Unit 4

Describing Functions

Functions can be represented using tables, graphs, equations, and words. In this unit, you will explore what makes a relationship a function and how functions can model situations and tell stories. You will use function notation to describe key features of functions, compare different functions, and define functions. Finally, you will explore new types of functions that can model situations that have different rules for different inputs.

Essential Questions

- What are the characteristics of a function and how can a function tell a story?
- What are key features of functions and how can you describe them?
- How can you use function notation as a tool to communicate precisely?

Summary | Lesson 1

We can represent rules with verbal descriptions or as a table of *inputs* and *outputs*. All sets of inputs and outputs are called *relations*. A **function** is a special kind of relation that assigns exactly one output to each possible input.

You can determine whether a rule is a function by organizing the inputs and outputs into a table. If one input has multiple possible outputs, then the rule is not a function.

Here are two examples.

Rule A takes an integer and outputs an integer that is one less.

Input	Output
1	0
2	1
2	1
4	3

In this relationship, Rule A is a function because each input has exactly one output.

Rule B takes a number and outputs a random number that is greater.

Input	Output
0	2
0	10
-2	0
-1.6	-1.2

In this relationship, Rule B is *not* a function because each input has multiple outputs.

Try This

This rule takes any value and either multiplies or divides it by 2.

Is this rule a function? Explain your thinking.

Input	Output
2	4
10	20
3	6
2	1

Summary | Lesson 2

Function notation is a way to write the inputs and outputs of a function. For example, $f(4) = 9$ is a statement written in function notation. It says that when the input of the function f is 4, the output is 9. In other words, when the value of the *independent variable* is 4, the value of the *dependent variable* is 9.

Here is another example. We can use a function to determine the price of a slice of pizza based on the number of toppings.

Menu

Slice of Pizza $1.75 plus $0.50 per topping

This table shows some input-output pairs for the function $s(t)$.

Number of Toppings	Price ($)
0	1.75
1	2.25
2	2.75

$s(2) = 2.75$ is a statement written in function notation.

- $s(2)$ can be read as "s of two."
- For this situation, the number of toppings is the *independent variable*, t, and the price of a slice of pizza is the *dependent variable*, $s(t)$.
- $s(2) = 2.75$ means the price of a slice of pizza with 2 toppings is $2.75.

Try This

An ice cream shop serves ice cream in either a waffle cone or a bowl.

$w(x) = 2.25x + 3.5$ represents the cost of ordering a waffle cone, where x is the number of scoops of ice cream.

a What is the value of $w(2)$?

b What does $w(4) = 12.5$ mean in this situation?

Summary | Lesson 3

You can represent function rules with equations, verbal descriptions, and tables.

For example, the function $s(t)$ describes the relationship between the cost of a slice of pizza and the number of toppings, t. Let's represent this function rule with an equation and a table.

Description

Menu
Slice of Pizza
$1.75 plus $0.50
per topping

Table

Number of Toppings	Price ($)
0	1.75
1	2.25
2	2.75

Equation

$s(t) = 1.75 + 0.50t$

You can use the equation to determine different values of the function.
Let's determine the value of $s(4)$:
$s(4) = 1.75 + 0.50(4)$
$s(4) = 3.75$
This means the price of a slice of pizza with 4 toppings is $3.75.

Try This

An ice cream shop serves ice cream in either a waffle cone or a bowl.

A bowl of ice cream costs $1.75 plus $2.25 for each scoop of ice cream.

a Write an equation for $b(x)$ where b represents the cost of a bowl of ice cream and x represents the number of scoops.

b What is the value of $b(3)$?

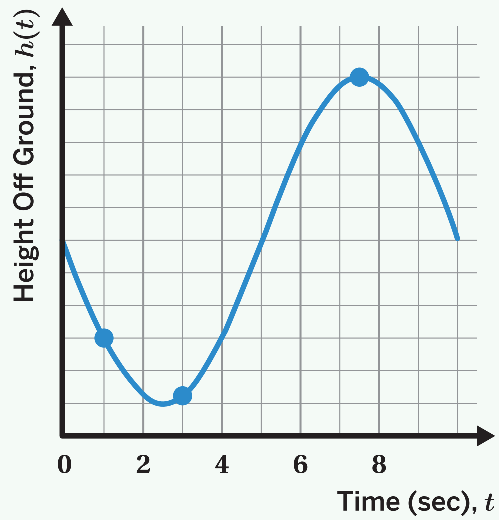
Summary | Lesson 4

A graph can reveal in more detail what is happening during a situation. Here is an example.

The function $h(t)$ represents the height of the cart on the Ferris wheel at time t.

We can use the graph to describe many parts of the situation. For example:

- At around 7.5 seconds, the Ferris wheel cart is at its maximum height.
- $h(1)$ is greater than $h(3)$. This means the Ferris wheel cart was higher off the ground at 1 second than at 3 seconds.

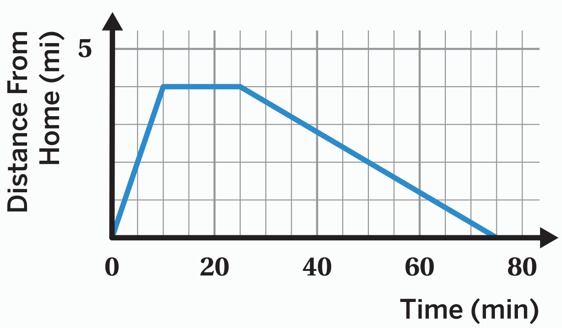


While we can use the graph to describe many things, there are lots of things the graph cannot describe. For example:

- How much fun the people are having
- How many people are riding the Ferris wheel

Try This

Jasmine takes the bus to the library and walks home. This graph shows Jasmine's distance from home $d(t)$.



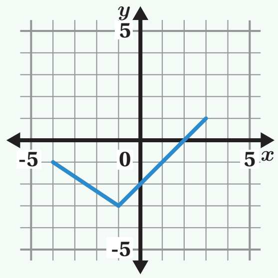
a What does $d(15) = d(20)$ represent in this situation?

b What does $d(50)$ represent in this situation?

Summary | Lesson 5

We can use the key features of a graph to help us describe a function or sketch a possible graph of a function. Here is an example. Let's analyze the graph of this function.

Minimum: The lowest point on a graph.	(-1, -3)
Maximum: The highest point on a graph.	(3, 1)
Positive: The x-values where the function has positive outputs; the graph is *above* the x-axis.	$x > 2$
Negative: The x-values where the function has negative outputs; the graph is *below* the x-axis.	$x < 2$
Increasing: The x-values where the graph is sloping upward as you read the graph from left to right. As the inputs increase, the outputs also increase.	$x > -1$
Decreasing: The x-values where the graph is sloping downward as you read the graph from left to right. As the inputs increase, the outputs decrease.	$x < -1$

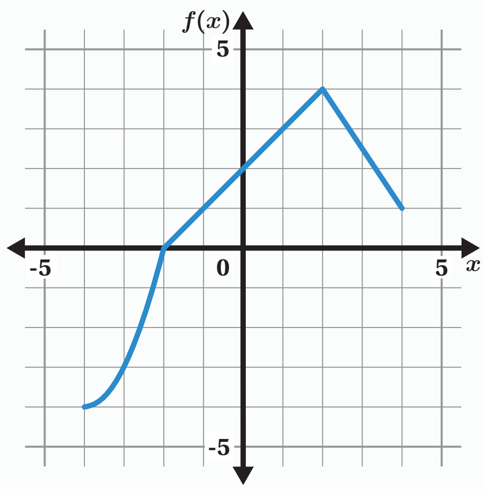


Try This

Select *all* the statements that are true for $f(x)$.

$f(x)$ . . .

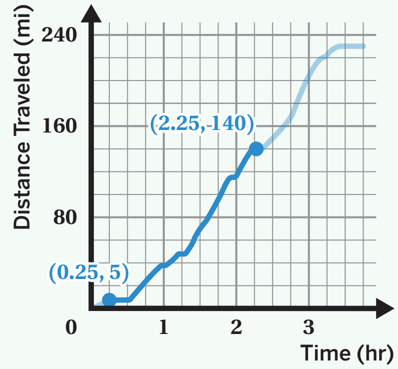
- ☐ **A.** Is positive when $x > -2$.
- ☐ **B.** Is increasing when $x > -2$.
- ☐ **C.** Is decreasing when $x > 2$.
- ☐ **D.** Has a minimum at $(-4, -4)$.
- ☐ **E.** Has a maximum at $(0, 4)$.



Summary | Lesson 6

Functions can have different rates of change over different intervals. The **average rate of change** is equivalent to the *slope* of the line between two points.

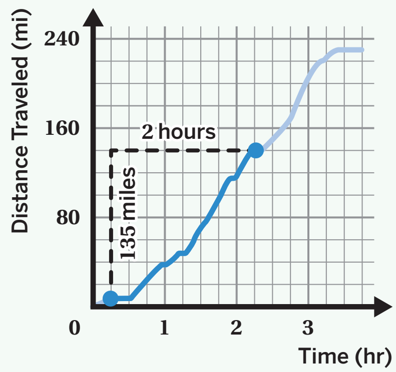
This graph represents Troy's car trip.



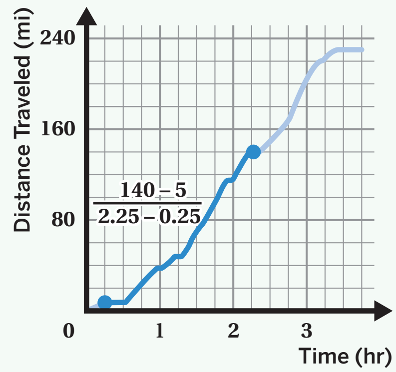
We can calculate the average rate of change over an **interval**, a specific length between two points, like the interval from 0.25 to 2.25 hours.

