Amplify Desmos Math FLORIDA

Teacher Edition

ACC 7

Volume 1

Amplify Desmos Math FLORIDA

Accelerated 7

Volume 1: Units 1–5

Teacher Edition

About Amplify

Amplify is dedicated to collaborating with educators to create learning experiences that are rigorous and riveting for all students. Amplify creates K–12 core and supplemental curriculum, assessment, and intervention programs for today's students.

A pioneer in K–12 education since 2000, Amplify is leading the way in next-generation curriculum and assessment. All of our programs provide teachers with powerful tools that help them understand and respond to the needs of every student.

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Amplify gratefully acknowledges the work of distinguished program advisors from English Learners Success Forum (ELSF), who have been integral in the development of Amplify Desmos Math. ELSF is a 501(c)(3) nonprofit organization whose mission is to expand educational equity for multilingual learners by increasing the supply of high-quality instructional materials that center their cultural and linguistic assets.

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Dear Teacher,

Welcome to Amplify Desmos Math Florida! We're so excited your school has chosen our program this year. Your hard work plays an essential role in your students' math experience, so we wanted to reach out to introduce ourselves.

We developed the program around the idea that a structured approach to problem-based instruction builds on students' curiosity to develop lasting grade-level understanding for each and every member of your classroom. Each lesson offers opportunities for you to build on students' understanding, connect their ideas, develop their skill fluency, and empower them to ask questions, explore, and make discoveries. This year, our mission is for your students to learn math — and to love learning math!

Here's what you can expect:

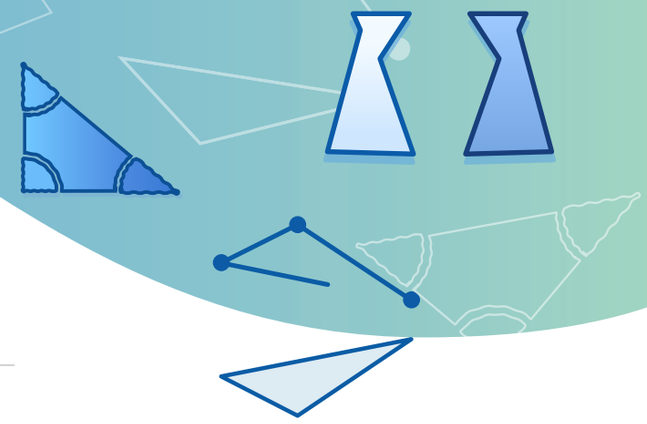
- A program designed around the **Florida B.E.S.T. Standards for Mathematics**, grounded in the strands, standards, and benchmarks, and the importance of student exploration and investigation.
- **Deep and intentional alignment** to the Florida B.E.S.T. standards, Florida English Language Arts standards for K-12 Mathematics, and Florida English Language Development Standards related to the Language of Mathematics.
- **Interactive lessons** that blend paper-based and digital learning, including:
  - » Lessons that drive classroom discussions so students can work toward a shared understanding and sense of community.
  - » Manipulatives, visual models, discussions, and drawings to encourage perseverance and revision.
  - » Easy-to-follow lesson plans, with clear teaching suggestions, strategies, and Math Language Routines.
- **Lesson practice** to support fluency, help students review previous topics, and gain experience with FAST Assessment problem types.
- **Recommended differentiation moves** that meet the needs of all learners.
- **Diagnostic, formative, and summative assessments** along with lesson-level checks for understanding.
- **A caregiver resource** for each unit that includes explanations of key math concepts and problems to try with your student.

We hope your students enjoy using technology to explore math, working with classmates to solve problems, and learning new and interesting concepts. We also hope you love experiencing it with them!

—The Amplify Desmos Math Florida team

Unit 1 Rigid Transformations and Congruence

Students investigate translations, reflections, and rotations, and use these transformations to make informal arguments about congruence. They also explore angle relationships and the triangle sum theorem.



Pre-Unit Check 1L

Sub-Unit 1 Rigid Transformations 2

				B.E.S.T. Benchmark Connection
1.01	Spinning, Flipping, Sliding	Naming Transformations	3A	1.01: MA.8.GR.2.1, MTR.4.1
1.02	Moving Day	Describing Transformations Precisely	10A	1.02: MA.8.GR.2.1, MTR.4.1, MTR.5.1
1.03	Getting Coordinated, Part 1	Coordinates of Translations and Reflections	17A	1.03: MA.8.GR.2.1, MA.8.GR.2.3, MTR.3.1, MTR.4.1
1.04	Getting Coordinated, Part 2	Coordinates of Transformations	24A	1.04: MA.8.GR.2.1, MA.8.GR.2.3, MTR.3.1, MTR.4.1

Sub-Unit Quiz 30B

Sub-Unit 2 Congruence 31–32

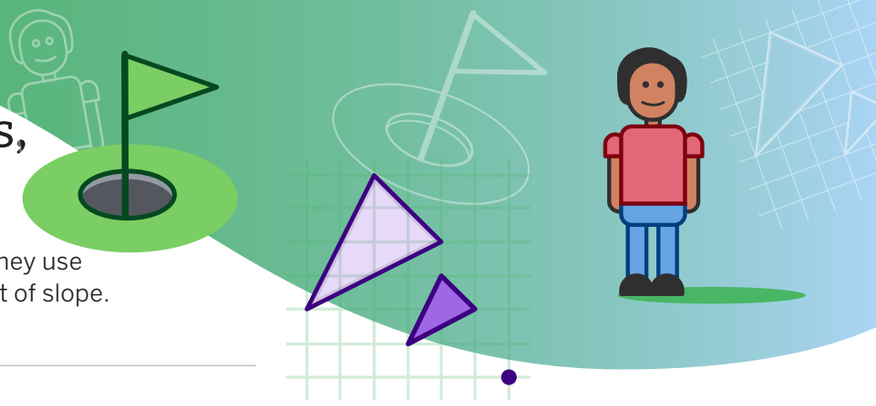
				B.E.S.T. Benchmark Connection
1.05	No Bending, No Stretching	Rigid Transformations	33A	1.05: MA.8.GR.2.1, MA.8.GR.2.3, MTR.4.1, MTR.5.1
1.06	Are They the Same?	Defining Congruence	39A	1.06: MA.8.GR.2.1, MTR.4.1
1.07	Are They Congruent?	Rigid Transformations and Congruent Figures	47A	1.07: MA.8.GR.2.1, MA.8.GR.2.3, MTR.4.1

Sub-Unit 3 Angle Relationships 54

				B.E.S.T. Benchmark Connection
1.08	Tearing It Up	Angle Sums in Triangles	55A	1.08: MA.8.GR.1.5, MTR.3.1
1.09	Friendly Angles	Complementary, Supplementary, Vertical, and Adjacent Angles	62A	1.09: MA.8.AR.2.1, MA.8.GR.1.4, MTR.1.1, MTR.3.1
1.10	Sum It Up	Exploring Angle Sums in Polygons	72A	1.10: MA.8.GR.1.6, MTR.5.1

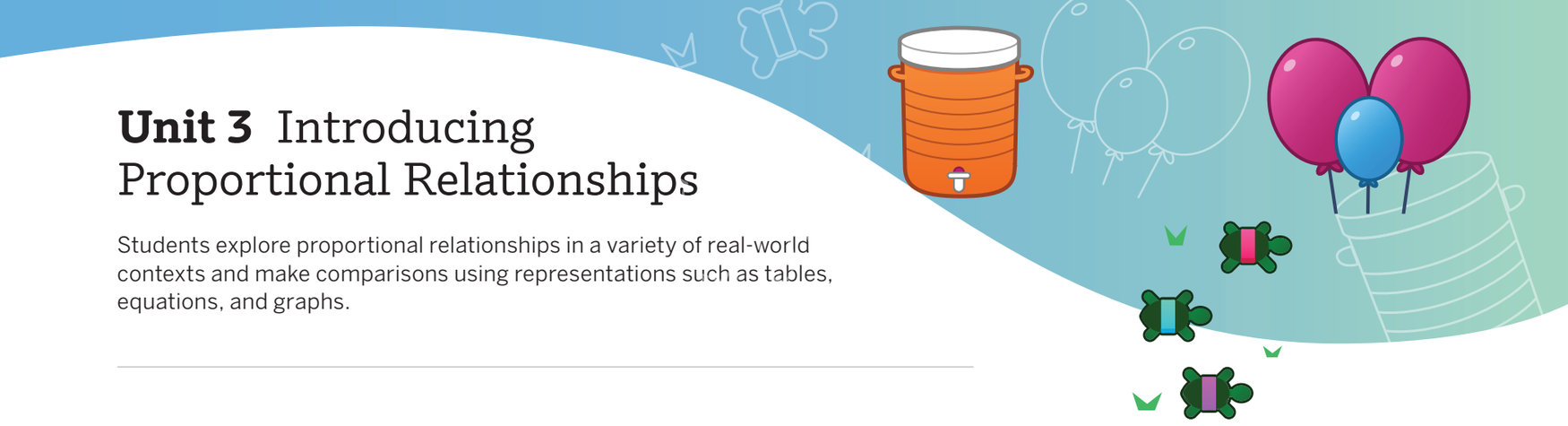
End-of-Unit Assessment 78B

Unit 2 Scale Factors, Dilations, Similarity, and Slope



Students study scaled copies, dilations, and similar figures. They use similar triangles to explain, understand, and apply the concept of slope.

			B.E.S.T. Benchmark Connection
	Pre-Unit Check	79N	
Sub-Unit 1	Scaled Copies	80	
2.01	Scaling Machines | What Are Scaled Copies?	81A	**2.01: MA.7.GR.1.5,** MTR.4.1, MTR.5.1
2.02	Scaling Robots | Lengths and Scaled Copies	90A	**2.02: MA.7.GR.1.5,** MTR.4.1, MTR.5.1
2.03	Make It Scale | Drawing Scaled Copies	97A	**2.03: MA.7.GR.1.5,** MTR.4.1
2.04	Scale Factor Challenges | Effects of Scale Factors	104A	**2.04: MA.7.GR.1.5,** MTR.4.1, MTR.5.1
2.05	Tiles | Scaling and Area	112A	**2.05: MA.7.GR.1.5,** MTR.1.1, MTR.3.1, MTR.4.1
2.06	Scale Drawings | Comparing Scale Factor and Scale	120A	**2.06: MA.7.GR.1.5,** MTR.2.1, MTR.6.1, MTR.7.1
	Sub-Unit Quiz	125A	
Sub-Unit 2	Dilations	126	**B.E.S.T. Benchmark Connection**
2.07	Sketchy Dilations | Introducing Dilations	127A	**2.07: MA.8.GR.2.2,** MTR.4.1
2.08	Dilation Mini Golf | Dilations With No Grid	133A	**2.08: MA.8.GR.2.2,** MTR.4.1, MTR.7.1
2.09	Make It a Combo! | Combining a Rigid Transformation and a Dilation	140A	**2.09: MA.8.GR.2.2,** MTR.1.1, MTR.4.1
2.10	Match My Dilation | Dilations on a Square Grid	146A	**2.10: MA.8.GR.2.2, MA.8.GR.2.3,** MTR.4.1, MTR.5.1
2.11	Dilations on a Plane | Dilations With Coordinates	153A	**2.11: MA.8.GR.2.2, MA.8.GR.2.3,** MTR.4.1, MTR.5.1
	Sub-Unit Quiz	159A	
Sub-Unit 3	Similarity	160	**B.E.S.T. Benchmark Connection**
2.12	Scavenger Hunt | Similar Figures, Sides, and Angles	161A	**2.12: MA.8.GR.2.2, MA.8.GR.2.3,** MTR.4.1
2.13	Are Angles Enough? | Similar Triangles	168A	**2.13: MA.8.GR.1.4, MA.8.GR.1.5,** MTR.4.1
2.14	Shadows | Side Length Relationships in Similar Triangles	175A	**2.14: MA.8.GR.2.4,** MTR.2.1
Sub-Unit 4	Slope	182	**B.E.S.T. Benchmark Connection**
2.15	Water Slide | Similarity and Slope	183A	**2.15: MA.8.AR.3.2, MA.8.GR.2.4,** MTR.2.1, MTR.4.1, MTR.5.1
2.16	Slope Challenges | Developing Fluency with Slope	191A	**2.16: MA.8.AR.3.2,** MTR.3.1, MTR.4.1
	End-of-Unit Assessment	198B	



Unit 3 Introducing Proportional Relationships

Students explore proportional relationships in a variety of real-world contexts and make comparisons using representations such as tables, equations, and graphs.

Pre-Unit Check 199L

Sub-Unit 1 Proportional Relationships in Tables 200

3.01 Balloon Float | Introducing Proportional Relationships With Tables 201A

3.02 Sugar, Spice, and Everything Rice | Constant of Proportionality 209A

3.03 Pen Pals | Converting Units 215A

Sub-Unit 2 Proportional Relationships in Equations 222

3.04 Robot Factory | Proportional Relationships and Equations 223A

3.05 Snapshots | More Equations of Proportional Relationships 229A

3.06 Two and Two | Two Equations for Each Relationship 236A

3.07 All Kinds of Equations | Equations of Proportional Relationships 243A

Sub-Unit Quiz 249A

Sub-Unit 3 Proportional Relationships in Graphs 250

3.08 DinoPops | Introducing Graphs of Proportional Relationships 251A

3.09 Gallon Challenge | Interpreting Graphs of Proportional Relationships 259A

3.10 Three Turtles | Proportional Relationships and Graphs 266A

3.11 Four Representations | Connecting Descriptions, Tables, Graphs, and Equations 274A

End-of-Unit Assessment 279A

B.E.S.T. Benchmark Connection
3.01: MA.7.AR.4.1, MA.7.AR.4.2, MA.8.AR.3.1, MTR.4.1, MTR.5.1
3.02: MA.7.AR.4.1, MA.7.AR.4.2, MA.8.AR.3.1, MTR.2.1, MTR.6.1
3.03: MA.7.AR.3.3, MTR.2.1, MTR.4.1

B.E.S.T. Benchmark Connection
3.04: MA.7.AR.4.2, MA.7.AR.4.4, MTR.2.1, MTR.3.1
3.05: MA.7.AR.4.2, MA.7.AR.4.4, MA.7.AR.4.5, MTR.2.1, MTR.7.1
3.06: MA.7.AR.4.2, MA.7.AR.4.4, MA.7.AR.4.5, MTR.2.1, MTR.5.1
3.07: MA.7.AR.4.1, MA.7.AR.4.2, MA.7.AR.4.4, MA.7.AR.4.5, MA.8.AR.3.1, MTR.1.1, MTR.4.1, MTR.5.1

B.E.S.T. Benchmark Connection
3.08: MA.7.AR.4.1, MTR.2.1, MTR.5.1
3.09: MA.7.AR.4.2, MTR.3.1, MTR.7.1
3.10: MA.7.AR.4.1, MA.7.AR.4.2, MA.7.AR.4.4, MA.7.AR.4.5, MA.8.AR.3.1, MTR.1.1, MTR.2.1, MTR.4.1
3.11: MA.7.AR.4.2, MA.7.AR.4.3, MA.7.AR.4.4, MTR.2.1, MTR.7.1

Unit 4 Linear Relationships and Equations

Students compare representations of linear relationships and interpret these representations in context. Students also write equations for lines and determine solutions to two-variable linear equations.

Pre-Unit Check 281L

Sub-Unit 1 Linear Relationships 282

B.E.S.T. Benchmark Connection
4.01 Flags | Introduction to Linear Relationships ..... 283A — **MA.8.AR.3.1, MA.8.AR.3.2,** MTR.1.1, MTR.2.1, MTR.4.1
4.02 Water Cooler | Slopes Don't Have to Be Positive ..... 291A — **MA.8.AR.3.2, MA.8.AR.3.5,** MTR.1.1, MTR.2.1, MTR.3.1
4.03 Ups and Downs | Analyzing Linear Relationships ..... 299A — **MA.8.AR.3.2, MA.8.AR.3.3, MA.8.AR.3.4, MA.8.AR.3.5,** MTR.2.1, MTR.4.1
4.04 Stacking Cups | More With Linear Relationships ..... 306A — **MA.8.AR.3.2, MA.8.AR.3.4, MA.8.AR.3.5,** MTR.5.1, MTR.7.1

Sub-Unit Quiz 313A

Sub-Unit 2 Linear Equations and Points 314

B.E.S.T. Benchmark Connection
4.05 Landing Planes | Calculating Slope ..... 315A — **MA.8.AR.3.2,** MTR.3.1, MTR.5.1
4.06 Coin Capture | Horizontal and Vertical Lines ..... 322A — **MA.8.AR.3.2, MA.8.AR.3.3,** MTR.4.1, MTR.5.1
4.07 Why Intercepts? | Equations of All Kinds of Lines ..... 328A — **MA.8.AR.3.2, MA.8.AR.3.3, MA.8.AR.3.4,** MTR.1.1, MTR.4.1
4.08 Pennies and Quarters | Using Linear Relationships to Solve Problems ..... 336A — **MA.8.AR.3.3, MA.8.AR.3.4, MA.8.AR.3.5,** MTR.1.1, MTR.5.1, MTR.7.1

End-of-Unit Assessment 343A

Unit 5 Linear Equations and Linear Systems

Students solve linear equations with rational coefficients and determine the number of possible solutions. They also solve systems of two linear equations algebraically and graphically.

Pre-Unit Check 345L

Sub-Unit 1 Two-Step Equations 346

				B.E.S.T. Benchmark Connection
5.01	Balancing Equations	Solving Equations With Balanced Hangers	347A	5.01: MA.7.NSO.2.2, MA.7.AR.1.2, MA.7.AR.2.2, MTR.2.1, MTR.4.1, MTR.5.1
5.02	Keeping It True	Solving Equations	354A	5.02: MA.7.NSO.2.2, MA.7.AR.2.2, MTR.2.1, MTR.4.1, MTR.5.1
5.03	Factoring and Expanding	Options for Solving One Equation	360A	5.03: MA.7.AR.2.2, MTR.2.1, MTR.4.1, MTR.5.1
5.04	Pass the Equation	Solving Equations Multiple Ways	367A	5.04: MA.7.NSO.2.2, MA.7.AR.2.2, MTR.2.1, MTR.3.1, MTR.5.1
5.05	Community Day	Using Equations to Solve Problems	373A	5.05: MA.7.AR.2.2, MTR.2.1, MTR.4.1, MTR.7.1

Sub-Unit 2 Multi-Step Equations 380

				B.E.S.T. Benchmark Connection
5.06	Equation Roundtable	Analyzing Unbalanced Equations	381A	5.06: MA.8.AR.1.2, MA.8.AR.2.1, MTR.4.1
5.07	All, Some, or None? Part 1	How Many Solutions?	387A	5.07: MA.8.AR.1.2, MA.8.AR.2.1, MTR.4.1, MTR.5.1
5.08	Strategic Solving	Solving Many Types of Equations	395A	5.08: MA.8.AR.1.2, MA.8.AR.2.1, MTR.1.1, MTR.5.1
5.09	When Will They Meet?	Solving Linear Equations in Context	402A	5.09: MA.8.AR.1.2, MA.8.AR.2.1, MA.8.AR.3.3, MTR.2.1
5.10	Do the Two-Step	Solving Two-Step Linear Inequalities	410A	5.10: MA.8.AR.1.2, MA.8.AR.2.2, MTR.5.1, MTR.7.1

Sub-Unit Quiz 417A

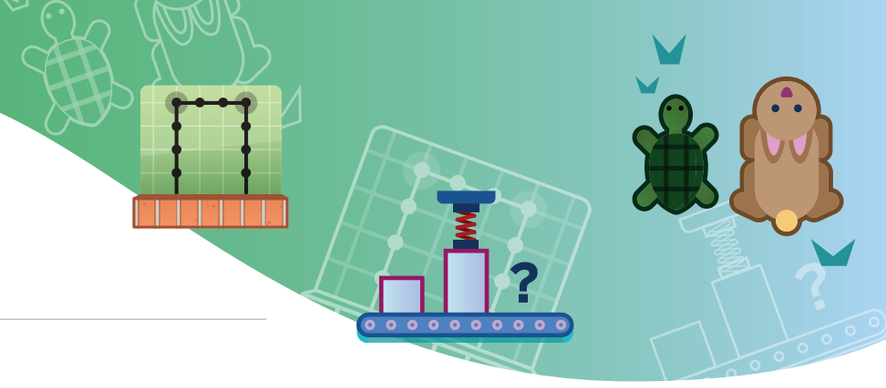
Sub-Unit 3 Systems of Linear Equations 418

				B.E.S.T. Benchmark Connection
5.11	On or Off the Line?	Interpreting Points On or Off the Line	419A	5.11: MA.8.AR.4.1, MTR.2.1, MTR.4.1
5.12	On Both Lines	Representing Systems of Linear Equations	426A	5.12: MA.8.AR.4.1, MA.8.AR.4.3, MTR.1.1, MTR.4.1
5.13	Make Them Balance	Graphing Systems of Linear Equations	432A	5.13: MA.8.AR.4.1, MA.8.AR.4.2, MA.8.AR.4.3, MTR.5.1
5.14	Line Zapper	Solving Systems of Linear Equations	440A	5.14: MA.8.AR.4.1, MA.8.AR.4.3, MTR.4.1, MTR.5.1
5.15	All, Some, or None? Part 2	Systems of Equations with No, One, or Many Solutions	447A	5.15: MA.8.AR.4.2, MA.8.AR.4.3, MTR.5.1

End-of-Unit Assessment 454B

Unit 6 Functions

Students learn about functions, analyze representations of functions, and examine functions in different contexts.



Pre-Unit Check 455L

Sub-Unit 1 Introduction to Functions 456

6.01	Guess My Rule | Introduction to Functions	457A	**6.01: MA.8.F.1.1,** MTR.4.1
6.02	The Ins and Outs | Determining Domain and Range	465A	**6.02: MA.8.F.1.1,** MTR.2.1
6.03	Function or Not? | Graphs of Functions and Non-Functions	474A	**6.03: MA.8.F.1.1,** MTR.3.1, MTR.4.1
6.04	Window Frames | Functions and Equations	482A	**6.04: MA.8.AR.2.1, MA.8.AR.3.3, MA.8.AR.3.5, MA.8.F.1.1,** MTR.4.1, MTR.7.1

Sub-Unit Quiz 489A

Sub-Unit 2 Representing and Interpreting Functions 490

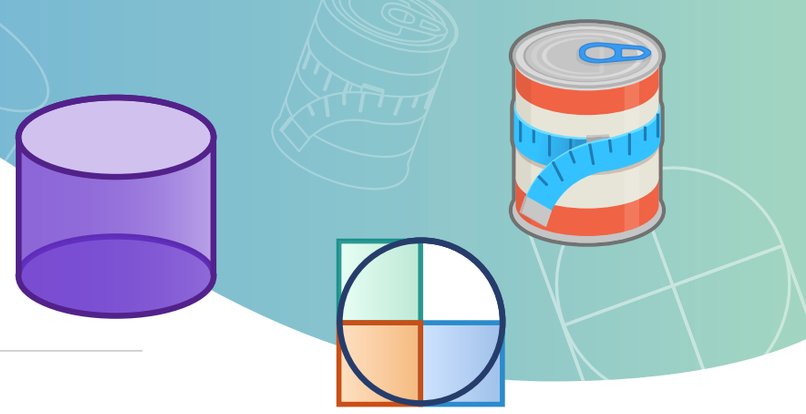
6.05	The Tortoise and the Hare | Interpreting Graphs of Functions	491A	**6.05: MA.8.AR.3.4, MA.8.AR.3.5, MA.8.F.1.3,** MTR.4.1
6.06	Elevation Examination | Analyzing Function Representations That Are Linear or Nonlinear	498A	**6.06: MA.8.AR.3.1, MA.8.F.1.1, MA.8.F.1.2,** MTR.2.1, MTR.4.1
6.07	Comparing Linear Functions | Comparing Representations of Functions	505A	**6.07: MA.8.AR.3.5, MA.8.F.1.2, MA.8.F.1.3,** MTR.2.1, MTR.4.1

End-of-Unit Assessment 512B

B.E.S.T. Benchmark Connection

Unit 7 Circles and Cylinders

Students explore the relationships between the radius, diameter, circumference, and area of a circle. They also explore the volume and surface area of cylinders.



Pre-Unit Check 513L

Sub-Unit 1 Circles 514

			B.E.S.T. Benchmark Connection
7.01	Is It a Circle?	Parts of a Circle 515A	7.01: MA.7.GR.1.3, MTR.4.1
7.02	Measuring Around	Estimating and Calculating Circumference 522A	7.02: MA.7.GR.1.3, MTR.1.1, MTR.4.1, MTR.6.1
7.03	Radius Squares	Exploring Circle Area 529A	7.03: MA.7.GR.1.4, MTR.4.1, MTR.5.1, MTR.6.1
7.04	Circle Area	Calculating Circle Area 536A	7.04: MA.7.GR.1.4, MTR.1.1, MTR.3.1, MTR.4.1
7.05	Area Challenges	Calculating Areas of Complex Shapes 542A	7.05: MA.7.GR.1.4, MA.7.GR.1.5, MTR.1.1, MTR.4.1, MTR.5.1
7.06	Circle vs. Square	Calculating Area Given Perimeter 548A	7.06: MA.7.GR.1.4, MA.7.GR.1.5, MTR.1.1, MTR.3.1

Sub-Unit Quiz 553A

Sub-Unit 2 Solid Geometry 554

			B.E.S.T. Benchmark Connection
7.07	Cylinders	Volumes of Cylinders 555A	7.07: MA.7.GR.2.3, MTR.2.1, MTR.3.1, MTR.4.1
7.08	Scaling Cylinders	Applying Volumes of Cylinders 563A	7.08: MA.7.GR.2.3, MTR.1.1, MTR.5.1, MTR.7.1
7.09	From Flat to Round	Surface Area of Cylinders 570A	7.09: MA.7.GR.2.1, MA.7.GR.2.2, MTR.1.1, MTR.2.1, MTR.7.1

End-of-Unit Assessment 577A

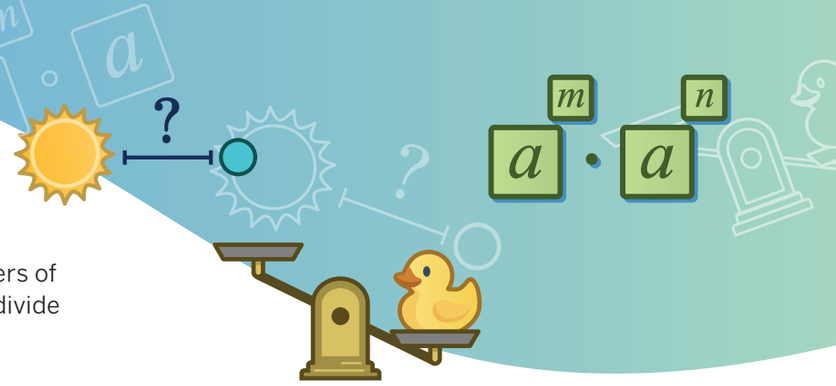
Unit 8 Associations in Data

Students decide the best display for numerical and categorical data. Then they use scatter plots and fitted lines to analyze numerical data with two variables, identify associations, and make predictions.

			B.E.S.T. Benchmark Connection
	Pre-Unit Check	579L	
Sub-Unit 1	Displaying Data	580	**B.E.S.T. Benchmark Connection**
8.01	The County Fair | Creating Circle Graphs	581A	**8.01: MA.7.DP.1.4,** MTR.4.1, MTR.5.1
8.02	Choices, Choices | Choosing an Appropriate Representation for Data	589A	**8.02: MA.7.DP.1.4, MA.7.DP.1.5,** MTR.2.1, MTR.4.1, MTR.7.1
8.03	On Display | Creating an Appropriate Representation for Data	596A	**8.03: MA.7.DP.1.4, MA.7.DP.1.5,** MTR.2.1, MTR.7.1
Sub-Unit 2	Organizing Numerical Data	604	**B.E.S.T. Benchmark Connection**
8.04	Click Battle | Organizing Data	605A	**8.04: MA.8.DP.1.1,** MTR.2.1, MTR.4.1, MTR.6.1, MTR.7.1
8.05	Birds, Birds, Birds | Plotting Data	613A	**8.05: MA.8.DP.1.1,** MTR.2.1, MTR.4.1, MTR.7.1
8.06	Robots | What a Point on a Scatter Plot Means	619A	**8.06: MA.8.DP.1.1,** MTR.2.1, MTR.4.1, MTR.6.1, MTR.7.1
	Sub-Unit Quiz	625A	
Sub-Unit 3	Analyzing Numerical Data	626	**B.E.S.T. Benchmark Connection**
8.07	Dapper Cats | Linear Models and Outliers	627A	**8.07: MA.8.DP.1.2, MA.8.DP.1.3,** MTR.4.1, MTR.6.1, MTR.7.1
8.08	Interpreting Scatter Plots | Showing Patterns in Data	634A	**8.08: MA.8.DP.1.1, MA.8.DP.1.3,** MTR.4.1, MTR.6.1, MTR.7.1
8.09	Find the Fit | Fitting a Line to Data	641A	**8.09: MA.8.DP.1.3,** MTR.4.1, MTR.6.1
8.10	Interpreting Slopes | The Slope of a Linear Model	649A	**8.10: MA.8.AR.3.4, MA.8.AR.3.5, MA.8.DP.1.2, MA.8.DP.1.3,** MTR.4.1, MTR.5.1, MTR.6.1, MTR.7.1
8.11	Scatter Plot City | Observing More Patterns in Scatter Plots	658A	**8.11: MA.8.DP.1.1, MA.8.DP.1.2, MA.8.DP.1.3,** MTR.4.1, MTR.5.1
8.12	Animal Brains | Analyzing Bivariate Data	664A	**8.12: MA.8.AR.3.5, MA.8.DP.1.1, MA.8.DP.1.2, MA.8.DP.1.3,** MTR.2.1, MTR.5.1, MTR.6.1, MTR.7.1
	End-of-Unit Assessment	671A	

Unit 9 Exponents and Scientific Notation

Students develop fluency with expressions involving exponents, powers of 10, and scientific notation. They will also add, subtract, multiply, and divide numbers written in scientific notation.



Pre-Unit Check 673N

Sub-Unit 1 Exponent Properties 674

		B.E.S.T. Benchmark Connection
9.01	Circles | Exponent Notation 675A	**9.01: MA.7.NSO.1.1,** MTR.2.1, MTR.3.1, MTR.5.1
9.02	Combining Exponents | Equivalent Expressions With Exponents 682A	**9.02: MA.7.NSO.1.1, MA.7.NSO.2.1,** MTR.2.1, MTR.5.1, MTR.6.1
9.03	Power Pairs | Multiplying Powers and Powers of Powers 689A	**9.03: MA.7.NSO.1.1,** MTR.2.1, MTR.5.1, MTR.6.1
9.04	Rewriting Powers | Rewriting Exponential Expressions as a Single Power 695A	**9.04: MA.7.NSO.1.1,** MTR.2.1, MTR.5.1, MTR.6.1
9.05	Exponent Zero | Using Patterns to Understand Zero Exponents 702A	**9.05: MA.7.NSO.1.1,** MTR.4.1, MTR.5.1
9.06	Negative Bases | Powers With Negative Bases 708A	**9.06: MA.7.NSO.1.1,** MTR.4.1, MTR.5.1

Sub-Unit 2 Algebraic Expressions 716

		B.E.S.T. Benchmark Connection
9.07	Negative Exponents | Understanding Negative Exponents and Expressions with Integer Exponents 717A	**9.07: MA.8.NSO.1.3, MA.8.AR.1.1, MA.8.AR.1.2,** MTR.2.1, MTR.4.1, MTR.5.1
9.08	Write a Rule | Generalizing Exponent Properties 725A	**9.08: MA.8.NSO.1.3, MA.8.AR.1.1,** MTR.4.1, MTR.5.1
9.09	Algebraic Expressions With Exponents | Using Rules of Exponents to Evaluate and Simplify Expressions 731A	**9.09: MA.8.NSO.1.3, MA.8.AR.1.1,** MTR.3.1, MTR.4.1, MTR.5.1
9.10	Multiplying Linear Expressions | Multiplying Monomials and Binomials 739A	**9.10: MA.8.NSO.1.3, MA.8.AR.1.2,** MTR.3.1, MTR.4.1
9.11	Fabulous Factors | Factoring Algebraic Expressions 746A	**9.11: MA.8.NSO.1.3, MA.8.AR.1.3,** MTR.4.1

Sub-Unit Quiz 752B

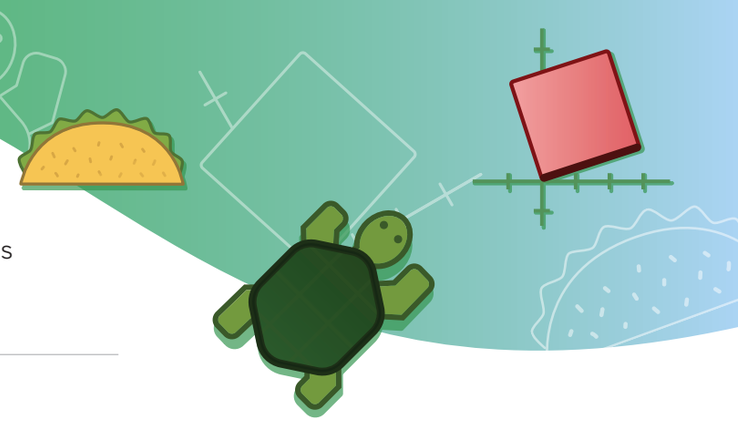
Sub-Unit 3 Scientific Notation 753–754

		B.E.S.T. Benchmark Connection
9.12	Scales and Weights, Part 1 | Describing Large Numbers Using Powers of 10 ..... 755A	**9.12: MA.8.NSO.1.4, MA.8.NSO.1.6,** MTR.2.1, MTR.5.1, MTR.7.1
9.13	Scales and Weights, Part 2 | Describing Small Numbers Using Powers of 10 ..... 762A	**9.13: MA.8.NSO.1.3, MA.8.NSO.1.4, MA.8.NSO.1.6,** MTR.2.1, MTR.3.1, MTR.4.1, MTR.5.1
9.14	Specific and Scientific | Definition of Scientific Notation 771A	**9.14: MA.8.NSO.1.4, MA.8.NSO.1.6,** MTR.2.1, MTR.3.1, MTR.5.1
9.15	Multiplying and Dividing | Multiplying and Dividing With Scientific Notation 777A	**9.15: MA.8.NSO.1.3, MA.8.NSO.1.4, MA.8.NSO.1.5, MA.8.NSO.1.6,** MTR.2.1, MTR.4.1, MTR.5.1
9.16	Balance the Scale | Multiplying, Dividing, and Estimating with Scientific Notation 784A	**9.16: MA.8.NSO.1.3, MA.8.NSO.1.4, MA.8.NSO.1.5, MA.8.NSO.1.6,** MTR.5.1, MTR.6.1, MTR.7.1
9.17	Use Your Powers | Applications of Arithmetic With Scientific Notation 791A	**9.17: MA.8.NSO.1.3, MA.8.NSO.1.4, MA.8.NSO.1.5, MA.8.NSO.1.6,** MTR.5.1, MTR.6.1, MTR.7.1
9.18	City Lights | Adding and Subtracting With Scientific Notation 797A	**9.18: MA.8.NSO.1.3, MA.8.NSO.1.4, MA.8.NSO.1.5, MA.8.NSO.1.6,** MTR.5.1, MTR.6.1, MTR.7.1

End-of-Unit Assessment 804B

Unit 10 The Pythagorean Theorem and Irrational Numbers

Students explore the Pythagorean theorem and different types of numbers (square roots and cube roots, rational and irrational numbers).



Pre-Unit Check 805L

Sub-Unit 1 Square Roots and Cube Roots 806

			B.E.S.T. Benchmark Connection
10.01	Tilted Squares	The Areas of Tilted Squares 807A	**10.01: Building Toward: MA.8.AR.2.3,** MTR.2.1, MTR.4.1, MTR.5.1
10.02	From Squares to Roots	Side Lengths and Areas 814A	**10.02: MA.8.NSO.1.1, MA.8.NSO.1.2, MA.8.AR.2.3,** MTR.2.1, MTR.4.1, MTR.5.1
10.03	Between Squares	Approximating Square Roots 821A	**10.03: MA.8.NSO.1.1, MA.8.NSO.1.2, MA.8.AR.2.3,** MTR.2.1, MTR.4.1, MTR.5.1, MTR.6.1
10.04	Root Down	Reasoning About Square Roots 828A	**10.04: MA.8.NSO.1.1, MA.8.NSO.1.2, MA.8.AR.2.3,** MTR.5.1, MTR.6.1
10.05	Filling Cubes	Edge Lengths, Volumes, and Cube Roots 835A	**10.05: MA.8.NSO.1.1, MA.8.NSO.1.2, MA.8.AR.2.3,** MTR.2.1, MTR.5.1, MTR.6.1, MTR.7.1

Sub-Unit Quiz 842B

Sub-Unit 2 The Pythagorean Theorem 843–844

			B.E.S.T. Benchmark Connection
10.06	The Pythagorean Theorem	Squares of Side Lengths 845A	**10.06: MA.8.NSO.1.1, MA.8.GR.1.1,** MTR.2.1, MTR.5.1
10.07	Pictures to Prove It	A Proof of the Pythagorean Theorem 851A	**10.07: MA.8.NSO.1.1, MA.8.GR.1.1,** MTR.2.1, MTR.4.1, MTR.5.1
10.08	Triangle-Tracing Turtle	Finding Unknown Side Lengths 858A	**10.08: MA.8.NSO.1.1, MA.8.GR.1.1,** MTR.4.1, MTR.6.1, MTR.7.1
10.09	Make It Right	The Converse of the Pythagorean Theorem 865A	**10.09: MA.8.NSO.1.1, MA.8.GR.1.3,** MTR.2.1, MTR.5.1, MTR.6.1
10.10	Can You Build It?	The Triangle Inequality 873A	**10.10: MA.8.NSO.1.1, MA.8.GR.1.3,** MTR.3.1, MTR.5.1
10.11	Taco Truck	Applications of the Pythagorean Theorem 881A	**10.11: MA.8.NSO.1.1, MA.8.AR.3.2, MA.8.GR.1.1,** MTR.4.1, MTR.6.1, MTR.7.1
10.12	Pond Hopper	Finding Distances on the Coordinate Plane 887A	**10.12: MA.8.NSO.1.1, MA.8.GR.1.2,** MTR.5.1, MTR.6.1

Sub-Unit 3 Rational and Irrational Numbers 894

			B.E.S.T. Benchmark Connection
10.13	Code Breaking	Equivalent Forms 895A	**10.13: MA.7.NSO.1.2,** MTR.3.1, MTR.4.1
10.14	Hit the Target	Rational and Irrational Numbers 902A	**10.14: MA.8.NSO.1.1, MA.8.NSO.1.2,** MTR.5.1, MTR.6.1
10.15	Road Trip!	Solving Problems Using Order of Operations 909A	**10.15: MA.8.NSO.1.7,** MTR.5.1, MTR.7.1
10.16	Simplify It!	Expressions With Exponents and Radicals 917A	**10.16: MA.8.NSO.1.7,** MTR.4.1, MTR.5.1

End-of-Unit Assessment 924B

Unit 11 Probability

Students learn ways to display all of the possible outcomes for a repeated experiment. They also learn the difference between theoretical and experimental probability and calculate them.

Pre-Unit Check 925L

Sub-Unit 1 Theoretical Probability 926

11.01 What Can Happen? | Understanding Sample Spaces 927A

11.02 Representing Sample Spaces | Tables, Tree Diagrams, and Descriptions 934A

11.03 A Likely Story | Theoretical Probability 941A

11.04 We're on a Roll! | Understanding Experimental Probability 949A

11.05 What Could Happen? | Using Probability to Make Predictions 957A

End-of-Unit Assessment 963A

B.E.S.T. Benchmark Connection
11.01: MA.8.DP.2.1, MTR.5.1
11.02: MA.8.DP.2.1, MTR.2.1
11.03: MA.8.DP.2.1, MA.8.DP.2.2, MTR.5.1
11.04: MA.8.DP.2.1, MA.8.DP.2.2, MA.8.DP.2.3, MTR.4.1
11.05: MA.8.DP.2.3, MTR.5.1, MTR.7.1

A curiosity-driven program that builds students' lifelong math proficiency

As we developed Amplify Desmos Math Florida, we asked ourselves: how can we support teachers in creating a collaborative classroom of learners excited about math?

With that question in mind, we built the program around four core tenets:

A structured approach to problem-based learning

The program thoughtfully combines conceptual understanding, procedural fluency, and application. Each lesson is designed to tell a story by posing problems that invite a variety of approaches before guiding students to synthesize their understanding of the learning goals.

The Teacher Edition provides guidance for teachers to anticipate and monitor strategies students may use, select and sequence students' ideas, and orchestrate productive discussions to help students make connections between their own ideas and those of their classmates.[1]

Proficiency Progression

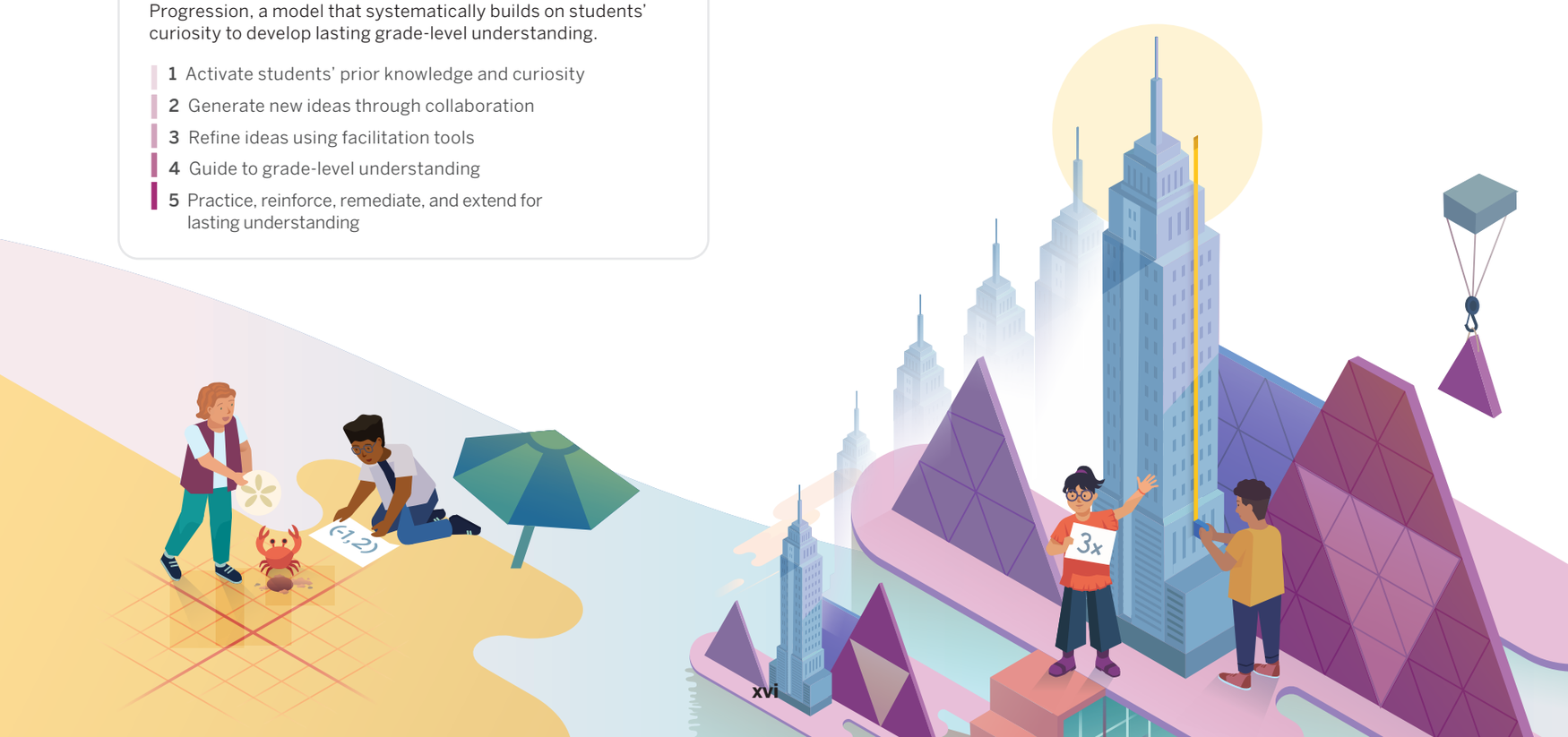
Lessons are designed around what we call the Proficiency Progression, a model that systematically builds on students' curiosity to develop lasting grade-level understanding.

1. Activate students' prior knowledge and curiosity
2. Generate new ideas through collaboration
3. Refine ideas using facilitation tools
4. Guide to grade-level understanding
5. Practice, reinforce, remediate, and extend for lasting understanding

Access to grade-level math for every student, every day

Tasks in each lesson are thoughtfully sequenced so that all students can engage with the math each day without any roadblocks. Every lesson includes suggestions for accessibility and differentiation to support, strengthen, and stretch students' understanding.

We also provide additional resources that integrate seamlessly with core instruction, including a suite of assessments, tailored practice resources that adjust to students' learning, and other intervention solutions. Cohesive differentiation and intervention resources support and challenge students on their path toward deeper understanding of the learning goals, ensuring all students a path toward a deeper understanding of the learning goals of the course.



Student thinking is valuable and can be made evident

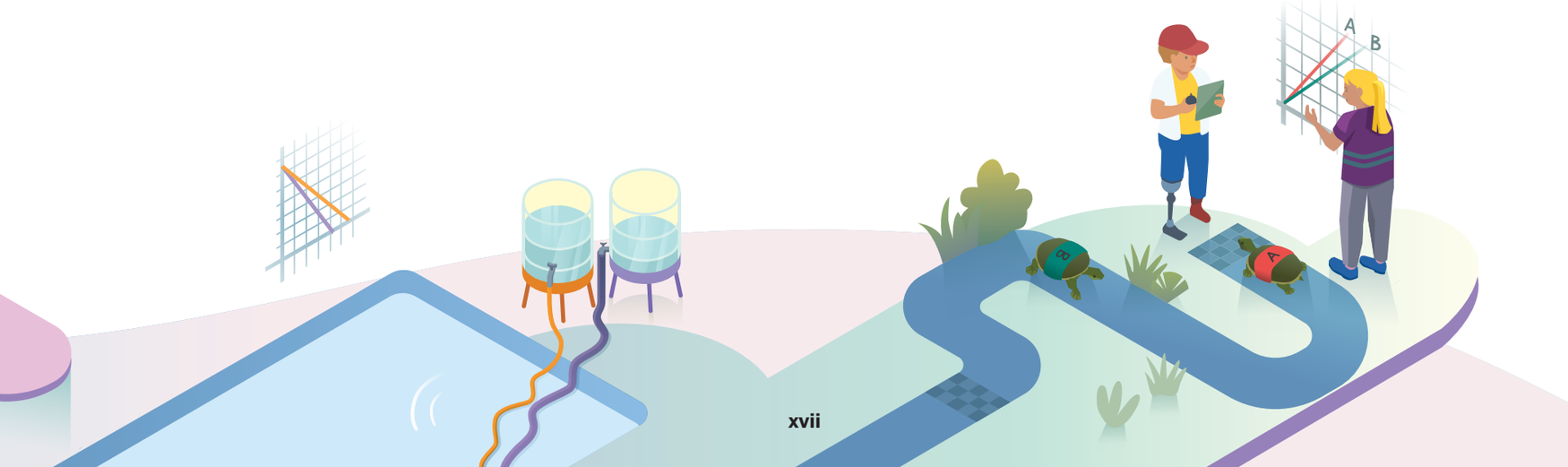
Students take an active role in developing their own ideas first and then synthesize them as a class. To guide the learning process, students see each other's thinking, engage in conversations, and connect to each other through the understanding that they can use math to make sense of the world. This fuels classroom conversations and a shared understanding of math.

Math that motivates students

Picture a classroom where students are so eagerly engaged in a lesson, they wish it would not end. It is buzzing with the sounds of natural curiosity.

Our curriculum supports collaborative classrooms, invites mathematical creativity, and evokes wonder, empowering students to see themselves and their classmates as having brilliant mathematical ideas.

[1] Smith, M.S., & Stein, M.K. (2018). *5 practices for orchestrating productive mathematics discussions* (2nd ed.). SAGE Publications.



Scope and Sequence | Grade 6–Accelerated 7

Volume 1

	Unit 1	Unit 2	Unit 3	Unit 4
Grade 6 127 days	Area and Surface Area 8 instructional days 3 assessment days	Introducing Ratios 12 instructional days 3 assessment days, Math in Action	Unit Rates and Percentages 13 instructional days 3 assessment days	Multiplying and Dividing Fractions 15 instructional days 3 assessment days, Math in Action
Grade 7 116 days	Scale Drawings 10 instructional days 3 assessment days	Introducing Proportional Relationships 13 instructional days 4 assessment days, Math in Action	Measuring Circles 6 instructional days 3 assessment days	Proportional Relationships and Percentages 12 instructional days 4 assessment days, Math in Action
Grade 8 120 days	Rigid Transformations and Congruence 10 instructional days 3 assessment days	Dilations, Similarity, and Slope 10 instructional days 4 assessment days, Math in Action	Proportional and Linear Relationships 11 instructional days 3 assessment days	Linear Equations and Linear Systems 10 instructional days 4 assessment days, Math in Action
Algebra 1 130 days	Linear Equations and Inequalities 18 instructional days 4 assessment days, Math in Action	Describing Data 17 instructional days 3 assessment days	Describing Functions 12 instructional days 4 assessment days, Math in Action	Describing Functions 10 instructional days 3 assessment days

Volume 1

	Unit 1	Unit 2	Unit 3	Unit 4	Unit 5
Accelerated 6 160 days	Area and Surface Area 11 instructional days 3 assessment days 14 days total	Introducing Ratios 12 instructional days 4 assessment days, Math in Action 16 days total	Unit Rates and Percentages 13 instructional days 3 assessment days 16 days total	Multiplying and Dividing Fractions 15 instructional days 4 assessment days, Math in Action 19 days total	Decimal Arithmetic 10 instructional days 4 assessment days 14 days total
Accelerated 7 164 days	Rigid Transformations and Congruence 10 instructional days 3 assessment Days 13 days total	Scale Factors, Dilations, Similarity, and Slope 16 instructional days 5 assessment days, Math in Action 21 days total	Introducing Proportional Relationships 11 instructional days 3 assessment Days 14 days total	Linear Relationships and Equations 8 instructional days 4 assessment days, Math in Action 12 days total	Linear Equations and Linear Systems 15 instructional days 3 assessment Days 18 days total

Volume 2

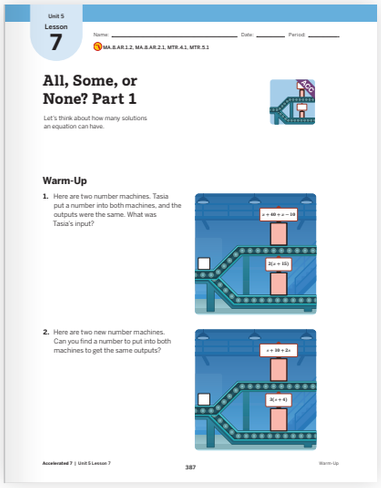
Unit 5	Unit 6	Unit 7	Unit 8	Unit 9
Decimal Arithmetic 10 instructional days 3 assessment Days	Expressions and Equations 11 instructional days 4 assessment Days, Math in Action	Positive and Negative Numbers 18 instructional days 3 assessment Days	Describing Data 12 instructional days 4 assessment Days, Math in Action	
Positive and Negative Rational Numbers 10 instructional days 3 assessment days	Equations, Expressions, and Inequalities 16 instructional days 4 assessment days, Math in Action	Polygons and Cylinders 8 instructional days 3 assessment days	Probability and Sampling 13 instructional days 4 assessment days, Math in Action	
Functions 7 instructional days 3 assessment days	Associations in Data 9 instructional days 4 assessment days, Math in Action	Exponents and Scientific Notation 13 instructional days 3 assessment days	The Pythagorean Theorem and Irrational Numbers 15 instructional days 4 assessment days, Math in Action	Probability 5 instructional days 2 assessment days
Exponential Functions 15 instructional days 4 assessment days, Math in Action	Quadratic Functions 15 instructional days 3 assessment days	Quadratic Equations 18 instructional days 4 assessment days, Math in Action		

Volume 2

Unit 6	Unit 7	Unit 8	Unit 9	Unit 10	Unit 11
Expressions and Equations 17 instructional days 4 assessment days, Math in Action 21 days total	Positive and Negative Numbers 19 instructional days 3 assessment days 22 days total	Equations and Inequalities with Integers 7 instructional days 4 assessment days, Math in Action 11 days total	Positive and Negative Numbers 17 instructional days 3 assessment days 20 days total	Describing Data 5 instructional days 2 assessment days 7 days total	
Functions 7 instructional days 4 assessment days, Math in Action 11 days total	Circles and Cylinders 9 instructional days 3 assessment Days 12 days total	Data 12 instructional days 4 assessment days, Math in Action 16 days total	The Pythagorean Theorem and Irrational Numbers 18 instructional days 3 assessment Days 21 days total	Polygons and Cylinders 16 instructional days 3 assessment days 19 days total	Probability 5 instructional days 2 assessment Days 7 days total

Program Resources

For Students


Unit 5
Lesson 7

Name: _____ Date: _____ Period: _____

MA.8.AR.1.2, MA.8.AR.2.1, MTR.4.1, MTR.5.1

All, Some, or None? Part 1

Let's think about how many solutions an equation can have.

Warm-Up

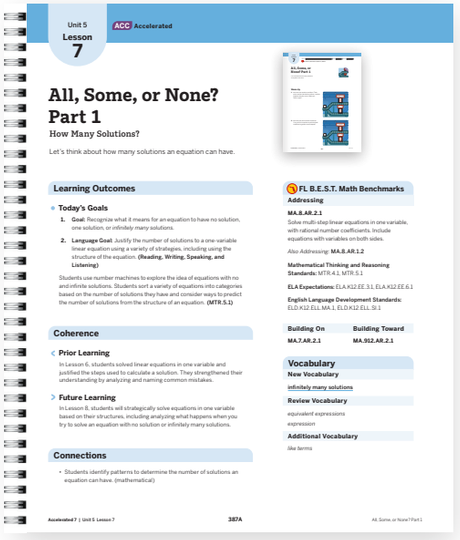
1. Here are two number machines. Tasia put a number into both machines, and the outputs were the same. What was Tasia's input?

2. Here are two new number machines. Can you find a number to put into both machines to get the same outputs?

Accelerated 7 | Unit 5 Lesson 7 387 Warm-Up

- Student Edition (two-volume)

For Teachers


Unit 5
Lesson 7

ACC Accelerated

All, Some, or None? Part 1

How Many Solutions?

Let's think about how many solutions an equation can have.

Learning Outcomes

Today's Goals

1. **Goal:** Recognize what it means for an equation to have no solution, one solution, or infinitely many solutions.
2. **Language Goal:** Justify the number of solutions to a one-variable linear equation using a variety of strategies, including using the structure of the equation. **(Reading, Writing, Speaking, and Listening)**

Students use number machines to explore the idea of equations with no and infinite solutions. Students sort a variety of equations into categories based on the number of solutions they have and consider ways to predict the number of solutions from the structure of an equation. **(MTR.5.1)**

Coherence

Prior Learning

In Lesson 6, students solved linear equations in one variable and justified the steps used to calculate a solution. They strengthened their understanding by analyzing and naming common mistakes.

Future Learning

In Lesson 8, students will strategically solve equations in one variable based on their structures, including analyzing what happens when you try to solve an equation with no solution or infinitely many solutions.

Connections

- Students identify patterns to determine the number of solutions an equation can have. (mathematical)

FL B.E.S.T. Math Benchmarks

Addressing

MA.8.AR.2.1

Solve multi-step linear equations in one variable, with rational number coefficients. Include equations with variables on both sides.

Also Addressing: **MA.8.AR.1.2**

Mathematical Thinking and Reasoning Standards: MTR.4.1, MTR.5.1

ELA Expectations: ELA.K12.EE.3.1, ELA.K12.EE.6.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.7.AR.2.1	MA.912.AR.2.1

Vocabulary

New Vocabulary

infinitely many solutions

Review Vocabulary

equivalent expressions

expression

Additional Vocabulary

like terms

Accelerated 7 | Unit 5 Lesson 7 387A All, Some, or None? Part 1

- Teacher Edition (two-volume)

Additional Resources

Assessment and Lesson Resources

- Unit Assessments
- Show What You Know Lesson Assessments
- Rubrics and Teacher Answer Keys
- Activity Sheets and Cards

Intervention and Extension Resources

- Mini-Lessons
- Prior grade Mini-Lessons
- Extensions

Optional: Manipulative Kits

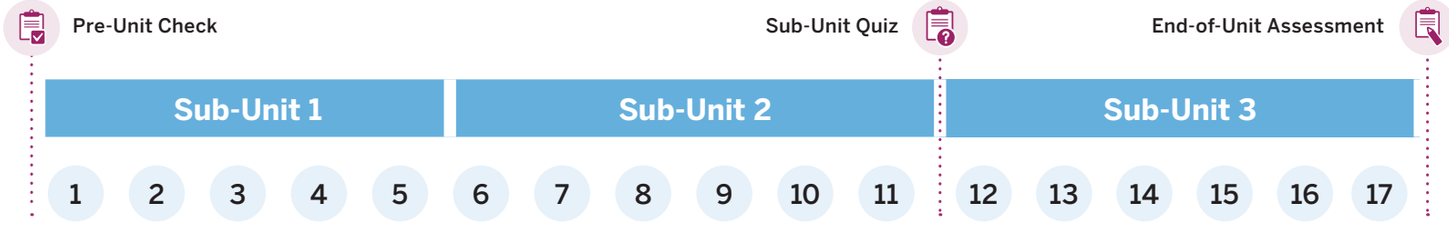
hand2mind

Program Architecture

Course

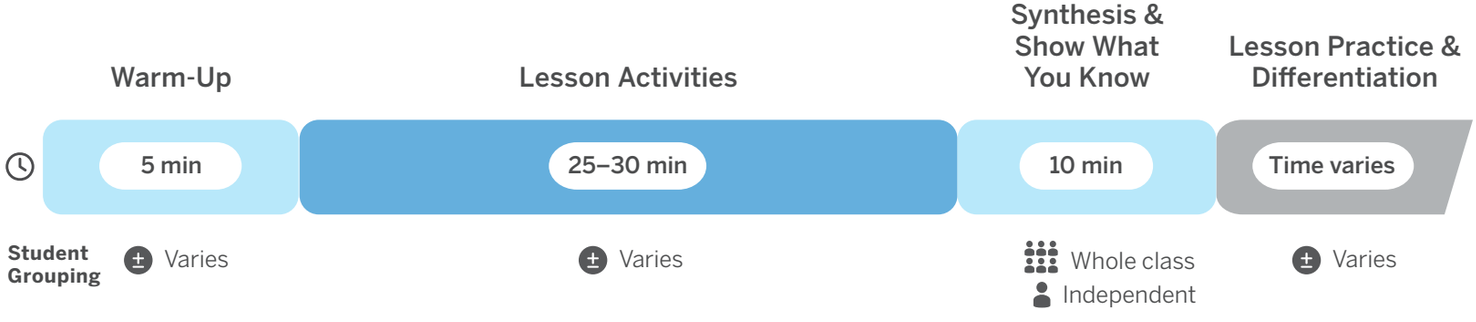
Note: The number of lessons varies from unit to unit. See Scope and Sequence pages for the full program scope.

Unit



Note: The number of sub-units, quizzes, and lessons within each unit varies. This depiction shows the general structure of a unit. See the course Table of Contents for more details.

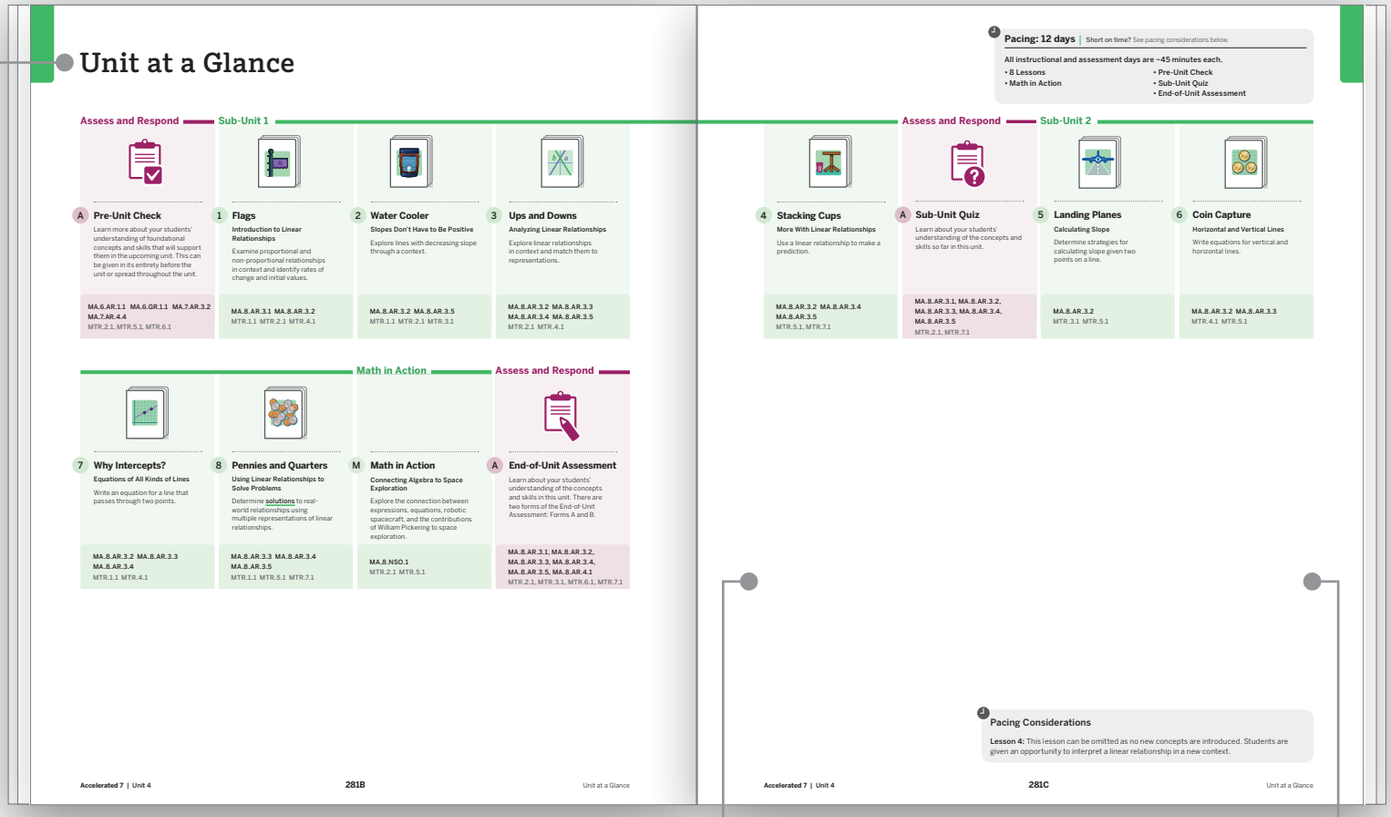
Lesson



Navigating This Program

Unit & Sub-Unit Resources

Each unit includes a range of resources designed to support teachers in thinking through the progression of mathematics that students will engage with over the course of the unit. These resources can support teachers in their unit planning, as well as choices they make in response to students' thinking, strengths, and needs that arise over the course of the unit.



Unit at a Glance

Assess and Respond — Sub-Unit 1

A Pre-Unit Check
Learn more about your students' understanding of foundational concepts and skills that will support them in the upcoming unit. This can be given in its entirety before the unit or spread throughout the unit.
MA.6.AR.1.1 MA.6.GR.1.1 MA.7.AR.3.2 MA.7.AR.4.4
MTR.2.1, MTR.5.1, MTR.6.1

1 Flags
Introduction to Linear Relationships
Examine proportional and non-proportional relationships in context and identify rates of change and initial values.
MA.8.AR.3.1 MA.8.AR.3.2
MTR.1.1 MTR.2.1 MTR.4.1

2 Water Cooler
Slopes Don't Have to Be Positive
Explore lines with decreasing slope through a context.
MA.8.AR.3.2 MA.8.AR.3.5
MTR.1.1 MTR.2.1 MTR.3.1

3 Ups and Downs
Analyzing Linear Relationships
Explore linear relationships in context and match them to representations.
MA.8.AR.3.2 MA.8.AR.3.3 MA.8.AR.3.4 MA.8.AR.3.5
MTR.2.1 MTR.4.1

Math in Action — Assess and Respond

7 Why Intercepts?
Equations of All Kinds of Lines
Write an equation for a line that passes through two points.
MA.8.AR.3.2 MA.8.AR.3.3 MA.8.AR.3.4
MTR.1.1 MTR.4.1

8 Pennies and Quarters
Using Linear Relationships to Solve Problems
Determine solutions to real-world relationships using multiple representations of linear relationships.
MA.8.AR.3.3 MA.8.AR.3.4 MA.8.AR.3.5
MTR.1.1 MTR.5.1 MTR.7.1

M Math in Action
Connecting Algebra to Space Exploration
Explore the connection between expressions, equations, robotic spacecraft, and the contributions of William Pickering to space exploration.
MA.8.NSO.1
MTR.2.1 MTR.5.1

A End-of-Unit Assessment
Learn about your students' understanding of the concepts and skills in this unit. There are two forms of the End-of-Unit Assessment: Forms A and B.
MA.8.AR.3.1, MA.8.AR.3.2, MA.8.AR.3.3, MA.8.AR.3.4, MA.8.AR.3.5, MA.8.AR.4.1
MTR.2.1, MTR.3.1, MTR.6.1, MTR.7.1

Accelerated 7 | Unit 4 281B Unit at a Glance

Pacing: 12 days | Short on time? See pacing considerations below.
All instructional and assessment days are ~45 minutes each.
• 8 Lessons
• Math in Action
• Pre-Unit Check
• Sub-Unit Quiz
• End-of-Unit Assessment

Assess and Respond — Sub-Unit 2

4 Stacking Cups
More With Linear Relationships
Use a linear relationship to make a prediction.
MA.8.AR.3.2 MA.8.AR.3.4 MA.8.AR.3.5
MTR.5.1, MTR.7.1

A Sub-Unit Quiz
Learn about your students' understanding of the concepts and skills so far in this unit.
MA.8.AR.3.1, MA.8.AR.3.2, MA.8.AR.3.3, MA.8.AR.3.4, MA.8.AR.3.5
MTR.2.1, MTR.7.1

5 Landing Planes
Calculating Slope
Determine strategies for calculating slope given two points on a line.
MA.8.AR.3.2
MTR.3.1 MTR.5.1

6 Coin Capture
Horizontal and Vertical Lines
Write equations for vertical and horizontal lines.
MA.8.AR.3.2 MA.8.AR.3.3
MTR.4.1 MTR.5.1

Pacing Considerations
Lesson 4: This lesson can be omitted as no new concepts are introduced. Students are given an opportunity to interpret a linear relationship in a new context.

Accelerated 7 | Unit 4 281C Unit at a Glance

Every unit has a **Unit at a Glance** page that shows teachers everything they need to know to get started planning out their upcoming unit.

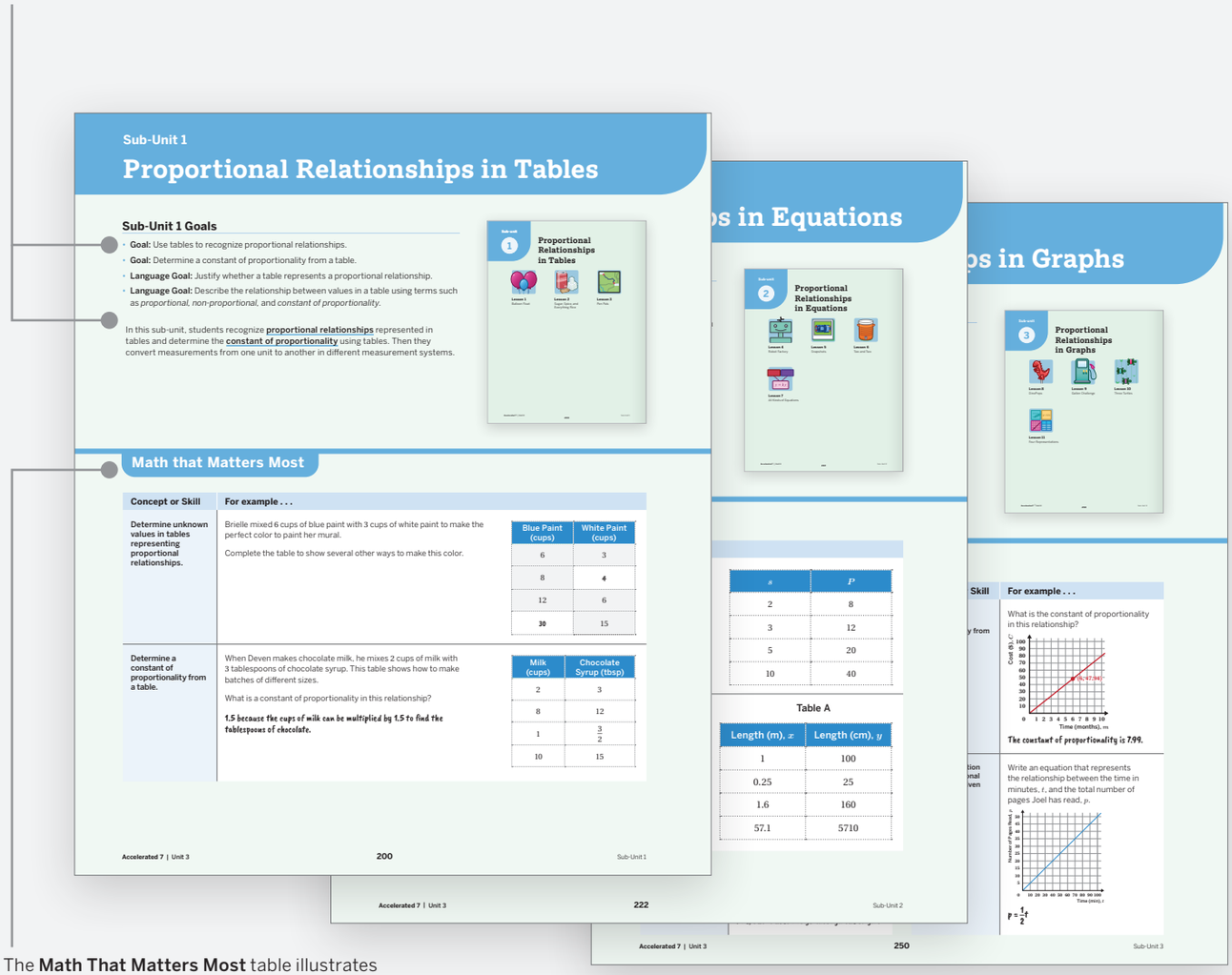
This overview of **concept development** supports teachers in making decisions about potential intervention points during the unit. It indicates at which point students have had multiple opportunities to focus on each key concept or skill. It does not necessarily mean there are no further opportunities to strengthen and develop these skills.

Teachers are provided with thoughtful **pacing considerations** for how they can adjust the pacing of the unit as needed without compromising unit learning goals.

Unit Overview pages

Teachers will find a comprehensive set of resources for each unit, including an overview of the math of the unit, a visual summary of the Unit at a Glance, a preview of each of the unit assessments, and guidance about differentiation, accessibility, language development, materials, technology, and connections to future learning. Each Unit Overview also includes a professional development activity and a formative Pre-Unit Check that teachers can use to learn more about students' understanding of foundational concepts and skills that will support them in the upcoming unit.

The **Sub-Unit Overview** page summarizes the **key goals** and **vocabulary** for this part of the unit.



The **Math That Matters Most** table illustrates for teachers the most important progressions of strategies, skills, and language that develop over the course of the sub-unit.

Sub-Unit Overview pages

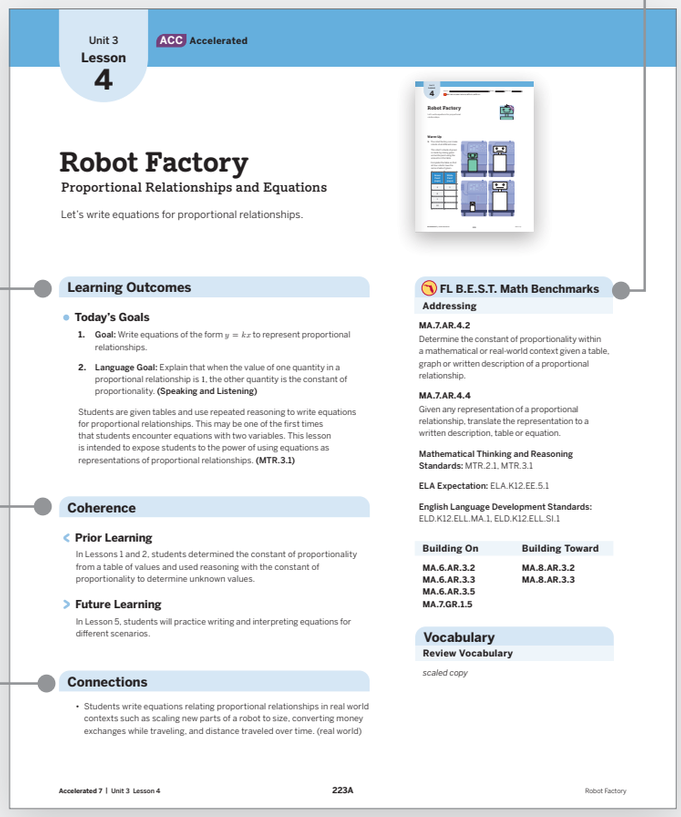
The lessons within each unit are grouped into sub-units that address a related group of concepts. All sub-units include lesson practice and Show What You Know formative assessments, and targeted sub-units include a quiz.

Navigating This Program

Lesson Supports

Throughout this Teacher Edition, lesson guidance for teachers is organized clearly and consistently so that they have all of the information they need at their fingertips.

The **Florida B.E.S.T. Math Benchmarks** section will list all standards addressed in this lesson, including the **Mathematical Thinking and Reasoning Standards,** the **ELA Expectations,** and the **English Language Development Standards.** This section shows the standards the lesson builds on and the standards the lesson builds toward.



Unit 3
Lesson
4

ACC Accelerated

Robot Factory

Proportional Relationships and Equations

Let's write equations for proportional relationships.

Learning Outcomes

Today's Goals

1. **Goal:** Write equations of the form $y = kx$ to represent proportional relationships.
2. **Language Goal:** Explain that when the value of one quantity in a proportional relationship is 1, the other quantity is the constant of proportionality. **(Speaking and Listening)**

Students are given tables and use repeated reasoning to write equations for proportional relationships. This may be one of the first times that students encounter equations with two variables. This lesson is intended to expose students to the power of using equations as representations of proportional relationships. **(MTR.3.1)**

Coherence

Prior Learning

In Lessons 1 and 2, students determined the constant of proportionality from a table of values and used reasoning with the constant of proportionality to determine unknown values.

Future Learning

In Lesson 5, students will practice writing and interpreting equations for different scenarios.

Connections

- Students write equations relating proportional relationships in real world contexts such as scaling new parts of a robot to size, converting money exchanges while traveling, and distance traveled over time. (real world)

FL B.E.S.T. Math Benchmarks

Addressing

MA.7.AR.4.2
Determine the constant of proportionality within a mathematical or real-world context given a table, graph or written description of a proportional relationship.

MA.7.AR.4.4
Given any representation of a proportional relationship, translate the representation to a written description, table or equation.

Mathematical Thinking and Reasoning Standards: MTR.2.1, MTR.3.1

ELA Expectation: ELA.K12.EE.5.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.6.AR.3.2	MA.8.AR.3.2
MA.6.AR.3.3	MA.8.AR.3.3
MA.6.AR.3.5	
MA.7.GR.1.5	

Vocabulary

Review Vocabulary

scaled copy

Accelerated 7 | Unit 3 Lesson 4 223A Robot Factory

The **Learning Outcomes** section is where teachers will find the goals of the lesson, including language goals

In the **Coherence** section, teachers will find information on prior learning that has built to the math in this lesson, as well as future learning that this lesson is helping build to.

In the **Connections** section, teachers will find opportunities to help their students make connections cross-curricularly, to the real world, or to other mathematical learning.

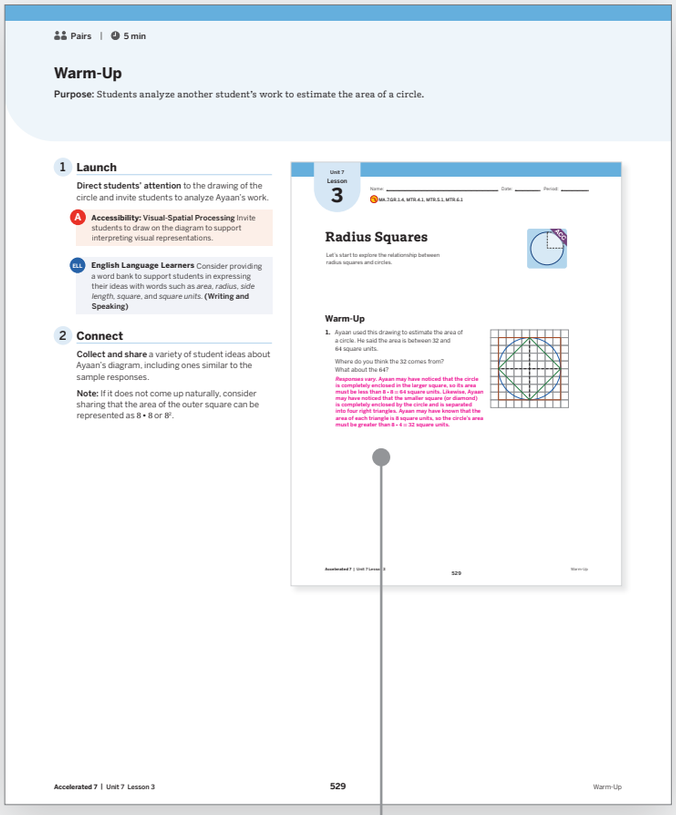
Lesson Overview

This introductory page orients teachers to the topic, standards, and key learning goals of the lesson, including any new vocabulary terms that will be introduced.

The **Key Takeaways** and **Lesson Takeaway** summarize the most important ideas in the lesson.

The **time frame** and suggested **student grouping** is listed for each part of the lesson.

The **Prep Checklist** displays all materials students will need for the lesson.



Sample student responses to the Warm-Up prompt are provided to help teachers prepare to facilitate the conversation.

Lesson at a Glance

The Lesson at a Glance page describes the purpose of the Warm-Up, Activities, Synthesis, and Show What You Know. Teachers will find suggested timing for each part of the lesson, as well as guidance on whether students should work individually, in pairs, in small groups, or with the whole class. The page also lists any hands-on materials that may be needed.

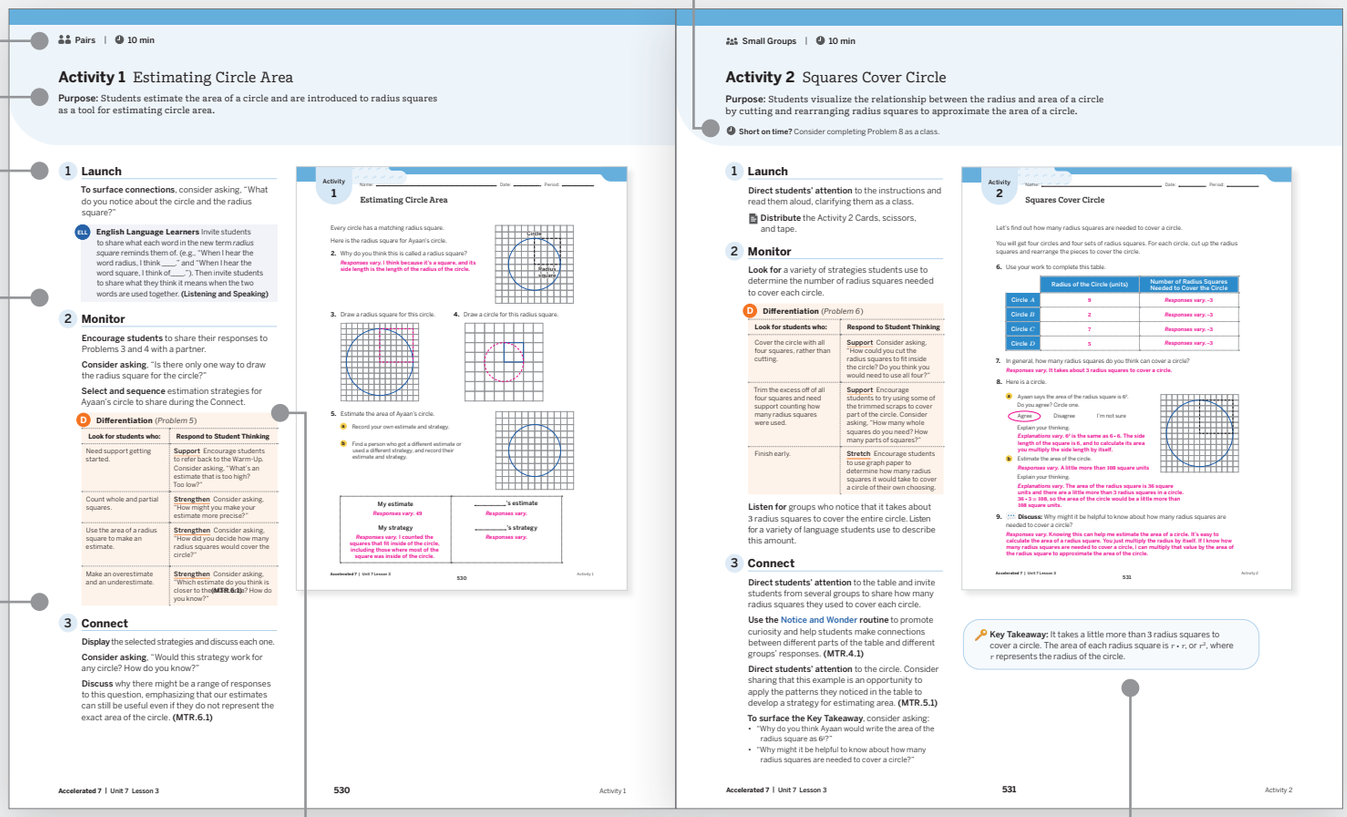
Warm-Up

Every Amplify Desmos Math Florida lesson begins with a Warm-Up to help draw students into the lesson. It might elicit information from their personal experience, remind them of a context they have seen before, invite them to think about the previous lesson, or preview a calculation that will appear in the current lesson.

Navigating This Program

The **Purpose** of each activity is highlighted here, as well as suggestions for the **student grouping** and **time frame** for the activity.

Teachers are provided with thoughtful **Short on time?** suggestions for how they can adjust their facilitation of the lesson as needed without compromising lesson goals.



Pairs | 10 min

Activity 1 Estimating Circle Area

Purpose: Students estimate the area of a circle and are introduced to radius squares as a tool for estimating circle area.

1 Launch

To surface connections, consider asking, "What do you notice about the circle and the radius square?"

ELL **English Language Learners** Invite students to share what each word in the new term radius square reminds them of, (e.g., "When I hear the word radius, I think ____" and "When I hear the word square, I think of ____"). Then invite students to share what they think it means when the two words are used together. **(Listening and Speaking)**

2 Monitor

Encourage students to share their responses to Problems 3 and 4 with a partner.

Consider asking, "Is there only one way to draw the radius square for the circle?"

Select and sequence estimation strategies for Ayaan's circle to share during the Connect.

D **Differentiation** (*Problem 5*)

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Encourage students to refer back to the Warm-Up. Consider asking, "What's an estimate that is too high? Too low?"
Count whole and partial squares.	**Strengthen** Consider asking, "How might you make your estimate more precise?"
Use the area of a radius square to make an estimate.	**Strengthen** Consider asking, "How did you decide how many radius squares would cover the circle?"
Make an overestimate and an underestimate.	**Strengthen** Consider asking, "Which estimate do you think is closer to this (**MTR.6.3**)? How do you know?"

3 Connect

Display the selected strategies and discuss each one.

Consider asking, "Would this strategy work for any circle? How do you know?"

Discuss why there might be a range of responses to this question, emphasizing that our estimates can still be useful even if they do not represent the exact area of the circle. **(MTR.6.1)**

Accelerated 7 | Unit 7 Lesson 3 530 Activity 1

Activity 1 — Estimating Circle Area

Every circle has a matching radius square.

Here is the radius square for Ayaan's circle.

2. Why do you think this is called a radius square?

Responses vary. I think because it's a square, and its side length is the length of the radius of the circle.

3. Draw a radius square for this circle.

4. Draw a circle for this radius square.

5. Estimate the area of Ayaan's circle.

Record your own estimate and strategy.

Find a person who got a different estimate or used a different strategy, and record their estimate and strategy.

My estimate	____'s estimate
Responses vary. 49	*Responses vary.*
My strategy	**____'s strategy**
Responses vary. I counted the squares that fit inside of the circle, including those where most of the square was inside of the circle.	*Responses vary.*

Small Groups | 10 min

Activity 2 Squares Cover Circle

Purpose: Students visualize the relationship between the radius and area of a circle by cutting and rearranging radius squares to approximate the area of a circle.

Short on time? Consider completing Problem 8 as a class.

1 Launch

Direct students' attention to the instructions and read them aloud, clarifying them as a class.

Distribute the Activity 2 Cards, scissors, and tape.

2 Monitor

Look for a variety of strategies students use to determine the number of radius squares needed to cover each circle.

D **Differentiation** (*Problem 6*)

Look for students who:	Respond to Student Thinking
Cover the circle with all four squares, rather than cutting.	**Support** Consider asking, "How could you cut the radius squares to fit inside the circle? Do you think you would need to use all four?"
Trim the excess off of all four squares and need support counting how many radius squares were used.	**Support** Encourage students to try using some of the trimmed scraps to cover part of the circle. Consider asking, "How many whole squares do you need? How many parts of squares?"
Finish early.	**Stretch** Encourage students to use graph paper to determine how many radius squares it would take to cover a circle of their own choosing.

Listen for groups who notice that it takes about 3 radius squares to cover the entire circle. Listen for a variety of language students use to describe this amount.

3 Connect

Direct students' attention to the table and invite students from several groups to share how many radius squares they used to cover each circle.

Use the Notice and Wonder routine to promote curiosity and help students make connections between different parts of the table and different groups' responses. **(MTR.4.1)**

Direct students' attention to the circle. Consider sharing that this example is an opportunity to apply the patterns they noticed in the table to develop a strategy for estimating area. **(MTR.5.1)**

To surface the Key Takeaway, consider asking:

- "Why do you think Ayaan would write the area of the radius square as 6²?"
- "Why might it be helpful to know about how many radius squares are needed to cover a circle?"

Accelerated 7 | Unit 7 Lesson 3 531 Activity 2

Activity 2 — Squares Cover Circle

Let's find out how many radius squares are needed to cover a circle.

You will get four circles and four sets of radius squares. For each circle, cut up the radius squares and rearrange the pieces to cover the circle.

6. Use your work to complete this table.

	Radius of the Circle (units)	Number of Radius Squares Needed to Cover the Circle
Circle *A*	9	*Responses vary.*
Circle *B*	2	*Responses vary.*
Circle *C*	7	*Responses vary.*
Circle *D*	5	*Responses vary.*

7. In general, how many radius squares do you think can cover a circle?

Responses vary. It takes about 3 radius squares to cover a circle.

8. Here is a circle.

Ayaan says the area of the radius square is 6². Do you agree? Circle one.

Agree Disagree I'm not sure

Explain your thinking.

Explanations vary. 6² is the same as 6 • 6. The side length of the square is 6, and to calculate its area you multiply the side length by itself.

Estimate the area of the circle.

Responses vary. A little more than 108 square units

Explain your thinking.

Explanations vary. The area of the radius square is 36 square units and there are a little more than 3 radius squares in a circle. 36 • 3 = 108, so the area of the circle would be a little more than 108 square units.

9. **Discuss:** Why might it be helpful to know about how many radius squares are needed to cover a circle?

Responses vary. Knowing this can help me estimate the area of a circle. It's easy to calculate the area of a radius square. You just multiply the radius by itself. If I know how many radius squares are needed to cover a circle, I can multiply that value by the area of the radius square to approximate the area of the circle.

Key Takeaway: It takes a little more than 3 radius squares to cover a circle. The area of each radius square is $r \cdot r$, or r^2, where r represents the radius of the circle.

In the **Launch**, **Monitor**, **Connect** guidance, teachers will find ways to help students get started, suggested facilitation moves, and discussion questions.

The guidance for every lesson includes a D **Differentiation** table to support teachers in meeting the needs of all students during the activity. This table can help teachers anticipate the ways students may approach the activity, and provides prompts that they can use during the lesson to **Support**, **Strengthen**, and **Stretch** individual students in their thinking.

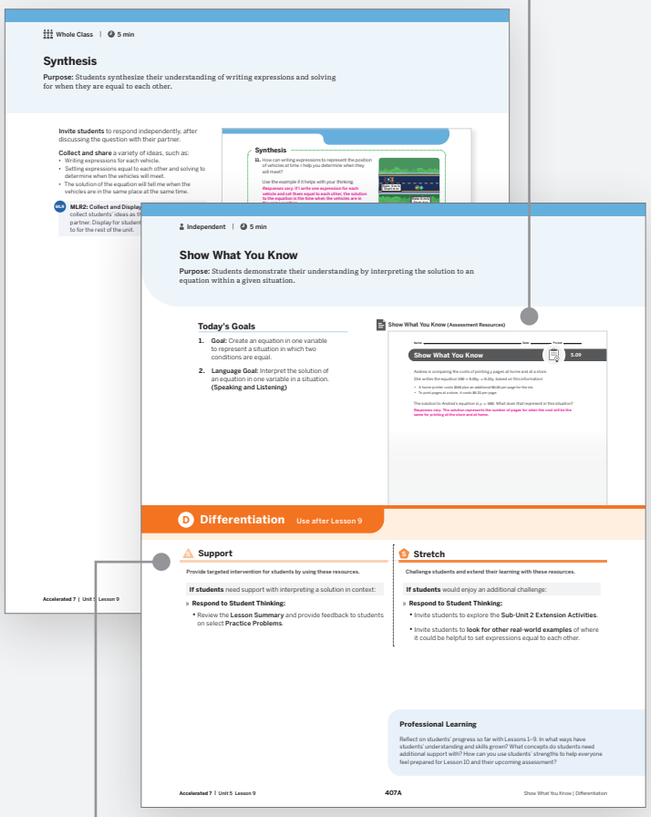
The **Key Takeaway** is called out to highlight the learning goal of the activity and provide teachers with an example of how to frame the big idea of the activity for students.

Activities

Each lesson includes one, two, or three activities. These activities are the heart of each lesson. Students notice, wonder, explore, calculate, predict, measure, explain their thinking, settle disputes, create challenges for their classmates, and more.

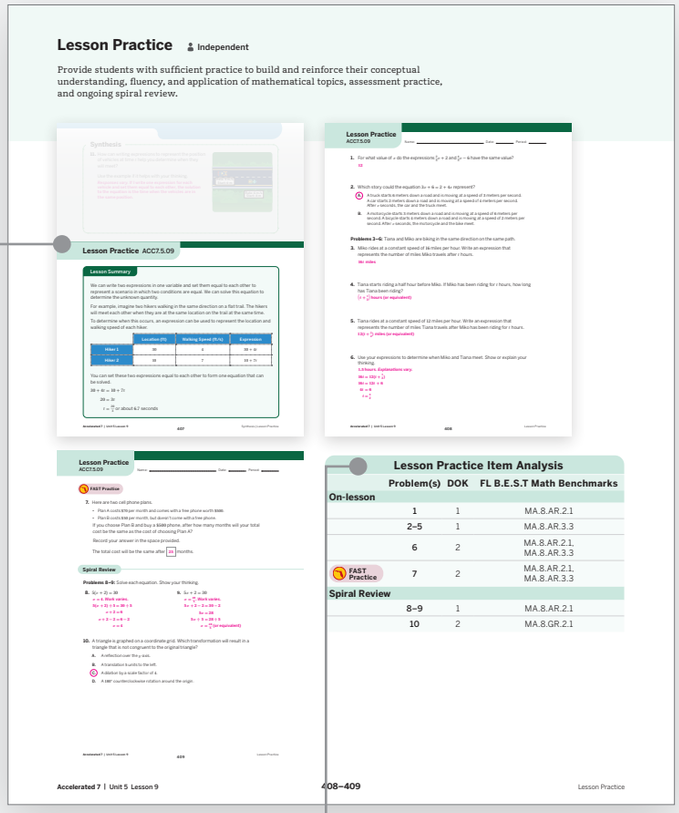
Guidance is provided to help teachers Launch, Monitor, and Connect students' thinking over the course of each activity. Teachers will also find suggestions for pacing, facilitation moves, discussion questions, examples of early student thinking, ideas for students who may enjoy a challenge, and ways to increase access to the language and content. The suggestions for facilitation provide comprehensive support for teachers as they prepare for and teach lessons.

Lessons conclude with the **Show What You Know**, which presents an opportunity for students to reflect on the main learning goals of the lesson. This is a great way for both students and teachers to access a formative check for understanding.



Each Show What You Know is accompanied by a table with suggestions in three categories: students who need **support**, would benefit from more practice to **strengthen** their understanding, or are interested in a **stretch** to deepen or extend their thinking.

Each **Lesson Practice** begins with a **Summary** of the big ideas in the lesson, often including a worked example. Students can highlight parts of the summary or share it with a caregiver or classmate.



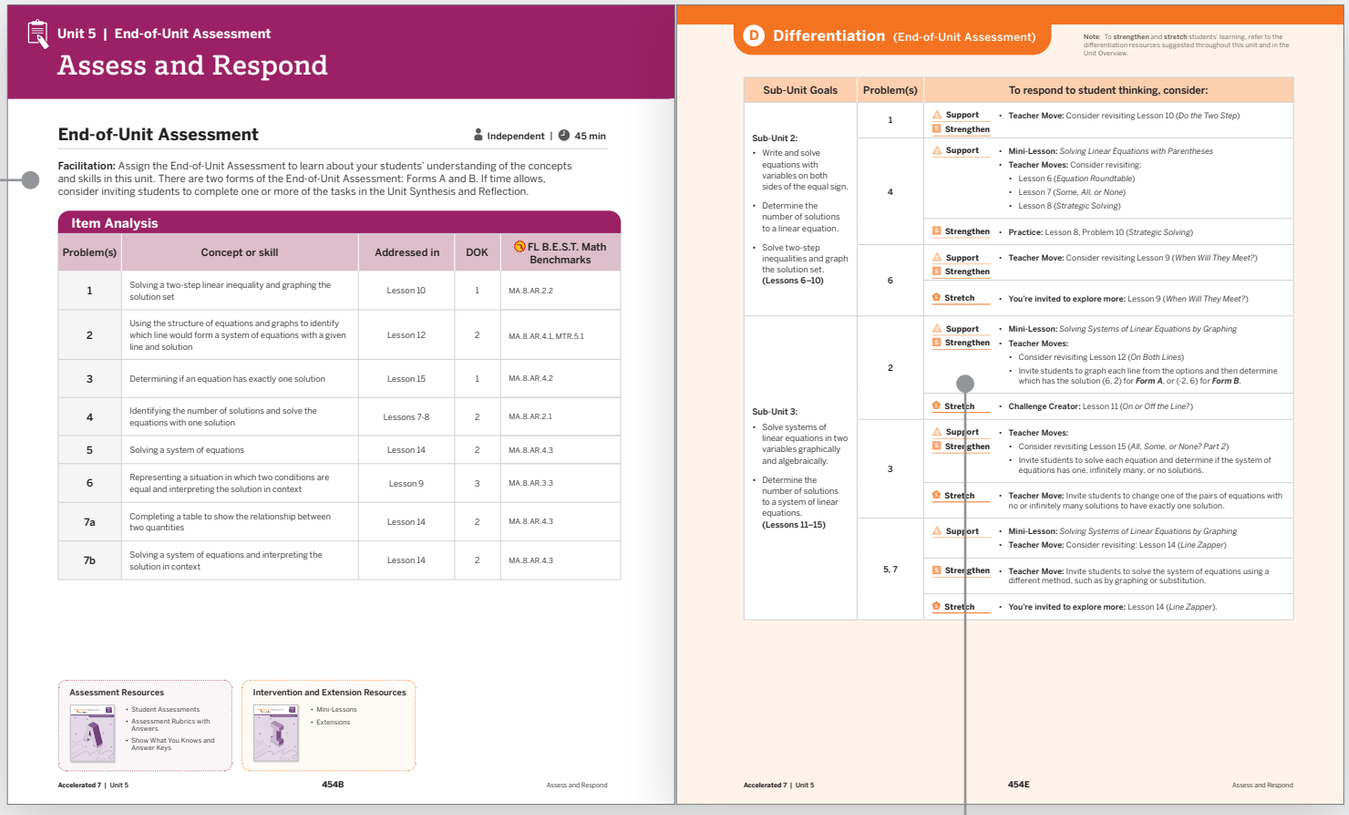
A **Practice Problem Item Analysis** table breaks down the problems by type, Depth of Knowledge (DOK), and corresponding B.E.S.T. Math Benchmarks.

Synthesis

The Synthesis is an opportunity for students to put the key ideas from the lesson into their own words. There is typically an open-ended prompt followed by a discussion for students to consolidate and refine their ideas about the learning goals. If time allows, it is also an opportunity for students to revise their responses after the discussion.

Lesson Practice

Daily practice problems for the day's lesson are included both online and in the print Student Edition, including Fluency, FAST Practice, and Spiral Review.



Unit 5 | End-of-Unit Assessment

Assess and Respond

End-of-Unit Assessment

Independent | 45 min

Facilitation: Assign the End-of-Unit Assessment to learn about your students' understanding of the concepts and skills in this unit. There are two forms of the End-of-Unit Assessment: Forms A and B. If time allows, consider inviting students to complete one or more of the tasks in the Unit Synthesis and Reflection.

Item Analysis

Problem(s)	Concept or skill	Addressed in	DOK	FL B.E.S.T. Math Benchmarks
1	Solving a two-step linear inequality and graphing the solution set	Lesson 10	1	MA.8.AR.2.2
2	Using the structure of equations and graphs to identify which line would form a system of equations with a given line and solution	Lesson 12	2	MA.8.AR.4.1, MTR.5.1
3	Determining if an equation has exactly one solution	Lesson 15	1	MA.8.AR.4.2
4	Identifying the number of solutions and solve the equations with one solution	Lessons 7-8	2	MA.8.AR.2.1
5	Solving a system of equations	Lesson 14	2	MA.8.AR.4.3
6	Representing a situation in which two conditions are equal and interpreting the solution in context	Lesson 9	3	MA.8.AR.3.3
7a	Completing a table to show the relationship between two quantities	Lesson 14	2	MA.8.AR.4.3
7b	Solving a system of equations and interpreting the solution in context	Lesson 14	2	MA.8.AR.4.3

Assessment Resources
- Student Assessments
- Assessment Rubrics with Answers
- Show What You Knows and Answer Keys

Intervention and Extension Resources
- Mini-Lessons
- Extensions

Accelerated 7 | Unit 5 454B Assess and Respond

D Differentiation (End-of-Unit Assessment)

Note: To **strengthen** and **stretch** students' learning, refer to the differentiation resources suggested throughout this unit and in the Unit Overview.

Sub-Unit Goals	Problem(s)	To respond to student thinking, consider:
Sub-Unit 2: • Write and solve equations with variables on both sides of the equal sign. • Determine the number of solutions to a linear equation. • Solve two-step inequalities and graph the solution set. **(Lessons 6–10)**	1	Support, Strengthen: **Teacher Move:** Consider revisiting Lesson 10 (*Do the Two Step*)
	4	Support: **Mini-Lesson:** *Solving Linear Equations with Parentheses* **Teacher Moves:** Consider revisiting: • Lesson 6 (*Equation Roundtable*) • Lesson 7 (*Some, All, or None*) • Lesson 8 (*Strategic Solving*)
		Strengthen: **Practice:** Lesson 8, Problem 10 (*Strategic Solving*)
	6	Support, Strengthen: **Teacher Move:** Consider revisiting Lesson 9 (*When Will They Meet?*)
		Stretch: **You're invited to explore more:** Lesson 9 (*When Will They Meet?*)
Sub-Unit 3: • Solve systems of linear equations in two variables graphically and algebraically. • Determine the number of solutions to a system of linear equations. **(Lessons 11–15)**	2	Support, Strengthen: **Mini-Lesson:** *Solving Systems of Linear Equations by Graphing* **Teacher Moves:** • Consider revisiting Lesson 12 (*On Both Lines*) • Invite students to graph each line from the options and then determine which has the solution (6, 2) for ***Form A***, or (-2, 6) for ***Form B***.
		Stretch: **Challenge Creator:** Lesson 11 (*On or Off the Line?*)
	3	Support, Strengthen: **Teacher Moves:** • Consider revisiting Lesson 15 (*All, Some, or None? Part 2*) • Invite students to solve each equation and determine if the system of equations has one, infinitely many, or no solutions.
		Stretch: **Teacher Move:** Invite students to change one of the pairs of equations with no or infinitely many solutions to have exactly one solution.
	5, 7	Support: **Mini-Lesson:** *Solving Systems of Linear Equations by Graphing* **Teacher Move:** Consider revisiting: Lesson 14 (*Line Zapper*)
		Strengthen: **Teacher Move:** Invite students to solve the system of equations using a different method, such as by graphing or substitution.
		Stretch: **You're invited to explore more:** Lesson 14 (*Line Zapper*).

Accelerated 7 | Unit 5 454E Assess and Respond

An **Item Analysis** is provided to show what concepts and skills are assessed in each problem.

A **Differentiation** table suggests specific materials from the unit to **Support**, **Strengthen**, or **Stretch** students' understanding based on their responses to different assessment problems.

Assess and Respond

Each unit typically includes one or two Sub-Unit Quizzes. Quizzes are designed for students to show what they know and can do based on what they have learned so far in the unit. Each unit includes Assess and Respond guidance for the Pre-Unit Check, Sub-Unit Quizzes, and End-of-Unit Assessment.

Facilitating Lesson Activities

Launch, Monitor, Connect

Amplify Desmos Math Florida is designed with a structured approach to problem-based learning that systematically builds on students' curiosity. Students are first invited to explore problems that create an intellectual need for new mathematical ideas. Then the teacher builds on strategies used by students and connects their ideas to the learning goals of the lesson. This approach is an interpretation of Smith and Stein's *5 Practices for Orchestrating Productive Mathematics Discussions*.

1 Launch

The Launch is a short, whole-class conversation that creates a need or excitement, provides clarity, or helps students connect to their prior knowledge or personal experience, which ensures that everyone has access to the upcoming work.

Considerations for Launching

- Try to keep it short. Set students up to get started with a clear and catchy invitation to the math.
- Wait to model a specific way to solve. Leave space for a variety of different student approaches.
- Pair up. Encourage small groups of students to talk through their thinking as they work.

2 Monitor

As students work individually, in pairs, or in groups, teachers explore students' thinking, ask questions, and provide support to help move the conversations closer to the intended math learning goal.

Considerations for Monitoring

- Look for students' strategies and check in with students as they work.
- Ask questions to learn what students are thinking. The suggested differentiation moves can be used to support, strengthen, and stretch their ideas.
- Use the Differentiation table to help select and sequence student ideas to highlight during the Connect discussion.

3 Connect

Teachers connect students' ideas to the key learning goals of the lesson, facilitating class discussions that help students synthesize and solidify the big ideas.

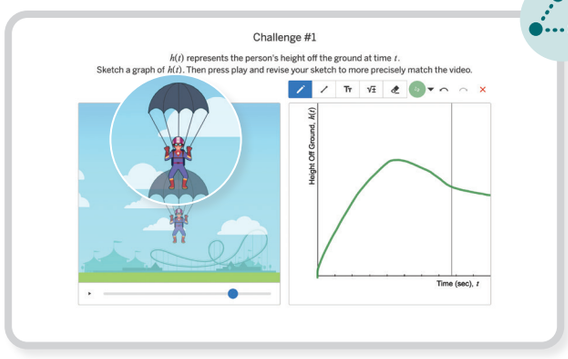
Considerations for Connecting

- Save a few minutes at the end of each lesson activity to bring students back together to discuss.
- It is okay to start the class discussion before all students have finished the activity.
- Students will be able to contribute to the discussion and learn from their classmates based on their in-progress work.
- Center discussions on students' ideas by displaying one or more of their responses and connecting the responses to the Key Takeaway of the activity.

Smith, M.S., & Stein, M.K. (2018). *5 practices for orchestrating productive mathematics discussions* (2nd ed.). SAGE Publications.

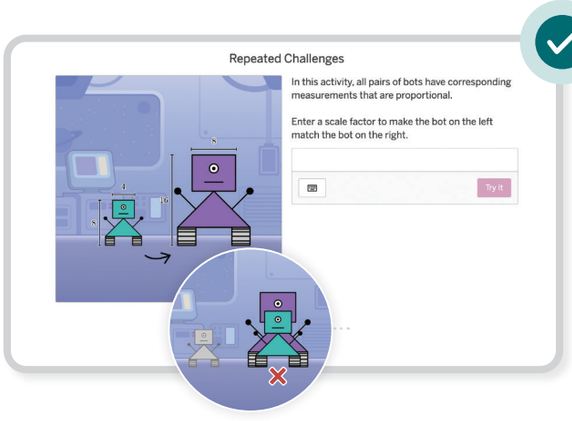
The Power of Digital

Amplify Desmos Math Florida provides the option for teachers to deliver lessons using the power of digital technology. This includes the use of visual and dynamic interactions that pique students' interest and invite all students to engage in the mathematics. These interactions and animations allow students to test predictions, get feedback, share ideas, and connect representations.



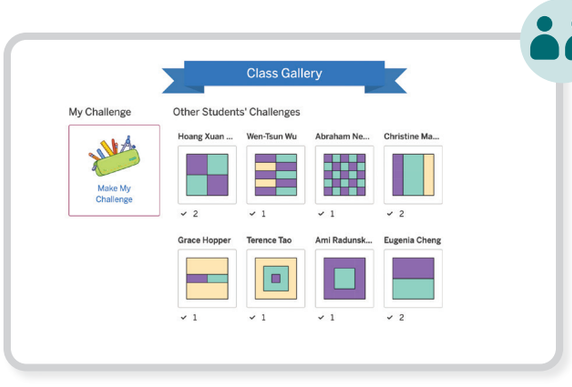
Delightful, engaging interactions

The digital interactions included in lesson activities are designed to elicit students' thinking in a way that feels fun and inviting. As students play and explore math concepts, teachers can highlight the ideas that students share, connect those ideas to other students' ideas, and build on their thinking through productive class discussion.



Responsive Feedback™

In Amplify Desmos Math Florida, students are invited to try their thinking out — even if that thinking is still in the "rough draft" phase. As students interact with the digital screens, they see visuals and simulations that respond faithfully to their inputs. This meaningful feedback allows students to experience the joy of causing the animation to react to their mathematical ideas. As a result, students may notice interesting things about both correct and incorrect answers more readily.

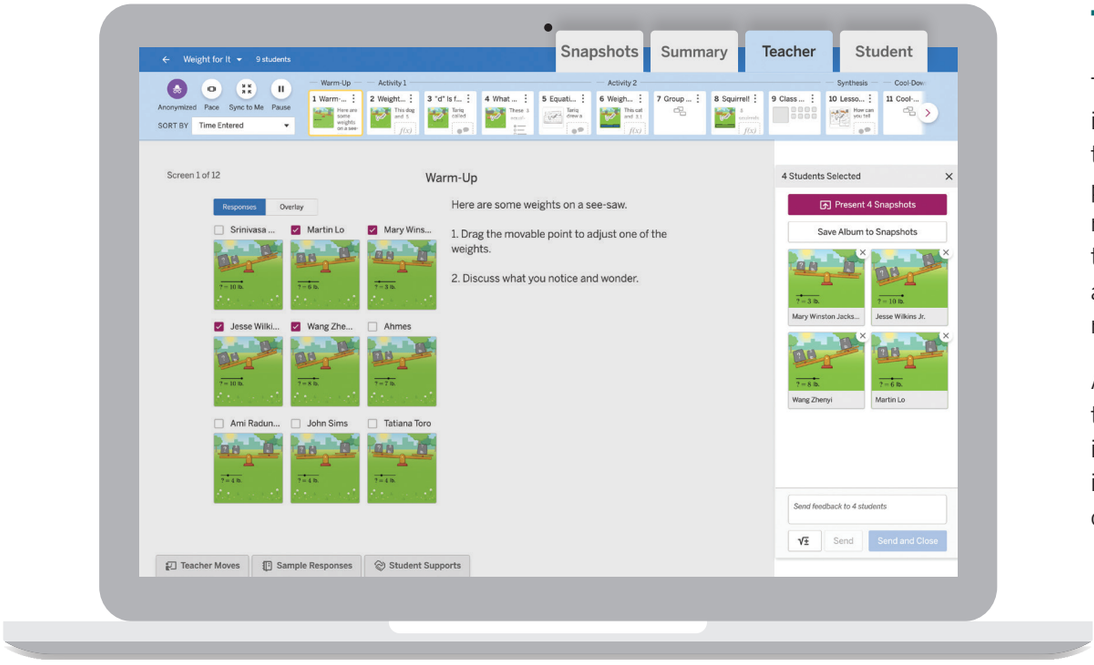


Collaborative experiences

Digital tools allow students to interact with each other's mathematical ideas in a variety of ways. Students can use the "Share With Class" button to exchange mathematical thinking with each other directly on an activity screen. Activities like Challenge Creator and Polygraph offer fun ways for students to play with the math together. Whether working solo or in tandem on devices, students are never alone as they work through activities.

Digital Facilitation Tools

Amplify Desmos Math Florida includes a suite of digital facilitation tools. These tools foster collaborative classrooms and help teachers share their students' thinking.



Teacher Dashboard

The dashboard gives the teacher insight into students' thinking in real time. Teachers can zoom in on a particular student or view all students' responses at once. This can help to identify students that may need additional support and those who are ready for extensions.

After reviewing students' thinking, teachers can select and display specific ideas or the distribution of responses to invite students into productive, student-centered discussions.

Snapshot View

Select and sequence students' work to connect mathematical ideas. Teachers can even add their own questions to prompt further thinking.

Summary View

Monitor students' progress or accuracy for a set of screens. Click into any box to see a specific student's work on that screen.

Teacher View

Answer questions such as:

- How did all students answer this question?
- What answers were most common?

Student View

Preview screens for students. For example, the teacher might work through a screen with the class or talk through upcoming screens before students work on their own.

A powerful conversation toolkit

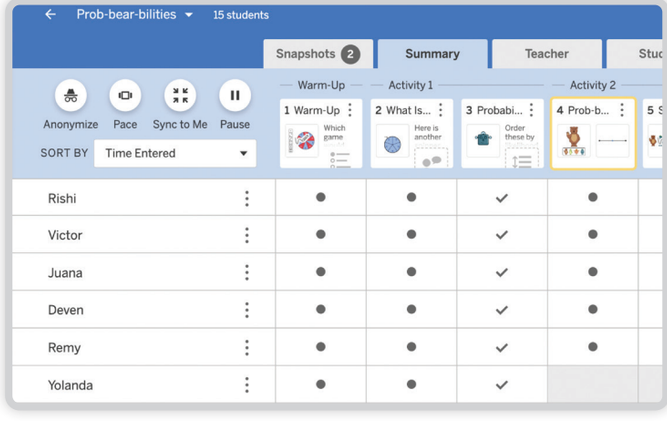
Pacing, pausing, and sync tools can support teachers as they encourage mathematical discourse and collaborative thinking in their classes.

Anonymize swaps out students' names to help them feel more comfortable sharing their ideas. Students' names are replaced by the names of famous mathematicians.

Pace allows the teacher to make a certain number of screens available for students to work on.

Sync to Me enables the teacher to bring all students to the same screen.

Pause stops students from working so that the teacher can gather everyone's attention for discussion.



Math in Action

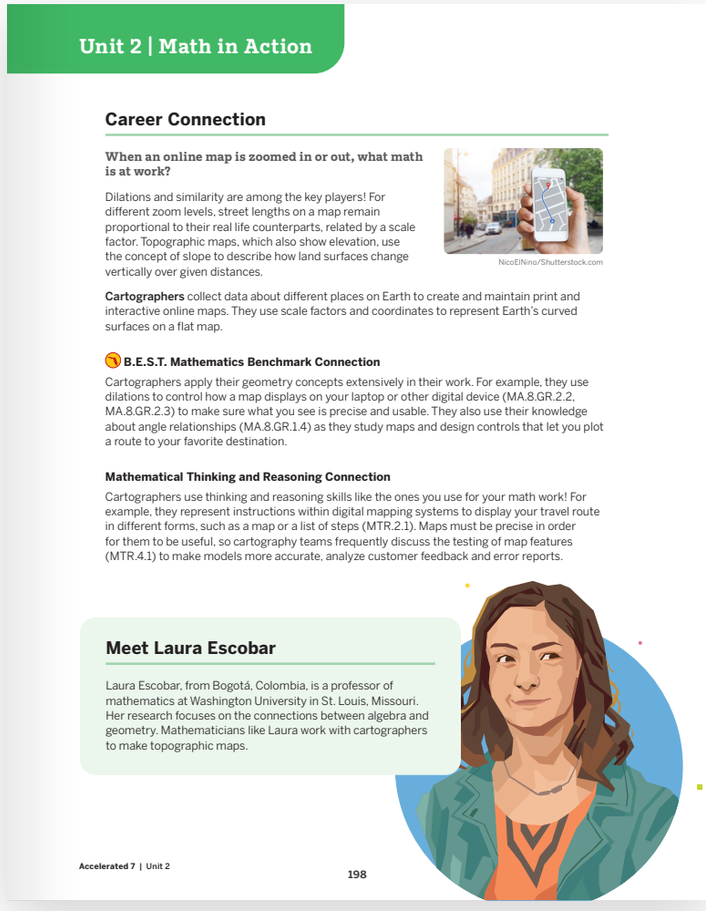
Each Math in Action feature highlights an individual whose work in the field of mathematics, or a connected field, have made an impact on American society. These features can be used to help make connections between the mathematical content of the grade and how math shows up in a wide variety of careers.

Career Connections

Amplify Desmos Math Florida intentionally celebrates traditional mathematics fields as well as elevates work that engages mathematics in unique ways.

Representation of Mathematicians

Students can read about the contributions and work of a variety of individuals.



Unit 2 | Math in Action

Career Connection

When an online map is zoomed in or out, what math is at work?

Dilations and similarity are among the key players! For different zoom levels, street lengths on a map remain proportional to their real life counterparts, related by a scale factor. Topographic maps, which also show elevation, use the concept of slope to describe how land surfaces change vertically over given distances.

NicoElNino/Shutterstock.com

Cartographers collect data about different places on Earth to create and maintain print and interactive online maps. They use scale factors and coordinates to represent Earth's curved surfaces on a flat map.

B.E.S.T. Mathematics Benchmark Connection

Cartographers apply their geometry concepts extensively in their work. For example, they use dilations to control how a map displays on your laptop or other digital device (MA.8.GR.2.2, MA.8.GR.2.3) to make sure what you see is precise and usable. They also use their knowledge about angle relationships (MA.8.GR.1.4) as they study maps and design controls that let you plot a route to your favorite destination.

Mathematical Thinking and Reasoning Connection

Cartographers use thinking and reasoning skills like the ones you use for your math work! For example, they represent instructions within digital mapping systems to display your travel route in different forms, such as a map or a list of steps (MTR.2.1). Maps must be precise in order for them to be useful, so cartography teams frequently discuss the testing of map features (MTR.4.1) to make models more accurate, analyze customer feedback and error reports.

Meet Laura Escobar

Laura Escobar, from Bogotá, Colombia, is a professor of mathematics at Washington University in St. Louis, Missouri. Her research focuses on the connections between algebra and geometry. Mathematicians like Laura work with cartographers to make topographic maps.

Accelerated 7 | Unit 2

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Laura Escobar

Mathematics and cartography? Laura Escobar found a way to combine her interest in both by connecting algebra and geometry with cartography to make topographic maps.

William Pickering

Mathematics and space exploration? William Pickering found a way to combine both, working at NASA to help launch the first U.S. satellite and contributing to robotics that were used for missions to the Moon, Venus, and Mars.

Eugenia Cheng

Mathematics and playing the piano? Eugenia Cheng found a way to combine both, using the rhythmic pattern of notes to create pieces that are lively and relaxed.

Kimberly F. Sellers

Mathematics and protein? Kimberly F. Sellers found a way to combine both by using statistics to understand proteins.

Facilitating Meaningful Conversations

Amplify Desmos Math Florida is designed to support interactive and collaborative classrooms, where students make sense of mathematics by sharing their thinking in paired, small-group, and whole-class conversations.

Classroom conversations are an opportunity for students to connect and share their experiences, strengths, and knowledge for the benefit of the entire class. Students are encouraged to share their early thinking, consider the ideas of others, and appreciate different approaches and changing one's mind. Lessons celebrate math as a source of joy and as a tool for analyzing and making sense of the world. Teachers will find structures and tips to support them in facilitating meaningful conversations throughout each lesson.

Centering Personal Connections

Teachers are encouraged to consider the experiences of the students in their classes and what connections they might have to various math topics and contexts. The Teacher Edition offers support for building conversations around the real-world and mathematical experiences that students bring to the classroom.

Use the Tell a Story routine to help students make sense of the turtle race animation. Invite students to write a story about the animation using their experiences, perspectives, and creativity.

In the lesson **Turtle Time Trials**, students are invited to use the Tell a Story routine to create their own narratives about what the turtles in the animation are doing and why. This is a chance for students to use their creativity while engaging deeply in the math.

1 Launch

Display the images of the rice dishes.

To surface connections, consider asking, "What is your favorite rice dish? When or where do you eat it?"

In the lesson **Rice Ratios**, students are introduced to the concept of equivalent ratios in the context of making rice. The Teacher Edition provides teachers with prompts they can use to help students connect the rice dishes in the activities to their own lives.

Supporting Student Collaboration

Different kinds of **groupings** are suggested for different phases of a lesson, including **whole-class** discussions and **pair** work. The conversations that these groupings encourage are opportunities for students to share their thinking and listen to each others' perspectives.

How will you support student collaboration?

- Discuss what supporting each other without giving answers could look or sound like.
- Invite a student from each group to read the task aloud.

The Teacher Edition provides suggestions for intentional groupings to support access and language development and help students support their partners without simply sharing answers. Digital activities usually suggest that pairs of students share a single device. This 2:1 structure allows students to talk through tasks together in ways that inform each other's thinking.

Celebrating Different Perspectives

Many lesson activities invite students to consider a variety of different responses, including early or unfinished thinking.

Use the Which One Doesn't Belong? routine to support students in comparing structures of two-variable equations.

Encourage students to look for more than one possibility.

In the lesson **Subway Seats**, students use the Which One Doesn't Belong? routine to begin exploring two-variable equations. Classroom conversations about activities like this are a way for teachers to elicit different kinds of student brilliance. The Teacher Edition provides suggestions for how teachers can engage students in conversation and reinforce the idea that each studenthas something uniquely valuable to contribute.

Accessibility

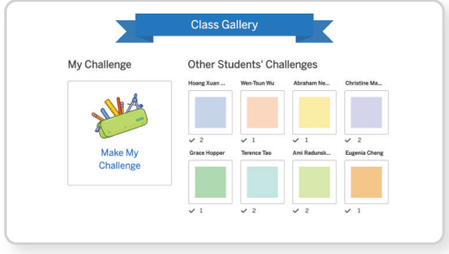
Amplify Desmos Math Florida is built to support all students in accessing and participating in meaningful and challenging learning. This support is incorporated into the curriculum structure, lesson-level guidance, and digital tools.

Universal Design for Learning

Each lesson incorporates opportunities for engagement, representation, action, and expression based on the guidelines of Universal Design for Learning (UDL). UDL is a research-based framework designed to ensure meaningful learning experiences for all students.[1]

Multiple Means of Engagement

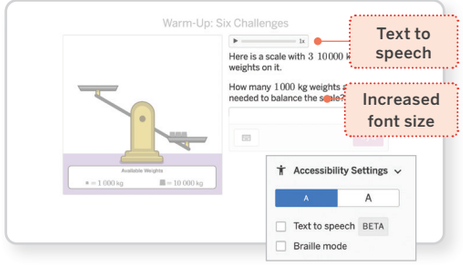
Individuals are each motivated in different ways, at different times, and in different contexts. Lessons are designed to welcome student interests, support sustaining effort and persistence, and develop an awareness of others.



Sustaining Effort and Persistence: Students are invited to build their own challenge for other students to solve, which provides opportunities for choice and autonomy, as well as joy and play.

Multiple Means of Representation

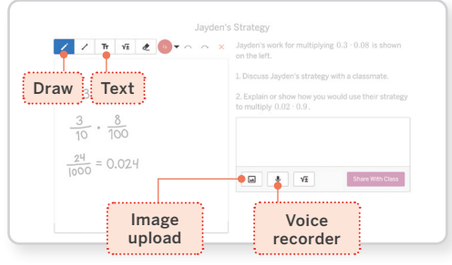
Learners make sense of information differently. Amplify Desmos Math Florida includes options for presenting information in multiple ways to support comprehension and understanding of language.



Present Information in Multiple Ways: Students and teachers can use digital tools such as text to speech, enlarged font, and dynamic visuals.

Multiple Means of Action and Expression

Learners differ in how they navigate learning environments and express what they know. Amplify Desmos Math Florida ensures that materials and interactions are accessible, support multiple means of students' expression and communication, and scaffold executive functioning.



Support Multiple Means of Student Expression and Communication: Students can communicate their ideas in multiple ways, including sketching, uploading photos, or responding with audio.

Lesson Facilitation Supports

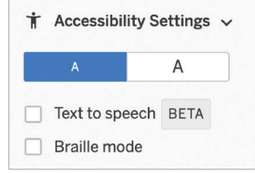
Every lesson includes at least one specific suggestion the teacher can use to increase access to the lesson without reducing the mathematical demand of the tasks. These suggestions address the following areas:

- Conceptual Processing
- Visual-Spatial Processing
- Executive Functioning
- Memory and Attention
- Fine Motor Skills
- Affective Functioning

A **Accessibility: Memory and Attention** To support keeping track of information, invite students to record the dimensions they try and whether or not they created a scaled copy.

Accessibility Tools

Students have the ability to control accessibility tools so that each learning experience is customized to their individual needs. In many instances, these tools can be turned on or off at any point of instruction.



Text to speech: Reads text instructions to students in multiple languages.

Enlarged font: Increases the size of all text on screen.

Braille mode: Includes narration of digital interactions.

Language selection: Toggles between languages.

[1] https://udlguidelines.cast.org/

Differentiation

Differentiation in Amplify Desmos Math Florida enables teachers to ensure all students have access to grade-level math content. This support is organized for teachers into three categories:

Support

Provide targeted intervention for students by using these resources.

Strengthen

Reinforce students' understanding of the concepts assessed by using these resources.

Stretch

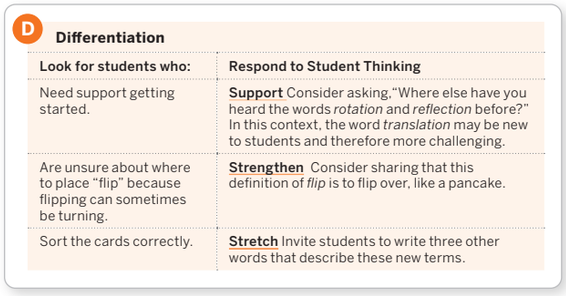
Challenge students and extend their learning with these resources.

Differentiation: In-Lesson Teacher Moves

Within every lesson activity, teachers can use the **Differentiation** suggestions to provide **in-the-moment instructional support** to learners as they engage in the work of the lesson.

Teachers are provided with clear student actions and understandings to look for, each matched with immediately usable suggestions for how to respond to the student thinking illustrated in each row of the table.

In addition to using these suggestions in the moment as teachers monitor student work, teachers can review the Differentiation table in advance to help them anticipate how students are likely to approach the activity.



Differentiation

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Consider asking, "Where else have you heard the words *rotation* and *reflection* before?" In this context, the word *translation* may be new to students and therefore more challenging.
Are unsure about where to place "flip" because flipping can sometimes be turning.	**Strengthen** Consider sharing that this definition of *flip* is to flip over, like a pancake.
Sort the cards correctly.	**Stretch** Invite students to write three other words that describe these new terms.

Differentiation: Beyond the Lesson

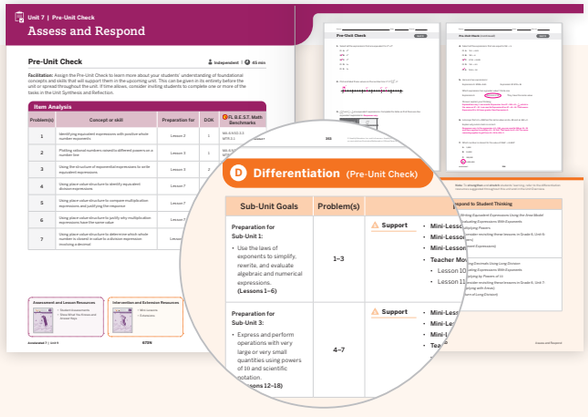
In each lesson, students' understanding of the learning goals is broken down for teachers into three categories: students who need support to understand the learning goals, students who need to continue strengthening their understanding of the learning goals, and students who are ready to stretch their learning.

Each **Support**, **Strengthen**, and **Stretch** resource is designed to take 15 minutes:

- **Mini-Lessons:** Targeted intervention for students who need additional support or need more time.
- **Extensions:** Small group or independent challenges.

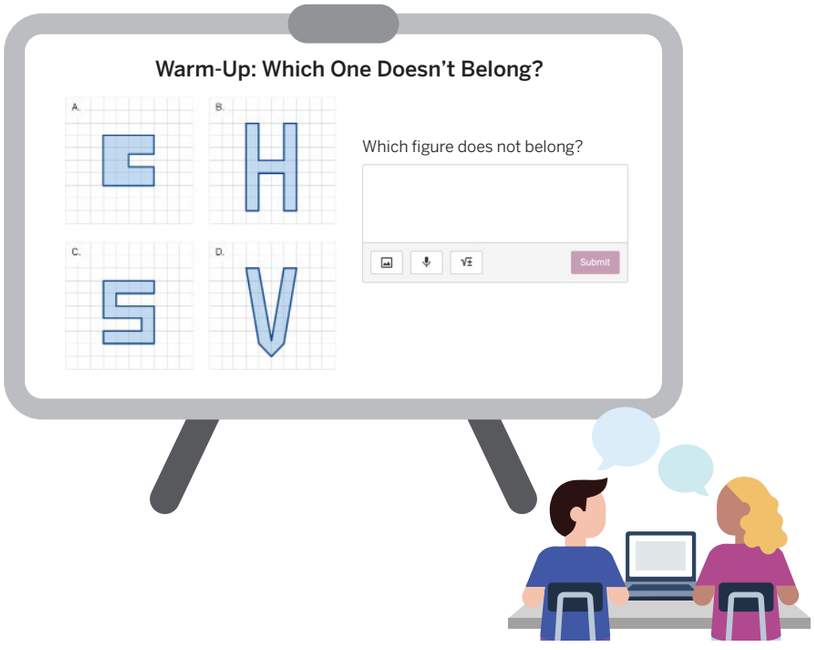
Differentiation: Assess and Respond

At each assessment point in a unit, teachers have the opportunity to respond to student understanding. For each assessment item, teachers are provided with clear suggestions for how to support, strengthen, or stretch student understanding of the corresponding concept based on their assessment responses.



Instructional Routines

Instructional routines that are used repeatedly create efficiencies for teachers so that they can attend to student thinking and communicate what is important in their classroom. Instead of focusing on the directions of an activity, students can focus on making sense of and communicating about the math.



Here are some examples:

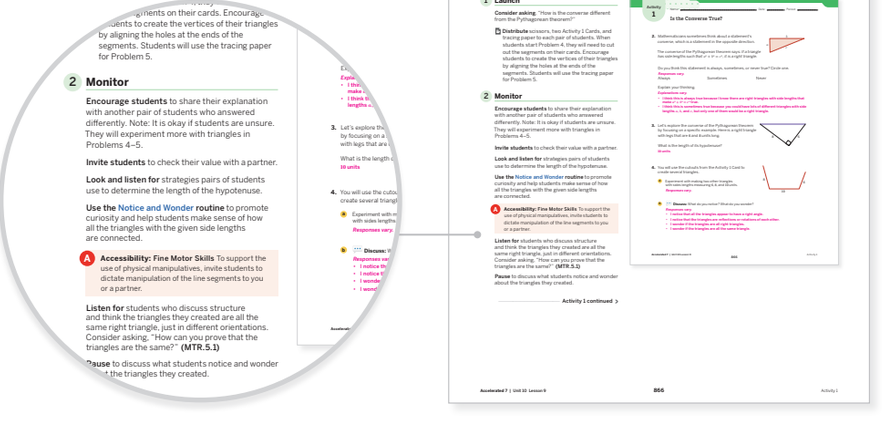
- The **Which One Doesn't Belong?** routine communicates to students that their ideas have value, that there are many ways to be correct in math, and that they can learn math by sharing their math thinking with each other.
- The **Stronger and Clearer Each Time** routine communicates the importance of feedback and creates an opportunity for students to learn from each other as they construct and refine their viable arguments.

Each instructional routine included in an Amplify Desmos Math Florida lesson creates opportunities for conversations and supports meaningful discussion. Implementing these routines can be a practical tool for establishing a classroom learning community that values students' mathematical thinking.

Instructional Routines Embedded in the Curriculum

Instructional routines can be found throughout each lesson in the Teacher Edition. Here is a list of the instructional routines used in the Amplify Desmos Math Florida curriculum:

- **MLR1:** Stronger and Clearer Each Time
- **MLR2:** Collect and Display
- **MLR3:** Critique, Correct, Clarify
- **MLR5:** Co-Craft Questions
- **MLR6:** Three Reads
- **MLR7:** Compare and Connect
- Decide and Defend
- Notice and Wonder
- Number Talk
- Tell a Story
- Think-Pair-Share
- Which One Doesn't Belong?



For more information about the Instructional Routines embedded in the Amplify Desmos Math Florida curriculum, see the Routine Facilitation Guide in the Professional Development Library.

Math Language Development

At Amplify Desmos Math Florida, we acknowledge that there is a strong interconnection between mathematical content, practices, and language. We believe that developing mathematical language is critical for all learners, while recognizing and supporting the unique needs of English Language Learners.

Our approach to math language development focuses on when, how, and why students are using language to make sense of and share their mathematical ideas. Every lesson in Amplify Desmos Math Florida includes opportunities for all students to develop mathematical language as they experience the content, while providing intentional support for English Language Learners. We purposefully progress language throughout the units by cultivating students' language and supporting students in making their arguments and explanations stronger, clearer, and more precise as they progress from lesson to lesson.

Here are four ways we support students with developing mathematical language:

Vocabulary

Units and lessons build connections between students' language and the new vocabulary for that unit. This honors the language assets that students bring into their learning.

Some **new vocabulary** is marked as *optional*; these are terms students may encounter in other settings, but are not essential for them to access the lesson. Vocabulary is *review* if has been introduced and defined in a prior unit or course. The Unit Overview also lists **contextual vocabulary**, which are context-specific non-mathematical terms critical for students to understand the lesson.

Create class definitions for the terms **pre-image** and **image**, and support students in making sense of *prime notation* ($'$). To help generate student language, consider asking:

- "How are the labels different on the two figures?"
- "What does *pre* mean? How is that connected to the term *pre-image*?"
- "Do you think the placement of A', B', C', and D' on the image matters?"

Language Goals

Language goals attend to the mathematics students are learning and are written through the lens of one or more of four language modalities: **Reading**, **Writing**, **Speaking**, and **Listening**. They describe how students will grow their ability to analyze, compare, critique, describe, discuss, explain, interpret, and justify. Language goals appear at both the lesson and sub-unit level.

- **Today's Goals**
  1. **Goal:** Calculate the area of a parallelogram on a grid using a variety of strategies.
  2. **Language Goal:** Compare and contrast strategies for determining the area of a parallelogram. **(Writing, Speaking, and Listening)**
  3. **Language Goal:** Describe parallelograms using precise mathematical language. **(Speaking and Listening)**

Math Language Routines (MLR)

MLR **Math Language Routines**[1] are used within lessons to do one or more of the following: highlight student-developed language and ideas, cultivate conversation, support mathematical sense-making, and promote meta-cognition. Tips for facilitating MLRs are included when they would be helpful within lessons.

MLR **MLR7: Compare and Connect** Invite students to analyze each strategy, compare them, and then connect to their strategy. Consider asking:

- "Who used a strategy similar to Omari's? Ivory's?"
- "Where is the constant difference in each strategy?"
- "Why did different strategies lead to the same outcome?"

English Language Learners Support

Supports for ELL **English Language Learners** are called out at intentional points within each lesson. These suggested supports are specific, targeted actions that are beneficial for English Language Learners. They often describe a modification to increase access to the task or support with contextual or mathematical language development that can often be supportive of all learners. English Language Learner supports may also be attached to MLRs.

ELL **English Language Learners** Encourage students to paraphrase each other's ideas to help them make connections and incorporate vocabulary from the unit. **(Speaking and Listening)**

[1] Zwiers, J., Dieckmann, J., Rutherford-Quach, S., Daro, V., Skarin, R., Weiss, S., & Malamut, J. (2017). Principles for the Design of Mathematics Curricula: Promoting Language and Content Development. Retrieved from Stanford University, UL/SCALE website: http://ell.stanford.edu/content/mathematics-resources-additional-resources

Developing Fluency

Procedural fluency is the ability to apply procedures efficiently, flexibly, and accurately; to transfer procedures to different problems and contexts; to build or modify procedures from other procedures; and to recognize when one strategy or procedure is more appropriate to apply than another*.

In Accelerated 7, Florida students are expected to perform all four operations with integers, positive decimals and positive fractions with procedural fluency.

Expectation

Exploration

To develop understanding through the use of manipulatives, visual models, discussions, estimation and drawings.

Procedural Reliability

To utilize skills to develop an accurate, reliable method that aligns with the student's understanding and learning style.

Procedural Fluency

To utilize skills to become fluent with an efficient, generalizable and accurate procedure, including a standard algorithm.

How Amplify Desmos Math Florida Supports the Expectation

- Manipulatives and visual models and diagrams are used extensively to support student's exploration of new concepts.
- Discussions are strongly encouraged in each lesson, including 💬 Discuss prompts in the student edition as well as suggested discussion prompts in the teacher's edition.
- Students are encouraged to estimate and compare estimations to actual answers to support reasonableness of problem solving solutions.
- Students are often asked to analyze a drawing or to make a drawing to explain their reasoning and strengthen their understanding of the math concepts being studied.

- Frequently in each course, students are asked to analyze and evaluate work presented in their student edition that represents work done by other students for accuracy, preference to learning style, and for efficiency.

- Program features such as practice, spiral reviews, mini lessons, and pre-unit assessments are all designed to assist students with achieving procedural fluency.

*National Research Council [2001, 2005, 2012]; Star [2005]

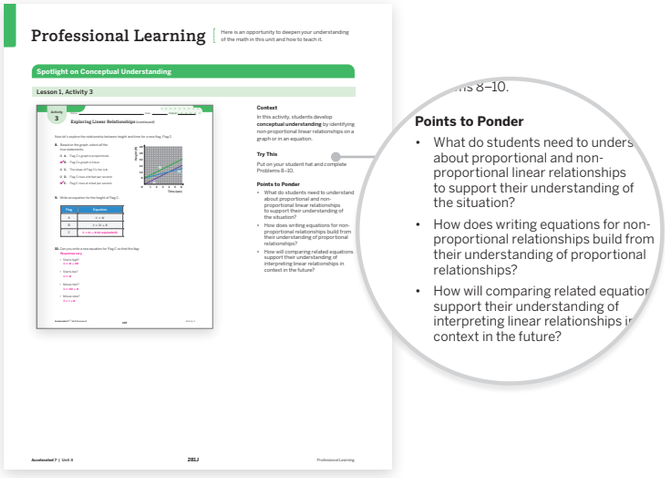
Professional Learning

The Amplify Desmos Math Florida program includes regular opportunities for teachers to deepen their understanding while they teach.

Embedded Professional Development

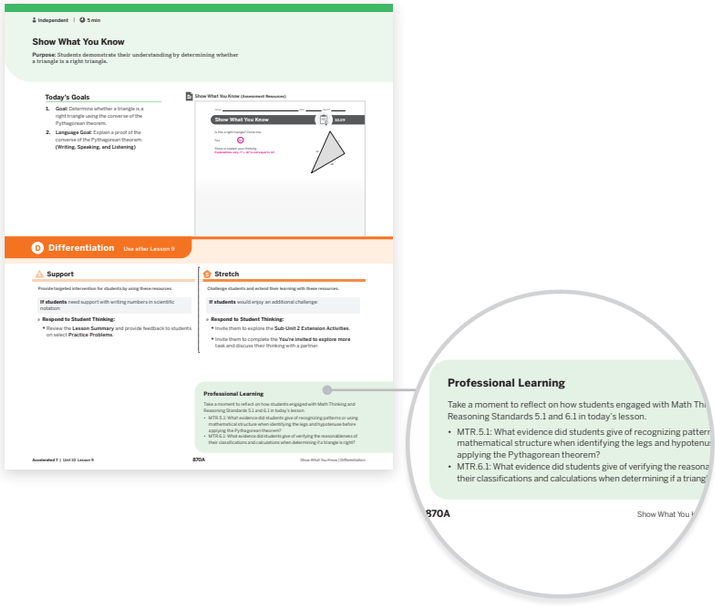
Unit-Level Professional Development

The Unit Overview for each unit includes a Professional Development page, which lists opportunities for teachers to deepen their understanding of the math in the unit and how to teach it.



Lesson-Level Professional Development

Each lesson provides a moment of professional learning as an opportunity for teachers to reflect on how well the lesson went. These professional learning moments can relate to principles, instruction, pedagogy, or program content.



Professional Development Sessions

Amplify's professional development team supports both teachers and administrators throughout their entire implementation journey. These sessions are available as on-site or virtual options.

Launch

Launch sessions introduce Amplify programs and support strong implementation. After learning about the program's foundational principles, you'll practice implementing the program in a collaborative environment.

Strengthen

Strengthen sessions deepen understanding of the program. These offerings are targeted and are meant to take your practice – and your students' learning to the next level.

Coach

These sessions are tailored to elevate instructional practices and meet the unique needs of teachers and leaders. These sessions can be customized leveraging our menu of supports.

Professional learning on your schedule

The PD Library serves as a centralized location where educators can access an ever-growing collection of professional learning resources and courses designed to support the implementation of Amplify Desmos Math Florida. Educators will find bite-sized how to videos, unit and lesson planning guides and templates, resources for leaders, and so much more!

Program Assessments

A variety of performance data in Amplify Desmos Math Florida provides evidence of student learning while helping students bolster their skills and understanding.

Throughout lessons, units, and the entire program, you will find summative and formative assessments meant to provide insights into students' conceptual understandings. Student learning is never a surprise at the end of a unit — with Amplify Desmos Math Florida, understanding is made continually visible.

Lesson-Level Assessments

Amplify Desmos Math lessons are centered around sense-making and in-the-moment feedback. Daily moments of assessment provide insights into students' understandings of concepts and skills.

Show What You Know

Each lesson has a daily formative assessment focused on one of the **key concepts** in the lesson. Show What You Know moments are opportunities for students to show their teacher what they understand and what they are still learning.

Unit-Level Assessments

Our **embedded unit assessments** offer key insights into students' conceptual understanding of math. These assessments provide regular, actionable information about how students are thinking about and processing math, with both **auto-scoring** and **in-depth rubrics** that help teachers anticipate and respond to students' learning needs.

Pre-Unit Check

Each unit begins with a check to determine students' **proficiency** with **prerequisite skills** that are helpful for success in the upcoming unit. This check serves as an affirmation of the knowledge and skills with which students come into the unit.

Sub-Unit Quizzes

In Sub-Unit Quizzes, students are assessed on a subset of **conceptual understandings and procedural fluencies** from the unit, with rubrics that help illuminate students' current understanding and provides guidance for responding to student thinking.

End-of-Unit Assessment

Students engage with rigorous grade-level mathematics through a variety of formats and tasks in the End-of-Unit Assessment. A combination of auto-scored and rubric-scored items provide deep conceptual insight. Two forms are included to help facilitate assessment administration in the classroom.

Florida's B.E.S.T. Standards for Mathematics

The following correlations show the alignment of Amplify Desmos Math Florida to the FL B.E.S.T. Mathematics Standards for Accelerated 7.

Benchmark	B.E.S.T Mathematics Benchmark	Amplify Desmos Math Lesson(s)
Number Sense and Operations		
MA.7.NSO.1.1	Know and apply the Laws of Exponents to evaluate numerical expressions and generate equivalent numerical expressions, limited to whole-number exponents and rational number bases.	**Unit 9** Lessons 1, 2, 3, 4, 5, 6, 12
MA.7.NSO.1.2	Rewrite rational numbers in different but equivalent forms including fractions, mixed numbers, repeating decimals and percentages to solve mathematical and real-world problems.	**Unit 10** Lesson 13
MA.8.NSO.1.1	Extend previous understanding of rational numbers to define irrational numbers within the real number system. Locate an approximate value of a numerical expression involving irrational numbers on a number line.	**Unit 10** Lessons 2, 3, 4, 5, 6, 7, 8, 9, 10, 11,12, 14
MA.8.NSO.1.2	Plot, order and compare rational and irrational numbers, represented in various forms.	**Unit 10** Lessons 2, 3, 4, 5, 14
MA.8.NSO.1.3	Extend previous understanding of the Laws of Exponents to include integer exponents. Apply the Laws of Exponents to evaluate numerical expressions and generate equivalent numerical expressions, limited to integer exponents and rational number bases, with procedural fluency.	**Unit 9** Lessons 7, 8, 9, 10, 11, 13, 15, 16, 17, 18
MA.8.NSO.1.4	Express numbers in scientific notation to represent and approximate very large or very small quantities. Determine how many times larger or smaller one number is compared to a second number.	**Unit 9** Lessons 12, 13, 14, 15, 16, 17, 18
MA.8.NSO.1.5	Add, subtract, multiply and divide numbers expressed in scientific notation with procedural fluency.	**Unit 9** Lessons 15, 16, 17, 18
MA.8.NSO.1.6	Solve real-world problems involving operations with numbers expressed in scientific notation.	**Unit 9** Lessons 12, 13, 14, 15, 16, 17, 18
MA.8.NSO.1.7	Solve multi-step mathematical and real-world problems involving the order of operations with rational numbers including exponents and radicals.	**Unit 10** Lessons 15, 16
Algebraic Reasoning		
MA.7.AR.2.2	Write and solve two-step equations in one variable within a mathematical or real-world context, where all terms are rational numbers.	**Unit 5** Lessons 1, 2, 3, 4, 5
MA.7.AR.3.3	Solve mathematical and real-world problems involving the conversion of units across different measurement systems.	**Unit 3** Lesson 3
MA.7.AR.4.1	Determine whether two quantities have a proportional relationship by examining a table, graph or written description.	**Unit 3** Lessons 1, 2, 7, 8, 10
MA.7.AR.4.2	Determine the constant of proportionality within a mathematical or real-world context given a table, graph or written description of a proportional relationship.	**Unit 3** Lessons 1, 2, 4, 5, 6, 7, 9, 10, 11
MA.7.AR.4.3	Given a mathematical or real-world context, graph proportional relationships from a table, equation or a written description.	**Unit 3** Lesson 11
MA.7.AR.4.4	Given any representation of a proportional relationship, translate the representation to a written description, table or equation.	**Unit 3** Lessons 4, 5, 6, 7, 10, 11

Florida's B.E.S.T. Standards for Mathematics

MA.7.AR.4.5	Solve real-world problems involving proportional relationships.	**Unit 3** Lessons 5, 6, 7, 10
MA.8.AR.1.1	Apply the Laws of Exponents to generate equivalent algebraic expressions, limited to integer exponents and monomial bases.	**Unit 9** Lessons 7, 8, 9
MA.8.AR.1.2	Apply properties of operations to multiply two linear expressions with rational coefficients.	**Unit 5** Lessons 6, 7, 8, 9, 10 **Unit 9** Lessons 7, 10
MA.8.AR.1.3	Rewrite the sum of two algebraic expressions having a common monomial factor as a common factor multiplied by the sum of two algebraic expressions.	**Unit 9** Lesson 11
MA.8.AR.2.1	Solve multi-step linear equations in one variable, with rational number coefficients. Include equations with variables on both sides.	**Unit 1** Lesson 9 **Unit 5** Lessons 6, 7, 8, 9 **Unit 6** Lesson 4
MA.8.AR.2.2	Solve two-step linear inequalities in one variable and represent solutions algebraically and graphically.	**Unit 5** Lesson 10
MA.8.AR.2.3	Given an equation in the form of $x^2 = p$ and $x^3 = q$, where p is a whole number and q is an integer, determine the real solutions.	**Unit 10** Lessons 1, 2, 3, 4, 5
MA.8.AR.3.1	Determine if a linear relationship is also a proportional relationship.	**Unit 3** Lessons 1, 2, 7, 8, 10 **Unit 4** Lesson 1 **Unit 6** Lesson 6
MA.8.AR.3.2	Given a table, graph or written description of a linear relationship, determine the slope.	**Unit 2** Lessons 15, 16 **Unit 4** Lessons 1, 2, 3, 4, 5, 6, 7 **Unit 10** Lesson 11
MA.8.AR.3.3	Given a table, graph or written description of a linear relationship, write an equation in slope-intercept form.	**Unit 4** Lessons 3, 6, 7, 8 **Unit 5** Lesson 9 **Unit 6** Lesson 4
MA.8.AR.3.4	Given a mathematical or real-world context, graph a two-variable linear equation from a written description, a table or an equation in slope-intercept form.	**Unit 4** Lessons 3, 4, 7, 8 **Unit 6** Lesson 5
MA.8.AR.3.5	Given a real-world context, determine and interpret the slope and y-intercept of a two-variable linear equation from a written description, a table, a graph or an equation in slope-intercept form.	**Unit 4** Lessons 2, 3, 4, 8 **Unit 6** Lessons 4, 5, 7 **Unit 8** Lessons 10, 12
MA.8.AR.4.1	Given a system of two linear equations and a specified set of possible solutions, determine which ordered pairs satisfy the system of linear equations.	**Unit 5** Lessons 11, 12, 13, 14
MA.8.AR.4.2	Given a system of two linear equations represented graphically on the same coordinate plane, determine whether there is one solution, no solution or infinitely many solutions.	**Unit 5** Lessons 13, 15
MA.8.AR.4.3	Given a mathematical or real-world context, solve systems of two linear equations by graphing.	**Unit 5** Lessons 12, 13, 14, 15
	Functions	
MA.8.F.1.1	Given a set of ordered pairs, a table, a graph or mapping diagram, determine whether the relationship is a function. Identify the domain and range of the relation.	**Unit 6** Lessons 1, 2, 3, 4, 6

Florida's B.E.S.T. Standards for Mathematics

MA.8.F.1.2	Given a function defined by a graph or an equation, determine whether the function is a linear function. Given an input-output table, determine whether it could represent a linear function.	**Unit 6** Lessons 6, 7
MA.8.F.1.3	Analyze a real-world written description or graphical representation of a functional relationship between two quantities and identify where the function is increasing, decreasing or constant.	**Unit 6** Lessons 5, 7
	Geometric Reasoning	
MA.7.GR.1.3	Explore the proportional relationship between circumferences and diameters of circles. Apply a formula for the circumference of a circle to solve mathematical and real-world problems.	**Unit 7** Lessons 1, 2
MA.7.GR.1.4	Explore and apply a formula to find the area of a circle to solve mathematical and real-world problems.	**Unit 7** Lessons 3, 4, 5, 6
MA.7.GR.1.5	Solve mathematical and real-world problems involving dimensions and areas of geometric figures, including scale drawings and scale factors.	**Unit 2** Lessons 1, 2, 3, 4, 5, 6 **Unit 7** Lessons 5, 6
MA.7.GR.2.1	Given a mathematical or real-world context, find the surface area of a right circular cylinder using the figure's net.	**Unit 7** Lesson 9
MA.7.GR.2.2	Solve real-world problems involving surface area of right circular cylinders.	**Unit 7** Lesson 9
MA.7.GR.2.3	Solve mathematical and real-world problems involving volume of right circular cylinders.	**Unit 7** Lessons 7, 8
MA.8.GR.1.1	Apply the Pythagorean Theorem to solve mathematical and real-world problems involving unknown side lengths in right triangles.	**Unit 10** Lessons 6, 7, 8, 10, 11
MA.8.GR.1.2	Apply the Pythagorean Theorem to solve mathematical and real-world problems involving the distance between two points in a coordinate plane.	**Unit 10** Lesson 12
MA.8.GR.1.3	Use the Triangle Inequality Theorem to determine if a triangle can be formed from a given set of sides. Use the converse of the Pythagorean Theorem to determine if a right triangle can be formed from a given set of sides.	**Unit 10** Lessons 9, 10
MA.8.GR.1.4	Solve mathematical problems involving the relationships between supplementary, complementary, vertical or adjacent angles.	**Unit 1** Lesson 9 **Unit 2** Lesson 13
MA.8.GR.1.5	Solve problems involving the relationships of interior and exterior angles of a triangle.	**Unit 1** Lesson 8 **Unit 2** Lesson 13
MA.8.GR.1.6	Develop and use formulas for the sums of the interior angles of regular polygons by decomposing them into triangles.	**Unit 1** Lesson 10
MA.8.GR.2.1	Given a preimage and image generated by a single transformation, identify the transformation that describes the relationship.	**Unit 1** Lessons 1, 2, 3, 4, 5, 6, 7
MA.8.GR.2.2	Given a preimage and image generated by a single dilation, identify the scale factor that describes the relationship.	**Unit 2** Lessons 7, 8, 9, 10, 11, 12

Florida's B.E.S.T. Standards for Mathematics

MA.8.GR.2.3	Describe and apply the effect of a single transformation on two-dimensional figures using coordinates and the coordinate plane.	**Unit 1** Lessons 3, 4, 5, 7 **Unit 2** Lessons 10, 11, 12
MA.8.GR.2.4	Solve mathematical and real-world problems involving proportional relationships between similar triangles.	**Unit 2** Lessons 14, 15
Data Analysis and Probability		
MA.7.DP.1.4	Use proportional reasoning to construct, display and interpret data in circle graphs.	**Unit 8** Lessons 1, 2, 3
MA.7.DP.1.5	Given a real-world numerical or categorical data set, choose and create an appropriate graphical representation.	**Unit 8** Lessons 2, 3
MA.8.DP.1.1	Given a set of real-world bivariate numerical data, construct a scatter plot or a line graph as appropriate for the context.	**Unit 8** Lessons 4, 5, 6, 8, 12
MA.8.DP.1.2	Given a scatter plot within a real-world context, describe patterns of association.	**Unit 8** Lessons 7, 10, 11, 12
MA.8.DP.1.3	Given a scatter plot with a linear association, informally fit a straight line.	**Unit 8** Lessons 7, 8, 9, 10, 12
MA.8.DP.2.1	Determine the sample space for a repeated experiment.	**Unit 11** Lessons 1, 2, 3, 4
MA.8.DP.2.2	Find the theoretical probability of an event related to a repeated experiment.	**Unit 11** Lessons 3, 4
MA.8.DP.2.3	Solve real-world problems involving probabilities related to single or repeated experiments, including making predictions based on theoretical probability.	**Unit 11** Lessons 4, 5

Mathematical Thinking and Reasoning Standards

The following correlations show the alignment of Amplify Desmos Math Florida to the Mathematical Thinking and Reasoning Standards.

MA.K12.MTR.1.1 Actively participate in effortful learning both individually and collectively.

Unit 1, Lesson 9
Unit 2, Lessons 5, 9
Unit 3, Lessons 7,10
Unit 4, Lessons 1, 2, 7, 8
Unit 5, Lessons 8, 12
Unit 7, Lessons 2, 4, 5, 6, 8, 9

MA.K12.MTR.2.1 Demonstrate understanding by representing problems in multiple ways.

Unit 2, Lessons 6, 14, 15
Unit 3, Lessons 2, 3, 4, 5, 6, 8, 10, 11
Unit 4, Lessons 1, 2, 3
Unit 5, Lessons 1, 2, 3, 4, 5, 9, 11
Unit 6, Lessons 2, 6, 7
Unit 7, Lessons 7, 9
Unit 8, Lessons 2, 3, 4, 5, 6, 12
Unit 9, Lessons 1, 2, 3, 4, 7, 12, 13, 14, 15
Unit 10, Lessons 1, 2, 3, 5, 6, 7, 9
Unit 11, Lesson 2

MA.K12.MTR.3.1 Complete tasks with mathematical fluency.

Unit 1, Lessons 3, 4, 8, 9
Unit 2, Lessons 5, 16
Unit 3, Lessons 4, 9
Unit 4, Lessons 2, 5
Unit 5, Lesson 4
Unit 6, Lesson 3
Unit 7, Lessons 4, 6, 7
Unit 9, Lessons 1, 9, 10, 13, 14
Unit 10, Lessons 10, 13

MA.K12.MTR.4.1 Engage in discussions that reflect on the mathematical thinking of self and others.

Unit 1, Lessons 1, 2, 3, 4, 5, 6, 7
Unit 2, Lessons 1, 2, 3, 4, 5, 7, 8, 9, 10, 11, 12, 13, 15, 16
Unit 3, Lessons 1, 3, 7, 10
Unit 4, Lessons 1, 3, 6, 7
Unit 5, Lessons 1, 2, 3, 5, 6, 7, 11, 12, 14
Unit 6, Lessons 1, 3, 4, 5, 6, 7
Unit 7, Lessons 1, 2, 3, 4, 5, 7
Unit 8, Lessons 1, 2, 4, 5, 6, 7, 8, 9, 10, 11
Unit 9, Lessons 5, 6, 7, 8, 9, 10, 11, 13, 15
Unit 10, Lessons 1, 2, 3, 7, 8, 11, 13, 16
Unit 11, Lesson 4

MA.K12.MTR.5.1 Use patterns and structure to help understand and connect mathematical concepts.

Unit 1, Lessons 2, 5, 10
Unit 2, Lessons 1, 2, 4, 10, 11, 15
Unit 3, Lessons 1, 6, 7, 8
Unit 4, Lessons 4, 5, 6, 8
Unit 5, Lessons 1, 2, 3, 4, 7, 8, 10, 13, 14, 15
Unit 7, Lessons 3, 5, 8
Unit 8, Lessons 1, 10, 11, 12
Unit 9, Lessons 1, 2, 3, 4, 5, 6, 8, 9, 12, 13, 14, 15, 16, 17, 18
Unit 10, Lessons 1, 2, 3, 4, 5, 6, 7, 9, 10, 12, 14, 15, 16
Unit 11, Lessons 1, 3, 5

MA.K12.MTR.6.1 Assess the reasonableness of solutions.

Unit 2, Lesson 6
Unit 3, Lesson 2
Unit 7, Lessons 2, 3
Unit 8, Lessons 4, 6, 7, 8, 9, 10, 12
Unit 9, Lessons 2, 3, 4, 16, 17, 18
Unit 10, Lessons 3, 4, 5, 8, 9, 11, 12, 14

MA.K12.MTR.7.1 Apply mathematics to real-world contexts.

Unit 2, Lessons 6, 8
Unit 3, Lessons 5, 9, 11
Unit 4, Lessons 4, 8
Unit 5, Lessons 5, 10
Unit 6, Lesson 4
Unit 7, Lessons 8, 9
Unit 8, Lessons 2, 3, 4, 5, 6, 7, 8, 10, 12
Unit 9, Lessons 12, 16, 17, 18
Unit 10, Lessons 5, 8, 11, 15
Unit 11, Lesson 5

Florida's B.E.S.T. Standards for English Language Arts

ELA.K12.EE.1.1 Cite evidence to explain and justify reasoning.

Unit 1, Lessons 1, 3, 4, 6, 7, 9
Unit 2, Lessons 1, 2, 3, 5, 9, 12, 16
Unit 3, Lessons 5, 6, 7, 10, 11
Unit 4, Lessons 1, 6, 7, 8
Unit 5, Lessons 1, 2, 3, 9, 11, 12
Unit 6, Lessons 3, 6
Unit 7, Lessons 2, 6, 9
Unit 8, Lessons 1, 2, 4, 6, 9, 10
Unit 9, Lessons 1, 2, 6, 9, 12, 16
Unit 10, Lessons 3, 4, 6, 8, 9, 10, 12, 13
Unit 11, Lessons 2, 3, 4, 5

ELA.K12.EE.2.1 Read and comprehend grade-level complex texts proficiently.

Unit 3, Lesson 7
Unit 4, Lesson 8
Unit 5, Lesson 5
Unit 10, Lesson 15

ELA.K12.EE.3.1 Make inferences to support comprehension.

Unit 1, Lessons 1, 2, 3, 4, 5, 10
Unit 2, Lessons 1, 2, 4, 6, 7, 11, 14, 15, 16
Unit 3, Lessons 1, 2, 3, 5, 6, 7, 8, 9, 10
Unit 4, Lessons 3, 4, 5, 6
Unit 5, Lessons 2, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15
Unit 6, Lessons 2, 3, 4, 5, 6
Unit 7, Lessons 2, 3, 4
Unit 8, Lessons 4, 5, 7, 8, 10, 11, 12
Unit 9, Lessons 1, 2, 3, 5, 7, 8, 9, 10, 12, 14
Unit 10, Lessons 2, 3, 4, 5, 14
Unit 11, Lessons 2, 4, 5

ELA.K12.EE.4.1 Use appropriate collaborative techniques and active listening skills when engaging in discussions in a variety of situations.

Unit 1, Lessons 2, 5
Unit 2, Lessons 2, 5, 8, 9, 10, 12, 13, 15
Unit 3, Lessons 3, 9
Unit 4, Lessons 2, 3, 7
Unit 5, Lessons 2, 4, 8, 10, 15
Unit 6, Lessons 1, 2, 5
Unit 7, Lessons 1, 4, 8, 9
Unit 8, Lessons 2, 3, 6, 7, 8
Unit 9, Lessons 4, 5, 6, 7, 9
Unit 10, Lessons 5, 8, 9, 10, 11, 12, 16
Unit 11, Lesson 3

ELA.K12.EE.5.1 Use the accepted rules governing a specific format to create quality work.

Unit 1, Lessons 8, 10
Unit 2, Lessons 1, 4, 5, 8, 10, 14, 15, 16
Unit 3, Lessons 1, 2, 3, 4, 9
Unit 4, Lessons 1, 2, 5, 7
Unit 5, Lessons 1, 3, 4, 6, 12, 15
Unit 6, Lessons 6, 7
Unit 7, Lessons 5, 7, 8
Unit 8, Lesson 3
Unit 9, Lessons 3, 7, 10, 11, 13, 15, 17, 18
Unit 10, Lessons 1, 5, 7, 8, 13, 14, 15, 16
Unit 11, Lessons 3, 5

ELA.K12.EE.6.1 Use appropriate voice and tone when speaking or writing.

Unit 1, Lessons 2, 7, 9
Unit 2, Lessons 6, 13
Unit 3, Lessons 5, 11
Unit 5, Lessons 7, 13
Unit 6, Lessons 1, 2, 4, 5
Unit 7, Lesson 1
Unit 8, Lessons 1, 2, 5, 9, 10, 11, 12
Unit 9, Lesson 8
Unit 10, Lessons 1, 2, 3, 6, 10, 16
Unit 11, Lesson 1

English Language Development Standards

ELD.K12.ELL.MA.1 English language learners communicate information, ideas and concepts necessary for academic success in the content area of Mathematics.

Unit 1, Lessons - All lessons
Unit 2, Lessons - All lessons
Unit 3, Lessons - All lessons
Unit 4, Lessons - All lessons
Unit 5, Lessons - All lessons
Unit 6, Lessons - All lessons
Unit 7, Lessons - All lessons
Unit 8, Lessons - All lessons
Unit 9, Lessons - All lessons
Unit 10, Lessons - All lessons
Unit 11, Lessons - All lessons

ELD.K12.ELL.SI.1 English language learners communicate for social and instructional purposes within the school setting.

Unit 1, Lessons - All lessons
Unit 2, Lessons - All lessons
Unit 3, Lessons - All lessons
Unit 4, Lessons - All lessons
Unit 5, Lessons - All lessons
Unit 6, Lessons - All lessons
Unit 7, Lessons - All lessons
Unit 8, Lessons - All lessons
Unit 9, Lessons - All lessons
Unit 10, Lessons - All lessons
Unit 11, Lessons - All lessons

UNIT 1

Rigid Transformations and Congruence

Students investigate translations, reflections, and rotations, and use these transformations to make informal arguments about congruence. They also explore angle relationships and the triangle sum theorem.

Essential Questions

- What are different ways to transform a figure?
- How can we use rigid transformations to decide whether two figures are congruent?
- How can transformations help make sense of angle relationships?

Sub-Unit Goals

Sub-Unit 1

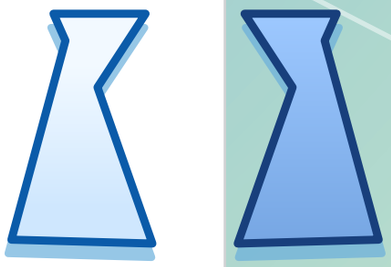
- Perform translations, reflections, and rotations on and off a grid.

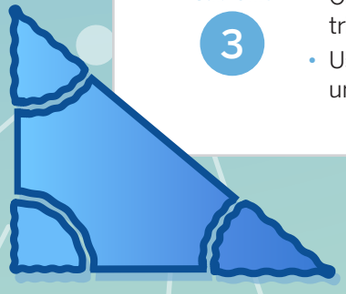
Sub-Unit 2

- Determine whether two figures are congruent using rigid transformations.

Sub-Unit 3

- Observe that the sum of angle measures in a triangle is 180 degrees.
- Use special angle relationships to determine unknown angle measures.





The Math of the Unit

Unit Outcomes Telling a story that narrows and deepens with the grade

Florida B.E.S.T. Standards for Mathematics

Understand similarity and congruence using models and transformations. **(MA.8.GR.2)**

Develop an understanding of the Pythagorean Theorem and angle relationships involving triangles. **(MA.8.GR.1)**

Solve multi-step one-variable equations and inequalities. **(MA.8.AR.2)**

Benchmarks Addressed in This Unit: MA.8.GR.1.4 MA.8.GR.1.5 MA.8.GR.1.6 MA.8.GR.2.1 MA.8.GR.2.3 MA.8.AR.2.1

Mathematical Thinking and Reasoning Standards

- Actively participate in effortful learning both individually and collectively. **(MTR.1.1)**
- Complete tasks with mathematical fluency. **(MTR.3.1)**
- Engage in discussions that reflect on the mathematical thinking of self and others. **(MTR.4.1)**
- Use patterns and structure to help understand and connect mathematical concepts. **(MTR.5.1)**

Coherence Telling a story within and across grades

In this Unit

Students learn to visualize, describe, and perform translations, rotations, and reflections on a grid. They learn and use the word *congruent* and use rigid transformations to determine whether figures are congruent. Students also learn that the sum of the interior angle measures in a triangle is $180°$ and use special angle relationships to solve mathematical problems.

Prior Learning

- Angle measures and unknown angles **(MA.4.GR.1.2, MA.4.GR.1.3)**
- Classifying triangles and quadrilaterals **(MA.5.GR.1.1)**
- Graphing points in the plane **(MA.6.GR.1.1)**
- Areas of triangles and quadrilaterals **(MA.6.GR.2.2)**
- Decomposing polygons and composite figures to determine area **(MA.7.GR.1.2)**
- Solving problems involving dimensions and areas of figures **(MA.7.GR.1.5)**

Future Learning

- **Unit 2:** Similarity and dilations **(MA.8.GR.2.2, MA.8.GR.2.3)**
- Function transformations **(MA.912.F.2)**
- Triangle congruence theorems **(MA.912.GR.1, MA.912.GR.2)**

See the Connections to Future Learning page for more information, including explanations and examples.

Rigor and Balance Integrating the aspects of rigor in ways that support one another

Here are examples of the aspects of rigor and balance in this unit. See the first page of each lesson's Teacher Edition for more information.

Conceptual Understanding

In **Lessons 1–2**, students visualize and describe the movements of translations, reflections, and rotations. In **Lesson 6**, they make sense of congruence.

Procedural Fluency

In **Lessons 2–4**, students perform rigid transformations both on and off a grid.

Productive Disposition

In **Lesson 1**, students have opportunities to develop a productive disposition as they see their own language as mathematical.

Unit at a Glance

Assess and Respond

A Pre-Unit Check

Learn more about your students' understanding of foundational concepts and skills that will support them in the upcoming unit. This can be given in its entirety before the unit or spread throughout the unit.

MA.4.GR.1.3 MA.4.GR.2.1 MA.5.GR.4.2 MA.6.GR.1.1 MA.6.GR.1.2
MTR.4.1 MTR.5.1

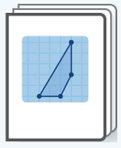
Sub-Unit 1

1 Spinning, Flipping, Sliding

Naming Transformations

Determine whether a **transformation** is a **translation**, **rotation**, or **reflection** and describe how each affects a figure.

MA.8.GR.2.1
MTR.4.1

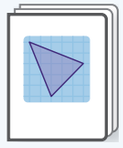


2 Moving Day

Describing Transformations Precisely

Translate, reflect, and rotate figures by hand on a square grid.

MA.8.GR.2.1
MTR.4.1 MTR.5.1



3 Getting Coordinated, Part 1

Coordinates of Translations and Reflections

Investigate the effects of a translation or reflection on the coordinates of a transformed image.

MA.8.GR.2.1 MA.8.GR.2.3
MTR.3.1 MTR.4.1

7 Are They Congruent?

Rigid Transformation and Congruent Figures

Use a rigid transformation to show two figures are congruent.

MA.8.GR.2.1 MA.8.GR.2.3
MTR.4.1

8 Tearing It Up

Angle Sums in Triangles

Observe that the sum of the interior angles of a triangle is always $180°$ and investigate the relationship between an **exterior angle** of a triangle and the **remote interior angles**.

MA.8.GR.1.5
MTR.3.1

9 Friendly Angles

Complementary, Supplementary, Vertical, and Adjacent Angles

Continue to explore angle measurements and their relationships.

MA.8.GR.1.4 MA.8.AR.2.1
MTR.1.1 MTR.3.1

10 Sum It Up

Exploring Angle Sums in Polygons

Use knowledge of the angle sum of triangles and decomposition to determine the angle sums of a quadrilateral, pentagon, and hexagon.

MA.8.GR.1.6
MTR.5.1

Pacing: 13 days | **Short on time?** See pacing considerations below.

All instructional and assessment days are ~45 minutes each.

- 10 Lessons
- Pre-Unit Check
- Sub-Unit Quiz
- End-of-Unit Assessment

Sub-Unit 2

4 Getting Coordinated, Part 2

Coordinates of Transformations

Continue to investigate the effects of transformations on coordinates on a plane.

MA.8.GR.2.1 MA.8.GR.2.3
MTR.3.1 MTR.4.1

Assess and Respond

A Sub-Unit Quiz

Learn about your students' understanding of the concepts and skills so far in this unit.

MA.8.GR.2.1, MA.8.GR.2.3
MTR.2.1, MTR.5.1

5 No Bending, No Stretching

Rigid Transformations

Introduce the term **rigid transformations** and see that they preserve lengths and angle measures.

MA.8.GR.2.1 MA.8.GR.2.3
MTR.4.1 MTR.5.1

6 Are They the Same?

Defining Congruence

Explore what it means for figures to be the "same" and learn the term **congruent**.

MA.8.GR.2.1
MTR.4.1

Assess and Respond

End-of-Unit Assessment

Learn about your students' understanding of the concepts and skills in this unit. There are two forms of the End-of-Unit Assessment: Forms A and B.

MA.8.GR.1.4 MA.8.GR.1.5
MA.8.GR.2.1 MA.8.GR.2.3
MTR.4.1 MTR.5.1

Pacing Considerations

Lessons 3–4: These lessons can be condensed into one lesson if students show a strong understanding of coordinates from the Pre-Unit Check. If condensed, make sure to discuss how each type of transformation affects the coordinates of a figure.

Lessons 8–9: These lessons can be condensed into one lesson if students show a strong understanding of angle relationships in earlier lessons or in the Pre-Unit Check.

Lesson Materials and Prep

Here are the materials and resources needed within each lesson, how to prepare them, and where to find them.

Lesson 1

- **Copy:** Show What You Know, Activity 1 Sheet *(one per student)*
- **Materials:** tracing paper *(as needed)*

Lesson 2

- **Copy:** Show What You Know
- **Cut:** Activity 2 Cards *(one set per pair)*
- **Materials:** straightedges, tracing paper, coloring tools *(optional)*

Lesson 3

- **Copy:** Show What You Know
- **Materials:** tracing paper *(as needed)*

Lesson 4

- **Copy:** Show What You Know, Activity 2 Sheet *(one per student)*
- **Materials:** tracing paper *(as needed)*

Lesson 5

- **Copy:** Show What You Know
- **Cut:** Activity 2 Cards *(one set per pair)*
- **Materials:** protractors, rulers, tracing paper, coloring tools, highlighters *(as needed)*, notecards *(as needed)*

Lesson 6

- **Copy:** Show What You Know
- **Materials:** tracing paper *(as needed)*

Lesson 7

- **Copy:** Show What You Know
- **Materials:** tracing paper *(as needed)*

Lesson 8

- **Copy:** Show What You Know, Activity 3 sheet *(one per student)*
- **Cut:** Activity 1 Cards *(one per student)*
- **Materials:** blank paper *(one per student)*, straightedges, tracing paper *(as needed)*

Lesson 9

- **Copy:** Show What You Know

Lesson 10

- **Copy:** Show What You Know

Resource Books

Assessment and Lesson Resources

- Pre-Unit Check
- Sub-Unit Quiz
- End-of-Unit Assessment, Forms A & B
- Assessment Rubrics
- Show What You Knows
- Activity Sheets and Cards

Intervention and Extension Resources

- Mini-Lessons
- Prior Grade Mini-Lessons
- Extensions

Caregiver Support

These resources can be shared with students' caregivers. They provide background on the mathematics in this unit, as well as suggestions for supporting students at home.

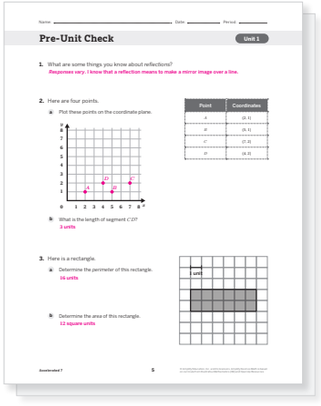
Assessments

Every unit includes a variety of formative and summative assessments designed to highlight students' understanding of a variety of concepts and skills.

Pre-Unit Check

Assign the **Pre-Unit Check** to learn more about your students' understanding of foundational concepts and skills that will support them in the upcoming unit. This can be given in its entirety before the unit or spread throughout the unit.

Standards assessed: MA.4.GR.1.3, MA.4.GR.2.1, MA.5.GR.4.2, MA.6.GR.1.1, MA.6 GR.1.2


Pre-Unit Check

Unit 1

1. What are some things you know about reflections?

Responses vary. I know that a reflection means to make a mirror image over a line.

2. Here are four points.

a. Plot these points on the coordinate plane.

Point	Coordinates
A	(2, 1)
B	(5, 1)
C	(7, 4)
D	(4, 4)

b. What is the length of segment $C'D'$?

3 units

3. There is a rectangle.

a. Determine the perimeter of this rectangle.

16 units

b. Determine the area of this rectangle.

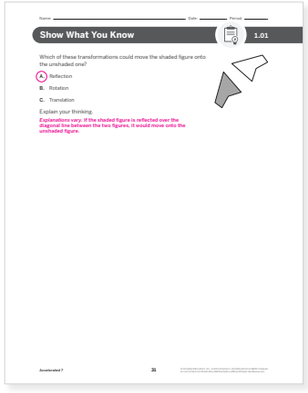
12 square units

Accelerated 7 5

Lesson | Show What You Know

Use the **Show What You Know** for each lesson to learn more about your students' progress toward the lesson goal(s). Here is one example from this unit.

Standard assessed on Lesson 1 Show What You Know: MA.8.GR.2.1


Show What You Know

1.01

Which of these transformations could move the shaded figure onto the unshaded one?

A. Reflection

B. Rotation

C. Translation

Explain your thinking.

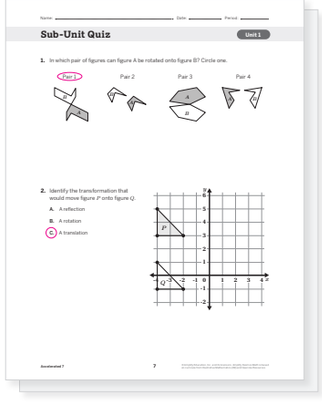
Explanations vary. If the shaded figure is reflected over the diagonal line between the two figures, it would move onto the unshaded figure.

Accelerated 7 31

Sub-Unit Quiz

Assign the **Sub-Unit Quiz** to learn about your students' understanding of the concepts and skills so far in this unit.

Standards assessed: MA.8.GR.2.1, MA.8.GR.2.3


Sub-Unit Quiz

Unit 1

1. In which pair of figures can figure A be rotated onto figure B? Circle one.

Pair 1 Pair 2 Pair 3 Pair 4

2. Identify the transformation that would move figure P onto figure Q.

A. A reflection

B. A rotation

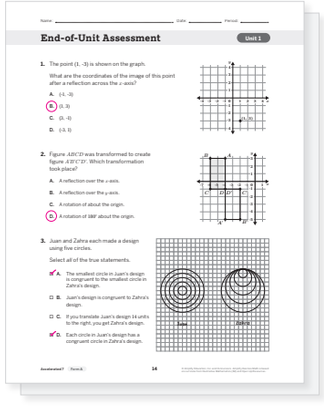
C. A translation

Accelerated 7 7

End-of-Unit Assessment

Assign the **End-of-Unit Assessment** to learn about your students' understanding of the concepts and skills in this unit. There are two forms of the End-of-Unit Assessment: Forms A and B.

Standards assessed: MA.8.GR.1.5, MA.8.GR.2.1, MA.8.GR.2.3


End-of-Unit Assessment

Unit 1

1. The point (1, -3) is shown on the graph.

What are the coordinates of the image of this point after a reflection across the x-axis?

A. (-1, -3)

B. (1, 3)

C. (3, -1)

D. (-3, 1)

2. Figure $ABCD$ was transformed to create figure $A'B'C'D'$. Which transformation took place?

A. A reflection over the x-axis.

B. A reflection over the y-axis.

C. A rotation of about the origin.

D. A rotation of 180° about the origin.

3. Juan and Zahra each made a design using five circles.

Select all of the true statements.

A. The smallest circle in Juan's design is congruent to the smallest circle in Zahra's design.

B. Juan's design is congruent to Zahra's design.

C. If you translate Juan's design 14 units to the right, you get Zahra's design.

D. Each circle in Juan's design has a congruent circle in Zahra's design.

Accelerated 7 Form A 14

Differentiation

Every unit includes additional resources to support, strengthen, and stretch student learning.

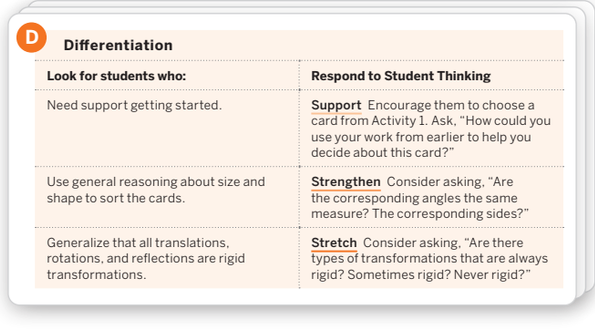
Differentiation Across the Unit

	Support	Strengthen	Stretch
	Provide targeted intervention for students by using these resources.	Reinforce students' understanding of the concepts assessed by using these resources.	Challenge students and extend their learning with these resources.
Sub-Unit 1 (Lessons 1–4) • Perform translations, reflections, and rotations on and off a grid.	**Mini-Lesson** • *Determining Coordinates After a Rotation*	**Challenge Creator** • Lesson 4 (*Getting Coordinated, Part 2*) **Practice** • Lessons 1–4	
Sub-Unit 2 (Lessons 5–7) • Determine whether two figures are congruent using rigid transformations.	**Mini-Lesson** • *Justifying Congruent Figures*	**Repeated Challenges** • Lesson 6 (*Are They the Same?*) **Practice** • Lessons 5–7	
Sub-Unit 3 (Lessons 8–10) • Observe that the sum of angle measures in a triangle is 180 degrees. • Use special angle relationships to determine unknown angle measures.	**Mini-Lesson** • *Determining Unknown Angle Measures*	**Practice** • Lessons 8–10	**DOK3 Practice** • Lessons 8 and 10

Differentiation Within the Lesson

Each Teacher Edition includes point-of-use differentiation suggestions to modify or adjust instruction during the lesson to accommodate the needs of all learners.

Here is one example from **Lesson 5**, where students are making arguments about rigid transformations.



Differentiation

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Encourage them to choose a card from Activity 1. Ask, "How could you use your work from earlier to help you decide about this card?"
Use general reasoning about size and shape to sort the cards.	**Strengthen** Consider asking, "Are the corresponding angles the same measure? The corresponding sides?"
Generalize that all translations, rotations, and reflections are rigid transformations.	**Stretch** Consider asking, "Are there types of transformations that are always rigid? Sometimes rigid? Never rigid?"

Accessibility

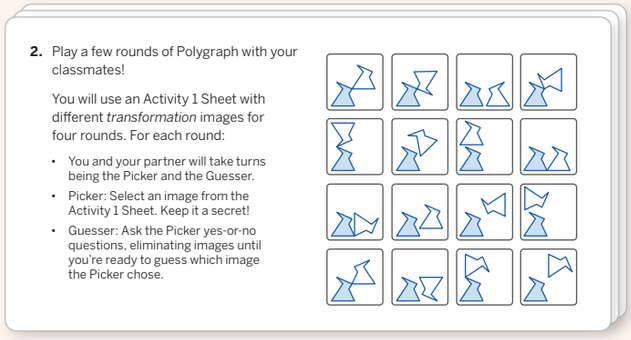
Every lesson is designed using the principles of Universal Design for Learning. Every lesson also includes support suggestions that build on students' strengths.

Universal Design for Learning

Here is one example from a lesson designed to support Engagement.

Spotlight on Sustaining Effort & Persistence . . .

Lesson 1: Students are randomly partnered to play a game of Polygraph. Each player has the opportunity to give, apply, and interpret feedback, which helps foster collaboration and community as they make sense of vocabulary about transformations.



2. Play a few rounds of Polygraph with your classmates!

You will use an Activity 1 Sheet with different *transformation* images for four rounds. For each round:

- You and your partner will take turns being the Picker and the Guesser.
- Picker: Select an image from the Activity 1 Sheet. Keep it a secret!
- Guesser: Ask the Picker yes-or-no questions, eliminating images until you're ready to guess which image the Picker chose.

Just-In-Time Accommodations and Supports

Each Teacher Edition includes suggested supports based on the areas of cognitive function from the Education Development Center (EDC).

Spotlight on Visual-Spatial Processing . . .

Here are two suggestions in this unit for supporting students with interpreting visual representations.

Lesson 4, Activity 1

A **Accessibility: Conceptual Processing** Invite students to revisit their work from the previous problem to support students in generalizing.

Lesson 5, Activity 1

A **Accessibility: Visual-Spatial Processing** Invite students to use coloring tools or highlighters and annotate the corresponding side lengths and angles to support interpreting visual representations.

Language Development

Every lesson includes opportunities for all students to develop fluency with mathematical language.

Vocabulary

New Vocabulary	Lesson(s)
reflection **rotation** **transformation** **translation**	1
clockwise **correspond** **counterclockwise** **image** **pre-image**	2
rigid transformation	5
congruent	6
exterior angle **interior angle** **remote interior angles**	8
adjacent angles **complementary angles** **supplementary angles** **vertical angles**	9

Review Vocabulary
origin

Additional Vocabulary
prime notation

Math Language Routines and Language Goals

Math Language Routine	Lesson(s)
MLR1: Stronger and Clearer Each Time	3
MLR2: Collect and Display	1, 2, 5, 7, 9, and 10
MLR3: Critique, Correct, Clarify	10
MLR7: Compare and Connect	8

Sub-Unit Language Goals

- Describe translations, reflections, and rotations.
- Justify whether two figures are congruent using language such as *rigid transformations*, *translation*, *reflection*, *rotation*, *pre-image*, and *image*.
- Describe the characteristics of congruent figures.
- Describe what it means for angles to be *complementary* or *supplementary*.

English Language Learners Support

Every Teacher Edition includes suggestions for supporting the needs of students learning English.

Here is one example from **Lesson 2**, where students are learning the term *pre-image*.

ELL **English Language Learners** Invite students to share what a new term reminds them of (e.g., "When I hear *pre-image*, I think . . ."). This may help surface multiple meanings and encourage students to connect to their prior knowledge. **(Reading and Listening)**

Connections to Future Learning

Here is how the content in this unit connects to where your students are headed in their math journeys.

Similarity and Dilations

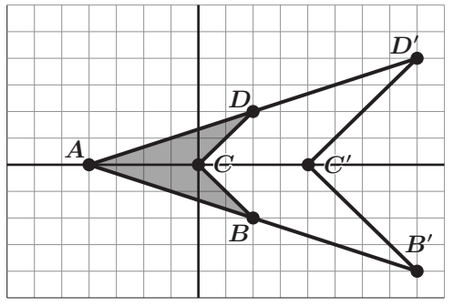
In this unit, students learn about congruent figures and rigid transformations. **In Unit 2**, students will apply this to understand similar figures and dilations, which are transformations that do change side lengths.

FL B.E.S.T. Math Benchmarks:
MA.8.GR.2.2, MA.8.GR.2.3

Example:

Each point in figure $AB'C'D'$ is twice as far from the center of dilation (point A) as it is in figure $ABCD$.

The scale factor from figure $ABCD$ to figure $AB'C'D'$ is 2, and figure $ABCD$ is similar to figure $AB'C'D'$.



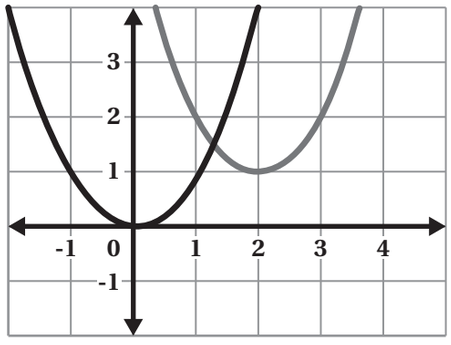
Function Transformations

In this unit, students perform transformations on figures. **In a future course**, students will perform transformations on functions.

FL B.E.S.T. Math Benchmark:
MA.912.F.2

Example:

The equation of the solid parabola is $f(x) = x^2$. By translating the parabola 2 units right and 1 unit up, the equation of the gray parabola is $g(x) = (x - 2)^2 + 1$.



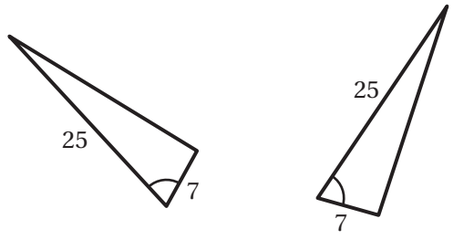
Triangle Congruence Theorems

In this unit, students use rigid transformations to understand what it means for two polygons to be congruent. **In a future course**, students will use rigid transformations to prove theorems and identify conditions for triangle congruence.

FL B.E.S.T. Math Benchmarks:
MA.912.GR.1, MA.912.GR.2

Example:

Here are two triangles. Two pairs of corresponding sides and the angle between them are congruent.



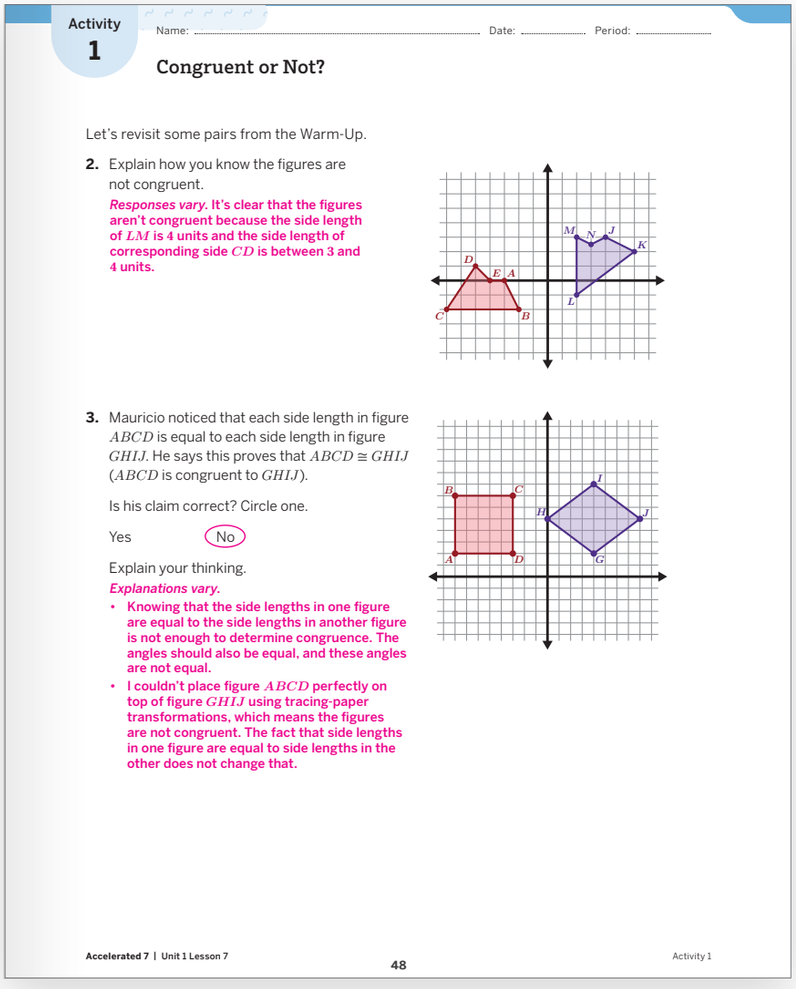
This turns out to be enough information to know that the triangles are congruent, which means that all corresponding sides and angles have the same measures on both triangles.

Professional Learning

Here is an opportunity to deepen your understanding of the math in this unit and how to teach it.

Spotlight on Connecting Representations

Lesson 7, Activity 1



Activity 1

Name: Date: Period:

Congruent or Not?

Let's revisit some pairs from the Warm-Up.

2. Explain how you know the figures are not congruent.

Responses vary. It's clear that the figures aren't congruent because the side length of LM is 4 units and the side length of corresponding side CD is between 3 and 4 units.

3. Mauricio noticed that each side length in figure $ABCD$ is equal to each side length in figure $GHIJ$. He says this proves that $ABCD \cong GHIJ$ ($ABCD$ is congruent to $GHIJ$).

Is his claim correct? Circle one.

Yes (No)

Explain your thinking.

Explanations vary.

- Knowing that the side lengths in one figure are equal to the side lengths in another figure is not enough to determine congruence. The angles should also be equal, and these angles are not equal.
- I couldn't place figure $ABCD$ perfectly on top of figure $GHIJ$ using tracing-paper transformations, which means the figures are not congruent. The fact that side lengths in one figure are equal to side lengths in the other does not change that.

Accelerated 7 | Unit 1 Lesson 7 48 Activity 1

Context

In this activity, students apply their knowledge of rigid transformations and congruence to justify whether two figures are congruent.

Try This

Put on your student hat and complete Problems 2 and 3.

Points to Ponder

- What do students need to understand about rigid transformations and congruent figures to support their reasoning for why two figures are congruent?
- Some students may think that the figures in Problem 3 are congruent when they are not. How might you support students in making connections to rigid transformations?

Notes:

Assess and Respond

Pre-Unit Check

Independent | 45 min

Facilitation: Assign the Pre-Unit Check to learn more about your students' understanding of foundational concepts and skills that will support them in the upcoming unit. This can be given in its entirety before the unit or spread throughout the unit. If time allows, consider inviting students to complete one or more of the tasks in the Unit Synthesis and Reflection.

Item Analysis

Problem(s)	Concept or skill	Preparation for	DOK	FL B.E.S.T. Math Benchmarks
1	Recalling prior knowledge of reflections	Lesson 1	2	MA.6.GR.1.1
2	Plotting points on a coordinate grid and finding the distance between them	Lesson 3	1	MA.5.GR.4.2, MA.6.GR.1.2
3	Determining perimeter and area of a rectangle using a grid	Lesson 5	1	MA.4.GR.2.1, MTR.5.1
4	Using properties of straight angles to find a missing angle measure and justifying the response	Lesson 8	2	MA.4.GR.1.3 MTR.4.1

Assessment and Lesson Resources

- Student Assessments
- Show What You Know and Answer Keys

Intervention and Extension Resources

- Mini-Lessons
- Extensions

Name: ..... Date: ..... Period: .....

Pre-Unit Check

Unit 1

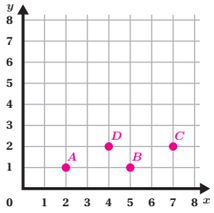
1. What are some things you know about *reflections*?

Responses vary. **I know that a reflection means to make a mirror image over a line.**

2. Here are four points.

Point	Coordinates
A	(2, 1)
B	(5, 1)
C	(7, 2)
D	(4, 2)

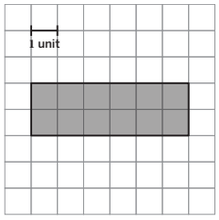
a Plot these points on the coordinate plane.



b What is the length of segment CD?

3 units

3. Here is a rectangle.



a Determine the *perimeter* of this rectangle.

16 units

b Determine the *area* of this rectangle.

12 square units

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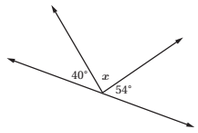
Name: ..... Date: ..... Period: .....

Pre-Unit Check (continued)

Unit 1

4. Determine the measure of angle x.

86°



Explain your thinking.

Explanations vary. **All of the angles that make up the line must sum to 180°, and $180 - 40 - 54 = 86$.**

D Differentiation (Pre-Unit Check)

Note: To **strengthen** and **stretch** students' learning, refer to the differentiation resources suggested throughout this unit and in the Unit Overview.

Sub-Unit Goals	Problem(s)	Respond to Student Thinking
Preparation for Sub-Unit 1: Perform translations, reflections, and rotations on and off a grid. **(Lessons 1–4)**	1	**Support** • **Teacher Move:** After **Lesson 1** (*Spinning, Flipping, Sliding*), consider revisiting Problem 1 and invite students to revise their work.
	2	**Support** • **Mini-Lesson:** Plotting Points on the Coordinate Grid • **Teacher Move:** During **Lesson 3** (*Getting Coordinated, Part 1*), Activity 2, look for students swapping coordinates, and invite those students to label their axes and coordinates with x and y.
	3	**Support** • **Teacher Move:** Before **Lesson 7** (*Are They Congruent?*), invite students to review **Problem 3** and create an anchor chart of student strategies for finding the area and perimeter.
	4	**Support** • **Mini-Lesson:** Identifying Acute, Obtuse, Right, and Straight Angles • **Teacher Move:** During **Lesson 8** (*Tearing It Up*) Activity 2, invite students to describe connections between angles that combine to form a straight line and the degree measure of a straight line.

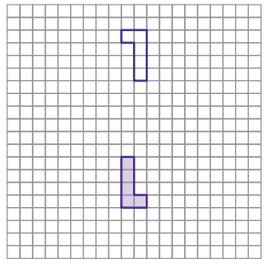
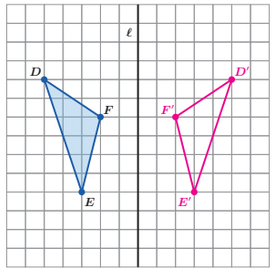
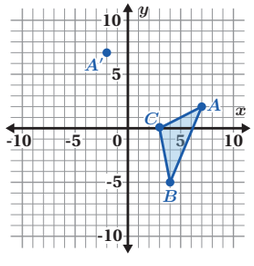
Rigid Transformations

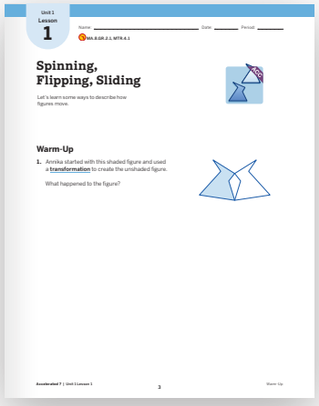
Sub-Unit 1 Goals

- **Goal:** Perform translations, reflections, and rotations on and off a grid.
- **Language Goal:** Describe translations, reflections, and rotations.

In this sub-unit, students visualize, describe, and perform **translations**, **reflections**, and **rotations** on and off a grid. They begin by using their own words to describe these moves and their characteristics and transition to describing **transformations** more precisely as the sub-unit progresses. They also consider the effects that transformations have on figures on the coordinate plane.

Math That Matters Most

Concept or Skill	For example . . .
Identify and describe translations, rotations, and reflections.	Describe a transformation to complete this challenge. • *Rotate 180° around the center of the grid.* 
Translate, rotate, or reflect a figure on a grid.	Reflect triangle DEF over line ℓ. 
Determine the coordinates of a transformed polygon after a translation, reflection, or rotation.	Determine the coordinates for the image of figure ABC after a rotation 90° counterclockwise around center $(0, 0)$. Pre-Image Coordinates / Image Coordinates $(7, 2)$ — $(-2, 7)$ $(4, -5)$ — **(5, 4)** $(3, 0)$ — **(0, 3)** 


Unit 1
Lesson
1

Spinning,
Flipping, Sliding

Let's learn some ways to describe how figures move.

Warm-Up

1. Annika started with this shaded figure and used a transformation to create the unshaded figure.

What happened to the figure?

Spinning, Flipping, Sliding

Naming Transformations

Let's learn some ways to describe how figures move.

Learning Outcomes

Today's Goals

1. **Goal:** Identify *translations*, *rotations*, and *reflections*.
2. **Language Goal:** Describe how *translations*, *rotations*, and *reflections* affect a figure. **(Speaking, Listening, and Writing)**

Students discover a need for more precise language when describing **transformations** by playing Polygraph. They determine whether a transformation is a **translation**, **rotation**, or **reflection** and describe how each transformation affects a figure.

Coherence

Prior Learning

In a previous course, students plotted ordered pairs on a coordinate plane and identified the x- or y-axis as the line of reflection when ordered pairs have opposite x-coordinates or opposite y-coordinates.

Future Learning

In Lesson 2, students will translate, reflect, and rotate figures by hand. In Lessons 3 and 4, students will generalize patterns for how translations, reflections, and rotations affect the coordinates of a figure.

Connections

- Students recognize the importance of common vocabulary when communicating with others about transformations. (mathematical, cross-curricular)

FL B.E.S.T. Math Benchmarks

Addressing

MA.8.GR.2.1

Given a preimage and image generated by a single transformation, identify the transformation that describes the relationship.

Mathematical Thinking and Reasoning Standard: MTR.4.1

ELA Expectations: ELA.K12.EE.1.1, ELA.K12.EE.3.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.6.GR.1.1	MA.912.F.2 MA.912.GR.2

Vocabulary

New Vocabulary

reflection	rotation
transformation	translation

Lesson at a Glance

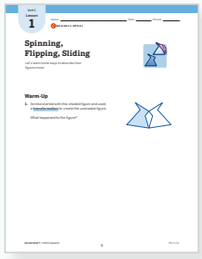
~ 45 min

MA.8.GR.2.1

Warm-Up

Pairs | 5 min

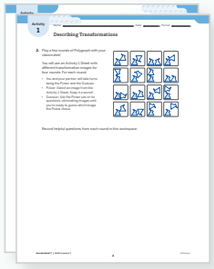
Students describe how a figure moves to prepare for Polygraph in Activity 1. They are also introduced to the term **transformation**.



Activity 1

Pairs | 20 min

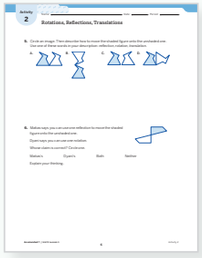
Students play Polygraph to discover a need for more precise language when describing transformations. Students connect everyday language to the terms **translation**, **reflection**, and **rotation**. **(MTR.4.1)**



Activity 2

Pairs | 10 min

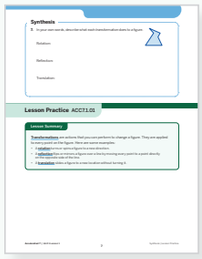
Students use the terms *reflection*, *rotation*, and *translation* to construct arguments about how to move one figure onto another. **Routine:** Decide and Defend **(MTR.4.1)**



Synthesis

Whole Class | 5 min

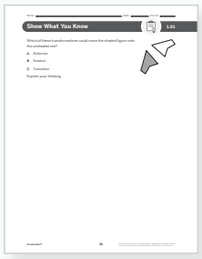
Students synthesize their understanding of reflections, rotations, and translations.
MLR2: Collect and Display



Show What You Know

Independent | 5 min

Students demonstrate their understanding by identifying and explaining the transformation that could move one figure onto another.



Prep Checklist

Prepare Activity 1 Sheets.

This lesson includes:

- Student Edition
- Show What You Know
- Activity 1 Sheet, one per student
  - Tracing paper (as needed)

Pairs | 5 min

Warm-Up

Purpose: Students describe how a figure moves to prepare for Polygraph in Activity 1. They are also introduced to the term **transformation**.

1 Launch

Invite students to use their own language to describe what happened to the figure.

ELL **English Language Learners** Invite students to share what the term *transformation* reminds them of (e.g., "When I hear *transformation*, I think . . ."). This may help surface multiple meanings and encourage students to connect to their prior knowledge. **(Reading and Listening)**

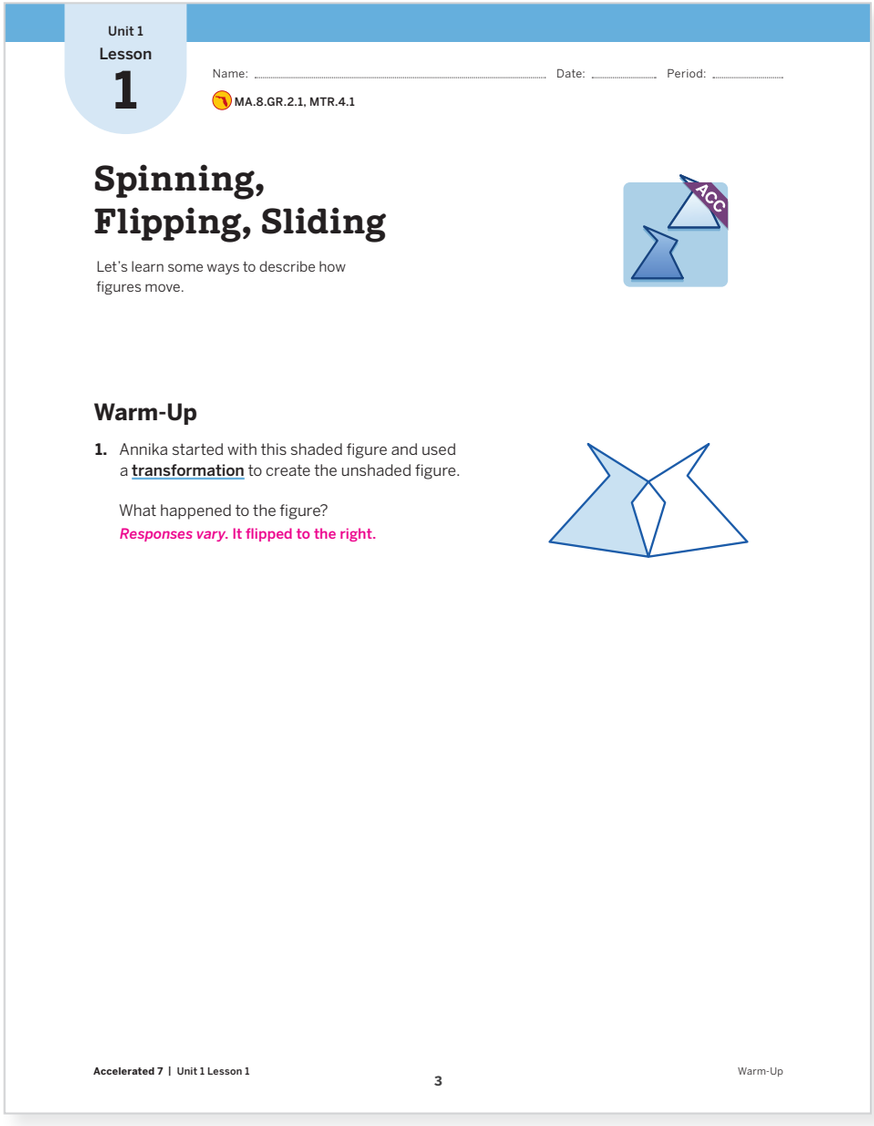
Note: It is okay if students do not yet have a precise definition of *transformation* at this stage. They will deepen their understanding throughout this lesson.

2 Connect

Consider asking, "How would you describe what the term **transformation** means in your own words?"

Look for and share student responses that might be useful in the Polygraph.

Note: There is no need to introduce the terms *translation*, *rotation*, and *reflection* at this time. Students will discover the need for precise language as they play Polygraph.



Unit 1
Lesson
1

Name: ..... Date: ..... Period: .....

MA.8.GR.2.1, MTR.4.1

Spinning, Flipping, Sliding

Let's learn some ways to describe how figures move.

Warm-Up

1. Annika started with this shaded figure and used a **transformation** to create the unshaded figure.

   What happened to the figure?

   Responses vary. It flipped to the right.

Pairs | 20 min

Activity 1 Describing Transformations

Purpose: Students play Polygraph to discover a need for more precise language when describing transformations. Students connect everyday language to the terms **translation**, **reflection**, and **rotation**. **(MTR.4.1)**

Short on time? Consider omitting Problem 4 or doing it as a class.

1 Launch

Invite students to find a partner in a different location to encourage movement.

Students will need one or more partners. Distribute the Activity 1 Sheet so that one partner has Set A and one has Set B. Invite one partner to be the Picker and the other to be the Guesser in the first round, and then switch roles for the next round.

2 Monitor

Listen for interesting or effective questions that use students' language to represent transformations as well as questions that could be improved with more precise vocabulary.

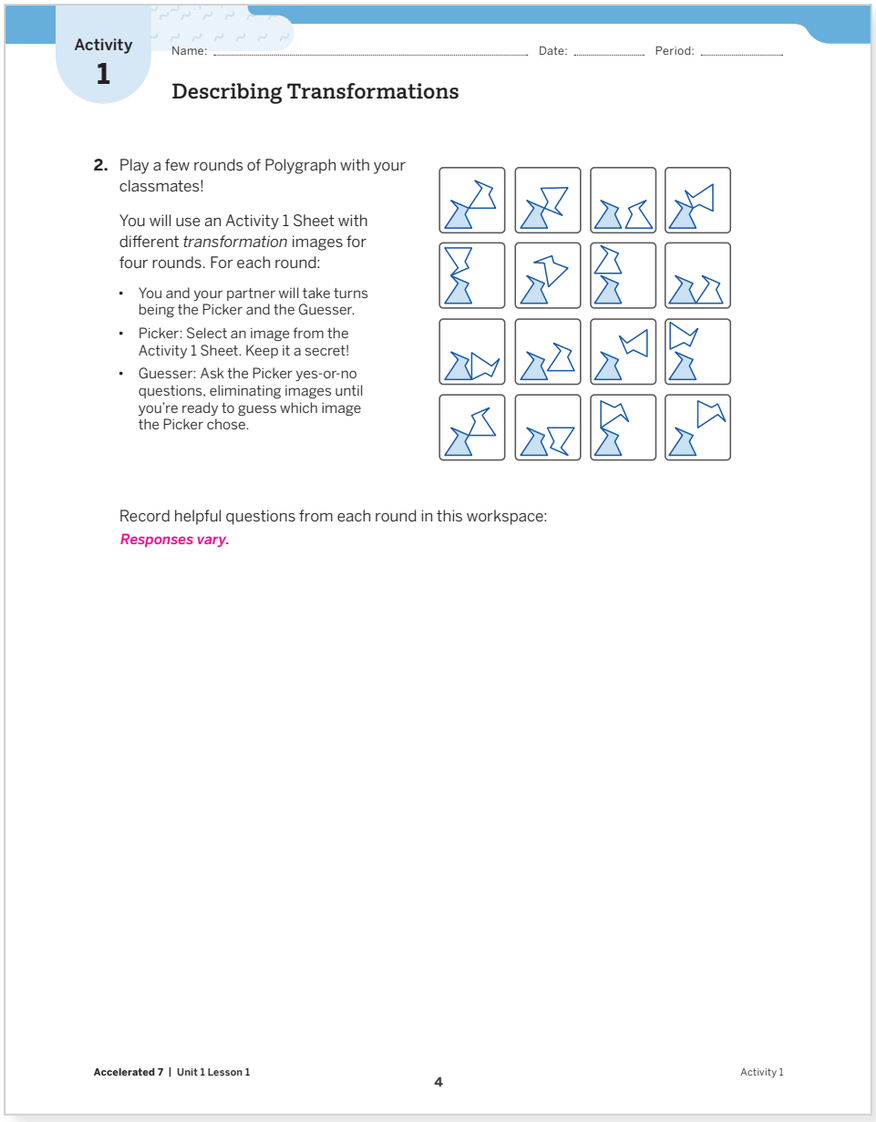
ELL **English Language Learners** Consider pausing to share words or phrases that pairs are using in their questions. **(Writing and Speaking)**

Pause to share the student questions and highlight places where precise vocabulary could have helped students understand their partners' thinking.

Consider asking:

- "What were you paying attention to as you wrote your questions?"
- "What words or phrases did you find particularly helpful?"

Activity 1 continued >



Activity 1

Name: _____ Date: _____ Period: _____

Describing Transformations

2. Play a few rounds of Polygraph with your classmates!

You will use an Activity 1 Sheet with different *transformation* images for four rounds. For each round:

- You and your partner will take turns being the Picker and the Guesser.
- Picker: Select an image from the Activity 1 Sheet. Keep it a secret!
- Guesser: Ask the Picker yes-or-no questions, eliminating images until you're ready to guess which image the Picker chose.

Record helpful questions from each round in this workspace:

Responses vary.

Accelerated 7 | Unit 1 Lesson 1 4 Activity 1

Pairs | 20 min

Activity 1 Describing Transformations (continued)

Purpose: Students play Polygraph to discover a need for more precise language when describing transformations. Students connect everyday language to the terms **translation**, **reflection**, and **rotation**. **(MTR.4.1)**

Short on time? Consider omitting Problem 4 or doing it as a class.

2 Monitor

Pause to discuss the first round of play from the activity sheet. Without exposing the chosen shapes, ask students to share the synonyms they have used for the different transformations.

D Differentiation

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Consider asking, "Where else have you heard the words *rotation* and *reflection* before?" In this context, the word *translation* may be new to students and therefore more challenging.
Are unsure about where to place "flip" because flipping can sometimes be turning.	**Strengthen** Consider sharing that this definition of *flip* is to flip over, like a pancake.
Sort the cards correctly.	**Stretch** Invite students to write three other words that describe these new terms.

Invite students to share their thinking with a partner as they choose a transformation for each pair of shapes.

A Accessibility: Executive Functioning To support students in carrying out multiple steps, chunk this activity into more manageable parts by inviting students to choose one pair of shapes at a time and decide which word best describes it.

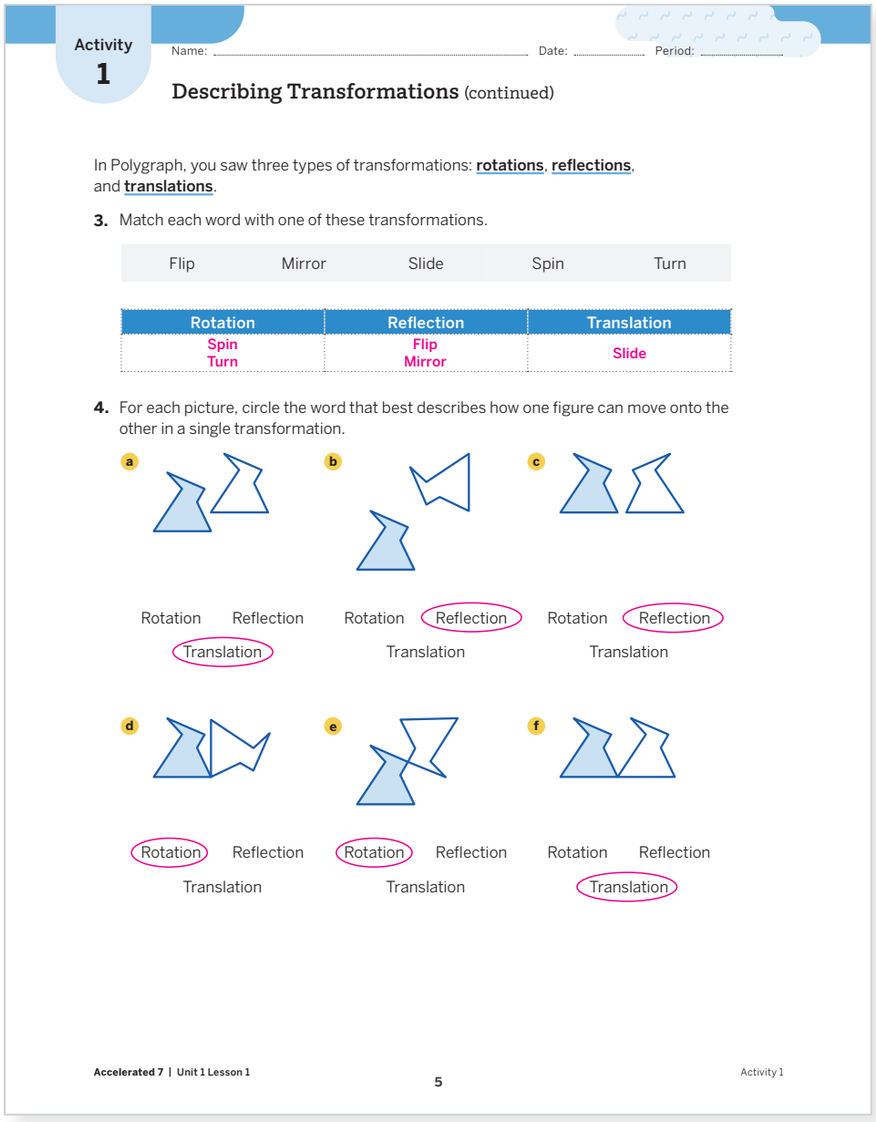
3 Connect

Invite students to share their strategies for sorting the words and images.

Create a class definition for the terms **reflection**, **rotation**, and **translation** using the pairs of shapes, students' language, and the glossary.

ELL English Language Learners Invite students to research how to say the new terms in other languages.

To surface the Key Takeaway, ask students, "How could we have used this vocabulary to help us in Polygraph?" **(MTR.4.1)**



Activity 1

Name: _____ Date: _____ Period: _____

Describing Transformations (continued)

In Polygraph, you saw three types of transformations: **rotations**, **reflections**, and **translations**.

3. Match each word with one of these transformations.

Flip Mirror Slide Spin Turn

Rotation	Reflection	Translation
Spin Turn	Flip Mirror	Slide

4. For each picture, circle the word that best describes how one figure can move onto the other in a single transformation.

a. Rotation Reflection Translation

b. Rotation Reflection Translation

c. Rotation Reflection Translation

d. Rotation Reflection Translation

e. Rotation Reflection Translation

f. Rotation Reflection Translation

Accelerated 7 | Unit 1 Lesson 1 5 Activity 1

Key Takeaway: The words **reflection**, **rotation**, and **translation** have specific definitions that allow us to communicate precisely about how figures move.

Pairs | 10 min

Activity 2 Rotations, Reflections, Translations

Purpose: Students use the terms *reflection*, *rotation*, and *translation* to construct arguments about how to move one figure onto another. **(MTR.4.1)**

1 Launch

For Problem 5, **Invite students** to use the new vocabulary to describe the transformations.

2 Monitor

Select and sequence student responses to share during the Connect.

D Differentiation *(Problem 6)*

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Ask students to explain the transformation using their own words or the drawing, then connect that language to the terms *reflect* and *rotate*.
Think Matias is correct.	**Support** Invite them to choose a specific point on the figure and use tracing paper or their hand to visualize where that point would be after the reflection.
Explain that Dyani is correct because the figure has been rotated.	**Strengthen** Consider asking, "Is there any way to use a reflection to move the shaded figure to the unshaded one?"
Explain that Dyani is correct because two reflections would be necessary for Matias to be correct.	**Stretch** Encourage students to create their own picture with two figures where both students will be correct.

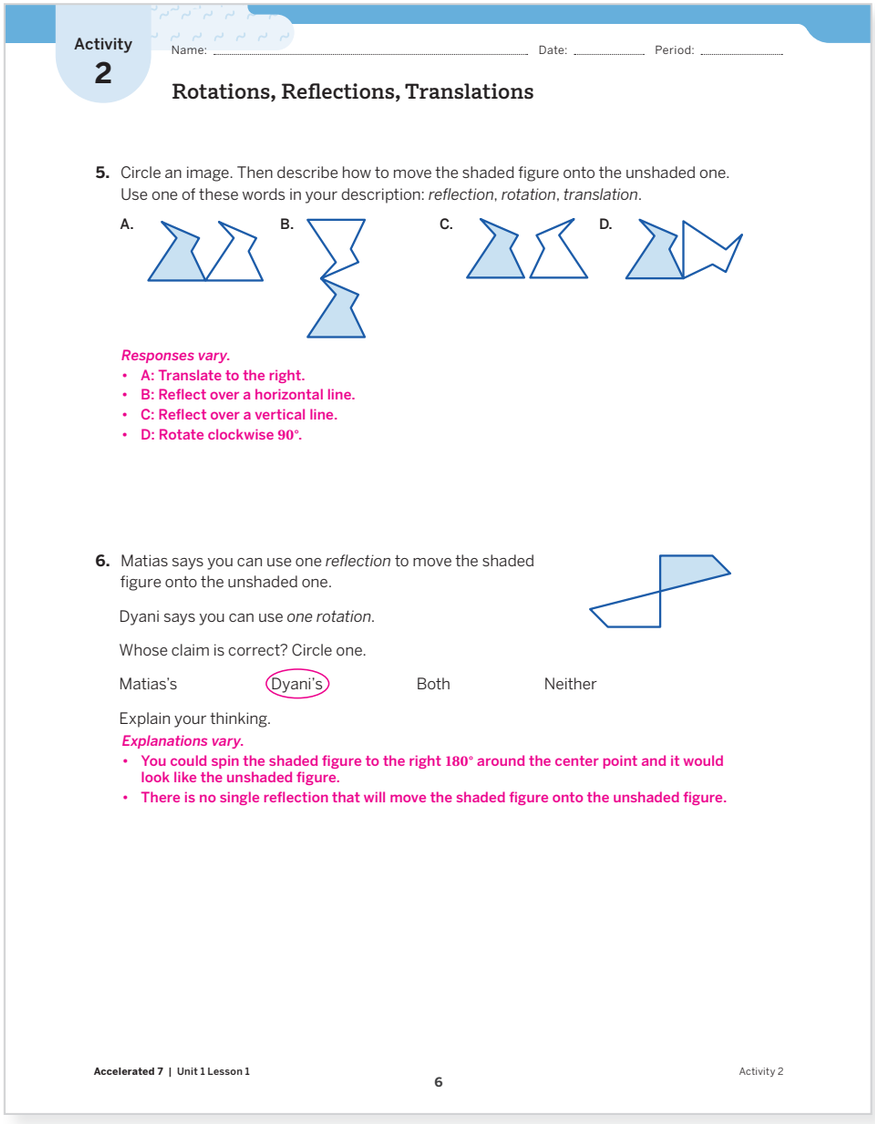
3 Connect

Use the Decide and Defend routine to support students in strengthening their ability to make arguments and to critique the reasoning of others. **(MTR.4.1)**

Invite students who made different choices to use reasoning to convince their classmates.

Consider asking:

- "What reasoning or evidence supports your claim?"
- "What evidence or language was most convincing to you?"



Activity 2

Name: _____ Date: _____ Period: _____

Rotations, Reflections, Translations

5. Circle an image. Then describe how to move the shaded figure onto the unshaded one. Use one of these words in your description: *reflection*, *rotation*, *translation*.

A. B. C. D.

Responses vary.

- A: Translate to the right.
- B: Reflect over a horizontal line.
- C: Reflect over a vertical line.
- D: Rotate clockwise 90°.

6. Matias says you can use one *reflection* to move the shaded figure onto the unshaded one.

Dyani says you can use *one rotation*.

Whose claim is correct? Circle one.

Matias's (Dyani's) Both Neither

Explain your thinking.

Explanations vary.

- You could spin the shaded figure to the right 180° around the center point and it would look like the unshaded figure.
- There is no single reflection that will move the shaded figure onto the unshaded figure.

Accelerated 7 | Unit 1 Lesson 1 6 Activity 2

Whole Class | 5 min

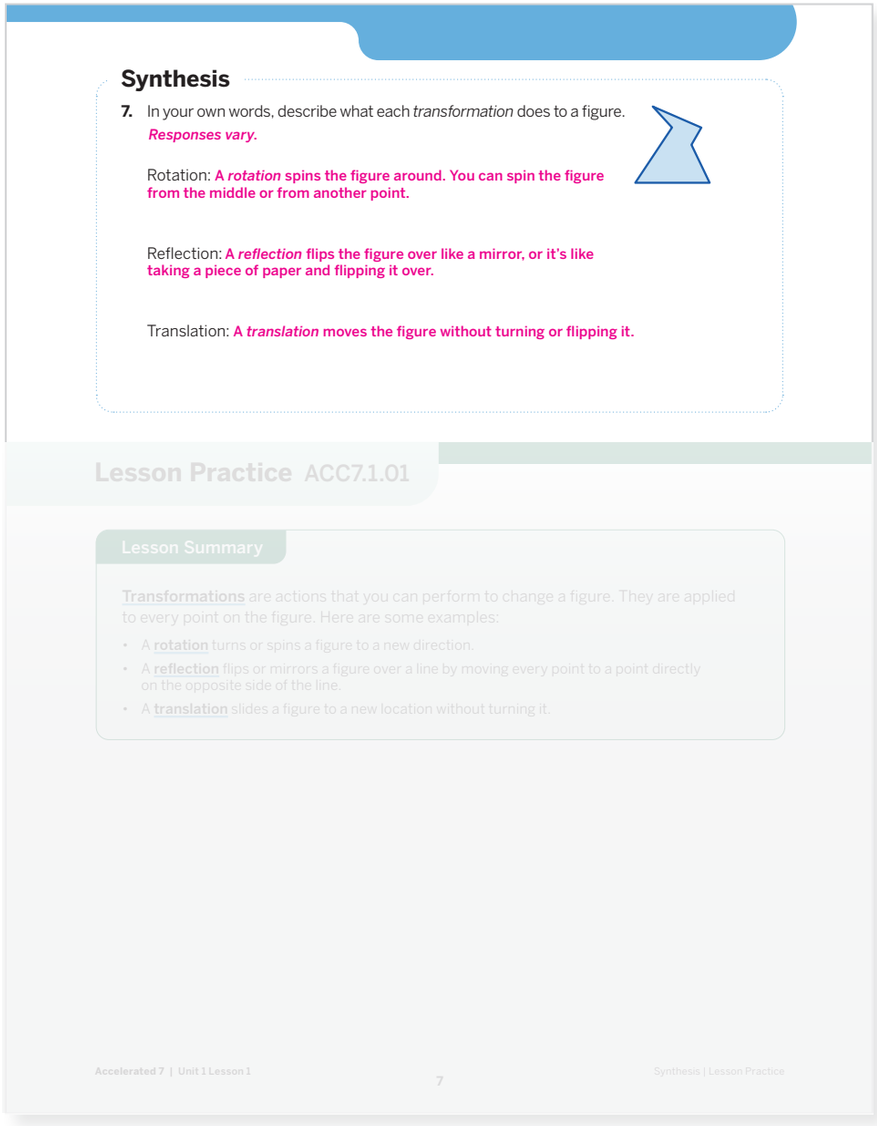
Synthesis

Purpose: Students synthesize their understanding of reflections, rotations, and translations.

Invite students to respond independently, and then share their thinking with a partner.

MLR **MLR2: Collect and Display** Use this routine to formalize the definitions of *reflection*, *rotation*, and *translation* using students' language. Display any new language, examples, or diagrams for students to reference later.

Note: If time allows, invite students to revise their previous thinking based on the discussion.



Synthesis

7. In your own words, describe what each *transformation* does to a figure.

Responses vary.

Rotation: **A *rotation* spins the figure around. You can spin the figure from the middle or from another point.**

Reflection: **A *reflection* flips the figure over like a mirror, or it's like taking a piece of paper and flipping it over.**

Translation: **A *translation* moves the figure without turning or flipping it.**

Lesson Practice ACC7.1.01

Lesson Summary

Transformations are actions that you can perform to change a figure. They are applied to every point on the figure. Here are some examples:

- A rotation turns or spins a figure to a new direction.
- A reflection flips or mirrors a figure over a line by moving every point to a point directly on the opposite side of the line.
- A translation slides a figure to a new location without turning it.

Accelerated 7 | Unit 1 Lesson 1 7 Synthesis | Lesson Practice

Lesson Takeaway: The words **reflection**, **rotation**, and **translation** have specific definitions that allow us to communicate precisely about how figures move. A *reflection* flips or mirrors a figure, a *rotation* turns or spins it, and a *translation* slides it.

Independent | 5 min

Show What You Know

Purpose: Students demonstrate their understanding by identifying and explaining the transformation that could move one figure onto another.

Today's Goals

1. **Goal:** Identify *translations*, *rotations*, and *reflections*.
2. **Language Goal:** Describe how *translations*, *rotations*, and *reflections* affect a figure. **(Speaking, Listening, and Writing)**

Show What You Know (Assessment Resources)

Name: _____ Date: _____ Period: _____

Show What You Know 1.01

Which of these transformations could move the shaded figure onto the unshaded one?

A. Reflection
B. Rotation
C. Translation

Explain your thinking.

Explanations vary. If the shaded figure is reflected over the diagonal line between the two figures, it would move onto the unshaded figure.

Differentiation Use after Lesson 1

Support

Provide targeted intervention for students by using these resources.

If students need support understanding the difference between the three transformations:

» **Respond to Student Thinking:**

- Revisit the **Warm-Up** as a class and invite students to discuss how each of the three transformations would affect the figure.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 1 Extension Activities**.

Professional Learning

In this lesson, students discovered a need for precise vocabulary through the game Polygraph. How did seeing and describing the pairs of figures develop a need for precise vocabulary? How will this experience support students' learning throughout the unit?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

7. In your own words, describe what each *transformation* does to a figure.

Responses vary.

Rotation: A *rotation* spins the figure around. You can spin the figure from the middle or from another point.

Reflection: A *reflection* flips the figure over like a mirror, or it's like taking a piece of paper and flipping it over.

Translation: A *translation* moves the figure without turning or flipping it.

Lesson Practice ACC7.1.01

Lesson Summary

Transformations are actions that you can perform to change a figure. They are applied to every point on the figure. Here are some examples:

- A **rotation** turns or spins a figure to a new direction.
- A **reflection** flips or mirrors a figure over a line by moving every point to a point directly on the opposite side of the line.
- A **translation** slides a figure to a new location without turning it.

Lesson Practice
ACC7.1.01

Name: _____ Date: _____ Period: _____

1. These five frames show a figure's different positions.

Describe how the figure moves to get from its position in each frame to the next.

From	To	Description of Moves
Frame 1	Frame 2	The figure is translated to the right.
Frame 2	Frame 3	The figure reflects over a horizontal line.
Frame 3	Frame 4	The figure is translated to the left.
Frame 4	Frame 5	The figure reflects over a horizontal line, returning to its original position in Frame 1.

Problems 2–4: Determine whether each transformation shows a translation, reflection, or rotation.

2. Translation

3. Rotation

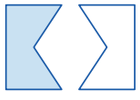
4. Reflection

Lesson Practice
ACC7.1.01

Name: _____ Date: _____ Period: _____

FAST Practice

5. Which transformation could move the shaded figure to the unshaded one? Select *all* that apply.



- ☑ A. Reflection
- ☑ B. Rotation
- ☐ C. Translation
- ☐ D. None of these

Spiral Review

Problems 6–9: Evaluate each expression.

6. $-5 \cdot (-2.4)$ 12

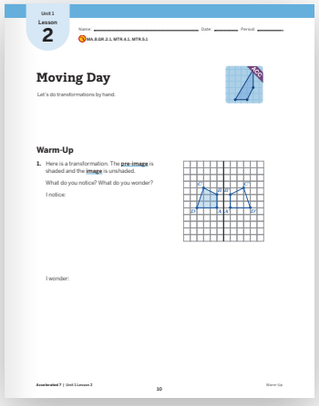
7. $-7.4 \div 10$ -0.74 (or equivalent)

8. $\frac{4}{7} \div (-2)$ $\frac{2}{7}$ (or equivalent)

9. $4 \cdot \left(\frac{3}{8}\right)$ $\frac{3}{2}$ (or equivalent)

Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1	2	MA.8.GR.2.1
	2–4	1	MA.8.GR.2.1
FAST Practice	**5**	2	MA.8.GR.2.1
Spiral Review			
	6–9	1	MA.7.NSO.2.2


Unit 1
Lesson 2
Name _____ Date _____ Period _____
MA.8.GR.2.1, MTR.4.1, MTR.5.1
Moving Day
Let's do transformations by hand.
Warm-Up
1. Here is a transformation. The pre-image is shaded and the image is unshaded.
What do you notice? What do you wonder?
I notice:
I wonder:
Accelerated 7 | Unit 1 Lesson 2
10
Warm-Up

Moving Day

Describing Transformations Precisely

Let's do transformations by hand.

Learning Outcomes

Today's Goals

1. **Goal:** Translate, reflect, or rotate a figure on a grid.
2. **Language Goal:** Describe translations, reflections, and rotations on a grid. **(Reading, Writing, Speaking, and Listening)**

Students translate, reflect, and rotate figures by hand on a square grid. They practice interpreting and using precise language for transformations as they describe a transformation for a classmate to draw.

Coherence

Prior Learning

In Lesson 1, students were introduced to the terms *translation*, *reflection*, and *rotation* to describe the way figures can be transformed.

Future Learning

Lessons 3 and 4, students will generalize how translations, reflections, and rotations affect the coordinates of a figure.

Connections

- Students use precise language when describing rigid transformations in a partner activity. (mathematical, cross-curricular)

FL B.E.S.T. Math Benchmarks

Addressing

MA.8.GR.2.1
Given a preimage and image generated by a single transformation, identify the transformation that describes the relationship.

Mathematical Thinking and Reasoning Standards: MTR.4.1, MTR.5.1

ELA Expectations: ELA.K12.EE.3.1, ELA.K12.EE.4.1, ELA.K12.EE.6.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.6.GR.1.1	MA.912.F.2 MA.912.GR.2

Vocabulary

New Vocabulary

clockwise
counterclockwise
preimage
correspond
image

Additional Vocabulary

prime notation

Lesson at a Glance

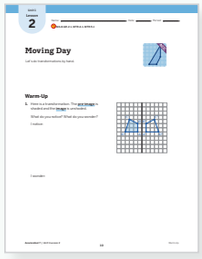
~ 45 min

MA.8.GR.2.1

Warm-Up

Whole Class | 5 min

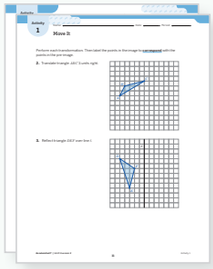
Students use the **Notice and Wonder** routine to make sense of *prime notation* (′) and the terms **pre-image** and **image**.



Activity 1

Pairs | 15 min

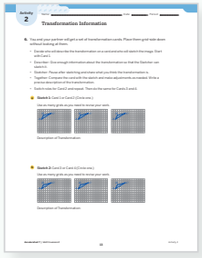
Students build fluency in precisely transforming figures by hand on a grid and are introduced to the terms **correspond**, **clockwise**, and **counterclockwise**.
Routine: Think-Pair-Share **(MTR.5.1)**



Activity 2

Pairs | 15 min

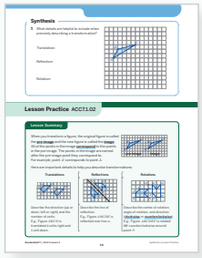
Students discover a need for precise communication by describing a transformation to their partner who will attempt to draw it, then revise their work based on the result. **(MTR 4.1)**



Synthesis

Whole Class | 5 min

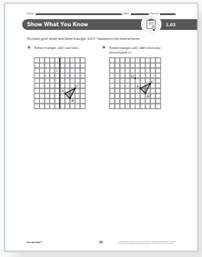
Students synthesize their understanding of what information is needed to describe each type of transformation precisely.
MLR2: Collect and Display



Show What You Know

Independent | 5 min

Students demonstrate their understanding by reflecting and rotating a figure on a square grid.



Prep Checklist

Prepare the Activity 2 Cards. Invite students to use their Student Edition.

This lesson includes:

- Student Edition
- Show What You Know
- Activity 2 Cards, one set per pair, pre-cut
- Straightedges, Tracing paper, Coloring tools (optional)

Whole Class | 5 min

Warm-Up

Purpose: Students use the Notice and Wonder routine to make sense of *prime notation* (′) and the terms **pre-image** and **image**.

1 Launch

Direct students' attention to the transformation.

Use the Notice and Wonder routine to promote curiosity and help students make sense of the transformation and the new notation.

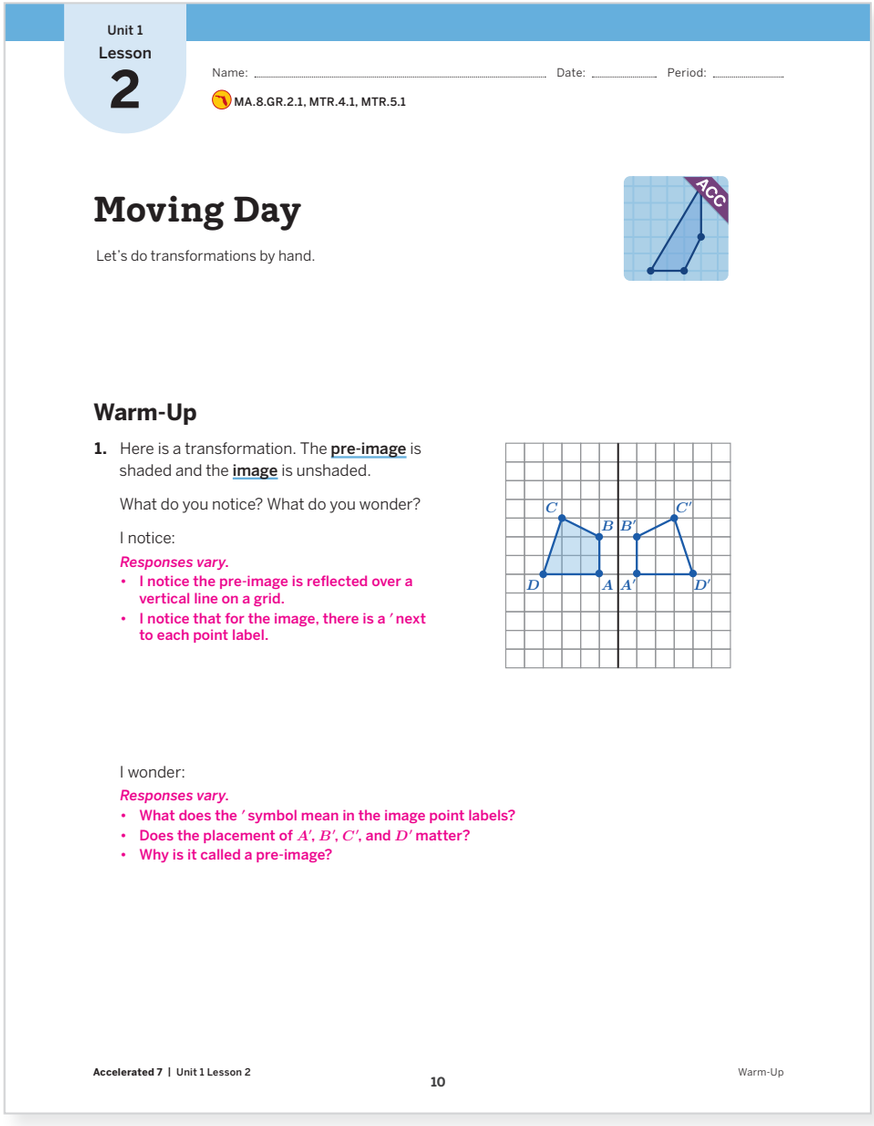
2 Connect

Create a record of what students noticed and wondered, along with the names of the students who shared them.

Create class definitions for the terms **pre-image** and **image**, and support students in making sense of *prime notation* (′). To help generate student language, consider asking:

- "How are the labels different on the two figures?"
- "What does "pre" mean? How is that connected to the term *pre-image*?"
- "Do you think the placement of A', B', C', and D' on the image matters?"

ELL **English Language Learners** Invite students to share what a new term reminds them of (e.g., "When I hear *pre-image*, I think . . ."). This may help surface multiple meanings and encourage students to connect to their prior knowledge. **(Reading and Listening)**



Unit 1
Lesson
2

Name: _____ Date: _____ Period: _____

MA.8.GR.2.1, MTR.4.1, MTR.5.1

Moving Day

Let's do transformations by hand.

Warm-Up

1. Here is a transformation. The **pre-image** is shaded and the **image** is unshaded.

What do you notice? What do you wonder?

I notice:

Responses vary.

- I notice the pre-image is reflected over a vertical line on a grid.
- I notice that for the image, there is a ′ next to each point label.

I wonder:

Responses vary.

- What does the ′ symbol mean in the image point labels?
- Does the placement of A', B', C', and D' matter?
- Why is it called a pre-image?

Accelerated 7 | Unit 1 Lesson 2 10 Warm-Up

Key Takeaway: An **image** is a new figure created after a **pre-image** is transformed. The image is labeled with the symbol ′.

Pairs | 15 min

Activity 1 Move It

Purpose: Students build fluency in precisely transforming figures by hand on a grid and are introduced to the terms **correspond**, **clockwise**, and **counterclockwise**. **(MTR.5.1)**

1 Launch

Use the Think-Pair-Share routine to help students shape their ideas about how geometry tools may be helpful.

Create a class definition for the term **correspond** using students' language and the glossary. Record it for students to refer to later in the unit.

Distribute geometry tools for students to choose from, including tracing paper and straightedges.

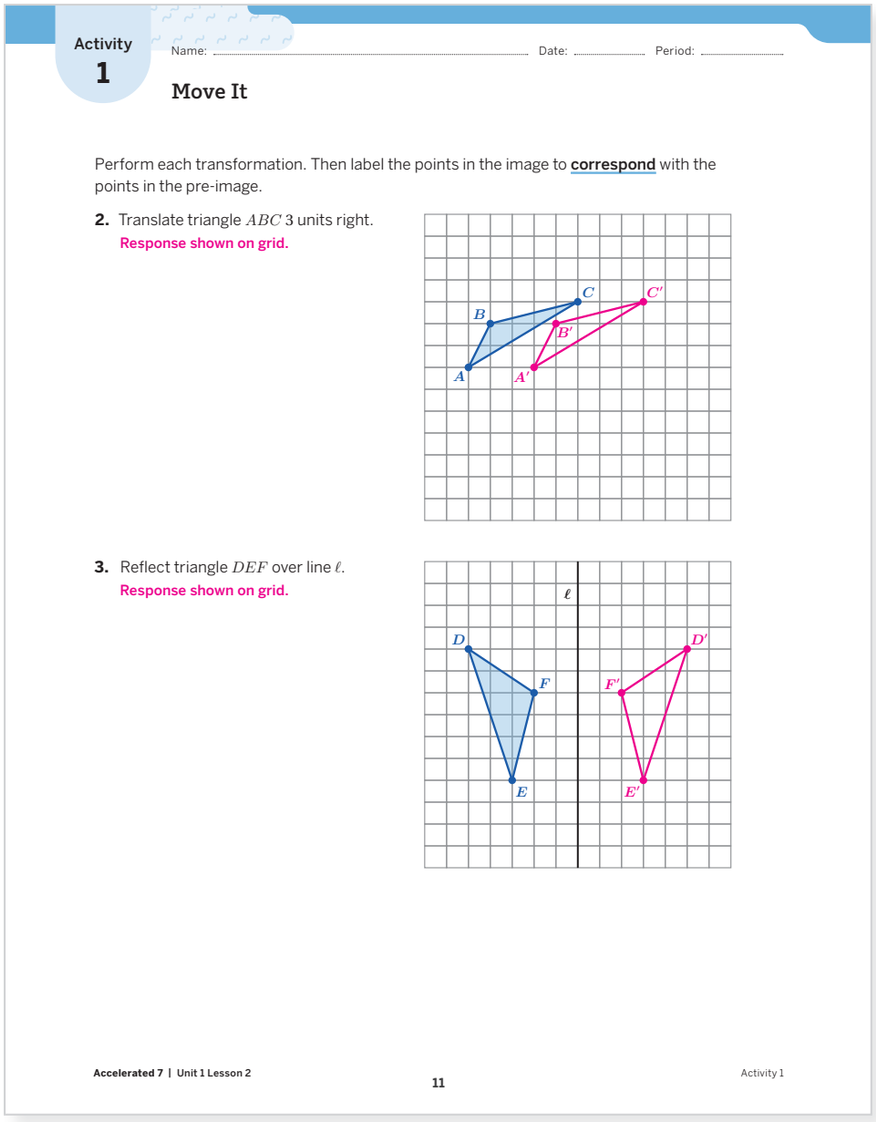
2 Monitor

Look for a variety of strategies to share during the Connect.

D Differentiation *(Problems 1–5)*

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Invite students to start by considering what the transformation is and performing it on just one point.
End up with an image that doesn't match the pre-image.	**Support** Consider asking, "How could you check that your transformation is correct?" Remind students that they can use tracing paper to check their work.
Perform the transformation but do not label the points.	**Strengthen** Consider asking, "How was the image labeled in the Warm-Up? How can you replicate that in your work?"
Need support with rotating the triangle by hand. *(Problem 4)*	**Strengthen** Consider demonstrating how to hold the pencil on center O then rotate the tracing paper.
Use a variety of strategies, such as: • Using tracing paper. • Using a ruler to measure distance between corresponding points. • Using the grid to help decide where to draw the transformed figures.	**Strengthen** Invite these students to think about how they would share their strategies during the Connect.
Would like to explore transformations more.	**Stretch** Invite them to try different transformations on the figures.

Activity 1 continued >



Activity 1

Name: Date: Period:

Move It

Perform each transformation. Then label the points in the image to **correspond** with the points in the pre-image.

2. Translate triangle ABC 3 units right.
Response shown on grid.

3. Reflect triangle DEF over line ℓ.
Response shown on grid.

Accelerated 7 | Unit 1 Lesson 2 11 Activity 1

Pairs | 15 min

Activity 1 Move It (continued)

Purpose: Students build fluency in precisely transforming figures by hand on a grid and are introduced to the terms **correspond**, **clockwise**, and **counterclockwise**. **(MTR.5.1)**

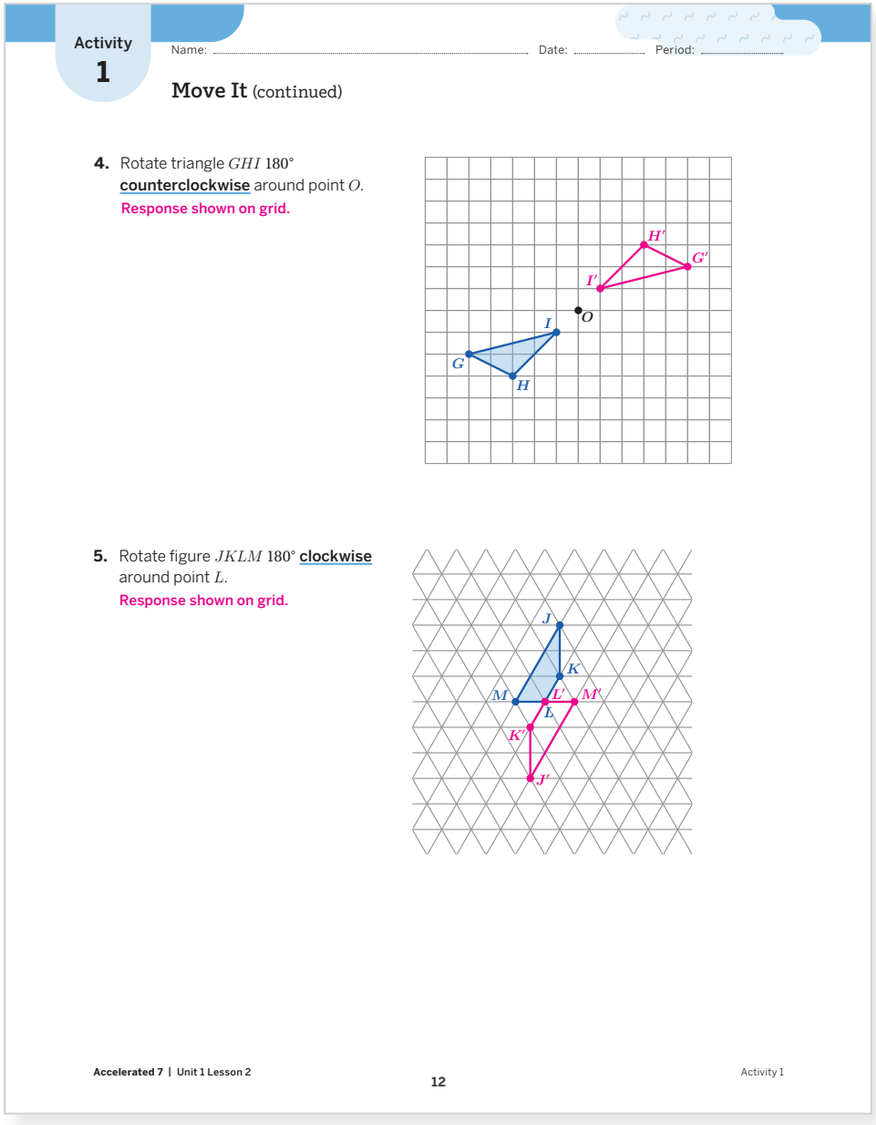
2 Monitor

Consider asking, "What do **clockwise** and **counterclockwise** mean? When have you seen these terms?"

3 Connect

Invite students to share the strategies they used to transform the figures. **(MTR.5.1)**

Consider asking, "What do you think is important to remember when performing transformations by hand?"



Activity 1

Name: Date: Period:

Move It (continued)

4. Rotate triangle GHI 180° **counterclockwise** around point O.

Response shown on grid.

5. Rotate figure $JKLM$ 180° **clockwise** around point L.

Response shown on grid.

Accelerated 7 | Unit 1 Lesson 2 12 Activity 1

Pairs | 15 min

Activity 2 Transformation Information

Purpose: Students discover a need for precise communication by describing a transformation to their partner who will attempt to draw it, then revise their work based on the result. **(MTR.4.1)**

Short on time? Consider asking students to describe the transformations on Cards 3 and 4 as a class.

1 Launch

Direct students' attention to the instructions and invite pairs to choose one person to be the Describer and one person to be the Sketcher for the first round.

Distribute the Activity 2 Cards and invite students to keep them face down.

Consider sharing that the Describer should let the Sketcher finish their sketch before giving another description.

A **Accessibility: Conceptual Processing** Invite students to revisit their work in Activity 1 to help apply concepts to new situations.

2 Monitor

Invite students to spend a couple of minutes on the first card with their partner and to use the space provided to revise their work once they receive more information from their partner.

Pause briefly to discuss what information students needed to provide their partner in order for them to perform the transformation from the first card.

ELL **English Language Learners** Provide a word bank to support students in expressing their ideas with words such as: *rotate*, *reflect*, *translate*, *point*, *degrees*, *line*, *clockwise*, *counterclockwise*, *units*, etc. **(Writing and Speaking)**

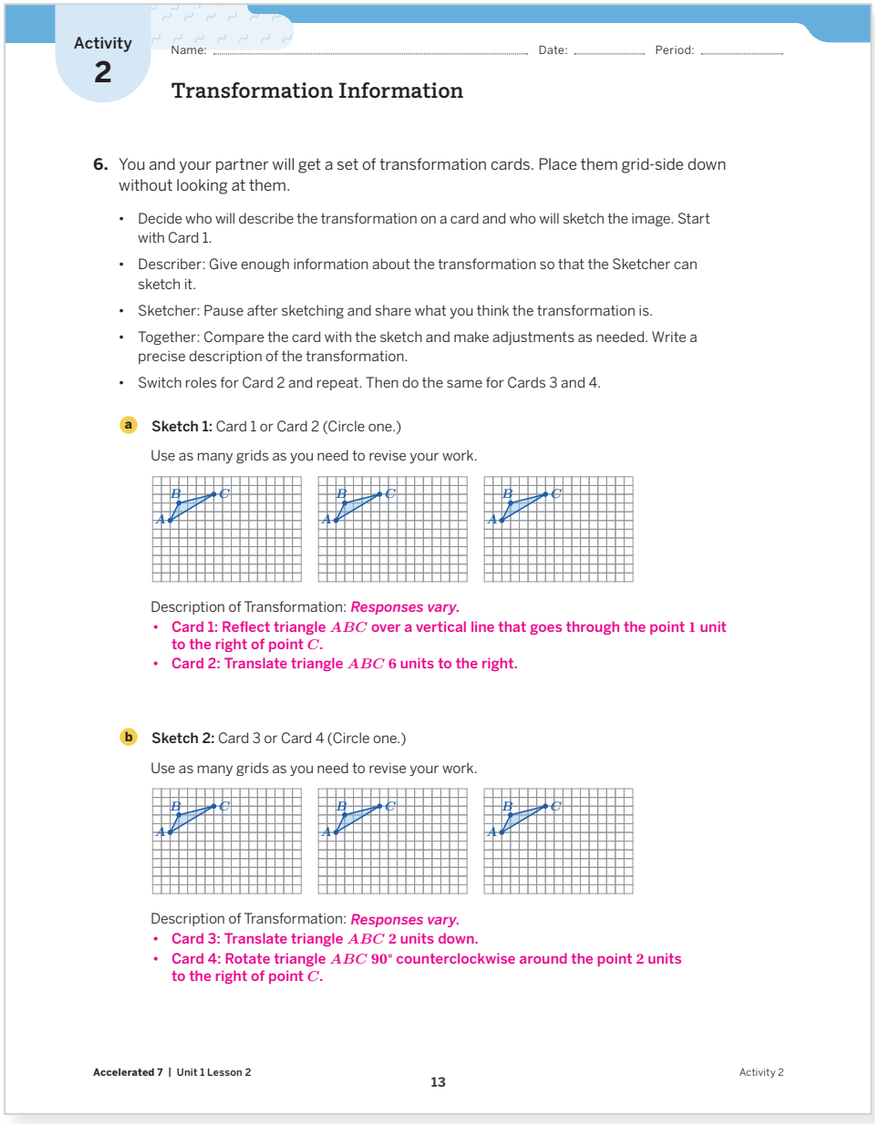
Invite students to complete the rest of the cards with their partner.

Look for precise descriptions of each of the transformations to share during the Connect.

3 Connect

Invite students to share descriptions of the transformations that allowed their partner to draw the transformation accurately. **(MTR.4.1)**

Consider asking:
- "What was challenging about this activity?"
- "How did you revise your work?"



Activity 2

Name: Date: Period:

Transformation Information

6. You and your partner will get a set of transformation cards. Place them grid-side down without looking at them.

- Decide who will describe the transformation on a card and who will sketch the image. Start with Card 1.
- Describer: Give enough information about the transformation so that the Sketcher can sketch it.
- Sketcher: Pause after sketching and share what you think the transformation is.
- Together: Compare the card with the sketch and make adjustments as needed. Write a precise description of the transformation.
- Switch roles for Card 2 and repeat. Then do the same for Cards 3 and 4.

a **Sketch 1:** Card 1 or Card 2 (Circle one.)

Use as many grids as you need to revise your work.

Description of Transformation: ***Responses vary.***
- **Card 1: Reflect triangle ABC over a vertical line that goes through the point 1 unit to the right of point C.**
- **Card 2: Translate triangle ABC 6 units to the right.**

b **Sketch 2:** Card 3 or Card 4 (Circle one.)

Use as many grids as you need to revise your work.

Description of Transformation: ***Responses vary.***
- **Card 3: Translate triangle ABC 2 units down.**
- **Card 4: Rotate triangle ABC 90° counterclockwise around the point 2 units to the right of point C.**

Accelerated 7 | Unit 1 Lesson 2 13 Activity 2

Whole Class | 5 min

Synthesis

Purpose: Students synthesize their understanding of what information is needed to describe each type of transformation precisely.

Invite students to respond independently, and then share their thinking with a partner.

Collect and share a variety of ideas, such as:

- Translations require a direction and a number of units.
- Reflections require a line, including the orientation of the line and its relationship to the pre-image.
- Rotations require a center, angle, and direction of rotation.

MLR **MLR2: Collect and Display** Use this routine to summarize the information needed to precisely describe each transformation using students' language. Display any new language, examples, or diagrams publicly for students to reference later.

Synthesis

7. What details are helpful to include when precisely describing a transformation?

Responses vary.

Translation: **Describe the direction and distance of the translation.**

Reflection: **Describe the line of reflection, including the orientation of the line and its location in relation to the pre-image.**

Rotation: **Describe the center, angle, and direction of the rotation.**

Lesson Practice ACC7.1.02

Lesson Summary

When you transform a figure, the original figure is called the **pre-image** and the new figure is called the **image**. All of the points in the image **correspond** to the points in the pre-image. The points in the image are named after the pre-image point they correspond to. For example, point A' corresponds to point A.

Here are important details to help you describe transformations:

Translations

Describe the direction (up or down, left or right) and the number of units.
E.g., Figure $ABCD$ is translated 4 units right and 1 unit down.

Reflections

Describe the line of reflection.
E.g., Figure $ABCDE$ is reflected over line m.

Rotations

Describe the center of rotation, angle of rotation, and direction (**clockwise** or **counterclockwise**).
E.g., Figure $ABCDEF$ is rotated 90° counterclockwise around point P.

Accelerated 7 | Unit 1 Lesson 2 14 Synthesis | Lesson Practice

Lesson Takeaway: In order to precisely describe a transformation, it is helpful to include different pieces of information, like lines, points, and number of degrees.

Independent | 5 min

Show What You Know

Purpose: Students demonstrate their understanding by reflecting and rotating a figure on a square grid.

Today's Goals

1. **Goal:** Translate, reflect, or rotate a figure on a grid.
2. **Language Goal:** Describe translations, reflections, and rotations on a grid. **(Reading, Writing, Speaking, and Listening)**

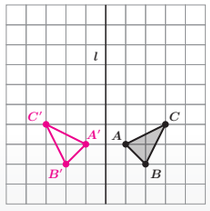
Show What You Know (Assessment Resources)

Name: _____ Date: _____ Period: _____

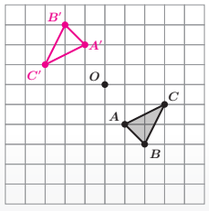
Show What You Know 1.02

On each grid, draw and label triangle $A'B'C'$ based on the instructions.

a. Reflect triangle ABC over line l.



b. Rotate triangle ABC 180° clockwise around point O.



D Differentiation Use after Lesson 2

Support

Provide targeted intervention for students by using these resources.

If students need support understanding how to transform figures on a grid:

» **Respond to Student Thinking:**

- Revisit **Activity 1** before providing individualized feedback on the relevant **Practice Problems**.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite them to create their own figures and challenge their classmates to transform them.
- Invite students to explore the **Sub-Unit 1 Extension Activities**.

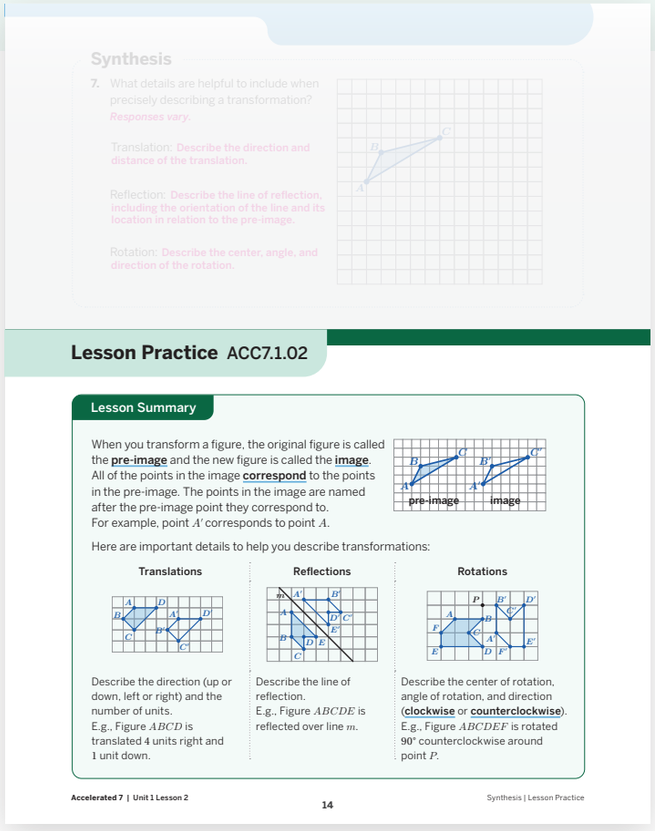
Professional Learning

Reflect on how students revised their work today. How did they talk about mistakes? Were they able to notice and correct their work? How can you support students in recognizing that revision is an important part of learning?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.



Synthesis

7. What details are helpful to include when precisely describing a transformation?

Responses vary.

Translation: Describe the direction and distance of the translation.

Reflection: Describe the line of reflection, including the orientation of the line and its location in relation to the pre-image.

Rotation: Describe the center, angle, and direction of the rotation.

Lesson Practice ACC7.1.02

Lesson Summary

When you transform a figure, the original figure is called the **pre-image** and the new figure is called the **image**. All of the points in the image **correspond** to the points in the pre-image. The points in the image are named after the pre-image point they correspond to. For example, point A' corresponds to point A.

pre-image image

Here are important details to help you describe transformations:

Translations

Describe the direction (up or down, left or right) and the number of units.
E.g., Figure $ABCD$ is translated 4 units right and 1 unit down.

Reflections

Describe the line of reflection.
E.g., Figure $ABCDE$ is reflected over line m.

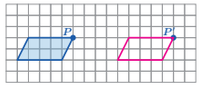
Rotations

Describe the center of rotation, angle of rotation, and direction (**clockwise** or **counterclockwise**).
E.g., Figure $ABCDEF$ is rotated 90° counterclockwise around point P.

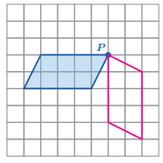
Lesson Practice
ACC7.1.02

Name: ..... Date: ..... Period: .....

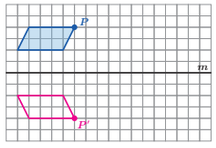
1. Translate this figure to take point P to P'.



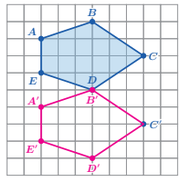
2. Rotate this figure 90° counterclockwise around point P.



3. Reflect this figure over line m.



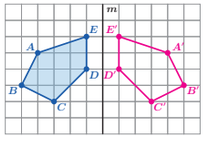
4. Translate figure $ABCDE$ to take point C to C'.



Lesson Practice
ACC7.1.02

Name: ..... Date: ..... Period: .....

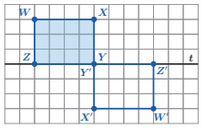
5. Reflect figure $ABCDE$ over line m.



FAST Practice

6. Figure $WXYZ$ was transformed to create figure $W'X'Y'Z'$. Which transformation took place?

A. A reflection over line t.
B. A translation 6 units down.
C. A rotation 90° clockwise around point Y.
D. A rotation 180° clockwise around point Y.



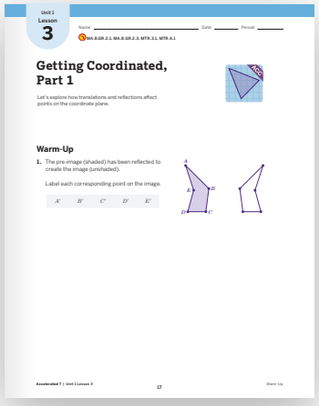
Spiral Review

7. Match each expression with an equivalent expression.

Expressions	Equivalent Expressions
a. $-3x - 7$	b. $-0.9 + 5.7x$
b. $-3.4 + 5.7x + 2.5$	d. $-\frac{1}{2}(6x - 14)$
c. $1.8x - 5.9 + 3.9x$	c. $5.7x - 5.9$
d. $-3x + 7$	a. $\left(\frac{7}{2} - \frac{3}{2}x\right) \cdot 2$

Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1–5	2	MA.8.GR.2.1
FAST Practice	6	2	MA.8.GR.2.1
Spiral Review			
	7	1	MA.7.AR.1.1



Getting Coordinated, Part 1

Coordinates of Translations and Reflections

Let's explore how translations and reflections affect points on the coordinate plane.

Learning Outcomes

Today's Goals

1. **Goal:** Determine the coordinates of a transformed image after a translation or reflection.
2. **Language Goal:** Explain how a translation or reflection impacts the coordinates of a transformed image. **(Speaking, Listening, Reading, and Writing)**

Students investigate how a translation or reflection impacts the coordinates of a transformed image. They first compare a reflection with and without a coordinate grid, describing how the grid enables them to be more precise. Then they generalize the impact of translations and reflections on the coordinates using patterns in a table. **(MTR.4.1)**

Coherence

Prior Learning

In Lesson 2, students described and drew transformations on a grid without coordinates.

Future Learning

In Lesson 4, students will investigate the effects of rotations on a coordinate plane.

Connections

- Students expand their use of the coordinate plane to translate and reflect figures. (mathematical)

FL B.E.S.T. Math Benchmarks

Addressing

MA.8.GR.2.1
Given a preimage and image generated by a single transformation, identify the transformation that describes the relationship.

MA.8.GR.2.3
Describe and apply the effect of a single transformation on two-dimensional figures using coordinates and the coordinate plane.

Mathematical Thinking and Reasoning Standards: MTR.3.1, MTR.4.1

ELA Expectations: ELA.K12.EE.1.1, ELA.K12.EE.3.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.6.GR.1.1	**MA.912.F.2** **MA.912.GR.2.4**

Lesson at a Glance

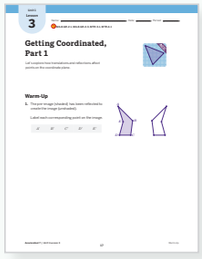
~ 45 min

MA.8.GR.2.1, MA.8.GR.2.3

Warm-Up

Pairs | 5 min

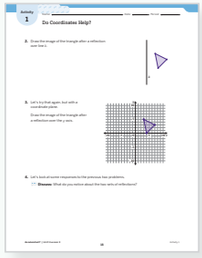
Students use prime notation to label the corresponding points of an image given the pre-image.



Activity 1

Pairs | 8 min

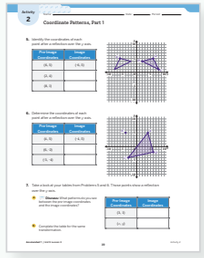
Students discover the usefulness of the coordinate plane by reflecting a figure with and without a coordinate plane and comparing the accuracy of each.



Activity 2

Pairs | 10 min

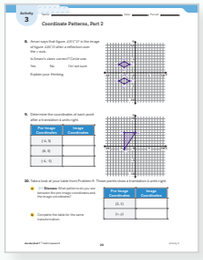
Students generalize the patterns between pre-image and image coordinates after a reflection. **(MTR.4.1)**



Activity 3

Pairs | 12 min

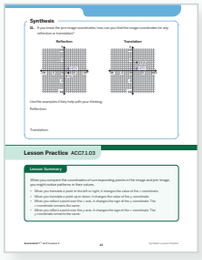
Students use the **Decide and Defend** routine to examine the differences between translations and reflections. They then generalize the patterns between pre-image and image coordinates after a translation. **(MTR.3.1, MTR.4.1)**



Synthesis

Whole Class | 5 min

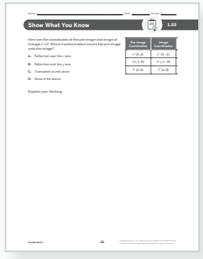
Students synthesize their understanding of how to determine the image coordinates for any reflection or translation. **MLR1: Stronger and Clearer Each Time** **(MTR.3.1)**



Show What You Know

Independent | 5 min

Students demonstrate their understanding by determining which transformation was performed given the pre-image and image coordinates.



Prep Checklist

This lesson includes:

- Student Edition
- Show What You Know
- Tracing paper (as needed)

Pairs | 5 min

Warm-Up

Purpose: Students use prime notation to label the corresponding points of an image given the pre-image.

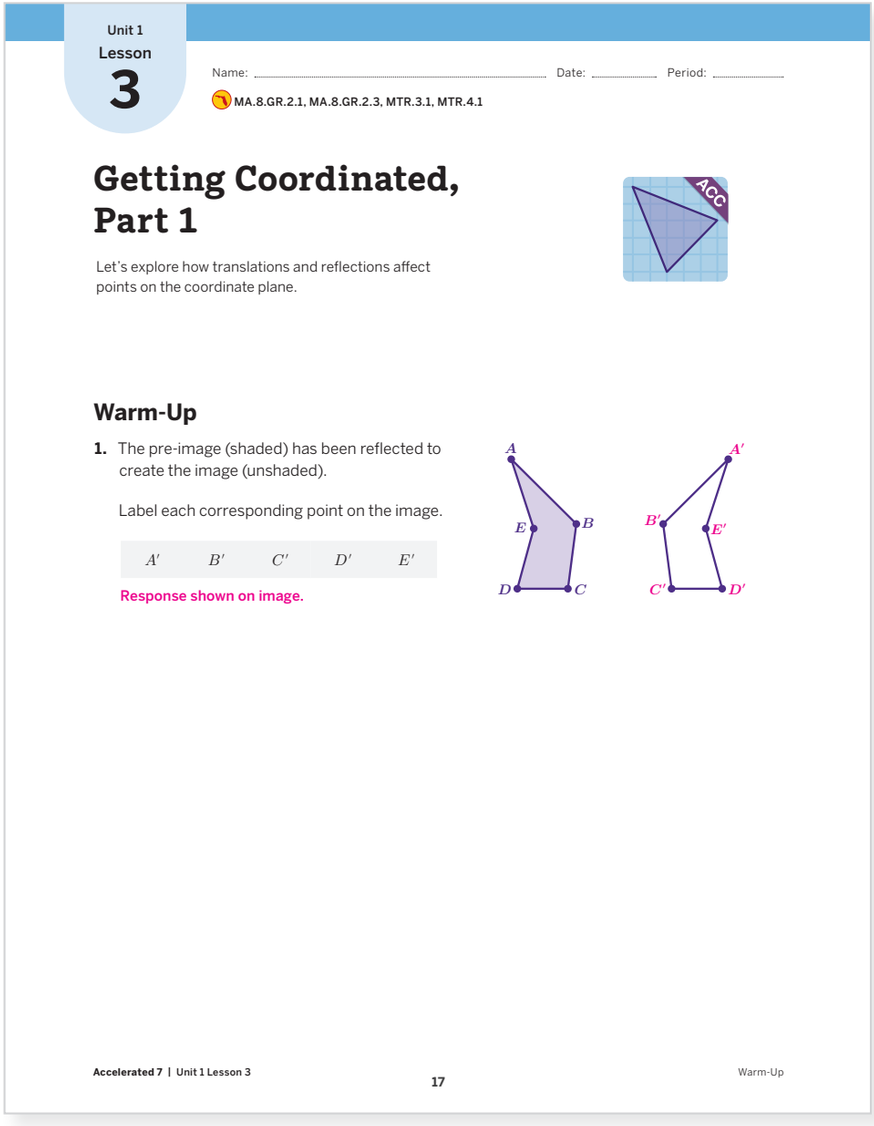
1 Launch

To surface prior knowledge, consider discussing what students remember about vocabulary from the prior lesson: *pre-image*, *image*, *corresponding*, and *prime notation*.

Invite students to label the corresponding points.

2 Connect

Invite students to share their responses and encourage them to explain their reasoning.



Unit 1
Lesson
3

Name: _____ Date: _____ Period: _____

MA.8.GR.2.1, MA.8.GR.2.3, MTR.3.1, MTR.4.1

Getting Coordinated, Part 1

Let's explore how translations and reflections affect points on the coordinate plane.

ACC

Warm-Up

1. The pre-image (shaded) has been reflected to create the image (unshaded).

Label each corresponding point on the image.

A'	B'	C'	D'	E'

Response shown on image.

Accelerated 7 | Unit 1 Lesson 3 17 Warm-Up

Pairs | 8 min

Activity 1 Do Coordinates Help?

Purpose: Students discover the usefulness of the coordinate plane by reflecting a figure with and without a coordinate plane and comparing the accuracy of each.

1 Launch

Consider sharing that students will be doing the same transformation twice, once without a coordinate plane and once with.

2 Monitor

Note: It is okay if students are not precise in their thinking off the grid. The purpose of this activity is to highlight the power of the grid for precision.

Invite students to share their translations with a partner.

3 Connect

Invite students to share their reflections with the class.

Consider asking:

- "Which reflection produced more accurate images?"
- "Why do you think that is?"

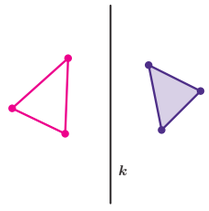
Activity 1

Name: ..... Date: ..... Period: .....

Do Coordinates Help?

2. Draw the image of the triangle after a reflection over line k.

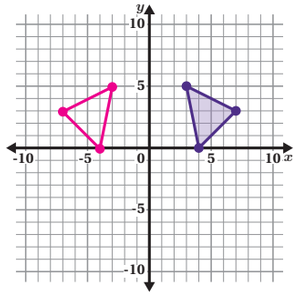
Sample shown on image.



3. Let's try that again, but with a coordinate plane.

Draw the image of the triangle after a reflection over the y-axis.

Response shown on image.



4. Let's look at some responses to the previous two problems.

Discuss: What do you notice about the two sets of reflections?

Responses vary.

- Without a grid, the transformed triangles are much more spread out.
- With a grid, the transformed triangles are much more aligned and accurate.
- The grid makes it easier to place each point at its precise location.

Accelerated 7 | Unit 1 Lesson 3 18 Activity 1

Pairs | 10 min

Activity 2 Coordinate Patterns, Part 1

Purpose: Students generalize the patterns between pre-image and image coordinates after a reflection. **(MTR.4.1)**

Short on time? Consider omitting Problem 6.

1 Launch

Consider sharing that the purpose of this activity is to notice patterns and generalize how coordinates change when a figure is reflected.

To surface prior knowledge, consider asking students what they know about the y-axis.

2 Monitor

Encourage students to seek feedback from classmates on their work and use it to revise their work.

D Differentiation *(Problems 5-6)*

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Encourage them to draw the points first, then identify the coordinates.
Count by ones as they move points along the grid to the other side of the y-axis.	**Support** Consider asking, "How can the coordinates help us find the right place for each point on the other side of the y-axis?"
Are not precise in their description of which coordinates change and how.	**Support** Consider asking, "Where on the graph should the image be? What sign would the coordinates be in that quadrant?"

Listen for patterns students notice in how coordinates change when a figure is reflected.

ELL English Language Learners Provide a word bank to support students in expressing their ideas with words such as: x- and y-*coordinate*, *reflection*, *pre-image*, *opposite*, *negative*, *sign*, etc.

3 Connect

Invite students to share the patterns they noticed, including ones similar to the Differentiation table above.

Consider asking:

- "What generalizations can you make based on the examples you worked on?"
- "Why does the x-coordinate change for a reflection over the y-axis?"
- "What would happen if the figure was reflected over the x-axis?"

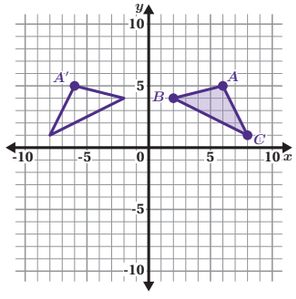
Activity 2

Name: _____ Date: _____ Period: _____

Coordinate Patterns, Part 1

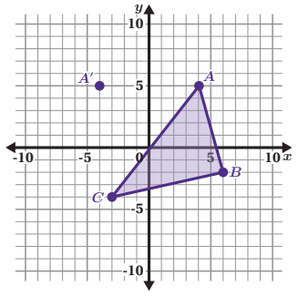
5. Identify the coordinates of each point after a reflection over the y-axis.

Pre-Image Coordinates	Image Coordinates
(6, 5)	(-6, 5)
(2, 4)	(-2, 4)
(8, 1)	(-8, 1)



6. Determine the coordinates of each point after a reflection over the y-axis.

Pre-Image Coordinates	Image Coordinates
(4, 5)	(-4, 5)
(6, -2)	(-6, -2)
(-3, -4)	(3, -4)



7. Take a look at your tables from Problems 5 and 6. Those points show a reflection over the y-axis.

a. **Discuss:** What patterns do you see between the pre-image coordinates and the image coordinates?

Responses vary. I noticed that the sign of the x-coordinate changed, but the sign of the y-coordinate stayed the same.

b. Complete the table for the same transformation.

Pre-Image Coordinates	Image Coordinates
(3, 1)	(-3, 1)
(x, y)	$(-x, y)$

Accelerated 7 | Unit 1 Lesson 3 19 Activity 2

Key Takeaway: When a figure is reflected over the y-axis, the x-coordinate changes sign. When a figure is reflected over the x-axis, the y-coordinate changes sign.

Pairs | 12 min

Activity 3 Coordinate Patterns, Part 2

Purpose: Students use the Decide and Defend routine to examine the differences between translations and reflections. They then generalize the patterns between pre-image and image coordinates after a translation. **(MTR.3.1, MTR.4.1)**

1 Launch

Invite students to choose their response and explain their thinking.

Use the Decide and Defend routine to support students in making arguments and critiquing the reasoning of others. **(MTR.4.1)**

2 Monitor

Invite students to look for patterns and encourage them to use feedback from classmates to revise their work.

Select and sequence students' patterns to share during the Connect.

D Differentiation *(Problem 9)*

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Encourage them to draw the image first, then look for the coordinates.
Think there is not a pattern or haven't found one yet.	**Support** Invite them to think about how they think the x- or y-coordinate should change based on the given transformation.
Identify the coordinates by the number of units translated in each direction.	**Strengthen** Consider asking, "How would the coordinates change for a translation 4 units left?"
Fill in the table precisely.	**Stretch** Consider asking, "How would the table change if the translation were left instead of right ?"

A Accessibility: Conceptual Processing Invite students to describe their thinking in the previous problem and apply this same strategy to this problem to encourage abstract thinking.

3 Connect

To surface the Key Takeaway, consider asking:

- "What pattern(s) do you see in the table?"
- "Do you think this pattern will always be true?"

Activity 3

Name: _____ Date: _____ Period: _____

Coordinate Patterns, Part 2

8. Amari says that figure $A'B'C'D'$ is the image of figure $ABCD$ after a reflection over the x-axis.

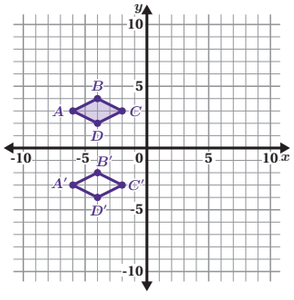
Is Amari's claim correct? Circle one.

Yes (No) I'm not sure

Explain your thinking.

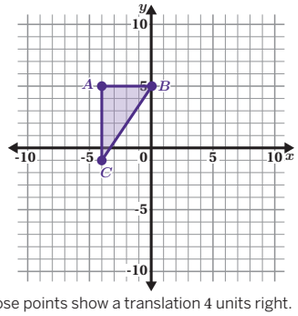
Explanations vary.

- The corresponding points don't align.
- It looks the same, but I think if you try to reflect point D over the x-axis, it would land where point B' is right now.
- This is a translation, not a reflection.



9. Determine the coordinates of each point after a translation 4 units right.

Pre-Image Coordinates	Image Coordinates
(-4, 5)	(0, 5)
(0, 5)	(4, 5)
(-4, -1)	(0, -1)



10. Take a look at your table from Problem 9. Those points show a translation 4 units right.

a. **Discuss:** What patterns do you see between the pre-image coordinates and the image coordinates?

Responses vary. I notice that the y-coordinate stayed the same and the x-coordinate increased by 4.

b. Complete the table for the same transformation.

Pre-Image Coordinates	Image Coordinates
(3, 1)	(7, 1)
(x, y)	$(x + 4, y)$

Accelerated 7 | Unit 1 Lesson 3 20 Activity 3

Key Takeaway: When a point is translated left or right, the x-coordinate increases or decreases by the number of units. When a point is translated up or down, the y-coordinate increases or decreases by the number of units.

Whole Class | 5 min

Synthesis

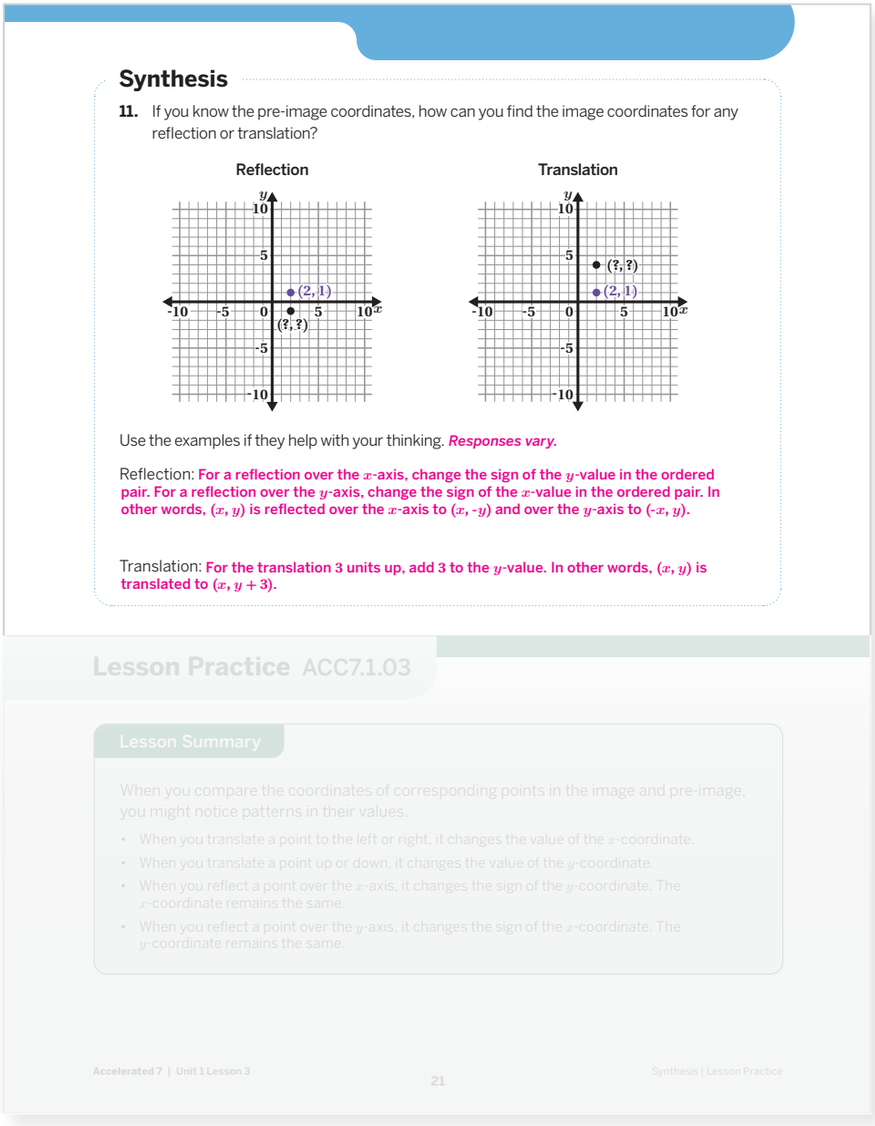
Purpose: Students synthesize their understanding of how to determine the image coordinates for any reflection or translation.

Invite students to respond independently, and then share their thinking with a partner.

To strengthen understanding of both concepts, invite students to switch between practicing reflection and translation examples.

Collect and share multiple student responses for both a reflection and a translation.

MLR **MLR1: Stronger and Clearer Each Time**
Consider using this routine to support students in developing precise language. Invite students to meet with a partner, give feedback on each other's drafts, then use the feedback to write a second clearer draft. See the Routine Facilitation Guide for more information.



Synthesis

11. If you know the pre-image coordinates, how can you find the image coordinates for any reflection or translation?

Reflection

Translation

Use the examples if they help with your thinking. ***Responses vary.***

Reflection: **For a reflection over the x-axis, change the sign of the y-value in the ordered pair. For a reflection over the y-axis, change the sign of the x-value in the ordered pair. In other words, (x, y) is reflected over the x-axis to $(x, -y)$ and over the y-axis to $(-x, y)$.**

Translation: **For the translation 3 units up, add 3 to the y-value. In other words, (x, y) is translated to $(x, y + 3)$.**

Lesson Practice ACC7.1.03

Lesson Summary

When you compare the coordinates of corresponding points in the image and pre-image, you might notice patterns in their values.

- When you translate a point to the left or right, it changes the value of the x-coordinate.
- When you translate a point up or down, it changes the value of the y-coordinate.
- When you reflect a point over the x-axis, it changes the sign of the y-coordinate. The x-coordinate remains the same.
- When you reflect a point over the y-axis, it changes the sign of the x-coordinate. The y-coordinate remains the same.

Accelerated 7 | Unit 1 Lesson 3 21 Synthesis | Lesson Practice

Lesson Takeaway: Reflections over the axes and translations have patterns in the coordinates we can use to perform transformations accurately.

Independent | 5 min

Show What You Know

Purpose: Students demonstrate their understanding by determining which transformation was performed given the pre-image and image coordinates.

Today's Goals

1. **Goal:** Determine the coordinates of a transformed image after a translation or reflection.
2. **Language Goal:** Explain how a translation or reflection impacts the coordinates of a transformed image. **(Speaking, Listening, Reading, and Writing)**

Show What You Know (Assessment Resources)

Name: _____ Date: _____ Period: _____

Show What You Know 1.03

Here are the coordinates of the pre-image and image of triangle CAT. Which transformation moves the pre-image onto the image?

Pre-Image Coordinates	Image Coordinates
C (5, 2)	C' (5, -2)
A (-1, 8)	A' (-1, -8)
T (4, 0)	T' (4, 0)

A. Reflection over the x-axis
B. Reflection over the y-axis
C. Translation 4 units down
D. None of the above

Explain your thinking.

Explanations vary. When reflecting over the x-axis, the sign of the y-coordinate changes.

D Differentiation Use after Lesson 3

Support

Provide targeted intervention for students by using these resources.

If students need support understanding coordinate planes:

» **Respond to Student Thinking:**

- Use the ***Plotting Points on the Coordinate Plane*** **Mini-Lesson**.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 1 Extension Activities**.

Professional Learning

Reflect on your facilitation of the **Decide and Defend** routine in this lesson. How did students defend their argument? What about how you facilitated the routine supported students in constructing an argument?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

11. If you know the pre-image coordinates, how can you find the image coordinates for any reflection or translation?

Reflection

Translation

Use the examples if they help with your thinking. *Responses vary.*

Reflection: For a reflection over the x-axis, change the sign of the y-value in the ordered pair. For a reflection over the y-axis, change the sign of the x-value in the ordered pair. In other words, (x, y) is reflected over the x-axis to $(x, -y)$ and over the y-axis to $(-x, y)$.

Translation: For the translation 3 units up, add 3 to the y-value. In other words, (x, y) is translated to $(x, y + 3)$.

Lesson Practice ACC7.1.03

Lesson Summary

When you compare the coordinates of corresponding points in the image and pre-image, you might notice patterns in their values.

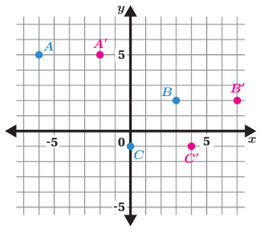
- When you translate a point to the left or right, it changes the value of the x-coordinate.
- When you translate a point up or down, it changes the value of the y-coordinate.
- When you reflect a point over the x-axis, it changes the sign of the y-coordinate. The x-coordinate remains the same.
- When you reflect a point over the y-axis, it changes the sign of the x-coordinate. The y-coordinate remains the same.

Lesson Practice
ACC7.1.03

Name: _____ Date: _____ Period: _____

1. Plot the location of points A, B, and C after a translation 4 units to the right. Label the points A', B', and C'. Then write the coordinates in the table.

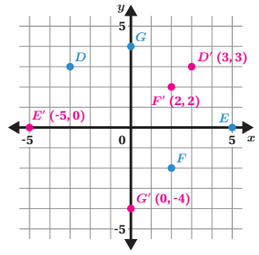
Point	Coordinates
A'	(-2, 5)
B'	(7, 2)
C'	(4, -1)



Problems 2–3: Points D, E, F, and G are plotted on the coordinate plane.

2. Plot the coordinates of points D and E after a reflection over the y-axis. Label the images D' and E'. Include the coordinates.

Responses shown on graph.



3. Plot the coordinates of points F and G after a reflection over the x-axis. Label the images F' and G'. Include the coordinates.

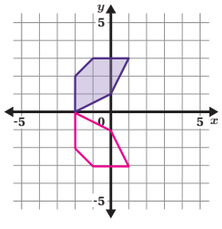
Responses shown on graph.

Lesson Practice
ACC7.1.03

Name: _____ Date: _____ Period: _____

4. Determine the coordinates of each point after the shaded pre-image is reflected over the x-axis. Use the graph if it helps with your thinking.

Pre-Image Coordinates	Image Coordinates
(-2, 0)	(-2, 0)
(-2, 2)	(-2, -2)
(-1, 3)	(-1, -3)
(1, 3)	(1, -3)
(0, 1)	(0, -1)



FAST Practice

5. Point $H(5, 3)$ is translated 4 units down to get point H'. Which of the following are the coordinates of point H'?

A. (1, -1) **B.** (9, 3) **C.** (1, 3) **D.** (5, -1)

Spiral Review

Problems 6–11: Compare each of these values using the symbols <, =, or >.

6. -11 ___>___ -15

7. 8.01 ___>___ .8

8. $\frac{2}{3}$ ___>___ $\frac{3}{2}$

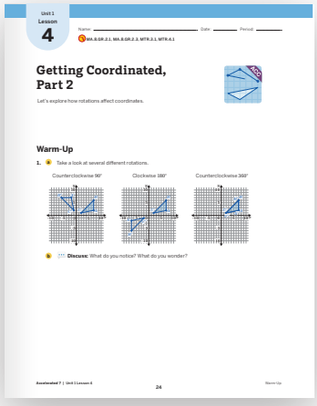
9. -8.01 ___<___ -8

10. -(-6) ___=___ 6

11. -2.5 ___=___ $\frac{10}{4}$

Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1–3	1	MA.8.GR.2.1, MA.8.GR.2.3
	4	2	MA.8.GR.2.1, MA.8.GR.2.3
FAST Practice	5	2	MA.8.GR.2.3
Spiral Review			
	6–11	1	MA.6.NSO.1.1


Unit 1
Lesson
4

Getting Coordinated, Part 2

Let's explore how rotations affect coordinates.

Warm-Up

1. Take a look at several different rotations.

Counterclockwise 90° Clockwise 180° Counterclockwise 360°

Discuss: What do you notice? What do you wonder?

Accelerated 7 | Unit 1 Lesson 4 24

Getting Coordinated, Part 2

Coordinates of Transformations

Let's explore how rotations affect coordinates.

Learning Outcomes

Today's Goals

1. **Goal:** Determine the coordinates of a transformed figure after a translation, reflection, or rotation.
2. **Language Goal:** Explain how to rotate points on the coordinate plane using vocabulary from this unit. **(Writing, Speaking, and Listening)**

Students continue to investigate the effects of transformations on coordinates on a plane. They describe how rotations affect the coordinates of an image and create a challenge to develop fluency in how different transformations affect the coordinates of a pre-image. **(MTR.3.1)**

Coherence

Prior Learning

In Lesson 3, students analyzed how the coordinates of an image change after a reflection or translation on the coordinate plane.

Future Learning

In Lessons 5–7, students will explore whether two figures are congruent using rigid transformations.

Connections

- Students continue to expand their use of the coordinate plane to rotate figures. (mathematical)

FL B.E.S.T. Math Benchmarks

Addressing

MA.8.GR.2.1
Given a preimage and image generated by a single transformation, identify the transformation that describes the relationship.

MA.8.GR.2.3
Describe and apply the effect of a single transformation on two-dimensional figures using coordinates and the coordinate plane.

Mathematical Thinking and Reasoning Standards: MTR.3.1, MTR.4.1

ELA Expectations: ELA.K12.EE.1.1, ELA.K12.EE.3.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.6.GR.1.1	**MA.912.F.2** **MA.912.GR.2.4**

Vocabulary

Review Vocabulary

origin

Lesson at a Glance

~ 45 min

MA.8.GR.2.1, MA.8.GR.2.3

Warm-Up

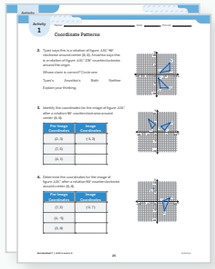
Pairs | 5 min

Students explore multiple rotations and discuss using the Notice and Wonder routine.

Activity 1

Pairs | 20 min

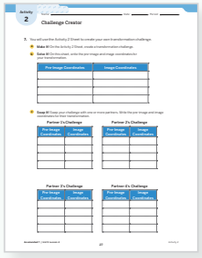
Students examine and generalize how the coordinates of a figure change after rotations of 90, 180, and 270 degrees. **(MTR.4.1)**



Activity 2

Pairs | 10 min

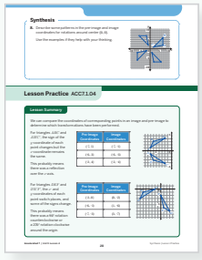
Students practice performing transformations with coordinates by designing a challenge and solving their and others' challenges. **(MTR.3.1)**



Synthesis

Whole Class | 5 min

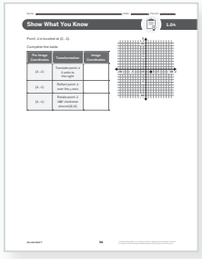
Students synthesize their understanding of how rotations affect the coordinates of an image.



Show What You Know

Independent | 5 min

Students demonstrate their understanding by determining the coordinates of an image after a translation, reflection, and rotation.



Prep Checklist

Prepare the Activity 2 Sheets.

This lesson includes:

- Student Edition
- Show What You Know
- Activity 2 Sheet (one per student)
  - Tracing paper (as needed)

Pairs | 5 min

Warm-Up

Purpose: Students explore multiple rotations and discuss using the Notice and Wonder routine.

1 Launch

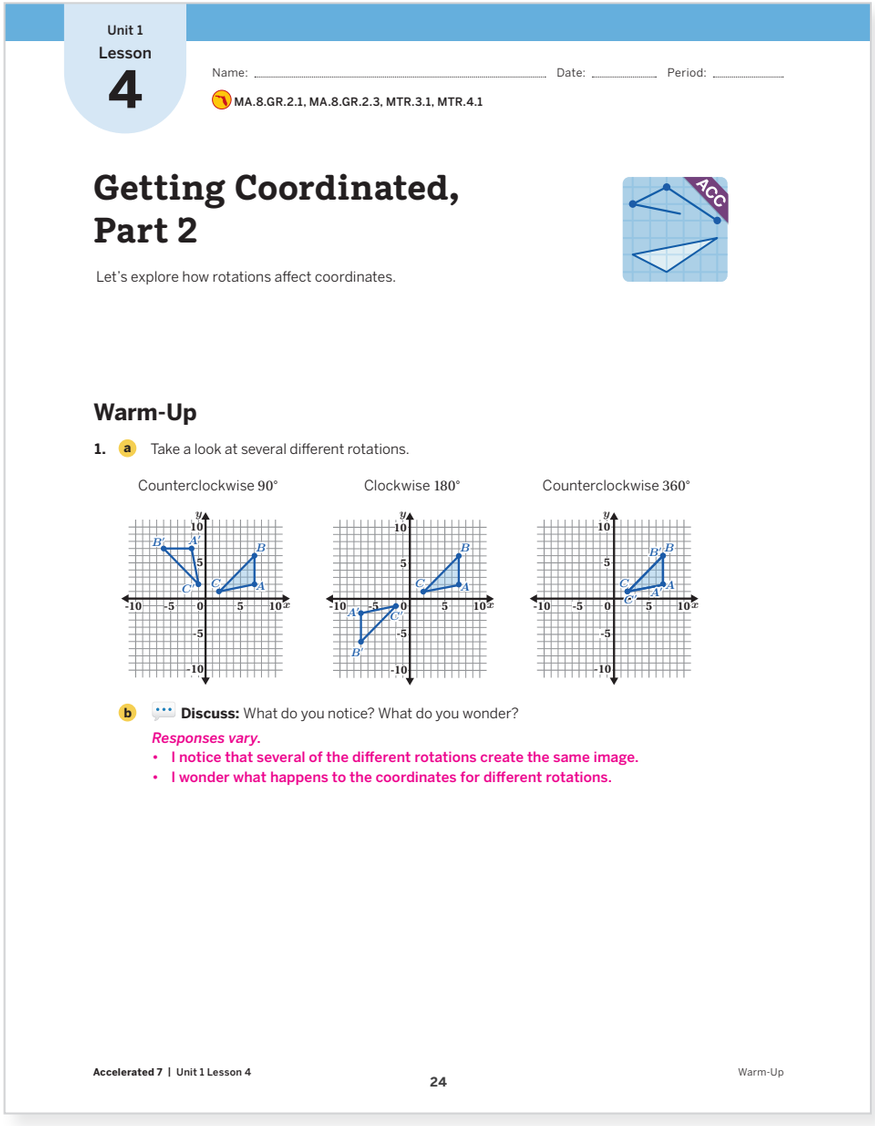
To encourage playfulness, invite students to explore many rotations in both directions and all degree measures.

Use the Notice and Wonder routine to promote curiosity and help students make sense of rotations. **(MTR.3.1)**

2 Connect

Invite students to share what they noticed and wondered from their explorations.

Note: If it doesn't come up naturally, consider asking, "How do the angle measures relate to a circle?"



Unit 1
Lesson
4

Name: _____ Date: _____ Period: _____

MA.8.GR.2.1, MA.8.GR.2.3, MTR.3.1, MTR.4.1

Getting Coordinated, Part 2

Let's explore how rotations affect coordinates.

ACC

Warm-Up

1. a Take a look at several different rotations.

Counterclockwise 90° | Clockwise 180° | Counterclockwise 360°

b **Discuss:** What do you notice? What do you wonder?

Responses vary.

- I notice that several of the different rotations create the same image.
- I wonder what happens to the coordinates for different rotations.

Accelerated 7 | Unit 1 Lesson 4 24 Warm-Up

Pairs | 20 min

Activity 1 Coordinate Patterns

Purpose: Students examine and generalize how the coordinates of a figure change after rotations of 90, 180, and 270 degrees. **(MTR.4.1)**

Short on time? Consider omitting Problem 4.

1 Launch

Invite students to use their noticings from the Warm-Up to decide whose claim is correct.

Pause to discuss student responses.

2 Monitor

Invite students to explore Problems 3–6.

Look for a variety of student strategies and offer support as needed.

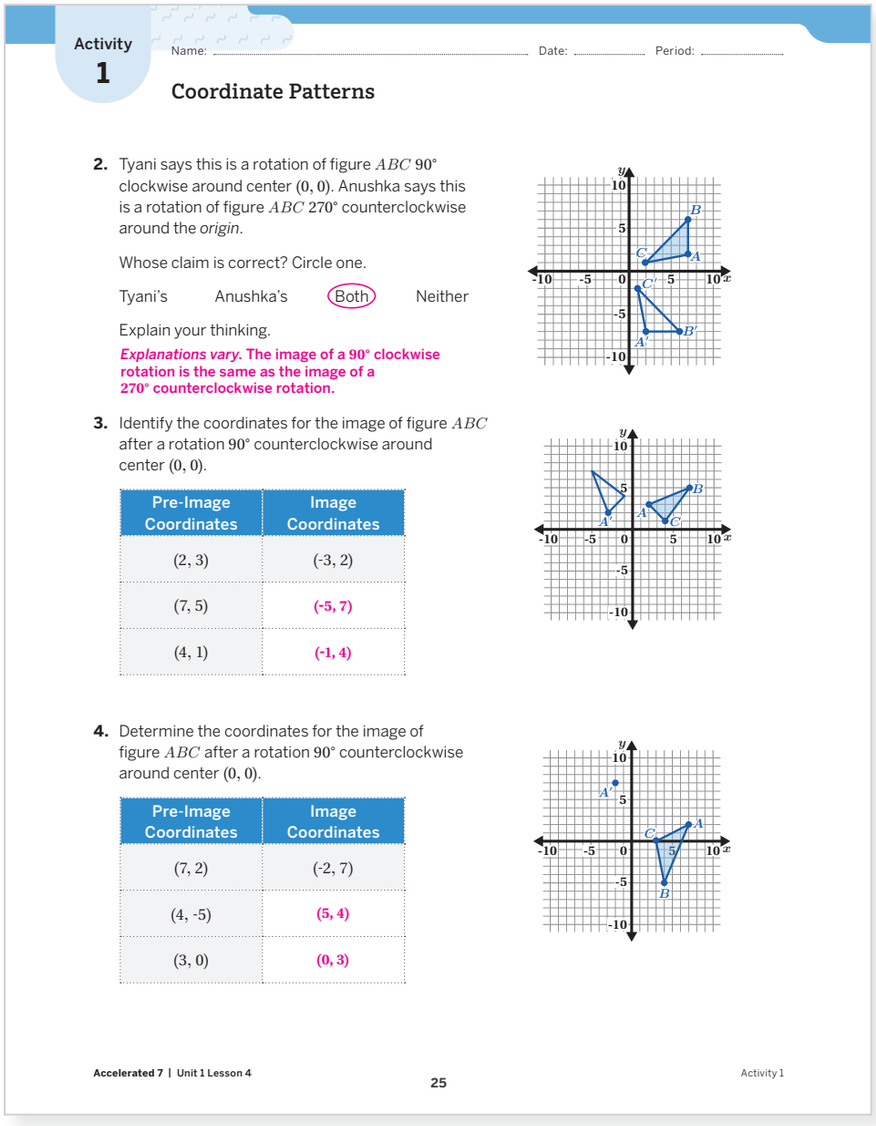
D Differentiation

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Consider asking, "What pattern do you notice in the coordinates from the previous problem?"
Notice that there is a pattern in changing the order and sign of the coordinates but struggle to apply it accurately or consistently.	**Support** Consider asking, "What quadrant is the pre-image point in? What does that tell us about its signs? What quadrant will the image point be in? What does that tell us about its signs?"
Accurately apply the pattern.	**Strengthen** Consider asking, "Why does this strategy work? Would it work for rotations of other degree measures?"
Would like to explore patterns in coordinates after a rotation further.	**Stretch** Consider asking, "What pattern, if any, would there be if the figure were rotated around a point other than the origin?"

A Accessibility: Conceptual Processing Invite students to revisit their work from the previous problem to support students in generalizing.

A Accessibility: Visual-Spatial Processing As students interpret visual representations, provide tracing paper that students can use to rotate the points.

Activity 1 continued >



Activity 1

Name: _____ Date: _____ Period: _____

Coordinate Patterns

2. Tyani says this is a rotation of figure ABC 90° clockwise around center $(0, 0)$. Anushka says this is a rotation of figure ABC 270° counterclockwise around the *origin*.

Whose claim is correct? Circle one.

Tyani's Anushka's (Both) Neither

Explain your thinking.

Explanations vary. The image of a 90° clockwise rotation is the same as the image of a 270° counterclockwise rotation.

3. Identify the coordinates for the image of figure ABC after a rotation 90° counterclockwise around center $(0, 0)$.

Pre-Image Coordinates	Image Coordinates
(2, 3)	(-3, 2)
(7, 5)	(-5, 7)
(4, 1)	(-1, 4)

4. Determine the coordinates for the image of figure ABC after a rotation 90° counterclockwise around center $(0, 0)$.

Pre-Image Coordinates	Image Coordinates
(7, 2)	(-2, 7)
(4, -5)	(5, 4)
(3, 0)	(0, 3)

Accelerated 7 | Unit 1 Lesson 4 25 Activity 1

Pairs | 20 min

Activity 1 Coordinate Patterns (continued)

Purpose: Students examine and generalize how the coordinates of a figure change after rotations of 90, 180, and 270 degrees. **(MTR.4.1)**

2 Monitor

Invite students to make arguments and justify their reasoning. **(MTR.4.1)**

ELL **English Language Learners** Provide sentence frames to help students explain their reasoning (e.g., "I agree or disagree with Binta/Chloe because _____.").

Look and listen for students' strategies, particularly students who are showing repeated reasoning. **(MTR.4.1)**

D **Differentiation** *(Problem 6)*

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Encourage students to use tracing paper to draw the rotation.
Use the same pattern as a 90-degree rotation.	**Support** Consider asking, "How is this rotation alike and different from the 90 degree rotation?"
Change one coordinate sign but not both.	**Support** Consider asking, "What transformation have we seen with that pattern? How is this one different?"

A **Accessibility: Visual-Spatial Processing** As students interpret visual representations, provide tracing paper that students can use to rotate the points.

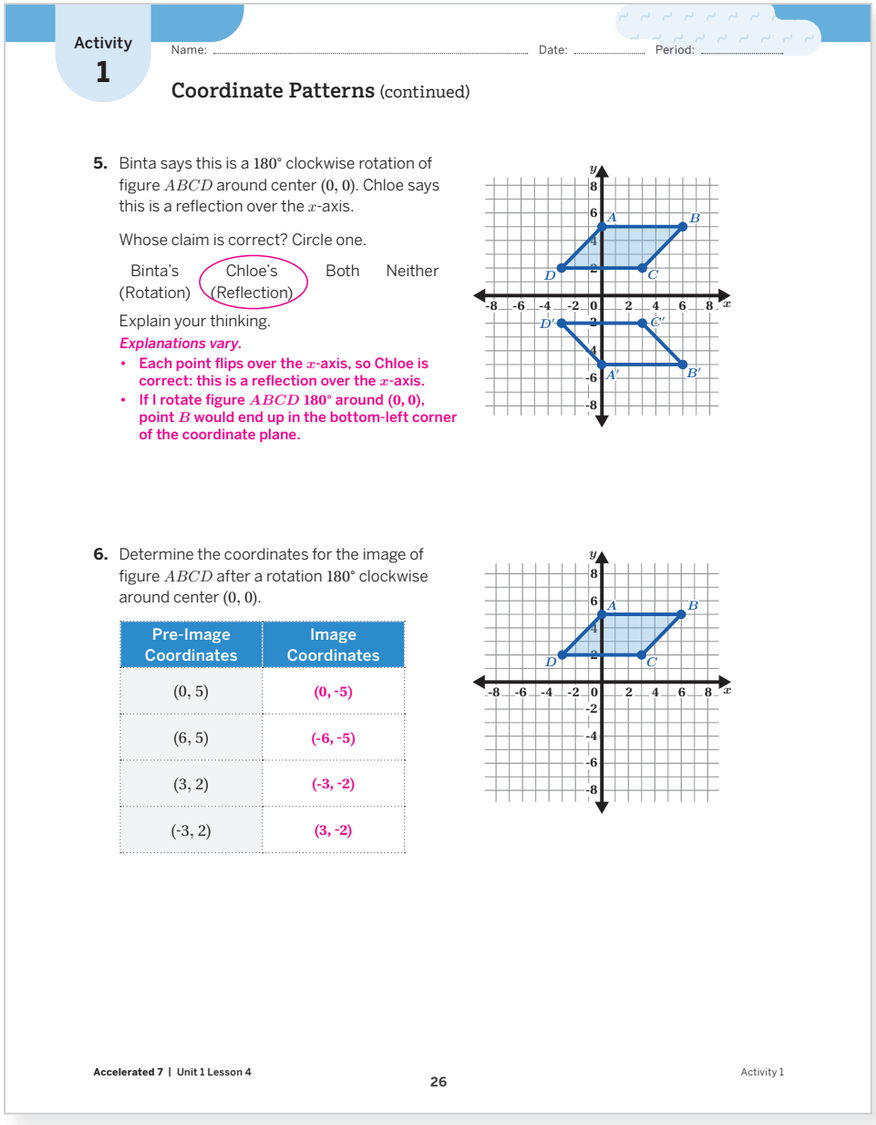
3 Connect

Invite students to share their strategies for determining the coordinates of the image after the rotation. Students should recognize and use these patterns but do not need to memorize them.

To surface the Key Takeaway, consider asking:

- "How is rotating by 180 degrees alike and different from a 90 degree rotation?"
- "Will this strategy work for any 180-degree rotation?"
- "What would change if it were a 270 or 360-degree rotation?"

Note: If time allows, use the **Decide and Defend** routine to discuss responses from Problem 5.



Activity 1

Name: Date: Period:

Coordinate Patterns (continued)

5. Binta says this is a 180° clockwise rotation of figure $ABCD$ around center $(0, 0)$. Chloe says this is a reflection over the x-axis.

Whose claim is correct? Circle one.

Binta's (Rotation) Chloe's (Reflection) Both Neither

Explain your thinking.

Explanations vary.

- Each point flips over the x-axis, so Chloe is correct: this is a reflection over the x-axis.
- If I rotate figure $ABCD$ 180° around $(0, 0)$, point B would end up in the bottom-left corner of the coordinate plane.

6. Determine the coordinates for the image of figure $ABCD$ after a rotation 180° clockwise around $(0, 0)$.

Pre-Image Coordinates	Image Coordinates
(0, 5)	(0, -5)
(6, 5)	(-6, -5)
(3, 2)	(-3, -2)
(-3, 2)	(3, -2)

Accelerated 7 | Unit 1 Lesson 4 26 Activity 1

Key Takeaway: We can use patterns in how coordinates change for rotations to help us precisely rotate figures around $(0, 0)$ by 90, 180, 270, or 360 degrees. For example, a 180-degree rotation around the *origin* changes the sign of the x- and y-coordinates.

Pairs | 10 min

Activity 2 Challenge Creator

Purpose: Students practice performing transformations with coordinates by designing a challenge and solving their and others' challenges. **(MTR.3.1)**

1 Launch

Consider sharing that students will create their own transformation challenge. Distribute the Activity 2 Sheet so they can create their transformation challenge.

To encourage creativity, invite students to make a challenge they think will be different from ones their classmates create.

2 Monitor

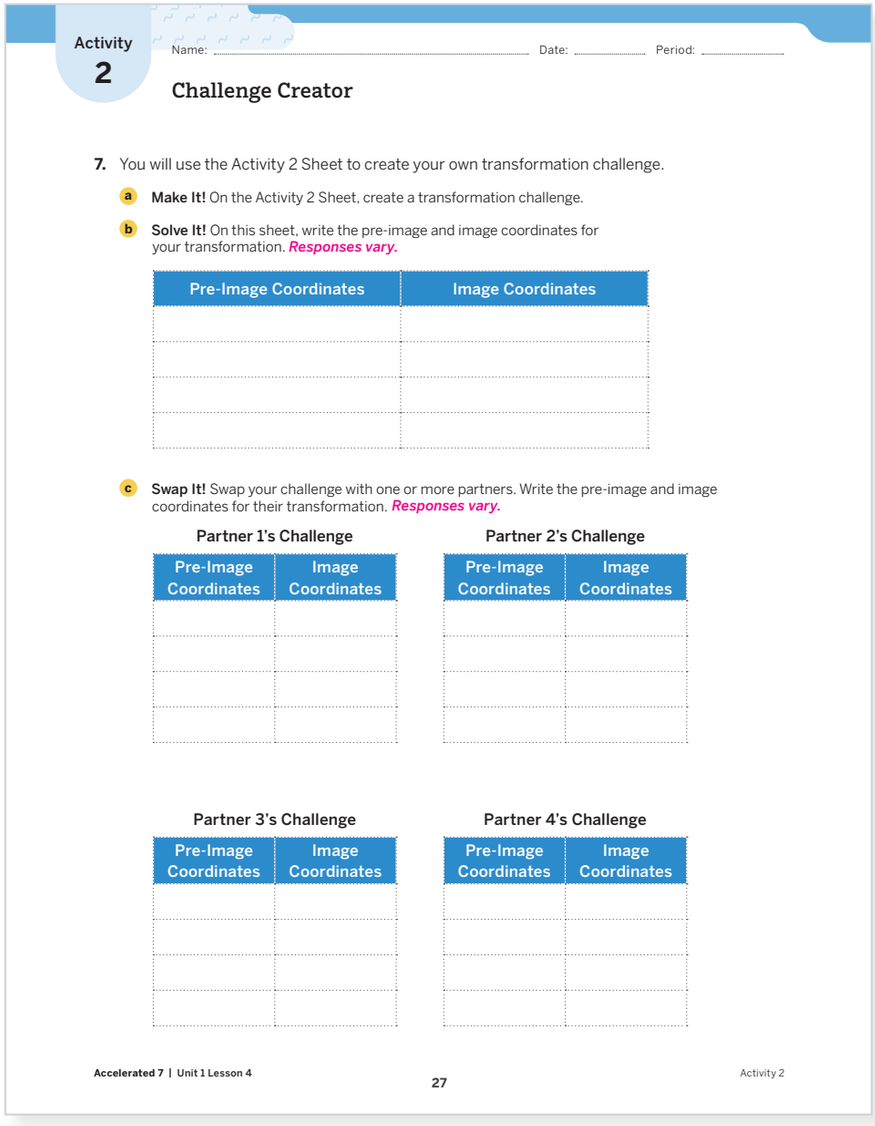
Listen for strategies students use when discussing how to solve the problems, such as using their knowledge of how each transformation affects the image coordinates. **(MTR.3.1)**

Collect several creative challenges and solutions to share during the Connect.

A **Accessibility: Conceptual Processing** Invite students to revisit their work from the previous lesson to help apply concepts to new situations.

3 Connect

Share the transformation challenges. Invite students to share/discuss how the coordinates changed as a result of the transformation.



Activity 2

Name: _____ Date: _____ Period: _____

Challenge Creator

7. You will use the Activity 2 Sheet to create your own transformation challenge.

a **Make It!** On the Activity 2 Sheet, create a transformation challenge.

b **Solve It!** On this sheet, write the pre-image and image coordinates for your transformation. ***Responses vary.***

Pre-Image Coordinates	Image Coordinates

c **Swap It!** Swap your challenge with one or more partners. Write the pre-image and image coordinates for their transformation. ***Responses vary.***

Partner 1's Challenge

Pre-Image Coordinates	Image Coordinates

Partner 2's Challenge

Pre-Image Coordinates	Image Coordinates

Partner 3's Challenge

Pre-Image Coordinates	Image Coordinates

Partner 4's Challenge

Pre-Image Coordinates	Image Coordinates

Accelerated 7 | Unit 1 Lesson 4 27 Activity 2

Whole Class | 5 min

Synthesis

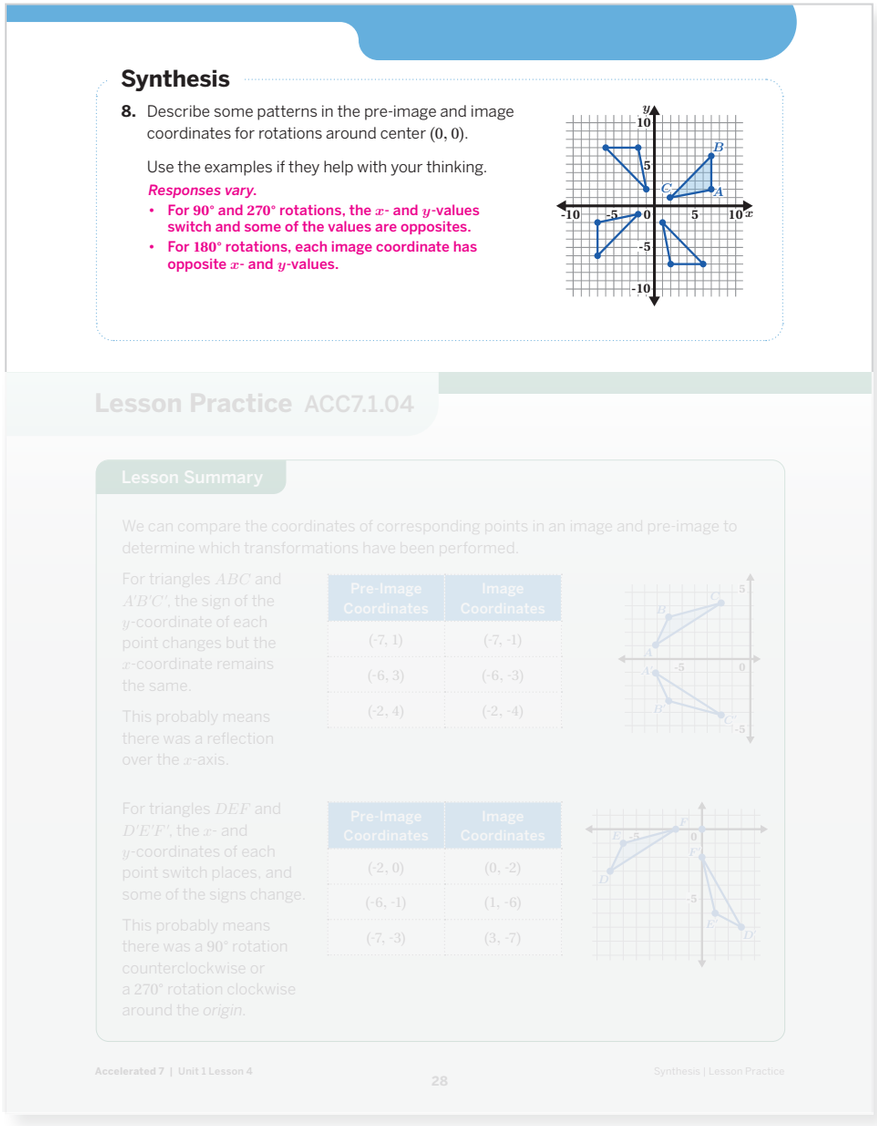
Purpose: Students synthesize their understanding of how rotations affect the coordinates of an image.

Invite students to respond independently, and then share their thinking with a partner.

Collect and share a variety of ideas, such as:

- Relating the positive and negative coordinates to specific quadrants.
- Switching the x- and y-coordinates.

Note: If time allows, invite students to share with a partner what makes sense about each strategy and what questions they still have.



Synthesis

8. Describe some patterns in the pre-image and image coordinates for rotations around center $(0, 0)$.

Use the examples if they help with your thinking.

Responses vary.

- For 90° and 270° rotations, the x- and y-values switch and some of the values are opposites.
- For 180° rotations, each image coordinate has opposite x- and y-values.

Lesson Practice ACC7.1.04

Lesson Summary

We can compare the coordinates of corresponding points in an image and pre-image to determine which transformations have been performed.

For triangles ABC and $A'B'C'$, the sign of the y-coordinate of each point changes but the x-coordinate remains the same.

This probably means there was a reflection over the x-axis.

Pre-Image Coordinates	Image Coordinates
(-7, 1)	(-7, -1)
(-6, 3)	(-6, -3)
(-2, 4)	(-2, -4)

For triangles DEF and $D'E'F'$, the x- and y-coordinates of each point switch places, and some of the signs change.

This probably means there was a 90° rotation counterclockwise or a 270° rotation clockwise around the *origin*.

Pre-Image Coordinates	Image Coordinates
(-2, 0)	(0, -2)
(-6, -1)	(1, -6)
(-7, -3)	(3, -7)

Accelerated 7 | Unit 1 Lesson 4 28 Synthesis | Lesson Practice

Lesson Takeaway: We can use patterns in how rotations affect coordinates to help us precisely rotate figures around $(0, 0)$ by 90, 180, 270, or 360 degrees. For a 90- or 270-degree rotation, the x- and y-coordinates switch and one changes sign. For a 180-degree rotation, both coordinates change signs. For a 360-degree rotation, both coordinates stay the same.

Independent | 5 min

Show What You Know

Purpose: Students demonstrate their understanding by determining the coordinates of an image after a translation, reflection, and rotation.

Today's Goals

1. **Goal:** Determine the coordinates of a transformed figure after a translation, reflection, or rotation.
2. **Language Goal:** Explain how to rotate points on the coordinate plane using vocabulary from this unit. **(Writing, Speaking, and Listening)**

Show What You Know (Assessment Resources)

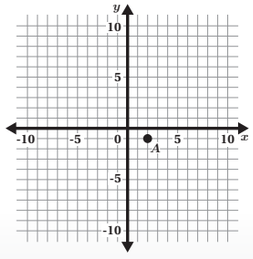
Name: _____ Date: _____ Period: _____

Show What You Know 1.04

Point A is located at (2, -1).

Complete the table.

Pre-Image Coordinates	Transformation	Image Coordinates
(2, -1)	Translate point A 2 units to the right	(4, -1)
(2, -1)	Reflect point A over the y-axis	(-2, -1)
(2, -1)	Rotate point A 180° clockwise around (0, 0)	(-2, 1)



D Differentiation Use after Lesson 4

Support

Provide targeted intervention for students by using these resources.

If students need support understanding how to determine the coordinates of an image after a transformation:

» **Respond to Student Thinking:**

- Use the ***Determining Coordinates After a Rotation*** **Mini-Lesson**.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 1 Extension Activities**.

Professional Learning

How did students' understanding of transformations on the coordinate plane grow between last lesson and this lesson? What opportunities were there to build on or solidify their previous knowledge? What opportunities might there be in future lessons and units to continue to build these understandings?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

8. Describe some patterns in the pre-image and image coordinates for rotations around center $(0, 0)$.

Use the examples if they help with your thinking.

Responses vary.

- For 90° and 270° rotations, the x- and y-values switch and some of the values are opposites.
- For 180° rotations, each image coordinate has opposite x- and y-values.

Lesson Practice ACC7.1.04

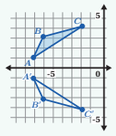
Lesson Summary

We can compare the coordinates of corresponding points in an image and pre-image to determine which transformations have been performed.

For triangles ABC and $A'B'C'$, the sign of the y-coordinate of each point changes but the x-coordinate remains the same.

This probably means there was a reflection over the x-axis.

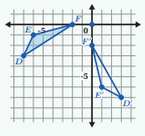
Pre-Image Coordinates	Image Coordinates
(-7, 1)	(-7, -1)
(-6, 3)	(-6, -3)
(-2, 4)	(-2, -4)



For triangles DEF and $D'E'F'$, the x- and y-coordinates of each point switch places, and some of the signs change.

This probably means there was a 90° rotation counterclockwise or a 270° rotation clockwise around the *origin*.

Pre-Image Coordinates	Image Coordinates
(-2, 0)	(0, -2)
(-6, -1)	(1, -6)
(-7, -3)	(3, -7)

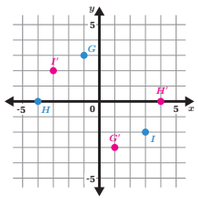


Lesson Practice
ACC7.1.04

Name: _____ Date: _____ Period: _____

1. Plot the location of points G, H, and I after a 180° rotation around the origin. Label the points G', H', and I'. Then write the coordinates in the table.

Point	Coordinates
G'	(1, -3)
H'	(4, 0)
I'	(-3, 2)



2. Point $P(5, 3)$ is rotated 90° counterclockwise around the origin and the image is labeled P'. Which of the following are the coordinates of point P'?

A. (1, -3) **B.** (-3, -5) **C.** (3, -5) **D.** (-3, 5)

3. Here are the pre-image and image coordinates of points on a graph. Describe the transformation.

Responses vary.

- 90° clockwise rotation around the origin
- 270° counterclockwise rotation around the origin

Pre-Image Coordinates	Image Coordinates
(0, 5)	(5, 0)
(-2, 1)	(1, 2)
(4, 3)	(3, -4)
(6, 0)	(0, -6)
(-5, -1)	(-1, 5)

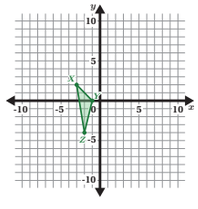
Lesson Practice
ACC7.1.04

Name: _____ Date: _____ Period: _____

FAST Practice

4. Triangle XYZ is rotated 90° clockwise around the origin. What are the coordinates of point X'?

A. (3, 2)
B. (2, 3)
C. (-2, -3)
D. (-3, -2)



Spiral Review

Problems 5–9: Write an expression that has a value of $\frac{3}{5}$ based on the given rule.

