rods that are either 1 unit or 2 units long. The table and image below show how many different ways he can build rods of length 1, 2, and 3 using 1-unit and 2-unit rods.

Length of the line	Number of ways
1	1
2	2
3	3
...	...



a Continue the sequence to find how many different ways Antwon can make a rod that is 10 units long.

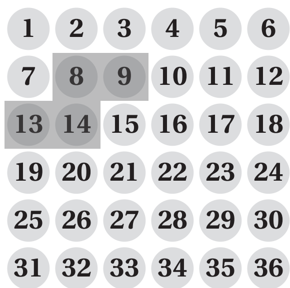
b Jalen created a number sequence using the same rule that Antwon discovered in the "Number of ways" column from part a. In Jalen's sequence, the second number is 4 and the fourth number is 11. Can you determine the first number in his sequence?

2

Here is a 6×6 number grid. Ava and Bao are playing the game of *Guess my Number* by placing a Z shape to cover four of the numbers. Once Bao covers the numbers, he tells Ava the sum for her to guess the numbers. Ava has a strategy of representing the top right number with n.

a What do you think is the rest of her strategy to determine the numbers? Try with several numbers.

b How will Ava's strategy be different if the number grid is 5×5?

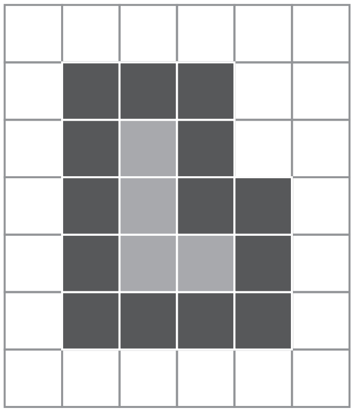


For example, the sum of the covered numbers is
$8 + 9 + 13 + 14 = 44$.

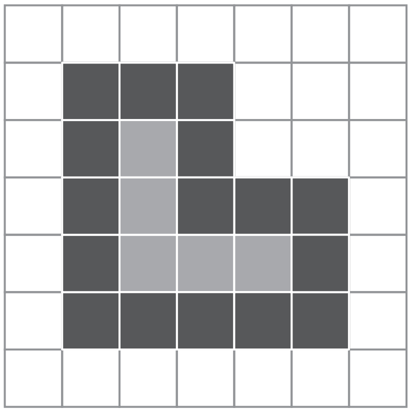
Equivalent Expressions (continued)

Name: .... Date: .... Period: ....

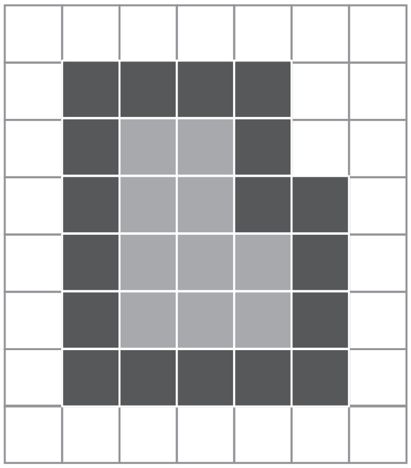
3 Here is an L-shaped pool with the size of 2×3. It has 14 border tiles.



a How many border tiles are needed to surround a 3×3 L-shaped pool?



b How many border tiles are needed to surround a 3×4 L-shaped pool?



c How many border tiles are needed to surround a 12×16 L-shaped pool?

Equivalent Expressions

Assign problems to students who want to extend their thinking.

Problems 1, 2 and 3 can be solved in any order. If time allows, consider sharing Problems 2 and 3 with all students.

Problem 1

Students will extend their understanding of algebraic expressions by exploring the Fibonacci sequence.

Provide students with the following hint if additional scaffolding is needed.

- **Hint:** Form the 4 and 5 units long lines and determine the number of different ways. What do you notice?

a. 89 different ways. *Explanations vary*.

Length	Number of ways	
4	5	1 1 1 1; 1 1 2; 2 1 1; 2 2; 1 2 1
5	8	1 1 1 1 1; 1 1 2 1; 1 1 1 2; 2 2 1; 2 1 1 1; 1 2 2; 1 2 1 1; 2 1 2

So the sequence is $1, 2, 3, 5, 8$. The next term will be the sum of the two previous terms. The 6th term (6 units long) is $5 + 8 = 13$, The 7th term is $8 + 13 = 21$, The 8th term is $13 + 21 = 34$, The 9th term is $34 + 21 = 55$, and 10th term is $34 + 55 = 89$.

b. 3. *Explanations vary*. Jalen's sequence is $x, 4, (x + 4), (x + 4 + 4)$ and because the fourth term is 11, x is $11 - 8 = 3$

Problem 2

Students will extend their understanding of algebraic expressions by exploring the sums on a grid.

Provide students with the following hints if additional scaffolding is needed.

- **Hint 1:** How can you express the number just before n using n?
- **Hint 2:** Look at the numbers in any column. How do they increase? How will they increase if it is a 10×10 grid?

a. *Explanations vary*. Ava represents the top right with n. Then the rest of the numbers can also be expressed in terms of n. The sum is $n + (n - 1) + (n + 4) + (n + 5) = 4n + 8$

	$n - 1$	n
$n + 4$	$n + 5$	

When Bao tells her the sum, 44, she writes the equation and solves for n, $4n + 8 = 44$, so $n = 9$. Then she determines the rest of the numbers as $8, 9, 13, 14$.

b. Then the number underneath x will be 5 more than it instead of 6. So the numbers before it will be $n + 4$ and $n + 3$

	$n - 1$	n
$n + 3$	$n + 4$	

Continued next page ...

Problem 3

Students will extend their understanding of properties of equivalent expressions by solving the border tiles problem.

Provide students with the following hint if additional scaffolding is needed.

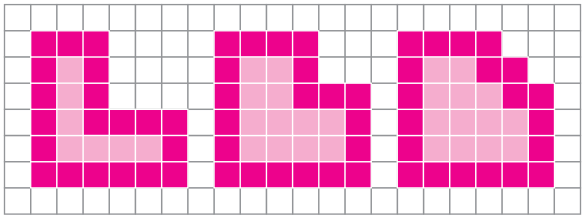
- **Hint:** The pool thickness in part b is different then part a. You may start with exploring if the thickness of the L shape makes a difference in the number of border tiles.

a. 16 tiles

b. 18 tiles

c. 56 tiles. *Explanations vary.*

I first tried 4×4 L-shaped pools with different thicknesses.



They all have the same number of border tiles: 20.

Pool size	Size of the border rectangle	Number of border tiles	
2×3	4×5	14	$2(4 + 5) - 4$
3×3	5×5	16	$2(5 + 5) - 4$
4×3	6×5	18	$2(6 + 5) - 4$
$n \times m$	$n + 2 \times m + 2$	$2n + 2m + 4$ or $2(m + m + 2)$	$2(n + m + 4) - 4$

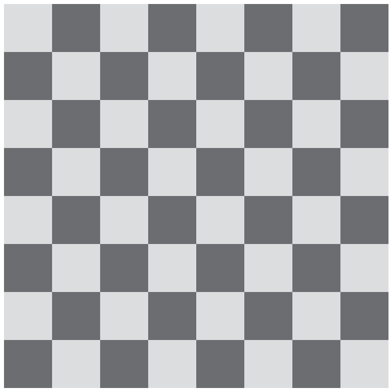
So no matter the thickness of the 12×16 pool, the border rectangle will have size of 14×18. The number of border tiles will be $2(14 + 16) - 4 = 56$ tiles.

Expressions Involving Exponents

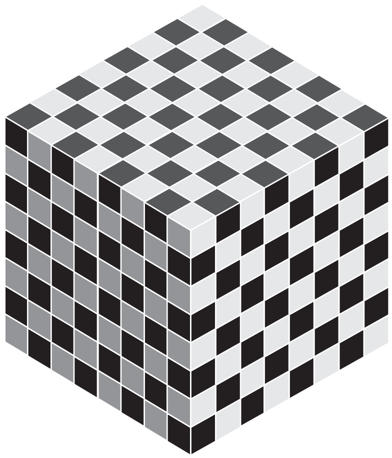
Name: .... Date: .... Period: ....

Student Choice Remember to show or explain your thinking.

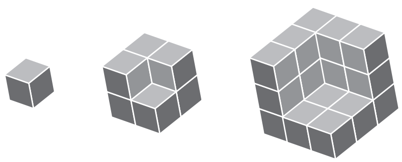
1 How many squares are there on an 8×8 checkerboard? (It is not 64!)



2 How many cubes are there in an $8 \times 8 \times 8$ cube?



3 How many cubes are there in the 10th figure?



Expressions Involving Exponents (continued)

Name: .... Date: .... Period: ....

4 Lagrange's square number theorem states that every number can be expressed as the sum of, at most, four square numbers. For example, $12 = 9 + 1 + 1 + 1$. Let's try:

a 13

b 15

c 178

5 Complete each of the following problems.

a Consider the equation $\square^2 + \square^2 = \square^2$.

One example of three different whole numbers that could go in the boxes are 3, 4, and 5, because $3^2 + 4^2 = 5^2$. Can you find a different set of three whole numbers that make the equation true?

b How many sets of three different whole numbers can you find?

c Can you find a set of three different whole numbers that make the equation true?

$$\square^3 + \square^3 = \square^3$$

d Can you find a set of three different whole numbers that make the equation true?

$$\square^4 + \square^4 = \square^4$$

Expressions Involving Exponents

Assign problems to students who want to extend their thinking.