Here are two different strategies.

Strategy 1



Strategy 2



The average rate of change for the interval 0.25 to 2.25 hours is 67.5.

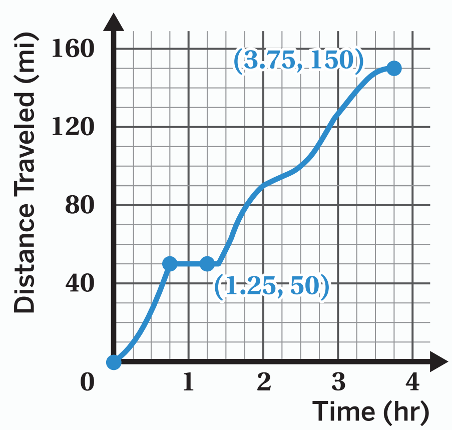
That means that Troy's average speed was 67.5 miles per hour in that interval.

Try This

Oscar took the train to attend his friend's birthday. Here is a graph of his trip.

Determine Oscar's average rate of change for the interval 1.25 to 3.75 hours.

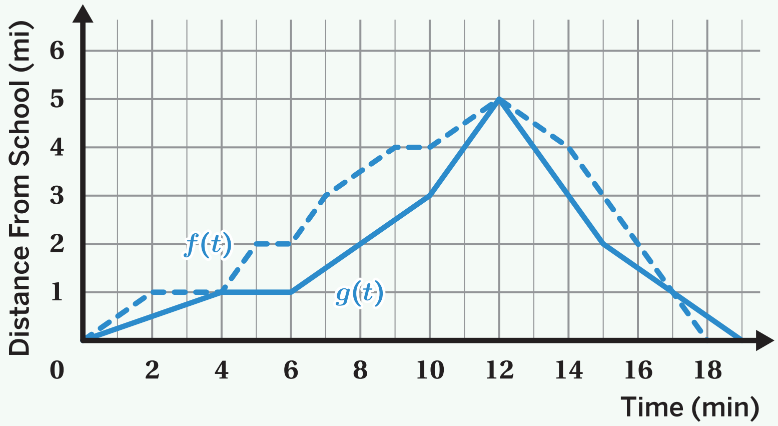
Show or explain your thinking.



Summary | Lesson 7

We can analyze functions by comparing the key features of different intervals on a graph, then using function notation to describe them.

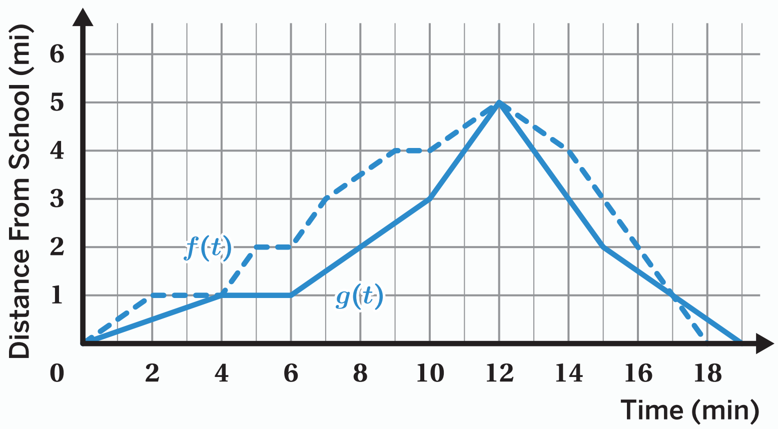
For instance, here are some true statements about these two graphs:



- When $t = 4$, $f(t) = g(t)$.
- $f(8) > g(8)$
- $f(12) = g(12)$
- $f(15) > g(15)$
- $f(t)$ and $g(t)$ have the same maximum.
- $f(t)$ and $g(t)$ are both decreasing from 12 to 15 minutes.
- $f(t)$ and $g(t)$ have the same average rate of change from 5 to 6 minutes.

Try This

A school has two buses that take different routes to drop students off. $f(t)$ and $g(t)$ represent the distance of each bus from school (in miles) after t minutes.



Select *all* the true statements:

- ☐ **A.** $f(6) = g(6)$
- ☐ **B.** $f(10) > g(10)$
- ☐ **C.** $f(17) = g(17)$
- ☐ **D.** $g(5) = 1$
- ☐ **E.** $f(18) > g(18)$

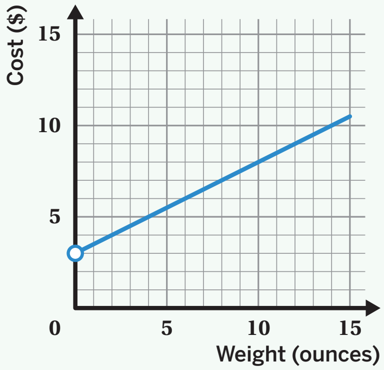
Summary | Lesson 8

Domain and **range** are used to describe the inputs and outputs of a function. The domain is the set of all possible input values (or all possible values for the *independent variable*). The range is the set of all possible output values (or all possible values for the *dependent variable*).

Having context for what the function is describing helps to make sense of possible inputs and outputs. Some domains and ranges are *discrete* while others are *continuous*. *Discrete* means that the possible values aren't connected or *continuous*, like whole numbers. *Continuous* means that the possible values are all connected, like the values along the graph of a line.

You can find the domain by looking at the x-values as you read the graph from left to right. You can determine the range by looking at the y-values as you read the graph from bottom to top.

Here the function $f(w) = 3 + 0.5w$ represents the cost of a frozen yogurt that weighs w ounces.



To determine the domain, consider which inputs make sense for the situation:

- Someone could buy 0.5 and 2 ounces of frozen yogurt.
- Someone cannot buy 0 or -2 ounces of frozen yogurt.

The domain of this function is all numbers greater than 0.

To determine the range, consider which outputs make sense.

- Some possible outputs are \$4, \$6, and \$6.25.
- Someone cannot pay a negative amount of dollars for frozen yogurt. It also doesn't make sense to have a cost of \$3 since that would mean a customer purchased a yogurt weighing 0 ounces.

The range of this function is all the dollar amounts greater than 3 up to the nearest cent.

Try This

A local mechanic sells and replaces tires. The total cost of replacing tires is \$50 for labor plus \$100 for each tire.

The function $r(t) = 50 + 100t$ represents the total cost for t tires.

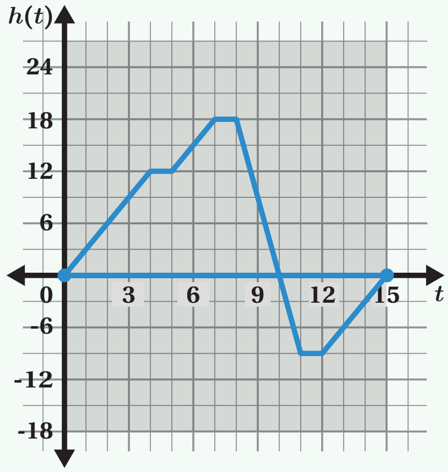
a Select *all* the values that are in the domain of $r(t)$.

☐ **A.** -2 ☐ **B.** 0 ☐ **C.** 1 ☐ **D.** 4 ☐ **E.** 1.5

b Describe the domain of $r(t)$.

Summary | Lesson 9

The *domain* and *range* of a function can each be described using a **compound inequality**, which is two or more inequalities joined together. You can write a compound inequality using symbols or using the words "and" or "or".



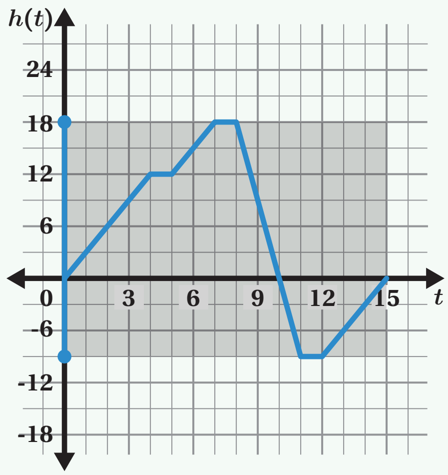
A graph can help you visualize the domain and range of a function, making it easier to describe them using compound inequalities.

Domain: The domain describes the width of the function, or how far left and right the function goes. The domain is also the set of all inputs to the function, or all values of the independent variable. The domain of this function is all the t values from 0 to 15.

$$t \geq 0 \text{ and } t \leq 15$$

$$0 \leq t \text{ and } 15 \geq t$$

$$0 \leq t \leq 15$$



Range: The range describes the height of the function, or how far up and down the function goes. The range is also the set of all outputs of the function, or all values of the dependent variable. The range of this function is all the $h(t)$ values from -9 to 18.

$$h(t) \geq -9 \text{ and } h(t) \leq 18$$

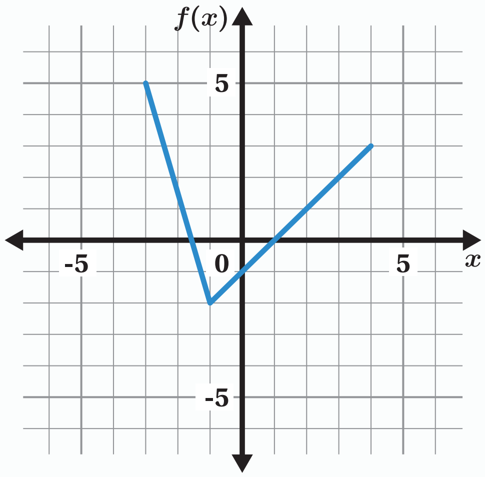
$$-9 \leq h(t) \text{ and } 18 \geq h(t)$$

$$-9 \leq h(t) \leq 18$$

Try This

Match the domain and range of $f(x)$ with one of these compound inequalities:

$-5 \leq x \leq 4$	$-2 \leq f(x) \leq 5$
$-3 \leq f(x) \leq 7$	$-3 \leq x \leq 4$
$-3 \leq x \leq 5$	



Domain: ________________

Range: ________________

Summary | Lesson 10

You can restrict a function's domain or range to highlight specific portions of a graph. Inequalities are one way to represent these restrictions symbolically. One strategy you can use to create a precise domain and range restriction is to use the ordered pairs at the boundaries of the interval.

