Unit 3

Measuring Circles

Circles are round and come in many different sizes. One way to compare circles is by measuring the distance around the circle. In this unit, you will measure circles in a variety of ways and describe their size using these measurements.

Essential Questions

- How do we measure circles when all of our tools are straight?
- What is π and what does it have to do with circles?
- How can squares help you measure the space inside circles?

© Amplify Education, Inc and its licensors. Amplify Desmos Math is based on curricula from Illustrative Mathematics (IM).

Summary | Lesson 1

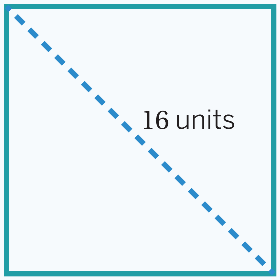
Proportional relationships can appear in geometry when you look at scaled copies of shapes.

Some examples of these proportional relationships include:

- The side lengths of a square and its perimeter
- The diagonal length of a square and its perimeter
- The diagonal length of an octagon and its perimeter

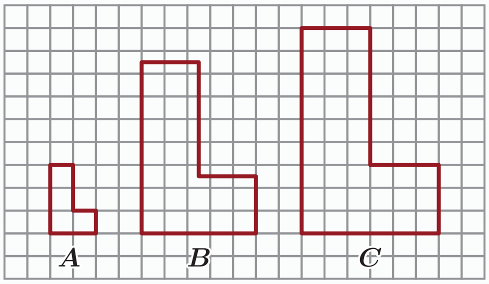
Because the relationship between the diagonal length and the perimeter of a square is proportional, you can determine one of the measurements if you know the other.

For example, to determine the perimeter of a square whose diagonal length is 16 units, you can multiply by the constant of proportionality, approximately 2.83. $16 \cdot 2.83 \approx 45$ units.



Try This

Figures A, B, and C are scaled copies. This table shows the relationship between the height and perimeter of each figure.



a Is the relationship proportional? Explain your thinking.

b What would be the perimeter of a scaled copy with a height of 12 units?

Height (units)	Perimeter (units)
3	10
7.5	25
9	30
12	

Summary | Lesson 2

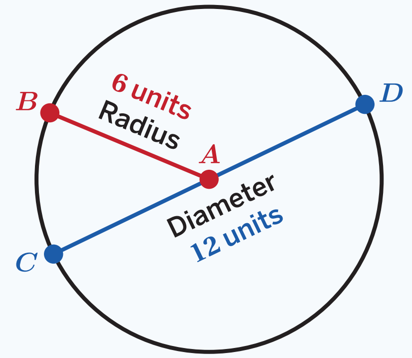
A *circle* is a shape made out of all the points that are the same distance from a center point.

You can measure a circle with its **radius**. A radius is a line segment that connects the center of a circle with a point on the circle. Every radius of a circle is the same length.

You can also measure a circle with its **diameter**, which is the distance from one point on a circle through the center to another point on the circle. It is also the longest distance across the circle.

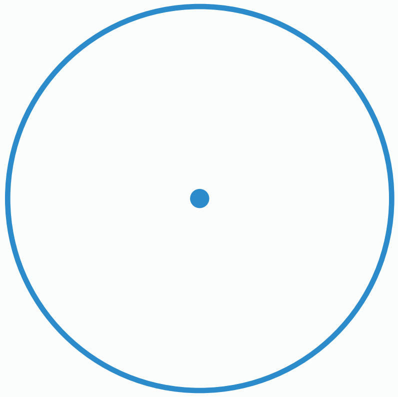
For any circle, the length of the diameter is two times the length of the radius.

For example, here is a circle. The radius AB is 6 units. The diameter CD is 12 units.



Try This

Here is a circle. Draw and label a radius and a diameter.

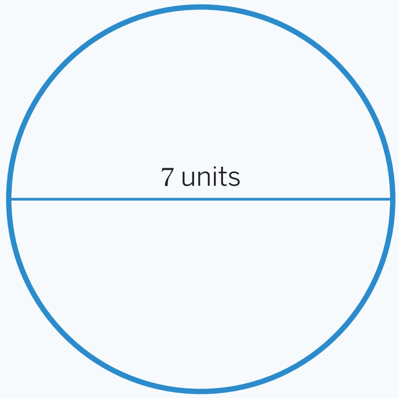


Summary | Lesson 3

The distance around a circle is called the **circumference**. There is a proportional relationship between the *diameter* of a circle and its circumference. The *constant of proportionality* for that relationship is π (**pi**). π is often approximated as 3.14 or $\frac{22}{7}$.