Problems 1, 3, and 4 can be solved in any order. Assign Problem 3 to students who have solved Problem 2. Assign Problem 5 to students who have solved Problem 4. If time allows, consider sharing Problems 1–5 with all students.

Problem 1

Students will extend their understanding of exponents by determining the total number of squares in a chessboard.

Provide students with the following hint if additional scaffolding is needed.

- **Hint:** How many 1×1 squares are there on a checkerboard? 2×2? 3×3?

204 squares. *Explanations vary.*

On the checkerboard, there are 64, or 8^2, 1×1 *squares,* 49, or 7^2, 2×2 *squares,* 36, or 6^2, 3×3 *squares . . . so in total there are* $64 + 49 + 36 + 25 + 16 + 9 + 4 + 1 = 204$ *squares.*

1×1	2×2	3×3	4×4	5×5	6×6	7×7	8×8
8^2	7^2	6^2	5^2	4^2	3^2	2^2	1^2

Problem 2

Students will extend their understanding of exponents by determining the total number of cubes in a $8 \times 8 \times 8$ cube.

Provide students with the following hint if additional scaffolding is needed.

- **Hint:** How many 1×1 cubes are there? 2×2? 3×3?

1296 cubes. *Explanations vary.*

On the $8 \times 8 \times 8$ *cube, there are* 512, or 8^3, 1×1 *cubes,* 343, or 7^3, 2×2 *cubes,* 216, or 6^3, 3×3 *cubes. . . so in total there are* $512 + 343 + 216 + 125 + 64 + 27 + 8 + 1 = 1296$ *cubes*

$1 \times 1 \times 1$	$2 \times 2 \times 2$	$3 \times 3 \times 3$	$4 \times 4 \times 4$	$5 \times 5 \times 5$	$6 \times 6 \times 6$	$7 \times 7 \times 7$	$8 \times 8 \times 8$
8^3	7^3	6^3	5^3	4^3	3^3	2^3	1^3

Problem 3

Students will extend their understanding of exponents by determining the 10th term of the sequence.

Provide students with the following hint if additional scaffolding is needed.

- **Hint:** Investigate each figure closely. How do you describe the figures?

271 cubes. *Explanations vary.*

Figure 1	Figure 2	Figure 3	Figure 4	Figure 5
1	7	19	37	61
$1^3 - 0^3$	$2^3 - 1^3$	$3^3 - 2^3$	$4^3 - 3^3$	$5^3 - 4^3$

So Figure 10 will have $10^3 - 9^3 = 1000 - 729 = 271$ cubes.

Continued next page ...

Expressions Involving Exponents (continued)

Problem 4	
Students will extend their understanding of exponents by finding examples to Lagrange's square number theorem. **Provide students with the following hint** if additional scaffolding is needed. • **Hint:** Consider listing perfect square numbers to determine the sums.	a. *Responses vary.* $13 = 9 + 4 = 3^2 + 2^2$ b. *Responses vary.* $15 = 9 + 4 + 1 + 1 = 3^2 + 2^2 + 1^2 + 1^2$ c. *Responses vary.* $178 = 144 + 25 + 9 = 12^2 + 5^2 + 3^2$

Problem 5	
Students will extend their understanding of evaluating expressions with exponents by attempting to find solutions for various equations with exponents. **Provide students with the following hint** if additional scaffolding is needed. • **Hint:** In part a, consider investigating the multiples of 3, 4, and 5 as one of the examples.	a. *Responses vary.* $\{6, 8, 10\}, \{5, 12, 13\}$ b. *Responses vary*. There are an infinite number of triples. c. No, it is not possible. d. No, it is not possible.

Applying Percentages

Name: .... Date: .... Period: ....

Student Choice Remember to show or explain your thinking.

1 Amy sold two sofas at a price of $\$1{,}200$ a piece. She made a 25% profit on the first sofa and a 20% loss on the other. She assumed that she still made a profit on the combined sale. Is she correct?

2

a Dinner at a restaurant costs $\$23.74$. The sales tax is 10% and the customer plans to tip 20%. How much more expensive is it to pay the tip with the tax already included, rather than pay the tip first without the tax included?

b The cost of a meal at the restaurant is c dollars, the sales tax is 10% and the customer plans to tip 20%. Write an expression to represent how much more expensive it would be for the customer to pay the tip on the price the tax, instead of the cost of the meal without the tax.

Applying Percentages (continued)

Name: .... Date: .... Period: ....

3 From 22 kg of fresh mushrooms, 2.5 kilogram of dried mushrooms are obtained, containing 12% water. What is the percentage of water in the fresh mushrooms?

Applying Percentages

Assign problems to students who want to extend their thinking.

Assign Problem 2 to students who have solved Problem 1. If time allows, consider sharing Problems 2 and 3 with all students.

Problem 1

Students will extend their understanding of using equations to solve problems about percent change by representing multiple percent changes with expressions.

Provide students with the following hints if additional scaffolding is needed.

- **Hint:** How can you determine the cost price of the first sofa? The second sofa?
- **Hint:** What is the total cost price of two sofas?

The pricing of the first sofa:

$x \cdot \frac{125}{100} = 1.25x$, $1.25x = 1{,}200$. The cost of the first sofa is $960.

The pricing of the second sofa: $y \cdot \frac{80}{100} = 0.8x$, $0.8x = 1{,}200$. The cost of the first sofa is $1,500.

Total cost price is $960 + 1500 = 2460$, total selling price is $1200 + 1200 = 2400$. To determine if there was a profit or loss, subtract the total cost price from the total selling price Total selling price − Total cost price $= 2400 - 2460 =$ -60. A negative result indicates a loss. Therefore, Amy actually incurred a loss of $60 on the combined sale.

Problem 2

Students will extend their understanding of sales tax and tips by comparing the cost of tipping before and after sales tax.

Provide students with the following hint if additional scaffolding is needed.

- **Hint:** What is the cost of the dinner including tax?

a. $0.48. *Explanations vary.*
The cost of the meal with the tip on top of the tax can be calculated by $23.74 \cdot 1.1 \cdot 1.2$, which results in a total cost of $31.34.

The cost of the meal with the tip determined before the tax can be calculated by $23.74 \cdot 0.2 + 23.74 \cdot 1.1$, which results in about $30.86.

The difference is $30.86 - 31.34$.

b. 0.02c. *Explanations vary*. The cost of the meal with the tip on top of the tax can be calculated by $c \cdot 1.1 \cdot 1.2 = 1.32c$.

The cost of the meal with the tip determined before the tax can be calculated by $c \cdot 0.2 + c \cdot 1.1 = 1.3c$. The difference is $1.32c - 1.3c = 0.02c$.

Continued next page ...

Problem 3

Students will extend their understanding of solving problems about percent change with multiple percent changes.

Provide students with the following hint if additional scaffolding is needed.

- **Hint:** Determine the percent of dried mushrooms that can be obtained from fresh mushrooms.

$2.5 \cdot 0.88 = 2.2$ kg completely dried mushrooms.

Explanations vary. I checked what % of fresh mushrooms is dried by multiplying $2.2 \cdot \frac{100}{22} = 10$, so 10% of the mushroom is dried and 90% of the mushroom is water.

Unit 7

Extensions

Signed Numbers and Absolute Values

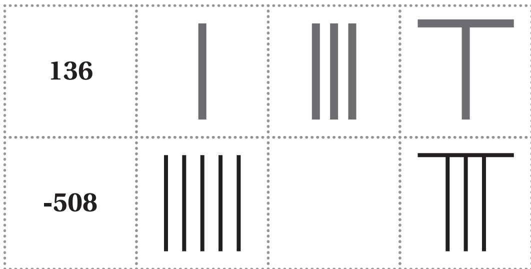
Name: .... Date: .... Period: ....

Student Choice Remember to show or explain your thinking.

1

One of the first known records of negative numbers being used comes from the Chinese number rod system seen here, which dates to around 200 BCE. The need for positive and negative numbers came from taxes — receiving and paying currency.

Use the image to decipher and determine how to write each of the following numbers using Chinese number rods.



a 27

b 4,852

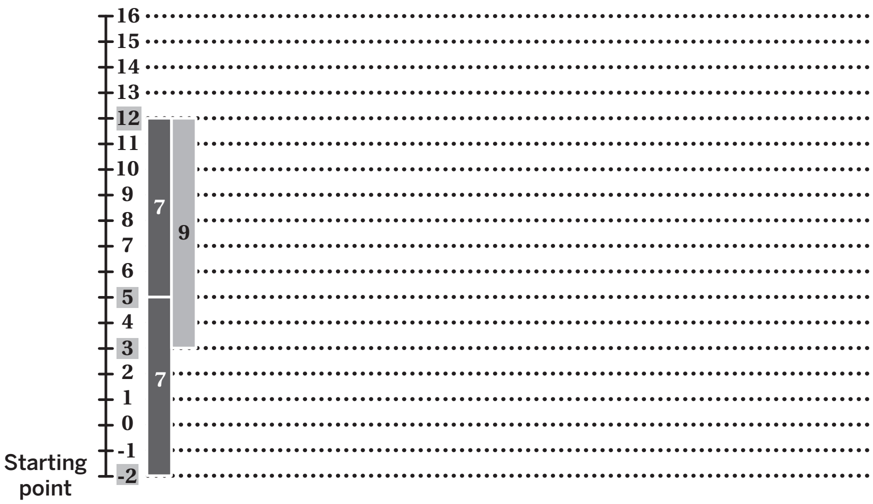
c -3,906

2

The only elevator of an 18-floor building has only two buttons: and where you can either go 7 floors UP or 9 floors DOWN.