Here is an example. Let's restrict the domain and range of $h(x)$ to highlight the interval from $(-3, 7)$ to $(6, 1)$.

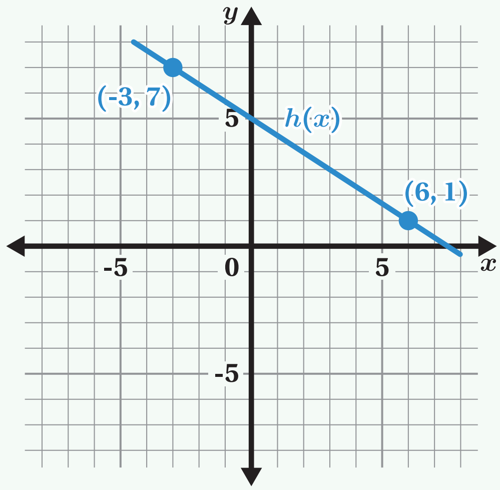
To restrict the domain, use the x-values of each ordered pair:

Domain: $-3 \leq x \leq 6$

To restrict the range, use the y-values of each ordered pair:

Range: $1 \leq h(x) \leq 7$

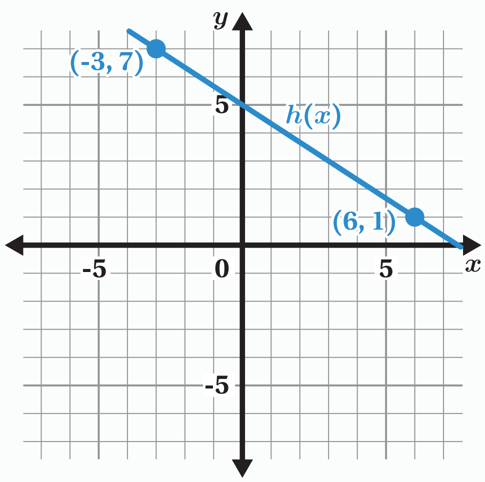
When you restrict the domain, you are restricting the x-values, which should be included when you write the inequality. When you restrict the range, you are restricting the y-values, or the output values of $h(x)$.



Try This

a Write a domain that restricts the graph of $h(x)$ from $(-3, 7)$ to $(6, 1)$.

b Write a range that restricts the graph of $h(x)$ from $(-3, 7)$ to $(6, 1)$.



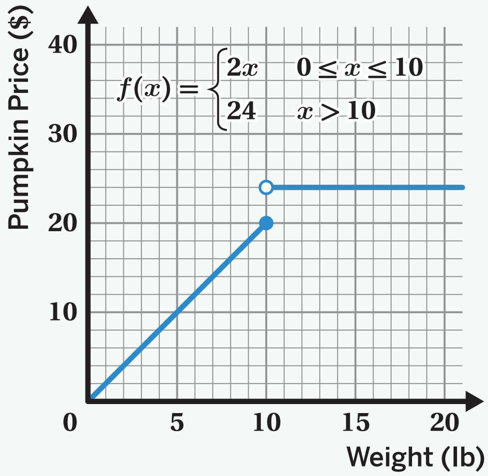
A **piecewise-defined function** is a function in which different rules apply to different intervals in its domain. You can use a graph or an equation to evaluate a piecewise-defined function.

Let's look at an example. The function $f(x)$ represents the price of a pumpkin with a weight of x pounds.

You can use the graph to evaluate $f(4)$ and $f(15)$.

- The point $(4, 8)$ is on the graph, so $f(4) = 8$.
- The point $(15, 24)$ is on the graph, so $f(15) = 24$.

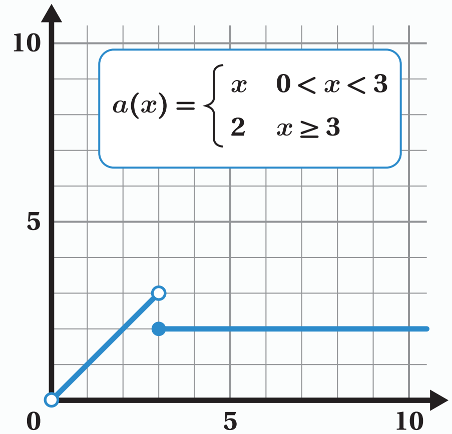
You can also use the equation to evaluate $f(4)$ and $f(15)$.



Value	Domain Interval	Equation	Evaluate
$f(4)$	$x = 4$ is in $0 \le x \le 10$	$f(x) = 2x$	$f(4) = 2(4)$ $= 8$
$f(15)$	$x = 15$ is in $x > 10$	$f(x) = 24$	$f(15) = 24$

Try This

Here is a graph of a piecewise-defined function.



a Why is this a piecewise-defined function?

b What is $a(1)$?

c What is $a(13)$?

Summary | Lesson 12

You can use *piecewise-defined functions* to represent situations. A **step function** is one special kind of piecewise-defined function, where every section of the graph is a point or a horizontal line at a constant value.

Here are some helpful ways to write the equations of a piecewise-defined function:

- The number of conditions in an equation is equal to the number of pieces in the function and graph.
- In the piecewise equation, each piece represents one condition and has its own domain.
- You can write the domain as an inequality.
  - $\geq$ or $\leq$ means to include that value.
  - $>$ or $<$ means to exclude that value.
- You can represent each condition by a section of the graph with open boundary points ($>$ or $<$) or closed boundary points ($\geq$ or $\leq$).

Graphing your piecewise-defined function might also help in making sense of the situation.

Try This

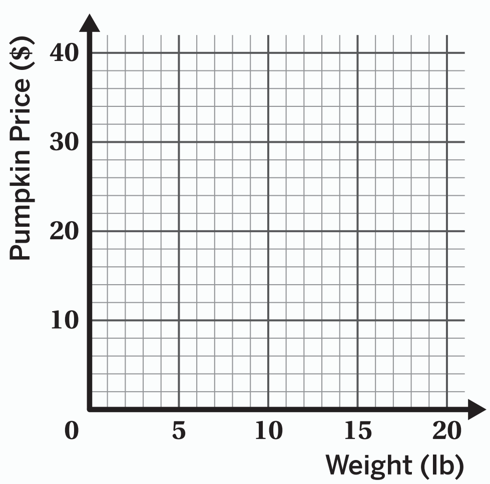
Here are the prices for pumpkins at a local farm:

- Pumpkins less than 5 pounds: $10
- Pumpkins greater than or equal to 15 pounds: $30
- All other pumpkins: $20

a Complete the piecewise-defined function to represent the price of pumpkins at this farm.

$$f(x) = \begin{cases} 10 & \square \\ \square & 5 \leq x < 15 \\ 30 & \square \end{cases}$$

b Sketch a graph of the function.



Summary | Lesson 13

There are several ways we can define, or describe, a *sequence*. When you define a sequence recursively, you are determining each term using the previous term.

You can define a sequence recursively by identifying the first term of the sequence and writing a rule for how the sequence changes between terms by either a *constant ratio* or a *constant difference.* When writing the rule in function notation, you can write the recursive definition by referencing the previous term, which can be written as $f(n-1)$.

Here is an example of a recursive definition for the sequence $32, 16, 8, 4, 2, 1$ written in function notation.

$$f(n) = \begin{cases} 32 & n = 1 \\ 0.5\, f(n-1) & n \geq 2 \end{cases}$$

The first term is 32, and the sequence changes by a constant ratio of 0.5. The rule is to multiply the previous term $f(n-1)$ by 0.5.

Try This

Determine which recursive definition matches this sequence:

$2, 5, 8, 11, 14, 17, 20, 23$

A. $f(1) = 2$

$f(n) = 3f(n-1)$

B. $f(1) = 23$

$f(n) = f(n-1) + 3$

C. $f(1) = 2$

$f(n) = f(n-1) + 3$

D. $f(1) = 5$

$f(n) = 2f(n-1)$

You can define a sequence explicitly by writing a function where the input is the term number and the output is the value of that term in the sequence. It can be helpful to look for the pattern between the terms (for example, if they have a *constant difference* or a *constant ratio*). Then determine the first term in the sequence and/or the sequence value when term n is 0.

Here are two sequences defined recursively and explicitly.

