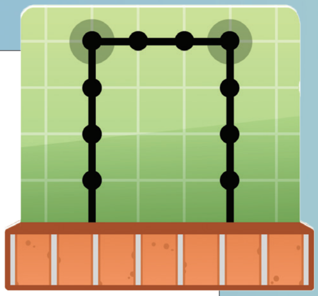


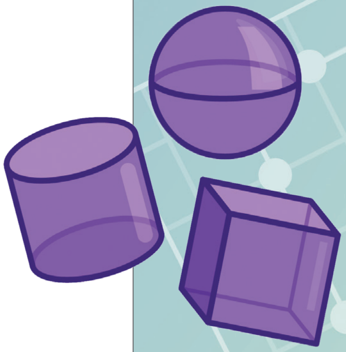
Unit 5

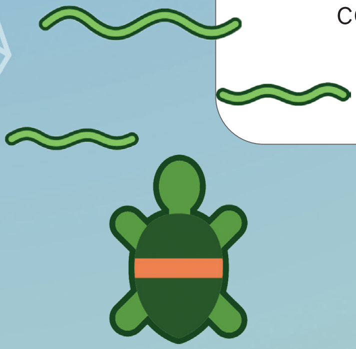
Functions and Volume

You will learn about functions for the first time. You will analyze representations of functions, and examine functions in the context of the volume of cylinders, cones, and spheres.

Essential Questions

- What makes a relationship a function?
- How are functions useful in representing situations?
- What are some relationships between a cylinder, a cone, and a sphere with common dimensions?

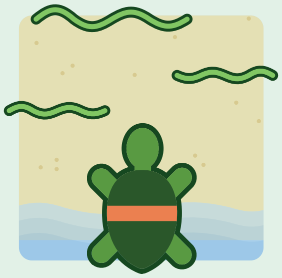




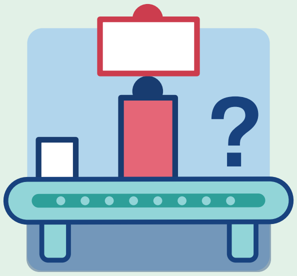
Sub-Unit

1

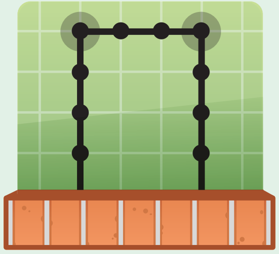
Introduction to Functions



Lesson 1
Turtle Crossing



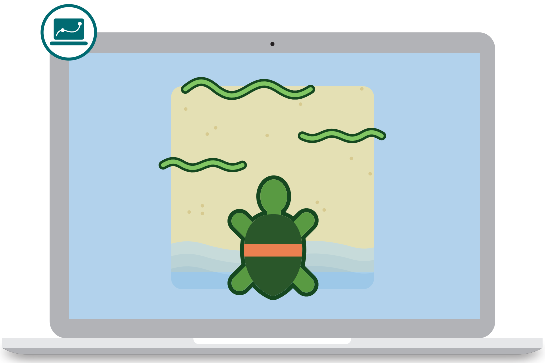
Lesson 2
Guess My Rule



Lesson 3
Function or Not?

Lesson 4
Dependence Day

Name: ____________ Date: ________ Period: ________

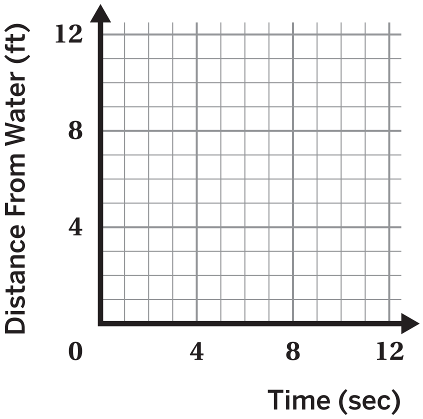


Turtle Crossing

Let's make sense of graphs.

Warm-Up

1 **a** Draw a distance vs. time graph to represent a turtle's journey across the sand.



b What story does your graph tell about the turtle's journey?

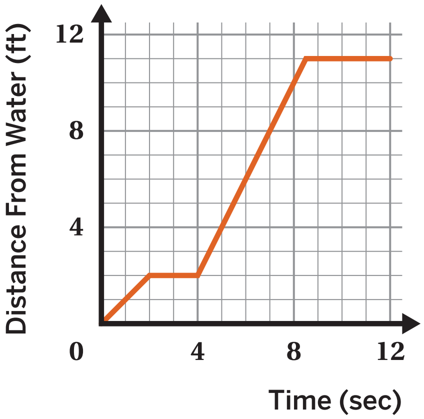
Activity
1

Name: .... Date: .... Period: ....

Turtle Graphs

2 Kris drew this graph to represent a turtle's journey.

What story does the graph tell about the turtle's journey?



3 Let's watch an animation to see what Kris's turtle did.

💬 **Discuss:** How does this animation compare to your story?

4 Use Kris's graph to answer each question.

a At 8 seconds, how far is the turtle from the water?

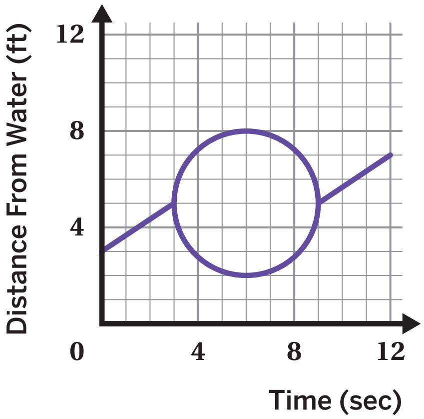
b When is the turtle 4 feet from the water?

Name: .... Date: .... Period: ....

Turtle Graphs (continued)

5 Arnav drew this graph to represent a new turtle.

What story does the graph tell about the turtle's journey?



6 Let's watch an animation to see what Arnav's turtle did.

💬 **Discuss:** How does this animation compare to your story?

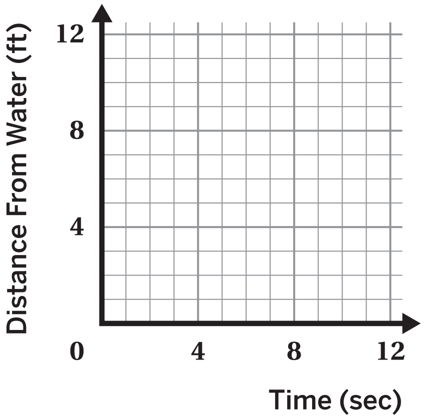
Name: .... Date: .... Period: ....

Dangerous Crossings

7 Let's watch the animation of another turtle crossing.

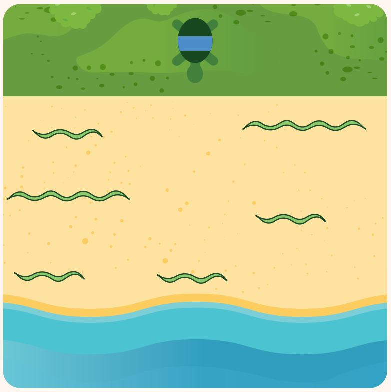
Discuss: What do you notice? What do you wonder?

8 Draw a graph of the turtle's distance from the water over time based on the animation from the previous problem.



Explore More

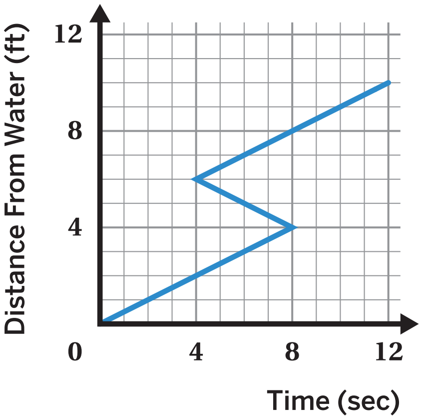
9 Use the digital activity to draw a new graph to help the turtle go from the grass to the water while avoiding the snakes.



10 Synthesis

Citlalli drew this graph to represent a new turtle.

Explain what story the graph is telling at 6 seconds. Does the turtle's journey seem realistic?



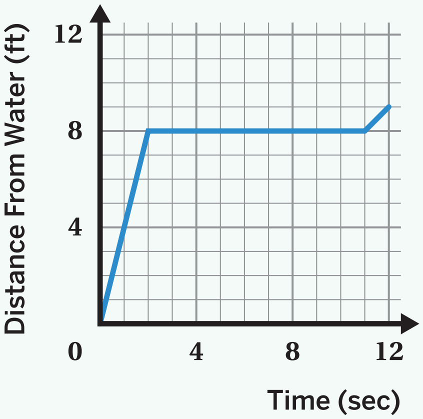
Lesson Practice 8.5.01

Lesson Summary

You can use a graph to represent a situation. Analyzing a point on a graph or pieces of a graph can help you interpret part of the situation.

For example, this graph represents a turtle's journey across sand. A turtle walks for 2 seconds until it is 8 feet from the water. It stops for 9 seconds and then continues walking away from the water.

The point $(6, 8)$ represents the turtle's distance of 8 feet from the water after 6 seconds.

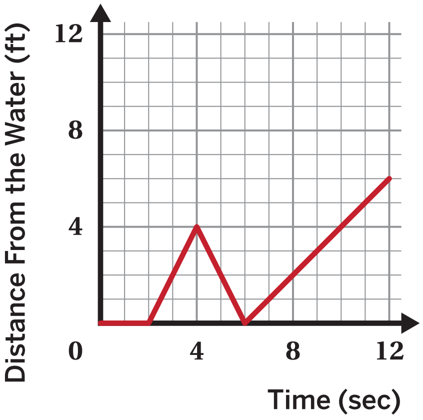


Lesson Practice

8.5.01

Name: Date: Period:

Problems 1–3: This graph represents a turtle walking across the sand.

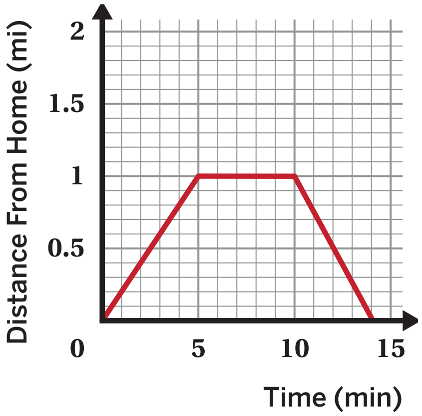


1. What story does the graph tell about the turtle's journey?

2. How far was the turtle from the water after 8 seconds?

3. After how many seconds is the turtle's distance 2 feet from the water?

Problems 4–6: This graph shows Maki's distance from home as time passes. Determine whether each statement is true or false.



4. Maki was 1 mile from home at 5 minutes.

5. Maki was 10 miles from home at 1 minute.

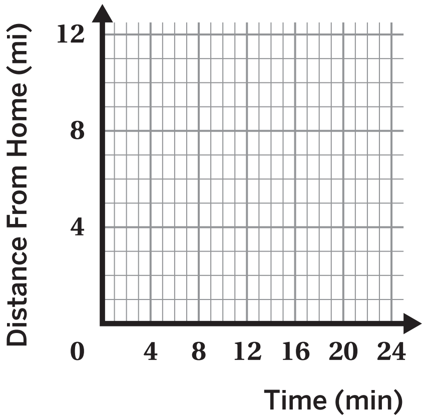
6. Maki's distance from home didn't change from 5 to 10 minutes.

Lesson Practice
8.5.01

Name: .... Date: .... Period: ....

7. Markayla's family went on vacation. First, they drove away from their house. After several minutes, they stopped to get gas. When Markayla's family left the gas station, they realized they forgot something and drove back to their house. After a few minutes, they drove away from their house again.

Sketch a graph that could represent Markayla's family's distance from home vs. time.



Spiral Review

8. Solve this system of linear equations. Show or explain your thinking.

$$\begin{cases} y = x - 4 \\ y = 6x - 10 \end{cases}$$

Reflection

1. Circle the problem you're most interested in knowing more about.

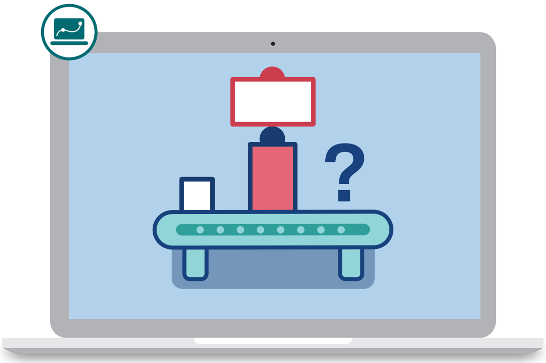
2. Use this space to ask a question or share something you're proud of.

Unit 5

Lesson

2

Name: ________ Date: ________ Period: ________



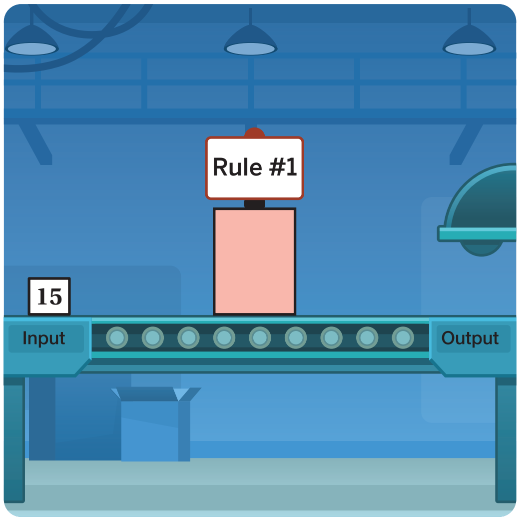
Guess My Rule

Let's explore rules to develop the concept of a function.

Warm-Up

1 This machine uses a secret rule, Rule #1, to turn inputs into outputs.

Rule #1 allows *all integers* as inputs.



a Let's watch this machine at work.

b What could Rule #1 be? Select *all* that apply.

- ☐ **A.** Divide by 2, then add 5.
- ☐ **B.** Divide by 3.
- ☐ **C.** Take the ones digit.
- ☐ **D.** Multiply by 3.
- ☐ **E.** Subtract 10.

2 Let's enter a different integer to help you decide what Rule #1 is.

What could Rule #1 be now? Select *all* that apply.

- ☐ **A.** Divide by 2, then add 5.
- ☐ **B.** Divide by 3.
- ☐ **C.** Take the ones digit.
- ☐ **D.** Multiply by 3.
- ☐ **E.** Subtract 10.

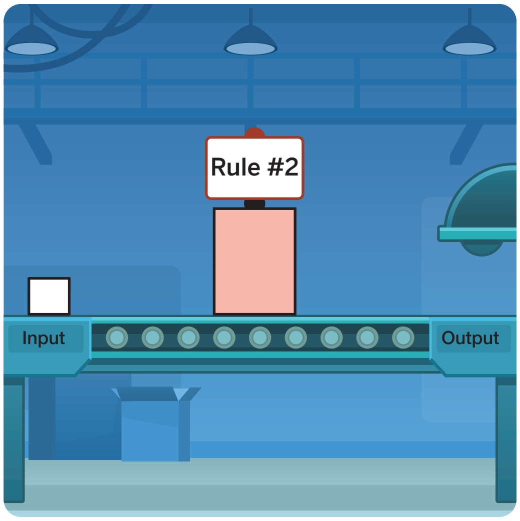
Name: .... Date: .... Period: ....

Guess My Rule

3 This machine uses a new rule called Rule #2.

Rule #2 allows *all numbers* as inputs.

Let's test several inputs to see how Rule #2 works.



4 Here are some inputs and outputs from other students.

If you input 6 into Rule #2, what do you think the output will be?

Explain your thinking.

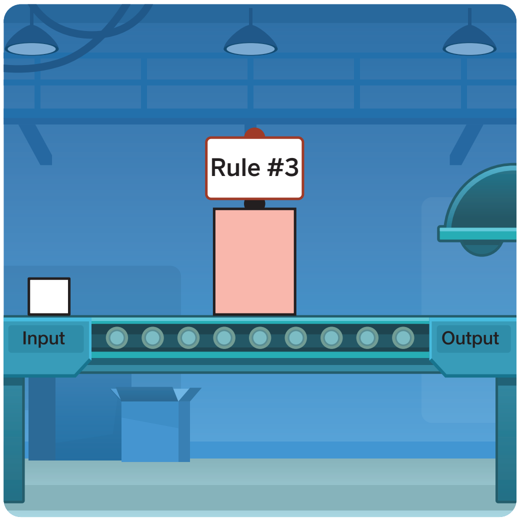
Rule #2

Input	Output
-13	7
-61	7
81	7
-100	7
60	7

5 Rule #3 allows *single words* as inputs.

a Let's test several inputs to see how Rule #3 works.

b **Discuss:** What do you think the rule might be?



Name: ____________________ Date: __________ Period: __________

Guess My Rule (continued)

6 Here are some inputs and outputs from other students.

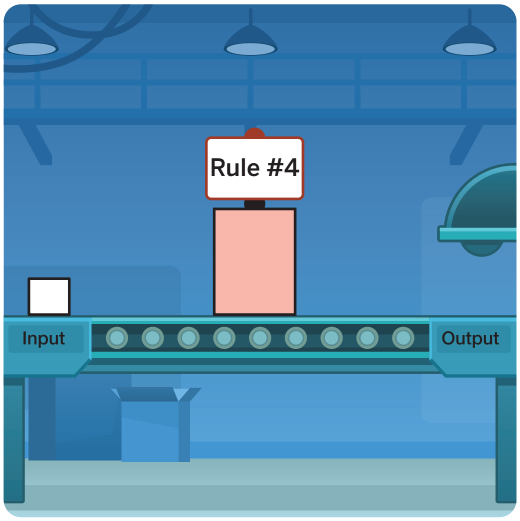
Test your understanding of Rule #3 by completing the table.

Rule #3

Input	Output
mint	u
hen	o
clear	s
friend	
wallet	
friend	
party	

7 Rule #4 allows *single letters* (like "A") as inputs.

Let's test several inputs to see how Rule #4 works.



8 Here are some inputs and outputs from other students.

What do you think the output will be if you input "A" into Rule #4? Explain your thinking.

Then compare your response with your classmates' responses.

Input	Output
W	William
A	Anand
A	Adam

Activity
2

Name: ______ Date: ______ Period: ______

What Is a Function?

9 Rules #1, #2, and #3 all represent **functions**.

Rule #4 does *not* represent a function.

a What do you think makes a rule a function?

b Compare your response with a classmate's. Then revise your response to make it stronger and clearer.

Rule #1: Function

Input	Output
35	25
723	713
-4	-14
53	43
723	713

Rule #2: Function

Input	Output
15	7
18	7
262	7
-3	7
82.3	7

Rule #3: Function

Input	Output
hi	J
my	Z
name	F
is	T
Arturo	P

Rule #4: Not a Function

Input	Output
H	Hailey
J	Jada
M	Mai
H	Hamza
M	Madison

10 Determine whether each table represents a function.

a

Button Selected (Input)	Drink Received (Output)
A	Water
B	Seltzer
C	Juice
D	Water

b

Money Spent (Input)	Number of Items (Output)
$1	2
$8	12
$7	1
$1	3

c

Height in Feet (Input)	Height in Inches (Output)
5	60
4.5	54
6	72
5.4	64.8

d

Time in Seconds (Input)	Musical Note (Output)
1.5	D
1.25	D
1.5	E
2	D

11 This table does *not* represent a function.

a Change the fewest numbers so the table could represent a function.

b Explain your thinking.

Input	Output
1	5
2	10
3	15
2	20
1	24

12 Synthesis

How can you determine whether a table could represent a function?

Use the examples if they help with your thinking.

Rule #1: Function

Input	Output
35	25
723	713
-4	-14
53	43
723	713

Rule #2: Function

Input	Output
15	7
18	7
262	7
-3	7
82.3	7

Rule #3: Function

Input	Output
hi	J
my	Z
name	F
is	T
Arturo	P

Rule #4: Not a Function

Input	Output
H	Hailey
J	Jada
M	Mai
H	Hamza
M	Madison

Lesson Practice 8.5.02

Lesson Summary

A function is a rule that assigns exactly one output for each possible input. Another way to say this is that the output is a function of the input.

Function

Input	Output
15	7
10	7
20	8
5	9

In this function table, each input appears with exactly one output. Even if an input is repeated in the table, the input should give the same output as previously seen within the table.

Not a function

Input	Output
10	6
10	7
20	8
5	9

Notice in this table, the input of 10 appears twice with two different outputs.

Lesson Practice

8.5.02

Name: .... Date: .... Period: ....

Problems 1–2: Complete each table based on its rule.

1. Rule: Divide by 4 and then add 2.

Input	Output
0	
2	
4	
6	
8	

2. Rule: If odd, write 1. If even, write 0.

Input	Output
1	
2	
3	
7	
12	

Problems 3–4: Determine whether each table could represent a function. Explain your thinking.

3.

Input	Output
4	-2
1	-1
0	0
1	1
4	2

4.

Input	Output
-2	4
-1	1
0	0
1	1
2	4

5. Which of the following tables could represent a function?

A.