For any circle, we can calculate the circumference, C, using the equation $C = \pi d$.

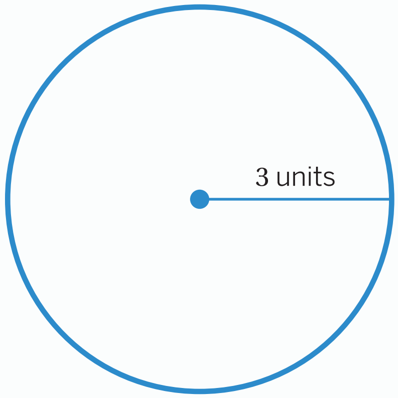
For example, if the diameter of a circle is 7 units, the circumference of the circle can be calculated approximately as $7 \cdot 3.14 = 21.98$. More accurately, $C = 7\pi$ units.



If we know the *radius* of a circle, we can calculate the circumference by first determining the diameter, then using the equation $C = \pi d$.

Try This

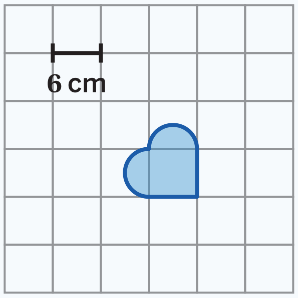
What is the circumference of a circle with a radius of 3 units?



Summary | Lesson 4

You can use what you know about the perimeter of squares and rectangles and the circumference of circles to find the perimeter of complex shapes.

- Determine what pieces it is made up of such as semicircles, quarter circles, and straight pieces.
- Determine the length of each straight piece and the radius or diameter of each partial circle.
- Determine the perimeter or circumference of each piece.
- Add them together to get the total perimeter.



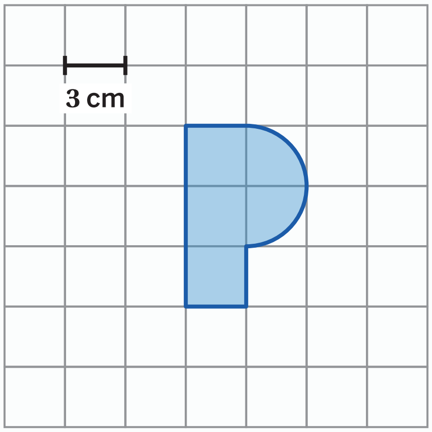
The perimeter of the heart shape is made up of 2 semicircles and 2 straight pieces. The *scale* is 6 centimeters.

Each semicircle has a diameter of 6 centimeters. Together they make up one entire circle with a diameter of 6 centimeters and a circumference of $6 \cdot \pi = 6\pi$ centimeters. Each straight edge is 6 centimeters long.

The total perimeter is $6\pi + 12$ centimeters.

Try This

Determine the perimeter of this shape.

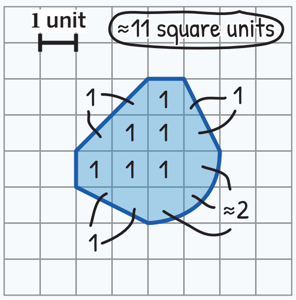


Summary | Lesson 5

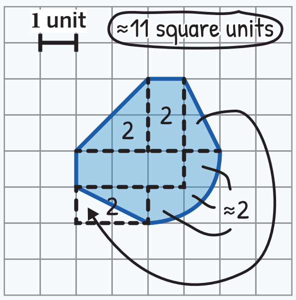
There are many ways you can find the *area* of a complex shape on a grid.

Here are some strategies you can use:

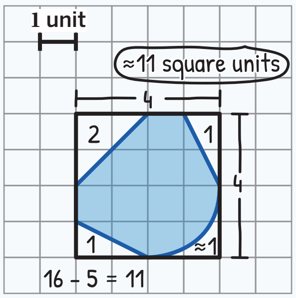
Counting whole and partial squares



Decomposing and rearranging

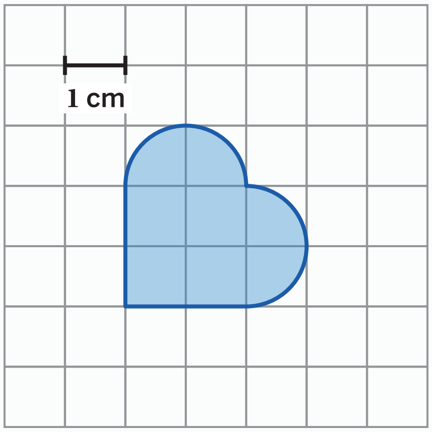


Surrounding and subtracting



Try This

Estimate the area of this shape.