Sequence	Recursive Definition	Explicit Definition
$15, 12, 9, 6, 3, \ldots$	$a(1) = 15$ $a(n) = a(n-1) - 3$	$a(n) = 18 - 3n$ or $a(n) = 15 - 3(n-1)$
Term, n / $b(n)$: 1 / 20; 2 / 10; 3 / 5; 4 / 2.5	$b(1) = 20$ $b(n) = \frac{1}{2} \cdot b(n-1)$	$b(n) = 40 \cdot \left(\frac{1}{2}\right)^n$ or $b(n) = 20 \cdot \left(\frac{1}{2}\right)^{n-1}$

Term, n	$b(n)$
1	20
2	10
3	5
4	2.5

Try This

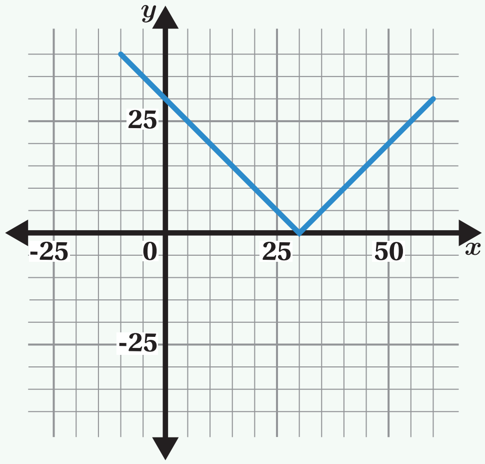
Write a recursive definition and an explicit definition for the sequence $f(n)$.

Sequence	Recursive Definition	Explicit Definition
(see table below)	$f(1) =$ ____________ $f(n) = f(n-1)$ ____________	$f(n) =$ ____________

Term, n	$f(n)$
1	2
2	5
3	8
4	11

Summary | Lesson 15

The output of an **absolute value function** is the distance of its input from a given value. The equation of an absolute value function is defined using absolute value symbols, and its graph forms the shape of a V. We can write absolute value functions in the form $f(x) = |x - h|$, where $f(x)$ gives the distance of any input, x, from h. Let's look at an example.



Mr. DeAndre asked his students to guess a mystery number and gave each student a score. Their score was how far away their guess was from his mystery number, 30.

Here is the graph of the function $f(x) = |x - 30|$, which gives the score for each guess, x.

We can use the equation to determine the value of $f(25)$ and interpret its meaning.

$$\begin{aligned} f(25) &= |25 - 30| \\ &= |\text{-}5| \\ &= 5 \end{aligned}$$

This means a student who guessed 25 was 5 away from the mystery number.

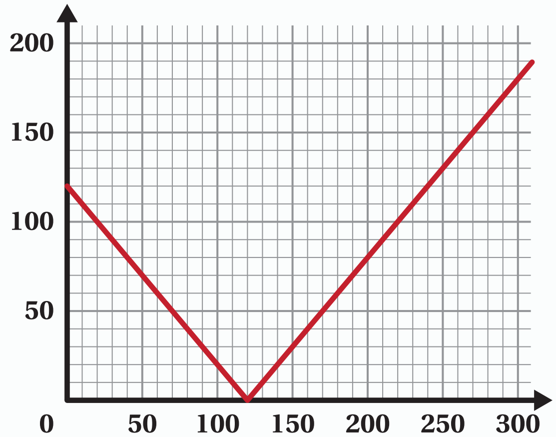
Try This

A carnival offers a prize for guessing the correct number of candies in a jar.

The function $a(x) = |x - 119|$ represents a person's score for a guess of x candies.

a. What is the value of $a(120)$?

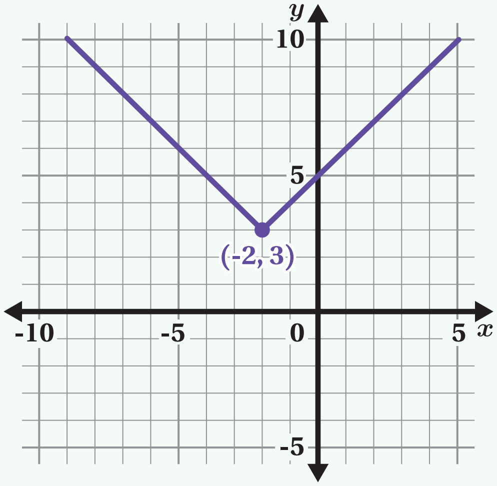
b. What does $a(120)$ mean in this situation?



Summary | Lesson 16

You can determine key features of the graph of an *absolute value function* by analyzing its table or equation, which are both helpful in sketching its graph.

Here are the graph and table for the absolute value function $f(x) = |x + 2| + 3$.



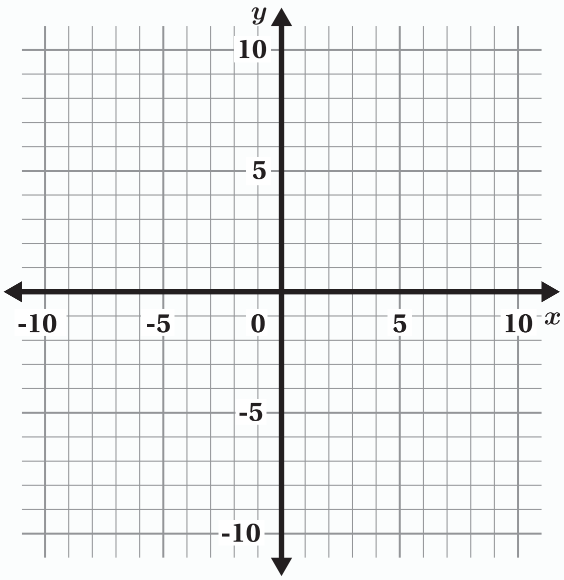
x	$f(x) = \lvert x + 2 \rvert + 3$
-4	5
-2	3
0	5
2	7

Evaluating $f(x)$ at $x = -2$ makes the equation equal to 3. This means that when the input of the function is -2, the output is 3 and a point on the graph of $f(x)$ is (-2, 3). The values in the table show that there is symmetry around the point (-2, 3). This tells us that (-2, 3) is the minimum value of the function, which we can see by looking at the graph.

Try This

a Make a graph of the function $a(x) = |x + 3| + 1$. Create a table if it helps with your thinking.

x	$a(x) = \lvert x + 3 \rvert + 1$



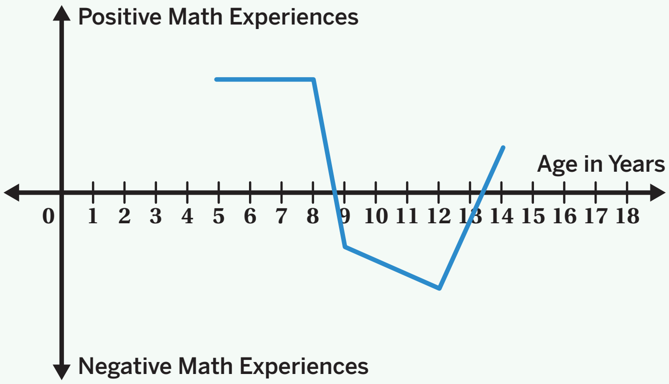
b What is the minimum value of this function? Explain your thinking.

Storytelling is a powerful way to learn more about others and to reflect on your own journey. Graphing a story can help us see interesting self-discoveries and have deeper discussions.

When you use equations, tables, words, graphs, and their key features to represent real-world relationships, pay close attention to the scale and units. You can look for the maximums or minimums; intercepts; intervals where the graph increases, decreases, or remains constant; and domain and range to make sense of the situation or someone's story. Let's look at an example.

Here's a graph of someone's math experiences over time. From the graph, we can learn that:

- Most of their positive math experiences are from ages 5 to 8, and their most negative math experience was at age 12.
- Their math experiences decreased from age 8 to 12 and increased after age 12.
- This person graphed their experiences from age 5 to age 14. It's possible they drew this graph at age 14.

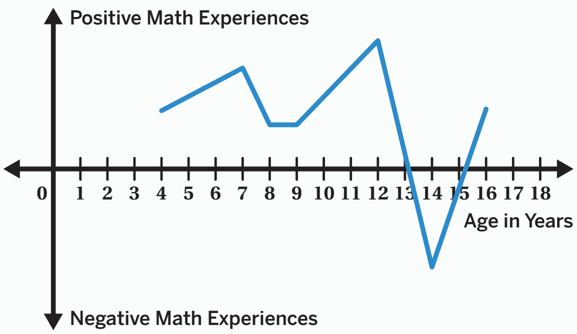


There is also a lot we can't tell from the graph of someone's story. For example, we can't tell what the positive or negative math experiences were, or what emotions they were feeling at the time. The graph gives us only a window into someone else's story, not the full image.

Try This

Joel is an 11th grade student. He graphed his math experiences, $f(x)$, as a function of age, x.

Write a story about Joel's math experiences using some of these terms:



Word Bank		
increase	decrease	minimum
domain	range	maximum

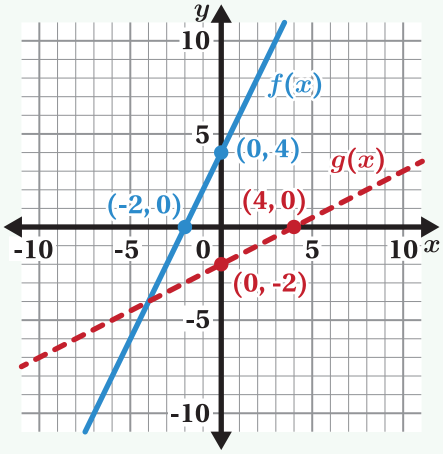
Summary | Lesson 18

Two functions are **inverses** of each other if their input-output pairs are reversed, and the graph of one function can be reflected onto the other over the line $y = x$. In general, if a function has a point at (h, k), then the inverse function has a point at (k, h).

Here is a linear function $f(x) = 2x + 4$.

One strategy you can use to determine the inverse of $f(x)$ is:

1. Choose two points on $f(x)$ and switch the x- and y-values of each point.
2. Then draw a line that goes through those points, and determine the slope and y-intercept.



