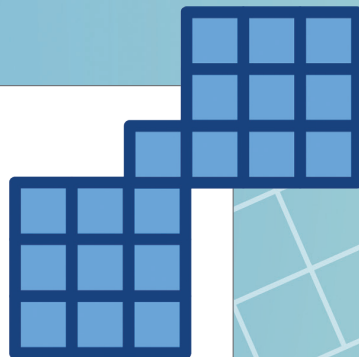


Unit **7**

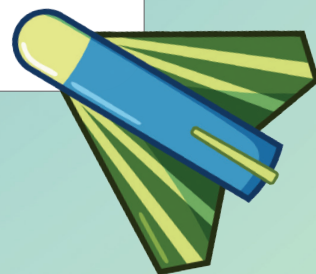
# Quadratic Functions



What does a ball flying through the air have in common with a business estimating how much money they can make selling their products? Both situations can be modeled by quadratic functions. In this unit, you will learn about how quadratic relationships are different from linear and exponential relationships, and create equations in different forms to represent them. You will model situations with quadratic functions and make predictions and decisions based on your models.

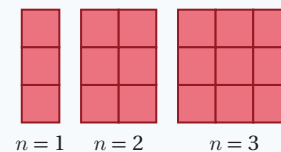
## Essential Questions

- What are the important features of quadratic relationships?
- How can we graph and write equations for quadratic functions in different forms?
- What situations can be modeled by quadratic functions?

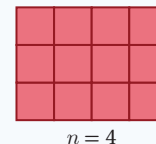


You can investigate visual patterns to determine how to build a particular figure or write an expression to represent the pattern.

Here is an example of the first three steps of a visual pattern.



You can describe what changes and what stays the same to help you draw the next figure, where  $n = 4$ .



Describing how you see a visual pattern changing can help you write a rule or expression to determine the other values in the pattern.

Here are two ways you may determine value of the tenth step of this pattern, or how many tiles are in  $n = 10$ .

- If you see the pattern increasing by a column of 3 each step, you could add 3 more until you reach  $n = 10$ .
- If you see each step in the pattern as a rectangle that has the dimensions 3-by- $n$ , you could multiply 3 by 10 to find the total number of tiles.
- In both cases, you would determine that there will be 30 tiles when  $n = 10$ .

$n$	Number of Tiles
1	3
2	6
3	9
...	...
10	30

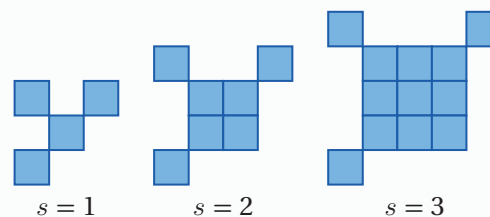
Arrows indicate an increase of +3 between steps 1, 2, 3, and 10. A curved arrow from 10 to 30 is labeled  $\times 3$ .

There are many ways to see and describe visual patterns accurately.

## Try This

Here are the first three steps of a pattern.

- a** Describe how the pattern changes.



- b** Determine the number of tiles when  $s = 10$ .

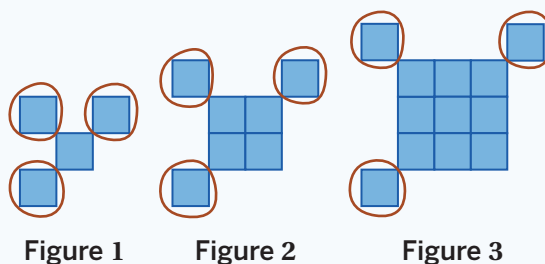
You can determine if a visual pattern represents a linear, exponential, or **quadratic relationship**, or something else.

If you can observe a square that is changing throughout a pattern, the relationship might be quadratic and you can use a squared term to write a quadratic expression.

Here are some strategies you might use to write an expression to represent a visual pattern:

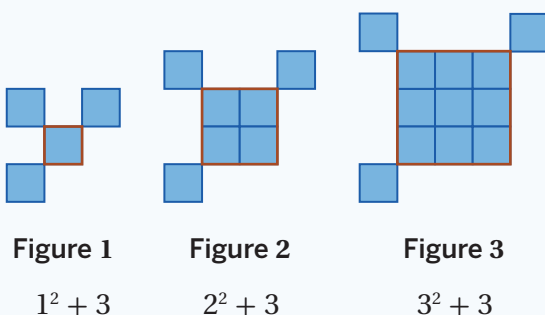
You can look for what is changing and staying the same.

- The 3 outside tiles stay the same.
- The interior square is growing from 1 to 4 to 9.



You can look for where you see the figure number,  $n$ , in each diagram.

- You can see the figure number as the side length of the growing square.
- You can write each number of tiles as an expression.

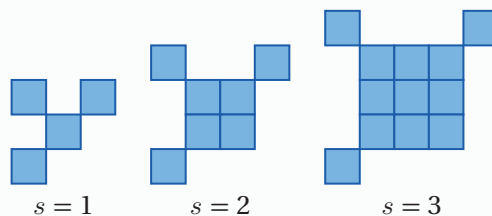


You can write a quadratic expression to represent the number of tiles in Figure  $n$  as  $n^2 + 3$ .

## Try This

Here are the first three steps of a pattern.

- a** Does this pattern show a quadratic relationship? Explain your thinking.



- b** Write an expression for the number of tiles in terms of  $s$ .

## Summary | Lesson 3

You can analyze a table of values, a pattern, or an equation to help you determine whether a relationship is linear, quadratic, exponential, or something else.

- Linear relationships have a constant first difference in the  $y$ -values when the  $x$ -values change by a constant value.
- Quadratic relationships change by a constant **second difference**, the difference between the first differences. A **quadratic function** is a function that represents a quadratic relationship.
- Exponential relationships have a constant ratio.

Here are some examples.

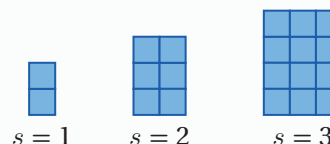
	Linear	Quadratic	Exponential																														
Table	<table><thead><tr><th><math>x</math></th><th><math>f(x)</math></th></tr></thead><tbody><tr><td>1</td><td>3</td></tr><tr><td>2</td><td>6</td></tr><tr><td>3</td><td>9</td></tr><tr><td>4</td><td>12</td></tr></tbody></table> <div><div><math>\curvearrowright +3</math></div><div><math>\curvearrowright +3</math></div><div><math>\curvearrowright +3</math></div></div>	$x$	$f(x)$	1	3	2	6	3	9	4	12	<table><thead><tr><th><math>x</math></th><th><math>g(x)</math></th></tr></thead><tbody><tr><td>1</td><td>2</td></tr><tr><td>2</td><td>5</td></tr><tr><td>3</td><td>10</td></tr><tr><td>4</td><td>17</td></tr></tbody></table> <div><div><math>\curvearrowright +3</math></div><div><math>\curvearrowright +5</math></div><div><math>\curvearrowright +7</math></div><div><math>\curvearrowright +2</math></div><div><math>\curvearrowright +2</math></div></div>	$x$	$g(x)$	1	2	2	5	3	10	4	17	<table><thead><tr><th><math>x</math></th><th><math>h(x)</math></th></tr></thead><tbody><tr><td>1</td><td>6</td></tr><tr><td>2</td><td>12</td></tr><tr><td>3</td><td>24</td></tr><tr><td>4</td><td>48</td></tr></tbody></table> <div><div><math>\curvearrowright \cdot 2</math></div><div><math>\curvearrowright \cdot 2</math></div><div><math>\curvearrowright \cdot 2</math></div></div>	$x$	$h(x)$	1	6	2	12	3	24	4	48
$x$	$f(x)$																																
1	3																																
2	6																																
3	9																																
4	12																																
$x$	$g(x)$																																
1	2																																
2	5																																
3	10																																
4	17																																
$x$	$h(x)$																																
1	6																																
2	12																																
3	24																																
4	48																																
Equation	$f(x) = 3x$	$g(x) = x^2 + 1$	$h(x) = 3 \cdot 2^x$																														

## Try This

Here are the first three steps of a pattern.