For example, when you press two times and then press one time, you can visit floors 5, 12 and 3.

a Is it possible to reach all the floors of the building using the elevator?



b In which order have you visited the floors?

c How many times did you press the buttons to try to visit all the floors?

Signed Numbers and Absolute Values (continued)

Name: .... Date: .... Period: ....

3 If the absolute value of x is greater than the absolute value of y, then x is greater than y.

a Determine whether each set of values for x and y supports or does not support this claim.

x	y	Supports	Does not support
8	7	☐	☐
-4	0	☐	☐
-2.1	-2.05	☐	☐
$\frac{3}{4}$	$\frac{1}{5}$	☐	☐

b Can you generalize for which numbers does the inequality $|x| > |y|$ imply that $x > y$? And for which numbers does it not?

4 Determine the number that makes each of the following statements true.

a The difference between the absolute value of a number and the number is 8. What is the number?

b The sum of a number and its absolute value is 8. What is the number?

c What if the result from each problem was 6? 5? Can you generalize your findings?

Signed Numbers and Absolute Values

Assign problems to students who want to extend their thinking.

Problems 1, 2, and 3 can be solved in any order. Assign Problem 4 to students who have solved Problem 3. If time allows, consider sharing Problems 1 and 2 with all students.

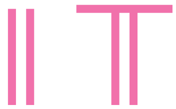
Problem 1

Students will extend their understanding of the ancient Chinese number rod system by rewriting numbers using the rod system.

Provide students with the following hint if additional scaffolding is needed.

- **Hint:** Think about the transition between 5 and 6 in the Chinese number rod system. How can you write 7 using this pattern shift?

a.



b.

c.

Note: Positive numbers represented by thicker gray lines in parts a and b. The negative number in part c is represented by the thinner black lines.

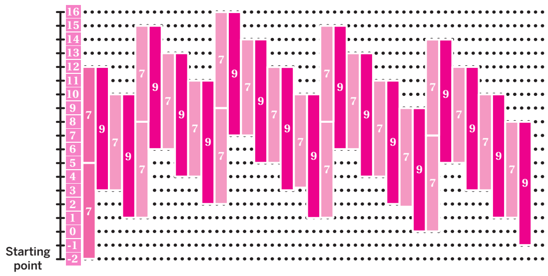
Problem 2

Students will extend their understanding of negative and positive numbers by exploring the elevator puzzle.

Provide students with the following hint if additional scaffolding is needed.

- **Hint:** After you press two times and then press one time, you can visit floors 5, 12 and 3. At floor 3, can you press again?

a. Yes. Sample response:



b. *Responses vary.*
5, 12, 3, 10, 1, 8, 15, 6, 13, 4, 11, 2, 9, 16, 7, 14, 5, 12, 3, 10, 1, 8, 15, 6, 13, 4, 11, 2, 9, 0, 7, 14, 5, 12, 3, 10, 1, 8, -1

c. *Responses vary.*
39 times for the sequence above.

Continued next page ...

Signed Numbers and Absolute Values (continued)

Problem 3

Students will extend their understanding of absolute value by reasoning about statements that contain absolute value.

Provide students with the following hint if additional scaffolding is needed.

- **Hint:** In part a, you can create a table with the columns $x > y$ and $x > y$ to see if each pair of numbers support or does not support the claim.

a.

x	y	$\|x\| > \|y\|$	$x > y$	Supports	Does not support
8	7	$\|8\| > \|7\|$	$8 > 7$	☑	☐
-4	0	$\|-4\| > \|0\|$	$-4 \not> 0$	☐	☑
-2.1	-2.05	$\|-2.1\| > \|-2.05\|$	$-2.1 \not> -2.05$	☐	☑
$\frac{3}{4}$	$\frac{1}{5}$	$\left\|\frac{3}{4}\right\| > \left\|\frac{1}{5}\right\|$	$\frac{3}{4} > \frac{1}{5}$	☑	☐

b. *Explanations vary*. $|x| > |y|$ implies $|x| > |y|$ only when x and y are both non-negative, i.e., When $|x| = x$, and $|y| = y$, $|x| > |y|$ implies $x > y$

Problem 4

Students will extend their understanding of absolute value by reasoning about statements that contain absolute value.

Provide students with the following hint if additional scaffolding is needed.

- **Hint:** How could you rewrite each of the problems using algebraic expressions?

a. $x = -4$
Explanations vary.
$|x| - x = 8$ cannot be true when x is positive in this case, because the expression $|x| - x$ would always be equal to 0, not 8. Therefore x is negative; $-2x = 8$, $x = -4$

b. $x = 4$
Explanations vary.
$|x| + x = 8$ cannot be true when x is negative in this case, because the expression $|x| + x$ would always be equal to 0, not 8. Therefore x is positive; $2x = 8$, $x = 4$

c. For any value of n, where $|x| - x = n$ and $|x| + x = n$, the value of x will be $-\frac{n}{2}$ for the difference, and $\frac{n}{2}$ for the sum.

Operations With Rational Numbers

Name: .... Date: .... Period: ....

Student Choice Remember to show or explain your thinking.

1

Here are magic squares with positive and negative integers. Determine the unknown numbers.

a

		-1
2	0	
1	-4	

b

-4		
	-1	
	-3	2

c

0	-7	
-5		
-4		

2

Use the numbers -2, -1, 0, 1, 2 to complete the Kenken puzzle.

- Every row and column must use one of -2, -1, 0, 1, 2 only once.
- Each region must have the indicated result at the top left corner.
- For example, the *difference* (operation after the number) of the top left L-shape made up of three squares must be 0.

0-		-3+		0÷
0x		1+	3-	
	0x			3+
0-		1	0x	
		0		-1

3

Determine the sums. What patterns do you notice?

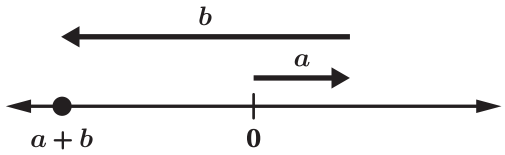
a $-1 + 2 - 3 + 4 - 5 + 6 - 7 + 8 \ldots -97 + 98 - 99 + 100$

b $1 - 2 + 3 - 4 + 5 - 6 + 7 - 8 \ldots + 97 - 98 + 99 - 100$

Name: .... Date: .... Period: ....

4

Consider the following figure. For the numbers a and b, which expression is equivalent to $|a + b|$?



- **a** $|a| + |b|$
- **b** $|a| - |b|$
- **c** $|b| - |a|$

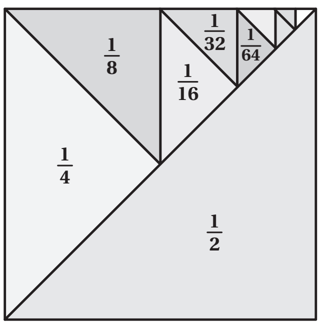
5

Determine the following sums.

- **a** $\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16}$
- **b** $\frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \frac{1}{81}$
- **c** $\frac{1}{4} + \frac{1}{16} + \frac{1}{64} + \frac{1}{256}$
- **d** Following the pattern in parts a–c, determine the value of $\frac{1}{5} + \frac{1}{25} + \frac{1}{125} + \frac{1}{625}$.

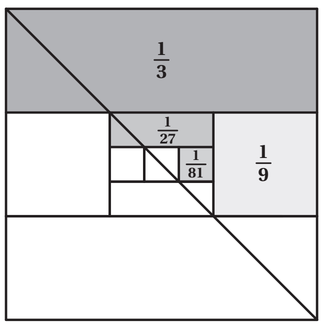
Continue the patterns to estimate each sum if it continues infinitely.

e



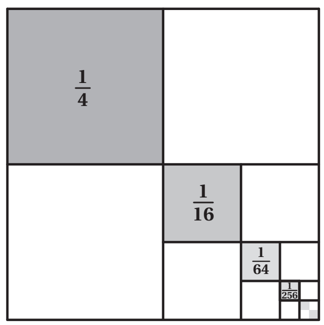
$\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32} + \ldots$

f



$\frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \frac{1}{81} + \frac{1}{243} + \ldots$

g



$\frac{1}{4} + \frac{1}{16} + \frac{1}{64} + \frac{1}{256} + \frac{1}{1024} + \ldots$

- **h** What do you notice about the infinite sums in parts e–g? Can you estimate the sum of $\frac{1}{n} + \frac{1}{n^2} + \frac{1}{n^3} + \frac{1}{n^4} + \ldots$

Operations With Rational Numbers

Assign problems to students who want to extend their thinking.

Problems 1 and 3–5 can be solved in any order. Assign Problem 2 to students who have solved Problem 1. If time allows, consider sharing Problems 2 and 5 with all students.

Problem 1

Students will extend their understanding of adding and subtracting integers by solving magic square puzzles.

Provide students with the following hint if additional scaffolding is needed.

- **Hint:** Determine the sum of each row, column, and diagonal for each of the puzzles.

a.

-3	4	-1
2	0	-2
1	-4	3

b.

-4	1	0
3	-1	-5
-2	-3	2

c.

0	-7	-2
-5	-3	-1
-4	1	-6

Problem 2

Students will extend their understanding of adding and subtracting integers by solving a KenKen puzzle.