Points on $f(x)$	Points on $g(x)$	Calculate the slope and y-intercept.	Write the equation of the inverse, $g(x)$.
(-2, 0)	(0, -2)	$\frac{-2 - 0}{0 - 4} = \frac{1}{2}$	The inverse of $f(x)$ is $g(x) = \frac{1}{2}x - 2$.
(0, 4)	(4, 0)	y-intercept: (0, -2)	

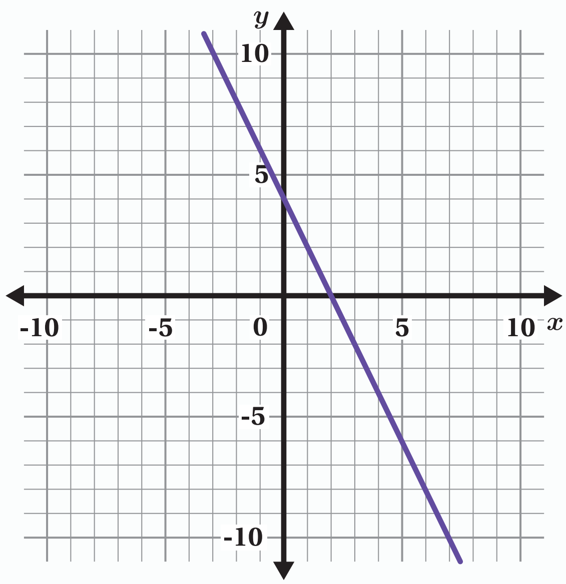
Try This

Here is the graph of $f(x) = -2x + 4$.

Use the graph to write an equation for the inverse function.

Draw a graph of the inverse function if it helps your thinking.

$g(x) =$ ____________________



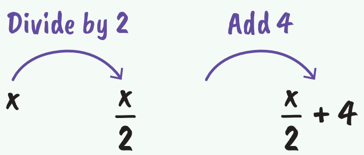
Summary

One strategy you can use to determine the equation of an inverse function is to determine the operations of the original function, and then apply the inverse operations in reverse order.

For example, here is the function $f(x) = \frac{x}{2} + 6$. $f(x)$ and $g(x)$ are inverses.

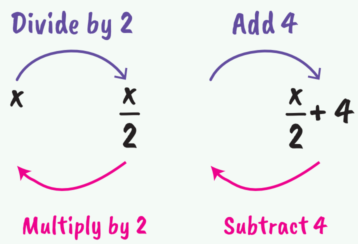
To find $g(x)$, first you can determine the operations acting on $f(x)$.

$$f(x) = \frac{x}{2} + 4$$



Then you can determine the inverse operations in reverse order.

$$f(x) = \frac{x}{2} + 4$$



The inverse function $g(x)$ will first subtract by 4, then multiply by 2. The inverse of $f(x)$ is $g(x) = 2(x - 4)$.

Try This

$h(x) = 3x + 9$.

$g(x)$ is the inverse of $h(x)$.

Determine the equation for $g(x)$.

$g(x) =$ ____________________

Lesson 1

No. *Explanations vary*. The input of 2 has multiple possible outputs (4 and 1), which means this is not a function.

Lesson 2

a $w(2) = 8$. [One strategy is to substitute 2 for x in the equation.
$w(2) = 2.25(2) + 3.5$
$w(2) = 4.5 + 3.5$
$w(2) = 8$]

b *Responses vary*. A waffle cone with 4 scoops of ice cream costs $\$12.50$.

Lesson 3

a $b(x) = 1.75 + 2.25x$

b $\$8.50$.

[One strategy is to substitute 3 for x in the equation $b(x) = 1.75 + 2.25x$.

$b(3) = 1.75 + 2.25(3)$

$b(3) = 1.75 + 6.75$

$b(3) = 8.5$]

Lesson 4

a Jasmine's distance from home is the same at 15 minutes as at 20 minutes because she is in the library.

b $d(50)$ represents the distance Jasmine was from home after 50 minutes, which was 2 miles.

Lesson 5

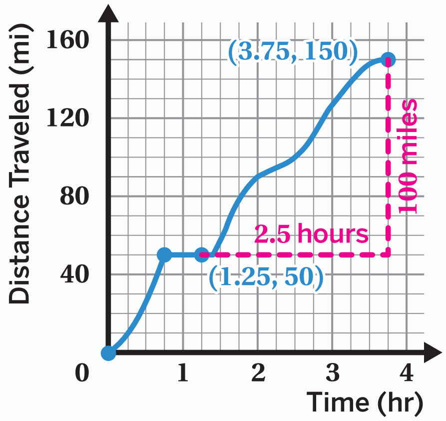
A. Is positive when $x > -2$.

C. Is decreasing when $x > 2$.

D. Has a minimum at $(-4, -4)$.

Lesson 6

40 miles per hour. *Explanations vary.* Here is one strategy for determining the average rate of change:



$\frac{100}{2.5} = 40$

Lesson 7

B. $f(10) > g(10)$

C. $f(17) = g(17)$

D. $g(5) = 1$

Lesson 8

a. C. 1

D. 4

b. *Responses vary.* Whole numbers greater than or equal to 1.

Lesson 9

Domain: $-3 \leq x \leq 4$

Range: $-2 \leq f(x) \leq 5$

Lesson 10

a $-3 \le x \le 6$

b $1 \le h(x) \le 7$

Lesson 11

a *Responses vary.* Different rules apply to different intervals of the function's domain. When x is between 0 and 3, $a(x) = x$. But when x is 3 or above, $a(x) = 2$.

b 1. [One strategy is to look at the graph where $x = 1$. At $x = 1$, the graph passes through the point $(1, 1)$, so $a(1) = 1$.]

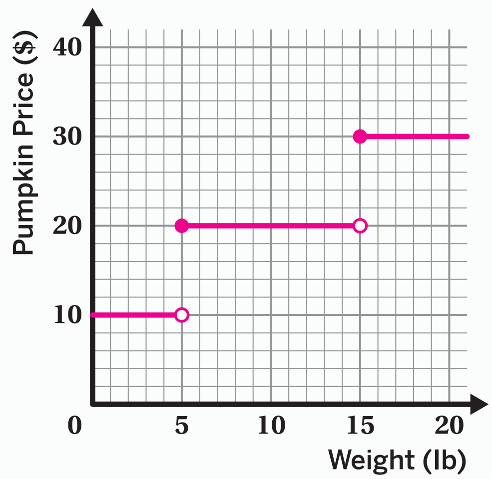
c 2. [One strategy is to use the equation. For values of x greater than or equal to 3, the function is equal to 2.]

Lesson 12

a

$$f(x) = \begin{cases} 10 & x < 5 \\ 20 & 5 \le x < 15 \\ 30 & x \ge 15 \end{cases}$$

b



Lesson 13

C. $f(1) = 2$

$f(n) = f(n - 1) + 3$

Lesson 14

Sequence	Recursive Definition	Explicit Definition
<table><tr><th>Term, n</th><th>$f(n)$</th></tr><tr><td>1</td><td>2</td></tr><tr><td>2</td><td>5</td></tr><tr><td>3</td><td>8</td></tr><tr><td>4</td><td>11</td></tr></table>	$f(1) = \mathbf{2}$ $f(n) = f(n-1)\ \mathbf{+ 3}$	$f(n) = \mathbf{3n - 1}$

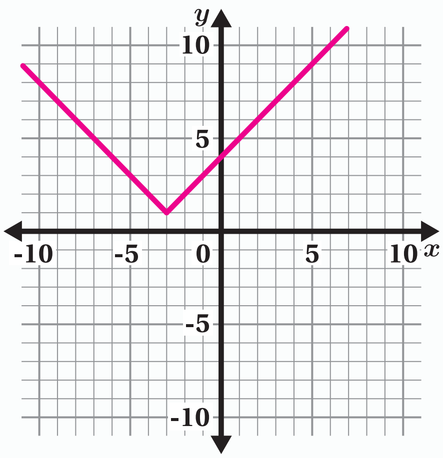
Lesson 15

a $a(120) = 1$

b *Responses vary*. If someone guessed 120, they were 1 away from the correct number of candies in the jar.

Lesson 16

a



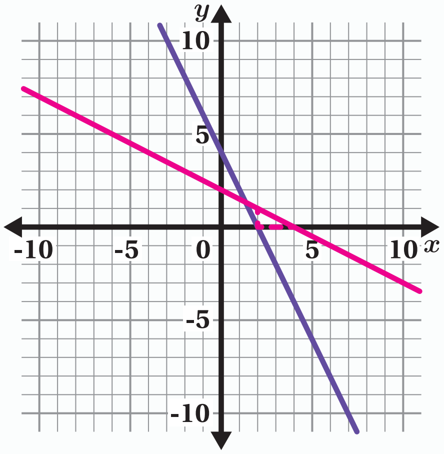
b (-3, 1). *Explanations vary*. The minimum value is (-3, 1) because substituting -3 for x into $a(x)$ makes the absolute value expression equal to 0. When the input is -3, the output is 1 which tells us (-3, 1) is the minimum of the graph.

Lesson 17

Responses vary. Joel had positive math experiences from ages 4 to 13, with a maximum at age 12 (which is when he had his best math experience). His positive math experiences increased from ages 4 to 7 and 9 to 12. They decreased from ages 12 to 14, with a minimum at age 14 (which is when he had his worst math experience). The domain of the graph is from 4 to 16, which represents all the ages that Joel had math class so far.

Lesson 18

$g(x) = -\frac{1}{2}x + 2$.
[Here is one strategy: Two points on $f(x)$ are $(1, 2)$ and $(0, 4)$. Switching the x- and y- values gives two points on $g(x)$: $(2, 1)$ and $(4, 0)$. We can draw a line through those points on the graph:



To determine the slope, we can calculate the change in y-coordinates divided by the change in x-coordinates: $\frac{(1-0)}{(2-4)} = \frac{1}{-2} = -\frac{1}{2}$.

