Amplify Desmos Math
FLORIDA

Student Edition

6

Volume 2

Amplify Desmos Math FLORIDA

Grade 6

Volume 2: Units 5–8

Student Edition

About Amplify

Amplify is dedicated to collaborating with educators to create learning experiences that are rigorous and riveting for all students. Amplify creates K–12 core and supplemental curriculum, assessment, and intervention programs for today's students.

A pioneer in K–12 education since 2000, Amplify is leading the way in next-generation curriculum and assessment. All of our programs provide teachers with powerful tools that help them understand and respond to the needs of every student.

Amplify gratefully acknowledges the work of distinguished program advisors from English Learners Success Forum (ELSF), who have been integral in the development of Amplify Desmos Math. ELSF is a 501(c)(3) nonprofit organization whose mission is to expand educational equity for multilingual learners by increasing the supply of high-quality instructional materials that center their cultural and linguistic assets.

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Dear Student,

Welcome to Amplify Desmos Math Florida! We are excited to be partnering with you this year. You play an essential role in math class, so we wanted to reach out to introduce ourselves and tell you a bit about who we are.

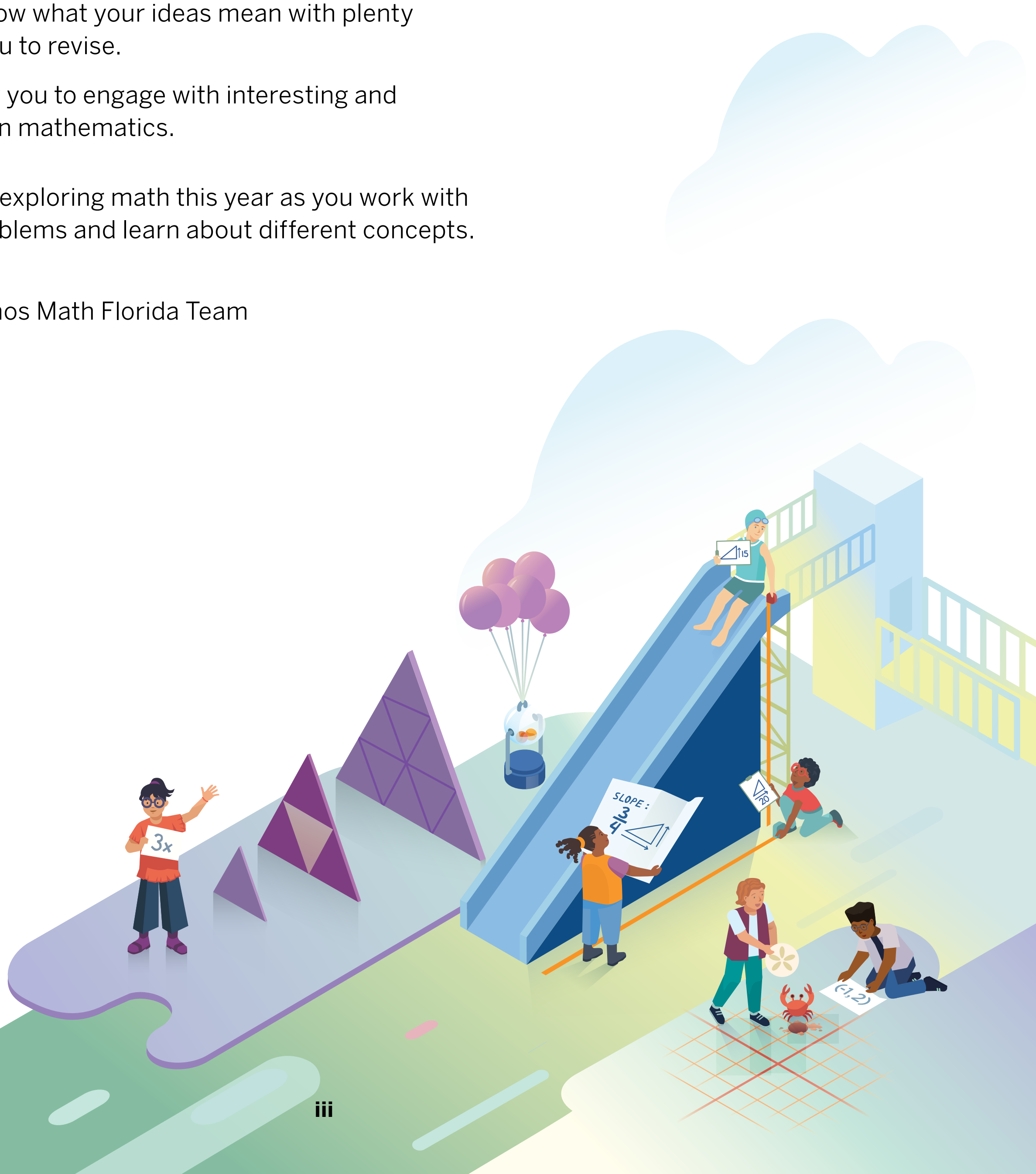
Amplify Desmos Math Florida is a team of math educators on a mission to support you and your classmates in learning math. We hope each lesson inspires you to use your creativity, ask questions, and discover connections between math concepts and the world around us.

Here is what you can expect this year:

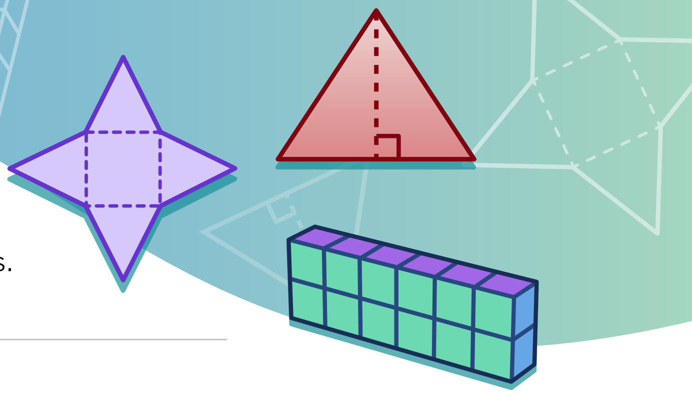
- Lessons that encourage you to ask questions, explore, settle disputes, create challenges for your classmates, and more!
- Activities that show what your ideas mean with plenty of chances for you to revise.
- Opportunities for you to engage with interesting and important ideas in mathematics.

We hope you enjoy exploring math this year as you work with friends to solve problems and learn about different concepts.

–The Amplify Desmos Math Florida Team



Unit 1 Area and Surface Area



In this unit, you will learn to calculate areas of polygons. You will also represent polyhedra with nets and calculate their surface areas.

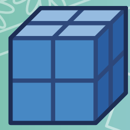
Unit 2 Introducing Ratios

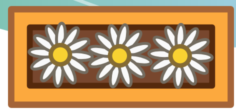
In this unit, you will be introduced to the concept of ratios. You will also represent ratios using double number lines, tables, and tape diagrams, and use ratio reasoning to solve problems.

Unit 3 Unit Rates and Percentages

In this unit, you will apply ratio reasoning from Unit 2 to unit rates and recognize that equivalent ratios have the same unit rates. You will also use a variety of strategies and representations of percentages to determine missing percentages, parts, and wholes.

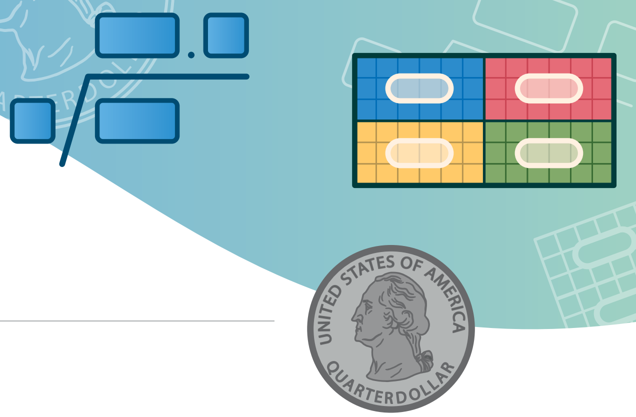
Unit 4 Multiplying and Dividing Fractions





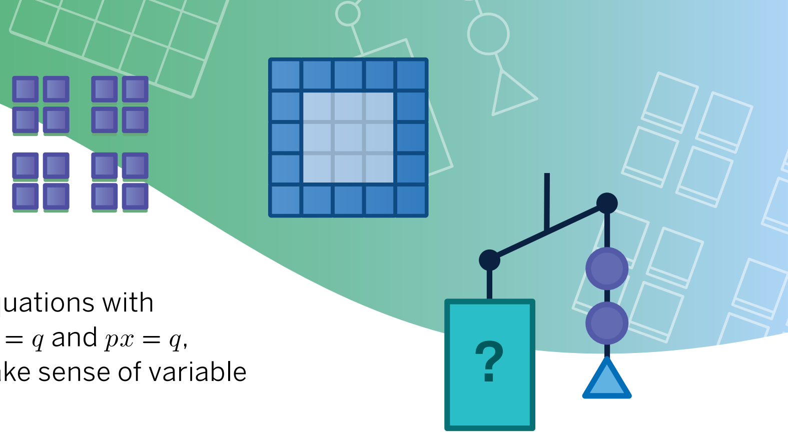
In this unit, you will extend what you learned about multiplying and dividing whole numbers to multiply and divide fractions by fractions. You will learn a variety of strategies, including making tape diagrams, creating common denominators, and rewriting equivalent multiplication problems using the reciprocal.

Unit 5 Decimal Arithmetic



In this unit, you will develop and use a variety of strategies for multiplying and dividing multi-digit decimals.

Unit 6 Expressions and Equations



In this unit, you will reason about expressions and equations with variables. You will solve equations of the forms $x + p = q$ and $px = q$, write equivalent expressions using variables, and make sense of variable expressions involving exponents.

Unit 7 Positive and Negative Numbers

In this unit, you will explore positive and negative numbers in several contexts: on a number line, with inequalities, on the coordinate plane, and in equations.

Unit 8 Describing Data

In this unit, you will visualize data using dot plots, histograms, and box plots. You will also calculate measures of center and spread.

Florida’s B.E.S.T. Standards for Mathematics

Benchmark	B.E.S.T Mathematics Benchmark
Number Sense and Operations	
MA.6.NSO.1.1	Extend previous understanding of numbers to define rational numbers. Plot, order and compare rational numbers.
MA.6.NSO.1.2	Given a mathematical or real-world context, represent quantities that have opposite direction using rational numbers. Compare them on a number line and explain the meaning of zero within its context.
MA.6.NSO.1.3	Given a mathematical or real-world context, interpret the absolute value of a number as the distance from zero on a number line. Find the absolute value of rational numbers.
MA.6.NSO.1.4	Solve mathematical and real-world problems involving absolute value, including the comparison of absolute value.
MA.6.NSO.2.1	Multiply and divide positive multi-digit numbers with decimals to the thousandths, including using a standard algorithm with procedural fluency.
MA.6.NSO.2.2	Extend previous understanding of multiplication and division to compute products and quotients of positive fractions by positive fractions, including mixed numbers, with procedural fluency.
MA.6.NSO.2.3	Solve multi-step real-world problems involving any of the four operations with positive multi-digit decimals or positive fractions, including mixed numbers.
MA.6.NSO.3.1	Given a mathematical or real-world context, find the greatest common factor and least common multiple of two whole numbers.
MA.6.NSO.3.2	Rewrite the sum of two composite whole numbers having a common factor, as a common factor multiplied by the sum of two whole numbers.
MA.6.NSO.3.3	Evaluate positive rational numbers and integers with natural number exponents.
MA.6.NSO.3.4	Express composite whole numbers as a product of prime factors with natural number exponents.
MA.6.NSO.3.5	Rewrite positive rational numbers in different but equivalent forms including fractions, terminating decimals and percentages.
MA.6.NSO.4.1	Apply and extend previous understandings of operations with whole numbers to add and subtract integers with procedural fluency.
MA.6.NSO.4.2	Apply and extend previous understandings of operations with whole numbers to multiply and divide integers with procedural fluency.
Algebraic Reasoning	
MA.6.AR.1.1	Given a mathematical or real-world context, translate written descriptions into algebraic expressions and translate algebraic expressions into written descriptions.
MA.6.AR.1.2	Translate a real-world written description into an algebraic inequality in the form of $x > a$, $x < a$, $x \geq a$ or $x \leq a$. Represent the inequality on a number line.
MA.6.AR.1.3	Evaluate algebraic expressions using substitution and order of operations.
MA.6.AR.1.4	Apply the properties of operations to generate equivalent algebraic expressions with integer coefficients.
MA.6.AR.2.1	Given an equation or inequality and a specified set of integer values, determine which values make the equation or inequality true or false.
MA.6.AR.2.2	Write and solve one-step equations in one variable within a mathematical or real-world context using addition and subtraction, where all terms and solutions are integers.

Florida's B.E.S.T. Standards for Mathematics

MA.6.AR.2.3	Write and solve one-step equations in one variable within a mathematical or real-world context using multiplication and division, where all terms and solutions are integers.
MA.6.AR.2.4	Determine the unknown decimal or fraction in an equation involving any of the four operations, relating three numbers, with the unknown in any position.
MA.6.AR.3.1	Given a real-world context, write and interpret ratios to show the relative sizes of two quantities using appropriate notation: a/b, a to b, or $a{:}b$ where $b \neq 0$.
MA.6.AR.3.2	Given a real-world context, determine a rate for a ratio of quantities with different units. Calculate and interpret the corresponding unit rate.
MA.6.AR.3.3	Extend previous understanding of fractions and numerical patterns to generate or complete a two- or three-column table to display equivalent part-to-part ratios and part-to-part-to-whole ratios.
MA.6.AR.3.4	Apply ratio relationships to solve mathematical and real-world problems involving percentages using the relationship between two quantities.
MA.6.AR.3.5	Solve mathematical and real-world problems involving ratios, rates and unit rates, including comparisons, mixtures, ratios of lengths and conversions within the same measurement system.
Geometric Reasoning	
MA.6.GR.1.1	Extend previous understanding of the coordinate plane to plot rational number ordered pairs in all four quadrants and on both axes. Identify the x- or y-axis as the line of reflection when two ordered pairs have an opposite x- or y-coordinate.
MA.6.GR.1.2	Find distances between ordered pairs, limited to the same x-coordinate or the same y-coordinate, represented on the coordinate plane.
MA.6.GR.1.3	Solve mathematical and real-world problems by plotting points on a coordinate plane, including finding the perimeter or area of a rectangle.
MA.6.GR.2.1	Derive a formula for the area of a right triangle using a rectangle. Apply a formula to find the area of a triangle.
MA.6.GR.2.2	Solve mathematical and real-world problems involving the area of quadrilaterals and composite figures by decomposing them into triangles or rectangles.
MA.6.GR.2.3	Solve mathematical and real-world problems involving the volume of right rectangular prisms with positive rational number edge lengths using a visual model and a formula.
MA.6.GR.2.4	Given a mathematical or real-world context, find the surface area of right rectangular prisms and right rectangular pyramids using the figure's net.
Data Analysis and Probability	
MA.6.DP.1.1	Recognize and formulate a statistical question that would generate numerical data.
MA.6.DP.1.2	Given a numerical data set within a real-world context, find and interpret mean, median, mode and range.
MA.6.DP.1.3	Given a box plot within a real-world context, determine the minimum, the lower quartile, the median, the upper quartile and the maximum. Use this summary of the data to describe the spread and distribution of the data.
MA.6.DP.1.4	Given a histogram or line plot within a real-world context, qualitatively describe and interpret the spread and distribution of the data, including any symmetry, skewness, gaps, clusters, outliers and the range.
MA.6.DP.1.5	Create box plots and histograms to represent sets of numerical data within real-world contexts.
MA.6.DP.1.6	Given a real-world scenario, determine and describe how changes in data values impact measures of center and variation.

Mathematical Thinking and Reasoning Standards

MA.K12.MTR.1.1	Actively participate in effortful learning both individually and collectively.
MA.K12.MTR.2.1	Demonstrate understanding by representing problems in multiple ways.
MA.K12.MTR.3.1	Complete tasks with mathematical fluency.
MA.K12.MTR.4.1	Engage in discussions that reflect on the mathematical thinking of self and others.
MA.K12.MTR.5.1	Use patterns and structure to help understand and connect mathematical concepts.
MA.K12.MTR.6.1	Assess the reasonableness of solutions.
MA.K12.MTR.7.1	Apply mathematics to real-world contexts.

English Language Arts B.E.S.T. Standards

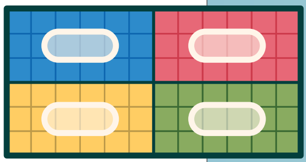
ELA.K12.EE.1.1	Cite evidence to explain and justify reasoning.
ELA.K12.EE.2.1	Read and comprehend grade-level complex texts proficiently.
ELA.K12.EE.3.1	Make inferences to support comprehension.
ELA.K12.EE.4.1	Use appropriate collaborative techniques and active listening skills when engaging in discussions in a variety of situations.
ELA.K12.EE.5.1	Use the accepted rules governing a specific format to create quality work.
ELA.K12.EE.6.1	Use appropriate voice and tone when speaking or writing.

English Language Development Standards

ELD.K12.ELL.MA.1	English language learners communicate information, ideas and concepts necessary for academic success in the content area of Mathematics.
ELD.K12.ELL.SI.1	English language learners communicate for social and instructional purposes within the school setting.

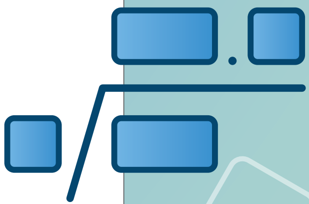
Unit 5

Decimal Arithmetic



When you multiply and divide numbers with decimal places, instead of just estimating, you can get far more precise answers. Doing so can help you make sense of all sorts of real-world situations — like working with money, comparing grocery prices, and even buying a car!

Essential Questions

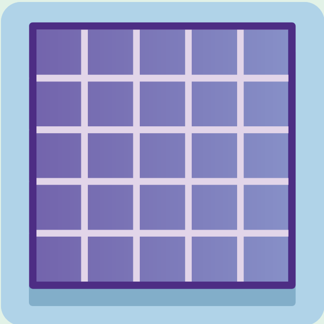


- How do the place values in each decimal number in a calculation affect the place value of the result?
- How are strategies for multiplying decimals like strategies for dividing decimals? How are they different?

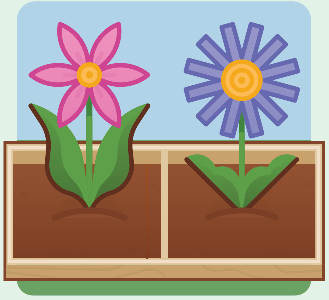
Sub-unit

1

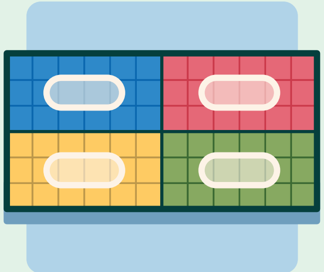
Multiplying Decimals



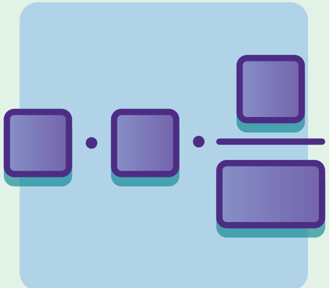
Lesson 1
Decimal Multiplication



Lesson 2
Garden Arrangements



Lesson 3
Multiplying With Areas



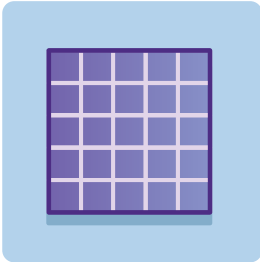
Lesson 4
Multiplication Methods

Name: ________ Date: ________ Period: ________

MA.6.NSO.2.1, MTR.2.1

Decimal Multiplication

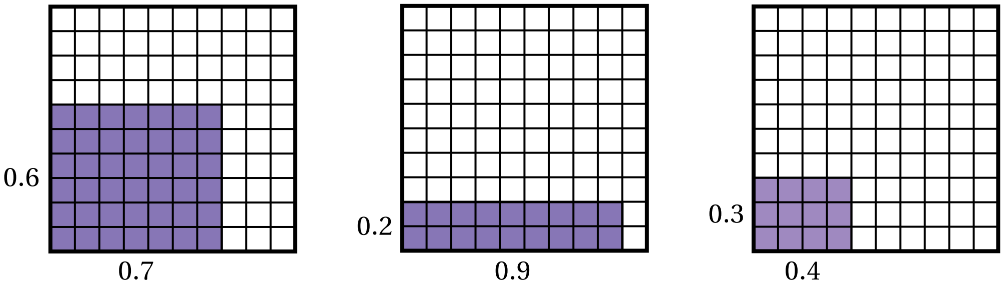
Let's explore decimal multiplication using place value.



Warm-Up

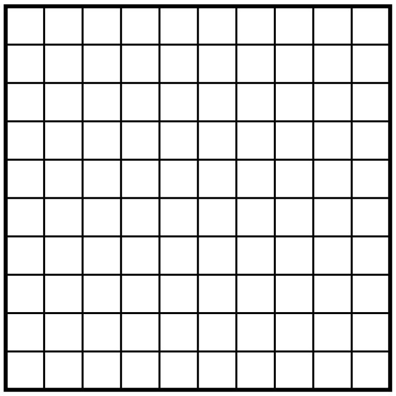
1. The large rectangle has a length and a width of one.

a Take a look at these shaded rectangles with different lengths and widths.



b Try to create shaded rectangles with the following areas:

Length	Width	Area
		0.16
		0.24
		0.30



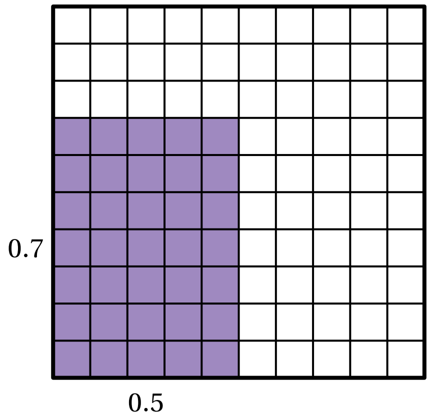
Activity 1

Name: ________ Date: ________ Period: ________

Keepin' It One Hundredth

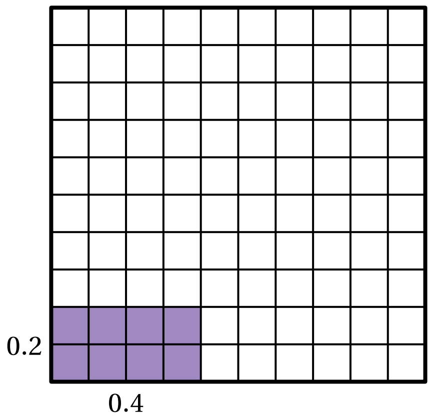
2. Makayla says that $0.5 \cdot 0.7 = 0.35$.

Use the diagram to show or explain why this makes sense.



3. Multiply $0.4 \cdot 0.2$.

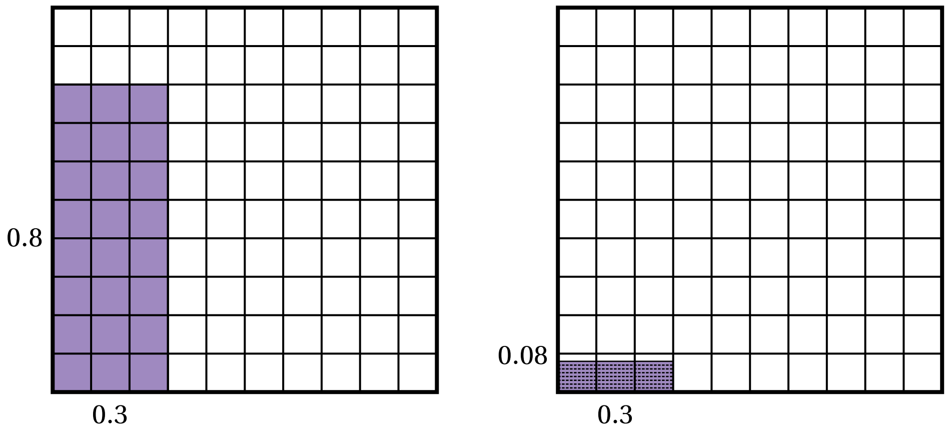
Write your answer as a decimal.



Name: .... Date: .... Period: ....

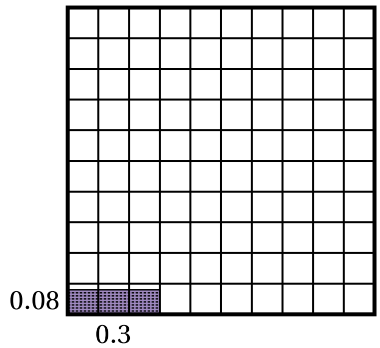
Keepin' It One Hundredth (continued)

4. Here are two diagrams that represent different multiplication problems.



Discuss: How are the diagrams alike? How are they different?

5. What is $0.3 \cdot 0.08$?



6. This is Jayden's work for multiplying $0.3 \cdot 0.08$.

$$0.3 \cdot 0.08$$
$$\frac{3}{10} \cdot \frac{8}{100}$$
$$\frac{24}{1000} = 0.024$$

a **Discuss:** What is Jayden's strategy?

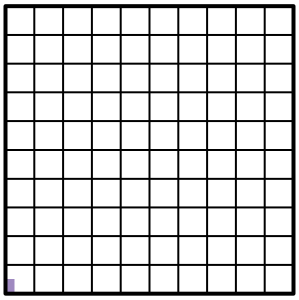
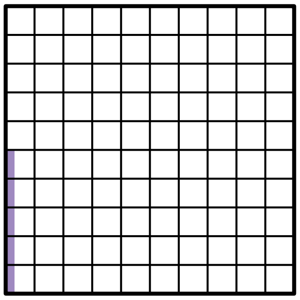
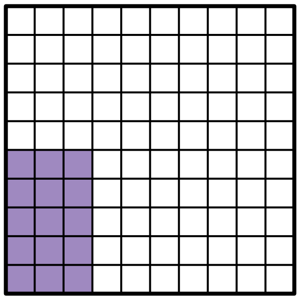
b Show or explain how you would use his strategy to multiply $0.02 \cdot 0.9$.

Name: ______ Date: ______ Period: ______

Clicking Into Place Value

7. Match each area with its equivalent expressions.

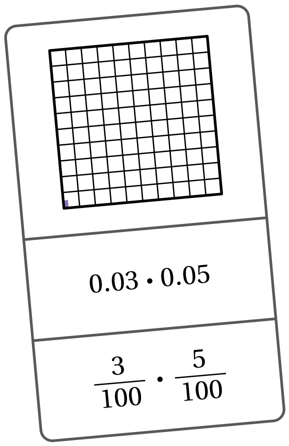
$\frac{15}{100}$	$0.3 \cdot 0.5$	0.015
$0.03 \cdot 0.05$	$\frac{3}{100} \cdot \frac{5}{100}$	$0.03 \cdot 0.5$

Area 1	Area 2	Area 3
		

8. Jayden matched these choices in the previous problem.

He noticed that the product was missing.

Calculate $0.03 \cdot 0.05$.



Activity
3

Name: .... Date: .... Period: ....

Repeated Challenges

9. Solve as many challenges as you have time for.

a $0.04 \bullet 0.6$

b $0.03 \bullet 0.02$

c $0.2 \bullet 0.007$

d $0.003 \bullet 0.3$

e $0.5 \bullet 0.4$

f $0.001 \bullet 0.08$

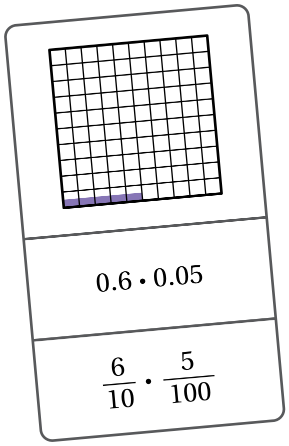
g $1 \bullet 0.05$

h $0.4 \bullet 0.005$

Synthesis

10. Show or describe how decimals, fractions, and the hundredths chart are related.

Use the example if it helps with your thinking.



Lesson Practice 5.01

Lesson Summary

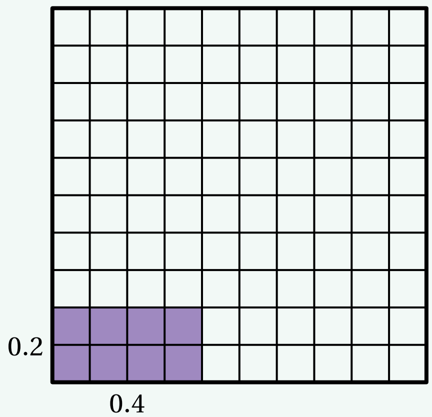
Two strategies for multiplying decimals are:

- Using an area model.
- Writing the decimals as equivalent fractions.

One advantage to using an area model is that you can visualize the product. For example, you can represent $0.4 \cdot 0.2$ as a rectangle with a length of 0.4 and a width of 0.2. On a hundredths chart, you can count the shaded boxes, each representing $\frac{1}{100}$, to determine the product.

It can, however, be challenging to use an area model to represent decimals smaller than tenths or hundredths.

Your other option is to convert decimals to equivalent fractions. $0.4 \cdot 0.2$ can be written as $\frac{4}{10} \cdot \frac{2}{10}$, which equals $\frac{8}{100}$ or 0.08.



Area = length • width

$= 0.4 \cdot 0.2$

$= 0.08$

Lesson Practice
5.01

Name: .... Date: .... Period: ....

Problems 1–4: Determine the value of each expression.

1. $20 \cdot 40$

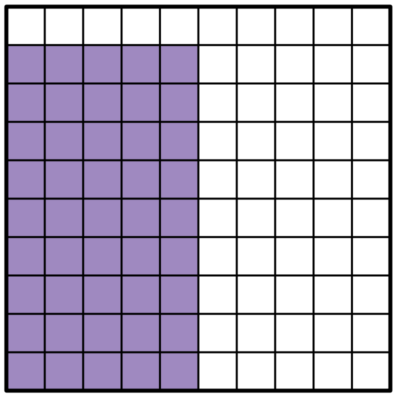
2. $200 \cdot 40$

3. $2 \cdot 40$

4. $2 \cdot 0.4$

Problems 5–6: Here is an expression: $0.5 \cdot 0.9$.

5. Explain why the diagram represents $0.5 \cdot 0.9$.



6. What is the value of $0.5 \cdot 0.9$?

Problems 7–8: Emiliano attempted to calculate $0.003 \cdot 0.007$. Here is his work.

$$0.003 \cdot 0.007 = \frac{3}{1000} \cdot \frac{7}{1000}$$
$$= \frac{21}{1000} = 0.021$$

7. What do you think Emiliano's mistake might be?

8. What is the value of $0.003 \cdot 0.007$?

Name: ________ Date: ________ Period: ________

Problems 9–10: Determine the value of each expression.

9. $0.3 \cdot 0.2$

10. $1.2 \cdot 5$

FAST Practice

11. Select *all* the expressions that have the same value as $0.05 \cdot 0.6$.

- ☐ **A.** $5 \cdot \frac{1}{100} \cdot 6 \cdot \frac{1}{10}$
- ☐ **B.** $5 \cdot 6 \cdot \frac{1}{1000}$
- ☐ **C.** $5 \cdot 0.001 \cdot 6 \cdot 0.01$
- ☐ **D.** 0.03
- ☐ **E.** 0.003

Spiral Review

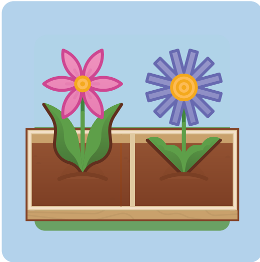
12. A plumber has 52.2 meters of PVC pipe to use on a job. On the first day, she used 21.863 meters of the pipe. How many meters of pipe does she have left after the first day?

Ⓐ 30.663　Ⓑ 30.337　Ⓒ 30.763　Ⓓ 30.237

Name: .... Date: .... Period: ....

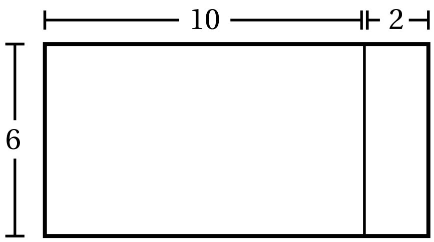
MA.6.NSO.3.2, MTR.2.1

Garden Arrangements



Let's create expressions using area models.

Warm-Up



1. What is the area of this rectangle?

2. Write as many expressions as you can to represent the area of this rectangle.

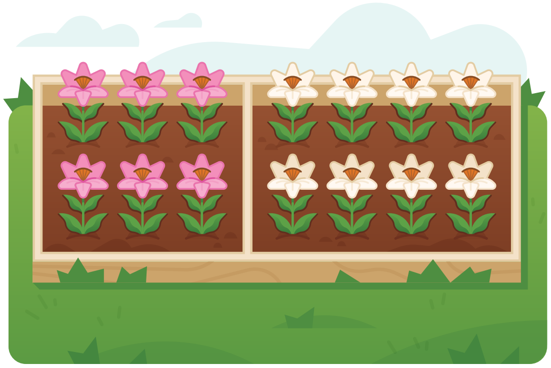
Activity 1

Name: .... Date: .... Period: ....

Planting Flowers

3. Sai planted pink and white lilies in his family garden. Sai and his brother, Ichiro, wrote two different expressions to represent the total number of lilies.

Sai	Ichiro
$2(3 + 4)$	$6 + 8$



a **Discuss:** Select *one* brother and explain how you think he came up with his expression.

b Are the values of the expressions the same? Show or explain your thinking.

4. Select *all* the expressions that represent the total number of tulips.

- ☐ **A.** $3(5) + 3(4)$
- ☐ **B.** $15 + 12$
- ☐ **C.** $3 \cdot 5 \cdot 4$
- ☐ **D.** $3(5 + 4)$
- ☐ **E.** $3(9)$

Activity
2

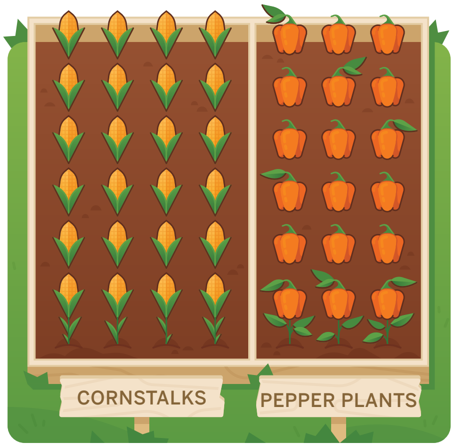
Name: .... Date: .... Period: ....

Garden Arrangements

Sai's family also has a vegetable garden. Sai wants to plant 24 cornstalks and 18 pepper plants. He wants to have the same number of rows for both types of plant.

5. Sai created the plan shown as one option for organizing his garden.

Create as many other options as you can of ways Sai can create a garden with equally-spaced plants.



Name: .... Date: .... Period: ....

Garden Arrangements (continued)

6. Choose *one* of the options that you created. Write as many expressions as you can to represent that garden arrangement.

7. Fill in the blanks to show *two* different ways to represent the total number of cornstalks and pepper plants in Sai's garden.

3	4	6	18	24

$\_\_\_\_ + \_\_\_\_ = \_\_\_\_ (\_\_\_\_ + \_\_\_\_)$

8. **Discuss:**

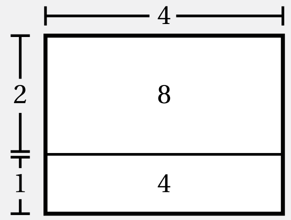
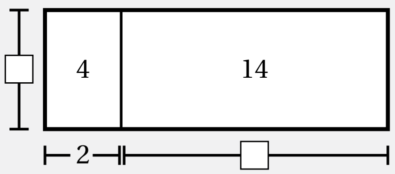
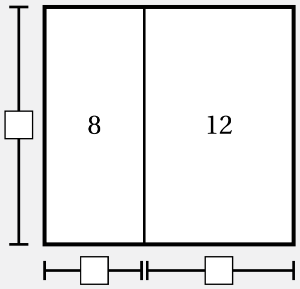
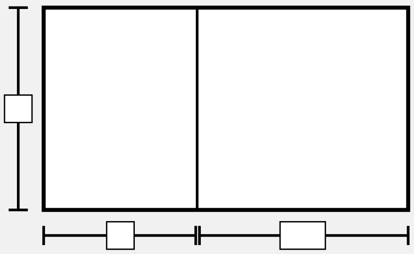
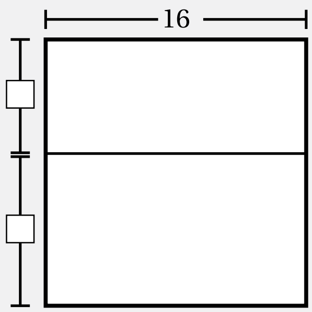
- What does the left side of the equation tell you about Sai's garden? The right side?
- When would it be helpful to use the left side of the equation? The right side?

Activity 3

Name: ____________ Date: ________ Period: ________

Two Expressions, Same Area

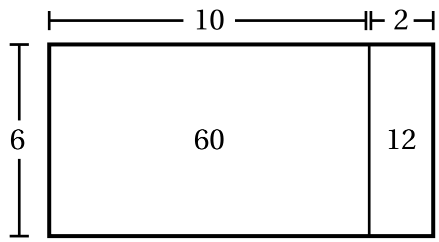
9. Fill in the missing values of each area model. Write two expressions to represent the total area.

Area Model	Expression 1	Expression 2
	$8 + 4$	$4(2 + 1)$
a 		
b 		
c 		$7(8 + 13)$
d 	$48 + 64$	

Synthesis

10. Explain how you can write multiple expressions to describe the area of a rectangle.

Use this rectangle if it helps with your thinking.



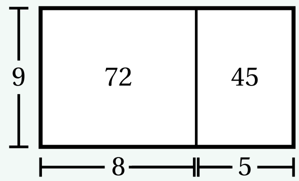
Lesson Practice 5.02

Lesson Summary

We can use common factors to create equivalent expressions.

For example, since 7 is a common factor of each term in the expression $21 + 14$, you can create the equivalent expression $7(3 + 2)$. This is an example of the *distributive property*.

We can use area models to determine equivalent expressions.



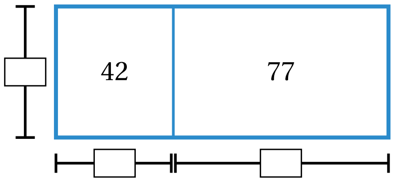
Let’s write two expressions for this area model.

- $72 + 45$ represents the sum of the areas of the smaller rectangles.
- $9(8 + 5)$ represents the length of the whole rectangle multiplied by the width. This expression shows us that 9 is a common factor of 72 and 45.

Name: .... Date: .... Period: ....

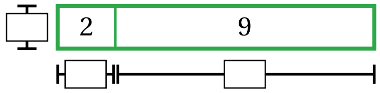
Problems 1–4: Use the area models to rewrite each expression using parentheses.

1. $42 + 77$



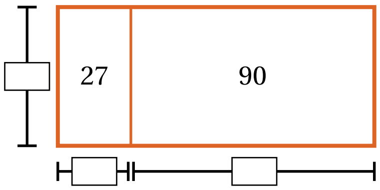
.... (........ + ...)

2. $2 + 9$



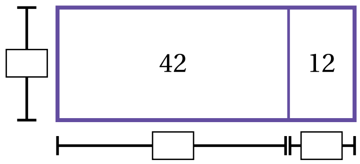
.... (........ + ...)

3. $27 + 90$



.... (........ + ...)

4. $42 + 12$

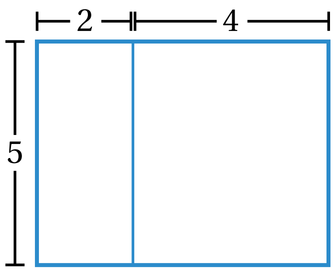


.... (........ + ...)

5. Select *all* the expressions that represent the area of the largest rectangle in figure A.

- ☐ **A.** $5(2 + 4)$
- ☐ **B.** $5 \cdot 2 + 4$
- ☐ **C.** $5 \cdot 2 + 5 \cdot 4$
- ☐ **D.** $5 \cdot 2 \cdot 4$
- ☐ **E.** $5 + 2 + 4$
- ☐ **F.** $5 \cdot 6$

Figure A



Lesson Practice
5.02

Name: .... Date: .... Period: ....

FAST Practice

6. Which expression is equivalent to $54 + 66$ *and* uses the greatest common factor of the two numbers being added in the expression?

A. $2(27 + 33)$

B. $6(9 + 11)$

C. $3(18 + 22)$

D. $11(5 + 6)$

Spiral Review

Problems 7–10: Evaluate each expression.

7. $3.05 + 0.028$

8. $6.15 - 1.045$

9. $3.4 - 0.005$

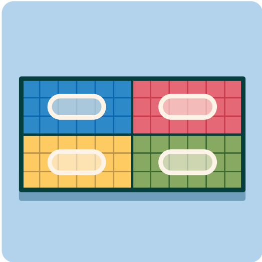
10. $4.5 + 0.009$

Name: ______ Date: ______ Period: ______

MA.6.NSO.2.1, MTR.2.1, MTR.6.1

Multiplying With Areas

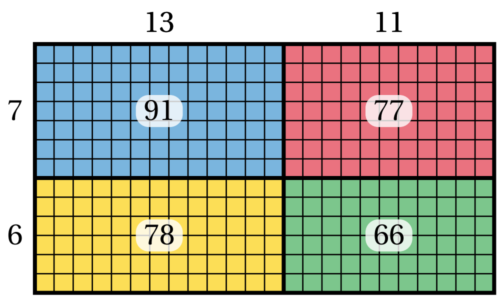
Let's use area models to multiply decimals.



Warm-Up

1. Diego likes using area models to multiply whole numbers.

a Let's look at a way to split this 24-by-13 rectangle.



b What do you notice? What do you wonder?

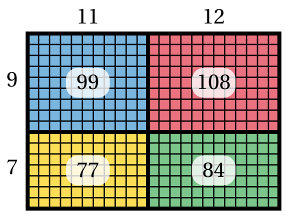
I notice:

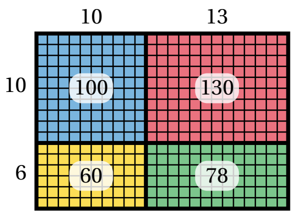
I wonder:

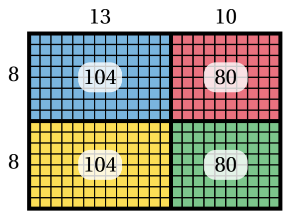
Name: Date: Period:

Creating an Area Model

2. Here are some new area models. Use them to multiply 23 • 16.

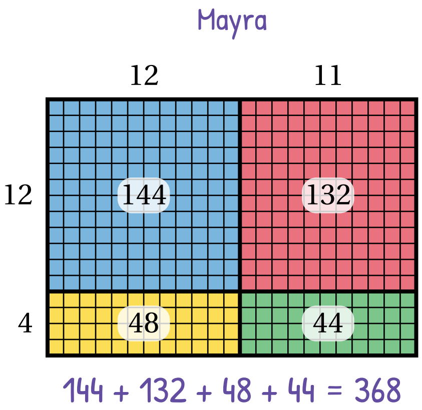


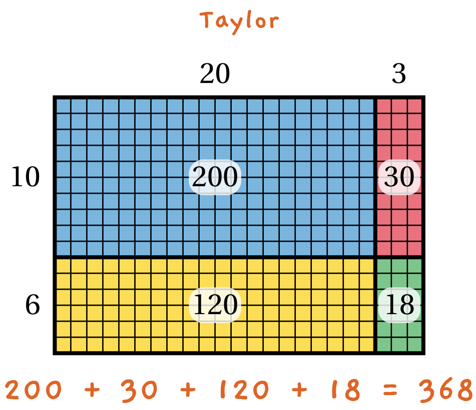




3. Mayra and Taylor made different area models to multiply 23 • 16.

Discuss: Which area model is more helpful? Explain your thinking.





Name: ________ Date: ________ Period: ________

Creating an Area Model (continued)

4. Diego wonders if area models could also help him multiply decimals like $2.3 \cdot 1.6$.

a Show how you would split this rectangle into smaller parts to multiply $2.3 \cdot 1.6$.

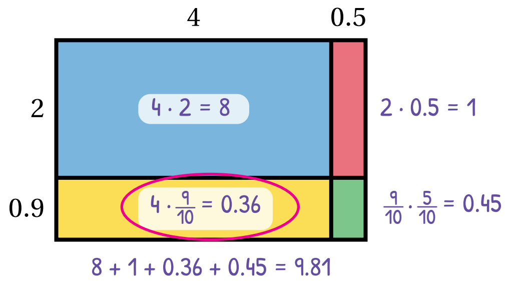
b Explain why you split it that way.

5. Use your area model to calculate $2.3 \cdot 1.6$.

Name: .... Date: .... Period: ....

Calculating With an Area Model

6. Diego made an error while multiplying $4.5 \cdot 2.9$.



a Circle the error in Diego's work.

b What would you say to help him understand his mistake?

7. Multiply $3.4 \cdot 2.8$.

Use the diagram if it helps with your thinking.

8. Multiply $5.2 \cdot 0.42$.

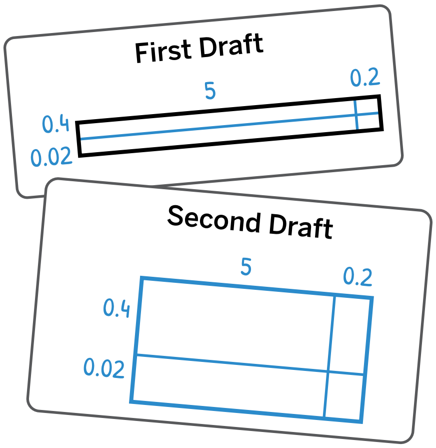
Use the diagram if it helps with your thinking.

Name: .... Date: .... Period: ....

Calculating With an Area Model (continued)

9. Lucia made two drafts to calculate $5.2 \cdot 0.42$.

What are the advantages and disadvantages of Lucia's second draft?

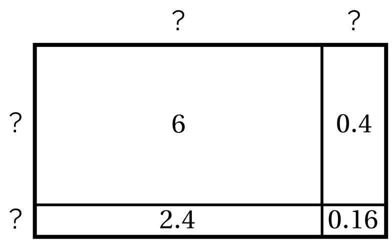


Advantages:

Disadvantages:

10. Here is an area model that is missing some labels.

What multiplication problem could this help you solve? Explain your thinking.



Synthesis

11. Describe how you can use an area model to multiply decimals like $2.7 \cdot 1.4$.

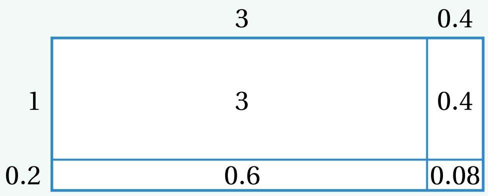
Draw on the diagram if it helps with your thinking.

Lesson Practice 5.03

Lesson Summary

One way to multiply two decimals is to use an area model.

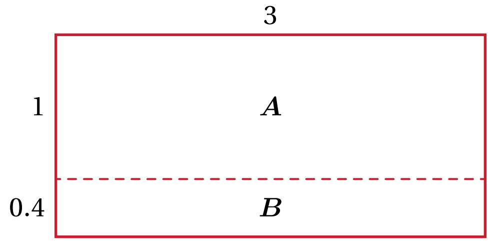
To use an area model, separate the decimals into parts. This rectangle has side lengths measuring 3.4 and 1.2 units. Each side length has been split apart by place value: 3.4 has been split into $3 + 0.4$ and 1.2 has been split into $1 + 0.2$.



The total area of the rectangle is equal to the sum of the areas of the four smaller rectangles: $3.4 \cdot 1.2 = 3 + 0.4 + 0.6 + 0.08 = 4.08$.

Name: ________ Date: ________ Period: ________

Problems 1–2: Here is a diagram that represents $3 \cdot 1.4$.

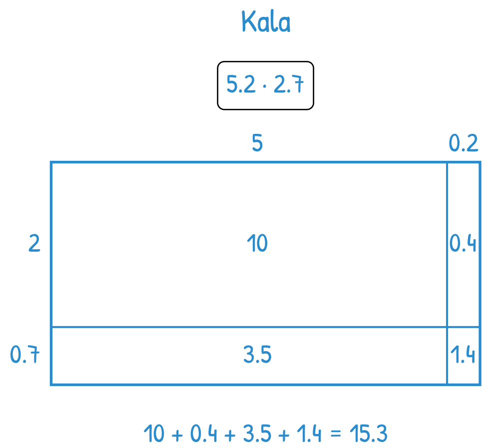


1. Determine the areas of A and B.

A	B

2. What is the value of $3 \cdot 1.4$?

Problems 3–4: Here is Kala's work for determining $5.2 \cdot 2.7$.



3. Kala made an error. Circle what you think the error is. Explain your thinking.

4. What is the value of $5.2 \cdot 2.7$? Show or explain your thinking.

5. Draw an area diagram that represents $2.5 \cdot 1.4$.

6. Determine the product of $2.5 \cdot 1.4$. Use your area diagram from Problem 5 if it helps with your thinking.

Lesson Practice
5.03

Name: .... Date: .... Period: ....

FAST Practice

7. Determine the value of $0.34 \cdot 0.02$.

$\square$

Spiral Review

8. Tariq buys a granola bar that costs $1.59. He pays with seven quarters, for a total of $1.75. How much change should he get?

Problems 9–10: Fill in the blanks with the missing digits that make each problem true.

9.

$$\begin{array}{r} 4\,.\,3\,\square \\ +\ \square\,.\,1\,5 \\ \hline 6\,.\,\square\,2 \end{array}$$

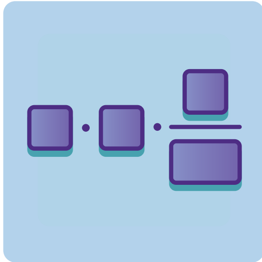
10.

$$\begin{array}{r} 1\,.\,5\,\square \\ +\ \square\,.\,3\,8 \\ \hline 1\,.\,\square\,4 \end{array}$$

Name: .... Date: .... Period: ....

MA.6.NSO.2.1, MTR.1.1, MTR.2.1, MTR.3.1, MTR.4.1

Multiplication Methods



Let's use place value to multiply decimals vertically.

Warm-Up

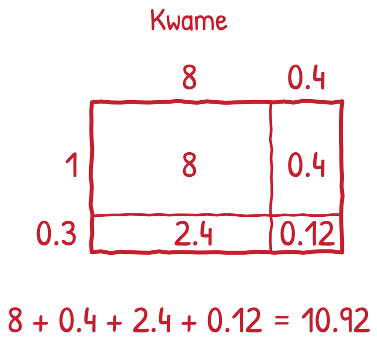
1. Calculate $84 \cdot 13$. Show your thinking.

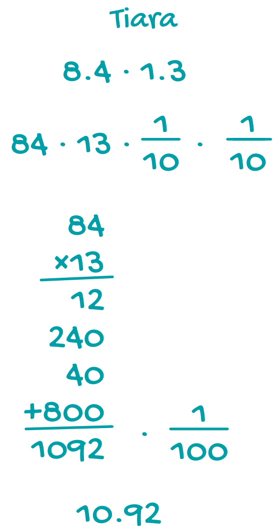
2. Find someone who used a different strategy than you did. Discuss how you each calculated $84 \cdot 13$.

Name: .... Date: .... Period: ....

Multiple Methods

Kwame and Tiara used different strategies to multiply $8.4 \cdot 1.3$.





3. **Discuss:** How are their strategies alike? How are they different?

4. Show how each student might set up $8.4 \cdot 0.13$.

Kwame **Tiara**

5. Use either strategy to finish calculating $8.4 \cdot 0.13$.

6. Calculate $0.352 \cdot 0.25$.

Name: .... Date: .... Period: ....

Multiple Multiplication Methods

7. Tiara wrote this expression to help her calculate $2.9 \cdot 0.015$.

$$29 \cdot 15 \cdot \frac{1}{10} \cdot \frac{1}{1000}$$

If $29 \cdot 15 = 435$, then what is $2.9 \cdot 0.015$?

A. 4.35 **B.** 0.435 **C.** 0.0435 **D.** 0.00435

8. If $165 \cdot 12 = 1980$, then what is $16.5 \cdot 1.2$? Explain your thinking.

9. Select all the expressions that have a product of 0.024.

- ☐ **A.** $0.06 \cdot 0.4$
- ☐ **B.** $0.6 \cdot 0.04$
- ☐ **C.** $0.04 \cdot 0.06$
- ☐ **D.** $2 \cdot 0.012$
- ☐ **E.** $1.2 \cdot 0.02$

10. Write another expression that has a product of 0.024.

Name: ________ Date: ________ Period: ________

Scavenger Hunt

11. Start with any of the scavenger hunt sheets.

- Record the sheet shape, solve the problem, and write your answer.
- Look for your answer at the top of another scavenger hunt sheet. Solve that problem.
- Repeat until you make it back to your starting sheet.

Sheet: ________ **Answer** ________ →	**Sheet:** ________ ↙ **Answer** ________
Sheet: ________ **Answer** ________ →	**Sheet:** ________ ↙ **Answer** ________
Sheet: ________ **Answer** ________ →	**Sheet:** ________ **Answer** ________

Synthesis

12. Describe a strategy that helps you multiply decimals like $0.039 \cdot 3.2$.

Lesson Practice 5.04

Lesson Summary

There are several different strategies you can use to multiply decimals, such as area models and fractions. You can even convert the decimals to whole numbers and then use place value reasoning. Depending on the problem, one strategy might be more helpful than another.

Let's solve $2.4 \cdot 0.03$ using two strategies: converting fractions and using whole numbers with place value reasoning.

Strategy 1: Converting Fractions

Rewrite each value as an equivalent fraction.

$$2.4 \cdot 0.03 = \frac{24}{10} \cdot \frac{3}{100}$$

Multiply the fractions.

$$\frac{24}{10} \cdot \frac{3}{100} = \frac{72}{1000}$$

Use the denominator to determine the place value.

$\frac{72}{1000}$ is 72 thousandths.

$$\frac{72}{1000} = 0.072$$

Strategy 2: Whole Numbers With Place Value Reasoning

Think of each term as a whole number, then multiply.

$$2.4 \cdot 0.03 \longrightarrow 24 \cdot 3$$
$$24 \cdot 3 = 72$$

Think about the place value of each term.

2.4 is 24 tenths.
0.03 is 3 hundredths.

Determine the appropriate place value of the product.

tenths times hundredths = thousandths

$2.4 \cdot 0.03 = 72$ thousandths
$2.4 \cdot 0.03 = 0.072$

Lesson Practice
5.04

Name: .... Date: .... Period: ....

1. Explain how you could use $3 \cdot 65 = 195$ to determine $0.003 \cdot 0.65$.

2. Maia wrote this expression to help her calculate $4.5 \cdot 0.17$.

$$45 \cdot 17 \cdot \frac{1}{10} \cdot \frac{1}{100}$$

If $45 \cdot 17 = 765$, then what is $4.5 \cdot 0.17$?

3. Select *all* the expressions that have a product of 0.0042.

- ☐ **A.** $0.007 \cdot 0.6$
- ☐ **B.** $0.07 \cdot 0.06$
- ☐ **C.** $0.007 \cdot 0.06$
- ☐ **D.** $0.7 \cdot 0.06$
- ☐ **E.** $0.21 \cdot 0.02$

Problems 4–5: Determine the value of each expression using any strategy. Show or explain your thinking.

4. $5.4 \cdot 2.4$

5. $1.01 \cdot 0.00035$

Lesson Practice
5.04

Name: .... Date: .... Period: ....

FAST Practice

6. A pound of blueberries costs $3.50 and a pound of clementines costs $2.50. What is the total cost of 0.6 pounds of blueberries and 1.8 pounds of clementines? Explain your thinking.

The cost of 0.6 pounds of blueberries is $ ☐, and the cost of 1.8 pounds of clementines is $ ☐. So the total cost is $ ☐.

Spiral Review

Problems 7–10: Determine the value of each expression.

7. $20 \cdot 5$

8. $20 \cdot 0.8$

9. $20 \cdot 0.04$

10. $20 \cdot 5.84$

Problems 11–12: Amari bought 12 mini muffins for $5.40.

11. At this rate, what is the price of 4 mini muffins? Show or explain your thinking.

12. How many mini muffins can Amari buy with $4.00? Explain your thinking.

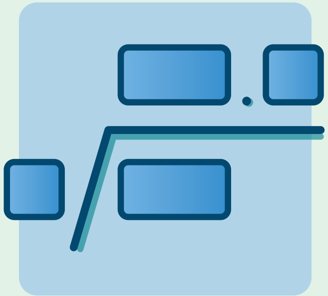
Number of Mini Muffins	Price ($)
12	5.40

Sub-unit

2

Dividing Decimals

Lesson 5
Division Diagrams



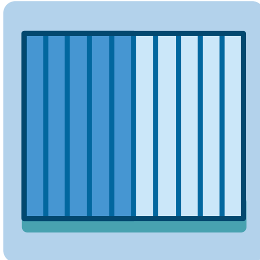
Lesson 6
Long Division and Other Strategies

Lesson 7
Movie Time

Name: .... Date: .... Period: ....

MA.6.NSO.2.1, MTR.2.1

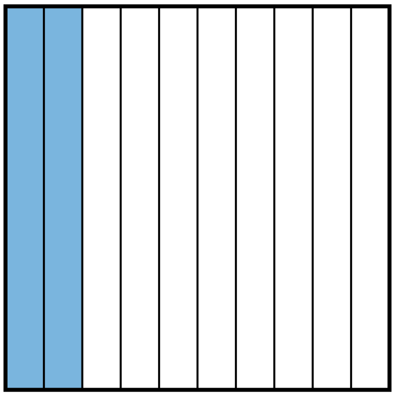
Division Diagrams



Let's divide decimals using hundredths charts and new expressions.

Warm-Up

1. This large square represents 1.



a Select *all* the equations you could use to determine how many blue pieces you need to fill the large square.

- ☐ **A.** $1 \div 0.2 = ?$
- ☐ **B.** $0.2 \div 1 = ?$
- ☐ **C.** $10 \div 2 = ?$
- ☐ **D.** $0.2 \cdot ? = 1$
- ☐ **E.** $2 \div 10 = ?$

b Pick *one* equation and explain how it represents the diagram.

Activity 1

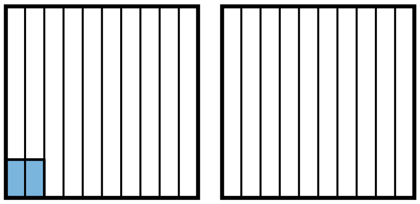
Name: ________ Date: ________ Period: ________

Decimal Division Strategies

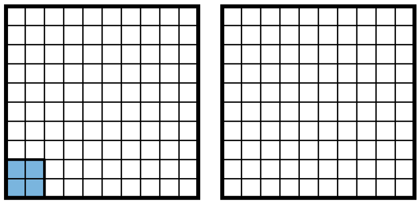
2. Each large square represents 1.

Determine the value of $2 \div 0.04$. Use the tenths or hundredths chart if it helps with your thinking.

Tenths

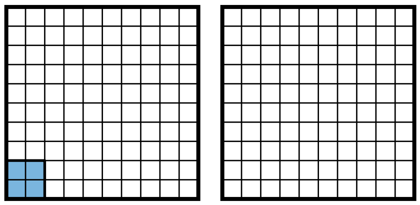


Hundredths



3. Diamond claims that $2 \div 0.04$ has the same value as $200 \div 4$.

Explain why this makes sense.



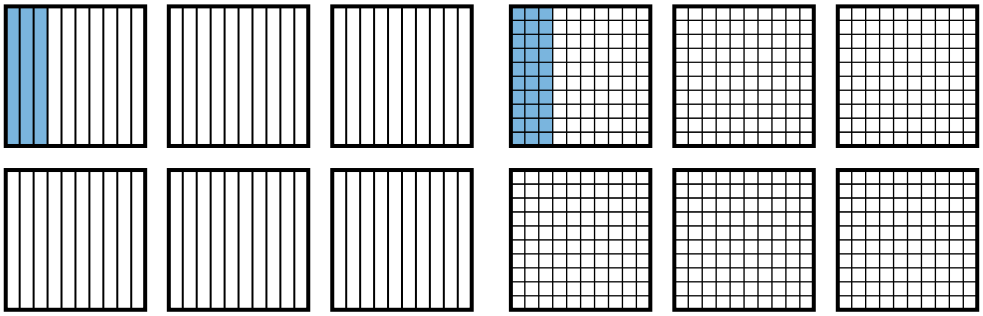
Name: .... Date: .... Period: ....

Decimal Division Strategies (continued)

4. Each large square represents 1.

Determine the value of $6 \div 0.3$. Use the tenths or hundredths chart if it helps with your thinking.

Tenths **Hundredths**



5. Here is how Arjun determined the value of $6 \div 0.3$.

$$6 \div 0.3 = \frac{6}{1} \div \frac{3}{10}$$
$$= \frac{60}{10} \div \frac{3}{10}$$
$$= 60 \div 3$$
$$= 20$$

a **Discuss:** Describe Arjun's strategy.

b Use his strategy to determine the value of $5 \div 0.02$.

Activity 2

Name: ________ Date: ________ Period: ________

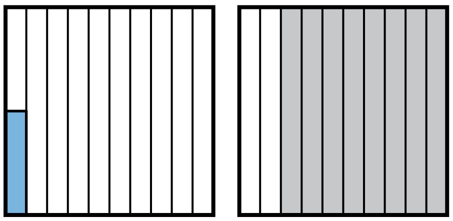
Different Expression, Same Value

6. Select *all* the expressions that have the same value as $1.2 \div 0.05$.

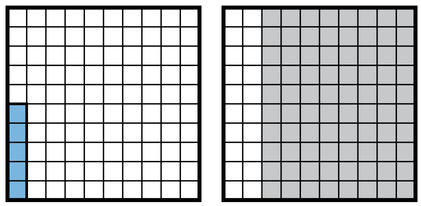
Use the tenths or hundredths chart if it helps with your thinking.

- ☐ **A.** $12 \div 5$
- ☐ **B.** $12 \div 0.5$
- ☐ **C.** $120 \div 5$
- ☐ **D.** $\frac{120}{100} \div \frac{5}{100}$
- ☐ **E.** $\frac{12}{100} \div \frac{5}{100}$

Tenths



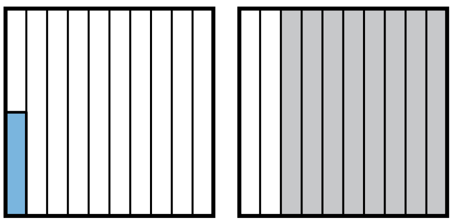
Hundredths



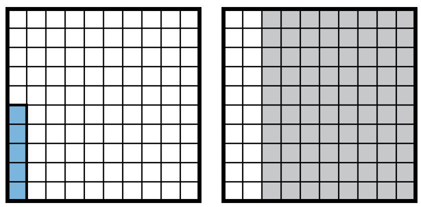
7. Calculate $1.2 \div 0.05$.

Use the tenths or hundredths chart if it helps with your thinking.

Tenths



Hundredths



Name: .... Date: .... Period: ....

Different Expression, Same Value (continued)

8. Write an expression with the same value as $3.6 \div 0.012$ using fractions.

9. Here is Raven's work for $3.6 \div 0.012$.

What would you say to help her understand her mistake?

$$3.6 \div 0.012 = \frac{36}{10} \div \frac{12}{1000}$$
$$= 36 \div 12$$
$$= 3$$

Activity 3

Name: ______ Date: ______ Period: ______

Card Sort

10. Match each division problem with its equivalent representations.

a $\frac{240}{100} \div \frac{8}{100}$

b $\frac{240}{100} \div \frac{80}{100}$

c $\frac{2400}{1000} \div \frac{8}{1000}$

d 30

e 300

f $\frac{2400}{1000} \div \frac{800}{1000}$

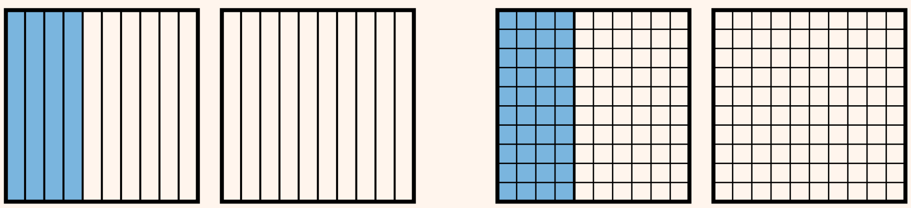
$2.4 \div 0.8$	$2.4 \div 0.08$	$2.4 \div 0.008$

You're invited to explore more.

11. The value of $2 \div 0.4$ is 5.

How many other decimal division expressions can you write that have a value of 5?

Tenths **Hundredths**



Record a dividend and divisor for each expression.

Dividend	2				
Divisor	0.4				

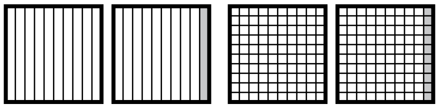
Synthesis

12. Circle an expression that has the same value as $1.9 \div 0.1$.

$\frac{19}{10} \div \frac{1}{10}$　　　$19 \div 1$　　　$190 \div 10$

Explain how you know it has the same value. Use the diagrams if they help with your thinking.

Tenths　　　**Hundredths**

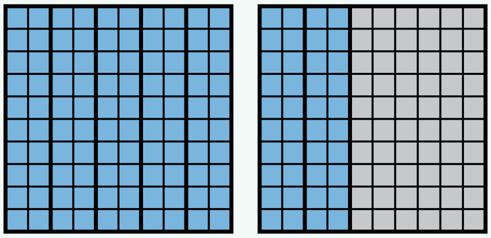


Lesson Practice 5.05

Lesson Summary

We can use hundredths charts, thousandths charts, and fractions to help us visualize and divide decimals.

This diagram represents the expression $1.4 \div 0.2$.



$$1.4 \div 0.2 = \frac{14}{10} \div \frac{2}{10}$$
$$= 14 \div 2$$
$$= 7$$

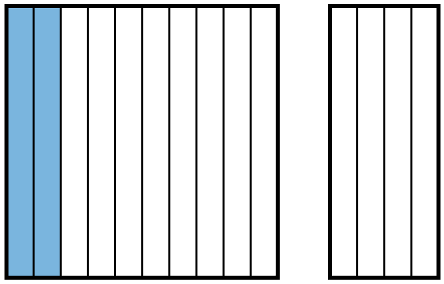
- Using the hundredths chart, you can count the number of groups of 2 tenths needed to fill the 1 whole and 4 tenths. It takes 7 groups, so $1.4 \div 0.2 = 7$.
- You can rewrite each decimal as an equivalent fraction, so 1.4 becomes $\frac{14}{10}$ and 0.2 becomes $\frac{2}{10}$. Now you can use your knowledge of fraction division to calculate the quotient.

Sometimes you will need to use common denominators to solve expressions with fractions. For example, $1.5 \div 0.03 = \frac{15}{10} \div \frac{3}{100} = \frac{150}{100} \div \frac{3}{100} = 150 \div 3 = 50$.

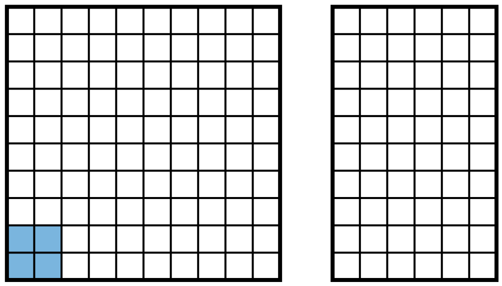
Lesson Practice
5.05

Name: ______ Date: ______ Period: ______

1. Determine the value of $1.4 \div 0.2$.



2. Determine the value of $1.6 \div 0.04$.



3. Select *all* the expressions that are equivalent to $3.5 \div 0.005$.

☐ **A.** $35 \div 5$

☐ **B.** $3500 \div 5$

☐ **C.** $35 \div 0.05$

☐ **D.** $\frac{35}{10} \div \frac{5}{100}$

☐ **E.** $\frac{35}{10} \div \frac{5}{1000}$

4. Calculate $3.5 \div 0.005$.

Problems 5–6: Remy says: *To determine the value of* $0.27 \div 0.003$, *I can divide* 270 by 3.