Provide students with the following hints if additional scaffolding is needed.

- **Hint 1:** Start with the single cells in the bottom rows.
- **Hint 2:** The top right rectangle has $0 \div$. In which cell must 0 be at?
- **Hint 3:** When inserting the zeroes for $0 \times$, remember that each row and column must have each number once.

0- 1	2	-3+ -2	-1	0÷ 0
0x 0	-1	1+ 2	3- 1	-2
2	0x 0	-1	-2	3+ 1
0- -1	-2	1 1	0x 0	2
-2	1	0 0	2	-1 -1

Continued next page ...

Problem 3

Students will extend their understanding of evaluating expressions with integers by evaluating a complex sum.

Provide students with the following hint if additional scaffolding is needed.

- **Hint:** Determine the sum of each consecutive pair. How many pairs are there in each sum?

a. 50. *Explanations vary*. I noticed that the difference in each pair, $-1 + 2$, $-3 + 4$, is 1. Because the expression starts with -1 and ends with 100, I know there are 50 pairs. The sum of the 50 pairs of 1 is 50.

b. -50. *Explanations vary*. I noticed that the difference in each pair, $1 - 2$, $3 - 4$, is -1. Because the expression starts with +1 and ends with -100, I know there are 50 pairs. The sum of the 50 pairs of -1 is -50.

Problem 4

Students will extend their understanding of adding positive and negative numbers by determining which absolute value expression is equivalent to the arrow diagram shown.

Provide students with the following hint if additional scaffolding is needed.

- **Hint:** Consider substituting a and b with positive and negative numbers.

c. *Explanations vary*.

I substituted the values $b = -10$ and $a = 3$ to make sense of the diagram. Therefore $a + b = -7$ and $|a + b| = 7$. The expression that results in 7 is $|b| - |a|$ because $|a| + |b| = 13$ and $|a| - |b| = -7$.

Problem 5

Students will extend their understanding of adding and subtracting rational numbers by estimating the infinite sums of fractions.

Provide students with the following hint if additional scaffolding is needed.

- **Hint:** In parts e–g, consider that the addends go to infinity. How much of the remaining region will be shaded.

a. $\frac{15}{16}$

b. $\frac{40}{81}$

c. $\frac{85}{256}$

d. $\frac{156}{625}$

e. 1. *Explanations vary*. If we keep adding fractions with the denominators of powers of 2, I think that the entire square will be covered.

f. $\frac{1}{2}$. *Explanations vary*. If we keep adding fractions with the denominators of powers of 3, I think a half of the square will be covered.

g. $\frac{1}{3}$. *Explanations vary*. If we keep adding fractions with the denominators of powers of 4, I think that the $\frac{1}{3}$ of the square will be covered.

h. $\frac{1}{n-1}$. *Explanations vary*. By looking at the sums in parts e–g, the ratio of the total shaded regions by the fractions is always 1 less than the denominator.

The Coordinate Plane

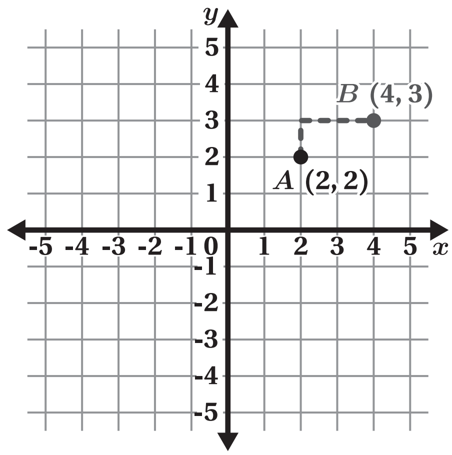
Name: .... Date: .... Period: ....

Student Choice Start with any problem. Remember to show or explain your thinking.

1

Taxicab geometry is a form of geometry in which you can only move along the lines of a grid.

Point $A(2, 2)$ is given on the coordinate plane. Point $B(4, 3)$ is 3 units away from A.



a Determine all the other points that are 3 units away from the point A.

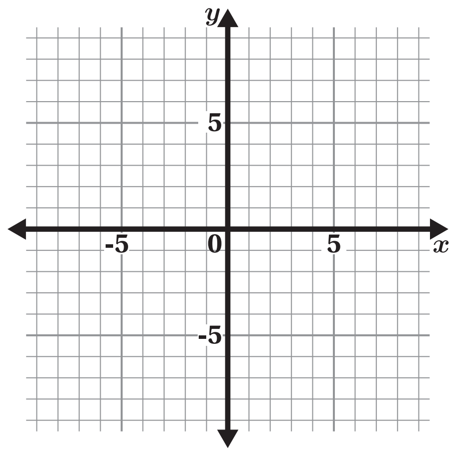
b What do you notice?

2

a Determine as many coordinate points as possible whose x- and y-coordinates have a sum of 5.

b Graph the coordinate points on the coordinate plane. What do you notice?

c Try another value to sum the coordinates to. What do you notice?



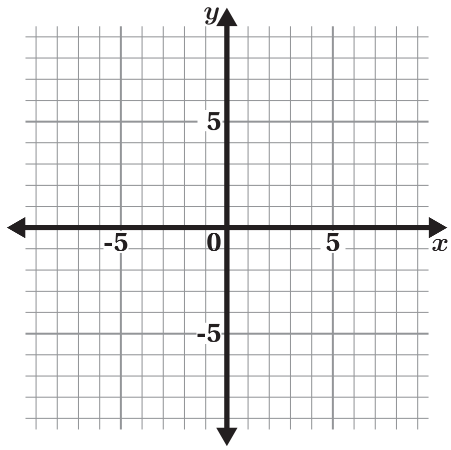
The Coordinate Plane (continued)

Name: .... Date: .... Period: ....

3 Choose any point and plot it on the coordinate plane.

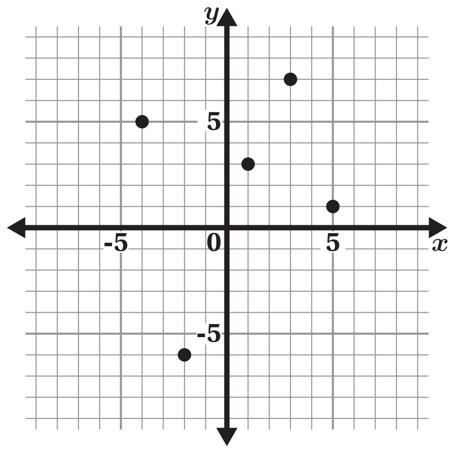
a Reflect the point across the x-axis, then across the y-axis, then across the x-axis again, and across the y-axis again.

b What do you notice? Why do you think that happened?



4 There are five points in the coordinate plane. You can choose any starting point and travel along the gridlines.

Draw the shortest path that visits all five points and returns to your original point. State the number of total units traveled.



The Coordinate Plane

Assign problems to students who want to extend their thinking.

Problems 1–4 can be solved in any order. If time allows, consider sharing Problems 2 and 4 with all students.

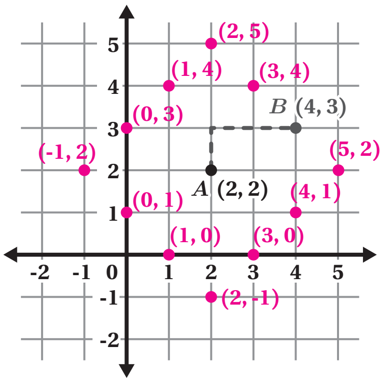
Problem 1

Students will extend their understanding of distance on the coordinate plane by learning about the taxicab geometry.

Provide students with the following hints if additional scaffolding is needed.

- **Hint 1:** Determine the coordinates of the two points horizontally 3 units away from A.
- **Hint 2:** Determine the coordinates of the two points vertically 3 units away from A.

a.



b. *Responses vary.*

- The points form a square.
- All the points that are 3 units away from A have coordinates that differ 3 units from the coordinate for point A. For example $(2, 5)$'s x-coordinate is the same as the x-coordinate for Point A, but its y-coordinate differs by 3. $(0, 1)$'s x-coordinate differs by 2 and y-coordinate differs by 1 for a total of 3 again.

Problem 2

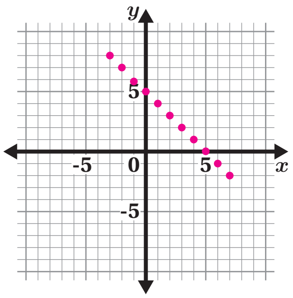
Students will extend their understanding of points on the coordinate plane by determining different ordered pairs that sum to 5, graphing the ordered pairs, and noting their observations.

Provide students with the following hint if additional scaffolding is needed.

- **Hint:** In part a, you may want to start writing the coordinates systematically. For example, determine the value of y-coordinate if $x = 0, x = 1, x = 2 \ldots,$ then consider using the negative values of x.

a. *Responses vary.*

$(0, 5), (5, 0), (1, 4), (4, 1), (2, 3), (3, 2), (-1, 6), (6, -1), (-2, 7), (7, -2), (-3, 8), (8, -3)$



b. *Responses vary.*

The points create a line segment that extends through $(0, 5)$ to $(5, 0)$.

c. For any sum, the coordinates always create a line.

Continued next page ...

Problem 3

Students will extend their understanding of reflections of points in the coordinate plane, by completing a series of reflections that result in the original point.

Provide students with the following hint if additional scaffolding is needed.