Input	Output
1	0
2	5
3	2.5
4	5
5	8

B.

Input	Output
0	1
5	2
2.5	3
5	4
8	5

C.

Input	Output
3	-8
3	-2
3	-1
3	6
3	12

D.

Input	Output
-1	0
-2	8
2	2
-1	-1
-2	9

Lesson Practice

8.5.02

Name: .... Date: .... Period: ....

6. Complete the table so that it could represent a function.

Input	Output
-2	0
0	
	10
4	

7. Many people consider mathematician Ada Lovelace to be the world's first computer programmer. Lovelace used functions to write programs for an early computing machine. Certain inputs (such as pressing a certain key on a keyboard) caused certain outputs.

   Here's a table that shows how pressing different keys causes different movements in a video game character. Could this table represent a function? Explain your thinking.

Key Pressed	Movement
Right arrow	Walk right
Left arrow	Walk left
Up arrow	Walk right
Down arrow	Jump

Spiral Review

Problems 8–10: Determine whether each ordered pair is a solution for the equation $2x + 4y = 16$.

8. $(1, 3)$

9. $(6, 1)$

10. $(0, 4)$

Reflection

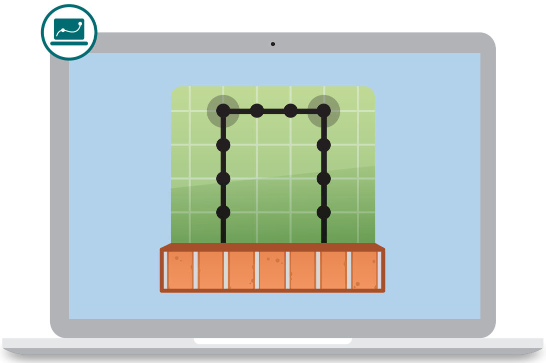
1. Put a star next to the problem you understood best.
2. Use this space to ask a question or share something you're proud of.

Unit 5

Lesson

3

Name: .... Date: .... Period: ....



Function or Not?

Let's determine whether a graph represents a function.

Warm-Up

1 We learned that a function is a rule that assigns exactly one output to each possible input.

That means each input determines a single output.

Complete the table so that y is *not* a function of x.

x	y
3	
5	4
	6

Name: Date: Period:

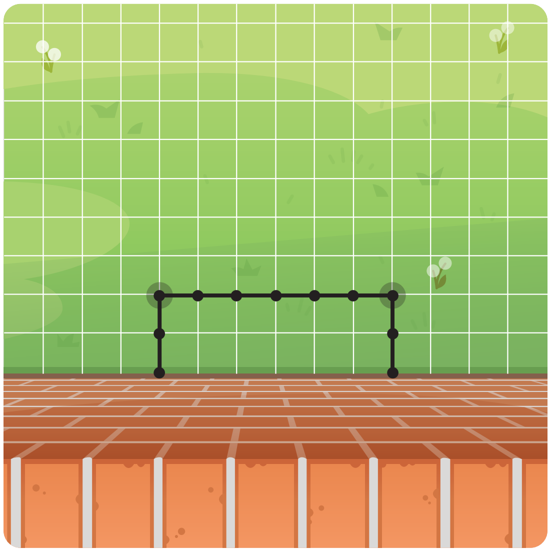
Rectangular Pen

2 Emma is hiring builders to create a rectangular pen, a closed space where animals can be kept. Emma wants to build three sides of fencing against a brick wall.

Let's observe the relationship between the area and the amount of fencing.

Enter an amount of fencing and the area for a pen that you've observed.

Amount of Fencing (m)	Area (sq. m)



3 Here is a table that shows the amount of fencing and the area for several pens.

Emma gave the builders 12 meters of fencing to build the pen.

Is it possible for the builders to determine the area of Emma's pen?

Yes No I'm not sure

Explain your thinking.

Amount of Fencing (m)	Area (sq. m)
8	8
12	10
14	24
12	16
16	24
20	32

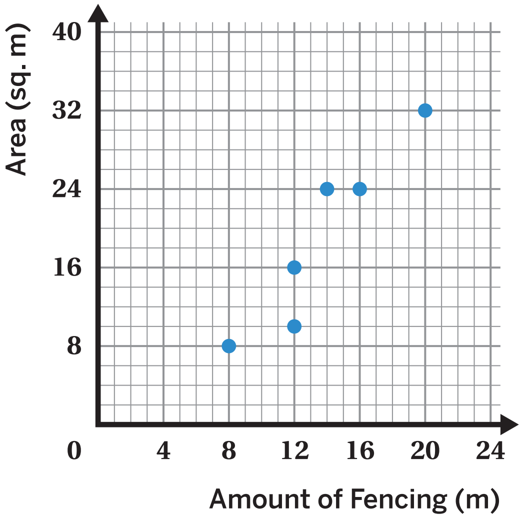
Name: ______________________ Date: __________ Period: __________

Rectangular Pen (continued)

4 Here is a graph that shows the relationship between area and amount of fencing for several pens.

How can you use the graph to quickly determine that area is not a function of the amount of fencing?

Area is not a function of the amount of fencing because . . .



Activity
2

Name: .... Date: .... Period: ....

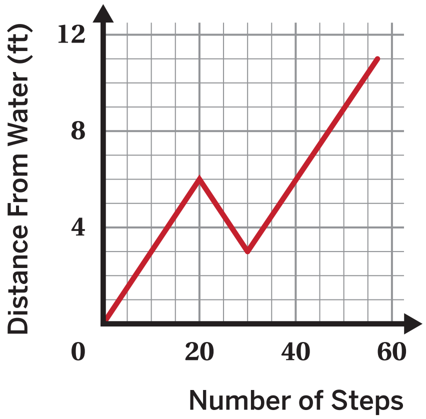
Turtle Crossing

5 Let's watch an animation of a turtle's journey across the sand.

Write the turtle's number of steps and distance from the water for one point in time.

Number of Steps	Distance from Water (ft)

6 How can you use this graph to decide whether distance from the water is a function of the number of steps?

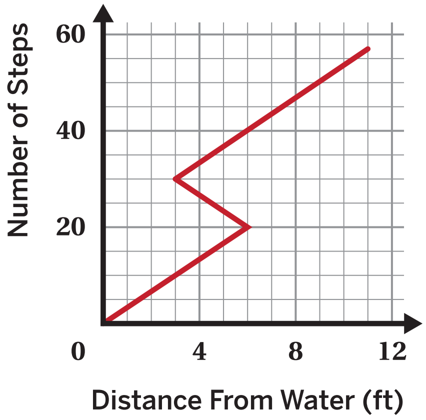


7 Here is the relationship in reverse.

Is the number of steps a function of distance? Circle one.

Yes No

Explain your thinking.

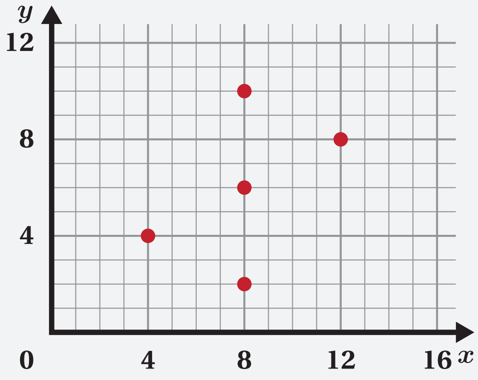


Name: ______________________ Date: __________ Period: __________

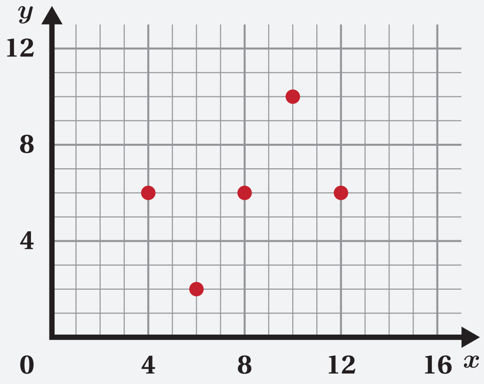
Turtle Crossing (continued)

8 Determine whether each graph represents a function or not.

Graph A

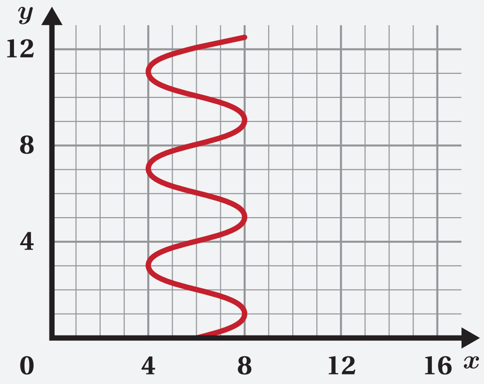


Graph B



Graph C

Graph D



Function	Not a function

Explore More

9 **a** Use the digital activity to produce a graph that is surprising in some way.

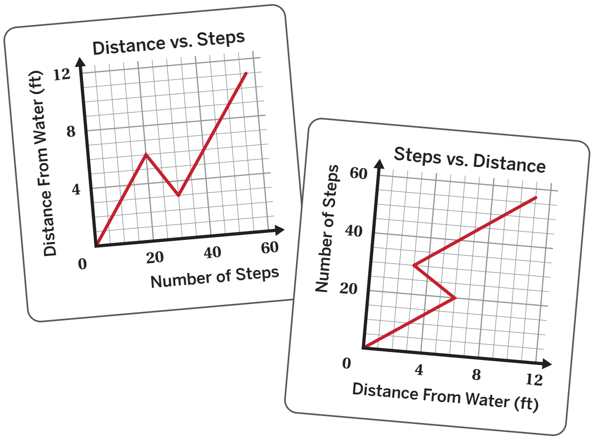
b **Discuss:**

- Does this graph represent a function?
- What does the surprising part of this graph represent in context?

10 Synthesis

How can you determine whether a graph represents a function?

Use the examples if it helps with your thinking.

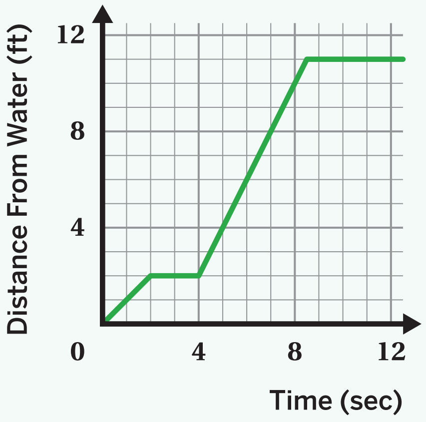


Lesson Practice 8.5.03

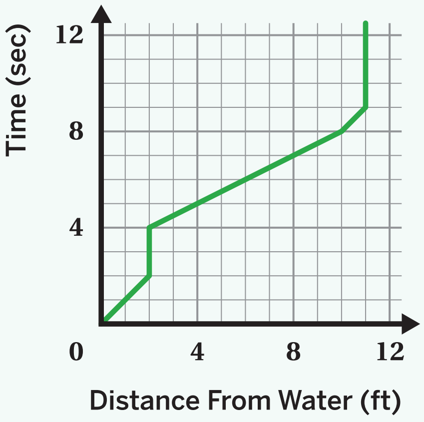
Lesson Summary

A graph represents a function when each x-value, or input, only has one corresponding y-value, or output. If a graph has multiple y-values for the same x-value, it does not represent a function.

Here are two graphs of the same turtle's journey.



This graph represents a function because for every second, x, the turtle is at only one corresponding distance, y.



This graph does not represent a function because at both 2 feet and 11 feet, the turtle has multiple corresponding times.

Lesson Practice
8.5.03

Name: ________________ Date: ________ Period: ________

Problems 1–3: A group of students are timed while sprinting 100 meters.

Time (seconds)	Speed (meters per second)
13.8	7.246
15.9	6.289
16.3	6.135
17.1	5.848
18.2	5.495

1. Is speed a function of time? Explain your thinking.

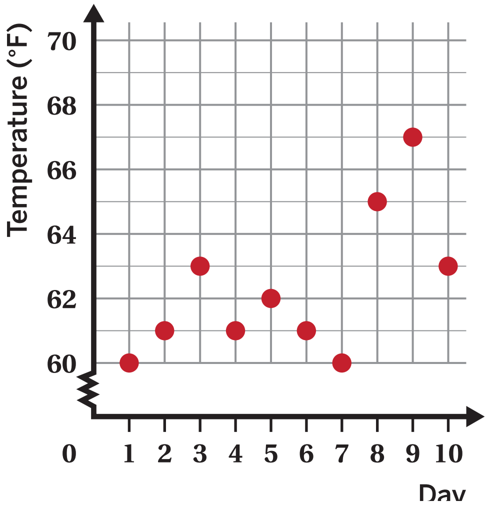
2. Is distance a function of time? Explain your thinking.

Time (seconds)	Distance (meters)
13.8	100
15.9	100
16.3	100
17.1	100
18.2	100

3. Is time a function of distance? Explain your thinking.

Distance (meters)	Time (seconds)
100	13.8
100	15.9
100	16.3
100	17.1
100	18.2

4. This graph represents a city's high temperatures over a 10-day period. Determine whether this statement is true or false: *The high temperature is a function of the day*. Explain your thinking.



Lesson Practice
8.5.03

Name: .... Date: .... Period: ....

Problems 5–6: Determine whether each table could represent a function. Explain your thinking.

5.

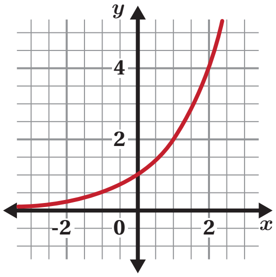
Input	Output
1	0
2	0
3	0
4	0
5	0

6.

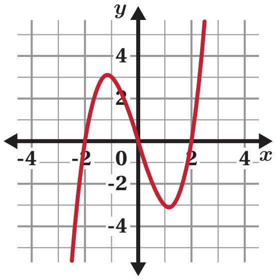
Input	Output
0	1
0	2
0	3
0	4
0	5

7. Which graph does *not* represent y as a function of x?

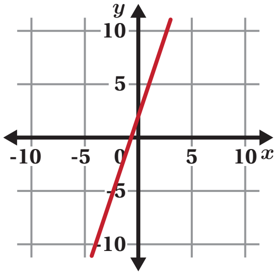
A.



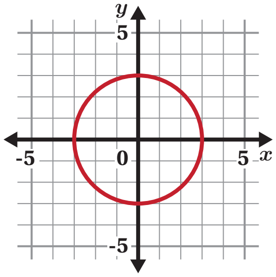
B.



C.



D.



Spiral Review

Problems 8–10: Determine whether each system of equations has one solution, no solution, or infinitely many solutions.

8. $\begin{cases} y = x + 4 \\ y = x + 4 \end{cases}$

9. $\begin{cases} y = -\frac{4}{5}x + 7 \\ y = \frac{4}{5}x - 2 \end{cases}$

10. $\begin{cases} y = 2x + \frac{1}{5} \\ y = 2x + 42 \end{cases}$

Reflection

1. Put a question mark next to a problem you're feeling unsure of.
2. Use this space to ask a question or share something you're proud of.

Unit 5
Lesson
4

Name: .... Date: .... Period: ....

Dependence Day

Let's explore the relationships between variables of functions.

Warm-Up

The Metropolis Events Committee is in charge of planning events for the city.

1. What might they need to consider as part of the planning process?

2. The Events Committee wants to hire a taco truck to provide food for an event. The taco truck charges $\$150$ for set-up and $\$10$ for each person.

   How much would the taco truck cost if 80 people attend the event? Show or explain your thinking.

Activity

1

Name: ______ Date: ______ Period: ______

Party Planning

3. **Situation A:** The Events Committee is thinking about hiring the taco truck again. The taco truck always charges $\$150$ for set-up and $\$10$ for each person.

a Complete the table to show the cost of the taco truck for different numbers of attendees.

Number of Attendees	Food Cost ($)
80	
120	
180	
200	

b Write an equation to calculate the food cost, f, of *any* number of attendees, a.

c Ari thinks the relationship between food cost and attendees represents a function because each input has only one possible output.

Do you agree with Ari's claim? Explain your thinking.

Name: .... Date: .... Period: ....

Party Planning (continued)

4. **Situation B:** The Events Committee has a food budget, which is the maximum food cost, for each event.

a The budget for the next event is $\$1{,}000$. What is the maximum number of people they can feed?

b Complete the table to show the number of possible attendees for other budgets.

Food Cost ($)	Number of Attendees
1,000	
1,200	
1,500	
2,000	

c Describe in words how to determine the number of attendees for any food cost.

d Write an equation to calculate the number of attendees, a, based on the budget for the food cost, f.

e Does the relationship between food cost and attendees represent a function? Explain your thinking.

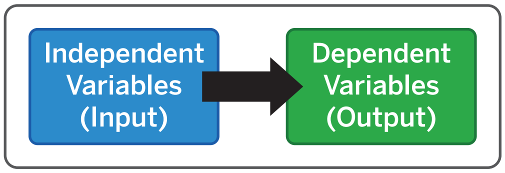
Activity
2

Name: ______________________ Date: __________ Period: __________

Independent and Dependent Variables

In some relationships between two variables, one is called the **independent variable** and the other is called the **dependent variable**.

The independent variable is an input. The dependent variable, or output, depends on the input.



5. Situation A: The Events Committee is using the equation $f = 10 \cdot a + 150$ to calculate possible food costs for an event with a attendees. Explain why it would make sense to call the food cost, f, the dependent variable.

6. Situation B: The Events Committee has raised money for the food cost for their next event and wants to know how many attendees they can host.

a In this situation, what do you think they should use as the independent and dependent variable?

Independent variable: ______________________

Dependent variable: ______________________

Explain your thinking.

b What equation might they use? Circle one.

$a = \dfrac{f - 150}{10}$ $\qquad$ $f = 10 \cdot a + 150$

c **Discuss:** How did you decide which equation they might use?

Name: ______ Date: ______ Period: ______

Winter Carnival

The Events Committee is planning a Winter Carnival. At the carnival, attendees will get tickets that they can exchange to play games. Each game uses 3 tickets.

7. Use the bank to fill in the table. Not all variables and equations will be used and some may be used twice.

Number of Tickets, t	Ticket Cost, c		Number of Games, g
$c = 3 + t$	$t = 3g$	$g = \frac{t}{3}$	$t = c - 3$

	How many games can you play with 28 tickets?	How many tickets should you purchase if you want to play 9 games?
Independent Variable		
Dependent Variable		
Equation		

8. 💬 **Discuss:** How did you choose which variables were independent and dependent?

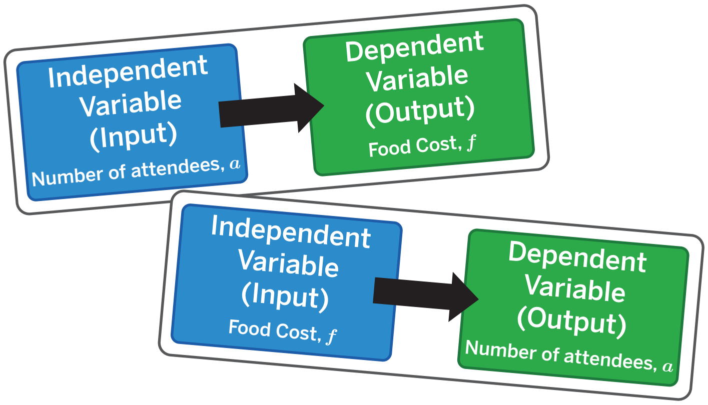
Explore More

9. Create your own situation where the dependent variable is the amount of space needed to hold an event.

Synthesis

10. How can you determine which variable is independent in a situation? Which is dependent?

Use the example if it helps with your thinking.



Lesson Practice 8.5.04

Lesson Summary

In a situation represented by a function, the input is often called the **independent variable** and the output is called the **dependent variable**. The independent variable and dependent variable can switch depending on the problem you are trying to solve.

The independent variable is an input. The dependent variable, or output, depends on the input.

For example, in this situation, m represents the total number of miles walked and d represents the number of days of walking for someone who walks 2 miles a day.

Question	Independent and Dependent Variable	Equation	Explanation
How many miles have I walked, m, after d days?	Independent: Days, d Dependent: Miles, m	$m = 2d$	The number of miles walked depends on the number of days of walking.
How many days, d, will it take me to walk m miles?	Independent: Miles, m Dependent: Days, d	$d = \frac{m}{2}$	The number of days depends on the number of miles.

Lesson Practice

8.5.04

Name: ______________________ Date: __________ Period: __________

Problems 1–2: Write an equation that expresses the output as a function of the input. Then determine the independent and dependent variables.