We can then use the graph to see that the y-intercept is at $(0, 2)$. This means that the equation of the line is $g(x) = -\frac{1}{2}x + 2$.]

Lesson 19

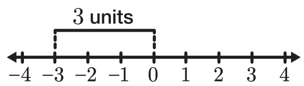
$g(x) = \frac{(x-9)}{3}$. [The operations of the original function are to first multiply by 3, then add 9. Applying the inverse operations in reverse order results in first subtracting 9, then dividing by 3.]

Algebra 1 Unit 4 Glossary/Álgebra 1 Unidad 4 Glosario

A

English

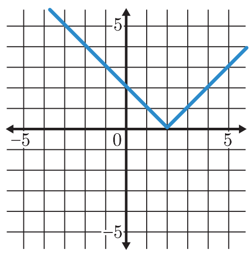
absolute value The absolute value of a number is its distance from 0 on the number line.



For example, the absolute value of -3 is 3 because -3 is 3 units away from 0. This is written as |-3| $= 3$.

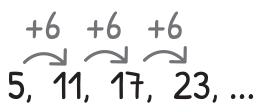
|4| $= 4$ and |-4| $= 4$. They are both 4 units away from 0.

absolute value function A function that is defined using absolute value symbols. When written in the form $f(x) = |x - h|$, its output is the distance of its input from a given value, h.



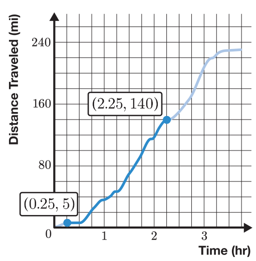
For example, $f(x) = |x - 2|$ outputs the distance from 2 for every input value.

arithmetic sequence A sequence that changes by a constant difference.



In this arithmetic sequence, the first term is 5 and the constant difference is 6.

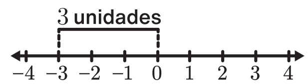
average rate of change A measure of how much a function changes, on average, over an interval. To calculate the average rate of change over an interval, find the slope between the point on the graph of the function where the interval begins and the point where it ends.



To calculate the average rate of change over the interval from 0.25 hours to 2.25 hours, calculate the change in y-values $(140 - 5)$ and divide by the change in x-values $(2.25 - 0.25)$. The average rate of change from 0.25 to 2.25 is 67.5, which means the average speed on that trip was 67.5 miles per hour in that interval.

Español

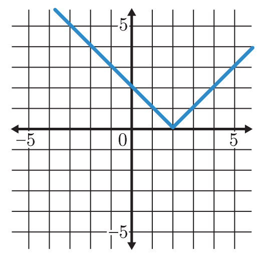
valor absoluto El valor absoluto de un número es su distancia al 0 en la recta numérica.



Por ejemplo, el valor absoluto de -3 es 3 porque -3 está a 3 unidades del 0. Esto se escribe |-3| $= 3$.

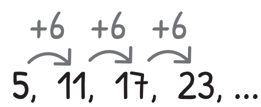
|4| $= 4$ y |-4| $= 4$. Ambos están a 4 unidades del 0.

función de valor absoluto Una función que se define utilizando símbolos de valor absoluto. Cuando se escribe en la forma $f(x) = |x - h|$, su salida es la distancia de su entrada a un valor determinado, h.



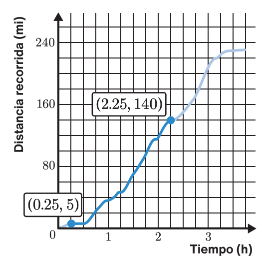
Por ejemplo, $f(x) = |x - 2|$ arroja la distancia de cada valor de entrada al 2.

secuencia aritmética Una secuencia que cambia con una diferencia constante.



En esta secuencia aritmética, el primer término es 5 y la diferencia constante es 6.

tasa de cambio promedio Una medida de cuánto cambia una función, en promedio, en un intervalo. Para calcular la tasa de cambio promedio a lo largo de un intervalo, se halla la pendiente entre el punto donde empieza el intervalo y el punto donde termina en la gráfica de la función.

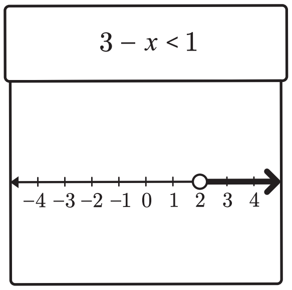


Para calcular la tasa de cambio promedio en el intervalo de 0.25 horas a 2.25 horas, se calcula el cambio en los valores de y $(140 - 5)$ y se divide por el cambio en los valores de x $(2.25 - 0.25)$. La tasa de cambio promedio de 0.25 a 2.25 es 67.5, lo que significa que la velocidad media en ese trayecto fue de 67.5 millas por hora en ese intervalo.

English

B

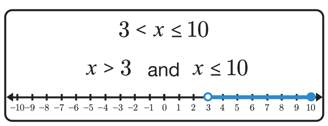
boundary point The value that separates the solution set of an inequality from non-solutions. A solid boundary point indicates that the point is included in the solution set. An empty boundary point indicates the point is not included in the solution set.



The solution set to $3 - x < 1$ has a boundary point at $x = 2$. The point at 2 is empty because 2 is not included in the solution set.

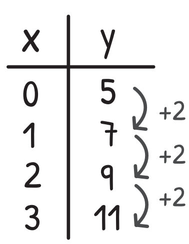
C

compound inequality Two or more inequalities joined together. A compound inequality can be written using symbols or the words "and" or "or."



The numbers greater than 3 and less than or equal to 10 can be written as:
$x > 3$ and $x \leq 10$ or $3 < x \leq 10$

constant difference When the difference between any two consecutive values in a pattern is the same, there is a constant difference.

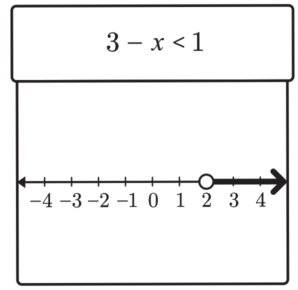


The pattern in the table has a constant difference of 2.

Español

B

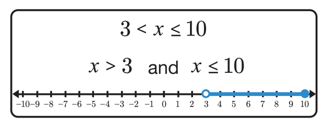
punto límite El valor que separa el conjunto de soluciones de una desigualdad de todos los valores que no son soluciones. Un punto límite sólido (rellenado) indica que el punto está incluido en el conjunto de soluciones. Un punto límite vacío indica que el punto no está incluido en el conjunto de soluciones.



El conjunto de soluciones de $3 - x < 1$ tiene un punto límite en $x = 2$. El punto en 2 no está relleno porque 2 no está incluido en el conjunto de soluciones.

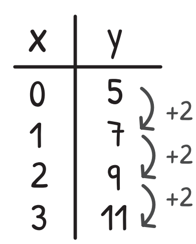
C

desigualdad compuesta Dos o más desigualdades juntas. Una desigualdad compuesta puede escribirse con símbolos o las palabras "y" u "o".



Los números mayores que 3 y menores o iguales que 10 pueden escribirse de la siguiente forma:
$x > 3$ y $x \leq 10$ o $3 < x \leq 10$

diferencia constante Cuando la diferencia entre dos valores consecutivos cualesquiera en un patrón permanece igual, hay una diferencia constante.



El patrón en la tabla tiene una diferencia constante de 2.

Algebra 1 Unit 4 Glossary/Álgebra 1 Unidad 4 Glosario

English

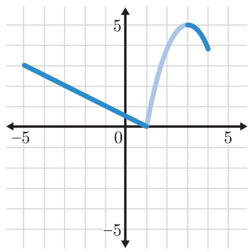
constant ratio When the ratio between any two consecutive values in a pattern is the same, there is a constant ratio.

x	y	
0	1	· 3
1	3	· 3
2	9	· 3
3	27	

The pattern in the table has a constant ratio of 3.

D

decreasing (interval or function) A function is decreasing when its outputs decrease as its inputs increase. A function can be decreasing for its entire domain or over an interval.



For example, the function $f(x)$ is decreasing when -5 $< x < 1$ and when $3 < x < 4$.

dependent variable The value of a dependent variable is based on the value of another variable or set of variables. In a function, the value of the dependent variable represents the output. The dependent variable is typically on the vertical axis of a graph and in the right-hand column of a table.

discrete A set of values is discrete if there is separation between the values. A graph can be described as discrete when it consists of unconnected points or intervals.

The set of numbers 1, 2, 3 is discrete, while the set of all values between 1 and 3 is not discrete.

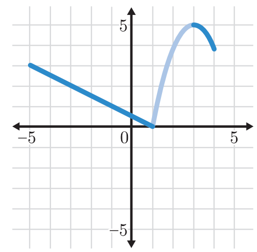
Español

razón constante Cuando la razón entre dos valores consecutivos cualesquiera en un patrón permanece igual, hay una razón constante.

x	y	
0	1	· 3
1	3	· 3
2	9	· 3
3	27	

El patrón en la tabla tiene una razón constante de 3.

decreciente (intervalo o función) Una función es decreciente cuando sus salidas decrecen a medida que crecen sus entradas. Una función puede ser decreciente en todo su dominio o a lo largo de un intervalo.



Por ejemplo, la función $f(x)$ es decreciente cuando -5 $< x < 1$ y cuando $3 < x < 4$.