Responses vary.

5. An expression that is a sum.

$\frac{1}{5}+\frac{2}{5}$

6. An expression that is a difference.

$1\frac{2}{5}-\frac{4}{5}$

7. An expression that is a product.

$3\cdot\frac{1}{5}$

8. An expression that is a quotient.

$6\div 10$

9. An expression that involves at least two operations.

$2\left(\frac{1}{10}+\frac{2}{10}\right)$

Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1–2	2	MA.8.GR.2.3
	3	2	MA.8.GR.2.1, MA.8.GR.2.3
FAST Practice	4	2	MA.8.GR.2.3
Spiral Review			
	5–9	2	MA.6.NSO.2

Assess and Respond

Sub-Unit Quiz 1

Independent | 45 min

Facilitation: Assign the Sub-Unit Quiz to learn about your students' understanding of the concepts and skills so far in this unit. If time allows, consider inviting students to complete one or more of the tasks in the Unit Synthesis and Reflection.

Item Analysis

Problem(s)	Concept or skill	Addressed in	DOK	FL B.E.S.T. Math Benchmarks
1	Recognizing properties of rotations	Lesson 1	1	MA.8.GR.2.1
2	Identifying the transformation that maps one figure onto another in a coordinate plane	Lesson 2	2	MA.8.GR.2.3 MTR.2.1
3	Identifying the transformation that maps one figure onto another in a coordinate plane	Lesson 2	1	MA.8.GR.2.3
4	Using the structure of a coordinate plane and a transformation to determine the coordinates of a point after a transformation	Lesson 3	2	MA.8.GR.2.3 MTR.5.1
5	Drawing the image of a figure after a rotation	Lesson 4	2	MA.8.GR.2.3
6	Using the structure of a coordinate plane to describe a transformation that maps one figure onto another	Lesson 4	2	MA.8.GR.2.3 MTR.2.1 MTR.5.1

Assessment and Lesson Resources

- Student Assessments
- Assessment Rubrics with Answers
- Show What You Knows and Answer Keys

Intervention and Extension Resources

- Mini-Lessons
- Extensions

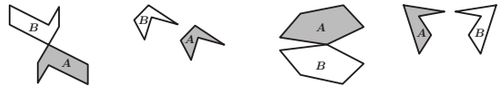
Name: ..... Date: ..... Period: .....

Sub-Unit Quiz

Unit 1

1. In which pair of figures can figure A be rotated onto figure B? Circle one.

Pair 1 Pair 2 Pair 3 Pair 4

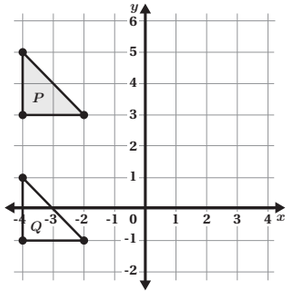


2. Identify the transformation that would move figure P onto figure Q.

A. A reflection

B. A rotation

C. A translation



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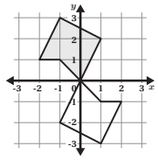
Name: ..... Date: ..... Period: .....

Sub-Unit Quiz (continued)

Unit 1

3. Here are three transformations.

a. Which transformation moves figure E onto figure F?

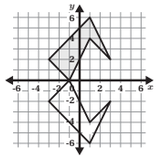


A. Translation

B. Rotation

C. Reflection

b. Which transformation moves figure G onto figure H?

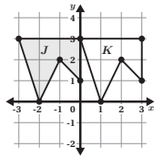


A. Translation

B. Rotation

C. Reflection

c. Which transformation moves figure J onto figure K?

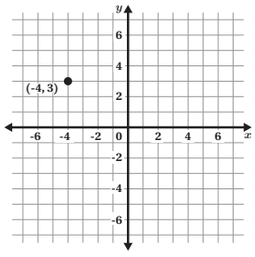


A. Translation

B. Rotation

C. Reflection

4. The point $(-4, 3)$ is shown on the graph. Write the coordinates of $(-4, 3)$ after each transformation.



a. The image of $(-4, 3)$ after a rotation of $180°$ around point $(0, 0)$.
$(4, -3)$

b. The image of $(-4, 3)$ after a translation 2 units to the right.
$(-2, 3)$

c. The image of $(-4, 3)$ after a reflection over the y-axis.
$(4, 3)$

D Differentiation (Sub-Unit Quiz 1)

Note: To **strengthen** and **stretch** students' learning, refer to the differentiation resources suggested throughout this unit and in the Unit Overview.

Sub-Unit Goals	Problem(s)	To respond to student thinking, consider:	
• Identifying and describing translations, reflections, and rotations on and off a grid. • Using the structure of a coordinate plane to describe a transformation that maps one figure onto another **or** to determine the coordinates of a point after a transformation. **(Lessons 1–4)**	1	S **Support** S **Strengthen**	• **Teacher Move:** Consider revisiting Lesson 1 (*Spinning, Flipping, Sliding*)
	2, 3	S **Support** S **Strengthen**	• **Teacher Move:** Consider revisiting Lesson 2 (*Moving Day*)

	4, 5	S **Support**	• **Mini-Lesson:** *Determining Coordinates After a Rotation*
			• **Teacher Move:** Consider revisiting:
			  • Lesson 3 *(Getting Coordinated, Part 1)*
			  • Lesson 4 *(Getting Coordinated, Part 2)*

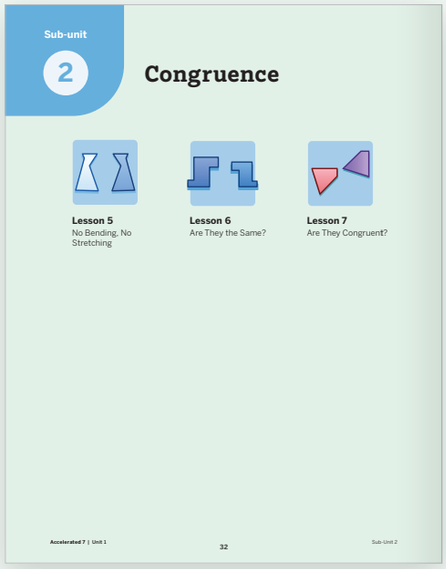
		S **Strengthen**	• **Challenge Creator:** Lesson 4 *(Getting Coordinated, Part 2)*

Congruence

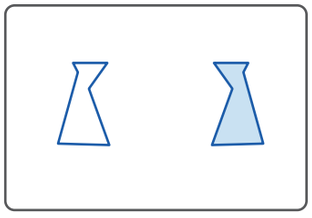
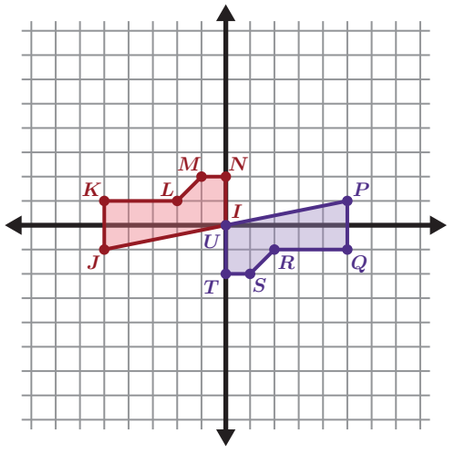
Sub-Unit 2 Goals

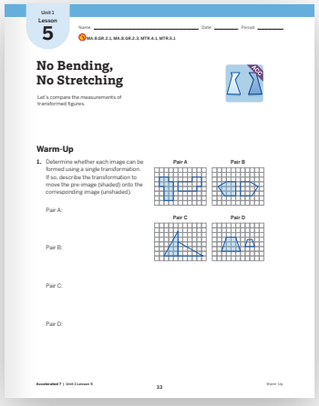
- **Goal:** Determine whether two figures are congruent using rigid transformations.
- **Language Goal:** Justify whether two figures are congruent using language such as *rigid transformations, translation, reflection, rotation, pre-image,* and *image*.
- **Language Goal:** Describe the characteristics of congruent figures.

In this sub-unit, students learn and use the word **congruent** and come to understand that two figures are congruent if one figure can be moved onto the other using a **rigid transformation**. In a future course, students will extend this understanding to prove theorems and shortcuts for determining triangle congruence.



Math That Matters Most

Concept or Skill	For example . . .
Understand that rigid transformations preserve the side lengths and angle measures of a figure.	Devon says that this card shows a rigid transformation. Is her claim correct? *Yes* Explain your thinking. *The corresponding side lengths and angle measurements are the same.* 
Justify whether two figures are congruent using rigid transformations.	Describe a rigid transformation that will convince your classmates that the figures from Pair D are congruent. • *Rotate figure PQRSTU 180° counterclockwise around (0, 0).* • *Transform figure PQRSTU by rotating it around point U moving point P onto point J.* 



No Bending, No Stretching

Rigid Transformations

Let's compare measurements of transformed figures.

Learning Outcomes

Today's Goals

1. **Goal:** Understand that *rigid transformations* preserve the side lengths and angle measures of a figure.
2. **Language Goal:** Justify whether a *rigid transformation* took place using language such as *corresponding side lengths, corresponding angles, pre-image*, and *image*. **(Writing, Speaking, and Listening)**

Students begin to see that translations, reflections, and rotations preserve lengths and angle measures and categorize transformations as *rigid* or *non-rigid*. They use the structure of the grid and geometric tools to measure corresponding sides and angles in a polygon and its image after translations, reflections, or rotations to generalize that these types of transformation (all rigid) do not change side lengths or angle measures of a pre-image. **(MTR.4.1, MTR.5.1)**

Coherence

Prior Learning

In Lessons 3 and 4, students talked about corresponding points on a pre-image and its image after a transformation. This helps build toward understanding corresponding side lengths and angle measures in this lesson.

Future Learning

In Lesson 6, students will learn that they can call two figures *congruent* if one can be moved onto the other using a rigid transformation.

Connections

- Students use a variety of tools and strategies to compare side lengths and angle measures of figures formed by rigid transformations. (mathematical)

FL B.E.S.T. Math Benchmarks

Addressing

MA.8.GR.2.1
Given a preimage and image generated by a single transformation, identify the transformation that describes the relationship.

MA.8.GR.2.3
Describe and apply the effect of a single transformation on two-dimensional figures using coordinates and the coordinate plane.

Mathematical Thinking and Reasoning Standards: MTR.4.1, MTR.5.1

ELA Expectations: ELA.K12.EE.3.1, ELA.K12.EE.4.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.6.GR.1.1	MA.912.F.2 MA.912.GR.2

Vocabulary

New Vocabulary

rigid transformation

Lesson at a Glance

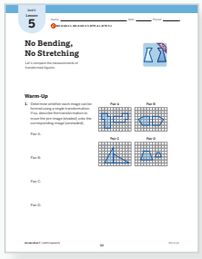
~ 45 min

MA.8.GR.2.1, MA.8.GR.2.3

Warm-Up

Pairs | 5 min

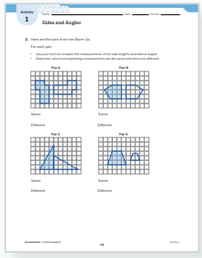
Students use the **Think-Pair-Share** routine to determine whether a transformation can be used to map one figure onto another. These figures will be used again in Activity 1.



Activity 1

Pairs | 20 min

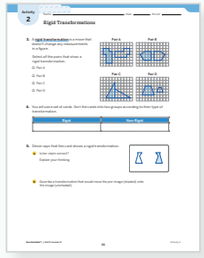
Students use a variety of tools to measure the side lengths and angles of pairs of figures to generalize about these measurements in translations, rotations, and reflections. **(MTR.5.1)**



Activity 2

Pairs | 10 min

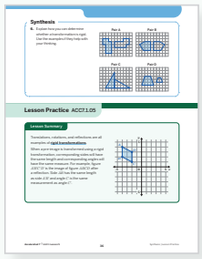
Students learn the term **rigid transformation** and justify whether or not transformations are rigid or non-rigid. **(MTR.4.1)**



Synthesis

Whole Class | 5 min

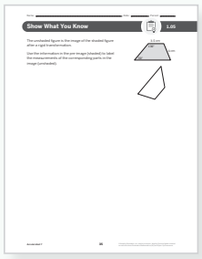
Students synthesize their understanding of rigid transformations as transformations that preserve side lengths and angle measures.
MLR2: Collect and Display



Show What You Know

Independent | 5 min

Students demonstrate their understanding by using measurements on a pre-image to determine corresponding measurements on an image after a rigid transformation.



Prep Checklist

Prepare the Activity 2 Cards.

This lesson includes:

- Student Edition
- Show What You Know
- Activity 2 Cards, one set per pair, pre-cut
- Protractors, Rulers, Tracing paper, Coloring tools, Highlighters (as needed)
- Notecards (as needed)

Pairs | 5 min

Warm-Up

Purpose: Students use the Think-Pair-Share routine to determine whether a transformation can be used to map one figure onto another. These figures will be used again in Activity 1.

1 Launch

Direct students' attention to the four pairs of figures.

To surface prior knowledge, consider asking, "Which transformations have you learned about?"

ELL **English Language Learners** Provide a word bank to support students in expressing their ideas with words such as: *pre-image, image, rotate, reflect, translate, clockwise, counterclockwise, over the ____ line*, etc. It may be helpful to provide full sentences such as, "Rotate the pre-image ____ degrees clockwise/counterclockwise around the point ____." **(Writing and Speaking)**

Look for students who use a variety of precision in their descriptions to highlight during the Connect.

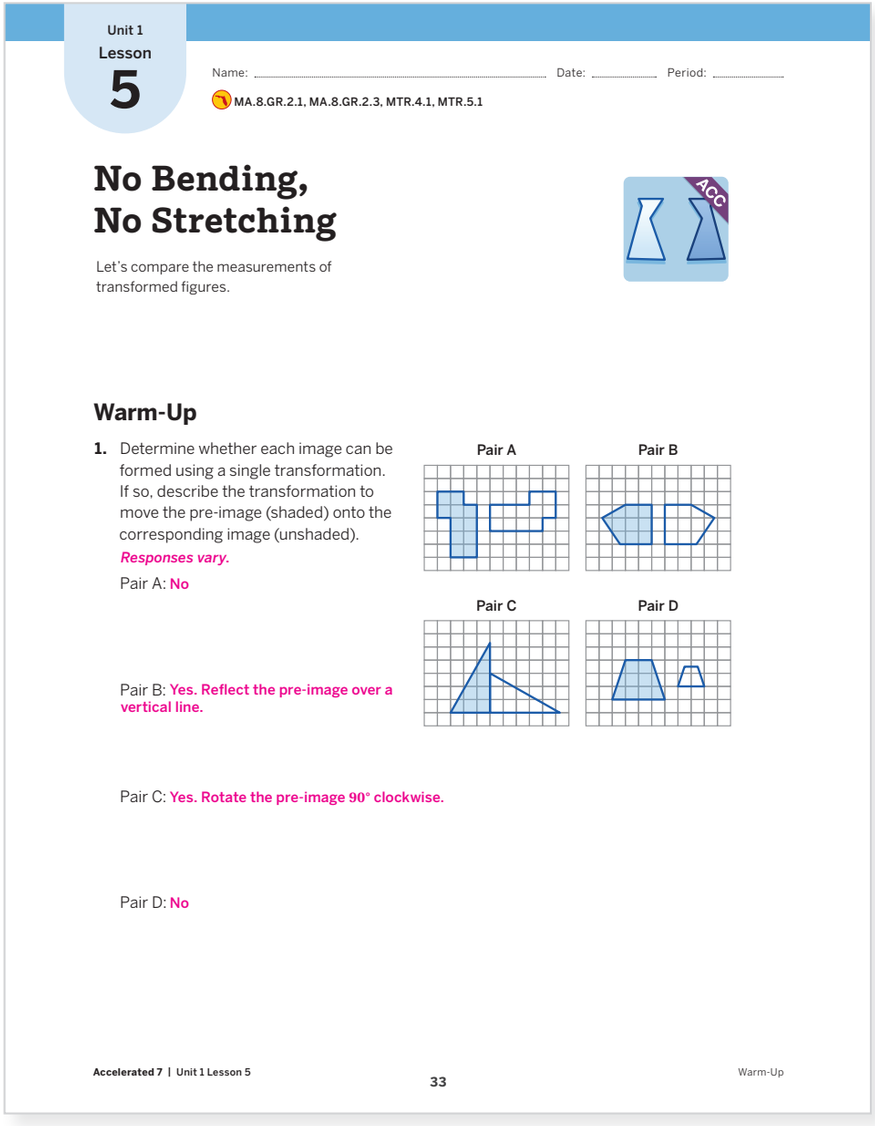
2 Connect

Invite students to share their transformations for each pair of figures.

Note: Students may notice that the figures in Pair D are different sizes. They will be more formally introduced to dilations in Unit 2, so they can use whatever language feels most comfortable for them when describing Pair D.

Consider asking:

- "What could we do to make this description stronger and clearer?"
- "How do you think _____ knew that this was a translation/reflection/rotation?"



Unit 1
Lesson
5

Name: _____ Date: _____ Period: _____

MA.8.GR.2.1, MA.8.GR.2.3, MTR.4.1, MTR.5.1

No Bending, No Stretching

Let's compare the measurements of transformed figures.

ACC

Warm-Up

1. Determine whether each image can be formed using a single transformation. If so, describe the transformation to move the pre-image (shaded) onto the corresponding image (unshaded).

Responses vary.

Pair A: No

Pair B: Yes. Reflect the pre-image over a vertical line.

Pair C: Yes. Rotate the pre-image 90° clockwise.

Pair D: No

Pair A Pair B Pair C Pair D

Accelerated 7 | Unit 1 Lesson 5 33 Warm-Up

Pairs | 20 min

Activity 1 Sides and Angles

Purpose: Students use a variety of tools to measure the side lengths and angles of pairs of figures to generalize about these measurements in translations, rotations, and reflections. **(MTR.5.1)**

Short on time? Invite students in each pair to share the measuring tasks so each student only measures four figures (two pairs of figures).

1 Launch

Note that the pairs of figures are the same ones from the Warm-Up.

Discuss what tools students think they would use to compare the side lengths and interior angles of each figure.

Distribute materials that students can choose from to measure side lengths and interior angles, including tracing paper and rulers. If needed, demonstrate how to measure using a ruler, a protractor, or discuss how to make a custom ruler using an index card or tracing paper.

2 Monitor

Look for a variety of tools and strategies students use to compare the side lengths and angles of the figures in each pair. Students may measure or use tracing paper to support them in their thinking.

D Differentiation

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Encourage them to identify a side on the pre-image and its corresponding side on the image. Ask, "Are these the same length? How can you tell?"
Recognize equal lengths in vertical and horizontal sides but not diagonal sides.	**Support** Consider asking, "How could we use a ruler or tracing paper to compare these lengths?"
Use the structure of translations, rotations, and reflections to reason about why the measurements are the same. **(MTR.5.1)**	**Stretch** Consider asking, "What makes Pair D different from the other pairs?"

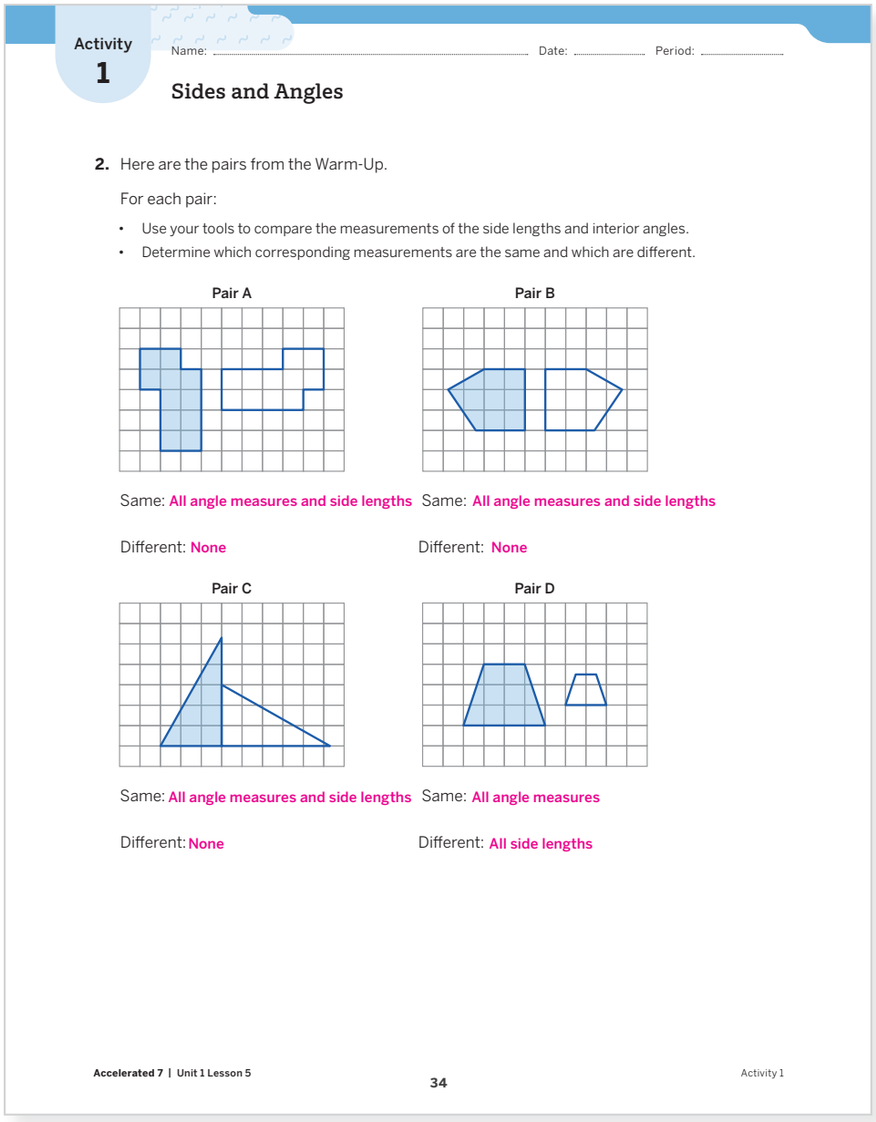
A Accessibility: Visual-Spatial Processing Invite students to use coloring tools or highlighters and annotate the corresponding side lengths and angles to support interpreting visual representations.

3 Connect

Direct students' attention to the Student Edition and discuss whether the measurements were the same or not for each pair.

To surface the Key Takeaway, consider asking:

- "In which pairs did the side lengths stay the same? The angle measures? Why does this make sense to you?"
- "Do you think side lengths and angle measures stay the same after any translation, rotation, or reflection? Why?" **(MTR.5.1)**



Activity 1

Name: Date: Period:

Sides and Angles

2. Here are the pairs from the Warm-Up.

For each pair:

- Use your tools to compare the measurements of the side lengths and interior angles.
- Determine which corresponding measurements are the same and which are different.

Pair A

Same: All angle measures and side lengths

Different: None

Pair B

Same: All angle measures and side lengths

Different: None

Pair C

Same: All angle measures and side lengths

Different: None

Pair D

Same: All angle measures

Different: All side lengths

Accelerated 7 | Unit 1 Lesson 5 34 Activity 1

Key Takeaway: Rotating, translating, and reflecting does not change the measurements of side lengths or angles.

Pairs | 10 min

Activity 2 Rigid Transformations

Purpose: Students learn the term **rigid transformation** and justify whether or not transformations are rigid or non-rigid. **(MTR.4.1)**

1 Launch

Direct students' attention to the definition of a **rigid transformation**. Invite students to read it and rephrase it in their own words.

Note: While Pair A does show a rigid transformation, it results from combining a rotation and a translation. Students will explore sequences of rigid transformations in a future course.

ELL **English Language Learners** Invite students to share what a new term reminds them of (e.g., "When I hear *rigid transformation*, I think . . ."). This may help surface multiple meanings. **(Reading and Listening)**

Discuss which of the pairs from Activity 1 show a rigid transformation before students begin the card sort.

Distribute the Activity 2 cards to each pair of students. Encourage them to work together to reach an agreement about how to sort each card.

2 Monitor

Listen for a variety of students' arguments to share during the Connect. **(MTR.4.1)**

D **Differentiation**

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Encourage them to choose a card from Activity 1. Ask, "How could you use your work from earlier to help you decide about this card?"
Use general reasoning about size and shape to sort the cards.	**Strengthen** Consider asking, "Are the corresponding angles the same measure? The corresponding sides?"
Generalize that all translations, rotations, and reflections are rigid transformations.	**Stretch** Consider asking, "Are there types of transformations that are always rigid? Sometimes rigid? Never rigid?"

A **Accessibility: Executive Function** Invite students to focus on one card at a time and continue revisiting the definition of *rigid transformation* to support students in carrying out a transformation.

3 Connect

To surface the Key Takeaway, consider asking:

- "Why did you think Card ___showed a *rigid transformation?*"**(MTR.4.1)**
- "What transformation would move the pre-image onto the image?"

Activity 2

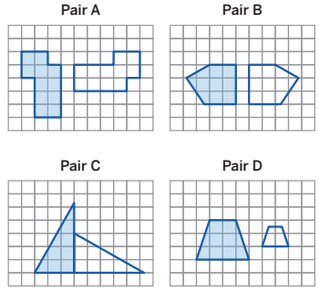
Name: Date: Period:

Rigid Transformations

3. A **rigid transformation** is a move that doesn't change any measurements in a figure.

Select *all* the pairs that show a rigid transformation.

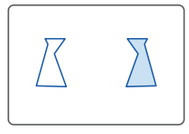
- ☐ Pair A
- ☑ Pair B
- ☑ Pair C
- ☐ Pair D



4. You will use a set of cards. Sort the cards into two groups according to their type of transformation.

Rigid	Non-Rigid
A, D, F, H	B, C, E, G

5. Devon says that this card shows a rigid transformation.



a. Is her claim correct? Yes

Explain your thinking.
Explanations vary. The corresponding side lengths and angle measurements are the same.

b. Describe a transformation that would move the pre-image (shaded) onto the image (unshaded).
Responses vary. A reflection over a vertical line.

Accelerated 7 | Unit 1 Lesson 5 35 Activity 2

Key Takeaway: Translations, rotations, and reflections are all **rigid transformations**.

Whole Class | 5 min

Synthesis

Purpose: Students synthesize their understanding of rigid transformations as transformations that preserve side lengths and angle measures.

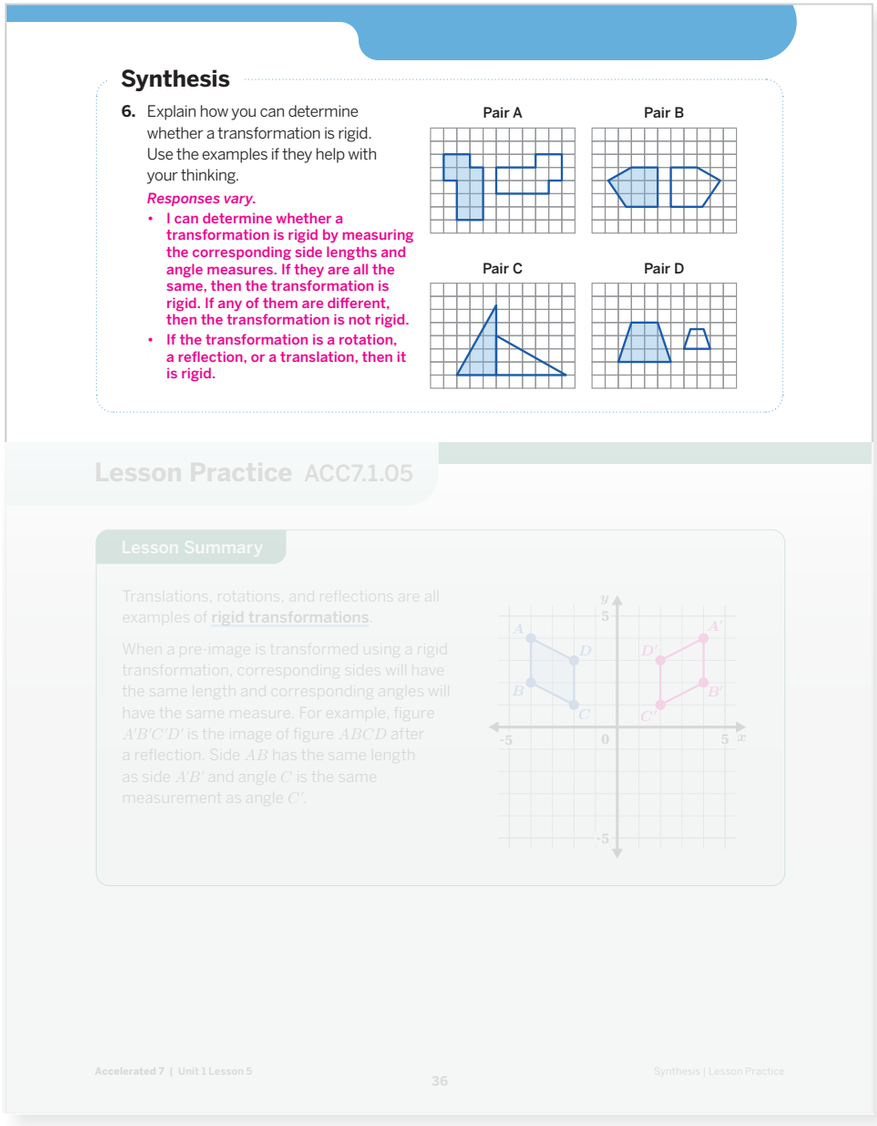
Invite students to respond independently, and then share their thinking with a partner.

Collect and share a variety of ideas, such as:

- Rigid transformations do not change angle measurements or side lengths.
- All the transformations we have learned so far are rigid.
- When a pre-image changes size or shape, the transformation is not rigid.

MLR **MLR2: Collect and Display** Use this routine to create an anchor chart for the term *rigid transformation* using students' language. Display any new language, examples, or diagrams for students to reference later.

Note: If time allows, invite students to revise their previous thinking based on the discussion.



Synthesis

6. Explain how you can determine whether a transformation is rigid. Use the examples if they help with your thinking.

Responses vary.

- I can determine whether a transformation is rigid by measuring the corresponding side lengths and angle measures. If they are all the same, then the transformation is rigid. If any of them are different, then the transformation is not rigid.
- If the transformation is a rotation, a reflection, or a translation, then it is rigid.

Pair A

Pair B

Pair C

Pair D

Lesson Practice ACC7.1.05

Lesson Summary

Translations, rotations, and reflections are all examples of rigid transformations.

When a pre-image is transformed using a rigid transformation, corresponding sides will have the same length and corresponding angles will have the same measure. For example, figure $A'B'C'D'$ is the image of figure $ABCD$ after a reflection. Side AB has the same length as side $A'B'$ and angle C is the same measurement as angle C'.

Accelerated 7 | Unit 1 Lesson 5 36 Synthesis | Lesson Practice

Lesson Takeaway: Rigid transformations maintain the side lengths and angle measurements of a pre-image. Translations, rotations, and reflections are all rigid transformations.

Independent | 5 min

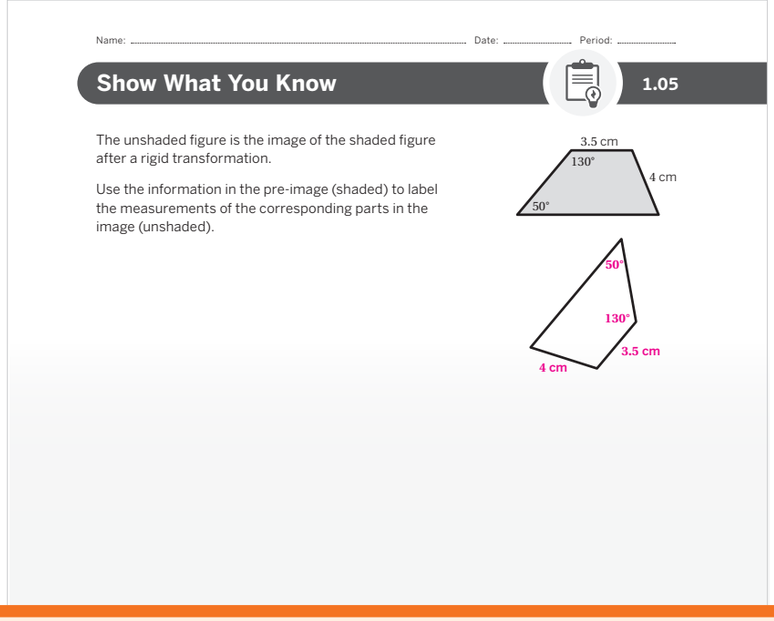
Show What You Know

Purpose: Students demonstrate their understanding by using measurements on a pre-image to determine corresponding measurements on an image after a rigid transformation.

Today's Goals

1. **Goal:** Understand that *rigid transformations* preserve the side lengths and angle measures of a figure.
2. **Language Goal:** Justify whether a *rigid transformation* took place using language such as *corresponding side lengths*, *corresponding angles*, *pre-image*, and *image*. **(Writing, Speaking, and Listening)**

Show What You Know (Assessment Resources)



Name: _____ Date: _____ Period: _____

Show What You Know 1.05

The unshaded figure is the image of the shaded figure after a rigid transformation.

Use the information in the pre-image (shaded) to label the measurements of the corresponding parts in the image (unshaded).

D Differentiation Use after Lesson 5

Support

Provide targeted intervention for students by using these resources.

If students need support understanding how to find side lengths and angle measurements after a rigid transformation:

» **Respond to Student Thinking:**

- Review the **Lesson Summary** and label the congruent side lengths and angles.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 2 Extension Activities**.

Professional Learning

Take a moment to reflect on Activity 2 of this lesson, where students worked together to justify whether cards showed a rigid transformation or not. What structures supported students to learn with and from each other? What might support students in working together and sharing their reasoning aloud?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

6. Explain how you can determine whether a transformation is rigid. Use the examples if they help with your thinking.

Responses vary.

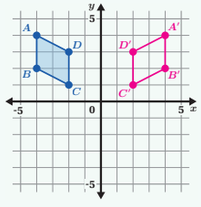
- I can determine whether a transformation is rigid by measuring the corresponding side lengths and angle measures. If they are all the same, then the transformation is rigid. If any of them are different, then the transformation is not rigid.
- If the transformation is a rotation, a reflection, or a translation, then it is rigid.

Lesson Practice ACC7.1.05

Lesson Summary

Translations, rotations, and reflections are all examples of **rigid transformations**.

When a pre-image is transformed using a rigid transformation, corresponding sides will have the same length and corresponding angles will have the same measure. For example, figure $A'B'C'D'$ is the image of figure $ABCD$ after a reflection. Side AB has the same length as side $A'B'$ and angle C is the same measurement as angle C'.



Lesson Practice
ACC7.1.05

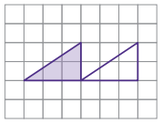
Name: _____ Date: _____ Period: _____

1. Is there a rigid transformation that can move the shaded figure onto the unshaded figure? **No**

Explain your thinking.

Explanations vary. **The angle measures of the two figures are different, and a rigid transformation must preserve all lengths and angle measures.**

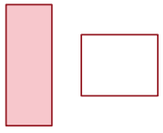
2. Describe a rigid transformation that moves the shaded figure onto the unshaded figure.



Responses vary. **Translate 3 units right.**

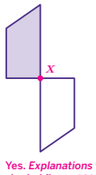
Problems 3–6: Determine whether a rigid transformation can move the shaded figure onto the unshaded figure. If so, describe the rigid transformation.

3.



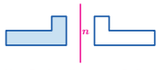
No, a rigid transformation cannot be performed.

4.



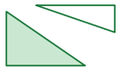
Yes. *Explanations vary.* Rotate the shaded figure 180° around point X.

5.



Yes. *Explanations vary.* Reflect the shaded figure over line n.

6.

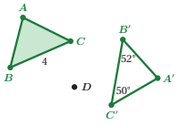


No, a rigid transformation cannot be performed.

Lesson Practice
ACC7.1.05

Name: _____ Date: _____ Period: _____

Problems 7–9: Triangle $A'B'C'$ is an image of triangle ABC after a rotation. The center of rotation is point D, which lies at the origin.



7. What is the length of side $B'C'$?
4 units

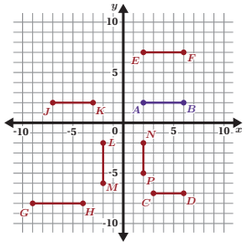
8. What is the measure of angle B?
52°

9. What is the measure of angle C?
50°

FAST Practice

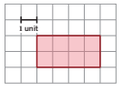
10. Which of the following segments are the result of a single rigid transformation of segment AB? Select *all* that apply.

- ☐ **A.** Line segment CD
- ☑ **B.** Line segment KJ
- ☐ **C.** Line segment NP
- ☑ **D.** Line segment LM
- ☐ **E.** Line segment GH
- ☑ **F.** Line segment EF



Spiral Review

Problems 11–12: Here is a rectangle.

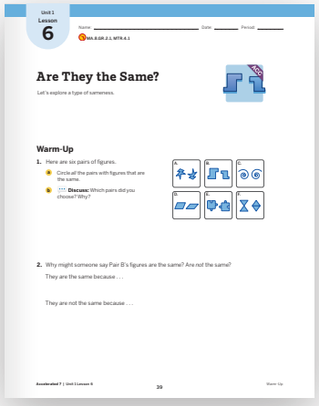


11. What is the area of the rectangle?
8 square units

12. What is the perimeter of the rectangle?
12 units

Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1–9	2	MA.8.GR.2.1
FAST Practice	10	2	MA.8.GR.2.3
Spiral Review			
	11–12	1	MA.4.GR.2.1


Unit 1
Lesson
6

Name _____ Date _____ Period _____

MA.8.GR.2.1, MTR.4.1

Are They the Same?

Let's explore a type of sameness.

Warm-Up

1. Here are six pairs of figures.
   - Circle all the pairs with figures that are the same.
   - **Discuss:** Which pairs did you choose? Why?

2. Why might someone say Pair B's figures are the same? Are not the same?
   They are the same because . . .
   They are not the same because . . .

Accelerated 7 | Unit 1 Lesson 6 39 Warm-Up

Are They the Same?

Defining Congruence

Let's explore a type of sameness.

Learning Outcomes

Today's Goals

1. **Goal:** Recognize that two figures are *congruent* if the second can be obtained from the first by a translation, reflection, or rotation.
2. **Language Goal:** Justify whether two figures are *congruent* using a rigid transformation. **(Reading, Writing, Speaking, and Listening)**

Students explore what it means for figures to be the "same" and learn that the term **congruent** is a mathematical way to talk about figures being the "same." They discover that two figures are congruent if there is a rigid transformation that moves one onto the other. **(MTR.4.1)**

Coherence

Prior Learning

In Lesson 5, students learned that translations, reflections, and rotations are examples of rigid transformations. They saw that rigid transformations preserve side lengths and angle measures.

Future Learning

In Lesson 7, students will develop and analyze arguments for whether two figures are congruent.

Connections

- Students apply their understanding of rigid transformations to verify if two figures are congruent. (mathematical)

FL B.E.S.T. Math Benchmarks

Addressing

MA.8.GR.2.1
Given a preimage and image generated by a single transformation, identify the transformation that describes the relationship.

Mathematical Thinking and Reasoning Standard: MTR.4.1

ELA Expectation: ELA.K12.EE.1.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.6.GR.1.1	MA.912.F.2 MA.912.GR.2

Vocabulary

New Vocabulary

congruent

Lesson at a Glance

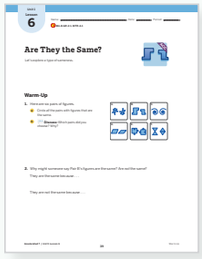
~ 45 min

MA.8.GR.2.1

Warm-Up

Pairs | 5 min

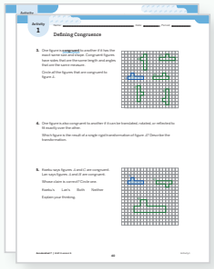
Students discover a need for a precise meaning of sameness by deciding whether a series of figures are the same. **(MTR.4.1)**



Activity 1

Pairs | 20 min

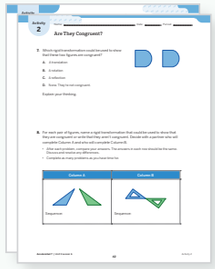
Students explore the idea of congruence by identifying and justifying whether pairs of figures are **congruent**. **(MTR.4.1)**



Activity 2

Pairs | 10 min

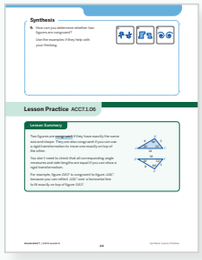
Students practice justifying congruence by identifying a rigid transformation that moves one figure onto another.



Synthesis

Whole Class | 5 min

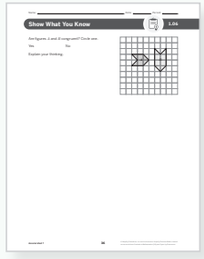
Students synthesize their understanding of how to determine whether two figures are congruent using rigid transformations.



Show What You Know

Independent | 5 min

Students demonstrate their understanding by identifying and justifying if two figures are congruent. **(MTR.4.1)**



Prep Checklist

This lesson includes:

- Student Edition
- Show What You Know
- Tracing paper (as needed)

Pairs | 5 min

Warm-Up

Purpose: Students discover a need for a precise meaning of sameness by deciding whether a series of figures are the same. **(MTR.4.1)**

1 Launch

Consider asking, "What are ways that two things can be the same?"

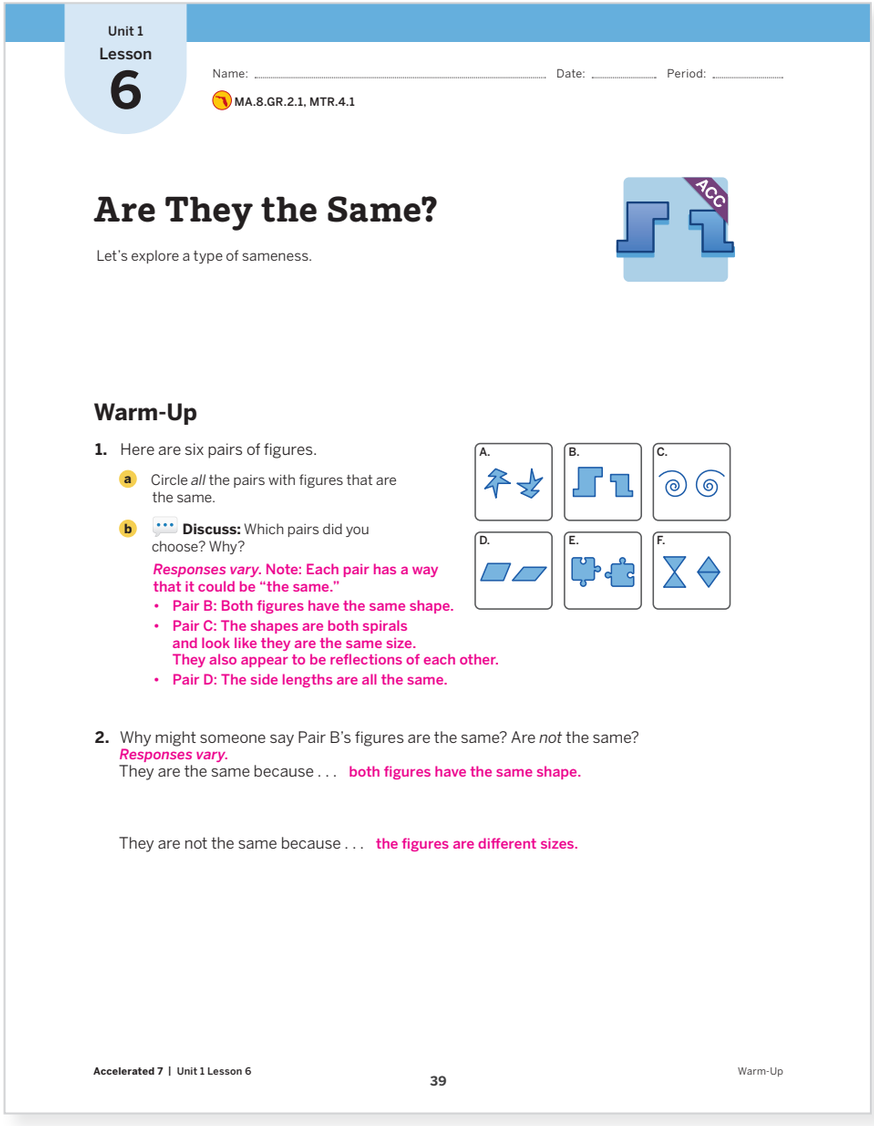
Invite students to discuss their thinking about whether each pair is the same with a partner.

2 Connect

Invite students to share their responses. It is okay — even desirable — to lack consensus at this stage. Students will focus their understanding on one type of sameness (congruence) in the next activity.

Invite students to make arguments and justify their reasoning for why these figures are and are not the same. **(MTR.4.1)**

Consider asking, "Do you think the fact that these are different sizes means that they are not the same?"



Unit 1
Lesson
6

Name: ..... Date: ..... Period: .....

MA.8.GR.2.1, MTR.4.1

Are They the Same?

Let's explore a type of sameness.

ACC

Warm-Up

1. Here are six pairs of figures.

   a. Circle *all* the pairs with figures that are the same.

   b. **Discuss:** Which pairs did you choose? Why?

   Responses vary. Note: Each pair has a way that it could be "the same."
   - Pair B: Both figures have the same shape.
   - Pair C: The shapes are both spirals and look like they are the same size. They also appear to be reflections of each other.
   - Pair D: The side lengths are all the same.

A. B. C. D. E. F.

2. Why might someone say Pair B's figures are the same? Are *not* the same?
   Responses vary.
   They are the same because . . . both figures have the same shape.

   They are not the same because . . . the figures are different sizes.

Accelerated 7 | Unit 1 Lesson 6 39 Warm-Up

Pairs | 20 min

Activity 1 Defining Congruence

Purpose: Students explore the idea of congruence by identifying and justifying whether pairs of figures are **congruent**. **(MTR.4.1)**

1 Launch

Consider sharing that it can be helpful to have a specific way to define figures that are the "same."

Invite students to read the definition of **congruent** and rephrase it in their own words.

ELL **English Language Learners** Invite students to research how to say *congruent* in other languages. Create an anchor chart with the term in multiple languages, along with examples. **(Reading and Listening)**

2 Monitor

Look for a variety of student thinking.

D **Differentiation** *(Problem 4)*

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Encourage them to draw or use tracing paper to show how one figure would move onto the other.
Need support with describing a transformation precisely.	**Support** Provide sentence stems such as "translate ___ units up/down, left/right," "rotate ___ degrees around point ___," and "reflect ___ over the line ___."
Precisely describe a transformation.	**Stretch** Invite students to explain why one of the figures they did not circle is *not* congruent to figure A.

Collect a variety of student responses.

A **Accessibility: Conceptual Processing** Invite students to share how this problem is alike and different from the previous problem to help them apply concepts to new situations. Consider asking, "How are figures G and H and the figures you said were congruent in the previous problems alike? How are they different?"

Activity 1 continued >

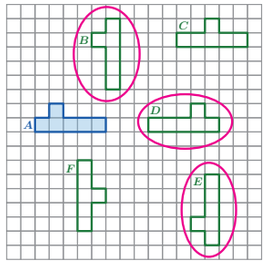
Activity 1

Name: _____ Date: _____ Period: _____

Defining Congruence

3. One figure is **congruent** to another if it has the exact same size and shape. Congruent figures have sides that are the same length and angles that are the same measure.

Circle *all* the figures that are congruent to figure A.



4. One figure is also congruent to another if it can be translated, rotated, or reflected to fit exactly over the other.

Which figure is the result of a single rigid transformation of figure A? Describe the transformation.

Figure D is a reflection of figure A over a vertical line.

5. Kweku says figures A and G are congruent. Lan says figures A and H are congruent.

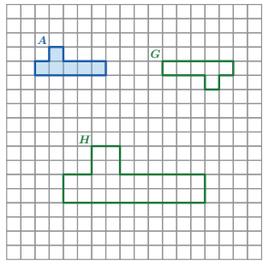
Whose claim is correct? Circle one.

(Kweku's) Lan's Both Neither

Explain your thinking.

Explanations vary.

- If you rotate figure A 180°, you can make figure A land directly on top of figure G so that they match perfectly.
- Figure H is the same shape as figure A but a different size. Congruent figures must be the same size *and* shape.



Accelerated 7 | Unit 1 Lesson 6 40 Activity 1

Activity 1 Defining Congruence (continued)

Purpose: Students explore the idea of congruence by identifying and justifying whether pairs of figures are **congruent**. **(MTR.4.1)**

2 Monitor

Encourage students to share their reasoning with a partner and work together to reach an agreement about how to sort the cards.

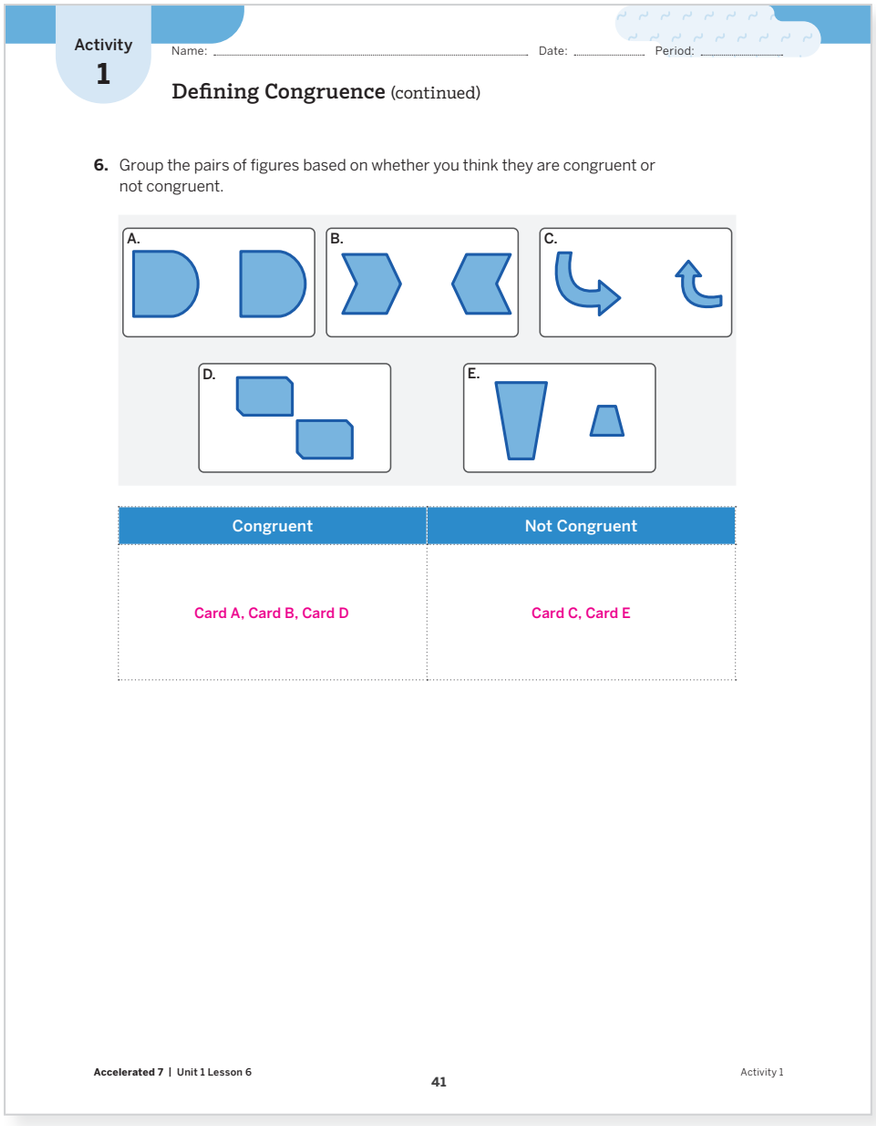
3 Connect

Use the Decide and Defend routine to support students in making arguments and critiquing the reasoning of others. **(MTR.4.1)**

Note: If everyone is in agreement, consider making a class argument using different students' reasoning.

To surface the Key Takeaway, consider asking:

- "How did you decide who was correct?"
- "For Problem 5, What would you say to someone who said figures A and H are *congruent* because they are the same shape?"**(MTR.4.1)**



Activity 1

Name: _____ Date: _____ Period: _____

Defining Congruence (continued)

6. Group the pairs of figures based on whether you think they are congruent or not congruent.

A. B. C. D. E.

Congruent	Not Congruent
Card A, Card B, Card D	Card C, Card E

Accelerated 7 | Unit 1 Lesson 6 41 Activity 1

Key Takeaway: Two figures are **congruent** if there is a rigid transformation that can move one figure onto the other. Congruent figures have the exact same size and shape.

Pairs | 10 min

Activity 2 Are They Congruent?

Purpose: Students practice justifying congruence by identifying a rigid transformation that moves one figure onto another.

Short on time? Consider inviting students to complete fewer challenges in Problem 8.

1 Launch

Invite students to discuss their reasoning with a partner as they choose a transformation.

2 Monitor

Encourage students to solve as many challenges as they have time for. Share that sense-making is more important than speed.

Students will need a partner for this activity. They will solve and compare their solutions to problems.

Listen for strategies students use in discussing how to solve the problems, such as how they describe rigid transformations.

A **Accessibility: Conceptual Processing** Invite students to revisit their work in the previous problem to help apply concepts to new situations.

Activity 2 continued >

Activity 2

Name: Date: Period:

Are They Congruent?

7. Which rigid transformation could be used to show that these two figures are congruent?

(A.) A translation

B. A rotation

C. A reflection

D. None. They're not congruent.

Explain your thinking.

Explanations vary.

- A translation because the figure can be slid to the right to move onto the other figure.

8. For each pair of figures, name a rigid transformation that could be used to show that they are congruent or write that they aren't congruent. Decide with a partner who will complete Column A and who will complete Column B.

- After each problem, compare your answers. The answers in each row should be the same. Discuss and resolve any differences.
- Complete as many problems as you have time for.

Responses vary.

Column A	Column B
Sequence: A reflection	Sequence: A rotation

Accelerated 7 | Unit 1 Lesson 6 42 Activity 2

Pairs | 10 min

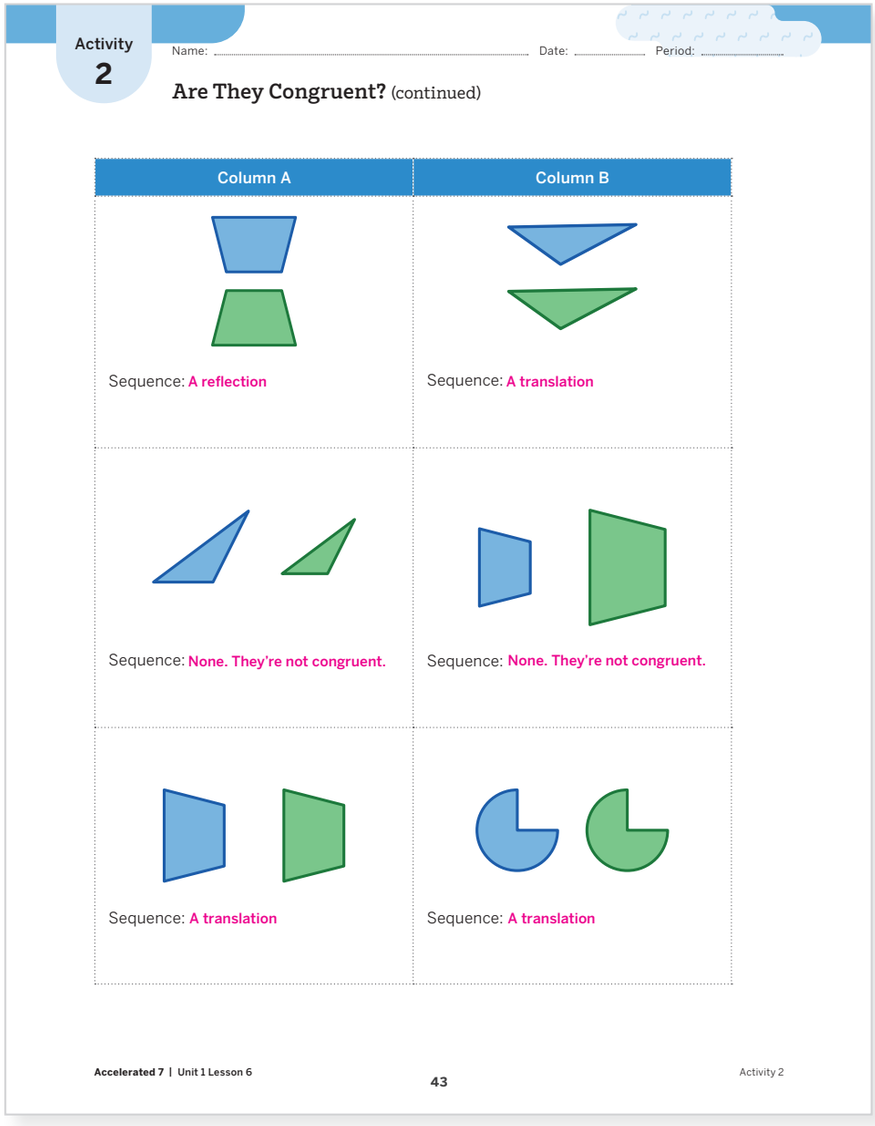
Activity 2 Are They Congruent? (continued)

Purpose: Students practice justifying congruence by identifying a rigid transformation that moves one figure onto another.

Short on time? Consider inviting students to complete fewer challenges in Problem 8.

3 Connect

Invite students to share strategies that they found helpful or what they learned from the mistakes they made while solving these challenges.



Activity 2

Name: Date: Period:

Are They Congruent? (continued)

Column A	Column B
Sequence: A reflection	Sequence: A translation
Sequence: None. They're not congruent.	Sequence: None. They're not congruent.
Sequence: A translation	Sequence: A translation

Accelerated 7 | Unit 1 Lesson 6 43 Activity 2

Whole Class | 5 min

Synthesis

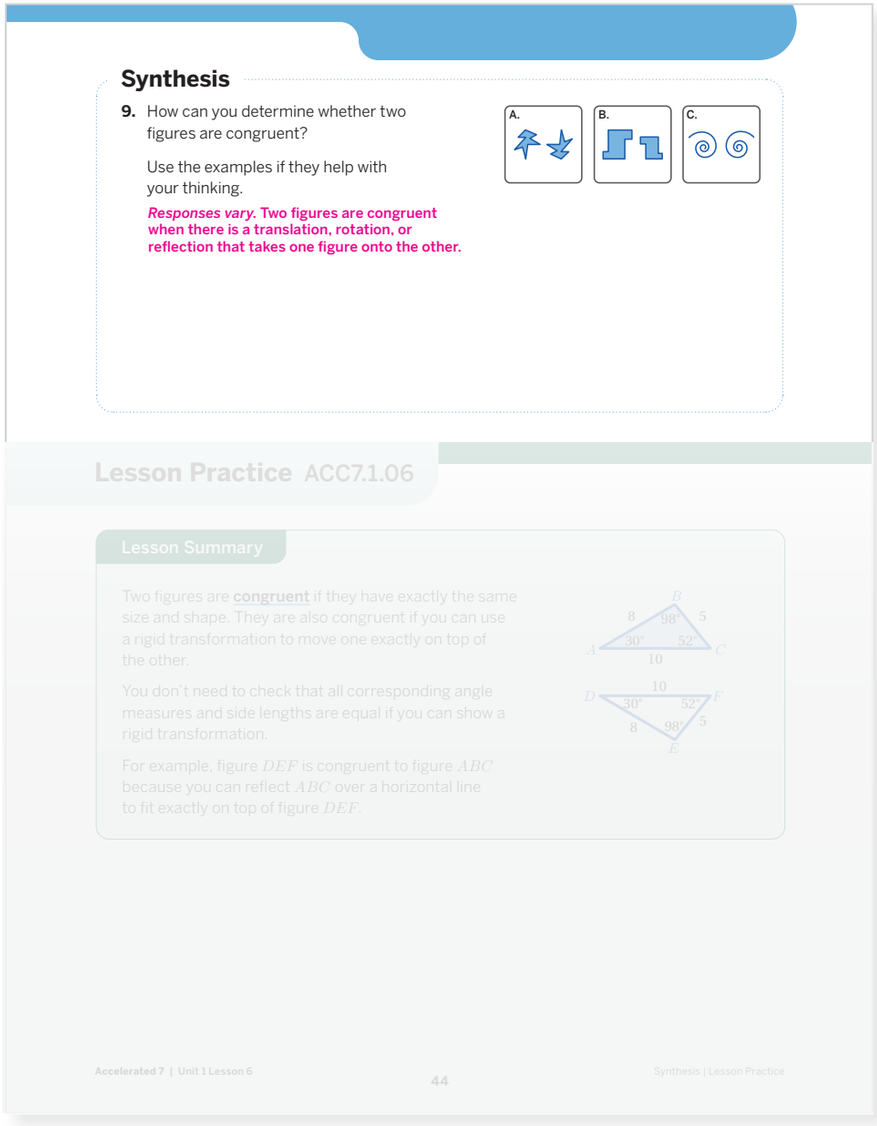
Purpose: Students synthesize their understanding of how to determine whether two figures are congruent using rigid transformations.

Invite students to respond independently, and then share their thinking with a partner.

Collect and share a variety of ideas, such as:

- Using tracing paper to perform rigid transformations.
- Looking at the orientation of a figure and deciding if a translation, reflection, or rotation would move it onto another figure.

Note: If time allows, invite students to revise their previous thinking based on the discussion.



Synthesis

9. How can you determine whether two figures are congruent?

Use the examples if they help with your thinking.

Responses vary. Two figures are congruent when there is a translation, rotation, or reflection that takes one figure onto the other.

Lesson Practice ACC7.1.06

Lesson Summary

Two figures are **congruent** if they have exactly the same size and shape. They are also congruent if you can use a rigid transformation to move one exactly on top of the other.

You don't need to check that all corresponding angle measures and side lengths are equal if you can show a rigid transformation.

For example, figure DEF is congruent to figure ABC because you can reflect ABC over a horizontal line to fit exactly on top of figure DEF.

Accelerated 7 | Unit 1 Lesson 6 44 Synthesis | Lesson Practice

Lesson Takeaway: Two figures are **congruent** if there is a rigid transformation that can move one figure onto the other. We can identify this transformation by looking at where the figures are located and how they are oriented.

Independent | 5 min

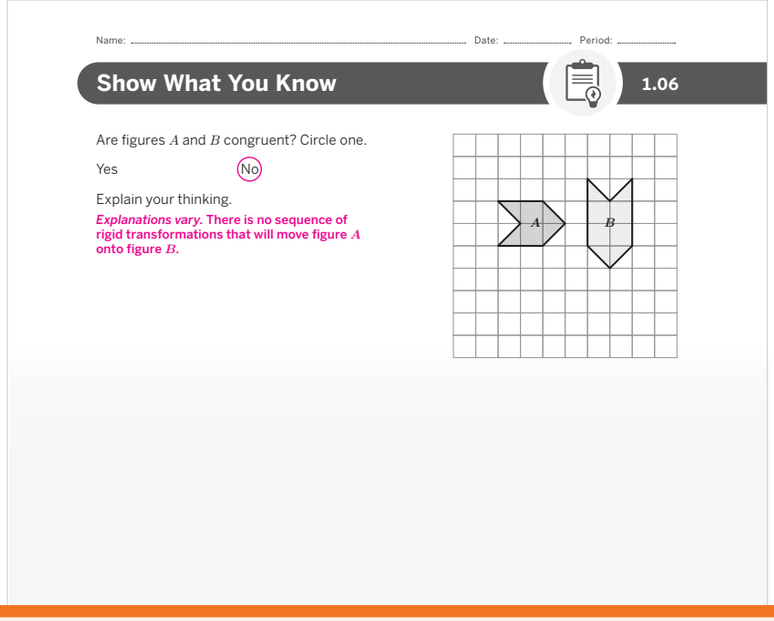
Show What You Know

Purpose: Students demonstrate their understanding by identifying and justifying if two figures are congruent. **(MTR.4.1)**

Today's Goals

1. **Goal:** Recognize that two figures are *congruent* if the second can be obtained from the first by a translation, reflection, or rotation.
2. **Language Goal:** Justify whether two figures are *congruent* using a rigid transformation. **(Reading, Writing, Speaking, and Listening)**

Show What You Know (Assessment Resources)



Name: _____ Date: _____ Period: _____

Show What You Know 1.06

Are figures A and B congruent? Circle one.

Yes (No)

Explain your thinking.

Explanations vary. There is no sequence of rigid transformations that will move figure A onto figure B.

D Differentiation Use after Lesson 6

Support

Provide targeted intervention for students by using these resources.

If students need support understanding what makes figures congruent:

» **Respond to Student Thinking:**

- Use the ***Justifying Congruent Figures*** **Mini-Lesson**.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 2 Extension Activities**.

Professional Learning

Reflect on how students collaborated with a partner as they solved problems today. How did they talk about mistakes? Were they able to notice the correct and incorrect parts of each other's work? How can you support students in valuing learning from both correct and incorrect thinking? How can you support them in respecting their classmates as they share disagreements or corrections?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

9. How can you determine whether two figures are congruent?

Use the examples if they help with your thinking.

Responses vary. Two figures are congruent when there is a translation, rotation, or reflection that takes one figure onto the other.

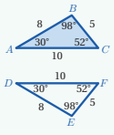
Lesson Practice ACC7.1.06

Lesson Summary

Two figures are **congruent** if they have exactly the same size and shape. They are also congruent if you can use a rigid transformation to move one exactly on top of the other.

You don't need to check that all corresponding angle measures and side lengths are equal if you can show a rigid transformation.

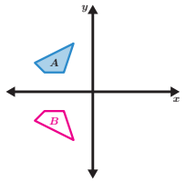
For example, figure DEF is congruent to figure ABC because you can reflect ABC over a horizontal line to fit exactly on top of figure DEF.



Lesson Practice ACC7.1.06

Name: Date: Period:

Problems 1–2: This coordinate plane shows figure A.



1. Reflect figure A over the x-axis. Label the image B.

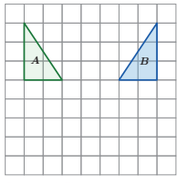
Sample shown on graph.

2. Are figures A and B congruent? Explain your thinking.

Yes. *Explanations vary.* Because there is a rigid transformation taking quadrilateral A to quadrilateral B, the two figures are congruent.

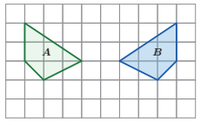
Problems 3–6: Determine whether figure A is congruent to figure B. Explain your thinking.

3.



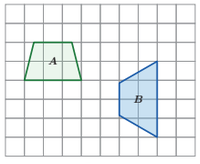
Congruent. Figure A can be reflected over a vertical line halfway between figures A and B.

4.



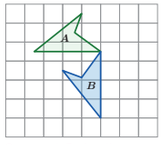
Congruent. Figure A can be reflected over a vertical line halfway between figures A and B.

5.



Not congruent. *Explanations vary.* There is no rigid transformation that will move figure A onto figure B.

6.



Congruent. *Explanations vary.* Figure A can be rotated 90° counterclockwise with the vertex shared by the two figures as the center of rotation.

Lesson Practice ACC7.1.06

Name: Date: Period:

FAST Practice

7. Pentagon $ABCDE$ was graphed on a coordinate plane and then rotated 90° counterclockwise around the origin to form pentagon $A'B'C'D'E'$. Which statement is true?

A. Pentagon $A'B'C'D'E'$ is not congruent to pentagon $ABCDE$.

B. The area of pentagon $A'B'C'D'E'$ is not equal to the area of pentagon $ABCDE$.

C. The angle measures of pentagon $A'B'C'D'E'$ are congruent to the corresponding angle measures of pentagon $ABCDE$.

D. The perimeter of pentagon $A'B'C'D'E'$ is greater than the perimeter of pentagon $ABCDE$.

Spiral Review

Problems 8–9: Determine the coordinates of the image of point A (2, -5) after each transformation.

8. A reflection over the x-axis.

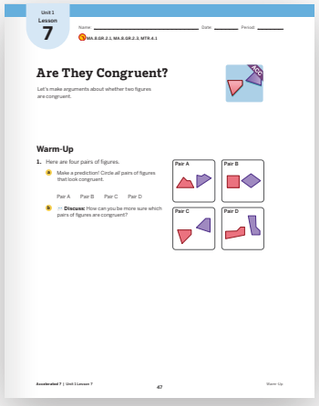
(2, 5)

9. A reflection over the y-axis.

(-2, -5)

Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1–6	2	MA.8.GR.2.1
FAST Practice	7	2	MA.8.GR.2.1
Spiral Review			
	8–9	1	MA.8.GR.2.3


Unit 1
Lesson
7

Are They Congruent?

Let's make arguments about whether two figures are congruent.

Warm-Up

1. Here are four pairs of figures.

Pair A Pair B Pair C Pair D

Pair A

Pair B

Pair C

Pair D

Are They Congruent?

Rigid Transformations and Congruent Figures

Let's make arguments about whether two figures are congruent.

Learning Outcomes

Today's Goals

1. **Goal:** Use rigid transformations to determine whether two figures on a grid are congruent.
2. **Language Goal:** Justify whether two figures are congruent using vocabulary from this unit. **(Reading, Writing, Speaking, and Listening)**

Students use rigid transformations to show two figures are congruent and construct arguments for why two figures are *not* congruent. They come to understand that, for many figures, simply having corresponding side lengths that are equal or equal areas will *not* guarantee the figures are congruent. They evaluate the strength of arguments that two figures are congruent. **(MTR.4.1)**

Coherence

Prior Learning

In Lesson 6, students defined what it means for two figures to be congruent and began to apply this meaning to determine whether two figures were congruent.

Future Learning

In Unit 2, students will analyze dilations with or without a grid and determine scale factor, using it to calculate related measures.

Connections

- Students use evidence to support a claim when determining if two figures are congruent. (mathematical, cross-curricular)

FL B.E.S.T. Math Benchmarks

Addressing

MA.8.GR.2.1
Given a preimage and image generated by a single transformation, identify the transformation that describes the relationship.

MA.8.GR.2.3
Describe and apply the effect of a single transformation on two-dimensional figures using coordinates and the coordinate plane.

Mathematical Thinking and Reasoning Standard: MTR.4.1

ELA Expectations: ELA.K12.EE.1.1, ELA.K12.EE.6.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.6.GR.1.1	MA.912.F.2 MA.912.GR.2.4

Lesson at a Glance

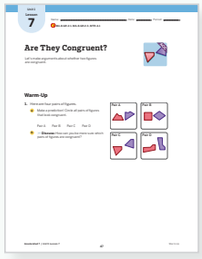
~ 45 min

MA.8.GR.2.1, MA.8.GR.2.3

Warm-Up

Pairs | 5 min

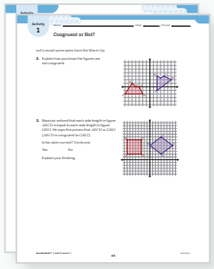
Students make predictions about which figures are congruent and will further investigate in the rest of the lesson.



Activity 1

Pairs | 15 min

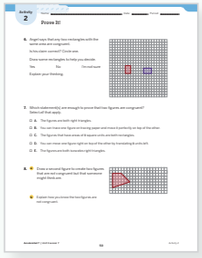
Students examine whether equal side lengths, equal area, and transformations are convincing arguments to prove two figures are congruent. **(MTR.4.1)**



Activity 2

Pairs | 15 min

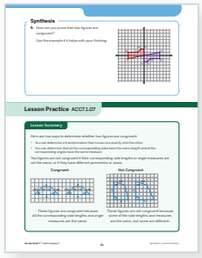
Students evaluate arguments about whether two figures are congruent and create their own set of non-congruent figures. **Routine:** Decide and Defend **(MTR.4.1)**



Synthesis

Whole Class | 5 min

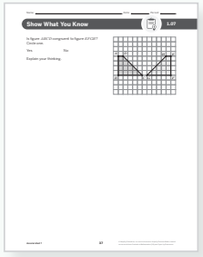
Students synthesize their understanding of how to prove two figures are congruent.
MLR2: Collect and Display



Show What You Know

Independent | 5 min

Students demonstrate their understanding by determining whether two figures are congruent.



Prep Checklist

This lesson includes:

- Student Edition
- Show What You Know
- Tracing paper (as needed)

Pairs | 5 min

Warm-Up

Purpose: Students make predictions about which figures are congruent and will further investigate in the rest of the lesson.

1 Launch

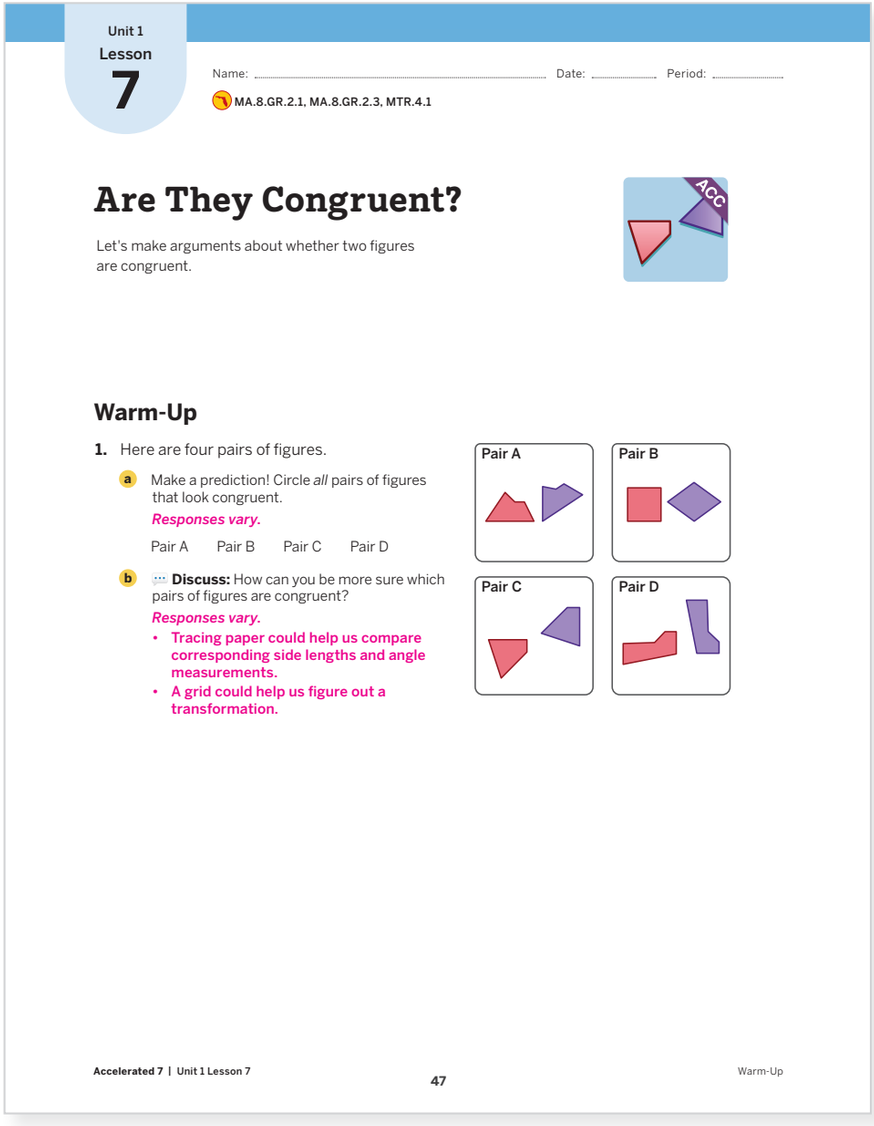
Invite students to predict which pairs of figures will be congruent and discuss what would convince them the figures are congruent with a partner.

Note: It is okay — even desirable — to lack consensus at this stage. Activity 1 will build toward consensus when students investigate each pair individually.

2 Connect

Consider asking:

- "What do you remember about congruence from the previous lesson?"
- "How could someone convince you that two figures are congruent?"



Unit 1
Lesson
7

Name: ..... Date: ..... Period: .....

MA.8.GR.2.1, MA.8.GR.2.3, MTR.4.1

Are They Congruent?

Let's make arguments about whether two figures are congruent.

ACC

Warm-Up

1. Here are four pairs of figures.

a. Make a prediction! Circle *all* pairs of figures that look congruent.

Responses vary.

Pair A Pair B Pair C Pair D

b. **Discuss:** How can you be more sure which pairs of figures are congruent?

Responses vary.

- Tracing paper could help us compare corresponding side lengths and angle measurements.
- A grid could help us figure out a transformation.

Accelerated 7 | Unit 1 Lesson 7 47 Warm-Up

Pairs | 15 min

Activity 1 Congruent or Not?

Purpose: Students examine whether equal side lengths, equal area, and transformations are convincing arguments to prove two figures are congruent. **(MTR.4.1)**

1 Launch

Invite students to think independently before discussing their thoughts with a partner.

2 Monitor

In Problem 3, **Pause** briefly to discuss the congruence symbol.

Consider asking:

- "What does this symbol remind you of?"
- "In what situations might it be helpful to use this symbol? When might it make more sense to write out the words?"

A **Accessibility: Conceptual Processing** Invite students to revisit their work in the previous problem to help apply concepts to new situations.

Activity 1 continued >

Activity 1

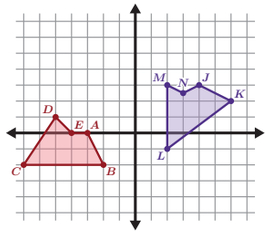
Name: Date: Period:

Congruent or Not?

Let's revisit some pairs from the Warm-Up.

2. Explain how you know the figures are not congruent.

Responses vary. It's clear that the figures aren't congruent because the side length of LM is 4 units and the side length of corresponding side CD is between 3 and 4 units.



3. Mauricio noticed that each side length in figure $ABCD$ is equal to each side length in figure $GHIJ$. He says this proves that $ABCD \cong GHIJ$ ($ABCD$ is congruent to $GHIJ$).

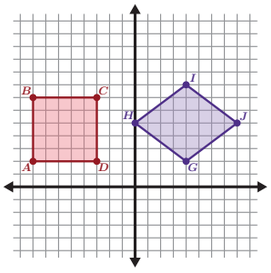
Is his claim correct? Circle one.

Yes (No)

Explain your thinking.

Explanations vary.

- Knowing that the side lengths in one figure are equal to the side lengths in another figure is not enough to determine congruence. The angles should also be equal, and these angles are not equal.
- I couldn't place figure $ABCD$ perfectly on top of figure $GHIJ$ using tracing-paper transformations, which means the figures are not congruent. The fact that side lengths in one figure are equal to side lengths in the other does not change that.



Accelerated 7 | Unit 1 Lesson 7 48 Activity 1

Pairs | 15 min

Activity 1 Congruent or Not? (continued)

Purpose: Students examine whether equal side lengths, equal area, and transformations are convincing arguments to prove two figures are congruent. **(MTR.4.1)**

2 Monitor

Look for a variety of student responses.

D Differentiation *(Problem 4)*

Look for students who:	Respond to Student Thinking
Believe figures are congruent if the side lengths or areas are the same.	**Support** Consider asking, "What does *congruence* mean?" Invite students to use tracing paper to see if one figure can be moved onto the other.
Choose the correct response but need support explaining their thinking.	**Support** Consider asking, "Why was this argument the most convincing to you?"
Explain that they chose their response because it explains the transformation that proves the side lengths and angles do not change.	**Strengthen** Invite students to share their thinking during the Connect.

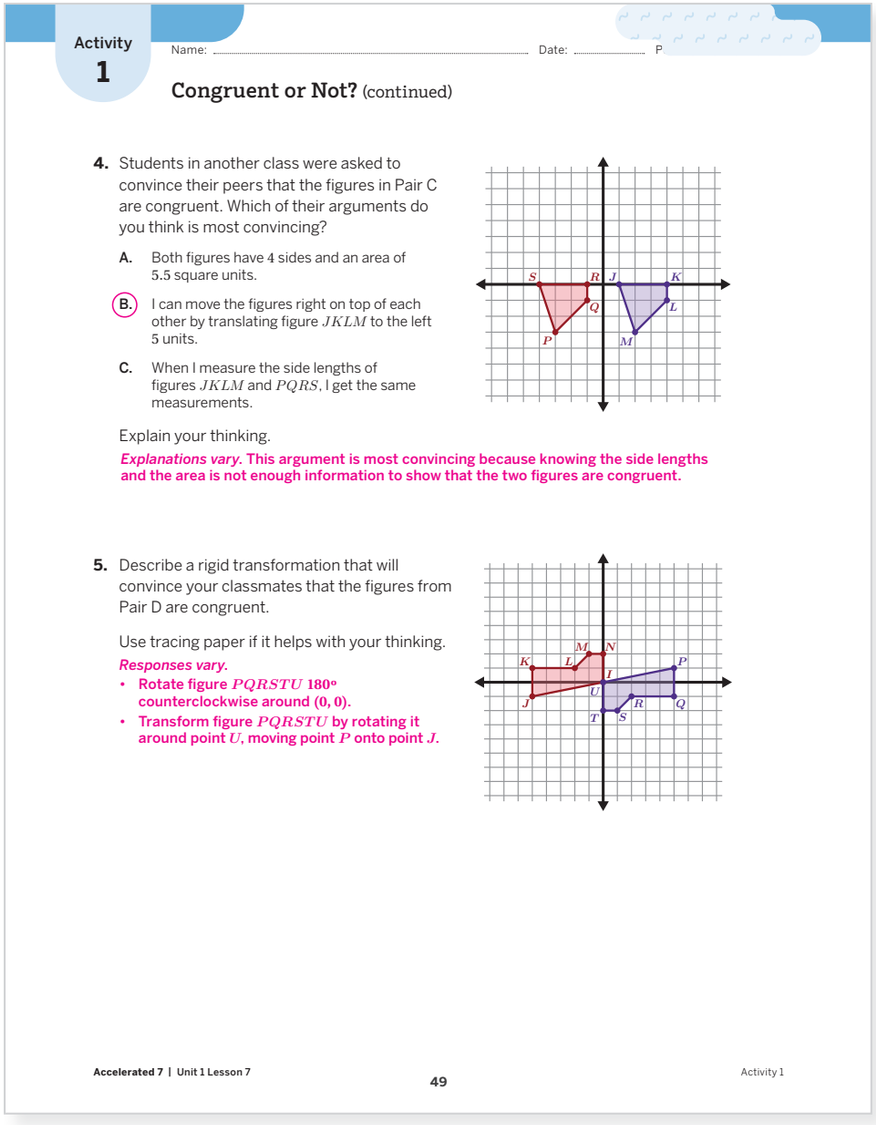
ELL English Language Learners Provide sentence frames to help students explain their strategy (e.g., "This argument is most convincing because ____."). **(Writing and Speaking)**

To support students getting started, encourage them to refer back to the anchor chart created previously with what information is needed to precisely describe each transformation.

3 Connect

For Problem 4, **Discuss** students' explanations of which argument they found most convincing.

To surface the Key Takeaway, consider asking, "Why can't having equal side lengths or area alone prove that two figures are congruent?"



Activity 1

Name: ..... Date: ..... P

Congruent or Not? (continued)

4. Students in another class were asked to convince their peers that the figures in Pair C are congruent. Which of their arguments do you think is most convincing?

A. Both figures have 4 sides and an area of 5.5 square units.

B. I can move the figures right on top of each other by translating figure $JKLM$ to the left 5 units.

C. When I measure the side lengths of figures $JKLM$ and $PQRS$, I get the same measurements.

Explain your thinking.

Explanations vary. This argument is most convincing because knowing the side lengths and the area is not enough information to show that the two figures are congruent.

5. Describe a rigid transformation that will convince your classmates that the figures from Pair D are congruent.

Use tracing paper if it helps with your thinking.

Responses vary.

- Rotate figure $PQRSTU$ 180° counterclockwise around (0, 0).
- Transform figure $PQRSTU$ by rotating it around point U, moving point P onto point J.

Accelerated 7 | Unit 1 Lesson 7 49 Activity 1

Key Takeaway: Two figures having equal side lengths and/or equal areas is not sufficient to prove the figures are congruent. A transformation that moves one figure onto the other is sufficient to prove two figures are congruent.

Pairs | 15 min

Activity 2 Prove It!

Purpose: Students evaluate arguments about whether two figures are congruent and create their own set of non-congruent figures. **(MTR.4.1)**

Short on time? Consider omitting Problem 8.

1 Launch

To support students getting started, encourage them to create two new rectangles with the same area.

Pause to share the rectangles students created.

Use the Decide and Defend routine to support students in making arguments and critiquing the reasoning of others. **(MTR.4.1)**

2 Monitor

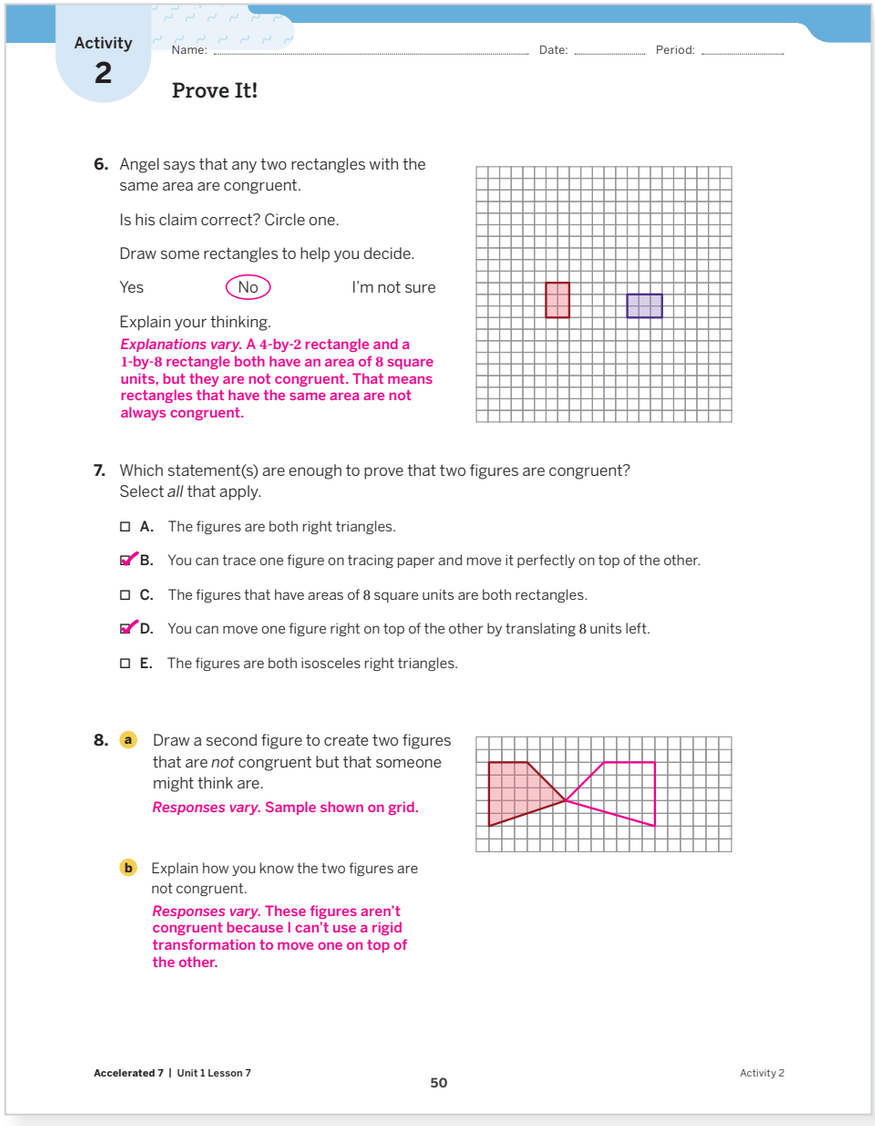
For Problem 7, **Invite students** to share the choices they made and explain their reasoning.

For Problem 8, **To encourage creativity**, challenge students to make a pair of figures that are different from their classmates'.

3 Connect

Invite students to explain their reasons for selecting or not selecting each choice.

Consider asking, "Can you give an example of two figures that meet the criteria and are not congruent?"



Activity 2

Name: Date: Period:

Prove It!

6. Angel says that any two rectangles with the same area are congruent.

Is his claim correct? Circle one.

Draw some rectangles to help you decide.

Yes (No) I'm not sure

Explain your thinking.

Explanations vary. A 4-by-2 rectangle and a 1-by-8 rectangle both have an area of 8 square units, but they are not congruent. That means rectangles that have the same area are not always congruent.

7. Which statement(s) are enough to prove that two figures are congruent? Select *all* that apply.

- ☐ **A.** The figures are both right triangles.
- ☑ **B.** You can trace one figure on tracing paper and move it perfectly on top of the other.
- ☐ **C.** The figures that have areas of 8 square units are both rectangles.
- ☑ **D.** You can move one figure right on top of the other by translating 8 units left.
- ☐ **E.** The figures are both isosceles right triangles.

8. a. Draw a second figure to create two figures that are *not* congruent but that someone might think are.

Responses vary. Sample shown on grid.

b. Explain how you know the two figures are not congruent.

Responses vary. These figures aren't congruent because I can't use a rigid transformation to move one on top of the other.

Accelerated 7 | Unit 1 Lesson 7 50 Activity 2

Whole Class | 5 min

Synthesis

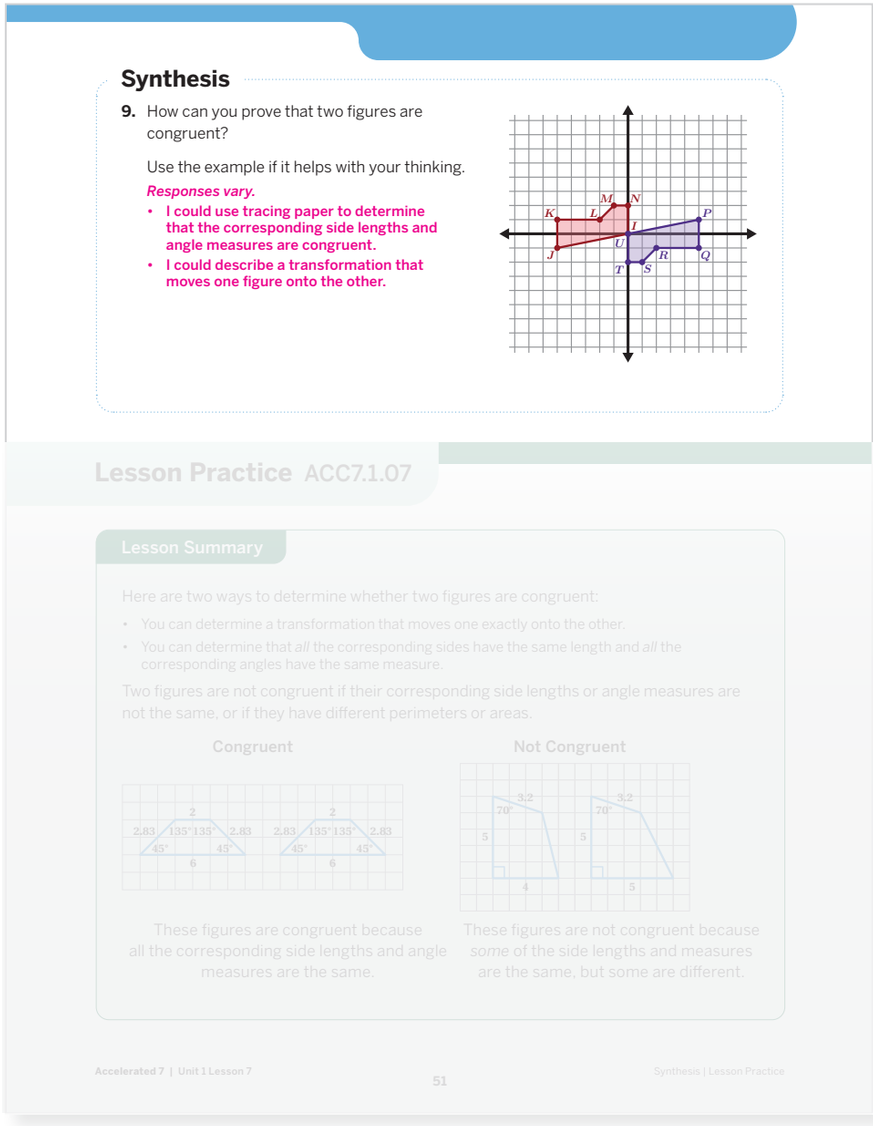
Purpose: Students synthesize their understanding of how to prove two figures are congruent.

Invite students to respond independently, and then share their thinking with a partner.

Collect and share a variety of ideas, such as:
- Using tools such as tracing paper.
- Describing sequences of transformations.

MLR **MLR2: Collect and Display** Use this routine to add to the definition of *congruence* from the previous lesson using students' language. Display any new language, examples, or diagrams for students to reference later.

Note: If time allows, invite students to revise their previous thinking based on the discussion.



Synthesis

9. How can you prove that two figures are congruent?

Use the example if it helps with your thinking.

Responses vary.
- I could use tracing paper to determine that the corresponding side lengths and angle measures are congruent.
- I could describe a transformation that moves one figure onto the other.

Lesson Practice ACC7.1.07

Lesson Summary

Here are two ways to determine whether two figures are congruent:
- You can determine a transformation that moves one exactly onto the other.
- You can determine that *all* the corresponding sides have the same length and *all* the corresponding angles have the same measure.

Two figures are not congruent if their corresponding side lengths or angle measures are not the same, or if they have different perimeters or areas.

Congruent

These figures are congruent because all the corresponding side lengths and angle measures are the same.

Not Congruent

These figures are not congruent because *some* of the side lengths and measures are the same, but some are different.

Accelerated 7 | Unit 1 Lesson 7 51 Synthesis | Lesson Practice

Lesson Takeaway: Two figures are congruent if there is a transformation that moves one onto the other. Other properties of the figures such as their side lengths or area alone are not sufficient to prove the figures are congruent.

Independent | 5 min

Show What You Know

Purpose: Students demonstrate their understanding by determining whether two figures are congruent.

Today's Goals

1. **Goal:** Use rigid transformations to determine whether two figures on a grid are congruent.
2. **Language Goal:** Justify whether two figures are congruent using vocabulary from this unit. **(Reading, Writing, Speaking, and Listening)**

Show What You Know (Assessment Resources)

Name: _____ Date: _____ Period: _____

Show What You Know 1.07

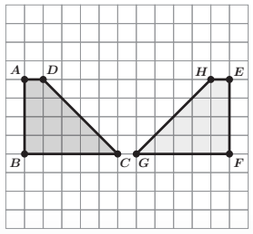
Is figure $ABCD$ congruent to figure $EFGH$? Circle one.

(Yes) No

Explain your thinking.

Explanations vary.

- Figure $ABCD$ is reflected over a vertical line. This moves figure $ABCD$ onto figure $EFGH$.
- The corresponding side lengths and angle measures are equal.



D Differentiation Use after Lesson 7

Support

Provide targeted intervention for students by using these resources.

If students need support understanding how to use transformations to prove figures are congruent:

» **Respond to Student Thinking:**

- Use the ***Justifying Congruent Figures*** **Mini-Lesson**.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 2 Extension Activities**.

Professional Learning

Reflect on your facilitation of the Decide and Defend routine from this lesson and the previous lessons. What about the routine felt successful? What are areas for growth next time? How has your facilitation grown throughout this unit?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

9. How can you prove that two figures are congruent?

Use the example if it helps with your thinking.

Responses vary.

- I could use tracing paper to determine that the corresponding side lengths and angle measures are congruent.
- I could describe a transformation that moves one figure onto the other.

Lesson Practice ACC7.1.07

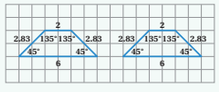
Lesson Summary

Here are two ways to determine whether two figures are congruent:

- You can determine a transformation that moves one exactly onto the other.
- You can determine that *all* the corresponding sides have the same length and *all* the corresponding angles have the same measure.

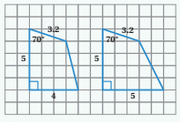
Two figures are not congruent if their corresponding side lengths or angle measures are not the same, or if they have different perimeters or areas.

Congruent



These figures are congruent because all the corresponding side lengths and angle measures are the same.

Not Congruent



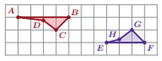
These figures are not congruent because *some* of the side lengths and measures are the same, but some are different.

Lesson Practice
ACC7.1.07

Name: _____ Date: _____ Period: _____

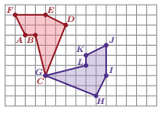
Problems 1–2: For each problem, determine whether the figures are congruent and explain how you know. ***Explanations vary.***

1.



No. Figure $ABCD$ is not congruent to figure $EFGH$ because the side length of AB is 4 units and the side length of EF is 3 units. AB and EF are the longest sides of each figure and need to be the same length for the figures to be congruent.

2.



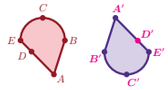
Yes. When figure $ABCDEF$ is rotated 90° clockwise around point C, it will move onto figure $KLGHIJ$.

3. If two rectangles have the same perimeter, will they always be congruent? Explain your thinking.

No. *Explanations vary.* A rectangle with a length of 3 feet and width of 2 feet will have the same perimeter as a rectangle with a length of 4 feet and a width of 1 foot. However, their corresponding sides do not have the same length, so the rectangles will not be congruent.

Problems 4–6: Here are two congruent figures.

Figure 1 Figure 2



4. Label Figure 2 with points A', B', C', and E' so that they correspond to points A, B, C, and E in Figure 1.

5. If segment AB is 2 centimeters long, how long is segment $A'B'$? Explain your thinking.

2 centimeters. *Explanations vary.* The figures are congruent, so the corresponding segments of congruent figures are also congruent.

6. Plot point D' where you think it is located. Explain your thinking.

Explanations vary. Because the figures are congruent, point D' will be on the corresponding side. It will be in the same location on $A'E'$ as D is on AE.

Lesson Practice
ACC7.1.07

Name: _____ Date: _____ Period: _____

FAST Practice

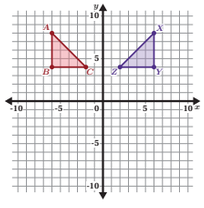
7. Triangle ABC and triangle XYZ are congruent. Which sequence of transformations will move triangle ABC onto triangle XYZ?

A. A reflection over the y-axis.

B. A reflection over the x-axis.

C. A translation 4 units right.

D. A translation 8 units right.



Spiral Review

8. Isaiah buys a shirt that costs $25.00. He has a coupon for 15% off. The sales tax is 8%. What is the total cost of the shirt after tax?

$22.95

9. Sydney invests $500 in a savings account. The simple interest rate is 3.75%. How much will Sydney have in the account after 3 years?

$556.25

10. A manufacturer decreased the amount of shampoo in a bottle from 25 fluid ounces to 22.5 fluid ounces. What percent decrease is this?

10%

Lesson Practice Item Analysis

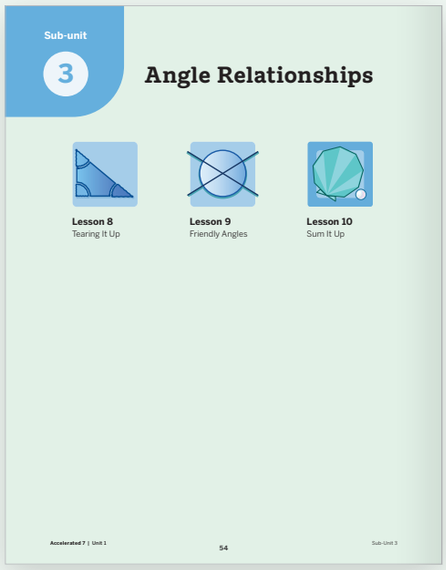
	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1–3	2	MA.8.GR.2.3
	4–6	2	MA.8.GR.2.1
FAST Practice	7	2	MA.8.GR.2.3
Spiral Review			
	8–10	1	MA.7.AR.3.1

Angle Relationships

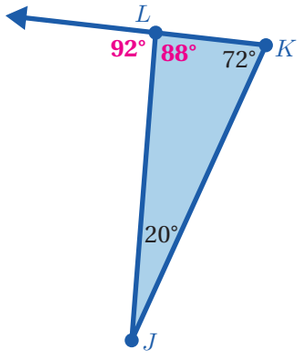
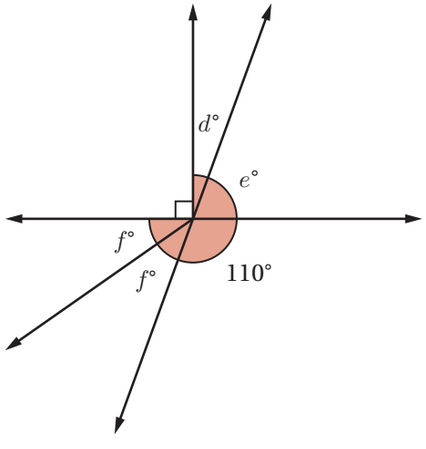
Sub-Unit 3 Goals

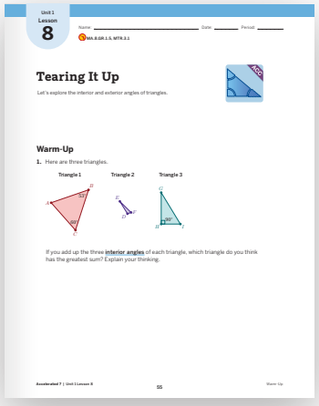
- **Goal:** Observe that the sum of the angle measures in a triangle is 180 degrees.
- **Goal:** Use special angle relationships to determine unknown angle measures.
- **Language Goal:** Describe what it means for angles to be *complementary* or *supplementary*.

In this sub-unit, students observe that the sum of the interior angle measures in a triangle is always 180°, and use rigid transformations to explain why. Students then learn about special angle relationships, such as **complementary** and **supplementary angles**, and use these angle relationships to determine unknown angle measures. Students explore the interior angle sums of different polygons and use observations to generalize.



Math That Matters Most

Concept or Skill	For example . . .
Solve problems involving the relationships of interior and exterior angles of a triangle.	Determine the measure of the missing interior and exterior angles. 
Solve mathematical problems involving the relationships between supplementary, complementary, vertical, or adjacent angles.	Here is a new angle puzzle. Determine the values of d, e, and f. *d = 20, e = 70, f = 35* 


Unit 1
Lesson
8
Name _____ Date _____ Period _____
MA.8.GR.1.5, MTR.3.1
Tearing It Up
Let's explore the interior and exterior angles of triangles.
Warm-Up
1. Here are three triangles.
Triangle 1
Triangle 2
Triangle 3
If you add up the three interior angles of each triangle, which triangle do you think has the greatest sum? Explain your thinking.
Accelerated 7 | Unit 1 Lesson 8
55
Warm-Up

Tearing It Up

Angle Sums in Triangles

Let's explore the interior and exterior angles of triangles.

Learning Outcomes

Today's Goals

1. **Goal:** Observe that the sum of the angle measures in a triangle is 180 degrees.
2. **Goal:** Observe that the sum of the measures of the *remote interior angles* of a triangle is equal to the measure of the *exterior angle*.
3. **Language Goal:** Explain the relationship between the sum of a triangle's *interior angle* measures and the angle of a straight line. **(Speaking, Listening, and Writing)**

Students explore different triangles and observe that the sum of the **interior angles** of a triangle is always 180°. Then, they generalize that if a triangle is decomposed into its three angles, the three angles can form a straight line that has an angle measure of 180°. Finally, they observe that the **remote interior angles** of a triangle can be put together to have the same measure as the **exterior angle**. **(MTR.3.1)**

Coherence

Prior Learning

In previous courses, students learned about angle measures as an attribute of polygons and identified angles in triangles as acute, right, or obtuse.

Future Learning

In Lesson 9, students will use their understanding of the sum of the angle measures in a triangle to develop and use formulas for the sums of the interior angles of other polygons.

Connections

- Students relate the measure of a straight angle and the interior angle measure sum of a triangle in a hands-on activity. (mathematical)

FL B.E.S.T. Math Benchmarks

Addressing

MA.8.GR.1.5
Solve problems involving the relationships of interior and exterior angles of a triangle.

Mathematical Thinking and Reasoning Standard: MTR.3.1

ELA Expectation: ELA.K12.EE.5.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.4.GR.1.2 **MA.4.GR.1.3** **MA.5.GR.1.1**	**MA.912.GR.1.3**

Vocabulary

New Vocabulary

exterior angle
interior angle
remote interior angles

Lesson at a Glance

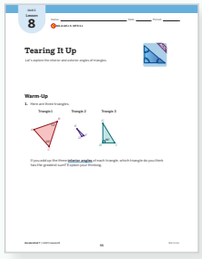
~ 45 min

MA.8.GR.1.5

Warm-Up

Pairs | 5 min

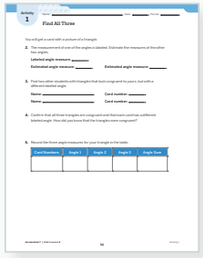
Students make predictions about the sum of **interior angles** in three different triangles.



Activity 1

Small Groups | 10 min

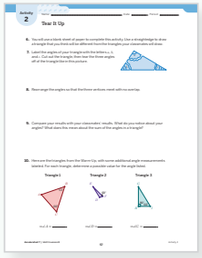
Students experiment with the interior angle sum in a triangle by matching three congruent triangles that have different angles labeled.



Activity 2

Pairs | 10 min

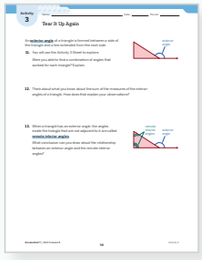
Students generalize that the three angles in any triangle can be used to form a straight angle, which always measures $180°$. MLR7: Compare and Connect



Activity 3

Pairs | 10 min

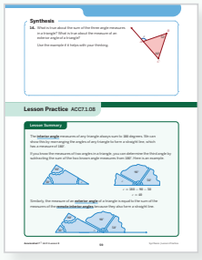
Students generalize that the measure of an **exterior angle** of a triangle is equal to the sum of the measures of the **remote interior angles**.



Synthesis

Whole Class | 5 min

Students synthesize their understanding of the relationship between the interior angles in a triangle and between an exterior angle and the remote interior angles of a triangle.



Show What You Know

Independent | 5 min

Students demonstrate their understanding by selecting three angle measures that could be the interior angles of a triangle.

Prep Checklist

Prepare the Activity 1 Cards.
Copy Activity 3 Sheets

This lesson includes:

- Student Edition
- Show What You Know
- Activity 1 Cards, one per student, pre-cut
  - Straightedges
  - Blank sheet of paper, one per student
- Activity 3 Sheet, one per pair
  - Tracing paper (as needed)

Pairs | 5 min

Warm-Up

Purpose: Students make predictions about the sum of **interior angles** in three different triangles.

1 Launch

Direct students' attention to the images of the three triangles.

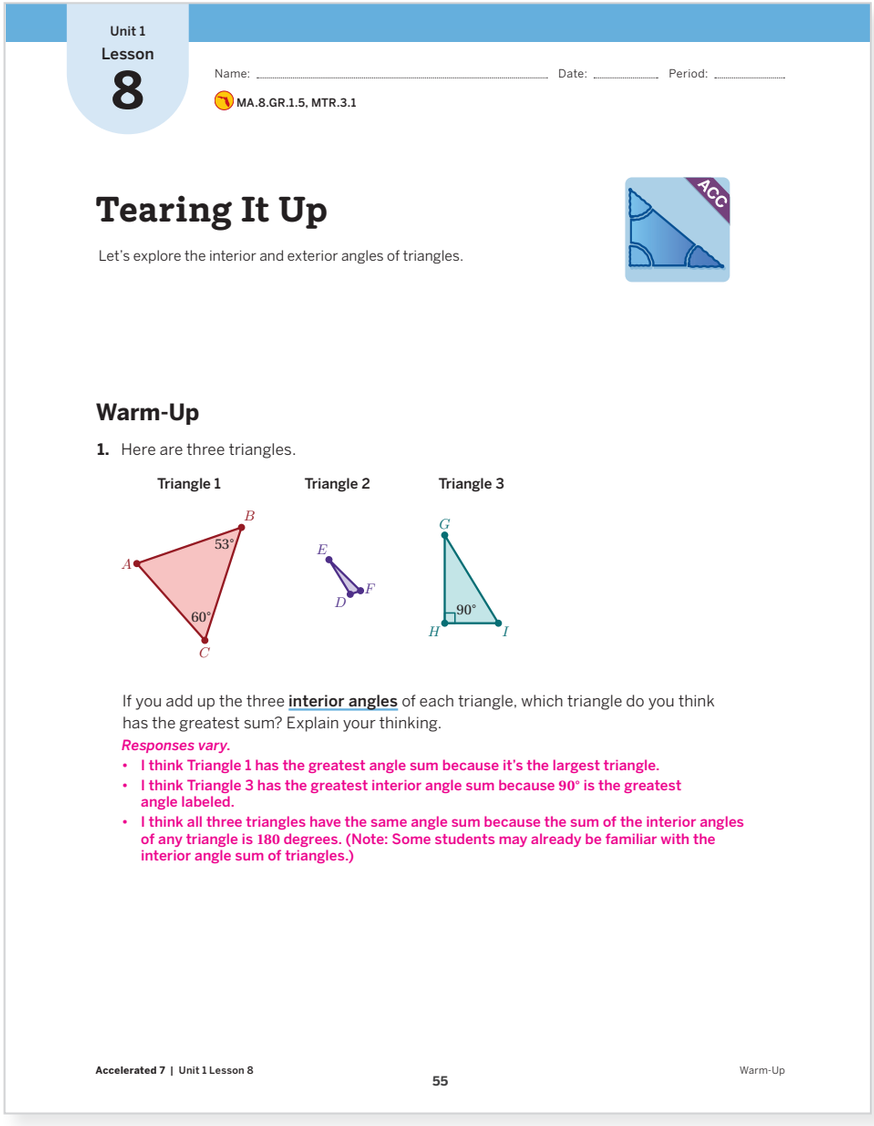
To support students getting started, consider asking, "What do you think **interior angles** *of each triangle* means?"

ELL **English Language Learners** Invite students to share what a new term reminds them of (e.g., "When I hear *interior angles*, I think . . ."). This may help surface multiple meanings and encourage students to connect to their prior knowledge. **(Reading and Listening)**

2 Connect

Invite students to share which triangle they thought had the greatest sum by polling the class and inviting them to explain their thinking.

Note: Emphasize the range of student responses. It's okay — even desirable — to lack consensus. Students will learn that the sum of the interior angles is always 180° later in the lesson.



Unit 1
Lesson
8

Name: _____ Date: _____ Period: _____

MA.8.GR.1.5, MTR.3.1

Tearing It Up

Let's explore the interior and exterior angles of triangles.

ACC

Warm-Up

1. Here are three triangles.

Triangle 1

Triangle 2

Triangle 3

If you add up the three **interior angles** of each triangle, which triangle do you think has the greatest sum? Explain your thinking.

Responses vary.

- I think Triangle 1 has the greatest angle sum because it's the largest triangle.
- I think Triangle 3 has the greatest interior angle sum because 90° is the greatest angle labeled.
- I think all three triangles have the same angle sum because the sum of the interior angles of any triangle is 180 degrees. (Note: Some students may already be familiar with the interior angle sum of triangles.)

Accelerated 7 | Unit 1 Lesson 8 55 Warm-Up

Small Groups | 10 min

Activity 1 Find All Three

Purpose: Students experiment with the interior angle sum in a triangle by matching three congruent triangles that have different angles labeled.

1 Launch

Direct students' attention to the directions for the activity and consider reading them aloud.

To surface prior knowledge, consider asking, "How might we decide if two triangles are congruent?"

Distribute one card to each student. Make tracing paper and other geometry tools available for students to use as needed.

A **Accessibility: Executive Functioning** To support this multi-step activity, invite two students to play out an example of determining whether or not their triangles are congruent.

2 Monitor

Look for a variety of strategies students use to decide which of their classmates' triangles are congruent to theirs.

D **Differentiation** *(Problems 3–4)*

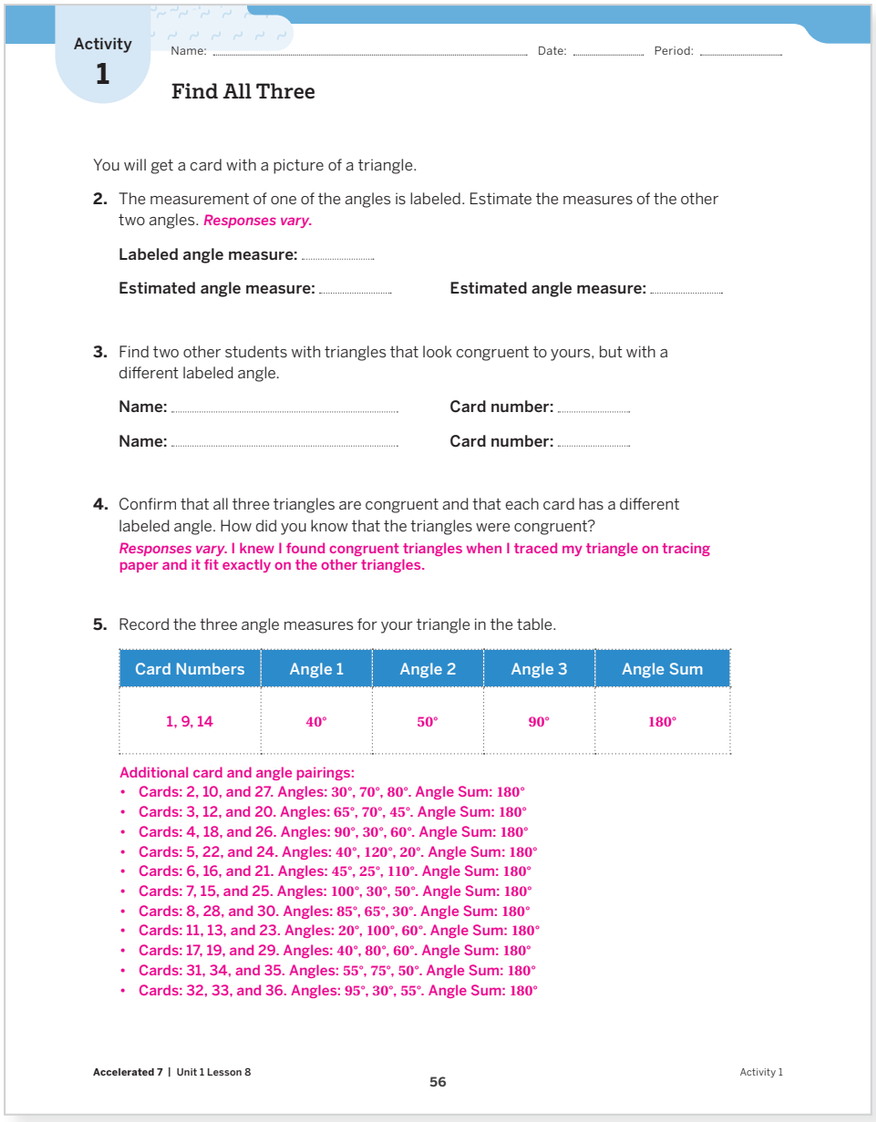
Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Encourage them to estimate each of the missing angle measures in their triangle.
Decide two triangles are congruent because they have the same angle measure (e.g., both have a 30° angle).	**Support** Consider asking, "How can you check if these triangles are congruent?"
Use a variety of strategies to check for congruence, such as: • Using tracing paper or laying the triangles on top of one another. • Describing transformations that would move one onto the other.	**Stretch** Invite students to share how they would convince a classmate that their triangles are congruent.

3 Connect

Invite groups to share their card numbers, angle measures, and the sum of their angle measures.

Consider asking:

- "What do you notice about the sum of the three angle measures of other groups' triangles?"
- "Do you think this pattern is true for any triangle? Why or why not?" **(MTR.3.1)**



Activity 1

Name: _____ Date: _____ Period: _____

Find All Three

You will get a card with a picture of a triangle.

2. The measurement of one of the angles is labeled. Estimate the measures of the other two angles. *Responses vary.*

   Labeled angle measure: _____

   Estimated angle measure: _____ **Estimated angle measure:** _____

3. Find two other students with triangles that look congruent to yours, but with a different labeled angle.

   Name: _____ **Card number:** _____

   Name: _____ **Card number:** _____

4. Confirm that all three triangles are congruent and that each card has a different labeled angle. How did you know that the triangles were congruent?

   Responses vary. I knew I found congruent triangles when I traced my triangle on tracing paper and it fit exactly on the other triangles.

5. Record the three angle measures for your triangle in the table.

Card Numbers	Angle 1	Angle 2	Angle 3	Angle Sum
1, 9, 14	40°	50°	90°	180°

Additional card and angle pairings:

- Cards: 2, 10, and 27. Angles: 30°, 70°, 80°. Angle Sum: 180°
- Cards: 3, 12, and 20. Angles: 65°, 70°, 45°. Angle Sum: 180°
- Cards: 4, 18, and 26. Angles: 90°, 30°, 60°. Angle Sum: 180°
- Cards: 5, 22, and 24. Angles: 40°, 120°, 20°. Angle Sum: 180°
- Cards: 6, 16, and 21. Angles: 45°, 25°, 110°. Angle Sum: 180°
- Cards: 7, 15, and 25. Angles: 100°, 30°, 50°. Angle Sum: 180°
- Cards: 8, 28, and 30. Angles: 85°, 65°, 30°. Angle Sum: 180°
- Cards: 11, 13, and 23. Angles: 20°, 100°, 60°. Angle Sum: 180°
- Cards: 17, 19, and 29. Angles: 40°, 80°, 60°. Angle Sum: 180°
- Cards: 31, 34, and 35. Angles: 55°, 75°, 50°. Angle Sum: 180°
- Cards: 32, 33, and 36. Angles: 95°, 30°, 55°. Angle Sum: 180°

Accelerated 7 | Unit 1 Lesson 8 56 Activity 1

Pairs | 10 min

Activity 2 Tear It Up

Purpose: Students generalize that the three angles in any triangle can be used to form a straight angle, which always measures 180°.

Short on time? Consider completing this activity as a whole class instead of each student making their own triangle.

1 Launch

Direct students' attention to the image of the triangle and the directions.

Invite a student volunteer to demonstrate the steps of the activity for the class to see. If possible, do not give away that the three angles will form a line so that students can make that connection with their own triangle.

2 Monitor

Look and listen for students who make a variety of connections about the angles in a triangle.

D Differentiation *(Problems 5–8)*

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Consider modeling with a different triangle what it means to "rearrange the angles so that the three vertices meet with no overlap."
Recognize that the angles make a straight line but do not connect this with 180°.	**Strengthen** Consider asking, "What do you know about angles that combine to make a straight line? How does that connect to what we did earlier?"
Make connections between Activity 1 and that straight lines measure 180°.	**Strengthen** Invite them to think about how they might share their thinking during the Connect.

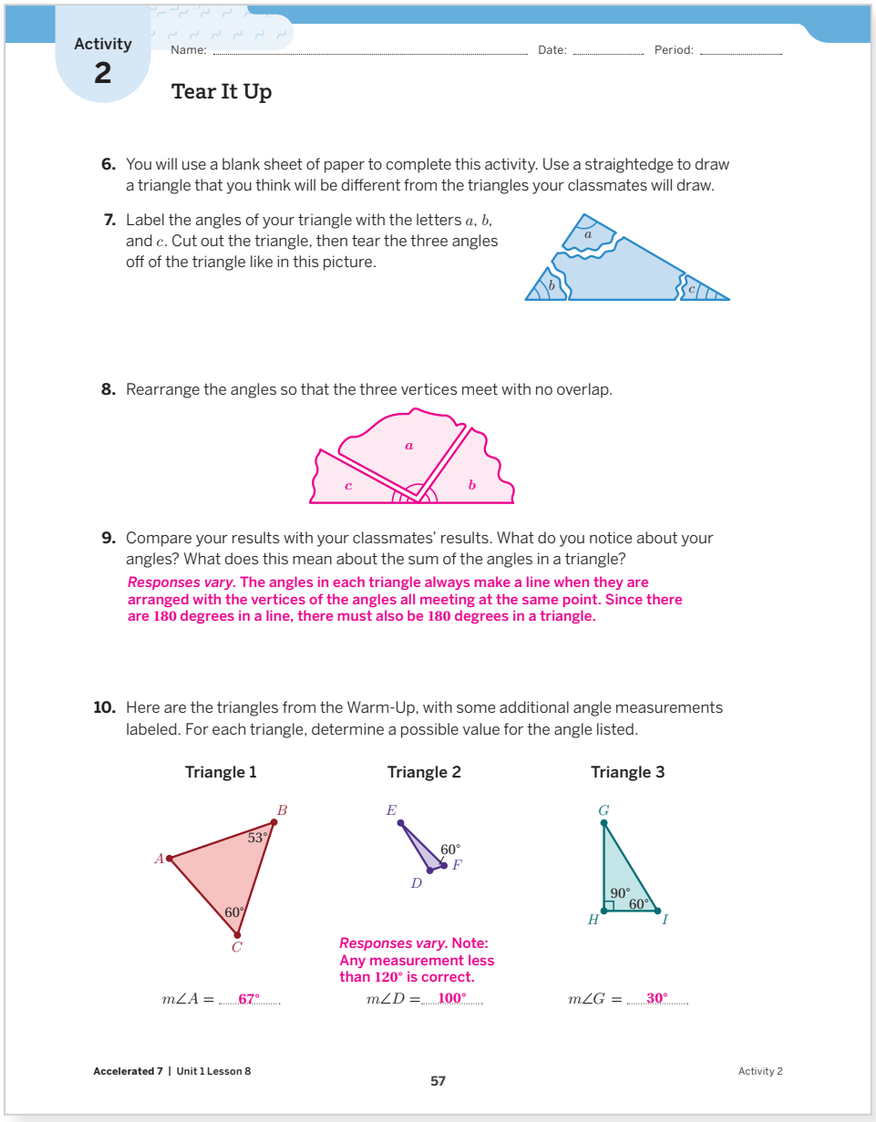
3 Connect

Invite students to form small groups to compare triangles.

MLR MLR7: Compare and Connect Invite students to analyze and compare their triangle with their classmates, and then connect to what they noticed in Activity 1. See the Routine Facilitation Guide for more information.

To surface the Key Takeaway, consider displaying the triangles from Problem 10, asking:

- "What was in common about the angle relationship between your triangle and your classmates' triangles? How does this relate to what we noticed in Activity 1?"
- "How did you use what you know about angles in a triangle to determine the unknown angles?"
- "Do you think the experiments we have done are enough to show that this relationship is always true? What might we do to be more sure?"



Activity 2

Name: _____ Date: _____ Period: _____

Tear It Up

6. You will use a blank sheet of paper to complete this activity. Use a straightedge to draw a triangle that you think will be different from the triangles your classmates will draw.

7. Label the angles of your triangle with the letters a, b, and c. Cut out the triangle, then tear the three angles off of the triangle like in this picture.

8. Rearrange the angles so that the three vertices meet with no overlap.

9. Compare your results with your classmates' results. What do you notice about your angles? What does this mean about the sum of the angles in a triangle?

Responses vary. The angles in each triangle always make a line when they are arranged with the vertices of the angles all meeting at the same point. Since there are 180 degrees in a line, there must also be 180 degrees in a triangle.

10. Here are the triangles from the Warm-Up, with some additional angle measurements labeled. For each triangle, determine a possible value for the angle listed.

Triangle 1

$m\angle A =$ 67°

Triangle 2

Responses vary. Note: Any measurement less than 120° is correct.

$m\angle D =$ 100°

Triangle 3

$m\angle G =$ 30°

Accelerated 7 | Unit 1 Lesson 8 57 Activity 2

Key Takeaway: When the angles in a triangle are rearranged with a common vertex, they make a straight angle, which measures 180°. This fact is useful for determining the measures of unknown angles in a triangle.

Pairs | 10 min

Activity 3 Tear It Up Again

Purpose: Students generalize that the measure of an **exterior angle** of a triangle is equal to the sum of the measures of the **remote interior angles**.

1 Launch

Direct students' attention to the definition of an **exterior angle**.

Consider asking, "What type of angles were we looking at in the previous activities?"

Distribute the Activity 3 Sheet to each pair. Encourage them to work together to determine how the interior angles can cover the exterior angle for each triangle.

2 Monitor

Invite students to share their ideas.

ELL **English Language Learners** Encourage students to paraphrase each other's ideas to help them make connections and incorporate vocabulary from the unit. **(Speaking and Listening)**

Listen for students who make a connection between the fact that the sum of the measures of the interior angles is $180°$ and that the measures of the adjacent interior and exterior angles add up to $180°$.

3 Connect

To surface the Key Takeaway, consider asking:

- "Did you get the same result for both exterior angles?"
- "If you drew a different exterior angle on the triangle, do you think you would get the same result?"

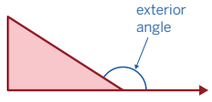
D **Differentiation: (Strengthen)** Invite students to draw other exterior angles on the triangles and test their theories.

Activity 3

Name: _____ Date: _____ Period: _____

Tear It Up Again

An **exterior angle** of a triangle is formed between a side of the triangle and a line extended from the next side.



11. You will use the Activity 3 Sheet to explore.

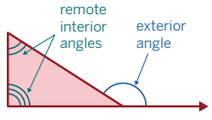
Were you able to find a combination of angles that worked for each triangle? Explain.

Responses vary. **Yes, I found that angle x and angle y covered the exterior angle for both triangles.**

12. Think about what you know about the sum of the measures of the interior angles of a triangle. How does that explain your observations?

Responses vary. **The sum of the measures of the interior angles is $180°$, which is a straight line. Angle z and the exterior angle also make a straight line, so it makes sense that angles x, y, and z together make a straight line.**

13. When a triangle has an exterior angle, the angles inside the triangle that are not adjacent to it are called **remote interior angles**.



What conclusion can you draw about the relationship between an exterior angle and the remote interior angles?

Responses vary. **The measures of the remote interior angles add up to the measure of the exterior angle.**

Accelerated 7 | Unit 1 Lesson 8 58 Activity 3

Key Takeaway: When you put the **remote interior angles** of a triangle together, they are the same size as the **exterior angle**.

Whole Class | 5 min

Synthesis

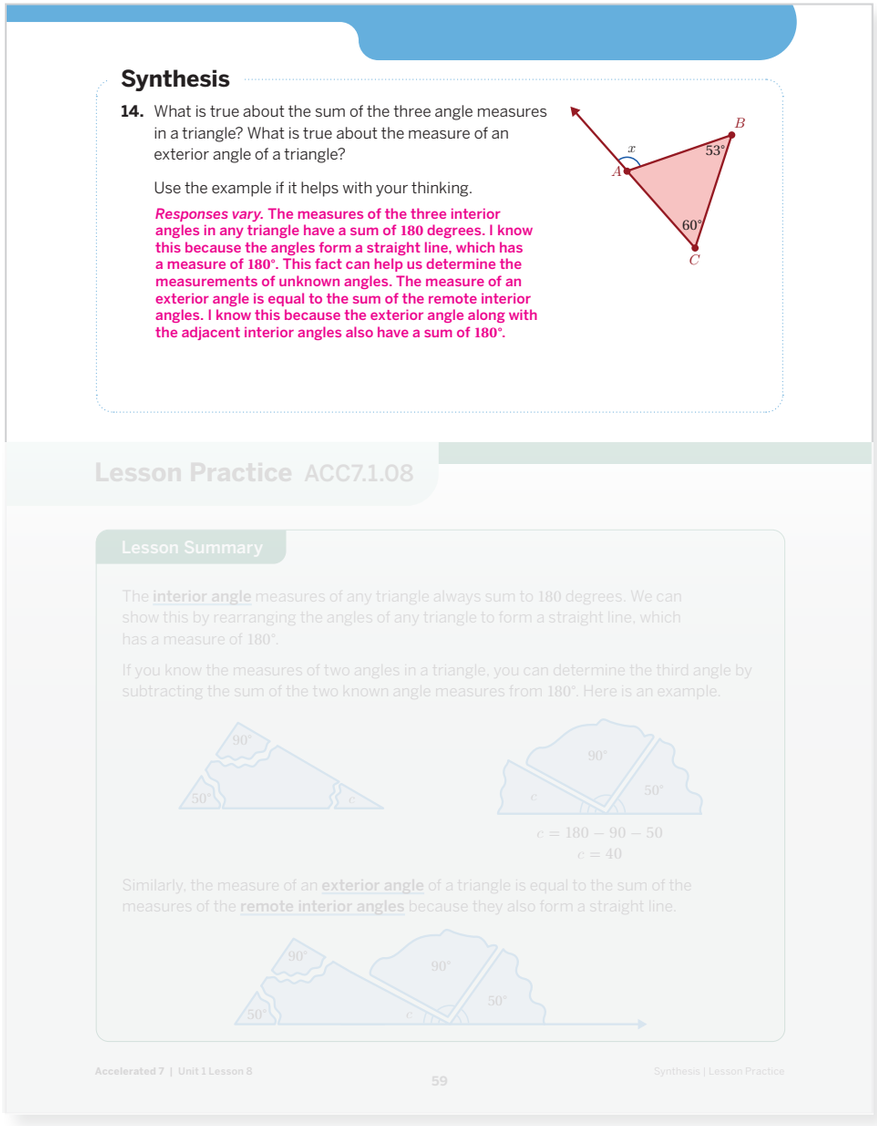
Purpose: Students synthesize their understanding of the relationship between the interior angles in a triangle and between an exterior angle and the remote interior angles of a triangle.

Invite students to respond independently, and then share their thinking with a partner.

Collect and share a variety of ideas, such as:

- The sum of the angles in a triangle is 180 degrees.
- The three angle measures in a triangle form a straight line or straight angle, which is 180 degrees.
- Using the relationship between angles in a triangle to determine unknown angles.
- The measure of an exterior angle of a triangle is equal to the sum of the measures of the remote interior angles.

Note: If time allows, invite students to revise their previous thinking based on the discussion.



Synthesis

14. What is true about the sum of the three angle measures in a triangle? What is true about the measure of an exterior angle of a triangle?

Use the example if it helps with your thinking.

Responses vary. The measures of the three interior angles in any triangle have a sum of 180 degrees. I know this because the angles form a straight line, which has a measure of 180°. This fact can help us determine the measurements of unknown angles. The measure of an exterior angle is equal to the sum of the remote interior angles. I know this because the exterior angle along with the adjacent interior angles also have a sum of 180°.

Lesson Practice ACC7.1.08

Lesson Summary

The interior angle measures of any triangle always sum to 180 degrees. We can show this by rearranging the angles of any triangle to form a straight line, which has a measure of 180°.

If you know the measures of two angles in a triangle, you can determine the third angle by subtracting the sum of the two known angle measures from 180°. Here is an example.

$c = 180 - 90 - 50$

$c = 40$

Similarly, the measure of an exterior angle of a triangle is equal to the sum of the measures of the remote interior angles because they also form a straight line.

Accelerated 7 | Unit 1 Lesson 8 59 Synthesis | Lesson Practice

Lesson Takeaway: The sum of the three **interior angles** in a triangle is 180°. One way to see this is to rearrange the angles in a triangle to form a straight line, which has an angle measure of 180°. This also shows that the measure of an **exterior angle** of a triangle is equal to the sum of the measures of the **remote interior angles**.

Independent | 5 min

Show What You Know

Purpose: Students demonstrate their understanding by selecting three angle measures that could be the interior angles of a triangle.

Today's Goals

1. **Goal:** Observe that the sum of the angle measures in a triangle is 180 degrees.
2. **Goal:** Observe that the sum of the measures of the *remote interior angles* of a triangle is equal to the measure of the *exterior angle*.
3. **Language Goal:** Explain the relationship between the sum of a triangle's *interior angle* measures and the angle of a straight line. **(Speaking, Listening, and Writing)**

Show What You Know (Assessment Resources)

Name: _____ Date: _____ Period: _____

Show What You Know 1.08

Select *three* angle measures that could be angles in the same triangle.

- ☑ A. 42°
- ☐ B. 180°
- ☐ C. 90°
- ☑ D. 18°
- ☑ E. 120°

Explain your reasoning.

Explanations vary. The three angle measures 42°, 18°, and 120° have a sum of 180 degrees, and all triangles have an interior angle sum of 180 degrees.

D Differentiation Use after Lesson 8

Support

Provide targeted intervention for students by using these resources.

If students need support understanding how to determine which angles create a triangle:

» **Respond to Student Thinking:**

- Review the **Lesson Summary** and create several examples of angles in a triangle as a group.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 3 Extension Activities**.

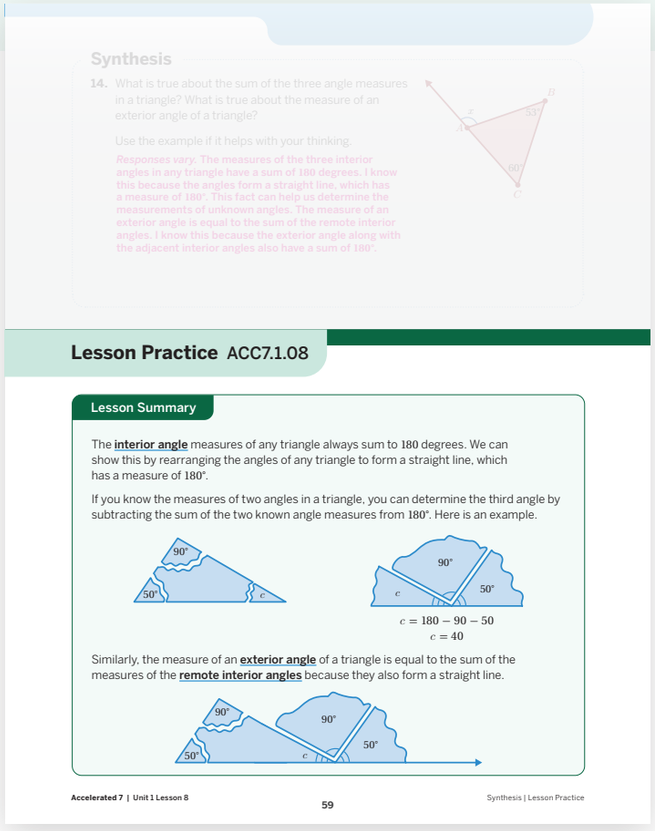
Professional Learning

Take a moment to reflect on this lesson. In this lesson, students had opportunities to notice the relationship between the angle measures in a triangle. How might having built this understanding slowly throughout the lesson support them in using this relationship in Lesson 9?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.



Synthesis

14. What is true about the sum of the three angle measures in a triangle? What is true about the measure of an exterior angle of a triangle?

Use the example if it helps with your thinking.

Responses vary. The measures of the three interior angles in any triangle have a sum of 180 degrees. I know this because the angles form a straight line, which has a measure of 180°. This fact can help us determine the measurements of unknown angles. The measure of an exterior angle is equal to the sum of the remote interior angles. I know this because the exterior angle along with the adjacent interior angles also have a sum of 180°.

Lesson Practice ACC7.1.08

Lesson Summary

The **interior angle** measures of any triangle always sum to 180 degrees. We can show this by rearranging the angles of any triangle to form a straight line, which has a measure of 180°.

If you know the measures of two angles in a triangle, you can determine the third angle by subtracting the sum of the two known angle measures from 180°. Here is an example.

$c = 180 - 90 - 50$

$c = 40$

Similarly, the measure of an **exterior angle** of a triangle is equal to the sum of the measures of the **remote interior angles** because they also form a straight line.

Lesson Practice
ACC7.1.08

Name: _____ Date: _____ Period: _____

1. If triangle ABC is a right triangle and $m\angle A = 40°$, what are possible measures for angles B and C?

Responses vary.
- $m\angle B = 50°$ and $m\angle C = 90°$
- $m\angle B = 90°$ and $m\angle C = 50°$

2. Triangle ABC is an isosceles triangle and the measure of one of the angles is 40°. List a set of possible angle measures in this triangle.

Responses vary.
- 70°, 70°, 40°
- 40°, 40°, 100°

3. Can there be a triangle with two right angles? Explain your thinking.

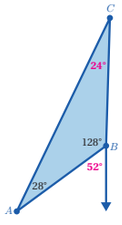
No. *Explanations vary.* The sum of the three interior angles of a triangle is 180 degrees. Because two right angles already have a sum of 180 degrees, the measure of the third angle must be 0°, which is not possible for a triangle.

4. Angle A in triangle ABC is obtuse. Can angle B or angle C be obtuse? Explain your thinking.

No. *Explanations vary.* The angles in a triangle need to add to 180 degrees. If two of the angles are obtuse, then their sum is already more than 180 degrees.

Problems 5–6: Determine the measure of each missing interior and exterior angle.

5.



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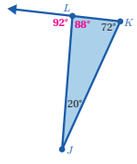


Lesson Practice
ACC7.1.08

Name: _____ Date: _____ Period: _____

FAST Practice

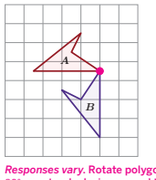
7. Select *all* of the sets of angles that could make a triangle.

- ☑ **A.** 60°, 60°, 60°
- ☐ **B.** 90°, 90°, 45°
- ☐ **C.** 30°, 40°, 50°
- ☑ **D.** 90°, 45°, 45°
- ☑ **E.** 120°, 30°, 30°

Spiral Review

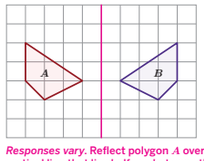
Problems 8–9: For each pair of polygons, describe a transformation that shows polygon A is congruent to polygon B.

8.



Responses vary. Rotate polygon A 90° counterclockwise around the bottom-right vertex of polygon A.

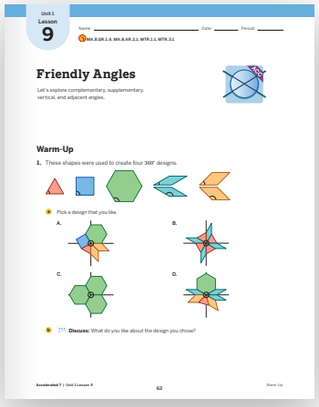
9.



Responses vary. Reflect polygon A over a vertical line that lies halfway between the two polygons.

Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1, 3–4	2	MA.8.GR.1.5
	2	3	MA.8.GR.1.5
	5–6	1	MA.8.GR.1.5
FAST Practice	7	2	MA.8.GR.1.5
Spiral Review			
	8–9	2	MA.8.GR.2.1


Unit 1
Lesson
9
Name _____ Date _____ Period _____
MA.8.GR.1.4, MA.8.AR.2.1, MTR.1.1, MTR.3.1
Friendly Angles
Let's explore complementary, supplementary, vertical, and adjacent angles.
Warm-Up
1. These shapes were used to create four 360° designs.
Pick a design that you like.
A.
B.
C.
D.
Discuss: What do you like about the design you chose?
Accelerated 7 | Unit 1 Lesson 9
62
Warm-Up

Friendly Angles

Complementary, Supplementary, Vertical, and Adjacent Angles

Let's explore complementary, supplementary, vertical, and adjacent angles.

Learning Outcomes

Today's Goals

1. **Goal:** Determine unknown angle measures by reasoning about *complementary*, *supplementary*, *vertical*, and *adjacent* angles.
2. **Goal:** Use angle relationships to write equations and determine unknown angle measures.
3. **Language Goal:** Describe what it means for angles to be *complementary* or *supplementary*. **(Writing)**
4. **Language Goal:** Explain how the parts of an equation represent an angle diagram. **(Writing, Speaking, and Listening)**

Students continue to explore angle measurements and their relationships. They use their creativity and understanding of new angle relationships to determine angle measures. Students identify and describe specific angle relationships: **complementary angles**, **supplementary angles**, **vertical angles**, and **adjacent angles**.

Coherence

Prior Learning

In Lesson 8, students explored the interior angles of triangles, coming to understand that they always add up to $180°$.

Future Learning

In Lesson 10, students will explore angle relationships in other polygons.

Connections

- Students explore special angle pair relationships through artistic designs composed of angles and lines (mathematical, cross-curricular)
- Students relate solving equations to determining the measure of angles formed by intersecting lines and rays. (mathematical)

FL B.E.S.T. Math Benchmarks

Addressing

MA.8.GR.1.4
Solve mathematical problems involving the relationships between supplementary, complementary, vertical or adjacent angles.

Also Addressing: **MA.8.AR.2.1**

Mathematical Thinking and Reasoning Standards: MTR.1.1, MTR.3.1

ELA Expectations: ELA.K12.EE.1.1, ELA.K12.EE.6.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.4.GR.1	**MA.912.GR.1.1**
MA.5.GR.1.1	

Vocabulary

New Vocabulary

- adjacent angles
- complementary angles
- supplementary angles
- vertical angles

Lesson at a Glance

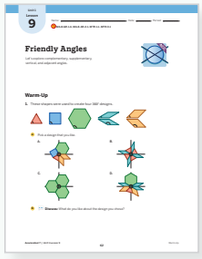
~ 45 min

MA.8.GR.1.4, MA.8.AR.2.1

Warm-Up

Pairs | 5 min

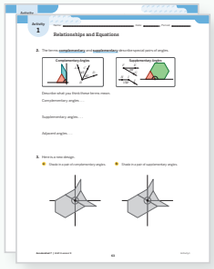
Students express their creativity and familiarize themselves with the shapes they will investigate in the lesson.



Activity 1

Pairs | 10 min

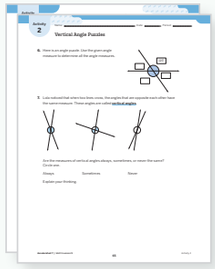
Students learn and use the precise terms **complementary angles** and **supplementary angles** while making connections between angle diagrams and equations.



Activity 2

Pairs | 10 min

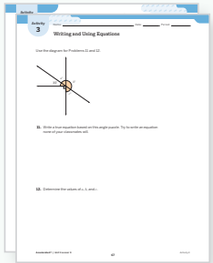
Students explain why the measures of **vertical angles** are equal. They also use vertical angles to solve angle puzzles. **Routine:** Decide and Defend **(MTR.4.1)**



Activity 3

Pairs | 10 min

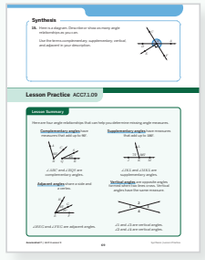
Students strengthen their understanding of the connections between equations and angle diagrams. Students fluently write equations that represent angle diagrams and also make sense of other students' equations. **(MTR.3.1)**



Synthesis

Whole Class | 5 min

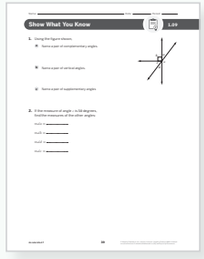
Students synthesize their description of angle relationships using the new terms *complementary*, *supplementary*, *vertical*, and *adjacent*. MLR2: Collect and Display



Show What You Know

Independent | 5 min

Students demonstrate their understanding by using angle relationships in a diagram to determine missing angle measures.



Prep Checklist

This lesson includes:

Student Edition
Show What You Know

Pairs | 5 min

Warm-Up

Purpose: Students express their creativity and familiarize themselves with the shapes they will investigate in the lesson.

1 Launch

Students will explore four premade designs.

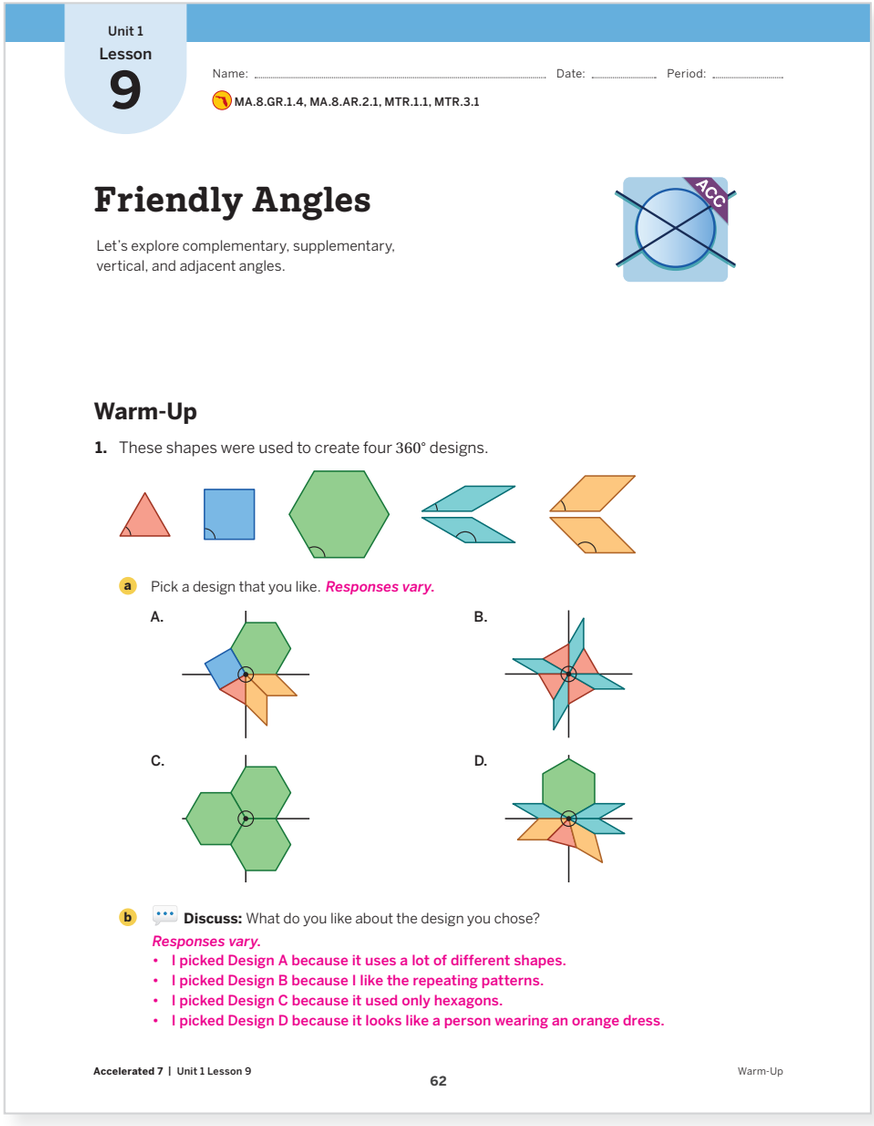
ELL **English Language Learners** Invite students to share what they know about the polygons and their angles to increase access to the task. **(Reading and Listening)**

Listen for interesting observations to share during the Connect.

2 Connect

To encourage playfulness, invite students to share unique reasons for why they liked the design they chose.

To surface connections, consider asking, "How are these designs alike and different from each other?" Some students may notice that B and C have repeated patterns. Others may notice that none of the four designs have any gaps meaning the angles at the center add up to $360°$.



Unit 1
Lesson
9

Name: ..... Date: ..... Period: .....

MA.8.GR.1.4, MA.8.AR.2.1, MTR.1.1, MTR.3.1

Friendly Angles

Let's explore complementary, supplementary, vertical, and adjacent angles.

Warm-Up

1. These shapes were used to create four $360°$ designs.

a Pick a design that you like. *Responses vary.*

A. B. C. D.

b **Discuss:** What do you like about the design you chose?

Responses vary.

- I picked Design A because it uses a lot of different shapes.
- I picked Design B because I like the repeating patterns.
- I picked Design C because it used only hexagons.
- I picked Design D because it looks like a person wearing an orange dress.

Accelerated 7 | Unit 1 Lesson 9 62 Warm-Up

Pairs | 10 min

Activity 1 Relationships and Equations

Purpose: Students learn and use the precise terms **complementary angles** and **supplementary angles** while making connections between angle diagrams and equations.

1 Launch

Consider asking, "What do you notice and wonder about each diagram?"

Create a class definition for **complementary angles** and **supplementary angles** using students' language and the glossary.

2 Monitor

In Problem 2, **Look for** correct responses that use **adjacent angles** as well as correct responses that use non-adjacent angles. Consider mentioning to students that complementary angles and supplementary angles may or may not be adjacent (i.e., they may or may not share a vertex and have a common side). These words describe the sum of the angle measures rather than their positions.

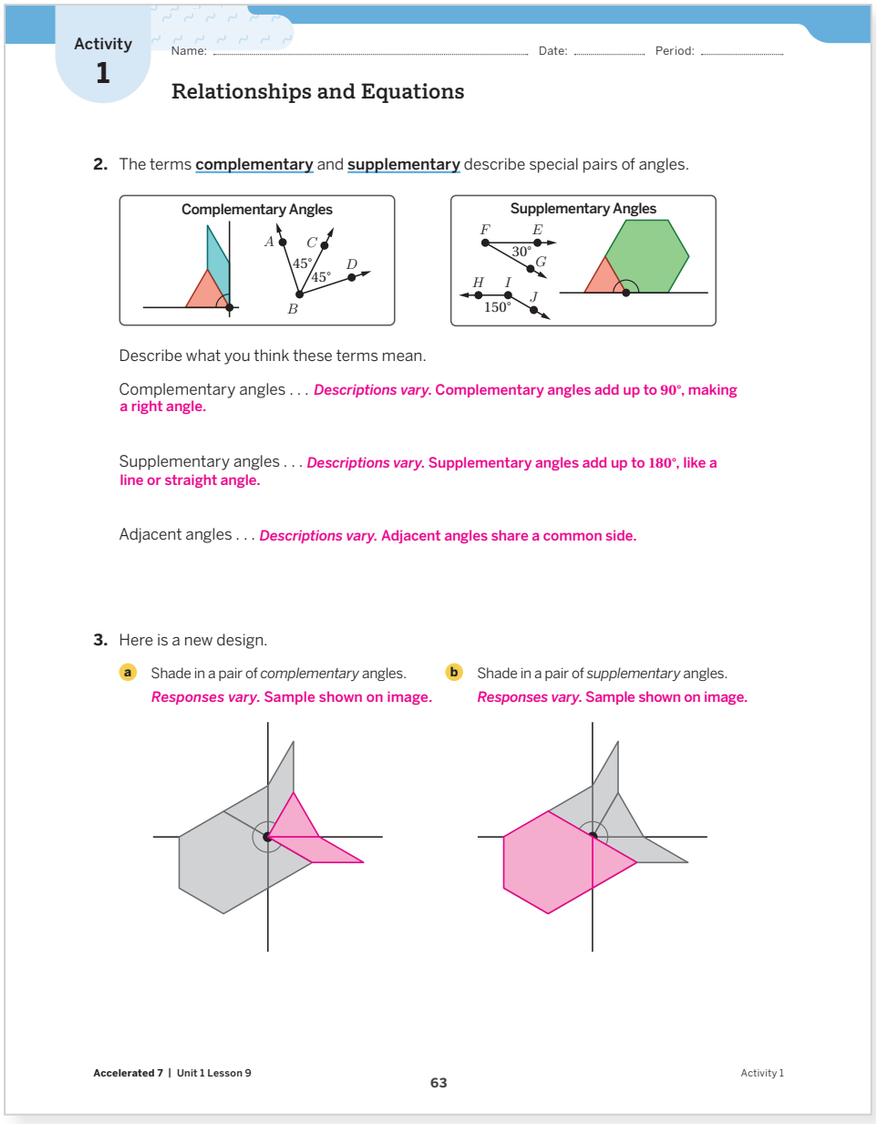
Pause briefly to discuss angle relationships.

Consider asking:

- "How do you know if a pair of angles is complementary, supplementary, or neither?"
- "Can you identify a different pair of complementary angles? Can you identify a different pair of supplementary angles?"

Note: Complementary angles and supplementary angles technically describe pairs of angles. Look for students who select three or more angles that correctly sum to 180° for the supplementary angle question.

Activity 1 continued >



Activity 1

Name: ..... Date: ..... Period: .....

Relationships and Equations

2. The terms **complementary** and **supplementary** describe special pairs of angles.

Complementary Angles

Supplementary Angles

Describe what you think these terms mean.

Complementary angles . . . ***Descriptions vary.*** **Complementary angles add up to 90°, making a right angle.**

Supplementary angles . . . ***Descriptions vary.*** **Supplementary angles add up to 180°, like a line or straight angle.**

Adjacent angles . . . ***Descriptions vary.*** **Adjacent angles share a common side.**

3. Here is a new design.

a Shade in a pair of *complementary* angles.
Responses vary. **Sample shown on image.**

b Shade in a pair of *supplementary* angles.
Responses vary. **Sample shown on image.**

Accelerated 7 | Unit 1 Lesson 9 63 Activity 1

Pairs | 10 min

Activity 1 Relationships and Equations (continued)

Purpose: Students learn and use the precise terms **complementary angles** and **supplementary angles** while making connections between angle diagrams and equations.

2 Monitor

For Problem 4, **Invite students** to sketch in their Student Edition to show their thinking if it helps them with their explanations.

A **Accessibility: Conceptual Processing** Invite students to highlight where they see the $2x$ and the 120 on the diagram to support connecting the equation and diagram.

For Problem 5, **Encourage students** to share their reasoning for each equation with a partner and to work together to reach an agreement about which equations represent parts of the diagram. **(MTR.1.1)**

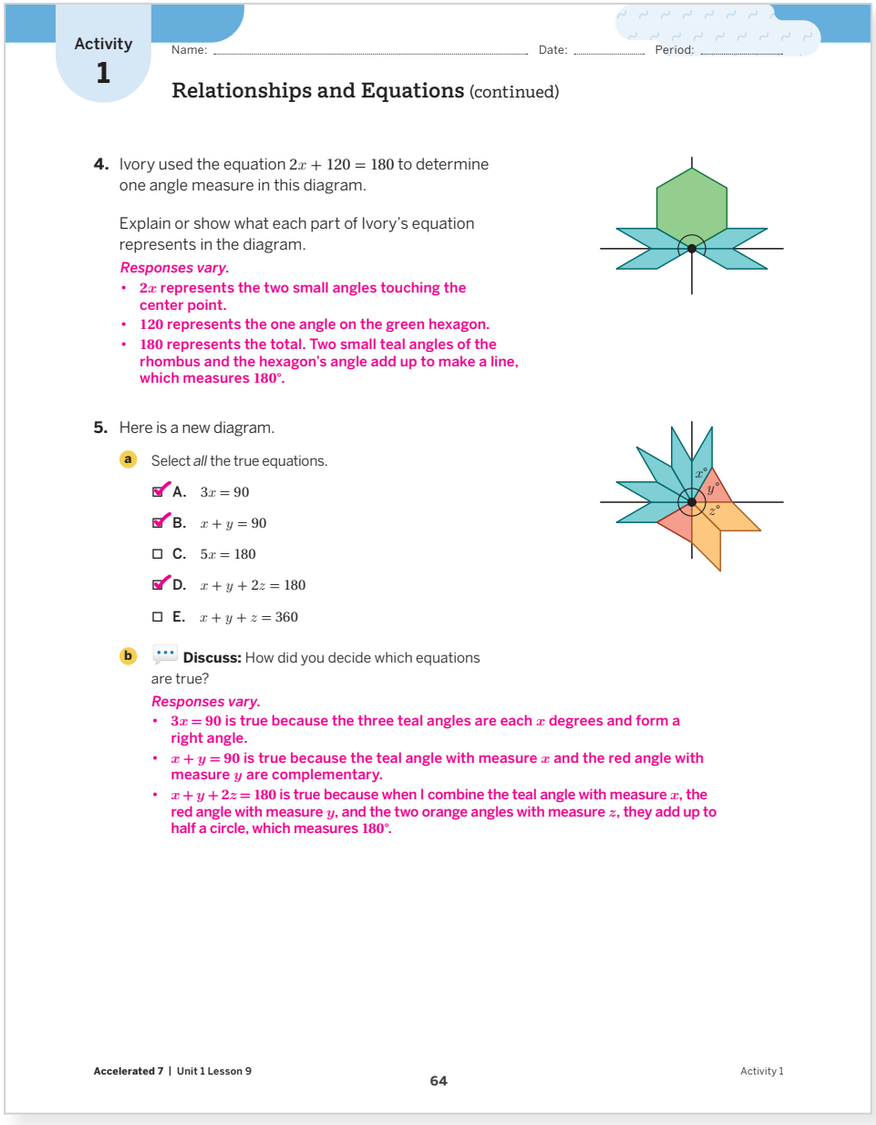
D **Differentiation** *(Problems 4 and 5)*

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Encourage them to estimate the size of the green angle relative to the $180°$ measure of a straight angle.
Use a variety of strategies to determine if equations are true, such as: • Using friendly and familiar angles. • Rearranging figures. • Writing repetitive addition as multiplication. • Adding different figures together.	**Strengthen** Invite students to share during the Connect. Consider asking, "What is another true equation that is not listed?"

3 Connect

Discuss each equation. Invite a different student to explain their reasoning for selecting each equation. Then discuss whether the equation is true or false and why. If it does not come up naturally, consider asking students what strategies they have for determining if angles make a straight angle (e.g., holding up a piece of paper).

To surface the Key Takeaway, consider asking, "How could you change an equation to make it true?"



Activity 1

Name: _____ Date: _____ Period: _____

Relationships and Equations (continued)

4. Ivory used the equation $2x + 120 = 180$ to determine one angle measure in this diagram.

Explain or show what each part of Ivory's equation represents in the diagram.

Responses vary.

- $2x$ represents the two small angles touching the center point.
- 120 represents the one angle on the green hexagon.
- 180 represents the total. Two small teal angles of the rhombus and the hexagon's angle add up to make a line, which measures $180°$.

5. Here is a new diagram.

a. Select *all* the true equations.

- ☑ A. $3x = 90$
- ☑ B. $x + y = 90$
- ☐ C. $5x = 180$
- ☑ D. $x + y + 2z = 180$
- ☐ E. $x + y + z = 360$

b. **Discuss:** How did you decide which equations are true?

Responses vary.

- $3x = 90$ is true because the three teal angles are each x degrees and form a right angle.
- $x + y = 90$ is true because the teal angle with measure x and the red angle with measure y are complementary.
- $x + y + 2z = 180$ is true because when I combine the teal angle with measure x, the red angle with measure y, and the two orange angles with measure z, they add up to half a circle, which measures $180°$.

Accelerated 7 | Unit 1 Lesson 9 64 Activity 1

Key Takeaway: Certain angle relationships have special names. **Complementary angles** are pairs of angles that add to $90°$. **Supplementary angles** are pairs of angles that add to $180°$.

Pairs | 10 min

Activity 2 Vertical Angle Puzzles

Purpose: Students explain why the measures of **vertical angles** are equal. They also use vertical angles to solve angle puzzles.

Short on time? Consider discussing an angle to reveal as a class for Problem 9.

1 Launch

To support students getting started, consider asking:

- "How are the angles related?"
- "What special relationships can you identify?"

2 Monitor

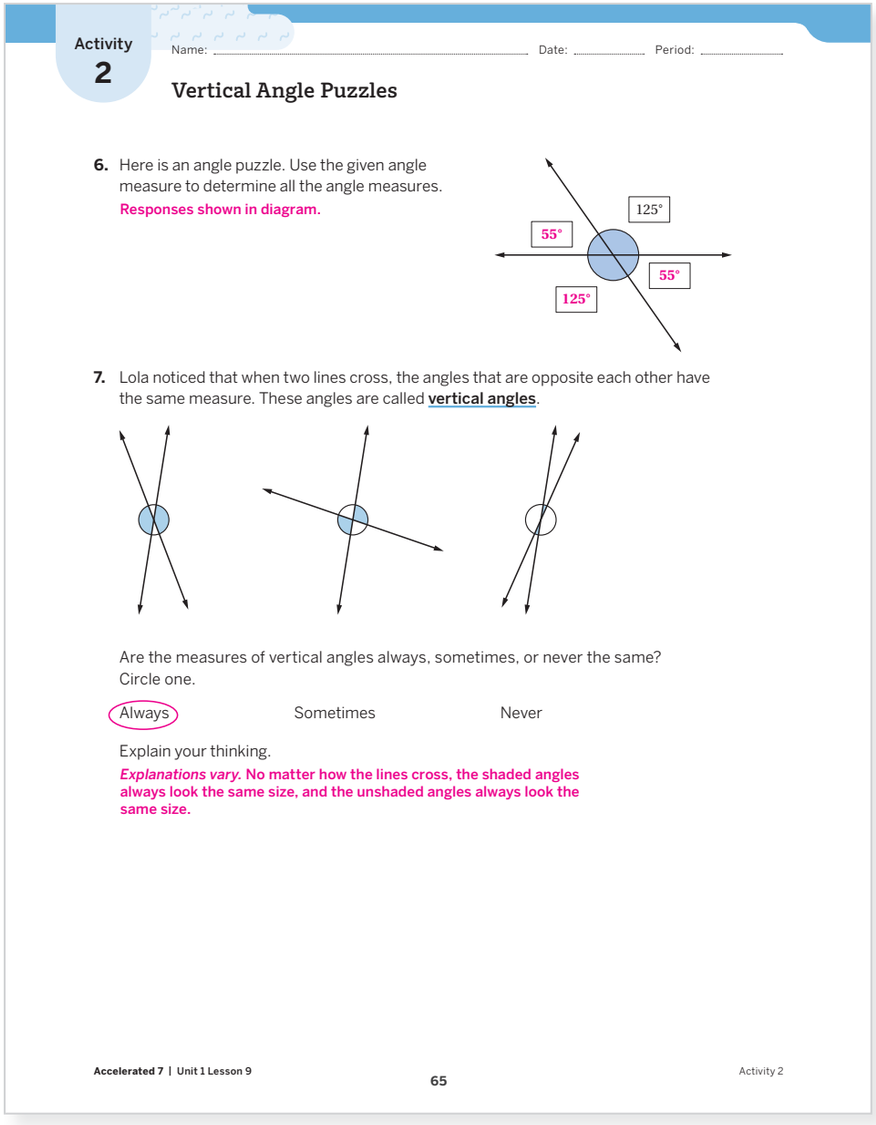
Look for students who are precise in naming familiar angle relationships including complementary angles and supplementary angles.

Consider asking, "What do you notice and wonder about the angle relationships and measures that you discovered?"

Students are given three examples of vertical angles. Encourage them to draw additional examples if it helps with their thinking.

ELL **English Language Learners** Provide sentence frames to help students explain their reasoning (e.g., Vertical angles are always/sometimes/never the same size because _____.).

Activity 2 continued >



Activity 2

Name: _____ Date: _____ Period: _____

Vertical Angle Puzzles

6. Here is an angle puzzle. Use the given angle measure to determine all the angle measures.

Responses shown in diagram.

7. Lola noticed that when two lines cross, the angles that are opposite each other have the same measure. These angles are called **vertical angles**.

Are the measures of vertical angles always, sometimes, or never the same? Circle one.

Always Sometimes Never

Explain your thinking.

Explanations vary. No matter how the lines cross, the shaded angles always look the same size, and the unshaded angles always look the same size.

Accelerated 7 | Unit 1 Lesson 9 65 Activity 2

Pairs | 10 min

Activity 2 Vertical Angle Puzzles (continued)

Purpose: Students explain why the measures of **vertical angles** are equal. They also use vertical angles to solve angle puzzles.

Short on time? Consider discussing an angle to reveal as a class for Problem 9.

2 Monitor

To surface connections, encourage students to revisit the previous page to review vertical angles. Discuss each pair of angles. Invite a student who selected each pair to explain their reasoning. Then discuss whether the angles definitely have the same angle measures. Consider recording any ideas that emerge for all students to reference throughout the unit.

Consider asking, "If you rotated the diagonal line, what would happen to angles 2, 3, and 5? How can this help you decide if 2 and 3 or 3 and 5 are vertical angles?"

To support students getting started, consider asking, "What angle would be the most helpful to know?"

Invite students to share with a classmate which angle(s) they chose and why.

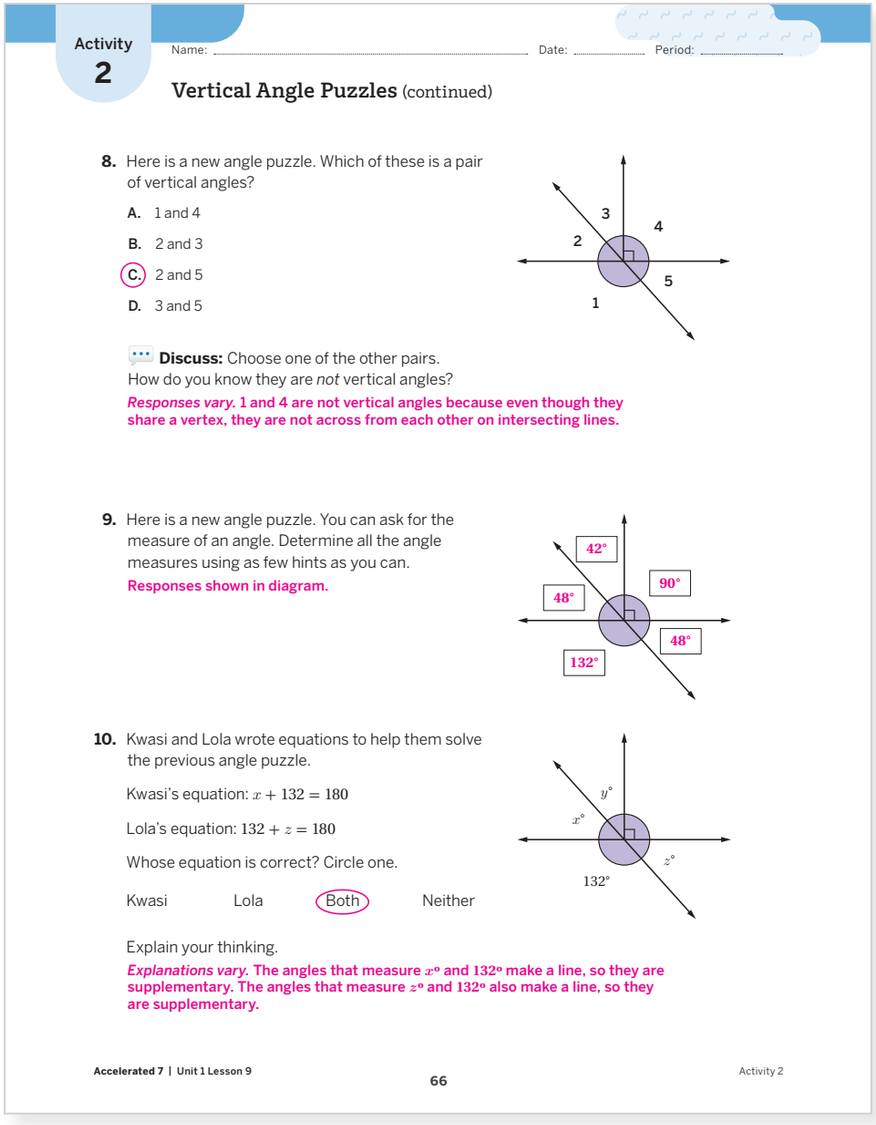
For Problem 5, **Students** will need to request an angle from the teacher. Encourage students to think strategically as they request an angle measure and to try to ask for as few angle measures as they can.

To support students getting started, invite them to identify and mark familiar angle relationships they notice.

3 Connect

Use the Decide and Defend routine to support students in making arguments and critiquing the reasoning of others. **(MTR.4.1)**

To surface the Key Takeaway, create a class definition for **vertical angles** using students' language and the glossary.



Activity 2

Name: _____ Date: _____ Period: _____

Vertical Angle Puzzles (continued)

8. Here is a new angle puzzle. Which of these is a pair of vertical angles?

A. 1 and 4
B. 2 and 3
C. 2 and 5
D. 3 and 5

Discuss: Choose one of the other pairs. How do you know they are *not* vertical angles?

Responses vary. 1 and 4 are not vertical angles because even though they share a vertex, they are not across from each other on intersecting lines.

9. Here is a new angle puzzle. You can ask for the measure of an angle. Determine all the angle measures using as few hints as you can.

Responses shown in diagram.

10. Kwasi and Lola wrote equations to help them solve the previous angle puzzle.

Kwasi's equation: $x + 132 = 180$

Lola's equation: $132 + z = 180$

Whose equation is correct? Circle one.

Kwasi Lola Both Neither

Explain your thinking.

Explanations vary. The angles that measure $x°$ and $132°$ make a line, so they are supplementary. The angles that measure $z°$ and $132°$ also make a line, so they are supplementary.

Accelerated 7 | Unit 1 Lesson 9 66 Activity 2

Key Takeaway: **Vertical angles** are on opposite sides of intersecting lines and are always the same measure.

Pairs | 10 min

Activity 3 Writing and Using Equations

Purpose: Students strengthen their understanding of the connections between equations and angle diagrams. Students fluently write equations that represent angle diagrams and also make sense of other students' equations. **(MTR.3.1)**

Short on time? Consider inviting students to write their equations on the board for other classmates to use.

1 Launch

Share that there are many different equations that represent this diagram.

Invite students to identify familiar angle relationships.

2 Monitor

To support students getting started, consider asking them to describe the relationship between two angles in words (e.g., a and b make a line). Then work together to translate their words into an equation.

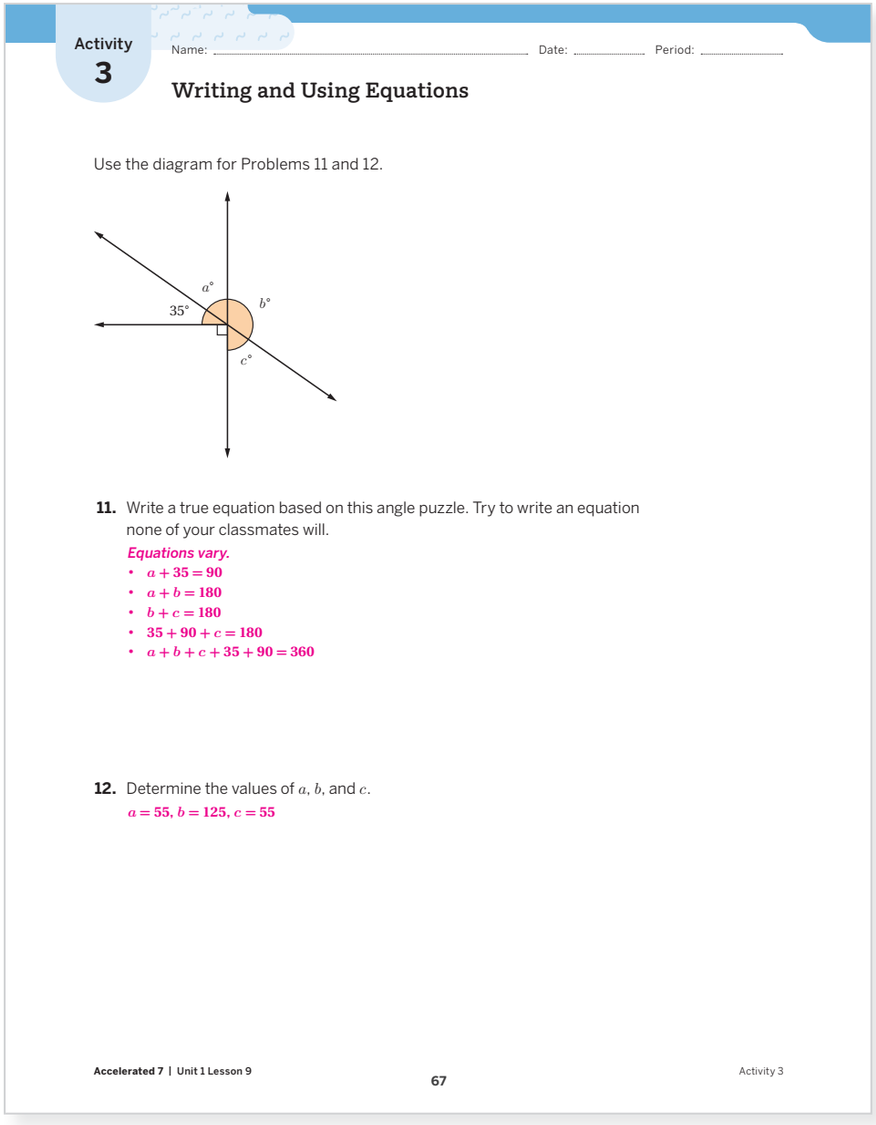
Accessibility: Conceptual Processing Invite students to revisit how they used angles within triangles to write equations in Lesson 8 to help apply concepts to new situations.

Consider asking, "Will the equation you wrote for the previous problem help you to determine all of the missing values?"

Encourage students to revisit the previous problem to write additional equations and discuss with a classmate. Some students may notice that equations with only one variable are easier to solve. By starting with these equations, they will obtain new information that can be used to write and solve a new equation.

Accessibility: Executive Functioning Consider chunking this activity by inviting students to choose one value to solve for at a time.

Activity 3 continued >



Activity 3

Name: _____ Date: _____ Period: _____

Writing and Using Equations

Use the diagram for Problems 11 and 12.

11. Write a true equation based on this angle puzzle. Try to write an equation none of your classmates will.

Equations vary.

- $a + 35 = 90$
- $a + b = 180$
- $b + c = 180$
- $35 + 90 + c = 180$
- $a + b + c + 35 + 90 = 360$

12. Determine the values of a, b, and c.

$a = 55, b = 125, c = 55$

Accelerated 7 | Unit 1 Lesson 9 67 Activity 3

Pairs | 10 min

Activity 3 Writing and Using Equations (continued)

Purpose: Students strengthen their understanding of the connections between equations and angle diagrams. Students fluently write equations that represent angle diagrams and also make sense of other students' equations. **(MTR.3.1)**

Short on time? Consider inviting students to write their equations on the board for other classmates to use.

2 Monitor

Listen for students who are precise in naming angle relationships including vertical angles.

Look for students who write and solve equations to determine the missing values.

Select and sequence several true and false student equations. Consider highlighting unique or creative equations.

D Differentiation *(Problems 13 and 14)*

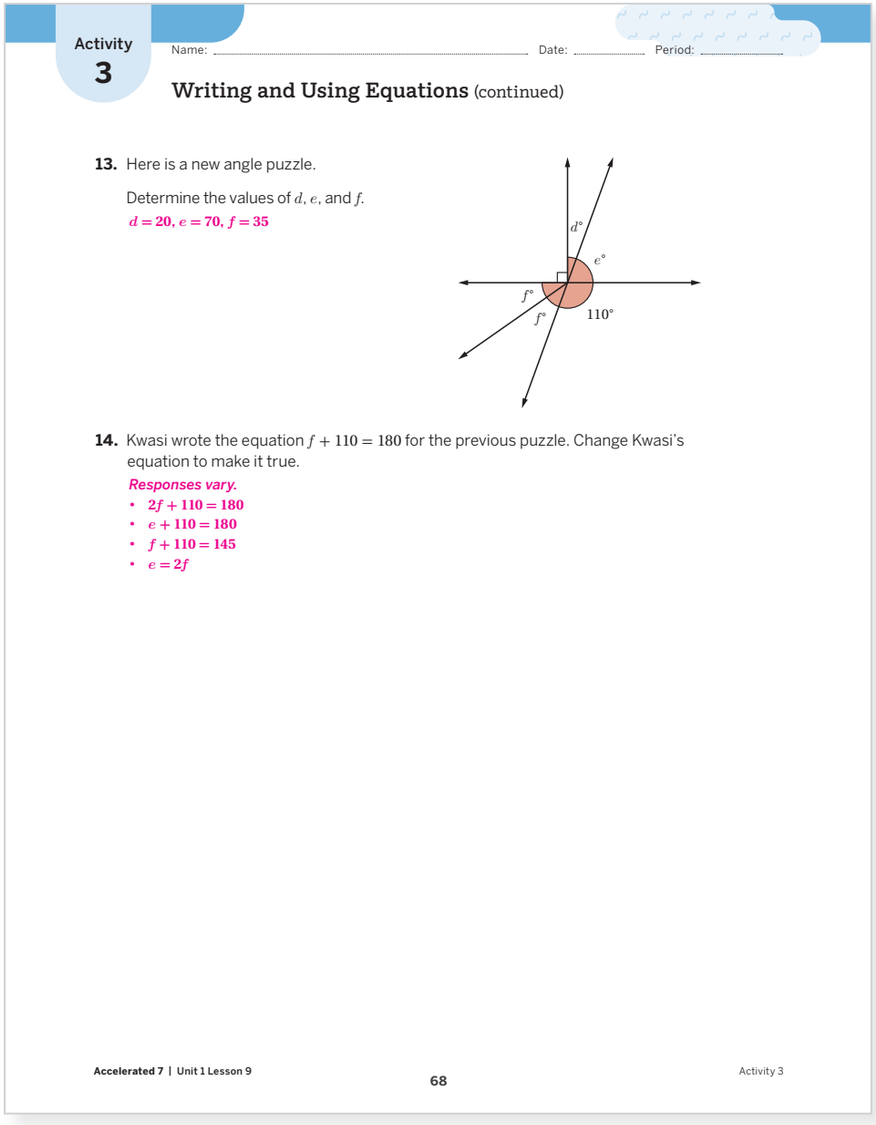
Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Encourage them to revisit their work on Problem 7.
Write a false equation, such as: • $f = e$ • $2f = e + d$ • $d + e = 180$	**Support** Consider asking, "What angle relationship is represented by this equation?"
Write a true equation, such as: • $2f = e$ • $2f + 110 = 180$ • $a + b = 90$	**Strengthen** Encourage students to write multiple equations representing different types of angle relationships.

3 Connect

Consider asking, "Does this equation represent the diagram? How many different true equations can you write?"

Encourage students to justify their reasoning.

To surface the Key Takeaway, invite students to share their equations. Discuss an example equation using each of the angle relationships: complementary, supplementary, and vertical. Share with students that there are many equations that can be written to solve for missing information. Also, equations can be rewritten using fewer terms.



Activity 3

Name: _____ Date: _____ Period: _____

Writing and Using Equations (continued)

13. Here is a new angle puzzle.

Determine the values of d, e, and f.

$d = 20$, $e = 70$, $f = 35$

14. Kwasi wrote the equation $f + 110 = 180$ for the previous puzzle. Change Kwasi's equation to make it true.

Responses vary.

- $2f + 110 = 180$
- $e + 110 = 180$
- $f + 110 = 145$
- $e = 2f$

Accelerated 7 | Unit 1 Lesson 9 68 Activity 3

Key Takeaway: Identifying angle relationships helps to write equations and determine unknown angle measures.

Whole Class | 5 min

Synthesis

Purpose: Students synthesize their description of angle relationships using the new terms *complementary*, *supplementary*, *vertical*, and *adjacent*.

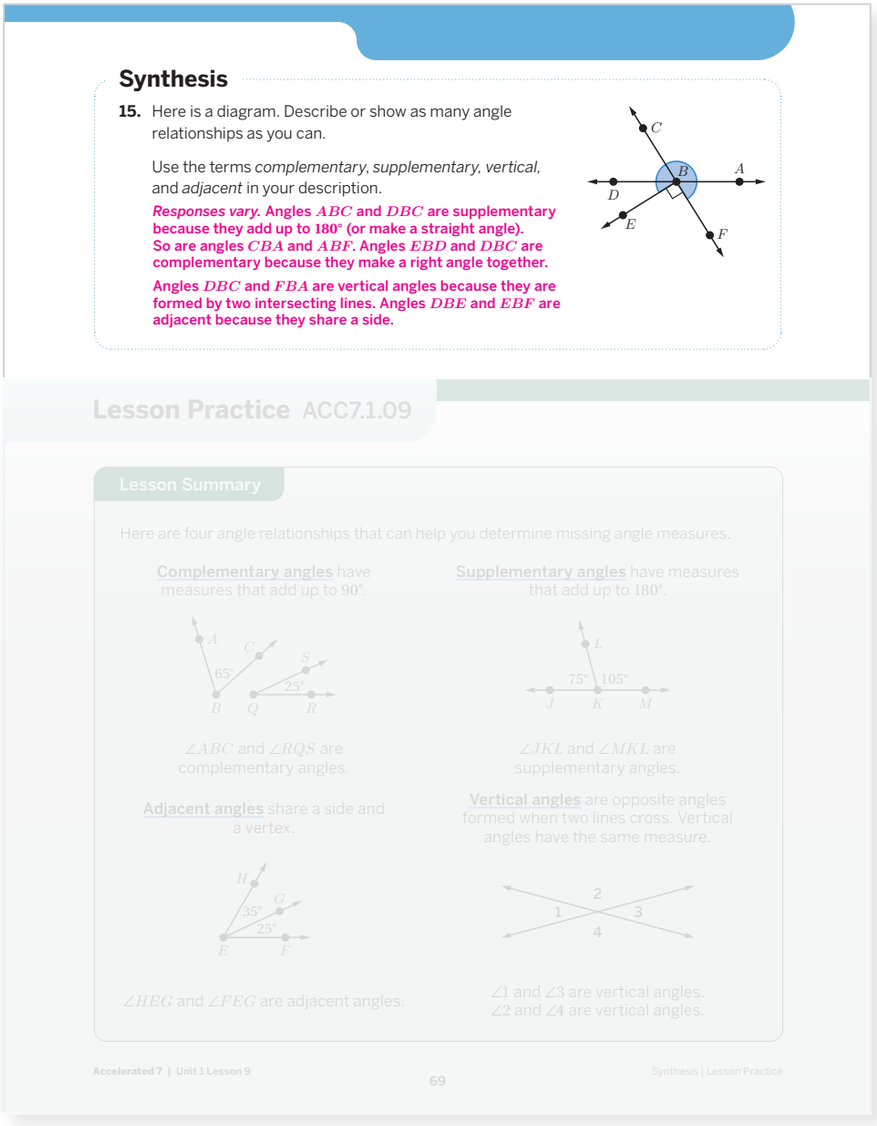
Invite students to respond independently, and then share their thinking with a partner.

Collect and share a variety of ideas, such as:

- Identifying adjacent angle pairs that form a straight line or right angle.
- Using substitution, $\angle DBC$ and $\angle ABF$ must have the same angle measure, so if $\angle EBD$ and $\angle DBC$ are complementary, then $\angle EBD$ and $\angle ABF$ are complementary.
- Using variables and writing equations to represent their thinking, if $m\angle EBD + 90 + m\angle ABF = 180$, then $m\angle EBD + m\angle ABF = 90$, so $\angle EBD$ and $\angle ABF$ are complementary.

MLR **MLR2: Collect and Display** Use this routine to formalize the definitions of *adjacent angles*, *complementary angles*, *supplementary angles*, and *vertical angles* using students' language. Display any new language, examples, or diagrams for students to reference later.

Note: If time allows, invite students to revise their previous thinking based on the discussion.



Synthesis

15. Here is a diagram. Describe or show as many angle relationships as you can.

Use the terms *complementary*, *supplementary*, *vertical*, and *adjacent* in your description.

Responses vary. Angles ABC and DBC are supplementary because they add up to 180° (or make a straight angle). So are angles CBA and ABF. Angles EBD and DBC are complementary because they make a right angle together.

Angles DBC and FBA are vertical angles because they are formed by two intersecting lines. Angles DBE and EBF are adjacent because they share a side.

Lesson Practice ACC7.1.09

Lesson Summary

Here are four angle relationships that can help you determine missing angle measures.

Complementary angles have measures that add up to 90°.

$\angle ABC$ and $\angle RQS$ are complementary angles.

Supplementary angles have measures that add up to 180°.

$\angle JKL$ and $\angle MKL$ are supplementary angles.

Adjacent angles share a side and a vertex.

$\angle HEG$ and $\angle FEG$ are adjacent angles.

Vertical angles are opposite angles formed when two lines cross. Vertical angles have the same measure.

$\angle 1$ and $\angle 3$ are vertical angles.
$\angle 2$ and $\angle 4$ are vertical angles.

Accelerated 7 | Unit 1 Lesson 9 69 Synthesis | Lesson Practice

Lesson Takeaway: We can write equations based on angle relationships to help solve for missing angle measures.

Independent | 5 min

Show What You Know

Purpose: Students demonstrate their understanding by using angle relationships in a diagram to determine missing angle measures.

Today's Goals

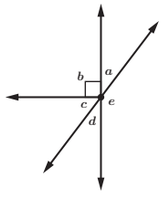
1. **Goal:** Determine unknown angle measures by reasoning about *complementary*, *supplementary*, *vertical*, and *adjacent* angles.
2. **Goal:** Use angle relationships to write equations and determine unknown angle measures.
3. **Language Goal:** Describe what it means for angles to be *complementary* or *supplementary*. **(Writing)**
4. **Language Goal:** Explain how the parts of an equation represent an angle diagram. **(Writing, Speaking, and Listening)**

Show What You Know (Assessment Resources)

Name: _____ Date: _____ Period: _____

Show What You Know 1.09

1. Using the figure shown,



   a. Name a pair of complementary angles.
   Angles c and d

   b. Name a pair of vertical angles.
   Angles a and

   c. Name a pair of supplementary angles
   Angles a and e

2. If the measure of angle c is 50 degrees, find the measures of the other angles:

$m\angle a =$ $m\angle a = 40°$

$m\angle b =$ $m\angle b = 90°$

$m\angle d =$ $m\angle d = 40°$

$m\angle e =$ $m\angle e = 140°$

D Differentiation Use after Lesson 9

Support

Provide targeted intervention for students by using these resources.

If students need support understanding how to calculate unknown angle measures using angle relationships:

» **Respond to Student Thinking:**

- Revisit the equations in **Activity 3** and invite students to share strategies for writing and solving equations to determine unknown angle measures.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 3 Extension Activities** | 15 min

Professional Learning

How did students tap into their creativity today? How did you recognize and encourage students' creativity?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

15. Here is a diagram. Describe or show as many angle relationships as you can.

Use the terms *complementary, supplementary, vertical,* and *adjacent* in your description.

Responses vary. Angles ABC and DBC are supplementary because they add up to 180° (or make a straight angle). So are angles CBA and ABF. Angles EBD and DBC are complementary because they make a right angle together.

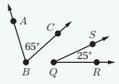
Angles DBC and FBA are vertical angles because they are formed by two intersecting lines. Angles DBE and EBF are adjacent because they share a side.

Lesson Practice ACC7.1.09

Lesson Summary

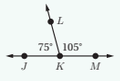
Here are four angle relationships that can help you determine missing angle measures.

Complementary angles have measures that add up to 90°.



$\angle ABC$ and $\angle RQS$ are complementary angles.

Supplementary angles have measures that add up to 180°.

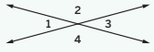


$\angle JKL$ and $\angle MKL$ are supplementary angles.

Adjacent angles share a side and a vertex.

$\angle HEG$ and $\angle FEG$ are adjacent angles.

Vertical angles are opposite angles formed when two lines cross. Vertical angles have the same measure.

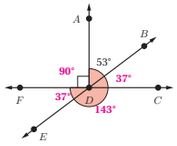


$\angle 1$ and $\angle 3$ are vertical angles.
$\angle 2$ and $\angle 4$ are vertical angles.

Lesson Practice ACC7.1.09

Name: _____ Date: _____ Period: _____

Problems 1–3: Here is a diagram.



1. Determine the measure of each angle.

Angle	Measure (degrees)
ADB	53
BDC	37
CDE	143
FDE	37
FDA	90

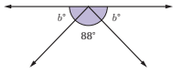
2. Identify one pair of vertical angles, one pair of complementary angles, and one pair of supplementary angles in the diagram.

Responses vary. Angles BDC and FDE are vertical angles. Angles ADB and CDB are complementary angles. Angles FDE and CDE are supplementary angles.

3. Explain how you know if a pair of angles are vertical angles.

Responses vary. I know they are vertical angles if they are angles opposite each other where two lines cross.

Problems 4–5: Here is a new diagram.



4. Which equation represents the relationship between the angles in the figure?

A. $88 + b = 90$

B. $88 + b = 180$

C. $2b + 88 = 90$

D. $2b + 88 = 180$

5. Dakota says that the angles marked b are vertical angles. Eva disagrees. Who is correct? Explain your thinking.

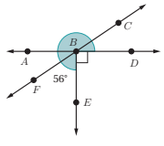
Eva. *Explanations vary.* The angles are equal but not vertical. The angles are made from one line and two segments meeting at a point. To be vertical angles, they would need to be made from two lines crossing.

Lesson Practice ACC7.1.09

Name: _____ Date: _____ Period: _____

FAST Practice

6. This diagram includes supplementary, complementary, adjacent, and vertical angles. Determine each of the missing angle measures. Record your answers in the spaces provided.



Angle	Measure (degrees)
ABC	146
CBD	34
DBE	90
FBA	34

Spiral Review

Problems 7–10: Solve each equation.

7. $x + 40 = 180$ $x = 140$

8. $x - 25 = 90$ $x = 115$

9. $2x = 180$ $x = 90$

10. $\frac{2}{3}x = 40$ $x = 60$

Problems 11–12: A small dog gets fed $\frac{3}{4}$ of a cup of dog food twice a day.

11. Write an equation representing the relationship between the number of days, d, and the number of cups of food, f.

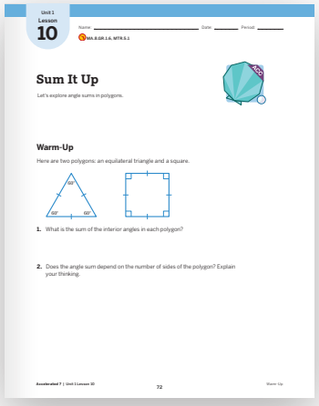
$f = 1.5d$ (or equivalent)

12. How many days will a large bag of dog food last if a new bag contains 210 cups of food?

140 days

Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1, 4	2	MA.8.GR.1.4, MA.8.AR.2.1
	2–3, 5	1	MA.8.GR.1.4
FAST Practice	6	1	MA.8.GR.1.4, MA.8.AR.2.1
Spiral Review			
	7–10	1	MA.6.AR.2.2, MA.6.AR.2.3
	11–12	2	MA.6.AR.2.3


Unit 1
Lesson
10

Name: _____ Date: _____ Period: _____

MA.8.GR.1.6, MTR.5.1

Sum It Up

Let's explore angle sums in polygons.

Warm-Up

Here are two polygons: an equilateral triangle and a square.

1. What is the sum of the interior angles in each polygon?

2. Does the angle sum depend on the number of sides of the polygon? Explain your thinking.

Accelerated 7 | Unit 1 Lesson 10 72 Warm-Up

Sum It Up

Exploring Angle Sums in Polygons

Let's explore angle sums in polygons.

Learning Outcomes

Today's Goals

1. **Goal:** Determine the interior angle sums of polygons by decomposing them into triangles.
2. **Goal:** Develop a formula that can be used to determine the interior angle sum of any polygon.
3. **Language Goal:** Describe how decomposition into triangles can be used to calculate the sum of the interior angles of a polygon. **(Reading, Writing, and Speaking)**

Students use their knowledge of the angle sum of triangles and decomposition to determine the angle sums of a quadrilateral, pentagon, and hexagon. Then they observe patterns and generalize to develop the formula $(n-2) \times 180°$. **(MTR.5.1)**

Coherence

Prior Learning

In Lesson 8, students were introduced to angle measures and interior angle sums of triangles. They also used the angle sum theorem to determine a missing angle measure.

Future Learning

In future courses, students will use this knowledge to prove relationships and theorems about triangles, parallelograms, trapezoids, and other polygons.

Connections

- Students build upon their understanding of decomposing a polygon into smaller figures to determine the interior angle measure sum of a triangle. (mathematical)

FL B.E.S.T. Math Benchmarks

Addressing

MA.8.GR.1.6
Develop and use formulas for the sums of the interior angles of regular polygons by decomposing them into triangles.

Mathematical Thinking and Reasoning Standard: MTR.5.1

ELA Expectations: ELA.K12. EE.3.1. ELA.K12.EE.5.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.6.GR.2.2	MA.912.GR.1.3
MA.7.GR.1.2	MA.912.GR.1.4
	MA.912.GR.1.5

Lesson at a Glance

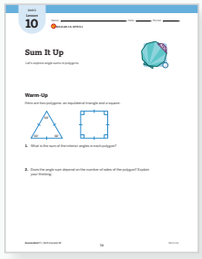
~ 45 min

MA.8.GR.1.6

Warm-Up

Pairs | 5 min

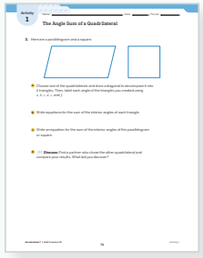
Students determine angle sums in a triangle and square and think about whether or not the angle sum depends on the number of sides.
Routine(s): Think-Pair-Share



Activity 1

Small Groups | 10 min

Students use decomposition of a parallelogram or square into triangles to determine the angle sum.

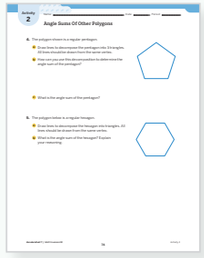


Activity 2

Pairs | 10 min

Students apply what they have learned to determine the angle sums of a regular pentagon and regular hexagon. **(MTR.5.1)**

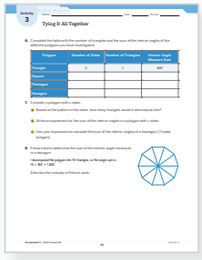
Key Takeaway: The angle sum of a polygon can be found by decomposing the shape into triangles.



Activity 3

Pairs | 10 min

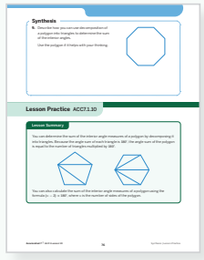
Students use the Notice and Wonder routine to identify a pattern between the number of sides and the angle sum of a polygon and develop a formula.
MLR3: Critique, Correct, Clarify **(MTR.5.1)**



Synthesis

Whole Class | 5 min

Students synthesize their understanding about the sum of the interior angles of a polygon.
MLR2: Collect and Display **(MLR2)**



Show What You Know

Independent | 5 min

Students demonstrate their understanding by critiquing strategies of another for calculating the sum of the interior angles of a polygon.
(MTR.4.1)

Prep Checklist

This lesson includes:

- Student Edition
- Show What You Know

Pairs | 5 min

Warm-Up

Purpose: Students determine angle sums in a triangle and square and think about whether or not the angle sum depends on the number of sides.

1 Launch

To support students getting started, consider asking them to describe how the angle measures are shown in the triangle and square.

Use the **Think-Pair-Share** routine to facilitate a discussion about the angle measures in the triangle and square. Invite students to think independently and then share how they determined the angle sum in each polygon.

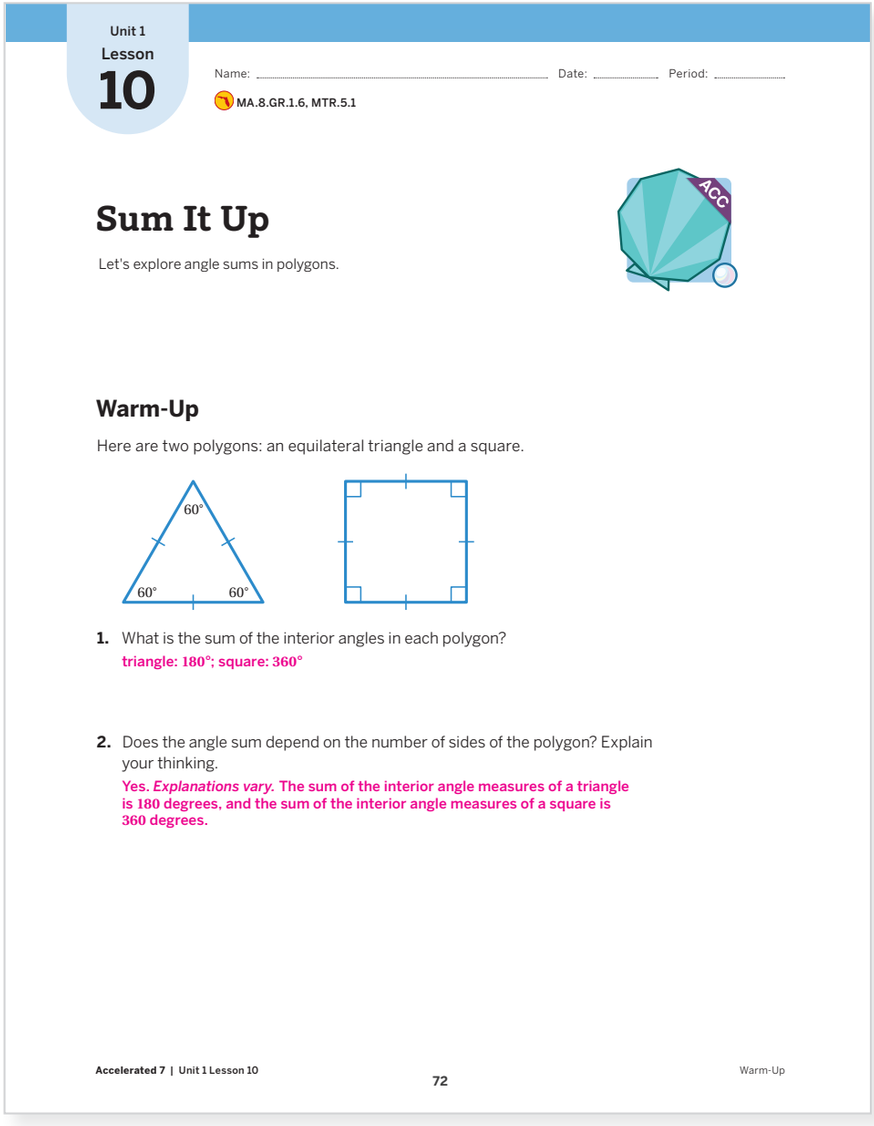
A **Accessibility: Memory and Attention**
Encourage students who struggle with understanding how the drawings show the angle measures to circle each angle measure in the triangle and write 90° next to each interior angle in the square.

Collect and share students' answers, inviting each pair to share their method for solving.

2 Connect

Consider asking:

- "Do the polygons have different numbers of sides?"
- "Do the polygons have different numbers of angles?"
- "Do the polygons have different or the same angle sums?"



Unit 1
Lesson
10

Name: _____ Date: _____ Period: _____

MA.8.GR.1.6, MTR.5.1

Sum It Up

Let's explore angle sums in polygons.

ACC

Warm-Up

Here are two polygons: an equilateral triangle and a square.

60° 60° 60°

1. What is the sum of the interior angles in each polygon?
   triangle: 180°; square: 360°

2. Does the angle sum depend on the number of sides of the polygon? Explain your thinking.
   Yes. *Explanations vary.* The sum of the interior angle measures of a triangle is 180 degrees, and the sum of the interior angle measures of a square is 360 degrees.

Accelerated 7 | Unit 1 Lesson 10 72 Warm-Up

Small Groups | 10 min

Activity 1 The Angle Sum of a Quadrilateral

Purpose: Students use decomposition of a parallelogram or square into triangles to determine the angle sum.

1 Launch

To surface prior knowledge, consider asking:

- "What is a diagonal?"
- "Is there more than one way to draw a diagonal on your shape?"
- "What does it mean to decompose a shape?"

2 Monitor

For Problem 3d, **Encourage students** to find a partner who chose the same quadrilateral and compare their work.

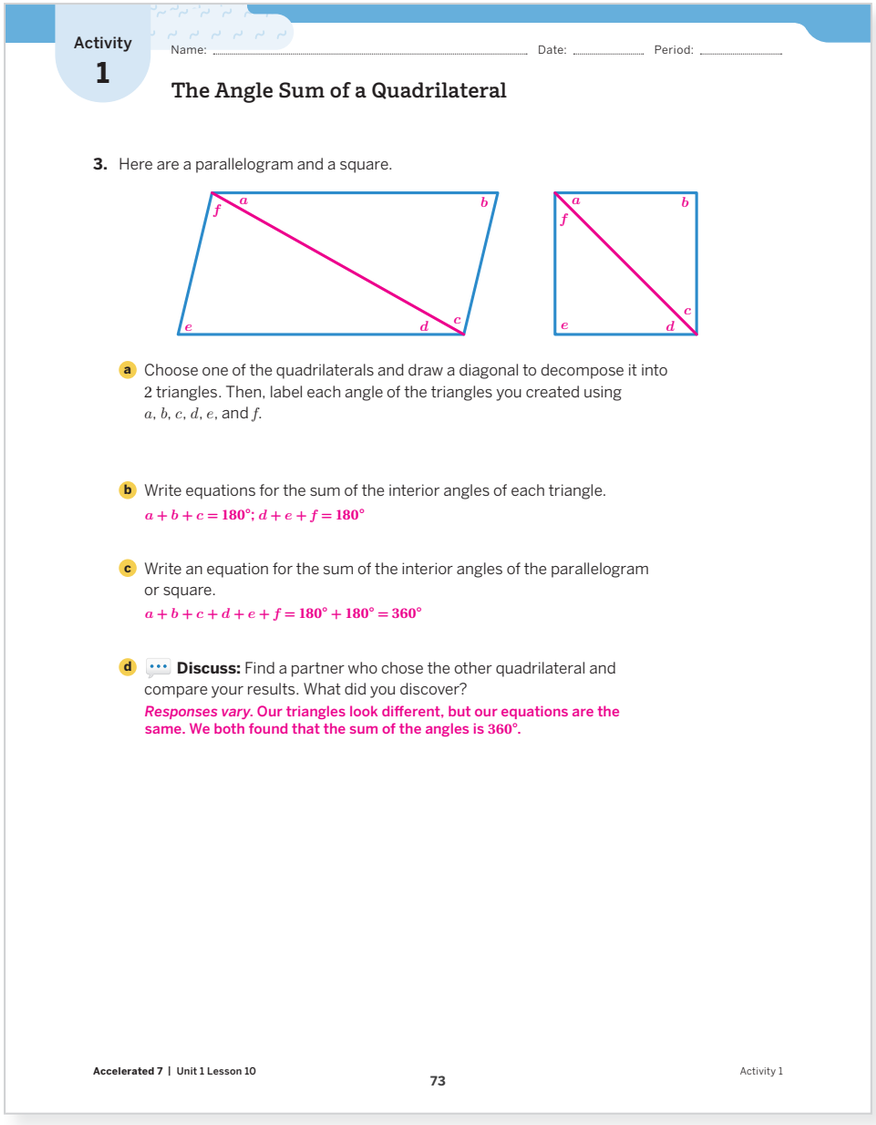
A **Accessibility: Visual-Spatial Processing** To support processing of complex visual information, invite students to focus on one triangle at a time by covering up the other triangle.

Pause to have students discuss how they wrote their equations if time allows.

3 Connect

Invite students to debate whether or not the parallelogram and the square have the same interior angle sum.

ELL **English Language Learners** Invite students to share what the phrase *sum of the interior angles* means in their own words. Encourage them to use examples from the Warm-Up to support their understanding. **(Reading and Speaking)**



Activity 1

Name: _____ Date: _____ Period: _____

The Angle Sum of a Quadrilateral

3. Here are a parallelogram and a square.

a. Choose one of the quadrilaterals and draw a diagonal to decompose it into 2 triangles. Then, label each angle of the triangles you created using a, b, c, d, e, and f.

b. Write equations for the sum of the interior angles of each triangle.
$a + b + c = 180°$; $d + e + f = 180°$

c. Write an equation for the sum of the interior angles of the parallelogram or square.
$a + b + c + d + e + f = 180° + 180° = 360°$

d. **Discuss:** Find a partner who chose the other quadrilateral and compare your results. What did you discover?
Responses vary. Our triangles look different, but our equations are the same. We both found that the sum of the angles is 360°.

Accelerated 7 | Unit 1 Lesson 10 73 Activity 1

Pairs | 10 min

Activity 2 Angle Sum of Other Polygons

Purpose: Students apply what they have learned to determine the angle sums of a regular pentagon and regular hexagon. **(MTR.5.1)**

Short on time? Consider omitting Problem 5.

1 Launch

Invite students to find a partner in a different location to encourage movement.

Look for students who do not decompose the pentagon correctly. Remind students that all lines should be drawn from the same vertex.

2 Monitor

Encourage students to discuss what they notice about the number of sides of the pentagon and the number of triangles formed. Then have them discuss what they notice about the process used to determine the angle sum of the pentagon. Consider asking, "How does the decomposition show you the sum of the interior angles in the pentagon?" **(MTR.5.1)**

Create a class summary statement that describes why decomposition of a polygon into triangles by drawing lines from a single vertex can be used to determine the sum of the interior angles for a polygon.

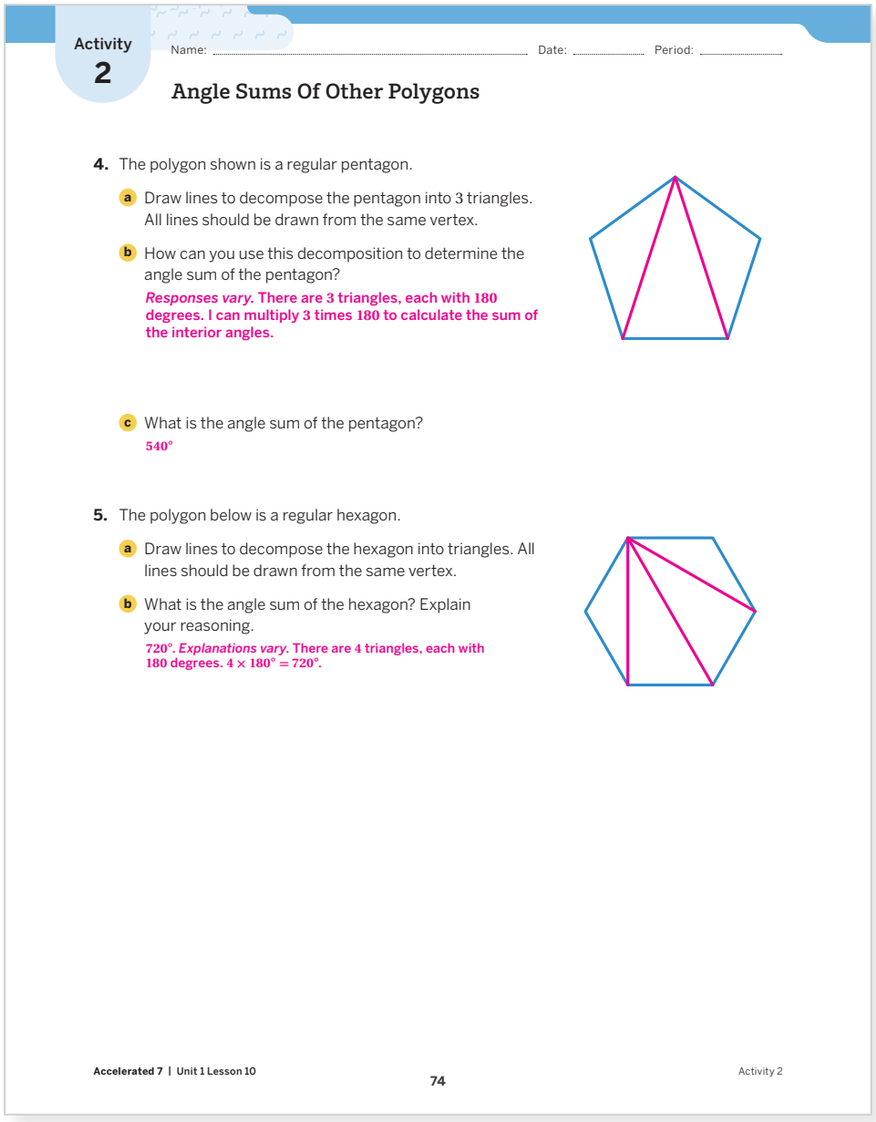
3 Connect

Listen for students who notice that the angle sum of the hexagon is $180°$ more than the angle sum of the pentagon.

Note: If time permits, have students label the angles of each triangle and write an equation like they did in the previous activity.

To surface the Key Takeaway, consider asking:

- "How can you use decomposition into triangles to determine the angle sum?"
- "Once you decompose the polygon into triangles, what is your next step to determine the angle sum?"



Activity 2

Name: _____ Date: _____ Period: _____

Angle Sums Of Other Polygons

4. The polygon shown is a regular pentagon.

a. Draw lines to decompose the pentagon into 3 triangles. All lines should be drawn from the same vertex.

b. How can you use this decomposition to determine the angle sum of the pentagon?

Responses vary. There are 3 triangles, each with 180 degrees. I can multiply 3 times 180 to calculate the sum of the interior angles.

c. What is the angle sum of the pentagon?

540°

5. The polygon below is a regular hexagon.

a. Draw lines to decompose the hexagon into triangles. All lines should be drawn from the same vertex.

b. What is the angle sum of the hexagon? Explain your reasoning.

720°. *Explanations vary.* There are 4 triangles, each with 180 degrees. $4 \times 180° = 720°$.

Accelerated 7 | Unit 1 Lesson 10 74 Activity 2

Key Takeaway: The angle sum of a polygon can be found by decomposing the shape into triangles.

Pairs | 10 min

Activity 3 Tying It All Together

Purpose: Students use the Notice and Wonder routine to identify a pattern between the number of sides and the angle sum of a polygon and develop a formula. **(MTR.5.1)**

1 Launch

Use the Notice and Wonder routine to encourage students to look for patterns in the table. Consider asking, "How do the numbers in the first two columns compare?"

2 Monitor

Listen for students who use the connections in the table to write a formula. **(MTR.5.1)**

D Differentiation *(for Problem 7)*

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Encourage them to look back at the table and imagine the variable n in the first column. Consider asking, "How is the number of sides related to the number of triangles?"
Struggle to write the formula in part b.	**Strengthen** Consider asking, "How is the number of triangles related to the angle sum?"
Write the correct formula for the angle sum.	**Stretch** Consider asking, "If a polygon has an angle sum of 3,240°, how many sides does it have?"

3 Connect

To surface the Key Takeaway, invite students to describe the formula in their own words. Consider asking, "Why do you subtract 2 from n?"

Listen for students who realize that some of the angles on the decomposed triangles are in the interior of the polygon and are not included in the interior angle sum.

If time allows, challenge students to write an equation for the angle sum using the 10 decomposed triangles and explain why it works. Answers vary. The equation $10 \times 180° - 360° = 1{,}440°$ takes the total of the interior angles of all 10 triangles and subtracts the 360° in the center of the polygon.

MLR MLR3: Critique, Correct, Clarify For Problem 8, Use this routine to help students describe errors in Polina's work. Invite students to share their responses from the activity. See the Routine Facilitation Guide for more information.

Activity 3

Name: _____ Date: _____ Period: _____

Tying It All Together

6. Complete the table with the number of triangles and the sum of the interior angles of the different polygons you have investigated.

Polygon	Number of Sides	Number of Triangles	Interior Angle Measure Sum
Triangle	3	1	180°
Square	4	2	360°
Pentagon	5	3	540°
Hexagon	6	4	720°

7. Consider a polygon with n sides.

a. Based on the pattern in the table, how many triangles would it decompose into?
$n - 2$

b. Write an expression for the sum of the interior angles in a polygon with n sides.
$(n - 2) \times 180°$

c. Use your expression to calculate the sum of the interior angles of a heptagon (7-sided polygon).
$(7 - 2) \times 180° = 900°$

8. Polina tried to determine the sum of the interior angle measures in a decagon.

I decomposed the polygon into 10 triangles, so the angle sum is 10 × 180° = 1,800°.

Describe the mistake in Polina's work.
Responses vary. Polina did not decompose the figure by drawing all lines from the same vertex. Some of the angles in the triangles are in the center of the figure, so their measures are not included in the sum of the interior angles.

Accelerated 7 | Unit 1 Lesson 10 75 Activity 3

Key Takeaway: The angle sum of a polygon is equal to 180° times 2 less than the number of sides.

Whole Class | 5 min

Synthesis

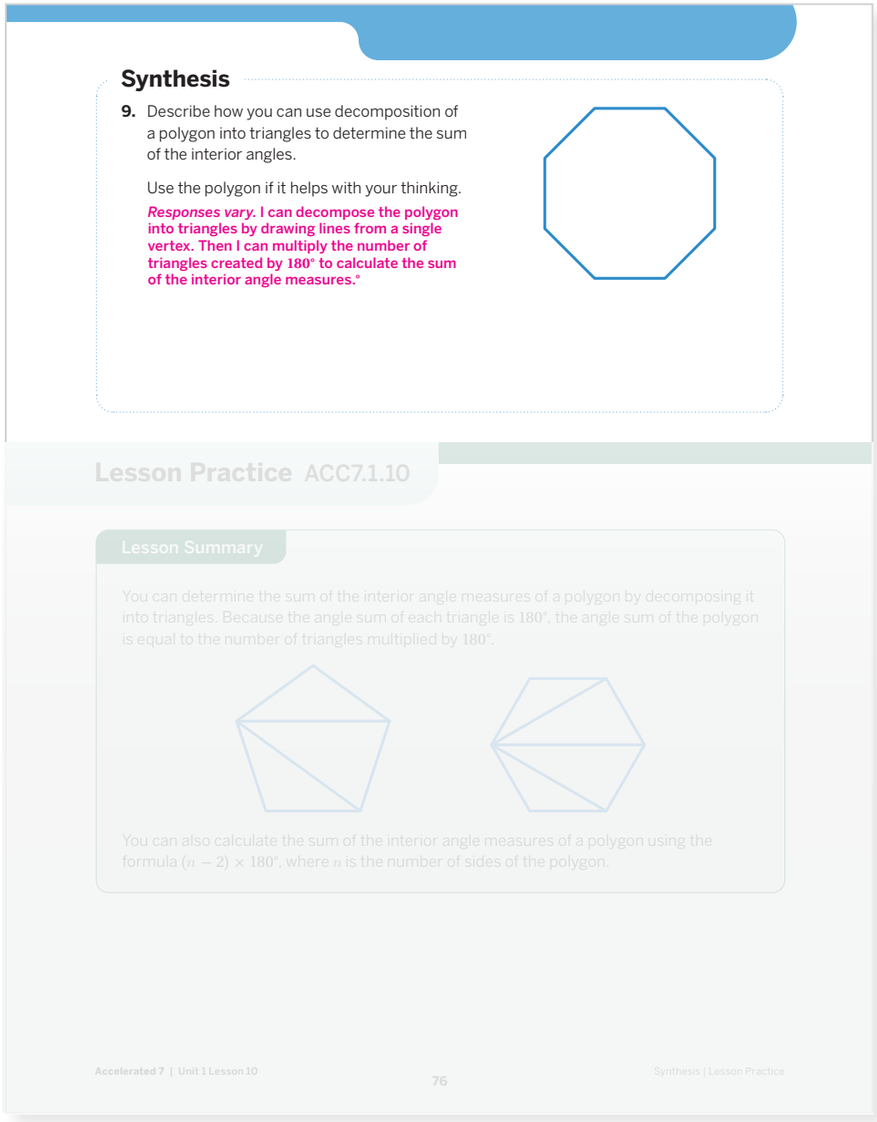
Purpose: Students synthesize their understanding about the sum of the interior angles of a polygon.

Invite students to respond independently, and then share their thinking with a partner.

Collect and share students' ideas about calculating the sum of the interior angles of a regular polygon using a formula and the decomposition into triangles method.

MLR **MLR2: Collect and Display** Use this routine to collect students' ideas about calculating the sum of the interior angles of a regular polygon. Display for students to reference and add to for the rest of the unit.

Note: If time allows, invite students to discuss when it might be better to use the formula than decomposition to determine an angle sum.



Synthesis

9. Describe how you can use decomposition of a polygon into triangles to determine the sum of the interior angles.

Use the polygon if it helps with your thinking.

Responses vary. I can decompose the polygon into triangles by drawing lines from a single vertex. Then I can multiply the number of triangles created by $180°$ to calculate the sum of the interior angle measures.°

Lesson Practice ACC7.1.10

Lesson Summary

You can determine the sum of the interior angle measures of a polygon by decomposing it into triangles. Because the angle sum of each triangle is $180°$, the angle sum of the polygon is equal to the number of triangles multiplied by $180°$.

You can also calculate the sum of the interior angle measures of a polygon using the formula $(n - 2) \times 180°$, where n is the number of sides of the polygon.

Accelerated 7 | Unit 1 Lesson 10 76 Synthesis | Lesson Practice

Lesson Takeaway: The sum of the interior angles of a polygon depends on the number of sides of the polygon.

Independent | 5 min

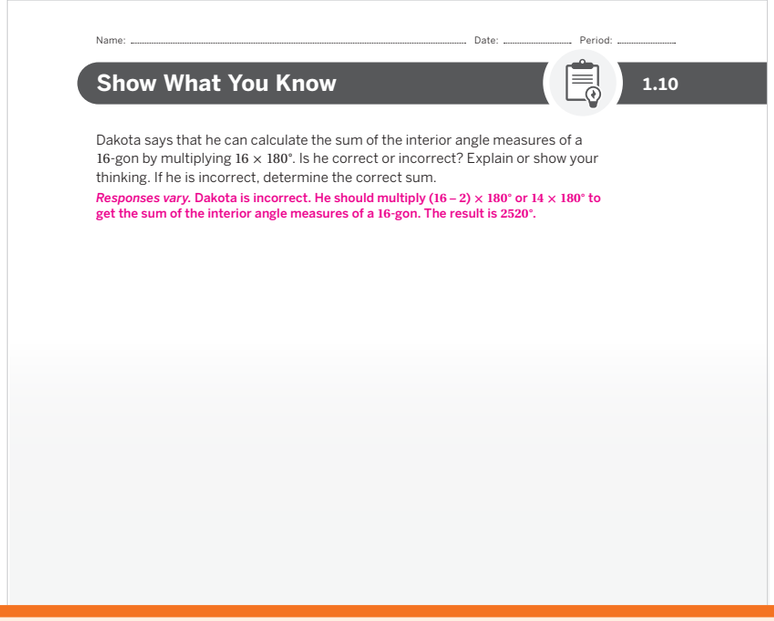
Show What You Know

Purpose: Students demonstrate their understanding by critiquing strategies for calculating the sum of the interior angles of a polygon. **(MTR.4.1).**

Today's Goals

1. **Goal:** Determine the interior angle sums of polygons by decomposing them into triangles.
2. **Goal:** Develop a formula that can be used to determine the interior angle sum of any polygon.
3. **Language Goal:** Describe how decomposition into triangles can be used to calculate the sum of the interior angles of a polygon. **(Reading, Writing, and Speaking)**

Show What You Know (Assessment Resources)



Name: _____ Date: _____ Period: _____

Show What You Know 1.10

Dakota says that he can calculate the sum of the interior angle measures of a 16-gon by multiplying $16 \times 180°$. Is he correct or incorrect? Explain or show your thinking. If he is incorrect, determine the correct sum.

Responses vary. Dakota is incorrect. He should multiply $(16 - 2) \times 180°$ or $14 \times 180°$ to get the sum of the interior angle measures of a 16-gon. The result is 2520°.

D Differentiation Use after Lesson 10

Support

Provide targeted intervention for students by using these resources.

If students need support understanding how to find the sum of the interior angle measures of a polygon:

» **Respond to Student Thinking:**

- Revisit Problem 6 in **Activity 3** and invite students to draw several examples of each polygon, identify the number of sides and the number of triangles in each figure. Then have them practice writing the formula and replacing the unknowns to calculate the sum of the interior angles.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 3 Extension Activities** | 15 min

Professional Learning

Take a moment to reflect on this lesson. In this lesson, students connected their previous understanding of the angle measure of a triangle to determine the angle sum of various polygons. How will that work support their understanding of similar triangles in Unit 2?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

9. Describe how you can use decomposition of a polygon into triangles to determine the sum of the interior angles.

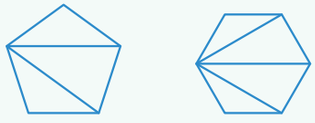
Use the polygon if it helps with your thinking.

Responses vary. I can decompose the polygon into triangles by drawing lines from a single vertex. Then I can multiply the number of triangles created by 180° to calculate the sum of the interior angle measures.°

Lesson Practice ACC7.1.10

Lesson Summary

You can determine the sum of the interior angle measures of a polygon by decomposing it into triangles. Because the angle sum of each triangle is 180°, the angle sum of the polygon is equal to the number of triangles multiplied by 180°.



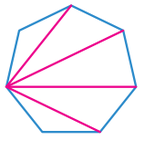
You can also calculate the sum of the interior angle measures of a polygon using the formula $(n - 2) \times 180°$, where n is the number of sides of the polygon.

Lesson Practice ACC7.1.10

Name: ..... Date: ..... Period: .....

Problems 1–2: Decompose each polygon into triangles by drawing lines from the same vertex. Then calculate the sum of the interior angle measures.

1. Heptagon



There are 5 triangles.
$5 \times 180° = 900°$

2. Nonagon

There are 7 triangles.
$7 \times 180° = 1{,}260°$

3. A regular polygon has sides that are the same length and angles that are the same measure. A regular dodecagon has 12 sides. Determine the measure of each interior angle. Explain your reasoning.

The total measure of the interior angles is $(12 - 2) \times 180° = 10 \times 180° = 1{,}800°$.
Since each angle has the same measure, each angle measures $1{,}800° \div 12 = 150°$.

4. Jamar says that he can calculate the sum of the interior angle measures of a 15-gon by multiplying $15 \times 180°$.

Do you agree with him? Explain your thinking.

Responses vary. I don't agree with him. He should multiply 180° by 13 to get 2,340°.

FAST Practice

5. What is the sum of the interior angle measures of a polygon with 20 sides?

A. 180°
B. 2,000°
C. 3,240°
D. 3,600°

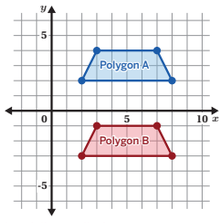
Lesson Practice ACC7.1.10

Name: ..... Date: ..... Period: .....

Spiral Review

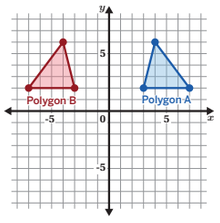
Problems 6–9: For each pair of polygons, describe the transformation that maps polygon A onto polygon B.

6.



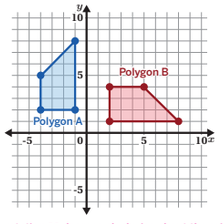
translation 5 units down

7.



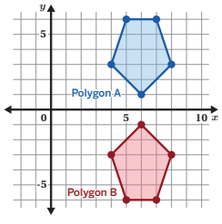
reflection across the y-axis

8.



rotation 90 degrees clockwise about the origin

9.



reflection across the x-axis

Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1–2	2	MA.8.GR.1.6
	3	3	MA.8.GR.1.6
	4	2	MA.8.GR.1.6
FAST Practice	5	2	MA.8.GR.1.6
Spiral Review			
	6–9	2	MA.8.GR.2.1

Assess and Respond

End-of-Unit Assessment

Independent | 45 min

Facilitation: Assign the End-of-Unit Assessment to learn about your students' understanding of the concepts and skills in this unit. There are two forms of the End-of-Unit Assessment: Forms A and B. If time allows, consider inviting students to complete one or more of the tasks in the Unit Synthesis and Reflection.

Item Analysis, Forms A and B

Problem(s)	Concept or skill	Addressed in	DOK	FL B.E.S.T. Math Benchmarks
1	Reflecting a single point on a coordinate plane	Lesson 3	1	MA.8.GR.2.3
2	Identifying which rigid transformation took place given a preimage and image of a rectangle on a coordinate plane	Lesson 4	1	MA.8.GR.2.3
3	Using properties of congruent figures	Lesson 5	2	MA.8.GR.2.1
4	Using rigid transformation to show congruence	Lesson 7	2	MA.8.GR.2.3
5	Explaining whether two figures are congruent on a coordinate plane	Lesson 7	2	MA.8.GR.2.3
6a	Using angle relationships to solve for unknown angle measures	Lesson 9	2	MA.8.GR.1.4
6b	Critiquing the reasoning of another who inaccurately wrote an equation relating angle measures.	Lesson 9	3	MA.8.GR.1.4
7a	Recognizing congruent sides given a diagram of transformed triangles	Lesson 6	2	MA.8.GR.2.1
7b	Recognizing congruent triangles given a diagram of transformed triangles	Lesson 7	1	MA.8.GR.2.1
7c	Determining angle measurements using angle relationships in triangles	Lesson 8	2	MA.8.GR.1.5

Assessment Resources

- Student Assessments (Forms A and B)
- Assessment Rubrics with Answers
- Show What You Knows and Answer Keys

Intervention and Extension Resources

- Mini-Lessons
- Extensions

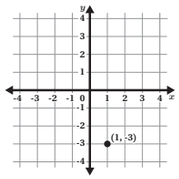
Name: ..... Date: ..... Period: .....

End-of-Unit Assessment

Unit 1

1. The point $(1, -3)$ is shown on the graph.

What are the coordinates of the image of this point after a reflection across the x-axis?



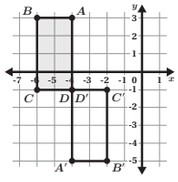
A. $(-1, -3)$

B. $(1, 3)$

C. $(3, -1)$

D. $(-3, 1)$

2. Figure $ABCD$ was transformed to create figure $A'B'C'D'$. Which transformation took place?



A. A reflection over the x-axis.

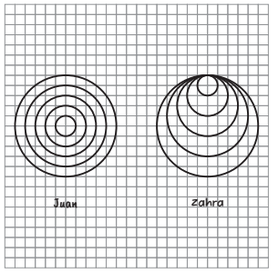
B. A reflection over the y-axis.

C. A rotation of about the origin.

D. A rotation of $180°$ about the origin.

3. Juan and Zahra each made a design using five circles.

Select *all* of the true statements.



☑ A. The smallest circle in Juan's design is congruent to the smallest circle in Zahra's design.

☐ B. Juan's design is congruent to Zahra's design.

☐ C. If you translate Juan's design 14 units to the right, you get Zahra's design.

☑ D. Each circle in Juan's design has a congruent circle in Zahra's design.

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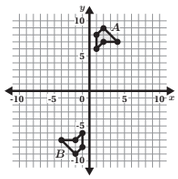
Name: ..... Date: ..... Period: .....

End-of-Unit Assessment (continued)

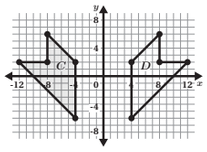
Unit 1

4. Use translations, rotations, or reflections to explain how you know polygon A is congruent to polygon B.



Responses vary. Polygon A is congruent to Polygon B because it can be mapped onto Polygon B using a $180°$ rotation around the origin.

5. Here are two sets of figures.



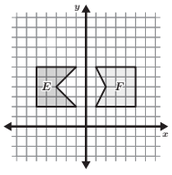
a. Is figure C congruent to figure D?

Yes

Explain your thinking.

Explanations vary. I can reflect figure C across the y-axis onto figure D.

b. Is figure E congruent to figure F?



No

Explain your thinking.

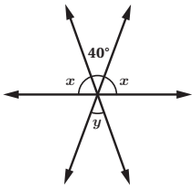
Explanations vary. The figures look congruent, but when figure E is moved on top of figure F using a reflection across the y-axis, they do not match up.

Name: ..... Date: ..... Period: .....

End-of-Unit Assessment (continued)

Unit 1

6. Here is a diagram.



a. Determine the values of x and y.

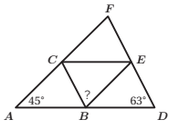
$x = 70°$; $y = 40°$

b. Haruto wrote the equation $x + y + 40 = 180$. Change Haruto's equation to make it true.

Responses vary. $x + x + 40 = 180$ (*or equivalent*)

7. To create this diagram:

- Triangle ABC was translated so that point A moved onto point C.
- Triangle ABC was then translated so that point A moved onto point B.



a. Identify at least *two pairs* of congruent sides in the figure. Explain how you know they are congruent.

Responses vary. Side AB is congruent to sides CE and BD because triangle CEF and triangle BDE are images of triangle ABC after a rigid transformation.

b. Name a triangle that is congruent to triangle CBE.

Responses vary.
- Triangle EFC
- Triangle BCA
- Triangle DEB

c. What is the measure of angle CBE? Show or explain your thinking.

$72°$. *Explanations vary.* The three angles with vertices at B must sum to $180°$ since they make a line. I know that angle EBD is the same measure as angle CAB, $45°$, and angle ABC is the same measure as angle BDE, $63°$. Angle CBE must be $180 - 45 - 63 = 72°$.

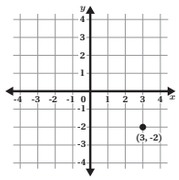
Name: Date: Period:

End-of-Unit Assessment

Unit 1

1. The point (3, -2) is shown on the graph.

What are the coordinates of the image of this point after a reflection across the y-axis?



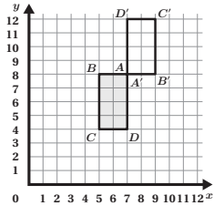
A. (-2, -3)

B. (2, -3)

C. (3, 2)

D. (-3, -2)

2. Figure $ABCD$ was transformed to create figure $A'B'C'D'$. Which transformation took place?



A. A reflection over the x-axis.

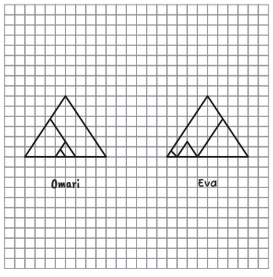
B. A reflection over the y-axis.

C. A rotation of 90° about the origin.

D. A rotation of 180° about the origin.

3. Omari and Eva each made a design using four triangles.

Select *all* of the true statements.



☑ **A.** The largest triangle in Omari's design is congruent to the largest triangle in Eva's design.

☑ **B.** Each triangle in Omari's design has a congruent triangle in Eva's design.

☐ **C.** If you translate Omari's design 10 units to the right, you get Eva's design.

☐ **D.** Omari's design is congruent to Eva's design.

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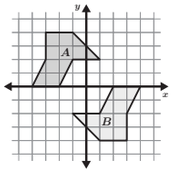
Name: Date: Period:

End-of-Unit Assessment (continued)

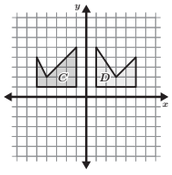
Unit 1

4. Use translations, rotations, or reflections to explain how you know polygon A is congruent to polygon B.

Responses vary. If polygon A is rotated 180° about the origin, it matches up perfectly with polygon B.



5. Here are two sets of figures.



a Is figure C congruent to figure D?

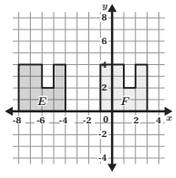
No

Explain your thinking.

Explanations vary. Figure C is not congruent to figure D. The figures look congruent, but when figure C is moved on top of figure D using reflections or rotations, they do not match up.

b Is figure E congruent to figure F?

Yes



Explain your thinking.

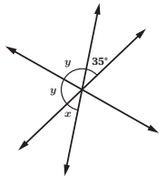
Explanations vary. Figure E is congruent to figure F. I can translate figure E 7 units right onto figure F.

Name: Date: Period:

End-of-Unit Assessment (continued)

Unit 1

6. Here is a diagram.



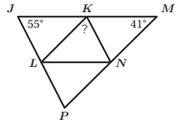
a Determine the values of x and y.

$x = 35°$; $y = 52.5°$

b River wrote the equation $x + y + y + 35 = 180$. Change River's equation to make it true.

Responses vary. $y + y + 35 = 180$ or $35 + y + y = 180$ (or equivalent)

7. To create this diagram:



- Triangle JKL was translated so that point J moved onto point L.
- Triangle JKL was then translated so that point J moved onto point K.

a Identify at least *two pairs* of congruent sides in the figure. Explain how you know they are congruent.

Responses vary. Side JK is congruent to sides LN and KM because triangle LNP and triangle KMN are images of triangle JKL after a rigid transformation.

b Name a triangle that is congruent to triangle LKN.

Responses vary.
- Triangle NPL
- Triangle KLJ
- Triangle MNK

c What is the measure of angle LKN? Show or explain your thinking.

84°. *Explanations vary.* The three angles with vertices at K must sum to 180° since they make a line. I know that angle NKM is the same measure as angle LJK, 55°, and angle JKL is the same measure as angle KMN, 41°. Angle LKN must be $180 - 55 - 41 = 84°$.

D Differentiation (End-of-Unit Assessment)

Note: To **strengthen** and **stretch** students' learning, refer to the differentiation resources suggested throughout this unit and in the Unit Overview.

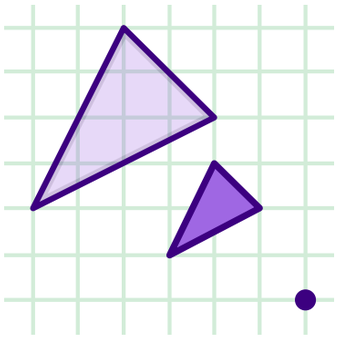
Sub-Unit Goals	Problem(s)	To respond to student thinking, consider:	
Sub-Unit 1: • Perform translations, reflections, and rotations on and off a grid. • Describe translations, reflections, and rotations **(Lessons 1–4)**	1	**Support**	• **Teacher Move:** Consider revisiting Lesson 3 (*Getting Coordinated, Part 1*)
		Strengthen	• **Challenge Creator:** Lesson 4 (*Getting Coordinated, Part 2*)
	2	**Support**	• **Mini-Lesson:** *Determining Coordinates After a Rotation* • **Teacher Move:** Consider revisiting Unit 1, Lesson 4 (*Getting Coordinated, Part 2*)
		Strengthen	• **Teacher Move:** Consider revisiting **Activity 1** in Unit 1, Lesson 1 (*Spinning, Flipping, Sliding*) to have students practice using the terminology for describing transformations.
		Stretch	• **Challenge Creator:** Lesson 4 (*Getting Coordinated, Part 2*)
Sub-Unit 2: • Determine whether two figures are congruent using rigid transformations. • Justify whether two figures are congruent using language such as rigid transformations, translation, reflection, rotation, pre-image, and image. • Describe the characteristics of congruent figures using vocabulary **(Lessons 5–7)**	3	**Support** **Strengthen**	• **Teacher Move:** Consider revisiting Lesson 5 (*No Bending, No Stretching*)
	4, 5a, 5b	**Support** **Strengthen**	• **Mini-Lesson:** *Justifying Congruent Figures* • **Teacher Move:** Consider revisiting Unit 1, Lesson 7 (*Are They Congruent?*)

	7a, 7b	**Support**
		Strengthen
			• **Teacher Move:** Consider Revisiting:
			  • Lesson 6 (*Are They The Same?*)
			  • Lesson 7 (*Are They Congruent?*)

Sub-Unit 3: • Observe that the sum of the angle measures in a triangle is 180 degrees. • Use special angle relationships to determine unknown angle measures. • Describe what it means for angles to be complementary or supplementary. **(Lessons 8–10)**	7c	**Support** **Strengthen**	• **Teacher Move:** Consider revisiting Unit 1, Lesson 8 (*Tear It Up*)
	6a, 6b	**Support** **Strengthen**	• **Mini-Lesson:** *Determining Unknown Angle Measures* • **Teacher Move:** Consider revisiting Lesson 9 (*Friendly Angles*)
		Stretch	• **Teacher Move:** Consider revisiting the **Warm-Up Activity** in Lesson 9 (*Friendly Angles*). Have students determine the angle measures of the designs in Problem 1a or from their own creation. • **Teacher Move:** Invite students to design an angle puzzle that uses triangles and straight angles to solve.

UNIT 2

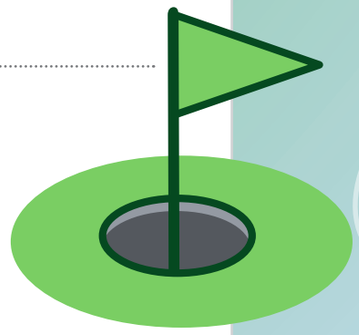
Scale Factors, Dilations, Similarity, and Slope



Students study scaled copies, dilations, and similar figures. They use similar triangles to explain, understand, and apply the concept of slope.

Essential Questions

- What does it mean to dilate a figure?
- How can transformations be used to decide whether two figures are similar?
- How can similar triangles be used to determine the slope of a line?



Sub-Unit Goals

Sub-Unit 1

- Describe how scaling affects lengths, angles, and areas in scaled copies.
- Use scale factors to create and compare scaled copies.

Sub-Unit 2

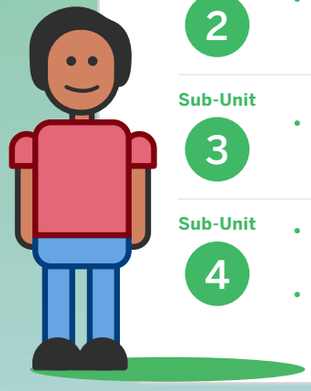
- Apply dilations to figures on and off a coordinate grid.

Sub-Unit 3

- Use transformations to determine whether two figures are similar.

Sub-Unit 4

- Make connections between slope and similar triangles on a line.
- Determine the slopes of lines.



The Math of the Unit

Unit Outcomes Telling a story that narrows and deepens with the grade

Florida B.E.S.T. Standards for Mathematics

Solve problems involving two-dimensional figures, including circles. **(MA.7.GR.1)**

Develop an understanding of the Pythagorean Theorem and angle relationships involving triangles. **(MA.8.AR.GR.1)**

Understand similarity and congruence using models and transformations. **(MA.8.AR.GR.2)**

Extend understanding of proportional relationships to two-variable linear equations. **(MA.8.AR.3)**

Benchmarks Addressed in This Unit: MA.7.GR.1.5 MA.8.GR.1.4 MA.8.GR.1.5 MA.8.GR.2.2 MA.8.GR.2.3 MA.8.GR.2.4 MA.8.AR.3.2

Mathematical Thinking and Reasoning Standards

- Demonstrate understanding by representing problems in multiple ways. **(MTR.2.1)**
- Engage in discussions that reflect on the mathematical thinking of self and others. **(MTR.4.1)**
- Use patterns and structure to help understand and connect mathematical concepts. **(MTR.5.1)**

Also Addressed: **MTR.1.1, MTR.3.1, MTR.6.1, MTR.7.1**

Coherence Telling a story within and across grades

In this Unit

Students examine and create scaled copies of figures, as well as explore scale drawings of real-life objects. They perform dilations on and off the coordinate plane using a scale factor and center of dilation. Students use dilations and previously learned transformations to justify that two triangles are similar. Using similar triangles, students learn about the concept of slope triangles, which help them determine the slope of a given line.

Prior Learning

- Multiplying and dividing fractions **(MA.6.NSO.2.2)**
- Ratio reasoning **(MA.6.AR.3.1, MA.6.AR.3.2, MA.6.AR.3.3)**
- Graphing points **(MA.6.GR.1.1)**
- **Unit 1:** Rigid transformations and congruence **(MA.8.GR.2.1, MA.8.GR.2.3)**

Future Learning

- **Unit 4:** Linear relationships **(MA.8.AR.3)**
- Trigonometry and similar triangles **(MA.912.GR.1, MA.912.T.1)**
- Average rate of change **(MA.912.F.1.3)**

See the Connections to Future Learning page for more information, including explanations and examples.

Rigor and Balance Integrating the aspects of rigor in ways that support one other

Here are examples of the aspects of rigor and balance in this unit. See the first page of each lesson's Teacher Edition for more information.

Conceptual Understanding

In **Lessons 7–11**, students understand how translations, reflections, rotations, and dilations combine to move one figure onto another. In **Lesson 15**, students understand what slope means, making connections to similar triangles.

Strategic Competence

In **Lesson 9,** students use a variety of strategies to determine unknown lengths in similar triangles with a particular slope.

Procedural Fluency

In **Lessons 10–11**, students practice dilating figures on and off the grid using distance measurements. In **Lesson 10**, they determine slope in a series of repeated challenges.

Adaptive Reasoning

In **Lesson 8**, students reflect on and revise their strategies for dilating points in the mini-golf game.

Application

In **Lesson 14**, students apply their understanding of similar triangles and proportional relationships in the context of shadows.

Productive Disposition

In **Lesson 7**, students use their creativity to make drawings that include dilations.

Unit at a Glance

Assess and Respond

A Pre-Unit Check

Learn more about your students' understanding of foundational concepts and skills that will support them in the upcoming unit. This can be given in its entirety before the unit or spread throughout the unit.

MA.4.M.1.2 MA.4.FR.2.4 MA.6.AR.3.3 MA.6.AR.3.5 MA.6.GR.2.2
MTR.4.1 MTR.5.1 MTR.6.1

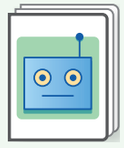
Sub-Unit 1

1 Scaling Machines

What Are Scaled Copies?

Describe characteristics of **scaled copies** and create scaled copies using equivalent ratios.

MA.7.GR.1.5
MTR.4.1 MTR.5.1



2 Scaling Robots

Lengths and Scaled Copies

Scale figures by applying a **scale factor** to each length in the figure.

MA.7.GR.1.5
MTR.4.1 MTR.5.1

6 Scale Drawings

Comparing Scale Factor and Scale

Explore how **scale** is used to represent distances in the real world.

MA.7.GR.1.5
MTR.2.1 MTR.6.1 MTR.7.1

Assess and Respond

A Sub-Unit Quiz

Learn about your students' understanding of the concepts and skills so far in this unit.

MA.7.GR.1.5
MTR.3.1 MTR.4.1 MTR.5.1

Sub-Unit 2

7 Sketchy Dilations

Introducing Dilations

Explore the concept of **dilation** as a process that produces scaled copies of an original figure.

MA.8.GR.2.2
MTR.4.1

11 Dilations on a Plane

Dilations With Coordinates

Dilate figures using the coordinate plane.

MA.8.GR.2.2 MA.8.GR.2.3
MTR.4.1 MTR.5.1

Assess and Respond

A Sub-Unit Quiz

Learn about your students' understanding of the concepts and skills so far in this unit.

MA.8.GR.2.2, MA.8.GR.2.3

Sub-Unit 3

12 Scavenger Hunt

Similar Figures, Sides, and Angles

Justify whether two figures are **similar**, *congruent*, or neither using transformations.

MA.8.GR.2.2 MA.8.GR.2.3
MTR.4.1

Pacing: 21 days | **Short on time?** See pacing considerations below.

All instructional and assessment days are ~45 minutes each.

- 16 Lessons
- Math in Action
- Pre-Unit Check
- Sub-Unit Quizzes (2)
- End-of-Unit Assessment

3 Make It Scale

Drawing Scaled Copies

Draw accurate scaled copies on a grid by applying knowledge of scale factor.

MA.7.GR.1.5
MTR.4.1

4 Scale Factor Challenges

Effects of Scale Factors

Determine which scale factors create a copy that is larger, smaller, or the same size as the original.

MA.7.GR.1.5
MTR.4.1 MTR.5.1

5 Tiles

Scaling and Area

Explore how different scale factors affect the area of scaled copies.

MA.7.GR.1.5
MTR.1.1 MTR.3.1 MTR.4.1

8 Dilation Mini Golf

Dilations With No Grid

Perform dilations using measurement tools without a grid.

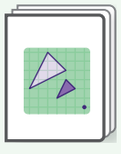
MA.8.GR.2.2
MTR.4.1 MTR.7.1

9 Make It a Combo!

Combining a Rigid Transformation and a Dilation

Perform *transformations*, including dilations, to move one figure onto another.

MA.8.GR.2.2
MTR.1.1 MTR.4.1



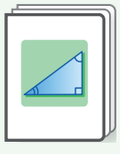
10 Match My Dilation

Dilations on a Square Grid

Dilate figures using a square grid to support measuring distances.

MA.8.GR.2.2 MA.8.GR.2.3
MTR.4.1 MTR.5.1

Sub-Unit 4



13 Are Angles Enough?

Similar Triangles

Justify that two triangles are similar using their angle measures.

MA.8.GR.1.4 MA.8.GR.1.5
MTR.4.1

14 Shadows

Side Length Relationships in Similar Triangles

Determine missing side lengths of similar triangles using equivalent ratio relationships.

MA.8.GR.2.4
MTR.2.1

15 Water Slide

Similarity and Slope

Use similar triangles and their height-to-base ratio to make sense of a given line's **slope**.

MA.8.AR.3.2 MA.8.GR.2.4
MTR.2.1 MTR.4.1 MTR.5.1

Unit at a Glance

continued

Math in Action

16 Slope Challengess

Developing Fluency with Slope

Use **slope triangles** to determine slopes of lines on the coordinate plane.

MA.8.AR.3.2
MTR.3.1 MTR.4.1

M Math in Action

Connecting Dilations and Similarity to Maps

Explore the connection between dilations, similarity, map making, and the contributions of Laura Escobar to the field of cartography.

MA.8.GR.1.4 MA.8.GR.2.2 MA.8.GR.2.3
MTR.2.1 MTR.4.1

Assess and Respond

A End-of-Unit Assessment

Learn about your students' understanding of the concepts and skills in this unit. There are two forms of the End-of-Unit Assessment: Forms A and B.

MA.8.GR.1.5 MA.8.GR.2.2 MA.8.GR.2.3
MA.8.GR.2.4 MA.8.GR.3.2
MTR.4.1 MTR.5.1

Pacing Considerations

Lesson 7: This lesson introduces students to concepts of dilation and similarity, which will be addressed in more depth in upcoming lessons. If most students demonstrate a strong understanding of dilations and similarity, this lesson may be omitted.

Lesson 9: This lesson can be omitted because it is another opportunity for students to explore dilations and does not introduce new content.

Lesson 10: This lesson supports students in developing fluency with precisely communicating the information needed to perform a dilation on paper. If students show a strong understanding working with dilations in earlier lessons, this lesson may be omitted. If omitted, be sure to discuss how to describe and apply dilations to a polygon on a grid given the coordinates of its vertices and the center of dilation.

Lesson Materials and Prep

Here are the materials and resources needed within each lesson, how to prepare them, and where to find them.

Lesson 1	Lesson 2	Lesson 3	Lesson 4
Copy: Show What You Know	**Copy:** Show What You Know	**Copy:** Show What You Know **Materials:** graph paper *(as needed)*, index cards	**Copy:** Show What You Know, Activity 3 You're Invited to Explore More Sheet *(one per student, as needed)*
Lesson 5	**Lesson 6**	**Lesson 7**	**Lesson 8**
Copy: Show What You Know	**Copy:** Show What You Know, Activity 2 You're Invited to Explore More Sheet *(one per student, as needed)* **Materials:** calculators, measuring tools *(in centimeters)*	**Copy:** Show What You Know **Materials:** straightedges *(optional)*	**Copy:** Show What You Know **Materials:** rulers or straightedges
Lesson 9	**Lesson 10**	**Lesson 11**	**Lesson 12**
Copy: Show What You Know **Materials:** straightedges, tracing paper	**Copy:** Show What You Know, Activity 3 Sheet *(one per student)* **Materials:** straightedges, graph paper *(as needed)*	**Copy:** Show What You Know **Materials:** coloring tools	**Copy:** Show What You Know **Cut:** Activity 2 Cards *(one card per student)* **Materials:** protractors, rulers, straightedges, tracing paper
Lesson 13	**Lesson 14**	**Lesson 15**	**Lesson 16**
Copy: Show What You Know **Materials:** tracing paper *(as needed)*, meter stick	**Copy:** Show What You Know **Materials:** coloring tools, straightedges	**Copy:** Show What You Know, Warm-Up Activity Sheet *(one per pair)* **Materials:** scissors, straightedges, graph paper *(as needed)*	**Copy:** Show What You Know **Materials:** straightedges *(as needed)*

Resource Books

Assessment and Lesson Resources

- Pre-Unit Check
- Sub-Unit Quiz
- End-of-Unit Assessment, Forms A & B
- Assessment Rubrics
- Show What You Knows
- Activity Sheets and Cards

Intervention and Extension Resources

- Mini-Lessons
- Prior Grade Mini-Lessons
- Extensions

Caregiver Support

These resources can be shared with students' caregivers. They provide background on the mathematics in this unit, as well as suggestions for supporting students at home.

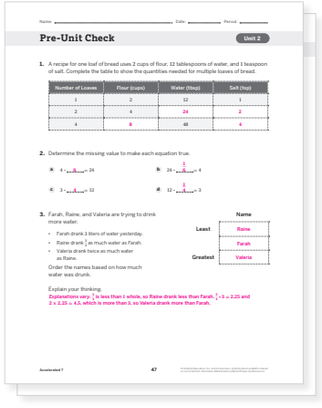
Assessments

Every unit includes a variety of formative and summative assessments designed to highlight students' understanding of a variety of concepts and skills.

Pre-Unit Check

Assign the **Pre-Unit Check** to learn more about your students' understanding of foundational concepts and skills that will support them in the upcoming unit. This can be given in its entirety before the unit or spread throughout the unit.

FL B.E.S.T. Math Benchmarks: MA.4.M.1.2, MA.4.FR.2.4, MA.6.AR.3.3, MA.6.AR.3.5, MA.6.GR.2.2


Name _____ Date _____ Period _____

Pre-Unit Check Unit 2

1. A recipe for one loaf of bread uses 2 cups of flour, 12 tablespoons of water, and 1 teaspoon of salt. Complete the table to show the quantities needed for multiple loaves of bread.

Number of Loaves	Flour (cups)	Water (tbsp)	Salt (tsp)
1	2	12	1
2	4	24	2
4	8	48	4

2. Determine the missing value to make each equation true.

a. $4 \cdot \square = 24$ b. $24 \cdot \square = 4$

c. $2 \cdot \square = 12$ d. $12 \cdot \square = 3$

3. Farah, Raine, and Valeria are trying to drink more water.
- Farah drank 3 liters of water yesterday.
- Raine drank $\frac{3}{4}$ as much water as Farah.
- Valeria drank twice as much water as Raine.

Order the names based on how much water was drunk.

	Name
Least	Raine
	Farah
Greatest	Valeria

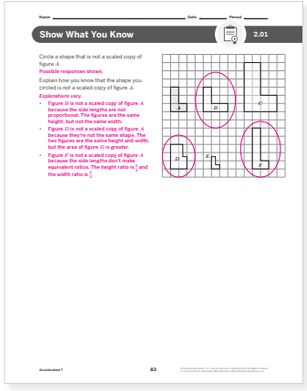
Explain your thinking:
Explanations vary: $\frac{3}{4}$ is less than 1 whole, so Raine drank less than Farah. $\frac{3}{4} \cdot 3 = 2.25$ and $2 \cdot 2.25 = 4.5$, which is more than 3, so Valeria drank more than Farah.

Accelerated 7 47

Lesson | Show What You Know

Use the **Show What You Know** for each lesson to learn more about your students' progress toward the lesson goal(s). Here is one example from this unit.

FL B.E.S.T. Math Benchmarks on Lesson 1 Show What You Know: MA.8.AR.2.2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Show What You Know 2.01

Circle a shape that is not a scaled copy of figure *A*.
Possible responses shown.

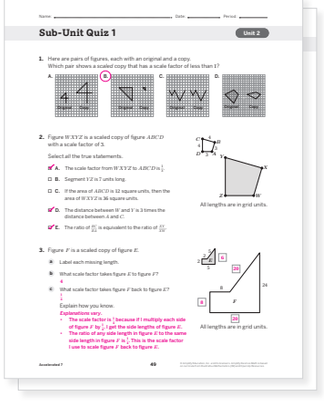
Explain how you know that the shape you circled is *not* a scaled copy of figure *A*.
Explanations vary:
- Figure *B* is not a scaled copy of figure *A* because the side lengths are not proportional. The figures are the same height, but not the same width.
- Figure *D* is not a scaled copy of figure *A* because they're not the same shape. The two figures are the same height and width, but the area of figure *D* is greater.
- Figure *F* is not a scaled copy of figure *A* because the side lengths don't make equivalent ratios. The height ratio is $\frac{1}{1}$ and the width ratio is $\frac{2}{3}$.

Accelerated 7 83

Sub-Unit Quizzes 1 & 2

Assign the **Sub-Unit Quizzes** to learn about your students' understanding of the concepts and skills so far in this unit.

FL B.E.S.T. Math Benchmarks: MA.7.GR.1.5, MA.8.GR.2.2, MA.8.GR.2.3


Name _____ Date _____ Period _____

Sub-Unit Quiz 1 Unit 2

1. Here are pairs of figures, each with an original and a copy. Which pair shows a scaled copy that has a scale factor of less than 1?

A. B. C. D.

2. Figure $WXYZ$ is a scaled copy of figure $ABCD$ with a scale factor of 3.

Select all the true statements.
- ☑ A. The scale factor from $WXYZ$ to $ABCD$ is $\frac{1}{3}$.
- ☐ B. Segment YZ is 7 units long.
- ☐ C. If the area of $ABCD$ is 12 square units, then the area of $WXYZ$ is 36 square units.
- ☑ D. The distance between W and Y is 3 times the distance between A and C.
- ☑ E. The ratio of $\frac{W}{Y}$ is equivalent to the ratio of $\frac{XY}{BC}$.

All lengths are in grid units.

3. Figure F is a scaled copy of figure E.
a. Label each missing length.
b. What scale factor takes figure E to figure F? 4
c. What scale factor takes figure F back to figure E? $\frac{1}{4}$
Explain how you know.
Explanations vary:
- The scale factor is $\frac{1}{4}$ because if I multiply each side of figure F by $\frac{1}{4}$ I get the side lengths of figure E.
- The ratio of any side length in figure E to the same side length in figure F is $\frac{4}{1}$. This is the scale factor I use to scale figure F back to figure E.

All lengths are in grid units.

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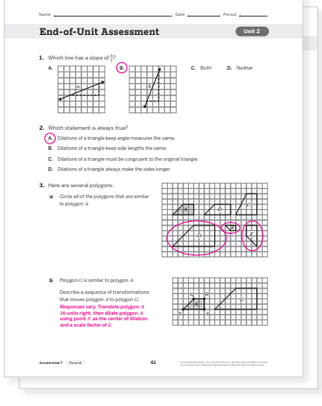
End-of-Unit Assessment

Assign the **End-of-Unit Assessment** to learn about your students' understanding of the concepts and skills in this unit. There are two forms of the End-of-Unit Assessment: Forms A and B.

FL B.E.S.T. Math Benchmarks: MA.8.GR.1.5, MA.8.GR.2.2, MA.8.GR.2.3, MA.8.GR.2.4, MA.8.GR.3.2


Name _____ Date _____ Period _____

End-of-Unit Assessment Unit 2

1. Which line has a slope of $\frac{2}{3}$?

A. B. C. Both D. Neither

2. Which statement is always true?
A. Dilations of a triangle keep angle measures the same.
B. Dilations of a triangle keep side lengths the same.
C. Dilations of a triangle must be congruent to the original triangle.
D. Dilations of a triangle always make the sides longer.

3. Here are several polygons.

a. Circle all of the polygons that are similar to polygon *A*.

b. Polygon *G* is similar to polygon *A*.
Describe a sequence of transformations that moves polygon *A* to polygon *G*.
Responses vary. Translate polygon *A* 10 units right, then dilate polygon *A* using point *S* as the center of dilation and a scale factor of 2.

Accelerated 7 Form A 61

Differentiation

Every unit includes additional resources to support, strengthen, and stretch student learning.

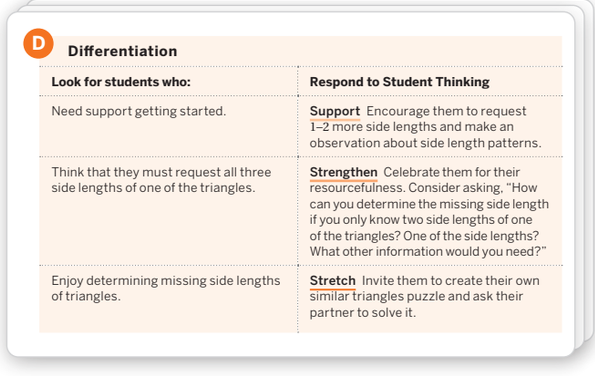
Differentiation Across the Unit

	Support	Strengthen	Stretch
	Provide targeted intervention for students by using these resources.	Reinforce students' understanding of the concepts assessed by using these resources.	Challenge students and extend their learning with these resources.
Sub-Unit 1 (Lessons 1–6) • Describe how scaling affects lengths, angles, and areas in scaled copies. • Use scale factors to create and compare scaled copies.	**Mini-Lesson** • *Connecting Scale Factors to Scaled Copies*	**Repeated Challenges** • Lesson 4 (*Scale Factor Challenges*) **Practice** • Lessons 1–6	**You're invited to explore more.** • Lesson 3 (*Make It Scale*) • Lesson 4 (*Scale Factor Challenges*) • Lesson 6 (*Scale Drawings*) **DOK3 Practice** • Lessons 2–5
Sub-Unit 2 (Lessons 7–11) • Apply dilations to figures on and off a coordinate grid.	**Mini-Lesson** • *Dilating Figures on a Square Grid*	**Challenge Creator** • Lesson 4 (*Match My Dilation*) **Practice** • Lessons 7–11	**DOK3 Practice** • Lesson 7
Sub-Unit 3 (Lessons 12–14) • Use transformations to determine whether two figures are similar.	**Mini-Lesson** • *Determining Missing Side Lengths in Similar Triangles*	**Practice** • Lessons 12–14	**DOK3 Practice** • Lessons 12 and 13
Sub-Unit 4 (Lessons 15–16) • Make connections between slope and similar triangles on a line. • Determine the slopes of lines.	**Mini-Lesson** • *Calculating Slope By Drawing Triangles on a Coordinate Plane*	**Repeated Challenges** • Lesson 10 (*Slope Challenges*) **Practice** • Lessons 15–16	**DOK3 Practice** • Lesson 16

Differentiation Within the Lesson

Each Teacher Edition includes point-of-use differentiation suggestions to modify or adjust instruction during the lesson to accommodate the needs of all learners.

Here is one example from **Lesson 14**, where students determine missing side lengths of similar triangles.



Differentiation

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Encourage them to request 1–2 more side lengths and make an observation about side length patterns.
Think that they must request all three side lengths of one of the triangles.	**Strengthen** Celebrate them for their resourcefulness. Consider asking, "How can you determine the missing side length if you only know two side lengths of one of the triangles? One of the side lengths? What other information would you need?"
Enjoy determining missing side lengths of triangles.	**Stretch** Invite them to create their own similar triangles puzzle and ask their partner to solve it.

Accessibility

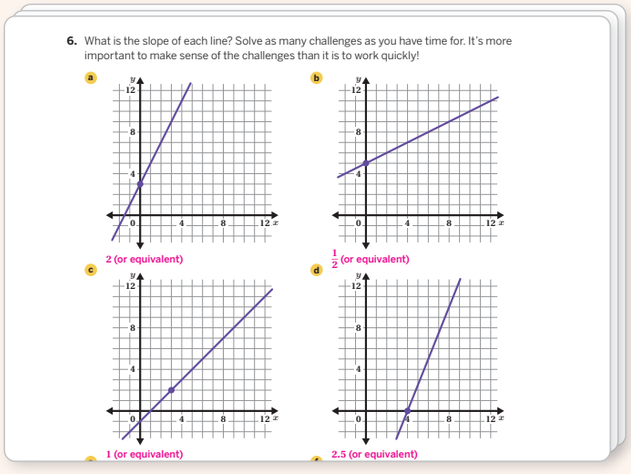
Every lesson is designed using the principles of Universal Design for Learning. Every lesson also includes support suggestions that build on students' strengths.

Universal Design for Learning

Here is one example from a lesson designed to support Action & Expression.

Spotlight on Action & Expression . . .

Lesson 16: In this lesson, students build fluency with calculating slope when solving repeated challenges. The repeated challenges allow them to access problems at different levels of rigor with responsive feedback.



Just-In-Time Accommodations and Supports

Each Teacher Edition includes suggested supports based on the areas of cognitive function from the Education Development Center (EDC).

Spotlight on Executive Functioning . . .

Here are two suggestions in this unit for supporting students with managing multi-step processes and complex information.

Lesson 12, Activity 2

A **Accessibility: Executive Functioning** To support this multi-step activity, invite two students to play an example round for the class.

Lesson 15, Activity 2

A **Accessibility: Executive Functioning** Consider providing a T-chart for students to record what they notice and wonder.

Language Development

Every lesson includes opportunities for all students to develop fluency with mathematical language.

Vocabulary

New Vocabulary	Lesson(s)
proportional **scaled copy**	1
corresponding parts **scale factor**	2
scale **scale drawing**	6
dilation	7
center of dilation	8
similar	12
slope	15
slope triangle	16

Review Vocabulary
area
congruent
equivalent ratios
image
origin
pre-image
ratio
sequence of transformations

Math Language Routines and Language Goals

Math Language Routine	Lesson(s)
MLR1: Stronger and Clearer Each Time	13
MLR2: Collect and Display	1, 5–8, 10, and 15
MLR3: Critique, Correct, Clarify	4 and 15–16
MLR5: Co-craft Questions	14
MLR7: Compare and Connect	14

Sub-Unit Language Goals

- Analyze how scaling affects lengths, angles, and areas in scaled copies using mathematically precise language.
- Describe dilations precisely using terms such as *pre-image, center of dilation,* and *scale factor.*
- Justify whether two figures are similar using terms such as *translation, rotation, reflection, dilation, pre-image,* and *image.*
- Explain how to calculate the slope of a line.

English Language Learners Support

Every Teacher Edition includes suggestions for supporting the needs of students learning English.

Here is one example from **Lesson 14**, where students determine missing sides of similar triangles and explain their strategies.

ELL **English Language Learners** Provide sentence frames to help students explain their strategy (e.g., "I noticed that _____, so I _____." or "First, I _____ because _____.") **(Writing and Speaking)**

Connections to Future Learning

Here is how the content in this unit connects to where your students are headed in their math journeys.

Linear Relationships

In this unit, students learn about slope in the context of similar triangles. **In Unit 4**, students will make connections between slope and the constant of proportionality and the rate of change of a linear relationship.

FL B.E.S.T. Math Benchmark:
MA.8.AR.3

Example:

The slope of the line in the graph on the right shows the relationship between the number of rides taken and the change in the value of a public transit card.

The slope is $-\frac{40}{16} = -2.5$, or $-\$2.50$ added to the card per ride.

Trigonometry and Similar Triangles

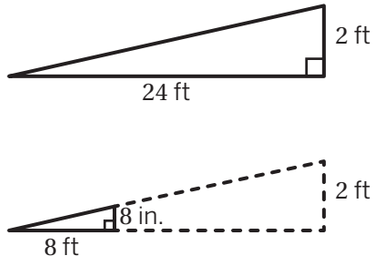
In this unit, students experimentally verify that corresponding angles and ratios of side lengths in similar figures must be equivalent. **In a future course**, students will prove this fact and use it to develop trigonometric ratios such as sine, cosine, and tangent.

FL B.E.S.T. Math Benchmarks:
MA.912.GR.1, MA.912.T.1

Example:

For example, the maximum height-to-length ratio for an accessibility ramp is $1:12$.

We can write this relationship using the *tangent function* as $tan(x°) = \frac{opposite}{adjacent} = \frac{1}{12}$. Solving for x, we see that any ramp with an angle of $= 4.67°$ or less will meet the requirement.



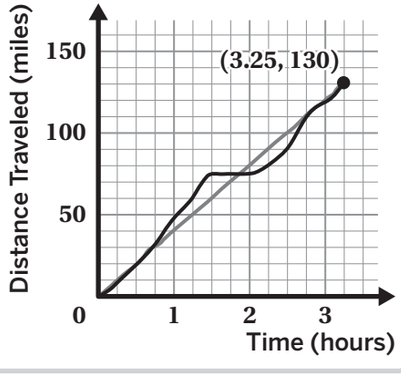
Average Rate of Change

In this unit, students explore the slope of lines through the lens of dilations and scale factors. **In a future course**, students will use slopes to calculate the average rate of change of functions over specific intervals.

FL B.E.S.T. Math Benchmark:
MA.912.F.1.3

Example:

The graph shows the distance Arjun's family traveled on a trip as a function of time. Because of traffic and stops along the way, Arjun's family does not travel at a constant speed. The slope of the line between the starting and ending points can help determine Arjun's family's average speed for the trip: $\frac{130}{3.25} = 40$ miles per hour.

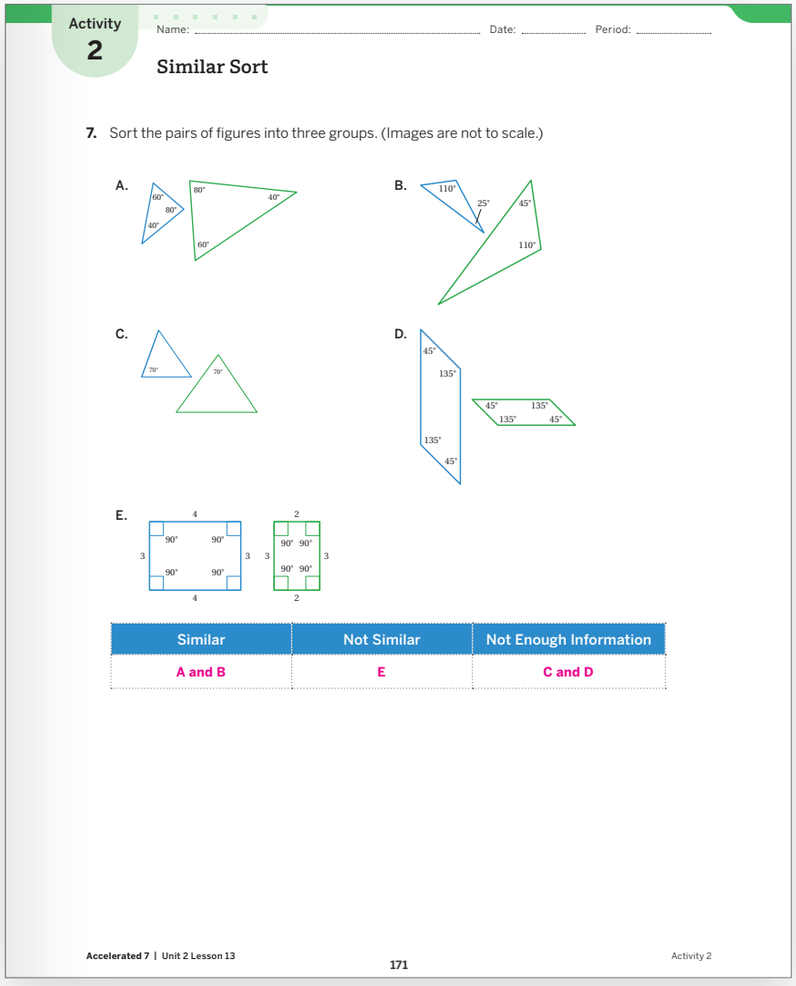


Professional Learning

Here is an opportunity to deepen your understanding of the math in this unit and how to teach it.

Spotlight on Strategic Competence

Lesson 13, Activity 2



Activity 2

Name: _____ Date: _____ Period: _____

Similar Sort

7. Sort the pairs of figures into three groups. (Images are not to scale.)

A.

B.

C.

D.

E.

Similar	Not Similar	Not Enough Information
A and B	E	C and D

Accelerated 7 | Unit 2 Lesson 13 171 Activity 2

Context

In this activity, students apply what they know about the properties of similarity to sort pairs of polygons as *similar*, *not similar*, or *not enough information*.

Try This

Put on your student hat and complete Problem 7.

Points to Ponder

- What do students need to understand about similarity to determine if they have enough information to classify each pair?
- Some students may sort pair D as *similar*. How might you support students to think critically in identifying what information is needed to make a classification?
- How does similarity build from their understanding of proportionality?

Notes:

Assess and Respond

Pre-Unit Check

Independent | 45 min

Facilitation: Assign the Pre-Unit Check to learn more about your students' understanding of foundational concepts and skills that will support them in the upcoming unit. This can be given in its entirety before the unit or spread throughout the unit. If time allows, consider inviting students to complete one or more of the tasks in the Unit Synthesis and Reflection.

Item Analysis

Problem(s)	Concept or skill	Preparation for	DOK	FL B.E.S.T. Math Benchmarks
1	Completing a part-to-part-to-part ratio table.	Lesson 2	1	MA.6.AR.3.3
2	Determining unknown values of equations by understanding the effects of multiplying by a whole number versus a fraction	Lesson 3	1	MA.4.FR.2.4
3	Interpreting contexts involving multiplying numbers by values less than or greater than 1	Lesson 4	2	MA.6.FR.2.4 MTR.6.1
4	Calculating the area of a figure on a grid composed of triangles and rectangles	Lesson 5	2	MA.6.GR.2.2, MTR.4.1. MTR.5.1
5	Ordering different units of length within the same measurement system	Lesson 6	1	MA.4.M.1.2
6	Using a table to convert between different units of measure	Lesson 7	1	MA.6.AR.3.5

Assessment and Lesson Resources

- Student Assessments
- Show What You Knows and Answer Keys

Intervention and Extension Resources

- Mini-Lessons
- Extensions

Name: Date: Period:

Pre-Unit Check

Unit 2

1. A recipe for one loaf of bread uses 2 cups of flour, 12 tablespoons of water, and 1 teaspoon of salt. Complete the table to show the quantities needed for multiple loaves of bread.

Number of Loaves	Flour (cups)	Water (tbsp)	Salt (tsp)
1	2	12	1
2	4	24	2
4	8	48	4

2. Determine the missing value to make each equation true.

a. $4 \cdot \underline{6} = 24$

b. $24 \cdot \underline{\frac{1}{6}} = 4$

c. $3 \cdot \underline{4} = 12$

d. $12 \cdot \underline{\frac{1}{4}} = 3$

3. Farah, Raine, and Valeria are trying to drink more water.

- Farah drank 3 liters of water yesterday.
- Raine drank $\frac{3}{4}$ as much water as Farah.
- Valeria drank twice as much water as Raine.

Order the names based on how much water was drunk.

	Name
Least	Raine
	Farah
Greatest	Valeria

Explain your thinking.

Explanations vary. $\frac{3}{4}$ is less than 1 whole, so Raine drank less than Farah. $\frac{3}{4} \cdot 3 = 2.25$ and $2 \times 2.25 = 4.5$, which is more than 3, so Valeria drank more than Farah.

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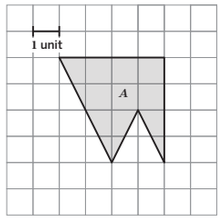
Name: Date: Period:

Pre-Unit Check (continued)

Unit 2

4. What is the area of figure A?

10 square units



Explain or show your thinking.

Explanations vary. Figure A can be surrounded by a 4-by-4-unit square with two triangles removed. Those triangles have areas of 4 square units and 2 square units. The area of figure A is 10 square units because $16 - 4 - 2 = 10$.

5. Order these units by length.

1 mile | 1 inch | 1 yard | 1 foot

1 inch	1 foot	1 yard	1 mile
Least			Greatest

6. Marine biologists use Wiffle balls in photos to measure corals and other objects. This measurement is a Wiffle.

Complete the table of Wiffles and inches.

Distance (Wiffles)	Distance (inches)
2	7
6	21
3	10.5

D Differentiation (Pre-Unit Check)

Note: To **strengthen** and **stretch** students' learning, refer to the differentiation resources suggested throughout this unit and in the Unit Overview.

Sub-Unit Goals	Problem(s)	Respond to Student Thinking
Preparation for Sub-Unit 1: • Calculate the area of rectangles and triangles. • Calculate the area of composite figures by decomposing into rectangles and triangles or by surrounding and subtracting. **(Lessons 1–5)**	1	**Support** • **Teacher Move:** Before **Lesson 1**, consider inviting students to share strategies for completing the table.
	2	**Support** • **Teacher Move:** During the Synthesis in **Lesson 2**, consider writing the equation $10 \cdot _____ = 15$ and discussing solving strategies.
	3	**Support** • **Mini-Lesson:** *Comparing Products Without Multiplying* • **Teacher Move:** After **Lesson 4**, consider revisiting this problem and inviting students to share their reasoning. If students do not make connections to scale factors, consider asking, "If Farah is the original, what would Rebecca's scale factor be? Valeria's?"
	4	**Support** • **Mini-Lesson:** *Calculating Areas of Triangles* • **Teacher Move:** Before **Lesson 5**, consider inviting students to share strategies for how they calculated the area of this composite figure.
	5	**Support** • **Teacher Move:** Before **Lesson 7**, consider creating an anchor chart to review units of measure for length. List common metric and customary units of length in order from smallest to largest.
	6	**Support** • **Teacher Move:** After **Lesson 6**, consider reviewing this problem and discussing strategies for finding the missing values.

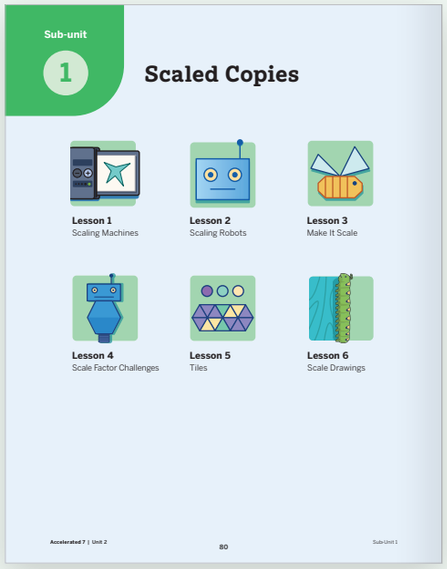
Sub-Unit 1

Scaled Copies

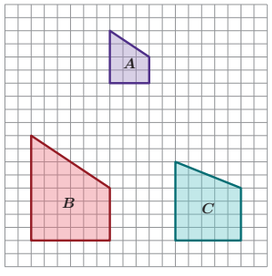
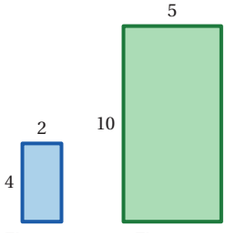
Sub-Unit 1 Goals

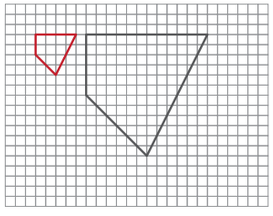
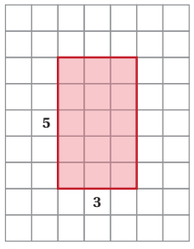
- **Goal:** Describe how scaling affects lengths, angles, and areas in scaled copies.
- **Goal:** Use scale factors to create and compare scaled copies.
- **Language Goal:** Analyze how scaling affects lengths, angles, and areas in scaled copies using mathematically precise language.

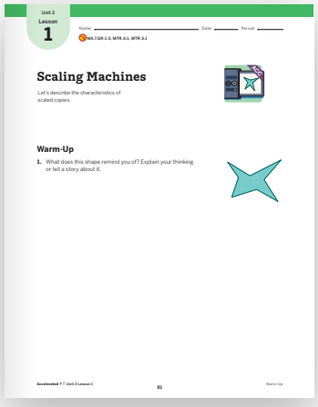
In this sub-unit, students explore objects that are and are not **scaled copies** of one another to build understanding of the characteristics of a scaled copy. They describe how scaling affects lengths, angles, and areas in scaled copies, and use **scale factors** to create and compare scaled copies.



Math That Matters Most

Concept or Skill	For example . . .
Determine whether or not two figures are scaled copies.	Here is figure A and two other figures. Which is a scaled copy of figure A?  Figure B is a scaled copy of figure A because it has the same angles and all the lengths are multiplied by 2.
Determine the scale factor between two scaled copies.	What scale factor makes figure A match figure B?  Figure A Figure B The scale factor 2.5 makes figure A match figure B.

Concept or Skill	For example . . .
Create a scaled copy using a given scale factor.	Draw a scaled copy of the polygon using a scale factor of 3.  The gray polygon is a scaled copy of the red polygon using a scale factor of 3.
Calculate the area of a scaled copy with a given scale factor.	Imagine scaling this rectangle using a scale factor of 4. What is the area of the scaled copy?  The area of the scaled copy is 240 square units. The dimensions of the copy are $5 \cdot 4 = 20$ units and $3 \cdot 4 = 12$ units, and $20 \cdot 12 = 240$.


Unit 2
Lesson
1
Name: Date: Period:
MA.7.GR.1.5, MTR.4.1, MTR.5.1
Scaling Machines
Let's describe the characteristics of scaled copies.
Warm-Up
1. What does this shape remind you of? Explain your thinking or tell a story about it.
81
Warm-Up

Scaling Machines

What Are Scaled Copies?

Let's describe the characteristics of scaled copies.

Learning Outcomes

Today's Goals

1. **Goal:** Determine whether one figure is a *scaled copy* of another.
2. **Goal:** Use equivalent ratios to create a *scaled copy* of a figure.
3. **Language Goal:** Justify whether two shapes are scaled copies of one another. **(Speaking and Listening)**

Students develop an understanding of what **scaled copies** are, describe characteristics of scaled copies, and use *equivalent ratios* to create several scaled copies of a figure. The lesson is designed to be accessible to all students regardless of prior knowledge, and to encourage students to see math as playful and exploratory.

Coherence

Prior Learning

In a previous course, students recognized and created equivalent ratios.

Future Learning

Later in the unit, students will continue to reason quantitatively about scaling, including learning that scaled copies are related by a scale factor. They will analyze and create precise scaled copies.

Connections

- Within the context of copies made on a working printer and a broken printer, students explore the criteria for what makes one figure a scaled copy of the other. (real world)

FL B.E.S.T. Math Benchmarks

Addressing

MA.7.GR.1.5
Solve mathematical and real-world problems involving dimensions and areas of geometric figures, including scale drawings and scale factors.

Mathematical Thinking and Reasoning Standards: MTR.4.1, MTR.5.1

ELA Expectations: ELA.K12.EE.1.1, ELA.K12.EE.3.1, ELA.K12.EE.5.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.6.AR.3.5	MA.8.GR.2.2
MA.6.GR.1.3	MA.912.GR.4.3

Vocabulary

New Vocabulary

scaled copy
proportional

Review Vocabulary

equivalent ratios

Lesson at a Glance

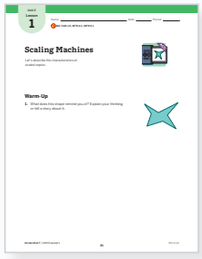
~ 45 min

MA.7.GR.1.5

Warm-Up

Independent | 5 min

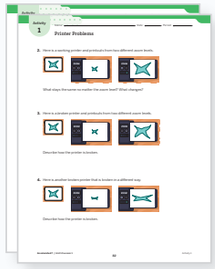
Students analyze a shape in preparation for identifying scaled copies of the shape.



Activity 1

Pairs | 15 min

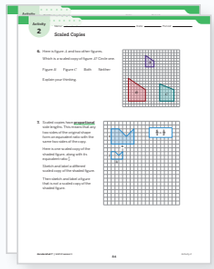
Students explore properties of **scaled copies** by identifying patterns in examples and non-examples. **(MTR.5.1)**



Activity 2

Pairs | 10 min

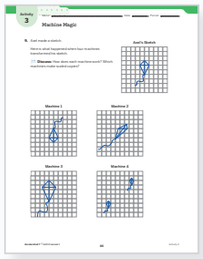
Students determine whether or not shapes are scaled copies by reasoning about *equivalent ratios* between sides.



Activity 3

Pairs | 5 min

Students discover ways that a shape can be changed to create or not create a scaled copy.

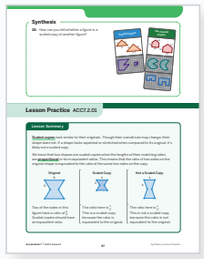


Synthesis

Whole Class | 5 min

Students synthesize their understanding of the important features of scaled copies.

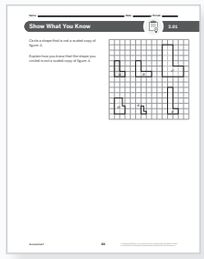
MLR2: Collect and Display



Show What You Know

Independent | 5 min

Students demonstrate their understanding by determining whether shapes are scaled copies.



Prep Checklist

This lesson includes:

- Student Edition
- Show What You Know

Independent | 5 min

Warm-Up

Purpose: Students analyze a shape in preparation for identifying scaled copies of the shape.

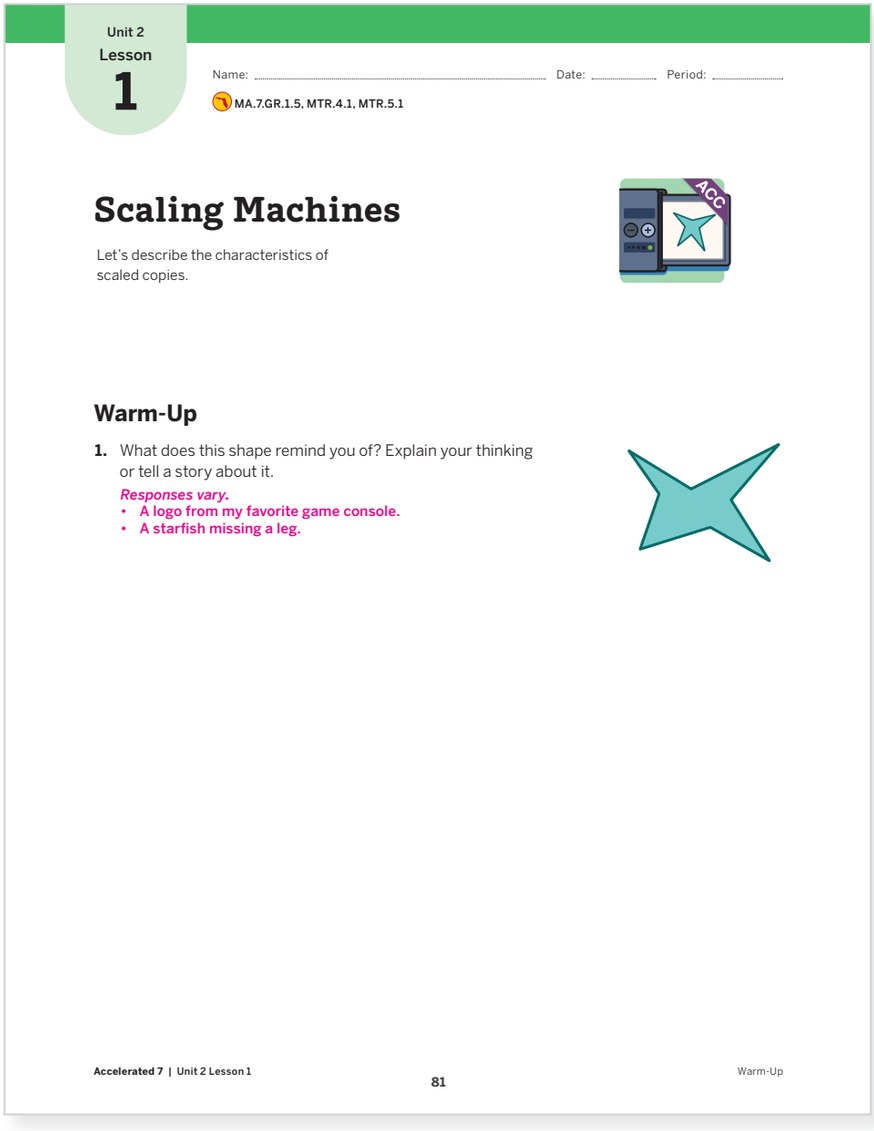
1 Launch

Invite students to share their observations.

2 Connect

Consider sharing that students will be using this shape in the next activity.

ELL **English Language Learners** Invite students to share in their primary languages. Encourage all students to use gestures and/or visuals as they share. **(Speaking and Listening)**



Unit 2
Lesson
1

Name: _____ Date: _____ Period: _____

MA.7.GR.1.5, MTR.4.1, MTR.5.1

Scaling Machines

Let's describe the characteristics of scaled copies.

Warm-Up

1. What does this shape remind you of? Explain your thinking or tell a story about it.

Responses vary.
- A logo from my favorite game console.
- A starfish missing a leg.

Accelerated 7 | Unit 2 Lesson 1 81 Warm-Up

Pairs | 15 min

Activity 1 Printer Problems

Purpose: Students explore properties of **scaled copies** by identifying patterns in examples and non-examples. **(MTR.5.1)**

1 Launch

To surface prior knowledge, consider asking students what experiences they have had with printers or copiers.

Consider asking:

- "Have you ever used the zoom feature on a web page?"
- "What happens to the picture or text when you zoom in or zoom out?"

2 Monitor

Listen for informal language students use to describe the working printer, such as "same shape, but different size."

Note: Allow students to use their own language to describe what they see. The term *scaled copy* will be introduced later in the activity.

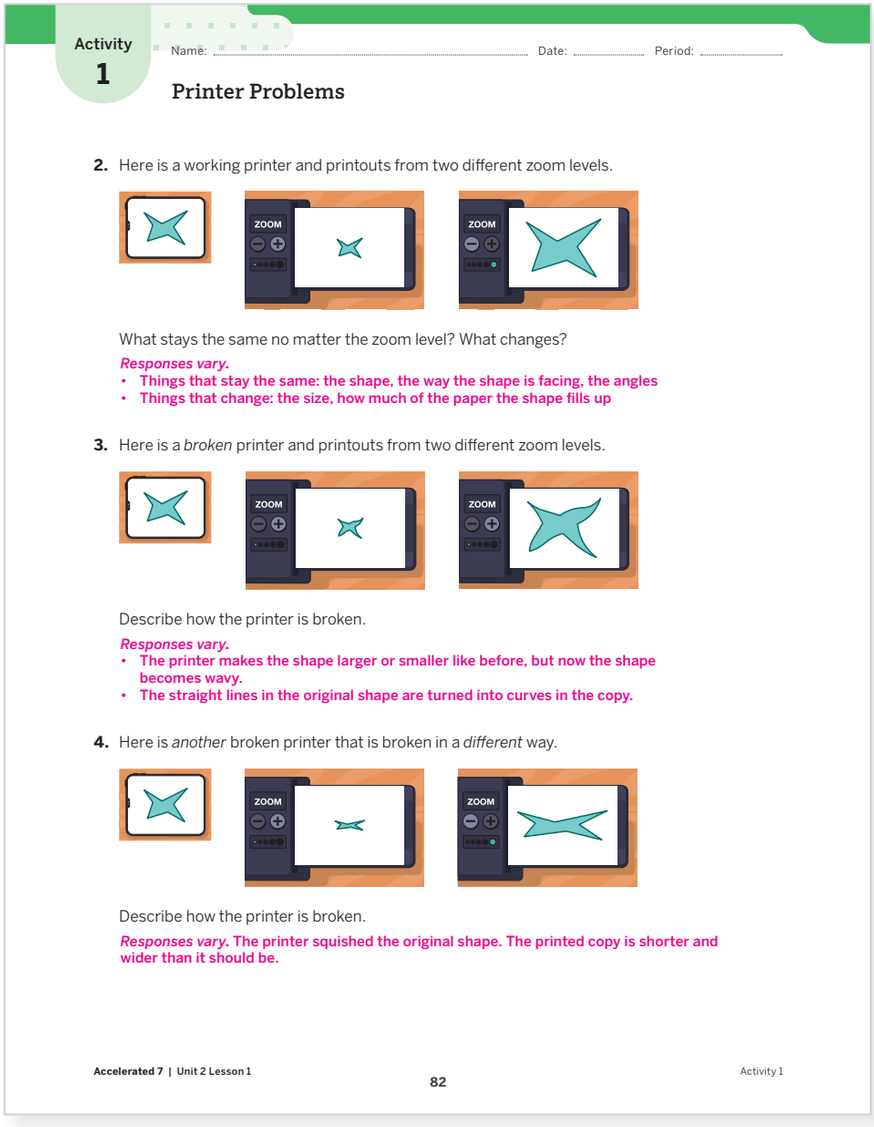
Consider asking, "How is this similar to the working printer? How is it different?" **(MTR.5.1)**

Responses vary.

- All the printers make the shape larger or smaller depending on the zoom level.
- The broken printer makes the shape wavy. *(Problem 3)*
- The broken printer squishes the shape by making it shorter or wider than it should be. *(Problem 4)*

Listen to partner discussions for student language and ideas that might be useful to share during the Connect.

Activity 1 continued >



Activity 1

Name: Date: Period:

Printer Problems

2. Here is a working printer and printouts from two different zoom levels.

What stays the same no matter the zoom level? What changes?

Responses vary.

- **Things that stay the same: the shape, the way the shape is facing, the angles**
- **Things that change: the size, how much of the paper the shape fills up**

3. Here is a *broken* printer and printouts from two different zoom levels.

Describe how the printer is broken.

Responses vary.

- **The printer makes the shape larger or smaller like before, but now the shape becomes wavy.**
- **The straight lines in the original shape are turned into curves in the copy.**

4. Here is *another* broken printer that is broken in a *different* way.

Describe how the printer is broken.

***Responses vary.* The printer squished the original shape. The printed copy is shorter and wider than it should be.**

Accelerated 7 | Unit 2 Lesson 1 82 Activity 1

Pairs | 15 min

Activity 1 Printer Problems (continued)

Purpose: Students explore properties of **scaled copies** by identifying patterns in examples and non-examples. **(MTR.5.1)**

2 Monitor

MLR **MLR1: Stronger and Clearer Each Time (optional)** Consider using this routine to support students in developing precise language. Invite students to meet with a partner, give feedback on each other's drafts, then use the feedback to write a second clearer draft. See the Routine Facilitation Guide for more information. **(MTR.4.1)**

Select and sequence student responses to share during the Connect.

D Differentiation

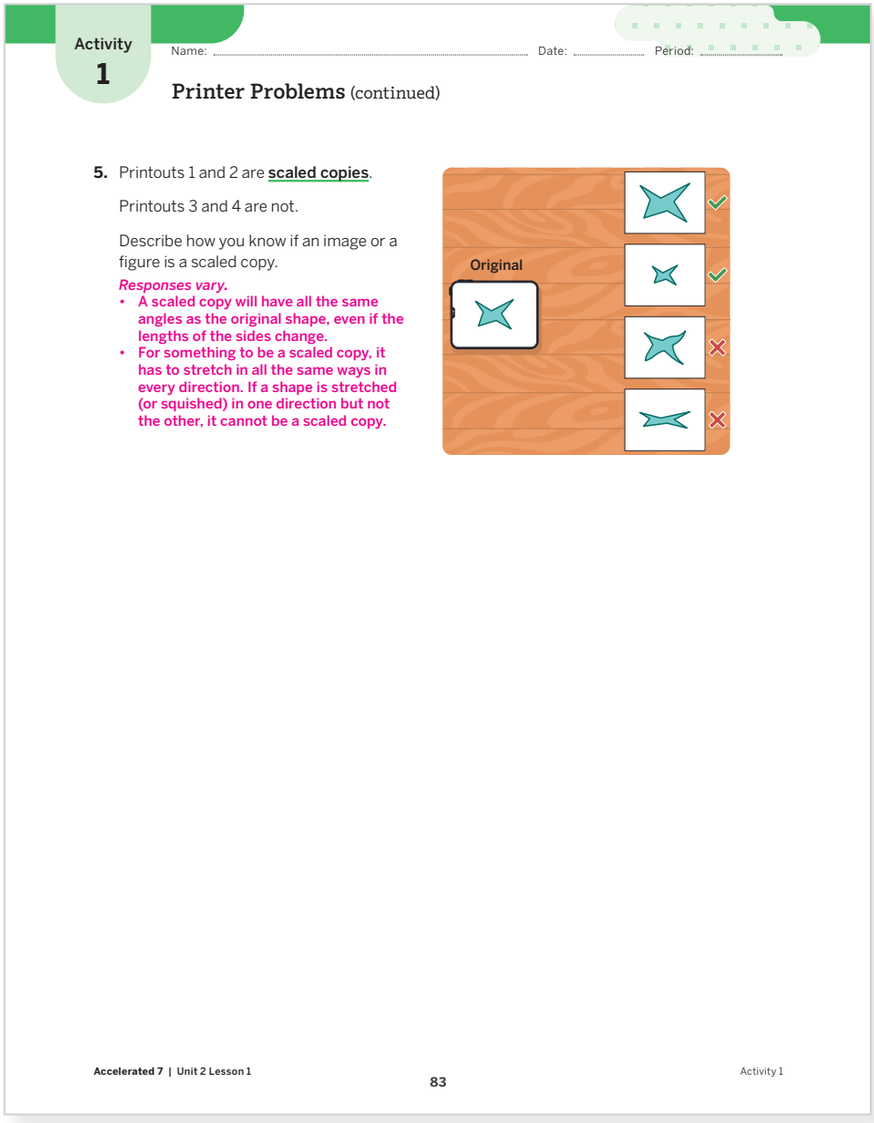
Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Encourage students to revisit their thinking on Problems 2–4 and make a connection to the explanations they wrote there.
Write or say that scaled copies are "larger or smaller than the original."	**Support** Consider asking, "Printouts 3 and 4 are larger and smaller but are not scaled copies. What do 1 and 2 have in common that make them scaled copies?"
Write or say that scaled copies have the "same shape."	**Strengthen** Invite students to say more. Consider asking, "Are all triangles scaled copies of one another? What about all rectangles?"
Use a variety of informal language to describe scaled copies, such as: • Larger, smaller, or the same size. • Same angles. • Different length. • Different area.	**Strengthen** Capture these strategies to share during the Connect. Consider asking, "What are some ways to create copies that are not scaled copies?"

3 Connect

Direct students' attention to the captured responses and invite students to share their thinking.

Create a class definition of the term **scaled copy** using students' informal language. Consider including non-examples to help emphasize what features a scaled copy always has.

Note: This lesson provides an introduction to the term *scaled copy*, which will be defined more precisely in Lesson 2.



Activity 1

Name: Date: Period:

Printer Problems (continued)

5. Printouts 1 and 2 are **scaled copies**.

Printouts 3 and 4 are not.

Describe how you know if an image or a figure is a scaled copy.

Responses vary.
- A scaled copy will have all the same angles as the original shape, even if the lengths of the sides change.
- For something to be a scaled copy, it has to stretch in all the same ways in every direction. If a shape is stretched (or squished) in one direction but not the other, it cannot be a scaled copy.

Accelerated 7 | Unit 2 Lesson 1 83 Activity 1

Key Takeaway: A **scaled copy** has the same angles and the same overall shape as the original figure but can be the same size, larger, or smaller.

Pairs | 10 min

Activity 2 Scaled Copies

Purpose: Students determine whether or not shapes are scaled copies by reasoning about *equivalent ratios* between sides.

Short on time? Consider omitting Problem 6.

1 Launch

Invite students to make and justify a choice.

D **Differentiation: (Support)** To support students getting started, consider asking, "Which copy has the same angles as the original?"

Note: It is okay if students are unsure. They will have more opportunities to explore what makes a scaled copy.

2 Monitor

Invite students to read the text aloud to one another and to explain what they think **proportional** means in their own words.

Consider asking, "What strategies will you use to draw a scaled copy of the figure and draw a figure that is not a scaled copy?"

A **Accessibility: Memory and Attention** To support keeping track of information, invite students to record the dimensions they try and whether or not they created a scaled copy.

Note: If further discussion about scaled copies is needed, consider pausing briefly to show a shape with a height of 6 and a length of 8. Consider asking, "Why might someone think this is a scaled copy?"

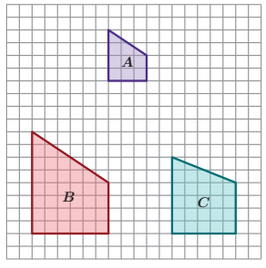
Activity 2 continued >

Activity 2

Name: Date: Period:

Scaled Copies

6. Here is figure A and two other figures.



Which is a scaled copy of figure A? Circle one.

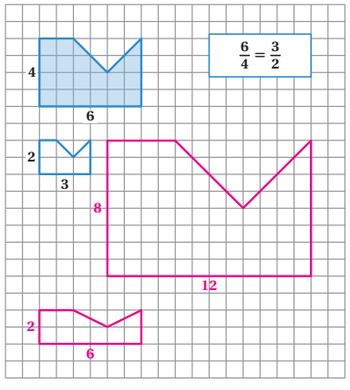
(Figure B) Figure C Both Neither

Explain your thinking.

Explanations vary.

- Figure B is the only one that has the same shape as figure A. Figure C is close to the same shape, but its top side is a little flatter.
- Figure B has the same angles as figure A, and the side lengths are all 2 times longer. Figure C doesn't have the same angles as figure A.

7. Scaled copies have **proportional** side lengths. This means that any two sides of the original shape form an *equivalent ratio* with the same two sides of the copy.



Here is one scaled copy of the shaded figure, along with its equivalent ratio $\frac{3}{2}$.

Sketch and label a *different* scaled copy of the shaded figure.

Then sketch and label a figure that is *not* a scaled copy of the shaded figure.

Responses vary.

Accelerated 7 | Unit 2 Lesson 1 84 Activity 2

Pairs | 10 min

Activity 2 Scaled Copies (continued)

Purpose: Students determine whether or not shapes are scaled copies by reasoning about *equivalent ratios* between sides.

Short on time? Consider omitting Problem 6.

2 Monitor

Encourage students to share their reasoning with a partner and work together to reach an agreement about whether the pairs of shapes are scaled copies. **(MTR.4.1)**

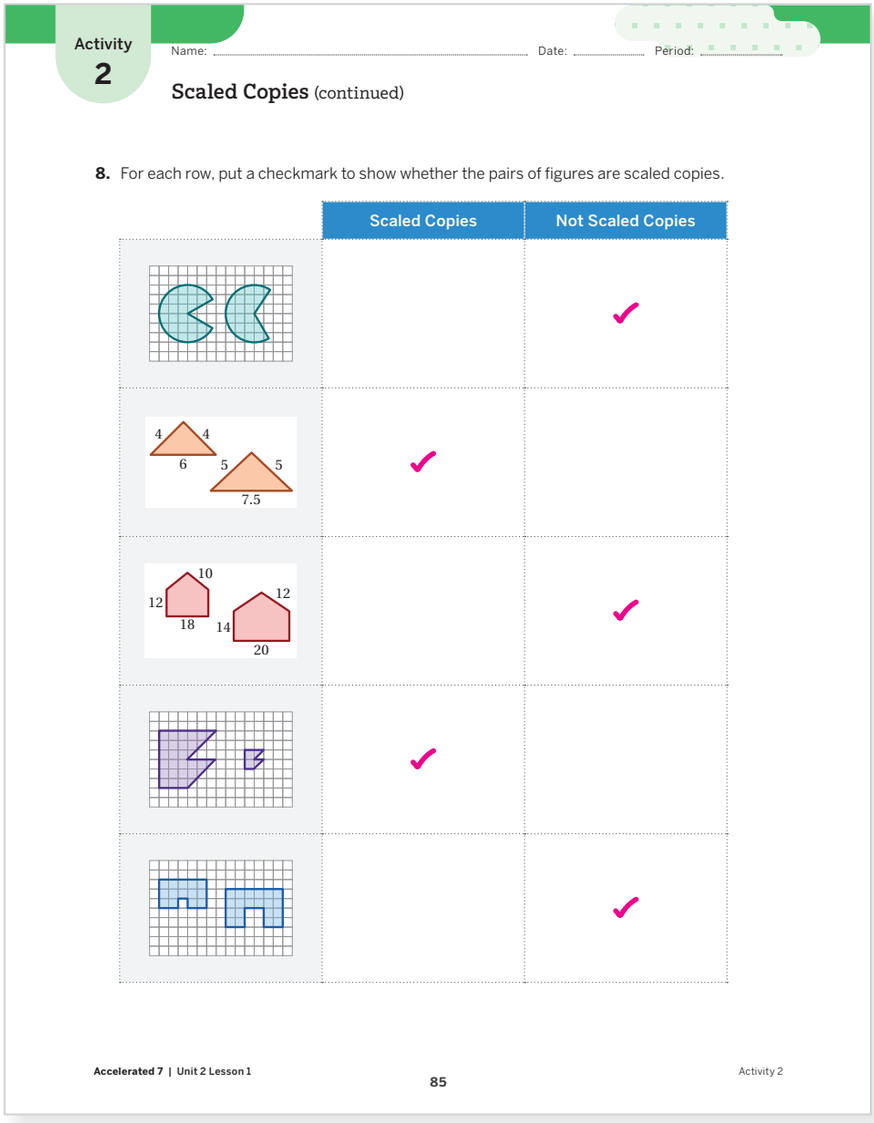
ELL **English Language Learners** Provide sentence frames to help students explain their strategy (e.g., "I can tell these are scaled copies because _____."). **(Writing and Speaking)**

Listen for a variety of student strategies for categorizing the pairs of shapes including both using *equivalent ratios* and inferring using the overall shape of each figure.

3 Connect

Invite students to share explanations and strategies for how they categorized the pairs of shapes.

To surface the Key Takeaway, consider asking, "How do the gridlines or measurements help you determine whether the shapes are scaled copies?"



Activity 2

Name: Date: Period:

Scaled Copies (continued)

8. For each row, put a checkmark to show whether the pairs of figures are scaled copies.

	Scaled Copies	Not Scaled Copies
		✓
4, 4, 6, 5, 5, 7.5	✓	
10, 12, 18, 12, 14, 20		✓
	✓	
		✓

Accelerated 7 | Unit 2 Lesson 1 85 Activity 2

Key Takeaway: Scaled copies have **proportional** side lengths. This means that any two sides of one shape form an *equivalent ratio* with the two matching sides of the other.

Pairs | 5 min

Activity 3 Machine Magic

Purpose: Students discover ways that a shape can be changed to create or not create a scaled copy.

Short on time? Consider omitting Activity 3.

1 Launch

Direct students' attention to Axel's sketch. Invite students to share their observations about the characteristics of the kite.

To generate excitement, consider asking, "Have you ever flown a kite?"

2 Monitor

Listen for a variety of formal and informal language students use to describe the machines.

ELL **English Language Learners** Encourage students to paraphrase each other's ideas to help them make connections and incorporate vocabulary from the unit. **(Speaking and Listening)**

Select and sequence student ideas to share during the Connect.

D Differentiation

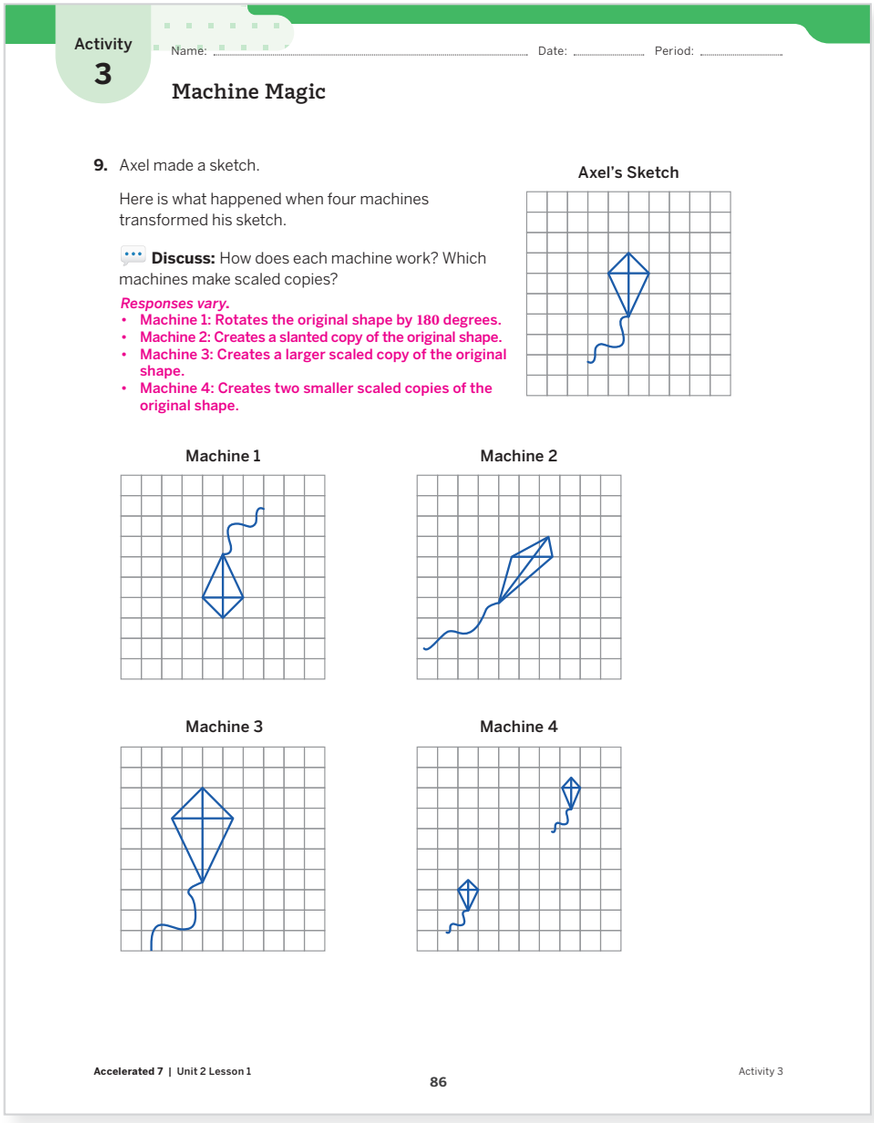
Look for students who:	Respond to Student Thinking
Think Machine 1 does not create scaled copies because it rotates the sketch.	**Support** Consider asking, "What are the important characteristics of scaled copies?"
Actively listen to their partners and build on each other's ideas or language.	**Strengthen** Celebrate these students one-on-one or publicly.
Identify that Machines 1, 3, and 4 create scaled copies.	**Stretch** Consider asking, "What do those three machines have in common?"

Listen for student strategies and invite students to share more about their explanations and ideas.

3 Connect

Discuss the characteristics students generated to describe which machines make scaled copies.

Consider asking, "What do the machines that create scaled copies have in common?"



Activity 3

Name: Date: Period:

Machine Magic

9. Axel made a sketch.

Here is what happened when four machines transformed his sketch.

Discuss: How does each machine work? Which machines make scaled copies?

Responses vary.
- Machine 1: Rotates the original shape by 180 degrees.
- Machine 2: Creates a slanted copy of the original shape.
- Machine 3: Creates a larger scaled copy of the original shape.
- Machine 4: Creates two smaller scaled copies of the original shape.

Axel's Sketch

Machine 1

Machine 2

Machine 3

Machine 4

Accelerated 7 | Unit 2 Lesson 1 86 Activity 3

Whole Class | 5 min

Synthesis

Purpose: Students synthesize their understanding of the important features of scaled copies.

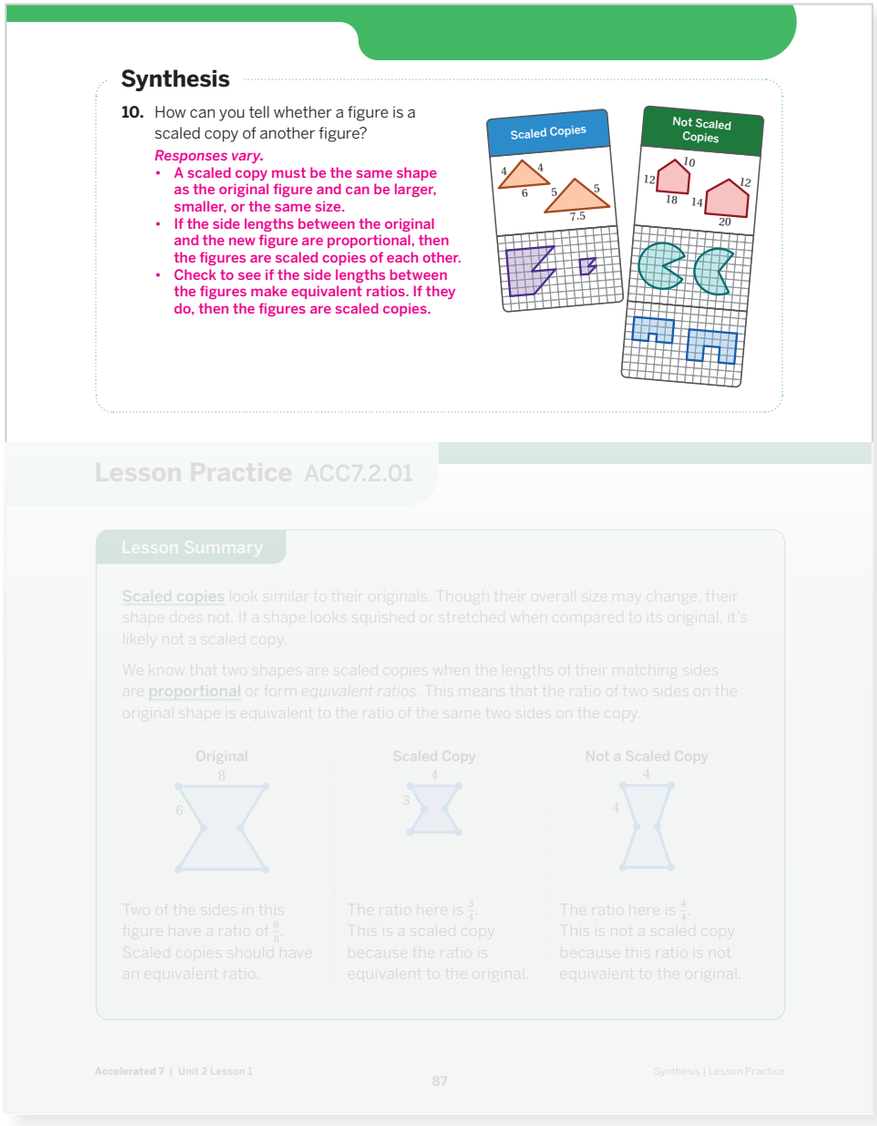
Invite students to respond independently, and then share their thinking with a partner.

Collect and share a variety of ideas, such as:

- Scaled copies are the same overall shape, but can be different sizes.
- You can check whether two shapes are scaled copies by looking for equivalent ratios between the sides (checking whether the side lengths are proportional).
- Figures that are not scaled copies of each other might be distorted. They have lines or angles that do not look like the original.

MLR **MLR2: Collect and Display** Use this routine to formalize the definitions of *scaled copies* and *proportional* using students' language. Display any new language, examples, or diagrams for students to reference later.

Note: If time allows, invite students to restate the definitions to a partner in their own words, and share what questions they still have.



Synthesis

10. How can you tell whether a figure is a scaled copy of another figure?

Responses vary.

- A scaled copy must be the same shape as the original figure and can be larger, smaller, or the same size.
- If the side lengths between the original and the new figure are proportional, then the figures are scaled copies of each other.
- Check to see if the side lengths between the figures make equivalent ratios. If they do, then the figures are scaled copies.

Lesson Practice ACC7.2.01

Lesson Summary

Scaled copies look similar to their originals. Though their overall size may change, their shape does not. If a shape looks squished or stretched when compared to its original, it's likely not a scaled copy.

We know that two shapes are scaled copies when the lengths of their matching sides are proportional or form *equivalent ratios*. This means that the ratio of two sides on the original shape is equivalent to the ratio of the same two sides on the copy.

Original	Scaled Copy	Not a Scaled Copy
8, 6	4, 3	4, 4
Two of the sides in this figure have a ratio of $\frac{6}{8}$. Scaled copies should have an equivalent ratio.	The ratio here is $\frac{3}{4}$. This is a scaled copy because the ratio is equivalent to the original.	The ratio here is $\frac{4}{4}$. This is not a scaled copy because this ratio is not equivalent to the original.

Accelerated 7 | Unit 2 Lesson 1 87 Synthesis | Lesson Practice

Lesson Takeaway: A **scaled copy** is a copy with the same overall shape as the original figure. It can be the same size, larger, or smaller. The two figures must have **proportional** side lengths, meaning they form *equivalent ratios*.

Independent | 5 min

Show What You Know

Purpose: Students demonstrate their understanding by determining whether shapes are scaled copies.

Today's Goals

1. **Goal:** Determine whether one figure is a *scaled copy* of another.
2. **Goal:** Use equivalent ratios to create a *scaled copy* of a figure.
3. **Language Goal:** Justify whether two shapes are *scaled copies* of one another. **(Speaking and Listening)**

Show What You Know (Assessment Resources)

Name: _____ Date: _____ Period: _____

Show What You Know 2.01

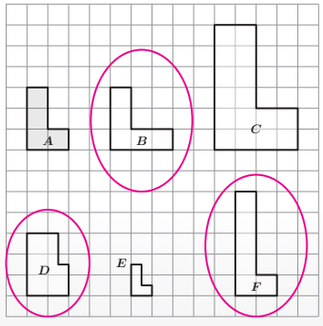
Circle a shape that is not a scaled copy of figure A.

Possible responses shown.

Explain how you know that the shape you circled is *not* a scaled copy of figure A.

Explanations vary.

- **Figure B is not a scaled copy of figure A because the side lengths are not proportional. The figures are the same height, but not the same width.**
- **Figure D is not a scaled copy of figure A because they're not the same shape. The two figures are the same height and width, but the area of figure D is greater.**
- **Figure F is not a scaled copy of figure A because the side lengths don't make equivalent ratios. The height ratio is $\frac{5}{3}$ and the width ratio is $\frac{2}{2}$.**



D Differentiation Use after Lesson 1

Support

Provide targeted intervention for students by using these resources.

If students need support understanding how to determine if figures are scaled copies:

» **Respond to Student Thinking:**

- Invite students to discuss how to use equivalent ratios to determine if two shapes are scaled copies with a partner.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 1 Extension Activities**.

Professional Learning

Reflect on getting to know your class today. What did you learn about your students?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

10. How can you tell whether a figure is a scaled copy of another figure?

Responses vary.

- A scaled copy must be the same shape as the original figure and can be larger, smaller, or the same size.
- If the side lengths between the original and the new figure are proportional, then the figures are scaled copies of each other.
- Check to see if the side lengths between the figures make equivalent ratios. If they do, then the figures are scaled copies.

Lesson Practice ACC7.2.01

Lesson Summary

Scaled copies look similar to their originals. Though their overall size may change, their shape does not. If a shape looks squished or stretched when compared to its original, it's likely not a scaled copy.

We know that two shapes are scaled copies when the lengths of their matching sides are **proportional** or form *equivalent ratios*. This means that the ratio of two sides on the original shape is equivalent to the ratio of the same two sides on the copy.

Original	Scaled Copy	Not a Scaled Copy
Two of the sides in this figure have a ratio of $\frac{6}{8}$. Scaled copies should have an equivalent ratio.	The ratio here is $\frac{3}{4}$. This is a scaled copy because the ratio is equivalent to the original.	The ratio here is $\frac{4}{4}$. This is not a scaled copy because this ratio is not equivalent to the original.

Lesson Practice
ACC7.2.01

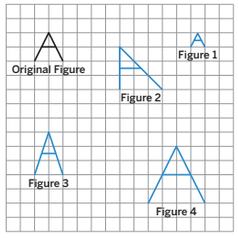
Name: _____ Date: _____ Period: _____

1. Andre's grandma ordered school pictures. She thought she would receive the original portrait, but instead she got the following images. How is each image different from the original portrait of Andre?

Original Portrait	Image A	Image B	Image C	Image D

Responses vary. Image A is "squished" from the sides. Image B looks like a smaller copy of the original. Image C is slanted. Image D is "squished" from the top.

Problems 2–4: Here is the original figure along with several other figures.



2. Which figures are scaled copies of the original figure?

Figures 1 and 4

3. How does each scaled copy compare to the original?

Responses vary.

- Figure 1 is a scaled copy of the original because it's the same shape, the lengths form equivalent ratios, and the angles have the same measure.
- Figure 4 is a scaled copy of the original because its dimensions are proportional to the original figure. It is twice as tall and twice as wide as the original figure.

4. How do you know a figure is *not* a scaled copy?

Responses vary.

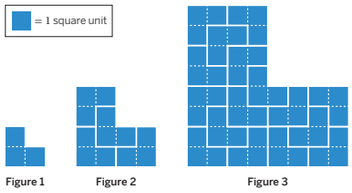
- Figure 2 is not a scaled copy of the original because it is not the same shape as the original figure. The angles at the top of the "A" are different from the original.
- Figure 3 is not a scaled copy of the original because its height and width aren't proportional to the original.

Lesson Practice
ACC7.2.01

Name: _____ Date: _____ Period: _____

FAST Practice

Problems 5–7: Figures 1, 2, and 3 are scaled copies of each other. Each small square has a side length of 1 unit and an area of 1 square unit.



5. Complete the table.

	Figure 1	Figure 2	Figure 3
Perimeter (units)	8	16	32
Area (sq. units)	3	12	48

6. Describe any patterns you see.

Responses vary. The perimeter of each figure that follows is twice the perimeter of the previous figure. The area of each figure that follows is four times greater than the area of the previous figure.

7. If the pattern continues, what will be the perimeter and area of Figure 4?

Perimeter: 64 units Area: 192 sq. units

Spiral Review

Problems 8–11: Complete each equation with a number that makes it true.

8. $5 \cdot \boxed{3} = 15$

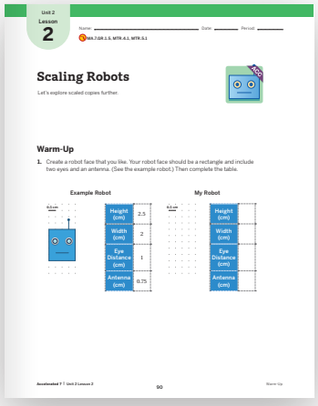
9. $6 \cdot \boxed{1.5} = 9$

10. $\boxed{8} \cdot 4 = 32$

11. $\boxed{\frac{1}{4}} \cdot 12 = 3$

Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1–4	2	MA.7.GR.1.5
FAST Practice	5–7	2	MA.7.GR.1.5
Spiral Review			
	8–11	1	MA.5.FR.2.2


Unit 2
Lesson 2
Scaling Robots
Let's explore scaled copies further.
Warm-Up
1. Create a robot face that you like. Your robot face should be a rectangle and include two eyes and an antenna. (See the example robot.) Then complete the table.
Example Robot
My Robot
Height (cm)
Width (cm)
Eye Distance (cm)
Antenna (cm)
2.5
2
1
0.75
90

Scaling Robots

Lengths and Scaled Copies

Let's explore scaled copies further.

Learning Outcomes

Today's Goals

1. **Goal:** Recognize that a *scale factor* is the number multiplied by each side length in an original figure to create a scaled copy.
2. **Goal:** Understand that there is a proportional relationship between lengths in a figure and corresponding lengths in a scaled copy.
3. **Language Goal:** Explain how scaling affects side lengths in scaled copies. **(Speaking and Listening)**

Students scale images, make connections between **scale factor** and equivalent ratios, and analyze other students' thinking about scaled copies. This lesson provides opportunities to communicate to students that incorrect, imprecise, and unfinished thinking are valuable.

Coherence

Prior Learning

In Lesson 1, students learned what scaled copies are and described their characteristics.