- a** Is the relationship between the step number and the number of tiles in this pattern quadratic, linear, or exponential?

Explain your thinking.



- b** Write an expression for the number of tiles in terms of  $s$ .

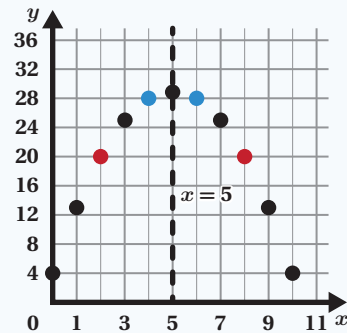
The graph of a quadratic function is called a **parabola**. Parabolas have a **line of symmetry** that goes through the maximum or minimum point. If you fold a parabola along this line, you get two identical halves. Here is a table and graph that represent a quadratic relationship.

Table

$x$	$y$
2	20
4	28
5	29
6	28
8	20

You can see the points are symmetrical across the line of symmetry at  $x = 5$ .

Graph

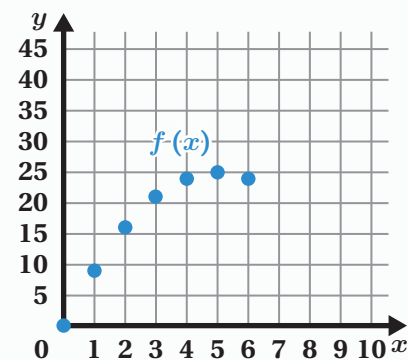


You can see the points create the shape of a parabola and are symmetrical across the line of symmetry at  $x = 5$ .

## Try This

Here are some points that belong to a quadratic function,  $f(x)$ .

- Draw and label the line of symmetry where you think it is located on this parabola.
- Plot four other points that will be part of this parabola. Explain your thinking.



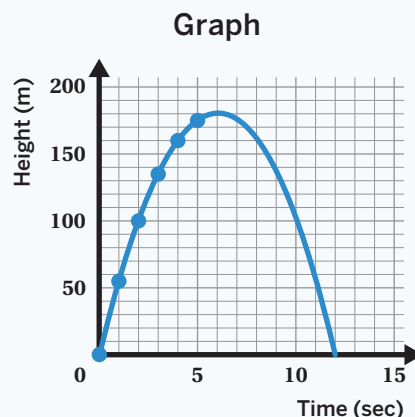
You can use tables and graphs to make predictions about quadratic relationships in context.

This table and graph show the height of a stomp rocket at various times.

**Table**

Time (sec)	Height (m)
0	0
1	55
2	100
3	135
4	160
5	175

$\begin{matrix} +55 \\ +45 \\ +35 \\ +25 \\ +15 \end{matrix}$ 
 $\begin{matrix} -10 \\ -10 \\ -10 \\ -10 \end{matrix}$



You can extend the pattern of the table using the second difference.

From the table, you can see how the rocket is 160 meters high after 4 seconds and 175 meters high after 5 seconds.

You can use the graph to determine the maximum height of the rocket by looking for the highest point on the parabola.

The maximum height of the rocket is 180 meters at 6 seconds.

You can also see from the graph that it takes 12 seconds for the rocket to land.

## Try This

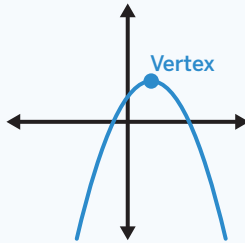
This table shows the height of a ball dropped from the Leaning Tower of Pisa.

- How high was the ball after 3 seconds?
- Will the ball hit the ground before or after 4 seconds? Explain your thinking.

Time (sec)	Height (ft)
0	190
1	174
2	126

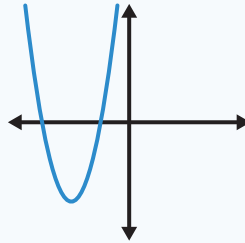
You can describe key features of *parabolas* with terms like: vertex, concave up, or concave down.

Vertex



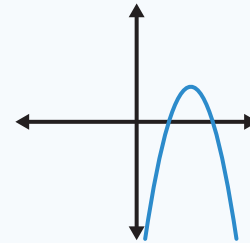
The vertex is the *maximum* or *minimum* point on a parabola (where a parabola changes from increasing to decreasing, or vice versa).

Concave Up



A parabola that opens upward is concave up.

Concave Down



A parabola that opens downward is concave down.

## Try This

Describe each parabola using the terms in the word bank.

$x$ -intercept

vertex

minimum

concave up

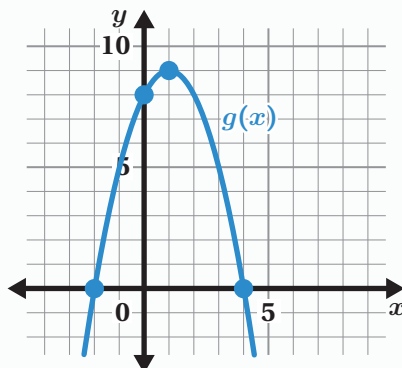
$y$ -intercept

axis of symmetry

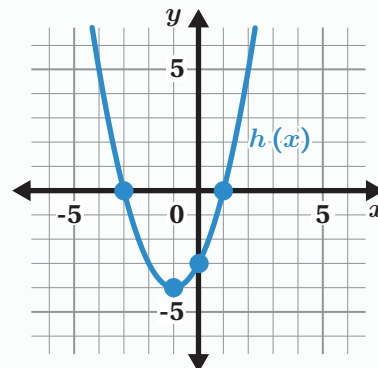
maximum

concave down

a



b



The graph of a *quadratic function* is a parabola. You can identify key features of a quadratic function from a graph, table, or equation to help you interpret quadratic functions in a context.

For example, this graph models the path of a stomp rocket.

You can use the  $y$ -intercept to describe the starting height of the stomp rocket.

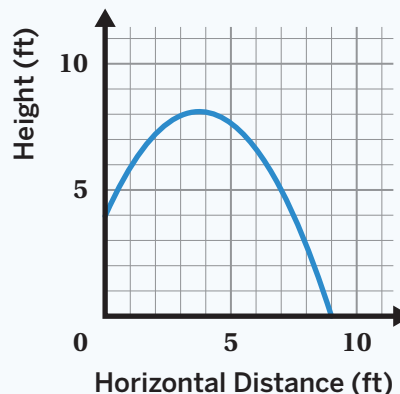
- The  $y$ -intercept is  $(0, 4)$ .
- This means that at the start, or when the horizontal distance was 0 feet, the height was 4 feet.

You can use the *vertex* to describe the maximum height of the stomp rocket.

- The vertex is about  $(3.5, 8)$ .
- This means that the stomp rocket reached a maximum height of 8 feet.

You can use the  $x$ -intercept to describe where the stomp rocket lands.

- The  $x$ -intercept on the graph is  $(9, 0)$ .
- This means that the horizontal distance was 9 feet when the rocket landed, or when the height was 0 feet.