Summary | Lesson 6

You can use radius squares to estimate the area of a circle.

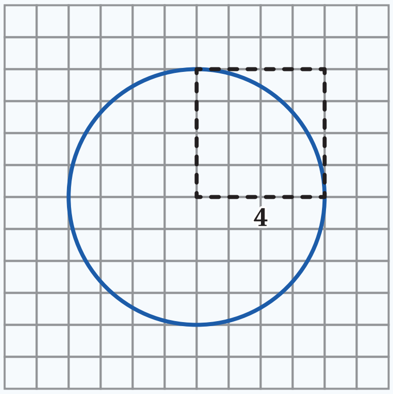
A radius square is a square whose side length is the same as the radius of the circle.

When you break apart and rearrange the radius squares, it takes a little more than 3 radius squares to cover a circle. This means that the area of the circle is a little more than three times the area of the radius square.

In the example, the radius is 4 units.

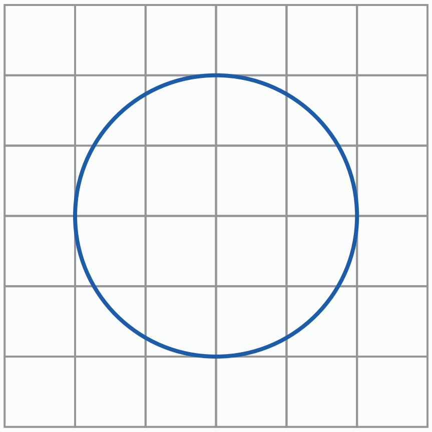
The area of the radius square is $4 \cdot 4$ or 4^2, which is equal to 16 square units.

Since it takes a little more than 3 radius squares to cover a circle, the area of the circle is a little more than $3 \cdot 16$, or a little more than 48 square units.



Try This

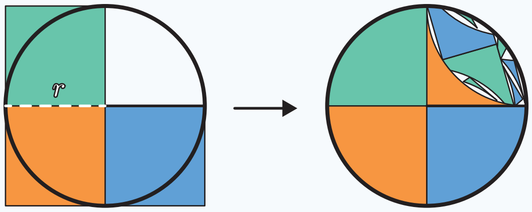
a Draw a radius square for this circle. Then determine the area of the radius square.



b Estimate the area of the circle. Explain your thinking.

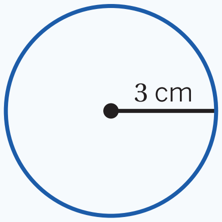
You can find the area of a circle if you know the length of its radius, r.

The *approximate* area of a circle is equal to the area of a little more than 3 radius squares. Each radius square has an area of r^2.



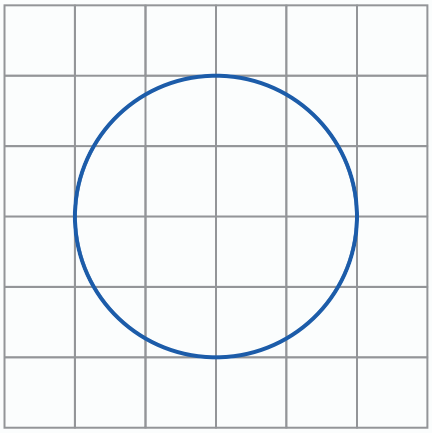
The *exact* area of a circle is equal to the area of π radius squares. You can express this with the formula $A = \pi \cdot r^2$.

For example, to find the area of a circle with a radius of 3 centimeters, you can calculate 3^2, then multiply the result by π. The area of the circle is 9π square centimeters.