- **Hint:** Reflect $(2, 3)$ across the x-axis. Record how the coordinates change. Then reflect the new point across the y-axis to record how the coordinates change.

a. *Graphs vary*. The resulting point should reflect back to the original point chosen.

b. *Explanations vary*. After the reflections, the resulting point is back in the original location.

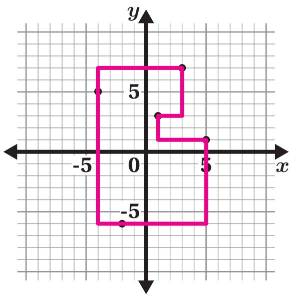
Problem 4

Students will extend their understanding of distance on the coordinate plane by experimenting with different paths and calculating the distance for each path.

Provide students with the following hint if additional scaffolding is needed.

- **Hint:** After you pick your starting point, select your second point to go to. Think about the possible ways to travel between these two points. Does the path selection affect the length of the path?

a. *Graphs vary*. 48 units



Unit 8

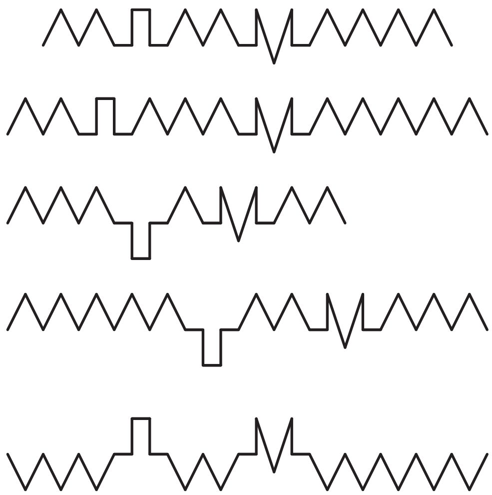
Extensions

Equations

Name: .... Date: .... Period: ....

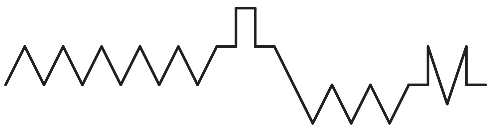
Student Choice Start with any problem. Remember to show or explain your thinking.

1 Astronomers have sent messages like this into outer space in order to establish communication with intelligent life on other planets. Researchers hope that even if such alien life forms cannot understand our written language, they might use radio for communication and be adept at mathematics.



a Can you decipher this mathematical message?

b Complete the message below



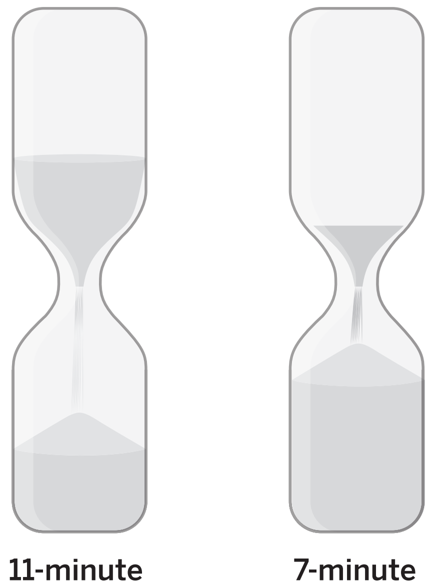
2 Complete the magic square using only the negative and positive integers from -8 to 8, so that the sum of every row and column, and the two diagonals equal zero.

	3		-4
5			
-8			
4	-2		

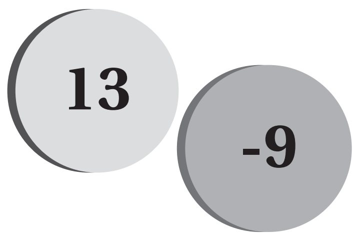
Equations (continued)

Name: .... Date: .... Period: ....

3 Can you measure exactly 15 minutes using nothing but an 11-minute hourglass and a 7-minute hourglass?



4 A faraway country uses a currency system with just two coins: worth 13 cents and -9 cents respectively. Is it possible to purchase an item that costs 19 cents?



Equations

Assign problems to students who want to extend their thinking.

Problems can be solved in any order. If time allows, consider sharing Problems 1 and 3 with all students.

Problem 1

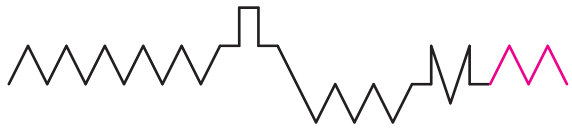
Students will extend their understanding of four operations with rational numbers by deciphering a mathematical message

Provide students with the following hint if additional scaffolding is needed.

- **Hint:** The first line is an addition equation.

a. Line 1: $2 + 2 = 4$
Line 2: $2 + 3 = 5$
Line 3: $3 - 1 = 2$
Line 4: $5 - 2 = 3$
Line 5: $-3 + (-2) = -5$

b. $5 + (-3) = 2$



Problem 2

Students will extend their understanding of four operations with rational numbers by filling a fully blank magic square.

Provide students with the following hint if additional scaffolding is needed.

- **Hint:** One possible answer has a diagonal of 1, -7, 7, 1.

Responses vary.

-1	3	2	-4
5	-7	-6	8
-8	6	7	-5
4	-2	-3	1

Continued next page ...

Equations (continued)

Problem 3

Students will extend their understanding of four operations with rational numbers by solving a version of famous hourglass puzzles

Provide students with the following hint if additional scaffolding is needed.

- **Hint:** You can run each hourglass more than once.

***Responses vary*.**
Flip over both hourglasses to start them.
As soon as the 7-minute hourglass runs out, flip it over and restart it.

As soon as the 11-minute hourglass runs out, flip over the 7-minute hourglass (even though it hasn't finished yet). The 7-minute hourglass has been running for 4 minutes ($11 - 7$), so it will run for 4 more minutes after flipping it over.

Once the 7-minute hourglass runs out for the second time, 15 minutes ($11 + 4$) are over.

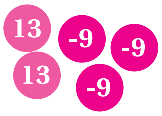
Problem 4

Students will extend their understanding of operations with rational numbers by solving a negative coin puzzle.

Provide students with the following hints if additional scaffolding is needed.

- **Hint 1:** If you can create 1 cent, you can create any amount.
- **Hint 2:** To create 1 cent, you might consider creating -1 cent first

Yes. *Explanations vary*.
First, notice that you can make -1 cent using three -9 cent coins and two 13 cent coins, since
$3 \cdot (-9) + 2 \cdot 13 = -1$.



Next, we can make 1 cent by adding a 13 cent coin and twelve of these -1 cent combinations:
$13 + 12[3 \cdot (-9) + 2 \cdot 13] = 1$
$13 + 12\,(-1) = 1$

Finally, we just need 19 copies of this 1 cent combination:
$19 \cdot \{13 + [12 \cdot 3 \cdot (-9) + 2 \cdot 13]\}$.

In fact, any positive or negative whole number can be created using combinations of these coins.

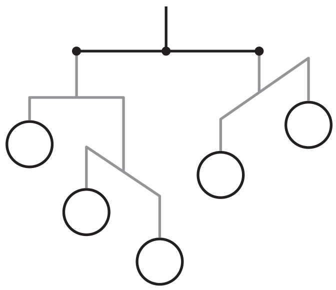
Inequalities

Name: .... Date: .... Period: ....

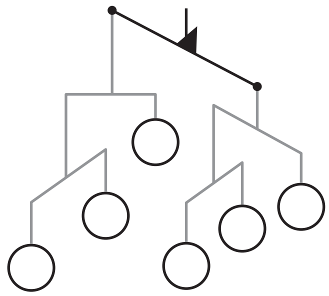
Student Choice Start with any problem. Remember to show or explain your thinking.

1 Keep the unbalanced hanger diagrams as they are by inserting the given numbers.

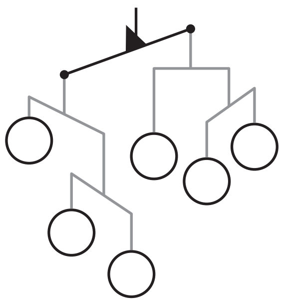
a -3, -2, 0, 2, 3



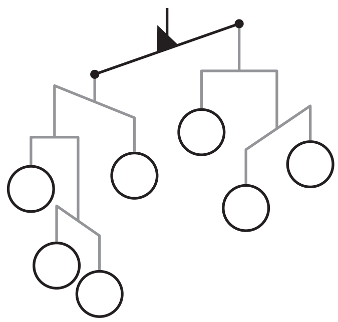
b -3, -2, 1, 2, 5, 6



c -4, -2, 0, 2, 6, 8



d Pick your own numbers!



Inequalities (continued)

Name: .... Date: .... Period: ....

2 $|x| < 4$ is given.

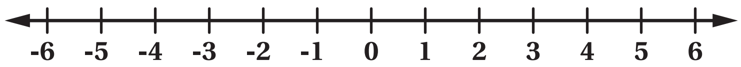
a What are all the possible integer values of x?

b Give three non-integer examples of x that satisfy the inequality.

c Give an example of x that satisfies the inequality and is larger than 3.

d Give an example of x that satisfies the inequality and is smaller than -3.

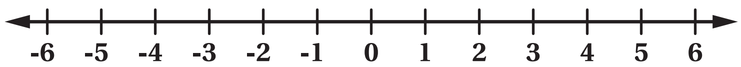
e Graph the inequality using your answers in parts $a - d$.



f Graph the inequality -$4 < x < 4$. What did you notice?

g The inequalities $|x| < 6$ and $a < x < b$ have the same solution sets. What are the values of a and b?

h Graph the inequality $|x| > 4$.



i Write two inequalities that when combined they have the same solution set as $|x| > 4$.