## Try This

Here is the graph of a balloon's trajectory.

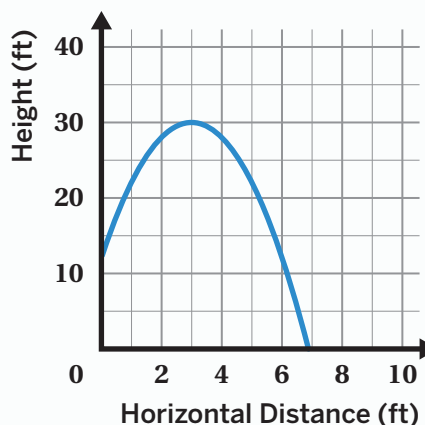
- a** Which key feature represents the starting height of the balloon? Circle one.

$x$ -intercept       $y$ -intercept      vertex

- b** Which key feature represents the horizontal distance where the balloon touches the ground? Circle one.

$x$ -intercept       $y$ -intercept      vertex

- c** What is the maximum height the balloon reaches?





You can graph a quadratic function by evaluating its equation at different  $x$ -values. A table can be helpful to organize your thinking for quadratics written in standard or factored form.

Here are two examples. You can use the table to determine the value of each function when  $x = -3$  and  $x = 1$ , and determine the corresponding points that are on the graph.

$$f(x) = 2x^2 + 8x - 10$$

$x$	$2x^2$	$8x$	$-10$	$2x^2 + 8x - 10$	Point on Graph
-3	$2(-3)^2 = 18$	$8(-3) = -24$	-10	$18 - 24 - 10 = -16$	$(-3, -16)$
1	$2(1)^2 = 2$	$8(1) = 8$	-10	$2 + 8 - 10 = 0$	$(1, 0)$

$$g(x) = (3x - 1)(x + 5)$$

$x$	$(3x - 1)$	$(x + 5)$	$(3x - 1)(x + 5)$	Point on Graph
-3	$3(-3) - 1 = -10$	$-3 + 5 = 2$	$(-10)(2) = -20$	$(-3, -20)$
1	$3(1) - 1 = 2$	$1 + 5 = 6$	$(2)(6) = 12$	$(1, 12)$

## Try This

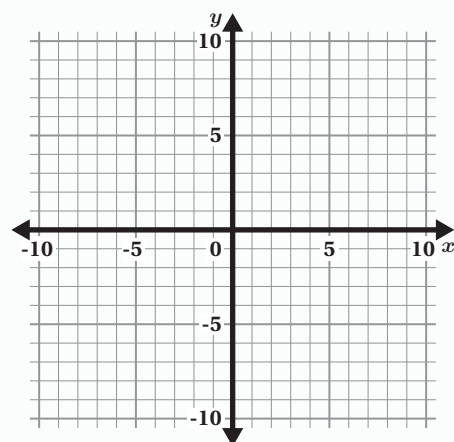
Here is a function:  $p(x) = (x - 3)(x + 3)$ .

- a** Determine three points on the graph of  $p(x)$ .

Use the table if it helps with your thinking.

$x$			

- b** Draw the graph of  $p(x)$ .



Quadratic equations can be written in many forms. Two of them are **standard form** and **factored form**.

The standard form of a quadratic equation has a squared term, and can have linear and/or constant terms added or subtracted.

The factored form of a quadratic equation has two factors multiplied together.

Here are some examples of functions written in different forms. In the last column,  $c(x)$  is a quadratic function written in a different form, and  $f(x)$  is an exponential function.

Factored Form	Standard Form	Neither/Not Quadratic
$a(x) = x^2 - 5x + 6$	$b(x) = \frac{1}{2}(x + 1)(x + 5)$	$c(x) = (x + 3)^2 - 1$
$d(x) = 2x^2 + 8x - 10$	$e(x) = (2x - 2)(x + 3)$	$f(x) = 3^x$

## Try This

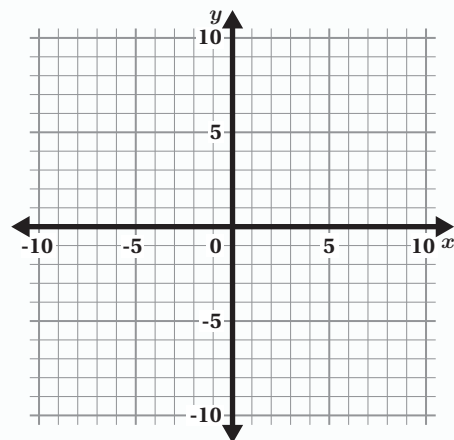
Here is a function:  $h(x) = x^2 - x + 2$ .

- a** Complete the table for  $h(x)$ .

$x$	$h(x)$

- b** Did the form of the equation affect the points you chose? Explain your thinking.

- c** Draw the graph of  $h(x)$ .



Different forms of quadratic equations help us see different key features of a parabola.

In a standard form quadratic equation, the *y-intercept* is the constant term in the equation.

In a factored form quadratic equation, the *x-intercepts* are the values that make each factor equal to 0.

Here is an example of the same function written in standard and factored form. We can use the different forms to determine key features and graph the parabola.

## Standard Form

$$f(x) = 2x^2 - 4x - 6$$

$$f(x) = 2x^2 - 4x - 6$$

The *y*-intercept is (0, -6).

## Factored Form

$$f(x) = (2x + 2)(x - 3)$$

<i>x</i>	$(2x + 2)$	$(x - 3)$	$(2x + 2)(x - 3)$
-1	$2(-1) + 2 = 0$	$(-1) - 3 = -4$	$(0)(-4) = 0$
3	$2(3) + 2 = 8$	$(3) - 3 = 0$	$(8)(0) = 0$

The *x*-intercepts are (-1, 0) and (3, 0).

## Try This

Here is the same quadratic function written in two forms.

Standard Form	Factored Form
$f(x) = 2x^2 + 8x - 10$	$f(x) = (2x - 2)(x + 5)$

**a** Determine the *x*-intercept(s) of the function.

**b** Determine the *y*-intercept of the function.

We can use key features of a quadratic function to create graphs.

Here is an example. Determine the  $x$ -intercepts and the vertex of the function

$$f(x) = (x + 5)(x - 1).$$

The  $x$ -intercepts of the graph are the  $x$ -values that make each factor equal to 0.

$x$	$(x + 5)$	$(x - 1)$	$(x + 5)(x - 1)$
-5	$(-5) + 5 = 0$	$(-5) - 1 = -6$	$(0)(-6) = 0$
1	$(1) + 5 = 6$	$(1) - 1 = 0$	$(6)(0) = 0$

The  $x$ -intercepts are  $(-5, 0)$  and  $(1, 0)$ .

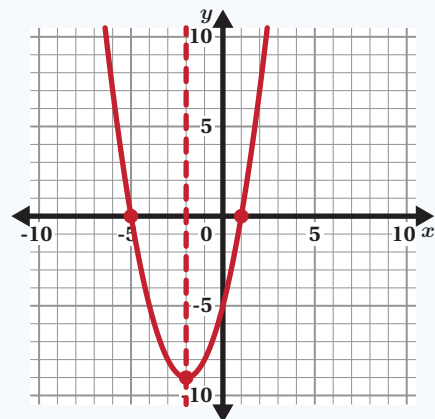
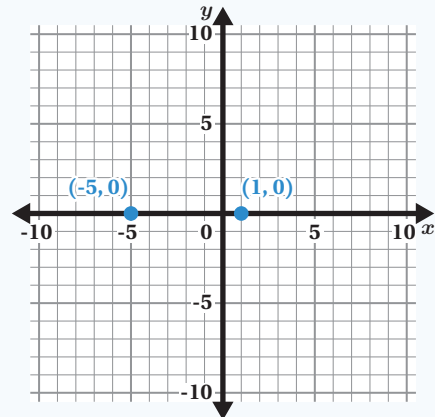
The vertex is always directly between the two  $x$ -intercepts.