1. The perimeter, p, of a square with side length s:

Equation: ______________________

Independent variable: ______________________

Dependent variable: ______________________

2. The total cost, c, after a sales tax of 7% is applied to the cost of a purchase, p.

Equation: ______________________

Independent variable: ______________________

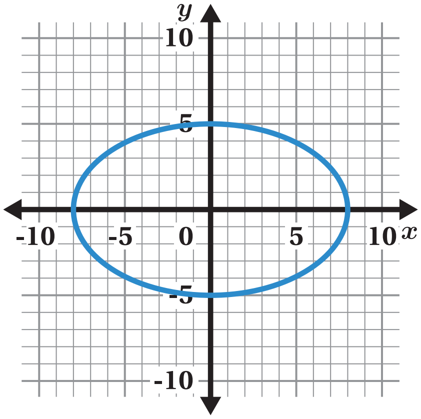
Dependent variable: ______________________

3. Which of these could represent a function? Select *all* that apply.

☐ **A.** $y = \frac{2}{3}x - 5$

☐ **B.** $x = 4$

☐ **C.**



☐ **D.**

x	y
-1	7
-3	7
-2	7
-1	7

Lesson Practice

8.5.04

Name: .... Date: .... Period: ....

Problems 4–7: Rafael earns $\$10.50$ per hour helping his neighbor with chores.

4. Is the amount he earns a function of the number of hours he works? Explain your thinking.

5. Is the number of hours he works a function of the amount he earns? Explain your thinking.

6. Write an equation that describes the situation. Use x to represent the independent variable and y to represent the dependent variable.

7. How much will Rafael earn if he works 3 hours each weekday next week? Show or explain your thinking.

Spiral Review

Problems 8–9: Solve each system of equations. Show your thinking.

8. $\begin{cases} y = 7x + 10 \\ y = \text{-}4x - 23 \end{cases}$

9. $\begin{cases} y = 3x - 6 \\ y = \text{-}2x - 1 \end{cases}$

Reflection

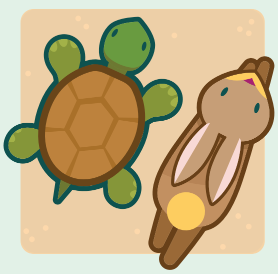
1. Put a heart next to the problem you feel most confident about.

2. Use this space to ask a question or share something you're proud of.

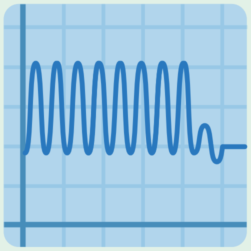
Sub-Unit

2

Representing and Interpreting Functions

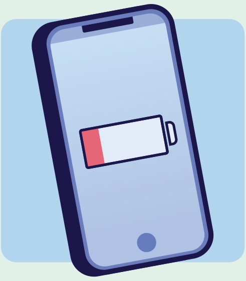


Lesson 5
The Tortoise and the Hare

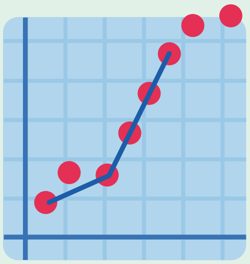


Lesson 6
Graphing Stories

Lesson 7
Comparing Linear Functions



Lesson 8
Charge!



Lesson 9
Piecing It Together

Unit 5

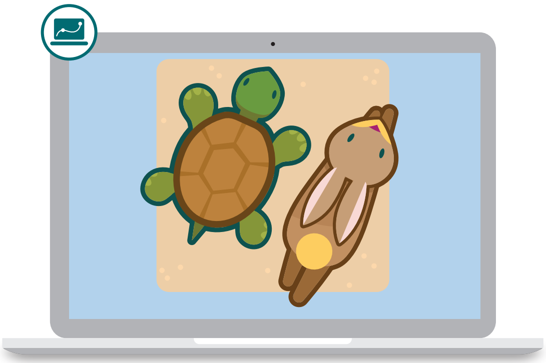
Lesson

5

Name: Date: Period:

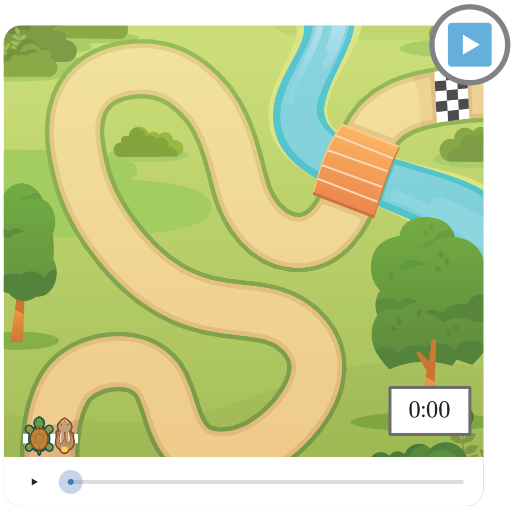
The Tortoise and the Hare

Let's interpret the graph of a function in context.



Warm-Up

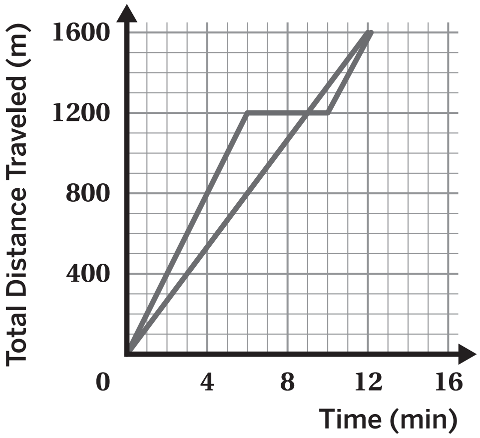
1 Let's watch an animation of the tortoise and the hare. Then tell a story based on what you see.



2 This graph shows two functions representing the relationship between distance and time: one for the tortoise and one for the hare.

a Which animal does each function represent? Label the graph with "Tortoise" and "Hare."

b Explain your thinking.



Activity
1

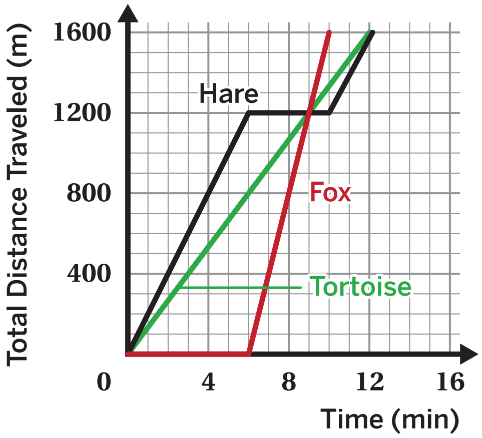
Name: .... Date: .... Period: ....

The Tortoise, the Hare, and the Fox

3 This graph shows the relationship between distance and time for a third animal, the fox.

Tell a story about the fox's journey during the race.

Include specific details about time and distance.



4 Let's watch an animation of the race.

Discuss: How does your story compare to the actual race?

5 Here are five statements about the race. Select *all* the true statements.

☐ **A.** The fox's distance was always increasing.

☐ **B.** Between 6 and 10 minutes, the fox was traveling faster than the other animals.

☐ **C.** When the hare reached 800 meters, the fox was still at the starting line.

☐ **D.** The graph of the tortoise represents a function, but the other two graphs do not represent functions.

☐ **E.** All three graphs represent functions.

Activity
2

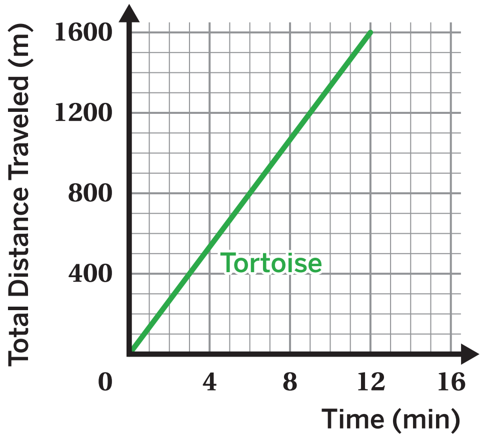
Name: .... Date: .... Period: ....

The Tortoise and the Dog

6 Next, the tortoise raced a dog.

Draw a graph representing the relationship between distance and time for a dog that makes *all* of these statements true:

- The dog got a head start but lost the race.
- The dog and tortoise were tied at 800 meters.
- The dog's distance was decreasing for 3 minutes.



7 Let's watch an animation of the race described in Problem 6 and a graph that was made to represent it.

Revise your graph for Problem 6, as needed.

8 The blue graph shows what Ash drew to represent the dog's race.

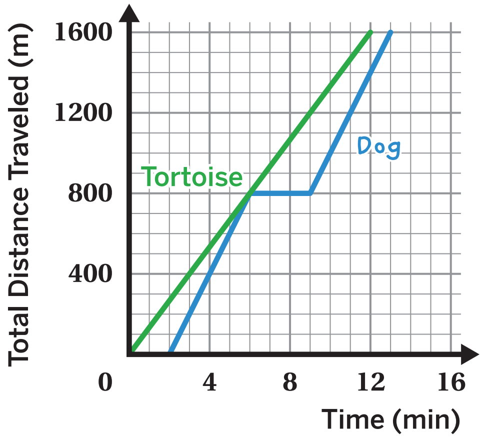
At least one of the following statements is false. Circle a false statement.

A. The dog got a head start but lost the race.

B. The dog and tortoise were tied at 800 meters.

C. The dog's distance was decreasing for 3 minutes.

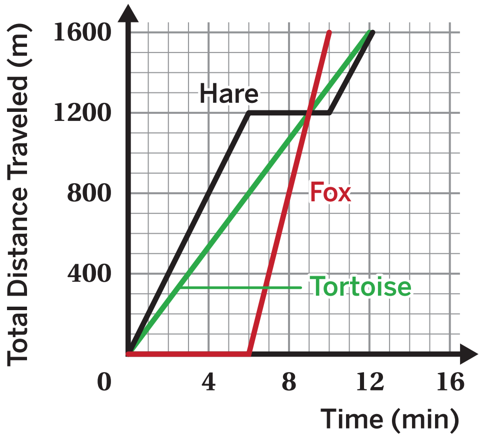
Explain your thinking.



9 Synthesis

What can a function's graph tell us about a situation?

Use the example if it helps with your thinking.

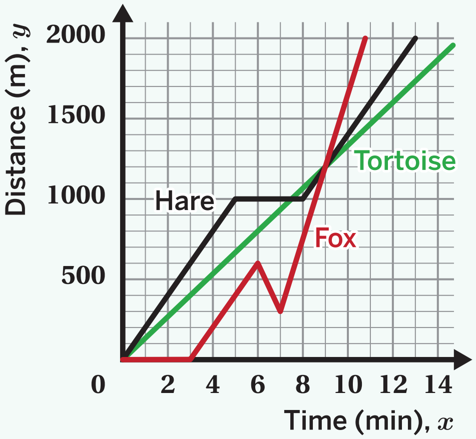


Lesson Practice 8.5.05

Lesson Summary

A graph can be helpful when comparing multiple functions in a situation, such as by comparing the initial value, slope, and points of intersection.

For example, this graph represents a race between a hare, a tortoise, and a fox. From 0 to 5 minutes, the hare is moving at a steady pace of 200 meters per minute and is in first place. At 9 minutes, the race is tied. The fox does not begin the race until three minutes have passed, but it speeds up at 7 minutes to a pace of 450 meters per minute. The fox wins the race at about 11 minutes.

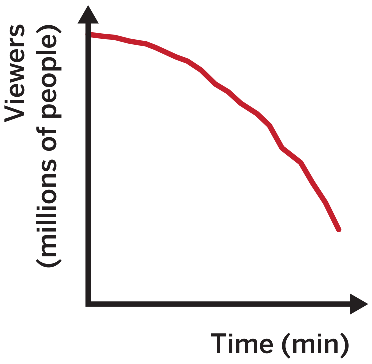


Lesson Practice
8.5.05

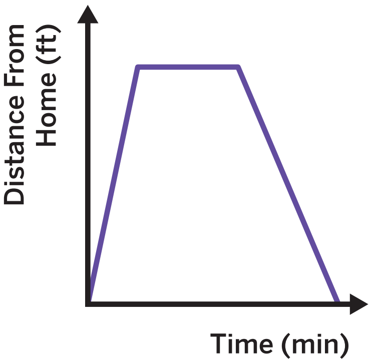
Name: .... Date: .... Period: ....

Problems 1–2: For each graph, write a story that the graph tells about each situation.

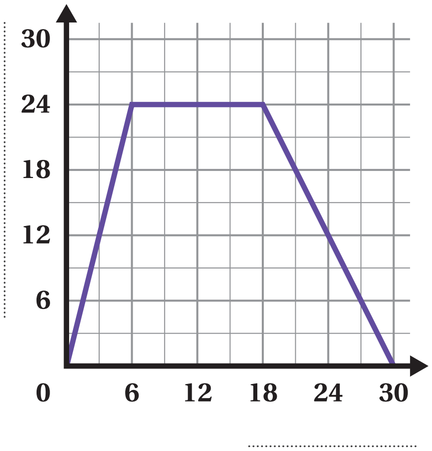
1. The relationship between number of viewers of a short video and time.



2. The distance of a cat from its home as a function of time.



Problems 3–8: Charlie filled up the tub and gave the family dog a bath. Then Charlie let the water in the tub drain. The graph shows the amount of water in the tub, in gallons, as a function of time, in minutes.



3. Label the axes.

4. When did Charlie turn off the water faucet?

5. How much water was in the tub when Charlie bathed the dog?

6. How long did it take the tub to drain completely?

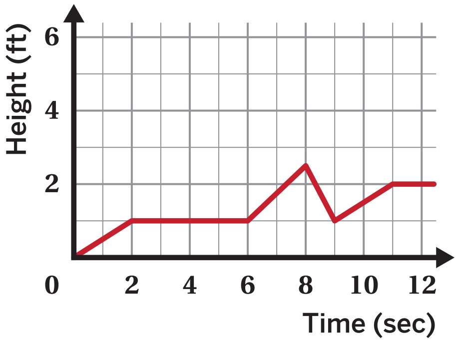
7. At what rate did the faucet fill the tub?

8. At what rate did the water drain from the tub?

Lesson Practice
8.5.05

Name: .... Date: .... Period: ....

9. This graph shows the height, in feet, that a butterfly is flying above the ground over time. At what interval is the butterfly's rate of change 0 feet per second? Select *all* that apply.



☐ **A.** Between 0 and 2 seconds

☐ **B.** Between 2 and 6 seconds

☐ **C.** Between 6 and 8 seconds

☐ **D.** Between 9 and 11 seconds

☐ **E.** Between 11 and 12 seconds

Spiral Review

10. A car is traveling at a speed of either 55 miles per hour or 35 miles per hour, depending on the speed limits, until it reaches its destination 200 miles away. Let x represent the amount of time, in hours, that the car is traveling at 55 miles per hour. Let y represent the amount of time, in hours, that the car is traveling at 35 miles per hour. The equation $55x + 35y = 200$ represents this situation.

If the car spends 2.5 hours traveling at 35 miles per hour on the trip, how long does it spend traveling at 55 miles per hour? Show or explain your thinking.

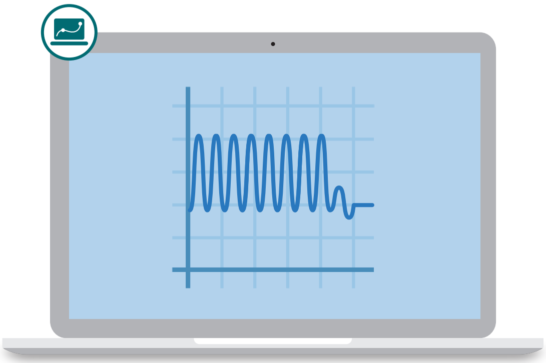
Reflection

1. Circle a problem you want to talk to a classmate about.

2. Use this space to ask a question or share something you're proud of.

Unit 5
Lesson
6

Name: Date: Period:



Graphing Stories

Let’s make connections between scenarios and the graphs that represent them.

Warm-Up

1 Clem loves to play on the playground. Let’s watch a short video of Clem on the swings. What different quantities are changing in this video?

2 Let’s look at two possible graphs to represent this situation.

💬 **Discuss:** How are these graphs similar? How are they different?

Activity

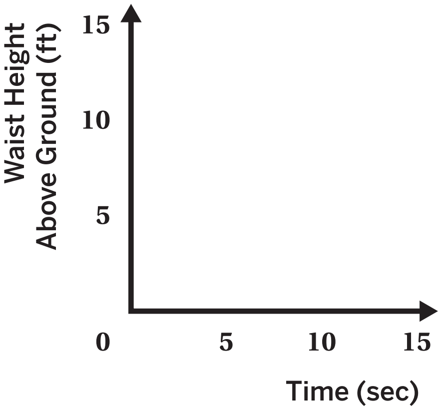
1

Name: ______ Date: ______ Period: ______

Tyler on the Slide

3 Let's watch an animation of Tyler on the slide.

Sketch a graph representing the relationship between Tyler's waist height and time.

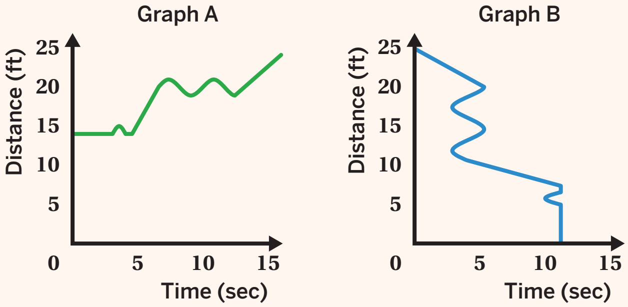


4 Let's watch an animation of a graph representing the relationship between Tyler's waist height above the ground and time.

Discuss: How does this graph represent the situation? How might you revise this graph?

Explore More

5 Use the digital activity to watch the video of Tyler again. Which graph could represent the relationship between Tyler's distance from the right edge of the screen and time?



Explain your thinking.

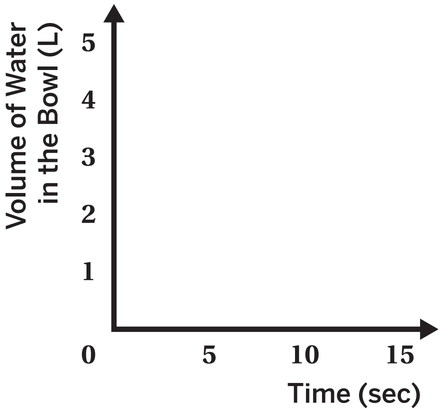
Activity
2

Name: .... Date: .... Period: ....

Water in a Bowl

6 Let's watch an animation of water in a bowl.

Sketch a graph representing the relationship between the volume of water in the 5-liter bowl and time.



7 Let's watch an animation of Cielo's graph representing the relationship between the volume of water in the bowl and time.

💬 **Discuss:** How does Cielo's graph represent the situation? How might you revise this graph?

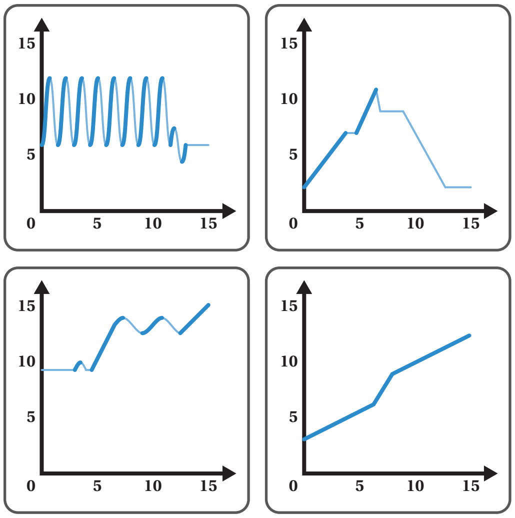
Activity
3

Name: .... Date: .... Period: ....

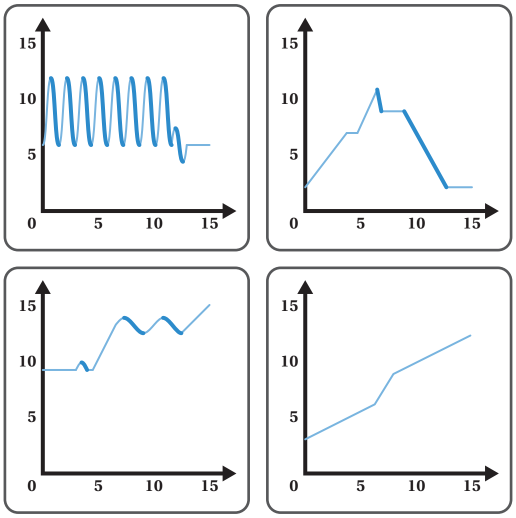
Describing Graphs

8 Here are some graphs from this lesson. Parts of the graphs are bolded to show where they are either increasing, decreasing, *linear*, or **non-linear**.