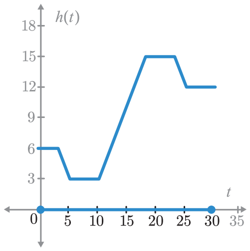
variable dependiente El valor de una variable dependiente se basa en el valor de otra variable o conjunto de variables. En una función, el valor de la variable dependiente representa la salida. La variable dependiente suele estar en el eje vertical de una gráfica y en la columna derecha de una tabla.

discreto Un conjunto de valores es discreto si hay separación entre los valores. Una gráfica puede describirse como discreta cuando consta de puntos o intervalos que no se conectan.

El conjunto de números 1, 2, 3 es discreto, mientras que el conjunto de todos los valores entre 1 y 3 no es discreto.

English

domain The set of all possible input values for a function or relation. The domain can be described in words or as an inequality.



The domain of this graph can be described as:
In words: All numbers from 0 to 30.
Compound inequality: $0 \le t \le 30$.

E

explicit definition A sequence can be defined recursively or explicitly. An explicit definition is a formula that determines the value of any term in a sequence using the term number. An explicit definition of a sequence can be written as an expression, an equation, a function, or in words.

Example Sequence:
$4, 6, 9, 13.5, \ldots$
Explicit Definition:
Let n represent the term number.
$f(n) = 4 \cdot 1.5^{(n-1)}$

F

function A rule, or relation, that assigns exactly one output to each possible input. Every function is a relation, but not every relation is a function. In a function, the value of the output variable depends on the value of the input variable.

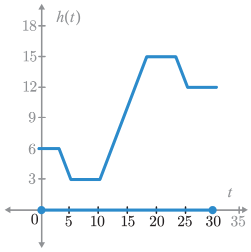
function notation A way of writing about the inputs and outputs of a function.

$$f(x) = 2x + 1$$
$$f(4) = 9$$

For example, $f(4) = 9$ is a statement written in function notation. It says that when the input of the function f is 4, the output is 9. In other words, when the value of the independent variable is 4, the value of the dependent variable is 9.

Español

dominio El conjunto de todos los valores de entrada posibles de una función o relación. El dominio puede describirse con palabras o como una desigualdad.



El dominio de esta gráfica puede describirse de la siguiente manera: Con palabras: Todos los números del 0 al 30. Desigualdad compuesta: $0 \le t \le 30$.

E

definición explícita Una secuencia puede definirse de forma recursiva o explícita. Una definición explícita es una fórmula que determina el valor de cualquier término de una secuencia utilizando el número del término. Una definición explícita de una secuencia puede escribirse como una expresión, una ecuación, una función o con palabras.

Ejemplo de secuencia:
$4, 6, 9, 13.5, \ldots$
Definición explícita:
Sea n el número término.
$f(n) = 4 \cdot 1.5^{(n-1)}$

F

función Una regla, o relación, que asigna exactamente una salida a cada entrada posible. Toda función es una relación, pero no toda relación es una función. En una función, el valor de la variable de salida depende del valor de la variable de entrada.

notación de funciones Una forma de representar de manera escrita las entradas y salidas de una función.

$$f(x) = 2x + 1$$
$$f(4) = 9$$

Por ejemplo, $f(4) = 9$ es una expresión escrita en notación de funciones. Indica que cuando la entrada de la función f es 4, la salida es 9.
En otras palabras, cuando el valor de la variable independiente es 4, el valor de la variable dependiente es 9.

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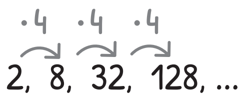
English

G

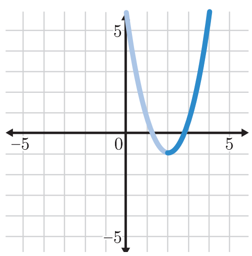
geometric sequence A sequence that changes by a constant ratio.



In this geometric sequence, the first term is 2 and the constant ratio is 4.

I

increasing (interval or function) A function is increasing when its outputs increase as its inputs increase. A function can be increasing for its entire domain or over an interval.



For example, the function $f(x)$, whose graph is shown, is increasing when $x > 2$.

independent variable The value of an independent variable is not based on the value of any other variable. In a function, the value of the independent variable represents the input. The independent variable is typically on the horizontal axis of a graph and in the left-hand column of a table.

inequality A comparison statement that uses the symbols <, >, ≤, or ≥. Inequalities are used to represent the relationship between numbers, variables, or expressions that are not always equal.

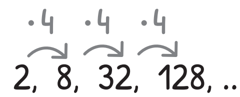
For example, the inequality $y + 2 \geq 30$ means that the value of the expression $y + 2$ will be greater than or equal to 30.

input In a function, any value that you substitute for x is called an input. The input is sometimes called the independent variable. The input typically appears on the horizontal axis of a graph and in the left-hand column of a table.

Español

G

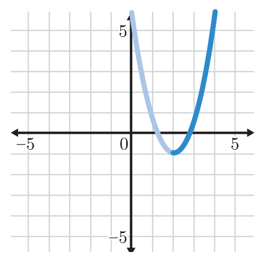
secuencia geométrica Una secuencia que cambia según una razón constante.



En esta secuencia geométrica, el primer término es 2 y la razón constante es 4.

I

creciente (intervalo o función) Una función es creciente cuando sus salidas aumentan a medida que aumentan sus entradas. Una función puede ser creciente en todo su dominio o a lo largo de un intervalo.



Por ejemplo, la función $f(x)$, cuya gráfica se muestra, es creciente cuando $x > 2$.

variable independiente El valor de una variable independiente no depende del valor de ninguna otra variable. En una función, el valor de la variable independiente representa la entrada. La variable independiente suele estar en el eje horizontal de una gráfica y en la columna izquierda de una tabla.

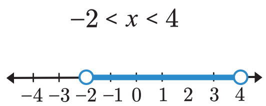
desigualdad Un enunciado de comparación que utiliza los símbolos <, >, ≤ o ≥. Las desigualdades se usan para representar la relación entre números, variables o expresiones que no siempre son iguales.

Por ejemplo, la desigualdad $y + 2 \geq 30$ significa que el valor de la expresión $y + 2$ será mayor o igual que 30.

entrada En una función, todo valor que sustituya a x se denomina entrada. La entrada a veces se denomina variable independiente. La entrada suele estar en el eje horizontal de una gráfica y en la columna izquierda de una tabla.

English

interval A set of values between two points.



In words: All numbers between -2 and 4.
Inequality: $-2 < x < 4$

inverse (of a function) Two functions are inverses of each other if their input-output pairs are reversed, and the graph of one function is a reflection of the other over the line $y = x$. If a function has a point (h, k), then the inverse function has a point at (k, h).

$a(x)$

Input	Output
5	16
10	26
1	8

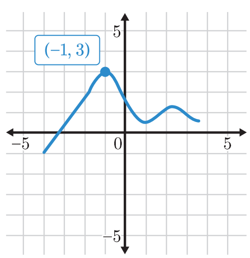
$b(x)$

Input	Output
16	5
26	10
8	1

For example, $a(x) = 2x + 6$ and $b(x)=\frac{(x-6)}{2}$ are inverses because the input-output pairs are reversed.

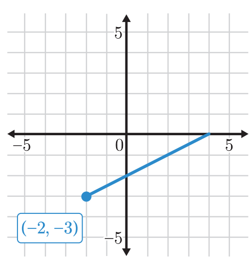
M

maximum (of a function) The highest point on the graph.



The maximum of this function is (-1, 3).

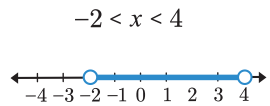
minimum (of a function) The lowest point on the graph.



The minimum of this function is (-2, -3).

Español

intervalo Un conjunto de valores entre dos puntos.



Con palabras: Todos los números entre el -2 y el 4.
Desigualdad: $-2 < x < 4$.

inverso (de una función) Dos funciones son inversas entre si sus pares de entrada-salida son inversos, y la gráfica de una función es un reflexión de la otra sobre la recta $y = x$. Si una función tiene un punto en (h, k), entonces la función inversa tiene un punto en (k, h).

$a(x)$

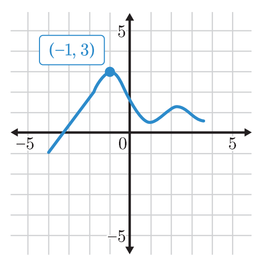
Entrada	Salida
5	16
10	26
1	8

$b(x)$

Entrada	Salida
16	5
26	10
8	1

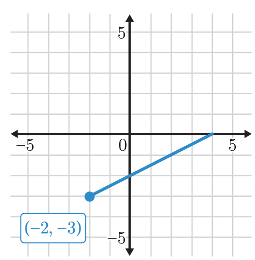
Por ejemplo, $a(x) = 2x + 6$ y $b(x)=\frac{(x-6)}{2}$ son inversos porque los pares de entrada y salida están invertidos.

máximo (de una función) El punto más alto en una gráfica.



El máximo de esta función está en (-1, 3).

mínimo (de una función) El punto más bajo en una gráfica.



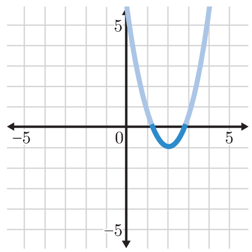
El mínimo de esta función está en (-2, -3).

Algebra 1 Unit 4 Glossary/Álgebra 1 Unidad 4 Glosario

N

English

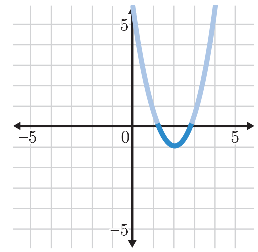
negative (interval or function) A function is negative when its outputs are negative and its graph is below the x-axis. A function can be negative for its entire domain or over an interval.