The  $x$ -value of the vertex is -2 because -2 is exactly in the middle of the two  $x$ -intercepts.

We substitute  $x = -2$  into the equation to determine the  $y$ -value.

$x$	$(x + 5)$	$(x - 1)$	$(x + 5)(x - 1)$
-2	$(-2) + 5 = 3$	$(-2) - 1 = -3$	$(3)(-3) = -9$

The vertex is at the point  $(-2, 9)$ .

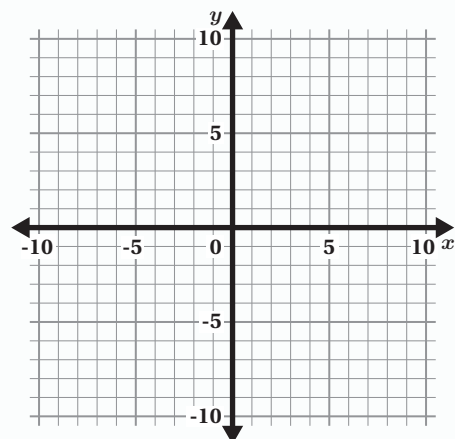


## Try This

Draw the graph of the function

$$k(x) = (4x - 4)(x - 3).$$

Include the  $x$ -intercepts and vertex.

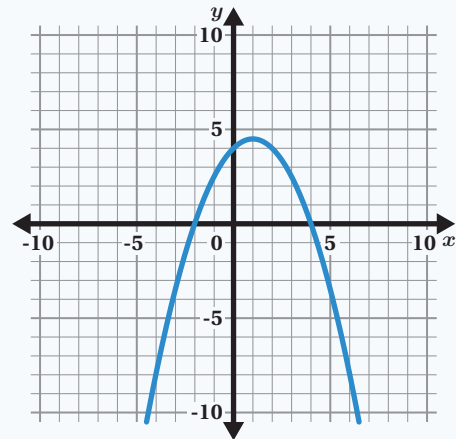


We can write quadratic equations to match graphs and key features. One way to do this is in the form  $y = a(x - m)(x - n)$ .

- Use the  $x$ -intercepts to write the *factors* of the equation.
- If the parabola is concave down, make the  $a$ -value negative.
- Adjust the  $a$ -value to match the vertical position of the vertex and the  $y$ -intercept.

Here is an example strategy for writing an equation to match this graph.

- The  $x$ -intercepts are  $(-2, 0)$  and  $(4, 0)$ . You can write the factors as  $y = (x + 2)(x - 4)$ .
- Since the parabola is concave down, multiply by a negative number, like  $y = -(x + 2)(x - 4)$ .
- Adjust the  $a$ -value to make the vertical position of the vertex match the graph, like  $y = -\frac{1}{2}(x + 2)(x - 4)$ .

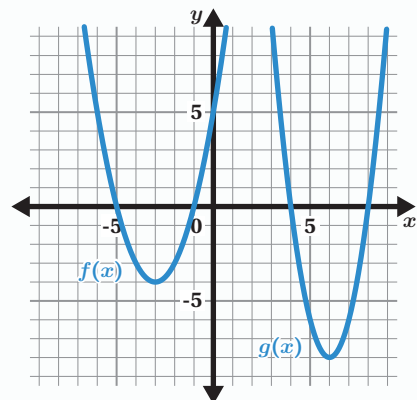


## Try This

Write a quadratic equation to match each graph.

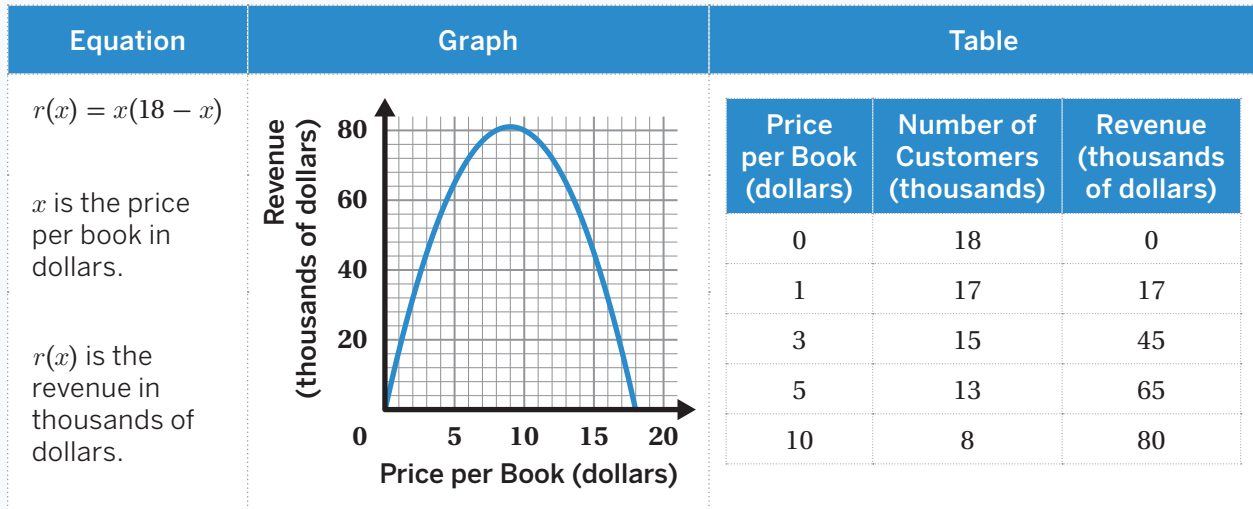
$f(x) =$

$g(x) =$



You can use quadratic functions to make sense of the amount of money made from selling an item, also called *revenue*. We can assume that if an item is more expensive, fewer people will buy it.

Here is an example of selling books.



You can see from the vertex on the graph that selling books for \$9 will earn a maximum revenue.

You can use the equation to determine the maximum revenue by evaluating  $r(x)$  when  $x = 9$ .

$$r(x) = x(18 - x)$$

$$r(9) = 9(18 - 9)$$

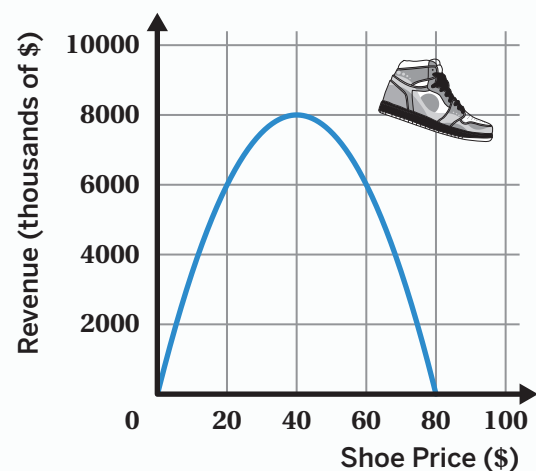
$$r(9) = 81 \quad \text{So the maximum revenue will be \$81,000.}$$

## Try This

Here is a graph that models the relationship between revenue and shoe price for a new shoe at the Lace Up Shoe Company.