Future Learning

In Lesson 3, students will draw and analyze scaled copies with greater precision.

Connections

- Students extend their understanding of scaled figures to use corresponding parts to determine if a figure is a scaled copy of the other. (mathematical)
- Students critique student thinking and provide feedback for how to improve their understanding about how to make a scaled copy of an object. (mathematical, cross-curricular)

FL B.E.S.T. Math Benchmarks

Addressing

MA.7.GR.1.5
Solve mathematical and real-world problems involving dimensions and areas of geometric figures, including scale drawings and scale factors.

Mathematical Thinking and Reasoning Standards: MTR.4.1, MTR.5.1

ELA Expectations: ELA.K12.EE.1.1, ELA.K12.EE.3.1, ELA.K12.EE.4.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.6.AR.3.5	MA.8.GR.2.2
MA.6.GR.1.3	MA.912.GR.4.3

Vocabulary

New Vocabulary

scale factor
corresponding parts

Lesson at a Glance

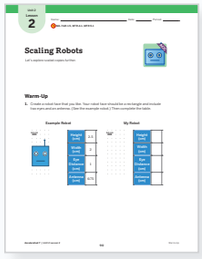
~ 45 min

MA.7.GR.1.5

Warm-Up

Independent | 5 min

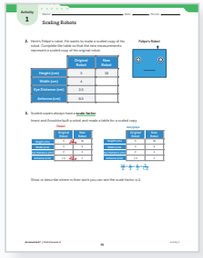
Students express their creativity as they design a robot.



Activity 1

Pairs | 15 min

Students scale a robot and look for patterns between matching lengths, identifying two ways to conceptualize scale factor. **(MTR.5.1)**



Activity 2

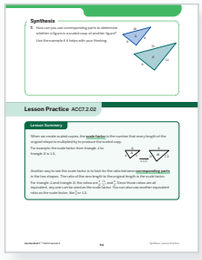
Pairs | 15 min

Students revise other students' work using their understanding of scale factor.

Synthesis

Whole Class | 5 min

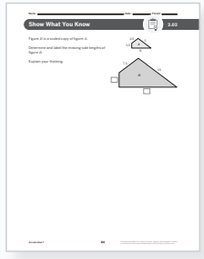
Students synthesize their understanding of scale factor and how it relates to lengths in scaled copies.



Show What You Know

Independent | 5 min

Students demonstrate their understanding of scale factor by determining the side lengths of a scaled copy.



Prep Checklist

This lesson includes:

- Student Edition
- Show What You Know

Independent | 5 min

Warm-Up

Purpose: Students express their creativity as they design a robot.

1 Launch

Share that in today's lesson, students will explore scaled copies using robots.

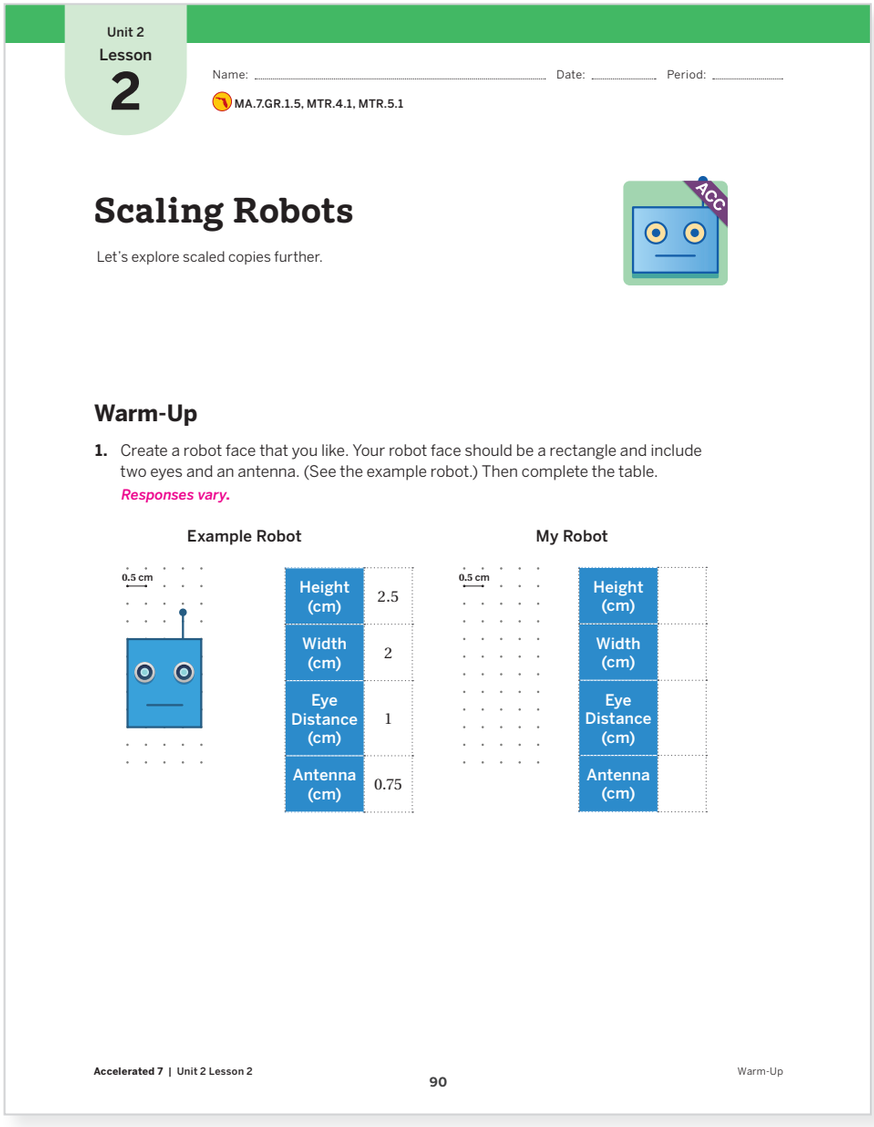
To encourage playfulness, invite students to make up a name for, or a story about, their robot.

2 Connect

Consider using the Think-Pair-Share routine (optional) to support student-to-student discourse and connection. Invite students to think independently and then take turns sharing their robot and its name or story with a partner. Invite several students to share about their or their partner's robot.

ELL **English Language Learners** Consider personally inviting students to share if they feel comfortable, including encouraging them to use their primary languages. **(Speaking and Listening)**

To surface connections, consider asking "How do you think we are going to use what we learned in the previous lesson with the robots you designed?"



Unit 2
Lesson
2

Name: _____ Date: _____ Period: _____

MA.7.GR.1.5, MTR.4.1, MTR.5.1

Scaling Robots

Let's explore scaled copies further.

Warm-Up

1. Create a robot face that you like. Your robot face should be a rectangle and include two eyes and an antenna. (See the example robot.) Then complete the table.

Responses vary.

Example Robot

Height (cm)	2.5
Width (cm)	2
Eye Distance (cm)	1
Antenna (cm)	0.75

My Robot

Height (cm)	
Width (cm)	
Eye Distance (cm)	
Antenna (cm)	

Pairs | 15 min

Activity 1 Scaling Robots

Purpose: Students scale a robot and look for patterns between matching lengths, identifying two ways to conceptualize scale factor. **(MTR.5.1)**

1 Launch

Consider asking, "What happens if only some parts are resized and not others? Would it still look like the same robot?"

2 Monitor

Invite students to write a response independently, then discuss their ideas with a partner.

MLR **MLR1: Stronger and Clearer Each Time (optional)** Consider using this routine to support students in developing precise language. Invite students to meet with a partner, give feedback on each other's drafts, then use the feedback to write a second clearer draft. See the Routine Facilitation Guide for more information. **(MTR.4.1)**

Select and sequence student explanations to share during the Connect. Consider including imprecise or unfinished explanations.

D Differentiation

Look for students who:	Respond to Student Thinking
Explain only Imani's strategy.	**Support** Encourage them to describe what Anushka wrote. Consider asking, "Why might Anushka have paired 10 and 5 together in a ratio?"
Describe Anushka's strategy without identifying the scale factor.	**Strengthen** Consider asking, "How do the ratios Anushka wrote connect to the scale factor of 2?"
Include words such as equivalent ratio and proportional in their explanation.	**Stretch** Consider inviting these students to share during the Connect or celebrating them publicly for attending to precision.

3 Connect

Invite students to share their explanations.

To surface the Key Takeaway, create a class definition of the term **scale factor** using students' language and the glossary. Record it for students to refer to later in the unit. Consider updating the class definition of *scaled copy* to reflect students' growing understanding of the term.

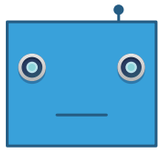
Activity 1

Name: Date: Period:

Scaling Robots

2. Here's Felipe's robot. He wants to make a scaled copy of his robot. Complete the table so that the new measurements represent a scaled copy of his original robot.

Felipe's Robot



	Original Robot	New Robot
Height (cm)	5	10
Width (cm)	4	8
Eye Distance (cm)	2.5	5
Antenna (cm)	0.5	1

3. Scaled copies always have a **scale factor**.

Imani and Anushka built a robot and made a table for a scaled copy.

Imani

	Original Robot	New Robot
Height (cm)	5	10
Width (cm)	3	6
Eye Distance (cm)	2	4
Antenna (cm)	1.5	3

Anushka

	Original Robot	New Robot
Height (cm)	5	10
Width (cm)	3	6
Eye Distance (cm)	2	4
Antenna (cm)	1.5	3

$\frac{10}{5} = \frac{6}{3} = \frac{4}{2} = \frac{3}{1.5}$

Show or describe where in their work you can see the scale factor is 2.

Responses vary.

- Imani's work shows the scale factor as the number you need to multiply the values in the first column by to get the values in the second column. For example, $5 \cdot 2 = 10$ and $3 \cdot 2 = 6$.
- Anushka wrote all the lengths as ratios, and those ratios are equivalent. They all have the same value, 2, which must be the scale factor.

Accelerated 7 | Unit 2 Lesson 2 91 Activity 1

Key Takeaway: Scaled copies are related by the **scale factor**. We can think of scale factor in two ways: it is the number all side lengths are multiplied by to create a scaled copy, and it is the ratio between matching sides of a scaled copy and an original figure.

Pairs | 15 min

Activity 2 Analyzing Robots

Purpose: Students revise other students' work using their understanding of scale factor.

Short on time? Consider omitting Problem 6.

1 Launch

Invite students to inspect Imani's new robot's dimensions in the table and continue to the next problem when they have made and justified a choice.

ELL **English Language Learners** Invite students to read the prompt aloud and paraphrase what it means. Consider reviewing the class definition of scaled copy. **(Reading)**

Note: Some students may believe that the new robot is a scaled copy because each measurement is added by 6. It's okay — even desirable — to lack consensus at this stage.

2 Monitor

Invite students to work independently for a few minutes, then share their responses with a partner.

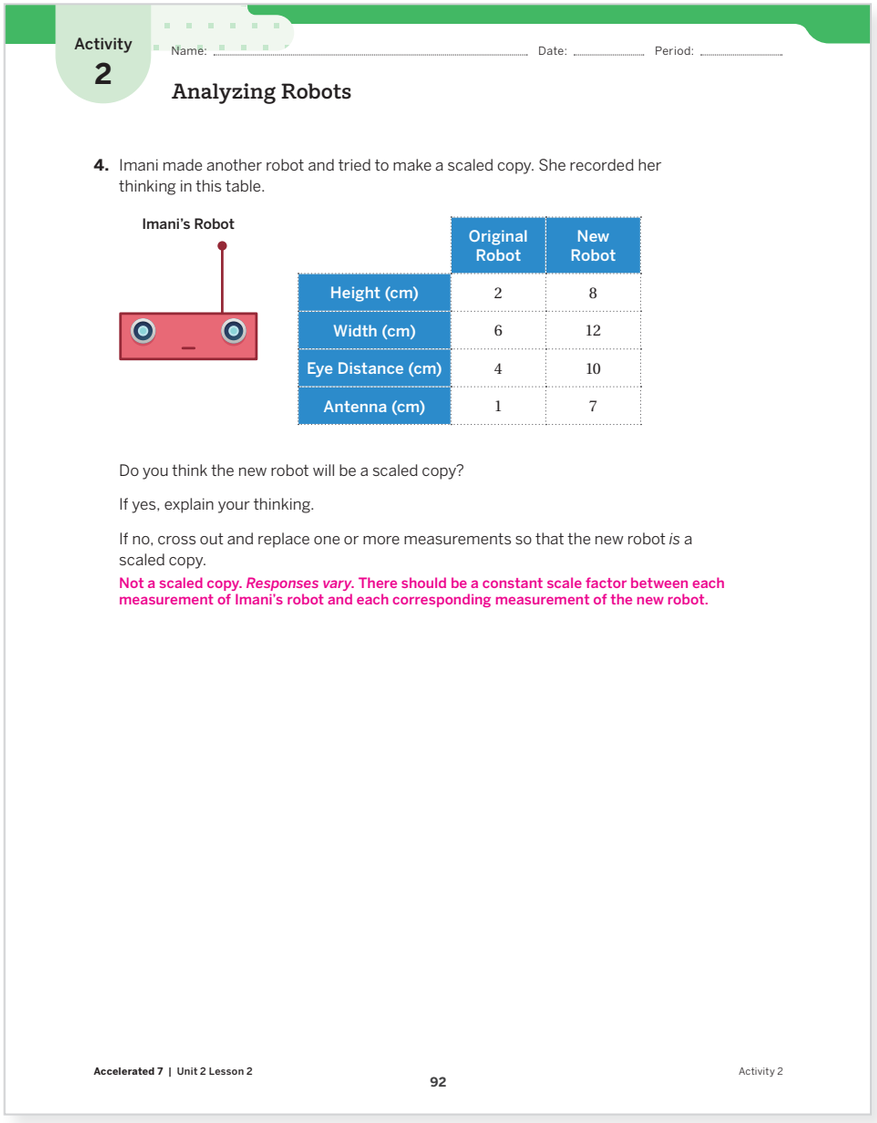
Encourage students who finish early to try creating a robot with a different scale factor.

Select and Sequence several student tables that have different scale factors. Consider including some tables that do not create scaled copies.

Pause briefly and display share examples of some students' tables. Consider asking:

- "Does this table represent a scaled copy?"
- "What scale factor does this table use?"

Activity 2 continued >



Activity 2

Name: Date: Period:

Analyzing Robots

4. Imani made another robot and tried to make a scaled copy. She recorded her thinking in this table.

Imani's Robot

	Original Robot	New Robot
Height (cm)	2	8
Width (cm)	6	12
Eye Distance (cm)	4	10
Antenna (cm)	1	7

Do you think the new robot will be a scaled copy?

If yes, explain your thinking.

If no, cross out and replace one or more measurements so that the new robot *is* a scaled copy.

Not a scaled copy. *Responses vary.* There should be a constant scale factor between each measurement of Imani's robot and each corresponding measurement of the new robot.

Accelerated 7 | Unit 2 Lesson 2 92 Activity 2

Pairs | 15 min

Activity 2 Analyzing Robots (continued)

Purpose: Students revise other students' work using their understanding of scale factor.

Short on time? Consider omitting Problem 6.

2 Monitor

Invite students to read the definition of **corresponding parts** and identify them in Anushka's robots.

D **Differentiation: (Support)** Consider asking, "How can you tell the two figures are not scaled copies?"

A **Accessibility: Visual-Spatial Processing** Invite students to use coloring tools to identify pairs of corresponding sides.

Encourage students to read others' responses and discuss their ideas with a partner.

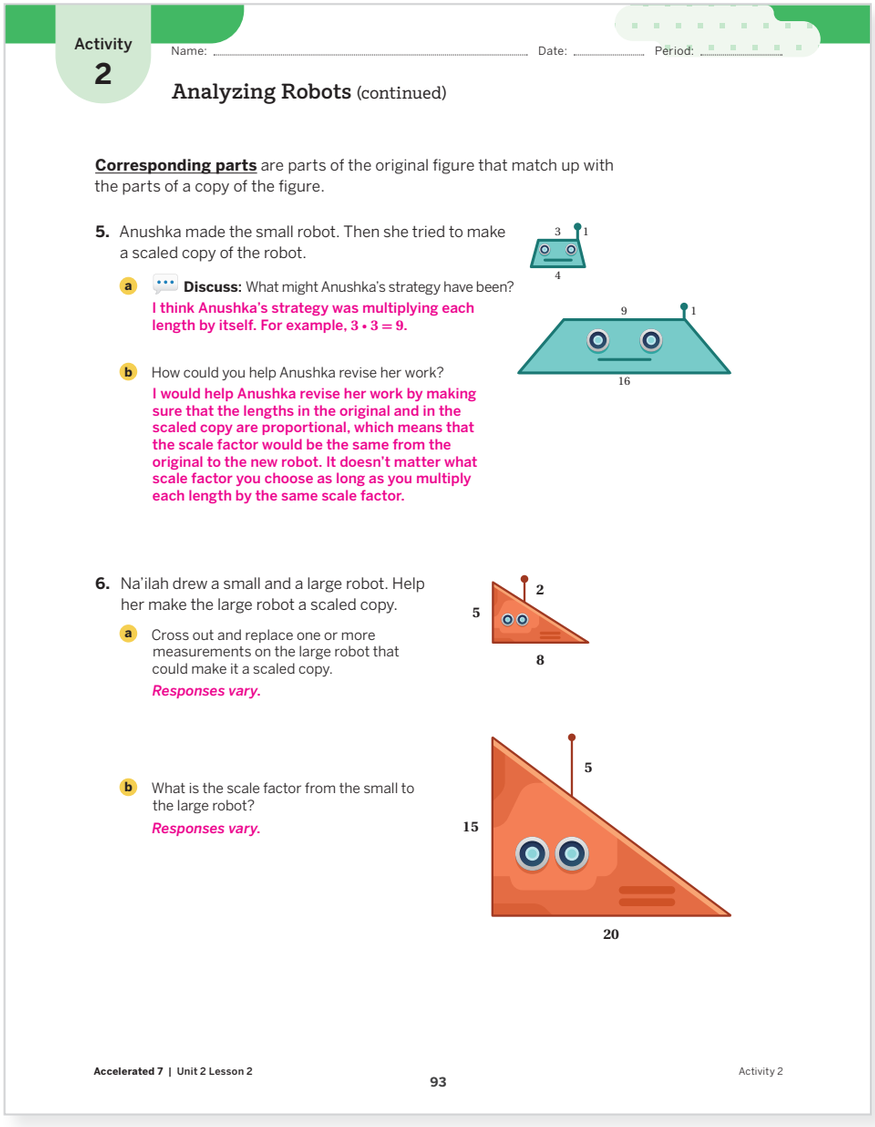
D **Differentiation: (Stretch)** To encourage exploration, consider asking, "If the antenna of the scaled copy was 5.5, what would the measurements of the height and width need to be?"

Look for strategies to share during the Connect. Look for a variety of solutions.

3 Connect

Share several captured solutions. As a class, consider creating a list of all the scaled copies students generated along with their accompanying scale factors.

To surface the Key Takeaway, consider asking, "Is it possible to create a scaled copy by only changing the width? Why or why not?" No, because the height and antenna would still have two different scale factors.



Activity 2

Name: Date: Period:

Analyzing Robots (continued)

Corresponding parts are parts of the original figure that match up with the parts of a copy of the figure.

5. Anushka made the small robot. Then she tried to make a scaled copy of the robot.

a. Discuss: What might Anushka's strategy have been?

I think Anushka's strategy was multiplying each length by itself. For example, $3 \cdot 3 = 9$.

b. How could you help Anushka revise her work?

I would help Anushka revise her work by making sure that the lengths in the original and in the scaled copy are proportional, which means that the scale factor would be the same from the original to the new robot. It doesn't matter what scale factor you choose as long as you multiply each length by the same scale factor.

6. Na'ilah drew a small and a large robot. Help her make the large robot a scaled copy.

a. Cross out and replace one or more measurements on the large robot that could make it a scaled copy.

Responses vary.

b. What is the scale factor from the small to the large robot?

Responses vary.

Accelerated 7 | Unit 2 Lesson 2 93 Activity 2

Key Takeaway: To create a scaled copy, every length in the original figure is multiplied by the same scale factor. If lengths are multiplied by different scale factors, the result is not a scaled copy.

Whole Class | 5 min

Synthesis

Purpose: Students synthesize their understanding of scale factor and how it relates to lengths in scaled copies.

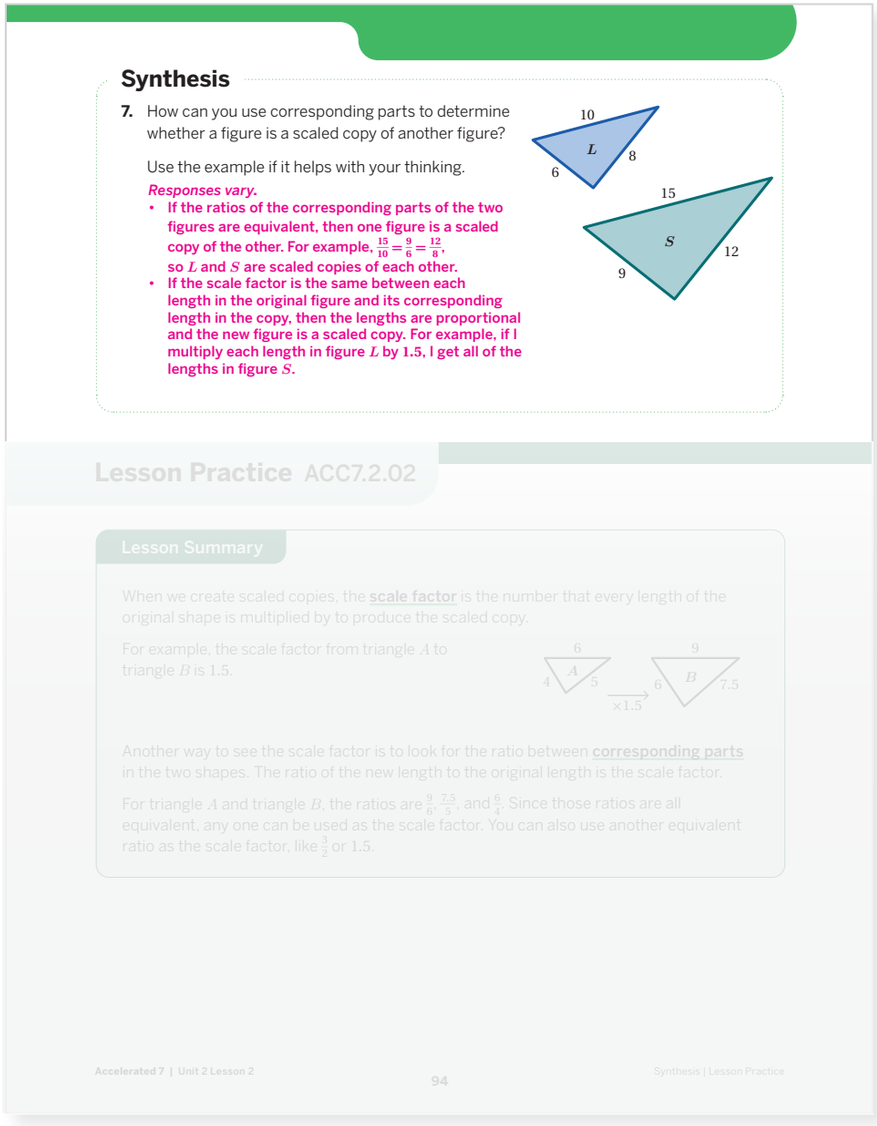
Invite students to respond independently, and then share their thinking with a partner.

Note: Encourage students to use a calculator to test ideas they have about the relationship between the side lengths.

Collect and share a variety of ideas, such as:

- Looking for a multiplicative pattern between corresponding lengths. In the example, the lengths in L are multiplied by 1.5 to create the lengths in S.
- Checking the ratios of corresponding side lengths. If they are equivalent, the figure is a scaled copy. In the example, $\frac{15}{10} = \frac{12}{8} = \frac{9}{6}$.

Note: If time allows, invite students to revise their previous response based on the discussion.



Synthesis

7. How can you use corresponding parts to determine whether a figure is a scaled copy of another figure?

Use the example if it helps with your thinking.

Responses vary.

- If the ratios of the corresponding parts of the two figures are equivalent, then one figure is a scaled copy of the other. For example, $\frac{15}{10} = \frac{9}{6} = \frac{12}{8}$, so L and S are scaled copies of each other.
- If the scale factor is the same between each length in the original figure and its corresponding length in the copy, then the lengths are proportional and the new figure is a scaled copy. For example, if I multiply each length in figure L by 1.5, I get all of the lengths in figure S.

Lesson Practice ACC7.2.02

Lesson Summary

When we create scaled copies, the scale factor is the number that every length of the original shape is multiplied by to produce the scaled copy.

For example, the scale factor from triangle A to triangle B is 1.5.

Another way to see the scale factor is to look for the ratio between corresponding parts in the two shapes. The ratio of the new length to the original length is the scale factor.

For triangle A and triangle B, the ratios are $\frac{9}{6}$, $\frac{7.5}{5}$, and $\frac{6}{4}$. Since those ratios are all equivalent, any one can be used as the scale factor. You can also use another equivalent ratio as the scale factor, like $\frac{3}{2}$ or 1.5.

Accelerated 7 | Unit 2 Lesson 2 94 Synthesis | Lesson Practice

Lesson Takeaway: The lengths in an original figure and in a scaled copy are proportional and are related by the **scale factor**.

Independent | 5 min

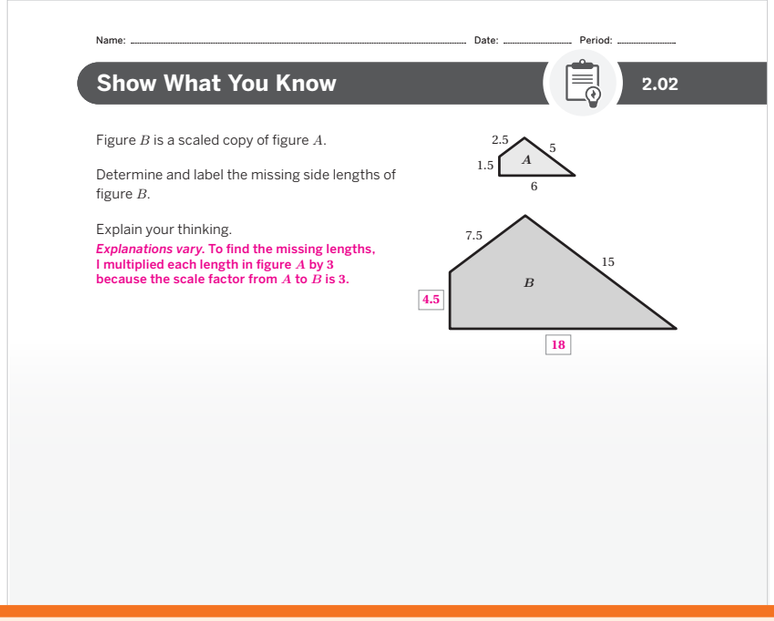
Show What You Know

Purpose: Students demonstrate their understanding of scale factor by determining the side lengths of a scaled copy.

Today's Goals

1. **Goal:** Recognize that a *scale factor* is the number multiplied by each side length in an original figure to create a scaled copy.
2. **Goal:** Understand that there is a proportional relationship between lengths in a figure and corresponding lengths in a scaled copy.
3. **Language Goal:** Explain how scaling affects side lengths in scaled copies. **(Speaking and Listening)**

Show What You Know (Assessment Resources)



Name: _____ Date: _____ Period: _____

Show What You Know 2.02

Figure B is a scaled copy of figure A.

Determine and label the missing side lengths of figure B.

Explain your thinking.

Explanations vary. To find the missing lengths, I multiplied each length in figure A by 3 because the scale factor from A to B is 3.

Differentiation Use after Lesson 2

Support

Provide targeted intervention for students by using these resources.

If students need additional support understanding the effects of scaling on side lengths:

» **Respond to Student Thinking:**

- Review Imani and Anushka's methods from Activity 1.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 1 Extension Activities**.

Professional Learning

Reflect on the strategies students used today. What opportunities were there to share specific students' strategies with the class and celebrate them? What facilitation techniques (e.g., naming strategies after the students) can help make this a part of your classroom culture? How might you continue to reference student strategies from today in the future?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

7. How can you use corresponding parts to determine whether a figure is a scaled copy of another figure?

Use the example if it helps with your thinking.

Responses vary.

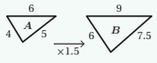
- If the ratios of the corresponding parts of the two figures are equivalent, then one figure is a scaled copy of the other. For example, $\frac{15}{10} = \frac{9}{6} = \frac{12}{8}$, so L and S are scaled copies of each other.
- If the scale factor is the same between each length in the original figure and its corresponding length in the copy, then the lengths are proportional and the new figure is a scaled copy. For example, if I multiply each length in figure L by 1.5, I get all of the lengths in figure S.

Lesson Practice ACC7.2.02

Lesson Summary

When we create scaled copies, the **scale factor** is the number that every length of the original shape is multiplied by to produce the scaled copy.

For example, the scale factor from triangle A to triangle B is 1.5.



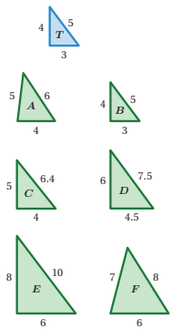
Another way to see the scale factor is to look for the ratio between **corresponding parts** in the two shapes. The ratio of the new length to the original length is the scale factor.

For triangle A and triangle B, the ratios are $\frac{9}{6}$, $\frac{7.5}{5}$, and $\frac{6}{4}$. Since those ratios are all equivalent, any one can be used as the scale factor. You can also use another equivalent ratio as the scale factor, like $\frac{3}{2}$ or 1.5.

Lesson Practice
ACC7.2.02

Name: _____ Date: _____ Period: _____

Problems 1–3: Here are seven triangles.



1. Which triangles are scaled copies of triangle T?

   Triangles B, D, and E

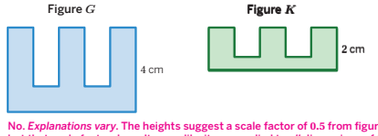
2. For each scaled copy, write the scale factor that takes triangle T to that triangle. Leave blank if it is not a scaled copy.

Triangle	Scale Factor
A	
B	1
C	
D	1.5
E	2
F	

3. List two triangles whose lengths can be represented by the equivalent ratios $\frac{4}{8} = \frac{5}{10} = \frac{3}{6}$.

   Triangles T and E or triangles B and E

4. Tyler says that figure K is a scaled copy of figure G. Is Tyler correct? Explain your thinking.

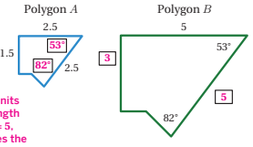


No. *Explanations vary.* The heights suggest a scale factor of 0.5 from figure G to K, but that scale factor doesn't seem like it was applied to *all* dimensions of figure G.

Lesson Practice
ACC7.2.02

Name: _____ Date: _____ Period: _____

Problems 5–7: Polygon B is a scaled copy of polygon A.



5. What is the scale factor that takes polygon A to polygon B? Explain your thinking.

   2. *Explanations vary.* The side length of 5 units in polygon B corresponds with the side length of 2.5 units in polygon A. Because $2.5 \cdot 2 = 5$, all the side lengths in polygon B are 2 times the corresponding side lengths in polygon A.

6. Enter the missing lengths in polygon B.

   Responses shown on diagram.

7. Enter the missing angle measurements in polygon A.

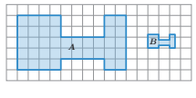
   Responses shown on diagram.

FAST Practice

8. Figures A and B are scaled copies.

   What scale factor takes A to B?

   $\frac{1}{4}$.



9. Using the digits 0 to 9 without repetition, fill in the blanks to make rectangles that are scaled copies.

   Responses vary.

Spiral Review

Problems 10–13: Evaluate each expression.

10. $\frac{1}{4} \cdot 32 = 8$
11. $\frac{1}{4} \cdot 5.6 = 1.4$
12. $7.2 \cdot \frac{1}{9} = 0.8$
13. $2 \div \frac{1}{4} = 8$

Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1–2	1	MA.7.GR.1.5
	3–7	2	MA.7.GR.1.5
	8	3	MA.7.GR.1.5
FAST Practice	9	2	MA.7.GR.1.5
Spiral Review			
	10–13	1	MA.5.FR.2.2

Make It Scale

Drawing Scaled Copies

Let's draw scaled copies.

Learning Outcomes

Today's Goals

1. **Goal:** Draw a scaled copy of a figure using a given scale factor.
2. **Language Goal:** Describe and critique strategies for drawing scaled copies of a figure. **(Writing, Speaking, and Listening)**

Students apply their knowledge of scale factors to draw scaled copies on a grid. They explain strategies for creating accurate scaled copies, including ensuring all side lengths are multiplied by the same factor and all angle measurements stay the same.

Coherence

Prior Learning

In Lesson 2, students learned the term scale factor. They studied two ways of thinking about scale factor: as the multiplier applied to side lengths when scaling a figure, and as the ratio between corresponding sides in scaled copies.

Future Learning

In Lesson 4, students will study scale factors less than, equal to, and greater than 1 and notice patterns in how the scale factor affects the size of a scaled copy.

Connections

- Students draw scaled copies of figures on grids with different scale factors. (mathematical, cross-curricular)
- Students analyze samples of student thinking in an error analysis activity in which they determine the scale factor used to create a sketch from an original figure. (mathematical, cross-curricular)

FL B.E.S.T. Math Benchmarks

Addressing

MA.7.GR.1.5
Solve mathematical and real-world problems involving dimensions and areas of geometric figures, including scale drawings and scale factors.

Mathematical Thinking and Reasoning Standard: MTR.4.1

ELA Expectation: ELA.K12.EE.1.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.6.AR.3.5	MA.8.GR.2.2
MA.6.GR.1.3	MA.912.GR.4.3

Lesson at a Glance

~ 45 min

MA.7.GR.1.5

Warm-Up

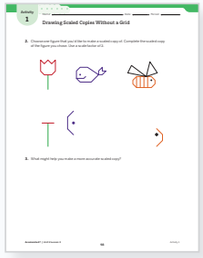
Independent | 5 min

Students use the **Which One Doesn't Belong?** routine to surface their understanding of lengths and angles in scaled copies.

Activity 1

Pairs | 15 min

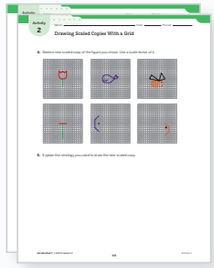
Students attempt to draw a scaled copy without measuring tools and consider what tools would be helpful in drawing a precise scaled copy.



Activity 2

Pairs | 15 min

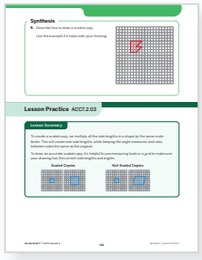
Students strategically use a grid to draw precise scaled copies and analyze the strategies of other students. **(MTR.4.1)**



Synthesis

Whole Class | 5 min

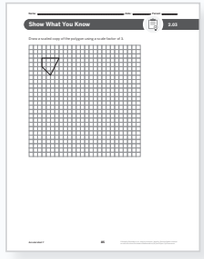
Students synthesize their understanding of how to draw a scaled copy, attending to both side lengths and angle measurements.



Show What You Know

Independent | 5 min

Students demonstrate their understanding by using a grid to create a scaled copy of a polygon.



Prep Checklist

This lesson includes:

- Student Edition
- Show What You Know
- Graph paper (as needed)
- Index cards

Independent | 5 min

Warm-Up

Purpose: Students use the Which One Doesn't Belong? routine to surface their understanding of lengths and angles in scaled copies.

1 Launch

Use the Which One Doesn't Belong? routine to support students in noticing and precisely describing properties of scaled copies.

Draw students' attention to all four pairs of shapes. Invite students to choose one they believe doesn't belong and justify their choice. **(MTR.4.1)**

Consider sharing that each pair does not belong in some way. Encourage students to look for more than one possibility.

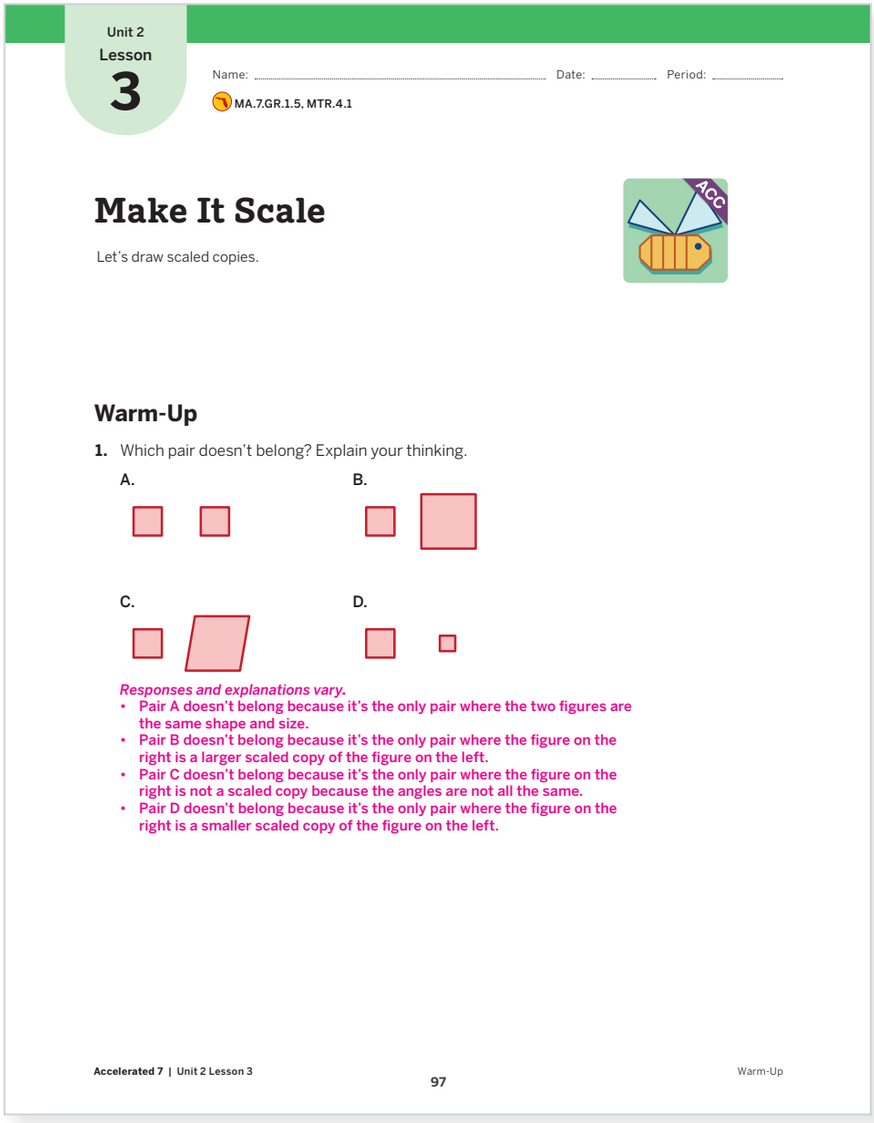
2 Connect

Invite students to share at least one reason why *each* pair doesn't belong.

ELL **English Language Learners** Provide sentence frames (e.g., "_____doesn't belong because _____." or "_____is the only one that has/ doesn't have_____."). **(Writing and Speaking)**

To surface the Key Takeaway, encourage students to explain how they know that pairs are or are not scaled copies.

Note: If it does not come up naturally, consider mentioning the relationship between the angles in the original and the copy for the bottom left pair.



Unit 2
Lesson
3

Name: _____ Date: _____ Period: _____

MA.7.GR.1.5, MTR.4.1

Make It Scale

Let's draw scaled copies.

Warm-Up

1. Which pair doesn't belong? Explain your thinking.

A.

B.

C.

D.

Responses and explanations vary.

- Pair A doesn't belong because it's the only pair where the two figures are the same shape and size.
- Pair B doesn't belong because it's the only pair where the figure on the right is a larger scaled copy of the figure on the left.
- Pair C doesn't belong because it's the only pair where the figure on the right is not a scaled copy because the angles are not all the same.
- Pair D doesn't belong because it's the only pair where the figure on the right is a smaller scaled copy of the figure on the left.

Accelerated 7 | Unit 2 Lesson 3 97 Warm-Up

Key Takeaway: Scale factor can change the side lengths of a figure, but scaled copies always have the same angle measurements as the original.

Pairs | 15 min

Activity 1 Drawing Scaled Copies Without a Grid

Purpose: Students attempt to draw a scaled copy without measuring tools and consider what tools would be helpful in drawing a precise scaled copy.

Short on time? Consider omitting the discussion on Problem 3 and encouraging students to read others' responses instead.

1 Launch

Invite students to choose an image that interests them.

Distribute index cards to students. Consider modeling using the index card as a straightedge to draw a segment.

2 Monitor

Encourage students to not spend too much time on this drawing. Explain that they will have the opportunity to create a more precise drawing later in the lesson.

A **Accessibility: Fine Motor Skills** To reduce anxiety about precision, share that the goal is to create a rough draft, rather than to be precise when using the sketch tool. Consider encouraging students to choose the flower, which is the figure with the fewest segments.

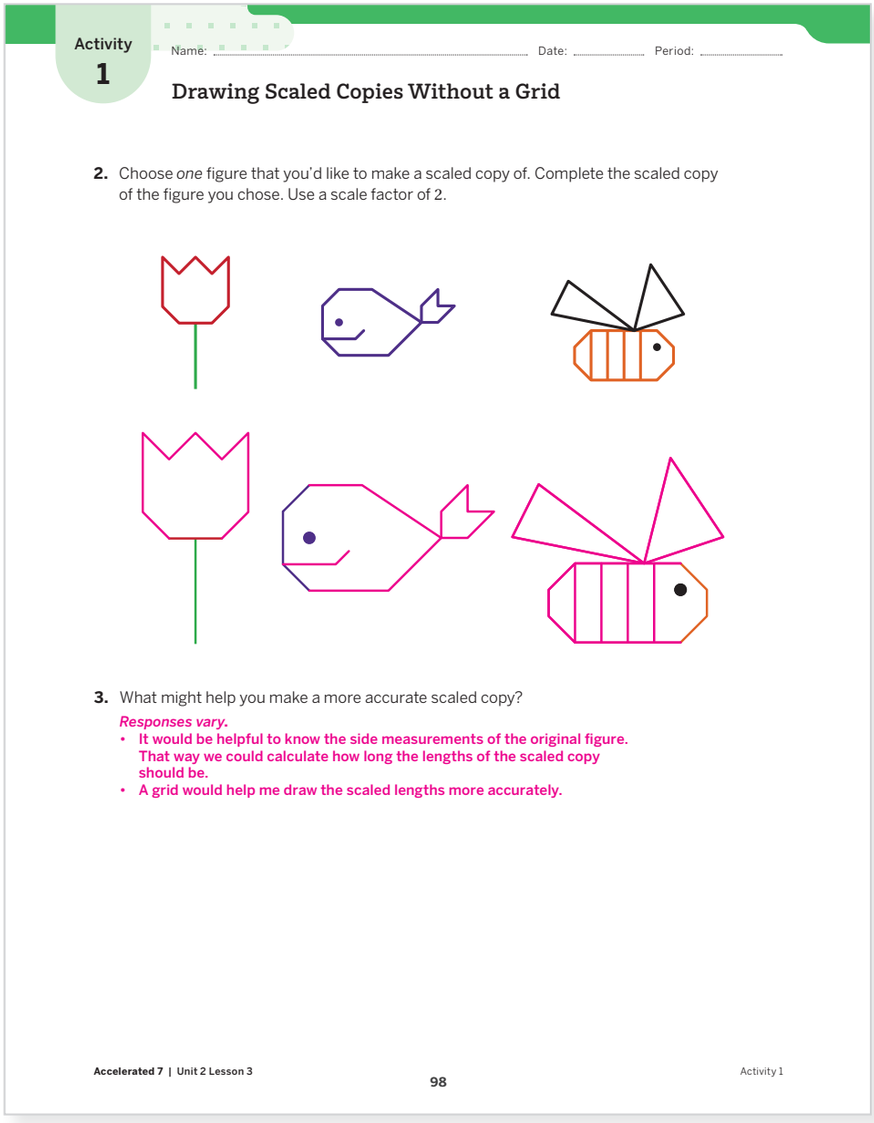
Listen for explanations to share during the Connect. Look for students whose responses mention lengths, angles, or measuring tools.

3 Connect

Share the captured explanations. Consider asking, "Were your thoughts similar or different?"

ELL **English Language Learners** Encourage students to paraphrase each other's ideas to help them make connections and incorporate vocabulary from the unit. **(Speaking and Listening)**

To surface the Key Takeaway, consider asking, "How did it feel to create a scale drawing without any measurements or measuring tools?"



Activity 1

Name: Date: Period:

Drawing Scaled Copies Without a Grid

2. Choose *one* figure that you'd like to make a scaled copy of. Complete the scaled copy of the figure you chose. Use a scale factor of 2.

3. What might help you make a more accurate scaled copy?

Responses vary.
- It would be helpful to know the side measurements of the original figure. That way we could calculate how long the lengths of the scaled copy should be.
- A grid would help me draw the scaled lengths more accurately.

Accelerated 7 | Unit 2 Lesson 3 98 Activity 1

Key Takeaway: Creating an accurate scale drawing requires keeping the original angle measurements and knowing precise side lengths so you can apply the scale factor.

Pairs | 15 min

Activity 2 Drawing Scaled Copies With a Grid

Purpose: Students strategically use a grid to draw precise scaled copies and analyze the strategies of other students. **(MTR.4.1)**

1 Launch

Share that students will now have the opportunity to recreate their scaled copy using a grid. Encourage students to strive for a more precise copy than their previous attempt.

2 Monitor

Look for students' progress on their scaled copies.

D Differentiation

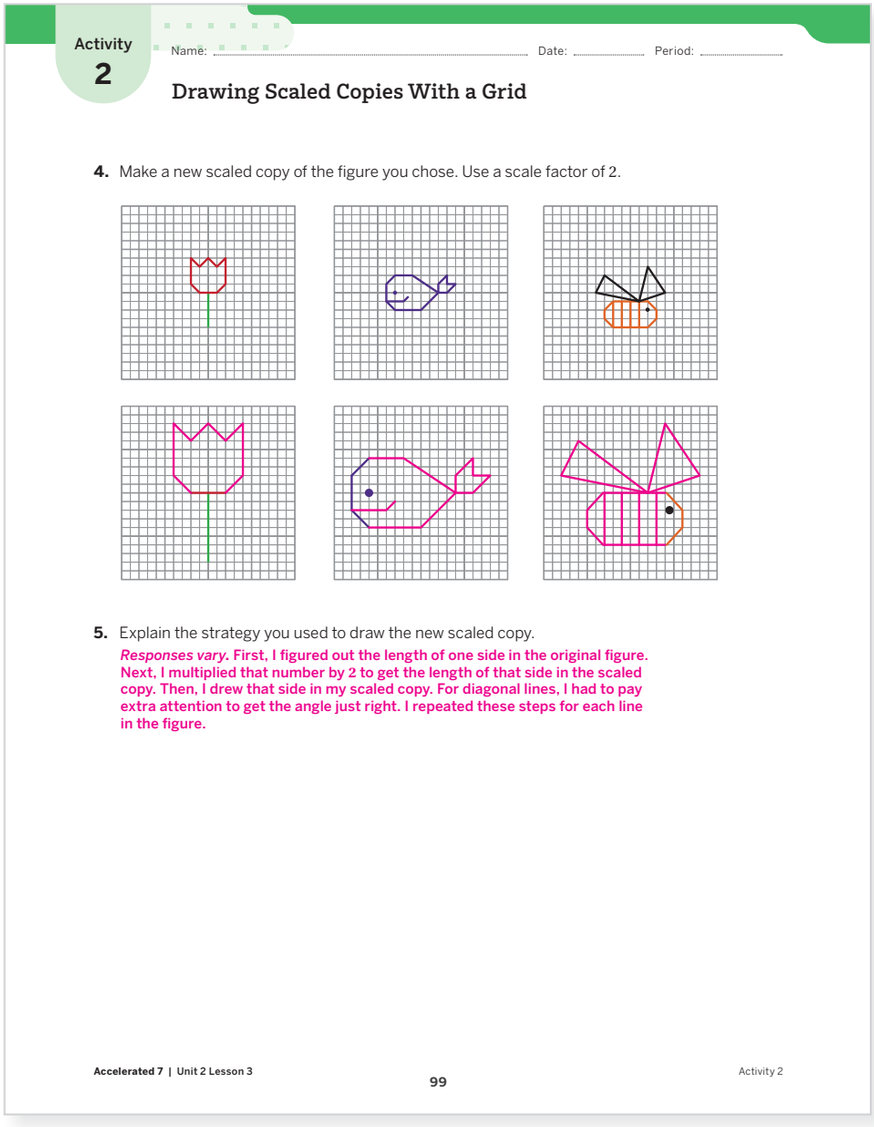
Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Invite students to choose a segment in the original figure and count its length. Consider asking, "How would the length change if we applied a scale factor of 2?"
Scale side lengths accurately, but do not consider the size of corresponding angles.	**Support** Consider asking, "What strategies can you try for drawing diagonal lengths?"
Check their work after drawing each segment.	**Strengthen** Consider asking, "What pattern do you notice between the original segment and the copy? What do you predict will be the new length of the next segment?"

Look for a variety of student strategies to share. Consider including strategies that are incomplete or imprecise, then using the discussion to strengthen these explanations as a class.

Pause briefly to share examples of student's strategies. Invite students to reflect on which strategies they used or would like to try.

Note: If it does not come up naturally, consider asking, "What were your strategies for drawing diagonal lines?" *Responses vary.* I counted the number of units down and to the right in the original and multiplied those by the scale factor.

Activity 2 continued >



Activity 2

Name: Date: Period:

Drawing Scaled Copies With a Grid

4. Make a new scaled copy of the figure you chose. Use a scale factor of 2.

5. Explain the strategy you used to draw the new scaled copy.

Responses vary. First, I figured out the length of one side in the original figure. Next, I multiplied that number by 2 to get the length of that side in the scaled copy. Then, I drew that side in my scaled copy. For diagonal lines, I had to pay extra attention to get the angle just right. I repeated these steps for each line in the figure.

Accelerated 7 | Unit 2 Lesson 3 99 Activity 2

Pairs | 15 min

Activity 2 Drawing Scaled Copies With a Grid (continued)

Purpose: Students strategically use a grid to draw precise scaled copies and analyze the strategies of other students. **(MTR.4.1)**

2 Monitor

Encourage students to apply the strategies they wrote about in Problem 5.

Invite students to discuss their thinking with a partner.

Listen to partner discussions for student ideas to highlight during the Connect.

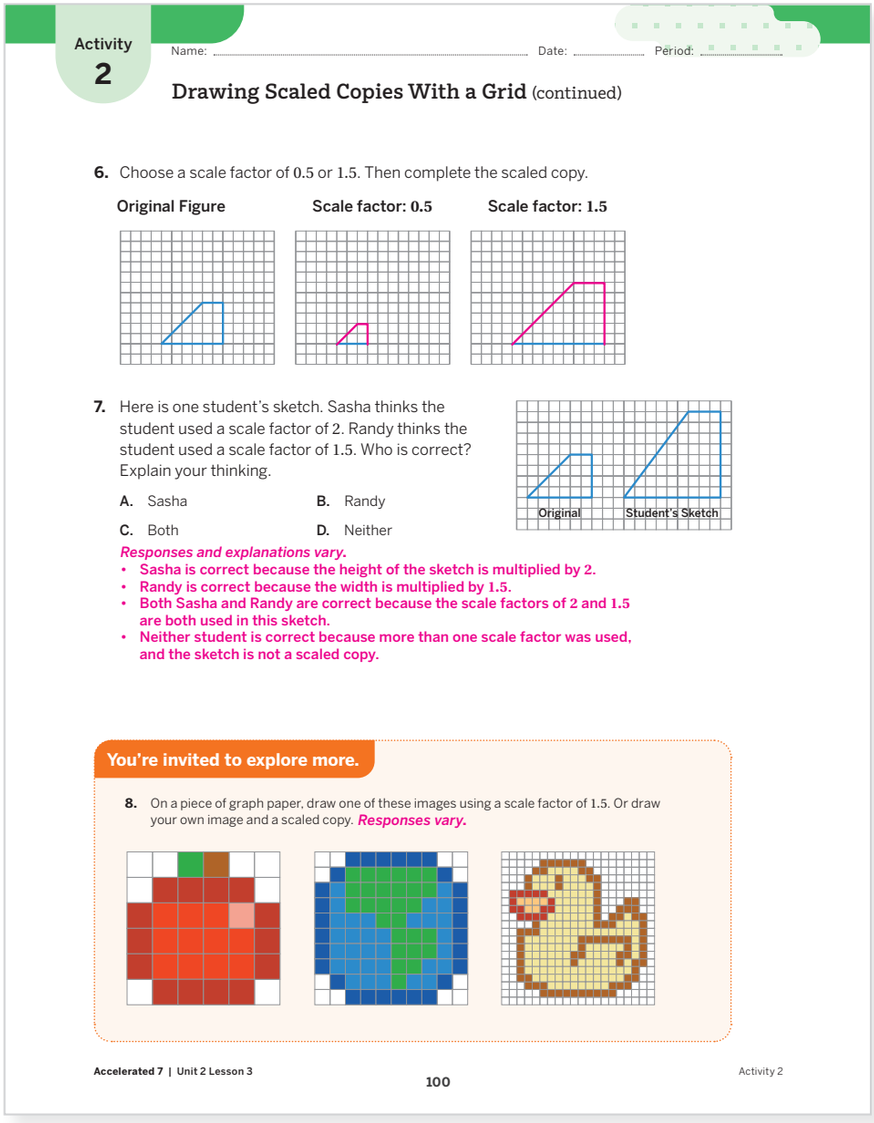
D **Differentiation: You're invited to explore more. (Stretch)** Invite students who would like to explore scale drawings further to complete this optional task. Encourage them to discuss their thinking with a partner.
Note: Students will need graph paper.

3 Connect

Invite students to share their responses, calling attention to any conflict or consensus.

Consider using the Decide and Defend routine (optional) to support students in making arguments and critiquing the reasoning of others. See the Routine Facilitation Guide for more information. **(MTR.4.1)**

To surface the Key Takeaway, consider asking:
- "Where do you see Sasha's scale factor in the sketch?"
- "Where do you see Randy's scale factor?"
- "Did this student create a scaled copy?"



Activity 2

Name: Date: Period:

Drawing Scaled Copies With a Grid (continued)

6. Choose a scale factor of 0.5 or 1.5. Then complete the scaled copy.

Original Figure	Scale factor: 0.5	Scale factor: 1.5

7. Here is one student's sketch. Sasha thinks the student used a scale factor of 2. Randy thinks the student used a scale factor of 1.5. Who is correct? Explain your thinking.

Original Student's Sketch

A. Sasha B. Randy
C. Both D. Neither

Responses and explanations vary.
- Sasha is correct because the height of the sketch is multiplied by 2.
- Randy is correct because the width is multiplied by 1.5.
- Both Sasha and Randy are correct because the scale factors of 2 and 1.5 are both used in this sketch.
- Neither student is correct because more than one scale factor was used, and the sketch is not a scaled copy.

You're invited to explore more.

8. On a piece of graph paper, draw one of these images using a scale factor of 1.5. Or draw your own image and a scaled copy. *Responses vary.*

Accelerated 7 | Unit 2 Lesson 3 100 Activity 2

Key Takeaway: A measuring tool, like a grid, is helpful to create scaled copies. It can help us precisely measure lengths and make sure that every length is multiplied by the same scale factor.

Whole Class | 5 min

Synthesis

Purpose: Students synthesize their understanding of how to draw a scaled copy, attending to both side lengths and angle measurements.

Invite students to respond independently, and then share their thinking with a partner.

Collect and share a variety of ideas, such as:

- Multiplying all side lengths by the same scale factor.
- Drawing horizontal and vertical lines first.
- Using different strategies for checking that angle measurements remain the same from the original figure to the copy.

Note: If time allows, invite students to share with a partner what makes sense about each strategy and what questions they still have.

Synthesis

9. Describe how to draw a scaled copy.

Use the example if it helps with your thinking.

Responses vary.

- **When making a scaled copy, make sure all the side lengths are multiplied by the same scale factor.**
- **When drawing the new figure, keep in mind how you draw the angles so that the angles remain the same in both figures.**

Lesson Practice ACC7.2.03

Lesson Summary

To create a scaled copy, we multiply all the side lengths in a shape by the same *scale factor*. This will create new side lengths, while keeping the angle measures and ratio between sides the same as the original.

To draw an accurate scaled copy, it's helpful to use measuring tools or a grid to make sure your drawing has the correct side lengths and angles.

Scaled Copies | Not Scaled Copies

Accelerated 7 | Unit 2 Lesson 3 101 Synthesis | Lesson Practice

Lesson Takeaway: In a scaled copy, all the side lengths are multiplied by the scale factor, but the angle measures remain the same. Using a grid can help you create a precise scaled copy.

Independent | 5 min

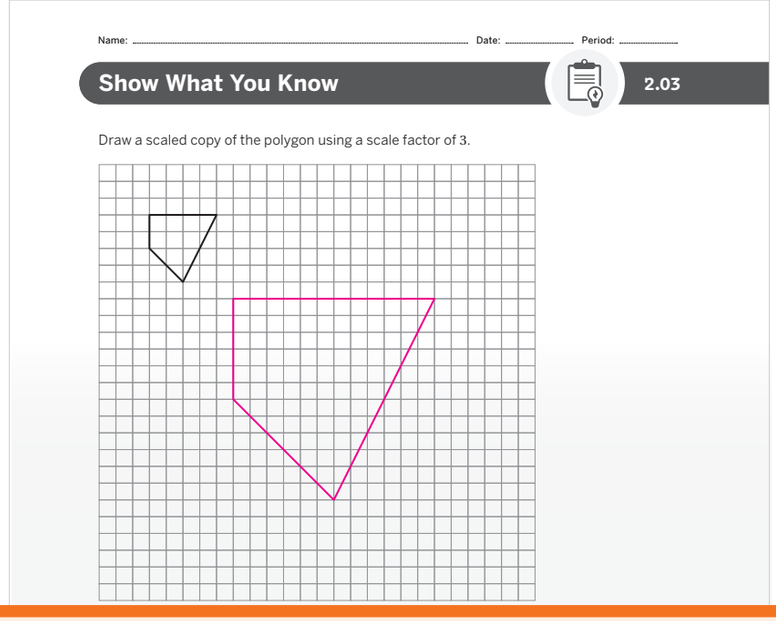
Show What You Know

Purpose: Students demonstrate their understanding by using a grid to create a scaled copy of a polygon.

Today's Goals

1. **Goal:** Draw a scaled copy of a figure using a given scale factor.
2. **Language Goal:** Describe and critique strategies for drawing scaled copies of a figure. **(Writing, Speaking, and Listening)**

Show What You Know (Assessment Resources)


Name: _____ Date: _____ Period: _____

Show What You Know 2.03

Draw a scaled copy of the polygon using a scale factor of 3.

D Differentiation Use after Lesson 3

Support

Provide targeted intervention for students by using these resources.

If students need support understanding how to draw a scaled copy of a figure:

» **Respond to Student Thinking:**

- Revisit the Lesson Summary and provide targeted feedback on select Practice Problems.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 1 Extension Activities**.
- Invite students to investigate the **You're Invited to Explore More** problem.

Professional Learning

Reflect on the student thinking that was shared in this lesson. Who participated and who did not participate in sharing? What trends do you see in participation?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

9. Describe how to draw a scaled copy.

Use the example if it helps with your thinking.

Responses vary.

- When making a scaled copy, make sure all the side lengths are multiplied by the same scale factor.
- When drawing the new figure, keep in mind how you draw the angles so that the angles remain the same in both figures.

Lesson Practice ACC7.2.03

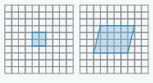
Lesson Summary

To create a scaled copy, we multiply all the side lengths in a shape by the same *scale factor*. This will create new side lengths, while keeping the angle measures and ratio between sides the same as the original.

To draw an accurate scaled copy, it's helpful to use measuring tools or a grid to make sure your drawing has the correct side lengths and angles.

Scaled Copies

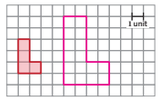
Not Scaled Copies



Lesson Practice ACC7.2.03

Name: _____ Date: _____ Period: _____

Problems 1–2: Here is a polygon.



1. Draw a scaled copy of the polygon using a scale factor of 2.

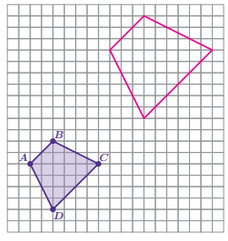
Sample shown.

2. What is the area and perimeter of your scaled copy?

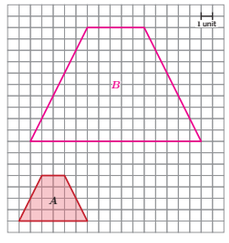
The area is 16 square units. The perimeter is 20 units.

3. Draw a scaled copy of figure $ABCD$ using a scale factor of 1.5.

Sample shown.



Problems 4–5: Imagine there is a quadrilateral B that is a scaled copy of quadrilateral A. Its shortest side is 5 units long.



4. What is the scale factor from quadrilateral A to B?

2.5

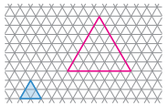
5. Draw quadrilateral B.

Sample shown.

Lesson Practice ACC7.2.03

Name: _____ Date: _____ Period: _____

Problems 6–7: Here is an equilateral triangle.



6. Draw a scaled copy of this equilateral triangle using a scale factor of 3.

Responses vary.

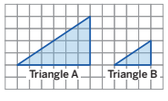
7. Equilateral triangles are always scaled copies. What other shapes are always scaled copies? **squares, circles, or any regular shape (shapes where all sides and all angles are equal)**

FAST Practice

8. Triangle B is a scaled copy of triangle A. Gabriela says the scale factor is 0.5. Is she correct?

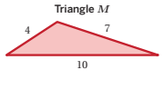
Select **ONE** correct answer in each box to complete the sentences.

Gabriela is (A.) correct **B.** incorrect. The height and base of triangle A is multiplied by (A.) 0.5 **B.** 2.



Spiral Review

9. Triangle Z is a scaled copy of triangle M. Select *all* sets of values that could be the side lengths of triangle Z.



☐ **A.** 8, 11, and 14 ☑ **B.** 10, 17.5, and 25

☐ **C.** 6, 9, and 11 ☑ **D.** 6, 10.5, and 15

☑ **E.** 8, 14, and 20

Problems 10–12: Solve each equation. Show or explain your thinking.

10. $6x = 156$

$x = 26$

Work varies.

$6x \div 6 = 156 \div 6$

$x = 26$

11. $16x = 8$

$x = \frac{1}{2}$

Work varies.

$16x \div 16 = 8 \div 16$

$x = \frac{1}{2}$

12. $\frac{1}{5}x = 1$

$x = 5$

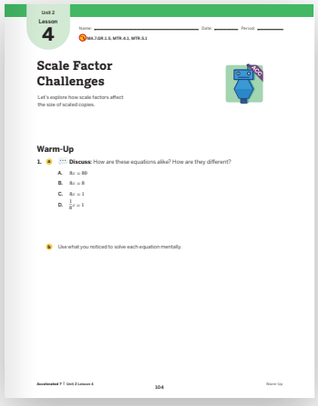
Work varies.

$\frac{1}{5}x \div \frac{1}{5} = 1 \div \frac{1}{5}$

$x = 5$

Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1–5	1	MA.7.GR.1.5
	6–7	3	MA.7.GR.1.5
FAST Practice	8	2	MA.7.GR.1.5
Spiral Review			
	9	2	MA.7.GR.1.5
	10–12	1	MA.6.AR.2.3


Unit 2
Lesson
4

Scale Factor Challenges

Let's explore how scale factors affect the size of scaled copies.

Warm-Up

1. Discuss: How are these equations alike? How are they different?
   A. $8x = 80$
   B. $8x = 8$
   C. $8x = 1$
   D. $\frac{1}{8}x = 1$

Use what you noticed to solve each equation mentally.

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Scale Factor Challenges

Effects of Scale Factors

Let's explore how scale factors affect the size of scaled copies.

Learning Outcomes

Today's Goals

1. **Goal:** Determine the effect of scale factors greater than 1, between 0 and 1, or equal to 1 on lengths in scaled copies.
2. **Goal:** Understand how the scale factor from figure A to figure B relates to the scale factor from figure B to figure A.
3. **Language Goal:** Explain how different scale factors affect lengths in scaled copies. **(Speaking and Listening)**

Students determine which scale factors create a scaled copy that is larger than, smaller than, or the same size as the original. They discover patterns in how scale factors affect the size of a scaled copy and apply these patterns by using scale factors that take a scaled copy back to the original figure. **(MTR.5.1)**

Coherence

Prior Learning

In Lesson 3, students used grids to create scaled copies with a given scale factor.

Future Learning

In Lesson 5, students will continue to explore how scaling a figure affects its size, noticing that area does not scale in the same way as lengths.

Connections

- Students determine if a scaled copy of a robot will be larger than, smaller than, or the same size as the original using the scale factor. (mathematical)

FL B.E.S.T. Math Benchmarks

Addressing

MA.7.GR.1.5
Solve mathematical and real-world problems involving dimensions and areas of geometric figures, including scale drawings and scale factors.

Mathematical Thinking and Reasoning Standards: MTR.4.1, MTR.5.1

ELA Expectations: ELA.K12.EE.3.1, ELA.K12.EE.5.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.6.AR.3.5	MA.8.GR.2.2
MA.6.GR.1.3	MA.912.GR.4.3

Lesson at a Glance

~ 45 min

MA.7.GR.1.5

Warm-Up

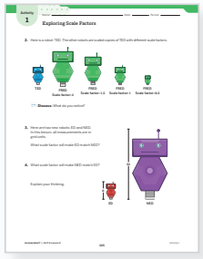
Pairs | 5 min

Students think about pairs of numbers that multiply to make 1.

Activity 1

Pairs | 10 min

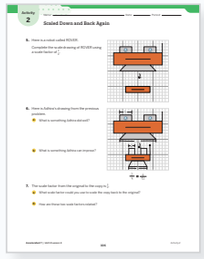
Students discover how different scale factors affect the size of a scaled copy.



Activity 2

Pairs | 10 min

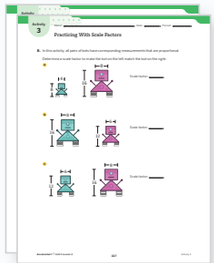
Students reflect on the relationship between the scale factors that take an original to its scaled copy and back. **Routine:** MLR3: Critique, Correct, Clarify



Activity 3

Pairs | 10 min

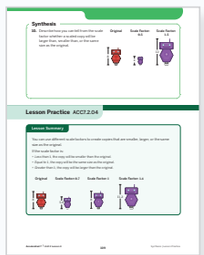
Students use proportional reasoning to determine the scale factor from one figure to another.



Synthesis

Whole Class | 5 min

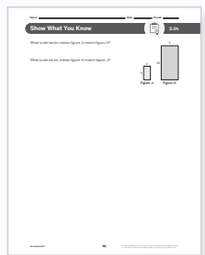
Students synthesize their understanding of how scale factor affects the size of a scaled copy compared to its original.



Show What You Know

Independent | 5 min

Students demonstrate their understanding by using proportional reasoning to determine the scale factors between two figures.



Prep Checklist

Prepare Activity 3 sheets.

This lesson includes:

- Student Edition
- Show What You Know
- Activity 3 You're Invited to Explore More Sheet, one per student (as needed)

Pairs | 5 min

Warm-Up

Purpose: Students think about pairs of numbers that multiply to make 1.

1 Launch

Direct students' attention to the equations and invite students to discuss how they think they are alike and different.

Listen for students who notice a connection between equations C and D. Consider inviting these students to share their ideas during the Connect.

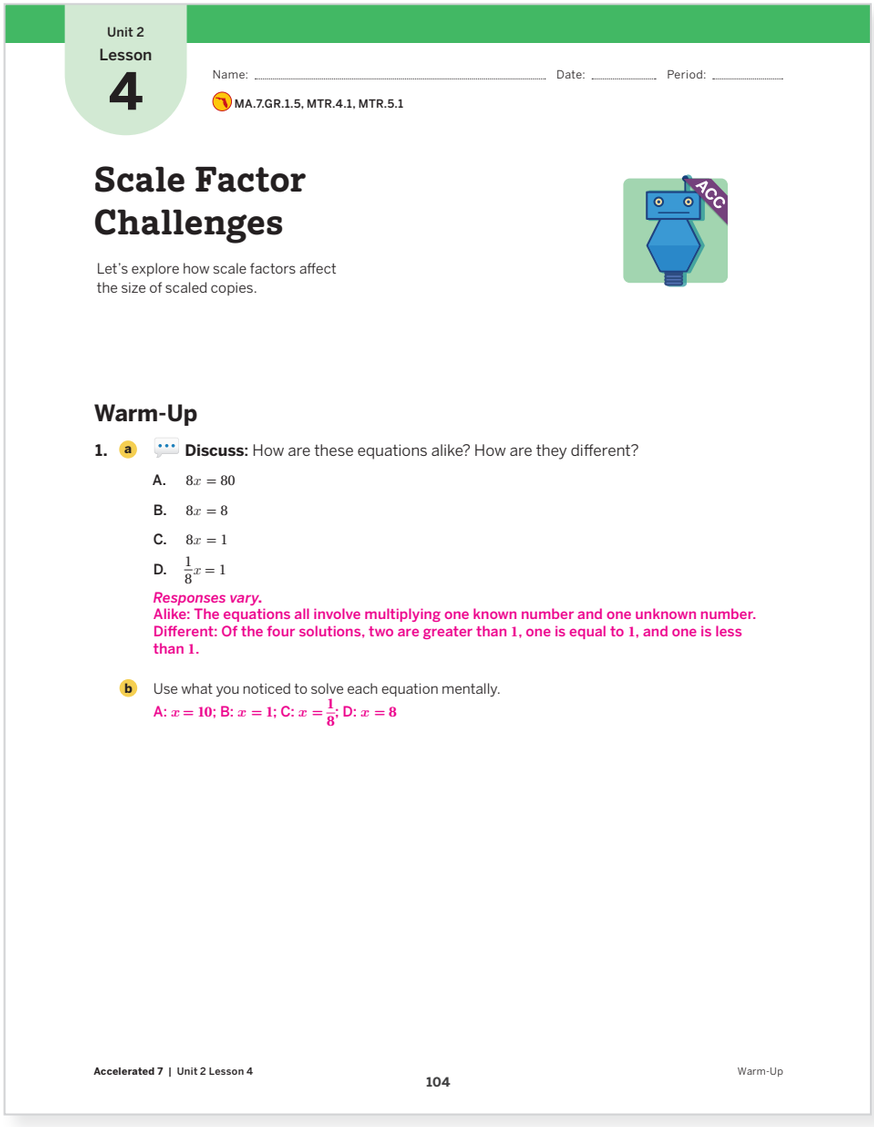
2 Connect

Invite students to share their responses.

To surface connections, consider showing how to verify the solutions to the equations using substitution. If it does not come up naturally, consider highlighting that the last two equations both show 8 and $\frac{1}{8}$ multiplying to make 1.

Consider asking, "What other pairs of numbers multiply to make 1?"

Note: The word *reciprocal* is not required in this lesson or unit, but you may choose to introduce it if it would be helpful to describe this relationship.



Unit 2
Lesson
4

Name: ..... Date: ..... Period: .....

MA.7.GR.1.5, MTR.4.1, MTR.5.1

Scale Factor Challenges

Let's explore how scale factors affect the size of scaled copies.

Warm-Up

1. a **Discuss:** How are these equations alike? How are they different?

   A. $8x = 80$

   B. $8x = 8$

   C. $8x = 1$

   D. $\frac{1}{8}x = 1$

   Responses vary.
   Alike: The equations all involve multiplying one known number and one unknown number.
   Different: Of the four solutions, two are greater than 1, one is equal to 1, and one is less than 1.

   b Use what you noticed to solve each equation mentally.
   A: $x = 10$; B: $x = 1$; C: $x = \frac{1}{8}$; D: $x = 8$

Accelerated 7 | Unit 2 Lesson 4 104 Warm-Up

Pairs | 10 min

Activity 1 Exploring Scale Factors

Purpose: Students discover how different scale factors affect the size of a scaled copy.

1 Launch

Share that the purpose of this activity is to explore how different scale factors affect the size of a scaled copy.

Consider using the Notice and Wonder routine (optional) to promote curiosity and help students make sense of different scale factors. See the Routine Facilitation Guide for more information. **(MTR.5.1)**

2 Monitor

Look for students who try scale factors between 0 and 1 or less than 0. Consider inviting these students to share what they discovered during the Connect.

Look for student ideas to share. Look for students who make connections between Problems 3 and 4.

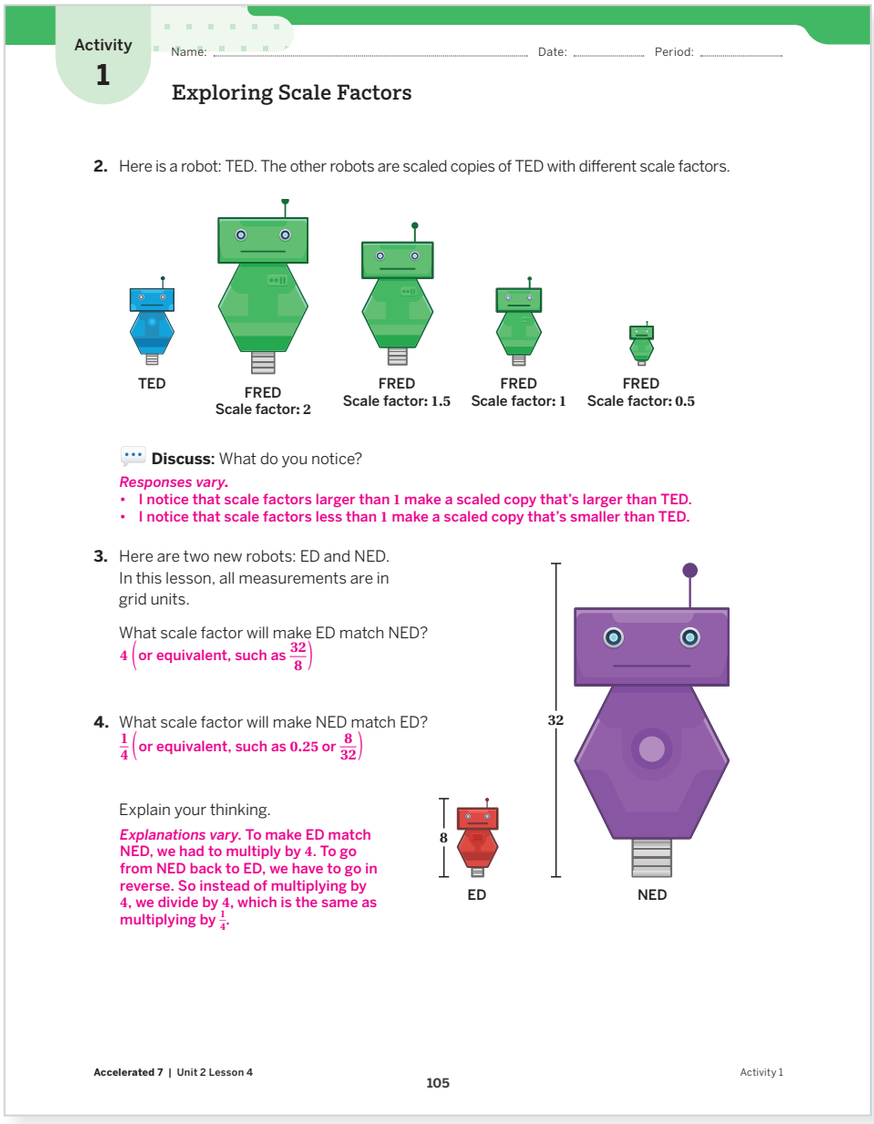
D Differentiation

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Consider asking, "What kinds of scale factors result in a smaller scaled copy?"
Determine that we can make NED match ED by dividing by 4, but do not generate a scale factor.	**Strengthen** Celebrate the structure they noticed. Invite these students to reflect by asking, "Is there a way to express division by 4 as multiplication?" **(MTR.5.1)**
Identify a scale factor of $\frac{1}{4}$ or equivalent.	**Stretch** Invite these students to create a general strategy for determining the scale factor when given the original height and the scaled copy height. **(MTR.5.1)**

3 Connect

Share the captured responses. Consider asking:

- "What happens when you scale a figure using a scale factor between 0 and 1?"
- "When a scaled copy is larger than the original, what must be true about the scale factor? What if the scaled copy is the same size?"



Activity 1

Name: Date: Period:

Exploring Scale Factors

2. Here is a robot: TED. The other robots are scaled copies of TED with different scale factors.

TED

FRED
Scale factor: 2

FRED
Scale factor: 1.5

FRED
Scale factor: 1

FRED
Scale factor: 0.5

Discuss: What do you notice?

Responses vary.
- I notice that scale factors larger than 1 make a scaled copy that's larger than TED.
- I notice that scale factors less than 1 make a scaled copy that's smaller than TED.

3. Here are two new robots: ED and NED. In this lesson, all measurements are in grid units.

What scale factor will make ED match NED?

4 (or equivalent, such as $\frac{32}{8}$)

4. What scale factor will make NED match ED?

$\frac{1}{4}$ (or equivalent, such as 0.25 or $\frac{8}{32}$)

Explain your thinking.

Explanations vary. To make ED match NED, we had to multiply by 4. To go from NED back to ED, we have to go in reverse. So instead of multiplying by 4, we divide by 4, which is the same as multiplying by $\frac{1}{4}$.

8

ED

32

NED

Accelerated 7 | Unit 2 Lesson 4

105

Activity 1

Pairs | 10 min

Activity 2 Scaled Down and Back Again

Purpose: Students reflect on the relationship between the scale factors that take an original to its scaled copy and back.

Short on time? Consider omitting Problem 6.

1 Launch

Invite students to explore Problem 5–7.

Encourage students to draw the outline of the robot as accurately as they can. Shading the interior and drawing the eyes is optional.

2 Monitor

Differentiation: (Support) If students are struggling to start, consider asking, "What happens to the lengths in the drawing when we use the scale factor $\frac{1}{2}$?" The lengths are smaller than the original. All the lengths are divided by 2.

MLR3: Critique, Correct, Clarify Use this routine to help students describe strengths, errors, and ambiguities in Adhira's work. Then invite students to:

a. Discuss what is *correct* about the response.
b. Discuss what is *incorrect* or *unclear*.
c. Write a question to help Adhira see how their work can be improved, or create a revised response. **(MTR.4.1)**

Select and sequence student explanations to share during the Connect.

Look for students who make connections between the scale factors $\frac{1}{2}$ and 2.

3 Connect

Share the captured responses. Invite students to identify which parts of each explanation they find to be valuable.

Consider asking:

- "When you knew the scale factor from the original to the copy, what strategy helped you determine the scale factor to get back to the original?"
- "How is this related to the work we did with 8 and $\frac{1}{8}$ in the Warm-Up?" In the Warm-Up, we made 1. We can multiply the scale factors of 4 and $\frac{1}{4}$ to make 1.

Invite students to revise their response to make it stronger and clearer.

Consider sharing that the two scale factors multiply to make 1.

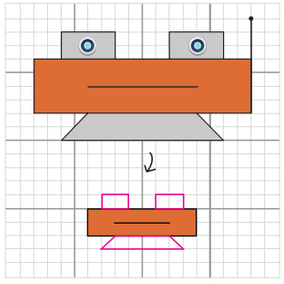
Activity 2

Name: Date: Period:

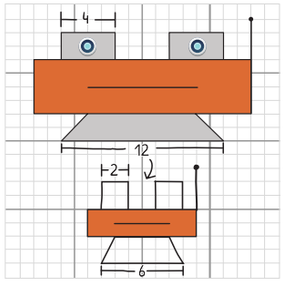
Scaled Down and Back Again

5. Here is a robot called ROVER.

Complete the scale drawing of ROVER using a scale factor of $\frac{1}{2}$.



6. Here is Adhira's drawing from the previous problem.



$\frac{2}{4} = \frac{6}{12}$

a. What is something Adhira did well?

Responses vary. Adhira used equivalent ratios $\left(\frac{2}{4} = \frac{6}{12}\right)$ to figure out how wide to make everything in the new drawing.

b. What is something Adhira can improve?

Responses vary. Adhira forgot to apply the scale factor to how tall things are. The eyes and trapezoid body should each be 1 unit tall (instead of 2), and the antenna should be $3 \cdot \frac{1}{2} = 1.5$ units tall (instead of 3).

7. The scale factor from the original to the copy is $\frac{1}{2}$.

a. What scale factor could you use to scale the copy back to the original?

2

b. How are these two scale factors related?

Responses vary. The scale factors multiply to make 1.

Accelerated 7 | Unit 2 Lesson 4 106 Activity 2

Key Takeaway: Between a pair of scaled copies, there are two possible scale factors. One scale factor takes the original to the scaled copy, and the other takes the copy back to the original. These two scale factors always multiply to make 1.

Pairs | 10 min

Activity 3 Practicing With Scale Factors

Purpose: Students use proportional reasoning to determine the scale factor from one figure to another.

Short on time? Consider sharing with students that it is okay if they do not complete all the challenges.

1 Launch

Invite students to solve as many challenges as they have time for. Share that sense-making is more important than speed.

A **Accessibility: Memory and Attention** Invite students who would benefit from having more time to think deeply to complete fewer challenges.

2 Monitor

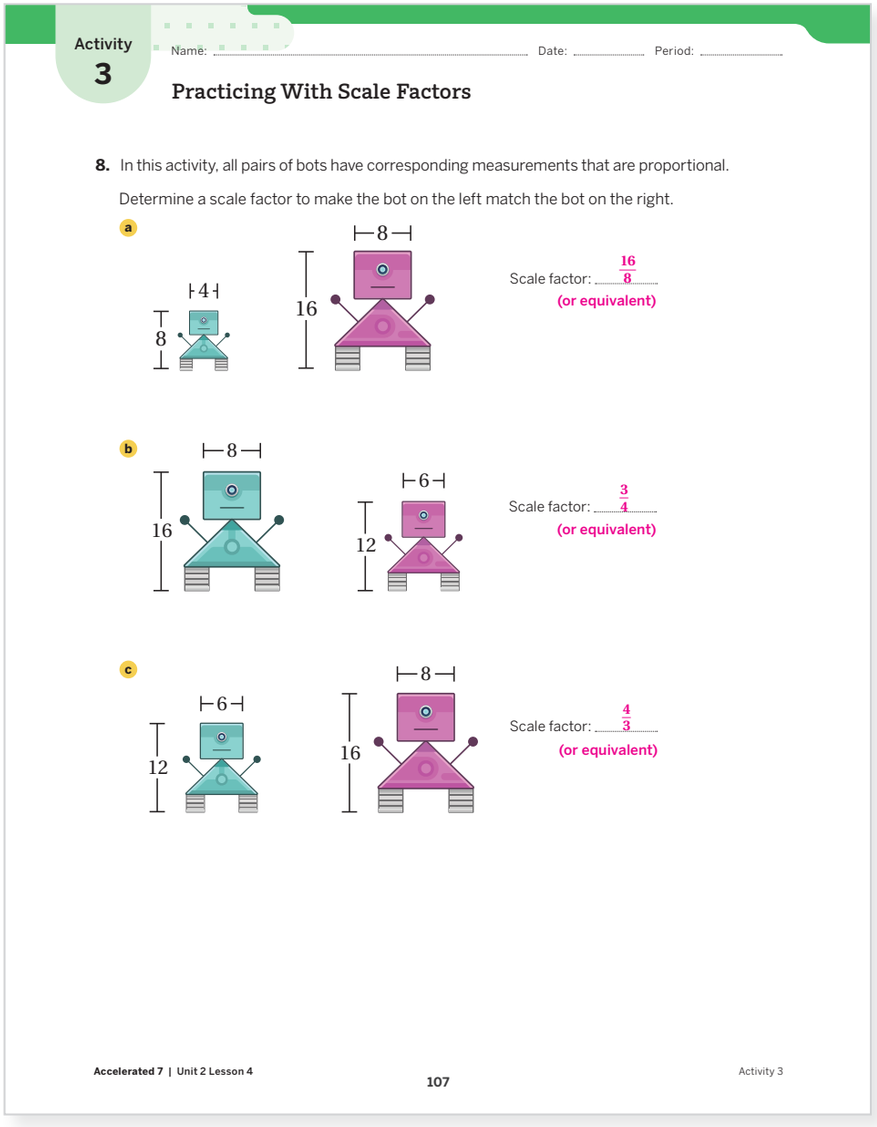
Listen for student strategies and partner discussions, offering help or encouragement where needed.

D **Differentiation**

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Encourage students to identify pairs of matching sides and look for a relationship between them.
Choose a scale factor greater than 1 when the scaled copy is smaller than the original.	**Support** Consider asking, "What can you tell about the scale factor when the scaled copy is smaller than the original?"
Recognize that the scale factor is the ratio $\frac{\text{new height}}{\text{original height}}$.	**Strengthen** Invite students to think about how they would share their strategies during the Connect.
Use estimation to determine the scale factor.	**Stretch** Encourage these students to look for structure. Consider asking, "How could you set up an equation or expression to determine the scale factor?"

Look for a variety of strategies to share, including strategies that identify the scale factor as a ratio of new to old lengths.

Activity 3 continued >



Activity 3

Name: _____ Date: _____ Period: _____

Practicing With Scale Factors

8. In this activity, all pairs of bots have corresponding measurements that are proportional.

Determine a scale factor to make the bot on the left match the bot on the right.

a

Scale factor: $\frac{16}{8}$ (or equivalent)

b

Scale factor: $\frac{3}{4}$ (or equivalent)

c

Scale factor: $\frac{4}{3}$ (or equivalent)

Pairs | 10 min

Activity 3 Practicing With Scale Factors (continued)

Purpose: Students use proportional reasoning to determine the scale factor from one figure to another.

Short on time? Consider sharing with students that it is okay if they do not complete all the challenges.

2 Monitor

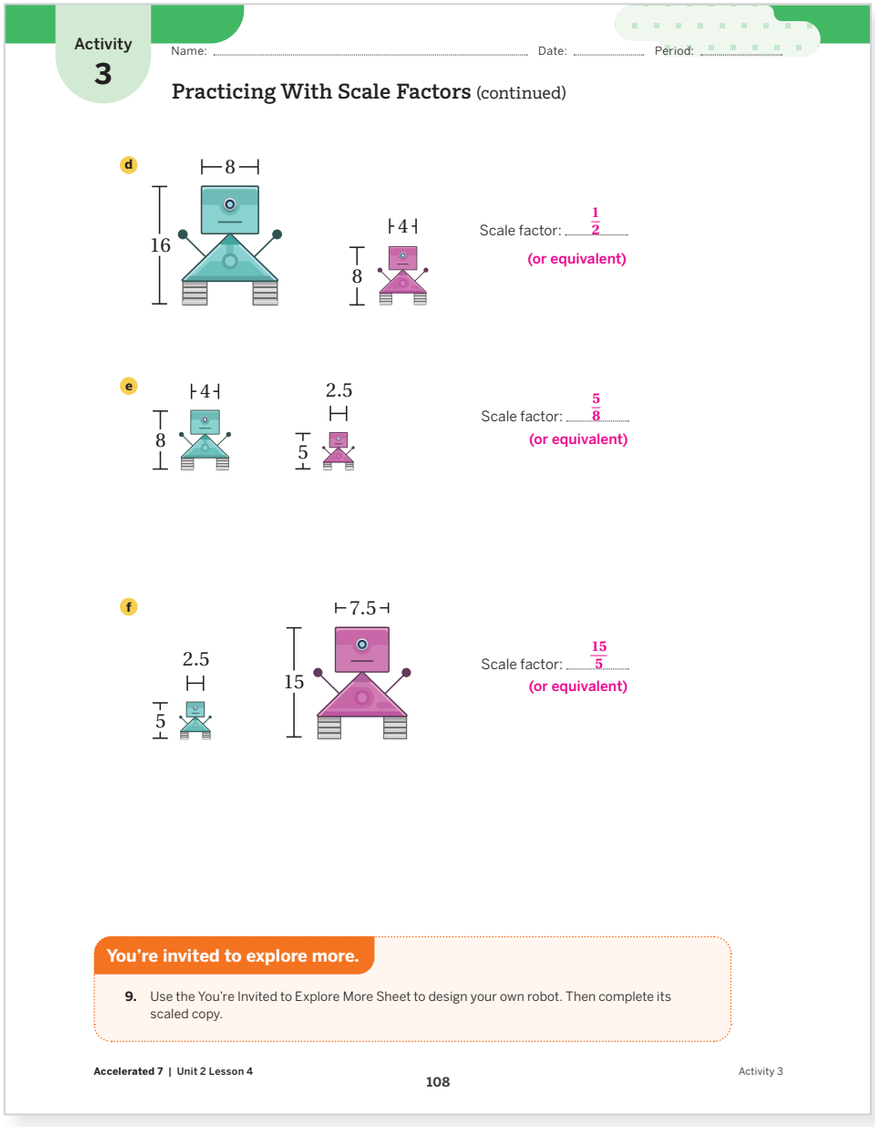
D **Differentiation: You're invited to explore more. (Stretch)** Invite students who want to explore using scale factors further to complete this optional task. Encourage them to discuss their thinking with a partner.
Note: Students will need copies of the You're Invited to Explore More Sheet.

3 Connect

Invite students to share strategies they found helpful or what they learned from mistakes they made while solving these challenges.

ELL **English Language Learners** Invite students to share strategies in their primary languages. Encourage all students to use gestures and/or visuals as they share. **(Speaking and Listening)**

Consider sharing the student-created robot designs.



Activity 3

Name: Date: Period:

Practicing With Scale Factors (continued)

d

Scale factor: $\frac{1}{2}$ (or equivalent)

e

Scale factor: $\frac{5}{8}$ (or equivalent)

f

Scale factor: $\frac{15}{5}$ (or equivalent)

You're invited to explore more.

9. Use the You're Invited to Explore More Sheet to design your own robot. Then complete its scaled copy.

Accelerated 7 | Unit 2 Lesson 4 108 Activity 3

Whole Class | 5 min

Synthesis

Purpose: Students synthesize their understanding of how the scale factor affects the size of a scaled copy compared to its original.

Invite students to respond independently, and then share their thinking with a partner.

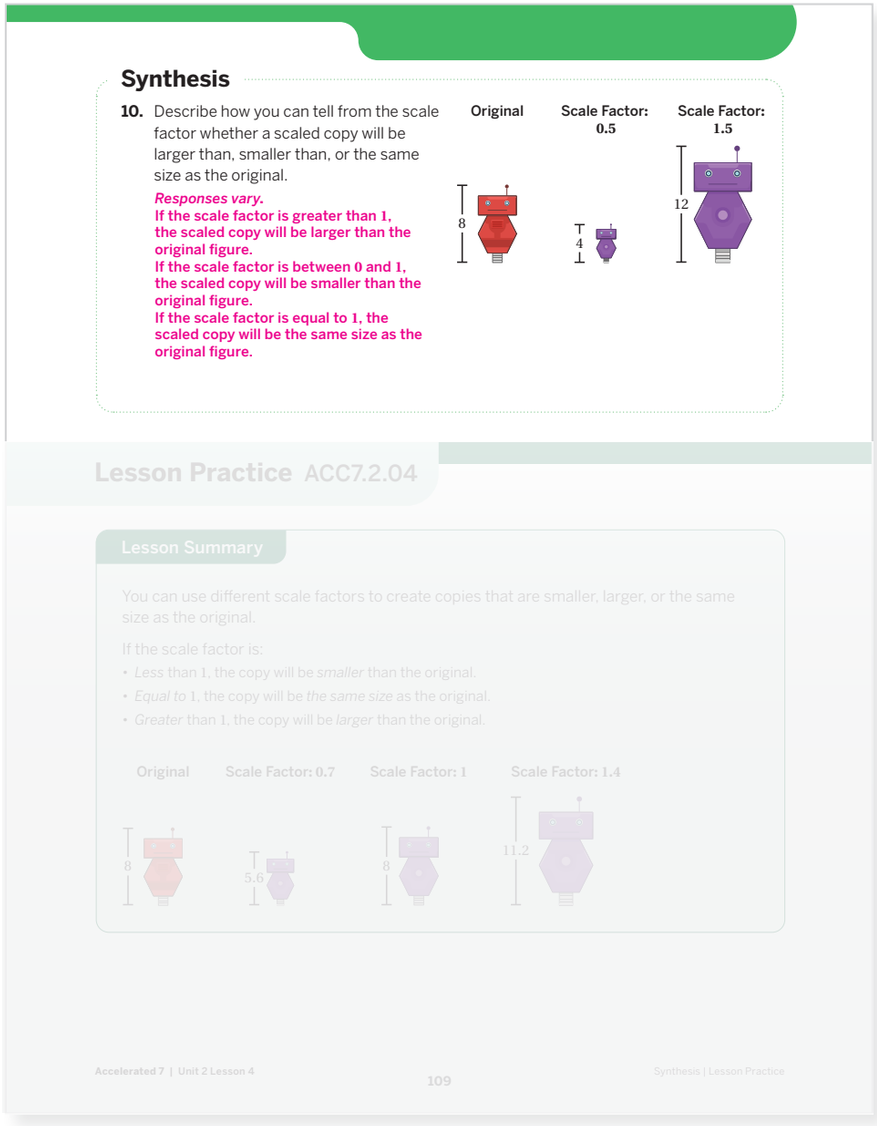
Collect and share a variety of ideas, such as:

- Scale factors between 0 and 1 create smaller copies.
- Scale factors like $\frac{1}{2}$ have the same effect as dividing by 2, so they create smaller copies.
- Scale factors greater than 1 create bigger copies.
- Using a scale factor of 1 creates the same size copy, since multiplying the lengths by 1 does not change them.

ELL **English Language Learners** Use gestures to illustrate how different scale factors affect the size of the resulting copy.

Invite students to share the connections they see between the responses.

Note: If time allows, invite students to revise their previous thinking, strengthening or clarifying their response based on the discussion.



Synthesis

10. Describe how you can tell from the scale factor whether a scaled copy will be larger than, smaller than, or the same size as the original.

Responses vary.
If the scale factor is greater than 1, the scaled copy will be larger than the original figure.
If the scale factor is between 0 and 1, the scaled copy will be smaller than the original figure.
If the scale factor is equal to 1, the scaled copy will be the same size as the original figure.

Lesson Practice ACC7.2.04

Lesson Summary

You can use different scale factors to create copies that are smaller, larger, or the same size as the original.

If the scale factor is:

- *Less* than 1, the copy will be *smaller* than the original.
- *Equal to* 1, the copy will be *the same size* as the original.
- *Greater* than 1, the copy will be *larger* than the original.

Accelerated 7 | Unit 2 Lesson 4 109 Synthesis | Lesson Practice

Lesson Takeaway: The scale factor affects the size of the scaled copy. Scale factors greater than 1 create larger copies, while scale factors between 0 and 1 create smaller copies. A scale factor of 1 creates a copy the same size as the original.

Independent | 5 min

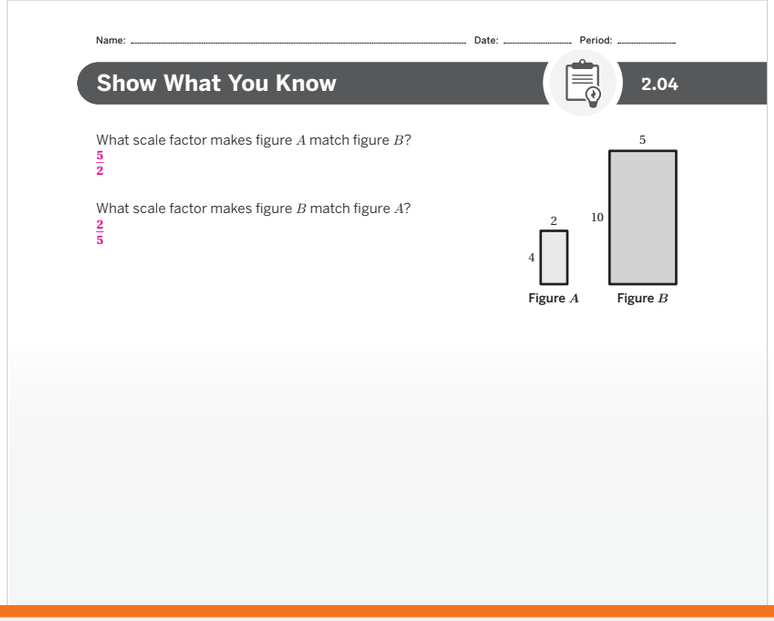
Show What You Know

Purpose: Students demonstrate their understanding by using proportional reasoning to determine the scale factors between two figures.

Today's Goals

1. **Goal:** Determine the effect of scale factors greater than 1, between 0 and 1, or equal to 1 on lengths in scaled copies.
2. **Goal:** Understand how the scale factor from figure A to figure B relates to the scale factor from figure B to figure A.
3. **Language Goal:** Explain how different scale factors affect lengths in scaled copies. **(Speaking and Listening)**

Show What You Know (Assessment Resources)



Name: _____ Date: _____ Period: _____

Show What You Know 2.04

What scale factor makes figure A match figure B?

$\frac{5}{2}$

What scale factor makes figure B match figure A?

$\frac{2}{5}$

Differentiation Use after Lesson 4

Support

Provide targeted intervention for students by using these resources.

If students need support understanding the effect of scale factors on lengths in scaled copies:

» **Respond to Student Thinking:**

- Use the ***Connecting Scale Factors to Scaled Copies*** **Mini-lesson**.
- Invite students to discuss how different scale factors affect lengths in scaled copies with a partner.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 1 Extension Activities**.
- Invite students to investigate the **You're Invited to Explore More** problem.

Professional Learning

Reflect on students' use of the grid in Activity 2. How did sketching support students with communicating their ideas or participating in the lesson? What support might students need to use grids effectively in the future?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

10. Describe how you can tell from the scale factor whether a scaled copy will be larger than, smaller than, or the same size as the original.

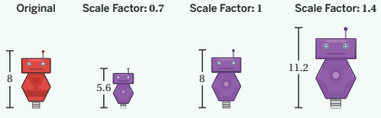
Responses vary.
If the scale factor is greater than 1, the scaled copy will be larger than the original figure.
If the scale factor is between 0 and 1, the scaled copy will be smaller than the original figure.
If the scale factor is equal to 1, the scaled copy will be the same size as the original figure.

Lesson Practice ACC7.2.04

Lesson Summary

You can use different scale factors to create copies that are smaller, larger, or the same size as the original.

If the scale factor is:
- *Less* than 1, the copy will be *smaller* than the original.
- *Equal to* 1, the copy will be *the same size* as the original.
- *Greater* than 1, the copy will be *larger* than the original.



Lesson Practice ACC7.2.04

Name: _____ Date: _____ Period: _____

1. Rectangles P, Q, R, and S are scaled copies of one another. For each pair, state whether the scale factor that takes one figure to another is greater than 1, equal to 1, or less than 1.

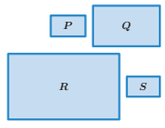
Rectangle P to rectangle R Greater than 1

Rectangle Q to rectangle S Less than 1

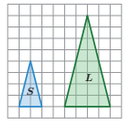
Rectangle Q to rectangle R Greater than 1

Rectangle S to rectangle P Equal to 1

Rectangle R to rectangle P Less than 1



Problems 2–4: Triangle S and triangle L are scaled copies of one another.

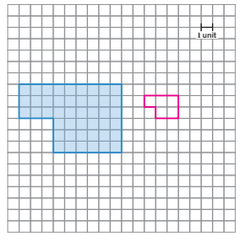


2. What is the scale factor that takes triangle S to triangle L?
2

3. What is the scale factor that takes triangle L to triangle S?
$\frac{1}{2}$

4. Triangle M (not shown) is also a scaled copy of triangle S. The scale factor that takes triangle S to triangle M is $\frac{3}{2}$. What is the scale factor that takes triangle M to triangle S?
$\frac{2}{3}$

Problems 5–7: Here is a polygon.



5. Draw a scaled copy of the polygon that has a perimeter of 10 units.
Sample shown.

6. What is the scale factor from the original polygon to your scaled copy?
$\frac{1}{3}$

7. What is the scale factor from the scaled copy back to the original polygon?
3

Lesson Practice ACC7.2.04

Name: _____ Date: _____ Period: _____

8. Will any two squares always be scaled copies of one another?
Yes

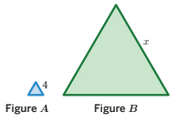
Explain your thinking.
Explanations vary. To be scaled copies, two figures need to have the same angles as one another and proportional sides. In any square, all the angles are 90° and the ratios between the sides are always equal to 1. Therefore, any two squares are scaled copies of one another.

FAST Practice

9. Figure B is a scaled copy of figure A with a scale factor of $5\frac{1}{2}$.

What is the value of x?

$x =$ 22



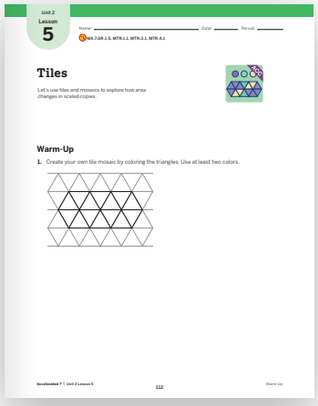
Spiral Review

10. Select *all* the ratios that are equivalent to 12 : 3.

- ☐ A. 6 : 1
- ☐ B. 1 : 4
- ☑ C. 4 : 1
- ☑ D. 24 : 6
- ☐ E. 15 : 6
- ☑ F. 1,200 : 300
- ☐ G. 112 : 13

Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1, 6–7	1	MA.7.GR.1.5
	2–5	2	MA.7.GR.1.5
	8	3	MA.7.GR.1.5
FAST Practice	9	1	MA.7.GR.1.5
Spiral Review			
	10	1	MA.6.AR.3.5



Tiles

Scaling and Area

Let's use tiles and mosaics to explore how area changes in scaled copies.

Learning Outcomes

Today's Goals

1. **Goal:** Calculate the area of a scaled copy.
2. **Language Goal:** Explain how scaling affects lengths and areas in scaled copies. **(Reading, Writing, Speaking, and Listening)**

Students explore how different scale factors affect the *area* of scaled copies. They notice that when you create a scaled copy, the area is not scaled by the same scale factor as the side lengths. They then use repeated reasoning to generate strategies for determining the area of a scaled copy. **(MTR.3.1)**

Coherence

Prior Learning

In Lesson 4, students analyzed how scale factors less than, equal to, or greater than 1 affect the size of a scaled copy compared to the original figure.

Future Learning

In Lesson 6, students will use scales to estimate the lengths of very small or very large real-world objects represented in scale drawings.

Connections

- Students apply their understanding of scale factor to identify patterns in mosaics and create their own designs. (real world)

FL B.E.S.T. Math Benchmarks

Addressing

MA.7.GR.1.5
Solve mathematical and real-world problems involving dimensions and areas of geometric figures, including scale drawings and scale factors.

Mathematical Thinking and Reasoning Standards: MTR.1.1, MTR.3.1, MTR.4.1

ELA Expectations: ELA.K12.EE.1.1, ELA.K12.EE.4.1, ELA.K12.EE.5.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.6.AR.3.5	**MA.8.GR.2.2**
MA.6.GR.1.3	**MA.912.GR.3.4**
MA.6.GR.2.2	**MA.912.GR.4.3**
	MA.912.GR.4.4

Vocabulary

Review Vocabulary

area

Lesson at a Glance

~ 45 min

MA.7.GR.1.5

Warm-Up

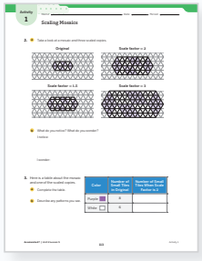
Pairs | 5 min

Students create a mosaic they will use to explore how scaling affects *area*.

Activity 1

Pairs | 15 min

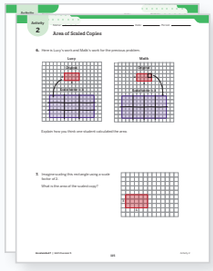
Students use a mosaic to make sense of how the area changes when a figure is scaled by a scale factor. **Routines:** Notice and Wonder, Decide and Defend **(MTR.1.1)**



Activity 2

Pairs | 15 min

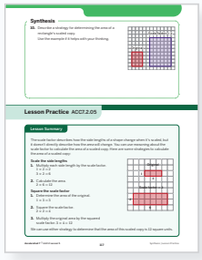
Students develop and use strategies for calculating the area of a scaled copy.
Routine: Think-Pair-Share



Synthesis

Whole Class | 5 min

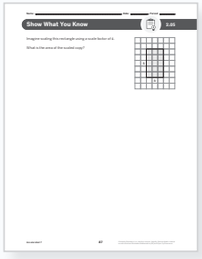
Students synthesize their understanding of the relationship between the areas of scaled copies.
MLR2: Collect and Display



Show What You Know

Independent | 5 min

Students demonstrate their understanding by calculating the area of a scaled rectangle.



Prep Checklist

This lesson includes:

- Student Edition
- Show What You Know

Pairs | 5 min

Warm-Up

Purpose: Students create a mosaic they will use to explore how scaling affects *area*.

1 Launch

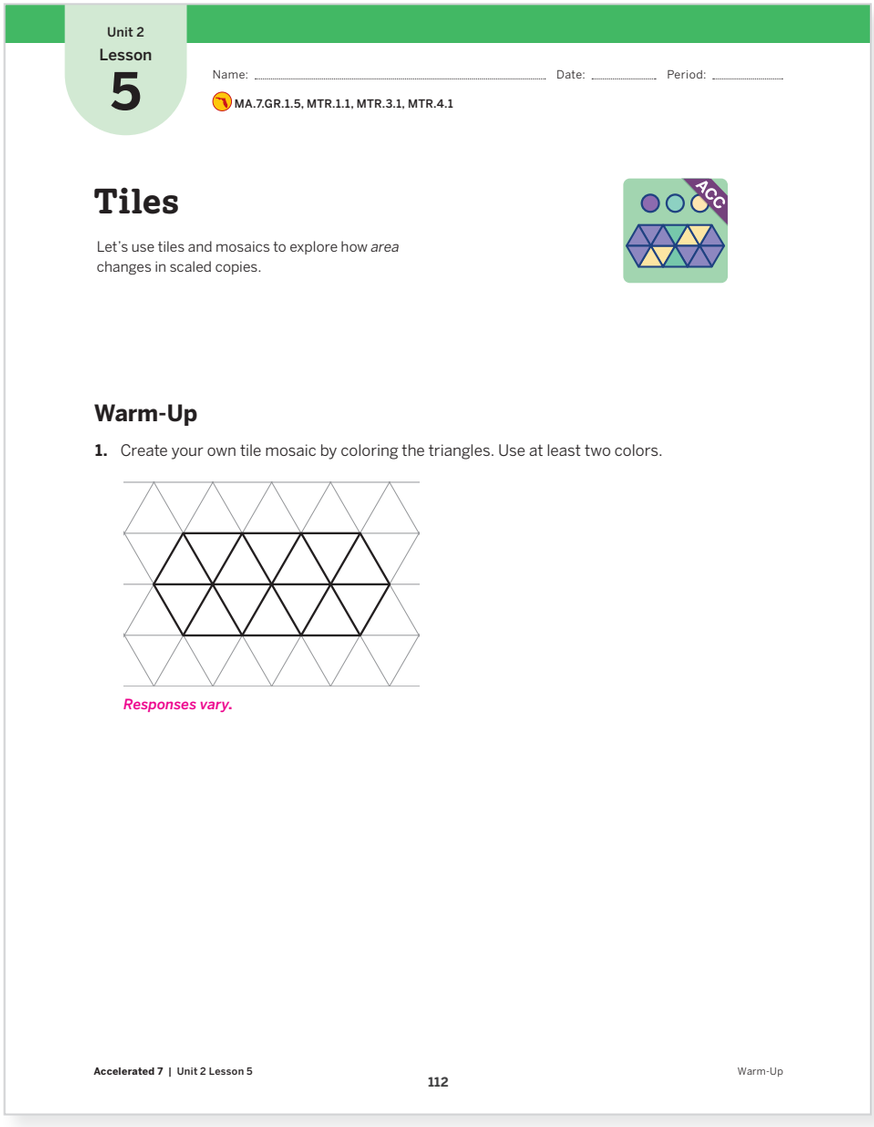
To surface connections, invite students to share with a partner all the things they could measure in their mosaic.

Consider sharing that students will use a similar mosaic in the next activity.

To encourage creativity, invite students to create a mosaic they like.

2 Connect

Invite students to share the mosaics they created.



Unit 2
Lesson
5

Name: ..... Date: ..... Period: .....

MA.7.GR.1.5, MTR.1.1, MTR.3.1, MTR.4.1

Tiles

Let's use tiles and mosaics to explore how *area* changes in scaled copies.

ACC

Warm-Up

1. Create your own tile mosaic by coloring the triangles. Use at least two colors.

Responses vary.

Accelerated 7 | Unit 2 Lesson 5 112 Warm-Up

Pairs | 15 min

Activity 1 Scaling Mosaics

Purpose: Students use a mosaic to make sense of how the area changes when a figure is scaled by a scale factor. **(MTR.1.1)**

1 Launch

Use the Notice and Wonder routine to promote curiosity and help students make sense of how scaling affects *area*.

A **Accessibility: Visual-Spatial Processing** Invite students to work with a partner to count the shaded tiles at different scale factors.

2 Monitor

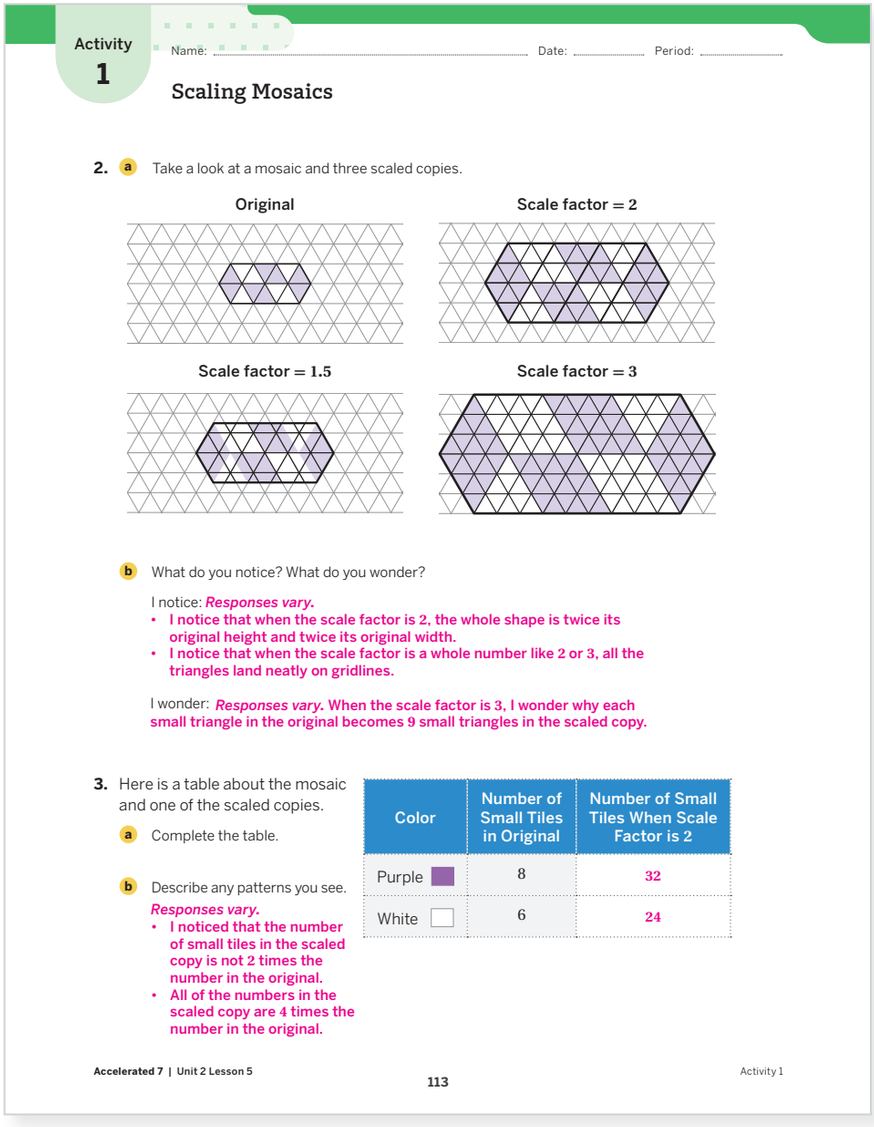
Look for students who notice a multiplicative pattern in the table. Encourage students to identify the factor by which the number of tiles is multiplied.

ELL **English Language Learners** Provide a word bank to support students in expressing their ideas with words such as: area, length, scale factor, and scaled copy. **(Writing and Speaking)**

A **Accessibility: Executive Functioning** To support students who would benefit from more processing time, provide a pre-completed table and have students focus on describing patterns. Consider asking, "How did the side lengths change from the original to the scaled copy? How did the number of tiles change?"

Note: Consider waiting until most students have completed Problem 3 to discuss as a whole class. It is okay if students do not yet see the relationship between the scale factor and the change in number of tiles.

Activity 1 continued >



Activity 1

Name: Date: Period:

Scaling Mosaics

2. a Take a look at a mosaic and three scaled copies.

Original

Scale factor = 2

Scale factor = 1.5

Scale factor = 3

b What do you notice? What do you wonder?

I notice: *Responses vary.*
- I notice that when the scale factor is 2, the whole shape is twice its original height and twice its original width.
- I notice that when the scale factor is a whole number like 2 or 3, all the triangles land neatly on gridlines.

I wonder: *Responses vary.* When the scale factor is 3, I wonder why each small triangle in the original becomes 9 small triangles in the scaled copy.

3. Here is a table about the mosaic and one of the scaled copies.

a Complete the table.

Color	Number of Small Tiles in Original	Number of Small Tiles When Scale Factor is 2
Purple	8	32
White	6	24

b Describe any patterns you see.

Responses vary.
- I noticed that the number of small tiles in the scaled copy is not 2 times the number in the original.
- All of the numbers in the scaled copy are 4 times the number in the original.

Accelerated 7 | Unit 2 Lesson 5 113 Activity 1

Pairs | 15 min

Activity 1 Scaling Mosaics (continued)

Purpose: Students use a mosaic to make sense of how the area changes when a figure is scaled by a scale factor. **(MTR.1.1)**

2 Monitor

Encourage students to sketch to show their thinking.

Listen for a variety of student ideas to highlight during the Connect.

D Differentiation

Look for students who:	Respond to Student Thinking
Believe the number of tiles will be 3 times the original, because the scale factor is 3.	**Support** Encourage students to choose a color and count the number of tiles of that color in both figures.
Notice that the number of tiles is not multiplied by the scale factor.	**Strengthen** Consider asking, "How would you describe how the number of tiles changed from the original to the scaled copy?"
Notice that the scaled copy has 9 times as many tiles as the original.	**Stretch** Consider asking, "How many tiles would there be if the scale factor were 4 instead of 3?"

Invite students to use as many different ways as they can to calculate the *area* of the large rectangle.

Note: Consider waiting to discuss Problem 5 until Activity 2. Students will see two strategies for this same problem on the following page.

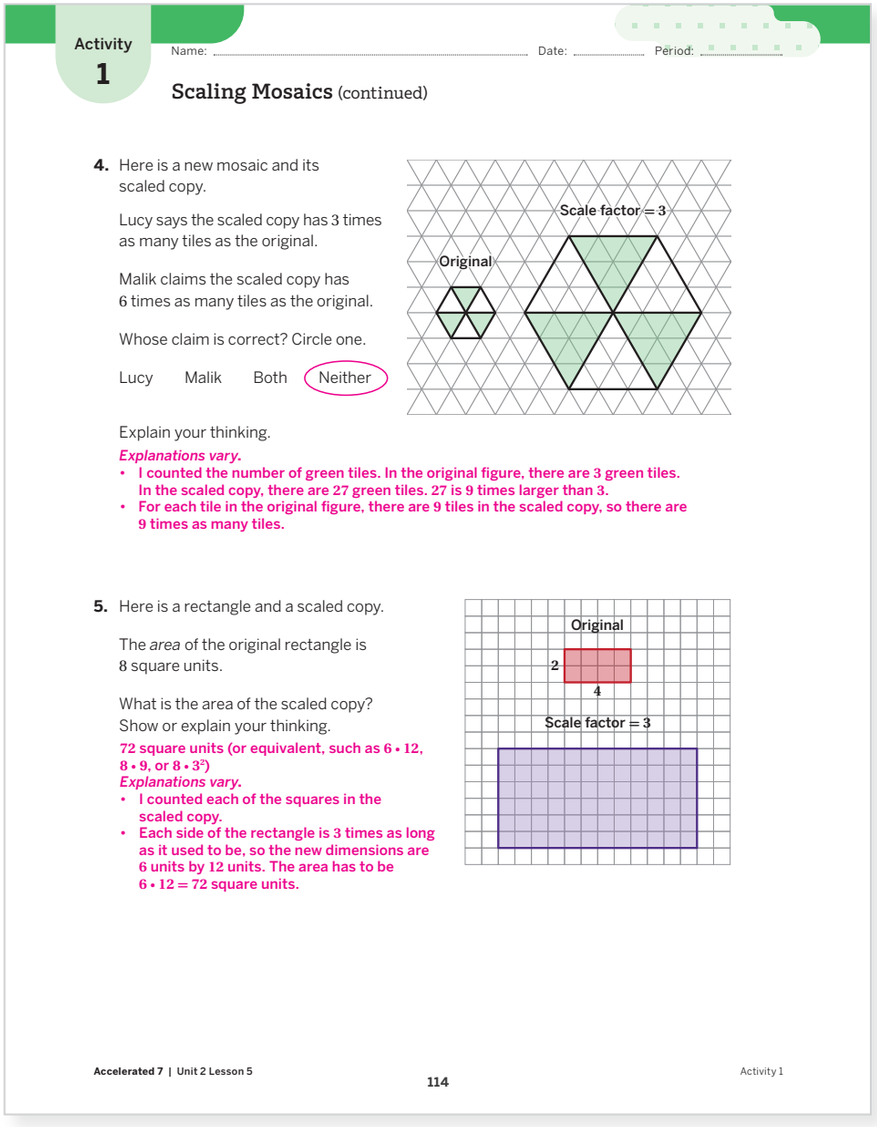
3 Connect

Invite students to share their work.

Use the Decide and Defend routine to support students in making arguments and critiquing the reasoning of others. See the Routine Facilitation Guide for more information. **(MTR.4.1)**

To surface the Key Takeaway, consider asking:

- "Where do you see the scale factor of 3 in the scaled copy?" Each side is 3 times as large as the original figure.
- "How does scaling affect the number of tiles?" The number of tiles increases. It is not 3 times as large like when we scale the sides.



Activity 1

Name: Date: Period:

Scaling Mosaics (continued)

4. Here is a new mosaic and its scaled copy.

Lucy says the scaled copy has 3 times as many tiles as the original.

Malik claims the scaled copy has 6 times as many tiles as the original.

Whose claim is correct? Circle one.

Lucy Malik Both Neither

Explain your thinking.

Explanations vary.
- I counted the number of green tiles. In the original figure, there are 3 green tiles. In the scaled copy, there are 27 green tiles. 27 is 9 times larger than 3.
- For each tile in the original figure, there are 9 tiles in the scaled copy, so there are 9 times as many tiles.

5. Here is a rectangle and a scaled copy.

The *area* of the original rectangle is 8 square units.

What is the area of the scaled copy? Show or explain your thinking.

72 square units (or equivalent, such as $6 \cdot 12$, $8 \cdot 9$, or $8 \cdot 3^2$)

Explanations vary.
- I counted each of the squares in the scaled copy.
- Each side of the rectangle is 3 times as long as it used to be, so the new dimensions are 6 units by 12 units. The area has to be $6 \cdot 12 = 72$ square units.

Accelerated 7 | Unit 2 Lesson 5 114 Activity 1

Key Takeaway: When a figure is scaled by a scale factor, the *area* is *not* scaled by the same scale factor.

Pairs | 15 min

Activity 2 Area of Scaled Copies

Purpose: Students develop and use strategies for calculating the area of a scaled copy.

Short on time? Consider omitting Problems 8 and 9 and focusing the discussion on student strategies from Problem 7.

1 Launch

Consider sharing that both Lucy and Malik entered $8 \cdot 9$ for the area.

Use the Think-Pair-Share routine to help students shape their ideas about Lucy's and Malik's strategies. See the Routine Facilitation Guide for more information.

Encourage students to make connections between the expression $8 \cdot 9$ and the sketches Malik and Lucy created.

Discuss each strategy, including students' ideas from Problem 5. Consider polling the class by asking:

- "Who saw it the way Malik saw it? How would you describe what he did?"
- "Who saw it the way Lucy saw it? What did she do?"
- "Who saw it a different way? How would you describe your strategy?"

Consider creating a record or anchor chart of strategies so students can refer back to them during the rest of the activity.

2 Monitor

Invite students to use the strategies from Problem 6 or a new strategy to determine the area of the scaled copy.

Look for students who draw sketches or write area expressions that suggest they are noticing structure, such as:

- $15 \cdot 4$
- $3 \cdot 5 \cdot 2^2$

D **Differentiation: (Support)** Students may not yet be confident applying the strategies discussed in the launch. Encourage these students to sketch the scaled copy, then look for a pattern or connection they can apply to future problems. **(MTR.3.1)**

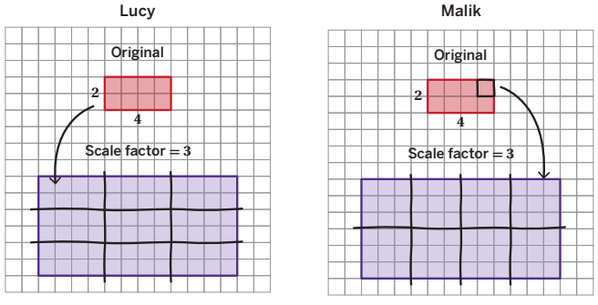
Activity 2 continued >

Activity 2

Name: Date: Period:

Area of Scaled Copies

6. Here is Lucy's work and Malik's work for the previous problem.



Explain how you think one student calculated the area.

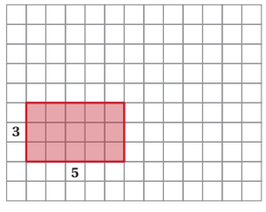
Responses vary.

- Lucy's strategy: Lucy created 9 copies of the original rectangle. Since the original rectangle had an area of 8 square units, the total area is $8 \cdot 9 = 72$ square units.
- Malik's strategy: Malik realized that each tile in the original became a 3-by-3 square in the scaled copy. There are 8 groups of 9 tiles in the scaled copy, so the area is $8 \cdot 9 = 72$ square units.

7. Imagine scaling this rectangle using a scale factor of 2.

What is the area of the scaled copy?

60 square units



Accelerated 7 | Unit 2 Lesson 5 115 Activity 2

Pairs | 15 min

Activity 2 Area of Scaled Copies (continued)

Purpose: Students develop and use strategies for calculating the area of a scaled copy.

Short on time? Consider omitting Problems 8 and 9 and focusing the discussion on student strategies from Problem 7.

2 Monitor

Select and sequence student strategies to share during the Connect.

D Differentiation

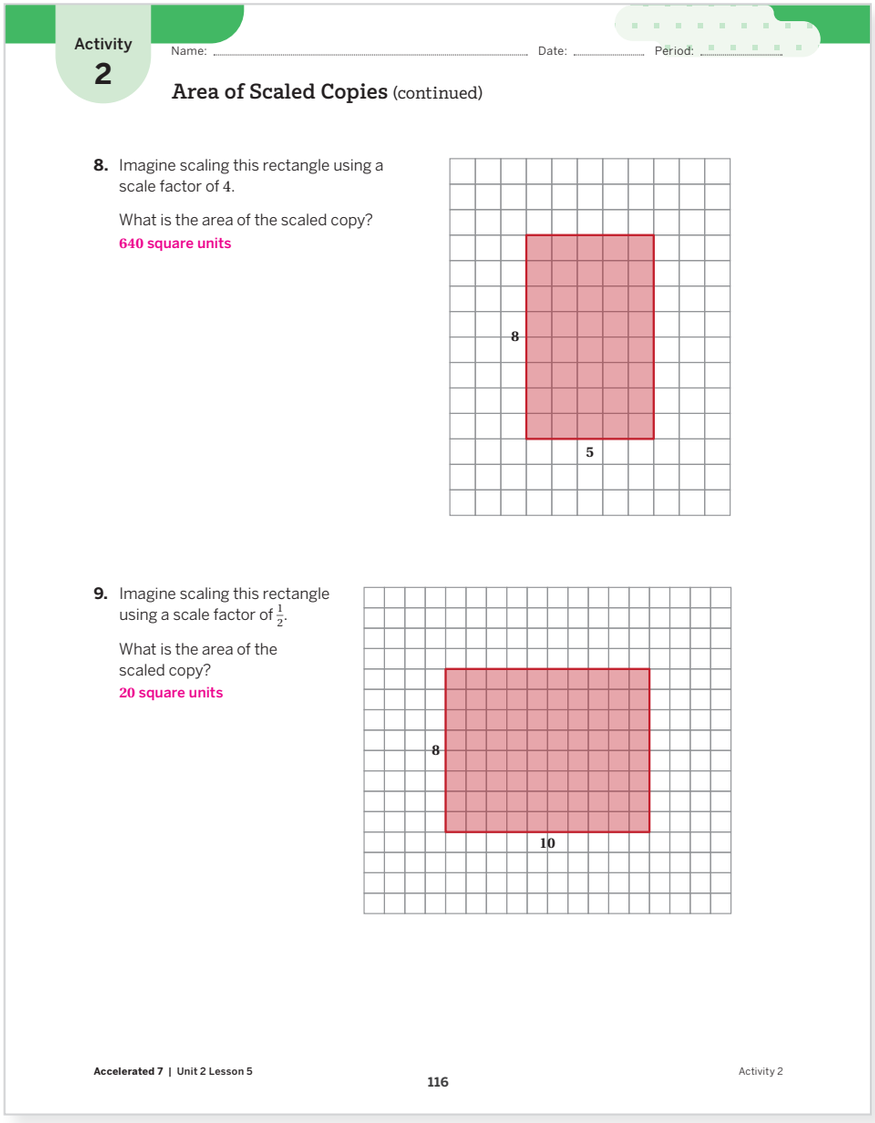
Look for students who:	Respond to Student Thinking
Create a sketch of the scaled rectangle.	**Support** Encourage students to try to determine the scaled lengths using *scale factor*.
Calculate the area of the original rectangle, then multiply by the scale factor.	**Support** Consider asking, "What are the length and width of the scaled copy? What is the area of a rectangle with those dimensions?"
Use a variety of strategies to determine the area, including: • Multiplying each length by the scale factor, then using those lengths to calculate area (e.g., $(8 \cdot 4) \cdot (5 \cdot 4)$). • Strategies similar to Lucy's or Malik's.	**Strengthen** Consider inviting these students to share their strategies during the Connect.
Apply the square of the scale factor, or multiply by the scale factor twice, to calculate the area.	**Stretch** Consider asking, "What relationship do you see between scale factor and area?"

3 Connect

Invite students to make connections between different strategies.

Consider asking, "What is similar or different about these strategies?"

If students generate expressions or sketches that represent the square of the scale factor, consider asking, "How is the 16 in this expression connected to the scale factor of 4?"



Activity 2

Name: Date: Period:

Area of Scaled Copies (continued)

8. Imagine scaling this rectangle using a scale factor of 4.

What is the area of the scaled copy?

640 square units

9. Imagine scaling this rectangle using a scale factor of $\frac{1}{2}$.

What is the area of the scaled copy?

20 square units

Accelerated 7 | Unit 2 Lesson 5 116 Activity 2

Whole Class | 5 min

Synthesis

Purpose: Students synthesize their understanding of the relationship between the areas of scaled copies.

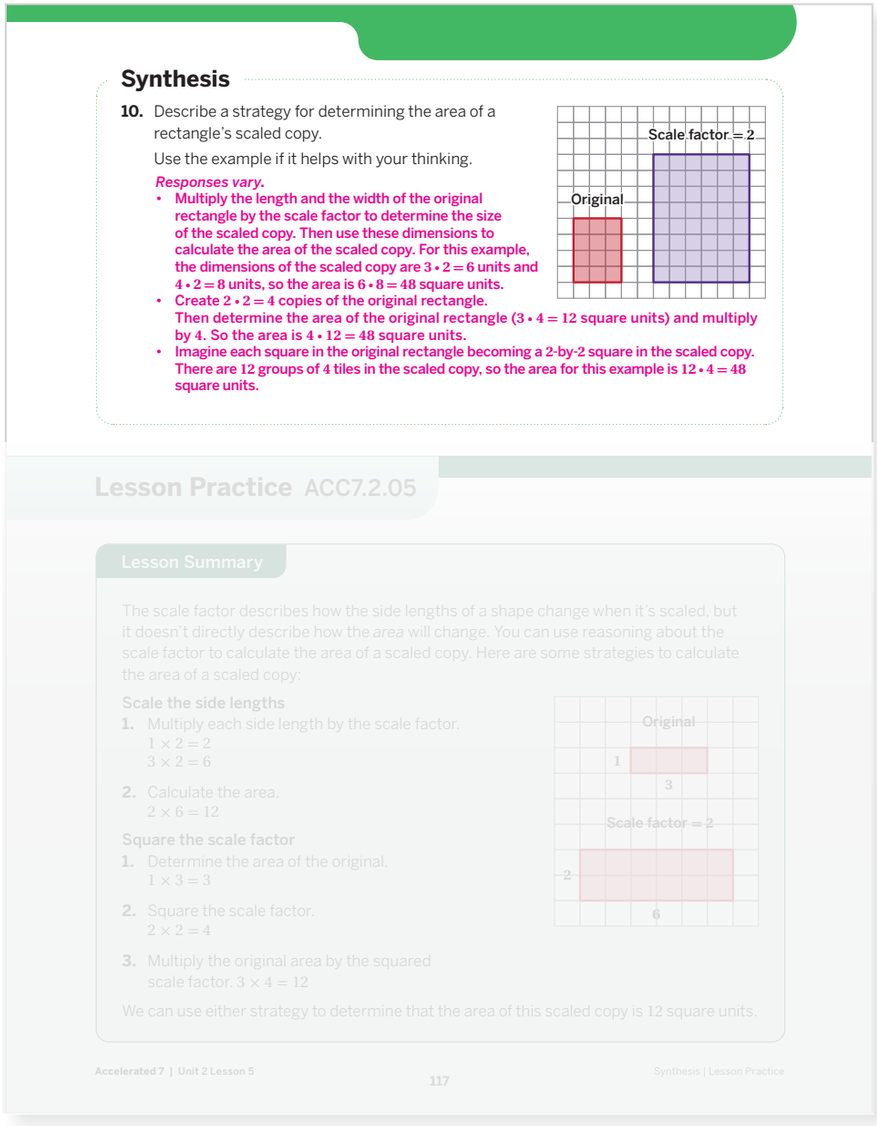
Invite students to respond independently, and then share their thinking with a partner.

Collect and share a variety of ideas, such as:

- Multiplying both length and width by the scale factor, then using these dimensions to calculate area.
- Creating 4 copies of the original rectangle, determining the original area, and multiplying by 4.
- Imagining each square in the original rectangle becoming a 2-by-2 square in the scaled copy. There would be 12 of these 2-by-2 squares, so the area is $12 \cdot 4 = 48$.

MLR **MLR2: Collect and Display** Use this routine to collect students' strategies as they discuss with a partner. Display for students to reference and add to for the rest of the unit.

Note: If time allows, invite students to share with a partner what makes sense about each strategy and what questions they still have.



Synthesis

10. Describe a strategy for determining the area of a rectangle's scaled copy.

Use the example if it helps with your thinking.

Responses vary.

- Multiply the length and the width of the original rectangle by the scale factor to determine the size of the scaled copy. Then use these dimensions to calculate the area of the scaled copy. For this example, the dimensions of the scaled copy are $3 \cdot 2 = 6$ units and $4 \cdot 2 = 8$ units, so the area is $6 \cdot 8 = 48$ square units.
- Create $2 \cdot 2 = 4$ copies of the original rectangle. Then determine the area of the original rectangle ($3 \cdot 4 = 12$ square units) and multiply by 4. So the area is $4 \cdot 12 = 48$ square units.
- Imagine each square in the original rectangle becoming a 2-by-2 square in the scaled copy. There are 12 groups of 4 tiles in the scaled copy, so the area for this example is $12 \cdot 4 = 48$ square units.

Lesson Practice ACC7.2.05

Lesson Summary

The scale factor describes how the side lengths of a shape change when it's scaled, but it doesn't directly describe how the *area* will change. You can use reasoning about the scale factor to calculate the area of a scaled copy. Here are some strategies to calculate the area of a scaled copy:

Scale the side lengths

1. Multiply each side length by the scale factor.
   $1 \times 2 = 2$
   $3 \times 2 = 6$
2. Calculate the area.
   $2 \times 6 = 12$

Square the scale factor

1. Determine the area of the original.
   $1 \times 3 = 3$
2. Square the scale factor.
   $2 \times 2 = 4$
3. Multiply the original area by the squared scale factor. $3 \times 4 = 12$

We can use either strategy to determine that the area of this scaled copy is 12 square units.

Accelerated 7 | Unit 2 Lesson 5 117 Synthesis | Lesson Practice

Lesson Takeaway: When a figure is scaled, its lengths are all scaled by the scale factor, but the *area* is not. However, you can still use the scale factor to calculate the area of a scaled copy.

Independent | 5 min

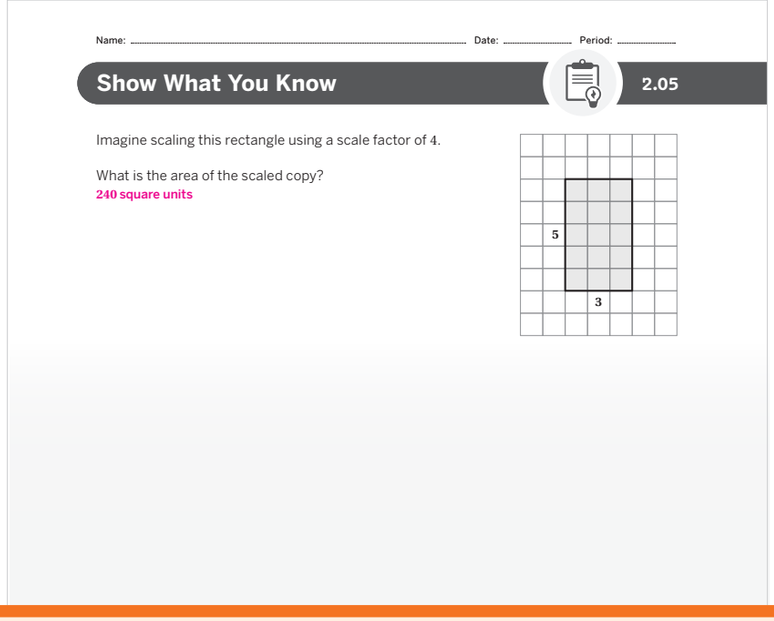
Show What You Know

Purpose: Students demonstrate their understanding by calculating the area of a scaled rectangle.

Today's Goals

1. **Goal:** Calculate the area of a scaled copy.
2. **Language Goal:** Explain how scaling affects lengths and areas in scaled copies. **(Reading, Writing, Speaking, and Listening)**

Show What You Know (Assessment Resources)



Name: _____ Date: _____ Period: _____

Show What You Know 2.05

Imagine scaling this rectangle using a scale factor of 4.

What is the area of the scaled copy?

240 square units

Differentiation Use after Lesson 5

Support

Provide targeted intervention for students by using these resources.

If students need support understanding how to calculate the area of a scaled copies:

» **Respond to Student Thinking:**

- Invite students to discuss how to determine the length and width of a scaled rectangle with a partner.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 1 Extension Activities**.

Professional Learning

Take a moment to reflect on an instructional routine from this lesson. Students will use these routines throughout the year. What about the routine felt successful? What are areas for growth next time? Consider consulting the routine facilitation guide for more ideas.

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

10. Describe a strategy for determining the area of a rectangle's scaled copy.

Use the example if it helps with your thinking.

Responses vary.

- Multiply the length and the width of the original rectangle by the scale factor to determine the size of the scaled copy. Then use these dimensions to calculate the area of the scaled copy. For this example, the dimensions of the scaled copy are $3 \cdot 2 = 6$ units and $4 \cdot 2 = 8$ units, so the area is $6 \cdot 8 = 48$ square units.
- Create $2 \cdot 2 = 4$ copies of the original rectangle. Then determine the area of the original rectangle ($3 \cdot 4 = 12$ square units) and multiply by 4. So the area is $4 \cdot 12 = 48$ square units.
- Imagine each square in the original rectangle becoming a 2-by-2 square in the scaled copy. There are 12 groups of 4 tiles in the scaled copy, so the area for this example is $12 \cdot 4 = 48$ square units.

Lesson Practice ACC7.2.05

Lesson Summary

The scale factor describes how the side lengths of a shape change when it's scaled, but it doesn't directly describe how the *area* will change. You can use reasoning about the scale factor to calculate the area of a scaled copy. Here are some strategies to calculate the area of a scaled copy:

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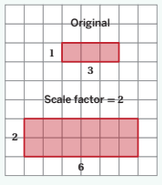
2. Calculate the area.
   $2 \times 6 = 12$

Square the scale factor

1. Determine the area of the original.
   $1 \times 3 = 3$
2. Square the scale factor.
   $2 \times 2 = 4$
3. Multiply the original area by the squared scale factor. $3 \times 4 = 12$

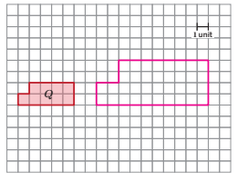


We can use either strategy to determine that the area of this scaled copy is 12 square units.

Lesson Practice
ACC7.2.05

Name: _____ Date: _____ Period: _____

Problems 1–4: Here is polygon Q.



1. Draw a scaled copy of polygon Q using a scale factor of 2.
Sample shown.

2. What is the area of the scaled copy?
36 square units

3. Complete the table for different scaled copies of polygon Q.

Scale Factor	Perimeter (units)	Area (sq. units)
1	14	9
2	28	**36**
3	**42**	**81**

4. How is the relationship between the scale factor and perimeter different from the relationship between the scale factor and area?
Responses vary. **The perimeter of a scaled copy grows by the scale factor. For example, a scaled copy of polygon Q using a scale factor of 3 has a perimeter of 42 ($14 \cdot 3 = 42$). The area doesn't grow by the scale factor because $9 \cdot 3 \neq 81$.**

5. Imagine a right triangle with an area of 36 square units.

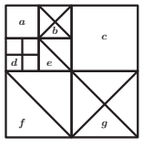
Scaled copies are made using different scale factors. Complete the table with the area of each scaled copy.

Scale Factor	Area (sq. units)
1	36
2	**144**
3	**324**
5	**900**
$\frac{1}{2}$	**9**
$\frac{2}{3}$	**16**

Lesson Practice
ACC7.2.05

Name: _____ Date: _____ Period: _____

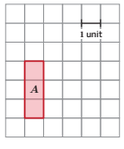
6. The square shown is made up of scaled copies of a triangle and a square. The area of region d is 1 square unit. Determine as many different areas as you can.



Region	a	b	c	d	e	f	g
Area (sq. units)	4	1	16	1	2	8	4

FAST Practice

Problems 7–8: Ivory created a scaled copy of rectangle A. The area of Ivory's scaled copy is 48 square units.



7. How many times greater is the area of Ivory's scaled copy than the area of rectangle A?

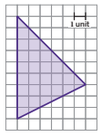
16 times greater

8. What scale factor did Ivory use to create their scaled copy?

4

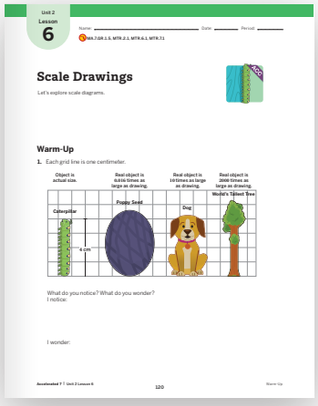
Spiral Review

9. Determine the area of this triangle.
27 square units



Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1–5	2	MA.7.GR.1.5
	6	3	MA.7.GR.1.5
FAST Practice	7–8	2	MA.7.GR.1.5
Spiral Review			
	9	2	MA.6.GR.2.1


Unit 2
Lesson 6
Scale Drawings
Let's explore scale diagrams.
Warm-Up
1. Each grid line is one centimeter.
Object is actual size.
Real object is 0.016 times as large as drawing.
Real object is 10 times as large as drawing.
Real object is 2000 times as large as drawing.
Caterpillar
Poppy Seed
Dog
World's Tallest Tree
4 cm
What do you notice? What do you wonder?
I notice:
I wonder:
120

Scale Drawings

Comparing Scale Factor and Scale

Let's explore scale diagrams.

Learning Outcomes

Today's Goals

1. **Goal:** Recognize that a *scale* is a ratio of distance in a drawing to the corresponding distance in an object in the real world.
2. **Goal:** Interpret the *scale* of a drawing.
3. **Language Goal:** Explain how a *scale* helps to represent actual distances. **(Reading, Writing, Speaking, and Listening)**

Students are introduced to the term **scale**, and begin to explore why scales are useful for representing very large or very small real-world objects. They describe how using a scale is similar or different to using a scale factor. They use scales to estimate actual lengths of objects.

Coherence

Prior Learning

In Lessons 1–5, students used scale factors to create and compare scaled copies.

Future Learning

In Lessons 7–11, students will extend their understanding of scale factor to perform dilations.

Connections

- Students apply their understanding of scale to estimate the measurements of real-world objects. (real world)

FL B.E.S.T. Math Benchmarks

Addressing

MA.7.GR.1.5
Solve mathematical and real-world problems involving dimensions and areas of geometric figures, including scale drawings and scale factors.

Mathematical Thinking and Reasoning Standards: MTR.2.1, MTR.6.1, MTR.7.1

ELA Expectations: ELA.K12.EE.3.1, ELA.K12.EE.6.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.6.AR.3.5	MA.8.GR.2.2
MA.6.GR.1.3	MA.912.GR.4.3

Vocabulary

New Vocabulary

scale	scale drawings

Lesson at a Glance

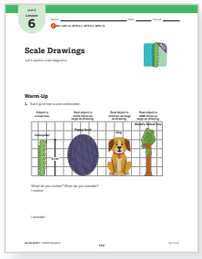
~ 45 min

MA.7.GR.1.5

Warm-Up

Pairs | 5 min

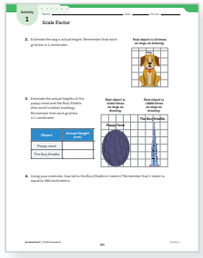
Students notice and wonder about very large or small objects represented on a grid with a scale factor. **Routine:** Notice and Wonder



Activity 1

Pairs | 15 min

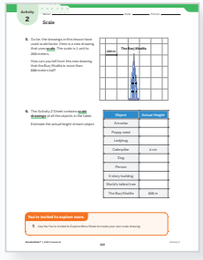
Students use scale factors to estimate the actual height of objects.



Activity 2

Pairs | 15 min

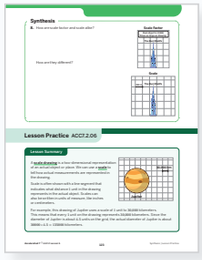
Students use **scale** to interpret drawings of real-world objects and to estimate the height of objects. **(MTR.2.1)**



Synthesis

Whole Class | 5 min

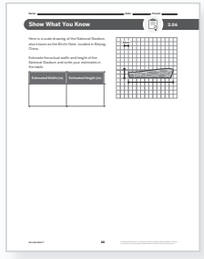
Students synthesize their understanding of scale. MLR2: Collect and Display



Show What You Know

Independent | 5 min

Students demonstrate their understanding of scale by estimating the actual width and height of National Stadium.



Prep Checklist

Prepare Activity 2 Sheets.

This lesson includes:

- Student Edition
- Show What You Know
- Activity 2 Sheet, one per student
  - Calculators
- Activity 2 You're Invited to Explore More Sheet, one per student (as needed)
  - Measuring tools (in centimeters)

Pairs | 5 min

Warm-Up

Purpose: Students notice and wonder about very large or small objects represented on a grid with a scale factor.

1 Launch

Consider sharing that students will examine drawings of actual objects in the world in this lesson and for the rest of the unit.

2 Connect

Use the Notice and Wonder routine to promote curiosity and help students make sense of the visual.

A **Accessibility: Visual-Spatial Processing**
Invite students to answer a more concrete question first to support interpreting visual representations. Consider asking, "What does the sentence tell us about the size of the actual dog?"

Invite students to share their observations. Consider asking, "Where have you seen a poppy seed in real life? Is it bigger or smaller than a caterpillar?"

Note: If the following ideas do not arise naturally during the discussion, consider sharing them with the class:

- The number in the sentence "Real object is 10 times as large" means that the scale factor from this dog to the actual dog is 10.
- The grid is 1 cm by 1 cm based on the given information in the problem.

Unit 2
Lesson
6

Name: ..... Date: ..... Period: .....

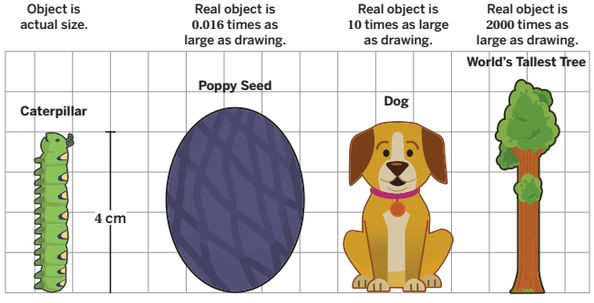
MA.7.GR.1.5, MTR.2.1, MTR.6.1, MTR.7.1

Scale Drawings

Let's explore scale diagrams.

Warm-Up

1. Each grid line is one centimeter.



What do you notice? What do you wonder?
I notice: *Responses vary.*

- I notice that the drawing of the caterpillar is actual size, but the other objects are not.
- I notice that the scale factor is different for each object.
- I notice that the caterpillar is 4 centimeters tall, which means the gridlines are 1 centimeter apart.

I wonder: *Responses vary.*

- I wonder where that giant tree is.
- I wonder how tall these objects are.

Accelerated 7 | Unit 2 Lesson 6 120 Warm-Up

Pairs | 15 min

Activity 1 Scale Factor

Purpose: Students use scale factors to estimate the actual height of objects.

Short on time? Consider omitting Problem 4.

1 Launch

Consider asking, "What is the scale factor from the drawing to the actual dog?"

Invite students to check in with a partner before moving on.

2 Monitor

Select and sequence student strategies for using the scale factor, including those that show the operations used.

D Differentiation

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Consider asking, "How tall does the object appear to be?"
Use the caterpillar as a reference point for the size of the poppy seed or Burj Khalifa.	**Support** Consider asking, "What does this sentence tell you about the scale factor? How does the grid help us make sense of the size?"
Count the units of each object on the grid and multiply by the scale factor.	**Stretch** Consider asking, "What would change about the scale factor if the grid were made up of smaller squares?"
Mention how small or large the numbers are for the poppy seed and the Burj Khalifa.	**Stretch** Consider asking, "How else could you measure the height of those objects?"

Encourage students who finish early to determine how tall the Burj Khalifa is in kilometers. Consider asking, "Which unit of measurement makes the most sense?" **(MTR.6.1)**

Invite students to share their estimates with their partner for the height of the Burj Khalifa before moving on to converting between centimeters and meters.

3 Connect

Invite students to discuss their strategies. Consider asking, "What might be challenging about using scale factor to estimate the size of real objects?" **(MTR.7.1)**

ELL English Language Learners

Encourage students to paraphrase each other's ideas to help make connections and incorporate vocabulary from the unit. **(Speaking and Listening)**

Activity 1

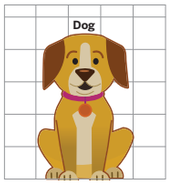
Name: Date: Period:

Scale Factor

2. Estimate the dog's actual height. Remember that each grid line is 1 centimeter.

Responses vary. 45 centimeters

Real object is 10 times as large as drawing.



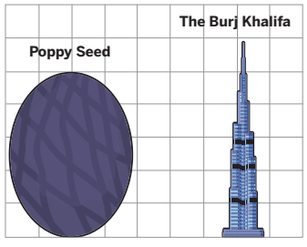
3. Estimate the actual heights of the poppy seed and the Burj Khalifa (the world's tallest building). Remember that each grid line is 1 centimeter.

Responses vary.

Object	Actual Height (cm)
Poppy seed	0.08
The Burj Khalifa	82800

Real object is 0.016 times as large as drawing.

Real object is 14000 times as large as drawing.



4. Using your estimate, how tall is the Burj Khalifa in meters? Remember that 1 meter is equal to 100 centimeters.

Responses vary. 828 meters

Accelerated 7 | Unit 2 Lesson 6 121 Activity 1

Pairs | 15 min

Activity 2 Scale

Purpose: Students use **scale** to interpret drawings of real-world objects and to estimate the height of objects. **(MTR.2.1)**

1 Launch

Invite students to read the problem and then pause to ask, "How is the scale on the grid similar or different from what you know about scale factor?" **(MTR.2.1)**

Create class definitions of the terms **scale** and **scale drawing** using students' language and the glossary. Encourage students to note the differences between scale and scale factor.

2 Monitor

Distribute the Activity 2 Sheet to students.

A **Accessibility: Executive Functioning** Chunk this activity into more manageable parts (e.g., presenting one problem at a time), which will aid students who benefit from support with organizational skills.

Look for students who count units on the grid and multiply by the scale. Capture these expressions to share during the Connect.

Note: Students may not have seen the symbol μ. Consider telling students that a meter is one million times as large as a micrometer.

D **Differentiation: You're invited to explore more. (Stretch)** Invite students who would like to explore using scale to represent the height of real-world objects further to complete this optional task. Encourage them to discuss their thinking with a partner.
Note: Students will need copies of the You're Invited to Explore More Sheet.

3 Connect

Invite students to share how they determined the actual height of a few objects. Consider displaying the multiplicative expressions in addition to the height to ask about what each part of the multiplication statement represents in the situation. **(MTR.2.1)**

To surface the Key Takeaway, consider asking, "Why might it be useful for a scale to involve different units?"

Activity 2

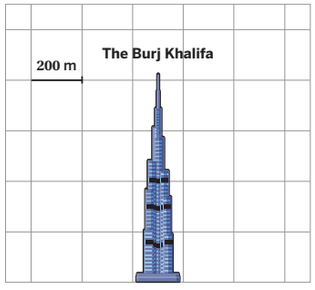
Name: Date: Period:

Scale

5. So far, the drawings in this lesson have used scale factor. Here is a new drawing that uses **scale**. The scale is 1 unit to 200 meters.

How can you tell from this new drawing that the Burj Khalifa is more than 800 meters tall?

Responses vary.
- The Burj Khalifa is a little more than 4 units tall, which means it's a little more than $4 \cdot 200 = 800$ meters tall.
- Count the gridlines up from the bottom of the drawing: 200, 400, 600, 800. This means the Burj Khalifa is a little more than 800 meters tall.



6. The Activity 2 Sheet contains **scale drawings** of all the objects in the table.

Estimate the actual height of each object.

Responses vary. Samples shown in table.

Object	Actual Height
Amoeba	140 μm
Poppy seed	800 μm
Ladybug	5.5 mm
Caterpillar	4 cm
Dog	45 cm
Person	180 cm
5-story building	15 m
World's tallest tree	115 m
The Burj Khalifa	828 m

You're invited to explore more.

7. Use the You're Invited to Explore More Sheet to create your own scale drawing.

Accelerated 7 | Unit 2 Lesson 6 122 Activity 2

Key Takeaway: A **scale drawing** is a drawing that represents an actual place, object, or person. A **scale** can be used to show how lengths on a drawing relate to the lengths on the actual object. All measurements in a scale drawing correspond to the measurements of the actual object by the same scale.

Whole Class | 5 min

Synthesis

Purpose: Students synthesize their understanding of scale.

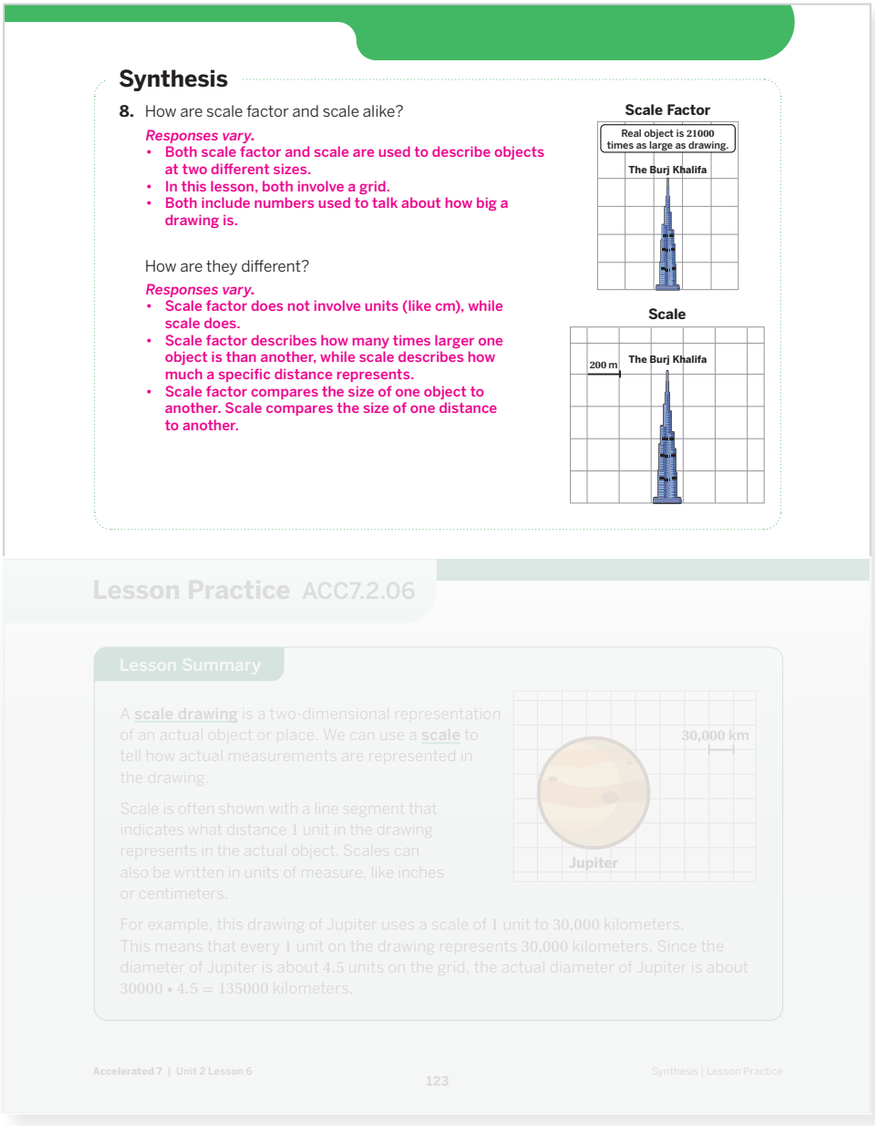
Invite students to respond independently, and then share their thinking with a partner.

Collect and share a variety of ideas, such as:

- Both scale factor and scale are used to describe objects at two different sizes.
- Both include values used to talk about how big or small a drawing is in comparison to the real object.
- Scale factor does not involve units, but scale does.
- Scale factor describes how many times larger one object is than another, while scale describes how much a specific distance represents.

MLR **MLR2: Collect and Display** Use this routine to collect students' ideas about similarities and differences between *scale factor* and *scale* as they share. Display for students to reference and add to for the rest of the unit.

Note: If time allows, invite students to revise their previous thinking based on the discussion.



Synthesis

8. How are scale factor and scale alike?

Responses vary.

- Both scale factor and scale are used to describe objects at two different sizes.
- In this lesson, both involve a grid.
- Both include numbers used to talk about how big a drawing is.

How are they different?

Responses vary.

- Scale factor does not involve units (like cm), while scale does.
- Scale factor describes how many times larger one object is than another, while scale describes how much a specific distance represents.
- Scale factor compares the size of one object to another. Scale compares the size of one distance to another.

Lesson Practice ACC7.2.06

Lesson Summary

A scale drawing is a two-dimensional representation of an actual object or place. We can use a scale to tell how actual measurements are represented in the drawing.

Scale is often shown with a line segment that indicates what distance 1 unit in the drawing represents in the actual object. Scales can also be written in units of measure, like inches or centimeters.

For example, this drawing of Jupiter uses a scale of 1 unit to 30,000 kilometers. This means that every 1 unit on the drawing represents 30,000 kilometers. Since the diameter of Jupiter is about 4.5 units on the grid, the actual diameter of Jupiter is about $30000 \cdot 4.5 = 135000$ kilometers.

Accelerated 7 | Unit 2 Lesson 6 123 Synthesis | Lesson Practice

Lesson Takeaway: A scale describes how much a specific distance in a drawing represents on the real object. Scales can be written in units of measure, e.g., 1 unit on the grid represents 2 miles.

Independent | 5 min

Show What You Know

Purpose: Students demonstrate their understanding of scale by estimating the actual width and height of National Stadium.

Today's Goals

1. **Goal:** Recognize that a *scale* is a ratio of distance in a drawing to the corresponding distance in an object in the real world.
2. **Goal:** Interpret the *scale* of a drawing.
3. **Language Goal:** Explain how a *scale* helps to represent actual distances. **(Reading, Writing, Speaking, and Listening)**

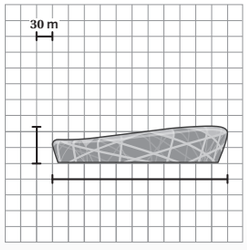
Show What You Know (Assessment Resources)

Name: _____ Date: _____ Period: _____

Show What You Know 2.06

Here is a scale drawing of the National Stadium, also known as the Bird's Nest, located in Beijing, China.

Estimate the actual width and height of the National Stadium and write your estimates in the table.



Estimated Width (m)	Estimated Height (m)
Responses between 330 and 340 meters are considered correct.	Responses between 60 and 80 meters are considered correct.

Differentiation Use after Lesson 6

Support

Provide targeted intervention for students by using these resources.

If students need support understanding how the scale of a drawing corresponds to real world objects:

» **Respond to Student Thinking:**

- Students will also have more opportunities to develop this concept in future lessons, so intervention is not necessary at this time.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 2 Extension Activities**.
- Invite students to investigate the **You're Invited to Explore More** problem.

Professional Learning

Take a moment to reflect on how students' understanding of the words *scale*, *scale factor*, *scale drawing*, and *scaled copy* have developed in this unit. How are students using these words to talk about scale drawings?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

8. How are scale factor and scale alike?

Responses vary.

- Both scale factor and scale are used to describe objects at two different sizes.
- In this lesson, both involve a grid.
- Both include numbers used to talk about how big a drawing is.

How are they different?

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Lesson Practice ACC7.2.06

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A **scale drawing** is a two-dimensional representation of an actual object or place. We can use a **scale** to tell how actual measurements are represented in the drawing.

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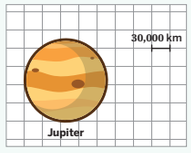


For example, this drawing of Jupiter uses a scale of 1 unit to 30,000 kilometers. This means that every 1 unit on the drawing represents 30,000 kilometers. Since the diameter of Jupiter is about 4.5 units on the grid, the actual diameter of Jupiter is about $30000 \cdot 4.5 = 135000$ kilometers.

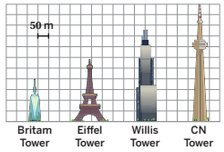
Lesson Practice ACC7.2.06

Name: _____ Date: _____ Period: _____

1. Here is a scale drawing of some towers in different cities. Use the scale to estimate how tall each tower is.

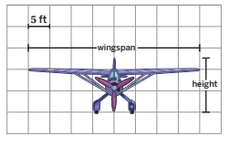
Responses vary.

Tower	Estimated Height (m)
Britam Tower	200
Eiffel Tower	300
Willis Tower	440
CN Tower	550



Problems 2–3: The Westland Lysander was an aircraft used by the Royal Air Force in the 1930s. Here is a scale drawing that shows the Lysander.

Approximate the actual lengths.



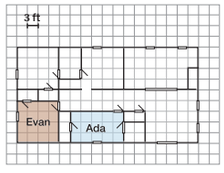
2. The wingspan of the plane:

About 40 feet

3. The height of the plane:

About 12 feet

Ada and Evan are siblings. Here is a scale drawing of their house that shows their rooms.



4. Complete the table with the dimensions of their rooms.

	Length (ft)	Width (ft)
Ada's Room	15	9
Evan's Room	12	12

Lesson Practice ACC7.2.06

Name: _____ Date: _____ Period: _____

FAST Practice

5. Here is a map of a town, with a scale at the bottom. Estimate how far Aki's house is from the pizza place.

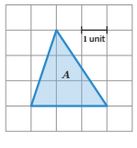
About 400 feet

Spiral Review

6. Complete the table so that the two values in each row are related by the same ratio.

3	12
1.5	6
4	16
0.25	1

Problems 7–8: Tyrone created a scaled copy of triangle A with an area of 72 square units.



7. How many times greater is the area of the scaled copy compared to the area of triangle A?

16 times greater

8. What scale factor did Tyrone apply to triangle A to create the scaled copy?

4

Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1–4	2	MA.7.GR.1.5
FAST Practice	5	2	MA.7.GR.1.5
Spiral Review			
	6	1	MA.6.AR.3.3
	7–8	2	MA.7.GR.1.5

Assess and Respond

Sub-Unit Quiz 1

Independent | 45 min

Facilitation: Assign the Sub-Unit Quiz to learn about your students' understanding of the concepts and skills so far in this unit. If time allows, consider inviting students to complete one or more of the tasks in the Unit Synthesis and Reflection.

Item Analysis

Problem(s)	Concept or skill	Addressed in	DOK	FL B.E.S.T. Math Benchmarks
1	Identifying scaled copies that have a scale factor of less than 1	Lesson 1	1	MA.7.GR.1.5 MTR.5.1
2	Comparing lengths and areas in original figures and scaled copies	Lesson 2	1	MA.7.GR.1.5
3a	Calculating missing lengths in scaled copies	Lesson 2	1	MA.7.GR.1.5
3b	Determining scale factors between original figures and scaled copies	Lesson 3	1	MA.7.GR.1.5
3c	Determining and justifying scale factors between original figures and scaled copies	Lesson 4	2	MA.7.GR.1.5
4a	Determining and justifying if the side lengths in two figures are proportional	Lesson 2	2	MA.7.GR.1.5 MTR.3.1
4b	Drawing scaled copies	Lesson 3	2	MA.7.GR.1.5
5a	Connecting scaled copies and area	Lesson 5	3	MA.7.GR.1.5
5b	Determining and justifying scale factors	Lesson 5	2	MA.7.GR.1.5 MTR.4.1

Assessment and Lesson Resources

- Student Assessments
- Assessment Rubrics with Answers
- Show What You Knows and Answer Keys

Intervention and Extension Resources

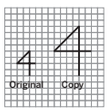
- Mini-Lessons
- Extensions

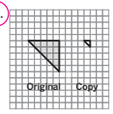
Name: ..... Date: ..... Period: .....

Sub-Unit Quiz 1

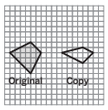
Unit 2

1. Here are pairs of figures, each with an original and a copy.
Which pair shows a *scaled* copy that has a scale factor of less than 1?

A. 

B. 

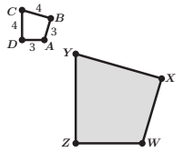
C.

D. 

2. Figure $WXYZ$ is a scaled copy of figure $ABCD$ with a scale factor of 3.

Select *all* the true statements.

- ☑ **A.** The scale factor from $WXYZ$ to $ABCD$ is $\frac{1}{3}$.
- ☐ **B.** Segment YZ is 7 units long.
- ☐ **C.** If the area of $ABCD$ is 12 square units, then the area of $WXYZ$ is 36 square units.
- ☑ **D.** The distance between W and Y is 3 times the distance between A and C.
- ☑ **E.** The ratio of $\frac{BC}{BA}$ is equivalent to the ratio of $\frac{XY}{XW}$.



All lengths are in grid units.

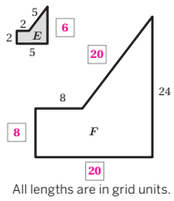
3. Figure F is a scaled copy of figure E.

a. Label each missing length.

b. What scale factor takes figure E to figure F?
4

c. What scale factor takes figure F back to figure E?
$\frac{1}{4}$

Explain how you know.
Explanations vary.
- The scale factor is $\frac{1}{4}$ because if I multiply each side of figure F by $\frac{1}{4}$, I get the side lengths of figure E.
- The ratio of any side length in figure E to the same side length in figure F is $\frac{1}{4}$. This is the scale factor I use to scale figure F back to figure E.



All lengths are in grid units.

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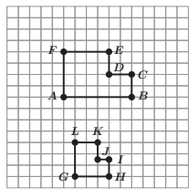
Name: ..... Date: ..... Period: .....

Sub-Unit Quiz 1 (continued)

Unit 2

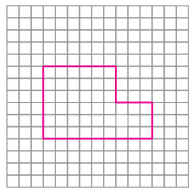
4. Here are figures $ABCDEF$ and $GHIJKL$.



a. Are the side lengths in $ABCDEF$ proportional to the side lengths in $GHIJKL$?
No

Explain how you know.
Explanations vary.
- Even though the width of $GHIJKL$ is half the width of $ABCDEF$, its height is not half. So the side lengths of the figures are not proportional.
- The side lengths of the figures are not proportional because they don't have equivalent ratios. For example, the widths form a ratio of $\frac{6}{3} = 2$, but the heights form a ratio of $\frac{4}{3}$, which is not equal to 2.

b. Draw a scaled copy of figure $ABCDEF$ using a scale factor of $\frac{3}{2}$.

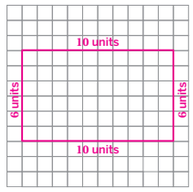


5. Rectangle S is 3 units by 5 units.

a. Draw a scaled copy of rectangle S with an area of 60 square units. Label each side length of the copy.

b. What is the scale factor between rectangle S and your copy?
2

Explain how you know.
Explanations vary. Student response depends on their response in Problem 5a.
If their response in Problem 5a is correct, here is a sample response: All of the lengths of the scaled copy are twice the lengths of the original figure. For example, the height of the original figure is 3 units, while the height of the scaled copy is $3 \cdot 2 = 6$ units.



D Differentiation (Sub-Unit Quiz 1)

Note: To **strengthen** and **stretch** students' learning, refer to the differentiation resources suggested throughout this unit and in the Unit Overview.

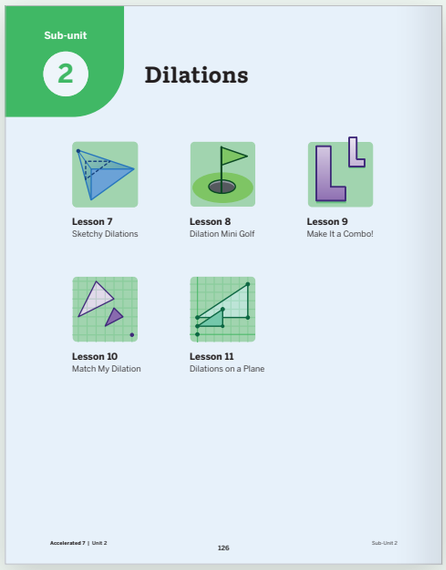
Sub-Unit Goals	Problem(s)	To respond to student thinking, consider:	
Sub-Unit 1: • Describe how scaling affects lengths, angles, and areas in scaled copies. • Use scale factors to create and compare scaled copies. **(Lessons 1–6)**	1, 4b	**Support**	• **Teacher Move:** Consider revisiting Unit 1, Lesson 1 (Scaling Machines) and Unit 1, Lesson 4 (Scale Factor Challenges)
		Strengthen	• **Practicing With Scale Factors:** Lesson 4 (Scale Factor Challenges)
		Stretch	• **You're invited to explore more:** Lesson 3 (Make It Scale)
	3	**Support**	• **Mini-Lesson:** Connecting Scale Factors to Scaled Copies (ML 1.04)
		Strengthen	• **Teacher Move:** Consider asking, "Which side measurements should be used to determine a scale factor between two figures?"
		Stretch	• **You're invited to explore more:** Lesson 4 (Scale Factor Challenges)
	2, 4a, 5	**Support**	• **Teacher Move:** Consider revisiting Unit 1, Lesson 3 (Make It Scale) and Unit 1, Lesson 5 (Tiles)
		Strengthen	• **Teacher Move:** Consider asking, "How does scaling impact the area of a scaled copy? How does it impact the side lengths?"
		Stretch	• **You're invited to explore more:** Lesson 5 (Back in My Day)

Dilations

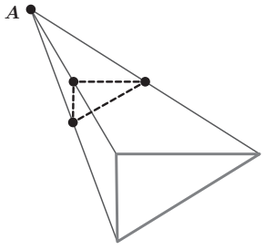
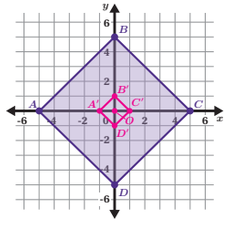
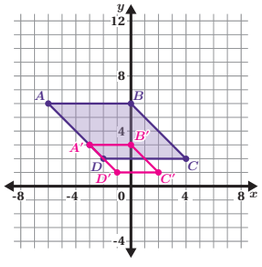
Sub-Unit 2 Goals

- **Goal:** Apply dilations to figures on and off a coordinate grid.
- **Language Goal:** Describe dilations precisely using terms such as *pre-image, center of dilation*, and *scale factor*.

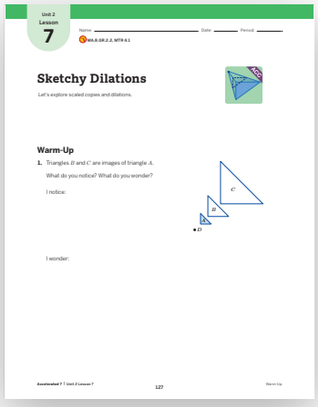
In this sub-unit, students visualize and perform **dilations** using **scale factors** and **centers of dilation** on and off a grid. They determine how the size of a scale factor affects the size of an image. Students develop and describe strategies for performing dilations.



Math That Matters Most

Concept or Skill	For example . . .
Describe the effects that dilations have on a figure.	Describe how dilations work. Use this example if it helps with your thinking. *Dilations create scaled copies. They require a point, and then lines extend from that point through the vertices of the figure being dilated. The dilated figure could be anywhere along those lines.* 
Identify the center of dilation and scale factor in a dilation.	Description of transformation: *Dilate figure ABCD by using (0, 0) as the center of dilation and a scale factor of $\frac{1}{5}$.* 
Dilate a figure using its coordinates, a center of dilation, and a scale factor.	**a** Dilate parallelogram $ABCD$ using $(0, 0)$ as the center of dilation and a scale factor of $\frac{1}{2}$. **b** Write the image coordinates in the table. 

Pre-Image Coordinates	Image Coordinates
(-6, 6)	**(-3, 3)**
(0, 6)	**(0, 3)**
(4, 2)	**(2, 1)**
(-2, 2)	**(-1, 1)**


Unit 2
Lesson
7

Sketchy Dilations

Let's explore scaled copies and dilations.

Warm-Up

1. Triangles B and C are images of triangle A.
What do you notice? What do you wonder?

I notice:

I wonder:

127

Sketchy Dilations

Introducing Dilations

Let's explore scaled copies and dilations.

Learning Outcomes

Today's Goals

1. **Goal:** Make connections between dilations, scaled copies, and stretching machines.
2. **Language Goal:** Describe the effects that dilations have on a figure. **(Reading, Writing, Speaking, and Listening)**

Students are introduced to the concept of **dilation** in the context of stretching machines. In particular, students explore dilation as a process that produces scaled copies of an original figure using lines drawn from a center point.

Coherence

Prior Learning

In Unit 1, students learned about three types of rigid transformations: translations, reflections, and rotations. In Lessons 1–6, students learned about scaled copies of figures and scale drawings.

Future Learning

In Lesson 8, students will use the terms *center of dilation* and *scale factor* to describe distances of points in a dilation.

Connections

- Students use a stretching machine analogy to connect their understanding of a scaled copy of a figure to dilations. (mathematical)
- Students relate dilations to a scaled copy of a figure, which is useful in model designs, maps, and blueprints. (mathematical, real world)

FL B.E.S.T. Math Benchmarks

Addressing

MA.8.GR.2.2
Given a preimage and image generated by a single dilation, identify the scale factor that describes the relationship.

Mathematical Thinking and Reasoning Standard: MTR.4.1

ELA Expectation: ELA.K12.EE.3.1

English Language Development Standards: ELL.MA.1, ELL.SI.1

Building On	Building Toward
MA.7.GR.1.5	MA.912.F.2 MA.912.GR.2

Vocabulary

New Vocabulary

dilation

Review Vocabulary

image *pre-image*

Lesson at a Glance

~ 45 min

MA.8.GR.2.2

Warm-Up

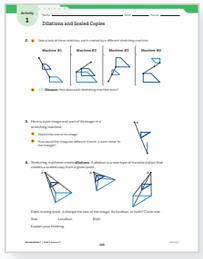
Pairs | 5 min

Students use the **Notice and Wonder** routine to revisit the concept of scaled copies of a triangle.

Activity 1

Pairs | 15 min

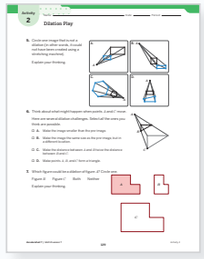
Students are introduced to **dilations** as transformations that create scaled copies by exploring stretching machines.



Activity 2

Pairs | 15 min

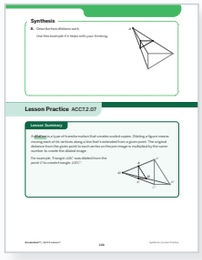
Students continue to make sense of dilations using stretching machines and consider whether or not a figure could be the result of a dilation. **(MTR.4.1)**



Synthesis

Whole Class | 5 min

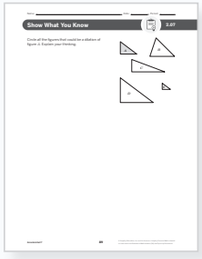
Students synthesize their understanding of how dilations work. **MLR2: Collect and Display**



Show What You Know

Independent | 5 min

Students demonstrate their understanding by deciding whether or not a figure is the *image* of a triangle after a dilation.



Prep Checklist

This lesson includes:

- Student Edition
- Show What You Know
- Straightedges (optional)

Pairs | 5 min

Warm-Up

Purpose: Students use the Notice and Wonder routine to revisit the concept of scaled copies of a triangle.

1 Launch

To surface connections, consider asking:

- "In math class, what does the word *image* make you think of?"
- "If triangles B and C are images, what would we call triangle A?"

Encourage students to sketch on the image if it helps with their thinking. Look for students who draw lines connecting corresponding points on the triangles or other interesting sketches.

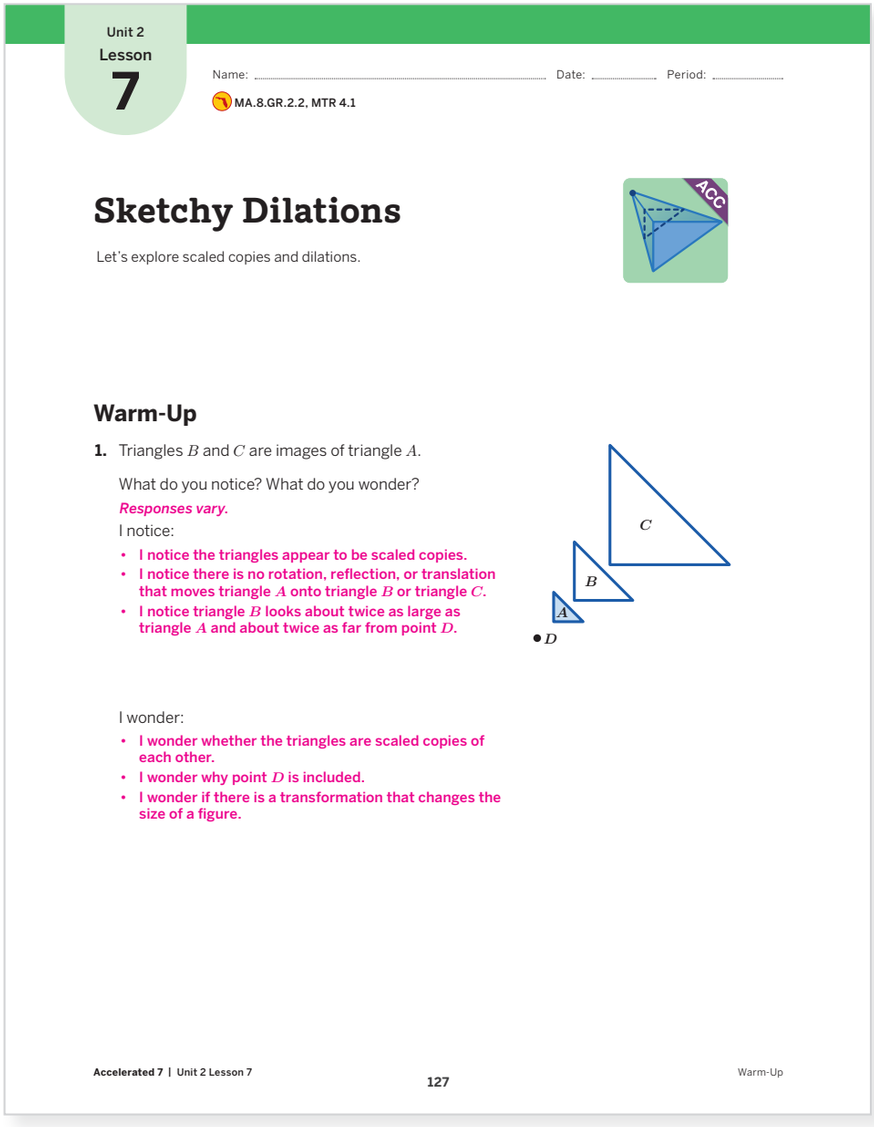
ELL **English Language Learners** Provide a word bank to support students in expressing their ideas with words such as: *pre-image*, *image*, *scaled copy*, *triangle*, *point*, etc. **(Writing and Speaking)**

2 Connect

Use the Notice and Wonder routine to promote curiosity and help students notice properties of a dilation, like that it produces scaled copies that are along lines from a point.

Note: If it doesn't come up naturally, consider asking:

- "Are the images scaled copies of the pre-image? Why or why not?"
- "How do you think the point in the bottom left corner is related to the pre-image and images?"



Unit 2
Lesson
7

Name: ..... Date: ..... Period: .....

MA.8.GR.2.2, MTR 4.1

Sketchy Dilations

Let's explore scaled copies and dilations.

Warm-Up

1. Triangles B and C are images of triangle A.

What do you notice? What do you wonder?

Responses vary.

I notice:

- I notice the triangles appear to be scaled copies.
- I notice there is no rotation, reflection, or translation that moves triangle A onto triangle B or triangle C.
- I notice triangle B looks about twice as large as triangle A and about twice as far from point D.

I wonder:

- I wonder whether the triangles are scaled copies of each other.
- I wonder why point D is included.
- I wonder if there is a transformation that changes the size of a figure.

Accelerated 7 | Unit 2 Lesson 7 127 Warm-Up

Pairs | 15 min

Activity 1 Dilations and Scaled Copies

Purpose: Students are introduced to **dilations** as transformations that create scaled copies by exploring stretching machines.

Short on time? Consider discussing the machines in the first image as a class, then continuing to the rest of the activity.

1 Launch

To encourage creativity, invite students to imagine a picture on a computer. Consider asking, "What happens if you drag one side? What happens if you drag the corner?"

Invite students to imagine how each machine would work with a different sketch.

Pause to discuss how the stretching machines work. Consider asking, "How are all the stretching machines alike?"

2 Monitor

Invite students to explore Problems 3–4.

A **Accessibility: Fine Motor Skills** To support the use of manipulatives, invite students to dictate what they would like to sketch to you or a partner, then describe what they notice.

Look and listen for ideas to surface during the Connect.

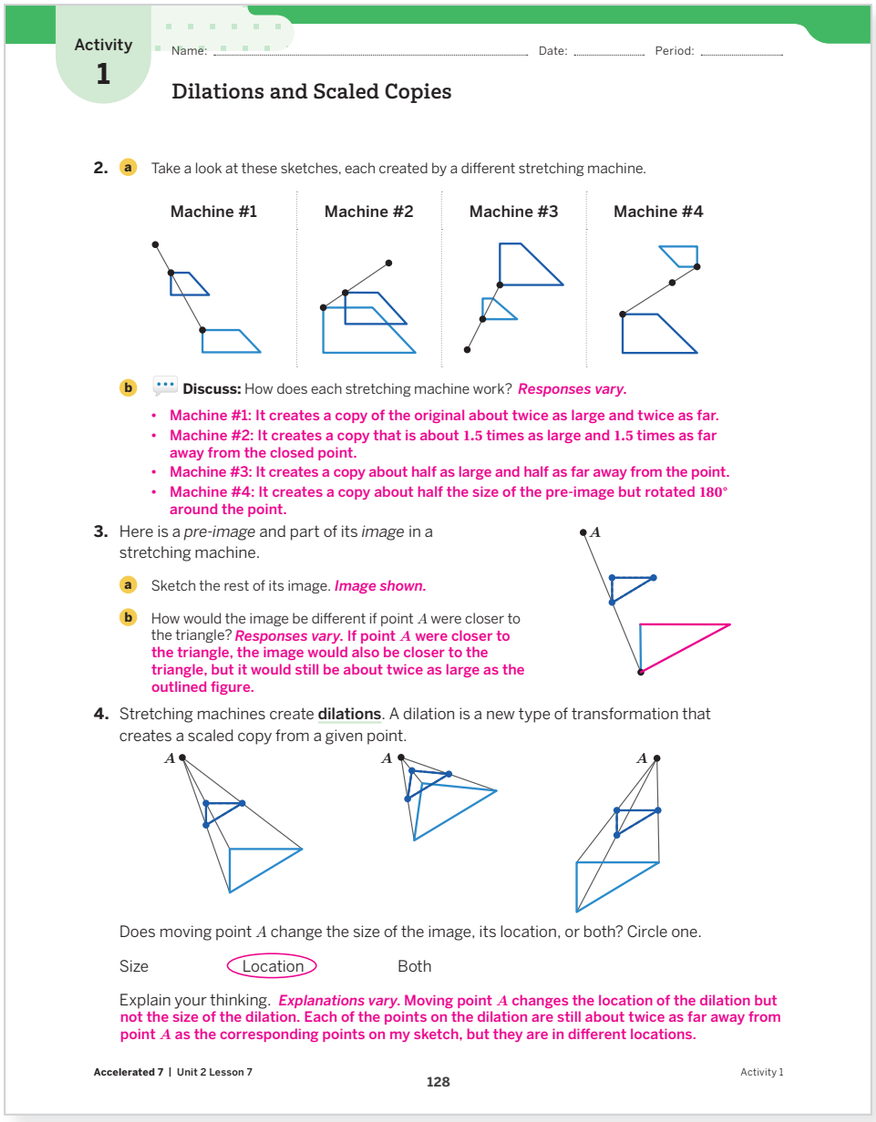
D **Differentiation** *(Problems 2–4)*

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Encourage them to focus on just size, then on just location.
Say that moving point A changes size and location.	**Support** Invite them to use their fingers or a straightedge to measure the size of the image before and after moving the point.
Explain that moving the point changes the location but not size.	**Stretch** Consider asking, "What would change the size of the image?"

3 Connect

To surface the Key Takeaway, consider asking:

- "How would you explain what a **dilation** is in your own words?"
- "In a stretching machine, how are point A, the pre-image, and the image related?"



Activity 1

Name: _____ Date: _____ Period: _____

Dilations and Scaled Copies

2. a Take a look at these sketches, each created by a different stretching machine.

Machine #1 | Machine #2 | Machine #3 | Machine #4

b **Discuss:** How does each stretching machine work? *Responses vary.*

- Machine #1: It creates a copy of the original about twice as large and twice as far.
- Machine #2: It creates a copy that is about 1.5 times as large and 1.5 times as far away from the closed point.
- Machine #3: It creates a copy about half as large and half as far away from the point.
- Machine #4: It creates a copy about half the size of the pre-image but rotated 180° around the point.

3. Here is a *pre-image* and part of its *image* in a stretching machine.

a Sketch the rest of its image. *Image shown.*

b How would the image be different if point A were closer to the triangle? *Responses vary.* If point A were closer to the triangle, the image would also be closer to the triangle, but it would still be about twice as large as the outlined figure.

4. Stretching machines create **dilations**. A dilation is a new type of transformation that creates a scaled copy from a given point.

Does moving point A change the size of the image, its location, or both? Circle one.

Size (Location) Both

Explain your thinking. *Explanations vary.* Moving point A changes the location of the dilation but not the size of the dilation. Each of the points on the dilation are still about twice as far away from point A as the corresponding points on my sketch, but they are in different locations.

Accelerated 7 | Unit 2 Lesson 7 128 Activity 1

Key Takeaway: **Dilations** are a type of transformation that create scaled copies of a *pre-image* using lines coming from a central point. Moving the central point changes the location of the pre-image but not its size.

Pairs | 15 min

Activity 2 Dilation Play

Purpose: Students continue to make sense of dilations using stretching machines and consider whether or not a figure could be the result of a dilation. **(MTR.4.1)**

1 Launch

Encourage students to discuss what they notice about each possible dilation with a partner before choosing one they think is not a dilation.

2 Monitor

Look and listen for students who discuss that a dilation must create a scaled copy or that in a dilation, there should be straight lines from the point to the pre-image to the image. Both strategies may surface again during the Connect.

Pause briefly to discuss students' ideas about how moving the points may affect the image.

Select and sequence student explanations to share during the Connect.

D Differentiation *(Problem 7)*

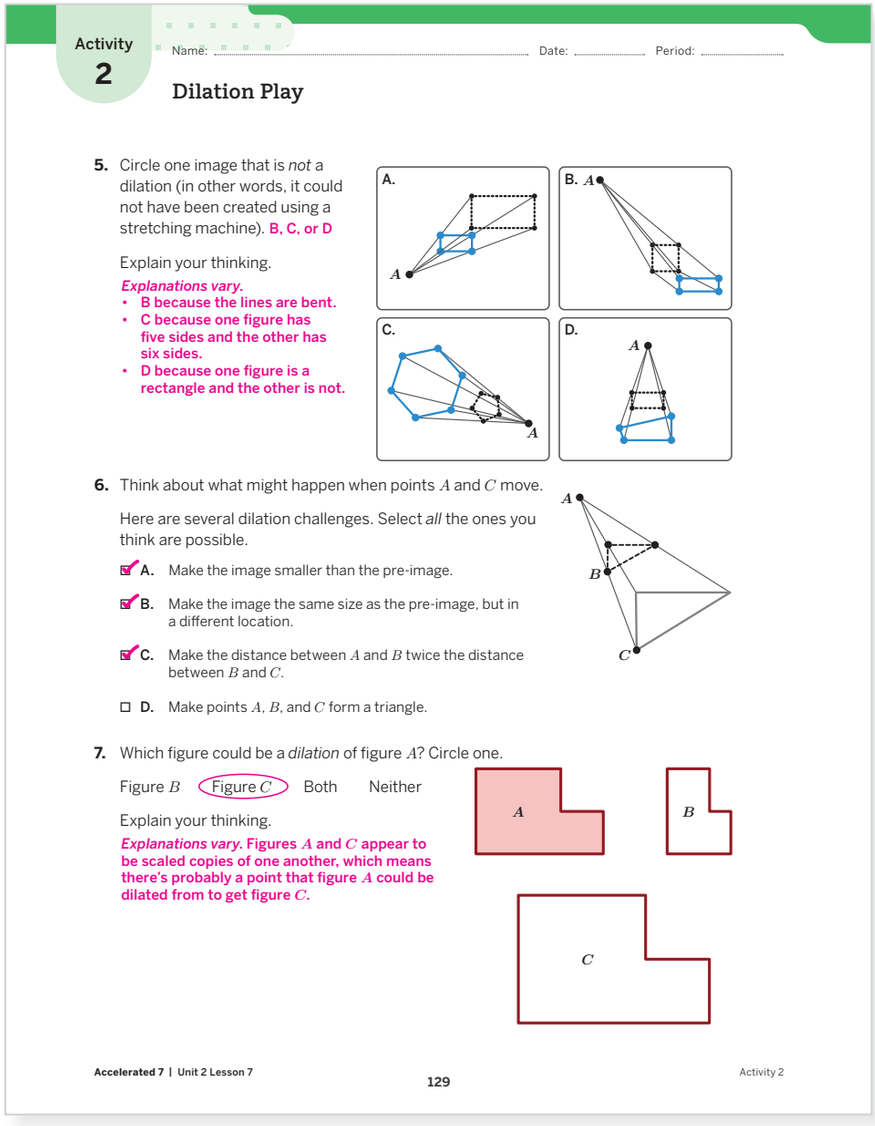
Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Encourage them to revisit previous problems and ask, "How can you tell if a figure is a *dilation*?"
Select "Neither" because there is no center point or lines between the figures.	**Support** Encourage them to try to draw imaginary lines between figure A and each figure like in previous problems.
Select figure C and mention that these are scaled copies.	**Strengthen** Consider asking, "How do you know figure C is a scaled copy of figure A?"

3 Connect

Invite students to share their thinking and justify their reasoning. **(MTR.4.1)**

Consider asking, "How can you decide if a figure is a dilation or not?"

Note: If time allows, return to the previous problem and discuss how points A, B, and C always form a line, so they cannot form a triangle. This will be helpful in future lessons in this unit.



Activity 2

Name: Date: Period:

Dilation Play

5. Circle one image that is *not* a dilation (in other words, it could not have been created using a stretching machine). **B, C, or D**

A. B. C. D.

Explain your thinking.

Explanations vary.
- **B because the lines are bent.**
- **C because one figure has five sides and the other has six sides.**
- **D because one figure is a rectangle and the other is not.**

6. Think about what might happen when points A and C move.

Here are several dilation challenges. Select *all* the ones you think are possible.

- ☑ **A.** Make the image smaller than the pre-image.
- ☑ **B.** Make the image the same size as the pre-image, but in a different location.
- ☑ **C.** Make the distance between A and B twice the distance between B and C.
- ☐ **D.** Make points A, B, and C form a triangle.

7. Which figure could be a *dilation* of figure A? Circle one.

Figure B (Figure C) Both Neither

Explain your thinking.

Explanations vary. **Figures A and C appear to be scaled copies of one another, which means there's probably a point that figure A could be dilated from to get figure C.**

Accelerated 7 | Unit 2 Lesson 7 129 Activity 2

Whole Class | 5 min

Synthesis

Purpose: Students synthesize their understanding of how dilations work.

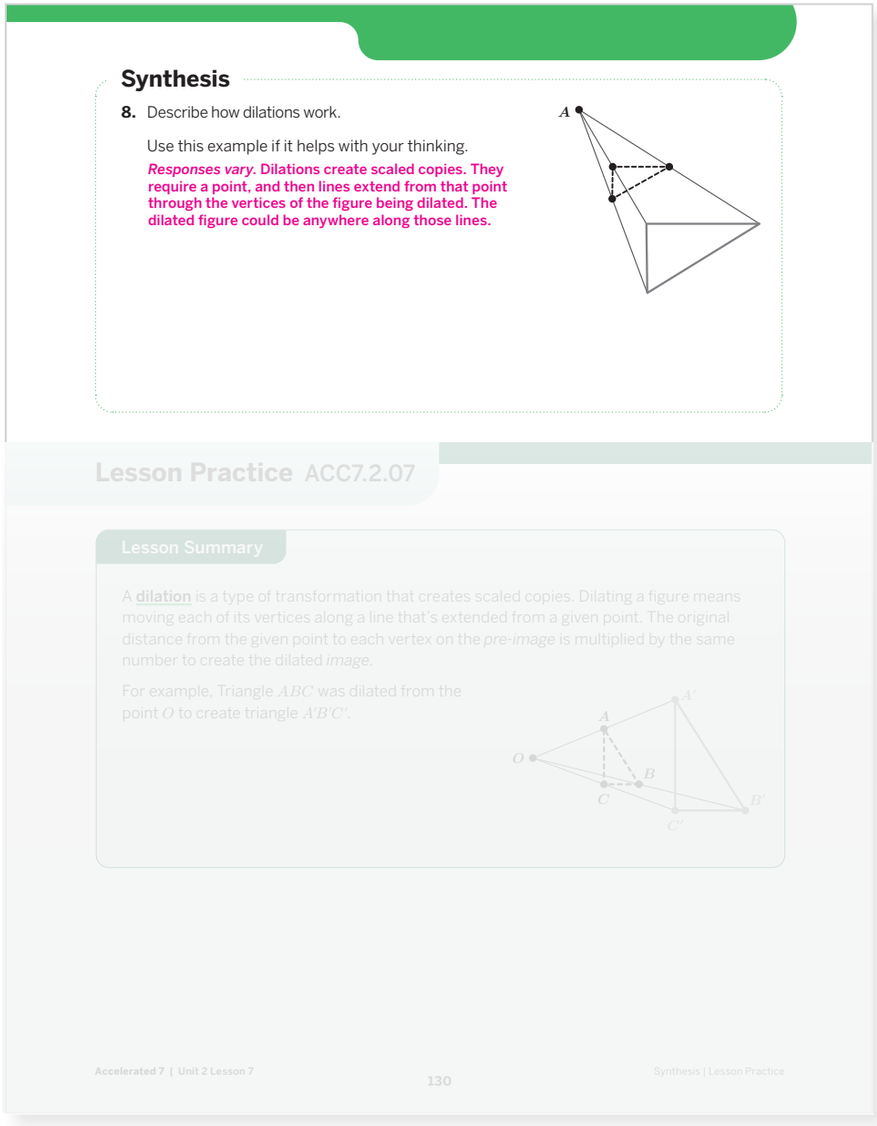
Invite students to respond independently, and then share their thinking with a partner.

Collect and share a variety of ideas, such as:

- Dilations make scaled copies of a pre-image.
- Dilations use lines that come from a point, through the points on a pre-image, and then extend to make the image.
- The image can be any distance away along the line from the center point.

MLR **MLR2: Collect and Display** Use this routine to collect students' language, examples, and diagrams of dilations. Create a class definition of *dilations* using students' language and the glossary and display for students to reference and add to for the rest of the unit.

Note: If time allows, invite students to revise their previous thinking based on the discussion, including language like *image*, *pre-image*, *point*, *lines*, etc.



Synthesis

8. Describe how dilations work.

Use this example if it helps with your thinking.

Responses vary. **Dilations create scaled copies. They require a point, and then lines extend from that point through the vertices of the figure being dilated. The dilated figure could be anywhere along those lines.**

Lesson Practice ACC7.2.07

Lesson Summary

A dilation is a type of transformation that creates scaled copies. Dilating a figure means moving each of its vertices along a line that's extended from a given point. The original distance from the given point to each vertex on the *pre-image* is multiplied by the same number to create the dilated *image*.

For example, Triangle ABC was dilated from the point O to create triangle $A'B'C'$.

Accelerated 7 | Unit 2 Lesson 7 130 Synthesis | Lesson Practice

Lesson Takeaway: **Dilations** are a type of transformation that create scaled copies using lines from a point through the *pre-image*. Adjusting the location of the point changes the location of the *image*, and adjusting the distance between the pre-image and the image changes its size.

Independent | 5 min

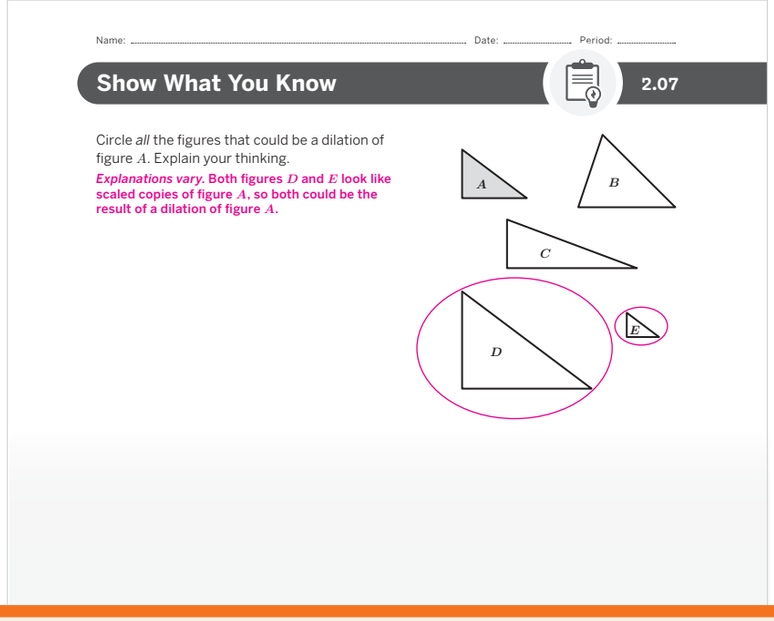
Show What You Know

Purpose: Students demonstrate their understanding by deciding whether or not a figure is the *image* of a triangle after a dilation.

Today's Goals

1. **Goal:** Make connections between dilations, scaled copies, and stretching machines.
2. **Language Goal:** Describe the effects that dilations have on a figure. **(Reading, Writing, Speaking, and Listening)**

Show What You Know (Assessment Resources)



Name: _____ Date: _____ Period: _____

Show What You Know 2.07

Circle *all* the figures that could be a dilation of figure A. Explain your thinking.

Explanations vary. Both figures D and E look like scaled copies of figure A, so both could be the result of a dilation of figure A.

D Differentiation Use after Lesson 7

Support

Provide targeted intervention for students by using these resources.

If students need support with identifying images after a dilation:

» **Respond to Student Thinking:**

- Provide opportunities for students to practice visualizing the reduction and enlargement based on the scale factors.

Strengthen

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 2 Extension Activities**.

Professional Learning

Reflect on your facilitation of the Notice and Wonder routine in the Warm-Up. How did students engage in the routine? How were you able to honor a variety of student ideas? What moves might support more students to share their thinking in the future?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

8. Describe how dilations work.

Use this example if it helps with your thinking.

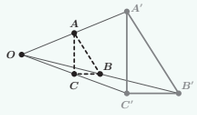
Responses vary. Dilations create scaled copies. They require a point, and then lines extend from that point through the vertices of the figure being dilated. The dilated figure could be anywhere along those lines.

Lesson Practice ACC7.2.07

Lesson Summary

A **dilation** is a type of transformation that creates scaled copies. Dilating a figure means moving each of its vertices along a line that's extended from a given point. The original distance from the given point to each vertex on the *pre-image* is multiplied by the same number to create the dilated *image*.

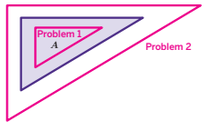
For example, Triangle ABC was dilated from the point O to create triangle $A'B'C'$.



Lesson Practice ACC7.2.07

Name: _____ Date: _____ Period: _____

Problems 1–2: Here is triangle A.



1. Draw a dilation of triangle A where the image has a smaller area. *Responses vary.*

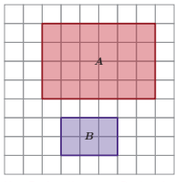
2. Draw a dilation of triangle A where the image has a larger area. *Responses vary.*

3. Rectangle A is 12 centimeters by 3 centimeters. Rectangle B is a scaled copy of rectangle A. Select *all* the pairs that could be the dimensions of rectangle B.

- ☑ **A.** 6 centimeters by 1.5 centimeters
- ☐ **B.** 10 centimeters by 1 centimeter
- ☑ **C.** 18 centimeters by 4.5 centimeters
- ☐ **D.** 6 centimeters by 1 centimeter
- ☑ **E.** 80 centimeters by 20 centimeters

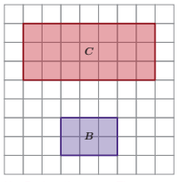
Problems 4–5: Determine whether each pair of figures represents an original figure and its scaled copy. Explain your thinking.

4.



Yes. *Explanations vary.* Rectangle A is double the height and width of rectangle B.

5.



No. *Explanations vary.* The corresponding side lengths of the rectangles are not proportional.

Lesson Practice ACC7.2.07

Name: _____ Date: _____ Period: _____

FAST Practice

6. Select *all* the figures that could be a scaled copy of triangle F.

- ☐ **A.**
- ☑ **B.**
- ☐ **C.**
- ☐ **D.**
- ☑ **E.**

Spiral Review

7. Which of these sets of angle measures could be the three interior angle measures of a triangle?

A. 40°, 50°, 60° **B.** 50°, 60°, 70°

C. 60°, 70°, 80° **D.** 70°, 80°, 90°

8. What makes two figures congruent? Explain your thinking.

Two figures are congruent if one can move onto the other using translations, rotations, or reflections. Congruent figures are the same size and shape.

Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1–2	1	MA.8.GR.2.2
	3	2	MA.8.GR.2.2
	4–5	3	MA.8.GR.2.2
FAST Practice	6	2	MA.8.GR.2.2
Spiral Review			
	7	1	MA.8.GR.1.5
	8	2	MA.8.GR.2.1

Unit 2
Lesson
8
Dilation Mini Golf
Let's dilate points using measurement tools.
Warm-Up
1. Welcome to Dilation Mini Golf.
Point A is the starting point of the golf ball. After a dilation, the ball is in the hole.
Discuss:
In this situation, what is the pre-image? What is the image?
What do you think scale factor: 4 means?
Scale Factor: 4
Scale Factor: 4
133

Dilation Mini Golf

Dilations With No Grid

Let's dilate points using measurement tools.

Learning Outcomes

Today's Goals

1. **Goal:** Identify the center and scale factor in a dilation.
2. **Goal:** Dilate a point or figure using distance measurements.
3. **Language Goal:** Describe strategies for dilating a point. **(Speaking and Listening)**

Students use the context of mini golf to make sense of how the components of a dilation are related, namely the pre-image, image, center of dilation, and scale factor. They attend to precision as they use distances to dilate points instead of the structure of a grid. Students will first see dilations of points and then build to line segments and eventually figures. **(MTR.7.1)**

Coherence

Prior Learning

In Lesson 7, students were introduced to dilation as a method for producing scaled copies of figures.

Future Learning

In Lessons 9–11, students will explore dilations of figures on a square grid and using coordinates.

Connections

- Within the context of mini-golf, student explore how distances can be applied to dilate points. (real world)

FL B.E.S.T. Math Benchmarks

Addressing

MA.8.GR.2.2
Given a preimage and image generated by a single dilation, identify the scale factor that describes the relationship.

Mathematical Thinking and Reasoning Standards: MTR.4.1, MTR.7.1

ELA Expectations: ELA.K12.EE.4.1, ELA.K12.EE.5.1

English Language Development Standards: ELL.MA.1, ELL.SI.1

Building On	Building Toward
MA.7.GR.1.5	MA.912.F.2 MA.912.GR.2

Vocabulary

New Vocabulary

center of dilation

Lesson at a Glance

~ 45 min

MA.8.GR.2.2

Warm-Up

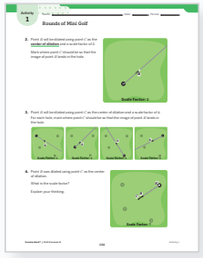
Pairs | 5 min

Students develop an intuition for dilating points in the context of playing mini golf.
Routine: Think-Pair-Share

Activity 1

Pairs | 10 min

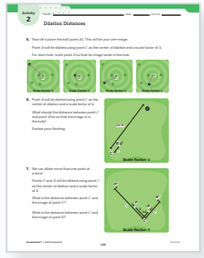
Students explore how the **center of dilation** and the scale factor impact the relationship between the pre-image and the image, creating a need for using measurements. **(MTR.4.1)**



Activity 2

Pairs | 10 min

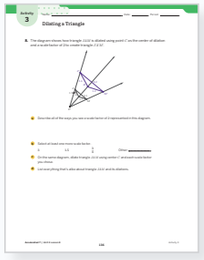
Students use distance measurements to dilate points and lines given a center of dilation and a scale factor.



Activity 3

Independent | 10 min

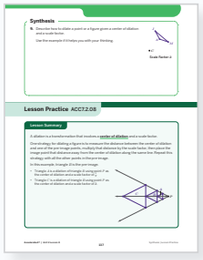
Students dilate figures not on a grid using paper and a straightedge or ruler. **(MTR.7.1)**



Synthesis

Whole Class | 5 min

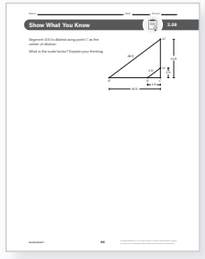
Students synthesize their strategies for dilating a point using distances, a center of dilation, and a scale factor. **MLR2: Collect and Display**



Show What You Know

Independent | 5 min

Students demonstrate their understanding by identifying the scale factor used in a dilation.



Prep Checklist

This lesson includes:

- Student Edition
- Show What You Know
- Rulers or straightedges

Pairs | 5 min

Warm-Up

Purpose: Students develop an intuition for dilating points in the context of playing mini golf.

1 Launch

To surface connections, consider asking:

- "What are your favorite games to play with your friends and family?"
- "Have you ever played or seen someone play a game like this?"

Use the Think-Pair-Share routine to help students shape their ideas about how the game is connected to a dilation. See the Routine Facilitation Guide for more information.

2 Connect

Invite students to share what they think the pre-image and image are.

Consider asking, "How are this game and the stretching machines from the last lesson alike?"

Note: The class will create a shared definition of *scale factor* in the Connect of Activity 1.

Unit 2
Lesson
8

Name: ..... Date: ..... Period: .....

MA.8.GR.2.2, MTR.4.1, MTR.7.1

Dilation Mini Golf

Let's dilate points using measurement tools.

Warm-Up

1. Welcome to Dilation Mini Golf.

Point B is the starting point of the golf ball. After a dilation, the ball is in the hole.

Discuss:

- In this situation, what is the pre-image? What is the image?
- What do you think *scale factor:* 4 means?

Responses vary.

- Point B is the pre-image and the ball in the hole is the image.
- *Scale factor:* 4 represents the number of segments between the center and the hole.
- *Scale factor:* 4 is the number of times the distance between point B and point C is multiplied.

Accelerated 7 | Unit 2 Lesson 8 133 Warm-Up

Pairs | 10 min

Activity 1 Rounds of Mini Golf

Purpose: Students explore how the **center of dilation** and the scale factor impact the relationship between the pre-image and the image, creating a need for using measurements. **(MTR.4.1)**

1 Launch

To surface connections, consider reading the description aloud and inviting students to paraphrase it or ask clarifying questions.

ELL **English Language Learners** Consider modeling or inviting a student to model strategies for annotating the text to make sense of it. **(Reading)**

Consider asking:

- "Why do you think point C is called a *center of dilation*?
- "Have you heard the term *scale factor* before? If so, where?"

2 Monitor

Listen for students who look for connections between the *Scale Factor*: 2 and the distances in the game.

Encourage students to take turns placing the center of dilation so that point B' is in the hole. Students will dilate on four separate images, one for each hole.

Select and sequence student explanations to share during the Connect. **(MTR.4.1)**

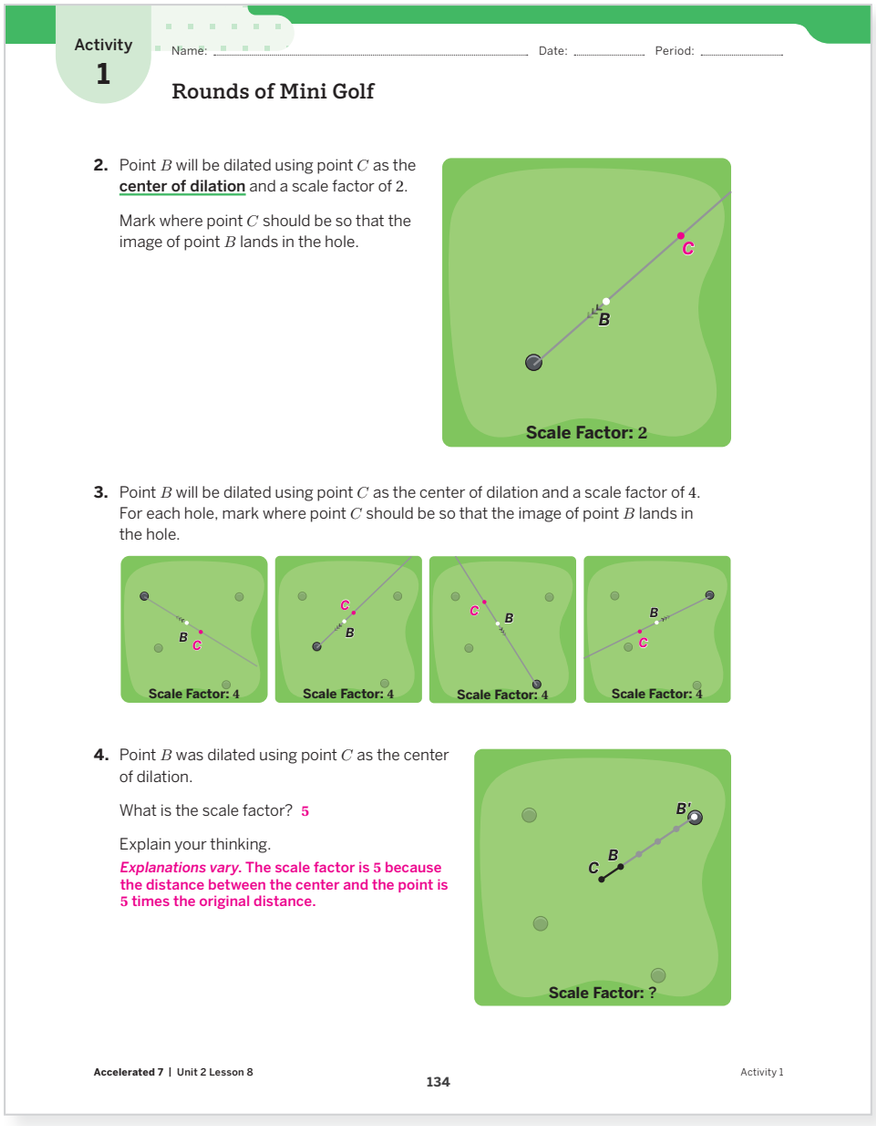
D **Differentiation** *(Problem 4)*

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Encourage them to revisit the previous two problems and connect the scale factor and the distances.
Write "4" because there are four gray segments.	**Support** Consider asking, "How many times longer is that distance from the center to the hole than the distance between the center and the pre-image?"
Explain that the scale factor is 5 because the new distance is 5 times the original distance.	**Stretch** Consider asking, "How would the situation be different if the scale factor were 6?"

3 Connect

Invite students to justify why they chose different values for the scale factor.

To surface the Key Takeaway, create a class definition of the term **center of dilation** using students' language and the glossary. Record it for students to refer to later in the unit.



Activity 1

Name: _____ Date: _____ Period: _____

Rounds of Mini Golf

2. Point B will be dilated using point C as the **center of dilation** and a scale factor of 2.

Mark where point C should be so that the image of point B lands in the hole.

3. Point B will be dilated using point C as the center of dilation and a scale factor of 4. For each hole, mark where point C should be so that the image of point B lands in the hole.

4. Point B was dilated using point C as the center of dilation.

What is the scale factor? 5

Explain your thinking.

Explanations vary. The scale factor is 5 because the distance between the center and the point is 5 times the original distance.

Accelerated 7 | Unit 2 Lesson 8 134 Activity 1

Key Takeaway: The **center of dilation** is the point that distances are measured from in a dilation. The scale factor is the value that the distance between the center and the pre-image is multiplied by to get the distance between the center and the image.

Pairs | 10 min

Activity 2 Dilation Distances

Purpose: Students use distance measurements to dilate points and lines given a center of dilation and a scale factor.

Short on time? Consider completing Problem 7 as a class.

1 Launch

To support connections, consider reading the prompt aloud and asking, "How is this task like the task in Activity 1? How is it different?"

2 Monitor

Listen for students who notice that the scale factor is 3 and use that to help them determine distances.

Encourage students to read others' responses and decide how others' strategies compare to their own.

Look for a variety of strategies students use to determine where each point should be after the dilation.

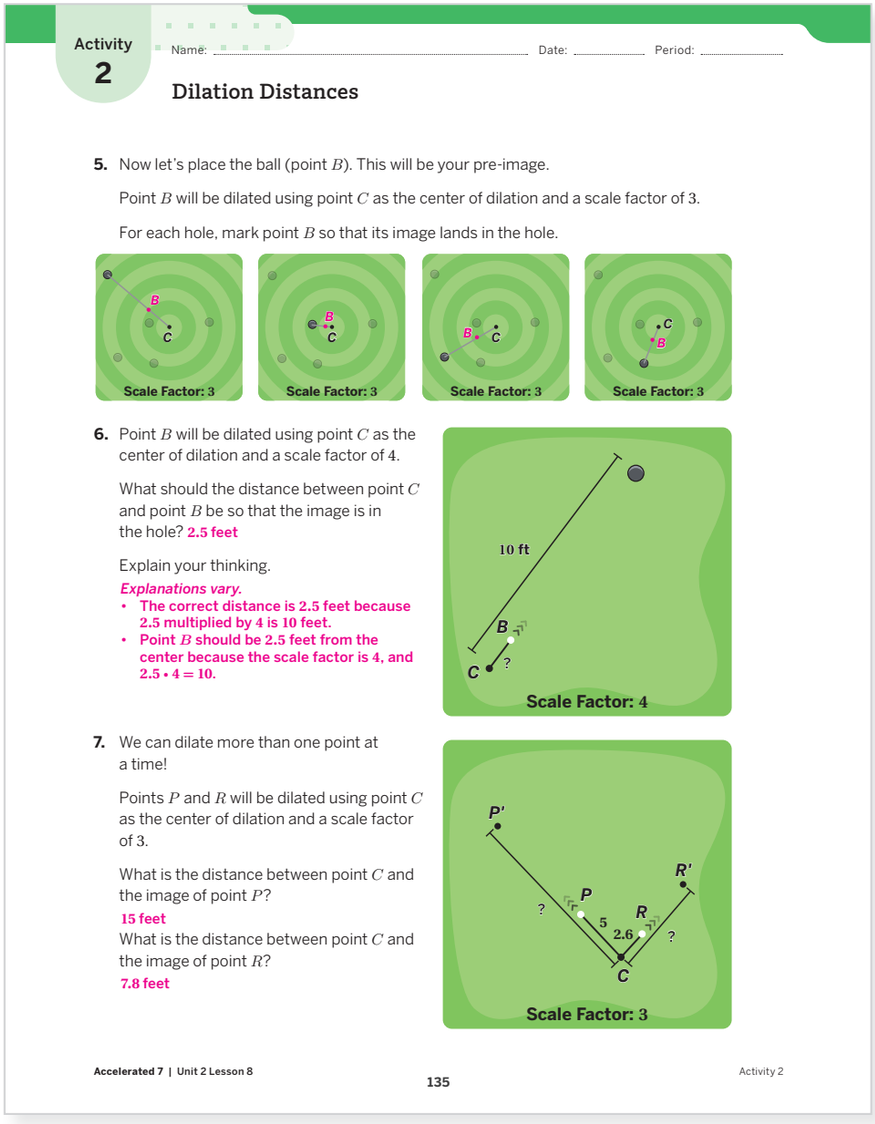
D Differentiation *(Problem 7)*

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Consider asking, "How could we use the scale factor to help us?"
Say 20 and 10.4 because they are measuring from the pre-image instead of the center.	**Support** Consider asking, "How could we use the scale factor to help us?"
Use a variety of strategies to determine the distances, including: • Multiplying the original distance by the scale factor. • Using a straightedge to mark units of distance.	**Strengthen** Invite students to think about how they might share their strategy during the Connect. **Stretch** Consider asking, "What do you think would happen if we connected points P and R?"

3 Connect

Invite students to share their strategies for determining where points P' and R' should be.

To surface the Key Takeaway, consider asking:
- "How can knowing distances help with dilations?"
- "How are the distances between the center of dilation and the pre-image and the center of dilation and the image related?"
- "How could we use what we know about dilating points to dilate a line or a figure?"



Activity 2

Name: _____ Date: _____ Period: _____

Dilation Distances

5. Now let's place the ball (point B). This will be your pre-image.

Point B will be dilated using point C as the center of dilation and a scale factor of 3.

For each hole, mark point B so that its image lands in the hole.

6. Point B will be dilated using point C as the center of dilation and a scale factor of 4.

What should the distance between point C and point B be so that the image is in the hole? **2.5 feet**

Explain your thinking.

Explanations vary.
- **The correct distance is 2.5 feet because 2.5 multiplied by 4 is 10 feet.**
- **Point B should be 2.5 feet from the center because the scale factor is 4, and $2.5 \cdot 4 = 10$.**

7. We can dilate more than one point at a time!

Points P and R will be dilated using point C as the center of dilation and a scale factor of 3.

What is the distance between point C and the image of point P?

15 feet

What is the distance between point C and the image of point R?

7.8 feet

Accelerated 7 | Unit 2 Lesson 8 135 Activity 2

Key Takeaway: To dilate a point, measure the distance between the **center of dilation** and the pre-image, multiply that distance by the scale factor, and place the image the new distance from the center along the same line.

Independent | 10 min

Activity 3 Dilating a Triangle

Purpose: Students dilate figures not on a grid using paper and a straightedge or ruler. **(MTR.7.1)**

Short on time? Consider inviting students to choose only one scale factor.

1 Launch

To surface connections, consider asking, "How is dilating a figure like dilating a point? How is it different?"

Note: Make sure all students have access to a ruler or straightedge.

2 Monitor

Look for students who use a variety of precision in their dilations and respond individually.

D Differentiation

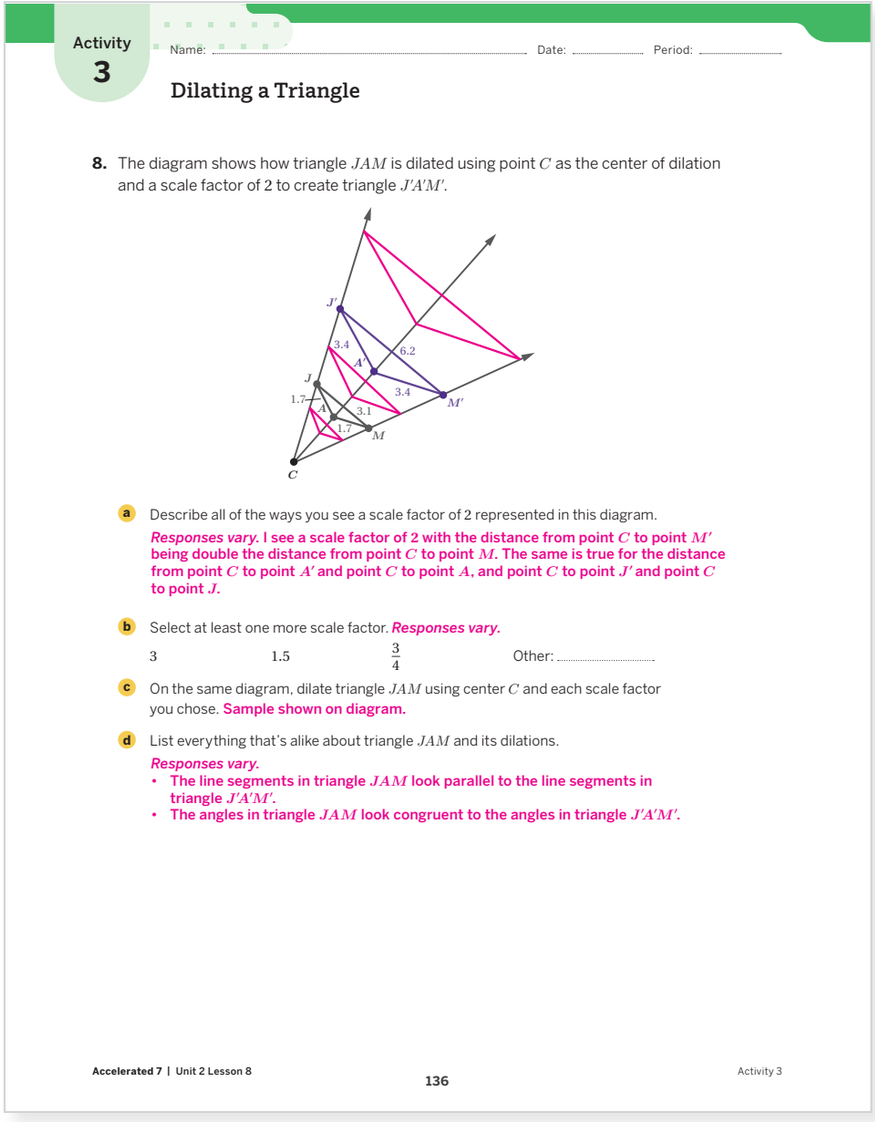
Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Encourage them to measure and compare the distances between points J and C and points J' and C.
Estimate the measurements for vertices of the image.	**Strengthen** Consider asking, "How could we use measurements to be more precise?"
Notice that the image is kind of like the pre-image, but a different size.	**Strengthen** Consider asking, "What makes the figures alike? Are there any side lengths or angle measurements that look congruent?"
Would like to explore dilations by hand further.	**Stretch** Encourage them to try more than one scale factor and compare all three triangles.

A Accessibility: Fine Motor Skills

To support students measuring and to reduce anxiety about precision, consider pairing students – once students agree on a strategy for dilating the point, one student performs the dilation by hand.

3 Connect

Invite students to share the strategies they used and challenges they had with dilating the triangle.



Activity 3

Name: _____ Date: _____ Period: _____

Dilating a Triangle

8. The diagram shows how triangle JAM is dilated using point C as the center of dilation and a scale factor of 2 to create triangle $J'A'M'$.

a. Describe all of the ways you see a scale factor of 2 represented in this diagram.

Responses vary. I see a scale factor of 2 with the distance from point C to point M' being double the distance from point C to point M. The same is true for the distance from point C to point A' and point C to point A, and point C to point J' and point C to point J.

b. Select at least one more scale factor. *Responses vary.*

3 1.5 $\frac{3}{4}$ Other: _____

c. On the same diagram, dilate triangle JAM using center C and each scale factor you chose. Sample shown on diagram.

d. List everything that's alike about triangle JAM and its dilations.

Responses vary.

- The line segments in triangle JAM look parallel to the line segments in triangle $J'A'M'$.
- The angles in triangle JAM look congruent to the angles in triangle $J'A'M'$.

Accelerated 7 | Unit 2 Lesson 8 136 Activity 3

Whole Class | 5 min

Synthesis

Purpose: Students synthesize their strategies for dilating a point using distances, a center of dilation, and a scale factor.

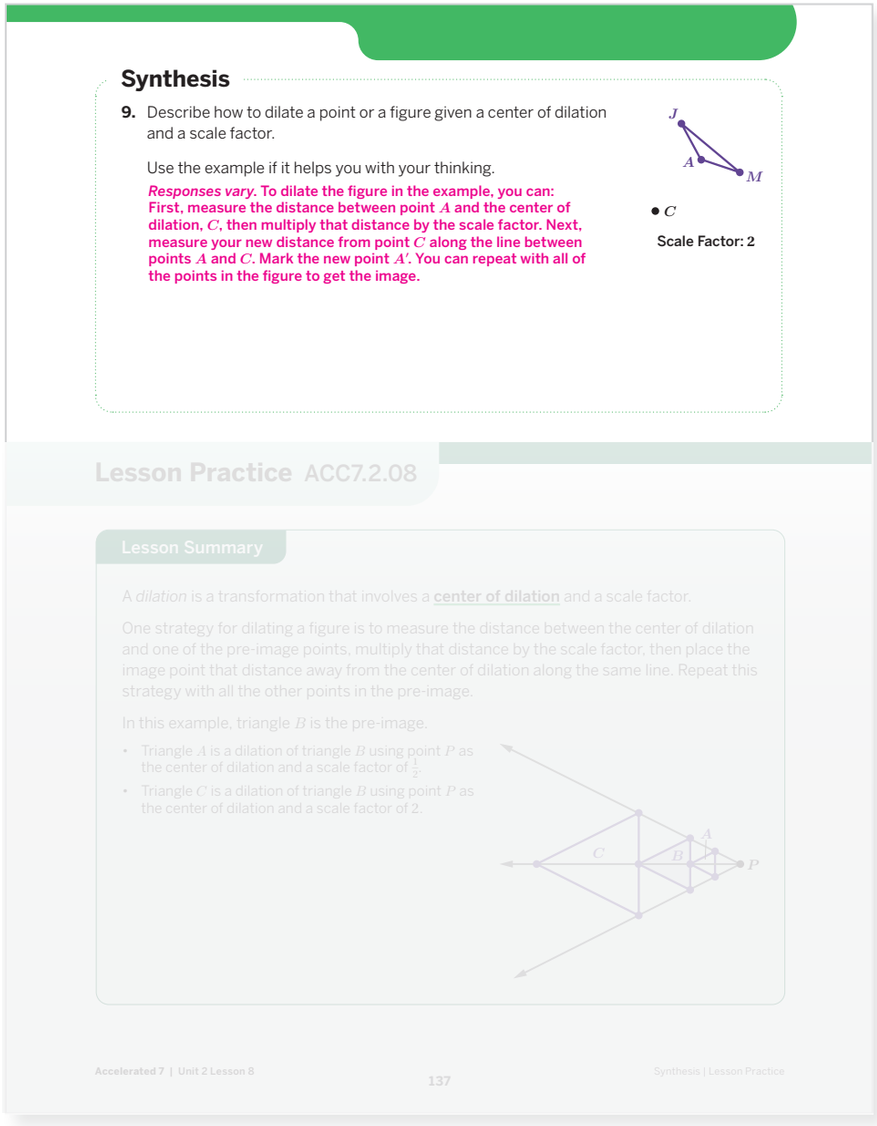
Invite students to respond independently, and then share their thinking with a partner.

Collect and share a variety of ideas, such as:

- Dilating each vertex/point one at a time.
- Measuring the distance between the pre-image point and the center of dilation.
- Multiplying the original distance by the scale factor, then measuring from the center again along the line between the center of dilation and the pre-image point.

MLR **MLR2: Collect and Display** Use this routine to collect students' strategies as they share during the Connect. Display for students to reference and add to for the rest of the unit.

Note: If time allows, invite students to share with a partner what makes sense about each strategy and what questions they still have.



Synthesis

9. Describe how to dilate a point or a figure given a center of dilation and a scale factor.

Use the example if it helps you with your thinking.

Responses vary. To dilate the figure in the example, you can: First, measure the distance between point A and the center of dilation, C, then multiply that distance by the scale factor. Next, measure your new distance from point C along the line between points A and C. Mark the new point A'. You can repeat with all of the points in the figure to get the image.

Lesson Practice ACC7.2.08

Lesson Summary

A *dilation* is a transformation that involves a center of dilation and a scale factor.

One strategy for dilating a figure is to measure the distance between the center of dilation and one of the pre-image points, multiply that distance by the scale factor, then place the image point that distance away from the center of dilation along the same line. Repeat this strategy with all the other points in the pre-image.

In this example, triangle B is the pre-image.

- Triangle A is a dilation of triangle B using point P as the center of dilation and a scale factor of $\frac{1}{2}$.
- Triangle C is a dilation of triangle B using point P as the center of dilation and a scale factor of 2.

Accelerated 7 | Unit 2 Lesson 8 137 Synthesis | Lesson Practice

Lesson Takeaway: We can dilate a figure by dilating each of its vertices one by one. To dilate a point, measure the distance between the **center of dilation** and the pre-image, multiply that distance by the scale factor, and place the image the new distance from the center along the same line.

Independent | 5 min

Show What You Know

Purpose: Students demonstrate their understanding by identifying the scale factor used in a dilation.

Today's Goals

1. **Goal:** Identify the center and scale factor in a dilation.
2. **Goal:** Dilate a point or figure using distance measurements.
3. **Language Goal:** Describe strategies for dilating a point. **(Speaking and Listening)**

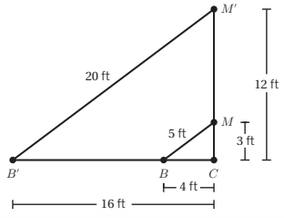
Show What You Know (Assessment Resources)

Name: _____ Date: _____ Period: _____

Show What You Know 2.08

Segment BM is dilated using point C as the center of dilation.

What is the scale factor? Explain your thinking.



4. *Explanations vary.*
   - The scale factor is 4 because the distance between points C and M' is 4 times the distance between points C and M.
   - The scale factor is 4 because segment BM is 5 feet and segment $B'M'$ is 20 feet, and $5 \cdot 4 = 20$.

D Differentiation Use after Lesson 8

Support

Provide targeted intervention for students by using these resources.

If students need support with identifying the scale factor in a dilation:

» **Respond to Student Thinking**

- Use the ***Connecting Scale Factors to Scaled Copies*** **Mini-Lesson**.
- Revisit the **Lesson Summary** with students before providing individualized feedback on the relevant **Practice Problems**.
- Revisit **Activity 3** and invite students to choose other scale factors to dilate triangle JAM.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 2 Extension Activities.**

Professional Learning

Take a moment to reflect on this lesson. In this lesson, students developed early ideas about the terms *center of dilation* and *scale factor* and how to use them to dilate a figure. How will their understanding of these ideas build over the next several lessons? How can you build on ideas students shared today to support them in Lessons 9–11?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

9. Describe how to dilate a point or a figure given a center of dilation and a scale factor.

Use the example if it helps you with your thinking.

Responses vary. To dilate the figure in the example, you can: First, measure the distance between point A and the center of dilation, C, then multiply that distance by the scale factor. Next, measure your new distance from point C along the line between points A and C. Mark the new point A'. You can repeat with all of the points in the figure to get the image.

Scale Factor: 2

Lesson Practice ACC7.2.08

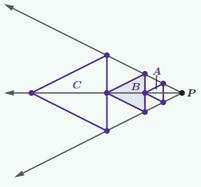
Lesson Summary

A *dilation* is a transformation that involves a **center of dilation** and a scale factor.

One strategy for dilating a figure is to measure the distance between the center of dilation and one of the pre-image points, multiply that distance by the scale factor, then place the image point that distance away from the center of dilation along the same line. Repeat this strategy with all the other points in the pre-image.

In this example, triangle B is the pre-image.

- Triangle A is a dilation of triangle B using point P as the center of dilation and a scale factor of $\frac{1}{2}$.
- Triangle C is a dilation of triangle B using point P as the center of dilation and a scale factor of 2.



Lesson Practice ACC7.2.08

Name: _____ Date: _____ Period: _____

Problems 1–2: Segment AB is 3 centimeters long. Point O is the center of dilation.

1. How long is the image of segment AB after a dilation with a scale factor of 5?
15 centimeters

2. How long is the image of segment AB after a dilation with a scale factor of 3.7?
11.1 centimeters

Problems 3–6: Here are points A and B. Plot the points after each dilation.
Responses shown on diagram.

3. Point C is the image of point B using point A as the center of dilation and a scale factor of 2.

4. Point D is the image of point A using point B as the center of dilation and a scale factor of 2.

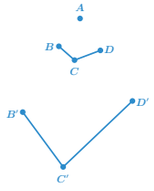
5. Point E is the image of point B using point A as the center of dilation and a scale factor of $\frac{1}{2}$.

6. Point F is the image of point A using point B as the center of dilation and a scale factor of $\frac{1}{2}$.

7. Isaiah claims that figure $B'C'D'$ is a dilation of figure BCD using point A as the center of dilation. Is Isaiah's claim correct? **No**

Explain your thinking.

Explanations vary. **I drew a ray from point A through points B and B' and then drew another ray from point A through point C to point C'. The ratio of the distance between points A and B' to the distance between points A and B is not the same as the ratio of the distance between points A and C' to the distance between points A and C.**



Lesson Practice ACC7.2.08

Name: _____ Date: _____ Period: _____

FAST Practice

Problems 8–9: Here is a diagram.

8. Point H is the image of point G using point F as the center of dilation. What is the scale factor?
6

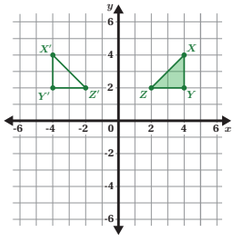
9. If the distance from point F to point H is 9 units, what is the distance from point F to point G? Record your answer in the space provided.

1.5 units

Spiral Review

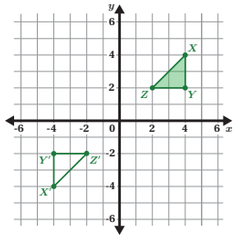
Problems 10–11: Determine whether each statement is *true* or *false*. If false, write a correct statement about the transformation.

10. Triangle XYZ is translated 4 units to the left to form triangle $X'Y'Z'$.



False; Triangle XYZ is reflected across the y-axis to form triangle $X'Y'Z'$.

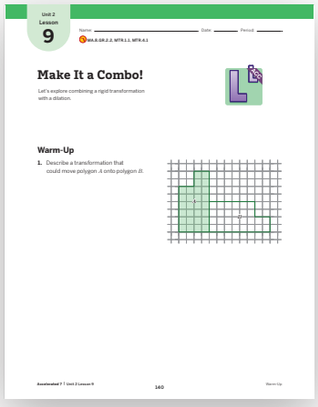
11. Triangle XYZ is rotated 180° about the origin to form triangle $X'Y'Z'$.



True

Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1–2	1	MA.8.GR.2.2
	3–6	2	MA.8.GR.2.2
	7	3	MA.8.GR.2.2
FAST Practice	**8–9**	2	MA.8.GR.2.2
Spiral Review			
	10–11	2	MA.8.GR.2.1


Unit 2
Lesson 9
Name _____ Date _____ Period _____
MA.8.GR.2.2, MTR.1.1, MTR.4.1
Make It a Combo!
Let's explore combining a rigid transformation with a dilation.
Warm-Up
1. Describe a transformation that could move polygon A onto polygon B.
140

Make It a Combo!

Combining a Rigid Transformation and a Dilation

Let's explore combining a rigid transformation with a dilation.

Learning Outcomes

Today's Goals

1. **Goal:** Use transformations, including dilations, to move one figure onto another.
2. **Language Goal:** Describe transformations using terms such as *dilate*, *translate*, *reflect*, *rotate*, *image*, and *pre-image*. **(Writing, Speaking, and Listening)**

Students explore translations, reflections, rotations, and dilations to move one figure onto another. They experiment with dilations, including the effects of the location of the center of dilation on the resulting image. This lesson previews the effect of scale factors less than 1, which students will explore more in the next lesson.

Coherence

Prior Learning

In Lesson 8, students dilated points and figures using a center of dilation and a scale factor without using a grid.

Future Learning

In Lesson 10, students will more precisely dilate figures on a grid and explore the effects of scale factors greater than 1, between 0 and 1, and equal to 1.

Connections

- Students build upon their understanding of rigid transformations on the coordinate plane by exploring combinations of transformations that include dilations. (mathematical)
- Students use precise language when moving one figure onto another using a combination of transformations. (mathematical, cross-curricular)

FL B.E.S.T. Math Benchmarks

Addressing

MA.8.GR.2.2
Given a preimage and image generated by a single dilation, identify the scale factor that describes the relationship.

Mathematical Thinking and Reasoning Standards: MTR.1.1, MTR.4.1

ELA Expectations: ELA.K12.EE.1.1, ELA.K12.EE.4.1

English Language Development Standards: ELL.MA.1, ELL.SI.1

Building On	Building Toward
MA.7.GR.1.5	MA.912.F.2 MA.912.GR.2

Lesson at a Glance

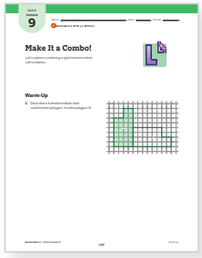
~ 45 min

MA.8.GR.2.2

Warm-Up

Pairs | 5 min

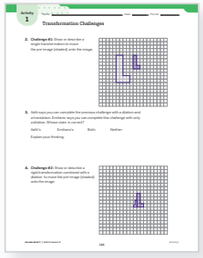
Students use their knowledge of translations, reflections, and rotations from Unit 1.



Activity 1

Pairs | 15 min

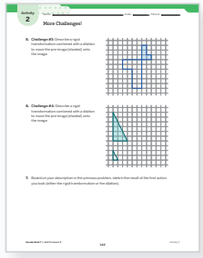
Students complete challenges using different transformations. They use the **Decide and Defend** routine to explore how the center of dilation affects a transformation. **(MTR.4.1)**



Activity 2

Pairs | 15 min

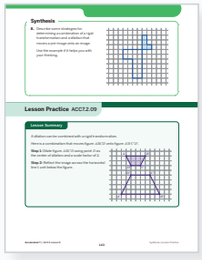
Students complete more challenges that combine a rigid transformation with a dilation.
Routine: Think-Pair-Share **(MTR.1.1)**



Synthesis

Whole Class | 5 min

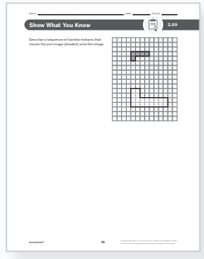
Students synthesize their understanding by describing strategies to move one figure onto another, including when to use a dilation.



Show What You Know

Independent | 5 min

Students demonstrate their understanding by describing a sequence of transformations that move a pre-image onto an image on a grid.



Prep Checklist

This lesson includes:

- Student Edition
- Show What You Know

- Straightedges
- Tracing paper

Pairs | 5 min

Warm-Up

Purpose: Students use their knowledge of translations, reflections, and rotations from Unit 1.

1 Launch

To surface prior knowledge, consider asking, "What are some things you remember about the transformations we learned in Unit 1?"

A **Accessibility: Memory and Attention** Invite students to refer back to the anchor chart created previously or their notes on translations, reflections, and rotations in Unit 1 to activate prior knowledge.

Invite students to think about the position or orientation of the pre-image compared to the image.

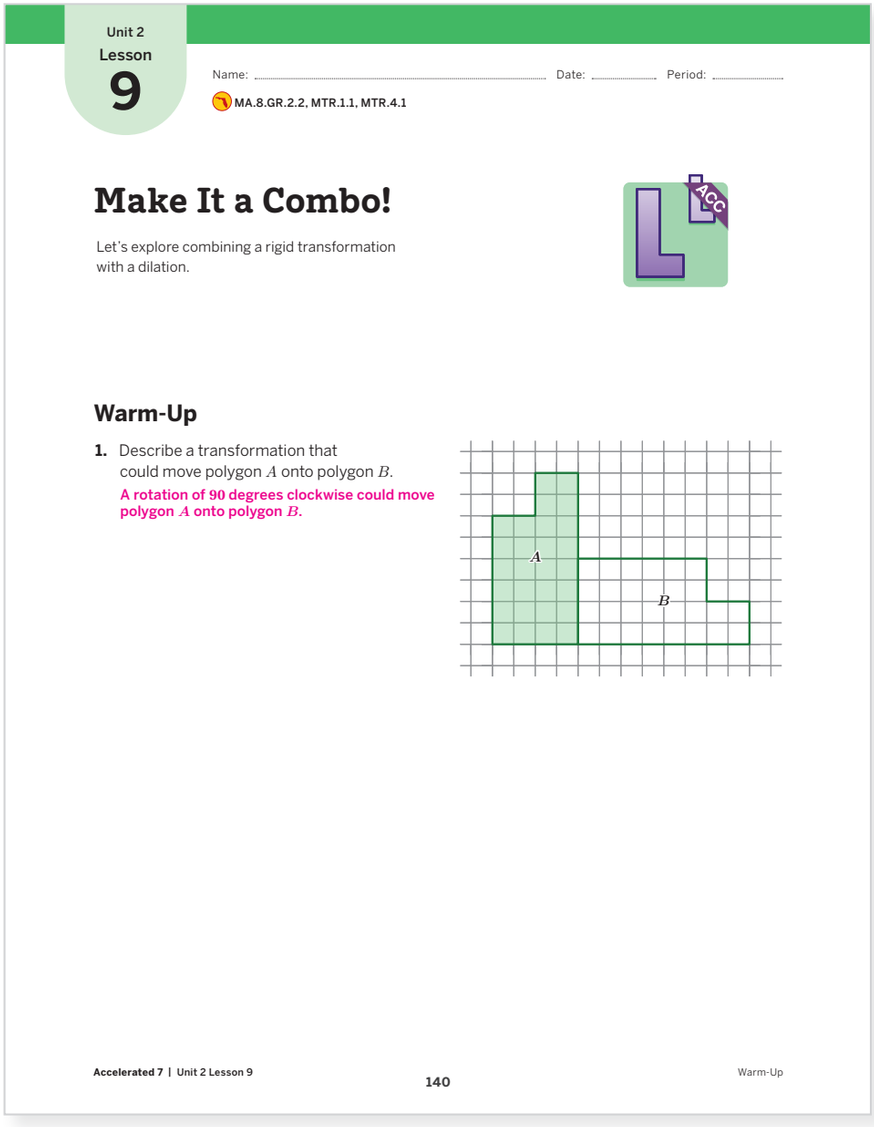
2 Connect

Invite students to share their description of the transformation.

Consider asking:

- "How did you decide which transformation to use?"
- "What is helpful about this description? How could we make it stronger or clearer?"
- "How would the image be different if the transformation were combined with a dilation?"

Note: If it does not come up naturally, consider discussing that for a rotation, you need to know the center of rotation.



Unit 2
Lesson
9

Name: ..... Date: ..... Period: .....

MA.8.GR.2.2, MTR.1.1, MTR.4.1

Make It a Combo!

Let's explore combining a rigid transformation with a dilation.

Warm-Up

1. Describe a transformation that could move polygon A onto polygon B.

   A rotation of 90 degrees clockwise could move polygon A onto polygon B.

Accelerated 7 | Unit 2 Lesson 9 140 Warm-Up

Pairs | 15 min

Activity 1 Transformation Challenges

Purpose: Students complete challenges using different transformations. They use the Decide and Defend routine to explore how the center of dilation affects a transformation. **(MTR.4.1)**

1 Launch

Invite students to use tracing paper and a straightedge to identify the transformation.

2 Monitor

Look for students who identify the center of dilation and scale factor.

Look for student responses that support each option to highlight during the Connect.

Look for a variety of different transformations students use to solve this challenge and respond individually.

D Differentiation *(Problem 4)*

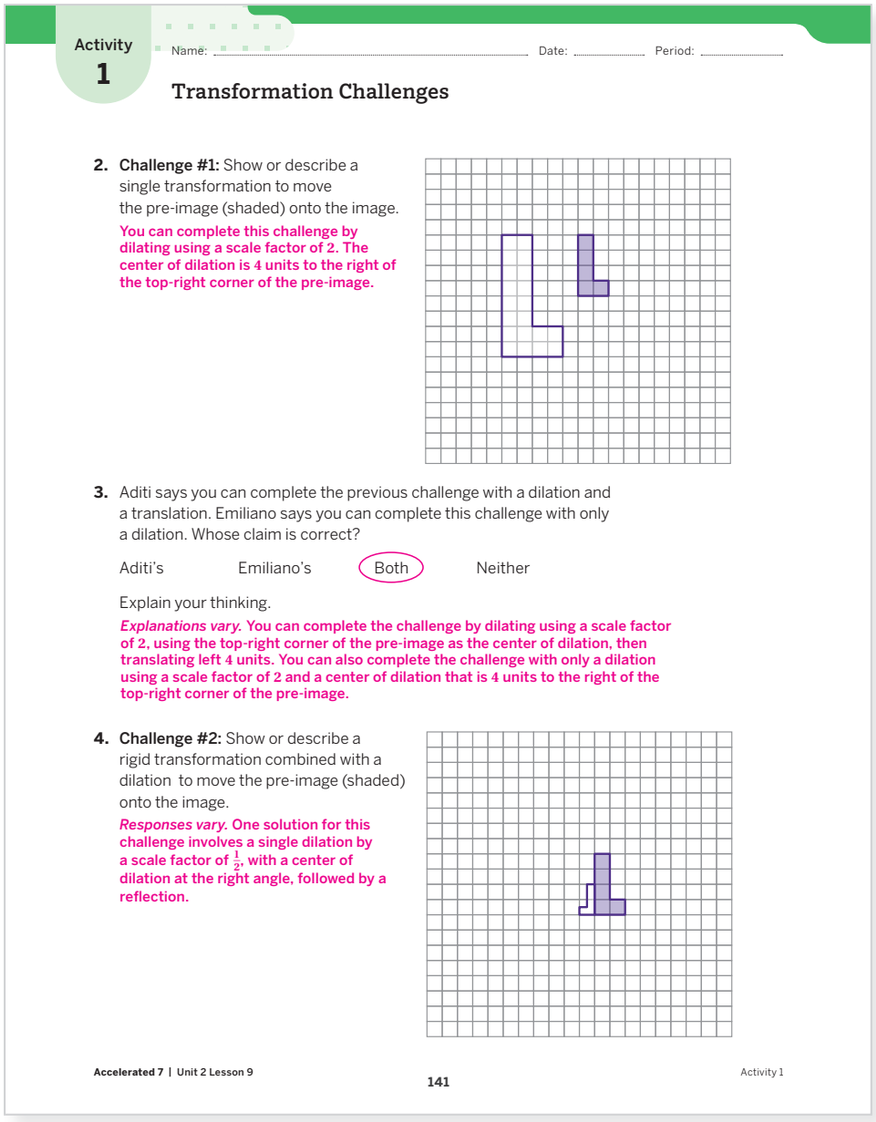
Look for students who:	Respond to Student Thinking
Need support creating an image that is smaller than the pre-image.	**Support** Encourage students to try a variety of scale factors. Consider asking, "What could I multiply a number by to get a smaller number?"
Use the correct types of transformations without precision.	**Strengthen** Consider asking, "What is the center of dilation? Line of reflection? How does that impact where the image will be?"
Use precise descriptions of a reflection and dilation.	**Stretch** Consider asking, "Could you have completed this challenge using fewer moves? Why or why not?"

3 Connect

Use the Decide and Defend routine to support students in making arguments and critiquing the reasoning of others. **(MTR.4.1)**

To surface the Key Takeaway, consider asking, "How did Aditi's and Emiliano's center of dilation impact whether or not they also needed a translation?"

Note: If time allows, invite students to share the transformations they used on this challenge.



Activity 1

Name: _____ Date: _____ Period: _____

Transformation Challenges

2. Challenge #1: Show or describe a single transformation to move the pre-image (shaded) onto the image.

You can complete this challenge by dilating using a scale factor of 2. The center of dilation is 4 units to the right of the top-right corner of the pre-image.

3. Aditi says you can complete the previous challenge with a dilation and a translation. Emiliano says you can complete this challenge with only a dilation. Whose claim is correct?

Aditi's Emiliano's Both Neither

Explain your thinking.

Explanations vary. You can complete the challenge by dilating using a scale factor of 2, using the top-right corner of the pre-image as the center of dilation, then translating left 4 units. You can also complete the challenge with only a dilation using a scale factor of 2 and a center of dilation that is 4 units to the right of the top-right corner of the pre-image.

4. Challenge #2: Show or describe a rigid transformation combined with a dilation to move the pre-image (shaded) onto the image.

Responses vary. One solution for this challenge involves a single dilation by a scale factor of $\frac{1}{2}$, with a center of dilation at the right angle, followed by a reflection.

Accelerated 7 | Unit 2 Lesson 9 141 Activity 1

Key Takeaway: Dilations can be combined with translations, reflections, and rotations to move a pre-image onto an image. The center of dilation is important because it affects the location of the image after the dilation.

Pairs | 15 min

Activity 2 More Challenges!

Purpose: Students complete more challenges that combine a rigid transformation with a dilation. **(MTR.1.1)**

Short on time? Consider omitting the sketch in Problem 7 and having students verbally discuss their descriptions instead.

1 Launch

Invite students to sketch out different types of transformations they could use. Consider asking, "What would a translation/rotation/reflection look like?"

2 Monitor

Listen for and celebrate students who discuss what the center of dilation, center of rotation, or scale factor should be.

Collect a variety of descriptions to share during Connect. **(MTR.1.1)**

D Differentiation *(Problem 6)*

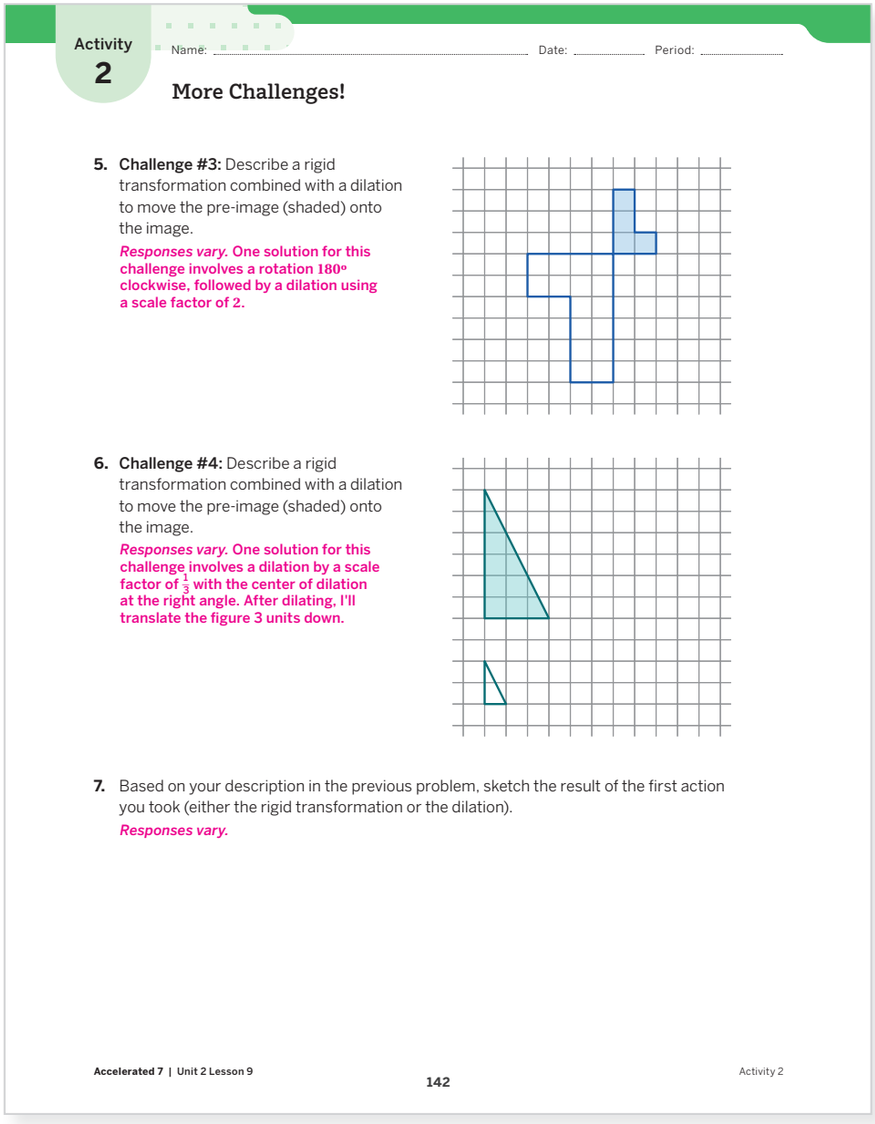
Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Consider asking, "How could you make the larger triangle shrink so that it's the same size as the smaller triangle?"
Choose a transformation but don't include the necessary information.	**Support** Consider asking, "What other information would someone need to be able to sketch the transformation correctly?"
Precisely describe the transformation, using language like *scale factor*, *center of dilation*, *rotation*, etc.	**Stretch** Consider asking, "What would happen if the scale factor increased or decreased?"

ELL English Language Learners Provide a word bank to support students in expressing their ideas with words such as: *dilation*, *scale factor*, *center of dilation*, *translation*, *reflection*, *rotation*, etc. **(Writing and Speaking)**

3 Connect

Pause to share the captured descriptions. Use the Think-Pair-Share routine to support student-to-student discourse deciding if each description will work. See the Routine Facilitation Guide for more information.

Consider asking, "What could help make these descriptions more clear?"



Activity 2

Name: _____ Date: _____ Period: _____

More Challenges!

5. Challenge #3: Describe a rigid transformation combined with a dilation to move the pre-image (shaded) onto the image.

Responses vary. One solution for this challenge involves a rotation 180° clockwise, followed by a dilation using a scale factor of 2.

6. Challenge #4: Describe a rigid transformation combined with a dilation to move the pre-image (shaded) onto the image.

Responses vary. One solution for this challenge involves a dilation by a scale factor of $\frac{1}{3}$ with the center of dilation at the right angle. After dilating, I'll translate the figure 3 units down.

7. Based on your description in the previous problem, sketch the result of the first action you took (either the rigid transformation or the dilation).

Responses vary.

Accelerated 7 | Unit 2 Lesson 9 142 Activity 2

Whole Class | 5 min

Synthesis

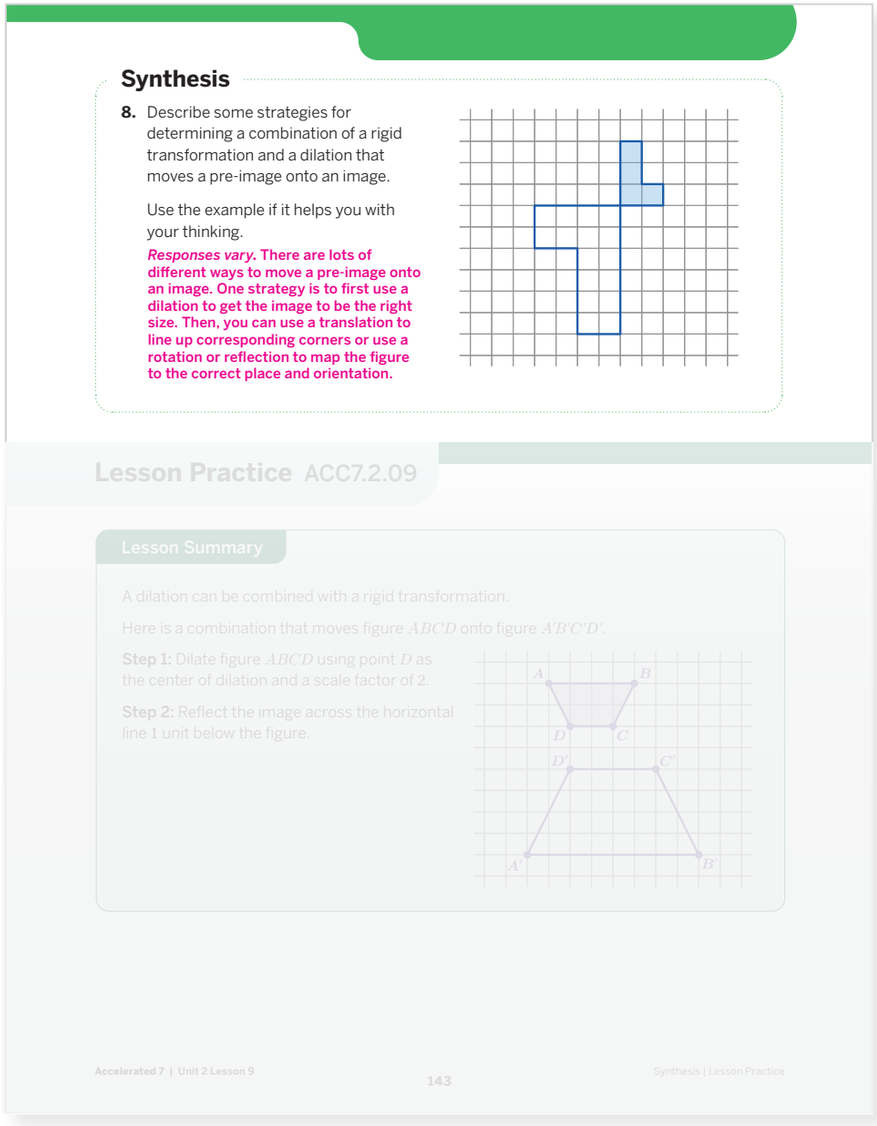
Purpose: Students synthesize their understanding by describing strategies to move one figure onto another, including when to use a dilation.

Invite students to respond independently, and then share their thinking with a partner.

Collect and share a variety of ideas, such as:

- Saying that sometimes dilations are not needed.
- Mentioning where the center of rotation should be.
- Using a reflection instead of a rotation for changing the orientation of the figure.

Note: If time allows, invite students to revise their previous thinking based on the discussion.



Synthesis

8. Describe some strategies for determining a combination of a rigid transformation and a dilation that moves a pre-image onto an image.

Use the example if it helps you with your thinking.

Responses vary. **There are lots of different ways to move a pre-image onto an image. One strategy is to first use a dilation to get the image to be the right size. Then, you can use a translation to line up corresponding corners or use a rotation or reflection to map the figure to the correct place and orientation.**

Lesson Practice ACC7.2.09

Lesson Summary

A dilation can be combined with a rigid transformation.

Here is a combination that moves figure $ABCD$ onto figure $A'B'C'D'$.

Step 1: Dilate figure $ABCD$ using point D as the center of dilation and a scale factor of 2.

Step 2: Reflect the image across the horizontal line 1 unit below the figure.

Accelerated 7 | Unit 2 Lesson 9 143 Synthesis | Lesson Practice

Lesson Takeaway: We can use transformations, including translations, reflections, rotations, and dilations, to move one figure onto another. The center of dilation and scale factor are both important pieces of information in a dilation.

Independent | 5 min

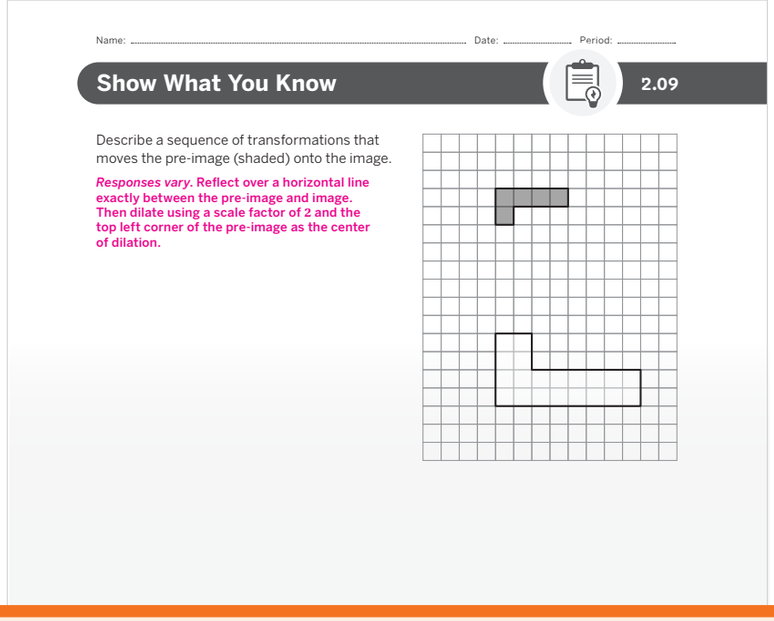
Show What You Know

Purpose: Students demonstrate their understanding by describing a sequence of transformations that move a pre-image onto an image on a grid.

Today's Goals

1. **Goal:** Use a sequence of transformations, including dilations, to move one figure onto another.
2. **Language Goal:** Describe a sequence of transformations using terms such as *dilate*, *translate*, *reflect*, *rotate*, *image*, and *pre-image*. **(Writing, Speaking, and Listening)**

Show What You Know (Assessment Resources)



Name: _____ Date: _____ Period: _____

Show What You Know 2.09

Describe a sequence of transformations that moves the pre-image (shaded) onto the image.

Responses vary. Reflect over a horizontal line exactly between the pre-image and image. Then dilate using a scale factor of 2 and the top left corner of the pre-image as the center of dilation.

D Differentiation Use after Lesson 9

Support

Provide targeted intervention for students by using these resources.

If students need support with identifying and describing transformations:

» **Respond to Student Thinking**

- Revisit the **Synthesis** and give students individualized feedback.
- Select a few related **Practice** Problems and give individualized feedback.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking**

- Invite students to explore the **Sub-Unit 2 Extension Activities.**

Professional Learning

In what ways are students taking risks when trying out strategies for solving transformation challenges? What questions might encourage students to continue using creativity as they strengthen their understanding of dilations and other transformations?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

8. Describe some strategies for determining a combination of a rigid transformation and a dilation that moves a pre-image onto an image.

Use the example if it helps you with your thinking.

Responses vary. There are lots of different ways to move a pre-image onto an image. One strategy is to first use a dilation to get the image to be the right size. Then, you can use a translation to line up corresponding corners or use a rotation or reflection to map the figure to the correct place and orientation.

Lesson Practice ACC7.2.09

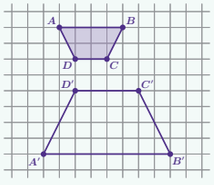
Lesson Summary

A dilation can be combined with a rigid transformation.

Here is a combination that moves figure $ABCD$ onto figure $A'B'C'D'$.

Step 1: Dilate figure $ABCD$ using point D as the center of dilation and a scale factor of 2.

Step 2: Reflect the image across the horizontal line 1 unit below the figure.

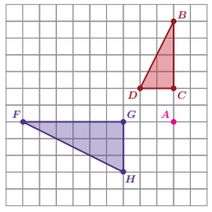


Lesson Practice ACC7.2.09

Name: _____ Date: _____ Period: _____

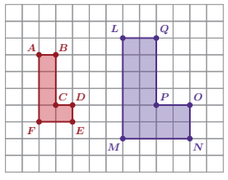
Problems 1–4: Is there a single rigid transformation or a rigid transformation combined with a dilation that moves one figure onto the other? (Write *yes* or *no*.) If you wrote *yes*, describe the transformation(s). If you wrote *no*, describe how you know it's not possible.

1.



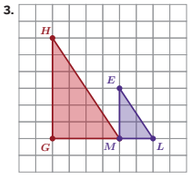
Yes. *Explanations vary.* Rotate triangle BCD 90° counterclockwise around point A and then dilate using A as the center of dilation and a scale factor of 1.5.

2.



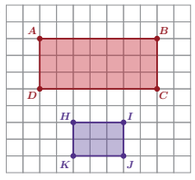
No. *Explanations vary.* The corresponding sides have different scale factors, so there is no transformation that moves one figure onto the other.

3.



Yes. *Explanations vary.* Dilate triangle ELM using point M as the center of dilation with a scale factor of 2 and then translate the image 4 units to the left.

4.



No. *Explanations vary.* The corresponding sides have different scale factors, so there is no transformation that moves one figure onto the other.

Lesson Practice ACC7.2.09

Name: _____ Date: _____ Period: _____

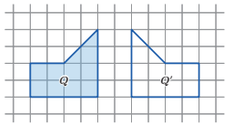
5. Caasi says that dilating a figure by a scale factor of $\frac{3}{2}$ will produce an image that is smaller than the pre-image. Is Caasi's claim correct? Explain your thinking.

No. *Explanations vary.* $\frac{3}{2}$ is a scale factor that is greater than 1, so the image will be larger, not smaller.

FAST Practice

6. Which transformation will move the pre-image onto the image?

A. Translating to the right.
B. Rotating 90°.
C. Dilating by a scale factor of 4.
D. Reflecting across a vertical line.



Spiral Review

Problems 7–9: Point G is located at (5, -2). Determine the coordinates of the image after each transformation.

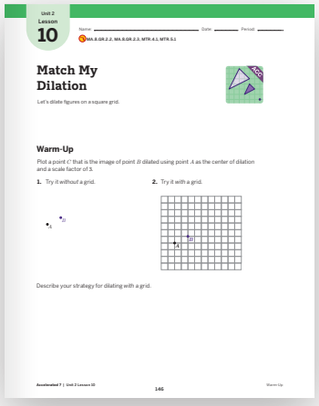
7. A reflection over the y-axis.
(-5, -2)

8. A reflection over the x-axis.
(5, 2)

9. A 180° rotation around the origin.
(-5, 2)

Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1–5	2	MA.8.GR.2.2
FAST Practice	6	2	MA.8.GR.2.2
Spiral Review			
	7–9	1	MA.8.GR.2.3


Unit 2
Lesson
10
Name: Date: Period:
MA.8.GR.2.2, MA.8.GR.2.3, MTR.4.1, MTR.5.1
Match My Dilation
Let's dilate figures on a square grid.
Warm-Up
Plot a point C that is the image of point B dilated using point A as the center of dilation and a scale factor of 3.
1. Try it without a grid.
2. Try it with a grid.
Describe your strategy for dilating with a grid.
Accelerated 7 | Unit 2 Lesson 10
146
Warm-Up

Match My Dilation

Dilations on a Square Grid

Let's dilate figures on a square grid.

Learning Outcomes

Today's Goals

1. **Goal:** Dilate figures on a grid using a center of dilation and scale factor.
2. **Language Goal:** Describe the effects of scale factors greater than 1, between 0 and 1, and equal to 1 on an image. **(Writing, Speaking, and Listening)**

Students explore the effects of scale factors greater than 1, between 0 and 1, and equal to 1 and dilate figures on a grid by dilating each point. The grid offers a way of measuring distances between points, especially points that lie at the intersection of grid lines. This lesson continues to emphasize that scale factors affect both the size of the image and its distance from the center of dilation.

Coherence

Prior Learning

In Lessons 8–9, students learned how to use a center of dilation and a scale factor to create dilations by measuring distances.

Future Learning

In Lessons 11–12, students will perform dilations, translations, reflections, and rotations to transform figures on and off of a coordinate plane.

Connections

- Students explore how the scale factor of a dilation can be used to predict its size and location on a grid. (mathematical)

FL B.E.S.T. Math Benchmarks

Addressing

MA.8.GR.2.2
Given a preimage and image generated by a single dilation, identify the scale factor that describes the relationship.

MA.8.GR.2.3
Describe and apply the effect of a single transformation on two-dimensional figures using coordinates and the coordinate plane.

Mathematical Thinking and Reasoning Standards: MTR.4.1, MTR.5.1

ELA Expectations: ELA.K12.EE.4.1, ELA.K12.EE.5.1

English Language Development Standards: ELL.MA.1, ELL.SI.1

Building On	Building Toward
MA.6.GR.1.1	MA.912.F.2
MA.7.GR.1.5	MA.912.GR.2.4

Lesson at a Glance

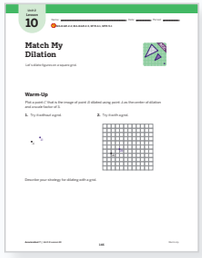
~ 45 min

MA.8.GR.2.2, MA.8.GR.2.3

Warm-Up

Pairs | 5 min

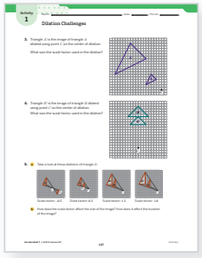
Students use the **Think-Pair-Share** routine to highlight the usefulness of a grid in identifying the precise location of the image.



Activity 1

Pairs | 10 min

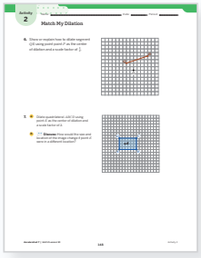
Students use the structure of a grid to perform dilations with a variety of scale factors and to draw attention to the features of dilations. **(MTR.5.1)**



Activity 2

Pairs | 10 min

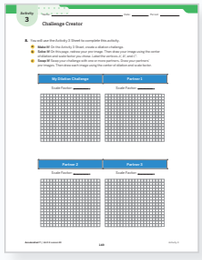
Students develop strategies for dilating figures. **(MTR.4.1)**



Activity 3

Pairs | 10 min

Students develop fluency in dilating figures given a center of dilation and scale factor.

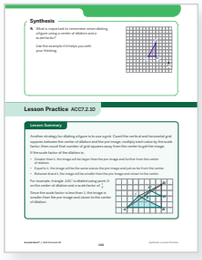


Synthesis

Whole Class | 5 min

Students synthesize what they learned about strategies used to dilate pre-images using various scale factors and centers of dilation.

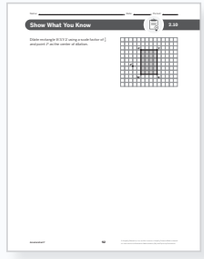
MLR2: Collect and Display



Show What You Know

Independent | 5 min

Students demonstrate their understanding by dilating a rectangle on a square grid given a center of dilation and a scale factor.



Prep Checklist

This lesson includes:

- Student Edition
- Show What You Know
- Activity 3 Sheet, one per student
- Straightedges
- Graph paper (as needed)

Pairs | 5 min

Warm-Up

Purpose: Students use the Think-Pair-Share routine to highlight the usefulness of a grid in identifying the precise location of the image.

1 Launch

To surface prior knowledge, consider reading the prompt aloud and asking, "What do you remember about the term *center of dilation*? *Scale factor*?"

Encourage students to make their best guess about where point C would be with and without a grid. Invite students to estimate instead of measuring.

2 Connect

Use the Think-Pair-Share routine to support student-to-student discourse. See the Routine Facilitation Guide for more information.

Consider asking, "How was doing the dilation different with and without the grid? Which did you prefer? What did you notice about your and your partner's dilation on Problem 1 compared to Problem 2?"

Unit 2
Lesson
10

Name: _____ Date: _____ Period: _____

MA.8.GR.2.2, MA.8.GR.2.3, MTR.4.1, MTR.5.1

Match My Dilation

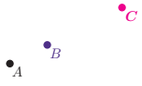
Let's dilate figures on a square grid.

Warm-Up

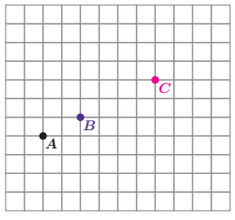
Plot a point C that is the image of point B dilated using point A as the center of dilation and a scale factor of 3.

1. Try it *without* a grid.

Responses vary.



2. Try it *with* a grid.



Describe your strategy for dilating *with* a grid.

Responses vary.

- The distance between points A and C should be 3 times the distance from point A to point B. I used the grid to count squares. To move from point A to point B, I went 1 square up and 2 squares to the right. So point C should be $1 \cdot 3 = 3$ squares up and $2 \cdot 3 = 6$ squares to the right of point A.
- To get from point A to point B on the grid, I went up 1 and right 2. To get from point B to point C, I went up 1 and right 2 two more times.

Accelerated 7 | Unit 2 Lesson 10 146 Warm-Up

Pairs | 10 min

Activity 1 Dilation Challenges

Purpose: Students use the structure of a grid to perform dilations with a variety of scale factors and to draw attention to the features of dilations. **(MTR.5.1)**

1 Launch

To surface connections, consider asking, "How might the grid be helpful here?"

Invite students to use straightedges to support them in dilating figures throughout the lesson.

2 Monitor

Invite students to write their responses and then share their thinking with a partner.

Look and listen for students who use a variety of strategies to determine the scale factor.

D Differentiation *(Problem 4)*

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Consider asking, "What kind of scale factor might create an image that is smaller than the pre-image?" **Support** Encourage students to try different types of numbers, observe the results, and revise if necessary.
Select a scale factor larger than 1 or a negative scale factor.	**Strengthen** Invite them to share what happened when they tried this strategy. Why do they think this happened?
Select $\frac{3}{4}$ (or equivalent).	**Stretch** Consider asking, "What kind of rule can we generalize for dilations?"

Select and sequence responses to share during the Connect.

ELL English Language Learners Provide sentence frames to help students explain their reasoning (e.g. "Scale factor tells us _____."). **(Reading, Writing, Speaking, and Listening)**

3 Connect

Discuss how scale factor affects the size and location of the dilated image.

To surface the Key Takeaway, display selected student responses and consider asking:

- "What information does the scale factor tell us?"
- "How might we annotate the grid to show the scale factor?" **(MTR.5.1)**

Activity 1

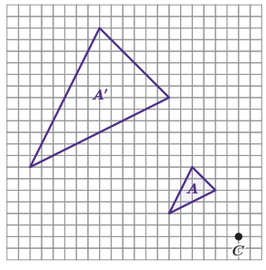
Name: _____ Date: _____ Period: _____

Dilation Challenges

3. Triangle A' is the image of triangle A dilated using point C as the center of dilation.

What was the scale factor used in the dilation?

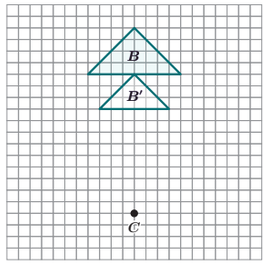
3



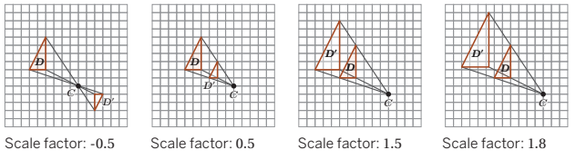
4. Triangle B' is the image of triangle B dilated using point C as the center of dilation.

What was the scale factor used in the dilation?

$\frac{3}{4}$ (or equivalent)



5. a. Take a look at these dilations of triangle D.



Scale factor: -0.5 Scale factor: 0.5 Scale factor: 1.5 Scale factor: 1.8

b. How does the scale factor affect the size of the image? How does it affect the location of the image?

Responses vary. For scale factors greater than 1, the image is bigger than the pre-image and further from the center of dilation. For scale factors between 0 and 1, the image is smaller than the pre-image and in between the pre-image and center of dilation. For scale factors less than 0, the image and pre-image are on opposite sides of the center of dilation, and the image is also flipped.

Accelerated 7 | Unit 2 Lesson 10 147 Activity 1

Key Takeaway: We can predict the relative size and location of an image using the scale factor, which helps us when performing dilations.

Pairs | 10 min

Activity 2 Match My Dilation

Purpose: Students develop strategies for dilating figures. **(MTR.4.1)**

1 Launch

Consider asking, "What looks different about this dilation compared to previous ones? What looks the same?"

Encourage students to sketch their dilations directly on the grid.

2 Monitor

Invite students to read their strategies out loud to one another before moving on to the next dilation.

A **Accessibility: Conceptual Processing** Invite students to use different colors on the activity to color-code the steps of their work.

Look and listen for a variety of strategies. **(MTR.4.1)**

D **Differentiation** *(Problem 7)*

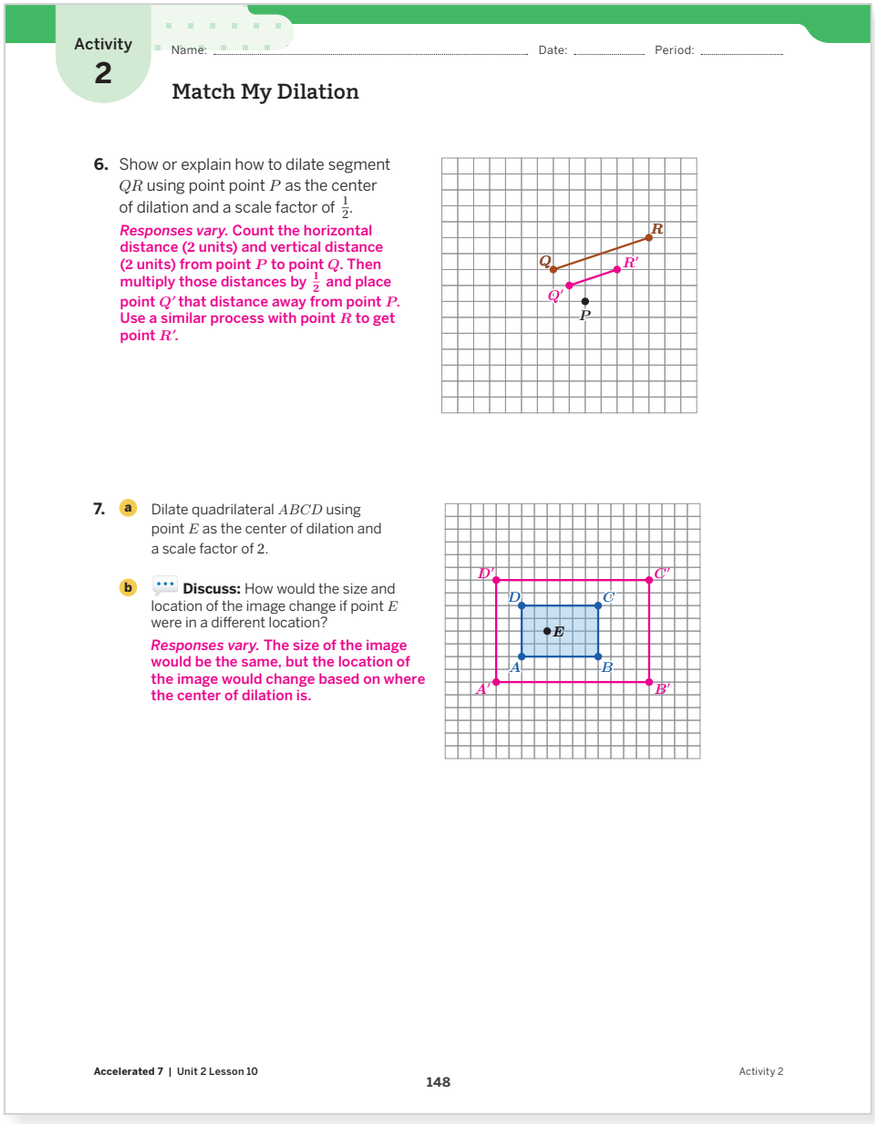
Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Encourage them to try to draw lines between the pre-image and the image.
Make a dilation that does not use a scale factor of 2.	**Support** Consider asking, "How might we count the grid units to check our work?"
Use strategies such as: • Counting horizontal and vertical units on the grid. • Drawing auxiliary lines.	**Stretch** Identify other transformations that would move quadrilateral $ABCD$ onto quadrilateral $A'B'C'D'$, making connections to prior learning.

Listen for conjectures students make during the discussion about point E if point E were in a different location to highlight during the Connect.

3 Connect

To surface connections, consider asking:

- "How might we summarize our process for dilating a figure on a grid?"
- "How does the center of dilation impact the location and size of the image?"



Activity 2

Name: _____ Date: _____ Period: _____

Match My Dilation

6. Show or explain how to dilate segment QR using point point P as the center of dilation and a scale factor of $\frac{1}{2}$.

Responses vary. Count the horizontal distance (2 units) and vertical distance (2 units) from point P to point Q. Then multiply those distances by $\frac{1}{2}$ and place point Q' that distance away from point P. Use a similar process with point R to get point R'.

7. a. Dilate quadrilateral $ABCD$ using point E as the center of dilation and a scale factor of 2.

b. **Discuss:** How would the size and location of the image change if point E were in a different location?

Responses vary. The size of the image would be the same, but the location of the image would change based on where the center of dilation is.

Accelerated 7 | Unit 2 Lesson 10 148 Activity 2

Pairs | 10 min

Activity 3 Challenge Creator

Purpose: Students develop fluency in dilating figures given a center of dilation and scale factor.

Short on time? Consider skipping or reducing time for this activity.

1 Launch

Consider sharing that students will create their own pre-images for other students to dilate using a given scale factor and center of dilation.

Distribute the Activity 3 Sheet so they can create their dilation challenge.

To encourage creativity, invite students to make a challenge they think will be different from ones their classmates create.

2 Monitor

Listen for strategies students use when discussing how to solve the problems, such as how they used the grid to apply the scale factor for a particular challenge.

Look for several creative challenges and solutions to share during the Connect.

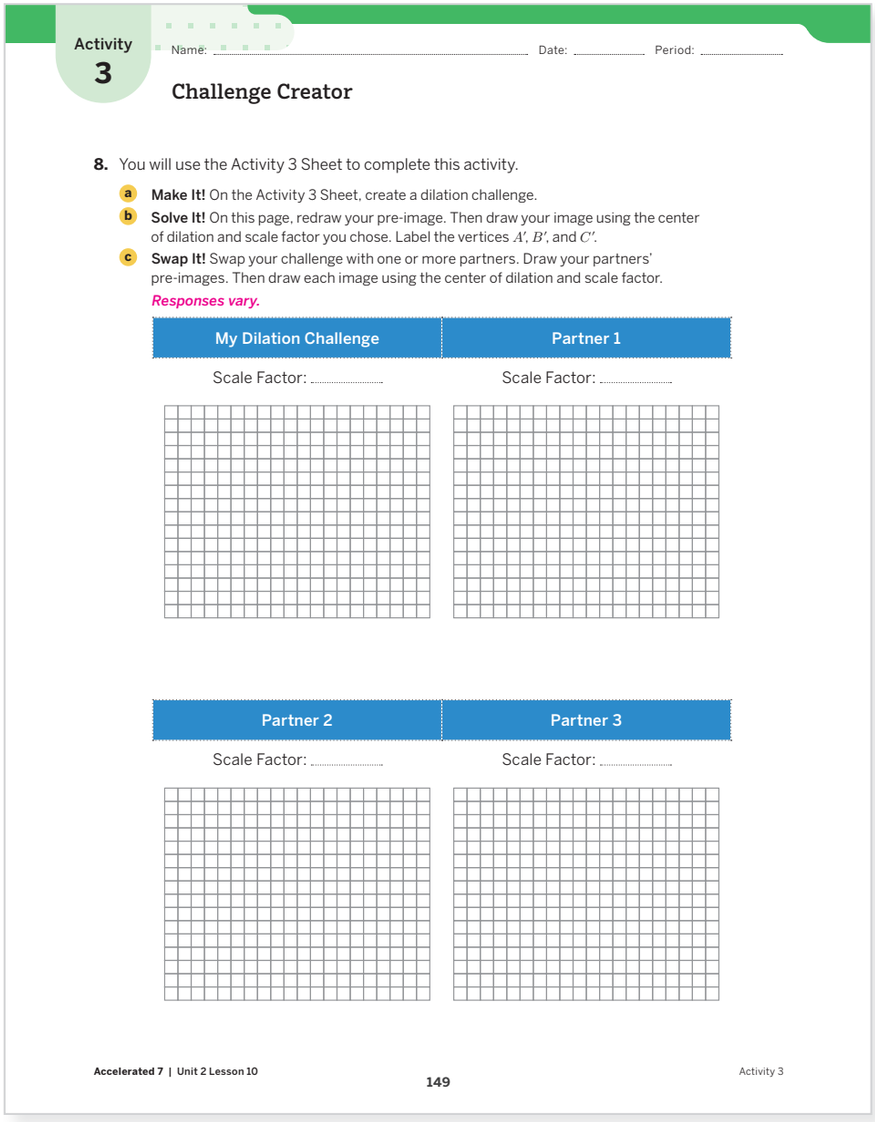
A **Accessibility: Executive Functioning** To support students initiating complex tasks, identify a challenge that would be a good place to start.

3 Connect

Invite students to share their strategies for dilating different pre-images.

Consider asking:

- "How did you pick the challenges you wanted to solve?"
- "What made some challenges more complex than others?"
- "Was there a strategy you heard that surprised you?"



Activity 3

Name: _____ Date: _____ Period: _____

Challenge Creator

8. You will use the Activity 3 Sheet to complete this activity.

a. **Make It!** On the Activity 3 Sheet, create a dilation challenge.

b. **Solve It!** On this page, redraw your pre-image. Then draw your image using the center of dilation and scale factor you chose. Label the vertices A', B', and C'.

c. **Swap It!** Swap your challenge with one or more partners. Draw your partners' pre-images. Then draw each image using the center of dilation and scale factor.

Responses vary.

My Dilation Challenge	Partner 1
Scale Factor: _____	Scale Factor: _____

Partner 2	Partner 3
Scale Factor: _____	Scale Factor: _____

Accelerated 7 | Unit 2 Lesson 10 149 Activity 3

Whole Class | 5 min

Synthesis

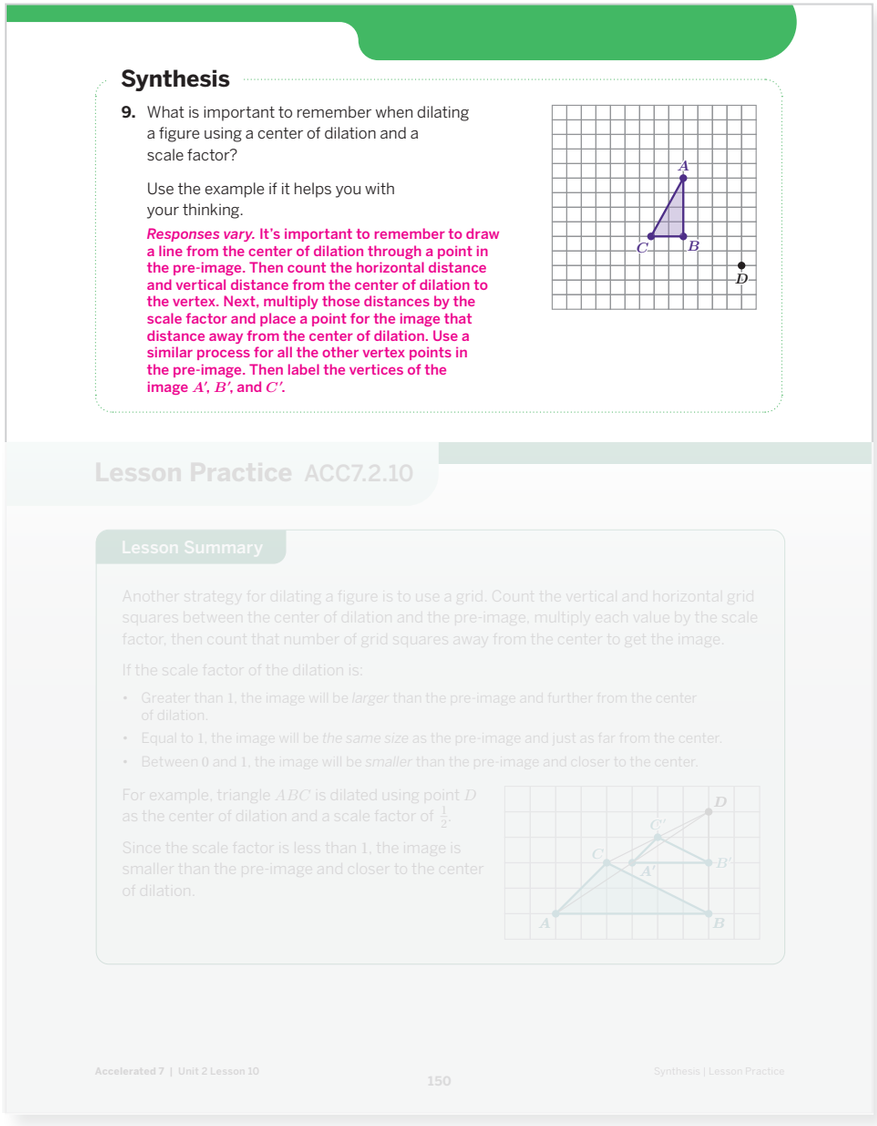
Purpose: Students synthesize what they learned about strategies used to dilate pre-images using various scale factors and centers of dilation.

Invite students to respond independently, and then share their thinking with a partner.

Collect and share a variety of ideas, such as:

- It is helpful to draw a line from the center of dilation through the vertex in the pre-image.
- Another strategy is to use a grid to count the grid squares between the center of dilation and each point.
- The created image will be the same shape as the pre-image, but will be a different size.
- The scale factor can help you predict what size the image will be and where it will be placed on the grid.

MLR **MLR2: Collect and Display** Use this routine to collect students' strategies as they discuss with a partner or share during the Connect. Display for students to reference and add to for the rest of the unit.



Synthesis

9. What is important to remember when dilating a figure using a center of dilation and a scale factor?

Use the example if it helps you with your thinking.

Responses vary. It's important to remember to draw a line from the center of dilation through a point in the pre-image. Then count the horizontal distance and vertical distance from the center of dilation to the vertex. Next, multiply those distances by the scale factor and place a point for the image that distance away from the center of dilation. Use a similar process for all the other vertex points in the pre-image. Then label the vertices of the image A', B', and C'.

Lesson Practice ACC7.2.10

Lesson Summary

Another strategy for dilating a figure is to use a grid. Count the vertical and horizontal grid squares between the center of dilation and the pre-image, multiply each value by the scale factor, then count that number of grid squares away from the center to get the image.

If the scale factor of the dilation is:

- Greater than 1, the image will be *larger* than the pre-image and further from the center of dilation.
- Equal to 1, the image will be *the same size* as the pre-image and just as far from the center.
- Between 0 and 1, the image will be *smaller* than the pre-image and closer to the center.

For example, triangle ABC is dilated using point D as the center of dilation and a scale factor of $\frac{1}{2}$.

Since the scale factor is less than 1, the image is smaller than the pre-image and closer to the center of dilation.

Accelerated 7 | Unit 2 Lesson 10 150 Synthesis | Lesson Practice

Lesson Takeaway: In order to perform a dilation, measure the distance between the center of dilation and the pre-image point, multiply that distance by the scale factor, and place the image point that distance from the center of dilation along the same line.

Independent | 5 min

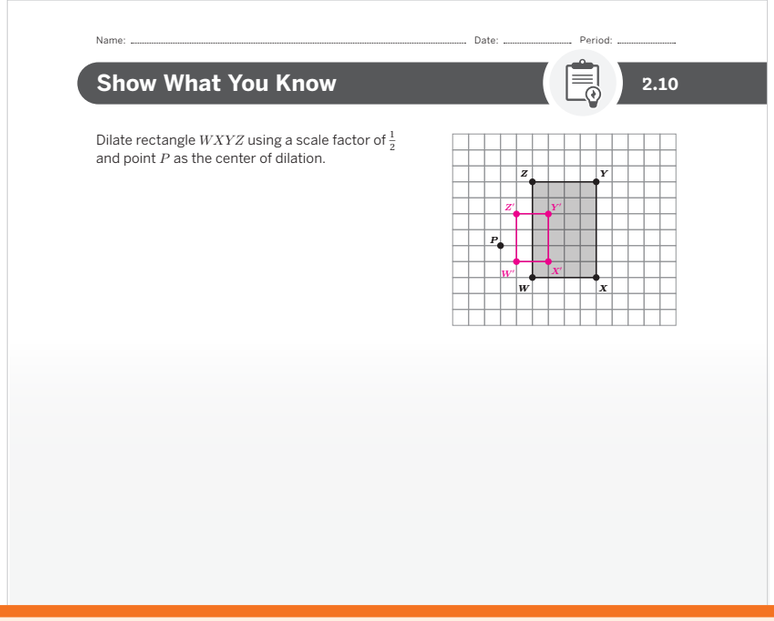
Show What You Know

Purpose: Students demonstrate their understanding by dilating a rectangle on a square grid given a center of dilation and a scale factor.

Today's Goals

1. **Goal:** Dilate figures on a grid using a center of dilation and scale factor.
2. **Language Goal:** Describe the effects of scale factors greater than 1, between 0 and 1, and equal to 1 on an image. **(Writing, Speaking, and Listening)**

Show What You Know (Assessment Resources)



Name: _____ Date: _____ Period: _____

Show What You Know 2.10

Dilate rectangle $WXYZ$ using a scale factor of $\frac{1}{2}$ and point P as the center of dilation.

Differentiation Use after Lesson 10

Support

Provide targeted intervention for students by using these resources.

If students need support with dilating shapes on a coordinate grid:

» **Respond to Student Thinking:**

- Use the ***Dilating Figures on a Square*** **Grid Mini-Lesson.**
- Revisit the **Lesson 8 Summary** and invite students to share strategies for how to use the center of dilation.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 2 Extension Activities.**

Professional Learning

Take a moment to reflect on the strategies students used in this lesson. What opportunities were there to celebrate specific students' strategies for performing dilations? What opportunities might there be in future lessons to connect back to the strategies students shared?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

9. What is important to remember when dilating a figure using a center of dilation and a scale factor?

Use the example if it helps you with your thinking.

Responses vary. It's important to remember to draw a line from the center of dilation through a point in the pre-image. Then count the horizontal distance and vertical distance from the center of dilation to the vertex. Next, multiply those distances by the scale factor and place a point for the image that distance away from the center of dilation. Use a similar process for all the other vertex points in the pre-image. Then label the vertices of the image A', B', and C'.

Lesson Practice ACC7.2.10

Lesson Summary

Another strategy for dilating a figure is to use a grid. Count the vertical and horizontal grid squares between the center of dilation and the pre-image, multiply each value by the scale factor, then count that number of grid squares away from the center to get the image.

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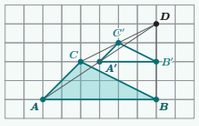
For example, triangle ABC is dilated using point D as the center of dilation and a scale factor of $\frac{1}{2}$.

Since the scale factor is less than 1, the image is smaller than the pre-image and closer to the center of dilation.



Accelerated 7 | Unit 2 Lesson 10 150 Synthesis | Lesson Practice

Lesson Practice
ACC7.2.10

Name: Date: Period:

Problems 1–2: Segment AB measures 3 centimeters. Point O is the center of dilation.

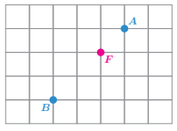
1. How long is the image of AB after a dilation with a scale factor of $\frac{1}{5}$?

$\frac{3}{5}$ centimeters

2. How long is the image of AB after a dilation with a scale factor of s?

$3s$ centimeters

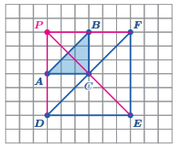
3. Here are points A and B. Point F is the image of point B using point A as the center of dilation and a scale factor of $\frac{1}{3}$. Plot point F.



4. Triangle ABC was transformed into triangle DFE using a dilation.

Label the center of dilation P. Then determine the scale factor.

Center of dilation P shown on diagram.
The scale factor is 2.

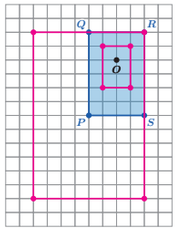


Problems 5–6: Here is rectangle $PQRS$.

Draw the image of rectangle $PQRS$ after each dilation.

5. A dilation using point R as the center of dilation and a scale factor of 2.

6. A dilation using point O as the center of dilation and a scale factor of $\frac{1}{2}$.



Accelerated 7 | Unit 2 Lesson 10 151 Lesson Practice

Lesson Practice
ACC7.2.10

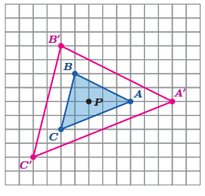
Name: Date: Period:

FAST Practice

Problems 7–9: Here is triangle ABC.

7. Dilate each vertex of triangle ABC using point P as the center of dilation and a scale factor of 2. Draw the image and label the vertices $A'B'C'$.

Response shown on grid.



8. If the length of side AB is 4.5 units and the length of side BC is 4.1 units, what are the lengths of side $A'B'$ and side $B'C'$?

- Side $A'B'$ is 9 units long.
- Side $B'C'$ is 8.2 units long.

9. How would the image be affected if point P were in a different location? Select *all* the statements that are true.

- ☑ **A.** The location of the image would change based on where the center of dilation is located.
- ☑ **B.** The distance from point A' to point P will always be twice the distance from point A to point P.
- ☑ **C.** The size of the image would remain unchanged regardless of the new position of point P.
- ☐ **D.** The size of the image would change proportionally to the distance from the center of dilation.
- ☐ **E.** The location of the image would remain fixed even if point P were moved.

Spiral Review

Problems 10–11: Triangle ABC has vertices $A(3, 5)$, $B(6, 1)$, and $C(2, -2)$. Determine the coordinates of the vertices of the image for each transformation.

10. Triangle ABC is translated up 10 units.

$A'(3, 15)$, $B'(6, 11)$, and $C'(2, 8)$.

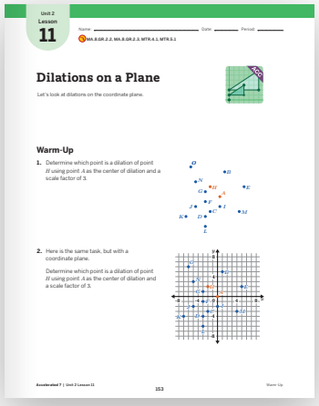
11. Triangle ABC is rotated 90° clockwise around the origin.

$A'(5, -3)$, $B'(1, -6)$, and $C'(-2, -2)$.

Accelerated 7 | Unit 2 Lesson 10 152 Lesson Practice

Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1–2	1	MA.8.GR.2.2
	3–4	2	MA.8.GR.2.2, MA.8.GR.2.3
	5–6	1	MA.8.GR.2.2, MA.8.GR.2.3
FAST Practice	7–9	2	MA.8.GR.2.2, MA.8.GR.2.3
Spiral Review			
	10–11	1	MA.8.GR.2.3


Unit 2
Lesson
11

Dilations on a Plane

Let's look at dilations on the coordinate plane.

Warm-Up

1. Determine which point is a dilation of point M using point A as the center of dilation and a scale factor of 3.

2. Here is the same task, but with a coordinate plane.

   Determine which point is a dilation of point M using point A as the center of dilation and a scale factor of 3.

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Dilations on a Plane

Dilations With Coordinates

Let's look at dilations on the coordinate plane.

Learning Outcomes

Today's Goals

1. **Goal:** Dilate a figure using its coordinates, a center of dilation, and a scale factor.
2. **Language Goal:** Explain how dilations affect the coordinates of a figure when the center of dilation is the origin. **(Speaking and Writing)**

Students describe and perform dilations on a coordinate plane. The coordinates allow for more precise descriptions of dilations. Students also practice using precise language for dilations as they describe them to a partner.

Coherence

Prior Learning

Students performed dilations using a straightedge in Lesson 8 and using a square grid without coordinates in Lesson 10.

Future Learning

In Lesson 12, students will define *similarity* using a combination of a rigid transformation and a dilation.

Connections

- Students use precise language when describing dilations in a partner activity. (mathematical, cross-curricular)
- Students continue to expand their use of the coordinate plane to dilate figures. (mathematical)

FL B.E.S.T. Math Benchmarks

Addressing

MA.8.GR.2.2
Given a preimage and image generated by a single dilation, identify the scale factor that describes the relationship.

MA.8.GR.2.3
Describe and apply the effect of a single transformation on two-dimensional figures using coordinates and the coordinate plane.

Mathematical Thinking and Reasoning Standards: MTR.4.1, MTR.5.1

ELA Expectation: ELA.K12.EE.3.1

English Language Development Standards: ELL.MA.1, ELL.SI.1

Building On	Building Toward
MA.7.GR.1.5	MA.9.12.F.2 MA.912.GR.2.4

Vocabulary

Review Vocabulary

origin

Lesson at a Glance

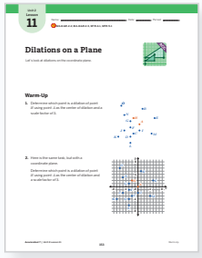
~ 45 min

MA.8.GR.2.2, MA.8.GR.2.3

Warm-Up

Pairs | 5 min

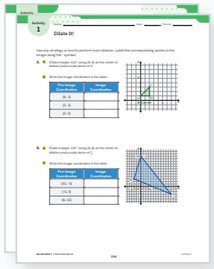
Students dilate a point with and without a coordinate plane to surface strategies that can be used with this new structure.



Activity 1

Pairs | 15 min

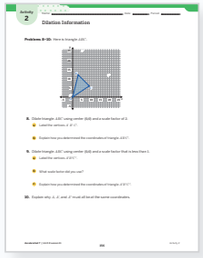
Students develop fluency with dilating figures on the coordinate plane using the *origin* as the center of dilation. **(MTR.5.1)**



Activity 2

Pairs | 15 min

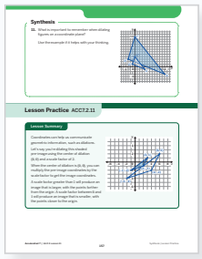
Students precisely describe the effect of scale factor on a figure within the coordinate plane. **(MTR.5.1)**



Synthesis

Whole Class | 5 min

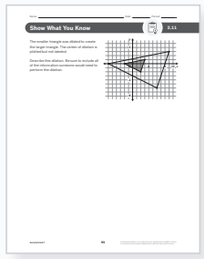
Students synthesize their understanding of performing dilations precisely on the coordinate plane. **(MTR.4.1)**



Show What You Know

Independent | 5 min

Students demonstrate their understanding of performing and reading precise dilations.



Prep Checklist

This lesson includes:

- Student Edition
- Show What You Know
- Coloring tools

Pairs | 5 min

Warm-Up

Purpose: Students dilate a point with and without a coordinate plane to surface strategies that can be used with this new structure.

1 Launch

Direct students' attention to the points without a grid and highlight point A as the center of dilation.

Consider asking, "What do you remember about dilating points?"

A **Accessibility: Visual-Spatial Processing**
Invite students to use coloring tools to support interpreting the points shown with and without a grid.

Invite students to move on to the second problem.

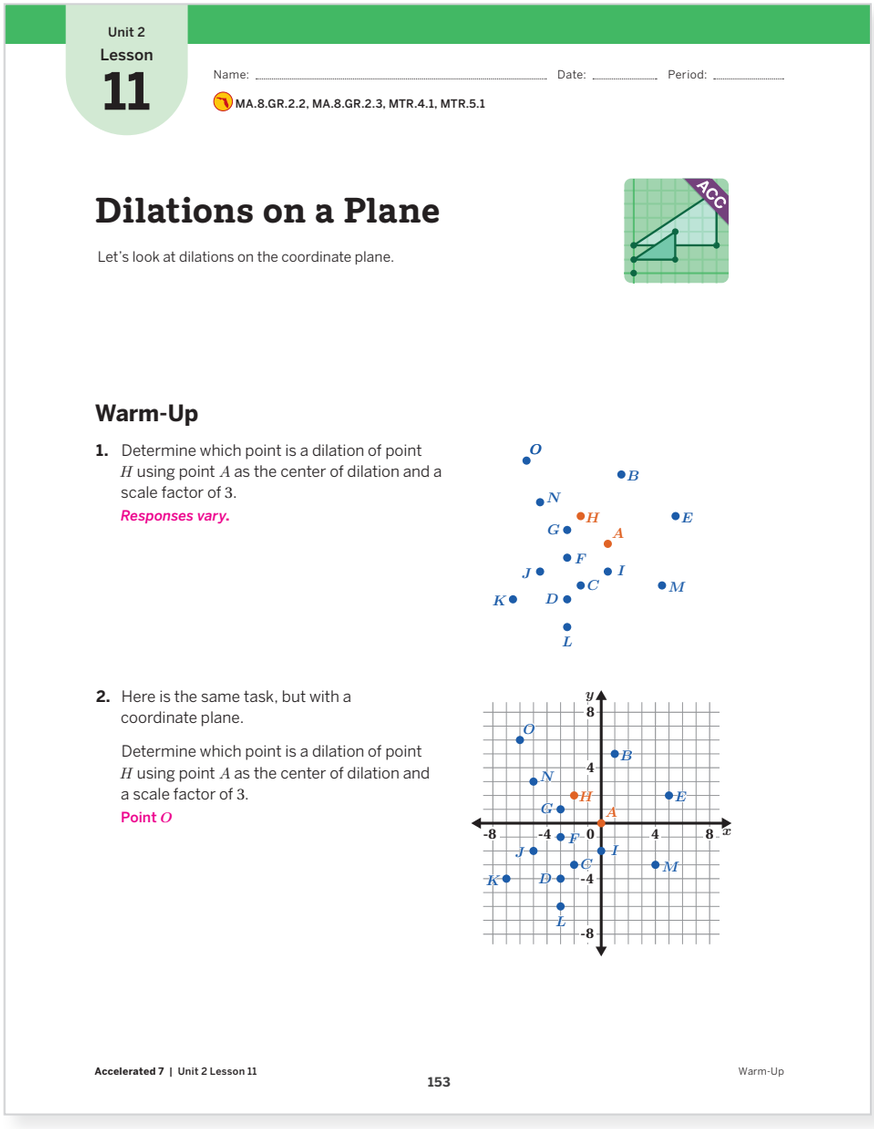
2 Connect

Direct students' attention to Problem 2 to discuss student strategies for dilating point H on the coordinate plane.

To surface prior knowledge, invite students to share how they used the units of the grid to determine distance between points.

Consider asking, "How did the coordinate plane change your strategy?"

Note: If it does not come up naturally, consider discussing how coordinates can be helpful for dilating points.



Unit 2
Lesson
11

Name: _____ Date: _____ Period: _____

MA.8.GR.2.2, MA.8.GR.2.3, MTR.4.1, MTR.5.1

Dilations on a Plane

Let's look at dilations on the coordinate plane.

Warm-Up

1. Determine which point is a dilation of point H using point A as the center of dilation and a scale factor of 3.
Responses vary.

2. Here is the same task, but with a coordinate plane.

Determine which point is a dilation of point H using point A as the center of dilation and a scale factor of 3.
Point O

Accelerated 7 | Unit 2 Lesson 11 153 Warm-Up

Pairs | 15 min

Activity 1 Dilate it!

Purpose: Students develop fluency with dilating figures on the coordinate plane using the *origin* as the center of dilation. **(MTR.5.1)**

Short on time? Consider omitting Problems 5 and 6.

1 Launch

Direct the students' attention to the first dilation and invite students to note any key pieces of information in the directions, such as the location of the center of dilation.

To support students getting started, discuss what it means to "label the corresponding points in the image using the ′ symbol."

A **Accessibility: Executive Functioning** To support students initiating complex tasks, identify possible places to start. For example, focus on dilating one point at a time.

2 Monitor

Encourage students to annotate any helpful points or distances.

Select dilation strategies to share during the Connect. Listen for patterns and relationships students identify among the coordinates, the grid, and the scale factor.

D **Differentiation** *(Problems 3–6)*

Look for students who:	Respond to Student Thinking
Need support with reading coordinates.	**Support** Encourage them to label the pre-image coordinates and focus on one point at a time.
Plot imprecise points.	**Support** Celebrate the strategies they already used to plot image points. Invite them to draw auxiliary lines connecting points to the center of dilation and then measure distances using the grid.
Determine an algorithm to determine the image coordinates.	**Stretch** Consider asking, "Does this strategy always work?
Dilate figures using a strategy that works consistently.	**Stretch** To preview similarity, consider asking, "What's the relationship between the pre-image and the image? What is the same about these figures? What is different?"

Activity 1 continued >

Activity 1

Name: Date: Period:

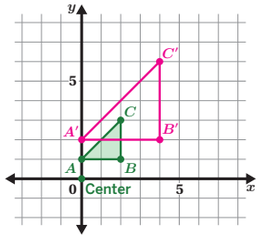
Dilate It!

Use any strategy or tool to perform each dilation. Label the corresponding points in the image using the ′ symbol.

3. a Dilate triangle ABC using $(0, 0)$ as the center of dilation and a scale factor of 2.

b Write the image coordinates in the table.

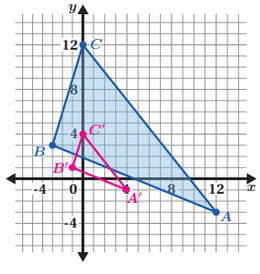
Pre-Image Coordinates	Image Coordinates
(0, 1)	(0, 2)
(2, 1)	(4, 2)
(2, 3)	(4, 6)



4. a Dilate triangle ABC using $(0, 0)$ as the center of dilation and a scale factor of $\frac{1}{3}$.

b Write the image coordinates in the table.

Pre-Image Coordinates	Image Coordinates
(12, -3)	(4, -1)
(-3, 3)	(-1, 1)
(0, 12)	(0, 4)



Accelerated 7 | Unit 2 Lesson 11 154 Activity 1

Pairs | 15 min

Activity 1 Dilate it! (continued)

Purpose: Students develop fluency with dilating figures on the coordinate plane using the *origin* as the center of dilation. **(MTR.5.1)**

Short on time? Consider omitting Problems 5 and 6.

2 Monitor

ELL **English Language Learners** Encourage students to paraphrase each other's ideas as they perform each dilation to help make connections and incorporate vocabulary from the unit. **(Speaking and Listening)**

3 Connect

Invite a few students to share their strategies and ideas.

To surface connections, consider asking:

- "What strategies were helpful for these dilations?"
- "How was dilating using a coordinate plane similar to dilating using a grid? How was it different?" **(MTR.5.1)**

Note: If it does not come up naturally, discuss the pattern students may have noticed involving multiplying pre-image coordinates by the scale factor to dilate figures.

Activity 1

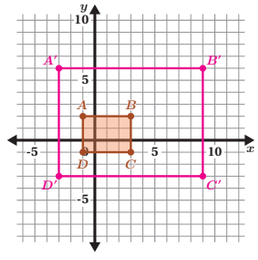
Name: Date: Period:

Dilate It! (continued)

5. a Dilate rectangle $ABCD$ using $(0, 0)$ as the center of dilation and a scale factor of 3.

b Write the image coordinates in the table.

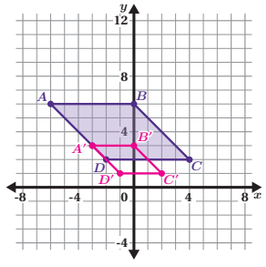
Pre-Image Coordinates	Image Coordinates
(-1, 2)	(-3, 6)
(3, 2)	(9, 6)
(3, -1)	(9, -3)
(-1, -1)	(-3, -3)



6. a Dilate parallelogram $ABCD$ using $(0, 0)$ as the center of dilation and a scale factor of $\frac{1}{2}$.

b Write the image coordinates in the table.

Pre-Image Coordinates	Image Coordinates
(-6, 6)	(-3, 3)
(0, 6)	(0, 3)
(4, 2)	(2, 1)
(-2, 2)	(-1, 1)



7. Review your work for Problems 2–6.

Discuss: What effect does the scale factor have on the coordinates of a figure?

Responses vary. The coordinates of the image of a dilation on the coordinate plane are a product of the scale factor and the x- and y- coordinates of the pre-image.

Accelerated 7 | Unit 2 Lesson 11 155 Activity 1

Pairs | 15 min

Activity 2 Dilation Information

Purpose: Students precisely describe the effect of scale factor on a figure within the coordinate plane. **(MTR.5.1)**

1 Launch

Direct students' attention to the figure in the coordinate plane.

Consider demonstrating how to make a table to organize the coordinates for the pre-image and the images from Problems 8 and 9.

2 Monitor

Listen for a variety of descriptions and questions, particularly students who use precise language, like *center of dilation* and *scale factor*. Invite them to share during the Connect.

A **Accessibility: Visual-Spatial Processing**
Invite students to use coloring tools to support interpreting visual represntation.

3 Connect

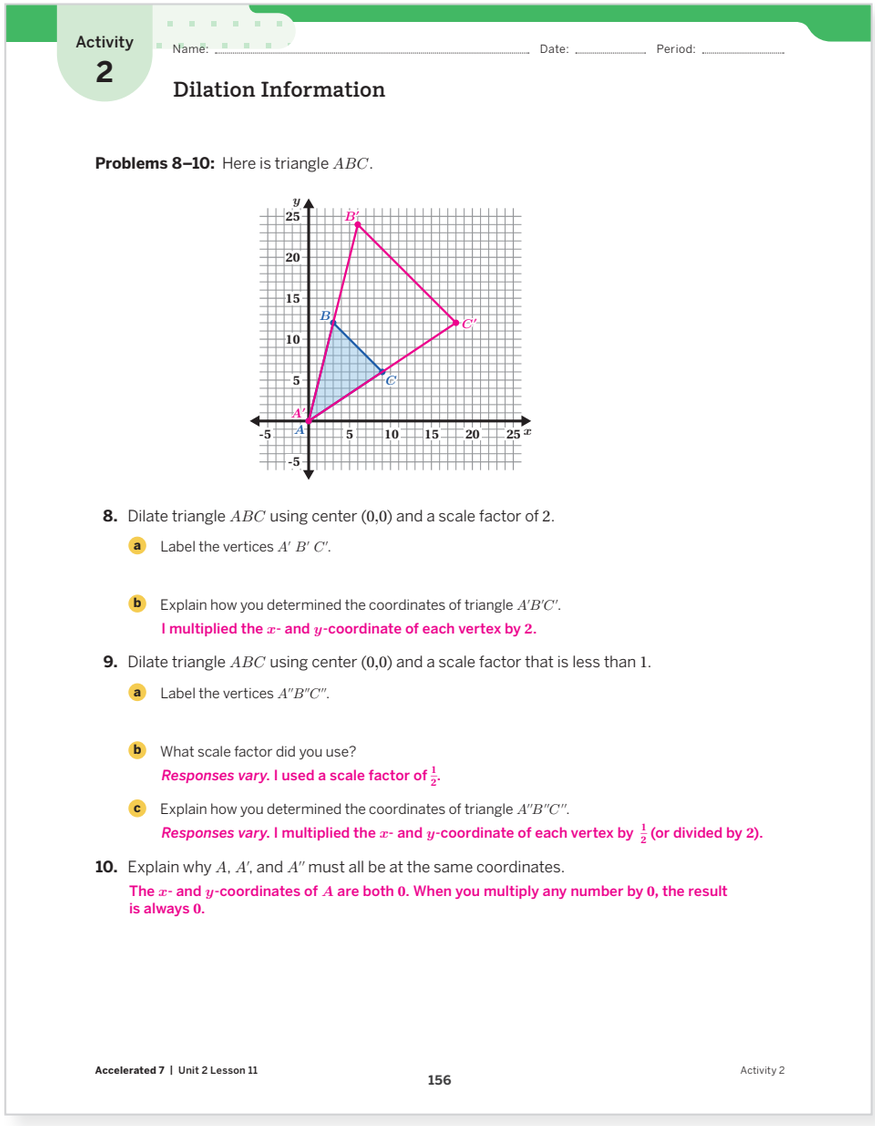
Invite students to share the scale factors they used for Problem 9 and any related observations.

Create a record of student dilation descriptions.

Invite students to share their thoughts about Problem 10.

To surface the Key Takeaway, consider asking:
- "What information do you need to perform a dilation on the coordinate plane?"
- "What patterns can you generalize about the relationship between the coordinates of the pre-image, the scale factor, and the coordinates of the image?" **(MTR.5.1)**

Encourage students to add ideas to a class poster of recorded responses.



Activity 2

Name: Date: Period:

Dilation Information

Problems 8–10: Here is triangle ABC.

8. Dilate triangle ABC using center $(0,0)$ and a scale factor of 2.

a. Label the vertices A' B' C'.

b. Explain how you determined the coordinates of triangle $A'B'C'$.
I multiplied the x- and y-coordinate of each vertex by 2.

9. Dilate triangle ABC using center $(0,0)$ and a scale factor that is less than 1.

a. Label the vertices $A''B''C''$.

b. What scale factor did you use?
Responses vary. I used a scale factor of $\frac{1}{2}$.

c. Explain how you determined the coordinates of triangle $A''B''C''$.
Responses vary. I multiplied the x- and y-coordinate of each vertex by $\frac{1}{2}$ (or divided by 2).

10. Explain why A, A', and A'' must all be at the same coordinates.
The x- and y-coordinates of A are both 0. When you multiply any number by 0, the result is always 0.

Accelerated 7 | Unit 2 Lesson 11 156 Activity 2

Key Takeaway: When the center of dilation is the *origin*, multiply the pre-image coordiantes by the scale factor to dilate the figure.

Whole Class | 5 min

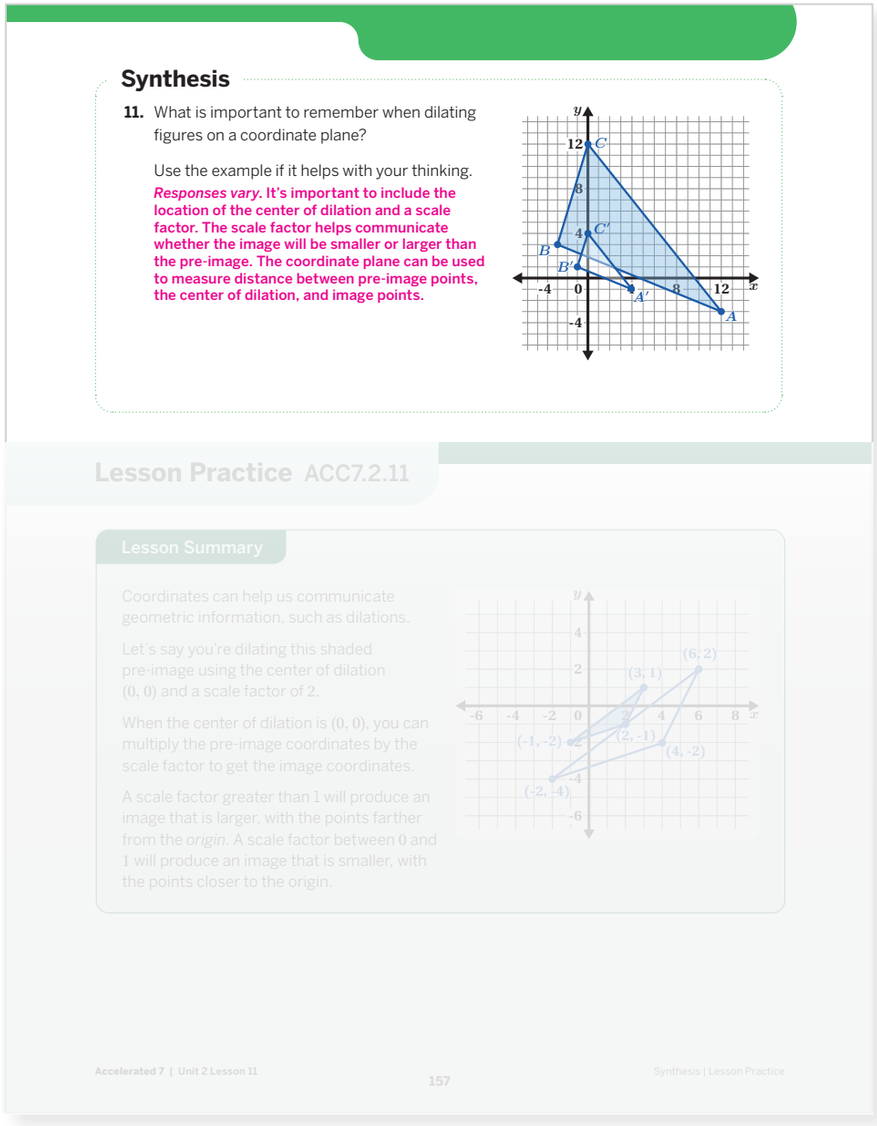
Synthesis

Purpose: Students synthesize their understanding of performing dilations precisely on the coordinate plane. **(MTR.4.1)**

Invite students to respond independently, and then share their thinking with a partner.