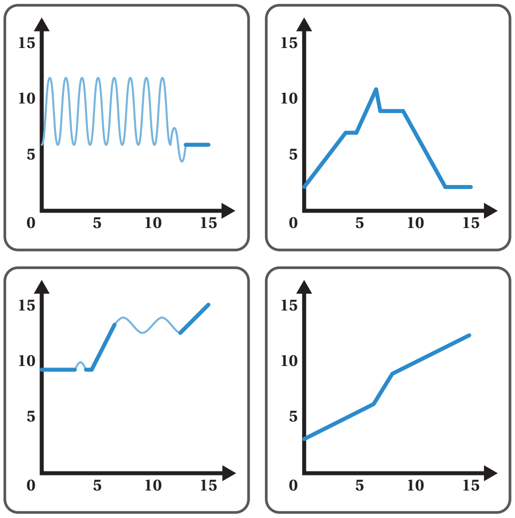
Increasing



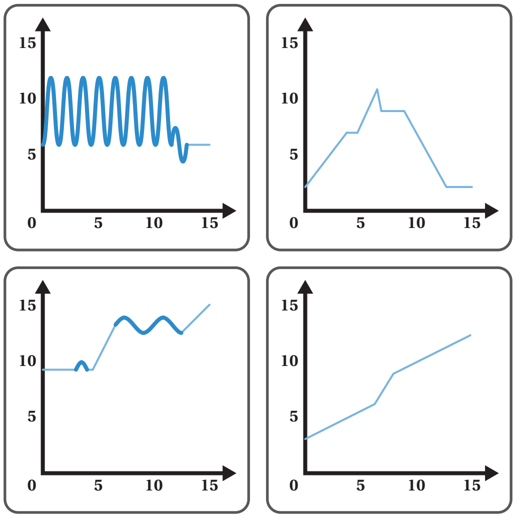
Decreasing



Linear



Non-Linear

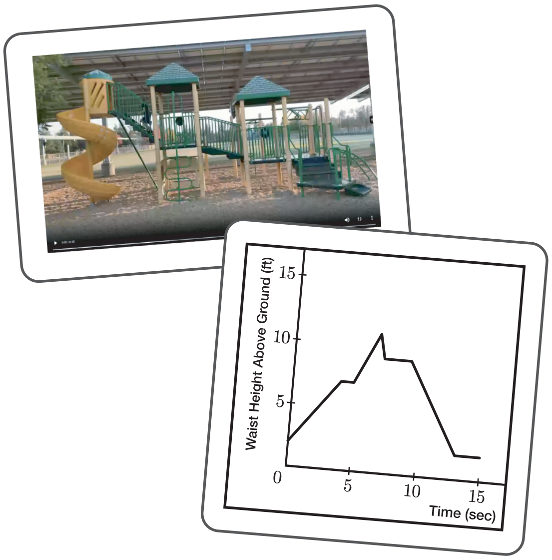


Discuss: What do you think each term means?

9 Synthesis

What can be helpful to consider when graphing a function that represents a real-world situation?

Use the example if it helps with your thinking.



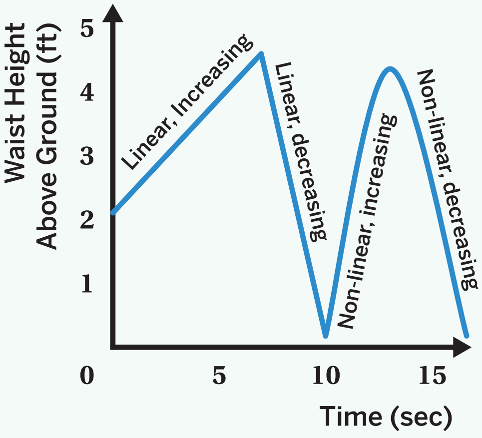
Lesson Practice 8.5.06

Lesson Summary

We can use graphs to represent a story. When drawing a graph, it can be helpful to identify the variables involved so that you can label the axes. Depending on the independent and dependent variables, different graphs can represent distinct details of the same story. It may also be helpful to identify key points in the story according to these chosen variables to help you sketch these features.

The function is:

- **Increasing** when part of the graph is going up from left to right.
- **Decreasing** when part of the graph is going down from left to right.
- *Linear* when part of the graph is a straight line. (Note: A vertical line is not a function.)
- **Non-linear** when part of the graph is not a straight line.



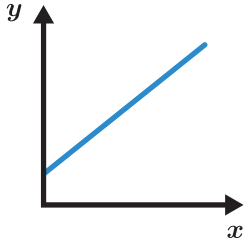
Lesson Practice
8.5.06

Name: ______________ Date: ________ Period: ________

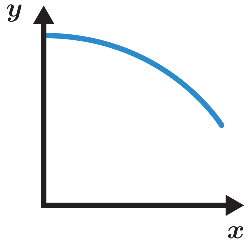
1. Determine which graph best represents the description.

Description	**Graph**
a Linear and decreasing	________
b Non-linear and increasing	________
c Linear and increasing	________
d Non-linear and decreasing	________

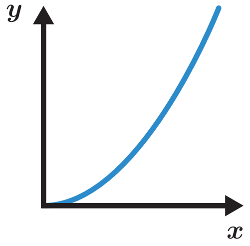
Graph A



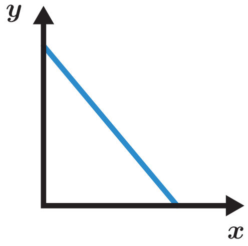
Graph B



Graph C

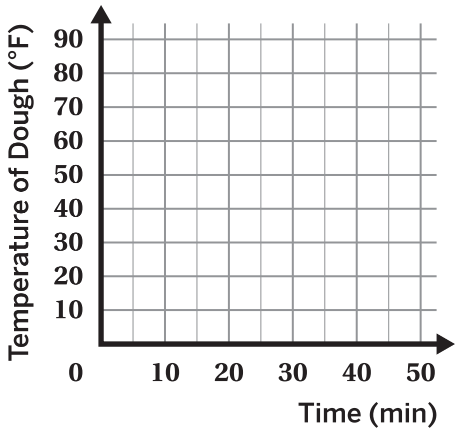


Graph D

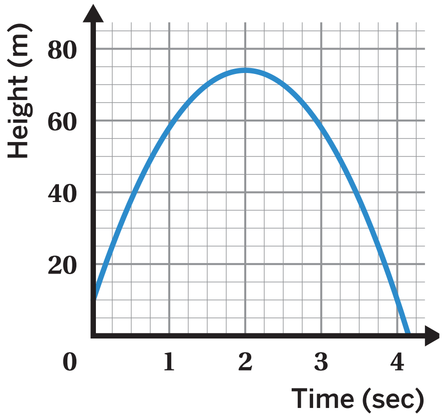


2. Avi places a batch of homemade pretzels in the refrigerator. The dough takes 15 minutes to cool from 70°F to 40°F. Once it is cool, the dough stays in the refrigerator for another 30 minutes. Avi then places the pretzels in the oven to bake. After 5 minutes in the oven, the temperature of the pretzel dough is 80°F.

Sketch a graph that represents this situation.



Problems 3–6: This graph represents the height of an object that is launched upwards from a tower and then falls to the ground.



3. How tall is the tower from which the object was launched?

4. Plot the point that represents the greatest height of the object and the time it took the object to reach that height.

5. Determine one time interval when the height of the object was increasing.

6. Determine one time interval when the height of the object was decreasing.

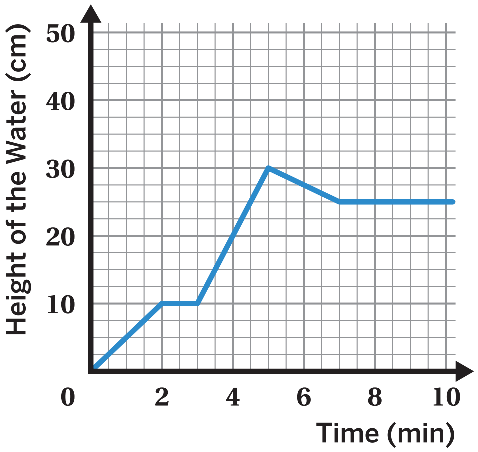
Lesson Practice

8.5.06

Name: ____________ Date: ________ Period: ________

7. Kimaya fills an aquarium with water. This graph shows the height of the water in the aquarium vs. time.

Tell a story about how Kimaya fills the aquarium based on what you see. Include specific heights and times.



Spiral Review

Problems 8–9: Solve each equation. Show your thinking.

8. $-(-2x + 1) = 9 - 14x$

9. $3x + \frac{3}{5} = \frac{1}{3}(5x + 5)$

Reflection

1. Put a star next to a problem that looked more difficult than it really was.

2. Use this space to ask a question or share something you're proud of.

Name: ______ Date: ______ Period: ______

Comparing Linear Functions

Let's compare linear functions represented in different ways.

Warm-Up

Jada has $50 in a savings account and saves $7 per week.

1. What could the independent variable and dependent variable be in this situation? Explain your thinking.

2. Write an equation representing this situation. Be sure to define the variables that you use.

3. Is your relationship from Problem 2 a function? Explain your thinking.

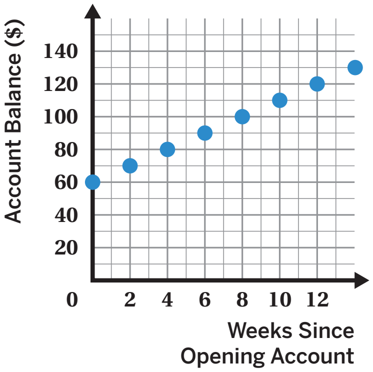
Activity
1

Name: .... Date: .... Period: ....

Which Is Growing Faster?

Noah and Azul both opened a savings account on the same day. Each person's account balance is a function of the weeks since the account was opened. Here is some information about each account.

Noah's Account



Azul's Account

$a = 8w + 60$, where w is the number of weeks since the account was opened, and a is the account balance

4. Who started with more money in their account? Circle one.

Noah Azul They started with the same amount.

Explain your thinking.

5. Who is saving money at a faster rate? Circle one.

Noah Azul They are saving at the same rate.

Explain your thinking.

6. How much will Noah save over the course of a year if he does not make any withdrawals?

(Note: There are 52 weeks in a year.) Show or explain your thinking.

7. How long will it take Azul to save the same amount? Show or explain your thinking.

Activity 2

Name: .... Date: .... Period: ....

Making Deposits vs. Withdrawals

Take a look at the accounts of four customers. They each have an account balance, a, measured over w weeks.

Account A

The account balance, a, is represented by the function $a = 65 + 10w$, where w represents the number of weeks since the account was opened.

Account B

The account balance starts at $\$40$ and decreases by $\$8.50$ per week.

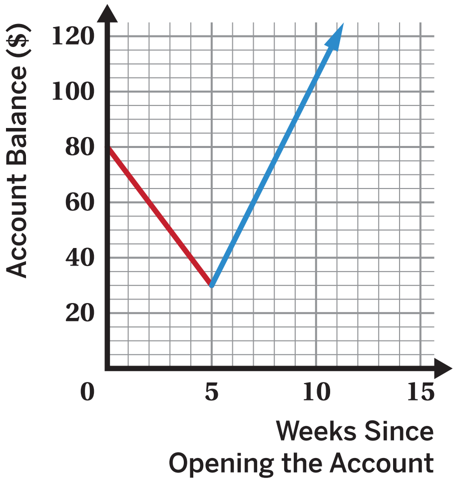
Account C

The account balance is represented by this table:

Number of Weeks	Account Balance ($)
1	71
3	48
7	2

Account D

The account balance is represented by this graph:



8. **Discuss:**

a Which account(s) show customers making deposits? Explain your thinking.

b Which account(s) show customers making withdrawals? Explain your thinking.

Name: .... Date: .... Period: ....

Making Deposits vs. Withdrawals (continued)

9. Kiana says that all four relationships are **linear functions** and that each situation can be represented with one equation in the form $y = mx + b$. Is Kiana's claim correct? Explain your thinking.

10. Which account will have the most money at the end of a year? Explain your thinking.

11. **a** How much money did Account A have after one year?

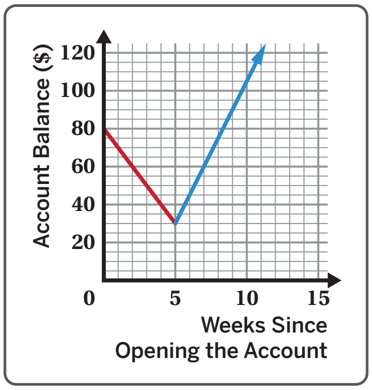
b How long did it take Account D to have the same amount?

Synthesis

12. How can you compare linear functions shown using different representations?

Number of Weeks	Account Balance ($)
1	71
3	48
7	2

$a = 65 + 10w$

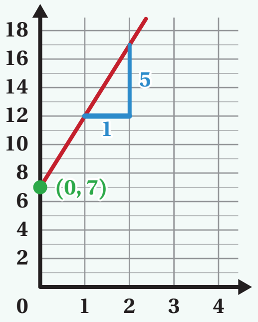


Lesson Practice 8.5.07

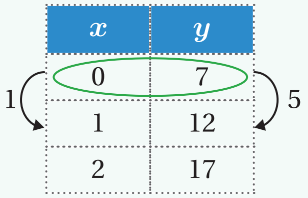
Lesson Summary

Linear relationships that have exactly one output for every possible input are called **linear functions**. All linear functions can be represented with a graph, a table, and an equation in the form $y = mx + b$, where m is the rate of change and b is the initial value.

Graph



Table



x	y
0	7
1	12
2	17

Equation

$y = 5x + 7$

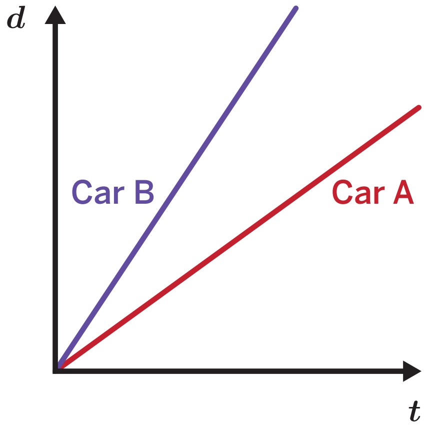
The slope, or rate of change, is a ratio between the difference of the y-values and the difference of the x-values. In a graph, you can use slope triangles to find the rate of change. In the equation $y = mx + b$, it is the coefficient of the independent variable. In this example, the slope is $\frac{5}{1} = 5$.

The initial value, or y-intercept, is the dependent value when the independent value is 0. In the equation $y = mx + b$, the y-intercept is the constant, b. In this example, the y-intercept is 7.

Lesson Practice
8.5.07

Name: .... Date: .... Period: ....

1. Two cars are traveling on the same highway in the same direction. The graphs show the distance, d, of each car from the starting point as a function of time, t. Which car is traveling faster? Explain your thinking.



Problems 2–4: Kiri and Remy race each other home from school. They run at the same speed, but Kiri's house is slightly closer to the school than Remy's house. Suppose there is a graph that shows their distances from home, in meters, as a function of the time, in seconds, from when they began the race.

2. If you were to read the graphs from left to right, would you expect the lines to increase or decrease? Explain your thinking.

3. How would you expect the lines representing Kiri's run and Remy's run to be different? Explain your thinking.

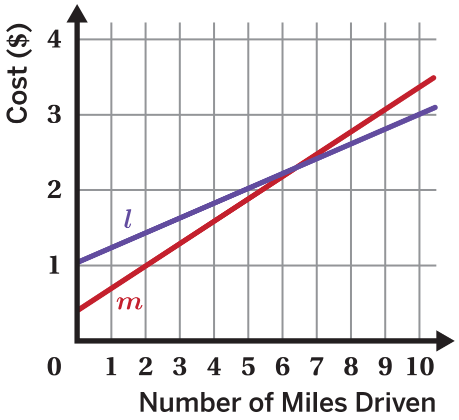
4. How would you expect the lines representing Kiri's run and Remy's run to be alike? Explain your thinking.

Lesson Practice

8.5.07

Name: .... Date: .... Period: ....

Problems 5–6: Two car services offer to pick up a customer and take them to their destination. Service A charges a flat fee of $\$0.40$ plus $\$0.30$ for each mile of the trip. Service B charges a flat fee of $\$1.10$ plus c dollars for each mile of their trip.



5. Match the services with the lines l and m.

6. For Service B, is the additional charge per mile greater than or less than $\$0.30$ for each mile of the trip? Explain your thinking.

Spiral Review

7. Write an equation for each line shown on the graph.

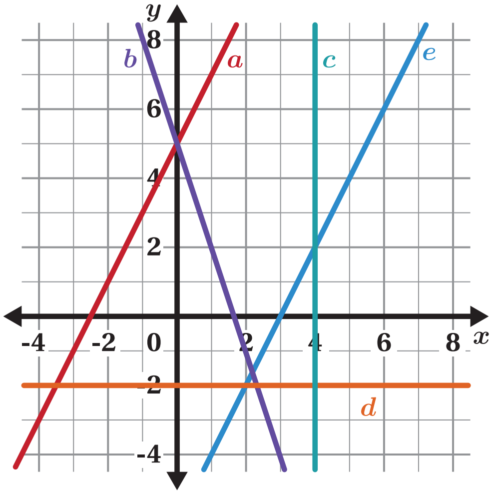
Line a: ....

Line b: ....

Line c: ....

Line d: ....

Line e: ....

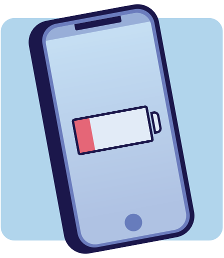


Reflection

1. Put a question mark next to a problem you were feeling stuck on.

2. Use this space to ask a question or share something you're proud of.

Name: ____________ Date: ________ Period: ________

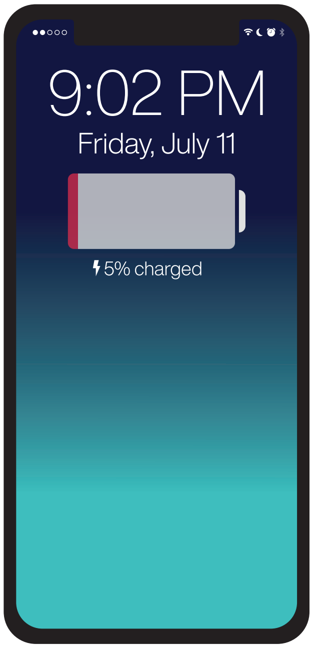


Charge!

Let's use linear functions to model a real-world situation.

Warm-Up

1. Tell a story about this image.



Name: .... Date: .... Period: ....

Charge!

2. Estimate the time when the phone will be fully charged. Explain your thinking.

3. **a** What relevant information do you know that would help to answer this question?

b What additional information would be helpful?

4. Let's look at some screens with additional information. Record any information that you think is relevant.

5. **Discuss:** What mathematical representations might help you make a more precise prediction for when the phone will be fully charged? Explain your thinking.

6. Use the additional information to revise your estimate for when the phone will be fully charged. Show or explain your thinking using *at least two* mathematical representations.

Activity

2

Name: .... Date: .... Period: ....

Is it Linear?

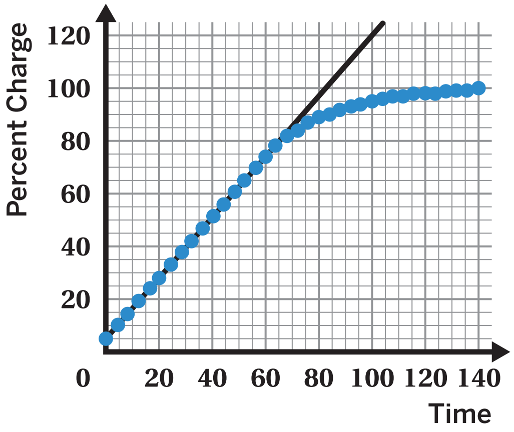
Let’s look at a graph that reveals when the phone is fully charged.

7. Is the relationship between the percent charge and time a function? Explain your thinking.

8. When might it be appropriate to use a linear function to model the data that describes the percent charge and the time? When might it *not* be appropriate?

Synthesis

9. How might a linear function be helpful to model a situation? What are some limitations of using linear models?



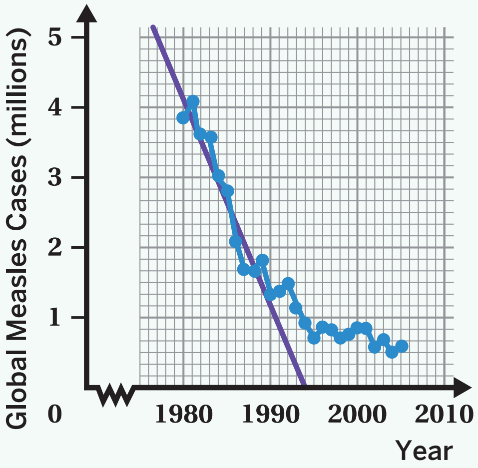
Lesson Practice 8.5.08

Lesson Summary

Sometimes a situation can be modeled by a linear function. Even if a function is non-linear, parts of its data can be modeled by a linear function, which can be used to help make predictions.

For example, you can use a linear function to model a section of this data on global measles cases from 1980 to 1990.

You can use this model to estimate that there would have been approximately 2.5 million cases of measles in 1985.