- According to the model, what is the maximum predicted revenue for the Lace Up shoe?
- The Peak Sneaks shoe company uses the revenue model  $r(s) = s(20000 - 200s)$ , where  $r(s)$  represents predicted revenue and  $s$  represents shoe price. Which company would make the most revenue from selling their shoe for \$60?

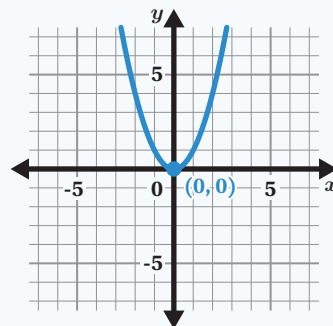
Explain your thinking.



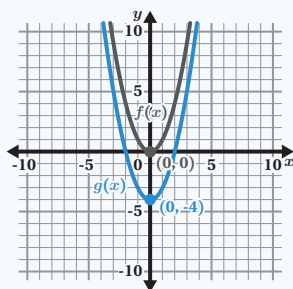
Here is the graph of  $f(x) = x^2$ .

Functions can be translated horizontally and vertically.

Here are three examples of *translations* of  $f(x)$ . The equations for these translations are written in **vertex form**, which highlights the coordinates of the vertex in the equation.



**Vertical Translations**

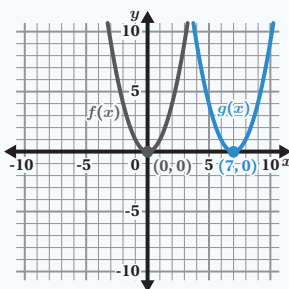


$f(x)$  is translated 4 units down.

Its equation is

$$g(x) = x^2 - 4.$$

**Horizontal Translations**

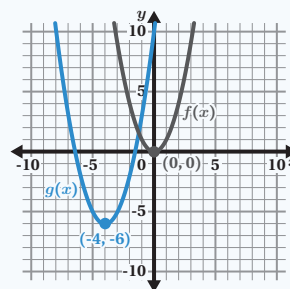


$f(x)$  is translated 7 units right.

Its equation is

$$h(x) = (x - 7)^2.$$

**Vertical and Horizontal Translations**



$f(x)$  is translated 4 units left and 6 units down.

Its equation is

$$j(x) = (x + 4)^2 - 6.$$

## Try This

Describe the translations of each function when compared to  $f(x) = x^2$ .

**a**  $g(x) = (x + 1)^2$

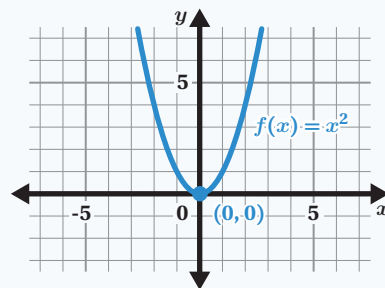
**b**  $h(x) = x^2 + 2$

**c**  $j(x) = (x + 3)^2 - 5$

Quadratic functions like  $f(x) = a(x - h)^2 + k$  are written in vertex form, where  $(h, k)$  is the vertex of the parabola, and  $a$  shows the **vertical stretch**.

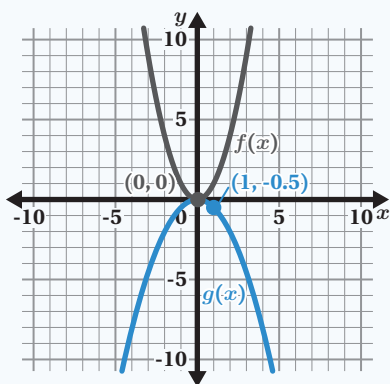
The  $a$ -value:

- Multiplies each output of the function by a constant value.
- Identifies the amount of vertical stretch in the  $y$ -direction.
- Tells whether a parabola is concave up or concave down.



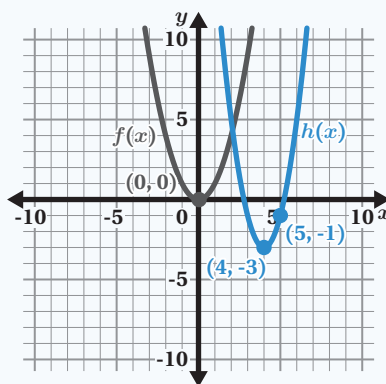
Let's look at some examples of quadratic functions with a vertical stretch:

**Vertical Stretch**



Here  $f(x)$  was vertically stretched by a factor of  $-\frac{1}{2}$  to make  $g(x) = -\frac{1}{2}x^2$ . This made the parabola wider and concave down.

**Vertical Stretch and Translations**

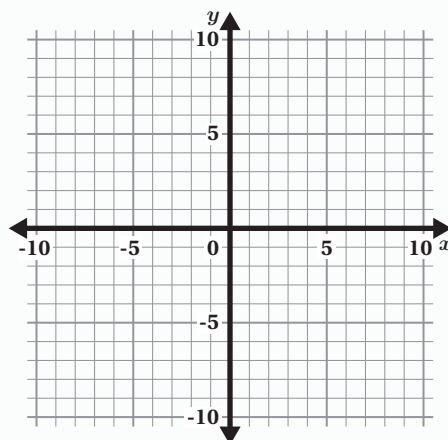


Here  $f(x)$  was vertically stretched by a factor of 2 and translated right 4 units and down 3 units to make  $h(x) = 2(x - 4)^2 - 3$ .

## Try This

Here is a function:  $f(x) = -3(x + 2)^2 + 5$ .

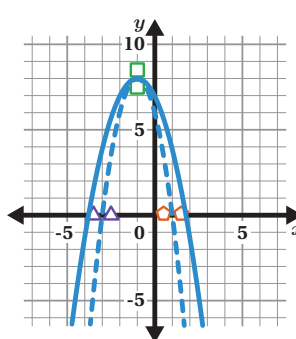
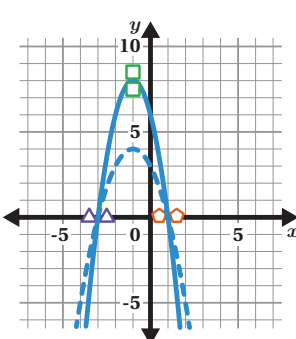
- Determine the vertex of  $f(x)$ .
- Determine the vertical stretch of  $f(x)$ .
- Draw the graph of  $f(x)$ .





You can use key features to write quadratic functions in standard, factored, or vertex form.

Here are two strategies for using key features to write an equation of a parabola that will pass through these gates:

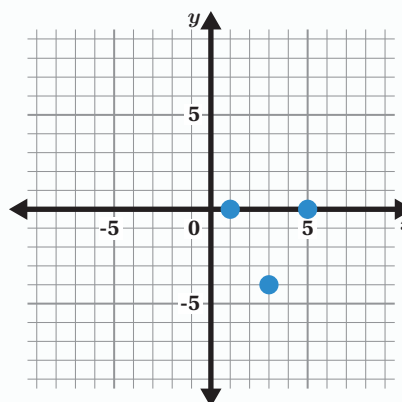
Vertex Form: $f(x) = a(x - h)^2 + k$	Factored Form: $f(x) = a(x - b)(x - c)$
<p>Identify a possible vertex: <math>(-1, 8)</math></p> <p><math>f(x) = a(x + 1) + 8</math></p>  <p>Since the parabola is concave down, the <math>a</math>-value needs to be negative. Then I can adjust the value to vertically stretch the parabola to match the <math>x</math>-intercepts.</p> <p><math>f(x) = -2(x + 1) + 8</math></p>	<p>Identify possible <math>x</math>-intercepts: <math>(-3, 0)</math> and <math>(1, 0)</math> Substitute the <math>x</math>-values of the <math>x</math>-intercepts into <math>b</math> and <math>c</math>:</p> <p><math>f(x) = a(x + 3)(x - 1)</math></p>  <p>Since the parabola is concave down, the <math>a</math>-value needs to be negative. Then I can adjust the value to vertically stretch the parabola to adjust the vertical position of the vertex.</p> <p><math>f(x) = -2(x + 3)(x - 1)</math></p>

## Try This

Write equations in each form for a parabola that passes through these points.