Collect and share a variety of ideas, such as:

- When the center of dilation is the origin, we can multiply the scale factor by the pre-image coordinates to get the image coordinates.
- The scale factor tells us how large the image will be compared to the pre-image.



Synthesis

11. What is important to remember when dilating figures on a coordinate plane?

Use the example if it helps with your thinking.

Responses vary. **It's important to include the location of the center of dilation and a scale factor. The scale factor helps communicate whether the image will be smaller or larger than the pre-image. The coordinate plane can be used to measure distance between pre-image points, the center of dilation, and image points.**

Lesson Practice ACC7.2.11

Lesson Summary

Coordinates can help us communicate geometric information, such as dilations.

Let's say you're dilating this shaded pre-image using the center of dilation (0, 0) and a scale factor of 2.

When the center of dilation is (0, 0), you can multiply the pre-image coordinates by the scale factor to get the image coordinates.

A scale factor greater than 1 will produce an image that is larger, with the points farther from the *origin*. A scale factor between 0 and 1 will produce an image that is smaller, with the points closer to the origin.

Accelerated 7 | Unit 2 Lesson 11 157 Synthesis | Lesson Practice

Lesson Takeaway: We use coordinates to perform dilations precisely. When the center of dilation is the *origin*, the scale factor can be multiplied by the figure's coordinates.

Independent | 5 min

Show What You Know

Purpose: Students demonstrate their understanding of performing and reading precise dilations.

Today's Goals

1. **Goal:** Dilate a figure using its coordinates, a center of dilation, and a scale factor.
2. **Language Goal:** Explain how dilations affect the coordinates of a figure when the center of dilation is the origin. **(Speaking and Writing)**

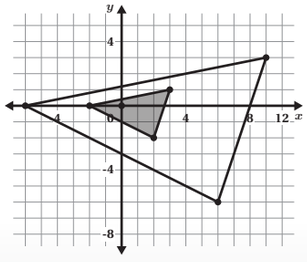
Show What You Know (Assessment Resources)

Name: _____ Date: _____ Period: _____

Show What You Know 2.11

The smaller triangle was dilated to create the larger triangle. The center of dilation is plotted but not labeled.

Describe this dilation. Be sure to include all of the information someone would need to perform the dilation.



Responses vary. Responses should include:

- The center of dilation is (0, 0) because the distance between the pre-image points and (0, 0) grows with the same scale factor for all points in the original figure.
- The triangle being dilated has vertices at (-2, 0), (3, 1), and (2, -2).
- The scale factor is 3 because the distance between each vertex and the center of dilation grows by a factor of 3. For example, the vertex (-2, 0) is 2 units away from the center of dilation. The corresponding vertex in the image is (-6, 0), which is 6 units away from the same point. This distance is multiplied by a factor of 3.

Differentiation Use after Lesson 11

Support

Provide targeted intervention for students by using these resources.

If students need support with dilations in the coordinate plane:

» **Respond to Student Thinking**

- Use the ***Dilating Figures on a Square Grid*** **Mini-Lesson**.
- Revisit ***Activity 1*** (especially **Problems 5 and 6**). Encourage students to determine if multiplying coordinates and scale factors always, sometimes, or never works.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 2 Extension Activities.**

Professional Learning

Take a moment to reflect on this lesson. How did the way in which you learned about dilations inform how you planned for the lesson? How did it impact how you taught the lesson?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

11. What is important to remember when dilating figures on a coordinate plane?

Use the example if it helps with your thinking.

Responses vary. It's important to include the location of the center of dilation and a scale factor. The scale factor helps communicate whether the image will be smaller or larger than the pre-image. The coordinate plane can be used to measure distance between pre-image points, the center of dilation, and image points.

Lesson Practice ACC7.2.11

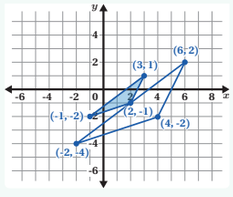
Lesson Summary

Coordinates can help us communicate geometric information, such as dilations.

Let's say you're dilating this shaded pre-image using the center of dilation **(0, 0)** and a scale factor of 2.

When the center of dilation is **(0, 0)**, you can multiply the pre-image coordinates by the scale factor to get the image coordinates.

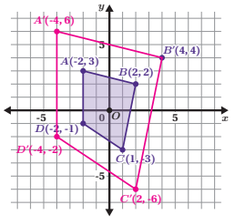
A scale factor greater than 1 will produce an image that is larger, with the points farther from the *origin*. A scale factor between **0** and **1** will produce an image that is smaller, with the points closer to the origin.



Lesson Practice
ACC7.2.11

Name: _____ Date: _____ Period: _____

Problems 1–3: Quadrilateral $ABCD$ is dilated with the origin as the center of dilation, moving point B onto point B'.

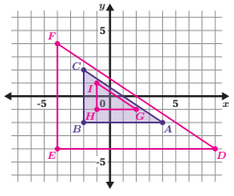


1. What is the scale factor of the dilation?
The scale factor is 2.

2. Draw quadrilateral $A'B'C'D'$.
Response shown on graph.

3. Label the coordinate points for A', C', and D'.
Response shown on graph.

Problems 4–6: Here is triangle ABC on the coordinate plane.



4. Using the origin as the center and a scale factor of 2, draw the dilation of triangle ABC. Label the image triangle DEF.
Response shown on graph.

5. Using the origin as the center and a scale factor of $\frac{1}{2}$, draw the dilation of triangle ABC. Label the image triangle GHI.
Response shown on graph.

6. Triangle GHI is a dilation of triangle DEF. Identify the center of dilation and the scale factor.
The center of dilation is the origin and the scale factor is $\frac{1}{4}$.

Lesson Practice
ACC7.2.11

Name: _____ Date: _____ Period: _____

FAST Practice

7. Here are the pre-image and image coordinates of points on a graph. Complete the statement so that it correctly describes the transformation.

Pre-Image Coordinates	Image Coordinates
(0, 4)	(0, 12)
(-1, 3)	(-3, 9)
(5, -6)	(15, -18)

Record your answers in the spaces provided.

This is a dilation with (0,0) as the center of dilation and a scale factor of 3.

Spiral Review

Problems 8–9: Use what you know about the interior angle measures of triangles to complete these problems.

8. Triangle JKL is a right triangle, and the measure of angle J is 28°. What are the measures of the other two angles?
The measures of the other two angles are 90° and 62°.

9. Triangle PQR is an obtuse triangle, and the measure of angle Q is 72°. What are possible measures of the other two angles?
Responses vary. The sum of the other two angle measures should equal 108°, and one angle measure should be greater than 90°.

Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1–6	2	MA.8.GR.2.2, MA.8.GR.2.3
FAST Practice	7	2	MA.8.GR.2.2
Spiral Review			
	8–9	2	MA.8.GR.1.5

Assess and Respond

Sub-Unit Quiz 1

Independent | 45 min

Facilitation: Assign the Sub-Unit Quiz to learn about your students' understanding of the concepts and skills so far in this unit. If time allows, consider inviting students to complete one or more of the tasks in the Unit Synthesis and Reflection.

Item Analysis

Problem(s)	Concept or skill	Addressed in	DOK	FL B.E.S.T. Math Benchmarks
1	Identifying a dilation from an image and pre-image	Lesson 1	2	MA.8.GR.2.2
2	Using the structure of dilations to identify pairs of polygons that show a pre-image and a dilated image	Lesson 2	1	MA.8.GR.2.2
3a	Using the structure of dilations to determine the scale factor of a dilated image	Lesson 2	1	MA.8.GR.2.2
3b	Using the structure of dilations to determine a side measurement of a dilated image	Lesson 3	1	MA.8.GR.2.2
4	Using the structure of dilations to precisely describing a dilation that maps a smaller figure onto a larger figure	Lesson 4	1	MA.8.GR.2.3
5	Drawing an image after a dilation	Lesson 5	2	MA.8.GR.2.3

Assessment Resources

- Student Assessments
- Assessment Rubrics with Answers
- Show What You Knows and Answer Keys

Intervention and Extension Resources

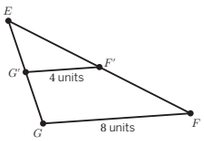
- Mini-Lessons
- Extensions

Name: ..... Date: ..... Period: .....

Sub-Unit Quiz 2

Unit 2

1. Which dilation moves triangle GEF onto triangle $G'EF'$?



A. Center of dilation: point G
Scale factor: 2

B. Center of dilation: point G
Scale factor: $\frac{1}{2}$

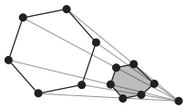
C. Center of dilation: point E
Scale factor: 2

D. Center of dilation: point E
Scale factor: $\frac{1}{2}$

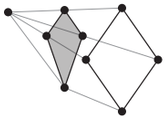
2. Select *all* of the pairs of polygons where the image appears to be the result of a dilation of the pre-image.

☑ **A.** Pair 1

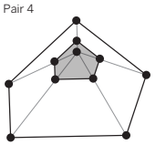
☐ **B.** Pair 2



☐ **C.** Pair 3



☑ **D.** Pair 4



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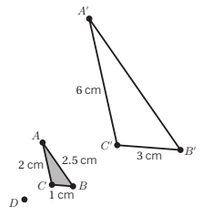
Name: ..... Date: ..... Period: .....

Sub-Unit Quiz 2 (continued)

Unit 2

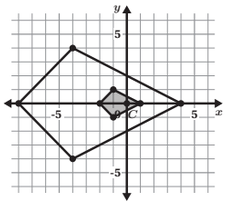
3. Triangle ABC is dilated using point D as the center of dilation and a scale factor to form triangle $A'B'C'$.



a What is the scale factor?
3

b What is the measurement of side $A'B'$?
7.5 centimeters

4. Precisely describe a dilation that takes the smaller figure to the larger figure.
Responses vary. **The smaller figure is dilated using point C at the origin as the center of dilation and a scale factor of 4.**



D Differentiation (Sub-Unit Quiz 2)

Note: To **strengthen** and **stretch** students' learning, refer to the differentiation resources suggested throughout this unit and in the Unit Overview.

Sub-Unit Goals	Problem(s)	To respond to student thinking, consider:	
• Describe dilations precisely using terms such as pre-image, center of dilation, and scale factor. • Apply dilations to figures on and off a coordinate grid. **(Lessons 7–11)**	1	S **Support** S **Strengthen**	• **Teacher Move:** Consider revisiting Lesson 7 (*Sketchy Dilations*)
	2, 3	S **Support** S **Strengthen**	• **Teacher Move:** Consider revisiting: • Lesson 8 (*Dilation Mini Golf*)
	4	S **Support**	• **Mini-Lesson:** *Dilating Figures on a Square Grid* • **Teacher Move:** Consider revisiting Lesson 10 (*Match My Dilation*)
		S **Strengthen**	• **Challenge Creator:** Lesson 10 (*Match My Dilation*)
	5	S **Support** S **Strengthen**	• **Teacher Move:** Consider revisiting: • Lesson 11 (*Dilations on a Plane*)

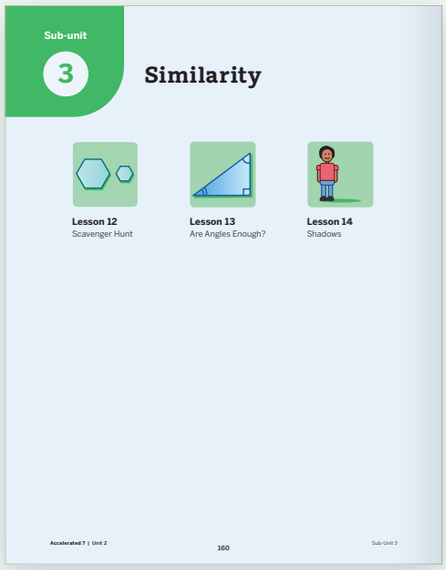
Sub-Unit 3

Similarity

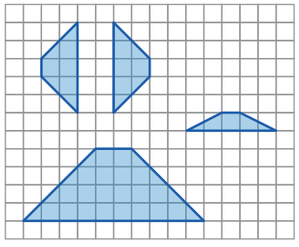
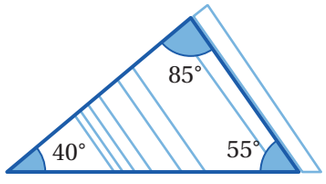
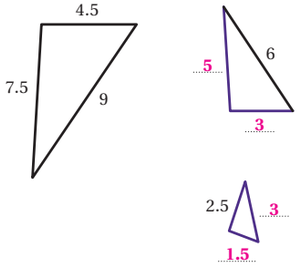
Sub-Unit 3 Goals

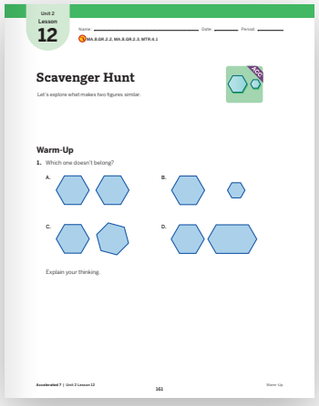
- **Goal:** Use transformations to determine whether two figures are similar.
- **Language Goal:** Justify whether two figures are similar using terms such as *translation, reflection, rotation, dilation, pre-image,* and *image*.

In this sub-unit, students determine if a pre-image can be moved onto an image using a dilation or by combining a dilation with a rigid transformation. They identify such figures as **similar**. Students also identify similarity in triangles using congruent corresponding angles and explore how equivalent ratios represent corresponding side lengths in similar triangles. Students use these ratios to help determine missing side lengths of similar triangles.



Math That Matters Most

Concept or Skill	For example . . .
Similar figures have congruent corresponding angles and corresponding side lengths with a common scale factor.	What does it mean for two figures to be similar? What does it tell you about their corresponding side lengths and angle measures? Use the example if it helps with your thinking. *Two figures are similar when one can be mapped to the other using a rigid transformation combined with a dilation. If figures are similar, their corresponding angles are all congruent and their corresponding sides are all proportional (related by the same scale factor).* 
Justify that two triangles are similar using their angle measures.	Here are some triangles that all have 40°, 85°, and 55° angles. Discuss: Are these triangles all similar? *Yes, all of the triangles are similar because if you dilate one of the triangles it fits onto the other triangles.* 
Describe and apply equivalent ratios involved in similar triangles to determine missing side lengths.	Here are three similar triangles. Determine all the side lengths using as few hints as you can. You can ask for the measure of two or three side lengths, if needed. What was your strategy? *I noticed that the longest side of the biggest triangle was 1.5 times the length of the corresponding side in the medium triangle. I used this ratio to determine the missing side lengths of the medium triangle. Then, I noticed that the middle side of the smallest triangle was half the length of the corresponding side in the medium triangle. I used this ratio to calculate the other missing side lengths.* 


Unit 2
Lesson
12
Scavenger Hunt
Let's explore what makes two figures similar.
Warm-Up
1. Which one doesn't belong?
A.
B.
C.
D.
Explain your thinking.

Scavenger Hunt

Similar Figures, Sides, and Angles

Let's explore what makes two figures similar.

Learning Outcomes

Today's Goals

1. **Goal:** Recognize that two figures are similar if there is a combination of a rigid transformation and a dilation that moves one onto another.
2. **Goal:** Recognize that similar figures have congruent corresponding angles and corresponding side lengths with a common scale factor.
3. **Language Goal:** Justify whether two figures are similar, congruent, or neither using the language of transformations. **(Writing, Speaking, and Listening)**

Students think about **similar** figures in two ways — using transformations to show whether two figures are similar and thinking about their side lengths and angle measures. They come to understand that similar figures have *congruent* angles (because none of the four transformations change angle measures) and proportional side lengths (connected to the scale factor of the dilation).

Coherence

Prior Learning

In Lessons 7–11, students performed transformations by hand. In Unit 1, students learned properties of congruent figures.

Future Learning

In Lesson 13, students will explore whether or not angle measures are sufficient for determining if two triangles are similar.

Connections

- Students collaborate in a scavenger hunt activity to identify patterns to develop a precise definition of similar figures. (mathematical)
- Students apply similar figures in the field of fashion design. (real world)

FL B.E.S.T. Math Benchmarks

Addressing

MA.8.GR.2.2
Given a preimage and image generated by a single dilation, identify the scale factor that describes the relationship.

MA.8.GR.2.3
Describe and apply the effect of a single transformation on two-dimensional figures using coordinates and the coordinate plane.

Mathematical Thinking and Reasoning Standard: MTR.4.1

ELA Expectations: ELA.K12.EE.1.1, ELA.K12.EE.4.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.6.GR.1.1	MA.912.F.2
MA.7.GR.1.5	MA.912.GR.2
	MA.912.GR.2.4

Vocabulary

New Vocabulary

similar

Review Vocabulary

congruent

Lesson at a Glance

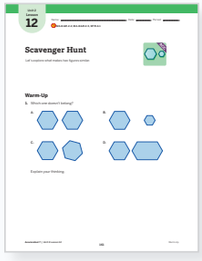
~ 45 min

MA.8.GR.2.2, MA.8.GR.2.3

Warm-Up

Whole Class | 5 min

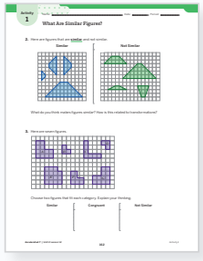
Students use the **Which One Doesn't Belong?** routine to name characteristics of pairs of figures. **(MTR.4.1)**



Activity 1

Pairs | 10 min

Students explore what makes figures **similar** and use transformations to determine whether pairs of figures are similar, *congruent*, or neither. **Routine:** Think-Pair-Share



Activity 2

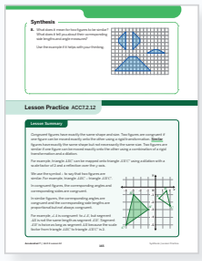
Pairs | 20 min

Students use a scavenger hunt to explore relationships between corresponding side lengths and angles in similar figures.

Synthesis

Whole Class | 5 min

Students synthesize their understanding of what makes two figures similar.



Show What You Know

Independent | 5 min

Students demonstrate their understanding by determining if two figures are similar using their angle and side length measurements.

Prep Checklist

Prepare the Activity 2 Cards.

This lesson includes:

- Student Edition
- Show What You Know
- Activity 2 Cards, one card per student, pre-cut
- Protractors
- Rulers
- Straightedges
- Tracing paper

Whole Class | 5 min

Warm-Up

Purpose: Students use the Which One Doesn't Belong? routine to name characteristics of pairs of figures. **(MTR.4.1)**

1 Launch

Direct students' attention to the four pairs of figures.

Use the Which One Doesn't Belong? routine to support students in noticing and using precise language to describe pairs of figures. This will support students as they determine if pairs of figures are congruent, similar, or neither throughout the lesson. **(MTR.4.1)**

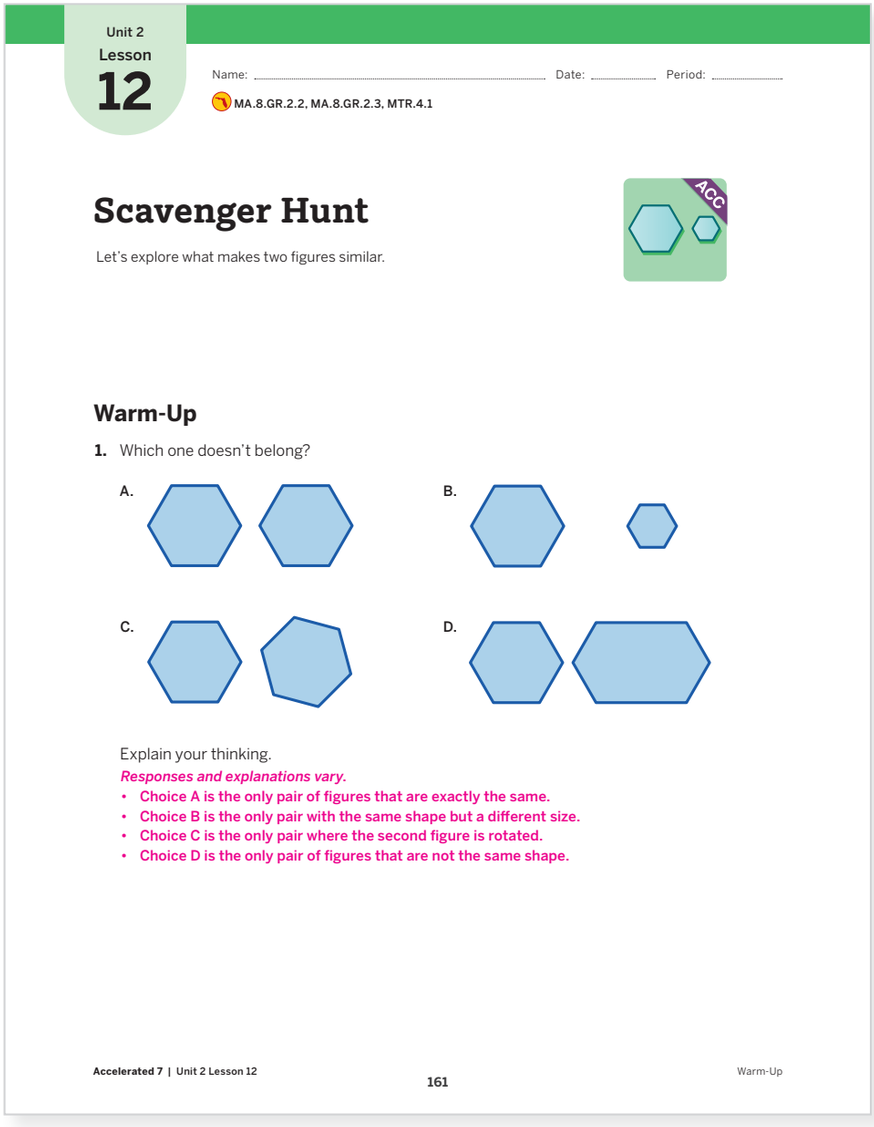
Consider sharing that each pair does not belong in some way. Encourage students to look for more than one possibility.

2 Connect

Invite students to share at least one reason why *each* pair doesn't belong.

Consider asking:

- "What is another way to say why pair ___ doesn't belong?"
- "Which pairs could be the pre-image and image after a transformation? Which transformation(s)?"
- "Are any of the pairs of figures *congruent*? How do you know?"



Unit 2
Lesson
12

Name: _____ Date: _____ Period: _____

MA.8.GR.2.2, MA.8.GR.2.3, MTR.4.1

Scavenger Hunt

Let's explore what makes two figures similar.

ACC

Warm-Up

1. Which one doesn't belong?

A.

B.

C.

D.

Explain your thinking.

Responses and explanations vary.

- Choice A is the only pair of figures that are exactly the same.
- Choice B is the only pair with the same shape but a different size.
- Choice C is the only pair where the second figure is rotated.
- Choice D is the only pair of figures that are not the same shape.

Accelerated 7 | Unit 2 Lesson 12 161 Warm-Up

Pairs | 10 min

Activity 1 What Are Similar Figures?

Purpose: Students explore what makes figures **similar** and use transformations to determine whether pairs of figures are similar, *congruent*, or neither.

1 Launch

Direct students' attention to the examples of shapes that are similar and not similar.

Use the Think-Pair-Share routine to help students shape their ideas about similar figures. See the Routine Facilitation Guide for more information.

Create a class definition of the term **similar** using students' language and the glossary. Record it for students to refer to later in the unit.

ELL **English Language Learners** Invite students to share different meanings of the term *similar*. Clarify which meaning this word will have for this class. **(Reading and Listening)**

2 Monitor

Encourage students to share their reasoning with a partner and work together to reach an agreement before choosing pairs of figures.

Look and listen for arguments students create about why pairs of figures are congruent, similar, or neither. **(MTR.4.1)**

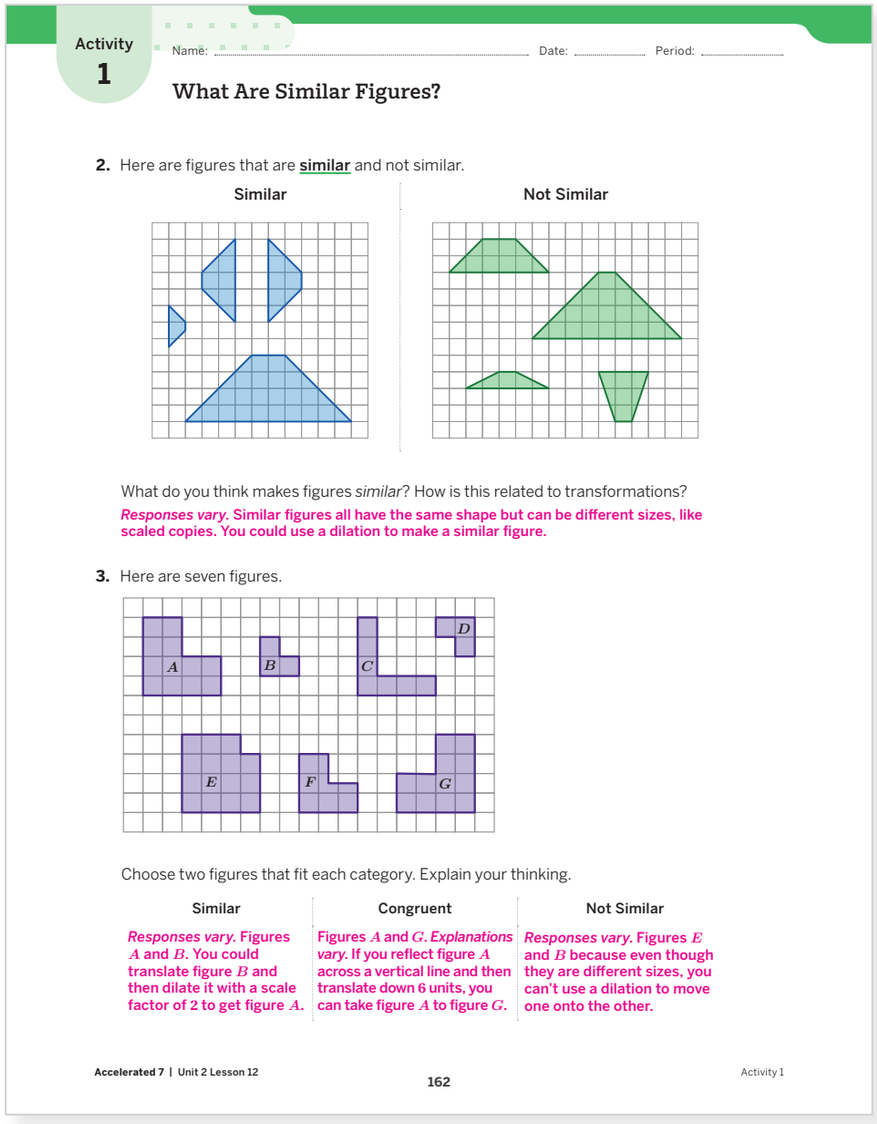
D **Differentiation** *(Problem 3)*

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Encourage them to choose one figure and ask, "Are there any other figures you could move this onto using transformations? Which ones?"
Say figures A and C are similar or congruent because they both look like an L.	**Support** Consider asking, "Which transformations would you use to move figure A onto figure C?"
Say figures A and B are similar with a vague explanation.	**Strengthen** Consider asking, "How could you use transformations to convince someone that these are similar?"

3 Connect

Direct students' attention to Problem 3 and invite students to make arguments for different pairs of figures and justify their reasoning. **(MTR.4.1)**

To surface the Key Takeaway, consider asking, "How is the strategy for deciding that figures are similar like deciding they are congruent? How is it different?"



Activity 1

Name: _____ Date: _____ Period: _____

What Are Similar Figures?

2. Here are figures that are **similar** and not similar.

Similar

Not Similar

What do you think makes figures *similar*? How is this related to transformations?

Responses vary. Similar figures all have the same shape but can be different sizes, like scaled copies. You could use a dilation to make a similar figure.

3. Here are seven figures.

Choose two figures that fit each category. Explain your thinking.

Similar	Congruent	Not Similar
Responses vary. Figures A and B. You could translate figure B and then dilate it with a scale factor of 2 to get figure A.	Figures A and G. *Explanations vary.* If you reflect figure A across a vertical line and then translate down 6 units, you can take figure A to figure G.	*Responses vary.* Figures E and B because even though they are different sizes, you can't use a dilation to move one onto the other.

Accelerated 7 | Unit 2 Lesson 12 162 Activity 1

Key Takeaway: Figures are **similar** if one can be moved onto the other using a combination of a rigid transformation and a dilation.

Pairs | 20 min

Activity 2 Scavenger Hunt

Purpose: Students use a scavenger hunt to explore relationships between corresponding side lengths and angles in similar figures.

Short on time? Consider omitting Rounds 1 and 4.

1 Launch

To surface prior knowledge, invite students to paraphrase each question in their own words. It may be helpful to discuss what corresponding sides and angles are.

Direct students' attention to the instructions for how to find a partner. Consider asking, "What are some things that figures might have in common?"

A **Accessibility: Executive Functioning** To support this multi-step activity, invite two students to play an example round for the class.

Distribute one card per student and invite them to look at their card, then to find a partner based on the criteria.

Note: If students are unable to find a partner (i.e., there is an odd number of students, they think none of the figures meet the requirement, etc.), invite them to sketch a figure that would be a yes for three or more of the questions.

2 Monitor

Look and listen for strategies students use, such as laying the figures on top of each other, using a ruler or protractor to find precise measurements, or using tracing paper.

Direct students' attention to the next set of criteria, invite students to find a new partner, and repeat.

Direct students' attention to the pair of figures and invite students to work with their current partner to analyze them.

D **Differentiation: (Support)** Look for students who say that the figures are similar or congruent because the sides are congruent. Consider asking them, "Which transformations would you use to move one onto the other?"

Pause to invite students to share their thinking for each question by polling the class. Invite students to make arguments and justify their reasoning for each response if there is no consensus. **(MTR.4.1)**

Consider asking, "What would need to change for the figures to be similar? Congruent?"

Activity 2 continued >

Activity 2

Name: _____ Date: _____ Period: _____

Scavenger Hunt

4. You will be given a card. For each round, you will have instructions for finding a partner. Once you have a partner, compare your cards and answer each question. ***Responses vary.***

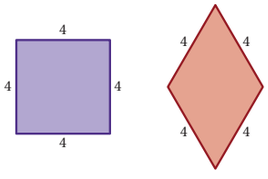
Round 1

Questions	Yes	No
Can a sequence of transformations move one figure onto the other?		
Are the figures similar?		
Are the figures congruent?		
Are the corresponding sides congruent?		
Are the corresponding angles congruent?		

Round 2

Questions	Yes	No
Can a sequence of transformations move one figure onto the other?		
Are the figures similar?		
Are the figures congruent?		
Are the corresponding sides congruent?		
Are the corresponding angles congruent?		

5. Here are two figures. Decide whether each statement is true or false.



a. Their corresponding sides are congruent.
True

b. Their corresponding angles are congruent.
False

c. The figures are similar.
False

d. The figures are congruent.
False

Accelerated 7 | Unit 2 Lesson 12 163 Activity 2

Pairs | 20 min

Activity 2 Scavenger Hunt (continued)

Purpose: Students use a scavenger hunt to explore relationships between corresponding side lengths and angles in similar figures.

Short on time? Consider omitting Rounds 1 and 4.

2 Monitor

Direct students' attention to the next set of criteria, invite students to find a new partner, and repeat.

Note: If time allows, consider pausing and asking, "Does anyone have figures that are similar but not congruent? Congruent but not similar?"

Invite students to analyze the four statements in Problem 7.

D Differentiation *(Problem 7)*

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Encourage them to think of or sketch an example from the scavenger hunt to help with their thinking.
Say that if figures are similar, they are never congruent or they never have congruent corresponding sides.	**Strengthen** Consider sketching two congruent figures and asking, "Are these figures similar? How could you use transformations to show it?"
Clearly explain their thinking.	**Stretch** Consider inviting them to write their own statements that are always, sometimes, or never true.

A Accessibility: Conceptual Processing Invite students to answer a more concrete question first to support abstract thinking, like, "Draw two similar figures. Are these also congruent?"

Note: Students may need support with statement d. Here are two alternate descriptions:

- "The ratio of a side in one figure to its corresponding side in the other figure is the same for all sides."
- "You can use a single number to multiply all of the side lengths in one figure to get the corresponding side lengths in the other."

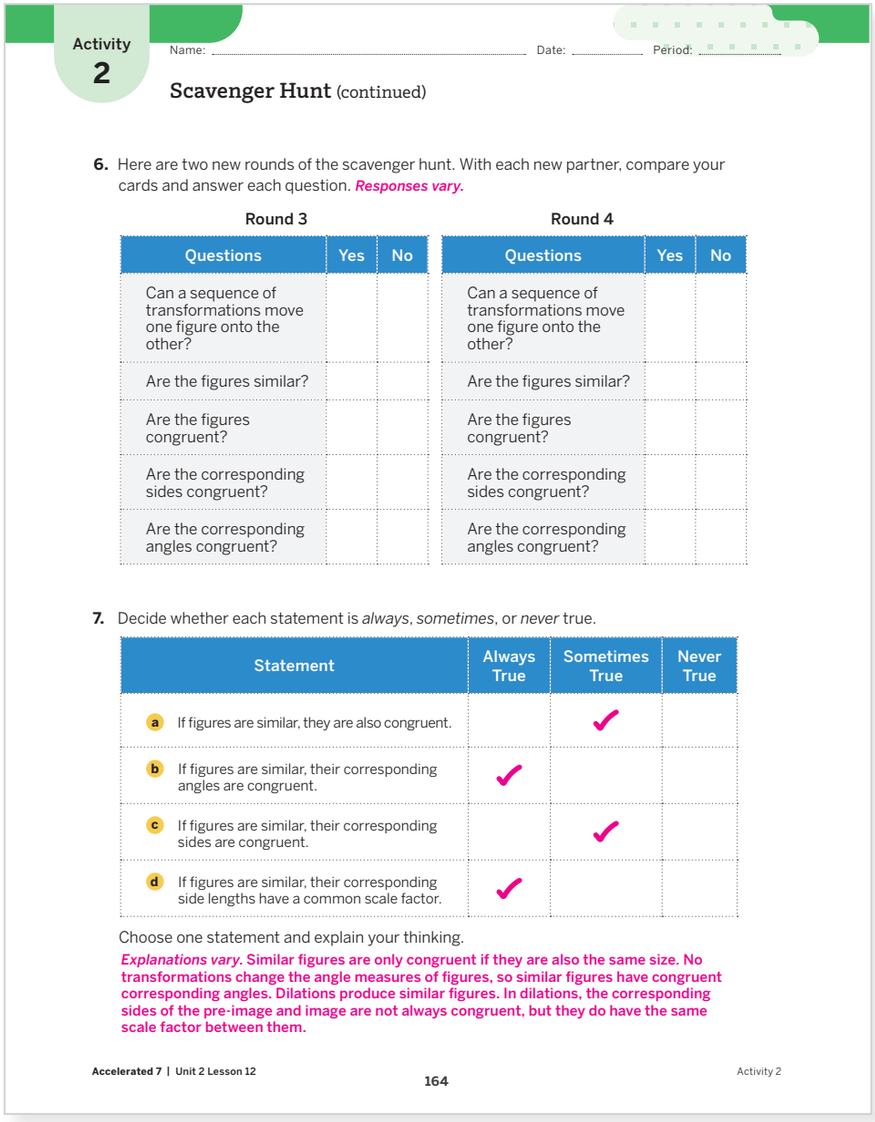
3 Connect

In Problem 7, **Invite students** to discuss each statement one by one.

To surface the Key Takeaway, consider asking:

- "What did you notice about the corresponding angles of similar figures? How does this compare to congruent figures?"
- "What did you notice about the corresponding sides of similar figures? How does this compare to congruent figures?"

Discuss what the class might add to the definition or anchor chart made earlier about similar figures.



Activity 2

Name: _____ Date: _____ Period: _____

Scavenger Hunt (continued)

6. Here are two new rounds of the scavenger hunt. With each new partner, compare your cards and answer each question. ***Responses vary.***

Round 3

Questions	Yes	No
Can a sequence of transformations move one figure onto the other?		
Are the figures similar?		
Are the figures congruent?		
Are the corresponding sides congruent?		
Are the corresponding angles congruent?		

Round 4

Questions	Yes	No
Can a sequence of transformations move one figure onto the other?		
Are the figures similar?		
Are the figures congruent?		
Are the corresponding sides congruent?		
Are the corresponding angles congruent?		

7. Decide whether each statement is *always*, *sometimes*, or *never* true.

Statement	Always True	Sometimes True	Never True
a. If figures are similar, they are also congruent.		✓	
b. If figures are similar, their corresponding angles are congruent.	✓		
c. If figures are similar, their corresponding sides are congruent.		✓	
d. If figures are similar, their corresponding side lengths have a common scale factor.	✓		

Choose one statement and explain your thinking.

Explanations vary. **Similar figures are only congruent if they are also the same size. No transformations change the angle measures of figures, so similar figures have congruent corresponding angles. Dilations produce similar figures. In dilations, the corresponding sides of the pre-image and image are not always congruent, but they do have the same scale factor between them.**

Accelerated 7 | Unit 2 Lesson 12 164 Activity 2

Key Takeaway: Similar figures and *congruent* figures both have congruent corresponding angles. Congruent figures have congruent corresponding sides and similar figures have corresponding side lengths with a common scale factor.

Whole Class | 5 min

Synthesis

Purpose: Students synthesize their understanding of what makes two figures similar.

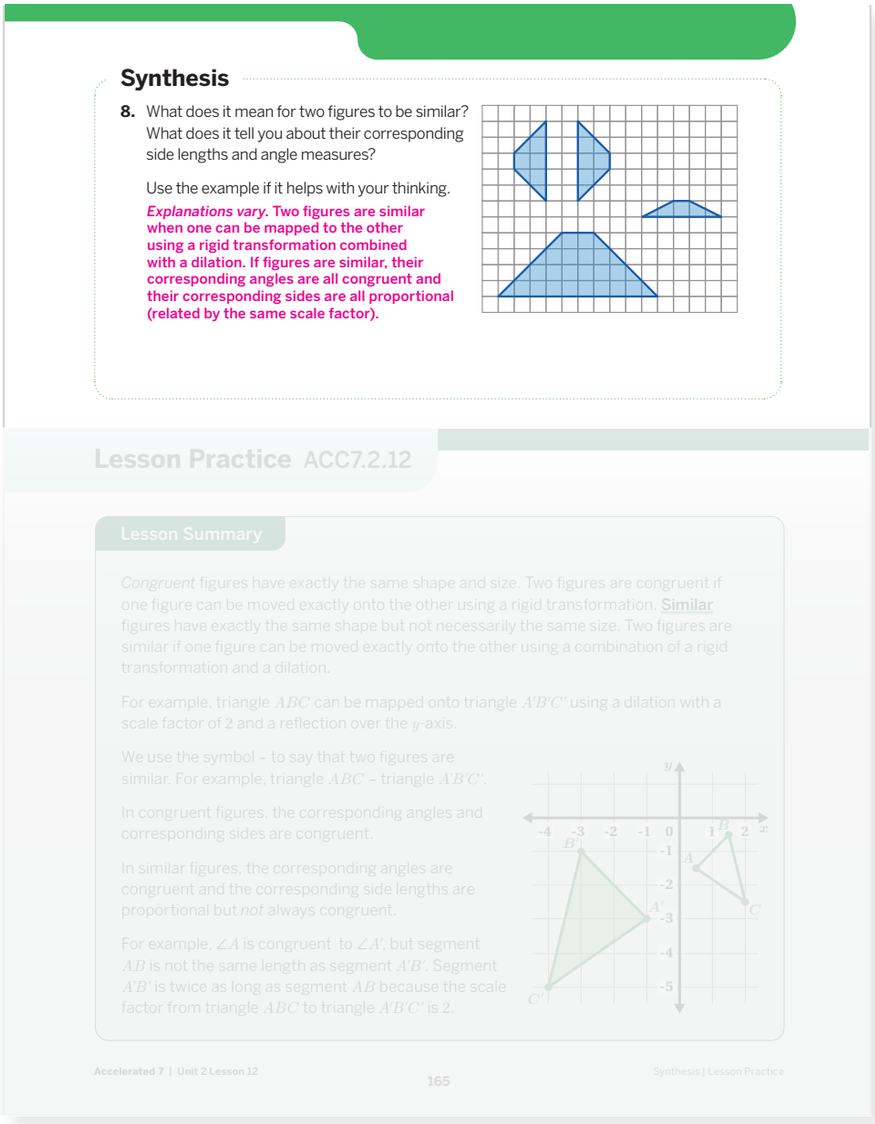
Invite students to respond independently, and then share their thinking with a partner.

Collect and share a variety of ideas, such as:

- Figures are similar if you can use a combination of a rigid transformation and a dilation to move one onto the other.
- Corresponding side lengths in similar figures are proportional (have the same scale factor between them).
- Corresponding angles in similar figures are congruent.

Create or add to an anchor chart about similar figures developed throughout class.

Note: If time allows, invite students to revise their description based on the discussion.



Synthesis

8. What does it mean for two figures to be similar? What does it tell you about their corresponding side lengths and angle measures?

Use the example if it helps with your thinking.

Explanations vary. **Two figures are similar when one can be mapped to the other using a rigid transformation combined with a dilation. If figures are similar, their corresponding angles are all congruent and their corresponding sides are all proportional (related by the same scale factor).**

Lesson Takeaway: Two figures are **similar** if one figure can be moved exactly onto the other using a combination of a rigid transformation and a dilation. In similar figures, the corresponding angles are *congruent* and the corresponding side lengths are proportional but not always congruent.

Independent | 5 min

Show What You Know

Purpose: Students demonstrate their understanding by determining if two figures are similar using their angle and side length measurements.

Today's Goals

1. **Goal:** Recognize that two figures are similar if there is a combination of a rigid transformation and a dilation that moves one onto another.
2. **Goal:** Recognize that similar figures have congruent corresponding angles and corresponding side lengths with a common scale factor.
3. **Language Goal:** Justify whether two figures are similar, congruent, or neither using the language of transformations. **(Writing, Speaking, and Listening)**

Show What You Know (Assessment Resources)

Name: _____ Date: _____ Period: _____

Show What You Know 2.12

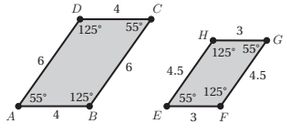
Is quadrilateral $ABCD$ similar to quadrilateral $EFGH$?

Yes quadrilateral $ABCD$ is similar to quadrilateral $EFGH$.

Explain your thinking.

Explanations vary.

- The corresponding angles are congruent, and there is a common scale factor of $\frac{3}{4}$ between the corresponding sides.



Differentiation Use after Lesson 12

Support

Provide targeted intervention for students by using these resources.

If students need support with identifying similar figures:

» **Respond to Student Thinking:**

- Review the **Lesson Summary** with students before providing individualized feedback on the relevant **Practice Problems**.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 3 Extension Activities.**

Professional Learning

Take a moment to reflect on how students' understanding of how the meaning of similar has developed over the course of this lesson. How might students use their different ideas about similarity in Lesson 13, where they decide what information is needed to show similarity, and in Lessons 14–16, where they use the side length relationships in similar triangles to determine unknown side lengths and explore slope?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

8. What does it mean for two figures to be similar? What does it tell you about their corresponding side lengths and angle measures?

Use the example if it helps with your thinking.

Explanations vary. Two figures are similar when one can be mapped to the other using a rigid transformation combined with a dilation. If figures are similar, their corresponding angles are all congruent and their corresponding sides are all proportional (related by the same scale factor).

Lesson Practice ACC7.2.12

Lesson Summary

Congruent figures have exactly the same shape and size. Two figures are congruent if one figure can be moved exactly onto the other using a rigid transformation. **Similar** figures have exactly the same shape but not necessarily the same size. Two figures are similar if one figure can be moved exactly onto the other using a combination of a rigid transformation and a dilation.

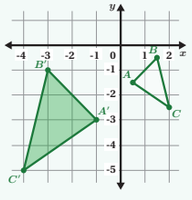
For example, triangle ABC can be mapped onto triangle $A'B'C'$ using a dilation with a scale factor of 2 and a reflection over the y-axis.

We use the symbol ~ to say that two figures are similar. For example, triangle ABC ~ triangle $A'B'C'$.

In congruent figures, the corresponding angles and corresponding sides are congruent.

In similar figures, the corresponding angles are congruent and the corresponding side lengths are proportional but *not* always congruent.

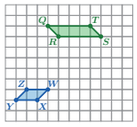
For example, $\angle A$ is congruent to $\angle A'$, but segment AB is not the same length as segment $A'B'$. Segment $A'B'$ is twice as long as segment AB because the scale factor from triangle ABC to triangle $A'B'C'$ is 2.

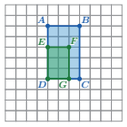


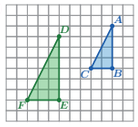
Lesson Practice ACC7.2.12

Name: _____ Date: _____ Period: _____

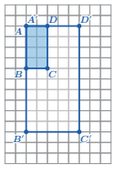
1. Which graph shows a pair of similar figures?

A. 

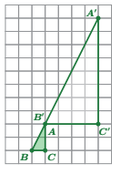
B. 

C. 

Problems 2–3: Show that the figures are similar by describing the rigid transformation, dilation, or rigid transformation and dilation that moves the shaded pre-image onto the unshaded image.

2. 

Responses vary. Dilate $ABCD$ using point A as the center of dilation and a scale factor of 2.5.

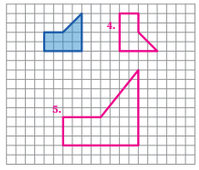
3. 

Responses vary. Translate triangle ABC so that point B moves onto point A. Dilate the image using point A as the center of dilation and a scale factor of 4.

Problems 4–5: Here is a polygon. *Responses vary.* Samples shown on graph.

4. Draw a similar polygon that could be mistaken for being *not* similar.

5. Draw a polygon that is not similar, but could be mistaken for a similar polygon.



Lesson Practice ACC7.2.12

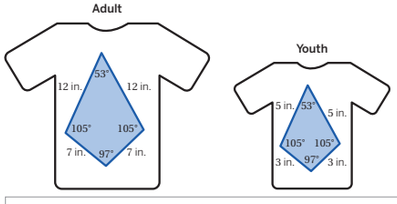
Name: _____ Date: _____ Period: _____

FAST Practice

6. A fashion designer is working with a supplier to print her design on different-sized shirts. She checks the quality of the shirts by using congruence and similarity.

Based on the measurements, do you think the designer should continue to work with the supplier? Explain your thinking using congruence and similarity.

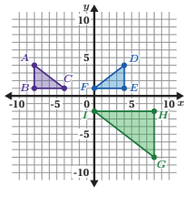
Record your answer in the space provided.



Responses vary. The designer should not continue to work with the supplier because the designs on the shirts are not similar. The corresponding angles are congruent, but there isn't a common scale factor between the corresponding sides.

Spiral Review

Problems 7–8: Here are some triangles on a graph.



7. Describe a single transformation that shows that triangle DEF is congruent to triangle ABC.

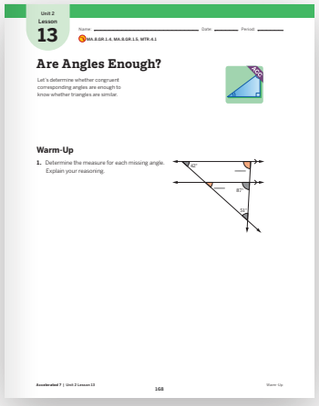
Responses vary. Reflect over the line $x = -2$.

8. Describe a sequence of transformations that shows that triangle DEF is similar to triangle GHI.

Responses vary. Dilate by a scale factor of 2 using center (0, 0), and then reflect over the x-axis.

Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1	1	MA.8.GR.2.3
	2–3	2	MA.8.GR.2.2, MA.8.GR.2.3
	4–5	2	MA.8.GR.2.3
FAST Practice	6	3	MA.8.GR.2.2, MA.8.GR.2.3
Spiral Review			
	7	2	MA.8.GR.2.1
	8	2	MA.8.GR.2.2


Unit 2
Lesson
13
Are Angles Enough?
Let's determine whether congruent corresponding angles are enough to know whether triangles are similar.
Warm-Up
1. Determine the measure for each missing angle. Explain your reasoning.

Are Angles Enough?

Similar Triangles

Let's determine whether congruent corresponding angles are enough to know whether triangles are similar.

Learning Outcomes

Today's Goals

1. **Goal:** Know that two pairs of congruent corresponding angles are enough to determine that triangles are similar.
2. **Language Goal:** Justify that two triangles are similar using their angle measures. **(Writing, Speaking, and Listening)**

Students determine whether two figures are similar by looking only at the corresponding angle measures. They see that having congruent corresponding angles is not sufficient to determine quadrilaterals are similar, but is enough to determine that triangles are similar. They build on their knowledge from Unit 1 that the sum of the angle measures in a triangle is 180 degrees, so two corresponding angle measures can help determine the third angle measure and is enough to show triangles are similar.

Coherence

Prior Learning

In Lesson 12, students saw that two figures are similar if there is a dilation or a combination of a rigid transformation and a dilation that can be applied to move one onto another. They also learned that corresponding angles in similar figures are congruent.

Future Learning

In Lesson 14, students will use the properties of similar triangles to determine missing side lengths. This builds toward slope in Lessons 15 and 16.

Connections

- Students use sufficient evidence to prove a claim that figures are similar. (mathematical, cross-curricular)

FL B.E.S.T. Math Benchmarks

Addressing

MA.8.GR.1.4
Solve mathematical problems involving the relationships between supplementary, complementary, vertical or adjacent angles.

MA.8.GR.1.5
Solve problems involving the relationships of interior and exterior angles of a triangle.

Mathematical Thinking and Reasoning Standard: MTR.4.1

ELA Expectations: ELA.K12.EE.4.1, ELA.K12.EE.6.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.4.GR.1.2	MA.912.GR.1.1
MA.4.GR.1.3	MA.912.GR.1.3
MA.5.GR.1.1	

Lesson at a Glance

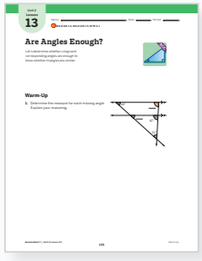
~ 45 min

MA.8.GR.1.4, MA.8.GR.1.5

Warm-Up

Pairs | 5 min

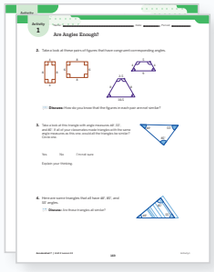
Students surface their prior knowledge of angle relationships from Unit 1 to support them in this lesson.



Activity 1

Pairs | 20 min

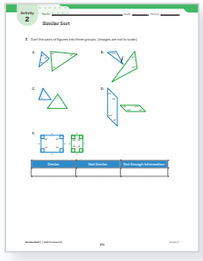
Students analyze figures to show that congruent corresponding angles are not enough evidence to show that quadrilaterals are similar but is enough to show that triangles are. **Routine:** Think-Pair-Share



Activity 2

Pairs | 10 min

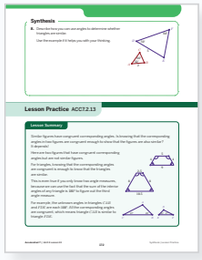
Students apply what they just learned about similar triangles and generalize that two angles are sufficient to determine triangle similarity. **(MTR.4.1)**



Synthesis

Whole Class | 5 min

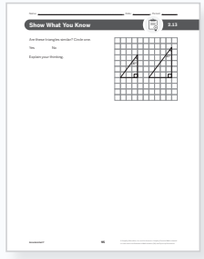
Students synthesize their understanding that two pairs of congruent angle measures are enough to show that triangles are similar. MLR1: Stronger and Clearer Each Time



Show What You Know

Independent | 5 min

Students demonstrate their understanding by determining whether two triangles are similar using their angle measures.



Prep Checklist

This lesson includes:

- Student Edition
- Show What You Know
- Tracing paper (as needed)
- Meter stick

Pairs | 5 min

Warm-Up

Purpose: Students surface their prior knowledge of angle relationships from Unit 1 to support them in this lesson.

Short on time? Consider doing this Warm-Up as a class or omitting it and discussing angle relationships as needed.

1 Launch

To surface prior knowledge, consider inviting the class to list everything they remember about angle relationships. This reference may be useful throughout this lesson.

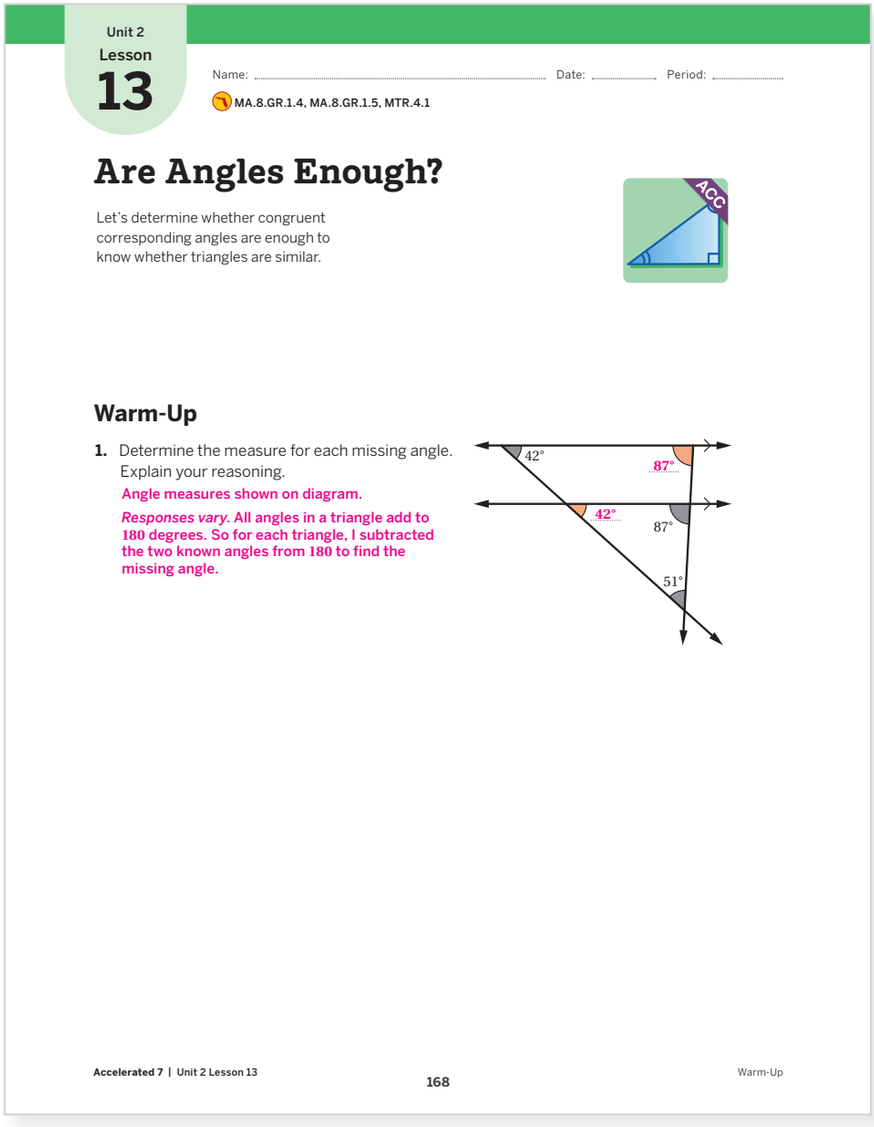
2 Connect

Invite students to share which angle measure they determined first, along with their strategies for determining how to use angle relationships in triangles to determine each of the unknown angles.

Consider asking:

- "Which angle relationships did you find most helpful?"
- "Which missing angle measure was most challenging? Why?"

ELL **English Language Learners** Provide sentence frames to help students explain their strategy. (e.g., "I know the angles in a triangle _____. So I _____.")



Unit 2
Lesson
13

Name: Date: Period:

MA.8.GR.1.4, MA.8.GR.1.5, MTR.4.1

Are Angles Enough?

Let's determine whether congruent corresponding angles are enough to know whether triangles are similar.

Warm-Up

1. Determine the measure for each missing angle. Explain your reasoning.

   Angle measures shown on diagram.

   Responses vary. All angles in a triangle add to 180 degrees. So for each triangle, I subtracted the two known angles from 180 to find the missing angle.

Accelerated 7 | Unit 2 Lesson 13 168 Warm-Up

Pairs | 20 min

Activity 1 Are Angles Enough?

Purpose: Students analyze figures to show that congruent corresponding angles are not enough evidence to show that quadrilaterals are similar but is enough to show that triangles are.

1 Launch

To surface prior knowledge, consider asking "Where do you see congruent corresponding angles in these images?"

Use the Think-Pair-Share routine to support student-to-student discourse about why these figures are not similar. To encourage movement, invite students to find a partner in a different location. See the Routine Facilitation Guide for more information.

2 Monitor

Invite students to share their reasoning to support their claims.

A **Accessibility: Fine Motor Skills** To support use of manipulatives, invite students to draw their ideas on paper to keep a log of their thought process.

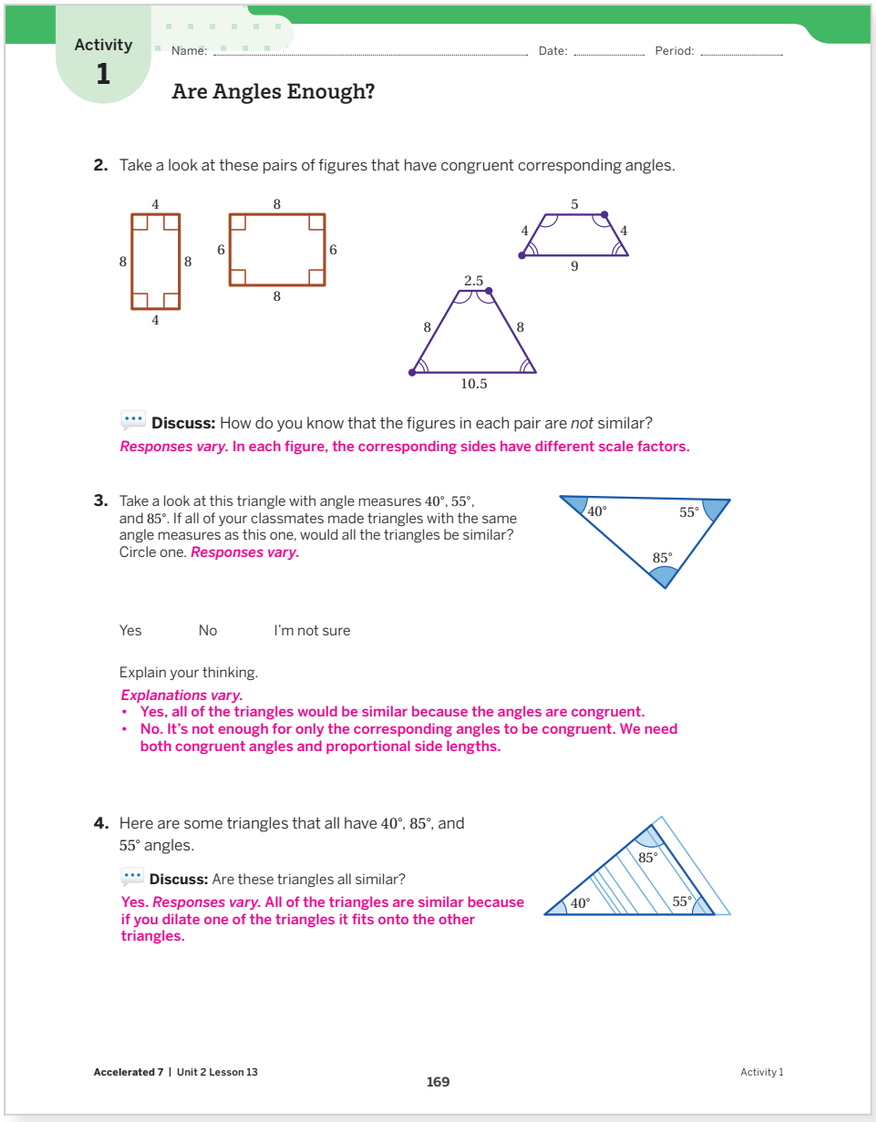
Note: It's okay — even desirable — to lack consensus at this stage. This will build toward consensus in the next problem.

Listen for students who notice that all of the triangles are similar and use a variety of strategies to explain why to highlight during the Pause.

Pause to demonstrate how these similar triangles are formed using a meter stick to demonstrate the changing side location that generates the similar triangles.

Invite students to share what they noticed and justify their thinking. **(MTR.4.1)**

Activity 1 continued >



Activity 1

Name: _____ Date: _____ Period: _____

Are Angles Enough?

2. Take a look at these pairs of figures that have congruent corresponding angles.

Discuss: How do you know that the figures in each pair are *not* similar?

Responses vary. **In each figure, the corresponding sides have different scale factors.**

3. Take a look at this triangle with angle measures 40°, 55°, and 85°. If all of your classmates made triangles with the same angle measures as this one, would all the triangles be similar? Circle one. ***Responses vary.***

Yes No I'm not sure

Explain your thinking.

Explanations vary.

- **Yes, all of the triangles would be similar because the angles are congruent.**
- **No. It's not enough for only the corresponding angles to be congruent. We need both congruent angles and proportional side lengths.**

4. Here are some triangles that all have 40°, 85°, and 55° angles.

Discuss: Are these triangles all similar?

Yes. *Responses vary.* All of the triangles are similar because if you dilate one of the triangles it fits onto the other triangles.

Accelerated 7 | Unit 2 Lesson 13 169 Activity 1

Pairs | 20 min

Activity 1 Are Angles Enough? (continued)

Purpose: Students analyze figures to show that congruent corresponding angles are not enough evidence to show that quadrilaterals are similar but is enough to show that triangles are.

2 Monitor

Invite students to explore the rest of the activity. Consider asking, "How is this situation different from the one before?"

Look and listen for a variety of student triangles.

D Differentiation *(Problems 5 and 6)*

Look for students who:	Respond to Student Thinking
Try to create a triangle where all the angle measures are 40°.	**Support** Consider asking, "What do you know about the sum of the angle measures in a triangle?"
Say "I'm not sure" to whether the triangles will be similar.	**Strengthen** Consider asking, "What would you want to know to be more sure?"
Draw or describe a counterexample to show that it is not enough information.	**Stretch** Consider asking, "Is there an angle measure that would be enough to know all triangles with that angle are similar? Why or why not?" No, because one angle measure does not give us enough information to know the measure of the other two angles.

Listen for comparisons students make between knowing one angle measure and knowing all three angle measures to highlight during the Connect.

3 Connect

To surface the Key Takeaway, consider asking:

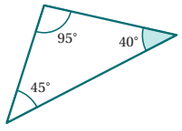
- "Do you think three pairs of congruent corresponding angles is enough to know that triangles are similar? What about two pairs?" **(MTR.4.1)**
- "Why do you think congruent corresponding angles are enough to show triangles are similar, but not other figures?"

Activity 1

Name: ..... Date: ..... Period: .....

Are Angles Enough? (continued)

5. Take a look at this triangle with one angle measuring 40°. If all of your classmates made triangles with one angle measuring 40°, would all the triangles be similar? Circle one. ***Responses vary.***



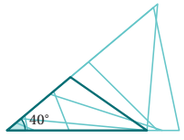
Yes No I'm not sure

Explain your thinking.

Explanations vary.

- **Yes, all of the triangles would be similar because one angle in everyone's triangle would be congruent.**
- **No, one angle being the same in each triangle is not enough to make all the triangles similar.**

6. Here are some triangles that all have a 40° angle.



Discuss: Are these triangles all similar?

No. *Responses vary.* They are not similar because even though they share one angle, which is congruent to itself, there is no way to determine a second pair of congruent angles.

Accelerated 7 | Unit 2 Lesson 13 170 Activity 1

Key Takeaway: One pair of congruent corresponding angles is not enough information to show that two triangles are similar because there is no way to know what the measures of the other pairs of angles are. You can show that two triangles are similar with three pairs of congruent corresponding angles (or two pairs of congruent corresponding angles).

Pairs | 10 min

Activity 2 Similar Sort

Purpose: Students apply what they just learned about similar triangles and generalize that two angles are sufficient to determine triangle similarity. **(MTR.4.1)**

1 Launch

Invite students to spend a minute noticing and wondering about the pairs of figures before sorting them.

2 Monitor

Encourage students to share their reasoning with a partner and work together to reach an agreement about how to sort the pairs of figures.

ELL **English Language Learners** Invite students to ensure that everyone in the pair can justify their thinking. Vary who is called on to represent several pairs' thinking during the Connect. **(Writing, Speaking, and Listening)**

Listen for a variety of strategies to highlight during the Connect.

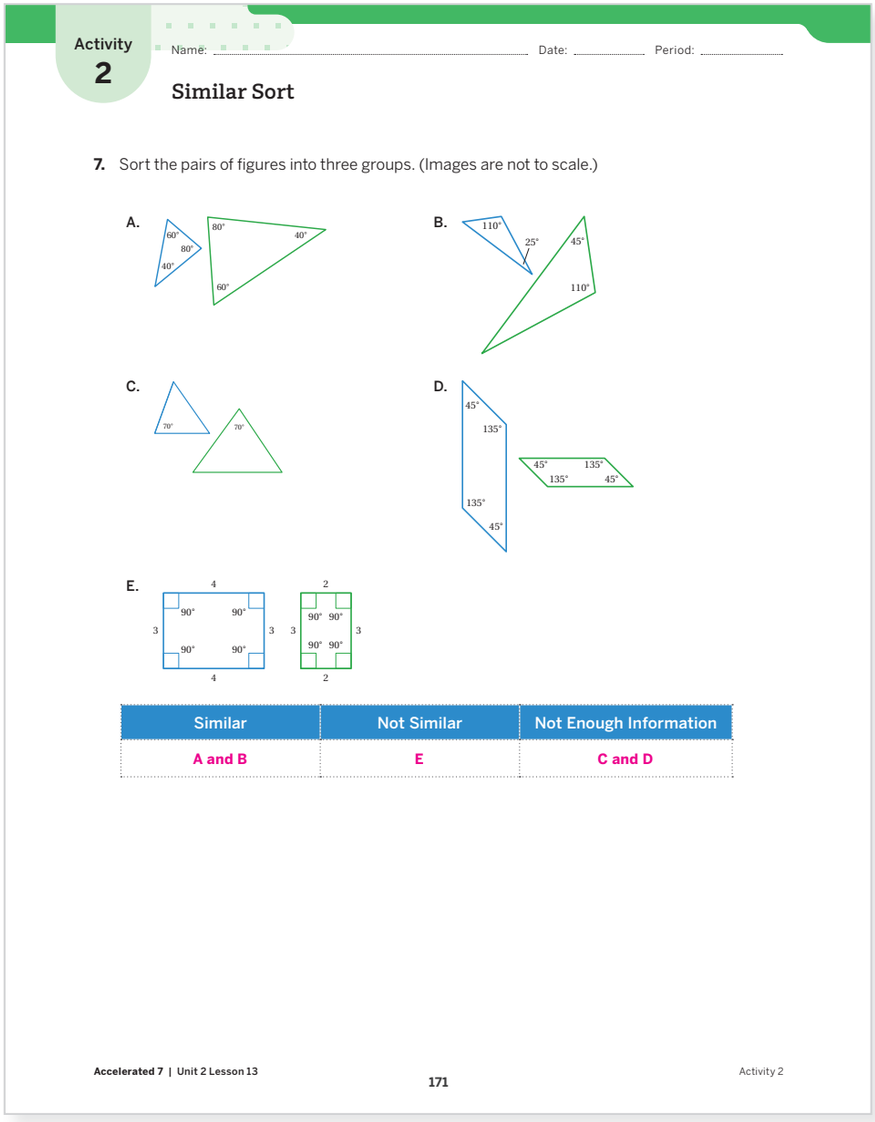
D Differentiation

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Encourage them to use tracing paper or consider asking, "How might what we talked about in Activity 1 help here?"
Say that the tilted quadrilaterals in pair D are similar because they have congruent angle measures.	**Support** Consider drawing a really long quadrilateral with those angle measures and asking, "Is this also similar? How do you know?"
Say that there is not enough information for pair B because two congruent pairs are not shown.	**Strengthen** Consider asking, "What is the third angle measure of the smaller triangle? How can you figure it out?"

Note: If time allows, invite pairs to compare their groups, justify their thinking, and make revisions based on their conversation.

3 Connect

Invite students to justify their thinking for each card one by one and critique each other's reasoning. **(MTR.4.1)**



Activity 2

Name: _____ Date: _____ Period: _____

Similar Sort

7. Sort the pairs of figures into three groups. (Images are not to scale.)

A.

B.

C.

D.

E.

Similar	Not Similar	Not Enough Information
A and B	E	C and D

Accelerated 7 | Unit 2 Lesson 13 171 Activity 2

Whole Class | 5 min

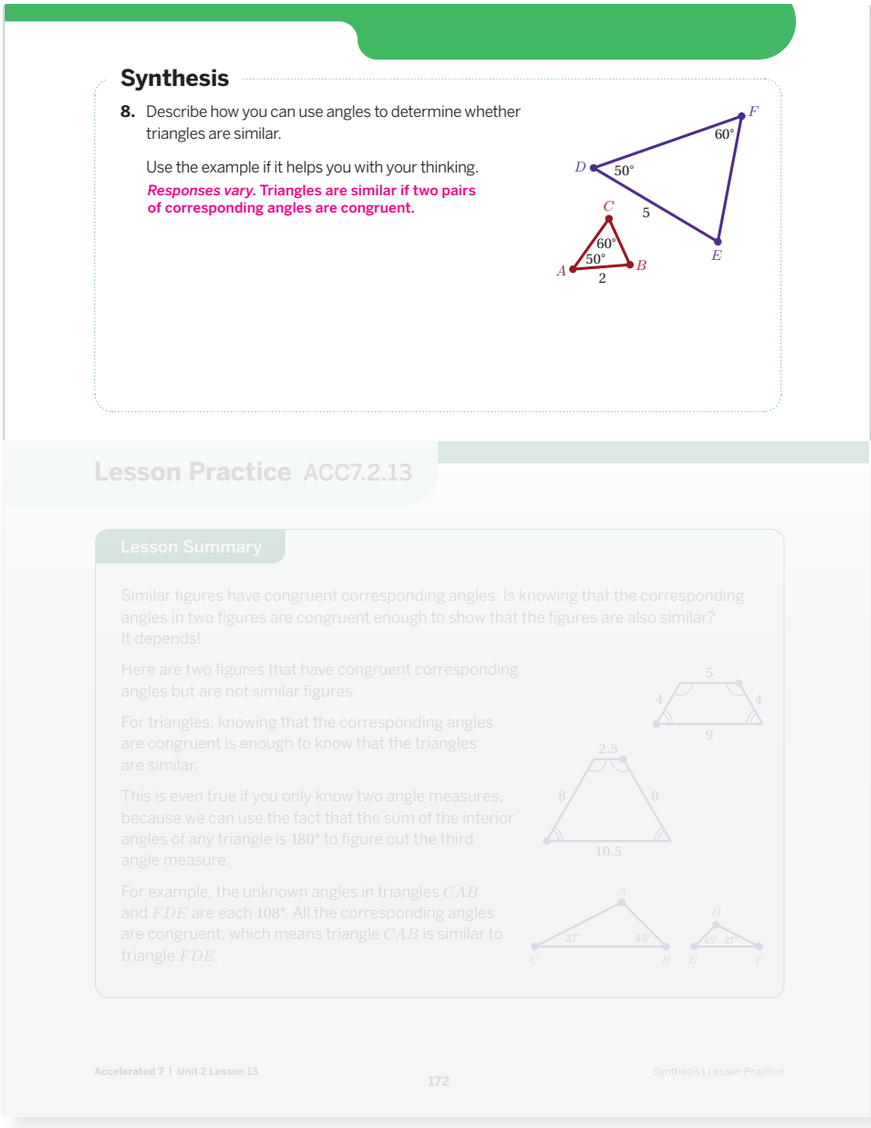
Synthesis

Purpose: Students synthesize their understanding that two pairs of congruent angle measures are enough to show that triangles are similar.

Invite students to respond independently, and then share their thinking with a partner.

Collect and share a variety of ideas, such as:
- Using pairs of congruent corresponding angles.
- Two pairs of angles being enough information because you can determine the measure of the third pair of angles.

MLR **MLR1: Stronger and Clearer Each Time** Use this routine to support students in developing precise language. Invite students to meet with 1–2 different partners, give feedback on each other's drafts, then use the feedback to write a second clearer draft. See the Routine Facilitation Guide for more information.



Synthesis

8. Describe how you can use angles to determine whether triangles are similar.

Use the example if it helps you with your thinking.

Responses vary. **Triangles are similar if two pairs of corresponding angles are congruent.**

Lesson Practice ACC7.2.13

Lesson Summary

Similar figures have congruent corresponding angles. Is knowing that the corresponding angles in two figures are congruent enough to show that the figures are also similar? It depends!

Here are two figures that have congruent corresponding angles but are not similar figures.

For triangles, knowing that the corresponding angles are congruent is enough to know that the triangles are similar.

This is even true if you only know two angle measures, because we can use the fact that the sum of the interior angles of any triangle is 180° to figure out the third angle measure.

For example, the unknown angles in triangles CAB and FDE are each 108°. All the corresponding angles are congruent, which means triangle CAB is similar to triangle FDE.

Accelerated 7 | Unit 2 Lesson 13 172 Synthesis | Lesson Practice

Lesson Takeaway: Two congruent corresponding angle pairs are sufficient evidence for showing that two triangles are similar.

Independent | 5 min

Show What You Know

Purpose: Students demonstrate their understanding by determining whether two triangles are similar using their angle measures.

Today's Goals

1. **Goal:** Know that two pairs of congruent corresponding angles are enough to determine that triangles are similar.
2. **Language Goal:** Justify that two triangles are similar using their angle measures. **(Writing, Speaking, and Listening)**

Show What You Know (Assessment Resources)

Name: _____ Date: _____ Period: _____

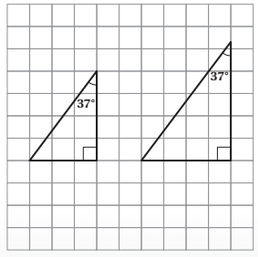
Show What You Know 2.13

Are these triangles similar? Circle one.

(Yes) No

Explain your thinking.

Explanations vary. These triangles are similar because two pairs of corresponding angles are congruent.



D Differentiation Use after Lesson 13

Support

Provide targeted intervention for students by using these resources.

If students need support with determining whether triangles are similar from their angle measures:

» **Respond to Student Thinking:**

- Select a few related **Practice Problems** and give students individualized feedback.
- Revisit the sorting task in **Activity 2** and give students individualized feedback.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 3 Extension Activities**.

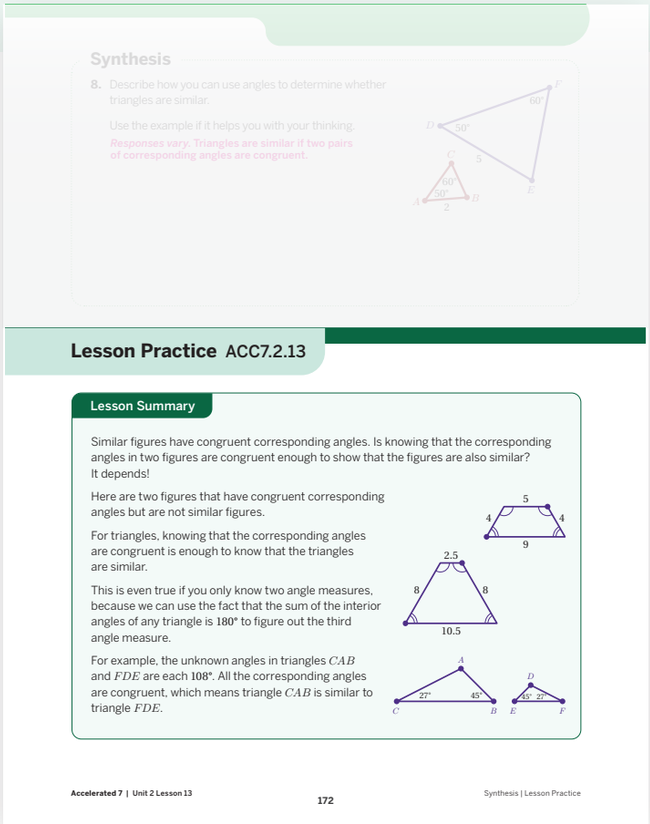
Professional Learning

Reflect on the lesson you just taught. How did your use of the Think-Pair-Share routine or paired student interactions support your facilitation today? How did you decide what to show and when to support students as they worked and during whole class conversations?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.



Synthesis

8. Describe how you can use angles to determine whether triangles are similar.

Use the example if it helps you with your thinking.

Responses vary. Triangles are similar if two pairs of corresponding angles are congruent.

Lesson Practice ACC7.2.13

Lesson Summary

Similar figures have congruent corresponding angles. Is knowing that the corresponding angles in two figures are congruent enough to show that the figures are also similar? It depends!

Here are two figures that have congruent corresponding angles but are not similar figures.

For triangles, knowing that the corresponding angles are congruent is enough to know that the triangles are similar.

This is even true if you only know two angle measures, because we can use the fact that the sum of the interior angles of any triangle is 180° to figure out the third angle measure.

For example, the unknown angles in triangles CAB and FDE are each 108°. All the corresponding angles are congruent, which means triangle CAB is similar to triangle FDE.

Lesson Practice ACC7.2.13

Name: _____ Date: _____ Period: _____

Problems 1–3: Here are some angle measures for different pairs of triangles. Determine whether each pair is *similar*, *not similar*, or if there is *not enough information* to know.

1. Triangle A: 53°, 71°
   Triangle B: 53°, 71°
   Similar

2. Triangle E: 63°, 45°
   Triangle F: 14°, 71°
   Not Similar

3. Triangle G: 100°
   Triangle H: 70°
   Not enough information

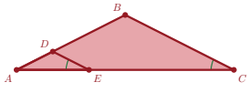
4. Draw two equilateral triangles to fit each category in the table. If it's not possible, write "Not possible."

Not Congruent	Not Similar
Triangles vary.	Not possible

5. Do you think two equilateral triangles will be *always*, *sometimes*, or *never similar*? Explain your thinking.

   Always. *Explanations vary.* An equilateral triangle will always have three angles that measure 60°, so two equilateral triangles will always have at least two angles that have the same measurements (in fact, they'll always have three). That means they will always be similar.

6. In this figure, $\angle AED$ and $\angle ACB$ are congruent. How do you know that triangle ABC is similar to triangle ADE?



Responses vary. It is given that $\angle AED$ and $\angle ACB$ are congruent. $\angle DAE$ is an angle for both triangles, and it is congruent to itself. The sum of interior angles of any triangle is 180°, so $\angle ABC$ must also be congruent to $\angle ADE$.

Both triangles have at least two congruent angles, which means they are similar.

Lesson Practice ACC7.2.13

Name: _____ Date: _____ Period: _____

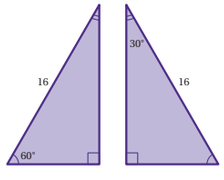
7. Triangle A has two angles that measure 90° and 33°. Triangle B has two angles that measure 90° and 57°. Determine whether the triangles are *similar* or *not similar*. Explain your thinking.

   The triangles are similar. *Explanations vary.* The sum of the interior angle measures of any triangle is 180°, so the third angle measure must be 57° for triangle A and 33° for triangle B. There are three pairs of corresponding angles that are congruent, which means the triangles are similar.

FAST Practice

8. Which of the following *best* describes the triangles shown?

   A. Congruent but not similar
   B. Similar but not congruent
   C. Both similar and congruent
   D. Neither similar nor congruent



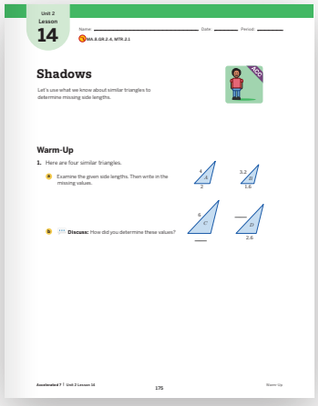
Spiral Review

9. A figure is translated 3 units to the right and then dilated by a scale factor of 3. Determine whether the pre-image and image are *congruent*, *similar*, or *neither*.

   Similar

Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1–5	2	MA.8.GR.1.5
	6–7	3	MA.8.GR.1.4, MA.8.GR.1.5
FAST Practice	8	2	MA.8.GR.1.5
Spiral Review			
	9	1	MA.8.GR.2.2


Unit 2
Lesson
14

Shadows

Let's use what we know about similar triangles to determine missing side lengths.

Warm-Up

1. Here are four similar triangles.

Shadows

Side Length Relationships in Similar Triangles

Let's use what we know about similar triangles to determine missing side lengths.

Learning Outcomes

Today's Goals

1. **Goal:** Recognize that corresponding side lengths of similar triangles have equal ratios to each other and within each triangle.
2. **Goal:** Determine missing side lengths in pairs of similar triangles.
3. **Language Goal:** Describe strategies for determining missing side lengths in similar triangles. **(Writing, Speaking, and Listening)**

Students discover that similar triangles share common proportions between and among their corresponding side lengths and use these ratios to solve for missing side lengths. They apply their knowledge of similarity to reason abstractly about similar triangles in the context of shadows for different real life objects. Students use their knowledge of similar triangles to create models that help determine missing side lengths. **(MTR.2.1)**

Coherence

Prior Learning

In Lesson 13, students discovered that two pairs of congruent corresponding angles are sufficient to know that triangles are similar.

Future Learning

In Sub-Unit 4, students will use similar triangles to make sense of the term *slope* and how to determine the slope of a line.

Connections

- Students use shadows and mirrors to find the heights of tall objects when determining the side lengths of similar figures. (real world)

FL B.E.S.T. Math Benchmarks

Addressing

MA.8.GR.2.4
Solve mathematical and real-world problems involving proportional relationships between similar triangles.

Mathematical Thinking and Reasoning Standard: MTR.2.1

ELA Expectations: ELA.K12.EE.3.1, ELA.K12.EE.5.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.7.GR.1.5	**MA.912.GR.1.2** **MA.912.GR.1.3** **MA.912.GR.1.6**

Lesson at a Glance

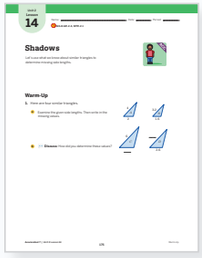
~ 45 min

MA.8.GR.2.4

Warm-Up

Pairs | 5 min

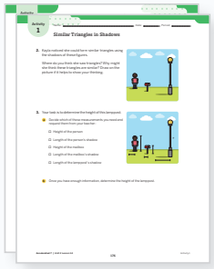
Students use the Notice and Wonder routine to surface proportional reasoning strategies as they determine missing side lengths of similar triangles.



Activity 1

Pairs | 25 min

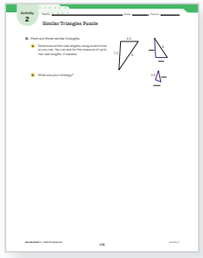
Students apply their knowledge of similar triangles to objects and their shadows. They generate and make sense of at least two strategies for determining missing side lengths. **MLR5: Co-Craft Questions, MLR7: Compare and Connect** **(MTR.2.1)**



Activity 2

Pairs | 5 min

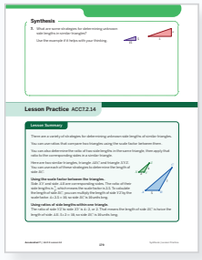
Students build fluency with using relevant expressions to model and solve for missing side lengths of similar triangles. **(MTR.2.1)**



Synthesis

Whole Class | 5 min

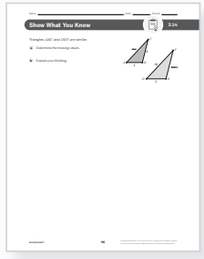
Students synthesize their understanding of the relationships between and among side lengths in similar triangles.



Show What You Know

Independent | 5 min

Students demonstrate their understanding by determining missing side lengths of similar triangles and explaining their strategies.



Prep Checklist

This lesson includes:

- Student Edition
- Show What You Know
- Coloring tools
- Straightedges

Pairs | 5 min

Warm-Up

Purpose: Students surface proportional reasoning strategies as they determine missing side lengths of similar triangles.

1 Launch

Invite students to make observations out loud about the figures before completing the problem.

Invite students to annotate the problem directly with observations.

A **Accessibility: Visual-Spatial Processing**
Provide access to coloring tools so students can show which sides are corresponding.

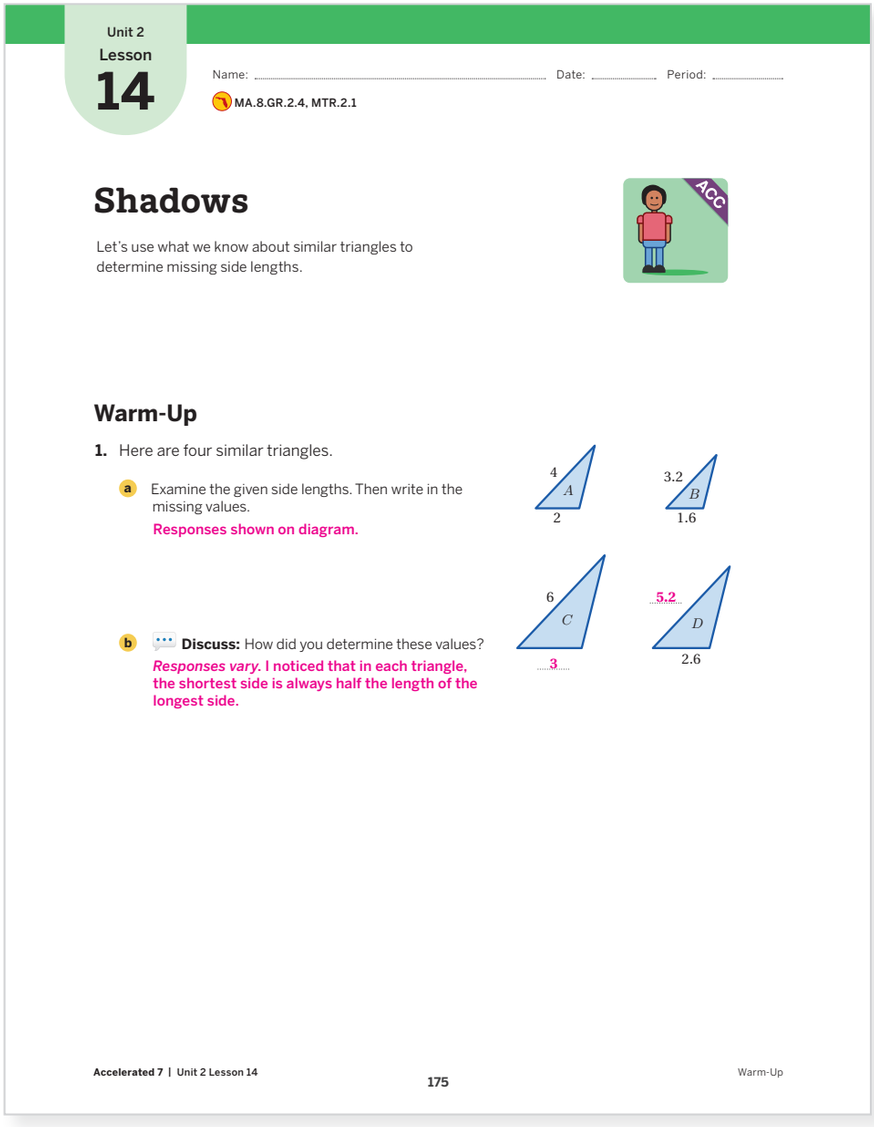
Note: If time allows, consider using the **Notice and Wonder** routine to promote curiosity and make sense of different ways to visualize proportions in similar triangles.

2 Connect

Invite students to share their strategies for determining missing values.

Discuss the different ways students visualized the proportional relationships.

Note: If it doesn't come up naturally, consider asking:
- "Which sides are corresponding?"
- "How do you know?"
- "How might scale factor be used here?"



Unit 2
Lesson
14

Name: ..... Date: ..... Period: .....

MA.8.GR.2.4, MTR.2.1

Shadows

Let's use what we know about similar triangles to determine missing side lengths.

ACC

Warm-Up

1. Here are four similar triangles.

a. Examine the given side lengths. Then write in the missing values.
Responses shown on diagram.

b. **Discuss:** How did you determine these values?
Responses vary. I noticed that in each triangle, the shortest side is always half the length of the longest side.

Accelerated 7 | Unit 2 Lesson 14 175 Warm-Up

Pairs | 25 min

Activity 1 Similar Triangles in Shadows

Purpose: Students apply their knowledge of similar triangles to objects and their shadows. They generate and make sense of at least two strategies for determining missing side lengths. **(MTR.2.1)**

1 Launch

To generate excitement, invite students to reflect on what they've noticed about shadows and the sun.

Note: If time allows, consider taking students outside to look for their shadows and make observations about lengths of their shadows and the position of the sun.

MLR **MLR5: Co-Craft Questions** Use this routine to support students in making sense of this context involving shadows, by asking questions without the pressure to produce answers.

2 Monitor

Encourage students to make connections to the Warm-Up.

Look and listen for a variety of student strategies to solve for the missing height of the lamppost.

D **Differentiation** *(Problem 3)*

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Encourage them to request 1–2 missing lengths and make observations.
Create ratios that link non-corresponding side lengths.	**Support** Invite them to mark which side lengths are corresponding.
Solve for the missing height by requesting all of the unknown side lengths.	**Strengthen** Consider asking, "How could you have solved for the missing height with less information?"
Use ratios to correctly solve for missing side lengths.	**Stretch** Invite students to think about what other relationships show up in the diagram.

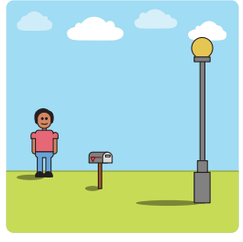
Activity 1 continued >

Activity 1

Name: _____ Date: _____ Period: _____

Similar Triangles in Shadows

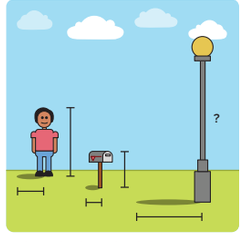
2. Kayla noticed she could form similar triangles using the shadows of these figures.



Where do you think she saw triangles? Why might she think these triangles are similar? Draw on the picture if it helps to show your thinking.

Responses vary. Each object and its shadow form two sides of a right triangle. The triangles are all similar because the shadows have the same scale factor based on the time of day.

3. Your task is to determine the height of this lamppost.



a. Decide which of these measurements you need and request them from your teacher:

- ☐ Height of the person
- ☐ Length of the person's shadow
- ☐ Height of the mailbox
- ☐ Length of the mailbox's shadow
- ☐ Length of the lamppost's shadow

Selections vary. To answer part b, students need to select the length of the lamppost's shadow, and both the height and length of at least one of the other objects.

b. Once you have enough information, determine the height of the lamppost.

180 inches (or equivalent)

Accelerated 7 | Unit 2 Lesson 14 176 Activity 1

Pairs | 25 min

Activity 1 Similar Triangles in Shadows (continued)

Purpose: Students apply their knowledge of similar triangles to objects and their shadows. They generate and make sense of at least two strategies for determining missing side lengths. **(MTR.2.1)**

2 Monitor

Pause briefly to discuss strategies students used to solve for the missing height in the previous problem.

MLR **MLR7: Compare and Connect** Invite students to analyze and compare Ama's and Neena's strategies, and then connect to their own strategy for determining the missing height of the lamppost. **(MTR.2.1)**

Consider asking, "What did each student do? How does each strategy compare to your own?"

Invite students to explore Problem 5.

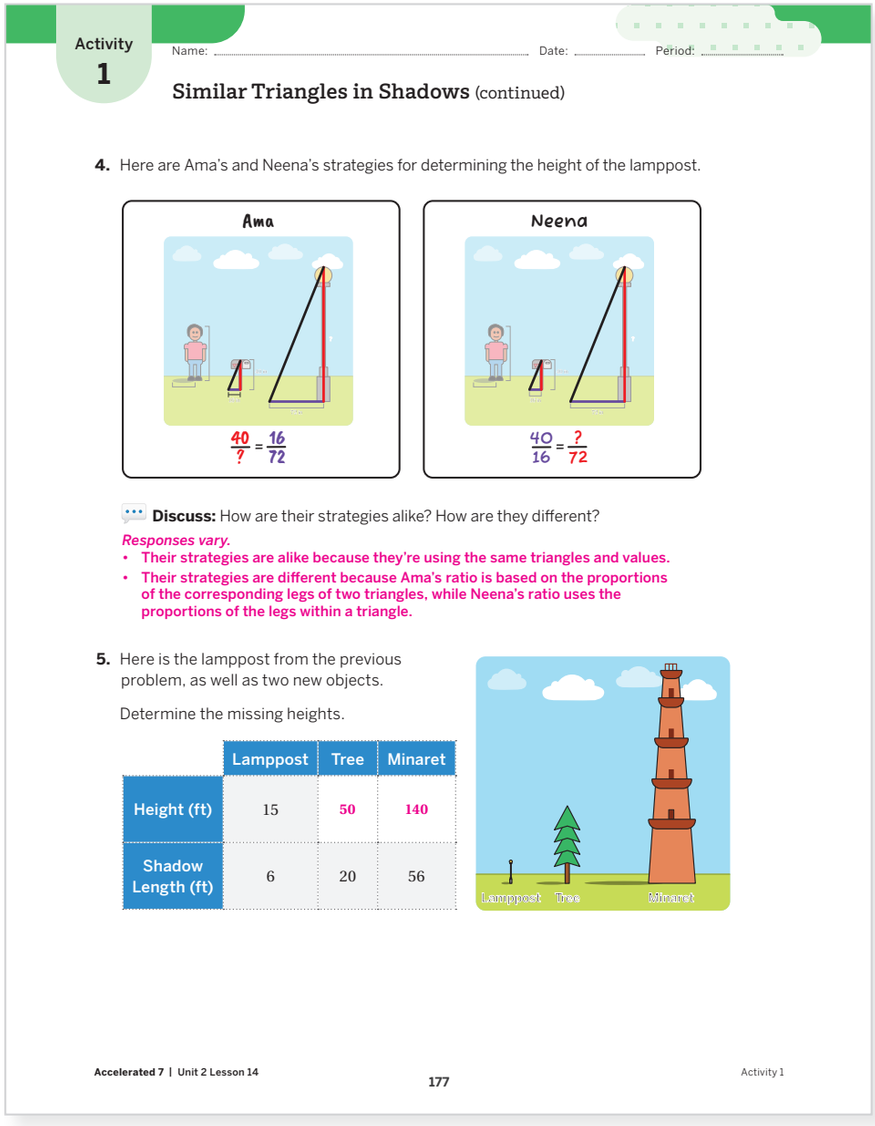
3 Connect

Encourage students to refer to the strategies explored in Problems 3–4.

Invite students to annotate patterns and relationships between height and shadow length. They should have access to coloring tools and straightedges to annotate relationships.

To surface the Key Takeaway, consider creating an anchor chart of different student strategies for determining missing side lengths, such as:

- Different ways to annotate similar triangle diagrams.
- Corresponding side lengths.
- Scale factors.
- Ratios comparing corresponding side lengths.
- Ratios comparing side lengths within one triangle.



Activity 1

Name: Date: Period:

Similar Triangles in Shadows (continued)

4. Here are Ama's and Neena's strategies for determining the height of the lamppost.

Ama

$\frac{40}{?} = \frac{16}{72}$

Neena

$\frac{40}{16} = \frac{?}{72}$

Discuss: How are their strategies alike? How are they different?

Responses vary.

- Their strategies are alike because they're using the same triangles and values.
- Their strategies are different because Ama's ratio is based on the proportions of the corresponding legs of two triangles, while Neena's ratio uses the proportions of the legs within a triangle.

5. Here is the lamppost from the previous problem, as well as two new objects.

Determine the missing heights.

	Lamppost	Tree	Minaret
Height (ft)	15	50	140
Shadow Length (ft)	6	20	56

Accelerated 7 | Unit 2 Lesson 14 177 Activity 1

Key Takeaway: We can solve for missing side lengths of similar triangles using a variety of strategies. These strategies include using the scale factor between similar triangles and ratios of side lengths within a triangle.

Pairs | 5 min

Activity 2 Similar Triangles Puzzle

Purpose: Students build fluency with using relevant expressions to model and solve for missing side lengths of similar triangles. **(MTR.2.1)**

Short on time? Consider omitting this activity.

1 Launch

To generate excitement, consider sharing that students will begin a puzzle, similar to previous challenges with determining angles.

2 Monitor

Select descriptions of student strategies to share during the Connect.

D Differentiation

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Encourage them to request 1–2 more side lengths and make an observation about side length patterns.
Think that they must request all three side lengths of one of the triangles.	**Support** Celebrate them for their resourcefulness. Consider asking, "How can you determine the missing side length if you only know two side lengths of one of the triangles? One of the side lengths? What other information would you need?"
Enjoy determining missing side lengths of triangles.	**Stretch** Invite them to create their own similar triangles puzzle and ask their partner to solve it.

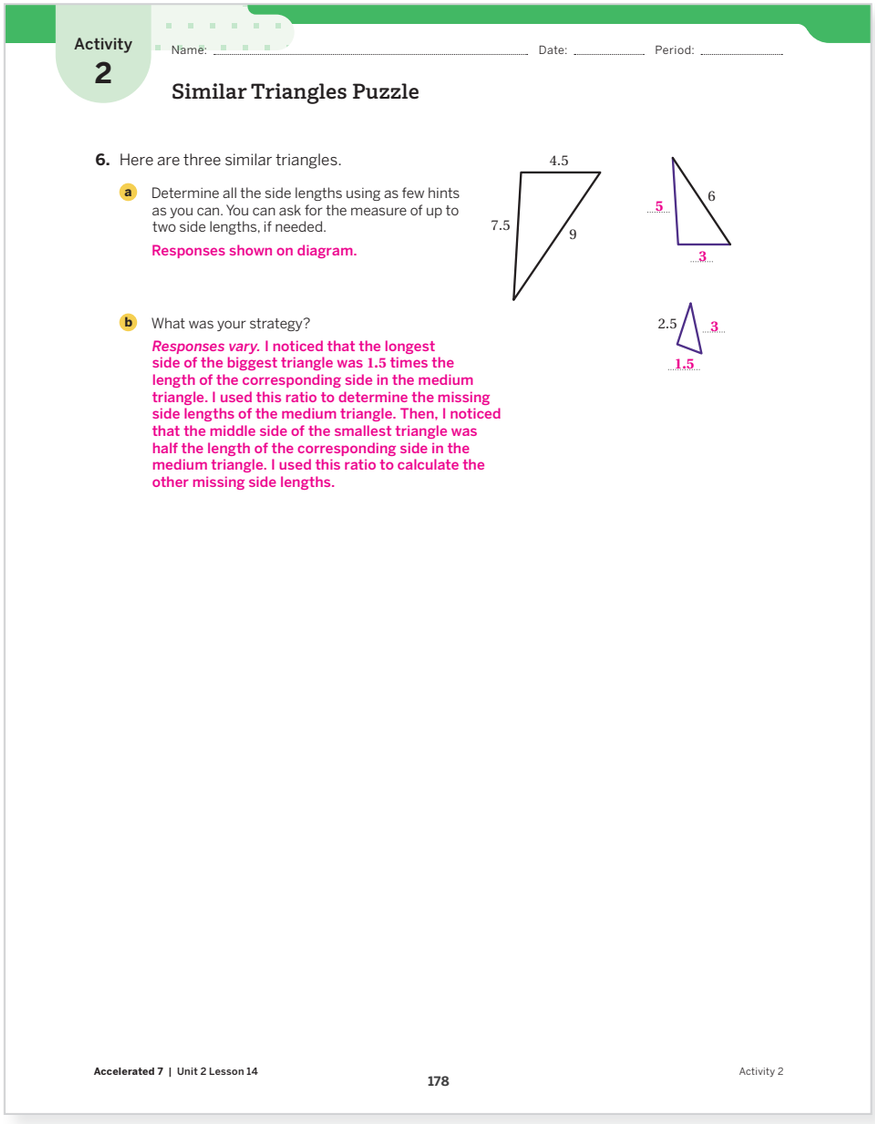
ELL **English Language Learners** Provide sentence frames to help students explain their strategy (e.g., "I noticed that _____, so I _____." or "First, I _____ because _____."). **(Writing and Speaking)**

3 Connect

Share and celebrate the captured strategy descriptions.

To surface connections, consider asking:

- "What strategies were helpful for determining the missing side lengths?"
- "How were these strategies similar to or different from the ones Ama and Neena used?"
- "How did you decide how many side lengths you needed?"



Activity 2

Name: _____ Date: _____ Period: _____

Similar Triangles Puzzle

6. Here are three similar triangles.

a. Determine all the side lengths using as few hints as you can. You can ask for the measure of up to two side lengths, if needed.

Responses shown on diagram.

b. What was your strategy?

Responses vary. I noticed that the longest side of the biggest triangle was 1.5 times the length of the corresponding side in the medium triangle. I used this ratio to determine the missing side lengths of the medium triangle. Then, I noticed that the middle side of the smallest triangle was half the length of the corresponding side in the medium triangle. I used this ratio to calculate the other missing side lengths.

Accelerated 7 | Unit 2 Lesson 14 178 Activity 2

Whole Class | 5 min

Synthesis

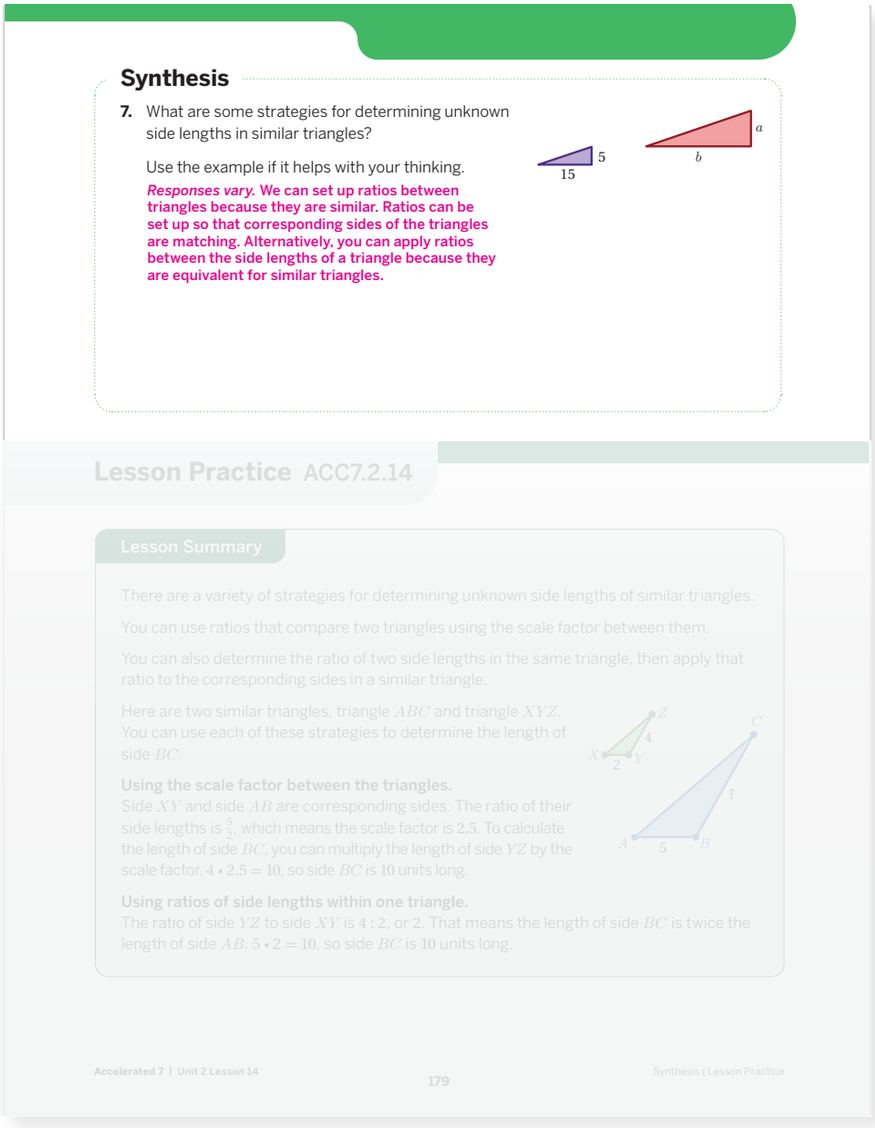
Purpose: Students synthesize their understanding of the relationships between and among side lengths in similar triangles.

Invite students to respond independently, and then share their thinking with a partner.

Collect and share a variety of ideas, such as:
- Strategies to identify corresponding side lengths between triangles.
- Ways to visualize proportions between triangles and within triangles.
- Scale factors used when solving for a corresponding side length in a similar triangle.

Invite a few students to share their work.

Note: If time allows, invite students to revise their previous thinking based on the discussion.



Synthesis

7. What are some strategies for determining unknown side lengths in similar triangles?

Use the example if it helps with your thinking.

Responses vary. We can set up ratios between triangles because they are similar. Ratios can be set up so that corresponding sides of the triangles are matching. Alternatively, you can apply ratios between the side lengths of a triangle because they are equivalent for similar triangles.

Lesson Practice ACC7.2.14

Lesson Summary

There are a variety of strategies for determining unknown side lengths of similar triangles.

You can use ratios that compare two triangles using the scale factor between them.

You can also determine the ratio of two side lengths in the same triangle, then apply that ratio to the corresponding sides in a similar triangle.

Here are two similar triangles, triangle ABC and triangle XYZ. You can use each of these strategies to determine the length of side BC.

Using the scale factor between the triangles.
Side XY and side AB are corresponding sides. The ratio of their side lengths is $\frac{5}{2}$, which means the scale factor is 2.5. To calculate the length of side BC, you can multiply the length of side YZ by the scale factor. $4 \cdot 2.5 = 10$, so side BC is 10 units long.

Using ratios of side lengths within one triangle.
The ratio of side YZ to side XY is 4 : 2, or 2. That means the length of side BC is twice the length of side AB. $5 \cdot 2 = 10$, so side BC is 10 units long.

Accelerated 7 | Unit 2 Lesson 14 179 Synthesis | Lesson Practice

Lesson Takeaway: There are a variety of strategies for determining unknown side lengths of similar triangles. You can use ratios that compare two triangles using the scale factor between them. Alternatively, you can determine the ratio of two side lengths in the same triangle and apply that ratio to corresponding sides in a similar triangle.

Independent | 5 min

Show What You Know

Purpose: Students demonstrate their understanding by determining missing side lengths of similar triangles and explaining their strategies.

Today's Goals

1. **Goal:** Recognize that corresponding side lengths of similar triangles have equal ratios to each other and within each triangle.
2. **Goal:** Determine missing side lengths in pairs of similar triangles.
3. **Language Goal:** Describe strategies for determining missing side lengths in similar triangles. **(Writing, Speaking, and Listening)**

Show What You Know (Assessment Resources)

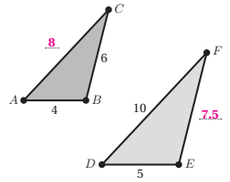
Name: _____ Date: _____ Period: _____

Show What You Know 2.14

Triangles ABC and DEF are similar.

a. Determine the missing values.
Responses shown on diagram.

b. Explain your thinking.
Responses vary. I saw that the ratio of the corresponding side AB to DE was 4:5, or 1:1.25. So, I multiplied the length of BC by the scale factor. $6 \times 1.25 = 7.5$, so side BC is 7.5 units long. Then, I divided the length of DF by the scale factor. $10 \div 1.25 = 8$, so side AC is 8 units long.



Differentiation Use after Lesson 14

Support

Provide targeted intervention for students by using these resources.

If students need support with determining missing side lengths of similar triangles:

» **Respond to Student Thinking:**

- Use the ***Determining Missing Side Lengths in Similar Triangles*** **Mini-Lesson**.
- Revisit **Unit 1, Lesson 5, Warm-Up** (No Bending, No Stretching) and invite students to identify corresponding sides.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 3 Extension Activities.**

Professional Learning

Take a moment to reflect on the context of shadows used in this lesson. Why was this context particularly useful in building students' understanding of similar triangles in this lesson?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

7. What are some strategies for determining unknown side lengths in similar triangles?

Use the example if it helps with your thinking.

Responses vary. We can set up ratios between triangles because they are similar. Ratios can be set up so that corresponding sides of the triangles are matching. Alternatively, you can apply ratios between the side lengths of a triangle because they are equivalent for similar triangles.

Lesson Practice ACC7.2.14

Lesson Summary

There are a variety of strategies for determining unknown side lengths of similar triangles.

You can use ratios that compare two triangles using the scale factor between them.

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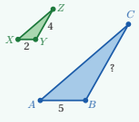
Here are two similar triangles, triangle ABC and triangle XYZ. You can use each of these strategies to determine the length of side BC.



Using the scale factor between the triangles.
Side XY and side AB are corresponding sides. The ratio of their side lengths is $\frac{5}{2}$, which means the scale factor is 2.5. To calculate the length of side BC, you can multiply the length of side YZ by the scale factor. $4 \cdot 2.5 = 10$, so side BC is 10 units long.

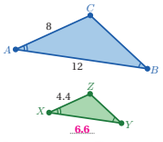
Using ratios of side lengths within one triangle.
The ratio of side YZ to side XY is 4 : 2, or 2. That means the length of side BC is twice the length of side AB. $5 \cdot 2 = 10$, so side BC is 10 units long.

Lesson Practice ACC7.2.14

Name: _____ Date: _____ Period: _____

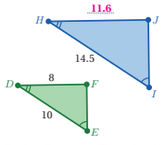
Problems 1–2: Triangle ABC is similar to triangle XYZ. Triangle DEF is similar to triangle HIJ. Determine the missing values. Explain your thinking. The figures may not be drawn to scale.

1.



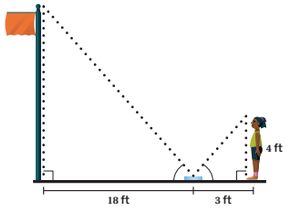
Explanations vary. The ratio of side AB to side AC is 12 : 8 or 1.5. That means the length of side XY is 1.5 times the length of side XZ. $4.4 \cdot 1.5 = 6.6$.

2.



Explanations vary. The ratio of the corresponding side lengths HI to DE is 14.5 : 10, so the scale factor is 1.45. I can multiply the length of side DF by the scale factor to determine the length of side HJ: $8 \cdot 1.45 = 11.6$.

3. Zoe uses a mirror to measure the height of a flagpole. She places the mirror flat on the ground, then walks backward until she can see the top of the flagpole in the mirror. As the diagram shows, the way that light reflects in the mirror creates a set of congruent angles. Now she can use her own height, the mirror's distance from the flagpole, and her distance from the mirror to calculate the height of the flagpole.



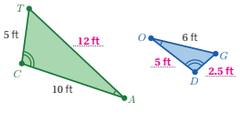
Use the measurements in the diagram to calculate the flagpole's height. Show or explain your thinking.

24 feet. *Explanations vary.* The ratio of Zoe's height to the distance from Zoe's feet to the mirror is the same as the ratio of the flagpole's height to the distance from the base of the flagpole to the mirror. $\frac{4}{3} = \frac{24}{18}$.

Lesson Practice ACC7.2.14

Name: _____ Date: _____ Period: _____

4. Triangle CAT is similar to triangle DOG. Triangle CAT is dilated by a scale factor of $\frac{1}{2}$, creating triangle DOG. Determine the missing side lengths. Show or explain your thinking.



Explanations vary.

$5 \cdot \frac{1}{2} = 2.5$

$10 \cdot \frac{1}{2} = 5$

$6 \div \frac{1}{2} = 12$

FAST Practice

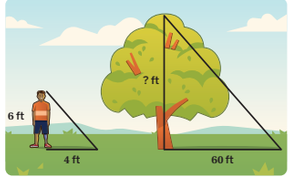
5. On a sunny day, a 6-foot-tall person has a shadow with a length of 4 feet. At the same time, a nearby tree has a shadow 60 feet long. What is the height of the tree?

A. 80 feet

B. 90 feet

C. 120 feet

D. 180 feet



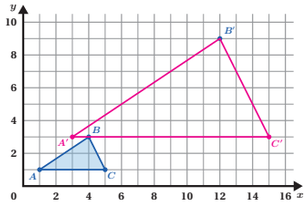
Spiral Review

Problems 6–7: Triangle ABC is dilated with the origin as the center of rotation, moving point B onto point B'.

6. What is the scale factor of the dilation?

3

7. Draw and label triangle $A'B'C'$.



Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1–4	2	MA.8.GR.2.4
FAST Practice	5	2	MA.8.GR.2.4
Spiral Review			
	6–7	2	MA.8.GR.2.2

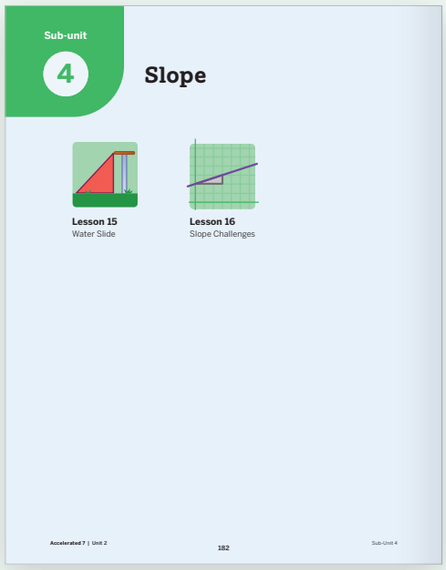
Sub-Unit 4

Slope

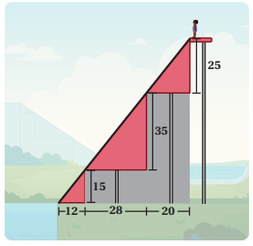
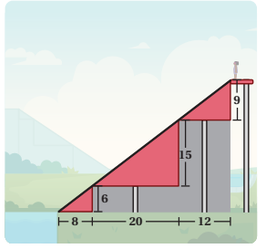
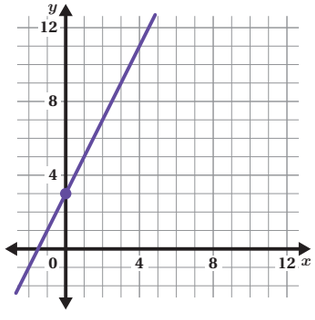
Sub-Unit 4 Goals

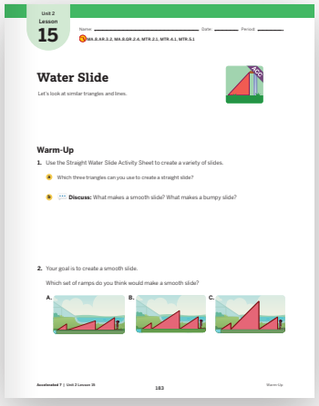
- Make connections between slope and similar triangles on a line.
- Determine the slopes of lines.
- **Language Goal:** Explain how to calculate the slope of a line.

In this sub-unit, students explore the connection between similar triangles and the **slope** of a line. They explain why all **slope triangles** on the same line are similar and they develop strategies for calculating slope using the height-to-base ratio of these triangles.



Math That Matters Most

Concept or Skill	For example . . .
Explain how similar triangles and slopes of lines are connected.	Slope measures the steepness of a line. This slide forms a line with a slope of $\frac{5}{4}$. How do you think slope is calculated? *You can calculate slope by finding the height-to-base ratio of one of the similar triangles.* 
Use similar slope triangles to determine the slope of a line.	What is the slope of this slide? $\frac{3}{4}$ *(or equivalent)* Explain your thinking. *I found the slope by calculating the height-to-base ratio of the smallest triangle, which was $\frac{6}{8}$ or $\frac{3}{4}$.* 
Describe a strategy for determining the slope of a line.	Describe a strategy for determining the slope of any line. Use the example if it helps you with your thinking. • *Draw a slope triangle between two points on the line. The slope is the height of the triangle over the base of the triangle.* • *To determine the slope of a line, find the height-to-base ratio of the slope triangle.* 


Unit 2
Lesson
15

Water Slide

Let's look at similar triangles and lines.

Warm-Up

1. Use the Straight Water Slide Activity Sheet to create a variety of slides.

2. Your goal is to create a smooth slide.

Which set of ramps do you think would make a smooth slide?

A. B. C.

183

Water Slide

Similarity and Slope

Let's look at similar triangles and lines.

Learning Outcomes

Today's Goals

1. **Goal:** Use similar triangles to reason about the *slope* of a line.
2. **Language Goal:** Explain how similar triangles and slopes of lines are connected. **(Writing, Speaking, and Listening)**

Students learn about the **slope** of a line in the context of water slides. They make connections between the height-to-base ratio in similar triangles, the slope of a line, and creating a smooth water slide. Students also use their creativity as they design their own slides and use them to create similar triangles.

Coherence

Prior Learning

In Lessons 12–14, students learned about the properties of similar triangles.

Future Learning

In Lesson 16, students will practice calculating slopes of lines using slope triangles.

Connections

- Students use slope triangles to design a smooth ride on a water slide. (real world)

FL B.E.S.T. Math Benchmarks

Addressing

MA.8.AR.3.2

Given a table, graph or written description of a linear relationship, determine the slope.

Also Addressing: **MA.8.GR.2.4**

Mathematical Thinking and Reasoning Standards: MTR.2.1, MTR.4.1, MTR.5.1

ELA Expectations: ELA.K12.EE.3.1, ELA.K12.EE.4.1, ELA.K12.EE.5.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.7.AR.4.2	MA.912.AR.2.2

Vocabulary

New Vocabulary

slope

Review Vocabulary

ratio

Lesson at a Glance

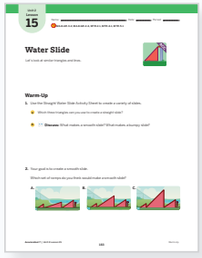
~ 45 min

MA.8.GR.2.4, MA.8.AR.3.2

Warm-Up

Pairs | 5 min

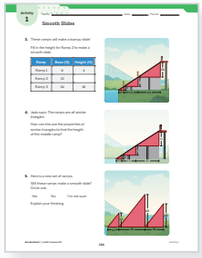
Students compare triangles that create smooth and bumpy slides and make conjectures about what makes a smooth slide.
Routine: Think-Pair-Share



Activity 1

Pairs | 10 min

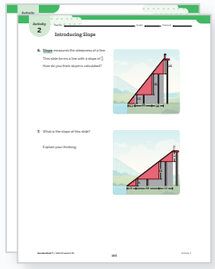
Students make connections between similar triangles and a smooth water slide.
Routine: Decide and Defend **(MTR.2.1)**



Activity 2

Pairs | 15 min

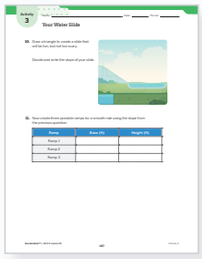
Students make sense of **slope** as the height-to-base *ratio* of similar triangles on a line, a structure to determine slope. They use this structure to analyze student work. MLR3: Critique, Correct, Clarify **(MTR.4.1)**



Activity 3

Pairs | 5 min

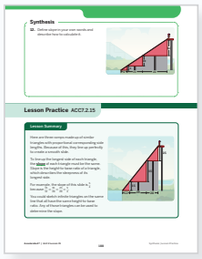
Students use their creativity to create their own smooth water slides and make use of this structure to determine their slopes. **(MTR.5.1)**



Synthesis

Whole Class | 5 min

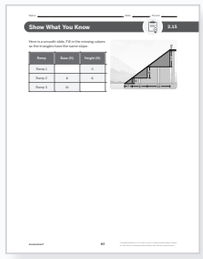
Students synthesize their understanding of slope and how it is connected to similar triangles. MLR2: Collect and Display



Show What You Know

Independent | 5 min

Students demonstrate their understanding by determining missing values in triangles with the same slope.



Prep Checklist

Prepare Warm Up Activity Sheets

This lesson includes:

- Student Edition
- Show What You Know
- Warm-Up Activity Sheet, 1 per pair
- Scissors
- Straightedges
- Graph paper (as needed)

Pairs | 5 min

Warm-Up

Purpose: Students compare triangles that create smooth and bumpy slides and make conjectures about what makes a smooth slide.

Short on time? Consider using a projector to create different smooth and bumpy slides together as a class.

1 Launch

To generate excitement, consider asking, "What's the biggest slide you've ever been on?"

Distribute the Straight Water Slide Activity Sheet and scissors.

ELL **English Language Learners** Invite students to act out, show videos/images, use gestures, or talk about smooth and bumpy slides before starting the task. **(Reading and Listening)**

Use the Think-Pair-Share routine to help students shape their ideas about smooth and bumpy slides. See the Routine Facilitation Guide for more information.

2 Connect

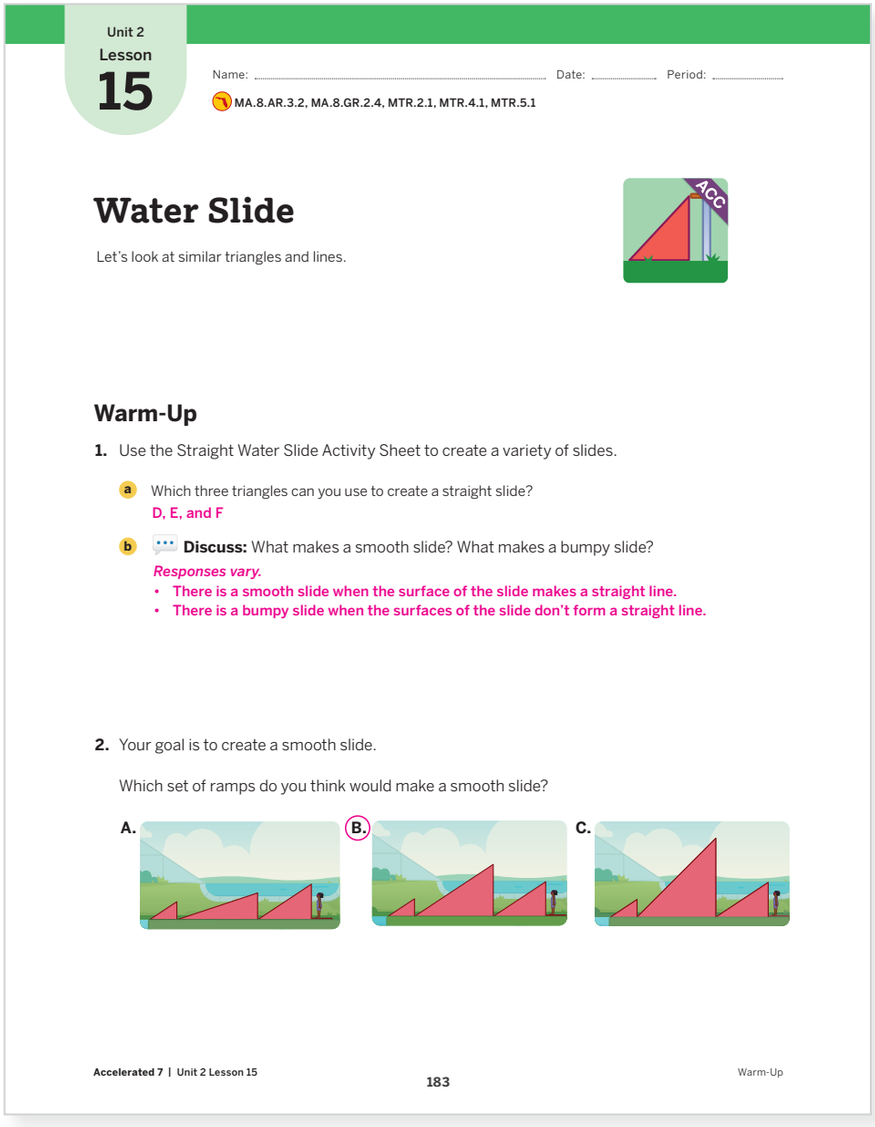
Encourage students to think about what kinds of triangles might make smooth slides.

Invite students to share their strategies for making a smooth slide.

Consider asking:

- "What's the same about the triangles on both kinds of slides?"
- "What is special about the triangles that make smooth slides?"
- "How could you use the grid lines in making your triangle ramp choices for a smooth slide?"

Note: If it does not come up naturally, consider asking whether students think the triangles that make a smooth slide are similar triangles.



Unit 2
Lesson
15

Name: ..... Date: ..... Period: .....

MA.8.AR.3.2, MA.8.GR.2.4, MTR.2.1, MTR.4.1, MTR.5.1

Water Slide

Let's look at similar triangles and lines.

Warm-Up

1. Use the Straight Water Slide Activity Sheet to create a variety of slides.

   a. Which three triangles can you use to create a straight slide?
   D, E, and F

   b. **Discuss:** What makes a smooth slide? What makes a bumpy slide?
   Responses vary.
   - There is a smooth slide when the surface of the slide makes a straight line.
   - There is a bumpy slide when the surfaces of the slide don't form a straight line.

2. Your goal is to create a smooth slide.

   Which set of ramps do you think would make a smooth slide?

   A. B. C.

Accelerated 7 | Unit 2 Lesson 15 183 Warm-Up

Pairs | 10 min

Activity 1 Smooth Slides

Purpose: Students make connections between similar triangles and a smooth water slide. **(MTR.2.1)**

1 Launch

Invite students to analyze the given ramp image and consider asking, "How is this related to the Warm-Up? What height might be too big? Too small?"

2 Monitor

Invite students to share feedback with their partner about whether or not their chosen height would produce a bumpy slide.

Encourage students to read others' responses and decide if these strategies were similar to or different from their own.

Select and sequence explanations to share during the Connect.

D Differentiation *(Problem 5)*

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Encourage them to use tracing paper or graph paper to draw the triangles and explore how the dimensions are related.
Reason that the triangles will not create a smooth slide.	**Support** Invite these students to find evidence using the heights and bases.
Say that the ramps will create a smooth slide but do not make connections to similarity.	**Strengthen** Encourage students to compare their explanation with a classmate's before the Connect.
Use the language of similar triangles in their response.	**Stretch** Invite students to make connections between this activity and performing dilations.

3 Connect

For Problem 5, **Invite students** to share and defend their response.

To surface the Key Takeaway, consider asking:
- "How can we convince someone the ramps will make a smooth slide?"
- "Are there other triangles that could possibly be a part of this smooth slide?" **(MTR.2.1)**

Note: If time allows, use the Decide and Defend routine to support students in making arguments and critiquing the reasoning of others.

Activity 1

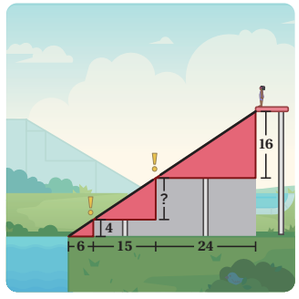
Name: _____ Date: _____ Period: _____

Smooth Slides

3. These ramps will make a bumpy slide!

Fill in the height for Ramp 2 to make a smooth slide.

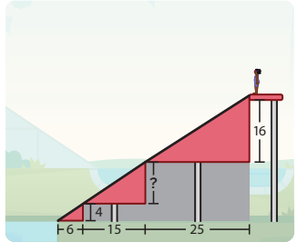
Ramp	Base (ft)	Height (ft)
Ramp 1	6	4
Ramp 2	15	10
Ramp 3	24	16



4. Jada says: *The ramps are all similar triangles.*

How can she use the properties of similar triangles to find the height of the middle ramp?

Responses vary. Jada can determine what scale factor, when multiplied by the base of Ramp 1, results in the base of Ramp 2. Then she can determine the height of Ramp 2 by multiplying that scale factor by the height of Ramp 1.



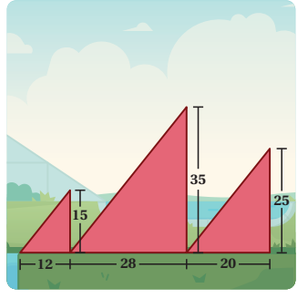
5. Here is a new set of ramps.

Will these ramps make a smooth slide? Circle one.

(Yes) No I'm not sure

Explain your thinking.

Explanations vary. These ramps will make a smooth slide because the triangles are similar. I can tell they're similar because the corresponding sides are proportional.



Accelerated 7 | Unit 2 Lesson 15 184 Activity 1

Key Takeaway: A line can be made up of similar right triangles with proportional heights and bases.

Pairs | 15 min

Activity 2 Introducing Slope

Purpose: Students make sense of **slope** as the height-to-base *ratio* of similar triangles on a line, a structure to determine slope. They use this structure to analyze student work. **(MTR.4.1)**

1 Launch

Share that **slope** measures the steepness of a line.

Invite students to write their ideas of how they think slope is calculated.

To surface connections, choose and share several responses showing the different ways students explained their thinking about slope.

Pause to create an anchor chart with helpful student definitions of *slope*, with explicit mention of how slope can be calculated by finding the height-to-base *ratio* of a right triangle created by a line.

ELL **English Language Learners** Invite students to research how to say *slope* and *ratio* in other languages. **(Reading and Listening)**

2 Monitor

Invite students to explore Problems 7–9. Look for different student responses to respond to in real time.

D **Differentiation**

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Invite students to share how this problem is different from and similar to the previous one.
Compute a ratio using side measures from different triangles.	**Support** Encourage them to refer to the anchor chart to remind them of the definition of *slope*.
Compute a slope using the base-to-height ratio (instead of height-to-base).	**Support** Invite students to compare their strategy with a classmate's.
Correctly compute the slope using the height-to-base ratio.	**Stretch** Encourage students to determine the side lengths of another triangle that would work under this slide.

Activity 2 continued >

Activity 2

Name: _____ Date: _____ Period: _____

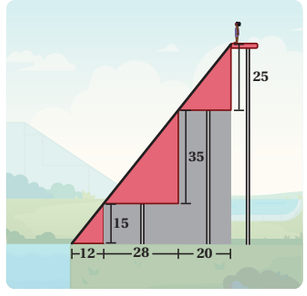
Introducing Slope

6. **Slope** measures the steepness of a line.

This slide forms a line with a slope of $\frac{5}{4}$.

How do you think slope is calculated?

Responses vary. You can calculate slope by finding the height-to-base ratio of one of the similar triangles.

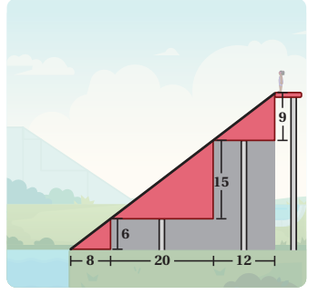


7. What is the slope of this slide?

$\frac{3}{4}$ (or equivalent)

Explain your thinking.

Explanations vary. I found the slope by calculating the height-to-base ratio of the smallest triangle, which was $\frac{6}{8}$ or $\frac{3}{4}$.



Accelerated 7 | Unit 2 Lesson 15 185 Activity 2

Pairs | 15 min

Activity 2 Introducing Slope (continued)

Purpose: Students make sense of **slope** as the height-to-base *ratio* of similar triangles on a line, a structure to determine slope. They use this structure to analyze student work. **(MTR.4.1)**

2 Monitor

Encourage students to revisit their work in Activity 1, Problem 3 to support their work here.

Pause briefly to allow students to visually process the image before answering the question.

A **Accessibility: Executive Functioning** Consider providing a T-chart for students to record what they notice and wonder.

Select student explanations to highlight during the Connect.

3 Connect

Share the selected explanations.

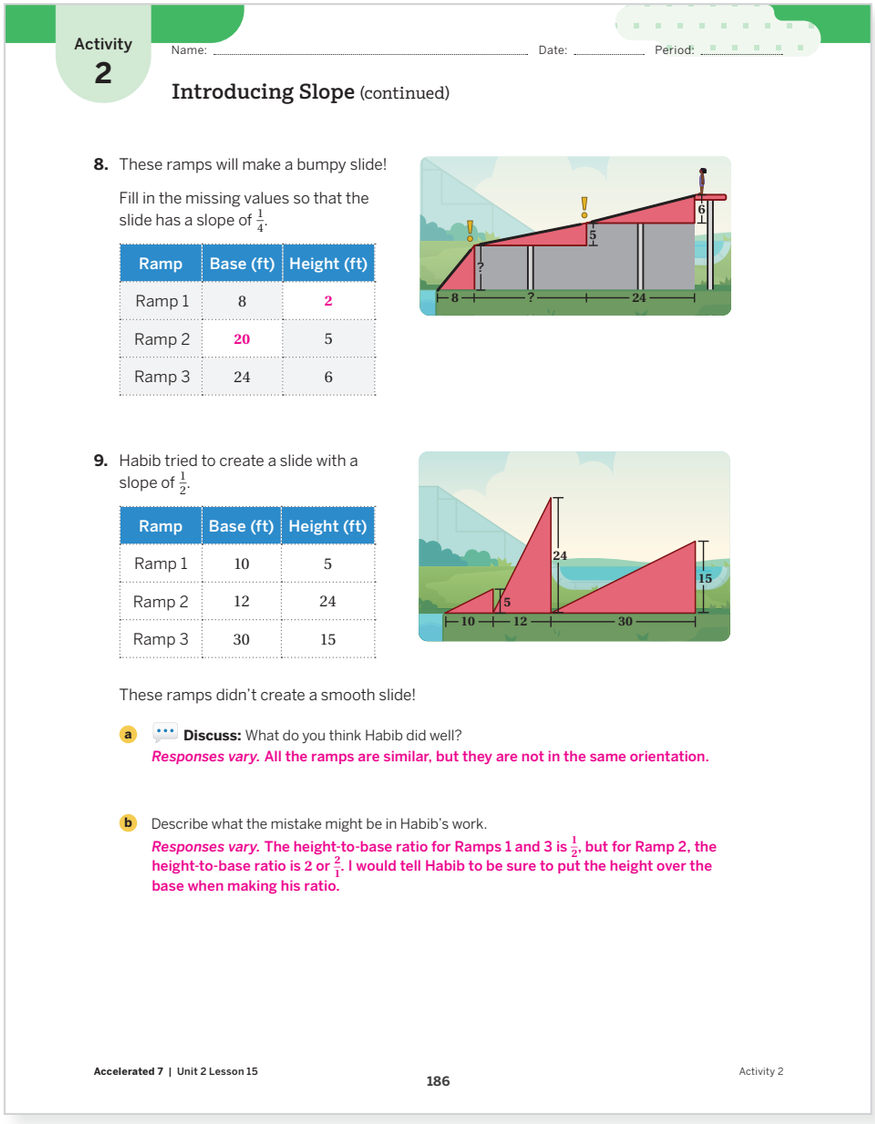
MLR **MLR3: Critique, Correct, Clarify** Use this routine to help students describe strengths, errors, and ambiguities in Habib's work. As a class:

- Discuss what is *correct* about his response.
- Discuss what is *incorrect* or *unclear*.
- Create a *correct and clear* solution.

To surface the Key Takeaway, consider recording:

- Correct elements of Habib's work that can support a class definition of *slope*.
- Any structures that help students connect slope as the height-to-base ratio of right triangles.

Note: Students need to leave this activity with a clear understanding that slope is the height-to-base ratio of the triangle, often written in the form of a fraction.



Activity 2

Name: _____ Date: _____ Period: _____

Introducing Slope (continued)

8. These ramps will make a bumpy slide!

Fill in the missing values so that the slide has a slope of $\frac{1}{4}$.

Ramp	Base (ft)	Height (ft)
Ramp 1	8	2
Ramp 2	20	5
Ramp 3	24	6

9. Habib tried to create a slide with a slope of $\frac{1}{2}$.

Ramp	Base (ft)	Height (ft)
Ramp 1	10	5
Ramp 2	12	24
Ramp 3	30	15

These ramps didn't create a smooth slide!

a. **Discuss:** What do you think Habib did well?
Responses vary. All the ramps are similar, but they are not in the same orientation.

b. Describe what the mistake might be in Habib's work.
Responses vary. The height-to-base ratio for Ramps 1 and 3 is $\frac{1}{2}$, but for Ramp 2, the height-to-base ratio is 2 or $\frac{2}{1}$. I would tell Habib to be sure to put the height over the base when making his ratio.

Accelerated 7 | Unit 2 Lesson 15 186 Activity 2

Key Takeaway: **Slope** measures the steepness of a line and can be determined by the height-to-base *ratio* of the triangles on the line.

Pairs | 5 min

Activity 3 Your Water Slide

Purpose: Students use their creativity to create their own smooth water slides and make use of this structure to determine their slopes. **(MTR.5.1)**

Short on time? Consider omitting this activity or designing a slide as a class.

1 Launch

Consider asking, "What is an example of a slide that would be too scary?"

To encourage creativity, prompt students to try a slope that they have not seen yet.

Invite students to use straightedges and graph paper to create slides that are reasonable given their slope.

2 Monitor

Look for unique water slides as you monitor student discourse and progress.

Encourage students to share their reasoning with a partner and work together to reach an agreement on the slope of their designed slide.

Select and sequence strategies to share during the Connect.

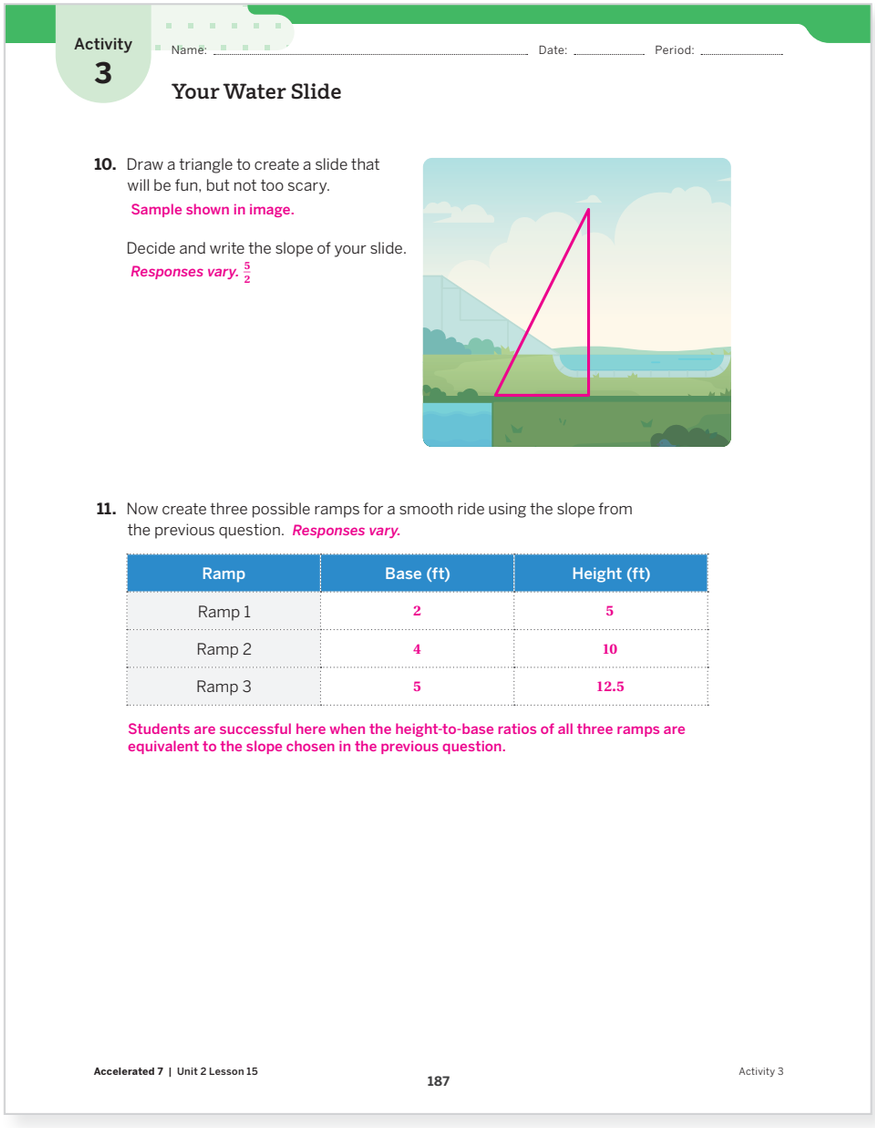
D Differentiation *(Problem 11)*

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Encourage students to guess-and-check side lengths to fill in the table.
Guess-and-check side lengths to create accurate triangles.	**Support** Celebrate their persistence. Invite these students to draw their triangles on graph paper. Consider asking, "What are some relationships you notice between the side lengths?"
Use proportional reasoning to solve for missing sides.	**Strengthen** Invite these students to share their thinking during the Synthesis.
Use scale factor, dilation, and similar triangles ideas from earlier in the unit.	**Stretch** Invite students to write a rule that can help someone calculate slope for other triangles.

3 Connect

Share the selected examples of student triangles and tables and invite students to identify any patterns they notice between the slopes and the bases and heights of the triangles. **(MTR.5.1)**

Consider asking, "How many different triangles do you think have this slope?"



Activity 3

Name: _____ Date: _____ Period: _____

Your Water Slide

10. Draw a triangle to create a slide that will be fun, but not too scary.

Sample shown in image.

Decide and write the slope of your slide.

Responses vary. $\frac{5}{2}$

11. Now create three possible ramps for a smooth ride using the slope from the previous question. *Responses vary.*

Ramp	Base (ft)	Height (ft)
Ramp 1	2	5
Ramp 2	4	10
Ramp 3	5	12.5

Students are successful here when the height-to-base ratios of all three ramps are equivalent to the slope chosen in the previous question.

Accelerated 7 | Unit 2 Lesson 15 187 Activity 3

Whole Class | 5 min

Synthesis

Purpose: Students synthesize their understanding of slope and how it is connected to similar triangles.

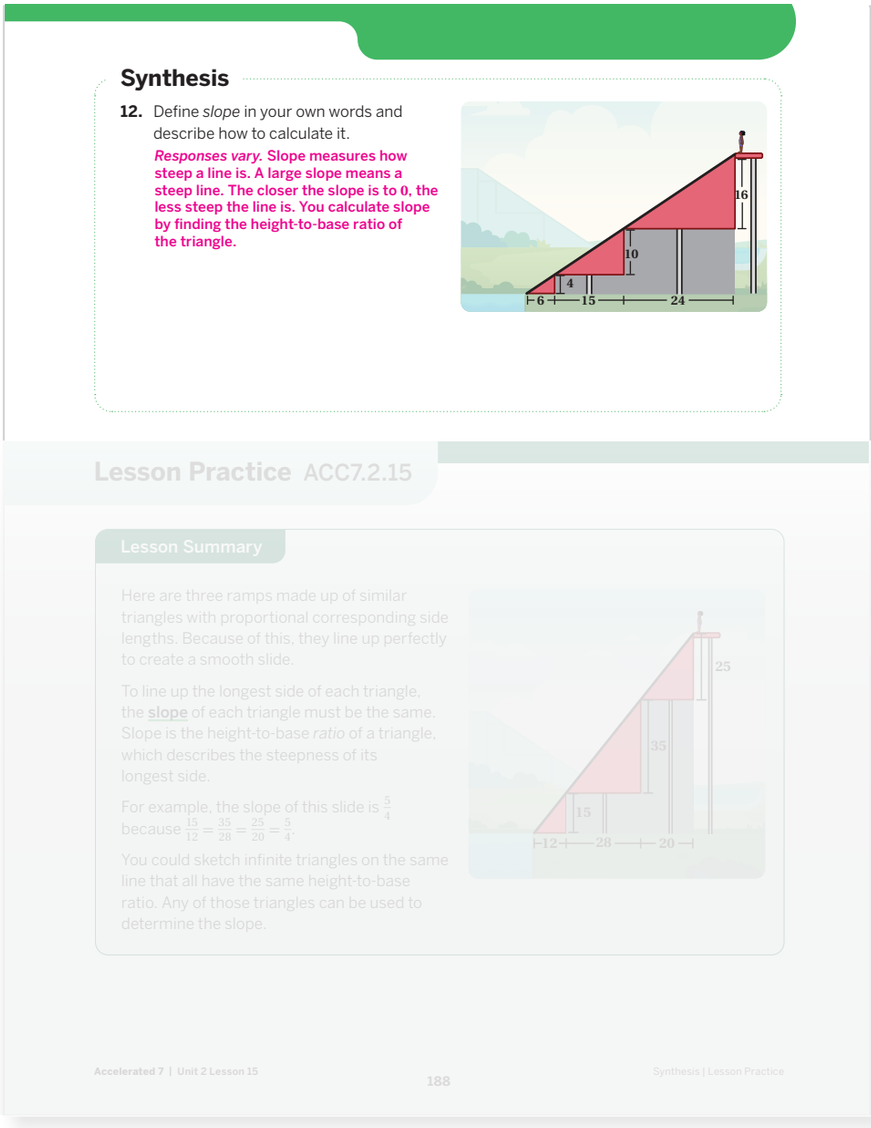
Invite students to respond independently, and then share their thinking with a partner.

Select and share a variety of ideas, such as:

- The height-to-base ratio of the similar right triangles is the same as the slope of the line.
- Slope measures the steepness of a line.

MLR **MLR2: Collect and Display** Use this routine to formalize the definition of *slope* using students' language. Display any new language, examples, or diagrams for students to reference later.

Note: If time allows, invite students to revise their previous thinking in Problem 7 based on the discussion.



Synthesis

12. Define *slope* in your own words and describe how to calculate it.

Responses vary. Slope measures how steep a line is. A large slope means a steep line. The closer the slope is to 0, the less steep the line is. You calculate slope by finding the height-to-base ratio of the triangle.

Lesson Practice ACC7.2.15

Lesson Summary

Here are three ramps made up of similar triangles with proportional corresponding side lengths. Because of this, they line up perfectly to create a smooth slide.

To line up the longest side of each triangle, the slope of each triangle must be the same. Slope is the height-to-base *ratio* of a triangle, which describes the steepness of its longest side.

For example, the slope of this slide is $\frac{5}{4}$ because $\frac{15}{12} = \frac{35}{28} = \frac{25}{20} = \frac{5}{4}$.

You could sketch infinite triangles on the same line that all have the same height-to-base ratio. Any of those triangles can be used to determine the slope.

Accelerated 7 | Unit 2 Lesson 15 188 Synthesis | Lesson Practice

Lesson Takeaway: There are infinite triangles on the same line that all have the same height-to-base *ratio*. This height-to-base ratio of the triangles is called the **slope** of the line.

Independent | 5 min

Show What You Know

Purpose: Students demonstrate their understanding by determining missing values in triangles with the same slope.

Today's Goal

1. **Goal:** Use similar triangles to reason about the *slope* of a line.
2. **Language Goal:** Explain how similar triangles and slopes of lines are connected. **(Writing, Speaking, and Listening)**

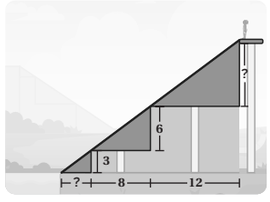
Show What You Know (Assessment Resources)

Name: _____ Date: _____ Period: _____

Show What You Know 2.15

Here is a smooth slide. Fill in the missing values so the triangles have the same slope.

Ramp	Base (ft)	Height (ft)
Ramp 1	4	3
Ramp 2	8	6
Ramp 3	12	9



D Differentiation Use after Lesson 15

Support

Provide targeted intervention for students by using these resources.

If students need support with reasoning with ratios that create the same slope

» **Respond to Student Thinking:**

- Use the ***Connecting Scale Factors to Scaled* Copies Mini-Lesson**.
- Revisit Jada's method in **Activity 1, Problem 4** and invite students to explain why it will create a smooth ride.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 4 Extension Activities**.

Professional Learning

Take a moment to reflect on this lesson. In this lesson, students developed early ideas about the term *slope*. How will their understanding of slope build over the next several lessons?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

12. Define *slope* in your own words and describe how to calculate it.

Responses vary. Slope measures how steep a line is. A large slope means a steep line. The closer the slope is to 0, the less steep the line is. You calculate slope by finding the height-to-base ratio of the triangle.

Lesson Practice ACC7.2.15

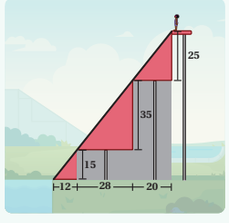
Lesson Summary

Here are three ramps made up of similar triangles with proportional corresponding side lengths. Because of this, they line up perfectly to create a smooth slide.

To line up the longest side of each triangle, the **slope** of each triangle must be the same. Slope is the height-to-base *ratio* of a triangle, which describes the steepness of its longest side.

For example, the slope of this slide is $\frac{5}{4}$ because $\frac{15}{12} = \frac{35}{28} = \frac{25}{20} = \frac{5}{4}$.

You could sketch infinite triangles on the same line that all have the same height-to-base ratio. Any of those triangles can be used to determine the slope.

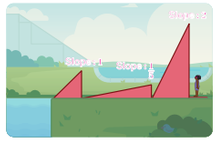


Lesson Practice
ACC7.2.15

Name: _____ Date: _____ Period: _____

1. Here are three ramps. One ramp has a slope of 1, one ramp has a slope of 2, and one ramp has a slope of $\frac{1}{5}$. Label each ramp with its slope.

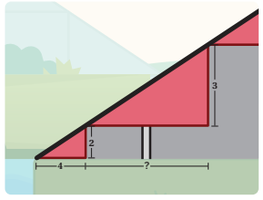
Responses shown in image.



2. Jaylin created a slide with a slope of $\frac{1}{3}$. LaShawn created a slide with a slope of $\frac{1}{2}$. Whose slide was steeper? Explain your thinking.

LaShawn's slide. ***Explanations vary.*** **Both slides have a slope with the same vertical distance, but Jaylin's slope has a greater horizontal distance, which means it's less steep than LaShawn's slide.**

Problems 3–5: Here are three ramps that make a smooth slide.



3. How long is the base of the second ramp?

6 units

4. Explain how you know the triangles that form the ramps are similar.

Explanations vary. **The second triangle ramp is a dilation of the first triangle ramp by a scale factor of 1.5 and then translated.**

5. Which statement about the slopes of the ramps is true?

A. The slope of the first ramp is $\frac{2}{3}$ the slope of the second ramp because the height of the first ramp is $\frac{2}{3}$ the length of the second ramp.

B. The slope of the first ramp is the same as the slope of the second ramp because their triangles are similar.

C. The slope of the first ramp is $\frac{4}{9}$ the slope of the second ramp because the area of the first ramp triangle is $\frac{4}{9}$ the area of the second ramp triangle.

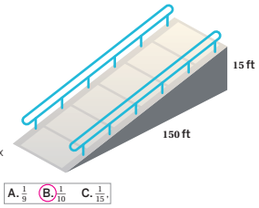
D. The slope of the second ramp is 2 more than the slope of the first ramp because the difference between the bases of the ramps is 2.

Lesson Practice
ACC7.2.15

Name: _____ Date: _____ Period: _____

FAST Practice

6. A builder is constructing a ramp for one of the entrances at the local library. He is checking the diagram before he starts construction. One requirement is that public wheelchair ramps must have a maximum slope of $\frac{1}{12}$.



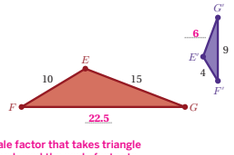
Does this ramp fit the requirement? Select ONE correct answer in each box to complete the sentences.

[A. Yes. (B.) No.] The slope of the line is [A. $\frac{1}{9}$ (B.) $\frac{1}{10}$ C. $\frac{1}{15}$,] which is [(A.) greater B. less] than $\frac{1}{12}$.

Spiral Review

7. Triangle EFG is similar to triangle $E'F'G'$. Determine the missing values and explain your thinking.

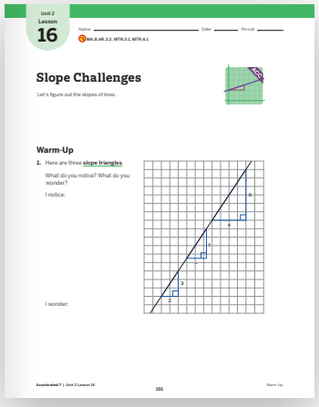
Explanations vary.



I used the corresponding sides to find the scale factor that takes triangle $E'F'G'$ to triangle EFG, which is $\frac{10}{4} = 2.5$. Then I used the scale factor to find the missing side lengths, $9 \cdot 2.5 = 22.5$ and $15 \div 2.5 = 6$, so the side lengths are 22.5 and 6.

Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1–5	1	MA.8.AR.3.2
FAST Practice	6	1	MA.8.AR.3.2
Spiral Review			
	7	2	MA.8.GR.2.4


Unit 2
Lesson
16

Slope Challenges

Let's figure out the slopes of lines.

Warm-Up

1. Here are three slope triangles.

What do you notice? What do you wonder?

I notice:

I wonder:

Slope Challenges

Developing Fluency with Slope

Let's figure out the slopes of lines.

Learning Outcomes

Today's Goals

1. **Goal:** Use similar *slope triangles* to determine the slope of a line.
2. **Language Goal:** Describe a strategy for determining the slope of a line. **(Reading, Writing, Speaking, and Listening)**

Students practice using **slope triangles** to determine slopes of lines on the coordinate plane. They develop fluency by both determining slopes and analyzing the work of fictitious students. In this lesson, students use repeated reasoning to generalize at least one strategy for determining the slope of a line, a skill they will need throughout the year. **(MTR.3.1)**

Coherence

Prior Learning

In Lesson 15, students used their knowledge to make sense of slope as the height-to-base ratio of similar triangles on a line.

Future Learning

In Unit 4, students will use their knowledge of slope to explore linear relationships, including using slope to write equations of lines. In Unit 8, students will calculate the slopes of lines of fit.

Connections

- Students analyze samples of student thinking in an error analysis activity in which they determine the slope of a line from its graph. (mathematical, cross-curricular)

FL B.E.S.T. Math Benchmarks

Addressing

MA.8.AR.3.2
Given a table, graph or written description of a linear relationship, determine the slope.

Mathematical Thinking and Reasoning Standards: MTR.3.1, MTR.4.1

ELA Expectations: ELA.K12.EE.1.1, ELA.K12.EE.3.1, ELA.K12.EE.5.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.7.AR.4.2	MA.912.AR.2.2

Vocabulary

New Vocabulary

slope triangle

Lesson at a Glance

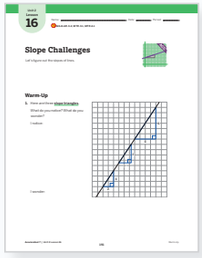
~ 45 min

MA.8.AR.3.2

Warm-Up

Pairs | 5 min

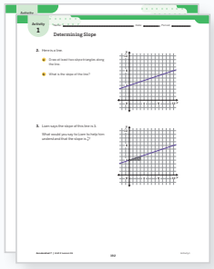
Students use the Notice and Wonder routine to make connections between slope triangles and what they learned in the previous lesson about slope.



Activity 1

Pairs | 15 min

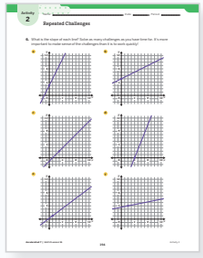
Students develop strategies for determining the slope of a line using slope triangles and by analyzing student work.
Routines: Decide and Defend; MLR3: Correct, Critique, Clarify **(MTR.4.1)**



Activity 2

Independent | 15 min

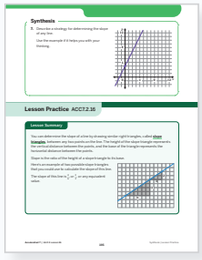
Students develop fluency with determining the slopes of lines in a series of repeated challenges. **(MTR.3.1)**



Synthesis

Whole Class | 5 min

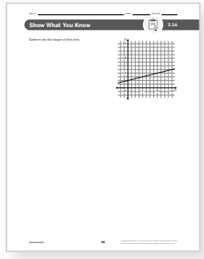
Students synthesize and generalize strategies for determining the slope of any line.



Show What You Know

Independent | 5 min

Students demonstrate their understanding by determining the slope of a line on the coordinate plane.



Prep Checklist

This lesson includes:

- Student Edition
- Show What You Know
- Straightedges (as needed)

Pairs | 5 min

Warm-Up

Purpose: Students use the Notice and Wonder routine to make connections between slope triangles and what they learned in the previous lesson about slope.

1 Launch

To surface prior knowledge, consider asking, "What do you remember about *slope* from earlier?"

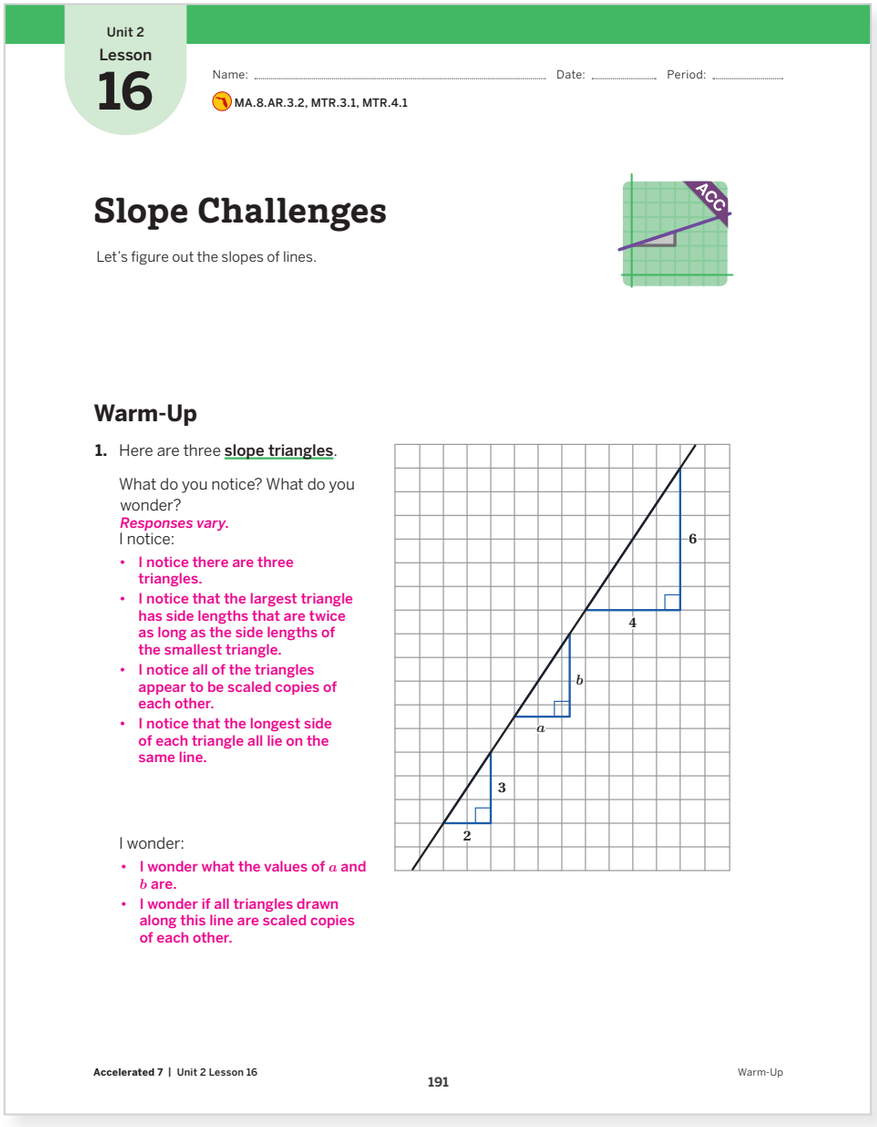
Use the Notice and Wonder routine to promote curiosity and help students make sense of **slope triangles**. **(MTR.4.1)**

2 Connect

Create a record of what students noticed and wondered, along with the name of the students who shared them. This may be useful to refer back to throughout the lesson.

Consider discussing:

- "Why do you think these are called *slope triangles*?"
- "How might we use these triangles to determine the slope?"



Unit 2
Lesson
16

Name: Date: Period:

MA.8.AR.3.2, MTR.3.1, MTR.4.1

Slope Challenges

Let's figure out the slopes of lines.

Warm-Up

1. Here are three slope triangles.

What do you notice? What do you wonder?
Responses vary.
I notice:

- I notice there are three triangles.
- I notice that the largest triangle has side lengths that are twice as long as the side lengths of the smallest triangle.
- I notice all of the triangles appear to be scaled copies of each other.
- I notice that the longest side of each triangle all lie on the same line.

I wonder:

- I wonder what the values of a and b are.
- I wonder if all triangles drawn along this line are scaled copies of each other.

Accelerated 7 | Unit 2 Lesson 16 191 Warm-Up

15 min

Activity 1 Determining Slope

Purpose: Students develop strategies for determining the slope of a line using slope triangles and by analyzing student work. **(MTR.4.1)**

1 Launch

Direct students' attention to the line.

To surface connections, consider asking, "How do you think these slope triangles will be different from the ones in the Warm-Up?"

2 Monitor

Invite students to check their slope with a partner.

Look and listen for a variety of student thinking, including thinking similar to Liam's on the next problem.

D **Differentiation** *(Problems 1–3)*

Look for students who:	Respond to Student Thinking
Need support getting started. Draw slope triangles that are not on intersection points.	**Support** Encourage them to return the Warm-Up and ask, "How can we use the grid lines to make the sides of the slope triangle? Where do you think the 6 and 4 in the slope triangle came from?"
Say that the slope is 3 or $\frac{6}{2}$.	**Support** Consider asking, "How did we use the height and base of the triangle to get the slope in the Warm-Up?"
Guess and check the slope of 3.	**Strengthen** Consider asking, "How are the height and base of your slope triangles related to the number 3?"
Recognize equivalent ratios, like $\frac{2}{6}$ and $\frac{1}{3}$.	**Stretch** Invite students to find the largest slope triangle they can and test if it has the same height-to-base ratio.

Pause to discuss Liam's thinking as a class after most students have responded.

MLR **MLR3: Critique, Correct, Clarify** Use this routine to help students describe strengths, errors, and ambiguities in Liam's work. As a class:

- Discuss what is *correct* about his response.
- Discuss what is *incorrect* or *unclear*.
- Invite students to share what they would say to Liam.

Activity 1 continued >

Activity 1

Name: _____ Date: _____ Period: _____

Determining Slope

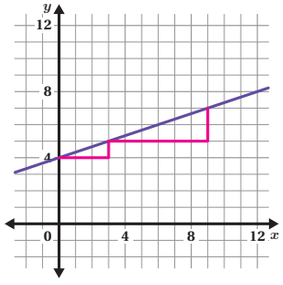
2. Here is a line.

a. Draw at least two *slope triangles* along the line.

Responses vary. Sample shown on graph.

b. What is the slope of the line?

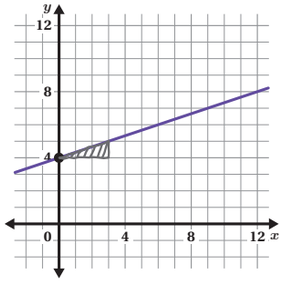
$\frac{1}{3}$ (or equivalent)



3. Liam says the slope of this line is 3.

What would you say to Liam to help him understand that the slope is $\frac{1}{3}$?

Responses vary. I would tell Liam that the slope of the line is the height-to-base ratio of the slope triangle. The height is 1 and the base is 3, so the slope is $\frac{1}{3}$.



Accelerated 7 | Unit 2 Lesson 16 192 Activity 1

Pairs | 15 min

Activity 1 Determining Slope (continued)

Purpose: Students develop strategies for determining the slope of a line using slope triangles and by analyzing student work. **(MTR.4.1)**

2 Monitor

Invite students to explore Problems 4–5.

Encourage students to draw slope triangles to help them with their thinking.

Look for students who write slopes similar to Kweku's and Liam's on the next problem.

A **Accessibility: Visual-Spatial Processing** Pair students and invite a partner to label the lengths of the slope triangle to support processing visual information.

Look for student responses and explanations to share during the Connect, including responses that disagree.

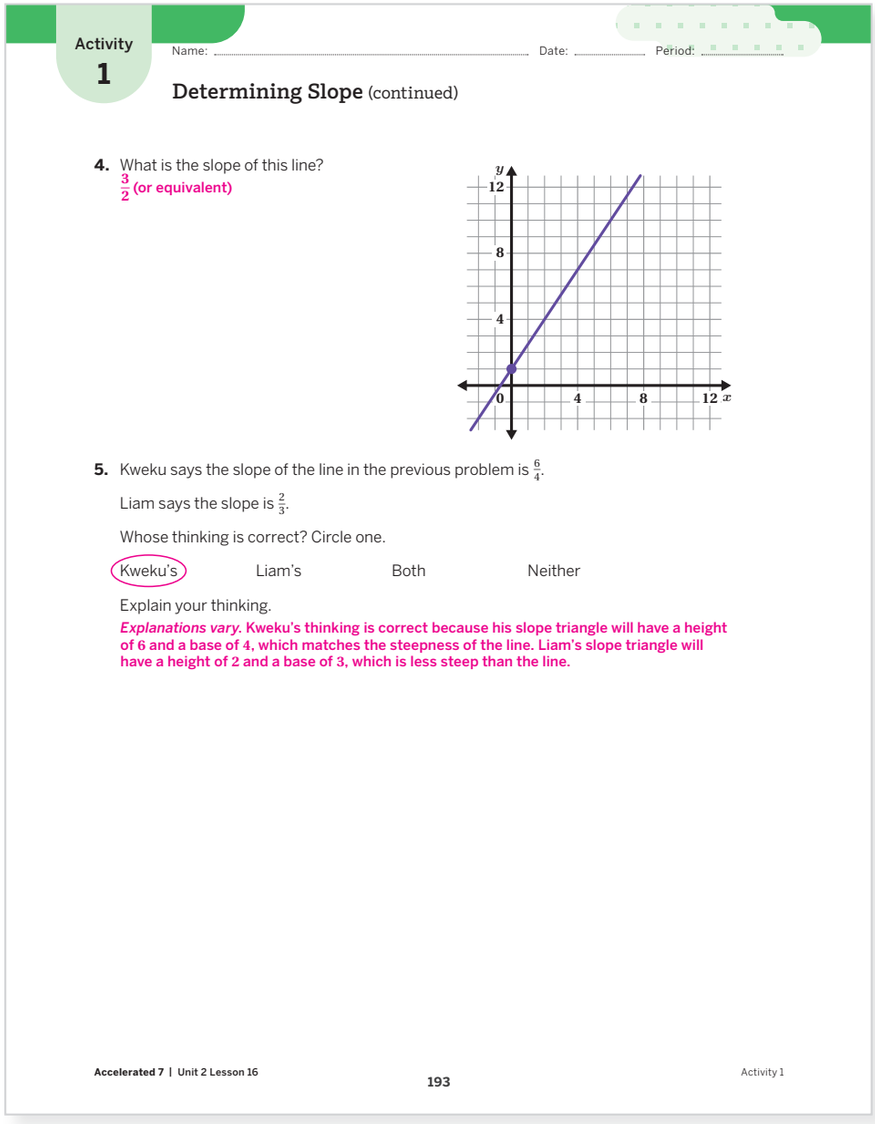
3 Connect

For Problem 5, **Invite** several students to share their responses.

Use the Decide and Defend routine to support students in making arguments and critiquing the reasoning of others. Invite students who made different choices to use reasoning to convince their classmates. **(MTR.4.1)**

To surface the Key Takeaway, consider inviting students to explain to a classmate how to determine the slope of a line, then sharing several explanations aloud.

Note: It may be helpful to record strategies for determining slope that students can refer to throughout the lesson.



Activity 1

Name: Date: Period:

Determining Slope (continued)

4. What is the slope of this line?

$\frac{3}{2}$ (or equivalent)

5. Kweku says the slope of the line in the previous problem is $\frac{6}{4}$.

Liam says the slope is $\frac{2}{3}$.

Whose thinking is correct? Circle one.

Kweku's Liam's Both Neither

Explain your thinking.

Explanations vary. Kweku's thinking is correct because his slope triangle will have a height of 6 and a base of 4, which matches the steepness of the line. Liam's slope triangle will have a height of 2 and a base of 3, which is less steep than the line.

Accelerated 7 | Unit 2 Lesson 16 193 Activity 1

Key Takeaway: The slope of a line is the ratio of the height to the base of the **slope triangles** on a line. A slope triangle with a height-to-base ratio of 6 to 4 has the same slope as a slope triangle with a height-to-base ratio of 3 to 2.

Independent | 15 min

Activity 2 Repeated Challenges

Purpose: Students develop fluency with determining the slopes of lines in a series of repeated challenges. **(MTR.3.1)**

Short on time? Consider shortening this activity or omitting and returning to it as students need more practice with slope in future units.

1 Launch

Students will need a partner for this activity. They will solve and compare their solutions to problems.

Invite students to solve as many challenges as they have time for. Share that it's more important to make sense of the challenges than it is to work quickly. **(MTR.3.1)**

A **Accessibility: Memory and Attention** Invite students who would benefit from having more time to think deeply to complete fewer challenges.

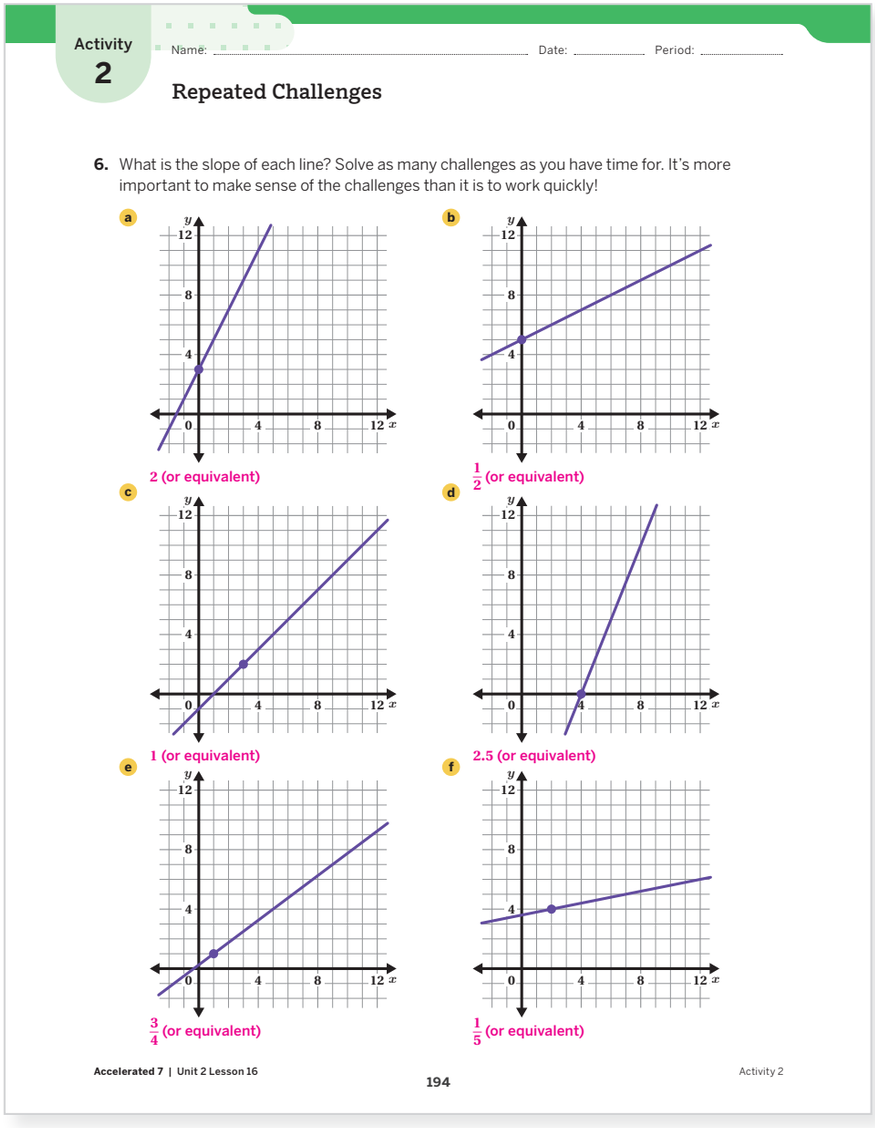
2 Monitor

Listen for strategies students use in discussing how to determine the slopes, such as drawing slope triangles, counting horizontal and vertical distances, or recognizing slopes written as equivalent ratios.

Note: Look at students' work to see how many challenges each student has completed correctly.

3 Connect

Invite students to share strategies that they found helpful or what they learned from the mistakes they made while solving these challenges.



Activity 2

Name: _____ Date: _____ Period: _____

Repeated Challenges

6. What is the slope of each line? Solve as many challenges as you have time for. It's more important to make sense of the challenges than it is to work quickly!

a. 2 (or equivalent)

b. $\frac{1}{2}$ (or equivalent)

c. 1 (or equivalent)

d. 2.5 (or equivalent)

e. $\frac{3}{4}$ (or equivalent)

f. $\frac{1}{5}$ (or equivalent)

Accelerated 7 | Unit 2 Lesson 16 194 Activity 2

Whole Class | 5 min

Synthesis

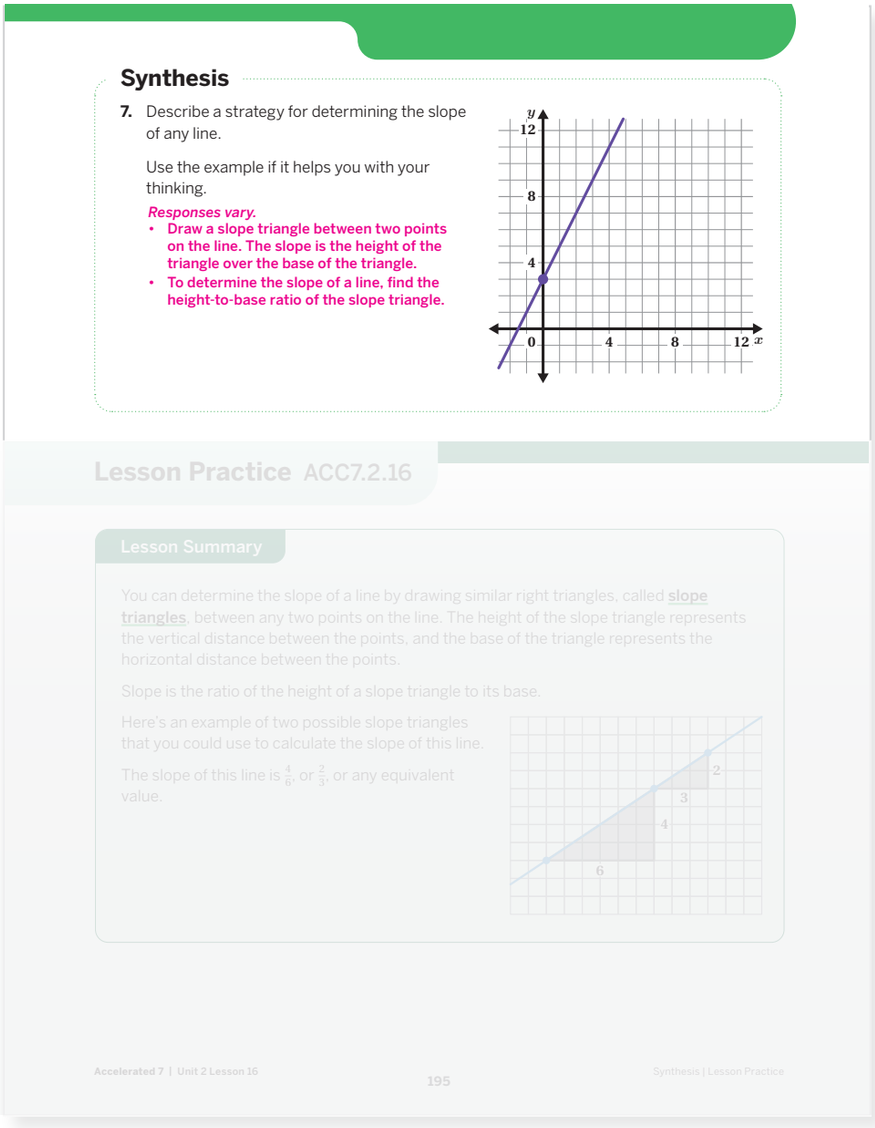
Purpose: Students synthesize and generalize strategies for determining the slope of any line.

Invite students to respond independently, and then share their thinking with a partner.

Collect and share a variety of ideas, such as:

- Drawing slope triangles and using the side lengths.
- Counting horizontal and vertical distances.
- Writing the slope as the ratio of the height to the base of a slope triangle.
- Thinking about the steepness of the line (a steeper line has a larger slope).

Note: If time allows, invite students to share with a partner what makes sense about each strategy and what questions they still have.



Synthesis

7. Describe a strategy for determining the slope of any line.

Use the example if it helps you with your thinking.

Responses vary.

- Draw a slope triangle between two points on the line. The slope is the height of the triangle over the base of the triangle.
- To determine the slope of a line, find the height-to-base ratio of the slope triangle.

Lesson Practice ACC7.2.16

Lesson Summary

You can determine the slope of a line by drawing similar right triangles, called slope triangles, between any two points on the line. The height of the slope triangle represents the vertical distance between the points, and the base of the triangle represents the horizontal distance between the points.

Slope is the ratio of the height of a slope triangle to its base.

Here's an example of two possible slope triangles that you could use to calculate the slope of this line.

The slope of this line is $\frac{4}{6}$, or $\frac{2}{3}$, or any equivalent value.

Accelerated 7 | Unit 2 Lesson 16 195 Synthesis | Lesson Practice

Lesson Takeaway: Drawing **slope triangles** is a helpful strategy for determining the slope of a line. The slope is the ratio of the height to the base of a slope triangle.

Independent | 5 min

Show What You Know

Purpose: Students demonstrate their understanding by determining the slope of a line on the coordinate plane.

Today's Goals

1. **Goal:** Use similar *slope triangles* to determine the slope of a line.
2. **Language Goal:** Describe a strategy for determining the slope of a line. **(Reading, Writing, Speaking, and Listening)**

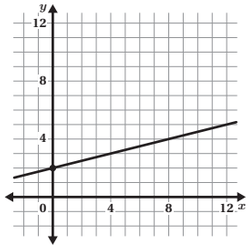
Show What You Know (Assessment Resources)

Name: _____ Date: _____ Period: _____

Show What You Know 2.16

Determine the slope of this line.

$\frac{1}{4}$ (or equivalent)



Differentiation Use after Lesson 16

Support

Provide targeted intervention for students by using these resources.

If students need support determining slopes of lines:

» **Respond to Student Thinking:**

- Use the ***Calculating Slope by Drawing Triangles on a Coordinate Plane*** **Mini-Lesson**.
- Use the ***Determining the Constant of Proportionality*** **Mini-Lesson**.
- Select a few related **Practice Problems** and give individualized feedback.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 4 Extension Activities**.

Professional Learning

Take a moment to reflect on the **Decide and Defend** routine in this lesson. How did you support students in making a clear and concise claim, asking clarifying questions, and posing counterarguments? What about the routine felt successful? What are areas for growth next time?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

7. Describe a strategy for determining the slope of any line.

Use the example if it helps you with your thinking.

Responses vary.

- Draw a slope triangle between two points on the line. The slope is the height of the triangle over the base of the triangle.
- To determine the slope of a line, find the height-to-base ratio of the slope triangle.

Lesson Practice ACC7.2.16

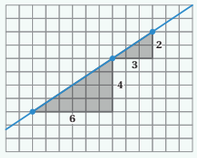
Lesson Summary

You can determine the slope of a line by drawing similar right triangles, called **slope triangles**, between any two points on the line. The height of the slope triangle represents the vertical distance between the points, and the base of the triangle represents the horizontal distance between the points.

Slope is the ratio of the height of a slope triangle to its base.

Here's an example of two possible slope triangles that you could use to calculate the slope of this line.

The slope of this line is $\frac{4}{6}$, or $\frac{2}{3}$, or any equivalent value.

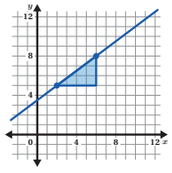


Lesson Practice ACC7.2.16

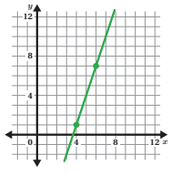
Name: _____ Date: _____ Period: _____

Problems 1–4: Determine the slope of each line.

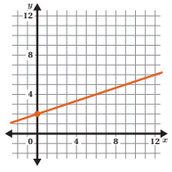
1. Slope: $\frac{3}{4}$ (or equivalent)



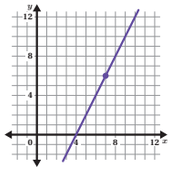
2. Slope: 3 (or equivalent)



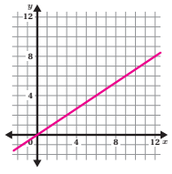
3. Slope: $\frac{1}{3}$ (or equivalent)



4. Slope: 2 (or equivalent)



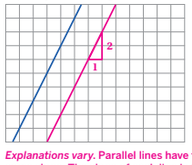
5. Draw a line with a slope of $\frac{2}{3}$.



Responses vary.

6. Here is a line. Draw a line that is parallel. What is the slope of each line?

Explain your thinking.



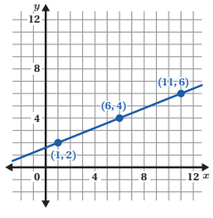
Explanations vary. Parallel lines have the same slope. The slope of each line is 2.

Lesson Practice ACC7.2.16

Name: _____ Date: _____ Period: _____

7. Fatima says the slope of this line is $\frac{2}{5}$. Daniela says the slope of this line is 2.5. Whose claim is correct? Explain your thinking.

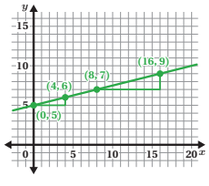
Fatima's. *Explanations vary.* If I draw a slope triangle, the vertical distance, 2, divided by the horizontal distance, 5, would be $\frac{2}{5}$.



FAST Practice

8. Select *all* the expressions that could represent the slope of this line.

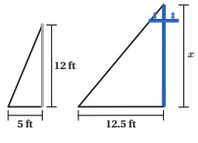
- [x] A. $\frac{1}{4}$
- [ ] B. $\frac{4}{6}$
- [ ] C. 4
- [x] D. $\frac{2}{8}$
- [ ] E. $\frac{8}{2}$



Spiral Review

9. A 12-foot flagpole casts a shadow that is 5 feet long. At the same time, a nearby telephone pole casts a shadow that is 12.5 feet long. What is the height of the telephone pole, x?

A. 5.2 feet
B. 15 feet
C. 18.5 feet
D. 30 feet



Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1–4	1	MA.8.AR.3.2
	5–7	2	MA.8.AR.3.2
FAST Practice	8	3	MA.8.AR.3.2
Spiral Review			
	9	2	MA.8.GR.2.4

Math in Action

Students consider the math they learned in this unit and how it applies to the workplace, their world, and communities.

In this Unit

Students studied dilations and similar figures. They used similar triangles to explain, understand, and apply the concept of slope.

Career Connection | Meet Laura Escobar

Invite students to read about how the zoom feature on an interactive online map uses dilations and similarity and how cartographers connect algebra and geometry to create and maintain maps. Consider showing a topographic map to students and invite them to explore its features. Then invite them to read about mathematician Laura Escobar whose research focuses on the connections between algebra and geometry.

Consider asking:

- "What are some other ways that algebra and geometry might be connected?"

B.E.S.T. Benchmark Connection

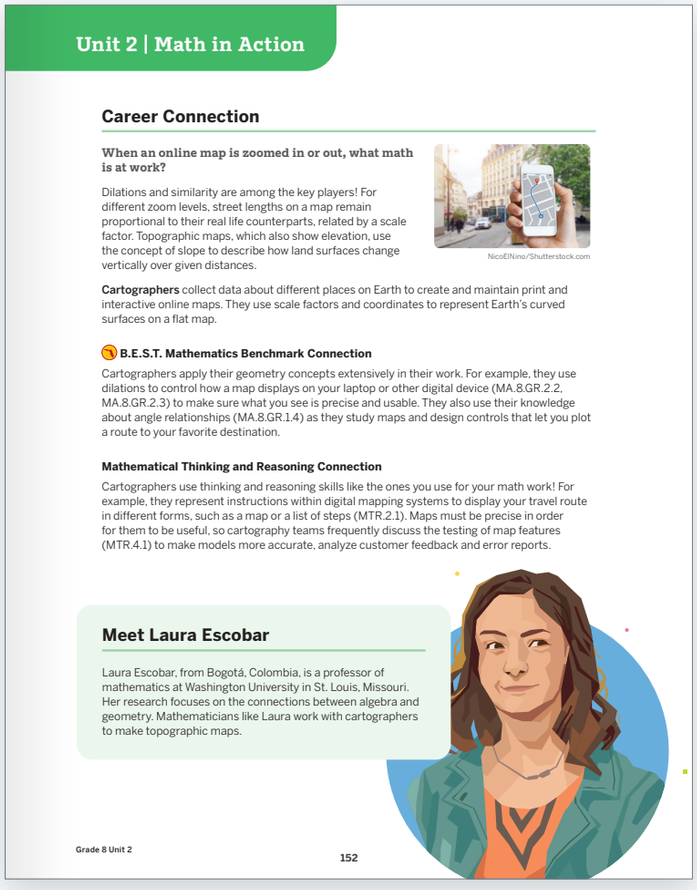
Consider asking:

- "What other math concepts have you studied in this unit could you apply to online maps?" Sample response: You can open an online map on a variety of devices and compare the scale factors used to change the displays. (MA.8.GR.2.4)

Mathematical Thinking and Reasoning Connections

Consider asking:

- "How would you explain to a classmate how cartographers and online mapping designers apply Mathematical Thinking and Reasoning standard MTR 7.1 in their work?" Sample response: People use maps on their phones and in their cars. So cartographers connect transformations, dilations, and similarity to everyday use of online map programs so that they work accurately and efficiently.



Unit 2 | Math in Action

Career Connection

When an online map is zoomed in or out, what math is at work?

Dilations and similarity are among the key players! For different zoom levels, street lengths on a map remain proportional to their real life counterparts, related by a scale factor. Topographic maps, which also show elevation, use the concept of slope to describe how land surfaces change vertically over given distances.

NicoElNino/Shutterstock.com

Cartographers collect data about different places on Earth to create and maintain print and interactive online maps. They use scale factors and coordinates to represent Earth's curved surfaces on a flat map.

B.E.S.T. Mathematics Benchmark Connection

Cartographers apply their geometry concepts extensively in their work. For example, they use dilations to control how a map displays on your laptop or other digital device (MA.8.GR.2.2, MA.8.GR.2.3) to make sure what you see is precise and usable. They also use their knowledge about angle relationships (MA.8.GR.1.4) as they study maps and design controls that let you plot a route to your favorite destination.

Mathematical Thinking and Reasoning Connection

Cartographers use thinking and reasoning skills like the ones you use for your math work! For example, they represent instructions within digital mapping systems to display your travel route in different forms, such as a map or a list of steps (MTR.2.1). Maps must be precise in order for them to be useful, so cartography teams frequently discuss the testing of map features (MTR.4.1) to make models more accurate, analyze customer feedback and error reports.

Meet Laura Escobar

Laura Escobar, from Bogotá, Colombia, is a professor of mathematics at Washington University in St. Louis, Missouri. Her research focuses on the connections between algebra and geometry. Mathematicians like Laura work with cartographers to make topographic maps.

Grade 8 Unit 2

152

Assess and Respond

End-of-Unit Assessment

Independent | 45 min

Facilitation: Assign the End-of-Unit Assessment to learn about your students' understanding of the concepts and skills in this unit. There are two forms of the End-of-Unit Assessment: Forms A and B. If time allows, consider inviting students to complete one or more of the tasks in the Unit Synthesis and Reflection.

Item Analysis, Forms A and B

Problem(s)	Concept or skill	Addressed in	DOK	FL B.E.S.T. Math Benchmarks
1	Using the structure of lines graphed on grids to identify a line by its slope	Lesson 16	1	MA.8.AR.3.2 MTR.5.1
2	Identifying properties of dilations	Lesson 10	2	MA.8.GR.2.2
3a	Using the structure of similar figures to identify similar polygons	Lesson 12	1	MA.8.GR.2.2 MTR.5.1
3b	Describing a sequence of transformations to map a polygon onto a similar polygon	Lesson 12	2	MA.8.GR.2.2
4	Determining the slope of a line	Lesson 16	1	MA.8.AR.3.2
5a	Determining angle measures and side lengths for similar figures	Lesson 12	1	MA.8.GR.2.2
5b	Using the structure of transformations to describe a sequence of transformations that show two figures are similar	Lesson 12	2	MA.8.GR.2.3
6	Using properties of similar figures to explain why two triangles might not be similar	Lesson 13	2	MA.8.GR.2.4 MTR.4.1
7a, 7b	Drawing the dilation of a figure given a center of dilation and a scale factor	Lesson 10	1	MA.8.GR.2.3 MTR.5.1
7c	Using the structure of dilations to explain why two figures are similar	Lesson 12	3	MA.8.GR.2.2 MTR.5.1

Assessment Resources

- Student Assessments
- Assessment Rubrics with Answers
- Show What You Knows and Answer Keys

Intervention and Extension Resources

- Mini-Lessons
- Extensions

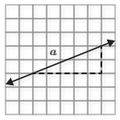
Name: Date: Period:

End-of-Unit Assessment

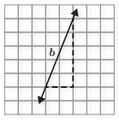
Unit 2

1. Which line has a slope of $\frac{5}{2}$?

A.



B.



C. Both **D.** Neither

2. Which statement is *always* true?

A. Dilations of a triangle keep angle measures the same.

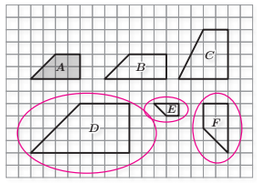
B. Dilations of a triangle keep side lengths the same.

C. Dilations of a triangle must be congruent to the original triangle.

D. Dilations of a triangle always make the sides longer.

3. Here are several polygons.

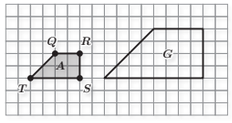
a Circle *all* of the polygons that are similar to polygon A.



b Polygon G is similar to polygon A.

Describe a sequence of transformations that moves polygon A to polygon G.

Responses vary. Translate polygon A 10 units right, then dilate polygon A using point S as the center of dilation and a scale factor of 2.



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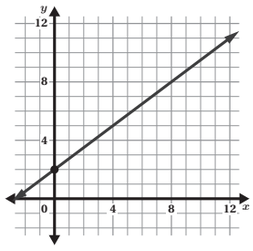
Name: Date: Period:

End-of-Unit Assessment (continued)

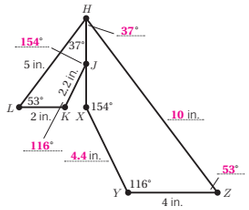
Unit 2

4. What is the slope of this line?

$\frac{3}{4}$ (or equivalent)



5. Figure $HJKL$ is similar to figure $HXYZ$.



a Label the missing side and angle measurements in both figures.

Responses shown in figure.

b Precisely describe a sequence of transformations to show that figure $HJKL$ is similar to figure $HXYZ$.

Responses vary. Reflect polygon $HJKL$ over line HJ, then dilate using a scale factor of 2 and point H as the center of dilation.

Name: Date: Period:

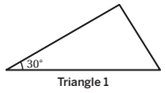
End-of-Unit Assessment (continued)

Unit 2

6. Here is Triangle 1. Triangle 2 also has a 30° angle.

Explain or show why Triangle 1 and Triangle 2 might *not* be similar.

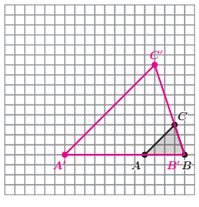
Responses vary. For two triangles to be similar, all three pairs of corresponding angles must be congruent. One pair of corresponding angles is 30°, but the remaining angles could be different in each triangle, so the triangles might not be similar.



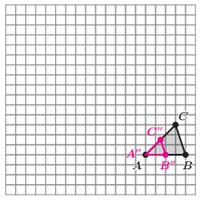
Triangle 1

7. Here is triangle ABC.

a Draw triangle $A'B'C'$, the image of triangle ABC, after a dilation using point B as the center of dilation and a scale factor of 3.



b Draw triangle $A''B''C''$, the image of triangle ABC, after a dilation using point A as the center of dilation and a scale factor of $\frac{1}{2}$.



c Explain why triangles $A'B'C'$ and $A''B''C''$ are similar.

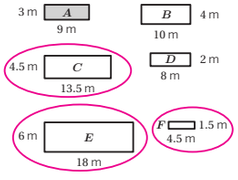
Responses vary. If triangle $A'B'C'$ is dilated using point B as the center of dilation and a scale factor of $\frac{1}{3}$, the result is triangle ABC. If triangle ABC is then dilated using point A as the center of dilation and a scale factor of $\frac{1}{2}$, the result is triangle $A''B''C''$.

Name: Date: Period:

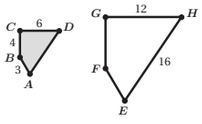
End-of-Unit Assessment (continued)

Unit 2

8. Circle *all* the scaled copies of rectangle A.

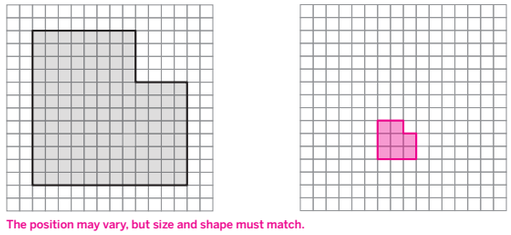


9. Figure $EFGH$ is a scaled copy of figure $ABCD$. Select *all* of the true statements.



- ☑ **A.** Segment EF is twice as long as segment AB.
- ☐ **B.** The ratio of $\frac{AB}{BC}$ is equivalent to the ratio of $\frac{EH}{HG}$.
- ☐ **C.** The scale factor from $EFGH$ to $ABCD$ is 2.
- ☑ **D.** The length of segment AD is 8 units.
- ☐ **E.** The area of $EFGH$ is twice the area of $ABCD$.

10. Draw a scaled copy of the polygon using a scale factor of $\frac{1}{4}$.



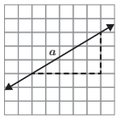
The position may vary, but size and shape must match.

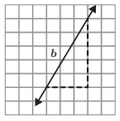
Name: Date: Period:

End-of-Unit Assessment

Unit 2

1. Which line has a slope of $\frac{3}{5}$?

A. 

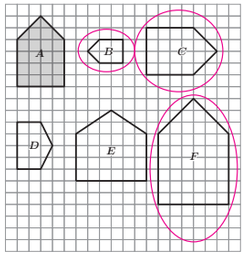
B. 

C. Both **D.** Neither

2. Which statement is *not* true?

A. Dilations of a line are always parallel to the original line.
B. Dilations of a triangle are always similar to the original triangle.
C. Dilations always keep side lengths the same.
D. Dilations always keep angle measures the same.

3. Here are several polygons.

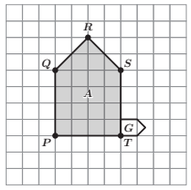


a Circle *all* of the polygons that are similar to polygon A.

b Polygon G is similar to polygon A.

Describe a sequence of transformations that moves polygon A to polygon G.

Responses vary. Rotate polygon A 90° clockwise around point T, then dilate polygon A using a scale factor of $\frac{1}{4}$ and point T as the center of dilation.



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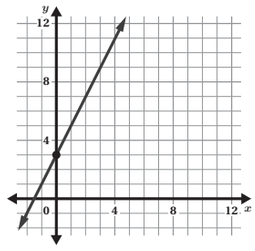
Name: Date: Period:

End-of-Unit Assessment (continued)

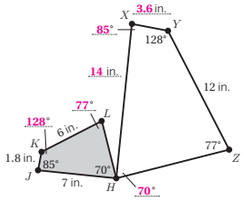
Unit 2

4. What is the slope of this line?

2 (or equivalent)



5. Figure $HJKL$ is similar to figure $HXYZ$.



a Label the missing side and angle measurements in both figures.

Responses shown in figure.

b Precisely describe a sequence of transformations to show that figure $HJKL$ is similar to figure $HXYZ$.

Responses vary. Rotate polygon $HJKL$ 90° clockwise around point H, then dilate it by a scale factor of 2 using point H as the center of dilation.

Name: Date: Period:

End-of-Unit Assessment (continued)

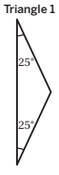
Unit 2

6. Here is Triangle 1. Triangle 2 also has two 25° angles.

Explain or show why Triangle 1 and Triangle 2 must be similar.

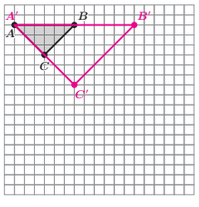
Responses vary. For two triangles to be similar, all three pairs of corresponding angles must be congruent. Two pairs of corresponding angles are 25°, and the angles in each triangle must sum to 180°, so the third pair of corresponding angles are both 130°.

Triangle 1

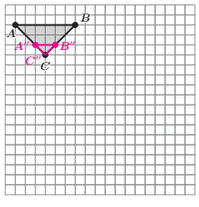


7. Here is triangle ABC.

a Draw triangle $A'B'C'$, the image of triangle ABC, after a dilation using point A as the center of dilation and a scale factor of 2.



b Draw triangle $A''B''C''$, the image of triangle ABC, after a dilation using point C as the center of dilation and a scale factor of $\frac{1}{3}$.



c Explain why triangles $A'B'C'$ and $A''B''C''$ are similar.

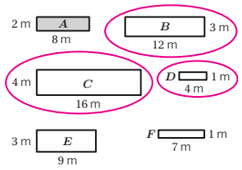
Responses vary. If triangle $A'B'C'$ is dilated using point A as the center of dilation and a scale factor of $\frac{1}{2}$, the result is triangle ABC. If triangle ABC is then dilated using point C as the center of dilation and a scale factor of $\frac{1}{3}$, the result is triangle $A''B''C''$.

Name: Date: Period:

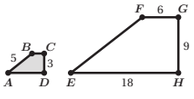
End-of-Unit Assessment (continued)

Unit 2

8. Circle *all* the scaled copies of rectangle A.

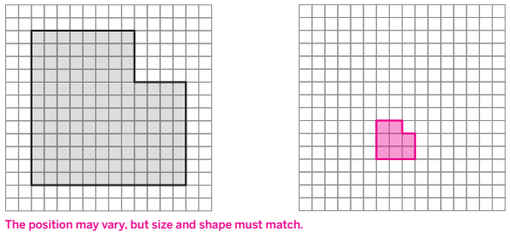


9. Figure $EFGH$ is a scaled copy of figure $ABCD$. Select *all* of the true statements.



- ☐ **A.** Segment GH is three times as long as segment AB.
- ☑ **B.** The ratio of $\frac{AB}{BC}$ is equivalent to the ratio of $\frac{EF}{FG}$.
- ☑ **C.** The scale factor from $EFGH$ to $ABCD$ is $\frac{1}{3}$.
- ☑ **D.** The length of segment BC is 2 units.
- ☐ **E.** The area of $EFGH$ is three times the area of $ABCD$.

10. Draw a scaled copy of the polygon using a scale factor of $\frac{1}{4}$.



The position may vary, but size and shape must match.

D Differentiation (End-of-Unit Assessment)

Note: To **strengthen** and **stretch** students' learning, refer to the differentiation resources suggested throughout this unit and in the Unit Overview.

Sub-Unit Goals	Problem(s)	To respond to student thinking, consider:	

Sub-Unit 2:
• Dilate figures on and off a coordinate grid.
• Identify the scale factor and the center of dilation.
(Lessons 7–11)
	2, 7	**Support**	• **Mini-Lesson:** *Dilating Figures on a Square Grid*
			• **Teacher Move:** Consider revisiting:
			  • Lesson 9 (*Make It a Combo!*)
			  • Lesson 9 (*Match My Dilation*)
			  • Lesson 11 (*Dilations on a Plane*)

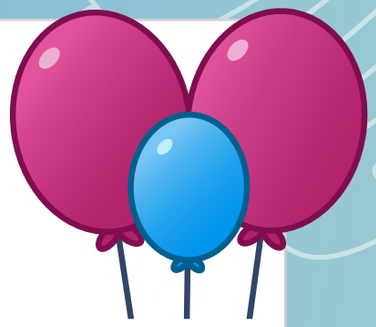
		Strengthen	• **Challenge Creator:** Lesson 4 (Match My Dilation)

Sub-Unit 3:
• Use transformations to determine whether two figures are similar.
• Use strategies to determine missing side lengths of similar triangles.
(Lessons 12–14)
	3, 5, 6	**Support**
		Strengthen
			• **Mini-Lesson:** *Determining Missing Side Lengths in Similar Triangles*
			• **Teacher Move:** Consider revisiting:
			  • Lesson 12 (*Scavenger Hunt*)
			  • Lesson 13 (*Are Angles Enough?*)

		Stretch	• **Teacher Move:** Invite students to draw two similar figures and include the minimum number of angles and side lengths such that someone could find all the missing angles and side lengths.
Sub-Unit 4: • Make connections between the slope of a line and similar triangles created by that line. • Determine the slopes of lines. **(Lessons 15–16)**	1, 4	**Support**	• **Mini Lesson:** *Calculating Slope by Drawing Triangles on a Coordinate Plane* • **Teacher Move:** Consider revisiting Lesson 16 (*Slope Challenges*)
		Strengthen	• **Repeated Challenges:** Lesson 16 (*Slope Challenges*)

UNIT 3

Introducing Proportional Relationships



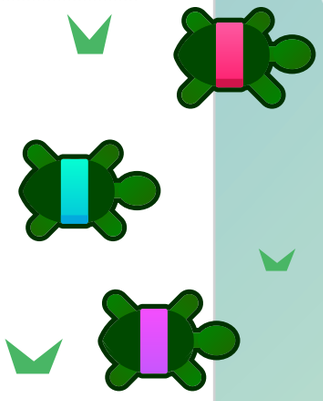
Students explore proportional relationships in a variety of real-world contexts and make comparisons using representations such as tables, equations, and graphs.

Essential Questions

- What does it mean for two things to be proportionally related? How can you tell?
- What are the different ways you can represent proportional relationships? How are the representations related?

Sub-Unit Goals

Sub-Unit 1	• Use tables to recognize proportional relationships. • Determine a constant of proportionality from a table.
Sub-Unit 2	• Write and use equations to describe proportional relationships. • Determine a constant of proportionality from an equation.
Sub-Unit 3	• Use graphs to recognize proportional relationships. • Determine a constant of proportionality from a graph.



The Math of the Unit

Unit Outcomes Telling a story that narrows and deepens with the grade

Florida B.E.S.T. Standards for Mathematics

Use percentages and proportional reasoning to solve problems. **(MA.7.AR.3)**

Analyze and represent two-variable proportional relationships. **(MA.7.AR.4)**

Extend understanding of proportional relationships to two-variable linear equations. **(MA.8.AR.3)**

Benchmarks Addressed in This Unit: MA.7.AR.3.3 MA.7.AR.4.1 MA.7.AR.4.2 MA.7.AR.4.3 MA.7.AR.4.4 MA.7.AR.4.5 MA.8.AR.3.1

Mathematical Thinking and Reasoning Standards

- Demonstrate understanding by representing problems in multiple ways. **(MTR.2.1)**
- Engage in discussions that reflect on the mathematical thinking of self and others. **(MTR.4.1)**
- Use patterns and structure to help understand and connect mathematical concepts. **(MTR.5.1)**
- Apply mathematics to real-world contexts. **(MTR.7.1)**

Also Addressed: **MTR.1.1, MTR.3.1, MTR.6.1**

Coherence Telling a story within and across grades

In this Unit

Students explore proportional relationships in a variety of real-world contexts and make comparisons using representations such as tables, equations, and graphs.

Prior Learning

- Write and interpret ratios **(MA.6.AR.3.1)**
- Equivalent ratios **(MA.6.AR.3.3, MA.6.AR.3.5)**
- Unit rates **(MA.6.AR.3.2, MA.6.AR.3.5)**
- Graph points in a coordinate plane **(MA.6.GR.1.1)**

Future Learning

- **Unit 4:** Linear relationships **(MA.8.AR.3.1, MA.8.AR.3.2, MA.8.AR.3.3)**

See the Connections to Future Learning page for more information, including explanations and examples.

Rigor and Balance Integrating the aspects of rigor in ways that support one another

Here are examples of the aspects of rigor and balance in this unit. See the first page of each lesson's Teacher Edition for more information.

Conceptual Understanding

In **Lessons 1–9**, students develop conceptual understanding of proportionality by analyzing multiple representations of proportional relationships.

Strategic Competence

In **Lesson 1**, students develop strategic competence as they use different strategies to determine unknown values in a table.

Procedural Fluency

In **Lessons 4–6**, students develop procedural fluency as they calculate constants of proportionality.

Adaptive Reasoning

In **Lesson 6**, students develop adaptive reasoning as they consider how one relationship can be represented in many ways.

Application

In **Lesson 10**, students apply their understanding of proportional relationships to compare the distance/time relationships of different turtles.

Productive Disposition

In **Lesson 8**, students have opportunities to develop a productive disposition as they problem solve in a situation like manufacturing Dinopops.

Unit at a Glance

Assess and Respond

A Pre-Unit Check

Learn more about your students' understanding of foundational concepts and skills that will support them in the upcoming unit. This can be given in its entirety before the unit or spread throughout the unit.

MA.5.GR.4.1 MA.6.AR.3.1 MA.6.AR.3.2 MA.6.AR.3.3 MA.6.AR.3.4 MA.6.AR.3.5
MTR.4.1 MTR.7.1

Sub-Unit 1

1 Balloon Float

Introducing Proportional Relationships With Tables

Explore **proportional relationships** and analyze tables of equivalent ratios.

MA.7.AR.4.1 MA.7.AR.4.2
MTR.4.1 MTR.5.1

2 Sugar, Spice, and Everything Rice

Constant of Proportionality

Determine, interpret, and use **constants of proportionality** to make sense of proportional relationships.

MA.7.AR.4.1 MA.7.AR.4.2 MA.8.AR.3.1
MTR.2.1 MTR.6.1

3 Pen Pals

Converting Units

Convert measurements from one unit to another in different measurement systems using ratios.

MA.7.AR.3.3
MTR.2.1 MTR.4.1

Assess and Respond

A Sub-Unit Quiz

Learn about your students' understanding of the concepts and skills so far in this unit.

MA.7.AR.4.1 MA.7.AR.4.2 MA.7.AR.4.4 MA.7.AR.4.5
MTR.5.1 MTR.6.1 MTR.7.1

Sub-Unit 3

8 DinoPops

Introducing Graphs of Proportional Relationships

Explore what a proportional relationship looks like graphically.

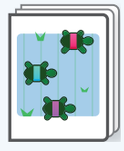
MA.7.AR.4.1
MTR.2.1 MTR.5.1

9 Gallon Challenge

Interpreting Graphs of Proportional Relationships

Use graphs to identify constants of proportionality.

MA.7.AR.4.2
MTR.3.1 MTR.7.1



10 Three Turtles

Proportional Relationships and Graphs

Use graphs and equations to compare proportional relationships.

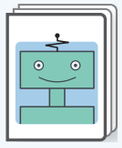
MA.7.AR.4.1 MA.7.AR.4.2 MA.7.AR.4.4 MA.7.AR.4.5 MA.8.AR.3.1
MTR.1.1 MTR.2.1 MTR.4.1

Pacing: 14 days | **Short on time?** See pacing considerations below.

All instructional and assessment days are ~45 minutes each.

- 11 Lessons
- Pre-Unit Check
- Sub-Unit Quiz
- End-of-Unit Assessment

Sub-Unit 2



4 Robot Factory

Proportional Relationships and Equations

Use repeated reasoning about tables to write equations for proportional relationships.

MA.7.AR.4.2 MA.7.AR.4.4
MTR.2.1 MTR.3.1

5 Snapshots

More Equations of Proportional Relationships

Write and interpret equations that represent proportional relationships.

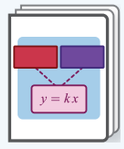
MA.7.AR.4.2 MA.7.AR.4.4 MA.7.AR.4.5
MTR.2.1 MTR.7.1

6 Two and Two

Two Equations for Each Relationship

Find the two constants of proportionality for a proportional relationship, discover they are **reciprocals**, and use them to write two equations representing the relationship.

MA.7.AR.4.2 MA.7.AR.4.4 MA.7.AR.4.5
MTR.2.1 MTR.5.1



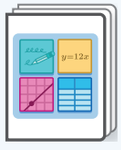
7 All Kinds of Equations

Equations of Proportional Relationships

Compare proportional and non-proportional relationships.

MA.7.AR.4.1 MA.7.AR.4.2 MA.7.AR.4.4 MA.7.AR.4.5 MA.8.AR.3.1
MTR.1.1 MTR.4.1 MTR.5.1

Assess and Respond



11 Four Representations

Connecting Descriptions, Tables, Graphs, and Equations

Examine the connection between verbal descriptions, tables, equations, and graphs of proportional relationships.

MA.7.AR.4.2 MA.7.AR.4.3 MA.7.AR.4.4
MTR.2.1 MTR.7.1

A End-of-Unit Assessment

Learn about your students' understanding of the concepts and skills in this unit. There are two forms of the End-of-Unit Assessment: Forms A and B.

MA.7.AR.4.1 MA.7.AR.4.2 MA.7.AR.4.4 MA.7.AR.4.5
MTR.3.1 MTR.4.1 MTR.6.1 MTR.7.1

Pacing Considerations

Lesson 5: This lesson supports students in developing fluency writing and using equations to make sense of proportional relationships. If students show a strong understanding working with equations of proportional relationships in earlier lessons, this lesson may be omitted. If omitted, be sure to discuss how each part of an equation of a proportional relationship relates to a situation in Lessons 6, 7, and 10.

Lesson 11: This lesson gives students an opportunity to connect descriptions, tables, equations, and graphs of proportional relationships. There is no new content introduced in this lesson.

Lesson Materials and Prep

Here are the materials and resources needed within each lesson, how to prepare them, and where to find them.

Lesson 1	Lesson 2	Lesson 3	Lesson 4
Copy: Show What You Know	**Copy:** Show What You Know **Materials:** scientific calculators *(as needed)*	**Copy:** Show What You Know	**Copy:** Show What You Know
Lesson 5	**Lesson 6**	**Lesson 7**	**Lesson 8**
Copy: Show What You Know, Activity 3 You're Invited to Explore More Sheet *(one per student, as needed)*	**Copy:** Show What You Know	**Copy:** Show What You Know, Activity 2 You're Invited to Explore More Sheet *(one per student, as needed)* **Materials:** calculators *(as needed)*	**Copy:** Show What You Know
Lesson 9	**Lesson 10**	**Lesson 11**	
Copy: Show What You Know	**Copy:** Show What You Know, Activity 3 You're Invited to Explore More Sheet *(one per student, as needed)* **Materials:** straightedges *(as needed)*	**Copy:** Show What You Know	

Resource Books

Assessment and Lesson Resources

- Pre-Unit Check
- Sub-Unit Quizzes
- End-of-Unit Assessment, Forms A & B
- Assessment Rubrics
- Show What You Knows
- Activity Sheets and Cards

Intervention and Extension Resources

- Mini-Lessons
- Prior Grade Mini-Lessons
- Extensions

Caregiver Support

These resources can be shared with students' caregivers. They provide background on the mathematics in this unit, as well as suggestions for supporting students at home.

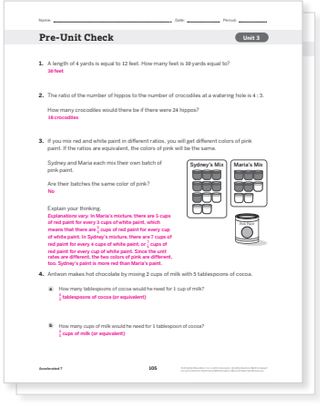
Assessments

Every unit includes a variety of formative and summative assessments designed to highlight students' understanding of a variety of concepts and skills.

Pre-Unit Check

Assign the **Pre-Unit Check** to learn more about your students' understanding of foundational concepts and skills that will support them in the upcoming unit. This can be given in its entirety before the unit or spread throughout the unit.

Standards assessed: MA.5.GR.4.1, MA.6.AR.3.1, MA.6.AR.3.2, MA.6.AR.3.3, MA.6.AR.3.4, MA.6.AR.3.5



Pre-Unit Check

Unit 3

1. A length of 4 yards is equal to 12 feet. How many feet is 10 yards equal to?
30 feet

2. The ratio of the number of hippos to the number of crocodiles at a watering hole is 4 : 3.
How many crocodiles would there be if there were 24 hippos?
18 crocodiles

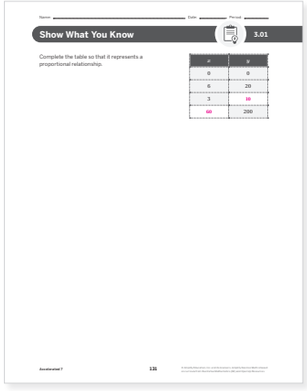
3. If you mix red and white paint in different ratios, you will get different colors of pink paint. If the ratios are equivalent, the colors of pink will be the same.
Sydney and Maria each mix their own batch of pink paint.
Are their batches the same color of pink?
No
Explain your thinking.
Explanations vary. In Maria's mixture, there are 3 cups of red paint for every 3 cups of white paint, which means that there are 1 cups of red paint for every cup of white paint. In Sydney's mixture, there are 7 cups of red paint for every 4 cups of white paint, or 1 3/4 cups of red paint for every cup of white paint. Since the unit rates are different, the two colors of pink are different, too. Sydney's paint is more red than Maria's paint.
Sydney's Mix
Maria's Mix
4. Amiwon makes hot chocolate by mixing 2 cups of milk with 5 tablespoons of cocoa.
a. How many tablespoons of cocoa would he need for 1 cup of milk?
2 1/2 tablespoons of cocoa (or equivalent)
b. How many cups of milk would he need for 1 tablespoon of cocoa?
2/5 cups of milk (or equivalent)

Accelerated 7 105

Lesson | Show What You Know

Use the **Show What You Know** for each lesson to learn more about your students' progress toward the lesson goal(s). Here is one example from this unit.

Standards assessed on Lesson 1 Show What You Know: MA.7.AR.3.2



Show What You Know

3.01

Complete the table so that it represents a proportional relationship.

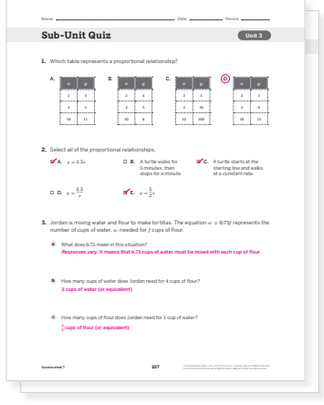
d	w
0	0
6	20
3	10
60	200

Accelerated 7 131

Sub-Unit Quiz

Assign the **Sub-Unit Quiz** to learn about your students' understanding of the concepts and skills so far in this unit.

Standards assessed: MA.7.AR.4.1, MA.7.AR.4.2, MA.7.AR.4.4, MA.7.AR.4.5



Sub-Unit Quiz

Unit 3

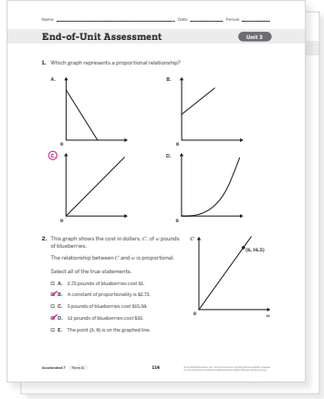
1. Which table represents a proportional relationship?
A. B. C. D.
2. Select all of the proportional relationships.
A. $y = 2.5x$
B. A turtle walks for 5 minutes, then stops for a minute.
C. A turtle starts at the starting line and walks at a constant rate.
D. $y = \frac{0.5}{x}$
E. $y = \frac{1}{2}x$
3. Jordan is mixing water and flour to make tortillas. The equation $w = 0.75f$ represents the number of cups of water, w, needed for f cups of flour.
a. What does 0.75 mean in this situation?
Responses vary. It means that 0.75 cups of water must be mixed with each cup of flour.
b. How many cups of water does Jordan need for 4 cups of flour?
3 cups of water (or equivalent)
c. How many cups of flour does Jordan need for 1 cup of water?
1 1/3 cups of flour (or equivalent)

Accelerated 7 107

End-of-Unit Assessment

Assign the **End-of-Unit Assessment** to learn about your students' understanding of the concepts and skills in this unit. There are two forms of the End-of-Unit Assessment: Forms A and B.

Standards assessed: MA.7.AR.4.1, MA.7.AR.4.2, MA.7.AR.4.4, MA.7.AR.4.5



End-of-Unit Assessment

Unit 3

1. Which graph represents a proportional relationship?
A. B. C. D.
2. This graph shows the cost in dollars, C, of w pounds of blueberries.
The relationship between C and w is proportional.
Select all of the true statements.
A. 2.75 pounds of blueberries cost $1.
B. A constant of proportionality is $2.75.
C. 5 pounds of blueberries cost $15.56.
D. 12 pounds of blueberries cost $33.
E. The point (3, 9) is on the graphed line.
(16, 44.1)

Accelerated 7 Form A 114

Differentiation

Every unit includes additional resources to support, strengthen, and stretch student learning.

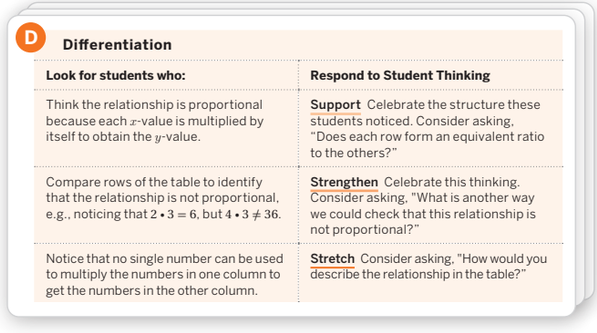
Differentiation Across the Unit

	Support	Strengthen	Stretch
	Provide targeted intervention for students by using these resources.	Reinforce students' understanding of the concepts assessed by using these resources.	Challenge students and extend their learning with these resources.
Sub-Unit 1 (Lessons 1–3) • Use tables to recognize proportional relationships. • Determine a constant of proportionality from a table.	**Mini-Lesson** • *Determining the Constant of Proportionality*	**Practice** • Lessons 1–3	**You're invited to explore more.** • Lesson 1 (*Balloon Float*) **DOK3 Practice** • Lesson 1
Sub-Unit 2 (Lessons 4–7) • Write and use equations to describe proportional relationships. • Determine a constant of proportionality from an equation.	**Mini-Lessons** • *Finding More Than One Equation to Represent Proportional Relationships* • *Representing Proportional Relationships*	**Turtle Challenges** • Lesson 10 (Three Turtles) **Practice** • Lessons 4–7	**You're invited to explore more.** • Lesson 5 (*Snapshots*) • Lesson 6 (*Two and Two*) • Lesson 7 (*All Kinds of Equations*) **DOK3 Practice** • Lessons 5 and 7
Sub-Unit 3 (Lessons 8–11) • Use graphs to recognize proportional relationships. • Determine a constant of proportionality from a graph.	**Mini-Lesson** • *Comparing Proportional Relationships With Graphs*	**Practice** • Lessons 8–11	**You're invited to explore more.** • Lesson 8 (*DinoPops*) • Lesson 10 (*Three Turtles*)

Differentiation Within the Lesson

Each Teacher Edition includes point-of-use differentiation suggestions to modify or adjust instruction during the lesson to accommodate the needs of all learners.

Here is one example from **Lesson 1**, where students analyze tables of equivalent ratios to build conceptual understanding of proportional relationships.



Differentiation

Look for students who:	Respond to Student Thinking
Think the relationship is proportional because each x-value is multiplied by itself to obtain the y-value.	**Support** Celebrate the structure these students noticed. Consider asking, "Does each row form an equivalent ratio to the others?"
Compare rows of the table to identify that the relationship is not proportional, e.g., noticing that $2 \cdot 3 = 6$, but $4 \cdot 3 \neq 36$.	**Strengthen** Celebrate this thinking. Consider asking, "What is another way we could check that this relationship is not proportional?"
Notice that no single number can be used to multiply the numbers in one column to get the numbers in the other column.	**Stretch** Consider asking, "How would you describe the relationship in the table?"

A Accessibility

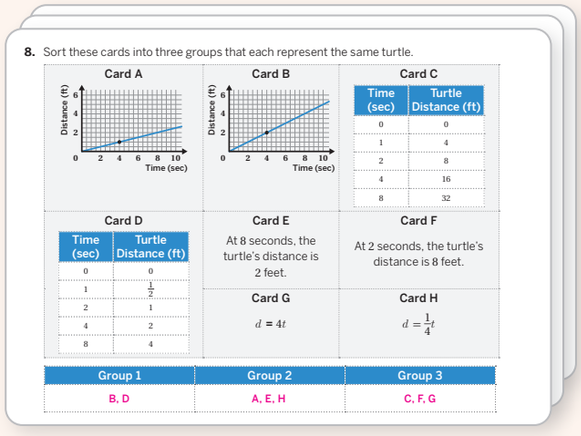
Every lesson is designed using the principles of Universal Design for Learning. Every lesson also includes support suggestions that build on students' strengths.

Universal Design for Learning

Here is one example from a lesson designed to support Representation.

Spotlight on Language and Symbols . . .

Lesson 10: In this lesson, students explore the relationship between graphs, equations, and tables, making explicit links between representations.



Just-In-Time Accommodations and Supports

Each Teacher Edition includes suggested supports based on the areas of cognitive function from the Education Development Center (EDC).

Spotlight on Executive Functioning . . .

Here are two suggestions in this unit for supporting students with initiating complex tasks.

Lesson 6, Activity 1

A **Accessibility: Executive Functioning** Consider chunking this activity by inviting students to choose one table at a time and look for an equation to match it.

Lesson 11, Activity 2

A **Accessibility: Executive Functioning** Consider pre-selecting two quantities from Activity 1 and providing a corresponding table of values for the relationship. This will allow students to focus on analyzing the connections between the table, graph, and equation.

Language Development

Every lesson includes opportunities for all students to develop fluency with mathematical language.

Vocabulary

New Vocabulary	Lesson(s)
proportional relationship	2
constant of proportionality	3
reciprocal	6

Review Vocabulary
coordinate plane
equivalent ratios
origin
scaled copy

Math Language Routines

Math Language Routine	Lesson(s)
MLR1: Stronger and Clearer Each Time	11
MLR2: Collect and Display	1–4, 7, and 9
MLR3: Critique, Correct, Clarify	4
MLR7: Compare and Connect	1, 3, and 9

Sub-Unit Language Goals

- Justify whether a table represents a proportional relationship.
- Describe the relationship between values in a table using terms such as *proportional, non-proportional,* and *constant of proportionality.*
- Describe relationships between values in an equation using terms such as *proportional, non-proportional,* and *constant of proportionality.*
- Describe the relationship between values in a graph using terms such as *proportional, non-proportional,* and *constant of proportionality.*

English Language Learners Support

Every Teacher Edition includes suggestions for supporting the needs of students learning English.

Here is one example from **Lesson 7**, where students are analyzing relationships in multiple representations to determine whether a relationship is proportional. Students are given sentence frames to support them in expressing their thinking about proportional relationships.

ELL **English Language Learners** Provide sentence frames to help students explain their strategy (e.g., "___ is proportional/not proportional because___."). **(Writing and Speaking)**

Connections to Future Learning

Here is how the content in this unit connects to where your students are headed in their math journeys.

Linear Relationships

In this unit, students write equations to describe proportional relationships. **In Unit 4,** students revisit proportional relationships and write equations for linear relationships that are not proportional.

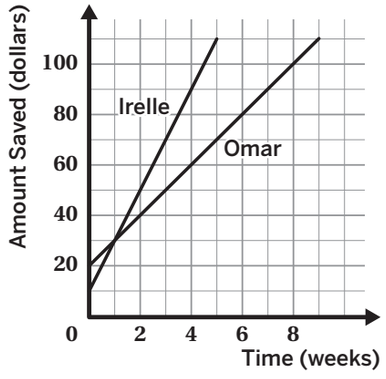
FL B.E.S.T. Math Benchmarks:
MA.8.AR.3.1, MA.8.AR.3.2, MA.8.AR.3.3

Example:

For example, Irelle and Omar save some of the money they earned. Let w represent the number of weeks passed. Let s represent the amount saved.

Equation for Irelle's savings: $s = 20w + 10$

Equation for Omar's savings: $s = 10w + 20$

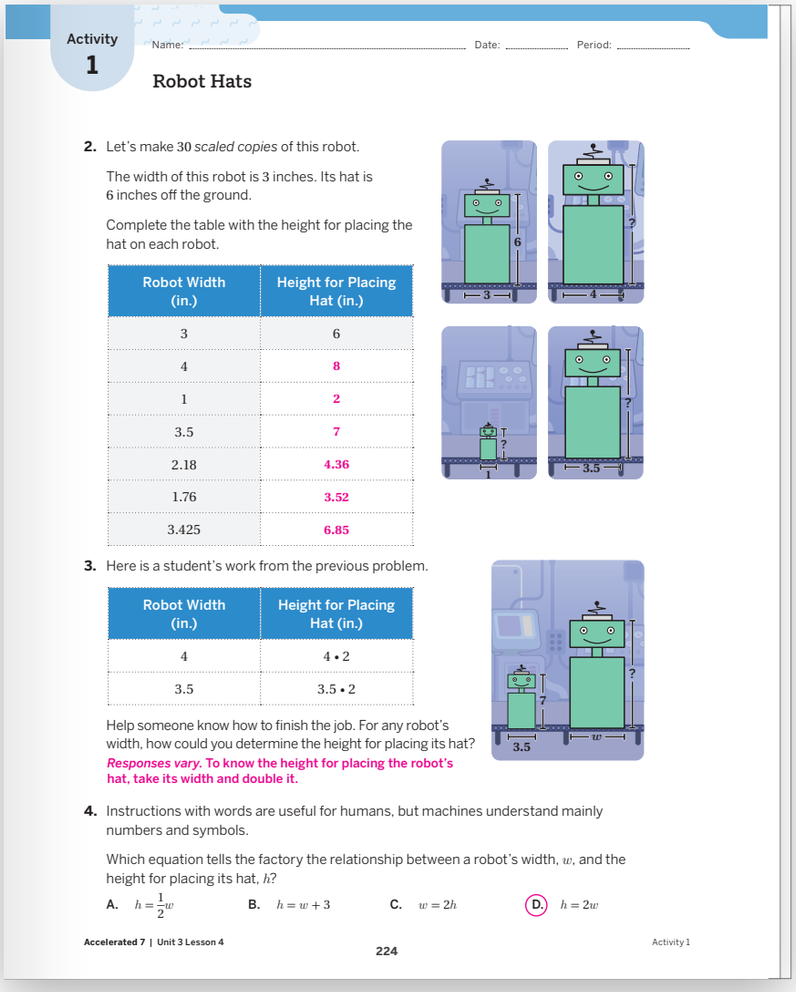


Professional Learning

Here is an opportunity to deepen your understanding of the math in this unit and how to teach it.

Spotlight on Conceptual Understanding

Lesson 4, Activity 1



Activity 1

Name: Date: Period:

Robot Hats

2. Let's make 30 *scaled copies* of this robot.

The width of this robot is 3 inches. Its hat is 6 inches off the ground.

Complete the table with the height for placing the hat on each robot.

Robot Width (in.)	Height for Placing Hat (in.)
3	6
4	8
1	2
3.5	7
2.18	4.36
1.76	3.52
3.425	6.85

3. Here is a student's work from the previous problem.

Robot Width (in.)	Height for Placing Hat (in.)
4	4 • 2
3.5	3.5 • 2

Help someone know how to finish the job. For any robot's width, how could you determine the height for placing its hat?

Responses vary. To know the height for placing the robot's hat, take its width and double it.

4. Instructions with words are useful for humans, but machines understand mainly numbers and symbols.

Which equation tells the factory the relationship between a robot's width, w, and the height for placing its hat, h?

A. $h = \frac{1}{2}w$ B. $h = w + 3$ C. $w = 2h$ D. $h = 2w$

Accelerated 7 | Unit 3 Lesson 4 224 Activity 1

Context

In this activity, students extend their understanding of proportional relationships in tables to write equations using the constant of proportionality.

Try This

Put on your student hat and complete Activity 1.

Points to Ponder

- In what ways will students be able to build on their prior knowledge of proportional relationships in tables to write equations?
- Students will deepen their understanding of how the constant of proportionality can be represented. What will be challenging for students as they try to move between the table and equation representations? How can you support them through these challenges?

Notes:

Assess and Respond

Pre-Unit Check

Independent | 45 min

Facilitation: Assign the Pre-Unit Check to learn more about your students' understanding of foundational concepts and skills that will support them in the upcoming unit. This can be given in its entirety before the unit or spread throughout the unit. If time allows, consider inviting students to complete one or more of the tasks in the Unit Synthesis and Reflection.

Item Analysis

Problem(s)	Concept or skill	Preparation for	DOK	FL B.E.S.T. Math Benchmarks
1, 2	Calculating equivalent ratios	Lesson 1	1	MA.6.AR.3.1, MA.6.AR.3.5, MTR.7.1
3	Explaining whether or not two ratios are equivalent	Lesson 1	2	MA.6.AR.3.1, MTR.4.1
4	Calculating two different unit rates for one relationship	Lesson 1	1	MA.6.AR.3.5, MTR.7.1
5	Scaling values in a table	Lesson 2	1	MA.6.AR.3.5, MTR.7.1
6	Calculating and using unit rates	Lesson 2	1	MA.6.AR.3.2, MA.6.AR.3.5 MTR.4.1
7	Graphing points in the coordinate plane	Lesson 8	1	MA.5.GR.4.1

Assessment and Lesson Resources

- Student Assessments
- Show What You Knows and Answer Keys

Intervention and Extension Resources

- Mini-Lessons
- Extensions

Name: Date: Period:

Pre-Unit Check

Unit 3

1. A length of 4 yards is equal to 12 feet. How many feet is 10 yards equal to?
   30 feet

2. The ratio of the number of hippos to the number of crocodiles at a watering hole is 4 : 3.

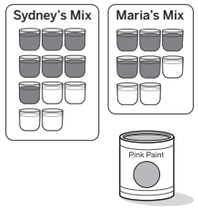
   How many crocodiles would there be if there were 24 hippos?
   18 crocodiles

3. If you mix red and white paint in different ratios, you will get different colors of pink paint. If the ratios are equivalent, the colors of pink will be the same.

   Sydney and Maria each mix their own batch of pink paint.

   Are their batches the same color of pink?
   No

   Explain your thinking.
   Explanations vary. **In Maria's mixture, there are 5 cups of red paint for every 3 cups of white paint, which means that there are $\frac{5}{3}$ cups of red paint for every cup of white paint. In Sydney's mixture, there are 7 cups of red paint for every 4 cups of white paint, or $\frac{7}{4}$ cups of red paint for every cup of white paint. Since the unit rates are different, the two colors of pink are different, too. Sydney's paint is more red than Maria's paint.**



4. Antwon makes hot chocolate by mixing 2 cups of milk with 5 tablespoons of cocoa.

   a. How many tablespoons of cocoa would he need for 1 cup of milk?
   $\frac{5}{2}$ tablespoons of cocoa (or equivalent)

   b. How many cups of milk would he need for 1 tablespoon of cocoa?
   $\frac{2}{5}$ cups of milk (or equivalent)

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Name: Date: Period:

Pre-Unit Check (continued)

Unit 3

5. An airplane flew across the Pacific Ocean at a constant speed. The table shows the amount of time and the distance traveled. Complete the table.

Time (hr)	Distance Traveled (mi)
2	**1,100**
3	1,650
6	**3,300**

6. Blueberries cost $4.00 per pound.

   a. How many pounds of blueberries can you buy for $1.00?
   0.25 pounds (or equivalent)

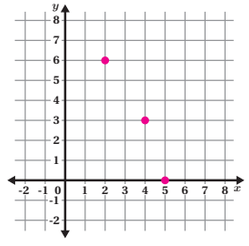
   b. How many pounds of blueberries can you buy for $13.00?
   3.25 pounds (or equivalent)

   Explain or show your thinking.
   Explanations vary. **For every $1, I can buy 0.25 pounds of blueberries. So for $13, I can buy 3.25 pounds of blueberries (because $13 \cdot 0.25 = 3.25$).**

7. The table shows some coordinate pairs.

x	y
4	3
2	6
5	0

   Plot these points in the coordinate plane.



D Differentiation (Pre-Unit Check)

Note: To **strengthen** and **stretch** students' learning, refer to the differentiation resources suggested throughout this unit and in the Unit Overview.

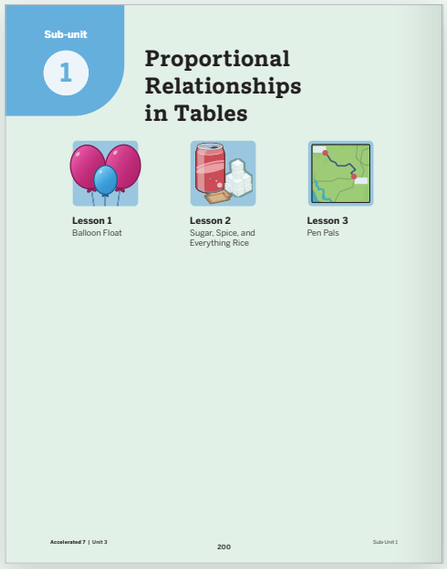
Sub-Unit Goals	Problem(s)	Respond to Student Thinking
Preparation for Sub-Unit 1: • Use tables to recognize proportional relationships. • Determine a constant of proportionality from a table. **(Lessons 1–3)**	1–3	**Support** • **Mini-Lesson:** *Generating Equivalent Ratios* • **Teacher Move:** During **Lesson 1**, Activity 1, consider reviewing how to use representations such as double number lines and tables of equivalent ratios.
	5, 6	**Support** • **Teacher Move:** After the **Lesson 1** Warm-Up, invite students to add pairs of values to the table.
	4	**Support** • **Mini-Lesson:** *Calculating Unit Rates* • **Teacher Move:** During the **Lesson 6** Synthesis, invite students to share strategies for determining the unit rates in this problem.
	7	**Support** • **Mini-Lesson:** *Plotting Points on the Coordinate Plane* • **Teacher Move:** During **Lesson 8**, Activity 1, consider using the movable point to review the meaning of the x and y coordinates.

Proportional Relationships in Tables

Sub-Unit 1 Goals

- **Goal:** Use tables to recognize proportional relationships.
- **Goal:** Determine a constant of proportionality from a table.
- **Language Goal:** Justify whether a table represents a proportional relationship.
- **Language Goal:** Describe the relationship between values in a table using terms such as *proportional*, *non-proportional*, and *constant of proportionality*.

In this sub-unit, students recognize **proportional relationships** represented in tables and determine the **constant of proportionality** using tables. Then they convert measurements from one unit to another in different measurement systems.



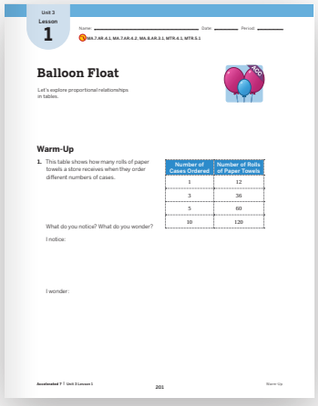
Math that Matters Most

Concept or Skill	For example . . .
Determine unknown values in tables representing proportional relationships.	Brielle mixed 6 cups of blue paint with 3 cups of white paint to make the perfect color to paint her mural. Complete the table to show several other ways to make this color.

Blue Paint (cups)	White Paint (cups)
6	3
8	4
12	6
30	15

Concept or Skill	For example . . .
Determine a constant of proportionality from a table.	When Deven makes chocolate milk, he mixes 2 cups of milk with 3 tablespoons of chocolate syrup. This table shows how to make batches of different sizes. What is a constant of proportionality in this relationship? 1.5 because the cups of milk can be multiplied by 1.5 to find the tablespoons of chocolate.

Milk (cups)	Chocolate Syrup (tbsp)
2	3
8	12
1	$\frac{3}{2}$
10	15


Unit 3
Lesson
1

Name: _____ Date: _____ Period: _____

MA.7.AR.4.1, MA.7.AR.4.2, MA.8.AR.3.1, MTR.4.1, MTR.5.1

Balloon Float

Let's explore proportional relationships in tables.

Warm-Up

1. This table shows how many rolls of paper towels a store receives when they order different numbers of cases.

Number of Cases Ordered	Number of Rolls of Paper Towels
1	12
3	36
5	60
10	120

What do you notice? What do you wonder?

I notice:

I wonder:

Accelerated 7 | Unit 3 Lesson 1 201 Warm-Up

Balloon Float

Introducing Proportional Relationships With Tables

Let's explore proportional relationships in tables.

Learning Outcomes

Today's Goals

1. **Goal:** Use a table to calculate unknown quantities in a *proportional relationship*.
2. **Goal:** Determine whether two quantities are in a *proportional relationship*.
3. **Language Goal:** Describe patterns in tables that represent *proportional relationships*. **(Reading, Writing, Speaking, and Listening)**

Students explore **proportional relationships** by analyzing tables of *equivalent ratios*. Students learn that they can use patterns of multiplicative strategies to determine unknown values for quantities that are in a proportional relationship. **(MTR.5.1)**

Coherence

Prior Learning

In a previous course, students used tables, tape diagrams, and double number line diagrams to solve problems with equivalent ratios.

Future Learning

In Lesson 2, students will learn about constants of proportionality and use them to compare relationships.

Connections

- Students apply proportional reasoning to determine if two quantities are in a proportional relationship in real world contexts such as the weight of an object and number of balloons needed to keep it afloat. (real world)

FL B.E.S.T. Math Benchmarks

Addressing

MA.7.AR.4.1
Determine whether two quantities have a proportional relationship by examining a table, graph or written description.

Also Addressing: **MA.7.AR.4.2, MA.8.AR.3.1**

Mathematical Thinking and Reasoning Standards: MTR.4.1, MTR.5.1

ELA Expectations: ELA.K12.EE.3.1, ELA.K12.EE.5.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.6.AR.3.3	**MA.7.AR.4.3**
MA.6.AR.3.5	**MA.7.AR.4.4**
	MA.7.AR.4.5

Vocabulary

New Vocabulary

proportional relationship

Review Vocabulary

equivalent ratios

Lesson at a Glance

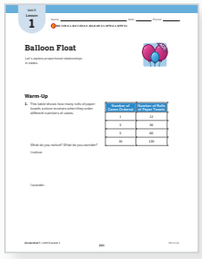
~ 45 min

MA.7.AR.4.1, MA.7.AR.4.2, MA.8.AR.3.1

Warm-Up

Pairs | 5 min

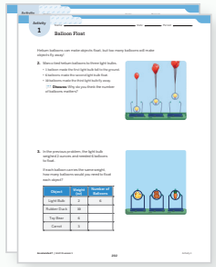
Students use the **Notice and Wonder** routine to identify patterns among related quantities in a table.



Activity 1

Pairs | 10 min

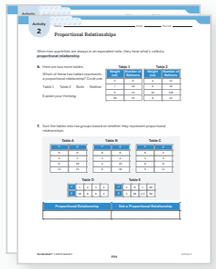
Students develop strategies for completing a table that represents a **proportional relationship**. **MLR7: Compare and Connect**



Activity 2

Pairs | 20 min

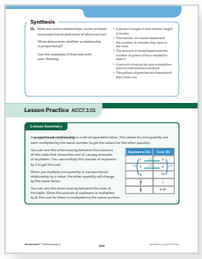
Students are introduced to the term *proportional relationship* and apply their understanding to analyze several tables. **MLR2: Collect and Display**



Synthesis

Whole Class | 5 min

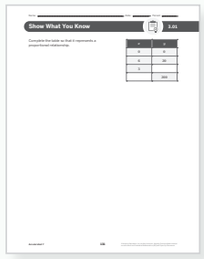
Students synthesize their understanding of how to determine whether a relationship is proportional.



Show What You Know

Independent | 5 min

Students demonstrate their understanding by completing a table to show a proportional relationship.



Prep Checklist

This lesson includes:

- Student Edition
- Show What You Know

Pairs | 5 min

Warm-Up

Purpose: Students use the Notice and Wonder routine to identify patterns among related quantities in a table.

Short on time? Consider omitting Problem 1 and using the discussion in Activity 1 Problem 2 as a Warm-Up for this lesson.

1 Launch

Use the Notice and Wonder routine to promote curiosity and help students make sense of the table. Encourage students to look for structure and patterns in the rows and columns of the table. **(MTR.5.1)**

2 Connect

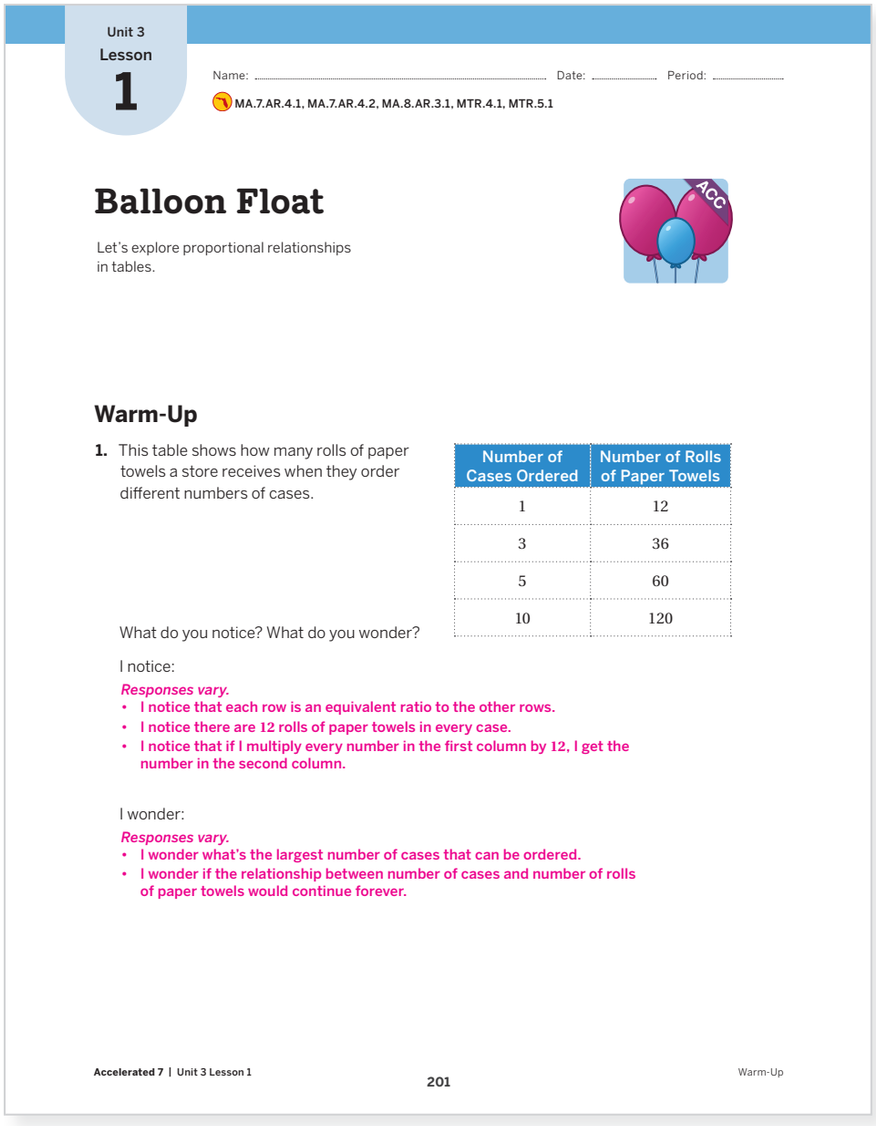
Invite students to share what they noticed and wondered.

Consider asking, "Did anyone notice that the number of cases increases as we move down the table? Does this make you wonder whether the table could be extended forever?"

Note: If it doesn't come up naturally, consider asking students what patterns they notice both *across* and *down* the table. Consider introducing one or more as noticings from a student in a previous class or year.

ELL **English Language Learners** Create a list of words used in reference to tables that students might not know, such as *row*, *column*, *horizontal*, and *vertical*. Include visuals or translations of each word in languages used by your students. **(Reading and Listening)**

To surface the Key Takeaway, consider sharing that this lesson will involve studying tables that represent a variety of contexts. Consider asking, "What are the key features you look at when you are analyzing a table?"



Unit 3
Lesson
1

Name: ..... Date: ..... Period: .....

MA.7.AR.4.1, MA.7.AR.4.2, MA.8.AR.3.1, MTR.4.1, MTR.5.1

Balloon Float

Let's explore proportional relationships in tables.

Warm-Up

1. This table shows how many rolls of paper towels a store receives when they order different numbers of cases.

Number of Cases Ordered	Number of Rolls of Paper Towels
1	12
3	36
5	60
10	120

What do you notice? What do you wonder?

I notice:

Responses vary.
- I notice that each row is an equivalent ratio to the other rows.
- I notice there are 12 rolls of paper towels in every case.
- I notice that if I multiply every number in the first column by 12, I get the number in the second column.

I wonder:

Responses vary.
- I wonder what's the largest number of cases that can be ordered.
- I wonder if the relationship between number of cases and number of rolls of paper towels would continue forever.

Accelerated 7 | Unit 3 Lesson 1 201 Warm-Up

Key Takeaway: You can use a table to represent a relationship between two quantities. When analyzing a table, you can look for patterns between the rows and between the columns.

Pairs | 10 min

Activity 1 Balloon Float

Purpose: Students develop strategies for completing a table that represents a **proportional relationship**.

1 Launch

To surface prior knowledge, consider asking students what experiences they have had with helium balloons.

Consider asking, "What do you think it means for an object to float?" Floating means the object will stay in place, not falling to the ground or flying away.

Invite students to describe what is happening in the picture.

2 Monitor

A Accessibility: Visual-Spatial Processing
Provide additional printed copies of the table for students to draw on or highlight.

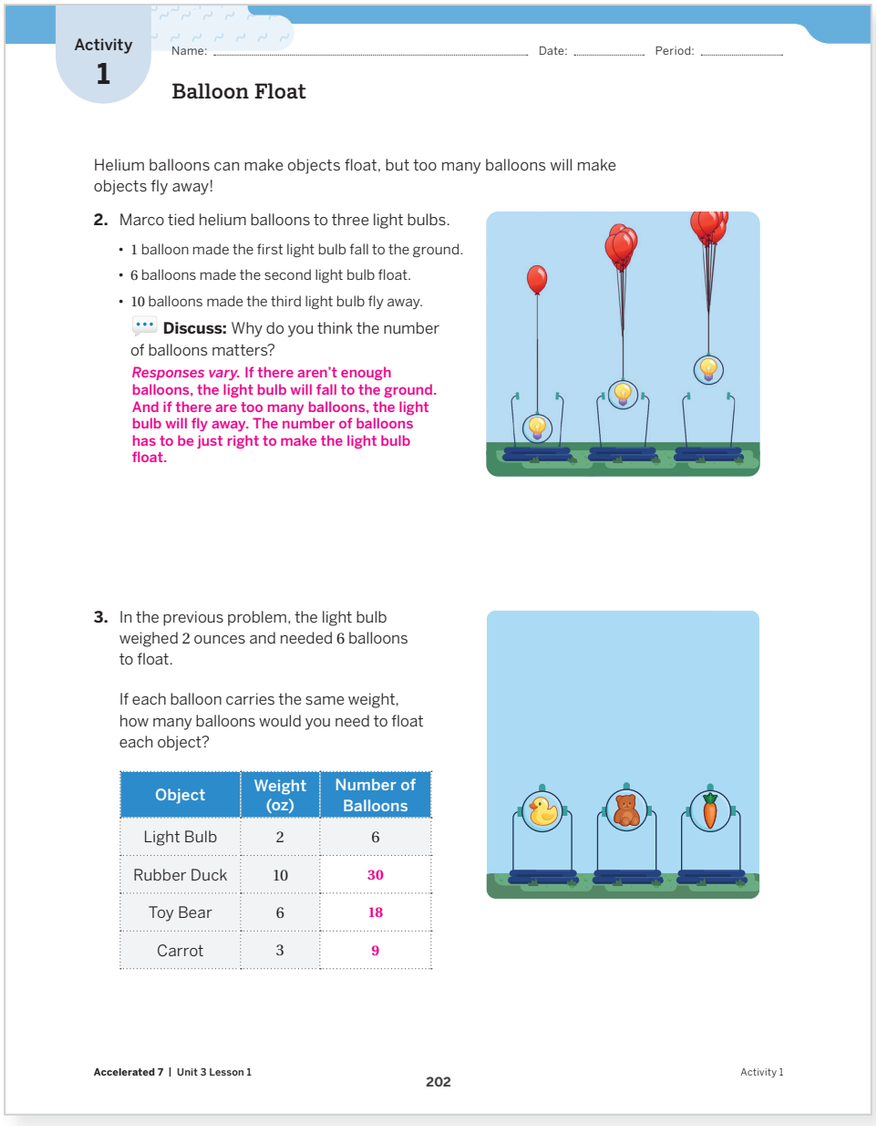
Look for a variety of student thinking.

D Differentiation

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Consider asking, "Which of the patterns we noticed in the Warm-Up might be useful here?"
Identify the relationship between the columns of the table by multiplying each weight by 3 to determine the number of balloons.	Consider capturing the work of these students as the basis for the Connect, rather than that of the sample students.
Identify a scale factor between rows of the table, then apply that scale factor to determine unknown weights.	**Stretch** Consider asking, "What if we saw an object that floated with 27 balloons, and we needed to determine its weight? How would you apply your strategy?"
Identify that an object weighing 1 ounce would need 3 balloons to float.	**Stretch** Consider asking, "Why is it helpful to determine the number of balloons needed for a 1 ounce object?"

ELL English Language Learners Encourage students to paraphrase each other's ideas to help make connections and incorporate vocabulary from the unit. **(Speaking and Listening)**

Activity 1 continued >



Activity 1

Name: _____ Date: _____ Period: _____

Balloon Float

Helium balloons can make objects float, but too many balloons will make objects fly away!

2. Marco tied helium balloons to three light bulbs.

- 1 balloon made the first light bulb fall to the ground.
- 6 balloons made the second light bulb float.
- 10 balloons made the third light bulb fly away.

Discuss: Why do you think the number of balloons matters?

Responses vary. If there aren't enough balloons, the light bulb will fall to the ground. And if there are too many balloons, the light bulb will fly away. The number of balloons has to be just right to make the light bulb float.

3. In the previous problem, the light bulb weighed 2 ounces and needed 6 balloons to float.

If each balloon carries the same weight, how many balloons would you need to float each object?

Object	Weight (oz)	Number of Balloons
Light Bulb	2	6
Rubber Duck	10	30
Toy Bear	6	18
Carrot	3	9

Accelerated 7 | Unit 3 Lesson 1 202 Activity 1

Pairs | 10 min

Activity 1 Balloon Float (continued)

Purpose: Students develop strategies for completing a table that represents a proportional relationship.

2 Monitor

Listen for a variety of student thinking to highlight during the Connect, including:

- Using equivalent ratios to represent the relationships between the columns or rows.
- Using multiplication (or division) to determine the number of balloons.

Note: Students might use terms such as *scale factor* or *unit rate*, but it is not necessary at this point. Vocabulary for describing these relationships will be formalized later in the lesson and the unit.

D **Differentiation: (Support)** Support students getting started by encouraging them to look back at Ariel's and Emma's strategies. Consider asking, "Could you use these strategies to complete the table? Which do you think would be more helpful?"

Look for students who move flexibly between different strategies to complete the table, such as scaling up the light bulb row to find the weight of the cake and multiplying the weight of the banana by 3.

3 Connect

MLR **MLR7: Compare and Connect** Invite students to analyze and compare each strategy, and then connect to their work in Problem 3. See the Routine Facilitation Guide for more information. **(MTR.4.1)**

To surface the Key Takeaway, consider asking:

- "Why does Ariel's strategy work? Emma's?"
- "How are Ariel's and Emma's strategies alike? How are they different?"
- "When might you use each strategy?"

Activity 1

Name: _____ Date: _____ Period: _____

Balloon Float (continued)

4. Here are two strategies for determining the number of balloons needed to make the rubber duck float.

Ariel

Object	Weight (oz)	Number of Balloons
Light Bulb	2	6
Rubber Duck	10	30
Toy Bear	6	
Carrot	3	

Emma

Object	Weight (oz)	Number of Balloons
Light Bulb	2	6
Rubber Duck	10	30
Toy Bear	6	
Carrot	3	

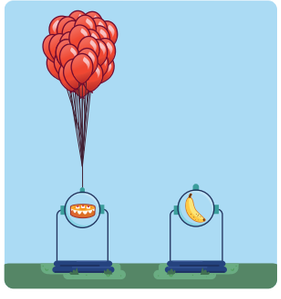
Discuss: How might Ariel and Emma use their strategies to finish their tables?

Responses vary.

- Ariel might finish the table by scaling the numbers in the first row (2 ounces and 6 balloons). To determine how many balloons would make the toy bear float, she would need to multiply by a scale factor of 3, which would give her 18 balloons. To determine the number of balloons for the carrot, she would need to multiply by a scale factor of 1.5, which would give her 9 balloons.
- Emma might finish the table by multiplying each weight by 3 to get the number of balloons. The unit rate (balloons per ounce) is 3, which tells us how many balloons are required to carry 1 ounce of weight.

5. Here are some new objects. Complete the table so that each object floats.

Object	Weight (oz)	Number of Balloons
Light Bulb	2	6
Cake	20	60
Banana	$3\frac{1}{3}$	10



Accelerated 7 | Unit 3 Lesson 1 203 Activity 1

Key Takeaway: To complete a table showing a proportional relationship between two quantities, we look for multiplicative patterns between the rows or columns of the table.

Pairs | 20 min

Activity 2 Proportional Relationships

Purpose: Students are introduced to the term *proportional relationship* and apply their understanding to analyze several tables.

1 Launch

Invite students to read the definition of **proportional relationship** and rephrase it in their own words. Consider sharing that the relationship between balloons and ounces in Activity 1 is proportional.

Encourage students to refer to the class definition of *equivalent ratios*.

Listen for a variety of ideas, such as:

- Equivalent ratios are created by multiplying, so we should look for a multiplicative pattern.
- Scaled copies also have equivalent ratios. A proportional relationship is like scaling.
- The relationship should stay the same. From one quantity to another, we always see the same ratio.

MLR **MLR2: Collect and Display** Use this routine to collect students' ideas and language as they discuss with a partner. Display for students to reference and add to for the rest of the unit.

D **Differentiation: (Support)** Students may think Table 1 shows a proportional relationship because the number of balloons is always 3 more than the weight. Encourage these students to check for equivalent ratios in each row of the table. Consider asking, "How many balloons do you need to hold each ounce?"

Invite students to share their reasoning.

Note: Spend adequate time on this discussion to ensure that all students notice the multiplicative relationships in the proportional table, including both the constant ratio between columns and the scaling relationship between rows.

2 Monitor

Encourage students to share their reasoning with a partner and work together to reach an agreement about how to sort the tables.

D **Differentiation: (Support)** Encourage students to relate the tables to the balloon and weight context. Consider asking, "If the columns were labeled ounces and balloons, what would each row mean? Do the values make sense for a proportional relationship?"

Activity 2 continued >

Activity 2

Name: _____ Date: _____ Period: _____

Proportional Relationships

When two quantities are always in an *equivalent ratio*, they have what's called a **proportional relationship**.

6. Here are two more tables.

Which of these two tables represents a proportional relationship? Circle one.

Table 1 (Table 2) Both Neither

Explain your thinking.

Explanations vary. The number of balloons is always 3 times the weight in ounces. That means they're always in a 3 to 1 ratio, which means their relationship is proportional.

Table 1

Weight (oz)	Number of Balloons
3	6
7	10
9	12
30	33

Table 2

Weight (oz)	Number of Balloons
4	12
6	18
42	126
8	24

7. Sort the tables into two groups based on whether they represent proportional relationships.

Table A

x	y
0	0
4	5
8	10
12	15

Table B

x	y
0	0
2	4
4	16
6	36

Table C

x	y
0	2
3	5
6	8
9	11

Table D

x	1	2	3	4
y	10	8	6	4

Table E

x	2	8	1	20
y	5	20	2.5	50

Proportional Relationship	Not a Proportional Relationship
Table A, Table E	Table B, Table C, Table D

Accelerated 7 | Unit 3 Lesson 1 204 Activity 2

Pairs | 20 min

Activity 2 Proportional Relationships (continued)

Purpose: Students are introduced to the term *proportional relationship* and apply their understanding to analyze several tables.

2 Monitor

Look for a variety of student thinking.

D Differentiation

Look for students who:	Respond to Student Thinking
Think the relationship is proportional because each x-value is multiplied by itself to obtain the y-value.	**Support** Celebrate the structure these students noticed. Consider asking, "Does each row form an equivalent ratio to the others?"
Compare rows of the table to identify that the relationship is not proportional, e.g., noticing that $2 \cdot 3 = 6$, but $4 \cdot 3 \neq 36$.	**Strengthen** Celebrate this thinking. Consider asking, "What is another way we could check that this relationship is not proportional?"
Notice that no single number can be used to multiply the numbers in one column to get the numbers in the other column.	**Stretch** Consider asking, "How would you describe the relationship in the table?"

Listen for the assumptions students make about each situation, such as that the marathon runner maintains a consistent pace during the race. Select and sequence student ideas to share during the Connect.

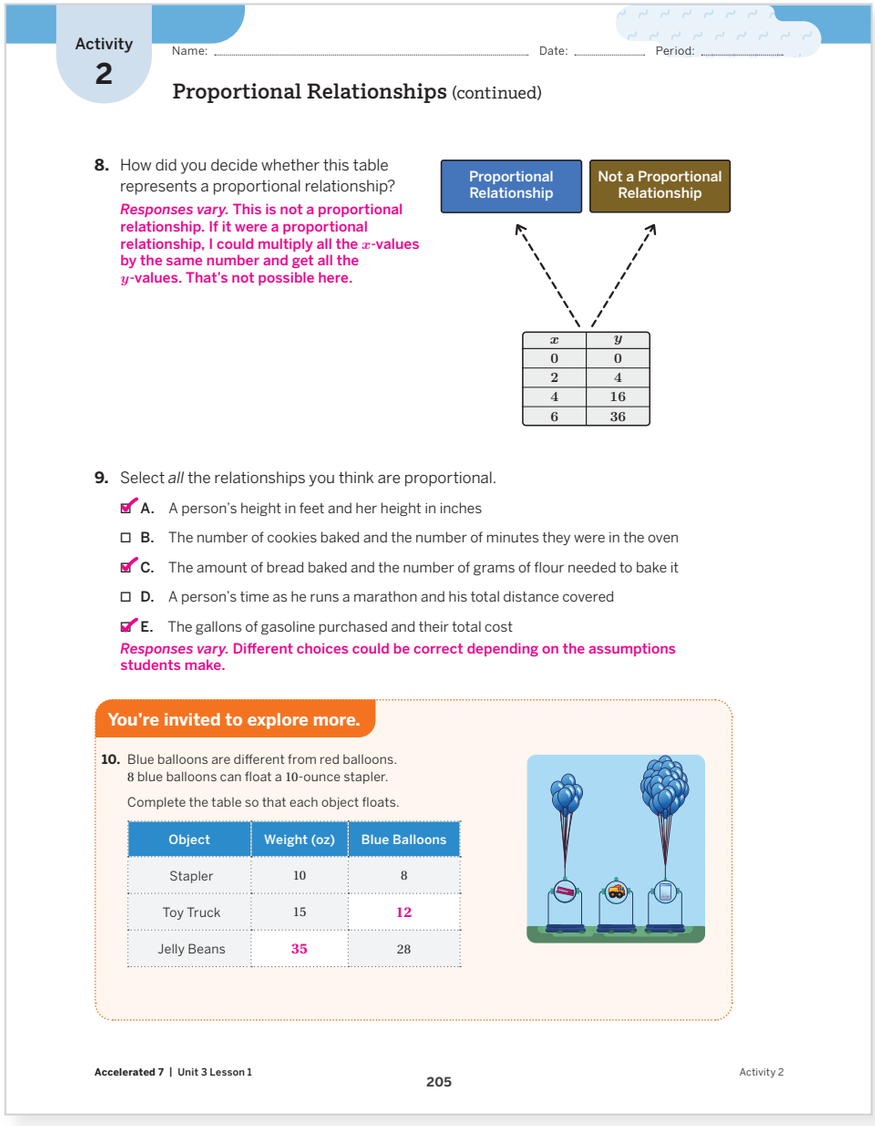
D Differentiation: You're invited to explore more. **(Stretch)** Invite students who would like to explore tables representing proportional relationships further to complete this optional task. Encourage them to discuss their thinking with a partner.

3 Connect

Invite students to share their responses and discuss each option.

Consider asking, "What criteria did you use to decide whether a relationship is proportional?" *Responses vary.* In a proportional relationship, there must be a consistent ratio. The relationship does not change over time.

Note: It is okay — even desirable — to lack consensus on every option. Use the discussion to surface what assumptions students made and clarify the essential features of a proportional relationship.



Activity 2

Name: Date: Period:

Proportional Relationships (continued)

8. How did you decide whether this table represents a proportional relationship?

Responses vary. This is not a proportional relationship. If it were a proportional relationship, I could multiply all the x-values by the same number and get all the y-values. That's not possible here.

Proportional Relationship | Not a Proportional Relationship

x	y
0	0
2	4
4	16
6	36

9. Select *all* the relationships you think are proportional.

- ☑ **A.** A person's height in feet and her height in inches
- ☐ **B.** The number of cookies baked and the number of minutes they were in the oven
- ☑ **C.** The amount of bread baked and the number of grams of flour needed to bake it
- ☐ **D.** A person's time as he runs a marathon and his total distance covered
- ☑ **E.** The gallons of gasoline purchased and their total cost

Responses vary. Different choices could be correct depending on the assumptions students make.

You're invited to explore more.

10. Blue balloons are different from red balloons. 8 blue balloons can float a 10-ounce stapler.

Complete the table so that each object floats.

Object	Weight (oz)	Blue Balloons
Stapler	10	8
Toy Truck	15	12
Jelly Beans	35	28

Accelerated 7 | Unit 3 Lesson 1 205 Activity 2

Key Takeaway: In a proportional relationship, the two quantities always form an *equivalent ratio*. If the ratio changes, the relationship is not proportional.

Whole Class | 5 min

Synthesis

Purpose: Students synthesize their understanding of how to determine whether a relationship is proportional.

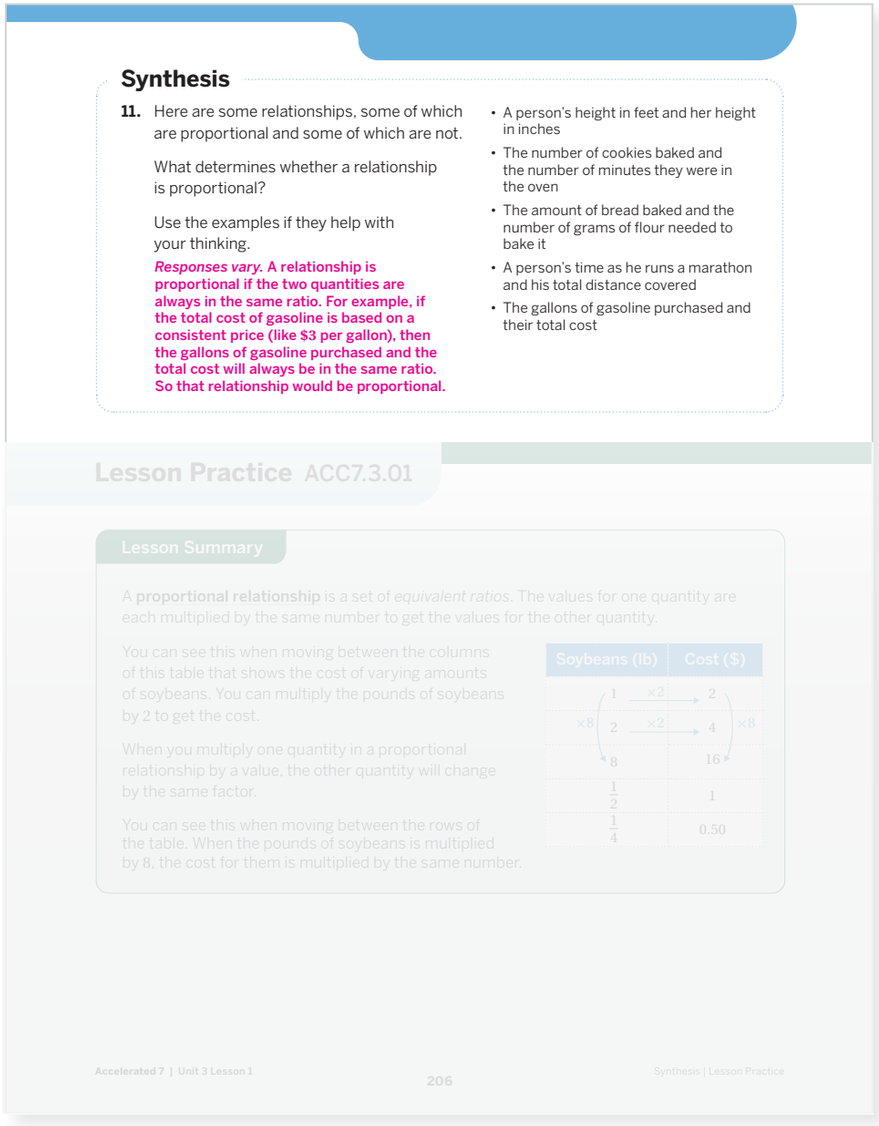
Invite students to respond independently, and then share their thinking with a partner.

Capture and share a variety of ideas, such as:

- A relationship is proportional if the two quantities are always in the same ratio.
- If the rate in a situation is changing, like a runner who takes a break to walk, the relationship is not proportional.
- A relationship is proportional if you can double one quantity and the other quantity also doubles.
- If one quantity stays the same while the other changes, the relationship is not proportional. When baking, if you decide to make more cookies, you don't need to change the oven temperature.

Encourage students to add their ideas and language to the display about proportional relationships.

Note: If time allows, invite students to revise their previous thinking based on the discussion.



Synthesis

11. Here are some relationships, some of which are proportional and some of which are not.

What determines whether a relationship is proportional?

Use the examples if they help with your thinking.

- A person's height in feet and her height in inches
- The number of cookies baked and the number of minutes they were in the oven
- The amount of bread baked and the number of grams of flour needed to bake it
- A person's time as he runs a marathon and his total distance covered
- The gallons of gasoline purchased and their total cost

Responses vary. **A relationship is proportional if the two quantities are always in the same ratio. For example, if the total cost of gasoline is based on a consistent price (like $3 per gallon), then the gallons of gasoline purchased and the total cost will always be in the same ratio. So that relationship would be proportional.**

Lesson Practice ACC7.3.01

Lesson Summary

A **proportional relationship** is a set of *equivalent ratios*. The values for one quantity are each multiplied by the same number to get the values for the other quantity.

You can see this when moving between the columns of this table that shows the cost of varying amounts of soybeans. You can multiply the pounds of soybeans by 2 to get the cost.

When you multiply one quantity in a proportional relationship by a value, the other quantity will change by the same factor.

You can see this when moving between the rows of the table. When the pounds of soybeans is multiplied by 8, the cost for them is multiplied by the same number.

Soybeans (lb)	Cost ($)
1	2
2	4
8	16
$\frac{1}{2}$	1
$\frac{1}{4}$	0.50

Accelerated 7 | Unit 3 Lesson 1 206 Synthesis | Lesson Practice

Lesson Takeaway: Two quantities are in a **proportional relationship** when they change together in a multiplicative pattern. When one quantity is multiplied, the other is multiplied by the same factor. This also creates *equivalent ratios*.

Independent | 5 min

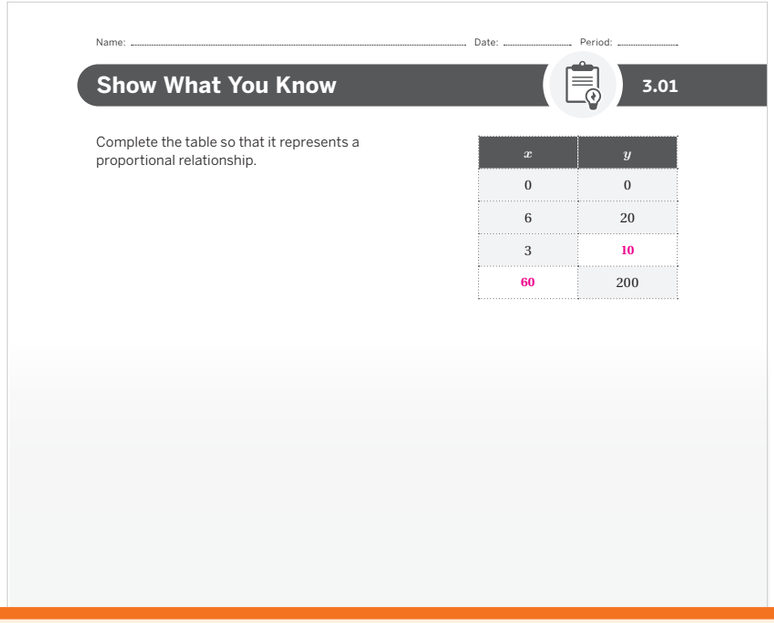
Show What You Know

Purpose: Students demonstrate their understanding by completing a table to show a proportional relationship.

Today's Goals

1. **Goal:** Use a table to calculate unknown quantities in a *proportional relationship*.
2. **Goal:** Determine whether two quantities are in a *proportional relationship*.
3. **Language Goal:** Describe patterns in tables that represent *proportional relationships*. **(Reading, Writing, Speaking, and Listening)**

Show What You Know (Assessment Resources)



Name: _____ Date: _____ Period: _____

Show What You Know 3.01

Complete the table so that it represents a proportional relationship.

x	y
0	0
6	20
3	10
60	200

Differentiation Use after Lesson 1

Support

Provide targeted intervention for students by using these resources.

If students need support understanding how to complete a table to show a proportional relationship:

» **Respond to Student Thinking:**

- Students will have more opportunities to develop their understanding of this concept in Lesson 3. You may choose to not intervene at this time

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 1 Extension Activities**.
- Invite students to complete the **Explore More** task and discuss their thinking with a partner.

Professional Learning

Reflect on your facilitation of the **MLR2: Collect and Display** routine in this lesson. How did students engage in the routine? How did your facilitation of the routine support student understanding of the term **proportional relationship**? How might you facilitate it differently in the future?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

11. Here are some relationships, some of which are proportional and some of which are not.

- A person's height in feet and her height in inches
- The number of cookies baked and the number of minutes they were in the oven
- The amount of bread baked and the number of grams of flour needed to bake it
- A person's time as he runs a marathon and his total distance covered
- The gallons of gasoline purchased and their total cost

What determines whether a relationship is proportional?

Use the examples if they help with your thinking.

Responses vary. A relationship is proportional if the two quantities are always in the same ratio. For example, if the total cost of gasoline is based on a consistent price (like $3 per gallon), then the gallons of gasoline purchased and the total cost will always be in the same ratio. So that relationship would be proportional.

Lesson Practice ACC7.3.01

Lesson Summary

A **proportional relationship** is a set of *equivalent ratios*. The values for one quantity are each multiplied by the same number to get the values for the other quantity.

You can see this when moving between the columns of this table that shows the cost of varying amounts of soybeans. You can multiply the pounds of soybeans by 2 to get the cost.

When you multiply one quantity in a proportional relationship by a value, the other quantity will change by the same factor.

You can see this when moving between the rows of the table. When the pounds of soybeans is multiplied by 8, the cost for them is multiplied by the same number.

Soybeans (lb)	Cost ($)
1 ×2	2
×8 2 ×2	4 ×8
8	16
$\frac{1}{2}$	1
$\frac{1}{4}$	0.50

Lesson Practice ACC7.3.01

Name: _____ Date: _____ Period: _____

Problems 1–4: Complete the tables so that the relationship is proportional.

1.

x	y
30	3
120	12
100	10

2.

x	y
1	1.5
3	4.5
8	12

3.

x	y
15	45
1	3
0	0

4.

x	y
0.2	1
1	5
4	20

5. Entrance to a state park costs $6 per vehicle, plus $2 per person. The table shows the entry cost for several recent groups of visitors. Is the relationship between the number of people and the total entrance cost a proportional relationship?

No

Explain your thinking.

Explanations vary. When I look at the ratio of people in the vehicle to the total entry cost, I can tell these are not equivalent ratios.

Number of People in Vehicle	Total Cost ($)
2	10
3	12
4	14
10	26

6. A store charges $4.80 for 16 ounces of bubble tea. Complete the table so that it shows a proportional relationship between ounces of tea and cost.

Response shown in table.

Tea (oz)	Cost ($)
16	4.80
20	6.00
24	7.20

Lesson Practice ACC7.3.01

Name: _____ Date: _____ Period: _____

FAST Practice

7. A bakery uses 8 tablespoons of honey for every 10 cups of flour to make bread dough. Complete the table so that it shows a proportional relationship between the amount of honey and the amount of flour.

Honey (tbsp)	Flour (cups)
8	10
20	25
13	16.25
10	12.5

Spiral Review

Problems 8–10: Solve each equation. Show your thinking. *Work varies.*

8. $\frac{1}{2} + x = 2$

$\frac{1}{2} + x - \frac{1}{2} = 2 - \frac{1}{2}$

$x = 1\frac{1}{2}$

9. $\frac{2}{3}y = 6$

$\frac{2}{3}y \div \frac{2}{3} = 6 \div \frac{2}{3}$

$y = 9$

10. $3 = \frac{1}{4}b$

$3 \div \frac{1}{4} = \frac{1}{4}b \div \frac{1}{4}$

$3 \cdot 4 = b$

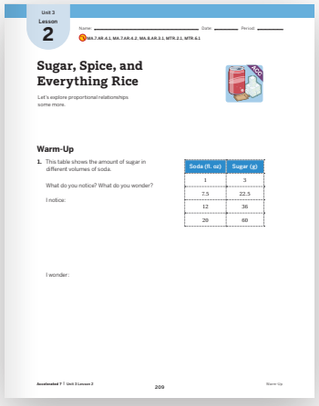
$12 = b$

11. A student makes a scaled copy of a rectangle. The dimensions of the original rectangle are 3.5 by 2.5 meters. Select *all* the possible dimensions of the scaled copy.

- ☑ **A.** 1.75 meters and 1.25 meters
- ☑ **B.** 7 meters and 5 meters
- ☐ **C.** 7 meters and 6 meters
- ☐ **D.** 10 meters and 2.5 meters
- ☑ **E.** 10.5 meters and 7.5 meters

Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1–4	1	MA.7.AR.4.1
	5–6	2	MA.7.AR.4.1
FAST Practice	7	2	MA.7.AR.4.1
Spiral Review			
	8–10	1	MA.6.AR.2.2, MA.6.AR.2.3
	11	1	MA.7.GR.1.5


Unit 3
Lesson 2

Sugar, Spice, and Everything Rice

Let's explore proportional relationships some more.

Warm-Up

1. This table shows the amount of sugar in different volumes of soda.

What do you notice? What do you wonder?

I notice:

Soda (fl. oz)	Sugar (g)
1	3
7.5	22.5
12	36
20	60

I wonder:

209

Sugar, Spice, and Everything Rice

Constant of Proportionality

Let's explore proportional relationships some more.

Learning Outcomes

Today's Goals

1. **Goal:** Determine the *constant of proportionality* for a proportional relationship represented in a table.
2. **Goal:** Use the *constant of proportionality* to determine values in a table.
3. **Language Goal:** Justify whether a table represents a proportional relationship, using terms such as *constant of proportionality*. **(Reading and Writing)**

Students determine, interpret, and use **constants of proportionality** to make sense of proportional relationships. They compare proportional relationships in context, using the constant of proportionality to support their reasoning. **(MTR.2.1)**

Coherence

Prior Learning

In Lesson 1, students observed equivalent ratios in tables and identified patterns representing proportional relationships. Students also used tables to determine unknown values for quantities in a proportional relationship.

Future Learning

In Lesson 3, students will convert measurements across different measurement systems using tables and double number lines.

Connections

- Students use the constant of proportionality to make sense of proportional relationships, such as comparing how "sugary" one drink is to another, modifying ingredients in a recipe, and more. (real world)

FL B.E.S.T. Math Benchmarks

Addressing

MA.7.AR.4.1
Determine whether two quantities have a proportional relationship by examining a table, graph or written description.

MA.7.AR.4.2
Determine the constant of proportionality within a mathematical or real-world context given a table, graph or written description of a proportional relationship.

Also Addressing: **MA.8.AR.3.1**

Mathematical Thinking and Reasoning Standards: MTR.2.1, MTR.6.1

ELA Expectations: ELA.K12.EE.3.1, ELA.K12.EE.5.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.6.AR.3.2	MA.7.AR.4.4
MA.6.AR.3.3	MA.7.AR.4.5
MA.6.AR.3.5	MA.8.AR.3.2

Vocabulary

New Vocabulary

constant of proportionality

Lesson at a Glance

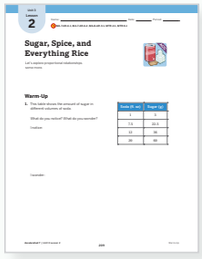
~ 45 min

MA.7.AR.4.1, MA.7.AR.4.2, MA.8.AR.3.1

Warm-Up

Independent | 5 min

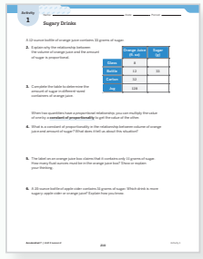
Students use the **Notice and Wonder** routine to make sense of the relationship between the volume of a soda and how many grams of sugar it contains.



Activity 1

Pairs | 15 min

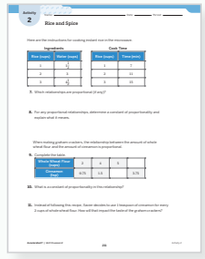
Students explore how the **constant of proportionality** can be helpful when analyzing a relationship. **MLR2: Collect and Display** **(MTR.2.1)**



Activity 2

Pairs | 15 min

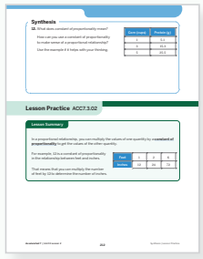
Students use constants of proportionality to compare cooking instructions and ingredients in other recipes.



Synthesis

Whole Class | 5 min

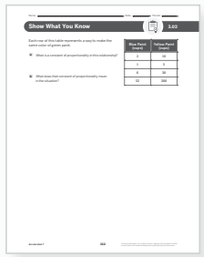
Students synthesize their understanding of using a constant of proportionality to make sense of relationships. **MLR2: Collect and Display**



Show What You Know

Independent | 5 min

Students demonstrate their understanding by determining and interpreting the constant of proportionality from a table.



Prep Checklist

This lesson includes:

- Student Edition
- Show What You Know
- Scientific calculators (as needed)

Independent | 5 min

Warm-Up

Purpose: Students use the Notice and Wonder routine to make sense of the relationship between the volume of a soda and how many grams of sugar it contains.

Short on time? Consider omitting the Warm-Up and using the Notice and Wonder routine to start Problem 2.

1 Launch

To surface connections, consider asking, "What do you know and wonder about soda?"

ELL **English Language Learners** Consider discussing the word soda and what other word(s) your students use to describe this product. In particular, invite students to respond in their primary languages. **(Writing and Speaking)**

Direct students' attention to the table.

Use the Notice and Wonder routine to promote curiosity and help students make sense of the relationship between grams of sugar in different volumes of soda.

2 Connect

Consider asking:

- "What do you notice and wonder when observing the table?"
- "What are you noticing and wondering about the relationship between sugar and ounces of soda?"

Note: If it doesn't come up naturally, consider asking, "What happens as the amount of soda gets bigger?"

Unit 3
Lesson
2

Name: _____ Date: _____ Period: _____

MA.7.AR.4.1, MA.7.AR.4.2, MA.8.AR.3.1, MTR.2.1, MTR.6.1

Sugar, Spice, and Everything Rice

Let's explore proportional relationships some more.

Warm-Up

1. This table shows the amount of sugar in different volumes of soda.

Soda (fl. oz)	Sugar (g)
1	3
7.5	22.5
12	36
20	60

What do you notice? What do you wonder?

I notice:

Responses vary.
- As the volume increases, the amount of sugar also increases.
- You can multiply the number of fluid ounces by 3 to get the number of grams of sugar.
- One fluid ounce of soda contains 3 grams of sugar.
- The relationship between the volume of soda and the amount of sugar is proportional.

I wonder:

Responses vary.
- What is a typical serving size for soda?
- Could you figure out how much sugar is in any amount of soda?
- How does the amount of sugar in soda compare to the amount of caffeine it has?
- How does the amount of sugar in soda compare to the amount of sugar in other drinks?

Accelerated 7 | Unit 3 Lesson 2 209 Warm-Up

Pairs | 15 min

Activity 1 Sugary Drinks

Purpose: Students explore how the **constant of proportionality** can be helpful when analyzing a relationship. **(MTR.2.1)**

1 Launch

To generate excitement, consider asking, "What do you think about orange juice?"

ELL **English Language Learners** Consider discussing the different containers and describing how they are connected to volume. **(Speaking, Writing, and Listening)**

2 Monitor

To support students getting started, consider asking, "How would the amount of sugar change as the bottle of orange juice gets bigger? Smaller?" **(MTR.6.1)**

A **Accessibility: Memory and Attention** Invite students to refer back to the previous lesson on proportional relationships to activate prior knowledge. Consider asking, "What do you remember from yesterday's lesson?"

Pause after most students have completed Problems 2 and 3, discuss the definition of **constant of proportionality**.

MLR **MLR2: Collect and Display** Use this routine to collect students' language as they consider the new term. Display for students to reference and add to for the rest of the unit.

ELL **English Language Learners** Invite students to share what the new term reminds them of (e.g., "When I hear *constant of proportionality*, I think . . ."). This may help surface multiple meanings and encourage students to connect to their prior knowledge. **(Reading and Listening)**

Look for students using different strategies to solve for and apply the constant of proportionality, such as:

- Utilizing the relationship across a row in a table to find a common multiplier.
- Solving for the amount of sugar for 1 ounce of orange juice and using this to scale up. **(MTR.2.1)**

3 Connect

Invite students to share their responses to Problem 6.

To surface the Key Takeaway, consider asking, "What can the constant of proportionality tell us about the taste of each beverage?"

Activity 1

Name: _____ Date: _____ Period: _____

Sugary Drinks

A 12-ounce bottle of orange juice contains 33 grams of sugar.

2. Explain why the relationship between the volume of orange juice and the amount of sugar is proportional.

Responses vary. No matter how much orange juice there is, the recipe, and the ratio between volume of juice and amount of sugar, will be the same.

3. Complete the table to determine the amount of sugar in different-sized containers of orange juice.

Response shown in table.

	Orange Juice (fl. oz)	Sugar (g)
Glass	8	22
Bottle	12	33
Carton	32	88
Jug	128	352

When two quantities have a *proportional relationship*, you can multiply the value of one by a **constant of proportionality** to get the value of the other.

4. What is a constant of proportionality in the relationship between volume of orange juice and amount of sugar? What does it tell us about this situation?

Responses vary.
- 2.75. There are 2.75 grams of sugar for each fluid ounce of orange juice.
- About 0.36. About 0.36 fluid ounces of orange juice contains 1 gram of sugar.

5. The label on an orange juice box claims that it contains only 11 grams of sugar. How many fluid ounces must be in the orange juice box? Show or explain your thinking.

4 fluid ounces. *Explanations vary.* I divided 11 grams of sugar by 2.75 grams per ounce.

6. A 25-ounce bottle of apple cider contains 31 grams of sugar. Which drink is more sugary: apple cider or orange juice? Explain how you know.

Orange juice. *Explanations vary.* $31 \div 25 = 1.24$, so there are 1.24 grams of sugar for each fluid ounce of apple cider. I already know there are 2.75 grams of sugar for each fluid ounce of orange juice. So the orange juice must be more sugary because it has more grams of sugar per fluid ounce.

Accelerated 7 | Unit 3 Lesson 2 210 Activity 1

Key Takeaway: The **constant of proportionality** is the multiplier between two quantities in a proportional relationship. Given one value in the relationship, you can multiply by the constant of proportionality to find the other value.

Pairs | 15 min

Activity 2 Rice and Spice

Purpose: Students use constants of proportionality to compare cooking instructions and ingredients in other recipes.

1 Launch

Direct students' attention to the two tables.

To surface connections, consider asking:
- "What quantities do these tables tell us about?"
- "What experience do you have with rice?"

A **Accessibility: Memory and Attention** Invite students to use a calculator to support them with performing calculations.

2 Monitor

D **Differentiation** *(Problems 7–10)*

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Consider asking, "What would *proportional* mean in each scenario?"
Think both tables are proportional. *(Problem 7)*	**Support** Invite them to revisit the class discussion on proportional relationships from the previous lesson.
Need support computing with fractions or decimals.	**Support** Consider asking, "What could you type in a calculator to determine that value?"
Use different strategies for determining and using the constant of proportionality, such as: • Utilizing the value that corresponds with 1 in the table. • Dividing values from the table to find a multiplier and confirming it is consistent.	**Strengthen** Invite these students to share during the Connect. Consider asking, "Can you think of an alternative strategy to solve for the constant of proportionality?" **(MTR.2.1)**

3 Connect

Invite students to share their solution for the constant of proportionality. Encourage students to make connections between the constants of proportionality and the taste of the graham crackers.

To surface connections, consider asking, "What is similar about our solutions? What is different?"

Activity 2

Name: _____ Date: _____ Period: _____

Rice and Spice

Here are the instructions for cooking instant rice in the microwave.

Ingredients

Rice (cups)	Water (cups)
1	$1\frac{1}{2}$
2	3
3	$4\frac{1}{2}$

Cook Time

Rice (cups)	Time (min)
1	7
2	11
3	15

7. Which relationships are proportional (if any)?

The relationship between the cups of rice and the cups of water is proportional.

8. For any proportional relationships, determine a constant of proportionality and explain what it means.

$1\frac{1}{2}$ or $\frac{2}{3}$. *Explanations vary.* You can multiply the number of cups of rice by $1\frac{1}{2}$ to get the number of cups of water. In other words, for each cup of rice you want to cook, you need $1\frac{1}{2}$ cups of water.

When making graham crackers, the relationship between the amount of whole wheat flour and the amount of cinnamon is proportional.

9. Complete the table.

Whole Wheat Flour (cups)	2	4	5	10
Cinnamon (tsp)	0.75	1.5	1.875	3.75

10. What is a constant of proportionality in this relationship?

0.375 or 2.667

11. Instead of following this recipe, Xavier decides to use 1 teaspoon of cinnamon for every 2 cups of whole wheat flour. How will that impact the taste of the graham crackers?

Responses vary. The original recipe calls for 0.375 teaspoons of cinnamon for each cup of whole wheat flour. Xavier's recipe, on the other hand, contains $1 \div 2 = 0.5$ teaspoons of cinnamon for each cup of whole wheat flour. Xavier's recipe has more cinnamon for each cup of flour, so his graham crackers will probably taste more cinnamony.

Accelerated 7 | Unit 3 Lesson 2 211 Activity 2

Whole Class | 5 min

Synthesis

Purpose: Students synthesize their understanding of using a constant of proportionality to make sense of relationships.

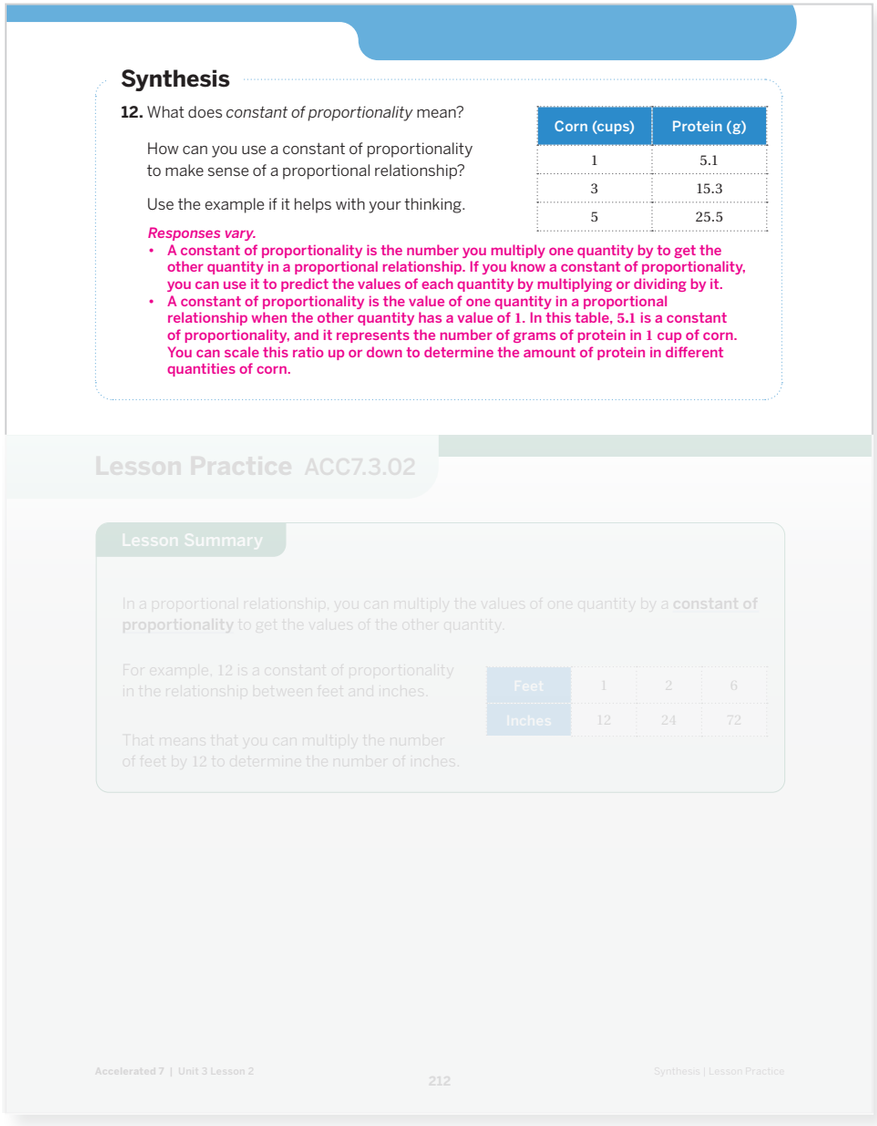
Invite students to respond independently, and then share their thinking with a partner.

Collect and share a variety of ideas about constants of proportionality, such as:

- It is the number you multiply one quantity by to get the other quantity.
- It is the value of one quantity when the other quantity has a value of 1.
- It can be used to find values in a proportional relationship.
- In this table it is 5.1, and it represents the number of grams of protein in 1 cup of corn.

MLR **MLR2: Collect and Display** Use this routine to revisit the class definition of *constant of proportionality*. Add any new ideas to the display for students to reference for the rest of the unit.

Note: If time allows, invite students to revise their responses based on the discussion.



Synthesis

12. What does *constant of proportionality* mean?

How can you use a constant of proportionality to make sense of a proportional relationship?

Use the example if it helps with your thinking.

Corn (cups)	Protein (g)
1	5.1
3	15.3
5	25.5

Responses vary.

- A constant of proportionality is the number you multiply one quantity by to get the other quantity in a proportional relationship. If you know a constant of proportionality, you can use it to predict the values of each quantity by multiplying or dividing by it.
- A constant of proportionality is the value of one quantity in a proportional relationship when the other quantity has a value of 1. In this table, 5.1 is a constant of proportionality, and it represents the number of grams of protein in 1 cup of corn. You can scale this ratio up or down to determine the amount of protein in different quantities of corn.

Lesson Practice ACC7.3.02

Lesson Summary

In a proportional relationship, you can multiply the values of one quantity by a **constant of proportionality** to get the values of the other quantity.

For example, 12 is a constant of proportionality in the relationship between feet and inches.

Feet	1	2	6
Inches	12	24	72

That means that you can multiply the number of feet by 12 to determine the number of inches.

Accelerated 7 | Unit 3 Lesson 2 212 Synthesis | Lesson Practice

Lesson Takeaway: You can use a table to justify whether a relationship is proportional by using strategies such as calculating the ratio for each pair of values to determine if there is a **constant of proportionality**.

Independent | 5 min

Show What You Know

Purpose: Students demonstrate their understanding by determining and interpreting the constant of proportionality from a table.

Today's Goals

1. **Goal:** Determine the *constant of proportionality* for a proportional relationship represented in a table.
2. **Goal:** Use the *constant of proportionality* to determine values in a table.
3. **Language Goal:** Justify whether a table represents a proportional relationship, using terms such as *constant of proportionality*. **(Reading and Writing)**

Show What You Know (Assessment Resources)

Name: _____ Date: _____ Period: _____

Show What You Know 3.02

Each row of this table represents a way to make the same color of green paint.

Blue Paint (cups)	Yellow Paint (cups)
2	10
1	5
6	30
52	260

a. What is a constant of proportionality in this relationship?
5 or $\frac{1}{5}$

b. What does that constant of proportionality mean in the situation?
Responses vary. The constant of proportionality 5 means that this particular color of paint uses 5 cups of yellow paint for every 1 cup of blue paint.

D Differentiation Use after Lesson 2

Support

Provide targeted intervention for students by using these resources.

If students need support understanding how to identify a constant of proportionality:

» **Respond to Student Thinking:**

- Review the Lesson Summary.
- Use the ***Determining the Constant of Proportionality*** **Mini-Lesson**.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 1 Extension Activities**.
- If students would like to explore nutrition in greater depth, invite them to research sodas, sport drinks, and juices and compare the amount of sugar or caffeine per ounce.

Professional Learning

Why was the context of sugar and cooking particularly useful in building students' understanding of constant of proportionality in this lesson?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

12. What does *constant of proportionality* mean?

How can you use a constant of proportionality to make sense of a proportional relationship?

Use the example if it helps with your thinking.

Corn (cups)	Protein (g)
1	5.1
3	15.3
5	25.5

Responses vary.

- A constant of proportionality is the number you multiply one quantity by to get the other quantity in a proportional relationship. If you know a constant of proportionality, you can use it to predict the values of each quantity by multiplying or dividing by it.
- A constant of proportionality is the value of one quantity in a proportional relationship when the other quantity has a value of 1. In this table, 5.1 is a constant of proportionality, and it represents the number of grams of protein in 1 cup of corn. You can scale this ratio up or down to determine the amount of protein in different quantities of corn.

Lesson Practice ACC7.3.02

Lesson Summary

In a proportional relationship, you can multiply the values of one quantity by a **constant of proportionality** to get the values of the other quantity.

For example, 12 is a constant of proportionality in the relationship between feet and inches.

Feet	1	2	6
Inches	12	24	72

That means that you can multiply the number of feet by 12 to determine the number of inches.

Lesson Practice
ACC7.3.02

Name: _____ Date: _____ Period: _____

Problems 1–2: When Deven makes chocolate milk, he mixes 2 cups of milk with 3 tablespoons of chocolate syrup. This table shows how to make batches of different sizes.

Milk (cups)	Chocolate Syrup (tbsp)
2	3
8	12
1	$\frac{3}{2}$
10	15

1. Is there a proportional relationship between cups of milk and tablespoons of chocolate syrup?

Yes

Show or explain your thinking.

Explanations vary. The relationship is proportional because there are 1.5 tablespoons of chocolate syrup for every cup of milk.

2. What is a constant of proportionality for this relationship?

1.5 or $\frac{2}{3}$

Problems 3–4: When you mix two colors of paint in equivalent ratios, the color you get is always the same.

3. Complete the table so that each row makes the same color purple.

Responses that are in a 1 : 3 ratio of blue cups to red cups are considered correct.

Blue (cups)	Red (cups)
2	6
1	3

4. What is a constant of proportionality for this relationship?

3 or $\frac{1}{3}$

Problems 5–7: Here is some information about the side lengths of two scaled copies, triangle A and triangle B.

Side Length of Triangle A (in.)	Side Length of Triangle B (in.)
1	6
$\frac{1}{2}$	3
$1\frac{1}{3}$	8

5. Complete the table to determine the missing side lengths of each triangle.

6. What is a constant of proportionality in this relationship?

6 or $\frac{1}{6}$

7. What does that constant of proportionality tell you about the triangles?

Responses vary. Each side length in triangle A is $\frac{1}{6}$ the length of the corresponding side in triangle B.

Lesson Practice
ACC7.3.02

Name: _____ Date: _____ Period: _____

8. Table A shows different amounts of blue and white paint that can be mixed together to create a color called "Blue Breeze." Sai wants to create a new color that is *bluer* than "Blue Breeze" and call it "Sea Glass Blue."

a. Complete the first row of Table B to create a possible mixture for "Sea Glass Blue."

Responses vary. Sample shown in table.

b. Complete the rest of Table B so that the relationship between cups of blue and cups of white is proportional.

Responses vary. Sample shown in table.

Table A Blue Breeze		Table B Sea Glass Blue	
Blue (cups)	White (cups)	Blue (cups)	White (cups)
1	3	4	8
2	6	7	14
5	15	0.5	1
8	24	12	24

FAST Practice

9. Sarah orders a small poster that is 100 square inches and costs $5.00. She then orders a large poster that is 500 square inches and costs $25.00. What does the constant of proportionality tell you about the costs of the posters?

The cost is $ 0.05 for each square inch of poster.

Spiral Review

10. Select *all* the scales that are equivalent to the scale 1 cm to 5 m.

- ☑ A. $\frac{1}{2}$ cm to $2\frac{1}{2}$ m
- ☐ B. 1 cm to 5 km
- ☐ C. 5 cm to 1 km
- ☑ D. 5 cm to 25 m
- ☑ E. 10 cm to 50 m

11. Select *all* of the ratios that are equivalent to 4 : 7.

- ☐ A. 8 : 15
- ☑ B. 16 : 28
- ☐ C. 7 : 4
- ☑ D. 20 : 35
- ☐ E. 6 : 9

Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1, 3, 5	2	MA.7.AR.4.1
	2, 4, 6	1	MA.7.AR.4.2
	7	2	MA.7.AR.4.2
	8	3	MA.7.AR.4.1
FAST Practice	9	2	MA.7.AR.4.2
Spiral Review			
	10	2	MA.7.GR.1.5
	11	1	MA.6.AR.3.1

Unit 3
Lesson 3
Name: Date: Period:
MA.7.AR.3.3, MTR.2.1, MTR.4.1
Pen Pals
Let's compare measurements in different systems.
Warm-Up
1. Four pen pals share letters with each other.
Discuss: What do you notice? What do you wonder?

Name	Eva	Ayaan	Thiago	Binta
Country	United States	India	Brazil	Liberia
Favorite Food	Bubble tea	Mango lassi	Quindim	Spaghetti
Favorite Animal	Horse	Dog	Horse	Bird
Favorite Sport	Football	Cricket	Futebol	Football

Accelerated 7 | Unit 3 Lesson 3
215
Warm-Up

Pen Pals

Converting Units

Let's compare measurements in different systems.

Learning Outcomes

Today's Goals

1. **Goal:** Convert measurements from one unit to another in different measurement systems.
2. **Language Goal:** Compare and contrast strategies for converting between units. **(Writing, Speaking, and Listening)**

Students convert measurements from one unit to another in different measurement systems. Students learn that they can represent unit conversion problems in multiple ways and that different representations can be useful in different situations. **(MTR.2.1)**

Coherence

Prior Learning

In a previous course, students converted measurements within the same system to solve mathematical and real-world problems. In Lessons 2 and 3, students created tables to help make sense of rates and proportional relationships.

Future Learning

In future courses, students will apply their understanding of unit conversions to solve real-world geometric problems.

Connections

- Students convert measurements from one unit to another using tables and double number-lines. (real world)
- Students describe strategies used by others to convert measurements to deepen their own understanding. (mathematical, cross curricular)

FL B.E.S.T. Math Benchmarks

Addressing

MA.7.AR.3.3
Solve mathematical and real-world problems involving the conversion of units across different measurement systems.

Mathematical Thinking and Reasoning Standards: MTR.2.1, MTR.4.1

ELA Expectations: ELA.K12.EE.3.1, ELA.K12.EE.4.1, ELA.K12.EE.5.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.6.AR.3.5	**MA.912.FL.2.4** **MA.912.T.2.1**

Lesson at a Glance

~ 45 min

MA.7.AR.3.3

Warm-Up

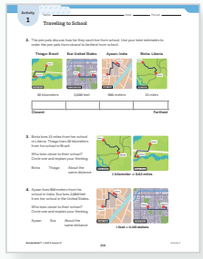
Pairs | 5 min

Students are introduced to the context of the lesson by comparing and contrasting some favorite things around the world.
Routine: Notice and Wonder

Activity 1

Pairs | 10 min

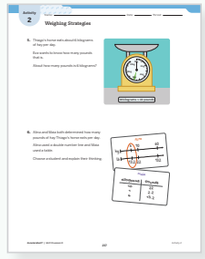
Students compare distances using conversion rates.
Routine: Think-Pair-Share



Activity 2

Pairs | 10 min

Students analyze different tools and strategies for converting measurements from one unit to another. MLR7: Compare and Connect (MTR.2.1)



Activity 3

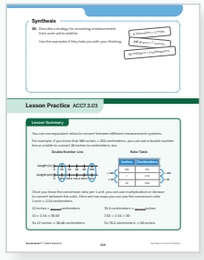
Pairs | 10 min

Students practice converting amounts of money from one currency to another.

Synthesis

Whole Class | 5 min

Students synthesize their understanding that ratio reasoning can be used to convert measurements from one unit to another in different measurement systems. MLR2: Collect and Display



Show What You Know

Independent | 5 min

Students demonstrate their understanding by converting among units of measurement in different systems using a ratio in terms other than 1.

Prep Checklist

This lesson includes:

- Student Edition
- Show What You Know

Pairs | 5 min

Warm-Up

Purpose: Students are introduced to the context of the lesson by comparing and contrasting some favorite things around the world.

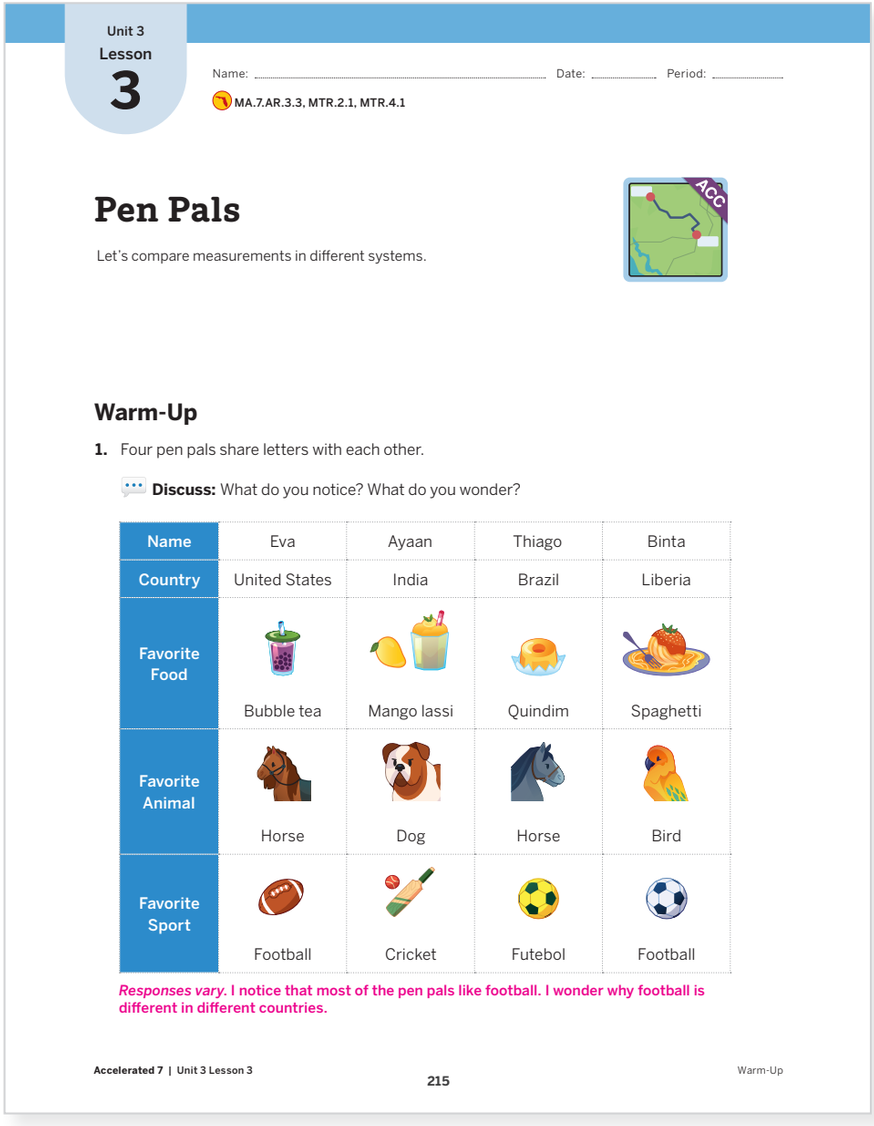
1 Launch

Use the Notice and Wonder routine to promote curiosity and help students make sense of the context. See the Routine Facilitation Guide for more information.

To surface connections, consider asking students what they know about pen pals, and whether anyone in the class has ever had a pen pal.

2 Connect

Listen for students making personal connections to the images of the favorite things.



Unit 3
Lesson
3

Name: _____ Date: _____ Period: _____

MA.7.AR.3.3, MTR.2.1, MTR.4.1

Pen Pals

Let's compare measurements in different systems.

Warm-Up

1. Four pen pals share letters with each other.

Discuss: What do you notice? What do you wonder?

Name	Eva	Ayaan	Thiago	Binta
Country	United States	India	Brazil	Liberia
Favorite Food	Bubble tea	Mango lassi	Quindim	Spaghetti
Favorite Animal	Horse	Dog	Horse	Bird
Favorite Sport	Football	Cricket	Futebol	Football

Responses vary. I notice that most of the pen pals like football. I wonder why football is different in different countries.

Accelerated 7 | Unit 3 Lesson 3 215 Warm-Up

Pairs | 10 min

Activity 1 Traveling to School

Purpose: Students compare distances using conversion rates.

1 Launch

Use the Think-Pair-Share routine to support student-to-student discourse. See the Routine Facilitation Guide for more information.

Invite a few students to share their work. It is okay — even desirable — to lack consensus at this stage. The activity will build toward consensus when students are given a ratio to use to convert between different units.

2 Monitor

Pause briefly to share with students that we'll be helping the pen pals convert distances from school from one unit of measurement to another.

Invite students to use a calculator or paper to help with their thinking throughout this lesson.

Look for a variety of strategies and explanations.

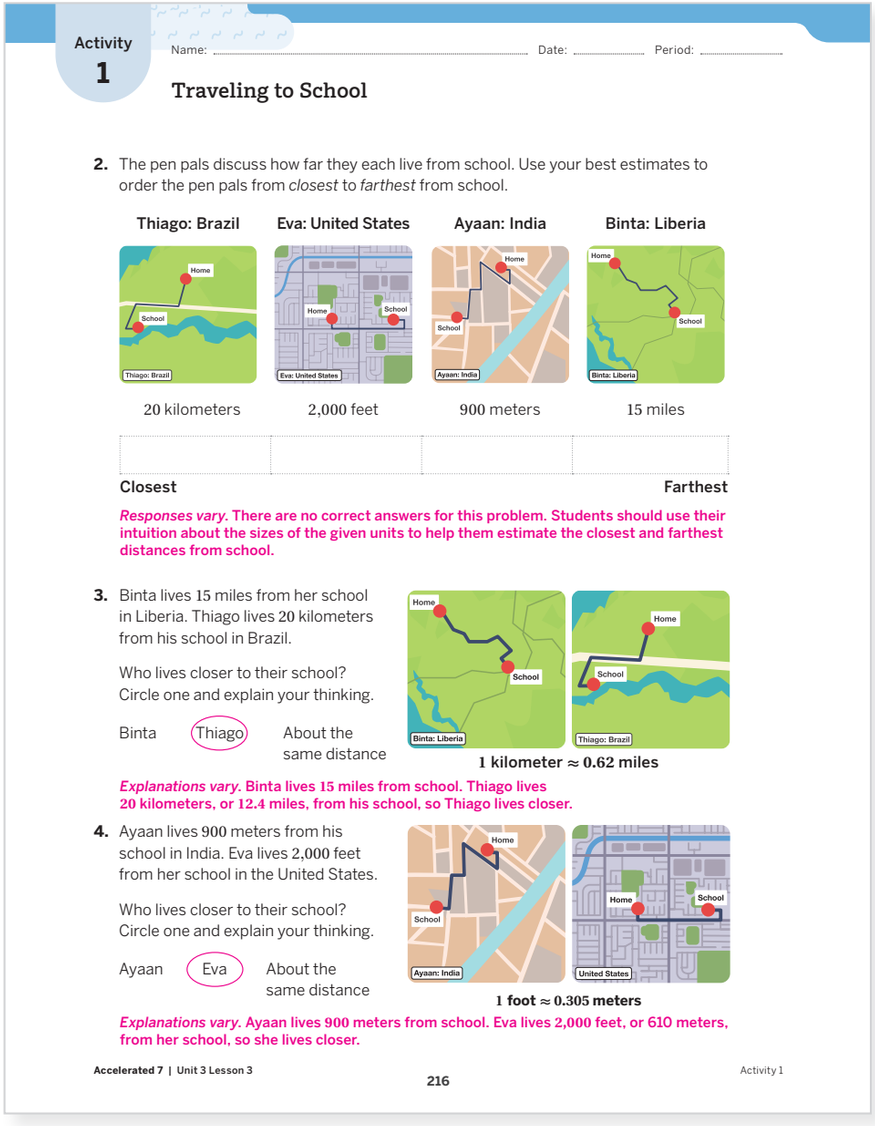
D Differentiation

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Consider asking, "What unit of measurement is longer? What is the relationship between the two?"
Only multiply or divide to do their conversion.	**Support** Invite them to use a double number line or table to support their ratio reasoning.
Use a table or double number line to support their ratio reasoning.	**Strengthen** Invite them to share their thinking during the Connect.

Note: This may be the first time students see the ≈ symbol used to represent an approximate equivalence.

3 Connect

Invite students to ask questions of each classmate after they share, including clarifying the strategies for making sense of the different units.



Activity 1

Name: Date: Period:

Traveling to School

2. The pen pals discuss how far they each live from school. Use your best estimates to order the pen pals from *closest* to *farthest* from school.

Thiago: Brazil	Eva: United States	Ayaan: India	Binta: Liberia
20 kilometers	2,000 feet	900 meters	15 miles

Closest Farthest

Responses vary. There are no correct answers for this problem. Students should use their intuition about the sizes of the given units to help them estimate the closest and farthest distances from school.

3. Binta lives 15 miles from her school in Liberia. Thiago lives 20 kilometers from his school in Brazil.

Who lives closer to their school? Circle one and explain your thinking.

Binta (Thiago) About the same distance

1 kilometer ≈ 0.62 miles

Explanations vary. Binta lives 15 miles from school. Thiago lives 20 kilometers, or 12.4 miles, from his school, so Thiago lives closer.

4. Ayaan lives 900 meters from his school in India. Eva lives 2,000 feet from her school in the United States.

Who lives closer to their school? Circle one and explain your thinking.

Ayaan (Eva) About the same distance

1 foot ≈ 0.305 meters

Explanations vary. Ayaan lives 900 meters from school. Eva lives 2,000 feet, or 610 meters, from her school, so she lives closer.

Accelerated 7 | Unit 3 Lesson 3 216 Activity 1

Pairs | 10 min

Activity 2 Weighing Strategies

Purpose: Students analyze different tools and strategies for converting measurements from one unit to another. **(MTR2.1)**

1 Launch

Direct students' attention to the scale and consider asking:

- "Which is heavier, 1 pound or 1 kilogram?"
- "How many pounds do you think is about the same as 5 kilograms?"

Invite students to check their responses with a partner.

2 Monitor

Look and listen for a variety of tools and strategies to share during the Connect. **(MTR.2.1)**

D Differentiation

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Consider asking, "What is a solution that is too high? Too low?"
Have an inaccurate response.	**Strengthen** Invite them to use a double number line or table to support their ratio reasoning.
Use strategies similar to Alina's or Maia's on the next problem or other creative strategies.	**Strengthen** Invite them to share their thinking during the Connect.

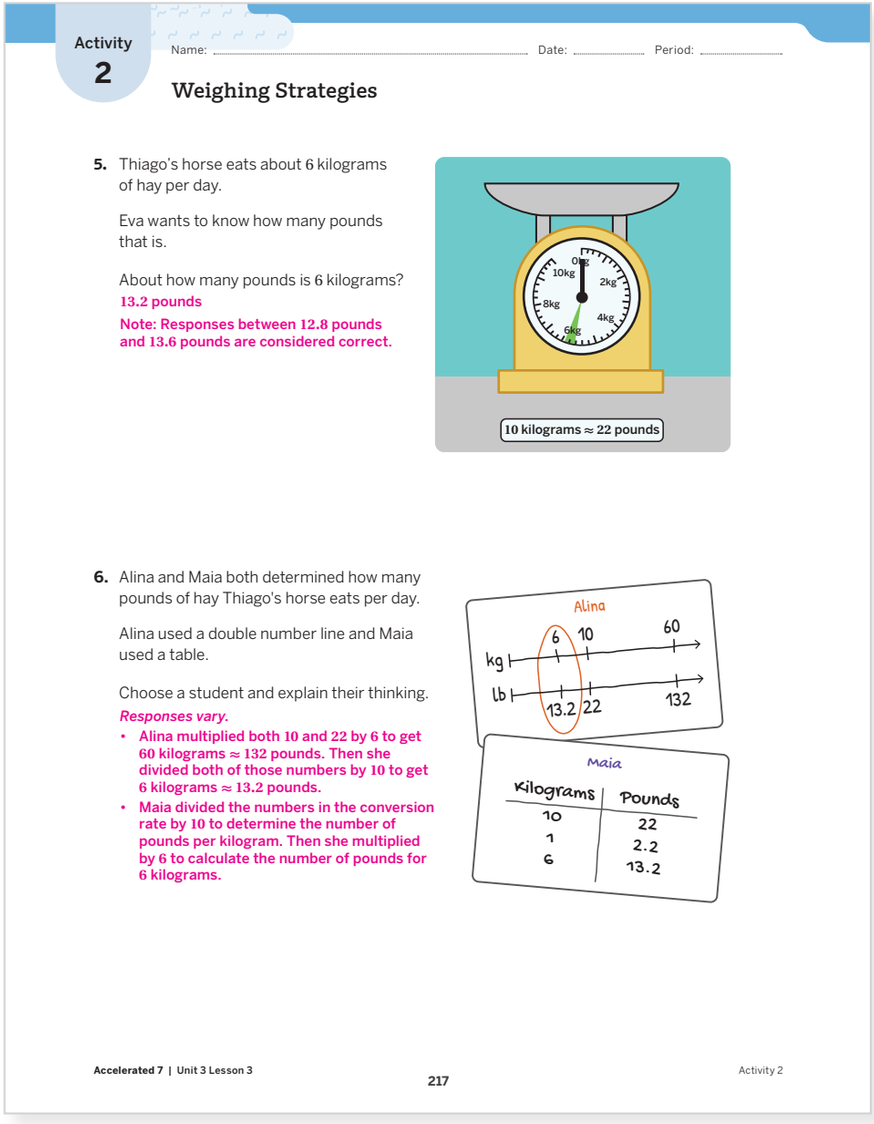
3 Connect

Invite students to select a strategy that makes the most sense to them.

MLR MLR7: Compare and Connect Invite students to analyze each representation, then find ways in which the representations are different (compare) and alike (connect). Consider asking:

- "How would you describe Alina's strategy in your own words? Maia's?"
- "Why do you think Maia used a 1 in her ratio table?"
- "What are the advantages of Alina's strategy? Maia's?"

Invite students that used similar or other creative strategies to share. Consider celebrating students who justify the strategy they used and compare the efficiency of their strategy to other strategies. **(MTR.4.1)**



Activity 2

Name: Date: Period:

Weighing Strategies

5. Thiago's horse eats about 6 kilograms of hay per day.

Eva wants to know how many pounds that is.

About how many pounds is 6 kilograms?

13.2 pounds

Note: Responses between 12.8 pounds and 13.6 pounds are considered correct.

6. Alina and Maia both determined how many pounds of hay Thiago's horse eats per day.

Alina used a double number line and Maia used a table.

Choose a student and explain their thinking.

Responses vary.

- **Alina multiplied both 10 and 22 by 6 to get 60 kilograms ≈ 132 pounds. Then she divided both of those numbers by 10 to get 6 kilograms ≈ 13.2 pounds.**
- **Maia divided the numbers in the conversion rate by 10 to determine the number of pounds per kilogram. Then she multiplied by 6 to calculate the number of pounds for 6 kilograms.**

Accelerated 7 | Unit 3 Lesson 3 217 Activity 2

Key Takeaway: When students are given a ratio to use to convert between different units, a table or double number line can be used to organize and find equivalent ratios.