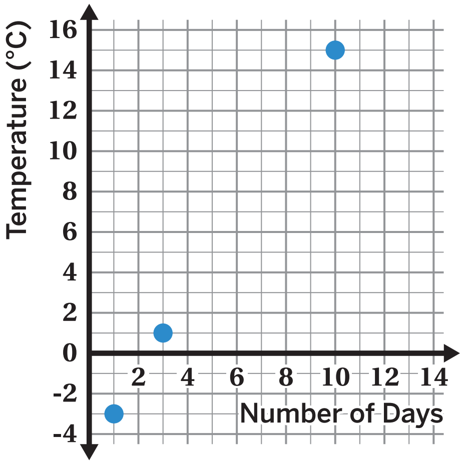
Source: *Our World in Data*

Lesson Practice

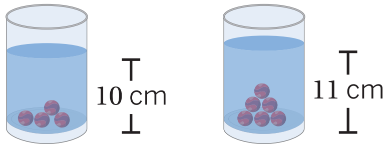
8.5.08

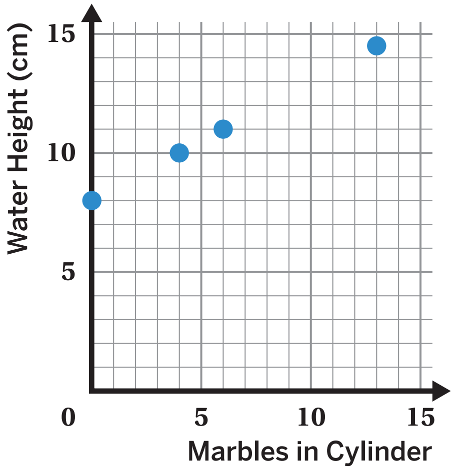
Name: .... Date: .... Period: ....

1. This graph shows the relationship between a city's high temperature, y, in degrees Celsius, and the number of days after the new year, x. Is the high temperature a linear function of the number of days after the new year? Explain your thinking.



Problems 2–5: In science class, Tiam uses a graduated cylinder filled partially with water to measure the change in the water's height as marbles are added. After dropping in 4 marbles, the height of the water is 10 centimeters. After dropping in 6 marbles, the height of the water is 11 centimeters.





2. How much does the height increase for each marble?

3. What was the height of the water in the cylinder before any marbles were dropped in?

4. What should the height of the water be after 13 marbles are dropped in?

5. Is the relationship between the height of the water and number of marbles a linear function? If so, what does the slope of the line represent? If not, explain why not.

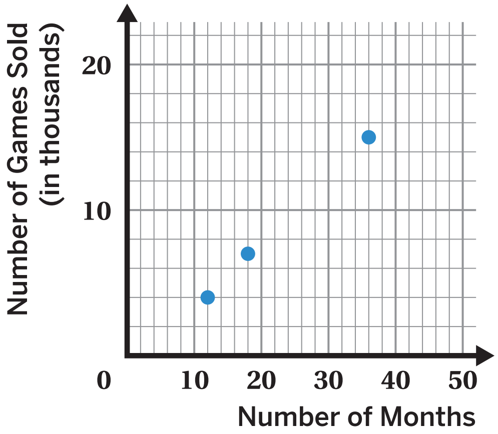
Lesson Practice

8.5.08

Name: .... Date: .... Period: ....

Problems 6–7: A board game company needs to know how many board games to produce. After the first 12 months, they sell 4,000 games. After 18 months, they sell 7,000 games. And after 36 months, they sell 15,000 games.

6. Do you think a single linear function could be used to estimate the number of games that will be sold after a certain number of months? If yes, draw a linear function to model the data. If no, explain your thinking.



7. Estimate the number of games sold after 48 months.

Spiral Review

Problems 8–9: Solve each equation. Show your thinking.

8. $5y + 14 = -43 - 3y$

9. $4(2a + 2) = 8(2 - 3a)$

Reflection

1. Put a heart next to a problem you understand well.

2. Use this space to ask a question or share something you're proud of.

Name: ________________ Date: ________ Period: ________

Piecing It Together

Let's create functions to model data sets.

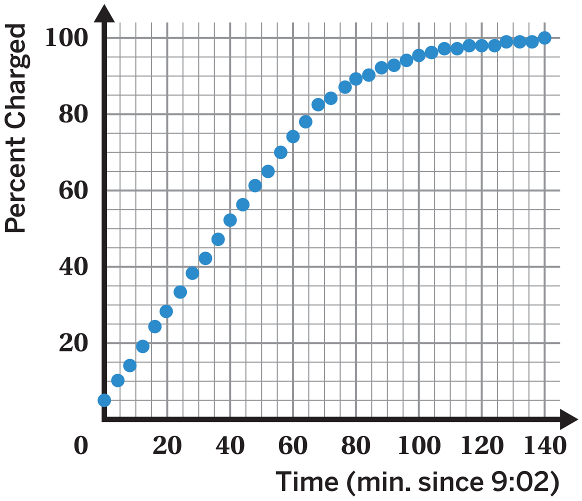
Warm-Up

1 Here is a data set for a phone charging over time.

A single linear function does not model this relationship very well.

a Sketch two connected line segments to model the relationship better.

b When was the phone charging the slowest? Explain your thinking.



Activity
1

Name: .... Date: .... Period: ....

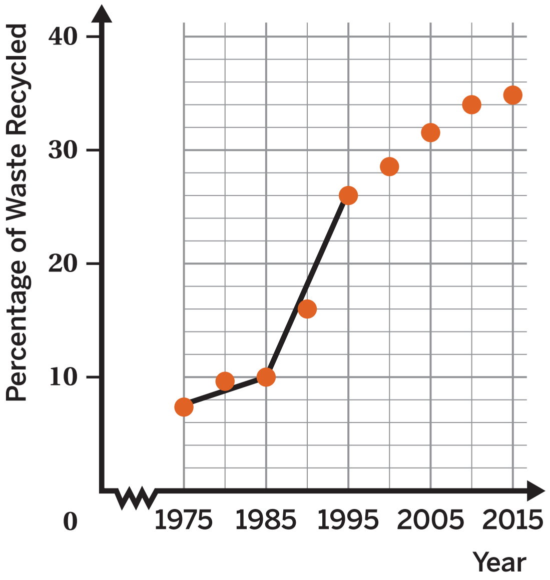
Recycling

2 This data set shows the percentage of waste produced in the United States that gets recycled over time.

A student started sketching a function to model this data.

a Sketch one more linear segment to complete the function.

b Approximate the slope of the segment you created.

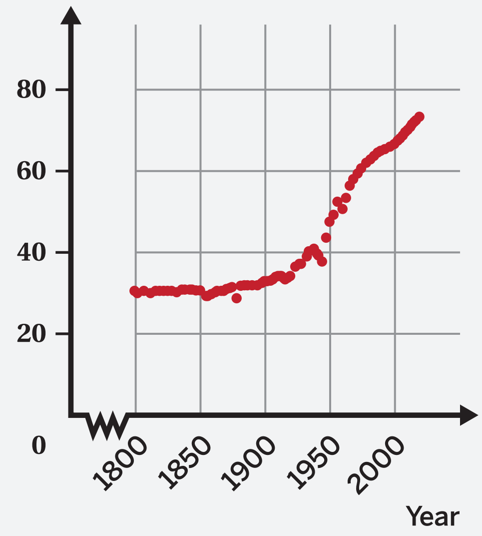


Explain what this number means about the percentage of waste recycled.

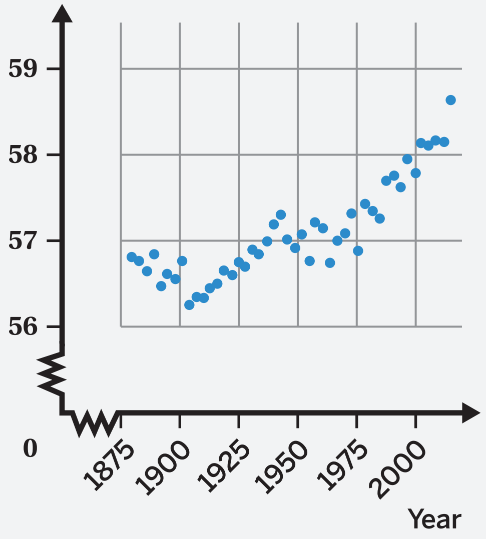
Name: .... Date: .... Period: ....

Four Data Sets

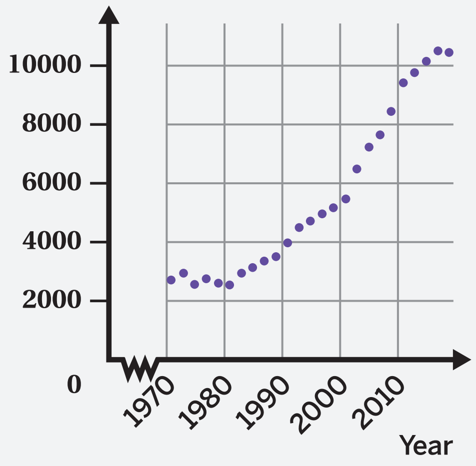
3 Here are four new data sets. Match each graph to the description you think it represents.



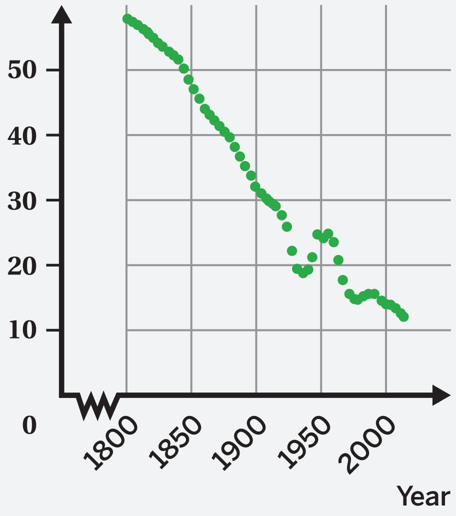
Graph A



Graph B



Graph C



Graph D

U.S. College Cost ($)	U.S. Births (per 1000 people)	Global Life Expectancy (age in years)	Global Temperature (°F)

Activity

3

Name: .... Date: .... Period: ....

Analyzing Data Sets

4 Look at the four data sets on the Activity 3 Sheet. Pick one that interests you.

Which data set are you choosing?

What do you notice? What do you wonder?

I notice:

I wonder:

5 **a** Using at least two line segments, sketch a function on the graph of the data set you chose on the Activity 3 Sheet to model the data.

b Describe your function using vocabulary from this unit.

linear	increasing
non-linear	decreasing

6 During which interval of time did the data seem to change the most?

Name: .... Date: .... Period: ....

Analyzing Data Sets (continued)

7 **a** Use your function to make a prediction for the year 2030.

b Do you think your function can be used to make an accurate prediction for the year 2050? Explain your thinking.

8 Share your findings with a student or a group that examined a different data set.

Discuss:

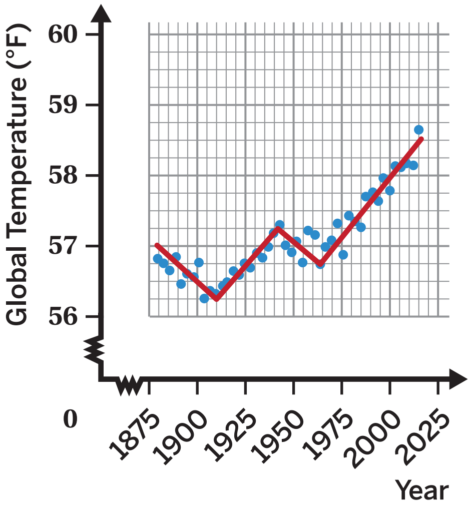
- What choices did you make as you sketched your function?
- What is your prediction for 2030, and how did you arrive at your prediction?
- Do you think your function can be used to make an accurate prediction for 2050?

9 Synthesis

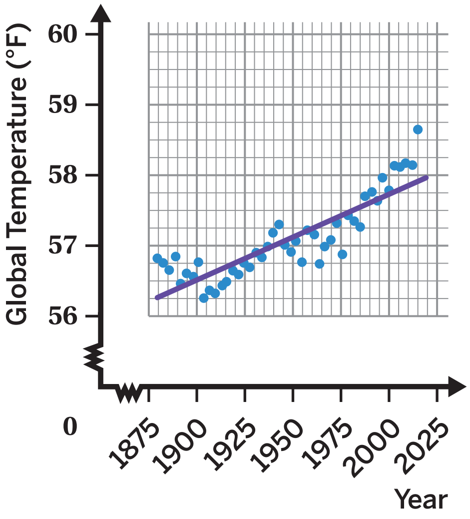
When is it helpful to model a data set with multiple linear segments instead of a single linear segment?

Use the examples if they help with your thinking.

Function A: Multiple Segments



Function B: Single Segment



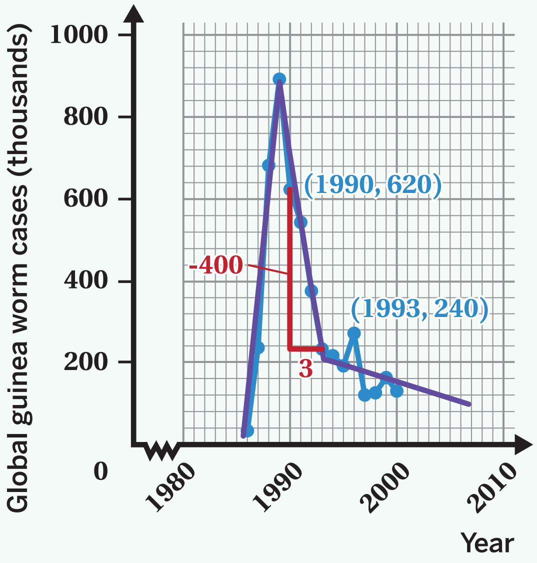
Lesson Practice 8.5.09

Lesson Summary

You can use one or more linear segments to represent a data set. Using multiple linear segments can help you precisely represent a data set.

For example, you can use multiple linear segments to model this data about cases of the Guinea Worm disease over time.

You can use this model to estimate that between 1990 and 1993, cases of Guinea Worm disease were changing at a rate of approximately -$\frac{400}{3}$ cases per year, or dropping by about 133.33 million cases per year.



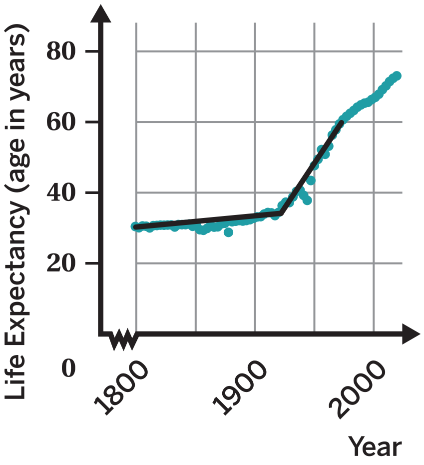
Source: *Our World in Data*

Lesson Practice

8.5.09

Name: .... Date: .... Period: ....

1. This graph shows global life expectancy over time. A student started to model the data with two linear functions. Sketch one more linear segment to complete the student's work.



Problems 2–3: On the first day after the new moon, 2% of the Moon's surface is illuminated. On the second day, 6% of the Moon's surface is illuminated.

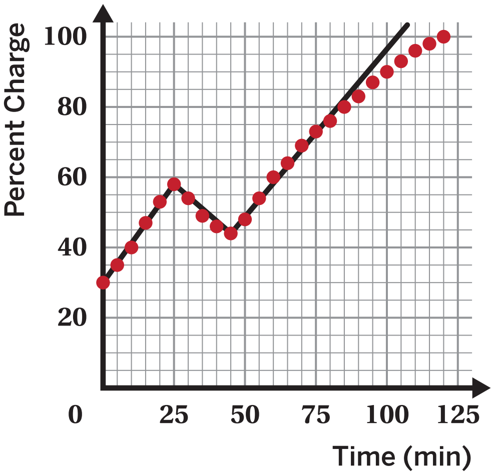
2. Assuming this data can be modeled with a linear function, complete the table. Round to the nearest day if necessary.

3. The Moon's surface is actually 100% illuminated on day 14. How appropriate is it to use a linear function for this data?

Day Number	Illumination
1	2%
2	6%
. . .	. . .
	50%
	100%

Problems 4–5: Elena is charging her laptop. After 25 minutes, she unplugs her laptop to complete her homework. After she completes her homework, she plugs in her laptop again until it is fully charged. This graph shows the percent charge of Elena's laptop over time.

4. Describe the function used to model the data.



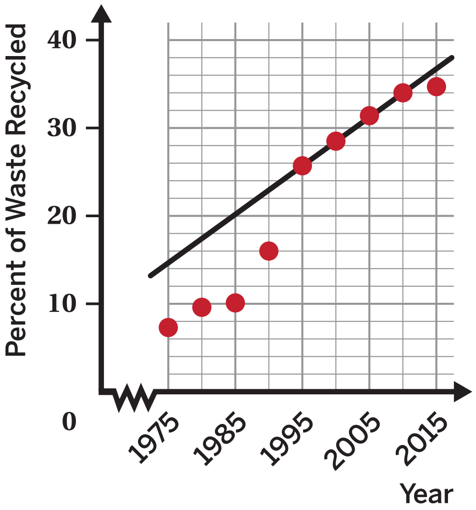
5. Which time interval is not modeled appropriately? Explain your thinking.

Lesson Practice

8.5.09

Name: .... Date: .... Period: ....

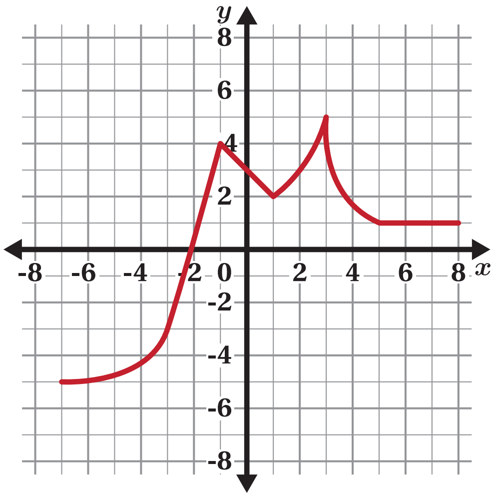
6. This graph shows the percentage of waste produced in the United States that gets recycled over time. A student draws a linear function that models the change from 1975 to 2015. For what years does the model make accurate predictions? For which years is it not as accurate?



Spiral Review

7. The graph shows y as a function of x. For which intervals is the function increasing? Select *all* that apply.

- ☐ **A.** From -7 to -3
- ☐ **B.** From -3 to -1
- ☐ **C.** From -1 to 1
- ☐ **D.** From 1 to 3
- ☐ **E.** From 3 to 5
- ☐ **F.** From 5 to 7



8. Which linear function has a greater rate of change? Explain your thinking.

Function A

$$y = \frac{1}{6}x + \frac{2}{5}$$

Function B

x	y
-9	3
1	5
21	9

Reflection

1. Circle one problem, word, or concept that you want to know more about.

2. Use this space to ask a question or share something you're proud of.

Name: .... Date: .... Period: ....

Practice Day 1

Let’s practice what you’ve learned so far in this unit!

You will use task cards for this Practice Day. Record all of your responses here.

Task A: Two Truths and a Lie

1. Circle one: A B C

Explanation:

2. Circle one: A B C

Explanation:

3. Circle one: A B C

Explanation:

Explore More

Situation 1 false statement:

Situation 2 false statement:

Situation 3 false statement:

Name: .... Date: .... Period: ....

Practice Day 1 (continued)

Task B: Function or Not?

1. Circle one:

Function Not a function

Explain if not a function:

2. Circle one:

Function Not a function

Explain if not a function:

3. Circle one:

Function Not a function

Explain if not a function:

4. Circle one:

Function Not a function

5. Circle one:

Function Not a function

6. Circle one:

Function Not a function

7. Circle one: Function Not a function

Explanation:

Explore More

Function:

Not a function:

Task C: Three Restaurants

1. Circle one: Yes or No

Explanation:

2. Circle one:

McDougal's Sandy's Burger Royalty

Explanation:

3. McDougal's: .... Sandy's: .... Burger Royalty: ....

4. Advantage:

Explore More

Segment 1: .... Segment 2: ....

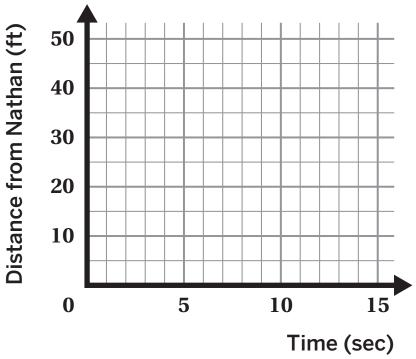
Segment 3: .... Segment 4: ....

Name: .... Date: .... Period: ....