Factored form:

Vertex form:



We can use quadratic equations, tables, and graphs to make sense of situations in society. These mathematical models can help inform the decisions we make about real-world issues.

Today we explored two models that represent the cost of housing. A community organization could use these models to determine a fairer price of rent.

### Try This

A cupcake shop created the model  $r(x) = x(200 - 20x)$ , where  $x$  is the price of a cupcake and  $r(x)$  is the predicted revenue.

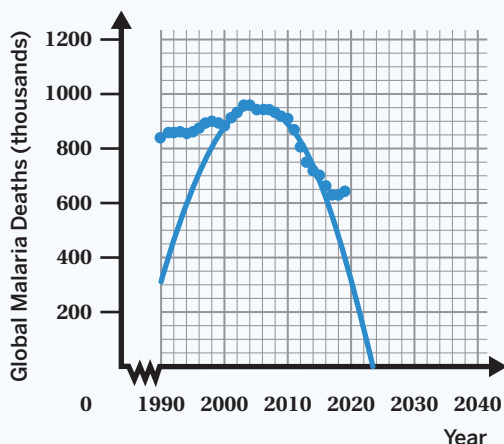
- a** Determine a fair price for a cupcake. Explain your thinking.
  
  
  
  
  
  
  
  
  
  
- b** Describe at least one disadvantage of this cupcake model.

Different types of functions can be used to model data and help us predict unknown data values. While models can be useful, they also have limitations. Some models may only be useful for predicting unknown values within a specific *domain*.

Let's look at two quadratic functions that could model the data for the number of global malaria deaths, measured in thousands, since 1990.

**Model A**

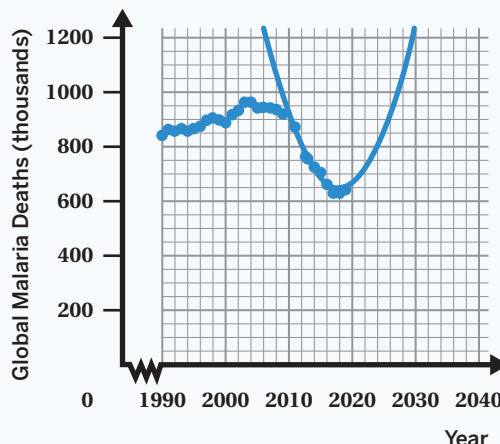
$$f(x) = -2.802(x - 28)^2 + 954$$



This model is a good fit for the data on the interval from 2000–2016. Beyond 2016, this model may not be useful because it suggests global malaria deaths reach zero by 2024, which is not true.

**Model B**

$$f(x) = 4.253(x - 15)^2 + 954$$



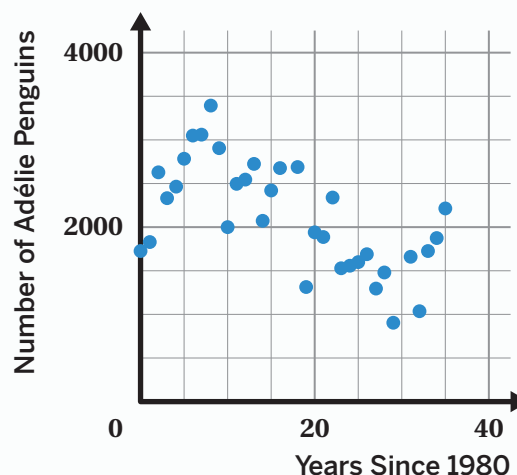
This model is a good fit for the data on the interval from 2010–2020. Beyond 2020, this model may not be useful because it suggests global malaria deaths will continue to increase forever, which may not be true.

## Try This

This graph shows how the Adélie penguin population on the South Orkney Islands has changed over time.

- Graph a quadratic function that would be useful in modeling part or all of this data.
- Do you think your model is useful for making a prediction about the Adélie penguin population in 2030 (50 years since 1980)?

Explain your thinking.



## Lesson 1

- a** *Responses vary.* The pattern forms a square in the middle surrounded by 1 tile in each of the three corners. The square in each step has a side length of  $s$ . The square's side length increases by 1 in each step.
- b** 103 tiles

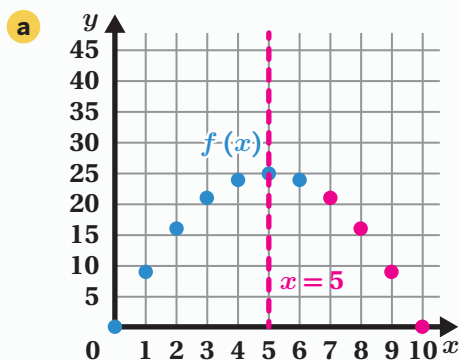
## Lesson 2

- a** *Yes. Explanations vary.* The pattern contains a square in the middle whose side length increases by 1 in each step.
- b**  $s^2 + 3$  (or equivalent)

## Lesson 3

- a** *Quadratic. Explanations vary.*
- There is a constant second difference of 4.
  - Each figure is made up of a square that is  $s$ -by- $s$  plus  $s$  tiles.
  - Quadratic relationships involve squaring a number.
- b**  $s^2 + s$  (or equivalent)

## Lesson 4



- b** Sample shown on graph. *Explanations vary.* I used the axis of symmetry to plot four more points on the parabola.

## Lesson 5

- a** 46 feet
- b** Before 4 seconds. *Explanations vary.* The second difference in the table is 32. This means that between 3 and 4 seconds, the ball would drop 112 feet. Since the ball is at 46 feet at 3 seconds, a fall of 112 feet between seconds 3 and 4 would mean the ball would reach the ground before 4 seconds.

Time (sec)	Height (ft)
0	190
1	174
2	126
3	46
4	-66

$190 - 174 = 16$   
 $174 - 126 = 48$   
 $126 - 46 = 80$   
 $46 - (-66) = 112$   
 $16 - 48 = -32$   
 $48 - 80 = -32$   
 $80 - 112 = -32$

## Lesson 6

- a** *Responses vary.* The parabola is concave down with a vertex at (1, 9). This vertex is a maximum. The axis of symmetry is  $x = 1$ . The  $y$ -intercept is (0, 8). The  $x$ -intercepts are (-2, 0) and (4, 0).
- b** *Responses vary.* The parabola is concave up with a vertex at (-1, -4). This vertex is a minimum. The axis of symmetry is  $x = -1$ . The  $y$ -intercept is (0, -3). The  $x$ -intercepts are (-3, 0) and (1, 0).