Pairs | 10 min

Activity 3 Favorite Things

Purpose: Students practice converting amounts of money from one currency to another.

Short on time? Consider omitting problem 7.

1 Launch

To surface connections, consider asking students if they have ever seen or used foreign currency, and what sorts of differences from U.S. currency they noticed.

A Accessibility: Visual-Spatial Processing
Provide graph paper, blank tables, or tape diagram templates for students to draw on to increase access to the task.

2 Monitor

Look and listen for students who may need individual support to be comfortable with at least one strategy.

D Differentiation

Look for students who:	Respond to Student Thinking
Need support getting started or use a guessing and checking strategy.	**Support** Invite students to either make a table or double number line. Consider asking, "Where would ____ go?"
Are able to find equivalent ratios that are close (e.g., $2 : 168.80$, $11 : 63.80$, and $16 : 3032.32$) to the desired ratio but are not sure what to do next.	**Strengthen** Celebrate their attempt as an estimation strategy. Consider asking, "Between what two values will the solution be?" Then encourage them to revisit Alina's and Maia's strategies.
Use a table, double number line, or other creative strategies to support their ratio reasoning.	**Strengthen** Invite them to share their thinking during the Connect. **Strengthen** Encourage students to try another strategy.

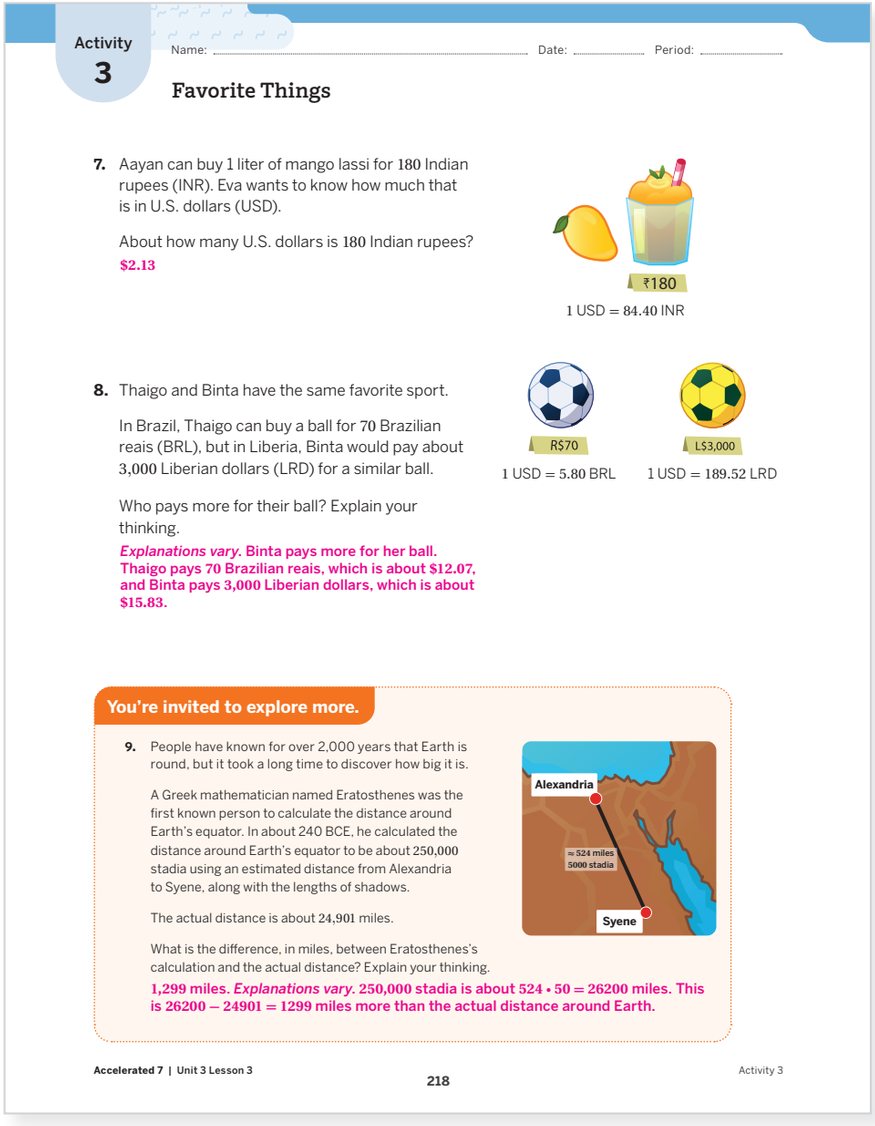
D Differentiation: You're invited to explore more. (Stretch) Invite students who would like to explore different units further to complete this optional task. Encourage them to discuss their thinking with a partner.

3 Connect

Invite students that had inaccuracies in their work to share their mistakes and their new, refined work or give the students an opportunity to ask the class for feedback.

Invite students that used tables, double number lines, or other creative strategies to share.

Consider asking, "How can equivalent ratios help us to convert units?"



Activity 3

Name: Date: Period:

Favorite Things

7. Aayan can buy 1 liter of mango lassi for 180 Indian rupees (INR). Eva wants to know how much that is in U.S. dollars (USD).

About how many U.S. dollars is 180 Indian rupees?

$2.13

1 USD = 84.40 INR

8. Thaigo and Binta have the same favorite sport.

In Brazil, Thaigo can buy a ball for 70 Brazilian reais (BRL), but in Liberia, Binta would pay about 3,000 Liberian dollars (LRD) for a similar ball.

Who pays more for their ball? Explain your thinking.

Explanations vary. Binta pays more for her ball. Thaigo pays 70 Brazilian reais, which is about $12.07, and Binta pays 3,000 Liberian dollars, which is about $15.83.

1 USD = 5.80 BRL

1 USD = 189.52 LRD

You're invited to explore more.

9. People have known for over 2,000 years that Earth is round, but it took a long time to discover how big it is.

A Greek mathematician named Eratosthenes was the first known person to calculate the distance around Earth's equator. In about 240 BCE, he calculated the distance around Earth's equator to be about 250,000 stadia using an estimated distance from Alexandria to Syene, along with the lengths of shadows.

The actual distance is about 24,901 miles.

What is the difference, in miles, between Eratosthenes's calculation and the actual distance? Explain your thinking.

1,299 miles. *Explanations vary.* 250,000 stadia is about 524 • 50 = 26200 miles. This is 26200 − 24901 = 1299 miles more than the actual distance around Earth.

Accelerated 7 | Unit 3 Lesson 3 218 Activity 3

Whole Class | 5 min

Synthesis

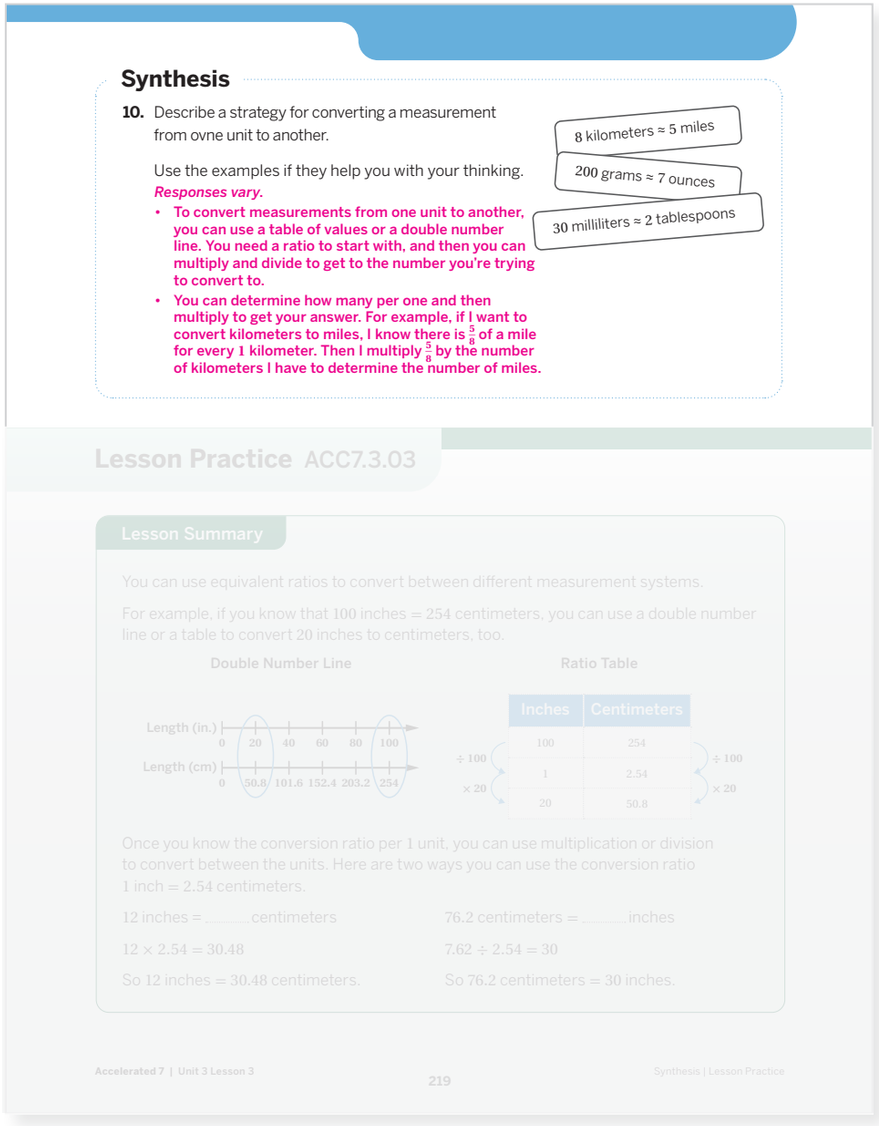
Purpose: Students synthesize their understanding that ratio reasoning can be used to convert measurements from one unit to another in different measurement systems.

Invite students to respond independently, and then share their thinking with a partner.

Collect and share a variety of ideas, such as:

- Writing your conversation rate as a ratio on a table or double number line to help organize and calculate equivalent ratios.
- Making an equivalent ratio in which one of the values is a 1 can be helpful to get your desired ratio.
- Using a common multiple of the original and desired values can be helpful when finding a ratio that can be used as a middle step.

MLR **MLR2: Collect and Display** Use this routine to formalize the strategies for converting units using students' language. Display any new language, examples, or diagrams for students to reference later.



Synthesis

10. Describe a strategy for converting a measurement from ovne unit to another.

Use the examples if they help you with your thinking.

8 kilometers ≈ 5 miles

200 grams ≈ 7 ounces

30 milliliters ≈ 2 tablespoons

Responses vary.

- To convert measurements from one unit to another, you can use a table of values or a double number line. You need a ratio to start with, and then you can multiply and divide to get to the number you're trying to convert to.
- You can determine how many per one and then multiply to get your answer. For example, if I want to convert kilometers to miles, I know there is $\frac{5}{8}$ of a mile for every 1 kilometer. Then I multiply $\frac{5}{8}$ by the number of kilometers I have to determine the number of miles.

Lesson Practice ACC7.3.03

Lesson Summary

You can use equivalent ratios to convert between different measurement systems.

For example, if you know that 100 inches = 254 centimeters, you can use a double number line or a table to convert 20 inches to centimeters, too.

Double Number Line

Length (in.): 0, 20, 40, 60, 80, 100

Length (cm): 0, 50.8, 101.6, 152.4, 203.2, 254

Ratio Table

Inches	Centimeters
100	254
1	2.54
20	50.8

÷ 100, × 20

Once you know the conversion ratio per 1 unit, you can use multiplication or division to convert between the units. Here are two ways you can use the conversion ratio 1 inch = 2.54 centimeters.

12 inches = _____ centimeters

12 × 2.54 = 30.48

So 12 inches = 30.48 centimeters.

76.2 centimeters = _____ inches

7.62 ÷ 2.54 = 30

So 76.2 centimeters = 30 inches.

Accelerated 7 | Unit 3 Lesson 3 219 Synthesis | Lesson Practice

Lesson Takeaway: Ratio reasoning can be used to convert measurements from one unit to another in different measurement systems. A table or double number line can be used to organize and find helpful equivalent ratios.

Independent | 5 min

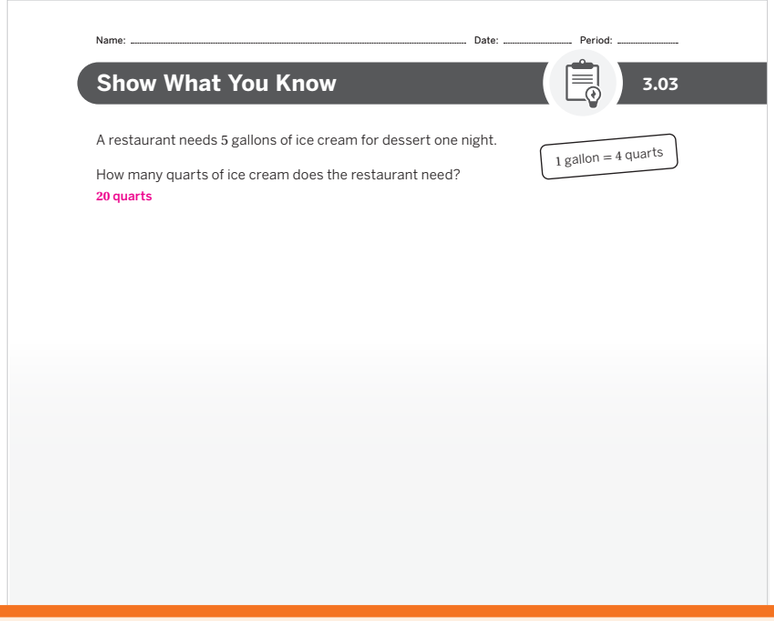
Show What You Know

Purpose: Students demonstrate their understanding by converting among units of measurement in different systems.

Today's Goals

1. **Goal:** Convert measurements from one unit to another in different measurement systems.
2. **Language Goal:** Compare and contrast strategies for converting between units. **(Writing, Speaking, and Listening)**

Show What You Know (Assessment Resources)



Name: _____ Date: _____ Period: _____

Show What You Know 3.03

A restaurant needs 5 gallons of ice cream for dessert one night.

How many quarts of ice cream does the restaurant need?

20 quarts

1 gallon = 4 quarts

D Differentiation Use after Lesson 3

Support

Provide targeted intervention for students by using these resources.

If students need support understanding how to convert among units of measurement in different systems using a ratio in terms other than 1:

» **Respond to Student Thinking:**

- Select a few related **Practice Problems** and give individualized feedback.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 1 Extension Activities**.

Professional Learning

Consider the proportional relationships you will teach in Units 8 and 11. How will students' work today in Pen Pals prepare them for using ratio reasoning to construct circle graphs and using the results of a probability experiment to make predictions. What are ways that you might call back to students' strategies from this lesson?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

10. Describe a strategy for converting a measurement from one unit to another.

Use the examples if they help you with your thinking.

8 kilometers ≈ 5 miles

200 grams ≈ 7 ounces

30 milliliters ≈ 2 tablespoons

Responses vary.

- To convert measurements from one unit to another, you can use a table of values or a double number line. You need a ratio to start with, and then you can multiply and divide to get to the number you're trying to convert to.
- You can determine how many per one and then multiply to get your answer. For example, if I want to convert kilometers to miles, I know there is $\frac{3}{5}$ of a mile for every 1 kilometer. Then I multiply $\frac{3}{5}$ by the number of kilometers I have to determine the number of miles.

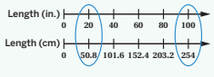
Lesson Practice ACC7.3.03

Lesson Summary

You can use equivalent ratios to convert between different measurement systems.

For example, if you know that 100 inches = 254 centimeters, you can use a double number line or a table to convert 20 inches to centimeters, too.

Double Number Line



Ratio Table

Inches	Centimeters
100	254
1	2.54
20	50.8

÷ 100, × 20 (Inches); ÷ 100, × 20 (Centimeters)

Once you know the conversion ratio per 1 unit, you can use multiplication or division to convert between the units. Here are two ways you can use the conversion ratio 1 inch = 2.54 centimeters.

12 inches = ... centimeters

$12 \times 2.54 = 30.48$

So 12 inches = 30.48 centimeters.

76.2 centimeters = ... inches

$7.62 \div 2.54 = 30$

So 76.2 centimeters = 30 inches.

Lesson Practice
ACC7.3.03

Name: ..... Date: ..... Period: .....

1. Malik is 57 inches tall. If 1 inch = 2.54 centimeters, which value is closest to his height in centimeters?

A. 22.4 centimeters

B. 57 centimeters

C. 144.8 centimeters

D. 3,551 centimeters

Problems 2–3: Use the conversion rate that makes the most sense to determine the approximate value of each missing quantity. Show or explain your thinking.

1 kilogram = 1000 grams	3 ounces ≈ 85 grams
1 pound ≈ 0.454 kilograms	4 kilograms ≈ 141 ounces

2. 15 ounces ≈ ..... 425 ..... grams

Explanations vary.

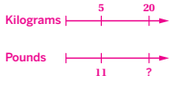
Ounces	Grams
3	85
15	?

×5 (Ounces), ×5 (Grams)

$85 \cdot 5 = 425$

3. 20 kilograms ≈ ..... 44 ..... pounds

Explanations vary.



$11 \cdot 4 = 44$

4. Dhruv's family exchanged 250 dollars for 4,250 pesos. Complete the table to determine the conversions between pesos and dollars.

Responses shown in table.

Dollars	Pesos
250	4,250
25	425
1	17
3	51
30	510

Lesson Practice
ACC7.3.03

Name: ..... Date: ..... Period: .....

FAST Practice

5. A yard is equal to 3 feet, and there are 1,760 yards in 1 mile. How many feet are there in 4 miles?

A. 3,520

B. 7,040

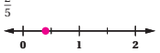
C. 5,280

D. 21,120

Spiral Review

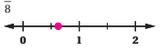
Problems 6–9: Plot each value on the number line.

6. $\frac{2}{5}$

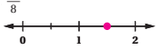


7. $1\frac{2}{5}$

8. $\frac{5}{8}$



9. $\frac{12}{8}$



Lesson Practice Item Analysis

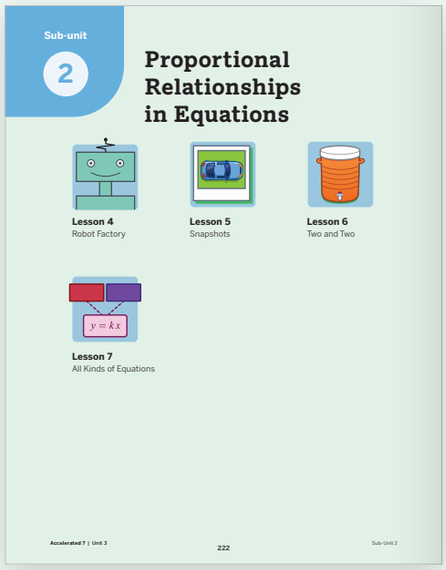
	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1–4	2	MA.7.AR.3.3
FAST Practice	5	2	MA.7.AR.3.3
Spiral Review			
		1	MA.6.NSO.1.1

Proportional Relationships in Equations

Sub-Unit 2 Goals

- **Goal:** Write and use equations to describe proportional relationships.
- **Goal:** Determine a constant of proportionality from an equation.
- **Language Goal:** Describe relationships represented with equations using terms such as *proportional*, *non-proportional*, and *constant of proportionality*.

In this sub-unit, students write equations describing proportional relationships and recognize proportional relationships represented with equations. They also determine the constant of proportionality using equations. Students write two equations that represent the same proportional relationship and notice that the two constants of proportionality are **reciprocals**.



Math that Matters Most

Concept or Skill	For example . . .
Write equations of the form $y = kx$ to represent proportional relationships.	The table represents a proportional relationship. Determine the constant of proportionality that completes the equation. $P =$ 4 s

s	P
2	8
3	12
5	20
10	40

Explore and interpret the concept of *reciprocals*.

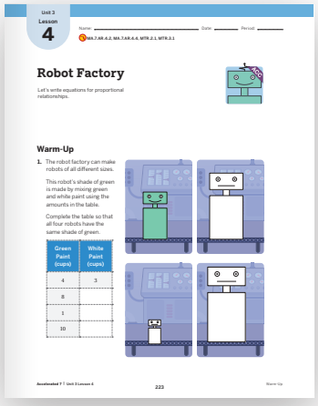
Alexis thinks that the equation $y = 100x$ matches Table A. Carlos thinks that the equation $x = \frac{1}{100}y$ matches Table A.

Whose claim is correct?

Both. The equation y = 100x is saying that you multiply each x-value by 100 to get the y-value. The equation $x = \frac{1}{100}y$ is saying that you multiply each y-value by $\frac{1}{100}$ to get the x-value.

Table A

Length (m), x	Length (cm), y
1	100
0.25	25
1.6	160
57.1	5710


Unit 3
Lesson
4

Name: _____ Date: _____ Period: _____

MA.7.AR.4.2, MA.7.AR.4.4, MTR.2.1, MTR.3.1

Robot Factory

Let's write equations for proportional relationships.

Warm-Up

1. The robot factory can make robots of all different sizes.

This robot's shade of green is made by mixing green and white paint using the amounts in the table.

Complete the table so that all four robots have the same shade of green.

Green Paint (cups)	White Paint (cups)
4	3
8	
1	
10	

Accelerated 7 | Unit 3 Lesson 4 223 Warm-Up

Robot Factory

Proportional Relationships and Equations

Let's write equations for proportional relationships.

Learning Outcomes

Today's Goals

1. **Goal:** Write equations of the form $y = kx$ to represent proportional relationships.
2. **Language Goal:** Explain that when the value of one quantity in a proportional relationship is 1, the other quantity is the constant of proportionality. **(Speaking and Listening)**

Students are given tables and use repeated reasoning to write equations for proportional relationships. This may be one of the first times that students encounter equations with two variables. This lesson is intended to expose students to the power of using equations as representations of proportional relationships. **(MTR.3.1)**

Coherence

Prior Learning

In Lessons 1 and 2, students determined the constant of proportionality from a table of values and used reasoning with the constant of proportionality to determine unknown values.

Future Learning

In Lesson 5, students will practice writing and interpreting equations for different scenarios.

Connections

- Students write equations relating proportional relationships in real world contexts such as scaling new parts of a robot to size, converting money exchanges while traveling, and distance traveled over time. (real world)

FL B.E.S.T. Math Benchmarks

Addressing

MA.7.AR.4.2
Determine the constant of proportionality within a mathematical or real-world context given a table, graph or written description of a proportional relationship.

MA.7.AR.4.4
Given any representation of a proportional relationship, translate the representation to a written description, table or equation.

Mathematical Thinking and Reasoning Standards: MTR.2.1, MTR.3.1

ELA Expectation: ELA.K12.EE.5.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.6.AR.3.2	**MA.8.AR.3.2**
MA.6.AR.3.3	**MA.8.AR.3.3**
MA.6.AR.3.5	
MA.7.GR.1.5	

Vocabulary

Review Vocabulary

scaled copy

Lesson at a Glance

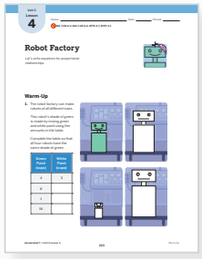
~ 45 min

MA.7.AR.4.2, MA.7.AR.4.4

Warm-Up

Pairs | 5 min

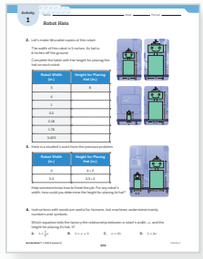
Students are introduced to the context of creating a robot and discuss what the constant of proportionality represents.



Activity 1

Pairs | 15 min

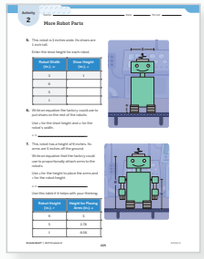
Students use repeated reasoning in tables to generate equations of the form $y = kx$ to produce robots fitting a design. MLR3: Critique, Correct, Clarify (MTR.3.1)



Activity 2

Pairs | 15 min

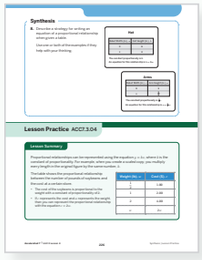
Students write equations to represent proportional relationships.



Synthesis

Whole Class | 5 min

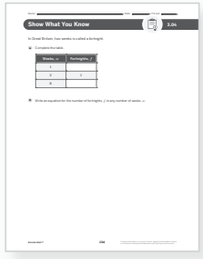
Students synthesize their understanding of using the constant of proportionality to write an equation that represents a proportional relationship. MLR2: Collect and Display



Show What You Know

Independent | 5 min

Students demonstrate their understanding by writing the equation of a proportional relationship.



Prep Checklist

This lesson includes:

- Student Edition
- Show What You Know

Pairs | 5 min

Warm-Up

Purpose: Students are introduced to the context of creating a robot and discuss what the constant of proportionality represents.

1 Launch

Direct students' attention to the robots of different sizes.

Consider asking, "What strategies can we use to determine the amount of white paint for each robot?"

Listen for students who use equivalent ratios, multiplicative relationships, or determine a constant of proportionality. **(MTR.2.1)**

D **Differentiation:** (Support) For students who need help getting started, consider asking, "How did we use a constant of proportionality in previous lessons to determine values in tables like this?"

Invite students to check their responses with a partner.

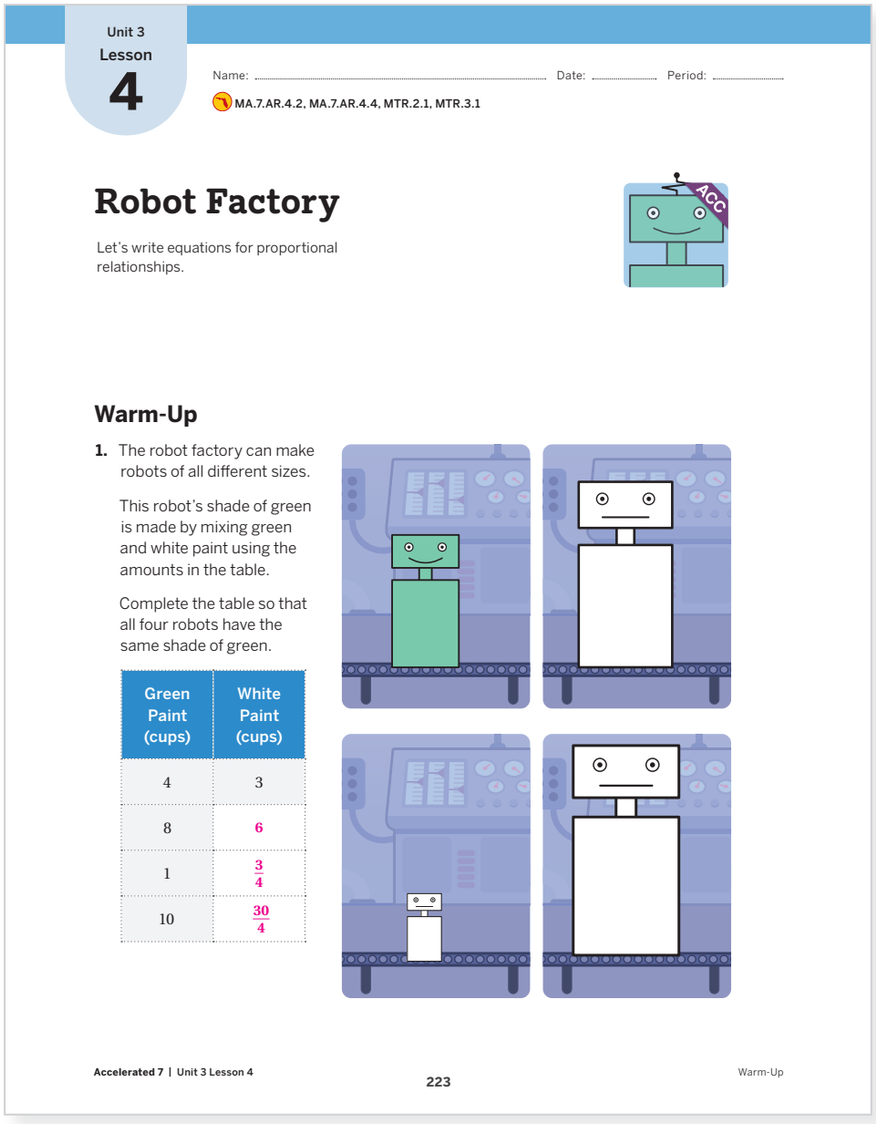
2 Connect

Invite students to share their strategies for completing the table.

Consider asking:

- "What is the constant of proportionality and where can you see it in the table?"**(MTR.2.1)**
- "What does the constant of proportionality represent in this situation?"

Note: If it does not come up naturally, consider discussing whether students believe it is a coincidence that the constant of proportionality is $\frac{3}{4}$ and the number paired with 1 is $\frac{3}{4}$.



Unit 3
Lesson
4

Name: ..... Date: ..... Period: .....

MA.7.AR.4.2, MA.7.AR.4.4, MTR.2.1, MTR.3.1

Robot Factory

Let's write equations for proportional relationships.

Warm-Up

1. The robot factory can make robots of all different sizes.

   This robot's shade of green is made by mixing green and white paint using the amounts in the table.

   Complete the table so that all four robots have the same shade of green.

Green Paint (cups)	White Paint (cups)
4	3
8	6
1	$\frac{3}{4}$
10	$\frac{30}{4}$

Accelerated 7 | Unit 3 Lesson 4 223 Warm-Up

Pairs | 15 min

Activity 1 Robot Hats

Purpose: Students use repeated reasoning in tables to generate equations of the form $y = kx$ to produce robots fitting a design. **(MTR.3.1)**

1 Launch

Consider sharing that students will be in charge of creating 30 *scaled copies* of their robot and they will be accessorized with hats, shoes, and arms.

2 Monitor

Select and sequence strategies for determining the height for placing the hat.

D Differentiation

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Invite students to form a strategy. Consider asking, "What relationships do you notice across or down the table?"
Attempt to use additive relationships to determine the height.	**Support** Consider asking, "What does it mean to be a scaled copy? How do the sides of scaled copies relate to each other?"
Represent the height by multiplying the width by 2.	**Stretch** Consider asking, "How could you represent your thinking in the table with an equation?"

MLR MLR3: Critique, Correct, Clarify Instead of using the discussion question in Problem 3, have students improve upon a statement, such as "Multiply by 2."
a. Discuss what is *correct* about the statement.
b. Discuss what is *incorrect* or *unclear.*
c. Write a question to help a student revise their statement.

3 Connect

Share and discuss the student strategies used to determine the height for placing the hat.

Invite students to discuss with a partner about which equation represents the relationship.

To surface the Key Takeaway, consider asking:
- "How would you explain what the equation $h = 2w$ means in a sentence?"
- "Why might someone think the equation $w = 2h$ is correct? What would you say in response?"
- "Why do we want to generate an equation?" **(MTR.3.1)**

Note: If time allows, consider using the equation $h = w + 3$ to highlight that an equation should successfully predict the height of all robots.

Activity 1

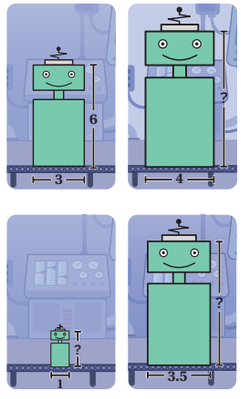
Name: _____ Date: _____ Period: _____

Robot Hats

2. Let's make 30 *scaled copies* of this robot.

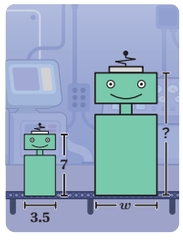
The width of this robot is 3 inches. Its hat is 6 inches off the ground.

Complete the table with the height for placing the hat on each robot.



Robot Width (in.)	Height for Placing Hat (in.)
3	6
4	8
1	2
3.5	7
2.18	4.36
1.76	3.52
3.425	6.85

3. Here is a student's work from the previous problem.



Robot Width (in.)	Height for Placing Hat (in.)
4	$4 \cdot 2$
3.5	$3.5 \cdot 2$

Help someone know how to finish the job. For any robot's width, how could you determine the height for placing its hat?

Responses vary. To know the height for placing the robot's hat, take its width and double it.

4. Instructions with words are useful for humans, but machines understand mainly numbers and symbols.

Which equation tells the factory the relationship between a robot's width, w, and the height for placing its hat, h?

A. $h = \frac{1}{2}w$ B. $h = w + 3$ C. $w = 2h$ D. $h = 2w$

Accelerated 7 | Unit 3 Lesson 4 224 Activity 1

Key Takeaway: You can use an equation to represent a proportional relationship that needs to be evaluated many times. It is helpful to determine the constant of proportionality to write the equation.

Pairs | 15 min

Activity 2 More Robot Parts

Purpose: Students write equations to represent proportional relationships.

Short on time? Consider omitting Problem 6. Students will have more opportunities with these types of equations in future lessons.

1 Launch

To generate excitement, consider asking, "What do you think your robot could be used for?"

2 Monitor

Look for students who use multiplication and division to represent the proportional relationships in the equations.

D Differentiation

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Consider asking, "What operations did we use in Activity 1 to write the equation? Can we use similar operations here?"
Divide the width by 3 or multiply the width by $\frac{1}{3}$ to write $d = \frac{w}{3}$ or $d = \frac{1}{3}w$. *(Problem 6)*	**Stretch** Consider asking, "How does the constant of proportionality show up in your equation?"
Try to use whole number values to represent the proportional relationship. *(Problem 7)*	**Support** Invite students to use decimal or fractional values when writing their equations.

Invite students to share their strategies for how they are determining the equations for their robot.

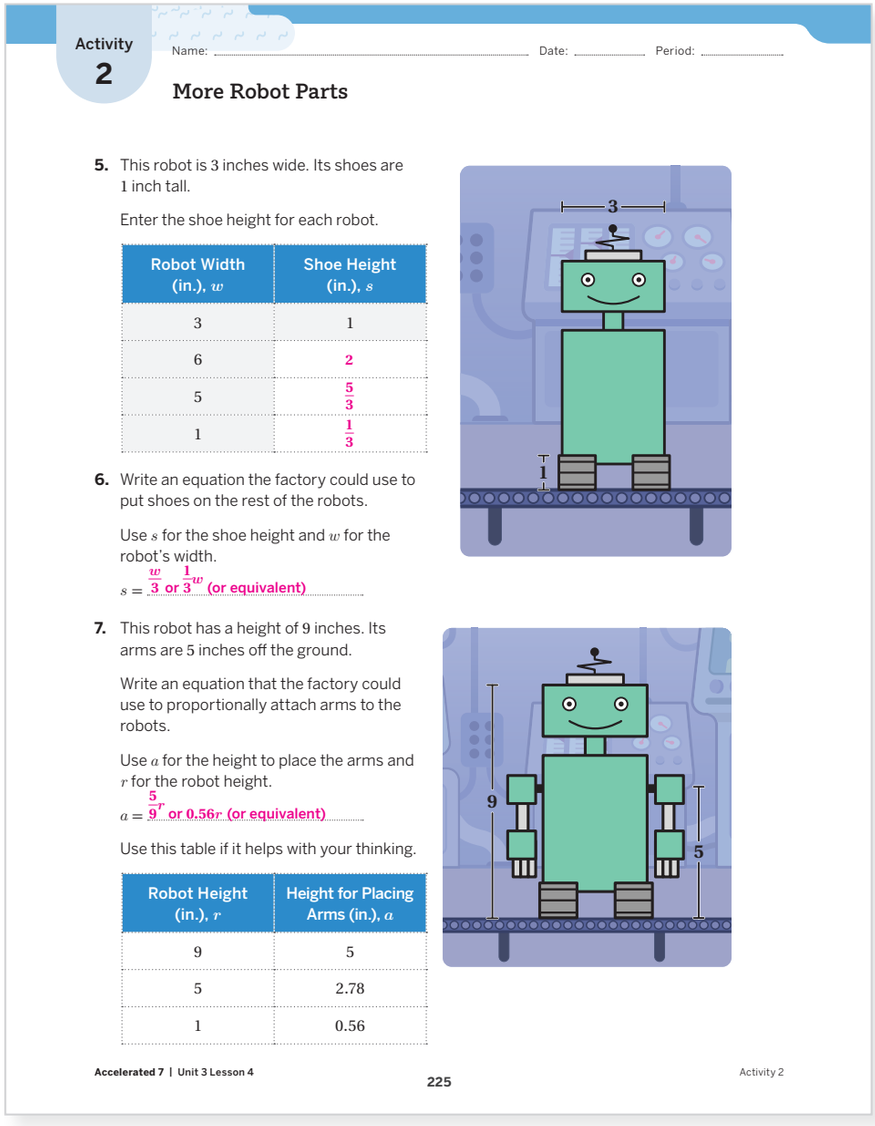
A **Accessibility: Memory and Attention** To activate prior knowledge, consider asking, "What information have we used earlier that might help us now?"

3 Connect

Share the captured equations and invite students to justify whether each equation will build the robots correctly.

Consider asking, "Where do we see the height of the robot and how far the arms are off the ground in the equation?"

Note: The relationship in Problem 7 is challenging and students may need support to write an equation to represent it. Students will have several more opportunities to practice over the subsequent lessons.



Activity 2

Name: Date: Period:

More Robot Parts

5. This robot is 3 inches wide. Its shoes are 1 inch tall.

Enter the shoe height for each robot.

Robot Width (in.), w	Shoe Height (in.), s
3	1
6	2
5	$\frac{5}{3}$
1	$\frac{1}{3}$

6. Write an equation the factory could use to put shoes on the rest of the robots.

Use s for the shoe height and w for the robot's width.

$s =$ $\frac{w}{3}$ or $\frac{1}{3}w$ (or equivalent)

7. This robot has a height of 9 inches. Its arms are 5 inches off the ground.

Write an equation that the factory could use to proportionally attach arms to the robots.

Use a for the height to place the arms and r for the robot height.

$a =$ $\frac{5}{9}r$ or $0.56r$ (or equivalent)

Use this table if it helps with your thinking.

Robot Height (in.), r	Height for Placing Arms (in.), a
9	5
5	2.78
1	0.56

Accelerated 7 | Unit 3 Lesson 4 225 Activity 2

Whole Class | 5 min

Synthesis

Purpose: Students synthesize their understanding of using the constant of proportionality to write an equation that represents a proportional relationship.

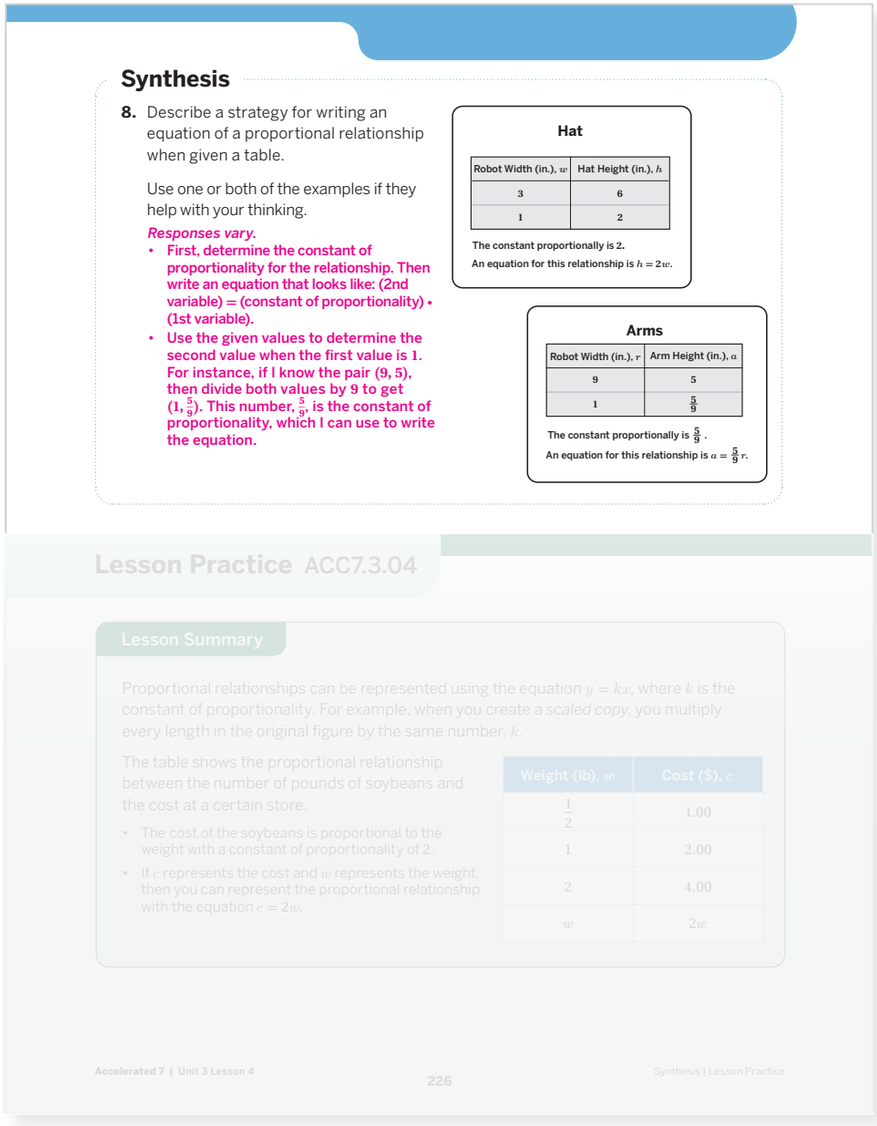
Invite students to respond independently, and then share their thinking with a partner.

Collect and share a variety of ideas, such as:

- Determining the constant of proportionality and using it to write the equation in the form $y = kx$.
- Determining the value of the second quantity when the first quantity is 1 (calculating the unit rate).
- Checking the equation and making sure that it works for all given values in the relationship.

MLR **MLR2: Collect and Display** Use this routine to collect students' strategies for writing an equation as they share. Display for students to reference and add to for the rest of the unit.

Note: If time allows, invite students to revise their previous thinking based on the discussion.



Synthesis

8. Describe a strategy for writing an equation of a proportional relationship when given a table.

Use one or both of the examples if they help with your thinking.

Responses vary.

- First, determine the constant of proportionality for the relationship. Then write an equation that looks like: (2nd variable) = (constant of proportionality) • (1st variable).
- Use the given values to determine the second value when the first value is 1. For instance, if I know the pair (9, 5), then divide both values by 9 to get $(1, \frac{5}{9})$. This number, $\frac{5}{9}$, is the constant of proportionality, which I can use to write the equation.

Hat

Robot Width (in.), w	Hat Height (in.), h
3	6
1	2

The constant proportionality is 2.

An equation for this relationship is $h = 2w$.

Arms

Robot Width (in.), r	Arm Height (in.), a
9	5
1	$\frac{5}{9}$

The constant proportionality is $\frac{5}{9}$.

An equation for this relationship is $a = \frac{5}{9}r$.

Lesson Practice ACC7.3.04

Lesson Summary

Proportional relationships can be represented using the equation $y = kx$, where k is the constant of proportionality. For example, when you create a *scaled copy*, you multiply every length in the original figure by the same number, k.

The table shows the proportional relationship between the number of pounds of soybeans and the cost at a certain store.

- The cost of the soybeans is proportional to the weight with a constant of proportionality of 2.
- If c represents the cost and w represents the weight, then you can represent the proportional relationship with the equation $c = 2w$.

Weight (lb), w	Cost ($), c
$\frac{1}{2}$	1.00
1	2.00
2	4.00
w	$2w$

Accelerated 7 | Unit 3 Lesson 4 226 Synthesis | Lesson Practice

Lesson Takeaway: It is helpful to determine the constant of proportionality when writing an equation that represents a proportional relationship. Once we have determined it, we can write the equation in the form $y = kx$, where k is the constant of proportionality.

Independent | 5 min

Show What You Know

Purpose: Students demonstrate their understanding by writing the equation of a proportional relationship.

Today's Goals

1. **Goal:** Write equations of the form $y = kx$ to represent proportional relationships.
2. **Language Goal:** Explain that when the value of one quantity in a proportional relationship is 1, the other quantity is the constant of proportionality. **(Speaking and Listening)**

Show What You Know (Assessment Resources)

Name: _____ Date: _____ Period: _____

Show What You Know 3.04

In Great Britain, two weeks is called a *fortnight*.

a. Complete the table.

Weeks, w	Fortnights, f
1	0.5
2	1
8	4

b. Write an equation for the number of fortnights, f, in any number of weeks, w.
$f = 0.5w$ (or equivalent)

D Differentiation Use after Lesson 4

Support

Provide targeted intervention for students by using these resources.

If students need support understanding how to determine the equation in a proportional relationship:

» **Respond to Student Thinking:**

- Consider reviewing the strategies for completing the table in Activity 1 before Lesson 5. Consider asking, "What relationships do you see between the rows and columns of the table?

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 2 Extension Activities**.

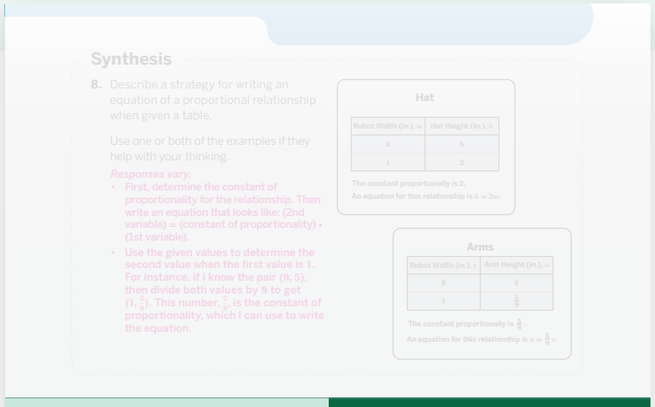
Professional Learning

Take a moment to reflect on how students' understanding of the term *constant of proportionality* has developed throughout this course. How has their language usage of *scaled copies*, *scale factor*, and *equivalent ratios* developed over the course? How do you hear students using these words to represent the multiplicative relationships that exist within proportional relationships?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.



Synthesis

8. Describe a strategy for writing an equation of a proportional relationship when given a table.

Use one or both of the examples if they help with your thinking.

Responses vary.

- First, determine the constant of proportionality for the relationship. Then write an equation that looks like: (2nd variable) = (constant of proportionality) • (1st variable).
- Use the given values to determine the second value when the first value is 1. For instance, if I know the pair (9, 5), then divide both values by 9 to get $(1, \frac{5}{9})$. This number, $\frac{5}{9}$, is the constant of proportionality, which I can use to write the equation.

Hat

Robot Width (in.), w	Hat Height (in.), h
3	6
1	2

The constant proportionality is 2.
An equation for this relationship is $h = 2w$.

Arms

Robot Width (in.), r	Arm Height (in.), a
9	5
1	$\frac{5}{9}$

The constant proportionality is $\frac{5}{9}$.
An equation for this relationship is $a = \frac{5}{9}r$.

Lesson Practice ACC7.3.04

Lesson Summary

Proportional relationships can be represented using the equation $y = kx$, where k is the constant of proportionality. For example, when you create a *scaled copy*, you multiply every length in the original figure by the same number, k.

The table shows the proportional relationship between the number of pounds of soybeans and the cost at a certain store.

- The cost of the soybeans is proportional to the weight with a constant of proportionality of 2.
- If c represents the cost and w represents the weight, then you can represent the proportional relationship with the equation $c = 2w$.

Weight (lb), w	Cost ($), c
$\frac{1}{2}$	1.00
1	2.00
2	4.00
w	$2w$

Lesson Practice
ACC7.3.04

Name: _____ Date: _____ Period: _____

1. The ceilings in many basements are made up of rectangular tiles. For one basement, each square meter of ceiling requires 10.75 tiles. Complete the table.

Area of Ceiling (sq. m)	Number of Tiles
1	10.75
10	107.5
5	53.75
x	$10.75x$

Problems 2–3: Each table represents a proportional relationship. Determine the constant of proportionality that completes each equation.

2.

s	P
2	8
3	12
5	20
10	40

$P =$ 4 s

3.

d	C
2	6.28
3	9.42
5	15.7
10	31.4

$C =$ 3.14 d

Problems 4–5: A plane flew at a constant speed between Denver and Chicago. It took the plane 1.5 hours to fly 915 miles.

4. Complete the table.

Time (hr)	Distance (mi)
1	610
1.5	915
2	1,220
2.5	1,525
t	$610t$

5. How far would the plane fly in 10 hours at this speed?

6,100 miles

Lesson Practice
ACC7.3.04

Name: _____ Date: _____ Period: _____

FAST Practice

Problems 6–8: While she was traveling in 2023, Mai received 342.50 Norwegian kroner in exchange for 50 Australian dollars.

6. How many kroner would Mai have received in exchange for 1 Australian dollar?

6.85 Norwegian kroner

7. Write an equation to represent the amount of kroner, k, received in exchange for a Australian dollars.

$k =$ $6.85a$

8. Determine the number of kroner Mai would receive in exchange for 120 Australian dollars.

822 Norwegian kroner

Spiral Review

Problems 9–10: A bicycle travels 21 meters in 3 seconds.

9. Complete the table.

Time (sec)	Distance (m)
3	21
$1\frac{1}{2}$	$10\frac{1}{2}$
$\frac{9}{10}$	$6\frac{3}{10}$

10. What is a constant of proportionality in this relationship?

7 or $\frac{1}{7}$ (or equivalent)

What does this represent in the situation?

Responses vary. It represents that the bicycle travels 7 meters every second.

Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1, 4–5	2	MA.7.AR.4.4
	2–3	1	MA.7.AR.4.2, MA.7.AR.4.4
FAST Practice	6–8	2	MA.7.AR.4.2 MA.7.AR.4.4
Spiral Review			
	9	2	MA.7.AR.4.1
	10	1	MA.7.AR.4.2

Unit 3
Lesson 5

Snapshots

Let's use equations to make sense of proportional relationships in the world.

Warm-Up

1. Write a story about this car's trip.

Snapshots

More Equations of Proportional Relationships

Let's use equations to make sense of proportional relationships in the world.

Learning Outcomes

Today's Goals

1. **Goal:** Connect each part of an equation of the form $y = kx$ to the situation it represents.
2. **Goal:** Create equations representing proportional relationships and use them to solve problems.
3. **Language Goal:** Interpret each part of an equation that represents a proportional relationship. **(Reading, Writing, Speaking, and Listening)**

Students develop fluency in writing and interpreting equations representing proportional relationships. Students explain the meaning of each part of an equation and solve problems in various contexts, moving between the abstract representation and concrete situation. **(MTR.2.1)**

Coherence

Prior Learning

In Lesson 4, students were introduced to equations with two variables and began writing equations to represent proportional relationships.

Future Learning

In Lesson 6, students will determine the two constants of proportionality for a proportional relationship and write two equations to represent the relationship.

Connections

- Students extend their understanding of linear equations to solve problems involving real world proportional relationships such as taking a trip, adjusting a recipe, and comparing different units of measure. (real world)

FL B.E.S.T. Math Benchmarks

Addressing

MA.7.AR.4.2
Determine the constant of proportionality within a mathematical or real-world context given a table, graph or written description of a proportional relationship.

MA.7.AR.4.4
Given any representation of a proportional relationship, translate the representation to a written description, table or equation.

Also Addressing: **MA.7.AR.4.5**

Mathematical Thinking and Reasoning Standards: MTR.2.1, MTR.7.1

ELA Expectations: ELA.K12.EE.1.1, ELA.K12.EE.3.1, ELA.K12.EE.6.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.6.AR.3.2	**MA.8.AR.3.2**
MA.6.AR.3.3	**MA.8.AR.3.3**
MA.6.AR.3.5	

Lesson at a Glance

~ 45 min

MA.7.AR.4.2, MA.7.AR.4.4, MA.7.AR.4.5

Warm-Up

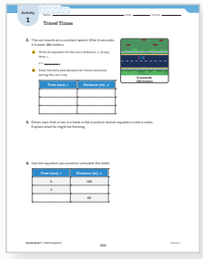
Pairs | 5 min

Students use the **Tell a Story** routine to explore how snapshots can provide information and invite questions about a situation.

Activity 1

Pairs | 10 min

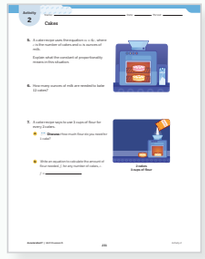
Students make connections between tables and equations and consider how each abstract representation relates to the situation.



Activity 2

Pairs | 10 min

Students practice using equations of the form $y = kx$ to reason about proportional relationships.

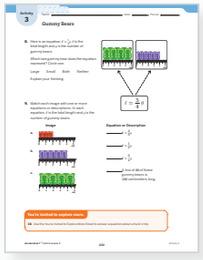


Activity 3

Pairs | 10 min

Students develop strategies to categorize descriptions, equations, and images that represent the same proportional relationship.

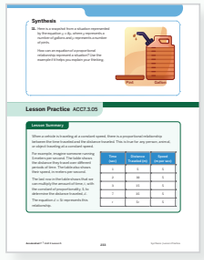
Routine: **Decide and Defend**



Synthesis

Whole Class | 5 min

Students synthesize their understanding of equations representing proportional relationships.



Show What You Know

Independent | 5 min

Students demonstrate their understanding by interpreting the meaning of one part of an equation in context.

Prep Checklist

Prepare Activity 3 sheets.

This lesson includes:

- Student Edition
- Show What You Know
- Activity 3 You're Invited to Explore More, one per student (as needed)

Pairs | 5 min

Warm-Up

Purpose: Students use the Tell a Story routine to explore how snapshots can provide information and invite questions about a situation.

1 Launch

To surface connections, consider asking students about their experience with taking trips in a car.

Invite students to share what they see in the snapshots.

Use the Tell a Story routine to help students make sense of the snapshots. Invite students to write a story about the images using their unique experiences and creativity. See the Routine Facilitation Guide for more information.

ELL **English Language Learners** Invite students to write stories in their primary language. **(Writing)**

Consider sharing that stories do not have to be complete or perfect to be valuable.

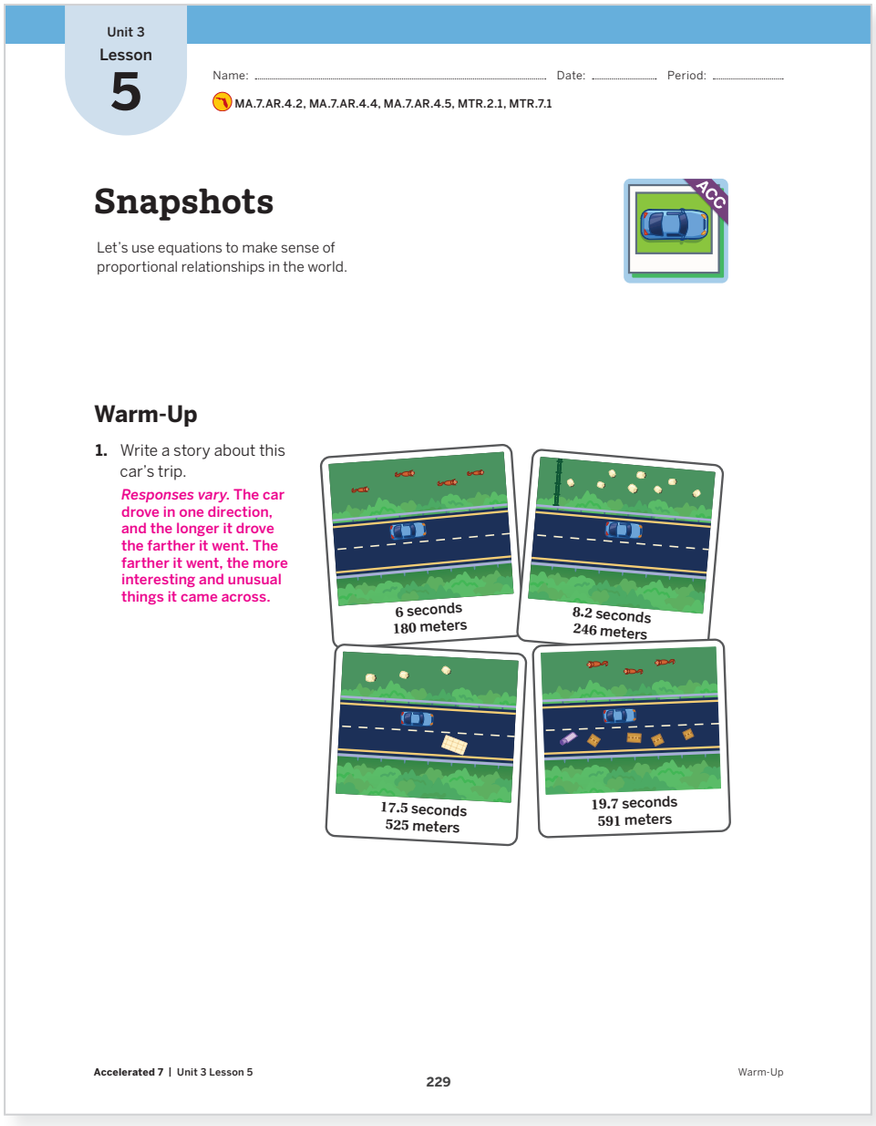
Listen for students who notice that the car is traveling at a constant speed. Consider inviting these students to share their thinking during Activity 1.

2 Connect

Invite students to share their stories.

Encourage students to ask questions about each other's stories. Consider asking:

- "What questions do you have about the car's trip after just looking at the pictures?"
- "Can we know the whole story from these snapshots? Why or why not?"



Unit 3
Lesson
5

Name: ..... Date: ..... Period: .....

MA.7.AR.4.2, MA.7.AR.4.4, MA.7.AR.4.5, MTR.2.1, MTR.7.1

Snapshots

Let's use equations to make sense of proportional relationships in the world.

ACC

Warm-Up

1. Write a story about this car's trip.

Responses vary. The car drove in one direction, and the longer it drove the farther it went. The farther it went, the more interesting and unusual things it came across.

Accelerated 7 | Unit 3 Lesson 5 229 Warm-Up

Pairs | 10 min

Activity 1 Travel Times

Purpose: Students make connections between tables and equations and consider how each abstract representation relates to the situation. **(MTR.2.1)**

1 Launch

Invite students to read the problem aloud and restate it in their own words.

Consider asking:

- "What does the phrase *constant speed* mean to you?"
- "How does knowing that the car has a constant speed help you write an equation?" **(MTR.7.1)**

Pause briefly once most students have written a correct equation and invite students to share their equations.

Note: Make sure that every student is able to write a correct equation and then complete the table. **(MTR.2.1)**

2 Monitor

Capture a variety of explanations, including some that are incomplete or unclear.

MLR **MLR1: Stronger and Clearer Each Time (optional)** Consider using this routine to support students in developing precise language. Invite students to meet with a partner, give feedback on each other's drafts, then use the feedback to write a second clearer draft. See the Routine Facilitation Guide for more information.

A **Accessibility: Conceptual Processing** Invite students to read their equation aloud with variable definitions instead of d and t (e.g., "The distance is equal to 30 times the number of seconds.").

Look for a variety of student thinking.

D **Differentiation**

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Encourage students to try substituting the given values into their equation.
Multiply a given distance by 30 to calculate a value for time.	**Support** Consider asking, "Which variable is multiplied by 30 in your equation?"
Scale rows in the table to find unknown values.	**Strengthen** Consider asking, "How can you check your work using the equation you wrote?"

3 Connect

Discuss the captured explanations.

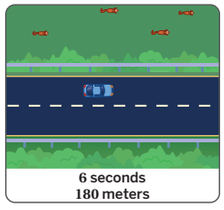
Consider creating an anchor chart to compare and contrast when a table or equation is a more useful representation.

Activity 1

Name: _____ Date: _____ Period: _____

Travel Times

2. The car travels at a constant speed. After 6 seconds, it travels 180 meters.



a. Write an equation for the car's distance, d, at any time, t.

$d =$ $30t$

b. Enter the time and distance for three moments during the car's trip.
Responses vary.

Time (sec), t	Distance (m), d
6	180
8.2	246
17.5	525

3. Ethan says that a row in a table is like a picture and an equation is like a video. Explain what he might be thinking.
Responses vary.

- A row in a table is just one pair of values in a relationship and an equation can tell you about all possible pairs of values.
- A row in a table is a single moment. An equation can provide all of the moments.

4. Use the equation you wrote to complete the table.

Time (sec), t	Distance (m), d
6	180
3	90
2	60

Accelerated 7 | Unit 3 Lesson 5 230 Activity 1

Key Takeaway: You can use the equation representing a relationship to predict any pair of values in that relationship.

Pairs | 10 min

Activity 2 Cakes

Purpose: Students practice using equations of the form $y = kx$ to reason about proportional relationships.

1 Launch

To surface connections, consider asking students about their experience with baking cakes.

Discuss each part of the equation $m = 6c$.

Note: Students may interpret this equation to mean that 1 ounce of milk makes 6 cakes. Consider rewriting the equation as $m = 6 \cdot c$ to emphasize that c is a quantity being multiplied, not a label of units.

2 Monitor

Look for students who write the expression $6 \cdot 12$.

ELL **English Language Learners** Encourage students to discuss with their partners. Provide sentence frames to help students explain their strategy (e.g., "I know that _____, so I _____."). **(Speaking)**

Listen for a variety of student thinking.

D Differentiation

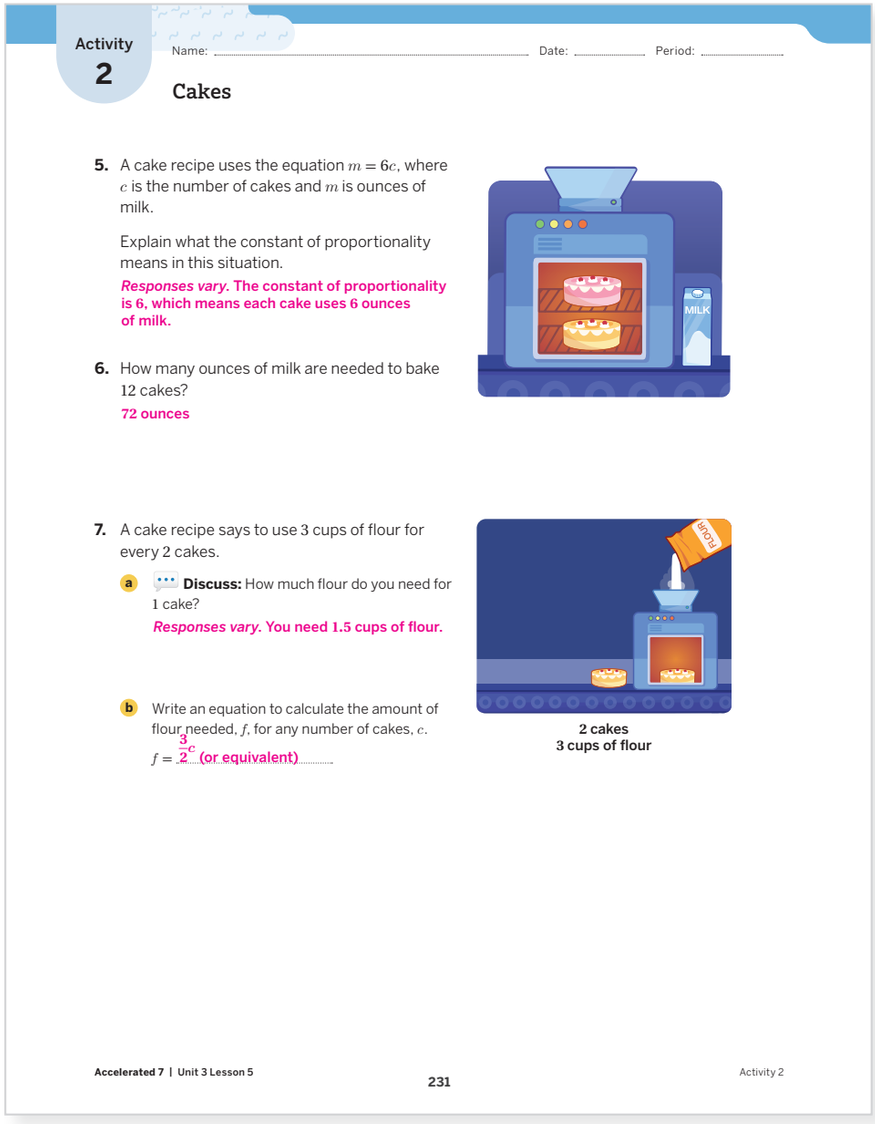
Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Encourage students to make a table of values to help them find the constant of proportionality.
Write $3f = 2c$.	**Support** Encourage students to make a table of values and use substitution in their equation to check their work.
Write $f = \frac{2}{3}c$.	**Support** Encourage students to write an equivalent ratio using the quantities in the original recipe to check whether their equation is correct.
Write $f = 1.5c$ or equivalent.	**Strengthen** Consider asking, "How could you use this equation to find the amount of flour for any number of cakes?" **(MTR.7.1)**

Capture student equations to share during the Connect.

3 Connect

Display the captured equations and invite students to explain what each part of the equation represents.

To surface the Key Takeaway, consider asking, "How did you find the constant of proportionality in this situation?"



Activity 2

Name: _____ Date: _____ Period: _____

Cakes

5. A cake recipe uses the equation $m = 6c$, where c is the number of cakes and m is ounces of milk.

Explain what the constant of proportionality means in this situation.

Responses vary. The constant of proportionality is 6, which means each cake uses 6 ounces of milk.

6. How many ounces of milk are needed to bake 12 cakes?

72 ounces

7. A cake recipe says to use 3 cups of flour for every 2 cakes.

a. **Discuss:** How much flour do you need for 1 cake?

Responses vary. You need 1.5 cups of flour.

b. Write an equation to calculate the amount of flour needed, f, for any number of cakes, c.

$f = \frac{3}{2}c$ (or equivalent)

2 cakes
3 cups of flour

Accelerated 7 | Unit 3 Lesson 5 231 Activity 2

Key Takeaway: In an equation of the form $y = kx$, k is the constant of proportionality. To find the constant of proportionality, you can divide any x-value and the y-value it is paired with.

Pairs | 10 min

Activity 3 Gummy Bears

Purpose: Students develop strategies to categorize descriptions, equations, and images that represent the same proportional relationship.

Short on time? Consider omitting Problem 9 and discussing Problem 8 during the Connect.

1 Launch

Use the Decide and Defend routine to support students in making arguments and critiquing the reasoning of others.

Invite students to share their responses.

Consider asking, "What does $\frac{5}{4}$ mean in this situation?"

2 Monitor

Encourage students to share their reasoning with a partner and work together to reach an agreement about how to match the images with the equations and description.

Look for a variety of student thinking.

D Differentiation

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Encourage students to identify the constant of proportionality in one equation. Consider asking, "Which image has the same constant of proportionality?"
Think that the equation $l = \frac{4}{5}g$ means that 4 gummy bears will have a length of 5 centimeters.	**Support** Encourage students to check their work by substituting those values into the equation.
Interpret the constant of proportionality as the length of one gummy bear.	**Stretch** Consider asking, "How did you decide where to group the description?" Invite students to create their own descriptions for groups without one.

D Differentiation: You're invited to explore more. (Stretch) Invite students who would like to explore proportional relationships further to complete this optional task. Encourage them to discuss their ideas with a partner.
Note: Students will need copies of the You're Invited to Explore More Sheet.

3 Connect

Invite students to share their matches. For incorrect matches, discuss why someone might make a match in this way.

Activity 3

Name: _____ Date: _____ Period: _____

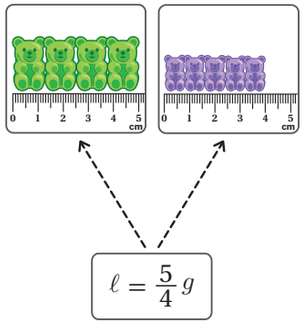
Gummy Bears

8. Here is an equation: $\ell = \frac{5}{4}g$. ℓ is the total length and g is the number of gummy bears.

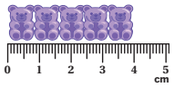
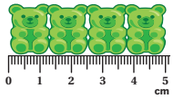
Which size gummy bear does the equation represent? Circle one.

(Large) Small Both Neither

Explain your thinking. *Explanations vary. The equation represents a situation where every gummy bear is $\frac{5}{4}$ inches long. The large gummy bear looks to be that length.*



9. Match each image with one or more equations or descriptions. In each equation, ℓ is the total length and g is the number of gummy bears.

Image	Equation or Description
a.	b $\ell = \frac{4}{5}g$
b. 	a $\ell = \frac{1}{2}g$
c. 	c $\ell = \frac{5}{4}g$
	c A line of 80 of these gummy bears is 100 centimeters long.

You're invited to explore more.

10. Use the You're Invited to Explore More Sheet to answer a question about a truck's trip.

Whole Class | 5 min

Synthesis

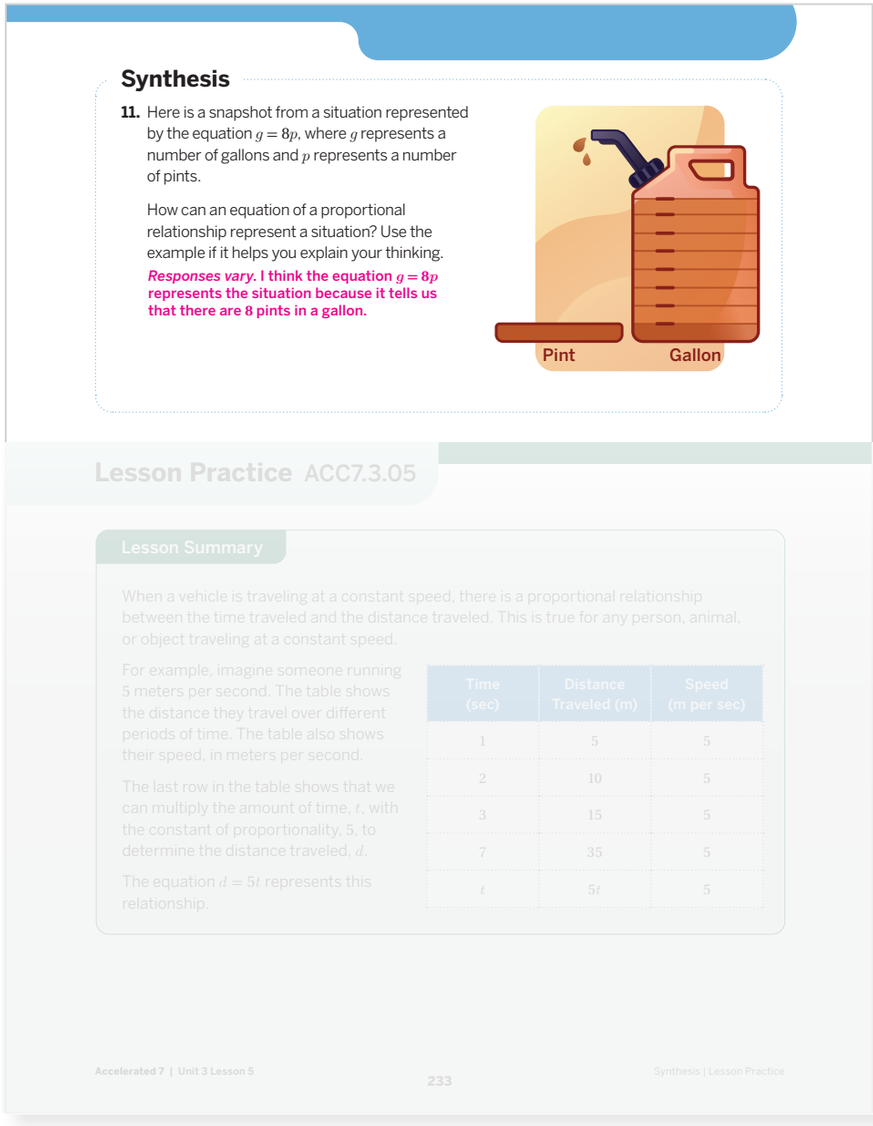
Purpose: Students synthesize their understanding of equations representing proportional relationships.

Invite students to respond independently, and then share their thinking with a partner.

Collect and share a variety of ideas, such as:

- Variables represent the quantities in the relationship.
- Recognizing the number in the equation is the constant of proportionality and multiplying it by one variable to find the value of the other variable.
- Using the equation to find points from the relationship and creating a table.

Note: If time allows, invite students to revise their previous thinking based on the discussion.



Synthesis

11. Here is a snapshot from a situation represented by the equation $g = 8p$, where g represents a number of gallons and p represents a number of pints.

How can an equation of a proportional relationship represent a situation? Use the example if it helps you explain your thinking.

Responses vary. **I think the equation $g = 8p$ represents the situation because it tells us that there are 8 pints in a gallon.**

Lesson Practice ACC7.3.05

Lesson Summary

When a vehicle is traveling at a constant speed, there is a proportional relationship between the time traveled and the distance traveled. This is true for any person, animal, or object traveling at a constant speed.

For example, imagine someone running 5 meters per second. The table shows the distance they travel over different periods of time. The table also shows their speed, in meters per second.

Time (sec)	Distance Traveled (m)	Speed (m per sec)
1	5	5
2	10	5
3	15	5
7	35	5
t	$5t$	5

The last row in the table shows that we can multiply the amount of time, t, with the constant of proportionality, 5, to determine the distance traveled, d.

The equation $d = 5t$ represents this relationship.

Accelerated 7 | Unit 3 Lesson 5 233 Synthesis | Lesson Practice

Lesson Takeaway: Tables and equations are both tools for making sense of proportional relationships in different situations. You can use an equation of the form $y = kx$ to create a table or solve problems once you make sense of what the pieces of the equation mean.

Independent | 5 min

Show What You Know

Purpose: Students demonstrate their understanding by interpreting the meaning of one part of an equation in context.

Today's Goals

1. **Goal:** Connect each part of an equation of the form $y = kx$ to the situation it represents.
2. **Goal:** Create equations representing proportional relationships and use them to solve problems.
3. **Language Goal:** Interpret each part of an equation that represents a proportional relationship. **(Reading, Writing, Speaking, and Listening)**

Show What You Know (Assessment Resources)

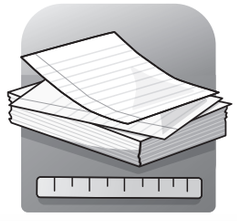
Name: _____ Date: _____ Period: _____

Show What You Know 3.05

The equation $h = 2.5r$ represents the height in inches, h, of r reams of paper.

What does 2.5 mean in this situation?

Responses vary. 2.5 inches is how tall one ream of paper is.



D Differentiation Use after Lesson 5

Support

Provide targeted intervention for students by using these resources.

If students need support understanding how to interpret the parts of an equation of the form $y = kx$:

» **Respond to Student Thinking:**

- Encourage students to practice reading each equation using variable definitions (e.g., read $m = 6c$ as "The number of ounces of milk is 6 times the number of cakes.").

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 2 Extension Activities**.
- Invite students to complete the **Explore More** task and discuss their thinking with a partner.

Professional Learning

In this lesson, students continue developing their understanding of equations with two variables. How does this model integrate with other representations of proportional relationships? How did you see students making sense of the connections between equations, tables, and situations?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

11. Here is a snapshot from a situation represented by the equation $g = 8p$, where g represents a number of gallons and p represents a number of pints.

How can an equation of a proportional relationship represent a situation? Use the example if it helps you explain your thinking.

Responses vary. I think the equation $g = 8p$ represents the situation because it tells us that there are 8 pints in a gallon.

Lesson Practice ACC7.3.05

Lesson Summary

When a vehicle is traveling at a constant speed, there is a proportional relationship between the time traveled and the distance traveled. This is true for any person, animal, or object traveling at a constant speed.

For example, imagine someone running 5 meters per second. The table shows the distance they travel over different periods of time. The table also shows their speed, in meters per second.

Time (sec)	Distance Traveled (m)	Speed (m per sec)
1	5	5
2	10	5
3	15	5
7	35	5
t	$5t$	5

The last row in the table shows that we can multiply the amount of time, t, with the constant of proportionality, 5, to determine the distance traveled, d.

The equation $d = 5t$ represents this relationship.

Lesson Practice
ACC7.3.05

Name: _____ Date: _____ Period: _____

Problems 1–3: A performer expects to sell 5,000 tickets for an upcoming concert. He plans to make $311,000 from the sales of these tickets. Assume that all tickets have the same price.

1. What is the price for one ticket? Show or explain your thinking.
$62.20. *Explanations vary.* I divided $311,000 by 5,000 to determine the price for one ticket.

2. Write an equation to represent the relationship between the number of tickets sold, x, and the total amount of money (in dollars) that he makes, y.
$y = 62.2x$

3. If he makes $379,420, how many tickets has he sold?
6,100 tickets

Problems 4–5: A car is traveling on a highway at a constant speed. The equation that represents the distance traveled in miles, d, for t hours is $d = 65t$.

4. What does the value 65 represent in this situation?
***Responses vary.* The value 65 is the speed, or constant of proportionality. The car travels 65 miles every hour.**

5. At this rate, how many miles will the car travel in 1.5 hours? Show your thinking.
97.5 miles. *Work varies.*
$d = 65t$
$d = 65(1.5)$
$d = 97.5$

Problems 6–7: On its way from New York to San Diego, a plane flew at a constant speed over Pittsburgh, Saint Louis, Albuquerque, and Phoenix.

6. This table shows the flight time and distance traveled for each segment of the flight. Complete the table.

Segment	Time (hr)	Distance (mi)	Speed (mph)
Pittsburgh to Saint Louis	1	550	**550**
Saint Louis to Albuquerque	1.7	**935**	**550**
Albuquerque to Phoenix	**0.6**	330	**550**

7. Let t represent the time in hours and d represent the distance in miles. Write an equation that represents the distance traveled for t hours.
$d = 550t$ (or equivalent)

Lesson Practice
ACC7.3.05

Name: _____ Date: _____ Period: _____

8. Na'ilah is making a pitcher of a lemon-flavored sports drink. The drink mix container says to mix $\frac{1}{4}$ cups of powdered drink mix with 2 quarts of water. She prefers her sports drink to taste *more* lemon-y than the recipe on the container. Complete the equation to represent a mixture of cups of drink mix, c, and quarts of water, w, that would be *more* lemon-y than the original mixture.
Responses where the constant of proportionality is greater than $\frac{1}{8}$ are considered correct.
$c =$ **$\frac{1}{5}$** w

FAST Practice

9. A train travels at a constant speed between Springfield and Chicago. The train travels $100\frac{1}{2}$ miles in $\frac{3}{4}$ hours. Complete the equation to represent the distance, d, that the train travels at any time, t.

$d =$ **134** t

Spiral Review

10. Select *all* the tables that represent a proportional relationship between x and y.

☑ **A.**

x	0	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{16}$
y	0	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$

☐ **B.**

x	0	$\frac{1}{2}$	2	3
y	0	$\frac{1}{4}$	4	9

☑ **C.**

x	0	4	8	12
y	0	2	4	6

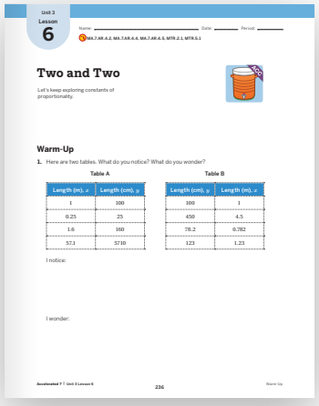
☑ **D.**

x	0	1	2	3
y	0	4	8	12

11. Is 4.5 a solution to the equation $1.5 + x = 6$?
Yes

Lesson Practice Item Analysis

Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson		
1, 4	1	MA.7.AR.4.2
2, 7	2	MA.7.AR.4.4
3	2	MA.7.AR.4.5
5	2	MA.7.AR.4.4, MA.7.AR.4.5
6	2	MA.7.AR.4.2
8	3	MA.7.AR.4.4, MA.7.AR.4.5
FAST Practice **9**	3	MA.7.AR.4.2, MA.7.AR.4.4
Spiral Review		
10	2	MA.7.AR.4.1
11	1	MA.6.AR.2.1


Unit 3
Lesson 6

Two and Two

Let's keep exploring constants of proportionality.

Warm-Up

1. Here are two tables. What do you notice? What do you wonder?

Table A

Length (m), x	Length (cm), y
1	100
0.25	25
1.6	160
57.1	5710

Table B

Length (cm), y	Length (m), x
100	1
450	4.5
78.2	0.782
123	1.23

I notice:

I wonder:

Accelerated 7 | Unit 3 Lesson 6 236 Warm-Up

Two and Two

Two Equations for Each Relationship

Let's keep exploring constants of proportionality.

Learning Outcomes

Today's Goals

1. **Goal:** Write two equations that represent the same proportional relationship, such as $y = kx$ and $x = \frac{1}{k}y$.
2. **Language Goal:** Explain the relationship between the two constants of proportionality in a proportional relationship using terms such as *reciprocal*. **(Reading and Writing)**

Students determine the two constants of proportionality for a proportional relationship and discover they are **reciprocals**. They interpret these constants of proportionality in context and use them to write two equations that represent each relationship. **(MTR.2.1)**

Coherence

Prior Learning

In a prior course, students discovered reciprocals. In Lesson 5, students developed fluency in writing and using equations to make sense of proportional relationships in various contexts.

Future Learning

In Lesson 8, students will represent proportional relationships graphically.

Connections

- Students apply linear equations to real-world proportional relationships such as the amount of time needed to fill a cooler based on the rate of water flow. (real world)

FL B.E.S.T. Math Benchmarks

Addressing

MA.7.AR.4.2
Determine the constant of proportionality within a mathematical or real-world context given a table, graph or written description of a proportional relationship.

MA.7.AR.4.4
Given any representation of a proportional relationship, translate the representation to a written description, table or equation.

Also Addressing: **MA.7.AR.4.5**

Mathematical Thinking and Reasoning Standards: MTR.2.1, MTR.5.1

ELA Expectations: ELA.K12.EE.1.1, ELA.K12.EE.3.1, ELA.K12.EE.6.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.6.AR.3.2	**MA.8.AR.3.2**
MA.6.AR.3.3	**MA.8.AR.3.3**
MA.6.AR.3.5	

Vocabulary

New Vocabulary

reciprocal

Lesson at a Glance

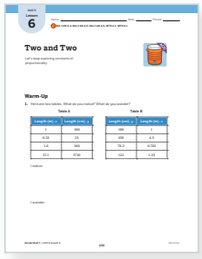
~ 45 min

MA.7.AR.4.2, MA.7.AR.4.4, MA.7.AR.4.5

Warm-Up

Whole Class | 5 min

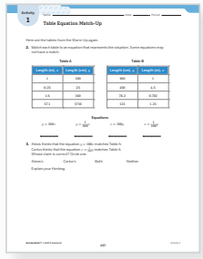
Students use the **Notice and Wonder** routine to compare two tables representing the relationship between meters and centimeters.



Activity 1

Pairs | 10 min

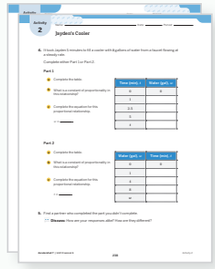
Students make connections between tables and equations representing the same proportional relationship. **Routine: Decide and Defend (MTR.2.1)**



Activity 2

Pairs | 20 min

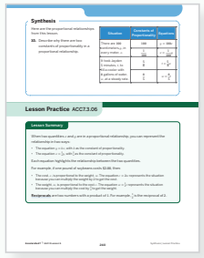
Students determine the two constants of proportionality for a relationship and discuss the concept of **reciprocals** to enrich their understanding.



Synthesis

Whole Class | 5 min

Students synthesize their understanding of why there are two constants of proportionality in a proportional relationship.



Show What You Know

Independent | 5 min

Students demonstrate their understanding by writing two equations to represent the same proportional relationship.

Prep Checklist

This lesson includes:

- Student Edition
- Show What You Know

Whole Class | 5 min

Warm-Up

Purpose: Students use the Notice and Wonder routine to compare two tables representing the relationship between meters and centimeters.

1 Launch

Direct students' attention to the two tables.

Use the Notice and Wonder routine to promote curiosity and help students make sense of the relationship between the two tables.

2 Connect

Invite students to share what they noticed and wondered.

ELL **English Language Learners** Encourage students to paraphrase each other's ideas to help them make connections and incorporate vocabulary from the unit. **(Speaking and Listening)**

Consider asking, "Did anyone notice that the column labels on the tables are the same, but reversed? Does this make you wonder about whether we could combine these tables?" **(MTR.5.1)**

Note: If it doesn't come up naturally, consider sharing that both tables show the relationship between meters and centimeters.

Invite students to use gestures such as holding out their arms and pinching their fingers to demonstrate the approximate length of 1 meter and 1 centimeter.

Unit 3
Lesson
6

Name: _____ Date: _____ Period: _____

MA.7.AR.4.2, MA.7.AR.4.4, MA.7.AR.4.5, MTR.2.1, MTR.5.1

Two and Two

Let's keep exploring constants of proportionality.

Warm-Up

1. Here are two tables. What do you notice? What do you wonder?

Table A

Length (m), x	Length (cm), y
1	100
0.25	25
1.6	160
57.1	5710

Table B

Length (cm), y	Length (m), x
100	1
450	4.5
78.2	0.782
123	1.23

I notice:

Responses vary.
- Both tables are about meters and centimeters, but the order of the columns is switched around.
- Both tables have a row that matches up 1 m to 100 cm.
- In Table A, the larger values are on the right, while in Table B, the larger values are on the left.
- The tables are proportional.

I wonder:

Responses vary.
- Could the information in these two tables be combined into just one table?
- Are there advantages to organizing the information about length in one of these ways compared to the other?

Accelerated 7 | Unit 3 Lesson 6 236 Warm-Up

Pairs | 10 min

Activity 1 Table Equation Match-Up

Purpose: Students make connections between tables and equations representing the same proportional relationship. **(MTR.2.1)**

Short on time? Problem 2 may be omitted.

1 Launch

Direct students' attention to the equations.

To surface connections, consider asking, "How are these equations alike? How are they different?" **(MTR.2.1)**

2 Monitor

Listen for a variety of student thinking.

D Differentiation *(Problem 2)*

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Encourage them to look for a constant of proportionality in each table by checking for a multiplier from one column to the other.
Determine that 100 is a constant of proportionality in Table A because it is paired with the value 1.	**Strengthen** Consider asking, "How did you decide whether $y = 100x$ or $x = 100y$ best represents the relationship you noticed?"
Substitute values from the tables into each equation to determine a match.	**Strengthen** Celebrate this strategy. Consider asking, "What is the constant of proportionality in each table? Where do you see it in the equations?"

A Accessibility: Executive Functioning Consider chunking this activity by inviting students to choose one table at a time and look for an equation to match it.

3 Connect

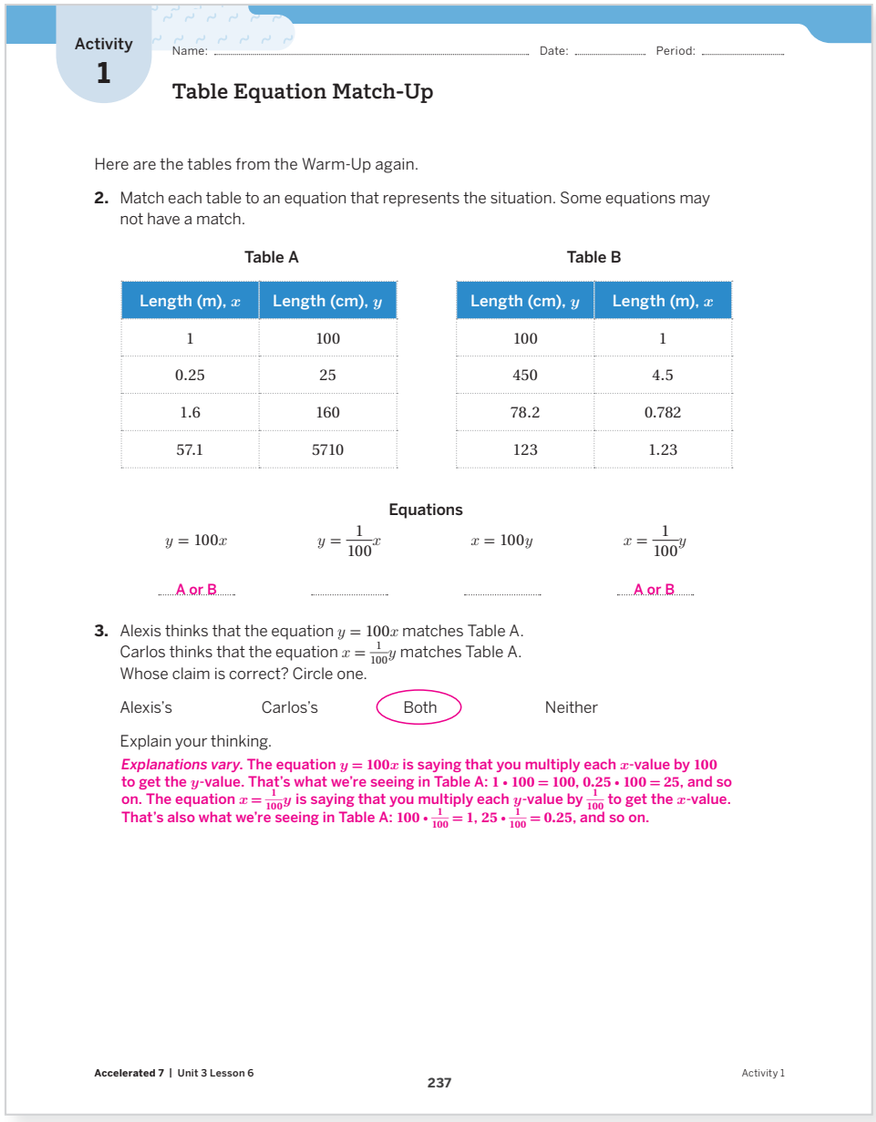
Use the Decide and Defend routine to support students in making arguments and critiquing the reasoning of others.

Display the distribution of responses by polling the class.

Invite students who made different choices to use reasoning to convince their classmates.

Consider asking, "What evidence or language was most convincing to you?"

To surface the Key Takeaway, consider asking:
- "What is the same about the two equations for Table A?"
- "What is different?" **(MTR.2.1)**



Activity 1

Name: Date: Period:

Table Equation Match-Up

Here are the tables from the Warm-Up again.

2. Match each table to an equation that represents the situation. Some equations may not have a match.

Table A

Length (m), x	Length (cm), y
1	100
0.25	25
1.6	160
57.1	5710

Table B

Length (cm), y	Length (m), x
100	1
450	4.5
78.2	0.782
123	1.23

Equations

$y = 100x$ — A or B

$y = \frac{1}{100}x$

$x = 100y$

$x = \frac{1}{100}y$ — A or B

3. Alexis thinks that the equation $y = 100x$ matches Table A. Carlos thinks that the equation $x = \frac{1}{100}y$ matches Table A. Whose claim is correct? Circle one.

Alexis's Carlos's Both Neither

Explain your thinking.

Explanations vary. The equation $y = 100x$ is saying that you multiply each x-value by 100 to get the y-value. That's what we're seeing in Table A: $1 \cdot 100 = 100$, $0.25 \cdot 100 = 25$, and so on. The equation $x = \frac{1}{100}y$ is saying that you multiply each y-value by $\frac{1}{100}$ to get the x-value. That's also what we're seeing in Table A: $100 \cdot \frac{1}{100} = 1$, $25 \cdot \frac{1}{100} = 0.25$, and so on.

Accelerated 7 | Unit 3 Lesson 6 237 Activity 1

Key Takeaway: You can write two equations to represent the same proportional relationship. The equations use the same variables, but they have different constants of proportionality.

Pairs | 20 min

Activity 2 Jayden's Cooler

Purpose: Students determine the two constants of proportionality for a relationship and discuss the concept of **reciprocals** to enrich their understanding.

1 Launch

Consider sharing the context for the activity and inviting students to use gestures to show the approximate size of a gallon of water.

ELL **English Language Learners** Display the image of the cooler being filled to increase access to the task. **(Reading and Listening)**

Direct students' attention to the instructions and invite students to complete Part 1 or Part 2. Encourage students to discuss their thinking with a partner completing the same part.

Note: Later in this activity, students will pair up with a classmate who completed the other part. Consider ensuring that about half of the class completes each part.

2 Monitor

Look for a variety of student thinking.

D **Differentiation** *(Problem 4)*

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Encourage students to restate the information they know about Jayden, then look for a row in the table they can fill in.
Need support determining a constant of proportionality.	**Support** Invite students to reflect on strategies they have previously used to determine constants of proportionality.
Need support writing an equation.	**Strengthen** Consider asking, "Where do you see the constant of proportionality in this table?"
Finish Problem 4 early.	**Stretch** Invite students to complete the other part in preparation for actively listening to and comparing with a partner.

Direct students' attention to the discussion question.

Invite students to find a partner who completed the other part and complete Problems 5–8 together.

Activity 2 continued >

Activity 2

Name: _____ Date: _____ Period: _____

Jayden's Cooler

4. It took Jayden 5 minutes to fill a cooler with 8 gallons of water from a faucet flowing at a steady rate.

Complete either Part 1 or Part 2.

Part 1

a. Complete the table.

b. What is a constant of proportionality in this relationship?

$\frac{8}{5}$

c. Complete the equation for this proportional relationship.

$w = \frac{8}{5}t$

Time (min), t	Water (gal), w
0	0
1	$\frac{8}{5}$
2.5	4
5	8
t	$\frac{8}{5}t$

Part 2

a. Complete the table.

b. What is a constant of proportionality in this relationship?

$\frac{5}{8}$

c. Complete the equation for this proportional relationship.

$t = \frac{5}{8}w$

Water (gal), w	Time (min), t
0	0
1	$\frac{5}{8}$
4	2.5
8	5
w	$\frac{5}{8}w$

5. Find a partner who completed the part you *didn't* complete.

Discuss: How are your responses alike? How are they different?

Responses vary. Our tables are both proportional and have two pairs of values that are the same: $t = 2.5$ and $w = 4$, $t = 5$ and $w = 8$. Our constants of proportionality are not identical but they are related: the numerator and denominator are just switched. The equations have the same structure but different constants of proportionality, and the variables are switched.

Accelerated 7 | Unit 3 Lesson 6 238 Activity 2

Pairs | 20 min

Activity 2 Jayden's Cooler (continued)

Purpose: Students determine the two constants of proportionality for a relationship and discuss the concept of **reciprocals** to enrich their understanding.

2 Monitor

Collect a variety of language and ideas students use to describe the relationship between the two constants of proportionality, such as:

- They have the same numbers.
- The numerator and denominator are switched.
- They multiply to make 1.
- They are *reciprocals*.

Listen for and celebrate the different ways that students explain what each constant of proportionality represents in context.

D **Differentiation: You're invited to explore more. (Stretch)** Invite students who would like to explore constants of proportionality further to complete this optional task. Encourage them to discuss their thinking with a partner.

3 Connect

Direct students' attention to Problem 4 and invite students to share the constants of proportionality they generated for each table.

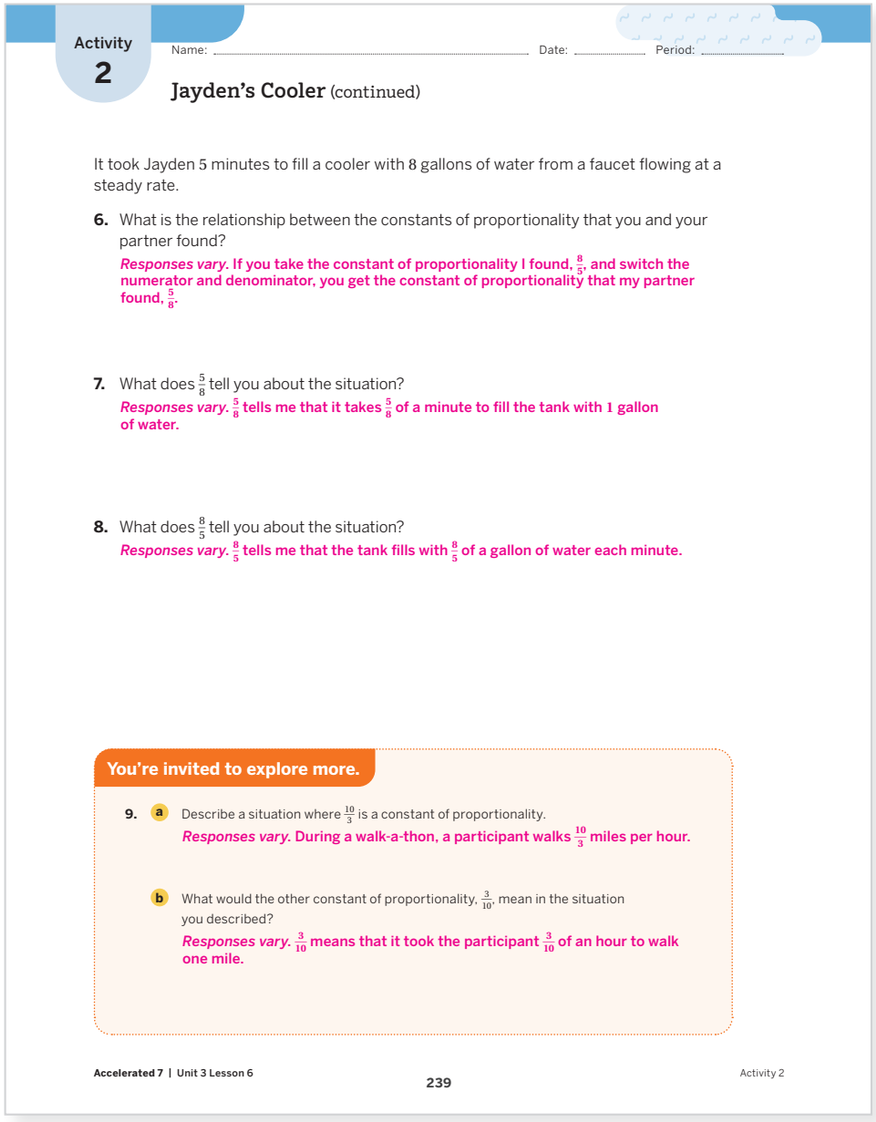
Share student examples and invite students to discuss their work further.

Consider asking:

- "How do you see the relationship between the two constants of proportionality?"
- "Where do you see the constant of proportionality in the table? In the equation?"
- "What does each constant of proportionality tell us about the situation?" **(MTR.2.1)**

Share that the relationship between the two constants of proportionality can be described using the term *reciprocals*.

Create a class definition of the term **reciprocal** using students' language, examples of equations, and the glossary. Record it for students to refer to later in the unit.



Activity 2

Name: _____ Date: _____ Period: _____

Jayden's Cooler (continued)

It took Jayden 5 minutes to fill a cooler with 8 gallons of water from a faucet flowing at a steady rate.

6. What is the relationship between the constants of proportionality that you and your partner found?

Responses vary. If you take the constant of proportionality I found, $\frac{8}{5}$, and switch the numerator and denominator, you get the constant of proportionality that my partner found, $\frac{5}{8}$.

7. What does $\frac{5}{8}$ tell you about the situation?

Responses vary. $\frac{5}{8}$ tells me that it takes $\frac{5}{8}$ of a minute to fill the tank with 1 gallon of water.

8. What does $\frac{8}{5}$ tell you about the situation?

Responses vary. $\frac{8}{5}$ tells me that the tank fills with $\frac{8}{5}$ of a gallon of water each minute.

You're invited to explore more.

9. a. Describe a situation where $\frac{10}{3}$ is a constant of proportionality.

Responses vary. During a walk-a-thon, a participant walks $\frac{10}{3}$ miles per hour.

b. What would the other constant of proportionality, $\frac{3}{10}$, mean in the situation you described?

Responses vary. $\frac{3}{10}$ means that it took the participant $\frac{3}{10}$ of an hour to walk one mile.

Accelerated 7 | Unit 3 Lesson 6 239 Activity 2

Key Takeaway: For every proportional relationship, there are two constants of proportionality that are **reciprocals** of each other.

Whole Class | 5 min

Synthesis

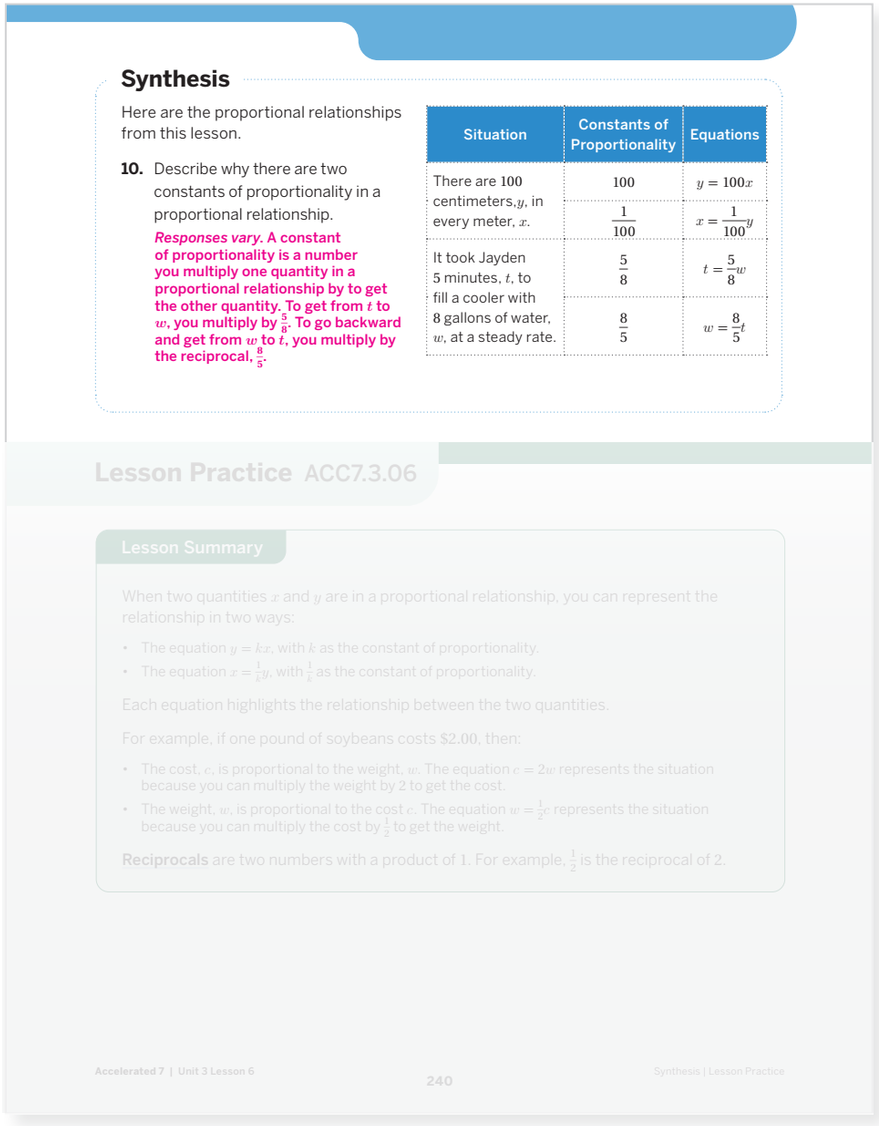
Purpose: Students synthesize their understanding of why there are two constants of proportionality in a proportional relationship.

Invite students to respond independently, and then share their thinking with a partner.

Collect and share a variety of ideas, such as:

- The constant of proportionality tells you what to multiply to get from one quantity to another. There are two because you need to go back and forth between two quantities.
- The two constants of proportionality are like the two scale factors between scaled copies.
- In Jayden's example, the constants of proportionality represent how much water he used per minute or how many minutes it took to add one gallon of water to the cooler.
- In the measurement example, it is possible to convert from centimeters to meters or from meters to centimeters.

Consider asking, "If you knew one constant of proportionality, how might you figure out the other?"



Synthesis

Here are the proportional relationships from this lesson.

10. Describe why there are two constants of proportionality in a proportional relationship.

Responses vary. A constant of proportionality is a number you multiply one quantity in a proportional relationship by to get the other quantity. To get from t to w, you multiply by $\frac{5}{8}$. To go backward and get from w to t, you multiply by the reciprocal, $\frac{8}{5}$.

Situation	Constants of Proportionality	Equations
There are 100 centimeters, y, in every meter, x.	100	$y = 100x$
	$\frac{1}{100}$	$x = \frac{1}{100}y$
It took Jayden 5 minutes, t, to fill a cooler with 8 gallons of water, w, at a steady rate.	$\frac{5}{8}$	$t = \frac{5}{8}w$
	$\frac{8}{5}$	$w = \frac{8}{5}t$

Lesson Practice ACC7.3.06

Lesson Summary

When two quantities x and y are in a proportional relationship, you can represent the relationship in two ways:

- The equation $y = kx$, with k as the constant of proportionality.
- The equation $x = \frac{1}{k}y$, with $\frac{1}{k}$ as the constant of proportionality.

Each equation highlights the relationship between the two quantities.

For example, if one pound of soybeans costs $2.00, then:

- The cost, c, is proportional to the weight, w. The equation $c = 2w$ represents the situation because you can multiply the weight by 2 to get the cost.
- The weight, w, is proportional to the cost c. The equation $w = \frac{1}{2}c$ represents the situation because you can multiply the cost by $\frac{1}{2}$ to get the weight.

Reciprocals are two numbers with a product of 1. For example, $\frac{1}{2}$ is the reciprocal of 2.

Accelerated 7 | Unit 3 Lesson 6 240 Synthesis | Lesson Practice

Lesson Takeaway: For every proportional relationship, there are two constants of proportionality that are **reciprocals** of each other. Two equations can be written to represent each proportional relationship.

Independent | 5 min

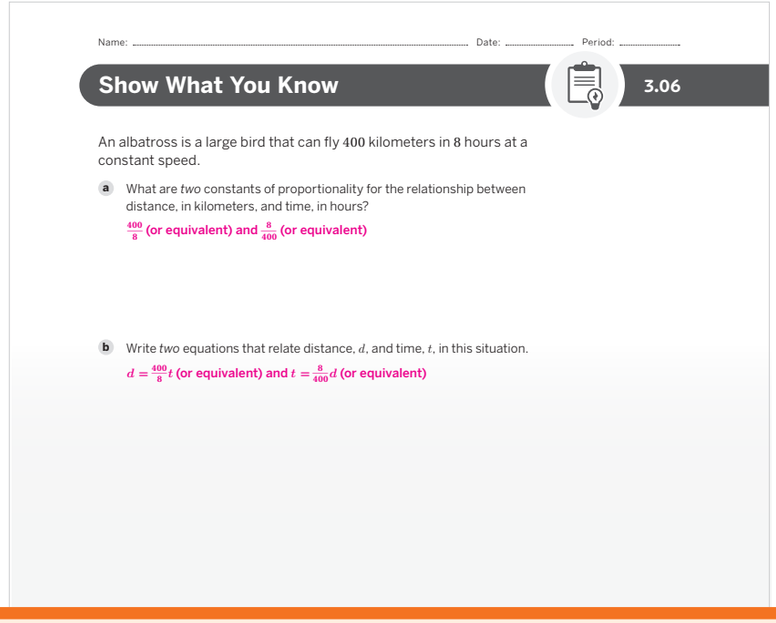
Show What You Know

Purpose: Students demonstrate their understanding by writing two equations to represent the same proportional relationship.

Today's Goals

1. **Goal:** Write two equations that represent the same proportional relationship, such as $y = kx$ and $x = \frac{1}{k}y$.
2. **Language Goal:** Explain the relationship between the two constants of proportionality in a proportional relationship using terms such as *reciprocal*. **(Reading and Writing)**

Show What You Know (Assessment Resources)



Name: _____ Date: _____ Period: _____

Show What You Know 3.06

An albatross is a large bird that can fly 400 kilometers in 8 hours at a constant speed.

a. What are *two* constants of proportionality for the relationship between distance, in kilometers, and time, in hours?

$\frac{400}{8}$ (or equivalent) and $\frac{8}{400}$ (or equivalent)

b. Write *two* equations that relate distance, d, and time, t, in this situation.

$d = \frac{400}{8}t$ (or equivalent) and $t = \frac{8}{400}d$ (or equivalent)

D Differentiation Use after Lesson 6

Support

Provide targeted intervention for students by using these resources.

If students need support understanding how to identify both constants of proportionality in a relationship:

» **Respond to Student Thinking:**

- Use the ***Finding More Than One Equation to Represent Proportional Relationships*** **Mini-Lesson.**

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 2 Extension Activities**.
- Invite students to complete the **Explore More** task and discuss their thinking with a partner

Professional Learning

Take a moment to reflect on students' work in pairs today. What trends do you see in participation? How might you invite students to celebrate each other's brilliance as they work with partners in the future?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

Here are the proportional relationships from this lesson.

Situation	Constants of Proportionality	Equations
There are 100 centimeters, y, in every meter, x.	100	$y = 100x$
	$\frac{1}{100}$	$x = \frac{1}{100}y$
It took Jayden 5 minutes, t, to fill a cooler with 8 gallons of water, w, at a steady rate.	$\frac{5}{8}$	$t = \frac{5}{8}w$
	$\frac{8}{5}$	$w = \frac{8}{5}t$

10. Describe why there are two constants of proportionality in a proportional relationship.

Responses vary. A constant of proportionality is a number you multiply one quantity in a proportional relationship by to get the other quantity. To get from t to w, you multiply by $\frac{5}{8}$. To go backward and get from w to t, you multiply by the reciprocal, $\frac{8}{5}$.

Lesson Practice ACC7.3.06

Lesson Summary

When two quantities x and y are in a proportional relationship, you can represent the relationship in two ways:

- The equation $y = kx$, with k as the constant of proportionality.
- The equation $x = \frac{1}{k}y$, with $\frac{1}{k}$ as the constant of proportionality.

Each equation highlights the relationship between the two quantities.

For example, if one pound of soybeans costs $2.00, then:

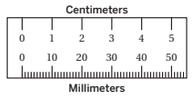
- The cost, c, is proportional to the weight, w. The equation $c = 2w$ represents the situation because you can multiply the weight by 2 to get the cost.
- The weight, w, is proportional to the cost c. The equation $w = \frac{1}{2}c$ represents the situation because you can multiply the cost by $\frac{1}{2}$ to get the weight.

Reciprocals are two numbers with a product of 1. For example, $\frac{1}{2}$ is the reciprocal of 2.

Lesson Practice
ACC7.3.06

Name: _____ Date: _____ Period: _____

Problems 1–3: There is a proportional relationship between a length measured in centimeters and the same length measured in millimeters.



1. Complete the table and then determine a constant of proportionality.

Length (cm)	Length (mm)
4	40
12	120
50	500
88.49	884.9

Constant of proportionality: 10 or $\frac{1}{10}$ (or equivalent)

2. Complete the table and then determine a constant of proportionality.

Length (mm)	Length (cm)
25	2.5
240	24
4	0.4
699.1	69.91

Constant of proportionality: $\frac{1}{10}$ or 10 (or equivalent)

3. How are these two constants of proportionality related to each other?

Responses vary. They are reciprocals of each other.

4. Amoli and Emika are converting measurements between inches and feet.

Amoli says that the constant of proportionality for the relationship between inches and feet is 12. Emika says it is $\frac{1}{12}$.

Whose claim is correct?

A. Amoli's B. Emika's C. Both D. Neither

Explain your thinking.

Explanations vary. Amoli is saying that there are 12 inches for every 1 foot. Emika is saying that there is $\frac{1}{12}$ of a foot for every 1 inch.

Lesson Practice
ACC7.3.06

Name: _____ Date: _____ Period: _____

FAST Practice

Problems 5–8: A recipe for granola calls for $\frac{5}{2}$ cups of rolled oats and $\frac{3}{4}$ cups of shredded coconut.

5. Complete the table to show how many cups of coconut should be used for different amounts of oats.

Oats (cups)	Coconut (cups)
1	$\frac{3}{10}$ or equivalent
5	$1\frac{1}{2}$ or equivalent
12	$3\frac{3}{5}$ or equivalent

6. What is a constant of proportionality in this relationship? What does it represent?

$\frac{3}{10}$ (or $\frac{10}{3}$). *Responses vary.* For each cup of oats, you should use $\frac{3}{10}$ cups of coconut.

7. What is the *other* constant of proportionality? What does it represent?

$\frac{10}{3}$ (or $\frac{3}{10}$). *Responses vary.* For each cup of coconut, you should use $\frac{10}{3}$ cups of oats.

8. In a recipe, t represents the number of cups of oats, and c represents the number of cups of coconut. Select *all* equations that represent the relationship between t and c.

- ☑ A. $c = \frac{3}{10}t$
- ☐ B. $t = \frac{3}{10}c$
- ☐ C. $c = \frac{10}{3}t$
- ☑ D. $t = \frac{10}{3}c$
- ☐ E. $c = \frac{5}{2}t$
- ☐ F. $t = \frac{2}{5}c$

Spiral Review

9. The Wrangell-St. Elias National Park and Preserve in Alaska has an area of 20,625 square miles. A scale drawing of the Wrangell-St. Elias National Park and Preserve has an area of 8.25 square inches. What is the scale of the drawing?

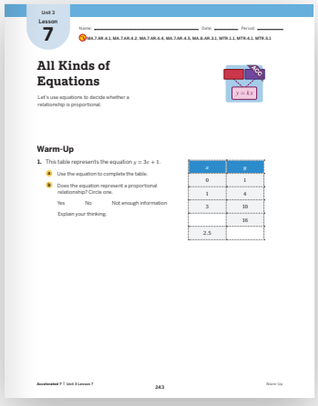
1 inch to 50 miles (or equivalent)

10. Which equation has a constant of proportionality equal to 5?

A. $5y = 5x$ B. $5y = 15x$ C. $3y = 5x$ D. $3y = 15x$

Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1–3	1	MA.7.AR.4.2, MA.7.AR.4.4
	4	2	MA.7.AR.4.2
FAST Practice	5–8	2	MA.7.AR.4.2 MA.7.AR.4.4
Spiral Review			
	9	2	MA.7.GR.1.5
	10	1	MA.7.AR.4.2



All Kinds of Equations

Equations of Proportional Relationships

Let's use equations to decide whether a relationship is proportional.

Learning Outcomes

Today's Goals

1. **Goal:** Understand that equations of the form $y = kx$ represent proportional relationships.
2. **Language Goal:** Justify whether a relationship is proportional or not by looking at the structure of an equation. **(Reading, Writing, Speaking, and Listening)**
3. **Language Goal:** Compare and contrast equations that do and do not represent proportional relationships. **(Speaking and Listening)**

Students return to comparing proportional and non-proportional relationships, focusing on the connection between the structure of the equation and the type of relationship it represents. **(MTR.5.1)**

Coherence

Prior Learning

In Lesson 6, students found the two constants of proportionality for a proportional relationship, explained how they are related to each other, and used them to write two equations representing the relationship.

Future Learning

In Lesson 8, students will explore what a proportional relationship looks like graphically.

Connections

- Students use the constant of proportionality to determine if real world relationships are proportional such as hours worked and money earned. (real world)

FL B.E.S.T. Math Benchmarks

Addressing

MA.7.AR.4.1
Determine whether two quantities have a proportional relationship by examining a table, graph or written description.

MA.7.AR.4.4
Given any representation of a proportional relationship, translate the representation to a written description, table or equation.

Also Addressing: **MA.7.AR.4.2, MA.7.AR.4.5, MA.8.AR.3.1**

Mathematical Thinking and Reasoning Standards: MTR.1.1, MTR.4.1, MTR.5.1

ELA Expectations: ELA.K12.EE.1.1, ELA.K12.EE.2.1, ELA.K12.EE.3.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.6.AR.3.2	**MA.8.AR.3.2**
MA.6.AR.3.3	**MA.8.AR.3.3**
MA.6.AR.3.5	

Lesson at a Glance

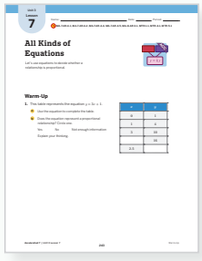
~ 45 min

MA.7.AR.4.1, MA.7.AR.4.2, MA.7.AR.4.4, MA.7.AR.4.5, MA.8.AR.3.1

Warm-Up

Pairs | 5 min

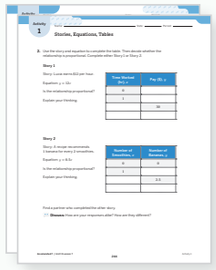
Students complete and use a table to describe whether a given relationship is proportional.



Activity 1

Pairs | 15 min

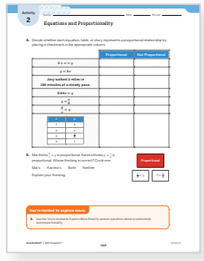
Students analyze multiple representations of relationships with a partner to surface strategies for deciding if an equation represents a proportional relationship.



Activity 2

Pairs | 15 min

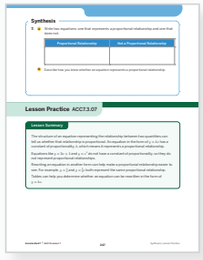
Students analyze relationships in multiple representations and use the **Decide and Defend** routine to construct viable arguments about whether a relationship is proportional.



Synthesis

Whole Class | 5 min

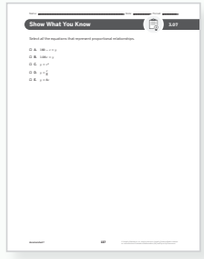
Students synthesize their understanding of proportional relationships by writing an equation that represents a proportional relationship and another that does not. **MLR2: Collect and Display**



Show What You Know

Independent | 5 min

Students demonstrate their understanding by selecting equations that represent proportional relationships.



Prep Checklist

Prepare copies of Activity 2 sheets.

This lesson includes:

- Student Edition
- Show What You Know
- Activity 2 You're Invited to Explore More Sheet, one per student (as needed)
- Calculators (as needed)

Pairs | 5 min

Warm-Up

Purpose: Students complete and use a table to describe whether a given relationship is proportional.

1 Launch

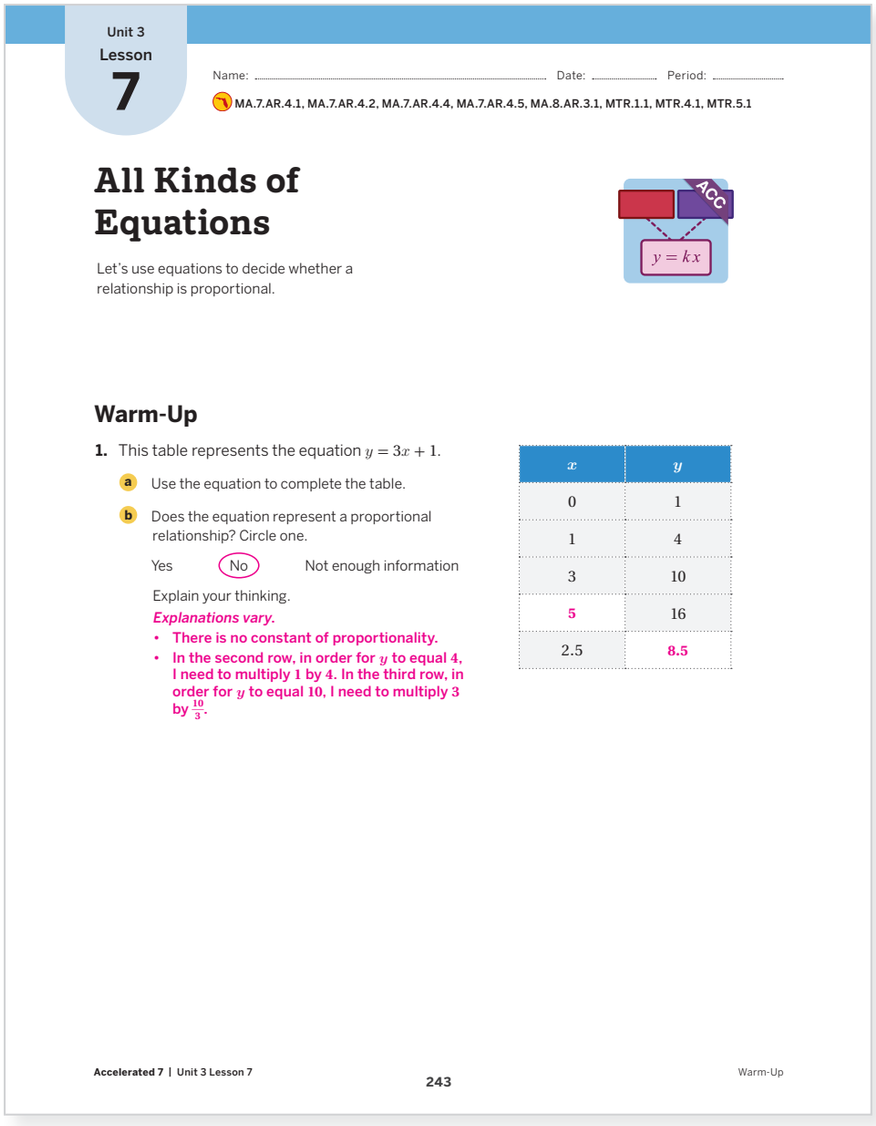
To surface prior knowledge, consider asking "What does it mean for a relationship to be proportional?"

A **Accessibility: Memory and Attention** Invite students to use a calculator to support them with performing calculations throughout this lesson.

2 Connect

Collect and share several different student responses for proportional and not proportional.

Consider asking, "How did you decide if the relationship was proportional or not proportional?"



Unit 3
Lesson
7

Name: _____ Date: _____ Period: _____

MA.7.AR.4.1, MA.7.AR.4.2, MA.7.AR.4.4, MA.7.AR.4.5, MA.8.AR.3.1, MTR.1.1, MTR.4.1, MTR.5.1

All Kinds of Equations

Let's use equations to decide whether a relationship is proportional.

Warm-Up

1. This table represents the equation $y = 3x + 1$.

x	y
0	1
1	4
3	10
5	16
2.5	8.5

a. Use the equation to complete the table.

b. Does the equation represent a proportional relationship? Circle one.

Yes (No) Not enough information

Explain your thinking.

Explanations vary.

- There is no constant of proportionality.
- In the second row, in order for y to equal 4, I need to multiply 1 by 4. In the third row, in order for y to equal 10, I need to multiply 3 by $\frac{10}{3}$.

Accelerated 7 | Unit 3 Lesson 7 243 Warm-Up

Pairs | 15 min

Activity 1 Stories, Equations, Tables

Purpose: Students analyze multiple representations of relationships with a partner to surface strategies for deciding if an equation represents a proportional relationship.

Short on time? Consider omitting the movement of connecting with a partner and instead hold the discussion briefly as a class.

1 Launch

To surface connections, consider asking, "What representations can we use to capture a story?"

Direct students' attention to the directions for the activity. Students will complete Story 1 or Story 2 and then compare responses.

ML/EL **MLR6: Three Reads (optional)** Consider inviting students to read the story to themselves multiple times, first focusing on what the situation is about, then what the quantities are, and finally the question, in order to support them in making sense of the relationship it represents. See the Routine Facilitation Guide for more information. **(MTR.1.1)**

2 Monitor

Look for a variety of student strategies.

D **Differentiation** *(Stories 1–2)*

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Consider asking, "How can you use the equation to determine values in tables like this?"
Think the relationships are not proportional.	**Support** Encourage students to compare with classmates that worked on the same column to hear their strategies.
Keep increasing the x-values by 1 without analyzing the given y-value in the 3rd row.	**Support** Considering asking, "How can you use the equation to confirm values in each row of your table?"

Pause after most students have discussed Stories 1 and 2. Invite students to share whether their relationships were proportional and how they knew.

Activity 1 continued >

Activity 1

Name: _____ Date: _____ Period: _____

Stories, Equations, Tables

2. Use the story and equation to complete the table. Then decide whether the relationship is proportional. Complete either Story 1 or Story 2.

Story 1

Story: Lucia earns \$12 per hour.

Equation: $y = 12x$

Is the relationship proportional?
Yes
Explain your thinking.
Explanations vary. The constant of proportionality is 12.

Time Worked (hr), x	Pay (\$), y
0	0
1	12
2.5	30
3	36

Responses in the last row vary. Sample shown.

Story 2

Story: A recipe recommends 1 banana for every 2 smoothies.

Equation: $y = 0.5x$

Is the relationship proportional?
Yes
Explain your thinking.
Explanations vary. The constant of proportionality is 0.5.

Number of Smoothies, x	Number of Bananas, y
0	0
1	0.5
5	2.5
8	4

Responses in the last row vary. Sample shown.

Find a partner who completed the other story.

Discuss: How are your responses alike? How are they different?

Responses vary. The equations, tables, and stories were different, but both relationships were proportional even though they had different constants of proportionality.

Accelerated 7 | Unit 3 Lesson 7 244 Activity 1

Pairs | 15 min

Activity 1 Stories, Equations, Tables (continued)

Purpose: Students analyze multiple representations of relationships with a partner to surface strategies for deciding if an equation represents a proportional relationship.

2 Monitor

Invite students to complete Story 3 or Story 4.

D Differentiation *(Stories 3–4)*

Look for students who:	Respond to Student Thinking
Need support with remembering how to work with exponents. *(Story 4)*	**Support** Remind them another way to write x^2 is $x \cdot x$. Consider asking, "How does the story about the area help us determine the missing values?"
Thinking that the situation is proportional because there is a rate of $35 per month. *(Story 3)*	**Support** Invite them to revisit other problems identified as proportional and consider asking, "What was the relationship between the left column and right column in the table?"
Would like to explore relationships in stories further.	**Stretch** Consider asking, "Can you write another story that would be represented by the same equation and table?" **Stretch** Consider asking, "How could you describe the relationship between constants of proportionality and coefficients to a classmate?"

Invite students to share their thinking after comparing with a partner.

Listen for student explanations to highlight during the Connect, such as:
- Identifying the constant of proportionality.
- Looking at the equation and making generalizations about the structure, e.g., "It has the form $y =$ (something)x." **(MTR.5.1)**

3 Connect

Share the captured student explanations to highlight different strategies for deciding if an equation represents a proportional relationship.

To surface the Key Takeaway, consider asking:
- "How can you tell from the equation if a relationship is proportional?"
- "Where can you see the constant of proportionality in the equation?"
- "What does the constant of proportionality tell us about the story?"

Activity 1

Name: Date: Period:

Stories, Equations, Tables (continued)

Complete either Story 3 or Story 4.

Story 3

Story: A cell phone costs $500, plus $35 per month for the plan.

Equation: $y = 35x + 500$

Is the relationship proportional?
No
Explain your thinking.
There is no constant of proportionality.

Number of Months, x	Total Cost ($), y
0	500
1	535
3	605
5	675

Responses in the last row vary. Sample shown.

Story 4

Story: The area of a square is the side length multiplied by itself.

Equation: $y = x^2$

Is the relationship proportional?
No
Explain your thinking.
There is no constant of proportionality.

Side Length (units), x	Area (sq. units), y
0	0
1	1
10	100
5	25

Responses in the last row vary. Sample shown.

3. Here are the equations that represent the four stories.

a. Select *all* the equations that represent a proportional relationship.

- ☑ A. $y = 12x$
- ☐ B. $y = 500 + 35x$
- ☑ C. $y = \frac{1}{2}x$
- ☐ D. $y = x^2$

b. Explain one way to decide if an equation represents a proportional relationship.
Explanations vary. One way to decide if a relationship is proportional is to look at the equation and see if it has the form $y =$ (something)x.

Accelerated 7 | Unit 3 Lesson 7 245 Activity 1

Key Takeaway: If an equation represents a proportional relationship, then one variable is multiplied or divided by the constant of proportionality.

Pairs | 15 min

Activity 2 Equations and Proportionality

Purpose: Students analyze relationships in multiple representations and use the Decide and Defend routine to construct viable arguments about whether a relationship is proportional.

1 Launch

Invite students to make sense of the relationships independently. Encourage students to share their reasoning with a partner and work together to reach an agreement about how to determine whether each equation, table, or story represents a proportional relationship.

2 Monitor

Listen for different reasoning students give for why representations are proportional or not.

A **Accessibility: Executive Function** Invite students to focus on one representation at a time to support students in carrying out multiple steps.

D **Differentiation: (Support)** Students may struggle with equations where the y is not isolated. Invite them to work with a partner to rewrite the equation or create a table of values.

Listen and look for a variety of student responses.

ELL **English Language Learners** Provide sentence frames to help students explain their strategy (e.g., "___ is proportional/not proportional because ___."). **(Writing and Speaking)**

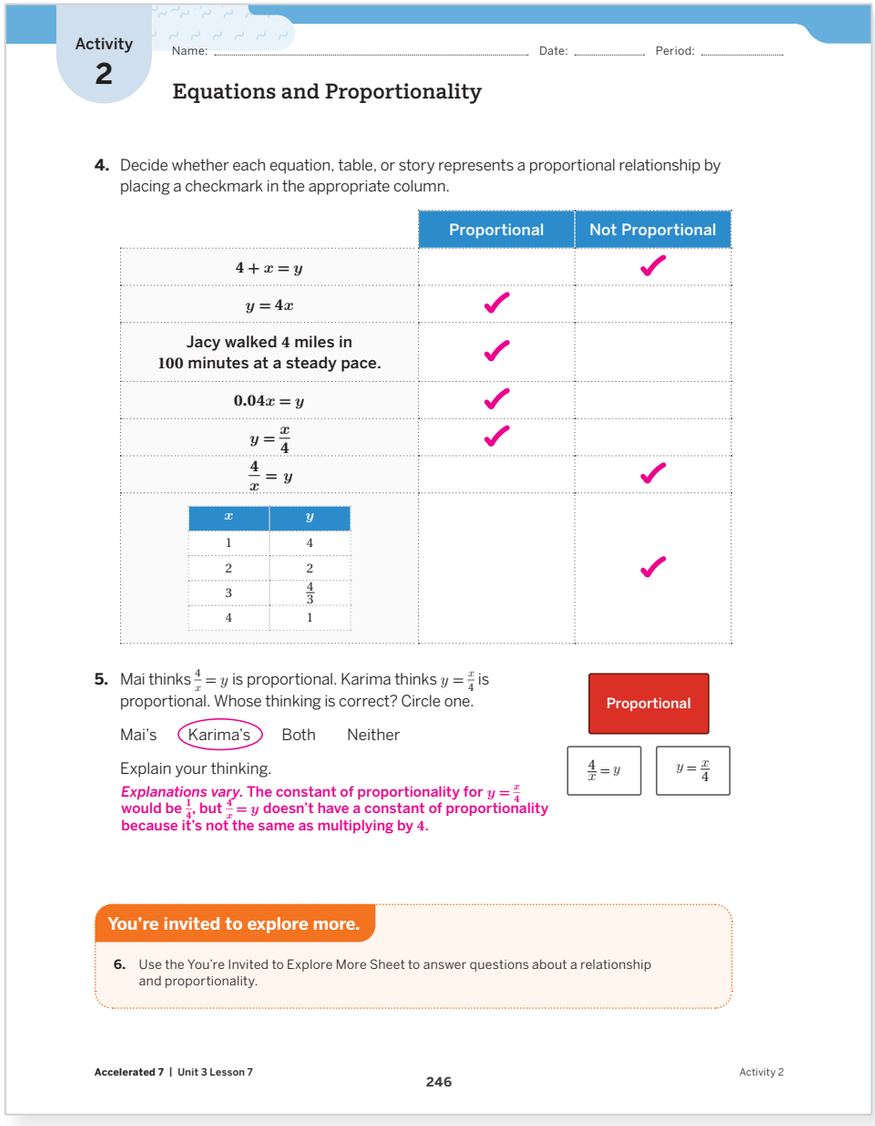
D **Differentiation: You're invited to explore more. (Stretch)** Invite students who want to explore proportional and not proportional situations further to complete this optional task. Encourage them to discuss their thinking with a partner. **Note:** Students will need copies of the You're Invited to Explore More Sheet.

3 Connect

Invite students to share their responses.

Use the Decide and Defend routine to support students in making arguments and critiquing the reasoning of others. **(MTR.4.1)**

Invite students to share their reasoning for each choice. Consider asking, "How are each of these like other equations we have used today? How are they different?"



Activity 2

Name: _____ Date: _____ Period: _____

Equations and Proportionality

4. Decide whether each equation, table, or story represents a proportional relationship by placing a checkmark in the appropriate column.

	Proportional	Not Proportional
$4 + x = y$		✓
$y = 4x$	✓	
Jacy walked 4 miles in 100 minutes at a steady pace.	✓	
$0.04x = y$	✓	
$y = \frac{x}{4}$	✓	
$\frac{4}{x} = y$		✓
x | y 1 | 4 2 | 2 3 | $\frac{4}{3}$ 4 | 1		✓

5. Mai thinks $\frac{4}{x} = y$ is proportional. Karima thinks $y = \frac{x}{4}$ is proportional. Whose thinking is correct? Circle one.

Mai's (Karima's) Both Neither

Explain your thinking.

Explanations vary. The constant of proportionality for $y = \frac{x}{4}$ would be $\frac{1}{4}$, but $\frac{4}{x} = y$ doesn't have a constant of proportionality because it's not the same as multiplying by 4.

Proportional

$\frac{4}{x} = y$ | $y = \frac{x}{4}$

You're invited to explore more.

6. Use the You're Invited to Explore More Sheet to answer questions about a relationship and proportionality.

Accelerated 7 | Unit 3 Lesson 7 246 Activity 2

Whole Class | 5 min

Synthesis

Purpose: Students synthesize their understanding of proportional relationships by writing an equation that represents a proportional relationship and another that does not.

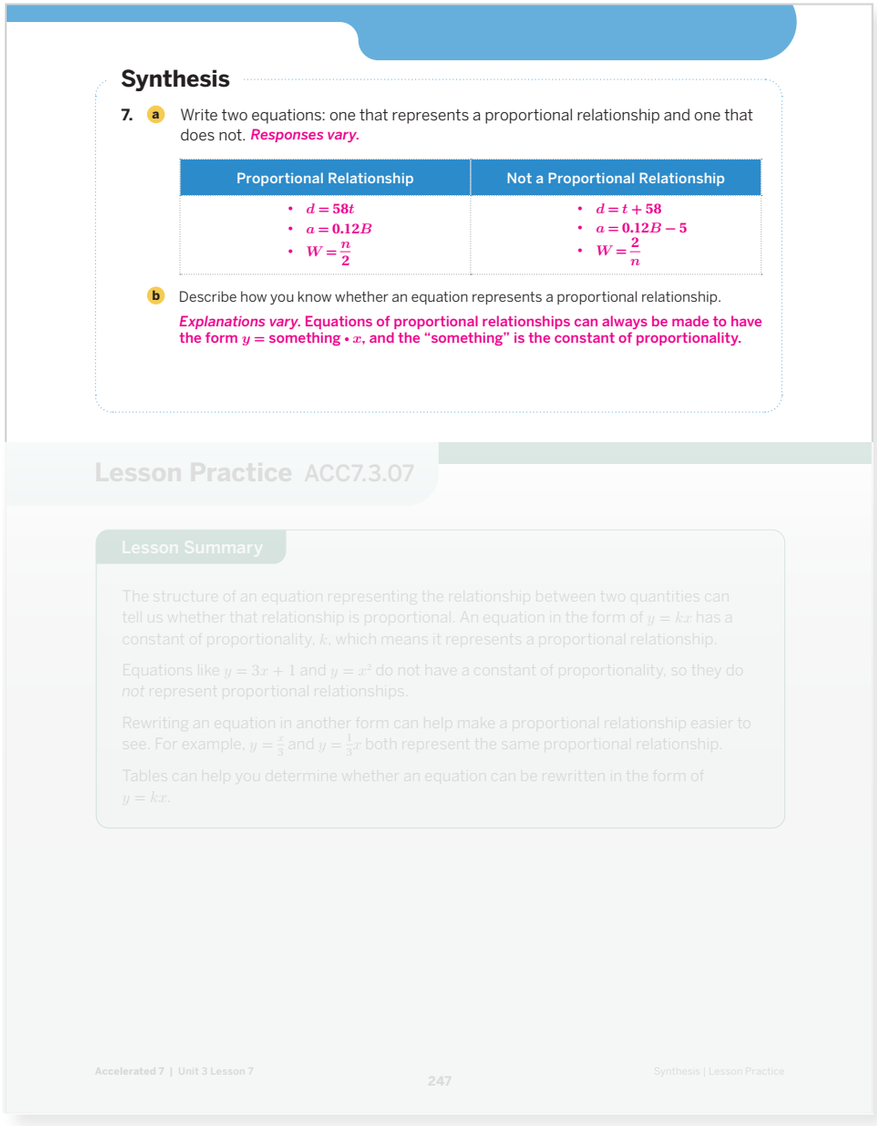
Invite students to respond independently, and then share their thinking with a partner.

Collect and share a variety of equations and ideas, such as:

- Proportional equations look like $y =$ (something)x.
- Non-proportional equations sometimes have a plus in them.
- Non-proportional equations sometimes have exponents.

MLR **MLR2: Collect and Display** Use this routine to collect students' equations as they generate them with a partner. Add examples of proportional and non-proportional relationships to the class anchor chart. Consider displaying the chart for students to reference and add to for the rest of the unit.

Note: If time allows, invite students to add to their previous thinking for the term *constant of proportionality*, based on the discussion about equations.



Synthesis

7. a Write two equations: one that represents a proportional relationship and one that does not. ***Responses vary.***

Proportional Relationship	Not a Proportional Relationship
• $d = 58t$ • $a = 0.12B$ • $W = \frac{n}{2}$	• $d = t + 58$ • $a = 0.12B - 5$ • $W = \frac{2}{n}$

b Describe how you know whether an equation represents a proportional relationship.

Explanations vary. **Equations of proportional relationships can always be made to have the form $y =$ something $\cdot x$, and the "something" is the constant of proportionality.**

Lesson Practice ACC7.3.07

Lesson Summary

The structure of an equation representing the relationship between two quantities can tell us whether that relationship is proportional. An equation in the form of $y = kx$ has a constant of proportionality, k, which means it represents a proportional relationship.

Equations like $y = 3x + 1$ and $y = x^2$ do not have a constant of proportionality, so they do *not* represent proportional relationships.

Rewriting an equation in another form can help make a proportional relationship easier to see. For example, $y = \frac{x}{3}$ and $y = \frac{1}{3}x$ both represent the same proportional relationship.

Tables can help you determine whether an equation can be rewritten in the form of $y = kx$.

Accelerated 7 | Unit 3 Lesson 7 247 Synthesis | Lesson Practice

Lesson Takeaway: Proportional relationships, in equation form, can always be rewritten to have the form $y = k \cdot x$, where k is the constant of proportionality.

Independent | 5 min

Show What You Know

Purpose: Students demonstrate their understanding by selecting equations that represent proportional relationships.

Today's Goals

1. **Goal:** Understand that equations of the form $y = kx$ represent proportional relationships.
2. **Language Goal:** Justify whether a relationship is proportional or not by looking at the structure of an equation. **(Reading, Writing, Speaking, and Listening)**
3. **Language Goal:** Compare and contrast equations that do and do not represent proportional relationships. **(Speaking and Listening)**

Show What You Know (Assessment Resources)

Name: _____ Date: _____ Period: _____

Show What You Know 3.07

Select *all* the equations that represent proportional relationships.

- ☐ A. $180 - x = y$
- ☑ B. $1.08x = y$
- ☐ C. $y = x^8$
- ☑ D. $y = \frac{x}{8}$
- ☑ E. $y = 8x$

Differentiation Use after Lesson 7

Support

Provide targeted intervention for students by using these resources.

If students need support understanding how to identify equations that represent a proportional relationship:

» **Respond to Student Thinking:**

- Revisit Activity 2 Problem 4 as a class and invite students to share strategies for identifying equations that represent a proportional relationship.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 2 Extension Activities**.

Professional Learning

Reflect on students' progress with proportional relationships. In what ways have students' understanding and skills grown? What concepts do students need additional support with? How can you use students' strengths to help everyone feel prepared for their upcoming assessment?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

7. a. Write two equations: one that represents a proportional relationship and one that does not. *Responses vary.*

Proportional Relationship	Not a Proportional Relationship
• $d = 58t$ • $a = 0.12B$ • $W = \frac{n}{2}$	• $d = t + 58$ • $a = 0.12B - 5$ • $W = \frac{2}{n}$

b. Describe how you know whether an equation represents a proportional relationship.

Explanations vary. Equations of proportional relationships can always be made to have the form $y =$ something $\cdot x$, and the "something" is the constant of proportionality.

Lesson Practice ACC7.3.07

Lesson Summary

The structure of an equation representing the relationship between two quantities can tell us whether that relationship is proportional. An equation in the form of $y = kx$ has a constant of proportionality, k, which means it represents a proportional relationship.

Equations like $y = 3x + 1$ and $y = x^2$ do not have a constant of proportionality, so they do *not* represent proportional relationships.

Rewriting an equation in another form can help make a proportional relationship easier to see. For example, $y = \frac{x}{3}$ and $y = \frac{1}{3}x$ both represent the same proportional relationship.

Tables can help you determine whether an equation can be rewritten in the form of $y = kx$.

Lesson Practice
ACC7.3.07

Name: _____ Date: _____ Period: _____

Problems 1–2: The relationship between a distance in yards, y, and the same distance in miles, x, is represented by the equation $y = 1760x$.

1. Complete the table.

Distance (mi), x	Distance (yd), y
1	1,760
5	8,800
2	3,520
10	17,600

2. Is there a proportional relationship between a distance in yards and the same distance in miles?

Yes

Explain your thinking.

Explanations vary. The equation is of the form $y = kx$, where $k = 1760$.

Problems 3–4: Determine whether or not each relationship is proportional. Explain your thinking.

3.

x	y
2	5
3	7.5
6	15

Yes. *Explanations vary.* It could be proportional because $k = \frac{5}{2}$ for each pair of values listed in the table.

4. $y = 3.2x + 5$

No. *Explanations vary.* It's not proportional because the equation is not of the form $y = kx$.

Problems 5–6: Determine whether each relationship is proportional or not proportional. Explain your thinking.

5. The weight of a stack of standard 8.5-by-11-inch paper and the number of sheets of paper.

Proportional. *Explanations vary.* The weight of each sheet of paper is constant.

6. The weight of a stack of different-sized books (where each book weighs a different amount) and the number of books in a stack.

Not proportional. *Explanations vary.* Each book weighs a different amount, so there is no constant weight.

Lesson Practice
ACC7.3.07

Name: _____ Date: _____ Period: _____

7. Determine whether or not each relationship is proportional.

	Proportional	Not Proportional
The remaining length, L, of a 120-inch rope after x inches have been cut off: $120 - x = L$.	☐	☑
The total cost, t, after 8% sales tax is added to an item's price p: $1.08p = t$.	☑	☐
The number of marbles, x, each sister gets after m marbles are shared equally among four sisters: $x = \frac{m}{4}$.	☑	☐
The volume, V, of a rectangular prism whose height is 12 centimeters and whose base is a square with side lengths of s centimeters: $V = 12s^2$.	☐	☑

FAST Practice

8. Liam and Sadia are running a 60-meter race. Each of their distances can be represented by an equation in the form $y = kx$ where y is the distance in meters and x is the time in seconds.

Use this information to complete the table.

- Liam's distance is represented with the equation $y = 6x$.
- At 6 seconds, Liam is 12 meters ahead of Sadia.

Time (sec)	Liam's Distance (m)	Sadia's Distance (m)
0	0	0
2	12	8
4	24	16
6	36	24

Spiral Review

9. The equation $y = 3.5x$ can be used to determine the total cost, y, in dollars, of x ounces of blueberries. What does the number 3.5 represent in the equation?

A. The number of blueberries that \$1 can buy.

B. The number of blueberries in x ounces.

C. The cost of 1 ounce of blueberries.

D. The cost of x ounces of blueberries.

Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1–2	2	MA.7.AR.4.1, MA.7.AR.4.4
	3–4, 7	1	MA.7.AR.4.1
	5–6	2	MA.7.AR.4.1
FAST Practice	8	3	MA.7.AR.4.1, MA.7.AR.4.4
Spiral Review			
	9	2	MA.7.AR.4.2

Assess and Respond

Sub-Unit Quiz 1

Independent | 45 min

Facilitation: Assign the Sub-Unit Quiz to learn about your students' understanding of the concepts and skills so far in this unit. If time allows, consider inviting students to complete one or more of the tasks in the Unit Synthesis and Reflection.

Item Analysis

Problem(s)	Concept or skill	Addressed in	DOK	FL B.E.S.T. Math Benchmarks
1	Determining whether quantities are in a proportional relationship from a table	Lesson 1	2	MA.7.AR.4.1, MTR.5.1
2	Determining whether quantities are in a proportional relationship from an equation or situation	Lesson 4	2	MA.7.AR.4.1, MA.7.AR.4.4
3a	Explaining the meaning of a constant of proportionality	Lesson 2	2	MA.7.AR.4.5, MTR.6.1
3b, 3c	Solving problems using equations of proportional relationships	Lesson 5	2	MA.7.AR.4.5
4a	Determining values in a table of a proportional relationship	Lesson 4	2	MA.8.AR.3.1
4b	Determining a constant of proportionality for a proportional relationship represented in a table	Lesson 6	2	MA.7.AR.4.2
4c	Writing an equation for a proportional relationship	Lesson 5	2	MA.7.AR.4.4, MTR.7.1
5a	Determining two constants of proportionality for a proportional relationship	Lesson 6	2	MA.7.AR.4.2
5b	Writing two equations for a proportional relationship	Lesson 7	2	MA.7.AR.4.4, MTR.7.1
5c	Solving problems using equations of proportional relationships	Lesson 7	2	MA.7.AR.4.5

Assessment Resources

- Student Assessments
- Assessment Rubrics with Answers
- Show What You Knows and Answer Keys

Intervention and Extension Resources

- Mini-Lessons
- Extensions

Name: Date: Period:

Sub-Unit Quiz

Unit 3

1. Which table represents a proportional relationship?

A.

x	y
2	3
4	5
10	11

B.

x	y
2	4
4	5
10	6

C.

x	y
2	4
4	16
10	100

D.

x	y
2	3
4	6
10	15

2. Select *all* of the proportional relationships.

- ☑ A. $y = 2.5x$
- ☐ B. A turtle walks for 5 minutes, then stops for a minute.
- ☑ C. A turtle starts at the starting line and walks at a constant rate.
- ☐ D. $y = \frac{2.5}{x}$
- ☑ E. $y = \frac{5}{2}x$

3. Jordan is mixing water and flour to make tortillas. The equation $w = 0.75f$ represents the number of cups of water, w, needed for f cups of flour.

a. What does 0.75 mean in this situation?

Responses vary. It means that 0.75 cups of water must be mixed with each cup of flour.

b. How many cups of water does Jordan need for 4 cups of flour?

3 cups of water (or equivalent)

c. How many cups of flour does Jordan need for 1 cup of water?

$\frac{4}{3}$ cups of flour (or equivalent)

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Name: Date: Period:

Sub-Unit Quiz (continued)

Unit 3

4. When you mix two colors of paint in equivalent ratios, the result is always the same color.

a. Complete the table so that there is a proportional relationship between cups of blue paint and cups of red paint.

Blue Paint (cups), b	Red Paint (cups), r
1	$\frac{3}{2}$ (or equivalent)
2	3
$\frac{14}{3}$ (or equivalent)	7
10	15 (or equivalent)

b. What is a constant of proportionality for this relationship?

$\frac{3}{2}$ (or equivalent) or $\frac{2}{3}$ (or equivalent)

What does it represent in this situation?

Explanations vary.

- The constant of proportionality $\frac{3}{2}$ represents the number of cups of red paint needed for each cup of blue paint.
- The constant of proportionality $\frac{2}{3}$ represents the number of cups of blue paint needed for each cup of red paint.

c. Write an equation for the relationship between the number of cups of blue paint, b, and the number of cups of red paint, r.

$r = \frac{3}{2}b$ (or equivalent) or $b = \frac{2}{3}r$ (or equivalent)

5. Titus took 30 minutes to walk 2 miles at a constant rate.

- d is the distance Titus walks, in miles.
- t is the time it takes Titus to walk, in minutes.

a. Write *two* different constants of proportionality for this relationship.

15 (or equivalent), $\frac{1}{15}$ (or equivalent)

b. Write *two* different equations to represent the relationship between d and t.

$t = 15d$ (or equivalent), $d = \frac{1}{15}t$ (or equivalent)

c. If Titus walked 7.5 miles at this same rate, how many minutes would it take? Explain or show your thinking.

112.5 minutes. *Explanations vary.* I multiplied 7.5 miles by the constant of proportionality, 15, to get $7.5 \cdot 15 = 112.5$ minutes.

D Differentiation (Sub-Unit Quiz 1)

Note: To **strengthen** and **stretch** students' learning, refer to the differentiation resources suggested throughout this unit and in the Unit Overview.

Sub-Unit Goals	Problem(s)	To respond to student thinking, consider:	
Sub-Unit 1: • Use tables to recognize proportional relationships • Determine a constant of proportionality from a table **(Lessons 1–3)**	1, 4a, 4b	**Support**	• **Mini-Lesson: Determining the Constant of Proportionality**
		Strengthen	• **Teacher Move:** Consider revisiting Lesson 2, Activity 2 and discuss Problems 7 and 8. Ask students to share with a partner their thinking for the constant of proportionality found.
		Stretch	• **You're invited to explore more:** Lesson 2 (*Balloon Float*)
Sub-Unit 2: • Write and use equations to describe proportional relationships. • Determine a constant of proportionality from an equation. **(Lessons 4–7)**	3, 4c	**Support** **Strengthen**	• **Mini-Lesson: Finding More Than One Equation to Represent Proportional Relationships**
		Stretch	• **Teacher Move:** Consider revisiting Lesson 7, Activity 2 and discuss Problem 4. Ask students to share with a partner their thinking for each given situation.
	2, 5	**Support**	• **Teacher Move:** Consider revisiting Lesson 5, Activity 2 and asking, "How can you use a hanger diagram to solve the equation?" • **Challenge Creator:** Lesson 4 (Balancing Moves)
		Strengthen	• **Teacher Move:** Consider asking, "How does the diagram connect with the expression and situation? What is known and unknown?" Then invite students to collaborate and revise Problem 5.
		Stretch	• **You're invited to explore more:** Lesson 7 (*All Kinds of Equations*)

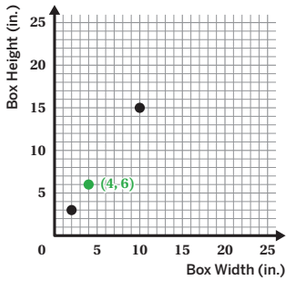
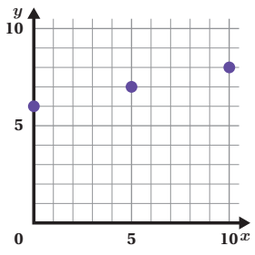
Proportional Relationships in Graphs

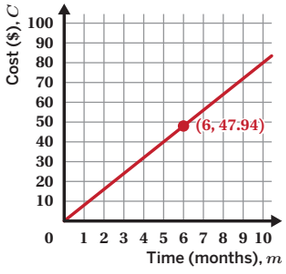
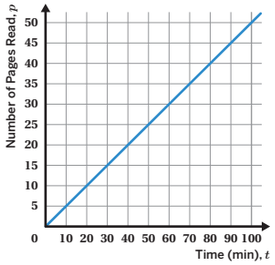
Sub-Unit 3 Goals

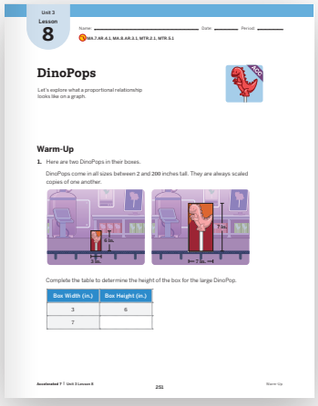
- **Goal:** Use graphs to recognize proportional relationships.
- **Goal:** Determine a constant of proportionality from a graph.
- **Language Goal:** Describe relationships represented with graphs using terms such as *proportional, non-proportional,* and *constant of proportionality.*

In this sub-unit, students recognize proportional relationships represented in graphs and determine the constant of proportionality using graphs. They learn that a proportional relationship can be represented by points on a line that passes through the origin on a coordinate plane.

Math that Matters Most

Concept or Skill	For example . . .
Use a graph of a proportional relationship to predict other points in the same relationship.	The graphed point represents a TriceraPop in a box. Add at least two more points to the graph to represent other boxes that fit a TriceraPop. 
Identify whether a relationship is proportional by studying a graph.	Determine whether the graph represents a proportional relationship. Explain your thinking.  *It is not proportional. The points are on a line, but it does not go through the origin.*

Concept or Skill	For example . . .
Determine a constant of proportionality from a graph.	What is the constant of proportionality in this relationship?  *The constant of proportionality is 7.99.*
Write an equation for a proportional relationship given a graph.	Write an equation that represents the relationship between the time in minutes, t, and the total number of pages Joel has read, p.  $p = \frac{1}{2}t$


Unit 3
Lesson 8
DinoPops
Let's explore what a proportional relationship looks like on a graph.
Warm-Up
1. Here are two DinoPops in their boxes.
DinoPops come in all sizes between 2 and 200 inches tall. They are always scaled copies of one another.
Complete the table to determine the height of the box for the large DinoPop.
Box Width (in.) | Box Height (in.)
3 | 6
7 |
251

DinoPops

Introducing Graphs of Proportional Relationships

Let's explore what a proportional relationship looks like on a graph.

Learning Outcomes

Today's Goals

1. **Goal:** Use a graph that represents a proportional relationship to predict other points in that relationship.
2. **Goal:** Understand that a proportional relationship can be represented by points on a line that passes through the origin.
3. **Language Goal:** Justify whether a graph represents a proportional relationship. **(Reading, Writing, Speaking, and Listening)**

Students explore what a proportional relationship looks like graphically. This is the first time in Grade 7 that students encounter a *coordinate plane*. Students reason abstractly and quantitatively about how lollipop boxes can be represented as points on a coordinate plane.

Coherence

Prior Learning

In Lesson 7, students developed strategies for deciding whether a relationship is proportional based on the structure of its equation.

Future Learning

In Lesson 9, students will work with graphs of proportional relationships and identify the constant of proportionality using a graph.

Connections

- Within the context of comparing box sizes of Dinopops, students determine if a relationship is proportional by its graph. (mathematical)

FL B.E.S.T. Math Benchmarks

Addressing

MA.7.AR.4.1
Determine whether two quantities have a proportional relationship by examining a table, graph or written description.

Also Addressing: **MA.8.AR.3.1**

Mathematical Thinking and Reasoning Standards: MTR.2.1, MTR.5.1

ELA Expectation: ELA.K12.EE.3.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.6.AR.3.5 **MA.6.GR.1.1**	**MA.8.AR.3.2**

Vocabulary

Review Vocabulary

coordinate plane

origin

Lesson at a Glance

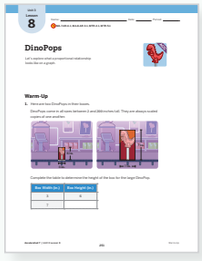
~ 45 min

MA.7.AR.4.1, MA.8.AR.3.1

Warm-Up

Pairs | 5 min

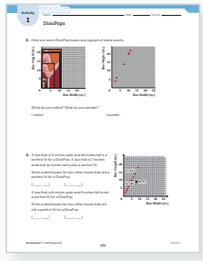
Students familiarize themselves with the lesson context and establish that the relationship between lollipop box width and box height is proportional.



Activity 1

Pairs | 8 min

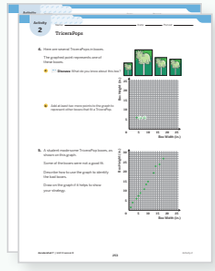
Students use the **Notice and Wonder** routine to explore graphs of proportional relationships, including their connection to the *origin*.



Activity 2

Pairs | 12 min

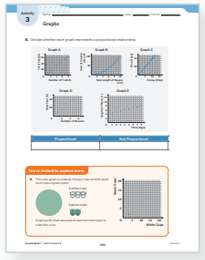
Students continue to graph proportional relationships and discuss the key features that allow them to determine proportionality from a graph.



Activity 3

Pairs | 10 min

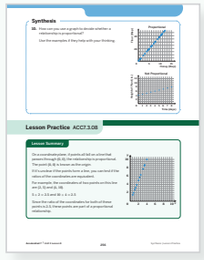
Students interpret graphs and discuss whether or not they represent a proportional relationship.



Synthesis

Whole Class | 5 min

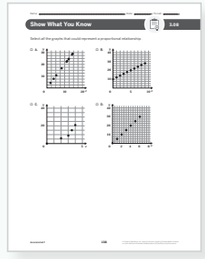
Students synthesize their understanding of representing proportional relationships using graphs.



Show What You Know

Independent | 5 min

Students demonstrate their understanding by identifying graphs of proportional relationships.



Prep Checklist

This lesson includes:

- Student Edition
- Show What You Know

Pairs | 5 min

Warm-Up

Purpose: Students familiarize themselves with the lesson context and establish that the relationship between lollipop box width and box height is proportional.

1 Launch

To generate excitement, consider asking students to guess the height of the record-holding world's tallest lollipop. (See's Candies made a lollipop that was approximately 199 inches tall in 2012.)

Invite students to determine measurements for other boxes that would fit a DinoPop.

2 Connect

Consider asking, "Is the relationship between box width and box height proportional? How can you tell?" *Responses vary.*

- Yes. The problem says they are scaled copies, and scaled copies are always proportional.
- Yes. In the table, you can always multiply the width by 2 to get the height. There is a constant of proportionality of 2.
- Yes. It could be represented by the equation $h = 2w$.

Unit 3
Lesson
8

Name: _____ Date: _____ Period: _____

MA.7.AR.4.1, MA.8.AR.3.1, MTR.2.1, MTR.5.1

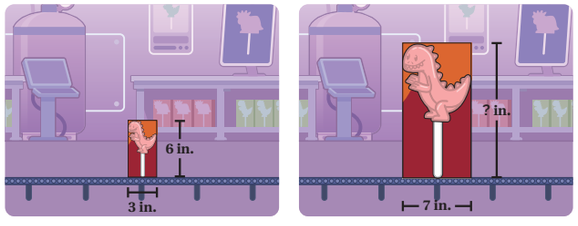
DinoPops

Let's explore what a proportional relationship looks like on a graph.

Warm-Up

1. Here are two DinoPops in their boxes.

DinoPops come in all sizes between 2 and 200 inches tall. They are always scaled copies of one another.



Complete the table to determine the height of the box for the large DinoPop.

Box Width (in.)	Box Height (in.)
3	6
7	14

Accelerated 7 | Unit 3 Lesson 8 251 Warm-Up

Pairs | 8 min

Activity 1 DinoPops

Purpose: Students use the Notice and Wonder routine to explore graphs of proportional relationships, including their connection to the *origin*.

1 Launch

Direct students' attention to the DinoPop boxes and the graph.

Use the Notice and Wonder routine to promote curiosity and help students make sense of the graph. **(MTR.2.1)**

Create a record of what students noticed and wondered, along with the names of the students who shared.

Note: If it does not come up naturally, consider sharing that the graph shows points on a *coordinate plane*. Invite students to read the axis labels to identify the quantities represented.

2 Monitor

Invite students to write ordered pairs for boxes that are a perfect fit for a DinoPop and for boxes that are not a perfect fit for a DinoPop.

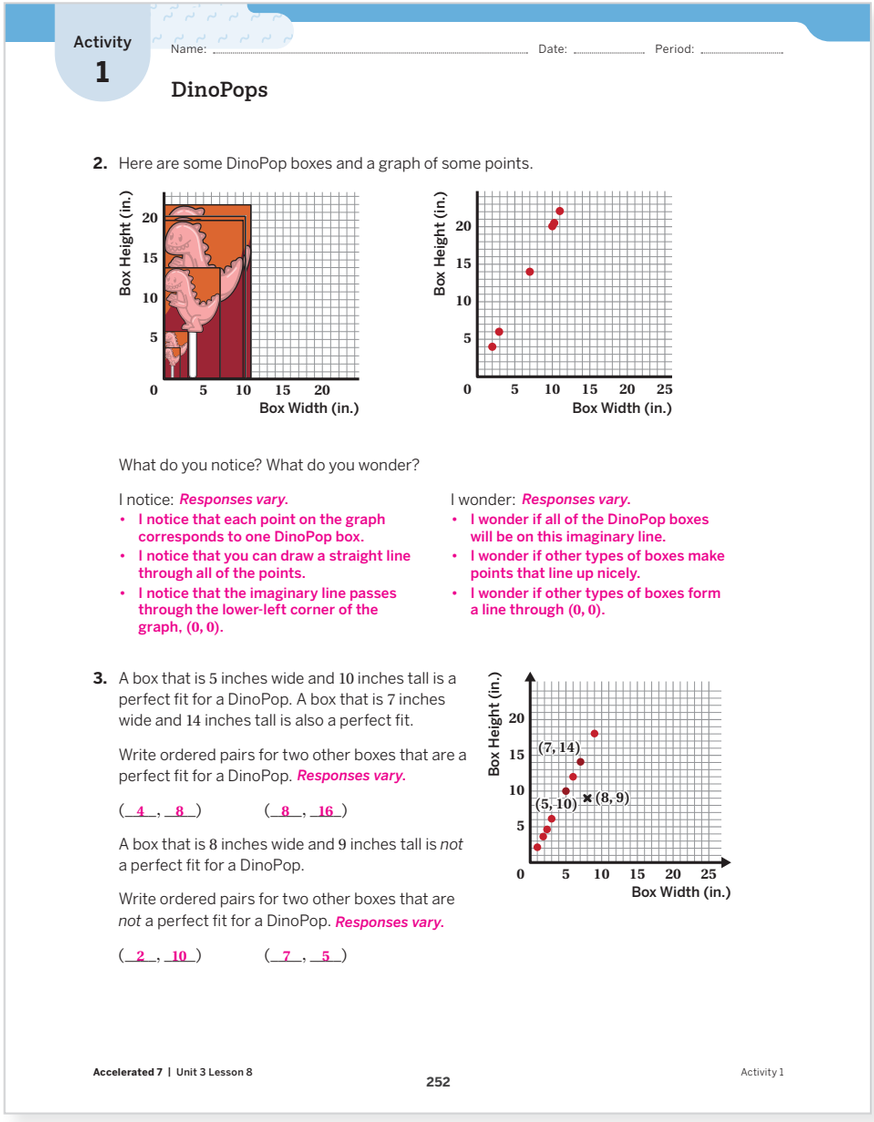
Listen for a variety of informal language students use to describe the graph.

3 Connect

To surface the Key Takeaway, consider asking, "What do you notice about the points representing a 'perfect fit' for a DinoPop?" **(MTR.5.1)**

Note: If it does not come up naturally, consider sharing that you could draw a straight line through the points, starting at $(0, 0)$. Share that this point is called the *origin*.

Consider asking, "Do you think a point at the origin represents a perfect DinoPops box? Why or why not?"



Activity 1

Name: Date: Period:

DinoPops

2. Here are some DinoPop boxes and a graph of some points.

What do you notice? What do you wonder?

I notice: *Responses vary.*
- I notice that each point on the graph corresponds to one DinoPop box.
- I notice that you can draw a straight line through all of the points.
- I notice that the imaginary line passes through the lower-left corner of the graph, (0, 0).

I wonder: *Responses vary.*
- I wonder if all of the DinoPop boxes will be on this imaginary line.
- I wonder if other types of boxes make points that line up nicely.
- I wonder if other types of boxes form a line through (0, 0).

3. A box that is 5 inches wide and 10 inches tall is a perfect fit for a DinoPop. A box that is 7 inches wide and 14 inches tall is also a perfect fit.

Write ordered pairs for two other boxes that are a perfect fit for a DinoPop. *Responses vary.*

(4, 8) (8, 16)

A box that is 8 inches wide and 9 inches tall is *not* a perfect fit for a DinoPop.

Write ordered pairs for two other boxes that are *not* a perfect fit for a DinoPop. *Responses vary.*

(2, 10) (7, 5)

Accelerated 7 | Unit 3 Lesson 8 252 Activity 1

Key Takeaway: You can represent a proportional relationship on a graph with a straight line passing through the point $(0, 0)$, which is called the *origin*.

Pairs | 12 min

Activity 2 TriceraPops

Purpose: Students continue to graph proportional relationships and discuss the key features that allow them to determine proportionality from a graph.

Short on time? Consider omitting Problem 6 and instead using it as a differentiation support for students needing an access point for Problem 7.

1 Launch

Invite students to discuss and add points to the graph.

D **Differentiation: (Support)** To support students getting started, consider asking, "What do you think each coordinate in the point $(4, 6)$ means?" **(MTR.2.1)**

Consider asking, "What strategies did you use to predict other points that would create a perfect TriceraPops box?"

A **Accessibility: Memory and Attention** Invite students to record strategies as they are shared to support students with keeping track of information. Students can use these strategies as they complete Problem 5.

2 Monitor

Invite students to find a few boxes that fit a TriceraPop, then continue to the next problem.

Listen for explanations of how students determined which boxes were not a perfect fit, including students who drew a line through the origin.

Pause briefly to invite students to share their work. Consider asking, "How can we recognize a proportional relationship on a graph?"

Activity 2 continued >

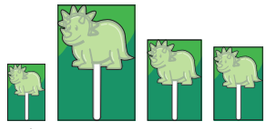
Activity 2

Name: Date: Period:

TriceraPops

4. Here are several TriceraPops in boxes.

The graphed point represents *one* of these boxes.

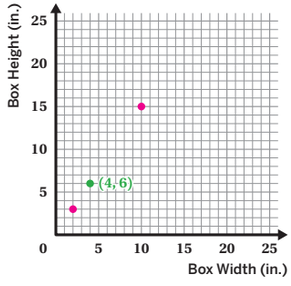


a. **Discuss:** What do you know about this box?

Responses vary. I know that this box has a width of 4 inches and a height of 6 inches.

b. Add at *least two* more points to the graph to represent other boxes that fit a TriceraPop.

Responses vary. Sample shown on graph.



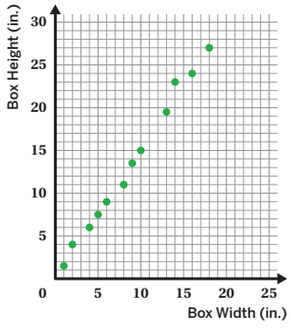
5. A student made some TriceraPop boxes, as shown on this graph.

Some of the boxes were not a good fit.

Describe how to use the graph to identify the bad boxes.

Draw on the graph if it helps to show your strategy.

Responses vary. Sketch a line that goes through the origin and most of the other points. The few points that do not fall on this line are the bad boxes.



Accelerated 7 | Unit 3 Lesson 8 253 Activity 2

Pairs | 12 min

Activity 2 TriceraPops (continued)

Purpose: Students continue to graph proportional relationships and discuss the key features that allow them to determine proportionality from a graph.

Short on time? Consider omitting Problem 6 and instead using it as a differentiation support for students needing an access point for Problem 7.

2 Monitor

Look for a variety of student thinking.

D Differentiation

Look for students who:	Respond to Student Thinking
Select "I'm not sure."	**Support** Encourage students to create a table of values and check for proportionality using their table.
Think the relationship is proportional because the points fall along a straight line.	**Support** Encourage students to test the width to height ratio for two boxes on the graph. Consider asking, "Do the two boxes form equivalent ratios?"
Think the relationship is proportional because the points are evenly spaced.	**Support** Invite students to refer to the strategies captured in Activity 1. Consider asking, "What is similar or different about this relationship?"
Determine that the relationship is not proportional because the points lie on a line that doesn't include the origin.	**Strengthen** Consider asking, "How else could you verify your answer?"

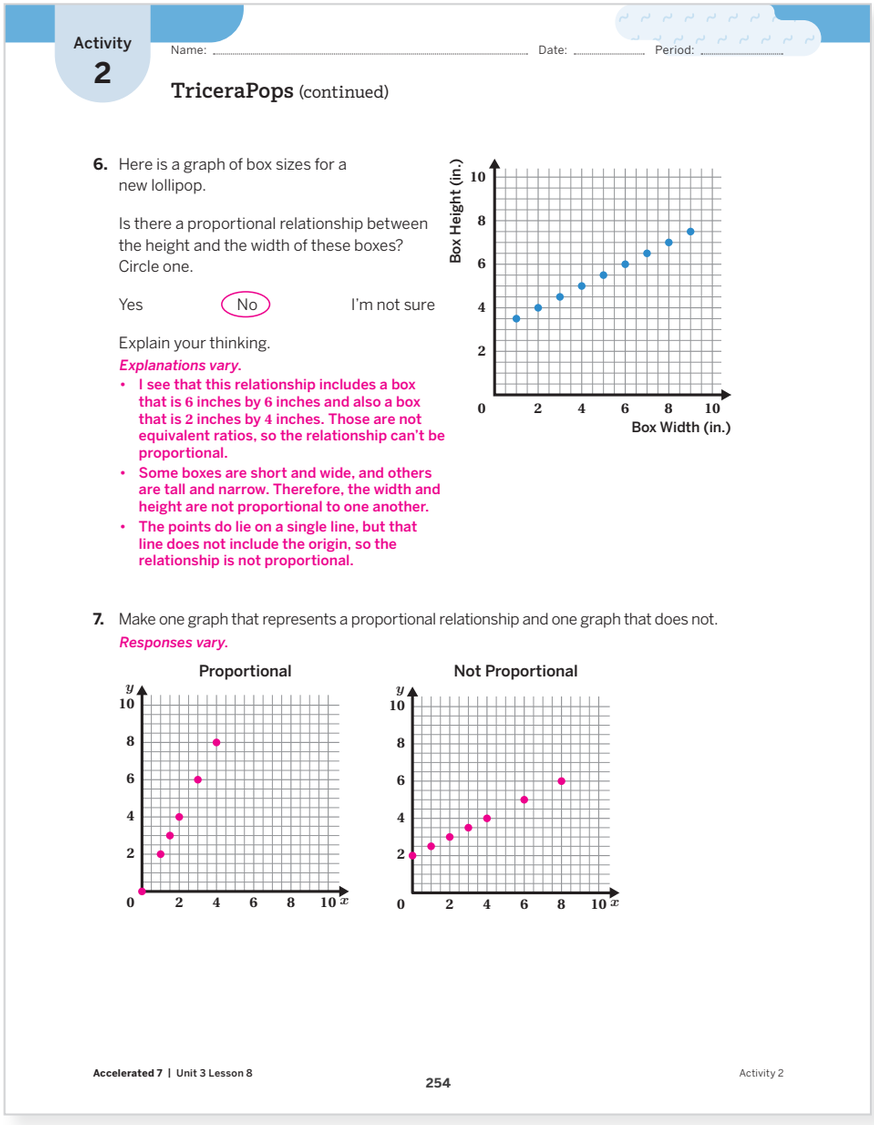
Select and sequence a variety of correct and incorrect responses to share during the Connect, such as:

- Points that lie along a curve.
- Points that are evenly or unevenly spaced.
- Points that lie along a straight line through the origin.
- Points that lie along a straight line not through the origin.

3 Connect

Consider asking, "What features do the proportional graphs have in common? What about the non-proportional graphs?" **(MTR.5.1)**

Create a class anchor chart showing the characteristics of proportional and non-proportional graphs based on the discussion.



Activity 2

Name: Date: Period:

TriceraPops (continued)

6. Here is a graph of box sizes for a new lollipop.

Is there a proportional relationship between the height and the width of these boxes? Circle one.

Yes (No) I'm not sure

Explain your thinking.

Explanations vary.

- I see that this relationship includes a box that is 6 inches by 6 inches and also a box that is 2 inches by 4 inches. Those are not equivalent ratios, so the relationship can't be proportional.
- Some boxes are short and wide, and others are tall and narrow. Therefore, the width and height are not proportional to one another.
- The points do lie on a single line, but that line does not include the origin, so the relationship is not proportional.

7. Make one graph that represents a proportional relationship and one graph that does not.

Responses vary.

Proportional

Not Proportional

Accelerated 7 | Unit 3 Lesson 8 254 Activity 2

Pairs | 10 min

Activity 3 Graphs

Purpose: Students interpret graphs and discuss whether or not they represent a proportional relationship.

1 Launch

Invite students to make sense of the relationships and share their reasoning with a partner to reach an agreement about whether each graph represents a proportional relationship.

ELL **English Language Learners** Provide a word bank to support students in expressing their ideas with words such as: origin, coordinate plane, graph, proportional, etc. **(Speaking)**

2 Monitor

Look for a variety of student thinking.

D **Differentiation**

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Encourage them to refer to the anchor chart showing the features of proportional graphs.
Associate proportionality with even spacing of points, e.g., think Graph E is proportional and Graph C is not.	**Support** Consider asking, "Do these graphs show every possible pair of values for the relationship? How can we check for proportionality even with limited information?"
Think any graph with points that fall along a straight line, e.g., Graph E, is proportional.	**Support** Encourage students to identify the coordinates of two points and check for equivalent ratios to verify their work.
Use strategies identified earlier in the lesson.	**Strengthen** Celebrate this thinking. Consider inviting these students to share during the Connect.

D **Differentiation: You're invited to explore more. (Stretch)** Invite students who would like to explore graphs of proportional relationships further to complete this optional task. Encourage them to discuss their thinking with a partner.

3 Connect

Invite students to share the strategies they used to determine whether each graph represents a proportional relationship.

Consider asking:

- "Which graphs were the most challenging? How did you decide whether the graph represents a proportional relationship?"
- "How would you describe the points in the graphs you sorted as proportional?" **(MTR.2.1)**

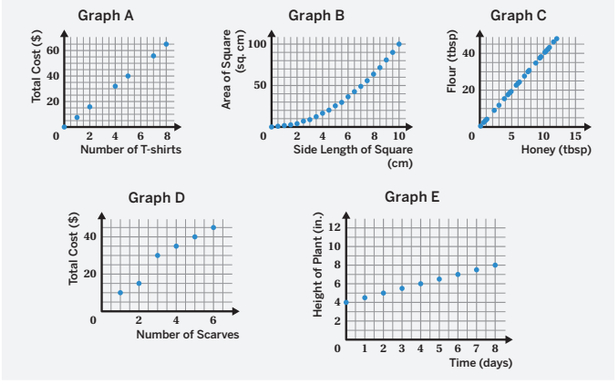
Note: If time allows, invite pairs to compare their groups, justify their thinking, and make revisions based on their conversation.

Activity 3

Name: Date: Period:

Graphs

8. Decide whether each graph represents a proportional relationship.

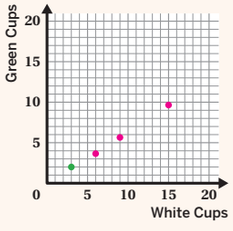


Proportional	Not Proportional
Graph A, Graph C	Graph B, Graph D, and Graph E

You're invited to explore more.

9. This color green is made by mixing 3 cups of white paint and 2 cups of green paint.

Graph points that represent *at least two* more ways to make this color.



Responses vary. Sample shown on graph.

Whole Class | 5 min

Synthesis

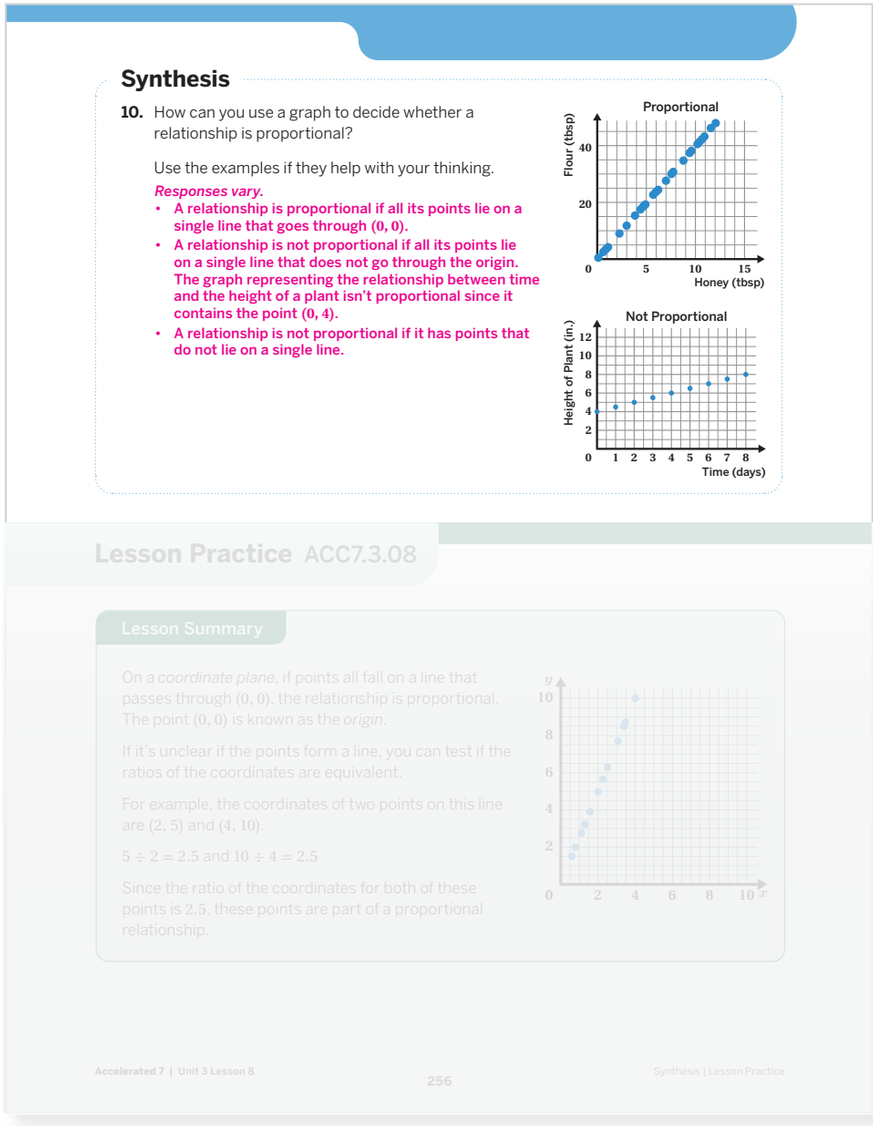
Purpose: Students synthesize their understanding of representing proportional relationships using graphs.

Invite students to respond independently, and then share their thinking with a partner.

Collect and share a variety of ideas, such as:

- Proportional relationships can be represented as points that lie on the same line.
- If the line passing through the points does not include the origin, the relationship is not proportional.
- The points on a graph of a proportional relationship do not need to be evenly spaced, as long as they lie on the same line.
- In a proportional relationship, the coordinates of points on the graph form equivalent ratios.

Note: If time allows, make additions to the anchor chart for the class and invite students to revise their previous thinking based on the discussion.



Synthesis

10. How can you use a graph to decide whether a relationship is proportional?

Use the examples if they help with your thinking.

Responses vary.

- A relationship is proportional if all its points lie on a single line that goes through (0, 0).
- A relationship is not proportional if all its points lie on a single line that does not go through the origin. The graph representing the relationship between time and the height of a plant isn't proportional since it contains the point (0, 4).
- A relationship is not proportional if it has points that do not lie on a single line.

Lesson Practice ACC7.3.08

Lesson Summary

On a *coordinate plane*, if points all fall on a line that passes through (0, 0), the relationship is proportional. The point (0, 0) is known as the *origin*.

If it's unclear if the points form a line, you can test if the ratios of the coordinates are equivalent.

For example, the coordinates of two points on this line are (2, 5) and (4, 10).

$5 \div 2 = 2.5$ and $10 \div 4 = 2.5$

Since the ratio of the coordinates for both of these points is 2.5, these points are part of a proportional relationship.

Accelerated 7 | Unit 3 Lesson 8 256 Synthesis | Lesson Practice

Lesson Takeaway: On a coordinate plane, if points all fall on a line that passes through the *origin*, the relationship is proportional.

Independent | 5 min

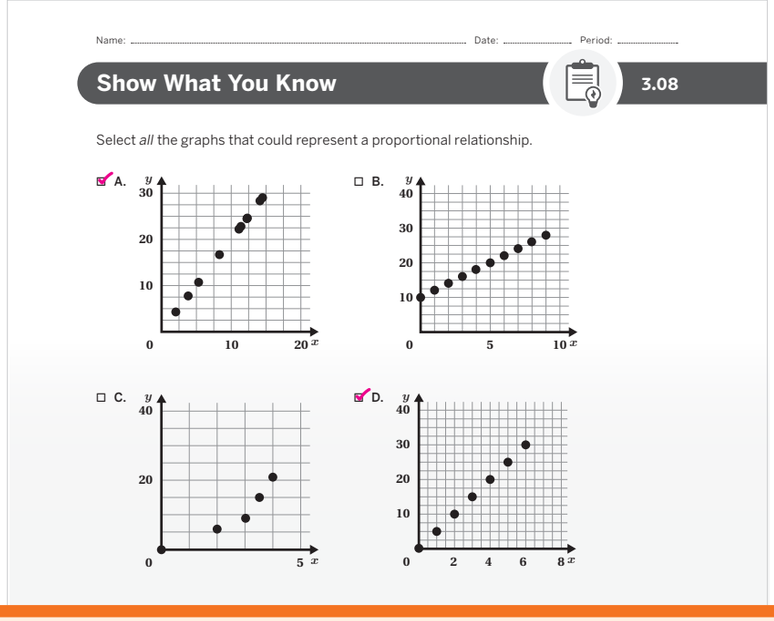
Show What You Know

Purpose: Students demonstrate their understanding by identifying graphs of proportional relationships.

Today's Goals

1. **Goal:** Use a graph that represents a proportional relationship to predict other points in that relationship.
2. **Goal:** Understand that a proportional relationship can be represented by points on a line that passes through the *origin*.
3. **Language Goal:** Justify whether a graph represents a proportional relationship. **(Reading, Writing, Speaking, and Listening)**

Show What You Know (Assessment Resources)



Name: _____ Date: _____ Period: _____

Show What You Know 3.08

Select *all* the graphs that could represent a proportional relationship.

☑ A.

☐ B.

☐ C.

☑ D.

D Differentiation Use after Lesson 8

Support

Provide targeted intervention for students by using these resources.

If students need support understanding how to determine proportionality from a graph:

» **Respond to Student Thinking:**

- Students will have more opportunities to develop their understanding of this concept in Lessons 9–10. You may choose to not intervene now.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 3 Extension Activities**.
- Invite students to complete the **Explore More** task and discuss their thinking with a partner.

Professional Learning

Take a moment to reflect on the strategies students used to sort the graphs in Activity 3. How did students use proportional relationships to justify their reasoning? How can you connect these strategies to other representations of proportional relationships?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

10. How can you use a graph to decide whether a relationship is proportional?

Use the examples if they help with your thinking.

Responses vary.

- A relationship is proportional if all its points lie on a single line that goes through (0, 0).
- A relationship is not proportional if all its points lie on a single line that does not go through the origin. The graph representing the relationship between time and the height of a plant isn't proportional since it contains the point (0, 4).
- A relationship is not proportional if it has points that do not lie on a single line.

Lesson Practice ACC7.3.08

Lesson Summary

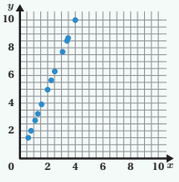
On a *coordinate plane*, if points all fall on a line that passes through (0, 0), the relationship is proportional. The point (0, 0) is known as the *origin*.

If it's unclear if the points form a line, you can test if the ratios of the coordinates are equivalent.

For example, the coordinates of two points on this line are (2, 5) and (4, 10).

$5 \div 2 = 2.5$ and $10 \div 4 = 2.5$

Since the ratio of the coordinates for both of these points is 2.5, these points are part of a proportional relationship.

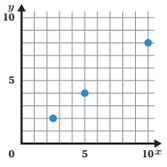


Lesson Practice ACC7.3.08

Name: _____ Date: _____ Period: _____

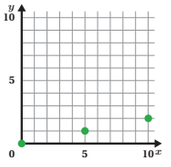
Problems 1–4: Determine if each graph represents a proportional relationship. Explain your thinking.

1.



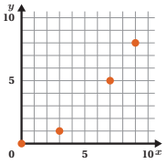
Proportional. *Explanations vary.* The line going through the points passes through the origin.

2.



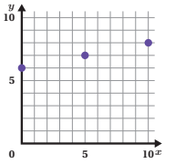
Proportional. *Explanations vary.* The line going through the points passes through the origin.

3.



Not proportional. *Explanations vary.* The points lie along a curve.

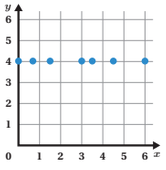
4.



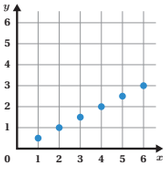
Not proportional. *Explanations vary.* The line passing through the points does not include the origin.

5. Select *all* the representations that show a proportional relationship.

☐ A.



☑ B.



☐ C.

x	0	2	4	6
y	0	3	9	27

☑ D.

x	0	2	4	6
y	0	12	24	36

Lesson Practice ACC7.3.08

Name: _____ Date: _____ Period: _____

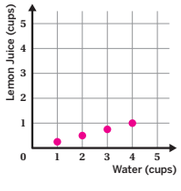
FAST Practice

6. A lemonade recipe calls for a $\frac{1}{4}$ cup of lemon juice for every cup of water. The table shows different amounts of water and lemon juice you can use to make this recipe. Graph the ordered pairs to determine whether the relationship between water and lemon juice is proportional.

Water (cups)	Lemon Juice (cups)
1	$\frac{1}{4}$
2	$\frac{1}{2}$
3	$\frac{3}{4}$
4	1

Select **ONE** correct answer in each box to complete the sentences.

The relationship [**A.** is] [**B.** is not] proportional. The points [**A.** fall] [**B.** do not fall] on a line that passes through the origin.



Spiral Review

7. Arturo earns $33.00 for babysitting for 4 hours. At this rate, how much will he earn if he babysits for 7 hours? Explain your thinking.

$57.75. *Explanations vary.* Arturo's rate is $8.25 per hour. 7 hours at that rate is $57.75.

8. A turtle is walking away from a rock. x represents the time in minutes that the turtle is walking. y represents the distance in meters between the rock and the turtle. If x and y are in a proportional relationship, select *all* the true statements.

☑ **A.** The equation $y = 3x$ could represent the distance that the turtle walks.

☐ **B.** The turtle walks for a bit and then stops for a minute before walking again.

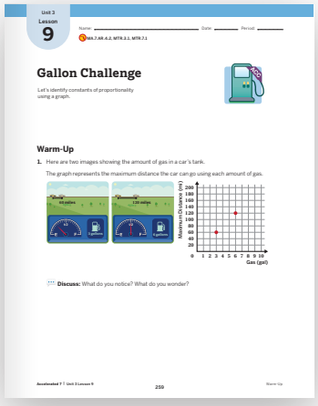
☑ **C.** The turtle walks away from the rock at a constant rate.

☐ **D.** The equation $y = x + 3$ could represent the distance that the turtle walks.

☐ **E.** After 6 minutes, the turtle walks 18 meters, and after 10 minutes, the turtle walks 20 meters.

Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1–4	1	MA.7.AR.4.1
	5	2	MA.7.AR.4.1
FAST Practice	6	2	MA.7.AR.4.1
Spiral Review			
	7	2	MA.7.AR.3.2
	8	2	MA.7.AR.4.4 MA.7.AR.4.5


Unit 3
Lesson 9
Name: Date: Period:
MA.7.AR.4.2, MTR.3.1, MTR.7.1
Gallon Challenge
Let's identify constants of proportionality using a graph.
Warm-Up
1. Here are two images showing the amount of gas in a car's tank.
The graph represents the maximum distance the car can go using each amount of gas.
Maximum Distance (mi)
Gas (gal)
Discuss: What do you notice? What do you wonder?
Accelerated 7 | Unit 3 Lesson 9
259
Warm-Up

Gallon Challenge

Interpreting Graphs of Proportional Relationships

Let's identify constants of proportionality using a graph.

Learning Outcomes

Today's Goals

1. **Goal:** Identify the constant of proportionality from the graph of a proportional relationship.
2. **Language Goal:** Interpret points on the graph of a proportional relationship in context, including $(1, k)$ where k is the constant of proportionality. **(Reading and Writing)**

Students continue to work with graphs of proportional relationships and use them to identify constants of proportionality. They analyze graphs of different vehicles' gas mileage and reason about how the concept of gas mileage is connected to the constant of proportionality.

Coherence

Prior Learning

In Lesson 8, students explored what a proportional relationship looks like graphically.

Future Learning

In Lesson 10, students will make comparisons of proportional relationships using graphs and equations. Students will also connect different representations of proportional relationships of the same situation.

Connections

- Students analyze graphs of different vehicles' gas mileage and relate gas mileage to the constant of proportionality. (real world)

FL B.E.S.T. Math Benchmarks

Addressing

MA.7.AR.4.2
Determine the constant of proportionality within a mathematical or real-world context given a table, graph or written description of a proportional relationship.

Mathematical Thinking and Reasoning Standards: MTR.3.1, MTR.7.1

ELA Expectations: ELA.K12.EE.3.1, ELA.K12.EE.4.1, ELA.K12.EE.5.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.6.AR.3.5	MA.7.AR.4.3
MA.7.AR.4.1	MA.8.AR.3.2

Lesson at a Glance

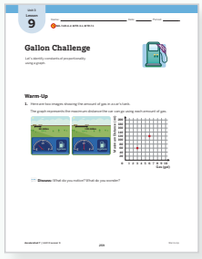
~ 45 min

MA.7.AR.4.2

Warm-Up

Pairs | 5 min

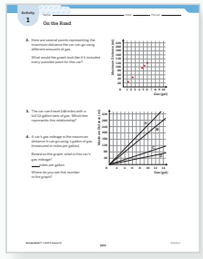
Students explore how values in a proportional relationship translate to points on a graph.
Routine: Notice and Wonder



Activity 1

Pairs | 15 min

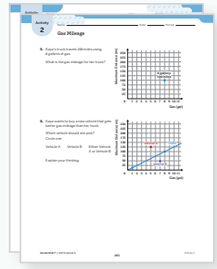
Students develop strategies for determining the gas mileage of a vehicle using its graph.
Routines: Think-Pair-Share, MLR7: Compare and Connect



Activity 2

Pairs | 15 min

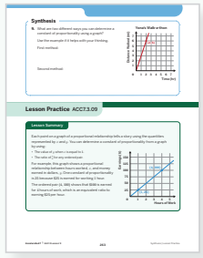
Students generalize strategies for calculating the constant of proportionality from a graph by finding the gas mileage of different vehicles.
MLR2: Collect and Display **(MTR.3.1)**



Synthesis

Whole Class | 5 min

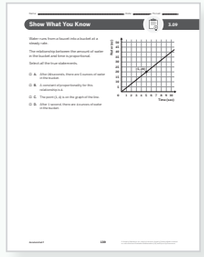
Students synthesize their understanding of determining constants of proportionality using graphs of proportional relationships.



Show What You Know

Independent | 5 min

Students demonstrate their understanding by interpreting points on a graph of a proportional relationship, including the point $(1, k)$.



Prep Checklist

This lesson includes:

- Student Edition
- Show What You Know

Pairs | 5 min

Warm-Up

Purpose: Students explore how values in a proportional relationship translate to points on a graph.

1 Launch

To surface connections, consider asking students what they know about gas in cars.

Invite students to analyze the images and the related graph.

Use the Notice and Wonder routine to promote curiosity and help students make sense of the relationship between gallons of gas and maximum distance.

2 Connect

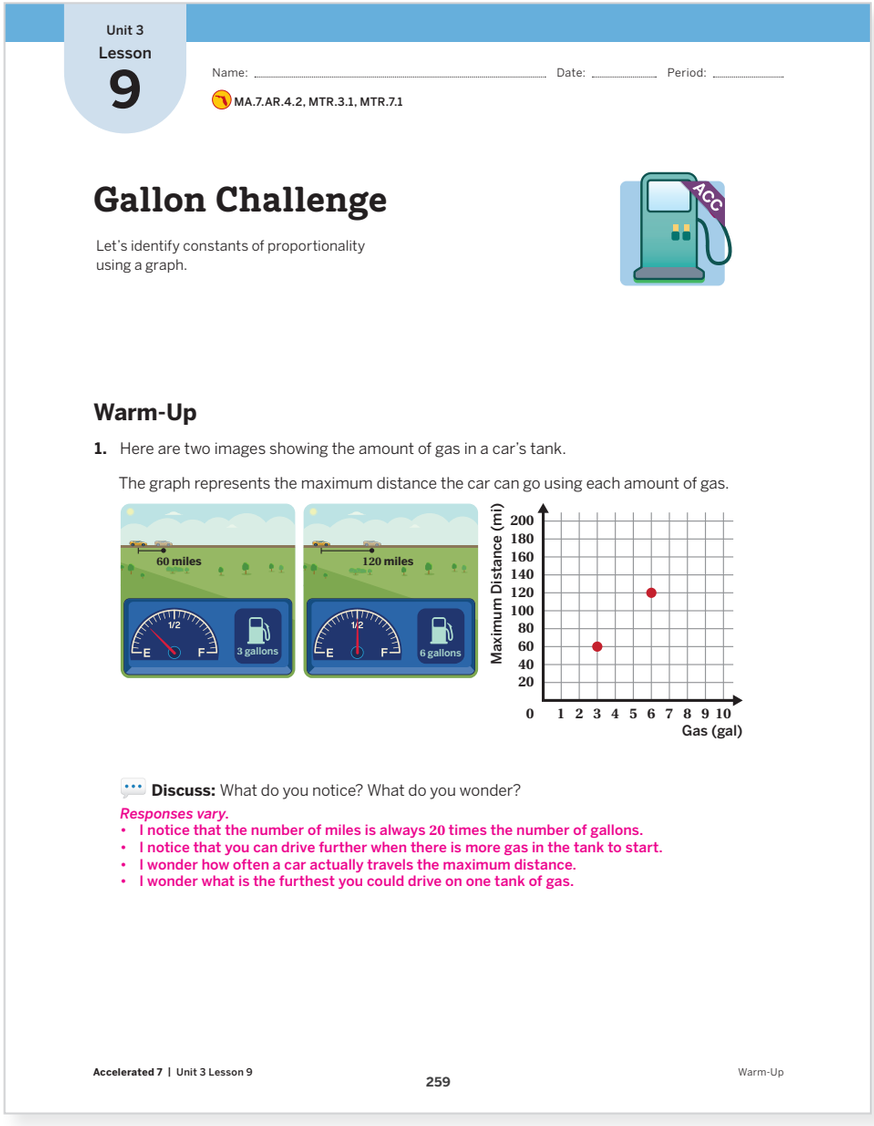
Invite students to share what they noticed and wondered.

ELL **English Language Learners** Provide sentence frames to help students explain their thinking (e.g., "I notice . . ." or "I wonder . . ."). **(Writing and Speaking)**

Note: If it doesn't come up naturally, consider asking, "What do you think the phrase *maximum distance* means?" Maximum distance is how far the car can travel before it runs out of gas, if it is running in the most efficient way possible.

To surface the Key Takeaway, consider asking:

- "How might we use the graph to predict how far the car can travel with more than 6 gallons of gas?"
- "What might be another point on the graph that fits the pattern?"



Unit 3
Lesson
9

Name: _____ Date: _____ Period: _____

MA.7.AR.4.2, MTR.3.1, MTR.7.1

Gallon Challenge

Let's identify constants of proportionality using a graph.

Warm-Up

1. Here are two images showing the amount of gas in a car's tank.

   The graph represents the maximum distance the car can go using each amount of gas.

Discuss: What do you notice? What do you wonder?

Responses vary.

- I notice that the number of miles is always 20 times the number of gallons.
- I notice that you can drive further when there is more gas in the tank to start.
- I wonder how often a car actually travels the maximum distance.
- I wonder what is the furthest you could drive on one tank of gas.

Accelerated 7 | Unit 3 Lesson 9 259 Warm-Up

Key Takeaway: A graph is a way to collect all the possible points that make a relationship true.

Pairs | 15 min

Activity 1 On the Road

Purpose: Students develop strategies for determining the gas mileage of a vehicle using its graph.

Short on time? Consider omitting the Think-Pair-Share routine and inviting students to explore Problems 2–4 in partners.

1 Launch

Use the Think-Pair-Share routine to help students shape their ideas about representing relationships with graphs.

ELL **English Language Learners** Invite students to share strategies in their primary languages. Encourage all students to use gestures and/or visuals as they share. **(Speaking and Listening)**

2 Monitor

Listen for students who mention the importance of the point $(12, 240)$.

Select and sequence explanations to share during the Connect.

D Differentiation

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Consider asking, "Where on the graph might you find information about 1 gallon of gas?"
Look for the maximum distance when the number of gallons is 1.	Select and sequence these to share during the Connect.
Divide the maximum distance by the number of gallons, e.g., $\frac{240}{12}$.	**Strengthen** Consider asking, "What other points on the graph could you use to find the gas mileage?"

3 Connect

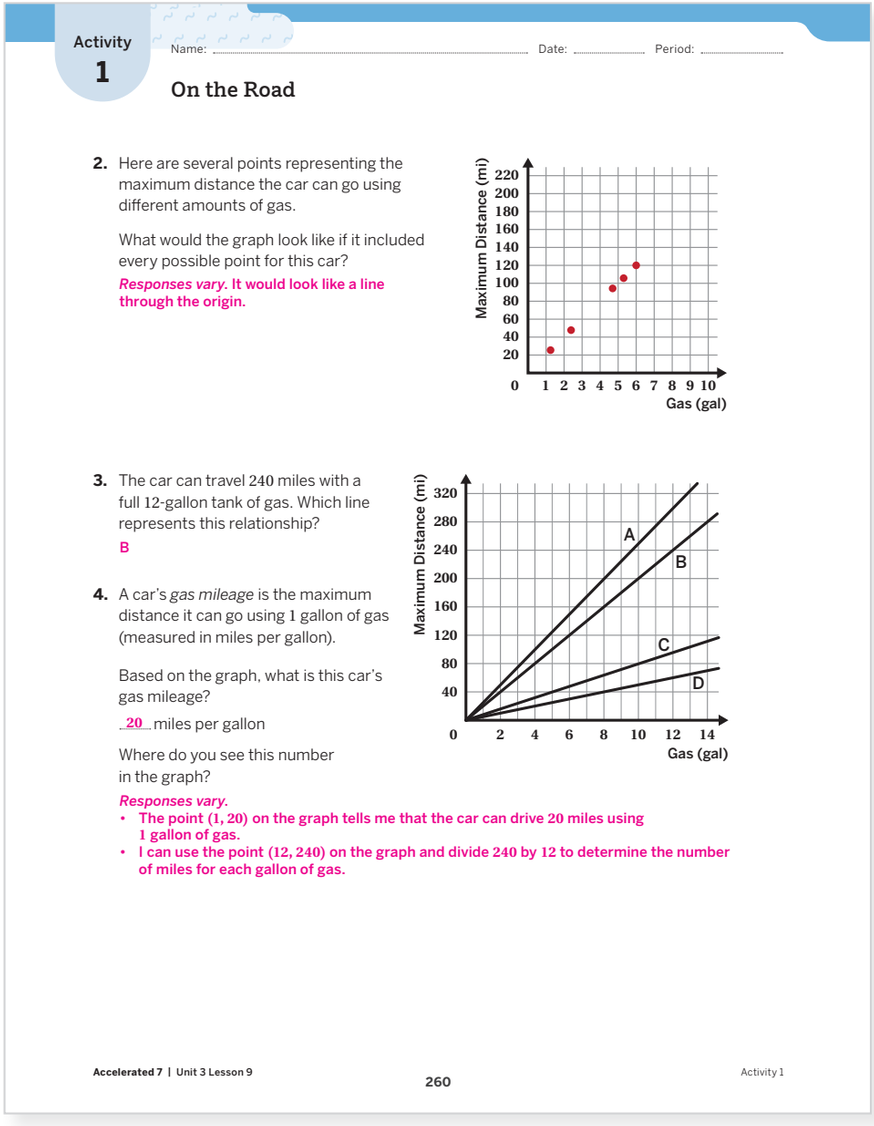
Invite students to share their strategies, ensuring at least two different strategies are showcased.

MLR **MLR7: Compare and Connect** Invite students to analyze and compare each strategy, and then connect to their own strategy.

ELL **English Language Learners** To support students in sharing their thinking, consider displaying sentence frames such as, "First, they _____. Next, they _____. Finally, _____." or "I see ____ in both strategies." **(Writing and Speaking)**

Consider asking:

- "How are these strategies alike? How are they different?"
- "Why did the two strategies lead to the same outcome?"



Activity 1

Name: Date: Period:

On the Road

2. Here are several points representing the maximum distance the car can go using different amounts of gas.

What would the graph look like if it included every possible point for this car?

Responses vary. It would look like a line through the origin.

3. The car can travel 240 miles with a full 12-gallon tank of gas. Which line represents this relationship?

B

4. A car's *gas mileage* is the maximum distance it can go using 1 gallon of gas (measured in miles per gallon).

Based on the graph, what is this car's gas mileage?

20 miles per gallon

Where do you see this number in the graph?

Responses vary.

- The point (1, 20) on the graph tells me that the car can drive 20 miles using 1 gallon of gas.
- I can use the point (12, 240) on the graph and divide 240 by 12 to determine the number of miles for each gallon of gas.

Accelerated 7 | Unit 3 Lesson 9 260 Activity 1

Key Takeaway: You can use different strategies to find the constant of proportionality from a graph. You can look for the quantity of y when x is 1, or choose a point and divide the y-coordinate by the x-coordinate.

Pairs | 15 min

Activity 2 Gas Mileage

Purpose: Students generalize strategies for calculating the constant of proportionality from a graph by finding the gas mileage of different vehicles. **(MTR.3.1)**

1 Launch

To surface connections, share that in this activity, students will compare the gas mileage of different vehicles.

Invite students to think back to the previous activity and restate the meaning of gas mileage in their own words.

Consider asking:

- "What does it mean for one vehicle to have better gas mileage than another?"
- "What kind of vehicles would you expect to have the best gas mileage? What about the worst?"

2 Monitor

Encourage students to use a strategy from Activity 1 to calculate the gas mileage of Kaya's truck. **(MTR.7.1)**

Look for a variety of student strategies.

D Differentiation

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Consider asking, "What are the coordinates of the point for Vehicle A? What can that tell you about the gas mileage?"
Think that either vehicle will be better than the truck because the points appear to the left of (8, 100).	**Support** Celebrate the structure these students noticed in the x-coordinates of the points. Encourage them to consider how the y-coordinate of each point affects gas mileage.
Think that Vehicle A has better gas mileage because the point is above the line for the truck.	**Strengthen** Celebrate the structure these students noticed. Consider asking, "How do you know that a point higher than the line represents better gas mileage?"
Draw a line to represent each vehicle and compare the maximum distances for the same number of gallons.	**Stretch** Celebrate this strategy. Consider asking, "How might you calculate the exact gas mileage for each vehicle?"

Activity 2 continued >

Activity 2

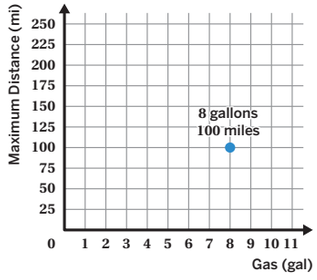
Name: Date: Period:

Gas Mileage

5. Kaya's truck travels 100 miles using 8 gallons of gas.

What is the gas mileage for her truck?

12.5 miles per gallon



6. Kaya wants to buy a new vehicle that gets better gas mileage than her truck.

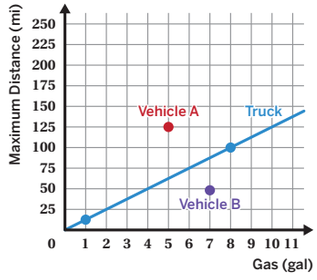
Which vehicle should she pick? Circle one.

(Vehicle A) Vehicle B Either Vehicle A or Vehicle B

Explain your thinking.

Explanations vary.

- Vehicle A's gas mileage is 25 miles per gallon. Vehicle B's gas mileage is less than 25 miles per gallon because the line for Vehicle B goes through a point lower on the graph than (1, 25).
- Using Vehicle A, you can go 125 miles using 5 gallons of gas. Using Vehicle B, you can't even go 75 miles.



Accelerated 7 | Unit 3 Lesson 9 261 Activity 2

Pairs | 15 min

Activity 2 Gas Mileage (continued)

Purpose: Students generalize strategies for calculating the constant of proportionality from a graph by finding the gas mileage of different vehicles. **(MTR.3.1)**

2 Monitor

Encourage students to try different strategies until they find one that is consistently successful, and to verify their work by finding the constant of proportionality in multiple ways. **(MTR.3.1)**

A **Accessibility: Conceptual Processing** Invite students to describe what they have tried to support self-monitoring of understanding.

Look for students who find the constant of proportionality by looking for the maximum distance corresponding to 1 gallon of gas or by dividing the y-value of a point on the line by the x-value of the same point. Consider inviting these students to share during the Connect.

D **Differentiation: (Stretch)** Invite students who would like to explore comparing proportional relationships further to compare the fuel efficiencies of the three vehicles to a car that gets 20 miles per gallon and carries 1–4 people.

3 Connect

Invite students to share their strategies for finding constants of proportionality given a graph.

ML/EL **MLR2: Collect and Display** Use this routine to collect students' ideas, language, and strategies as they share during the Connect. Display for students to reference and add to for the rest of the unit.

Consider asking:

- "How is the constant of proportionality like the gas mileage in this situation?"
- "Who used a different strategy and arrived at the same constant of proportionality?"
- "Which strategy was easier to use for each vehicle? Why?"

Activity 2

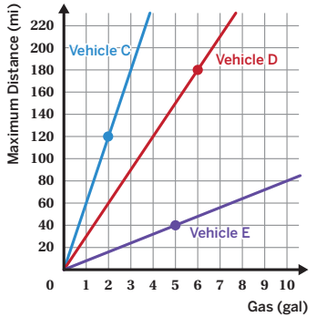
Name: Date: Period:

Gas Mileage (continued)

7. In the relationship between amount of gas and maximum distance, the vehicle's gas mileage is a constant of proportionality.

What is that constant of proportionality for each vehicle?

Vehicle	Constant of Proportionality (miles per gallon)
C	60
D	30
E	8



8. Here is the maximum distance traveled by three vehicles using a certain amount of gas.

Plane	Cruise Ship	Train
Note: The average plane has 100 passengers.	Note: The average cruise ship has 3000 passenger.	Note: The average train has 300 passengers.

Compare the fuel efficiency for each vehicle.

Responses vary. The plane gets 0.5 miles per gallon and carries 100 passengers. The cruise ship gets 0.02 miles per gallon and carries 3000 passengers. The train gets 0.2 miles per gallon and carries 300 passengers. That means a full cruise ship and train can carry passengers the same distance per gallon of gas, while the plane is less fuel efficient.

Accelerated 7 | Unit 3 Lesson 9 262 Activity 2

Whole Class | 5 min

Synthesis

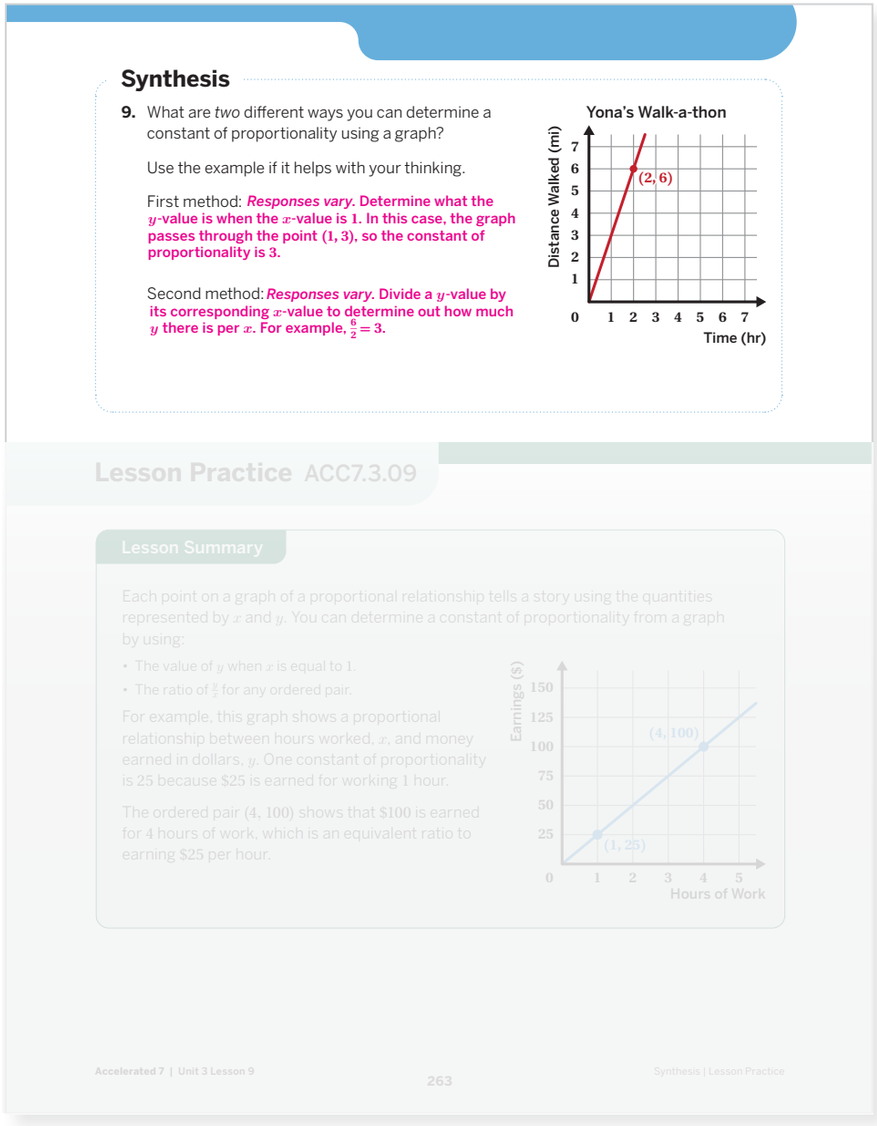
Purpose: Students synthesize their understanding of determining constants of proportionality using graphs of proportional relationships.

Invite students to respond independently, and then share their thinking with a partner.

Collect and share a variety of ideas, such as:

- Looking for the y-value when the x-value is 1.
- Choosing a point and dividing the y-value by the x-value.

Note: If time allows, invite students to share with a partner what makes sense about each strategy and what questions they still have.



Synthesis

9. What are *two* different ways you can determine a constant of proportionality using a graph?

Use the example if it helps with your thinking.

First method: ***Responses vary.*** **Determine what the y-value is when the x-value is 1. In this case, the graph passes through the point (1, 3), so the constant of proportionality is 3.**

Second method: ***Responses vary.*** **Divide a y-value by its corresponding x-value to determine out how much y there is per x. For example, $\frac{6}{2} = 3$.**

Lesson Practice ACC7.3.09

Lesson Summary

Each point on a graph of a proportional relationship tells a story using the quantities represented by x and y. You can determine a constant of proportionality from a graph by using:

- The value of y when x is equal to 1.
- The ratio of $\frac{y}{x}$ for any ordered pair.

For example, this graph shows a proportional relationship between hours worked, x, and money earned in dollars, y. One constant of proportionality is 25 because \$25 is earned for working 1 hour.

The ordered pair (4, 100) shows that \$100 is earned for 4 hours of work, which is an equivalent ratio to earning \$25 per hour.

Accelerated 7 | Unit 3 Lesson 9 263 Synthesis | Lesson Practice

Lesson Takeaway: A graph can be used to determine the constant of proportionality and make comparisons among quantities of proportional relationships.

Independent | 5 min

Show What You Know

Purpose: Students demonstrate their understanding by interpreting points on a graph of a proportional relationship, including the point $(1, k)$.

Today's Goals

1. **Goal:** Identify the constant of proportionality from the graph of a proportional relationship.
2. **Language Goal:** Interpret points on the graph of a proportional relationship in context, including $(1, k)$ where k is the constant of proportionality. **(Reading and Writing)**

Show What You Know (Assessment Resources)

Name: _____ Date: _____ Period: _____

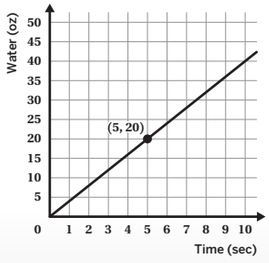
Show What You Know 3.09

Water runs from a faucet into a bucket at a steady rate.

The relationship between the amount of water in the bucket and time is proportional.

Select *all* the true statements.

- ☐ A. After 20 seconds, there are 5 ounces of water in the bucket.
- ☑ B. A constant of proportionality for this relationship is 4.
- ☑ C. The point $(1, 4)$ is on the graph of the line.
- ☑ D. After 1 second, there are 4 ounces of water in the bucket.



Differentiation Use after Lesson 9

Support

Provide targeted intervention for students by using these resources.

If students need support understanding how to determine constants of proportionality from graphs:

» **Respond to Student Thinking:**

- Select a few related **Practice Problems** and give individualized feedback.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 3 Extension Activities**.
- If students would like to explore the context further, Invite them to research real cars and consider whether the relationship between gas and distance traveled is always proportional.

Professional Learning

In this lesson, students are introduced to graphs of proportional relationships as lines rather than collections of points. Why was the context of gas mileage particularly useful in building students' understanding of graphs? How did it support students deepening their understanding of constants of proportionality?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

9. What are *two* different ways you can determine a constant of proportionality using a graph?

Use the example if it helps with your thinking.

First method: *Responses vary.* Determine what the y-value is when the x-value is 1. In this case, the graph passes through the point $(1, 3)$, so the constant of proportionality is 3.

Second method: *Responses vary.* Divide a y-value by its corresponding x-value to determine out how much y there is per x. For example, $\frac{6}{2} = 3$.

Lesson Practice ACC7.3.09

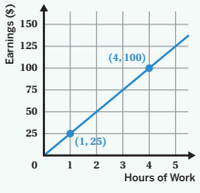
Lesson Summary

Each point on a graph of a proportional relationship tells a story using the quantities represented by x and y. You can determine a constant of proportionality from a graph by using:

- The value of y when x is equal to 1.
- The ratio of $\frac{y}{x}$ for any ordered pair.

For example, this graph shows a proportional relationship between hours worked, x, and money earned in dollars, y. One constant of proportionality is 25 because \$25 is earned for working 1 hour.

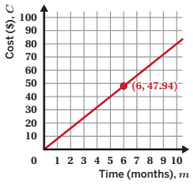
The ordered pair $(4, 100)$ shows that \$100 is earned for 4 hours of work, which is an equivalent ratio to earning \$25 per hour.



Lesson Practice ACC7.3.09

Name: ..... Date: ..... Period: .....

Problems 1–2: Here is a graph that shows a proportional relationship between the number of months Tiara had a streaming service subscription and the total amount of money she paid for the subscription.



After 6 months, Tiara paid \$47.94.

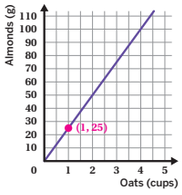
1. What is a constant of proportionality in this relationship?

7.99 (or equivalent) or $\frac{1}{7.99}$ (or equivalent)

2. Write an equation that represents the relationship between C, the total cost of the subscription, and m, the number of months.

$C = 7.99m$ or $m = \frac{1}{7.99}C$ (or equivalent)

Problems 3–4: A recipe for granola calls for a mix of almonds and oats. The graph shows the amount of almonds, in grams, for different amounts of oats, in cups.



3. Determine a constant of proportionality for this relationship, then explain its meaning.

25 or $\frac{25}{1}$. *Explanations vary.* The graph contains the point (2, 50), which gives the ratio $\frac{50}{2} = 25$. This means 25 grams of almonds are needed for every 1 cup of oats.

4. Label one place you see that constant of proportionality on the graph.

5. The graph shows the cost for two different varieties of apples. Which variety of apples has a higher cost per pound?

Gala

Explain your thinking.

Explanations vary. A constant of proportionality for Gala apples is 3 and a constant of proportionality for Red Delicious apples is 1. That means Gala apples have a higher unit price (\$3 per pound) than Red Delicious apples (\$1 per pound).

Lesson Practice ACC7.3.09

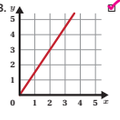
Name: ..... Date: ..... Period: .....

FAST Practice

6. Which of these relationships has the same constant of proportionality as the equation $y = \frac{2}{3}x$? Select *all* that apply.

☑ A.

☐ B.



☑ C.

x	y
0	0
3	2
6	4
9	6

☐ D.

x	y
0	0
2	3
4	6
6	9

Spiral Review

7. Write an expression that is equivalent to $8(n + 6)$.

Responses vary. $8n + 48$

Problems 8–10: Haru and Irene were running laps around the track. Their coach recorded their times at the end of laps 2, 4, and 6.

Haru's Run

Distance (laps)	Time (min)	Minutes Per Lap
2	4	2
4	9	2.25
6	15	2.5

Irene's Run

Distance (laps)	Time (min)	Minutes Per Lap
2	5	2.5
4	10	2.5
6	15	2.5

8. Complete the tables with the minutes per lap for each run.

9. Based on the table, is Haru running at a constant speed? Explain your thinking.

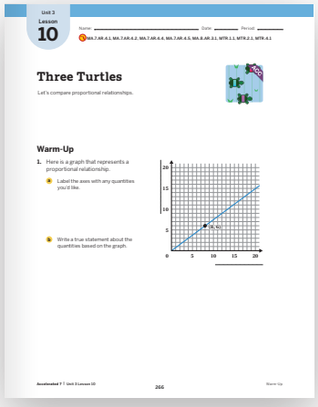
No. *Explanations vary.* Haru is not running at a constant speed because the number of minutes per lap is not the same for each row in his table.

10. Based on the table, is Irene running at a constant speed? Explain your thinking.

Yes. *Explanations vary.* Irene may be running at a constant speed because the number of minutes per lap is the same value for each row of her table. To be more certain, I'd also want to know her times for laps 1, 3, and 5.

Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1–5	2	MA.7.AR.4.2
FAST Practice	6	2	MA.7.AR.4.2
Spiral Review			
	7	1	MA.6.AR.1.4
	8	1	MA.6.AR.3.2
	9–10	2	MA.7.AR.4.2



Three Turtles

Proportional Relationships and Graphs

Let's compare proportional relationships.

Learning Outcomes

Today's Goals

1. **Goal:** Write an equation of a proportional relationship given a point on a graph.
2. **Language Goal:** Compare proportional relationships using their graphs and equations. **(Reading and Writing)**

Students use graphs and equations to compare proportional relationships. They make connections between graphs, equations, and the context of turtles in a race. They reason abstractly as they use graphs and equations to make claims about the turtles based on the steepness of a line or a constant of proportionality. **(MTR.2.1)**

Coherence

Prior Learning

In Lessons 8 and 9, students identified graphs of proportional relationships and connected graphs of these relationships to constants of proportionality.

Future Learning

In Lesson 11, students will create their own representations of proportional relationships (equations, tables, graphs, and descriptions).

Connections

- Students compare and contrast different representations of proportional relationships in real world contexts such as analyzing the results of distance/time relationships of different runners. (real world)

FL B.E.S.T. Math Benchmarks

Addressing

MA.7.AR.4.4
Given any representation of a proportional relationship, translate the representation to a written description, table or equation.

MA.7.AR.4.5
Solve real-world problems involving proportional relationships.

Also Addressing: **MA.7.AR.4.1, MA.7.AR.4.2, MA.8.AR.3.1**

Mathematical Thinking and Reasoning Standards: MTR.1.1, MTR.2.1, MTR.4.1

ELA Expectations: ELA.K12.EE.1.1, ELA.K12.EE.3.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.6.AR.1.1	**MA.8.AR.3.2**
MA.6.AR.3.2	**MA.8.AR.3.3**

Lesson at a Glance

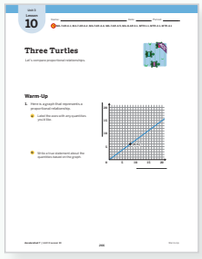
~ 45 min

MA.7.AR.4.1, MA.7.AR.4.2, MA.7.AR.4.4, MA.7.AR.4.5, MA.8.AR.3.1

Warm-Up

Pairs | 5 min

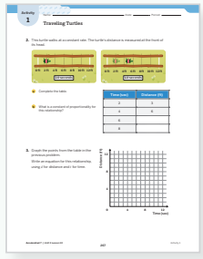
Students use their creativity to revisit the meaning of a point on a graph of a proportional relationship.



Activity 1

Pairs | 5 min

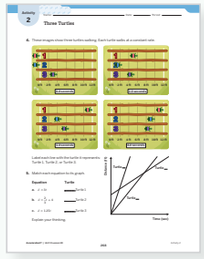
Students connect information in the graph, table, and equation of a single proportional relationship.



Activity 2

Pairs | 10 min

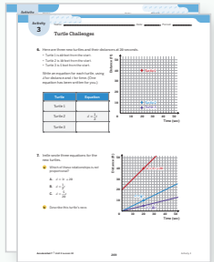
Students use graphs and equations to decontextualize a situation about three turtles in a race. **Routine:** Notice and Wonder **(MTR.2.1)**



Activity 3

Pairs | 15 min

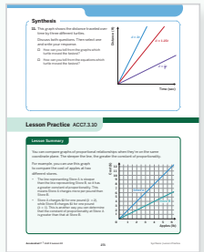
Students create and compare representations of proportional relationships. They also revisit how to differentiate between relationships that are proportional and not proportional.



Synthesis

Whole Class | 5 min

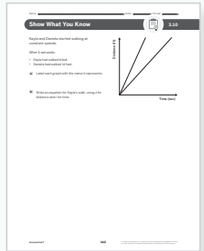
Students synthesize their ideas about using graphs and equations to compare proportional relationships.



Show What You Know

Independent | 5 min

Students demonstrate their understanding by labeling two relationships on a graph and writing an equation.



Prep Checklist

Prepare Activity 3 sheets.

This lesson includes:

- Student Edition
- Show What You Know
- Activity 3 You're Invited to Explore More Sheet, one per student (as needed)
- Straightedges (as needed)

Pairs | 5 min

Warm-Up

Purpose: Students use their creativity to revisit the meaning of a point on a graph of a proportional relationship.

Short on time? Consider omitting this Warm-Up or doing it as a class.

1 Launch

To surface prior knowledge, consider asking, "Is this a graph of a proportional relationship? How do you know?"

To encourage creativity, invite students to think of axes labels related to something they care about or different from what they think their classmates will choose.

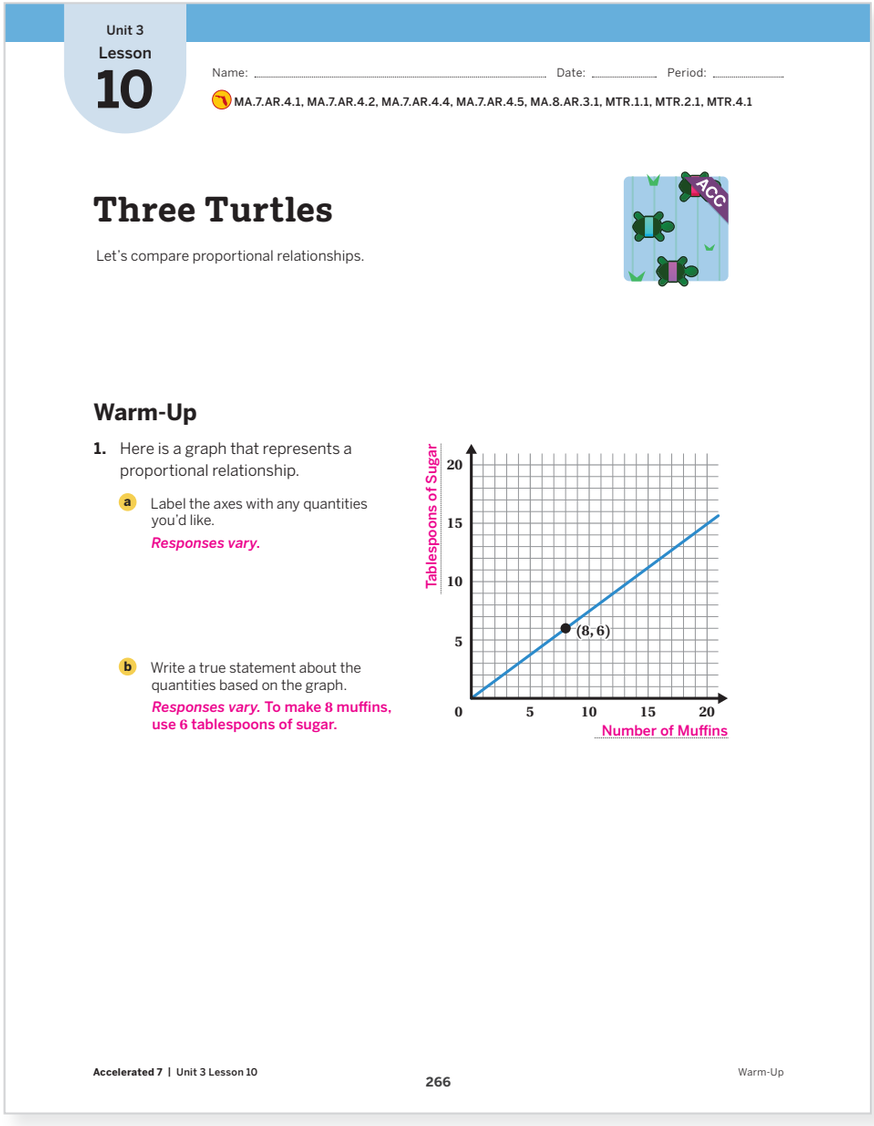
2 Connect

Look for a variety of graphs and statements, including ones that mention a constant of proportionality ($\frac{3}{4}$ or $\frac{4}{3}$).

Invite students to share their statements and to explain their reasoning.

Consider asking:

- "Where do you see the 8 and the 6 in _____'s statement? What do they mean?"
- "How were our statements alike? Different?"



Unit 3
Lesson
10

Name: _____ Date: _____ Period: _____

MA.7.AR.4.1, MA.7.AR.4.2, MA.7.AR.4.4, MA.7.AR.4.5, MA.8.AR.3.1, MTR.1.1, MTR.2.1, MTR.4.1

Three Turtles

Let's compare proportional relationships.

Warm-Up

1. Here is a graph that represents a proportional relationship.

   a. Label the axes with any quantities you'd like.

   Responses vary.

   b. Write a true statement about the quantities based on the graph.

   Responses vary. To make 8 muffins, use 6 tablespoons of sugar.

Accelerated 7 | Unit 3 Lesson 10 266 Warm-Up

Pairs | 5 min

Activity 1 Traveling Turtles

Purpose: Students connect information in the graph, table, and equation of a single proportional relationship.

1 Launch

Direct students' attention to the turtle's position in the image.

Invite students to measure the turtle's distance at 2 and 4 seconds.

2 Monitor

Look for students who write each of the two constants of proportionality ($\frac{3}{2}$ and $\frac{2}{3}$).

Encourage students to look for connections between the table and the graph before writing their equation. **(MTR.2.1)**

D Differentiation

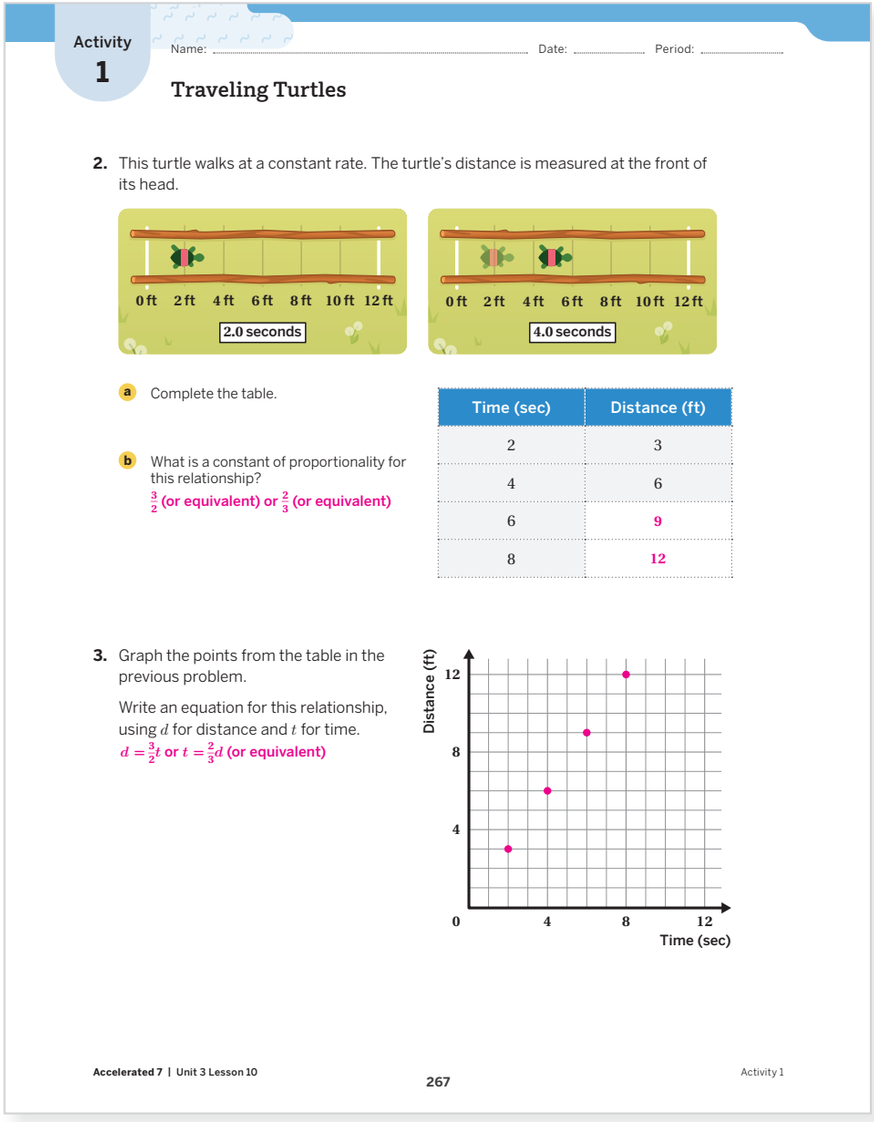
Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Encourage them to revisit their work from earlier lessons. Consider asking, "What do equations of proportional relationships typically look like?"
Write $t = \frac{3}{2}d$ or a different equation with the variables swapped.	**Support** Invite them to test the points from the table or graph in the equation, then to revise their thinking.
Use a variety of strategies, such as: • Noticing that every 2 seconds, the turtle travels 3 feet. • Using a constant of proportionality in their equation. • Testing the equation using the table or graph.	**Strengthen** Invite students to think about how they would share their strategy during the Connect. **Stretch** Invite students to write a *different* equation for this proportional relationship.

3 Connect

Invite students to share connections they see between the representations of the turtle situation.

Consider asking:

- "Where do you see the $\frac{3}{2}$ in the table? In the equation? In the graph?"
- "What might the graph and equation look like for a turtle who is moving faster? What about a turtle who is moving more slowly?"



Activity 1

Name: Date: Period:

Traveling Turtles

2. This turtle walks at a constant rate. The turtle's distance is measured at the front of its head.

a. Complete the table.

b. What is a constant of proportionality for this relationship?
$\frac{3}{2}$ (or equivalent) or $\frac{2}{3}$ (or equivalent)

Time (sec)	Distance (ft)
2	3
4	6
6	9
8	12

3. Graph the points from the table in the previous problem.

Write an equation for this relationship, using d for distance and t for time.
$d = \frac{3}{2}t$ or $t = \frac{2}{3}d$ (or equivalent)

Accelerated 7 | Unit 3 Lesson 10 267 Activity 1

Pairs | 10 min

Activity 2 Three Turtles

Purpose: Students use graphs and equations to decontextualize a situation about three turtles in a race. **(MTR.2.1)**

1 Launch

Use the Notice and Wonder routine to promote curiosity and help students notice features of the turtle race that will support them as they connect the movement of the turtles with graphs and equations. **(MTR.1.1)**

Note: This may be the first time students have seen a coordinate plane without a grid.

2 Monitor

Listen for students who make connections between the steepness of the graph and the speed of the turtle or the starting point of the turtle and the value at 0 on the graph.

A **Accessibility: Visual-Spatial Processing** Invite students to trace each graph with a pencil, then discuss how they are alike and different.

Select and sequence graphs and explanations to share during the Connect.

D **Differentiation**

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Encourage them to test out different values of t in each equation, like $t = 0$ and $t = 1$.
Swap the two proportional equations and graphs.	**Support** Consider asking, "How could you test to see if this equation makes sense with the graphs and turtles?"
Make connections between the steepness of the graph and the constant of proportionality.	**Strengthen** Consider inviting them to revise their response to convince a friend who made different matches.
Recognize that the turtle with a head start does not represent a proportional relationship.	**Stretch** Consider asking, "What would the turtle's race whose equation is $d = \frac{2}{3}t$ look like?

3 Connect

Share student graphs and explanations. If there is conflict, consider sharing 1–2 responses for each choice, and invite those students to share their reasoning.

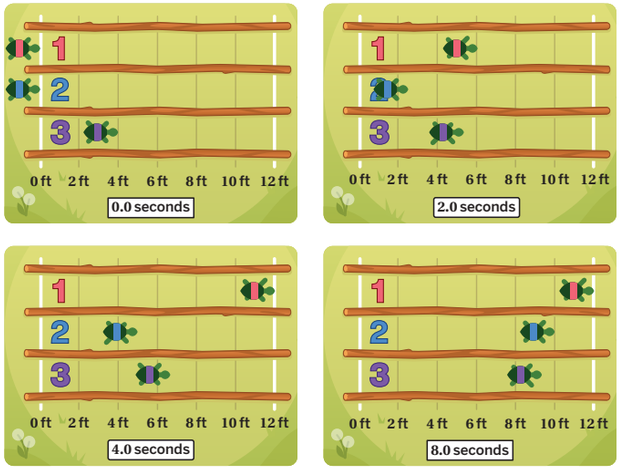
To surface the Key Takeaway, consider asking, "How do you see the information from each equation in its graph?"

Activity 2

Name: _____ Date: _____ Period: _____

Three Turtles

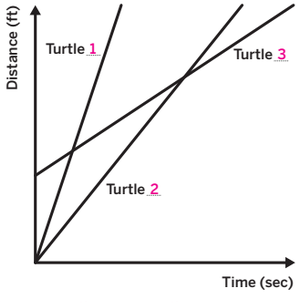
4. These images show three turtles walking. Each turtle walks at a constant rate.



Label each line with the turtle it represents: Turtle 1, Turtle 2, or Turtle 3.

5. Match each equation to its graph.

Equation	Turtle
a. $d = 3t$	a Turtle 1
b. $d = \frac{2}{3}t + 4$	c Turtle 2
c. $d = 1.25t$	b Turtle 3



Explain your thinking.

Explanations vary. The turtle with a head start has an equation representing a relationship that is not proportional. The slower proportional turtle matches the equation that has the smaller constant of proportionality.

Accelerated 7 | Unit 3 Lesson 10 268 Activity 2

Key Takeaway: We can use graphs and equations to represent situations in context. Steeper graphs and equations with larger constants of proportionality represent a faster rate. Graphs that do not start at $(0, 0)$ or have equations that look like $d =$ _____ $t +$ _____ represent a head start.

Pairs | 15 min

Activity 3 Turtle Challenges

Purpose: Students create and compare representations of proportional relationships. They also revisit how to differentiate between relationships that are proportional and not proportional.

Short on time? Consider omitting Problem 7.

1 Launch

Direct students' attention to the instructions and consider clarifying them as a class.

To surface prior knowledge, consider asking, "Where can you see on the graph that Turtle 1 is 40 feet from the start after 20 seconds?" **(MTR.2.1)**

2 Monitor

Listen for a variety of strategies students use to write their equations and respond individually.

D Differentiation

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Encourage them to sketch a line they think represents Turtle 1. Consider asking, "Where is the constant of proportionality in this line?"
Write an equation that does not go through the point on the graph.	**Support** Consider asking, "How could you test to see if this equation makes sense with the graphs and turtles?"
Use strategies such as: • Drawing a line and looking for $(1, k)$. • Calculating a constant of proportionality. • Writing the equation of a line not through the origin.	**Strengthen** Consider asking, "Is this the only equation of a line that could represent this turtle?"

Encourage students to read others' responses and decide if others' strategies were similar to or different from their own.

Activity 3 continued >

Activity 3

Name: Date: Period:

Turtle Challenges

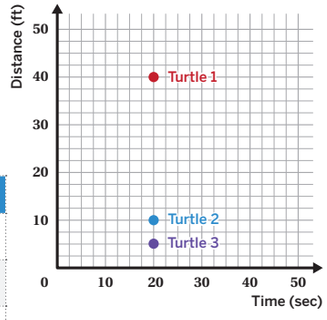
6. Here are three new turtles and their distances at 20 seconds.

- Turtle 1 is 40 feet from the start.
- Turtle 2 is 10 feet from the start.
- Turtle 3 is 5 feet from the start.

Write an equation for each turtle, using d for distance and t for time. (One equation has been written for you.)

Responses vary.

Turtle	Equation
Turtle 1	$d = 2t$ $d = 30 + \frac{t}{2}$
Turtle 2	$d = \frac{1}{2}t$
Turtle 3	$d = \frac{1}{4}t$ $d = 25 - t$



7. Irelle wrote three equations for the new turtles.

a. Which of these relationships is *not* proportional?

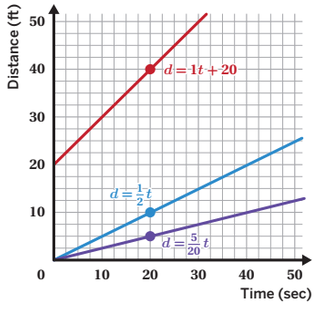
A. $d = 1t + 20$

B. $d = \frac{1}{2}t$

C. $d = \frac{5}{20}t$

b. Describe this turtle's race.

Responses vary. This turtle began the race with a 20-foot head start, and then traveled 1 foot per second for 30 seconds.



Accelerated 7 | Unit 3 Lesson 10 269 Activity 3

Pairs | 15 min

Activity 3 Turtle Challenges (continued)

Purpose: Students create and compare representations of proportional relationships. They also revisit how to differentiate between relationships that are proportional and not proportional.

2 Monitor

Encourage students to share their reasoning with a partner and work together to reach an agreement about how to sort the representations.

A **Accessibility: Executive Function** Invite students to focus on one representation at a time to support students in carrying out multiple steps.

ELL **English Language Learners** Provide a word bank to support students in expressing their ideas with words such as: *table, graph, equation, constant of proportionality*, etc. **(Speaking)**

Note: If time allows, invite pairs to compare their groups, justify their thinking, and make revisions based on their conversation. **(MTR.4.1)**

Encourage students to use straightedges to sketch their own lines to meet the criteria. They can add more than 3 lines or erase and draw new lines if they would like.

To encourage creativity, consider inviting students to imagine a turtle race and then try to represent it on their graphs.

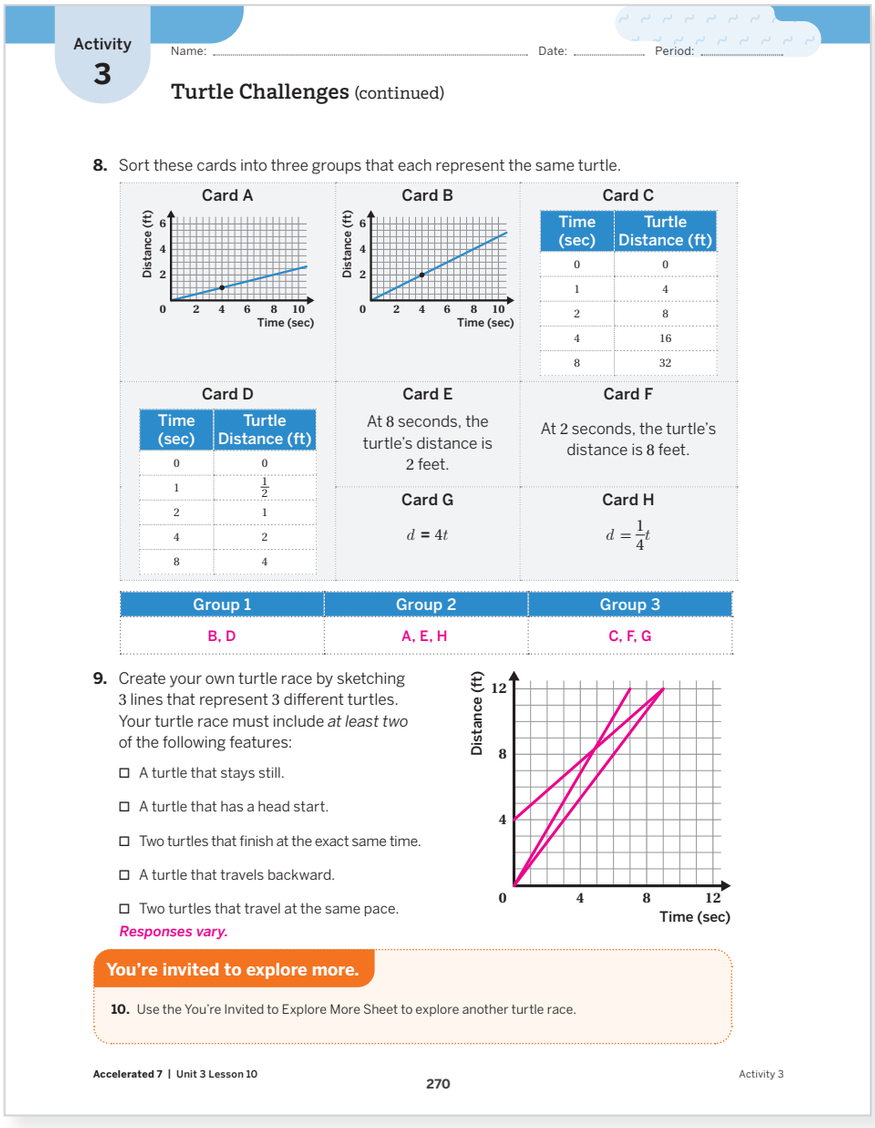
D **Differentiation: You're invited to explore more.** (Stretch) Invite students who would like to explore the relationship between turtle speed and graphs further to complete this optional task. Encourage them to discuss their thinking with a partner. **Note:** Students will need copies of the You're Invited to Explore More Sheet.

3 Connect

Invite students to share their groupings and consider discussing each group one at a time.

Consider asking:

- "Do _____ and _____ represent the same relationship? How do you know?"
- "Which was the last card you sorted? How did you decide where it should go?"
- "What strategies were helpful for deciding which representations were in the same group?"



Activity 3

Name: Date: Period:

Turtle Challenges (continued)

8. Sort these cards into three groups that each represent the same turtle.

Card A

Card B

Card C

Time (sec)	Turtle Distance (ft)
0	0
1	4
2	8
4	16
8	32

Card D

Time (sec)	Turtle Distance (ft)
0	0
1	$\frac{1}{2}$
2	1
4	2
8	4

Card E

At 8 seconds, the turtle's distance is 2 feet.

Card F

At 2 seconds, the turtle's distance is 8 feet.

Card G

$d = 4t$

Card H

$d = \frac{1}{4}t$

Group 1	Group 2	Group 3
B, D	A, E, H	C, F, G

9. Create your own turtle race by sketching 3 lines that represent 3 different turtles. Your turtle race must include *at least two* of the following features:

- ☐ A turtle that stays still.
- ☐ A turtle that has a head start.
- ☐ Two turtles that finish at the exact same time.
- ☐ A turtle that travels backward.
- ☐ Two turtles that travel at the same pace.

Responses vary.

You're invited to explore more.

10. Use the You're Invited to Explore More Sheet to explore another turtle race.

Accelerated 7 | Unit 3 Lesson 10 270 Activity 3

Whole Class | 5 min

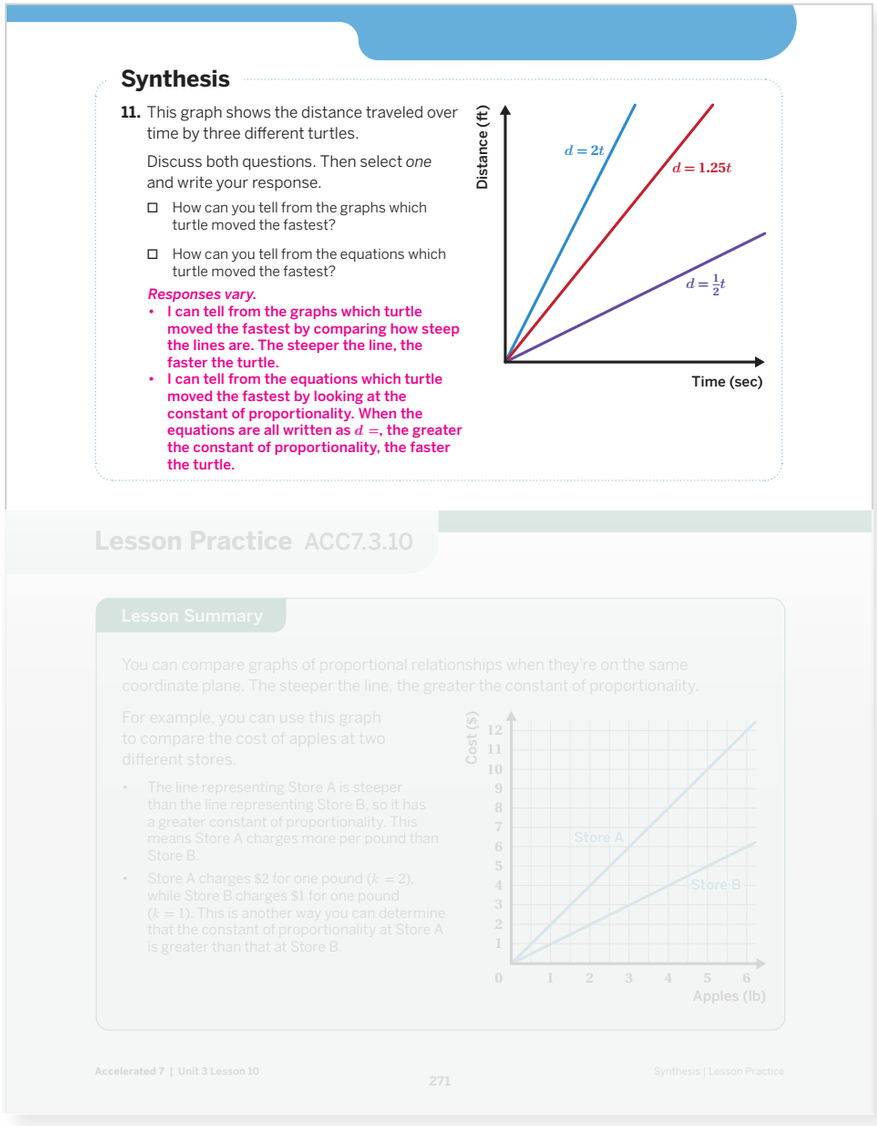
Synthesis

Purpose: Students synthesize their ideas about using graphs and equations to compare proportional relationships.

Invite students to respond independently, and then share their thinking with a partner.

Collect and share a variety of ideas, such as:
- Steeper lines represent faster turtles.
- The coefficient in the equation is also a constant of proportionality.
- The larger the constant of proportionality, the faster the turtle.

Note: If time allows, invite students to revise their previous thinking based on the discussion.



Synthesis

11. This graph shows the distance traveled over time by three different turtles.

Discuss both questions. Then select *one* and write your response.

- ☐ How can you tell from the graphs which turtle moved the fastest?
- ☐ How can you tell from the equations which turtle moved the fastest?

Responses vary.
- I can tell from the graphs which turtle moved the fastest by comparing how steep the lines are. The steeper the line, the faster the turtle.
- I can tell from the equations which turtle moved the fastest by looking at the constant of proportionality. When the equations are all written as $d =$, the greater the constant of proportionality, the faster the turtle.

Lesson Practice ACC7.3.10

Lesson Summary

You can compare graphs of proportional relationships when they're on the same coordinate plane. The steeper the line, the greater the constant of proportionality.

For example, you can use this graph to compare the cost of apples at two different stores.

- The line representing Store A is steeper than the line representing Store B, so it has a greater constant of proportionality. This means Store A charges more per pound than Store B.
- Store A charges $2 for one pound ($k = 2$), while Store B charges $1 for one pound ($k = 1$). This is another way you can determine that the constant of proportionality at Store A is greater than that at Store B.

Accelerated 7 | Unit 3 Lesson 10 271 Synthesis | Lesson Practice

Lesson Takeaway: The constant of proportionality in an equation of a proportional relationship is connected to the steepness of the graph of its line and the rate in a situation.

Independent | 5 min

Show What You Know

Purpose: Students demonstrate their understanding by labeling two relationships on a graph and writing an equation.

Today's Goals

1. **Goal:** Write an equation of a proportional relationship given a point on a graph.
2. **Language Goal:** Compare proportional relationships using their graphs and equations. **(Reading and Writing)**

Show What You Know (Assessment Resources)

Name: _____ Date: _____ Period: _____

Show What You Know 3.10

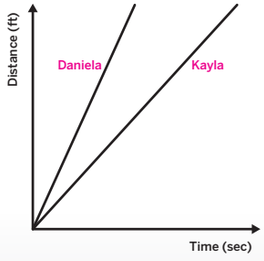
Kayla and Daniela started walking at constant speeds.

After 3 seconds:

- Kayla had walked 6 feet.
- Daniela had walked 12 feet.

a. Label each graph with the name it represents.

b. Write an equation for Kayla's walk, using d for distance and t for time.
$d = 2t$ (or equivalent)



Differentiation Use after Lesson 10

Support

Provide targeted intervention for students by using these resources.

If students need support understanding how to connect situations, graphs, and equations:

» **Respond to Student Thinking:**

- Use the ***Comparing Proportional Relationships with Graphs*** **Mini-Lesson**.
- Revisiting Activity 3's card sort as a class and inviting students to share strategies for making connections between representations.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 3 Extension Activities**.
- Invite students to complete the **Explore More** task and discuss their thinking with a partner.

Professional Learning

Take a moment to reflect on the strategies students used in this lesson. What opportunities were there to celebrate specific students' strategies for connecting representations and writing equations? What opportunities might there be in future lessons to connect back to the strategies students used today?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

11. This graph shows the distance traveled over time by three different turtles.

Discuss both questions. Then select *one* and write your response.

- ☐ How can you tell from the graphs which turtle moved the fastest?
- ☐ How can you tell from the equations which turtle moved the fastest?

Responses vary.

- I can tell from the graphs which turtle moved the fastest by comparing how steep the lines are. The steeper the line, the faster the turtle.
- I can tell from the equations which turtle moved the fastest by looking at the constant of proportionality. When the equations are all written as $d = \ldots$, the greater the constant of proportionality, the faster the turtle.

Lesson Practice ACC7.3.10

Lesson Summary

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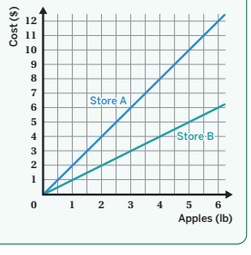
For example, you can use this graph to compare the cost of apples at two different stores.

- The line representing Store A is steeper than the line representing Store B, so it has a greater constant of proportionality. This means Store A charges more per pound than Store B.
- Store A charges \$2 for one pound ($k = 2$), while Store B charges \$1 for one pound ($k = 1$). This is another way you can determine that the constant of proportionality at Store A is greater than that at Store B.



Lesson Practice
ACC7.3.10

Name: _____ Date: _____ Period: _____

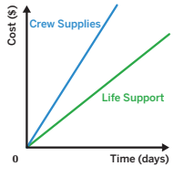
1. Meteoroid Perseid 245 and Asteroid X travel through the solar system. This graph shows how much distance they travel over time.

Does Asteroid X travel faster or slower than Perseid 245? **Faster**

Explain your thinking.

Explanations vary. **Asteroid X travels faster because its line is steeper. So it takes less time for Asteroid X to travel the same distance as Perseid 245.**

2. Having a crew on the International Space Station requires life support and crew supplies. The graph shows the relationship between the cost of life support and crew supplies, and the number of days spent in space.



Which costs less, life support or crew supplies?

Life support

Explain your thinking.

Explanations vary. **Life support costs less because its line is less steep than the line for crew supplies. If I draw a vertical line starting from the horizontal axis, it will intersect the life support line before the crew supplies line.**

Problems 3–6: At a supermarket, you can fill your own honey container and pay by the ounce. A customer buys 12 ounces of honey for \$5.40.

3. How much does the honey cost per ounce? Show your thinking.

\$0.45 per ounce. $\frac{5.40}{12} = 0.45$

4. How much honey can be bought per dollar? Show your thinking.

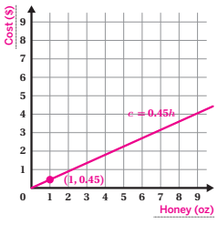
About 2.2 ounces per dollar. $\frac{12}{5.40} \approx 2.2$

5. Write *two* different equations representing this situation. Use h for ounces of honey and c for cost in dollars.

$c = 0.45h$ and $h = 2.2c$ (or equivalent)

6. Choose *one* equation and draw its graph on the coordinate plane. Be sure to label the axes.

Sample shown on graph.

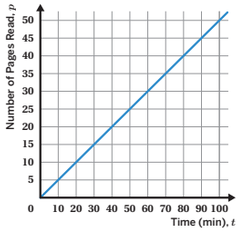


Lesson Practice
ACC7.3.10

Name: _____ Date: _____ Period: _____

FAST Practice

Problems 7–10: The graph shows the relationship between time in minutes, t, and the total number of pages Joel has read, p.



7. Is this relationship proportional?

Explain your thinking.

Yes. *Explanations vary.* The graph is a straight line that passes through the origin.

8. What is a constant of proportionality in this relationship?

$\frac{1}{2}$ (or equivalent) or 2 (or equivalent)

9. What does this number mean in this situation?

Responses vary. **Joel is reading a book at a constant rate of a $\frac{1}{2}$ page per minute.**

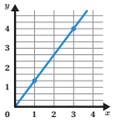
10. Write an equation that represents the relationship between the time in minutes, t, and the total number of pages Joel has read, p.

Equation: **$p = \frac{1}{2}t$ or $t = 2p$** (or equivalent)

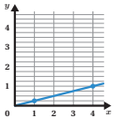
Spiral Review

Problems 11–14: Match each equation with its graph.

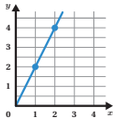
Graph A



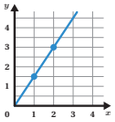
Graph B



Graph C



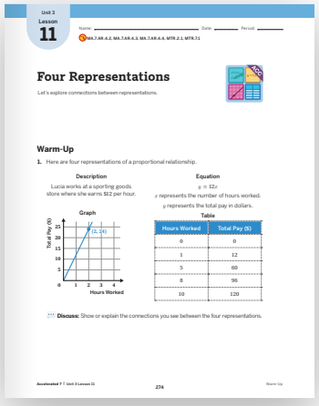
Graph D



11. $y = \frac{1}{4}x$ **B** **12.** $y = \frac{3}{2}x$ **D** **13.** $y = 2x$ **C** **14.** $y = \frac{4}{3}x$ **A**

Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1–4	2	MA.7.AR.4.5
	5–6	2	MA.7.AR.4.4
FAST Practice	**7– 10**	2	MA.7.AR.4.1, MA.7.AR.4.2, MA.7.AR.4.4
Spiral Review			
	11–14	1	MA.7.AR.4.4


Unit 3
Lesson
11

Four Representations

Let's explore connections between representations.

Warm-Up

1. Here are four representations of a proportional relationship.

Description

Lucia works at a sporting goods store where she earns $12 per hour.

Equation

$y = 12x$

x represents the number of hours worked.

y represents the total pay in dollars.

Graph

Table

Hours Worked	Total Pay ($)
0	0
1	12
5	60
8	96
10	120

Discuss: Show or explain the connections you see between the four representations.

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Four Representations

Connecting Descriptions, Tables, Graphs, and Equations

Let's explore connections between representations.

Learning Outcomes

Today's Goals

1. **Goal:** Create four different representations of a proportional relationship: description, table, graph, and equation.
2. **Language Goal:** Interpret proportional relationships in context using tables, equations, graphs, and verbal descriptions. **(Writing)**

Students examine the connections between verbal descriptions, tables, equations, and graphs of proportional relationships. As they work, students pay close attention to the variables as they build contextual meaning of the constant of proportionality.

Coherence

Prior Learning

Throughout this unit, students studied various representations of proportional relationships and how to find the constant of proportionality.

Future Learning

In Unit 4, students will apply their understanding of proportional relationships to explore non-proportional linear relationships.

Connections

- Students continue to build their understanding of different representations of proportional relationships in real world contexts such as total pay per hours worked and converting between different units of measure. (real world)

FL B.E.S.T. Math Benchmarks

Addressing

MA.7.AR.4.3
Given a mathematical or real-world context, graph proportional relationships from a table, equation or a written description.

MA.7.AR.4.4
Given any representation of a proportional relationship, translate the representation to a written description, table or equation.

Also Addressing: **MA.7.AR.4.2**

Mathematical Thinking and Reasoning Standards: MTR.2.1, MTR.7.1

ELA Expectations: ELA.K12.EE.1.1, ELA.K12.EE.6.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.6.AR.3.2	**MA.8.AR.3.2**
MA.6.AR.3.3	**MA.8.AR.3.3**
MA.6.AR.3.5	**MA.8.AR.3.4**
MA.6.GR.1.1	

Lesson at a Glance

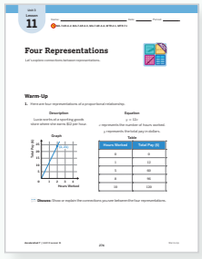
~ 45 min

MA.7.AR.4.2, MA.7.AR.4.3, MA.7.AR.4.4

Warm-Up

Pairs | 5 min

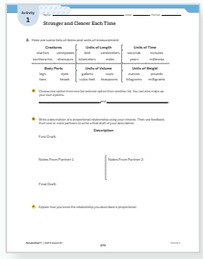
Students describe the similarities and differences between the four different representations of a proportional relationship.



Activity 1

Pairs | 10 min

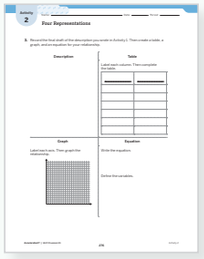
Students write and revise a description of a relationship, then justify whether it is proportional. MLR1: Stronger and Clearer Each Time



Activity 2

Pairs | 20 min

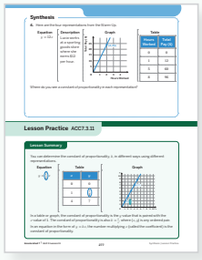
Students define, develop, describe, and represent their own proportional relationship and think about what quantities are related.



Synthesis

Whole Class | 5 min

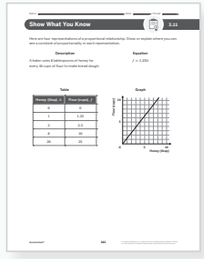
Students synthesize their understanding of the different representations and their uses for describing a proportional relationship.



Show What You Know

Independent | 5 min

Students demonstrate their understanding by identifying the constant of proportionality in different representations.



Prep Checklist

This lesson includes:

- Student Edition
- Show What You Know

Pairs | 5 min

Warm-Up

Purpose: Students describe the similarities and differences between the four different representations of a proportional relationship.

1 Launch

Direct student's attention to the four representations of the proportional relationship.

To surface prior knowledge, consider inviting students to try to find the same information in all four representations. **(MTR.2.1)**

ELL **English Language Learners** Encourage students to paraphrase each other's ideas to help them make connections and incorporate vocabulary from the unit. **(Speaking and Listening)**

2 Connect

Invite students to share the connections they see between the four representations.

Consider asking, "Which of these representations is the most useful?" Each representation is useful to highlight a different property of the function. For example, in the equation, it is easy to see the exact value of the constant of proportionality.

Unit 3
Lesson
11

Name: _____ Date: _____ Period: _____

MA.7.AR.4.2, MA.7.AR.4.3, MA.7.AR.4.4, MTR.2.1, MTR.7.1

Four Representations

Let's explore connections between representations.

Warm-Up

1. Here are four representations of a proportional relationship.

Description

Lucia works at a sporting goods store where she earns $12 per hour.

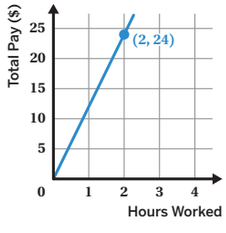
Equation

$y = 12x$

x represents the number of hours worked.

y represents the total pay in dollars.

Graph



Table

Hours Worked	Total Pay ($)
0	0
1	12
5	60
8	96
10	120

Discuss: Show or explain the connections you see between the four representations.

Responses vary. The constant of proportionality in the equation is 12. I also see this in the second row of the table, which says 1 hour worked and 12 dollars of pay. The point (2, 24) is plotted on the graph, and $24 \div 2 = 12$. This means the graph has a constant of proportionality of 12, too. The description mentions the same constant of proportionality, noting that Lucy earns $12 per hour.

Accelerated 7 | Unit 3 Lesson 11 274 Warm-Up

Pairs | 10 min

Activity 1 Stronger and Clearer Each Time

Purpose: Students write and revise a description of a relationship, then justify whether it is proportional.

Short on time? Consider limiting the rounds of feedback on the proportional relationships to two.

1 Launch

Direct student's attention to the list of items. To encourage playfulness, consider asking, "Which two options on these lists would be surprising if they were paired together?"

Invite students to choose a pair of options with their partner. Encourage students to choose a pair that no one else in class will choose.

2 Monitor

MLR **MLR1: Stronger and Clearer Each Time** Invite students to write a first draft and use feedback to make their thinking stronger and clearer.

D Differentiation

Look for students who:	Respond to Student Thinking
Are overwhelmed with the choices from the table.	**Support** Consider showing them how the Warm-Up related two quantities.
Need support with giving useful feedback.	**Support** Invite these students to look for clear values and units in each relationship. Consider having them practice giving feedback with their partner before giving feedback to other groups.
Finish early.	**Stretch** Invite them to come up with their own categories or units.

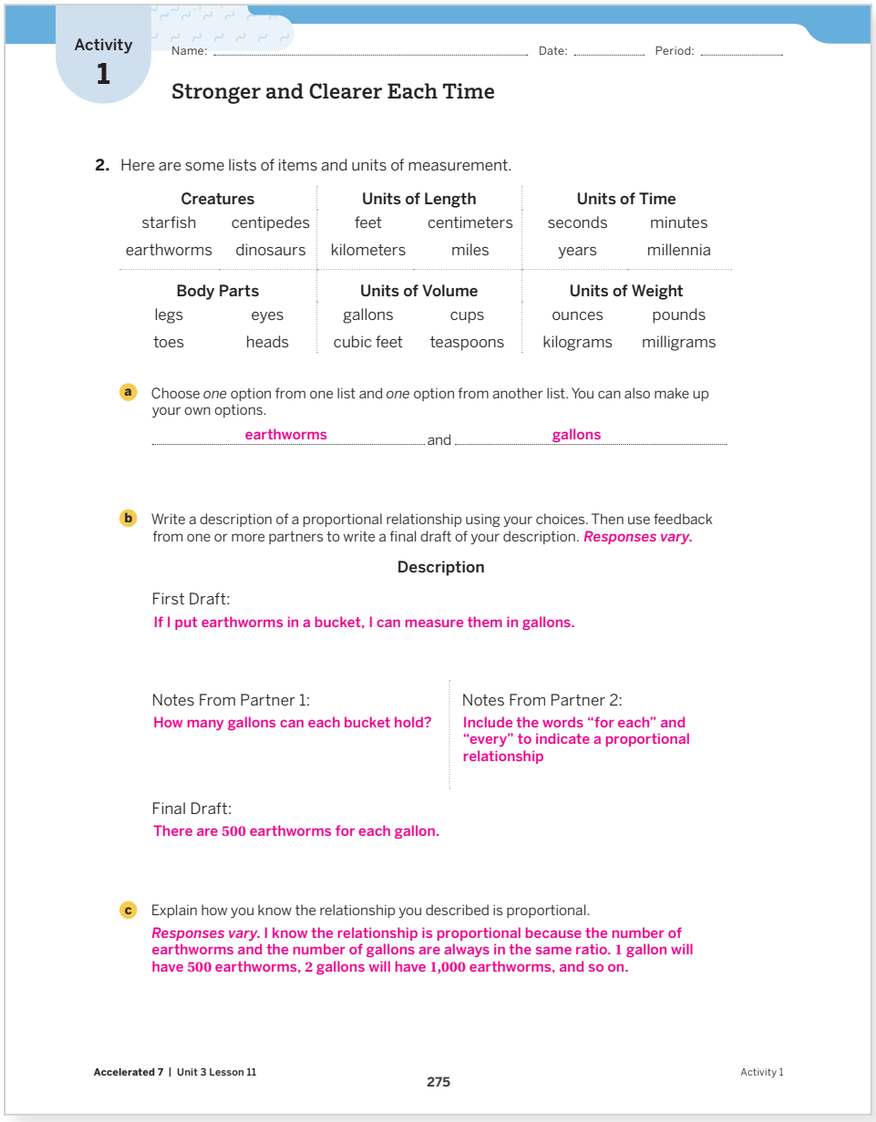
Invite students to share their drafts with two other pairs of partners and give each other feedback.

Encourage students to use the feedback they got from their partners to write a second draft that is stronger and clearer (with more precision, organization, etc.).

3 Connect

Share student drafts that were made stronger through feedback. Consider asking, "What specific pieces of feedback made the descriptions stronger and clearer?" **(MTR.7.1)**

To surface the Key Takeaway, consider asking, "How do you know that you have created a proportional relationship?"



Activity 1

Name: _____ Date: _____ Period: _____

Stronger and Clearer Each Time

2. Here are some lists of items and units of measurement.

Creatures		Units of Length		Units of Time	
starfish	centipedes	feet	centimeters	seconds	minutes
earthworms	dinosaurs	kilometers	miles	years	millennia
Body Parts		**Units of Volume**		**Units of Weight**	
legs	eyes	gallons	cups	ounces	pounds
toes	heads	cubic feet	teaspoons	kilograms	milligrams

a. Choose *one* option from one list and *one* option from another list. You can also make up your own options.

earthworms and gallons

b. Write a description of a proportional relationship using your choices. Then use feedback from one or more partners to write a final draft of your description. *Responses vary.*

Description

First Draft:

If I put earthworms in a bucket, I can measure them in gallons.

Notes From Partner 1:

How many gallons can each bucket hold?

Notes From Partner 2:

Include the words "for each" and "every" to indicate a proportional relationship

Final Draft:

There are 500 earthworms for each gallon.

c. Explain how you know the relationship you described is proportional.

Responses vary. I know the relationship is proportional because the number of earthworms and the number of gallons are always in the same ratio. 1 gallon will have 500 earthworms, 2 gallons will have 1,000 earthworms, and so on.

Accelerated 7 | Unit 3 Lesson 11 275 Activity 1

Key Takeaway: A strong description of a proportional relationship includes the value and units for how the two quantities change together. This often is represented by the constant of proportionality.

Pairs | 20 min

Activity 2 Four Representations

Purpose: Students define, develop, describe, and represent their own proportional relationship and think about what quantities are related.

1 Launch

Direct student's attention to the four representations from the Warm-Up. Share with students that they will create a graph, table, and an equation for their proportional relationship. **(MTR.2.1)**

A **Accessibility: Executive Functioning** Consider pre-selecting two quantities from Activity 1 and providing a corresponding table of values for the relationship. This will allow students to focus on analyzing the connections between the table, graph, and equation.

2 Monitor

Select and sequence examples that clearly show how the four representations are related to share during the Connect.

D **Differentiation**

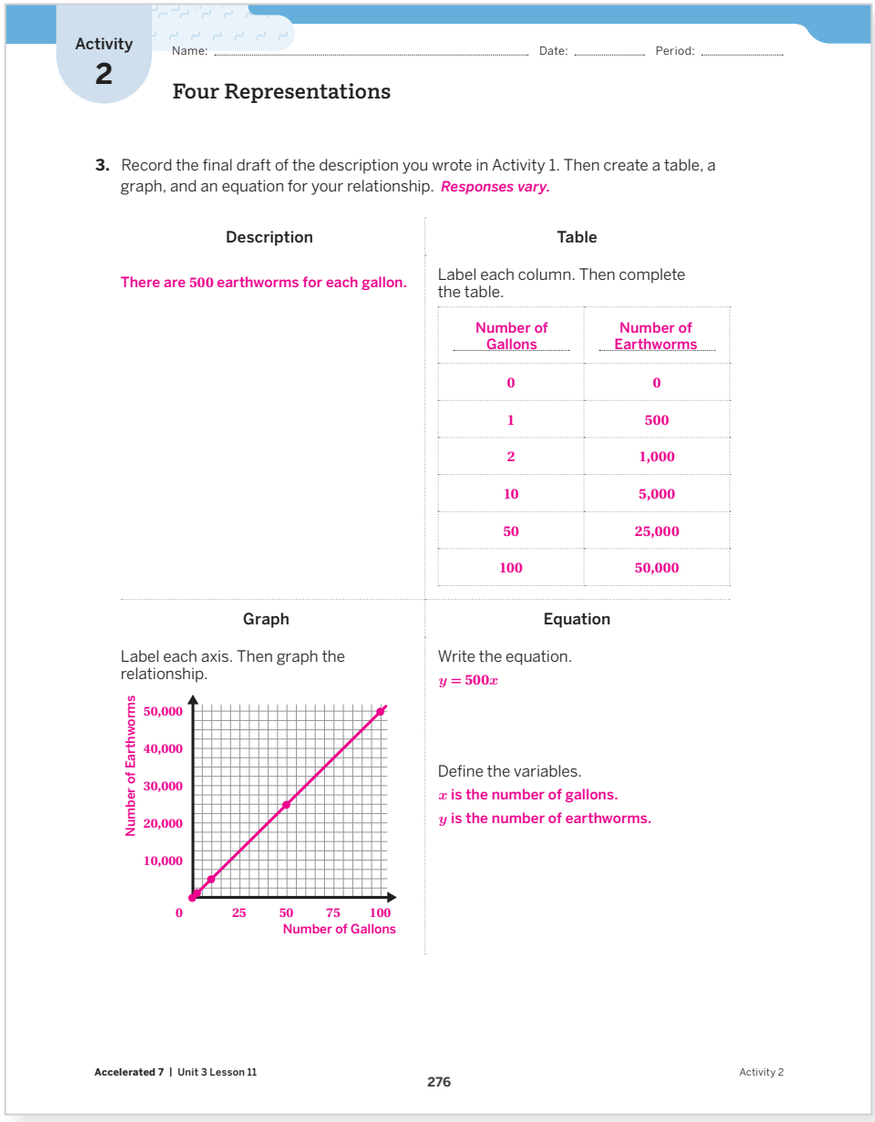
Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Encourage them to start by creating a table of values that represents their relationship.
Use different values in their tables and graphs.	**Support** Consider asking, "How can you clearly show the connection between the tables and the graph?"
Create tables or graphs without labels.	**Strengthen** Invite them to be precise with the values along the axes and the labels for the columns and axes.
Clearly identify the constant of proportionality in all of the representations.	**Strengthen** Consider asking, "What would change about each of your representations if you made your constant of proportionality larger?"

3 Connect

Share and discuss the examples that clearly show how the four representations are connected. Invite students to share how they can see the constant of proportionality in each representation. **(MTR.2.1)**

To surface the Key Takeaway, consider asking:

- "What do the different representations clearly show about your relationship? (For instance, what does a graph clearly show? A table? An equation?)"
- "Why would someone want to have representations about the relationship you have chosen?" **(MTR.7.1)**



Activity 2

Name: _____ Date: _____ Period: _____

Four Representations

3. Record the final draft of the description you wrote in Activity 1. Then create a table, a graph, and an equation for your relationship. *Responses vary.*

Description

There are 500 earthworms for each gallon.

Table

Label each column. Then complete the table.

Number of Gallons	Number of Earthworms
0	0
1	500
2	1,000
10	5,000
50	25,000
100	50,000

Graph

Label each axis. Then graph the relationship.

Equation

Write the equation.

$y = 500x$

Define the variables.

x is the number of gallons.

y is the number of earthworms.

Key Takeaway: Each representation communicates something very clearly about the proportional relationship. In each, you can determine the constant of proportionality.

Whole Class | 5 min

Synthesis

Purpose: Students synthesize their understanding of the different representations and their uses for describing a proportional relationship.

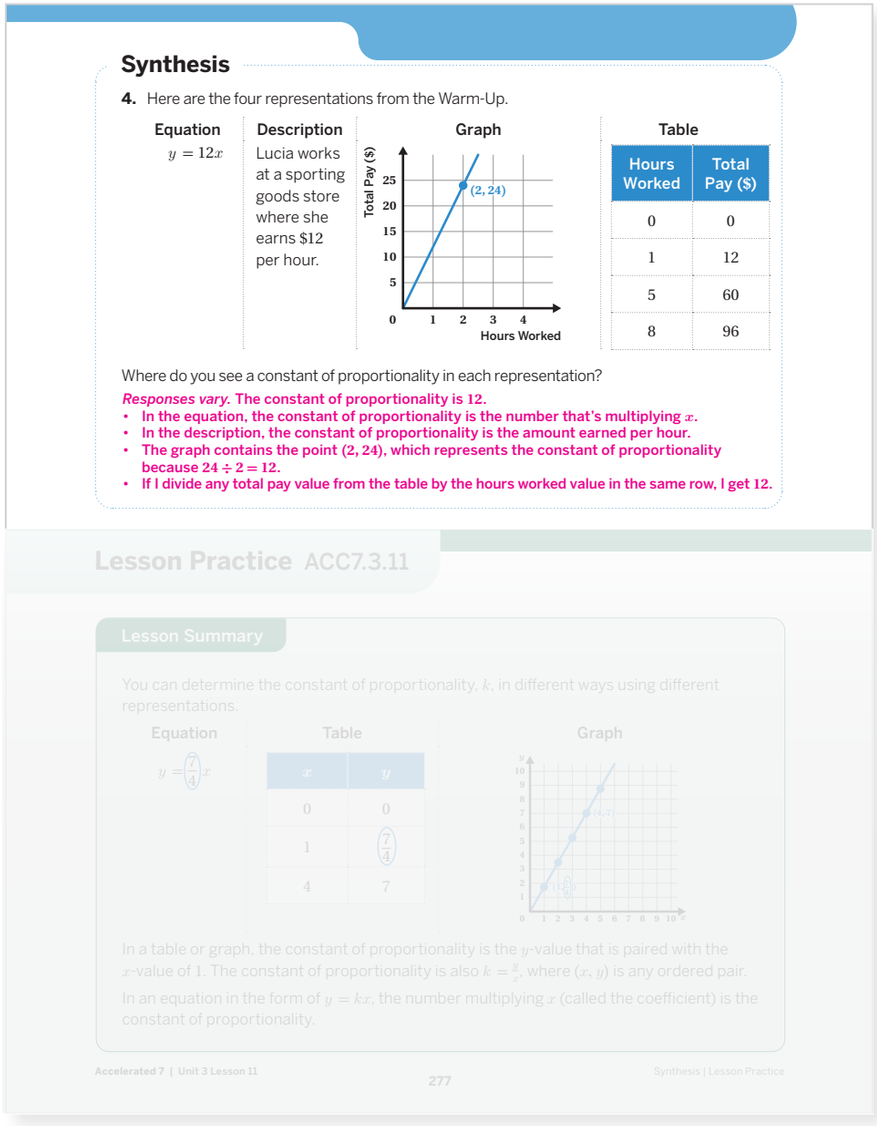
Invite students to respond independently, and then share their thinking with a partner.

Collect and share a variety of ideas, such as:

- The description often has a "per" or "for each" description that describes the constant of proportionality.
- The equation is in the form $y = kx$ and the constant of proportionality is k.
- The steepness of the graph shows the constant of proportionality.
- The value that is paired with 1 hour worked is the constant of proportionality.

Create a chart that highlights how students can identify the constant of proportionality in each representation.

Note: If time allows, invite students to share with a partner what makes sense about each representation and what questions they still have.



Synthesis

4. Here are the four representations from the Warm-Up.

Equation

$y = 12x$

Description

Lucia works at a sporting goods store where she earns $12 per hour.

Graph

Table

Hours Worked	Total Pay ($)
0	0
1	12
5	60
8	96

Where do you see a constant of proportionality in each representation?

Responses vary. The constant of proportionality is 12.

- In the equation, the constant of proportionality is the number that's multiplying x.
- In the description, the constant of proportionality is the amount earned per hour.
- The graph contains the point (2, 24), which represents the constant of proportionality because $24 \div 2 = 12$.
- If I divide any total pay value from the table by the hours worked value in the same row, I get 12.

Lesson Practice ACC7.3.11

Lesson Summary

You can determine the constant of proportionality, k, in different ways using different representations.

Equation

$y = \frac{7}{4}x$

Table

x	y
0	0
1	$\frac{7}{4}$
4	7

Graph

In a table or graph, the constant of proportionality is the y-value that is paired with the x-value of 1. The constant of proportionality is also $k = \frac{y}{x}$, where (x, y) is any ordered pair.

In an equation in the form of $y = kx$, the number multiplying x (called the coefficient) is the constant of proportionality.

Accelerated 7 | Unit 3 Lesson 11 277 Synthesis | Lesson Practice

Lesson Takeaway: Each representation of a proportional relationship can help us to determine the constant of proportionality. It is the k value in an equation of the form $y = kx$, the y-value paired with 1 in a table or graph, or the value associated with "for each" or "per" in a description.

Independent | 5 min

Show What You Know

Purpose: Students demonstrate their understanding by identifying the constant of proportionality in different representations.

Today's Goals

1. **Goal:** Create four different representations of a proportional relationship: description, table, graph, and equation.
2. **Language Goal:** Interpret proportional relationships in context using tables, equations, graphs, and verbal descriptions. **(Writing)**

Show What You Know (Assessment Resources)

Name: _____ Date: _____ Period: _____

Show What You Know 3.11

Here are four representations of a proportional relationship. Show or explain where you can see a constant of proportionality in each representation.

Description

A baker uses 8 tablespoons of honey for every 10 cups of flour to make bread dough.

Responses vary. A constant of proportionality is the number of cups of flour for each tablespoon of honey: $\frac{10}{8}$.

Equation

$f = 1.25h$

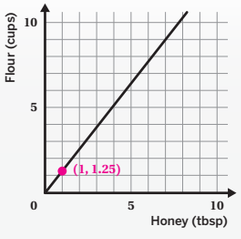
Responses vary. A constant of proportionality is k in the equation $f = kh$, so it's 1.25.

Table

Honey (tbsp), h	Flour (cups), f
0	0
1	1.25
2	2.5
8	10
20	25

Responses vary. A constant of proportionality is 1.25, which is the y-value paired with the x-value of 1.

Graph



Differentiation Use after Lesson 11

Support

Provide targeted intervention for students by using these resources.

If students need support understanding how to identify the constant of proportionality in one of the representations:

» **Respond to Student Thinking:**

- Use the ***Representing Proportional Relationships*** **Mini-lesson.**
- Revisit Activity 2 and give individualized feedback.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 3 Extension Activities**.

Professional Learning

Reflect on how students gave and received feedback to each other. How did they talk about mistakes? Were they able to notice the correct and incorrect parts of each student's descriptions? How can you support students in giving and receiving feedback to each other in the future?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

4. Here are the four representations from the Warm-Up.

Equation

$y = 12x$

Description

Lucia works at a sporting goods store where she earns $12 per hour.

Graph

Table

Hours Worked	Total Pay ($)
0	0
1	12
5	60
8	96

Where do you see a constant of proportionality in each representation?

Responses vary. The constant of proportionality is 12.

- In the equation, the constant of proportionality is the number that's multiplying x.
- In the description, the constant of proportionality is the amount earned per hour.
- The graph contains the point (2, 24), which represents the constant of proportionality because $24 \div 2 = 12$.
- If I divide any total pay value from the table by the hours worked value in the same row, I get 12.

Lesson Practice ACC7.3.11

Lesson Summary

You can determine the constant of proportionality, k, in different ways using different representations.

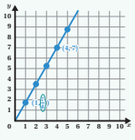
Equation

$y = \left(\frac{7}{4}\right)x$

Table

x	y
0	0
1	$\frac{7}{4}$
4	7

Graph



In a table or graph, the constant of proportionality is the y-value that is paired with the x-value of 1. The constant of proportionality is also $k = \frac{y}{x}$, where (x, y) is any ordered pair.

In an equation in the form of $y = kx$, the number multiplying x (called the coefficient) is the constant of proportionality.

Lesson Practice ACC7.3.11

Name: _____ Date: _____ Period: _____

Problems 1–4: Determine a constant of proportionality in each representation. Show your thinking.

1. David walked 15 feet in 3 seconds.

Responses vary. $\frac{1}{5}$ or 5. *Work varies.* $k = 15 \div 3 = 5$ or $k = 3 \div 15 = \frac{1}{5}$

2. $C = 4.8n$

Explanations vary. 4.8. The coefficient in the equation is 4.8.

3.

x	y
0	0
$\frac{7}{10}$	$\frac{28}{15}$

Responses vary. $\frac{8}{3}$ or $\frac{3}{8}$. *Work varies.*

$k = \frac{28}{15} \div \frac{7}{10} = \frac{28}{15} \cdot \frac{10}{7} = \frac{8}{3}$

or $k = \frac{7}{10} \div \frac{28}{15} = \frac{7}{10} \cdot \frac{15}{28} = \frac{3}{8}$

4.

Work varies. 2.75 or 0.3636 . . .

$k = 5.5 \div 2 = 2.75$ or $k = 2 \div 5.5 = 0.3636\ldots$

5. Here is a description, a graph, and a table.

Description

Yosef has to work 12 hours in a week to earn $180.

Graph

Table

Hours Worked	Total Pay ($)
0	0
2	25
6	75
10	125
20	250

Which one represents a different relationship than the others? Explain your thinking.

Explanations vary. The table. The description has a constant of proportionality of $180 \div 12$, which is equal to 15. The graph has a constant of proportionality of $60 \div 4$, which is also equal to 15. The table has a constant of proportionality of $25 \div 2$, which is equal to 12.5.

Lesson Practice ACC7.3.11

Name: _____ Date: _____ Period: _____

Problems 6–8: The equation $c = 2.95g$ gives the cost of gas on a particular day, where c represents the cost in dollars and g represents the number of gallons of gas purchased at a neighborhood gas station.

6. Write four ordered pairs that fit this relationship, where the x-value is the number of gallons of gas and the y-value is the cost.

Responses vary. (0, 0), (1, 2.95), (2, 5.90), (3, 8.85)

7. What does the value 2.95 represent in this situation?

Responses vary. It's the price of each gallon of gas.

8. Hailey's mom remarks, "You can get about a third of a gallon of gas for a dollar." Is she correct? Explain your thinking.

Yes. *Explanations vary.* The reciprocal of 2.95 is $\frac{1}{2.95} \approx 0.339$, which is about a third.

FAST Practice

9. $\left(\frac{1}{2}, 3\right)$ is a point on a line that represents a proportional relationship. Select *all* the points that are also on this line.

☐ A. $\left(9, \frac{3}{2}\right)$ ☑ B. $(1, 6)$ ☐ C. $(1, 3)$ ☐ D. $\left(\frac{3}{2}, \frac{1}{4}\right)$ ☑ E. $\left(\frac{7}{2}, 21\right)$

Spiral Review

10. Select *all* the tables that represent a proportional relationship between x and y.

☑ A.

x	y
0	0
$\frac{1}{4}$	$\frac{1}{8}$
$\frac{2}{4}$	$\frac{2}{8}$
$\frac{3}{4}$	$\frac{3}{8}$

☐ B.

x	y
0	0
$\frac{1}{2}$	$\frac{1}{4}$
$\frac{1}{3}$	$\frac{1}{5}$
$\frac{1}{4}$	$\frac{1}{6}$

☐ C.

x	y
0	0
1	1
2	4
4	16

☑ D.

x	y
0	0
1	4
3	12
4	16

Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1–4	2	MA.7.AR.4.2
	5	3	MA.7.AR.4.2
	6–8	2	MA.7.AR.4.2, MA.7.AR.4.4
FAST Practice	9	2	MA.7.AR.4.2
Spiral Review			
	10	2	MA.7.AR.4.1

Assess and Respond

End-of-Unit Assessment

Independent | 45 min

Facilitation: Assign the End-of-Unit Assessment to learn about your students' understanding of the concepts and skills in this unit. There are two forms of the End-of-Unit Assessment: Forms A and B. If time allows, consider inviting students to complete one or more of the tasks in the Unit Synthesis and Reflection.

Item Analysis, Forms A and B

Problem(s)	Concept or skill	Addressed in	DOK	FL B.E.S.T. Math Benchmarks
1	Identifying graphs that are proportional	Lesson 8	1	MA.7.AR.4.1
2	Determining a constant of proportionality and using it to answer questions	Lesson 9	2	MA.7.AR.4.2, MTR.6.1
3	Creating equations for proportional situations	Lesson 10	2	MA.7.AR.4.4, MTR.7.1
4	Comparing and interpreting graphs of proportional relationships and justifying a claim	Lesson 10	2	MA.7.AR.4.5, MTR.4.1
5	Determining values in a table of a proportional relationship	Lesson 3	2	MA.8.AR.3.1
6	Determining whether or not equations represent a proportional relationship	Lesson 7	2	MA.8.AR.3.1, MTR.3.1
7a	Determining how to label the x-axis and y-axis from a situation and a coordinate pair	Lesson 10	3	MA.8.AR.3.1
7b	Writing an equation to represent a proportional relationship when given a situation and a coordinate pair	Lesson 10	3	MA.7.AR.4.4
7c	Explaining what a point in the form (1, r) means in a proportional relationship	Lesson 10	3	MA.7.AR.4.5, MTR.6.1

Assessment Resources

- Student Assessments (Forms A and B)
- Assessment Rubrics with Answers
- Show What You Knows and Answer Keys

Intervention and Extension Resources

- Mini-Lessons
- Extensions

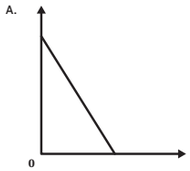
Name: Date: Period:

End-of-Unit Assessment

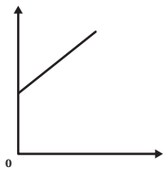
Unit 3

1. Which graph represents a proportional relationship?

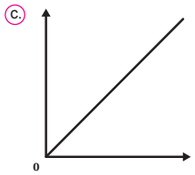
A.



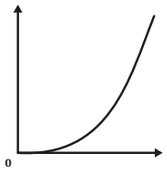
B.



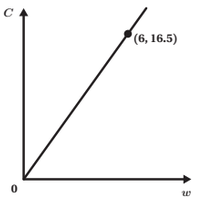
C.



D.



2. This graph shows the cost in dollars, C, of w pounds of blueberries.



The relationship between C and w is proportional.

Select *all* of the true statements.

- ☐ **A.** 2.75 pounds of blueberries cost \$1.
- ☑ **B.** A constant of proportionality is \$2.75.
- ☐ **C.** 5 pounds of blueberries cost \$15.50.
- ☑ **D.** 12 pounds of blueberries cost \$33.
- ☐ **E.** The point (3, 9) is on the graphed line.

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Name: Date: Period:

End-of-Unit Assessment (continued)

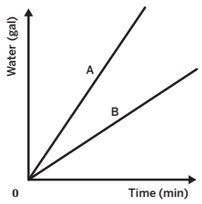
Unit 3

3. Kadeem rode his bike at a constant speed. He rode 1 mile in 5 minutes.

Which equation represents the amount of time in minutes, t, that it took for him to ride a distance of d miles?

A. $t = 5d$ **B.** $t = \frac{1}{5}d$ **C.** $t = d + 4$ **D.** $t = d - 4$

4. These two lines represent the amount of water filling two tanks of the same size over time.



Which tank is filling more quickly? Circle one.

Tank A Tank B Not enough information

Explain or show your thinking.

Explanations vary. I know Tank A is filling more quickly because it's represented by the steeper graph. I can also choose a time and see how much water is in the two tanks at that moment.

5. This table shows the weight of 100 raspberries at a market.

Complete the table so there is a proportional relationship between the number of raspberries and their weight.

Number of Raspberries	Weight (kg)
40	0.16 (or equivalent)
100	0.40
300	1.2 (or equivalent)

Name: Date: Period:

End-of-Unit Assessment (continued)

Unit 3

6. The equation $F = \frac{9}{5}C + 32$ represents the relationship between temperature in degrees Celsius, C, and temperature in degrees Fahrenheit, F.

Is there a proportional relationship between C and F?

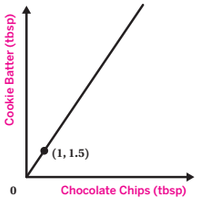
No

Explain or show your thinking.

Explanations vary. The equation $F = \frac{9}{5}C + 32$ is not in the form $y =$ (something)x.

7. A cookie recipe uses 3 tablespoons of cookie batter for every 2 tablespoons of chocolate chips.

This line represents the relationship between tablespoons of cookie batter and tablespoons of chocolate chips in the recipe. The point (1, 1.5) is on the line.



a Label the axes to represent the situation.

b Write an equation for the line.

Use b for the number of tablespoons of cookie batter and c for the number of tablespoons of chocolate chips.

$b = 1.5c$ (or equivalent)

c Explain what the point (1, 1.5) means in this situation.

Explanations vary. The point (1, 1.5) tells me that the recipe calls for 1 tablespoon of chocolate chips for every 1.5 tablespoons of cookie batter. This point gives a unit rate.

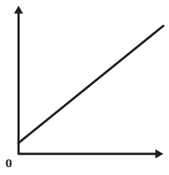
Name: Date: Period:

End-of-Unit Assessment

Unit 3

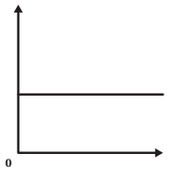
1. Which graph represents a proportional relationship?

A.

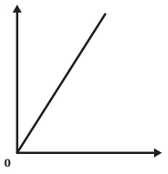


B.

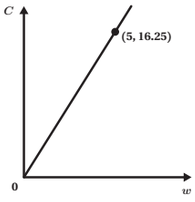
C.



D.



2. This graph shows the cost in dollars, C, of w pounds of strawberries.



The relationship between C and w is proportional.

Select *all* of the true statements.

- ☐ **A.** 3.25 pounds of strawberries cost \$1.
- ☑ **B.** A constant of proportionality is \$3.25.
- ☑ **C.** 4 pounds of strawberries cost \$13.
- ☐ **D.** 10 pounds of strawberries cost \$32.
- ☐ **E.** The point (3, 10) is on the graphed line.

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Name: Date: Period:

End-of-Unit Assessment (continued)

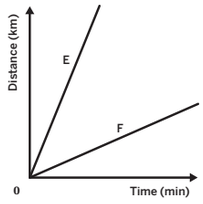
Unit 3

3. Abdullah walked to school at a constant speed. He walked 1 mile in 15 minutes.

Which equation represents the amount of time in minutes, t, that it took for him to walk a distance of d miles?

A. $t = d + 14$ B. $t = d - 14$ C. $t = 15d$ D. $t = \frac{1}{15}d$

4. These two lines represent the distance that two runners ran in a race.



Which runner ran more slowly? Circle one.

Runner E Runner F Not enough information

Explain or show your thinking.

Explanations vary. I know that Runner F ran slower because Runner F's graph is less steep. Another way to figure out which person ran slower is to choose a time and see which runner traveled the shorter distance in that time.

5. This table shows the weight of 100 raspberries at a market.

Complete the table so there is a proportional relationship between the number of blueberries and their weight.

Number of Blueberries	Weight (kg)
60	0.36 (or equivalent)
100	0.60
400	2.4 (or equivalent)

Name: Date: Period:

End-of-Unit Assessment (continued)

Unit 3

6. The equation $p = 2.2x$ represents the relationship between mass in pounds, p, and mass in kilograms, x.

Is there a proportional relationship between p and x?

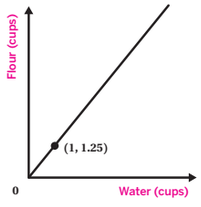
Yes

Explain or show your thinking.

Explanations vary. One way to decide if a relationship is proportional is to look at the equation and see if it has the form $y =$ (something)x. The "something" is the constant of proportionality. In this case, 2.22 is the constant of proportionality.

7. A recipe for tortillas uses 5 cups of flour for every 4 cups of water.

This line represents the relationship between the amount of flour and the amount of water needed for this recipe. The point (1, 1.25) is on the line.



a. Label the axes to represent the situation.

b. Write an equation for the line.

Use f for the number of cups of flour and w for the number of cups of water.

$f = 1.25w$ (or equivalent)

c. Explain what the point (1, 1.25) means in this situation.

Explanations vary. The point (1, 1.25) indicates that the recipe uses 1 cup of water for every 1.25 cups of flour.

D Differentiation (End-of-Unit Assessment)

Note: To **strengthen** and **stretch** students' learning, refer to the differentiation resources suggested throughout this unit and in the Unit Overview.

Sub-Unit Goals	Problem(s)	To respond to student thinking, consider:	
Sub-Unit 1: • Use table to Recognize proportional Relationships. • Determine a constant of proportionality from a table. **(Lessons 1–3)**	5	**Support**	• **Mini Lesson: Determining the Constant of Proportionality** • **Teacher Move:** Consider revisiting Lesson 2 (*Sugar, Spice, and Everything Rice*)
		Stretch	• **You're invited to explore more:** Lesson 1 (*Balloon Float*)
Sub-Unit 2: • Write and use equations to describe Proportional Relationships. • Determine a constant of proportionality from an equation. **(Lessons 4–7)**	6	**Support**	• **Mini Lesson: Finding More Than One Equation to Represent Proportional Relationships**
		Strengthen	• **Teacher Move:** Consider revisiting Lesson 7 (*All Kinds of Equations*)
		Stretch	• **You're invited to explore more:** Lesson 7 (*All Kinds of Equations*)
Sub-Unit 3: • Use graphs to Recognize proportional Relationships. • Determine a constant of proportionality from a graph. **(Lessons 8–11)**	1, 2	**Support**	• **Teacher Move:** Consider revisiting Lesson 8 (*DinoPops*) and Lesson 9 (*Gallon Challenge*)
		Strengthen	• Invite students to create a poster using multiple representations to display their mathematical thinking.
		Stretch	• **You're invited to explore more:** Lesson 8 (*DinoPops*).
	3, 4,7	**Support**	• **Mini-Lesson: Comparing Proportional Relationships With Graphs** • **Teacher Move:** Consider revisiting Lesson 9 (*Gallon Challenge*) and Lesson 10 (*Three Turtles*)
		Strengthen	• **Turtle Challenges:** Lesson 10 (*Three Turtles*)
		Stretch	• **You're invited to explore more:** Lesson 10 (*Three Turtles*)

UNIT 4

Linear Relationships and Equations

Students compare representations of linear relationships and interpret them in context. Students also write equations for lines and determine solutions to two-variable linear equations.

Essential Questions

- What can proportional relationships and slope teach you about linear relationships?
- How do equations, tables, and graphs of linear relationships connect to one another?
- What does it mean for an ordered pair to be a solution to a linear equation?

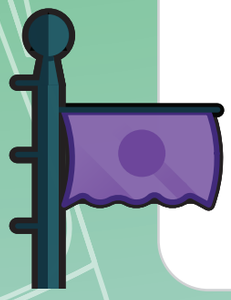
Sub-Unit Goals

Sub-Unit 1

- Represent non-proportional linear relationships with equations, tables, and graphs.
- Interpret the vertical intercept and slope of a linear relationship in context.

Sub-Unit 2

- Write an equation for a linear relationship given two points.
- Make connections between graphs and solutions of two-variable linear equations.
- Analyze strategies for calculating the slope and vertical intercept of a line through two points.



The Math of the Unit

Unit Outcomes Telling a story that narrows and deepens with the grade

Florida B.E.S.T. Standards for Mathematics

Extend understanding of proportional relationships to two-variable linear equations. **(MA.8.AR.3)**

Benchmarks Addressed in This Unit: MA.8.AR.3.1 MA.8.AR.3.2 MA.8.AR.3.3 MA.8.AR.3.4 MA.8.AR.3.5

Mathematical Thinking and Reasoning Standards

- Actively participate in effortful learning both individually and collectively. **(MTR.1.1)**
- Demonstrate understanding by representing problems in multiple ways. **(MTR.2.1)**
- Engage in discussions that reflect on the mathematical thinking of self and others. **(MTR.4.1)**
- Use patterns and structure to help understand and connect mathematical concepts. **(MTR.5.1)**

Also addressed: **MTR.3.1, MTR.7.1**

Coherence Telling a story within and across grades

In this Unit

Students make connections between non-proportional linear relationships, lines, and linear equations.

Prior Learning

- Proportional relationships in tables, equations, and graphs **(MA.7.AR.4)**
- **Unit 2:** Calculating slope **(MA.8.AR.3.2)**

Future Learning

- **Unit 5:** Systems of linear equations **(MA.8.AR.4)**
- **Unit 6:** Linear functions and non-linear functions **(MA.8.F.1.2, MA.912.F.1.6)**

See the Connections to Future Learning page for more information, including explanations and examples.

Rigor and Balance Integrating the aspects of rigor in ways that support one another

Here are examples of the aspects of rigor and balance in this unit. See the first page of each lesson's Teacher Edition for more information.

Conceptual Understanding

In **Lessons 1–4**, students make sense of unit rate, slope, proportional relationships, and non-proportional relationships. They analyze situations, graphs, and equations.

Strategic Competence

In **Lesson 4**, students use different representations of linear relationships to make predictions.

Procedural Fluency

In **Lessons 5–8** students calculate the slope of a line given two points and write equations of lines given various information.

Adaptive Reasoning

In **Lesson 5**, students navigate different values of the coordinate plane when determining slope.

Unit at a Glance

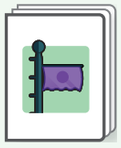
Assess and Respond

A Pre-Unit Check

Learn more about your students' understanding of foundational concepts and skills that will support them in the upcoming unit. This can be given in its entirety before the unit or spread throughout the unit.

MA.6.AR.1.1 MA.6.GR.1.1 MA.7.AR.3.2 MA.7.AR.4.4
MTR.2.1, MTR.5.1, MTR.6.1

Sub-Unit 1



1 Flags

Introduction to Linear Relationships

Examine proportional and non-proportional relationships in context and identify rates of change and initial values.

MA.8.AR.3.1 MA.8.AR.3.2
MTR.1.1 MTR.2.1 MTR.4.1

2 Water Cooler

Slopes Don't Have to Be Positive

Explore lines with decreasing slope through a context.

MA.8.AR.3.2 MA.8.AR.3.5
MTR.1.1 MTR.2.1 MTR.3.1

3 Ups and Downs

Analyzing Linear Relationships

Explore linear relationships in context and match them to representations.

MA.8.AR.3.2 MA.8.AR.3.3 MA.8.AR.3.4 MA.8.AR.3.5
MTR.2.1 MTR.4.1

Math in Action

7 Why Intercepts?

Equations of All Kinds of Lines

Write an equation for a line that passes through two points.

MA.8.AR.3.2 MA.8.AR.3.3 MA.8.AR.3.4
MTR.1.1 MTR.4.1

8 Pennies and Quarters

Using Linear Relationships to Solve Problems

Determine solutions to real-world relationships using multiple representations of linear relationships.

MA.8.AR.3.3 MA.8.AR.3.4 MA.8.AR.3.5
MTR.1.1 MTR.5.1 MTR.7.1

M Math in Action

Connecting Algebra to Space Exploration

Explore the connection between expressions, equations, robotic spacecraft, and the contributions of William Pickering to space exploration.

MA.8.NSO.1
MTR.2.1 MTR.5.1

Assess and Respond

A End-of-Unit Assessment

Learn about your students' understanding of the concepts and skills in this unit. There are two forms of the End-of-Unit Assessment: Forms A and B.

MA.8.AR.3.1, MA.8.AR.3.2, MA.8.AR.3.3, MA.8.AR.3.4, MA.8.AR.3.5, MA.8.AR.4.1
MTR.2.1, MTR.3.1, MTR.6.1, MTR.7.1

Pacing: 12 days | **Short on time?** See pacing considerations below.

All instructional and assessment days are ~45 minutes each.

- 8 Lessons
- Math in Action
- Pre-Unit Check
- Sub-Unit Quiz
- End-of-Unit Assessment

Assess and Respond

Sub-Unit 2

4 Stacking Cups

More With Linear Relationships

Use a linear relationship to make a prediction.

MA.8.AR.3.2 MA.8.AR.3.4 MA.8.AR.3.5
MTR.5.1, MTR.7.1

A Sub-Unit Quiz

Learn about your students' understanding of the concepts and skills so far in this unit.

MA.8.AR.3.1, MA.8.AR.3.2, MA.8.AR.3.3, MA.8.AR.3.4, MA.8.AR.3.5
MTR.2.1, MTR.7.1

5 Landing Planes

Calculating Slope

Determine strategies for calculating slope given two points on a line.

MA.8.AR.3.2
MTR.3.1 MTR.5.1

6 Coin Capture

Horizontal and Vertical Lines

Write equations for vertical and horizontal lines.

MA.8.AR.3.2 MA.8.AR.3.3
MTR.4.1 MTR.5.1

Pacing Considerations

Lesson 4: This lesson can be omitted as no new concepts are introduced. Students are given an opportunity to interpret a linear relationship in a new context.

Lesson Materials and Prep

Here are the materials and resources needed within each lesson, how to prepare them, and where to find them.

Lesson 1

- **Copy:** Show What You Know
- **Materials:** coloring tools, straightedges

Lesson 2

- **Copy:** Show What You Know
- **Materials:** straightedges, coloring tools or highlighters *(as needed)*

Lesson 3

- **Copy:** Show What You Know
- **Cut:** Activity 2 Cards *(one set per pair)*
- **Materials:** coloring tools *(as needed)*

Lesson 4

- **Copy:** Show What You Know
- **Materials:** graph paper *(as needed)*, stackable cups with a lid, like foam cups *(as needed)*

Lesson 5

- **Copy:** Show What You Know
- **Materials:** graph paper, straightedges

Lesson 6

- **Copy:** Show What You Know, Activity 2 Sheet *(one per student)*
- **Materials:** straightedges, calculators *(as needed)*

Lesson 7

- **Copy:** Show What You Know

Lesson 8

- **Copy:** Show What You Know
- **Materials:** calculators *(optional)*, U.S. coins, photos of U.S. coins, or coin manipulatives *(as needed)*

Resource Books

Assessment and Lesson Resources

- Pre-Unit Check
- Sub-Unit Quiz
- End-of-Unit Assessment, Forms A & B
- Assessment Rubrics
- Show What You Knows
- Activity Sheets and Cards

Intervention and Extension Resources

- Mini-Lessons
- Prior Grade Mini-Lessons
- Extensions

Caregiver Support

These resources can be shared with students' caregivers. They provide background on the mathematics in this unit, as well as suggestions for supporting students at home.

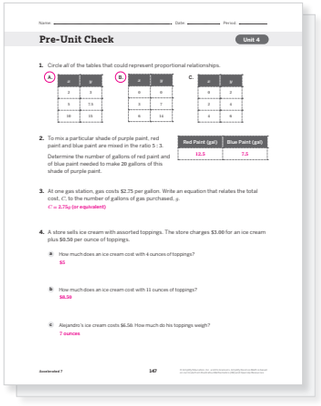
Assessments

Every unit includes a variety of formative and summative assessments designed to highlight students' understanding of a variety of concepts and skills.

Pre-Unit Check

Assign the **Pre-Unit Check** to learn more about your students' understanding of foundational concepts and skills that will support them in the upcoming unit. This can be given in its entirety before the unit or spread throughout the unit.

FL B.E.S.T. Math Benchmarks: MA.6.AR.1.1, MA.6.GR.1.1, MA.7.AR.3.2, MA.7.AR.4.4


Name _____ Date _____ Period _____

Pre-Unit Check Unit 4

1. Circle all of the tables that could represent proportional relationships.

A

x	y
2	1
3	7.5
10	35

B

x	y
10	14
4	10
14	28

C

x	y
4	2
6	3
2	4

2. To mix a particular shade of purple paint, red paint and blue paint are mixed in the ratio 5 : 3.
Determine the number of gallons of red paint and of blue paint needed to make 20 gallons of this shade of purple paint.

Red Paint (gal)	Blue Paint (gal)
12.5	7.5

3. At one gas station, gas costs $2.75 per gallon. Write an equation that relates the total cost, C, to the number of gallons of gas purchased, g.
$C = 2.75g$ (or equivalent)

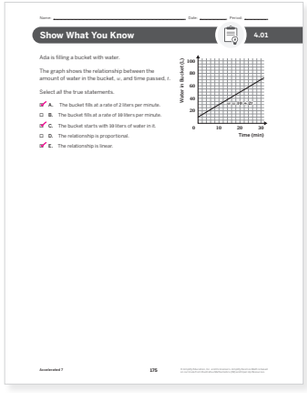
4. A store sells ice cream with assorted toppings. The store charges $3.00 for an ice cream plus $0.50 per ounce of toppings.
   a. How much does an ice cream cost with 4 ounces of toppings?
   $5
   b. How much does an ice cream cost with 11 ounces of toppings?
   $8.50
   c. Alejandro's ice cream costs $6.56. How much do his toppings weigh?
   7 ounces

Accelerated 7 147

Lesson | Show What You Know

Use the **Show What You Know** for each lesson to learn more about your students' progress toward the lesson goal(s). Here is one example from this unit.

FL B.E.S.T. Math Benchmarks on Lesson 1 Show What You Know: MA.8.AR.3.1


Name _____ Date _____ Period _____

Show What You Know 4.01

Ada is filling a bucket with water.

The graph shows the relationship between the amount of water in the bucket, w, and time passed, t.

Select all the true statements.

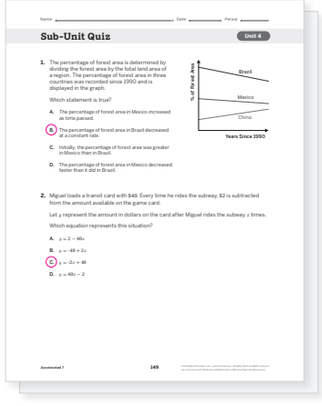
- [x] A. The bucket fills at a rate of 2 liters per minute.
- [ ] B. The bucket fills at a rate of 30 liters per minute.
- [x] C. The bucket starts with 30 liters of water in it.
- [ ] D. The relationship is proportional.
- [x] E. The relationship is linear.

Accelerated 7 175

Sub-Unit Quiz

Assign the **Sub-Unit Quiz** to learn about your students' understanding of the concepts and skills so far in this unit.

FL B.E.S.T. Math Benchmarks: MA.8.AR.3.1, MA.8.AR.3.2, MA.8.AR.3.3, MA.8.AR.3.4, MA.8.AR.3.5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Sub-Unit Quiz Unit 4

1. The percentage of forest area is determined by dividing the forest area by the total land area of a region. The percentage of forest area in three countries was recorded since 1990 and is displayed in the graph.

Which statement is true?

A. The percentage of forest area in Mexico increased as time passed.
B. The percentage of forest area in Brazil decreased at a constant rate.
C. Initially, the percentage of forest area was greater in Mexico than in Brazil.
D. The percentage of forest area in Mexico decreased faster than it did in Brazil.

2. Miguel loads a transit card with $48. Every time he rides the subway, $2 is subtracted from the amount available on the game card.
Let y represent the amount in dollars on the card after Miguel rides the subway x times.
Which equation represents this situation?

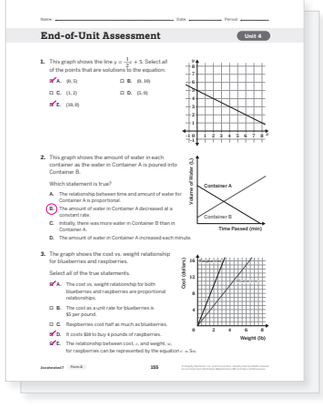
A. $y = 2 - 48x$
B. $y = -48 + 2x$
C. $y = -2x + 48$
D. $y = 48x - 2$

Accelerated 7 149

End-of-Unit Assessment

Assign the **End-of-Unit Assessment** to learn about your students' understanding of the concepts and skills in this unit. There are two forms of the End-of-Unit Assessment: Forms A and B.

FL B.E.S.T. Math Benchmarks: MA.8.AR.3.1, MA.8.AR.3.2, MA.8.AR.3.3, MA.8.AR.3.4, MA.8.AR.3.5, MA.8.AR.4.1


Name _____ Date _____ Period _____

End-of-Unit Assessment Unit 4

1. This graph shows the line $y = \frac{1}{2}x + 5$. Select all of the points that are solutions to the equation.

- [x] A. (0, 5)
- [ ] B. (6, 18)
- [ ] C. (1, 2)
- [ ] D. (5, 0)
- [x] E. (16, 13)

2. This graph shows the amount of water in each container as the water in Container A is poured into Container B.

Which statement is true?

A. The relationship between time and amount of water for Container A is proportional.
B. The amount of water in Container A decreased at a constant rate.
C. Initially, there was more water in Container B than in Container A.
D. The amount of water in Container A increased each minute.

3. The graph shows the cost vs. weight relationship for blueberries and raspberries.

Select all of the true statements.

- [x] A. The cost vs. weight relationship for both blueberries and raspberries are proportional relationships.
- [ ] B. The cost as a unit rate for blueberries is $5 per pound.
- [ ] C. Raspberries cost half as much as blueberries.
- [x] D. It costs $59 to buy 4 pounds of raspberries.
- [x] E. The relationship between cost, c, and weight, w, for raspberries can be represented by the equation $c = 5w$.

Accelerated 7 Form A 155

Differentiation

Every unit includes additional resources to support, strengthen, and stretch student learning.

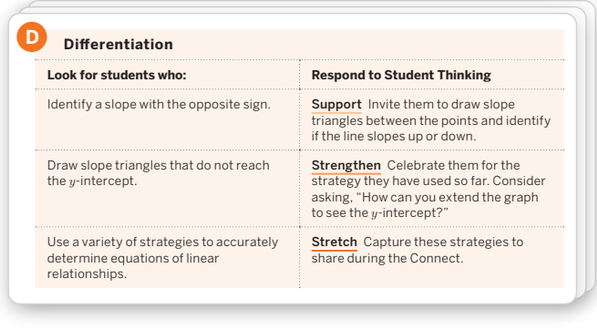
Differentiation Across the Unit

	Support	Strengthen	Stretch
	Provide targeted intervention for students by using these resources.	Reinforce students' understanding of the concepts assessed by using these resources.	Challenge students and extend their learning with these resources.
Sub-Unit 1 (Lessons 1–4) • Represent non-proportional linear relationships with equations, tables, and graphs. • Interpret the vertical intercept and slope of a linear relationship in context.	**Mini-Lesson** • *Interpreting Slope and Intercepts of Linear Relationships* **Previous Lessons** • Unit 3, Lesson 5 (*Snapshots*) • Unit 3, Lesson 11 (*Four Representations*)	**Practice** • Lessons 1–4	**You're invited to explore more.** • Lesson 2 (*Water Cooler*) • Lesson 4 (*Stacking Cups*) **DOK3 Practice** • Lesson 2
Sub-Unit 2 (Lessons 5–8) • Write an equation for a linear relationship given two points. • Make connections between graphs and solutions of two-variable linear equations. • Analyze strategies for calculating the slope and vertical intercept of a line through two points.	**Mini-Lesson** • *Calculating Slope Given Two Points* **Previous Lessons** • Unit 2, Lesson 15 (*Water Slide*) • Unit 2, Lesson 16 (*Slope Challenges*)	**Repeated Challenges** • Lesson 7 (*Why Intercepts?*) **Challenge Creator** • Lessons 5 and 6 **Practice** • Lessons 5–8	**You're invited to explore more.** • Lesson 5 (*Landing Planes*) • Lesson 8 (*Pennies and Quarters*) **DOK3 Practice** • Lesson 8

Differentiation Within the Lesson

Each Teacher Edition includes point-of-use differentiation suggestions to modify or adjust instruction during the lesson to accommodate the needs of all learners.

Here is one example from **Lesson 7**, where students write an equation of a line given two points.



Differentiation

Look for students who:	Respond to Student Thinking
Identify a slope with the opposite sign.	**Support** Invite them to draw slope triangles between the points and identify if the line slopes up or down.
Draw slope triangles that do not reach the y-intercept.	**Strengthen** Celebrate them for the strategy they have used so far. Consider asking, "How can you extend the graph to see the y-intercept?"
Use a variety of strategies to accurately determine equations of linear relationships.	**Stretch** Capture these strategies to share during the Connect.

Accessibility

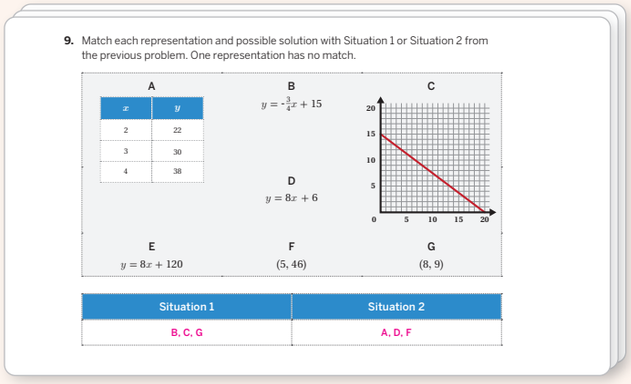
Every lesson is designed using the principles of Universal Design for Learning.

Universal Design for Learning

Here is one example from a lesson designed to support Engagement.

Spotlight on Language & Symbols . . .

Lesson 8: In this lesson, students explore equations, tables, and graphs of the same situation making explicit links between representations.



9. Match each representation and possible solution with Situation 1 or Situation 2 from the previous problem. One representation has no match.

A

x	y
2	22
3	30
4	38

B

$y = -\frac{3}{4}x + 15$

C

D

$y = 8x + 6$

E

$y = 8x + 120$

F

$(5, 46)$

G

$(8, 9)$

Situation 1	Situation 2
B, C, G	A, D, F

Just-In-Time Accommodations and Supports

Each Teacher Edition includes suggested supports based on the areas of cognitive function from the Education Development Center (EDC).

Spotlight on Memory and Attention . . .

Here are two suggestions in this unit for supporting students with maintaining working memory and sustaining focus.

Lesson 2, Activity 1

Accessibility: Memory and Attention Invite students to use additional paper to support them with performing calculations.

Lesson 4, Activity 3

Accessibility: Memory and Attention Invite students to refer back to notes they took earlier on slope and y-intercept or an anchor chart to activate prior knowledge.

Language Development

Every lesson includes opportunities for all students to develop fluency with mathematical language.

Vocabulary

New Vocabulary	Lesson(s)
linear relationship	1
horizontal intercept (x-intercept) **vertical intercept (y-intercept)**	2
solution to an equation	8

Review Vocabulary
proportional relationship
unit rate

Additional Vocabulary
undefined

Math Language Routines and Language Goals

Math Language Routine	Lesson(s)
MLR1: Stronger and Clearer Each Time	2
MLR2: Collect and Display	2, 3, and 5–7
MLR3: Critique, Correct, Clarify	1 and 7
MLR6: Three Reads	8
MLR7: Compare and Connect	5 and 7

Sub-Unit Language Goals

- Interpret the vertical intercept and slope of a linear relationship in context.
- Analyze strategies for calculating the slope and vertical intercept of a line through two points.

English Language Learners Support

Every Teacher Edition includes suggestions for supporting the needs of students learning English.

Here is one example from **Lesson 5**, where students explore the context of planes landing on a runway to support engaging with slope.

ELL **English Language Learners** Invite students to act out scenarios, show images, use gestures, or talk about landing planes before starting the task. **(Reading and Listening)**

Connections to Future Learning

Here is how the content in this unit connects to where your students are headed in their math journeys.

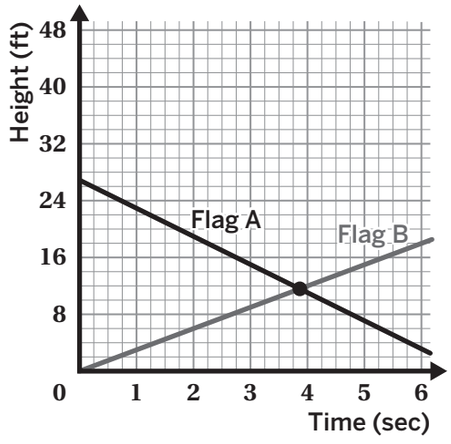
Systems of Linear Equations

In this unit, students explore linear equations. **In Unit 5**, students will solve systems of two linear equations.

FL B.E.S.T. Math Benchmark:
MA.8.AR.4.1, MA.8.AR.4.2, MA.8.AR.4.3

Example:

Flag A starts at a height of 27 feet and is being lowered at a speed of 4 feet per second. Flag B starts at the bottom of the flagpole and is being raised at a speed of 3 feet per second. When will the flags be at the same height? What height will that be? The answer to both questions is the point of intersection of the graphs of the equations, at about 11.5 feet, just before 4 seconds have passed.



Linear Functions and Nonlinear Functions

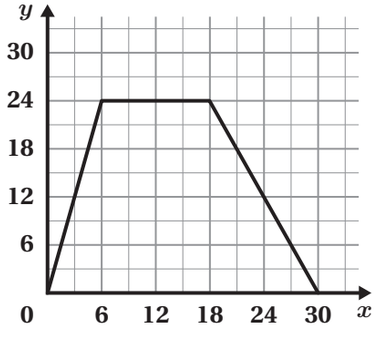
In this unit, students represent linear relationships. **In Unit 6**, students will determine whether or not linear relationships and non-linear relationships represent functions.

FL B.E.S.T. Math Benchmarks:
MA.8.F.1.1, MA.8.F.1.2, MA.8.F.1.3

Example:

In this graph, each piece of the function can be represented by a different linear equation.

This graph represents a function because each input has exactly one output. For every x-value, the graph has one unique y-value.