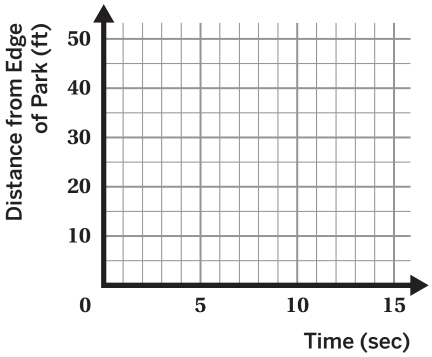
Practice Day 1 (continued)

Task D: Graphing Stories

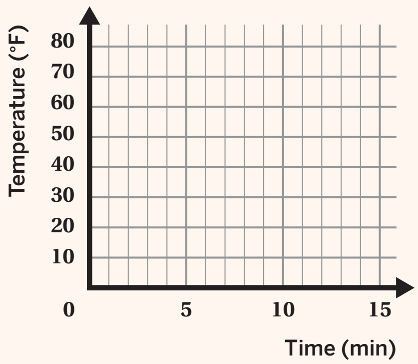
1.



2.



Explore More



Task E: Linear or Not?

1. Circle one: Linear Not Linear

2. Circle one: Linear Not Linear

3. Circle one: Linear Not Linear

4. Circle one: Linear Not Linear

Explore More

Situation 1:

Situation 2:

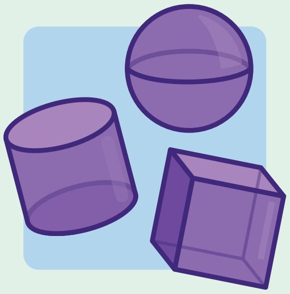
Situation 3:

Situation 4:

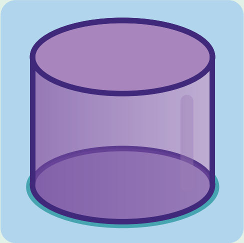
Sub-Unit

3

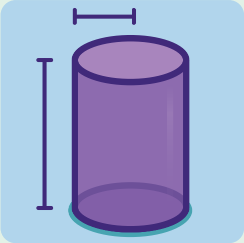
Volume



Lesson 10
Volume Lab

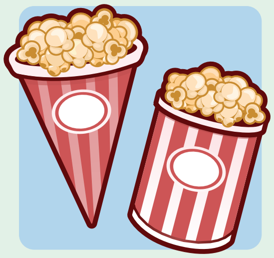


Lesson 11
Cylinders

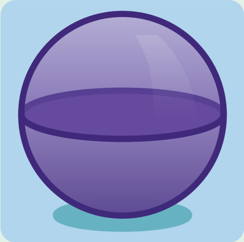


Lesson 12
Scaling Cylinders

Lesson 13
Cones



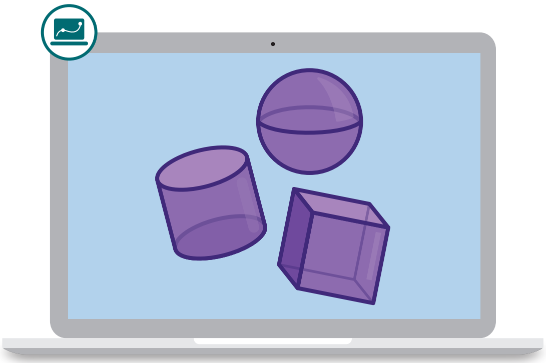
Lesson 14
Unknown Dimensions



Lesson 15
Spheres

Unit 5
Lesson
10

Name: ________ Date: ________ Period: ________



Volume Lab

Let's estimate the volume of three-dimensional solids.

Warm-Up

1 Which one doesn't belong? Explain your thinking.

Figure A

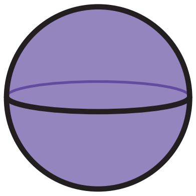


Figure B

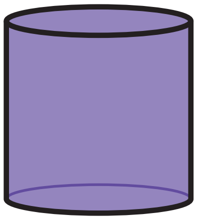


Figure C

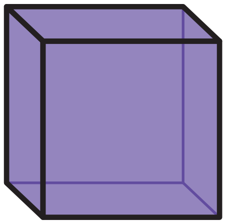
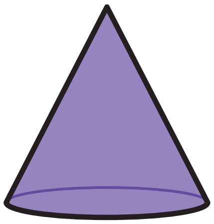


Figure D



Activity

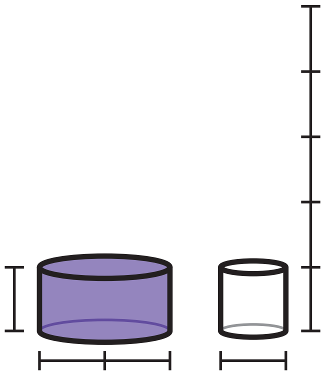
1

Name: ______ Date: ______ Period: ______

Comparing Volumes

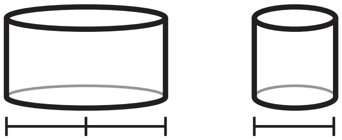
2 Here are two **cylinders**.

Draw a new height for the cylinder on the right so that both cylinders have the same *volume*.



3 Here are two cylinders with the same height.

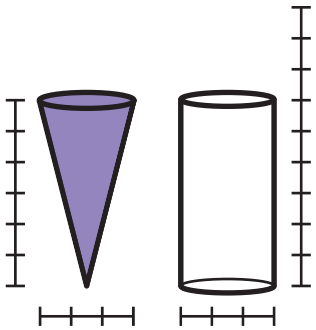
How many small cylinders do you think it would take to fill the large cylinder?



Name: ________ Date: ________ Period: ________

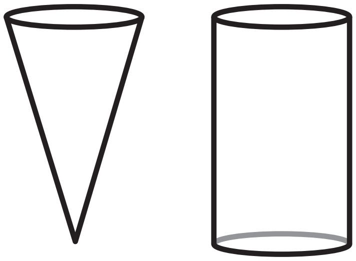
Comparing Volumes (continued)

4 Draw a new height for the cylinder so that both objects have the same volume.



5 Here is a **cone** and a cylinder with the same height and *diameter*.

How many cones would it take to fill the cylinder?



Activity
2

Name: .... Date: .... Period: ....

Volume Lab

6 Let’s use Screen 6 of the digital activity to explore some volume relationships!

a Select any *two* objects and adjust their dimensions. Then press “Compare.” Repeat this with several pairs of objects. Draw or record something that you found interesting or surprising.

b Two cones have equal diameters. The height of one cone is 2 times as large as the height of the other cone. How are the volumes of the cones related?

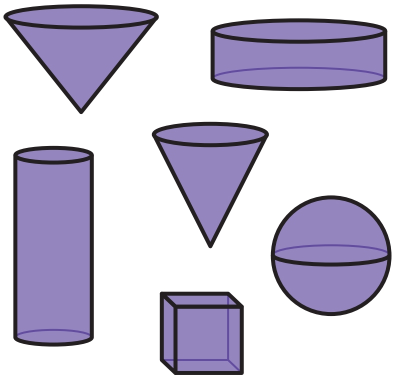
c Two cylinders have the same height. The diameter of one cylinder is 3 times as large as the diameter of the other cylinder. How are the volumes of the cylinders related?

d Describe the relationship between two different objects, where one has twice the volume of the other.

e Describe another interesting relationship between the volumes of two different objects.

7 Synthesis

Describe one volume relationship you discovered during this lesson.



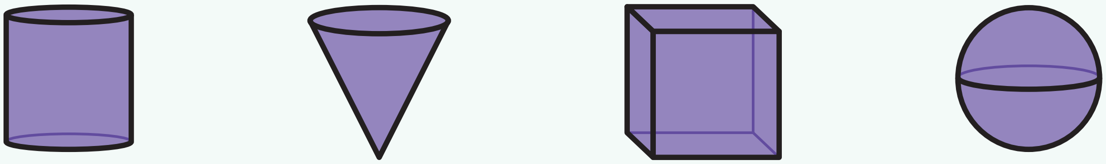
Lesson Practice 8.5.10

Lesson Summary

The *volume* of an object is the number of cubic units that fill its three-dimensional region without any gaps or overlaps.

You can often determine relationships between the volumes of different figures with similar measurements. For example, if the base of a **cone** and a **cylinder** have the same *diameter* and height, then the cylinder will have a volume that is three times greater than the cone.

There are also relationships between the volumes of the same figure with different measurements. For example, if the diameter of a **sphere** is doubled, or if the side length of a cube is doubled, the original volume of these figures will be multiplied by 8.

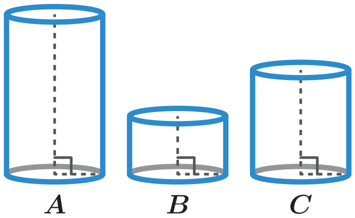


Lesson Practice

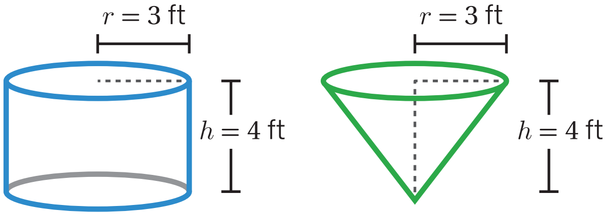
8.5.10

Name: .... Date: .... Period: ....

1. Cylinders A, B, and C have the same radius but different heights. Order the cylinders from *least* volume to *greatest* volume.



2. Here is a cylinder and a cone with the same base and height. How much more water would you need to fill the cylinder than the cone?



A. 3 times as much

B. 2 times as much

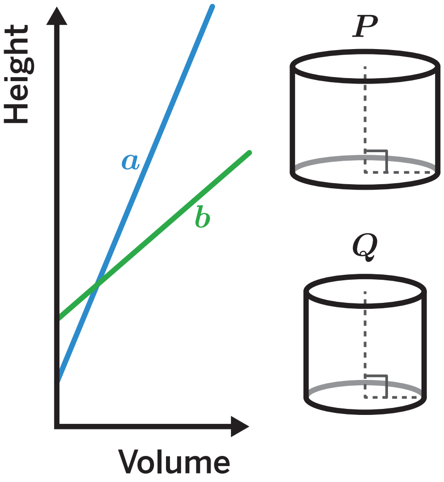
C. 5 times as much

D. 4 times as much

Problems 3–4: Cylinders P and Q have the same height. Each starts off filled with different amounts of water. The graph shows the height of the water in each cylinder as the volume of water increases.

3. Match lines a and b to cylinders P and Q.

Cylinder	Line
P	
Q	



4. Describe what the slopes of lines a and b represent in this situation.

Lesson Practice

8.5.10

Name: .... Date: .... Period: ....

Spiral Review

5. The area of a circle is approximately 201.06 square inches. What is its radius in inches?

6. Match each circle with its area.

- Circle A has a radius of 4 units.
- Circle B has a radius of 10 units.
- Circle C has a radius of 8 units.

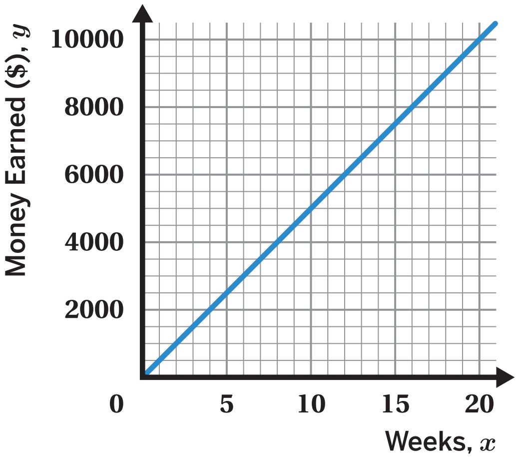
Area of the Circle	Circle
About 314 square units	
64π square units	
16π square units	

7. Here are two expressions that represent the volume of liquid in two different containers after t seconds:

- $1250 - 25t$ represents the volume of liquid in Container A.
- $50t + 250$ represents the volume of liquid in Container B.

What does the equation $1250 - 25t = 50t + 250$ represent in this situation?

8. Mai earns $\$1,710$ every 3 weeks by working as a freelance photographer. Jayla is also a freelance photographer whose earnings are represented by the graph. Who earns more per week, and how much more? Show or explain your thinking.



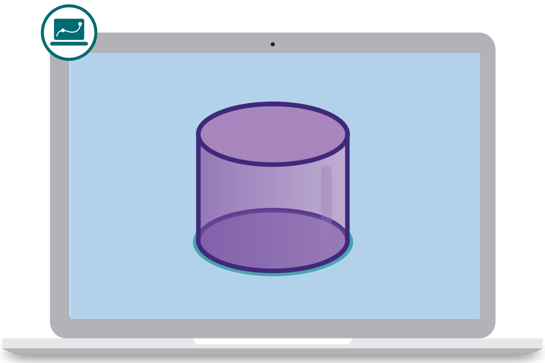
Reflection

1. Put a star next to a problem you want to understand better.

2. Use this space to ask a question or share something you're proud of.

Unit 5
Lesson
11

Name: .... Date: .... Period: ....

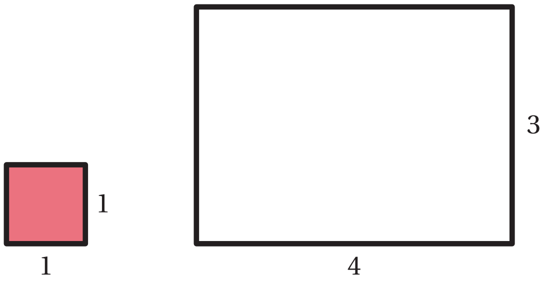


Cylinders

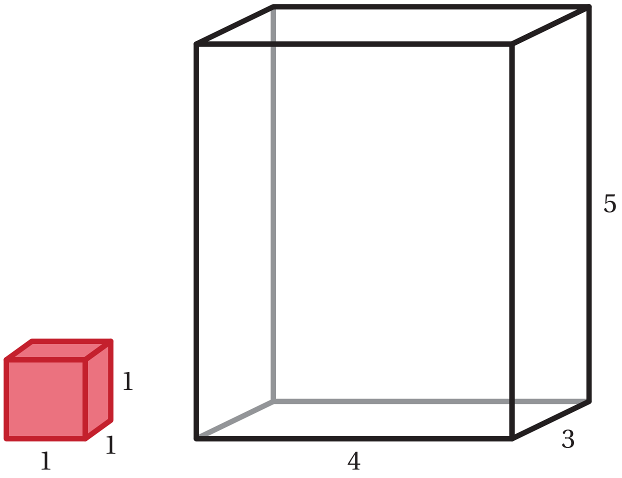
Let’s calculate the volume of cylinders.

Warm-Up

1 How many unit squares are needed to cover this rectangle?



2 How many unit cubes are needed to fill this rectangular prism?



Name: .... Date: .... Period: ....

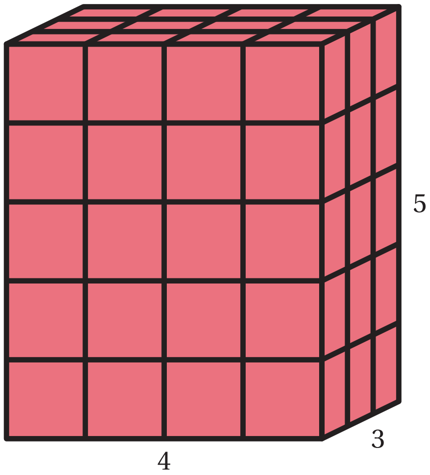
Volume Strategies

3 DeAndre calculated the volume in three ways:

- $V = 12 \cdot 5$
- $V = 15 \cdot 4$
- $V = 20 \cdot 3$

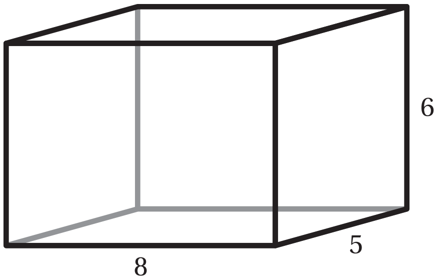
Let's focus on DeAndre's first equation.

Explain what 12 and 5 represent in the diagram.



4 Here is a new rectangular prism.

Use DeAndre's strategy to write an expression for its volume.

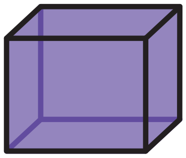


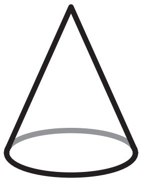
Name: .... Date: .... Period: ....

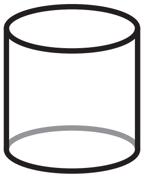
Volume Strategies (continued)

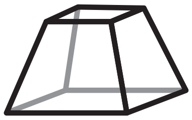
5 DeAndre described his strategy for calculating the volume of a solid.

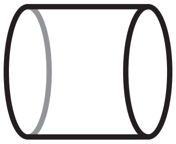
- First, find the area of the base.
- Then multiply that area by the height of the object.

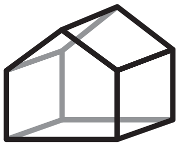

A


B


C


D


E

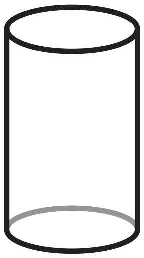

F

a Circle *all* of the objects for which DeAndre's strategy will work.

b **Discuss:** How did you decide which objects to choose?

6 DeAndre's strategy for calculating volume works for cylinders.

What information would you need to calculate the volume of this cylinder?

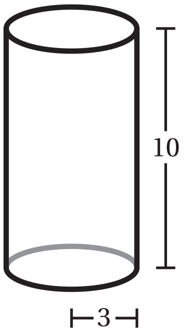


Name: .... Date: .... Period: ....

Cylinder Volumes

7 This cylinder has a height of 10 units and a *radius* of 3 units.

Calculate the volume of the cylinder.



8 Caasi incorrectly determined the volume of the cylinder with this calculation:

$V = 3 \cdot 3 \cdot 10 = 90$

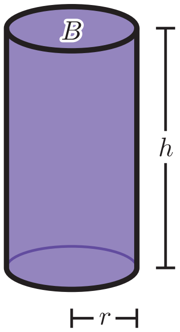
What did Caasi do well and what mistake did she make?

9 Here are two formulas for the volume of a cylinder:

$V = \pi r^2 \cdot h$

$V = B \cdot h$

Describe how the formulas are related.



Activity
3

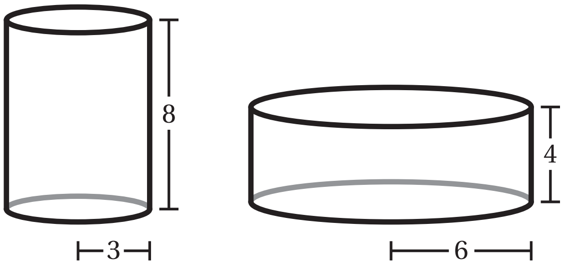
Name: .... Date: .... Period: ....

Calculating Cylindrical Volume

10 Calculate the volume of each cylinder.

Tall cylinder:

Short cylinder:



11 Sketch two different cylinders that have the same volume. What is the volume of each cylinder?

Cylinder A

Cylinder B

Explore More

12 **a** Write dimensions for a rectangular prism and a cylinder with volumes that are close to equal.

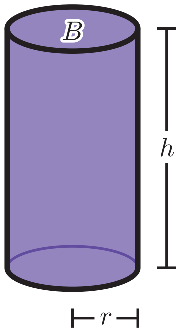
Rectangular prism:

Cylinder:

b Explain why they have close to equal volumes.

13 Synthesis

Describe a strategy for determining the volume of a cylinder given its radius and height.

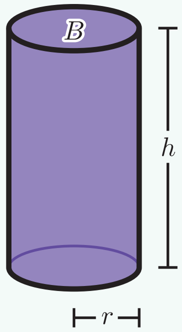


Lesson Practice 8.5.11

Lesson Summary

A prism has two congruent bases connected by perpendicular lines. Its volume can be determined by multiplying the area of its base by its height. A cylinder has two congruent circles for its base and the sides are perpendicular to the bases. This means you can also determine the volume of a cylinder by using the area of its base multiplied by its height.

If you know the radius and height of a cylinder, then you can determine the volume of the cylinder. The base area is determined using the expression $\pi \cdot r^2$. The volume, in cubic units, can be determined by multiplying the base area by the height, h. The formula for the volume of a cylinder is $V = \pi r^2 \cdot h$.



$V = \pi r^2 \cdot h$

Lesson Practice

8.5.11

Name: .... Date: .... Period: ....

Problems 1–3: Draw a cylinder.

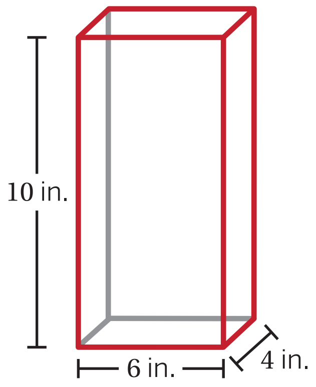
1. Label the radius 3 units and the height 10 units.

2. Determine the area of the base. Write your response in terms of π.

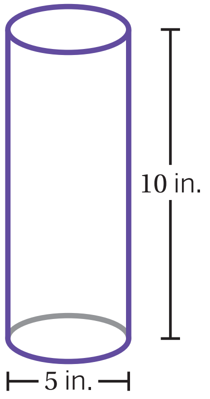
3. Determine the volume of the cylinder. Write your response in terms of π.