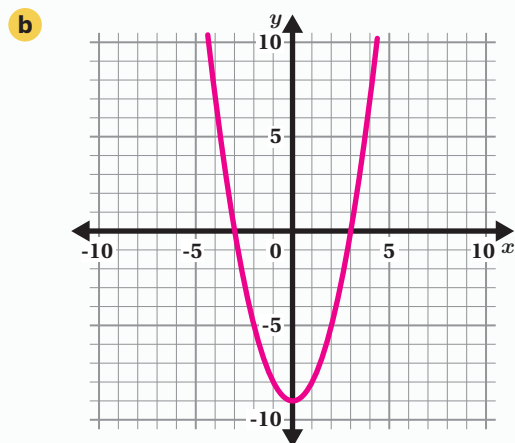
## Lesson 7

- a**  $y$ -intercept
- b**  $x$ -intercept
- c** The maximum height the balloon reaches is 30 feet.

## Lesson 8

- a** *Responses vary.*

$x$	$(x - 3)$	$(x + 3)$	$(x - 3)(x + 3)$
-3	-6	0	0
0	-3	3	-9
2	-1	5	-5

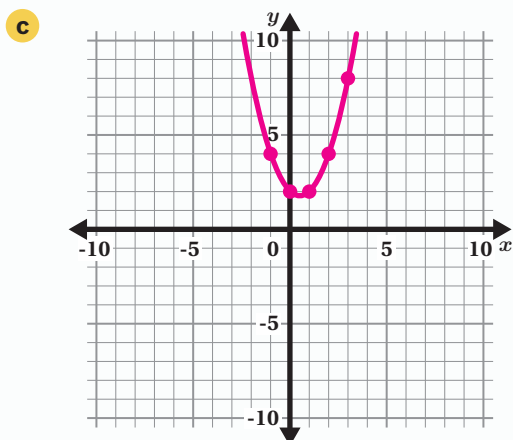


## Lesson 9

**a** Responses vary.

$x$	$h(x)$
-3	14
-1	4
0	2
1	2
3	8

**b** Responses vary. The equation is written in standard form. It was important to choose positive and negative  $x$ -values to attempt to capture the full shape of the parabola. Choosing 0 as an input allowed me to find the  $y$ -intercept of the graph.

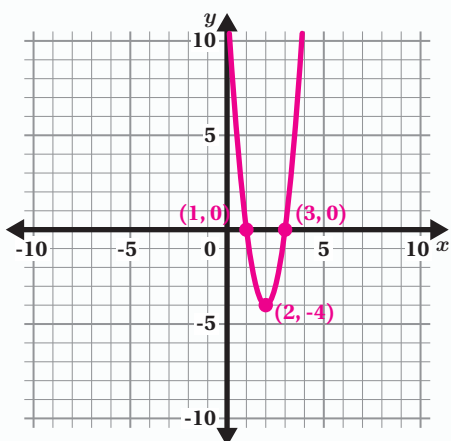


## Lesson 10

**a**  $(-5, 0)$  and  $(1, 0)$

**b**  $(0, -10)$

## Lesson 11



## Lesson 12

- a  $f(x) = (x + 5)(x + 1)$  (or equivalent)
- b  $g(x) = 2(x - 4)(x - 8)$  (or equivalent)

## Lesson 13

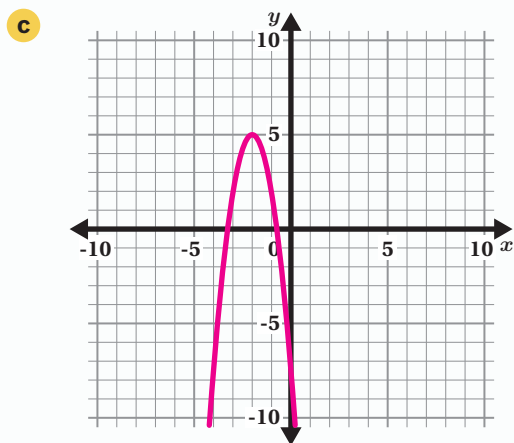
- a \$8,000,000
- b Lace Up Shoe Company. *Explanations vary.* At a price of \$60, the Lace Up Shoe company would generate \$6,000,000 in revenue, while Peak Sneaks would generate only \$480,000 in revenue.

## Lesson 14

- a  $g(x)$  is translated 1 unit to the left.
- b  $h(x)$  is translated 2 units up.
- c  $j(x)$  is translated 3 units to the left and 5 units down.

## Lesson 15

- a  $(-2, 5)$
- b The vertical stretch is -3.



## Lesson 16

Factored form:  $f(x) = (x - 1)(x - 5)$

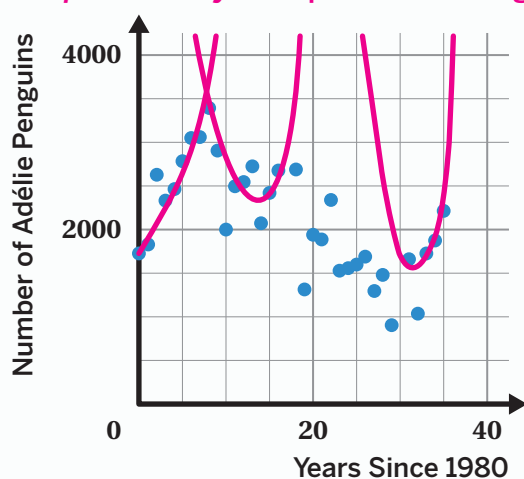
Vertex form:  $f(x) = (x - 3)^2 - 4$

## Lesson 17

- a *Responses vary.* \$5 is the price that would generate the most revenue, but it might make sense to charge less so that more people can afford the cupcakes.
- b *Responses vary.* This model doesn't consider other factors that may affect revenue of the cupcakes, like store location and dieting trends. The model assumes that all cupcakes will be priced the same and doesn't allow for bundles/packages of cupcakes.

## Lesson 18

- a *Responses vary.* Samples shown on graph.



- b *No. Explanations vary.* The data is not likely to continue on the same quadratic trend for more than 5 years.



# Algebra 1 Unit 7 Glossary/Álgebra 1 Unidad 7 Glosario

## English

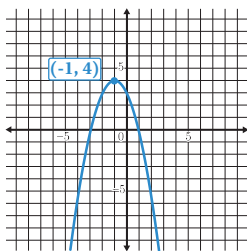
## Español

### C

#### concave down

A parabola that opens downward is described as concave down.

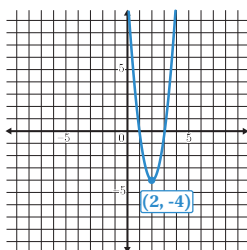
A function with a negative  $a$ -value will produce a concave down parabola.



This parabola is concave down. Two ways to write the equation are  $f(x) = -1(x + 1)^2 + 4$  and  $f(x) = -x^2 - 2x + 3$ .

#### concave up

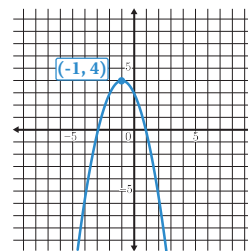
A parabola that opens upward is described as concave up. A function with a positive  $a$ -value will produce a concave up parabola.



This parabola is concave up. Two ways to write the equation are  $f(x) = 4(x - 2)^2 - 4$  and  $f(x) = 4x^2 - 16x + 12$ .

#### cóncava hacia abajo

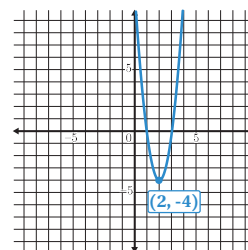
Una parábola que se abre hacia abajo se describe como cóncava hacia abajo. Una función con un valor  $a$  negativo producirá una parábola cóncava hacia abajo.



Esta parábola es cóncava hacia abajo. Dos formas de escribir la ecuación son  $f(x) = -1(x + 1)^2 + 4$  y  $f(x) = -x^2 - 2x + 3$ .