5. Is Remy correct? Explain your reasoning.

6. Calculate $0.27 \div 0.003$. Show or explain your thinking.

Lesson Practice
5.05

Name: .... Date: .... Period: ....

FAST Practice

7. Calculate $0.225 \div 0.005$.

Quotient: ☐

Spiral Review

Problems 8–9: Xavier is multiplying $1.5 \cdot 0.82$. He knows that $15 \cdot 82 = 1230$.

8. What is $1.5 \cdot 0.82$? Show or explain your thinking.

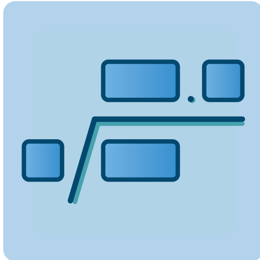
A. 0.0123 B. 0.123 C. 1.23 D. 12.3

9. What is $0.15 \cdot 0.82$?

Name: ________ Date: ________ Period: ________

MA.6.NSO.2.1, MTR.2.1

Long Division and Other Strategies



Let's use long division to divide decimals.

Warm-Up

1. **Discuss:** What is the value of each expression?

a. $0.8 \div 4$

b. $0.24 \div 3$

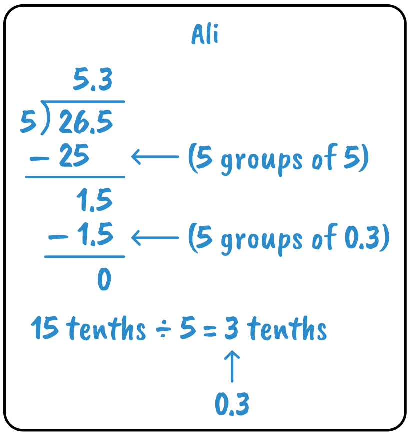
c. $0.035 \div 5$

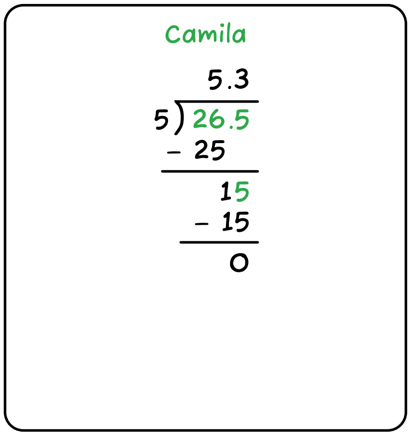
Name: ______ Date: ______ Period: ______

Ali’s and Camila’s Strategies

2. Use any strategy to calculate $26.5 \div 5$.

3. Let’s look at Ali’s and Camila’s **long division** strategies for calculating $26.5 \div 5$.

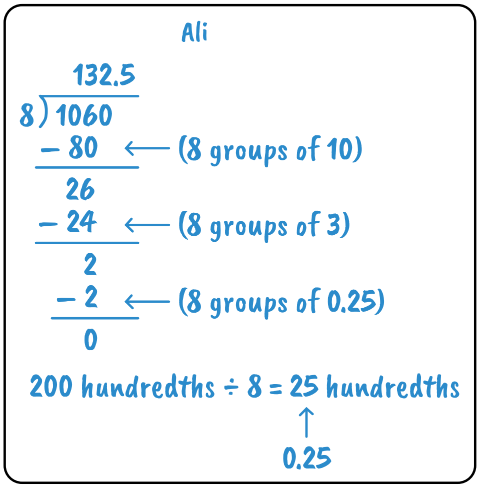


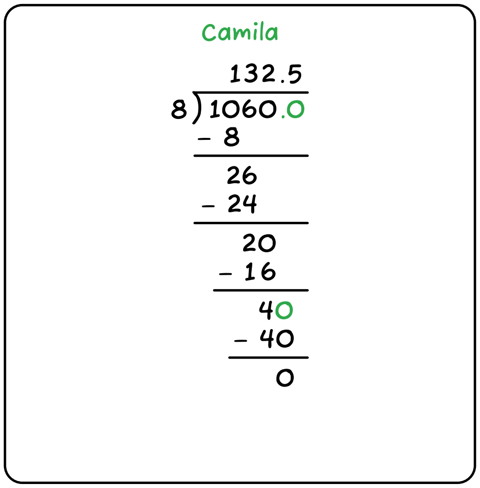


Discuss: How are their strategies alike? How are they different?

4. Use any strategy to calculate $106 \div 0.8$.

5. Let’s look at Ali’s and Camila’s strategies for calculating $106 \div 0.8$.





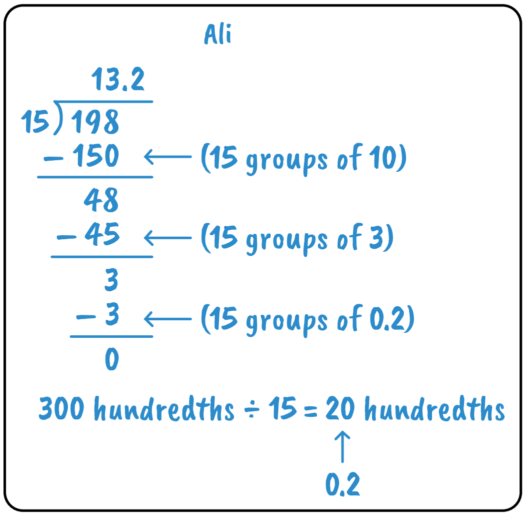
Discuss: Why was it helpful to rewrite this expression as $1060 \div 8$?

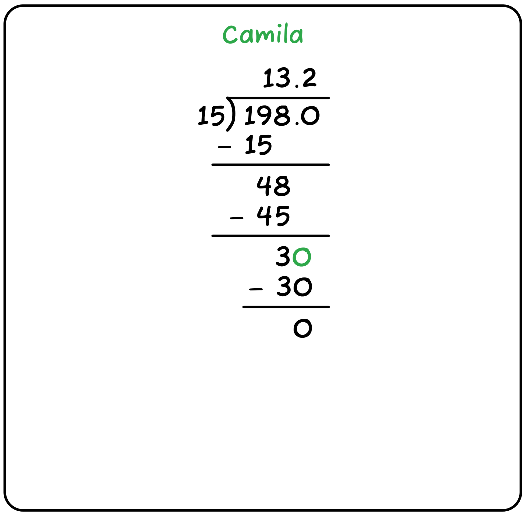
Name: .... Date: .... Period: ....

Ali’s and Camila’s Strategies (continued)

6. Calculate $19.8 \div 1.5$.

7. Let’s look at Ali’s and Camila’s strategies for calculating $19.8 \div 1.5$.





Discuss: Why did they each write $198 \div 15$ instead of $1980 \div 15$?

8. Use any strategy to show that these equations are true.

a $0.7 \div 0.4 = 1.75$

b $22.5 \div 0.04 = 562.5$

Activity 2

Name: ______ Date: ______ Period: ______

Finding Expressions

You will use the Activity 2 Cards to complete this table. You can use each card more than once.

9. Write down at least *one* expression that . . .

. . . includes division by a number	. . . has a quotient
Greater than 1.	Less than 1.
Less than 1.	Greater than 15.
In the hundredths place.	Close to 10.

10. Work with a partner to calculate the value of at least three expressions each. Show all of your thinking. Make sure you and your partner select different expressions. When you're finished, compare your thinking with your partner.

Expression: ______ Expression: ______ Expression: ______

My work: My work: My work:

Synthesis

11. What are some things you think are important to remember when dividing with decimals?

Use the examples if they help with your thinking.

$26.5 \div 5$

$57 \div 1.5$

$5.11 \div 0.05$

Lesson Practice 5.06

Lesson Summary

When dividing by a decimal, it can be helpful to rewrite the expression using whole numbers by multiplying by a power of 10.

For example, you can rewrite $7.65 \div 1.2$ as $765 \div 120$.

$$
\begin{aligned}
7.65 \div 1.2 &= \frac{765}{100} \div \frac{12}{10} \\
&= \frac{765}{100} \div \frac{120}{100} \\
&= 765 \div 120
\end{aligned}
$$

Once you have an expression with whole numbers, you can use **long division** to calculate the quotient one digit at a time from left to right.

$$
\begin{array}{r}
6.375 \\
120\overline{)765.000} \\
-720 \\
\hline
45\,0 \\
-36\,0 \\
\hline
9\,00 \\
-8\,40 \\
\hline
600 \\
-600 \\
\hline
0
\end{array}
$$

Lesson Practice
5.06

Name: .... Date: .... Period: ....

1. Select *all* the expressions that are equivalent to $4.5 \div 0.08$.

☐ **A.** $\frac{45}{100} \div \frac{8}{100}$ ☐ **B.** $45 \div 8$ ☐ **C.** $\frac{450}{100} \div \frac{8}{100}$

☐ **D.** $450 \div 8$ ☐ **E.** $45 \div 0.8$

2. What is the value of $4.5 \div 0.08$?

Problems 3–5: Use long division to calculate each quotient. Show your thinking.

3. $7.89 \div 2$ **4.** $176 \div 0.5$ **5.** $199.8 \div 0.8$

6. Four students set up a lemonade stand. By the end of the day, they earned \$17.52. If they split the amount equally, how much money would each student get? Show or explain your thinking.

Name: .... Date: .... Period: ....

7. A bag of pennies weighs 5.1 kilograms. Each penny weighs 2.5 grams. Which of these is the best estimate for the number of pennies in the bag? Show or explain your thinking. (1 kilogram $= 1000$ grams)

A. 20 B. 200 C. $2{,}000$ D. $20{,}000$

FAST Practice

8. Determine the quotient of $33.8 \div 32.5$.

☐

Spiral Review

Problems 9–10: Fill in the blanks to make each subtraction problem true.

9.

$$\begin{array}{r} 5 \\ - \square\;.\;\square\,\square\,\square \\ \hline 4\;.\;3\;2\;9 \end{array}$$

10.

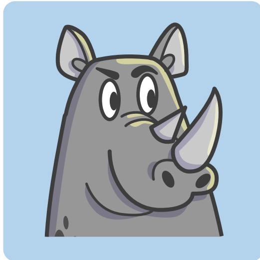
$$\begin{array}{r} 1 \\ - \square\;.\;\square\,\square\,\square \\ \hline 0\;.\;8\;6\;3 \end{array}$$

Unit 5
Lesson
7

Name: ____________ Date: ________ Period: ________

MA.6.NSO.2.1, MA.6.NSO.2.3, MTR.1.1, MTR.3.1, MTR.4.1

Movie Time



Let's use decimals to explore movies at different speeds.

Warm-Up

1. **a** Look at the digital settings for a movie viewing platform.

b What do you notice? What do you wonder?

I notice:

I wonder:

Activity 1

Name: ______ Date: ______ Period: ______

Speedy Cinema

2. A movie is 12 seconds long. Arnav wants to play the movie at a speed of 3x.

Which expression could he use to calculate how long it would take?

$12 - 3$ $12 \div 3$ $12 \cdot 3$ $3 \div 12$

Explain your thinking.

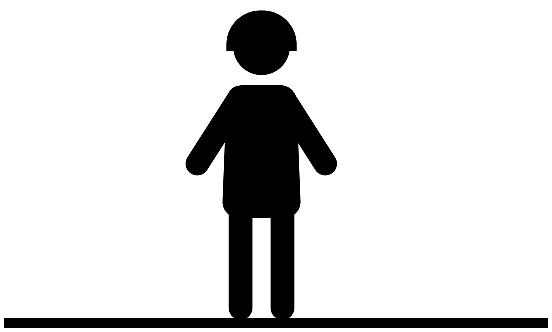
Speed	Duration (sec)
0.5x	24
1x	12
2x	6
4x	3
3x	?

3. A movie is 12 seconds long.

a. **Discuss:** Would it take more or less than 12 seconds to play the movie at a speed of 0.8x?

b. How long would it take to play the movie at 0.8x?

Speed: 0.8x



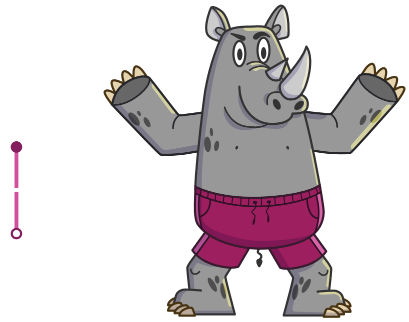
Activity 2

Name: ______ Date: ______ Period: ______

Make Your Own Movie

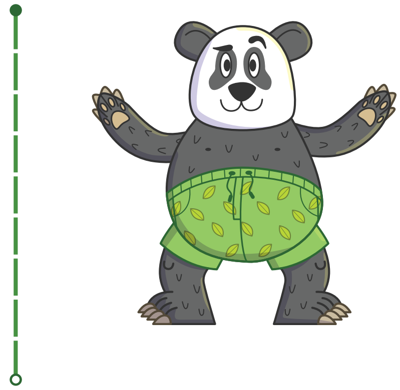
4. Roberto's movie is 2.4 seconds long. How long would it take to play his movie at 0.25x?

Playback speed: 0.25x



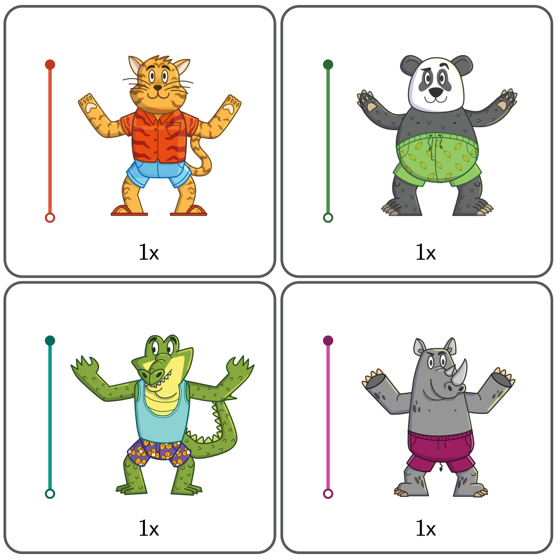
5. Dalia's movie is 12.25 seconds long. How long would it take to play her movie at 2.5x?

Playback speed: 2.5x



6. Here are four movies. Adjust the speeds so that all four movies take exactly 4.8 seconds to play.

Original Movie Length (sec)	Speed
1.2	
2.4	
4.8	
18	

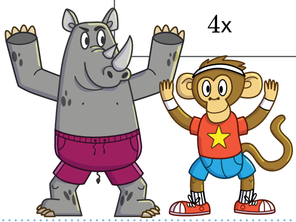


Synthesis

7. How can dividing be useful when thinking about movie durations and speeds? Use the numbers in the table if they help with your thinking.

Original movie length: 12 seconds

Speed	Duration
0.5x	24 seconds
1x	12 seconds
2x	6 seconds
4x	3 seconds



Lesson Practice 5.07

Lesson Summary

To determine how long it will take to play a movie at a certain speed, you can divide the length of the original movie by the playback speed.

Let's say a movie is 5.6 minutes long.

- When the playing speed is doubled, it will take 2.8 minutes to watch the movie: $5.6 \div 2 = 2.8$.
- When the playing speed is halved, it will take 11.2 minutes to watch the movie: $5.6 \div 0.5 = 11.2$.

Name: .... Date: .... Period: ....

1. Select *all* the expressions that have a value of 8.

- ☐ **A.** $3.2 \div 0.4$
- ☐ **B.** $0.32 \div 4$
- ☐ **C.** $\frac{32}{10} \div \frac{4}{10}$
- ☐ **D.** $32 \div \frac{4}{100}$
- ☐ **E.** $0.032 \div 0.04$

Problems 2–3: Naoki recorded a video of his dog catching a frisbee. The video is 10 seconds long.

2. Naoki plays the video in slow motion at 0.5x speed. How long does it take?

3. How long does the video take if Naoki plays it at 2.5x speed?

Problems 4–5: Farah is listening to an audiobook. To get through the book faster, she listens at 1.2x speed.

4. Complete the table to determine how long it will take to listen to each of the first three chapters.

	Duration at Regular Speed (min)	Duration at 1.2x Speed (min)
Chapter 1	12	
Chapter 2	10.5	
Chapter 3	39.3	

5. It takes Farah 8.5 minutes to listen to Chapter 4 at 1.2x speed. What is the length of Chapter 4 at regular speed?

- **A.** 10.2 minutes
- **B.** 9.7 minutes
- **C.** 7.3 minutes
- **D.** 7.08 minutes

Name: ________ Date: ________ Period: ________

6. Estimate whether $9 \cdot 0.8$ is greater than or less than 10. Explain your thinking.

7. Estimate whether $4.25 \cdot 2.5$ is greater than or less than 10. Explain your thinking.

FAST Practice

8. Estimate whether $3.9 \cdot 3.9$ is greater than or less than 10. Explain your thinking.

A. Less than 10. 3.9 is close to 3, and $3 \cdot 3 = 9$.

B. Less than 10. 3.9 is close to 4, and $4 \cdot 2 = 8$.

C. Greater than 10. 3.9 is close to 4, and $4 \cdot 4 = 16$.

D. Greater than 10. 3.9 is close to $3 + 10$, and $3 + 10 = 13$.

Spiral Review

9. Plant B is $6\frac{2}{3}$ inches tall. Plant C is $4\frac{4}{15}$ inches tall. Complete this sentence:

Plant C is ________ times as tall as Plant B.

Sub-unit

3

Solving Problems With Decimals

Lesson 8
Budget Vehicles

Lesson 9
Coining New Representations

Lesson 10
Grocery Prices

Name: ______ Date: ______ Period: ______

MA.6.NSO.2.1, MA.6.NSO.2.3, MTR.3.1, MTR.5.1, MTR.6.1, MTR.7.1

Budget Vehicles

Let's use decimals to compare car options.

Warm-Up

1. What might a family consider when buying a car?

Name: ________ Date: ________ Period: ________

Do Hybrid Cars Save Money?

2. Brielle's family wants to buy a brand new vehicle. They're choosing between the DesWagon, which is a gas-powered car, and the DesMobile, which is a hybrid car.

Use the Activity 1 Sheet to complete the table.

Problem	DesWagon 	DesMobile 
a Gas costs $3.50 per gallon in Brielle's neighborhood. How much would it cost to fill a tank of gas?		
b How much would it cost to drive 1 mile based on the cost of gas and the range of each vehicle?		
c How much would Brielle's family spend on gas yearly if they drove about 15,000 miles a year?		

3. If Brielle's family plans on driving the car for 5 years, which car do you think they should buy? Explain your thinking.

Name: .... Date: .... Period: ....

Do Electric Cars Save Money?

An advertisement for an electric vehicle claims that the savings on gas will make up for the higher sale price of the car.

Kweku is deciding between the DesWagon and the DesZip Electric. He determines that:

- The DesZip Electric costs $11,900 more than the DesWagon.
- On average, the cost of electricity for the DesZip Electric is $0.03 per mile.
- On average, the cost of gas for the DesWagon is $0.12 per mile.

Kweku drives about 18,000 miles a year.

4. How much money would he save in a year buying electricity for the DesZip Electric compared to buying gas for the DesWagon? Show your thinking.

5. About how long would it take to make up for the higher sale price of the DesZip Electric? Show your thinking.

6. You have been hired to design a new advertisement for either the DesZip, the DesWagon, or the DesMobile. Use your work from this lesson to create a persuasive and accurate advertisement. Include any additional information that you think is important.

Synthesis

7. In this lesson, you used decimal operations to help make an informed decision about buying a car. What other decisions might mathematics help us make?

Lesson Practice 5.08

Lesson Summary

Decimals show up in all sorts of real-world problems. We can use decimal operations to make well-informed decisions in these situations.

You might start by writing an equation to represent the situation, then determine its solution.

Problem	Equations
Apples cost $0.75 per pound. I have $4.00 to spend on apples. I need to know how many pounds of apples I can buy.	$4.00 \div 0.75 = 5\frac{1}{3}$ $5\frac{1}{3}$ pounds of apples
Callen is measuring wood for a wheelchair ramp. The ramp is 5.7 feet long, and Callen will place a nail every 6 inches. How many nails will she use?	$5.7 \cdot 12 = 68.4$ $68.4 \div 6 = 11.4$ 11 or 12 nails
Marco works at the grocery store and earns $11.50 per hour. Jaylene works at a pet store and earns $12.85 per hour. How much more money will Jaylene earn than Marco for a 5.5-hour shift?	$12.85 - 11.50 = 1.35$ $1.35 \cdot 5.5 = 7.425$ $7.43

Name: ______ Date: ______ Period: ______

Problems 1–2: A standard sheet of paper in the United States is 11 inches long and 8.5 inches wide. (12 inches = 1 foot)

1. To the nearest hundredth, how long and wide is a standard sheet of paper in feet?

Length (ft)	Width (ft)

2. A standard sheet of paper in Europe is 29.7 centimeters long and 21 centimeters wide. Which standard sheet of paper has a greater area? Circle one.

United States’ Europe’s They have the same area.

Problems 3–4: Tyler’s school is having a stair-climbing challenge. Tyler can climb 135 stairs in 90 seconds.

3. If Tyler climbs stairs at a constant rate, how many stairs can he climb per second? Show or explain your thinking.

4. Jin also participates in the challenge. She can climb 75 stairs in 1 minute. Who climbs at a faster rate? Show or explain your thinking.

5. Which polygon has the greatest area?

- **A.** A rectangle that is 3.25 inches wide and 6.1 inches long.
- **B.** A square with a side length of 4.6 inches.
- **C.** A parallelogram with a base of 5.875 inches and a height of 3.5 inches.
- **D.** A triangle with a base of 7.18 inches and a height of 5.4 inches.

FAST Practice

6. Mateo is using ribbon to wrap presents. He has a roll of ribbon that is 12 meters long. He plans to cut 9 pieces that are each 0.4 meters long. How much ribbon will he have left?

Mateo will have ☐ meter(s) of ribbon left.

Name: .... Date: .... Period: ....

Spiral Review

7. Annika walked $\frac{1}{8}$ of a 30-mile walking trail. How many miles did she walk?

Problems 8–9: Rope A is $6\frac{2}{3}$ inches long. Rope B is $4\frac{4}{15}$ inches long.

8. How much longer is Rope A than Rope B?

9. How many times longer is Rope A than Rope B?

Name: .... Date: .... Period: ....

MA.6.NSO.2.1, MA.6.NSO.2.3, MA.6.NSO.3.5, MTR.2.1, MTR.3.1

Coining New Representations

Let's explore the relationship between percentages, decimals, and fractions.

Warm-Up

1. Complete the table to show the values of these U.S. coins.

	Penny	Nickel	Dime	Quarter
Value (in cents)	1			
Fraction of $1	$\frac{1}{100}$			
Value (in dollars)	0.01			
Percent of $1	1%			

Activity 1

Name: ________ Date: ________ Period: ________

Percentages, Decimals, and Fractions

You will use the Activity 1 Cards for this activity.

2. Create groups of three by matching the *percent*, decimal, and fraction cards that are equivalent.

Percent	Decimal	Fraction

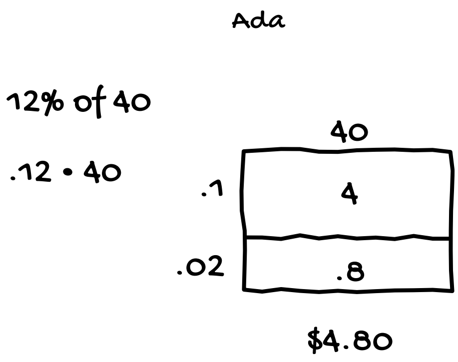
3. Choose any set of three cards and explain how you know they are equivalent.

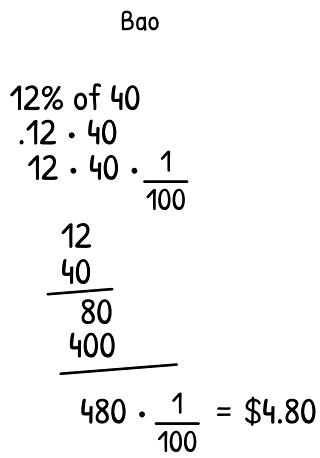
4. Imani makes a card with 8% on it. She says its decimal equivalent is 0.8. What could you say to Imani to help her understand her mistake?

Name: ______ Date: ______ Period: ______

Resale Value

5. Ada and Bao keep 12% of the price of each shirt as a profit. Here is how they each calculated 12% of 40.





Discuss: How are their strategies alike? How are they different?

6. Oscar says that he can calculate 40% of 70 by writing the expression $\frac{2}{5} \cdot 70$. Is his expression correct? Explain your thinking.

7. Match each expression with a question. One expression will have no match.

a $\frac{4}{100} \cdot 70$	**b** $0.04 \cdot 14$	**c** $0.4 \cdot 48$
d $\frac{4}{100} \cdot 14$	**e** $0.04 \cdot 70$	**f** $0.04 \cdot 48$

What is 4% of $\$14$?	What is 4% of $\$48$?	What is 4% of $\$70$?

Name: .... Date: .... Period: ....

Resale Value (continued)

8. Solve as many challenges as you have time for.

a The price of a space T-shirt is $\$22$. 16.5% of every sale goes to material cost. Calculate the material cost.

b The price of a striped button-up shirt is $\$30$. 9.4% of every sale goes to clothing company profit. Calculate the clothing company profit.

c The price of a striped long-sleeve T-shirt is $\$48$. 8.5% of every sale goes to transport cost. Calculate the transport cost.

d The price of a blue pair of shoes is $\$65$. 5.2% of every sale goes to factory profit. Calculate the factory profit.

Synthesis

9. Describe how you can convert between percentages, fractions, and decimals. Use the values in the chart if they help with your thinking.

Percent (%)	Fraction	Decimal
20	$\frac{20}{100}$	0.2
40	$\frac{2}{5}$	0.4
1.2	$\frac{1.2}{100}$	0.012

Lesson Practice 5.09

Lesson Summary

We can convert between *percentages*, decimals, and fractions to solve problems, including problems related to money.

We've learned that percent means "out of 100." This means that:

- We can write percentages as fractions with a denominator of 100. For example, 13% is equivalent to $\frac{13}{100}$.
- We can think of percentages as a number of hundredths, then determine an equivalent decimal value. For example, 13% is 13 hundredths, or 0.13.

It's important to think about place value when converting percentages into fractions and decimals. For example, 2.5% is equivalent to 2.5 hundredths $\left(\frac{2.5}{100}\right)$ or 25 thousandths $\left(\frac{25}{1000}\right)$, both of which can be represented by the decimal 0.025.

Lesson Practice
5.09

Name: .... Date: .... Period: ....

Problems 1–4: Determine the missing values in each row.

	Percent (%)	Decimal	Fraction
1.	5		
2.	400		
3.		0.5	
4.			$\frac{3}{5}$

5. Marc says that because percent means "out of 100," he needs to multiply 3.7 by $\frac{1}{100}$ to write a percentage that is equivalent to 3.7. Help Marc understand his mistake.

6. Match each percent with its decimal equivalent.

a 3.05%	.... 0.305
b 350%	.... 3.5
c 30.5%	.... 0.035
d 3.5%	.... 0.0305

FAST Practice

7. Which value is 0.2 expressed as a percentage?

A. 2%

B. 20%

C. 0.02%

D. 0.002%

Name: ______ Date: ______ Period: ______

Spiral Review

Problems 8–10: Mentally evaluate each quantity. Explain your thinking.

8. 15 is what percent of 30?

9. 3 is what percent of 12?

10. 6 is what percent of 10?

Unit 5
Lesson
10

Name: ________ Date: ________ Period: ________

MA.6.NSO.3.5, MTR.1.1, MTR.2.1, MTR.4.1

Grocery Prices

Let's explore the cost of groceries.

Warm-Up

A caterer has a budget of $\$1{,}500.00$ for appetizers, dinner, and dessert for an event.

1. **Discuss:** What percentage of the budget do you think the caterer might spend on these groceries for dessert for the event?

Sugar (5 lb)
Flour (5 lb)
Apples (3 lb)
Eggs (2 dozen)
Butter (32 oz)
Milk (1 gal)

2. Complete this statement:

   Buying the groceries from Groceryland, the caterer would spend ________ % of the budget on this grocery list.

3. Is this more or less than $\$100.00$? Explain your thinking.

Activity 1

Name: ______ Date: ______ Period: ______

Grocery Prices

Tyani and Anika were asked to calculate approximately how much the caterer would spend on these groceries at Groceryland. They each wrote expressions to help them. Here is their work.

Tyani	Anika
2.2% of 1500	2.2% of 1500
$= 0.22 \cdot 1500$	$= 0.022 \cdot 1500$

4. Whose work do you agree with? Explain your thinking.

5. Calculate 2.2% of $\$1,500.00$. Does this number make sense for this situation?

Here is the approximate budget for three of the caterer's other upcoming events.

Business Party	Business Dinner	Business Lunch
$\$1,700.00$	$\$2,100.00$	$\$700.00$

6. If the caterer spent 2.2% of the budget for each event on these groceries at Groceryland, how much would the caterer spend? Complete the table and show your thinking.

	Business Party	Business Dinner	Business Lunch
Expression for 2.2% of Budget			
Money Spent			

Name: ______ Date: ______ Period: ______

Grocery Prices (continued)

You'll use the Activities 1 & 2 Sheet to complete this activity.

7. For which events can the caterer buy all the groceries on the list at Groceryland using 2.2% or less of the budget? For which events is the caterer unable to do so?

8. **Discuss:**

a. Why do you think the cost of these groceries is different for different events?

b. What do you think the impact might be if the caterer spends more than 2.2% of the budget on the dessert groceries?

Activity 2

Name: .... Date: .... Period: ....

Bought Milk?

You'll use the Activities 1 & 2 Sheet to complete this activity.

9. How much does a gallon of milk cost at Groceryland?

10. What percentage of the total cost of the grocery list is that? Show your thinking.

A. 0.12% B. 1.2% C. 12% D. 0.012%

The caterer says: *Milk is too expensive at Groceries Plus. It's 14.2% of the total cost of that grocery list!*

11. Show or explain where the 14.2% comes from.

12. Do you agree with the caterer? If you do, explain what you think would be a fair price for milk at Groceries Plus. If you do not, explain why you think milk at Groceries Plus is priced fairly.

13. Choose a different store. What percentage of the total grocery bill is milk? Do you think milk is too expensive at this store? Show or explain your thinking.

You're invited to explore more.

14. Choose a different food that has a price that you think is too high. Use percentages, decimals, rates, or ratios to propose a different price for that food.

Synthesis

15. How can you determine the percent of a budget a particular item costs? Consider a budget of $\$100$ and a dinner choice priced at $\$17$ if it helps you with your thinking.

Lesson Practice 5.10

Lesson Summary

You've used ratios, double number lines, and tape diagrams to model and solve percent problems. Another strategy you can use is to convert the percent to a decimal, then multiply or divide.

Let's say a baker spent 4% of his monthly budget on fruit last week.
His monthly budget is $\$4{,}000.00$. How much money did he spend on one week's worth of fruit?

To determine the answer, first write 4% as a decimal: $4\% = 0.04$.

Then multiply by the total monthly budget: $0.04 \cdot 4000 = 160$.

That means the baker spent $\$160.00$ on fruit last week.

Lesson Practice
5.10

Name: .... Date: .... Period: ....

Problems 1–4: Jada's restaurant has a weekly profit of approximately $\$1,150.00$. She tries to spend no more than 40% of the weekly profit on groceries.

1. Write an expression to represent how much money she spends on the restaurant's weekly groceries.

2. How much money, at most, does Jada spend on groceries each week for the restaurant?

3. Jada puts 12% of the weekly profit into a savings account. How much money does Jada save from the restaurant each each week?

4. Jada's restaurant recently had to spend $\$184.00$ on an oven repair. What percent of the weekly profit did Jada spend on the oven repair?

   A. 0.16% B. 6.25% C. 16% D. 62.5%

5. Circle the expression that has a greater value.

   7% of 250 70% of 25 They have the same value.

FAST Practice

6. Oliver went to the store and purchased these items. Beef is the most expensive item. What percent of the total is it?

   ☐ %

Items	Cost ($)
Milk (1 gal)	$3.61
Beef (1 lb)	$7.10
Apples (1 lb)	$2.39
Bananas (1 lb)	$0.91
Oranges (1 lb)	$1.99
Potatoes (1 lb)	$1.75
Total	$17.75

Lesson Practice
5.10

Name: .... Date: .... Period: ....

Spiral Review

7. One serving of yogurt contains 5.2 grams of sugar. How many grams of sugar are in 14.25 servings of yogurt?

A. 7.41 grams **B.** 2.74 grams **C.** 19.45 grams **D.** 74.1 grams

Problems 8–11: Determine the value of each expression. Show your thinking.

8. $4.4 - 0.72$

9. $4 + 1.3 + 0.56$

10. $4.34 \div 0.7$

11. $0.32 \bullet 4.7$

Unit 6

Expressions and Equations

Up until now, an equal sign meant you were being asked to calculate an answer. In this unit, you'll learn about its other meaning: balance. When things are in balance, it becomes possible to know the unknown.

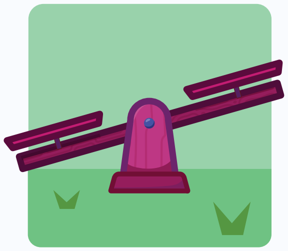
Essential Questions

- What does it mean for an equation to be true? Can an equation be false?
- What does it mean for two expressions to be equivalent?
- If multiplication is repeated addition, how do we represent repeated multiplication?

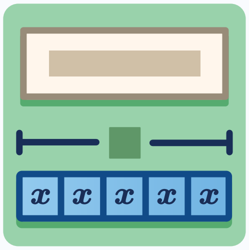
Sub-unit

1

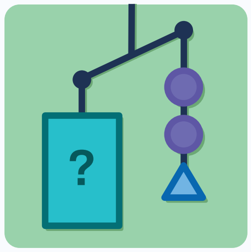
Solving Equations



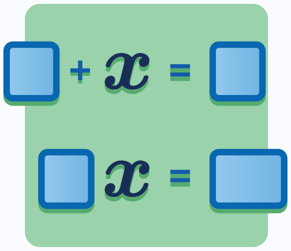
Lesson 1
Weight for It



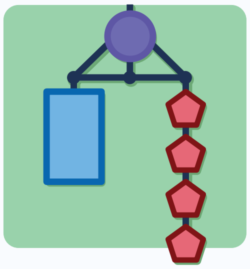
Lesson 2
Five Equations



Lesson 3
Hanging Around



Lesson 4
Swap and Solve



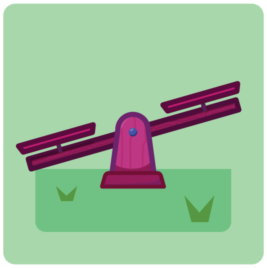
Lesson 5
Solutions of a Different Kind

Name: ______ Date: ______ Period: ______

MA.6.AR.2.2, MA.6.AR.2.3, MTR.2.1, MTR.4.1, MTR.5.1

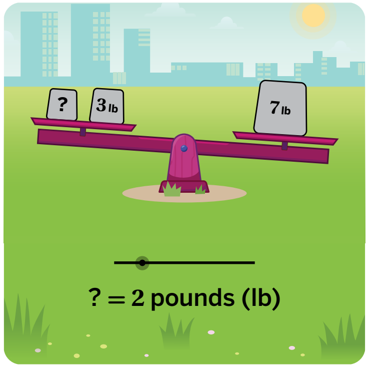
Weight for It

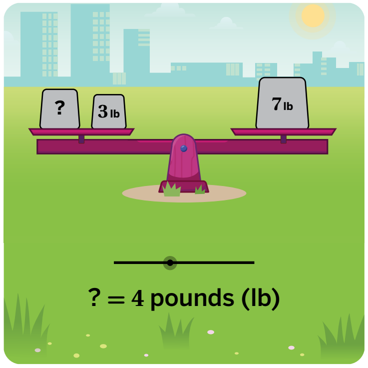
Let's use a seesaw to determine the weights of different animals.

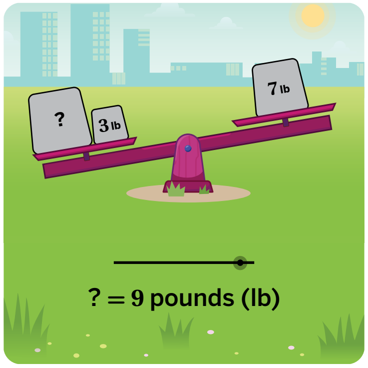


Warm-Up

1. **a** Take a look at some weights on a seesaw.







b What do you notice? What do you wonder?

I notice:

I wonder:

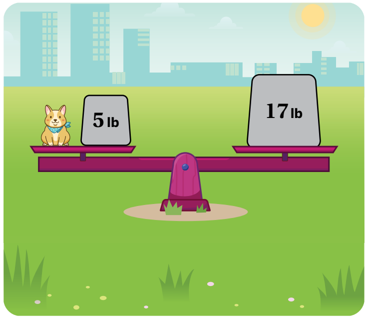
Activity 1

Name: .... Date: .... Period: ....

Equations and Tape Diagrams

2. This dog and a 5-pound weight balance a 17-pound weight.

How much does the dog weigh?



3. Tariq wrote an *equation* to represent the situation. He used the **variable** d to represent the dog's weight.

Explain how Tariq's equation is like the seesaw situation.

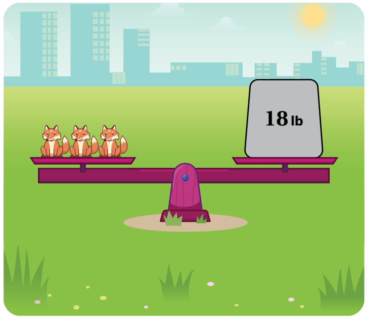
Tariq

$d + 5 = 17$

Name: .... Date: .... Period: ....

Equations and Tape Diagrams (continued)

4. These 3 foxes balance with an 18-pound weight. Each fox weighs the same amount.



a Choose an equation that represents this situation.

A. $3 + x = 18$

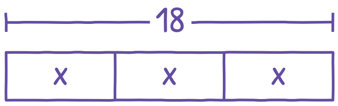
B. $3 \cdot x = 18$

C. $x + x + x = 18$

D. $3 + 18 = x$

b How much does each fox weigh?

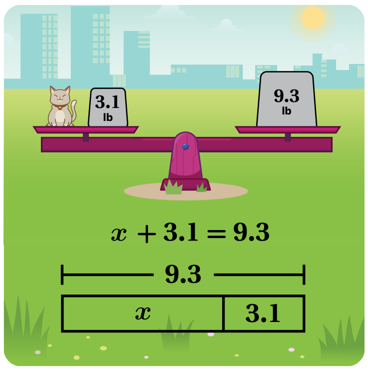
5. Tariq drew a tape diagram to determine the weight of each fox.



How are the tape diagram and the equation you chose in the previous problem alike?

6. This cat and a 3.1-pound weight balance a 9.3-pound weight.

How much does the cat weigh? Use the tape diagram if it helps with your thinking.

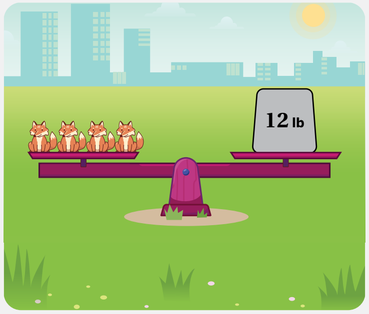
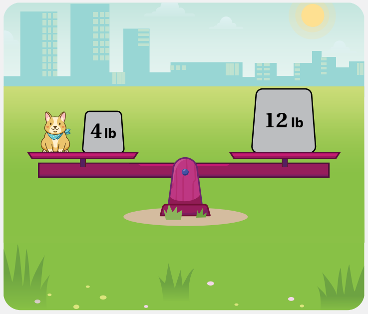


Activity 2

Name: ______ Date: ______ Period: ______

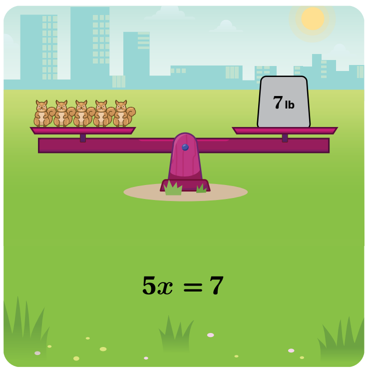
Determining Unknown Weights

7. For each equation or tape diagram, put a check mark under the balanced seesaw it represents.

		
$4 \cdot x = 12$	○	○
$4 + x = 12$	○	○
$x + x + x + x = 12$	○	○
$x = 8$	○	○
12 x x x x	○	○
$x = 3$	○	○
12 4 x	○	○

8. These 5 squirrels balance with a 7-pound weight. Each squirrel weighs the same amount.

How much does each squirrel weigh? Draw a tape diagram if it helps with your thinking.



Activity 3

Name: .... Date: .... Period: ....

Challenge Creator

9. You will use the Activity 3 Sheet to create your own seesaw challenge.

a **Make It!** On the Activity 3 Sheet, create a balanced seesaw challenge.

b **Solve It!** On this page, write an equation that represents your balanced seesaw problem. Then determine the weight of your animal. Draw a tape diagram if it helps with your thinking.

My Equation	Weight of My Animal

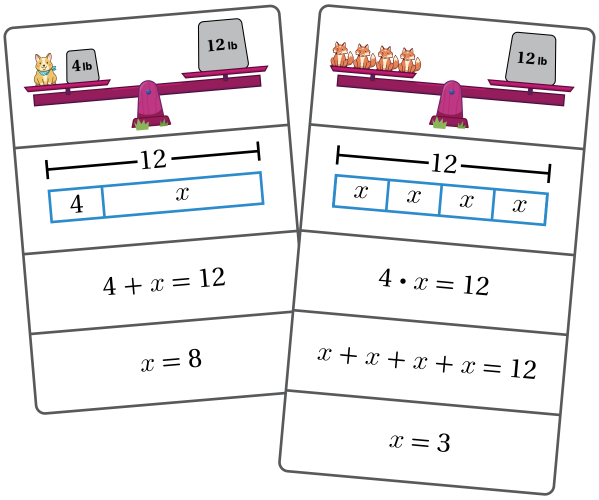
c **Swap It!** Swap your challenge with one or more partners. Write your partner's equation, then determine the weight of their animal. Draw a tape diagram if it helps with your thinking.

	Equation	Weight of One Animal
Partner 1		
Partner 2		
Partner 3		
Partner 4		

Synthesis

10. How can you tell if an equation and a tape diagram match?

I can tell if an equation and a tape diagram match . . .



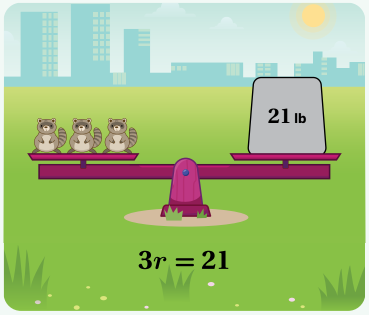
Lesson Practice 6.01

Lesson Summary

You can use seesaws and tape diagrams to represent *equations* and help determine unknown values.

We often use a letter, such as x or a, as a placeholder for an unknown number in tape diagrams and equations. This letter is called a **variable**.

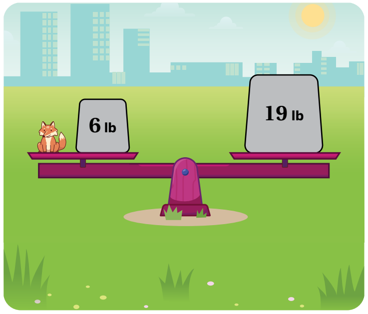
For example, if 3 equal-weight racoons weigh a total of 21 pounds, you can represent the weight of each raccoon with r and write the equation $3r = 21$.



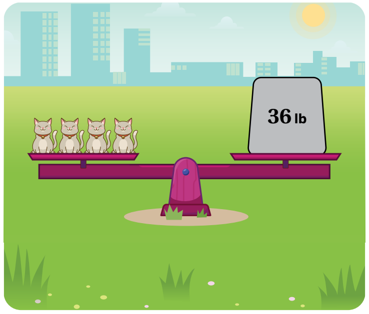
Lesson Practice
6.01

Name: ______________ Date: ________ Period: ________

1. Determine the weight of 1 fox.

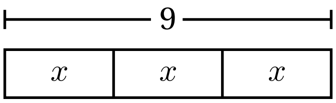


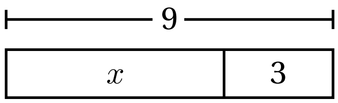
2. All 4 cats weigh the same amount. Determine the weight of 1 cat.



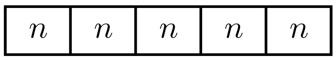
3. Match each equation to the tape diagram it represents.

$3 + x = 9$	$x + x + x = 9$	$x = 9 \div 3$	$3 \bullet x = 9$	$x = 9 - 3$





Problems 4–5: Kwabena is trying to determine the value of n in the equation $5 \bullet n = 35$. He begins drawing a tape diagram, but isn't sure how to complete it.



4. Complete Kwabena's tape diagram so it represents the equation $5 \bullet n = 35$.

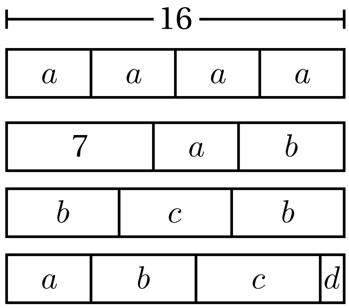
5. Determine the value of n.

Lesson Practice
6.01

Name: ______________ Date: ________ Period: ________

FAST Practice

6. Determine the value of a, b, c, and d.



$a =$ ☐

$b =$ ☐

$c =$ ☐

$d =$ ☐

Spiral Review

Problems 7–8: Calculate the price per pound for each item.

7. $2.52 for 4.5 pounds of potatoes

8. $7.75 for 2.5 pounds of broccoli

Problems 9–12: Fill in each box to create a true equation.

9. $7 + \square = 10$

10. $\square \bullet 5 = 45$

11. $23 - \square = 11$

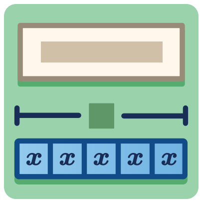
12. $\square \div 4 = 8$

Name: ______ Date: ______ Period: ______

MA.6.AR.2.1, MA.6.AR.2.2, MA.6.AR.2.3, MTR.2.1, MTR.4.1, MTR.5.1

Five Equations

Let's represent situations with equations and tape diagrams.



Warm-Up

1. Here is a situation. Let's make sense of it together as a class.

Four cats weigh 48 pounds total.

Each cat weighs the same, so they each weigh c pounds.

a **Discuss:** What is this situation about?

b Let's look at the information. What does the variable c represent?

c Create a tape diagram or sketch that represents this situation.

d Use your tape diagram or sketch to determine the value of c.

Activity 1

Name: .... Date: .... Period: ....

Equations and Tape Diagrams

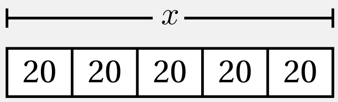
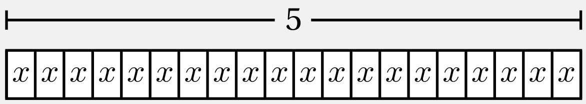
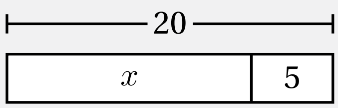
Here are five equations.

$x + 5 = 20$	$20 = x - 5$	$5 \cdot 20 = x$	$5x = 20$	$20x = 5$

2. Circle two equations that have something in common.

Discuss: How are these equations alike? How are they different?

3. Match each tape diagram with one of the equations. Two equations will not have matches.

Tape Diagram	Equation
	
	
	

4. Draw a tape diagram for an equation that did not have a match.

Tape Diagram	Equation

Activity 2

Name: ______ Date: ______ Period: ______

Which Equation?

4 cats weigh 48 pounds total. Each cat weighs the same, so they each weigh c pounds.

Equation	Solution to the Equation		Solution's Meaning
$4c = 48$	$c = 10$ NO	$c = 12$ YES	Each cat weighs 12 pounds.

5. What do you think a **solution to an equation** is?

You will use a set of description and situation cards.

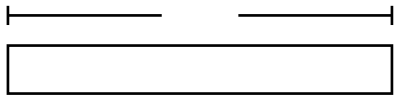
6. Match each card with the equation that represents it.

7. Determine the solution to each equation and write the solution's meaning for each situation.

	$x - 8 = 24$	$8x = 24$	$8 + x = 24$
Matching Cards			
Solution to the Equation			
Solution's Meaning			

Synthesis

8. How can you tell which equation represents a situation? Use the example if it helps with your thinking.

> Kwasi rides the subway 20 stops to get to work. After x stops, he has 5 stops left.
>
> 
>
> $x + 5 = 20$ $\qquad$ $5x = 20$

Lesson Practice 6.02

Lesson Summary

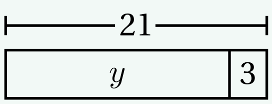
A tape diagram can help us visualize an equation and determine its solution. The **solution to an equation** is a value of the variable that makes the equation true.

When we work with an equation that represents a situation, it is important to determine what the variable represents when we determine the solution.

Here is an example.

Emmanuel needed $21 to buy a gift. He had $3 and borrowed the rest from his parents.

Consider the values $y = 16$ and $y = 18$. The value $y = 16$ is not a solution to the equation because it does not make the equation true. $3 + 16 = 19$ and $19 \neq 21$.

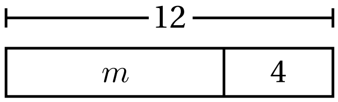
Equation	Tape Diagram	Solution to the Equation	Solution's Meaning
$3 + y = 21$		$y = 18$	Emmanuel borrowed $18 from his parents.

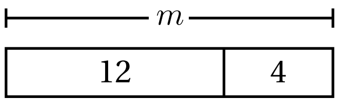
The value $y = 18$ makes the equation true. $3 + 18 = 21$.

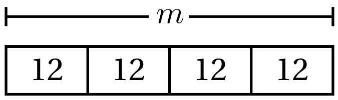
Lesson Practice
6.02

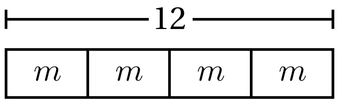
Name: ______ Date: ______ Period: ______

1. Match each equation to the tape diagram that represents it.

a


b


c


d


______ $12 = 4m$

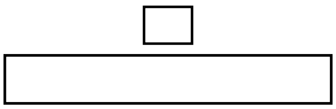
______ $m \div 4 = 12$

______ $12 + 4 = m$

______ $12 - 4 = m$

Problems 2–5: Aaliyah filled a water bottle with 24 ounces of water before school. She drank 15 ounces at lunch. There are x ounces of water left.

2. Draw a tape diagram to represent the situation.



3. Select *all* the equations that could represent this situation.

- ☐ **A.** $24 - 15 = x$
- ☐ **B.** $24 + 15 = x$
- ☐ **C.** $x + 15 = 24$
- ☐ **D.** $15x = 24$
- ☐ **E.** $24 \div 15 = x$

4. For each of the numbers below, determine which are solutions and which are not solutions to one of the equations you selected in Problem 3.

3 9 12 39

Solutions:

Not solutions:

5. Explain the solution's meaning in this situation.

Lesson Practice
6.02

Name: ______ Date: ______ Period: ______

FAST Practice

6. Each bag of beans weighs 32 ounces. A restaurant owner needs 224 ounces of beans. The equation $32b = 224$ represents the situation, where b represents the number of bags the owner needs. Which value makes the equation true?

A. 7 bags B. 8 bags C. 9 bags D. 10 bags

Spiral Review

Problems 7–9: Fill in each blank to create a true equation.

7. $2.83 - 1.6 = \square$ **8.** $\square + 2.1 = 7$ **9.** $\frac{3}{4} \cdot \square = 8$

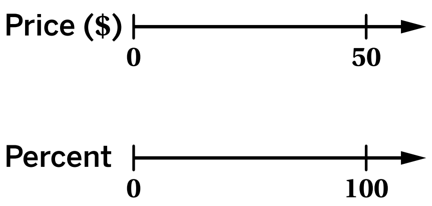
10. Fill in each blank using whole numbers from 0 to 9 so that x is the same value in each equation. Use each number only once.

$x = \square \cdot \square$ $x = \square + \square$ $x + \square = \square$

11. Select *all* the true equations.

- ☐ **A.** $5 + 0 = 0$
- ☐ **B.** $15 \cdot 0 = 0$
- ☐ **C.** $1.4 + 2.7 = 4.1$
- ☐ **D.** $\frac{2}{3} \cdot \frac{5}{9} = \frac{7}{12}$
- ☐ **E.** $4\frac{2}{3} = 5 - \frac{1}{3}$

12. Hailey paid \$40 for a jacket. The regular price was \$50. What percent of the regular price did Hailey pay? Use the double number line if it helps with your thinking.



Unit 6

Lesson

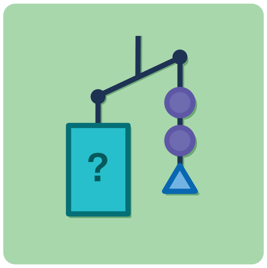
3

Name: ______________ Date: ________ Period: ________

MA.6.AR.2.2, MA.6.AR.2.3, MTR.2.1, MTR.4.1, MTR.5.1

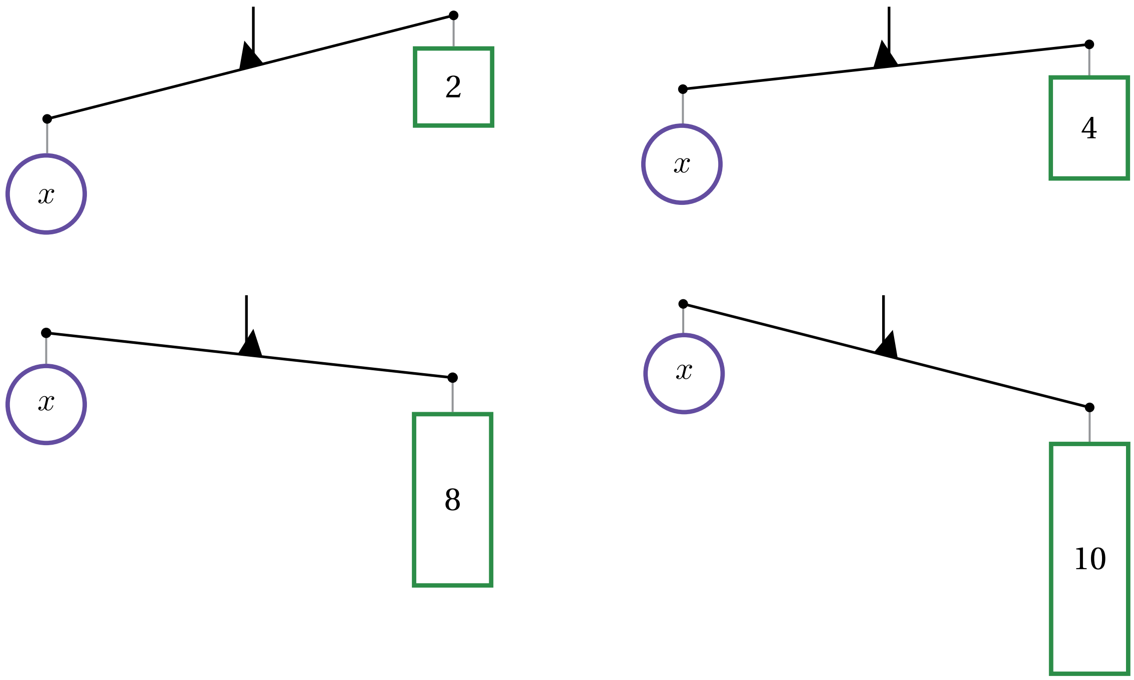
Hanging Around

Let's use balanced hangers to solve equations.



Warm-Up

1. **a** Take a look at the hangers with a circle of weight x on one side and a rectangle of different weights on the other side.



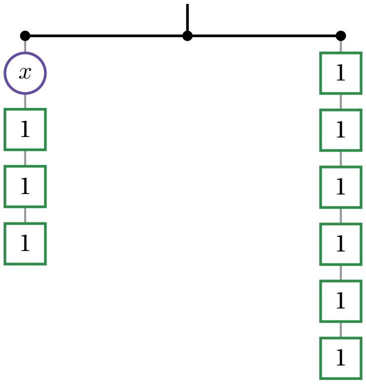
b **Discuss:** What is the weight of the circle? Explain your thinking.

Activity 1

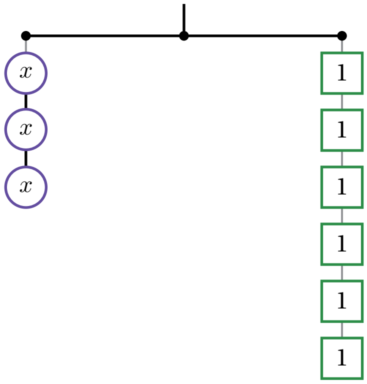
Name: .... Date: .... Period: ....

Connect It

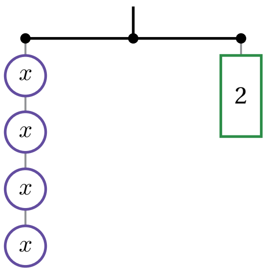
2. What value of x balances the hanger?



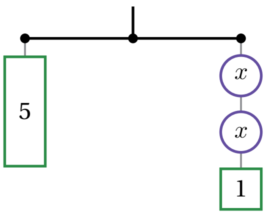
3. What value of x balances the hanger?



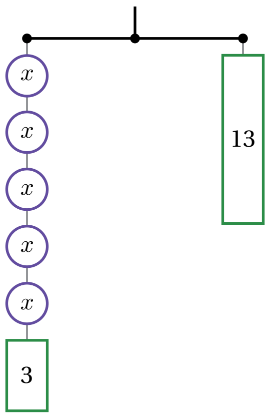
4. **a** Take a look at these hangers and the equations that represent them.



Equation: $4x = 2$



Equation: $5 = 2x + 1$



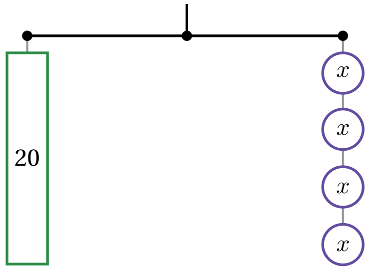
Equation: $5x + 3 = 13$

b Explain how an equation is like a hanger.

Name: .... Date: .... Period: ....

Make It, Solve It

5. Select an equation that represents this hanger.



A. $20 + x = 4$

B. $20 = 4x$

C. $20 = x + 4$

D. $20 = x + x + x + x$

Explain your thinking.

6. Use the hanger or the equation from the previous problem to determine the value of x that balances the hanger.

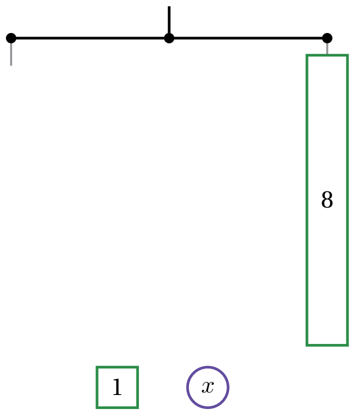
Activity 2

Name: .... Date: .... Period: ....

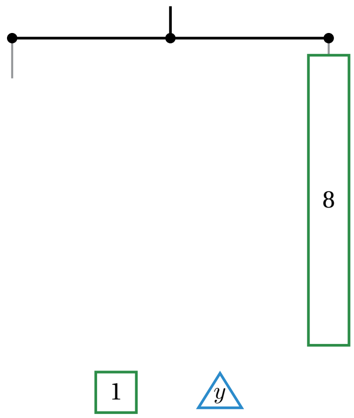
Make It, Solve It (continued)

7. Make a balanced hanger to represent each equation.

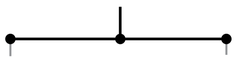
$5x = 8$



$5 + y = 8$



8. **a** Make a balanced hanger that represents $6 = x + 2$.



b What is the value of x that balances the hanger?

Activity 3

Name: .... Date: .... Period: ....

Challenge Creator

9. You will use the Activity 3 Sheet to create your own hanger challenge.

a **Make It!** On the Activity 3 Sheet, create your own balanced hanger challenge.

b **Solve It!** On this page, write the equation that represents your hanger and then determine the value of x that balances your hanger.

My Equation	Solution to My Equation

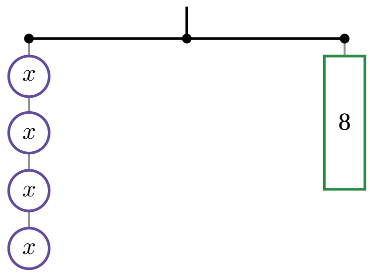
c **Swap It!** Swap your challenge with one or more partners. Write their equation, then determine the value of x that balances their hanger.

	Equation	Solution to Their Equation
Partner 1		
Partner 2		
Partner 3		
Partner 4		

Synthesis

10. How can a balanced hanger help determine the solution to an equation?

Use the hanger and equation if that helps you with your thinking.



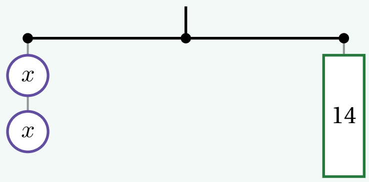
$4x = 8$

Lesson Practice 6.03

Lesson Summary

Hangers are a helpful way to represent equations. A hanger is balanced when the weight on both sides is equal.

Here is an example.

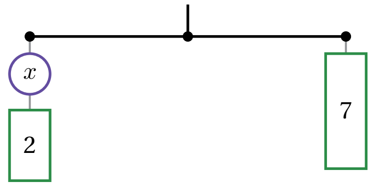


This hanger represents the equation $2x = 14$, or $x + x = 14$. The solution to this equation is the value of x that will keep the hanger balanced. The solution for this hanger is 7 because $7 + 7 = 14$ or $2(7) = 14$.

Lesson Practice
6.03

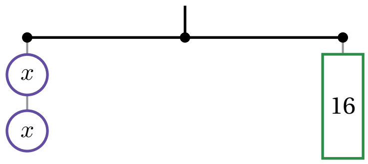
Name: .... Date: .... Period: ....

1. Anushka says that to balance this hanger the value of x must be 7. Is she correct? Explain your thinking.

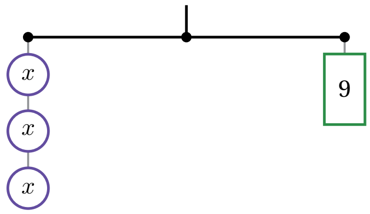


Problems 2–3: Determine the value of x that balances the hanger.

2.



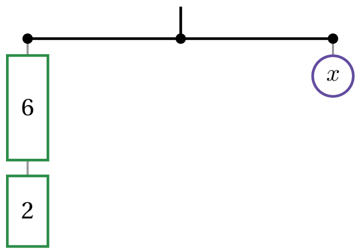
3.

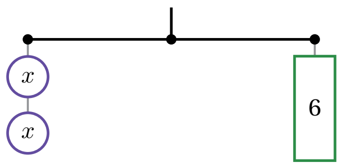


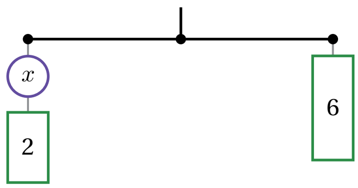
FAST Practice

4. Match each equation to the hanger it represents. One equation will have no match.

$2x = 6$	$2 + x = 6$	$6 + 2 = x$	$3 \cdot 2 = x$







Lesson Practice
6.03

Name: .... Date: .... Period: ....

Spiral Review

Problems 5–8: Determine the value of each expression.

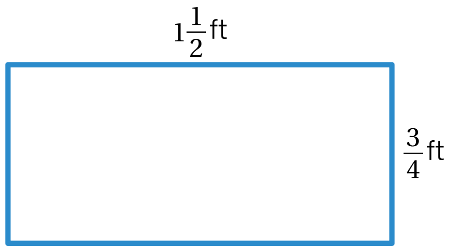
5. $12 + 2.4$

6. $12 \cdot 2.4$

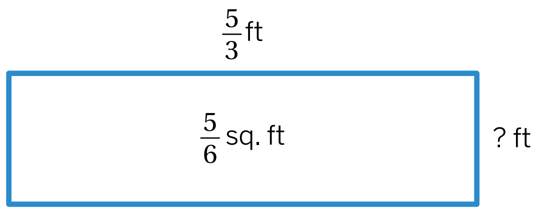
7. $12 - 2.4$

8. $12 \div 2.4$

9. Calculate the area of this rectangle.



10. Calculate the length of this rectangle.



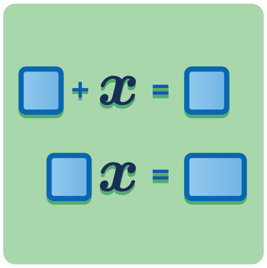
11. Precious set a goal to save $20 for a new game. Complete the table to show how much money Precious saved at different percentages of her goal.

Percentage of Goal (%)	Money Saved ($)
25	
75	
125	

Name: ______________ Date: ________ Period: ________

MA.6.AR.2.1, MA.6.AR.2.2, MA.6.AR.2.3, MTR.2.1, MTR.4.1, MTR.5.1

Swap and Solve



Let's write and solve equations.

Warm-Up

1. Here is a situation with hidden information. Let's make sense of it as a class.

Takeshi has ______ to spend on laundry. It costs ______ to wash and dry each load. Takeshi can wash ______ loads of laundry.

2. Dhruv and Nyanna each wrote an equation to represent this situation.

Dhruv	Nyanna
$p = 21 \cdot 3$	$3p = 21$

Whose equation is correct? How do you know?

Activity 1

Name: ______ Date: ______ Period: ______

Stronger and Clearer Each Time

Here is a set of equations we will use throughout this lesson.

$x + 4 = 8$	$4x = 8$	$\frac{x}{4} = 8$	$x - 4 = 8$
$2 + x = 20$	$2x = 20$	$20 \cdot 2 = x$	$x - 20 = 2$

3. Select an equation and solve it for x.

Equation	Solution

4. Write a first draft of a situation to match this equation. Make sure to include what the variable represents in your situation.

5. Meet with a partner to discuss your first drafts and provide feedback to each other.

6. Write a second draft that is stronger and clearer.

Activity 2

Name: ________ Date: ________ Period: ________

Trade and Solve

Takeshi has $21 to spend on laundry. It costs $3.00 to wash and dry each load. Takeshi can wash p loads of laundry.

Equation	Solution	Solution Check	Solution's Meaning
$3p = 21$	$p = 7$	$3 \cdot 7 = 21$	Takeshi can wash and dry 7 loads of laundry for $21.

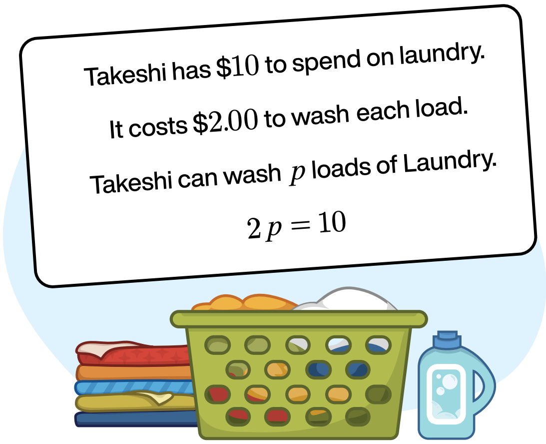
7. What do you think a solution's meaning is?

8. You will need several different partners for this activity. With each partner, trade the slips of paper with your situations and complete the table for their situation.

	Partner A	Partner B	Partner C
Partner's Name			
Equation			
Solution			
Solution Check			
Solution's Meaning			

Synthesis

9. What do you think is important to remember when writing equations to represent situations?



Lesson Practice 6.04

Lesson Summary

Writing an equation to match a situation is a helpful tool when trying to determine an unknown value. The equation can be solved using a variety of strategies such as tape diagrams, hangers, or inverse operations. We can check the solution to an equation by *substituting* the value of the variable to see if it makes the equation true. Once we have a solution to the equation, it's important to determine the meaning of the solution.

Here is an example.