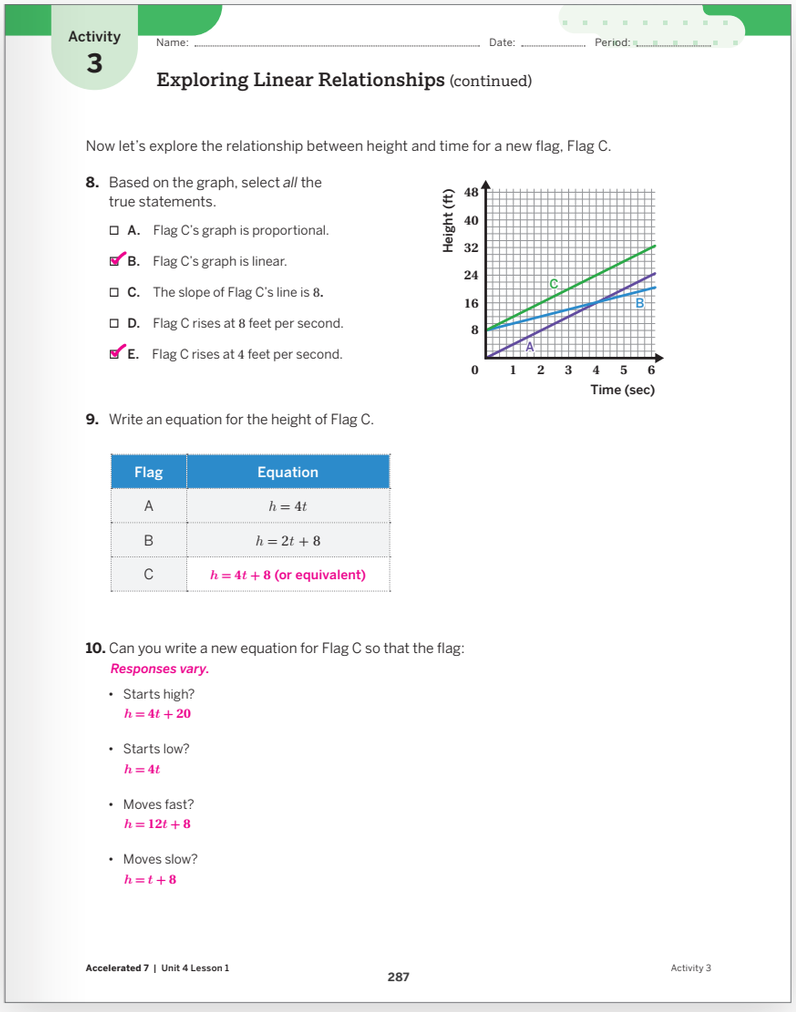


Professional Learning

Here is an opportunity to deepen your understanding of the math in this unit and how to teach it.

Spotlight on Conceptual Understanding

Lesson 1, Activity 3



Activity 3

Name: Date: Period:

Exploring Linear Relationships (continued)

Now let's explore the relationship between height and time for a new flag, Flag C.

8. Based on the graph, select *all* the true statements.

- ☐ **A.** Flag C's graph is proportional.
- ☑ **B.** Flag C's graph is linear.
- ☐ **C.** The slope of Flag C's line is 8.
- ☐ **D.** Flag C rises at 8 feet per second.
- ☑ **E.** Flag C rises at 4 feet per second.

9. Write an equation for the height of Flag C.

Flag	Equation
A	$h = 4t$
B	$h = 2t + 8$
C	$h = 4t + 8$ (or equivalent)

10. Can you write a new equation for Flag C so that the flag:

Responses vary.

- Starts high?
  $h = 4t + 20$
- Starts low?
  $h = 4t$
- Moves fast?
  $h = 12t + 8$
- Moves slow?
  $h = t + 8$

Accelerated 7 | Unit 4 Lesson 1 287 Activity 3

Context

In this activity, students develop **conceptual understanding** by identifying non-proportional linear relationships on a graph or in an equation.

Try This

Put on your student hat and complete Problems 8–10.

Points to Ponder

- What do students need to understand about proportional and non-proportional linear relationships to support their understanding of the situation?
- How does writing equations for non-proportional relationships build from their understanding of proportional relationships?
- How will comparing related equations support their understanding of interpreting linear relationships in context in the future?

Notes:

Assess and Respond

Pre-Unit Check

Independent | 45 min

Facilitation: Assign the Pre-Unit Check to learn more about your students' understanding of foundational concepts and skills that will support them in the upcoming unit. This can be given in its entirety before the unit or spread throughout the unit. If time allows, consider inviting students to complete one or more of the tasks in the Unit Synthesis and Reflection.

Item Analysis

Problem(s)	Concept or skill	Preparation for	DOK	FL B.E.S.T. Math Benchmarks
1	Determining if a relationship is proportional	Lesson 1	1	MA.7.AR.3.2 MTR.5.1
2	Solving problems with proportional relationships and ratios	Lesson 1	1	MA.7.AR.3.2 MTR.2.1
3	Writing equations to describe proportional relationships	Lesson 1	1	MA.7.AR.4.4
4	Solving problems that involve a unit rate and a starting value	Lesson 1	1	MA.6.AR.1.1 MTR.6.1
5	Plotting and describing points with the same x-coordinate	Lesson 6	1	MA.6.GR.1.1

Assessment and Lesson Resources

- Student Assessments
- Show What You Know and Answer Keys

Intervention and Extension Resources

- Mini-Lessons
- Extensions

Name: ..... Date: ..... Period: .....

Pre-Unit Check

Unit 4

1. Circle *all* of the tables that could represent proportional relationships.

A.

x	y
2	3
5	7.5
10	15

B.

x	y
0	0
3	7
6	14

C.

x	y
0	2
2	4
4	6

2. To mix a particular shade of purple paint, red paint and blue paint are mixed in the ratio 5 : 3.

Determine the number of gallons of red paint and of blue paint needed to make 20 gallons of this shade of purple paint.

Red Paint (gal)	Blue Paint (gal)
12.5	7.5

3. At one gas station, gas costs $2.75 per gallon. Write an equation that relates the total cost, C, to the number of gallons of gas purchased, g.

$C = 2.75g$ (or equivalent)

4. A store sells ice cream with assorted toppings. The store charges $3.00 for an ice cream plus $0.50 per ounce of toppings.

a. How much does an ice cream cost with 4 ounces of toppings?

$5

b. How much does an ice cream cost with 11 ounces of toppings?

$8.50

c. Alejandro's ice cream costs $6.50. How much do his toppings weigh?

7 ounces

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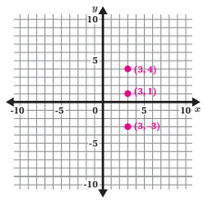
Name: ..... Date: ..... Period: .....

Pre-Unit Check (continued)

Unit 4

5. Here is a coordinate plane.



a. Plot and label three different points with an x-coordinate of 3.

Responses vary.

b. Describe *all* of the points with an x-coordinate of 3.

Responses vary.

- All the points could be described with the equation $x = 3$.
- All the points make a vertical line.

D Differentiation (Pre-Unit Check)

Note: To **strengthen** and **stretch** students' learning, refer to the differentiation resources suggested throughout this unit and in the Unit Overview.

Sub-Unit Goals	Problem(s)	Respond to Student Thinking
Preparation for Sub-Unit 1: • Represent non-proportional linear relationships with equations, tables, and graphs. **(Lessons 1–4)** **Preparation for Sub-Unit 2:** • Write an equation for a linear relationship given two points. Make connections between graphs and solutions of two-variable linear equations. **(Lessons 4–8)**	1, 2	**Support** • **Teacher Move:** Consider revisiting **Unit 3, Lesson 2** (*Sugar, Spice, and Everything Rice*). • **Teacher Move:** Throughout Sub-Unit 1, plan to focus on multiple ways to know that a relationship is proportional (e.g., graph with a line through the origin, equivalent ratios in tables).
	3	**Support** • **Mini-Lesson:** *Finding More Than One Equation to Represent Proportional Relationships* • **Teacher Move:** Consider revisiting **Unit 3, Lesson 5** (*Snapshots*).
	4	**Support** • **Teacher Move:** During **Lesson 2**, reinforce language about initial value and rate of change to support interpreting equations.
	5	**Support** • **Teacher Move:** Plan to spend extra time on the **Warm-Up of Lesson 7** discussing what is true about all of the points with the same x-value.

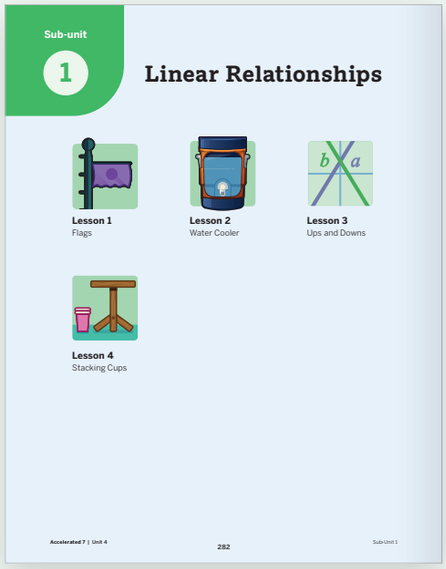
Sub-Unit 1

Linear Relationships

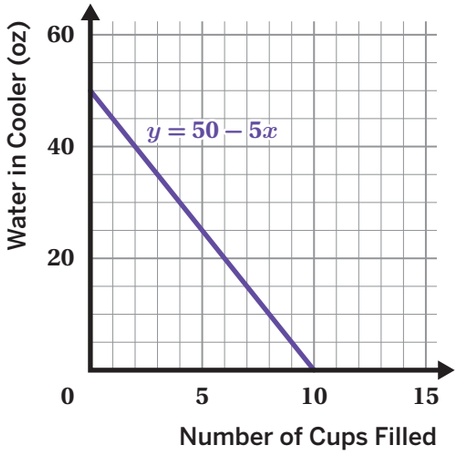
Sub-Unit 1 Goals

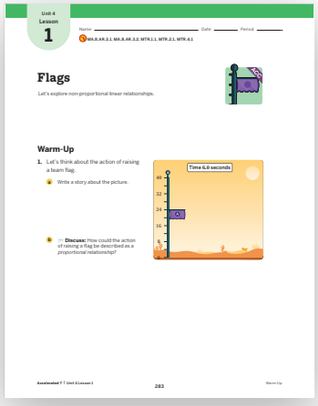
- **Goal:** Represent non-proportional linear relationships with equations, tables, and graphs.
- **Language Goal:** Interpret the vertical intercept and slope of a linear relationship in context.

In this sub-unit, students are introduced to **linear relationships** that are not proportional. They create and connect representations for linear relationships and interpret the slope, **vertical intercept**, and **horizontal intercept** of linear relationships in context.



Math That Matters Most

Concept or Skill	For example . . .
Write an equation for a linear relationship with a positive or negative slope.	A bike rental company charges a one-time fee of \$30 and then \$5 per hour to use a bike. Let y represent the total amount a customer pays in dollars after x hours. Write an equation that represents the situation. $y = 30 + 5x$
Interpret the slope and vertical intercept in context.	The line $y = 50 - 5x$ represents a different water cooler situation. Write a story that this graph could represent. A water cooler at a soccer practice started with 50 ounces of water in it. Each time someone filled a cup, the amount of water in the cooler decreased by 5 ounces. 


Unit 4
Lesson
1

Flags

Let's explore non-proportional linear relationships.

Warm-Up

1. Let's think about the action of raising a team flag.

Write a story about the picture.

Discuss: How could the action of raising a flag be described as a proportional relationship?

Time 6.0 seconds

283

Flags

Introduction to Linear Relationships

Let's explore non-proportional linear relationships.

Learning Outcomes

Today's Goals

1. **Goal:** Understand that there are *linear relationships* that are not proportional.
2. **Language Goal:** Describe how to identify non-proportional linear relationships from a graph or an equation. **(Reading, Writing, Speaking, and Listening)**

Students examine *proportional* and non-proportional relationships in context and identify rates of change and initial values. Students are also introduced to non-proportional **linear relationships** in this lesson. They describe these linear relationships using graphs and equations, and they engage in discussions about these supporting representations. Students make connections between representations and the context to write equations that represent non-proportional linear relationships. **(MTR.2.1, MTR.4.1)**

Coherence

Prior Learning

In Unit 3, students determined whether two quantities had a *proportional relationship* by examining tables, equations, and graphs.

Future Learning

In Lesson 2, students will explore similarities and differences in the directionality of lines and will reason with the idea of negative slope in context.

Connections

- Students use constant rate of change to interpret different heights an object has grown over time, as well as miles walked per hour. (real world)

FL B.E.S.T. Math Benchmarks

Addressing

MA.8.AR.3.1
Determine if a linear relationship is also a proportional relationship.

MA.8.AR.3.2
Given a table, graph or written description of a linear relationship, determine the slope.

Mathematical Thinking and Reasoning Standards: MTR.1.1, MTR.2.1, MTR.4.1

ELA Expectations: ELA.K12.EE.1.1, ELA.K12.EE.5.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.7.AR.4.1 **MA.7.AR.4.2**	**MA.912.AR.2.2**

Vocabulary

New Vocabulary

linear relationship

Additional Vocabulary

proportional relationship *unit rate*

Lesson at a Glance

~ 45 min

MA.8.AR.3.1, MA.8.AR.3.2

Warm-Up

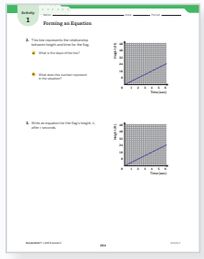
Pairs | 5 min

Students make sense of a situation and its connection to a linear representation.
Routine: Tell a Story **(MTR.1.1)**

Activity 1

Pairs | 5 min

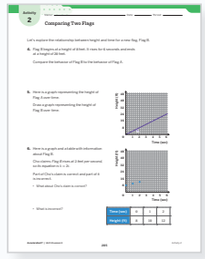
Students analyze a graph and contextualize its related equation. **(MTR.2.1)**



Activity 2

Pairs | 10 min

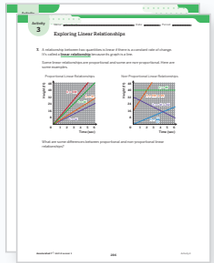
Students use the **MLR3: Critique, Correct, Clarify** routine to engage in discussions about representations of a context. **(MTR.4.1)**



Activity 3

Pairs | 15 min

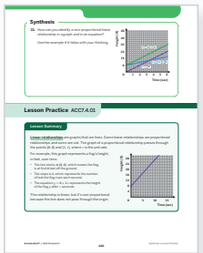
Students analyze several examples of graphs to engage in discussions about different types of **linear relationships**. **(MTR.4.1)**



Synthesis

Whole Class | 5 min

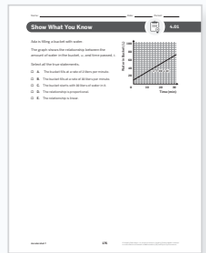
Students synthesize their understanding of *proportional* and non-proportional linear relationships.



Show What You Know

Independent | 5 min

Students demonstrate their understanding by interpreting the graph of a linear relationship.



Prep Checklist

This lesson includes:

- Student Edition
- Show What You Know
- Coloring tools
- Straightedges

Pairs | 5 min

Warm-Up

Purpose: Students make sense of a situation and its connection to a linear representation. **(MTR.1.1)**

1 Launch

Consider asking students to reflect on their own personal connections to flags before describing a situation about raising a flag.

Use the Tell a Story routine to help students make sense of the flag context. Invite students to write a story about the image using their unique experiences, perspectives, and creativity. **(MTR.1.1)**

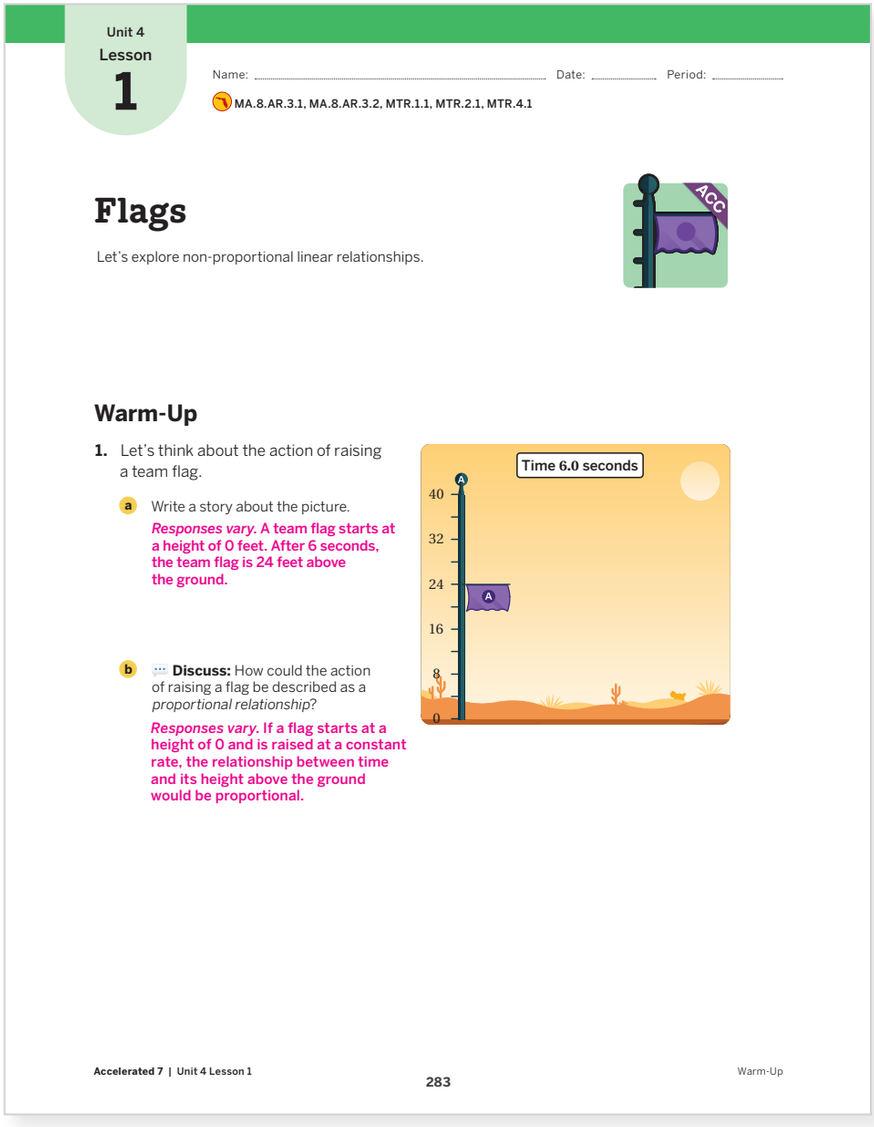
2 Connect

Invite students to share their observations about the image.

Create a record of the details students noticed and wondered, along with the name of the students who shared them.

To surface connections, consider asking:

- "How is this situation similar to ones we have seen before? How is it different?"
- "Where was the flag before it began to be raised? After it was raised? How do you know?"



Unit 4
Lesson
1

Name: Date: Period:

MA.8.AR.3.1, MA.8.AR.3.2, MTR.1.1, MTR.2.1, MTR.4.1

Flags

Let's explore non-proportional linear relationships.

Warm-Up

1. Let's think about the action of raising a team flag.

   a. Write a story about the picture.

   Responses vary. A team flag starts at a height of 0 feet. After 6 seconds, the team flag is 24 feet above the ground.

   b. **Discuss:** How could the action of raising a flag be described as a *proportional relationship*?

   Responses vary. If a flag starts at a height of 0 and is raised at a constant rate, the relationship between time and its height above the ground would be proportional.

Accelerated 7 | Unit 4 Lesson 1 283 Warm-Up

Pairs | 5 min

Activity 1 Forming an Equation

Purpose: Students analyze a graph and contextualize its related equation. **(MTR.2.1)**

1 Launch

To surface connections, invite students to share any words or images they think of when they hear the word *slope*.

2 Monitor

To support students getting started, consider asking, "How can we use the graph to help determine the slope?"

Listen for student slope understandings and equations to share out during the Connect.

D Differentiation *(Problems 2–3)*

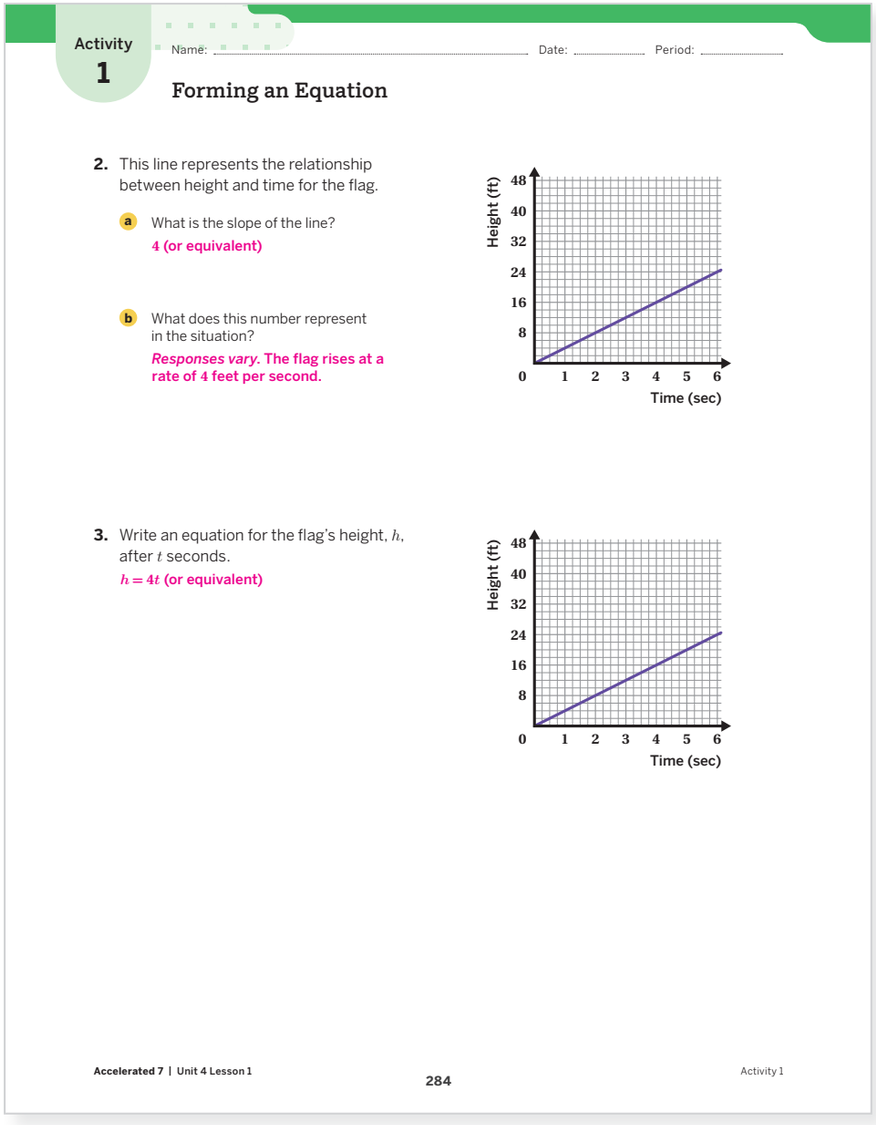
Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Invite them to create a slope triangle on the graph.
Represent slope triangles on the graph.	**Strengthen** Celebrate them for the strategies they have tried so far. Encourage them to use their triangles to determine a value for slope.
Write the equation as $h = \frac{1}{4}t$.	**Strengthen** Consider asking, "How do we determine slope? How did we do this in previous lessons?"
Write the equation as $h = 4t$.	**Stretch** Consider asking, "What might a graph or equation look like if the flag started at some height and was lowered to the ground?" Invite them to discuss and share with their partner.

3 Connect

Share out collected ideas about slope and equations, as well as strategies used to generate and understand these representations.

Consider asking:

- "How did our equation connect to this situation involving flags?"
- "How does the equation connect to the graph?"



Activity 1

Name: _____ Date: _____ Period: _____

Forming an Equation

2. This line represents the relationship between height and time for the flag.

a. What is the slope of the line?
4 (or equivalent)

b. What does this number represent in the situation?
Responses vary. **The flag rises at a rate of 4 feet per second.**

3. Write an equation for the flag's height, h, after t seconds.
$h = 4t$ (or equivalent)

Accelerated 7 | Unit 4 Lesson 1 284 Activity 1

Pairs | 10 min

Activity 2 Comparing Two Flags

Purpose: Students use the **Critique, Correct, Clarify** routine to engage in discussions about representations of a context. **(MTR.4.1)**

1 Launch

Direct students' attention to the description of Flag B.

Consider providing access to straightedges and coloring tools.

Consider asking, "What are some differences between Flag A and Flag B?"

ELL **English Language Learners** Encourage students to paraphrase each other's ideas to help make connections and incorporate vocabulary from the unit. **(Speaking and Listening)**

2 Monitor

For Problem 5, **Invite students** to check each other's work and provide feedback.

A **Accessibility: Visual-Spatial Processing** Encourage students to identify how specific moments in time connect with the graph.

Consider asking pairs how they might use each representation for evidence.

For Problem 6, **Look for** students who state that Cho's claim is incorrect because the argument doesn't take into account the flag starting 8 feet off the ground. Invite them to share their evidence during the Connect.

3 Connect

Display the representations for students to make sense of.

MLR **MLR3: Critique, Correct, Clarify** Use this routine to help students describe strengths, errors, and ambiguities in Cho's work. As a class:
- Discuss what is *correct* about her response.
- Discuss what is *incorrect* or *unclear*.
- Create a *correct* and *clear* solution.

To surface the Key Takeaway, consider asking:
- "How can we describe this relationship?"
- "Does the graph represent a *proportional relationship*? Why or why not?"

Activity 2

Name: Date: Period:

Comparing Two Flags

Let's explore the relationship between height and time for a new flag, Flag B.

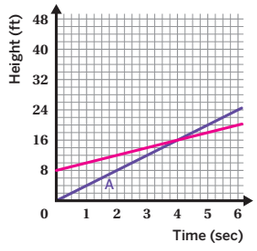
4. Flag B begins at a height of 8 feet. It rises for 6 seconds and ends at a height of 20 feet.

Compare the behavior of Flag B to the behavior of Flag A.

Responses vary.
- Flag B starts higher than Flag A, but ends lower.
- Both flags move up.

5. Here is a graph representing the height of Flag A over time.

Draw a graph representing the height of Flag B over time.



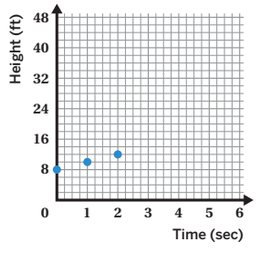
6. Here is a graph and a table with information about Flag B.

Cho claims: *Flag B rises at 2 feet per second, so its equation is* $h = 2t$.

Part of Cho's claim is correct and part of it is incorrect.

- What about Cho's claim is correct?

  Responses vary. Cho's claim is correct in that the flag rises 2 feet per second, so $2t$ will be part of the equation.



- What is incorrect?

  Responses vary. Cho's claim is incorrect because the equation $h = 2t$ means Flag B would start at 0 feet, instead of at 8 feet. The equation should be $h = 2t + 8$.

Time (sec)	0	1	2
Height (ft)	8	10	12

Accelerated 7 | Unit 4 Lesson 1 285 Activity 2

Key Takeaway: There are relationships that have a constant rate of change that do not go through the origin. An equation in the form $y = mx$ cannot represent these relationships.

Pairs | 15 min

Activity 3 Exploring Linear Relationships

Purpose: Students analyze several examples of graphs to engage in discussions about different types of **linear relationships**. **(MTR.4.1)**

Short on time? Consider facilitating Problem 10 as a class.

1 Launch

Invite students to explore both images and make observations with their partner.

Consider asking:

- "What do all of the proportional relationship lines have in common?"
- "What is different about the graphs of non-proportional linear relationships?"
- "How do those differences from the graphs appear in the equations?"

Create a record of ideas about the vocabulary term **linear relationship**. Encourage students to share ideas about both proportional and non-proportional relationships.

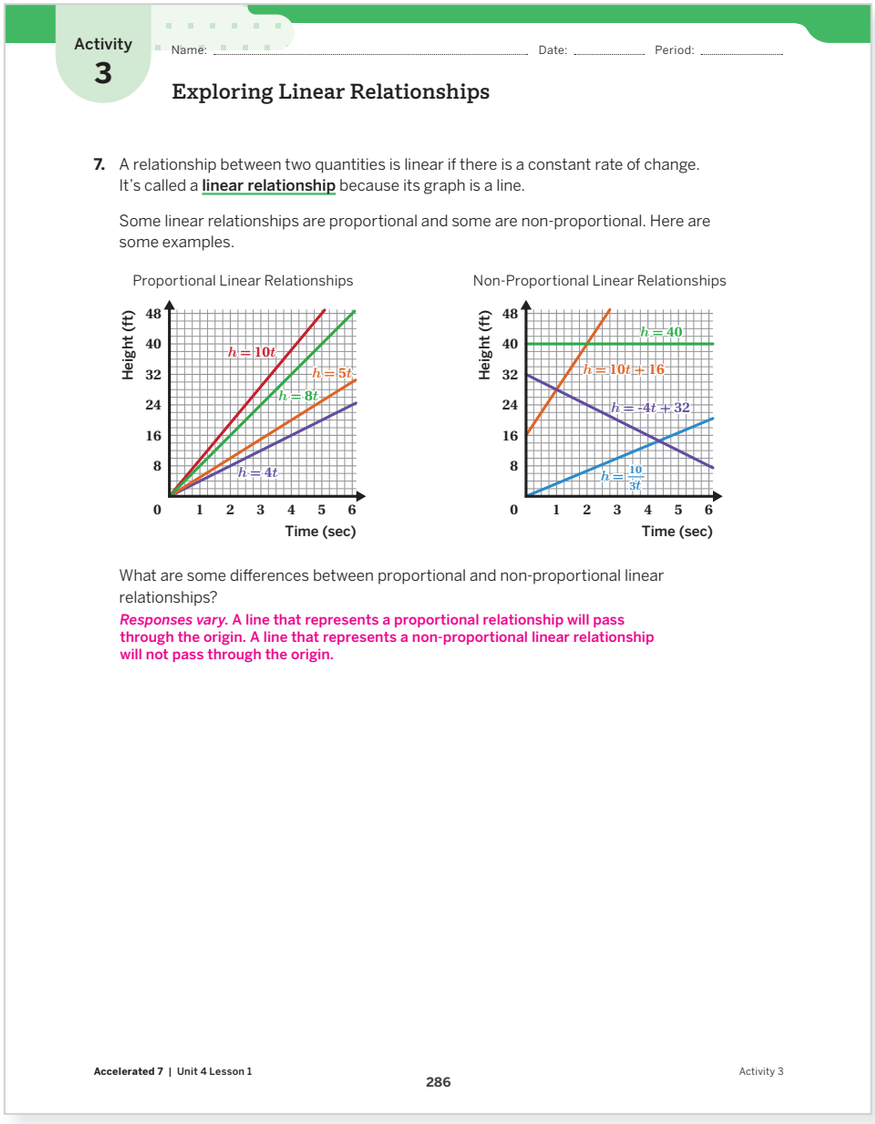
Note: The examples of proportional relationships here are intentionally all positive slopes so they fit in the first quadrant. Students will explore negative slopes formally in Lessons 2 and 3.

2 Monitor

Invite students to continue moving through the activity.

Look for conflict or consensus among student responses. Consider waiting to discuss this until the conversation on Problem 10.

Activity 3 continued >



Activity 3

Name: _____ Date: _____ Period: _____

Exploring Linear Relationships

7. A relationship between two quantities is linear if there is a constant rate of change. It's called a **linear relationship** because its graph is a line.

Some linear relationships are proportional and some are non-proportional. Here are some examples.

Proportional Linear Relationships

Non-Proportional Linear Relationships

What are some differences between proportional and non-proportional linear relationships?

Responses vary. A line that represents a proportional relationship will pass through the origin. A line that represents a non-proportional linear relationship will not pass through the origin.

Accelerated 7 | Unit 4 Lesson 1 286 Activity 3

Pairs | 15 min

Activity 3 Exploring Linear Relationships (continued)

Purpose: Students analyze several examples of graphs to engage in discussions about different types of linear relationships. **(MTR.4.1)**

Short on time? Consider facilitating Problem 10 as a class.

2 Monitor

Select and sequence student strategies to share during the Connect.

D Differentiation *(Problems 9–10)*

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Invite them to use the examples given to try equations and reflect on their attempts.
Identify individual points on the graph.	**Support** Celebrate their sense-making so far. Invite them to sketch slope triangles to support writing the equations.
Write an equation by guessing and checking different values.	**Strengthen** Invite them to consider how Flag C is similar to Flag B and Flag A. Consider asking, "How could you use those equations to help write an equation for Flag C?"
Want to explore non-proportional relationships further.	**Stretch** Invite them to sketch a graph and determine an equation for a flag that begins at 8 feet and lowers to the ground.

To encourage playfulness, invite students to think about a wide range of numbers in their equations.

Consider asking:
- "How can you make the flag start high or go low?"
- "How can you make the flag move fast or slow?"
- "What do you think a graph of a fast-moving flag that starts low to the ground would look like?"
- "What could be an equation for Flag C if it stays the same height the whole time?"

3 Connect

Share strategies collected during the Monitor.

Consider asking:
- "Which flag represents a proportional relationship? How do you know?"
- "Which flag represents a non-proportional relationship? How do you know?"
- "Which flags are linear?"

Note: If time allows, invite students to revise their previous thinking in Problem 7 based on the discussion.

Activity 3

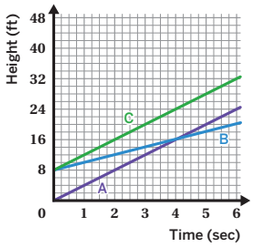
Name: _____ Date: _____ Period: _____

Exploring Linear Relationships (continued)

Now let's explore the relationship between height and time for a new flag, Flag C.

8. Based on the graph, select *all* the true statements.

- ☐ **A.** Flag C's graph is proportional.
- ☑ **B.** Flag C's graph is linear.
- ☐ **C.** The slope of Flag C's line is 8.
- ☐ **D.** Flag C rises at 8 feet per second.
- ☑ **E.** Flag C rises at 4 feet per second.



9. Write an equation for the height of Flag C.

Flag	Equation
A	$h = 4t$
B	$h = 2t + 8$
C	$h = 4t + 8$ (or equivalent)

10. Can you write a new equation for Flag C so that the flag:

Responses vary.

- Starts high?
  $h = 4t + 20$
- Starts low?
  $h = 4t$
- Moves fast?
  $h = 12t + 8$
- Moves slow?
  $h = t + 8$

Accelerated 7 | Unit 4 Lesson 1 287 Activity 3

Whole Class | 5 min

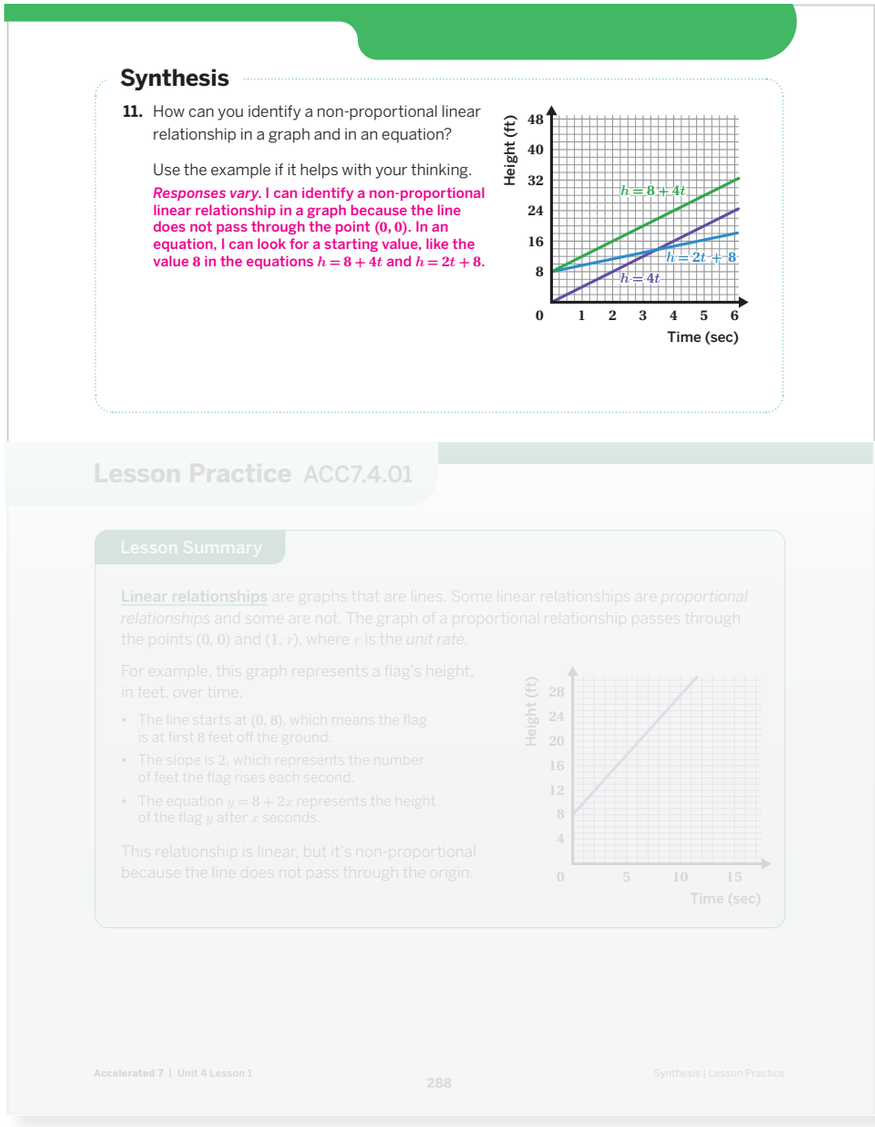
Synthesis

Purpose: Students synthesize their understanding of *proportional* and non-proportional linear relationships.

Invite students to respond independently, and then share their thinking with a partner.

Collect and share a variety of ideas, such as:
- A non-proportional linear relationship will not pass through the point $(0, 0)$.
- The equation of a non-proportional linear relationship has an initial value, such as 8 in $h = 8 + 4t$.
- All linear relationships have a slope, which can be shown by the steepness of a graphed line and the value of m in their equations.

To surface connections, consider asking:
- "What makes a non-proportional relationship different from a proportional relationship?"
- "Where does the initial value in an equation appear on the graph?"
- "How does slope appear in a linear relationship?"



Synthesis

11. How can you identify a non-proportional linear relationship in a graph and in an equation?

Use the example if it helps with your thinking.

Responses vary. **I can identify a non-proportional linear relationship in a graph because the line does not pass through the point (0, 0). In an equation, I can look for a starting value, like the value 8 in the equations $h = 8 + 4t$ and $h = 2t + 8$.**

Lesson Practice ACC7.4.01

Lesson Summary

Linear relationships are graphs that are lines. Some linear relationships are *proportional relationships* and some are not. The graph of a proportional relationship passes through the points $(0, 0)$ and $(1, r)$, where r is the *unit rate*.

For example, this graph represents a flag's height, in feet, over time.

- The line starts at $(0, 8)$, which means the flag is at first 8 feet off the ground.
- The slope is 2, which represents the number of feet the flag rises each second.
- The equation $y = 8 + 2x$ represents the height of the flag y after x seconds.

This relationship is linear, but it's non-proportional because the line does not pass through the origin.

Accelerated 7 | Unit 4 Lesson 1 288 Synthesis | Lesson Practice

Lesson Takeaway: Graphs that are lines represent **linear relationships**. A *proportional relationship* is a special type of linear relationship, but not all linear relationships are proportional. In particular, a graph of a non-proportional linear relationship will not pass through the origin, and its equation will have a non-zero initial value.

Independent | 5 min

Show What You Know

Purpose: Students demonstrate their understanding by interpreting the graph of a linear relationship.

Today's Goals

1. **Goal:** Understand that there are *linear relationships* that are not proportional.
2. **Language Goal:** Describe how to identify non-proportional linear relationships from a graph or an equation. **(Reading, Writing, Speaking, and Listening)**

Show What You Know (Assessment Resources)

Name: _____ Date: _____ Period: _____

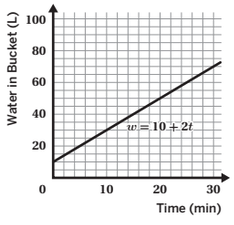
Show What You Know 4.01

Ada is filling a bucket with water.

The graph shows the relationship between the amount of water in the bucket, w, and time passed, t.

Select *all* the true statements.

- [x] **A.** The bucket fills at a rate of 2 liters per minute.
- [ ] **B.** The bucket fills at a rate of 10 liters per minute.
- [x] **C.** The bucket starts with 10 liters of water in it.
- [ ] **D.** The relationship is proportional.
- [x] **E.** The relationship is linear.



Differentiation Use after Lesson 1

Support

Provide targeted intervention for students by using these resources.

If students need support with linear relationships:

» **Respond to Student Thinking:**

- Use the ***Finding More Than One Equation to Represent Proportional Relationships*** **Mini-Lesson.**
- Review the **Lesson Summary** and discuss connections between the equation, graph, and situation.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 2 Extension Activities**.

Professional Learning

Take a moment to reflect on students' previous math experiences. What strengths did your students exhibit today? How can you build from those strengths as students continue learning about proportional relationships?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

11. How can you identify a non-proportional linear relationship in a graph and in an equation?

Use the example if it helps with your thinking.

Responses vary. I can identify a non-proportional linear relationship in a graph because the line does not pass through the point (0, 0). In an equation, I can look for a starting value, like the value 8 in the equations $h = 8 + 4t$ and $h = 2t + 8$.

Lesson Practice ACC7.4.01

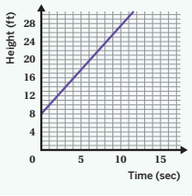
Lesson Summary

Linear relationships are graphs that are lines. Some linear relationships are *proportional relationships* and some are not. The graph of a proportional relationship passes through the points $(0, 0)$ and $(1, r)$, where r is the *unit rate*.

For example, this graph represents a flag's height, in feet, over time.

- The line starts at $(0, 8)$, which means the flag is at first 8 feet off the ground.
- The slope is 2, which represents the number of feet the flag rises each second.
- The equation $y = 8 + 2x$ represents the height of the flag y after x seconds.

This relationship is linear, but it's non-proportional because the line does not pass through the origin.



Lesson Practice ACC7.4.01

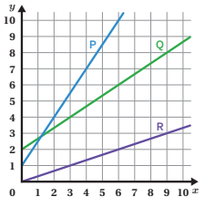
Name: ..... Date: ..... Period: .....

1. Match each equation with the graph of its line.

   a. $y = \frac{1}{3}x$ R

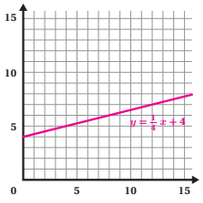
   b. $y = \frac{3}{2}x + 1$ P

   c. $y = \frac{2}{3}x + 2$ Q



2. Create a graph that shows a non-proportional linear relationship with a slope of $\frac{1}{4}$. Then write an equation that represents the graph.

   Responses vary.

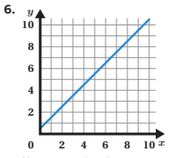


Problems 3–8: Determine whether each of these linear relationships is *proportional* or *non-proportional*.

3. From their initial step, a student walks at a constant speed of 4 miles per hour.
   Proportional

4. $y = 2x$
   Proportional

5. A giraffe at birth is 3 feet tall and grows 161 inches every month for a year.
   Non-proportional

6. 
   Non-proportional

7.

x	y
2	4
3	5
5	7

   Non-proportional

8. $y = 20 + 4x$
   Non-proportional

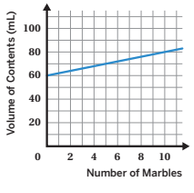
Lesson Practice ACC7.4.01

Name: ..... Date: ..... Period: .....

FAST Practice

9. In Aesop's fable "The Crow and the Pitcher," a thirsty crow discovers a pitcher of water but is unable to reach the water with its beak. So the crow adds pebbles to the pitcher to make the water rise higher and higher until it reaches the top. Then it can drink the water!

   Thiago recreates this situation using a cylinder containing water and a bunch of marbles. The line represents the volume of the cylinder's contents, y, after x marbles are added.



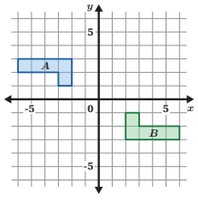
   Determine whether each statement is *true* or *false*.

	True	False
The slope of the line is 2.	☑	☐
The volume of the contents increases by 2 milliliters for every 1 marble added.	☑	☐
The equation representing the relationship between x and y is $y = 60x + 2$.	☐	☑

Spiral Review

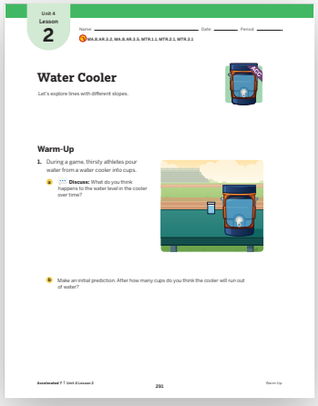
10. Are figures A and B congruent? Explain your thinking.

    Yes. *Explanations vary.* There's a rigid transformation that moves one figure onto the other, which means the figures are congruent. For example, figure A can be rotated 180° about the origin to move it onto figure B.



Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1	1	MA.8.AR.3.2
	2	2	MA.8.AR.3.1, MA.8.AR.3.2
	3–8	2	MA.8.AR.3.1
FAST Practice	9	2	MA.8.AR.3.1, MA.8.AR.3.2
Spiral Review			
	10	2	MA.8.GR.2.3


Unit 4
Lesson
2
Water Cooler
Let's explore lines with different slopes.
Warm-Up
1. During a game, thirsty athletes pour water from a water cooler into cups.
Discuss: What do you think happens to the water level in the cooler over time?
Make an initial prediction. After how many cups do you think the cooler will run out of water?
291

Water Cooler

Slopes Don't Have to Be Positive

Let's explore lines with different slopes.

Learning Outcomes

Today's Goals

1. **Goal:** Write an equation for a linear relationship with a negative slope by expressing regularity in repeated reasoning.
2. **Goal:** Connect representations of the slope and *vertical intercept* between a graph, equation, and situation.
3. **Language Goal:** Interpret the *horizontal* and *vertical intercepts* of a real-world linear relationship. **(Reading, Writing, Speaking, and Listening)**

Students are introduced to lines that visually slope downward from left to right. They explore a situation in which one quantity decreases at a constant rate in relation to a second quantity. Students repeatedly subtract a quantity to conceptualize negative slope. Then they use the graph of the situation to reason about the slope, **horizontal intercept**, and **vertical intercept** in terms of the situation. **(MTR.2.1, MTR.3.1)**

Coherence

Prior Learning

In Lesson 1, students were introduced to linear relationships and learned how to identify non-proportional linear relationships using graphs and equations.

Future Learning

In Lesson 3, students will interpret a negative slope in additional contexts and distinguish between situations with positive and negative slopes.

Connections

- Students utilize linear relationships to represent the amount of water remaining in a cooler over time, battery life remaining in a computer, and amount of money remaining in an account. (real world)

FL B.E.S.T. Math Benchmarks

Addressing

MA.8.AR.3.2
Given a table, graph, or written description of a linear relationship, determine the slope.

MA.8.AR.3.5
Given a real-world context, determine and interpret the slope and y-intercept of a two-variable linear equation from a written description, a table, a graph or an equation in slope-intercept form

Mathematical Thinking and Reasoning Standards: MTR.1.1, MTR.2.1, MTR.3.1

ELA Expectations: ELA.K12.EE.4.1, ELA.K12.EE.5.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.7.AR.4.2	MA.912.AR.2.2
MA.7.AR.4.5	MA.912.AR.2.5
	MA.912.F.1.5

Vocabulary

New Vocabulary

horizontal intercept (x-intercept)

vertical intercept (y-intercept)

Lesson at a Glance

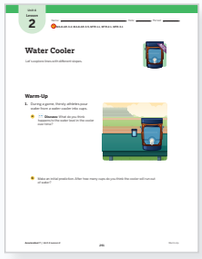
~ 45 min

MA.8.AR.3.2, MA.8.AR.3.5

Warm-Up

Pairs | 5 min

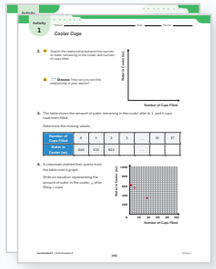
Students make sense of a water cooler situation in order to make a prediction.
Routine: Think-Pair-Share **(MTR.1.1)**



Activity 1

Pairs | 15 min

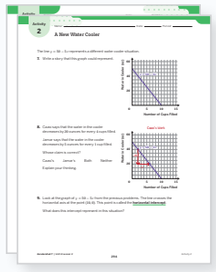
Students use multiple representations, namely a graph and table, to write an equation of a linear relationship with negative slope. **(MTR.2.1)**



Activity 2

Pairs | 15 min

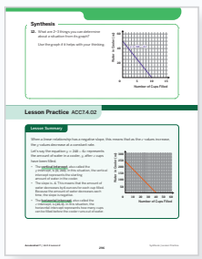
Students make sense of and interpret the slope and **horizontal intercept** of a linear relationship through multiple representations. MLR1: Stronger and Clearer Each Time **(MTR.2.1, MTR.3.1)**



Synthesis

Whole Class | 5 min

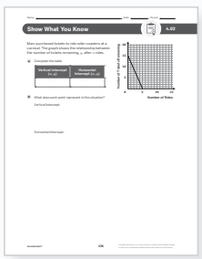
Students synthesize their understanding of how to interpret linear relationships with negative slopes. MLR2: Collect and Display



Show What You Know

Independent | 5 min

Students demonstrate their understanding by identifying and interpreting the vertical and horizontal intercepts from the graph of a linear relationship.



Prep Checklist

This lesson includes:

- Student Edition
- Show What You Know
- Straightedges
- Coloring tools or highlighters (as needed)

Pairs | 5 min

Warm-Up

Purpose: Students make sense of a water cooler situation in order to make a prediction. (MTR.1.1)

1 Launch

Use the Think-Pair-Share routine to promote curiosity and help students make sense of the water cooler context.

ELL **English Language Learners** Invite students to act out scenarios, show videos/images, use gestures, or talk about water coolers before starting the task. **(Reading and Listening)**

2 Connect

To surface connections, consider asking:

- "What questions come to mind when you see this water cooler?"
- "What quantities are involved in this situation?"
- "How would you describe the change happening in the water cooler?"
- "After how many cups do you think the cooler will run out of water? What did you use to make a prediction?"

Unit 4
Lesson
2

Name: _____ Date: _____ Period: _____

MA.8.AR.3.2, MA.8.AR.3.5, MTR.1.1, MTR.2.1, MTR.3.1

Water Cooler

Let's explore lines with different slopes.

Warm-Up

1. During a game, thirsty athletes pour water from a water cooler into cups.

a. **Discuss:** What do you think happens to the water level in the cooler over time?

Responses vary. **The water level in the cooler goes down as the cooler empties.**

b. Make an initial prediction. After how many cups do you think the cooler will run out of water?

Responses vary.

- **100 cups**
- **50 cups**
- **80 cups**

Accelerated 7 | Unit 4 Lesson 2 291 Warm-Up

Pairs | 15 min

Activity 1 Cooler Cups

Purpose: Students use multiple representations, namely a graph and table, to write an equation of a linear relationship with negative slope. **(MTR.2.1)**

1 Launch

Invite students to sketch the relationship between the ounces of water remaining and the number of cups filled.

Invite students to describe their sketches to their partner and to ask each other questions.

Pause briefly to share interesting and unique sketches.

2 Monitor

To support students getting started in Problem 3, consider asking, "How does the amount of water in the cooler change from 0 cups filled to 1 cup? From 1 cup to 2 cups?"

A **Accessibility: Memory and Attention** Invite students to use additional paper to support them with performing calculations.

Look for a variety of representations and strategies used.

D **Differentiation** *(Problem 4)*

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Encourage them to make sense of the table and identify any patterns with a partner.
Identified a slope for the equation but not an initial value.	**Strengthen** Consider asking, "How does this situation start? How much water is initially in the cooler?"
Identified an initial value for the equation but not slope.	**Strengthen** Consider asking, "How does this situation change for each cup filled? How can we determine the rate of change?"
Want to explore linear relationships with negative slopes further.	**Stretch** Encourage them to think of how their equations would change with a different size water cooler and cups.

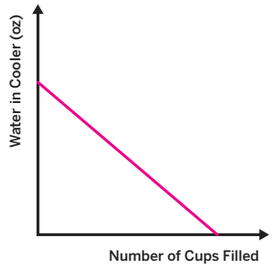
Activity 1 continued >

Activity 1

Name: _____ Date: _____ Period: _____

Cooler Cups

2. a Sketch the relationship between the ounces of water remaining in the cooler and number of cups filled.



b **Discuss:** How can you see this relationship in your sketch?

Responses vary. As the number of cups filled increases, the amount of water left in the cooler decreases. That's why my drawing shows a graph decreasing from left to right.

3. The table shows the amount of water remaining in the cooler after 0, 1, and 2 cups have been filled.

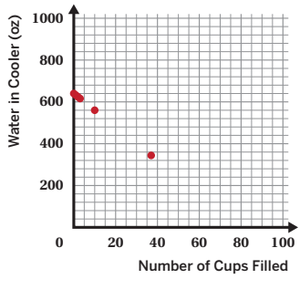
Determine the missing values.

Number of Cups Filled	0	1	2	3	. . .	10	37
Water in Cooler (oz)	640	632	624	616	. . .	560	344

4. A classmate plotted their points from the table onto a graph.

Write an equation representing the amount of water in the cooler, y, after filling x cups.

$y = 640 - 8x$ (or equivalent)



Accelerated 7 | Unit 4 Lesson 2 292 Activity 1

Pairs | 15 min

Activity 1 Cooler Cups (continued)

Purpose: Students use multiple representations, namely a graph and table, to write an equation of a linear relationship with negative slope. **(MTR.2.1)**

2 Monitor

Pause briefly to discuss the variety of predictions students made in Problem 5. Consider asking, "How did your estimate change? What made your estimate more precise in this problem?"

Listen for explanations and ideas about the terms **vertical intercept** and *slope* to share during the Connect.

D Differentiation *(Problem 6)*

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Invite students to identify the coordinate point for the vertical intercept on the graph. Consider asking, "How does this point relate to the situation?"
Identify the vertical intercept and slope from the graph or equation.	**Strengthen** Consider asking, "How can we use the axes to tell us what these values represent?"
Describe the vertical intercept and slope with precision.	**Stretch** Invite them to share their thinking with others in the class.

3 Connect

Share student ideas about vertical intercept and slope. Consider pausing to create an anchor chart of student ideas.

To surface the Key Takeaway, consider asking:

- "Where can you see the vertical intercept on a graph? In a table? In an equation?"
- "What does the vertical intercept represent in this situation?"
- "How is the slope different in this situation compared to others we have examined?"
- "What does slope represent?"

Activity 1

Name: _____ Date: _____ Period: _____

Cooler Cups (continued)

5. Make a final prediction:

After how many cups will the cooler run out of water?

80 cups

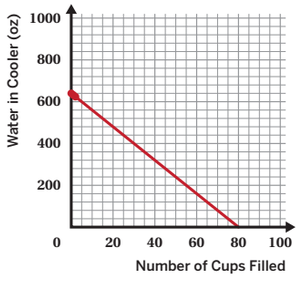
Explain your thinking.

Explanations vary. **I used the graph and found the value of x when y (the amount of water in the cooler) was 0.**

6. Here is the equation that a classmate wrote for the relationship between the water in the cooler and the number of cups:

$y = 640 - 8x$

The **vertical intercept** is the point where the graph of a line crosses the vertical axis.



a. What is the vertical intercept of this graph, and what does it represent in this situation?

Responses vary. **The vertical intercept is the point (0, 640), which represents how many ounces of water were in the cooler before any cups were filled.**

b. What is the slope of the line, and what does it represent in this situation?

Responses vary. **The slope is -8, which means the amount of water in the cooler decreases at a constant rate of 8 ounces each time a cup is filled.**

Accelerated 7 | Unit 4 Lesson 2 293 Activity 1

Key Takeaway: The **vertical intercept** represents the point on the graph where the line crosses the y-axis. Negative slope represents a decreasing constant rate of change for a linear relationship.

Pairs | 15 min

Activity 2 A New Water Cooler

Purpose: Students make sense of and interpret the slope and **horizontal intercept** of a linear relationship through multiple representations. **(MTR.2.1, MTR.3.1)**

Short on time? Consider drafting a story together as a class in Problem 10.

1 Launch

Use the Tell a Story routine to help students make sense of the graph. Invite students to write a story about the graph using their unique experiences, perspectives, and creativity.

A **Accessibility: Executive Functioning** To support initiating complex tasks, invite students to describe a possible first step in writing their story.

ELL **English Language Learners** Invite students to write first in their primary language and then translate it to English during the second draft. **(Writing)**

2 Monitor

Pause briefly to discuss Caasi's and Jamar's work in Problem 8. Consider asking:

- "What is similar between Caasi's work and Jamar's work? What is different?"
- "Could both of their claims be correct? How?"
- "How can we use Caasi's method to see Jamar's claim on the graph?"
- "How is this negative slope triangle similar or different to slope triangles you've seen before?"

Select and sequence student explanations of the **horizontal intercept** to share during the Connect.

Activity 2 continued >

Activity 2

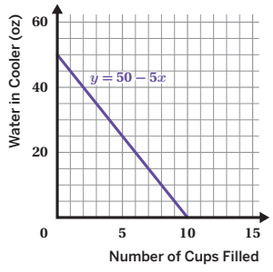
Name: Date: Period:

A New Water Cooler

The line $y = 50 - 5x$ represents a different water cooler situation.

7. Write a story that this graph could represent.

Responses vary. A water cooler at a soccer practice started with 50 ounces of water in it. Each time someone filled a cup, the amount of water in the cooler decreased.



8. Caasi says that the water in the cooler decreases by 20 ounces for every 4 cups filled.

Jamar says that the water in the cooler decreases by 5 ounces for every 1 cup filled.

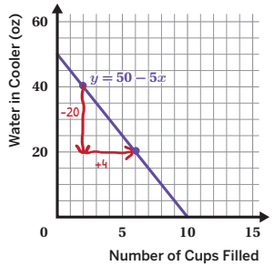
Whose claim is correct?

Caasi's Jamar's (Both) Neither

Explain your thinking.

Explanations vary. Caasi's and Jamar's claims are both correct because they're giving equivalent rates. The rates can both be represented by similar slope triangles on the graph.

Caasi's Work



9. Look at the graph of $y = 50 - 5x$ from the previous problems. The line crosses the horizontal axis at the point $(10, 0)$. This point is called the **horizontal intercept**.

What does this intercept represent in this situation?

Responses vary. $(10, 0)$ represents how many cups it takes to empty the cooler.

Accelerated 7 | Unit 4 Lesson 2 294 Activity 2

Pairs | 15 min

Activity 2 A New Water Cooler (continued)

Purpose: Students make sense of and interpret the slope and **horizontal intercept** of a linear relationship through multiple representations. **(MTR.2.1, MTR.3.1)**

Short on time? Consider drafting a story together as a class in Problem 10.

2 Monitor

D Differentiation

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Encourage them to revisit Problem 6 and make observations about what occurs when the graph approaches the horizontal intercept.
Make connections between the slope and the situation or between the vertical intercept and the situation.	**Strengthen** Celebrate them for their persistence so far. Encourage students to mark this point on the graph and to explain out loud to their partner what $(10, 0)$ represents using the axes labels.
Explain that the horizontal intercept represents how many cups it takes to empty the cooler.	**Stretch** Consider asking, "What is the difference between the horizontal intercept and the vertical intercept in this situation?" Invite these students to share their ideas during the Connect.

Invite students to review their earlier version of this story and reflect on how they can make the details stronger and more precise.

MLR **MLR1: Stronger and Clearer Each Time** Use this routine to support students in developing precise language. Invite students to meet with 1–2 different partners, give feedback on each other's drafts, then use the feedback to write a second clearer draft. See the Routine Facilitation Guide for more information.

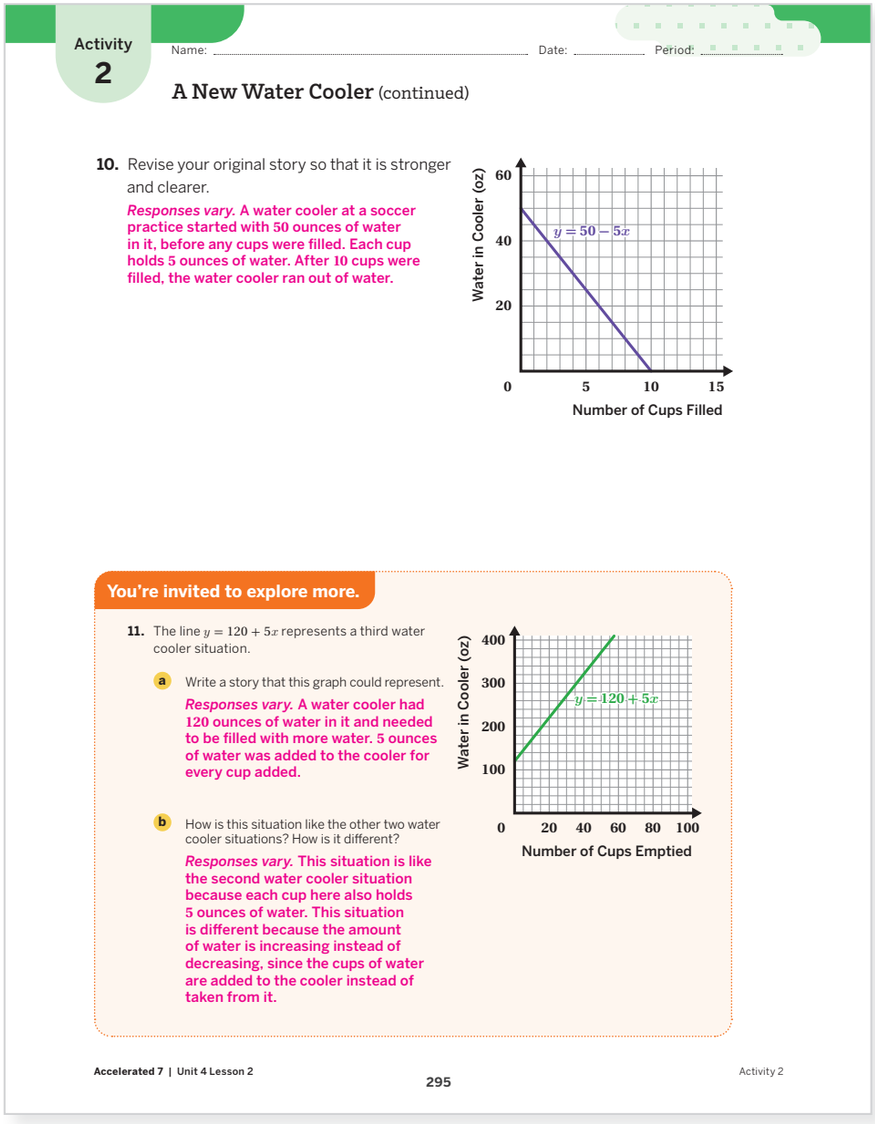
D **Differentiation: You're invited to explore more. (Stretch)** Invite students who would like to explore linear relationships with negative slopes further to complete this optional task. Encourage them to discuss their thinking with a partner.

3 Connect

Share selected student explanations for Problem 10.

To surface the Key Takeaway, consider asking:

- "How is the horizontal intercept different from the vertical intercept?"
- "What does the horizontal intercept represent?"
- "How is the water cooler in this activity different from the water cooler in the first situation? How can you see these differences in the graph and equation?"



Activity 2

Name: _____ Date: _____ Period: _____

A New Water Cooler (continued)

10. Revise your original story so that it is stronger and clearer.

Responses vary. A water cooler at a soccer practice started with 50 ounces of water in it, before any cups were filled. Each cup holds 5 ounces of water. After 10 cups were filled, the water cooler ran out of water.

You're invited to explore more.

11. The line $y = 120 + 5x$ represents a third water cooler situation.

a. Write a story that this graph could represent.

Responses vary. A water cooler had 120 ounces of water in it and needed to be filled with more water. 5 ounces of water was added to the cooler for every cup added.

b. How is this situation like the other two water cooler situations? How is it different?

Responses vary. This situation is like the second water cooler situation because each cup here also holds 5 ounces of water. This situation is different because the amount of water is increasing instead of decreasing, since the cups of water are added to the cooler instead of taken from it.

Accelerated 7 | Unit 4 Lesson 2 295 Activity 2

Key Takeaway: The **horizontal intercept** represents the point on the graph where the line crosses the x-axis.

Whole Class | 5 min

Synthesis

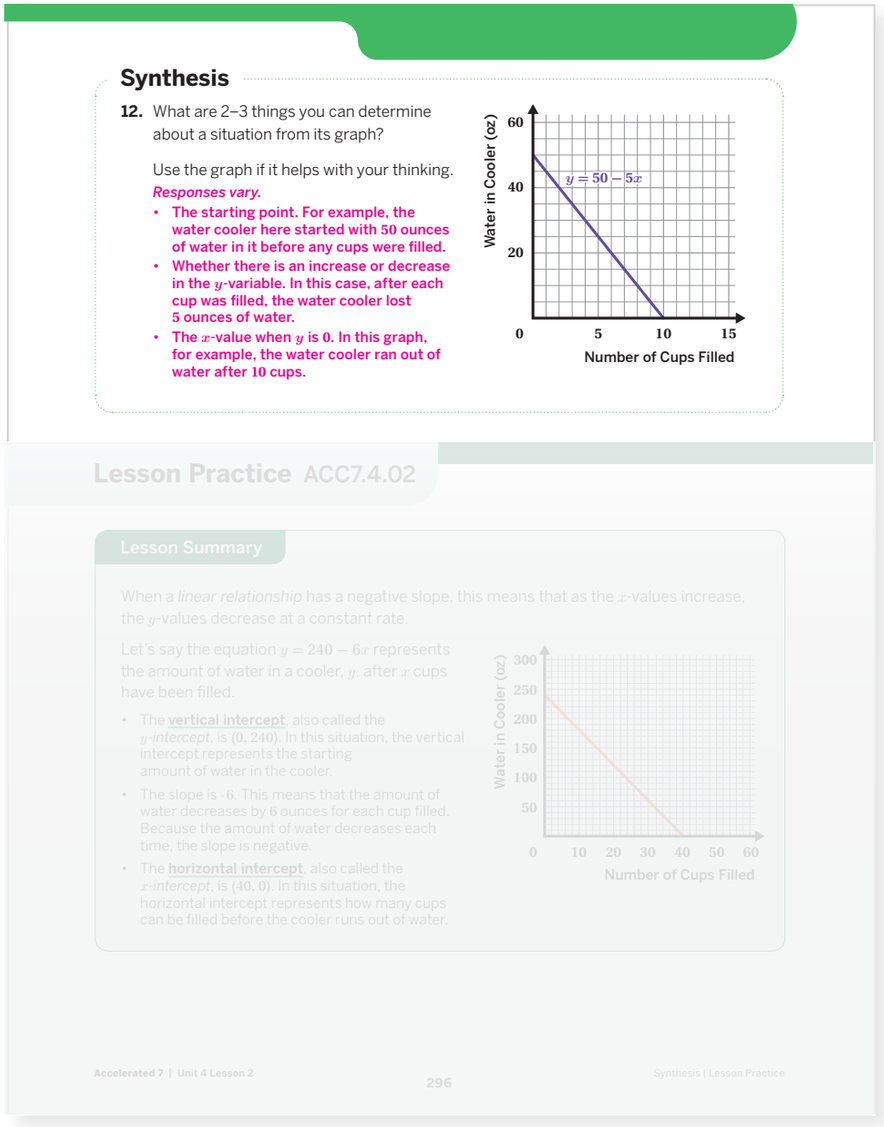
Purpose: Students synthesize their understanding of how to analyze linear relationships with negative slopes.

Invite students to respond independently, and then share their thinking with a partner.

Collect and share a variety of ideas, such as:

- The slope represents the rate of change for a situation. In this case, slope represents the amount of water emptied per cup filled.
- The vertical intercept can represent an initial value. In this case, it represents the initial amount of water in the cooler.
- The horizontal intercept can represent important information for a context. In this case, it can represent the number of cups of water filled when the water cooler is empty.
- In linear relationships with negative slope, as x values increase, the y values decrease at a constant rate.

MLR **MLR2: Collect and Display** Use this routine to collect students' ideas about what they can determine about a situation from a graph. Display for students to reference and add to for the rest of the unit.



Synthesis

12. What are 2–3 things you can determine about a situation from its graph?

Use the graph if it helps with your thinking.

Responses vary.

- The starting point. For example, the water cooler here started with 50 ounces of water in it before any cups were filled.
- Whether there is an increase or decrease in the y-variable. In this case, after each cup was filled, the water cooler lost 5 ounces of water.
- The x-value when y is 0. In this graph, for example, the water cooler ran out of water after 10 cups.

Lesson Practice ACC7.4.02

Lesson Summary

When a *linear relationship* has a negative slope, this means that as the x-values increase, the y-values decrease at a constant rate.

Let's say the equation $y = 240 - 6x$ represents the amount of water in a cooler, y, after x cups have been filled.

- The **vertical intercept**, also called the *y-intercept*, is $(0, 240)$. In this situation, the vertical intercept represents the starting amount of water in the cooler.
- The slope is -6. This means that the amount of water decreases by 6 ounces for each cup filled. Because the amount of water decreases each time, the slope is negative.
- The **horizontal intercept**, also called the *x-intercept*, is $(40, 0)$. In this situation, the horizontal intercept represents how many cups can be filled before the cooler runs out of water.

Accelerated 7 | Unit 4 Lesson 2 296 Synthesis | Lesson Practice

Lesson Takeaway: Linear equations represent information for a relationship. The slope represents the rate of change. The **vertical intercept** represents an initial value, where the graph crosses the vertical axis. The **horizontal intercept** is the point where the line crosses the horizontal axis.

Independent | 5 min

Show What You Know

Purpose: Students demonstrate their understanding by determining and interpreting the vertical and horizontal intercepts from the graph of a linear relationship.

Today's Goals

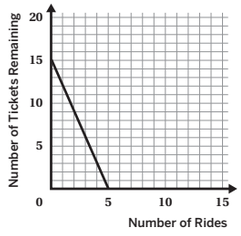
1. **Goal:** Write an equation for a linear relationship with a negative slope by expressing regularity in repeated reasoning.
2. **Goal:** Connect representations of the slope and *vertical intercept* between a graph, equation, and situation.
3. **Language Goal:** Interpret the *horizontal* and *vertical intercepts* of a real-world linear relationship. **(Reading, Writing, Speaking, and Listening)**

Show What You Know (Assessment Resources)

Name: _____ Date: _____ Period: _____

Show What You Know 4.02

Marc purchased tickets to ride roller coasters at a carnival. The graph shows the relationship between the number of tickets remaining, y, after x rides.



a. Complete the table.

Vertical Intercept (x, y)	Horizontal Intercept (x, y)
(0, 15)	(5, 0)

b. What does each point represent in this situation?

Vertical Intercept:

Responses vary. Marc started with 15 tickets.

Horizontal Intercept:

Responses vary. Marc went on 5 rides and then ran out of tickets.

D Differentiation Use after Lesson 2

Support

Provide targeted intervention for students by using these resources.

If students need support analyzing linear relationships with slopes:

» **Respond to Student Thinking:**

- Use the ***Interpreting Slope and Intercepts of Linear Relationships*** **Mini-Lesson**.
- Review the **Lesson Summary** with students before providing individualized feedback on the relevant **Practice Problems**.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 2 Extension Activities**.
- Invite them to research water coolers large enough to serve beverages to the entire class. Encourage them to write equations modeling the volume left after x students obtain a drink.

Professional Learning

Take a moment to reflect on this lesson. How did **MLR1: Stronger and Clearer Each Time** support students in interpreting the horizontal and vertical intercepts of a real-world linear relationship?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

12. What are 2–3 things you can determine about a situation from its graph?

Use the graph if it helps with your thinking.

Responses vary.

- The starting point. For example, the water cooler here started with 50 ounces of water in it before any cups were filled.
- Whether there is an increase or decrease in the y-variable. In this case, after each cup was filled, the water cooler lost 5 ounces of water.
- The x-value when y is 0. In this graph, for example, the water cooler ran out of water after 10 cups.

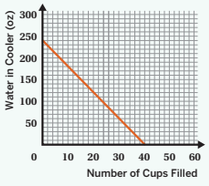
Lesson Practice ACC7.4.02

Lesson Summary

When a *linear relationship* has a negative slope, this means that as the x-values increase, the y-values decrease at a constant rate.

Let's say the equation $y = 240 - 6x$ represents the amount of water in a cooler, y, after x cups have been filled.

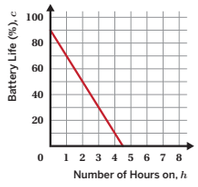
- The **vertical intercept**, also called the *y-intercept*, is (0, 240). In this situation, the vertical intercept represents the starting amount of water in the cooler.
- The slope is -6. This means that the amount of water decreases by 6 ounces for each cup filled. Because the amount of water decreases each time, the slope is negative.
- The **horizontal intercept**, also called the *x-intercept*, is (40, 0). In this situation, the horizontal intercept represents how many cups can be filled before the cooler runs out of water.



Lesson Practice ACC7.4.02

Name: ..... Date: ..... Period: .....

Problems 1–4: Tameeka is monitoring her computer's battery life. When the computer is on, the battery loses energy at a constant rate. This graph shows the percentage of battery life remaining, c, after the computer has been on for h hours.



1. What percent of battery life was left when Tameeka turned on the computer?

90%

2. Complete the table.

Number of Hours On	0	1	2	. . .	3.5
Battery Life, %	90	70	50	. . .	20

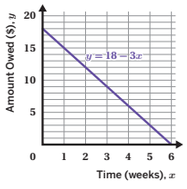
3. Write an equation that represents the percentage of battery life remaining, c, after the computer has been on for h hours.

$c = 90 - 20h$ (or equivalent)

4. After how many hours of being on will Tameeka's computer's battery life run out?

4.5 hours

Problems 5–6: Juliana borrowed some money from her brother. She pays him back by giving him the same amount every week. The graph shows how much she owes him, y, after each week, x.



5. What is the vertical intercept and what does it represent in this situation?

(0, 18). *Responses vary.* It represents the amount of money Juliana borrowed from her brother.

6. What is the horizontal intercept and what does it represent in this situation?

(6, 0). *Responses vary.* It represents the time, in weeks, that it takes for Juliana to pay her brother back.

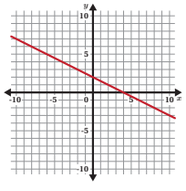
Lesson Practice ACC7.4.02

Name: ..... Date: ..... Period: .....

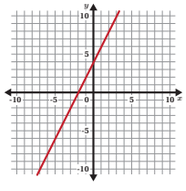
FAST Practice

7. Which graph has a slope of -2?

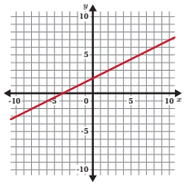
A.



B.



C.



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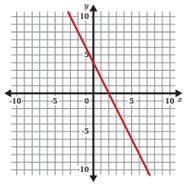


Spiral Review

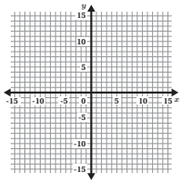
Problems 8–9: Determine the coordinates of the image after the point is dilated using the origin as the center of dilation and a scale factor of 2.5.

8. (1, 0)

(2.5, 0)

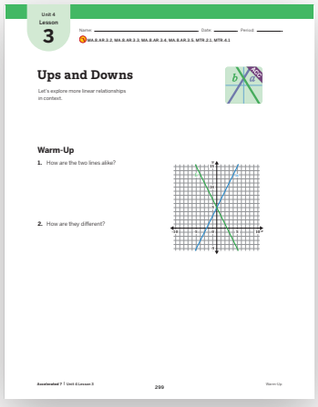
9. (-3, 6)

(-7.5, 15)



Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1–6	2	MA.8.AR.3.2, MA.8.AR.3.5
FAST Practice	7	3	MA.8.AR.3.2
Spiral Review			
	8–9	1	MA.8.GR.2.3



Ups and Downs

Analyzing Linear Relationships

Let's explore more linear relationships in context.

Learning Outcomes

Today's Goals

1. **Goal:** Create and connect representations for linear relationships.
2. **Language Goal:** Explain if the slope of a line is positive or negative, given different representations. **(Writing, Speaking, and Listening)**

Students further their understanding of linear relationships by matching situations to corresponding graphs. They contextualize each situation as they interpret the meaning of the slope and vertical intercept. Students determine whether the slope of a linear relationship is positive or negative and describe these situations in detail, translating among multiple representations of the same relationship. Students use this information to support contextualizing representations. **(MTR.2.1)**

Coherence

Prior Learning

In Lesson 1, students were formally introduced to non-proportional linear relationships with a positive slope. In Lesson 5, students were introduced to a context where the slope of a linear relationship was negative.

Future Learning

In Lesson 4, students will continue to interpret the slope and vertical intercept of a linear relationship in context.

Connections

- Students use linear relationships to represent real-world contexts that change at a positive or a negative constant rate. (real world)

FL B.E.S.T. Math Benchmarks

Addressing

MA.8.AR.3.3
Given a table, graph or written description of a linear relationship, write an equation in slope-intercept form.

MA.8.AR.3.4
Given a mathematical or real-world context, graph a two-variable linear equation from a written description, a table or an equation in slope-intercept form.

Also Addressing: **MA.8.AR.3.2, MA.8.AR.3.5**

Mathematical Thinking and Reasoning Standards: MTR.2.1, MTR.4.1

ELA Expectations: ELA.K12.EE.3.1, ELA.K12.EE.4.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.7.AR.4.3	MA.912.AR.2.2
MA.7.AR.4.4	MA.912.AR.2.4

Lesson at a Glance

~ 45 min

MA.8.AR.3.2, MA.8.AR.3.3, MA.8.AR.3.4, MA.8.AR.3.5

Warm-Up

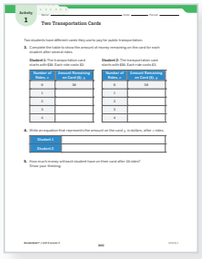
Pairs | 5 min

Students compare and contrast two linear equations with opposite slopes. **Routine:** Think-Pair-Share

Activity 1

Pairs | 15 min

Students reason about two linear situations in order to generate multiple representations. **(MTR.2.1)**



Activity 2

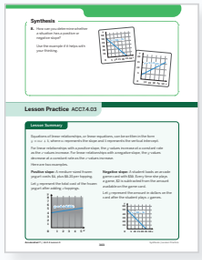
Pairs | 15 min

Students match graphs with situations and engage in discussions about the contextual details relevant for representations, such as slope and vertical intercept. **(MTR.4.1)**

Synthesis

Whole Class | 5 min

Students synthesize strategies for determining whether a situation has a positive or negative slope. MLR2: Collect and Display



Show What You Know

Independent | 5 min

Students demonstrate their understanding of positive and negative slope by determining which equation represents a linear relationship in context.

Prep Checklist

Prepare the Activity 2 Cards.

This lesson includes:

- Student Edition
- Show What You Know
- Activity 2 Cards, one set per pair, pre-cut
  - Coloring tools (as needed)

Pairs | 5 min

Warm-Up

Purpose: Students compare and contrast two linear equations with opposite slopes.

1 Launch

Direct students' attention to Problem 1 and invite students to reflect on similarities and differences between both lines.

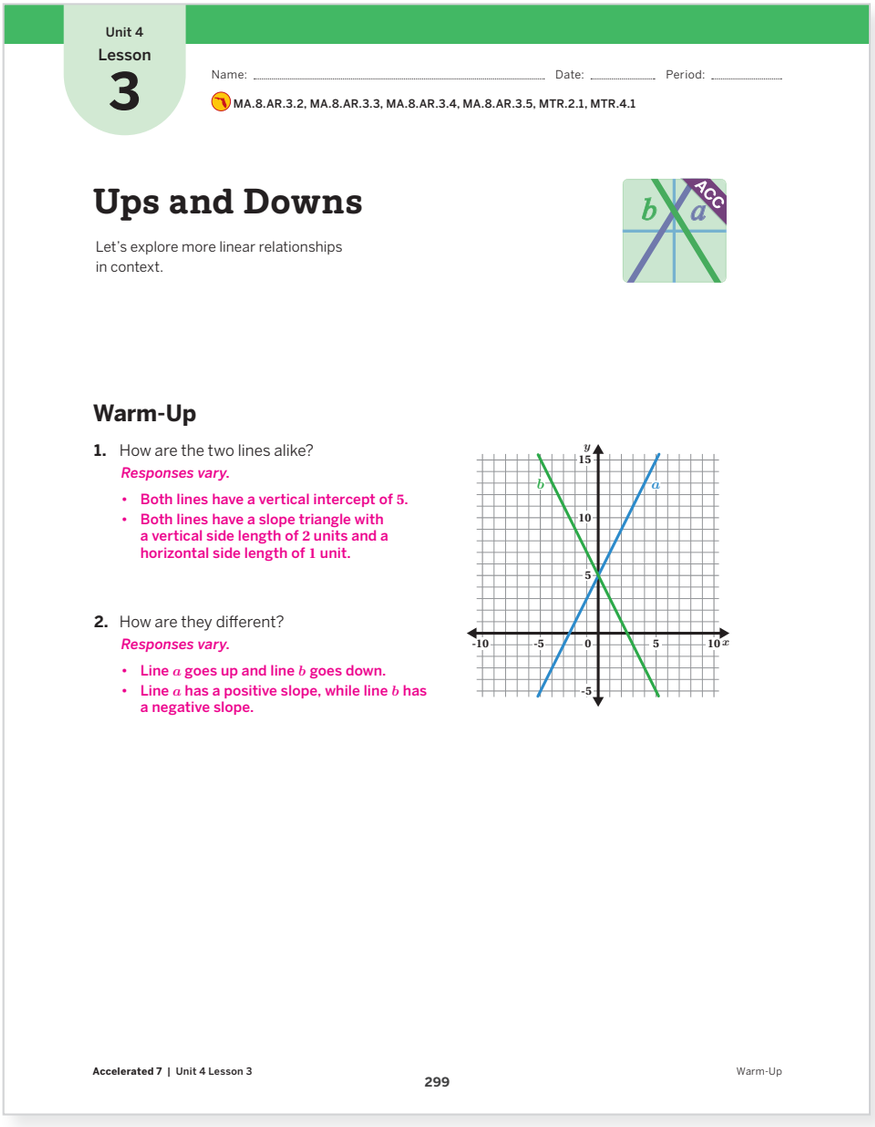
Use the Think-Pair-Share routine to help students shape their ideas about positive and negative slopes. See the Routine Facilitation Guide for more information.

A **Accessibility: Visual-Spatial Processing**
Add labels to the four different quadrants of the graph to support processing visual information.

2 Connect

Invite students to share their responses.

To surface the Key Takeaway, consider asking:
- "What did you notice about each line?"
- "How might you describe the slope of each line?"
- "What do you think the equations might look like?"



Unit 4
Lesson
3

Name: _____ Date: _____ Period: _____

MA.8.AR.3.2, MA.8.AR.3.3, MA.8.AR.3.4, MA.8.AR.3.5, MTR.2.1, MTR.4.1

Ups and Downs

Let's explore more linear relationships in context.

Warm-Up

1. How are the two lines alike?

Responses vary.

- Both lines have a vertical intercept of 5.
- Both lines have a slope triangle with a vertical side length of 2 units and a horizontal side length of 1 unit.

2. How are they different?

Responses vary.

- Line a goes up and line b goes down.
- Line a has a positive slope, while line b has a negative slope.

Key Takeaway: Lines with a positive slope increase from left to right. Lines with a negative slope decrease from left to right.

Pairs | 15 min

Activity 1 Two Transportation Cards

Purpose: Students reason about two linear situations in order to generate multiple representations. **(MTR.2.1)**

1 Launch

Direct students' attention to Problem 3 and invite students to share what they know about public transportation and how people use transportation cards to pay for their fares.

ELL **English Language Learners** Create a list of words about public transportation and fares that students might not know, along with visuals or translations of each word in languages used by your students. **(Reading and Listening)**

To surface prior knowledge, consider asking:

- "How can we show the value left on a transportation card after each ride?"
- "What might be some representations we can use to show these relationships?"

2 Monitor

Select and sequence equations and calculations generated by students to share during the Connect.

D **Differentiation** *(Problems 3–5)*

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Consider asking students to think aloud with their partner about how much money is left on the card after each use.
Determine the remaining amount by repeated subtraction.	**Strengthen** Celebrate them for their resourcefulness. Encourage them to reflect on the effectiveness of their method for a large number of rides, and to come up with another strategy that could be helpful.
Substitute 10 into their equation and correctly determine remaining balances on the fare cards.	**Stretch** Invite these students to share their ideas during the Connect. Encourage them to discuss with their partner how their representations would change if either student started with more money, less money, or had more expensive fares.

3 Connect

Invite students to share their strategies for each problem.

Consider asking, "How do we see information from the tables reflected in the equations?"

To surface the Key Takeaway, consider asking:

- "How could you show the slope in the table?"
- "What happens to the y-values as the x-values increase?"
- "Why do these patterns make sense in this context?"

Activity 1

Name: _____ Date: _____ Period: _____

Two Transportation Cards

Two students have different cards they use to pay for public transportation.

3. Complete the table to show the amount of money remaining on the card for each student after several rides.

Student 1: The transportation card starts with \$30. Each ride costs \$2.

Number of Rides, x	Amount Remaining on Card (\$), y
0	30
1	28
2	26
3	24
4	22

Student 2: The transportation card starts with \$50. Each ride costs \$3.

Number of Rides, x	Amount Remaining on Card (\$), y
0	50
1	47
2	44
3	41
4	38

4. Write an equation that represents the amount on the card y, in dollars, after x rides.

Student 1	$y = 30 - 2x$ (or equivalent)
Student 2	$y = 50 - 3x$ (or equivalent)

5. How much money will *each* student have on their card after 10 rides? Show your thinking.

Student 1 will have \$10 remaining; $30 - 2(10) = 10$.

Student 2 will have \$20 remaining; $50 - 3(10) = 20$.

Accelerated 7 | Unit 4 Lesson 3 300 Activity 1

Key Takeaway: For linear relationships where the x-values increase and the y-values decrease, the slope is negative.

Pairs | 15 min

Activity 2 Matchmaker

Purpose: Students match graphs with situations and engage in discussions about the relevant contextual details for representations, such as slope and vertical intercept. **(MTR.4.1)**

Short on time? Consider inviting students to only complete two rows in Problem 7.

1 Launch

Direct students' attention to Problem 6.

Distribute the Activity 2 Cards to each pair of students.

Consider distributing coloring tools to support sensemaking of representations.

A **Accessibility: Executive Functioning** Consider chunking this activity by inviting students to focus on one graph at a time.

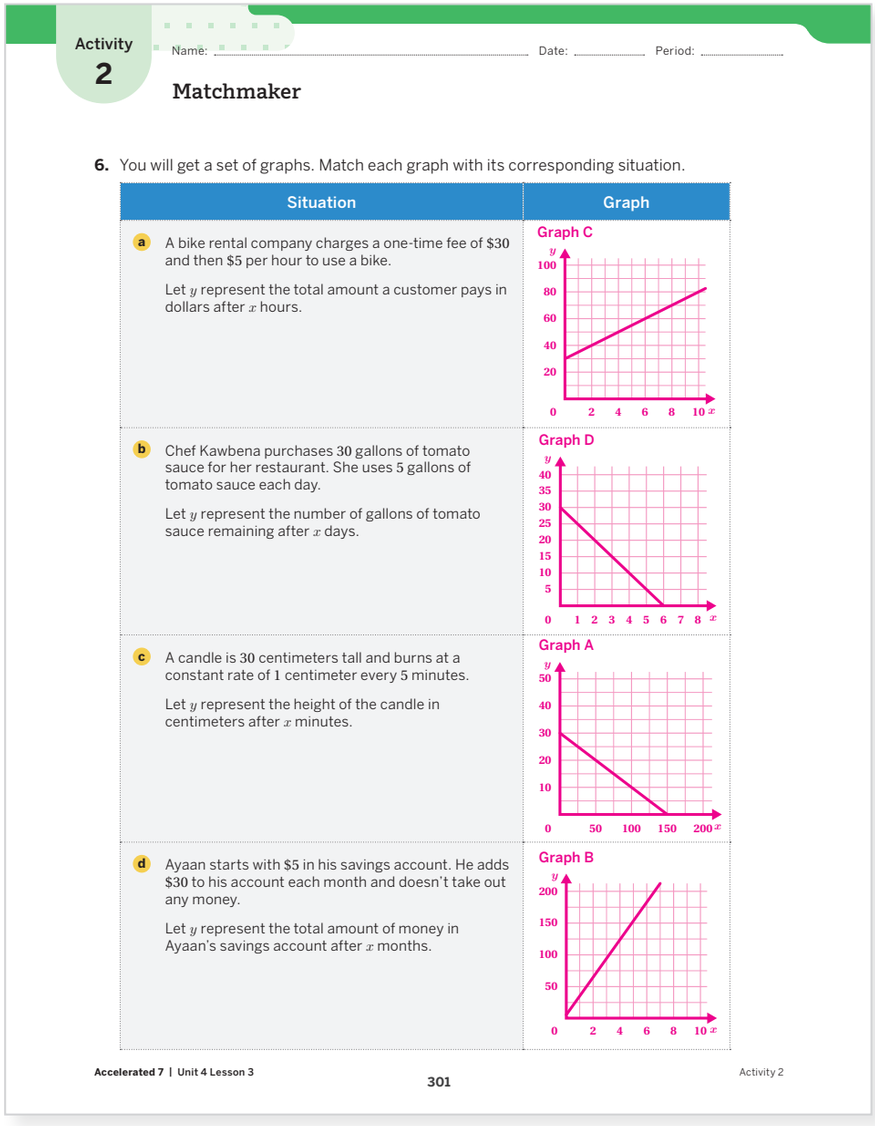
2 Monitor

Select and sequence student strategies and explanations used to justify matches.

D **Differentiation** *(Problems 6–7)*

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Encourage students to try a match and discuss with a partner if a table of values would help them confirm or reject the match.
Determine a slope that does not match the graph.	**Support** Invite students to draw slope triangles on the graphs and underline slope information in the scenarios.
Complete matches by paying attention to only slope or an initial value.	**Strengthen** Consider asking, "How can you use the vertical intercept or horizontal intercept to confirm if your matches are correct?"
Write equations to support matches.	**Stretch** Invite students to share their ideas during the Connect. Encourage them to extend their thinking further by interpreting different points in the graphs and using their equations to check their work.

Activity 2 continued >



Activity 2

Name: _____ Date: _____ Period: _____

Matchmaker

6. You will get a set of graphs. Match each graph with its corresponding situation.

Situation	Graph
a A bike rental company charges a one-time fee of \$30 and then \$5 per hour to use a bike. Let y represent the total amount a customer pays in dollars after x hours.	Graph C
b Chef Kawbena purchases 30 gallons of tomato sauce for her restaurant. She uses 5 gallons of tomato sauce each day. Let y represent the number of gallons of tomato sauce remaining after x days.	Graph D
c A candle is 30 centimeters tall and burns at a constant rate of 1 centimeter every 5 minutes. Let y represent the height of the candle in centimeters after x minutes.	Graph A
d Ayaan starts with \$5 in his savings account. He adds \$30 to his account each month and doesn't take out any money. Let y represent the total amount of money in Ayaan's savings account after x months.	Graph B

Accelerated 7 | Unit 4 Lesson 3 301 Activity 2

Pairs | 20 min

Activity 2 Matchmaker (continued)

Purpose: Students match graphs with situations and engage in discussions about the relevant contextual details for representations, such as slope and vertical intercept. **(MTR.4.1)**

Short on time? Consider inviting students to only complete two rows in Problem 7.

2 Monitor

Consider asking:

- "What strategies did you use to match each situation to its graph?"
- "How did you write about these strategies in the table?"

ELL **English Language Learners** Invite students to talk through their thinking with a partner before writing it down. **(Writing and Speaking)**

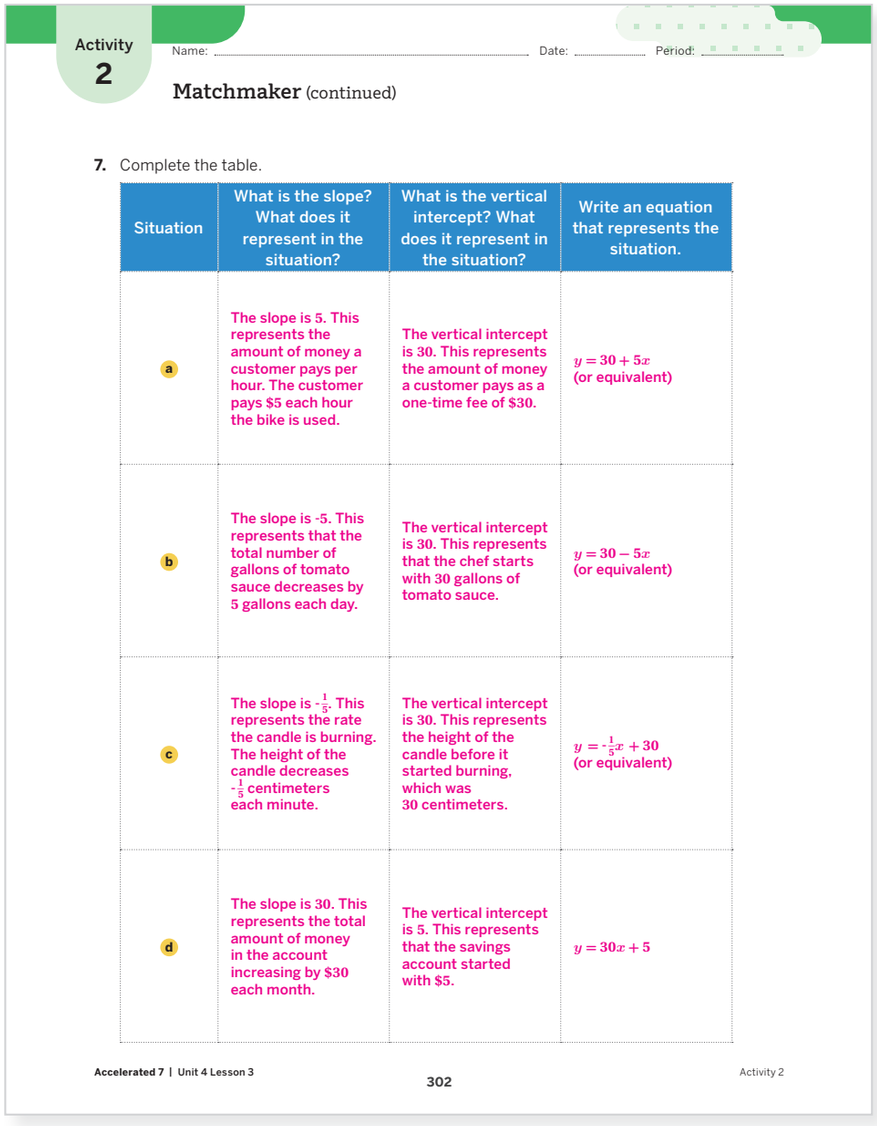
3 Connect

Invite students to share strategies and explanations for Problem 7. **(MTR.4.1)**

Note: If it does not come up naturally, consider asking:

To surface the Key Takeaway, consider asking:

- "How is the slope represented in the situation and in the graph?"
- "How is the vertical intercept in a graph represented in a situation?"
- "What equations did you create to represent the graphs?"
- "How could you tell if a situation was matched with a graph with a negative slope?"



Activity 2

Name: _____ Date: _____ Period: _____

Matchmaker (continued)

7. Complete the table.

Situation	What is the slope? What does it represent in the situation?	What is the vertical intercept? What does it represent in the situation?	Write an equation that represents the situation.
a	The slope is 5. This represents the amount of money a customer pays per hour. The customer pays \$5 each hour the bike is used.	The vertical intercept is 30. This represents the amount of money a customer pays as a one-time fee of \$30.	$y = 30 + 5x$ (or equivalent)
b	The slope is -5. This represents that the total number of gallons of tomato sauce decreases by 5 gallons each day.	The vertical intercept is 30. This represents that the chef starts with 30 gallons of tomato sauce.	$y = 30 - 5x$ (or equivalent)
c	The slope is $-\frac{1}{5}$. This represents the rate the candle is burning. The height of the candle decreases $-\frac{1}{5}$ centimeters each minute.	The vertical intercept is 30. This represents the height of the candle before it started burning, which was 30 centimeters.	$y = -\frac{1}{5}x + 30$ (or equivalent)
d	The slope is 30. This represents the total amount of money in the account increasing by \$30 each month.	The vertical intercept is 5. This represents that the savings account started with \$5.	$y = 30x + 5$

Accelerated 7 | Unit 4 Lesson 3 302 Activity 2

Whole Class | 5 min

Synthesis

Purpose: Students synthesize strategies for determining whether a situation has a positive or negative slope.

Direct students' attention to Problem 8 and invite students to respond independently, and then share their thinking with a partner.

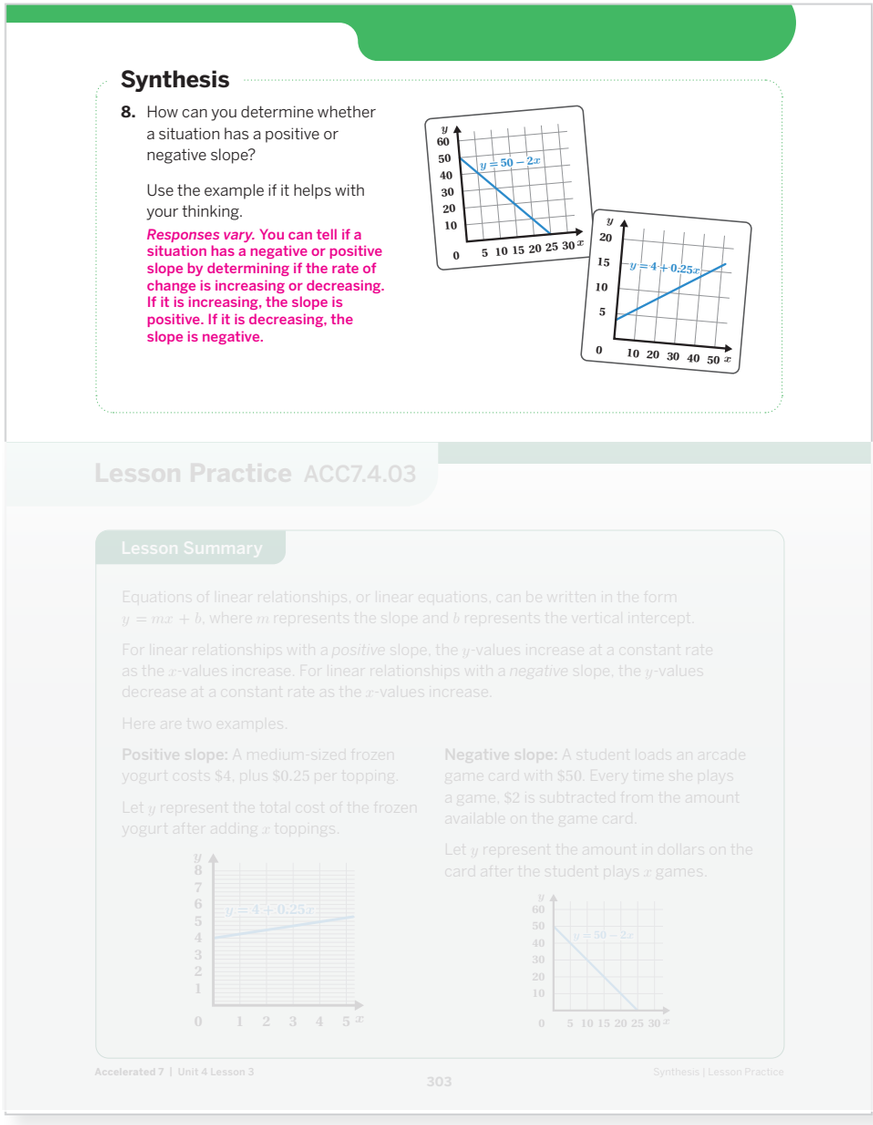
Collect and share a variety of ideas, such as:

- We can represent linear relationships with the equation $y = mx + b$, where m represents the slope and b represents the vertical intercept.
- If a linear relationship has y-values that increase as x-values also increase, the graph will increase from left to right, and the slope m will be positive.
- If a linear relationship has y-values that decrease as x-values increase, the graph will decrease from left to right, and the slope m will be negative.

Create a record of student ideas as they share.

MLR **MLR2: Collect and Display** Use this routine to collect students' ideas and examples as they share during the Connect. Display for students to reference and add to for the rest of the unit.

Note: If time allows, invite students to revise their work on Problem 7 based on the discussion.



Synthesis

8. How can you determine whether a situation has a positive or negative slope?

Use the example if it helps with your thinking.

Responses vary. **You can tell if a situation has a negative or positive slope by determining if the rate of change is increasing or decreasing. If it is increasing, the slope is positive. If it is decreasing, the slope is negative.**

Lesson Practice ACC7.4.03

Lesson Summary

Equations of linear relationships, or linear equations, can be written in the form $y = mx + b$, where m represents the slope and b represents the vertical intercept.

For linear relationships with a *positive* slope, the y-values increase at a constant rate as the x-values increase. For linear relationships with a *negative* slope, the y-values decrease at a constant rate as the x-values increase.

Here are two examples.

Positive slope: A medium-sized frozen yogurt costs \$4, plus \$0.25 per topping.

Let y represent the total cost of the frozen yogurt after adding x toppings.

Negative slope: A student loads an arcade game card with \$50. Every time she plays a game, \$2 is subtracted from the amount available on the game card.

Let y represent the amount in dollars on the card after the student plays x games.

Accelerated 7 | Unit 4 Lesson 3 303 Synthesis | Lesson Practice

Lesson Takeaway: For linear relationships with a positive slope, the y-values increase at a constant rate as the x-values increase. For linear relationships with a negative slope, the y-values decrease at a constant rate as the x-values increase.

Independent | 5 min

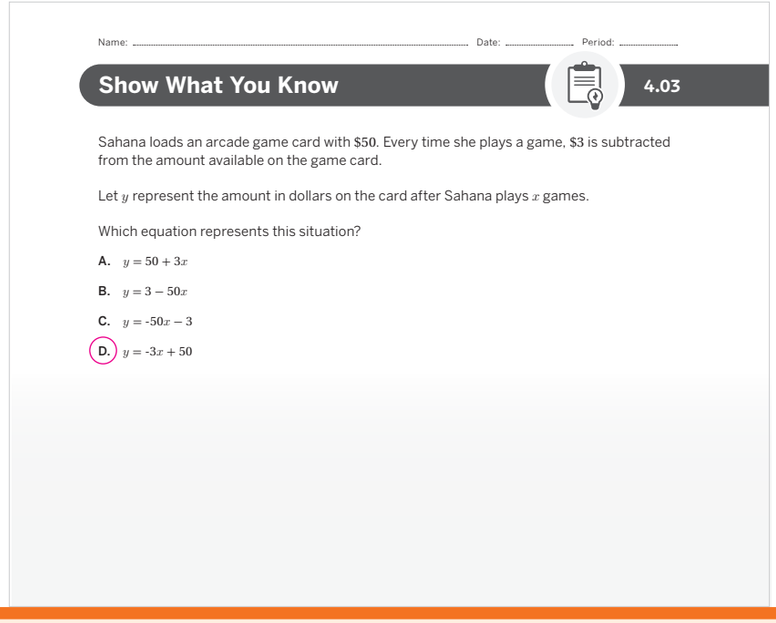
Show What You Know

Purpose: Students demonstrate their understanding of positive and negative slope by determining which equation represents a linear relationship in context.

Today's Goals

1. **Goal:** Create and connect representations for linear relationships.
2. **Language Goal:** Explain if the slope of a line is positive or negative, given different representations. **(Writing, Speaking, and Listening)**

Show What You Know (Assessment Resources)



Name: _____ Date: _____ Period: _____

Show What You Know 4.03

Sahana loads an arcade game card with $50. Every time she plays a game, $3 is subtracted from the amount available on the game card.

Let y represent the amount in dollars on the card after Sahana plays x games.

Which equation represents this situation?

A. $y = 50 + 3x$

B. $y = 3 - 50x$

C. $y = -50x - 3$

D. $y = -3x + 50$

D Differentiation Use after Lesson 3

Support

Provide targeted intervention for students by using these resources.

If students need support with analyzing linear relationships:

» **Respond to Student Thinking:**

- Revisit **Unit 2, Lesson 9, Activity 2** (*Water Slide*).
- Review the **Lesson Summary** with students before providing individualized feedback on the relevant **Practice Problems**.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 2 Extension Activities**.

Professional Learning

Take a moment to reflect on the strategies students used in this lesson. What opportunities were there to celebrate specific students' strategies for analyzing linear relationships? What opportunities might there be in future lessons to connect back to the strategies students shared?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

8. How can you determine whether a situation has a positive or negative slope?

Use the example if it helps with your thinking.

Responses vary. You can tell if a situation has a negative or positive slope by determining if the rate of change is increasing or decreasing. If it is increasing, the slope is positive. If it is decreasing, the slope is negative.

Lesson Practice ACC7.4.03

Lesson Summary

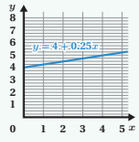
Equations of linear relationships, or linear equations, can be written in the form $y = mx + b$, where m represents the slope and b represents the vertical intercept.

For linear relationships with a *positive* slope, the y-values increase at a constant rate as the x-values increase. For linear relationships with a *negative* slope, the y-values decrease at a constant rate as the x-values increase.

Here are two examples.

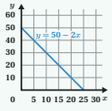
Positive slope: A medium-sized frozen yogurt costs \$4, plus \$0.25 per topping.

Let y represent the total cost of the frozen yogurt after adding x toppings.



Negative slope: A student loads an arcade game card with \$50. Every time she plays a game, \$2 is subtracted from the amount available on the game card.

Let y represent the amount in dollars on the card after the student plays x games.

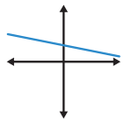


Lesson Practice ACC7.4.03

Name: ..... Date: ..... Period: .....

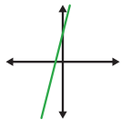
Problems 1–3: Determine whether the slope of each line is *positive* or *negative*.

1.



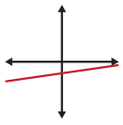
Negative

2.



Positive

3.



Positive

Problems 4–5: The elevation of an airplane and its time since takeoff can be modeled by a linear equation. The horizontal axis represents time in minutes and the vertical axis represents elevation in feet. For each situation, determine whether the slope is positive or negative.

4. The plane descends at a rate of 1,000 feet per minute.

Negative

5. The plane ascends at a rate of 2,000 feet per minute.

Positive

Problems 6–7: Ella borrowed \$40 from her brother. She pays him back by giving him \$5 each week.

6. Complete the table to show the amount Ella owes each week.

Time (weeks)	Amount Owed ($)
0	40
1	35
2	30
3	25

7. Write an equation that represents the amount that Ella owes, in dollars, after x weeks.

$y = -5x + 40$ (or equivalent)

Lesson Practice ACC7.4.03

Name: ..... Date: ..... Period: .....

8. y is the total cost of grapes after purchasing x ounces of grapes.

Sketch a graph that could represent this situation.

Responses vary.

FAST Practice

9. An acorn falls 20 feet from a tree to the ground at a rate of 4 feet every 2 seconds.

Determine whether each statement is *true* or *false*.

	True	False
The initial height of the acorn is 20 feet.	☑	☐
The acorn falls at a rate of $\frac{1}{2}$ foot every second.	☐	☑
The acorn is 5 feet above the ground after 10 seconds.	☐	☑

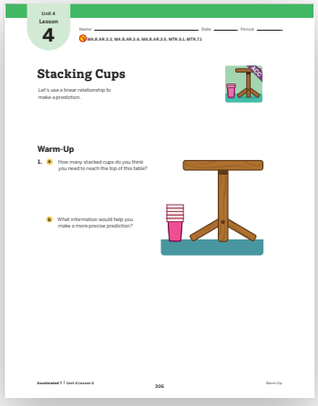
Spiral Review

10. The vertices of triangle ABC are $A(12,6)$, $B(1,3)$, and $C(18,0)$. Triangle ABC is dilated by a scale factor of $\frac{1}{3}$ using point $(0,0)$ as the center of dilation, resulting in triangle JKL. What are the coordinates of the vertices of triangle JKL?

$J(4,2)$, $K\left(\frac{1}{3},1\right)$, $L(6,0)$

Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1–3	1	MA.8.AR.3.2
	4–5	2	MA.8.AR.3.2
	6–7	2	MA.8.AR.3.3
	8	2	MA.8.AR.3.4
FAST Practice	9	2	MA.8.AR.3.5
Spiral Review			
	10	1	MA.8.GR.2.3


Unit 4
Lesson
4

Stacking Cups

Let's use a linear relationship to make a prediction.

Warm-Up

1. How many stacked cups do you think you need to reach the top of this table?

What information would help you make a more precise prediction?

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Stacking Cups

More With Linear Relationships

Let's use a linear relationship to make a prediction.

Learning Outcomes

Today's Goals

1. **Goal:** Use a linear relationship to make a prediction.
2. **Language Goal:** Explain the meaning of a line's slope and vertical intercept in a real-world situation. **(Reading, Writing, Speaking, and Listening)**

Students use a linear relationship to predict the number of stacked cups needed to reach different heights. They use the same tools they've been using to represent proportional relationships — graphs, tables, and equations — to represent a non-proportional linear relationship, and practice interpreting the meaning of the slope and vertical intercept in context.

Coherence

Prior Learning

In Lesson 3, students drew connections between different representations of linear relationships and distinguished between lines with positive and negative slopes. They also interpreted the meaning of the slope and vertical intercept of a linear relationship in context.

Future Learning

In a future course, students will solve problems using linear functions and interpret key parts of the graph based on the context.

Connections

- Student extend their understanding of representations of proportional relationships to non-proportional relationships. (mathematical)
- Students interpret the meaning of slope and vertical intercept of a linear relationship in real world contexts. (real world)

FL B.E.S.T. Math Benchmarks

Addressing

MA.8.AR.3.5
Given a real-world context, determine and interpret the slope and y-intercept of a two-variable linear equation from a written description, a table, a graph or an equation in slope-intercept form.

Also Addressing: **MA.8.AR.3.2, MA.8.AR.3.4**

Mathematical Thinking and Reasoning Standards: MTR.5.1, MTR.7.1

ELA Expectation: ELA.K12.EE.3.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.7.AR.4.5	**MA.912.AR.2.5**
	MA.912.F.1.5

Lesson at a Glance

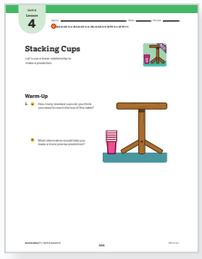
~ 45 min

MA.8.AR.3.2, MA.8.AR.3.4, MA.8.AR.3.5

Warm-Up

Independent | 5 min

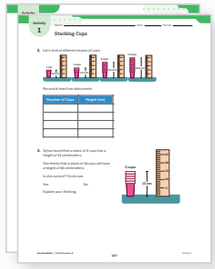
Students estimate the number of cups needed to reach a table, the problem they will explore in more detail in Activity 1.



Activity 1

Pairs | 15 min

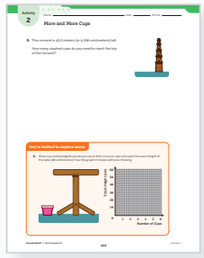
Students apply their knowledge of linear relationships to predict the number of stacked cups needed to reach the top of a table. **(MTR.7.1)**



Activity 2

Pairs | 10 min

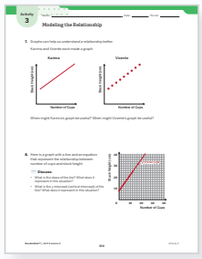
Students use the strategies that surfaced in Activity 1 or new strategies to predict the number of cups needed to reach a tall building (a minaret). **(MTR.7.1)**



Activity 3

Pairs | 5 min

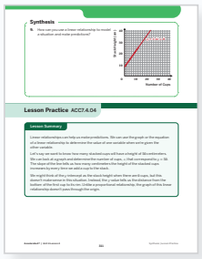
Students generalize to model the relationship between number of cups and stack height.



Synthesis

Whole Class | 5 min

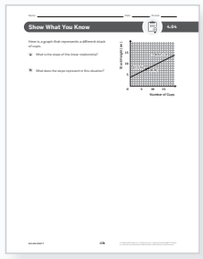
Students synthesize their understanding of how linear relationships can be used to model a situation and make predictions.



Show What You Know

Independent | 5 min

Students demonstrate their understanding by determining and interpreting a slope in a context.



Prep Checklist

This lesson includes:

- Student Edition
- Show What You Know

- Graph paper (as needed)
- Stackable cups with a lid, like foam cups (as needed)

Independent | 5 min

Warm-Up

Purpose: Students estimate the number of cups needed to reach a table, the problem they will explore in more detail in Activity 1.

1 Launch

To encourage playfulness, invite students to share an example of a time when they measured something with unexpected units (e.g., number of their hands, number of phone lengths, etc.).

Encourage students to make a prediction individually before sharing their thinking with a partner.

Note: Stacking cups end-over-end is not the same as cups being stacked by nesting one inside another. It is important for this lesson that the cups are nested.

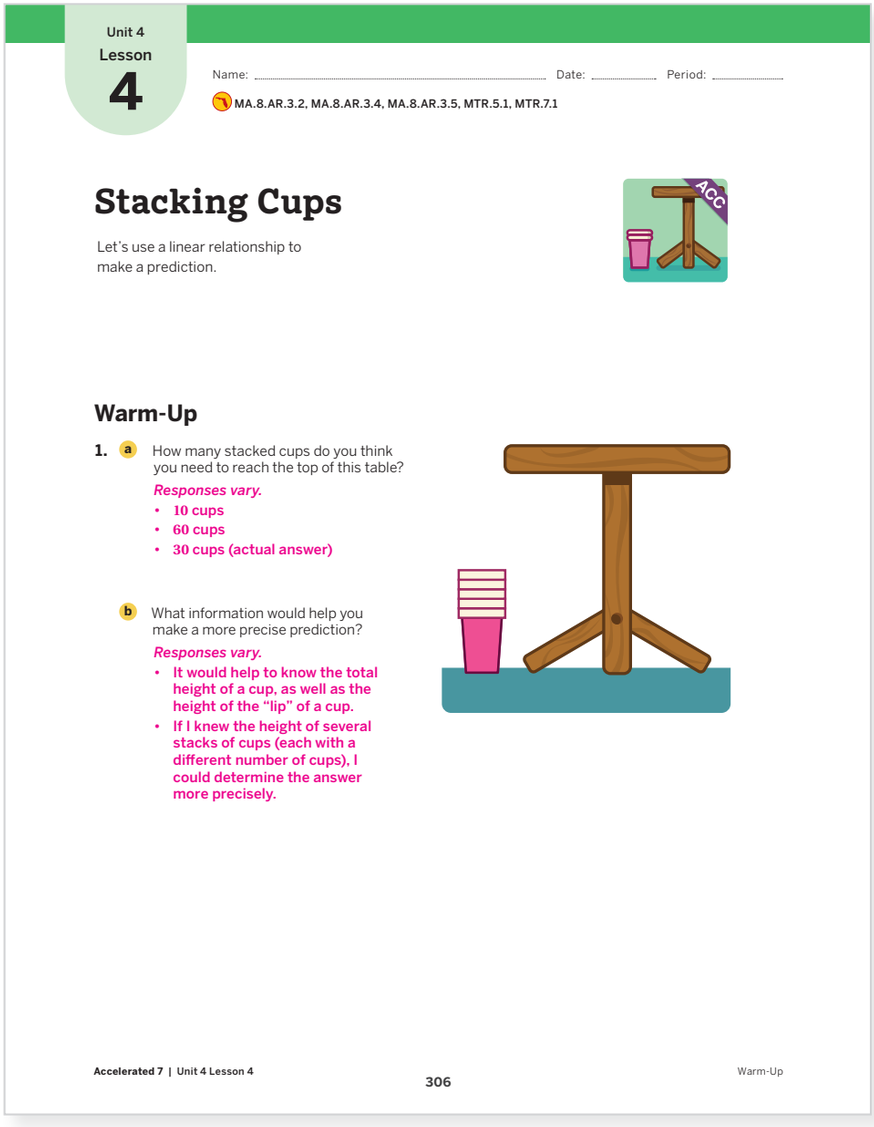
A **Accessibility: Visual-Spatial Processing and Conceptual Processing** Consider bringing in several cups that nest with a lip like the ones in the image to increase access to the task.

2 Connect

Invite students to share their strategies for determining the number of cups needed to build a stack that reaches the top of the table.

Consider asking:

- "What additional information would you need to find the number of cups?"
- "What structures or strategies might you use to help you determine the number of cups?"



Unit 4
Lesson
4

Name: ..... Date: ..... Period: .....

MA.8.AR.3.2, MA.8.AR.3.4, MA.8.AR.3.5, MTR.5.1, MTR.7.1

Stacking Cups

Let's use a linear relationship to make a prediction.

ACC

Warm-Up

1. a How many stacked cups do you think you need to reach the top of this table?

Responses vary.

- 10 cups
- 60 cups
- 30 cups (actual answer)

b What information would help you make a more precise prediction?

Responses vary.

- It would help to know the total height of a cup, as well as the height of the "lip" of a cup.
- If I knew the height of several stacks of cups (each with a different number of cups), I could determine the answer more precisely.

Accelerated 7 | Unit 4 Lesson 4 306 Warm-Up

Pairs | 15 min

Activity 1 Stacking Cups

Purpose: Students apply their knowledge of linear relationships to predict the number of stacked cups needed to reach the top of a table. **(MTR.7.1)**

Short on time? Consider omitting the discussion after Problem 3.

1 Launch

To surface prior knowledge, consider asking, "How might representing this situation using a table of values be helpful?"

2 Monitor

Look for students who choose different numbers of cups. It may be helpful to create one large table for students to add values to and reference throughout the lesson.

Encourage students to pause to discuss any patterns they see in the table before continuing to the next problem.

Look for students who use a variety of reasoning to justify why Sylvia is or is not correct.

D **Differentiation: (Support)** Some students may think that the relationship between stack height and number of cups is proportional. Invite those students to compare the height of 1 cup and the height of 3 cups as an example so they can see that tripling the number of cups does not triple the height.

Pause to discuss Sylvia's claim if in Problem 3, time allows. Invite several students to share their responses.

Consider asking:

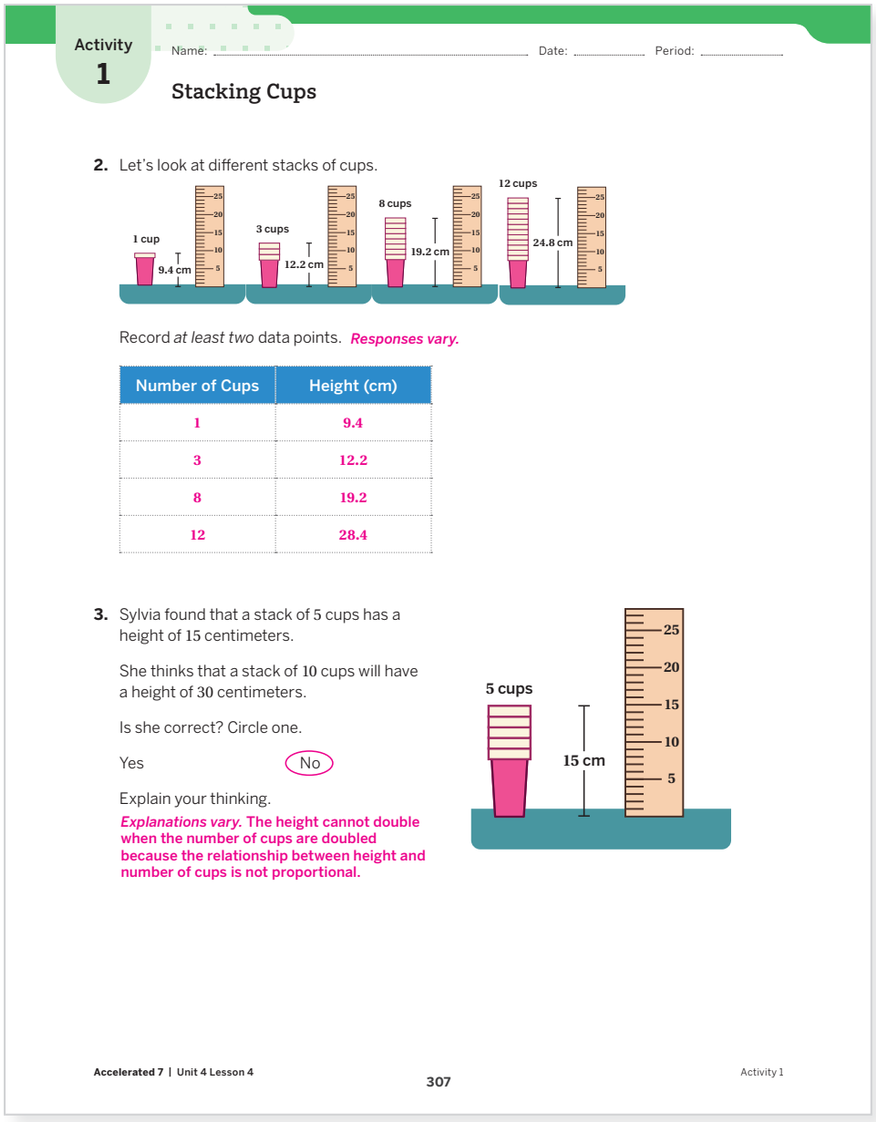
- "Is the relationship between number of cups and stack height proportional? Why or why not?"
- "What is the height of 10 cups? How do you know?" 22 cm, which can be calculated by multiplying 10 by 1.4 cm (the height of each lip) and adding it to 8 cm, the height of a cup with no lip.

ELL **English Language Learners** As students share in partner and whole-class discussions, provide sentence frames such as:

- She is/is not correct because . . .
- _____'s idea makes sense to me because . . .
- Another strategy would be _____ because . . .

(Writing and Speaking)

Activity 1 continued >



Activity 1

Name: _____ Date: _____ Period: _____

Stacking Cups

2. Let's look at different stacks of cups.

Record *at least two* data points. ***Responses vary.***

Number of Cups	Height (cm)
1	9.4
3	12.2
8	19.2
12	28.4

3. Sylvia found that a stack of 5 cups has a height of 15 centimeters.

She thinks that a stack of 10 cups will have a height of 30 centimeters.

Is she correct? Circle one.

Yes (No)

Explain your thinking.

Explanations vary. **The height cannot double when the number of cups are doubled because the relationship between height and number of cups is not proportional.**

Accelerated 7 | Unit 4 Lesson 4 307 Activity 1

Pairs | 15 min

Activity 1 Stacking Cups (continued)

Purpose: Students apply their knowledge of linear relationships to predict the number of stacked cups needed to reach the top of a table. **(MTR.7.1)**

Short on time? Consider omitting the discussion after Problem 3.

2 Monitor

Note: Ensure all students have access to blank paper or graph paper to record their thinking.

A **Accessibility: Conceptual Processing** Invite students to describe in words how they would figure out the height of any number of cups to encourage abstract thinking. It may also be helpful to use physical models or drawings.

Select and sequence strategies students use to model the situation to share during the Connect.

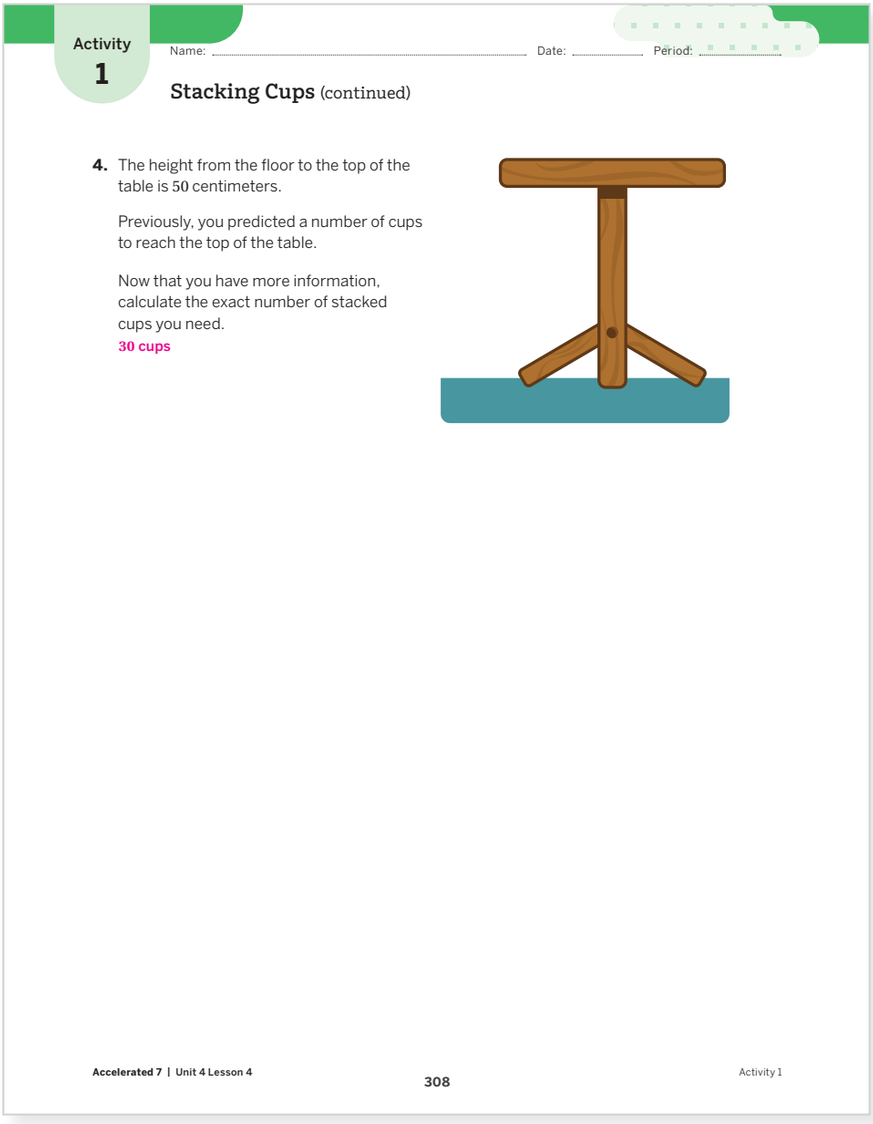
D **Differentiation** *(Problem 4)*

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Encourage them to look for patterns in the table they created earlier. Consider asking, "How much does the height grow by when there is 1 more cup added?"
Divide the height of the table by the height of one cup.	**Support** Consider sharing that their strategy would work if the relationship was proportional, asking, "How do we know that the relationship is not proportional?"
Use strategies such as: • Continuing the table until the height is 50 cm. • Making a graph from the table and looking for when the output is 50. • Writing an equation and solving for the number of cups when the height is 50 cm.	**Strengthen** Invite them to think about how they would share their strategy with the class. **Stretch** Consider asking, "Would your strategy still work if you were trying to reach a higher height? If not, how might you change your strategy?"

3 Connect

Invite students to share a variety of strategies for determining the number of cups it takes to reach the table.

To surface the Key Takeaway, consider asking, "Which representations we've learned in this unit were useful for figuring out the number of cups?"



Activity 1

Name: Date: Period:

Stacking Cups (continued)

4. The height from the floor to the top of the table is 50 centimeters.

Previously, you predicted a number of cups to reach the top of the table.

Now that you have more information, calculate the exact number of stacked cups you need.

30 cups

Key Takeaway: Linear relationships can model situations in which there is a constant value added every time. In this case, the stack height increases by the same number for every cup.

Pairs | 10 min

Activity 2 More and More Cups

Purpose: Students use the strategies that surfaced in Activity 1 or new strategies to predict the number of cups needed to reach a tall building (a minaret). **(MTR.7.1)**

1 Launch

To surface connections, consider asking:

- "What's the tallest building you have ever heard of? Ever seen?"
- "Have you ever seen a minaret? If so, where? What do you know about them?"

2 Monitor

Select and sequence strategies students use to model the situation to share during the Connect. **(MTR.7.1)**

D Differentiation *(Problem 5)*

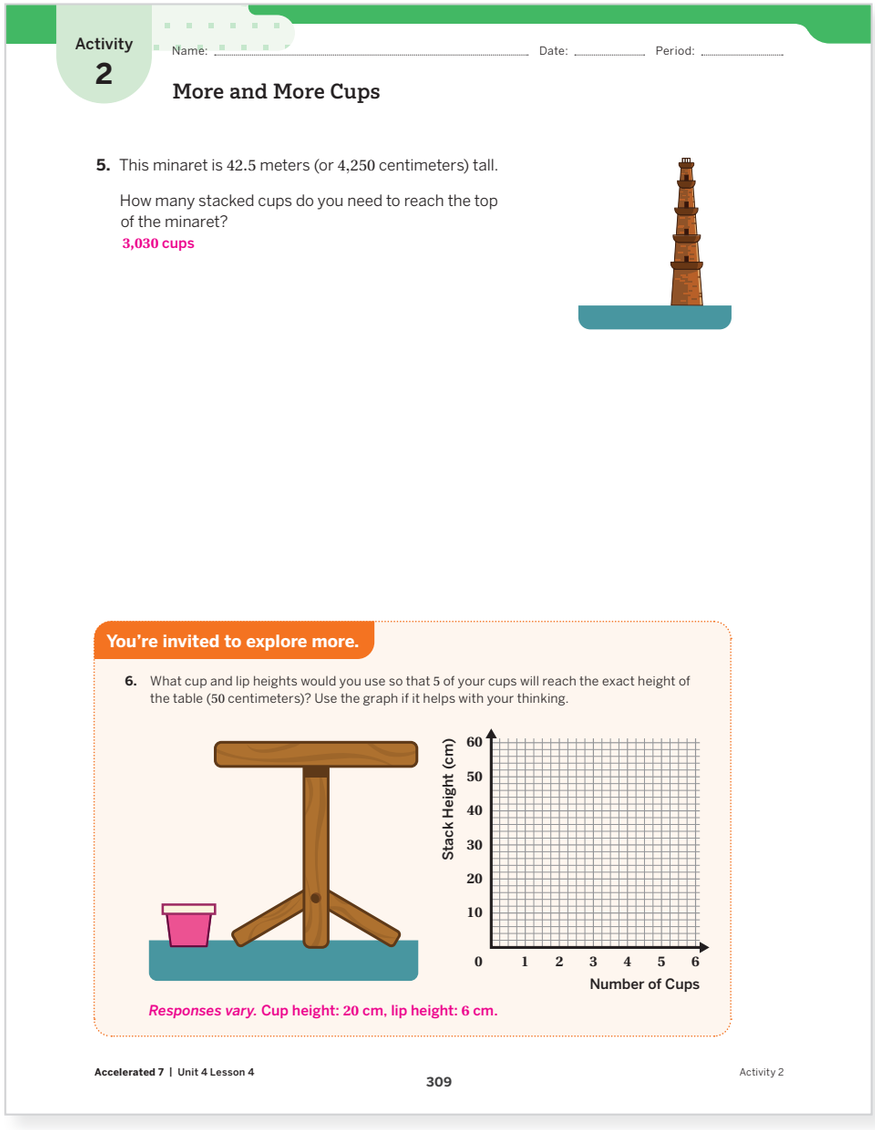
Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Encourage them to refer back to strategies surfaced during Activity 1.
Use strategies surfaced in Activity 1 or create their own.	**Strengthen** Consider asking students to reflect on which strategies were more or less helpful in this situation.
Want to explore modeling this situation further.	**Stretch** Invite students to predict whether they would need more cups to reach the top of their school and to estimate that number based on their calculations.

D Differentiation: You're invited to explore more. (Stretch) Invite students who would like to explore stacking cups further to complete this optional task. Encourage them to discuss their thinking with a partner.

3 Connect

Invite students to share a variety of strategies for determining the number of cups to reach the minaret.

Consider asking, "Which representations were most useful when the height got larger? Which became less useful?"



Activity 2

Name: _____ Date: _____ Period: _____

More and More Cups

5. This minaret is 42.5 meters (or 4,250 centimeters) tall.

How many stacked cups do you need to reach the top of the minaret?

3,030 cups

You're invited to explore more.

6. What cup and lip heights would you use so that 5 of your cups will reach the exact height of the table (50 centimeters)? Use the graph if it helps with your thinking.

Responses vary. **Cup height: 20 cm, lip height: 6 cm.**

Accelerated 7 | Unit 4 Lesson 4 309 Activity 2

Pairs | 5 min

Activity 3 Modeling the Relationship

Purpose: Students generalize to model the relationship between number of cups and stack height.

Short on time? Consider discussing both problems as a class in one discussion.

1 Launch

To surface prior knowledge, consider asking, "How are these graphs alike? How are they different?"

2 Monitor

Look for students who make connections between the individual points and the number of cups or between the continuous line and an equation or other model. These may surface again during the Connect.

ELL **English Language Learners** Invite students to talk through their thinking with a partner before writing it down. **(Writing and Speaking)**

Listen for a variety of ideas to highlight during the Connect.

D **Differentiation** *(Problem 8)*

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Consider asking, "What does each number in the equation tell us?"
Identify the slope and vertical intercept from the equation but need support interpreting them.	**Strengthen** Consider asking, "Where have we seen values like this in this lesson? What have they represented about the cups?"
Make connections between the slope and the height of the cup lip and the y-intercept and the initial cup height with no lip.	**Stretch** Consider asking, "How would the cups look different if the equation was $y = 8x + 1.4$?"

A **Accessibility: Memory and Attention** Invite students to refer back to notes they took earlier on slope and y-intercept or an anchor chart to activate prior knowledge.

3 Connect

Direct students' attention to the graph and create a model as a class to reference throughout the discussion.

To surface the Key Takeaway, consider asking:

- "When might a graph be a useful model? When might an equation be a useful model?"
- "Why do you think there is no point $(0, 8)$ on the graph but the model goes through that point?" The height of 0 cups is not 8 centimeters, so it makes sense that there is no point there. The equation $y = 1.4x + 8$ does have a solution of $(0, 8)$ so it makes sense that the model goes through that point.

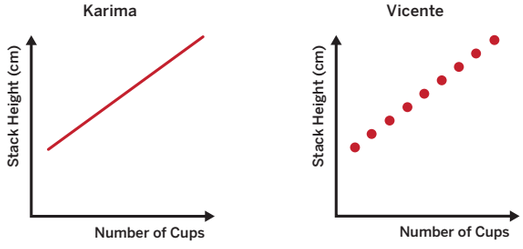
Activity 3

Name: Date: Period:

Modeling the Relationship

7. Graphs can help us understand a relationship better.

Karima and Vicente each made a graph.

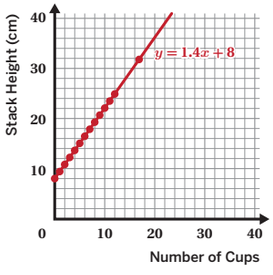


When might Karima's graph be useful? When might Vicente's graph be useful?

Responses vary.

- Karima's graph might be useful when you're thinking about an equation of a line that models this situation.
- Vicente's graph might be useful when you're thinking about each of the individual cups in this situation.

8. Here is a graph with a line and an equation that represent the relationship between number of cups and stack height.



Discuss:

- What is the slope of the line? What does it represent in this situation?
- What is the y-intercept (vertical intercept) of the line? What does it represent in this situation?

Responses vary.

- The slope is 1.4. It represents how much the height of the stack increases every time a cup gets added.
- The y-intercept is (0, 8). This represents the distance from the bottom of the stack to the lip of the first cup, which is 8 centimeters.

Accelerated 7 | Unit 4 Lesson 4 310 Activity 3

Key Takeaway: Graphs and equations are two ways to model a situation. When modeling, it is important to consider which values actually make sense in the situation and which ones do not.

Whole Class | 5 min

Synthesis

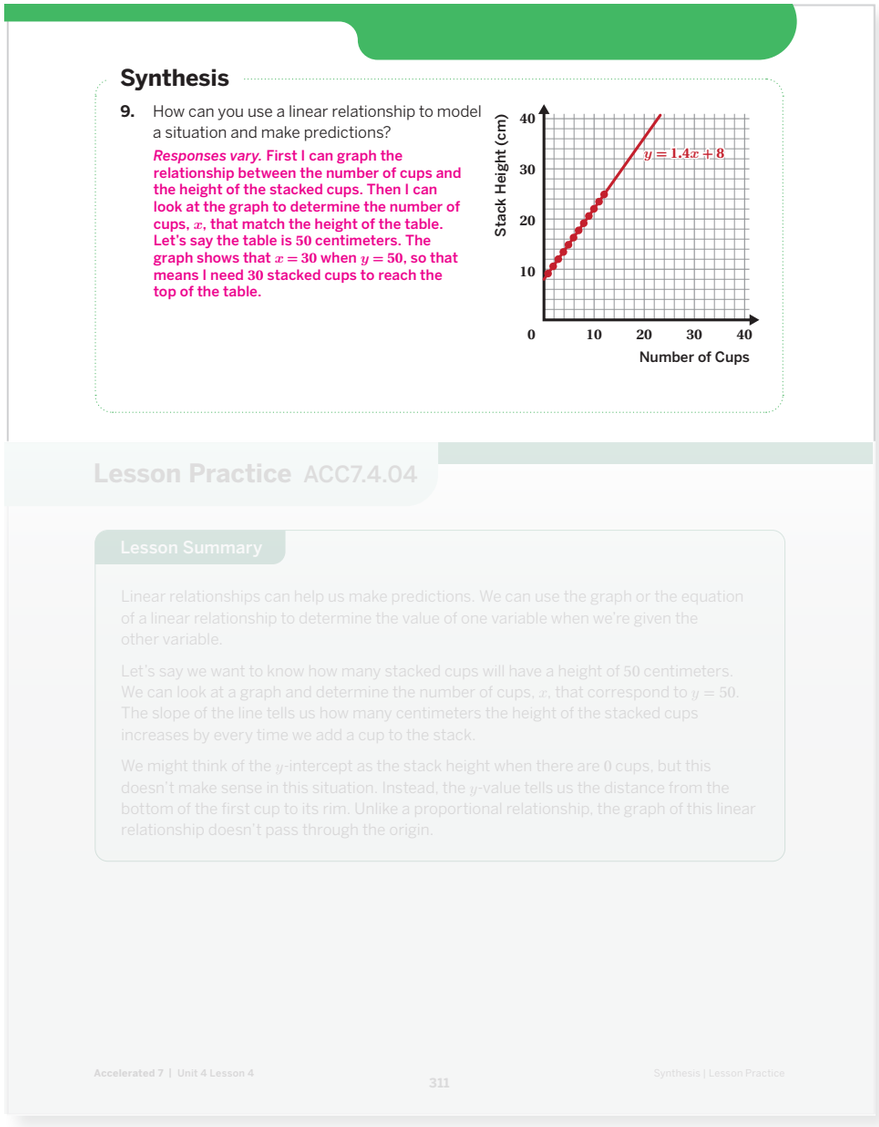
Purpose: Students synthesize their understanding of how linear relationships can be used to model a situation and make predictions.

Invite students to respond independently, and then share their thinking with a partner.

Collect and share a variety of ideas, such as:

- Tables can help us notice patterns and see the constant change in a linear relationship.
- Graphs can help us see trends and make predictions for other values.
- Equations can help us make predictions and answer questions about larger values not typically seen on a graph or in a table.

Note: If time allows, invite students to revise their previous thinking based on the discussion.



Synthesis

9. How can you use a linear relationship to model a situation and make predictions?

Responses vary. First I can graph the relationship between the number of cups and the height of the stacked cups. Then I can look at the graph to determine the number of cups, x, that match the height of the table. Let's say the table is 50 centimeters. The graph shows that $x = 30$ when $y = 50$, so that means I need 30 stacked cups to reach the top of the table.

Lesson Practice ACC7.4.04

Lesson Summary

Linear relationships can help us make predictions. We can use the graph or the equation of a linear relationship to determine the value of one variable when we're given the other variable.

Let's say we want to know how many stacked cups will have a height of 50 centimeters. We can look at a graph and determine the number of cups, x, that correspond to $y = 50$. The slope of the line tells us how many centimeters the height of the stacked cups increases by every time we add a cup to the stack.

We might think of the y-intercept as the stack height when there are 0 cups, but this doesn't make sense in this situation. Instead, the y-value tells us the distance from the bottom of the first cup to its rim. Unlike a proportional relationship, the graph of this linear relationship doesn't pass through the origin.

Accelerated 7 | Unit 4 Lesson 4 311 Synthesis | Lesson Practice

Lesson Takeaway: Linear relationships are useful for modeling situations and making predictions. When modeling, it is important to think about what values make sense and do not make sense in the situation.

Independent | 5 min

Show What You Know

Purpose: Students demonstrate their understanding by determining and interpreting a slope in a context.

Today's Goals

1. **Goal:** Use a linear relationship to make a prediction.
2. **Language Goal:** Explain the meaning of a line's slope and vertical intercept in a real-world situation. **(Reading, Writing, Speaking, and Listening)**

Show What You Know (Assessment Resources)

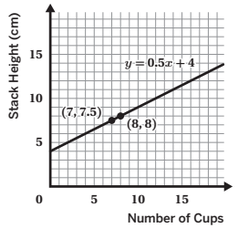
Name: _____ Date: _____ Period: _____

Show What You Know 4.04

Here is a graph that represents a different stack of cups.

a. What is the slope of this linear relationship?
0.5

b. What does the slope represent in this situation?
Responses vary. **The slope represents how much taller the stack becomes with every cup that's added.**



Differentiation Use after Lesson 4

Support

Provide targeted intervention for students by using these resources.

If students need support determining the slope of a line from an equation or graph, or interpreting the slope in context:

» **Respond to Student Thinking:**

- Use the ***Interpreting Slope and Intercepts of Linear Relationships*** **Mini-Lesson.**
- Review the **Lesson Summary** with students before providing individualized feedback on the relevant **Practice Problems**.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 2 Extension Activities**.
- Invite them to determine the number of identical cups needed to reach the top of a classroom table and a local landmark.

Professional Learning

Take a moment to reflect on how students engaged with the content in this lesson. Why was the context of stacking cups particularly useful in building students' understanding of modeling with linear relationships in this lesson? What are other contexts that might support students in creating representations and using them to make predictions?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

9. How can you use a linear relationship to model a situation and make predictions?

Responses vary. First I can graph the relationship between the number of cups and the height of the stacked cups. Then I can look at the graph to determine the number of cups, x, that match the height of the table. Let's say the table is 50 centimeters. The graph shows that $x = 30$ when $y = 50$, so that means I need 30 stacked cups to reach the top of the table.

Lesson Practice ACC7.4.04

Lesson Summary

Linear relationships can help us make predictions. We can use the graph or the equation of a linear relationship to determine the value of one variable when we're given the other variable.

Let's say we want to know how many stacked cups will have a height of 50 centimeters. We can look at a graph and determine the number of cups, x, that correspond to $y = 50$. The slope of the line tells us how many centimeters the height of the stacked cups increases by every time we add a cup to the stack.

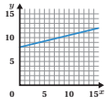
We might think of the y-intercept as the stack height when there are 0 cups, but this doesn't make sense in this situation. Instead, the y-value tells us the distance from the bottom of the first cup to its rim. Unlike a proportional relationship, the graph of this linear relationship doesn't pass through the origin.

Lesson Practice
ACC7.4.04

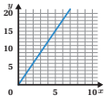
Name: Date: Period:

Problems 1–4: For each real-world situation, choose the graph that best represents it.

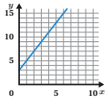
Graph A



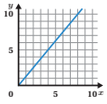
Graph B



Graph C



Graph D



1. y represents the perimeter of an equilateral triangle and x represents its side length. The slope of the line representing the relationship between x and y is 3.
Graph B

2. y represents the amount of money collected after x raffle tickets are purchased. The slope of the line representing the relationship between x and y is 0.25.
Graph A

3. y represents the number of chapters a student has read after x days. The slope of the line representing the relationship between x and y is $\frac{5}{4}$.
Graph D

4. y represents the total cost, in dollars, of x blueberry muffins, including a delivery fee. The slope of the line representing the relationship between x and y is 2.
Graph C

5. At a carnival, y represents the amount of money in a cash box after x tickets are purchased for carnival games. The line representing the relationship between x and y has a slope of $\frac{1}{4}$ and a y-intercept of 8. Explain what the slope and y-intercept represent in this situation.
Responses vary. The slope represents that each ticket costs $0.25. The y-intercept represents the amount, $8, that's already in the cash box before any tickets are sold.

Lesson Practice
ACC7.4.04

Name: Date: Period:

FAST Practice

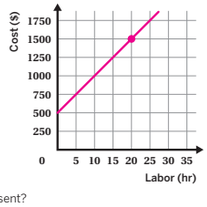
Problems 6–9: To paint a house, a painting company charges a flat rate of $500 for supplies, plus $50 for each hour of labor.

6. How much would the painting company charge to paint a house that needs 20 hours of labor? 50 hours of labor? Write your answers in the table.

Labor (hr)	Cost ($)
20	1500
50	3000

7. Draw a line representing the relationship between the number of hours of labor needed to paint the house and the total cost of paint.

8. Plot a point to show the cost of 20 hours of labor.



9. What is the slope of the line? What does it represent?

The slope of the line is (A.) 5 B. 50 C. 500.
It represents the (A.) cost per hour B. initial cost C. cost for 10 hours.

Spiral Review

10. Here is a table showing the values of a linear relationship. Write an equation to represent this relationship.
$y = -5x + 17$ (or equivalent)

x	0	1	2
y	17	12	7

11. Write the equation of a line that has a slope of 2 and a vertical intercept of 8.
$y = 2x + 8$ (or equivalent)

Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1–4	2	MA.8.AR.3.4
	5	2	MA.8.AR.3.5
FAST Practice	6–9	2	MA.8.AR.3.2, MA.8.AR.3.4, MA.8.AR.3.5
Spiral Review			
	10	2	MA.8.AR.3.3
	11	1	MA.8.AR.3.3

Assess and Respond

Sub-Unit Quiz 1

Independent | 45 min

Facilitation: Assign the Sub-Unit Quiz to learn about your students' understanding of the concepts and skills so far in this unit. If time allows, consider inviting students to complete one or more of the tasks in the Unit Synthesis and Reflection.

Item Analysis

Problem(s)	Concept or skill	Addressed in	DOK	FL B.E.S.T. Math Benchmarks
1	Interpreting lines with positive and negative slopes of non-proportional relationships that represent real-world situations	Lesson 5	2	MA.8.AR.3.5, MTR.7.1
2	Determining whether linear relationships are proportional or non-proportional	Lesson 1	1	MA.8.AR.3.1
3	Interpreting graphs of proportional relationships in context	Lessons 2, 3	3	MA.8.AR.3.2, MA.8.AR.3.3, MTR.7.1
4	Comparing proportional relationships given in different representations	Lesson 4	3	MA.8.AR.3.2, MA.8 AR.3.5, MTR.2.1
5a	Creating a graph with a negative slope that represents a real-world scenario	Lessons 4, 5	2	MA.8.AR.3.4, MTR.7.1
5b	Completing a table that represents a real-world scenario	Lesson 6	2	MA.8.AR.3.5, MTR.2.1, MTR.7.1
5c	Writing an equation to represent a real-world scenario	Lessons 6, 7	2	MA.8.AR.3.3, MTR.2.1, MTR.7.1

Assessment Resources

- Student Assessments
- Assessment Rubrics with Answers
- Show What You Knows and Answer Keys

Intervention and Extension Resources

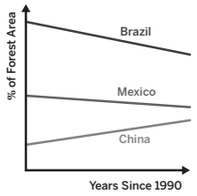
- Mini-Lessons
- Extensions

Name: ..... Date: ..... Period: .....

Sub-Unit Quiz

Unit 4

1. The percentage of forest area is determined by dividing the forest area by the total land area of a region. The percentage of forest area in three countries was recorded since 1990 and is displayed in the graph.



Which statement is true?

A. The percentage of forest area in Mexico increased as time passed.

B. The percentage of forest area in Brazil decreased at a constant rate.

C. Initially, the percentage of forest area was greater in Mexico than in Brazil.

D. The percentage of forest area in Mexico decreased faster than it did in Brazil.

2. Miguel loads a transit card with $40. Every time he rides the subway, $2 is subtracted from the amount available on the game card.

Let y represent the amount in dollars on the card after Miguel rides the subway x times.

Which equation represents this situation?

A. $y = 2 - 40x$

B. $y = -40 + 2x$

C. $y = -2x + 40$

D. $y = 40x - 2$

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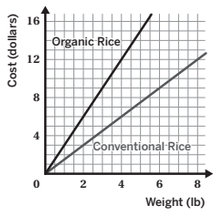
Name: ..... Date: ..... Period: .....

Sub-Unit Quiz (continued)

Unit 4

3. The graph shows the cost vs. weight relationship for two types of rice that can be bought at a bulk food store.



Select *all* of the true statements.

☑ A. The cost vs. weight relationships for both organic and conventional rice are proportional relationships.

☑ B. The cost as a unit rate for conventional rice is $1.50 per pound.

☐ C. Conventional rice costs twice as much per pound as organic rice.

☐ D. It costs $8 to buy 12 pounds of conventional rice.

☑ E. The relationship between cost, c, and weight, w, for organic rice can be represented by the equation $c = 3w$.

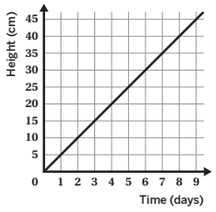
4. Nicolas planted three seeds. Each grew at a different constant rate. He measured the height of each plant every day and recorded his data below.

The table, the graph, and the equation show the relationship between time, t, in days and height, h, in centimeters for each of the plants.

Plant 1

Time (days)	Height (cm)
2	6
4	12
6	18
8	24
10	30

Plant 2



Plant 3

$h = 1.5t$

Which of the three plants grew the fastest? Explain your thinking.

Plant 2 grew the fastest. *Explanations vary.* The table shows that Plant 1 is 6 cm taller every 2 days, so Plant 1 grew 3 cm each day. The line in the graph has a slope of 5, so Plant 2 grew 5 cm per day. Plant 3's equation shows that it grew 1.5 cm each day.

D Differentiation (Sub-Unit Quiz 1)

Note: To **strengthen** and **stretch** students' learning, refer to the differentiation resources suggested throughout this unit and in the Unit Overview.

Sub-Unit Goals	Problem(s)	To respond to student thinking, consider:
• Identify if linear relationships are proportional or non-proportional. • Compare and interpret proportional relationships using multiple representations such as equations, tables, and graphs. **(Lessons 1–7)**	2	**Support** / **Strengthen** • **Teacher Move:** Consider revisiting Lesson 1 (*Turtle Time Trials*)
	3	**Support** / **Strengthen** • **Mini-Lesson:** *Writing Equations to Represent Proportional Relationships* • **Teacher Move:** Consider revisiting Lesson 2 (*Water Tank*)
	4	**Support** / **Strengthen** • **Teacher Move:** Consider revisiting Lesson 4 (*Flags*)
	1, 5a	**Support** / **Strengthen** • **Mini-Lesson:** *Interpreting Slope and Intercepts of Linear Relationships* • **Teacher Move:** Consider revisiting Lesson 5 (*Water Cooler*)
		Stretch • **You're invited to explore more:** Lesson 4 (*Proportional Posters*) • **You're invited to explore more:** Lesson 5 (*Water Cooler*)

	5b, 5c	**Support** / **Strengthen**
		• **Mini-Lesson:** *Interpreting Slope and Intercepts of Linear Relationships*
		• **Teacher Move:** Consider revisiting:
		  • Lesson 6 (*Ups and Downs*)
		  • Lesson 7 (*Stacking Cups*)

		Stretch • **You're invited to explore more:** Lesson 7 (*Stacking Cups*)

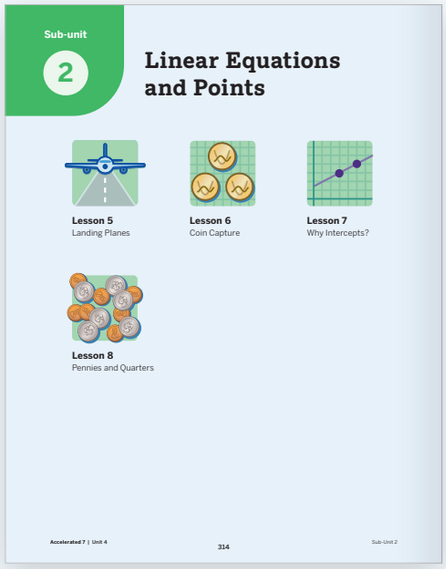
Sub-Unit 2

Linear Equations and Points

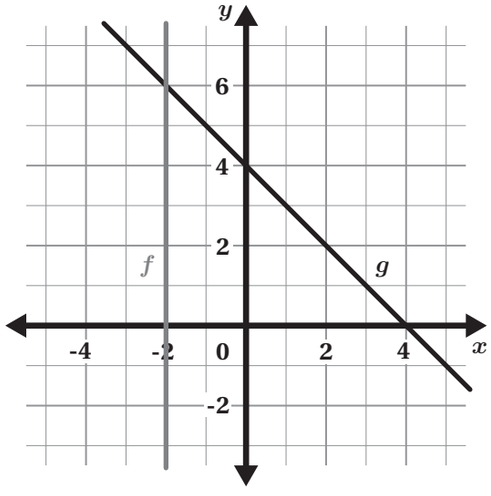
Sub-Unit 2 Goals

- **Goal:** Write an equation for a linear relationship given two points.
- **Goal:** Make connections between graphs and solutions of two-variable linear equations.
- **Language Goal:** Analyze strategies for calculating the slope and vertical intercept of a line through two points.

In this sub-unit, students generate a strategy to calculate the slope of a line through two given points and write equations for diagonal, vertical, and horizontal lines. Students also determine whether an ordered pair is a **solution** to a linear equation given in different forms.

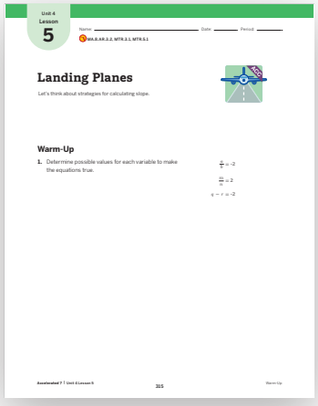


Math That Matters Most

Concept or Skill	For example . . .
Write an equation for a linear relationship given a graph or two points.	Write an equation for each line. Line f: $x = -2$ Line g: $y = -x + 4$ 
Write an equation for a linear relationship in the form $y = mx + b$.	The owner of a new restaurant is ordering tables and chairs. She wants to have only tables for 2 and tables for 4. The total number of people that can be seated in the restaurant is 120. Write an equation that represents the number of 2-seat tables, x, and the number of 4-seat tables, y, that the owner can order. $y = -\frac{1}{2}x + 30$

Unit 4
Lesson
5

ACC Accelerated


Unit 4
Lesson
5

Landing Planes

Let's think about strategies for calculating slope.

Warm-Up

1. Determine possible values for each variable to make the equations true.

$\frac{y}{3} = -2$

$\frac{z}{5} = 2$

$q - r = -2$

315

Landing Planes

Calculating Slope

Let's think about strategies for calculating slope.

Learning Outcomes

Today's Goals

1. **Goal:** Generate a strategy to calculate the slope of a line through two given points.
2. **Language Goal:** Compare and contrast different strategies for calculating slope. **(Reading, Writing, Speaking, and Listening)**

Students extend their work with slope triangles to develop a method for finding the slope of any line given the coordinates of two points on a line. They identify the sign of a slope and "land a plane" by calculating the slope of a plane's path. They make sense of different structures used to support finding the slope of a line. Then they create their own "runway" to share with their peers and compare their strategies. **(MTR.5.1)**

Coherence

Prior Learning

In Unit 2, Lesson 15, students were introduced to slope by making sense of similar triangles that shared a common line. In Lessons 1–4, students identified and contextualized a variety of slopes in different situations and also identified and analyzed slopes in a variety of situations.

Future Learning

In Lesson 6, students will consider the equations and slopes of horizontal and vertical lines in the context of a game where they capture coins. In Lesson 7, students will write linear equations using two points on the line.

Connections

- Students calculate the slope of a line (given two points) to land a plane. (real world)

FL B.E.S.T. Math Benchmarks

Addressing

MA.8.AR.3.2
Given a table, graph or written description of a linear relationship, determine the slope.

Mathematical Thinking and Reasoning Standards: MTR.3.1, MTR.5.1

ELA Expectations: ELA.K12.EE.3.1, ELA.K12.EE.5.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.7.AR.4.2	MA.912.AR.2.2

Lesson at a Glance

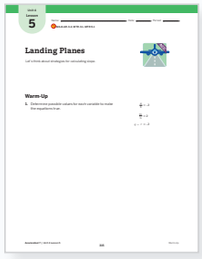
~ 45 min

MA.8.AR.3.2

Warm-Up

Whole Class | 5 min

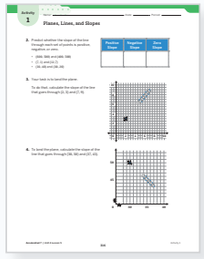
Students attend to the structure of expressions to support identifying possible signs of values in slope calculations. **Routine:** Number Talk **(MTR.5.1)**



Activity 1

Pairs | 10 min

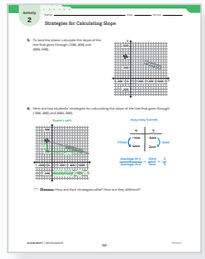
Students identify patterns in pairs of points whose slopes are positive, negative, and zero. **(MTR.5.1)**



Activity 2

Pairs | 10 min

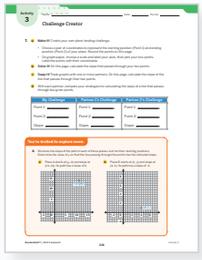
Students generalize strategies for calculating the slope between two points. **MLR7: Compare and Connect** **(MTR.5.1)**



Activity 3

Independent | 10 min

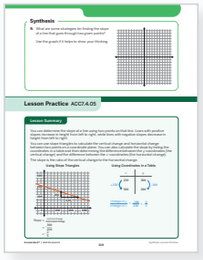
Students use their creativity to design their own airplane and landing strip, making use of this context to use structures that support slope calculations. **(MTR.3.1)**



Synthesis

Whole Class | 5 min

Students synthesize their strategies for determining the slope between two points. **MLR2: Collect and Display**



Show What You Know

Independent | 5 min

Students demonstrate their understanding by determining the slope between two points.

Prep Checklist

This lesson includes:

- Student Edition
- Show What You Know
- Graph paper
- Straightedges

Whole Class | 5 min

Warm-Up

Purpose: Students attend to the structure of expressions to support identifying possible signs of values in slope calculations. **(MTR.5.1)**

1 Launch

Use the Number Talk routine to support students in thinking flexibly about the signs in different operations involved in calculating slope. See the Routine Facilitation Guide for more information.

Invite students to identify values for the variables in the first equation independently. Then record a variety of possible values and ideas, along with the names of the students who shared them.

Consider asking, "Who thought about it in a different way?"

Note: This structure is repeated for each problem in the Number Talk.

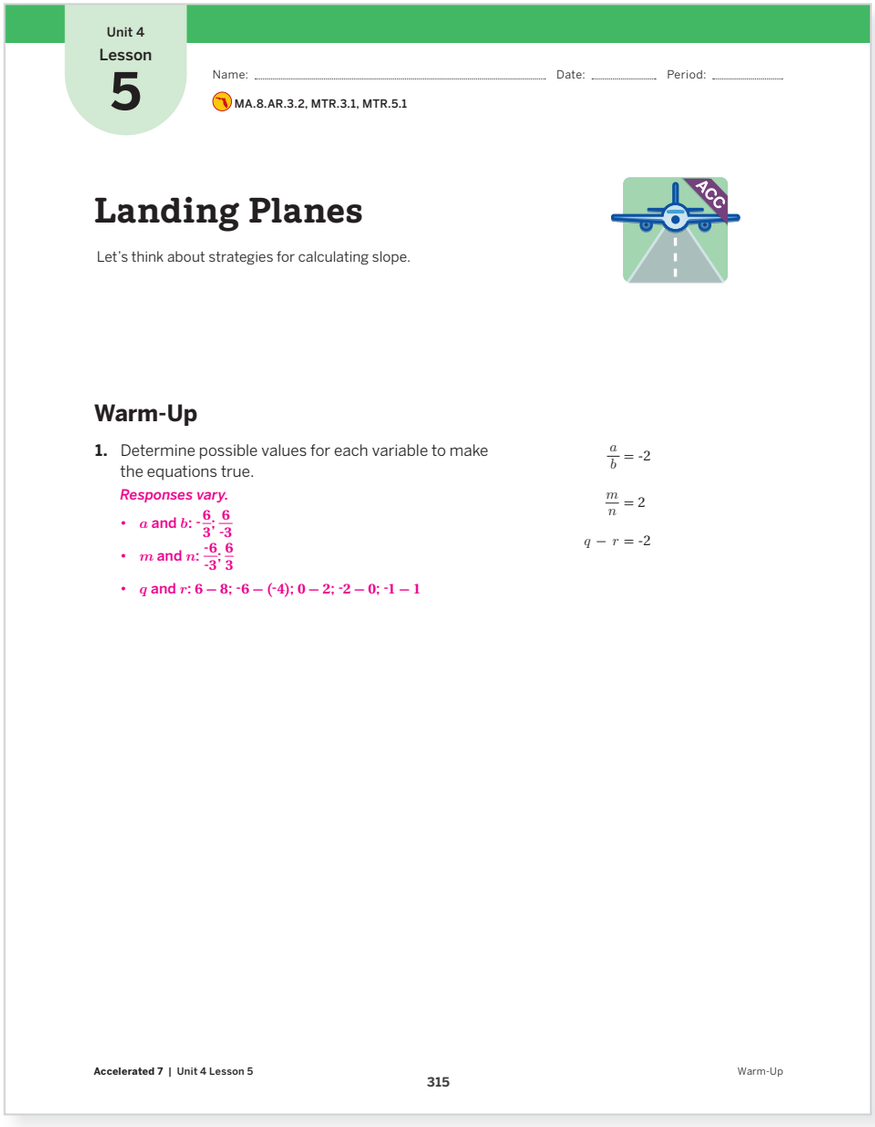
A **Accessibility: Executive Functioning** Consider chunking this activity by inviting students to choose one equation at a time.

2 Connect

Invite students to compare and contrast the different equations and to make connections to positive and negative slopes. **(MTR.5.1)**

Consider asking:

- "How do you know when to use a positive or negative value for each equation?"
- "When the quotient of two numbers is positive, what could be true about the signs of the numbers?"
- "When the quotient of two numbers is negative, what could be true about signs of the two numbers?"
- "When the difference between two numbers is negative, what does that tell you about the values of the numbers?"
- "How might these different operations connect to slope?"



Unit 4
Lesson
5

Name: _____ Date: _____ Period: _____

MA.8.AR.3.2, MTR.3.1, MTR.5.1

Landing Planes

Let's think about strategies for calculating slope.

Warm-Up

1. Determine possible values for each variable to make the equations true.

$\frac{a}{b} = -2$

$\frac{m}{n} = 2$

$q - r = -2$

Responses vary.

- a and b: $-\frac{6}{3}, \frac{6}{-3}$
- m and n: $\frac{-6}{-3}, \frac{6}{3}$
- q and r: $6 - 8$; $-6 - (-4)$; $0 - 2$; $-2 - 0$; $-1 - 1$

Accelerated 7 | Unit 4 Lesson 5 315 Warm-Up

Pairs | 10 min

Activity 1 Planes, Lines, and Slopes

Purpose: Students identify patterns in pairs of points whose slopes are positive, negative, and zero. **(MTR.5.1)**

Short on time? Consider facilitating Problem 2 together as a class.

1 Launch

Invite students to use gestures to show lines with positive, zero, and negative slope.

Consider sharing that students are making predictions, not calculating.

2 Monitor

Consider sharing that students will explore the context of planes landing on a runway.

ELL **English Language Learners** Invite students to act out scenarios, show images, use gestures, or talk about landing planes before starting the task. **(Reading and Listening)**

Listen for how students think about the sign and value of the slope of each line.

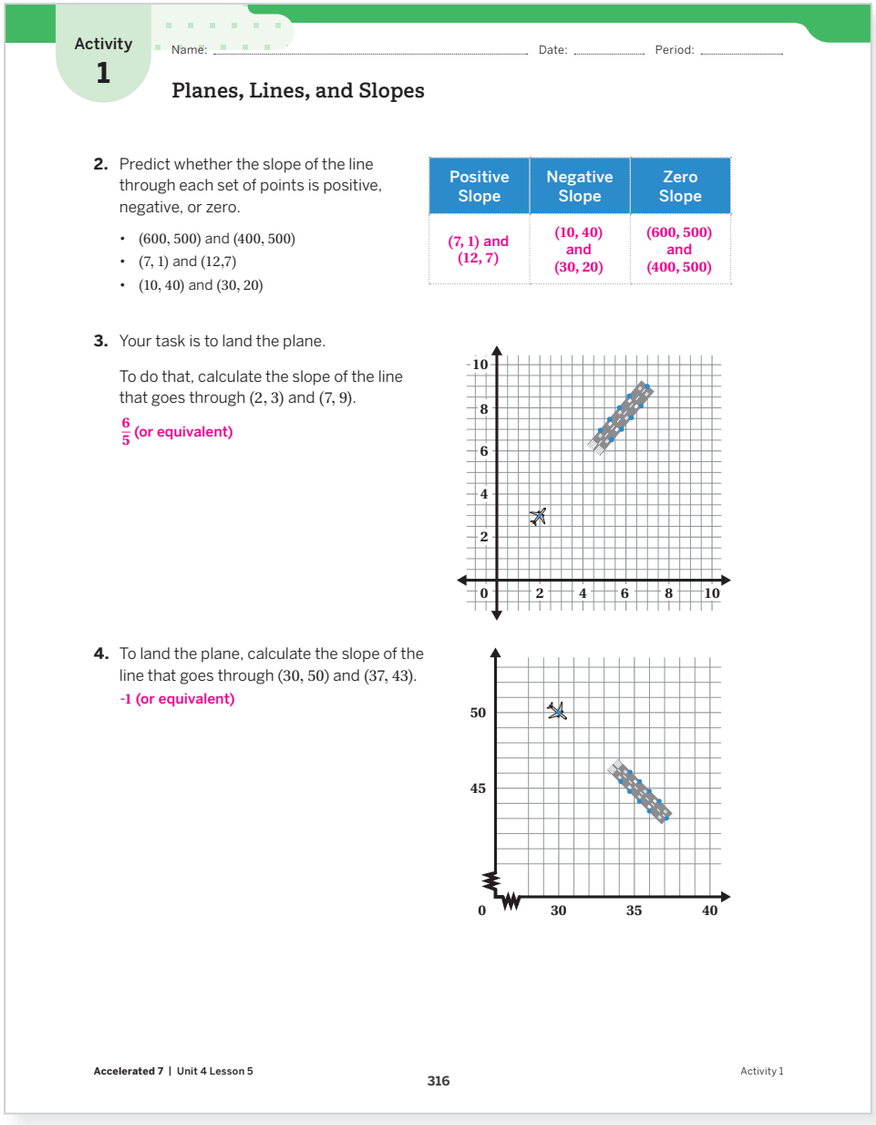
D Differentiation

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Invite them to try a few slopes and see if they can make a calculation.
Identified horizontal distance but did not accurately determine vertical distance.	**Support** Encourage them to sketch slope triangles. Consider asking, "How can you find the height of this slope triangle?"
Calculated both slopes as positive.	**Strengthen** Invite students to compare and contrast the slopes in each problem. Consider asking, "How do you know if the slope will be positive or negative?"
Accurately identified the value and sign of the slope.	**Stretch** Encourage them to think about strategies they can generalize to calculate the slope without the coordinate plane.

3 Connect

Share strategies students used to determine the sign of the slope.

Create a record of strategies, including predicting using the graph and identifying the sign of the change in x and the change in y.



Activity 1

Name: _____ Date: _____ Period: _____

Planes, Lines, and Slopes

2. Predict whether the slope of the line through each set of points is positive, negative, or zero.

- (600, 500) and (400, 500)
- (7, 1) and (12,7)
- (10, 40) and (30, 20)

Positive Slope	Negative Slope	Zero Slope
(7, 1) and (12, 7)	(10, 40) and (30, 20)	(600, 500) and (400, 500)

3. Your task is to land the plane.

To do that, calculate the slope of the line that goes through (2, 3) and (7, 9).

$\frac{6}{5}$ (or equivalent)

4. To land the plane, calculate the slope of the line that goes through (30, 50) and (37, 43).

-1 (or equivalent)

Accelerated 7 | Unit 4 Lesson 5 316 Activity 1

Pairs | 10 min

Activity 2 Strategies for Calculating Slope

Purpose: Students generalize strategies for calculating the slope between two points. **(MTR.5.1)**

1 Launch

To surface connections, before Problem 5 consider asking:

- "What is similar about this problem and the last few problems?"
- "What is different about this problem compared to the last few problems?"

Note: If it does not come up naturally, consider sharing that the axes are scaled differently.

2 Monitor

Listen for strategies used to calculate slope to share during the Connect.

D Differentiation *(Problem 5)*

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Encourage them to draw slope triangles.
Think that the vertical change between the two points is positive.	**Support** Invite them to annotate the diagram with the coordinates. Consider asking, "What could turn 400 into 200?"
Sketch a slope triangle and count the units on the grid.	**Strengthen** Celebrate them for the strategies they have tried so far. Consider asking, "How should we use the scale shown on the axes?"
Use a formula not introduced in the lesson to determine slope.	**Stretch** Celebrate students for making a connection to a strategy learned elsewhere. Encourage them to connect this formula with strategies seen earlier in the lesson.

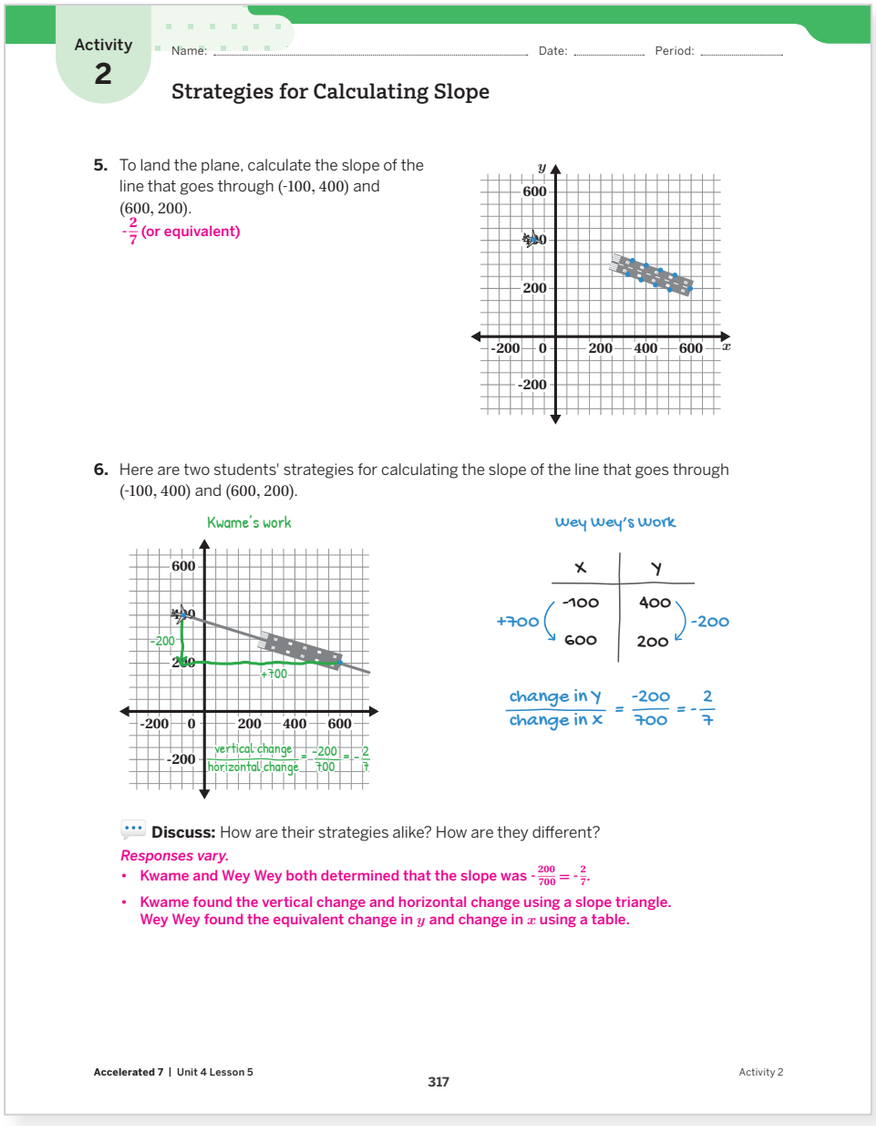
3 Connect

Draw students' attention to the two strategies shown in Problem 6.

MLR **MLR7: Compare and Connect** Invite students to analyze and compare each strategy, and then connect to their own strategy for determining slope.

To surface the Key Takeaway, consider asking:

- "How are the two strategies alike? How are they different?"
- "How did each student determine the vertical change and horizontal change?"
- "When might you use each strategy?"



Activity 2

Name: _____ Date: _____ Period: _____

Strategies for Calculating Slope

5. To land the plane, calculate the slope of the line that goes through $(-100, 400)$ and $(600, 200)$.

$-\frac{2}{7}$ (or equivalent)

6. Here are two students' strategies for calculating the slope of the line that goes through $(-100, 400)$ and $(600, 200)$.

Kwame's work

Wey Wey's work

x	y
-100	400
600	200

+700 -200

$\frac{\text{change in } y}{\text{change in } x} = \frac{-200}{700} = -\frac{2}{7}$

Discuss: How are their strategies alike? How are they different?

Responses vary.

- Kwame and Wey Wey both determined that the slope was $-\frac{200}{700} = -\frac{2}{7}$.
- Kwame found the vertical change and horizontal change using a slope triangle. Wey Wey found the equivalent change in y and change in x using a table.

Accelerated 7 | Unit 4 Lesson 5 317 Activity 2

Key Takeaway: In order to determine the slope of a line, we need to use two points on the line. Tables and graphs help us identify the vertical change and horizontal change between the points.

Independent | 10 min

Activity 3 Challenge Creator

Purpose: Students use their creativity to design their own airplane and landing strip, making use of this context to use structures that support slope calculations. **(MTR.3.1)**

Short on time? Consider omitting this activity.

1 Launch

To encourage playfulness, consider asking students to create unique "airport designs" that are distinct from the examples seen earlier. They will need graph paper and straightedges to sketch their challenges for their peers to complete.

2 Monitor

To support students getting started, consider asking, "Can you draw what you want your design to look like?"

A **Accessibility: Executive Functioning** Invite students to reflect on strategies they might use before responding to their classmates' challenges.

Capture a variety of unique challenges, particularly challenges where the x-axis and y-axis scales are different.

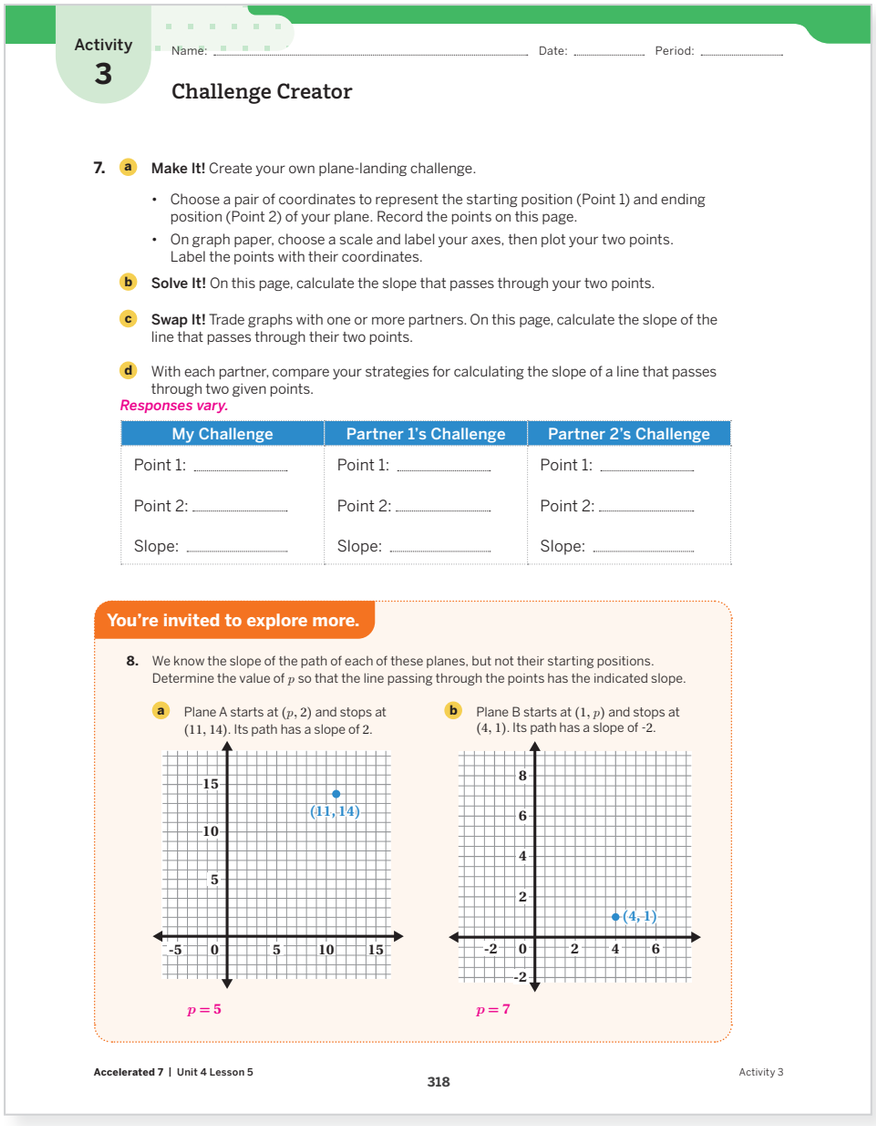
D **Differentiation: You're invited to explore more. (Stretch)** Invite students who would like to explore determining slope from two points further to complete this optional task. Encourage them to discuss their thinking with a partner.

3 Connect

Invite a few students to share their Challenges and invite students to reflect on what they learned from this experience.

Consider asking:

- "What strategies did you use to find the slope?"
- "How did you visualize the slope?"
- "Did order matter when subtracting different x- and y-values?"
- "How can you determine the slope when the scale on each axis is different?"



Activity 3

Name: _____ Date: _____ Period: _____

Challenge Creator

7. **a** **Make It!** Create your own plane-landing challenge.

- Choose a pair of coordinates to represent the starting position (Point 1) and ending position (Point 2) of your plane. Record the points on this page.
- On graph paper, choose a scale and label your axes, then plot your two points. Label the points with their coordinates.

b **Solve It!** On this page, calculate the slope that passes through your two points.

c **Swap It!** Trade graphs with one or more partners. On this page, calculate the slope of the line that passes through their two points.

d With each partner, compare your strategies for calculating the slope of a line that passes through two given points.

Responses vary.

My Challenge	Partner 1's Challenge	Partner 2's Challenge
Point 1: _____	Point 1: _____	Point 1: _____
Point 2: _____	Point 2: _____	Point 2: _____
Slope: _____	Slope: _____	Slope: _____

You're invited to explore more.

8. We know the slope of the path of each of these planes, but not their starting positions. Determine the value of p so that the line passing through the points has the indicated slope.

a Plane A starts at $(p, 2)$ and stops at $(11, 14)$. Its path has a slope of 2.

$p = 5$

b Plane B starts at $(1, p)$ and stops at $(4, 1)$. Its path has a slope of -2.

$p = 7$

Accelerated 7 | Unit 4 Lesson 5 318 Activity 3

Whole Class | 5 min | 6.NS.A.1, 6.NS.A

Synthesis

Purpose: Students synthesize their strategies for determining the slope between two points.

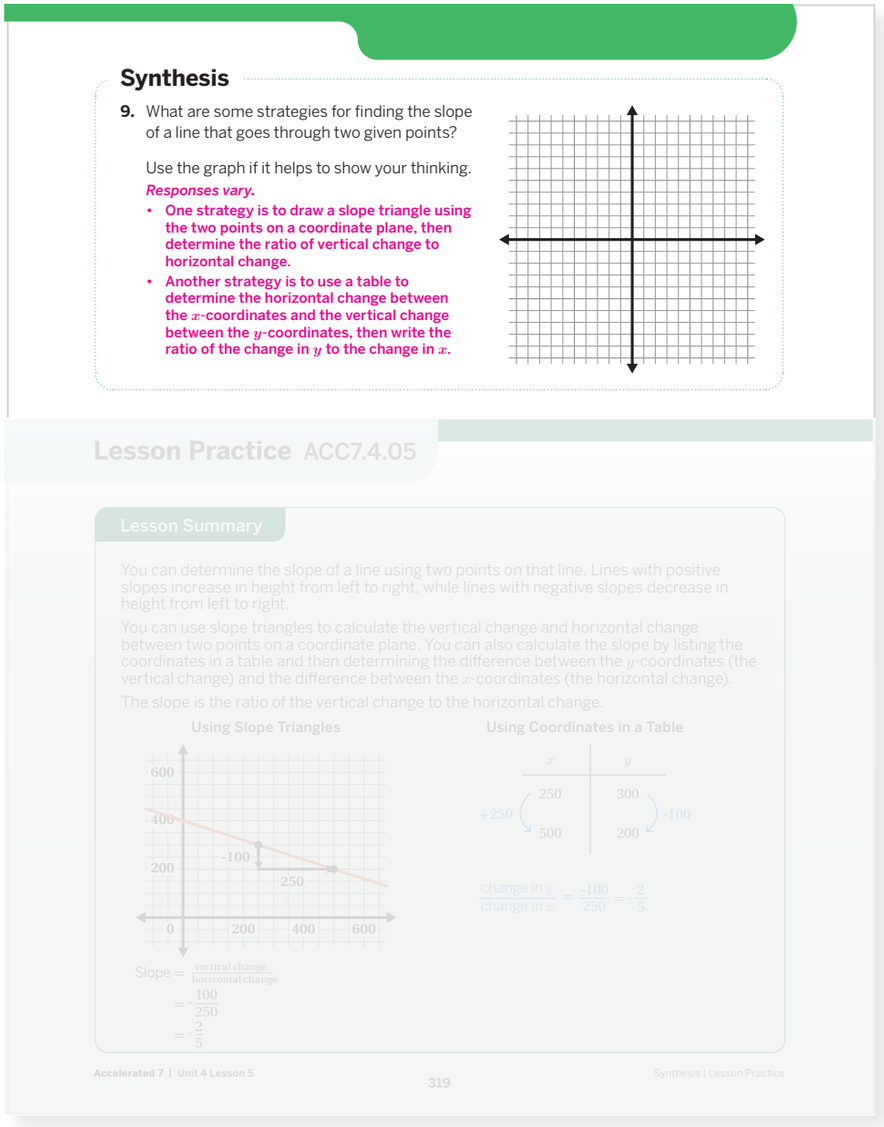
Invite students to respond independently, and then share their thinking with a partner.

Collect and share a variety of ideas, such as:

- Slope represents the constant rate of change for a linear relationship.
- Slope can be determined by finding the change in y-values and dividing by the change in x-values from two coordinate points.
- Slope can be determined by drawing slope triangles and dividing the vertical change by the horizontal change.

MLR **MLR2: Collect and Display** Use this routine to formalize the strategies for determining slope using students' language. Display any new language, examples, or diagrams for students to reference later.

Note: If time allows, invite students to share with a partner what makes sense about each strategy to determine slope and what questions they still have.



Synthesis

9. What are some strategies for finding the slope of a line that goes through two given points?

Use the graph if it helps to show your thinking.

Responses vary.

- One strategy is to draw a slope triangle using the two points on a coordinate plane, then determine the ratio of vertical change to horizontal change.
- Another strategy is to use a table to determine the horizontal change between the x-coordinates and the vertical change between the y-coordinates, then write the ratio of the change in y to the change in x.

Lesson Practice ACC7.4.05

Lesson Summary

You can determine the slope of a line using two points on that line. Lines with positive slopes increase in height from left to right, while lines with negative slopes decrease in height from left to right.

You can use slope triangles to calculate the vertical change and horizontal change between two points on a coordinate plane. You can also calculate the slope by listing the coordinates in a table and then determining the difference between the y-coordinates (the vertical change) and the difference between the x-coordinates (the horizontal change).

The slope is the ratio of the vertical change to the horizontal change.

Using Slope Triangles

600 400 200 0 200 400 600 -100 250

$\text{Slope} = \frac{\text{vertical change}}{\text{horizontal change}} = \frac{-100}{250} = -\frac{2}{5}$

Using Coordinates in a Table

x	y
250	300
500	200

+250 -100

$\frac{\text{change in } y}{\text{change in } x} = \frac{-100}{250} = -\frac{2}{5}$

Accelerated 7 | Unit 4 Lesson 5 319 Synthesis | Lesson Practice

Lesson Takeaway: When given two points, the change in x-values and the change in y-values can be identified using a graph or a table. The slope of the line is the ratio between the change in y-values and the change in x-values.

Independent | 5 min

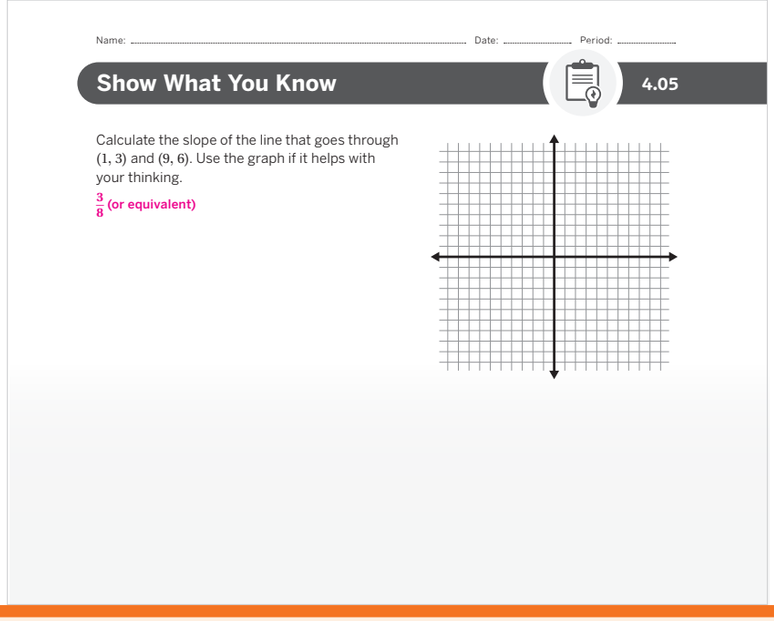
Show What You Know

Purpose: Students demonstrate their understanding by determining the slope between two points.

Today's Goals

1. **Goal:** Generate a strategy to calculate the slope of a line through two given points.
2. **Language Goal:** Compare and contrast different strategies for calculating slope. **(Reading, Writing, Speaking, and Listening)**

Show What You Know (Assessment Resources)



Name: Date: Period:

Show What You Know 4.05

Calculate the slope of the line that goes through (1, 3) and (9, 6). Use the graph if it helps with your thinking.

$\frac{3}{8}$ (or equivalent)

D Differentiation Use after Lesson 5

Support

Provide targeted intervention for students by using these resources.

If students need support with determining the slope between two points:

» **Respond to Student Thinking:**

- Use the ***Calculating Slope Given Two Points*** **Mini-Lesson.**
- Revisit **Lesson 1, Activity 1** (*Turtle Time Trials*) as a class and invite students to share strategies for interpreting and calculating slope in context.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 3 Extension Activities**.

Professional Learning

Consider the next several lessons you will teach. How will students' work today prepare them for writing equations of linear relationships using two points in Lesson 7? What are ways that you might call back to students' strategies from this lesson?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

9. What are some strategies for finding the slope of a line that goes through two given points?

Use the graph if it helps to show your thinking.

Responses vary.

- One strategy is to draw a slope triangle using the two points on a coordinate plane, then determine the ratio of vertical change to horizontal change.
- Another strategy is to use a table to determine the horizontal change between the x-coordinates and the vertical change between the y-coordinates, then write the ratio of the change in y to the change in x.

Lesson Practice ACC7.4.05

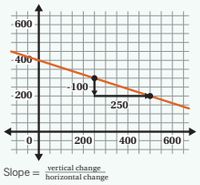
Lesson Summary

You can determine the slope of a line using two points on that line. Lines with positive slopes increase in height from left to right, while lines with negative slopes decrease in height from left to right.

You can use slope triangles to calculate the vertical change and horizontal change between two points on a coordinate plane. You can also calculate the slope by listing the coordinates in a table and then determining the difference between the y-coordinates (the vertical change) and the difference between the x-coordinates (the horizontal change).

The slope is the ratio of the vertical change to the horizontal change.

Using Slope Triangles



$$\text{Slope} = \frac{\text{vertical change}}{\text{horizontal change}} = \frac{-100}{250} = -\frac{2}{5}$$

Using Coordinates in a Table

x	y
250	300
+250 ↓	-100 ↓
500	200

$$\frac{\text{change in } y}{\text{change in } x} = \frac{-100}{250} = -\frac{2}{5}$$

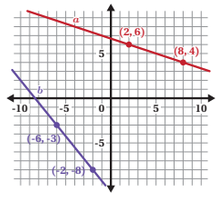
Lesson Practice ACC7.4.05

Name: _____ Date: _____ Period: _____

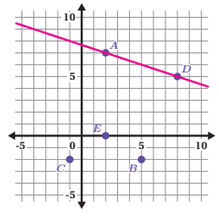
1. Calculate the slope of each line.

line a: $-\frac{1}{3}$ (or equivalent)

line b: $-\frac{5}{4}$ (or equivalent)



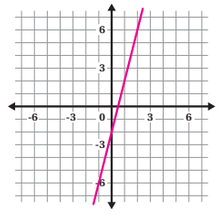
2. Draw a line with a slope of $-\frac{1}{3}$ that passes through point A.



What other point lies on that line?

Point D

Problems 3–4: Here is a blank graph.



3. Draw a line with a slope of 4 and a negative y-intercept.

Responses vary. Sample shown on graph.

4. Explain how you know the slope of your line is 4.

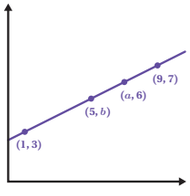
Responses vary. I can tell the slope is 4 because from (0, -2) to (1, 2) the change in y is +4 and the change in x is +1, and $\frac{4}{1} = 4$.

Lesson Practice ACC7.4.05

Name: _____ Date: _____ Period: _____

FAST Practice

Problems 5–7: All the points in this graph are on the same line.



5. What is the slope of the line? Explain your thinking.

$\frac{1}{2}$. *Explanations vary.* From (1, 3) to (9, 7), the vertical change is +4 and the horizontal change is +8, so the slope is $\frac{4}{8} = \frac{1}{2}$.

6. What are the values for a and b?

$a = 7, b = 5$

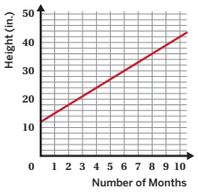
7. What is the x-value when $y = 0$?

Record your answer in the space provided.

$x =$ -5

Spiral Review

Problems 8–9: The graph shows the height of a bamboo plant, h, n months after it has been planted.



8. Write an equation that gives the bamboo's height, h, after n months.

$h = 3n + 12$ (or equivalent)

9. How many inches tall will the bamboo plant be after 18 months? Show your thinking.

66 inches. *Work varies.*

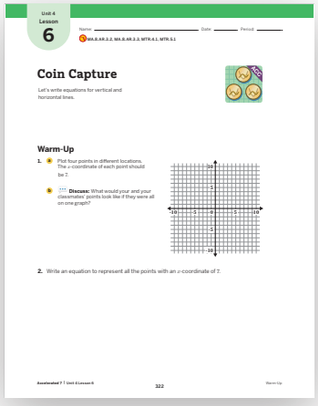
$h = 3(18) + 12$

$h = 54 + 12$

$h = 66$

Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1–4	2	MA.8.AR.3.2
FAST Practice	5–7	2	MA.8.AR.3.2
Spiral Review			
	8–9	2	MA.8.AR.3.3


Unit 4
Lesson
6

Coin Capture

Let's write equations for vertical and horizontal lines.

Warm-Up

1. Plot four points in different locations. The x-coordinate of each point should be 2.

Discuss: What would your and your classmates' points look like if they were all on one graph?

2. Write an equation to represent all the points with an x-coordinate of 7.

322

Coin Capture

Horizontal and Vertical Lines

Let's write equations for vertical and horizontal lines.

Learning Outcomes

Today's Goals

1. **Goal:** Write equations of horizontal and vertical lines.
2. **Language Goal:** Justify whether horizontal or vertical lines have a slope of 0. **(Reading and Writing)**

Students look for and make use of the structure of points to write the equations of vertical and horizontal lines. They attend to precision as they write equations of lines with different slopes to capture coins on a coordinate plane, and they justify whether horizontal or vertical lines have a slope of zero. **(MTR.4.1, MTR.5.1)**

Coherence

Prior Learning

In Lesson 5, students generated strategies for calculating the slope of a line given two points.

Future Learning

In Lesson 7, students will develop strategies for writing the equation of a line through two given points.

Connections

- Students collaborate to identify patterns and generalize their findings when writing the equations of horizontal and vertical lines. (mathematical)

FL B.E.S.T. Math Benchmarks

Addressing

MA.8.AR.3.3

Given a table, graph or written description of a linear relationship, write an equation in slope-intercept form.

Also Addressing: **MA.8.AR.3.2**

Mathematical Thinking and Reasoning Standards: MTR.4.1, MTR.5.1

ELA Expectations: ELA.K12.EE.1.1, ELA.K12.EE.3.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.7.AR.4.4	MA.912.AR.2.2

Vocabulary

Additional Vocabulary

undefined

Lesson at a Glance

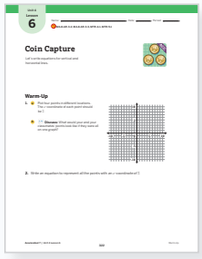
~ 45 min

MA.8.AR.3.2, MA.8.AR.3.3

Warm-Up

Independent | 5 min

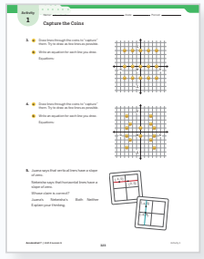
Students plot points with the same x-coordinate to make sense of the structure of equations of vertical lines. **(MTR.5.1)**



Activity 1

Pairs | 15 min

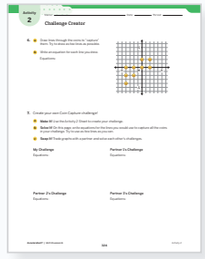
Students attend to precision when writing equations and analyze claims about the slopes of horizontal and vertical lines.
Routine: Decide and Defend **(MTR.4.1)**



Activity 2

Independent | 15 min

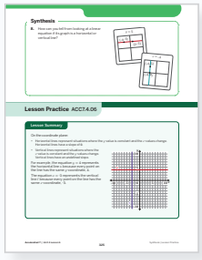
Students attend to precision with writing equations of lines in challenges designed by their classmates. **(MTR.4.1)**



Synthesis

Whole Class | 5 min

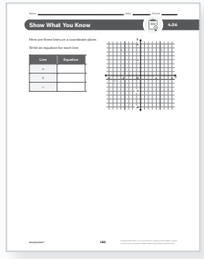
Students synthesize their strategies to identify equations of different types of lines.
MLR2: Collect and Display



Show What You Know

Independent | 5 min

Students demonstrate their understanding by writing equations of vertical and horizontal lines.



Prep Checklist

Prepare Activity 2 sheets.

This lesson includes:

- Student Edition
- Show What You Know
- Activity 2 Sheet, one per student
- Straightedges
- Calculators (as needed)

Independent | 5 min

Warm-Up

Purpose: Students plot points with the same x-coordinate to make sense of the structure of equations of vertical lines. **(MTR.5.1)**

1 Launch

Direct students' attention to the coordinate plane.

Invite students to predict what the points will look like on the graph before they plot them.

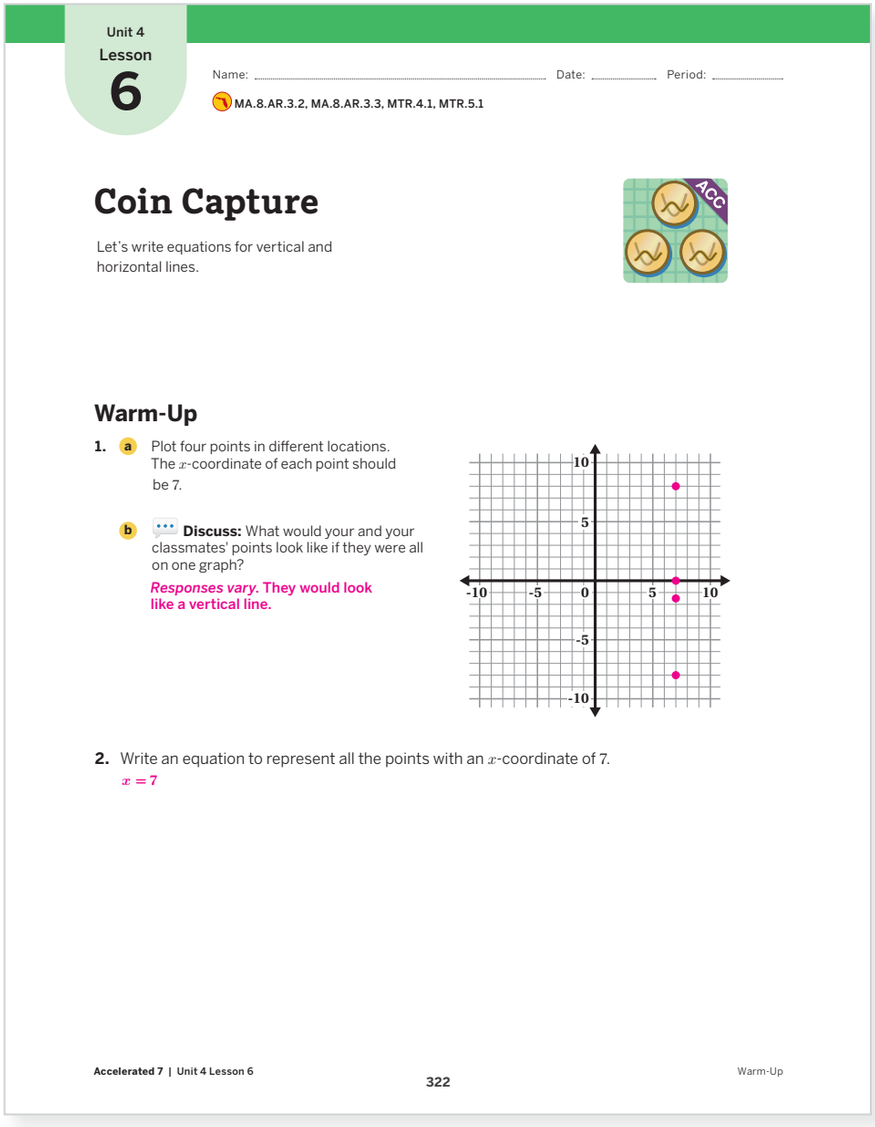
2 Connect

Encourage students to describe the pattern in their own words to a partner before trying the equation.

Listen for and share a variety of responses, including correct and incorrect equations.

To surface the Key Takeaway, consider asking:

- "Which equation makes sense and why?"
- "Why does the equation only contain the variable x?"
- "How can we describe the relationship between the quantities represented by x and y?"
- "Is it possible to have an equation without a y-value?"



Unit 4
Lesson
6

Name: ..... Date: ..... Period: .....

MA.8.AR.3.2, MA.8.AR.3.3, MTR.4.1, MTR.5.1

Coin Capture

Let's write equations for vertical and horizontal lines.

Warm-Up

1. a Plot four points in different locations. The x-coordinate of each point should be 7.

   b **Discuss:** What would your and your classmates' points look like if they were all on one graph?

   Responses vary. They would look like a vertical line.

2. Write an equation to represent all the points with an x-coordinate of 7.

   $x = 7$

Accelerated 7 | Unit 4 Lesson 6 322 Warm-Up

Key Takeaway: Vertical lines represent all of the points that have the same x-coordinate and different y-coordinates. Their equations can all be represented in the form $x = a$.

Pairs | 15 min

Activity 1 Capture the Coins

Purpose: Students attend to precision when writing equations and analyze claims about the slopes of horizontal and vertical lines. **(MTR.4.1)**

1 Launch

Invite students to think independently before discussing their ideas with a partner. Invite them to write equations to capture all of the coins.

Distribute straightedges to support sketching lines.

2 Monitor

Consider asking, "Is this the fewest number of lines you could use? What other lines might you input that could also work? What are different ways to write these equations?"

Note: The equations $x = 3$ and $x = -3$ will capture the edges of the coins on either side of these vertical lines.

Select and sequence a few explanations to share during the Connect.

D Differentiation *(Problem 5)*

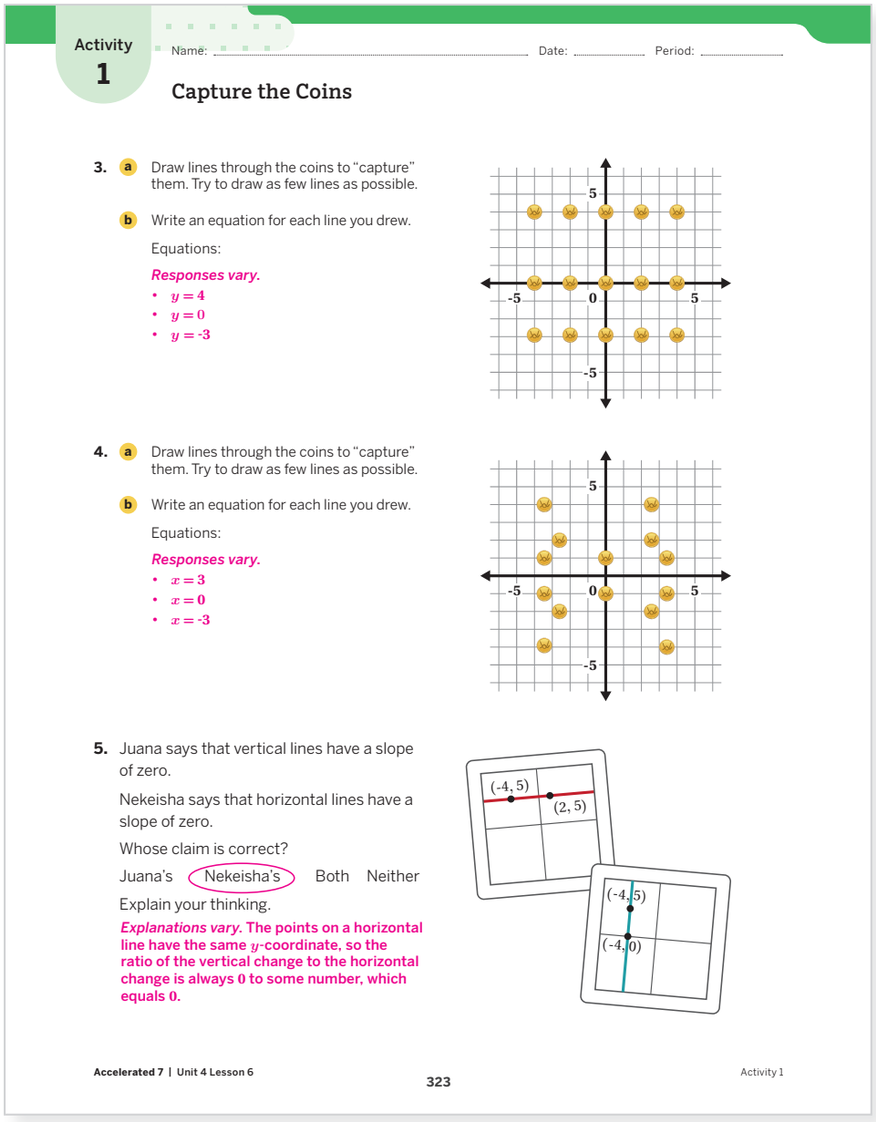
Look for students who:	Respond to Student Thinking
Agree with Juana.	**Support** Invite them to compute the slope of the lines using a calculator. Consider asking, "What happens when we divide a number by 0?"
Agree with Nekeisha without a clear explanation.	**Strengthen** Consider asking, "How do you know that Nekeisha's claim is correct? How do you know that Juana's claim is incorrect? How might you use evidence to support your claim?"
Agree with Nekeisha and support their claim with evidence.	**Stretch** Encourage students to discuss with their partner about generalizations we can make regarding horizontal and vertical lines.

3 Connect

Use the Decide and Defend routine to support students in making arguments and critiquing the reasoning of others. See the Routine Facilitation Guide. **(MTR.4.1)**

To surface the Key Takeaway, consider asking:
- "What happens when you divide a number by 0?"
- "Which lines have a slope of 0? How do we know?"

Note: If it does not come up naturally, consider sharing that vertical lines have an *undefined* slope because slope is determined by dividing the change in vertical distance by the change in horizontal distance. Because we cannot divide a quantity by 0, we consider this value undefined.



Activity 1

Name: _____ Date: _____ Period: _____

Capture the Coins

3. a. Draw lines through the coins to "capture" them. Try to draw as few lines as possible.

b. Write an equation for each line you drew.

Equations:

Responses vary.
- $y = 4$
- $y = 0$
- $y = -3$

4. a. Draw lines through the coins to "capture" them. Try to draw as few lines as possible.

b. Write an equation for each line you drew.

Equations:

Responses vary.
- $x = 3$
- $x = 0$
- $x = -3$

5. Juana says that vertical lines have a slope of zero.

Nekeisha says that horizontal lines have a slope of zero.

Whose claim is correct?

Juana's Nekeisha's Both Neither

Explain your thinking.

Explanations vary. The points on a horizontal line have the same y-coordinate, so the ratio of the vertical change to the horizontal change is always 0 to some number, which equals 0.

Accelerated 7 | Unit 4 Lesson 6 323 Activity 1

Key Takeaway: Equations of horizontal lines have a slope of 0 because a vertical change of 0 is divided by some horizontal change. Equations of vertical lines have an undefined slope because some vertical change is divided by 0 horizontal change.

Independent | 15 min

Activity 2 Challenge Creator

Purpose: Students attend to precision when writing equations of lines in challenges designed by their classmates. **(MTR.4.1)**

Short on time? Consider facilitating and completing Problem 6 as a class.

1 Launch

To encourage engagement, invite students to predict the fewest number of lines it will take to capture all of the coins. Consider asking, "What types of lines will capture the most coins?"

ELL **English Language Learners** Provide a reference that includes descriptions and visuals of different types of lines. **(Reading, Writing, Speaking, and Listening)**

2 Monitor

Consider sharing that students will create their own challenge and solve challenges from their classmates.

Students will need one or more partners for this activity. Distribute the Activity 2 Sheet so they can create their Coin Capture challenge.

A **Accessibility: Executive Functioning**
To support students initiating complex tasks, identify possible places to start.

Collect different examples to share during the Connect.

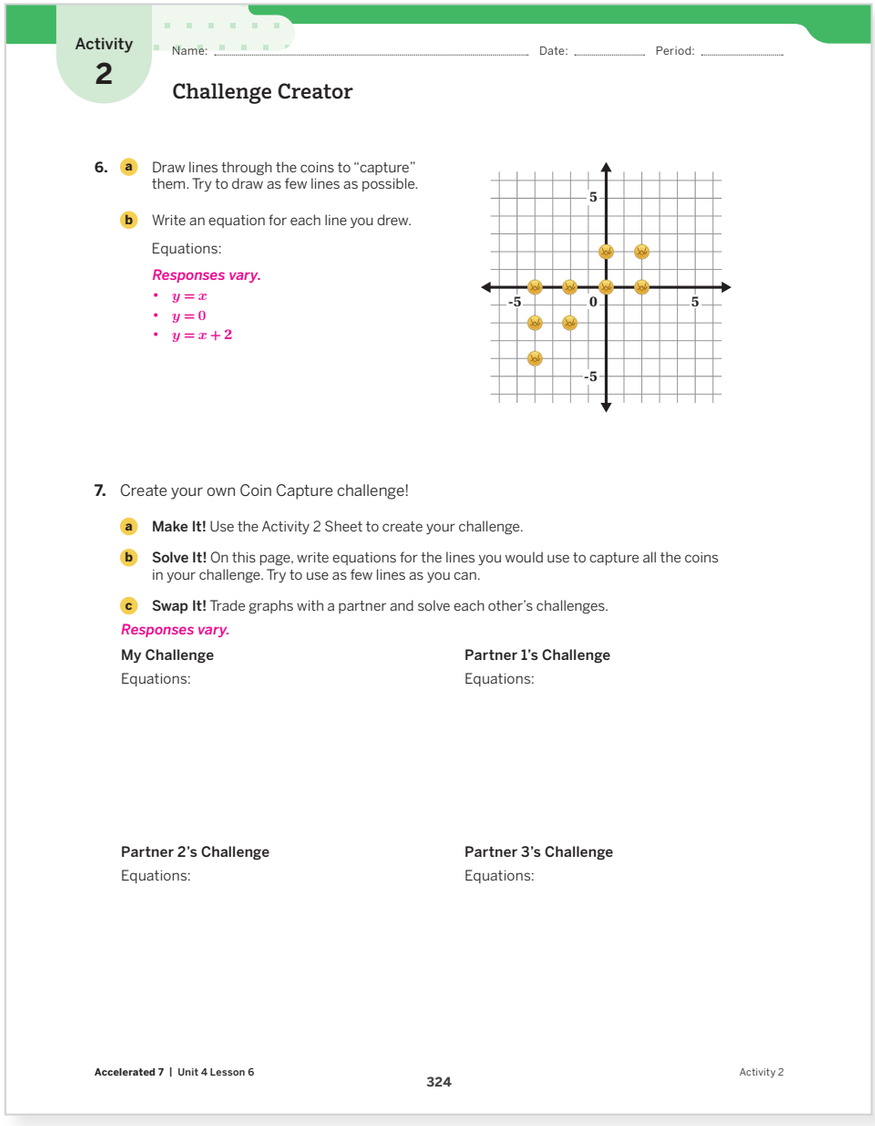
D **Differentiation** *(Problems 6–7)*

Look for students who:	Respond to Student Thinking
Need support writing an equation.	**Support** Invite them to reflect on which points could be captured with a horizontal or vertical line.
Generate equations that only capture one coin at a time.	**Strengthen** Consider asking, "Is there a line that can capture multiple coins? What would that line look like? Its equation?"
Want to explore writing equations of lines further.	**Stretch** Invite them to reflect on ways to use even fewer lines in the challenges they solved. Encourage students to try additional challenges using diagonal lines.

3 Connect

Display different challenges and invite students to make sense of strategies that helped them complete these challenges.

Consider asking:
- "How did you approach solving different challenges?"
- "When did it make sense to use horizontal lines? Vertical lines? Diagonal lines?"



Activity 2

Name: Date: Period:

Challenge Creator

6. a Draw lines through the coins to "capture" them. Try to draw as few lines as possible.

b Write an equation for each line you drew.

Equations:

Responses vary.
- $y = x$
- $y = 0$
- $y = x + 2$

7. Create your own Coin Capture challenge!

a **Make It!** Use the Activity 2 Sheet to create your challenge.

b **Solve It!** On this page, write equations for the lines you would use to capture all the coins in your challenge. Try to use as few lines as you can.

c **Swap It!** Trade graphs with a partner and solve each other's challenges.

Responses vary.

My Challenge
Equations:

Partner 1's Challenge
Equations:

Partner 2's Challenge
Equations:

Partner 3's Challenge
Equations:

Accelerated 7 | Unit 4 Lesson 6 324 Activity 2

Whole Class | 5 min

Synthesis

Purpose: Students synthesize their strategies for identifying equations of different types of lines.

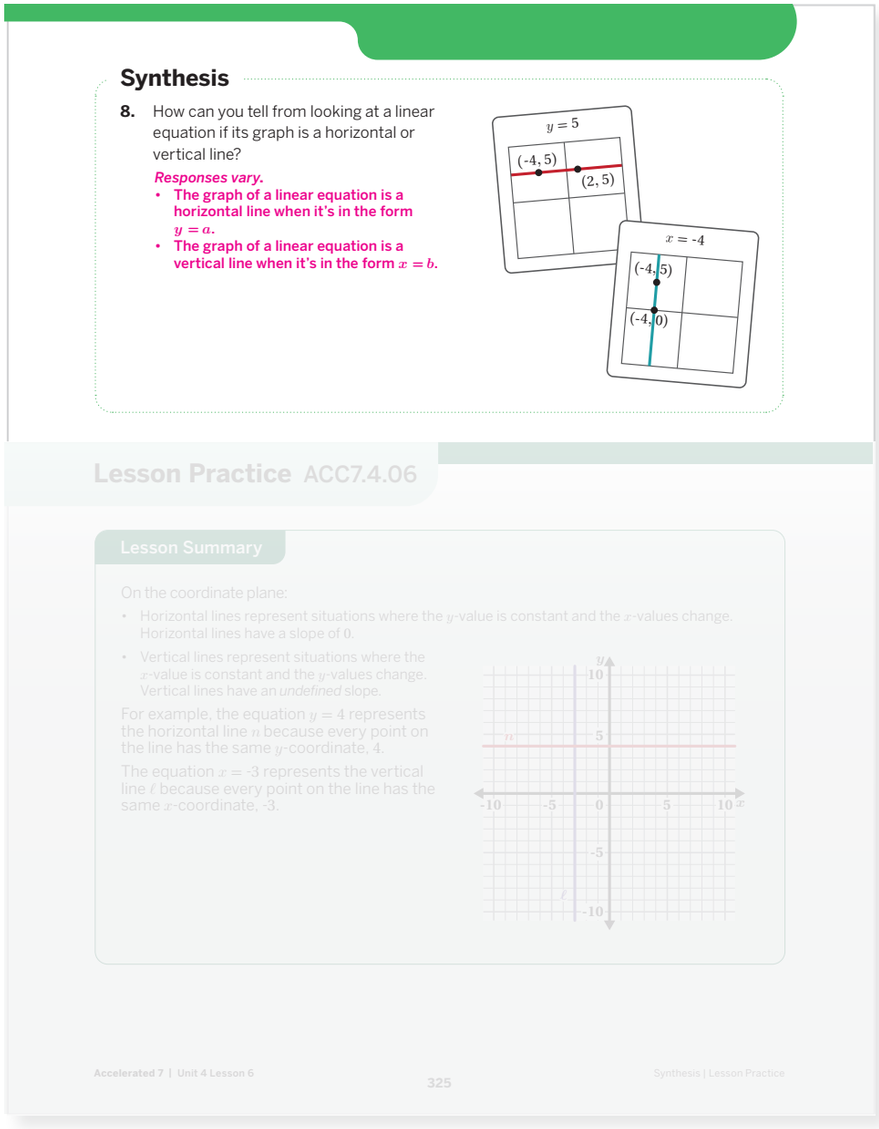
Invite students to respond independently, and then share their thinking with a partner.

Collect and share a variety of ideas, such as:

- Horizontal lines have equations in which y is equal to a constant.
- Vertical lines have equations in which x is equal to a constant.
- Horizontal lines have a slope of 0.
- Vertical lines have an undefined slope, represented by equations in the form $x = b$.

MLR **MLR2: Collect and Display** Use this routine to collect students' examples, as they share during the Connect. Display for students to reference and add to for the rest of the unit.

Note: If time allows, invite students to revise their previous thinking based on the discussion.



Synthesis

8. How can you tell from looking at a linear equation if its graph is a horizontal or vertical line?

Responses vary.

- The graph of a linear equation is a horizontal line when it's in the form $y = a$.
- The graph of a linear equation is a vertical line when it's in the form $x = b$.

Lesson Practice ACC7.4.06

Lesson Summary

On the coordinate plane:

- Horizontal lines represent situations where the y-value is constant and the x-values change. Horizontal lines have a slope of 0.
- Vertical lines represent situations where the x-value is constant and the y-values change. Vertical lines have an *undefined* slope.

For example, the equation $y = 4$ represents the horizontal line n because every point on the line has the same y-coordinate, 4.

The equation $x = -3$ represents the vertical line ℓ because every point on the line has the same x-coordinate, -3.

Accelerated 7 | Unit 4 Lesson 6 325 Synthesis | Lesson Practice

Lesson Takeaway: Horizontal lines represent situations where the y-value is constant and the x-values change. Their equations are written in the form $y = a$. Vertical lines represent situations where the x-value is constant and the y-values change. Their equations are written in the form $x = b$.

Independent | 5 min

Show What You Know

Purpose: Students demonstrate their understanding by writing equations of horizontal and vertical lines.

Today's Goals

1. **Goal:** Write equations of horizontal and vertical lines.
2. **Language Goal:** Justify whether horizontal or vertical lines have a slope of 0. **(Reading and Writing)**

Show What You Know (Assessment Resources)

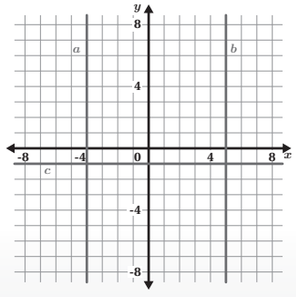
Name: _____ Date: _____ Period: _____

Show What You Know 4.06

Here are three lines on a coordinate plane.

Write an equation for each line.

Line	Equation
a	$x = -4$
b	$x = 5$
c	$y = -1$



D Differentiation Use after Lesson 6

Support

Provide targeted intervention for students by using these resources.

If students need support writing equations for horizontal and vertical lines:

» **Respond to Student Thinking:**

- Review the **Lesson Summary** with students before providing individualized feedback on the relevant **Practice Problems**.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 3 Extension Activities**.

Professional Learning

In this lesson, students learned how to generate equations of horizontal and vertical lines by playing a game of Coin Capture. How did playing a game support students in experiencing mathematics as delightful? When in the lesson did you see evidence of students experiencing math as joy?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

8. How can you tell from looking at a linear equation if its graph is a horizontal or vertical line?

Responses vary.

- The graph of a linear equation is a horizontal line when it's in the form $y = a$.
- The graph of a linear equation is a vertical line when it's in the form $x = b$.

Lesson Practice ACC7.4.06

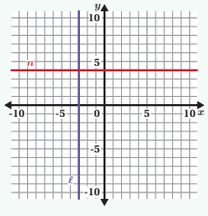
Lesson Summary

On the coordinate plane:

- Horizontal lines represent situations where the y-value is constant and the x-values change. Horizontal lines have a slope of 0.
- Vertical lines represent situations where the x-value is constant and the y-values change. Vertical lines have an *undefined* slope.

For example, the equation $y = 4$ represents the horizontal line n because every point on the line has the same y-coordinate, 4.

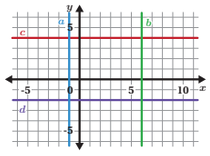
The equation $x = -3$ represents the vertical line ℓ because every point on the line has the same x-coordinate, -3.



Lesson Practice
ACC7.4.06

Name: _____ Date: _____ Period: _____

Problems 1–4: Here are four lines on a coordinate plane. Write an equation for each line.



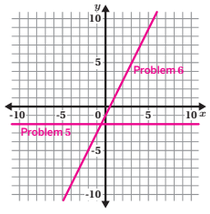
1. line a: $x = -1$

2. line b: $x = 6$

3. line c: $y = 4$

4. line d: $y = -2$

Problems 5–6: Use the coordinate plane to draw lines that meet the given slopes and y-intercepts. Then write an equation for each line.



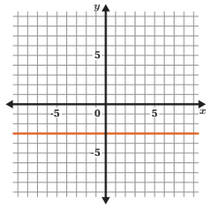
5. Slope: 0
y-intercept: -2

Equation: $y = -2$

6. Slope: 2
y-intercept: -1

Equation: $y = 2x - 1$

7. This graph represents a linear relationship. Choose the equation it matches.



A. $x = 3$

B. $x = -3$

C. $y = 3$

D. $y = -3$

Lesson Practice
ACC7.4.06

Name: _____ Date: _____ Period: _____

8. Write an equation for the line that passes through the points (4, 3) and (4, 15).

$x = 4$

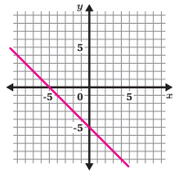
FAST Practice

9. Write an equation for the line that passes through the points (1, -6) and (-6, -6). Record your answer in the space provided.

$y = -6$

Spiral Review

10. Graph the equation $y = -x - 5$.

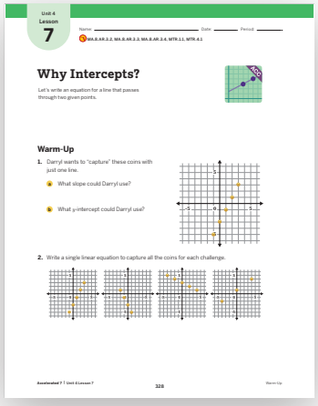


11. Select *all* the pairs of points that have lines passing through them with a slope of $\frac{2}{3}$.

- ☐ **A.** (0, 0) and (2, 3)
- ☑ **B.** (0, 0) and (3, 2)
- ☐ **C.** (1, 5) and (4, 2)
- ☑ **D.** (-2, -2) and (4, 2)
- ☐ **E.** (20, 30) and (-20, -30)

Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1–7	1	MA.8.AR.3.3
	8	2	MA.8.AR.3.2, MA.8.AR.3.3
FAST Practice	**9**	2	MA.8.AR.3.2, MA.8.AR.3.3
Spiral Review			
	10–11	1	MA.8.AR.3.4


Unit 4
Lesson
7

Why Intercepts?

Let's write an equation for a line that passes through two given points.

Warm-Up

1. Darryl wants to "capture" these coins with just one line.
   a. What slope could Darryl use?
   b. What y-intercept could Darryl use?
2. Write a single linear equation to capture all the coins for each challenge.

Why Intercepts?

Equations of All Kinds of Lines

Let's write an equation for a line that passes through two given points.

Learning Outcomes

Today's Goals

1. **Goal:** Write the equation that represents a linear relationship given two points.
2. **Language Goal:** Analyze strategies for determining the y-intercept of a line given two points. **(Reading, Writing, Speaking, and Listening)**

Students revisit calculating slope given two points. They develop strategies to write the equation of a line from two given points when the y-intercept is not visible on a graph. Students actively participate in learning as they apply their understanding in writing equations of lines with positive or negative slopes. **(MTR.1.1)**

Coherence

Prior Learning

In Lesson 5, students analyzed strategies for calculating slope. In Lesson 6, students focused on writing equations of horizontal and vertical lines.

Future Learning

In a future course, students will start exploring linear equations that are not written in $y = mx + b$ form and study the set of solutions to a linear equation — the set of all values of x and y that make the linear equation true.

Connections

- Students compare and contrast samples of student thinking for determining the y-intercept of a line when given two points using different strategies. (mathematical, cross-curricular)
- In an error analysis activity, students identify, explain, and correct student thinking in which they write the equation of a line given two points. (mathematical, cross-curricular)

FL B.E.S.T. Math Benchmarks

Addressing

MA.8.AR.3.3
Given a table, graph or written description of a linear relationship, write an equation in slope-intercept form.

Also Addressing: **MA.8.AR.3.2, MA.8.AR.3.4**

Mathematical Thinking and Reasoning Standards: MTR.1.1, MTR.4.1

ELA Expectations: ELA.K12.EE.1.1, ELA.K12.EE.4.1, ELA.K12.EE.5.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.7.AR.4.4	**MA.912.AR.2.2** **MA.912.AR.2.3**

Lesson at a Glance

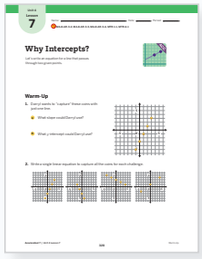
~ 45 min

MA.8.AR.3.2, MA.8.AR.3.3, MA.8.AR.3.4

Warm-Up

Pairs | 5 min

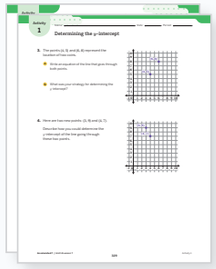
Students write equations of lines given the slope and y-intercept visible on a graph.
Routine: Think-Pair-Share



Activity 1

Pairs | 10 min

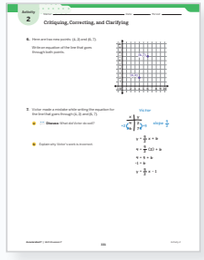
Students generate and analyze strategies for determining the y-intercept of linear equations given only two points. MLR7: Compare and Connect (MTR.1.1)



Activity 2

Pairs | 5 min

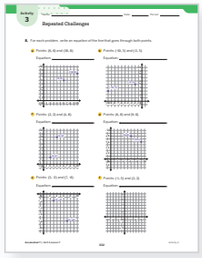
Students analyze student work to support writing an equation when the y-intercept isn't visible on a graph. MLR3: Critique, Correct, Clarify (MTR.4.1)



Activity 3

Independent | 15 min

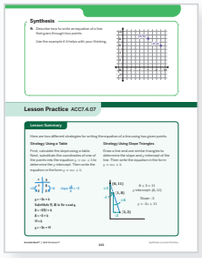
Students complete Repeated Challenges of writing equations of lines given two points.



Synthesis

Whole Class | 5 min

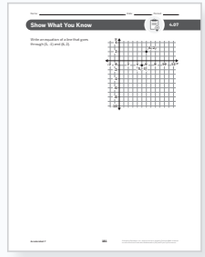
Students synthesize their strategies for writing equations of linear relationships given two points. MLR2: Collect and Display



Show What You Know

Independent | 5 min

Students demonstrate their understanding by writing the equation of a line given two points through which the line passes.



Prep Checklist

This lesson includes:

- Student Edition
- Show What You Know

Pairs | 5 min

Warm-Up

Purpose: Students write equations of lines given the slope and y-intercept visible on a graph.

Short on time? Consider omitting Problem 2.

1 Launch

Direct students' attention to Problem 1 and use the **Think-Pair-Share** routine to support collaborative learning. See the Routine Facilitation Guide for more information.

Invite students to think independently first before sharing with a partner.

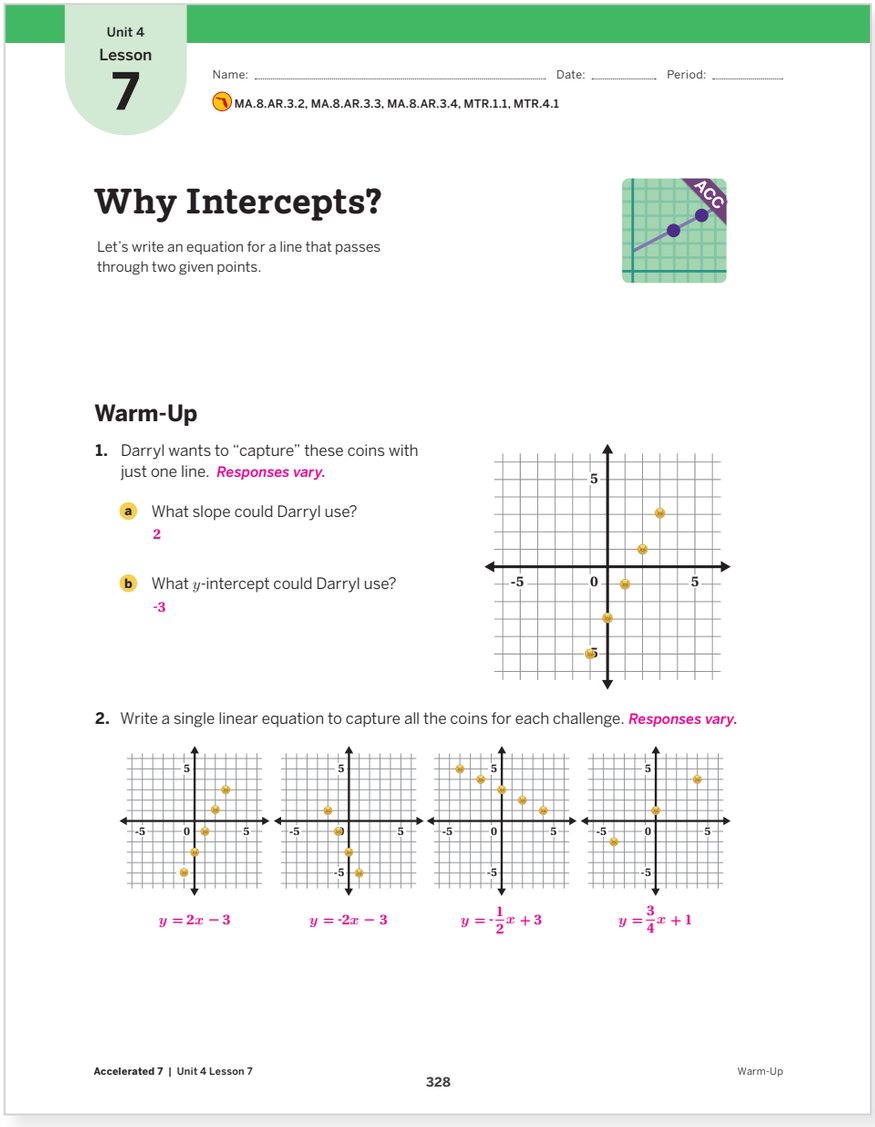
A **Accessibility: Visual-Spatial Processing** Invite students to use their own sketches or drawings to support interpreting visual representations such as the graphs.

2 Connect

Invite students to complete the four challenges. Make straightedges and coloring tools available for students to support interpreting the graphs.

To surface connections, consider asking,

- "What strategy did you use to identify the slope? The y-intercept?"
- "How did your strategies differ for each of the challenges?"



Unit 4
Lesson
7

Name: Date: Period:

MA.8.AR.3.2, MA.8.AR.3.3, MA.8.AR.3.4, MTR.1.1, MTR.4.1

Why Intercepts?

Let's write an equation for a line that passes through two given points.

ACC

Warm-Up

1. Darryl wants to "capture" these coins with just one line. *Responses vary.*

   a. What slope could Darryl use?
   2

   b. What y-intercept could Darryl use?
   -3

2. Write a single linear equation to capture all the coins for each challenge. *Responses vary.*

$y = 2x - 3$ $y = -2x - 3$ $y = -\frac{1}{2}x + 3$ $y = \frac{3}{4}x + 1$

Accelerated 7 | Unit 4 Lesson 7 328 Warm-Up

Pairs | 10 min

Activity 1 Determining the y-intercept

Purpose: Students generate and analyze strategies for determining the y-intercept of linear equations given only two points. **(MTR.1.1)**

1 Launch

Display Problem 3 and invite students to silently predict what the y-intercept will be for the line.

Consider asking, "How did you identify the y-intercept and slope?"

2 Monitor

Consider asking, "How is this problem different from the previous problems?"

Note: If it does not come up naturally, consider sharing that this is the first time where the y-intercept is not visible on the graph.

ELL **English Language Learners** Invite students to share strategies in their primary languages. Encourage all students to use gestures and/or visuals as they share. **(Speaking and Listening)**

Look and listen for student strategies similar to Tariq's and Nia's strategies in Problem 5 to make connections during the Connect. **(MTR.1.1)**

D **Differentiation** *(Problems 3–4)*

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Encourage them to draw a slope triangle between the points and identify any other slope triangles they could draw.
Counted the squares on the grid.	**Strengthen** Invite them to share their strategy with someone using a different strategy.
Identified the slope, but not the y-intercept.	**Strengthen** Consider asking, "How might we adjust or expand the grid to identify the y-intercept?"
Determined the equation correctly through a strategy of their choosing.	**Stretch** Encourage these students to use another strategy to determine the equation. Invite these students to share their ideas during the Connect.

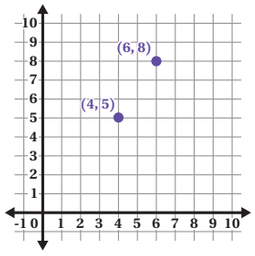
Activity 1 continued >

Activity 1

Name: _____ Date: _____ Period: _____

Determining the y-intercept

3. The points $(4, 5)$ and $(6, 8)$ represent the location of two coins.



a. Write an equation of the line that goes through both points.

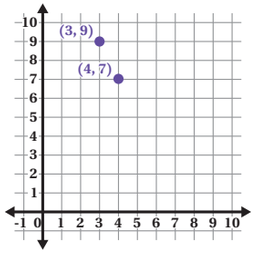
$y = \frac{3}{2}x - 1$ (or equivalent)

b. What was your strategy for determining the y-intercept?

Responses vary. I drew a line through the two points and saw that it crossed the y-axis at (0, -1).

4. Here are two new points: $(3, 9)$ and $(4, 7)$.

Describe how you could determine the y-intercept of the line going through these two points.



Responses vary.

- Draw a line through the two points and extend the grid.
- Use similar triangles to build up to the y-intercept.
- Write an equation using the slope, substitute in the x- and y-values of a point, then solve for the y-intercept.

Accelerated 7 | Unit 4 Lesson 7 329 Activity 1

Pairs | 10 min

Activity 1 Determining the y-intercept (continued)

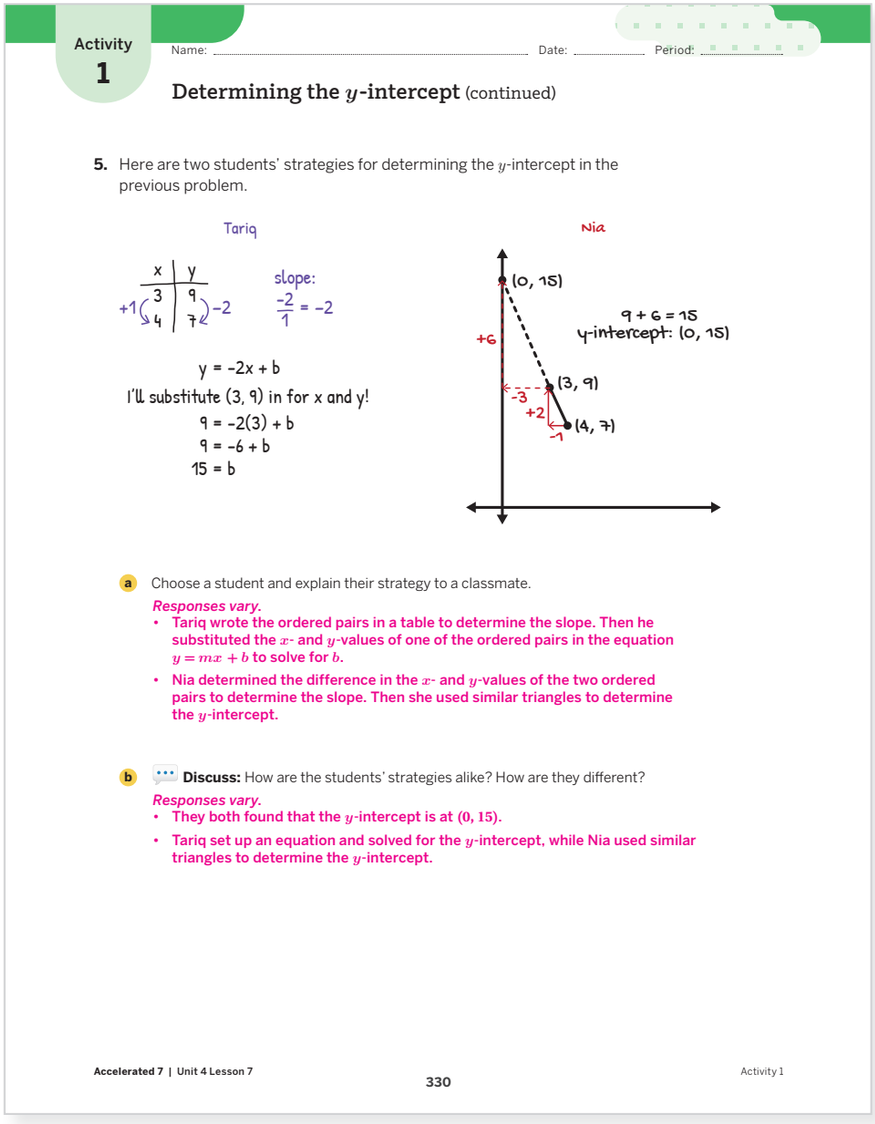
Purpose: Students generate and analyze strategies for determining the y-intercept of linear equations given only two points. **(MTR.1.1)**

3 Connect

Direct students' attention to the problem with Tariq's and Nia's work.

MLR **MLR7: Compare and Connect** Invite students to analyze and compare each strategy, and then connect to their own strategy. See the Routine Facilitation Guide for more information.

To surface the Key Takeaway, consider asking:
- "How did they calculate the y-intercept in each strategy?"
- "What was helpful about each strategy?"
- "When might you use each strategy?"



Activity 1

Name: Date: Period:

Determining the y-intercept (continued)

5. Here are two students' strategies for determining the y-intercept in the previous problem.

Tariq

x	y
3	9
4	7

+1, −2

slope: $\frac{-2}{1} = -2$

y = −2x + b
I'll substitute (3, 9) in for x and y!
9 = −2(3) + b
9 = −6 + b
15 = b

Nia

a. Choose a student and explain their strategy to a classmate.

Responses vary.
- Tariq wrote the ordered pairs in a table to determine the slope. Then he substituted the x- and y-values of one of the ordered pairs in the equation $y = mx + b$ to solve for b.
- Nia determined the difference in the x- and y-values of the two ordered pairs to determine the slope. Then she used similar triangles to determine the y-intercept.

b. **Discuss:** How are the students' strategies alike? How are they different?

Responses vary.
- They both found that the y-intercept is at (0, 15).
- Tariq set up an equation and solved for the y-intercept, while Nia used similar triangles to determine the y-intercept.

Accelerated 7 | Unit 4 Lesson 7 330 Activity 1

Key Takeaway: To determine the y-intercept of a line when it's not visible on a graph, you can use slope triangles to extend the line from given points to the y-axis or write an equation and solve for the y-intercept.

Pairs | 5 min

Activity 2 Critiquing, Correcting, and Clarifying

Purpose: Students analyze student work to support writing an equation when the y-intercept isn't visible on a graph. **(MTR.4.1)**

Short on time? Consider omitting Problem 7.

1 Launch

To surface connections, consider asking, "Which strategy might you choose to write this equation? Why?"

Invite students to discuss with their partner before choosing and using a strategy for Problem 6.

2 Monitor

Invite students to make connections out loud between Victor's work in Problem 7 and the graph before writing an explanation.

ELL **English Language Learners** Provide a word bank to support students in expressing their ideas, with words such as: *slope, y-intercept, substitute, solve, slope triangle*, etc. **(Writing and Speaking)**

Listen for and collect ideas to share during the Connect.

D **Differentiation** *(Problem 7)*

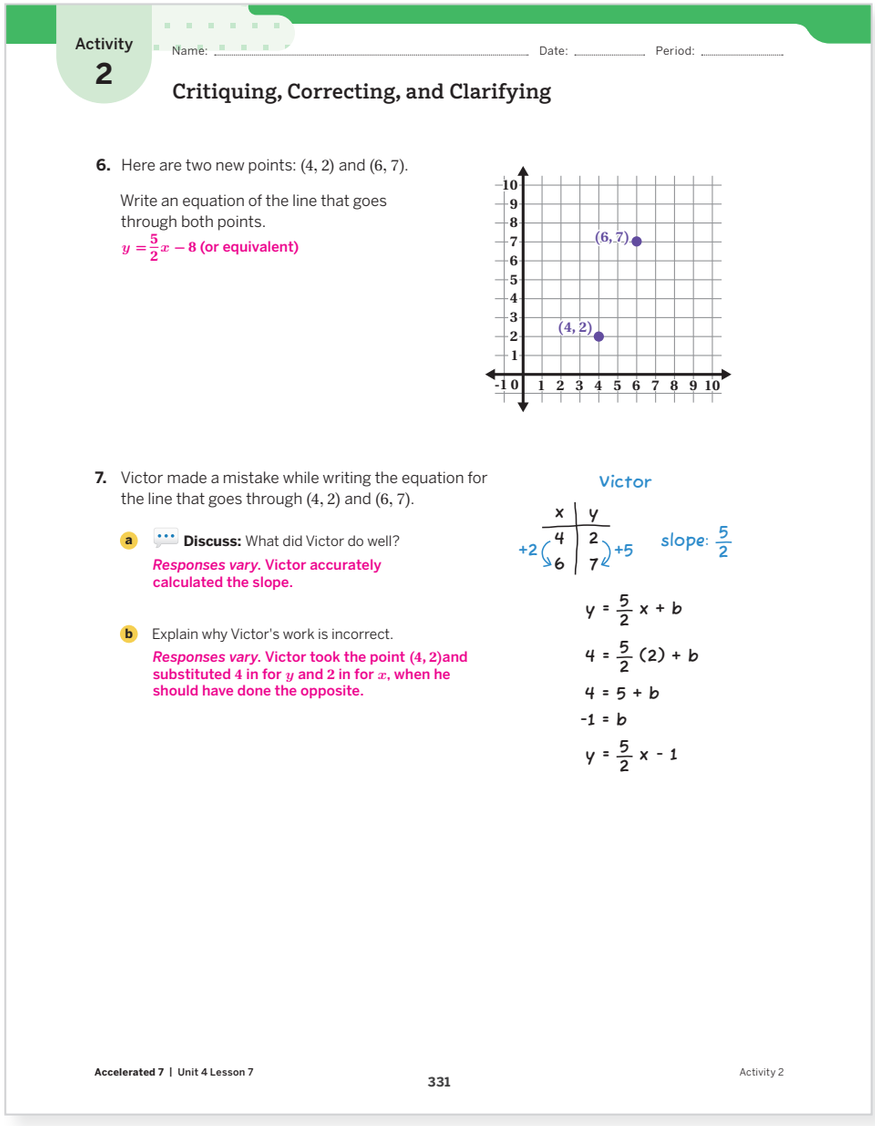
Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Consider asking, "What do you think Victor did first?"
Recognize that Victor's slope work is correct.	**Strengthen** Invite them to demonstrate that the slope is $\frac{5}{2}$ on the graph.
Think that Victor's substitution work is correct.	**Strengthen** Consider asking, "How could you use the graph to check Victor's y-intercept?"
Identify the correct slope work and recognize that the substitution for the x- and y-values were switched.	**Stretch** Encourage students to also use the equation to identify other points on the line.

3 Connect

Share a variety of student responses.

MLR **MLR3: Critique, Correct, Clarify** Use this routine to help students describe strengths, errors, and ambiguities in Victor's work. As a class:
- Discuss what is *correct* about his response.
- Discuss what is *incorrect* or *unclear*.
- Create a *correct* and *clear* solution. **(MTR.4.1)**

To surface connections, consider asking:
- "Which strategy did Victor try to use?"
- "How could Victor check his work with a different strategy?"



Activity 2

Name: Date: Period:

Critiquing, Correcting, and Clarifying

6. Here are two new points: $(4, 2)$ and $(6, 7)$.

Write an equation of the line that goes through both points.

$y = \frac{5}{2}x - 8$ (or equivalent)

7. Victor made a mistake while writing the equation for the line that goes through $(4, 2)$ and $(6, 7)$.

a. **Discuss:** What did Victor do well?

Responses vary. Victor accurately calculated the slope.

b. Explain why Victor's work is incorrect.

Responses vary. Victor took the point $(4, 2)$ and substituted 4 in for y and 2 in for x, when he should have done the opposite.

Victor

x	y
4	2
6	7

+2 +5 slope: $\frac{5}{2}$

$y = \frac{5}{2}x + b$

$4 = \frac{5}{2}(2) + b$

$4 = 5 + b$

$-1 = b$

$y = \frac{5}{2}x - 1$

Accelerated 7 | Unit 4 Lesson 7 331 Activity 2

Independent | 15 min

Activity 3 Repeated Challenges

Purpose: Students complete Repeated Challenges of writing equations of lines given two points.

Short on time? Consider completing fewer challenges or assigning Activity 3 as additional practice.

1 Launch

Consider sharing that Repeated Challenges is an opportunity to prioritize thinking deeply rather than completing as many problems as possible.

2 Monitor

Note: Review students' work to see how many challenges each student has completed correctly. It may be helpful for them to check their work with a partner.

A **Accessibility: Memory and Attention** For students who would benefit from having more time to think deeply, invite them to complete fewer challenges. To access the discussion in the Connect, it will be helpful for all students to interact with Challenge 3.

Select and sequence a variety of strategies and reflections used to write an equation.

D **Differentiation**

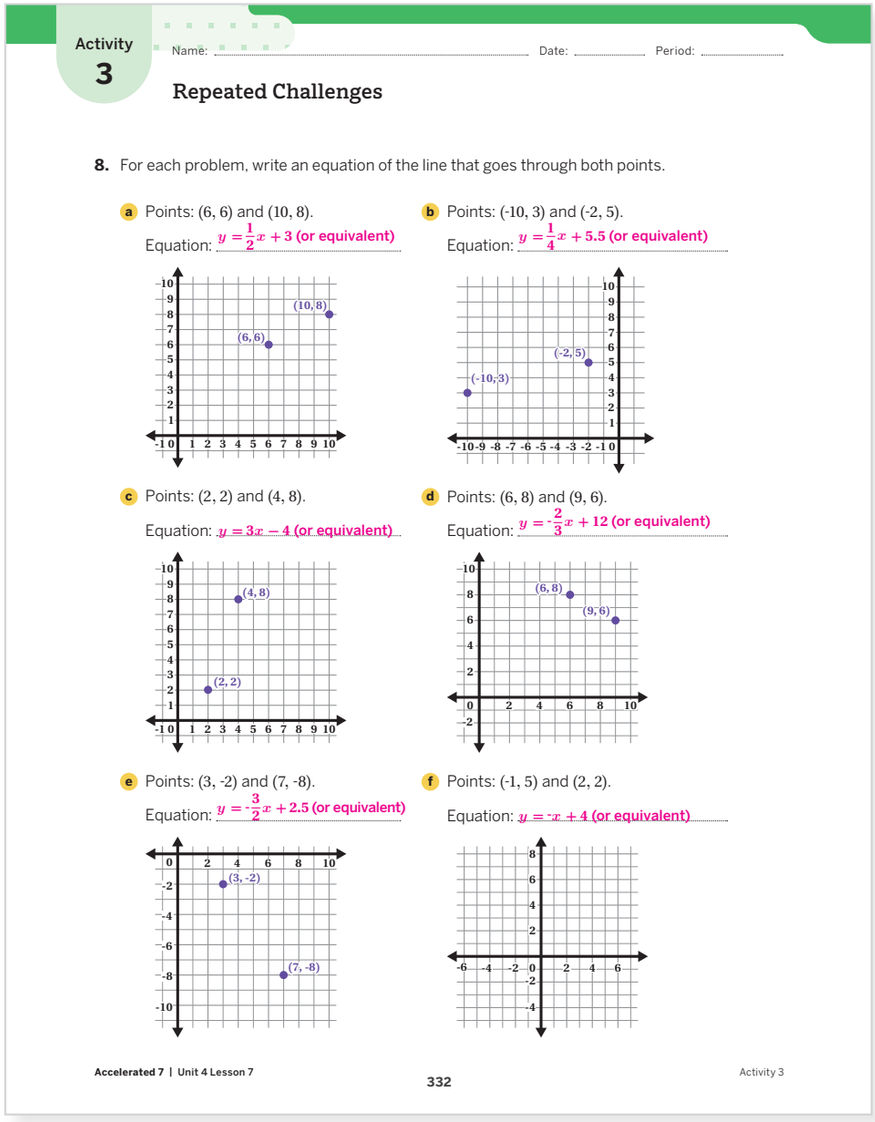
Look for students who:	Respond to Student Thinking
Identify a slope with the opposite sign.	**Support** Invite them to draw slope triangles between the points and identify if the line slopes up or down.
Draw slope triangles that do not reach the y-intercept.	**Strengthen** Celebrate them for the strategy they have used so far. Consider asking, "How can you extend the graph to see the y-intercept?"
Use a variety of strategies to accurately determine equations of linear relationships.	**Stretch** Capture these strategies to share during the Connect.

3 Connect

Share strategies used to complete the Repeated Challenges, particularly for Problem 8c.

To surface the Key Takeaway, consider asking:

- "What information did you need in order to write an equation?"
- "If you only have two points, how can you determine the equation?"



Activity 3

Name: Date: Period:

Repeated Challenges

8. For each problem, write an equation of the line that goes through both points.

a Points: (6, 6) and (10, 8).
Equation: $y = \frac{1}{2}x + 3$ (or equivalent)

b Points: (-10, 3) and (-2, 5).
Equation: $y = \frac{1}{4}x + 5.5$ (or equivalent)

c Points: (2, 2) and (4, 8).
Equation: $y = 3x - 4$ (or equivalent)

d Points: (6, 8) and (9, 6).
Equation: $y = -\frac{2}{3}x + 12$ (or equivalent)

e Points: (3, -2) and (7, -8).
Equation: $y = -\frac{3}{2}x + 2.5$ (or equivalent)

f Points: (-1, 5) and (2, 2).
Equation: $y = -x + 4$ (or equivalent)

Accelerated 7 | Unit 4 Lesson 7 332 Activity 3

Key Takeaway: We only need two points on a line in order to write the equation of a linear relationship.

Whole Class | 5 min

Synthesis

Purpose: Students synthesize their strategies for writing equations of linear relationships given two points.

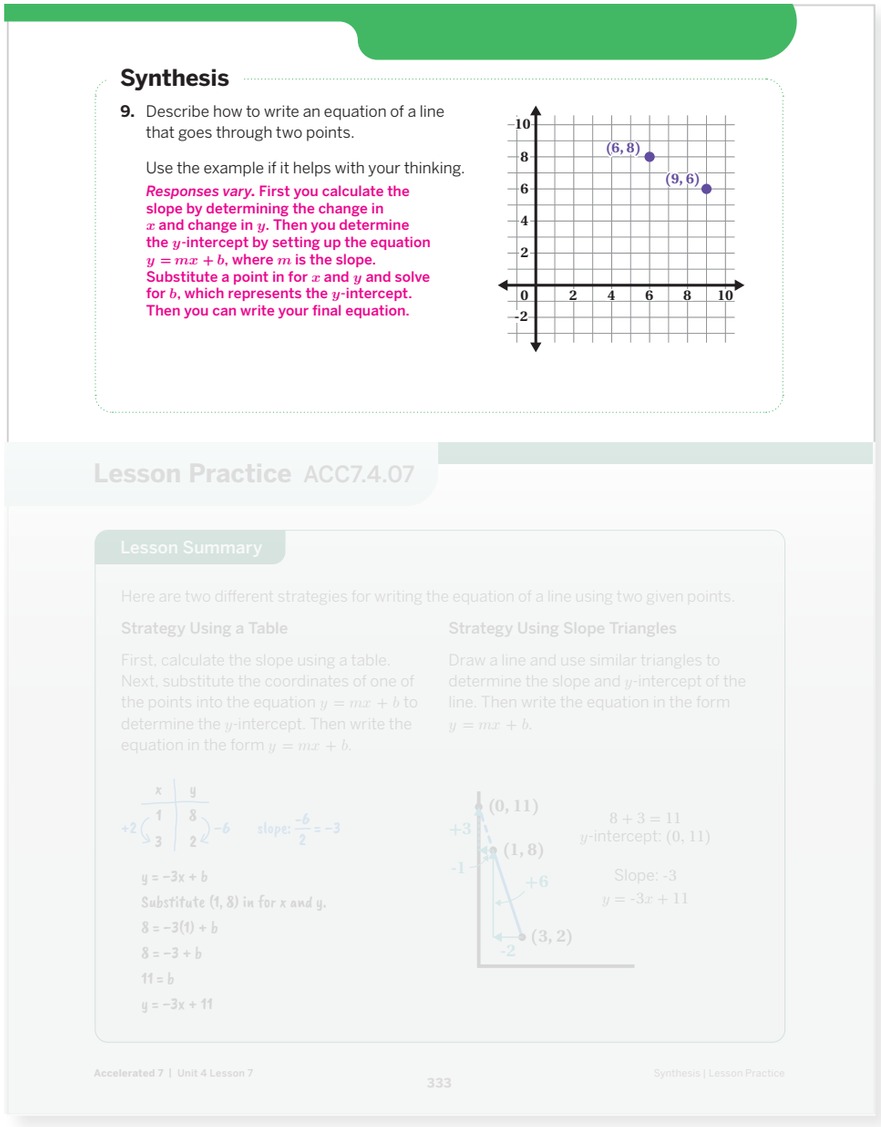
Invite students to respond independently, after discussing each question with their partner.

Capture and share a variety of ideas, such as:

- Two points are needed to write an equation of a linear relationship.
- Determine the slope using slope triangles or by analyzing constant rate of change using a table.
- Determine the y-intercept from a graph by extending slope triangles between two points.
- Solve for the y-intercept by substituting coordinates into $y = mx + b$ and solve for the b value.

MLR **MLR2: Collect and Display** Use this routine to formalize the strategies for writing an equation between two points using students' language. Display any new language, examples, or diagrams for students to reference later.

Note: If time allows, invite students to share with a partner what makes sense about each strategy and what questions they still have.



Synthesis

9. Describe how to write an equation of a line that goes through two points.

Use the example if it helps with your thinking.

Responses vary. First you calculate the slope by determining the change in x and change in y. Then you determine the y-intercept by setting up the equation $y = mx + b$, where m is the slope. Substitute a point in for x and y and solve for b, which represents the y-intercept. Then you can write your final equation.

Lesson Practice ACC7.4.07

Lesson Summary

Here are two different strategies for writing the equation of a line using two given points.

Strategy Using a Table

First, calculate the slope using a table. Next, substitute the coordinates of one of the points into the equation $y = mx + b$ to determine the y-intercept. Then write the equation in the form $y = mx + b$.

Strategy Using Slope Triangles

Draw a line and use similar triangles to determine the slope and y-intercept of the line. Then write the equation in the form $y = mx + b$.

Accelerated 7 | Unit 4 Lesson 7 333 Synthesis | Lesson Practice

Lesson Takeaway: When given two points, you can write an equation for the line through them in the form $y = mx + b$. There are multiple strategies that can be used to determine the slope, m, and the y-intercept, b.

Independent | 5 min

Show What You Know

Purpose: Students demonstrate their understanding by writing the equation of a line given two points through which the line passes.

Today's Goals

1. **Goal:** Write the equation that represents a linear relationship given two points.
2. **Language Goal:** Analyze strategies for determining the y-intercept of a line given two points. **(Reading, Writing, Speaking, and Listening)**

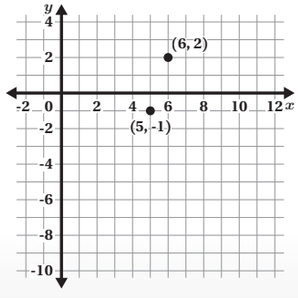
Show What You Know (Assessment Resources)

Name: _____ Date: _____ Period: _____

Show What You Know 4.07

Write an equation of a line that goes through (5, -1) and (6, 2).

$y = 3x - 16$



D Differentiation Use after Lesson 7

Support

Provide targeted intervention for students by using these resources.

If students need support writing equations of linear relationships:

» **Respond to Student Thinking:**

- Review **Activity 1** (especially Tariq's and Nia's strategies) before providing individualized feedback on the relevant **Practice Problems**.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 3 Extension Activities**.

Professional Learning

Reflect on how students analyzed student work and wrote equations of linear relationships today. How did they talk about mistakes? Were they able to notice the correct and incorrect parts of Victor's work? How can you support students in valuing learning from both correct and incorrect thinking?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

9. Describe how to write an equation of a line that goes through two points.

Use the example if it helps with your thinking.

Responses vary. First you calculate the slope by determining the change in x and change in y. Then you determine the y-intercept by setting up the equation $y = mx + b$, where m is the slope. Substitute a point in for x and y and solve for b, which represents the y-intercept. Then you can write your final equation.

Lesson Practice ACC7.4.07

Lesson Summary

Here are two different strategies for writing the equation of a line using two given points.

Strategy Using a Table

First, calculate the slope using a table. Next, substitute the coordinates of one of the points into the equation $y = mx + b$ to determine the y-intercept. Then write the equation in the form $y = mx + b$.

x	y
1	8
3	2

+2, −6 slope: $\frac{-6}{2} = -3$

$y = -3x + b$

Substitute (1, 8) in for x and y.

$8 = -3(1) + b$

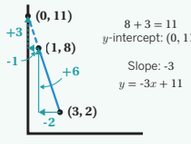
$8 = -3 + b$

$11 = b$

$y = -3x + 11$

Strategy Using Slope Triangles

Draw a line and use similar triangles to determine the slope and y-intercept of the line. Then write the equation in the form $y = mx + b$.



$8 + 3 = 11$

y-intercept: (0, 11)

Slope: -3

$y = -3x + 11$

Lesson Practice ACC7.4.07

Name: ..... Date: ..... Period: .....

1. Bao and Maia are each writing an equation of the line that passes through the points (2, 9) and (12, 14). They both calculate the slope as $\frac{1}{2}$.

- Bao substitutes the point (2, 9) to determine the y-intercept.
- Maia substitutes the point (12, 14) to determine the y-intercept.

Here is each student's work and their solutions for the y-intercept. Determine any mistakes in each student's work and explain how you would fix them.

Bao	Maia
$y = \frac{1}{2}x + b$	$y = \frac{1}{2}x + b$
$9 = \frac{1}{2}(2) + b$	$12 = \frac{1}{2}(14) + b$
$9 = 1 + b$	$12 = 7 + b$
$b = 8$	$b = 5$

Responses vary. Bao's work is accurate. Maia substituted the x- and y-values from the point (12, 14) incorrectly. She reversed the x- and y-coordinates.

Her calculations should be:

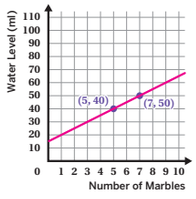
$y = \frac{1}{2}x + b$

$14 = \frac{1}{2}(12) + b$

$14 = 6 + b$

$8 = b$

2. Chloe added marbles to a container of water. When she added 5 marbles, the water level was 40 millimeters. When she added 7 marbles, the water level was 50 millimeters. Write an equation for the water level, y, after x marbles are added. Show or explain your thinking.



$y = 5x + 15$. *Explanations vary.*

Slope: 5

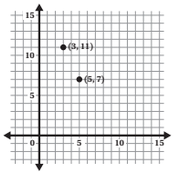
$40 = 5(5) + b$

$40 = 25 + b$

$b = 15$

$y = 5x + 15$

3. Here is a graph showing the points (3, 11) and (5, 7). What is the y-intercept of the line that passes through these points?



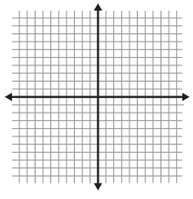
(0, 17)

Lesson Practice ACC7.4.07

Name: ..... Date: ..... Period: .....

Problems 4–5: Write the equation of the line that passes through each pair of points. Show your work, and use the coordinate plane if it helps with your thinking. *Work varies.*

4. (2, 14) and (6, 26) $y = 3x + 8$



Slope:

x	y
2	14
6	26

+4, +12 $\frac{12}{4} = 3$

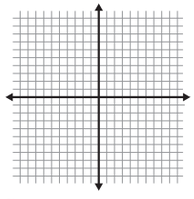
y-intercept:

$14 = 3(2) + b$

$14 = 6 + b$

$b = 8$

5. (-5, 7) and (1, 1) $y = -x + 2$



Slope:

x	y
-5	7
1	1

+6, -6 $\frac{-6}{6} = -1$

y-intercept:

$1 = -1(1) + b$

$1 = -1 + b$

$b = 2$

FAST Practice

6. A line has a slope of 2 and passes through the point (-6, 1). Which is the equation of that line?

A. $y = 2x + 4$ B. $y = 2x + 13$ C. $y = 2x + 8$ D. $y = 2x - 11$

Spiral Review

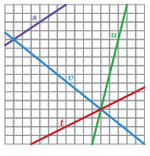
Problems 7–10: Determine if the slope of each line is positive or negative.

7. Line s
Positive

8. Line t
Positive

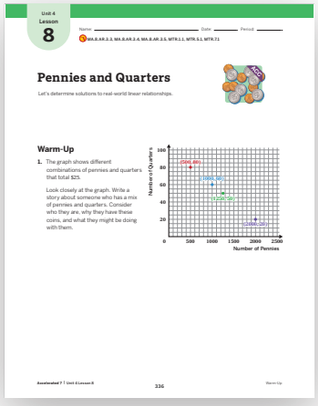
9. Line u
Positive

10. Line v
Negative



Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1–5	2	MA.8.AR.3.2, MA.8.AR.3.3
FAST Practice	6	2	MA.8.AR.3.2, MA.8.AR.3.3
Spiral Review			
	7–10	1	MA.8.AR.3.2



Pennies and Quarters

Using Linear Relationships to Solve Problems

Let's determine solutions to real-world linear relationships.

Learning Outcomes

Today's Goals

1. **Goal:** Connect situations with tables, equations, graphs, and solutions.
2. **Language Goal:** Describe how to determine solutions given a real-world situation. **(Speaking and Listening)**

Students use all of the representations of linear relationships they have studied to make sense of combinations of pennies and quarters worth a fixed amount. They write an equation that represents all possible combinations and interpret the real-world meaning of points that satisfy (and do not satisfy) that equation. Students also connect situations with representations of linear relationships of the form $y = mx + b$.

Coherence

Prior Learning

In Lesson 7, students explored linear equations in the form $y = mx + b$. They explored ways to find the slope and y-intercept of a line given two or more points on the line.

Future Learning

In Unit 5, students will use their knowledge of linear relationships to solve systems of linear equations algebraically and graphically.

Connections

- Students will use linear relationships to determine possible combinations of solutions that meet a specific constraint in real world contexts such as budgeting money, ordering food, and music. (real world)

FL B.E.S.T. Math Benchmarks

Addressing

MA.8.AR.3.3
Given a table, graph or written description of a linear relationship, write an equation in slope-intercept form.

MA.8.AR.3.5
Given a real-world context, determine and interpret the slope and y-intercept of a two-variable linear equation from a written description, a table, a graph or an equation in slope-intercept form.

Also Addressing: **MA.8.AR.3.4**

Mathematical Thinking and Reasoning Standards: MTR.1.1, MTR.5.1, MTR.7.1

ELA Expectations: ELA.K12.EE.1.1, ELA.K12.EE.2.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.7.AR.4.4	MA.912.AR.2.2, MA.912.AR.2.3

Vocabulary

New Vocabulary

solution to an equation

Lesson at a Glance

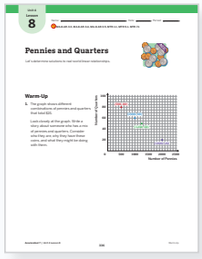
~ 45 min

MA.8.AR.3.3, MA.8.AR.3.4, MA.8.AR.3.5

Warm-Up

Independent | 5 min

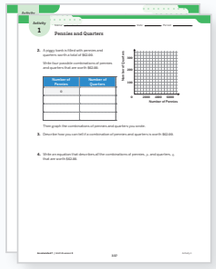
Students are introduced to the context they will explore in Activity 1. **Routine:** Tell a Story



Activity 1

Pairs | 20 min

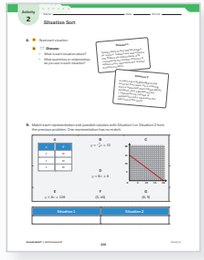
Students model a situation involving pennies and quarters with a table, equation, and graph, then use those to identify **solutions to the equation**. **(MTR.7.1)**



Activity 2

Pairs | 10 min

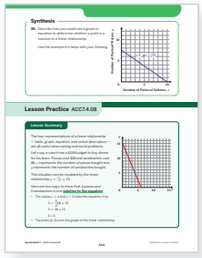
Students use structure to connect situations with the tables, equations, graphs, and ordered pairs that represent them. MLR6: Three Reads **(MTR.5.1)**



Synthesis

Whole Class | 5 min

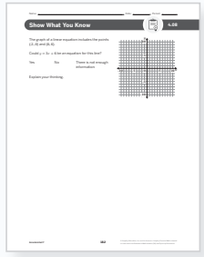
Students synthesize their understanding of how to determine whether a point is a solution from a context.



Show What You Know

Independent | 5 min

Students demonstrate their understanding by determining whether ordered pairs are solutions to a linear equation of the form $y = mx + b$.



Prep Checklist

This lesson includes:

- Student Edition
- Show What You Know
- Calculators (optional)
- U.S. coins, photos of U.S. coins, or coin manipulatives (as needed)

Independent | 5 min

Warm-Up

Purpose: Students are introduced to the context they will explore in Activity 1.

1 Launch

Consider using the Tell a Story routine to help students make sense of the graph. Invite students to write a story about the graph using their unique experiences, perspectives, and creativity.

Encourage creativity and remind students there is no right or wrong story. To encourage movement, invite students to work with a partner in a different location.

ELL **English Language Learners** Invite students to share what they know about the topic of saving money in piggy banks to increase access to the task. **(Reading and Listening)**

2 Connect

Invite students to share the stories that they wrote.

Consider asking:

- "What parts of your story connect to the graph?"
- "How does the graph help tell your story?"

Discuss how there are many stories that can be told about the graph. Consider sharing that the focus of today's lesson will be on how many pennies and quarters are in a piggy bank.

Unit 4
Lesson
8

Name: ..... Date: ..... Period: .....

MA.8.AR.3.3, MA.8.AR.3.4, MA.8.AR.3.5, MTR.1.1, MTR.5.1, MTR.7.1

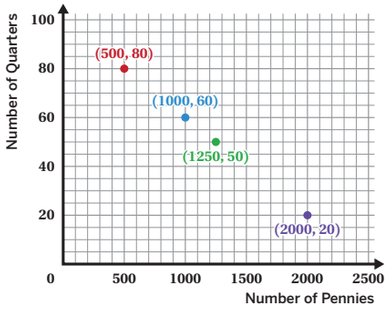
Pennies and Quarters

Let's determine solutions to real-world linear relationships.

Warm-Up

1. The graph shows different combinations of pennies and quarters that total $25.

   Look closely at the graph. Write a story about someone who has a mix of pennies and quarters. Consider who they are, why they have these coins, and what they might be doing with them.



Responses vary. Alex is saving up for a gift for his best friend's birthday. He opens his piggy bank, which is filled with pennies and quarters. Alex carefully sorts the coins and starts counting. He has 500 pennies and 80 quarters, which is equal to $25. This is enough to buy the art kit his friend always wanted.

Accelerated 7 | Unit 4 Lesson 8 336 Warm-Up

Pairs | 20 min

Activity 1 Pennies and Quarters

Purpose: Students model a situation involving pennies and quarters with a table, equation, and graph, then use those to identify **solutions to the equation**. **(MTR.7.1)**

1 Launch

To surface prior knowledge, consider asking:

- "What is the value of a penny? A quarter?"
- "What is a combination of pennies and quarters that are worth a total of $1?"

ELL **English Language Learners** It may be helpful to have examples of U.S. coins to support students who are new to this country's monetary system. **(Reading, Writing, Speaking, and Listening)**

D **Differentiation: (Support)** Invite students who need support getting started to use the scale of the graph to help them choose a number of pennies and quarters, check its value, then revise.

2 Monitor

Look for descriptions that connect to possible equations to highlight during the Pause.

Encourage students to read others' responses and decide if others' descriptions were alike or different from their own.

Capture equations to share during the Pause.

D **Differentiation** *(Problem 4)*

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Encourage them to take each piece of their description from the previous page and write it as part of an equation.
Write an equation with dollar values instead of variables. (e.g., $42 + 20 = 62$)	**Strengthen** Consider asking, "What values did you multiply to get each dollar value? Which one varies and which stays the same?"
Write an equation that models the situation.	**Stretch** Invite them to write an equivalent equation and explain what it means in the situation.

Pause to discuss equations students created one-at-a-time, recording them for students to refer to throughout the Activity. **(MTR.7.1)**

Activity 1 continued >

Activity 1

Name: Date: Period:

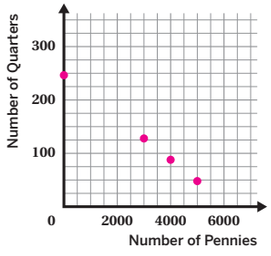
Pennies and Quarters

2. A piggy bank is filled with pennies and quarters worth a total of $62.00.

Write *four* possible combinations of pennies and quarters that are worth $62.00.

Responses vary.

Number of Pennies	Number of Quarters
0	248
3000	128
4000	88
5000	48



Then graph the combinations of pennies and quarters you wrote.

3. Describe how you can tell if a combination of pennies and quarters is worth $62.00.

Responses vary. You can multiply the number of pennies by 0.01, multiply the number of quarters by 0.25, add the values together, and then see if it's equal to 62.

4. Write an equation that describes *all* the combinations of pennies, p, and quarters, q, that are worth $62.00.

$q = -0.04p + 248$ (or equivalent)

Accelerated 7 | Unit 4 Lesson 8 337 Activity 1

Pairs | 20 min

Activity 1 Pennies and Quarters (continued)

Purpose: Students model a situation involving pennies and quarters with a table, equation, and graph, then use those to identify **solutions to the equation**. **(MTR.7.1)**

2 Monitor

To support connections, consider pausing the activity in Problem 5 and asking, "What does each part of Ava's equation represent in this situation?"

For Problem 6, **Look for** a variety of strategies and sketches students use to determine the number of pennies. Consider asking, "What does it mean to be a **solution to an equation**?

D Differentiation *(Problem 6)*

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Consider asking, "Could there be 100 pennies? Why or why not?"
Aren't sure their answer is correct.	**Strengthen** Invite them to test the values they found in the equation. Consider asking, "What might have been a more efficient strategy?"
Use strategies such as: • Substituting 200 for q and solving for p. • Finding where the line has a second coordinate of 200.	**Strengthen** Invite them to think about how they would describe their strategy. Consider asking, "How could you use the graph to check your thinking? The equation?"

D Differentiation: You're invited to explore more. (Stretch) Invite students who would like to explore linear relationships further to complete this optional task. Encourage them to discuss their thinking with a partner.

3 Connect

Invite students to share their strategies for determining the number of pennies, including using an equation and a graph.

To surface the Key Takeaway, consider asking:
- "How can you use the graph or the equation to decide how many pennies is a solution with 200 quarters?"
- "Is the point (1200, 200) a solution in this linear relationship? Why or why not?"

Activity 1

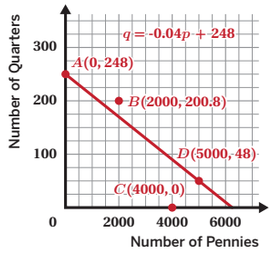
Name: _____ Date: _____ Period: _____

Pennies and Quarters (continued)

5. Ava wrote the equation $q = -0.04p + 248$ to represent all the combinations of pennies, p, and quarters, q, that are worth \$62.00.

Select *all* the points that are **solutions to the equation**.

- ☑ **A.** Point A
- ☐ **B.** Point B
- ☐ **C.** Point C
- ☑ **D.** Point D



6. If there are 200 quarters, how many pennies do you need for a total of \$62.00?

Use the graph if it helps with your thinking.

1200 pennies

You're invited to explore more.

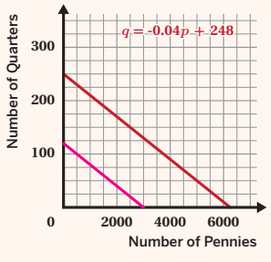
7. Here is a line and an equation describing all the combinations of pennies and quarters that are worth \$62.00.

A different pile of pennies and quarters is worth \$30.

a. Draw the graph of this situation on the same plane.

b. Describe the strategy you used.

Responses vary. **I graphed both intercepts and then drew a line connecting the two points.**



Accelerated 7 | Unit 4 Lesson 8 338 Activity 1

Key Takeaway: Solutions to linear equations can help answer questions about a situation.

Pairs | 10 min

Activity 2 Situation Sort

Purpose: Students use structure to connect situations with the tables, equations, graphs, and ordered pairs that represent them. **(MTR.5.1)**

Short on time? Consider completing Problem 8 or the entire activity as a class, or assigning at a later time.

1 Launch

MLR **MLR6: Three Reads** Use this routine to support students in making sense of the situations. Invite students to read each situation to themselves multiple times, first focusing on what the situation is about, then what the quantities are. See the Routine Facilitation Guide for more information. **(MTR.1.1)**

2 Monitor

Encourage students to share their reasoning with a partner and work together to reach an agreement about how to sort the representations.

Listen for students who make connections between the structure of each equation and the situation it represents. Students may interpret the ordered pairs with the given context to see if it makes sense in the situation. **(MTR.5.1)**

Note: If time allows, invite pairs to compare their groupings, justify their thinking, and make revisions based on their conversation.

A **Accessibility: Executive Function** Chunk this activity into more manageable parts by inviting students to choose one situation and determine whether each card could match.

ELL **English Language Learners** Provide a word bank to support students in expressing their ideas with words such as: *solution*, x-*value*, y-*value*, *equation*, *slope*, *intercept*, etc. **(Speaking)**

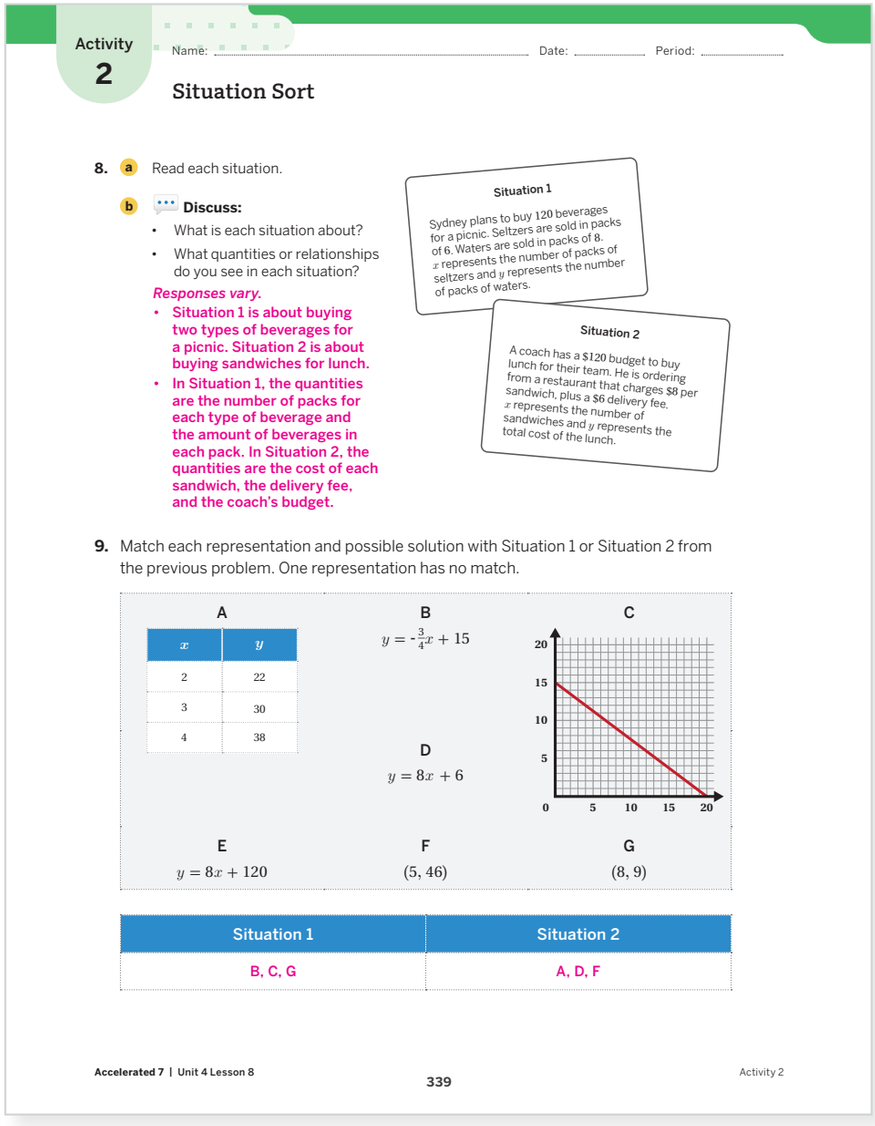
D **Differentiation: (Stretch)** Invite students to write another possible solution for each situation and explain what it means in context.

3 Connect

Invite students to share which representations and solutions match each situation in Problem 9.

Consider asking:

- "How did you decide which equation matched each situation?"
- "What do the values in a solution represent in the situation?"



Activity 2

Name: Date: Period:

Situation Sort

8. a Read each situation.

b **Discuss:**

- What is each situation about?
- What quantities or relationships do you see in each situation?

Responses vary.

- Situation 1 is about buying two types of beverages for a picnic. Situation 2 is about buying sandwiches for lunch.
- In Situation 1, the quantities are the number of packs for each type of beverage and the amount of beverages in each pack. In Situation 2, the quantities are the cost of each sandwich, the delivery fee, and the coach's budget.

Situation 1

Sydney plans to buy 120 beverages for a picnic. Seltzers are sold in packs of 6. Waters are sold in packs of 8. x represents the number of packs of seltzers and y represents the number of packs of waters.

Situation 2

A coach has a $120 budget to buy lunch for their team. He is ordering from a restaurant that charges $8 per sandwich, plus a $6 delivery fee. x represents the number of sandwiches and y represents the total cost of the lunch.

9. Match each representation and possible solution with Situation 1 or Situation 2 from the previous problem. One representation has no match.

A

x	y
2	22
3	30
4	38

B

$y = -\frac{3}{4}x + 15$

C

D

$y = 8x + 6$

E

$y = 8x + 120$

F

$(5, 46)$

G

$(8, 9)$

Situation 1	Situation 2
B, C, G	A, D, F

Accelerated 7 | Unit 4 Lesson 8 339 Activity 2

Whole Class | 5 min

Synthesis

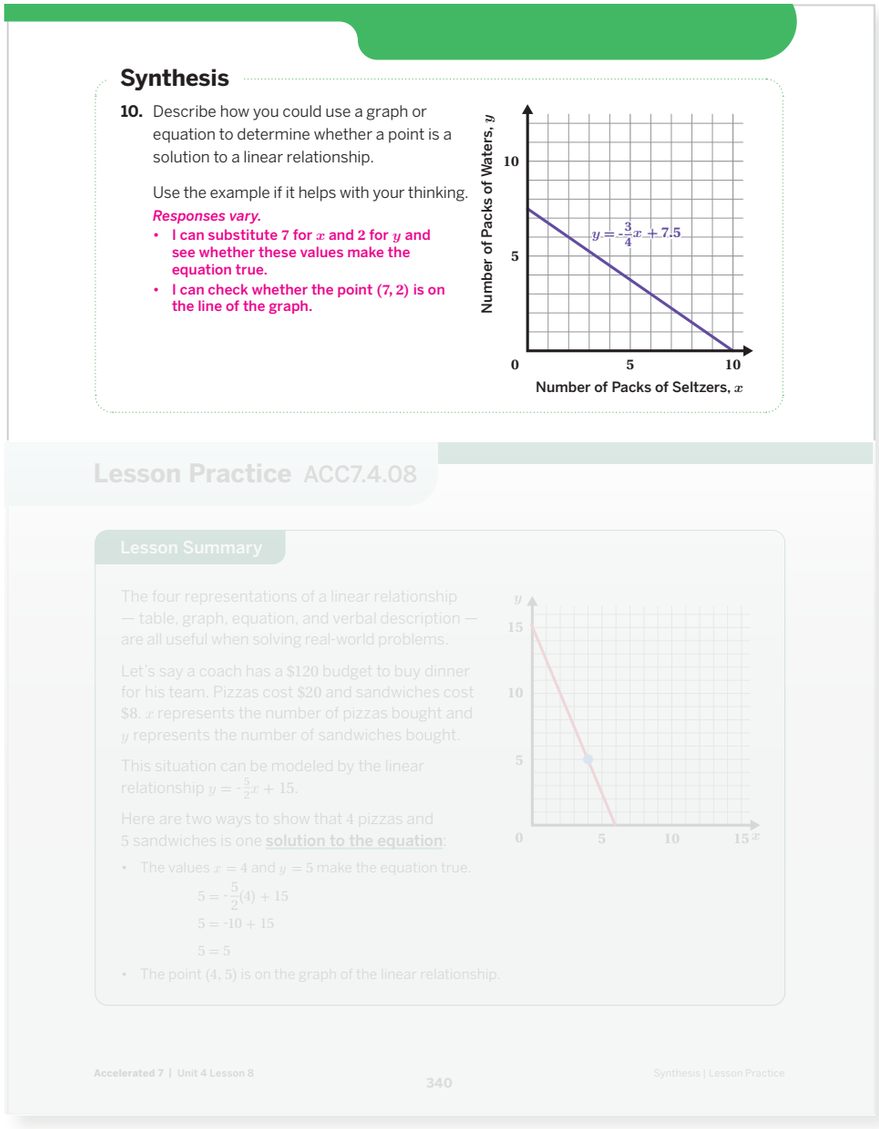
Purpose: Students synthesize their understanding of how to determine whether a point is a solution from a context.

Invite students to respond independently, and then share their thinking with a partner.

Collect and share a variety of ideas, such as:

- Checking to see if the values of the solution make sense for the quantities in the situation.
- Checking to see if the ordered pair of those values is a point on the graph of the equation.
- Writing an equation to model a situation, then substituting the values of the solution into that equation.

Note: If time allows, invite students to share with a partner what makes sense about each strategy and what questions they still have.



Synthesis

10. Describe how you could use a graph or equation to determine whether a point is a solution to a linear relationship.

Use the example if it helps with your thinking.

Responses vary.

- I can substitute 7 for x and 2 for y and see whether these values make the equation true.
- I can check whether the point (7, 2) is on the line of the graph.

Lesson Practice ACC7.4.08

Lesson Summary

The four representations of a linear relationship — table, graph, equation, and verbal description — are all useful when solving real-world problems.

Let's say a coach has a $120 budget to buy dinner for his team. Pizzas cost $20 and sandwiches cost $8. x represents the number of pizzas bought and y represents the number of sandwiches bought.

This situation can be modeled by the linear relationship $y = -\frac{3}{2}x + 15$.

Here are two ways to show that 4 pizzas and 5 sandwiches is one solution to the equation:

- The values $x = 4$ and $y = 5$ make the equation true.

$$5 = -\frac{5}{2}(4) + 15$$
$$5 = -10 + 15$$
$$5 = 5$$

- The point $(4, 5)$ is on the graph of the linear relationship.

Accelerated 7 | Unit 4 Lesson 8 340 Synthesis | Lesson Practice

Lesson Takeaway: Linear relationships of the form $y = mx + b$ can be used to model situations. In a situation, a solution can represent a combination of values that make sense in the situation.

Independent | 5 min

Show What You Know

Purpose: Students demonstrate their understanding by determining whether ordered pairs are solutions to a linear equation of the form $y = mx + b$.

Today's Goals

1. **Goal:** Connect situations with tables, equations, graphs, and solutions.
2. **Language Goal:** Describe how to determine solutions given a real-world situation. **(Speaking and Listening)**

Show What You Know (Assessment Resources)

Name: _____ Date: _____ Period: _____

Show What You Know 4.08

The graph of a linear equation includes the points $(-2, 0)$ and $(0, 6)$.

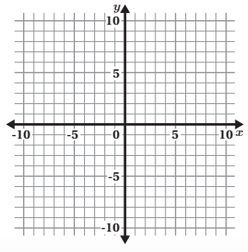
Could $y = 3x + 6$ be an equation for this line?

(Yes) No There is not enough information

Explain your thinking.

Explanations vary.

- The points $(-2, 0)$ and $(0, 6)$ both make the equation $y = 3x + 6$ true.
- The graph of $y = 3x + 6$ goes through both of the points.



Differentiation Use after Lesson 8

Support

Provide targeted intervention for students by using these resources.

If students need support deciding if an ordered pair is a solution to a linear equation:

» **Respond to Student Thinking:**

- Review the **Lesson Summary** with students before providing individualized feedback on the relevant **Practice Problems**.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 3 Extension Activities**.

Professional Learning

Take a moment to reflect on how students engaged with the modeling process in today's lesson. What variables did they consider when they were making sense of the pennies and quarters? What might you do next time students are modeling a situation?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

10. Describe how you could use a graph or equation to determine whether a point is a solution to a linear relationship.

Use the example if it helps with your thinking.

Responses vary.

- I can substitute 7 for x and 2 for y and see whether these values make the equation true.
- I can check whether the point (7, 2) is on the line of the graph.

Lesson Practice ACC7.4.08

Lesson Summary

The four representations of a linear relationship — table, graph, equation, and verbal description — are all useful when solving real-world problems.

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This situation can be modeled by the linear relationship $y = -\frac{5}{2}x + 15$.

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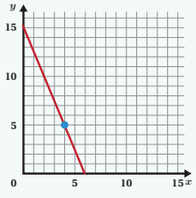
$$5 = -\frac{5}{2}(4) + 15$$
$$5 = -10 + 15$$
$$5 = 5$$

- The point $(4, 5)$ is on the graph of the linear relationship.



Lesson Practice
ACC7.4.08

Name: Date: Period:

Problems 1–5: The owner of a new restaurant is ordering tables and chairs. She wants to have only tables for 2 and tables for 4. The total number of people that can be seated in the restaurant is 120.

1. Complete the table to show some possible combinations of 2-seat tables and 4-seat tables that will seat 120 customers.

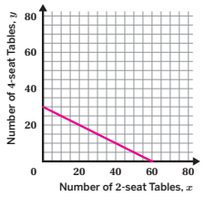
Responses vary.

Number of 2-Seat Tables	Number of 4-Seat Tables
0	30
10	25
40	10

2. Write an equation that represents the number of 2-seat tables, x, and the number of 4-seat tables, y, that the owner can order.

$y = -\frac{1}{2}x + 30$ (or equivalent)

3. Graph all the possible combinations of 2-seat and 4-seat tables that will seat 120 customers.



4. What is the slope of the line? What does it tell you about the situation?

$-\frac{1}{2}$ (or equivalent). *Responses vary.* For every 4-seat table taken away, two 2-seat tables can be added.

5. What are the x- and y-intercepts of the line? What do they represent in the situation?

The x-intercept is (60, 0) and the y-intercept is (0, 30). Responses vary. The x-intercept means that 60 tables are needed if only 2-seat tables are used. The y intercept means that 30 tables are needed if only 4-seat tables are used.

Lesson Practice
ACC7.4.08

Name: Date: Period:

FAST Practice

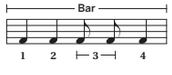
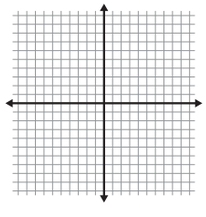
Problems 6–7: A bar is a section of music made up of notes. A quarter note is 1 beat long. An eighth note is $\frac{1}{2}$ a beat long. A music composer wants to create a bar that's 4 beats long.

Eighth Note = $\frac{1}{2}$ beat

Quarter Note = 1 beat

6. What are two possible combinations of eighth notes ($\frac{1}{2}$ a beat long) and quarter notes (1 beat long) that she could use to make a bar that's 4 beats long? Use a graph if it helps with your thinking.

Responses vary.

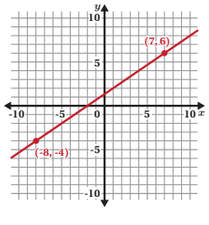
- 8 eighth notes and 0 quarter notes
- 4 eighth notes and 2 quarter notes



7. Write an equation to represent the number of eighth notes, x, and number of quarter notes, y, in a bar that has a total of 4 beats. Record your answer in the space provided.

$y = -\frac{1}{2}x + 4$ (or equivalent)

Spiral Review

Problems 8–10: Here is the graph of a linear relationship.



8. What is the y-intercept of the line?

A. $\frac{4}{3}$ B. $\frac{3}{4}$ C. $\frac{2}{3}$ D. $\frac{3}{2}$

9. What is the slope of the line?

A. $\frac{4}{3}$ B. $\frac{3}{4}$ C. $\frac{2}{3}$ D. $\frac{3}{2}$

10. Write an equation that represents the line.

$y = \frac{2}{3}x + \frac{4}{3}$ (or equivalent)

Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1–2, 4–5	2	MA.8.AR.3.3, MA.8.AR.3.5
	3	2	MA.8.AR.3.4
FAST Practice	6–7	3	MA.8.AR.3.3, MA.8.AR.3.5
Spiral Review			
	8–9	3	MA.8.AR.3.2
	10	2	MA.8.AR.3.3

Math in Action

Purpose: Students consider the math they learned in this unit and how it applies to the workplace, their world, and communities.

In this Unit

Students solved linear equations and determined the number of possible solutions. They also solved systems of two linear equations algebraically and graphically.

Career Connection | Meet William Pickering

Invite students to discuss how they see examples of robotics in their world.

Consider asking:

- "How do you benefit from the work done at science laboratories?" Sample response: Robotics are used in household devices such as the roomba. Surgeons use robotics to perform surgeries that require extreme precision, especially when accessing very small spaces.

B.E.S.T. Benchmark Connections

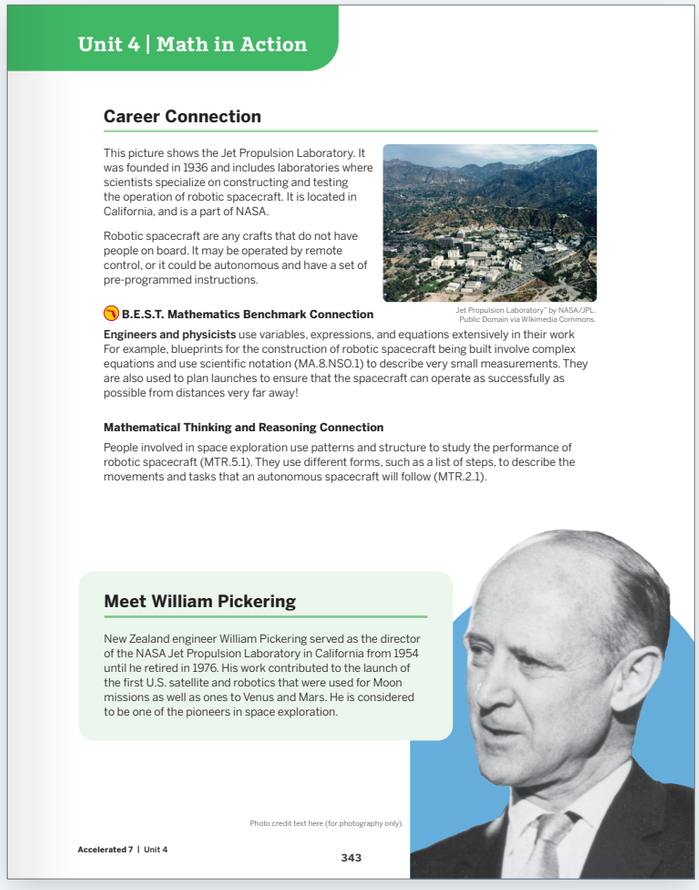
Consider asking:

- "What other math concepts have you studied in this unit could you apply to robotics?" Sample response: You can use equations and their solutions to program a robot for specific actions. (MA.8.GR.2.4)

Mathematical Thinking and Reasoning Connections

Consider asking:

- "How would you explain to a classmate how engineers and others involved in designing robots that are used in everyday life apply Mathematical Thinking and Reasoning standard MTR.6.1 in their work?" Sample response: People use robots for automated tasks such as cleaning a carpet or assembling the parts of a car. So engineers and those who are involved in building these robots study how the devices work and evaluate their results. These results may require changes or updates to the parts of the robot or the coding of the instructions to improve performance.



Unit 4 | Math in Action

Career Connection

This picture shows the Jet Propulsion Laboratory. It was founded in 1936 and includes laboratories where scientists specialize on constructing and testing the operation of robotic spacecraft. It is located in California, and is a part of NASA.

Robotic spacecraft are any crafts that do not have people on board. It may be operated by remote control, or it could be autonomous and have a set of pre-programmed instructions.

"Jet Propulsion Laboratory" by NASA/JPL. Public Domain via Wikimedia Commons.

B.E.S.T. Mathematics Benchmark Connection

Engineers and physicists use variables, expressions, and equations extensively in their work For example, blueprints for the construction of robotic spacecraft being built involve complex equations and use scientific notation (MA.8.NSO.1) to describe very small measurements. They are also used to plan launches to ensure that the spacecraft can operate as successfully as possible from distances very far away!

Mathematical Thinking and Reasoning Connection

People involved in space exploration use patterns and structure to study the performance of robotic spacecraft (MTR.5.1). They use different forms, such as a list of steps, to describe the movements and tasks that an autonomous spacecraft will follow (MTR.2.1).

Meet William Pickering

New Zealand engineer William Pickering served as the director of the NASA Jet Propulsion Laboratory in California from 1954 until he retired in 1976. His work contributed to the launch of the first U.S. satellite and robotics that were used for Moon missions as well as ones to Venus and Mars. He is considered to be one of the pioneers in space exploration.

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Accelerated 7 | Unit 4

343

Assess and Respond

End-of-Unit Assessment

Independent | 45 min

Facilitation: Assign the End-of-Unit Assessment to learn about your students' understanding of the concepts and skills in this unit. There are two forms of the End-of-Unit Assessment: Forms A and B. If time allows, consider inviting students to complete one or more of the tasks in the Unit Synthesis and Reflection.

Item Analysis, Forms A and B

Problem(s)	Concept or skill	Addressed in	DOK	FL B.E.S.T. Math Benchmarks
1	Identifying solutions to a linear equation written as $y = mx + b$ from an equation or graph	Lesson 11	1	MA.8.AR.4.1
2	Interpreting lines with positive and negative slopes that represent real-world situations	Lesson 5	1	MA.8.AR.3.5, MTR.6.1
3	Interpreting graphs of proportional relationships in context	Lessons 1, 2	2	MA.8.AR.3.1, MA.8.AR.3.2, MA.8.AR.3.3, MTR.3.1
4	Writing equations of lines with positive slope, negative slope, and horizontal and vertical lines	Lesson 8–10	1	MA.8.AR.3.3
5	Comparing different proportional relationships given different representations	Lesson 3	3	MA.8.AR.3.2, MTR.2.1
6a	Completing a table that represents a real-world scenario	Lesson 11	1	MA.8.AR.3.2, MTR.6.1
6b	Writing an equation that represents a real-world scenario	Lesson 11	1	MA.8.AR.3.3, MTR.2.1
6c	Creating a graph with a negative slope that represents a real-world scenario	Lesson 11	1	MA.8.AR.3.4, MTR.6.1
7a	Determining the value given a real-world scenario	Lesson 4	2	MA.8.AR.3.5, MTR.7.1
7b	Representing a situation with an initial value and rate of change with an equation and a graph	Lesson 4	2	MA.8.AR.3.3, MTR.7.1
7c	Representing a situation with an initial value and rate of change with an equation and a graph	Lesson 4	2	MA.8.AR.3.4, MTR.7.1
7d	Describing differences in graphs of real world situations, given their initial values and rates of change	Lesson 4	3	MA.8.AR.3.5, MTR.7.1

Assessment Resources

- Student Assessments
- Assessment Rubrics with Answers
- Show What You Knows and Answer Keys

Intervention and Extension Resources

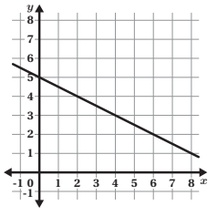
- Mini-Lessons
- Extensions

Name: Date: Period:

End-of-Unit Assessment

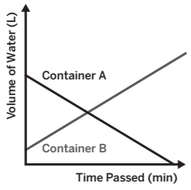
Unit 4

1. This graph shows the line $y = \frac{1}{2}x + 5$. Select *all* of the points that are solutions to the equation.



- [x] **A.** $(0, 5)$
- [ ] **B.** $(0, 10)$
- [ ] **C.** $(1, 2)$
- [ ] **D.** $(5, 0)$
- [x] **E.** $(10, 0)$

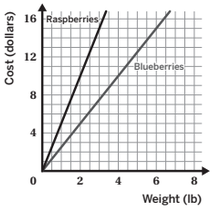
2. This graph shows the amount of water in each container as the water in Container A is poured into Container B.



Which statement is true?

- **A.** The relationship between time and amount of water for Container A is proportional.
- **B.** The amount of water in Container A decreased at a constant rate.
- **C.** Initially, there was more water in Container B than in Container A.
- **D.** The amount of water in Container A increased each minute.

3. The graph shows the cost vs. weight relationship for blueberries and raspberries.



Select *all* of the true statements.

- [x] **A.** The cost vs. weight relationship for both blueberries and raspberries are proportional relationships.
- [ ] **B.** The cost as a unit rate for blueberries is \$5 per pound.
- [ ] **C.** Raspberries cost half as much as blueberries.
- [x] **D.** It costs \$10 to buy 4 pounds of raspberries.
- [x] **E.** The relationship between cost, c, and weight, w, for raspberries can be represented by the equation $c = 5w$.

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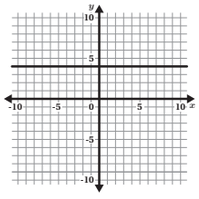
Name: Date: Period:

End-of-Unit Assessment (continued)

Unit 4

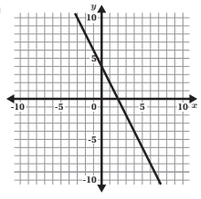
4. Write an equation for each line.

a



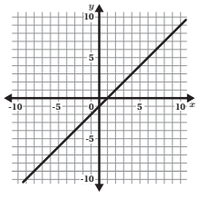
Equation: $y = 4$

b



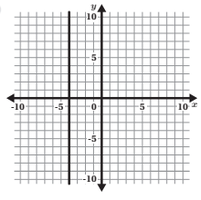
Equation: $y = 4 - 2x$ (or equivalent)

c



Equation: $y = x - 1$ (or equivalent)

d



Equation: $x = -4$

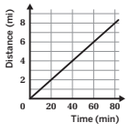
Name: Date: Period:

End-of-Unit Assessment (continued)

Unit 4

5. One day, three runners ran 10 miles, each at a different constant speed.

Runner 1



Runner 2

Time (min)	Distance (mi)
18	2
36	4
54	6
72	8
90	10

Runner 3

$d = \frac{1}{8}t$

t time (minutes)
d distance (miles)

Which runner ran the fastest? Explain your thinking.

Runner 3. *Explanations vary*. Runner 1's rate is 1 mile every 10 minutes. Runner 2's rate is 1 mile every 9 minutes. Runner 3's rate is 1 mile every 8 minutes.

6. Beans cost \$1.50 per pound. Rice costs \$1.00 per pound. Joel has \$7.50 for beans and rice.

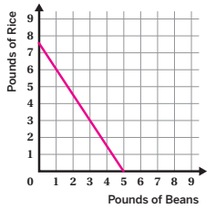
a Complete the table showing three ways Joel can spend exactly \$7.50 on beans and rice.

Pounds of Beans, b	Pounds of Rice, r
1	6
3	3
5	0

b Write an equation that represents the pounds of beans, b, and rice, r, Joel can buy for \$7.50.

$1.5b + 1r = 7.5$ (or equivalent)

c Draw a graph of the pounds of beans, b, and rice, r, Joel can buy for \$7.50.



Name: Date: Period:

End-of-Unit Assessment (continued)

Unit 4

7. A cell phone plan costs \$200 to start. Then there is a \$50 charge each month.

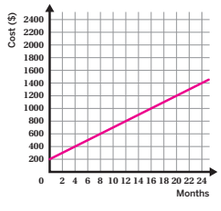
a What is the total cost (startup fee and monthly charge) to use the cell phone plan for 2 months?

\$300

b Write an equation to represent the total cost, y, to use the cell phone plan for x months.

$y = 200 + 50x$

c Graph the relationship between total cost and number of months.



d A new cell phone plan costs \$100 to start. Then there is a \$50 charge each month. Describe how the *graph* of this new plan would be the same and how it would be different.

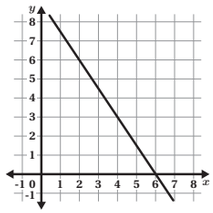
Responses vary. The graph of the new plan would have a different vertical intercept but the same slope as the graph of the first plan. The lines would be parallel to each other because their slopes are the same.

Name: Date: Period:

End-of-Unit Assessment

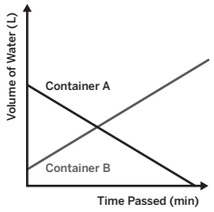
Unit 4

1. This graph shows the line $y = -\frac{3}{2}x + 9$. Select *all* of the points that are solutions to the equation.



- ☐ A. $(0, 6)$
- ☑ B. $(0, 9)$
- ☑ C. $(2, 6)$
- ☑ D. $(6, 0)$
- ☐ E. $(9, 0)$

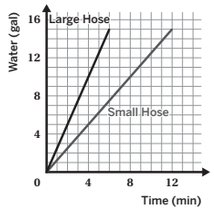
2. This graph shows the amount of water in each container as the water in Container A is poured into Container B.



Which statement is true?

- A. The amount of water in Container A increased each minute.
- B. The amount of water in Container B decreased at a constant rate.
- C. The relationship between time and amount of water for Container B is proportional.
- (D.) Initially, there was more water in Container A than in Container B.

3. The graph shows the amount of water in the pool vs. time relationship for filling a kiddie pool with a small hose and a large hose.



Select *all* of the true statements.

- ☑ A. The amount of water in the pool vs. time relationship for both the large and small hoses are proportional relationships.
- ☑ B. It takes 6 minutes to fill the pool with the large hose and 12 minutes with the small hose.
- ☑ C. The speed as a unit rate for filling the pool with the large hose is 2.5 gallons per minute.
- ☐ D. If the pool is filled with the small hose for 4 minutes it will have 10 gallons of water.
- ☐ E. The relationship between time, t, and the amount of water, w, for the small hose can be represented by the equation $w = 2.5t$.

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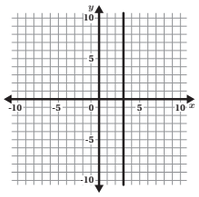
Name: Date: Period:

End-of-Unit Assessment (continued)

Unit 4

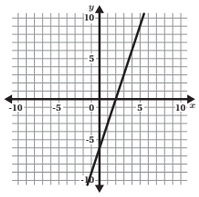
4. Write an equation for each line.

a



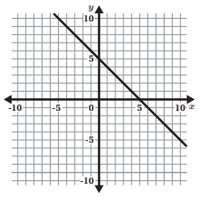
Equation: $x = 3$

b



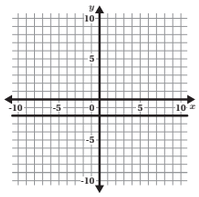
Equation: $y = -6 + 3x$ (or equivalent)

c



Equation: $y = -x + 5$ (or equivalent)

d



Equation: $y = -2$

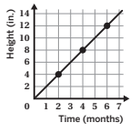
Name: Date: Period:

End-of-Unit Assessment (continued)

Unit 4

5. Amir is growing three plants. He keeps track of each plant's height over time.

Plant 1



Plant 2

Time (months)	Height (in.)
2	5
4	10
6	15
8	20
10	25

Plant 3

$h = 3.5t$

t time (months)
h height (in.)

Which plant is growing the slowest? Explain your thinking.

Plant 1. *Explanations vary*. Plant 1 grows 2 inches every month. Plant 2 grows 2.5 inches every month. Plant 3 grows 3.5 inches every month.

6. A truck delivers tables and chairs for a fair. The truck can carry 1,000 pounds total.

- Tables weigh 50 pounds each.
- Chairs weigh 10 pounds each.

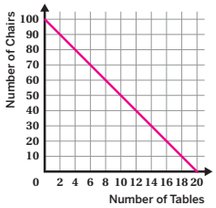
a Complete the table showing three ways the truck can be packed with tables and chairs.

Number of Tables, t	Number of Chairs, c
1	95
8	60
20	0

b Write an equation that represents the number of tables, t, and the number of chairs, c, that the truck can carry.

$50t + 10c = 1000$ (or equivalent)

c Draw a graph of the number of tables, t, and the number of chairs, c, that the truck can carry.



Name: Date: Period:

End-of-Unit Assessment (continued)

Unit 4

7. A restaurant charges $10 for each burrito, and a $5 delivery fee.

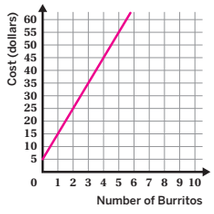
a What is the total cost to have 4 burritos delivered?

$45

b Write an equation to represent the total cost, y, to have x burritos delivered.

$y = 5 + 10x$

c Graph the relationship between total cost and number of burritos.



d A new restaurant charges $7 for each burrito, and a $5 delivery fee. Describe how the *graph* of this new restaurant's burrito delivery would be the same and how it would be different.

***Responses vary*. The graph for the new restaurant would have the same vertical intercept but a different slope from the graph of the first restaurant. The lines would not be parallel to each other because their slopes are different.**

D Differentiation (End-of-Unit Assessment)

Note: To **strengthen** and **stretch** students' learning, refer to the differentiation resources suggested throughout this unit and in the Unit Overview.

Sub-Unit Goals	Problem(s)	To respond to student thinking, consider:
Sub-Unit 1: • Represent non-proportional linear relationships with equations, tables, and graphs. • Interpret the vertical intercept and slope of a linear relationship in context. **(Lessons 1–4)**	2	**Support** **Strengthen** • **Mini-Lesson:** *Interpreting Slope and Intercepts of Linear Relationships* • **Teacher Move:** Consider revisiting Lesson 1 (*Flags*)
	3, 5, 7	**Support** **Strengthen** • **Mini-Lesson:** *Interpreting Slope and Intercepts of Linear Relationships* • **Teacher Move:** Consider revisiting Lesson 2 (*Water Cooler*)

		Stretch
		• **You're invited to explore more:**
		  • Lesson 2 (Water Cooler)
		  • Lesson 4 (Stacking Cups)

Sub-Unit 2: • Write an equation for a linear relationship given two points. • Make connections between graphs and solutions of two-variable linear equations. • Analyze strategies for calculating the slope and vertical intercept of a line through two points. **(Lessons 7–8)**	1, 6	**Support** **Strengthen** • **Teacher Moves:** Consider revisiting Lesson 8 (*Pennies and Quarters*)
		Stretch • **You're invited to explore more:** Lesson 8 (*Pennies and Quarters*)

	4	**Support**
		• **Mini-Lesson:** *Calculating Slope Given Two Points*
		• **Teacher Move:** Consider revisiting:
		  • Lesson 5 (*Landing Planes*)
		  • Lesson 6 (*Coin Capture*)
		  • Lesson 7 (*Why Intercepts?*)

		Strengthen
		• **Challenge Creator:**
		  • Lesson 5 (*Landing Planes*)
		  • Lesson 6 (*Coin Capture*)
		• **Repeated Challenges:** Lesson 7 (*Why Intercepts?*)

		Stretch • **You're invited to explore more:** Lesson 6 (*Landing Planes*)

UNIT 5

Linear Equations and Linear Systems

Students solve linear equations and determine the number of possible solutions. They also solve systems of two linear equations algebraically and graphically.

Essential Questions

- How can you solve an inequality that has two operations?
- How can systems of equations be used to represent situations and solve problems?
- What does it mean for an equation or system of equations to have *no*, *one*, or *infinitely many* solutions?

Sub-Unit Goals

Sub-Unit 1

- Solve equations of the form $px + q = r$ and $p(x + q) = r$ in real-world and mathematical problems.

Sub-Unit 2

- Determine the number of solutions to a linear equation.
- Solve two-step linear inequalities and graph the solutions on a number line.

Sub-Unit 3

- Solve systems of linear equations in two variables graphically and algebraically.

The Math of the Unit

Unit Outcomes Telling a story that narrows and deepens with the grade

Florida B.E.S.T. Standards for Mathematics

Write and solve equations and inequalities in one variable. **(MA.7.AR.2)**

Generate equivalent algebraic expressions. **(MA.8.AR.1)**

Solve multi-step one-variable equations and inequalities. **(MA.8.AR.2)**

Extend understanding of proportional relationships to two-variable linear equations. **(MA.8.AR.3)**

Develop an understanding of two-variable systems of equations. **(MA.8.AR.4)**

Benchmarks Addressed in This Unit: MA.7.AR.2.2 MA.8.AR.1.2 MA.8.AR.2.1 MA.8.AR.2.2 MA.8.AR.3.3 MA.8.AR.4.1 MA.8.AR.4.2 MA.8.AR.4.3

Mathematical Thinking and Reasoning Standards

- Actively participate in effortful learning both individually and collectively. **(MTR.1.1)**
- Demonstrate understanding by representing problems in multiple ways. **(MTR.2.1)**
- Engage in discussions that reflect on the mathematical thinking of self and others. **(MTR.4.1)**
- Use patterns and structure to help understand and connect mathematical concepts. **(MTR.5.1)**

Also Addressed: **MTR.3.1, MTR.7.1**

Coherence Telling a story within and across grades

In this Unit

Students solve linear equations and systems of two linear equations algebraically and graphically.

Prior Learning

- Rewriting algebraic expressions in equivalent forms **(MA.7.AR.1.1, MA.7.AR.1.2)**
- Writing and solving equations **(MA.6.AR.2.2)**
- Writing and solving inequalities **(MA.7.AR.2.1)**
- Interpreting solutions to equations **(MA.6.AR.2.1)**
- **Unit 4:** Linear relationships **(MA.8.AR.3)**

Future Learning

- **Unit 6:** Equations of functions **(MA.8.F.1.2)**
- Writing and solving systems of equations in context **(MA.912.AR.9.1)**
- Solving systems with non-linear equations **(MA.912.AR.9.2, MA.912.AR.9.3)**

See the Connections to Future Learning page for more information, including explanations and examples.

Rigor and Balance Integrating the aspects of rigor in ways that support one another

Here are examples of the aspects of rigor and balance in this unit. See the first page of each lesson's Teacher Edition for more information.

Conceptual Understanding

In **Lesson 13**, students compare representations of systems of equations with no solution, one solution, and infinitely many solutions.

Strategic Competence

In **Lessons 8 and 15**, students utilize and compare different solving strategies.

Procedural Fluency

In **Lesson 6**, students solve linear equations with parentheses and variables on both sides.

Adaptive Reasoning

In **Lesson 6**, students analyze equations to determine which moves unbalance an equation.

Application

In **Lesson 9**, students apply strategies for solving linear equations by setting two expressions equal to each other and solving for x to answer questions about a context.

Productive Disposition

In **Lesson 14**, students persevere in solving challenges as they zap lines to solve systems of linear equations.

Unit at a Glance

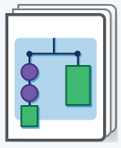
Assess and Respond

A Pre-Unit Check

Learn more about your students' understanding of foundational concepts and skills that will support them in the upcoming unit. This can be given in its entirety before the unit or spread throughout the unit.

MA.6.AR.1.1 MA.6.AR.1.4 MA.6.AR.2.1 MA.6.AR.2.2
MTR.2.1 MTR.3.1 MTR.5.1 MTR.7.1

Sub-Unit 1

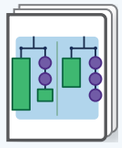


1 Balancing Equations

Solving Equations With Balanced Hangers

Understand strategies for solving an equation and connect those to balanced moves on a hanger.

MA.7.AR.2.2
MTR.2.1 MTR.4.1 MTR.5.1

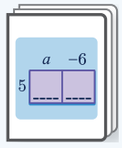


2 Keeping It True

Solving Equations

Describe and use strategies for solving equations of the forms $px + q = r$ or $p(x + q) = r$ that involve positive and negative numbers.

MA.7.AR.2.2
MTR.2.1 MTR.4.1 MTR.5.1



3 Factoring and Expanding

Options for Solving One Equation

Rewrite expressions by **factoring** and **expanding**, and then solve equations in multiple ways and compare the results.

MA.7.AR.2.2
MTR.2.1 MTR.4.1 MTR.5.1

8 Strategic Solving

Solving Many Types of Equations

Make sense of and solve equations with no solution, one solution, and infinitely many solutions.

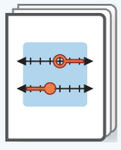
MA.8.AR.1.2 MA.8.AR.2.1
MTR.1.1 MTR.5.1

9 When Will They Meet?

Solving Linear Equations in Context

Apply knowledge of solving equations by considering two vehicles moving on a road.

MA.8.AR.1.2 MA.8.AR.2.1 MA.8.AR.3.3
MTR.2.1



10 Do the Two-Step

Solving Two-Step Linear Inequalities

Solve inequalities with positive and negative rational coefficients and connect the solutions of inequalities to their graphs.

MA.8.AR.1.2 MA.8.AR.2.2
MTR.5.1 MTR.7.1

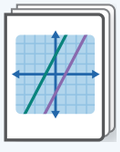
Assess and Respond

A Sub-Unit Quiz

Learn about your students' understanding of the concepts and skills so far in this unit.

MA.7.AR.2.2 MA.8.AR.2.1 MA.8.AR.2.2 MA.8.AR.3.3
MTR.4.1 MTR.6.1

Assess and Respond



15 All, Some, or None? Part 2

Systems of Equations With No, One, or Many Solutions

Connect algebraic and graphical representations of systems and determine the number of solutions from both representations.

MA.8.AR.4.2 MA.8.AR.4.3
MTR.5.1

A End-of-Unit Assessment

Learn about your students' understanding of the concepts and skills in this unit. There are two forms of the End-of-Unit Assessment: Forms A and B.

MA.8.AR.2.1 MA.8.AR.2.2 MA.8.AR.3.3 MA.8.AR.4.1 MA.8.AR.4.2 MA.8.AR.4.3
MTR.5.1

Pacing: 18 days | **Short on time?** See pacing considerations below.

All instructional and assessment days are ~45 minutes each.

- 15 Lessons
- Pre-Unit Check
- Sub-Unit Quiz
- End-of-Unit Assessment

Sub-Unit 2

4 Pass the Equation

Solving Equations Multiple Ways

Solve equations using a variety of strategies.

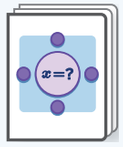
MA.7.AR.2.2
MTR.2.1 MTR.3.1 MTR.5.1

5 Community Day

Using Equations to Solve Problems

Represent and answer questions about situations in context.

MA.7.AR.2.2
MTR.2.1 MTR.4.1 MTR.7.1

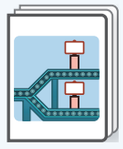


6 Equation Roundtable

Analyzing Unbalanced Equations

Develop fluency in solving equations by analyzing solving steps, including strategies for linear equations with rational number coefficients.

MA.8.AR.1.2 MA.8.AR.2.1
MTR.4.1



7 All, Some, or None? Part 1

How Many Solutions?

Explore the idea of equations with no solution and **infinitely many solutions** using number machines.

MA.8.AR.1.2 MA.8.AR.2.1
MTR.4.1 MTR.5.1

Sub-Unit 3

11 On or Off the Line?

Interpreting Points On or Off the Line

Consider pairs of linear equations in various contexts and interpret the meaning of points on and off the lines.

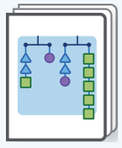
MA.8.AR.4.1
MTR.2.1 MTR.4.1

12 On Both Lines

Representing Systems of Linear Equations

Analyze two linear relationships and graphs of lines in context.

MA.8.AR.4.1 MA.8.AR.4.3
MTR.1.1 MTR.4.1

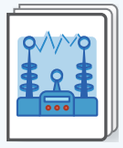


13 Make Them Balance

Graphing Systems of Linear Equations

Explore the concept of a **system of equations** and its solution(s) using the familiar context of balanced hangers.

MA.8.AR.4.1 MA.8.AR.4.2 MA.8.AR.4.3
MTR.5.1



14 Line Zapper

Solving Systems of Linear Equations

Continue to explore systems where both equations are of the form $y = mx + b$.

MA.8.AR.4.1 MA.8.AR.4.3
MTR.4.1 MTR.5.1

Pacing Considerations

Lesson 12: This lesson introduces a graph that represents two linear relationships in context, and invites students to make sense of the point of intersection. If students show a strong understanding of the relationship between solutions of a linear equation and its graph in earlier lessons, this lesson may be omitted. If omitted, be sure to discuss the meaning of the point of intersection of two linear relationships.

Lesson Materials and Prep

Here are the materials and resources needed within each lesson, how to prepare them, and where to find them.