Problems 4–6: Containers A and B hold oatmeal. Container A is a rectangular prism and Container B is a cylinder.

A



B

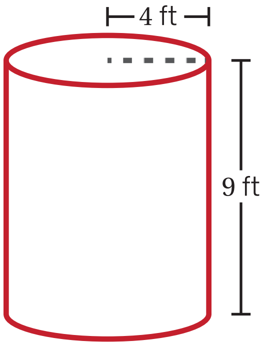


4. The diameter of Container B is 5 inches. What is its radius?

5. Which container's base has a larger area? Explain your thinking.

6. Which has a larger volume: Container A or B? Explain your thinking.

7. Here is a cylinder with a radius of 4 feet and a height of 9 feet. What is the volume of the cylinder in cubic feet? Round your answer to the nearest hundredth.



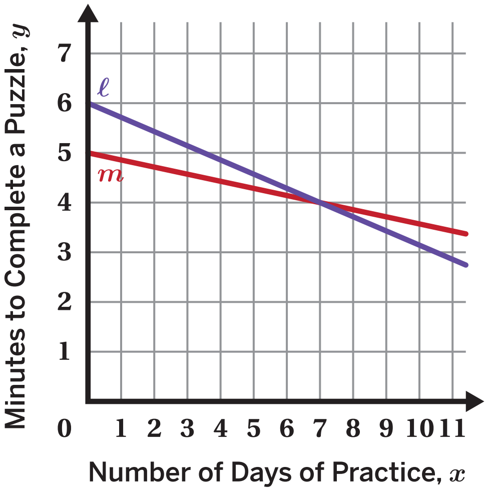
Lesson Practice

8.5.11

Name: ……………… Date: ……………… Period: ………………

Spiral Review

Problems 8–9: Two students join a puzzle-solving club, and they each improve their completion time as they practice. Student A improves their completion time at a faster rate than Student B.



8. Match each student with the line that represents their time.

9. Which student completed puzzles faster before practicing? Explain your thinking.

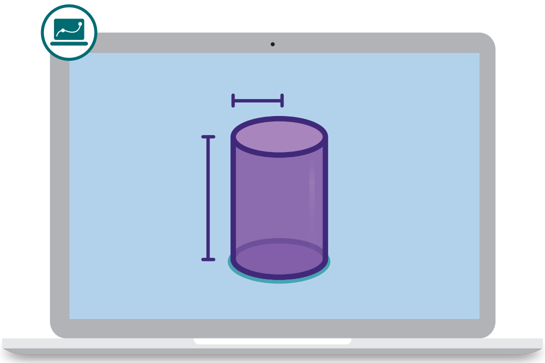
Reflection

1. Put a heart next to the problem you found most interesting.

2. Use this space to ask a question or share something you're proud of.

Unit 5
Lesson
12

Name: .... Date: .... Period: ....



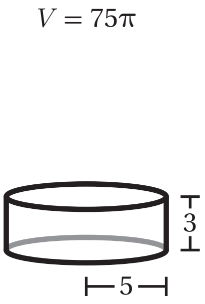
Scaling Cylinders

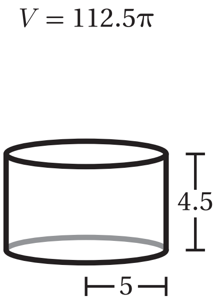
Let's see how changing a cylinder's radius or height impacts its volume.

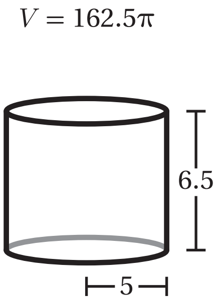
Warm-Up

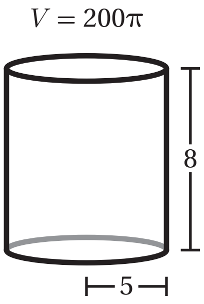
1 We learned that in a function, the independent variable represents the input and the dependent variable represents the output.

Here are several cylinders:









In this situation, what could the independent and dependent variables be?

Independent variable: ....

Dependent variable: ....

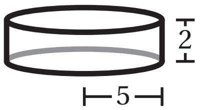
Activity
1

Name: .... Date: .... Period: ....

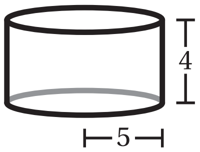
Changing the Height

2 Select the cylinder that represents the plotted point. Explain to a classmate how you chose.

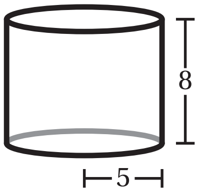
A.



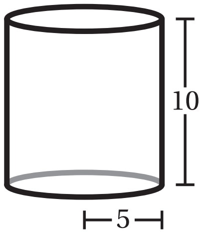
B.

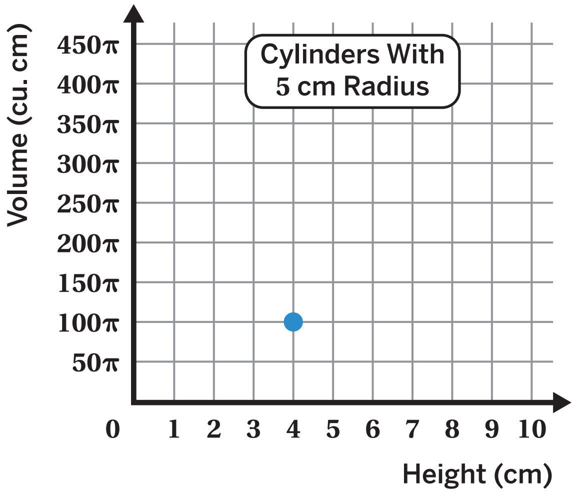


C.



D.





3 Let's watch an animation graphing the relationship between a cylinder's height and its volume.

Discuss: What do you notice? What do you wonder?

4 Let's look at a graph that represents the relationship between the height and the volume for cylinders with a radius of 5 centimeters.

Use the graph and the table to help you find the volume of each of the four cylinders.

Express each volume in terms of π.

Object	Height	Volume (cu. cm)
Cylinder A	2	
Cylinder B	4	100π
Cylinder C	8	
Cylinder D	16	

Activity
2

Name: .... Date: .... Period: ....

Changing the Radius

5 Let's see what happens if we keep the height of a cylinder constant but change the radius.

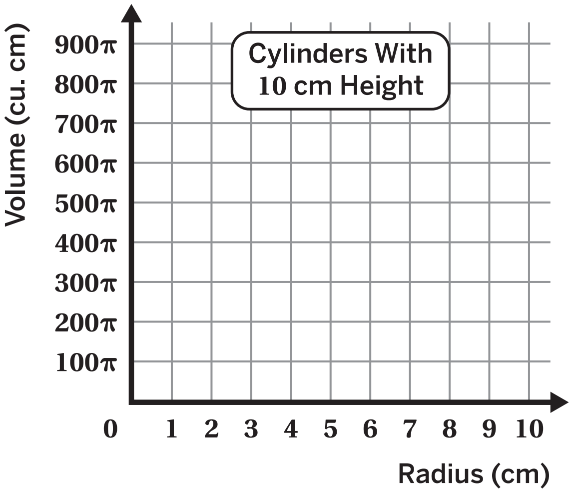
a Choose and record a radius for two different cylinders that each have a height of 10 centimeters.

	Radius (cm)	Volume (cu. cm)
Cylinder 1		
Cylinder 2		

b Calculate the volume for each cylinder. Express each volume in terms of π.

6 **a** Plot points to represent your two cylinders.

b Make a sketch of what you think the graph looks like for *all* cylinders with a height of 10 centimeters.



7 Let's watch an animation graphing the relationship between a cylinder's radius and its volume.

Discuss: Is this relationship a linear function? Explain your thinking.

Name: ________________ Date: ________ Period: ________

Changing the Radius (continued)

8 Let's look at a graph that represents the relationship between the radius and the volume for cylinders with a height of 10 centimeters.

Use the graph and the table to help you find the volume of each of the four cylinders.

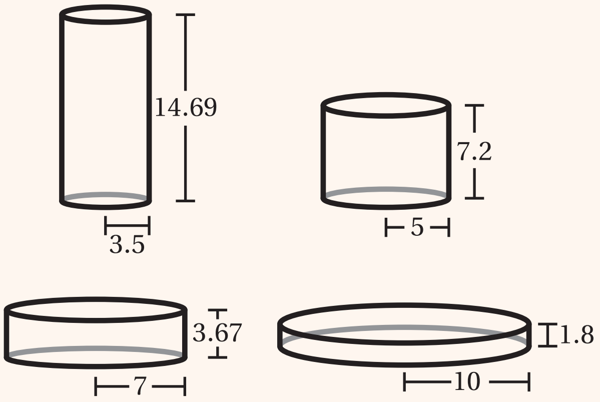
Express each volume in terms of π.

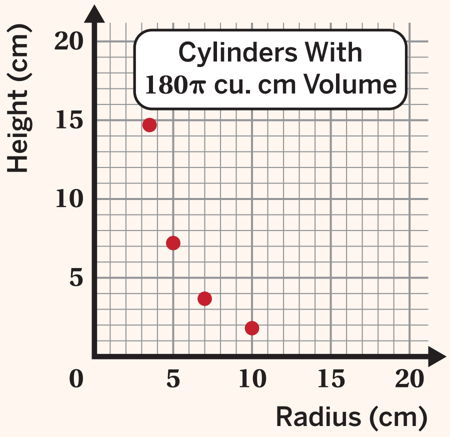
Object	Radius (cm)	Volume (cu. cm)
Cylinder E	2	
Cylinder F	4	
Cylinder G	8	
Cylinder H	16	

Explore More

9 Explore the relationship between radius and height when the volume of a cylinder is fixed. Here are several cylinders that all have a volume of 180π.

Discuss: What do you notice? What do you wonder?

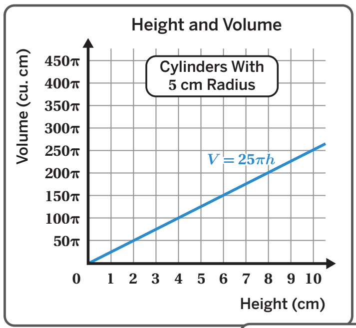


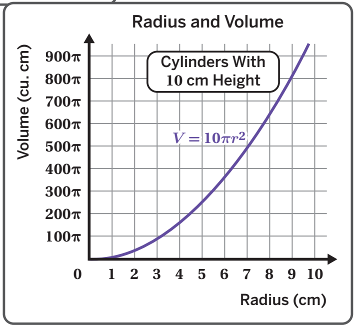


10 Synthesis

Here are the relationships we explored today.

How can you tell if the relationship between height and volume and the relationship between radius and volume are linear or non-linear?





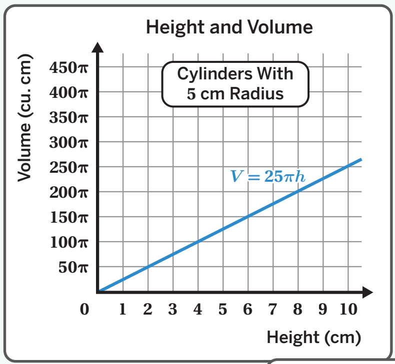
Lesson Practice 8.5.12

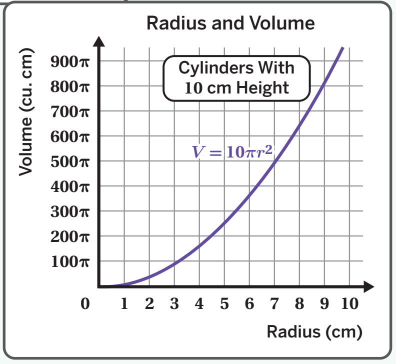
Lesson Summary

The volume of a cylinder depends on the cylinder's radius and height. The formula for the volume of a cylinder is $V = \pi r^2 h$, where r represents the radius and h represents the height.

When a cylinder's height, h, increases at a constant rate, the cylinder's volume, V, also increases at a constant rate. This means there is a proportional linear relationship between the height and volume. That's why we can represent the relationship between volume and height with a straight line.

On the other hand, we *cannot* represent the relationship between a cylinder's radius and volume with a line because the ratio of the volume to the radius changes as the radius increases. That's why the graph of the relationship between radius and volume is curved and non-linear.





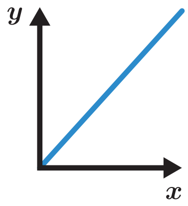
Lesson Practice
8.5.12

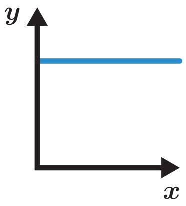
Name: ……………………… Date: ……………… Period: ………………

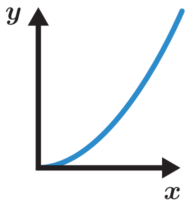
1. Each row of this table lists information about a specific cylinder. Complete the table.

Diameter (units)	Area of Base (sq. units)	Height (units)	Volume (cu. units)
4		10	
6			63π
	25π	6	

2. Which graph could represent the volume of water in a cylinder as a function of its height if the radius is held constant? Explain your thinking.

A.


B.


C.


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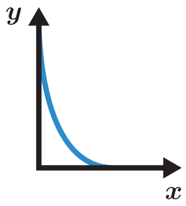


Problems 3–6: Imagine several cylinders that all have a height of 18 meters. Let r represent the radiuses of the cylinders, in meters, and V represent the volume of the cylinders, in cubic meters.

3. Write an equation that represents the relationship between the volume, V, and the radius, r, for all cylinders with a height of 18 meters.

4. Complete this table:

r (m)	1	2	3
V (cu. m)			

5. If the radius of a cylinder is doubled, does the volume double? Explain your thinking.

6. Is the graph representing the relationship between a cylinder's volume and its radius linear? Explain your thinking.

Lesson Practice

8.5.12

Name: .... Date: .... Period: ....

7. A cylinder has a volume of 48π cubic centimeters and a height represented by h.

Complete this table with the volumes of other cylinders that have the same radius but different heights.

Height (cm)	Volume (cu. cm)
h	48π
$2h$	
$5h$	
$\frac{h}{2}$	
$\frac{h}{5}$	

8. Which change do you think would increase the volume of a cylinder the most — doubling the radius or doubling the height? Explain your thinking.

Spiral Review

Problems 9–10: A gas company's delivery truck has a cylindrical tank with a diameter of 14 feet and a height of 40 feet.

9. Draw the tank, then label its radius and height.

10. How much gas can fit in the tank? Show or explain your thinking.

Reflection

1. Circle the problem you feel least confident about.

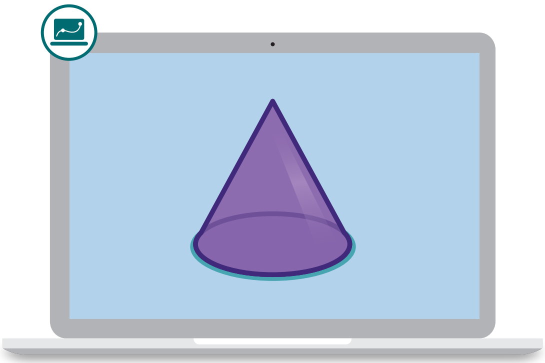
2. Use this space to ask a question or share something you're proud of.

Unit 5

Lesson

13

Name: ________________ Date: ________ Period: ________



Cones

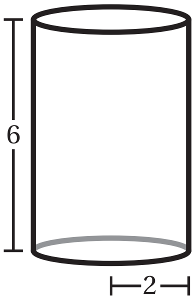
Let's explore cones and their volumes.

Warm-Up

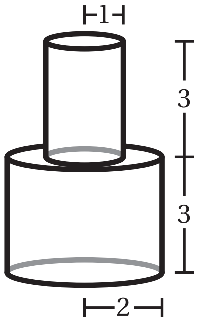
1 Determine the volume of figure A (a cylinder) and figure B (which is composed of two cylinders).

Write their volumes in terms of π.

Figure	Volume (cu. cm)
A	
B	



A



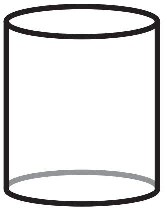
B

Name: ______________ Date: ______ Period: ______

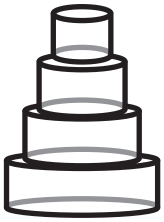
Estimating the Volume of a Cone

2 Here are four figures with the same height and the same radius for their largest bases.

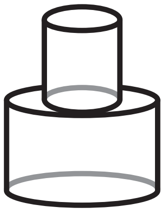
Cylinder



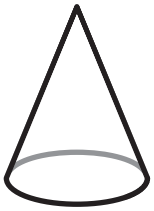
Four cylinders



Two cylinders



Cone



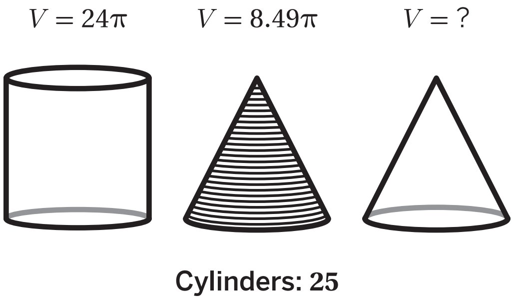
Order the figures by volume from *least* to *greatest*.

Least **Greatest**

3 Let's see what happens when we increase the number of cylinders.

What do you think the exact volume of the cone might be?

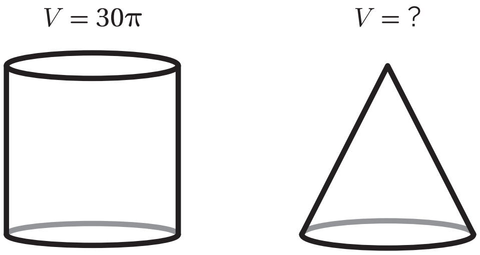
Explain your thinking.



4 Here is another set of figures.

a Estimate the volume of the cone.

b Let's see what happens when we increase the number of cylinders.



Discuss: What do you notice about the relationship between cylinders and cones?

Activity

2

Name: ______ Date: ______ Period: ______

Volume of a Cone

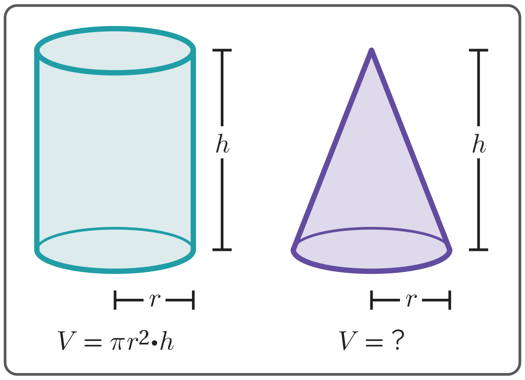
5 Each row of the table shows the volumes of a cylinder and a cone with the same height and radius.

Fill in the unknown values.

Volume of Cylinder (cu. cm)	Volume of Cone (cu. cm)
24π	8π
30π	10π
120π	
60π	
	15π

6 One way of writing a formula for the volume of a cylinder with radius r and height h is $V = \pi r^2 \cdot h$.

Write a formula for the volume of a cone.



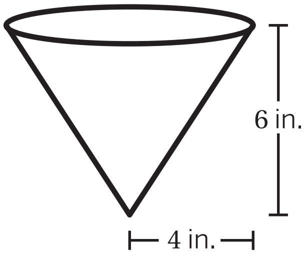
Activity
3

Name: .... Date: .... Period: ....

Comparing the Volume of a Cone

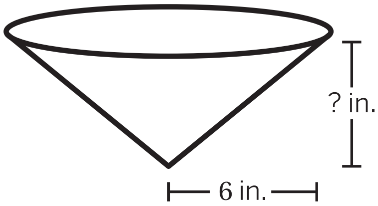
7 Let's look at one way of writing a formula for the volume of a cone with radius r and height h.

Calculate the volume of this cone.



8 The volume of this cone is 60π cubic inches.

What is the height of the cone?

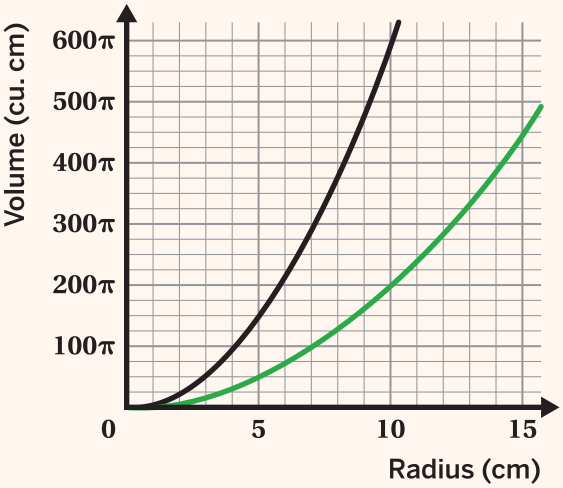


Explore More

9 This graph shows the relationship between radius and volume for cones and cylinders that have the same height.

a Label each curve to show which shape it represents.