Situation	Equation	Solution	Solution Check	Solution's Meaning
Adah has \$42 to spend on music downloads. Each download costs \$7. She can buy x downloads.	$7x = 42$	$x = 6$	$7 \cdot 6 = 42$	She can buy 6 music downloads.

In some cases, we may be given a set of possible solutions to an equation. We can check each possible solution using substitution to see if it makes the equation true or false. Only one value will make an equation true. All other values will create false statements, indicating that they are not solutions to the equation. The value $x = 7$ makes the above equation false because $7 \cdot 7 = 49$, so $x = 7$ is not the solution to the equation.

Lesson Practice
6.04

Name: .... Date: .... Period: ....

1. Tiara buys a pack of paper with 200 sheets. She divides the sheets of paper equally into 5 binders. Select *all* the equations that represent the number of sheets of paper in each binder, b.

- ☐ **A.** $b = 200 \div 5$
- ☐ **B.** $200 \div b = 5$
- ☐ **C.** $b = 5 \bullet 200$
- ☐ **D.** $b \div 5 = 200$
- ☐ **E.** $5b = 200$

Problems 2–3: Here is an equation: $2 + x = 6$.

2. Write a situation that the equation could represent.

3. Describe the meaning of the x in your situation.

Problems 4–5: A plant in Zahra's garden grows 3 inches taller each month. After x months, the plant has grown 12 inches.

4. Write an equation that could represent this situation.

5. Describe the meaning of the x in the situation.

6. Use the numbers 0 to 9 to complete each equation so that the value of x is the same. Use each number only once.

$\square x = \square$ $\qquad$ $x + \square = \square$ $\qquad$ $x - \square = \square$

Lesson Practice
6.04

Name: .... Date: .... Period: ....

FAST Practice

7. Anika buys 5 notebooks that contain 60 pages each. Select *all* the equations that represent the total number of pages, p.

- ☐ **A.** $p = 60 \div 5$
- ☐ **B.** $p \div 5 = 60$
- ☐ **C.** $5 + 60 = p$
- ☐ **D.** $5p = 60$
- ☐ **E.** $p = 5 \cdot 60$

Spiral Review

8. Select *all* the equations that have a solution of $c = 2$.

- ☐ **A.** $4c = 80$
- ☐ **B.** $c + 98 = 100$
- ☐ **C.** $c - 12 = 10$
- ☐ **D.** $6c = 12$
- ☐ **E.** $18c = 36$

Problems 9–11: Solve each equation.

9. $6m = 42$

10. $p + 7 = 11$

11. $n - 14 = 19$

12. Compare the information given about triangle C and triangle D.

Triangle C	**Triangle D**
Base = 12 inches	Base = 15 inches
Height = 8 inches	Height = 6.5 inches

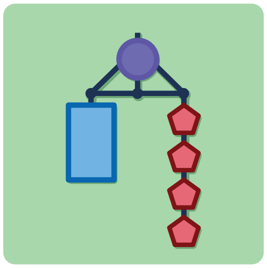
Which triangle has the greater area?

Explain your thinking.

Name: Date: Period:

MA.6.AR.2.4, MTR.3.1, MTR.4.1, MTR.5.1

Solutions of a Different Kind



Let's use a variety of strategies to determine the unknown in an equation with decimals or fractions.

Warm-Up

Determine the value of each expression mentally. Try to think of more than one strategy.

1. $5 - 2$

2. $5 - 2.1$

3. $5 - 2.17$

4. $5 - 2.017$

Activity 1

Name: .... Date: .... Period: ....

What's the Relationship?

5. Consider the equation $\frac{5}{8} + x = \frac{12}{8}$.

a. Draw a model to represent the equation.

b. Explain your thinking.

c. How can you use the relationship between the numbers to determine the solution to the equation?

6. Maia says that she can use the multiplication and division fact family $4, 6, 24$ to help solve the equation $0.24 = 6z$. The value of z is 0.4. Is she correct? Explain your thinking.

Activity 2

Name: ________ Date: ________ Period: ________

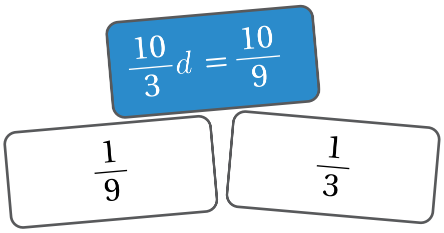
Solving and Solutions

7. Match each solution to its equation. Two solutions will not have a match.

0.5	0.1	0.8
$\frac{1}{9}$	$\frac{7}{10}$	$\frac{1}{3}$

$\frac{10}{3}d = \frac{10}{9}$	$12.6 = b + 12.1$	$c + \frac{3}{10} = 1$	$4 = \frac{a}{0.2}$

8. Imani and Deiondre solved this equation.



Imani said the solution is $d = \frac{1}{9}$.

Deiondre said the solution is $d = \frac{1}{3}$.

Whose solution is correct? Circle one.

Imani's Diondre's Both Neither

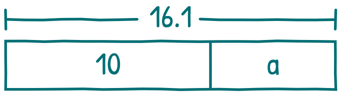
Explain your thinking.

Name: .... Date: .... Period: ....

Solving and Solutions (continued)

9. Here are Fabiana's and Alejandro's strategies for solving $10 + a = 16.1$.

Fabiana



The whole is 16.1. One part is 10.
I can remove 10 from 16.1.
I have 6.1 left.
So a is 6.1.

Alejandro

$10 + a = 16.1$

$10 + 6 = 16$

so . . .

$10 + 6.1 = 16.1$

Discuss: How are their strategies alike? How are they different?

10. Use either Fabiana's or Alejandro's strategy to solve the equation $3.02 = x + 2.01$.

Activity 3

Name: ____________ Date: ________ Period: ________

Repeated Challenges

11. • Decide with your partner who will complete Column A and who will complete Column B.
• The solutions in each row should be the same. Compare your solutions, then discuss and resolve any differences.
• Solve as many equations as you have time for. Sense-making is more important than speed.

	Column A	Column B
a	$3.6 = 4x$ $x =$ ________	$7x = 6.3$ $x =$ ________
b	$1.3 = x + 0.5$ $x =$ ________	$2.1 = x + 1.3$ $x =$ ________
c	$\frac{1}{3} = 2x$ $x =$ ________	$3x = \frac{1}{2}$ $x =$ ________
d	$x - 6.4 = 9$ $x =$ ________	$8.4 = x - 7$ $x =$ ________

Name: .... Date: .... Period: ....

Repeated Challenges (continued)

	Column A	Column B
e	$x + 1.8 = 14.7$ $x =$	$x + 5.3 = 18.2$ $x =$ ....
f	$\frac{1}{2}x = \frac{3}{4}$ $x =$	$\frac{3}{16} = \frac{1}{8}x$ $x =$ ....
g	$\frac{7}{8} = x + \frac{2}{8}$ $x =$	$x + \frac{6}{8} = \frac{11}{8}$ $x =$ ....

Synthesis

12. Describe a strategy for solving an equation with decimals or fractions.

Use the examples if they help with your thinking.

$\frac{1}{3}x = \frac{1}{18}$

$3 + x = 15.6$

Lesson Practice 6.05

Lesson Summary

There are many strategies to determine the unknown value in an equation with decimals or fractions, such as drawing models or using number sense to determine the value that makes an equation true.

Here are two examples that use number sense to solve an equation.

Equation	Explanation
$x + 1.5 = 3.8$	Original equation
$2 + 1 = 3$ and $3 + 5 = 8$	Use number sense and place value.
$x = 2.3$	The solution to this equation is 2.3.

Equation	Explanation
$\frac{5}{2}y = \frac{5}{8}$	Original equation
$5 \cdot 1 = 5$ and $2 \cdot 4 = 8$	Use the fact families $1, 5, 5$ and $2, 4, 8$.
$y = \frac{1}{4}$	The solution to this equation is $\frac{1}{4}$.

Lesson Practice
6.05

Name: .... Date: .... Period: ....

1. Select *all* the equations that have a solution of $n = 1.1$.

☐ **A.** $2n = 2.2$ ☐ **B.** $2.3 - n = 1.2$ ☐ **C.** $4n = 4.16$

☐ **D.** $n \div 1 = 1.1$ ☐ **E.** $n - 0.3 = 0.8$

Problems 2–7: Solve each equation. Use number sense or draw a diagram if it helps with your thinking.

2. $4m = 8$

3. $\frac{1}{2}a = \frac{5}{8}$

4. $10d = 32$

5. $w + 5.2 = 17$

6. $1.5x = 0.9$

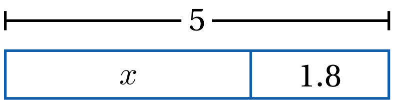
7. $24.6 = 6.1 + c$

FAST Practice

Problems 8–9: Use the tape diagram if it helps with your thinking.

8. Determine the value of x in the equation $x + 1.8 = 5$.

$x =$ ☐



9. Vihaan says the solution to $x + 1.8 = 5$ is $x = 6.8$. Explain how you know this is incorrect.

This is incorrect because $6.8 + 1.8$ would equal ☐ instead of ☐.

Lesson Practice
6.05

Name: .... Date: .... Period: ....

Spiral Review

10. Calculate each product.

Expression	Product
$212 \cdot 2$	
$21.2 \cdot 0.2$	
$21.2 \cdot 0.02$	

11. Kweku and Javier each used a different strategy to determine 25% of 60. Whose strategy is correct? Circle one.

Kweku
60×25

Javier
$60 \div 4$

Kweku's   Javier's   Both   Neither

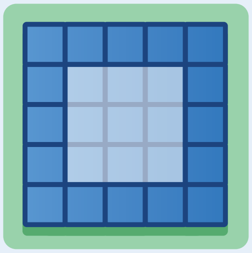
Explain your thinking.

Sub-unit

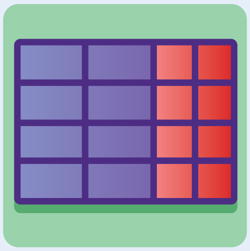
2

Equivalent Expressions

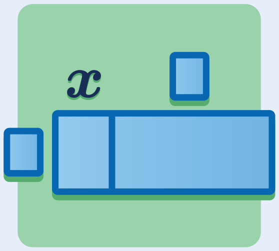
Lesson 6
Vari-apples



Lesson 7
Border Tiles



Lesson 8
Products and Sums



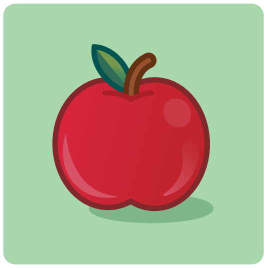
Lesson 9
Equivalent Expressions

Name: Date: Period:

MA.6.AR.1.1, MA.6.AR.1.3, MTR.2.1, MTR.4.1, MTR.5.1, MTR.7.1

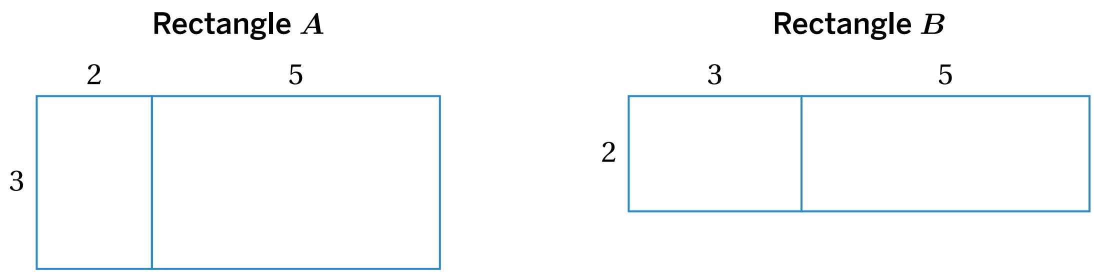
Vari-apples

Let’s use variable expressions to represent situations.



Warm-Up

1. Here are two rectangles.



Which rectangle has a greater area? Circle one.

Rectangle A Rectangle B They have the same area

Explain your thinking.

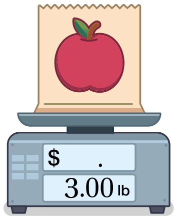
Activity 1

Name: ______ Date: ______ Period: ______

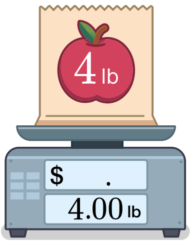
Intro to Variable Expressions

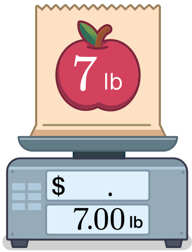
Apples at your store cost $1.50 per pound.

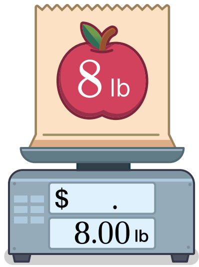
2. A customer orders 3 pounds of apples. How much should you charge them?



3. Here are three new orders. How much should you charge for each order?







Apples (lb)	Cost ($)
4	
7	
8	

4. Describe how to determine the cost of *any* number of pounds of apples.

Name: .... Date: .... Period: ....

Intro to Variable Expressions (continued)

5. Rudra and Sai each wrote an *expression* to describe the cost of p pounds of apples.

Rudra: $p + 1.50$

Sai: $1.50p$

Whose expression is correct? Circle one.

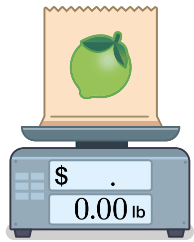
Rudra's Sai's Both Neither

Explain your thinking.

Apples (lb)	Cost ($)
3	4.50
4	6.00
7	10.50
8	12.00
p	

6. Limes at your store cost $\$2.40$ per pound.

How much should you charge for p pounds of limes?



7. Match each translation with the math expression it represents. One expression will have no match.

$y + 9$	$y - 9$	$9y$	$y \div 9$

A number decreased by 9	A number multiplied by 9	9 greater than a number

Name: ________ Date: ________ Period: ________

Comparing Variable Expressions

8. For $5, you can get your groceries delivered. What is the total cost for each of these grocery deliveries?

Cost of Groceries ($)	Total Cost ($)
37.95	42.95
50.86	
72.11	
87.94	

Grocery cost:	$37.95
Delivery fee:	$5.00
Total:	$42.95

9. Write an expression for how much you should charge for g dollars worth of groceries, including delivery.

10. **a** Translate each verbal expression into a math expression.

- The product of 10 and a number:
- The sum of a number and 10:
- 15 more than a number:
- 15 multiplied by a number:

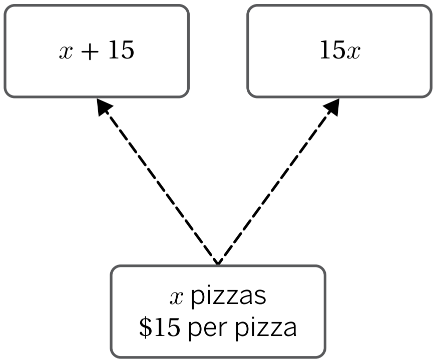
b Match each situation in the table with a math expression you wrote in part a to represent the total cost. Two expressions will have no match.

x pizzas $15 per pizza	x dollars of groceries $10 for delivery

Name: .... Date: .... Period: ....

Comparing Variable Expressions (continued)

11. Which expression represents this situation?



12. The expression $15x$ has one **term**. The expression $15 + x$ has two terms.

Select *all* the expressions that also have two terms.

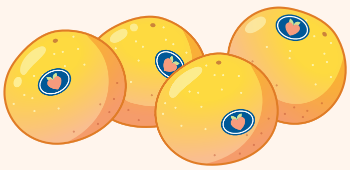
- ☐ **A.** $y - 6$
- ☐ **B.** $\frac{1}{2}x$
- ☐ **C.** $3x + 2$
- ☐ **D.** $x + y$
- ☐ **E.** $3 \cdot 2$

13. Choose one of the math expressions you selected from Problem 12 and write a verbal translation of the expression.

You're invited to explore more.

14. Describe a situation that could be represented by the expression $2p + 6$. Create a table if it helps with your thinking.

$2p + 6$



Name: .... Date: .... Period: ....

Evaluating Variable Expressions

15. The expression for the cost in dollars for p pineapples at your store is $4p$.

a. What is the cost of 1 pineapple? Explain your thinking.

b. What is the cost of 5 pineapples?

16. The expression for the cost in dollars of b containers of blueberries plus delivery is $3b + 5$.

Rudra and Sai each calculated the cost for the delivery of 4 containers of blueberries.

Rudra	Sai
3 • 4 + 5	3 • 4 + 5
12 + 5	3 • 9
$17	$27

Whose calculation is correct? Circle one.

Rudra's   Sai's   Neither

Explain your thinking.

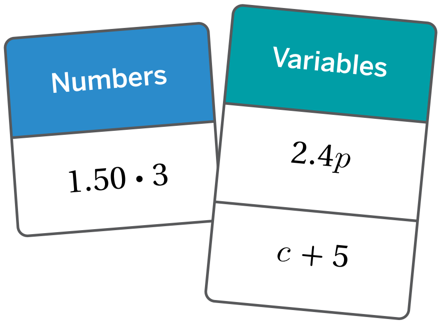
17. Evaluate $10x + 7y$ for $x = 5$ and $y = 2$.

Synthesis

18. Here are two types of expressions: expressions with numbers and expressions with variables.

When might each kind of expression be useful?

Expressions with numbers are useful when . . .



Expressions with variables are useful when . . .

Lesson Practice 6.06

Lesson Summary

We can use an *expression* with a variable to represent situations with known and unknown values. Each part in the expression represents a different value in the situation. Here are a few examples.

- The cost of 1 pound of grapes is $\$2.25$. If p represents pounds of grapes, the expression $2.25p$ can be used to calculate the total cost for any number of pounds of grapes. This expression only has one **term**.
- A grocery store adds a $\$10$ fee to the cost of groceries for delivery. If c represents the cost of groceries and 10 represents the delivery cost, the expression $c + 10$ can be used to calculate the total cost for groceries and delivery. This expression has two terms, c and 10.

We can also evaluate an expression with a variable for a known value of the variable. For example, if you spend $\$42$ on groceries and the cost for delivered groceries is $c + 10$, where c is the cost of groceries, you can calculate the cost to have them delivered using $c = 42$. Substitute the value for c in the expression and evaluate.

$$c + 10$$

$$42 + 10$$

$$52$$

So the cost for having your groceries delivered is $\$52$.

Lesson Practice
6.06

Name: .... Date: .... Period: ....

Problems 1–3: Oranges cost $1.25 per pound. How much would it cost to buy:

1. 2 pounds of oranges?
2. 5 pounds of oranges?
3. x pounds of oranges?

4. You need red and blue ribbon for a craft project. The instructions say that the red ribbon should be 7 inches longer than the blue ribbon.

   Complete the table to show how long the red ribbon should be for different lengths of blue ribbon.

Blue Ribbon (in.)	Red Ribbon (in.)
10	
27	
x	

Problems 5–6: The variable s represents the number of students in one class in your school.

5. What does $\frac{1}{2}s$ represent?
6. What does $s + 1$ represent?

Problems 7–9: Evaluate the expression $3m + 5$ for each value of m.

Example

$m = 7$

$3(7) + 5 = 26$

7. $m = 8$

8. $m = 12$

9. $m = 15$

Name: .... Date: .... Period: ....

10. Choose any value for a.

$a =$ ☐

Use your value to complete the puzzle.

$3a$	$+$	4	$=$					
$-$				$+$				$-$
a		$4a$	$+$	$4a$	$-$		$=$	$5a$
$-$		$+$		$=$				$=$
a					$-$	$2a$	$=$	
$-$		$=$				$+$		
a	$+$	$6a$	$-$		$=$	$4a$		
$=$		$+$				$=$		
		a		a	$+$		$=$	

FAST Practice

11. 35 riders are on a bus, and n riders get off at the same stop. In this scenario, what does the expression $35 - n$ represent?

$35 - n$ represents the number of riders that [A. get off B. are still on] the bus after n riders [A. get on B. get off] the bus at the same stop.

Spiral Review

12. LaShawn's class raised $500 for a fundraiser. They used 10% of the money to cover the cost of materials, saved 20% for the next fundraising project, and donated the rest. How much money did LaShawn's class donate?

Explain your thinking.

13. A garbage bin can hold 50 gallons of waste.

Complete the table to show what percent of the bin would be filled for different amounts of waste.

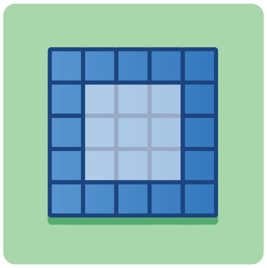
Waste (gal)	Percent Filled (%)
5	
30	
45	

Name: .... Date: .... Period: ....

MA.6.AR.1.4, MTR.2.1, MTR.4.1, MTR.5.1

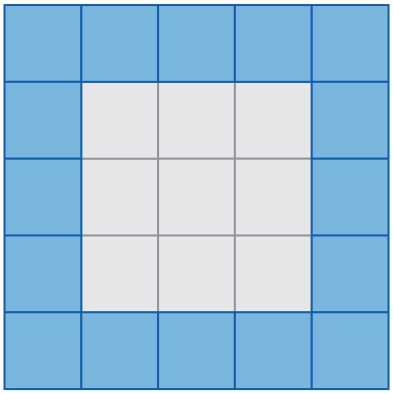
Border Tiles

Let's use diagrams to determine which expressions are equivalent.



Warm-Up

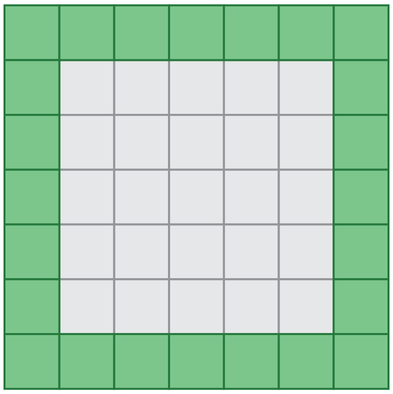
1. Here is a 3-by-3 square surrounded by border tiles.



a. Without counting one by one, how many border tiles are there?

b. Explain how you see it.

2. Here is a 5-by-5 square surrounded by border tiles. Without counting one by one, how many border tiles are there?

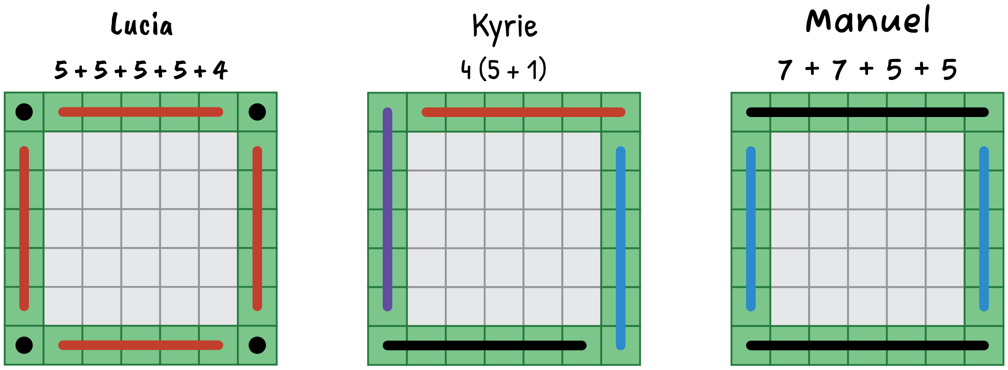


Activity 1

Name: ________ Date: ________ Period: ________

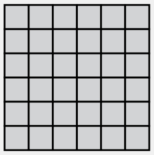
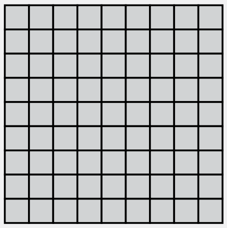
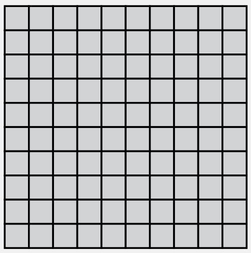
Border Tiles

3. **a** Take a look at Lucia's, Kyrie's, and Manuel's expressions for the 5-by-5 square.



b **Discuss:** How are all of their expressions alike?

4. Here are three new squares. Determine the number of border tiles for each square.

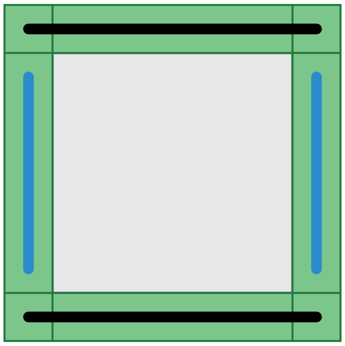
Model	Square	Border Tiles
	6-by-6	
	9-by-9	
	10-by-10	

5. How can you determine the number of border tiles for an n-by-n square? Use your table if it helps with your thinking.

Name: .... Date: .... Period: ....

Equivalent Expressions

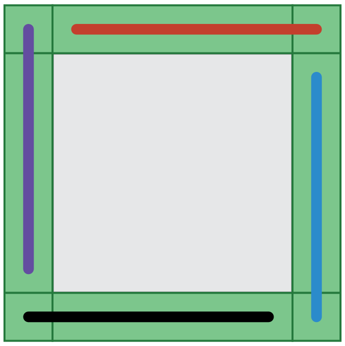
6. Manuel wrote this expression for the number of border tiles in an n-by-n square:



$(n + 2) + (n + 2) + n + n$

Show or explain how Manuel's expression is connected to his sketch.

7. Here is Kyrie's sketch for the same square.



What might Kyrie's expression be for the number of border tiles in an n-by-n square?

Name: Date: Period:

Equivalent Expressions (continued)

8. Equivalent expressions are expressions that are equal for every value of a variable.

Here are two expressions.

Kyrie: $4(n + 1)$

Manuel: $(n + 2) + (n + 2) + n + n$

a Use each expression to calculate the number of border tiles when $n = 8$.

Kyrie	Manuel
$4(n + 1)$	$(n + 2) + (n + 2) + n + n$

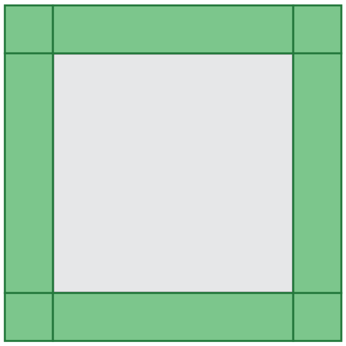
b **Discuss:** How can you tell that Kyrie's and Manuel's expressions are equivalent?

9. Which expression is also equivalent to Kyrie's and Manuel's expressions?

A. $(n + 1)(n + 1)$

B. $4n + 1$

C. $4n + 4$

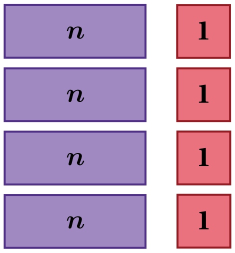


Name: ____________ Date: ________ Period: ________

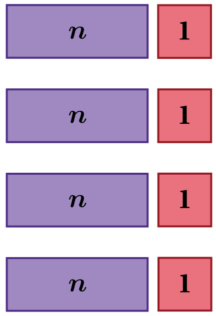
A New Diagram

10. Here is a new way of visualizing expressions.

$4n + 4$



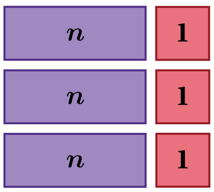
$4(n + 1)$

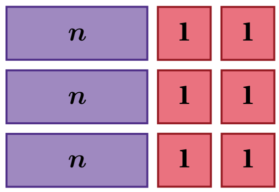


Discuss: How do these diagrams show that $4n + 4$ and $4(n + 1)$ are equivalent?

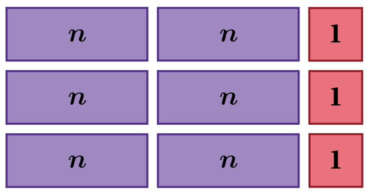
11. Match each expression with the diagram it represents. One expression will have no match.

$3(n + 1)$	$3n + 6$	$3n + 3$
$3(n + 3)$	$(n + 2) + (n + 2) + (n + 2)$	





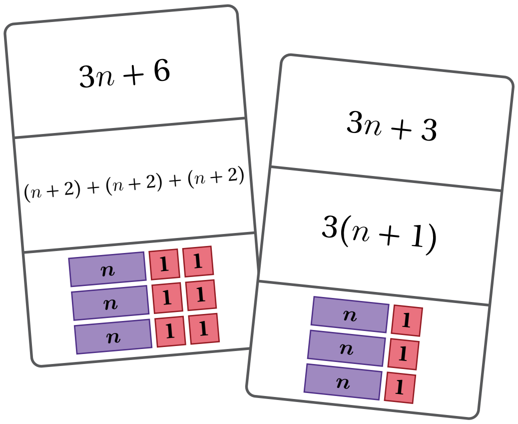
12. Write an expression to represent this new diagram. Try to write an expression you think none of your classmates will.



Synthesis

13. How can you decide if two expressions are equivalent?

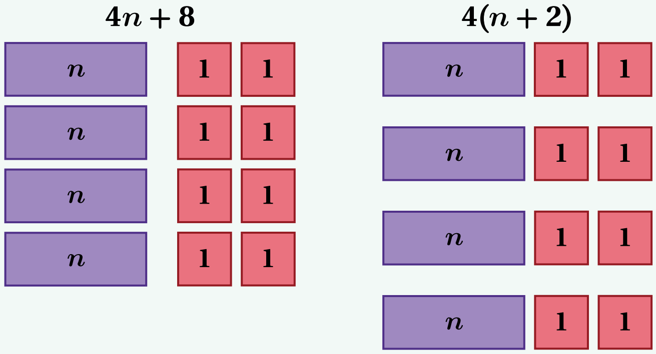
Use the examples if they help you explain your thinking.



Lesson Practice 6.07

Lesson Summary

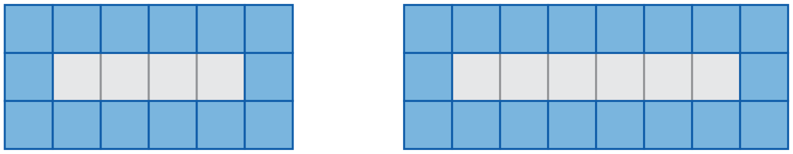
Equivalent expressions are expressions that are equal for every value of a variable, such as $4n + 8$ and $4(n + 2)$. Diagrams that represent these expressions can help us visually decide if the expressions are equivalent.



The diagrams for $4n + 8$ and $4(n + 2)$ both show 4 n-tiles and 8 one-tiles. Therefore, $4n + 8$ and $4(n + 2)$ are equivalent expressions because they are equal for every value of n.

Name: .... Date: .... Period: ....

Problems 1–3: Here are examples of an n-by-1 rectangle.



1. How many border tiles are there in the 4-by-1 rectangle?

2. How many border tiles are in the 6-by-1 rectangle?

3. Diego says $2n + 6$ represents the number of tiles needed for the border of an n-by-1 rectangle. Explain why his strategy is correct.

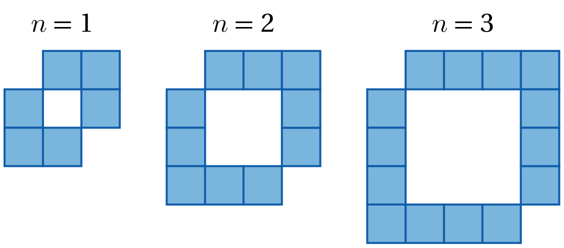
Problems 4–5: Here are five expressions.

$2 + 6n$	$2(n + 3)$	$n + 3$	$n + n + 6$	$(n + 3) + (n + 3)$

4. Write *all* of the expressions that are equivalent to $2n + 6$.

5. Choose an expression that is *not* equivalent to $2n + 6$. Explain how you know it is not equivalent.

Problems 6–7: Here is a pattern.



6. Write an expression that describes the number of tiles for any stage, n.

7. Write an equivalent expression.

Lesson Practice
6.07

Name: ______________ Date: ______ Period: ______

FAST Practice

8. Which expression is equivalent to $8n + 2$?

(A.) $4(2n + 2)$ (B.) $2(4n + 1)$ (C.) $4n + 2 + n + 2$ (D.) $n + 8 + 2$

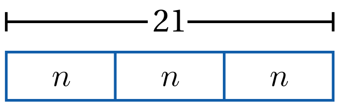
Spiral Review

Problems 9–10: Write an equation to represent each situation.

9. Aba's dog was 6 inches tall when it was a puppy but is now 14 inches tall. Aba's dog grew n inches.

10. Apples cost \$2 per pound. Darius bought x pounds of apples for a total cost of \$6.

Problems 11–12: Here is a tape diagram.



11. Write an equation to represent the tape diagram.

12. Determine the value of n.

Problems 13–16: Evaluate each expression for $b = 5$.

13. $3.5b$

14. $6b + 1$

15. $\frac{1}{4} + b$

16. $\frac{1}{2}b$

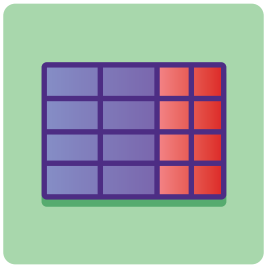
Unit 6
Lesson
8

Name: ________ Date: ________ Period: ________

MA.6.AR.1.4, MTR.4.1

Products and Sums

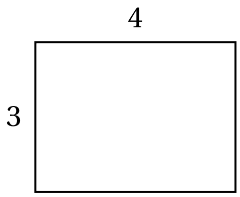
Let’s explore equivalent expressions using rectangle areas.



Warm-Up

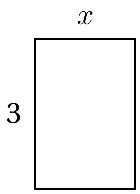
1. Write an expression for the area of each rectangle.

a



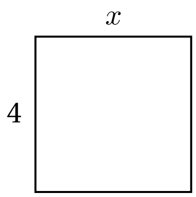
Expression: ________

b



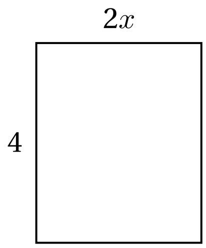
Expression: ________

c



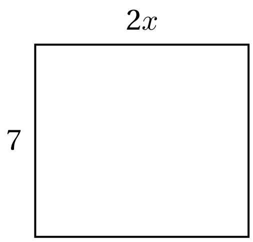
Expression: ________

d



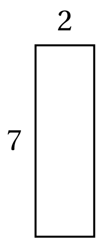
Expression: ________

e



Expression: ________

f

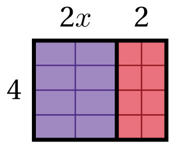
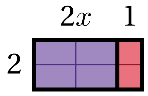
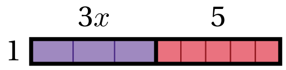
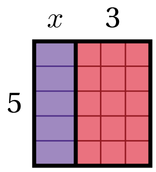


Expression: ________

Name: ________ Date: ________ Period: ________

Rectangles and Equivalent Expressions

2. Here are four rectangles and the *product* and *sum* expressions that represent their areas.

Rectangle				
Product	$4(2x + 2)$	$2(2x + 1)$	$1(3x + 5)$	$5(x + 3)$
Sum	$8x + 8$	$4x + 2$	$3x + 5$	$5x + 15$

What do you notice about the product expressions?

What do you notice about the sum expressions?

3. **a** Create a rectangle with an area of $2(3x + 4)$. Label the sides of your rectangle.

b Write an equivalent expression for the area.

4. How would you convince someone that $2(3x + 4)$ is *not* equivalent to $6x + 4$?

Name: .... Date: .... Period: ....

More Rectangles and Equivalent Expressions

5. Create a rectangle with an area of $6x + 12$.

6. Select *all* the expressions that are equivalent to $6x + 12$. Use the rectangle you created or draw a new rectangle if it helps with your thinking.

- ☐ **A.** $6(x + 2)$
- ☐ **B.** $6(x + 12)$
- ☐ **C.** $3(2x + 4)$
- ☐ **D.** $3(x + 4)$
- ☐ **E.** $2(3x + 6)$

7. **a** Here are three new expressions. Select the *two* expressions that are equivalent.

- ☐ **A.** $4(x + 2)$
- ☐ **B.** $4(x + 8)$
- ☐ **C.** $4x + 8$

b Create a drawing to convince someone that the two expressions you selected are equivalent.

Activity 3

Name: ________ Date: ________ Period: ________

Challenge Creator

8. You will use the Activity 3 Sheet to create your own rectangle challenge.

a. **Make It!** On the Activity 3 Sheet, create a rectangle challenge.

b. **Solve It!** On this page, write two expressions that represent your rectangle. Record one expression on your Activity 3 Sheet.

c. **Swap It!** Swap your challenge with one or more partners. Sketch your partner's rectangle and record their expression. Then create an equivalent expression.

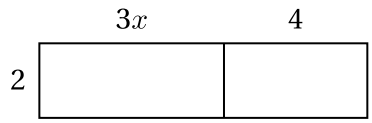
My First Expression	My Second Expression

	Sketch of Rectangle	Partner's Expression	Equivalent Expression
Partner 1			
Partner 2			
Partner 3			
Partner 4			

Synthesis

9. Describe how you can use the area of a rectangle to write two or more equivalent expressions.

Use the example if it helps to show your thinking.

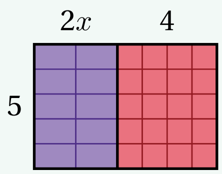


Lesson Practice 6.08

Lesson Summary

You can use areas of rectangles to write equivalent expressions. Two expressions that are equivalent are a *product* expression and a *sum* expression because they refer to the same area. No matter what value you substitute for the variable, the total area is the same.

Area Model



Product of Two Side Lengths

$5(2x + 4)$

$10x + 20$

Sum of Two Areas

$5 \cdot 2x + 5 \cdot 4$

$10x + 20$

Lesson Practice
6.08

Name: .... Date: .... Period: ....

1. Select *all* the expressions that are equivalent to $4b$.

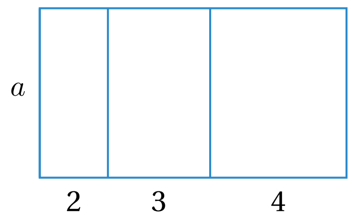
☐ **A.** $b + b + b + b$ ☐ **B.** $b + 4$ ☐ **C.** $b \bullet b \bullet b \bullet b$

☐ **D.** $2b + 2b$ ☐ **E.** $4 \bullet b$

2. Zola wrote the total area of the rectangle as $2a + 3a + 4a$.

Amir wrote the total area as $(2 + 3 + 4)a$.

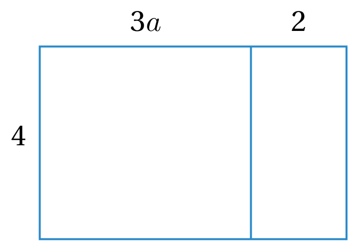
Explain why they are *both* correct.



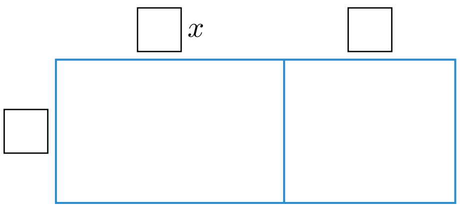
3. Write two equivalent expressions that could be used to represent the area of the rectangle.

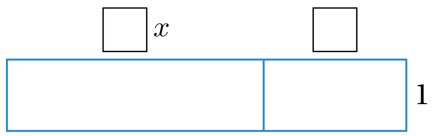
Expression 1: ....

Expression 2: ....



4. Use the numbers 0 to 9 to fill in each blank so that each rectangle has the same area. Use each number only once.



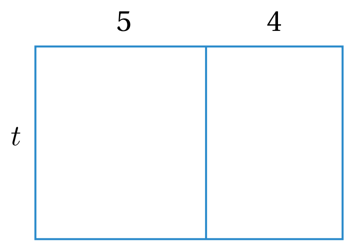


Lesson Practice
6.08

Name: .... Date: .... Period: ....

FAST Practice

5. Select *all* the expressions that represent the area of the rectangle.



- ☐ **A.** $t + 5 + 4$
- ☐ **B.** $5t + 4t$
- ☐ **C.** $9t$
- ☐ **D.** $4 \cdot 5 \cdot t$
- ☐ **E.** $(5 + 4)t$

Spiral Review

Problems 6–8: Titus's aunt is 17 years older than he is.

6. How old will his aunt be when Titus is 15 years old?

7. How old will his aunt be when Titus is 30 years old?

8. How old will his aunt be when Titus is x years old?

Problems 9–11: Solve each equation. Show your thinking.

9. $10m = 30$ **10.** $13 = h + 4$ **11.** $k + 15 = 41$

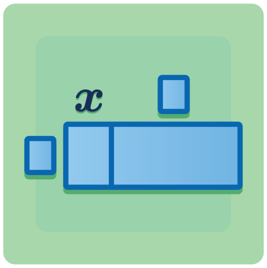
Unit 6
Lesson
9

Name: ______ Date: ______ Period: ______

MA.6.AR.1.4, MTR.4.1

Equivalent Expressions

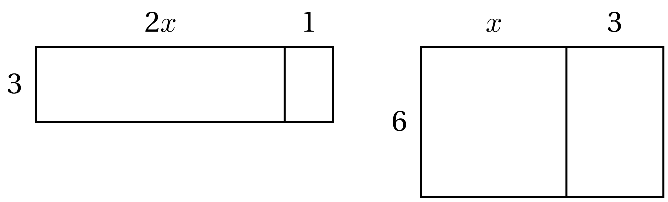
Let's explore equivalent expressions using area models and properties of operations.



Warm-Up

1. Here are two rectangles.

Rectangle A **Rectangle B**



a Which rectangle has an area of $6x + 3$?

Explain your thinking.

b What is the area of the other rectangle?

2. The **coefficient** of the expression $6x$ is 6.

Select *all* the expressions that also have a coefficient of 6.

☐ **A.** $6(2x)$

☐ **B.** $1(6x)$

☐ **C.** $2x \cdot 3$

☐ **D.** $2(4x)$

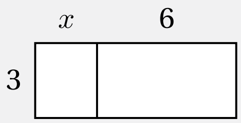
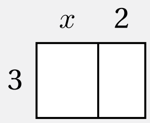
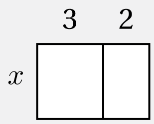
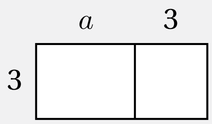
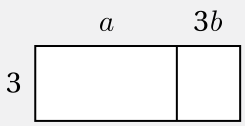
☐ **E.** $6 + x$

Name: ____________ Date: ________ Period: ________

Card Sort

3. You will use a set of cards for this activity.

- Match each product or sum to its representation. Three expressions will be missing.
- Write in each missing expression.

	Representation	Product	Sum
a			
b			
c			
d			
e			
f	The product of 3 and the sum of a and b.		

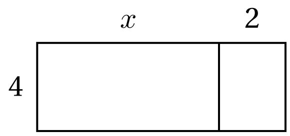
4. Explain how you determined one of the missing expressions.

Activity
2

Name: .... Date: .... Period: ....

Two Truths and a Lie

5. Two of these expressions represent the area of this rectangle.



a Which are they?

A. $3(x + 2) + 1(x + 2)$

B. $1 + 3(x + 2)$

C. $4(x + 2)$

b **Discuss:** Which expression is not equivalent to the others? How do you know?

6. In each row, two choices are equivalent and one is not. Circle the one that is *not* equivalent.

	Expression A	Expression B	Expression C
a	$6(2 + x)$	$2(6 + x)$	$6(x + 2)$
b	$16x$	$1(8 + x)$	$x + 8$
c	$3 + 2(3 + x)$	$5(3 + x)$	$9 + 2x$
d	The product of 5 and the sum of x and 2.	$5x + 2$	$5(x + 2)$
e	$4(6x + 3x)$	$36x$	$24x + 3x$
f	$7(x + 2)$	$3x + 4(x + 2)$	$7x + 8$

7. Pick one problem and explain how you decided which choice was not equivalent.

Synthesis

8. Explain how you can show that two expressions are equivalent.

Use these expressions if they help with your thinking.

Equivalent Expressions

$3x + 4(x + 2)$
$7x + 8$

Not Equivalent

$7(x + 2)$

Lesson Practice 6.09

Lesson Summary

The expression $8x + 2$ has two terms, and the term $8x$ has a **coefficient** of 8.
The expression $2(x + 1) + 3(2x)$ also has two terms, $2(x + 1)$ and $3(2x)$, but the terms are more complex.

To decide if two expressions are equivalent, you can draw models, substitute values, or rewrite the expressions. If the expressions are equivalent, you can use the distributive property and other operations to rewrite one expression to look like the other.

Here is an example: Determine whether $2(x + 1) + 3(2x)$ is equivalent to $8x + 2$.

$$\begin{aligned} 2(x + 1) + 3(2x) &= 2x + 2 + 6x \\ &= 2x + 6x + 2 \\ &= 8x + 2 \end{aligned}$$

$2(x + 1) + 3(2x)$ and $8x + 2$ are equivalent expressions because after using the distributive property and adding the *like terms*, the expressions are the same.

Lesson Practice
6.09

Name: .... Date: .... Period: ....

1. Complete the table.

Rectangle	$4 \quad x$ 2 [ ] [ ]	$m \quad 3$ 5 [ ] [ ]	$2a \quad b$ 3 [ ] [ ]
Product		$5(m+3)$	
Sum	$8+2x$		

2. Latifa and Joel are trying to rewrite $8y+24$ as a product expression.

Latifa	Joel
$8(y+3)$	$2(4y+12)$

Are Latifa's and Joel's expressions both equivalent to $8y+24$? Circle one.

Yes No

Explain your thinking.

Problems 3–5: Determine whether each pair of expressions are equivalent.

	Expression A	Expression B	Equivalent?
3.	$2(x+8)+3$	$2x+19$	Yes No
4.	$5+2(y+4)$	$7y+28$	Yes No
5.	$3(z+1)+3(z+1)$	$6(z+1)$	Yes No

Name: .... Date: .... Period: ....

6. Complete the table.

Product Expression	Sum Expression
	$4x + 8$
$(6 + 8)d$	
	$10m + 7m$
$3(2b + 5)$	
$6(u + 2t)$	

7. The area of a rectangle is $30 + 12x$.

List three possibilities for the length and the width of the rectangle.

Length	Width

FAST Practice

8. Select *all* the expressions that are equivalent to $4x + 8$.

☐ **A.** $4(x + 2)$

☐ **B.** $(4 + 8)x$

☐ **C.** $2(2x + 4)$

☐ **D.** $2(2x + 6)$

☐ **E.** $4 + 4(x + 1)$

Spiral Review

Problems 9–11: Solve each equation. Show your thinking.

9. $x + 5 = 11$

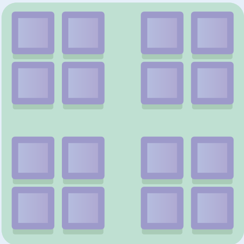
10. $6y = 18$

11. $\frac{w}{5} = 15$

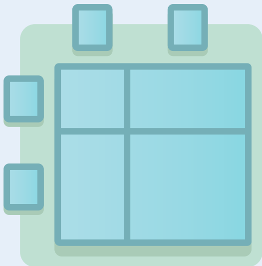
Sub-unit

3

Expressions Involving Exponents



Lesson 10
Powers



Lesson 11
Exponent Expressions

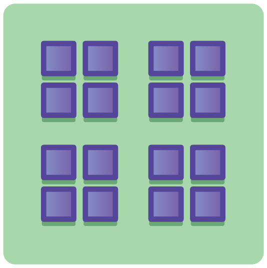
Unit 6
Lesson
10

Name: ____________ Date: ________ Period: ________

MA.6.NSO.3.3, MA.6.NSO.3.4, MTR.2.1, MTR.3.1

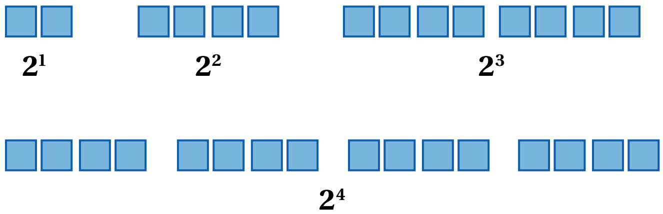
Powers

Let's see how exponents show repeated multiplication.



Warm-Up

1. Here are some images and their matching expressions.



Discuss: What do you notice? What do you wonder?

Activity 1

Name: ________ Date: ________ Period: ________

Powers of 2

2. The expression 2^4 ("2 to the power of 4") is equivalent to $2 \cdot 2 \cdot 2 \cdot 2$.

2^4

How could you determine the value of 2^5?

3. Write a number or expression that is equivalent to 2^5.

4. Group the equivalent expressions. One expression will not have a match.

$2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$	$2 + 2 + 2 + 2 + 2$		$2^4 \cdot 2$
2^5	$5 + 5$	$2 \cdot 5$	5^2

Group 1	Group 2

Name: .... Date: .... Period: ....

Powers of 2 (continued)

5. One expression in this group is *not* equivalent to the others. Which expression is it?

2^5
$2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$
$2 + 2 + 2 + 2 + 2$
$2^4 \cdot 2$

A. 2^5

B. $2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$

C. $2 + 2 + 2 + 2 + 2$

D. $2^4 \cdot 2$

Explain your thinking.

Name: .... Date: .... Period: ....

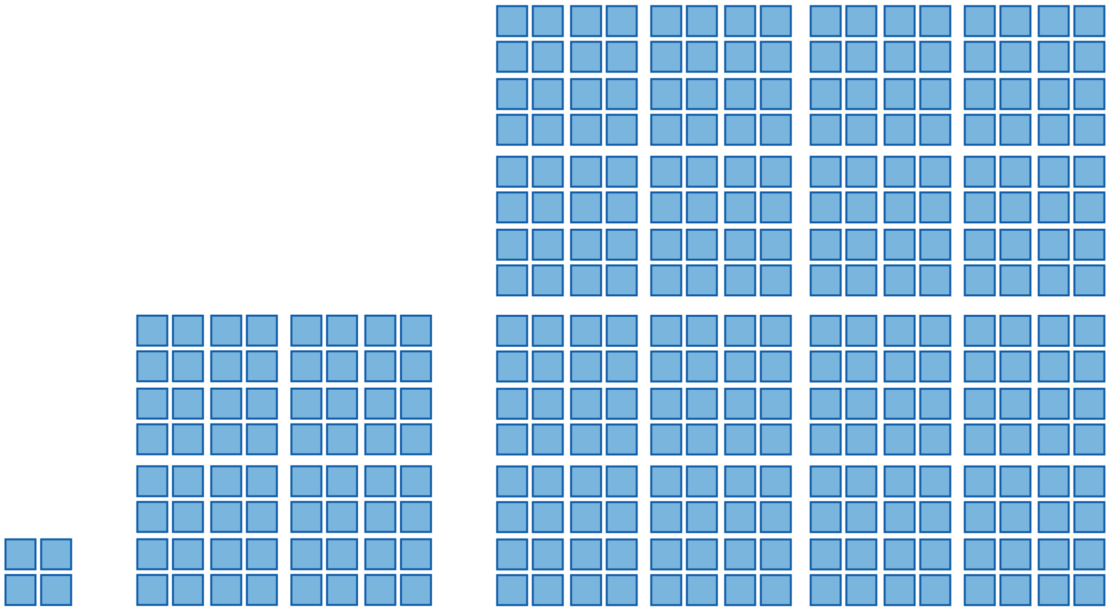
Exponents With Whole Number Bases

6. Select *one* expression that is equivalent to 3^4.

(A.) 12 (B.) $4 \cdot 4 \cdot 4$ (C.) $3 \cdot 3 \cdot 3 \cdot 3$ (D.) 81

Show or explain your thinking.

7. $4 \cdot 4$ is equivalent to 4^2, where 2 is the **exponent** and 4 is the **base**.



4^1 4^3 4^4

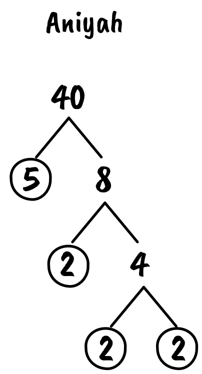
Write $4 \cdot 4 \cdot 4 \cdot 4 \cdot 4$ using at least one exponent.

8. Write a number or an expression that is equivalent to 4^3.

Name: ______ Date: ______ Period: ______

Exponents With Whole Number Bases (continued)

9. Aniyah says that 40 can be written as a product of **prime factors**.



a **Discuss:** What do you notice about Aniyah's factor tree?

b Use Aniyah's factor tree to rewrite 40 as a product of prime factors.

c How can you rewrite your expression using exponents?

Explain your thinking.

10. Which expression represents the prime factorization of 36?

A. $6 \cdot 6$ B. $2^2 \cdot 3^2$ C. $9 \cdot 2^2$ D. $2 \cdot 3 \cdot 6$

Show or explain your thinking.

Activity 3

Name: ______ Date: ______ Period: ______

Exponents With Fractional Bases

11. Here are some images and their matching expressions.

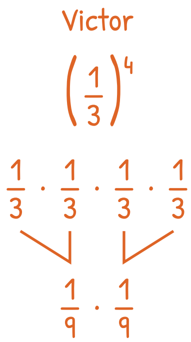
$\left(\frac{1}{2}\right)^1$ $\left(\frac{1}{2}\right)^2$ $\left(\frac{1}{2}\right)^3$

Write two things you know about $\left(\frac{1}{2}\right)^3$.

1.

2.

12. Victor wrote two expressions equivalent to $\left(\frac{1}{3}\right)^4$.



Write a different expression equivalent to $\left(\frac{1}{3}\right)^4$.

Explain your thinking.

13. Write an expression that is equivalent to $\left(\frac{1}{2}\right)^5$.

Name: .... Date: .... Period: ....

Exponents With Fractional Bases (continued)

14. Determine the value of each expression. Complete as many as you have time for.

a. 2^3

b. 4^2

c. $\left(\frac{1}{3}\right)^2$

d. 3^4

e. 5^1

f. $\left(\frac{1}{2}\right)^3$

g. 1^8

h. $\left(\frac{1}{4}\right)^3$

Synthesis

15. Without calculating, how can you tell whether expressions with exponents are equivalent?

$11 + 11 + 11 + 11 + 11$

$11^4 \cdot 11$

$11 \cdot 5$

$11 \cdot 11 \cdot 11 \cdot 11 \cdot 11$

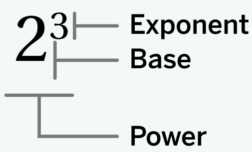
5^{11}

11^5

Lesson Practice 6.10

Lesson Summary

Exponents are used to represent repeated multiplication. In the expression 2^n, 2 is the **base**, and n is the **exponent**. If n is a positive whole number, it represents how many times 2 should be multiplied to determine the value of the expression.



Here are some examples.

$2^1 = 2$ $\quad\quad\quad$ $2^3 = 2 \cdot 2 \cdot 2$

There are several different ways to say "2^3."

- "Two to the power of three."
- "Two raised to the power of three."
- "Two to the third power."
- "Two cubed."

Exponents also can be used to represent the **prime factorization** of composite numbers.

For example, the number 24 can be rewritten using prime factors as $2 \cdot 2 \cdot 2 \cdot 3$ or $2^3 \cdot 3$.

Lesson Practice
6.10

Name: .... Date: .... Period: ....

1. Determine the value of each expression.

Expression	Value
$3 + 3 + 3 + 3$	
$3 \cdot 3 \cdot 3 \cdot 3$	
$4(3)$	
3^4	

2. Complete the table. The first row has been completed for you.

Expression With Exponent	Expression Without Exponent
3^5	$3 \cdot 3 \cdot 3 \cdot 3 \cdot 3$
	$2 \cdot 2 \cdot 2 \cdot 2$
4^3	
5^1	
	$\frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2}$
$\left(\frac{1}{3}\right)^4$	

3. Circle the two expressions that have the same value.

$6 + 6 + 6$ $\quad$ 6^3 $\quad$ 3^6 $\quad$ $3 \cdot 6$

Lesson Practice
6.10

Name: .... Date: .... Period: ....

4. Write the expression that represents each description, then determine its value.

Description	Expression	Value
Three to the third power		
Five to the second power		
Two to the power of five		
Three to the second power		

5. Rewrite 75 as a product of prime factors using exponents.

☐

FAST Practice

6. Select *all* the expressions that are equal to 16.

- ☐ **A.** 8^2
- ☐ **B.** 4^2
- ☐ **C.** 2^4
- ☐ **D.** 16^1
- ☐ **E.** 2^8

Spiral Review

Problems 7–9: Solve each equation.

7. $a - 2 = 5$

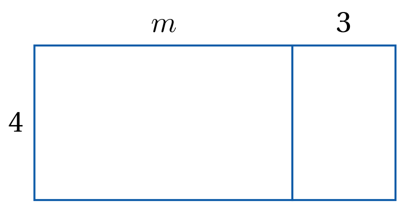
8. $b + 2 = 5$

9. $6c = 30$

10. Write two expressions for the area of the rectangle.

Product: ....

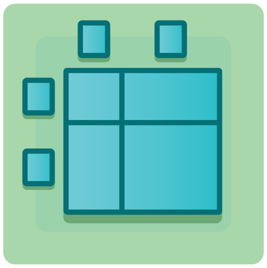
Sum: ....



Name: ______ Date: ______ Period: ______

MA.6.NSO.3.3, MTR.1.1, MTR.2.1, MTR.3.1, MTR.4.1

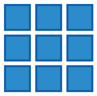
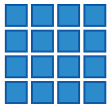
Exponent Expressions



Let's evaluate expressions with exponents.

Warm-Up

1. Here are some representations of the numbers 4, 9, and 16.

Number	4	9	16
Diagram			
Exponent Expression	2^2	3^2	4^2

a **Discuss:** What do you notice? What do you wonder?

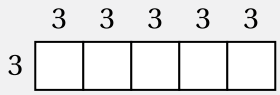
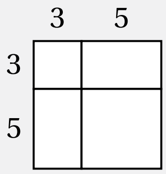
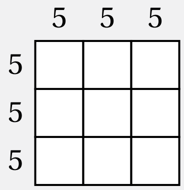
b Do you think 49 follows the same pattern as these numbers? Explain your thinking.

Name: .... Date: .... Period: ....

What's Missing?

2. You will use a set of cards for this activity.

- Work with a partner to group the matching diagrams, expressions, and values. There will be three blank spaces.
- Complete the table with the missing representations.

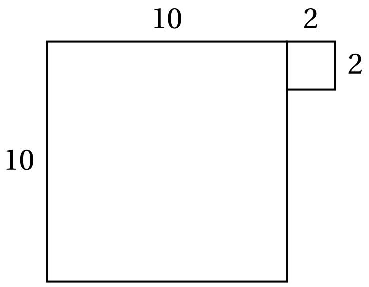
Diagram	Expression	Value
	$5 \cdot 3^2$	
		
		

Name: .... Date: .... Period: ....

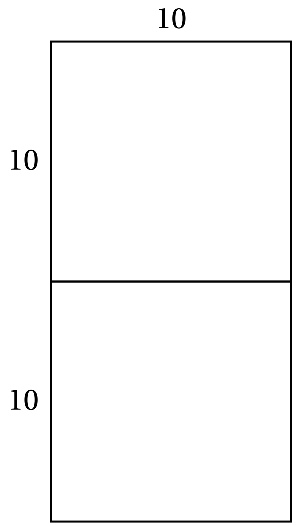
Evaluating Expressions

3. **Discuss:** Which diagram represents $2 + 10^2$?

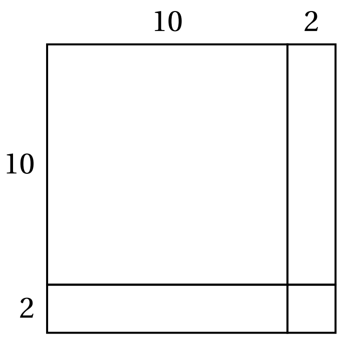
A.



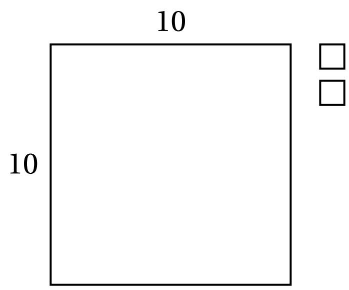
B.



C.



D.



4. Latifa and Nicolas each got a different value for $2 + 10^2$.

Latifa	Nicolas
$2 + 10^2$	$2 + 10^2$
12^2	$2 + 100$
144	102

Whose work is correct? Circle one.

Nicolas's Latifa's Both Neither

Explain your thinking.

Name: .... Date: .... Period: ....

Partner Problems

5. Decide with your partner who will complete Column A and who will complete Column B.

- Evaluate each expression when $a = 2$ and $b = 3$.
- The solutions in each row should be the same. Compare your solutions, then discuss and resolve any differences.

	Column A	Column B
a	$5^a + 4$	$2^a + 25$
b	$5^a - 4$	$25 - 2^a$
c	$\frac{4^2}{a}$	$\frac{a^5}{4}$
d	$(7 - a)^2 - b^2$	$\frac{1}{4}(1 + b)^3$
e	$\frac{8 - a^2}{4}$	$\frac{8 - 2(b)}{a}$
f	$\frac{2^4 - (b^2 - 5)}{2^a}$	$\frac{1^5 + (6^a - 10)}{b^2}$

You're invited to explore more.

6. Write an expression for your partner to evaluate. Swap problems, then write an expression with an exponent that has the same value as your partner's but uses different numbers.

Synthesis

7. What are some things to remember when determining the value of expressions with exponents?

Use these examples if they help with your thinking.

$5 \cdot 3^2$

$(3 + 5)^2$

$(3 \cdot 5)^2$

$5^2 + 3^2$

Lesson Practice 6.11

Lesson Summary

There is a specific *order of operations* we use to evaluate expressions with more than one operation, like $5 \cdot 2^4$ or $(5 \cdot 2)^4$.

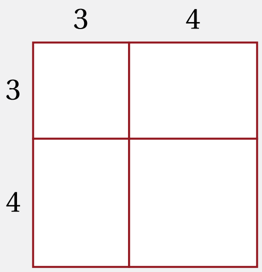
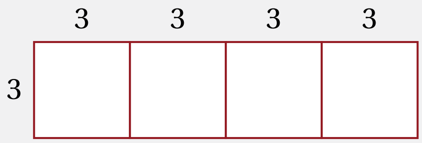
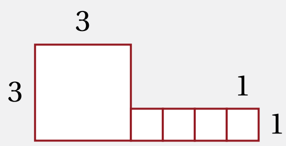
With Parentheses	**Without Parentheses**
Evaluate the operations in parentheses first:	Evaluate the term with the exponent first:
$(5 \cdot 2)^4$	$5 \cdot 2^4$
$(10)^4$	$5 \cdot (2 \cdot 2 \cdot 2 \cdot 2)$
$10 \cdot 10 \cdot 10 \cdot 10$	$5 \cdot 16$
$10{,}000$	80

Lesson Practice
6.11

Name: ____________ Date: ________ Period: ________

1. Match each expression with the diagram it represents. Evaluate the expression to determine the area of each diagram.

$4 \cdot 3^2$ $\quad$ $3^2 + 4$ $\quad$ $(3 + 4)^2$

Diagram	Expression	Area (sq. units)
		
		
		

Problems 2–3: Evaluate each expression for $f = 2$ and $g = 3$.

2.

Expression	Value
$5 + 4^f$	
$(g + f)^3$	
$f^2 \cdot 5$	
$8 \cdot \left(\frac{1}{2}\right)^f$	

3.

Expression	Value
$42 - 9 \cdot f^2$	
$\frac{12 - g^2}{6}$	
$\frac{g + (2 + 1)^f}{4}$	
$\frac{1}{4}(g - 1)^3$	

Lesson Practice
6.11

Name: ______________________ Date: __________ Period: __________

FAST Practice

4. What is the value of the expression $\frac{3 + 5^2}{4}$?

(A.) 16 (B.) 4 (C.) 7 (D.) $\frac{13}{4}$

Spiral Review

5. Use the numbers 0 to 9 to fill in each blank so that the values of the expressions are in order from *least* to *greatest*. Use each number only once.

$4^{\square}$	$\square^{4}$	$\square^{2}$	$3^{\square}$	$\square^{3}$	$2^{\square}$

Least **Greatest**

Problems 6–9: Determine the value of each expression.

6. $3(5)$

7. $3(5) + 2$

8. $3(5 + 2)$

9. $3(2 + 5)$

10. Select *all* the expressions that are equal to 3^4.

- ☐ **A.** $3 \cdot 3 \cdot 3 \cdot 3$
- ☐ **B.** 12
- ☐ **C.** $3 + 3 + 3 + 3$
- ☐ **D.** $9 \cdot 9$
- ☐ **E.** 81

Career Connection


temizyurek/Getty Images

Communication towers like the one shown transmit and receive radio waves between various devices and their networks. They are needed for our use of cell phones, as well as television broadcasting, radio transmission, and communications to keep the public safe.

Are there any towers like this one in your community? If so, where have you seen one?

Engineers and technicians design, construct, and maintain towers like this one to hold antennas and other communication equipment. They use expressions and equations to determine measurements and calculate costs for construction and transportation. Data collection and graphing will allow a telecommunications specialist to monitor usage and to ensure that the tower is working properly.

B.E.S.T. Mathematics Benchmark Connection

Engineers and technicians use many math concepts in their work. For example, they use simple and multi-step equations to design the parts for the tower (MA.6.NSO.2.3) to make sure what is built is precise and usable. Construction teams will evaluate formulas that include algebraic expressions (MA.6.AR.1.3) as they order and assemble materials to complete the towers.

Mathematical Thinking and Reasoning Connection

Scientists and engineers use thinking and reasoning skills like the ones you use for your math work! For example, they represent instructions in different forms, such as blueprints or a list of steps (MTR.2.1). Instructions and blueprint documents must be precise in order for them to be useful, so design teams frequently discuss the testing of the durability of materials and parts as they are assembled (MTR.4.1) to make towers more accurate, analyze their usage, and address errors.

Meet Sir Jagadish Chandra Bose

In 1895, Indian physicist Sir Jagadish Chandra Bose conducted experiments on certain properties of short radio waves. This led to an early form of radio detector. Sir Bose also constructed recorders that could automatically register very slight movements, which contributed to instruments used in physics, biology, and botany. Thanks to his achievements with radio waves, we can use our wireless devices for business and personal purposes.


"1920 Jagadish Chandra Bose" by Georges Chevalier, via Wikimedia Commons. CC BY-SA 4.0

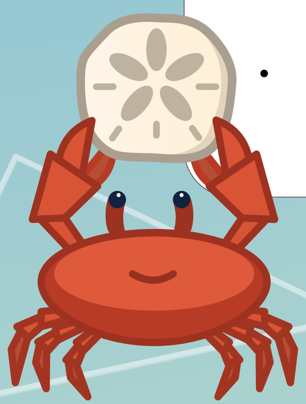
Unit 7

Positive and Negative Numbers

Think back to when you first learned about whole numbers and used them to count. Later, you saw there were numbers between them: fractions and decimals. Up until now, many numbers you've encountered have been greater than zero. There is an entire set of numbers (just as many, in fact), lurking on the other side of every number line.

Essential Questions

- What does it mean for a value to be less than zero?
- How are solving equations with positive numbers and with negative numbers alike and different?
- How do we represent all the numbers that are less than or greater than a value?
- How do we represent points with negative numbers on a coordinate plane?



Sub-unit

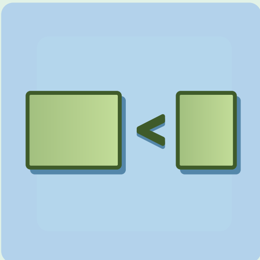
1

Signed Numbers and Absolute Values

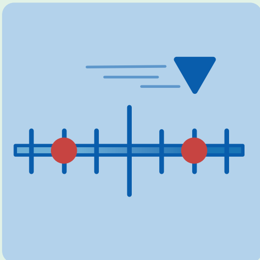
Lesson 1
Can You Dig It?