Lesson 1
- **Copy:** Show What You Know
- **Materials:** coloring tools *(as needed)*

Lesson 2
- **Copy:** Show What You Know

Lesson 3
- **Copy:** Show What You Know
- **Materials:** coloring tools *(as needed)*

Lesson 4
- **Copy:** Show What You Know

Lesson 5
- **Copy:** Show What You Know, Activity 2 Sheet *(one per student)*

Lesson 6
- **Copy:** Show What You Know
- **Cut:** Activity 2 Cards *(one set per group)*
- **Materials:** calculators *(as needed)*, coloring tools *(as needed)*

Lesson 7
- **Copy:** Show What You Know

Lesson 8
- **Copy:** Show What You Know
- **Materials:** calculators *(as needed)*, scrap paper *(as needed)*

Lesson 9
- **Copy:** Show What You Know

Lesson 10
- **Copy:** Show What You Know
- **Materials:** coloring tools *(as needed)*

Lesson 11
- **Copy:** Show What You Know, Activity 2 Sheet *(one per student)*
- **Materials:** calculator *(as needed)*, coins or coin manipulatives *(as needed)*

Lesson 12
- **Copy:** Show What You Know, Activity 1 Sheet *(one per group)*
- **Materials:** coloring tools, graph paper, poster paper, straightedges

Lesson 13
- **Copy:** Show What You Know
- **Materials:** calculators *(as needed)*

Lesson 14
- **Copy:** Show What You Know
- **Materials:** coloring tools or highlighters *(as needed)*

Lesson 15
- **Copy:** Show What You Know
- **Materials:** straightedges, graphing calculators *(as needed)*

Resource Books

Assessment and Lesson Resources

- Pre-Unit Check
- Sub-Unit Quizzes
- End-of-Unit Assessment, Forms A & B
- Assessment Rubrics
- Show What You Knows
- Activity Sheets and Cards

Intervention and Extension Resources

- Mini-Lessons
- Prior Grade Mini-Lessons
- Extensions

Caregiver Support

These resources can be shared with students' caregivers. They provide background on the mathematics in this unit, as well as suggestions for supporting students at home.

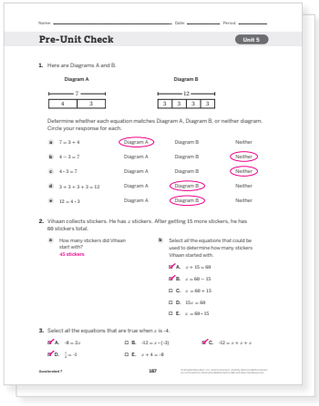
Assessments

Every unit includes a variety of formative and summative assessments designed to highlight students' understanding of a variety of concepts and skills.

Pre-Unit Check

Assign the **Pre-Unit Check** to learn more about your students' understanding of foundational concepts and skills that will support them in the upcoming unit. This can be given in its entirety before the unit or spread throughout the unit.

FL B.E.S.T. Math Benchmarks: MA.6.AR.1.1, MA.6.AR.1.4, MA.6.AR.2.1, MA.6.AR.2.2


Name _____ Date _____ Period _____

Pre-Unit Check Unit 5

1. Here are Diagrams A and B.

Diagram A

Diagram B

Determine whether each equation matches Diagram A, Diagram B, or neither diagram. Circle your response for each.

a. $7 = 3 + 4$	Diagram A	Diagram B	Neither
b. $4 = 3 = 7$	Diagram A	Diagram B	Neither
c. $4 + 3 = 7$	Diagram A	Diagram B	Neither
d. $3 + 3 + 3 + 3 = 12$	Diagram A	Diagram B	Neither
e. $12 = 4 + 3$	Diagram A	Diagram B	Neither

2. Vihaan collects stickers. He has x stickers. After getting 15 more stickers, he has 68 stickers total.

a. How many stickers did Vihaan start with?
43 stickers

b. Select all the equations that could be used to determine how many stickers Vihaan started with.
- [x] A. $x + 15 = 68$
- [x] B. $x = 68 - 15$
- [ ] C. $x = 68 + 15$
- [ ] D. $15x = 68$
- [ ] E. $x = 68 \div 15$

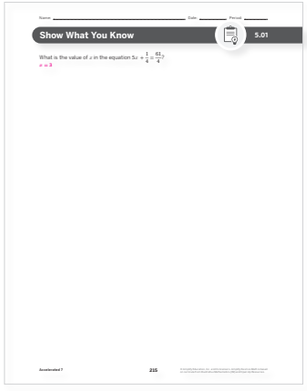
3. Select all the equations that are true when x is -4.
- [x] A. $-8 = 2x$
- [ ] B. $-12 = x + (-3)$
- [x] C. $42 = x + 7 + x$
- [x] D. $\frac{x}{4} = -1$
- [ ] E. $x + 4 = -8$

Accelerated 7 187

Lesson | Show What You Know

Use the **Show What You Know** for each lesson to learn more about your students' progress toward the lesson goal(s). Here is one example from this unit.

FL B.E.S.T. Math Benchmarks on Lesson 1 Show What You Know: MA.7.AR.2.2


Name _____ Date _____ Period _____

Show What You Know 5.01

What is the value of x in the equation $3x + \frac{1}{4} = \frac{61}{4}$?

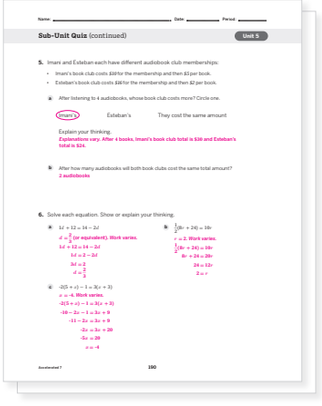
$x = 3$

Accelerated 7 215

Sub-Unit Quiz

Assign the **Sub-Unit Quiz** to learn about your students' understanding of the concepts and skills so far in this unit.

FL B.E.S.T. Math Benchmarks: MA.7.AR.2.2, MA.8.AR.2.1, MA.8.AR.2.2, MA.8.AR.3.3


Name _____ Date _____ Period _____

Sub-Unit Quiz (continued) Unit 5

5. Imani and Esteban each have different audiobook club memberships:
- Imani's book club costs $30 for the membership and then $3 per book.
- Esteban's book club costs $36 for the membership and then $2 per book.

a. After listening to 4 audiobooks, whose book club costs more? Circle one.

Imani's Esteban's They cost the same amount.

Explain your thinking.
Explanations vary. After 4 books, Imani's book club total is $30 and Esteban's total is $24.

b. After how many audiobooks will both book clubs cost the same total amount?
2 audiobooks

6. Solve each equation. Show or explain your thinking.

a. $1d + 12 = 14 - 2d$
$d = \frac{2}{3}$ (or equivalent). Work varies.
$1d + 12 = 14 - 2d$
$1d = 2 - 2d$
$3d = 2$
$d = \frac{2}{3}$

b. $\frac{1}{2}(6r + 24) = 16r$
$r = 2$. Work varies.
$\frac{1}{2}(6r + 24) = 16r$
$6r + 24 = 28r$
$24 = 12r$
$2 = r$

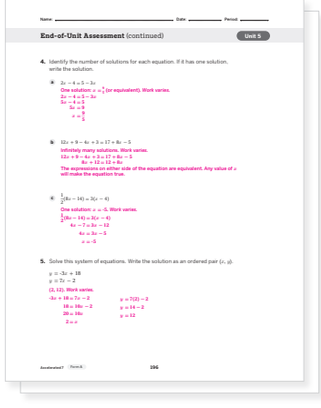
c. $-2(5 + r) - 1 = 3(r + 3)$
$r = 4$. Work varies.
$-2(5 + r) - 1 = 3(r + 3)$
$-10 - 2r - 1 = 3r + 9$
$-11 - 2r = 3r + 9$
$-2r = 3r + 20$
$-5r = 20$
$r = -4$

Accelerated 7 190

End-of-Unit Assessment

Assign the **End-of-Unit Assessment** to learn about your students' understanding of the concepts and skills in this unit. There are two forms of the End-of-Unit Assessment: Forms A and B.

FL B.E.S.T. Math Benchmarks: MA.8.AR.2.1, MA.8.AR.2.2, MA.8.AR.3.3, MA.8.AR.4.1, MA.8.AR.4.2, MA.8.AR.4.3


Name _____ Date _____ Period _____

End-of-Unit Assessment (continued) Unit 5

4. Identify the number of solutions for each equation. If it has one solution, write the solution.

a. $2x - 4 = 5 - 3x$
One solution: $x = \frac{9}{5}$ (or equivalent). Work varies.
$2x - 4 = 5 - 3x$
$5x - 4 = 5$
$5x = 9$
$x = \frac{9}{5}$

b. $12x + 9 - 4x + 3 = 17 + 8x - 5$
Infinitely many solutions. Work varies.
$12x + 9 - 4x + 3 = 17 + 8x - 5$
$8x + 12 = 12 + 8x$
The expressions on either side of the equation are equivalent. Any value of x will make the equation true.

c. $\frac{1}{2}(8x - 14) = 3(x - 4)$
One solution: $x = -5$. Work varies.
$\frac{1}{2}(8x - 14) = 3(x - 4)$
$4x - 7 = 3x - 12$
$4x = 3x - 5$
$x = -5$

5. Solve this system of equations. Write the solution as an ordered pair (x, y).

$y = -3x + 18$
$y = 7x - 2$
(3, 12). Work varies.
$-3x + 18 = 7x - 2$
$18 = 10x - 2$
$20 = 10x$
$2 = x$
$y = 7(2) - 2$
$y = 14 - 2$
$y = 12$

Accelerated 7 Form B 196

Differentiation

Every unit includes additional resources to support, strengthen, and stretch student learning.

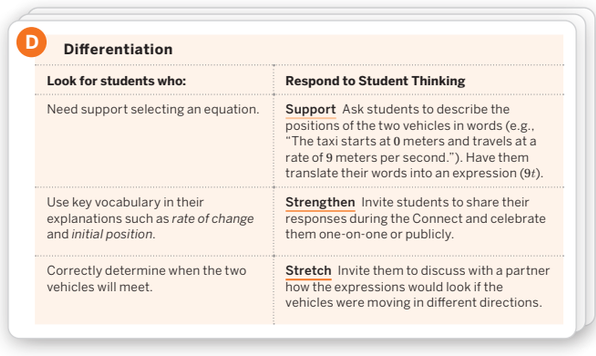
Differentiation Across the Unit

	Support	Strengthen	Stretch
	Provide targeted intervention for students by using these resources.	Reinforce students' understanding of the concepts assessed by using these resources.	Challenge students and extend their learning with these resources.
Sub-Unit 1 (Lessons 1–5) • Solve equations of the form $px + q = r$ and $p(x + q) = r$ in real-world and mathematical problems.	**Mini-Lesson** • *Solving Equations With Positive and Negative Numbers*	**Repeated Challenges** • Lesson 1 (*Balancing Equations*) **Practice** • Lessons 1–5	**DOK3 Practice** • Lesson 2
Sub-Unit 2 (Lessons 6–10) • Determine the number of solutions to a linear equation. • Solve two-step linear inequalities and graph the solutions on a number line.	**Mini-Lesson** • *Solving Linear Equations with Parentheses* **Previous Lesson** • Unit 4, Lesson 8 (*Pennies and Quarters*)	**Practice** • Lessons 6–10	**You're invited to explore more.** • Lesson 9 (*When Will They Meet?*) **DOK3 Practice** • Lessons 6, 7
Sub-Unit 3 (Lessons 11–15) • Solve systems of linear equations in two variables graphically and algebraically.	**Mini-Lesson** • *Solving Systems of Linear Equations by Graphing* **Previous Lessons** • Unit 4, Lesson 5 (*Landing Planes*) • Unit 4, Lesson 8 (*Pennies and Quarters*)	**Challenge Creator** • Lesson 11 (*On or Off the Line?*) **Practice** • Lessons 11–15	**You're invited to explore more.** • Lesson 14 (*Line Zapper*) **DOK3 Practice** • Lessons 12 and 14

Differentiation Within the Lesson

Each Teacher Edition includes point-of-use differentiation suggestions to modify or adjust instruction during the lesson to accommodate the needs of all learners.

Here is one example from **Lesson 9**, where students use equations to represent vehicles moving on a road.



Differentiation

Look for students who:	Respond to Student Thinking
Need support selecting an equation.	**Support** Ask students to describe the positions of the two vehicles in words (e.g., "The taxi starts at 0 meters and travels at a rate of 9 meters per second."). Have them translate their words into an expression ($9t$).
Use key vocabulary in their explanations such as *rate of change* and *initial position*.	**Strengthen** Invite students to share their responses during the Connect and celebrate them one-on-one or publicly.
Correctly determine when the two vehicles will meet.	**Stretch** Invite them to discuss with a partner how the expressions would look if the vehicles were moving in different directions.

Accessibility

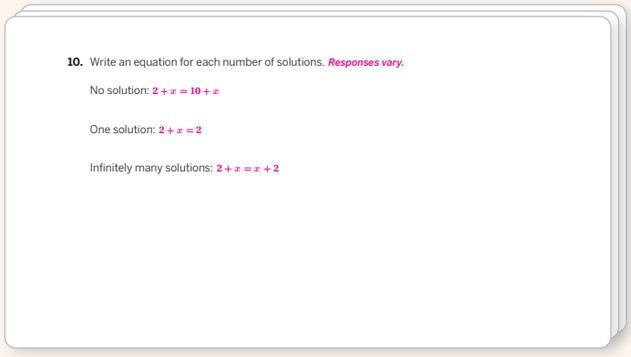
Every lesson is designed using the principles of Universal Design for Learning.

Universal Design for Learning

Here is one example from a lesson designed to support Engagement.

Spotlight on Providing Opportunities for Student Choice and Engagement . . .

Lesson 7: In this lesson, students are invited to write their own equations with zero, one, or infinite solutions which provides opportunities for individual choice.



10. Write an equation for each number of solutions. ***Responses vary.***

No solution: $2 + x = 10 + x$

One solution: $2 + x = 2$

Infinitely many solutions: $2 + x = x + 2$

Just-In-Time Accommodations and Supports

Each Teacher Edition includes suggested supports based on the areas of cognitive function from the Education Development Center (EDC).

Spotlight on Conceptual Processing . . .

Here are two suggestions in this unit for supporting students with abstract thinking.

Lesson 9, Activity 1

Accessibility: Conceptual Processing To support abstract thinking, invite students to write an expression for each row in the table.

Lesson 11, Activity 2

Accessibility: Conceptual Processing Invite students to share how each problem is different from the previous one to help apply concepts to new situations.

Language Development

Every lesson includes opportunities for all students to develop fluency with mathematical language.

Vocabulary

New Vocabulary	Lesson(s)
expand **factor**	3
infinitely many solutions	7
point of intersection	12
solution to a system of equations **system of equations**	13

Review Vocabulary
coefficient
distributive property
equivalent expressions
expression
inequality
rate
solution to an equation

Additional Vocabulary
inverse operations
like terms

Math Language Routines and Language Goals

Math Language Routine	Lesson(s)
MLR1: Stronger and Clearer Each Time	13
MLR2: Collect and Display	2–4 and 6–15
MLR3: Critique, Correct, Clarify	6
MLR6: Three Reads	5
MLR7: Compare and Connect	1, 3, and 15

Sub-Unit Language Goals

- Compare and contrast strategies for solving the same equation.
- Interpret the solution of an equation in one variable in context.
- Interpret the intersection point of two linear relationships in context given their graphs or equations.
- Describe strategies for solving systems of linear equations.

English Language Learners Support

Every Teacher Edition includes suggestions for supporting the needs of students learning English.

Here is one example from **Lesson 7**, where students clarify what it means for an equation to be true for no values, one value, or all values. This leads to an understanding of the term *infinitely many solutions*.

ELL **English Language Learners** Invite students to share what the phrases *True for One Value, True for No Values,* and *True for All Values* mean in their own words. Encourage them to use examples from earlier in the lesson to support their understanding.

Connections to Future Learning

Here is how the content in this unit connects to where your students are headed in their math journeys.

Equations of Functions

In this unit, students write linear equations that describe situations in context. **In Unit 6**, students will use equations to describe functions.

FL B.E.S.T. Math Benchmark: MA.8.F.1.3

Example:

Ricardo's company will cut 36 feet of wood trim into pieces of any size to create a window frame, as shown. Ricardo wants an equation to help him efficiently calculate the height, h, given any frame width, w. The equation $h = \frac{36-3w}{4}$ describes this relationship. The window frame's height is a function of its width.

Solving Systems with Elimination

In this unit, students solve systems of two linear equations using substitution and graphing. **In a future course**, they will learn to solve systems of linear equations using elimination, a method that generalizes to solve systems of more than two linear equations.

FL B.E.S.T. Math Benchmark: MA.912.AR.9.1

Example:

When solving a system of equations using elimination, we add or subtract equations (or multiples of equations) in order to eliminate one of the variables, which determines an equation in one variable. In this example, subtracting the two equations leaves $y = 3$. Substituting this value into either equation gives $x = 4$, so the solution to this system of equations is the ordered pair $(3, 4)$.

$$x + 2y = 10$$

$$x + y = 7$$

Solving Systems with Nonlinear Equations

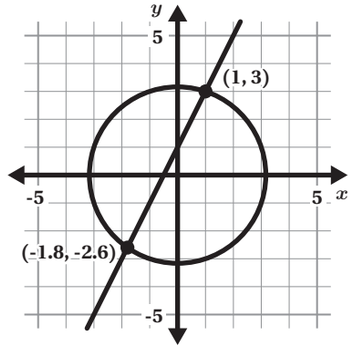
In this unit, students learn to solve systems of linear equations. **In a future course**, they will learn to solve systems of nonlinear equations.

FL B.E.S.T. Math Benchmarks: MA.912.AR.9.3

Example:

We want to know the solutions to the system of a line with the equation $y = 2x + 1$ and a circle with the equation $x^2 + y^2 = 10$.

Because there are two points of intersection, there are two solutions: $(1, 3)$ and $(-1.8, -2.6)$.

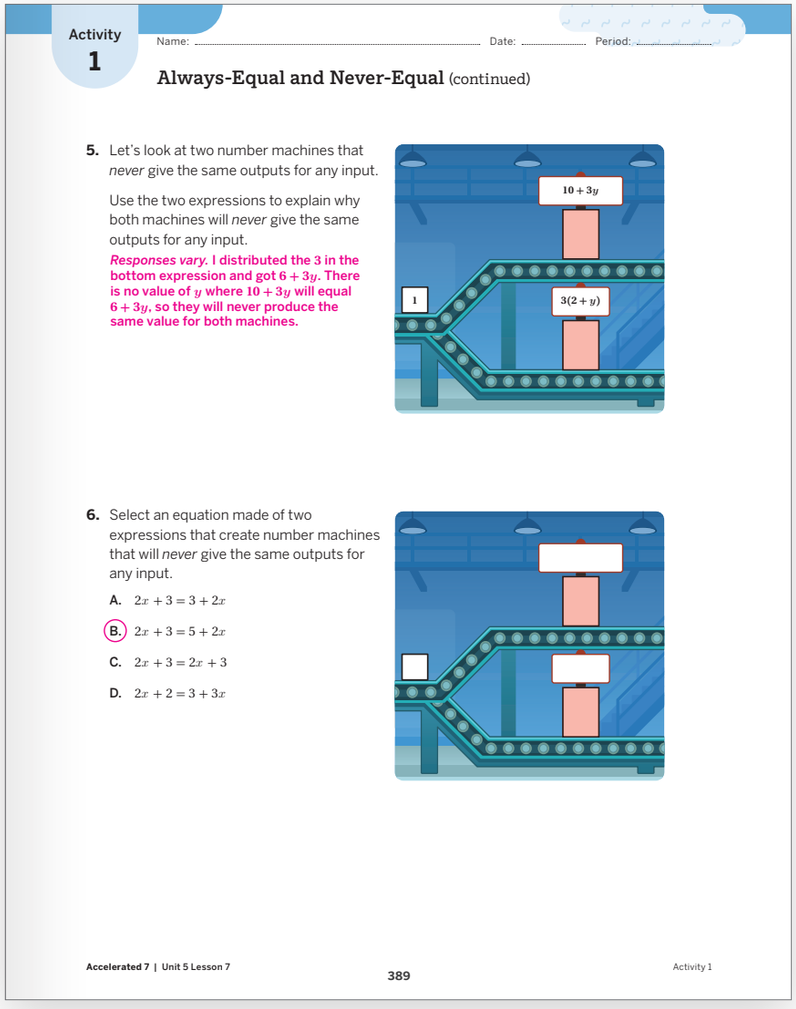


Professional Learning

Here is an opportunity to deepen your understanding of the math in this unit and how to teach it.

Spotlight on Connecting Representations

Lesson 7, Activity 1



Activity 1

Name: Date: Period:

Always-Equal and Never-Equal (continued)

5. Let's look at two number machines that *never* give the same outputs for any input.

Use the two expressions to explain why both machines will *never* give the same outputs for any input.

Responses vary. I distributed the 3 in the bottom expression and got $6 + 3y$. There is no value of y where $10 + 3y$ will equal $6 + 3y$, so they will never produce the same value for both machines.

6. Select an equation made of two expressions that create number machines that will *never* give the same outputs for any input.

A. $2x + 3 = 3 + 2x$

B. $2x + 3 = 5 + 2x$

C. $2x + 3 = 2x + 3$

D. $2x + 2 = 3 + 3x$

Accelerated 7 | Unit 5 Lesson 7 389 Activity 1

Context

In this activity, students develop conceptual understanding of the number of solutions a linear equation could have using number machines.

Try This

Put on your student hat and complete Problems 5 and 6.

Points to Ponder

- What do students need to understand about solutions to linear equations to support their conceptual understanding of no solution?
- How will the concept of no solution, one solution, or infinitely many solutions support their understanding of solutions to systems of equations in the future?

Notes:

Assess and Respond

Pre-Unit Check

Independent | 45 min

Facilitation: Assign the Pre-Unit Check to learn more about your students' understanding of foundational concepts and skills that will support them in the upcoming unit. This can be given in its entirety before the unit or spread throughout the unit. If time allows, consider inviting students to complete one or more of the tasks in the Unit Synthesis and Reflection.

Item Analysis

Problem(s)	Concept or skill	Preparation for	DOK	FL B.E.S.T. Math Benchmarks
1	Representing tape diagrams with equations	Lesson 1	1	MA.6.AR.1.1
2	Writing and solving one-step equations in one variable within a real-world context	Lesson 1	2	MA.6.AR.2.2 MTR.7.1
3	Determining which equations are true for a specified integer value	Lesson 1	2	MA.6.AR.2.1
4	Solving equations with one variable	Lesson 1	1	MA.6.AR.2.2, MA.6.AR.2.3 MTR.3.1
5	Applying the distributive property	Lesson 1	2	A.6.AR.1.4 MTR.2.1
6	Solving problems involving inequalities, including graphing inequalities	Lesson 10	2	MA.7.AR.2.1 MTR2.1, MTR.7.1

Assessment and Lesson Resources

- Student Assessments
- Show What You Knows and Answer Keys

Intervention and Extension Resources

- Mini-Lessons
- Extensions

Name: ..... Date: ..... Period: .....

Pre-Unit Check

Unit 5

1. Here are Diagrams A and B.

Diagram A

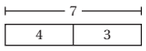
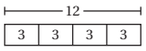


Diagram B



Determine whether each equation matches Diagram A, Diagram B, or neither diagram. Circle your response for each.

a	$7 = 3 + 4$	(Diagram A)	Diagram B	Neither
b	$4 - 3 = 7$	Diagram A	Diagram B	(Neither)
c	$4 \cdot 3 = 7$	Diagram A	Diagram B	(Neither)
d	$3 + 3 + 3 + 3 = 12$	Diagram A	(Diagram B)	Neither
e	$12 = 4 \cdot 3$	Diagram A	(Diagram B)	Neither

2. Vihaan collects stickers. He has x stickers. After getting 15 more stickers, he has 60 stickers total.

a. How many stickers did Vihaan start with?

45 stickers

b. Select *all* the equations that could be used to determine how many stickers Vihaan started with.

- ☑ A. $x + 15 = 60$
- ☑ B. $x = 60 - 15$
- ☐ C. $x = 60 + 15$
- ☐ D. $15x = 60$
- ☐ E. $x = 60 \cdot 15$

3. Select *all* the equations that are true when x is -4.

- ☑ A. $-8 = 2x$
- ☐ B. $-12 = x \cdot (-3)$
- ☑ C. $-12 = x + x + x$
- ☑ D. $\frac{x}{4} = -1$
- ☐ E. $x + 4 = -8$

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Name: ..... Date: ..... Period: .....

Pre-Unit Check (continued)

Unit 5

4. Solve each equation.

a. $p + 7 = 12$
$p = 5$

b. $90 = -20r$
$r = -4.5$
(or equivalent)

c. $\frac{1}{3}s = 7$
$s = 21$

5. Which expression is equivalent to $4(x + 2)$?

A. $12x$ B. $4x + 2$ C. $6x$ (D.) $4x + 8$

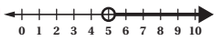
Explain your thinking.
Explanations vary. This is like 4 groups of $x + 2$, which is like $4 \cdot x + 4 \cdot 2$ or $4x + 8$.

6. Ishaan and Arnav are selling boxes of cookies.

a. Ishaan's goal is to sell more than 5 boxes. Which graph shows how many boxes Ishaan must sell in order to reach his goal?

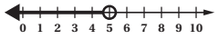
A.

(B.)



C.

D.



Explain your thinking.
Explanations vary. Ishaan wants to sell more than 5 boxes, so that's like shading to the right of 5. The number 5 doesn't get shaded because if he sells 5 boxes, he doesn't reach his goal.

b. Arnav sells each box for $3.75. His goal is to make more than $30. How many boxes of cookies could he sell to reach his goal?

9 or more boxes

Explain your thinking.
Explanations vary. To make exactly $30, Arnav needs to sell 8 boxes of cookies since $\frac{30}{3.75} = 8$. But he wants to make more than $30, so he needs to sell 9 or more boxes.

D Differentiation (Pre-Unit Check)

Note: To **strengthen** and **stretch** students' learning, refer to the differentiation resources suggested throughout this unit and in the Unit Overview.

Sub-Unit Goals	Problem(s)	Respond to Student Thinking
Preparation for Sub-Unit 1: • Solve equations of the form $px + q = r$ and $p(x + q) = r$ in real-world and mathematical problems. **(Lessons 1–5)**	1, 2	**Support** • **Teacher Move:** Consider revisiting Accelerated 6, Unit 6, Lesson 2 (*Five Equations*).
	3	**Support** • **Teacher Move:** Consider revisiting Accelerated 6, Unit 7, Lessons 8 and 9 (*More Floats and Anchors; Up, Up, and Away!*).
	4	**Support** • **Mini-Lesson:** *Solving Equations With Positive and Negative Numbers* • **Teacher Move:** Consider revisiting Lesson 1 (*Balancing Equations*).
	5	**Support** • **Mini-Lesson:** *Writing Equivalent Expressions Using the Area Model*
	6	**Support** • **Teacher Move:** Consider revisiting Accelerated 6, Unit 8, Lesson 3 (*Tunnel Travels*)

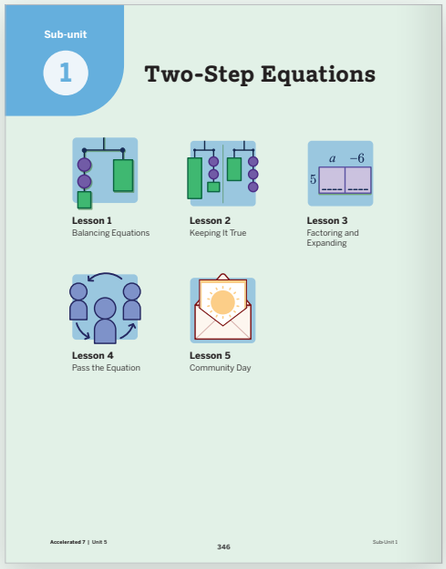
Sub-Unit 1

Two-Step Equations

Sub-Unit 1 Goals

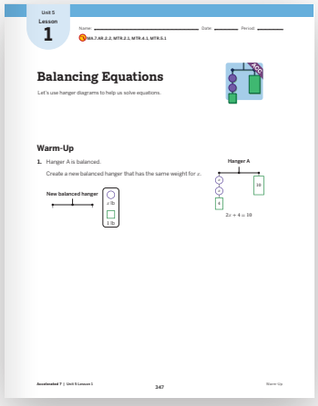
- **Goal:** Solve equations of the form $px + q = r$ and $p(x + q) = r$ in real-world and mathematical problems.
- **Language Goal:** Compare and contrast strategies for solving the same equation.

In this sub-unit, students learn how to solve equations of the form $px + q = r$ and $p(x + q) = r$. They recognize that there is more than one way to solve an equation that will lead to the same solution.



Math That Matters Most

Concept or Skill	For example . . .
Solve equations of the forms $px + q = r$ and $p(x + q) = r$ that involve positive and negative rational numbers.	Solve the equation $-4(x + 1) = -28$. $-4(x + 1) = -28$ $(x + 1) = 7$ $x = 6$
Write and solve equations to represent situations involving positive and negative numbers.	6 members of the Martinez family are going to their school's Community Day. They have a coupon for $\$4.50$ off each ticket. If they pay $\$40.50$ for all their tickets, how much does one ticket cost without the coupon? $6(x - 4.5) = 40.5$ $x = 11.25$ Each ticket costs $\$11.25$ without the coupon.


Unit 5
Lesson
1
Name: Date: Period:
MA.7.AR.2.2, MTR.2.1, MTR.4.1, MTR.5.1
Balancing Equations
Let's use hanger diagrams to help us solve equations.
Warm-Up
1. Hanger A is balanced.
Create a new balanced hanger that has the same weight for x.
New balanced hanger:
Hanger A:
$2x + 4 = 10$
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Warm-Up

Balancing Equations

Solving Equations With Balanced Hangers

Let's use hanger diagrams to help us solve equations.

Learning Outcomes

Today's Goals

1. **Goal:** Connect moves on hanger diagrams to moves that determine the value of a variable in an equation.
2. **Goal:** Solve equations of the form $px + q = r$ and $p(x + q) = r$ that involve positive rational numbers.
3. **Language Goal:** Describe how solving equations of the form $px + q = r$ and $p(x + q) = r$ connect to a hanger diagram. **(Reading, Writing, Speaking, and Listening)**

Students understand strategies for solving an equation and connect those to balanced moves on a hanger. Students also create their own hangers to match equations, and shift from solving for unknowns on hangers to solving for unknowns in equations.

Coherence

Prior Learning

In a previous course, students solved one-step equations in the form $x + p = q$ and $px = q$.

Future Learning

In Lesson 2, students will describe and use strategies for solving equations of the form $px + q = r$ or $p(x + q) = r$ that involve positive and negative numbers without a visual model.

Connections

- Students effectively connect the visual representation of a balanced hanger with the concept of an equation and its solution. (mathematical)

FL B.E.S.T. Math Benchmarks

Addressing

MA.7.AR.2.2
Write and solve two-step equations in one variable within a mathematical or real-world context, where all terms are rational numbers.

Also Addressing: **MA.7.NSO.2.2, MA.7.AR.1.2**

Mathematical Thinking and Reasoning Standards: MTR.2.1, MTR.4.1, MTR.5.1

ELA Expectations: ELA.K12.EE.1.1, ELA.K12.EE.5.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.6.AR.2.2 **MA.6.AR.2.3**	**MA.8.AR.2.1**

Lesson at a Glance

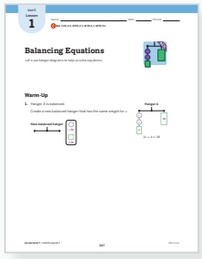
~ 45 min

MA.7.AR.2.2, MA.7.NSO.2.2, MA.7.AR.1.2,

Warm-Up

Pairs | 5 min

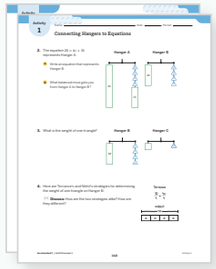
Students revisit balanced hangers and connect them to equations.



Activity 1

Pairs | 15 min

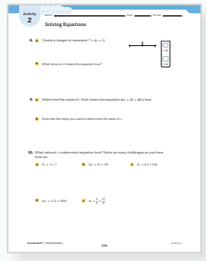
Students connect moves on hanger diagrams with moves used to determine the value of a variable in an equation. **Routines:** Decide and Defend, MLR7: Compare and Connect



Activity 2

Pairs | 15 min

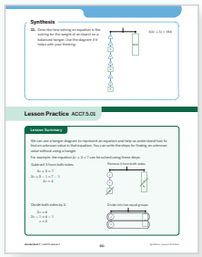
Students shift from solving for unknown weights of objects on hangers to solving for unknowns in equations.



Synthesis

Whole Class | 5 min

Students synthesize their understanding of the connections between moves on a hanger diagram and moves used to solve an equation.



Show What You Know

Independent | 5 min

Students demonstrate their understanding by determining the value of x in an equation.

Prep Checklist

This lesson includes:

- Student Edition
- Show What You Know
- Coloring tools (as needed)

Pairs | 5 min

Warm-Up

Purpose: Students revisit balanced hangers and connect them to equations.

1 Launch

Consider asking, "Where do you see each part of the equation in the hanger?" **(MTR.5.1)**

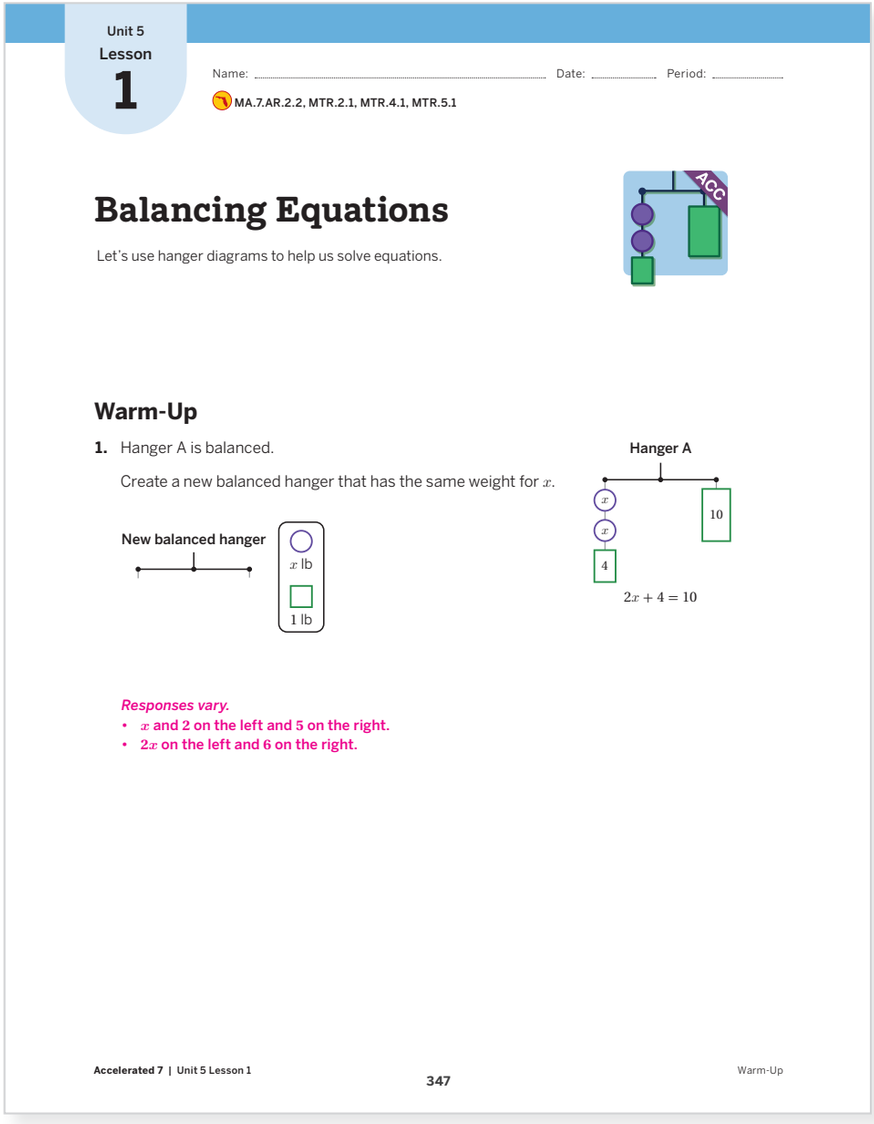
A **Accessibility: Conceptual Processing** Invite students to make connections to familiar contexts to support abstract thinking.

Invite students to share their hangers with a partner and give feedback on each other's work.

2 Connect

Invite students to share any connections they make between the hangers and their equations, or what they notice about each of the hangers that balance.

To surface the Key Takeaway, consider asking, "What value of x makes the hanger balanced?"



Unit 5
Lesson
1

Name: ..... Date: ..... Period: .....

MA.7.AR.2.2, MTR.2.1, MTR.4.1, MTR.5.1

Balancing Equations

Let's use hanger diagrams to help us solve equations.

ACC

Warm-Up

1. Hanger A is balanced.

Create a new balanced hanger that has the same weight for x.

New balanced hanger

x lb

1 lb

Hanger A

x x 4 10

$2x + 4 = 10$

Responses vary.

- x and 2 on the left and 5 on the right.
- $2x$ on the left and 6 on the right.

Accelerated 7 | Unit 5 Lesson 1 347 Warm-Up

Key Takeaway: Hanger diagrams can be used to represent equations where a value is unknown.

Pairs | 15 min

Activity 1 Connecting Hangers to Equations

Purpose: Students connect moves on hanger diagrams with moves used to determine the value of a variable in an equation.

1 Launch

To surface prior knowledge, consider asking, "How does a hanger stay balanced as new moves are made?" **(MTR.2.1)**

2 Monitor

To support students getting started, consider asking, "How are the two hangers related?"

D Differentiation

Look for students who:	Respond to Student Thinking
Think that the value of x is 4 because 4 triangles balance the rectangle.	**Support** Encourage them to determine the weight of the rectangle in Hanger C.
Divide 10 by 4.	**Strengthen** Invite them to share how this move keeps the hanger balanced.

Consider asking, "How can the weight you determined for one triangle be checked for accuracy in Hanger A and Hanger B?"

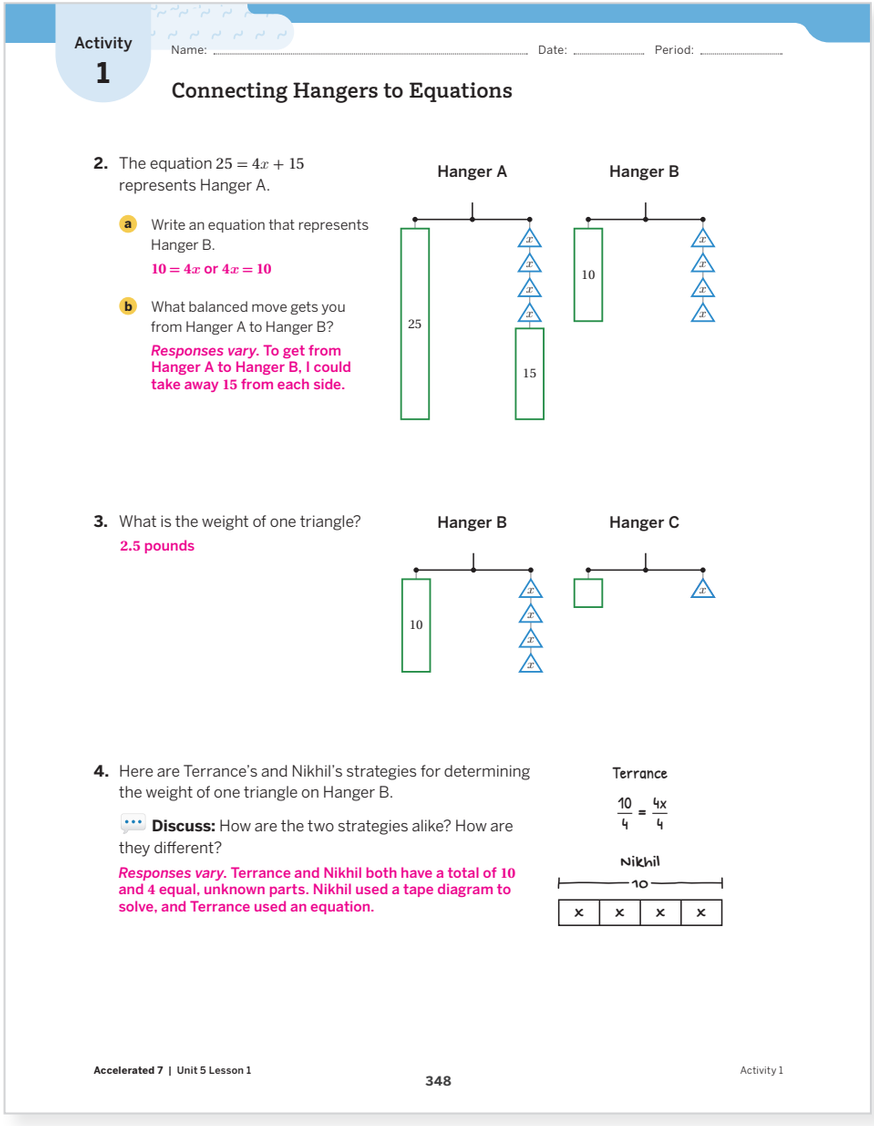
Listen for student responses mentioning the multiple representations.

MLR MLR7: Compare and Connect Invite students to analyze and compare each strategy, and then connect to their strategy. **(MTR.4.1)**

Consider asking:

- "What did each student do?"
- "Where do you see the same information in Terrance's and Nikhil's strategies?"
- "What do you think was helpful about each strategy?
- "How does each strategy compare to your own?"

Activity 1 continued >



Activity 1

Name: Date: Period:

Connecting Hangers to Equations

2. The equation $25 = 4x + 15$ represents Hanger A.

a. Write an equation that represents Hanger B.

$10 = 4x$ or $4x = 10$

b. What balanced move gets you from Hanger A to Hanger B?

Responses vary. To get from Hanger A to Hanger B, I could take away 15 from each side.

Hanger A

Hanger B

3. What is the weight of one triangle?

2.5 pounds

Hanger B

Hanger C

4. Here are Terrance's and Nikhil's strategies for determining the weight of one triangle on Hanger B.

Discuss: How are the two strategies alike? How are they different?

Responses vary. Terrance and Nikhil both have a total of 10 and 4 equal, unknown parts. Nikhil used a tape diagram to solve, and Terrance used an equation.

Terrance

$\frac{10}{4} = \frac{4x}{4}$

Nikhil

Accelerated 7 | Unit 5 Lesson 1 348 Activity 1

Pairs | 15 min

Activity 1 Connecting Hangers to Equations (continued)

Purpose: Students connect moves on hanger diagrams with moves used to determine the value of a variable in an equation.

2 Monitor

Support students getting started by asking, "What can you remove from the right and left side to keep this hanger balanced?" **(MTR.5.1)**

D Differentiation

Look for students who:	Respond to Student Thinking
Remove 6 from each side.	**Strengthen** Invite them to share how this move keeps the hanger balanced.
Split the hanger into two equal groups.	

Invite students to share their responses. Consider selecting and sequencing 1–2 responses for each of the most popular choices to display during the Connect.

Look for examples of student thinking to share during the Connect.

3 Connect

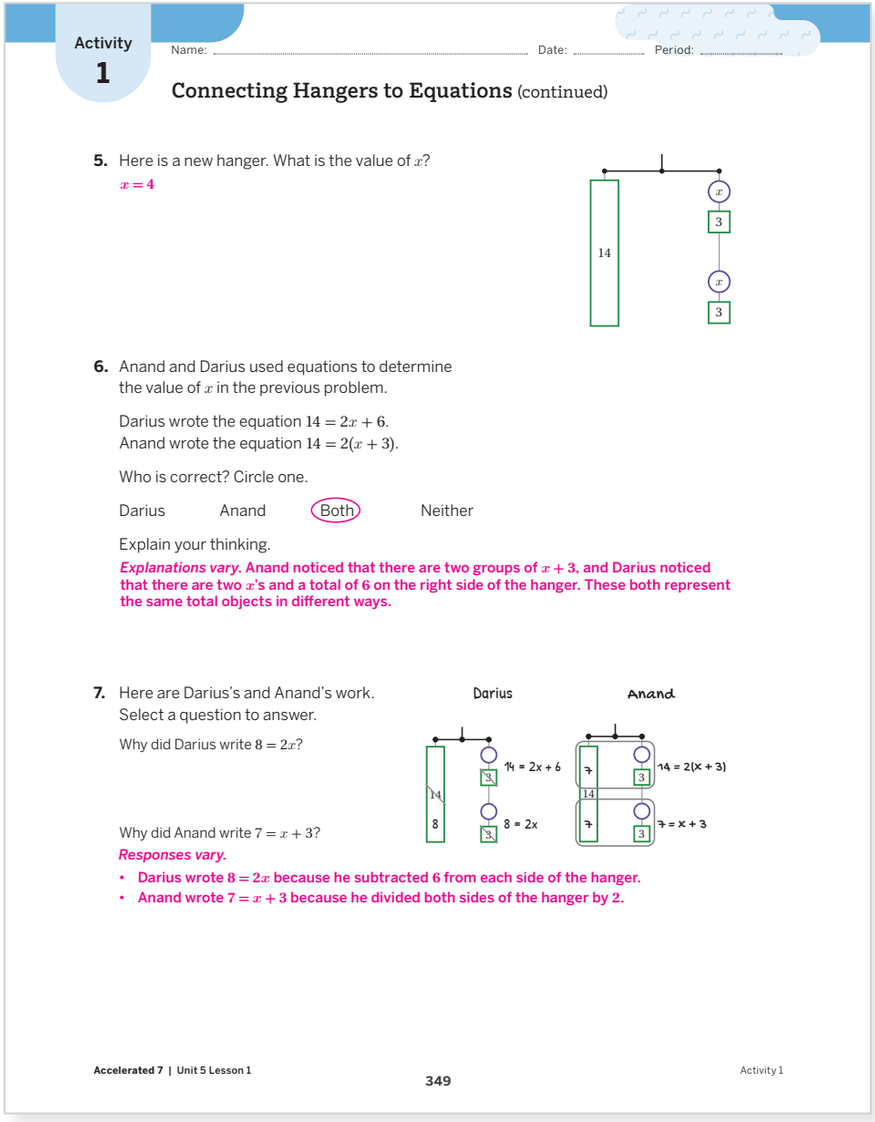
Use the Decide and Defend routine to support students in making arguments and critiquing the reasoning of others. **(MTR.4.1)**

Invite students who made different choices to use reasoning to convince their classmates.

Consider asking:
- "What reasoning or evidence supports your claim?"
- "What evidence or language was most convincing to you?"

Invite students to explain their reasoning for the question they chose to answer.

Consider asking, "What could each student's next step be to determine the weight of a circle? Why?"



Activity 1

Name: Date: Period:

Connecting Hangers to Equations (continued)

5. Here is a new hanger. What is the value of x?

$x = 4$

6. Anand and Darius used equations to determine the value of x in the previous problem.

Darius wrote the equation $14 = 2x + 6$.
Anand wrote the equation $14 = 2(x + 3)$.

Who is correct? Circle one.

Darius Anand (Both) Neither

Explain your thinking.

Explanations vary. Anand noticed that there are two groups of $x + 3$, and Darius noticed that there are two x's and a total of 6 on the right side of the hanger. These both represent the same total objects in different ways.

7. Here are Darius's and Anand's work. Select a question to answer.

Why did Darius write $8 = 2x$?

Why did Anand write $7 = x + 3$?

Responses vary.

- Darius wrote $8 = 2x$ because he subtracted 6 from each side of the hanger.
- Anand wrote $7 = x + 3$ because he divided both sides of the hanger by 2.

Accelerated 7 | Unit 5 Lesson 1 349 Activity 1

Pairs | 15 min

Activity 2 Solving Equations

Purpose: Students shift from solving for unknown weights of objects on hangers to solving for unknowns in equations.

Short on time? Consider inviting students to complete 1 or 2 problems on the Repeated Challenges.

1 Launch

Encourage students to think about how many of each shape are needed on each side of the hanger to represent the equation. **(MTR.2.1)**

Invite students to share their responses and consider asking, "Does it matter which side of the hanger the 7 is on?"

2 Monitor

Select and sequence student strategies to share during the Connect.

D Differentiation

Look for students who:	Respond to Student Thinking
Use a variety of strategies to determine the value of x such as: • Using guess-and-check. • Rewriting the equation as $4x + 8 = 40.4$. • Dividing both sides by 4 first.	**Strengthen** Invite these students to share their thinking.
Connect equations to hanger diagrams.	**Strengthen** Invite them to share how each equation would look if represented on a hanger diagram.
Would like to explore using hangers to solve equations further.	**Stretch** Invite students to write their own equation, draw a hanger to represent the equation, and then solve it to determine the value of the variable.

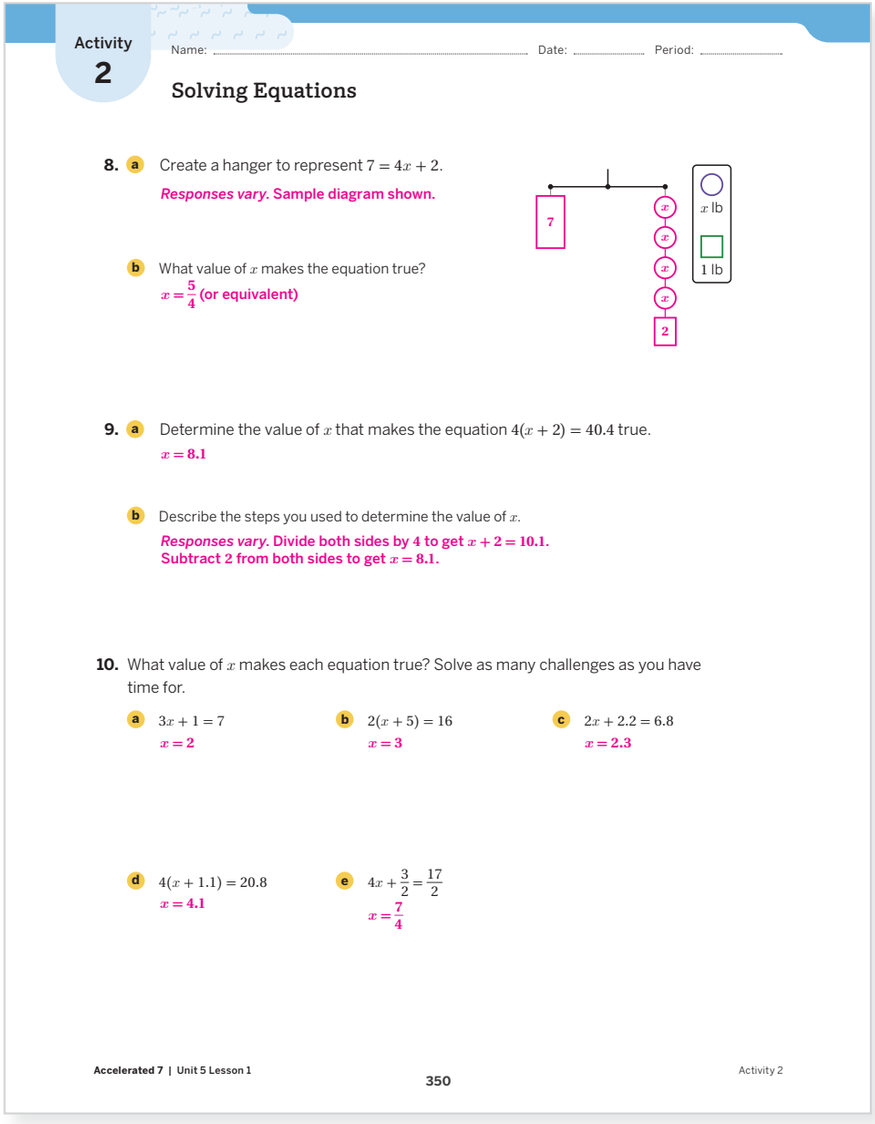
A Accessibility: Executive Functioning Invite students to draw a hanger diagram representing the equation and color-code corresponding features.

Look for how many challenges each student has completed correctly.

3 Connect

Invite students to discuss the steps they used to determine the value of x. Consider asking, "How can you check your solution to an equation?"

To surface the Key Takeaway, consider asking, "How are the equations $4x + 8 = 40.4$ and $4(x + 2) = 40.4$ related?" **(MTR.5.1)**



Activity 2

Name: Date: Period:

Solving Equations

8. a. Create a hanger to represent $7 = 4x + 2$.

Responses vary. Sample diagram shown.

b. What value of x makes the equation true?

$x = \frac{5}{4}$ (or equivalent)

9. a. Determine the value of x that makes the equation $4(x + 2) = 40.4$ true.

$x = 8.1$

b. Describe the steps you used to determine the value of x.

Responses vary. Divide both sides by 4 to get $x + 2 = 10.1$. Subtract 2 from both sides to get $x = 8.1$.

10. What value of x makes each equation true? Solve as many challenges as you have time for.

a. $3x + 1 = 7$, $x = 2$

b. $2(x + 5) = 16$, $x = 3$

c. $2x + 2.2 = 6.8$, $x = 2.3$

d. $4(x + 1.1) = 20.8$, $x = 4.1$

e. $4x + \frac{3}{2} = \frac{17}{2}$, $x = \frac{7}{4}$

Accelerated 7 | Unit 5 Lesson 1 350 Activity 2

Key Takeaway: There are multiple strategies to solve an equation such as connecting the moves on a hanger diagram and removing the same weight on both sides first or dividing the hanger into equal groups first.

Whole Class | 5 min

Synthesis

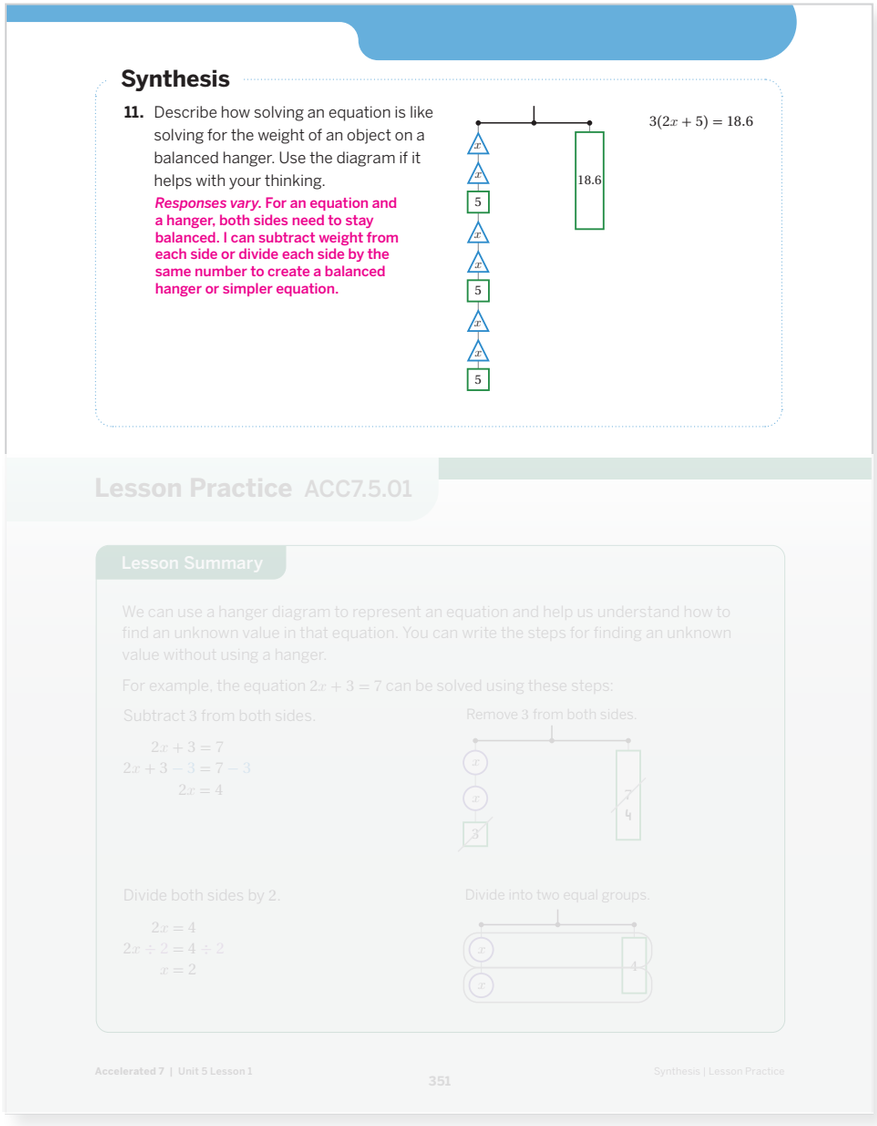
Purpose: Students synthesize their understanding of the connections between moves on a hanger diagram and moves used to solve an equation.

Invite students to respond independently, and then share their thinking with a partner.

Record and share a variety of ideas, such as:
- Applying the same move to both sides of the hanger and the equation.
- Keeping both sides of the hanger and the equation balanced.

Consider asking, "What connections do you see between these responses?"

Note: If time allows, invite students to share with a partner what makes sense about each response and what questions they still have.



Synthesis

11. Describe how solving an equation is like solving for the weight of an object on a balanced hanger. Use the diagram if it helps with your thinking.

Responses vary. For an equation and a hanger, both sides need to stay balanced. I can subtract weight from each side or divide each side by the same number to create a balanced hanger or simpler equation.

Lesson Practice ACC7.5.01

Lesson Summary

We can use a hanger diagram to represent an equation and help us understand how to find an unknown value in that equation. You can write the steps for finding an unknown value without using a hanger.

For example, the equation $2x + 3 = 7$ can be solved using these steps:

Subtract 3 from both sides.

$$2x + 3 = 7$$
$$2x + 3 - 3 = 7 - 3$$
$$2x = 4$$

Remove 3 from both sides.

Divide both sides by 2.

$$2x = 4$$
$$2x \div 2 = 4 \div 2$$
$$x = 2$$

Divide into two equal groups.

Accelerated 7 | Unit 5 Lesson 1 351 Synthesis | Lesson Practice

Lesson Takeaway: When solving an equation of the form $p(x + q) = r$ using a hanger diagram, different strategies can be helpful, such as removing weight on both sides first or dividing the hanger into equal groups.

Independent | 5 min

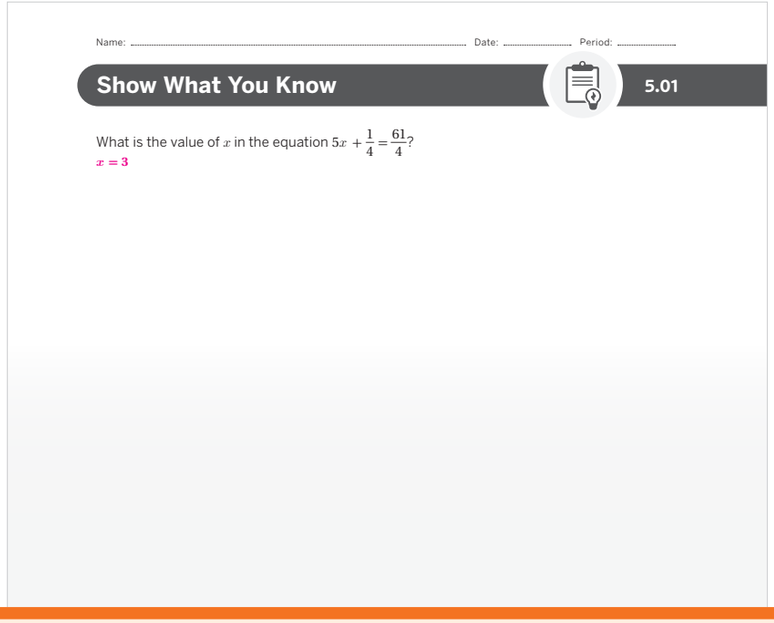
Show What You Know

Purpose: Students demonstrate their understanding by determining the value of x in an equation.

Today's Goals

1. **Goal:** Connect moves on hanger diagrams to moves that determine the value of a variable in an equation.
2. **Goal:** Solve equations of the form $px + q = r$ and $p(x + q) = r$ that involve positive rational numbers.
3. **Language Goal:** Describe how solving equations of the form $px + q = r$ and $p(x + q) = r$ connect to a hanger diagram. **(Reading, Writing, Speaking, and Listening)**

Show What You Know (Assessment Resources)



Name: _____ Date: _____ Period: _____

Show What You Know 5.01

What is the value of x in the equation $5x + \frac{1}{4} = \frac{61}{4}$?

$x = 3$

D Differentiation Use after Lesson 1

Support

Provide targeted intervention for students by using these resources.

If students need support understanding how to solve equations in the form $px + q = r$:

» **Respond to Student Thinking:**

- Review the Lesson Summary.
- Select a few related **Practice Problems** and give individualized feedback

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 1 Extension Activities**.

Professional Learning

Reflect on the facilitation of the **MLR7: Compare and Connect** routine in this lesson.

- What about the routine felt successful?
- What are some areas for growth?
- What other opportunities might there be in the next few lessons for inviting students to analyze and compare strategies for solving equations?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

11. Describe how solving an equation is like solving for the weight of an object on a balanced hanger. Use the diagram if it helps with your thinking.

Responses vary. For an equation and a hanger, both sides need to stay balanced. I can subtract weight from each side or divide each side by the same number to create a balanced hanger or simpler equation.

Lesson Practice ACC7.5.01

Lesson Summary

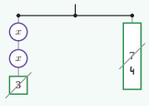
We can use a hanger diagram to represent an equation and help us understand how to find an unknown value in that equation. You can write the steps for finding an unknown value without using a hanger.

For example, the equation $2x + 3 = 7$ can be solved using these steps:

Subtract 3 from both sides.

$$2x + 3 = 7$$
$$2x + 3 - 3 = 7 - 3$$
$$2x = 4$$

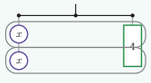
Remove 3 from both sides.



Divide both sides by 2.

$$2x = 4$$
$$2x \div 2 = 4 \div 2$$
$$x = 2$$

Divide into two equal groups.



Lesson Practice
ACC7.5.01

Name: _____ Date: _____ Period: _____

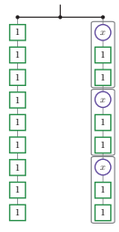
Problems 1–3: Solve each equation.

1. $x - 1 = 5$
$x = 6$

2. $2(x - 1) = 10$
$x = 6$

3. $500 = 100(x - 1)$
$x = 6$

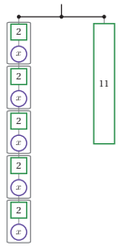
4. Explain how each part of the equation $9 = 3(x + 2)$ is represented in the hanger.



Responses vary.

- The circle has an unknown weight, so we use x to represent it.
- The 9 in the equation is represented by the left side of the hanger with 9 squares, each weighing 1 unit.
- There are 3 groups of $x + 2$ on the right side. This represents the $3(x + 2)$ in the equation.
- The equal sign means that the hanger is balanced.

Problems 5–6: Use the hanger.



5. Write an equation that represents this hanger.

Responses vary.

- $5(x + 2) = 11$
- $10 + 5x = 11$

6. What is the value of x that makes the equation true?

$x = \frac{1}{5}$ (or equivalent)

Lesson Practice
ACC7.5.01

Name: _____ Date: _____ Period: _____

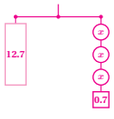
FAST Practice

Problems 7–8: Consider the equation $12.7 = 3x + 0.7$.

7. Draw a hanger to represent the equation in the space provided.

Sample diagram shown.

Your Diagram



8. What is the value of x that makes the equation true?

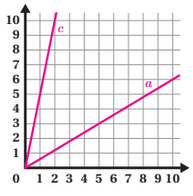
$x =$ 4

Spiral Review

Problems 9–10: Draw a line for each slope. Then label the line.

9. Line a has a slope of $\frac{3}{5}$.

10. Line c has a slope of 5.

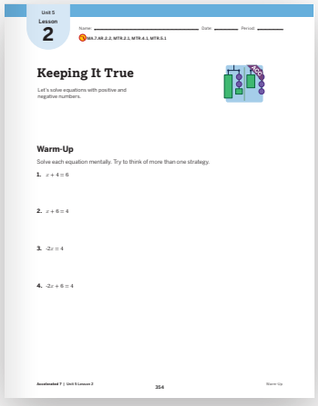


11. Which value is equivalent to $-9 + 5 + 3 - (-8)$?

A. -19 B. -9 C. -1 D. 7

Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1–3	1	MA.7.AR.2.2
	4–6	2	MA.7.AR.2.2
FAST Practice	7–8	2	MA.7.AR.2.2
Spiral Review			
	9–10	2	MA.8.AR.3.2
	11	2	MA.6.NSO.4.1


Unit 5
Lesson 2
Name: _____ Date: _____ Period: _____
MA.7.AR.2.2, MTR.2.1, MTR.4.1, MTR.5.1
Keeping It True
Let's solve equations with positive and negative numbers.
Warm-Up
Solve each equation mentally. Try to think of more than one strategy.
1. $x + 4 = 6$
2. $x + 6 = 4$
3. $2x = 4$
4. $-2x + 6 = 4$
Accelerated 7 | Unit 5 Lesson 2
354
Warm-Up

Keeping It True

Solving Equations

Let's solve equations with positive and negative numbers.

Learning Outcomes

Today's Goals

1. **Goal:** Solve equations of the form $px + q = r$ and $p(x + q) = r$ that involve positive and negative numbers.
2. **Language Goal:** Explain why a solution to an equation is accurate. **(Reading and Writing)**

Students describe and use strategies for solving equations of the form $px + q = r$ or $p(x + q) = r$ that involve positive and negative numbers. This is the first lesson where students are not given a visual model (either a tape diagram or a hanger diagram) to support their reasoning.

Coherence

Prior Learning

In Lesson 1, students connected moves on a hanger diagram to moves that determine the unknown value in an equation.

Future Learning

In Lesson 3, students will solve equations in multiple ways, often by dividing first or using the distributive property, and then compare the results.

Connections

- Students continue to effectively connect the visual representation of a balanced hanger with the concept of an equation and its solution. (mathematical)

FL B.E.S.T. Math Benchmarks

Addressing

MA.7.AR.2.2
Write and solve two-step equations in one variable within a mathematical or real-world context, where all terms are rational numbers.

Also Addressing: **MA.7.NSO.2.2**

Mathematical Thinking and Reasoning Standards: MTR.2.1, MTR.4.1, MTR.5.1

ELA Expectations: ELA.K12.EE.1.1, ELA.K12.EE.3.1, ELA.K12.EE.4.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On MA.6.AR.2.2, MA.6.AR.2.3

Building Toward

MA.8.AR.2.1

Vocabulary

Review Vocabulary

solution to an equation

Lesson at a Glance

~ 45 min

MA.7.AR.2.2, MA.7.NSO.2.2

Warm-Up

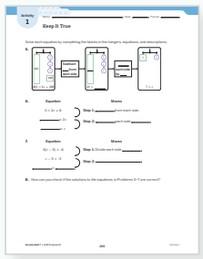
Independent | 7 min

Students surface strategies for solving equations, including those with negative numbers, using the **Number Talk** routine.

Activity 1

Small Groups | 10 min

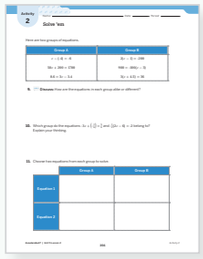
Students connect their understanding of balanced hangers from Lesson 1 to steps for solving an equation. **MLR2: Collect and Display**



Activity 2

Pairs | 18 min

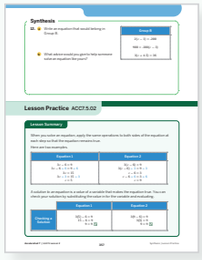
Students use the structure of equations to group them according to how they would be solved, then practice solving some of the equations. **Routine: Think-Pair-Share** **(MTR.5.1)**



Synthesis

Whole Class | 5 min

Students synthesize their understanding of the structure of equations by creating their own equation and providing advice on how it can be solved.



Show What You Know

Independent | 5 min

Students demonstrate their understanding by solving equations that require different first steps in solving.

Prep Checklist

This lesson includes:

- Student Edition
- Show What You Know

Independent | 7 min

Warm-Up

Purpose: Students surface strategies for solving equations, including those with negative numbers, using the **Number Talk** routine.

1 Launch

Use the Number Talk routine to support students in determining the value of x in an equation. See the Routine Facilitation Guide for more information. **(MTR.5.1)**

Invite students to solve the first equation independently. Then record a variety of strategies, along with the name of the student who shared them.

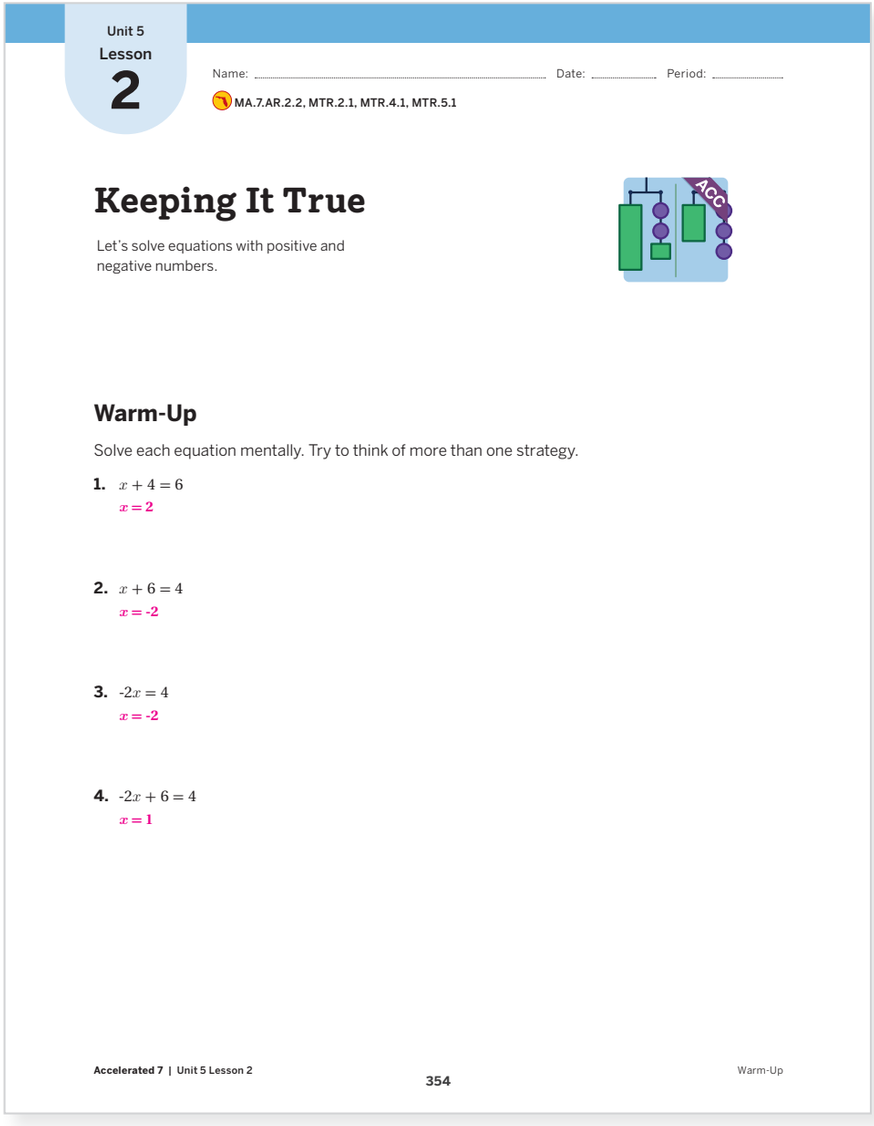
Consider asking, "Who thought about it in a different way?"

Note: This structure is repeated for each problem in the Number Talk.

2 Connect

Consider asking:

- "What does determining the *solution to an equation* mean?"
- "How can you tell whether a value for a variable in an equation makes the equation true?"



Unit 5
Lesson
2

Name: ..... Date: ..... Period: .....

MA.7.AR.2.2, MTR.2.1, MTR.4.1, MTR.5.1

Keeping It True

Let's solve equations with positive and negative numbers.

Warm-Up

Solve each equation mentally. Try to think of more than one strategy.

1. $x + 4 = 6$
   $x = 2$

2. $x + 6 = 4$
   $x = -2$

3. $-2x = 4$
   $x = -2$

4. $-2x + 6 = 4$
   $x = 1$

Accelerated 7 | Unit 5 Lesson 2 354 Warm-Up

Small Groups | 10 min

Activity 1 Keep It True

Purpose: Students connect their understanding of balanced hangers from Lesson 1 to steps for solving an equation.

Short on time? Consider discussing Problem 8 as a class.

1 Launch

Direct students' attention to the hanger and equation.

Invite students to share their thinking about why someone might subtract from both sides first. **(MTR.2.1)**

2 Monitor

Select and sequence student thinking to discuss during the Connect.

D Differentiation

Look for students who:	Respond to Student Thinking
Need support getting started. *(Problem 5)*	**Support** Reference the hanger diagram and ask, "What move was made to remove 180 from the right side of the diagram?"
Check their solution using substitution. *(Problems 6–7)*	**Strengthen** Invite students to share how this helps them.
Think about solving the equation using a different first step. *(Problem 7)*	**Stretch** Invite students to share how to solve the equation in a different way.

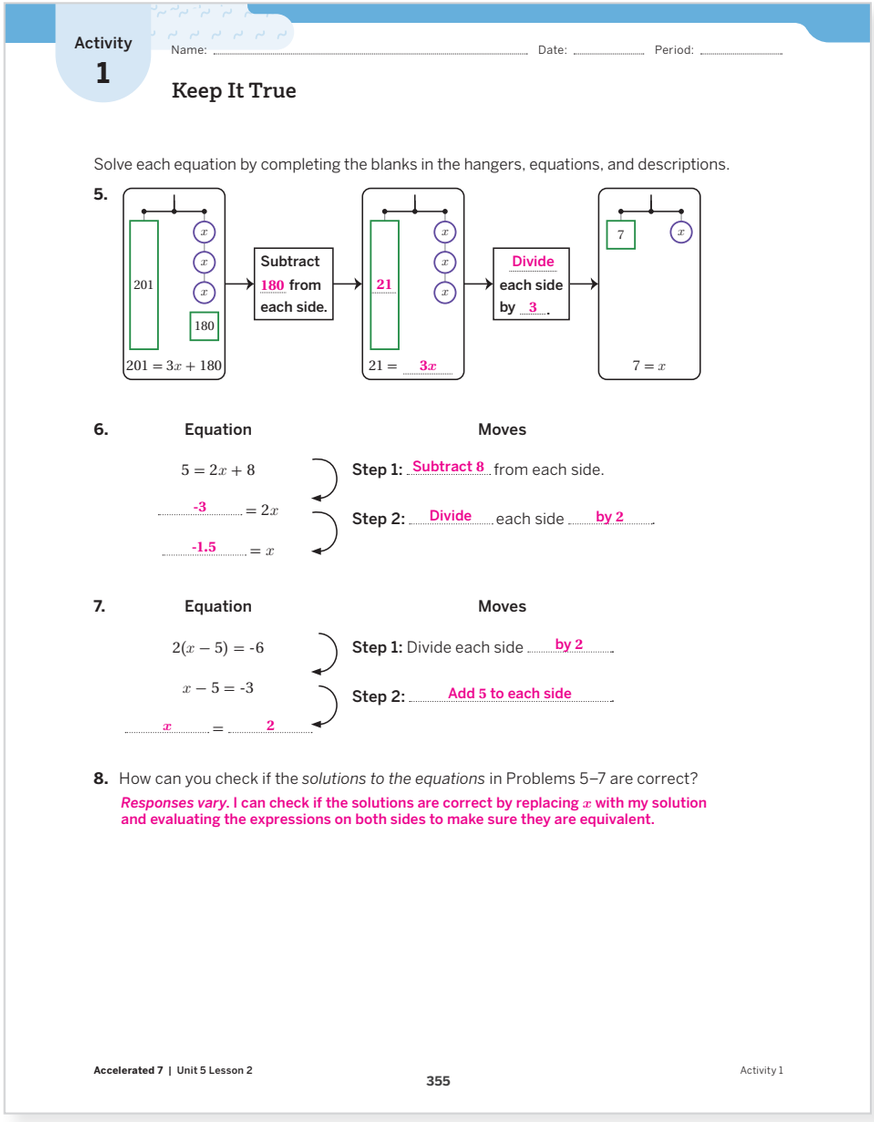
A Accessibility: Conceptual Processing Create or review an anchor chart that publicly displays possible steps for solving an equation.

3 Connect

Invite students to share their responses and strategies for each problem. Encourage students to share how to check their solution.

MLR MLR2: Collect and Display Use this routine to collect students' language, as they solve an equation. Display for students to reference and add to for the rest of the unit.

Direct students' attention to the equation and discuss the strategy of using the distributive property first to solve the equation.



Activity 1

Name: Date: Period:

Keep It True

Solve each equation by completing the blanks in the hangers, equations, and descriptions.

5.

6.

Equation	Moves
$5 = 2x + 8$	**Step 1:** Subtract 8 from each side.
-3 $= 2x$	**Step 2:** Divide each side by 2.
-1.5 $= x$	

7.

Equation	Moves
$2(x - 5) = -6$	**Step 1:** Divide each side by 2.
$x - 5 = -3$	**Step 2:** Add 5 to each side.
$x = 2$	

8. How can you check if the *solutions to the equations* in Problems 5–7 are correct?

Responses vary. I can check if the solutions are correct by replacing x with my solution and evaluating the expressions on both sides to make sure they are equivalent.

Accelerated 7 | Unit 5 Lesson 2 355 Activity 1

Pairs | 18 min

Activity 2 Solve 'em

Purpose: Students use the structure of equations to group them according to how they would be solved, then practice solving some of the equations. **(MTR.5.1)**

1 Launch

To surface prior knowledge, consider asking, "What operation is represented by a number outside of a set of parentheses?"

Use the Think-Pair-Share routine to support student-to-student discourse. Invite students to think independently and then take turns sharing with a partner. Invite several students to share their or their partner's thinking with the class.

Direct students' attention to the equations and invite them to share their partner's reasoning instead of their own. **(MTR.4.1)**

2 Monitor

Select and sequence student thinking to share during the Connect.

D Differentiation

Look for students who:	Respond to Student Thinking
Need support getting started. *(Problem 9)*	**Support** Consider asking, "What is a first step that you can take to solve each equation in Group A? Group B?"
Think the equation $-3x + \left(-\frac{1}{6}\right) = \frac{5}{6}$ belongs in Group B. *(Problem 10)*	**Support** Consider asking, "How is the use of parentheses different in the equations in Group A versus those in Group B?"
Check their solutions. *(Problem 11)*	**Strengthen** Invite students to share how they can ensure the accuracy of their solutions.

A Accessibility: Conceptual Processing Invite students to look at the relationship between the equations in each group to support identifying patterns.

3 Connect

Invite students to share how they used the structure of the equations to determine the placement of the two equations. **(MTR.5.1)**

To surface the Key Takeaway, consider asking:

- "What strategies were helpful to solve the equations in Group A?"
- "What strategies were helpful to solve the equations in Group B?"

Activity 2

Name: Date: Period:

Solve 'em

Here are two groups of equations.

Group A	Group B
$x - (-4) = -6$	$2(x - 1) = -200$
$50x + 200 = 1700$	$900 = -100(x - 3)$
$8.6 = 3x - 3.4$	$3(x + 4.5) = 36$

9. **Discuss:** How are the equations in each group alike or different?

Responses vary.

- Group A has equations that can be solved by adding or subtracting as the first step.
- Group B has equations that can be solved by dividing as the first step.
- Both groups have equations that contain positive numbers, negative numbers, fractions, and decimals.

10. Which group do the equations $-3x + \left(-\frac{1}{6}\right) = \frac{5}{6}$ and $-\frac{1}{2}(2x - 6) = -2$ belong to? Explain your thinking.

Responses vary. The equation $-3x + \left(-\frac{1}{6}\right) = \frac{5}{6}$ belongs in Group A because it could be solved by adding first. The equation $-\frac{1}{2}(2x - 6) = -2$ belongs in Group B because it could be solved by dividing first.

11. Choose two equations from *each* group to solve. *Responses vary.*

	Group A	Group B
Equation 1	$x - (-4) = -6, x = -10$ $50x + 200 = 1700, x = 30$	$2(x - 1) = -200, x = -99$ $900 = -100(x - 3), x = -6$
Equation 2	$8.6 = 3x - 3.4, x = 4$ $-3x + \left(-\frac{1}{6}\right) = \frac{5}{6}, x = -\frac{1}{3}$	$3(x + 4.5) = 36, x = 7.5$ $-\frac{1}{2}(2x - 6) = -2, x = 5$

Accelerated 7 | Unit 5 Lesson 2 356 Activity 2

Key Takeaway: The structure of an equation can help you determine how to solve it. For equations like $p(x + q) = r$, you might start by dividing by p or distributing p. For equations like $px + q = r$, you might add or subtract as a first step instead.

Whole Class | 5 min

Synthesis

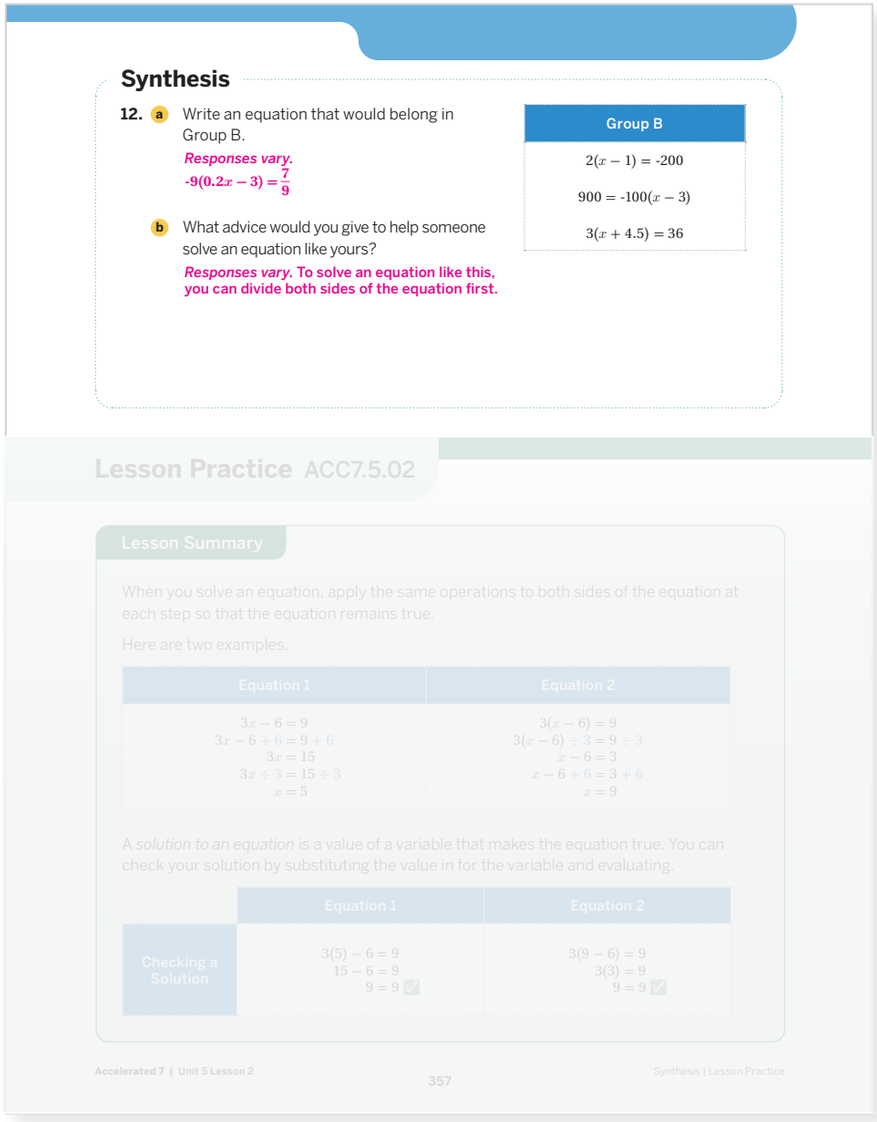
Purpose: Students synthesize their understanding of the structure of equations by creating their own equation and providing advice on how it can be solved.

Invite students to respond independently, and then share their thinking with a partner.

Collect and share a variety of ideas, such as:

- Giving the advice of dividing both sides of the equation first.
- Giving the advice of distributing first.

Note: If time allows, invite students to revise their previous thinking based on the discussion.



Synthesis

12. **a** Write an equation that would belong in Group B.

Responses vary.

$-9(0.2x - 3) = \frac{7}{9}$

Group B
$2(x - 1) = -200$
$900 = -100(x - 3)$
$3(x + 4.5) = 36$

b What advice would you give to help someone solve an equation like yours?

Responses vary. To solve an equation like this, you can divide both sides of the equation first.

Lesson Practice ACC7.5.02

Lesson Summary

When you solve an equation, apply the same operations to both sides of the equation at each step so that the equation remains true.

Here are two examples.

Equation 1	Equation 2
$3x - 6 = 9$ $3x - 6 + 6 = 9 + 6$ $3x = 15$ $3x \div 3 = 15 \div 3$ $x = 5$	$3(x - 6) = 9$ $3(x - 6) \div 3 = 9 \div 3$ $x - 6 = 3$ $x - 6 + 6 = 3 + 6$ $x = 9$

A *solution to an equation* is a value of a variable that makes the equation true. You can check your solution by substituting the value in for the variable and evaluating.

	Equation 1	Equation 2
Checking a Solution	$3(5) - 6 = 9$ $15 - 6 = 9$ $9 = 9$ ✓	$3(9 - 6) = 9$ $3(3) = 9$ $9 = 9$ ✓

Accelerated 7 | Unit 5 Lesson 2 357 Synthesis | Lesson Practice

Lesson Takeaway: The first step in solving equations may be different depending on the structure of the equation, but it is important that both sides of the equation remain balanced.

Independent | 5 min

Show What You Know

Purpose: Students demonstrate their understanding by solving equations that require different first steps in solving.

Today's Goals

1. **Goal:** Solve equations of the form $px + q = r$ and $p(x + q) = r$ that involve positive and negative numbers.
2. **Language Goal:** Explain why a solution to an equation is accurate. **(Reading and Writing)**

Show What You Know (Assessment Resources)

Name: _____ Date: _____ Period: _____

Show What You Know 5.02

Solve each equation. Show your thinking.

a. $-3x - 5 = 16$

$x = -7$

Work varies.

$-3x - 5 + 5 = 16 + 5$

$-3x = 21$

$-3x \div -3 = 21 \div -3$

$x = -7$

b. $12 = -4(x - 2)$

$x = -1$

Work varies.

$\frac{12}{-4} = \frac{-4}{-4}(x - 2)$

$-3 = x - 2$

$-3 + 2 = x - 2 + 2$

$-1 = x$

D Differentiation Use after Lesson 2

Support

Provide targeted intervention for students by using these resources.

If students need support understanding how to solve equations:

» **Respond to Student Thinking:**

- Use the ***Solving Equations With Positive and Negative Numbers*** **Mini-Lesson**.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 1 Extension Activities**.
- Invite students to write two equations with the same solution, one in the form $px + q = r$ and one in the form $p(x + q) = r$.

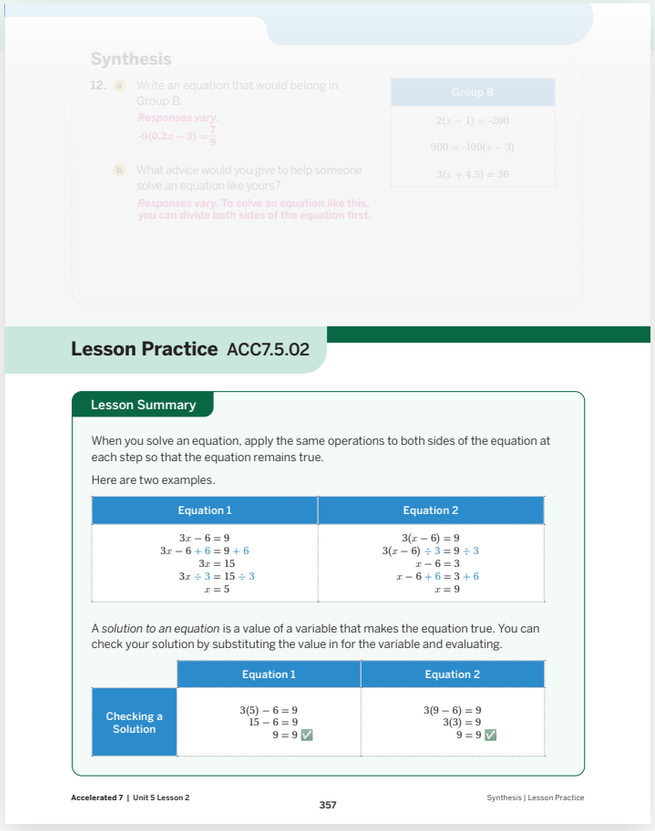
Professional Learning

What strengths did your students exhibit today? How can you build from those strengths as students continue learning about strategies for solving equations with and without visual models?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.



Synthesis

12. a. Write an equation that would belong in Group B.

Responses vary.

$-9(0.2x - 3) = \frac{7}{9}$

Group B
$2(x - 1) = -200$
$900 = -100(x - 3)$
$3(x + 4.5) = 36$

b. What advice would you give to help someone solve an equation like yours?

Responses vary. To solve an equation like this, you can divide both sides of the equation first.

Lesson Practice ACC7.5.02

Lesson Summary

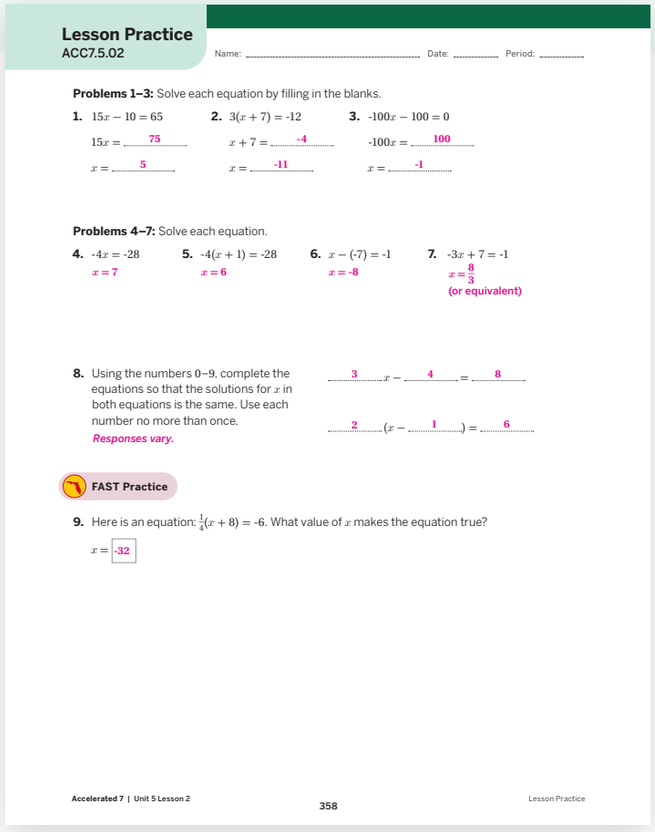
When you solve an equation, apply the same operations to both sides of the equation at each step so that the equation remains true.

Here are two examples.

Equation 1	Equation 2
$3x - 6 = 9$ $3x - 6 + 6 = 9 + 6$ $3x = 15$ $3x \div 3 = 15 \div 3$ $x = 5$	$3(x - 6) = 9$ $3(x - 6) \div 3 = 9 \div 3$ $x - 6 = 3$ $x - 6 + 6 = 3 + 6$ $x = 9$

A *solution to an equation* is a value of a variable that makes the equation true. You can check your solution by substituting the value in for the variable and evaluating.

	Equation 1	Equation 2
Checking a Solution	$3(5) - 6 = 9$ $15 - 6 = 9$ $9 = 9$ ✅	$3(9 - 6) = 9$ $3(3) = 9$ $9 = 9$ ✅



Lesson Practice
ACC7.5.02

Name: _____ Date: _____ Period: _____

Problems 1–3: Solve each equation by filling in the blanks.

1. $15x - 10 = 65$

   $15x =$ 75

   $x =$ 5

2. $3(x + 7) = -12$

   $x + 7 =$ -4

   $x =$ -11

3. $-100x - 100 = 0$

   $-100x =$ 100

   $x =$ -1

Problems 4–7: Solve each equation.

4. $-4x = -28$

   $x = 7$

5. $-4(x + 1) = -28$

   $x = 6$

6. $x - (-7) = -1$

   $x = -8$

7. $-3x + 7 = -1$

   $x = \frac{8}{3}$ (or equivalent)

8. Using the numbers 0–9, complete the equations so that the solutions for x in both equations is the same. Use each number no more than once.

   Responses vary.

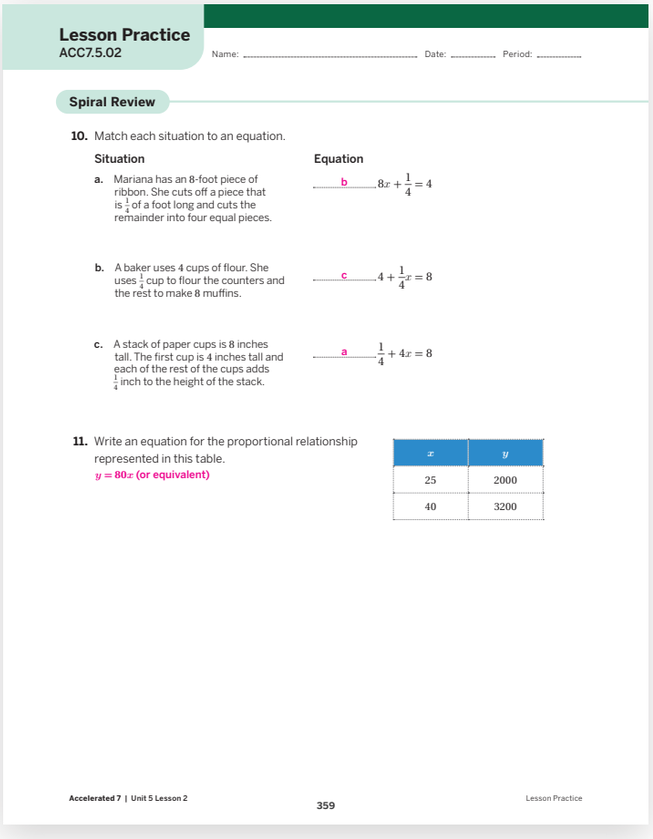
   3 $x -$ 4 $=$ 8

   2 $(x -$ 1 $) =$ 6

FAST Practice

9. Here is an equation: $\frac{1}{4}(x + 8) = -6$. What value of x makes the equation true?

   $x =$ -32



Lesson Practice
ACC7.5.02

Name: _____ Date: _____ Period: _____

Spiral Review

10. Match each situation to an equation.

Situation	Equation
a. Mariana has an 8-foot piece of ribbon. She cuts off a piece that is $\frac{1}{4}$ of a foot long and cuts the remainder into four equal pieces.	b. $8x + \frac{1}{4} = 4$
b. A baker uses 4 cups of flour. She uses $\frac{1}{4}$ cup to flour the counters and the rest to make 8 muffins.	c. $4 + \frac{1}{4}x = 8$
c. A stack of paper cups is 8 inches tall. The first cup is 4 inches tall and each of the rest of the cups adds $\frac{1}{4}$ inch to the height of the stack.	a. $\frac{1}{4} + 4x = 8$

11. Write an equation for the proportional relationship represented in this table.

$y = 80x$ (or equivalent)

x	y
25	2000
40	3200

Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1–7	2	MA.7.AR.2.2
	8	3	MA.7.AR.2.2
FAST Practice	9	2	MA.7.AR.2.2
Spiral Review			
	10	2	MA.7.AR.2.2
	11	2	MA.8.AR.3.3

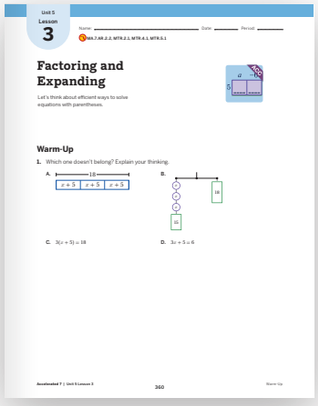
Unit 5
Lesson
3

ACC Accelerated

Factoring and Expanding

Options for Solving One Equation

Let's think about efficient ways to solve equations with parentheses.


Unit 5
Lesson
3
Factoring and Expanding
Let's think about efficient ways to solve equations with parentheses.
Warm-Up
1. Which one doesn't belong? Explain your thinking.
A.
B.
C. $3(x+5)=18$
D. $3x+5=6$
360

Learning Outcomes

Today's Goals

1. **Goal:** *Expand* and *factor* expressions that include positive and negative coefficients.
2. **Goal:** Solve equations that involve *expanding* an expression.
3. **Language Goal:** Analyze different strategies for solving the same equation. **(Reading and Writing)**

Students extend what they know about distributing and solving to **factor** and **expand** expressions, and then solve equations in multiple ways and compare the results.

Coherence

Prior Learning

In Lesson 2, students described and used strategies for solving equations of the form $px + q = r$ and $p(x + q) = r$ without a visual model.

Future Learning

In Lesson 4, students will begin solving more complex equations including adding and expanding expressions.

Connections

- Students use visual hanger diagrams to help understand the distributive property and balance. (mathematical)

FL B.E.S.T. Math Benchmarks

Addressing

MA.7.AR.2.2
Write and solve two-step equations in one variable within a mathematical or real-world context, where all terms are rational numbers.

Mathematical Thinking and Reasoning Standards: MTR.2.1, MTR.4.1, MTR.5.1

ELA Expectations: ELA.K12.EE.1.1, ELA.K12.EE.5.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.6.AR.1.4	**MA.8.AR.1.3**
MA.6.AR.2.2	**MA.8.AR.2.1**
MA.6.AR.2.3	

Vocabulary

New Vocabulary

expand	factor

Review Vocabulary

coefficient	*distributive property*

Lesson at a Glance

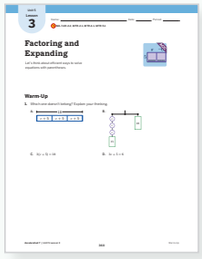
~ 45 min

MA.7.AR.2.2

Warm-Up

Pairs | 5 min

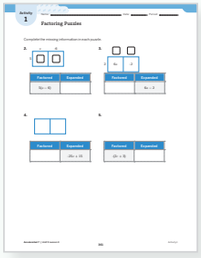
Students compare and contrast visual representations and equations in **factored** and **expanded** form using the Which One Doesn't Belong? routine.



Activity 1

Pairs | 10 min

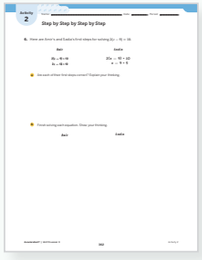
Students practice factoring and expanding expressions after analyzing diagrams using the Notice and Wonder routine.



Activity 2

Pairs | 10 min

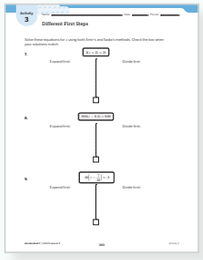
Students compare and contrast strategies to solve equations of the form $p(x + q) = r$ using the MLR7: Compare and Connect routine. **(MTR.4.1)**



Activity 3

Pairs | 10 min

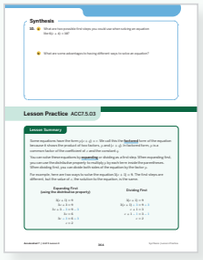
Students practice solving equations by expanding and dividing by a common factor.



Synthesis

Whole Class | 5 min

Students synthesize their understanding of solving equations of the form $p(x + q) = r$ by expanding first or by dividing first, and examine the advantages of both strategies.
MLR2: Collect and Display



Show What You Know

Independent | 5 min

Students demonstrate their understanding by solving an equation of the form $p(x + q) = r$.

Prep Checklist

This lesson includes:

- Student Edition
- Show What You Know
- Coloring tools (as needed)

Pairs | 5 min

Warm-Up

Purpose: Students compare and contrast visual representations and equations in factored and expanded form using the Which One Doesn't Belong? routine.

1 Launch

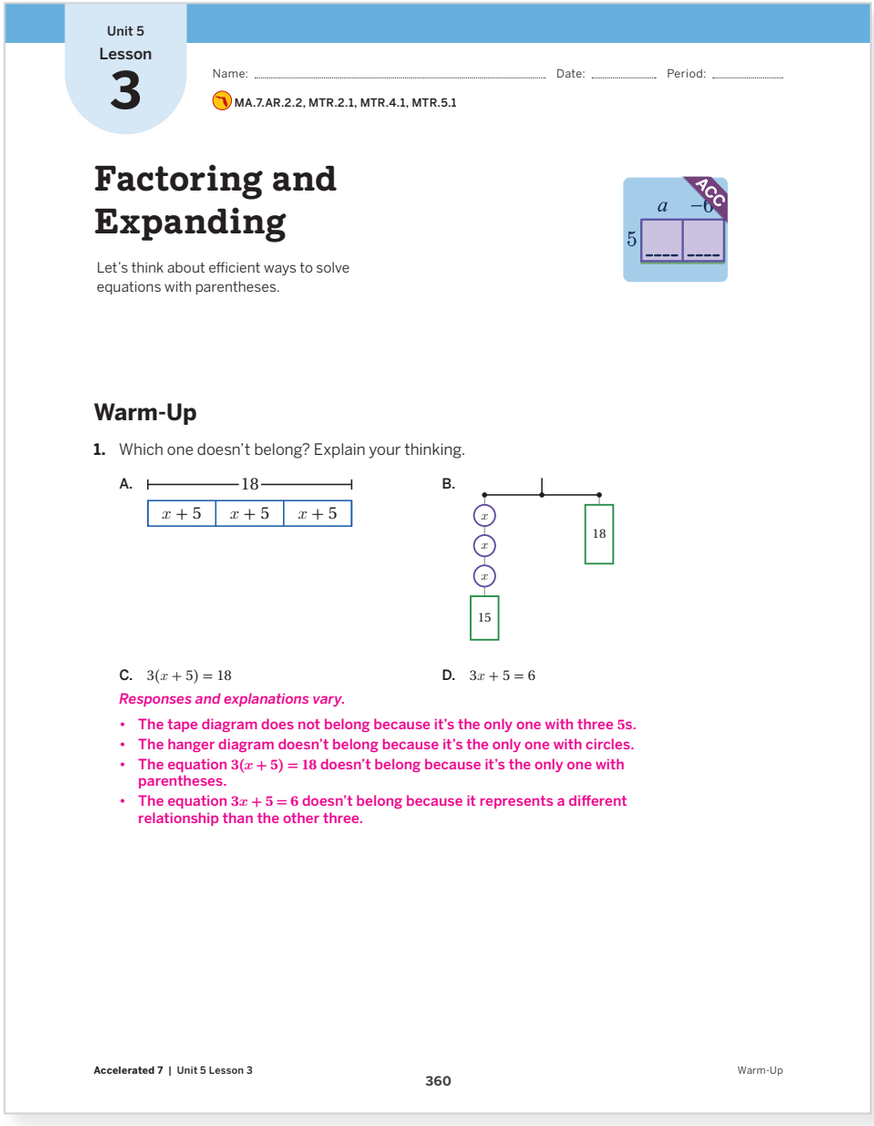
To surface prior knowledge, consider asking, "What are the diagrams in Choice A and Choice B called? Why are they used?"

Use the Which One Doesn't Belong? routine to support students in comparing and contrasting representations. See the Routine Facilitation Guide for more information. **(MTR.2.1)**

2 Connect

Invite students to share their responses.

Consider asking, "Which representation does not show the same relationship as the others? How do you know?"



Unit 5
Lesson
3

Name: ..... Date: ..... Period: .....

MA.7.AR.2.2, MTR.2.1, MTR.4.1, MTR.5.1

Factoring and Expanding

Let's think about efficient ways to solve equations with parentheses.

Warm-Up

1. Which one doesn't belong? Explain your thinking.

A.

B.

C. $3(x+5)=18$

D. $3x+5=6$

Responses and explanations vary.

- The tape diagram does not belong because it's the only one with three 5s.
- The hanger diagram doesn't belong because it's the only one with circles.
- The equation $3(x+5)=18$ doesn't belong because it's the only one with parentheses.
- The equation $3x+5=6$ doesn't belong because it represents a different relationship than the other three.

Accelerated 7 | Unit 5 Lesson 3 360 Warm-Up

Pairs | 10 min

Activity 1 Factoring Puzzles

Purpose: Students practice factoring and expanding expressions after analyzing diagrams using the Notice and Wonder routine.

Short on time? Consider having students complete only two puzzles.

1 Launch

Use the Notice and Wonder routine to promote curiosity and help students make sense of the two forms: factored and expanded. See the Routine Facilitation Guide for more information.

To surface connections between the diagrams and the area of a rectangle, consider asking, "How do the value(s) inside of the rectangular section relate to the area?" **(MTR.5.1)**

Create a class definition for the terms **factor** and **expand**.

2 Monitor

Select and sequence student strategies to share during the Connect.

D Differentiation *(Problems 2–5)*

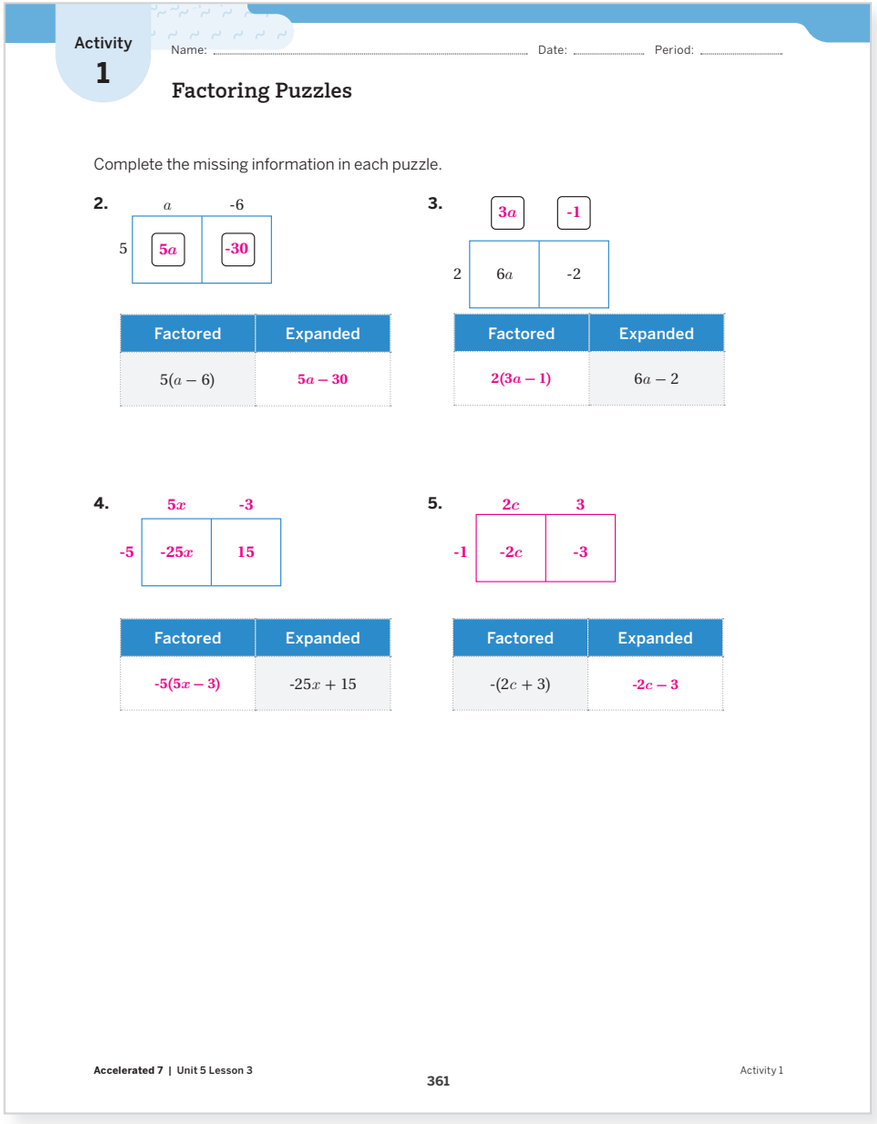
Look for students who:	Respond to Student Thinking
Combine numbers using addition or subtraction rather than multiplication.	**Support** Consider asking, "What does the operation 5(3) or 5(a) represent?"
Determine the correct values with incorrect signs.	**Support** Encourage students to review their work, looking specifically at the sign of each number to make sure that they make sense.
Form the connection that factoring is the same as "undoing" the *distributive property*.	**Strengthen** Invite students to share how factored form can be written as expanded form and vice versa.

A Accessibility: Visual-Spatial Processing To support interpreting visual representations, invite students to color-code parts of the rectangle and the corresponding parts in the expression.

3 Connect

Invite students to share their responses to the four problems and explain how the two forms are related.

To surface the Key Takeaway, consider asking, "How can you turn your factored-form expression into expanded form and vice versa?"



Activity 1

Name: Date: Period:

Factoring Puzzles

Complete the missing information in each puzzle.

2.

	a	-6
5	$5a$	-30

Factored	Expanded
$5(a-6)$	$5a-30$

3.

	$3a$	-1
2	$6a$	-2

Factored	Expanded
$2(3a-1)$	$6a-2$

4.

	$5x$	-3
-5	$-25x$	15

Factored	Expanded
$-5(5x-3)$	$-25x+15$

5.

	$2c$	3
-1	$-2c$	-3

Factored	Expanded
$-(2c+3)$	$-2c-3$

Accelerated 7 | Unit 5 Lesson 3 361 Activity 1

Key Takeaway: An expression can be written in **factored** form or **expanded** form, and the new expression is equivalent to the original expression.

Pairs | 10 min

Activity 2 Step by Step by Step by Step

Purpose: Students compare and contrast strategies to solve equations of the form $p(x + q) = r$ using the **MLR7: Compare and Connect** routine. **(MTR.4.1)**

1 Launch

Direct students' attention to the equation $2(x - 9) = 10$, and invite students to share how they would solve it.

MLR **MLR7: Compare and Connect** Invite students to analyze and compare each strategy, and then connect to their strategy. **(MTR.4.1)**

Consider asking:

- "What did each student do?"
- "Where do you see the same information in Amir's and Sadia's strategies?"
- "How does each strategy compare to your own?"
- "When might we use each strategy?"
- "Why did the different strategies lead to the same solution?"

2 Monitor

Select and sequence strategies to share during the Connect.

D **Differentiation** *(Problem 6)*

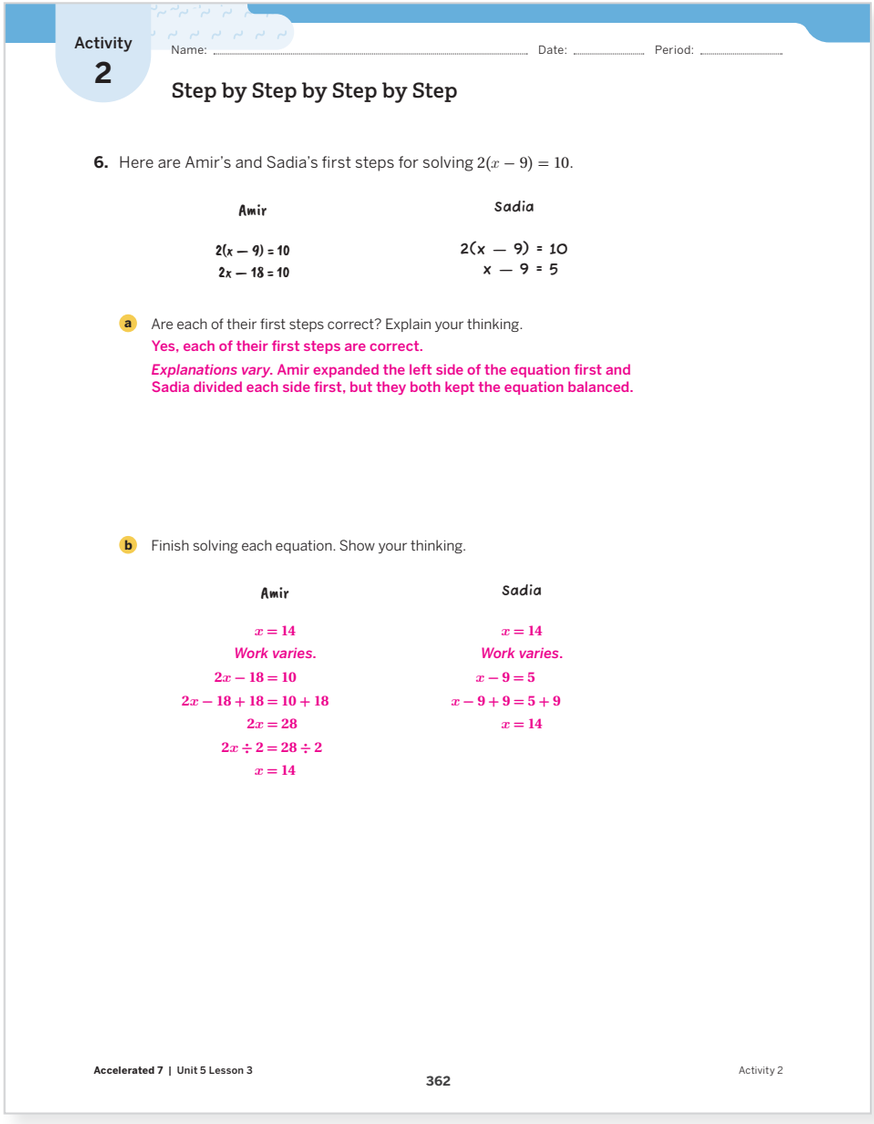
Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Consider asking, "What operation could we do to both sides of the equation to get $x - 9$ on the left and 5 on the right?"
Solve the equation on their own and substitute their response in Sadia's and Amir's first step.	**Strengthen** Invite students to share how this allows them to check the accuracy of the two equations.
Think that Sadia's strategy is the optimal method.	**Stretch** Invite students to solve the problem $5(x + 2) = 18$ using both methods and discuss what they notice.

ELL **English Language Learners** Provide sentence frames to help students explain their reasoning (e.g., These are/are not both correct because _____.). **(Speaking and Listening)**

3 Connect

Invite students to share how they finished solving the equations and what they noticed about the solutions.

To surface connections between the two approaches, consider asking, "Why do both strategies lead to the same solution?"



Activity 2

Name: _____ Date: _____ Period: _____

Step by Step by Step by Step

6. Here are Amir's and Sadia's first steps for solving $2(x - 9) = 10$.

Amir	Sadia
$2(x - 9) = 10$ $2x - 18 = 10$	$2(x - 9) = 10$ $x - 9 = 5$

a. Are each of their first steps correct? Explain your thinking.

Yes, each of their first steps are correct.

Explanations vary. Amir expanded the left side of the equation first and Sadia divided each side first, but they both kept the equation balanced.

b. Finish solving each equation. Show your thinking.

Amir	Sadia
$x = 14$ *Work varies.* $2x - 18 = 10$ $2x - 18 + 18 = 10 + 18$ $2x = 28$ $2x \div 2 = 28 \div 2$ $x = 14$	$x = 14$ *Work varies.* $x - 9 = 5$ $x - 9 + 9 = 5 + 9$ $x = 14$

Accelerated 7 | Unit 5 Lesson 3 362 Activity 2

Pairs | 10 min

Activity 3 Different First Steps

Purpose: Students practice solving equations by expanding and dividing by a common factor.

Short on time? Consider having students solve using one method while their partner solves using the other method. Ensure students practice each method.

1 Launch

Direct students' attention to the equation $3(x + 2) = 21$ and consider asking, "What does it mean to expand first? What does it mean to divide first?"

To surface connections to Sadia's and Amir's methods, invite students to discuss how to complete the first step of each strategy.

2 Monitor

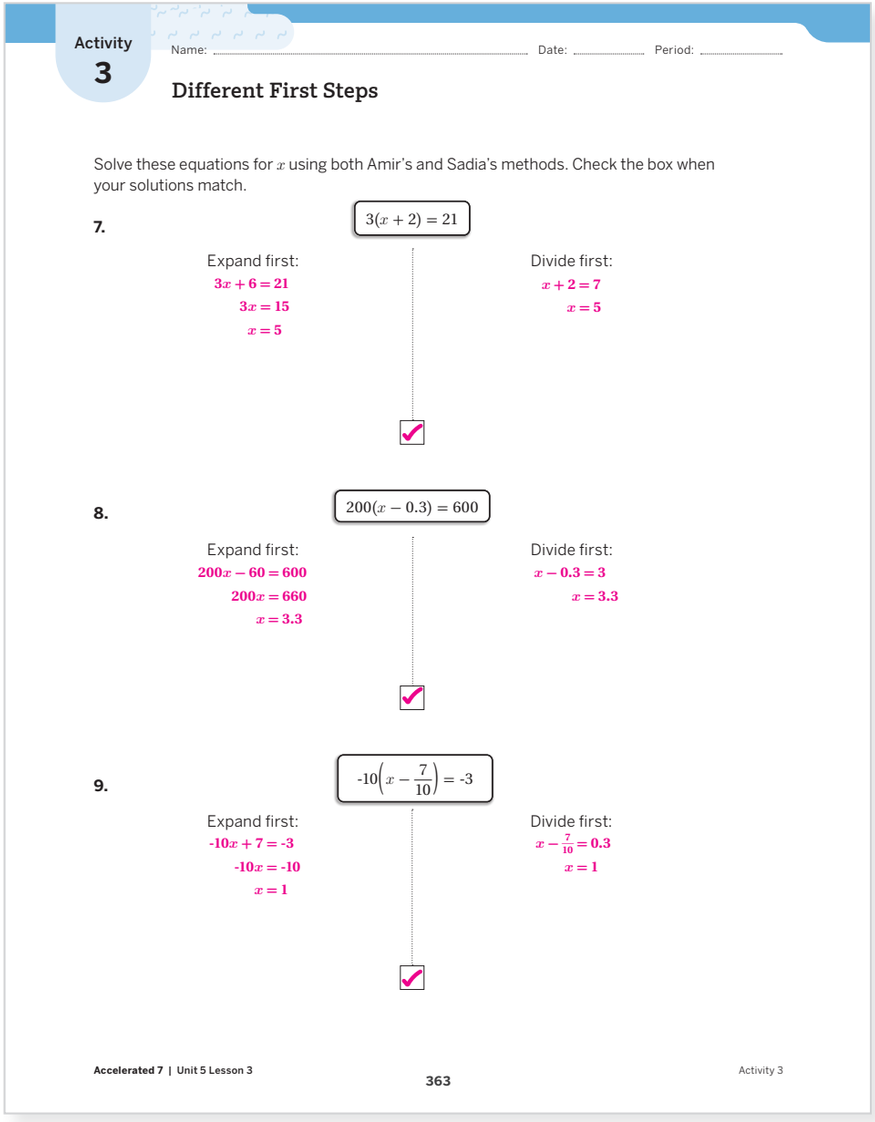
Select and sequence student strategies to share during the Connect.

D Differentiation *(Problems 7–9)*

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Consider asking, "How is this problem similar to Activity 2?"
Get solutions that don't match and substitute them into the original equation to assess which solution is incorrect.	**Strengthen** Invite students to share how this allows them to discover which method needs to be reworked.
Would like to explore solving equations involving expanding first or dividing first further.	**Stretch** Invite students to create an equation that they think would be more efficient to solve by expanding first and one that is more efficient to solve by dividing first, and then convince a classmate this is true.

3 Connect

Invite students to share which strategy they prefer and explain why.



Activity 3

Name: Date: Period:

Different First Steps

Solve these equations for x using both Amir's and Sadia's methods. Check the box when your solutions match.

7. $3(x + 2) = 21$

Expand first:
$3x + 6 = 21$
$3x = 15$
$x = 5$

Divide first:
$x + 2 = 7$
$x = 5$

8. $200(x - 0.3) = 600$

Expand first:
$200x - 60 = 600$
$200x = 660$
$x = 3.3$

Divide first:
$x - 0.3 = 3$
$x = 3.3$

9. $-10\left(x - \frac{7}{10}\right) = -3$

Expand first:
$-10x + 7 = -3$
$-10x = -10$
$x = 1$

Divide first:
$x - \frac{7}{10} = 0.3$
$x = 1$

Accelerated 7 | Unit 5 Lesson 3 363 Activity 3

Whole Class | 5 min

Synthesis

Purpose: Students synthesize their understanding of solving equations of the form $p(x + q) = r$ by expanding first or by dividing first, and examine the advantages of both strategies.

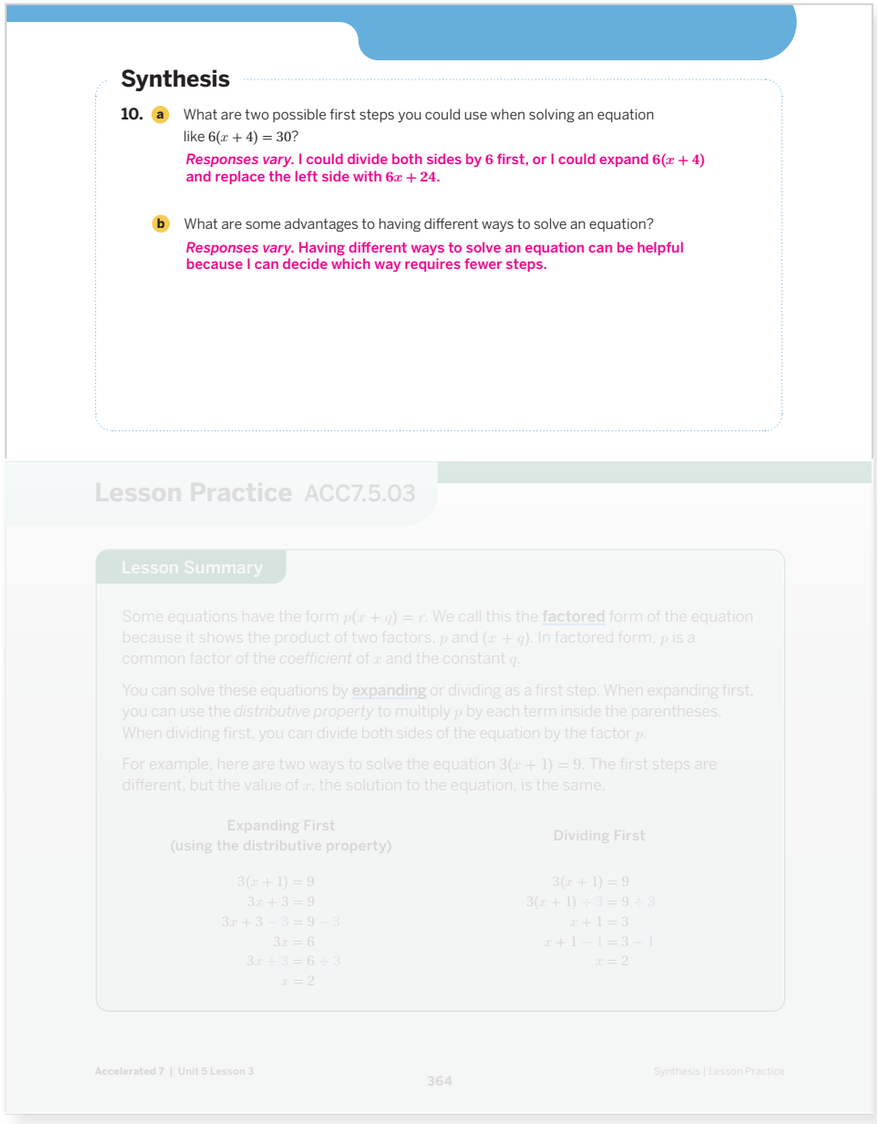
Invite students to respond independently, and then share their thinking with a partner.

Capture and share a variety of ideas, such as:

- Having fewer steps depending on the method used to solve.
- Using formal vocabulary like *expand* when describing their first step to solve.

MLR **MLR2: Collect and Display** Use this routine to formalize the definitions of *expand* and *factor* using students' language. Display any new language, examples, or diagrams for students to reference later.

Note: If time allows, invite students to share with a partner what makes sense about each strategy and what questions they still have.



Synthesis

10. a. What are two possible first steps you could use when solving an equation like $6(x + 4) = 30$?

Responses vary. I could divide both sides by 6 first, or I could expand $6(x + 4)$ and replace the left side with $6x + 24$.

b. What are some advantages to having different ways to solve an equation?

Responses vary. Having different ways to solve an equation can be helpful because I can decide which way requires fewer steps.

Lesson Practice ACC7.5.03

Lesson Summary

Some equations have the form $p(x + q) = r$. We call this the **factored** form of the equation because it shows the product of two factors, p and $(x + q)$. In factored form, p is a common factor of the *coefficient* of x and the constant q.

You can solve these equations by **expanding** or dividing as a first step. When expanding first, you can use the *distributive property* to multiply p by each term inside the parentheses. When dividing first, you can divide both sides of the equation by the factor p.

For example, here are two ways to solve the equation $3(x + 1) = 9$. The first steps are different, but the value of x, the solution to the equation, is the same.

Expanding First (using the distributive property)

$3(x + 1) = 9$
$3x + 3 = 9$
$3x + 3 - 3 = 9 - 3$
$3x = 6$
$3x \div 3 = 6 \div 3$
$x = 2$

Dividing First

$3(x + 1) = 9$
$3(x + 1) \div 3 = 9 \div 3$
$x + 1 = 3$
$x + 1 - 1 = 3 - 1$
$x = 2$

Accelerated 7 | Unit 5 Lesson 3 364 Synthesis | Lesson Practice

Lesson Takeaway: Equations of the form $p(x + q) = r$ can be solved by **expanding** first or dividing first, and there are advantages to both depending on the equation.

Independent | 5 min

Show What You Know

Purpose: Students demonstrate their understanding by solving an equation of the form $p(x + q) = r$.

Today's Goals

1. **Goal:** *Expand* and *factor* expressions that include positive and negative coefficients.
2. **Goal:** Solve equations that involve *expanding* an expression.
3. **Language Goal:** Analyze different strategies for solving the same equation. **(Reading and Writing)**

Show What You Know (Assessment Resources)

Name: _____ Date: _____ Period: _____

Show What You Know 5.03

Solve the equation. Show your thinking.

$8.88 = 4.44(x - 7)$

$x = 9$

Work varies.

$2 = x - 7$

$2 + 7 = x - 7 + 7$

$9 = x$

D Differentiation Use after Lesson 3

Support

Provide targeted intervention for students by using these resources.

If students need support understanding how to solve equations that involve expanding an expression:

» **Respond to Student Thinking:**

- Revisit Activity 2 and invite students to reflect on how they might apply Amir's and Sadia's strategies to other equations.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 1 Extension Activities**.
- Invite students to write an equation they would prefer to solve by factoring first, and another they would prefer to solve by dividing first.

Professional Learning

Students are asked to solve using Amir's and Sadia's methods. Are they comfortable with both methods, just one, or a different method entirely? Do any students need additional support with solving equations?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

10. a. What are two possible first steps you could use when solving an equation like $6(x + 4) = 30$?

Responses vary. I could divide both sides by 6 first, or I could expand $6(x + 4)$ and replace the left side with $6x + 24$.

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Responses vary. Having different ways to solve an equation can be helpful because I can decide which way requires fewer steps.

Lesson Practice ACC7.5.03

Lesson Summary

Some equations have the form $p(x + q) = r$. We call this the **factored** form of the equation because it shows the product of two factors, p and $(x + q)$. In factored form, p is a common factor of the *coefficient* of x and the constant q.

You can solve these equations by **expanding** or dividing as a first step. When expanding first, you can use the *distributive property* to multiply p by each term inside the parentheses. When dividing first, you can divide both sides of the equation by the factor p.

For example, here are two ways to solve the equation $3(x + 1) = 9$. The first steps are different, but the value of x, the solution to the equation, is the same.

Expanding First (using the distributive property)	Dividing First
$3(x + 1) = 9$	$3(x + 1) = 9$
$3x + 3 = 9$	$3(x + 1) \div 3 = 9 \div 3$
$3x + 3 - 3 = 9 - 3$	$x + 1 = 3$
$3x = 6$	$x + 1 - 1 = 3 - 1$
$3x \div 3 = 6 \div 3$	$x = 2$
$x = 2$	

Lesson Practice ACC7.5.03

Name: _____ Date: _____ Period: _____

Problems 1–3: Write each expression in expanded form.

1. $-2(-6) = 12$
2. $-2(-y) = 2y$
3. $-2(-6 - y) = 12 + 2y$

Problems 4–6: Complete the missing information in the puzzle and complete the table.

4.

	a	-5
4	$4a$	-20

Factored	Expanded
$4(a - 5)$	$4a - 20$

5.

	$-4a$	$\frac{1}{3}$
-3	$12a$	-1

Factored	Expanded
$-3\left(-4a + \frac{1}{3}\right)$	$12a - 1$

6.

	$3x$	-7
3	$9x$	-21

Factored	Expanded
$3(3x - 7)$	$9x - 21$

Responses vary.

7. Emmanuel and Mauricio started solving the equation $7(x - 2) = 91$. Finish solving each equation.

Emmanuel

$7(x - 2) = 91$
$7x - 14 = 91$
$7x = 105$
$x = 15$

Mauricio

$7(x - 2) = 91$
$x - 2 = 13$
$x = 15$

Lesson Practice ACC7.5.03

Name: _____ Date: _____ Period: _____

Problems 8–10: Solve each equation. Show your thinking.

8. $2(x - 3) = 14$
$2(x - 3) \div 2 = 14 \div 2$
$x - 3 = 7$
$x - 3 + 3 = 7 + 3$
$x = 10$

9. $-5(x - 1) = 40$
$-5(x - 1) \div (-5) = 40 \div (-5)$
$x - 1 = -8$
$x - 1 + 1 = -8 + 1$
$x = -7$

10. $\frac{5}{7}(x - 9) = 25$
$\frac{5}{7}(x - 9) \div \frac{5}{7} = 25 \div \frac{5}{7}$
$x - 9 = 35$
$x - 9 + 9 = 35 + 9$
$x = 44$

FAST Practice

11. A student incorrectly solved the equation $-5(x - 1) = 40$ as shown.

Step 1: $-5x + 5 = 40$

Step 2: $-5x = 45$

Step 3: $x = -9$

Select **ONE** correct answer in each box to complete the sentence.

The student made the first error in [A. Step 1 / (B.) Step 2]. The correct solution to the equation is [(A.) -7 / B. 9].

Spiral Review

12. Use long division to write $\frac{2}{5}$ as a decimal.

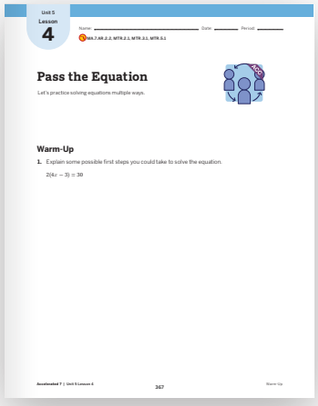
$$5\overline{)2.0}\quad 0.4,\ -20,\ 0$$

13. Which equation is true when $x = 2$?

A. $3x + 4 = 8$ B. $5x - 3 = 12$ (C.) $6x + 4 = 16$ D. $4x + 5 = 17$

Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1–3	1	MA.7.AR.1.2
	4–6	2	MA.7.AR.1.2
	7–10	2	MA.7.AR.2.2
FAST Practice	11	2	MA.7.AR.2.2
Spiral Review			
	12	1	MA.6.NSO.3.5
	13	2	MA.6.AR.2.1


Unit 5
Lesson
4

Pass the Equation

Let's practice solving equations multiple ways.

Warm-Up

1. Explain some possible first steps you could take to solve the equation.

$2(4x - 3) = 30$

367

Pass the Equation

Solving Equations Multiple Ways

Let's practice solving equations multiple ways.

Learning Outcomes

Today's Goals

1. **Goal:** Solve equations using a variety of strategies.
2. **Language Goal:** Compare and contrast strategies for solving the same equation. **(Reading, Writing, Speaking, and Listening)**

Students practice solving equations that involve adding terms and expanding expressions. They recognize that there are several possible sets of steps for the same equation that will lead to the same solution. This lesson is also an opportunity for students to practice converting between decimals and fractions as they solve equations.

Coherence

Prior Learning

In Lesson 3, students factored and expanded expressions and used different strategies to solve equations of the form $p(x + q) = r$.

Future Learning

In Lesson 5, students will write and solve equations. They will interpret the meaning of their solutions in context.

Connections

- Students solve equations using mutliple ways that provides different perspectives on the same problem. (mathematical)

FL B.E.S.T. Math Benchmarks

Addressing

MA.7.NSO.2.2
Add, subtract, multiply and divide rational numbers with procedural fluency.

MA.7.AR.2.2
Write and solve two-step equations in one variable within a mathematical or real-world context, where all terms are rational numbers.

Mathematical Thinking and Reasoning Standards: MTR.2.1, MTR.3.1, MTR.5.1

ELA Expectations: ELA.K12.EE.4.1, ELA.K12.EE.5.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.6.AR.2.2 **MA.6.AR.2.3**	**MA.8.AR.2.1**

Vocabulary

Additional Vocabulary

inverse operations

Lesson at a Glance

~ 45 min

MA.7.AR.2.2, MA.7.NSO.2.2

Warm-Up

Pairs | 5 min

Students surface several first steps for the same equation to show that there is often more than one useful first step. **Routine:** Think-Pair-Share

Unit 5 Lesson 4

Pass the Equation

Let's practice solving equations in multiple ways.

Warm-Up

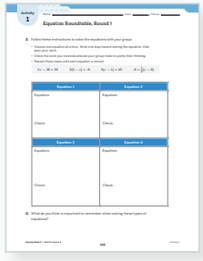
1. Explain some possible first steps you could take to solve the equation.

$2(x-3)=36$

Activity 1

Small Groups | 15 min

Students practice solving equations that have multiple viable solving pathways and get feedback from their peers to revise their thinking.


Activity 1

Equation Roundtable, Round 1

2. Follow these instructions to solve the equations with your group.
   - Assess one equation at a time. Write one step toward solving the equation. Then pass your work.
   - Check the work you received and add your group's work to justify their thinking.
   - Repeat these steps until each equation is solved.

$2x-36=36$ $\quad 3(2x-1)=x$ $\quad 5(x-1)=45$ $\quad -6=\frac{1}{2}(x-6)$

Equation 1	Equation 2
Equation:	Equation:
Check:	Check:
Equation 3	**Equation 4**
Equation:	Equation:
Check:	Check:

3. What do you think is important to remember when solving these types of equations?

Activity 2

Small Groups | 15 min

Students practice solving equations that involve both adding and expanding expressions.

Activity 2

Equation Roundtable, Round 2

4. Here are four new equations. Solve them using the same instructions as Activity 1.

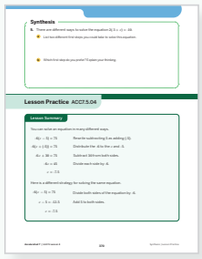
$2x-36=36$ $\quad 3(2x-1)=x$ $\quad 5(x-1)=45$ $\quad -6=\frac{1}{2}(x-6)$

Equation 1	Equation 2
Equation:	Equation:
Check:	Check:
Equation 3	**Equation 4**
Equation:	Equation:
Check:	Check:

Synthesis

Whole Class | 5 min

Students synthesize their understanding of a variety of first steps for solving complex equations. MLR2: Collect and Display


Synthesis

5. There are different ways to solve the equation $2(x-4)=36$.
   - List two different first steps you could take to solve the equation.
   - Which first step do you prefer? Explain your thinking.

Lesson Practice ACCT.5.04

Lesson Summary

You can solve an equation in many different ways.

$40x-10=75$ Reverse the multiplying 2 as adding (-5)
$40x+(-10)=75$ Distribute the -2 to the x and -5
$4x+36=75$ Subtract 36 from both sides.
$4x=40$ Divide each side by 4.
$x=10$

Here is a different strategy for solving the same equation.

$40x-10=75$ Divide both sides of the equation by 4.
$x-2.5=10$ Add 2.5 to both sides.
$x=12.5$

Show What You Know

Independent | 5 min

Students demonstrate their understanding by finding errors made in solving equations.

Show What You Know

Lin

Lin and Elena each started solving the equation $5(x-3)=45$.

Elena

$5(x-3)=45$
$x-3=9$

Lin

$5x-15=45$
$5x=30$

What was the error?

Prep Checklist

This lesson includes:

- Student Edition
- Show What You Know

Pairs | 5 min

Warm-Up

Purpose: Students surface several first steps for the same equation to show that there is often more than one useful first step.

1 Launch

Direct students' attention to the equation and consider asking, "What do you notice about the equation?"

Use the Think-Pair-Share routine to support student-to-student discourse. Invite students to think independently and then take turns sharing with a partner. Invite several students to share their or their partner's strategy with the class.

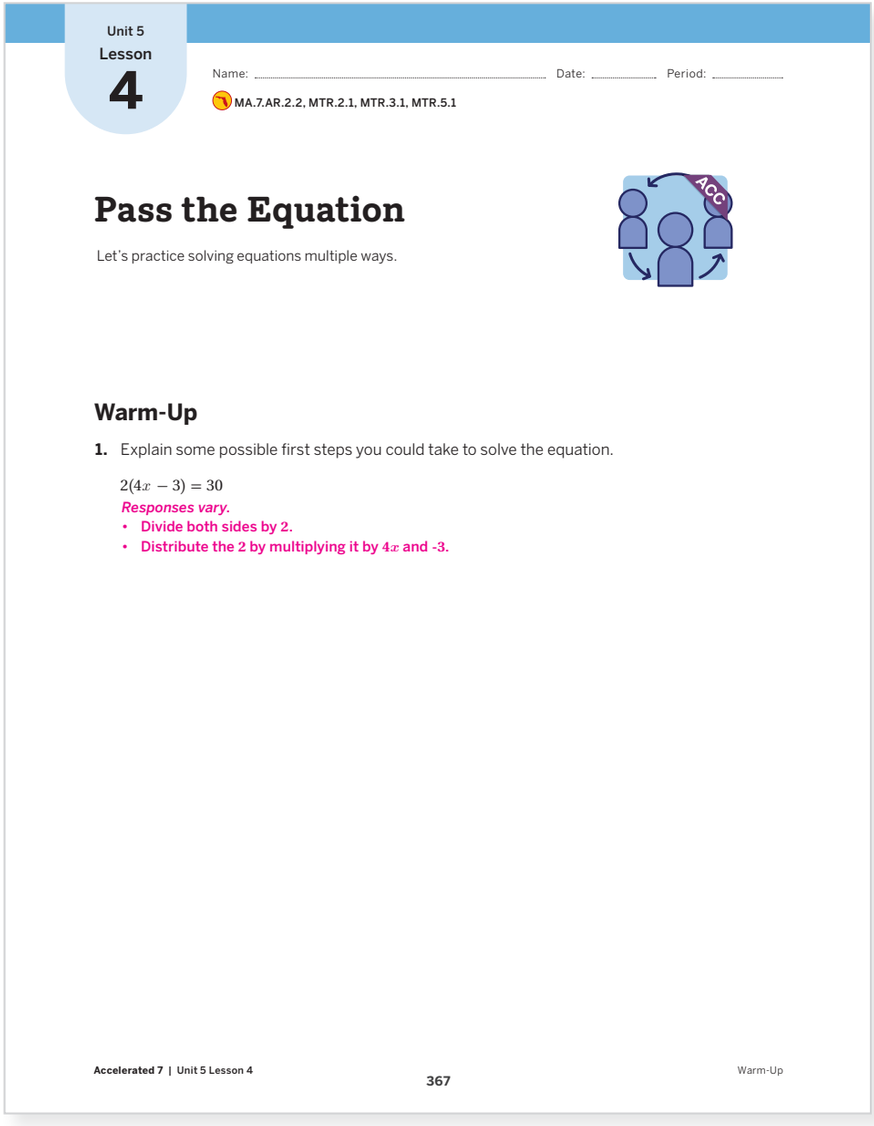
2 Connect

Invite students to share several possible first steps.

ELL **English Language Learners** Create or review an anchor chart that publicly displays common solving steps, along with vocabulary like *add*, *subtract*, *expand*, *factor*, *term*, and *equivalent expression* to aid students in explaining their reasoning.

Consider asking:

- "Which first step do you prefer? Why?"
- "Will you get the same solution regardless of which first step you choose?"



Unit 5
Lesson
4

Name: ..... Date: ..... Period: .....

MA.7.AR.2.2, MTR.2.1, MTR.3.1, MTR.5.1

Pass the Equation

Let's practice solving equations multiple ways.

Warm-Up

1. Explain some possible first steps you could take to solve the equation.

$2(4x - 3) = 30$

Responses vary.

- Divide both sides by 2.
- Distribute the 2 by multiplying it by $4x$ and -3.

Accelerated 7 | Unit 5 Lesson 4 367 Warm-Up

Small Groups | 15 min

Activity 1 Equation Roundtable, Round 1

Purpose: Students practice solving equations that have multiple viable solving pathways and get feedback from their peers to revise their thinking.

1 Launch

Direct students' attention to the Equation Roundtable instructions.

Invite students to read through the instructions and ask clarifying questions. It may be helpful to do a practice round as a class before students begin.

2 Monitor

A **Accessibility: Memory and Attention** Invite students to refer back to notes they took earlier on solving equations to activate prior knowledge.

D **Differentiation** *(Problem 2)*

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Encourage students to refer to the strategies from the Warm-Up.
Distribute to only one of the two terms in the parentheses.	**Support** Consider asking, "How could you check if this equation is equivalent to your first equation?"
Use strategies like: • Reasoning about the unknown value. • Distributing first. • Dividing both sides by the number outside the parentheses. • Using *inverse operations*. • Rewriting the equation with the variable on the left.	**Strengthen** Consider asking, "Why is the sign of a term important when distributing?" Select and sequence these strategies to share during the Connect.
Would like to explore this idea more.	**Stretch** Consider asking, "Will you get the same solution if you take a different first step?"

3 Connect

Direct students' attention to the four equations and invite the class to choose one to discuss.

Consider asking:

- "Did your partner ever make a move different than the one you expected them to? Describe it."
- "What advice would you give yourself or others when solving equations like these?"

Activity 1

Name: Date: Period:

Equation Roundtable, Round 1

2. Follow these instructions to solve the equations with your group.

- Choose one equation at a time. Write one step toward solving the equation, then pass your work.
- Check the work you received and ask your group mate to justify their thinking.
- Repeat these steps until each equation is solved.

$2x - 18 = 10$ $2(3 - x) = -8$ $5(x - 1) = 45$ $-6 = \frac{1}{2}(x - 8)$

Responses vary.

Equation 1	Equation 2
Equation: $2x - 18 = 10$ $2x = 28$ $x = 14$ Check: $2(14) - 18 = 10$ $28 - 18 = 10$ $10 = 10$	Equation: $2(3 - x) = -8$ $6 - 2x = -8$ $-2x = -14$ $x = 7$ Check: $2(3 - 7) = -8$ $2(-4) = -8$ $-8 = -8$
Equation 3	**Equation 4**
Equation: $5(x - 1) = 45$ $5x - 5 = 45$ $5x = 50$ $x = 10$ Check: $5(10 - 1) = 45$ $5(9) = 45$ $45 = 45$	Equation: $-6 = \frac{1}{2}(x - 8)$ $-6 = \frac{1}{2}x - 4$ $-2 = \frac{1}{2}x$ $-4 = x$ Check: $-6 = \frac{1}{2}(-4 - 8)$ $-6 = \frac{1}{2}(-12)$ $-6 = -6$

3. What do you think is important to remember when solving these types of equations?

Responses vary.

- When solving equations, you have to keep each side of the equation balanced.
- When solving equations, you can expand first if there are parentheses.

Accelerated 7 | Unit 5 Lesson 4 368 Activity 1

Key Takeaway: There are multiple ways to solve equations, including using the distributive property or using inverse operations on both sides of the equation. No matter which strategy you choose to solve an equation, the solution should be the same.

Small Groups | 15 min

Activity 2 Equation Roundtable, Round 2

Purpose: Students practice solving equations that involve both adding and expanding expressions.

Short on time? Consider omitting this activity or invite each student in the group to choose one equation to solve, then to take turns explaining their solving strategy to the group.

1 Launch

Direct students' attention to the instructions and invite students to share helpful strategies from Activity 1.

To surface connections, encourage students to compare and contrast the equations in Activity 1 and Activity 2. Students may notice that the structure of equations in Activity 2 requires combining like terms on the same side, which they practiced with expressions in a previous course. **(MTR.5.1)**

2 Monitor

Encourage students to explain how they know the equation is still balanced after each step.

D Differentiation *(Problem 4)*

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Encourage students to review strategies from Activity 1.
Combine unlike terms.	**Support** Encourage students to circle or highlight like terms on the same side before rewriting.
Use strategies like: • Using properties of integers and equations to correctly solve. • Rewriting subtraction as addition of a negative value. • Rewriting fractions and decimals in the same form.	**Strengthen** Consider asking, "How can you check that your solution is correct?"
Would like to explore this idea more.	**Stretch** Encourage groups to determine a solution path for each equation that uses the fewest number of steps. **(MTR.3.1)**

Activity 2

Name: Date: Period:

Equation Roundtable, Round 2

4. Here are four new equations. Solve them using the same instructions as Activity 1.

$2x - 18 = 10$ $\qquad$ $45 = 5(x - 1)$

$-10.5 = 6\left(x + \frac{1}{4}\right)$ $\qquad$ $-2.8(x - 3) = 9\frac{4}{5}$

Responses vary.

Equation 1

Equation: $2x - 18 = 10$; $2x = 28$; $x = 14$

Check: $2(14) - 18 = 10$; $28 - 18 = 10$; $10 = 10$

Equation 2

Equation: $45 = 5(x - 1)$; $45 = 5x - 5$; $50 = 5x$; $10 = x$

Check: $45 = 5(10 - 1)$; $45 = 5(9)$; $45 = 45$

Equation 3

Equation: $-10.5 = 6\left(x + \frac{1}{4}\right)$; $-\frac{21}{2} = 6x + \frac{6}{4}$; $-\frac{42}{4} = \frac{24}{4}x + \frac{6}{4}$; $-42 = 24x + 6$; $-48 = 24x$; $-2 = x$

Check: $-10.5 = 6\left(-2 + \frac{1}{4}\right)$; $-\frac{21}{2} = 6\left(-\frac{8}{4} + \frac{1}{4}\right)$; $-\frac{21}{2} = 6\left(-\frac{7}{4}\right)$; $-\frac{42}{4} = -\frac{42}{4}$

Equation 4

Equation: $-2.8(x - 3) = 9\frac{4}{5}$; $-2.8x + 8.4 = 9\frac{4}{5}$; $-2.8x + 8.4 = 9.8$; $-2.8x = 1.4$; $x = -0.5$ (or equivalent)

Check: $-2.8(-0.5 - 3) = 9\frac{4}{5}$; $-2.8(-3.5) = 9.8$; $9.8 = 9.8$

Accelerated 7 | Unit 5 Lesson 4 369 Activity 2

3 Connect

Invite students to compare and contrast the strategies their group used to solve the equations. Encourage students to share which first steps they prefer and their reasoning. **(MTR.2.1)**

To surface the Key Takeaway, consider asking, "What is helpful to remember when solving equations?"

Key Takeaway: There are multiple ways to solve equations, including using integer properties and writing equivalent expressions.

Whole Class | 5 min

Synthesis

Purpose: Students synthesize their understanding of a variety of first steps for solving complex equations.

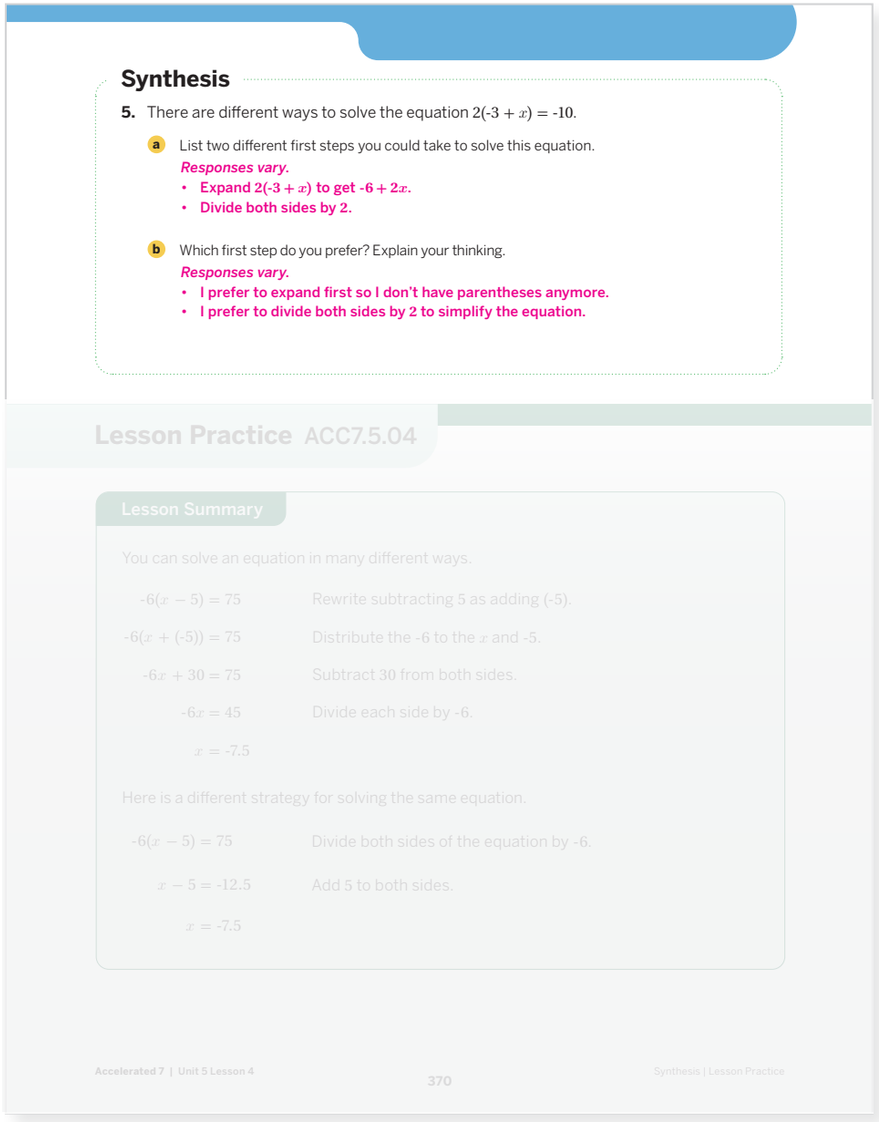
Direct students' attention to the equation and discussion questions.

Invite students to respond independently, and then share their thinking with a partner.

Collect and share a variety of ideas, such as:
- Dividing by 2 on both sides.
- Expanding the equation by distributing 2 to -3 and x.

MLR **MLR2: Collect and Display** Use this routine to collect students' ideas or strategies as they solve complex equations. Display for students to reference and add to for the rest of the unit.

Note: If time allows, invite students to share with a partner what makes sense about each strategy and what questions they still have.



Synthesis

5. There are different ways to solve the equation $2(-3 + x) = -10$.

a. List two different first steps you could take to solve this equation.

Responses vary.
- Expand $2(-3 + x)$ to get $-6 + 2x$.
- Divide both sides by 2.

b. Which first step do you prefer? Explain your thinking.

Responses vary.
- I prefer to expand first so I don't have parentheses anymore.
- I prefer to divide both sides by 2 to simplify the equation.

Lesson Practice ACC7.5.04

Lesson Summary

You can solve an equation in many different ways.

$-6(x - 5) = 75$	Rewrite subtracting 5 as adding (-5).
$-6(x + (-5)) = 75$	Distribute the -6 to the x and -5.
$-6x + 30 = 75$	Subtract 30 from both sides.
$-6x = 45$	Divide each side by -6.
$x = -7.5$	

Here is a different strategy for solving the same equation.

$-6(x - 5) = 75$	Divide both sides of the equation by -6.
$x - 5 = -12.5$	Add 5 to both sides.
$x = -7.5$	

Accelerated 7 | Unit 5 Lesson 4 370 Synthesis | Lesson Practice

Lesson Takeaway: There are many possible first steps for solving complex equations, including combining like terms and expanding expressions.

Independent | 5 min

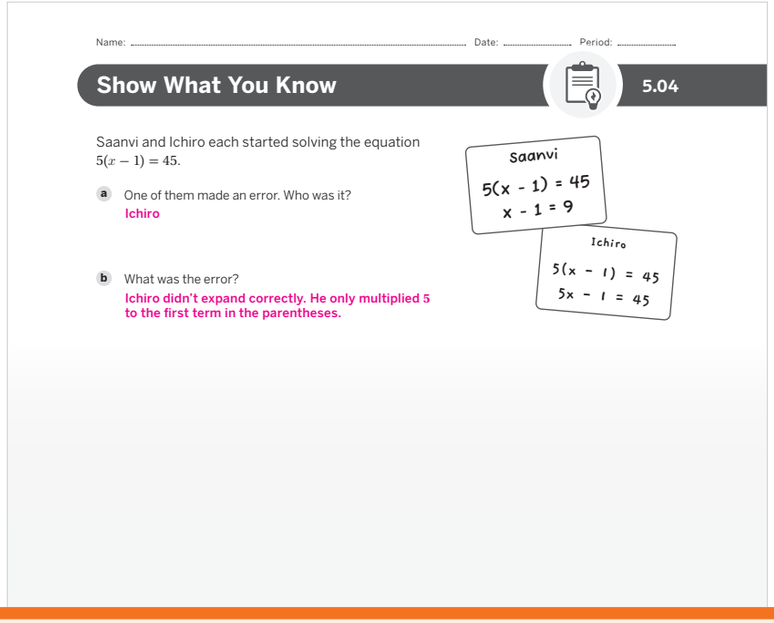
Show What You Know

Purpose: Students demonstrate their understanding by finding errors made in solving equations.

Today's Goals

1. **Goal:** Solve equations using a variety of strategies.
2. **Language Goal:** Compare and contrast strategies for solving the same equation. **(Reading, Writing, Speaking, and Listening)**

Show What You Know (Assessment Resources)



Name: _____ Date: _____ Period: _____

Show What You Know 5.04

Saanvi and Ichiro each started solving the equation $5(x - 1) = 45$.

a. One of them made an error. Who was it?
Ichiro

b. What was the error?
Ichiro didn't expand correctly. He only multiplied 5 to the first term in the parentheses.

Saanvi
5(x - 1) = 45
x - 1 = 9

Ichiro
5(x - 1) = 45
5x - 1 = 45

D Differentiation Use after Lesson 4

Support

Provide targeted intervention for students by using these resources.

If students need support understanding how to solve equations using a variety of strategies:

» **Respond to Student Thinking:**

- Revisit **Activity 1** as a class and inviting students to share strategies for how to solve equations.
- Select a few related **Practice Problems** and give individualized feedback.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 1 Extension Activities**.

Professional Learning

In what ways are students taking risks when trying out new strategies for solving equations? What types of questions might support students in exploring and testing new strategies?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

5. There are different ways to solve the equation $2(-3 + x) = -10$.

a. List two different first steps you could take to solve this equation.

Responses vary.

- Expand $2(-3 + x)$ to get $-6 + 2x$.
- Divide both sides by 2.

b. Which first step do you prefer? Explain your thinking.

Responses vary.

- I prefer to expand first so I don't have parentheses anymore.
- I prefer to divide both sides by 2 to simplify the equation.

Lesson Practice ACC7.5.04

Lesson Summary

You can solve an equation in many different ways.

$-6(x - 5) = 75$	Rewrite subtracting 5 as adding (-5).
$-6(x + (-5)) = 75$	Distribute the -6 to the x and -5.
$-6x + 30 = 75$	Subtract 30 from both sides.
$-6x = 45$	Divide each side by -6.
$x = -7.5$	

Here is a different strategy for solving the same equation.

$-6(x - 5) = 75$	Divide both sides of the equation by -6.
$x - 5 = -12.5$	Add 5 to both sides.
$x = -7.5$	

Lesson Practice
ACC7.5.04

Name: _____ Date: _____ Period: _____

Problems 1–3: Kwame and Maia each solved the same equation but used different strategies to do so. Show what they might have done to get from one step to another.

Kwame	Maia
$3.3(x - 10) = 66$	$3.3(x - 10) = 66$
$3.3x - 3.3 \cdot 10 = 66$	$3.3(x - 10) \div 3.3 = 66 \div 3.3$
$3.3x - 33 = 66$	$x - 10 = 20$
$3.3x - 33 + 33 = 66 + 33$	$x - 10 + 10 = 20 + 10$
$3.3x = 99$	$x = 30$
$3.3x \div 3.3 = 99 \div 3.3$	
$x = 30$	

1. How did Kwame begin? Maia?
   Kwame began by distributing first. Maia began by dividing each side by 3.3.
2. Complete the missing steps for each student.
   Sample shown in work.
3. Which method do you prefer to use for this equation? Why?
   Responses vary.

Problems 4–7: Solve each equation. Show your thinking.

4. $4(x - 3) = 16$
   $4(x - 3) \div (4) = 16 \div (4)$
   $x - 3 = 4$
   $x - 3 + 3 = 4 + 3$
   $x = 7$

5. $-5(x - 4) = 40$
   $-5(x - 4) \div (-5) = 40 \div (-5)$
   $x - 4 = -8$
   $x - 4 + 4 = -8 + 4$
   $x = -4$

6. $\frac{3}{7}(x - 8) = 15$
   $\frac{3}{7}(x - 8) \div \frac{3}{7} = 15 \div \frac{3}{7}$
   $x - 8 = 35$
   $x - 8 + 8 = 35 + 8$
   $x = 43$

7. $\frac{1}{6}(x + 6) = 11$
   $\frac{1}{6}(x + 6) \div \frac{1}{6} = 11 \div \frac{1}{6}$
   $x + 6 = 66$
   $x + 6 - 6 = 66 - 6$
   $x = 60$

Lesson Practice
ACC7.5.04

Name: _____ Date: _____ Period: _____

Problems 8–9: Alina and Naoki are each solving the equation $7(x + 2) = 91$.
Alina starts by using the distributive property.
Naoki starts by dividing each side by 7.

8. Show what Alina's and Naoki's full solution methods might look like.

Alina	Naoki
$7(x + 2) = 91$	$7(x + 2) = 91$
$7x + 14 = 91$	$7(x + 2) \div 7 = 91 \div 7$
$7x + 14 - 14 = 91 - 14$	$x + 2 = 13$
$7x = 77$	$x + 2 - 2 = 13 - 2$
$7x \div 7 = 77 \div 7$	$x = 11$
$x = 11$	

9. What is the same and what is different about their methods?
   Responses vary. Both divided each side by 7 (at different steps) and determined the solution $x = 11$. Alina subtracted 14 from each side while Naoki subtracted 2 from each side.

FAST Practice

10. What is the value of x in the equation $9 - \frac{x}{3} = 14$?
    $x =$ -15

Spiral Review

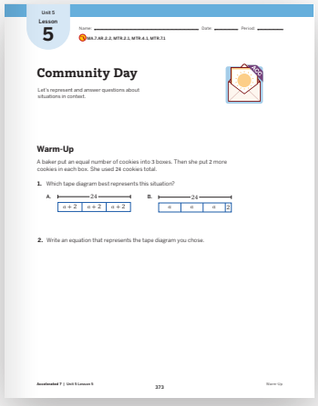
11. Vicente and Zwena are trying to write $9x - 2x + 4x$ using fewer terms.
    - Vicente says that $9x - 2x + 4x = 3x$ because the subtraction sign tells us to subtract everything that comes after $9x$.
    - Zwena says that $9x - 2x + 4x = 11x$ because the subtraction only applies to $2x$.

    Who is correct? Explain your thinking.
    Zwena. *Explanations vary.* Rewriting addition as subtraction gives us $9x + (-2x) + 4x$, which shows that the subtraction symbol in front of the $2x$ applies only to the $2x$ and not to the terms that come after it.

12. Choose the expression that is equivalent to $(9x + 5) + (-7x + 2)$.
    A. $-2x + 10$ B. $-63x + 10$ C. $16x + 7$ D. $2x + 7$

Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1–7	1	MA.7.AR.2.2
	8–9	2	MA.7.AR.2.2
FAST Practice	10	2	MA.7.AR.2.2
Spiral Review			
	11	2	MA.6.AR.1.4
	12	1	MA.7.AR.1.1


Unit 5
Lesson 5
Name: Date: Period:
MA.7.AR.2.2, MTR.2.1, MTR.4.1, MTR.7.1
Community Day
Let's represent and answer questions about situations in context.
Warm-Up
A baker put an equal number of cookies into 3 boxes. Then she put 2 more cookies in each box. She used 24 cookies total.
1. Which tape diagram best represents this situation?
A. 24
n + 2 | n + 2 | n + 2
B. 24
n | n | n | 2
2. Write an equation that represents the tape diagram you chose.
Accelerated 7 | Unit 5 Lesson 5
373
Warm-Up

Community Day

Using Equations to Solve Problems

Let's represent and answer questions about situations in context.

Learning Outcomes

Today's Goals

1. **Goal:** Write and solve equations to represent situations involving positive and negative numbers.
2. **Goal:** Connect visual representations, equations, and verbal descriptions of the same situation.
3. **Language Goal:** Compare and contrast situations using different visual representations and equations. **(Reading and Writing)**

Students use everything they have learned so far in the unit to create visuals and equations to represent and answer questions about situations in context. They make sense of verbal descriptions, and compare and contrast situations in similar contexts. **(MTR.2.1, MTR.7.1)**

Coherence

Prior Learning

In Lesson 4, students recognized that using a different set of steps to solve the same equation leads to the same solution.

Future Learning

In Lesson 6, students will analyze the solving steps for multi-step equations.

Connections

- Students use tape diagrams as a visual tool to represent and solve equations in real world situations. (mathematical, real world)

FL B.E.S.T. Math Benchmarks

Addressing

MA.7.AR.2.2
Write and solve two-step equations in one variable within a mathematical or real-world context, where all terms are rational numbers.

MA.7.NSO.2.3
Solve real-world problems involving any of the four operations with rational numbers.

Also Addressing: **MA.7.NSO.2.2**

Mathematical Thinking and Reasoning Standards: MTR.2.1, MTR.4.1, MTR.7.1

ELA Expectation: ELA.K12.EE.2.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.6.AR.2.2	**MA.8.AR.2.1**
MA.6.AR.2.3	

Lesson at a Glance

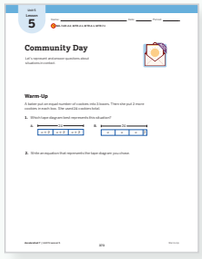
~ 45 min

MA.7.AR.2.2

Warm-Up

Pairs | 5 min

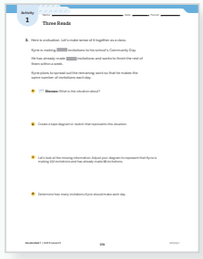
Students connect a visual representation and an equation to a description of a situation.



Activity 1

Pairs | 10 min

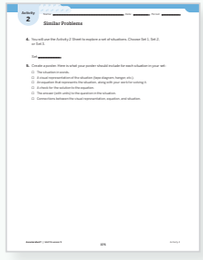
Students make sense of a situation described in words, represent the situation with a visual, and answer a question about it. MLR6: Three Reads (MTR.7.1)



Activity 2

Pairs | 20 min

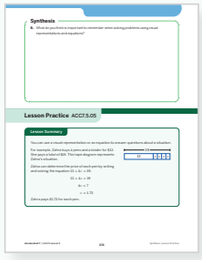
Students practice creating visuals in order to write and solve equations for situations in context. (MTR.2.1)



Synthesis

Whole Class | 5 min

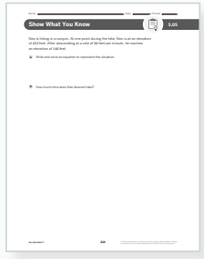
Students synthesize their understanding of what is important to remember when representing and answering questions about situations.



Show What You Know

Independent | 5 min

Students demonstrate their understanding by writing and solving an equation in context.



Prep Checklist

Prepare the Activity 2 Sheet.

This lesson includes:

- Student Edition
- Show What You Know
- Activity 2 Sheet, one per student

Pairs | 5 min

Warm-Up

Purpose: Students connect a visual representation and an equation to a description of a situation.

1 Launch

Direct students' attention to the situation and questions.

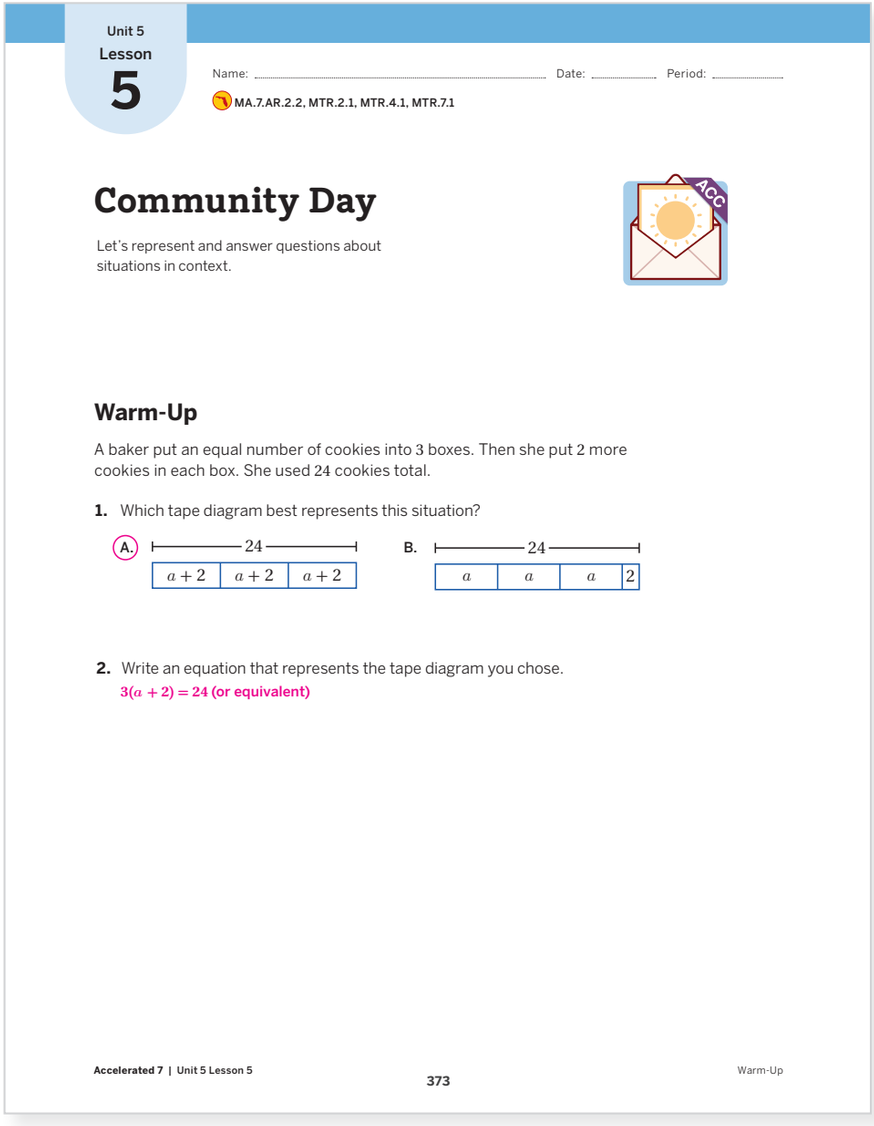
Invite a student to read the situation out loud and other students to paraphrase it in their own words.

2 Connect

To surface connections, consider asking:

- "What words in the situation helped you decide which diagram to choose?"
- "How do you know the other diagram does not represent this situation?"
- "What equation did you write? Are there any other equations that also represent the situation?"

D **Differentiation:** (Stretch) Encourage students to write a story that matches the tape diagram they did not select.



Unit 5
Lesson
5

Name: _____ Date: _____ Period: _____

MA.7.AR.2.2, MTR.2.1, MTR.4.1, MTR.7.1

Community Day

Let's represent and answer questions about situations in context.

Warm-Up

A baker put an equal number of cookies into 3 boxes. Then she put 2 more cookies in each box. She used 24 cookies total.

1. Which tape diagram best represents this situation?

A.

B.

2. Write an equation that represents the tape diagram you chose.

$3(a+2)=24$ (or equivalent)

Accelerated 7 | Unit 5 Lesson 5 373 Warm-Up

Pairs | 10 min

Activity 1 Three Reads

Purpose: Students make sense of a situation described in words, represent the situation with a visual, and answer a question about it. **(MTR.7.1)**

1 Launch

MLR **MLR6: Three Reads** Invite students to read the text three times, each with a particular focus, to support students in making sense of the situation. **(MTR.7.1)**

Direct students' attention to the situation with the hidden information and invite a student to read it aloud, then discuss the prompt in part a.

To surface connections, invite students to share a personal experience they have had with a Community Day.

Direct students' attention to part b and invite a student to read it aloud, then discuss the prompt.

Invite students to share their tape diagram or sketch.

To encourage creativity, celebrate the variety of diagrams and sketches students create.

Direct students' attention to parts c and d. After the third read, invite students to discuss the question with a partner, then as a class. **(MTR.4.1)**

2 Monitor

D **Differentiation** *(Problem 3)*

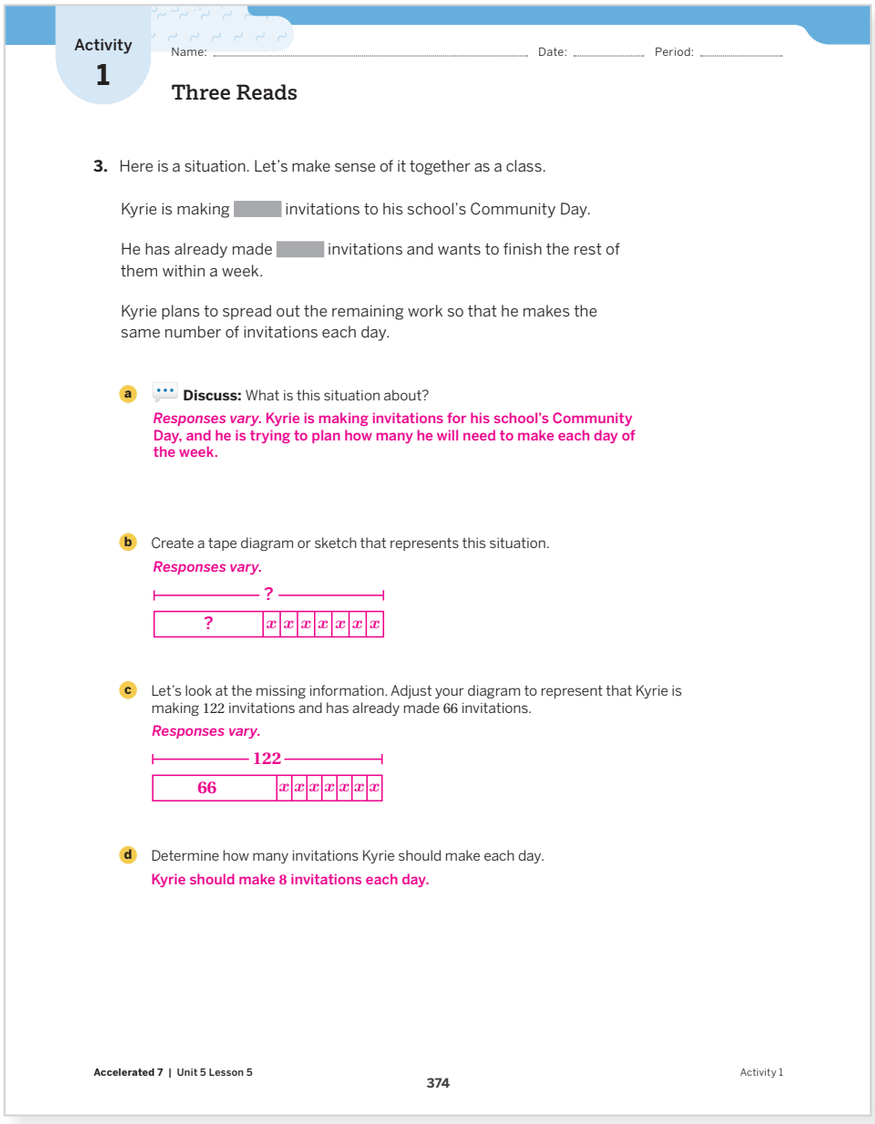
Look for students who:	Respond to Student Thinking
Need support interpreting the phrase "within a week."	**Support** Encourage students to choose a day of the week for the story and determine when Kyrie would be finished by.
Substitute the given values into the tape diagram.	**Strengthen** Consider asking, "What equation would represent this tape diagram and situation?"

A **Accessibility: Conceptual Processing** Invite students to answer a more concrete question first to support abstract thinking, like: *How can a tape diagram help you?*

3 Connect

Invite students to think of new values for this scenario, then to update their diagrams to match. **(MTR.2.1)**

To surface the Key Takeaway, consider asking, "How did you adjust your diagram to the new values?"



Activity 1

Name: Date: Period:

Three Reads

3. Here is a situation. Let's make sense of it together as a class.

Kyrie is making [redacted] invitations to his school's Community Day.

He has already made [redacted] invitations and wants to finish the rest of them within a week.

Kyrie plans to spread out the remaining work so that he makes the same number of invitations each day.

a. **Discuss:** What is this situation about?

Responses vary. Kyrie is making invitations for his school's Community Day, and he is trying to plan how many he will need to make each day of the week.

b. Create a tape diagram or sketch that represents this situation.

Responses vary.

c. Let's look at the missing information. Adjust your diagram to represent that Kyrie is making 122 invitations and has already made 66 invitations.

Responses vary.

d. Determine how many invitations Kyrie should make each day.

Kyrie should make 8 invitations each day.

Accelerated 7 | Unit 5 Lesson 5 374 Activity 1

Key Takeaway: Visuals and diagrams can help you represent situations and answer questions about them.

Pairs | 20 min

Activity 2 Similar Problems

Purpose: Students practice creating visuals in order to write and solve equations for situations in context. **(MTR.2.1)**

Short on time? Invite pairs to select one set. Have one partner work on Situation A and the other on Situation B and then compare their equations and solutions.

1 Launch

Direct students' attention to the instructions and review them as a class.

Distribute the Activity 2 sheet, one per student.

Consider sharing that the situations (A and B) in each set are very similar but not exactly the same.

Note: The sets go in order of difficulty. Invite students to choose one set that they believe would present a meaningful challenge to them.

2 Monitor

Look for student strategies, listen to small group discussions, and offer help or encouragement where needed.

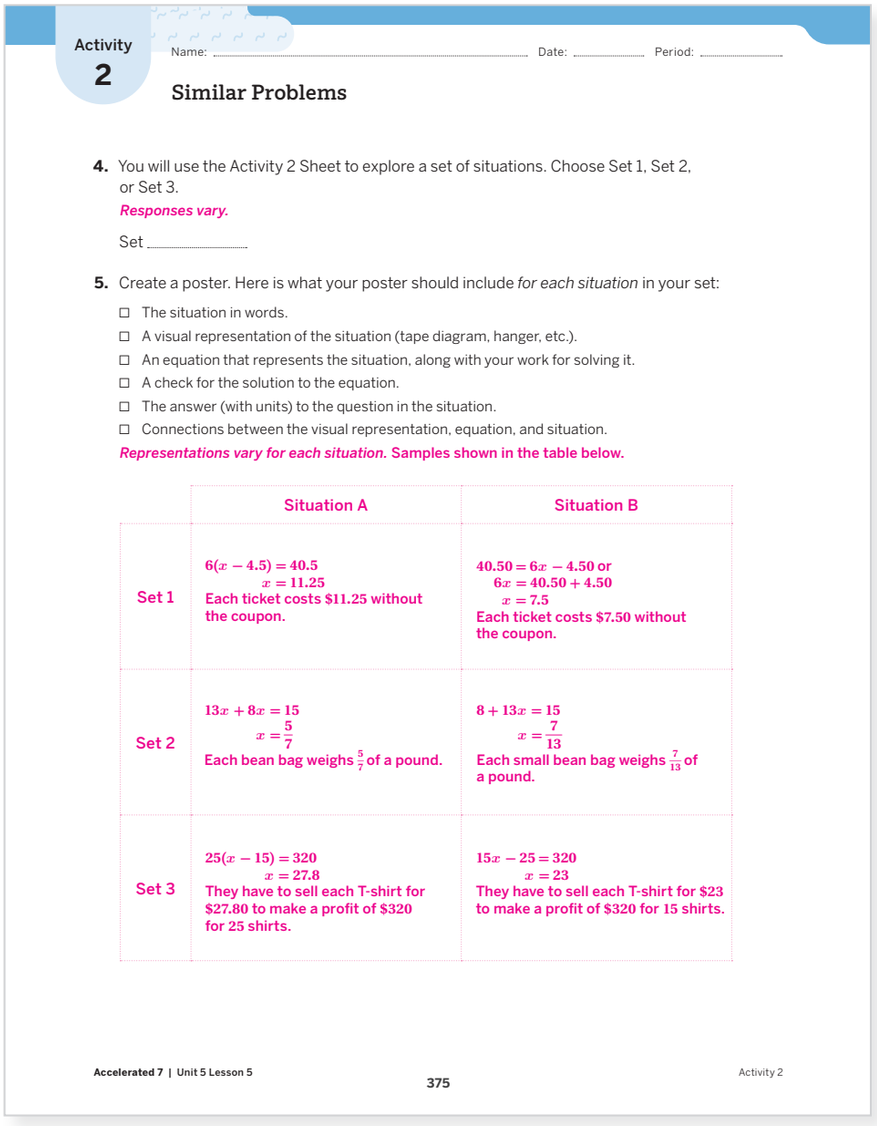
D Differentiation

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Encourage students to discuss specific words to help them determine the differences between the two situations.
Use their visual representation to solve. Use their equation to solve.	**Strengthen** Consider asking, "Would you get the same solution regardless of the representation or equation that you used to solve? Why or why not?"
Would like to explore real-world situations further.	**Stretch** Encourage students to create a new set of two related situations to represent.

3 Connect

Invite students to share their posters and explain connections between the visual representations, equations, and situations. Highlight students who created different visual representations or equations for the same situations. **(MTR.2.1)**

Consider asking, "What words did you find helpful to distinguish the two situations?" **(MTR.7.1)**



Activity 2

Name: _____ Date: _____ Period: _____

Similar Problems

4. You will use the Activity 2 Sheet to explore a set of situations. Choose Set 1, Set 2, or Set 3.

Responses vary.

Set _____

5. Create a poster. Here is what your poster should include *for each situation* in your set:

- ☐ The situation in words.
- ☐ A visual representation of the situation (tape diagram, hanger, etc.).
- ☐ An equation that represents the situation, along with your work for solving it.
- ☐ A check for the solution to the equation.
- ☐ The answer (with units) to the question in the situation.
- ☐ Connections between the visual representation, equation, and situation.

Representations vary for each situation. Samples shown in the table below.

	Situation A	Situation B
Set 1	$6(x - 4.5) = 40.5$ $x = 11.25$ Each ticket costs \$11.25 without the coupon.	$40.50 = 6x - 4.50$ or $6x = 40.50 + 4.50$ $x = 7.5$ Each ticket costs \$7.50 without the coupon.
Set 2	$13x + 8x = 15$ $x = \frac{5}{7}$ Each bean bag weighs $\frac{5}{7}$ of a pound.	$8 + 13x = 15$ $x = \frac{7}{13}$ Each small bean bag weighs $\frac{7}{13}$ of a pound.
Set 3	$25(x - 15) = 320$ $x = 27.8$ They have to sell each T-shirt for \$27.80 to make a profit of \$320 for 25 shirts.	$15x - 25 = 320$ $x = 23$ They have to sell each T-shirt for \$23 to make a profit of \$320 for 15 shirts.

Accelerated 7 | Unit 5 Lesson 5 375 Activity 2

Whole Class | 5 min

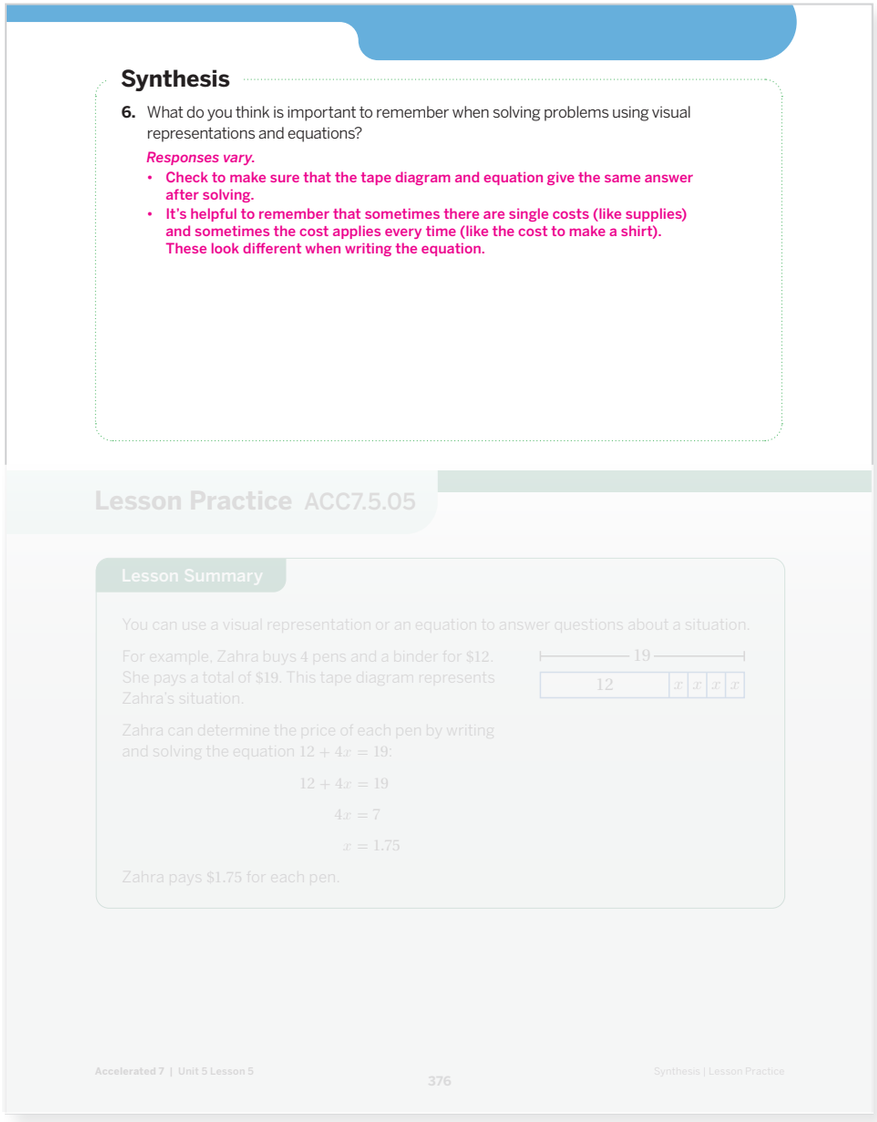
Synthesis

Purpose: Students synthesize their understanding of what is important to remember when representing and answering questions about situations.

Invite students to respond independently, and then share their thinking with a partner.

Collect and share a variety of ideas, such as:
- Using multiple representations to confirm their solution.
- Identifying the difference between constant values and coefficients.

Note: If time allows, invite students to revise their previous thinking based on the discussion.



Synthesis

6. What do you think is important to remember when solving problems using visual representations and equations?

Responses vary.
- **Check to make sure that the tape diagram and equation give the same answer after solving.**
- **It's helpful to remember that sometimes there are single costs (like supplies) and sometimes the cost applies every time (like the cost to make a shirt). These look different when writing the equation.**

Lesson Practice ACC7.5.05

Lesson Summary

You can use a visual representation or an equation to answer questions about a situation.

For example, Zahra buys 4 pens and a binder for \$12. She pays a total of \$19. This tape diagram represents Zahra's situation.

Zahra can determine the price of each pen by writing and solving the equation $12 + 4x = 19$:

$$12 + 4x = 19$$
$$4x = 7$$
$$x = 1.75$$

Zahra pays \$1.75 for each pen.

Accelerated 7 | Unit 5 Lesson 5 376 Synthesis | Lesson Practice

Lesson Takeaway: Creating tape diagrams and equations can help answer questions about situations. Key words like *same*, *each*, *every*, and *total* can help determine if a situation has a single value or a value that applies every time.

Independent | 5 min

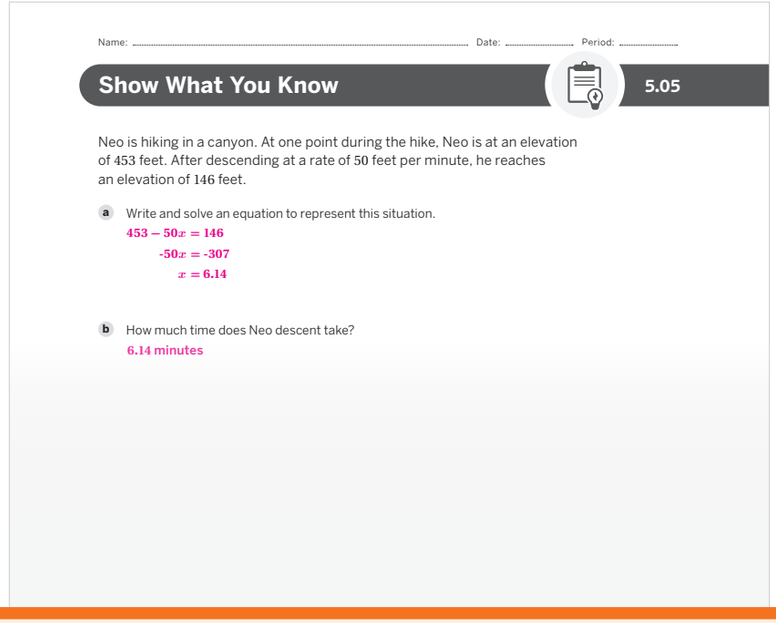
Show What You Know

Purpose: Students demonstrate their understanding by writing and solving an equation in context.

Today's Goals

1. **Goal:** Write and solve equations to represent situations involving positive and negative numbers.
2. **Goal:** Connect visual representations, equations, and verbal descriptions of the same situation.
3. **Language Goal:** Compare and contrast situations using different visual representations and equations. **(Reading and Writing)**

Show What You Know (Assessment Resources)



Name: _____ Date: _____ Period: _____

Show What You Know 5.05

Neo is hiking in a canyon. At one point during the hike, Neo is at an elevation of 453 feet. After descending at a rate of 50 feet per minute, he reaches an elevation of 146 feet.

a. Write and solve an equation to represent this situation.

$453 - 50x = 146$

$-50x = -307$

$x = 6.14$

b. How much time does Neo descent take?

6.14 minutes

Differentiation Use after Lesson 5

Support

Provide targeted intervention for students by using these resources.

If students need support understanding how to write equations to represent situations:

» **Respond to Student Thinking:**

- Revisit **Activity 1** and invite students to use the MLR6: Three Reads routine to make sense of a few related Practice Problems.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 1 Extension Activities**.
- Invite students to write an equation that could represent a real-world situation and determine the corresponding solutions.

Professional Learning

Why was the context of *Community Day* particularly useful in building student understanding of representing and answering questions about situations in context?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

6. What do you think is important to remember when solving problems using visual representations and equations?

Responses vary.

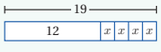
- Check to make sure that the tape diagram and equation give the same answer after solving.
- It's helpful to remember that sometimes there are single costs (like supplies) and sometimes the cost applies every time (like the cost to make a shirt). These look different when writing the equation.

Lesson Practice ACC7.5.05

Lesson Summary

You can use a visual representation or an equation to answer questions about a situation.

For example, Zahra buys 4 pens and a binder for $12. She pays a total of $19. This tape diagram represents Zahra's situation.



Zahra can determine the price of each pen by writing and solving the equation $12 + 4x = 19$:

$$12 + 4x = 19$$
$$4x = 7$$
$$x = 1.75$$

Zahra pays $1.75 for each pen.

Lesson Practice
ACC7.5.05

Name: _____ Date: _____ Period: _____

Problems 1–4: Select an equation to represent the situation. Then solve the equation and explain the solution's meaning in context.

$5x - 7 = 3$ $\quad 7 = 3(5 - x)$ $\quad 3x + 5 = -7$ $\quad x + 7 = 3 \cdot 5$

	Situation	Equation	Solution	Solution's Meaning
1.	The temperature outside is currently -7°C. Since midnight, the temperature tripled and then rose 5 degrees. What was the temperature at midnight?	$3x + 5 = -7$	$x = -4$	The temperature at midnight was -4°C.
2.	Ama has 7 pink roses plus some white roses. She gives all of her roses away by giving 5 roses to each of her 3 favorite teachers. How many white roses does Ama give away?	$x + 7 = 3 \cdot 5$	$x = 8$	Ama gives away 8 white roses.
3.	A family of 3 goes to a fair. Tickets cost $5 each, but each person has a coupon. They pay $7 altogether. How much money does each person save on buying their ticket?	$7 = 3(5 - x)$	$x = \frac{8}{3}$ (or equivalent)	Each person saved $\frac{8}{3}$ on their ticket cost, or $2.67.
4.	A club puts its members into 5 groups for an activity. 7 students leave early, so there are only 3 students left to finish the activity. How many students were in each group?	$5x - 7 = 3$	$x = 2$	2 students were in each group.

Lesson Practice
ACC7.5.05

Name: _____ Date: _____ Period: _____

FAST Practice

5. 6 soccer teams are practicing on a field. Each team has the same number of players, x. A coach asks 2 players from each team to leave the field to help move some equipment. Now there are 78 players on the field. Write and solve an equation whose solution is the number of players on each team.

Equation: $6(x - 2) = 78$ or $6x - 12 = 78$

Solution: $x =$ 15 players on each team

Spiral Review

Problems 6–9: Determine the value of the variable that makes each equation true.

6. $a \cdot 3 = -30$; $a = -10$
7. $-9 \cdot b = -45$; $b = 5$
8. $-89 \cdot 12 = c$; $c = -1068$
9. $d \cdot 88 = -88000$; $d = -1000$

10. Select *all* the expressions that show x increased by 35%.

- ☑ A. $1.35x$
- ☐ B. $\frac{35}{100}x$
- ☑ C. $x + \frac{35}{100}x$
- ☑ D. $(1 + 0.35)x$
- ☐ E. $(100 + 35)x$
- ☑ F. $\frac{100 + 35}{100}x$

11. Alma has a loan of $12,589. The loan requires Alma to pay 2.1% of the loan amount in interest at the end of each year. About how much interest will Alma pay on this loan at the end of the year?

A. $2,644 (B.) $264 C. $15,233 D. $12,853

Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T. Math Benchmarks
On-Lesson			
	1–4	2	MA.7.AR.2.2
FAST Practice	5	2	MA.7.AR.2.2
Spiral Review			
	6–9	1	MA.6.AR.2.3
	10–11	2	MA.7.AR.3.1

Sub-Unit 2

Multi-Step Equations

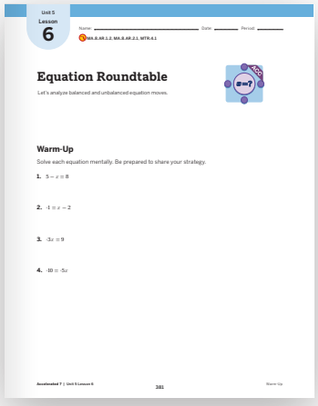
Sub-Unit 2 Goals

- **Goal:** Determine the number of solutions to a linear equation.
- **Goal:** Solve two-step linear inequalities and graph their solutions on a number line.
- **Language Goal:** Interpret the solution of an equation in one variable in context.

Students explore a variety of strategies to solve linear equations with variables on both sides of the equation (e.g., $3(x + 8) = 30 + x$). Along the way, they encounter equations that have no solution and ones that have **infinitely many solutions**. Students then apply what they have learned about equations to solve linear inequalities with two operations.

Math That Matters Most

Concept or Skill	For example . . .
Determine the number of solutions to a linear equation.	Write an equation for each number of solutions. • No solution: $2 + x = 10 + x$ • One solution: $2 + x = 2$ • Infinitely many solutions: $2 + x = x + 2$
Interpret a solution to an equation in context.	Here are two cell phone plans. Plan A costs \$70 per month and comes with a free phone worth \$500. Plan B costs \$50 per month, but doesn't come with a free phone. If you choose Plan B and buy a \$500 phone, after how many months will your total cost be the same as the cost of choosing Plan A? Show your thinking. 25 months. Let x be the number of months. Using Plan A, I spend 70x after x months. Using Plan B, I spend 50x + 500 after x months. $70x = 50x + 500$ $20x = 500$ $x = 25$ The plans will cost the same after 25 months.
Solve inequalities with 2 operations and graph the solution.	Solve the inequality. Graph the solution on a number line. $3 < \frac{1}{4}(w - 8)$ $12 < w - 8$ $20 < w$ or $w > 20$


Unit 5
Lesson 6
Name _____ Date _____ Period _____
MA.8.AR.1.2, MA.8.AR.2.1, MTR.4.1
Equation Roundtable
Let's analyze balanced and unbalanced equation moves.
Warm-Up
Solve each equation mentally. Be prepared to share your strategy.
1. $5 = x + 8$
2. $4 \cdot 0 \neq x - 2$
3. $-3x = 9$
4. $-10 = -5x$
Accelerated 7 | Unit 5 Lesson 6
381
Warm-Up

Equation Roundtable

Analyzing Unbalanced Equations

Let's analyze balanced and unbalanced equation moves.

Learning Outcomes

Today's Goals

1. **Goal:** Solve linear equations in one variable, including equations with rational coefficients.
2. **Language Goal:** Analyze solving steps to determine if they keep an equation balanced. **(Reading,Writing, Speaking, and Listening)**

Students develop fluency in solving equations by analyzing solving steps to determine if they keep an equation balanced, including strategies for solving linear equations with rational coefficients. They use a roundtable structure to collaboratively solve equations, checking to determine if their solution makes an equation true.

Coherence

Prior Learning

In Lessons 1–5, students developed fluency in writing and solving two-step equations using inverse operations.

Future Learning

In Lesson 7, students will explore equations that have no solution and infinitely many solutions.

Connections

- In an error analysis activity, students identify, explain, and correct student thinking in which they solve mult-step equations. (mathematical, cross-curricular)

FL B.E.S.T. Math Benchmarks

Addressing

MA.8.AR.2.1
Solve multi-step linear equations in one variable, with rational number coefficients. Include equations with variables on both sides.

Also Addressing: **MA.8.AR.1.2**

Mathematical Thinking and Reasoning Standard: MTR.4.1

ELA Expectations: ELA.K12.EE.3.1, ELA.K12.EE.5.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.7.AR.1.1	**MA.912.AR.2.1**
MA.7.AR.1.2	**MA.8.AR.4.3**
MA.7.AR.2.2	

Lesson at a Glance

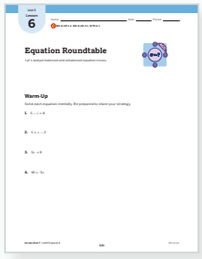
~ 45 min

MA.8.AR.1.2, MA.8.AR.2.1

Warm-Up

Whole Class | 5 min

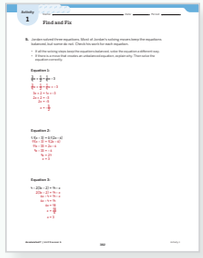
Students use the **Number Talk routine** to surface prior knowledge of balanced equation moves that can be used to solve equations with mental math.



Activity 1

Pairs | 15 min

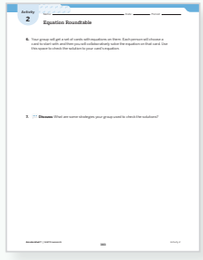
Students analyze equations to determine if the solving moves keep the equation balanced and surface strategies for solving equations with rational coefficients. **MLR3: Critique, Collect, Clarify** **(MTR.4.1)**



Activity 2

Small Groups | 15 min

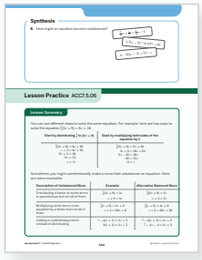
Students build fluency by collectively solving equations: determining the next step in solving an equation then passing to a peer to continue. **MLR2: Collect and Display** **(MTR.4.1)**



Synthesis

Whole Class | 5 min

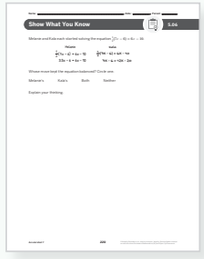
Students synthesize their understanding of moves that will make an equation unbalanced. **MLR2: Collect and Display**



Show What You Know

Independent | 5 min

Students demonstrate their understanding by analyzing the moves of two solving strategies to determine which keeps an equation balanced.



Prep Checklist

Prepare the Activity 2 Cards.

This lesson includes:

- Student Edition
- Show What You Know
- Activity 2 Cards, one per group, pre-cut.
- Calculators (as needed)
- Coloring tools (as needed)

Whole Class | 5 min

Warm-Up

Purpose: Students use the Number Talk routine to surface prior knowledge of balanced equation moves that can be used to solve equations with mental math.

Short on time? Consider omitting this Warm-Up.

1 Launch

Direct students' attention to each problem one at a time.

Use the Number Talk routine to surface prior knowledge about balanced equation moves.

Invite students to solve the first equation independently and then share their strategies aloud. Encourage them to consider how they can communicate their ideas in ways others can understand, and share that everyone is responsible for trying to understand each others' ideas.

Create a record of each strategy that students share, along with the name of the student who shared it.

Consider asking, "Who thought about it in a different way?"

ELL **English Language Learners** Encourage students to paraphrase each other's strategies. **(Speaking and Listening)**

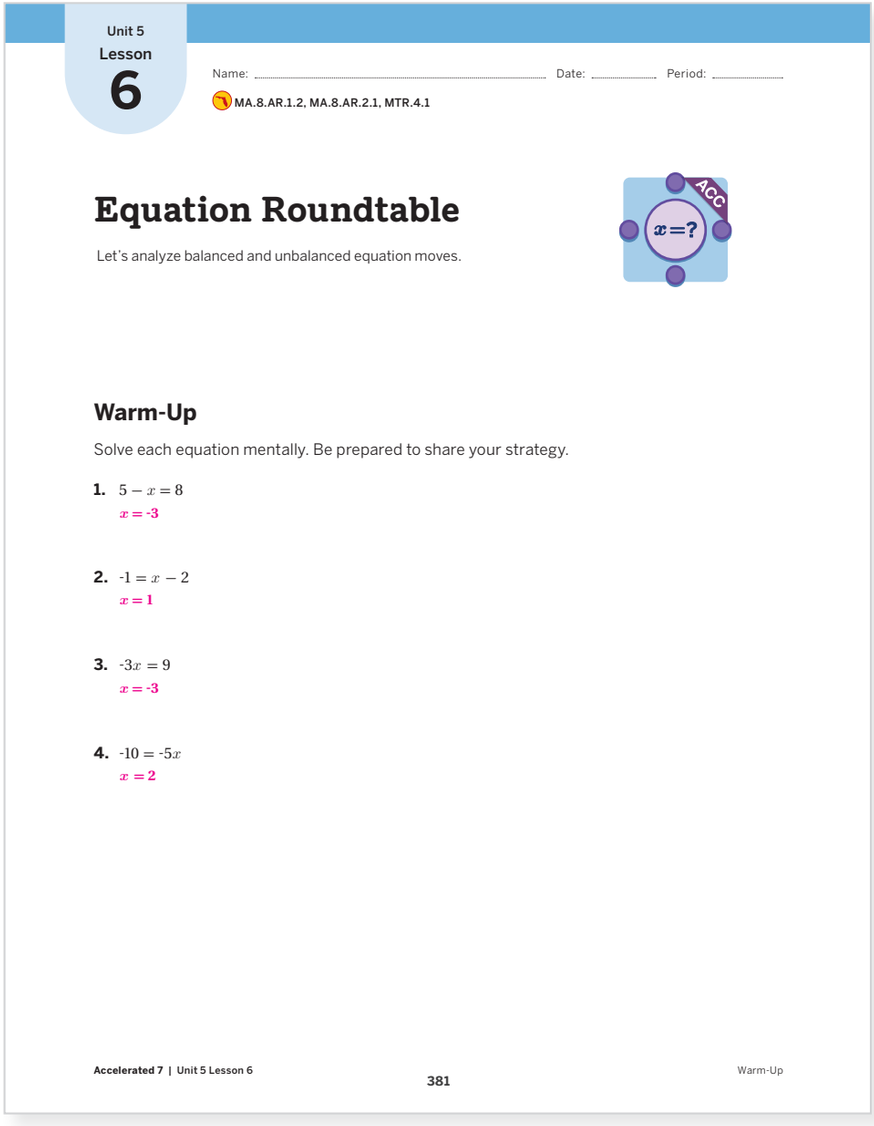
Note: Repeat this structure for each equation in the Number Talk.

2 Connect

Invite students to compare and contrast the different strategies.

Consider asking:

- "What connections do you notice among the strategies we've discussed?" *Responses vary.* All the strategies kept the equations balanced.
- "What moves were most helpful for the equations that had x multiplied by a number?" *Responses vary.* Dividing both sides of the equation by the coefficient, making sure to keep the equation balanced.



Unit 5
Lesson
6

Name: ..... Date: ..... Period: .....

MA.8.AR.1.2, MA.8.AR.2.1, MTR.4.1

Equation Roundtable

Let's analyze balanced and unbalanced equation moves.

Warm-Up

Solve each equation mentally. Be prepared to share your strategy.

1. $5 - x = 8$
   $x = -3$

2. $-1 = x - 2$
   $x = 1$

3. $-3x = 9$
   $x = -3$

4. $-10 = -5x$
   $x = 2$

Accelerated 7 | Unit 5 Lesson 6 381 Warm-Up

Pairs | 15 min

Activity 1 Find and Fix

Purpose: Students analyze equations to determine if the solving moves keep the equation balanced and surface strategies for solving equations with rational coefficients. **(MTR.4.1)**

1 Launch

Direct students' attention to the equation and invite them to discuss possible first steps to solve it.

2 Monitor

A **Accessibility: Executive Functioning** Consider chunking this activity by inviting students to use blank paper to cover up all the steps except the two they are analyzing and to use coloring tools to mark which moves they think make the equation unbalanced.

Look for a variety of strategies students use.

D **Differentiation** *(Problem 5)*

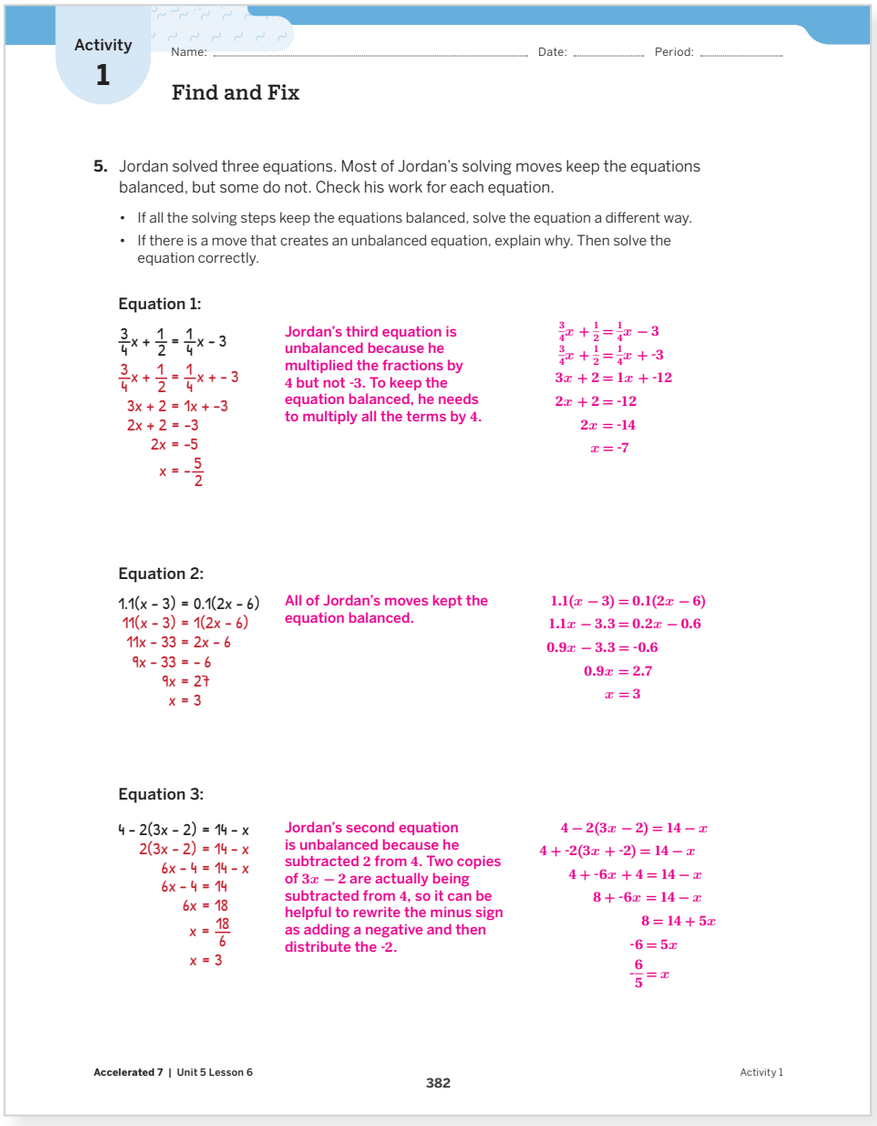
Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Consider asking, "What are some ways to check Jordan's work?"
Think all of Jordan's moves kept the equation balanced. *(Equation 1)*	**Support** Consider asking, "What did Jordan do to eliminate the fractions on both sides? When multiplying by a factor, what needs to happen to keep the equation balanced?"
Think Jordan removed the decimals in his first move. *(Equation 2)*	**Support** Consider asking, "What operation could you perform on 1.1 to get 11?"
Think calculations should be performed in order from left to right. *(Equation 3)*	**Support** Consider asking, "When there is a number outside of parentheses, what operation is implied?"

3 Connect

MLR **MLR3: Critique, Correct, Clarify** Use this routine to help students describe strengths and errors in Jordan's work. Invite students to share their responses from the activity. See the Routine Facilitation Guide for more information. **(MTR.4.1)**

To surface the Key Takeaway, consider asking:

- "What strategy did Jordan use to solve an equation with fractions or decimals?" *Responses vary.* Jordan multiplied both sides of the equation by a factor that would create a whole number.
- "What's another strategy for solving equations with fractions?"
- "What are some moves that make equations unbalanced?"



Activity 1

Name: Date: Period:

Find and Fix

5. Jordan solved three equations. Most of Jordan's solving moves keep the equations balanced, but some do not. Check his work for each equation.

- If all the solving steps keep the equations balanced, solve the equation a different way.
- If there is a move that creates an unbalanced equation, explain why. Then solve the equation correctly.

Equation 1:

$\frac{3}{4}x + \frac{1}{2} = \frac{1}{4}x - 3$
$\frac{3}{4}x + \frac{1}{2} = \frac{1}{4}x + -3$
$3x + 2 = 1x + -3$
$2x + 2 = -3$
$2x = -5$
$x = -\frac{5}{2}$

Jordan's third equation is unbalanced because he multiplied the fractions by 4 but not -3. To keep the equation balanced, he needs to multiply all the terms by 4.

$\frac{3}{4}x + \frac{1}{2} = \frac{1}{4}x - 3$
$\frac{3}{4}x + \frac{1}{2} = \frac{1}{4}x + -3$
$3x + 2 = 1x + -12$
$2x + 2 = -12$
$2x = -14$
$x = -7$

Equation 2:

$1.1(x - 3) = 0.1(2x - 6)$
$11(x - 3) = 1(2x - 6)$
$11x - 33 = 2x - 6$
$9x - 33 = -6$
$9x = 27$
$x = 3$

All of Jordan's moves kept the equation balanced.

$1.1(x - 3) = 0.1(2x - 6)$
$1.1x - 3.3 = 0.2x - 0.6$
$0.9x - 3.3 = -0.6$
$0.9x = 2.7$
$x = 3$

Equation 3:

$4 - 2(3x - 2) = 14 - x$
$2(3x - 2) = 14 - x$
$6x - 4 = 14 - x$
$6x - 4 = 14$
$6x = 18$
$x = \frac{18}{6}$
$x = 3$

Jordan's second equation is unbalanced because he subtracted 2 from 4. Two copies of $3x - 2$ are actually being subtracted from 4, so it can be helpful to rewrite the minus sign as adding a negative and then distribute the -2.

$4 - 2(3x - 2) = 14 - x$
$4 + -2(3x + -2) = 14 - x$
$4 + -6x + 4 = 14 - x$
$8 + -6x = 14 - x$
$8 = 14 + 5x$
$-6 = 5x$
$\frac{6}{5} = x$

Accelerated 7 | Unit 5 Lesson 6 382 Activity 1

Key Takeaway: One way to solve an equation with fractions is to multiply both sides of the equation by a factor that will result in whole number coefficients.

Small Groups | 15 min

Activity 2 Equation Roundtable

Purpose: Students build fluency by collectively solving equations: determining the next step in solving an equation then passing to a peer to continue. **(MTR.4.1)**

1 Launch

Direct students' attention to the instructions and review them as a class.

Note: There are four cards in each set, one for each student in a group of four. If students are in groups of three, invite them to skip one of the cards.

Distribute the Activity 2 Cards to each group.

MLR **MLR2: Collect and Display** Use this routine to revisit students' strategies for solving equations prior to beginning the Equation Roundtable. Add the new strategies for solving equations with rational coefficients to the display for students to reference as they solve the equations in this activity.

2 Monitor

A **Accessibility: Memory and Attention** Invite students to use a calculator to support them with performing calculations.

Look for a variety of strategies students use to solve the equations in their groups.

D **Differentiation** *(Problem 6)*

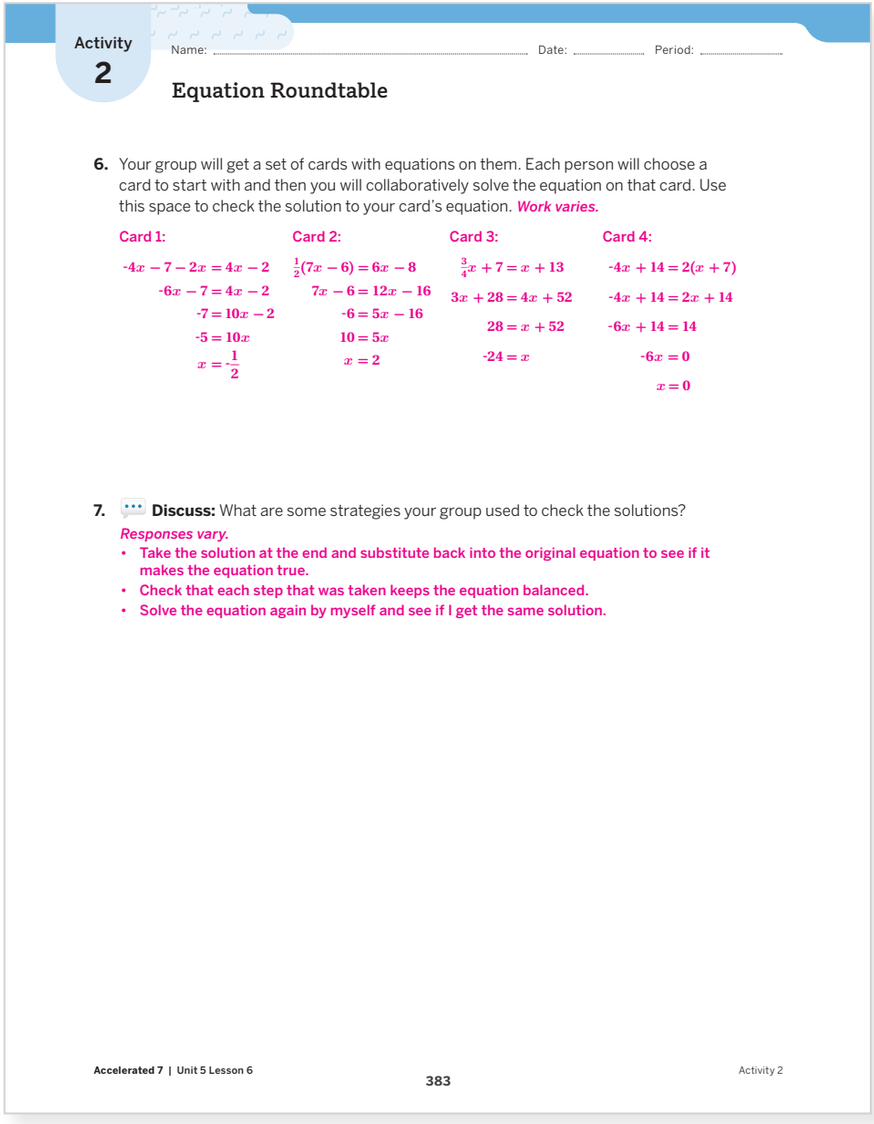
Look for students who:	Respond to Student Thinking
Need support getting started.	Support Encourage them to refer to the class list of strategies for solving equations. Consider asking, "What's a balanced equation move you can try?"
Multiply by a factor to eliminate fractions. (*Cards 2 and 3*)	Stretch Invite them to think about how they might share this strategy during the Connect.

Encourage students to check their solution to see if it makes the equation true.

3 Connect

Invite students to analyze their and their group's steps and consider whether there was a more efficient way to solve the equation. **(MTR.4.1)**

Consider asking, "How did starting from someone else's solving step feel different than starting from your own?"



Activity 2

Name: Date: Period:

Equation Roundtable

6. Your group will get a set of cards with equations on them. Each person will choose a card to start with and then you will collaboratively solve the equation on that card. Use this space to check the solution to your card's equation. *Work varies.*

Card 1:

$-4x - 7 - 2x = 4x - 2$
$-6x - 7 = 4x - 2$
$-7 = 10x - 2$
$-5 = 10x$
$x = -\frac{1}{2}$

Card 2:

$\frac{1}{2}(7x - 6) = 6x - 8$
$7x - 6 = 12x - 16$
$-6 = 5x - 16$
$10 = 5x$
$x = 2$

Card 3:

$\frac{3}{4}x + 7 = x + 13$
$3x + 28 = 4x + 52$
$28 = x + 52$
$-24 = x$

Card 4:

$-4x + 14 = 2(x + 7)$
$-4x + 14 = 2x + 14$
$-6x + 14 = 14$
$-6x = 0$
$x = 0$

7. **Discuss:** What are some strategies your group used to check the solutions?

Responses vary.

- Take the solution at the end and substitute back into the original equation to see if it makes the equation true.
- Check that each step that was taken keeps the equation balanced.
- Solve the equation again by myself and see if I get the same solution.

Accelerated 7 | Unit 5 Lesson 6 383 Activity 2

Whole Class | 5 min

Synthesis

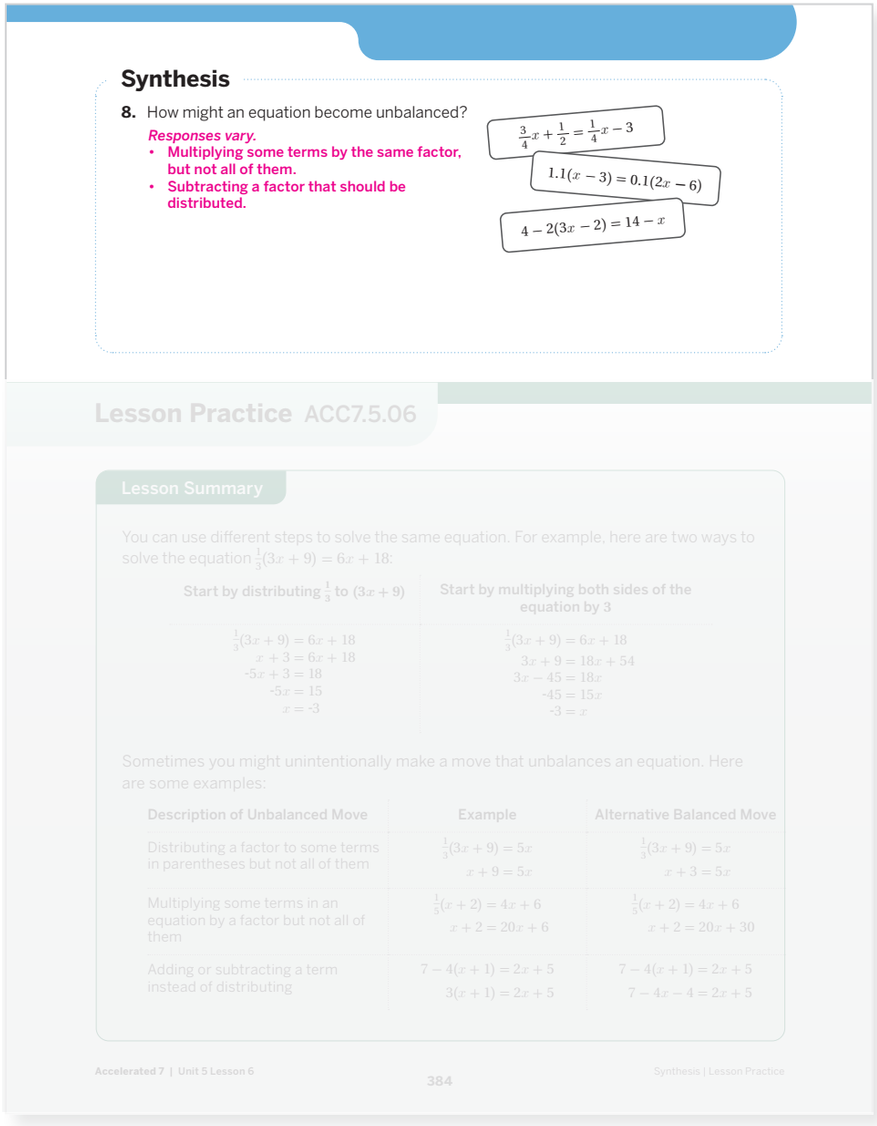
Purpose: Students synthesize their understanding of moves that will make an equation unbalanced.

Invite students to respond independently, and then share their thinking with a partner.

Collect and share a variety of ideas, such as:
- Multiplying or dividing only some of the terms by a number on both sides of the equation.
- Subtracting a number that should be distributed.

MLR **MLR2: Collect and Display** Use this routine to revisit students' strategies for solving linear equations. Add their ideas for moves that unintentionally create unbalanced equations to the display for students to reference for the rest of the unit.

Note: If time allows, invite students to share with a partner what makes sense about each strategy and what questions they still have.



Synthesis

8. How might an equation become unbalanced?

Responses vary.
- Multiplying some terms by the same factor, but not all of them.
- Subtracting a factor that should be distributed.

$\frac{3}{4}x + \frac{1}{2} = \frac{1}{4}x - 3$

$1.1(x - 3) = 0.1(2x - 6)$

$4 - 2(3x - 2) = 14 - x$

Lesson Practice ACC7.5.06

Lesson Summary

You can use different steps to solve the same equation. For example, here are two ways to solve the equation $\frac{1}{3}(3x + 9) = 6x + 18$:

Start by distributing $\frac{1}{3}$ to $(3x + 9)$	Start by multiplying both sides of the equation by 3
$\frac{1}{3}(3x + 9) = 6x + 18$ $x + 3 = 6x + 18$ $-5x + 3 = 18$ $-5x = 15$ $x = -3$	$\frac{1}{3}(3x + 9) = 6x + 18$ $3x + 9 = 18x + 54$ $3x - 45 = 18x$ $-45 = 15x$ $-3 = x$

Sometimes you might unintentionally make a move that unbalances an equation. Here are some examples:

Description of Unbalanced Move	Example	Alternative Balanced Move
Distributing a factor to some terms in parentheses but not all of them	$\frac{1}{3}(3x + 9) = 5x$ $x + 9 = 5x$	$\frac{1}{3}(3x + 9) = 5x$ $x + 3 = 5x$
Multiplying some terms in an equation by a factor but not all of them	$\frac{1}{5}(x + 2) = 4x + 6$ $x + 2 = 20x + 6$	$\frac{1}{5}(x + 2) = 4x + 6$ $x + 2 = 20x + 30$
Adding or subtracting a term instead of distributing	$7 - 4(x + 1) = 2x + 5$ $3(x + 1) = 2x + 5$	$7 - 4(x + 1) = 2x + 5$ $7 - 4x - 4 = 2x + 5$

Accelerated 7 | Unit 5 Lesson 6 384 Synthesis | Lesson Practice

Lesson Takeaway: There are many moves that you can make when solving that will make the equation unbalanced. You can avoid these moves by remembering to multiply a factor times all the terms on both sides of an equation and by rewriting negative coefficients as adding a negative, then distributing the negative factor.

Independent | 5 min

Show What You Know

Purpose: Students demonstrate their understanding by analyzing the moves of two solving strategies to determine which keeps an equation balanced.

Today's Goals

1. **Goal:** Solve linear equations in one variable, including equations with rational coefficients.
2. **Language Goal:** Analyze solving steps to determine if they keep an equation balanced. **(Reading,Writing, Speaking, and Listening)**

Show What You Know (Assessment Resources)

Name: _____ Date: _____ Period: _____

Show What You Know 5.06

Melanie and Kala each started solving the equation $\frac{1}{2}(7x - 6) = 6x - 10$.

Melanie	Kala
$\frac{1}{2}(7x - 6) = 6x - 10$	$\frac{1}{2}(7x - 6) = 6x - 10$
$3.5x - 6 = 6x - 10$	$7x - 6 = 12x - 20$

Whose move kept the equation balanced? Circle one.

Melanie's Kala's Both Neither

Explain your thinking.

Explanations vary. Kala's move kept the equation balanced because she multiplied both sides of the equation by 2. Melanie's move unbalanced the equation because she only distributed the $\frac{1}{2}$ to the first term in the parentheses instead of to both terms.

D Differentiation Use after Lesson 6

Support

Provide targeted intervention for students by using these resources.

If students need support with applying balanced equation moves to solve a linear equation:

» **Respond to Student Thinking:**

- Use the ***Solving Linear Equations With Parentheses*** **Mini-Lesson**.
- Review the **Lesson Summary** and provide feedback to students on select **Practice Problems**.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Revisit **Activity 2** as a class and invite students to solve the problems using a different strategy than they used before.
- Invite students to explore the **Sub-Unit 2 Extension Activities**.

Professional Learning

Take a moment to reflect on the strategies students used in this lesson. What opportunities were there to celebrate specific students' strategies for solving equations? What opportunities might there be in future lessons to connect back to the strategies students shared?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

8. How might an equation become unbalanced?

Responses vary.

- Multiplying some terms by the same factor, but not all of them.
- Subtracting a factor that should be distributed.

$\frac{1}{5}x + \frac{1}{5} = \frac{1}{4}x - 3$

$1.1(x - 3) = 0.1(2x - 6)$

$4 - 2(3x - 2) = 14 - x$

Lesson Practice ACC7.5.06

Lesson Summary

You can use different steps to solve the same equation. For example, here are two ways to solve the equation $\frac{1}{3}(3x + 9) = 6x + 18$:

Start by distributing $\frac{1}{3}$ to $(3x + 9)$	Start by multiplying both sides of the equation by 3
$\frac{1}{3}(3x + 9) = 6x + 18$ $x + 3 = 6x + 18$ $-5x + 3 = 18$ $-5x = 15$ $x = -3$	$\frac{1}{3}(3x + 9) = 6x + 18$ $3x + 9 = 18x + 54$ $3x - 45 = 18x$ $-45 = 15x$ $-3 = x$

Sometimes you might unintentionally make a move that unbalances an equation. Here are some examples:

Description of Unbalanced Move	Example	Alternative Balanced Move
Distributing a factor to some terms in parentheses but not all of them	$\frac{1}{3}(3x + 9) = 5x$ $x + 9 = 5x$	$\frac{1}{3}(3x + 9) = 5x$ $x + 3 = 5x$
Multiplying some terms in an equation by a factor but not all of them	$\frac{1}{5}(x + 2) = 4x + 6$ $x + 2 = 20x + 6$	$\frac{1}{5}(x + 2) = 4x + 6$ $x + 2 = 20x + 30$
Adding or subtracting a term instead of distributing	$7 - 4(x + 1) = 2x + 5$ $3(x + 1) = 2x + 5$	$7 - 4(x + 1) = 2x + 5$ $7 - 4x - 4 = 2x + 5$

Lesson Practice ACC7.5.06

Name: Date: Period:

Problems 1–3: Solve each equation. Show your thinking.

1. $2(x + 5) = 3x + 1$

$x = 9$. Work varies.

$2x + 10 = 3x + 1$
$10 = x + 1$
$9 = x$

2. $3y - 4 = 6 - 2y$

$y = 2$. Work varies.

$5y - 4 = 6$
$5y = 10$
$y = 2$

3. $3(n + 2) = 9(6 - n)$

$n = 4$. Work varies.

$3n + 6 = 54 - 9n$
$12n + 6 = 54$
$12n = 48$
$n = 4$

4. Here is how Gabriela solved an equation. Circle Gabriela's mistake(s) and explain what she did wrong.

Responses vary. Gabriela made two mistakes in the second line. She incorrectly multiplied 12 by 5, which should be 60, not 72. She also didn't distribute the negative sign correctly on the right side. It should be $4y - 5 + 9y$.

Gabriela

$12(5 + 2y) = 4y - (5 - 9y)$
$72 + 24y = 4y - 5 - 9y$
$72 + 24y = -5y - 5$
$24y = -5y - 77$
$29y = -77$
$y = -\frac{77}{29}$

What is the correct solution?

$y = -\frac{65}{11}$. *Work varies.*

$12(5 + 2y) = 4y - (5 - 9y)$
$60 + 24y = 4y - 5 + 9y$
$60 + 24y = 13y - 5$
$60 + 11y = -5$
$11y = -65$
$y = -\frac{65}{11}$

Problems 5–6: Solve each equation. Show your thinking.

5. $\frac{1}{9}(2m - 16) = \frac{1}{3}(2m + 4)$

$m = -7$. Work varies.

$\frac{1}{9}(2m - 16) \cdot 9 = \frac{1}{3}(2m + 4) \cdot 9$
$2m - 16 = 3(2m + 4)$
$2m - 16 = 6m + 12$
$-16 = 4m + 12$
$-28 = 4m$
$-7 = m$

6. $1.5(5 + 0.2y) = 0.4y - (0.6 - 0.9y)$

$y = 8.1$. Work varies.

$7.5 + 0.3y = 0.4y - 0.6 + 0.9y$
$75 + 3y = 4y - 6 + 9y$
$75 + 3y = 13y - 6$
$75 = 10y - 6$
$81 = 10y$
$8.1 = y$

Lesson Practice ACC7.5.06

Name: Date: Period:

7. Complete the equation so that the solution is $n = 8$. *Responses vary.*

$-4\left(\frac{2}{5}n - 6.1\right) =$ $\frac{1}{5}n + 10$

FAST Practice

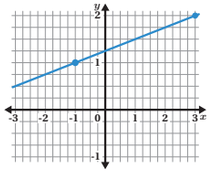
8. What is the solution to the equation $4.1(x + 2) = 8.1x - 1.8$?

A. $x = 1.6$
B. $x = 2.5$
C. $x = 0.4$
D. $x = -0.3$

Spiral Review

9. Here is the graph of a linear equation. Select *all* the true statements about this line and its equation.

- [x] A. One solution to the equation is the ordered pair (3, 2).
- [x] B. One solution to the equation is the ordered pair (-1, 1).
- [ ] C. One solution to the equation is the ordered pair $\left(\frac{3}{2}, 1\right)$.
- [ ] D. There are only two solutions.
- [x] E. The equation of the line is $y = \frac{1}{4}x + \frac{5}{4}$.



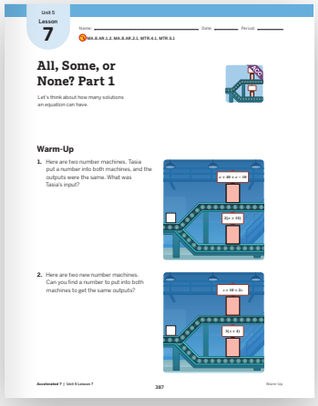
Problems 10–11: Evaluate each expression.

10. $8 + 5^2$ = 33

11. $(1 + 9)^3$ = 1000

Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T Math Benchmarks
On-lesson			
	1–6	2	MA.8.AR.2.1
	7	3	MA.8.AR.2.1
FAST Practice	8	2	MA.8.AR.2.1
Spiral Review			
	9	2	MA.8.AR.3.4
	10–11	2	MA.6.NSO.3.3



All, Some, or None? Part 1

How Many Solutions?

Let's think about how many solutions an equation can have.

Learning Outcomes

Today's Goals

1. **Goal:** Recognize what it means for an equation to have no solution, one solution, or *infinitely many solutions*.
2. **Language Goal:** Justify the number of solutions to a one-variable linear equation using a variety of strategies, including using the structure of the equation. **(Reading, Writing, Speaking, and Listening)**

Students use number machines to explore the idea of equations with no and infinite solutions. Students sort a variety of equations into categories based on the number of solutions they have and consider ways to predict the number of solutions from the structure of an equation. **(MTR.5.1)**

Coherence

Prior Learning

In Lesson 6, students solved linear equations in one variable and justified the steps used to calculate a solution. They strengthened their understanding by analyzing and naming common mistakes.

Future Learning

In Lesson 8, students will strategically solve equations in one variable based on their structures, including analyzing what happens when you try to solve an equation with no solution or infinitely many solutions.

Connections

- Students identify patterns to determine the number of solutions an equation can have. (mathematical)

FL B.E.S.T. Math Benchmarks

Addressing

MA.8.AR.2.1
Solve multi-step linear equations in one variable, with rational number coefficients. Include equations with variables on both sides.

Also Addressing: **MA.8.AR.1.2**

Mathematical Thinking and Reasoning Standards: MTR.4.1, MTR.5.1

ELA Expectations: ELA.K12.EE.3.1, ELA.K12.EE.6.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.7.AR.2.1	MA.912.AR.2.1

Vocabulary

New Vocabulary

infinitely many solutions

Review Vocabulary

equivalent expressions

expression

Additional Vocabulary

like terms

Lesson at a Glance

~ 45 min

MA.8.AR.1.2, MA.8.AR.2.1

Warm-Up

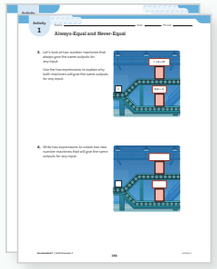
Pairs | 5 min

Students use number machines to examine pairs of *expressions* that result in outputs that are always equal or never equal for a given input. **Routine:** Notice and Wonder

Activity 1

Pairs | 15 min

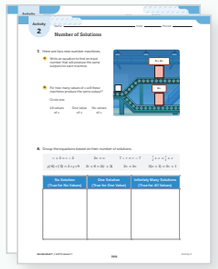
Students further explore number machines to determine whether two expressions are always equivalent or never equivalent.



Activity 2

Pairs | 15 min

Students apply what they learned in the previous activity to sort and write linear equations according to the number of solutions they have: one, none, or infinitely many. **(MTR.5.1)**



Synthesis

Whole Class | 5 min

Students synthesize their understanding of using an equation's structure to determine whether there are no solutions, one solution, or infinitely many solutions. MLR2: Collect and Display **(MTR.5.1)**

Show What You Know

Independent | 5 min

Students demonstrate their understanding by using the structure of equations to identify the number of solutions to three different equations.

Prep Checklist

This lesson includes:

- Student Edition
- Show What You Know

Pairs | 5 min

Warm-Up

Purpose: Students use number machines to examine pairs of *expressions* that result in outputs that are always equal or never equal for a given input.

1 Launch

Use the Notice and Wonder routine to promote curiosity and help students make sense of what looks familiar and different between the two machines. See the Routine Facilitation Guide for more information.

To encourage creativity, invite students to input numbers they think their classmates won't choose. Consider suggesting they use a table to organize their test values.

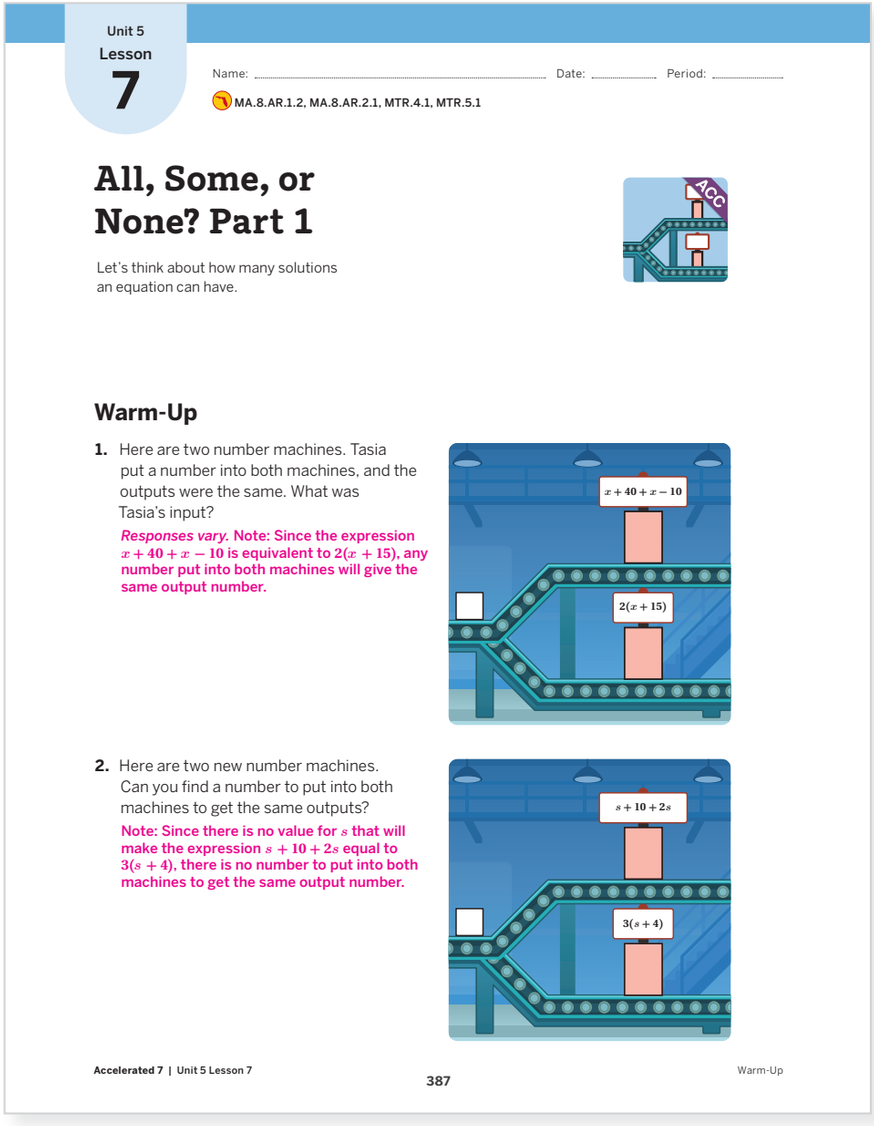
Capture student strategies to share during the Connect.

2 Connect

Invite several students to share responses for both problems.

Consider asking:

- "For the second set of machines, can anyone find an input that will make both machines have the same output?"
- "Why can we find an input that gives the same output for Problem 1 and not Problem 2?"



Unit 5
Lesson
7

Name: Date: Period:

MA.8.AR.1.2, MA.8.AR.2.1, MTR.4.1, MTR.5.1

All, Some, or None? Part 1

Let's think about how many solutions an equation can have.

ACC

Warm-Up

1. Here are two number machines. Tasia put a number into both machines, and the outputs were the same. What was Tasia's input?

   Responses vary. Note: Since the expression $x + 40 + x - 10$ is equivalent to $2(x + 15)$, any number put into both machines will give the same output number.

2. Here are two new number machines. Can you find a number to put into both machines to get the same outputs?

   Note: Since there is no value for s that will make the expression $s + 10 + 2s$ equal to $3(s + 4)$, there is no number to put into both machines to get the same output number.

Accelerated 7 | Unit 5 Lesson 7 387 Warm-Up

Pairs | 15 min

Activity 1 Always-Equal and Never-Equal

Purpose: Students further explore number machines to determine whether two expressions are always equivalent or never equivalent.

1 Launch

To surface connections, consider discussing which set of machines from the Warm-Up this is similar to.

Look and listen for responses that consider the equivalence of these expressions, or reference the distributive property and combining *like terms*.

Consider asking, "Why do you think this machine always produces outputs that are the same?"

A **Accessibility: Memory and Attention** Invite students to record each input and output pair to support students with keeping track of information.

2 Monitor

Look for a few interesting examples of two expressions that will produce the same outputs for any input.

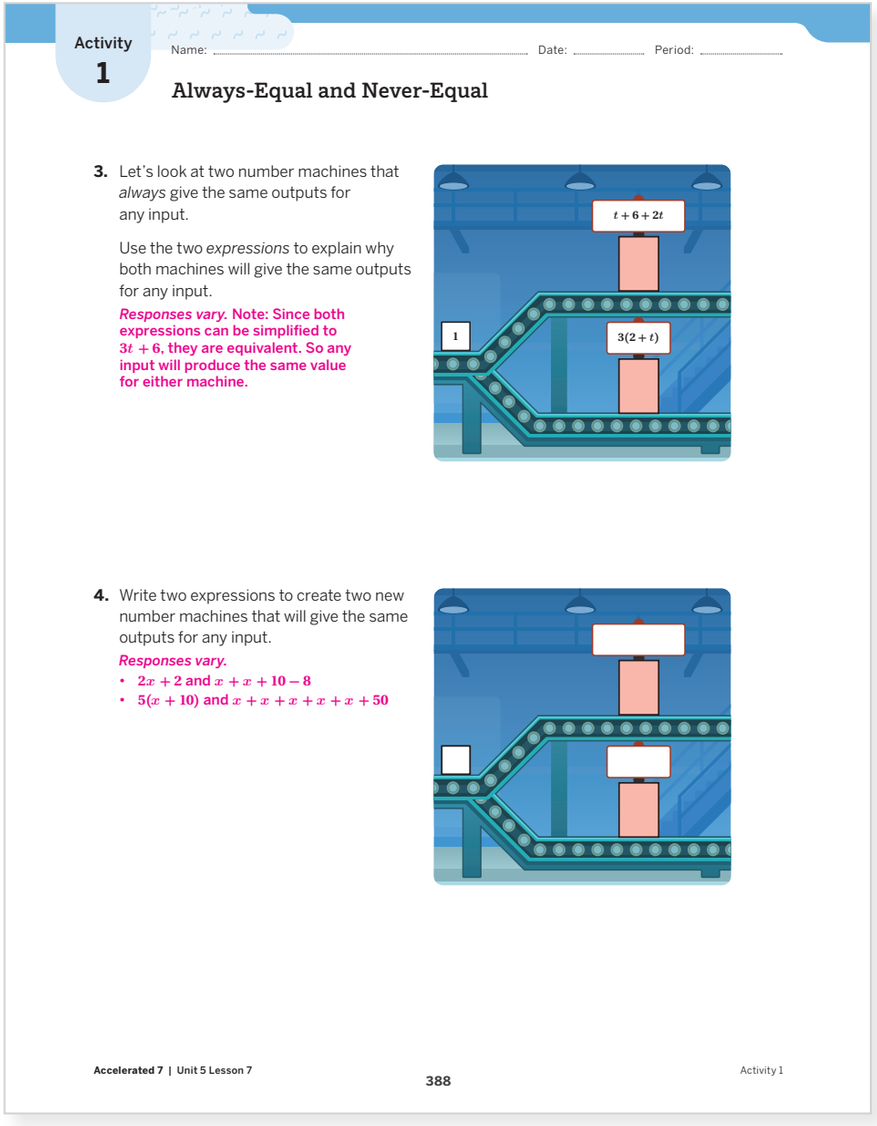
D **Differentiation** *(Problem 4)*

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Invite them to use the expressions in the previous problem as a starting point and change one number or variable.
Write two expressions that are not equivalent.	**Support** Invite students to visit peers and hear their strategies for writing two equivalent expressions.
Write the same expressions for always-equal machines.	**Strengthen** Encourage them to write two different expressions that are equivalent, using those in Problem 3 as an example.
Write unique expressions, particularly those that consider the structure of the expression or reference distributing the 3 in the bottom expression from Problem 3.	**Stretch** Invite students to share during the Connect. Consider asking, "Does your reasoning also apply to the expressions $4x + 5 + x$ and $5(x + 1)$?"

Pause and display examples of student's expressions. Discuss why *equivalent expressions* will always produce the same output. Consider asking:

- "What do you notice about the two expressions that are always equal?"
- "Why are they always equal?"

Activity 1 continued >



Activity 1

Name: Date: Period:

Always-Equal and Never-Equal

3. Let's look at two number machines that *always* give the same outputs for any input.

Use the two *expressions* to explain why both machines will give the same outputs for any input.

Responses vary. Note: Since both expressions can be simplified to $3t + 6$, they are equivalent. So any input will produce the same value for either machine.

4. Write two expressions to create two new number machines that will give the same outputs for any input.

Responses vary.

- $2x + 2$ and $x + x + 10 - 8$
- $5(x + 10)$ and $x + x + x + x + x + 50$

Pairs | 15 min

Activity 1 Always-Equal and Never-Equal (continued)

Purpose: Students further explore number machines to determine whether two expressions are always equivalent or never equivalent.

2 Monitor

Consider sharing that this next number machine will never give the same outputs for any input.

D Differentiation *(Problems 5-6)*

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Consider having students make a table of values to keep track of the inputs and outputs of each machine. Consider asking, "How do the outputs of the machines compare? How do you see that in the expressions for each machine?"
Conclude that the two expressions will never be equal after checking a few numbers.	**Strengthen** Invite students to consider the characteristics of the expressions themselves in their justification.
Write unique answers, particularly those that consider the difference between the outputs is always 4.	**Stretch** Invite students to share during the Connect. Consider asking, "Where do you see the difference of 4 when comparing the expressions?"

Consider asking, "For each equation, what two expressions will go in the number machine?"

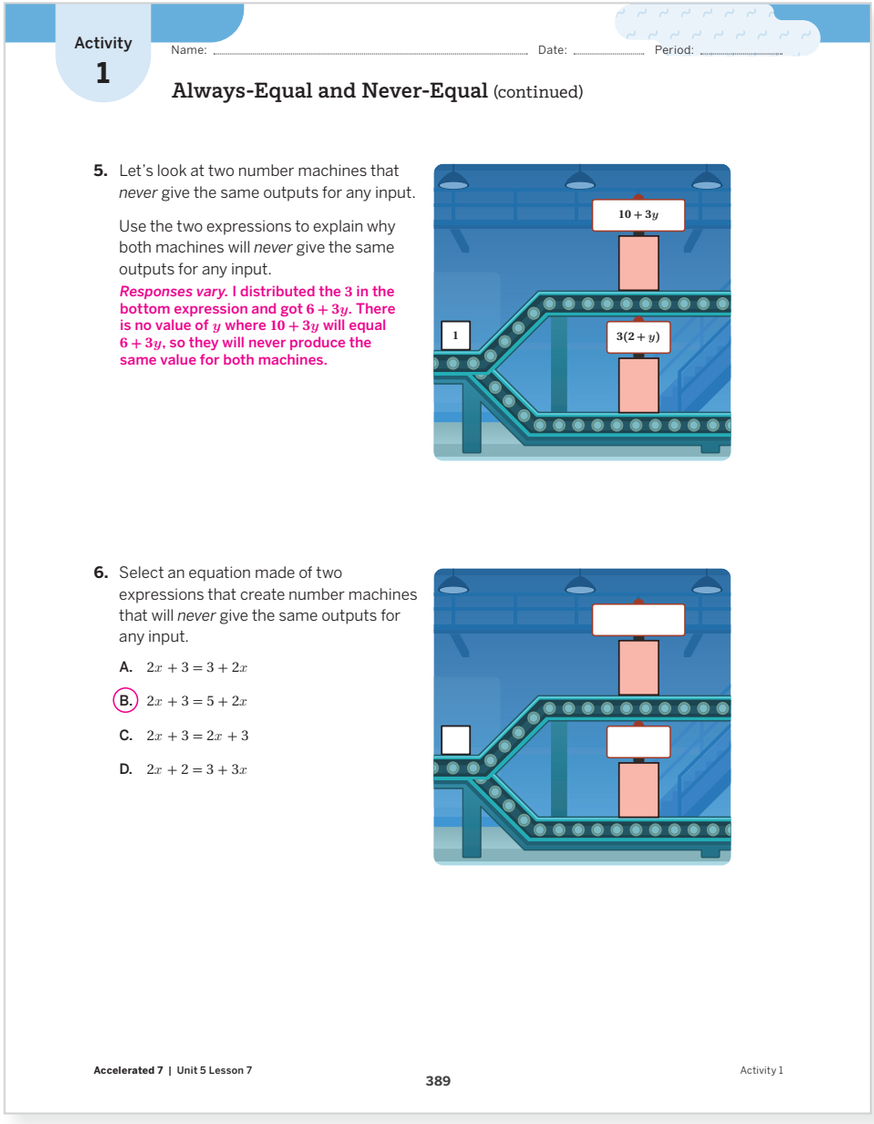
A Accessibility: Memory and Attention To support students in activating prior knowledge of equivalent expressions, invite them to describe what makes two expressions equivalent in their own words. Then ask them to create two expressions that are equivalent and two that are never equivalent, and explain why.

3 Connect

Invite students to share their responses, calling attention to any conflict or consensus.

To surface the Key Takeaway, consider asking:

- "What does the equal sign represent here?"
- "How do we know these machines will never be equal?"
- "How can you use the structure of the expressions to determine if they are always equal or never equal?"
- "Which of these equations would represent always equal machines?" Equation A



Activity 1

Name: Date: Period:

Always-Equal and Never-Equal (continued)

5. Let's look at two number machines that *never* give the same outputs for any input.

Use the two expressions to explain why both machines will *never* give the same outputs for any input.

Responses vary. **I distributed the 3 in the bottom expression and got $6 + 3y$. There is no value of y where $10 + 3y$ will equal $6 + 3y$, so they will never produce the same value for both machines.**

6. Select an equation made of two expressions that create number machines that will *never* give the same outputs for any input.

A. $2x + 3 = 3 + 2x$

B. $2x + 3 = 5 + 2x$

C. $2x + 3 = 2x + 3$

D. $2x + 2 = 3 + 3x$

Accelerated 7 | Unit 5 Lesson 7 389 Activity 1

Key Takeaway: When you set *expressions* equal to each other, look for an input (or inputs) that make the equation true. Some expressions will have outputs that are always equal, while others have outputs that are never equal.

Pairs | 15 min

Activity 2 Number of Solutions

Purpose: Students apply what they learned in the previous activity to sort and write linear equations according to the number of solutions they have: one, none, or infinitely many. (MTR.5.1)

Short on time? Consider omitting Problem 10.

1 Launch

Encourage students to try writing an equation independently before discussing with a partner.

Discuss strategies for determining the number of values of n that will produce the same output, and highlight why the equation $6 + 2n = 8n$ will only be true for one value of n.

Note: the language in the next problem is different from what students have seen already. Consider sharing that students will sort equations into these groups:

- All values of n (infinitely many solutions)
- One value of n (one solution)
- No values of n (no solution)

2 Monitor

Encourage students to share their reasoning with a partner and work together to reach an agreement about how to sort the equations in Problem 8.

A **Accessibility: Executive Function** Invite students to focus on one equation at a time to support students in carrying out multiple steps.

Note: If time allows, invite pairs to compare their groups, justify their thinking, and make revisions based on their conversation.

ELL **English Language Learners** Invite students to share what the phrases *True for One Value, True for No Values,* and *True for All Values* mean in their own words. Encourage them to use examples from earlier in the lesson to support their understanding. **(Reading and Speaking)**

D **Differentiation: (Stretch)** Encourage students who correctly sort all the equations to create their own equations and challenge their partner to decide if the equation will have one, none, or infinitely many solutions.

Look and listen for sorting strategies to share during the Connect, such as:

- Sorting equations with the same variable coefficient and different constant values as those with no solution.
- Rewriting expressions to be the same for equations with infinitely many solutions.

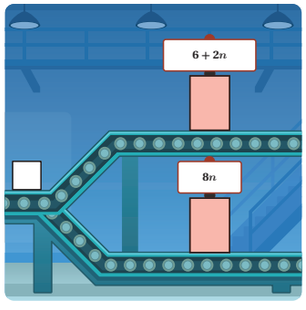
Activity 2 continued >

Activity 2

Name: Date: Period:

Number of Solutions

7. Here are two new number machines.



a. Write an equation to find an input number that will produce the same outputs for each machine.

$6 + 2n = 8n$ (or equivalent)

b. For how many values of n will these machines produce the same output?

Circle one.

All values of n | One value of n | No values of n

8. Group the equations based on their number of solutions.

$v + 2 = v - 2$	$2n = n$	$7 - r = r - 7$	$\frac{1}{2} + x = \frac{1}{3} + x$
$y(-6) \cdot (-3) = 2 \cdot y \cdot 9$	$2t + 6 = 2(t + 3)$	$2n = 2n$	$3(n + 1) = 3n + 1$

No Solution (True for No Values)	One Solution (True for One Value)	Infinitely Many Solutions (True for All Values)
$v + 2 = v - 2$ $\frac{1}{2} + x = \frac{1}{3} + x$ $3(n + 1) = 3n + 1$	$2n = n$ $7 - r = r - 7$	$y \cdot (-6) \cdot (-3) = 2 \cdot y \cdot 9$ $2t + 6 = 2(t + 3)$ $2n = 2n$

Accelerated 7 | Unit 5 Lesson 7 390 Activity 2

Pairs | 15 min

Activity 2 Number of Solutions (continued)

Purpose: Students apply what they learned in the previous activity to sort and write linear equations according to the number of solutions they have: one, none, or infinitely many. **(MTR.5.1)**

Short on time? Consider omitting Problem 10.

2 Monitor

Look for a variety of student responses.

D Differentiation

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Encourage them to choose any number for x and evaluate both expressions.
Solve the equation using balanced moves.	**Strengthen** Invite them to reach the same conclusion without writing anything. Consider asking, "How does the structure of the equation relate to your solution?"
Notice there are different constants being added to x.	**Strengthen** Invite students to think about how they might share their explanation during the Connect.

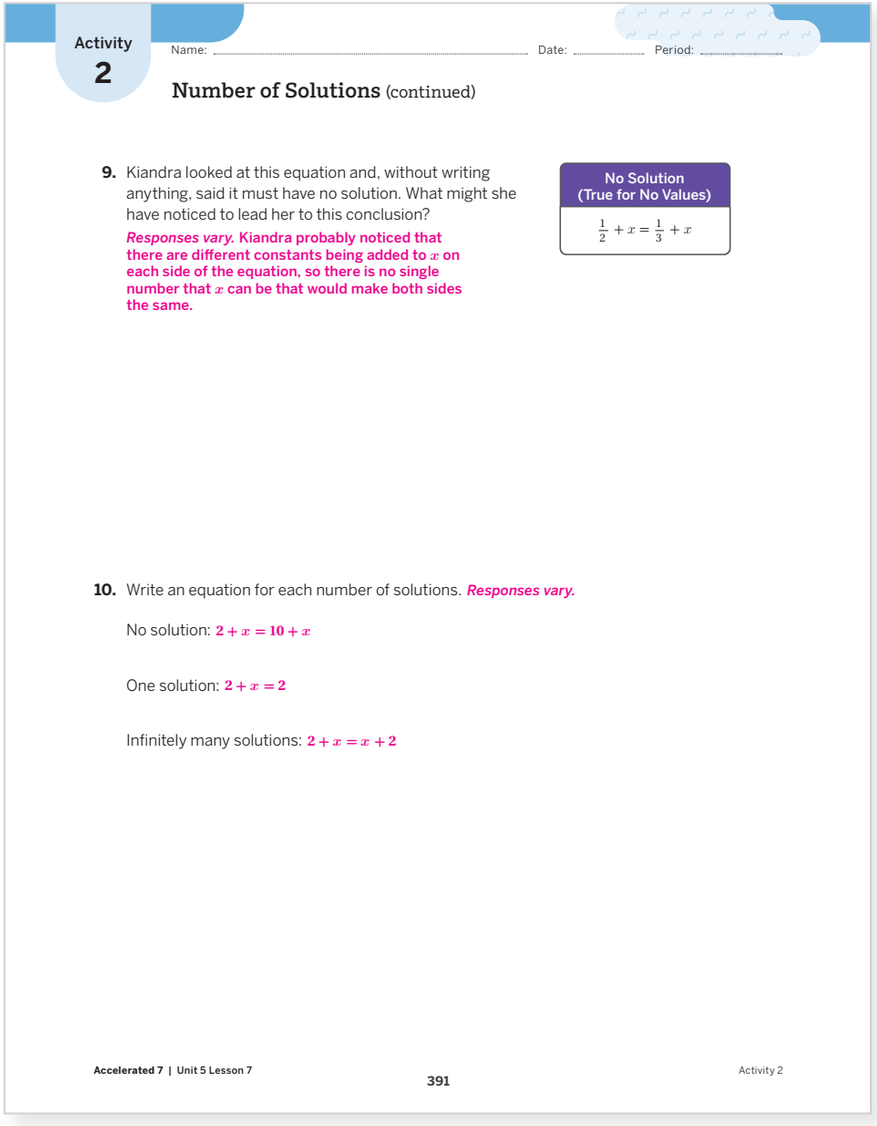
Look for responses to share during the Connect, especially ones that reference the structure of the equation to explain why this equation has no solution. **(MTR.5.1)**

To encourage creativity, invite students to write equations that are different from the ones in this activity. Highlight interesting examples such as those with fractions, the distributive property, or exponents.

3 Connect

To surface connections, consider asking students to justify their responses and critique Kiandra's reasoning in Problem 9. **(MTR.4.1)**

Consider asking, "How would you describe an equation with one solution? No solutions? Infinitely many solutions?" Invite students to share their descriptions with a partner and work together to reach an agreement about how to sort the equations.



Activity 2

Name: Date: Period:

Number of Solutions (continued)

9. Kiandra looked at this equation and, without writing anything, said it must have no solution. What might she have noticed to lead her to this conclusion?

Responses vary. Kiandra probably noticed that there are different constants being added to x on each side of the equation, so there is no single number that x can be that would make both sides the same.

No Solution (True for No Values)

$\frac{1}{2} + x = \frac{1}{3} + x$

10. Write an equation for each number of solutions. *Responses vary.*

No solution: $2 + x = 10 + x$

One solution: $2 + x = 2$

Infinitely many solutions: $2 + x = x + 2$

Accelerated 7 | Unit 5 Lesson 7 391 Activity 2

Whole Class | 5 min

Synthesis

Purpose: Students synthesize their understanding of using an equation's structure to determine whether there are no solutions, one solution, or infinitely many solutions. **(MTR.5.1)**

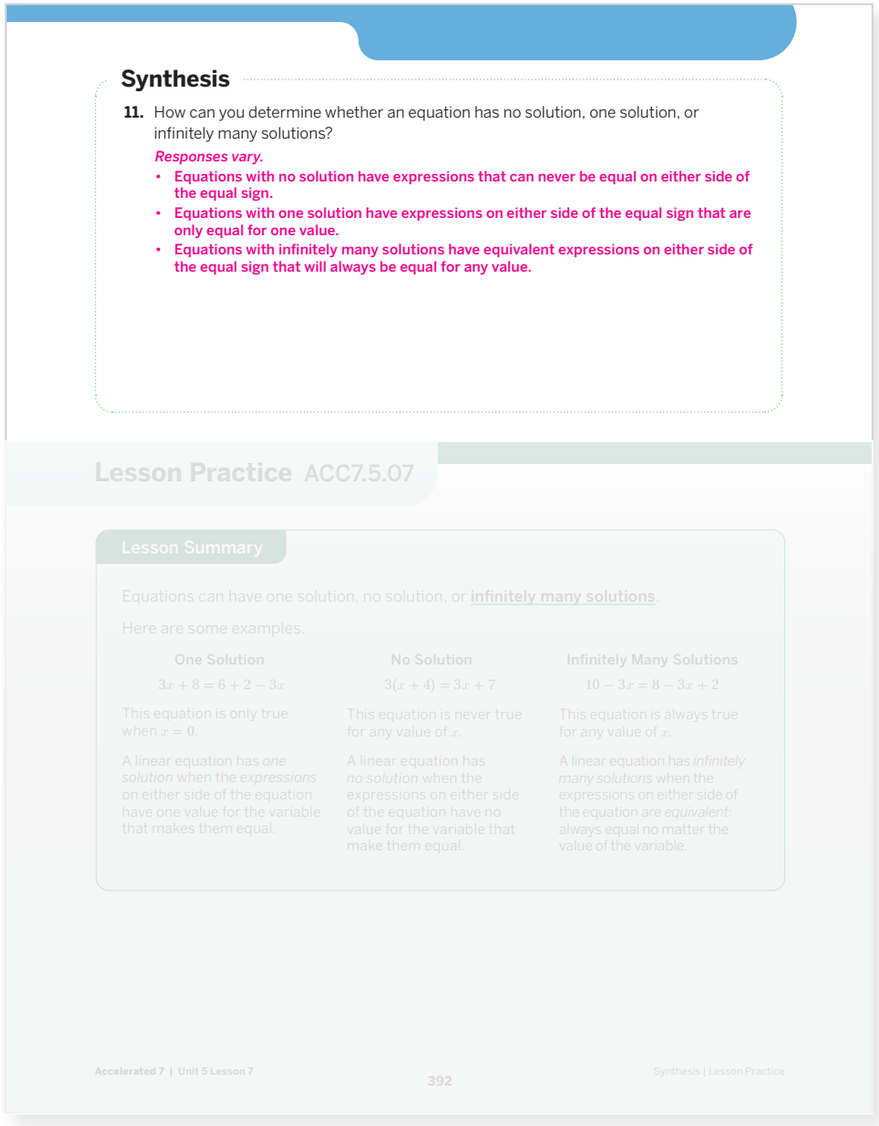
Invite students to respond independently, after discussing each situation with their partner.

Collect and share a variety of ideas, such as:

- Determining that an equation has no solution if both sides will never be equal.
- Determining that an equation has infinitely many solutions if both sides will always be equal.
- Determining that an equation has one solution if the equation is only true for one value.

MLR **MLR2: Collect and Display** Use this routine to collect students' ideas as they share during the Connect. Display for students to reference and add to for the rest of the unit. Consider using this routine to formalize the definition of *infinitely many solutions* using students' language. Display any new language, examples, or diagrams for students to reference later.

Note: If time allows, invite students to revise their previous thinking in Problem 10 based on the discussion.



Synthesis

11. How can you determine whether an equation has no solution, one solution, or infinitely many solutions?

Responses vary.

- Equations with no solution have expressions that can never be equal on either side of the equal sign.
- Equations with one solution have expressions on either side of the equal sign that are only equal for one value.
- Equations with infinitely many solutions have equivalent expressions on either side of the equal sign that will always be equal for any value.

Lesson Practice ACC7.5.07

Lesson Summary

Equations can have one solution, no solution, or infinitely many solutions.

Here are some examples.

One Solution	No Solution	Infinitely Many Solutions
$3x + 8 = 6 + 2 - 3x$	$3(x + 4) = 3x + 7$	$10 - 3x = 8 - 3x + 2$
This equation is only true when $x = 0$.	This equation is never true for any value of x.	This equation is always true for any value of x.
A linear equation has *one solution* when the expressions on either side of the equation have one value for the variable that makes them equal.	A linear equation has *no solution* when the expressions on either side of the equation have no value for the variable that make them equal.	A linear equation has *infinitely many solutions* when the expressions on either side of the equation are *equivalent*: always equal no matter the value of the variable.

Accelerated 7 | Unit 5 Lesson 7 392 Synthesis | Lesson Practice

Lesson Takeaway: Linear equations can have one, none, or **infinitely many solutions**. You can use the structure of the expressions on either side of the equal sign to determine the number of solutions for the equation. The number of solutions is based on whether the expressions on either side of the equal sign will never be equal, always be equal, or are equal for only one value.

Independent | 5 min

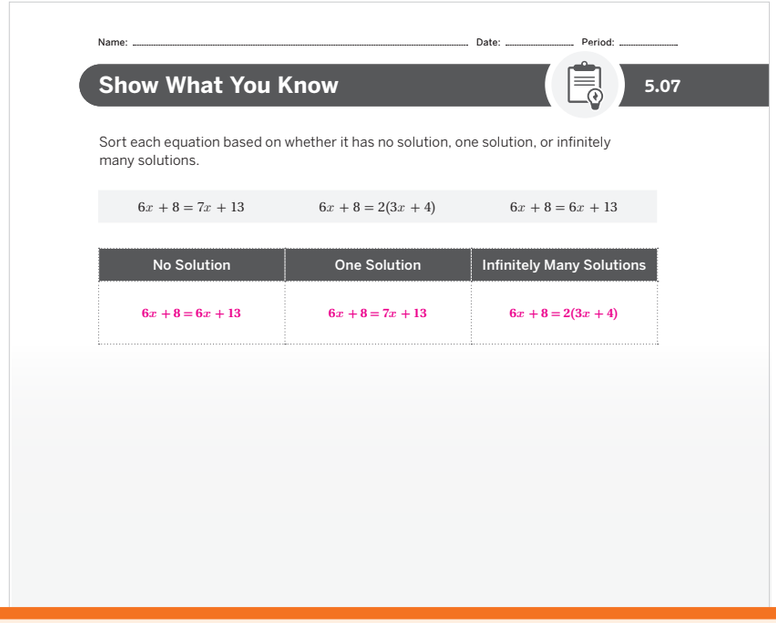
Show What You Know

Purpose: Students demonstrate their understanding by using the structure of equations to identify the number of solutions to three different equations.

Today's Goals

1. **Goal:** Recognize what it means for an equation to have no solution, one solution, or *infinitely many solutions*.
2. **Language Goal:** Justify the number of solutions to a one-variable linear equation using a variety of strategies, including using the structure of the equation. **(Reading, Writing, Speaking, and Listening)**

Show What You Know (Assessment Resources)



Name: _____ Date: _____ Period: _____

Show What You Know 5.07

Sort each equation based on whether it has no solution, one solution, or infinitely many solutions.

$6x + 8 = 7x + 13$ $6x + 8 = 2(3x + 4)$ $6x + 8 = 6x + 13$

No Solution	One Solution	Infinitely Many Solutions
$6x + 8 = 6x + 13$	$6x + 8 = 7x + 13$	$6x + 8 = 2(3x + 4)$

Differentiation Use after Lesson 7

Support

Provide targeted intervention for students by using these resources.

If students need support with determining whether equations have one, none, or infinitely many solutions:

» **Respond to Student Thinking:**

- Use the ***Solving Linear Equations With Parentheses*** **Mini-Lesson**.
- Review the **Lesson Summary** and provide feedback to students on select **Practice Problems**.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 2 Extension Activities**.

Professional Learning

Consider the next several lessons you will teach. How will students' work today in Activity 2 prepare them for strategically solving equations in Lesson 8, and solving systems of equations in Lesson 15? What are ways that you might call back to students' strategies from this lesson?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

11. How can you determine whether an equation has no solution, one solution, or infinitely many solutions?

Responses vary.

- Equations with no solution have expressions that can never be equal on either side of the equal sign.
- Equations with one solution have expressions on either side of the equal sign that are only equal for one value.
- Equations with infinitely many solutions have equivalent expressions on either side of the equal sign that will always be equal for any value.

Lesson Practice ACC7.5.07

Lesson Summary

Equations can have one solution, no solution, or **infinitely many solutions**.

Here are some examples.

One Solution	No Solution	Infinitely Many Solutions
$3x + 8 = 6 + 2 - 3x$	$3(x + 4) = 3x + 7$	$10 - 3x = 8 - 3x + 2$
This equation is only true when $x = 0$.	This equation is never true for any value of x.	This equation is always true for any value of x.
A linear equation has *one solution* when the expressions on either side of the equation have one value for the variable that makes them equal.	A linear equation has *no solution* when the expressions on either side of the equation have no value for the variable that make them equal.	A linear equation has *infinitely many solutions* when the expressions on either side of the equation are *equivalent*: always equal no matter the value of the variable.

Lesson Practice
ACC7.5.07

Name: _____ Date: _____ Period: _____

Problems 1–4: Decide whether the given equation has *one solution*, *no solution*, or *infinitely many solutions*. Explain your thinking.

1. $x - 13 = x + 1$

No solution. *Explanations vary.* There is no value for x that you can subtract 13 from and add 1 to and get the same thing.

2. $x + \frac{1}{2} = x - \frac{1}{2}$

No solution. *Explanations vary.* The expressions $x + \frac{1}{2}$ and $x - \frac{1}{2}$ will never be equal for any value of x.

3. $2(x + 3) = 5x + 6 - 3x$

Infinitely many solutions. *Explanations vary.* This equation is always true for any value of x because the expressions on either side of the equal sign are both equivalent to $2x + 6$.

4. $-(7 - 5x) = 6x - 3$

One solution. *Explanations vary.*

$-7 + 5x = 6x - 3$

$-7 + 3 = x$

$x = -4$

5. Ivory says that the equation $2x + 2 = x + 1$ has no solution because the left-hand side is double the right-hand side. Is Ivory's claim correct? Explain your thinking.

No. *Explanations vary.* Ivory is correct that the left-hand side of the equation is double the right-hand side, but I can solve this equation to find one value that makes the two sides equal.

$2x + 2 = x + 1$

$x + 2 = 1$

$x = -1$

6. Write the other side of this equation so that it is true for all values of x.

$\frac{1}{2}(6x - 10) - x =$ $2x - 5$ (or equivalent)

7. Write the other side of this equation so that it is true for no values of x. *Responses vary.*

$\frac{1}{2}(6x - 10) - x =$ _____ $2x + 5$ _____

Lesson Practice
ACC7.5.07

Name: _____ Date: _____ Period: _____

FAST Practice

8. Complete the statement so that it correctly identifies the number of solutions that exist for the given equation.

$5\left(0.2x + \frac{1}{25}\right) = x + \frac{1}{5}$

Select **ONE** correct answer in each box to complete the sentence.

The equation has **A.** no solution **B.** one solution **C.** infinitely many solutions.

Spiral Review

Problems 9–10: Solve each equation. Show your thinking.

9. $-4(r + 2) = 4(2 - 2r)$

$r = 4$

Work varies.

$-(r + 2) = 2 - 2r$

$-r - 2 = 2 - 2r$

$r - 2 = 2$

$r = 4$

10. $1.3 + 6d = 2.7 - 8d$

$d = 0.1$

Work varies.

$1.3 + 14d = 2.7$

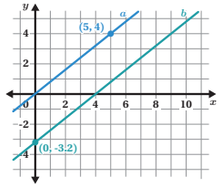
$14d = 1.4$

$d = 0.1$

11. These two lines are parallel. Write an equation for each line.

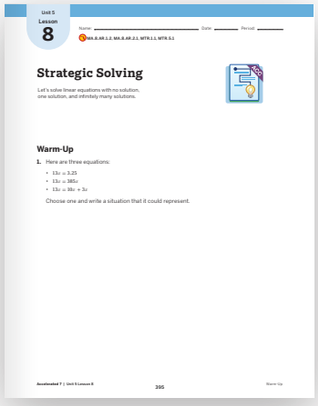
Line a: $y = 0.8x$ (or equivalent)

Line b: $y = 0.8x - 3.2$ (or equivalent)



Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T Math Benchmarks
On-lesson			
	1–2	1	MA.8.AR.2.1
	3–7	2	MA.8.AR.2.1
FAST Practice	8	3	MA.8.AR.2.1
Spiral Review			
	9–10	1	MA.8.AR.2.1
	11	2	MA.8.AR.3.3


Unit 5
Lesson 8
Name: _____ Date: _____ Period: _____
MA.8.AR.1.2, MA.8.AR.2.1, MTR.1.1, MTR.5.1
Strategic Solving
Let's solve linear equations with no solution, one solution, and infinitely many solutions.
Warm-Up
1. Here are three equations:
• 13x = 3.25
• 13x = 365x
• 13x = 30x + 3x
Choose one and write a situation that it could represent.
Accelerated 7 | Unit 5 Lesson 8
395
Warm-Up

Strategic Solving

Solving Many Types of Equations

Let's solve linear equations with no solution, one solution, and infinitely many solutions.

Learning Outcomes

Today's Goals

1. **Goal:** Solve linear equations in one variable, including equations with fractions, decimals, negative values, grouping symbols, and multiple terms.
2. **Language Goal:** Describe strategies for solving linear equations in one variable based on their structures. **(Speaking and Listening)**

Students make sense of and solve equations with no solution, one solution, and infinitely many solutions. They look for and make use of structure in equations to make predictions about their solutions and to determine appropriate solution strategies for solving different types of equations. **(MTR.5.1)**

Coherence

Prior Learning

In Lesson 6, students created equivalent equations and explored the steps for solving equations with one solution.

Future Learning

In Lesson 9, students will write and solve linear equations with variables on both sides in context.

Connections

- Students extend their understanding of predicting the number of solutions of an equation in a sorting activity. (mathematical)

FL B.E.S.T. Math Benchmarks

Addressing

MA.8.AR.2.1
Solve multi-step linear equations in one variable, with rational number coefficients. Include equations with variables on both sides.

Also Addressing: **MA.8.AR.1.2**

Mathematical Thinking and Reasoning Standards: MTR.1.1, MTR.5.1

ELA Expectations: ELA.K12.EE.3.1, ELA.K12.EE.4.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.7.AR.2.1	MA.912.AR.2.1

Lesson at a Glance

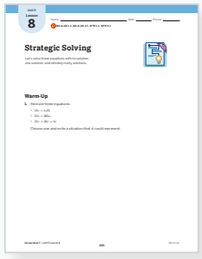
~ 45 min

Standards: MA.8.AR.1.2, MA.8.AR.2.1

Warm-Up

Pairs | 5 min

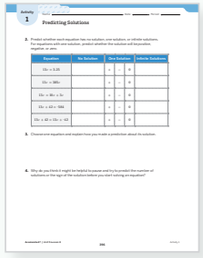
Students make sense of equations by writing a situation that could represent a given equation.



Activity 1

Pairs | 8 min

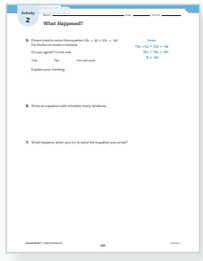
Students build their problem-solving skills by predicting the number and sign of solutions to equations and consider how these predictions can help them in solving. **(MTR.1.1)**



Activity 2

Pairs | 10 min

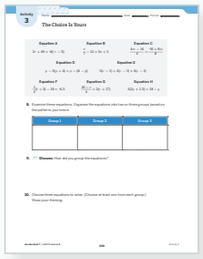
Students investigate the structure of equations with no solution or infinitely many solutions by examining student work and by writing and solving their own equation. **Routine:** Think-Pair-Share



Activity 3

Pairs | 12 min

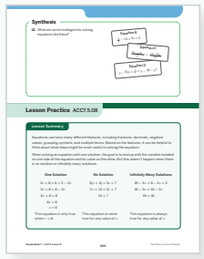
Students leverage the structure of equations to sort them into groups and solve. **(MTR.5.1)**



Synthesis

Whole Class | 5 min

Students synthesize strategies for solving equations with a variety of structures.
MLR2: Collect and Display



Show What You Know

Independent | 5 min

Students demonstrate their understanding by solving linear equations with different structures.

Prep Checklist

This lesson includes:

- Student Edition
- Show What You Know
- Calculators (as needed)
- Scrap paper (as needed)

Pairs | 5 min

Warm-Up

Purpose: Students make sense of equations by writing a situation that could represent a given equation.

Short on time? Consider omitting this Warm-Up or doing it as a class.

1 Launch

Direct students' attention to the equations.

To encourage creativity, invite students to brainstorm situations that could be modeled with an equation.

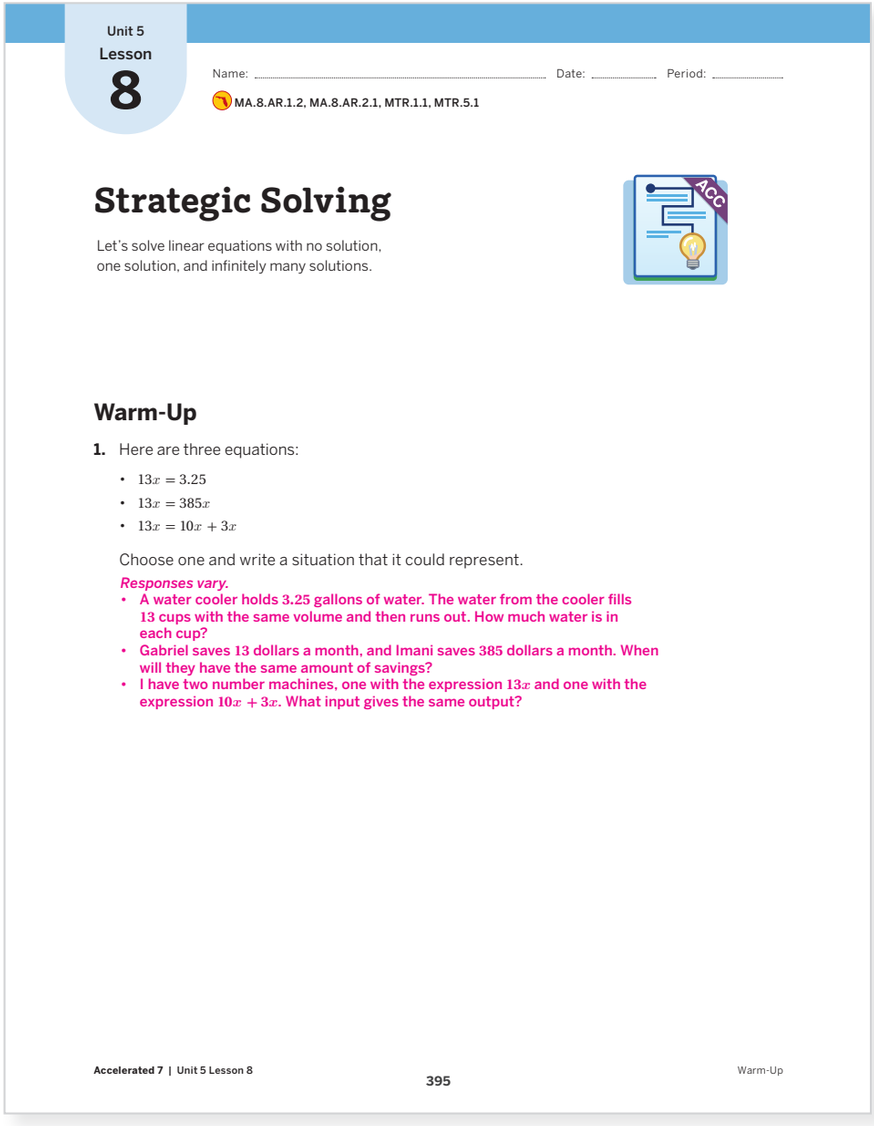
Invite students to write a situation that could be represented by one of the equations.

2 Connect

Invite students to share their situations for each equation.

Consider asking:

- "Which equation did you choose to represent and why?"
- "How do you know this situation can be modeled with this equation?"



Unit 5
Lesson
8

Name: Date: Period:

MA.8.AR.1.2, MA.8.AR.2.1, MTR.1.1, MTR.5.1

Strategic Solving

Let's solve linear equations with no solution, one solution, and infinitely many solutions.

Warm-Up

1. Here are three equations:

- $13x = 3.25$
- $13x = 385x$
- $13x = 10x + 3x$

Choose one and write a situation that it could represent.

Responses vary.

- A water cooler holds 3.25 gallons of water. The water from the cooler fills 13 cups with the same volume and then runs out. How much water is in each cup?
- Gabriel saves 13 dollars a month, and Imani saves 385 dollars a month. When will they have the same amount of savings?
- I have two number machines, one with the expression $13x$ and one with the expression $10x + 3x$. What input gives the same output?

Accelerated 7 | Unit 5 Lesson 8 395 Warm-Up

Pairs | 8 min

Activity 1 Predicting Solutions

Purpose: Students build their problem-solving skills by predicting the number and sign of solutions to equations and consider how these predictions can help them in solving. **(MTR.1.1)**

1 Launch

To surface prior knowledge, consider asking, "What are some features of equations with one solution, no solution, or infinitely many solutions?"

Encourage students to make their predictions without taking any steps to solve the equations.

2 Monitor

Look for a variety of student explanations about how they made their predictions to share during the Connect.

D **Differentiation:** (Support) Students may think they need to solve the equation first. Invite students to answer a more concrete question first to support abstract thinking, such as, "Do you think a number exists that we could multiply by 13 to get 3.25?"

A **Accessibility: Executive Functioning** Chunk this task into smaller, more manageable parts by having students first decide if there will be no solution, one solution, or infinitely many solutions and then consider whether the solution will be positive, negative, or zero.

ELL **English Language Learners** As students share in partner and whole-class discussions, provide sentence frames such as:

- I know that the solution will be ____ because...
- When I look at the structure of this equation I notice... **(Writing and Speaking)**

3 Connect

Invite students to share predictions and explanations for each equation.

Discuss why it might be helpful to try to make sense of an equation by predicting the number of solutions or the sign of the solution before you start solving an equation. **(MTR.1.1)**

Activity 1

Name: _____ Date: _____ Period: _____

Predicting Solutions

2. Predict whether each equation has no solution, one solution, or infinite solutions. For equations with one solution, predict whether the solution will be *positive*, *negative*, or *zero*.

Equation	No Solution	One Solution			Infinite Solutions
$13x = 3.25$		(+)	–	0	
$13x = 385x$		+	–	(0)	
$13x = 10x + 3x$		+	–	0	✔
$13x + 42 = -584$		+	(–)	0	
$13x + 42 = 13x + -42$	✔	+	–	0	

3. Choose one equation and explain how you made a prediction about its solution.

Responses vary. For $13x = 3.25$, I knew the equation had one positive solution because multiplying 13 by a positive number will give me another positive number.

4. Why do you think it might be helpful to pause and try to predict the number of solutions or the sign of the solution before you start solving an equation?

Responses vary. If I've made a prediction about the solution before I start solving, it can help me decide if my solution is reasonable once I'm done solving.

Accelerated 7 | Unit 5 Lesson 8 396 Activity 1

Pairs | 8 min

Activity 2 What Happened?

Purpose: Students investigate the structure of equations with no solution or infinitely many solutions by examining student work and by writing and solving their own equation.

1 Launch

Direct students' attention to Deven's work.

Use the Think-Pair-Share routine to help students shape their ideas about the steps Deven took to solve the equation. Invite students to think independently and then take turns sharing with a partner. Invite several students to share their or their partner's reasoning with the class.

Invite students to decide if they agree or disagree with Deven and explain their thinking.

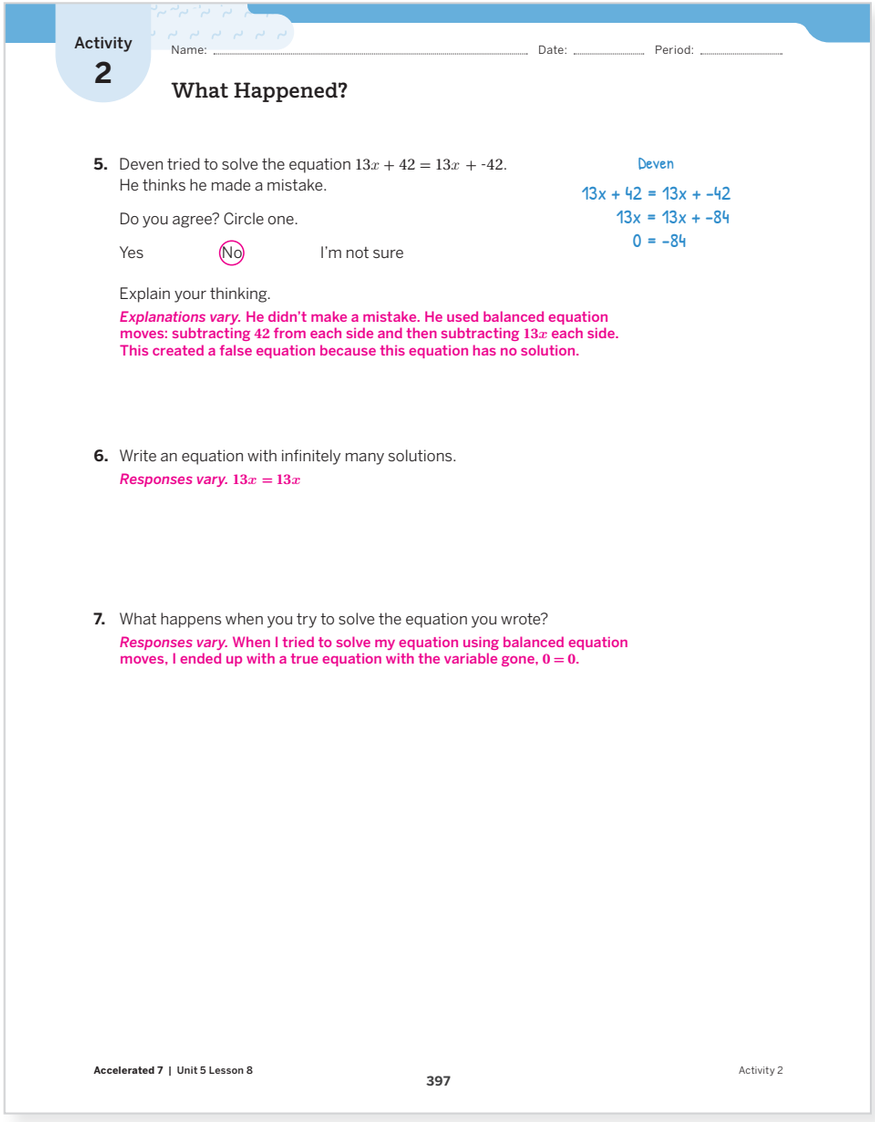
Consider asking, "Do you think Deven made a mistake? Why or why not?"

2 Monitor

Select and sequence a variety of equations and strategies used to solve them.

D Differentiation *(Problems 6–7)*

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Invite students to revisit their work from Activity 1 and what they know about the expressions in an equation with infinitely many solutions.
Write and solve an equation with infinitely many solutions.	**Strengthen** Consider asking, "How are the equivalent equations Deven produced alike and different from this one? What do you think that means?"
Would like to explore equations with infinitely many or no solutions further.	**Stretch** Invite students to come up with situations that could be represented by equations with infinitely many and no solutions.



Activity 2

Name: Date: Period:

What Happened?

5. Deven tried to solve the equation $13x + 42 = 13x + -42$. He thinks he made a mistake.

Deven
13x + 42 = 13x + -42
13x = 13x + -84
0 = -84

Do you agree? Circle one.

Yes (No) I'm not sure

Explain your thinking.

Explanations vary. He didn't make a mistake. He used balanced equation moves: subtracting 42 from each side and then subtracting $13x$ each side. This created a false equation because this equation has no solution.

6. Write an equation with infinitely many solutions.

Responses vary. $13x = 13x$

7. What happens when you try to solve the equation you wrote?

Responses vary. When I tried to solve my equation using balanced equation moves, I ended up with a true equation with the variable gone, $0 = 0$.

Accelerated 7 | Unit 5 Lesson 8 397 Activity 2

3 Connect

Share and discuss the equations students wrote and invite them to share how they know their equations have infinite solutions.

To surface the Key Takeaway, consider asking, "How is the result of attempting to solve an equation with infinitely many solutions similar to and different from attempting to solve an equation with no solution?"

Key Takeaway: When attempting to solve an equation with infinitely many or no solutions, creating an equivalent equation can help to determine whether the equation is always or never true.

Pairs | 12 min

Activity 3 The Choice Is Yours

Purpose: Students leverage the structure of equations to sort them into groups and solve. **(MTR.5.1)**

1 Launch

Direct students' attention to the equations and directions.

Consider sharing that there are many different ways to sort the equations and that the purpose of this sort is to analyze the structure of the equations, not to solve them yet. **(MTR.5.1)**

2 Monitor

A **Accessibility: Memory and Attention** Invite students to use a calculator to support them with performing calculations.

Select and sequence common and unique groupings and strategies for solving the equations in each group to share during the Connect.

D **Differentiation** *(Problem 10)*

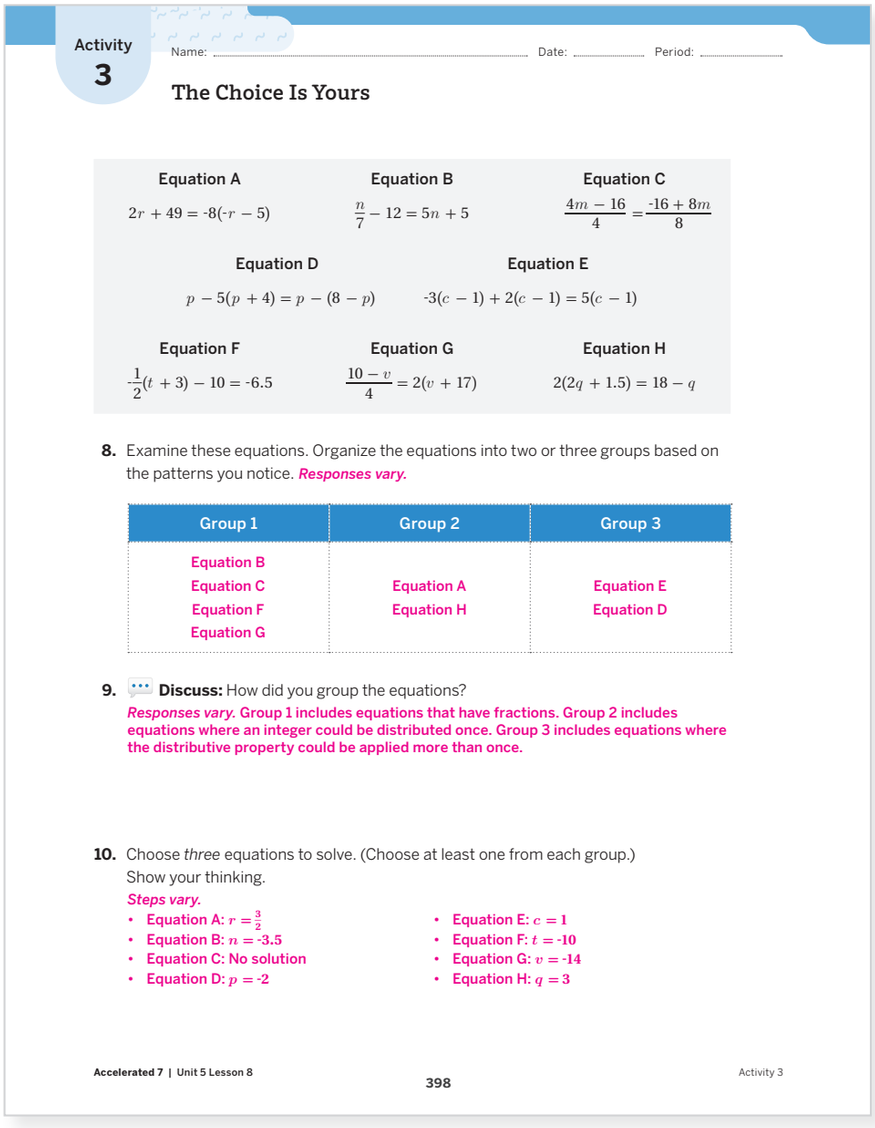
Look for students who:	Respond to Student Thinking
Identify a solution that does not make the equation true.	**Support** Encourage them to check their answers by substituting the solution into the original equation and evaluating it.
Applying the distributive property to only one term inside a set of parentheses.	**Support** Consider asking, "What do the parentheses mean in this equation?"
Multiplying only one side of the equation or only one term by a factor to eliminate fractions.	**Support** Consider asking, "Have you maintained equality? Is the resulting equation equivalent to the original? How do you know?"
Solve an equation from each group.	**Strengthen** Invite students to write another equation that would fit in each group and solve.
Would like to explore solving equations further.	**Stretch** Invite students to solve another equation from each group.

Note: Students may need additional paper to solve the equations.

3 Connect

Invite students to share how they grouped the equations, highlighting the structure that students noticed in each group. **(MTR.5.1)**

Consider asking, "What strategies were most helpful for solving the equations in each group?" *Responses vary.* For the group with fractions, it was helpful to multiply by a factor to eliminate the fractions before continuing to solve.



Activity 3

Name: Date: Period:

The Choice Is Yours

Equation A	Equation B	Equation C
$2r + 49 = -8(-r - 5)$	$\frac{n}{7} - 12 = 5n + 5$	$\frac{4m - 16}{4} = \frac{-16 + 8m}{8}$

Equation D	Equation E
$p - 5(p + 4) = p - (8 - p)$	$-3(c - 1) + 2(c - 1) = 5(c - 1)$

Equation F	Equation G	Equation H
$\frac{1}{2}(t + 3) - 10 = -6.5$	$\frac{10 - v}{4} = 2(v + 17)$	$2(2q + 1.5) = 18 - q$

8. Examine these equations. Organize the equations into two or three groups based on the patterns you notice. *Responses vary.*

Group 1	Group 2	Group 3
Equation B Equation C Equation F Equation G	Equation A Equation H	Equation E Equation D

9. **Discuss:** How did you group the equations?
Responses vary. Group 1 includes equations that have fractions. Group 2 includes equations where an integer could be distributed once. Group 3 includes equations where the distributive property could be applied more than once.

10. Choose *three* equations to solve. (Choose at least one from each group.) Show your thinking.
Steps vary.
- Equation A: $r = \frac{3}{2}$
- Equation B: $n = -3.5$
- Equation C: No solution
- Equation D: $p = -2$
- Equation E: $c = 1$
- Equation F: $t = -10$
- Equation G: $v = -14$
- Equation H: $q = 3$

Accelerated 7 | Unit 5 Lesson 8 398 Activity 3

Whole Class | 5 min

Synthesis

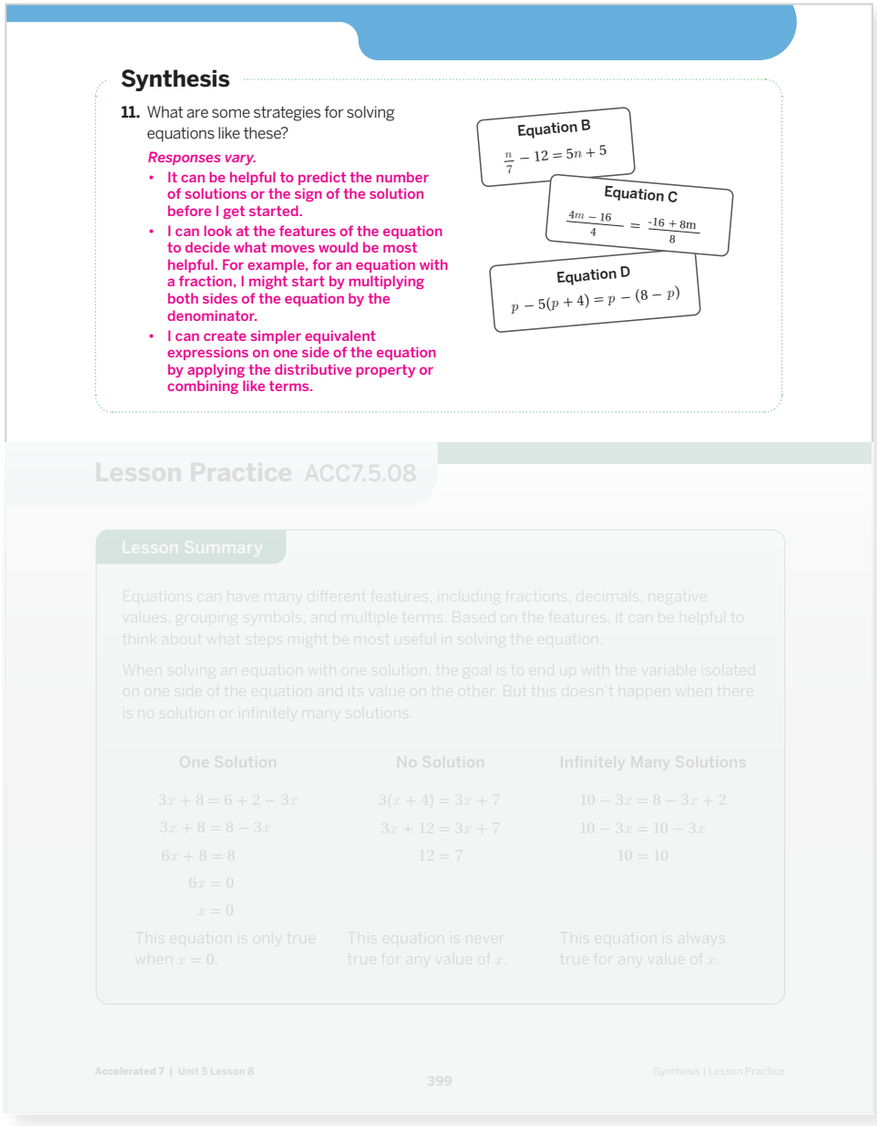
Purpose: Students synthesize strategies for solving equations with a variety of structures.

Invite students to respond independently, and then share their thinking with a partner.

Collect and share a variety of ideas, such as:
- Using predictions based on the structure of the equation.
- Applying the distributive property.
- Using multiplication to eliminate fractions.

MLR **MLR2: Collect and Display** Use this routine to revisit students' strategies for solving equations. Add any new strategies for students to reference for the rest of the unit.

Note: If time allows, invite students to share with a partner what makes sense about each strategy and what questions they still have.



Synthesis

11. What are some strategies for solving equations like these?

Equation B: $\frac{n}{7} - 12 = 5n + 5$

Equation C: $\frac{4m - 16}{4} = \frac{-16 + 8m}{8}$

Equation D: $p - 5(p + 4) = p - (8 - p)$

Responses vary.
- It can be helpful to predict the number of solutions or the sign of the solution before I get started.
- I can look at the features of the equation to decide what moves would be most helpful. For example, for an equation with a fraction, I might start by multiplying both sides of the equation by the denominator.
- I can create simpler equivalent expressions on one side of the equation by applying the distributive property or combining like terms.

Lesson Practice ACC7.5.08

Lesson Summary

Equations can have many different features, including fractions, decimals, negative values, grouping symbols, and multiple terms. Based on the features, it can be helpful to think about what steps might be most useful in solving the equation.

When solving an equation with one solution, the goal is to end up with the variable isolated on one side of the equation and its value on the other. But this doesn't happen when there is no solution or infinitely many solutions.

One Solution	No Solution	Infinitely Many Solutions
$3x + 8 = 6 + 2 - 3x$	$3(x + 4) = 3x + 7$	$10 - 3x = 8 - 3x + 2$
$3x + 8 = 8 - 3x$	$3x + 12 = 3x + 7$	$10 - 3x = 10 - 3x$
$6x + 8 = 8$	$12 = 7$	$10 = 10$
$6x = 0$		
$x = 0$		
This equation is only true when $x = 0$.	This equation is never true for any value of x.	This equation is always true for any value of x.

Accelerated 7 | Unit 5 Lesson 8 399 Synthesis | Lesson Practice

Lesson Takeaway: It can be helpful to look at the structure of an equation to choose which steps might be the most helpful in solving it, such as using the distributive property, combining like terms on each side of the equation, and multiplying to eliminate fractions.

Independent | 5 min

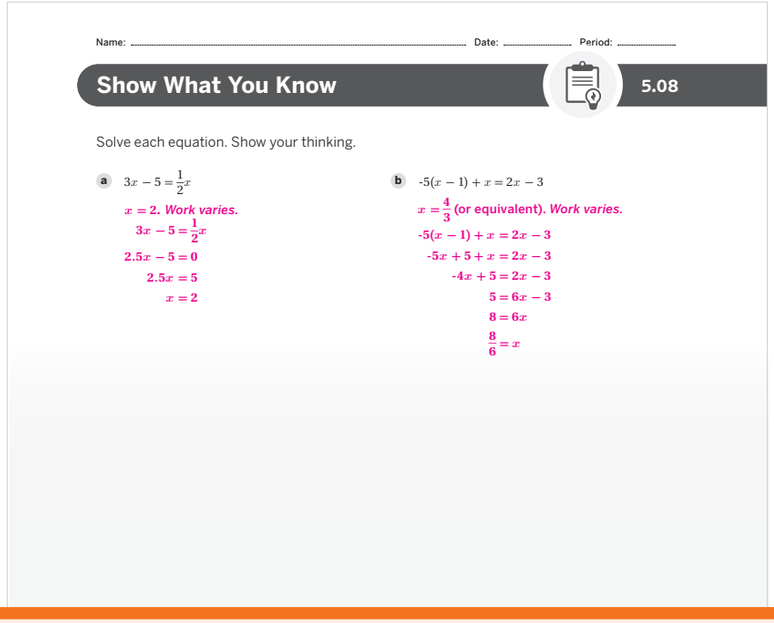
Show What You Know

Purpose: Students demonstrate their understanding by solving linear equations with different structures.

Today's Goals

1. **Goal:** Solve linear equations in one variable, including equations with fractions, decimals, negative values, grouping symbols, and multiple terms.
2. **Language Goal:** Describe strategies for solving linear equations in one variable based on their structures. **(Speaking and Listening)**

Show What You Know (Assessment Resources)



Name: _____ Date: _____ Period: _____

Show What You Know 5.08

Solve each equation. Show your thinking.

a $3x - 5 = \frac{1}{2}x$

$x = 2$. *Work varies.*

$3x - 5 = \frac{1}{2}x$

$2.5x - 5 = 0$

$2.5x = 5$

$x = 2$

b $-5(x - 1) + x = 2x - 3$

$x = \frac{4}{3}$ (or equivalent). *Work varies.*

$-5(x - 1) + x = 2x - 3$

$-5x + 5 + x = 2x - 3$

$-4x + 5 = 2x - 3$

$5 = 6x - 3$

$8 = 6x$

$\frac{8}{6} = x$

Differentiation Use after Lesson 8

Support

Provide targeted intervention for students by using these resources.

If students need support solving linear equations:

» **Respond to Student Thinking:**

- Use the ***Solving Linear Equations With Parentheses*** **Mini-Lesson**.
- Revisit **Activity 3** and invite students to share strategies for solving equations.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 2 Extension Activities**.
- Invite students to **choose two or more categories of equations** that they've solved so far, then write an example equation for each category with tips for solving it.

Professional Learning

Take a moment to reflect on how students engaged with looking for and making use of structure in today's lesson. How did students' understanding of the structure of equations contribute to their fluency in solving them?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

11. What are some strategies for solving equations like these?

Equation B: $\frac{n}{7} - 12 = 5n + 5$

Equation C: $\frac{4m - 16}{4} = \frac{-16 + 8m}{8}$

Equation D: $p - 5(p + 4) = p - (8 - p)$

Responses vary.

- It can be helpful to predict the number of solutions or the sign of the solution before I get started.
- I can look at the features of the equation to decide what moves would be most helpful. For example, for an equation with a fraction, I might start by multiplying both sides of the equation by the denominator.
- I can create simpler equivalent expressions on one side of the equation by applying the distributive property or combining like terms.

Lesson Practice ACC7.5.08

Lesson Summary

Equations can have many different features, including fractions, decimals, negative values, grouping symbols, and multiple terms. Based on the features, it can be helpful to think about what steps might be most useful in solving the equation.

When solving an equation with one solution, the goal is to end up with the variable isolated on one side of the equation and its value on the other. But this doesn't happen when there is no solution or infinitely many solutions.

One Solution	No Solution	Infinitely Many Solutions
$3x + 8 = 6 + 2 - 3x$	$3(x + 4) = 3x + 7$	$10 - 3x = 8 - 3x + 2$
$3x + 8 = 8 - 3x$	$3x + 12 = 3x + 7$	$10 - 3x = 10 - 3x$
$6x + 8 = 8$	$12 = 7$	$10 = 10$
$6x = 0$		
$x = 0$		
This equation is only true when $x = 0$.	This equation is never true for any value of x.	This equation is always true for any value of x.

Lesson Practice ACC7.5.08

Name: _____ Date: _____ Period: _____

Problems 1–3: Identify whether each equation has a solution that is *positive*, *negative*, or *zero* without solving.

1. $7x = 3.25$
Positive

2. $-7x = 32.5$
Negative

3. $3x + 11 = 11$
Zero

Problems 4–7: Solve each equation. Show your thinking.

4. $2b + 8 - 5b + 3 = -13 + 8b - 5$
$b = \frac{29}{11}$
Work varies.
$-3b + 11 = -18 + 8b$
$-11b + 11 = -18$
$-11b = -29$
$b = \frac{29}{11}$ (or equivalent)

5. $2x + 7 - 5x + 8 = 3(5 + 6x) - 12x$
$x = 0$
Work varies.
$-3x + 15 = 15 + 18x - 12x$
$-3x + 15 = 15 + 6x$
$-9x + 15 = 15$
$-9x = 0$
$x = 0$

6. $3(3 - 3x) = 2(x + 3) - 30$
$x = 3$
Work varies.
$9 - 9x = 2x + 6 - 30$
$9 - 9x = 2x - 24$
$9 = 11x - 24$
$33 = 11x$
$3 = x$

7. $\frac{1}{3}(z + 4) - 6 = \frac{2}{3}(5 - z)$
$z = 8$
Work varies.
$z + 4 - 18 = 2(5 - z)$
$z - 14 = 10 - 2z$
$3z - 14 = 10$
$3z = 24$
$z = 8$

8. Kadeem is solving the linear equation $x - 3(2 - 3x) = 2(5x + 3)$. Here are his final two steps. Select the statement that correctly describes Kadeem's solution.

Kadeem
$-6 + 10x = 10x + 6$
$-6 = 6$

A. The solution is the ordered pair $(-6, 6)$.
B. The solution is $x = 0$.
C. There are infinitely many solutions since $-6 = 6$ is a false statement.
(D.) There is no solution because $-6 = 6$ is a false statement.

Lesson Practice ACC7.5.08

Name: _____ Date: _____ Period: _____

FAST Practice

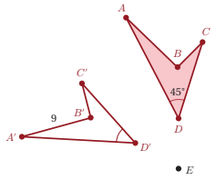
9. What is the solution to this equation?

$-\frac{1}{5}(5x - 15) + 18 = 6x$

A. $x = \frac{7}{20}$
B. $x = 2$
C. $x = \frac{20}{7}$
(D.) $x = 3$

Spiral Review

Problems 10–11: Figure $A'B'C'D'$ is the image of figure $ABCD$ after a rotation around point E.

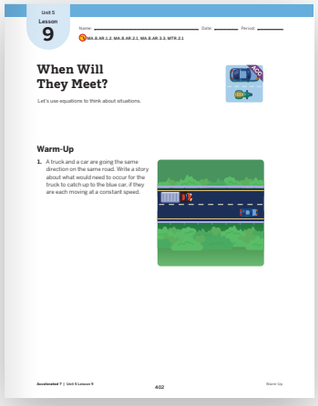


10. What is the length of side AB?
Explain your thinking.
9 units. *Explanations vary.* The corresponding side lengths of a rotated figure are congruent.

11. What is the measure of $\angle A'D'C'$?
Explain your thinking.
45°. *Explanations vary.* The corresponding angles of a rotated figure are congruent.

Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T Math Benchmarks
On-lesson			
	1–3	2	MA.8.AR.2.1
	4–7	1	MA.8.AR.1.2, MA.8.AR.2.1
	8	2	MA.8.AR.1.2, MA.8.AR.2.1
FAST Practice	9	2	MA.8.AR.1.2, MA.8.AR.2.1
Spiral Review			
	10–11	1	MA.8.GR.2.1



When Will They Meet?

Solving Linear Equations in Context

Let's use equations to think about situations.

Learning Outcomes

Today's Goals

1. **Goal:** Create an equation in one variable to represent a situation in which two conditions are equal.
2. **Language Goal:** Interpret the solution of an equation in one variable in context. **(Speaking and Listening)**

In this lesson, students apply their knowledge of solving equations by considering two vehicles moving on a road. Using the *rate* of speed and the initial position of the vehicles, they are asked to determine when the vehicles will meet. Students recognize that they can create two expressions and set them equal, solve the equation for the unknown, and interpret their solution in the context of the situation.

Coherence

Prior Learning

In Lesson 8, students solved linear equations in one variable, including equations with fractions, decimals, negative values, grouping symbols, and multiple terms. In Unit 4, students explored and interpreted linear relationships and their representations.

Future Learning

In Lesson 11, students will begin exploring systems of linear equations by considering graphs of equations and the meaning of the solution in context. In Lesson 13, students will be formally introduced to the term *system of linear equations*.

Connections

- Students apply constant rate to determine when vehicles (or hikers) will meet on the same road. (real world)

FL B.E.S.T. Math Benchmarks

Addressing

MA.8.AR.2.1
Solve multi-step linear equations in one variable, with rational number coefficients. Include equations with variables on both sides.

MA.8.AR.3.3
Given a table, graph or written description of a linear relationship, write an equation in slope-intercept form.

Also Addressing: **MA.8.AR.1.2**

Mathematical Thinking and Reasoning Standard: MTR.2.1

ELA Expectations: ELA.K12.EE.1.1, ELA.K12.EE.3.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.7.AR.1.1	**MA.912.AR.2.1**
MA.7.AR.2.2	**MA.912.AR.2.2**

Vocabulary

Review Vocabulary

rate

Lesson at a Glance

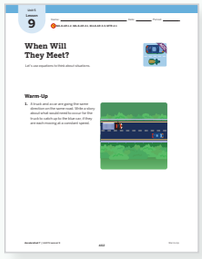
~ 45 min

MA.8.AR.1.2, MA.8.AR.2.1, MA.8.AR.3.3

Warm-Up

Pairs | 5 min

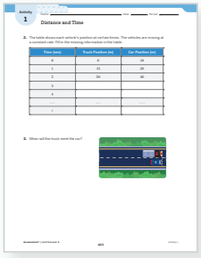
Students consider two vehicles moving at a constant *rate* to think about the different components in a situation, like speed and starting position. Routine: Tell a Story



Activity 1

Pairs | 7 min

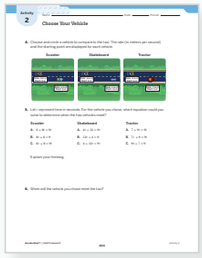
Students reason abstractly to generate expressions representing the position of two vehicles for a given time and determine when they will meet. **(MTR.2.1)**



Activity 2

Pairs | 10 min

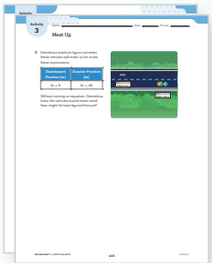
Students choose two expressions and set them equal to each other to determine when two new vehicles will meet.



Activity 3

Pairs | 13 min

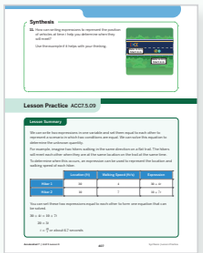
Students consider expressions representing the position of two vehicles for a given time and determine conditions under which they will eventually meet or never meet.



Synthesis

Whole Class | 5 min

Students synthesize their understanding of writing expressions and solving for when they are equal to each other. MLR2: Collect and Display



Show What You Know

Independent | 5 min

Students demonstrate their understanding by interpreting the solution to an equation within a given situation.

Prep Checklist

This lesson includes:

- Student Edition
- Show What You Know

Pairs | 5 min

Warm-Up

Purpose: Students consider two vehicles moving at a constant *rate* to think about the different components in this situation, like speed and starting position.

1 Launch

Direct student's attention to the image and invite students to share what they notice about the vehicles before making a prediction.

A **Accessibility: Affective Functioning** To support students in sharing their thinking, ask students in advance if they, you, or a classmate can share their thinking during the whole class discussion.

ELL **English Language Learners** Provide sentence frames (e.g., "The _____ must be traveling faster than the _____.") **(Writing and Speaking)**

Use the Tell a Story routine to help students make sense of the situation. Invite students to write a story about the picture using their unique experiences, perspectives, and creativity.

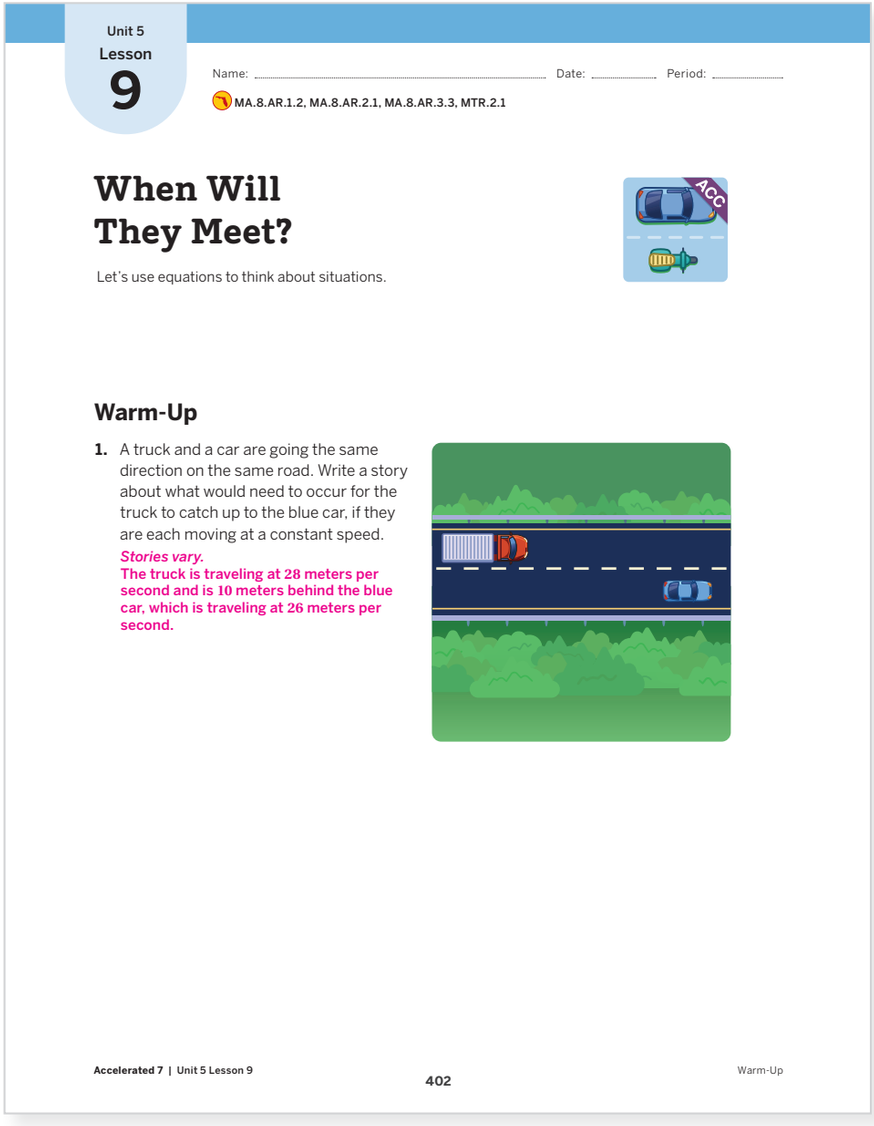
2 Connect

Invite several students to share their stories.

Consider asking:

- "How did you decide what numbers to use in your story?"
- "If both vehicles are moving at a constant speed, does that mean they are moving at the same speed?"

Note: If it does not come up naturally, consider sharing with the students what it means for a vehicle to be moving at a constant speed. Terms like *speeding up* or *slowing down* might help students to understand that constant speed means the vehicle is covering the same amount of distance in equal time intervals.



Unit 5
Lesson
9

Name: _____ Date: _____ Period: _____

MA.8.AR.1.2, MA.8.AR.2.1, MA.8.AR.3.3, MTR.2.1

When Will They Meet?

Let's use equations to think about situations.

Warm-Up

1. A truck and a car are going the same direction on the same road. Write a story about what would need to occur for the truck to catch up to the blue car, if they are each moving at a constant speed.

Stories vary.
The truck is traveling at 28 meters per second and is 10 meters behind the blue car, which is traveling at 26 meters per second.

Accelerated 7 | Unit 5 Lesson 9 402 Warm-Up

Pairs | 7 min

Activity 1 Distance and Time

Purpose: Students reason abstractly to generate expressions representing the position of two vehicles for a given time and determine when they will meet. **(MTR.2.1)**

1 Launch

Direct students' attention to the table and invite students to describe how it connects with the situation in the Warm-Up. Consider asking, "How can we see the constant rate in the table?"

Note: Ensure students understand that when two vehicles meet, the front of each vehicle will be in the same position.

A **Accessibility: Conceptual Processing** To support abstract thinking, invite students to write an expression for each row in the table.

Encourage students to explain their thinking as they fill in the rows of their table.

Consider asking, "What will it look like in the table when the vehicles meet?"

2 Monitor

Invite students to share their work with a partner and use any feedback given by their partner to revise their work.

Look for students who determine 4.5 seconds using the table or equations in Problem 3. Encourage them to practice sharing their strategy with a partner before sharing with the whole class during the Connect.

D **Differentiation** *(Problem 3)*

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Encourage them to guess and use partner feedback to check.
Determine the correct time by guessing and checking.	**Strengthen** Consider asking, "How could you use the table or expressions to determine this time?"
Extend or use their table to determine the meeting time.	**Strengthen** Invite them to share how the table helped them estimate to eventually get a precise answer.
Write and solve an equation to determine the meeting time.	**Strengthen** Invite them to justify the balanced moves they used to solve the equation.

3 Connect

Discuss and celebrate a variety of students' strategies.

To surface the Key Takeaway, consider asking:
- "How are each of these strategies alike?"
- "Which strategy do you prefer?"

Activity 1

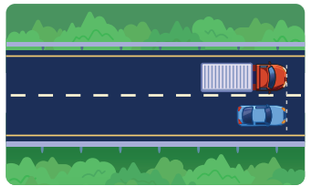
Name: _____ Date: _____ Period: _____

Distance and Time

2. The table shows each vehicle's position at certain times. The vehicles are moving at a constant *rate*. Fill in the missing information in the table.

Time (sec)	Truck Position (m)	Car Position (m)
0	0	18
1	15	29
2	30	40
3	45	51
4	60	62
...	...	...
t	$15t$	$11t + 18$

3. When will the truck meet the car?
4.5 seconds



Accelerated 7 | Unit 5 Lesson 9 403 Activity 1

Key Takeaway: Using representations of situations, like equations or tables, is helpful when answering questions about a situation.

Pairs | 10 min

Activity 2 Choose Your Vehicle

Purpose: Students choose two expressions and set them equal to each other to determine when two new vehicles will meet.

1 Launch

To surface connections, consider sharing that students will now choose their own vehicle matchups and determine when they will meet.

To encourage participation, consider asking, "What is the scenario in which a skateboard would be faster than a taxi? When will a tractor be faster than a taxi?"

2 Monitor

Encourage students to refer to the discussion from the end of Activity 1 to support their reasoning.

ELL **English Language Learners** Invite students to annotate images in their Student Editions with words in English and in their primary language. **(Reading, Writing, Listening, and Speaking)**

Listen for explanations of equation selections to share during the Connect.

D **Differentiation** *(Problems 5-6)*

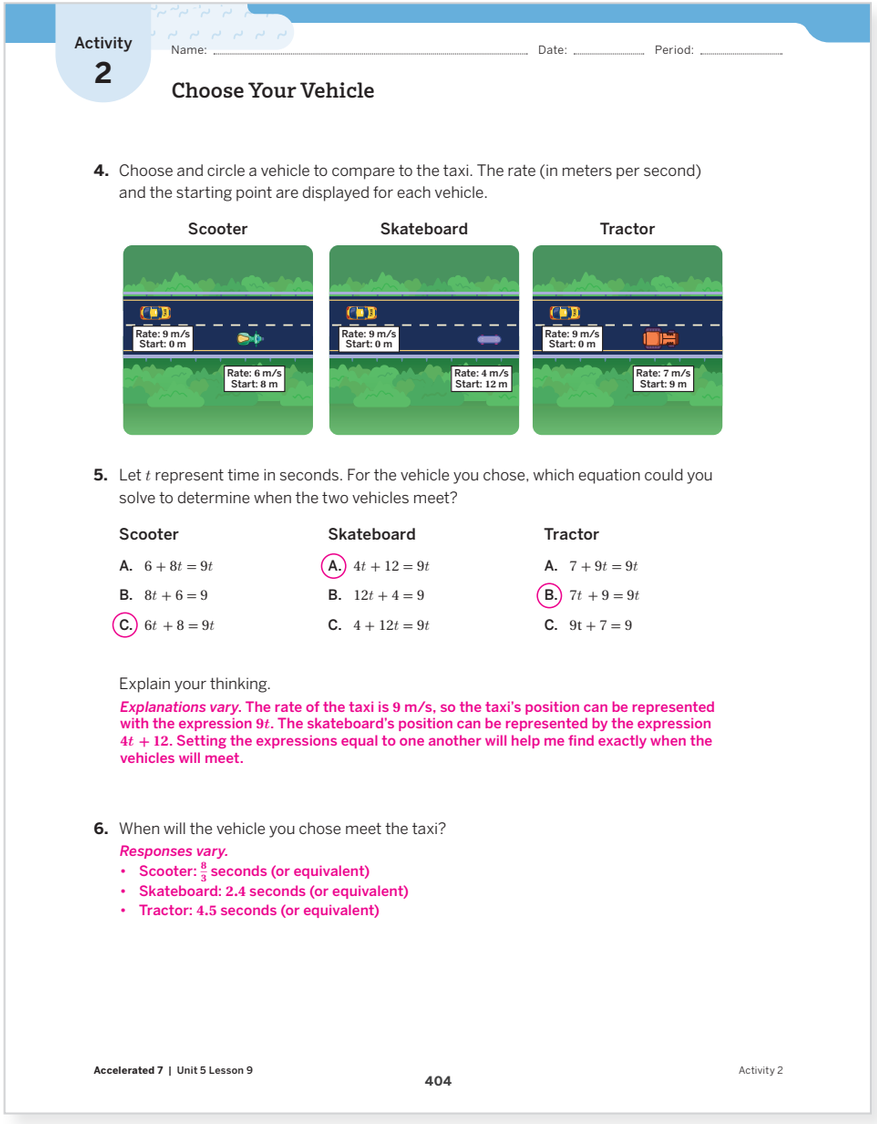
Look for students who:	Respond to Student Thinking
Need support selecting an equation.	**Support** Ask students to describe the positions of the two vehicles in words (e.g., "The taxi starts at 0 meters and travels at a rate of 9 meters per second."). Have them translate their words into an expression ($9t$).
Use key vocabulary in their explanations such as *rate of change* and *initial position.*	**Strengthen** Invite students to share their responses during the Connect and celebrate them one-on-one or publicly.
Correctly determine when the two vehicles will meet.	**Stretch** Invite them to discuss with a partner how the expressions would look if the vehicles were moving in different directions.