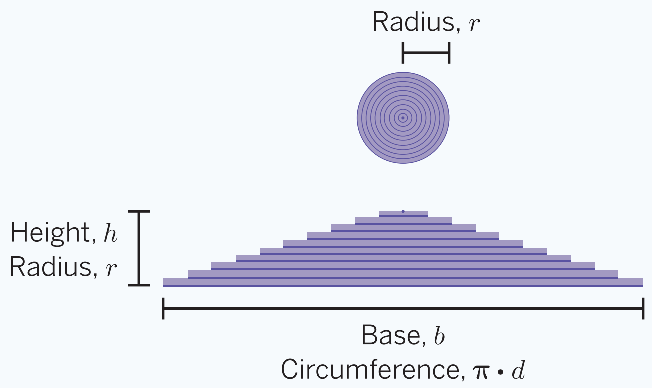
Try This

Determine the *exact* area of this circle.



If you take apart a circle and rearrange it to resemble a triangle, the formula for the area of a circle can be related to the formula for the area of the triangle.

This helps us make sense of each part of the formula for the area of a circle.



- The radius of a circle is r and the circumference is $\pi \cdot d$, so you can substitute those values into the equation for the area of a triangle.
- You can replace d with $2 \cdot r$.
- $\frac{1}{2} \cdot 2 = 1$, which leaves $A = \pi \cdot r^2$.

$$A = \frac{1}{2} \cdot b \cdot h$$

$$A = \frac{1}{2} \cdot (\pi \cdot \boldsymbol{d}) \cdot \boldsymbol{r}$$

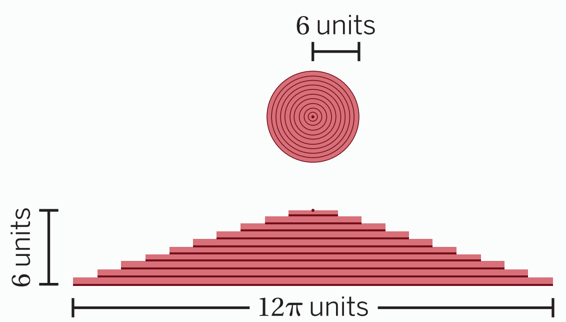
$$A = \frac{1}{2} \cdot \pi \cdot (2 \cdot r) \cdot r$$

$$A = \pi \cdot \boldsymbol{r} \cdot \boldsymbol{r}$$

$$A = \pi \cdot \boldsymbol{r}^2$$

Try This

Here is a circle cut into rings and unrolled into a triangle shape.



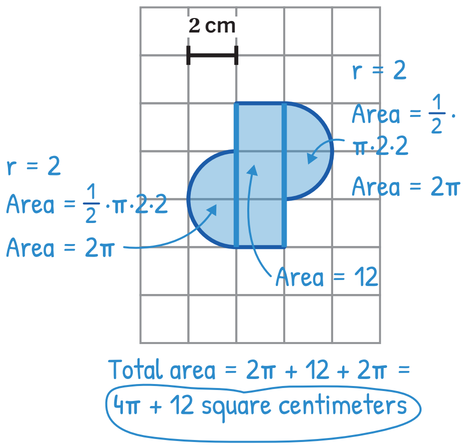
a Calculate the area of the circle.

b Calculate the area of the triangle.

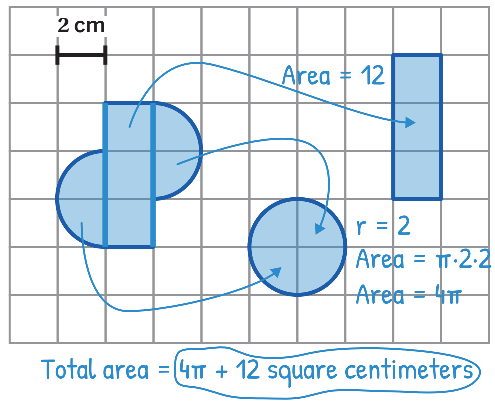
c How are the two areas related?

We can use different strategies and different ways of thinking to calculate the area of complex figures. Here are two ways we can determine the area of this shape:

Strategy 1



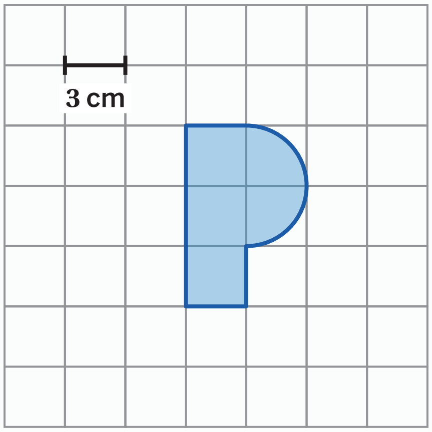
Strategy 2



The path to the solution may not be obvious at first, but by breaking the shape down into squares, circles, and parts of circles, we can figure things out!