Lesson 2
Digging Deeper

Lesson 3
Sub-Zero

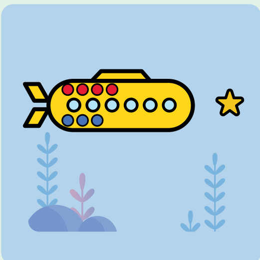


Lesson 4
Order in the Class

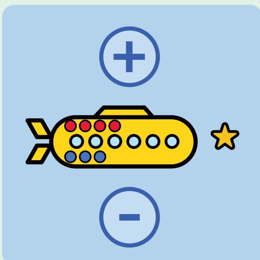


Lesson 5
Distance on the Number Line

Lesson 6
We've Got Game(s)



Lesson 7
Floats and Anchors



Lesson 8
More Floats and Anchors

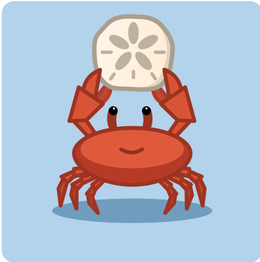
Lesson 9
Up, Up, and Away!

Name: .... Date: .... Period: ....

MA.6.NSO.1.1, MA.6.NSO.1.2, MA.6.NSO.4.1, MTR.1.1, MTR.5.1

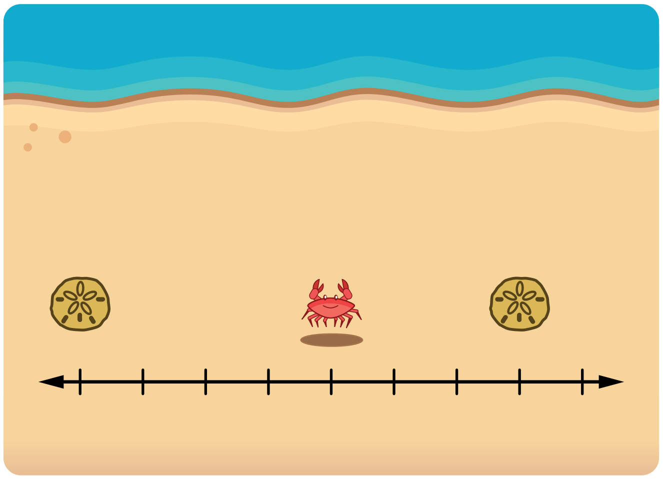
Can You Dig It?

Let's find the hidden sand dollars.



Warm-Up

1. Write a clue to help someone find the two sand dollars.



Activity 1

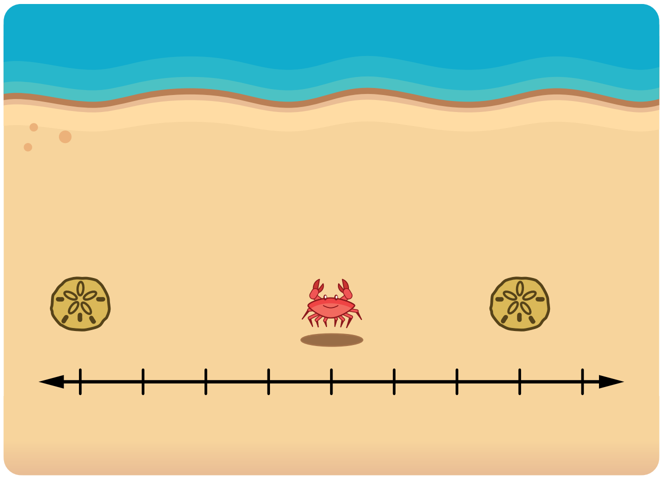
Name: ______ Date: ______ Period: ______

Finding Sand Dollars

2. Juliana and Kai each wrote correct clues.

Juliana: "One sand dollar is at positive 3 from the crab, and the other is at negative 4."

Kai: "Go 3 steps to the right of the crab to find the first sand dollar, then 7 steps to the left to find the other one."



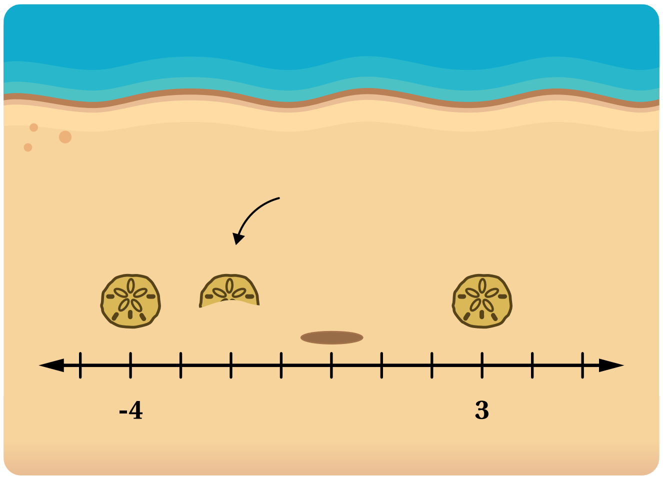
Discuss:

- How are their clues alike? How are they different?

- What do you think the "negative 4" in Juliana's clue means?

3. Here is a new sand dollar.

Write a clue that describes its location.

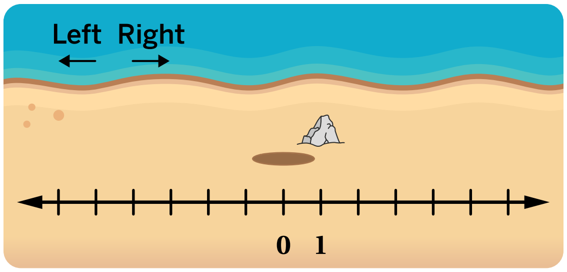


Name: .... Date: .... Period: ....

Sand Dollar Challenges

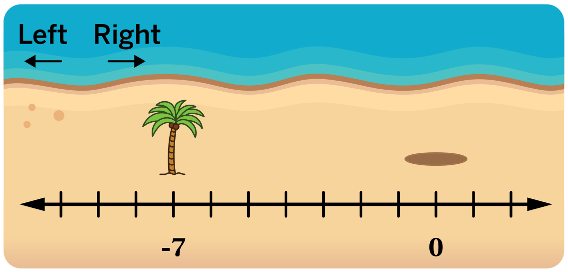
4. The rock is at 1 on the number line. A sand dollar is 4 units to the left of the rock.

Where is the sand dollar?



5. The palm tree is at -7 on the number line. A sand dollar is 3 units to the right of the palm tree.

Where is the sand dollar?



6. Juliana and Kai made mistakes on the previous problem.

Juliana said the sand dollar was at -5.

Kai said the sand dollar was at -10.

Circle your favorite mistake.

Juliana Kai

What could you tell this student to help them revise their answer?

Activity 3

Name: ________ Date: ________ Period: ________

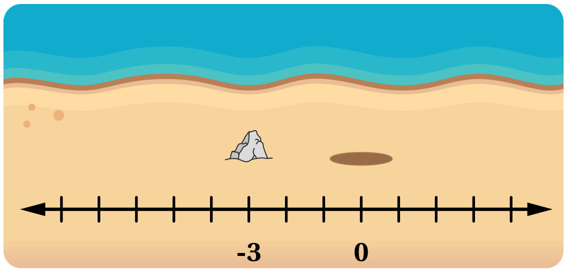
More Sand Dollar Challenges

7. The rock is at -3 on the number line. A sand dollar is 2 units away from the rock.

Lan says the sand dollar has to be at a **negative number**. Is she correct? Circle one.

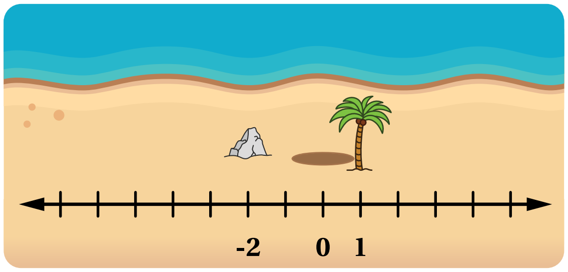
Yes No I'm not sure

Explain your thinking.



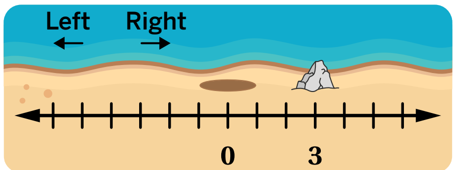
8. A sand dollar is 2 units away from the rock and 5 units away from the palm tree.

Where is the sand dollar?

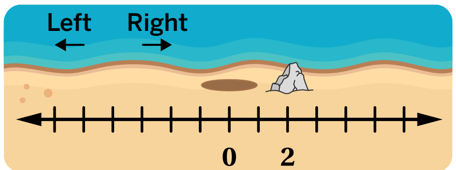


9. For each challenge, write the location of the sand dollar on the number line using the clue. Complete as many challenges as you have time for.

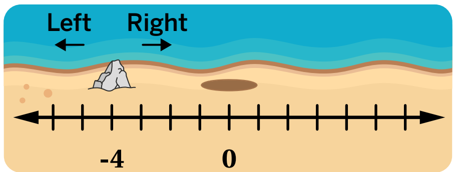
a 2 units to the right of the rock



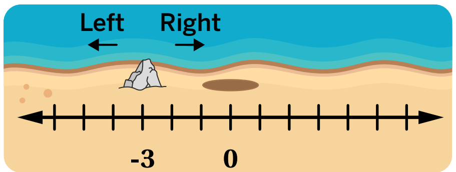
b 5 units to the left of the rock



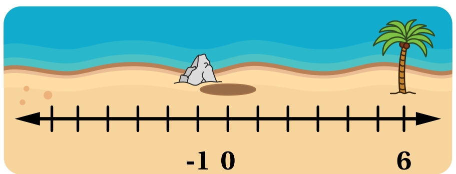
c 2 units to the left of the rock



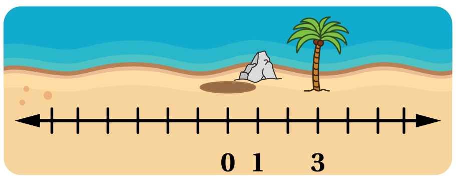
d 6 units to the right of the rock



e 3 units away from the rock and 4 units away from the palm tree



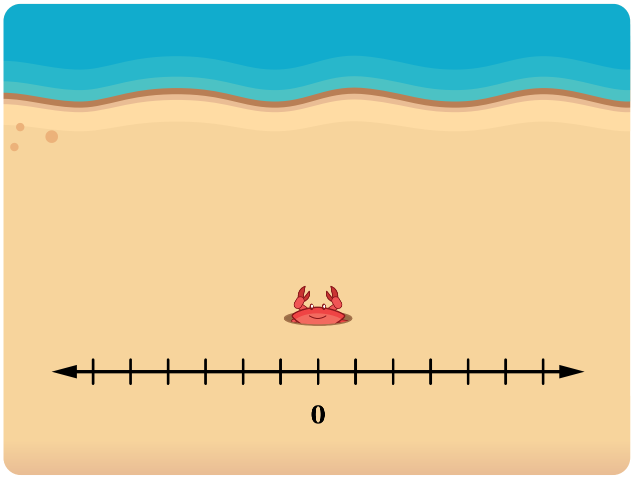
f 3 units away from the rock and 5 units away from the palm tree



Synthesis

10. List at least two things you know about positive and negative numbers on a number line.

Draw on the image if it helps to show your thinking.



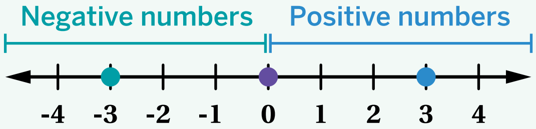
Lesson Practice 7.01

Lesson Summary

Positive numbers are numbers that are greater than 0. **Negative numbers** are numbers that are less than 0. Zero is neither positive nor negative.

You can extend a number line to the right of 0 to show positive numbers, and you can extend a number line to the left of 0 to show negative numbers.

For example:



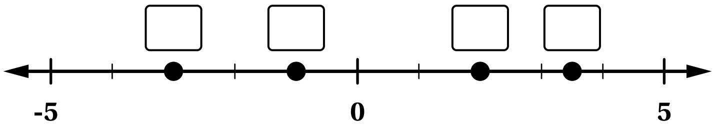
The number 3 is 3 units to the right of 0 on the number line.

The number -3 is 3 units to the left of 0 on the number line.

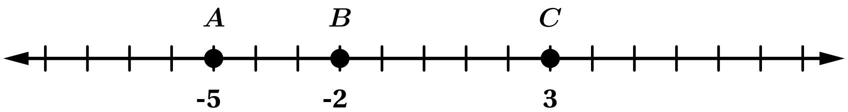
Lesson Practice
7.01

Name: .... Date: .... Period: ....

1. Fill in the blanks on the number line.



Problems 2–6: Here is a number line.



2. Describe where you would plot -100 on the number line.

3. Point D is 1 unit to the left of point A. Plot point D.

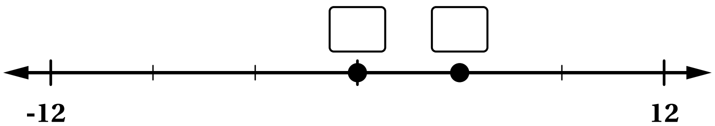
4. Point E is at 0. Plot point E.

5. List both locations that are 4 units away from point B.

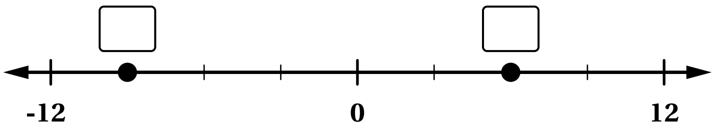
6. Point F is the same distance from point A and point C. Plot point F.

Problems 7–9: Fill in the blanks on the number lines.

7.



8.



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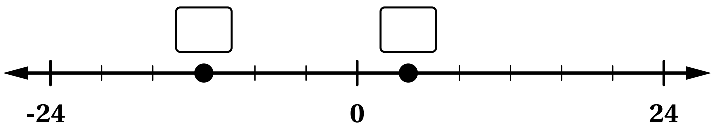


FAST Practice

10. Which pair of numbers are on opposite sides of zero on the number line?

A. 0 and 5 **B.** 2 and -5 **C.** -2 and -5 **D.** 2 and 5

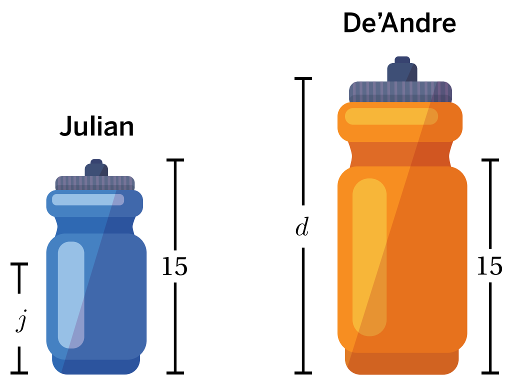
Name: ______ Date: ______ Period: ______

Spiral Review

11. A rectangle has an area of 24 square centimeters. If one side is $2\frac{2}{5}$ centimeters long, how long is the other side?

Explain your thinking.

Problems 12–14: Two friends at soccer practice are drinking from their water bottles. Julian drinks $\frac{3}{5}$ of his 15-ounce bottle. De'Andre drinks 15 ounces of water, which is $\frac{3}{5}$ of his bottle.



12. How are these two situations alike? How are they different?

13. Write an expression to represent the amount of water Julian drinks.

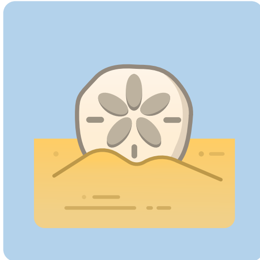
14. Write an expression to represent the total amount of water in De'Andre's bottle.

Name: ______________________ Date: ____________ Period: ____________

MA.6.NSO.1.1, MA.6.NSO.1.2, MTR.1.1, MTR.4.1

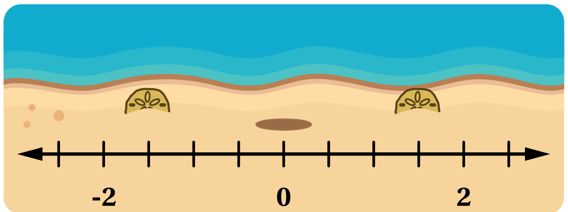
Digging Deeper

Let's plot positive and negative numbers on the number line.

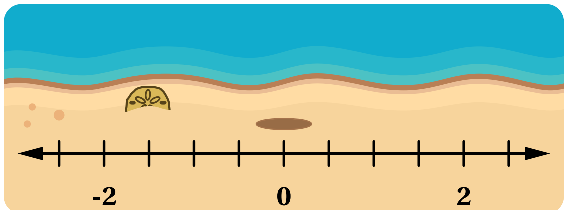


Warm-Up

1. Here are two buried sand dollars. Write clues to describe their locations.



2. Select *all* the numbers that describe the location of the sand dollar.



- ☐ **A.** $-\frac{3}{2}$
- ☐ **B.** -1.5
- ☐ **C.** $-1\frac{1}{2}$
- ☐ **D.** -2.5
- ☐ **E.** -3

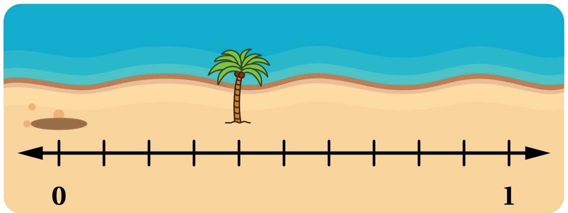
Activity 1

Name: .... Date: .... Period: ....

Rational Numbers and Their Opposites

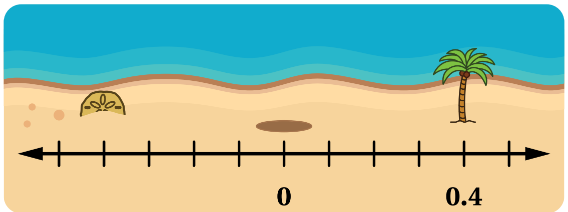
3. A sand dollar is buried under the tree.

Where is the sand dollar on the number line?



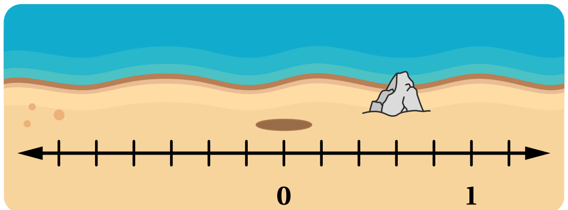
4. A new sand dollar is buried at the **opposite** of the palm tree.

What do you think opposite means here?



5. A new sand dollar is buried at the opposite of the rock.

Where is the new sand dollar?

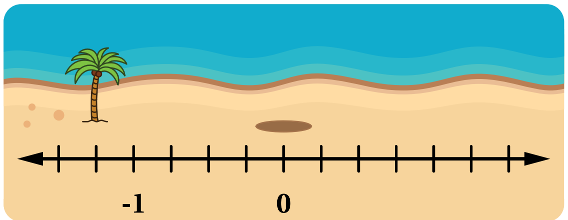


Name: .... Date: .... Period: ....

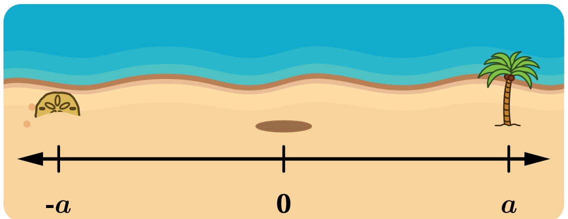
Rational Numbers and Their Opposites (continued)

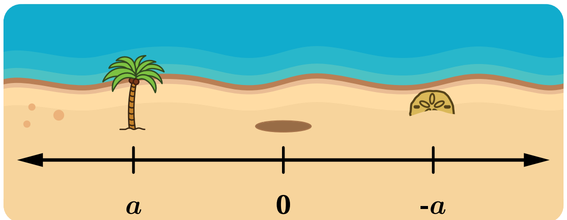
6. A new sand dollar is buried at the opposite of the palm tree.

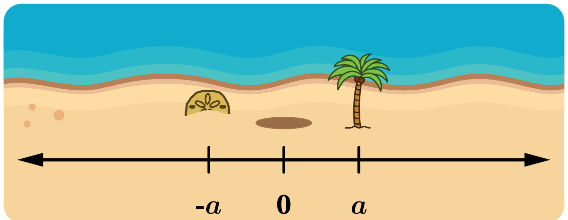
Where is the new sand dollar?

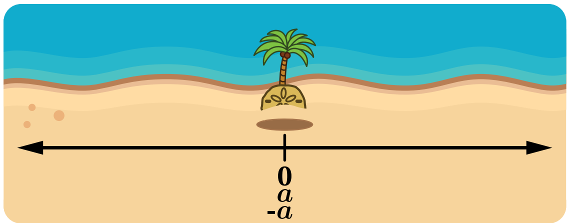


7. The palm tree is at a. The sand dollar is buried at the opposite of a, which you can write as -a. When a is in different locations, what do you notice about -a?









8. Pair opposite numbers together.

-(-5)	$\frac{3}{4}$	$\frac{4}{3}$	-0.75
$-1\frac{1}{3}$	-5	$\frac{1}{5}$	$-\frac{1}{5}$

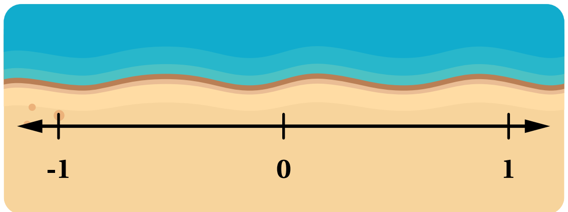
Pair 1	Pair 2	Pair 3	Pair 4

Activity 2

Name: .... Date: .... Period: ....

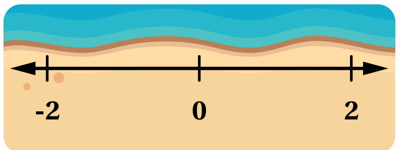
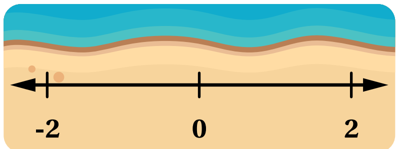
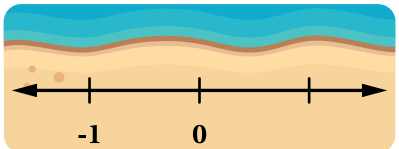
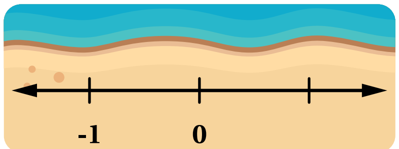
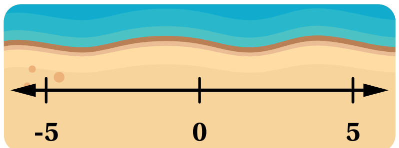
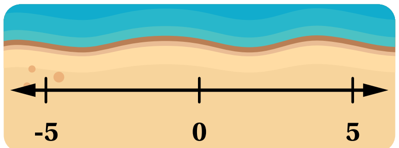
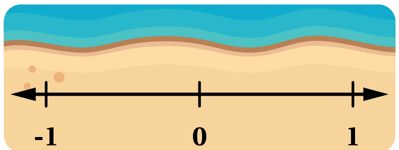
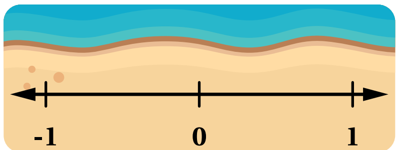
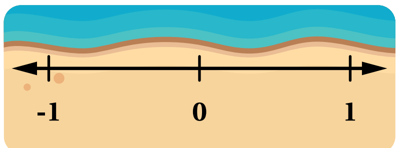
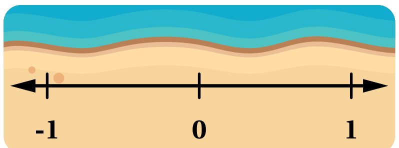
Rational Number Challenges

9. The crab is going to dig at $-\left(-\frac{3}{4}\right)$. Plot the sand dollar where the crab will find it.



10. Plot a point where the crab will dig to find the sand dollar.

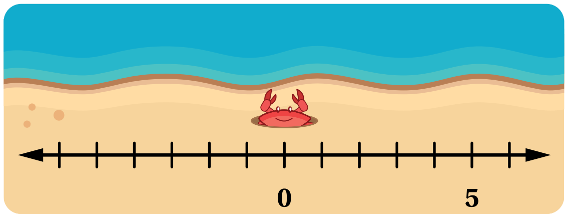
- Decide with your partner who will complete Column A and who will complete Column B.
- After plotting the location of each sand dollar, compare your solutions. The solutions in each row should be the same. Discuss and resolve any differences.

	Column A	Column B
a	The crab is going to dig at $\frac{3}{2}$. 	The crab is going to dig at 1.5. 
b	The crab is going to dig at -1.1. 	The crab is going to dig at the opposite of 1.1. 
c	The crab is going to dig at the opposite of -2. 	The crab is going to dig at -(-2). 
d	The crab is going to dig at -(-0.1). 	The crab is going to dig at 0.1. 
e	The crab is going to dig at the opposite of $-\frac{4}{5}$. 	The crab is going to dig at the opposite of -0.8. 

Synthesis

11. Describe where -3.2 is on a number line.

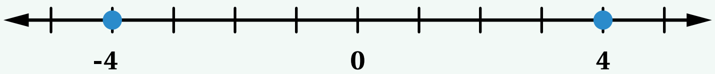
Draw on the image if it helps to show your thinking.



Lesson Practice 7.02

Lesson Summary

Two numbers are **opposites** if they are the same distance from 0 on different sides of the number line. For example, -4 and 4 are opposites because they are both 4 units away from 0.



Every number has an opposite, including fractions and decimals. 0 is its own opposite. The opposite of the opposite of a number is the number itself. For example, $-(-2) = 2$

All positive and negative whole numbers and 0 are a group of numbers called **integers**. All positive and negative numbers that can be written as fractions, including whole numbers, are called *rational numbers*.

2 and -2 are both integers and rational numbers.

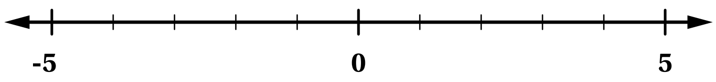
8.3, -8.3, $\frac{3}{2}$ and $-\frac{3}{2}$ are rational numbers, but *are not* integers.

Lesson Practice
7.02

Name: ________ Date: ________ Period: ________

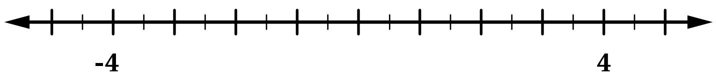
1. Plot each number in its approximate location on the number line.

-3	$\frac{3}{2}$	$-\frac{4}{3}$	$\frac{9}{4}$



2. Plot each number in its approximate location on the number line.

-2	$\frac{5}{4}$	-3.4	0



Problems 3–6: Complete each statement below.

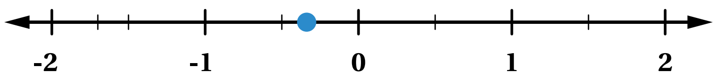
3. The opposite of -2 is ________ .

4. The opposite of $\frac{5}{4}$ is ________ .

5. The opposite of -3.4 is ________ .

6. The opposite of 0 is ________ .

Problems 7–8: Rebecca incorrectly plotted the point $-1\frac{7}{10}$ on the number line.



7. What question could you ask to help Rebecca understand her mistake?

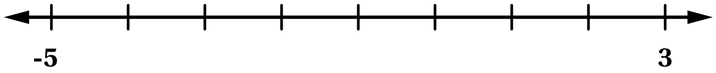
8. Plot $-1\frac{7}{10}$ in the correct location on the number line.

Lesson Practice
7.02

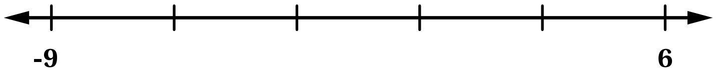
Name: ______ Date: ______ Period: ______

Problems 9–11: Plot and label where zero is located on each number line.

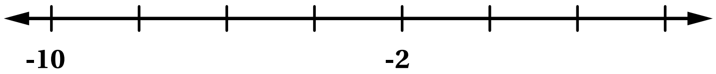
9.



10.

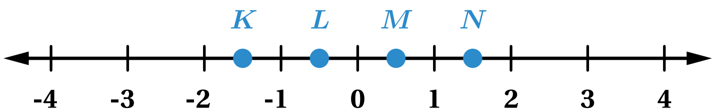


11.



FAST Practice

12. Points K, L, M, and N are plotted on the number line.



What point represents the location of -0.5?

A. Point K

B. Point L

C. Point M

D. Point N

Spiral Review

Problems 13–16: Solve each equation.

13. $3x = 6$

14. $\frac{1}{3}x = 6$

15. $\frac{1}{3} = 6x$

16. $\frac{1}{3} = \frac{1}{6}x$

Name: ______ Date: ______ Period: ______

MA.6.NSO.1.1, MA.6.NSO.1.2, MTR.2.1, MTR.3.1, MTR.6.1

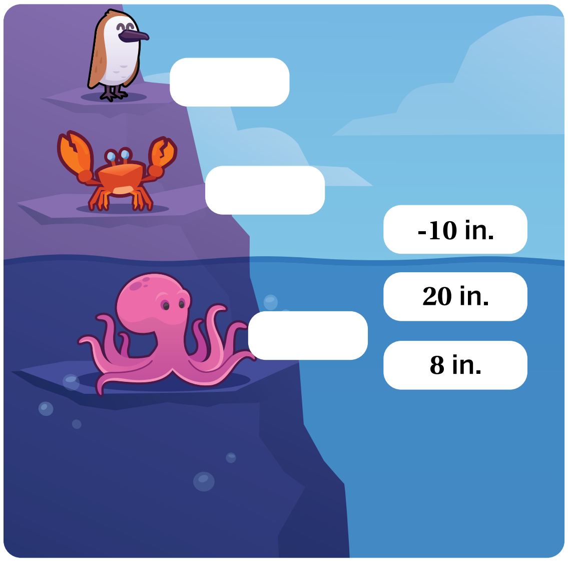
Sub-Zero

Let's use positive and negative numbers in context.

Warm-Up

1. Here are three animals at different elevations.

Match each elevation with the correct animal.



2. What do you think 0 inches represents in this situation?

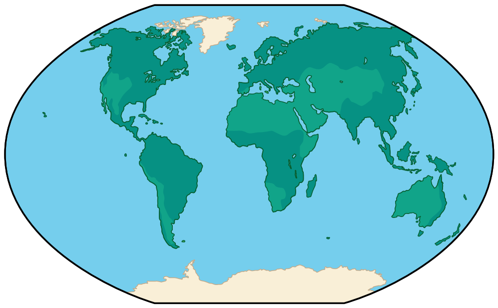
Activity 1

Name: .... Date: .... Period: ....

World Temperatures

3. Let's look at different average low temperatures around the world in January.

Choose a location you'd like to visit and explain your choice to a classmate.

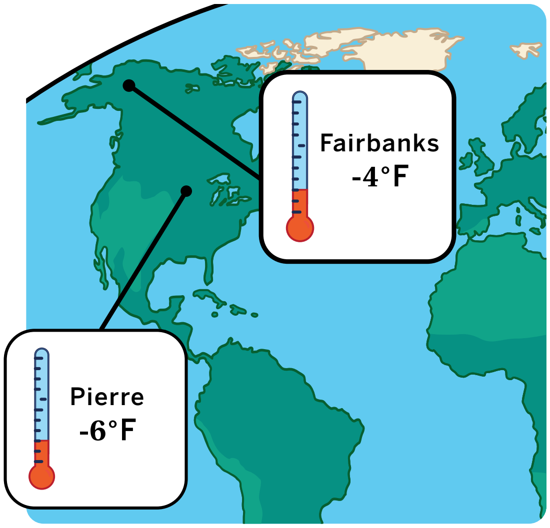


4. Neel says Fairbanks is warmer because $-4 > -6$. Mia says Pierre is warmer because 6 is greater than 4.

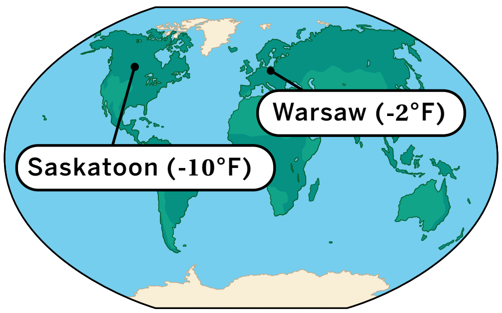
Whose thinking is correct? Circle one.

Neel's Mia's Both Neither

Explain your thinking.



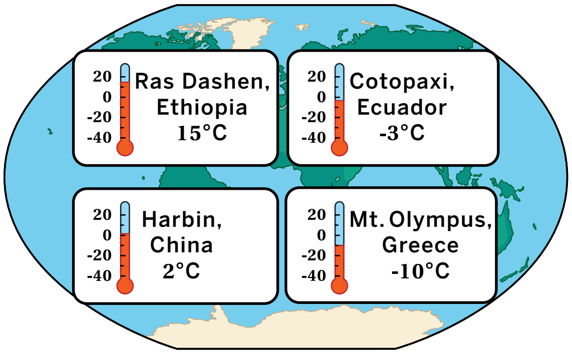
5. Write a temperature that is colder than Warsaw's and warmer than Saskatoon's.



Name: ______________________ Date: __________ Period: __________

Colder Temperatures

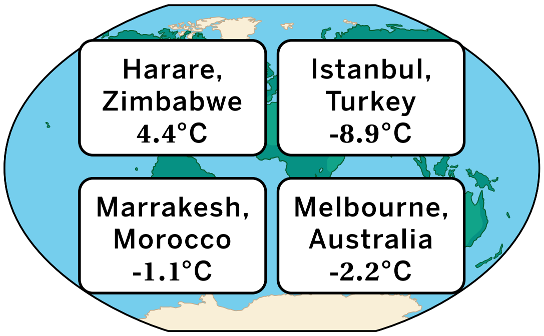
6. Here are the temperatures of different places around the world on a given day.



Select *all* of the true statements.

- ☐ **A.** Ras Dashen is warmer than Cotopaxi.
- ☐ **B.** Cotopaxi is -3 degrees below 0.
- ☐ **C.** Mt. Olympus is the coldest.
- ☐ **D.** Mt. Olympus is 12 degrees colder than Harbin.
- ☐ **E.** Cotopaxi is colder than Mt. Olympus.

7. Here are some of the coldest recorded temperatures for four cities.



Order these temperatures from *warmest* to *coldest*.

Warmest 🔥	
Coldest ❄️	

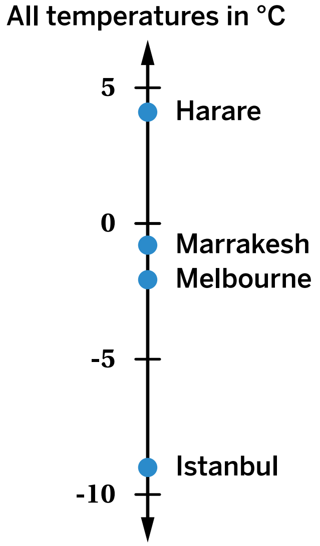
Name: ____________ Date: ____________ Period: ____________

Colder Temperatures (continued)

8. This number line shows some of the lowest recorded temperatures for the same four cities.

One of the lowest recorded temperatures in Rome, Italy, is -7.2°C.

Plot the point for Rome at its approximate location on the number line.



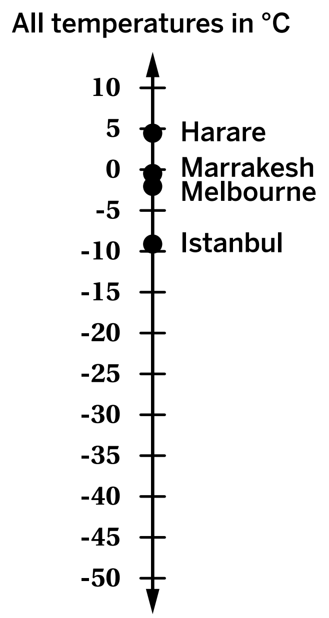
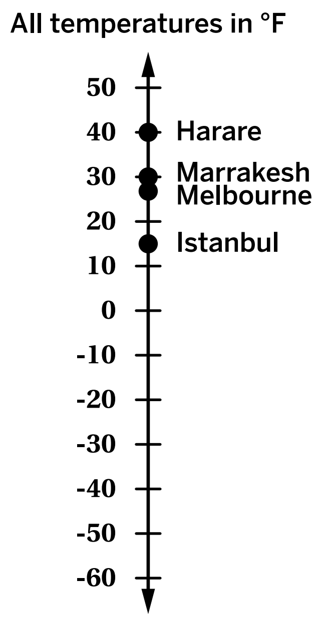
9. One way to write that Harare's temperature is warmer than Marrakesh's is $4.4 > -1.1$.

Write your own *inequality* comparing Rome's temperature to another city's temperature.

You're invited to explore more.

10. Look up the lowest recorded temperature in a city near you.

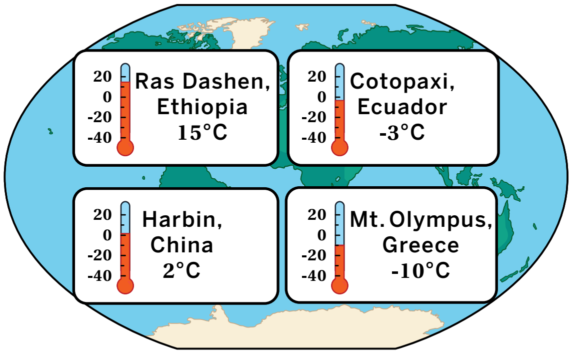
Plot and label a point to represent this temperature on one of the number lines.

Celsius	Fahrenheit
	

Synthesis

11. When comparing two temperatures, how can you tell which is warmer and which is colder?

Use the examples if they help with your thinking.



Lesson Practice 7.03

Lesson Summary

You can use a vertical number line to represent positive and negative numbers. On a vertical number line, points above 0 are positive and points below 0 are negative.

When talking about elevation, 0 feet represents sea level. This means that a positive elevation is above sea level and a negative elevation is below sea level.

When talking about temperature, 0°C means the temperature is freezing. If the temperature in Mt. Olympus is -10°C, that means it has a temperature of 10°C below 0°C, or below freezing.

If the temperature in Cotopaxi is -3°C, you can write the *inequality* $-10 < -3$, which means that it is colder in Mt. Olympus than it is in Cotopaxi.

Lesson Practice
7.03

Name: .... Date: .... Period: ....

Problems 1–3: Determine if each statement is true or false.

Statement	True	False
1. An elevation of 35 feet is the same as an elevation of -35 feet.	○	○
2. A city that has an elevation of -17 meters is closer to sea level than a city that has an elevation of -40 meters.	○	○
3. A temperature of -4°F is the same as a temperature 4°F below zero.	○	○

Problems 4–7: Here is a table that shows some elevations in several cities around the world.

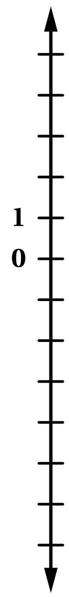
Location	Elevation (ft)
San Juan, Puerto Rico	-4
New Orleans, Louisiana	-7
Amsterdam, Netherlands	-2
Jakarta, Indonesia	3
Taipei City, Taiwan	5
Tunis, Tunisia	0

4. A city, not listed in the table, has a higher elevation than San Juan, Puerto Rico.

Select *all* the numbers that could represent the city's elevation in feet.

- ☐ **A.** 0
- ☐ **B.** 4
- ☐ **C.** -2
- ☐ **D.** -10
- ☐ **E.** -7

5. Plot all of the elevations on the number line.



6. Mio says: *I know that 2 is less than 4, so -2 must be less than -4. This means Amsterdam has a lower elevation than San Juan*. Is Mio correct? Explain your thinking.

7. Nasir says: 3 *is less than -7 because 3 is closer to 0 on the number line*. Is Nasir correct? Explain your thinking.

Name: ______ Date: ______ Period: ______

FAST Practice

Problems 8–10: Here is a table that shows some of the lowest temperatures recorded in five U.S. locations.

Location	Temperature (°F)
Boca, CA	-45
Coventry, CT	-32
Tallahassee, FL	-2
Mt. Carroll, IL	-38
CCC Fire Camp F-16, GA	-17

8. Which of these locations had the lowest record temperature?

A. Boca, CA

B. Tallahassee, FL

C. Coventry, CT

D. CCC Fire Camp F-16, GA

9. Which location had a lower record temperature: Tallahassee, FL, or CCC Fire Camp F-16, GA? Write a statement using $<$ or $>$ to compare the recorded temperatures.

10. How does the temperature in Coventry, CT compare to the temperature in Mt. Carroll, IL? Complete the statement using $<$ or $>$ to compare the recorded temperatures.

Temperature in Coventry, CT ☐ Temperature in Mt. Carroll, IL

Spiral Review

Problems 11–14: Determine the value of each expression.

11. $2^3 \cdot 4$

12. $\frac{2^3}{4}$

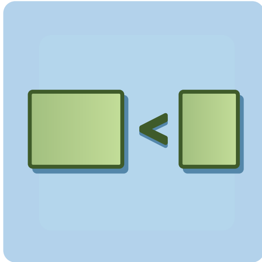
13. $2^4 - 4$

14. $2^3 + 4^3$

Name: .... Date: .... Period: ....

MA.6.NSO.1.1, MA.6.NSO.1.2, MA.6.NSO.3.5, MTR.1.1, MTR.4.1

Order in the Class



Let's compare positive and negative numbers.

Warm-Up

1. Which one doesn't belong?

A. $3 > \frac{5}{4}$

B. $-\frac{5}{4} < 3$

C. $\frac{5}{4} > -\frac{5}{4}$

D. $-\frac{5}{4} > -3$

Explain your thinking.

Activity 1

Name: ______ Date: ______ Period: ______

Greater Than?

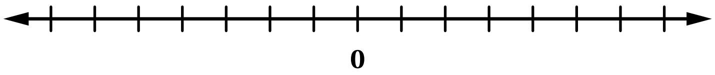
You will use your number card to compare numbers with your classmates.

My number: ______

2. For each round, compare your number with different classmates'. Use the number lines if they help with your thinking.

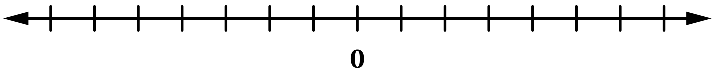
Round 1: Find a person whose number has a *different* **sign** than yours.

Partner's Name	Partner's Number	Comparison in Words	Comparison in Symbols
		______ is greater than ______.	______ > ______



Round 2: Find a person whose number is the opposite of yours.

Partner's Name	Partner's Number	Comparison in Words	Comparison in Symbols
		______ is greater than ______.	______ > ______



Activity 2

Name: ________________ Date: ________ Period: ________

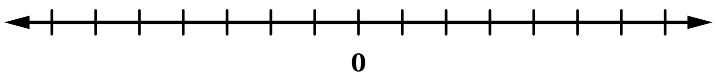
Least to Greatest

3. For each round, form a group of three or four people.

Round 3: Find three or four people whose number is the same form as yours (integer, fraction, or decimal). Record the values from each of your cards: ______, ______, ______, and ______. Write three different inequalities comparing these numbers.

Inequality 1	Inequality 2	Inequality 3
______ < ______	______ < ______	______ < ______

Plot and label each number at its approximate location on the number line.



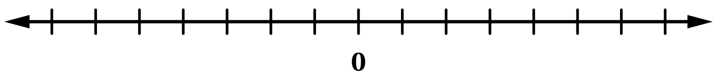
Order the values from *least* to *greatest*.

Least **Greatest**

Round 4: Find three or four people whose number is the same sign as yours *and* is the same form as yours (integer, fraction, or decimal). Record the values from each of your cards: ______, ______, ______, and ______. Write three different inequalities comparing these numbers.

Inequality 1	Inequality 2	Inequality 3
______ < ______	______ < ______	______ < ______

Plot and label each number at its approximate location on the number line.



Order the values from *least* to *greatest*.

Least **Greatest**

Activity 3

Name: ______ Date: ______ Period: ______

Ordering Numbers

4. Here are four more numbers.

-2	1.5	-1.5	-1

a Complete each number sentence using > or <.

-2 ______ 1.5 -1.5 ______ -1

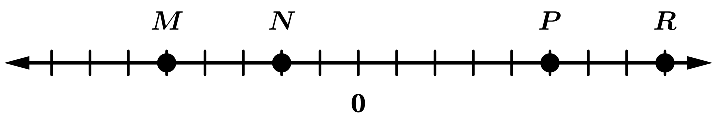
b Use two of the numbers to complete this sentence.

______ is the opposite of ______ because . . .

c Order the numbers from *least* to *greatest*.

Least **Greatest**

5. Use the number line to complete each sentence.



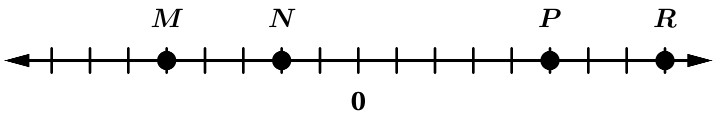
a ______ is the opposite of ______ because . . .

b ______ is greater than ______ because . . .

c ______ is the least of the numbers because . . .

Synthesis

6. Describe how to compare numbers on a number line. Use the example if it helps with your thinking.



Lesson Practice 7.04

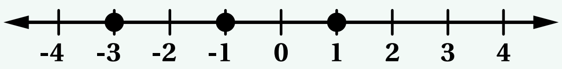
Lesson Summary

You can use a number line to compare numbers with different **signs** (like -4 and 3) or numbers with the same sign (like -4 and -3).

The order of numbers from least to greatest is the same order as they appear on the number line from left to right. This means that negative numbers farther from 0 are less than negative numbers that are closer to 0.

For example, let's say you want to compare -3 and -1. On a number line, -1 is to the right of -3. This means that -1 is greater than -3, or $-3 < -1$. This also makes sense because -1 is closer to 0 than -3 is.

A number line can also help you order numbers from least to greatest. 1 is greater than -1 and -3 because it is the farthest to the right on the number line.

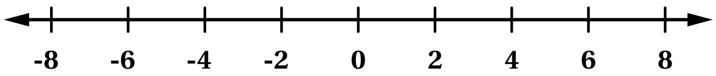


Lesson Practice
7.04

Name: .... Date: .... Period: ....

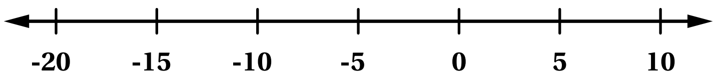
1. Complete each number sentence with a number that makes it true. Use the number line if it helps with your thinking.

a ☐ < 5 **b** ☐ < -5 **c** -5 < ☐ **d** -5 > ☐



2. Complete each number sentence with the symbol <, >, or =. Use the number line if it helps with your thinking.

a -5 .... 2 **b** 5 .... -5 **c** -12 .... -15 **d** -12.5 .... -12



3. Circle whether each statement is *true* or *false*.

Statement A: -8.4 is to the right of -8.7 on the number line. True False

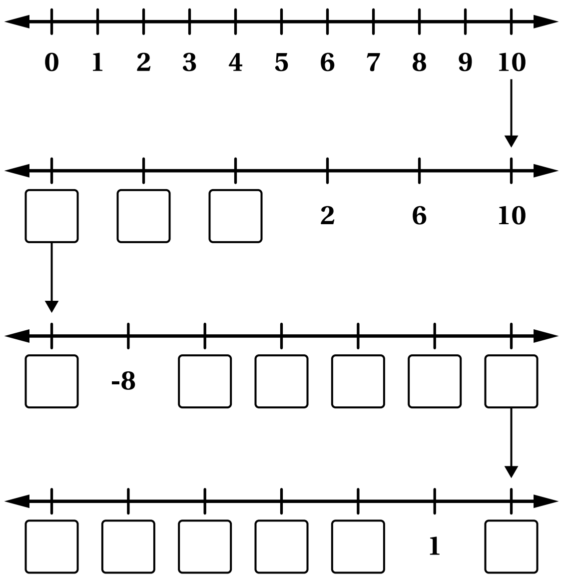
Statement B: -2.4 is greater than -2.3. True False

Statement C: $-\frac{11}{12} < -\frac{7}{12}$ True False

Choose one statement and explain your thinking.

4. Here is a number line maze.

Each number line has a different scale but at least one matching number, which is labeled with an arrow. Determine the missing values in this number line maze.



Name: ________ Date: ________ Period: ________

FAST Practice

5. Here are five numbers: $-\frac{2}{5}$, -1, $\frac{4}{3}$, 1, $-\frac{3}{2}$.

These numbers are plotted on a horizontal number line. Which statement about the locations of the numbers is true?

A. $-\frac{2}{5}$ is the farthest to the left, and 1 is farthest to the right.

B. -1 is the farthest to the left, and 1 is farthest to the right.

C. $\frac{4}{3}$ is the farthest to the left, and $-\frac{3}{2}$ is farthest to the right.

D. $-\frac{3}{2}$ is the farthest to the left, and $\frac{4}{3}$ is farthest to the right.

Spiral Review

6. Solve each equation. Write each solution as a fraction and as a decimal.

Equation	Solution (Fraction)	Solution (Decimal)
$2x = 3$		
$5y = 3$		
$0.3z = 0.09$		

Problems 7–10: Each lap around a track is 400 meters. How many meters will someone run in:

7. 2 laps?

8. 5 laps?

9. x laps?

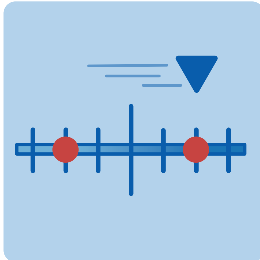
10. If Sol ran $7{,}600$ meters, how many laps did he run? Explain your thinking.

Name: ________ Date: ________ Period: ________

MA.6.NSO.1.1, MA.6.NSO.1.4, MTR.2.1, MTR.3.1

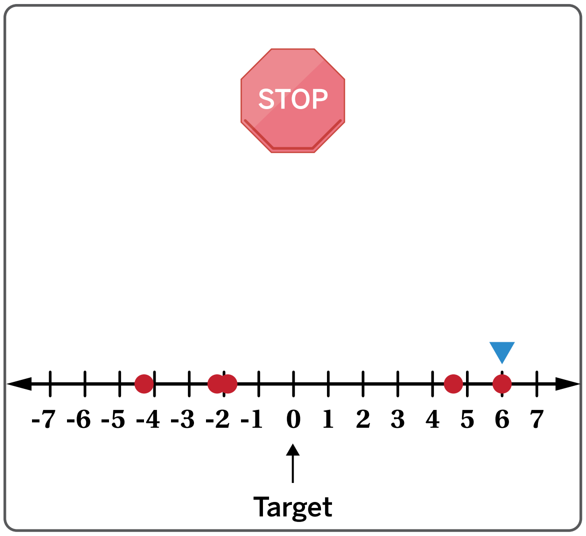
Distance on the Number Line

Let's explore the absolute value of rational numbers.



Warm-Up

1. Let's play a game to get a score.



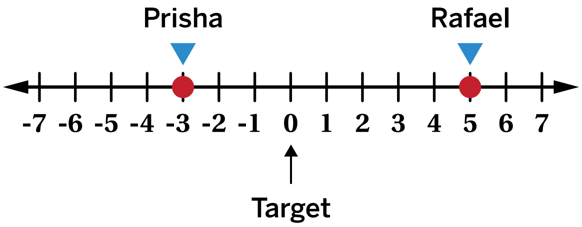
Number	Score
-2.2	2.2
-1.9	1.9
-4.3	4.3
4.6	4.6
6	6

Discuss: How do you think scores are determined?

What Is Absolute Value?

2. Prisha landed at -3. Rafael landed at 5. What was each student's score?

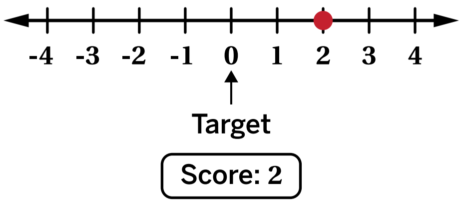
Student	Value
Prisha	
Rafael	



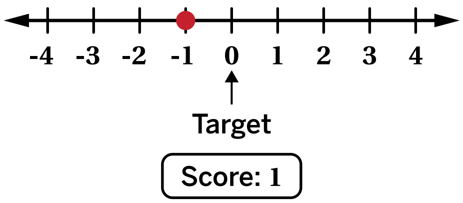
3. The game uses **absolute value** to calculate the score. We read |-2| as "the absolute value of negative 2."

a Take a look at some scores for different stopping points.

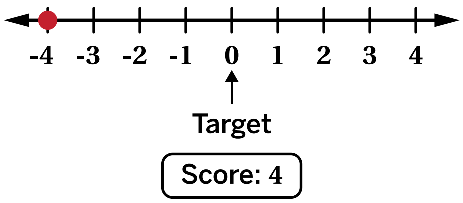
$|2| = 2$



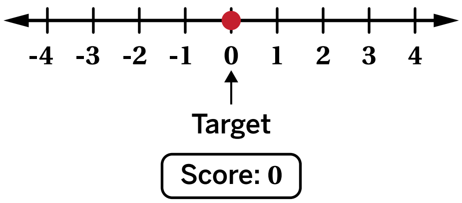
$|1| = 1$



$|-4| = 4$



$|0| = 0$



b Describe what you think absolute value means.

4. Prisha says $|x|$ means "the opposite of x." Rafael says $|x|$ means "how far x is from zero." Whose thinking is correct? Circle one.

Prisha's   Rafael's   Both   Neither

Explain your thinking.

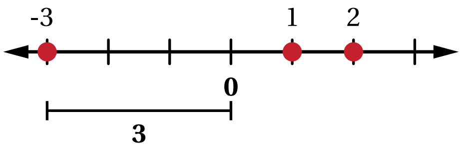
Activity 2

Name: .... Date: .... Period: ....

Comparing Absolute Values

5. Determine the value of each expression. Use the number line if it helps with your thinking.

Expression	Value
$\lvert -3 \rvert$	3
$\lvert 1 \rvert$	
$\lvert 2 \rvert$	



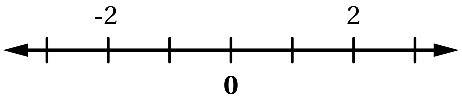
6. Determine whether each statement is true or false.

Statement	True	False
$\lvert 0 \rvert = 0$	◯	◯
$\lvert 6 \rvert = -6$	◯	◯
$\lvert 6 \rvert > \lvert -6 \rvert$	◯	◯
$-5 < -4$	◯	◯
$\lvert -5 \rvert < \lvert -4 \rvert$	◯	◯
$11 > -10$	◯	◯
$\lvert 11 \rvert > \lvert -10 \rvert$	◯	◯

7. Isaiah says -2 and 2 have the same absolute value.

Explain why Isaiah's claim is correct.

$\lvert -2 \rvert = \lvert 2 \rvert = 2$



Name: .... Date: .... Period: ....

Absolute Value Puzzles

8. **a** Make a true inequality by using each number at most once.

$$|\square| > \square$$

-2 | -1 | 1 | 2

b Explain how you know your inequality is true.

9. Make true statements by using each number exactly once.

$$|\square| > \square$$

$$\square = |\square|$$

3 | 4 | -4 | -5

10. Make true statements by using each number exactly once.

$$\square > |\square|$$

$$|\square| < \square$$

$$|\square| = \square$$

-1 | -2 | -4

2 | 3 | 4

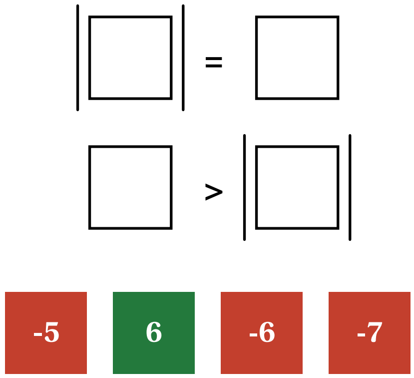
Name: .... Date: .... Period: ....

Absolute Value Puzzles (continued)

11. Is it possible to make true statements using each number exactly once? Circle one.

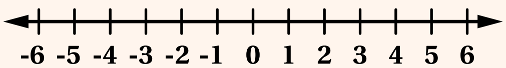
Yes No I'm not sure

Explain your thinking.



You're invited to explore more.

12. Use these clues to determine the values of A, B, and C. Use the number line to record your answers and to help you with your thinking.

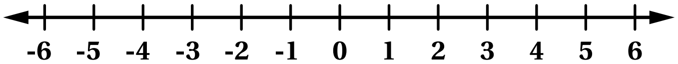


Clues

- The absolute value of A is 2.
- B is greater than A.
- Point B is closer to zero than point A is.
- C is positive.
- The distance between A and B is 1.
- The distance between B and C is 4.

Synthesis

13. Explain 2–3 things you know about absolute value. Use the number line if it helps to show your thinking.



Lesson Practice 7.05

Lesson Summary

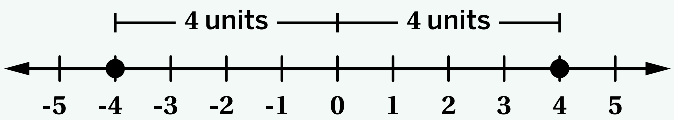
The **absolute value** of a number is a way to describe its distance from 0. For example:

The absolute value of -4 is 4 because it is 4 units away from 0.

$|-4| = 4$

The absolute value of 4 is also 4 because it is 4 units away from 0.

$|4| = 4$



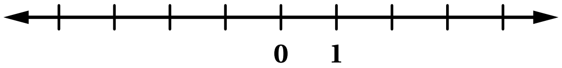
The distance from 0 to itself is 0, so $|0| = 0$.

Absolute values are helpful when you are interested in the size of a difference or measurement but its direction is not important.

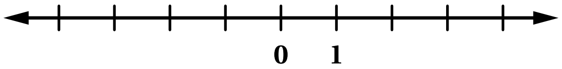
Lesson Practice
7.05

Name: ______ Date: ______ Period: ______

1. Plot and label all numbers that are 4 units away from 0.



2. Plot and label all numbers with an absolute value of 3.



3. Ivory claims that a number and its opposite will always have the same absolute value. Is Ivory correct? Explain your thinking.

Problems 4–11: Complete each number sentence with the symbol <, >, or =.

4. -2 ______ 1

5. |-2| ______ |1|

6. 2 ______ -1

7. |2| ______ |-1|

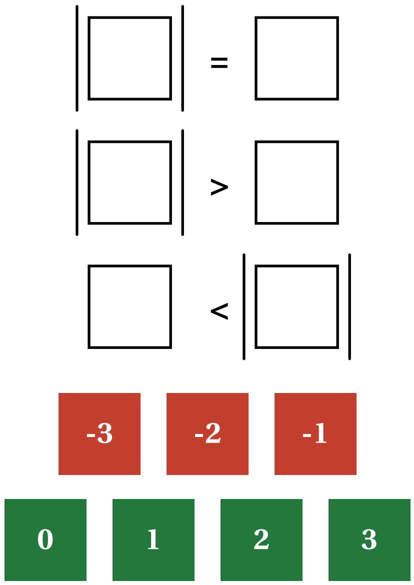
8. 3 ______ -3

9. |3| ______ |-3|

10. |-2| ______ |-4|

11. |-2| ______ -4

12. Make each statement true by using each number at most once.



Name: .... Date: .... Period: ....

FAST Practice

13. Which list of absolute value expressions are ordered from *least* to *greatest* value?

A. $|0|, |1|, |-5|, |-7|$

B. $|-7|, |-5|, |1|, |0|$

C. $|-7|, |-5|, |0|, |1|$

D. $|1|, |0|, |-5|, |-7|$

Spiral Review

Problems 14–17: Determine the value of each quotient.

14. $24 \div 15$

15. $0.24 \div 0.15$

16. $0.24 \div 0.015$

17. $0.024 \div 0.015$

Name: ____________ Date: ________ Period: ________

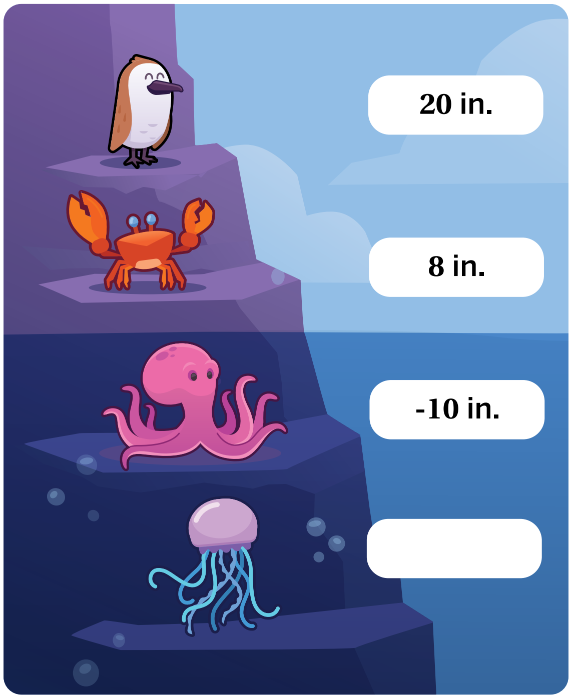
MA.6.NSO.1.1, MA.6.NSO.1.2, MA.6.NSO.1.3, MA.6.NSO.1.4, MTR.2.1, MTR.3.1

We've Got Game(s)

Let's compare rational numbers and absolute values to make sense of contexts.

Warm-Up

1. The jellyfish is farther from sea level than the bird is. What could its elevation be?



Explain your thinking.

Activity 1

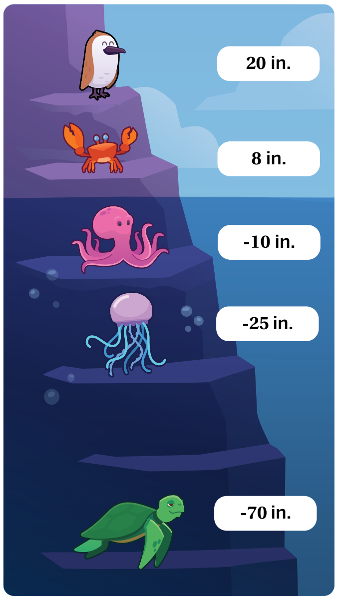
Name: .... Date: .... Period: ....

Comparing Distances From Sea Level

2. Which animal has the lowest elevation? The highest elevation?

Lowest elevation: ....

Highest elevation: ....



3. Which animal is farthest from sea level? Closest to sea level?

Farthest from sea level: ....

Closest to sea level: ....

4. Order the elevations of the animals from *closest* to sea level to *farthest* from sea level.

Closest to sea level **Farthest from sea level**

5. Here are two ways to compare the locations of the crab and the octopus. Match each inequality to the statement it represents.

a $-10 < 8$ .... The crab is closer to sea level than the octopus is.

b $|8| < |-10|$ .... The crab has a higher elevation than the octopus.

Activity 2

Name: .... Date: .... Period: ....

We've Got Game(s)

We've Got Game(s) sells board games.

6. The table shows the amount in the company's bank account(called an account balance) from January to June.

Discuss: What do you notice? What do you wonder?

Month	Account Balance ($)
January	-300
February	-200
March	-250
April	-100
May	0
June	50

7. Here is the account balance in July. Two students described it in different ways.

Month	Account Balance ($)
July	-100

Alma says: *The company had an account balance of* -$100.

Miko says: *The company had a debt of* $100.

Whose description is correct? Circle one.

Alma Miko Both Neither

Explain your thinking.

Name: Date: Period:

We've Got Game(s) (continued)

8. You will use a set of clue cards to estimate each account balance from August through December.

Month	Account Balance ($)
July	-100
August	
September	
October	
November	
December	

9. Alma said the account balance in December was -$200. Miko said it was $200 dollars. Explain why they are both correct.

You're invited to explore more.

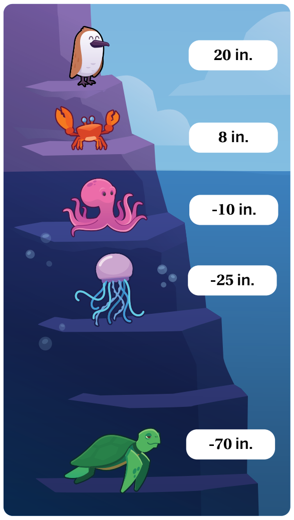
10. In the next year, the We've Got Game(s) account balance at the end of January was -$200. At the end of February it was $300. How much money did We've Got Game(s) make in February?

Explain your thinking.

Synthesis

11. When is it helpful to use absolute value to compare numbers in context?

Use this example if it helps with your thinking.



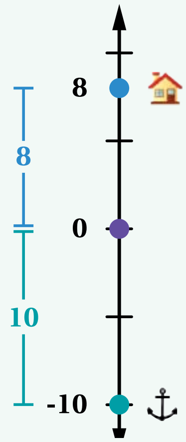
Lesson Practice 7.06

Lesson Summary

You can use absolute value to compare quantity changes or distances.

For example, suppose an anchor has an elevation of -10 meters and a house has an elevation of 8 meters.

- To compare their elevations and describe the anchor as having a lower elevation than the house, you can write $-10 < 8$.
- To compare their distances from sea level and describe the anchor as being farther away from sea level than the house, you can write $|-10| > |8|$.

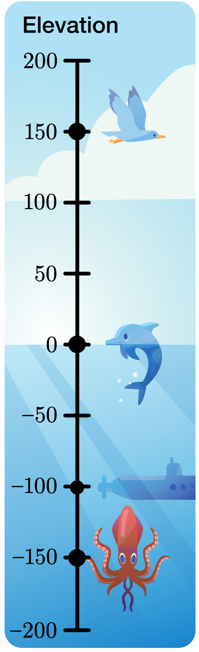


Lesson Practice
7.06

Name: .... Date: .... Period: ....

1. Use the number line to compare each statement. Determine which value is *greater*.

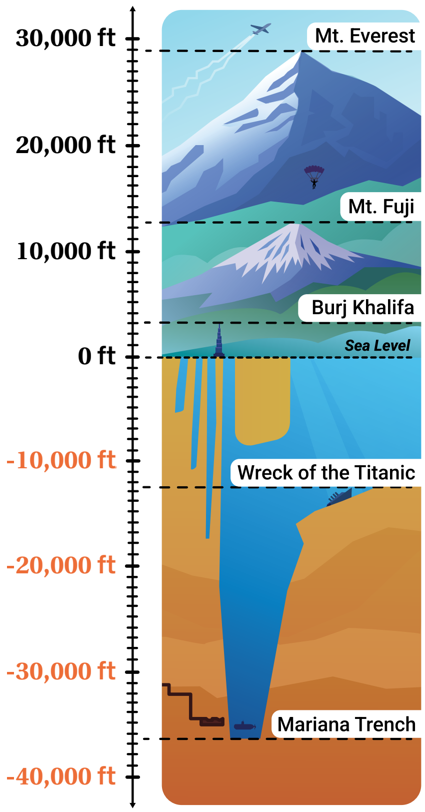
Statement A	Statement B	A	B	Same
The elevation of the submarine	The elevation of the giant squid	◯	◯	◯
The distance the submarine is from sea level	The distance the giant squid is from sea level	◯	◯	◯
The distance the seagull is from sea level	The distance the giant squid is from sea level	◯	◯	◯
Sea level	The elevation of the dolphin	◯	◯	◯



Problems 2–3: Here is a number line.

2. What is the approximate elevation of each location and its distance from sea level?

Location	Elevation (ft)	Distance from Sea Level (ft)
Mt. Everest		
Mt. Fuji		
Burj Khalifa		
Wreck of the Titanic		
Trench in Mariana Trench		



3. Select the two locations that are the same distance from sea level.

Name: ________ Date: ________ Period: ________

Problems 4–5: This table shows a bakery owner's bank account transactions.

Date	Item	Amount
May 3	Electricity	-$294
May 3	Catering order	$240
May 5	Baking supplies	-$147
May 6	Catering order	$428

4. What does the number -147 represent in this context?

5. Did the bakery owner receive more money or spend more money on May 3? Explain your thinking.

FAST Practice

6. Anushka thinks her freezer is broken, so she tracks the temperature every 2 hours. The table shows the temperature at different times of the day. At what time was her freezer's temperature closest to freezing level (0°C)?

A. 9 AM
B. 11 AM
C. 1 PM
D. 3 PM

Time	Temperature (°C)
9 AM	0.4
11 AM	-0.3
1 PM	0.1
3 PM	-0.7

Spiral Review

Problems 7–12: Use the symbols <, >, or = to make true comparisons.