3 Connect

Invite students to share the selected equation for each vehicle pairing.

Consider asking:

- "What was helpful about using an equation to determine the time when the vehicles met?"
- "What would have been different if you had used a table to solve the equation?"



Activity 2

Name: Date: Period:

Choose Your Vehicle

4. Choose and circle a vehicle to compare to the taxi. The rate (in meters per second) and the starting point are displayed for each vehicle.

Scooter | Skateboard | Tractor

5. Let t represent time in seconds. For the vehicle you chose, which equation could you solve to determine when the two vehicles meet?

Scooter	Skateboard	Tractor
A. $6 + 8t = 9t$	(A.) $4t + 12 = 9t$	A. $7 + 9t = 9t$
B. $8t + 6 = 9$	B. $12t + 4 = 9$	(B.) $7t + 9 = 9t$
(C.) $6t + 8 = 9t$	C. $4 + 12t = 9t$	C. $9t + 7 = 9$

Explain your thinking.

Explanations vary. The rate of the taxi is 9 m/s, so the taxi's position can be represented with the expression $9t$. The skateboard's position can be represented by the expression $4t + 12$. Setting the expressions equal to one another will help me find exactly when the vehicles will meet.

6. When will the vehicle you chose meet the taxi?

Responses vary.

- Scooter: $\frac{8}{3}$ seconds (or equivalent)
- Skateboard: 2.4 seconds (or equivalent)
- Tractor: 4.5 seconds (or equivalent)

Accelerated 7 | Unit 5 Lesson 9 404 Activity 2

Pairs | 13 min

Activity 3 Meet Up

Purpose: Students consider expressions representing the position of two vehicles for a given time and determine conditions under which they will eventually meet or never meet.

Short on time? Consider completing Problems 8 and 9 as a whole class.

1 Launch

Direct student's attention to the image and expressions and invite students to share connections to their work so far.

Invite students to think independently before discussing with their partner.

2 Monitor

Encourage students to identify the rate and starting position in each expression.

D Differentiation *(Problems 8-9)*

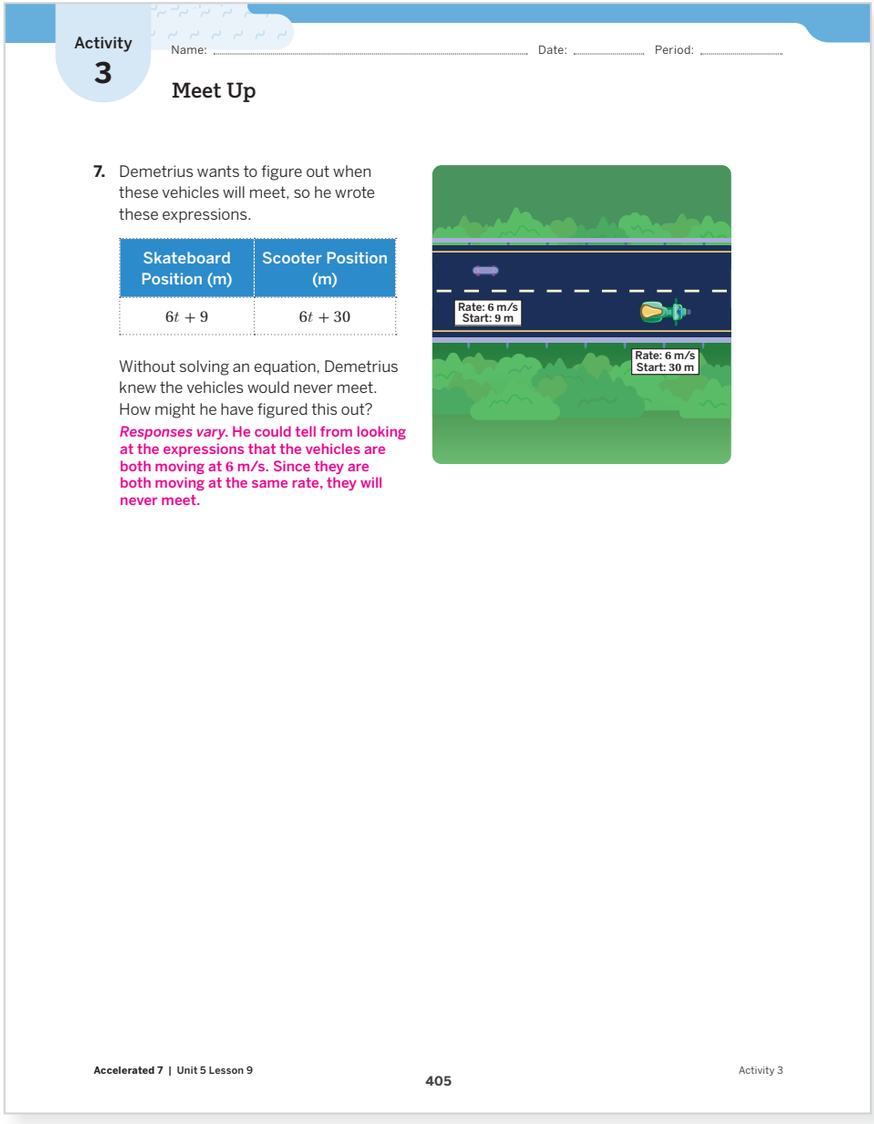
Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Invite them to recall the work they did during previous lessons with equations that are never true.
Say the skateboard and scooter have the same speed, but start at different places.	**Strengthen** Invite students to share their responses during the Connect and celebrate them one-on-one or publicly.
Would like to explore this concept further.	**Stretch** Invite them to share what they could change about one vehicle's expression to create a scenario where both vehicles are always in the same position.

ELL English Language Learners Invite students to talk through their thinking with a partner before writing it down. **(Writing and Speaking)**

Consider asking:

- "If the vehicles never meet, what does that tell us about the solution to the equation $6t + 9 = 6t + 30$?"
- "How does this equation connect to the structure of equations that have no solution?" **(MTR.5.1)**

Activity 3 continued >



Activity 3

Name: Date: Period:

Meet Up

7. Demetrius wants to figure out when these vehicles will meet, so he wrote these expressions.

Skateboard Position (m)	Scooter Position (m)
$6t + 9$	$6t + 30$

Without solving an equation, Demetrius knew the vehicles would never meet. How might he have figured this out?

Responses vary. He could tell from looking at the expressions that the vehicles are both moving at 6 m/s. Since they are both moving at the same rate, they will never meet.

Pairs | 13 min

Activity 3 Meet Up (continued)

Purpose: Students consider expressions representing the position of two vehicles for a given time and determine conditions under which they will eventually meet or never meet.

Short on time? Consider completing Problems 8 and 9 as a whole class.

2 Monitor

Look for students who use fractions, decimals, and negative numbers in their expressions.

D Differentiation

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Encourage them to try expressions from earlier in the lesson and change the numbers.
Have vehicles that will not meet.	**Support** Consider asking: • "Which vehicle is in front?" • "What does the vehicle in the back need to do in order to catch up to the other vehicle?" • "Which vehicle is moving faster?"
Solve their equation and determine a negative solution.	**Strengthen** Invite them to make sense of the solution in the context of the problem.
Use fractions, decimals, and negative numbers in their expressions.	**Strengthen** Invite them to share their responses during the Connect.

D Differentiation: You're invited to explore more. (Stretch) Invite students who would like to explore this concept further to complete this optional task. Encourage them to discuss their thinking with a partner.

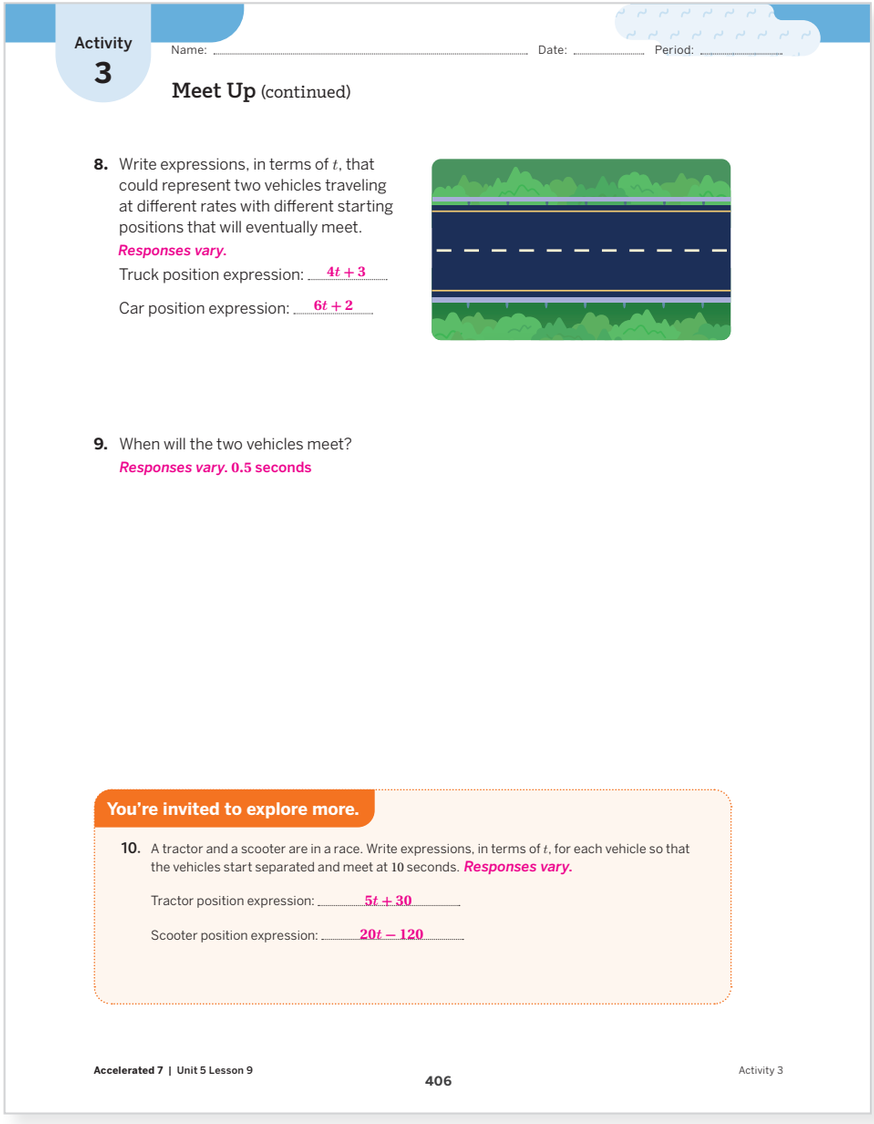
A Accessibility: Visual-Spatial Processing Invite students to draw a truck and car to support visualizing each vehicle's position.

Select and sequence expressions to share during the Connect.

3 Connect

To surface the Key Takeaway, consider asking:

- "What did you notice about the expressions for vehicles that didn't meet?"
- "How can setting the expressions equal to each other help us determine when the vehicles will meet?"
- "What do your solutions mean in this situation?"



Activity 3

Name: Date: Period:

Meet Up (continued)

8. Write expressions, in terms of t, that could represent two vehicles traveling at different rates with different starting positions that will eventually meet.

Responses vary.

Truck position expression: $4t + 3$

Car position expression: $6t + 2$

9. When will the two vehicles meet?

Responses vary. 0.5 seconds

You're invited to explore more.

10. A tractor and a scooter are in a race. Write expressions, in terms of t, for each vehicle so that the vehicles start separated and meet at 10 seconds. *Responses vary.*

Tractor position expression: $5t + 30$

Scooter position expression: $20t - 120$

Accelerated 7 | Unit 5 Lesson 9 406 Activity 3

Key Takeaway: You can set up equations to determine when vehicles traveling at different rates will meet. This equation can have one or no solution, depending on whether or not the vehicles will ever meet.

Whole Class | 5 min

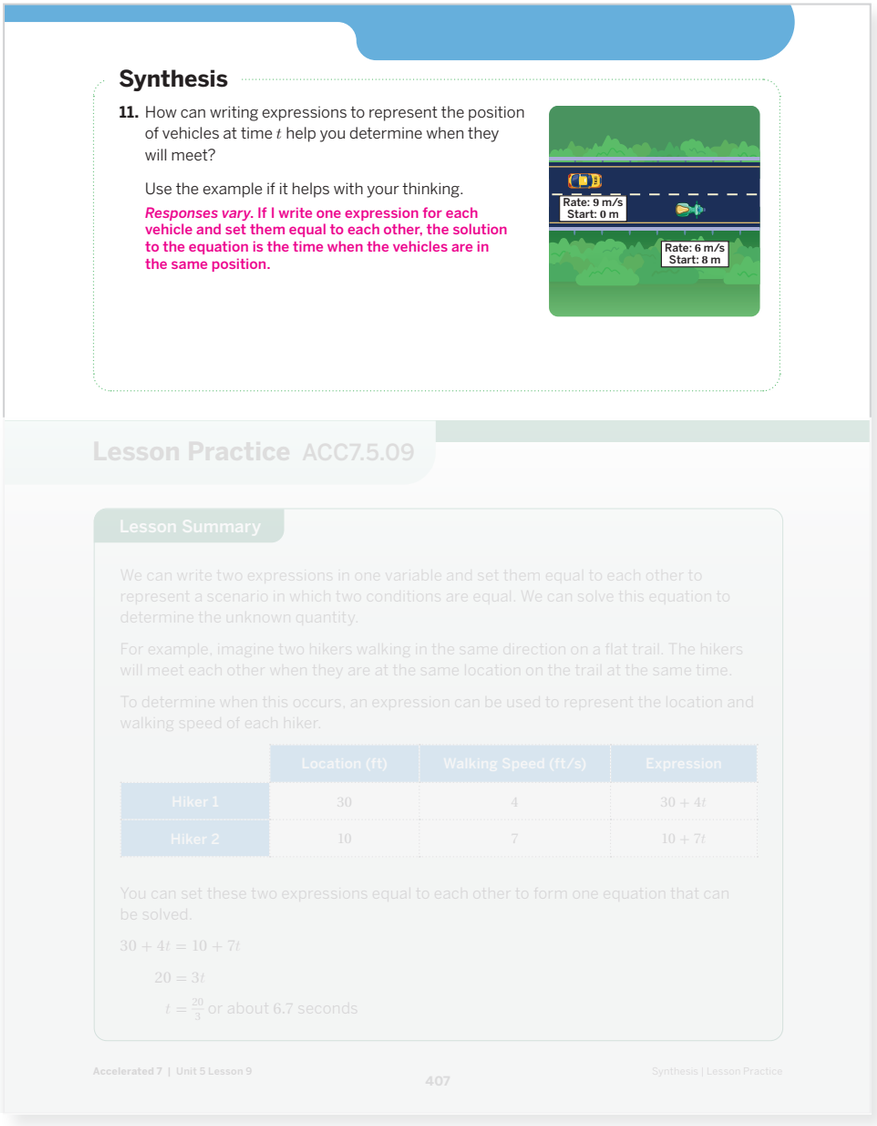
Synthesis

Purpose: Students synthesize their understanding of writing expressions and solving for when they are equal to each other.

Invite students to respond independently, after discussing the question with their partner.

Collect and share a variety of ideas, such as:
- Writing expressions for each vehicle.
- Setting expressions equal to each other and solving to determine when the vehicles will meet.
- The solution of the equation will tell me when the vehicles are in the same place at the same time.

MLR **MLR2: Collect and Display** Use this routine to collect students' ideas as they discuss with a partner. Display for students to reference and add to for the rest of the unit.



Synthesis

11. How can writing expressions to represent the position of vehicles at time t help you determine when they will meet?

Use the example if it helps with your thinking.

Responses vary. If I write one expression for each vehicle and set them equal to each other, the solution to the equation is the time when the vehicles are in the same position.

Lesson Practice ACC7.5.09

Lesson Summary

We can write two expressions in one variable and set them equal to each other to represent a scenario in which two conditions are equal. We can solve this equation to determine the unknown quantity.

For example, imagine two hikers walking in the same direction on a flat trail. The hikers will meet each other when they are at the same location on the trail at the same time.

To determine when this occurs, an expression can be used to represent the location and walking speed of each hiker.

	Location (ft)	Walking Speed (ft/s)	Expression
Hiker 1	30	4	$30 + 4t$
Hiker 2	10	7	$10 + 7t$

You can set these two expressions equal to each other to form one equation that can be solved.

$30 + 4t = 10 + 7t$

$20 = 3t$

$t = \frac{20}{3}$ or about 6.7 seconds

Accelerated 7 | Unit 5 Lesson 9 407 Synthesis | Lesson Practice

Lesson Takeaway: You can write two expressions to represent a real-world scenario and set those expressions equal to each other to help answer questions. Solving the equation helps to determine when the conditions are the same for the given context.

Independent | 5 min

Show What You Know

Purpose: Students demonstrate their understanding by interpreting the solution to an equation within a given situation.

Today's Goals

1. **Goal:** Create an equation in one variable to represent a situation in which two conditions are equal.
2. **Language Goal:** Interpret the solution of an equation in one variable in a situation. **(Speaking and Listening)**

Show What You Know (Assessment Resources)

Name: _____ Date: _____ Period: _____

Show What You Know 5.09

Andrea is comparing the costs of printing p pages at home and at a store.

She writes the equation $100 + 0.05p = 0.25p$ based on this information:

- A home printer costs $100 plus an additional $0.05 per page for the ink.
- To print pages at a store, it costs $0.25 per page.

The solution to Andrea's equation is $p = 500$. What does that represent in this situation?

Responses vary. The solution represents the number of pages for when the cost will be the same for printing at the store and at home.

Differentiation Use after Lesson 9

Support

Provide targeted intervention for students by using these resources.

If students need support with interpreting a solution in context:

» **Respond to Student Thinking:**

- Review the **Lesson Summary** and provide feedback to students on select **Practice Problems**.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 2 Extension Activities**.
- Invite students to **look for other real-world examples** of where it could be helpful to set expressions equal to each other.

Professional Learning

Reflect on students' progress so far with Lessons 1–9. In what ways have students' understanding and skills grown? What concepts do students need additional support with? How can you use students' strengths to help everyone feel prepared for Lesson 10 and their upcoming assessment?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

11. How can writing expressions to represent the position of vehicles at time t help you determine when they will meet?

Use the example if it helps with your thinking.

Responses vary. If I write one expression for each vehicle and set them equal to each other, the solution to the equation is the time when the vehicles are in the same position.

Lesson Practice ACC7.5.09

Lesson Summary

We can write two expressions in one variable and set them equal to each other to represent a scenario in which two conditions are equal. We can solve this equation to determine the unknown quantity.

For example, imagine two hikers walking in the same direction on a flat trail. The hikers will meet each other when they are at the same location on the trail at the same time.

To determine when this occurs, an expression can be used to represent the location and walking speed of each hiker.

	Location (ft)	Walking Speed (ft/s)	Expression
Hiker 1	30	4	$30 + 4t$
Hiker 2	10	7	$10 + 7t$

You can set these two expressions equal to each other to form one equation that can be solved.

$30 + 4t = 10 + 7t$

$20 = 3t$

$t = \frac{20}{3}$ or about 6.7 seconds

Lesson Practice
ACC7.5.09

Name: _____ Date: _____ Period: _____

1. For what value of x do the expressions $\frac{2}{3}x + 2$ and $\frac{4}{3}x - 6$ have the same value?

   12

2. Which story could the equation $3x + 6 = 2 + 4x$ represent?

   (A) A truck starts 6 meters down a road and is moving at a speed of 3 meters per second. A car starts 2 meters down a road and is moving at a speed of 4 meters per second. After x seconds, the car and the truck meet.

   B. A motorcycle starts 3 meters down a road and is moving at a speed of 6 meters per second. A bicycle starts 4 meters down a road and is moving at a speed of 2 meters per second. After x seconds, the motorcycle and the bike meet.

Problems 3–6: Tiana and Miko are biking in the same direction on the same path.

3. Miko rides at a constant speed of 16 miles per hour. Write an expression that represents the number of miles Miko travels after t hours.

   $16t$ miles

4. Tiana starts riding a half hour before Miko. If Miko has been riding for t hours, how long has Tiana been riding?

   $\left(t + \frac{1}{2}\right)$ hours (or equivalent)

5. Tiana rides at a constant speed of 12 miles per hour. Write an expression that represents the number of miles Tiana travels after Miko has been riding for t hours.

   $12\left(t + \frac{1}{2}\right)$ miles (or equivalent)

6. Use your expressions to determine when Miko and Tiana meet. Show or explain your thinking.

   1.5 hours. *Explanations vary.*

   $16t = 12\left(t + \frac{1}{2}\right)$

   $16t = 12t + 6$

   $4t = 6$

   $t = \frac{3}{2}$

Lesson Practice
ACC7.5.09

Name: _____ Date: _____ Period: _____

FAST Practice

7. Here are two cell phone plans.

   - Plan A costs $70 per month and comes with a free phone worth $500.
   - Plan B costs $50 per month, but doesn't come with a free phone.

   If you choose Plan B and buy a $500 phone, after how many months will your total cost be the same as the cost of choosing Plan A?

   Record your answer in the space provided.

   The total cost will be the same after [25] months.

Spiral Review

Problems 8–9: Solve each equation. Show your thinking.

8. $5(x + 2) = 30$

   $x = 4$. *Work varies.*

   $5(x + 2) \div 5 = 30 \div 5$

   $x + 2 = 6$

   $x + 2 - 2 = 6 - 2$

   $x = 4$

9. $5x + 2 = 30$

   $x = \frac{28}{5}$. *Work varies.*

   $5x + 2 - 2 = 30 - 2$

   $5x = 28$

   $5x \div 5 = 28 \div 5$

   $x = \frac{28}{5}$ (or equivalent)

10. A triangle is graphed on a coordinate grid. Which transformation will result in a triangle that is not congruent to the original triangle?

    A. A reflection over the y-axis.

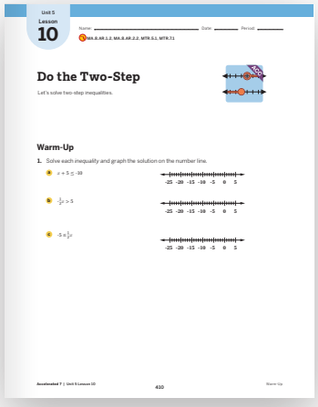
    B. A translation 5 units to the left.

    (C.) A dilation by a scale factor of 4.

    D. A 180° counterclockwise rotation around the origin.

Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T Math Benchmarks
On-lesson			
	1	1	MA.8.AR.2.1
	2–5	1	MA.8.AR.3.3
	6	2	MA.8.AR.2.1, MA.8.AR.3.3
FAST Practice	7	2	MA.8.AR.2.1, MA.8.AR.3.3
Spiral Review			
	8–9	1	MA.8.AR.2.1
	10	2	MA.8.GR.2.1



Do the Two-Step

Solving Two-Step Linear Inequalities

Let's solve two-step linear inequalities.

Learning Outcomes

Today's Goals

1. **Goal:** Solve two-step inequalities with positive and negative rational coefficients.
2. **Goal:** Represent the solutions to a two-step inequality graphically.
3. **Language Goal:** Compare and contrast the process for solving a two-step inequality with the process for solving a two-step equation. **(Writing, Speaking, and Listening)**

Students solve *inequalities* with positive and negative rational coefficients and connect the solutions of inequalities to their graphs. They apply what they know about inequalities to real-world situations, interpreting the solution set in context.

Coherence

Prior Learning

In a previous course, students solved one-step inequalities. In Lesson 8, students worked with different strategies to fluently solve linear equations.

Future Learning

In a future course, students will write, solve, and graph two-variable linear inequalities.

Connections

- Students extend their understanding of solving linear equations to solving linear inequalities. (mathematical)
- Students analyze and discuss the thinking of others to make sense of a strategy to solve linear inequalities. (mathematical, cross-curricular)

FL B.E.S.T. Math Benchmarks

Addressing

MA.8.AR.2.2
Solve two-step linear inequalities in one variable and represent solutions algebraically and graphically.

Also Addressing: **MA.8.AR.1.2**

Mathematical Thinking and Reasoning Standards: MTR.5.1, MTR.7.

ELA Expectations: ELA.K12.EE.3.1, ELA.K12.EE.4.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.7.AR.2.2	MA.912.AR.2.6

Vocabulary

Review Vocabulary

inequality

Lesson at a Glance

~ 45 min

MA.8.AR.1.2, MA.8.AR.2.2

Warm-Up

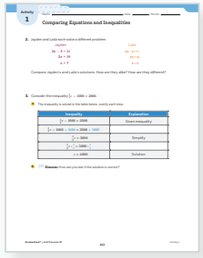
Pairs | 5 min

Students review solving and graphing one-step *inequalities*.
Routine: Think-Pair-Share

Activity 1

Pairs | 10 min

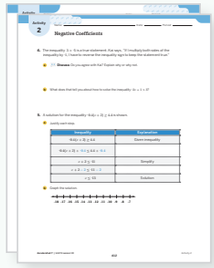
Students examine patterns and apply the structure of solving linear equations to solving linear inequalities. **(MTR.5.1)**



Activity 2

Pairs | 10 min

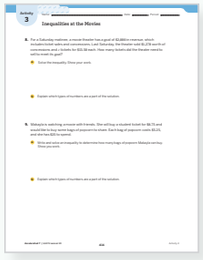
Students identify the impact of multiplying or dividing both sides of an inequality by a negative number.



Activity 3

Pairs | 10 min

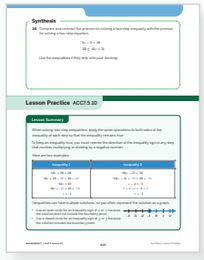
Students write and solve two-step linear inequalities to model real-world problems and interpret the solutions in context. **(MTR.7.1)**



Synthesis

Whole Class | 5 min

Students synthesize their understanding of solving two-step linear inequalities.
MLR2: Collect and Display



Show What You Know

Independent | 5 min

Students demonstrate their understanding by solving a two-step inequality and graphing the solution.

Prep Checklist

This lesson includes:

- Student Edition
- Show What You Know
- Coloring tools (as needed)

Pairs | 5 min

Warm-Up

Purpose: Students review solving and graphing one-step *inequalities*.

1 Launch

To surface prior knowledge, ask students what they recall about inequalities.

Use the Think-Pair-Share routine to support student-to-student discourse. Invite students to think independently and then take turns sharing with a partner. Invite several students to share their or their partner's strategy with the class.

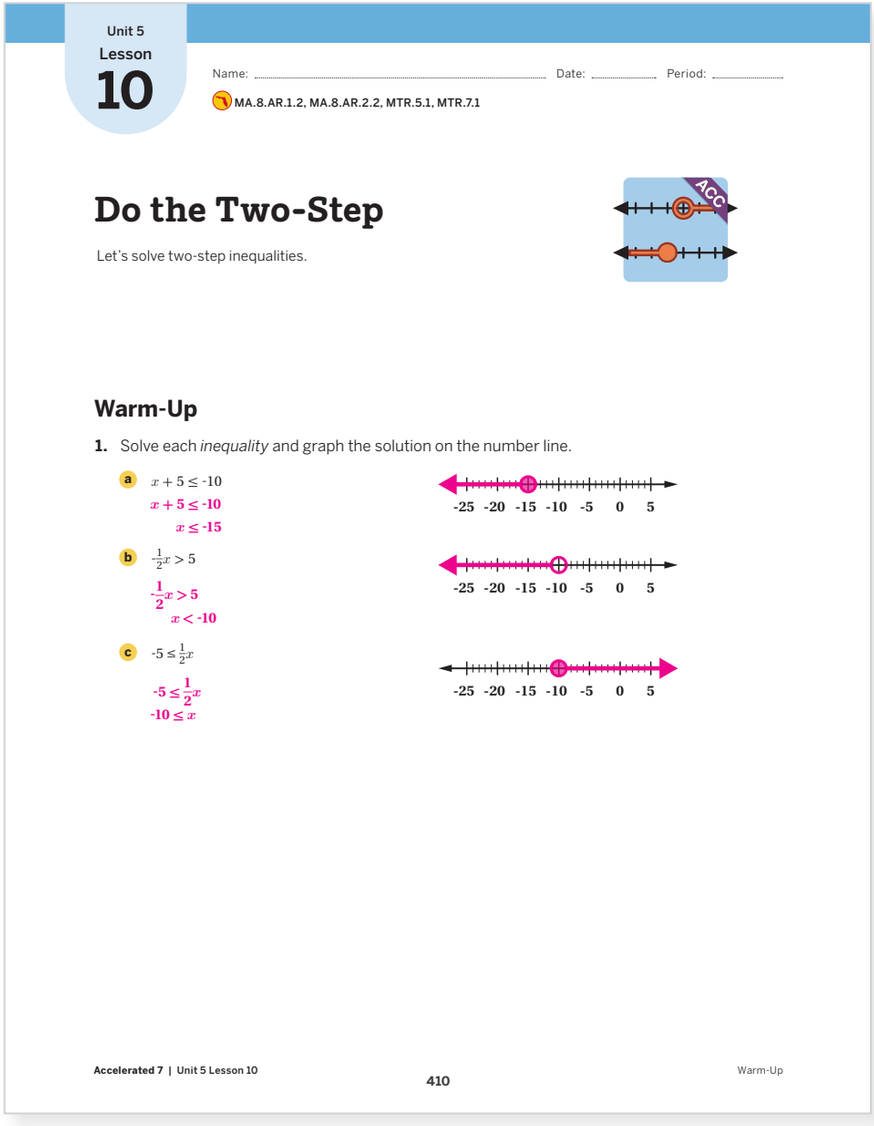
Listen for the ways the students explain how to solve the equations.

2 Connect

Invite students to share their strategies and graphs.

Note: If it doesn't come up naturally, consider asking:

- "What property of inequality can be used to solve the problem?"
- "How can you check to make sure the correct side of the graph is shaded?"
- "How do you know when to use an open circle or a closed circle?"



Unit 5
Lesson
10

Name: Date: Period:

MA.8.AR.1.2, MA.8.AR.2.2, MTR.5.1, MTR.7.1

Do the Two-Step

Let's solve two-step inequalities.

Warm-Up

1. Solve each *inequality* and graph the solution on the number line.

a. $x + 5 \leq -10$

$x + 5 \leq -10$

$x \leq -15$

b. $-\frac{1}{2}x > 5$

$-\frac{1}{2}x > 5$

$x < -10$

c. $-5 \leq \frac{1}{2}x$

$-5 \leq \frac{1}{2}x$

$-10 \leq x$

Accelerated 7 | Unit 5 Lesson 10 410 Warm-Up

Pairs | 10 min

Activity 1 Comparing Equations and Inequalities

Purpose: Students examine patterns and apply the structure of solving linear equations to solving linear inequalities. **(MTR.5.1)**

1 Launch

Direct students' attention to Jayden's and Laila's work.

To surface connections, consider asking:
- "What was Jayden's first step? His second?"
- "What steps did Laila take?"

A **Accessibility: Visual-Spatial Processing** Pair students and invite a partner to label each step to support processing visual information.

Discuss the properties of equality Jayden used for each step. Relate the properties of equality to the properties of inequality.

2 Monitor

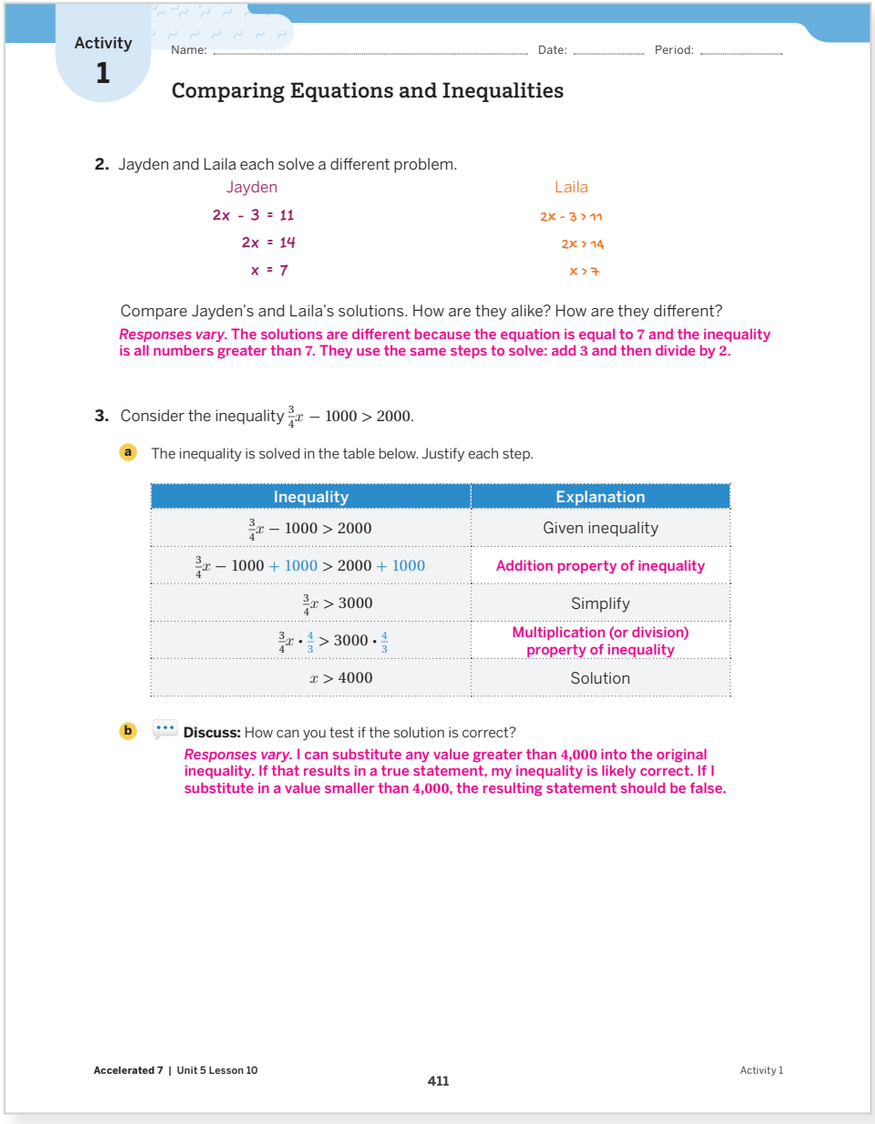
To support students getting started in Problem 3, encourage them to replace the inequality with an equal sign and perform the same steps they would if they were solving an equation.

Look and listen for explanations to share about how to determine values that are true for a linear inequality to share in the Connect.

3 Connect

Invite students to share their strategies for checking solutions to the inequality. Consider asking, "How is checking your solution to an inequality different than in an equation?"

To surface the Key Takeaway, consider asking, "How would your explanations change if it were an equation instead of an inequality?"



Activity 1

Name: Date: Period:

Comparing Equations and Inequalities

2. Jayden and Laila each solve a different problem.

Jayden	Laila
$2x - 3 = 11$	$2x - 3 > 11$
$2x = 14$	$2x > 14$
$x = 7$	$x > 7$

Compare Jayden's and Laila's solutions. How are they alike? How are they different?

Responses vary. The solutions are different because the equation is equal to 7 and the inequality is all numbers greater than 7. They use the same steps to solve: add 3 and then divide by 2.

3. Consider the inequality $\frac{3}{4}x - 1000 > 2000$.

a. The inequality is solved in the table below. Justify each step.

Inequality	Explanation
$\frac{3}{4}x - 1000 > 2000$	Given inequality
$\frac{3}{4}x - 1000 + 1000 > 2000 + 1000$	Addition property of inequality
$\frac{3}{4}x > 3000$	Simplify
$\frac{3}{4}x \cdot \frac{4}{3} > 3000 \cdot \frac{4}{3}$	Multiplication (or division) property of inequality
$x > 4000$	Solution

b. **Discuss:** How can you test if the solution is correct?

Responses vary. I can substitute any value greater than 4,000 into the original inequality. If that results in a true statement, my inequality is likely correct. If I substitute in a value smaller than 4,000, the resulting statement should be false.

Accelerated 7 | Unit 5 Lesson 10 411 Activity 1

Key Takeaway: Solving *inequalities* is similar to solving equations. The properties of inequality allow you to perform the same operation on both sides of the inequality while maintaining the truth of the statement.

Pairs | 10 min

Activity 2 Negative Coefficients

Purpose: Students identify the impact of multiplying or dividing both sides of an inequality by a negative number.

Short on time? Consider working Problem 7 as a whole class.

1 Launch

Invite students to consider Kai's statement in Problem 4.

Consider asking, "Does it matter if the inequality contains a variable?"

2 Monitor

Invite students to compare the steps in this Problem 5 with those in Problem 3.

D **Differentiation:** (Stretch) Challenge students to solve the inequality by first applying the distributive property on the left side. Have them reflect on how the solution and explanations change.

Invite students to compare their graph with a partner's to check their answer.

Activity 2 continued >

Activity 2

Name: Date: Period:

Negative Coefficients

4. The inequality $3 > -5$ is a true statement. Kai says, "If I multiply both sides of the inequality by -1, I have to reverse the inequality sign to keep the statement true."

a. **Discuss:** Do you agree with Kai? Explain why or why not.

Responses vary. I agree with Kai. When you multiply both sides of the inequality $3 > -5$ by -1, you get $-3 > 5$, which is false. If you reverse the inequality sign, the statement is true again, $-3 < 5$.

b. What does that tell you about how to solve the inequality $-2x + 1 > 3$?

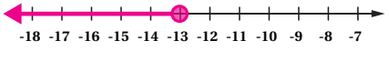
Responses vary. When I get to the step of dividing both sides by -2, I need to reverse the inequality sign.

5. A solution for the inequality $-0.4(x + 2) \geq 4.4$ is shown.

a. Justify each step.

Inequality	Explanation
$-0.4(x + 2) \geq 4.4$	Given inequality
$-0.4(x + 2) \div -0.4 \leq 4.4 \div -0.4$	Division property of inequality Reverse the inequality sign.
$x + 2 \leq -11$	Simplify
$x + 2 - 2 \leq -11 - 2$	Subtraction property of inequality
$x \leq -13$	Solution

b. Graph the solution.



Accelerated 7 | Unit 5 Lesson 10 412 Activity 2

Key Takeaway: The direction of the *inequality* symbol reverses when multiplying or dividing both sides of an inequality by a negative number.

Pairs | 10 min

Activity 2 Negative Coefficients (continued)

Purpose: Students identify the impact of multiplying or dividing both sides of an inequality by a negative number.

Short on time? Consider working Problem 7 as a whole class.

2 Monitor

Look and listen for explanations to share during the Connect.

D Differentiation *(Problem 6)*

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Invite students to analyze Rudra's work one line at a time. Consider asking, "How can we be sure this step maintains the inequality?"
Believe Rudra did not make a mistake.	**Support** Encourage students to substitute values greater than -12 into the original inequality to see if it makes the inequality true.
Identify Rudra's mistake.	**Strengthen** Invite them to write a rule to help Rudra identify when the inequality symbol should be reversed.

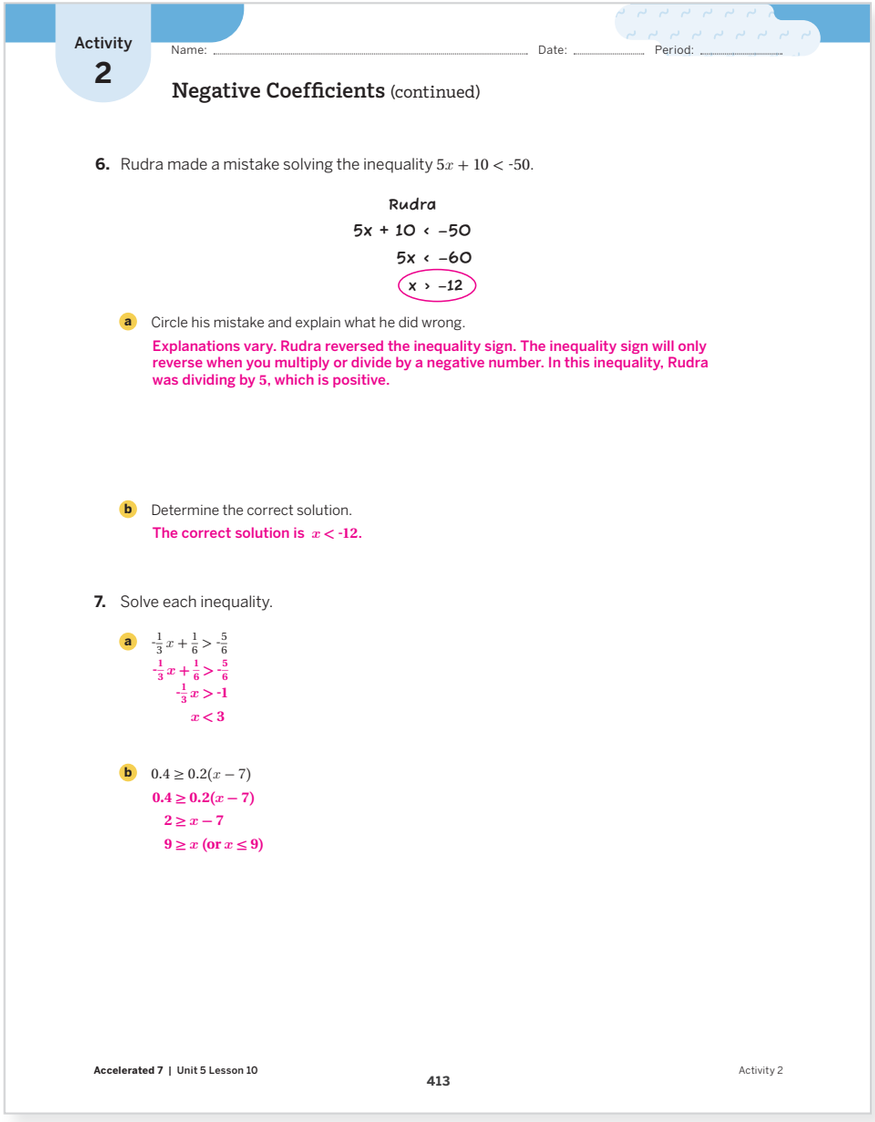
3 Connect

Invite students to share their explanations of Rudra's error. Consider sharing that it is a common misconception to think the inequality symbol changes direction if there is a negative number in the multiplication or division step but that the symbol will change only when the coefficient of the variable is negative.

Invite students to share their solutions to the inequalities in Problem 7. Encourage students to explain their reasoning.

Consider asking:

- "What operations did you use to solve the inequality?"
- "How did you test your solution?"
- "Is there another way to solve the inequality?"
- "Did the inequality sign change? Why or why not?"
- "What would a graph of the solution set look like on a number line?" **(MTR.3.1)**



Activity 2

Name: Date: Period:

Negative Coefficients (continued)

6. Rudra made a mistake solving the inequality $5x + 10 < -50$.

Rudra
$5x + 10 < -50$
$5x < -60$
$x > -12$

a. Circle his mistake and explain what he did wrong.

Explanations vary. Rudra reversed the inequality sign. The inequality sign will only reverse when you multiply or divide by a negative number. In this inequality, Rudra was dividing by 5, which is positive.

b. Determine the correct solution.

The correct solution is $x < -12$.

7. Solve each inequality.

a. $-\frac{1}{3}x + \frac{1}{6} > -\frac{5}{6}$
$-\frac{1}{3}x + \frac{1}{6} > -\frac{5}{6}$
$-\frac{1}{3}x > -1$
$x < 3$

b. $0.4 \geq 0.2(x - 7)$
$0.4 \geq 0.2(x - 7)$
$2 \geq x - 7$
$9 \geq x$ (or $x \leq 9$)

Accelerated 7 | Unit 5 Lesson 10 413 Activity 2

Key Takeaway: The direction of the *inequality* symbol reverses when multiplying or dividing both sides of an inequality by a negative number.

Pairs | 10 min

Activity 3 Inequalities at the Movies

Purpose: Students write and solve two-step linear inequalities to model real-world problems and interpret the solutions in context. **(MTR.7.1)**

Short on time? Consider omitting Problem 9

1 Launch

To generate excitement, consider asking, "What's the last movie you saw in a movie theater?"

Direct students' attention to the scenario in Problem 8. Read the problem as a class.

Discuss how the given inequality represents the situation.

Consider asking:
- "Where do each number and operation originate in the situation?"
- "Why is the greater than or equal to symbol used?"

Look and listen for student ideas and reasonings about which type of numbers belong in the solution to share during the Connect.

2 Monitor

Listen for students who understand how to deal with the remainder.

A **Accessibility: Conceptual Processing** Invite students to color-code features from the scenario that will be needed to write the inequality.

ELL **English Language Learners** Consider modeling or inviting a student to model strategies for annotating the text to make sense of it. **(Reading)**

Look for explanations on the correct steps used in solving the problem and interpreting the solution.

Connect

Capture and share a variety of responses, such as:
- Connecting each value and operation to the situation.
- Which inequality makes sense for the situation.
- Varied explanations as to which values are a part of the solution.

Invite students to share the justification for their choices and build on each other's reasoning.

Activity 3

Name: ..... Date: ..... Period: .....

Inequalities at the Movies

8. For a Saturday matinee, a movie theater has a goal of $2,000 in revenue, which includes ticket sales and concessions. Last Saturday, the theater sold $1,278 worth of concessions and x tickets for $13.50 each. How many tickets did the theater need to sell to meet its goal?

a. Solve the inequality. Show your work.

At least 54 tickets. *Work varies.*

$$13.5x + 1278 \geq 2000$$
$$13.5x \geq 722$$
$$x \geq 53.481481...$$

b. Explain which types of numbers are a part of the solution.

***Responses vary.* Only whole numbers 54 or greater can be part of the solution. Only whole numbers can be used because you cannot sell a fraction of a ticket.**

9. Makayla is watching a movie with friends. She will buy a student ticket for $8.75 and would like to buy some bags of popcorn to share. Each bag of popcorn costs $5.25, and she has $25 to spend.

a. Write and solve an inequality to determine how many bags of popcorn Makayla can buy. Show you work.

Makayla can buy up to and including 3 bags of popcorn. *Work varies.*

$$5.25x + 8.75 \leq 25$$
$$5.25x \leq 16.25$$
$$x \leq 3.095...$$

b. Explain which types of numbers are a part of the solution.

Responses vary. The solution includes whole numbers up to and including 3, so it is the set {0, 1, 2, 3}. Makayla cannot buy negative bags of popcorn or fractions of a bag, and 3 is the greatest number of bags she can afford.

Accelerated 7 | Unit 5 Lesson 10 414 Activity 3

Whole Class | 5 min

Synthesis

Purpose: Students synthesize their understanding of solving two-step linear *inequalities*.

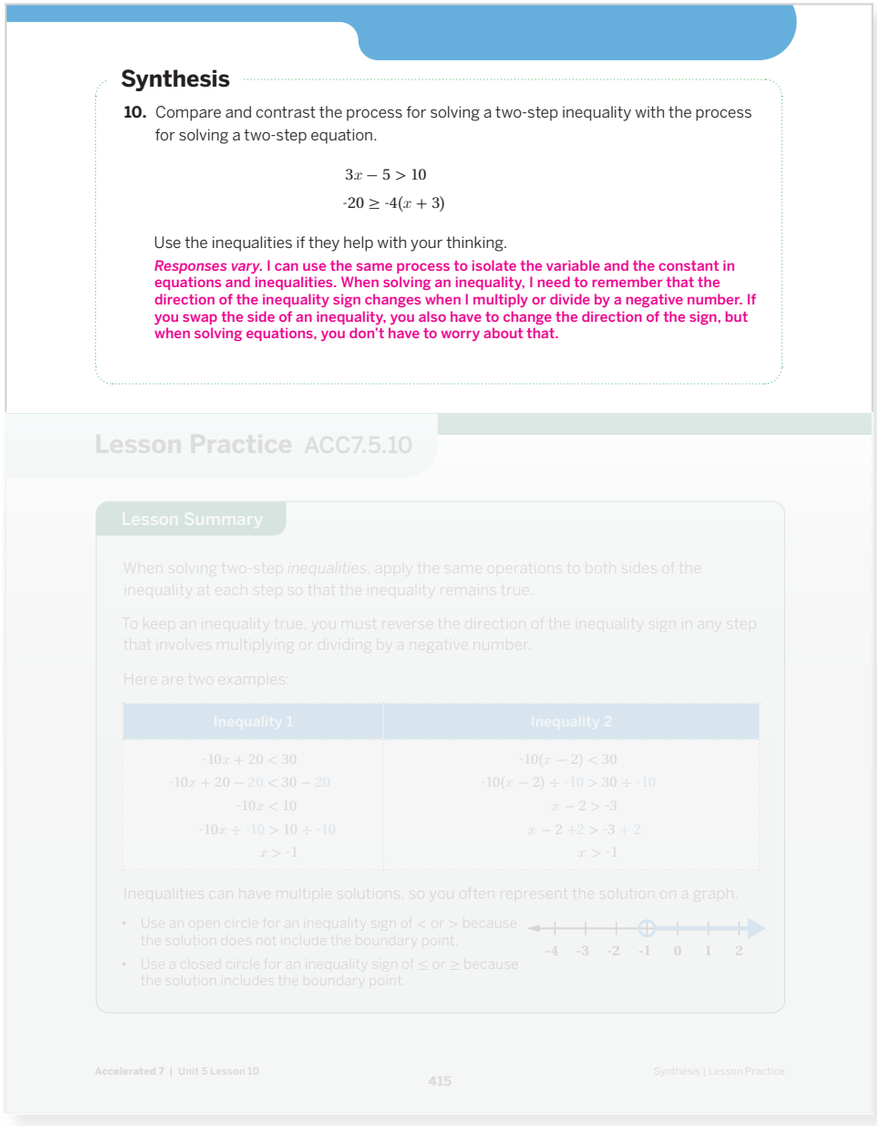
Invite students to respond independently, and then share their thinking with a partner.

Collect and share a variety of ideas, such as:

- The addition and subtraction properties of inequality work like those of equations.
- The multiplication and division properties of inequality work like those of equations.
- The distributive property works the same within inequalities and equations.
- Inequalities are unlike equations because you need to pay attention to the direction of the inequality symbol when you multiply or divide by negatives and when you transpose sides of the inequality.

MLR **MLR2: Collect and Display** Use this routine to collect students' ideas or strategies as they compare and contrast the process for solving a two-step inequality with that for solving a two-step equation. Display for students to reference later.

Note: If time allows, invite students to share their ideas with a partner and revise their thinking based on the discussion.



Synthesis

10. Compare and contrast the process for solving a two-step inequality with the process for solving a two-step equation.

$$3x - 5 > 10$$

$$-20 \geq -4(x + 3)$$

Use the inequalities if they help with your thinking.

Responses vary. I can use the same process to isolate the variable and the constant in equations and inequalities. When solving an inequality, I need to remember that the direction of the inequality sign changes when I multiply or divide by a negative number. If you swap the side of an inequality, you also have to change the direction of the sign, but when solving equations, you don't have to worry about that.

Lesson Practice ACC7.5.10

Lesson Summary

When solving two-step *inequalities*, apply the same operations to both sides of the inequality at each step so that the inequality remains true.

To keep an inequality true, you must reverse the direction of the inequality sign in any step that involves multiplying or dividing by a negative number.

Here are two examples:

Inequality 1	Inequality 2
$-10x + 20 < 30$	$-10(x - 2) < 30$
$-10x + 20 - 20 < 30 - 20$	$-10(x - 2) \div -10 > 30 \div -10$
$-10x < 10$	$x - 2 > -3$
$-10x \div -10 > 10 \div -10$	$x - 2 + 2 > -3 + 2$
$x > -1$	$x > -1$

Inequalities can have multiple solutions, so you often represent the solution on a graph.

- Use an open circle for an inequality sign of $<$ or $>$ because the solution does not include the boundary point.
- Use a closed circle for an inequality sign of $\leq$ or $\geq$ because the solution includes the boundary point.

-4 -3 -2 -1 0 1 2

Accelerated 7 | Unit 5 Lesson 10 415 Synthesis | Lesson Practice

Lesson Takeaway: Solving *inequalities* is similar to solving equations with one important exception: when an inequality is multiplied or divided by a negative, the inequality sign flips. The solution to an inequality can be tested by substituting a value from the solution into the original inequality.

Independent | 5 min

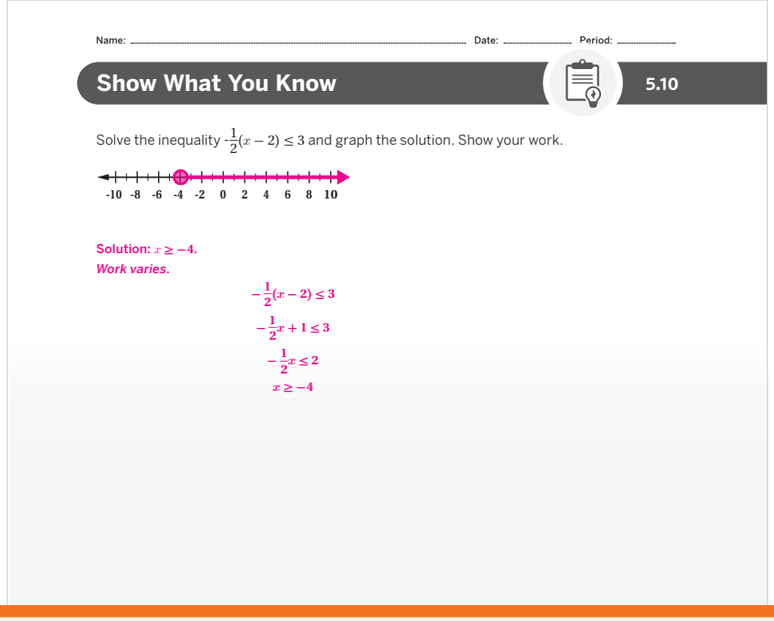
Show What You Know

Purpose: Students demonstrate their understanding by solving a two-step inequality and graphing the solution.

Today's Goals

1. **Goal:** Solve two-step inequalities with positive and negative rational coefficients.
2. **Goal:** Represent the solutions to a two-step inequality graphically.
3. **Language Goal:** Compare and contrast the process for solving a two-step inequality with the process for solving a two-step equation. **(Writing, Speaking, and Listening)**

Show What You Know (Assessment Resources)



Name: _____ Date: _____ Period: _____

Show What You Know 5.10

Solve the inequality $-\frac{1}{2}(x-2) \leq 3$ and graph the solution. Show your work.

Solution: $x \geq -4$.

Work varies.

$$-\frac{1}{2}(x-2) \leq 3$$
$$-\frac{1}{2}x + 1 \leq 3$$
$$-\frac{1}{2}x \leq 2$$
$$x \geq -4$$

D Differentiation Use after Lesson 10

Support

Provide targeted intervention for students by using these resources.

If students need support with solving two-step linear inequalities:

» **Respond to Student Thinking:**

- Review the **Lesson Summary** and provide feedback to students on select **Practice Problems**.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Create inequality problems that require the use of the distributive property.
- Create inequality problems that have a variable on both sides of the equation instead of constant terms.
- Invite students to explore the **Sub-Unit 2 Extension Activities**.

Professional Learning

Reflect on students' progress with solving linear equations and inequalities. In what ways have students' understanding and skills grown? What concepts do students need additional support with? How can you use students' strengths to help everyone feel prepared for their upcoming assessment?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

10. Compare and contrast the process for solving a two-step inequality with the process for solving a two-step equation.

$3x - 5 > 10$

$-20 \geq -4(x + 3)$

Use the inequalities if they help with your thinking.

Responses vary. I can use the same process to isolate the variable and the constant in equations and inequalities. When solving an inequality, I need to remember that the direction of the inequality sign changes when I multiply or divide by a negative number. If you swap the side of an inequality, you also have to change the direction of the sign, but when solving equations, you don't have to worry about that.

Lesson Practice ACC7.5.10

Lesson Summary

When solving two-step *inequalities*, apply the same operations to both sides of the inequality at each step so that the inequality remains true.

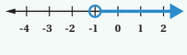
To keep an inequality true, you must reverse the direction of the inequality sign in any step that involves multiplying or dividing by a negative number.

Here are two examples:

Inequality 1	Inequality 2
$-10x + 20 < 30$	$-10(x - 2) < 30$
$-10x + 20 - 20 < 30 - 20$	$-10(x - 2) \div -10 > 30 \div -10$
$-10x < 10$	$x - 2 > -3$
$-10x \div -10 > 10 \div -10$	$x - 2 + 2 > -3 + 2$
$x > -1$	$x > -1$

Inequalities can have multiple solutions, so you often represent the solution on a graph.

- Use an open circle for an inequality sign of $<$ or $>$ because the solution does not include the boundary point.
- Use a closed circle for an inequality sign of $\leq$ or $\geq$ because the solution includes the boundary point.



Lesson Practice
ACC7.5.10

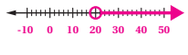
Name: _____ Date: _____ Period: _____

1. Explain the steps used to solve the inequality $-0.8x + 0.4 \geq 3.6$.

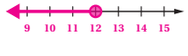
Inequality	Explanation
$-0.8x + 0.4 \geq 3.6$	Given inequality
$-0.8x + 0.4 - 0.4 \geq 3.6 - 0.4$	Subtraction property of inequality
$-0.8x \div -0.8 \leq 3.2 \div -0.8$	Division property of inequality
$x \leq -4$	Solution

Problems 2–4: Solve the inequality. Graph each solution on a number line.

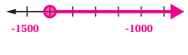
2. $3 < \frac{1}{4}(w - 8)$
   $12 < w - 8$
   $20 < w$ or
   $w > 20$



3. $-\frac{1}{3}x + 2 \geq -2$
   $\frac{1}{3}x \geq -4$
   $x \leq 12$



4. $0.25(y + 800) \geq -150$
   $y + 800 \geq -600$
   $y \geq -1400$



5. Tasia has $40 to spend at the fair. The admission is $12, and the rides are $3 each. Solve the inequality $3x + 12 \leq 40$ to determine the number of rides, x, she can ride. Explain the solution.
   $3x + 12 \leq 40$
   $3x \leq 28$
   $x \leq 9\frac{1}{3}$
   Explanations vary. Tasia can ride a maximum of 9 rides.

Lesson Practice
ACC7.5.10

Name: _____ Date: _____ Period: _____

6. A group of 8th graders has raised $2,500 for charity. If the group is selling raffle tickets for $5 each, how many tickets must the group sell to raise at least $4,000 in all? Write and solve an inequality to determine the number of tickets, x, the group must sell. Explain what types of numbers are included in the solution.
   $5x + 2500 \geq 4000$
   $5x \geq 1500$
   $x \geq 300$
   Explanations vary. The group must sell at least 300 tickets. The solution includes only whole numbers because a partial ticket cannot be sold.

FAST Practice

7. Faaria has $800 in her savings account and withdraws $50 each week. She wants to keep a minimum of $200 in the account. Solve the inequality $800 - 50x \geq 200$ to find the number of weeks she can withdraw money.

   A. $x \geq 12$
   B. $x \leq 12$
   C. $x \geq 20$
   D. $x \leq 20$

Spiral Review

Problems 8–9: Solve each of the following equations.

8. $5(t + 2) = 5t + 20$
   No real solutions

9. $7t - 20 = 4(t + 10)$
   $t = 20$

Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T Math Benchmarks
On-lesson			
	1	1	MA.8.AR.2.2
	2–6	1	MA.8.AR.2.2
FAST Practice	7	2	MA.8.AR.2.2
Spiral Review			
	8–9	1	MA.8.AR.2.1

Assess and Respond

Sub-Unit Quiz 1

Independent | 45 min

Facilitation: Assign the Sub-Unit Quiz to learn about your students' understanding of the concepts and skills so far in this unit. If time allows, consider inviting students to complete one or more of the tasks in the Unit Synthesis and Reflection.

Item Analysis

Problem(s)	Concept or skill	Addressed in	DOK	FL B.E.S.T. Math Benchmarks
1	Solving a two-step linear inequality and graphing the solution set	Lesson 10	1	MA.8.AR.2.2
2	Determining if an equation has infinitely many solutions	Lesson 7	2	MA.8.AR.2.1
3	Clearly explaining the first step students took to solve an equation	Lesson 6	2	MA.8.AR.2.1 MTR.4.1
4	Using structure to solve equations of the form $px + q = r$ and $p(x + q) = r$	Lesson 4	1	MA.7.AR.2.2
5	Determining when two conditions are equal in a situation that could be represented by an equation and interpreting the solution in context	Lesson 9	3	MA.8.AR.2.1, MA.8.AR.3.3 MTR.6.1
6	Solving multi-steps equations, including those with rational numbers	Lesson 8	2	MA.8.AR.2.1

Assessment Resources

- Student Assessments
- Assessment Rubrics with Answers
- Show What You Knows and Answer Keys

Intervention and Extension Resources

- Mini-Lessons
- Extensions

Name: Date: Period:

Sub-Unit Quiz

Unit 5

1. Solve $-2(x + 3) \geq -4$ and graph the solution on the number line.

$x \leq -1$

2. Select *all* the equations that have infinitely many solutions.

- ☑ **A.** $7x = 7x$
- ☐ **B.** $10 - 19 + 12x = 4x - 9 + 8x$
- ☐ **C.** $x \cdot 2 \cdot (-6) = x \cdot 3 \cdot 4$
- ☑ **D.** $\frac{1}{2}(6x + 5) = 3x + 2.5$
- ☐ **E.** $x - 6 = 6 - x$

3. Leah-James, Anika, and Sai are each solving the same equation for x. Describe the move they each made. *Responses vary.*

Original equation:
$12x + 4 = 20x - 12$

a The result of Leah-James's first step was:
$4 = 8x - 12$.
Leah-James subtracted $12x$ from each side.

b The result of Anika's first step was:
$3x + 1 = 5x - 3$.
Anika divided each term in the equation by 4.

c The result of Sai's first step was:
$12x + 16 = 20x$.
Sai added 12 to each side.

4. Solve each equation.

a $3x + 7 = 40$
$x = 11$

b $-2(x + 5) = 10$
$x = -10$

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Name: Date: Period:

Sub-Unit Quiz (continued)

Unit 5

5. Imani and Esteban each have different audiobook club memberships:

- Imani's book club costs *$10* for the membership and then *$5* per book.
- Esteban's book club costs *$16* for the membership and then *$2* per book.

a After listening to 4 audiobooks, whose book club costs more? Circle one.

(Imani's) Esteban's They cost the same amount

Explain your thinking.
***Explanations vary.* After 4 books, Imani's book club total is $30 and Esteban's total is $24.**

b After how many audiobooks will both book clubs cost the same total amount?
2 audiobooks

6. Solve each equation. Show or explain your thinking.

a $1d + 12 = 14 - 2d$
$d = \frac{2}{3}$ **(or equivalent).** ***Work varies.***
$1d + 12 = 14 - 2d$
$1d = 2 - 2d$
$3d = 2$
$d = \frac{2}{3}$

b $\frac{1}{2}(8r + 24) = 10r$
$r = 2$. ***Work varies.***
$\frac{1}{2}(8r + 24) = 10r$
$8r + 24 = 20r$
$24 = 12r$
$2 = r$

c $-2(5 + x) - 1 = 3(x + 3)$
$x = -4$. ***Work varies.***
$-2(5 + x) - 1 = 3(x + 3)$
$-10 - 2x - 1 = 3x + 9$
$-11 - 2x = 3x + 9$
$-2x = 3x + 20$
$-5x = 20$
$x = -4$

D Differentiation (Sub-Unit Quiz 1)

Note: To **strengthen** and **stretch** students' learning, refer to the differentiation resources suggested throughout this unit and in the Unit Overview.

Sub-Unit Goals	Problem(s)	To respond to student thinking, consider:
• Solve a two-step linear inequality and graph the solution set. • Write and solve equations with variables on both sides of the equal sign. • Determine the number of solutions to a linear equation. **(Lessons 6–10)**	1	**Support** / **Strengthen** • **Teacher Move:** Consider revisiting Lesson 10 (*Do the Two Step*)
	2	**Support** / **Strengthen** • **Teacher Move:** Consider revisiting Lesson 7 (*All, Some, None?*)
	3	**Support** / **Strengthen** • **Teacher Move:** Consider revisiting Lesson 6 (*Equation Roundtable*)

	4, 5	**Support** / **Strengthen**
		• **Mini-Lesson:** *Solving Linear Equations with Parentheses*
		• **Teacher Moves:** *Consider revisiting:*
		  • *Lesson 8 (Strategic Solving)*
		  • *Lesson 9 (When Will They Meet?)*

		Stretch • **You're invited to explore more:** Lesson 9 (*When Will They Meet?*)

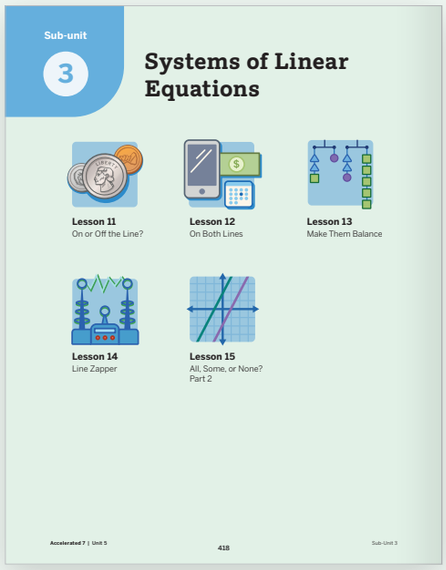
Sub-Unit 3

Solve Systems of Equations

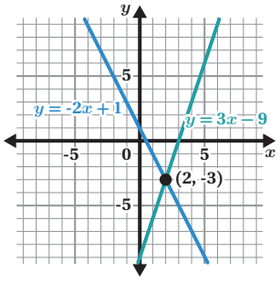
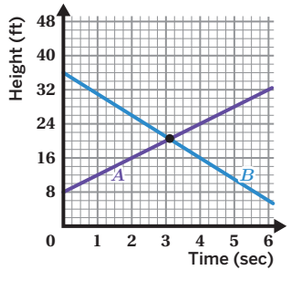
Sub-Unit 3 Goals

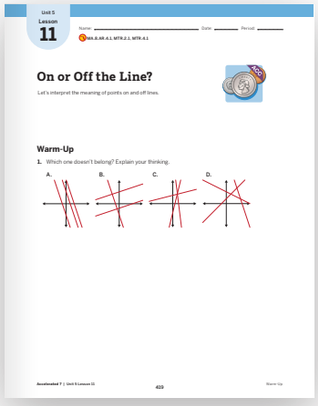
- **Goal:** Solve systems of linear equations in two variables graphically and algebraically.
- **Language Goal:** Interpret the intersection point of two linear relationships in context given their graphs or equations.
- **Language Goal:** Describe strategies for solving systems of linear equations.

In this sub-unit, students investigate **systems of linear equations** in two variables. They develop conceptual understanding of what the **solution to a system** means graphically and algebraically. They estimate solutions by graphing and move toward algebraic reasoning and solving by substitution. Like equations in one variable, students determine the number of possible solutions to a system.



Math That Matters Most

Concept or Skill	For example . . .
Solve systems of linear equations graphically and algebraically.	Solve the system of equations: $y = -2x + 1$ $y = 3x - 9$ $-2x + 1 = 3x - 9$ $\quad$ $y = -2(2) + 1$ $10 = 5x$ $\quad$ $y = -4 + 1$ $2 = x$ $\quad$ $y = -3$ (2, -3) is the solution 
Interpret a point of intersection in context.	The graph shows the height of two different flags over 6 seconds. What does the point of intersection represent in this situation? Both flags were at the same height: just over 18 feet, just after 3 seconds. 
Determine the number of solutions to a system of equations.	How many solutions does this system have? $y = \frac{1}{4}x + 2$ $y = 2x - 5$ Show or explain your thinking. This system will have one solution, (4, 3). $\frac{1}{4}x + 2 = 2x - 5$ $\quad$ $y = 2(4) - 5$ $7 = 1.75x$ $\quad$ $y = 8 - 5$ $x = 4$ $\quad$ $y = 3$


Unit 5
Lesson
11
Name _____ Date _____ Period _____
MA.8.AR.4.1, MTR.2.1, MTR.4.1
On or Off the Line?
Let's interpret the meaning of points on and off lines.
Warm-Up
1. Which one doesn't belong? Explain your thinking.
A.
B.
C.
D.
Accelerated 7 | Unit 5 Lesson 11
419
Warm-Up

On or Off the Line?

Interpreting Points On or Off the Line

Let's interpret the meaning of points on and off lines.

Learning Outcomes

Today's Goals

1. **Goal:** Determine a point that satisfies two linear relationships simultaneously using tables or graphs.
2. **Language Goal:** Interpret points on the graph of two simultaneous linear equations in context. **(Reading, Writing, Speaking, and Listening)**

Students consider pairs of linear equations in various contexts and interpret the meaning of points on and off the lines, including points on one, both, or neither lines. They investigate the mathematical structure of two simultaneous linear equations by using familiar representations and to develop the need for new solving strategies. **(MTR.5.1)**

Note: Students do not need to use the vocabulary term system of equations to be successful with this lesson. Students will be introduced to this term in Lesson 13.

Coherence

Prior Learning

In Unit 4, Lesson 8, students represented and determined solutions for a real-world situation involving coins. In Lesson 9, students applied their knowledge of solving equations with variables on both sides in context.

Future Learning

In Lesson 12, students will create a graph that represents two linear relationships in context and interpret the point of intersection.

Connections

- Students use linear relationships to determine possible combinations of solutions that meet speciifc monetary constraints. (real world)

FL B.E.S.T. Math Benchmarks

Addressing

MA.8.AR.4.1
Given a system of two linear equations and a specified set of possible solutions, determine which ordered pairs satisfy the system of linear equations.

Mathematical Thinking and Reasoning Standards: MTR.2.1, MTR.4.1

ELA Expectations: ELA.K12.EE.1.1, ELA.K12.EE.3.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.7.AR.4.2 **MA.7.AR.4.3**	**MA.912.AR.9.1**

Lesson at a Glance

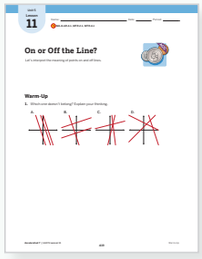
~ 45 min

MA.8.AR.4.1

Warm-Up

Pairs | 5 min

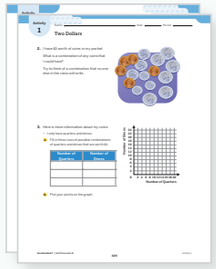
Students use the **Which One Doesn't Belong?** routine to describe different characteristics that arise when more than one line is graphed on a coordinate plane.



Activity 1

Pairs | 15 min

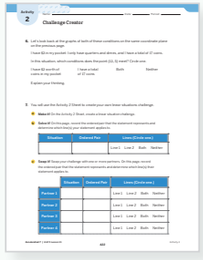
Students interpret the meaning of simultaneous linear equations by generating tables and graphs of linear equations in the context of coins. **(MTR.2.1)**



Activity 2

Independent | 15 min

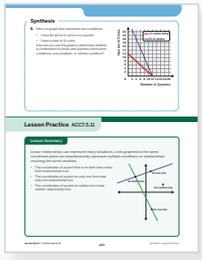
Students analyze two simultaneous lines in a real-world context to determine whether a point lies on one line, both lines, or neither line.
Routine: **Decide and Defend** **(MTR.4.1)**



Synthesis

Whole Class | 5 min

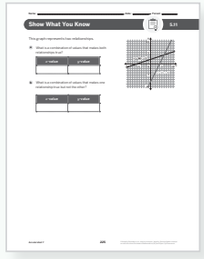
Students synthesize their understanding of how a graph can be a helpful tool in determining whether a point meets the conditions of two linear relationships. **MLR2: Collect and Display**



Show What You Know

Independent | 5 min

Students demonstrate their understanding by determining a point that makes one and both linear relationships true.



Prep Checklist

Prepare Activity 2 Sheets.

This lesson includes:

- Student Edition
- Show What You Know
- Activity 2 Sheet, one per pair
  - Calculators (as needed)
  - Coins or coin manipulatives (as needed)

Pairs | 5 min

Warm-Up

Purpose: Students use the Which One Doesn't Belong? routine to describe different characteristics that arise when more than one line is graphed on a coordinate plane.

1 Launch

Use the Which One Doesn't Belong? routine to support students in noticing and precisely describing properties of lines graphed together on a coordinate plane.

Consider sharing that each graph does not belong in some way. Encourage students to look for more than one possibility.

A **Accessibility: Executive Functioning** To support initiating complex tasks, consider identifying one graph and asking, "How is this one different from the rest?"

ELL **English Language Learners** Invite students to respond in their primary languages. Encourage all students to use gestures and/or visuals as they share. **(Writing, Speaking, and Listening)**

2 Connect

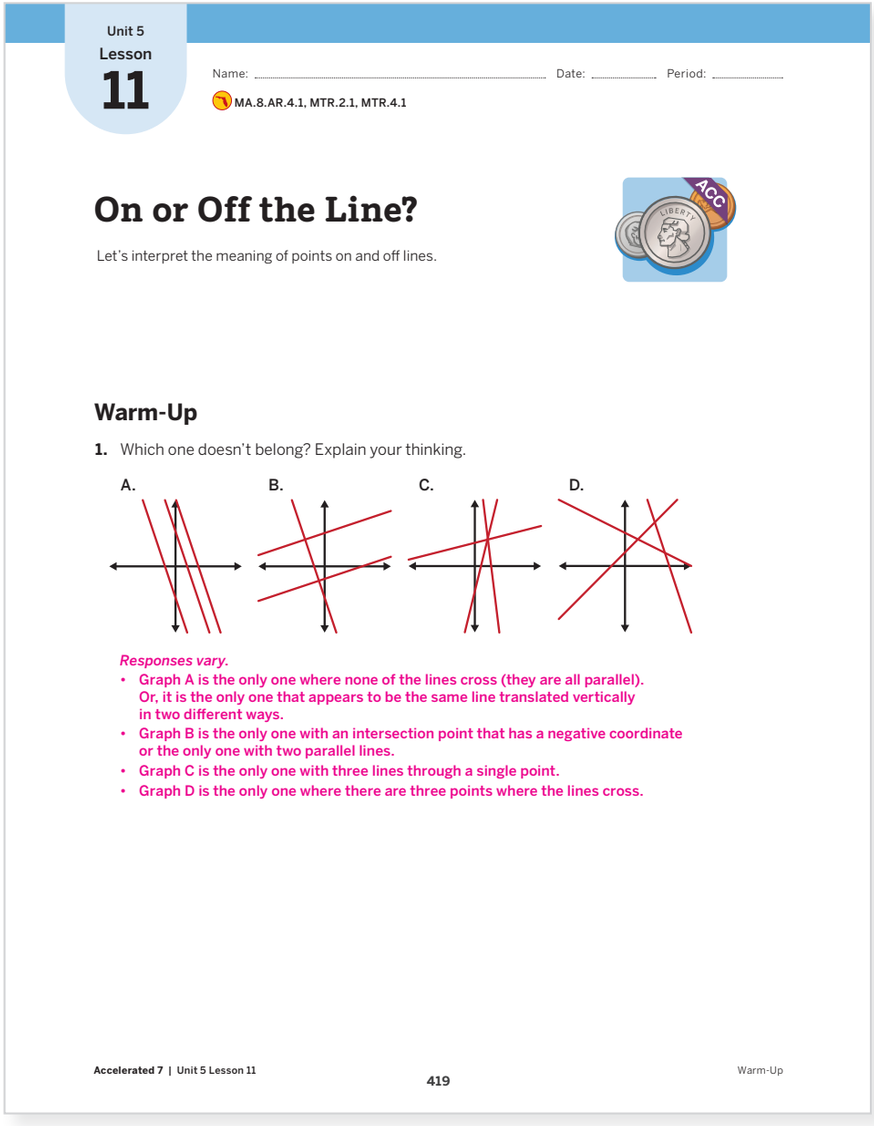
Invite students to share at least one reason why each graph doesn't belong.

Consider asking:

- "How is graph ___ like the other graphs? How is it different?"
- "What is another way to say why this graph doesn't belong?"
- "Is there another reason why this graph doesn't belong?"

Encourage students to use concepts and language introduced in previous lessons about lines such as *slope* and *intercept*.

Note: Students will be introduced to the term *point of intersection* in the next lesson. Some students may use the term naturally as they discuss what they see in this problem.



Unit 5
Lesson
11

Name: _____ Date: _____ Period: _____

MA.8.AR.4.1, MTR.2.1, MTR.4.1

On or Off the Line?

Let's interpret the meaning of points on and off lines.

Warm-Up

1. Which one doesn't belong? Explain your thinking.

A. B. C. D.

Responses vary.

- Graph A is the only one where none of the lines cross (they are all parallel). Or, it is the only one that appears to be the same line translated vertically in two different ways.
- Graph B is the only one with an intersection point that has a negative coordinate or the only one with two parallel lines.
- Graph C is the only one with three lines through a single point.
- Graph D is the only one where there are three points where the lines cross.

Accelerated 7 | Unit 5 Lesson 11 419 Warm-Up

Pairs | 15 min

Activity 1 Two Dollars

Purpose: Students interpret the meaning of simultaneous linear equations by generating tables and graphs of linear equations in the context of coins. **(MTR.2.1)**

1 Launch

To encourage participation, consider putting 15 dimes and 2 quarters in your pocket and ask students, "I have $2 in my pocket. What coins might I have in my pocket?"

Consider sharing that, in this lesson, students will be exploring different combinations of coins that equal $2.

A **Accessibility: Memory and Attention** To activate prior knowledge, consider providing coins or coin manipulatives and invite students to use them to create as many combinations of coins as they can to make $2.

ELL **English Language Learners** Bring in or show photos of a variety of U.S. coins (e.g., pennies, dimes, nickels, and quarters) and dollar bills. Label each coin and dollar with their name and value, including that one U.S. dollar is worth $1 and 100 cents. **(Reading and Listening)**

2 Monitor

To support students in getting started in Problem 3, consider asking, "How can we write the value of a quarter as a decimal? A dime?"

Invite students to respond to the prompt by considering different combinations.

Invite students to check their points with a partner.

A **Accessibility: Memory and Attention** Invite students to use a calculator to support them with performing calculations.

Look for a variety of combinations students make on this problem.

Activity 1 continued >

Activity 1

Name: Date: Period:

Two Dollars

2. I have $2 worth of coins in my pocket.

What is a combination of any coins that I could have?

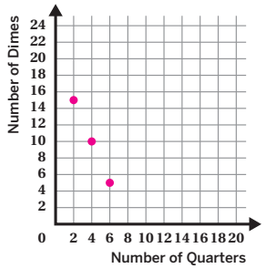
Try to think of a combination that no one else in the class will write.

Responses vary.
- 8 quarters
- 2 dimes, 1 nickel, and 7 quarters
- 200 pennies

3. Here is more information about my coins:
- I only have quarters and dimes.

a. Fill in three rows of possible combinations of quarters and dimes that are worth $2.
Responses vary.

Number of Quarters	Number of Dimes
2	15
4	10
6	5



b. Plot your points on the graph.
Responses vary.

Accelerated 7 | Unit 5 Lesson 11 420 Activity 1

Pairs | 15 min

Activity 1 Two Dollars (continued)

Purpose: Students interpret the meaning of simultaneous linear equations by generating tables and graphs of linear equations in the context of coins. **(MTR.2.1)**

2 Monitor

Look for a variety of combinations students write in this problem.

Note: Students can think of combinations on Problems 3 and 4 as if the two situations are independent (i.e., the combination of 17 coins does not also have to equal $2 in Problem 4).

Listen for students who notice that the different combinations make a linear pattern on the graph. Consider asking, "What do you notice about the pattern these combinations make on the graph? Why do you think that makes sense?"

Direct students' attention to the graph of both conditions on the same coordinate plane.

ELL **English Language Learners** Provide sentence frames to support students as they share their responses and defend them. For example, "There are ___ quarters and ___ dimes because . . . ". **(Reading, and Speaking)**

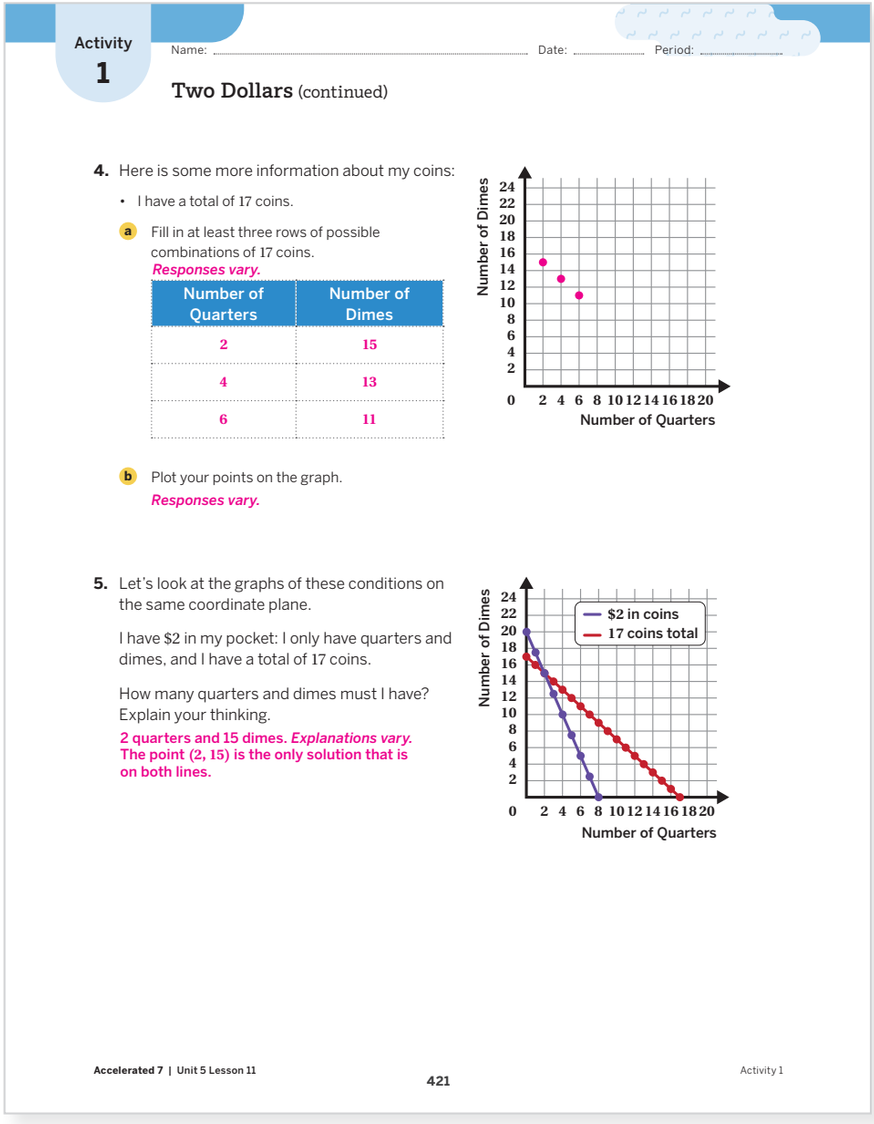
Listen for a variety of student explanations to share during the Connect, including some that make a connection between the solution and the lines on the graph.

3 Connect

Share and discuss the captured student explanations.

To surface the Key Takeaway, consider asking:

- "How can you tell from the graph that (2, 15) is a point that is on both lines? What does that mean about the point (2, 15) and the combination of quarters and dimes?" *Responses vary.* (2, 15) is the only point that has 17 coins that equal $2.
- "Are there any possible combinations besides 2 quarters and 15 dimes that will meet both conditions? How do you know?"
- "What can this graph tell us about the point (10, 2) in this situation?" **(MTR.4.1)**



Activity 1

Name: Date: Period:

Two Dollars (continued)

4. Here is some more information about my coins:

- I have a total of 17 coins.

a. Fill in at least three rows of possible combinations of 17 coins.
Responses vary.

Number of Quarters	Number of Dimes
2	15
4	13
6	11

b. Plot your points on the graph.
Responses vary.

5. Let's look at the graphs of these conditions on the same coordinate plane.

I have $2 in my pocket: I only have quarters and dimes, and I have a total of 17 coins.

How many quarters and dimes must I have? Explain your thinking.

2 quarters and 15 dimes. *Explanations vary.* The point (2, 15) is the only solution that is on both lines.

Accelerated 7 | Unit 5 Lesson 11 421 Activity 1

Key Takeaway: If a point is on two lines, it satisfies the conditions for both lines.

Independent | 15 min

Activity 2 Challenge Creator

Purpose: Students analyze two simultaneous lines in a real-world context to determine whether a point lies on one line, both lines, or neither line. **(MTR.4.1)**

Short on time? Consider inviting students to complete fewer of their classmates' challenges.

1 Launch

Encourage students to use the graph to determine their response.

Use the Decide and Defend routine to support students in making arguments and critiquing the reasoning of others. Invite students who made different choices to use reasoning to convince their classmates. **(MTR.4.1)**

Consider sharing that students will create their own challenge similar to the previous problem with a new situation. Demonstrate how students can create their challenge.

Encourage students to work with one or more partners for this activity. Distribute the Activity 2 Sheet so they can create their challenge.

To encourage creativity, invite students to make a challenge they think will be different from ones their classmates create.

2 Monitor

Listen for strategies students use in discussing how to solve the problems, such as plotting the point on the graph to determine which line, if any, it is on.

Look for several creative challenges and solutions to share during the Connect.

A **Accessibility: Conceptual Processing** Invite students to share how each problem is different from the previous one to help apply concepts to new situations.

3 Connect

Consider asking:

- "How did you pick the challenges you wanted to solve?"
- "How can you tell if the point is on one line? If it is on both lines?"
- "What strategies did you use to determine which line the statement applied to?"

Activity 2

Name: _____ Date: _____ Period: _____

Challenge Creator

6. Let's look back at the graphs of both of these conditions on the same coordinate plane on the previous page.

I have $2 in my pocket: I only have quarters and dimes, and I have a total of 17 coins.

In this situation, which conditions does the point (12, 5) meet? Circle one.

I have $2 worth of coins in my pocket | I have a total of 17 coins | Both | Neither

Explain your thinking.

Explanations vary. The point (12, 5) is only on the line representing 17 coins combined.

7. You will use the Activity 2 Sheet to create your own linear situations challenge.

a **Make it!** On the Activity 2 Sheet, create a linear situation challenge.

b **Solve it!** On this page, record the ordered pair that the statement represents and determine which line(s) your statement applies to. *Responses vary.*

Situation	Ordered Pair	Lines (Circle one.)
		Line 1 Line 2 Both Neither

c **Swap it!** Swap your challenge with one or more partners. On this page, record the ordered pair that the statement represents and determine which line(s) their statement applies to. *Responses vary.*

	Situation	Ordered Pair	Lines (Circle one.)
Partner 1			Line 1 Line 2 Both Neither
Partner 2			Line 1 Line 2 Both Neither
Partner 3			Line 1 Line 2 Both Neither
Partner 4			Line 1 Line 2 Both Neither

Accelerated 7 | Unit 5 Lesson 11 422 Activity 2

Whole Class | 5 min

Synthesis

Purpose: Students synthesize their understanding of how a graph can be a helpful tool in determining whether a point meets the conditions of two linear relationships.

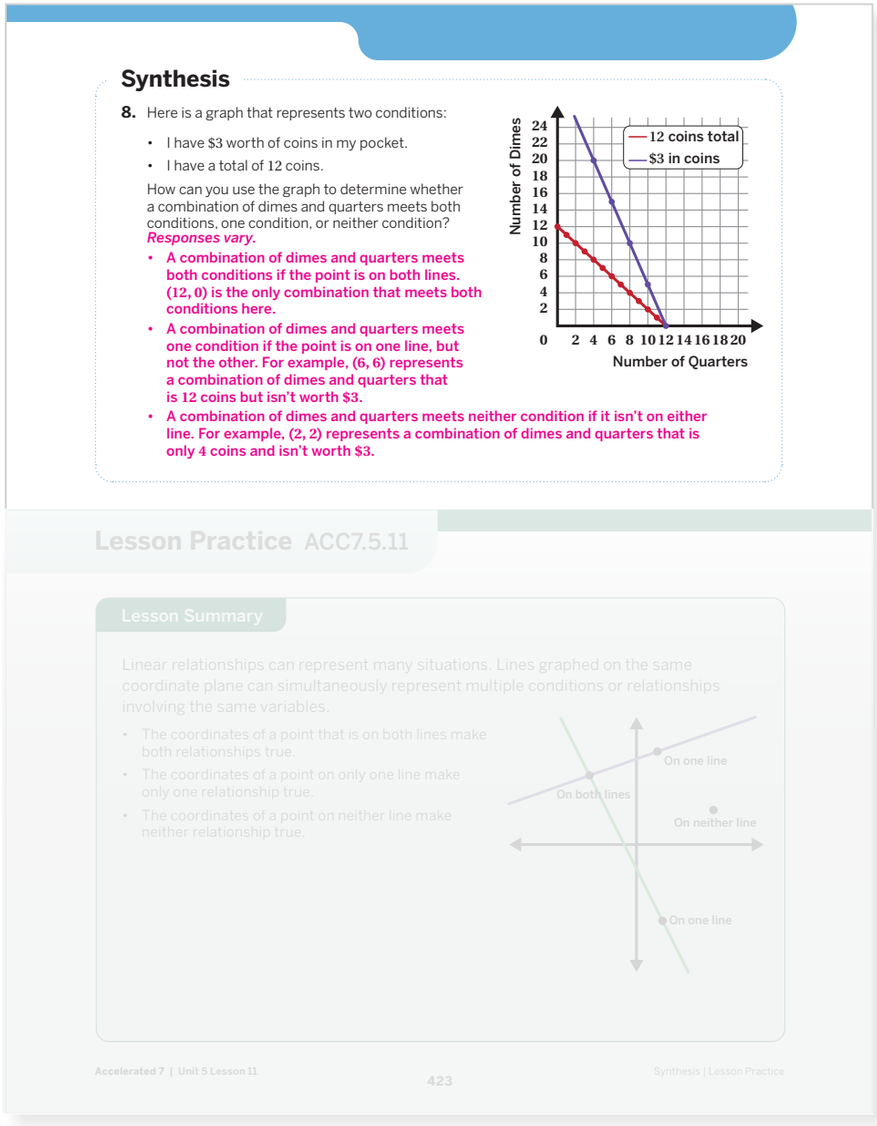
Invite students to respond independently, and then share their thinking with a partner.

Collect and share a variety of ideas, such as:

- Graphing the point to determine if it is on one line, both lines, or neither line.
- Determine if a point satisfies both conditions.
- Think of combinations for one condition, then use those combinations to find one that works for the second condition.

MLR **MLR2: Collect and Display** Use this routine to formalize the strategy for using a graph to understand whether points meet the conditions of linear relationships using students' language. Display any new language, examples, or diagrams for students to reference later.

Note: If time allows, invite students to revise their previous thinking based on the discussion.



Synthesis

8. Here is a graph that represents two conditions:

- I have \$3 worth of coins in my pocket.
- I have a total of 12 coins.

How can you use the graph to determine whether a combination of dimes and quarters meets both conditions, one condition, or neither condition?
Responses vary.

- A combination of dimes and quarters meets both conditions if the point is on both lines. (12, 0) is the only combination that meets both conditions here.
- A combination of dimes and quarters meets one condition if the point is on one line, but not the other. For example, (6, 6) represents a combination of dimes and quarters that is 12 coins but isn't worth \$3.
- A combination of dimes and quarters meets neither condition if it isn't on either line. For example, (2, 2) represents a combination of dimes and quarters that is only 4 coins and isn't worth \$3.

Lesson Practice ACC7.5.11

Lesson Summary

Linear relationships can represent many situations. Lines graphed on the same coordinate plane can simultaneously represent multiple conditions or relationships involving the same variables.

- The coordinates of a point that is on both lines make both relationships true.
- The coordinates of a point on only one line make only one relationship true.
- The coordinates of a point on neither line make neither relationship true.

Accelerated 7 | Unit 5 Lesson 11 423 Synthesis | Lesson Practice

Lesson Takeaway: You can tell if a point satisfies conditions of a linear relationship by examining the graph of that relationship. If a point is on one line, it satisfies the conditions of that relationship. If the point is on two lines, it satisfies the conditions of both relationships. If the point is not on either line, it does not satisfy the conditions of either relationship.

Independent | 5 min

Show What You Know

Purpose: Students demonstrate their understanding by determining a point that makes one and both linear relationships true.

Today's Goals

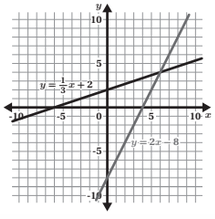
1. **Goal:** Determine a point that satisfies two linear relationships simultaneously using tables or graphs.
2. **Language Goal:** Interpret points on the graph of two simultaneous linear equations in context. **(Reading, Writing, Speaking, and Listening)**

Show What You Know (Assessment Resources)

Name: _____ Date: _____ Period: _____

Show What You Know 5.11

This graph represents two relationships.



a. What is a combination of values that makes both relationships true?

x-value	y-value
6	4

b. What is a combination of values that makes one relationship true but not the other?
Responses vary.

x-value	y-value
0	-8

D Differentiation Use after Lesson 11

Support

Provide targeted intervention for students by using these resources.

If students need support with interpreting graphs of two simultaneous linear equations:

» **Respond to Student Thinking:**

- Use the ***Solving Systems of Linear Equations by Graphing*** **Mini-Lesson**.
- Revisit **Activity 2** and invite students to share strategies for interpreting graphs of two simultaneous linear equations.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 3 Extension Activities**.
- Invite students to write equations for each of the relationships in the lesson.

Professional Learning

Take a moment to reflect on how your students engaged in today's lesson. What strengths did your students exhibit today? How can you build from those strengths as students continue learning about systems of linear equations?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

8. Here is a graph that represents two conditions:

- I have $3 worth of coins in my pocket.
- I have a total of 12 coins.

How can you use the graph to determine whether a combination of dimes and quarters meets both conditions, one condition, or neither condition?

Responses vary.

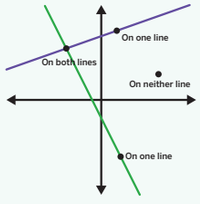
- A combination of dimes and quarters meets both conditions if the point is on both lines. (12, 0) is the only combination that meets both conditions here.
- A combination of dimes and quarters meets one condition if the point is on one line, but not the other. For example, (6, 6) represents a combination of dimes and quarters that is 12 coins but isn't worth $3.
- A combination of dimes and quarters meets neither condition if it isn't on either line. For example, (2, 2) represents a combination of dimes and quarters that is only 4 coins and isn't worth $3.

Lesson Practice ACC7.5.11

Lesson Summary

Linear relationships can represent many situations. Lines graphed on the same coordinate plane can simultaneously represent multiple conditions or relationships involving the same variables.

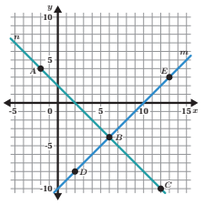
- The coordinates of a point that is on both lines make both relationships true.
- The coordinates of a point on only one line make only one relationship true.
- The coordinates of a point on neither line make neither relationship true.



Lesson Practice ACC7.5.11

Name: _____ Date: _____ Period: _____

Problems 1–5: Here is a coordinate plane.



1. Determine which line represents this condition: The coordinates of each point have a sum of 2.

Line n

2. Determine which line represents this condition: The y-coordinate of each point is 10 less than the x-coordinate.

Line m

3. Select *all* the points whose coordinates have a sum of 2.

- ☑ Point A
- ☑ Point B
- ☑ Point C
- ☐ Point D
- ☐ Point E

4. Select *all* the points whose y-coordinate is 10 less than the x-coordinate.

- ☐ Point A
- ☑ Point B
- ☐ Point C
- ☑ Point D
- ☑ Point E

5. Select *all* the points whose coordinates have a sum of 2 and the y-coordinate is 10 less than the x-coordinate.

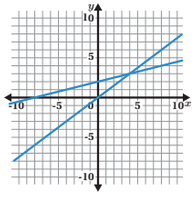
- ☐ Point A
- ☑ Point B
- ☐ Point C
- ☐ Point D
- ☐ Point E

Lesson Practice ACC7.5.11

Name: _____ Date: _____ Period: _____

FAST Practice

6. These two lines represent a system of equations. What is the y-coordinate of the point that makes both equations true?



A. 0
B. $\frac{1}{5}$
C. 3
D. 4

Spiral Review

Problems 7–9: Consider the equation $4x - 4 = 4x +$ _____. What value or expression could you write in the blank so that the equation is true for:

7. No values of x?

Responses vary. 7

8. All values of x?

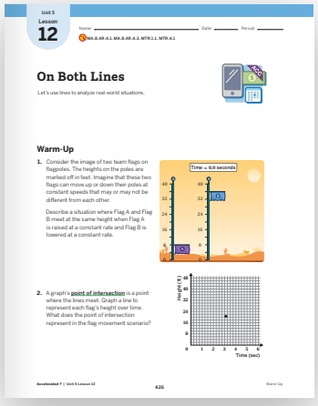
-4

9. One value of x?

Responses vary. 2x

Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T Math Benchmarks
On-lesson			
	1–5	2	MA.8.AR.4.1
FAST Practice	6	1	MA.8.AR.4.1
Spiral Review			
	7–9	2	MA.8.AR.2.1


Unit 5
Lesson
12
On Both Lines
Let's use lines to analyze real-world situations.
Warm-Up
1. Consider the image of two team flags on flagpoles. The heights on the poles are marked off in feet. Imagine that these two flags can move up or down their poles at constant speeds that may or may not be different from each other.
Describe a situation where Flag A and Flag B meet at the same height when Flag A is raised at a constant rate and Flag B is lowered at a constant rate.
Time = 6.5 seconds
2. A graph's point of intersection is a point where the lines meet. Graph a line to represent each flag's height over time. What does the point of intersection represent in the flag movement scenario?
Height (ft)
Time (sec)
426

On Both Lines

Representing Systems of Linear Equations

Let's use lines to analyze real-world situations.

Learning Outcomes

Today's Goals

1. **Goal:** Create a graph that represents two linear relationships in context and determine the *point of intersection*.
2. **Language Goal:** Interpret the *point of intersection* of two linear relationships in context given their graphs or equations. **(Writing, Speaking, and Listening)**

Students analyze two linear relationships and graphs of lines in context. They compare different representations of phone plans and interpret the point of intersection on a graph to draw a conclusion about which plan to choose. **(MTR.5.1)**

Coherence

Prior Learning

In Unit 4, students represented linear relationships with equations, tables, and graphs. In Lesson 11, students studied graphs of simultaneous equations and considered the meaning of points on and off the lines in context.

Future Learning

In Lesson 13, students will be introduced to the term *system of equations* and solve systems graphically. In Lesson 14, students will begin learning strategies for solving systems of linear equations algebraically.

Connections

- Students use systems of equations to compare the costs of different cell phone plans. (real world)

FL B.E.S.T. Math Benchmarks

Addressing

MA.8.AR.4.1
Given a system of two linear equations and a specified set of possible solutions, determine which ordered pairs satisfy the system of linear equations.

MA.8.AR.4.3
Given a mathematical or real-world context, solve systems of two linear equations by graphing.

Mathematical Thinking and Reasoning Standards: MTR.1.1, MTR.4.1

ELA Expectations: ELA.K12.EE.1.1, ELA.K12.EE.3.1, ELA.K12.EE.5.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.7.AR.4.2 **MA.7.AR.4.3**	**MA.912.AR.9.1**

Vocabulary

New Vocabulary

point of intersection

Lesson at a Glance

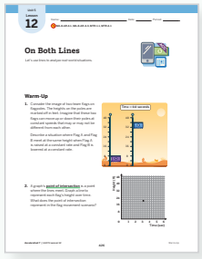
~ 45 min

MA.8.AR.4.1, MA.8.AR.4.3

Warm-Up

Pairs | 5 min

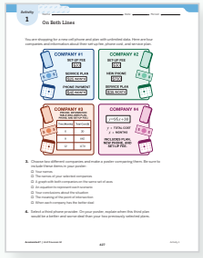
Students use the **Tell a Story** routine to make sense of two linear graphs on one set of axes in the context of flags. **(MTR.1.1)**



Activity 1

Small Groups | 20 min

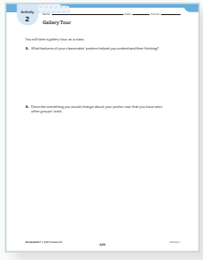
Students compare two simultaneous linear equations and interpret the **point of intersection** by deciding between two cell phone plans. **(MTR.1.1)**



Activity 2

Whole Class | 10 min

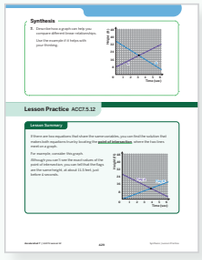
Students participate in a **Gallery Tour** to consider how graphs of simultaneous equations can clearly communicate their meaning. **MLR1: Stronger and Clearer Each Time** **(MTR.4.1)**



Synthesis

Whole Class | 5 min

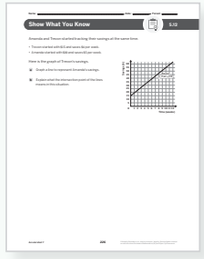
Students synthesize their understanding of how a graph can help compare different linear relationships. **MLR2: Collect and Display**



Show What You Know

Independent | 5 min

Students demonstrate their understanding by graphing and interpreting two linear relationships.



Prep Checklist

This lesson includes:

- Student Edition
- Show What You Know
- Activity 1 Sheet (answers)
- Coloring tools
- Graph paper
- Poster paper
- Straightedges

Pairs | 5 min

Warm-Up

Purpose: Students use the Tell a Story routine to make sense of two linear graphs on one set of axes in the context of flags. **(MTR.1.1)**

1 Launch

Direct students' attention to the image of two flags.

Use the Tell a Story routine to promote creativity and help students make sense of two linear relationships graphed simultaneously. **(MTR.1.1)**

Invite students to think independently before discussing their ideas with a partner.

2 Connect

Invite several students to share their stories.

Pause to give students time to complete their graphs.

Note: If it doesn't come up naturally, consider asking, "Why do you think the scale on the x-axis is from 0 to 6 and the scale on the y-axis is from 0 to 48"? *Responses vary.* The x-axis is measuring time in seconds, and all the flag movement happens within 6 seconds, so the graph only shows up to 6 on the x-axis. The y-axis is measuring the height of the flags in feet, with the top of the Flag A's height beginning at 8 feet and the top of Flag B's height beginning at 36 feet.

Consider asking, "When are the flags at the same height? Can you tell precisely from your graph? Why or why not?"

ELL **English Language Learners** Invite students to share what a new term reminds them of (e.g., "When I hear *point of intersection*, I think . . ."). This may help surface multiple meanings and encourage students to connect to their prior knowledge. **(Reading and Listening)**

To surface the Key Takeaway, create a class definition of the term **point of intersection** using the glossary and students' explanations of the meaning in context. Record it for students to refer to later in the unit.

Unit 5
Lesson
12

Name: _____ Date: _____ Period: _____

MA.8.AR.4.1, MA.8.AR.4.3, MTR.1.1, MTR.4.1

On Both Lines

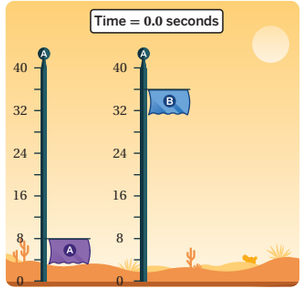
Let's use lines to analyze real-world situations.

Warm-Up

1. Consider the image of two team flags on flagpoles. The heights on the poles are marked off in feet. Imagine that these two flags can move up or down their poles at constant speeds that may or may not be different from each other.

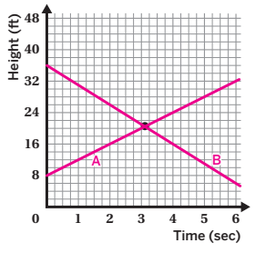
   Describe a situation where Flag A and Flag B meet at the same height when Flag A is raised at a constant rate and Flag B is lowered at a constant rate.

   Responses vary.
   - Flag A moves up the pole at one speed, and Flag B moves down the pole at another speed.
   - Flag A moves up the pole really fast, and Flag B moves very slowly down the pole.



2. A graph's point of intersection is a point where the lines meet. Graph a line to represent each flag's height over time. What does the point of intersection represent in the flag movement scenario?

   Responses vary. Just after 3 seconds, both flags were at the same height: just over 18 feet.



Accelerated 7 | Unit 5 Lesson 12 426 Warm-Up

Key Takeaway: The **point of intersection** is a point where two lines meet. It represents the x- and y-coordinates that make both equations true.

Small Groups | 20 min

Activity 1 On Both Lines

Purpose: Students compare two simultaneous linear equations and interpret the point of intersection by deciding between two cell phone plans. **(MTR.1.1)**

Short on time? Consider omitting Problem 4.

1 Launch

Direct students' attention to the instructions and phone company information.

To surface prior knowledge, consider asking, "What costs might there be when shopping for a new phone and phone plan?" *Responses vary.* The cost of a new phone, a phone plan, a set up fee, number of minutes used per month, etc.

Invite students to read about each company aloud as a class.

Distribute graph paper and tools for making a poster.

2 Monitor

A **Accessibility: Executive Functioning** Chunk this task into smaller, more manageable parts by having students choose one company and write an equation to represent it. Then have them do the same for a second company before creating a graph.

Look for a variety of strategies to compare the two companies. **(MTR.1.1)**

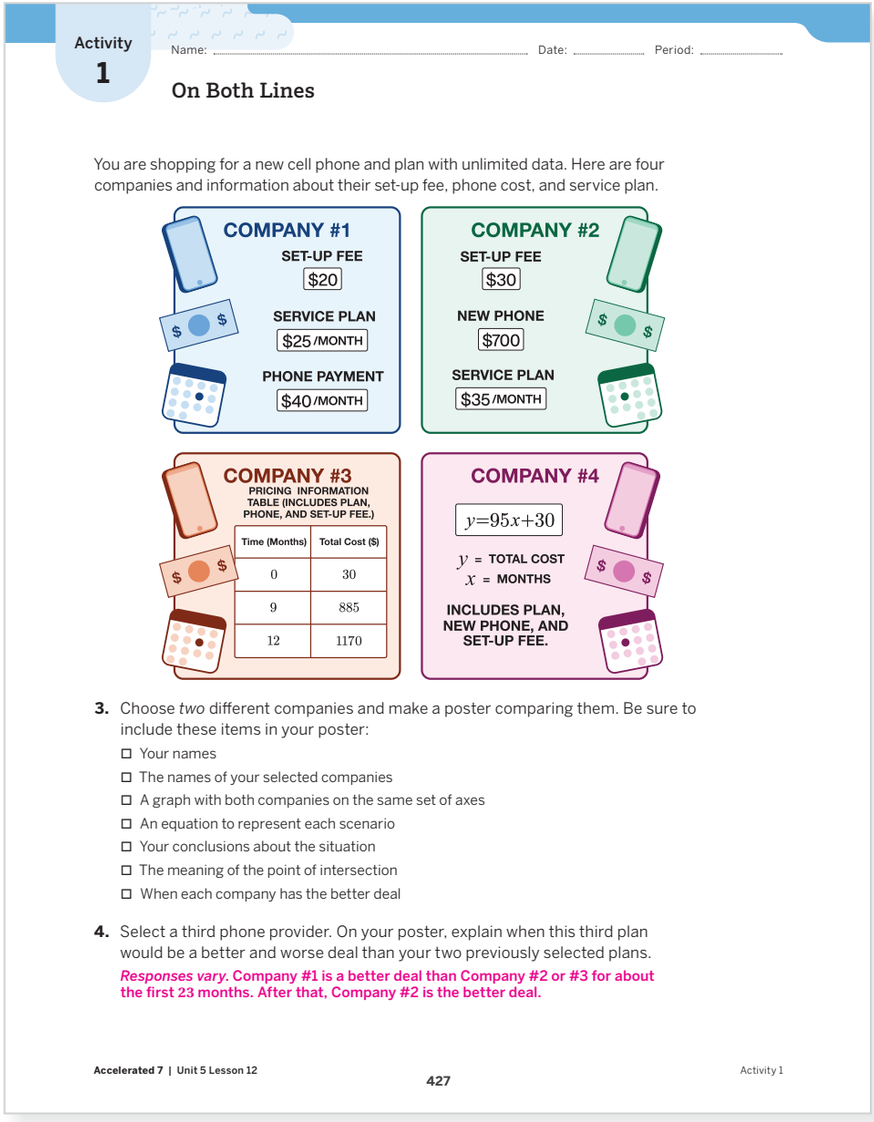
D **Differentiation** *(Problem 3)*

Look for students who:	Respond to Student Thinking
Need support determining axis labels and scales.	**Support** Consider asking, "How were the scales on the axes for the graph of the flags different? What do you think the range for your x- and y-axis variables will be?"
Notice that Company #3 and Company #4 have the same costs.	**Strengthen** Consider asking, "What will the graphs of these companies' costs look like? What will the points of intersection be?"

3 Connect

Invite students to share their conclusions about which company has the better deal.

To surface the Key Takeaway, consider asking, "What is the meaning of the term *point of intersection* in the context of the phone companies?"



Activity 1

Name: Date: Period:

On Both Lines

You are shopping for a new cell phone and plan with unlimited data. Here are four companies and information about their set-up fee, phone cost, and service plan.

COMPANY #1
SET-UP FEE $20
SERVICE PLAN $25/MONTH
PHONE PAYMENT $40/MONTH

COMPANY #2
SET-UP FEE $30
NEW PHONE $700
SERVICE PLAN $35/MONTH

COMPANY #3
PRICING INFORMATION TABLE (INCLUDES PLAN, PHONE, AND SET-UP FEE.)

Time (Months)	Total Cost ($)
0	30
9	885
12	1170

COMPANY #4
$y=95x+30$
y = TOTAL COST
x = MONTHS
INCLUDES PLAN, NEW PHONE, AND SET-UP FEE.

3. Choose *two* different companies and make a poster comparing them. Be sure to include these items in your poster:
   - ☐ Your names
   - ☐ The names of your selected companies
   - ☐ A graph with both companies on the same set of axes
   - ☐ An equation to represent each scenario
   - ☐ Your conclusions about the situation
   - ☐ The meaning of the point of intersection
   - ☐ When each company has the better deal

4. Select a third phone provider. On your poster, explain when this third plan would be a better and worse deal than your two previously selected plans.
   Responses vary. Company #1 is a better deal than Company #2 or #3 for about the first 23 months. After that, Company #2 is the better deal.

Accelerated 7 | Unit 5 Lesson 12 427 Activity 1

Key Takeaway: The point of intersection is one key feature that can be used to compare two graphs on the same axes. The point of intersection can tell you the x- and y-values that make both equations true. Graphing both equations on the same axes can show other information to help compare them, such as when one equation has a higher or lower y-value than the other.

Whole Class | 10 min

Activity 2 Gallery Tour

Purpose: Students participate in a Gallery Tour to consider how graphs of simultaneous equations can clearly communicate their meaning. **(MTR.4.1)**

Short on time? Consider inviting students to look at one or two other group's posters rather than doing a full Gallery Tour.

1 Launch

To generate excitement, invite each group to share something they are proud of about their poster.

Invite students to share their posters through a Gallery Tour. Encourage them to silently view their classmates' work and leave feedback. Then give each group time to review the feedback left on their work. **(MTR.4.1)**

Display a timer to support students in pacing their Gallery Tour so they have time to visit and leave feedback for each group.

A **Accessibility: Affective Functioning** To support students in sharing and receiving feedback, model giving and receiving constructive feedback as a class.

2 Monitor

Select and sequence multiple scales and ranges students used on the x- and y-axes of their graphs to share during the Connect.

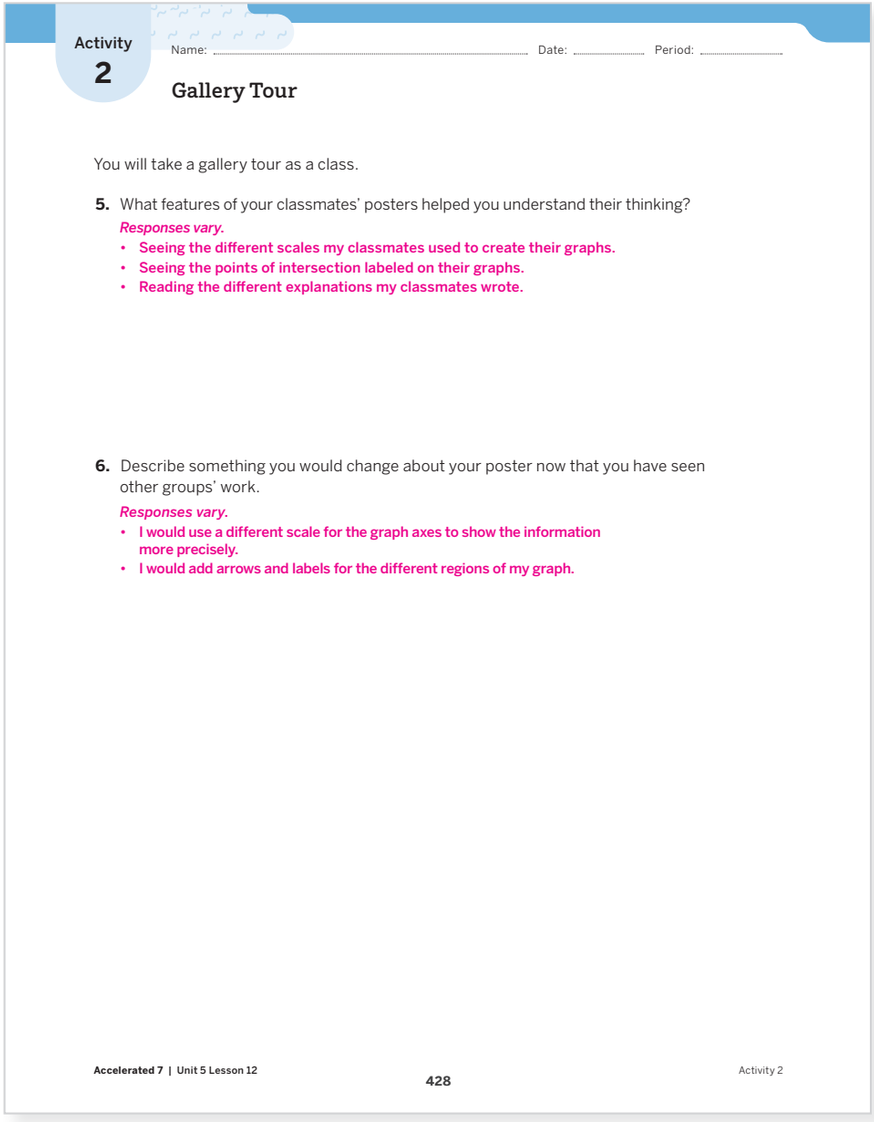
3 Connect

Display a few posters.

MLR **MLR1: Stronger and Clearer Each Time (optional)** Consider using this routine to support students in developing precise language. Invite students to use the feedback from the Gallery Tour to create a second clearer draft of their poster. See the Routine Facilitation Guide for more information.

To surface the Key Takeaway, consider asking:

- "What should you consider when designing a graph of linear equations?"
- "How can scales help us to be more precise in the way we compare two situations or graphs?"



Activity 2

Name: Date: Period:

Gallery Tour

You will take a gallery tour as a class.

5. What features of your classmates' posters helped you understand their thinking?

Responses vary.

- Seeing the different scales my classmates used to create their graphs.
- Seeing the points of intersection labeled on their graphs.
- Reading the different explanations my classmates wrote.

6. Describe something you would change about your poster now that you have seen other groups' work.

Responses vary.

- I would use a different scale for the graph axes to show the information more precisely.
- I would add arrows and labels for the different regions of my graph.

Accelerated 7 | Unit 5 Lesson 12 428 Activity 2

Key Takeaway: When designing a graph, it can be helpful to consider the range of the x- and y-values that make sense for the situation to determine what scales to use on the x- and y-axes. Choosing a precise scale can help to highlight key features, such as the point of intersection, and ensure the graph is clear.

Whole Class | 5 min

Synthesis

Purpose: Students synthesize their understanding of how a graph can help compare different linear relationships.

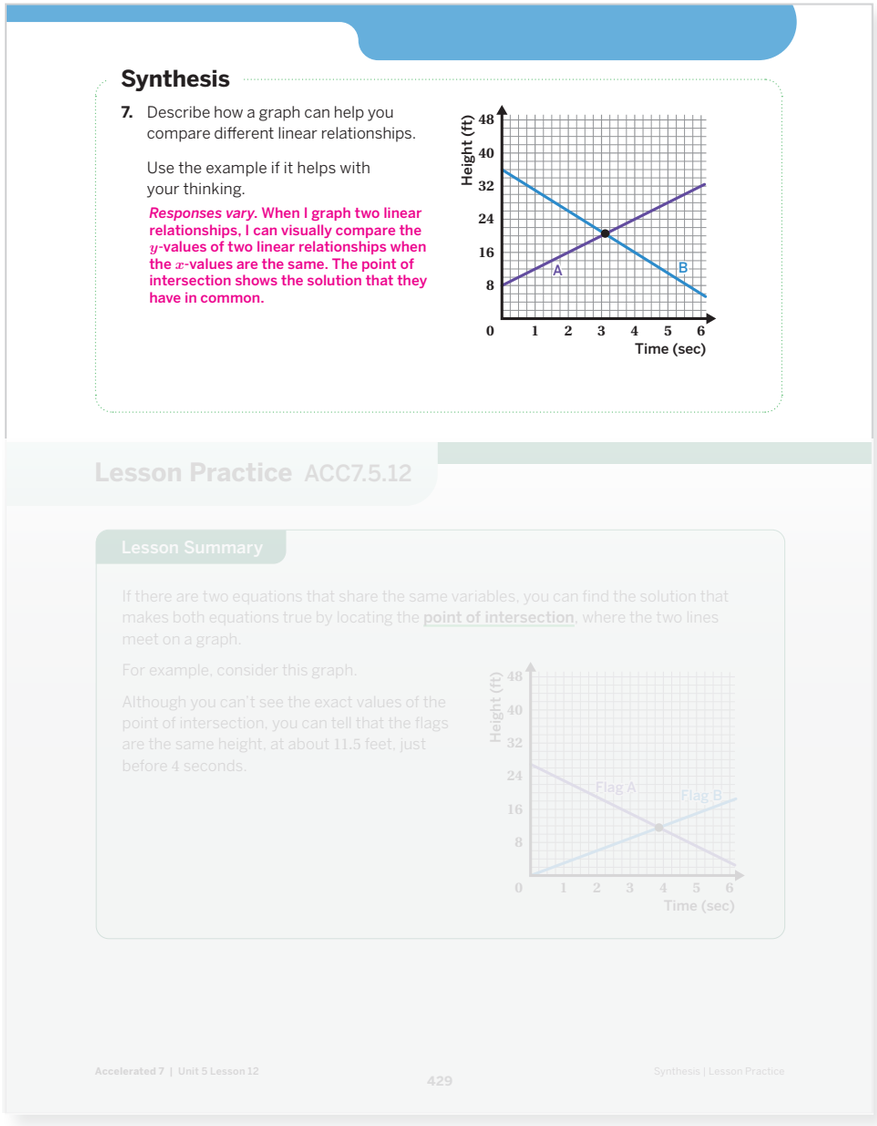
Invite students to respond independently, and then share their thinking with a partner.

Collect and share a variety of ideas, such as:

- The point of intersection on the graph is where the situations are the same.
- Using a scale that makes sense for the situation can help us compare the situations.

MLR **MLR2: Collect and Display** Use this routine to collect students' examples of how to use graphs to compare linear relationships as they share. Display for students to reference and add to for the rest of the unit.

Note: If time allows, invite students to revise their previous thinking based on the discussion.



Synthesis

7. Describe how a graph can help you compare different linear relationships.

Use the example if it helps with your thinking.

Responses vary. When I graph two linear relationships, I can visually compare the y-values of two linear relationships when the x-values are the same. The point of intersection shows the solution that they have in common.

Lesson Practice ACC7.5.12

Lesson Summary

If there are two equations that share the same variables, you can find the solution that makes both equations true by locating the **point of intersection**, where the two lines meet on a graph.

For example, consider this graph.

Although you can't see the exact values of the point of intersection, you can tell that the flags are the same height, at about 11.5 feet, just before 4 seconds.

Accelerated 7 | Unit 5 Lesson 12 429 Synthesis | Lesson Practice

Lesson Takeaway: A graph can be a useful tool for comparing two linear relationships. A graph visually helps compare the relationships at different x- and y-values. It can also show the **point of intersection**, or the point on the graph where the x- and y-values make both equations true, which might be approximate or precise.

Independent | 5 min

Show What You Know

Purpose: Students demonstrate their understanding by graphing and interpreting two linear relationships.

Today's Goals

1. **Goal:** Create a graph that represents two linear relationships in context and determine the *point of intersection*.
2. **Language Goal:** Interpret the *point of intersection* of two linear relationships in context given their graphs or equations. **(Writing, Speaking, and Listening)**

Show What You Know (Assessment Resources)

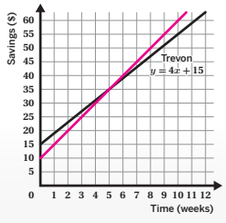
Name: _____ Date: _____ Period: _____

Show What You Know 5.12

Amanda and Trevon started tracking their savings at the same time.

- Trevon started with $15 and saves $4 per week.
- Amanda started with $10 and saves $5 per week.

Here is the graph of Trevon's savings.



a. Graph a line to represent Amanda's savings.

b. Explain what the intersection point of the lines means in this situation.

Responses vary. In this situation, the intersection at (5, 35) means that after 5 weeks, Trevon and Amanda each have $35.

D Differentiation Use after Lesson 12

Support

Provide targeted intervention for students by using these resources.

If students need support using a graph to determine the point of intersection between two linear relationships:

» **Respond to Student Thinking:**

- Use the ***Solving Systems of Linear Equations by Graphing*** **Mini-Lesson**.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 3 Extension Activities**.
- Invite students to research and create a similar problem in a different context, such as a gym membership or rental property, then use graphs to compare two options.

Professional Learning

Consider the next several lessons you will teach. Students will be introduced to the term *systems of equations* in the next lesson. How will students' work in this lesson prepare them for developing a conceptual understanding of systems of linear equations? What strategies might you revisit from this lesson?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

7. Describe how a graph can help you compare different linear relationships.

   Use the example if it helps with your thinking.

   Responses vary. When I graph two linear relationships, I can visually compare the y-values of two linear relationships when the x-values are the same. The point of intersection shows the solution that they have in common.

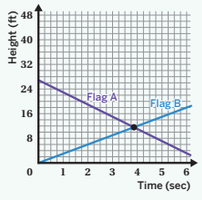
Lesson Practice ACC7.5.12

Lesson Summary

If there are two equations that share the same variables, you can find the solution that makes both equations true by locating the **point of intersection**, where the two lines meet on a graph.

For example, consider this graph.

Although you can't see the exact values of the point of intersection, you can tell that the flags are the same height, at about 11.5 feet, just before 4 seconds.

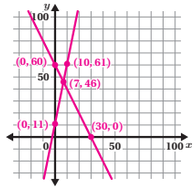


Lesson Practice ACC7.5.12

Name: _____ Date: _____ Period: _____

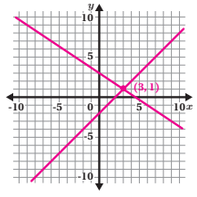
1. Jayden has $11 and begins saving $5 each week to buy a new phone. At the same time that Jayden begins saving, Aditi has $60 and begins spending $2 per week on supplies for her art class. Is there a week when they will have the same amount of money? Explain your thinking.

   Yes. *Explanations vary.* They will both have $46 after 7 weeks.



2. Draw a graph to determine x- and y-values that make both of the equations $y = -\frac{2}{3}x + 3$ and $y = x - 2$ true.

   (3, 1)



Problems 3–4: Prisha and Mia agree to go to the movies after they have earned the same amount of money for the same number of hours worked.

Mia earns $7 per hour mowing her neighbors' lawns. She also earned $14 one time for hauling away bags of recyclables.

Prisha babysits her neighbor's children. The table shows the amount of money, m, she earns in h hours.

h	m
1	$8.40
2	$16.80
3	$33.60

3. How many hours do they each have to work before they go to the movies?

   10 hours

4. How much will each of them have earned?

   $84

Lesson Practice ACC7.5.12

Name: _____ Date: _____ Period: _____

5. The graph of the equations $y = \square - x$ and $y = \square x - 3$ intersect at the point (2, 1). Determine the missing values in the equations. Show or explain your thinking.

   $y = 3 - x$ and $y = 2x - 3$. *Explanations vary.*

   I substituted the coordinates of the point (2, 1) into the first equation to determine the missing value, 3.

   I substituted the coordinates of the point (2, 1) into the second equation to determine the missing value, 2.

FAST Practice

6. The point where the graphs of two equations intersect has a y-coordinate of 2. One equation is $y = -3x + 5$.

   Determine the other equation if its graph has a slope of 1.

   Record your answer in the space provided.

   $y = x + 1$ (or equivalent)

Spiral Review

Problems 7–9: Determine whether each equation is true for one value, all values, or no values of x.

7. $10 + 3x = -4.2x + 9$

   One value

8. $5(4x + 1) - x = 19x + 5$

   All values

9. $-2(3x - 7) = -6x + 12$

   No values

Problems 10–11: Determine whether $x = -1$ is a solution to each equation.

10. $1.5x + 2 = 8.5x - 4$

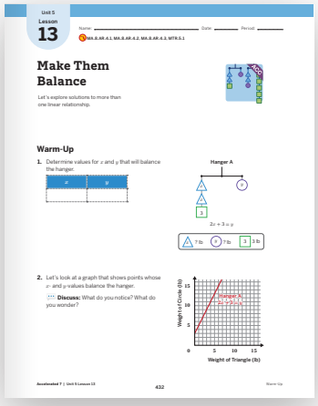
    No

11. $\frac{1}{2} + 2x = \frac{1}{2}x - 1$

    Yes

Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T Math Benchmarks
On-lesson			
	1–2	2	MA.8.AR.4.1
	3–4	2	MA.8.AR.4.1
	5	3	MA.8.AR.4.1
FAST Practice	6	3	MA.8.AR.4.1
Spiral Review			
	7–9	1	MA.8.AR.2.1
	10–11	2	MA.8.AR.2.1



Make Them Balance

Graphing Systems of Linear Equations

Let's explore solutions to more than one linear relationship.

Learning Outcomes

Today's Goals

1. **Goal:** Use hanger diagrams and graphs to determine the *solutions to systems of equations*.
2. **Language Goal:** Describe connections between the graph of a *system of equations* and its number of solutions. **(Reading, Writing, Speaking, and Listening)**

Students are formally introduced to the concept of a **system of equations** using the familiar context of balanced hangers. They explore what it means for a set of values to be a **solution to a system of equations** and begin to make sense of whether a system has no solution, one solution, or infinitely many solutions.

Coherence

Prior Learning

In Lessons 11–12, students studied graphs representing systems of linear equations and considered the meaning of the point of intersection of the lines in context. In Unit 4, students thought about ordered pairs as solutions to one linear equation.

Future Learning

In Lesson 14, students will explore systems where both equations are of the form $y = mx + b$ and determine solutions to systems of linear equations without a graph.

Connections

- Students build upon their understanding of balancing hangers to solutions of systems of two-variable equations and their graphs. (mathematical)

FL B.E.S.T. Math Benchmarks

Addressing

MA.8.AR.4.1
Given a system of two linear equations and a specified set of possible solutions, determine which ordered pairs satisfy the system of linear equations.

MA.8.AR.4.2
Given a system of two linear equations represented graphically on the same coordinate plane, determine whether there is one solution, no solution or infinitely many solutions.

Also Addressing: **MA.8.AR.4.3**

Mathematical Thinking and Reasoning Standard: MTR.5.1

ELA Expectations: ELA.K12.EE.3.1, ELA.K12.EE.6.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.7.AR.4.2	**MA.912.AR.9.1**
MA.7.AR.4.3	**MA.912.AR.9.4**
MA.7.AR.4.4	

Vocabulary

New Vocabulary

solution to a system of equations

system of equations

Lesson at a Glance

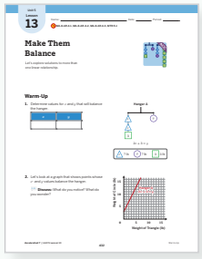
~ 45 min

MA.8.AR.4.1, MA.8.AR.4.2, MA.8.AR.4.3

Warm-Up

Pairs | 5 min

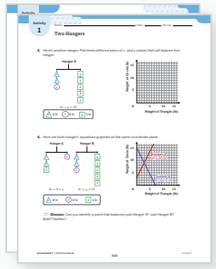
Students consider what it means for values to be a solution to an equation in two variables using a hanger and a graph.
Routine: Notice and Wonder



Activity 1

Pairs | 15 min

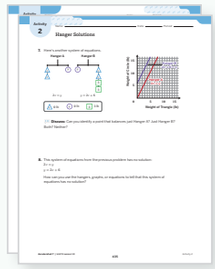
Students use hangers and graphs to represent **systems of equations** and discover strategies for determining the **solution to a system of equations**.



Activity 2

Pairs | 15 min

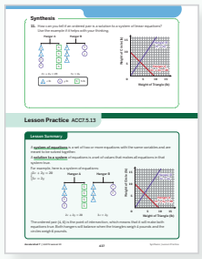
Students begin to make sense of graphs, hangers, and equations that represent systems of equations with no solution and infinitely many solutions. MLR1: Stronger and Clearer Each Time (MTR.5.1)



Synthesis

Whole Class | 5 min

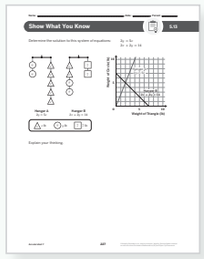
Students synthesize their understanding of deciding if an ordered pair is a solution to a system of equations. MLR2: Collect and Display



Show What You Know

Independent | 5 min

Students demonstrate their understanding by identifying the solution to a system of equations.



Prep Checklist

This lesson includes:

- Student Edition
- Show What You Know
- Calculators (as needed)

Pairs | 5 min

Warm-Up

Purpose: Students consider what it means for values to be a solution to an equation in two variables using a hanger and a graph.

1 Launch

To surface connections, consider asking, "How is this hanger like hangers we've seen before? How is it different?"

To support students getting started, consider presenting a pair of values that is not a solution and asking, "How can we tell this is not a solution using the hanger? The equation?"

A **Accessibility: Visual-Spatial Processing**
To support students in interpreting visual representations, invite pairs to describe in words or use concrete objects to model the hangers throughout this lesson.

2 Connect

Invite students to share several pairs of values. Some students may think in terms of the weights in the diagram, while others may look for sets of values that make the equation true.

Use the Notice and Wonder routine to help students make connections between the hanger, equation, and graph.

Create a record of the connections students noticed and wondered, along with the name of the students who shared them.

Note: If it doesn't come up naturally, consider asking, "Why do you think the points on the graph formed a line?"

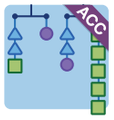
Unit 5
Lesson
13

Name: Date: Period:

MA.8.AR.4.1, MA.8.AR.4.2, MA.8.AR.4.3, MTR.5.1

Make Them Balance

Let's explore solutions to more than one linear relationship.

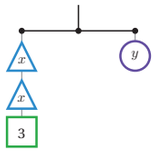


Warm-Up

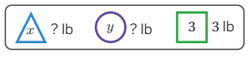
1. Determine values for x and y that will balance the hanger. *Responses vary.*

x	y
2	7

Hanger A



$2x + 3 = y$

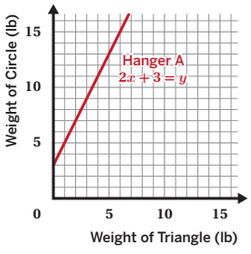


2. Let's look at a graph that shows points whose x- and y-values balance the hanger.

Discuss: What do you notice? What do you wonder?

Responses vary.

- I notice that the values I found for balancing the hanger match a point on the line.
- I wonder if any set of numbers that make the hanger balance will be on that line.



Accelerated 7 | Unit 5 Lesson 13 432 Warm-Up

Pairs | 15 min

Activity 1 Two Hangers

Purpose: Students use hangers and graphs to represent **systems of equations** and discover strategies for determining the **solution to a system of equations**.

Short on time? Consider omitting Problem 6 or completing it as a class during the Connect.

1 Launch

To surface connections, consider asking, "How is this hanger like the one in the Warm-Up? How is it different?"

2 Monitor

Look for students who identify points using the equation, the hanger, and the graph. This may be helpful in making connections to systems later in the Activity.

D **Differentiation: (Support)** Invite students who need support getting started to choose a point and use the equation or hanger to predict whether it will balance before testing, then use the result to revise their thinking.

Invite students to take turns sharing a point with a partner and inviting their partner to decide whether it balances one hanger, both hangers, or neither.

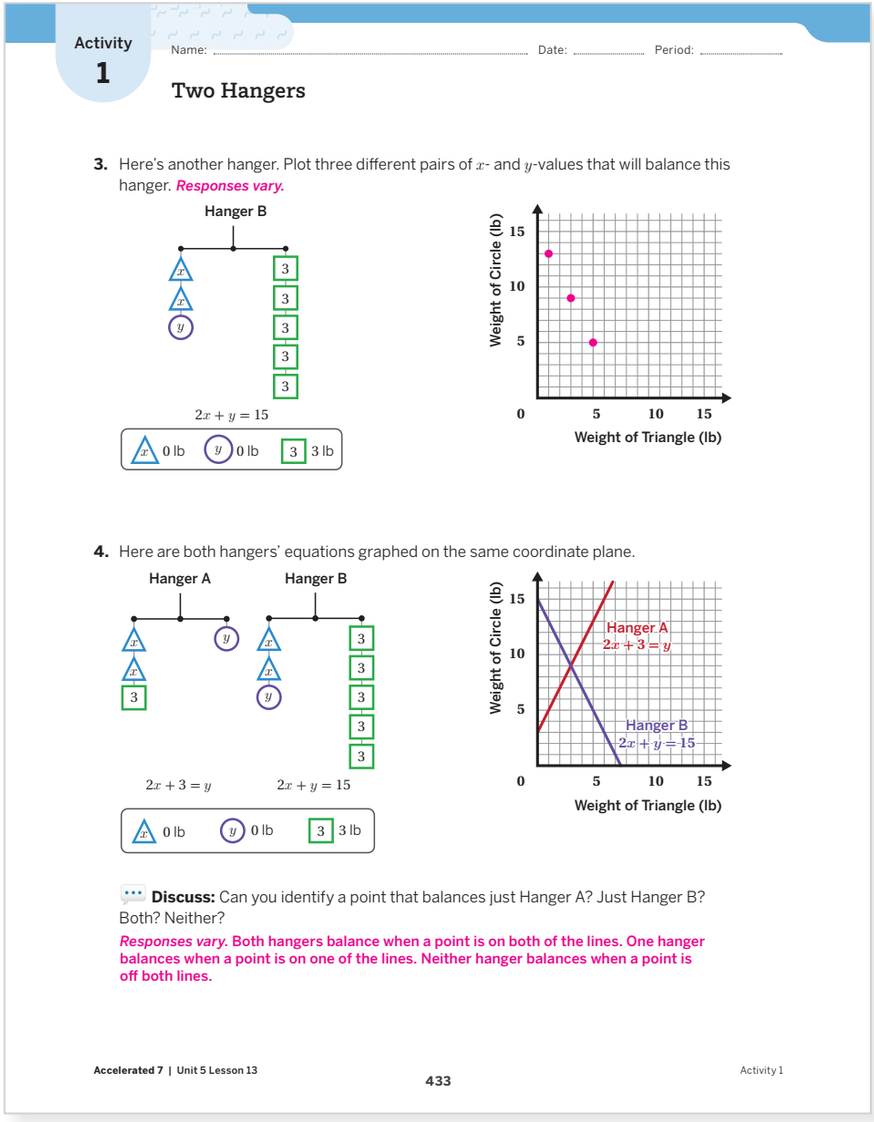
Encourage students to pause and reflect after each point to make connections between the location of the point and which hangers are balanced.

A **Accessibility: Memory and Attention** Provide students with a graphic organizer to support students with keeping track of the points they chose and which hangers were balanced.

Pause briefly to invite students to share points that they think balance one or more hangers and to justify their thinking. **(MTR.4.1)**

Consider asking, "How can you use the graph of each equation in Problem 4 to decide which hanger(s) will balance?"

Activity 1 continued >



Activity 1

Name: Date: Period:

Two Hangers

3. Here's another hanger. Plot three different pairs of x- and y-values that will balance this hanger. *Responses vary.*

Hanger B

$2x + y = 15$

x 0 lb y 0 lb 3 3 lb

4. Here are both hangers' equations graphed on the same coordinate plane.

Hanger A Hanger B

$2x + 3 = y$ $2x + y = 15$

x 0 lb y 0 lb 3 3 lb

Discuss: Can you identify a point that balances just Hanger A? Just Hanger B? Both? Neither?

Responses vary. **Both hangers balance when a point is on both of the lines. One hanger balances when a point is on one of the lines. Neither hanger balances when a point is off both lines.**

Accelerated 7 | Unit 5 Lesson 13 433 Activity 1

Pairs | 15 min

Activity 1 Two Hangers (continued)

Purpose: Students use hangers and graphs to represent **systems of equations** and discover strategies for determining the **solution to a system of equations.**

Short on time? Consider omitting Problem 6 or completing it as a class during the Connect.

2 Monitor

Invite students to read the definition of **solution to a system of equations** and rephrase it in their own words.

Create a class definition of a **system of equations** using students' language and the glossary. Record it for students to refer to later in the unit.

ELL **English Language Learners** Invite students to share what these new terms remind them of (e.g., "When I hear *system*, I think . . ."). This may help surface multiple meanings and encourage students to connect to their prior knowledge. **(Reading and Listening)**

Select and sequence explanations to share during the Connect.

D **Differentiation** *(Problem 6)*

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Encourage them to choose one point and test it in the equations or graphs.
Swap the x- and y-values.	**Support** Invite them to check their values in both equations and see what they notice.
Choose a solution to one but not both equations	
Choose $(4, 6)$ using either the graph, equations, or hangers.	**Strengthen** Consider asking, "How could including a different representation strengthen your argument?"
	Stretch Invite students to create a different system of equations that has the same solution.

3 Connect

Invite students to share their explanations and reasoning.

To surface the Key Takeaway, consider asking, "How can you use the graphs to tell that $(4, 6)$ is the solution to this system of equations? The hangers? The equations?"

Note: If it does not come up naturally, consider demonstrating how to check the x-and y-values in both equations to confirm that the point is a solution to the system of equations.

Activity 1

Name: Date: Period:

Two Hangers (continued)

5. You have been experimenting with two representations of this **system of equations**:

$2x + 3 = y$
$2x + y = 15$

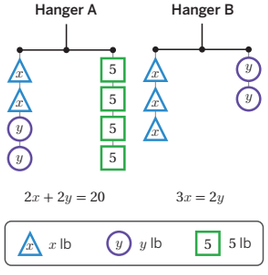
A **solution to a system of equations** is a set of values that makes both equations in that system true.

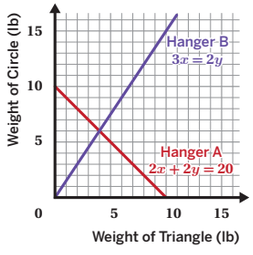
Write the solution to this system as an ordered pair. Explain your thinking.

(3, 9). *Explanations vary.* (3, 9) is the point of intersection of the two lines, which means that it is a solution to both equations.

6. This system represents another set of hangers:

$2x + 2y = 20$
$3x = 2y$





Which point is a solution to this system? Circle one.

$(1, 9)$ $(4, 6)$ $(6, 9)$ $(6, 4)$

Explain your thinking.

***Explanations vary.* I know that (4, 6) is the solution to the system of equations because that is the point where the lines intersect. I can substitute $x = 4$ and $y = 6$ into each equation and the values make both equations true.**

Accelerated 7 | Unit 5 Lesson 13 434 Activity 1

Key Takeaway: A **system of equations** is a set of two or more equations. The **solution to a system of equations** is a set of values that make all of the equations in the system true. The solution is also the point of intersection of the equations' graphs.

Pairs | 15 min

Activity 2 Hanger Solutions

Purpose: Students begin to make sense of graphs, hangers, and equations that represent systems of equations with no solution and infinitely many solutions. **(MTR.5.1)**

1 Launch

To support connections, consider inviting students to notice how the equations and hangers represent the same relationships.

2 Monitor

Invite students to take turns in Problem 7 and sharing a point with a partner and inviting their partner to decide whether it balances one, both, or neither hanger.

Look for students who reference the structure of the hangers (there is the same weight on one side but different weights on the other) or the structure of the graphs (they are parallel lines, so there will never be a point of intersection) in their explanations to surface during the Connect. **(MTR.5.1)**

Encourage students to read others' responses and decide if others' reasoning was similar to or different from their own.

Note: Students will continue to explore the number of solutions in a system of equations in Lesson 10.

MLR **MLR1: Stronger and Clearer Each Time** Use this routine to support students in developing precise language. Invite students to meet with a partner, give feedback on each other's drafts, then use the feedback to write a second clearer draft. See the Routine Facilitation Guide for more information.

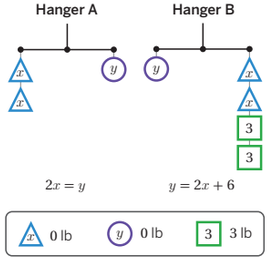
Activity 2 continued >

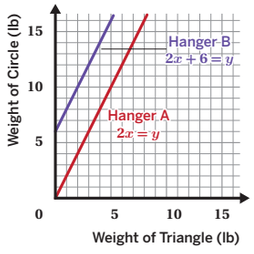
Activity 2

Name: Date: Period:

Hanger Solutions

7. Here's another system of equations.





Discuss: Can you identify a point that balances just Hanger A? Just Hanger B? Both? Neither?

Responses vary. Any point that is on one of the lines will make that hanger balance. Any point that is not on either of the lines will make neither hanger balance. It is not possible to make both hangers balance for this system of equations.

8. This system of equations from the previous problem has no solution:

$2x = y$

$y = 2x + 6$

How can you use the hangers, graphs, or equations to tell that this system of equations has no solution?

Responses vary. If I look at the graph, I can tell that the system has no solution because the lines are parallel and will never intersect. If I set the equations equal to each other and subtract $2x$ from each side, I get $0 = 6$, which is a false statement. This means that there is no solution to the equation $2x = 2x + 6$, so there is no solution to this system of equations.

Accelerated 7 | Unit 5 Lesson 13 435 Activity 2

Pairs | 15 min

Activity 2 Hanger Solutions (continued)

Purpose: Students begin to make sense of graphs, hangers, and equations that represent systems of equations with no solution and infinitely many solutions. **(MTR.5.1)**

2 Monitor

Listen for a variety of responses and explanations to highlight during the Connect.

D Differentiation *(Problem 9)*

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Encourage them to choose one point and test it in the equations or graphs.
Say that there is only one solution after testing one set of values.	**Strengthen** Consider asking, "Is this the only solution to this system? How do you know?"
Say *infinite solutions* by reasoning using the graph, equations, or hangers.	**Strengthen** Consider asking, "How could including a different representation strengthen your argument?"

Note: In Lesson 10, the concept of one solution, no solution, and infinitely many solutions will be abstracted to problems without context or diagrams. It may be useful to refer back to this problem for students who need support.

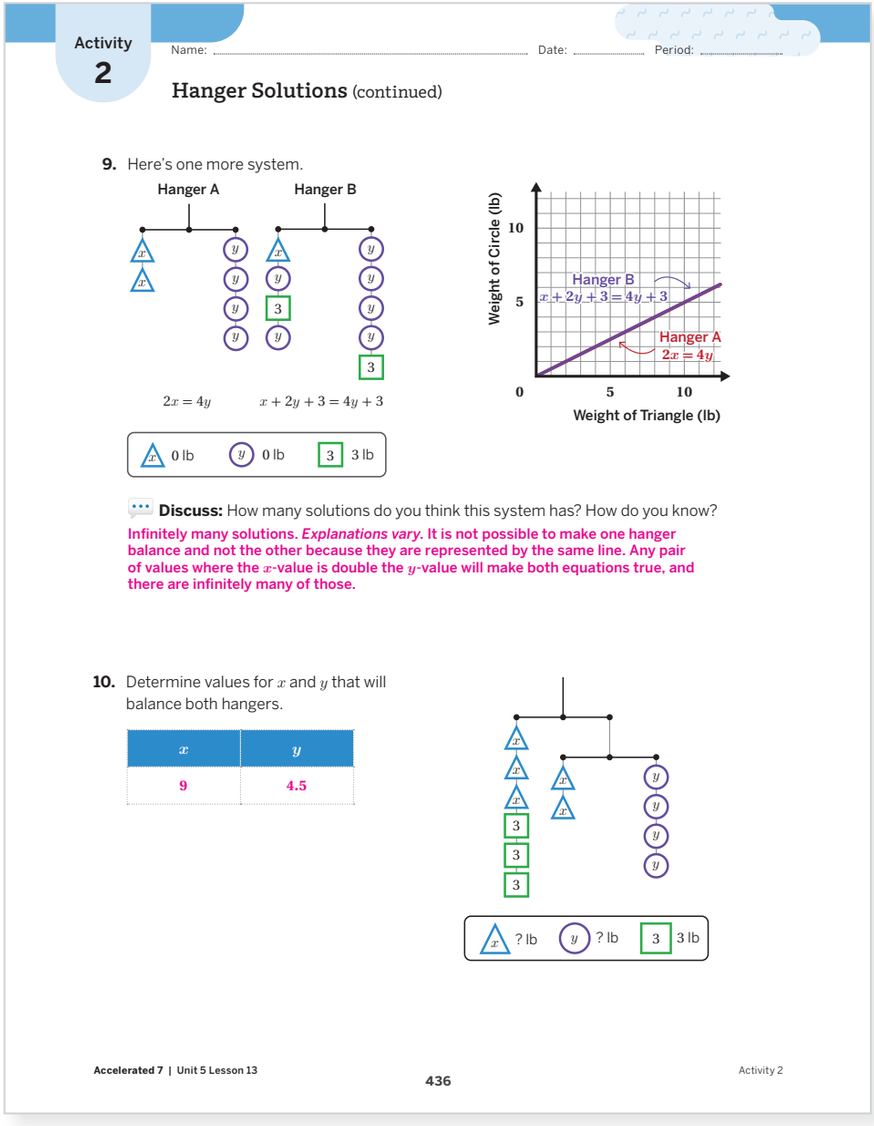
Look for students who are able to extend their thinking about systems of equations to rational values in Problem 10.

3 Connect

Invite students to share the number of solutions for Problem 9 and justify their reasoning.

To surface the Key Takeaway, consider asking:

- "How did you decide how many solutions this system of equations has?"
- "How can we tell this system has infinitely many solutions using the graphs? Equations? Hangers?"
- "What differences did you notice between the graphs of systems with one solution, no solution, and infinitely many solutions?"



Activity 2

Name: Date: Period:

Hanger Solutions (continued)

9. Here's one more system.

Hanger A: $2x = 4y$

Hanger B: $x + 2y + 3 = 4y + 3$

Discuss: How many solutions do you think this system has? How do you know?

Infinitely many solutions. *Explanations vary.* It is not possible to make one hanger balance and not the other because they are represented by the same line. Any pair of values where the x-value is double the y-value will make both equations true, and there are infinitely many of those.

10. Determine values for x and y that will balance both hangers.

x	y
9	4.5

Accelerated 7 | Unit 5 Lesson 13 436 Activity 2

Key Takeaway: **Systems of equations** can have one solution, no solution, or infinitely many solutions.

Whole Class | 5 min

Synthesis

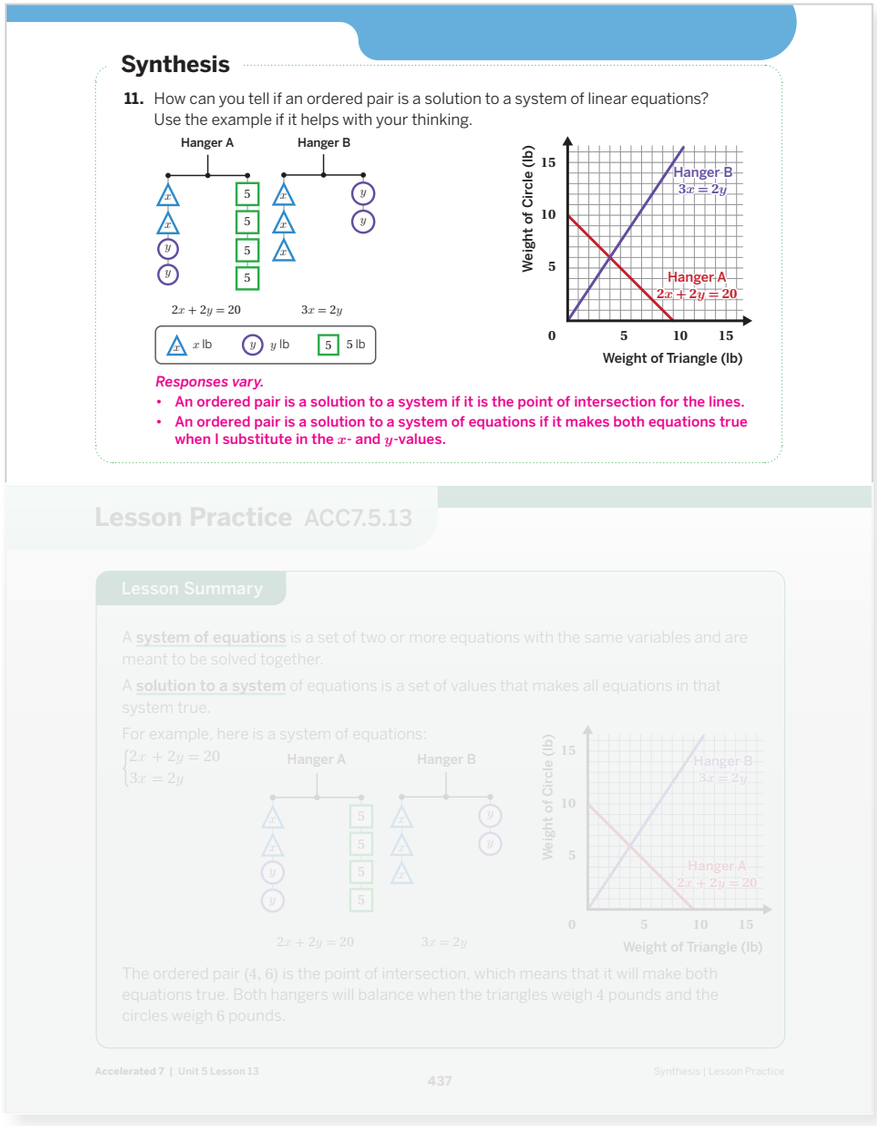
Purpose: Students synthesize their understanding of deciding if an ordered pair is a solution to a system of equations.

Invite students to respond independently, and then share their thinking with a partner.

Collect and share a variety of ideas, such as:
- A solution to a system of equations is at the point(s) where the graphs intersect.
- A solution makes both equations true (or both hangers balance).

MLR **MLR2: Collect and Display** Use this routine to formalize the definition of *solution to a system of equations* using students' language. Display any new language, examples, or diagrams for students to reference later.

Note: If time allows, invite students to revise their previous thinking based on the discussion.



Synthesis

11. How can you tell if an ordered pair is a solution to a system of linear equations? Use the example if it helps with your thinking.

Responses vary.
- An ordered pair is a solution to a system if it is the point of intersection for the lines.
- An ordered pair is a solution to a system of equations if it makes both equations true when I substitute in the x- and y-values.

Lesson Practice ACC7.5.13

Lesson Summary

A system of equations is a set of two or more equations with the same variables and are meant to be solved together.

A solution to a system of equations is a set of values that makes all equations in that system true.

For example, here is a system of equations:

$$\begin{cases} 2x + 2y = 20 \\ 3x = 2y \end{cases}$$

The ordered pair $(4, 6)$ is the point of intersection, which means that it will make both equations true. Both hangers will balance when the triangles weigh 4 pounds and the circles weigh 6 pounds.

Accelerated 7 | Unit 5 Lesson 13 437 Synthesis | Lesson Practice

Lesson Takeaway: Solutions to a system of two linear equations are points that make both equations true. A system can have no solution, one solution, or infinitely many solutions, which can be found at the point(s) of intersection of their graphs.

Independent | 5 min

Show What You Know

Purpose: Students demonstrate their understanding by identifying the solution to a system of equations.

Today's Goals

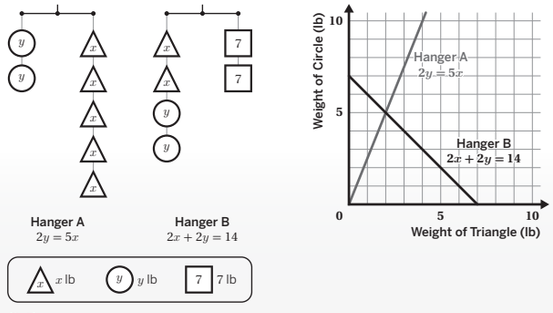
1. **Goal:** Use hanger diagrams and graphs to determine the *solutions to systems of equations*.
2. **Language Goal:** Describe connections between the graph of a *system of equations* and its number of solutions. **(Reading, Writing, Speaking, and Listening)**

Show What You Know (Assessment Resources)

Name: _____ Date: _____ Period: _____

Show What You Know 5.13

Determine the solution to this system of equations: $2y = 5x$, $2x + 2y = 14$



(2, 5)

Explain your thinking.

Explanations vary. The solution (2, 5) means that when the weight of the triangle is 2 and the weight of the circle is 5, both hangers will be balanced. This makes sense because substituting in those values makes each side of Hanger A weigh 10 pounds and each side of Hanger B weigh 14 pounds.

D Differentiation Use after Lesson 13

Support

Provide targeted intervention for students by using these resources.

If students need support identifying solutions to systems of equations from graphs or equations:

» **Respond to Student Thinking:**

- Use the Solv***ing Systems of Linear Equations by Graphing*** **Mini-Lesson**.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 3 Extension Activities**.
- Invite students to create their own hangers that represent systems of equations with infinitely many solutions.

Professional Learning

Take a moment to reflect on the strategies students used in this lesson. What opportunities were there to celebrate specific students' strategies for determining solutions? What opportunities might there be in future lessons to connect back to the strategies students shared today?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

11. How can you tell if an ordered pair is a solution to a system of linear equations? Use the example if it helps with your thinking.

Responses vary.

- An ordered pair is a solution to a system if it is the point of intersection for the lines.
- An ordered pair is a solution to a system of equations if it makes both equations true when I substitute in the x- and y-values.

Lesson Practice ACC7.5.13

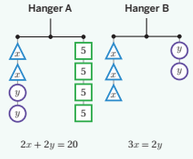
Lesson Summary

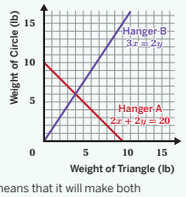
A **system of equations** is a set of two or more equations with the same variables and are meant to be solved together.

A **solution to a system** of equations is a set of values that makes all equations in that system true.

For example, here is a system of equations:

$\begin{cases} 2x + 2y = 20 \\ 3x = 2y \end{cases}$



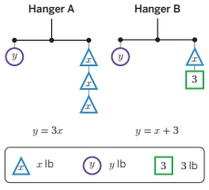


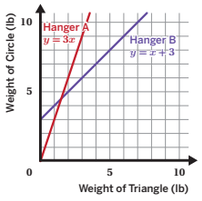
The ordered pair $(4, 6)$ is the point of intersection, which means that it will make both equations true. Both hangers will balance when the triangles weigh 4 pounds and the circles weigh 6 pounds.

Lesson Practice ACC7.5.13

Name: ..... Date: ..... Period: .....

Problems 1–2: The hangers and the graph represent the same system of equations.





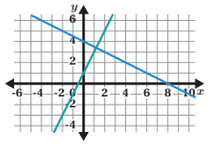
1. Determine the solution to the system of equations.

(1.5, 4.5)

2. What does the solution tell you about the weight of a triangle and the weight of a circle that will balance the hanger?

Responses vary. **The triangle weighs 1.5 pounds and the circle weighs 4.5 pounds.**

Problems 3–4: Here is a graph.



3. Write an equation that can represent each line.

$y = -\frac{1}{2}x + 4$ **and** $y = 2x + 1$

4. Estimate the solution to the system.

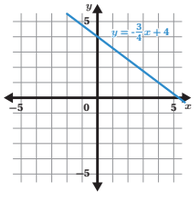
Responses vary. **(1.2, 3.4)**

Lesson Practice ACC7.5.13

Name: ..... Date: ..... Period: .....

FAST Practice

Problems 5–7: Here is a graph that represents one equation in a system of equations.



5. Write a second equation for the system so that it has infinitely many solutions.

$y = \frac{-3}{4}x + 4$ **(or equivalent)**

6. Write a second equation whose graph goes through $(0, 1)$ so that the system has no solution.

$y = \frac{-3}{4}x + 1$ **(or equivalent)**

7. Which equation creates a system with the graphed line that would have a solution of $(4, 1)$?

A. $y = \frac{3}{4}x + 6$ B. $y = \frac{1}{4}x + 6$ C. $y = \frac{1}{4}x - 4$ D. $y = \frac{1}{4}x + 2$

Spiral Review

8. Select *all* the equations that have no solution.

- ☐ A. $2 + 4(4x + 5) = 8x + 2x - 11$
- ☑ B. $-x + 3x - 7 = 2(x - 7)$
- ☑ C. $7 - 5x(\cdot 3) = 5(3x - 2)$
- ☐ D. $6x + 3(2x - 1) = 5x - 4 + 7x + 1$

Problems 9–10: Solve the equation. Show or explain your thinking.

9. $\frac{15(x-3)}{5} = 3(2x - 3)$

$x = 0$. ***Explanations vary.***

$\frac{15(x-3)}{5} = 3(2x - 3)$

$3(x - 3) = 3(2x - 3)$

$x - 3 = 2x - 3$

$-3 = x - 3$

$x = 0$

10. $0.4(x + 7) = 0.2(x + 40) - 5.2 + 0.2x$

Infinitely many solutions. ***Explanations vary.***

$0.4(x + 7) = 0.2(x + 40) - 5.2 + 0.2x$

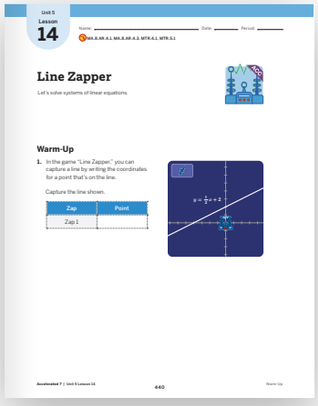
$0.4x + 2.8 = 0.2x + 8 - 5.2 + 0.2x$

$0.4x + 2.8 = 0.4x + 2.8$

There are infinitely many solutions because the coefficients and constants are the same on each side of the equal sign.

Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T Math Benchmarks
On-lesson			
	1–4	2	MA.8.AR.4.2, MA.8.AR.4.3
FAST Practice	5–7	2	MA.8.AR.4.1, MA.8.AR.4.2
Spiral Review			
	8–10	2	MA.8.AR.2.1


Unit 5
Lesson
14

Line Zapper

Let's solve systems of linear equations.

Warm-Up

1. In the game "Line Zapper," you can capture a line by writing the coordinates for a point that's on the line.

Capture the line shown.

Zap	Point
Zap 1	

440

Line Zapper

Solving Systems of Linear Equations

Let's solve systems of linear equations.

Learning Outcomes

Today's Goals

1. **Goal:** Solve a system of linear equations algebraically.
2. **Language Goal:** Explain how to determine the solution to a system of equations without using a graph. **(Speaking and Listening)**

Students continue to explore systems where both equations are of the form $y = mx + b$. They experience the need for solving systems of equations algebraically to determine the coordinates of intersection points on a graph. Students use equations to determine the coordinates of intersection points of lines without a graph.

Coherence

Prior Learning

In Lesson 13, students were formally introduced to systems of equations and identified their solutions graphically in the context of hanger diagrams.

Future Learning

In Lesson 15, students will analyze systems of linear equations with no solution and infinitely many solutions.

Connections

- Students extend their understanding of using graphs to solving a system of equations by playing a game called Line Zapper. (mathematical)

FL B.E.S.T. Math Benchmarks

Addressing

MA.8.AR.4.1
Given a system of two linear equations and a specified set of possible solutions, determine which ordered pairs satisfy the system of linear equations.

MA.8.AR.4.3
Given a mathematical or real-world context, solve systems of two linear equations by graphing.

Mathematical Thinking and Reasoning Standards: MTR.4.1, MTR.5.1

ELA Expectation: ELA.K12.EE.3.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.7.AR.4.2 **MA.7.AR.4.3**	**MA.912.AR.9.1**

Lesson at a Glance

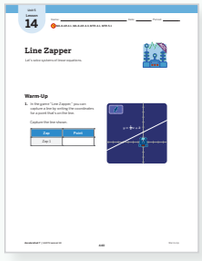
~ 45 min

MA.8.AR.4.1, MA.8.AR.4.3

Warm-Up

Pairs | 5 min

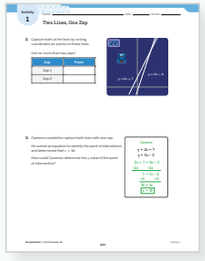
Students review the connection between representations of a linear equation and its points as solutions.



Activity 1

Pairs | 10 min

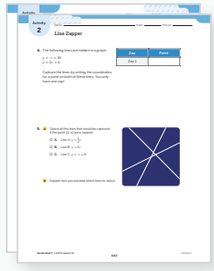
Students utilize the structure of equations in slope-intercept form to determine a point of intersection algebraically. **(MTR.5.1)**



Activity 2

Pairs | 20 min

Students use algebraic methods to precisely solve systems of equations. **(MTR.5.1)**

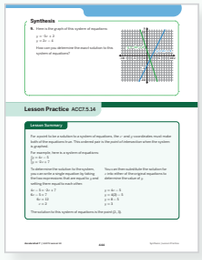


Synthesis

Whole Class | 5 min

Students synthesize their understanding of using algebraic reasoning to determine the precise solution to a system of linear equations.

MLR2: Collect and Display



Show What You Know

Independent | 5 min

Students demonstrate their understanding by solving a system of linear equations algebraically.

Prep Checklist

This lesson includes:

- Student Edition
- Show What You Know
- Coloring toos or highlighters (as needed)

Pairs | 5 min

Warm-Up

Purpose: Students review the connection between representations of a linear equation and its points as solutions.

1 Launch

To generate excitement, consider sharing that in this lesson, students will zap points on lines.

Invite students to write coordinates to try and capture the line.

ELL **English Language Learners** Explain that for this activity, the word zap means identifying a point that is on the line. **(Reading and Listening)**

Look for opportunities to recognize the brilliance of students who have not been heard from recently.

Look for a variety of student responses to share during the Connect, such as:
- The horizontal or vertical intercept.
- Coordinates that are outside the grid.

2 Connect

Share examples of student responses.

To surface connections to prior learning, highlight students that used the vertical or horizontal intercept as their coordinate point.

Consider asking, "How can you use the equation to prove your point is on the line?"

Unit 5
Lesson
14

Name: _____ Date: _____ Period: _____

MA.8.AR.4.1, MA.8.AR.4.3, MTR.4.1, MTR.5.1

Line Zapper

Let's solve systems of linear equations.

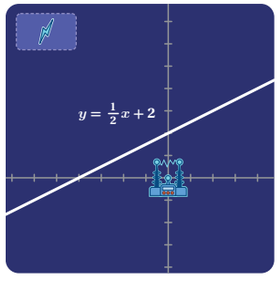
Warm-Up

1. In the game "Line Zapper," you can capture a line by writing the coordinates for a point that's on the line.

Capture the line shown.

Zap	Point
Zap 1	

Responses vary.
- (0, 2)
- (1.5, 2.75)
- (2, 3)



Accelerated 7 | Unit 5 Lesson 14 440 Warm-Up

Pairs | 10 min

Activity 1 Two Lines, One Zap

Purpose: Students utilize the structure of equations in slope-intercept form to determine a point of intersection algebraically. **(MTR.5.1)**

1 Launch

Consider sharing that now students will need to capture more than one line and will have more than one zap.

Note: All students should have and make use of scrap paper to determine points to zap for the remaining problems.

2 Monitor

Look for strategies for zapping lines to share during the Connect.

D **Differentiation** *(Problem 2)*

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Encourage them to focus on zapping one equation first.
Guess-and-check various coordinate pairs.	**Strengthen** Consider asking, "Is there a systematic way you could choose points to try?"
Set the expressions equal to each other.	**Strengthen** Consider asking, "How is the solution to your equation related to the graph?"
Write a point on the first line and another point on the second line.	**Stretch** Invite students to try to zap both lines with one point.
Use a key graphical feature like an intercept or the intersection point.	**Stretch** Invite students to identify those same features in the equations.

Select and sequence explanations to share during the Connect.

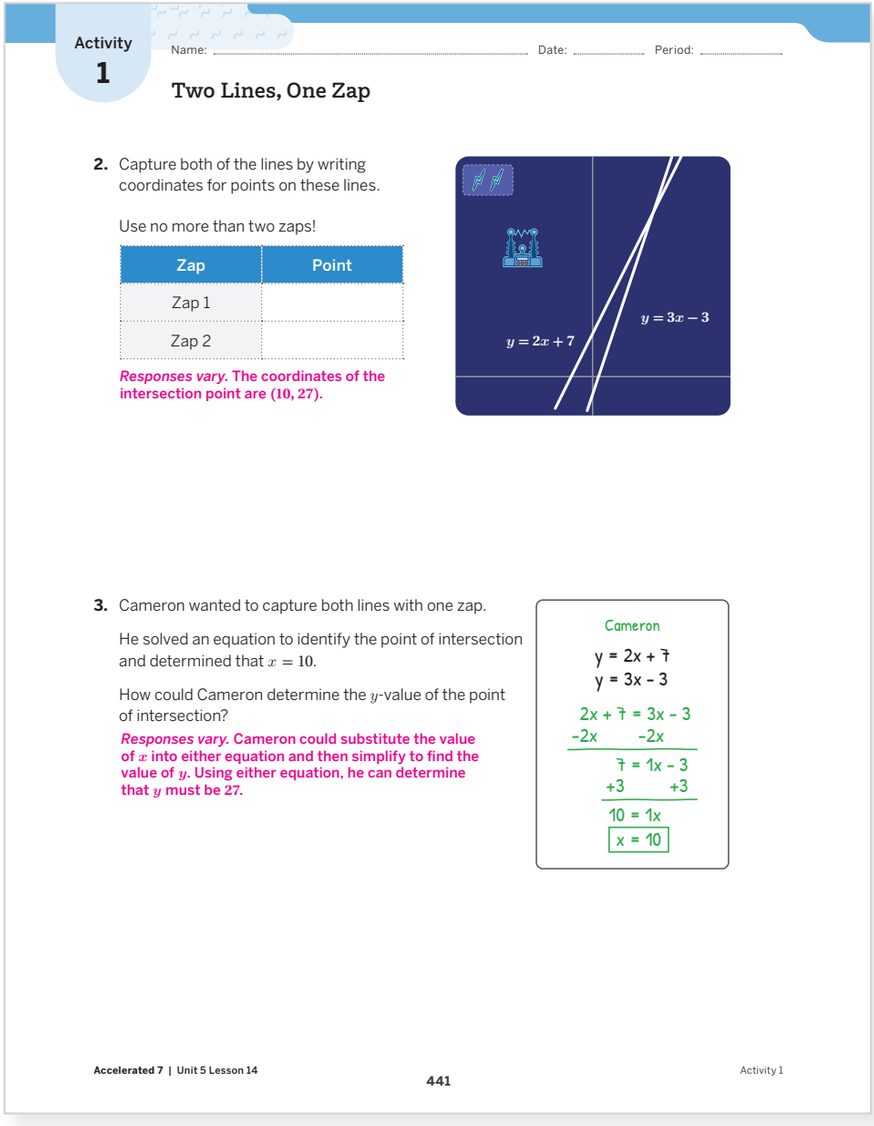
ELL **English Language Learners** Invite students to write first in their primary language and then translate it to English during a second draft. **(Writing)**

3 Connect

Share a variety of strategies for zapping lines and determining the y-value of the intersection point.

To surface the Key Takeaway, consider asking:

- "How can you use the equations to determine points to zap?"
- "What is helpful about Cameron's strategy?"
- "How is the structure of the equations related to Cameron's strategy?"



Activity 1

Name: Date: Period:

Two Lines, One Zap

2. Capture both of the lines by writing coordinates for points on these lines.

Use no more than two zaps!

Zap	Point
Zap 1	
Zap 2	

Responses vary. The coordinates of the intersection point are $(10, 27)$.

3. Cameron wanted to capture both lines with one zap.

He solved an equation to identify the point of intersection and determined that $x = 10$.

How could Cameron determine the y-value of the point of intersection?

Responses vary. Cameron could substitute the value of x into either equation and then simplify to find the value of y. Using either equation, he can determine that y must be 27.

Cameron

$y = 2x + 7$
$y = 3x - 3$

$2x + 7 = 3x - 3$
$-2x \quad -2x$
$7 = 1x - 3$
$+3 \quad +3$
$10 = 1x$
$x = 10$

Accelerated 7 | Unit 5 Lesson 14 441 Activity 1

Key Takeaway: To determine the point of intersection for two lines in the form $y = mx + b$, set the two equations equal to one another and solve for the x-coordinate. Then, substitute that value into either equation and determine the value of the y-coordinate.

Pairs | 20 min

Activity 2 Line Zapper

Purpose: Students use algebraic methods to precisely solve systems of equations. **(MTR.5.1)**

Short on time? Consider omitting Problem 7.

1 Launch

To surface connections, consider asking, "How is this line capture alike and different from the ones we've seen so far?"

2 Monitor

Look for and celebrate the different ways students zap lines.

A **Accessibility: Visual-Spatial Processing** Invite students to draw or highlight to support interpreting visual representations.

D **Differentiation** *(Problems 4-5)*

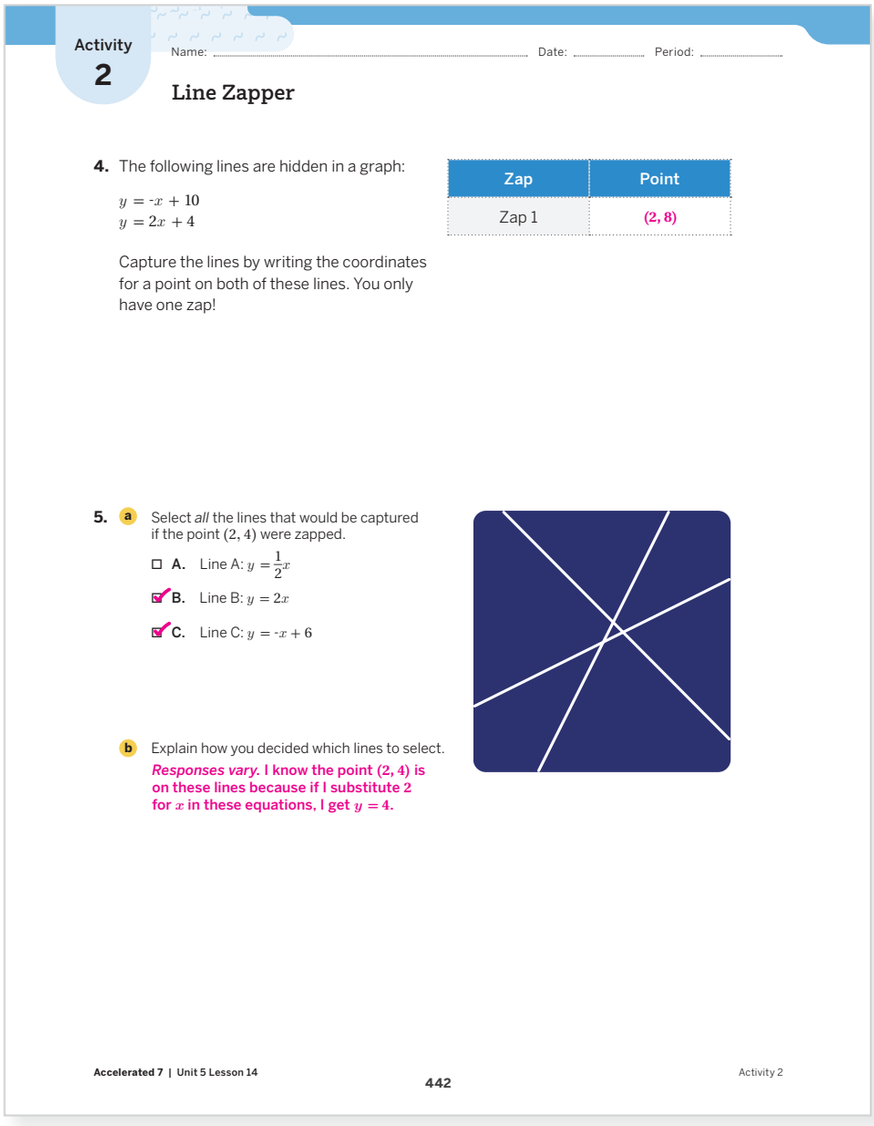
Look for students who:	Respond to Student Thinking
Zap one line at a time and need additional support to capture two lines at once.	Support Invite them to try Problem 5 and return to Problem 4 to apply their learning. Consider asking, "What can help you zap two lines with only one zap?"
Set the two equations equal to each other and solve for the intersection point.	Strengthen Invite them to share during the Connect.

Invite students, to think independently before discussing their ideas with a partner. **(MTR.4.1)**

Pause to discuss students' ideas if time allows.

ELL **English Language Learners** Provide sentence frames. (e.g., "I selected Line(s) _____ because _____." or "If the point (2, 4) was zapped I would capture Line(s) _____ because _____."). **(Writing and Speaking)**

Activity 2 continued >



Activity 2

Name: Date: Period:

Line Zapper

4. The following lines are hidden in a graph:

$y = -x + 10$
$y = 2x + 4$

Capture the lines by writing the coordinates for a point on both of these lines. You only have one zap!

Zap	Point
Zap 1	(2, 8)

5. a. Select *all* the lines that would be captured if the point (2, 4) were zapped.

- ☐ A. Line A: $y = \frac{1}{2}x$
- ☑ B. Line B: $y = 2x$
- ☑ C. Line C: $y = -x + 6$

b. Explain how you decided which lines to select.

Responses vary. I know the point (2, 4) is on these lines because if I substitute 2 for x in these equations, I get $y = 4$.

Accelerated 7 | Unit 5 Lesson 14 442 Activity 2

Pairs | 20 min

Activity 2 Line Zapper (continued)

Purpose: Students use algebraic methods to precisely solve systems of equations. **(MTR.5.1)**

Short on time? Consider omitting Problem 7.

2 Monitor

Look for a variety of strategies used in Problem 6 to share during the Connect.

A **Accessibility: Executive Functioning** Chunk this task into smaller, more manageable parts by having students first determine a point of intersection for two of the lines, and then determine a point of intersection for a different pair of lines.

D **Differentiation**

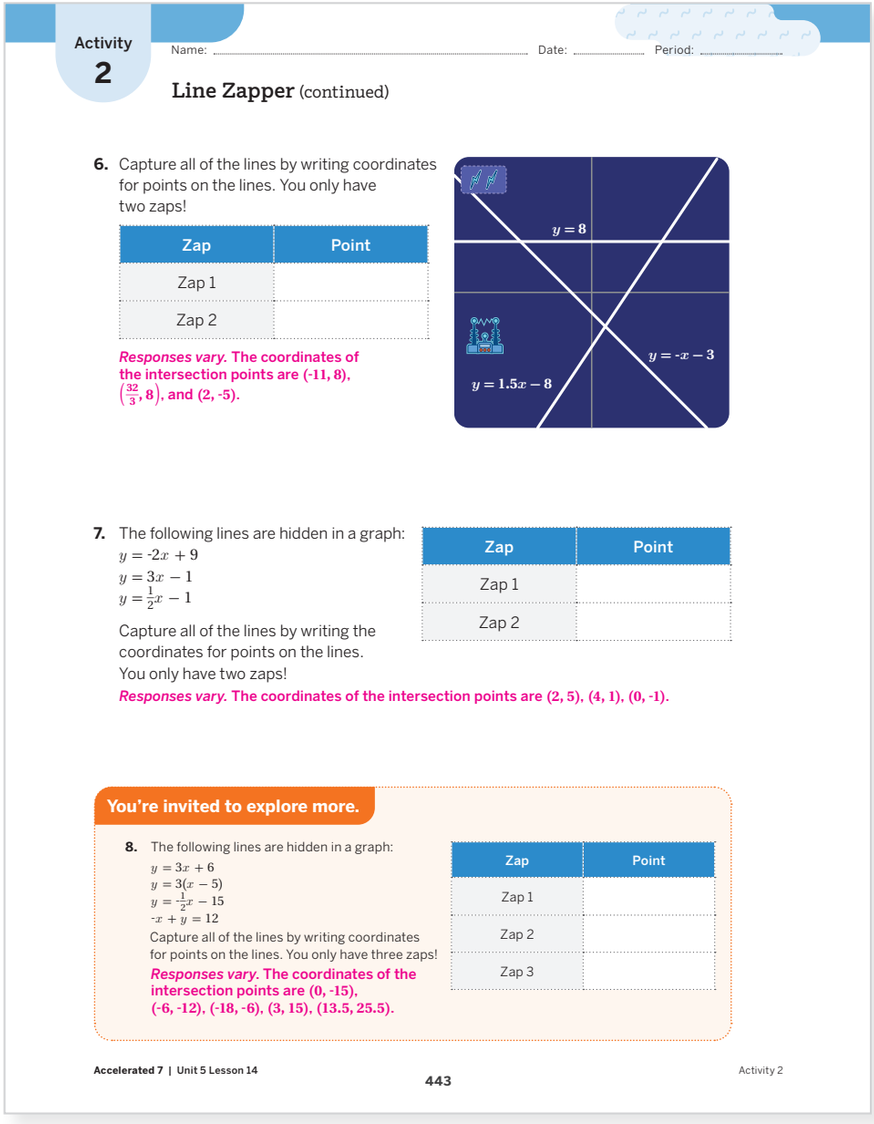
Look for students who:	Respond to Student Thinking
Determine a point or key feature for a single equation.	**Strengthen** Consider asking: • "How many lines did your zap capture?" • "How can you capture more than one line with a single point?"
Substitute a y-value of 8 in either of the two other equations.	**Strengthen** Consider asking, "How does this strategy help you use fewer zaps to capture the lines?"
Want to explore solving systems of equations further.	**Stretch** Consider asking, "Is it possible to light up all three lines using only one zap? Why or why not?"

Collect multiple student responses to share during the Connect.

3 Connect

Discuss strategies students used to capture the lines.

To surface the Key Takeaway, consider asking:
- "How did you decide which equation(s) to start with?"
- "How did you decide on a strategy?"
- "Why do you need multiple points to zap all three lines?"



Activity 2

Name: Date: Period:

Line Zapper (continued)

6. Capture all of the lines by writing coordinates for points on the lines. You only have two zaps!

Zap	Point
Zap 1	
Zap 2	

Responses vary. The coordinates of the intersection points are (-11, 8), $\left(\frac{32}{3}, 8\right)$, and (2, -5).

7. The following lines are hidden in a graph:
$y = -2x + 9$
$y = 3x - 1$
$y = \frac{1}{2}x - 1$

Capture all of the lines by writing the coordinates for points on the lines. You only have two zaps!

Zap	Point
Zap 1	
Zap 2	

Responses vary. The coordinates of the intersection points are (2, 5), (4, 1), (0, -1).

You're invited to explore more.

8. The following lines are hidden in a graph:
$y = 3x + 6$
$y = 3(x - 5)$
$y = \frac{-1}{2}x - 15$
$-x + y = 12$

Capture all of the lines by writing coordinates for points on the lines. You only have three zaps!

Zap	Point
Zap 1	
Zap 2	
Zap 3	

Responses vary. The coordinates of the intersection points are (0, -15), (-6, -12), (-18, -6), (3, 15), (13.5, 25.5).

Accelerated 7 | Unit 5 Lesson 14 443 Activity 2

Key Takeaway: The point of intersection between two lines is the solution to that system of equations because it is an ordered pair that is on both lines. Students can use points of intersection to more efficiently capture multiple lines.

Whole Class | 5 min

Synthesis

Purpose: Students synthesize their understanding of using algebraic reasoning to determine the precise solution to a system of linear equations.

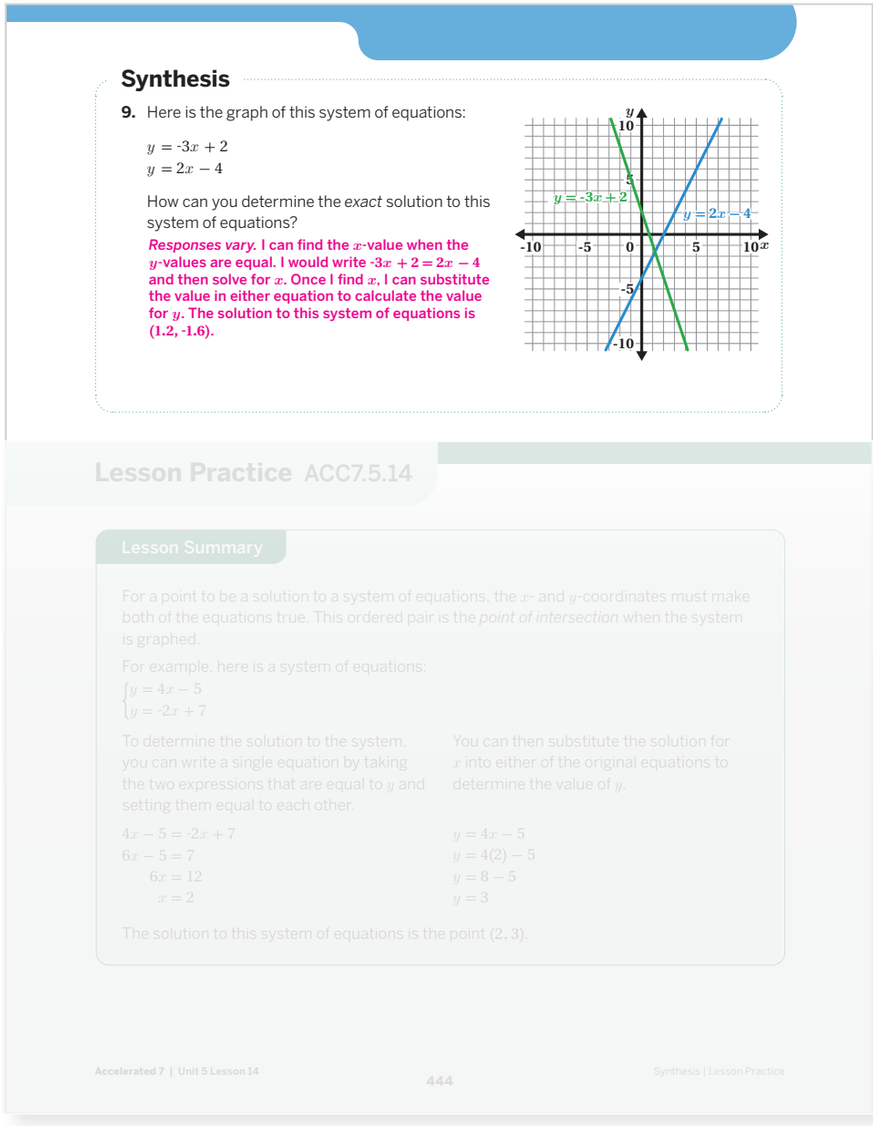
Invite students to respond independently, and then share their thinking with a partner.

Collect and share a variety of ideas, such as:

- Solving the system algebraically helps us determine the exact solution.
- When the equations are of the form $y = mx + b$, we can set the expressions equal to each other to solve for x, then substitute this value into one of the equations to determine y.

MLR **MLR2: Collect and Display** Use this routine to formalize the strategy for solving systems of linear equations algebraically, using students' language. Display any new language, examples, or diagrams for students to reference later.

Note: If time allows, invite students to revise their previous thinking based on the discussion.



Synthesis

9. Here is the graph of this system of equations:

$y = -3x + 2$
$y = 2x - 4$

How can you determine the *exact* solution to this system of equations?

Responses vary. **I can find the x-value when the y-values are equal. I would write $-3x + 2 = 2x - 4$ and then solve for x. Once I find x, I can substitute the value in either equation to calculate the value for y. The solution to this system of equations is (1.2, -1.6).**

Lesson Practice ACC7.5.14

Lesson Summary

For a point to be a solution to a system of equations, the x- and y-coordinates must make both of the equations true. This ordered pair is the *point of intersection* when the system is graphed.

For example, here is a system of equations:

$\begin{cases} y = 4x - 5 \\ y = -2x + 7 \end{cases}$

To determine the solution to the system, you can write a single equation by taking the two expressions that are equal to y and setting them equal to each other.

$4x - 5 = -2x + 7$
$6x - 5 = 7$
$6x = 12$
$x = 2$

You can then substitute the solution for x into either of the original equations to determine the value of y.

$y = 4x - 5$
$y = 4(2) - 5$
$y = 8 - 5$
$y = 3$

The solution to this system of equations is the point (2, 3).

Accelerated 7 | Unit 5 Lesson 14 444 Synthesis | Lesson Practice

Lesson Takeaway: You can use algebraic reasoning to determine the exact solution to a system of linear equations.

Independent | 5 min

Show What You Know

Purpose: Students demonstrate their understanding by solving a system of linear equations algebraically.

Today's Goals

1. **Goal:** Solve a system of linear equations algebraically.
2. **Language Goal:** Explain how to determine the solution to a system of equations without using a graph. **(Speaking and Listening)**

Show What You Know (Assessment Resources)

Name: _____ Date: _____ Period: _____

Show What You Know 5.14

What is the solution to this system of equations?

$y = 2x$
$y = 3x - 10$

Write your solution as an ordered pair.

(10, 20). *Work varies.*

$2x = 3x - 10$
$-x = -10$
$x = 10$

$y = 2(10)$
$y = 20$

Differentiation Use after Lesson 14

Support

Provide targeted intervention for students by using these resources.

If students need support solving systems of equations algebraically:

» **Respond to Student Thinking:**

- Revisit **Activity 2** with students before providing individualized feedback on the relevant **Practice Problems**.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 3 Extension Activities**.

Professional Learning

In this lesson, students learned how to solve systems of linear equations by playing Line Zapper. How did the activities in this lesson help students experience mathematics as engaging and enjoyable?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

9. Here is the graph of this system of equations:

$y = -3x + 2$
$y = 2x - 4$

How can you determine the *exact* solution to this system of equations?

Responses vary. I can find the x-value when the y-values are equal. I would write $-3x + 2 = 2x - 4$ and then solve for x. Once I find x, I can substitute the value in either equation to calculate the value for y. The solution to this system of equations is $(1.2, -1.6)$.

Lesson Practice ACC7.5.14

Lesson Summary

For a point to be a solution to a system of equations, the x- and y-coordinates must make both of the equations true. This ordered pair is the *point of intersection* when the system is graphed.

For example, here is a system of equations:

$\begin{cases} y = 4x - 5 \\ y = -2x + 7 \end{cases}$

To determine the solution to the system, you can write a single equation by taking the two expressions that are equal to y and setting them equal to each other.

$4x - 5 = -2x + 7$
$6x - 5 = 7$
$6x = 12$
$x = 2$

You can then substitute the solution for x into either of the original equations to determine the value of y.

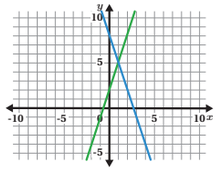
$y = 4x - 5$
$y = 4(2) - 5$
$y = 8 - 5$
$y = 3$

The solution to this system of equations is the point $(2, 3)$.

Lesson Practice ACC7.5.14

Name: _____ Date: _____ Period: _____

Problems 1–2: Here is the graph of this system of equations: $\begin{cases} y = -3x + 8 \\ y = 3x + 2 \end{cases}$



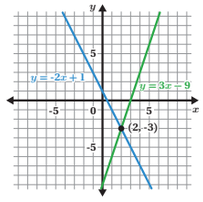
1. How can you determine the solution to this system of equations by looking at the graph?

***Responses vary.* The solution to the system is the point where both lines intersect.**

2. What is the solution to the system of equations?

(1, 5)

Problems 3–5: Use the lines in the graph to decide whether each statement is true or false.



3. The solution to the equation $-2x + 1 = 3x - 9$ is $x = 2$.

True

4. The point $(2, -3)$ is a solution to this system of equations: $\begin{cases} y = -2x + 1 \\ x = 2 \end{cases}$

True

5. The point $(0, 1)$ is a solution to the equation $y = -2x + 1$.

True

Lesson Practice ACC7.5.14

Name: _____ Date: _____ Period: _____

6. Here is how Haru tried to solve this system of equations. Did Haru solve the system of equations correctly? Explain your thinking.

Haru is incorrect. *Explanations vary.* The x-value is provided in the second equation, $x = 4$. Haru should have used this to determine the y-value by writing and solving the equation $y = -3(4) + 1$, which would result in $y = -11$. The x-value is 4 and the y-value is -11, so the solution should be $(4, -11)$.

Haru

$\begin{cases} y = -3x + 1 \\ x = 4 \end{cases}$

$-3x + 1 = 4$
$-3x = 3$
$x = -1$

$y = -3(-1) + 1$
$y = 4$

Solution: $(-1, 4)$

FAST Practice

7. What is the solution to the system of equations?

$\begin{cases} y = -4x + 2 \\ y = 3x - 5 \end{cases}$

A. $(-1, -2)$
B. $(-1, 2)$
C. $(1, -2)$
D. $(1, 2)$

Spiral Review

8. The temperature in degrees Fahrenheit, F, is related to the temperature in degrees Celsius, C, which is represented by the equation $F = \frac{9}{5}C + 32$. There is one temperature where the degrees Fahrenheit and degrees Celsius are the same, so that $C = F$. What is that temperature? Show or explain your thinking.

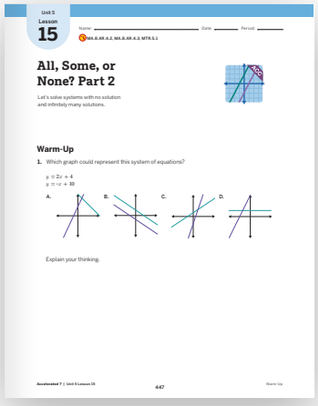
-40 degrees. *Explanations vary.*

$C = \frac{9}{5}C + 32$
$-\frac{4}{5}C = 32$
$C = -40$

At -40 °C, the temperature is also -40 °F.

Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T Math Benchmarks
On-lesson			
	1–2	2	MA.8.AR.4.3
	3–5	1	MA.8.AR.4.1
	6	3	MA.8.AR.4.3
FAST Practice	7	2	MA.8.AR.4.1
Spiral Review			
	8	2	MA.7.AR.2.2


Unit 5
Lesson
15
All, Some, or None? Part 2
Let's solve systems with no solution and infinitely many solutions.
Warm-Up
1. Which graph could represent this system of equations?
Explain your thinking.

All, Some, or None? Part 2

Systems of Equations With No, One, or Many Solutions

Let's solve systems with no solution and infinitely many solutions.

Learning Outcomes

Today's Goals

1. **Goal:** Determine whether systems of equations have no solution, one solution, or infinitely many solutions.
2. **Language Goal:** Justify whether a system of equations has no solution, one solution, or infinitely many solutions. **(Writing, Speaking, and Listening)**

Students connect algebraic and graphical representations of systems and determine the number of solutions from both representations. Students categorize systems to think strategically about what structures of linear equations will result in no solution, one solution, or infinitely many solutions. **(MTR.5.1)**

Coherence

Prior Learning

In Lesson 14, students algebraically solved systems where both equations were in the form $y = mx + b$. In Lesson 7, students were introduced to equations with no solution and infinitely many solutions.

Future Learning

In a future course, students will identify, describe, and employ strategies for solving systems of linear equations algebraically, including systems with no solution, one solution, and infinitely many solutions.

Connections

- Students make sense of the number of solutions for a system of equations by comparing the slopes and y-intercepts of the graphs of the lines. (mathematical)

FL B.E.S.T. Math Benchmarks

Addressing

MA.8.AR.4.2
Given a system of two linear equations represented graphically on the same coordinate plane, determine whether there is one solution, no solution or infinitely many solutions.

MA.8.AR.4.3
Given a mathematical or real-world context, solve systems of two linear equations by graphing.

Mathematical Thinking and Reasoning Standard: MTR.5.1

ELA Expectations: ELA.K12.EE.3.1, ELA.K12.EE.4.1, ELA.K12.EE.5.1

English Language Development Standards: ELD.K12.ELL.MA.1, ELD.K12.ELL.SI.1

Building On	Building Toward
MA.7.AR.4.3	**MA.912.AR.9.1**
MA.7.AR.4.4	**MA.912.AR.9.6**

Lesson at a Glance

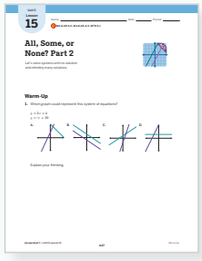
~ 45 min

FL. MA.8.AR.4.2, MA.8.AR.4.3

Warm-Up

Pairs | 5 min

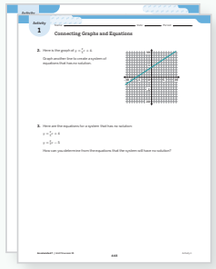
Students reason about a system of equations by looking at its structure and selecting a graph it could represent.
Routine: Think-Pair-Share **(MTR.5.1)**



Activity 1

Pairs | 15 min

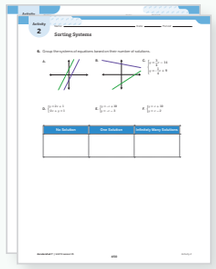
Students create graphs to draw connections between the features of graphed systems and their number of solutions.



Activity 2

Pairs | 15 min

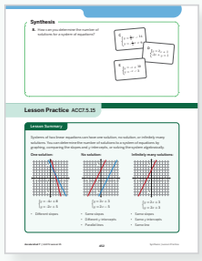
Students use the structure of equations in a system to categorize them by number of solutions. **MLR7: Compare and Connect** **(MTR.5.1)**



Synthesis

Whole Class | 5 min

Students synthesize their understanding of how to determine the number of solutions for a system of equations by using the structure of the equations or graphs. **MLR2: Collect and Display** **(MTR.5.1)**



Show What You Know

Independent | 5 min

Students demonstrate their understanding by using the structure of a system or its graph to determine the number of solutions. **(MTR.5.1)**

Prep Checklist

This lesson includes:

- Student Edition
- Show What You Know
- Straightedges
- Graphing calculators (as needed)

Pairs | 5 min

Warm-Up

Purpose: Students reason about a system of equations by looking at its structure and selecting a graph it could represent. **(MTR.5.1)**

1 Launch

Direct students' attention to the Warm-Up prompt.

To surface prior knowledge, consider asking, "In looking at an equation, what can you tell about its graph?"

A **Accessibility: Visual-Spatial Processing** To support distinguishing between graphs, invite pairs to interpret each graph aloud and discuss how they are similar and different.

Use the Think-Pair-Share routine to support student-to-student discourse. Invite students to think independently and then take turns sharing with a partner. Invite several students to share their or their partner's reasoning with the class.

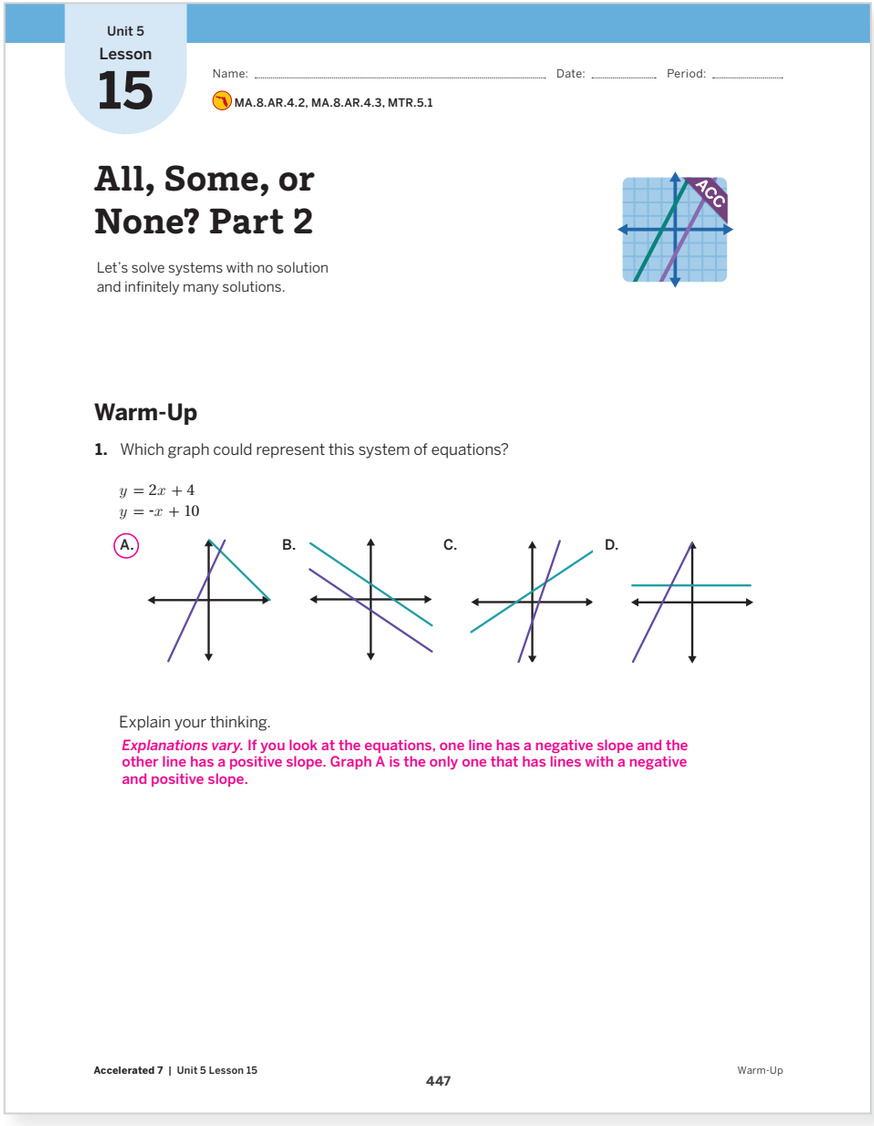
ELL **English Language Learners** Provide sentence frames to help students explain their strategy (e.g., "I noticed that _____, so I _____." or "First, I _____ because _____."). **(Writing and Speaking)**

2 Connect

Invite several students to share their responses and justify their thinking.

Consider asking, "What connections do you see between the equations and graphs?"

Note: If it does not come up naturally, consider highlighting how the slopes and y-intercepts in the equations are represented on the graph.



Unit 5
Lesson
15

Name: _____ Date: _____ Period: _____

MA.8.AR.4.2, MA.8.AR.4.3, MTR.5.1

All, Some, or None? Part 2

Let's solve systems with no solution and infinitely many solutions.

Warm-Up

1. Which graph could represent this system of equations?

$y = 2x + 4$
$y = -x + 10$

A. B. C. D.

Explain your thinking.

Explanations vary. If you look at the equations, one line has a negative slope and the other line has a positive slope. Graph A is the only one that has lines with a negative and positive slope.

Accelerated 7 | Unit 5 Lesson 15 447 Warm-Up

Pairs | 15 min

Activity 1 Connecting Graphs and Equations

Purpose: Students create graphs to draw connections between the features of graphed systems and their number of solutions.

1 Launch

To surface connections, share that students will continue to explore the relationship between the equations in a system, their graphs, and the number of solutions.

2 Monitor

Invite students to revise their work based on the feedback they receive from a peer.

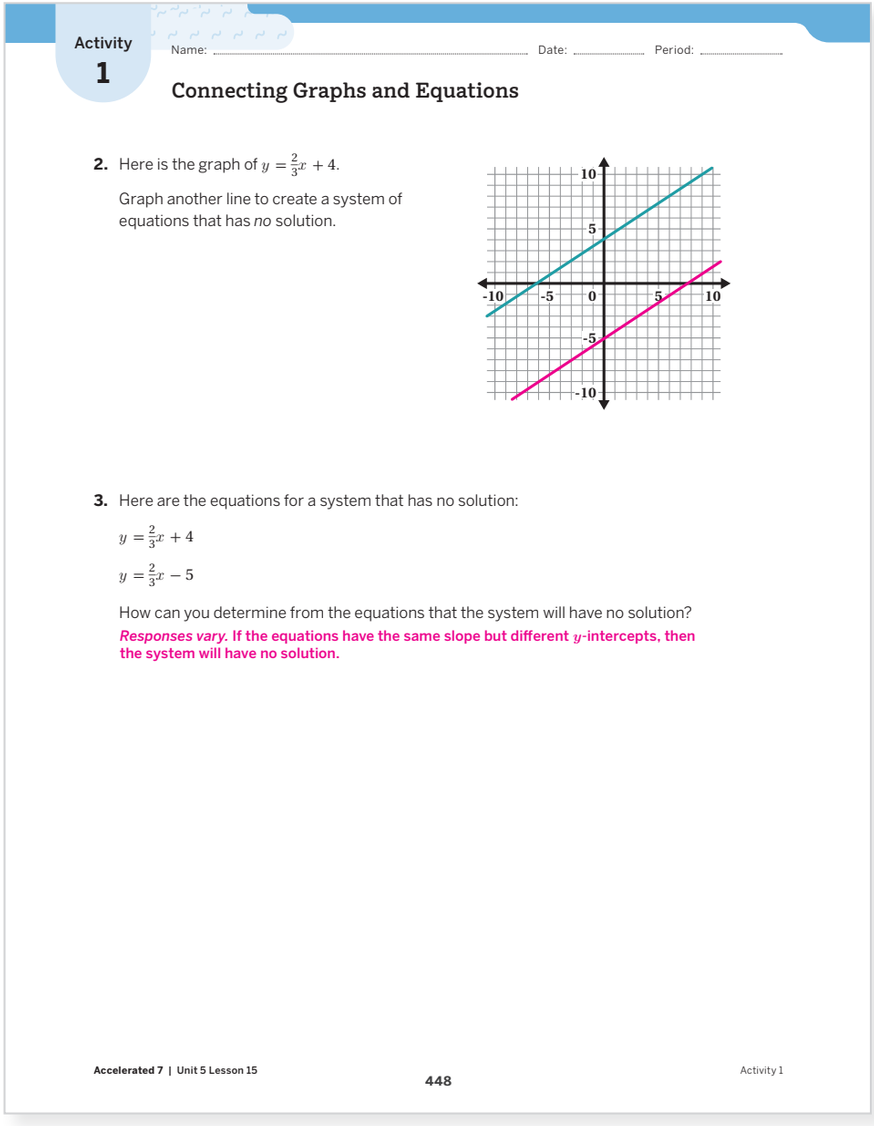
Encourage students to justify any changes they make by explaining to their partner why the revision is needed.

Students will need straightedges to support graphing.

Look for students who reference slopes in their explanation for Problems 2 or 3 and invite them to share during the Connect.

ELL **English Language Learners** Provide a word bank to support students in expressing their ideas with words such as: slope, y-intercept, point of intersection, etc. **(Writing and Speaking)**

Activity 1 continued >



Activity 1

Name: Date: Period:

Connecting Graphs and Equations

2. Here is the graph of $y = \frac{2}{3}x + 4$.

Graph another line to create a system of equations that has *no* solution.

3. Here are the equations for a system that has no solution:

$y = \frac{2}{3}x + 4$

$y = \frac{2}{3}x - 5$

How can you determine from the equations that the system will have no solution?

Responses vary. If the equations have the same slope but different y-intercepts, then the system will have no solution.

Accelerated 7 | Unit 5 Lesson 15 448 Activity 1

Pairs | 15 min

Activity 1 Connecting Graphs and Equations (continued)

Purpose: Students create graphs to draw connections between the features of graphed systems and their number of solutions.

2 Monitor

Consider asking, "What does it mean for an equation to have infinitely many solutions? How is this alike and different from a graphed system of equations?"

Responses vary. Equations with infinitely many solutions are true for all values, and systems with infinitely many equations are true for an infinite number of points, but not all points.

A **Accessibility: Visual-Spatial Processing**
Invite students to answer a more concrete question first to support interpreting visual representations. Consider asking, "What does it mean for the system of equations to have infinitely many solutions? What will that look like on a graph?"

Look and listen for explanations to share during the Connect.

D **Differentiation** *(Problem 5)*

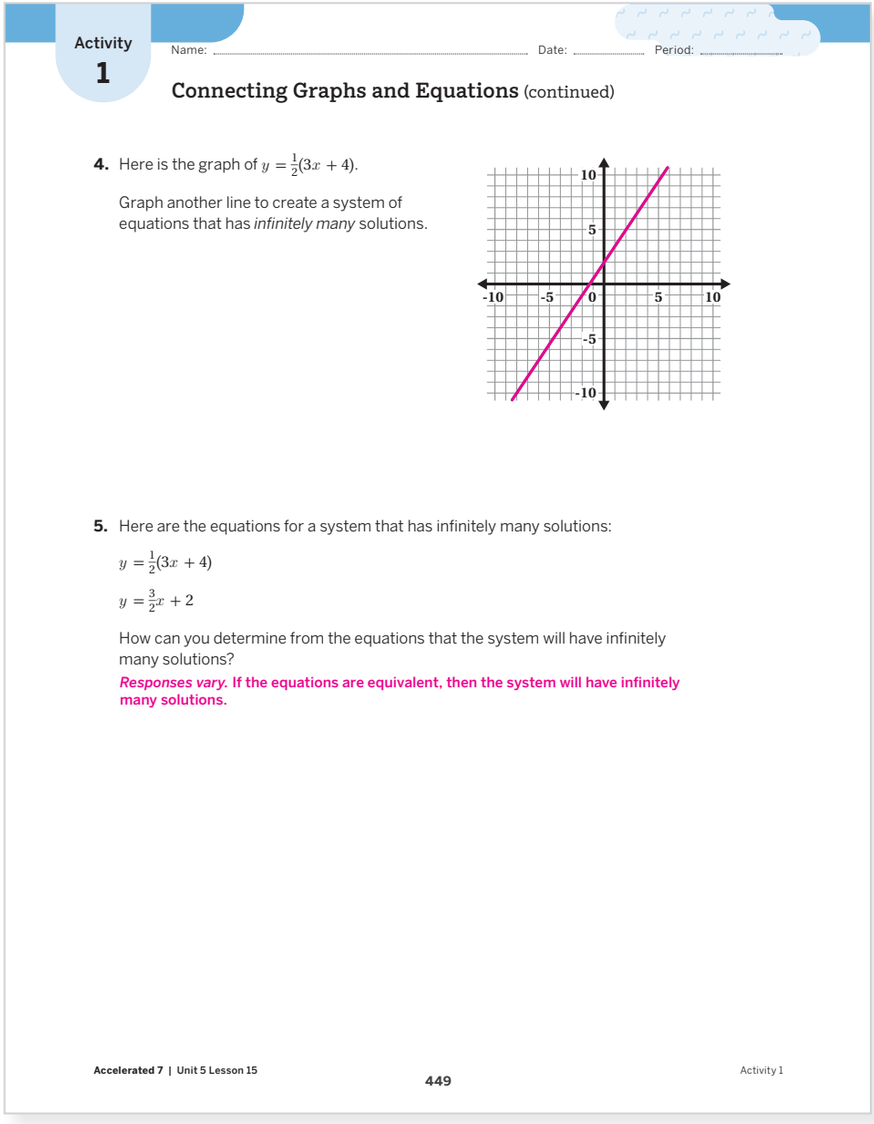
Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Consider asking: "How can you re-write the first equation?"
State that equations are equivalent in their explanation.	**Strengthen** Invite students to revise their response to make it stronger and clearer after the discussion.
Would like to explore systems with infinitely many solutions further.	**Stretch** Invite students to write another equivalent equation to create a system of three equations that have infinitely many solutions.

3 Connect

Invite students to share their responses from Problems 3 and 5.

To surface the Key Takeaway, consider asking:

- "How are the equations and the graph connected for a system with no solution?"
- "How are the equations and the graph connected for a system with infinitely many solutions?"
- "How are equations for systems with no and infinitely many solutions alike? How are they different?"



Activity 1

Name: Date: Period:

Connecting Graphs and Equations (continued)

4. Here is the graph of $y = \frac{1}{2}(3x + 4)$.

Graph another line to create a system of equations that has *infinitely many* solutions.

5. Here are the equations for a system that has infinitely many solutions:

$y = \frac{1}{2}(3x + 4)$

$y = \frac{3}{2}x + 2$

How can you determine from the equations that the system will have infinitely many solutions?

***Responses vary.* If the equations are equivalent, then the system will have infinitely many solutions.**

Accelerated 7 | Unit 5 Lesson 15 449 Activity 1

Key Takeaway: When a system of linear equations has no solution, the lines have the same slope and different y-intercepts. When there are infinitely many solutions, the lines have the same slope and the same y-intercept (i.e., the equations are equivalent.)

Pairs | 15 min

Activity 2 Sorting Systems

Purpose: Students use the structure of equations in a system to categorize them by number of solutions. **(MTR.5.1)**

Short on time? Consider facilitating Problem 6 as a class.

1 Launch

Consider sharing that students will sort systems of equations based on their number of solutions.

Students will need one or more partners for this activity.

2 Monitor

Encourage students to share their reasoning with a partner and work together to reach an agreement about how to sort the systems of equations.

A **Accessibility: Executive Functioning** Invite students to focus on one system at a time to support students in carrying out multiple steps.

ELL **English Language Learners** Provide sentence starters to help students explain their strategy (e.g., "I matched ___ with ___ because . . ." or "These two cards don't belong in the same group because . . ."). **(Writing and Speaking)**

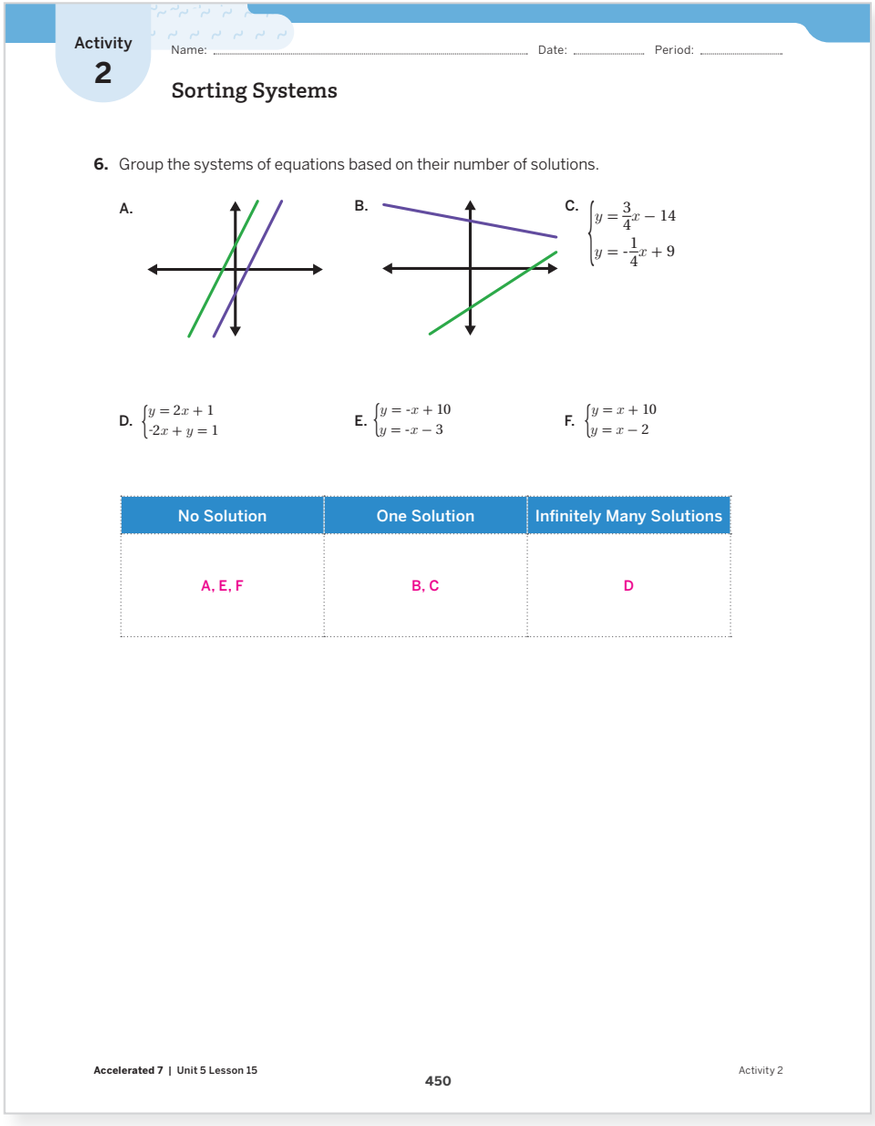
Note: If time allows, invite pairs to compare their groups, justify their thinking, and make revisions based on their conversation.

Look and listen for sorting strategies to share during the Connect, such as:

- Sorting systems based on their slopes and y-intercepts.
- Re-writing equations to show equivalence for systems with infinitely many solutions.

D **Differentiation: (Stretch)** Encourage students who correctly sort all of the systems to create their own systems and challenge their partner to decide if the system will have one, no, or infinitely many solutions.

Activity 2 continued >



Activity 2

Name: Date: Period:

Sorting Systems

6. Group the systems of equations based on their number of solutions.

A.

B.

C. $\begin{cases} y = \frac{3}{4}x - 14 \\ y = -\frac{1}{4}x + 9 \end{cases}$

D. $\begin{cases} y = 2x + 1 \\ -2x + y = 1 \end{cases}$

E. $\begin{cases} y = -x + 10 \\ y = -x - 3 \end{cases}$

F. $\begin{cases} y = x + 10 \\ y = x - 2 \end{cases}$

No Solution	One Solution	Infinitely Many Solutions
A, E, F	B, C	D

Accelerated 7 | Unit 5 Lesson 15 450 Activity 2

Pairs | 15 min

Activity 2 Sorting Systems (continued)

Purpose: Students use the structure of equations in a system to categorize them by number of solutions. **(MTR.5.1)**

1 Monitor

Display Oscar's and Melanie's strategies. Invite students to think independently before discussing their ideas with a partner.

Listen to partner discussions for student ideas to highlight in the Connect.

A **Accessibility: Memory and Attention** Invite students to record their ideas in their Student Edition or on paper.

2 Connect

Create a record of what is alike and different between the strategies, along with the names of the students who shared those ideas.

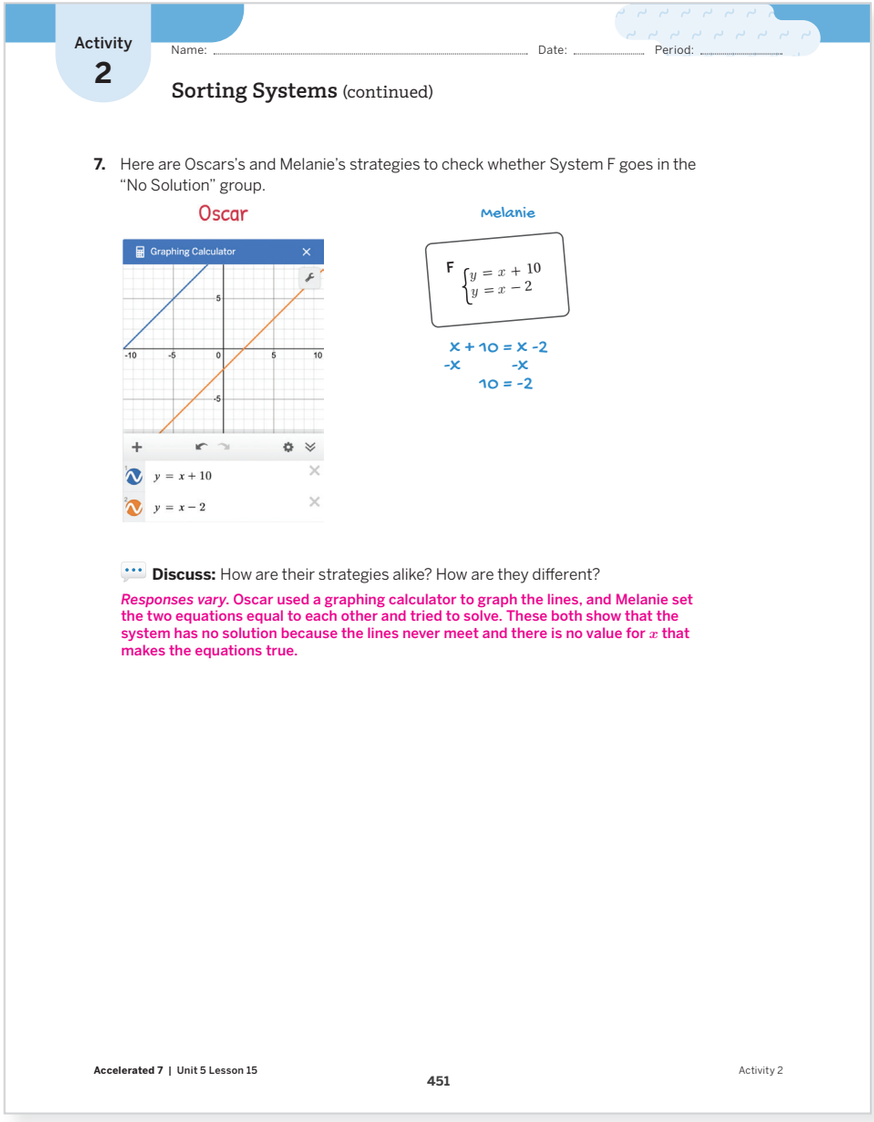
MLR **MLR7: Compare and Connect** Invite students to analyze and compare each strategy, and then connect to their strategies from Problem 7.

Consider asking:

- "What did each student do?"
- "How does each strategy compare to how you sorted equations?"
- "How does each student's work show that the system has no solution?"
- "What do you think was helpful about each strategy?"
- "When might you use each strategy?"
- "Why did the different strategies lead to the same outcome?"

ELL **English Language Learners** Provide sentence starters to help students explain their strategy (e.g., "The strategies are alike because . . ." or "The strategies are different because . . ."). **(Writing and Speaking)**

Note: If time allows, invite students to go back to Problem 6 and use the graphing calculator to check their sorting.



Activity 2

Name: Date: Period:

Sorting Systems (continued)

7. Here are Oscars's and Melanie's strategies to check whether System F goes in the "No Solution" group.

Oscar

Melanie

F $\begin{cases} y = x + 10 \\ y = x - 2 \end{cases}$

x + 10 = x - 2
-x -x
10 = -2

Discuss: How are their strategies alike? How are they different?

Responses vary. Oscar used a graphing calculator to graph the lines, and Melanie set the two equations equal to each other and tried to solve. These both show that the system has no solution because the lines never meet and there is no value for x that makes the equations true.

Accelerated 7 | Unit 5 Lesson 15 451 Activity 2

Whole Class | 5 min

Synthesis

Purpose: Students synthesize their understanding of how to determine the number of solutions for a system of equations by using the structure of the equations or graphs. **(MTR.5.1)**

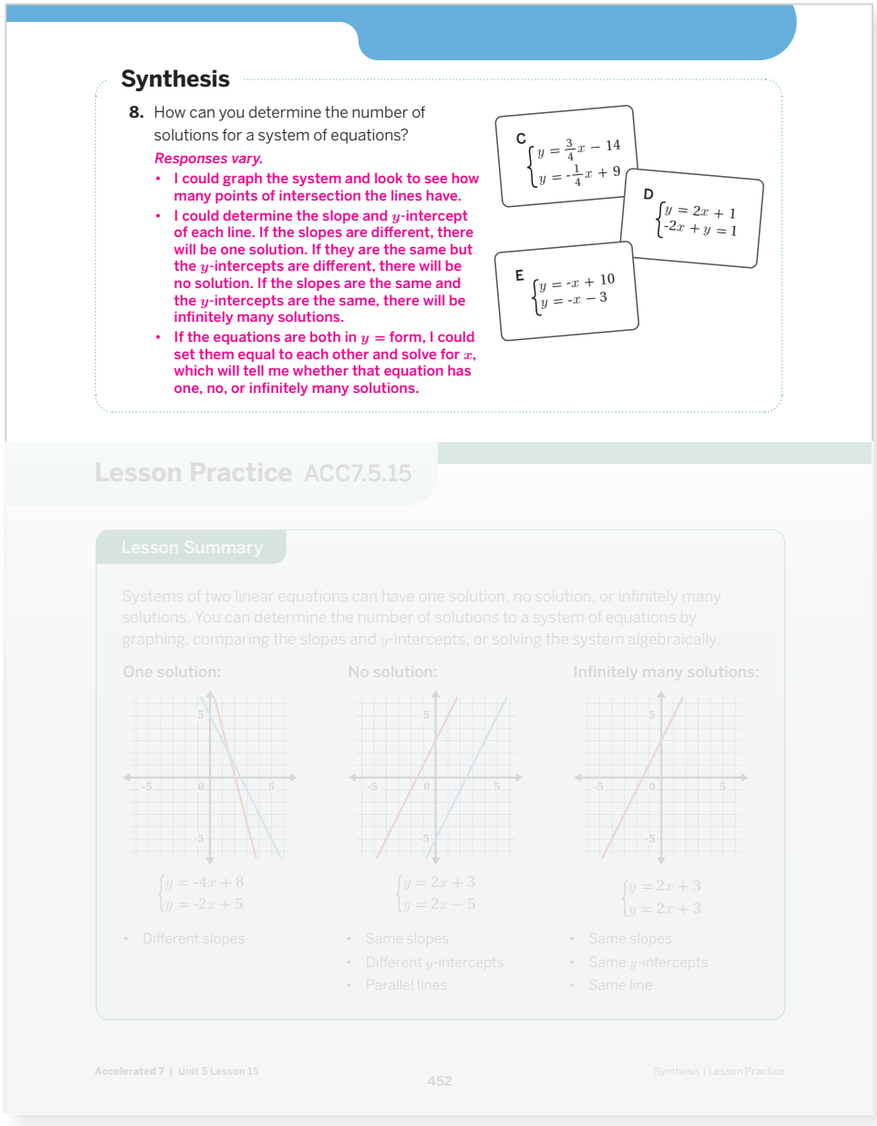
Invite students to respond independently, and then share their thinking with a partner.

Collect and share a variety of ideas, such as:

- Graphing the equations and determining the point(s) of intersection.
- Using the structure of the equations.
- Solving the system algebraically.

MLR **MLR2 Collect and Display** Use this routine to formalize the strategies for determining whether a system of linear equations has one, no, or infinitely many solutions using students' language. Display any new language, examples, or diagrams for students to reference later.

Note: If time allows, invite students to share with a partner what makes sense about each strategy and what questions they still have.



Synthesis

8. How can you determine the number of solutions for a system of equations?

Responses vary.

- I could graph the system and look to see how many points of intersection the lines have.
- I could determine the slope and y-intercept of each line. If the slopes are different, there will be one solution. If they are the same but the y-intercepts are different, there will be no solution. If the slopes are the same and the y-intercepts are the same, there will be infinitely many solutions.
- If the equations are both in $y =$ form, I could set them equal to each other and solve for x, which will tell me whether that equation has one, no, or infinitely many solutions.

C $\begin{cases} y = \frac{3}{4}x - 14 \\ y = -\frac{1}{4}x + 9 \end{cases}$

D $\begin{cases} y = 2x + 1 \\ -2x + y = 1 \end{cases}$

E $\begin{cases} y = -x + 10 \\ y = -x - 3 \end{cases}$

Lesson Practice ACC7.5.15

Lesson Summary

Systems of two linear equations can have one solution, no solution, or infinitely many solutions. You can determine the number of solutions to a system of equations by graphing, comparing the slopes and y-intercepts, or solving the system algebraically.

One solution:

$\begin{cases} y = -4x + 8 \\ y = -2x + 5 \end{cases}$

- Different slopes

No solution:

$\begin{cases} y = 2x + 3 \\ y = 2x - 5 \end{cases}$

- Same slopes
- Different y-intercepts
- Parallel lines

Infinitely many solutions:

$\begin{cases} y = 2x + 3 \\ y = 2x + 3 \end{cases}$

- Same slopes
- Same y-intercepts
- Same line

Accelerated 7 | Unit 5 Lesson 15 452 Synthesis | Lesson Practice

Lesson Takeaway: Systems of two linear equations can have no, one, or infinitely many solutions. You can determine the number of solutions to a system of equations by graphing, comparing the slopes and y-intercepts, or solving the system algebraically.

Independent | 5 min

Show What You Know

Purpose: Students demonstrate their understanding by using the structure of a system or its graph to determine the number of solutions. **(MTR.5.1)**

Today's Goals

1. **Goal:** Determine whether systems of equations have no solution, one solution, or infinitely many solutions.
2. **Language Goal:** Justify whether a system of equations has one solution, no solution, or infinitely many solutions. **(Writing, Speaking, and Listening)**

Show What You Know (Assessment Resources)

Name: _____ Date: _____ Period: _____

Show What You Know 5.15

How many solutions will this system of equations have?

$y = 3x + 5$
$y = 3x - 5$

(No solution) One solution Infinitely many solutions

Explain your thinking.

Explanations vary. The graphs of these equations will have the same slope but different y-intercepts. This means the lines will never meet.

D Differentiation Use after Lesson 15

Support

Provide targeted intervention for students by using these resources.

If students need support determining whether a system of linear equations has no solution, one solution, or infinitely many solutions:

» **Respond to Student Thinking:**

- Revisit **Activity 1** as a class and invite students to share strategies for determining the number of solutions to a system using equations.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**

- Invite students to explore the **Sub-Unit 3 Extension Activities**.
- Invite students to write systems of equations that have one solution, no solutions, and infinitely many solutions.

Professional Learning

Take a moment to reflect on this lesson. How did MLR7: Compare and Connect support students in determining and justifying whether a system of equations has no solution, one solution, or infinitely many solutions?

Lesson Practice

Independent

Provide students with sufficient practice to build and reinforce their conceptual understanding, fluency, and application of mathematical topics, assessment practice, and ongoing spiral review.

Synthesis

8. How can you determine the number of solutions for a system of equations?

Responses vary.

- I could graph the system and look to see how many points of intersection the lines have.
- I could determine the slope and y-intercept of each line. If the slopes are different, there will be one solution. If they are the same but the y-intercepts are different, there will be no solution. If the slopes are the same and the y-intercepts are the same, there will be infinitely many solutions.
- If the equations are both in $y =$ form, I could set them equal to each other and solve for x, which will tell me whether that equation has one, no, or infinitely many solutions.

C $\begin{cases} y = \frac{3}{4}x - 14 \\ y = -\frac{1}{4}x + 9 \end{cases}$

D $\begin{cases} y = 2x + 1 \\ -2x + y = 1 \end{cases}$

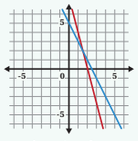
E $\begin{cases} y = -x + 10 \\ y = -x - 3 \end{cases}$

Lesson Practice ACC7.5.15

Lesson Summary

Systems of two linear equations can have one solution, no solution, or infinitely many solutions. You can determine the number of solutions to a system of equations by graphing, comparing the slopes and y-intercepts, or solving the system algebraically.

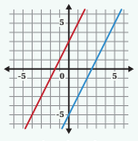
One solution:



$\begin{cases} y = -4x + 8 \\ y = -2x + 5 \end{cases}$

- Different slopes

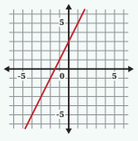
No solution:



$\begin{cases} y = 2x + 3 \\ y = 2x - 5 \end{cases}$

- Same slopes
- Different y-intercepts
- Parallel lines

Infinitely many solutions:



$\begin{cases} y = 2x + 3 \\ y = 2x + 3 \end{cases}$

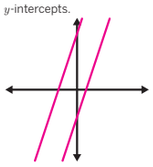
- Same slopes
- Same y-intercepts
- Same line

Lesson Practice ACC7.5.15

Name: _____ Date: _____ Period: _____

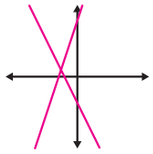
Problems 1–3: Sketch two lines that match each description. Then describe the number of solutions for each system of equations. *Graphs vary.*

1. Two lines with the same slope and different y-intercepts.



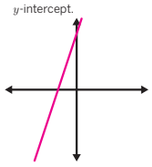
No solution

2. Two lines with different slopes.



One solution

3. Two lines with the same slope and same y-intercept.



Infinitely many solutions

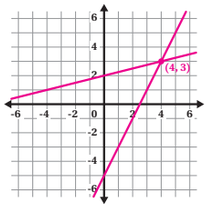
4. How many solutions does this system have?

$y = \frac{1}{4}x + 2$

$y = 2x - 5$

Show or explain your thinking.

One solution. *Explanations vary.* (4, 3) is the point of intersection.



Problems 5–6: Consider this system of equations: $\begin{cases} y = \frac{1}{6}x - \frac{2}{3} \\ y = \frac{1}{6}x - \frac{2}{9} \end{cases}$

5. Change one number to make a new system with one solution.

Responses vary. $\begin{cases} y = \frac{1}{3}x - \frac{2}{3} \\ y = \frac{1}{6}x - \frac{2}{9} \end{cases}$

6. Change one number to make a new system with an infinite number of solutions.

Responses vary. $\begin{cases} y = \frac{1}{6}x - \frac{2}{3} \\ y = \frac{1}{6}x - \frac{2}{3} \end{cases}$

Lesson Practice ACC7.5.15

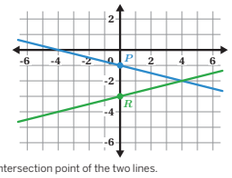
Name: _____ Date: _____ Period: _____

FAST Practice

Problems 7–8: Ali graphed this system:

$\begin{cases} y = \frac{1}{4}x - 1 \\ y = \frac{1}{2}x - 3 \end{cases}$

He marked its solutions with points P and R.



7. Which statement describes Ali's solutions?

A. His solutions are correct.

B. He marked the y-intercepts instead of the intersection point of the two lines.

C. He marked the y-intercepts instead of the x-intercepts.

D. He marked only the y-intercepts instead of the x- and y-intercepts.

8. What is the solution to the system of equations?

Record your answer in the space provided.

(4, -2)

Spiral Review

Problems 9–10: Solve each equation.

9. $\frac{3y - 6}{9} = \frac{4 - 2y}{-3}$

$y = 2$. *Work varies.*

$\frac{3y - 6}{9} \cdot 9 = \frac{4 - 2y}{-3} \cdot 9$

$3y - 6 = -3(4 - 2y)$

$3y - 6 = -12 + 6y$

$6 = 3y$

$2 = y$

10. $0.3(x - 10) - 1.8 = 2.7x$

$x = -2$. *Work varies.*

$0.3x - 3 - 1.8 = 2.7x$

$3x - 30 - 18 = 27x$

$3x - 48 = 27x$

$-48 = 24x$

$-2 = x$

Lesson Practice Item Analysis

	Problem(s)	DOK	FL B.E.S.T Math Benchmarks
On-lesson			
	1–3	1	MA.8.AR.4.2
	4–6	2	MA.8.AR.4.2
FAST Practice	7–8	2	MA.8.AR.4.3
Spiral Review			
	9–10	1	MA.8.AR.2.1

Assess and Respond

End-of-Unit Assessment

Independent | 45 min

Facilitation: Assign the End-of-Unit Assessment to learn about your students' understanding of the concepts and skills in this unit. There are two forms of the End-of-Unit Assessment: Forms A and B. If time allows, consider inviting students to complete one or more of the tasks in the Unit Synthesis and Reflection.

Item Analysis

Problem(s)	Concept or skill	Addressed in	DOK	FL B.E.S.T. Math Benchmarks
1	Solving a two-step linear inequality and graphing the solution set	Lesson 10	1	MA.8.AR.2.2
2	Using the structure of equations and graphs to identify which line would form a system of equations with a given line and solution	Lesson 12	2	MA.8.AR.4.1, MTR.5.1
3	Determining if an equation has exactly one solution	Lesson 15	1	MA.8.AR.4.2
4	Identifying the number of solutions and solve the equations with one solution	Lessons 7-8	2	MA.8.AR.2.1
5	Solving a system of equations	Lesson 14	2	MA.8.AR.4.3
6	Representing a situation in which two conditions are equal and interpreting the solution in context	Lesson 9	3	MA.8.AR.3.3
7a	Completing a table to show the relationship between two quantities	Lesson 14	2	MA.8.AR.4.3
7b	Solving a system of equations and interpreting the solution in context	Lesson 14	2	MA.8.AR.4.3

Assessment Resources

- Student Assessments
- Assessment Rubrics with Answers
- Show What You Knows and Answer Keys

Intervention and Extension Resources

- Mini-Lessons
- Extensions

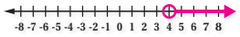
Name: Date: Period:

End-of-Unit Assessment

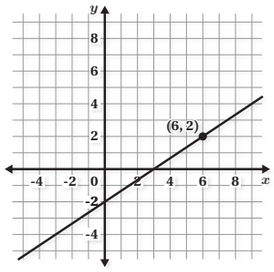
Unit 5

1. Solve $4(3 - x) < -4$ and graph the solution on the number line.

$x > 4$



2. A system of two equations has the solution (6, 2). Here is a graph of one of the equations.



What could the other equation be?

A. $y = 4x - 2$

B. $y = \frac{2}{3}x - 1$

C. $y = \frac{1}{2}x - 1$

D. $y = -\frac{3}{2}x + 6$

3. Which system of equations has exactly one solution?

A. $y = 3x + 1$, $y = -3x + 7$

B. $y = 3x + 1$, $y = 3x + 7$

C. $y = x + 10$, $y = x + 10$

D. $y = -x + 10$, $y = -x + 12$

Explain your thinking.

Explanations vary. Each line in this system of equations has a different slope, so I know they are not parallel and they are not the same line. They would intersect at exactly one point.

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Name: Date: Period:

End-of-Unit Assessment (continued)

Unit 5

4. Identify the number of solutions for each equation. If it has one solution, write the solution.

a $2x - 4 = 5 - 3x$

One solution: $x = \frac{9}{5}$ (or equivalent). *Work varies.*

$2x - 4 = 5 - 3x$
$5x - 4 = 5$
$5x = 9$
$x = \frac{9}{5}$

b $12x + 9 - 4x + 3 = 17 + 8x - 5$

Infinitely many solutions. *Work varies.*

$12x + 9 - 4x + 3 = 17 + 8x - 5$
$8x + 12 = 12 + 8x$

The expressions on either side of the equation are equivalent. Any value of x will make the equation true.

c $\frac{1}{2}(8x - 14) = 3(x - 4)$

One solution: $x = -5$. *Work varies.*

$\frac{1}{2}(8x - 14) = 3(x - 4)$
$4x - 7 = 3x - 12$
$4x = 3x - 5$
$x = -5$

5. Solve this system of equations. Write the solution as an ordered pair (x, y).

$y = -3x + 18$
$y = 7x - 2$

$(2, 12)$. *Work varies.*

$-3x + 18 = 7x - 2$
$18 = 10x - 2$
$20 = 10x$
$2 = x$

$y = 7(2) - 2$
$y = 14 - 2$
$y = 12$

Name: Date: Period:

End-of-Unit Assessment (continued)

Unit 5

6. Ari and Kiri are each saving money.

- Ari starts with $100 in savings and saves $5 each week.
- Kiri starts with $40 in savings and saves $10 each week.

a After 4 weeks, who has more money in savings? Circle one.

Ari Kiri They saved the same amount

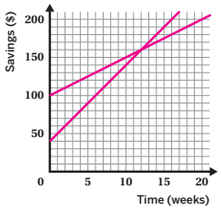
Explain your thinking.

Explanations vary. Ari has $120 because $100 + 4 \cdot 5$. Kiri has $80 because $80 = 40 + 4 \cdot 10$.

b After how many weeks will Ari and Kiri have the same amount of money in savings?

Show or explain your thinking.

Use the graph if it helps with your thinking.



12 weeks. *Explanations vary.* The solution to the equation $100 + 5n = 40 + 10n$ represents after how many weeks the amount of money in each account will be the same. The solution to this equation is $n = 12$.

Name: Date: Period:

End-of-Unit Assessment (continued)

Unit 5

7. Taylor is making jam. Her recipe calls for 3 strawberries for each apple.

a Complete the table so that each row matches Taylor's recipe.

Strawberries (lb), x	Apples (lb), y
3	10
5	6
8	0
6	4

b Taylor used 52 pieces of fruit altogether. She wrote two equations to represent the situation, where x is the number of apples and y is the number of strawberries:

$y = 3x$
$52 = x + y$

How many apples and strawberries did Taylor use?

13 apples and 39 strawberries

Show or explain your thinking.

Explanations vary. Solving the system of equations using substitution means $52 = x + (3x)$. Then $52 = 4x$, so $13 = x$. Substituting $13 = x$ into $y = 3x$, I found that $y = 39$.

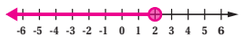
Name: Date: Period:

End-of-Unit Assessment

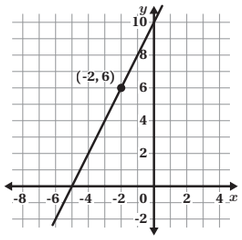
Unit 5

1. Solve $-3(x + 2) \geq -12$ and graph the solution on the number line.

$x \leq 2$



2. A system of two equations has the solution (-2, 6). Here is a graph of one of the equations.



What could the other equation be?

A. $y = 3x - 6$

B. $y = x + 8$

C. $y = 2x - 10$

D. $y = -2x + 6$

3. Which system of equations has exactly one solution?

A. $y = -x + 3$, $y = -x + 9$

B. $y = -2x + 3$, $y = 2x + 9$

C. $y = -2x + 3$, $y = -2x - 9$

D. $y = x + 3$, $y = x + 3$

Explain your thinking.

Explanations vary. Each line in this system of equations has a different slope, so I know they are not parallel and they are not the same line. They would intersect at exactly one point.

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Name: Date: Period:

End-of-Unit Assessment (continued)

Unit 5

4. Identify the number of solutions for each equation. If it has one solution, write the solution.

a. $-3x - 4 = x + 3$

One solution: $x = -\frac{7}{4}$ (or equivalent). *Work varies.*

$-3x - 4 = x + 3$
$-3x = x + 7$
$-4x = 7$
$x = -\frac{7}{4}$

b. $3x + 30 + x = 10 - x + 5x + 2$

No solution. *Work varies.*

$3x + 30 + x = 10 - x + 5x + 2$
$4x + 30 = 12 + 4x$
$30 = 12$

No value for x will make this equation true.

c. $3(x + 4) = \frac{1}{3}(6 + 12x)$

One solution: $x = 10$. *Work varies.*

$3(x + 4) = \frac{1}{3}(6 + 12x)$
$3x + 12 = 2 + 4x$
$12 = 2 + x$
$10 = x$

5. Solve this system of equations. Write the solution as an ordered pair (x, y).

$y = 5x - 6$
$y = -3x + 18$

$(3, 9)$. *Work varies.*

$5x - 6 = -3x + 18$
$8x - 6 = 18$
$8x = 24$
$x = 3$

$y = 5(3) - 6$
$y = 15 - 6$
$y = 9$

Name: Date: Period:

End-of-Unit Assessment (continued)

Unit 5

6. Aba just got a puppy and a kitten. She found out how much they will grow over the next year.

- The puppy weighs about 25 ounces and will gain about 5 ounces each month.
- The kitten weighs about 10 ounces and will gain about 10 ounces each month.

a. After 2 months, which pet will weigh more? Circle one.

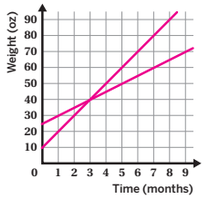
The puppy | The kitten | They will weigh the same amount

Explain your thinking.

Explanations vary. The puppy would weigh 35 ounces because $25 + 5 \cdot 2 = 35$. The kitten would weigh 30 ounces because $10 + 10 \cdot 2 = 30$.

b. After how many months will the puppy and the kitten weigh the same amount? Show or explain your thinking.

Use the graph if it helps with your thinking.



3 months. *Explanations vary.* The solution to the equation $25 + 5m = 10 + 10m$ represents after how many months the weights will be the same. The solution to this equation is $m = 3$.

Name: Date: Period:

End-of-Unit Assessment (continued)

Unit 5

7. Theo and Chey have $48 to spend on apples and strawberries to make jam. Apples cost $4 per pound and strawberries cost $6 per pound.

a. Complete the table so that each row costs $48.

Pounds of Apples, x	Pounds of Strawberries, y
9	2
6	4
0	8
12	0

b. Theo and Chey bought twice as many pounds of strawberries than pounds of apples. They wrote two equations, where x is pounds of apples and y is pounds of strawberries:

$48 = 4x + 6y$
$y = 2x$

How many pounds of each fruit did they buy?

6 pounds of apples and 3 pounds of strawberries

Show or explain your thinking.

Explanations vary. Solving the system of equations using substitution means $48 = 4x + 6(2x)$. Then $48 = 4x + 12x$, so $3 = x$. Substituting $3 = x$ into $y = 2x$, I found that $y = 6$.

D Differentiation (End-of-Unit Assessment)

Note: To **strengthen** and **stretch** students' learning, refer to the differentiation resources suggested throughout this unit and in the Unit Overview.

Sub-Unit Goals	Problem(s)	To respond to student thinking, consider:	
Sub-Unit 2: • Write and solve equations with variables on both sides of the equal sign. • Determine the number of solutions to a linear equation. • Solve two-step inequalities and graph the solution set. **(Lessons 6–10)**	1	**Support** **Strengthen**	• **Teacher Move:** Consider revisiting Lesson 10 (*Do the Two Step*)

	4	**Support**	• **Mini-Lesson:** *Solving Linear Equations with Parentheses*
			• **Teacher Moves:** Consider revisiting:
			  • Lesson 6 (*Equation Roundtable*)
			  • Lesson 7 (*Some, All, or None*)
			  • Lesson 8 (*Strategic Solving*)

		Strengthen	• **Practice:** Lesson 8, Problem 10 (*Strategic Solving*)
	6	**Support** **Strengthen**	• **Teacher Move:** Consider revisiting Lesson 9 (*When Will They Meet?*)
		Stretch	• **You're invited to explore more:** Lesson 9 (*When Will They Meet?*)

Sub-Unit 3:
• Solve systems of linear equations in two variables graphically and algebraically.
• Determine the number of solutions to a system of linear equations.
(Lessons 11–15)
	2	**Support**
		Strengthen
			• **Mini-Lesson:** *Solving Systems of Linear Equations by Graphing*
			• **Teacher Moves:**
			  • Consider revisiting Lesson 12 (*On Both Lines*)
			  • Invite students to graph each line from the options and then determine which has the solution (6, 2) for ***Form A***, or (-2, 6) for ***Form B***.

		Stretch	• **Challenge Creator:** Lesson 11 (*On or Off the Line?*)

	3	**Support**
		Strengthen
			• **Teacher Moves:**
			  • Consider revisiting Lesson 15 (*All, Some, or None? Part 2*)
			  • Invite students to solve each equation and determine if the system of equations has one, infinitely many, or no solutions.

		Stretch	• **Teacher Move:** Invite students to change one of the pairs of equations with no or infinitely many solutions to have exactly one solution.
	5, 7	**Support**	• **Mini-Lesson:** *Solving Systems of Linear Equations by Graphing* • **Teacher Move:** Consider revisiting: Lesson 14 (*Line Zapper*)
		Strengthen	• **Teacher Move:** Invite students to solve the system of equations using a different method, such as by graphing or substitution.
		Stretch	• **You're invited to explore more:** Lesson 14 (*Line Zapper*).

Glossary/Glosario

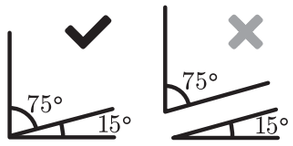
English

Español

A

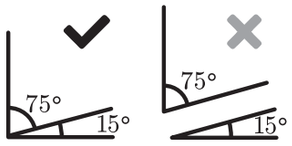
adjacent angles Angles that share a side and a vertex.

The image with the check mark shows a 75° angle and a 15° angle when they are adjacent.



ángulos adyacentes Ángulos que comparten un lado y un vértice.

La imagen con la marca de verificación muestra un ángulo de 75° y un ángulo de 15° cuando son adyacentes.



approximation A rounded value that you can use to represent a number that may be difficult to work with, such as an irrational number or a repeating decimal.

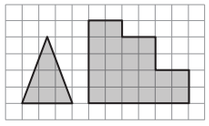
For example, the exact value of pi (π) is an irrational number, so we often use the approximate value of 3.14 in calculations involving pi.

aproximación Un valor redondeado que se puede usar para representar un número con el que podría ser complicado trabajar, como un número irracional o un decimal periódico.

Por ejemplo, el valor exacto de pi (π) es un número irracional, así que a menudo usamos el valor aproximado de 3.14 en cálculos que incluyen pi.

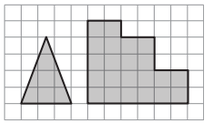
area The space inside a two-dimensional figure. It is expressed in square units.

The area of the triangle is 6 square units. The area of the other shape is 22 square units.



área El espacio dentro de una figura bidimensional. Se expresa en unidades cuadradas.

El área del triángulo mide 6 unidades cuadradas. El área de la otra figura mide 22 unidades cuadradas.



association If two variables are related to one another, we say that there is an association between them. Associations can be described as positive or negative, linear or nonlinear, and strong or weak.

Some examples of associations are:
Positive: One variable increases as the other also increases.
Negative: One variable decreases as the other increases.
Linear: Can be modeled by a straight line.
Nonlinear: Cannot be modeled by a straight line.

asociación Si dos variables están relacionadas entre sí, decimos que existe una asociación entre ambas. Las asociaciones pueden describirse como positivas o negativas, lineales o no lineales lineales y fuertes o débiles.

Algunos ejemplos de asociaciones son los siguientes:
Positiva: El valor de una variable aumenta a medida que el valor de la otra también aumenta. Negativa: El valor de una variable disminuye a medida que el valor de la otra aumenta.
Lineal: Puede modelarse con una línea recta.
No lineal: No puede modelarse con una línea recta.

B

bar chart A visual display of categorical data values that uses bars to show frequencies for each category.

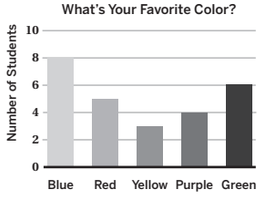
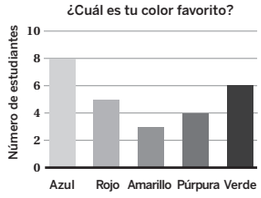


gráfico de barras Una representación visual de valores de datos categóricos que utiliza barras para mostrar frecuencias para cada categoría.



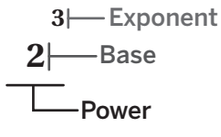
bar notation A way to represent the repeating digits of a decimal number where a small line is written over the digits that repeat.

For example, the repeating decimal $0.090909...$ can be written in bar notation as $0.\overline{09}$.

raya indicadora de decimales periódicos (vinculum) Una forma de representar los dígitos que se repiten en un número decimal. Sobre los dígitos que se repiten se traza una pequeña línea.

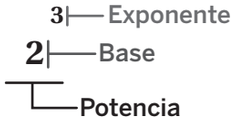
Por ejemplo, el decimal periódico $0.090909...$ puede escribirse con la raya indicadora de decimales periódicos (vinculum) como $0.\overline{09}$.

base (of a power) The number that is raised to an exponent. When determining the value of a power, the exponent tells you how many times the base should be multiplied.



In this example, 2 is the base.

base (de una potencia) El número elevado a un exponente. Al determinar el valor de una potencia, el exponente indica cuántas veces debe multiplicarse la base.



En este ejemplo, 2 es la base.

bivariate data Data that involves two variables. Each data point contains two pieces of information.

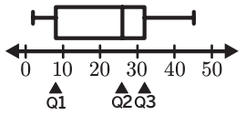
For example, a collection of students' heights and shoe sizes would be a bivariate data set.

datos bivariados Datos en los que se incluyen dos variables. Cada punto de datos contiene dos informaciones.

Por ejemplo, una colección de estaturas de estudiantes y tamaños de zapatos sería un conjunto de datos bivariados.

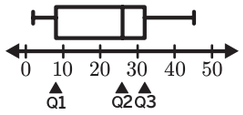
English

box plot A way to visualize numerical data sets. The data is divided into four sections using five values: the minimum, Q1, Q2 (or the median), Q3, and the maximum. A box is drawn between Q1 and Q3. The line inside the box represents the median.



Español

diagrama de caja Una forma de visualizar conjuntos de datos numéricos. Los datos se dividen en cuatro secciones utilizando cinco valores: el mínimo, Q1, Q2 (o la mediana), Q3 y el máximo. Se dibuja una caja entre Q1 y Q3. La línea dentro de la caja representa la mediana.



C

categorical data Data that can be sorted into categories instead of counted. Categorical data usually have values that are represented by words instead of numbers.

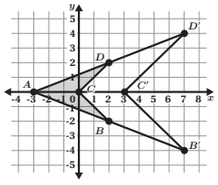
"What kind of pet do you have?" is a question that would result in categorical data.

datos categóricos Datos que pueden clasificarse en categorías en lugar de contarse. Los datos categóricos suelen tener valores que se representan mediante palabras en lugar de números.

"¿Qué tipo de mascota tienes?" es una pregunta que produciría datos categóricos.

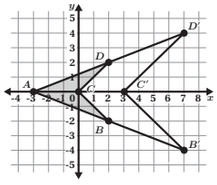
center of a dilation The point from which we measure distances in a dilation.

The center of dilation in this example is point A.



centro de una dilatación El punto desde el cual medimos las distancias en una dilatación.

El centro de dilatación en este ejemplo es el punto A.



circle A shape made out of all the points that are the same distance from a center point.

círculo Una figura formada por todos los puntos que están a la misma distancia de un punto central.

circle graph A visual display of categorical data. The whole set of data is represented by a circle and its interior. The categories are represented by fractional parts of the circle. Also called a pie chart.

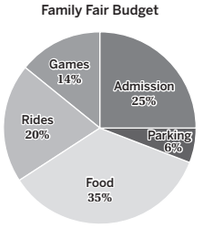
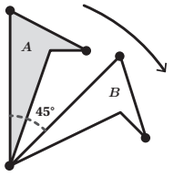


gráfico circular Una representación visual de datos categóricos. Todo el conjunto de datos está representado por un círculo y su interior. Las categorías están representadas por partes fraccionarias del círculo. También llamado gráfico de torta.

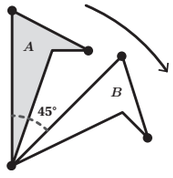
circumference The distance around a circle. If you imagine a circle as a piece of string, it is the length of the string. The circumference of a circle, C, can be calculated with the formula $C = \pi d$, where d is the diameter of the circle, or $C = 2\pi r$, where r is the radius.

circunferencia La distancia alrededor de un círculo. Si imaginamos el círculo como un trozo de cuerda, es la longitud de la cuerda. La circunferencia de un círculo, C, puede calcularse mediante la fórmula $C = \pi d$, donde d es el diámetro del círculo, o $C = 2\pi r$, donde r es el radio.

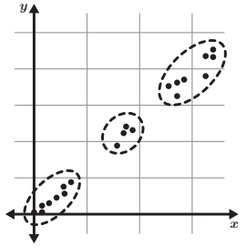
clockwise In the same direction as the hands of a clock; to the right (of a turn).



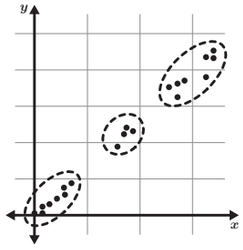
en el sentido de las manecillas del reloj Girar en la misma dirección que las manecillas de un reloj. En otras palabras, un giro a la derecha.



clusters Groups of data values that are close together.



agrupaciones (clusters) Grupos de valores de datos que están próximos entre sí.



Glossary/Glosario

English

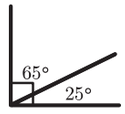
coefficient A number that is multiplied by a variable. Usually, there is no symbol between the coefficient and the variable.

In the expression $5x + 8$, 5 is the coefficient of x.

commutative property The property says $a + b = b + a$ and $a \cdot b = b \cdot a$. This means that expressions with addition or multiplication have the same sum or product no matter what order the numbers are in.

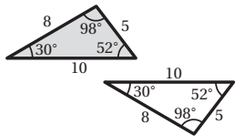
For example, $2 + 1 = 1 + 2$ or $3 \cdot 4 = 4 \cdot 3$.

complementary angles Two angles whose measures add up to $90°$.

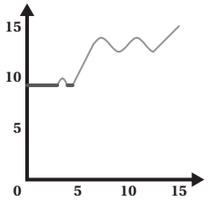


For example, a 65° angle and a 25° angle are complementary.

congruent One figure is congruent to another if they have exactly the same size and shape, or if one can be mapped to the other with a translation, rotation, or reflection.



constant (function) A function or interval of a function is constant if the y-values stay the same when the x-values go up.



The bolded parts of this function are constant.

constant of proportionality In a proportional relationship, the number used to multiply the values for one quantity to get the values for the other quantity is called the constant of proportionality.

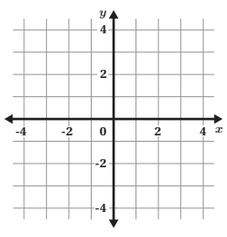
Carpet (sq. ft.)	Cost (dollars)
10	$\times\frac{2}{3}$ 15.00
20	$\times\frac{2}{3}$ 30.00
50	$\times\frac{2}{3}$ 75.00

In this table, one constant of proportionality is $\frac{2}{3}$.

converse A statement written in the opposite direction.

For example, the Pythagorean theorem states: If a triangle is right, it has side lengths such that $a^2 + b^2 = c^2$. The converse of the Pythagorean theorem is: If a triangle has side lengths such that $a^2 + b^2 = c^2$, it is a right triangle.

coordinate plane The coordinate plane consists of two axes that intersect at 0: one horizontal (often called the x-axis) and one vertical (often called the y-axis).



correspond (corresponding parts) To correspond is to match. When part of an original figure matches up with part of a copy, we call them corresponding parts. These could be points, segments, angles, or distances.

Español

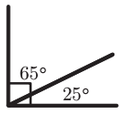
coeficiente Un número que se multiplica por una variable. Por lo general, no hay ningún símbolo entre el coeficiente y la variable.

En la expresión $5x + 8$, el coeficiente de x es 5.

propiedad conmutativa La propiedad indica que $a + b = b + a$ and $a \cdot b = b \cdot a$. Esto significa que las expresiones en las que se suma o se multiplica tienen la misma suma o el mismo producto independientemente del orden en el que estén los números.

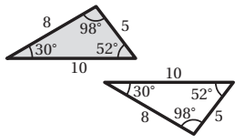
Por ejemplo, $2 + 1 = 1 + 2$ or $3 \cdot 4 = 4 \cdot 3$.

ángulos complementarios Dos ángulos cuyas medidas suman $90°$.

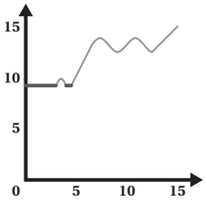


Por ejemplo, un ángulo de 65° y otro ángulo de 25° son complementarios.

congruente Una figura es congruente con otra si tienen exactamente el mismo tamaño y forma, o si una puede ser transformada en la otra mediante una traslación, rotación o reflexión.



constante (función) Una función o el intervalo de una función es constante si los valores de y se mantienen iguales cuando los valores de x aumentan.



Las partes resaltadas de esta función son constantes.

constante de proporcionalidad En una relación proporcional, una constante de proporcionalidad es el número que se puede usar para multiplicar los valores de una cantidad para obtener los valores de la otra cantidad.

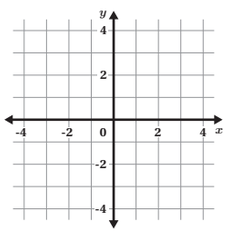
Alfombra (pies cuadrados)	Costo (dólares)
10	$\times\frac{2}{3}$ 15.00
20	$\times\frac{2}{3}$ 30.00
50	$\times\frac{2}{3}$ 75.00

En esta tabla, una constante de proporcionalidad es $\frac{2}{3}$.

recíproco Una expresión escrita en sentido contrario.

Por ejemplo, el teorema de Pitágoras indica lo siguiente: Si un triángulo es rectángulo, tiene longitudes de lado tales que $a^2 + b^2 = c^2$. El recíproco del teorema de Pitágoras sería el siguiente: Todo triángulo que tiene longitudes de lado tales que $a^2 + b^2 = c^2$, es un triángulo rectángulo.

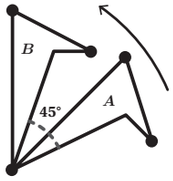
plano de coordenadas, plano cartesiano El plano de coordenadas consta de dos ejes que se intersecan en 0: uno horizontal (a menudo llamado el eje x) y uno vertical (a menudo llamado el eje y).



corresponder (partes correspondientes) Corresponder es coincidir. Cuando una parte de una figura original corresponde con una parte de una copia, se llaman partes correspondientes. Pueden ser puntos, segmentos, ángulos o distancias.

English

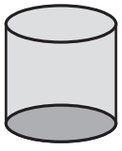
counterclockwise In the opposite direction as the hands of a clock; to the left (of a turn).



cube root The number that can be cubed to get n, written as $\sqrt[3]{n}$.
The cube root also describes the edge length of a cube with a volume of n.

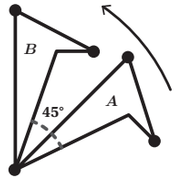
For example, the cube root of 64 ($\sqrt[3]{64}$) is 4 because 4^3 is 64. 4 is also the edge length of a cube that has a volume of 64 cubic units.

cylinder A 3-D figure that has two parallel circular bases connected by a curved surface.



Español

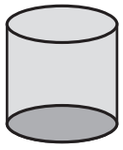
en sentido contrario a las manecillas del reloj Girar en la dirección opuesta a las manecillas de un reloj.



raíz cúbical El número que se puede elevar al cubo para obtener n. Se escribe $\sqrt[3]{n}$.
La raíz cúbica también describe la longitud de la arista de un cubo con un volumen de n.

Por ejemplo, la raíz cúbica de 64 ($\sqrt[3]{64}$) es 4 porque 4^3 es 64. 4 también es la longitud de lado de un cubo que tiene un volumen de 64 unidades cúbicas.

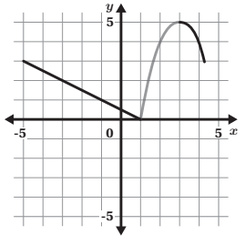
cilindro Una figura tridimensional que tiene dos bases circulares paralelas conectadas por una superficie curva.



D

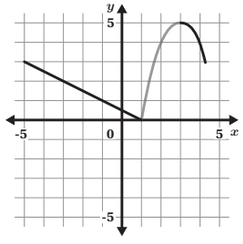
decreasing A function, or interval of a function, is decreasing if the y-values go down when the x-values go up.

The bolded parts of this function are decreasing.



decreciente Una función, o un intervalo de una función, es decreciente si los valores de y disminuyen cuando los valores de x aumentan.

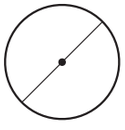
Las partes resaltadas de esta función son decrecientes.



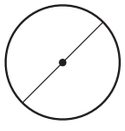
dependent variable (output) A variable whose value is based on the value of another variable or set of variables. The dependent variable is typically on the vertical axis of a graph and in the right-hand column of a table.
In a function, the dependent variable is sometimes called the output.

variable dependiente (salida) Una variable cuyo valor se basa en el valor de otra variable o un conjunto de variables. La variable dependiente suele estar en el eje vertical de una gráfica y en la columna derecha de una tabla.
En una función, la variable dependiente a veces se denomina la salida.

diameter The distance from one point on a circle through the center to another point on the circle. It is also the longest distance across the circle.

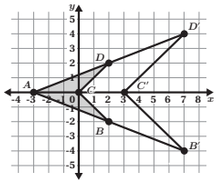


diámetro La distancia entre un punto y otro en un círculo, pasando por el centro. También es la distancia mayor entre un punto y otro en un círculo.



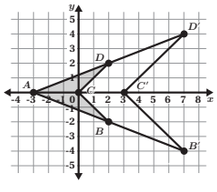
dilation A type of transformation that creates scaled copies. A dilation moves every point in a figure away from a center of dilation by the distance of a scale factor.

In this example, polygon $ABCD$ is dilated using point A as the center of dilation and a scale factor of 2. Point D' is twice as far away from point A as point D along the same ray.



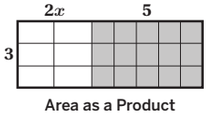
dilatación Un tipo de transformación que produce copias a escala. Una dilatación aleja cada punto de una figura del centro de dilatación de acuerdo con un factor de escala.

Aquí, el polígono $ABCD$ se dilata usando punto A como centro de dilatación y un factor de escala de 2. El punto D' está al doble de la distancia del punto A que el punto D en la misma semirrecta.



distributive property The distributive property says that multiplying a number by the sum of two or more terms is equal to multiplying the number by each term separately before adding them together.

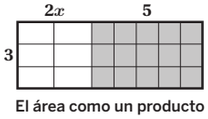
For example, $3(2x + 5) = 3 \cdot 2x + 3 \cdot 5 = 6x + 15$.



Area as a Product
$3(2x + 5)$

propiedad distributiva La propiedad distributiva indica que la multiplicación de un número por la suma de dos o más términos es igual a la suma de las multiplicaciones de dicho número por cada término individualmente.

Por ejemplo: $3(2x + 5) = 3 \cdot 2x + 3 \cdot 5 = 6x + 15$.

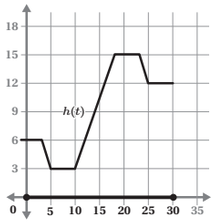


El área como un producto
$3(2x + 5)$

Glossary/Glosario

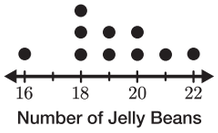
English

domain The set of all possible input values for a function or relation. The domain can be described in words or as an inequality.



The domain of this graph can be described as:
All numbers from 0 to 30 or $0 \leq t \leq 30$.

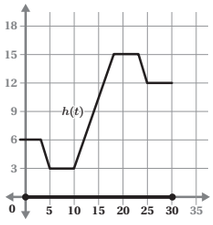
dot plot One way to visualize data. Every data point is shown as a dot above its value on a number line. Data sets with repeating values will have multiple dots aligned with the same value. A dot plot is sometimes called a line plot.



For example, this dot plot shows that 3 students guessed that there were 18 jelly beans in a jar.

Español

dominio El conjunto de todos los valores de entrada posibles de una función o relación. El dominio puede describirse con palabras o como una desigualdad.



El dominio de esta gráfica puede describirse de la siguiente manera:
Todos los números del 0 al 30 o $0 \leq t \leq 30$.

diagrama de puntos Una forma de visualizar datos. Cada punto de datos se muestra como un punto encima de su valor en una recta numérica. Los conjuntos de datos con valores repetidos tienen múltiples puntos alineados en un mismo valor.
Un diagrama de puntos a veces se llama gráfico de líneas.

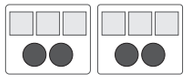
Por ejemplo, este diagrama de puntos muestra que 3 estudiantes estimaron que había 18 frijolitos de jalea en un tarro.

E

equivalent expressions Expressions that are equal for every value of a variable.

For example, $6x + 2x$ is equivalent to $5x + 3x$. No matter what value x is, the two expressions are always equal.

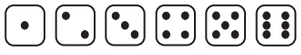
equivalent ratios Two ratios are equivalent if you can multiply each of the values in the first ratio by the same number to get the values in the second ratio.



3 : 2 is equivalent to 6 : 4 because $3 \cdot 2 = 6$ and $2 \cdot 2 = 4$.

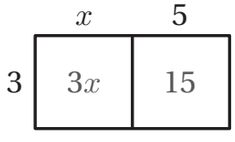
One lemonade recipe uses 3 cups of water and 2 lemons. Another uses 6 cups of water and 4 lemons. Both recipes will taste the same.

event A set of one or more outcomes in a chance experiment.



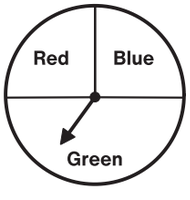
For example, if we roll a number cube, there are six possible outcomes.

expand To expand is to use the distributive property to multiply the factors in an expression and rewrite the expression as a sum. The new expression is equivalent to the original expression.



For example, we can expand $3(x + 5)$ to get the equivalent expression $3x + 15$.
The sum of the two boxes in the diagram show the expression in expanded form.

experiment An action or activity that you can do over and over again without knowing what the outcome will be each time.

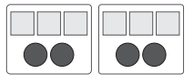


For example, each time you spin the spinner, it could land on red, blue, or green.

expresiones equivalentes Expresiones que son iguales para todos los valores de una variable.

Por ejemplo, $6x + 2x$ es equivalente a $5x + 3x$. Independientemente del valor de x, las dos expresiones son siempre iguales.

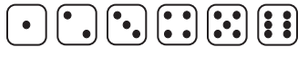
razónes equivalentes Dos razones son equivalentes si cada uno de los valores de la primera razón se puede multiplicar por el mismo número para obtener los valores de la segunda razón.



3 : 2 es equivalente a 6 : 4 porque $3 \cdot 2 = 6$ y $2 \cdot 2 = 4$.

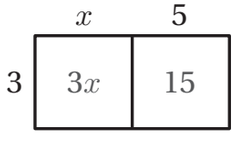
Una receta de limonada lleva 3 tazas de agua y 2 limones. Otra lleva 6 tazas de agua y 4 limones. Ambas recetas tendrán el mismo sabor.

evento Un conjunto de uno o más resultados en un experimento de azar.



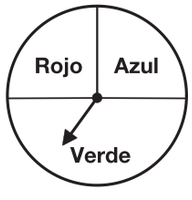
Por ejemplo, si tiramos un dado, hay seis resultados posibles.

expandir Expandir significa usar la propiedad distributiva para multiplicar los factores en una expresión y reescribir la expresión como una suma. La nueva expresión es equivalente a la expresión original.



Por ejemplo, podemos expandir $3(x + 5)$ para obtener la expresión equivalente $3x + 15$.
La suma de las dos casillas en el diagrama muestra la expresión en forma expandida.

experimento Una acción o actividad que se puede llevar a cabo una y otra vez sin saber cuál será el resultado en cada ocasión.



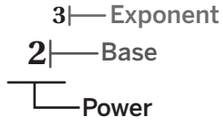
Por ejemplo, cada vez que se gira la ruleta, podría caer en rojo, azul o verde.

English

experimental probability Experimental probability is the ratio of the number of times an event occurs to the total number of trials conducted. It is determined through actual experiments rather than theoretical calculations.

exponent A number used to describe repeated multiplication.

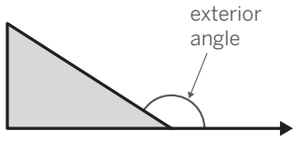
In this example, 3 is the exponent, which means $2^3 = 2 \cdot 2 \cdot 2$.



expression A set of numbers, symbols, and operators (such as + and •) grouped together to represent a value. Unlike equations, expressions do not include an equal sign (=).

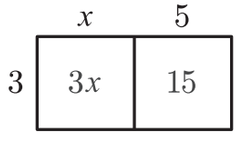
For example, $5x$, $m - 4$, $3y + 0.5$, and $2(n + 8)$ are all expressions.

exterior angle An exterior angle of a polygon is the angle that is formed between a side of the polygon and a line extended from the next side.



F

factor To factor is to use the distributive property to rewrite an expression as the product of two or more factors. The new expression is equivalent to the original expression.



For example, we can factor $3x + 15$ to get the equivalent expression $3(x + 5)$.
The product of the two sides in the diagram show the expression in factored form.

function A relationship that assigns exactly one output to each possible input.

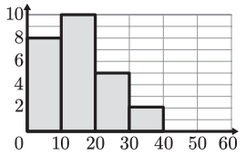
G

greatest common factor (GCF) The largest number that is a common factor of two numbers.

The common factors of 8 and 12 are 1, 2, and 4. The greatest common factor is 4.

H

histogram A way to visualize numerical data where the data is grouped into bins represented by rectangles. The height of each rectangle shows how many values are in that bin.



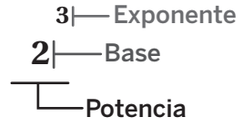
For example, this histogram shows that there are 8 data values from 0 up to (but not including) 10.

Español

probabilidad experimental La probabilidad experimental es la razón entre el número de veces que ocurre un suceso y el número total de ensayos realizados. Se determina mediante experimentos reales en vez de cálculos teóricos.

exponente Un número que se usa para describir multiplicaciones repetidas.

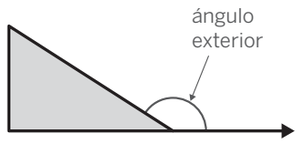
En este ejemplo, 3 es el exponente, lo cual significa que $2^3 = 2 \cdot 2 \cdot 2$.



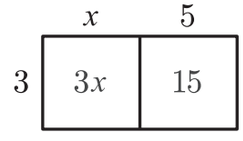
expresión Un conjunto de números, símbolos y operadores (como + y •) agrupados para representar un valor. A diferencia de las ecuaciones, las expresiones no incluyen un signo igual (=).

Por ejemplo, $5x$, $m - 4$, $3y + 0.5$, y $2(n + 8)$ son expresiones.

ángulo exterior Un ángulo exterior de un polígono es el ángulo que se forma entre un lado del polígono y una línea extendida desde el siguiente lado.



factorizar Factorizar significa usar la propiedad distributiva para reescribir una expresión como el producto de dos o más factores. La nueva expresión es equivalente a la expresión original.



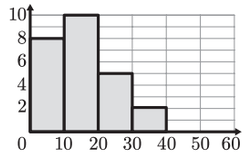
Por ejemplo, podemos factorizar $3x + 15$ para obtener la expresión equivalente $3(x + 5)$.
El producto de los dos lados del diagrama muestra la expresión en forma factorizada.

función Una relación que asigna exactamente una salida a cada entrada posible.

máximo común divisor (MCD) El número mayor que es factor común de dos números.

Los factores comunes de 8 y 12 son 1, 2 y 4. El máximo común divisor es 4.

histograma Una forma de visualizar datos numéricos en la que los datos se agrupan en intervalos que se representan con rectángulos. La altura de cada rectángulo muestra cuántos valores hay en ese intervalo.

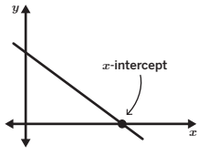


Por ejemplo, este histograma muestra que hay 8 valores de datos del 0 al 10 (sin incluirlo).

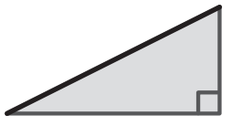
Glossary/Glosario

English

horizontal intercept The point where the graph of a line crosses the horizontal axis or when $y = 0$.
The horizontal intercept is sometimes called the x-intercept.

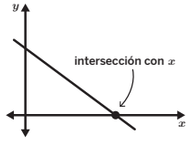


hypotenuse The side of a right triangle that is opposite the right angle. The hypotenuse is always the longest side of a right triangle.

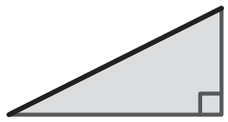


Español

intersección horizontal El punto donde la gráfica de una recta se cruza con el eje horizontal o cuando $y = 0$.
La intersección horizontal a veces se denomina intersección con el eje x.



hipotenusa El lado del triángulo rectángulo que está opuesto al ángulo recto. La hipotenusa siempre es el lado más largo de un triángulo rectángulo.



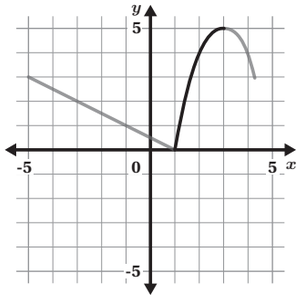
I

image A new figure that is created after a transformation of an original figure (called the pre-image). Every part of the pre-image moves in the same way to match up with a part of the image.

imagen Una nueva figura que se produce después de la transformación de una figura original (denominada preimagen). Todas las partes de la preimagen se mueven de la misma forma para coincidir con cada parte de la imagen.

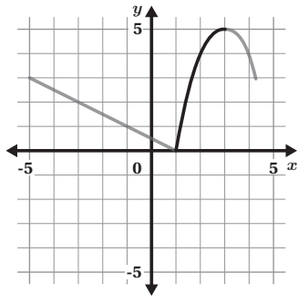
increasing A function, or interval of a function, is increasing if the y-values go up when the x-values go up.

The bolded part of this function is increasing.



creciente Una función, o un intervalo de una función, es creciente si los valores de y aumentan cuando los valores de x aumentan.

La parte resaltada de esta función es creciente.



independent variable (input) A variable whose value is not based on the value of any other variable. The independent variable is typically on the horizontal axis of a graph and in the left-hand column of a table.
In a function, the independent variable is sometimes called the input.

variable independiente (entrada) Una variable cuyo valor no depende del valor de ninguna otra variable. La variable independiente suele estar en el eje horizontal de una gráfica y en la columna izquierda de una tabla.
En una función, la variable independiente a veces se denomina la entrada.

inequality A comparison statement that uses the symbols $<$ or $>$. Inequalities are used to represent the relationship between numbers, variables, or expressions that are not always equal.

For example, the inequality $y > 30$ means that the value of the expression y is any number greater than 30.

desigualdad Un enunciado de comparación que utiliza los símbolos $<$ o $>$. Las desigualdades se usan para representar la relación entre números, variables o expresiones que no siempre son iguales.

Por ejemplo, la desigualdad $y > 30$ significa que el valor de la expresión y es cualquier número mayor que 30.

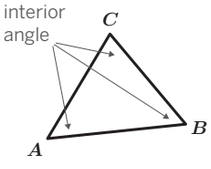
infinitely many solutions An equation has infinitely many solutions if it is true for any value of the variable. A system of equations has infinitely many solutions if the equations in the system are equivalent. Equivalent equations will create the same line on a graph, so every point on the line is a solution to each equation in the system.

For example, the equation $3x + 6 = 3(x + 2)$ has infinitely many solutions because the equation is true for any value of x.

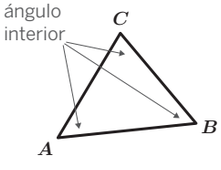
infinitas soluciones Una ecuación tiene un número infinito de soluciones si es verdadera independientemente del valor de la variable. Un sistema de ecuaciones tiene un número infinito de soluciones si las ecuaciones en el sistema son equivalentes. Las ecuaciones equivalentes producen la misma recta en una gráfica, por lo que cada punto en la recta es una solución de cada ecuación en el sistema.

Por ejemplo, la ecuación $3x + 6 = 3(x + 2)$ tiene un número infinito de soluciones porque la ecuación es verdadera para cualquier valor de x.

interior angle An angle between adjacent sides of a polygon, inside the polygon.



ángulo interior Un ángulo entre lados adyacentes de un polígono, dentro del polígono.



English

inverse operations Two operations that undo each other are called inverse operations.

Adding and subtracting are inverse operations. Multiplying and dividing are inverse operations.

irrational number A number that cannot be written as a fraction of two integers, where the denominator is not zero.

For example, 2 is a rational number because it can be written as $\frac{2}{1}$, whereas pi (π) is irrational because it cannot be written as a fraction of two non-zero integers.

Español

operaciones inversas Dos operaciones que se anulan entre sí se llaman operaciones inversas.

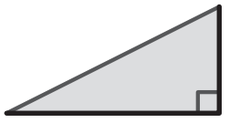
La suma y la resta son operaciones inversas. La multiplicación y la división son operaciones inversas.

número irracional Un número que no se puede escribir como una fracción de números enteros, donde el denominador es diferente de cero.

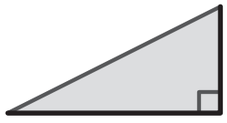
Por ejemplo, 2 es un número racional porque se puede escribir como $\frac{2}{1}$, mientras que pi (π) es irracional porque no se puede escribir como una fracción de dos números enteros distintos de cero.

L

legs The two sides of a right triangle that are not the hypotenuse. The legs are the sides that form the right angle.



catetos Los dos lados de un triángulo rectángulo que no son la hipotenusa. Los catetos son los lados que forman el ángulo recto.



like terms Two or more terms that have the same variables and exponent values. Numbers, decimals, and fractions are all like terms.

For example, $8x$ and $12x$ are like terms because they both have a variable of x. $8x$ and 12 are not like terms. $8x$ and $12x^2$ are also not like terms because they have different exponents.

términos semejantes Dos o más términos que tienen variables y valores de exponentes iguales. Los números enteros, los decimales y las fracciones son términos semejantes.

Por ejemplo, $8x$ y $12x$ son términos semejantes porque ambos tienen una variable x. $8x$ y 12 no son términos semejantes. $8x$ y $12x^2$ tampoco son términos semejantes porque tienen exponentes diferentes.

line graph A set of data points plotted on a coordinate plane and connected by lines. Line graphs allow us to investigate connections between two variables, including rates of change between consecutive points or groups of points.

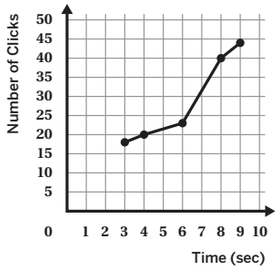
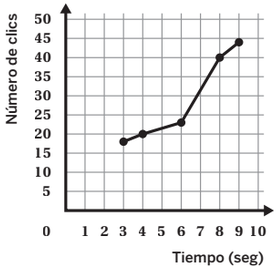


gráfico lineal Un conjunto de puntos de datos trazados en un plano de coordenadas y conectados por líneas. Los gráficos de líneas nos permiten investigar las conexiones entre dos variables, incluyendo las tasas de cambio entre puntos consecutivos o grupos de puntos.

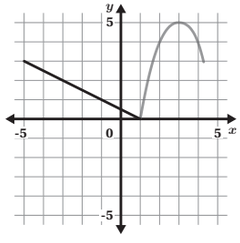


line of fit See *linear model*.

línea de ajuste Ver *modelo lineal*.

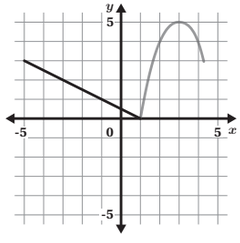
linear A function is linear when its graph is a straight, non-vertical line.

The bolded part of this function is linear.



lineal Una función es lineal cuando su gráfica es una línea recta que no es vertical.

La parte resaltada de esta función es lineal.



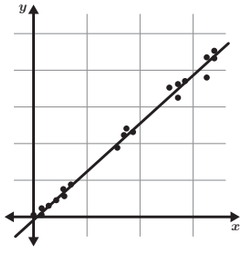
linear function A function that can be defined by an equation in the form $y = mx + b$, where m represents the slope and b represents the vertical intercept.
A vertical line is not a linear function because it has an input with different outputs.

función lineal Una función que puede definirse mediante una ecuación de la forma $y = mx + b$, donde m representa la pendiente y b representa la intersección vertical.
Una línea vertical no es una función lineal porque tiene una entrada con diferentes salidas.

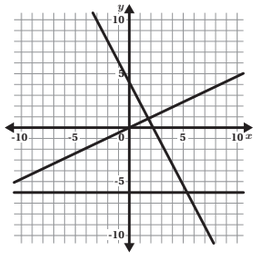
Glossary/Glosario

English

linear model A line that shows, or models, the general direction or trend of a group of points in a data set.
We can use linear models to make predictions about values related to a given data set.

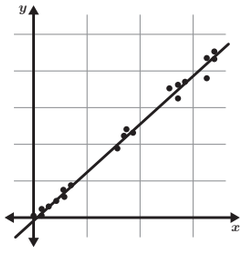


linear relationship A relationship is called linear when its graph is a line.

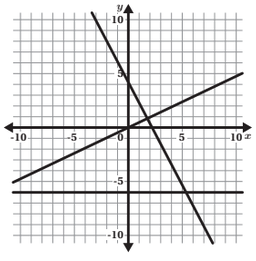


Español

modelo lineal Una línea que muestra, o modela, la dirección o tendencia general de un grupo de puntos en un conjunto de datos. Podemos emplear modelos lineales para hacer predicciones sobre valores relativos a un conjunto de datos determinado.



relación lineal Una relación se llama lineal cuando su gráfica es una recta.



M

mapping diagram A visual representation of a relation that shows a list of inputs and outputs and uses arrows to show how they are related.

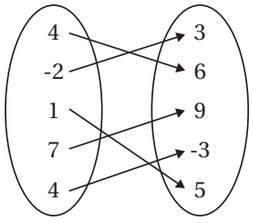
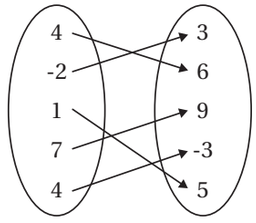


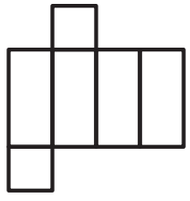
diagrama de mapeo Una representación visual de una relación que muestra una lista de entradas y salidas y utiliza flechas para mostrar cómo están relacionadas.



N

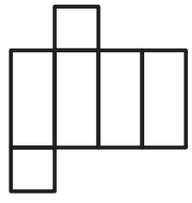
net A two-dimensional representation of a three-dimensional shape. It can be folded to make a polyhedron.

Here is a net for a rectangular prism.



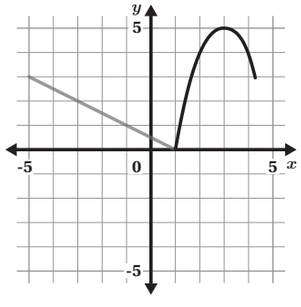
red Una representación bidimensional de una figura tridimensional. Puede plegarse para formar un poliedro.

Esta es una red de un prisma rectangular.



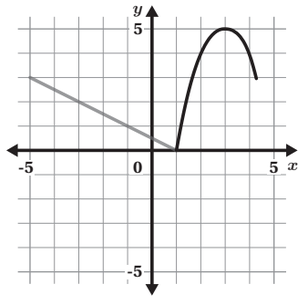
nonlinear A function is nonlinear when its graph is not a straight line.

The bolded part of this function is nonlinear.



no lineal Una función es no lineal cuando su gráfica no es una línea recta.

La parte resaltada de esta función es no lineal.



numerical data Data presented as numbers, quantities, or measurements that can be meaningfully compared. It is sometimes called quantitative data.

For example, data on a group of students' heights or arm spans.

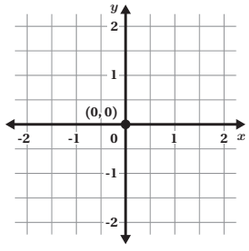
datos numéricos Datos que se presentan como números, cantidades o medidas que pueden compararse de forma significativa. A veces se denominan datos cuantitativos.

Por ejemplo, los datos de las estaturas o las longitudes de los brazos de un grupo de estudiantes.

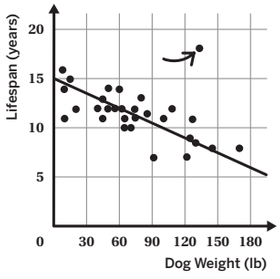
English

O

origin The point $(0, 0)$ on the coordinate plane. This is where the x-axis and the y-axis intersect.



outlier A data value that is far from the other values in the data set.



P

perfect cubes The cube of an integer is called a perfect cube.

For example, 27 is a perfect cube because $3 \cdot 3 \cdot 3 = 3^3$ and $3^3 = 27$.

perfect square The square of an integer is called a perfect square.

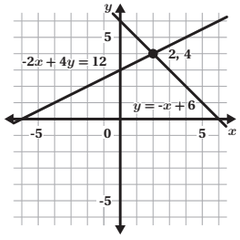
For example, 49 is a perfect square because $7 \cdot 7 = 7^2$ and $7^2 = 49$.

pi A number that represents the constant of proportionality between the diameter and circumference of any circle. The symbol for pi is π.

Some common approximations for π are 3.14 and $\frac{22}{7}$.

point of intersection A point where two lines meet.

For example, $(2, 4)$ is the point of intersection for the lines $y = -x + 6$ and $-2x + 4y = 12$.



power of ten A number written in the form 10^n, where n represents the number of times 10 is multiplied.

pre-image The name of a figure before any transformations are performed.

prime notation Using the prime symbol (′) to name points in an image that correspond to points in the pre-image.

For example, if point A is transformed, the corresponding point would be named A'.

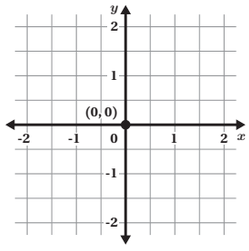
proportional Quantities are proportional if they form equivalent ratios.

For example, the heights and bases of these triangles are proportional because the ratio $6 : 12$ is equivalent to $4 : 8$.

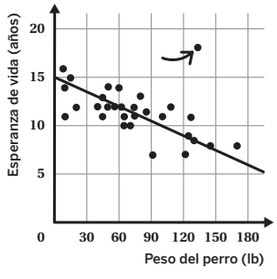
Español

O

origen El punto $(0, 0)$ en el plano de coordenadas. El punto en el que se intersecan el eje x y el eje y.



valor atípico Un valor que está lejos de los demás valores del conjunto de datos.



P

cubo perfecto El cubo de un número entero se denomina cubo perfecto.

Por ejemplo, 27 es un cubo perfecto porque $3 \cdot 3 \cdot 3 = 3^3$ y $3^3 = 27$.

cuadrado perfecto El cuadrado de un número entero se denomina cuadrado perfecto.

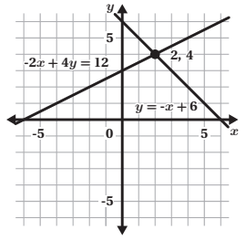
Por ejemplo, 49 es un cuadrado perfecto porque $7 \cdot 7 = 7^2$ y $7^2 = 49$.

pi Un número que representa la constante de proporcionalidad entre el diámetro y la circunferencia de cualquier círculo. El símbolo de pi es π.

Algunas aproximaciones comunes de π son 3.14 y $\frac{22}{7}$.

punto de intersección Un punto donde se cruzan dos rectas.

Por ejemplo, $(2, 4)$ es el punto de intersección de las rectas $y = -x + 6$ y $-2x + 4y = 12$.



potencia de diez Un número escrito de la forma 10^n, donde n representa el número de veces que se multiplica el 10.

preimagen El nombre de una figura antes de realizar una transformación.

notación prima Uso del signo prima (′) para denominar los puntos de una imagen que corresponden a puntos de la preimagen.

Por ejemplo, si el punto A se transforma, el punto correspondiente se denominaría A'.

proporcional Diferentes cantidades son proporcionales si forman razones equivalentes.

Por ejemplo, las alturas y las bases de estos triángulos son proporcionales porque la relación $6 : 12$ es equivalente a $4 : 8$.

Glossary/Glosario

English

proportional relationship A set of equivalent ratios. The values for one quantity are each multiplied by the same number to get the values for the other quantity.

Carpet (sq. ft.)	Cost (dollars)
10	15.00
20	30.00
50	75.00

(×1.5 from each row to the next)

For example, every cost in the table is equal to 1.5 times the number of square feet of carpet.

Pythagorean theorem The theorem that describes the relationship between the side lengths of a right triangle. The Pythagorean theorem says that the square of the hypotenuse is equal to the sum of the squares of the legs. We can write this as $a^2 + b^2 = c^2$.

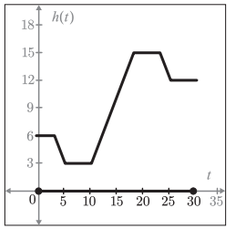
Q

quantitative data See *numerical data*.

R

radius A line segment that connects the center of a circle with a point on the circle. Every radius of a circle is the same length.

range (of a function) The set of all possible output values for a function or relation. The range can be described in words or as an inequality.



The range of this graph can be described as:
All numbers from 3 to 15.
$3 \leq h(t) \leq 15$.

rate A comparison, or ratio, that describes how two quantities change together.

rate of change See *slope*.

ratio A ratio $a : b$ is a relationship between two quantities. For every a of the first, there are b of the second.

If the ratio of apples to oranges in a fruit bowl is 2 : 3, then for every 2 apples, there are 3 oranges.
There are several ways to describe ratios.
- For every 3 squares, there are 2 circles.
- The ratio of squares to circles is 3 to 2.
- The ratio of squares to circles is 3 : 2.

rational number A number that can be written as a fraction of two integers, where the denominator is not zero.

Examples of rational numbers include 13, -74, 0, 0.2, and $\sqrt{9}$.

reciprocal The reciprocal of a fraction $\frac{a}{b}$ is $\frac{b}{a}$. The product of two fractions that are reciprocals of one another is 1.

For example, $\frac{3}{2}$ and $\frac{2}{3}$ are reciprocals because $\frac{3}{2} \times \frac{2}{3} = 1$.

Español

relación proporcional Un conjunto de razones equivalentes. Cada uno de los valores de una cantidad se multiplica por el mismo número para obtener los valores de la otra cantidad.

Alfombra (pies cuadrados)	Costo (dólares)
10	15.00
20	30.00
50	75.00

(×1.5 from each row to the next)

Por ejemplo, cada costo en la tabla es igual a 1.5 veces el número de pies cuadrados de alfombra.

teorema de Pitágoras El teorema que describe la relación entre las longitudes de lado de los triángulos rectángulos. El teorema de Pitágoras dice que el cuadrado de la hipotenusa es igual a la suma de los cuadrados de los catetos. Podemos escribirlo como $a^2 + b^2 = c^2$.

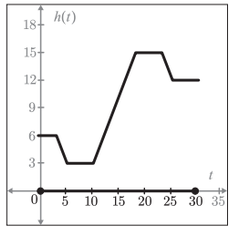
Q

datos cuantitativos Ver *datos numéricos*.

R

radio Un segmento de recta que conecta el centro de un círculo con un punto del círculo. Todos los radios de un círculo tienen la misma longitud.

rango (de una función) El conjunto de todos los posibles valores de salida de una función o relación. El rango puede describirse con palabras o como una desigualdad.

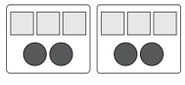


El rango de esta gráfica puede describirse de la siguiente manera:
Todos los números del 3 al 15.
$3 \leq h(t) \leq 15$

tasa Una comparación, o razón, que describe cómo cambian juntas dos cantidades.

tasa de cambio Véase *pendiente*.

razón Una razón $a : b$ es una relación entre dos cantidades. Por cada a del primero, hay b del segundo.



Si la razón de manzanas a naranjas en un frutero es 2 : 3, entonces por cada 2 manzanas hay 3 naranjas.
Hay varias formas de describir razones.
- Por cada 3 cuadrados hay 2 círculos.
- La razón de cuadrados a círculos es de 3 a 2.
- La razón de cuadrados a círculos es 3 : 2.

número racional Un número que se puede escribir como fracción de números enteros, donde el denominador es diferente de cero.

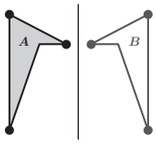
Algunos ejemplos de números racionales son 13, -74, 0, 0.2 y $\sqrt{9}$.

recíproco El recíproco de una fracción $\frac{a}{b}$ es $\frac{b}{a}$. El producto de dos fracciones que son recíprocas entre sí es 1.

Por ejemplo, $\frac{3}{2}$ y $\frac{2}{3}$ son recíprocos porque $\frac{3}{2} \times \frac{2}{3} = 1$.

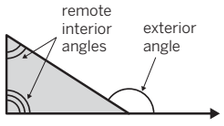
English

reflection A reflection across a line moves every point on a figure to a point directly on the opposite side of the line. The new point is the same distance from the line as it was in the original figure.



relation A way of creating input-output pairs. When a relation assigns exactly one output to every input, it is called a function.

remote interior angles The two angles that are inside the triangle and opposite from the exterior angle are the remote interior angles.



repeating decimal A decimal with one or more digits (not all zeros) that repeat forever. A repeating decimal can be written using bar notation over the digits that repeat or with the ellipses (...) at the end.

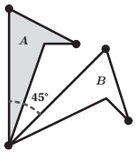
If the repeating digits are all zeros, it would be called a terminating decimal.

For example, the decimal representation of $\frac{1}{3}$ is $0.\overline{3}$, which means $0.33333...$

The decimal representation of $\frac{25}{22}$ is $1.1\overline{36}$, which means $1.1363636...$

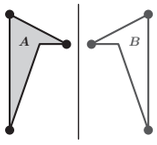
rigid transformation A move that does not change any measurements of a figure. Translations, rotations, and reflections (or any sequence of these) are rigid transformations.

rotation A rotation moves every point on a figure around a center by a given angle in a specific direction.



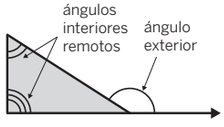
Español

reflexión Una reflexión con respecto a una línea mueve cada punto de una figura a un punto directamente en el lado opuesto de la línea. El nuevo punto está a la misma distancia de la línea que estaba en la figura original.



relación Una forma de establecer pares de entrada y salida. Cuando una relación asigna exactamente una salida a cada entrada, se denomina función.

ángulos interiores remotos Los dos ángulos que están dentro del triángulo y opuestos al ángulo exterior.



decimal periódico Un decimal con uno o más dígitos (no todos son ceros) que se repiten infinitamente. Un decimal periódico puede escribirse usando la raya indicadora de decimales periódicos (vinculum) encima de los dígitos que se repiten o con puntos suspensivos (...) al final.

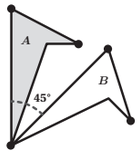
Si todos los dígitos periódicos son ceros, entonces se denomina decimal exacto.

Por ejemplo, la representación decimal de $\frac{1}{3}$ es $0.\overline{3}$, lo que significa $0.33333...$

La representación decimal de $\frac{25}{22}$ es $1.1\overline{36}$, o sea $1.1363636...$

transformación rígida Un movimiento que no cambia ninguna de las medidas de una figura. Las traslaciones, rotaciones y reflexiones son transformaciones rígidas, así como lo es cualquier secuencia de ellas.

rotación Una rotación mueve cada punto en una figura alrededor de un centro hacia una dirección específica y con un ángulo determinado.



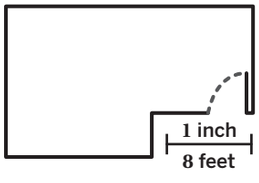
S

sample space The list of every possible outcome for a chance experiment.

For example, the sample space for tossing two coins is: heads-heads, tails-heads, heads-tails, tails-tails.

scale A scale tells us how the actual measurements of an object are represented in a drawing.

The scale of this floor plan tells us that 1 inch on the drawing represents 8 feet in the actual room.

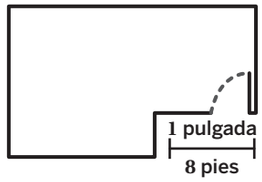


espacio muestral La lista de todos los posibles resultados de un experimento de azar.

Por ejemplo, el espacio muestral del lanzamiento de dos monedas es: cara-cara, sello-cara, cara-sello, sello-sello.

escala Una escala nos indica cómo están representadas en un dibujo o diagrama las medidas reales de un objeto.

La escala de este plano nos indica que 1 pulgada en el dibujo representa 8 pies en la habitación real.



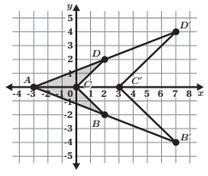
Glossary/Glosario

English

scale drawing A two-dimensional representation of an actual object or place. All the measurements in the drawing correspond to the measurements of the actual object by the same scale. Floor plans and maps are examples of scale drawings.

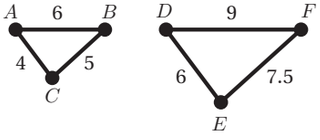
A scale may show that 1 centimeter on a map represents 30 miles on land. The distance marked on this map is 8 centimeters, or 240 miles.

scale factor The number used to create a dilation. All distances from the pre-image to the center of dilation are multiplied by the same number, called the *scale factor*.



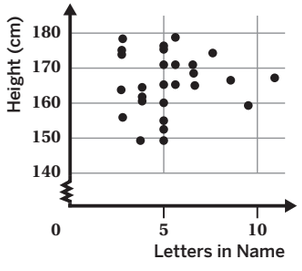
For example, the scale factor from polygon $ABCD$ to polygon $A'B'C'D'$ is 2.

scaled copy A copy of an image that may change in size, but always maintains the shape and angle measurements of the original. If a figure looks squished or stretched when compared to its original, it is not a scaled copy. To create a scaled copy, we multiply every length in the original figure by the same number.



For example, triangle DEF is a scaled copy of triangle ABC.

scatter plot A set of disconnected data points plotted on a coordinate plane. Scatter plots allow us to investigate connections between two variables.



scientific notation A way to write very large or very small numbers. In scientific notation, a number between 1 and 10 is multiplied by a power of 10.

For example, the number 425,000,000 in scientific notation is $4.25 \cdot 10^8$. The number 0.0000000783 in scientific notation is $7.83 \cdot 10^{-8}$.

significant digits The important digits in a measurement that show how precise it is. Zeros in a number are only considered significant when they are between two non-zero numbers or when they are trailing zeros in a number that has a decimal point.

For example, 450 has only two significant digits, but 450.0 has four.

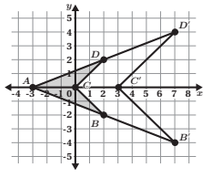
similar One figure is similar to another if translations, rotations, reflections, and dilations can be used to fit one exactly over the other. Two figures are similar if one is a scaled copy of the other.

Español

dibujo a escala Una representación bidimensional de un objeto o lugar reales. Todas las medidas del dibujo corresponden a las medidas del objeto real usando la misma escala. Los planos y los mapas son ejemplos de dibujos a escala.

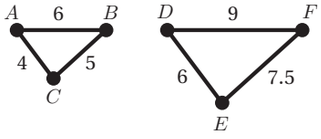
Una escala puede mostrar que 1 centímetro en un mapa representa 30 millas en tierra. La distancia mostrada en este mapa es 8 centímetros o 240 millas.

factor de escala El número que se usa para hacer una dilatación. Todas las distancias de la preimagen al centro de dilatación se multiplican por el mismo número, que se denomina *factor de escala*.



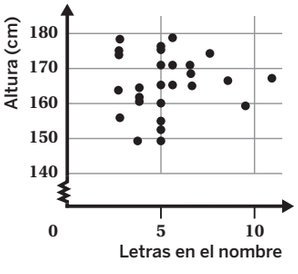
Por ejemplo, el factor de escala del polígono $ABCD$ al polígono $A'B'C'D'$ es 2.

copia a escala Una copia de una imagen que puede cambiar de tamaño, pero que siempre conserva la forma y las medidas de los ángulos de la imagen original. Si una figura luce aplastada o estirada en comparación con la original, no es una copia a escala. Para crear una copia a escala, multiplicamos todas las longitudes de la figura original por el mismo número.



Por ejemplo, el triángulo DEF es una copia a escala del triángulo ABC.

diagrama de dispersión Un conjunto de puntos de datos que no están conectados trazados en un plano de coordenadas. Los diagramas de dispersión nos permiten analizar las conexiones entre dos variables.



notación científica Una forma de escribir números muy grandes o muy pequeños. Cuando un número entre 1 y 10 está multiplicado por una potencia de 10, significa que está escrito en notación científica.

Por ejemplo, el número 425,000,000 en notación científica es $4.25 \cdot 10^8$. El número 0.0000000783 en notación científica es $7.83 \cdot 10^{-8}$.

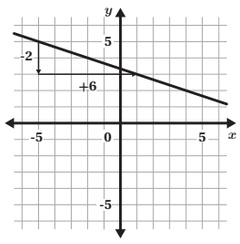
dígitos significativos Los dígitos importantes en una medición que muestran cuán precisa es. Los ceros en un número solo se consideran significativos cuando están entre dos números distintos de cero o cuando son ceros finales en un número que tiene un punto decimal.

Por ejemplo, 450 tiene solo dos dígitos significativos, pero 450.0 tiene cuatro.

semejante Una figura es semejante a otra si se pueden emplear traslaciones, rotaciones, reflexiones y dilataciones para que coincida exactamente con la otra. Dos figuras son semejantes si una es una copia a escala de la otra.

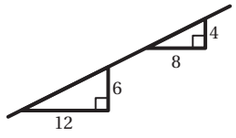
English

slope A number that describes the direction and steepness of a line. Slope represents the amount that y changes when x increases by 1. That's why the slope of a line is sometimes called a rate of change. To calculate the slope, divide the vertical distance between any two points on the line by the horizontal distance between those points.



The slope of this line is $-\frac{2}{6} = -\frac{1}{3}$.

slope triangle A triangle drawn to include two points on a line in order to determine the slope of that line. The longest side lies on the line and the other two sides are vertical and horizontal. The height of the slope triangle represents the vertical distance between the points, and the base of the triangle represents the horizontal distance between the points.



Both of these triangles are slope triangles.

solution to an equation The value or set of values that makes the equation true. A solution to an equation with two variables is a pair of values that makes the equation true, often written as an ordered pair, (x, y).

The solution to the equation $x + 15 = 8$ is $x = -7$ because $(-7) + 15 = 8$.
One solution to the equation $4x + 3y = 24$ is $(6, 0)$ because $4(6) + 3(0) = 24$.

solution to a system of equations A set of values that makes all equations in that system true.
When the equations are graphed, the solution to the system is the point of intersection.

For example, $(2, 4)$ is the solution to this system of equations, and the point of intersection on the graph.
$y = -x + 6$
$-2x + 4y = 12$

square root A positive number that can be squared to get n. Written as $\sqrt{n}$.
The square root is also the side length of a square with an area of n.

The square root of 16 ($\sqrt{16}$) is 4 because 4^2 is 16. The $\sqrt{16}$ is also the side length of a square that has an area of 16.

stem-and-leaf plot A table that organizes data by place value to compare data frequencies.

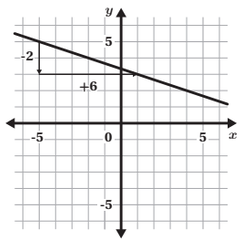
The stem-and-leaf plot shows the data set $(5, 8, 10, 14, 23, 26, 26, 30, 31, 31, 34, 35, 47, 48, 48, 61, 64)$.

Stem	Leaf
0	5 8
1	0 4
2	3 6 6
3	0 1 1 4 5
4	7 8 8
5	
6	1 4

Key: $1 \mid 0 = 10$

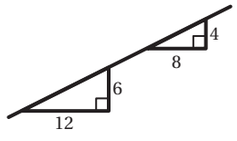
Español

pendiente Un número que describe la dirección e inclinación de una línea. La pendiente representa la cantidad en la que cambia y cuando x se incrementa en 1. Es por eso que la pendiente de una recta a veces se denomina tasa de cambio. Para calcular la pendiente, la distancia vertical entre dos puntos cualesquiera en la recta se divide entre la distancia horizontal entre dichos puntos.



La pendiente de esta recta es $-\frac{2}{6} = -\frac{1}{3}$.

triángulo de pendiente Un triángulo que se traza para incorporar dos puntos de una recta y así determinar la pendiente de dicha recta. El lado más largo se sitúa sobre la recta y los dos lados restantes son uno vertical y otro horizontal. La altura del triángulo de pendiente representa la distancia vertical entre los puntos, mientras que la base del triángulo representa la distancia horizontal entre los puntos.



Estos dos triángulos son triángulos de pendiente.

solución de una ecuación El valor o conjunto de valores que hacen que la ecuación sea verdadera. Una solución de una ecuación con dos variables es un par de valores que hacen que la ecuación sea verdadera, y a menudo se escribe como un par ordenado, (x, y).

La solución de la ecuación $x + 15 = 8$ es $x = -7$ porque $(-7) + 15 = 8$.
Una solución de la ecuación $4x + 3y = 24$ es $(6, 0)$ porque $4(6) + 3(0) = 24$.

solución de un sistema de ecuaciones Un conjunto de valores que hace que todas las ecuaciones de ese sistema sean verdaderas.
Al graficar las ecuaciones, la solución del sistema es el punto de intersección.

Por ejemplo, $(2, 4)$ es la solución de este sistema de ecuaciones y el punto de intersección en la gráfica.
$y = -x + 6$
$-2x + 4y = 12$

raíz cuadrada Un número positivo que se puede elevar al cuadrado para obtener n. Se escribe $\sqrt{n}$.
La raíz cuadrada también es la longitud de lado de un cuadrado con un área de n.

La raíz cuadrada de 16 ($\sqrt{16}$) es 4 porque 4^2 es 16. La $\sqrt{16}$ también es la longitud de lado de un cuadrado que tiene un área de 16.

diagrama de tallo y hojas Una tabla que organiza los datos por valor posicional para comparar frecuencias de datos.

El diagrama de tallo y hojas muestra el conjunto de datos $(5, 8, 10, 14, 23, 26, 26, 30, 31, 31, 34, 35, 47, 48, 48, 61, 64)$.

Tallo	Hoja
0	5 8
1	0 4
2	3 6 6
3	0 1 1 4 5
4	7 8 8
5	
6	1 4

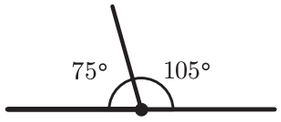
Leyenda: $1 \mid 0 = 10$

Glossary/Glosario

English

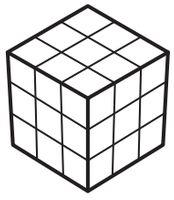
supplementary angles Two angles whose measures add up to $180°$.

For example, a $75°$ angle and a $105°$ angle are supplementary.



surface area The sum of the areas of the surfaces of a solid.

For example, the six faces of this cube each have an area of 9 square centimeters, so the surface area of the cube is $6 \cdot 9$, or 54 square centimeters.



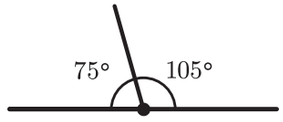
system of equations Two or more equations that represent the constraints on a shared set of variables.

For example, these equations make up a system of equations:
$x + y = -2$
$x - y = 12$

Español

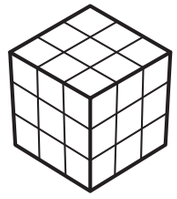
ángulos suplementarios Dos ángulos cuyas medidas suman $180°$.

Por ejemplo, un ángulo de $75°$ y otro ángulo de $105°$ son suplementarios.



área de superficie La suma de las áreas de las superficies de un sólido.

Por ejemplo, cada una de las seis caras de este cubo tiene un área de 9 centímetros cuadrados, por lo tanto, el área de superficie del cubo mide $6 \cdot 9$ o 54 centímetros cuadrados.



sistema de ecuaciones Dos o más ecuaciones que representan las restricciones en un conjunto compartido de variables.

Por ejemplo, estas ecuaciones forman un sistema de ecuaciones:
$x + y = -2$
$x - y = 12$

T

terminating decimal A decimal with a finite number of non-zero digits after the decimal point.

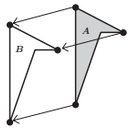
For example, 0.08, 1.5, and 0.2563 are all terminating decimals.

theoretical probability The probability that a certain outcome will occur, as determined through reasoning or calculation. It is equal to the ratio

$$\frac{\text{number of outcomes in the event}}{\text{number of outcomes in the sample space}}$$

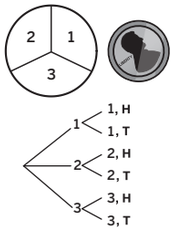
transformation An action or rule for moving or changing figures on a plane. Transformations include translations, reflections, rotations, and dilations.

translation A translation moves every point in a figure a given distance in a given direction.



tree diagram A diagram that represents all the possible outcomes in an experiment. It is often used to represent compound events.

For example, this tree diagram shows the outcomes of spinning a spinner and then flipping a coin.



triangle inequality theorem A theorem which states that in order for three line segments to form a triangle, the sum of the two shorter segments' lengths must be greater than the third segment's length.

For example, a triangle can have side lengths of 4, 5, and 6, because $4 + 5 > 6$.

decimal exacto Un decimal con un número finito de dígitos distintos de cero después del punto decimal.

Por ejemplo: 0.08, 1.5 y 0.2563 son decimales exactos.

probabilidad teórica La probabilidad de que ocurra un determinado resultado, determinada mediante razonamiento o cálculo. Es igual a la razón

$$\frac{\text{número de resultados en el suceso}}{\text{número de resultados en el espacio de muestra}}$$

transformación Una acción o regla para mover o cambiar figuras en un plano. Las transformaciones incluyen traslaciones, reflexiones, rotaciones y dilataciones.

traslación Una traslación mueve cada punto de una figura una determinada distancia en una determinada dirección.

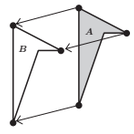
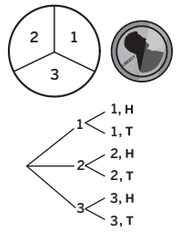


diagrama de árbol Diagrama que representa todos los resultados posibles de un experimento. A menudo se utiliza para representar eventos compuestos.

Por ejemplo, este diagrama de árbol muestra los resultados de girar una ruleta y luego lanzar una moneda.



teorema de la desigualdad triangular Un teorema que establece que, para que tres segmentos de recta formen un triángulo, la suma de las longitudes de los dos segmentos más cortos debe ser mayor que la longitud del tercer segmento.

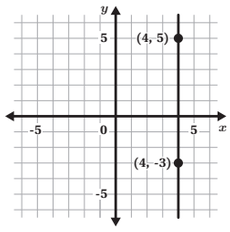
Por ejemplo, un triángulo puede tener lados de 4, 5 y 6, porque $4 + 5 > 6$.

English

Español

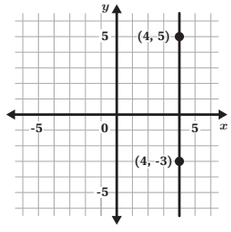
U

undefined An expression or term that has no understandable value. Dividing any (non-zero) value by 0 creates an undefined expression because we cannot divide by 0.



The slope of a vertical line is undefined because there is 0 horizonal change between any two points on the line. This means any vertical change would be divided by 0, which creates an undefined value.

indefinido Una expresión o un término que no tiene ningún valor comprensible. La división de un valor (diferente de cero) entre 0 produce una expresión indefinida porque no es posible dividir por 0.



La pendiente de una recta vertical es indefinida porque hay un cambio horizontal de 0 entre dos puntos cualesquiera en la recta. Esto significa que cualquier cambio vertical se dividiría por 0, lo cual produce un valor indefinido.

unit rate A rate that describes how one quantity changes when the other quantity changes by exactly 1 unit.

For example, if 12 people share 3 pizzas equally, then one unit rate is 4 people per pizza. Another unit rate in this situation is $\frac{1}{4}$ pizza per person.

tasa unitaria Una tasa que describe cómo cambia una cantidad cuando la otra cantidad cambia en exactamente 1 unidad.

Por ejemplo, si 12 personas se reparten 3 pizzas en partes iguales, entonces una tasa unitaria es 4 personas por pizza. Otra tasa unitaria en esta situación es $\frac{1}{4}$ de pizza por persona.

univariate data A data set that involves one variable. Each data point contains one piece of information.

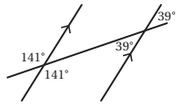
A collection of students' heights is a univariate data set.
Tables, dot plots, and bar graphs are useful for displaying univariate data.

datos univariados Un conjunto de datos que incluye una variable. Cada punto de datos contiene una información.

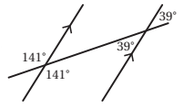
Una colección de las estaturas de estudiantes es un conjunto de datos univariado.
Las tablas, los diagramas de puntos y los diagramas de barras son útiles para mostrar datos univariados.

V

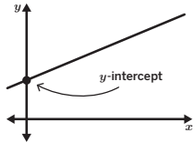
vertical angles Vertical angles are opposite angles that share the same vertex. They are formed by a pair of intersecting lines. Their angle measures are equal.



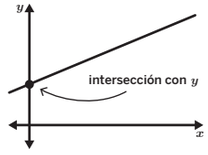
ángulos verticales Los ángulos verticales son ángulos opuestos que comparten el mismo vértice. Se forman con un par de rectas que se intersecan. Las medidas de sus ángulos son iguales.



vertical intercept The point where the graph of a line crosses the vertical axis or when $x = 0$. The vertical intercept is sometimes called the y-intercept.

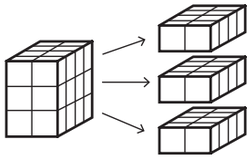


intersección vertical El punto donde la gráfica de una recta se cruza con el eje vertical o cuando $x = 0$. La intersección vertical a veces se denomina intersección con el eje y.



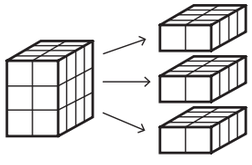
volume The number of cubic units that fill a 3-D region without any gaps or overlaps.

For example, the volume of this rectangular prism is 24 cubic units because it is made of 3 layers that are each 8 cubic units.



volumen La cantidad de unidades cúbicas que llenan un espacio tridimensional, sin vacíos ni superposiciones.

Por ejemplo, el volumen de este prisma rectangular mide 24 unidades cúbicas porque se compone de 3 capas de 8 unidades cúbicas cada una.



X

x-intercept See *horizontal intercept*.

intersección con el eje *x* Véase *intersección horizontal*.

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y-intercept See *vertical intercept*.

intersección con el eje *y* Véase *intersección vertical*.

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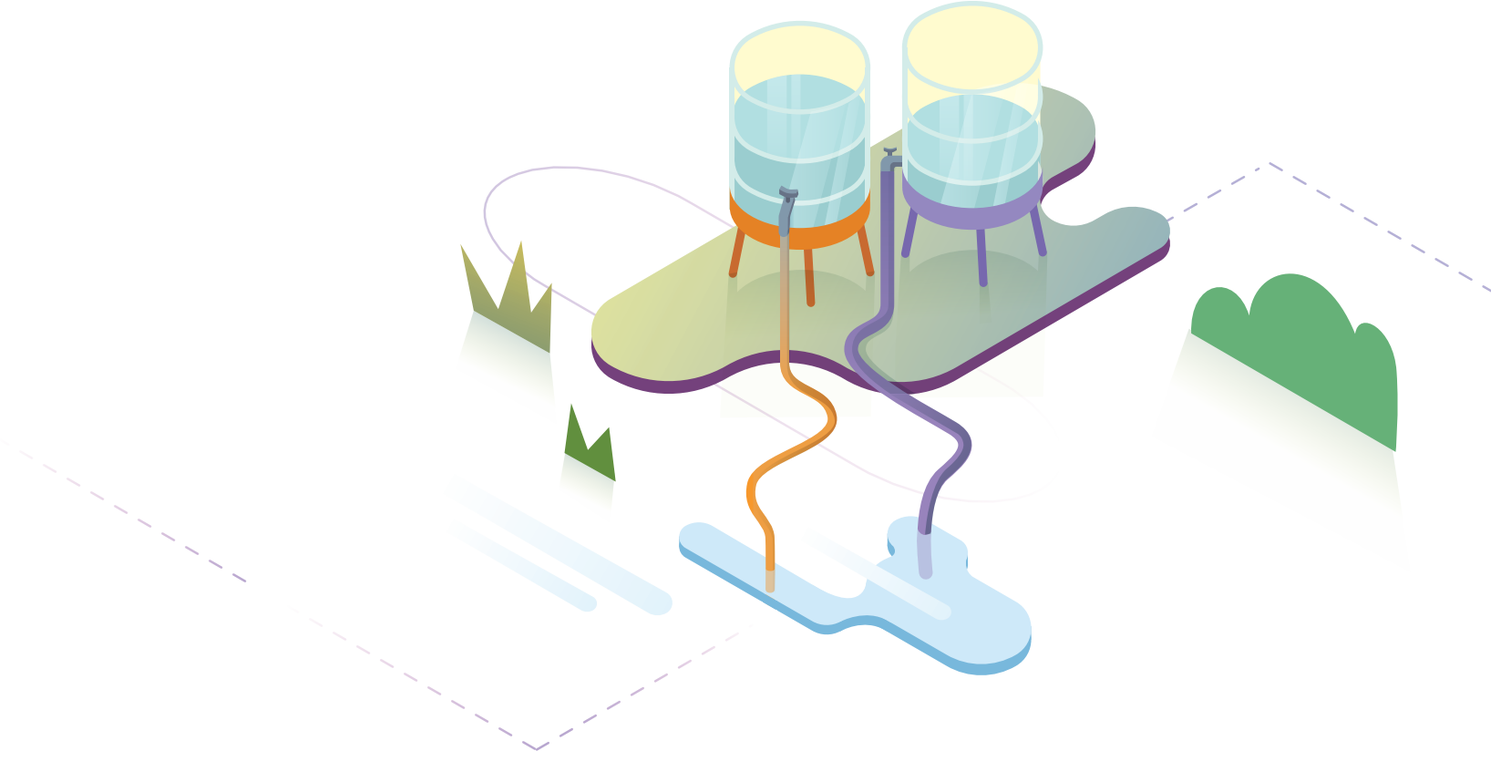

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