Try This

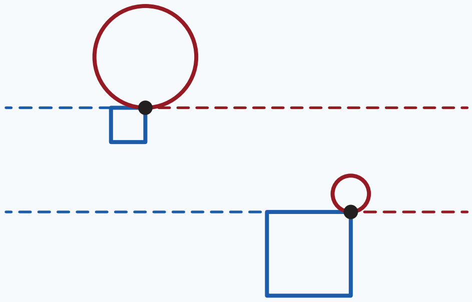
Determine the area of this shape.



Summary | Lesson 10

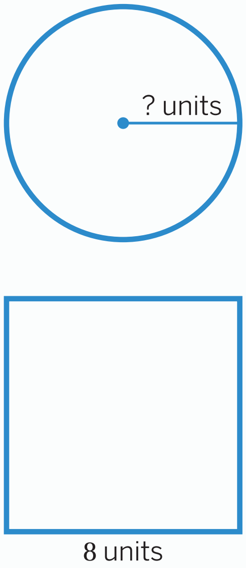
One of the fun things about math is how you can test predictions using calculations. You can make a guess about something, like when the total area of the square and circle will be greatest, and then determine how close your prediction is to the actual answer.

Sometimes your guess will turn out to be correct. Other times, you get to experience the surprise and wonder of discovering something unexpected!



Try This

The perimeter of this square is equal to the circumference of this circle.



a What is the radius of the circle?

b What is the *area* of the circle?

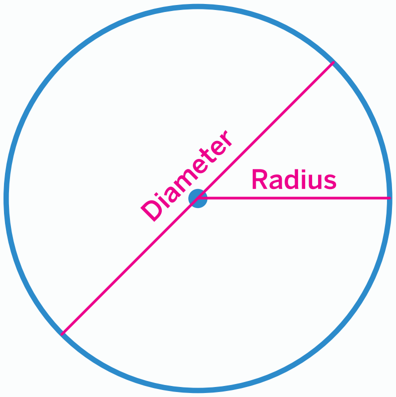
Lesson 1

a Yes. *Explanations vary*. The relationship is proportional because each height can be multiplied by $\frac{10}{3}$ to get the matching perimeter.

b 40 units. One strategy is to multiply the height by the scale factor $\frac{10}{3}$, and $12 \cdot \frac{10}{3} = 40$ units. Another strategy is to use the values from figure A. Since the height is 4 times as much $(3 \cdot 4 = 12)$, the perimeter will also be 4 times as much and $10 \cdot 4 = 40$ units.

Lesson 2

Drawings vary.



Lesson 3

6π units, 18.84 units, or $\frac{132}{7}$ units. *Responses vary as students may use π, 3.14, or $\frac{22}{7}$ in their calculations.*

Lesson 4

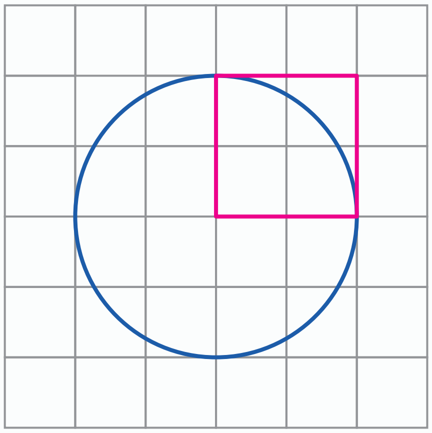
$3\pi + 18$, 27.42, or $\frac{192}{7}$ centimeters. *Responses vary as students may use π, 3.14, or $\frac{22}{7}$ in their calculations*. There are six 3-centimeter-long sides, which measure a total of 18 centimeters. There is also a semicircle with a diameter of 6 centimeters. If it were a whole circle, its circumference would be 6π centimeters. Since it's only half of the circle, the length of the curved side is 3π centimeters. If we add together the lengths of the curved side and the straight sides, we get a perimeter of $3\pi + 18$ centimeters.

Lesson 5

Responses vary. The area is a little more than 7 square centimeters. There is one 2-by-2-centimeter square with an area of 4 square centimeters. There are also two semicircles, which look like they have a combined area of a little more than 3 square centimeters.