7. -32 ________ |15|

8. |-32| ________ |15|

9. 32 ________ |-15|

10. 5 ________ -5

11. 5 ________ |-5|

12. -5 ________ |-5|

Name: ________________ Date: ________ Period: ________

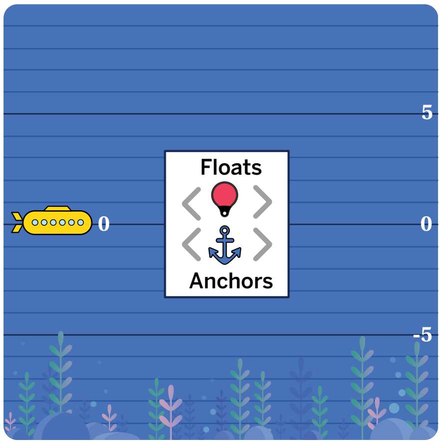
MA.6.NSO.4.1, MTR.1.1, MTR.4.1, MTR.5.1

Floats and Anchors

Let's use floats and anchors to represent values on a number line.

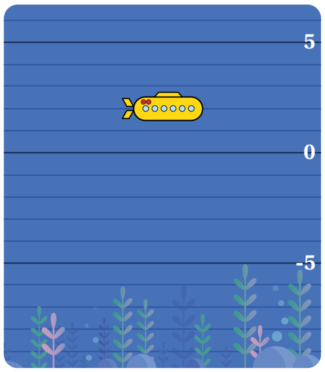
Warm-Up

1. This submarine is controlled by floats and anchors.

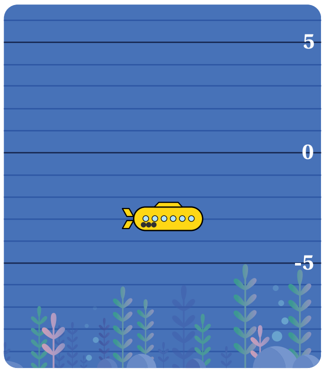


a Take a look at these different combinations.

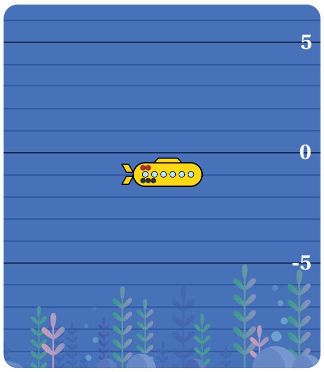
Add 2 floats



Add 3 anchors



Add 2 floats, Add 3 anchors



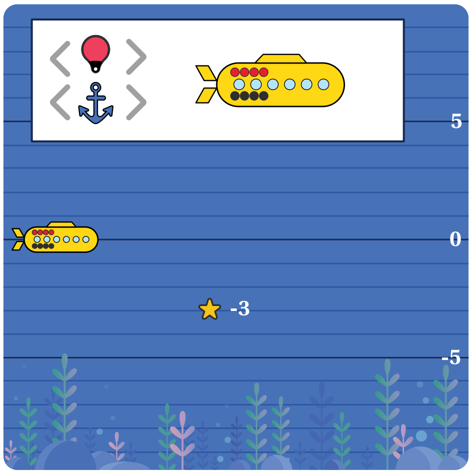
b **Discuss:** What do you notice? What do you wonder?

Activity 1

Name: .... Date: .... Period: ....

Collect the Star

2. This submarine starts with 4 floats and 4 anchors.

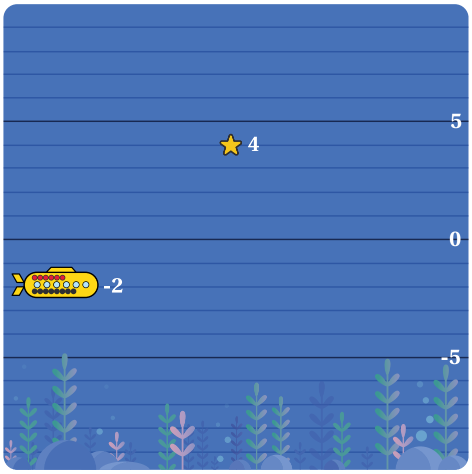


a **Discuss:** Why do you think this submarine's current position is at 0 units?

b The table shows one way to move the submarine to -3 to get the star. Write three more actions to get the submarine to -3.

Start	Action	Final
0	Remove 1 float Add 2 anchors	-3
0		-3
0		-3
0		-3

3. This submarine starts with 6 floats and 8 anchors.



The submarine has space for up to 10 floats and up to 10 anchors.

Write an action that could move the submarine to 4 units to collect the star.

Start	Action	Final
-2		4

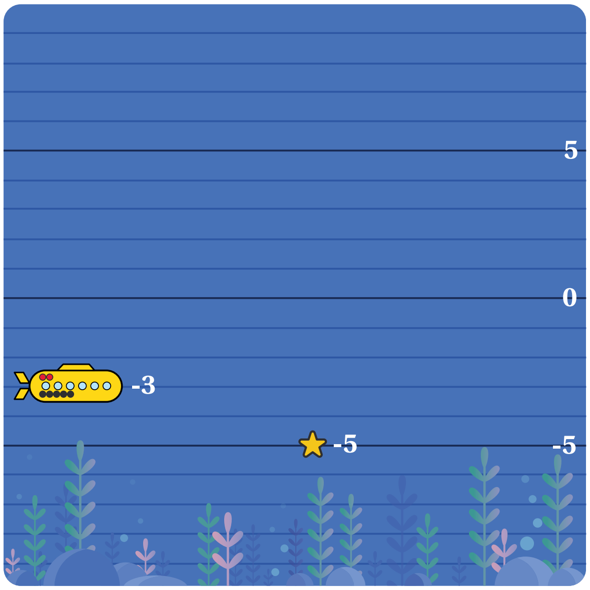
Name: Date: Period:

Collect the Star (continued)

4. This submarine starts at -3 units.

Select *all* the actions that would move it to -5 units.

- ☐ **A.** Add 2 floats
- ☐ **B.** Add 2 anchors
- ☐ **C.** Remove 1 float and add 1 anchor
- ☐ **D.** Add 3 floats and add 5 anchors
- ☐ **E.** Remove 2 floats and add 4 anchors



5. Imagine a new submarine. For each action, put a check for whether the submarine would go up, go down, or stay in the same position.

Action	Up	Same Position	Down
Add 3 floats Add 4 Anchors	○	○	○
Remove 10 anchors	○	○	○
Remove 5 floats Remove 5 anchors	○	○	○
Add 8 floats Remove 8 anchors	○	○	○
Add 6 floats Add 2 anchors	○	○	○
Remove 7 floats Add 3 anchors	○	○	○

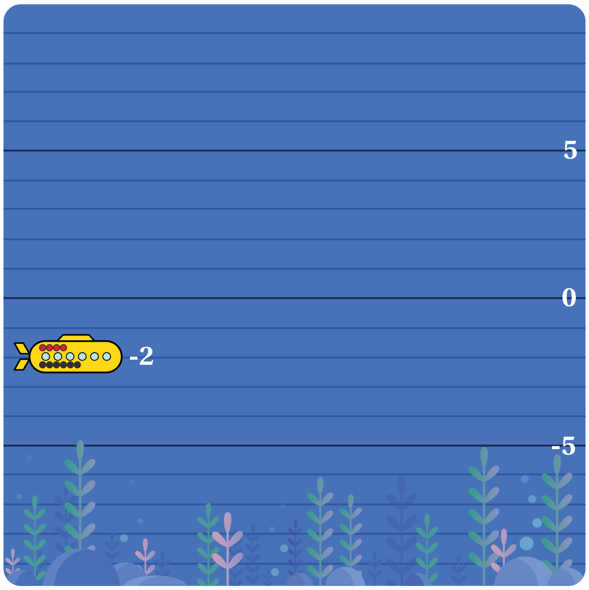
Name: .... Date: .... Period: ....

Sea-king Stars

6. The table shows the submarine's starting position and the action that will change its position.

What will be the submarine's final position?

Start	Action	Final
-2	Add 3 floats Remove 5 anchors	



7. Leo and Mai wrote expressions to answer the previous question.

Leo's expression: $-2 + 3 - 5$

Mai's expression: $-2 + 3 - (-5)$

Who wrote a correct expression? Circle one.

Leo Mai Both Neither

Explain your thinking.

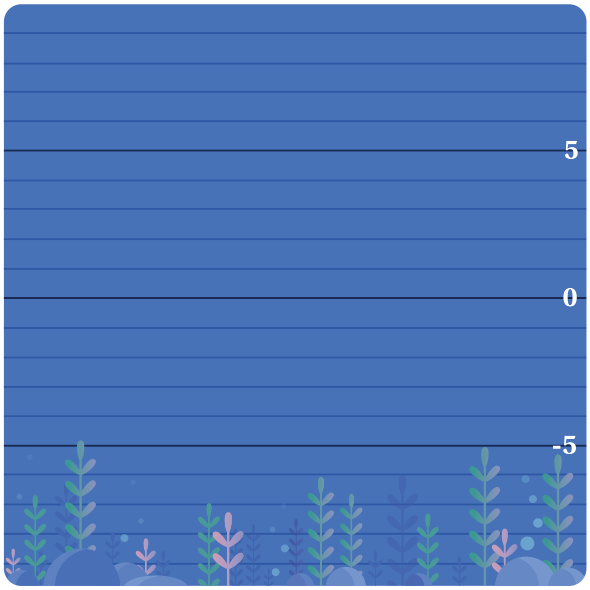
Activity 3

Name: .... Date: .... Period: ....

Captain's Challenge

8. What is the final position of each submarine? Complete as many challenges as you have time for. Use the image if it helps with your thinking.

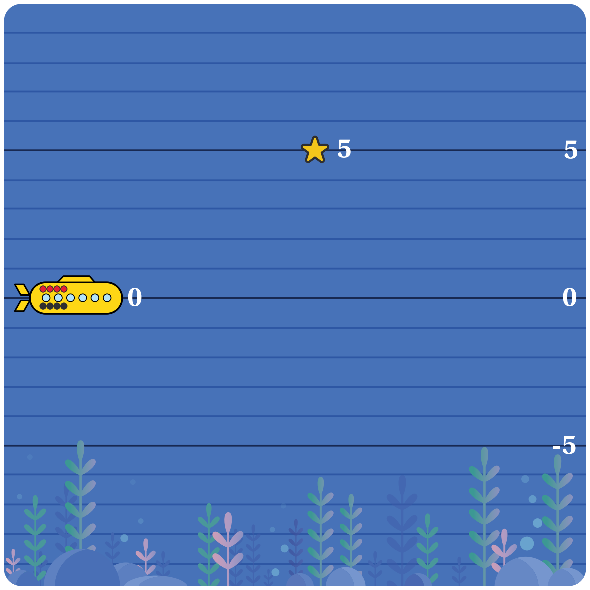
Start	Action	Final
0	Add 3 floats Add 7 anchors	
-9	Add 8 floats Remove 6 anchors	
1	Remove 3 floats Add 4 anchors	
-3	Remove 1 float Add 3 anchors	
-4	Remove 1 float	
5	Remove 8 floats Remove 3 anchors	
-2	Add 5 floats Remove 2 anchors	
-5	Add 1 float Remove 2 anchors	



Synthesis

9. Describe a set of actions that would allow this submarine to collect the star at 5 units.

Try to come up with something none of your classmates will.



Lesson Practice 7.07

Lesson Summary

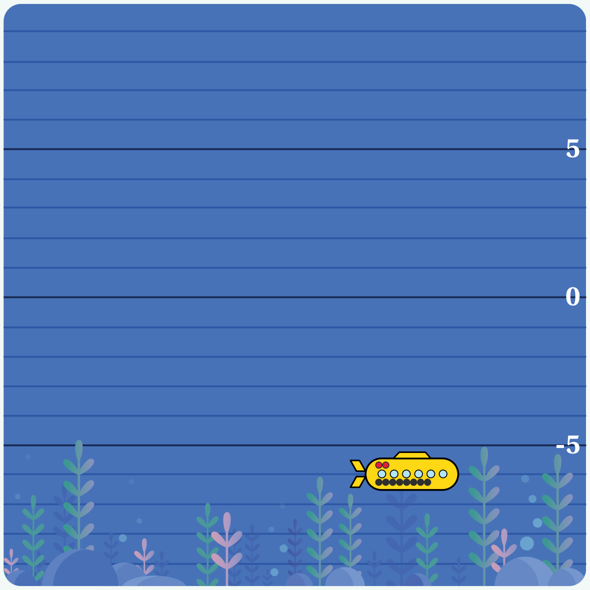
Using models such as floats and anchors on a vertical number line can be useful when representing addition and subtraction of positive and negative numbers.

For example, imagine a submarine whose position is at -6 units. The submarine will move from its position as 3 floats are added and 2 anchors are removed.

- 3 floats being added represents moving up 3 units or + 3.
- 2 anchors being removed represents moving up 2 units or -(-2) = + 2.

The submarine's new position would be -6 + 3 + 2 = -1 units.

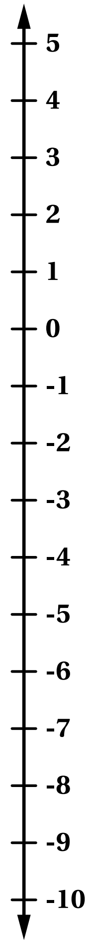
To move the submarine to 0 units from -1 units, 1 float can be added, represented by + 1. -1 and + 1 are an opposite pair, so they add to 0.



Lesson Practice
7.07

Name: .... Date: .... Period: ....

Problems 1–5: One moment in December, it was -8°C in Harbin, China, and -2°C in Beijing, China. Use the number line if it helps with your thinking.



1. Which city was colder?

2. At the same moment, it was 7 degrees warmer in Shanghai than it was in Beijing. What was the temperature in Shanghai?

3. How many degrees warmer was it in Shanghai than in Harbin?

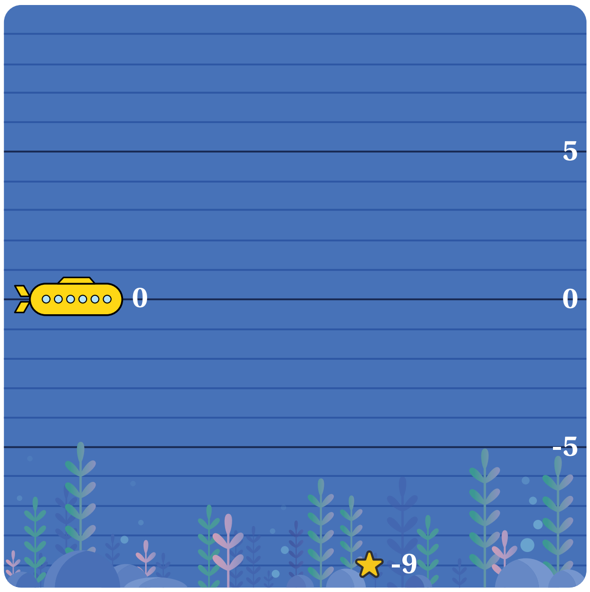
4. Later in the day, Beijing got 5 degrees colder. What was Beijing's new temperature?

5. Later in the day, Harbin's temperature was 1°C. By how much did Harbin's temperature change? Explain your thinking.

6. This submarine has 0 floats and 0 anchors. The submarine can hold up to 10 floats and 10 anchors.

a List *all* the combinations of floats and anchors that could collect the star at -9 units.

b How do you know there are no other combinations?



Lesson Practice
7.07

Name: .... Date: .... Period: ....

FAST Practice

7. A submarine is at the position -5 units. The expression -5 + (-2) represents the submarine's new position after adding 2 anchors. What is the submarine's new position?

A. -7 units **B.** -3 units **C.** 3 units **D.** 7 units

Spiral Review

8. Fill in each blank using the symbols >, <, or =.

3		-3
12		24
-12		-24
7		7.2
-7		-7.2

Problems 9–10: A color of green paint is made by mixing 2 cups of yellow paint with 3.5 cups of blue paint.

9. Complete the table to show how many cups of yellow and blue paint will make the same color of green but in a smaller amount.

Yellow Paint (cups)	Blue Paint (cups)

10. Complete the table to show how many cups of yellow and blue paint will make the same color of green but in a larger amount.

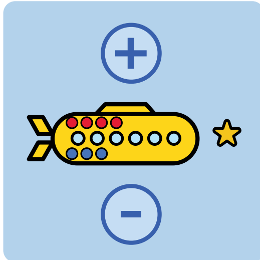
Yellow Paint (cups)	Blue Paint (cups)

Unit 7
Lesson
8

Name: ______ Date: ______ Period: ______

MA.6.NSO.4.1, MTR.1.1, MTR.3.1, MTR.5.1

More Floats and Anchors



Let's use floats and anchors to reason about adding and subtracting positive and negative numbers.

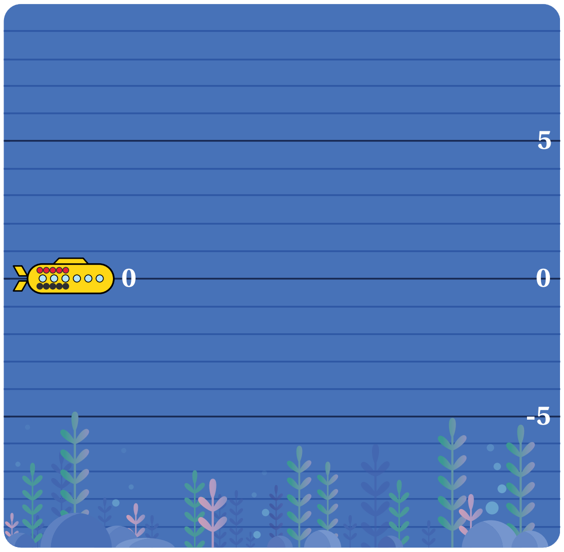
Warm-Up

1. *If you add some floats and remove some anchors, the submarine will go up.*

Is this statement always, sometimes, or never true? Circle one.

Always Sometimes Never

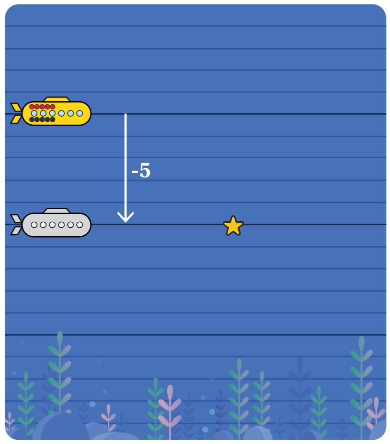
Explain your thinking.



Name: .... Date: .... Period: ....

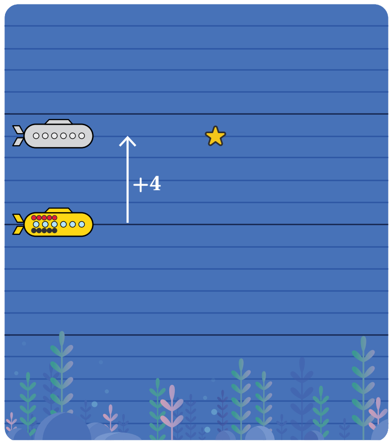
Ups and Downs

2. Select *all* the actions that would make this submarine go down 5 units.



- ☐ **A.** Add 5 floats
- ☐ **B.** Remove 5 floats
- ☐ **C.** Add 5 Anchors
- ☐ **D.** Remove 5 anchors
- ☐ **E.** Add 3 anchors and add 2 floats

3. Select *all* the actions that would make this submarine go up 4 units.



- ☐ **A.** Add 4 floats
- ☐ **B.** Remove 4 floats
- ☐ **C.** Add 4 Anchors
- ☐ **D.** Remove 4 anchors
- ☐ **E.** Add 3 floats and remove 1 anchor

4. Here are the details for four new submarine scenarios.

a Complete the table.

Start	Action	Final Expression	Final Value
3	Add 7 anchors	$3 + (-7)$	
	Remove 7 floats	$3 - 7$	
-2		$-2 + 8$	
		$-2 - (-8)$	

b **Discuss:** What do you notice? What do you wonder?

Activity 2

Name: .... Date: .... Period: ....

Depths of Understanding

5. Marc and Naoki are trying to evaluate 3 – (-2)

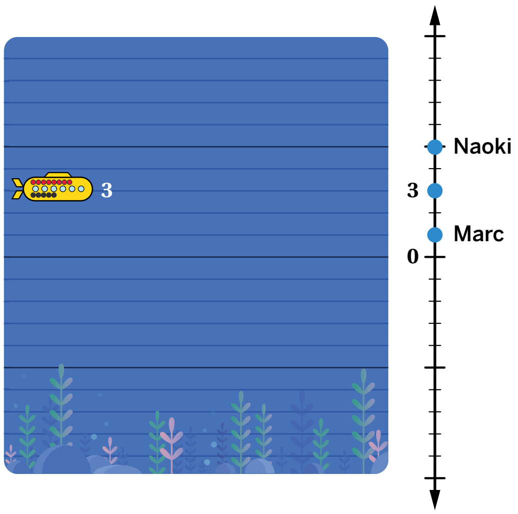
Marc says: *This is like adding* 2 *anchors, so the submarine goes down to* 1.

Naoki claims: *This is like removing* 2 *anchors, so the submarine goes up to* 5.

Whose thinking is correct? Circle one.

Marc (1)   Naoki (5)   Both   Neither

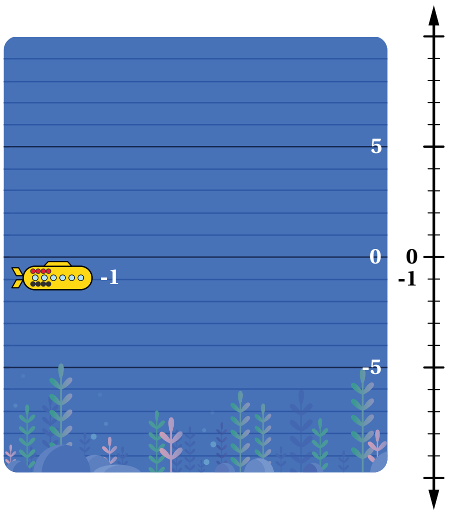
Explain your thinking.



6. What is the value of -1 + (-4)?

What is the value of -1 – (-4)?

Show or explain your thinking.



Name: .... Date: .... Period: ....

Depths of Understanding (continued)

7. Group the expressions into pairs that have the same value.

$-4 - (-10)$	$4 + (-10)$	$-4 + 10$	$-4 - 10$
$-4 + (-10)$	$4 + 10$	$4 - (-10)$	$4 - 10$

Pair 1	Pair 2

Pair 3	Pair 4

You're invited to explore more.

8. **a** Determine the value of each expression.

Expression	Value
1	
$1 - 2$	
$1 - 2 + 3$	
$1 - 2 + 3 - 4$	
$1 - 2 + 3 - 4 + 5$	

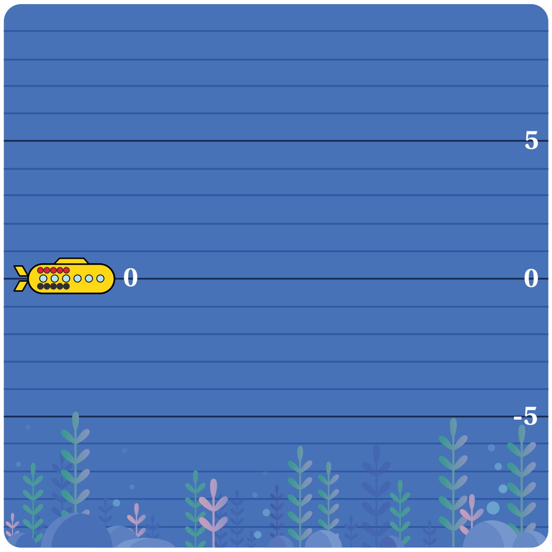
b Describe any patterns you notice.

c What is the value of the next expression? The 10th expression? The 100th expression?

Synthesis

9. Explain why it makes sense that $0 - (-5)$ is equivalent to $0 + 5$.

Use a floats and anchors situation if it helps with your thinking.



Lesson Practice 7.08

Lesson Summary

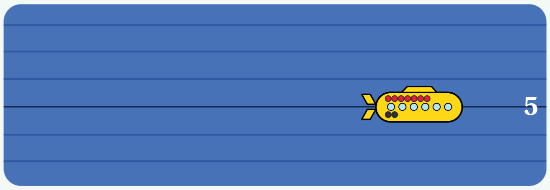
Different combinations of floats and anchors can give you the same result. Here are some examples:

- If a submarine starts at 4 units, adding 2 floats or removing 2 anchors will both result in the submarine moving up to 6 units. So adding a positive number is the same as subtracting a negative number.

Adding Floats	Removing Anchors
$4 + 2 = 6$	$4 - (-2) = 6$

- If a submarine starts at 5 units, removing 1 float or adding 1 anchor will both result in the submarine moving down to 4 units. So subtracting a number is the same as adding its opposite.

Removing Floats	Adding Anchors
$5 - 1 = 4$	$5 + (-1) = 4$



When you add two values that are opposites, the sum is always 0. These numbers are also called *additive inverses* of each other.

Name: .... Date: .... Period: ....

Problems 1–4: Determine the value of each expression.

1. $5 + (-3)$

2. $-5 + 3$

3. $-5 - 3$

4. $-5 - (-3)$

Problems 5–6: The table shows eight expressions.

5. Determine the value of each expression.

	Expression	Value
Expression 1	$1 + 2 - 3$	
Expression 2	$1 + 2 - 3 + 4$	
Expression 3	$1 + 2 - 3 + 4 - 5$	
Expression 4	$1 + 2 - 3 + 4 - 5 + 6$	
Expression 5	$1 + 2 - 3 + 4 - 5 + 6 - 7$	
Expression 6	$1 + 2 - 3 + 4 - 5 + 6 - 7 + 8$	
Expression 7	$1 + 2 - 3 + 4 - 5 + 6 - 7 + 8 - 9$	
Expression 8	$1 + 2 - 3 + 4 - 5 + 6 - 7 + 8 - 9 + 10$	

6. What is the value of the next expression? The 10th expression? The 20th expression?

7. A swimmer was 8 feet underwater. Then he swam 3 feet deeper. Riku wrote the expression $-8 - 3$. Charlie wrote the expression $-8 + (-3)$. Explain why both Riku and Charlie are correct.

8. The temperature was -13°F and then rose to 5°F. What was the *change* in temperature? Explain your thinking.

9. The temperature was 13°F and then dropped 5 degrees. What was the final temperature?

FAST Practice

10. The temperature was -13°F and then dropped 5 degrees. What was the final temperature?

☐ °F

Spiral Review

Problems 11–13: Complete each statement with a value that makes the statement true.

11. ________ < 13

12. ________ < -0.1

13. ________ > -2

Name: .... Date: .... Period: ....

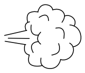
MA.6.NSO.4.2, MTR.2.1

Up, Up, and Away!

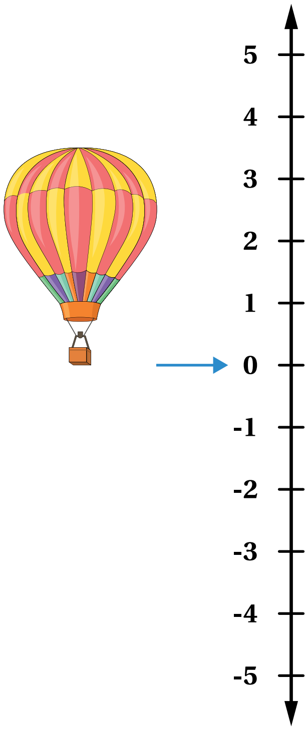
Let's take hot-air balloon rides to make sense of multiplying and dividing integers.

Warm-Up

1. Imagine that a hot-air balloon moves up or down according to these rules:

   Adding 1 puff of air  = Balloon moves *up* 1 unit and represents a positive direction

   Adding 1 sandbag  = Balloon moves *down* 1 unit and represents a negative direction



   a. The balloon starts at 0. How will the balloon move when you add 3 sandbags?

   b. Next, you add 4 puffs of air. How will the balloon move?

Activity 1

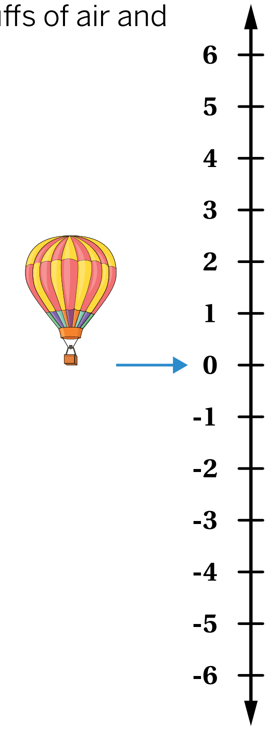
Name: .... Date: .... Period: ....

Ups and Downs

2. The table shows how the hot-air balloon moves, each time starting at 0. Puffs of air and sandbags are added in groups of 2.

For Movement 1, 3 groups of 2 puffs of air are added.

For Movement 2, 3 groups of 2 sandbags are added.



	Addition Expression	Multiplication Expression	Final Position
Movement 1	$2 + 2 + 2$	$3 \cdot (2)$	6
Movement 2	$(-2) + (-2) + (-2)$	$3 \cdot (-2)$	-6
Movement 3			4
Movement 4			-4
Movement 5			-8

a. Complete the table to match each final position.

b. **Discuss:** How can you describe each movement in terms of puffs of air or sandbags?

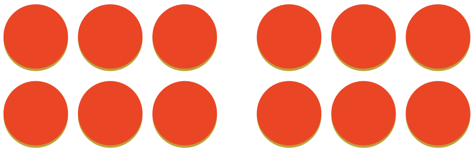
Activity 2

Name: ______ Date: ______ Period: ______

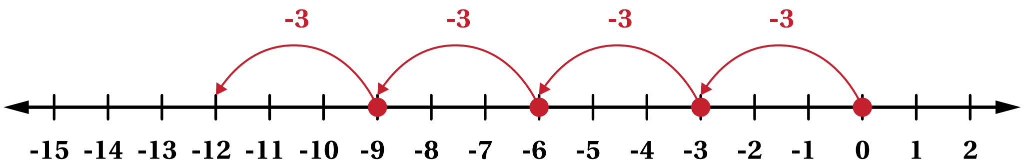
Modeling Integer Multiplication

3. Another hot-air balloon adds 4 groups of 3 sandbags. Ayaan and Emma each model the expression 4 • (-3) to determine the change in elevation.

Ayaan



Emma



a. What do you like about Ayaan's method?

b. What do you like about Emma's method?

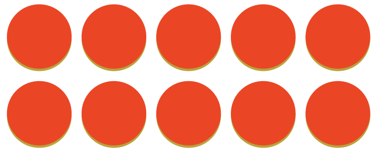
c. **Discuss:** What can you conclude about the product of a negative number and a positive number? Explain your thinking.

Name: ______ Date: ______ Period: ______

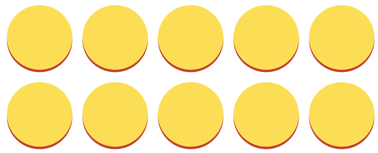
Modeling Integer Multiplication (continued)

4. Ayaan uses his counters to model the expression -2 • (-5).

Step 1: Model 2 • (-5)



Step 2: Flip over the counters to model the opposite of 2 • (-5).



Step 3: Determine the product.

-2 • (-5) = 10

a Explain why Ayaan models *the opposite of* 2 • (-5) in Step 2.

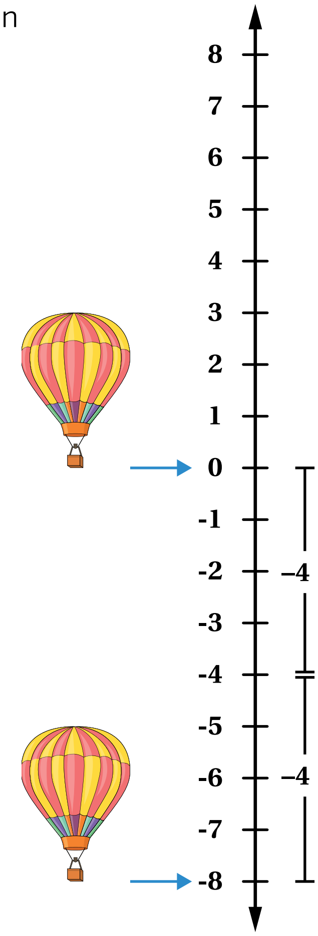
b How can you use counters to model -1 • (-6)?

c **Discuss:** Is the product of two negative numbers always positive? Explain your thinking.

Name: ________ Date: ________ Period: ________

Integer Division

5. The hot-air balloon starts at 0 and travels down to -8. The trip can be divided into 2 equal parts, as shown.



a Which division problem does the model represent?

- **A.** $8 \div 2 = 4$
- **B.** $-8 \div 2 = -4$
- **C.** $8 \div (-2) = -4$
- **D.** $-8 \div (-2) = 4$

b How can you model the problem with counters?

c **Discuss:** What can you conclude about the quotient of a negative number and a positive number?

6. Mohamed says that the quotients of $-\left(\frac{12}{6}\right)$, $\frac{-12}{6}$, and $\frac{12}{-6}$ are all equivalent. Do you agree? Explain your thinking.

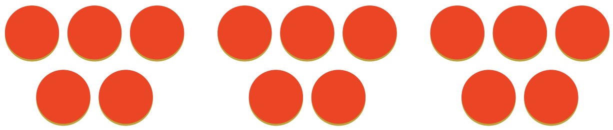
Activity 3

Name: .... Date: .... Period: ....

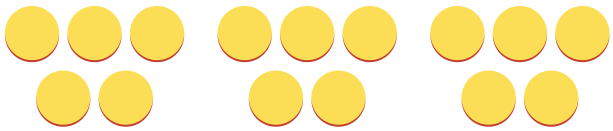
Integer Division (continued)

7. The counters below model the expression $-15 \div (-3)$.

Step 1: Model $-15 \div 3$.



Step 2: Flip over the counters to model the opposite of $-15 \div 3$.



a What is the quotient of $-15 \div (-3)$? Explain how the model shows the quotient.

b Is the quotient of two negative numbers always positive? Explain your thinking.

c **Discuss:** What patterns do you notice about the products and quotients of negative and positive integers?

Synthesis

8. For each problem, indicate whether the product or quotient of two integers is positive or negative.

Problem	Positive	Negative
Positive × Positive	◯	◯
Positive × Negative	◯	◯
Negative × Positive	◯	◯
Negative × Negative	◯	◯
Positive ÷ Positive	◯	◯
Positive ÷ Negative	◯	◯
Negative ÷ Positive	◯	◯
Negative ÷ Negative	◯	◯

Lesson Practice 7.09

Lesson Summary

The rules for multiplying and dividing integers are the same. When two integers have the same sign, their product or quotient is positive. When the signs of two integers differ, their product or quotient is negative.

Here are some examples:

$12 \cdot 4 = 48$

$12 \cdot -4 = -48$

$-12 \div 4 = -3$

$-12 \div -4 = 3$

Lesson Practice
7.09

Name: ______ Date: ______ Period: ______

Problems 1–4: Determine the value of each expression.

1. $10 \cdot (-2)$

2. $-4 \cdot (-6)$

3. $-9 \div (-3)$

4. $10 \div (2)$

Problems 5–6: Determine each missing number.

5. $-7 \cdot (?) = -28$

6. $-40 \div (?) = -8$

7. The temperature outside drops 12 degrees over 3 hours. What is the average temperature change per hour?

- **A.** 4 degrees
- **B.** 3 degrees
- **C.** -3 degrees
- **D.** -4 degrees

8. Is it *always*, *sometimes*, or *never* true that a negative number multiplied by a negative number is a positive number? Give an example to support your thinking.

FAST Practice

9. A business loses \$10 on each sale of a product that doesn't perform well. If they sell 3 of these products, what is the total loss? Choose the equation that represents the situation.

- **A.** $-\$10 \cdot (-3) = \$30$
- **B.** $-\$10 \cdot (3) = -\$30$
- **C.** $-\$10 \div (-3) = \$3.33$
- **D.** $-\$10 \div (3) = -\$3.33$

Name: .... Date: .... Period: ....

Spiral Review

Problems 10–11: Determine the value of each expression.

10. $6 + (-3)$

11. $-5 + (-3)$

12. A diver was 3 feet underwater. Then he dove 3 feet deeper. Ayaan and Emma wrote expressions to represent the situation.

- Ayaan wrote the expression $-3 - 3$.
- Emma wrote the expression $-3 + (-3)$.

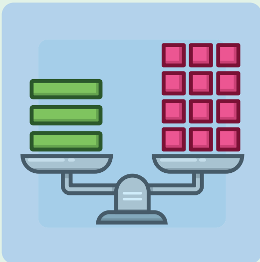
Whose expression is correct? Explain your thinking.

Sub-unit

2

Equations

Lesson 10
Plus or Minus



Lesson 11
Multiply, Divide, and Conquer

Name: ________ Date: ________ Period: ________

MA.6.AR.2.2, MTR.2.1, MTR.5.1, MTR.7.1

Plus or Minus

Let's use addition and subtraction of integers to write and solve one-step equations.

Warm-Up

1. Which one doesn't belong?

A. $8 = 4 + x$

B. $x - 4 = -8$

C. $4 - 8 = x$

D. $-4 + x = 8$

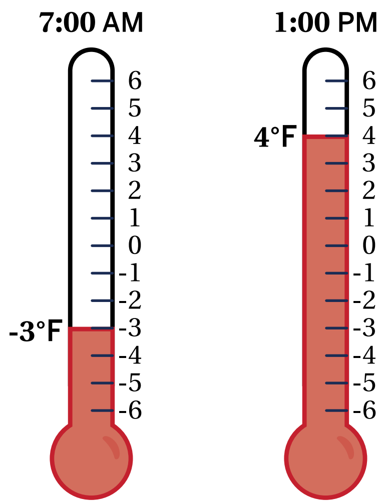
Explain your thinking.

Activity 1

Name: .... Date: .... Period: ....

Give or Take

2. At 7:00 AM, the temperature was -3°F. By 1:00 PM, the temperature was 4°F.



a Write as many equations as you can to represent the change in temperature, x.

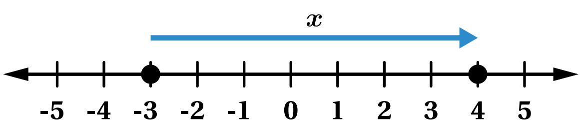
b Draw a model to represent one of the equations you wrote.

Name: .... Date: .... Period: ....

Give or Take (continued)

3. Sydney and Alejandro wrote the equation $-3 + x = 4$ to represent the situation.

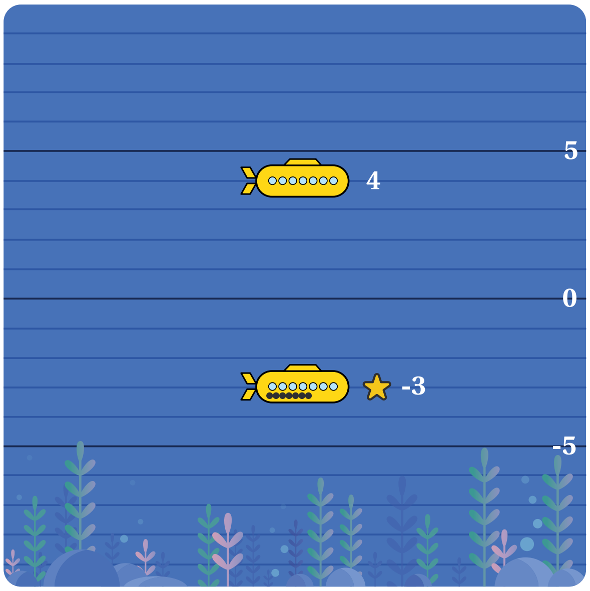
   a. Sydney used a number line to model the equation. Is Sydney correct? Explain your thinking.



   b. Alejandro used floats and anchors to model the equation.

   Alejandro says, "I started at 4. Then I added 7 anchors. This moved the submarine down 7, so I ended at -3."

   Is Alejandro correct? Explain your thinking.



Activity 2

Name: .... Date: .... Period: ....

Solve It!

4. Sydney and Alejandro solved their equation $-3 + x = 4$ using properties of equality and operations and got $x = 7$. Show or explain how they solved the equation.

5. Use Sydney and Alejandro's method to solve these equations.

a) $x - 5 = -12$

b) $23 = y - (-6)$

c) $n + (-7) = 2$

6. **Discuss:** What strategies can you use to determine whether the *solution to each equation* makes sense?

Synthesis

7. How can you apply operations and equality properties to solve addition and subtraction equations with integers?

   Use the examples if they help with your thinking.

   $x - 6 = \text{-}10$

   $4 = \text{-}5 + x$

   $x + (\text{-}3) = 9$

   $\text{-}11 = x - (\text{-}1)$

Lesson Practice 7.10

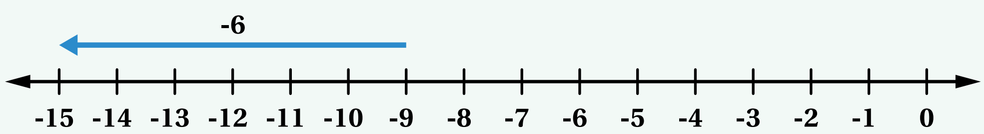
Lesson Summary

Equations can include positive and negative numbers. You can use equations and models such as tape diagrams or number lines to represent a mathematical situation.

You can use properties of operations and equality to solve addition and subtraction equations with integers.

For example, you can model this situation using a number line:

A rock drops 6 feet in a well and reaches an elevation of -15 feet. What was the rock's initial elevation?



Then, you can write the equation as $x + (\text{-}6) = \text{-}15$ and solve it using the subtraction property of equality. Subtracting a negative number is the same as adding a positive number, so $x = \text{-}9$.

$$x + (\text{-}6) = \text{-}15$$
$$x + (\text{-}6) - (\text{-}6) = \text{-}15 - (\text{-}6)$$
$$x = \text{-}15 + 6$$
$$x = \text{-}9$$

Name: .... Date: .... Period: ....

Problems 1–4: Seawater freezes at about -2°C. The temperature of a certain sea drops 13 degrees, and the water starts freezing. What was the original temperature of the sea?

1. Draw a model to represent the situation.

2. Select all equations that can represent the situation.

- ☐ **A.** $t = 13 + (\text{-}2)$
- ☐ **B.** $\text{-}2 + t = 13$
- ☐ **C.** $13 - (\text{-}2) = t$
- ☐ **D.** $\text{-}2 = t - 13$
- ☐ **E.** $t + (\text{-}13) = \text{-}2$

3. Choose one of the equations that can represent the situation and determine the solution.

4. Explain why your solution makes sense.

5. Solve the equation $\text{-}8 = x - (\text{-}12)$. Show or explain your thinking.

Name: .... Date: .... Period: ....

Problems 6–7: Write an equation for each situation.

6. The hiker's starting elevation in a river valley is 6 feet below sea level. The hiker reaches a height of 14 feet below sea level. What was the hiker's change in elevation?

7. The number -2 is 3 less than what number?

8. Solve the equation you wrote in Problem 7.

FAST Practice

9. Solve the equation $-12 + z = 13$.

$z =$ ☐

Spiral Review

10. Select *all* the expressions that are equivalent to -20.

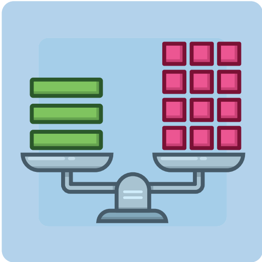
- ☐ **A.** $-10 \cdot (-2)$
- ☐ **B.** $-4 \cdot 5$
- ☐ **C.** $40 \div (-2)$
- ☐ **D.** $-100 \div (-50)$
- ☐ **E.** $1 \cdot (-20)$

11. Determine the missing number. $-3 \cdot (?) = (-15)$

Name: ____________ Date: ________ Period: ________

MA.6.AR.2.3, MTR.2.1, MTR.7.1

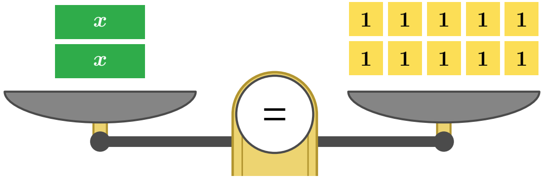
Multiply, Divide, and Conquer



Let's use balanced scales and tape diagrams to write and solve multiplication and division equations with integers.

Warm-Up

1. Take a look at this balanced scale with tiles that represent x on one side and tiles that represent 1 on the other side.

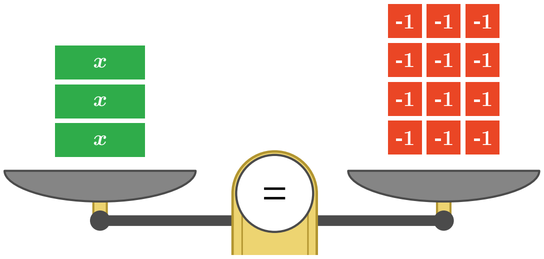


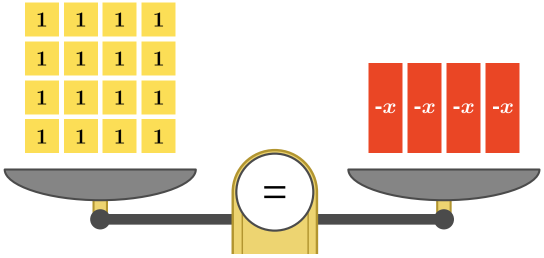
How can you write and solve an equation that the scale represents?

Name: .... Date: .... Period: ....

Multiply and Conquer

2. Consider these two scales.





a Write and solve an equation for the first scale.

b Write and solve an equation for the second scale

3. Arturo and Melanie both wrote equations to solve for x for this scale. Whose answer is correct? Explain your thinking.

Arturo

$$-5x = -20$$

$$\frac{-5x}{-5} = \frac{-20}{-5}$$

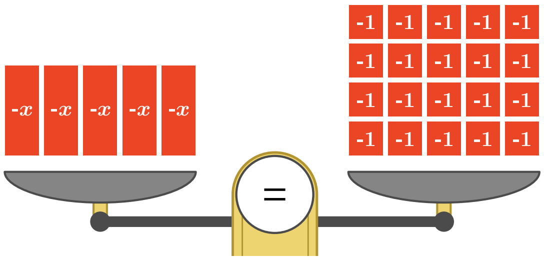
$$x = 4$$

Melanie

$$-5x = -20$$

$$\frac{-5x}{-5} = \frac{-20}{-5}$$

$$x = -4$$

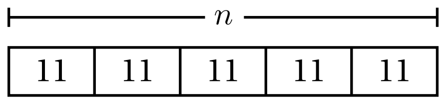


Activity
2

Name: .... Date: .... Period: ....

Divide and Conquer

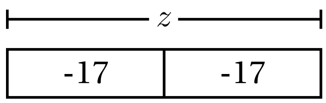
4. We can use tape diagrams to represent equations.



a. What value of n does the tape diagram represent?

b. How does the diagram represent $\frac{n}{5} = 11$? Explain your thinking.

5. a. What value of z does the tape diagram represent?



b. What equation does the tape diagram represent?
Explain your thinking.

Activity 3

Name: ______ Date: ______ Period: ______

Conquer the World

You will use a set of cards for this activity.

6. With your partner, match each situation with the equation and solution that show the same relationship.

Situation	Equation	Solution
A group of 5 teammates each takes 8 tennis balls from the bucket. What is the total number of tennis balls, x, that the group must return to the bucket at the end of practice?		
Annika regularly spends $5 after school at a nearby cafe. If she spends $40 in all, what number of days, x, has she visited the cafe?		
Over the course of 5 days, the temperature drops a total of 40 degrees. On average, how much did the temperature drop each day?		
A student lost a wallet during a nature walk. Five groups of people left the school to look for it. If each group backtracked 8 trail markers, how many trail markers did they check altogether?		

Synthesis

7. How does solving equations with integers compare with solving equations with whole numbers?

   Use the examples if they help with your thinking.

$4x = 24$

$\frac{x}{5} = 11$

$\text{-}6x = \text{-}42$

$\frac{x}{3} = \text{-}18$

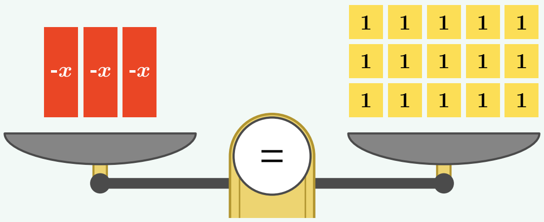
Lesson Practice 7.11

Lesson Summary

A balanced scale or tape diagram can help us visualize an equation with integers and determine its solution. We can apply what we know about integers to solve for the variable.

Here is an example.

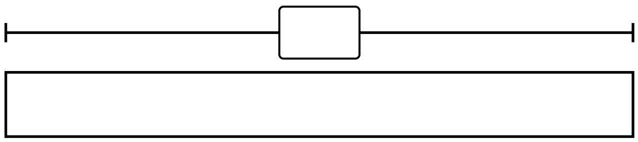
This balanced scale represents the equation $\text{-}3x = 15$ because the left side has 3 negative x-tiles and the right side has 15 1-tiles. The solution is the value of x that will keep the scale balanced. The solution to the equation is $x = \text{-}5$ because $\text{-}3(\text{-}5) = 15$.



Name: ________ Date: ________ Period: ________

Problems 1–4: A scuba diver goes from sea level to the ocean floor in 7 minutes. If he dives down 4 meters each minute, what is the elevation of the ocean floor?

1. Draw a tape diagram to represent the situation.



2. Select *all* the equations that can represent this situation.

- ☐ **A.** $7m = \text{-}4$
- ☐ **B.** $4m = \text{-}7$
- ☐ **C.** $\frac{m}{7} = \text{-}4$
- ☐ **D.** $\frac{m}{\text{-}4} = 7$
- ☐ **E.** $m = 7(\text{-}4)$

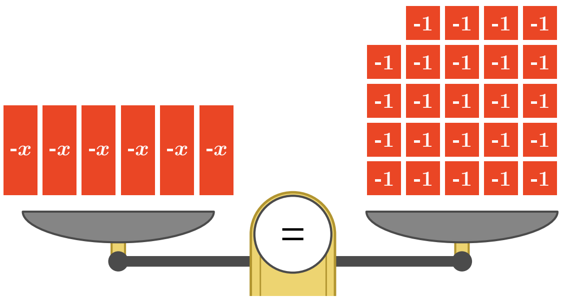
3. Determine the solution to one of the selected equations from Problem 2.

4. Explain the solution's meaning in this situation.

Problems 5–6: Consider the balanced scale.

5. Write an equation that the scale represents.

6. Solve the equation for the value of x.



Lesson Practice
7.11

Name: .... Date: .... Period: ....

FAST Practice

7. Solve the equation $\frac{y}{-8} = 6$.

$y = \square$

Spiral Review

Problems 8–11: Determine the value of each expression.

8. $-5 + 7$

9. $-5 - 7$

10. $-3 + 9 + (-7) + 4 + (-6) + (-5)$

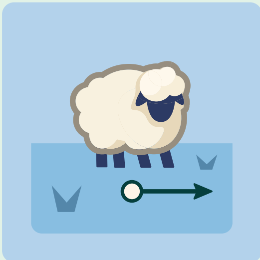
11. $-8 - (-7) + (-6) - 5 + 9$

Sub-unit

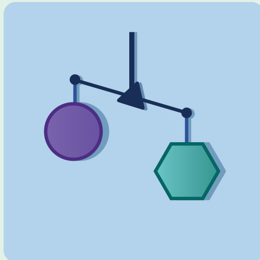
3

Inequalities

Lesson 12
Tunnel Travels



Lesson 13
Shira's Solutions



Lesson 14
Comparing Weights

Name: ______________________ Date: __________ Period: __________

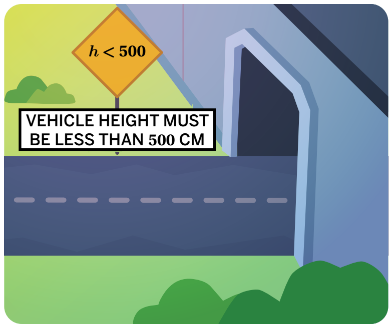
MA.6.AR.1.2, MTR.2.1, MTR.3.1

Tunnel Travels

Let's explore inequalities using words, symbols, and a number line.

Warm-Up

1. Select *all* of the vehicles that can fit in this tunnel.



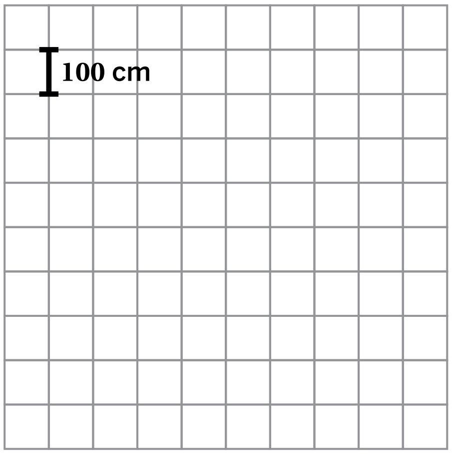
☐ **A.**

☐ **B.**

☐ **C.**

☐ **D.**

2. Sketch a vehicle that will fit in the tunnel.

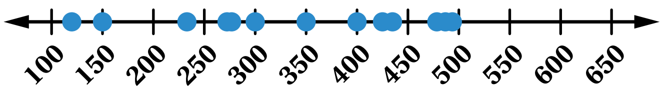


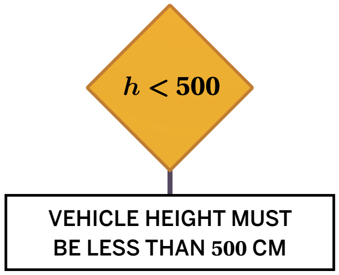
Activity 1

Name: .... Date: .... Period: ....

Tunnels, Garages, and More!

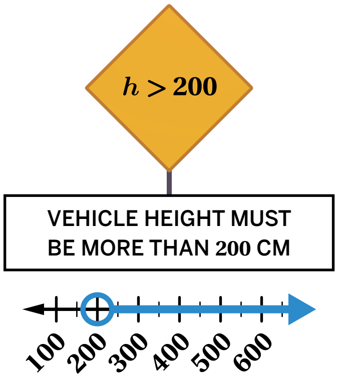
3. Here are the heights of several vehicles that fit in the tunnel.

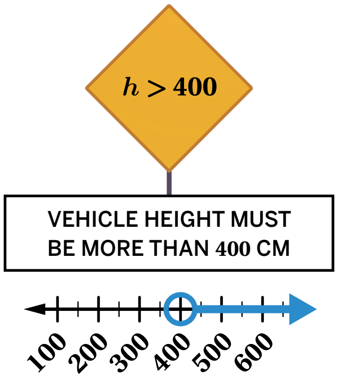


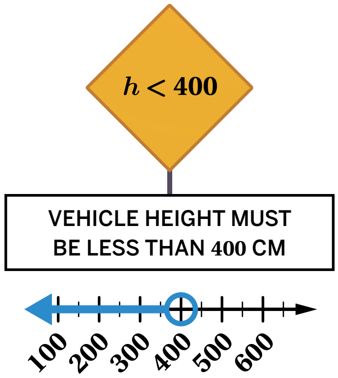


What do you think a graph of *all* the vehicle heights that fit would look like?

4. Here are three signs with inequalities and their number line graphs.



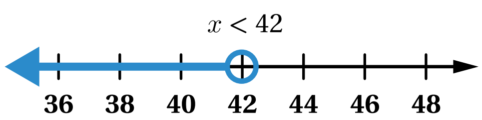


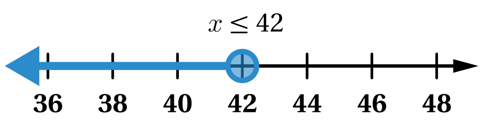


Discuss: What do you notice? What do you wonder?

5. Here are the graphs for $x < 42$ and $x \leq 42$.

Discuss: How are the graphs alike? How are they different?



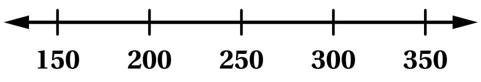


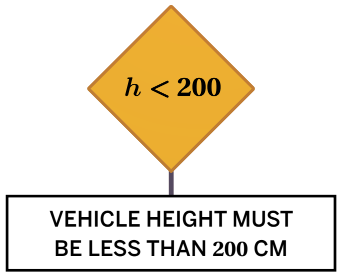
Name: .... Date: .... Period: ....

Tunnels, Garages, and More! (continued)

6. Norma Merrick Sklarek was the first African American woman to become a licensed architect in California. In 1975, she worked with with César Pelli to design the Pacific Design Center in Los Angeles, California.

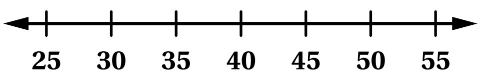
Graph all the possible vehicle heights that fit in this parking garage.

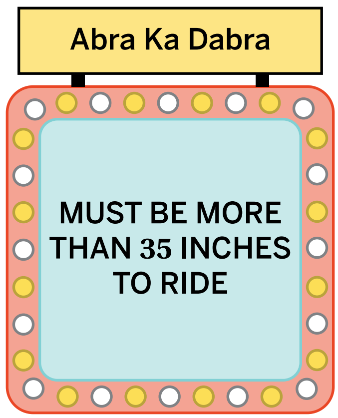




7. Fri Forjindam is co-owner and chief development officer of a company that develops theme parks. In 2016, she designed Bollywood Parks in Dubai. One ride at that park, Abra Ka Dabra, only allows passengers who are taller than 35 inches.

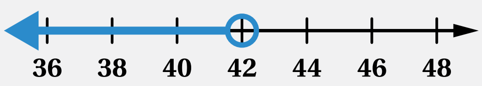
Graph all the possible heights for this ride.



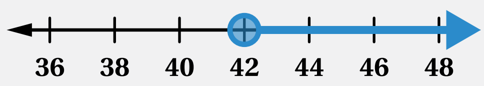


8. Group the choices that represent the same situation. One choice will have no match.

A.



B.



C. You must be over 42 inches tall to ride The Whipper.

D. You must be under 42 inches tall to ride the kiddie swings.

E. You must be at least 42 inches to ride the roller coaster.

F. $x < 42$

G. $x \geq 42$

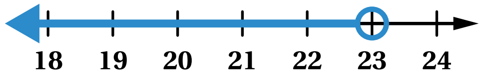
Group 1	Group 2

Activity 2

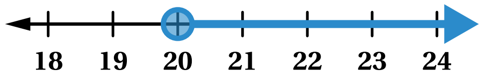
Name: .... Date: .... Period: ....

Inequalities Out of Context

9. Rewrite the inequality $x > 23$ so that it matches the graph.



10. To represent this graph:



- Martina wrote the inequality $20 \leq x$.
- Nasir wrote the inequality $x \leq 20$.

The symbol $\leq$ means **less than or equal to**.

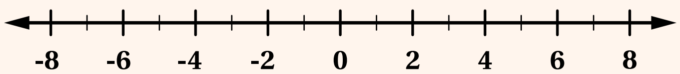
Whose inequality is correct? Circle one.

Martina's Nasir's Both Neither

Explain your thinking.

You're invited to explore more.

11. Here is a number line.



a. Determine three possible values for x if $|x| < 5$.

b. Plot these values on the number line.

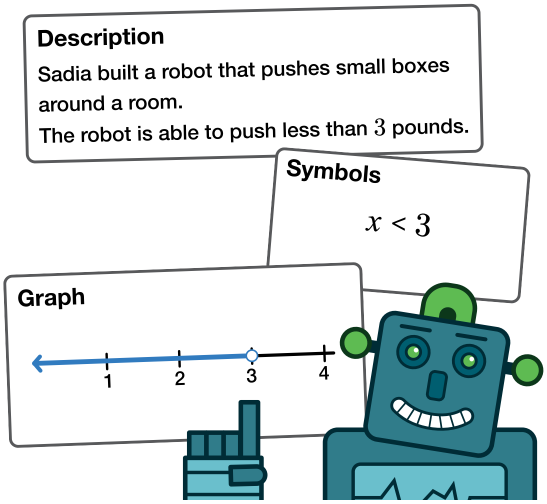
c. Plot as many other possible values for x as you can.

Synthesis

12. Circle one representation and explain how it shows that Sadia's robot can push a 2-pound box.

Description Symbols Graph

Explain your thinking.



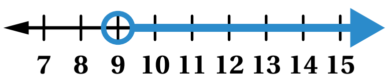
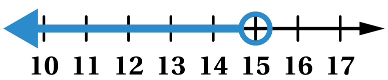
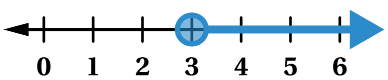
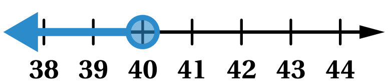
Lesson Practice 7.12

Lesson Summary

You can use variables, verbal descriptions, symbols, and number lines to represent inequalities related to real-world situations.

To represent an inequality on a number line, you can shade part of the number line to indicate that every point covered by the shaded region is a solution. Then draw an arrow on one end of the number line to show the possible solutions continue on in that direction.

Here are some examples.

Situation	Verbal Description	Inequality	Number Line
A two-year-old sleeps more than 9 hours a day.	Any value greater than 9.	$x > 9$	
A dog weighs less than 15 pounds.	Any value less than 15.	$x < 15$	
A violinist spends no less than 3 hours practicing each day.	Any value **greater than or equal to** 3.	$x \geq 3$	
The maximum speed of the car is 40 mph.	Any value **less than or equal to** 40.	$x \leq 40$	

Lesson Practice
7.12

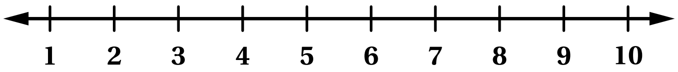
Name: .... Date: .... Period: ....

Problems 1–3: At a book sale, all books cost less than \$5.

1. List *three* possible prices for a book at this book sale.

2. Write an inequality to represent the cost of a book at the book sale, b.

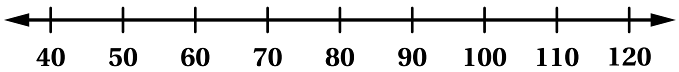
3. Graph all the possible prices of books at the sale.



Problems 4–7: n represents the number of eggs a sea turtle lays.

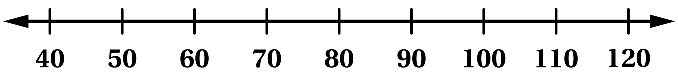
4. What does $50 < n$ mean in this situation?

5. Graph this inequality.



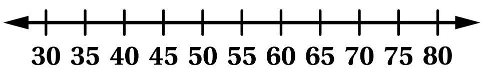
6. What does $n \leq 110$ mean in this situation?

7. Graph this inequality.

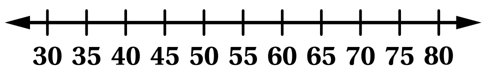


Problems 8–9: One day in Boston, the temperature was above 52°F and below 60°F.

8. Make two graphs, one to represent temperatures above 52°F and another to represent temperatures below 60°F.



Temperatures above 52°F



Temperatures below 60°F

9. Write two inequalities to represent the possible temperatures, T, on that day.

Lesson Practice
7.12

Name: .... Date: .... Period: ....

FAST Practice

10. The minimum amount of electricity used to run a computer is 30 watts. Write an inequality to represent the electricity, e, the computer needs.

Spiral Review

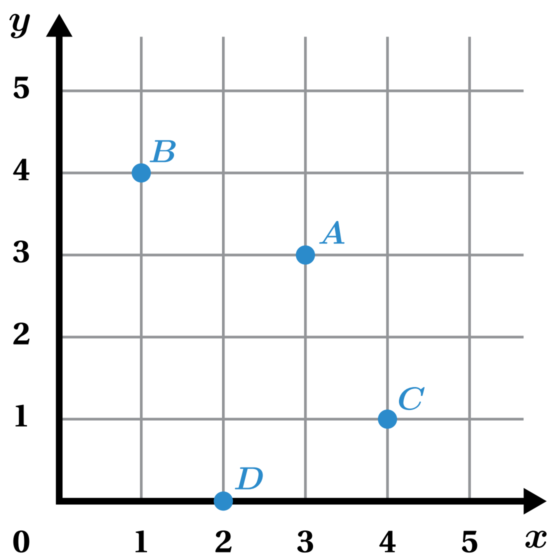
11. Select *all* of the temperatures that are warmer than -10°F.

- ☐ **A.** 10°F
- ☐ **B.** 0°F
- ☐ **C.** -5°F
- ☐ **D.** -11°F
- ☐ **E.** -20°F

12. Anika wrote the number sentence $|-5| = |5|$. Explain what it means in your own words.

13. Write the coordinates of each point shown on the graph.

Point	Coordinates
A	
B	
C	
D	

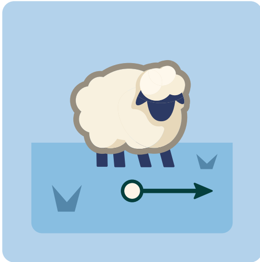


Name: ________________ Date: ________ Period: ________

MA.6.NSO.1.1, MA.6.AR.1.2, MA.6.AR.2.1, MTR.2.1, MTR.3.1

Shira's Solutions

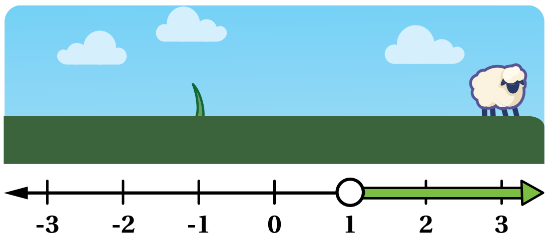
Let's find solutions to an inequality using a number line.



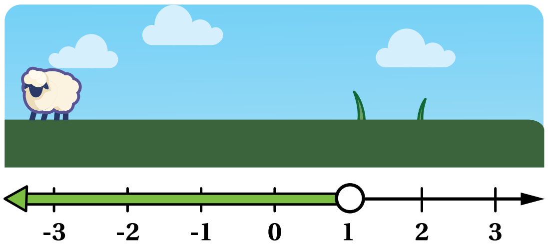
Warm-Up

1. Shira the Sheep loves eating all the blades of grass. These graphs show what happens when Shira eats grass based on different inequalities.

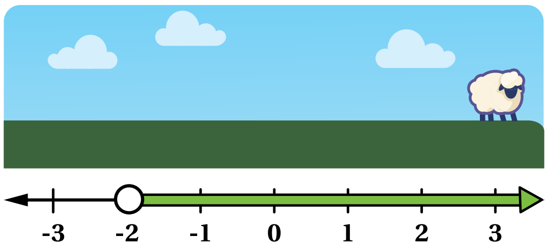
$x > 1$



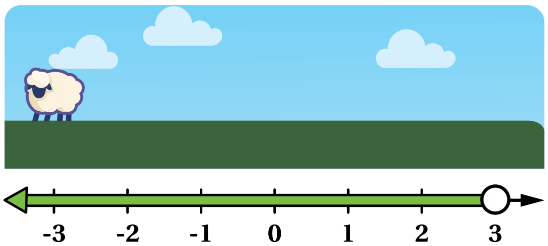
$x < 1$



$x > -2$



$x < 3$



Discuss: What do you notice?

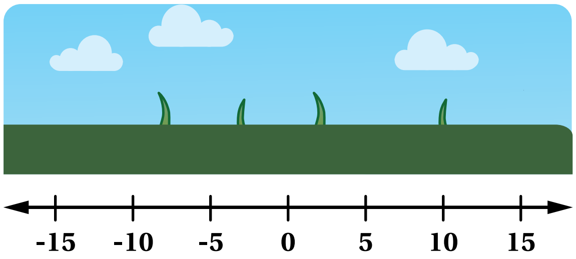
Name: .... Date: .... Period: ....

Connecting Graphs and Inequalities

2. Shira wants to eat these four blades of grass.

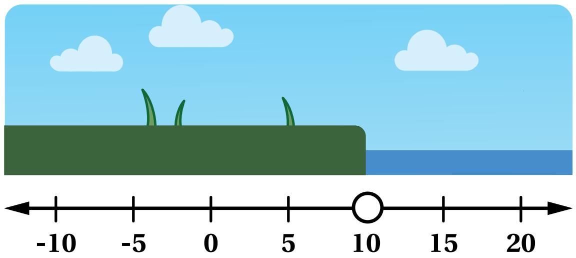
The inequality $11 < x$ did not work.

Fix this inequality to help Shira eat all the grass.