#### cóncava hacia arriba

Una parábola que se abre hacia arriba se describe como cóncava hacia arriba. Una función con un valor  $a$  positivo producirá una parábola cóncava hacia arriba.



Esta parábola es cóncava hacia arriba. Dos formas de escribir la ecuación son  $f(x) = 4(x - 2)^2 - 4$  y  $f(x) = 4x^2 - 16x + 12$ .

### F

**factored form** One of three common forms of a quadratic equation. A quadratic equation in factored form looks like:

$$f(x) = a(x - m)(x - n).$$

These equations are in factored form:

$$g(x) = x(x + 10)$$

$$2(x - 1)(x + 3) = y$$

$$y = (5x + 2)(3x - 1)$$

**forma factorizada** Una de las tres formas comunes de una ecuación cuadrática.

Una ecuación cuadrática en forma factorizada tiene el siguiente orden:

$$f(x) = a(x - m)(x - n).$$

Estas ecuaciones están en forma factorizada:

$$g(x) = x(x + 10)$$

$$2(x - 1)(x + 3) = y$$

$$y = (5x + 2)(3x - 1)$$

## English

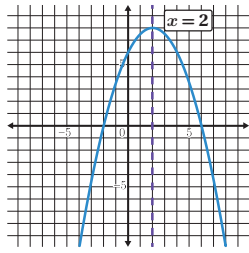
## Español

## I

**line of symmetry**

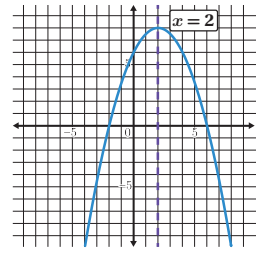
A line that divides a figure or the graph of a function into two halves. For every point (except the vertex), there is a corresponding point on the other side of the line that is the same distance from the line.

The equation of this line of symmetry is  $x = 2$ .


**eje de simetría**

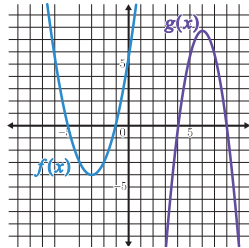
Una línea que divide una figura o la gráfica de una función en dos mitades. Cada punto (excepto el vértice) tiene un punto correspondiente en el otro lado de la línea, el cual está a la misma distancia de la línea.

La ecuación de este eje de simetría es  $x = 2$ .

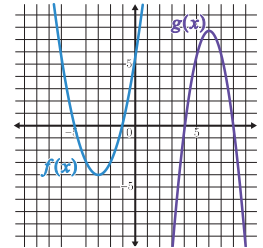


## P

**parabola** The graph of a quadratic function, which is a U-shaped curve.



**parábola** La gráfica de una función cuadrática, que es una curva en forma de U.



## Q

**quadratic function**

A function with output values that change by a constant second difference. Equations of quadratic functions have a squared term when written in standard form. The graph of a quadratic function is a parabola.

x	f(x)		
1	5		
2	8	+3	
3	14	+6	+3
4	23	+9	+3
5	35	+12	+3

**función cuadrática**

Una función con valores de salida que cambian de acuerdo con una segunda diferencia constante. Las ecuaciones de las funciones cuadráticas tienen un término elevado al cuadrado cuando se escriben en forma estándar. La gráfica de una función cuadrática es una parábola.

x	f(x)		
1	5		
2	8	+3	
3	14	+6	+3
4	23	+9	+3
5	35	+12	+3

**quadratic relationship** See *quadratic function*.

**relación cuadrática** Ver *función cuadrática*.

## R

**revenue** The amount of money generated by selling a product or service.

**ingresos** La cantidad de dinero que genera la venta de un producto o servicio.

# Algebra 1 Unit 7 Glossary/Álgebra 1 Unidad 7 Glosario

## English

## Español

### S

#### second difference

The differences between consecutive output values in the table of a function are called first differences. The differences between those values are called second differences. Quadratic functions have constant second differences.

x	f(x)
1	5
2	8
3	14
4	23
5	35

In this example, the first differences are 3, 6, 9, and 12. The second differences are constant at 3, so  $f(x)$  is a quadratic function.

#### standard form (of a quadratic equation)

One of three common forms of a quadratic equation. A quadratic equation in standard form looks like:  $f(x) = ax^2 + bx + c$ .

These equations are in standard form:

$$y = 2x^2 + 5x + 3$$

$$h(x) = x^2 + 3x$$

$$4x^2 - 7 = f(x)$$

#### segunda diferencia

Las diferencias entre valores de salida consecutivos en la tabla de una función se llaman primeras diferencias. Las diferencias entre dichos valores se llaman segundas diferencias. Las funciones cuadráticas tienen segundas diferencias constantes.

x	f(x)
1	5
2	8
3	14
4	23
5	35

En este ejemplo, las primeras diferencias son 3, 6, 9 y 12. Las segundas diferencias son constantes, de 3, por lo que  $f(x)$  es una función cuadrática.

#### forma estándar (de una ecuación cuadrática)

Una de las tres formas comunes de una ecuación cuadrática. Una ecuación cuadrática en forma estándar tiene el siguiente orden:  $f(x) = ax^2 + bx + c$ .

Estas ecuaciones están en forma estándar:

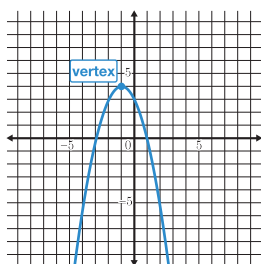
$$y = 2x^2 + 5x + 3$$

$$h(x) = x^2 + 3x$$

$$4x^2 - 7 = f(x)$$

### V

**vertex** On the graph of a quadratic or absolute value function, the vertex is the maximum or minimum point. The vertex is also where the function changes from increasing to decreasing, or vice versa.



**vertex form** One of three common forms of a quadratic equation. A quadratic equation in vertex form looks like:

$$f(x) = a(x - h)^2 + k.$$

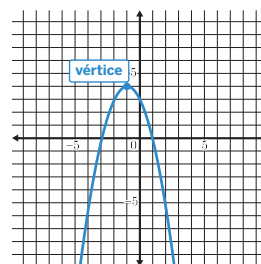
These equations are in vertex form:

$$(x - 3)^2 + 10 = g(x)$$

$$y = 2(x + 8)^2 - 1$$

$$f(x) = -(x - 6)^2 + 15$$

**vértice** En la gráfica de una función cuadrática o una función de valor absoluto, el vértice es el punto máximo o mínimo. El vértice también es donde la función cambia de creciente a decreciente, o viceversa.



**forma de vértice** Una de las tres formas comunes de una ecuación cuadrática. Una ecuación cuadrática en forma de vértice tiene el siguiente orden:  $f(x) = a(x - h)^2 + k$ .

Estas ecuaciones están en forma de vértice:

$$(x - 3)^2 + 10 = g(x)$$

$$y = 2(x + 8)^2 - 1$$

$$f(x) = -(x - 6)^2 + 15$$

## English

**vertical stretch** The result of multiplying the output values of a function by a factor. When a function is vertically stretched, the  $y$ -values of its graph move away from or toward the  $x$ -axis, but the  $x$ -values do not change.

## Español

**estiramiento vertical** El resultado de multiplicar los valores de salida de una función por un factor. Cuando una función tiene estiramiento vertical, los valores  $y$  de su gráfica se alejan del eje  $x$  o se acercan al eje  $x$ , pero los valores  $x$  no cambian.