Lesson 6

a



The area of the radius square is 4 square units.

b *Responses vary*. One radius square measures 4 square units, and it takes a little more than 3 radius squares to cover the full circle. Since $4 \cdot 3 = 12$, the area of the circle is a little more than 12 square units.

Lesson 7

4π square units. To determine the exact area, use the formula $A = \pi r^2$. The radius of this circle is 2 units, so the area is $\pi \cdot 2^2$, or 4π.

Lesson 8

a 36π square units. The formula for the area of a circle is $A = \pi r^2$. Since the radius is 6 units, the area is $\pi \cdot 6^2$, or 36π square units.

b 36π square units. To calculate the area of the triangle, multiply the base (12π units) by the height (6 units), then divide by 2. $12\pi \cdot 6 = 72\pi$, and $\frac{72\pi}{2} = 36\pi$.

c *Responses vary*. The areas are the same. This is because the triangle and circle cover the same total amount of space, just arranged in different ways.

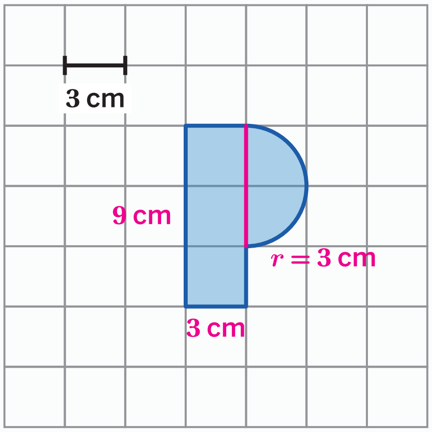
Lesson 9

$4.5\pi + 27$ square centimeters

One strategy is to split the shape into a rectangle and a semicircle, determine the area of each part, then add them together.

The rectangle has an area of $3 \cdot 9 = 27$ square centimeters.

The semicircle is half of a circle with a radius of 3 centimeters. The area of the whole circle would be 9π square centimeters, so the semicircle has an area of $\frac{9\pi}{2} = 4.5\pi$ square centimeters.



That means the total area of this shape is $4.5\pi + 27$ square centimeters.

Lesson 10

a $\frac{16}{\pi}$ units (about 5.09 units). *Responses vary as students may use* π, 3.14, *or* $\frac{22}{7}$ *in their calculations*. The square has a perimeter of $8 \cdot 4 = 32$ units. This is equal to the circumference of the circle. Since the formula for the circumference of a circle is $C = 2\pi r$, dividing the circumference by 2π will give us the circle's radius. $\frac{32}{2\pi} = \frac{16}{\pi} \approx 5.09$ units.

b $\frac{256}{\pi}$ units (about 81.53 units). *Responses vary as students may use* π, 3.14, *or* $\frac{22}{7}$ *in their calculations*. The circle has a radius of $\frac{16}{\pi}$ units. Since $A = \pi r^2$, the area of this circle is $\pi \times \left(\frac{16}{\pi}\right)^2 = \frac{256\pi}{\pi^2} = \frac{256}{\pi}$ square units.

Grade 7 Unit 3 Glossary/7.º grado Unidad 3 Glosario

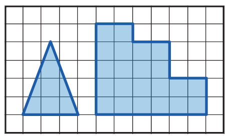
English

A

approximation A rounded value that you can use to represent a number that may be difficult to work with, such as an irrational number or a repeating decimal.

For example, the exact value of pi (π) is an irrational number, so we often use the approximate value of 3.14 in calculations involving pi.

area The space inside a two-dimensional figure. It is expressed in square units.



The area of the triangle is 6 square units. The area of the other shape is 22 square units.

C

circle A shape made out of all the points that are the same distance from a center point.

circumference The distance around a circle. If you imagine a circle as a piece of string, it is the length of the string. The circumference of a circle, C, can be calculated with the formula $C = \pi d$, where d is the diameter of the circle, or $C = 2\pi r$, where r is the radius.

constant of proportionality In a proportional relationship, the number used to multiply the values for one quantity to get the values for the other quantity is called the constant of proportionality.

Carpet (sq. ft)	Cost (dollars)
10 ←	$\times\frac{2}{3}$ 15.00
20 ←	$\times\frac{2}{3}$ 30.00
50 ←	$\times\frac{2}{3}$ 75.00

In this table, one constant of proportionality is $\frac{2}{3}$.