3. Shira the Sheep loves eating grass. She does not like water.

Write an inequality to help Shira the Sheep eat all the grass without falling in the water.

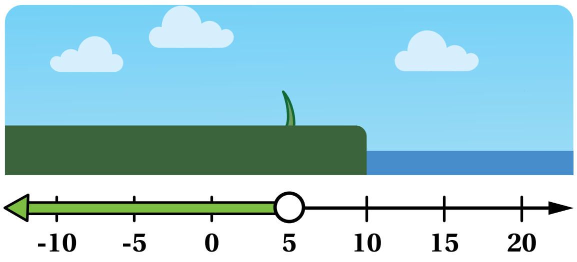


Explain your thinking.

Connecting Graphs and Inequalities (continued)

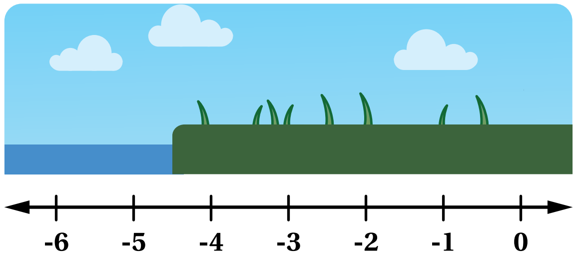
4. Kiana wrote $x < 5$ and was surprised that there was one blade of grass remaining.

Explain why 5 is not a **solution to the inequality** $x < 5$.

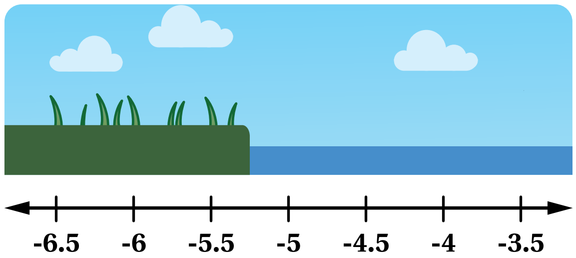


5. Write an inequality so that all the blades of grass are solutions and none of the water is.

Use the number line if it helps with your thinking.



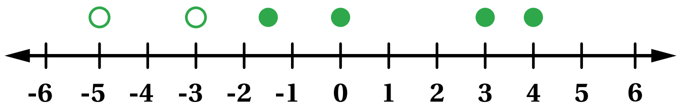
6. Write an inequality so that all the blades of grass are solutions and none of the water is.



Name: .... Date: .... Period: ....

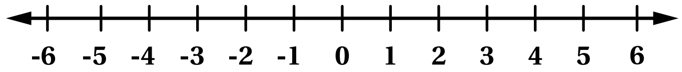
Solutions to an Inequality

7. Write an inequality so that all of the solid points are solutions and none of the open points are.



8. Write at least three solutions to the inequality $2.7 > x$.

Plot the solutions on the number line.



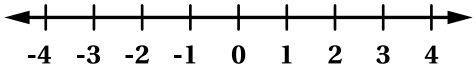
9. Let's look at other solutions to $2.7 > x$.

How many solutions does this inequality have? Explain your thinking.

Name: .... Date: .... Period: ....

Solutions to an Inequality (continued)

10. Make a graph of *all* the solutions to the inequality $x \geq -1.5$.



11. Match each inequality or solution with the graph that represents it.

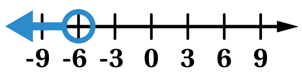
A. Some solutions: 5500, 6.5, -3

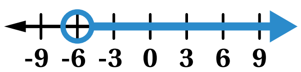
B. Some solutions: -100, 0.5, -6

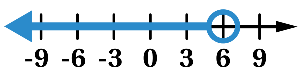
C. $x < -6$

D. $-6 > x$

E. $x < 6$

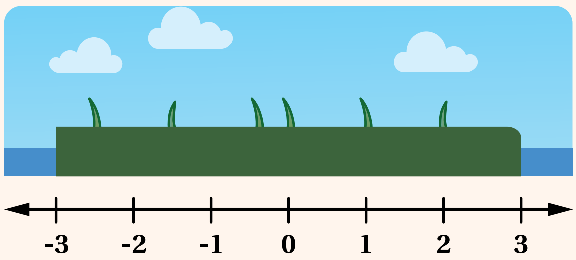






You're invited to explore more.

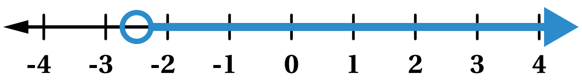
12. Write inequalities or describe how you can use inequalities to help Shira the Sheep eat all the blades of grass without falling in the water.



Synthesis

13. What does it mean for a number to be a solution to an inequality?

$x > -2.5$



Lesson Practice 7.13

Lesson Summary

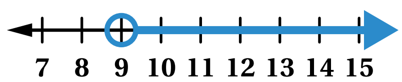
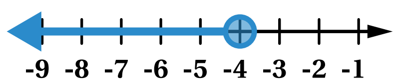
A **solution to an inequality** is any value that makes the inequality true. You can use a number line to represent the solutions to an inequality.

For example, for the inequality $c < 10$, you could say:

- 5 is a solution because $5 < 10$ is a true statement.
- 12 is not a solution because $12 < 10$ is not a true statement.

Some inequalities like $c < 10$ have an infinite number of solutions. We use inequality statements with variables and the symbols <, >, ≤, or ≥ to represent all the solutions.

Here are two examples.

Inequality	Description	Possible Solutions	Number Line
$x > 9$	Any value greater than 9.	9.75, 10, 11.3, 82	
$x \leq -4$	Any value less than or equal to -4.	-25, -13.7, -4.6, -4	

Lesson Practice
7.13

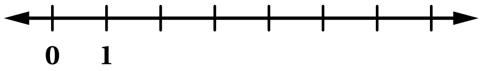
Name: .... Date: .... Period: ....

Problems 1–2: Here is the inequality $k > 5$.

1. Select *all* the values of k that are solutions to the inequality.

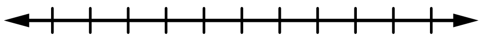
- ☐ **A.** 4.9
- ☐ **B.** 5
- ☐ **C.** 6
- ☐ **D.** 5.2
- ☐ **E.** -5.01

2. Make a graph of all the solutions to the inequality.

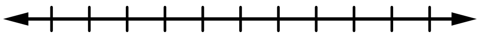


3. List three numbers that are solutions and three numbers that are not solutions to the inequality $x \le -2.25$. Use the number line if it helps with your thinking.

Solutions	Not Solutions

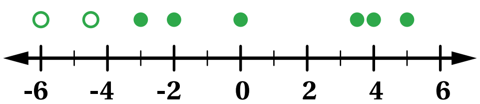


4. Write an inequality so that -0.75, -1.5, and -5 *are* solutions and 2, 0, and 100 *are not* solutions. Use the number line if it helps with your thinking.

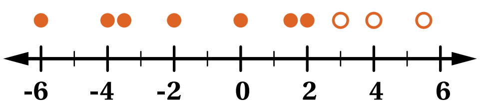


Problems 5–6: Write an inequality so that all of the solid points and none of the open points are solutions.

5.



6.



Name: ______ Date: ______ Period: ______

7. Use each number exactly once so that all of the statements are true.

-3   -2   -1   0   1   2   3

$x <$ ☐ **and** $x >$ ☐

Solutions to both inequalities:

☐ ☐ ☐

Not solutions to both inequalities:

☐ ☐

FAST Practice

8. Which value is a solution to both $-2.4 \geq x$ and $x < -3$?

(A.) -4.9   (B.) -3   (C.) -2.4   (D.) -1

Spiral Review

Problems 9–12: Complete each number sentence with the symbol <, >, or =.

9. $|-45|$ ______ -50

10. $|-50|$ ______ 45

11. $|-45|$ ______ 50

12. $|-45|$ ______ $|-50|$

13. Different numbers of hot sauce bottles and their costs are given in the table.

What is the unit rate in dollars per bottle?

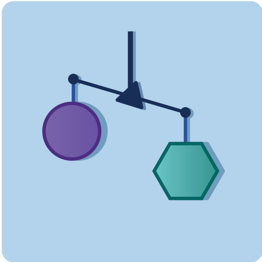
Number of Bottles	Cost
3	$10.62
5	$17.70
7	$24.78

Name: ______ Date: ______ Period: ______

MA.6.AR.1.2, MA.6.AR.2.1, MTR.1.1, MTR.4.1

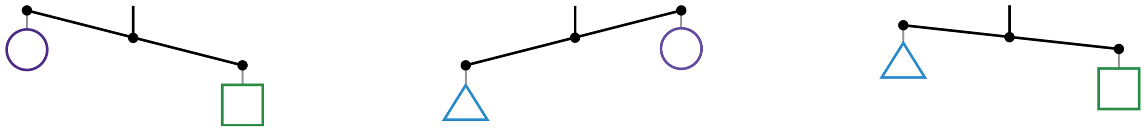
Comparing Weights

Let’s write and interpret inequalities to describe unbalanced hangers.



Warm-Up

1. Here are some hangers with different shapes.



Which shape is the heaviest? Circle one.

Circle Triangle Square

Explain your thinking.

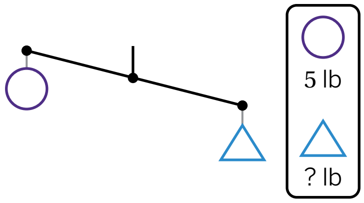
Activity 1

Name: .... Date: .... Period: ....

One-Variable Inequalities

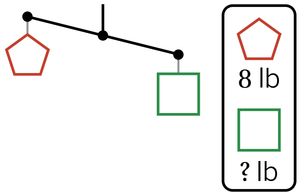
2. Here is an unbalanced hanger that Sora made. The circle weighs 5 pounds.

What is a possible weight for the triangle?



3. Here is an unbalanced hanger with a pentagon on one side and a square on the other.

The pentagon weighs 8 pounds.



Describe all the possible weights that will make the square *heavier* than the pentagon.

4. Jasmine and Terrance wrote inequalities to describe the possible weights, s, that make the square heavier than the pentagon.

- Jasmine: $8 < s$.
- Terrance: $s > 8$.

Whose inequality is correct? Circle one.

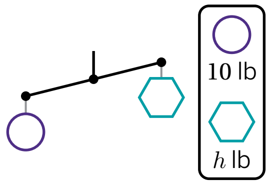
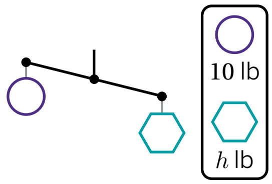
Jasmine's Terrance's Both Neither

Explain your thinking.

Name: ______ Date: ______ Period: ______

One-Variable Inequalities (continued)

5. Match each representation with one of the unbalanced hangers by placing a checkmark in the appropriate column.

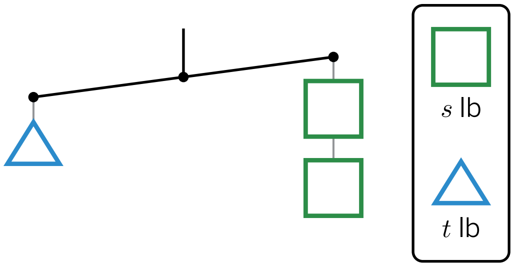
Representation		
10 pounds is less than the weight of a hexagon.		
One possible weight for this hexagon is 2 pounds.		
A hexagon weighs more than 10 pounds.		
10 pounds is greater than the weight of a hexagon.		
$10 > h$		
$10 < h$		
$h > 10$		

Activity 2

Name: .... Date: .... Period: ....

Many-Variable Inequalities

6. Which inequality represents this hanger?



A. $t < s$

B. $t < 2s$

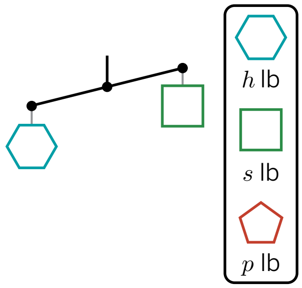
C. $t > 2s$

D. $t = 2s$

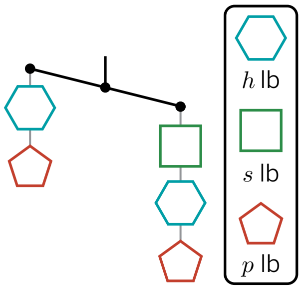
Explain your thinking.

7. Choose a hanger. Write an inequality that represents the hanger you chose.

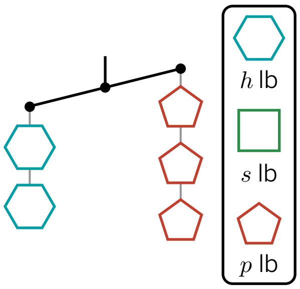
a



b



c



Name: ________ Date: ________ Period: ________

Challenge Creator

8. You will use the Activity 3 Sheet to create your own hanger challenge.

a **Make It!** On the Activity 3 Sheet, create a hanger challenge.

b **Solve It!** On this page, write an inequality that represents your hanger.

My Inequality

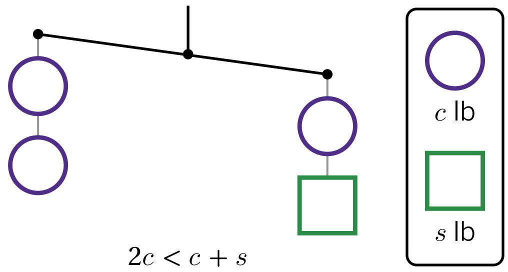
c **Swap It!** Swap your challenge with one or more partners. Then write an inequality for your partners' hanger.

	Inequality
Partner 1	
Partner 2	
Partner 3	
Partner 4	

Synthesis

9. Describe how an inequality is like an unbalanced hanger.

Use the diagram if it helps with your thinking.

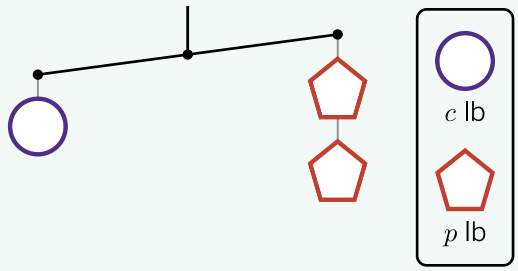


Lesson Practice 7.14

Lesson Summary

You can represent inequalities with unbalanced hangers. Inequalities may include a variable as a placeholder for an unknown value.

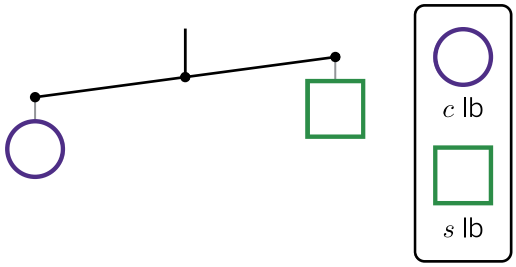
This hanger shows that the weight of the circle, c, is heavier than the weight of two pentagons, $2p$. This relationship can be represented by the inequality $c > 2p$ because these symbols mean that the circle has a greater weight than 2 pentagons.



Lesson Practice
7.14

Name: .... Date: .... Period: ....

Problems 1–4: Here is an unbalanced hanger.

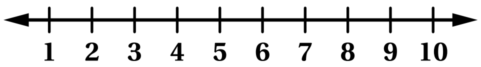


1. Write an inequality that compares the weights of the two shapes. Explain your inequality in words.

2. The square weighs 5 pounds. Rewrite your inequality to represent this.

3. List *three* possible weights for the circle. Use your inequality if it helps your thinking.

4. Graph all of the possible solutions for your inequality that represents the hanger.



Problems 5–7: There is leftover food that has been in Jin's refrigerator for d days.

5. What does the inequality $d < 7$ tell you about Jin's food?

6. What does the inequality $d > 0$ tell you about Jin's food?

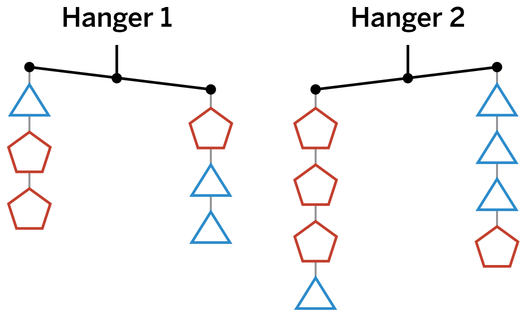
7. List *three* possible values of d that make both $d < 7$ and $d > 0$ true.

8. Gabriel is 12 years old. He has an older brother named Alejandro. Select *all* the inequalities that show the relationship between Gabriel's age, g, and Alejandro's age, a.

☐ **A.** $a < g$ ☐ **B.** $g < a$ ☐ **C.** $a > g$ ☐ **D.** $g > a$ ☐ **E.** $a > 12$

Name: .... Date: .... Period: ....

9. If Hanger 1 correctly shows how the pentagons and triangles balance, is it possible to create a hanger that looks like Hanger 2? Explain your thinking.



FAST Practice

10. Which of the following is a solution to the inequality $3c > 12$?

A. $c = 1$
B. $c = 3$
C. $c = 4$
D. $c = 9$

Spiral Review

Problems 11–12: Angel's family is driving to their grandmother's house, which is 325 miles away.

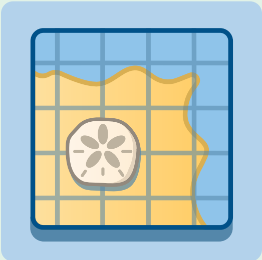
11. What percent of the distance have they traveled after driving 26 miles?

12. How far have they driven if they are 72% of the way to grandmother's house?

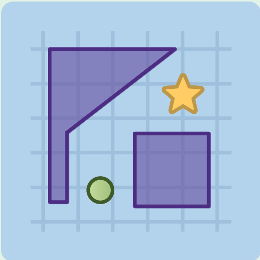
Sub-unit

4

The Coordinate Plane

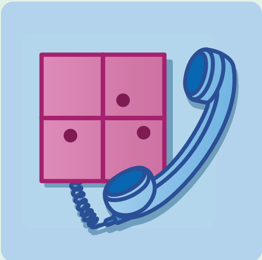


Lesson 15
Sand Dollar Search



Lesson 16
The A-maze-ing Coordinate Plane

Lesson 17
Polygon Maker

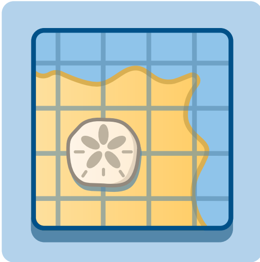


Lesson 18
Graph Telephone

Name: ______ Date: ______ Period: ______

MA.6.GR.1.1, MTR.1.1, MTR.4.1

Sand Dollar Search

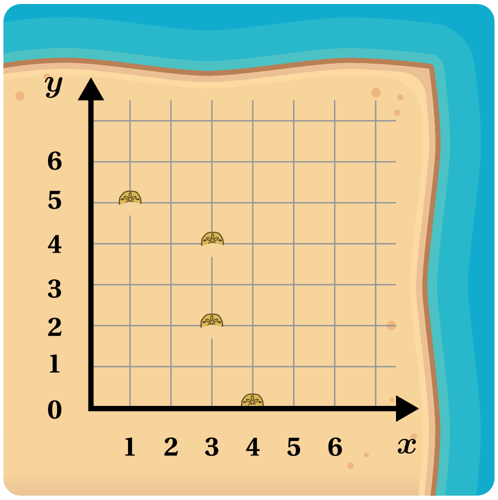


Let's explore negative numbers in the coordinate plane.

Warm-Up

1. Here is a map of part of an island.

Collect all four sand dollars by labeling each one with its coordinates.

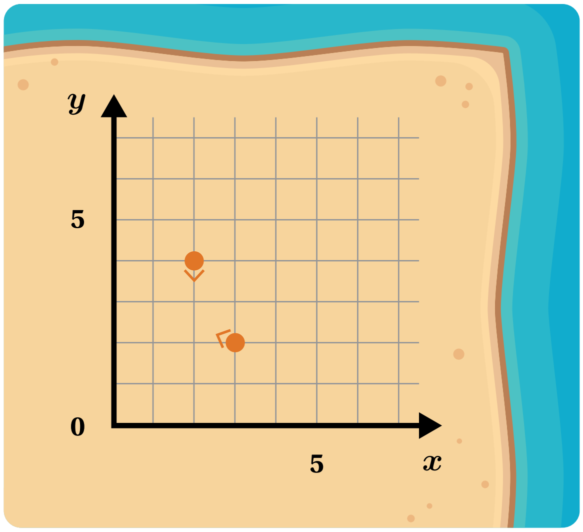


Name: .... Date: .... Period: ....

Find the Sand Dollar

2. Another sand dollar is buried on the island.

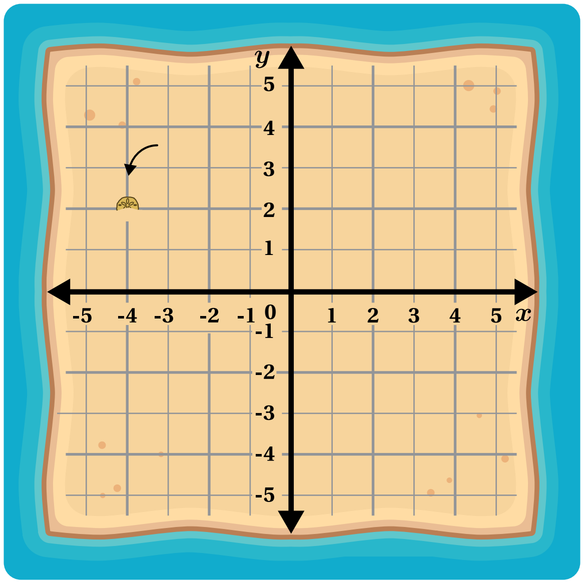
Here are two guesses for where the sand dollar is. Write coordinates for where you would search for it.



Coordinates (x, y)
$(3, 2)$
$(2, 4)$

3. You find a new map of the entire island.

A new sand dollar is visible. Write a clue that describes its location.



Activity 2

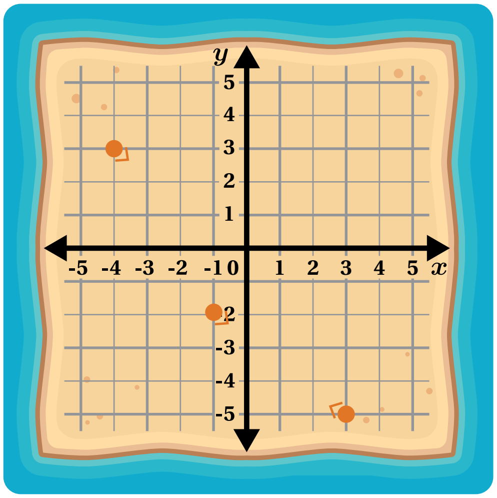
Name: ________ Date: ________ Period: ________

The Search Continues

4. There is another sand dollar buried somewhere on the island.

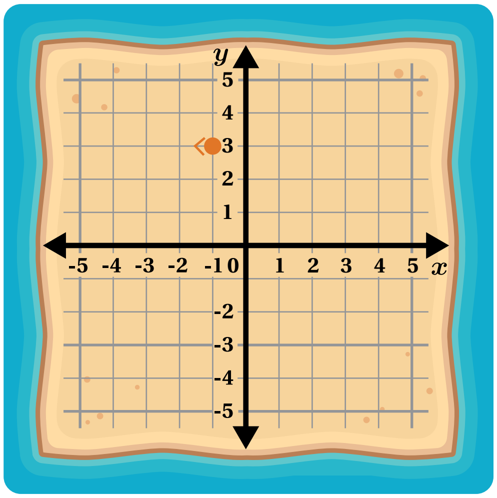
Here are three guesses for where the sand dollar is. Write coordinates for where you would search for it.

Coordinates (x, y)
(-4, 3)
(-1, -2)
(3, -5)



5. Esi was searching for another sand dollar. She entered (-1, 3) and the arrow pointed to the left.

What point would you recommend Esi try next? Explain your thinking.

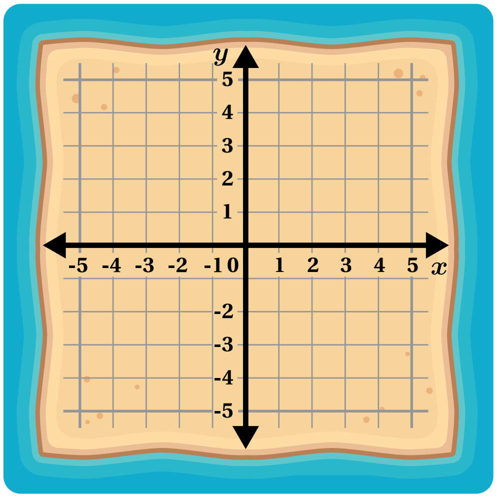


6. A sand dollar is hidden at a location that has two negative coordinates.

a Draw the sand dollar at a possible location.

Compare your point with a partner.

b **Discuss:** What do your points have in common?



Activity 3

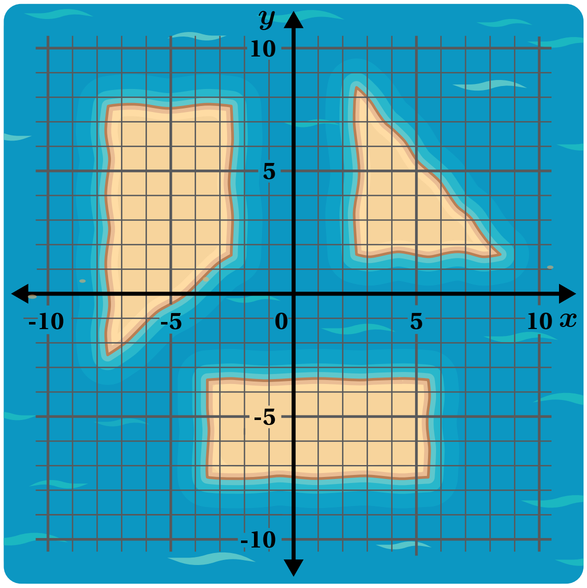
Name: ______ Date: ______ Period: ______

Find That Sand Dollar!

You will use support cards for this activity.

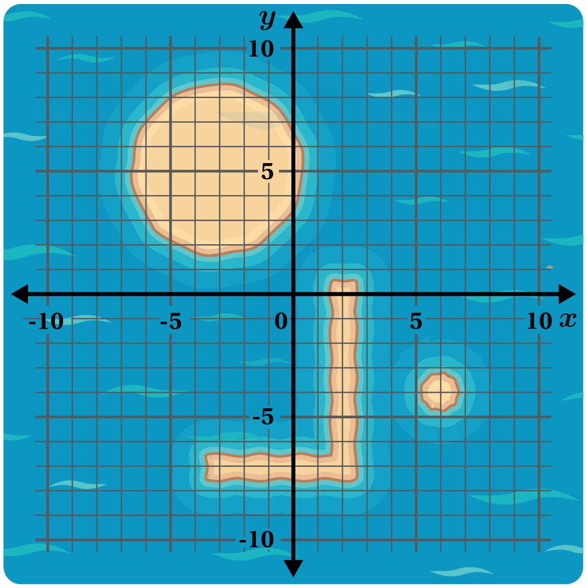
7. **Round 1:** There is a sand dollar on each of these islands.

   When it is your turn to find the sand dollar, label each guess and the feedback you get until you find all three sand dollars.



8. **Round 2:** Here are three new islands.

   Label each guess and the feedback you get until you find all three sand dollars.



Name: ______ Date: ______ Period: ______

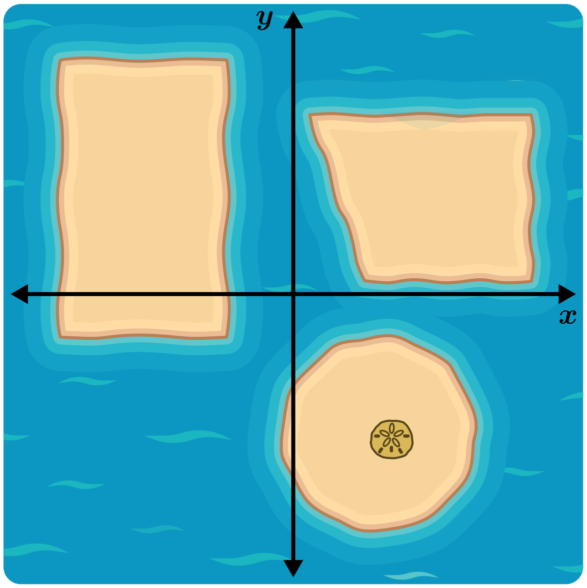
Find That Sand Dollar! (continued)

9. Which of these could be the location of the sand dollar?

A. $(4, 6)$ **B.** $(4, -6)$

C. $(-6, -4)$ **D.** $(-6, 4)$

Explain your thinking.



You're invited to explore more.

10. Ariel was searching for a sand dollar and the arrow pointed up.

a What must be true about the sand dollar's x-*coordinate*?

A. $x = -5$

B. $x < -5$

C. $x > -5$

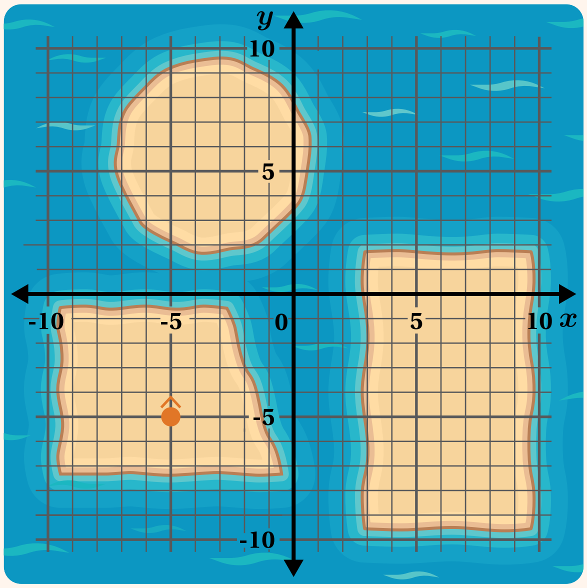
b What must be true about the sand dollar's y-*coordinate*?

A. $y = -5$

B. $y < -5$

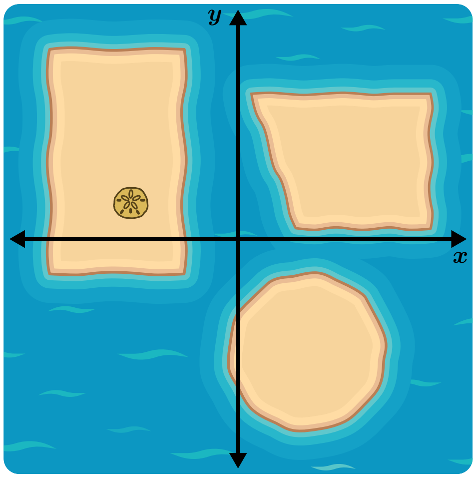
C. $y > -5$

Explain your thinking.



Synthesis

11. Explain what you know about the coordinates of this sand dollar.

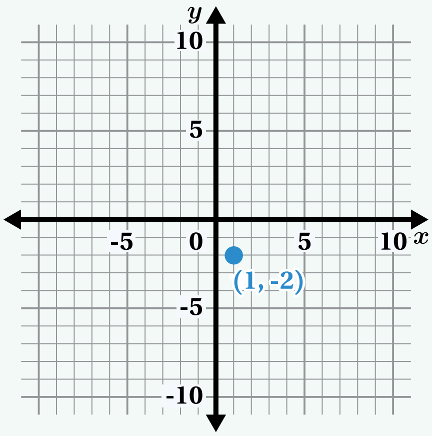


Lesson Practice 7.15

Lesson Summary

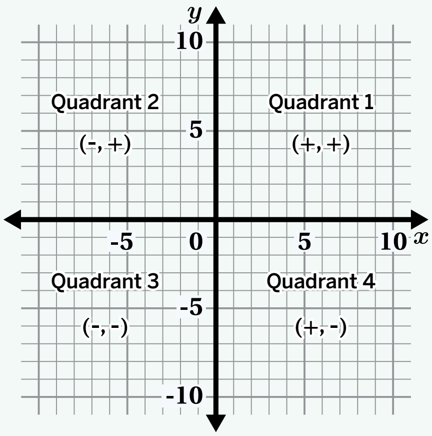
You can include positive and negative numbers along the x- and y-*axes*, just like on a number line. The x- and y-axes cross at the *origin*, or the point $(0, 0)$.

Ordered pairs are written (x, y), where the x-value is the horizontal location (left and right) and the y-value is the vertical location (up and down). For example, the point $(1, \text{-}2)$ is 1 unit to the right and 2 units down from the origin.



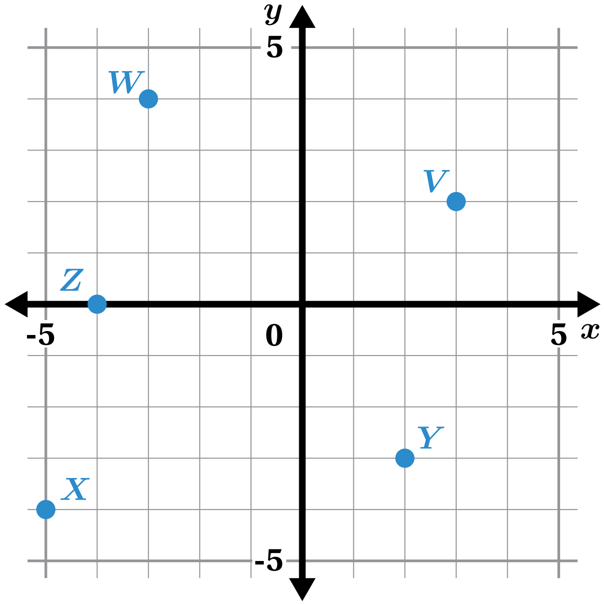
The four regions of the *coordinate plane* are called **quadrants**. They are numbered 1–4 starting with the top right quadrant and going in a circle counter-clockwise.

The image shows each quadrant, along with the sign of the x- and y-values in that quadrant.



Name: .... Date: .... Period: ....

Problems 1–5: Here is a coordinate plane.



1. Write the coordinates of each point in the table.

Point	Coordinates
V	
W	
X	
Y	
Z	

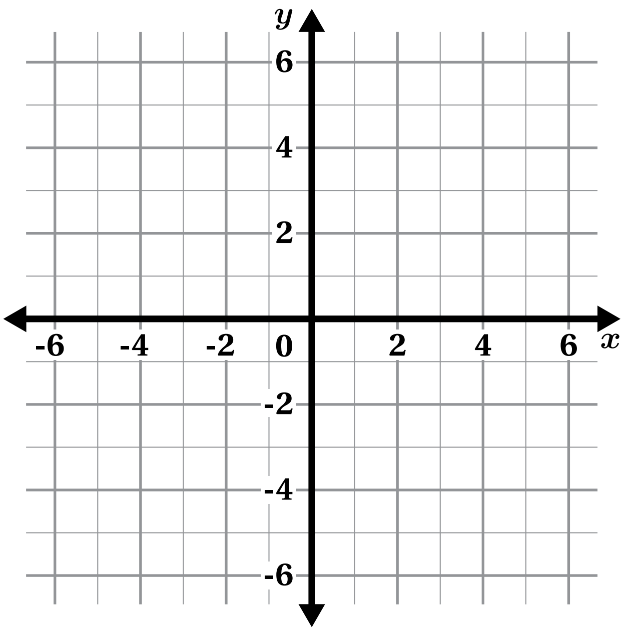
2. Plot and label point A at $(-2, 1)$.

3. Plot and label point P at $(1, -2)$.

4. Plot two points that are each 2 units away from point V. Label each point with its coordinates.

5. Point Q is more than 3 units directly to the left of point Y. Write at least one thing you know about the coordinates of point Q.

6. Draw the first letter of your name on the coordinate plane so that at least part of your drawing is in each quadrant.



Lesson Practice
7.15

Name: .... Date: .... Period: ....

FAST Practice

7. Select *all* the possible locations on the coordinate plane of the point $(x, -2)$, where $x \leq 0$.

- ☐ **A.** Quadrant 2
- ☐ **B.** Quadrant 3
- ☐ **C.** Quadrant 4
- ☐ **D.** x-axis
- ☐ **E.** y-axis

Spiral Review

8. The height requirement for an amusement park ride is written as $h > 42$, where h represents a rider's height in inches. Write a sentence or sketch a sign that describes these rules as clearly as possible.

9. Select *all* the values of x that are solutions to the inequality $x > -2$.

- ☐ **A.** -1
- ☐ **B.** -2
- ☐ **C.** -3
- ☐ **D.** -2.1
- ☐ **E.** -1.8

Problems 10–13: Solve each equation.

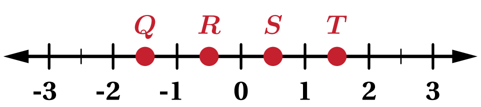
10. $\frac{3}{5}a = \frac{12}{5}$

11. $b + 8.3 = 8.9$

12. $1 = \frac{1}{4}c$

13. $2d = 6.4$

14. Here is a number line with points Q, R, S, and T. Which point represents $-1\frac{1}{2}$?



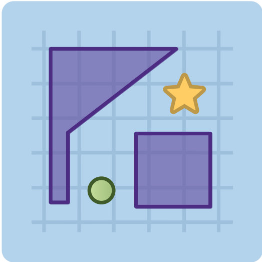
Unit 7
Lesson
16

Name: ______ Date: ______ Period: ______

MA.6.GR.1.1, MTR.1.1, MTR.4.1

The A-maze-ing Coordinate Plane

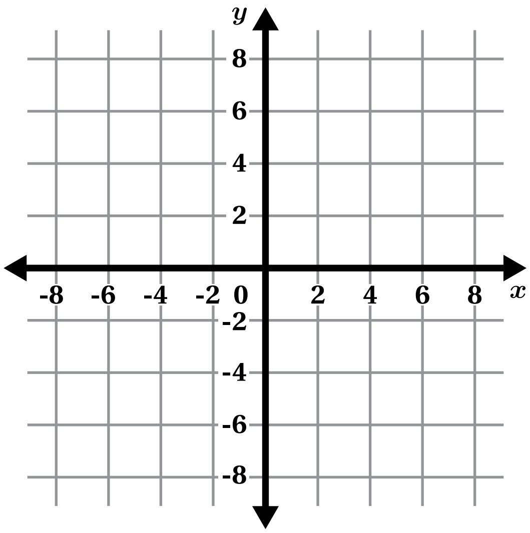
Let's practice with the coordinate plane using mazes.



Warm-Up

1. a Plot a point at the coordinates $(3, -1)$.

b Explain how you chose where to plot the point.



Activity 1

Name: ______________________ Date: __________ Period: __________

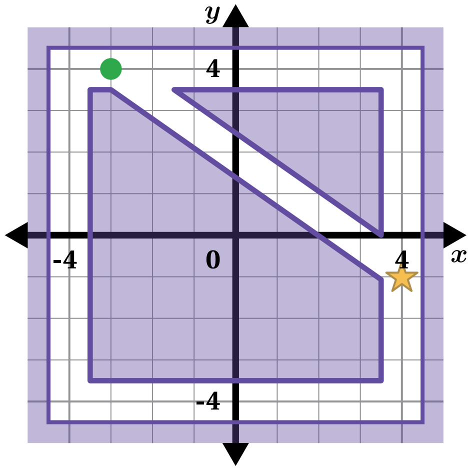
Maze Challenges

The goal of the maze is to move the ball to the star without hitting any walls.
For each challenge:

- Write the coordinates of a path to move the ball to the star. Your path can have several coordinate points.
- Pass your paper to a partner to draw the path using your coordinates.

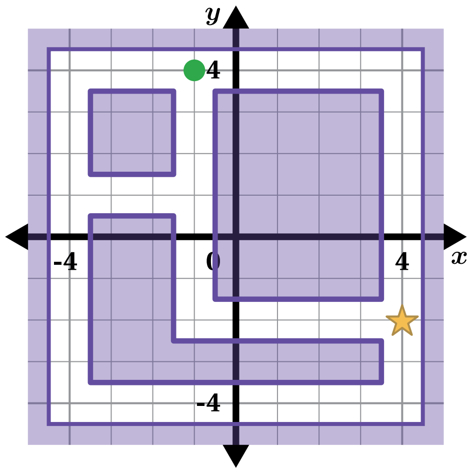
2. Write the coordinates of a path to move the ball to the star.

Your Path
(-3, 4)



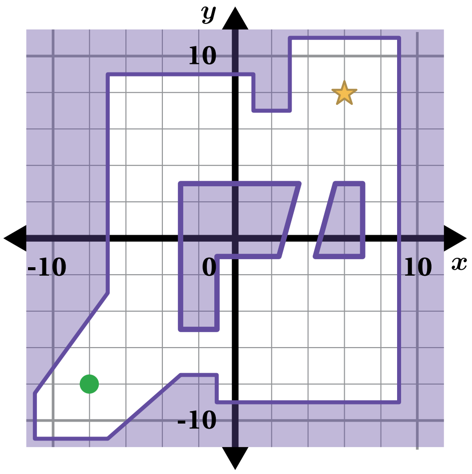
3. Write the coordinates of a path to move the ball to the star. Note: You do not need to use all the rows.

Your Path
(-1, 4)



4. Write the coordinates of a path to move the ball to the star.

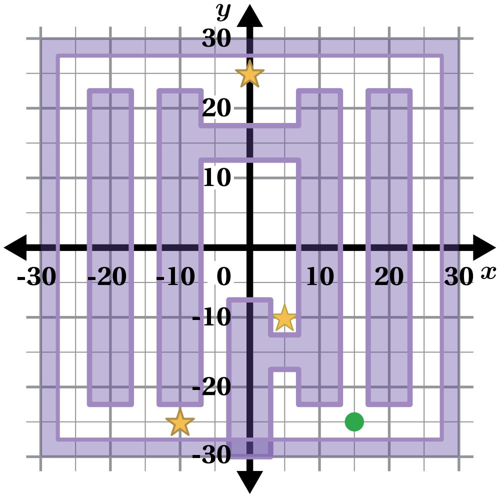
Your Path
(-8, -8)



Name: ______ Date: ______ Period: ______

More Mazes

5. In this maze, there are multiple stars.



a. Sketch a path to collect them all.

b. Write the coordinates that create your path.

Your Path
(15, -25)

Name: .... Date: .... Period: ....

More Mazes (continued)

6. Felipe and Annika were working on the previous maze.

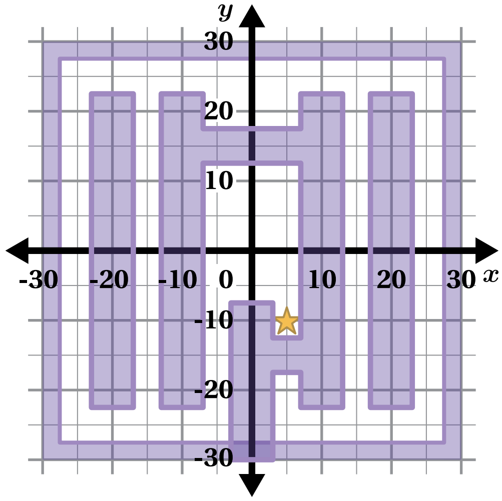
Felipe says: *The star's coordinates are* $(1, -2)$.

Annika says: *The coordinates are* $(1, -10)$.

Whose claim is correct? Circle one.

Felipe's Annika's Neither

Explain your thinking.

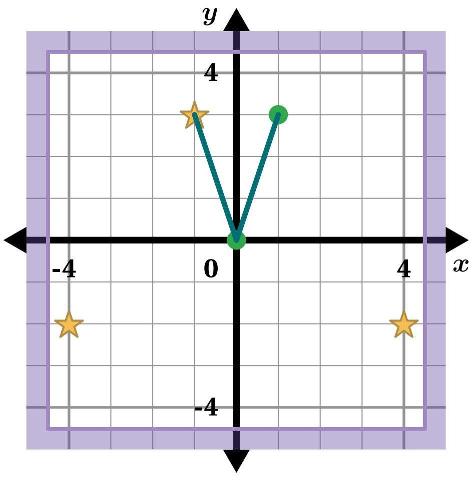


Name: Date: Period:

Reflections on the Plane

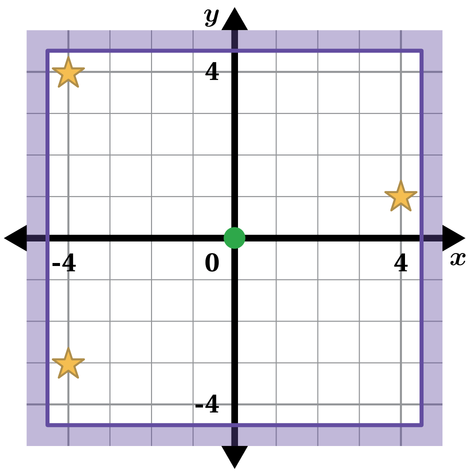
7. Complete the table to collect all the stars. Sketch the new paths on the graph.

Your Path	Mirrored Path
(0, 0)	(0, 0)
(1, 3)	(-1, 3)



8. The ball is mirrored across the y-axis. Write coordinates to collect all the stars. Choose your two points wisely!

Your Path	Mirrored Path
(0, 0)	(0, 0)



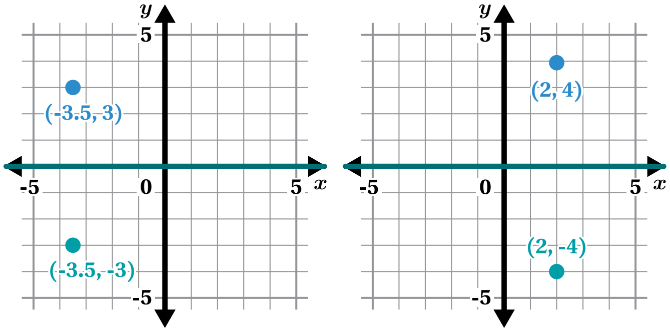
Name: ______________ Date: ________ Period: ________

Reflections on the Plane (continued)

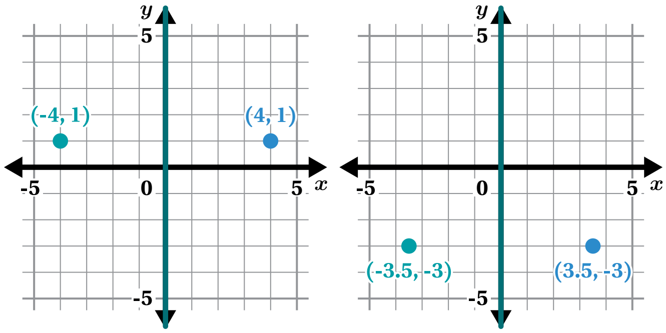
9. Here are several coordinates of a ball and its reflection.

Write at least two patterns about the coordinates that you found interesting.

Reflections Across the x-axis

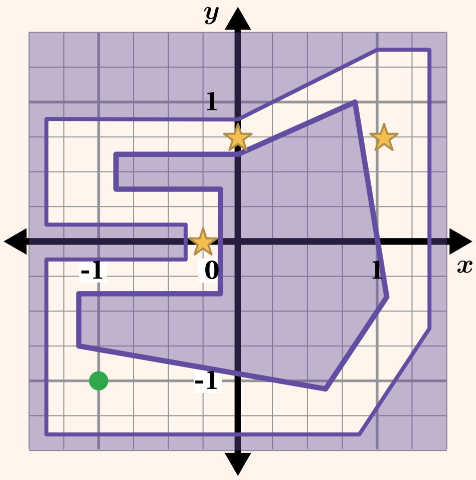


Reflections Across the y-axis



You're invited to explore more.

10. Sketch a path to collect all the stars. Then write the coordinates of your path.



Synthesis

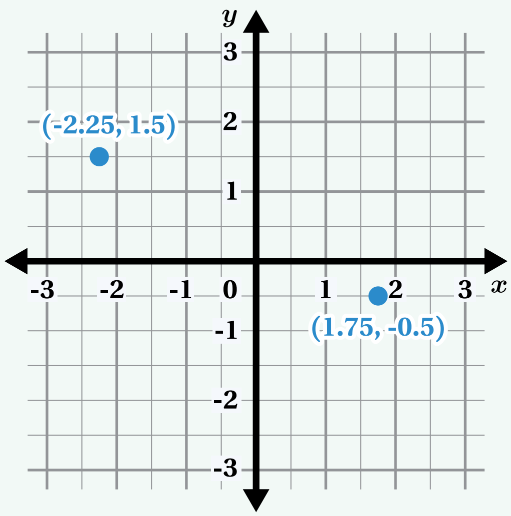
11. Look back over the challenges and select the one you are most proud of solving.

Write some advice for someone solving that challenge.

Lesson Practice 7.16

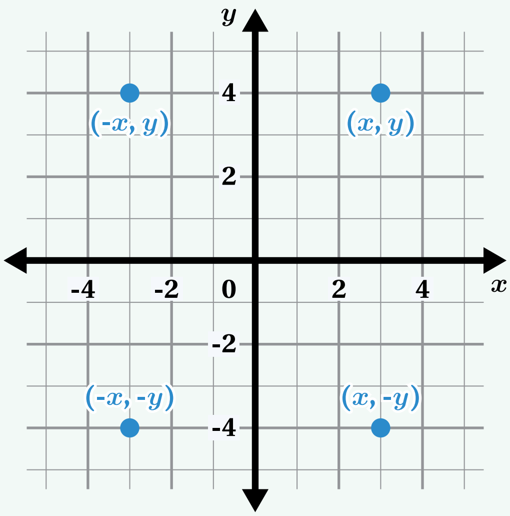
Lesson Summary

You can use different scales to show very big or very small numbers on a coordinate plane. In these cases, the interval is still consistent (e.g., goes by 2s or 0.5s). Sometimes points are plotted in between tick marks. Consider the points $(1.75, -0.5)$ and $(-2.25, 1.5)$ and where they appear on the graph shown.



The points $(3, 4)$ and $(3, -4)$ have the same x-coordinate and the y-coordinates only differ by their sign. We can see on the graph that those points are a *reflection*, or a mirror, of each other across the x-axis.

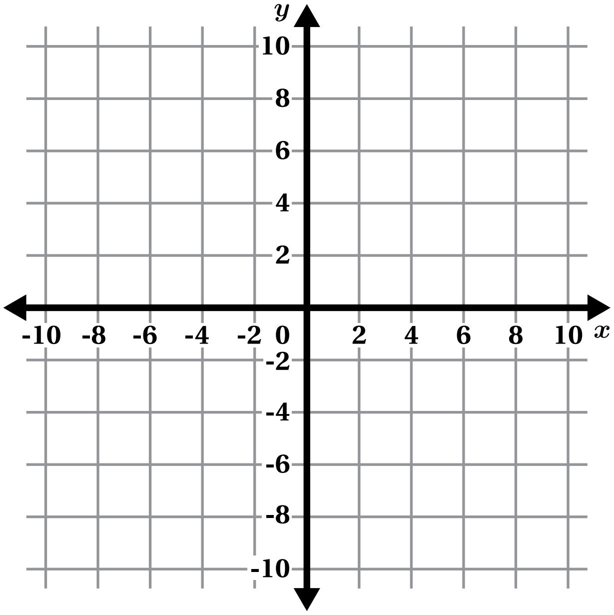
The points $(-3, -4)$ and $(3, -4)$ have the same y-coordinate and the x-coordinates only differ by their sign. We can see on the graph that those points are a reflection, or a mirror, of each other across the y-axis.



Lesson Practice
7.16

Name: .... Date: .... Period: ....

Problems 1–2: Here is a graph.

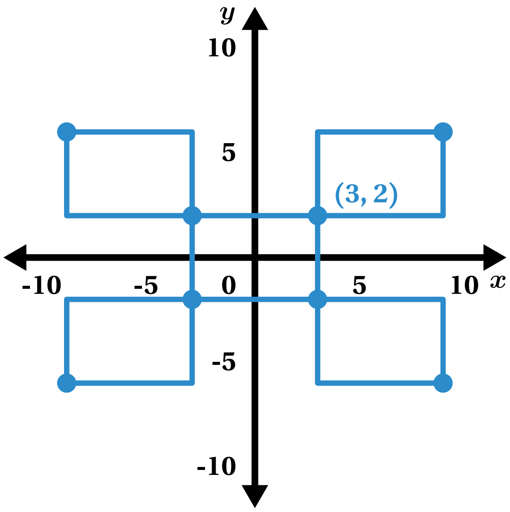


1. Plot and connect each point in order. Plot all of Column 1 first, then Column 2.

Column 1	Column 2
(-2, -9)	(-2, 4)
(-2, -3)	(0, 2)
(-4, -3)	(2, 4)
(-6, -1)	(2, -1)
(-6, 4)	(0, -3)
(-4, 2)	(-2, -3)

2. Describe your strategy for plotting the point (-2, -9) in Problem 1.

Problems 3–4: Here are identical rectangles on a coordinate plane. The origin is in the center of the middle rectangle.



3. Label the coordinates of the remaining points.

4. What patterns do you notice?

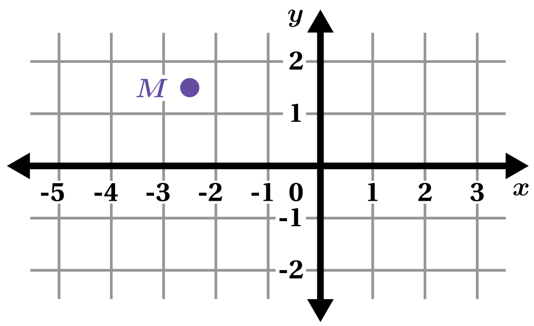
Name: ________ Date: ________ Period: ________

5. Point A is located at $(3, n)$ in the coordinate plane. Point B is located at $(-3, n)$. What do you know about points A and B?

FAST Practice

6. What is the value of the x-coordinate of point M? Your answer should be a decimal rounded to the nearest 0.5.

☐



Spiral Review

Problems 7–10: Complete each number sentence with the symbol <, >, or =. Use a number line if it helps with your thinking.

7. $\left|-\frac{3}{2}\right| \square -\frac{2}{3}$

8. $-\frac{3}{2} \square -\frac{2}{3}$

9. $-\frac{3}{2} \square \frac{2}{3}$

10. $\frac{3}{2} \square \left|-\frac{2}{3}\right|$

Problems 11–12: DeShawn's dog weighs 34 pounds. Jacy's dog weighs 12 pounds more than DeShawn's dog.

11. Select *all* the equations that show the weight of Jacy's dog, j.

- ☐ **A.** $j = 34 + 12$
- ☐ **B.** $j = 34 - 12$
- ☐ **C.** $j + 12 = 34$
- ☐ **D.** $j - 12 = 34$
- ☐ **E.** $j = 34 \cdot 12$

12. Determine how much Jacy's dog weighs.

Unit 7
Lesson
17

Name: ______ Date: ______ Period: ______

MA.6.GR.1.1, MA.6.GR.1.2, MA.6.GR.1.3, MTR.1.1, MTR.4.1

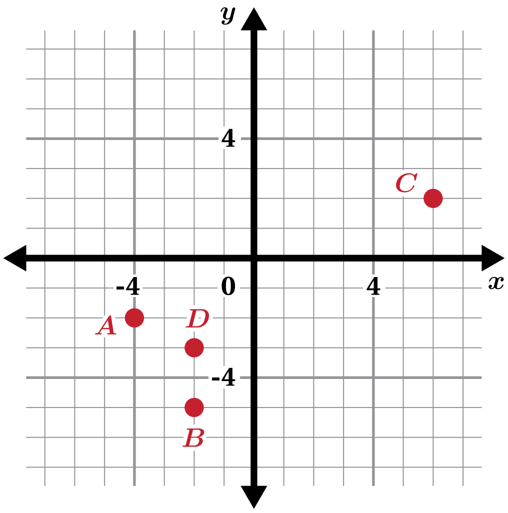
Polygon Maker

Let’s explore polygons on the coordinate plane.

Warm-Up

1. A Polygon Maker connects points on a coordinate plane to create a *polygon*.

Point	Coordinates
A	(-4, -2)
B	(-2, -5)
C	(6, 2)
D	(-2, -3)



a Connect the coordinates to create a polygon. Tell a classmate what the polygon reminds you of.

b Change one coordinate and connect the points to make a new polygon.

Activity 1

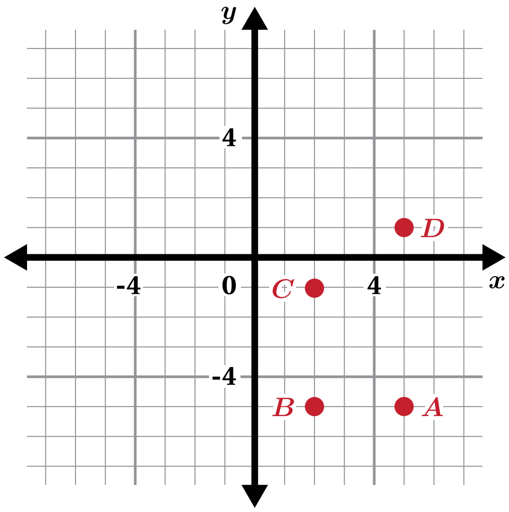
Name: ______ Date: ______ Period: ______

A Polygon

2. Melissa wants to make a rectangle.

Change some of her coordinates so that the Polygon Maker makes a rectangle.

Point	Coordinates
A	(5, -5)
B	(2, -5)
C	(2, -1)
D	(5, 1)



3. **a** Calculate the perimeter of the rectangle you made.

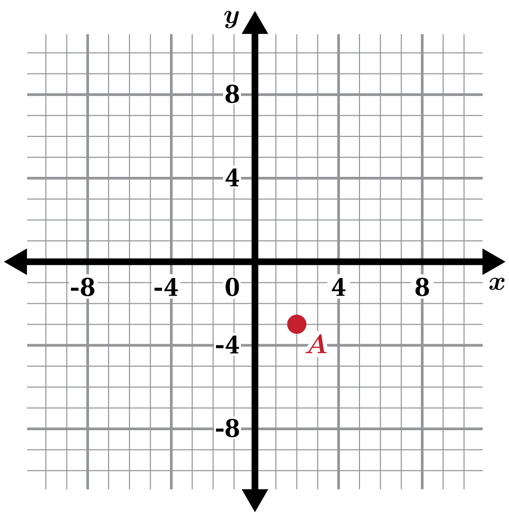
Explain your thinking.

b Calculate the area of the rectangle you made.

4. Hamza wants to make a square.

Complete the table with coordinates that will make a square. Sketch the square you made on the coordinate plane.

Point	Coordinates
A	(2, -3)
B	
C	
D	

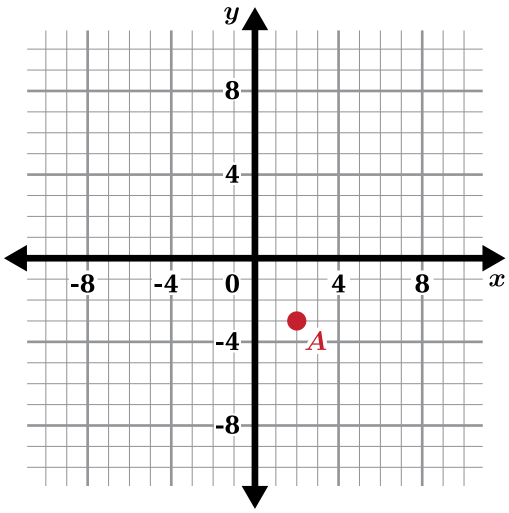


Name: .... Date: .... Period: ....

A Polygon (continued)

5. Here are Vicente's coordinates.

Point	Coordinates
A	(2, -3)
B	(2, 4)
C	(9, 4)
D	(9, -3)



Did he make a square? Circle one.

Yes No I'm not sure

Show or explain your thinking.

6. Vicente says: *My square has side lengths of 7.*

Sketch or describe where you see 7 in the table and graph.

Point	Coordinates
A	(2, -3)
B	(2, 4)
C	(9, 4)
D	(9, -3)

Activity 2

Name: .... Date: .... Period: ....

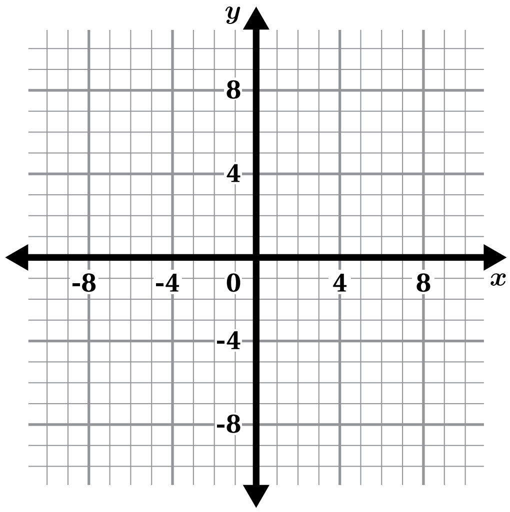
Make Your Own Polygon

7. Here are three descriptions:

- Looks like a house that's 15 units tall.
- Looks like a capital L that's 6 units wide.
- A rectangle with an area of 24 square units.

Choose one description. Write and sketch coordinates to make a polygon that matches that description.

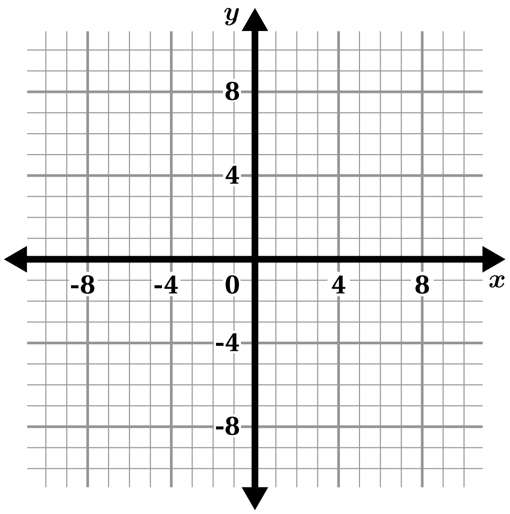
Point	Coordinates
A	
B	
C	
D	
E	



8. You will create your own polygon challenge.

- Write a description of a polygon on a slip of paper. Make it fun (like "looks like a pinecone") and include the size (like "must have a perimeter of 20 units").
- On this page, write the coordinates and sketch a polygon that matches your description.

Point	Coordinates
A	
B	
C	
D	



Name: .... Date: .... Period: ....

Make Your Own Polygon (continued)

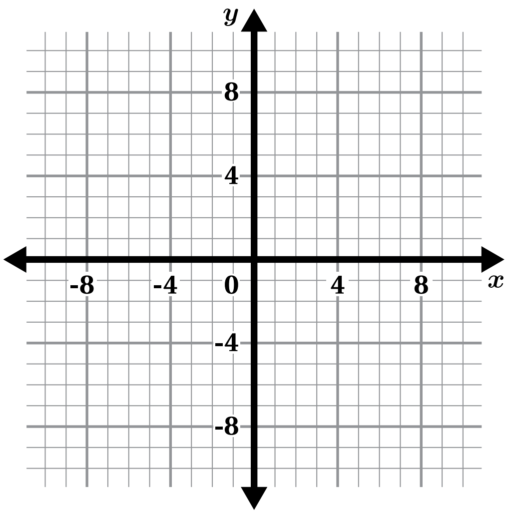
9. You will swap your polygon challenge with two classmates.

- Hand your slip of paper to a partner and invite them to use coordinates to create a polygon on their paper.
- When they finish, share your original polygon and compare.
- Write the coordinates and graph your partners' polygon challenges on this page.

Partner 1

Description:

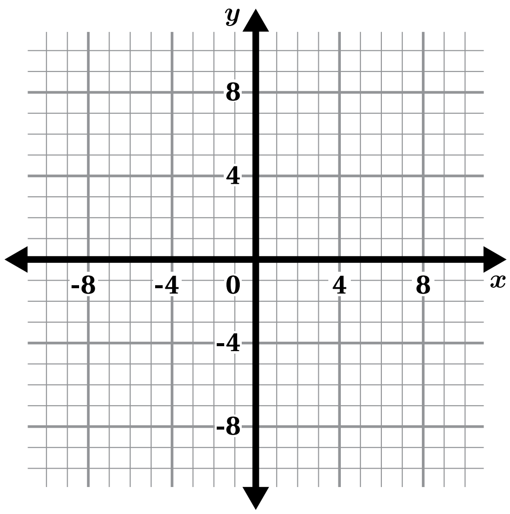
Point	Coordinates
A	
B	
C	
D	



Partner 2

Description:

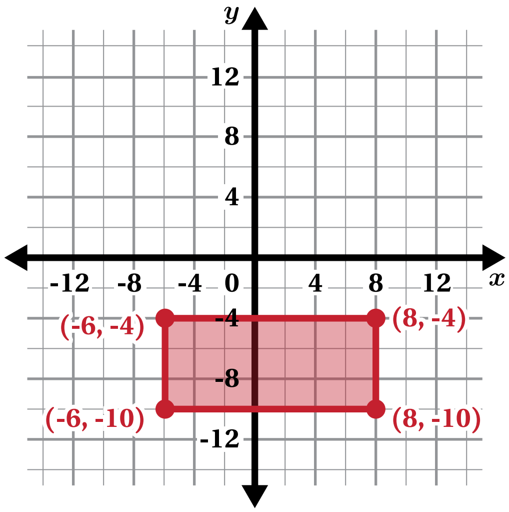
Point	Coordinates
A	
B	
C	
D	



Synthesis

10. Describe how you can use the coordinates to calculate the side lengths of a rectangle. Use the table and graph if they help with your explanation.

Point	Coordinates
A	(-6, -4)
B	(8, -4)
C	(8, -10)
D	(-6, -10)



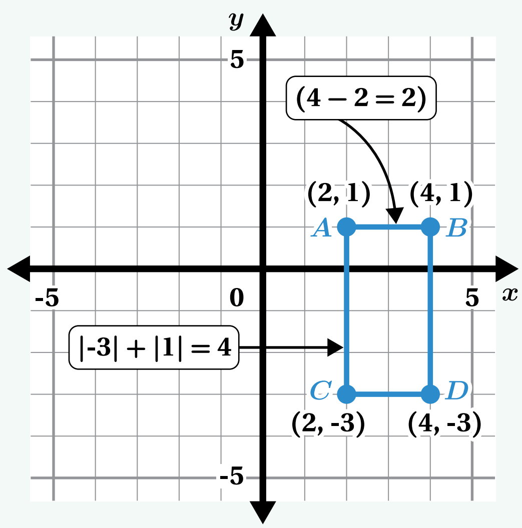
Lesson Practice 7.17

Lesson Summary

You can plot points on the coordinate plane to create *polygons*. When the vertices of a polygon are horizontally or vertically aligned in the graph, you can count the number of units between them to determine the length of that side.

You can also calculate the side lengths using the coordinates of each vertex. Here are two calculation strategies:

- If the coordinates are in the same quadrant, like points A and B, find the length by subtracting the coordinates that are different.
- If the coordinates are in different quadrants, like points A and C, use the absolute value to determine the distance each point is from the axis between them.



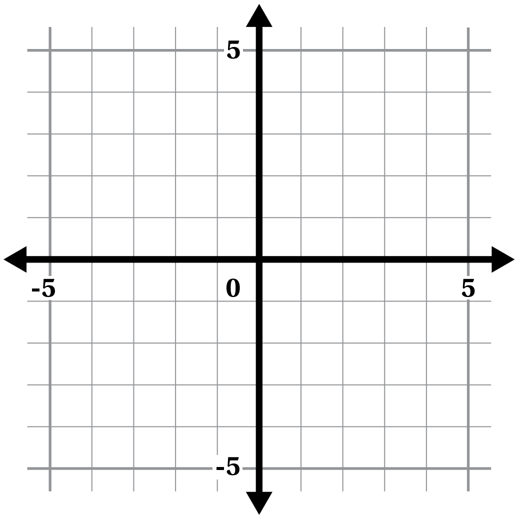
Lesson Practice
7.17

Name: .... Date: .... Period: ....

Problems 1–4: Here is a set of coordinates.