Inequalities

Assign problems to students who want to extend their thinking.

Problems 1 and 2 can be solved in any order. If time allows, consider sharing Problems 1 and 2 with all students.

Problem 1

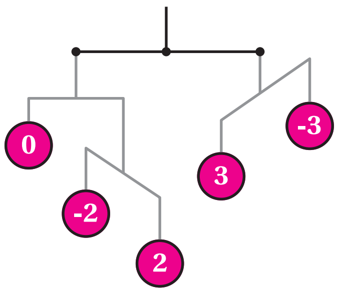
Students will extend their understanding of writing inequalities by solving the unbalanced hanger diagram puzzles.

Provide students with the following hint if additional scaffolding is needed.

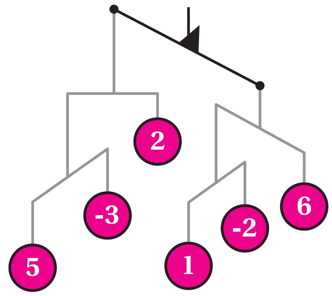
- **Hint:** In part d, start with the lowest hanger diagram to pick your numbers. Then determine their sum to write the value of the leftmost circle.

Responses vary.

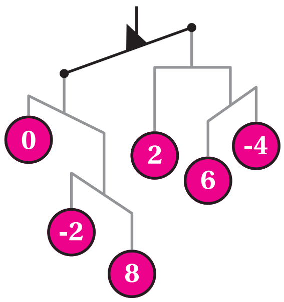
a.



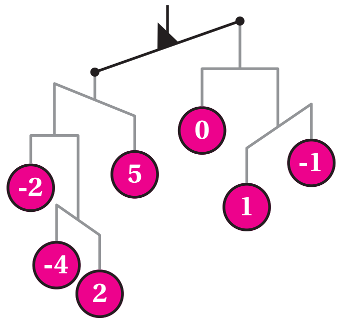
b.



c.



d. *Responses vary.*



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Problem 2

Students will extend their understanding of writing inequalities by solving an absolute value inequality using symbols, words, models, graphs

Provide students with the following hints if additional scaffolding is needed.

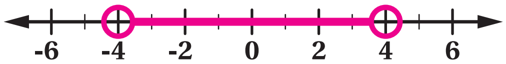
- **Hint 1:** In part a, what is the largest integer value that satisfies the inequality?
- **Hint 2:** In part a, what is the smallest integer value that satisfies the inequality?
- **Hint 3:** In part g, you can graph the inequality $|x| < 6$ to determine values of a and b.

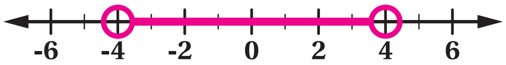
a. -3, -2, -1, 0, 1, 2, 3

b. *Responses vary.*
$-3.14, -2\frac{2}{7}, 0.66666\ldots$

c. *Responses vary.*
3.14

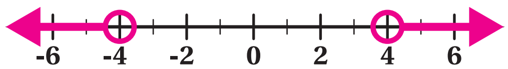
d. *Responses vary.*
$-3\frac{6}{11}$

e.


f.


I noticed that their solution sets are the same.

g. $a = -6$ and $b = 6$

h.


i. $x < -4$ and $x > 4$

Unit 9

Extensions

Unit 9
Sub-Unit 1
Extensions

Visualizing Data

Name: .... Date: .... Period: ....

Student Choice Start with any problem. Remember to show or explain your thinking.

1 A *stem-and-leaf plot* is another way to display data. In a stem-and-leaf plot, the greatest place value common to all the data values is usually used for the *stem*. The next greatest place value forms the *leaves*.

For example, here are the prices of 20 of the best rated cookies in Destown.

58¢	45¢	67¢	71¢	50¢
55¢	40¢	65¢	75¢	80¢
90¢	75¢	64¢	78¢	88¢
95¢	80¢	77¢	50¢	99¢

Stem-and-leaf plot of the prices of the best rated Cookies

4	0 5
5	0 0 5 8
6	4 5 7
7	1 5 5 7 8
8	0 0 8
9	0 5 9

4|0 means 40¢

a. What are some advantages of displaying data using a stem-and-leaf plot?

b. How is a stem-and-leaf plot similar to a dot plot? How is it different?

c. In which interval do most of the cookie prices lie?

d. Determine the prices that appear most frequently.

e. If Descafe wants to decide a price range for a new special cookie, what price range would you recommend for the cookie to have?

Visualizing Data (continued)

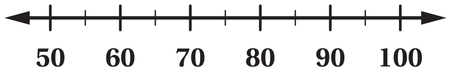
Name: .... Date: .... Period: ....

2

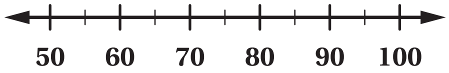
Ms. Lucas wants to create dot plots displaying the project grades of her three math classes. Here is some information about the classes:

- Each class has 10 students.
- All three classes have a symmetric distribution.
- The mean, mode, and median of each class is 80.
- There are more students who scored 80 in the second class than the first class.
- First two classes have the lowest score as 60 and highest score as 100. The third class has the lowest score as 75 and highest score as 85.

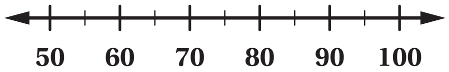
Help Ms. Lucas to create the dot plots.



Class 1 Project Grades



Class 2 Project Grades

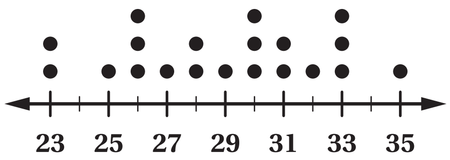


Class 3 Project Grades

3

In the 12th century, King Henry I of England fixed the *yard* as the distance from his nose to the thumb of his out-stretched arm. (Today it is 36 inches.)

Here is a dot plot showing the "yards" of 6th grade students in a school.



Length from nose to thumb (in.)

a Describe the shape of the dot plot. Are the dots evenly distributed or grouped on one side?

b Describe the center of the dot plot. What single plot would best represent the data?

c Describe the spread of the dot plot. Are there any outliers?

d Do you think it is reliable to use body measurements as the units of measurements?

Visualizing Data

Assign problems to students who want to extend their thinking.

Problems can be solved in any order. If time allows, consider sharing Problems 1, 2 and 3 with all students.

Problem 1

Students will extend their understanding of displaying data by learning about stem-and-leaf plots.

Provide students with the following hint if additional scaffolding is needed.

- **Hint:** In parts a and b, think about what kind of data sets are stem-and-leaf plots more manageable for: small sets or large sets.

a. *Responses vary.*
   - Unlike histograms, a stem-and-leaf plot retains the actual data values, allowing for a precise view of the data distribution.
   - It provides a clear visual of the data's distribution, making it easy to spot patterns like skewness, clustering, or gaps in the data.
   - Stem-and-leaf plots are relatively simple to create by hand, especially for small to moderate datasets.

b. *Responses vary.* They both display the distribution of data by showing individual data points. They both are useful for small data sets. A stem-and-leaf plot groups data using stems and focuses on the exact numerical values. A dot plot is simpler and visually represents each data point with a dot.

c. *Responses vary.* Between 50 and 80 cents here, or more specifically 70–79 cents.

d. 50, 75 and 80 cents.

e. *Responses vary.* I recommended a price between 70–90 cents per cookie.

Continued next page ...

Visualizing Data (continued)

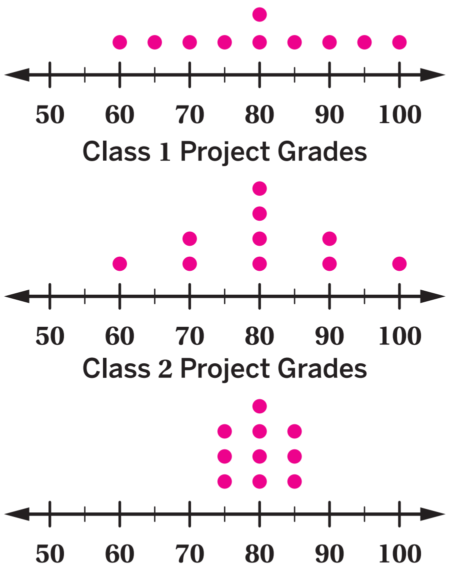
Problem 2

Students will extend their understanding of using dot plots to make claims about data by creating dot plots for three different data sets**.**

Provide students with the following hints if additional scaffolding is needed.

- **Hint 1:** *Mean* is the arithmetic average of all the scores in a data set. *Mode* is the most frequent score in a data set. When the values are in numerical order, the middle value of a data set is called the *median*.
- **Hint 2:** If you need additional information, search for information about symmetric data samples

Responses vary.



Class 1 Project Grades

Class 2 Project Grades

Class 3 Project Grades

Problem 3

Students will extend their understanding of using dot plots to make claims about data by describing the center, shape and the spread of the data.

Provide students with the following hints if additional scaffolding is needed.