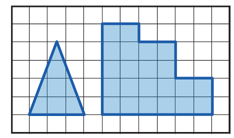
Español

A

aproximación Un valor redondeado que se puede usar para representar un número con el que podría ser complicado trabajar, como un número irracional o un decimal periódico.

Por ejemplo, el valor exacto de pi (π) es un número irracional, así que a menudo usamos el valor aproximado de 3.14 en cálculos que incluyen pi.

área El espacio dentro de una figura bidimensional. Se expresa en unidades cuadradas.



El área del triángulo mide 6 unidades cuadradas. El área de la otra figura mide 22 unidades cuadradas.

C

círculo Una figura formada por todos los puntos que están a la misma distancia de un punto central.

circunferencia La distancia alrededor de un círculo. Si imaginamos el círculo como un trozo de cuerda, es la longitud de la cuerda. La circunferencia de un círculo, C, puede calcularse mediante la fórmula $C = \pi d$, donde d es el diámetro del círculo, o $C = 2\pi r$, donde r es el radio.

constante de proporcionalidad En una relación proporcional, una constante de proporcionalidad es el número que se puede usar para multiplicar los valores de una cantidad para obtener los valores de la otra cantidad.

Alfombra (pies cuadrados)	Costo (dólares)
10 ←	$\times\frac{2}{3}$ 15.00
20 ←	$\times\frac{2}{3}$ 30.00
50 ←	$\times\frac{2}{3}$ 75.00

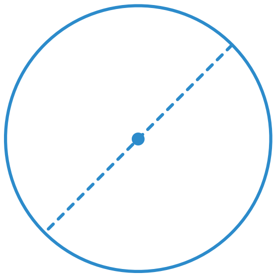
En esta tabla, una constante de proporcionalidad es $\frac{2}{3}$.

© Amplify Education, Inc and its licensors. Amplify Desmos Math is based on curricula from Illustrative Mathematics (IM).

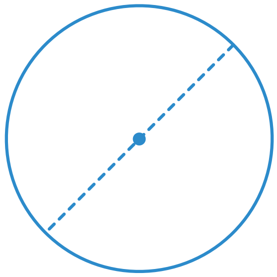
English	Español

D

diameter The distance from one point on a circle through the center to another point on the circle. It is also the longest distance across the circle.



diámetro La distancia entre un punto y otro en un círculo, pasando por el centro. También es la distancia mayor entre un punto y otro en un círculo.



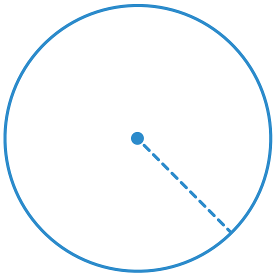
P

pi A number that represents the constant of proportionality between the diameter and circumference of any circle. The symbol for pi is π. Some common approximations for π are 3.14 and $\frac{22}{7}$.

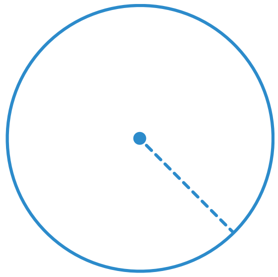
pi Un número que representa la constante de proporcionalidad entre el diámetro y la circunferencia de cualquier círculo. El símbolo de pi es π. Algunas aproximaciones comunes de π son 3.14 y $\frac{22}{7}$.

R

radius A line segment that connects the center of a circle with a point on the circle. Every radius of a circle is the same length.

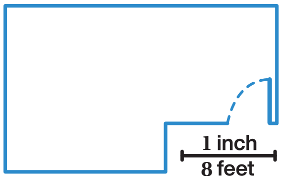


radio Un segmento de recta que conecta el centro de un círculo con un punto del círculo. Todos los radios de un círculo tienen la misma longitud.



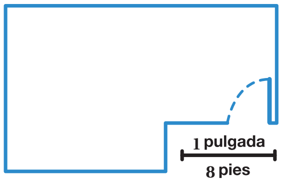
S

scale A scale tells us how the actual measurements of an object are represented in a drawing.



The scale of this floor plan tells us that 1 inch on the drawing represents 8 feet in the actual room.

escala Una escala nos indica cómo están representadas en un dibujo o diagrama las medidas reales de un objeto.



La escala de este plano nos indica que 1 pulgada en el dibujo representa 8 pies en la habitación real.