Point	Coordinates
A	$(-3, 1)$
B	$(3, 1)$
C	$(3, -4)$
D	$(-1, -4)$
E	$(-1, -2)$
F	$(-3, -2)$

1. Plot each point.



2. Connect the points to create polygon $ABCDEF$.

3. Determine the length of the segment between point A and point B.

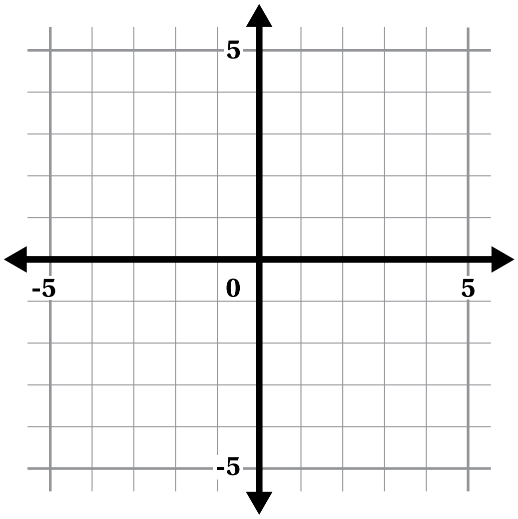
4. Determine the perimeter of polygon $ABCDEF$.

Problems 5–7: Three points of a rectangle are $(3, 0)$, $(3, -5)$, and $(-4, -5)$.

5. What are the coordinates of the missing point?

6. Sketch the rectangle and calculate its perimeter.

7. Calculate the area of the rectangle.



Lesson Practice
7.17

Name: .... Date: .... Period: ....

FAST Practice

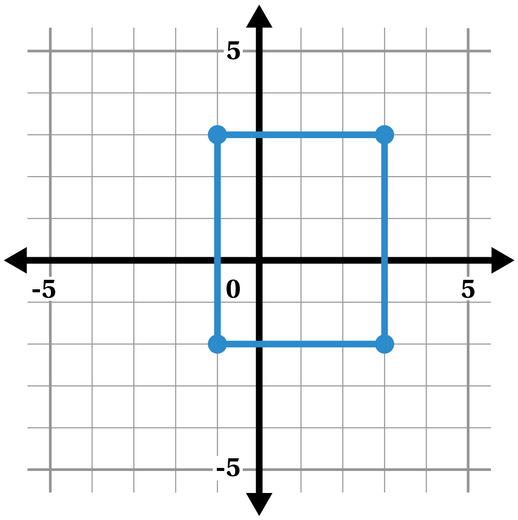
8. Gabriela drew this rectangle. Which statement is incorrect about Gabriela's drawing?

A. One of the coordinates of the rectangle is $(-1, -2)$.

B. The area of the rectangle is 20 square units.

C. The perimeter of the rectangle is 9 units.

D. The side lengths of the rectangle are 5 units and 4 units.



Spiral Review

9. Select *all* the values of x that are solutions to the inequality $-0.5 > x$.

- ☐ A. 0
- ☐ B. -1
- ☐ C. -0.40
- ☐ D. -0.6
- ☐ E. -0.55

10. Is $\frac{12}{5} \div \frac{3}{5}$ greater than, less than, or equal to 1?

11. What is the value of $\frac{9}{5} \div \frac{3}{5}$?

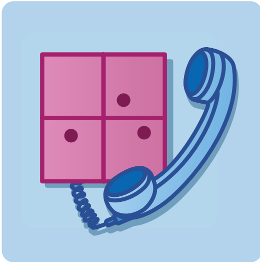
12. At top speed, an elephant can run 25 miles per hour and a giraffe can run 16 miles in $\frac{1}{2}$ hour. Which animal runs faster?

Unit 7
Lesson
18

Name: ________ Date: ________ Period: ________

MA.6.GR.1.1, MA.6.GR.1.3, MTR.4.1, MTR.5.1, MTR.7.1

Graph Telephone

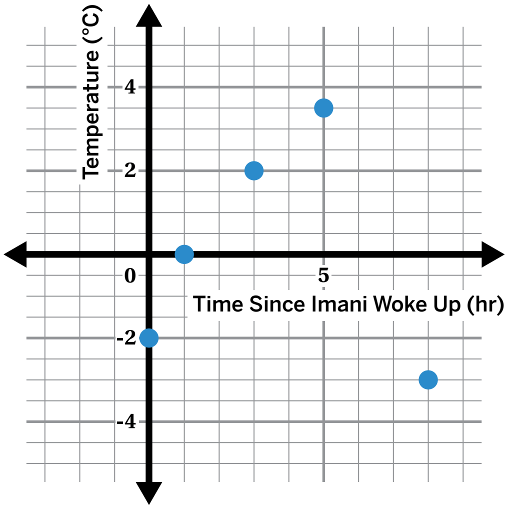


Let's plot and interpret coordinates to make sense of situations in context.

Warm-Up

Imani woke up at 9:30 AM and tracked the temperature outside over the course of the day.

9:30 AM	10:30 AM	12:30 PM	2:30 PM	5:30 PM
-2°C	0°C	2°C	3.5°C	-3°C



1. Write a story about the temperature over the course of the day.

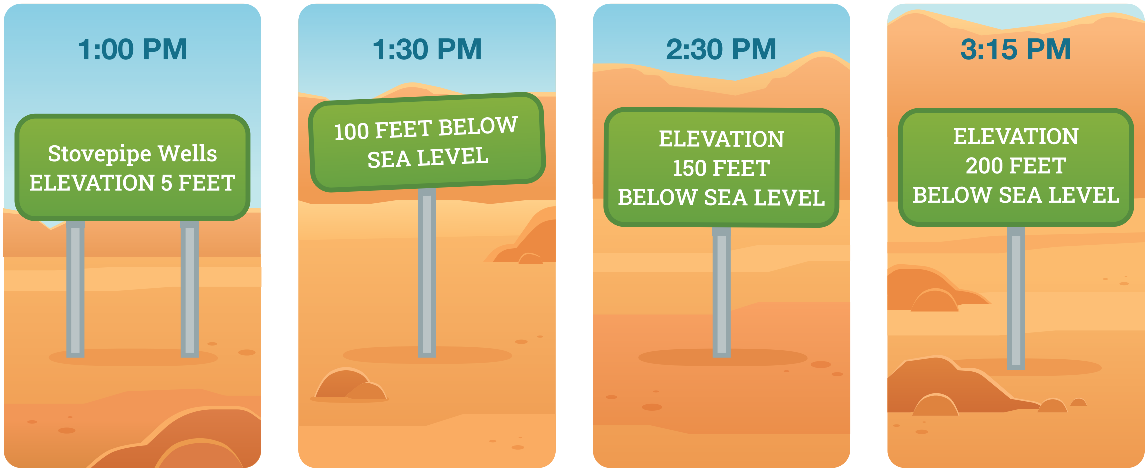
2. It was -4°C at 8:30 AM. Plot this information on the graph.

Activity 1

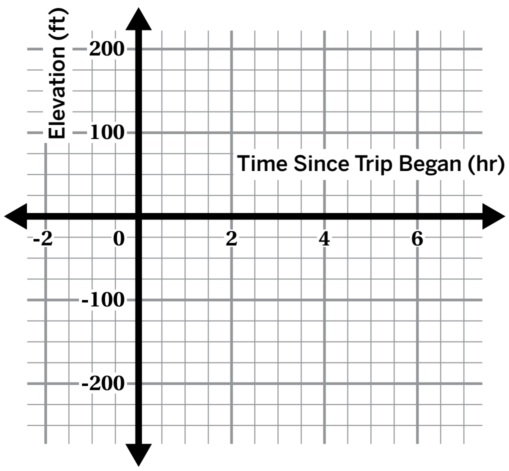
Name: .... Date: .... Period: ....

Elevation

Yona and her family took a road trip. One night, they stayed at a hotel where the elevation was 35 feet above sea level. They left their hotel at noon to drive through Death Valley, California. Yona took pictures along the way.



3. Plot five points on the graph to represent moments in Yona's trip. Include a point that represents when Yona left the hotel.



4. Plot at least three new points to represent Yona's trip back to the hotel.

5. Swap papers with a partner. Tell a story about their graph so that another person can understand it.

Activity 2

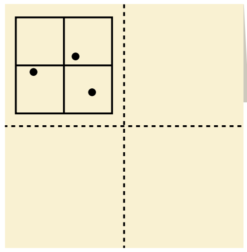
Name: ________ Date: ________ Period: ________

Graph Telephone

6. You will use one story from the Activity 2 Sheets.

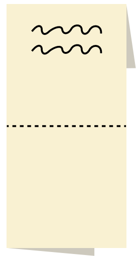
Round 1

- Read the story at the top of the page. Create a graph based on the story.
- When you're done, fold the paper to hide the story.
- When everyone is ready, pass your paper to the left.



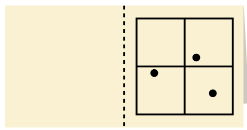
Round 2

- Write a story based on the graph your classmate made. Include enough details so that someone could recreate this graph.
- When you're done, fold the paper again so that only Rounds 2 and 3 are showing.
- When everyone is ready, pass your paper to the left.



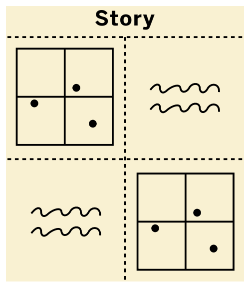
Round 3

- Create a graph based on the story your classmate wrote. When you are done, refold the paper so that only Rounds 3 and 4 are showing. Then pass your paper to the left.



Round 4

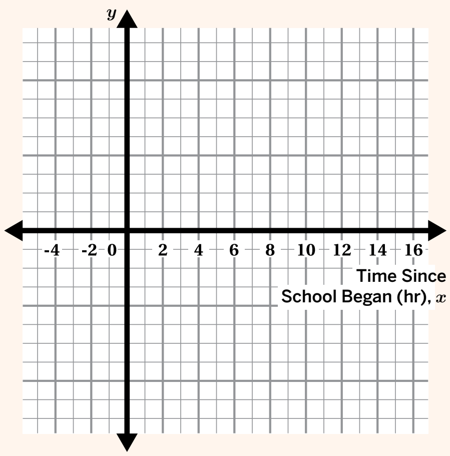
- Write a story based on the graph your classmate made in Round 3. When you're done, unfold the paper and look at how the story changed throughout the rounds. Share the stories and graphs.



You're invited to explore more.

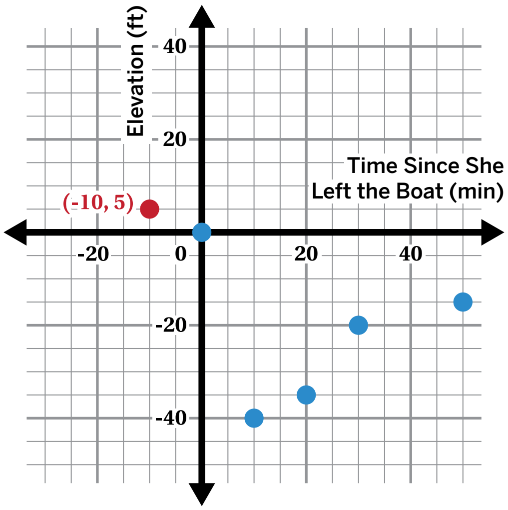
7. Think of a story about a school day, but don't write it down.

- Label and scale the y-axis based on your story.
- Create a graph to represent your story.
- Trade graphs with a classmate. Try to figure out what story their graph is telling.



Synthesis

8. Here is the graph of a situation when Ava jumped off a boat to go scuba diving. A point is added at (-10, 5).



a Describe what this new point means for the story.

b Describe how points on a coordinate plane can help us understand more about a situation.

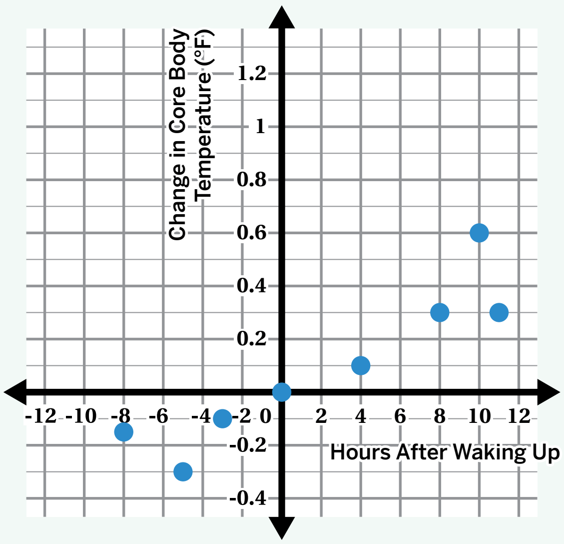
Lesson Practice 7.18

Lesson Summary

You can use points with positive and negative coordinates on a graph to make sense of different situations. Common contexts include elevation, time, temperature, and money.

Here's an example:

- A person used a device to track her change in temperature before and after waking up. She plotted her data on the graph.
- Time is represented on the x-axis. Temperature is represented on the y-axis.
- The points to the left of the y-axis represent changes in temperatures recorded while the person was sleeping. The points to the right of the y-axis represent changes in temperatures recorded after she woke up.

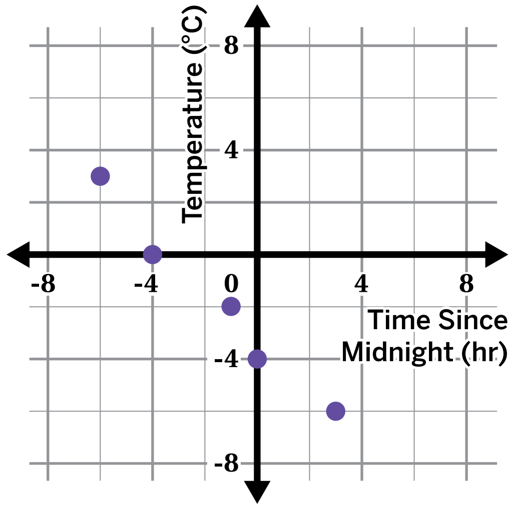


Coordinates on the graph tell different parts of the story. For example, the point (-5, -0.3) indicates that 5 hours before she woke up, her change in temperature was 0.3°F lower than when she woke up.

Lesson Practice
7.18

Name: .... Date: .... Period: ....

Problems 1–5: This graph shows the temperatures on one night in Minneapolis, Minnesota. The x-axis shows the amount of time since midnight, or 12 AM.



1. The point $(0, -4)$ is on the graph. What does this point tell you?

2. The point $(-4, 0)$ is also on the graph. What does this point tell you?

3. At 10:00 PM, it was -1°C. Plot this point on the graph.

4. What was the coldest temperature at night?

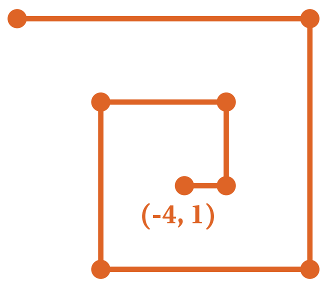
5. The temperature got warmer between 3:00 AM and 4:00 AM. At 4:00 AM, it was below freezing. Plot a point to represent a possible temperature at 4:00 AM.

Name: .... Date: .... Period: ....

FAST Practice

6. Felipe walked a path from point $(-4, 1)$ with these rules:

- Walk 1 meter and turn left.
- Walk 2 meters and turn left.
- Walk 3 meters and turn left.
- He followed this pattern 7 times.



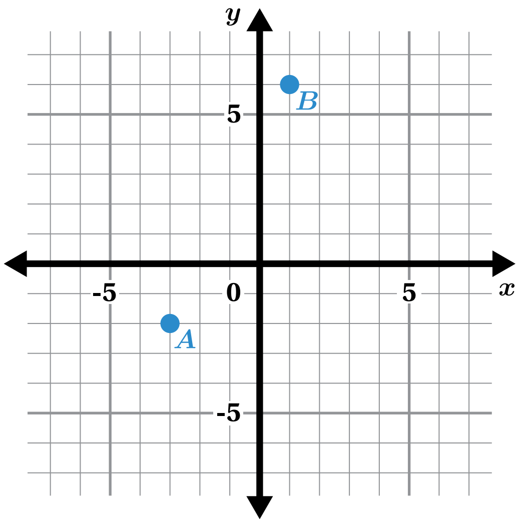
Felipe's path is shown. At what coordinates did Felipe end his walk?

(☐ , ☐)

Spiral Review

7. Titus wants to make a right triangle. Write a coordinate pair to make a right triangle.
Try to find as many different points as you can. (More than five are possible.)

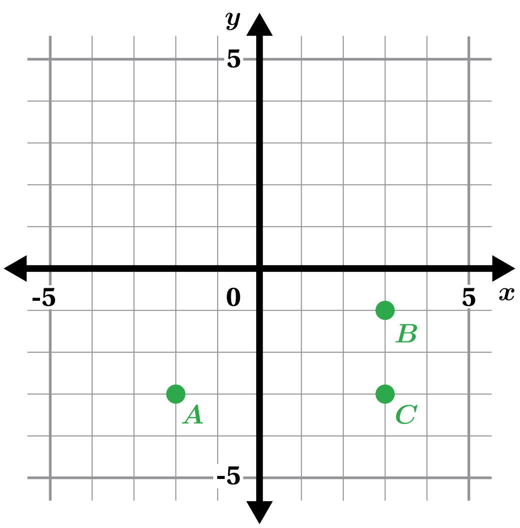
Point	Coordinates
A	(-3, -2)
B	(1, 6)
C	



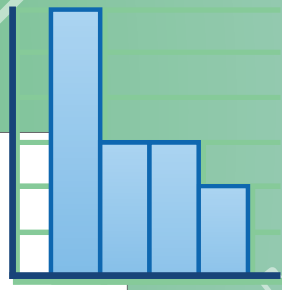
Problems 8–9: Here is a graph.

8. What is the distance between points B and C?

9. What is the distance between points A and C?



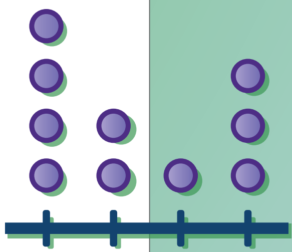
Unit 8



Describing Data

Statistics is the science of collecting and analyzing data. It is one of the most relevant aspects of mathematics in everyday life. In all cases, knowing what is typical is critical to understanding what is not.

Essential Questions



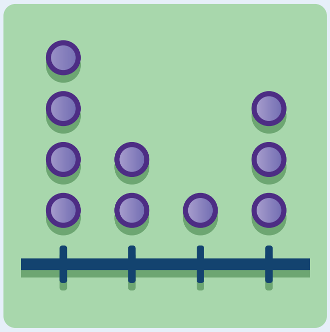
- What are statistical questions and how are they used?
- What are different ways to represent numerical data?
- How do we measure the center of a data set?
- How do we measure the spread of a data set?

Sub-unit

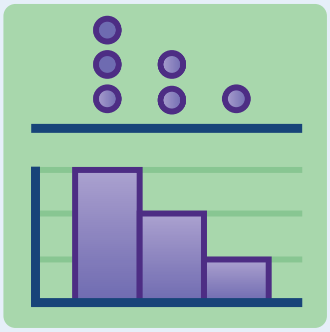
1

Visualizing Data

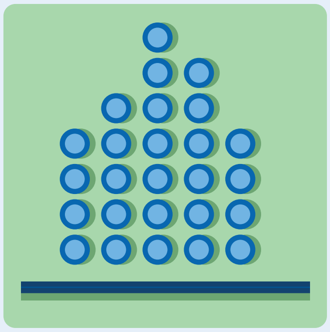
Lesson 1
Screen Time



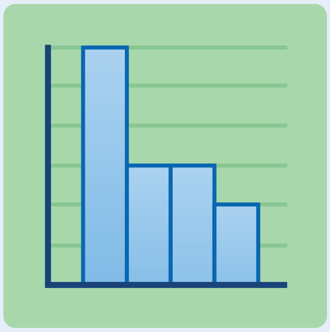
Lesson 2
Survey Says . . .



Lesson 3
The Plot Thickens



Lesson 4
What Does It All Mean?



Lesson 5
DIY Histograms

Name: ________ Date: ________ Period: ________

MA.6.DP.1.1, MTR.1.1, MTR.4.1

Screen Time

Let's learn about different types of data.

Warm-Up

1. Let's meet Antwon! Antwon is a college student. He was wondering: *How much time do my friends spend on their phones?* What could Antwon do to answer his question?

2. To collect data about his question, Antwon asked each friend: *What activity do you use your phone for the most?*

Discuss: How much time do you think Antwon's friends spend on their phones?

Friend	What activity do you use your phone for the most?
Maria	Texting
Julian	Watching videos
Carlos	Taking pictures
Laila	I don't have a phone
Troy	Playing games
Omar	I don't have a phone
Alina	Watching videos

Activity
1

Name: .... Date: .... Period: ....

Questions and Claims

3. Antwon collected more data by asking his friends different questions. Use Antwon's data to answer: *Do you agree that most of Antwon's friends spend more than* 4 *hours a day on their phones?* Circle one.

Agree Disagree I'm not sure

Friend	How many hours do you spend on your phone?	
	Each day	Each week
Maria	2	12
Julian	5	25
Carlos	4.5	24
Laila	0	0
Troy	8	40
Omar	0	0
Alina	5	28

4. Antwon claims most of his friends spend more than 4 hours a day on their phones.

Antwon is asking questions to determine if his claim is true. Order his questions from *most helpful* to *least helpful*.

Most helpful

Least helpful

a. What activity do you use your phone for the most?

b. How many hours are you on your phone each day?

c. How many hours are you on your phone each week?

d. Do you spend more than 4 hours a day on your phone?

Name: .... Date: .... Period: ....

Questions and Claims (continued)

5. Here are some more questions that Antwon asked his friends.

Friend	How many pets do you own?	What do you like to do after school?	How many years have you attended this college?	What's your favorite type of music?
Maria	1	Basketball	2	K-pop
Julian	0	Skateboard	2	Hip-hop
Carlos	2	Paint	2	Jazz
Laila	2	Bake	2	Rock
Troy	5	Video games	2	Hip-hop
Omar	1	Watch T.V.	2	Pop
Alina	0	Swim	2	Banda

Make two claims based on the data about Antwon's friends.

Claim 1:

Claim 2:

Name: .... Date: .... Period: ....

Categorical and Numerical Data

6. Asking different types of questions can produce different types of data.

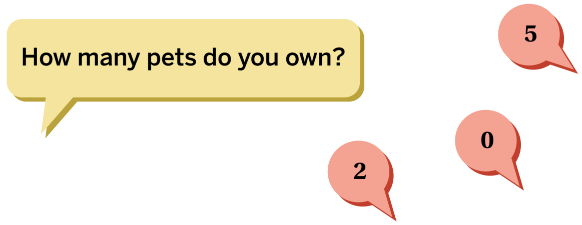
One of these questions produces **numerical data** and the other produces **categorical data**.

Describe what you think these terms mean.

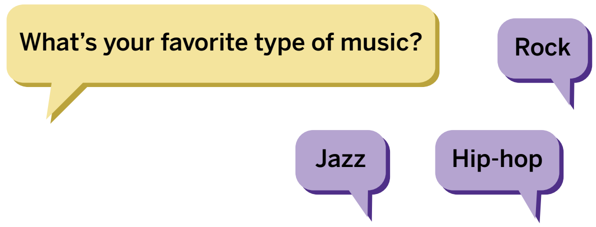
Numerical data:

Categorical data:

Numerical Data



Categorical Data



7. Match each question with the type of data it produces.

Question	Numerical Data	Categorical Data
Do you have 2 or more pets?	○	○
How many hours do you spend outdoors each day?	○	○
How old is the youngest person in your family?	○	○
How much does your backpack weigh?	○	○
What is your favorite vegetable?	○	○

Activity 3

Name: ____________ Date: ____________ Period: ____________

Challenge Creator

8. Write a statistical question that you can ask your classmates that will produce numerical data.

- **Make It!** Write a question for your classmates to answer.

My Question

- Explain why the data from your question will be numerical.

- **Solve It!** Ask some of your classmates your question and record their answers here.

Classmate's Name	Response to Question

Synthesis

9. Discuss both questions, then select *one* and write your response.

- What is something you learned today?
- What do you want to learn more about?

Lesson Practice 8.01

Lesson Summary

You can use surveys and measurements to collect data to answer questions about a topic. This data can be either **categorical data** or **numerical data**. Categorical data can be sorted into categories. Numerical data are numbers, quantities, or measurements that can be meaningfully compared. Some data that contain numbers, like addresses or dates, are categorical because the numbers are not quantities or measurements.

Here are some examples.

Categorical Data

- Favorite color
- Food people eat for lunch
- People's phone numbers

Numerical Data

- Number of people in each class
- Weights of dogs
- Ages of people in your school

Lesson Practice
8.01

Name: .... Date: .... Period: ....

1. Determine whether each question would produce numerical or categorical data.

Question	Numerical Data	Categorical Data
What is your favorite breakfast food?	◯	◯
How did you get to school this morning?	◯	◯
How many people live in your home?	◯	◯
What is the last thing you ate or drank?	◯	◯
How many minutes did it take you to get ready this morning, from waking up to leaving for school?	◯	◯

2. Choose one question from the table that would produce numerical data. Explain your thinking.

3. Choose one question from the table that would produce categorical data. Explain your thinking.

Problems 4–6: A pizza shop owner wants to improve her dinner service, so she collects data from her sales. Here is a copy of a receipt she is examining.

Booking type:	Online
Booking time:	5:30 PM
Server: Amir	Party Size: 4
1 × Pepperoni	12.00
2 × Vegetarian	21.00
1 × Chef's Special	13.50
Subtotal	46.50
Tax	4.65
Tip	10.00
Total	$61.15
Payment Method	Credit Card
Time	6:47 PM

4. Where do you see categorical data in this receipt?

5. Where do you see numerical data?

6. What questions could the pizza shop owner answer using the information from this receipt and others like it?

7. Lan claims that more students in her class walk to school than take the bus. Write a question that Lan could ask each of her classmates to investigate the claim.

FAST Practice

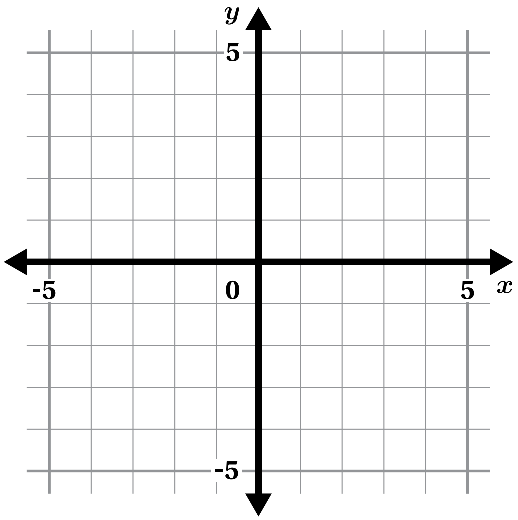
8. Select *all* the questions that can be answered using numerical data from receipts from the pizza shop.

- ☐ **A.** What types of pizzas did each party order?
- ☐ **B.** What was the tip for each order?
- ☐ **C.** How many pizzas did parties order?
- ☐ **D.** Who was the server for each order?
- ☐ **E.** What method of payment did each party use?

Spiral Review

Problems 9–10: Rectangle $CDEF$ has vertices $C = (-4, -1)$, $D = (-4, -4)$, $E = (-2, -4)$, and $F = (-2, -1)$.

9. Plot the rectangle on the coordinate plane and label the vertices.



10. What is the area of the rectangle?

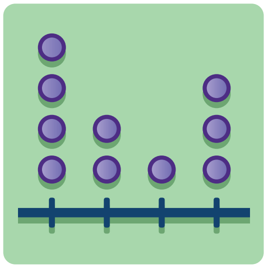
Unit 8
Lesson
2

Name: ________ Date: ________ Period: ________

MA.6.DP.1.1, MTR.1.1, MTR.4.1

Survey Says . . .

Let's explore statistical questions.



Warm-Up

1. Antwon used a survey to collect data about some of his friends.

What question might he have asked?

Friend	?
Maria	20
Julian	22
Carlos	17
Laila	10
Troy	45
Omar	15
Alina	20

Activity 1

Name: .... Date: .... Period: ....

Claims About Class Data

2. Meet Antwon's classmates!

Antwon claims that most of these students spend more than 4 hours on their phones each day.

Do you agree? Circle one.

Agree Disagree I'm not sure

Explain your thinking.

How many hours do you spend on your phone each day?

Terrance 0 hr	Shanice 3 hr	Dylan 4 hr	Felipe 0 hr	Farah 0 hr
Omar 0 hr	Maria 4 hr	Esteban 5 hr	Emma 6 hr	Alma 2 hr
Diamond 5 hr	Mauricio 6 hr	Eva 2 hr	Kai 9 hr	Troy 3 hr
Maria 3 hr	Julian 7 hr	Carlos 4 hr	Laila 0 hr	Alina 8 hr

3. Learn more about Antwon's classmates!

Antwon claims that most of these students have fewer than 3 pets.

Do you agree? Circle one.

Agree Disagree I'm not sure

Explain your thinking.

How many pets do you have?

Terrance 0	Shanice 2	Dylan 1	Felipe 2	Farah 9
Omar 1	Maria 0	Esteban 0	Emma 1	Alma 0
Diamond 3	Mauricio 2	Eva 3	Kai 6	Troy 1
Maria 1	Julian 1	Carlos 0	Laila 0	Alina 1

Activity 2

Name: ______ Date: ______ Period: ______

Questionable Data

4. Antwon surveyed his classmates about the number of siblings they each have and made a table of their responses.

Name	Number of Siblings	Name	Number of Siblings
Terance	0	Diamond	2
Shanice	1	Mauricio	2
Dylan	1	Eva	0
Felipe	3	Kai	0
Farah	2	Troy	1
Omar	2	Maria	3
Maria	3	Julian	1
Esteban	1	Carlos	5
Emma	6	Laila	2
Alma	1	Alina	0

Discuss: What are two things you learned about Antwon's classmates from this data?

5. Antwon surveyed his classmates about the number of books each student read this month and made a table of their responses.

Name	Number of Books Read	Name	Number of Books Read
Terance	7	Diamond	7
Shanice	5	Mauricio	0
Dylan	8	Eva	1
Felipe	7	Kai	6
Farah	5	Troy	7
Omar	6	Maria	1
Maria	7	Julian	5
Esteban	12	Carlos	4
Emma	4	Laila	3
Alma	5	Alina	8

Discuss: What are two things you learned about Antwon's classmates from this data?

Activity 2

Name: .... Date: .... Period: ....

Questionable Data (continued)

6. This table of data shows the number of books Antwon's classmates read this month.

Name	Number of Books Read	Name	Number of Books Read
Terance	7	Diamond	7
Shanice	5	Mauricio	0
Dylan	8	Eva	1
Felipe	7	Kai	6
Farah	5	Troy	7
Omar	6	Maria	1
Maria	7	Julian	5
Esteban	12	Carlos	4
Emma	4	Laila	3
Alma	5	Alina	8

Write a question that this table of data could help you answer. Organize the data as an ordered list if it helps with your thinking.

7. A **statistical question** requires more than one piece of data to answer it.

Select *all* the statistical questions.

- ☐ **A.** Have most students read more than 3 books this month?
- ☐ **B.** How many books has Farah read this month?
- ☐ **C.** Do students in this class prefer fiction or nonfiction?
- ☐ **D.** Is the book that Julian read fiction or nonfiction?
- ☐ **E.** How long are the books that Kai read?

Synthesis

8. How can a data set help you write a statistical question?

Use the example if it helps with your thinking.

Number of Posters Displayed in Classrooms

0, 3, 4, 0, 0, 0, 4, 5, 6, 2, 5, 6, 2, 9, 3, 3, 7, 4, 0, 8

Lesson Practice 8.02

Lesson Summary

A **statistical question** is a question that needs more than one piece of data to answer it.

Here is an example:

- "Which classroom in your school has the most books?" is a statistical question because you need to know the number of books in *each* classroom to answer it.
- "How many books are in your classroom?" is not a statistical question because you only need to know the number of books in *one* classroom to answer it.

You can organize data that answers a statistical question into a list or a table.

List

Number of Books in Classrooms

12, 12, 14, 15, 20, 20, 20, 22

Table

Classroom	Number of Books	Classroom	Number of Books
A	15	E	12
B	20	F	20
C	22	G	20
D	14	H	12

Lesson Practice
8.02

Name: .... Date: .... Period: ....

Problems 1–5: Five sixth-grade students at a school were each asked the following survey questions:

Question A: What grade are you in?

Question B: How many books did you read in the last year?

Question C: How many inches are in 1 foot?

Question D: How many dogs and cats do you have?

Their answers are shown in the table. Write the letter of the question that could have produced each line of data.

	Question	Diya	Peter	Tiana	Marc	Callen
1.		0	1	1	3	0
2.		12	12	12	12	12
3.		6	6	6	6	6
4.		11	5	18	20	9

5. How are Questions A and C different from the other questions?

Problems 6–7: Anya asked 10 students how many minutes it takes them to get to school each morning.

6. Which list could represent the data that Anya collected?

7. Which list could not represent Anya's data? Explain your thinking.

A. -5, 0, 2, 3, 3, 3, 4, 5, 5, 7

B. 8, 8, 10, 10, 10, 12, 13, 16, 18, 20

C. 120, 140, 140, 160, 180, 240, 260, 260, 280, 300

Name: .... Date: .... Period: ....

FAST Practice

8. Which question is a statistical question?

(A.) How old are you?

(B.) How old are the students in our school?

(C.) How old is your teacher?

(D.) What is the difference between your age and your teacher's age?

9. Choose an answer from Problem 8 that you did not select. Why do you think this is not a statistical question?

Spiral Review

10. Order these values from *least* to *greatest*.

$\lvert -17 \rvert$	$\lvert -18 \rvert$	-18	$\lvert 19 \rvert$	20

Least **Greatest**

Problems 11–13: Determine each quotient.

11. $45052 \div 28$

12. $6052 \div 17$

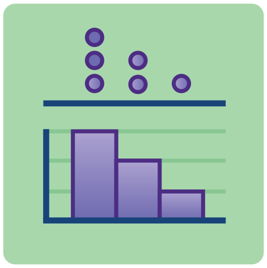
13. $60.52 \div 1.7$

Name: ______ Date: ______ Period: ______

MA.6.DP.1.5, MTR.4.1, MTR.6.1

The Plot Thickens

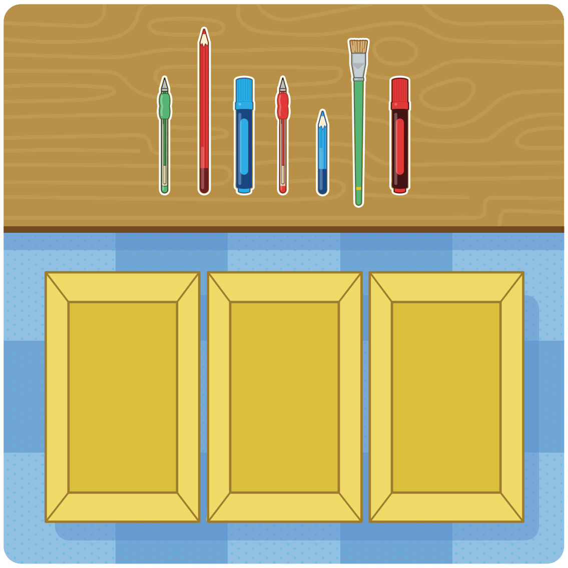
Let's use histograms to represent data sets.



Warm-Up

1. a Organize the art supplies into the bins.

   b Explain how you organized the supplies.



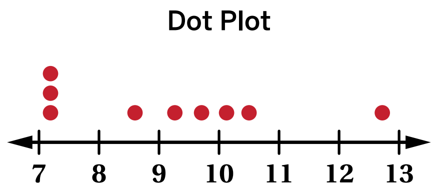
Activity 1

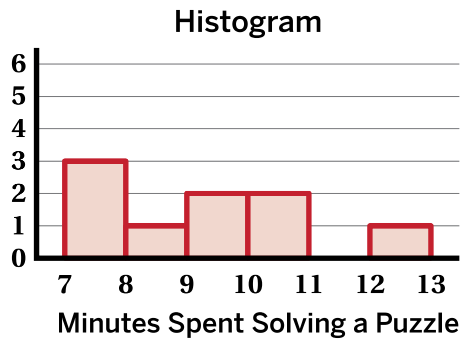
Name: ________ Date: ________ Period: ________

Introduction to Histograms

2. Here are two different ways to show the minutes students spend solving a puzzle.

Discuss: How do you think the histogram was made?



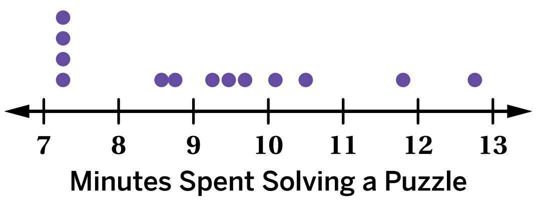


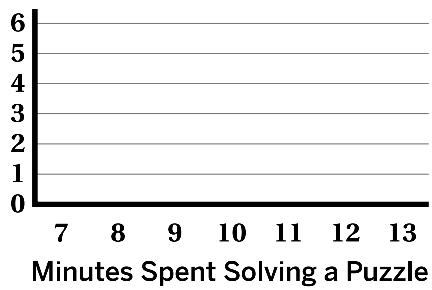
3. How are the histogram and the *dot plot* alike?

How are they different?

4. More times were added to the dot plot. Each bar in a histogram is called a **bin**.

Draw bins to make a histogram with this new data.





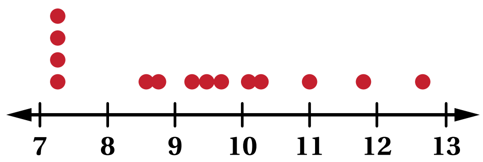
Activity 2

Name: ________ Date: ________ Period: ________

Creating Histograms

5. In statistics, each bin includes the smaller of the two numbers and goes up to but does not include the larger number. When a data point is on the edge between two bins, it goes into the bin with the larger values.

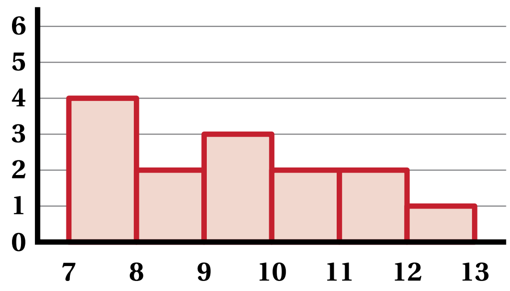
a 11 minutes was added to this dot plot. Notice in the histogram that 11 minutes goes into the bin for values 11 to 12.



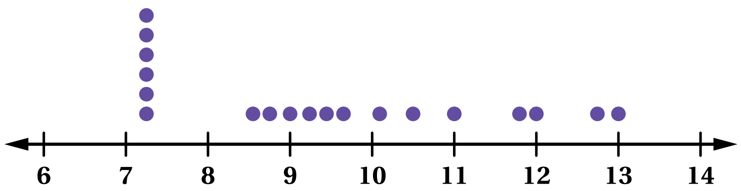
b Which bin would 8 minutes go into? Circle one.

7 to 8   8 to 9   Somewhere else

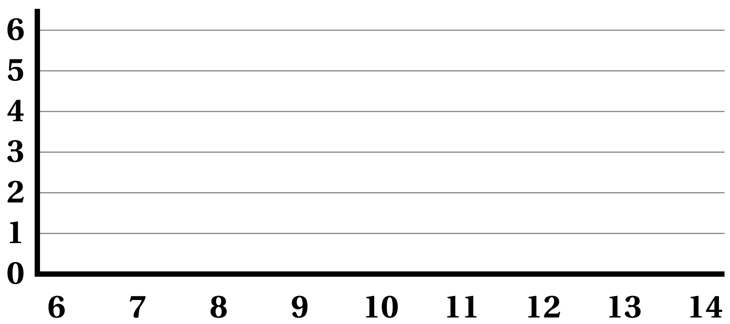
Explain your thinking.



6. So far, we've seen histograms with a bin size of 1. Here is a dot plot with some more times students spent solving a puzzle.



Make a histogram with a bin size of 2.



Activity **3**

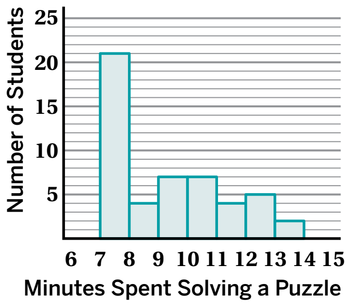
Name: ______________ Date: ______________ Period: ______________

Using Histograms

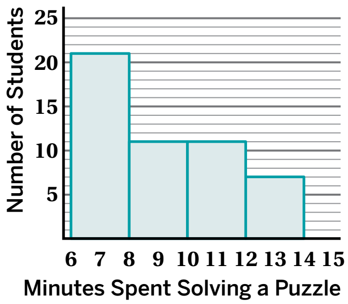
7. Let's look at the time spent solving a puzzle for 50 students. Why might these data points be difficult to visualize as a dot plot?

8. Each of these histograms shows the time spent solving a puzzle for 50 students.

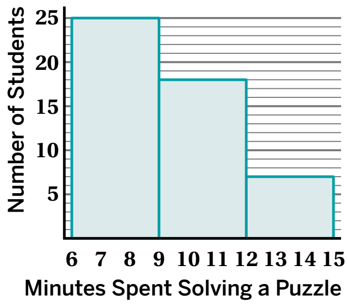
Bin Size 1



Bin Size 2



Bin Size 3



Discuss: What do you notice? What do you wonder?

9. Different visualizations like dot plots and histograms help answer different questions.

a Which of these questions can a histogram help answer?

(A.) Which student solved the puzzle the fastest?

(B.) How long did it take the fastest student to solve the puzzle?

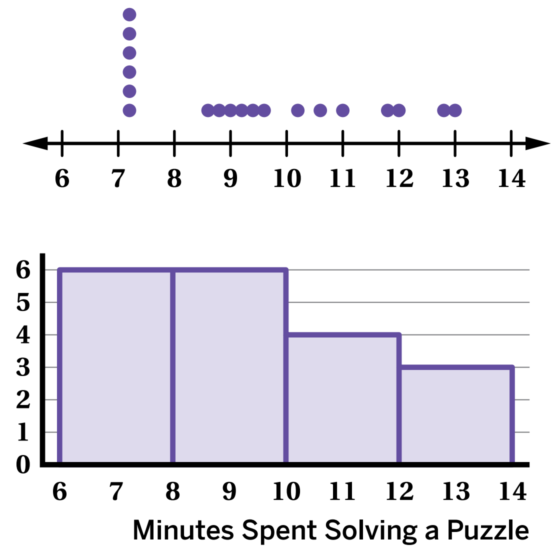
(C.) How many students solved the puzzle in 7 minutes 25 seconds?

(D.) How many students took 10 minutes or more to solve the puzzle?

b Answer the question you chose.

Synthesis

10. What is important to remember about a histogram?

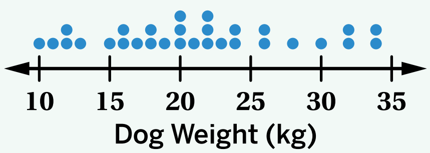


Lesson Practice 8.03

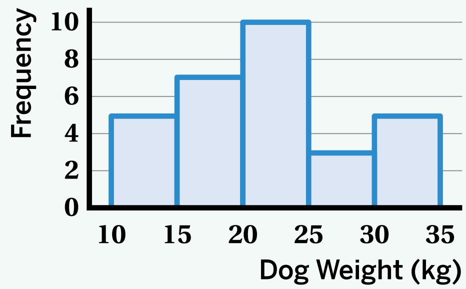
Lesson Summary

You can use *dot plots* and **histograms** to visualize numerical data. Here is an example of a data set of the weights of 30 dogs presented in a dot plot and in a histogram.

Dot Plot



Histogram



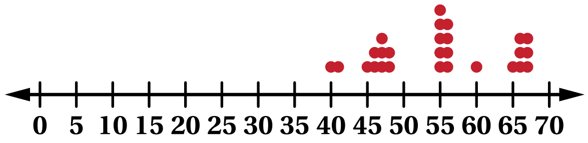
In a histogram, data values are grouped into bins that cover a range of values, and each **bin** has the same width. The height of each bar represents the total number of values in that range, including the left boundary (least value) but excluding the right boundary (greatest value). For example, the height of the tallest bar, from 20 to 25, represents weights of 20 kilograms up to (but not including) 25 kilograms.

Lesson Practice
8.03

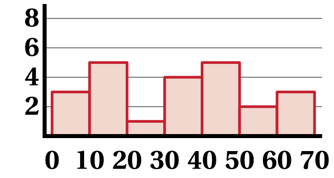
Name: .... Date: .... Period: ....

1. Match each histogram with the dot plot that represents the same data set.

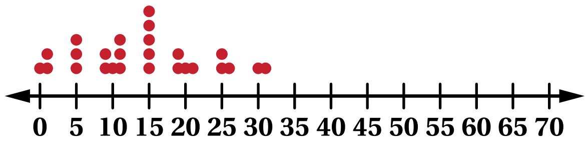
a



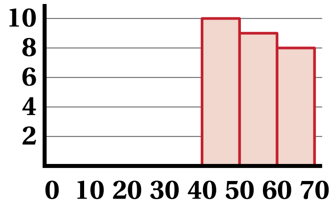
....



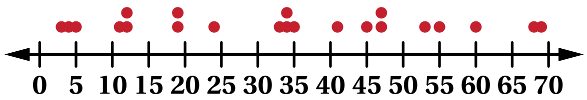
b



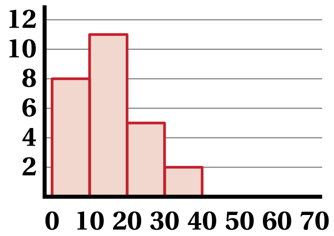
....



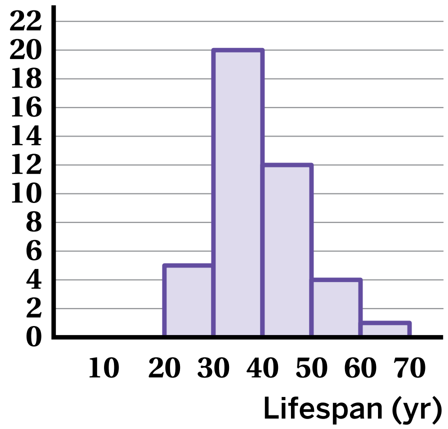
c



....



2. Here is a histogram showing the lifespans of 42 chimpanzees that lived in the wild. How many chimpanzees lived at least 50 years and less than 70 years?

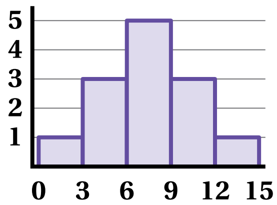


Problems 3–4: Here are three representations of the same data set.

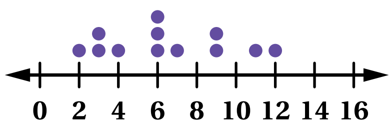
List

2, 3, 3, 4, 6, 6, 6,
7, 9, 9, 11, 12

Histogram



Dot Plot



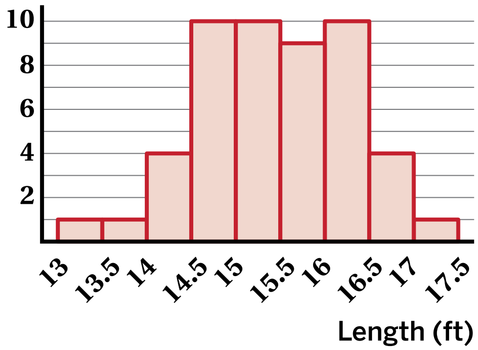
3. Circle the representation that doesn't match the other two.

4. Revise your circled representation to make it match the other two representations.

Name: .... Date: .... Period: ....

FAST Practice

5. A marine biologist is studying a group of sharks. She made a histogram of the lengths of a group of adult sharks.



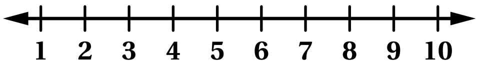
Select *all* of the false statements.

- ☐ **A.** A total of 9 sharks were measured.
- ☐ **B.** Two of the sharks are less than 14 feet long.
- ☐ **C.** A typical shark is about 14.5 to 16.5 feet long.
- ☐ **D.** The longest shark measured was 10 feet long.
- ☐ **E.** The smallest shark measured was 11 feet long.

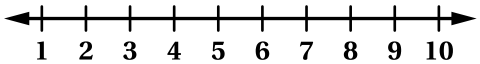
Spiral Review

Problems 6–7: Graph each inequality on a number line.

6. $m > 6$



7. $3 > n$



Problems 8–11: Determine the value of each expression.

8. $3.727 + 1.384$

9. $3.727 - 1.384$

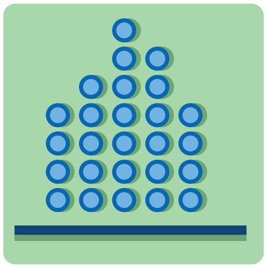
10. $5.01 \cdot 4.8$

11. $5.01 \div 4.8$

Name: ______________ Date: ________ Period: ________

MA.6.DP.1.4, MTR.4.1, MTR.7.1

What Does It All Mean?



Let's analyze real-world data using histograms and dot plots.

Warm-Up

1. Play a few rounds of Polygraph with your classmates!

You will use a Warm-Up Sheet with dot plots for four rounds. For each round:

- You and your partner will take turns being the Picker and the Guesser.
- Picker: Select a dot plot from the Warm-Up Sheet. Keep it a secret!
- Guesser: Ask the Picker yes-or-no questions, eliminating dot plots until you're ready to guess which dot plot the Picker chose.

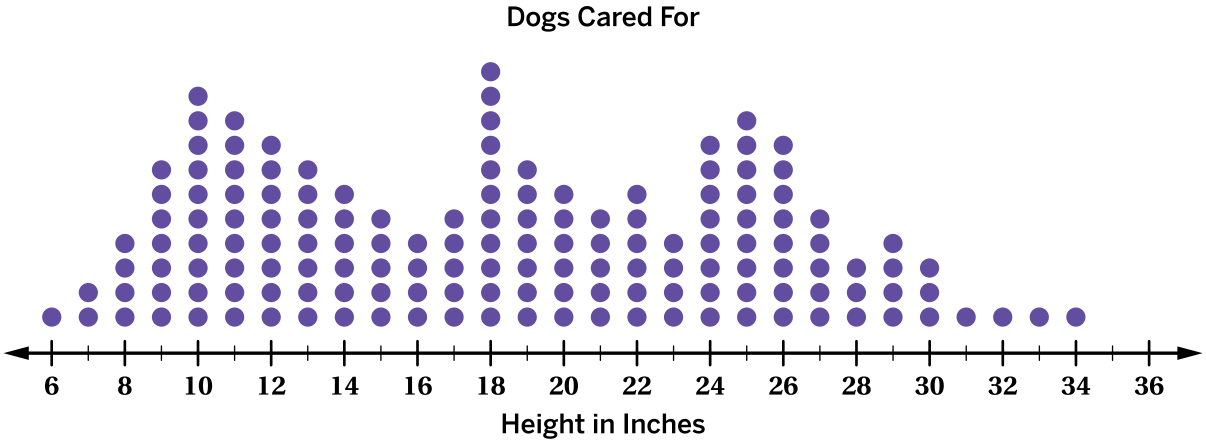
Record helpful questions from each round in this workspace:

Activity 1

Name: ______ Date: ______ Period: ______

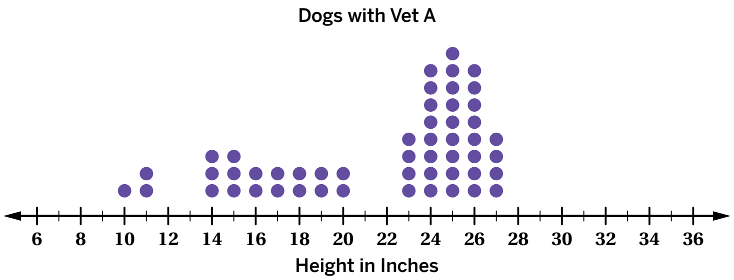
Describing Dot Plots and Histograms

2. The dot plot shows the heights of dogs cared for at a local veterinary clinic.

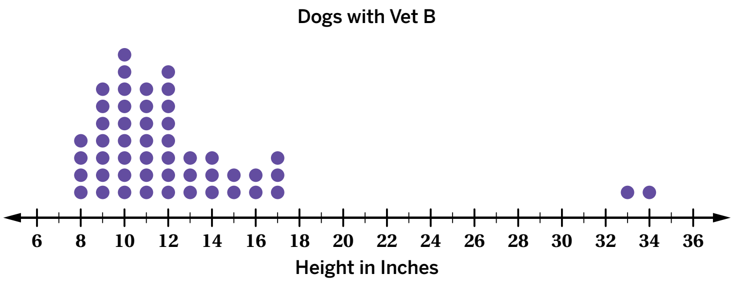


Discuss: How can you describe the dot plot?

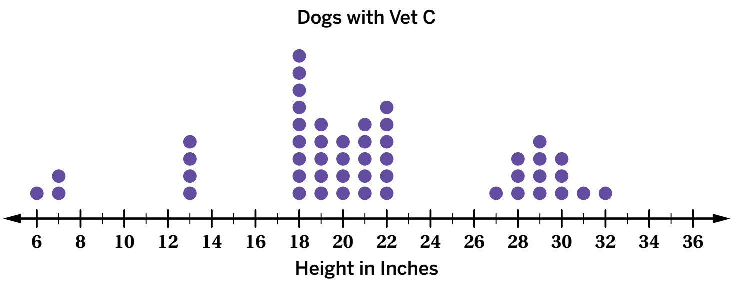
3. Three vets work at the clinic. Each vet made a dot plot for their own patients.



a The **range** of a data set is a measure of **spread**. Compare the ranges of the dot plots.



b The **shape** of the data for Vet A is **skewed**. Are either of the other dot plots skewed? Explain your thinking.

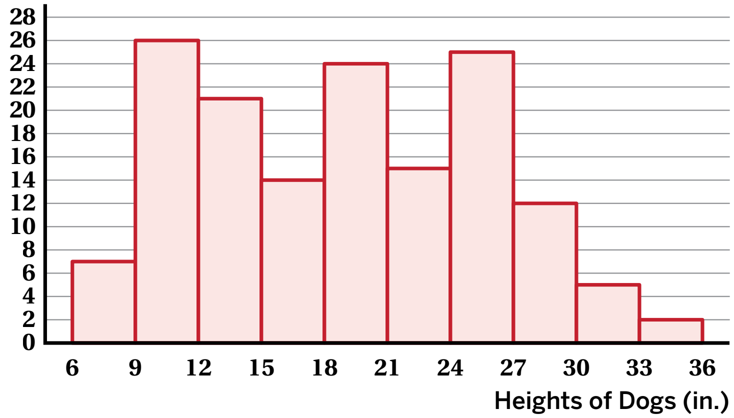


c Vet C appears to have **outliers** in his data at 6 inches and 7 inches. List any other outliers the dot plots have.

Name: ________ Date: ________ Period: ________

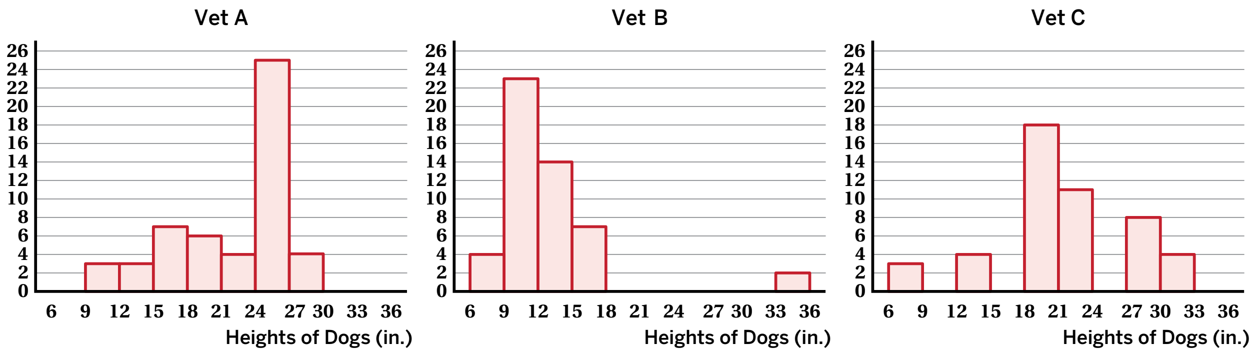
Describing Dot Plots and Histograms (continued)

4. This histogram shows the same information about dog heights at the veterinary clinic as in Problem 2.



Discuss: How are the data displays alike? How are they different?

5. The vets also made histograms for their patients.



a Sahana says the data for Vets B and C have **clusters** and gaps. Duri says the data for Vets B and C have gaps but only the data for Vet B has a cluster. Whose thinking is correct? Circle one.

Sahana's   Duri's   Neither

Explain your thinking.

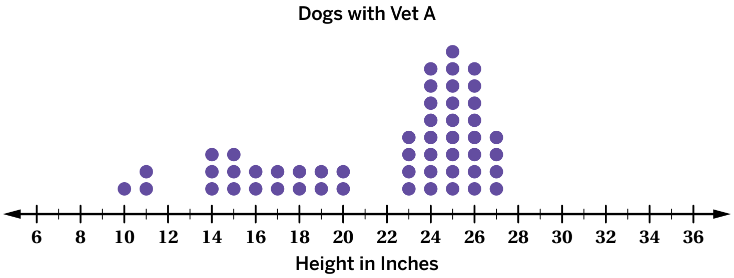
b Do any of these histograms have **symmetry**? Explain your thinking.

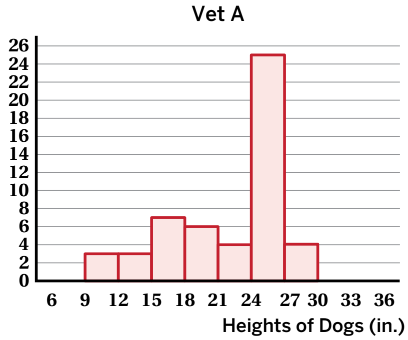
Activity 2

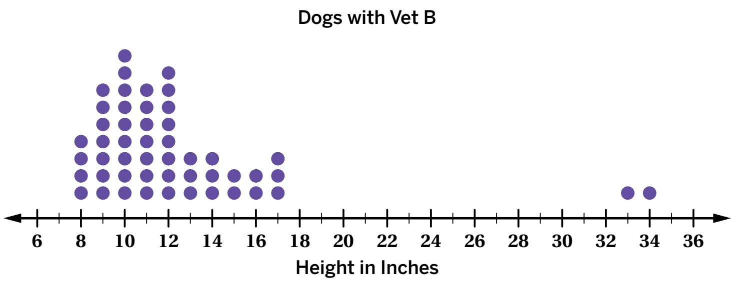
Name: .... Date: .... Period: ....

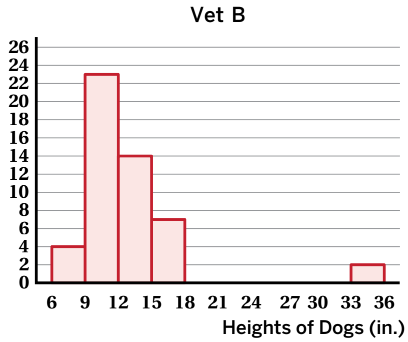
What It All Means

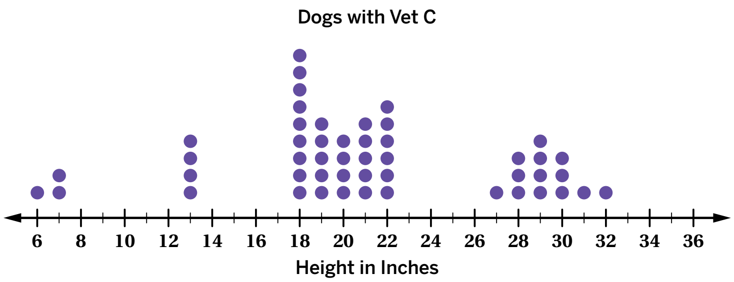
Let's interpret this data in the context of each veterinarian's patients.

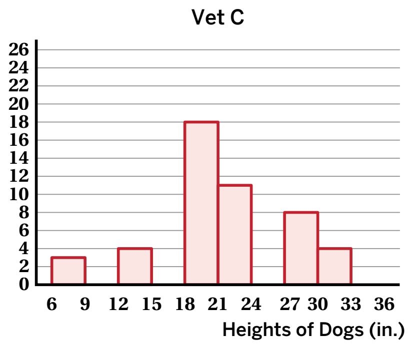












6. Group the choices that match each veterinarian. Choices may match more than one vet. One choice will have no match.

A. Sees more short dogs than tall dogs

B. Sees the most medium height dogs

C. Sees dogs with a wide variety of heights

D. Sees dogs with the smallest range of heights

E. Does not see any tall dogs

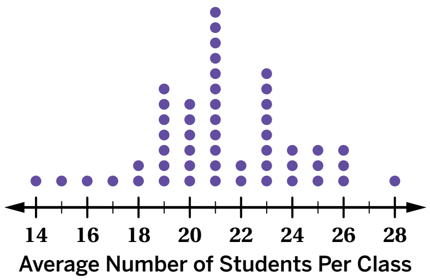
F. Does not see many short dogs

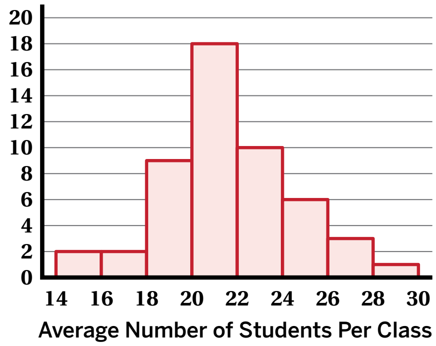
Vet A	Vet B	Vet C

Synthesis

7. What do spread and shape tell you about a dot plot or histogram?

Use the example if it helps with your thinking.





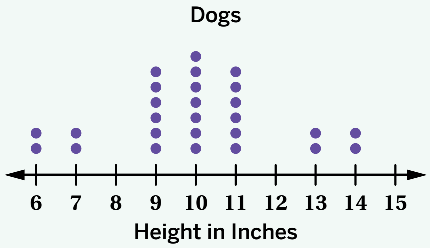
Lesson Practice 8.04

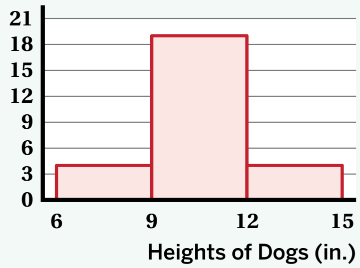
Lesson Summary

You can describe characteristics of dot plots and histograms and interpret their meaning in real-world situations.

The **range** describes the **spread** of the data as narrow or wide.

When the **shape** of a data display is **skewed**, more data values lie on one end, and few data values lie on the other end. A **symmetrical** display has more data values in the middle of the range and fewer data values on either end. You can interpret the shape of a data set in context. For example, in these displays, more dogs have a medium height while fewer dogs are shorter or taller.





You can identify different information using a dot plot or a histogram. For example, this dot plot shows two small gaps in the data, but the histogram does not convey this information.

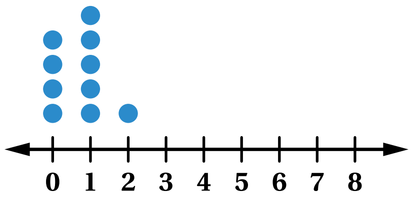
Some data displays show **clusters**, or groups of data values that are close together. **Outliers**, or data values far from other values in the set, indicate values unlike others in the set.

Lesson Practice
8.04

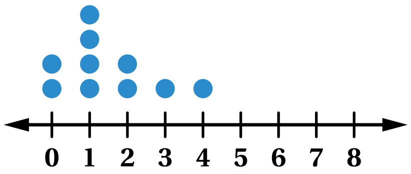
Name: .... Date: .... Period: ....

1. Which dot plot has the widest range?

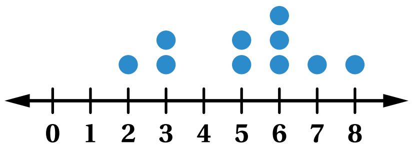
A.



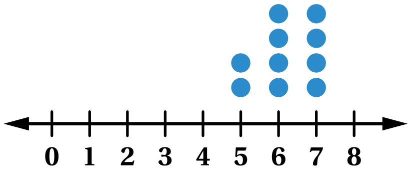
B.



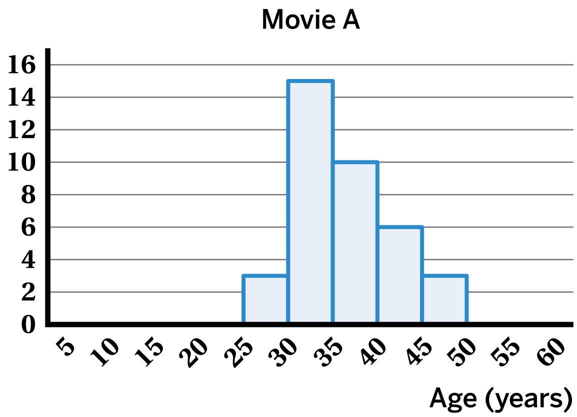
C.

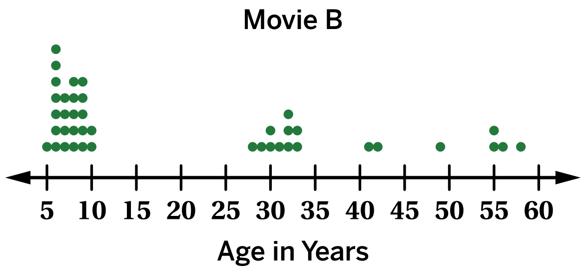


D.



Problems 2–5: These data displays represent ages of people who watched two different movies at a movie theater.





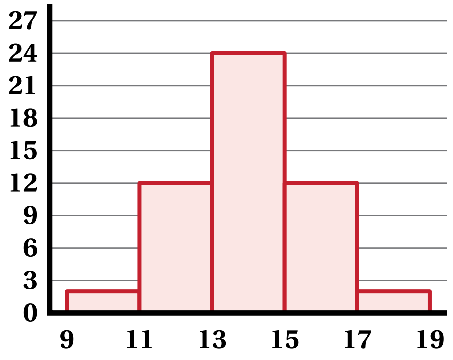
2. How are the data displays alike?

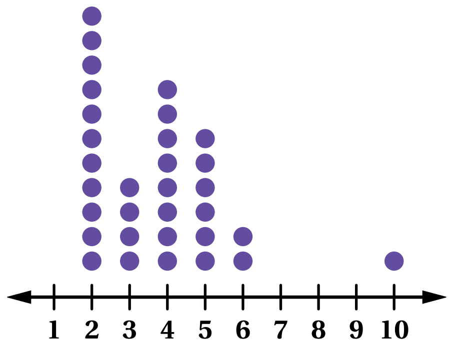
3. How are they different?

4. How does the typical age for Movie A compare to people watching Movie B?

5. What kind of movie do you think Movie B is? Explain your thinking.

Problems 6–8: Here are two displays of different data sets.





6. Describe the shape of each data display. Explain your thinking.

7. Which data display could represent the age in months that babies take their first steps? Explain your thinking.

FAST Practice

8. Describe the data display that shows an outlier. Select **ONE** correct answer in each box.

The [**A.** histogram **B.** dot plot] includes an outlier. The data value is [**A.** far from **B.** grouped with] other data values in the display.

Spiral Review

9. Select *all* of the numbers that are greater than 5.

☐ **A.** -9 ☐ **B.** 3 ☐ **C.** -6 ☐ **D.** $|-9|$ ☐ **E.** 7

Problems 10–11: Determine the value of each expression.

10. $6.05 \cdot 2.5$

11. $6.05 \div 2.5$

Name: .... Date: .... Period: ....

MA.6.DP.1.4, MA.6.DP.1.5, MTR.4.1, MTR.5.1, MTR.7.1

DIY Histograms

Let's create histograms to represent and compare classrooms.

Warm-Up

There is an average of 17 students per classroom at an elementary school.

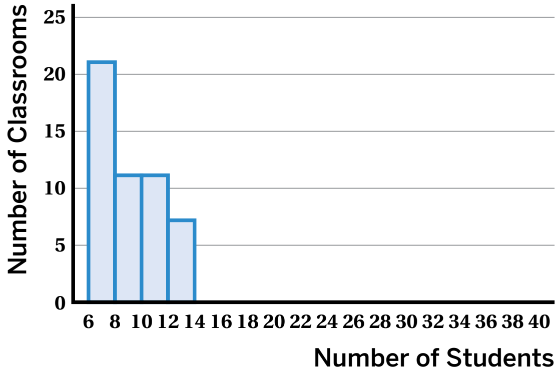
1. Make a prediction about the number of chairs in each classroom at this elementary school.