- **Hint 1:** What are the most frequent values of the data set? What are the values at the center?
- **Hint 2:** What are the smallest and the highest measures of yards?

a. *Responses vary*. The dot plot has the data clustering around certain values. It appears to be roughly symmetric, with more frequent values toward the middle (26 to 33).

b. *Responses vary*. Here, 30 and 33 appear frequently, but I think the single value that best represents the center is approximately 30 inches. This is where most dots are.

c. *Responses vary*. The data spans from 23 inches to 35 inches, showing a range of 12 inches with no extreme outliers.

d. *Responses vary*. Using body parts for measurement isn't very reliable because everyone's body is different. The data shows that the distance from the nose to the thumb ranges from 23 to 35 inches, which means it changes a lot between people. Today, we need fixed units like 36 inches for a yard so that measurements are the same for everyone.

Measuring Data: Mean, Median, Mode, IQR, and Range

Name: .... Date: .... Period: ....

Student Choice Remember to show or explain your thinking.

1 Mai is a sixth-grader who competes in the 100 meter hurdles. In eight track meets during the season, she recorded the following times, to the nearest one-hundredth of a second.

18.11, 31.23, 17.99, 18.25, 17.50, 35.55, 17.44, 17.85

a Explain what information you can determine by comparing the mean and the median.

b Why might Mai and her coach be interested in both the mean and the median times?

2 Suppose there is an error in the following data set:

3, 6, 8, 11, 11, 13, 14, 14, 14, 14, 16, 18, 20, 20, 20, 22, 24, 32, 36, 51

a What was the error if, when corrected, the minimum is the only value in the five-number summary that changes?

b What was the error if, when corrected, the minimum is not the only value in the five-number summary that changes? What other values change, and how?

Measuring Data: Mean, Median, Mode, IQR, and Range (continued)

Name: .... Date: .... Period: ....

3 Here is a list of seven numbers $x, 22, 1, 1, 1, 5, 10$.

a What value would make the mode < median < mean?

b What value would make the mean median and mode all the same value?

Measuring Data: Mean, Median, Mode, IQR, and Range

Assign problems to students who want to extend their thinking.

Problems 1 and 2 can be solved in any order. Assign Problem 3 to students who have solved Problems 1 and 2. If time allows, consider sharing Problem 3 with all students.

Problem 1

Students will extend their understanding of comparing the mean and the median by examining the measures of center for a data set with two outliers.

Provide students with the following hint if additional scaffolding is needed.

- **Hint:** Compare the mean and the median scores. Which one is higher? What does it tell you about Mai's scores?

a. *Responses vary*. The mean is 21.74 seconds and the median is 18.05 seconds. The mean is significantly higher than the median which tells me that some of the times are much higher than 18.05. One possible explanation is that she performs faster than 18.05 seconds at her typical or standard pace, but in some races she stumbles and falls on a hurdle, making her finishing time significantly higher.

b. *Responses vary*. The mean gives a sense of her overall performance, but may be skewed by extreme times, such as the 31.23 and 35.55, which are unusually slow. She might have tripped on a hurdle. I think removing these extremes and then calculating the mean can be more meaningful. On the other hand, the median provides a better idea of her more consistent, typical performance, discounting outliers.

   For improving performance, they might look closely at the median, while also examining the slow times (which inflate the mean) to identify and address any specific problems that led to those slower races.

Continued next page ...

Measuring Data: Mean, Median, Mode, IQR, and Range (continued)

Problem 2

Students will extend their understanding of the five-number summary by determining one change in the data set and how that impacts the five-number summary.

Provide students with the following hint if additional scaffolding is needed.

- **Hint:** Determine the current five number summary. Replace the minimum value, 3 with a number between the minimum and Q1, then replace with a number larger than Q1. Observe what happens.

a. 6 or higher (but less than 11)
Explanations vary. The current five-number summary is:

minimum	Q1	median	Q2	maximum
3	12	15	21	51

Since the minimum is currently 3, if I replace 3 with a larger value that is still smaller than the current Q1 (11), I can keep the other values as is. For instance, if the 3 was supposed to be a 7 (or any value between 1 and 11 except 3), the five-number summary would be the same except the minimum value.

b. *Explanations vary*. This time, I need to replace 3 by a value bigger than 11. Let's say 15. This correction would change the lower end of the data distribution, and the new five-number summary would be:

minimum	Q1	median	Q2	maximum
6	13.5	15.5	21	51

Q1 changed from 12 to 13.5 because the distribution of lower values has shifted.

Problem 3

Students will extend their understanding of comparison properties by solving the pentagon puzzle.

Provide students with the following hint if additional scaffolding is needed.

- **Hint:** Mode is the most frequent score in a data set. For example, In the data set 1, 2, 3, 4, 5, 4, the mode is 4.

a. Responses vary. When $x = 23$, mode is 1, median is 5, and the mean is 9.

b. Because the mode of the set is 1, both the median and the mean must also be 1. The value of x that makes mean, median, and mode all equal to 1 is -33.

With $x =$ -33, the list becomes, -33, 1, 1, 1, 5, 10, 22.

- Mode: 1
- Median: 1 (since the fourth value is 1)
- Mean: $\frac{-33 + 22 + 1 + 1 + 1 + 5 + 10}{7} = 1$

Sampling

Name: .... Date: .... Period: ....

Student Choice Start with any problem. Remember to show or explain your thinking.

1

Political parties often use samples to poll people about important issues. One common method is to call people and ask their opinions. In most places, though, they are not allowed to call cell phones. Explain how this restriction might lead to inaccurate samples of the population.

2

Here is 6 by 6 grid showing the ages of kids in a youth camp.

12	10	15	9	16	21
10	17	12	13	16	16
14	15	10	12	18	19
17	20	15	8	15	12
12	12	16	21	15	14
10	12	12	16	13	10

a Roll two dice showing the row and column number to pick 8 random numbers to calculate their mean.

b Compare each of these measures of center and spread for the population and the sample. What do you notice?

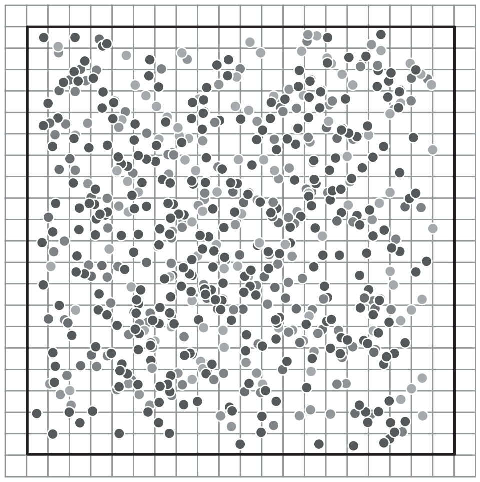
- Mean:
- Median:
- Mode:
- Range:
- IQR:

Sampling (continued)

Name: .... Date: .... Period: ....

3

How can you make an educational guess to determine the number of marbles in a 20 by 20 frame.



4

Here are the readings of a speed sensor in front of a school.

Sample data on the speeds of vehicles traveling

East (mph)	West (mph)
26, 30, 39, 34, 40, 21 25, 20, 24, 26, 22, 32	25, 35, 27, 34, 40, 22 20, 19, 18, 20, 19, 30

a Make a double box-and-whiskers diagram to represent the east and west end speeds (mph).

b Which measure would you use to convince the city council to add a stop sign to the east end? West end? Both?

Sampling

Assign problems to students who want to extend their thinking.

Problems 1–4 can be solved in any order. If time allows, consider sharing Problems 1–4 with all students.

Problem 1

Students will extend their understanding of the terms population and sample by critiquing a real-world scenario.

Provide students with the following hint if additional scaffolding is needed.

- **Hint:** Can all the people be reachable by their home phones?

Responses vary.

- Some people, especially younger people, may only have cell phones, so they will not be included in the sample. This may lead to more information being gathered from older people than younger people, and the information may not accurately represent everyone.
- If the polls are only calling home phones, they are only reaching individuals who work from home, do not work, or do not work during the day. This may lead to more information being gathered from this specific type of person, and may not accurately represent everyone.

Problem 2

Students will extend their understanding of sampling by comparing the center and spread measures of a random sample with the population.

Provide students with the following hints if additional scaffolding is needed.

- **Hint 1:** If the central measures of the entire grid and your sample are close, what does this tell you about the tendency of your sample?
- **Hint 2:** If the variability measures are close what does this tell you about the spread of your sample?

a. *Sample*

Responses vary.
I rolled for the sample 12, 10, 15, 10, 17, 21, 19, 12.

Mean: 14.5
Median: 13.5
Mode: 10, 12
Range: 11
IQR: 7

b. Entire Grid

Mean: ≈14.03
Median: 14
Mode: 12
Range: 13
IQR: 4

The mean of the sample, 14.5, is quite close to the mean of the full data set ≈14.03, suggesting that this sample is a reasonable representation of the average of the larger set. The range of the sample, 11, is slightly smaller than the range of the full set,13. This indicates that the sample doesn't fully capture the extreme values of the larger set but still represents a similar spread.

However, there are some minor differences in the median, mode, and IQR. The median and mode indicate a slight shift towards lower values in the sample, and the higher IQR suggests that there is a bit more variability in the sample compared to the middle values of the full set.

Continued next page ...

Problem 3

Students will extend their understanding of the purpose of sampling by exploring the methods of obtaining a representative sample.

Provide students with the following hint if additional scaffolding is needed.

- **Hint:** If you can insert a smaller square and count the number of marbles in it, how would you proceed to determine the total number of marbles? Where would you choose to insert the square?

Responses vary. Approximately 750 marbles.