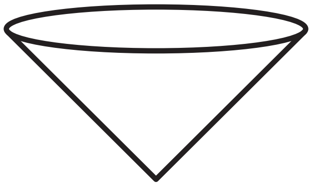
b What is the height of these cylinders and cones? Explain your thinking.



10 Synthesis

One way of writing the formula for volume of a cone is $V = \frac{1}{3}\pi r^2 \cdot h$.

What does each part of the formula represent?

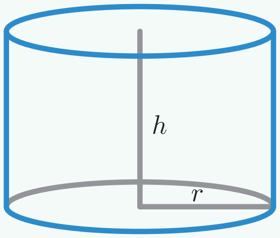


Lesson Practice 8.5.13

Lesson Summary

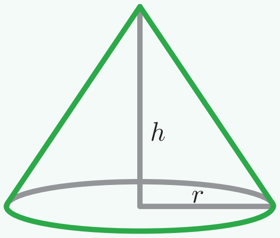
We learned that we can find the volume of a cylinder by calculating $V = \pi r^2 \cdot h$. If a cone and a cylinder have the same base and the same height, then the volume of the cone is one-third the volume of the cylinder.

If the radius and the height are known, we can determine the volume by using this formula for a cone: $V = \frac{1}{3}\pi r^2 \cdot h$.



Volume of a cylinder:

$V = \pi r^2 h$



Volume of a cone:

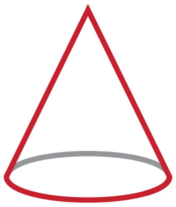
$V = \frac{1}{3}\pi r^2 h$

Lesson Practice

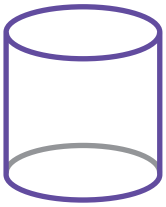
8.5.13

Name: .... Date: .... Period: ....

1. The volume of a cone is 36π cubic units. What is the volume of a cylinder with the same radius and the same height?

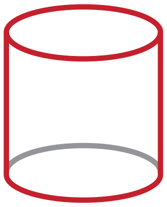


$V = 36\pi$

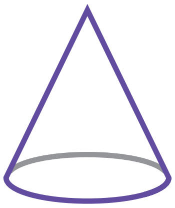


$V = ?$

2. The volume of a cylinder is 175π cubic units. What is the volume of a cone with the same radius and the same height?



$V = 175\pi$



$V = ?$

3. A cylinder and a cone have the same height and radius. The height of each is 5 centimeters, and the radius is 2 centimeters. Calculate the volume of the cylinder and the cone (rounded to the nearest tenth). Use 3.14 as an approximation for π.

Cylinder: ....

Cone: ....

Problems 4–6: This table shows the radiuses of four cones with a height of 18 meters.

4. Complete the table with the volume of each cone.

Radius (m)	Volume (cu. m)
1	
2	
3	
4	

5. Based on your table, if the radius of a cone doubles, does the volume also double? Explain your thinking.

6. Based on your table, is the relationship between the radius of a cone and its volume linear? Explain your thinking.

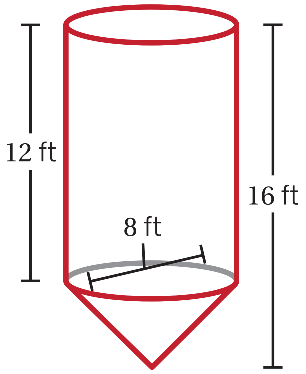
Lesson Practice

8.5.13

Name: .... Date: .... Period: ....

7. A silo is a large cylindrical container used on farms to hold grain. On Estaban's farm, a silo has a cone-shaped spout on the bottom to regulate the flow of grain going out. The diameter of the silo is 8 feet. The cylindrical part of the silo has a height of 12 feet, and the height of the entire silo is 16 feet.

Approximately how many cubic feet of grain can the entire silo hold? Explain your thinking.



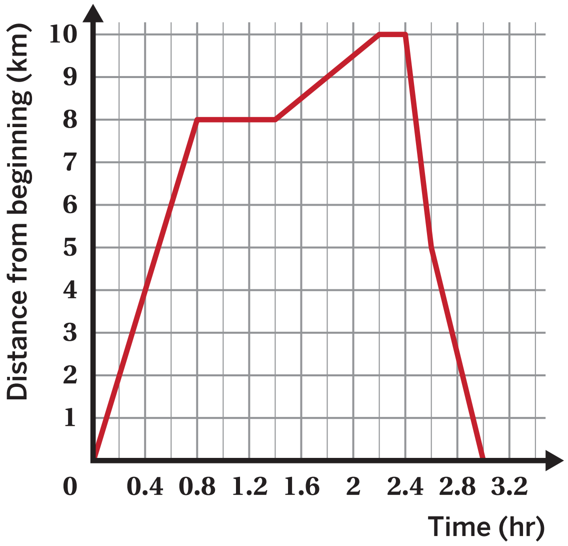
Spiral Review

Problems 8–10: This graph shows a trip on a bike trail.

8. When was the bike rider going the fastest?

9. During what times did the rider stop?

10. During what times was the rider going back toward the beginning of the trail?



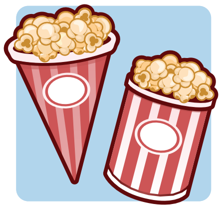
Reflection

1. Put a star next to a problem you could explain to a classmate.

2. Use this space to ask a question or share something you're proud of.

Name: .... Date: .... Period: ....

Unknown Dimensions

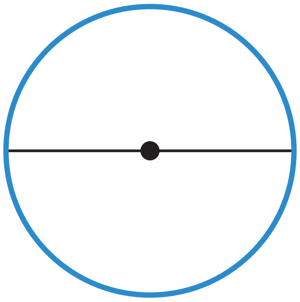


Let’s determine unknown dimensions.

Warm-Up

1. Here is a circle with an area of 81π square inches.

What is its diameter? Show or explain your thinking.



Activity
1

Name: .... Date: .... Period: ....

Cylinder Dimensions

Each row of this table lists the dimensions of a different cylinder.

2. Complete the table.

Diameter (units)	Radius (units)	Base Area (sq. units)	Height (units)	Cylinder Volume (cu. units)
		81π	2	
12				108π
			11	99π
		100π		20π
			h	$\pi \cdot r^2 \cdot h$

3. A cylinder has a diameter of 8 centimeters and a volume of 48π cubic centimeters.

a Draw the cylinder.

b Determine its height. Show or explain your thinking.

Activity
2

Name: ______ Date: ______ Period: ______

Cone Dimensions

Each row of this table lists the dimensions of a different cone.

4. Complete the table.

Diameter (units)	Radius (units)	Base Area (sq. units)	Height (units)	Cylinder Volume (cu. units)	Cone Volume (cu. units)
	4		3		
		36π	$\frac{1}{4}$		
20					200π
			12		64π

5. 💬 **Discuss:** How are determining the unknown dimensions of a cone and cylinder alike and different?

Activity

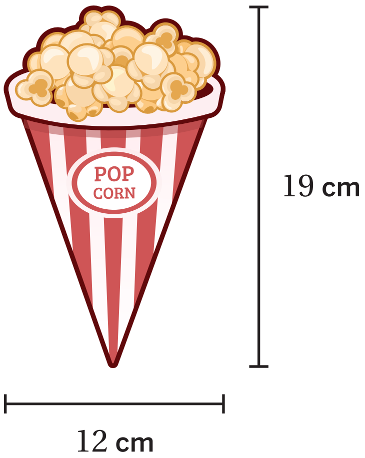
3

Name: .... Date: .... Period: ....

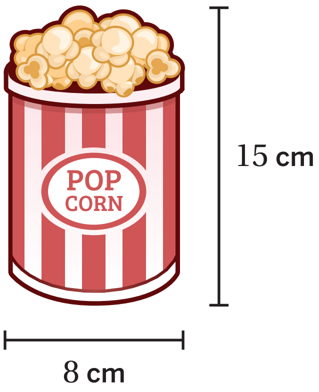
Which Is the Better Deal?

6. A movie theater offers two containers of popcorn for different prices:

$6.25



$6.50



Which container is the better deal? Show or explain your thinking.

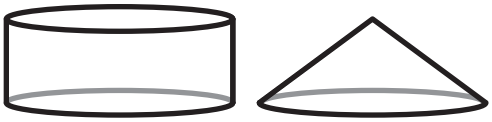
Explore More

7. Change either the diameter or the height of one of the popcorn containers so that they're an equally good deal.

Synthesis

8. Describe a strategy for determining an unknown dimension of a cylinder or cone.

Use the examples if they help with your thinking.



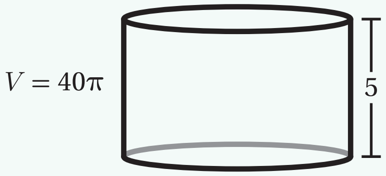
Lesson Practice 8.5.14

Lesson Summary

The volume of a cylinder and a cone depend on their radius and height. In both cases, if you know the radius and the height, you can determine the volume using the formula $V = \pi r^2 h$ (for cylinders) and $V = \frac{1}{3}\pi r^2 h$ (for cones).

And if you happen to know the volume and either the radius or the height, you can determine the other dimensions, too.

For example, if a cylinder has a height of 5 inches and a volume of 40π, you can calculate the area of the base by dividing the volume by the height: $\frac{40\pi}{5} = 8\pi$.



$$40\pi = B \cdot 5$$

$$8\pi = B$$

Lesson Practice
8.5.14

Name: .... Date: .... Period: ....

1. Draw a cylinder with a diameter of 6 centimeters and a volume of 36π cubic centimeters.

2. Determine the radius and the height of the cylinder from Problem 1. Show or explain your thinking. Then label your drawing with the cylinder's radius and height.

3. A cylinder has a diameter of 14 centimeters and a volume of $1{,}000$ cubic centimeters. What is its height? Express your answer to the nearest tenth of a centimeter.

4. Complete this table. The cylinder and cone in each row have the same dimensions.

Diameter (units)	Radius (units)	Base Area (sq. units)	Height (units)	Cylinder Volume (cu. units)	Cone Volume (cu. units)
	5		7		
6					40π
		36π			48π

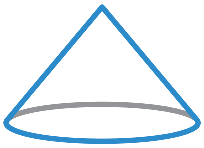
5. An ice cream shop offers two types of ice cream cones that hold the same volume. The waffle cone holds 12 ounces and is 5 inches tall. The sugar cone holds 12 ounces and is 8 inches tall. Which cone has a larger radius?

Waffle cone　　Sugar cone　　They have the same radius

Explain your thinking.

Name: ____________ Date: ________ Period: ________

6. The volume of this cone is 33π cubic units. What is the volume of a cylinder that has the same base area and the same height? Explain your thinking.



Spiral Review

Problems 7–8: A cone-shaped container is used to serve roasted almonds at a hockey game. The container has a diameter of 6 centimeters and a height of 7 centimeters.

7. Draw the cone. Label its height and radius.

8. If the container is filled completely with roasted almonds, how many cubic centimeters will it hold? Show or explain your thinking.

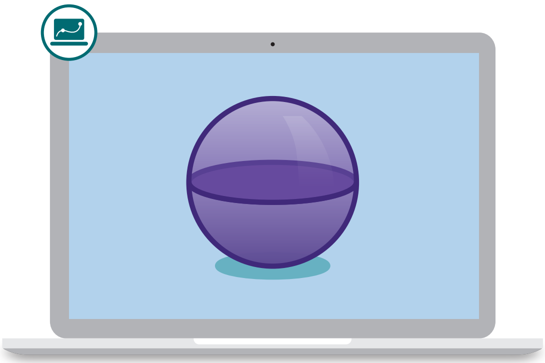
Reflection

1. Put a smiley face next to a problem you were stuck on and then figured out.

2. Use this space to ask a question or share something you're proud of.

Unit 5
Lesson
15

Name: ________ Date: ________ Period: ________



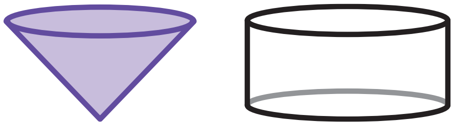
Spheres

Let’s develop a formula for the volume of a sphere.

Warm-Up

1 A cone and a cylinder have the same height and radius.

What fraction of the cylinder will be filled by the cone?



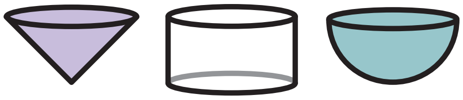
Name: ______ Date: ______ Period: ______

Hemispheres

2 What if we pour both a cone *and* a hemisphere into the cylinder?

Let's see what fraction of the cylinder will be filled.

Describe how the three volumes are related.



3 The volume of the cylinder is 27π cubic units.

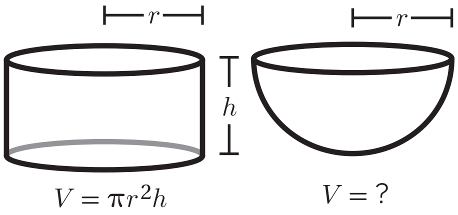
Write the volume of the cone and hemisphere.

Express the volumes in terms of π.

Object	Volume (cu. units)
Cone	
Cylinder	27π
Hemisphere	

4 One way of writing a formula for the volume of a cylinder is $V = \pi r^2 h$, where r is the radius and h is the height.

Write a formula for the volume of a hemisphere. Explain your thinking.



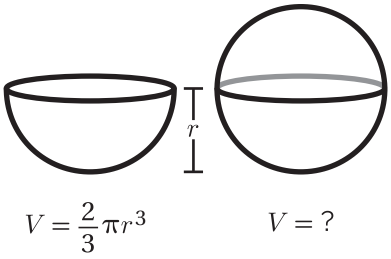
Activity
2

Name: ______ Date: ______ Period: ______

Finding Sphere Dimensions

5 One way of writing a formula for the volume of a hemisphere is $V = \frac{2}{3}\pi r^3$, where r is the radius.

Write a formula for the volume of a sphere. Explain your thinking.



6 Karima and Nasir are calculating the volume of a sphere with a radius of 2 units.

Karima

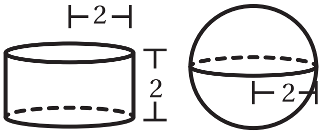
$V = \frac{4}{3}\pi r^3$ $r = 2$

$V = \frac{4}{3}\pi(2)^3$

$V = \frac{4}{3}\pi \cdot 8$

$V = \frac{32}{3}\pi$

Nasir



$V = \pi r^2 h$

$V = \pi(2)^2 \cdot 2$

$V = 8\pi$

$8\pi \cdot \frac{4}{3}$

$V = \frac{32}{3}\pi$

Discuss: How did each student determine the volume of a sphere?

Name: Date: Period:

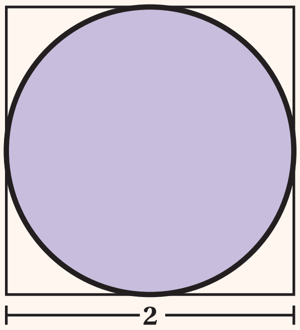
Finding Sphere Dimensions (continued)

7 Complete the table with the unknown dimensions of each sphere. Express your answers in terms of π.

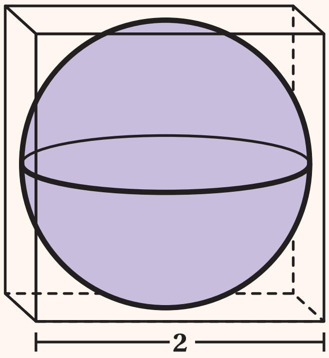
Diameter (units)	Radius (units)	Sphere Volume (cu. units)
	9	
8		
	3	
	6	
9		

Explore More

8 **a** What fraction of the square is filled by the circle?

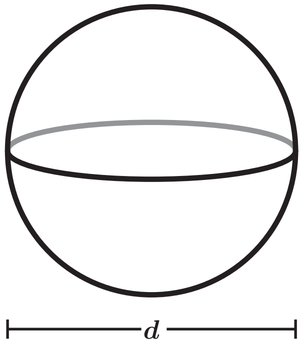


b What fraction of the cube is filled by the sphere?



9 Synthesis

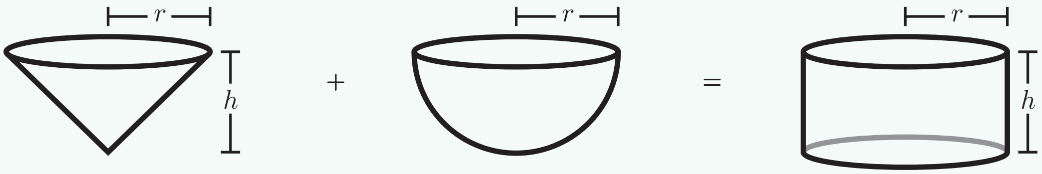
Describe a strategy for determining the volume of a sphere given its diameter.



Lesson Practice 8.5.15

Lesson Summary

You can determine the volume of a sphere using the formula $V = \frac{4}{3}\pi r^3$. If the radius and height of a cone, hemisphere, and cylinder are all the same, you can make sense of the volume formulas for each solid by seeing how much of the others they fill.



Volume of cone

$\left(\frac{1}{3} V \text{ of cylinder}\right)$

$V = \frac{1}{3}\pi \cdot r^2 h$

Volume of hemisphere

$\left(\frac{2}{3} V \text{ of cylinder}\right)$

$V = \frac{2}{3}\pi \cdot r^2 h$

Volume of cylinder

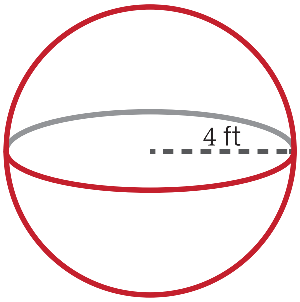
$V = \pi \cdot r^2 h$

The volume of the cone is $\frac{1}{3}$ of the volume of the cylinder. Since the volume of the cone and hemisphere together is equal to the cylinder, the volume of the hemisphere must be $1 - \frac{1}{3} = \frac{2}{3}$ of the volume of the cylinder. Then the volume of a sphere is twice the volume of a hemisphere, or $\frac{4}{3}$ the volume of the cylinder.

Lesson Practice
8.5.15

Name: .... Date: .... Period: ....

1. What is the volume of the sphere with a radius of 4 feet? Write your response in terms of π. Show or explain your thinking.



2. Calculate the volume of a sphere with a diameter of 6 inches. Write your answer both in terms of π and by using 3.14 to approximate π.

In terms of π: .... cubic inches

Using 3.14 as an approximation: .... cubic inches

3. Calculate the volume of a cylinder with a height of 6 inches and a diameter of 6 inches. Write your answer both in terms of π and by using 3.14 to approximate π.

In terms of π: .... cubic inches

Using 3.14 as an approximation: .... cubic inches

4. Calculate the volume of a cone with a height of 6 inches and a diameter of 6 inches. Write your answer both in terms of π and by using 3.14 to approximate π.

In terms of π: .... cubic inches

Using 3.14 as an approximation: .... cubic inches

5. In the previous three problems, you found the volumes of three solids with the same height and diameter. How are these volumes related?

Lesson Practice

8.5.15

Name: .... Date: .... Period: ....

6. A sphere has a radius of 5.1 centimeters. Determine the approximate volume of the sphere by using 3.14 for the value of π. Show or explain your thinking.

7. A soccer ball has a diameter of 22 centimeters. Which equation can be used to find V, the volume of the soccer ball in cubic centimeters?

A. $V = \frac{4}{3}\pi(22)^2$ **B.** $V = \frac{4}{3}\pi(11)^2$ **C.** $V = \frac{4}{3}\pi(22)^3$ **D.** $V = \frac{4}{3}\pi(11)^3$

Spiral Review

Problems 8–11: Evaluate each expression.

8. $\left(\frac{1}{2}\right)^2 \cdot 8$

9. $3^2 + 2^3$

10. $(2 \cdot 3)^2$

11. $\left(\frac{1}{3}\right)^2 \cdot 3^2$

Reflection

1. Circle the problem you think will help you most on the End-of-Unit Assessment.

2. Use this space to ask a question or share something you're proud of.

Name: Date: Period:

Practice Day 2

Let's practice what you've learned so far in this unit!

You will use task cards for this Practice Day. Record all of your responses here.

Task A: TV Snacks

1. Response:

2. Circle One: Yes No

Explain:

Explore More

Segment 1:

Segment 2:

Segment 3:

Segment 4:

Task B: Cylinders and Cones

1. Height:

2. Volume:

Explore More

Radius:

Height:

Name: Date: Period:

Practice Day 2 (continued)

Task C: Basketball

1. Circle One: Yes No

Explanation:

2. Time:

Explore More

Statement 1:

Statement 2:

Statement 3:

Task D: All the Spheres

1.

Radius (cm)	Volume of Sphere (cu. cm)
1	
2	
3	
4	

2. Circle One: Yes No

Explanation:

Explore More

Diameter:

Name: ________ Date: ________ Period: ________

Practice Day 2 (continued)

Task E: Missing-Dimension Detective

1. Height: ________

2. Radius: ________

Explore More

Circle One: Height Radius

Explanation:

Task F: Hydrate!

1. Milliliters of water: ________

2. Equation: ________

Explore More

Miles: ________