This function is negative when $1 < x < 3$.

Español

negativo (intervalo o función) Una función es negativa cuando sus salidas son negativas y su gráfica está por debajo del eje x. Una función puede ser negativa en todo su dominio o a lo largo de un intervalo.



Esta función es negativa cuando $1 < x < 3$.

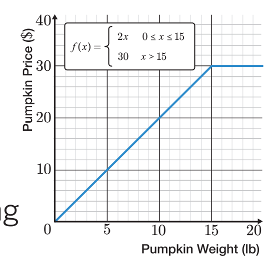
O

output The value of a function after it has been evaluated for an input value of x is called the output. The output is sometimes called the dependent variable. The output typically appears on the vertical axis of a graph and in the right-hand column of a table.

salida El valor de una función tras haber sido evaluada con un valor de entrada x se denomina salida. La salida a veces se denomina variable dependiente. La salida suele estar en el eje vertical de una gráfica y en la columna derecha de una tabla.

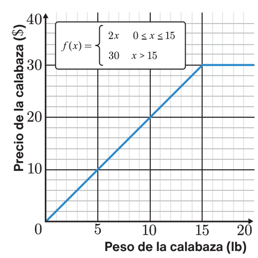
P

piecewise-defined function A function in which different rules apply to different intervals or values in its domain. When evaluating piecewise-defined functions, every input can only go with one rule.



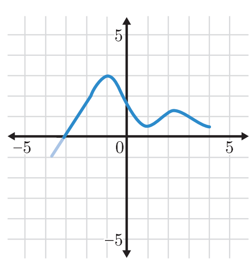
For example, when $0 \le x \le 15$, $f(x) = 2x$.
When $x > 15$, $f(x) = 30$.

función definida por partes Una función en la que se aplican distintas reglas a distintos intervalos o valores de su dominio. Al evaluar funciones definidas por partes, cada entrada solo puede ir con una regla.



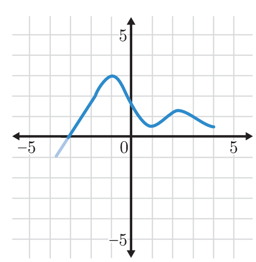
Por ejemplo, cuando $0 \le x \le 15$, $f(x) = 2x$.
Cuando $x > 15$, $f(x) = 30$.

positive (interval or function) A function is positive when its outputs are positive and its graph is above the x-axis. A function can be positive for its entire domain or over an interval.



This function is positive when $-3 < x < 4$.

positivo (intervalo o función) Una función es positiva cuando sus salidas son positivas y su gráfica está por encima del eje x. Una función puede ser positiva en todo su dominio o a lo largo de un intervalo.

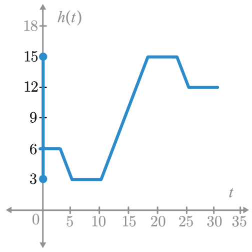


Esta función es positiva cuando $-3 < x < 4$.

English

R

range (of a function) The set of all possible output values for a function or relation. The range can be described in words or as an inequality.



The range of this graph can be described as:
In words: All numbers from 3 to 15.
Inequality: $3 \le h(t) \le 15$

rate of change The change in y divided by the change in x between any two points on the line. The rate of change in a linear relationship is also the slope of its graph.

recursive definition A sequence can be defined recursively or explicitly. Recursive definitions include at least the first term of the sequence and a rule for determining each term that follows. They can be written in function notation or in words.

Sequence:
$3, 4.5, 6, 7.5, 9, \ldots$

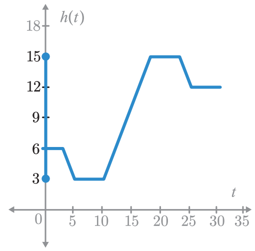
In Words:
First Term: 3
Rule: Add 1.5 to the previous term

In Function Notation:
$f(1) = 3$
$f(n) = f(n-1) + 1.5$
Let n represent the term number.

relation A way of creating input-output pairs. When a relation assigns exactly one output to every input, it is called a function.

Español

rango (de una función) El conjunto de todos los posibles valores de salida de una función o relación. El rango puede describirse con palabras o como una desigualdad.



El rango de esta gráfica puede describirse de la siguiente manera:
Todos los números del 3 al 15.
$3 \le h(t) \le 15$

tasa de cambio El cambio en y dividido por el cambio en x entre dos puntos cualesquiera de la línea. La tasa de cambio en una relación lineal también es la pendiente de su gráfica.

definición recursiva Una secuencia puede definirse de forma recursiva o explícita. Las definiciones recursivas incluyen por lo menos el primer término de la secuencia y una regla para determinar cada término que siga. Pueden escribirse en notación de funciones o en palabras.

Secuencia:
$3, 4.5, 6, 7.5, 9, \ldots$

En palabras:
Primer término: 3
Regla: Sumar 1.5 al término anterior

En notación de funciones:
$f(1) = 3$
$f(n) = f(n-1) + 1.5$
Sea n el número del término.

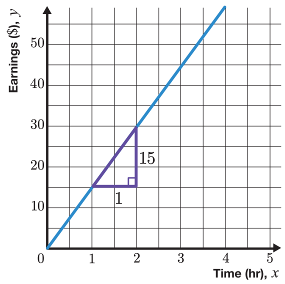
relación Una forma de establecer pares de entrada y salida. Cuando una relación asigna exactamente una salida a cada entrada, se denomina función.

Algebra 1 Unit 4 Glossary/Álgebra 1 Unidad 4 Glosario

English

S

slope A number that describes the direction and steepness of a line. Slope represents the amount that y changes when x increases by 1. Since the slope between any two points on a line will be the same, we can say that a line has a constant rate of change. One way to calculate slope is to divide the vertical distance between any two points on the line by the horizontal distance between those points.

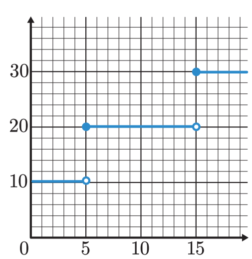


In this graph, y increases by 15 dollars when x increases by 1 hour. The slope of the line is 15, and the rate of change is 15 dollars per hour.

slope-intercept form A way to write a linear equation that highlights the slope and the y-intercept of the line it represents. Slope-intercept form equations are written as $y = mx + b$, where m represents the slope, b represents the y-intercept of the line, and x and y are variables.

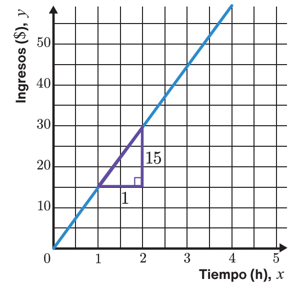
The equations $y = 2x + 4$ and $y = -5x - 10$ are in slope-intercept form. The equation $2x + 5y = 20$ is not in slope-intercept form.

step function A piecewise-defined function where every section of the graph is a point or a horizontal line at a constant value.



Español

pendiente Un número que describe la dirección e inclinación de una recta. La pendiente representa la cantidad en la que cambia y cuando x se incrementa en 1. Como la pendiente entre dos puntos cualesquiera de una recta es la misma, podemos decir que una recta tiene una tasa de cambio constante.Una manera de calcular la pendiente es dividir la distancia vertical entre dos puntos cualesquiera en la recta por la distancia horizontal entre dichos puntos.

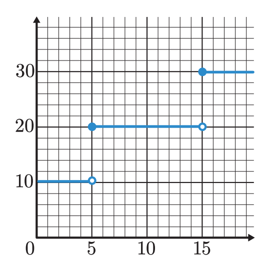


En esta gráfica, y incrementa en 15 dólares cuando x incrementa en 1 hora. La pendiente de la recta es 15 y la tasa de cambio es de 15 dólares por hora.

forma pendiente-intersección Una forma de escribir una ecuación lineal que destaca la pendiente y la intersección con el eje y de la recta que representa. Las ecuaciones en forma pendiente-intersección se escriben como $y = mx + b$, donde m representa la pendiente, b representa la intersección con el eje y de la recta, y tanto x como y son variables.

Las ecuaciones $y = 2x + 4$ y $y = -5x - 10$ están en forma pendiente-intersección. La ecuación $2x + 5y = 20$ no está en forma pendiente-intersección.

función escalonada Una función definida por partes en la que cada sección de la gráfica es un punto o una línea horizontal con un valor constante.



English | Español

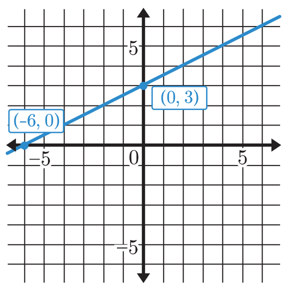
T

translation A transformation that moves every point in a function a given distance in a given direction. A translation changes the location of a function, but does not change its shape.

traslación Una transformación que mueve cada punto de una función una determinada distancia en una determinada dirección. Una traslación cambia la ubicación de una función, pero no su forma.

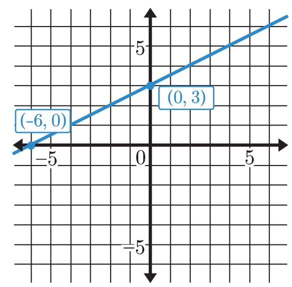
Y

y-intercept A point where the graph of an equation or function crosses the y-axis or when $x = 0$.



The y-intercept of the graph $-2x + 4y = 12$ is $(0, 3)$, or just 3.

intersección con el eje y Un punto donde la gráfica de una ecuación o función cruza el eje y, o cuando $x = 0$.



La intersección con el eje y de la gráfica de $-2x + 4y = 12$ es $(0, 3)$, o simplemente 3.