Explanations vary. I drew a 4 by 4 square in the 20 by 20 grid and counted the number of marbles. I chose the square in a region that I thought best represents the entire grid. There were 30 marbles.

Then I determined the ratio of my square to the entire grid; $\frac{4 \times 4}{20 \times 20} = \frac{1}{25}$, then I determined the total number of marbles by $30 \cdot 25 = 750$.

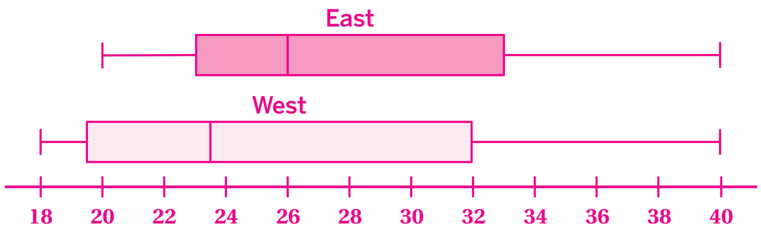
Problem 4

Students will extend their understanding of using measures of center and variability from random samples to draw conclusions by convincing the city council to add stop signs in front of a school.

Provide students with the following hint if additional scaffolding is needed.

- **Hint:** Determine the mean, mode, median, range, and IQR of both samples to compare.

a.



b. *Responses vary*.

- East End: I would use the mean speed of 28.25 miles per hour, which is higher than on the west side, and the presence of vehicles traveling at speeds up to 40 miles per hour.
- West End: I would use the range and IQR. Despite the lower mean and median, some vehicles are traveling at 40 miles per hour, and the IQR of 14 suggests more variability, with some drivers potentially posing a safety risk.
- Both Ends: I would use the extremes of 40 miles per hour in both directions, especially near a school zone. I argue for stop signs at both ends to ensure safer, slower traffic flow in both directions.

Unit 10

Extensions

Unit 10
Sub-Unit 1
Extensions

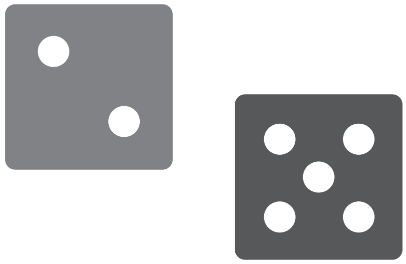
Probability

Name: .... Date: .... Period: ....

Student Choice Start with any problem. Remember to show or explain your thinking.

1 Two standard dice are rolled. What is the probability that the product of the numbers is

a odd? **b** even? **c** prime?

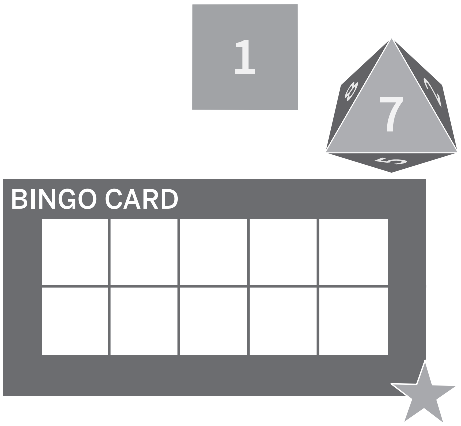


2 In a bingo game, each player will have a bingo card of 10 different numbers.

The caller rolls a cubic die (labeled 1–6) and an octahedron die (labeled 1–8), then finds the product to call out.

Players will cover the numbers on their bingo card as they are being called out. The one who covers all of the numbers first, wins.

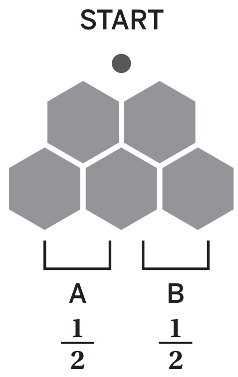
Pick the numbers on your bingo card to increase your chance of winning the game!



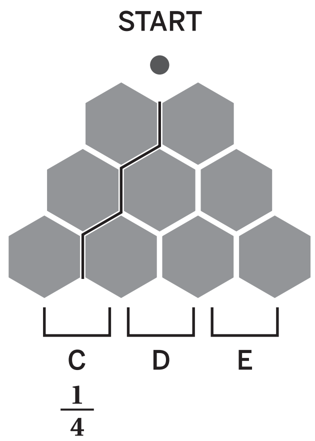
Probability (continued)

Name: .... Date: .... Period: ....

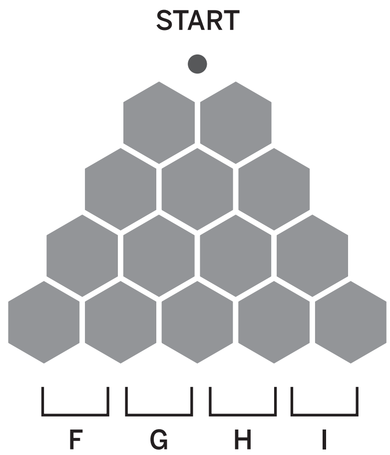
3 Here is a one layer example of a Galton board. A marble is rolled down from the start. It has a 50% chance of landing in bucket A and 50% chance of landing in bucket B.



a Another layer is added to the Galton Board. The probability of the marble ending in bucket C is $\frac{1}{4}$. What is the probability that the marble will land in bucket D? Bucket E?



b One more layer is added to the board. Determine the probabilities for marble to land in each bucket.



c How do the outcomes change as a new layer is added to the board? Can you estimate the outcomes in the next layer?

Probability

Assign problems to students who want to extend their thinking.

Problems can be solved in any order. If time allows, consider sharing Problems 1–3 with all students.

Materials
- Dice (optional) **(Problem 1)**

Problem 1

Students will extend their understanding of determining the probability of events by identifying the sample space of rolling two dice.

Provide students with the following hint if additional scaffolding is needed.

- **Hint:** In part c, a prime number is an integer greater than 1 that is only divisible by itself and 1.

a. $\frac{1}{4}$ **b.** $\frac{3}{4}$ **c.** $\frac{1}{6}$

Explanations vary.
I determined the sample space to calculate the probabilities.

×	⚀	⚁	⚂	⚃	⚄	⚅
⚀	1	2	3	4	5	6
⚁	2	4	6	8	10	12
⚂	3	6	9	12	15	18
⚃	4	8	12	16	20	24
⚄	5	10	15	20	25	30
⚅	6	12	18	24	30	36

Problem 2

Students will extend their understanding of determining the probability of events by identifying the sample space of rolling two dice.

Provide students with the following hint if additional scaffolding is needed.

- **Hint:** Determine the sample space to calculate which products have higher probability to be called out.

Responses vary. **I would choose 4, 6, 8, 12, 24 initially. Then for the rest of the numbers, I would choose among the set those which have the probability value of $\frac{1}{24}$ instead of $\frac{1}{48}$.**

Explanations vary.
I determined the sample space to calculate the probabilities. Then I chose the outcomes with the higher probabilities.

	1	2	3	4	5	6.	7	8
1	1	2	3	4	5	6	7	8
2	2	4	6	8	10	12	14	16
3	3	6	9	12	15	18	21	24
4	4	8	12	16	20	24	28	32
5	5	10	15	20	25	30	35	40
6.	6	12	18	24	30	36	42	48

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Problem 3

Students will extend their understanding of listing the sample space for a multi-step event by exploring the Galton board and normal distribution.

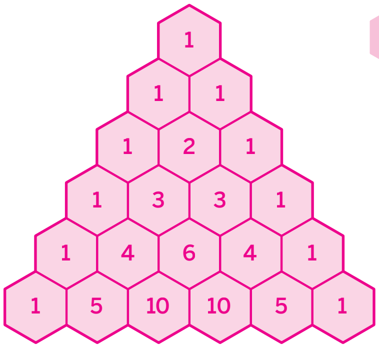
Provide students with the following hint if additional scaffolding is needed.

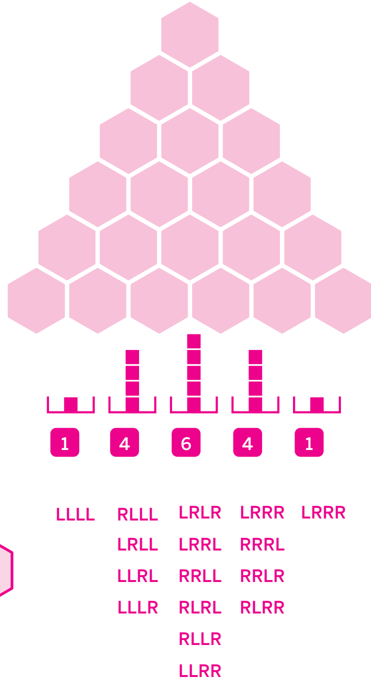
- **Hint:** How does the number of sample space increase every time a layer is added to the Galton board?

a. D: $\frac{2}{4}$ and E: $\frac{1}{4}$

b. F: $\frac{1}{8}$, G: $\frac{3}{8}$, H: $\frac{3}{8}$, and I: $\frac{1}{8}$

c. *Responses vary*. When a new layer is added, the total number of possible outcomes doubles.
Second layer: $2^2 = 4$ possible outcomes
Third layer: $2^3 = 8$ possible outcomes
The fourth layer will have $2^4 = 16$ possible outcomes
Because the distribution of the outcomes is determined according to the left or right path of the marble, the distribution of possibilities is the same for any two-outcome game, like flipping a coin.





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