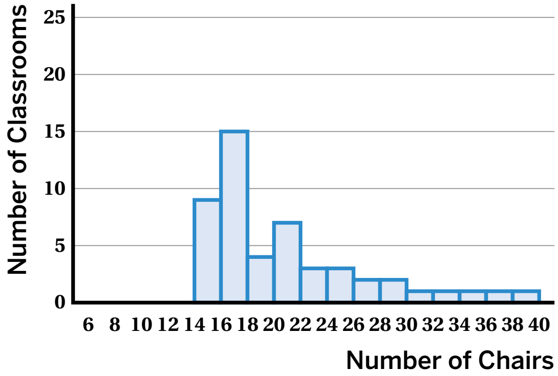
2. **Discuss:** How might the number of students compare to the number of chairs in each classroom?

Name: .... Date: .... Period: ....

Out of Reach

3. These histograms show the number of students and the number of chairs for 50 classrooms in a school district.





a How would you describe the shape of each histogram?

b What do you think is the **center** of the histogram showing the number of students? Explain what you think this number means.

c What do you think is the center of the histogram showing the number of chairs? Explain what you think this number means.

d Which histogram has a greater spread?

e Why do you think that might be the case?

Activity 2

Name: ________ Date: ________ Period: ________

Comparing Regions

4. Yosef started to make a *frequency table* for this data set.

Frequency Table

Number of Students	Frequency
6 to less than 8	𝍸
8 to less than 10	|||
10 to less than 12	

Number of Students		
7	11	9
7	11	7
8	11	7
7	9	10

a Complete the last row of Yosef's frequency table.

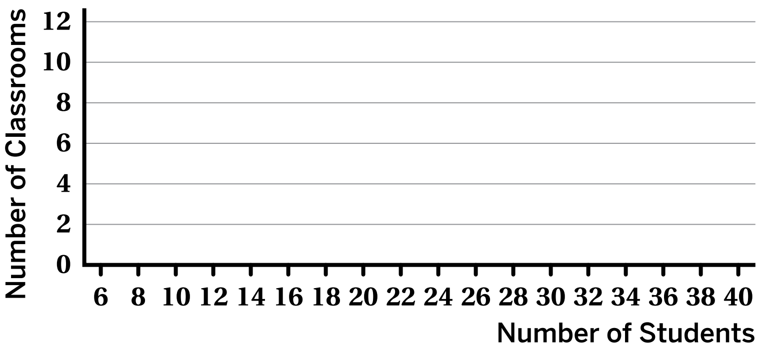
b How could a frequency table help Yosef make a histogram of this data?

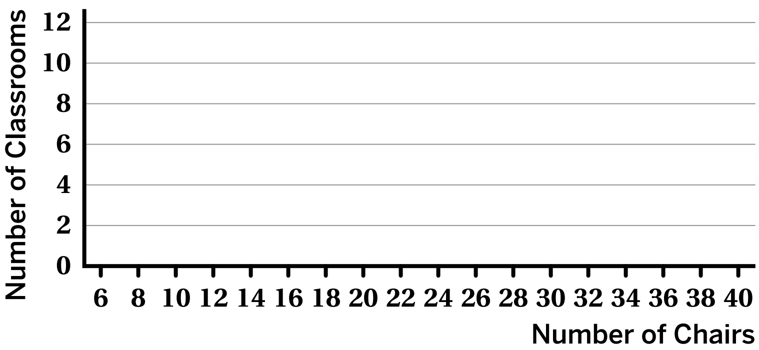
5. You'll use the Activity 2 Sheet to create two histograms with your partner.

Pick elementary school classrooms or high school classrooms. Then create:

- A histogram representing the number of students per classroom for that school.
- A second histogram representing the number of chairs per classroom for that school.

Make a frequency table if it helps with your thinking.





Name: .... Date: .... Period: ....

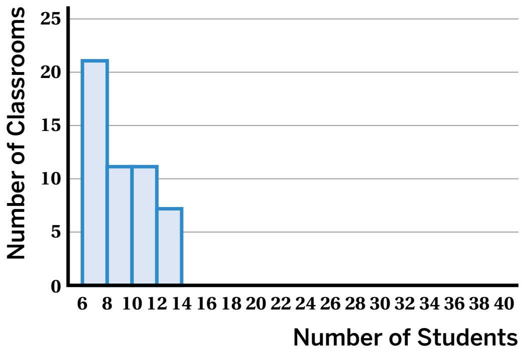
Comparing Regions (continued)

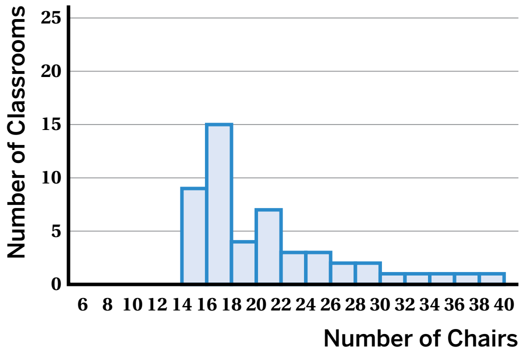
6. **Discuss:** How do the histograms for the elementary school compare to the histograms for the high school?

7. How could changing the bin size affect our understanding of the data? Do you think the bin size of these histograms should change or remain the same? Explain your thinking.

Synthesis

8. How does making histograms help us compare data sets?



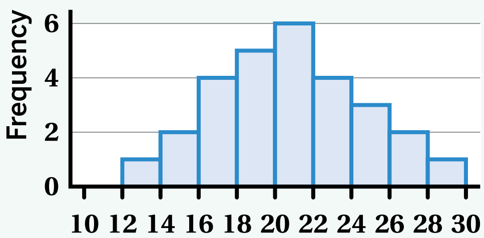


Lesson Practice 8.05

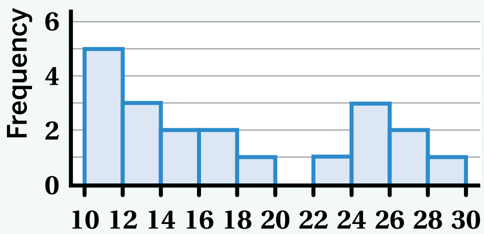
Lesson Summary

You can use histograms to compare numerical data sets using their shape, **center**, and spread. The center is the number that represents a typical value. Here are two histograms with very different shapes and features.

Histogram A



Histogram B



Shape	Histogram A is shaped like a mountain. It is symmetric with a peak around 21.	Histogram B has two clusters, with peaks around 11 and 25. There is a gap in the data between 20 and 22.
Center	The center is approximately 21.	The center is approximately 17.
Spread	Most values are in the middle and there are roughly the same amount on each side of 21.	Most values are on the left. Then, they get lower as it goes to the right.

Name: .... Date: .... Period: ....

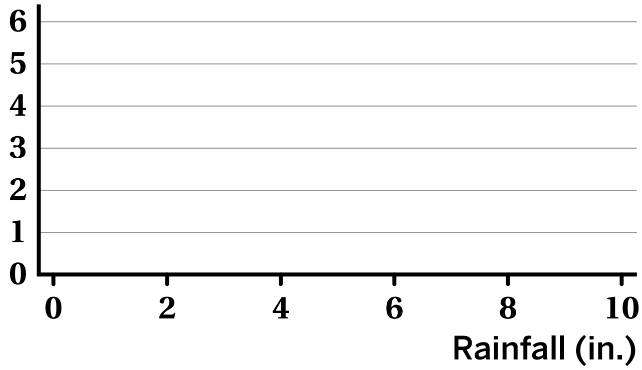
Problems 1–2: Here is the average amount of rainfall, in inches, for each month last year in Vicente's hometown.

Average Rainfall (in.)					
1.67	2	2.99	6.1	8.9	9.84
3.14	5.35	9.69	6	3.27	2.05

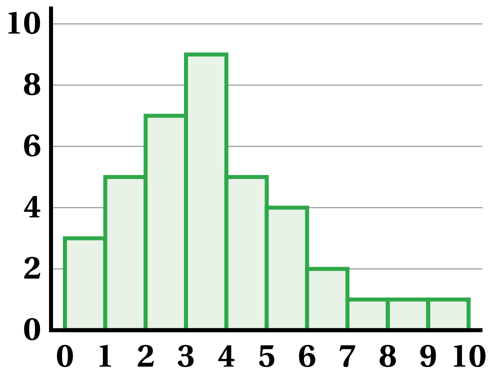
1. Complete the frequency table.

Rainfall (in.)	0 to less than 2	2 to less than 4	4 to less than 6	6 to less than 8	8 to less than 10
Frequency					

2. Create a histogram using the data about Vicente's hometown.



Problems 3–6: Here is a histogram.



3. Which of these situations could this histogram represent?

A. The height of a group of sixth-grade students (in inches).

B. The weight of students' backpacks (in pounds).

C. The temperature for 38 days in summer (in degrees).

D. The time it takes a group of students to walk to school (in hours).

Explain your thinking.

4. Describe the center of the histogram.

5. Describe the shape of the histogram.

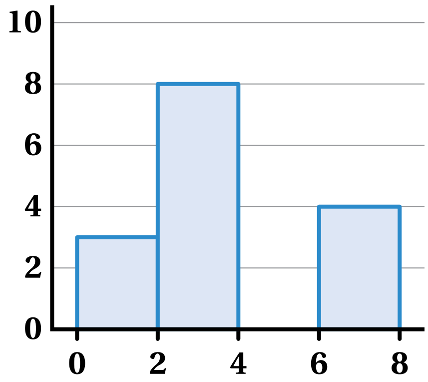
6. Describe the spread of the histogram.

Lesson Practice
8.05

Name: .... Date: .... Period: ....

FAST Practice

7. Which of these situations could this histogram represent?



- **A.** The weight of cars (in pounds)
- **B.** The height of trees in a forest (in feet)
- **C.** The time it takes to drive from a city in Florida to a city in Maine (in hours)
- **D.** The amount of water needed each week for a houseplant (in ounces)

Spiral Review

8. Order these numbers from *least* to *greatest*.

-6 0.6 0.16 6 -1.6

Least **Greatest**

9. Rebecca drank 12 ounces of water from her water bottle, which is 60% of the volume of the bottle. How much water can the bottle hold when full? Use the double number line if it helps with your thinking.

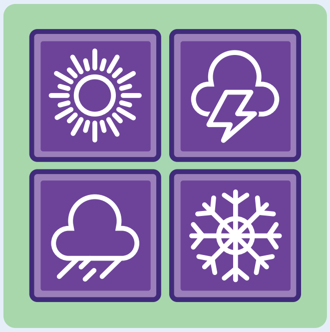
10. The point (-2, 3) is one vertex of a square on a coordinate plane. Name three points that could be the other vertices.

Sub-unit

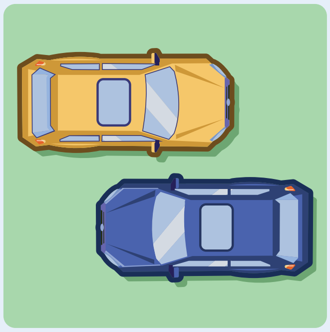
2

Measuring Data: Mean, Median, Mode, IQR, and Range

Lesson 6
Snack Time



Lesson 7
Modes of Weather

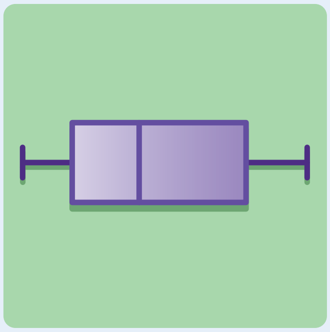


Lesson 8
Toy Cars

Lesson 9
In the News

Lesson 10
Hoops

Lesson 11
Pumpkin Patch



Lesson 12
Car, Plane, Bus, or Train?

Name: ____________ Date: ________ Period: ________

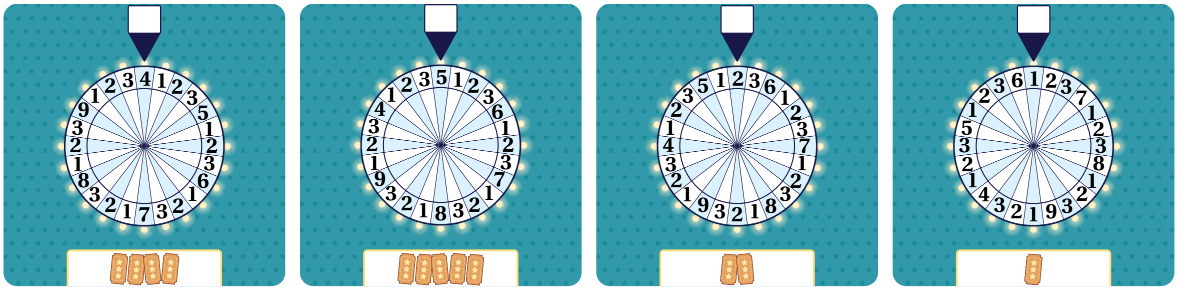
MA.6.DP.1.2, MTR.2.1, MTR.3.1, MTR.6.1

Snack Time

Let's explore the mean of a data set and what it tells us.

Warm-Up

1. Here are four images of an arcade game.



Discuss: What do you notice? What do you wonder?

Activity 1

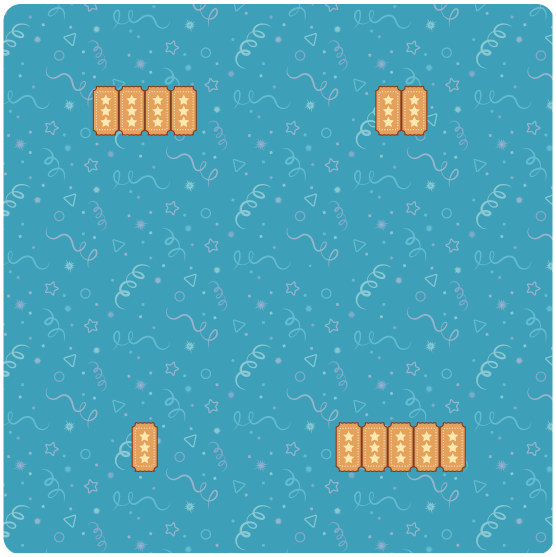
Name: .... Date: .... Period: ....

Mean as an Equal Share

2. 4 friends played this game at the arcade. Here are the tickets each friend won.

They decided to share the tickets equally. How many tickets should each friend get?

Explain your thinking.



3. Here is Ava's work for determining how many tickets each friend gets.

Discuss What was Ava's strategy?

Ava

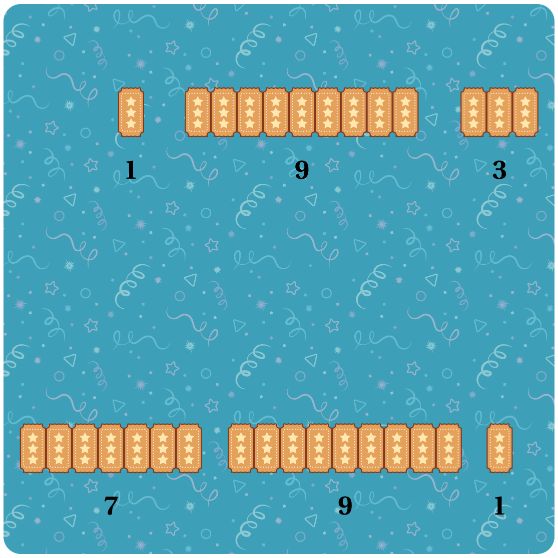
$4 + 2 + 1 + 5 = 12$

$12 \div 4$

4. The **mean**, or average, is the number of tickets each friend gets if the tickets are distributed equally.

Here are the tickets that 6 other friends won.

Calculate the mean number of tickets.



Activity 2

Name: .... Date: .... Period: ....

Mean as a Statistic

5. A **statistic** is a single number that measures something about a data set.

The mean is an example of a statistic.

What does the mean tell us about Ishaan's snacks?

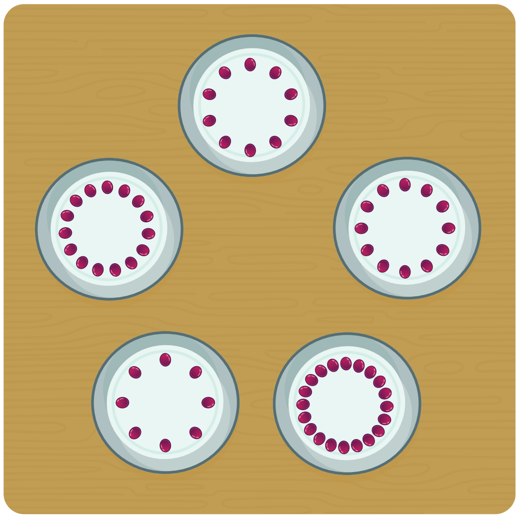
Ishaan's Snacks

Day	Number of Carrots
Monday	10
Tuesday	8
Wednesday	5
Thursday	12
Friday	10

Mean: 9 carrots

6. Here is how many grapes Joel ate each day last week. Calculate the mean of this data.

Day	Number of Grapes
Monday	10
Tuesday	12
Wednesday	20
Thursday	8
Friday	15



7. Ali and Makayla ate blueberries each day last week. Ali ate 12 blueberries per day. The table shows how many blueberries Makayla ate.

Who ate more blueberries per day on average? Circle one.

Ali Makayla They ate the same amount on average

Explain your thinking.

Ali's Snacks

12 blueberries per day

Makayla's Snacks

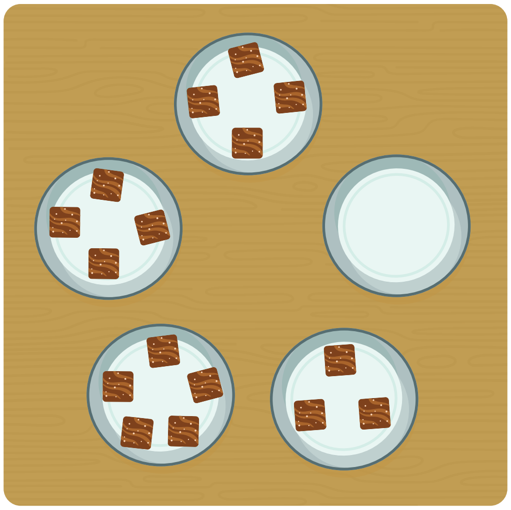
Day	Number of Blueberries
Monday	1
Tuesday	15
Wednesday	13
Thursday	11
Friday	15

Name: .... Date: .... Period: ....

Mean as a Statistic (continued)

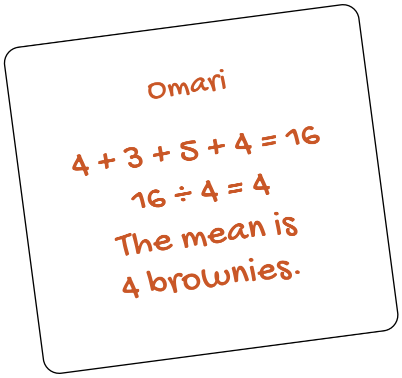
8. Here is how many brownies Omari ate.

Day	Number of Brownies
Monday	4
Tuesday	0
Wednesday	3
Thursday	5
Friday	4



Calculate the mean of this data.

9. Omari calculated the mean incorrectly.



Discuss:

- What is correct about Omari's work?
- What is incorrect about Omari's work?

Name: .... Date: .... Period: ....

Mean as a Measure of Center

10. For 5 days, Oliver recorded the number of cherries he had as a snack. The mean for his data is 11 cherries. Circle a data set that could *not* be Oliver's.

A. **Number of Cherries**

7, 8, 8, 9, 11

B. **Number of Cherries**

6, 9, 10, 11, 20

C. **Number of Cherries**

7, 9, 12, 13, 14

D. **Number of Cherries**

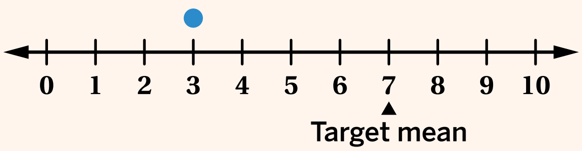
8, 9, 10, 11, 13

Explain your thinking.

You're invited to explore more.

11. Add at least *four* more points to create a dot plot that has a mean of 7. Then check your work.

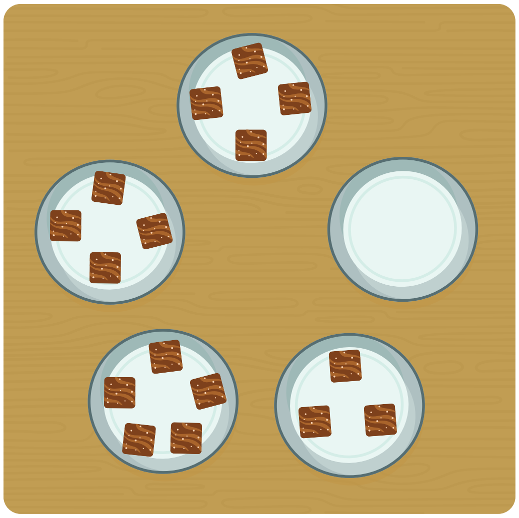
Create as many different dot plots as you have time for.



Synthesis

12. How can you determine the mean of a data set?

Use the example if it helps with your thinking.



Lesson Practice 8.06

Lesson Summary

A **statistic** is a single number that measures something about a data set. One way to measure the center of a data set is by determining the **mean**, or average, of all the data values. You can think of the mean as "an equal share."

For example, suppose this data set represents how many liters of water are in 5 bottles: $1, 4, 2, 3, 0$. To calculate the mean, you first add up all of the values to determine the total (10 liters), then divide that sum by the number of values (5 bottles). This example can be represented by the expression $(1 + 4 + 2 + 3 + 0) \div 5$, or $10 \div 5$. So, the mean amount of water in the 5 bottles is 2 liters (per bottle). The mean is a whole number in this example, but it is possible for the mean to be a rational number.

Lesson Practice
8.06

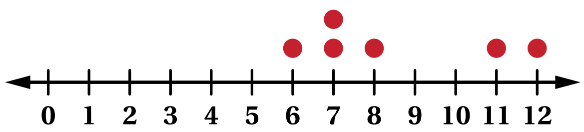
Name: .... Date: .... Period: ....

1. A preschool teacher plans to reorganize these 4 boxes of playing blocks so that each box contains an equal number of blocks. How could the teacher determine the number of blocks to put in each box?

Box 1	Box 2	Box 3	Box 4
32 blocks	18 blocks	41 blocks	9 blocks

2. Three classes worked together to raise money for their classroom libraries. They agreed to share the money equally. The first class raised $\$25.50$, the second class raised $\$49.75$, and the third class raised $\$37.25$. What is each class's equal share? Explain or show your thinking.

3. Kimaya guesses that 11 is the mean of this data set. Without calculating, determine if Kimaya's guess is correct. Explain your thinking.



Problems 4–5: For 12 days, Mai recorded how many minutes long her bus rides to school were. Here are the times she recorded.

Time on the Bus (min)											
9	12	6	9	10	7	6	12	9	8	10	10

4. Determine the mean for Mai's data. Explain or show your thinking.

5. What does the mean tell us about Mai's trip to school?

6. In her English class, Lan's teacher gives 4 quizzes, each worth 5 points. After 3 quizzes, Lan has scores of 4, 3, and 4. How many points does Lan need to get on the last quiz to have an average score of 4? Explain or show your thinking.

Name: ________ Date: ________ Period: ________

7. Select a data set that matches each statement. You can only select each data set once.

a The mean is 2. ________ 1, 1, 2, 2, 2, 4

b Only one value is greater than the mean. ________ 1, 1, 1, 1, 2, 3

c Two values are greater than the mean. ________ 0, 1, 2, 2, 3, 4

FAST Practice

8. In a round of mini golf, Angel records his number of putts on each hole.

Number of Putts								
2	3	1	4	5	2	3	4	3

What is his mean number of putts per hole?

☐ putts per hole

Spiral Review

Problems 9–11: Evaluate the expression $4x^3$ for each value of x.

9. 1

10. 2

11. -3

12. Select *all* of the values of x that are solutions to the inequality $x > -3$.

☐ **A.** -2

☐ **B.** |-4|

☐ **C.** -4

☐ **D.** 3.5

☐ **E.** -3.5

Name: ________ Date: ________ Period: ________

MA.6.DP.1.2, MTR.4.1, MTR.7.1

Modes of Weather

Let’s explore the mode of a data set and what it tells us.

Warm-Up

1. The calendar shows the number of hours of sunshine each day for a month.

What do you notice? What do you wonder?

Hours of Sunshine

S	M	T	W	T	F	S
		6	7	8	4	10
11	0	4	10	11	11	8
3	8	8	10	1	0	6
8	11	12	7	8	3	2
11	13	8	9			

Activity 1

Name: ______ Date: ______ Period: ______

A New Statistic

2. The table shows daily high temperatures for three weeks where Amara lives.

Day	Temperature (°F)	Day	Temperature (°F)	Day	Temperature (°F)
Monday	57	Monday	55	Monday	51
Tuesday	58	Tuesday	53	Tuesday	52
Wednesday	52	Wednesday	57	Wednesday	53
Thursday	44	Thursday	52	Thursday	52
Friday	45	Friday	52	Friday	56
Saturday	50	Saturday	50	Saturday	57
Sunday	52	Sunday	48	Sunday	58

a. Describe the data set. Which values in the data set repeat? How often does each value repeat?

b. The **mode** is a statistic that names the value found most often in a set of numbers. What is the mode of this data set?

c. **Discuss:** How can you display the data from the table to make it easier to determine the mode?

Activity 2

Name: .... Date: .... Period: ....

Identifying the Mode

This table shows the average wind speed each hour last Monday where Amara lives.

3. Identify the mode of the data set.

Show or explain your thinking.

4. What does the mode tell us about the wind speed last Monday where Amara lives?

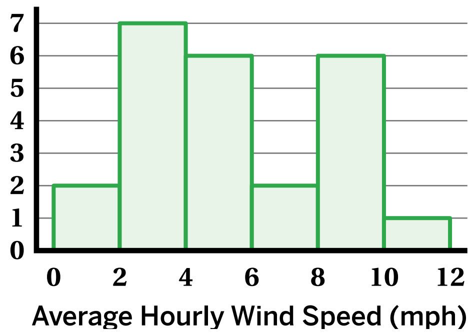
Average Hourly Wind Speed (mph)			
2	9	3	2
2	8	0	4
4	11	2	7
5	7	0	5
8	8	3	8
8	4	3	5

5. Amara and Dakota each displayed the wind speed data.

Amara

0, 0, 2, 2, 2, 2, 3, 3, 3, 4, 4, 4, 5, 5, 5, 7, 7, 8, 8, 8, 8, 8, 9, 11

Dakota



a How does your strategy for identifying the mode compare to Amara's and Dakota's strategies?

b What do you notice that is different between Amara's list and Dakota's histogram? Which display do you think represents the mode more accurately?

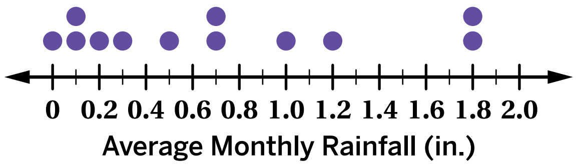
Activity 3

Name: .... Date: .... Period: ....

Interpreting the Mode

The dot plot shows the average monthly precipitation where Amara lives.

6. **a** Describe the frequency of the data values.



b What are the modes of the data set?

7. What do the modes tell us about the average monthly rainfall where Amara lives?

8. Amara calculates the mean for the monthly precipitation values. She says that the mean and the mode are the same for this data set.

Is Amara correct? Explain your thinking.

9. Amara's cousin lives in a different state. The average monthly inches of precipitation in her cousin's state are: 3.8, 1.6, 4.4, 4.2, 2.9, 1.5, 2.7, 2.2, 3.5, 2.1, 1.3, 3.0.

What is the mode of this data set? Explain your thinking.

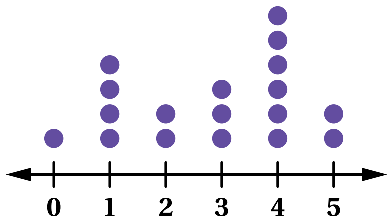
Synthesis

10. How can you determine the mode of a data set?

Use the examples if they help with your thinking.

Data Set

4, 1, 2, 3, 4, 4, 1, 1, 5, 3, 3, 4, 4, 5, 4, 2, 0, 1



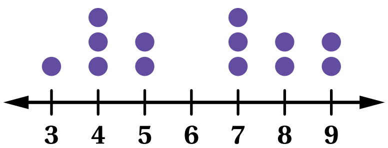
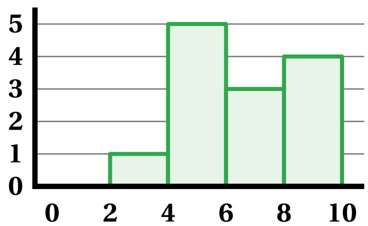
Lesson Practice 8.07

Lesson Summary

You can describe a data set using a measure of center called the **mode**. The mode is the data value that occurs most often in the data set. Some data sets have no mode. Some data sets have one mode. And other data sets have more than one mode.

To determine the mode, count how many times each data value appears in the data set. You can display the data set in a way that helps identify the mode. Some displays are more helpful than others.

Here is an example: 7, 9, 4, 4, 8, 8, 5, 3, 7, 9, 4, 7, 5.

Ordered List	Dot Plot	Histogram
3, 4, 4, 4, 5, 5, 7, 7, 7, 8, 8, 9, 9		

This data set has two modes, 4 and 7. The histogram with a bin size of 2 is not as helpful in accurately displaying the mode.

Name: ____________ Date: ________ Period: ________

Problems 1–4: Tyani kept track of how many miles she ran each day for 10 days. Here is the data.

Miles Run Each Day									
2.7	3.2	2.7	4.2	3.2	3.5	2.5	3.2	4.1	3.7

1. Determine the mode for Tyani's data. Show or explain your thinking.

2. What does the mode tell us about Tyani's running practice?

3. Basheera said that the mode of the data is 3. What could you say to help Basheera understand why her answer is incorrect?

4. How does the mode for this data set compare to the mean?

A. The mode is greater than the mean.

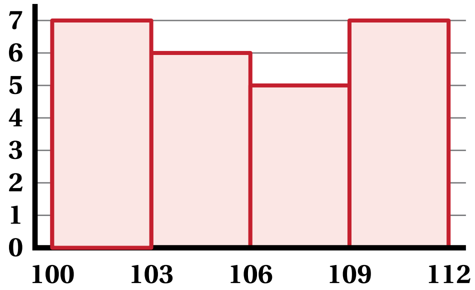
B. The mode is less than the mean.

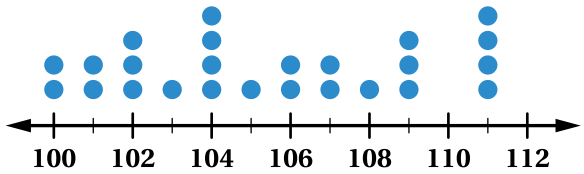
C. The mode is equal to the mean.

D. There is not enough information to compare.

Name: ________ Date: ________ Period: ________

Problems 5–6: Two students made data displays to understand the same data set.





5. Which data display can you use to determine the mode? Explain your thinking.

6. Determine the mode(s) for the data set.

7. Which data values must you remove from the data set so that the mode is $4\frac{1}{2}$?

$2\frac{3}{4}, 6\frac{1}{4}, 4\frac{1}{2}, 1, 2\frac{3}{4}, 5\frac{1}{2}, 4\frac{1}{2}, 1, \frac{1}{4}$

FAST Practice

8. What is the mode of the data set?

Pounds of Seafood									
4.8	5.4	5.2	4.4	5.4	3.9	5.2	6.0	4.5	5.2

Spiral Review

Problems 9–10: Determine the mean for each data set.

9. 0.4, 0.7, 0.5, 0.5, 0.1, 0.4, 0.2

10. $2\frac{1}{2}, 2\frac{1}{2}, 2\frac{1}{2}, 3, 4, 4, 4\frac{3}{4}, 4\frac{3}{4}$

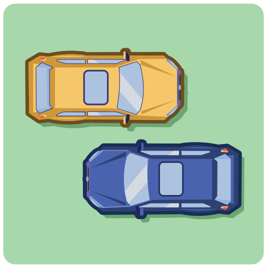
Unit 8
Lesson
8

Name: ______ Date: ______ Period: ______

MA.6.DP.1.2, MTR.2.1, MTR.3.1, MTR.6.1

Toy Cars

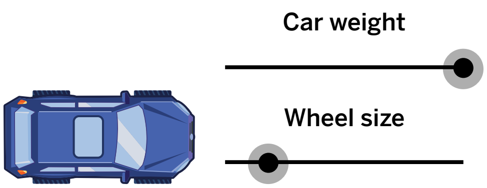
Let's explore the median of a data set and what it tells us.



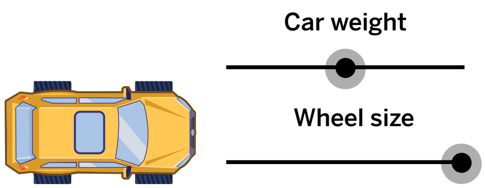
Warm-Up

1. Here are two toy cars with different colors, weights, and wheel sizes.

Car A

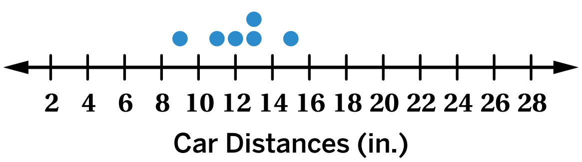


Car B

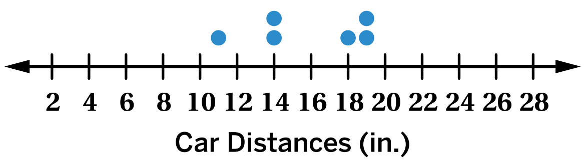


a Each car was launched 6 times. Compare their results.

Car A



Car B



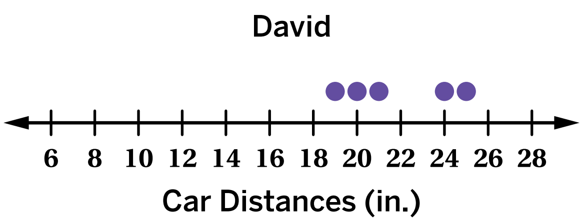
b **Discuss:** Which car generally travels farther? How do you know?

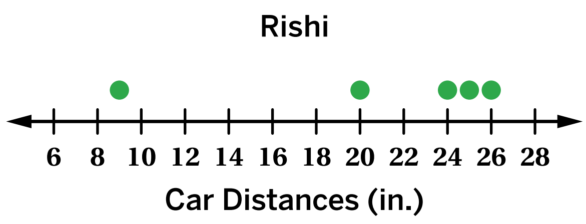
Activity 1

Name: ______ Date: ______ Period: ______

A New Measure of Center

2. David and Rishi launched their cars 5 times each.





Which car do you think generally travels farther? Circle one.

David's car   Rishi's car   I'm not sure

Explain your thinking.

3. David and Rishi each think their car travels farther.

David says: *When we compare the mean distance for each car, my car travels farther.*

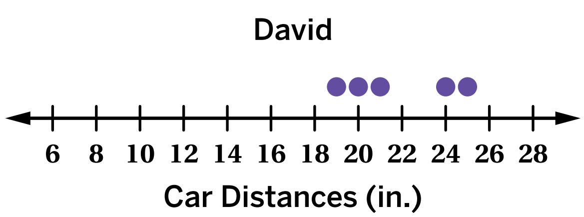
Rishi says: *When we compare the middle distance for each car, my car travels farther.*

Whose argument do you agree with? Circle one.

David's   Rishi's   Both   Neither

Explain your thinking.

4. The middle value of a data set is called the **median**. The median distance for Rishi's car is 24 inches. What is the median distance for David's car?



Activity
2

Name: ______ Date: ______ Period: ______

Many Medians

5. Yona launched her car 7 times and recorded the distances, in inches, on a notepad. She calculated the median but made a mistake.

Yona
~~19~~, ~~14~~, ~~18~~, (28), ~~21~~, ~~12~~, ~~14~~

a **Discuss:** What did Yona do well?

b What could you change to make all her work correct?

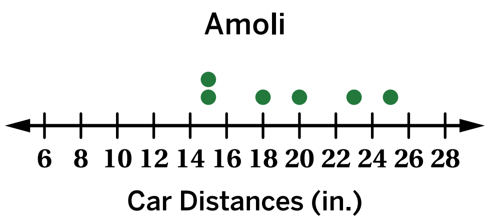
6. Here is Yona's data.

19, 14, 18, 28, 21, 12, 14

What is the median distance of this data set?

7. Amoli launched her car 6 times.

What do you think is the median of her data set? Explain your thinking.



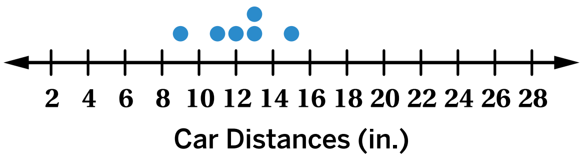
Name: .... Date: .... Period: ....

Many Medians (continued)

8. Statisticians agree that when there's an even number of data points, the median is the average of the middle two numbers.

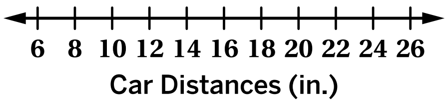
The median distance for Amoli's car is 19 inches because the average of 18 and 20 is 19.

The dot plot shows the data on Car A. What is the median distance that Car A traveled?



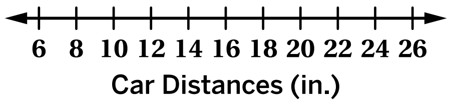
9. Calculate the median distance for each car. Complete as many problems as you have time for. Create a data display if it helps with your thinking.

a 14, 19, 15, 20, 17



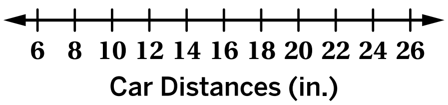
.... inches

b 25, 22, 19, 21, 14, 14



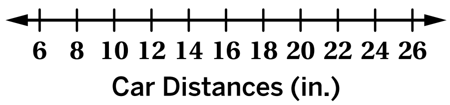
.... inches

c 18, 12, 24, 12, 18, 22, 22



.... inches

d 16, 6, 26, 14, 23, 27, 28, 19, 23, 17

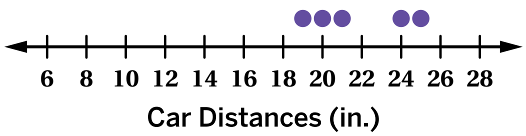


.... inches

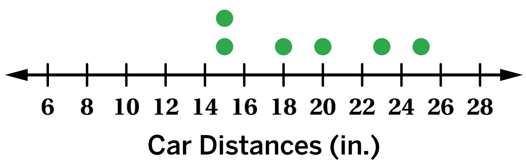
Synthesis

10. Describe how to determine the median of a data set. Use the examples if they help with your thinking.

David



Amoli



Lesson Practice 8.08

Lesson Summary

You can describe a data set using another measure of center called the **median**. The median is the "middle" value in a data set when the values are listed in order from least to greatest (or greatest to least). Half of the data values are less than or equal to the median, and half of the data values are greater than or equal to the median.

To determine the median from an ordered representation of the data, you can repeat a process of eliminating the pairs of least and greatest values.

Here are some examples.

Odd Number of Values

~~0~~ ~~1~~ ~~1~~ (2) ~~2~~ ~~4~~ ~~5~~

Once all pairs have been eliminated, only one value remains in the middle, making it the median.

Median: 2

Even Number of Values

~~0~~ ~~1~~ ~~1~~ (1 2) ~~2~~ ~~4~~ ~~5~~

Once all pairs have been eliminated, two values remain.

Their average is the median.

$(1 + 2) \div 2 = 1.5$

Median: 1.5

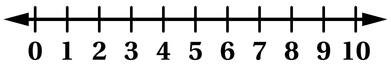
Lesson Practice
8.08

Name: ______________ Date: ________ Period: ________

Problems 1–2: Here is a data set.

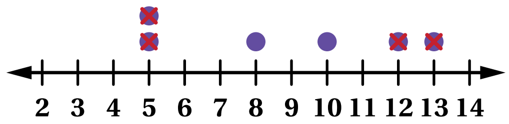
1	6	7	6	2	9	3

1. Create a dot plot for this data.



2. Determine the median of this data.

Problems 3–4: Kayla wants to determine the median of the data in this dot plot. She starts solving the problem but isn't sure what to do next.



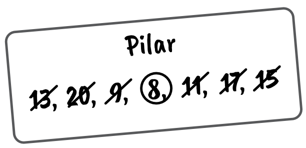
3. What could you say to help Kayla determine the median?

4. What is the median of this data?

5. The table shows Prisha's scores after attempting the first level of a video game 10 times. What is her median score?

130	150	120	170	130
120	160	160	190	140

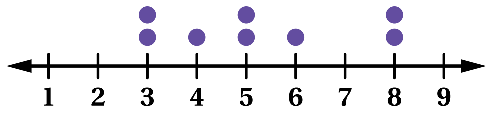
6. Pilar recorded the number of points she scored in her last 7 basketball games. She says that her median score was 8 points. Is she correct?



Explain your thinking.

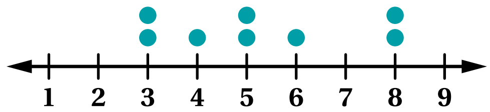
Name: .... Date: .... Period: ....

7. *Add* points to this dot plot to make the median 4.



FAST Practice

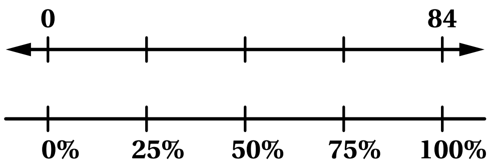
8. Cross out points to remove from this dot plot to make the median 4.



Spiral Review

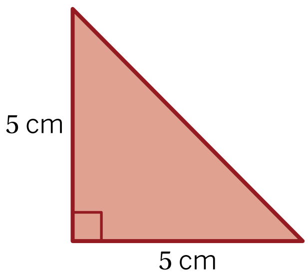
9. Determine 25%, 50%, and 75% of 84.

Use the double number line if it helps with your thinking.

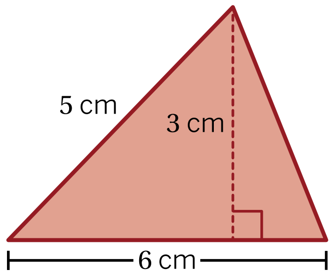


Problems 10–12: Calculate the area of each triangle.

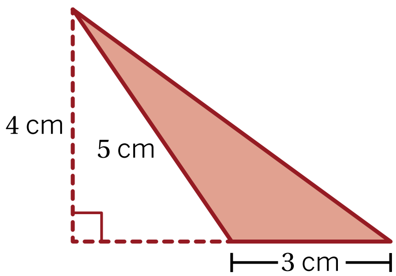
10.



11.



12.



Name: ______ Date: ______ Period: ______

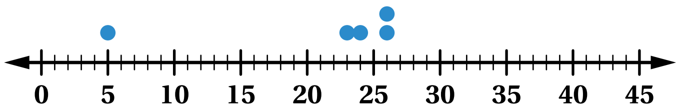
MA.6.DP.1.6, MTR.1.1, MTR.4.1

In the News

Let's compare the mean and median of data sets.

Warm-Up

1. Here is a data set.



a Which is greater, the mean or the median? Circle one.

Mean Median They are the same

b What could this data be about?

Name: .... Date: .... Period: ....

Mean vs. Median

2. Remy is throwing a party for her 12th birthday.

Here are the ages of the people who attend Remy's party.

Ages of Party Guests (yr)

11, 11, 11, 11, 11, 11, 11, 11, 11, 11, 11, 11, 11, 12, 12, 12, 12, 12, 12, 12, 12, 12, 12, 12, 12, 13, 13, 13, 13, 33, 34, 36, 40, 42, 43, 45

Median: 12 **Mean: 17**

What do you notice? What do you wonder?

I notice:

I wonder:

3. Here is a news headline about Remy's birthday party.

Let's look at the data the newspaper used.

Why is the headline misleading?

Name: .... Date: .... Period: ....

Mean vs. Median (continued)

4. Why do you think the mean is higher than the median?

5. Remy's aunt also threw a party.

Ages of Remy's Aunt's Party Guests (yr)

36, 36, 36, 37, 37, 37, 37, 37, 37, 38, 38, 39, 39, 39, 39, 40, 40, 40,
40, 40, 40, 41, 41, 41, 41, 41, 42, 42, 42, 43, 43, 44, 44, 44, 44, 45

Median: 40 **Mean: 40**

a Compare the data for Remy's party guests with the data for her aunt's party guests.

b **Discuss:** Why do you think the mean and median ages are closer together for Remy's aunt's party than Remy's party?

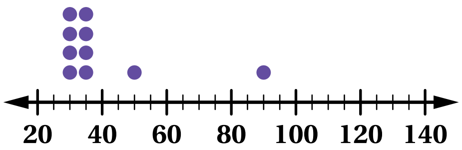
Activity 2

Name: .... Date: .... Period: ....

Which Would You Report?

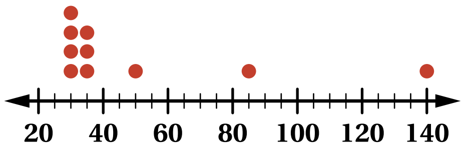
6. A reporter was wondering how salaries at the local grocery store have changed in recent years. She found data from five years ago and from today.

Five Years Ago



Employee Salaries (thousands of dollars)

Today



Employee Salaries (thousands of dollars)

What do you notice? What do you wonder?

I notice:

I wonder:

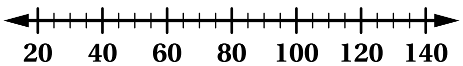
7. A local newspaper is considering two different headlines.

Headline 1

Headline 2

Discuss: Which headline do you think is more accurate? Why?

8. A different grocery store has a median salary that’s greater than the mean salary. Create a dot plot with at least 10 points to represent what the salaries at this grocery store could be.



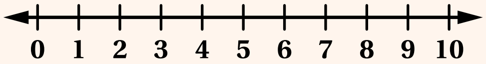
Employee Salaries (thousands of dollars)

Which Would You Report? (continued)

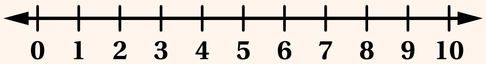
You're invited to explore more.

9. Create a dot plot that has:

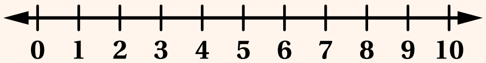
a
- A median of 7.
- A mean that is less than the median.



b
- A median of 6.
- A mean of 5.



c
- A median of 3.
- A mean of 5.

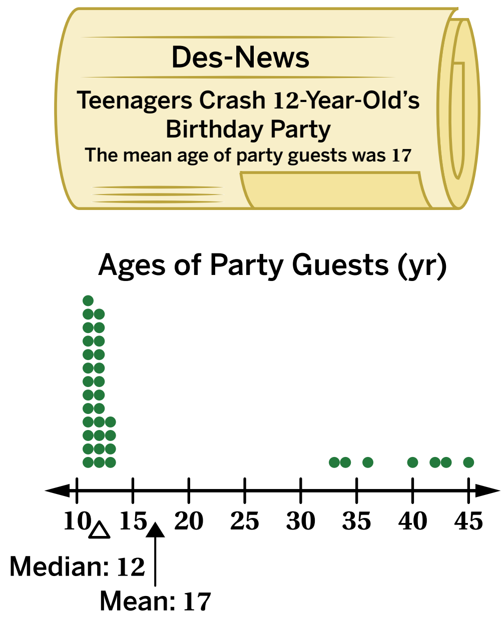


Synthesis

10. Discuss both questions below with a partner.

- When are the mean and median likely to be far apart?
- When might you choose to report the median instead of the mean?

Select *one* to answer. Use the example if it helps to show your thinking.

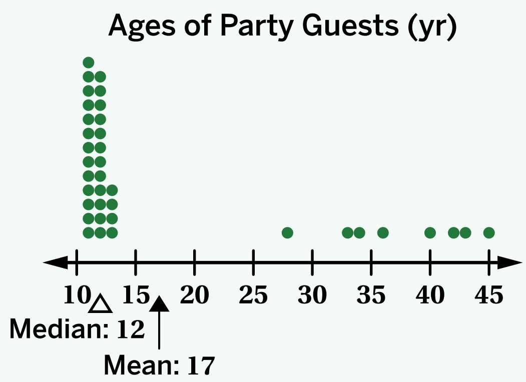


Lesson Practice 8.09

Lesson Summary

You can use mean and median to tell you different things about a data set. One measure might be more appropriate depending on the shape of the data and the situation.

Here is data from a 12-year-old's birthday party. The median of this data set is 12. The mean of this data set is 17. In this situation, the median is a better measure to represent the ages of the party guests because most guests are 11 to 13 years old. The mean has shifted away from where most of the data is because the older ages are added to the younger ages.

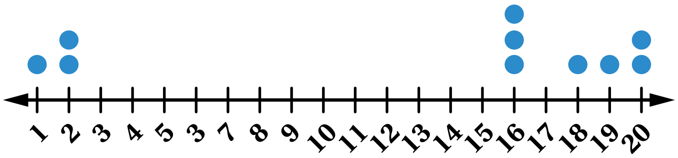


When most of the data is in one place but there are a few data points far away from the group (like in the dot plot shown), the mean and median are likely to be far apart.

Lesson Practice
8.09

Name: ______________ Date: ______________ Period: ______________

1. Here are two statistics related to this dot plot: 13 and 16.



Without calculating, determine which number is the median and which number is the mean.

Median: ______________ Mean: ______________

2. Here are the scores for Sahana's history homework.

100	100	100	100	95	100	90	100	0

The history teacher uses the mean to calculate an overall grade for homework. How might Sahana argue that the median is a better way to measure her work?

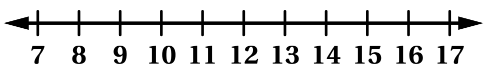
3. Ayaan competed in a bowling tournament. Here are his scores for each game.

Game 1	Game 2	Game 3	Game 4	Game 5	Game 6
300	250	275	120	243	290

Do you think the tournament judges should use the median or the mean to calculate Ayaan's final score? Justify your choice.

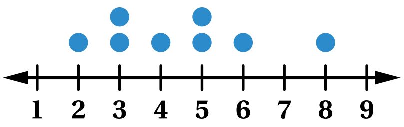
4. Create a dot plot with 5 data points that has:

- A median of 14.
- A mean less than 14.

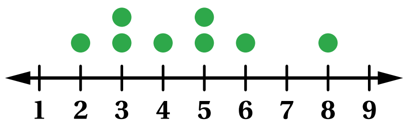


Name: .... Date: .... Period: ....

5. Add one point to this dot plot to make the median *greater* than the mean.

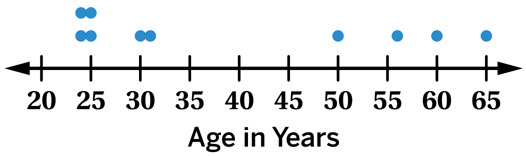


6. Add one point to this dot plot to make the median equal to the mean.



FAST Practice

7. Kweku surveyed 10 teachers at his school. The dot plot displays the ages of those teachers. Which of these statements is *true*?



A. The median is greater than the mean.

B. The median and the mean are the same.

C. The median is less than the mean.

D. The mean cannot be determined.

Spiral Review

Problems 8–11: Calculate each percentage.

8. 25% of 30

9. 75% of 30

10. 75% of 150

11. 58% of 18

Name: ______ Date: ______ Period: ______

MA.6.DP.1.2, MA.6.DP.1.6, MTR.4.1, MTR.5.1, MTR.7.1

Hoops

Let's explore how to measure the spread of a data set.

Warm-Up

1. You will use the Warm-Up Sheet for this activity.

How many baskets can you make?

- Flick a counter from each of the starting points. If the counter lands on top of the hoop, you made a basket.
- You get 8 shots per round.
- Try to make as many as you can!

Round 1: ______ baskets Round 2: ______ baskets

2. Let's look at the Round 1 and Round 2 results from the class.

Discuss: What do you notice?

Activity 1

Name: .... Date: .... Period: ....

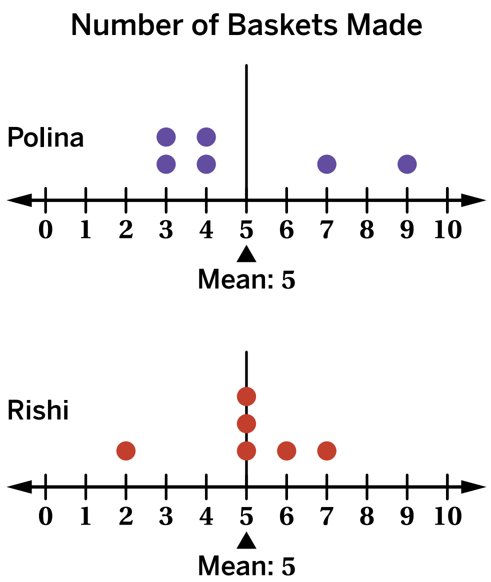
Range

3. Polina and Rishi practice 10 free throws everyday. Each dot plot shows the number of baskets they made on 6 different days.

Which player is more consistent? Circle one.

Polina Rishi

Explain your thinking.



4. The range is a statistic that measures the spread of a data set. Polina's data has a range of 6 baskets.

Is the range of Rishi's data larger than, smaller than, or equal to the range of Polina's data? Circle one.

Larger Smaller Equal

Explain your thinking.

Activity 2

Name: .... Date: .... Period: ....

Calculating Range

5. Deven also practices free throws everyday.

Number of Baskets	6	7	1	6	7	5	9	7

Calculate the range of Deven's data.

6. Deven practices free throws the next day.

Discuss: How would each of the following results affect the range of Deven's data?

- Deven makes fewer baskets than any other day.
- Deven makes the same number of baskets as another day.
- Deven makes more baskets than any other day.

7. Here are the number of baskets that a new player made.

Number of Baskets	3	3	4	5	5	5	7	8	9

Circle one data value to remove from the data set so the range is 5.

Explain your thinking.

Name: ______ Date: ______ Period: ______

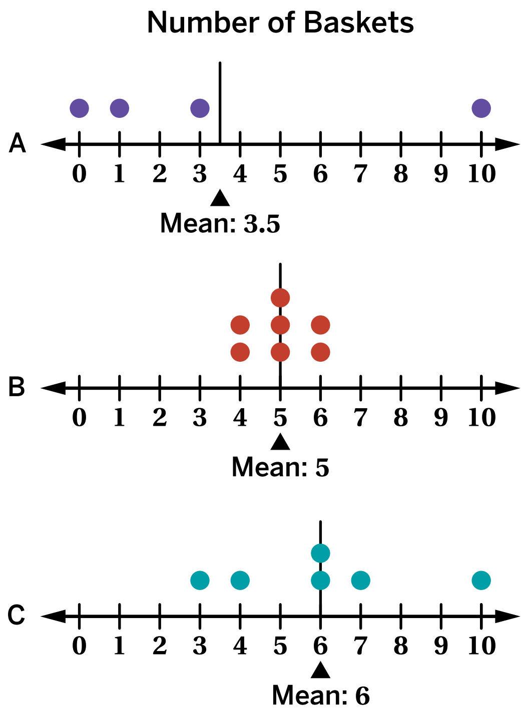
Calculating Range (continued)

8. Here are the number of baskets that three new players made.

Order the data sets from *smallest* to *largest* range.

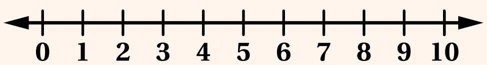
Smallest Range

Largest Range



You're invited to explore more.

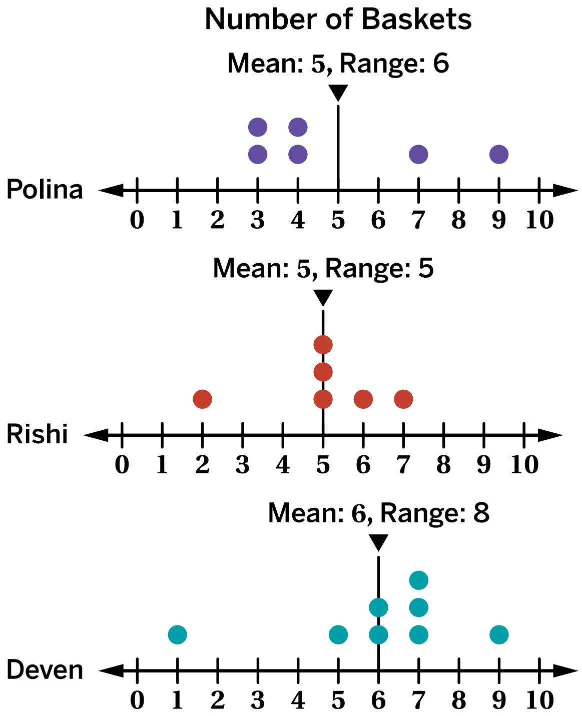
9. Create a dot plot with at least *four* points, a mean of 6, and a range of 4.



Synthesis

10. How does the range help you compare data sets?

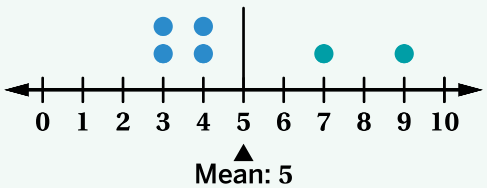
Use the examples if they help with your thinking.



Lesson Practice 8.10

Lesson Summary

You can describe how spread out the values in a data set are with a single number, the range. The range is calculated as the difference between the maximum value in a data set and the minimum value in a data set.



$9 - 3 = 6$

The range is 6.

The range is an example of a *measure of spread*. A measure of spread is a way to measure the consistency of the values in a data set. The smaller the value of the range, the less spread out the data points are and the more consistent the data is. The larger the range, the more spread out the data points are and the less consistent the data is.

Adding a data value less than or greater than the original values will increase the range. Removing a minimum or maximum that is not repeated will decrease the range.

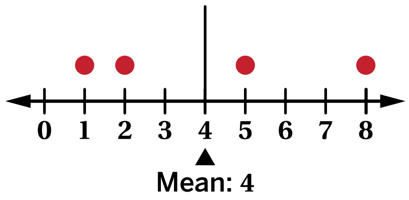
Name: .... Date: .... Period: ....

1. This table shows the amount of time it takes 6 students to get to school.

Time (minutes)	10	10	18	20	30	44

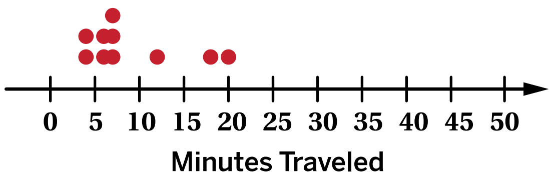
Calculate the range of this data set.

2. Calculate the range of this data set.

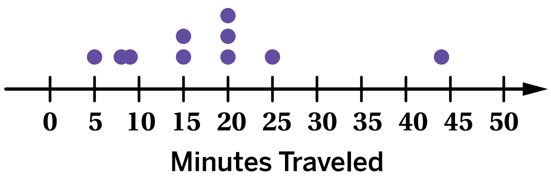


Problems 3–4: These dot plots show the travel times for 10 students from two cities.

City A



City B



3. The ranges have been calculated for you. Match each range to the correct city.

Range	City
16	
39	

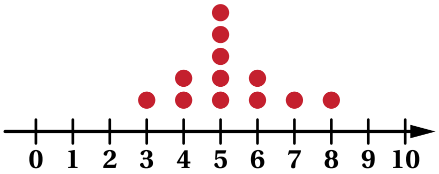
4. What data value can you add to City A so that the cities have the same range?

Lesson Practice
8.10

Name: .... Date: .... Period: ....

Problems 5–6: Nathan and Esi recorded the number of baskets they each made out of 10 attempts. They each collected 12 data points.

Nathan



Esi	7	8	9	0	1	2	1	3	2	10	9	8

5. Calculate the range of each player's data.

6. Which player is more consistent?

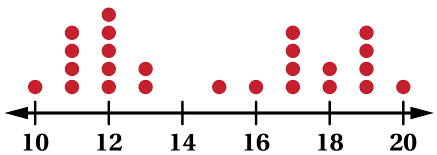
FAST Practice

7. Here are two dot plots showing the recorded speeds of two manatees.

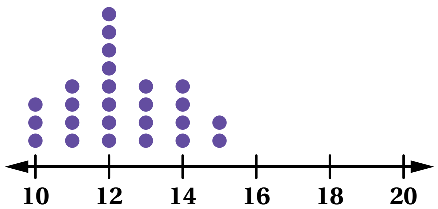
Which of these statements is true?

- **A.** The data for Dot Plot B is more spread out than the data for Dot Plot A.
- **B.** The manatee recorded in Dot Plot A swims at a more consistent speed.
- **C.** The range of Dot Plot A is greater than the range of Dot Plot B.
- **D.** Both dot plots have approximately the same mean.

Dot Plot A



Dot Plot B

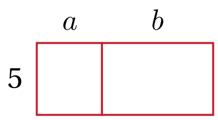


Spiral Review

8. Select *all* the expressions that are equivalent to $2(6 + 3x)$.

- ☐ **A.** $8 + 5x$
- ☐ **B.** $12x + 6$
- ☐ **C.** $12 + 6x$
- ☐ **D.** $6(2 + x)$
- ☐ **E.** $2(9x)$

9. Select *all* the expressions that represent the total area of the rectangle.



- ☐ **A.** $5(a + b)$
- ☐ **B.** $5 + ab$
- ☐ **C.** $5a + 5b$
- ☐ **D.** $2(5 + a + b)$
- ☐ **E.** $5ab$

Unit 8
Lesson
11

Name: ______ Date: ______ Period: ______

MA.6.DP.1.3, MTR.1.1, MTR.4.1

Pumpkin Patch

Let's determine and interpret the quartiles of a data set.

Warm-Up

1. A farmer sells different sizes of pumpkins.

A customer wants a medium-sized pumpkin.

Circle the pumpkins the farmer could sell to this customer.

Name: .... Date: .... Period: ....

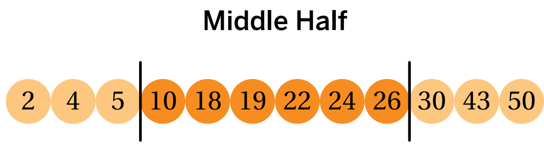
Introduction to Quartiles

2. Farmer Na'ilah always marks the middle half of her pumpkins as medium-sized.

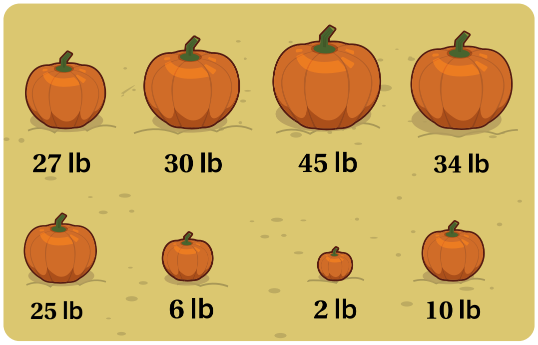
a Na'ilah made this diagram to show the pumpkins so that she could decide which pumpkins were in the middle half.

b Explain how she decided.

Pumpkin Weights (lb)



3. Here are some pumpkins on a different farm. Circle the pumpkins with weights that are in the middle half.

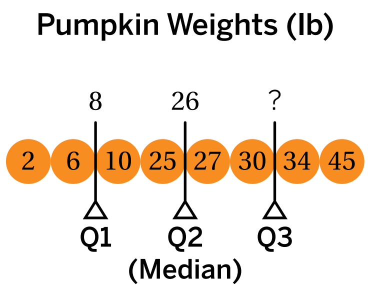


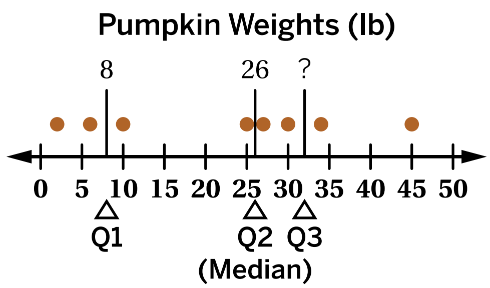
Name: .... Date: .... Period: ....

Introduction to Quartiles (continued)

4. Quartiles divide a data set into four sections. They can help us identify and describe the middle half of a data set.

Here are Quartile 1 (Q1) and Quartile 2 (Q2) for Na'ilah's pumpkins.





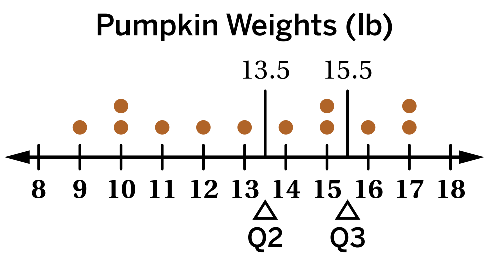
What do you think the value of Quartile 3 is?

Explain your thinking.

5. Quartile 1 is the median of the lower half of a data set.

Here are the weights of 12 pumpkins on Tasia's farm.

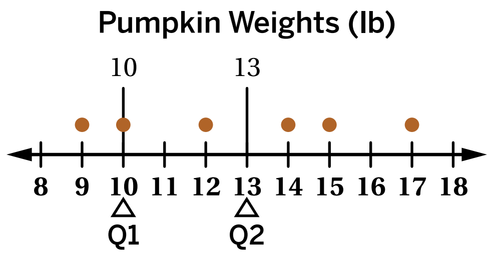
What is the value of Quartile 1?



6. Quartile 3 is the median of the upper half of a data set.

Here are the weights of 6 pumpkins on Tyler's farm.

What is the value of Quartile 3?

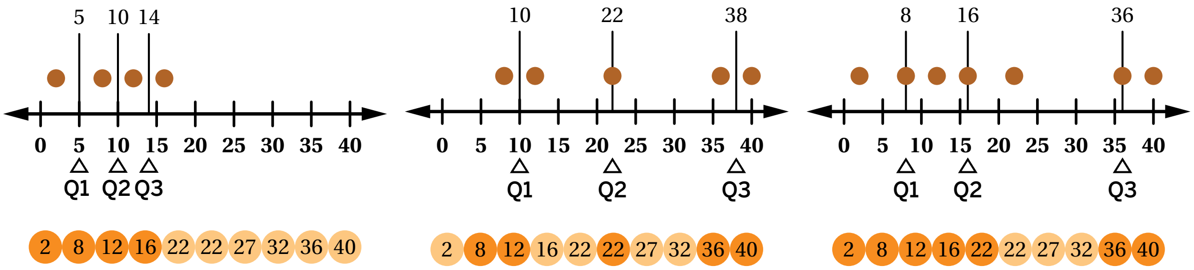


Activity 2

Name: ______ Date: ______ Period: ______

Reasoning About Quartiles

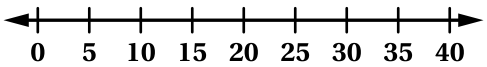
7. Here are some dot plots with different pumpkin weights.



4 pumpkins 5 pumpkins 7 pumpkins

a **Discuss:** What do you notice?

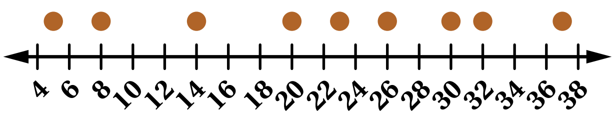
b Here are several challenges. Choose one challenge and create a dot plot that represents it using some or all of the pumpkins.



- ☐ Q2 is not equal to a pumpkin weight.
- ☐ Q1, Q2, and Q3 are equal to pumpkin weights.
- ☐ Q1, Q2, and Q3 are not equal to pumpkin weights.
- ☐ Q1 is equal to a pumpkin weight, but Q3 is not.
- ☐ Q1 and Q2 have the same value.

8. Here are 9 pumpkins on Adrian's farm. What do you think the values of Q1 and Q3 are? Explain your thinking. Use the dot plot if it helps to show your thinking.

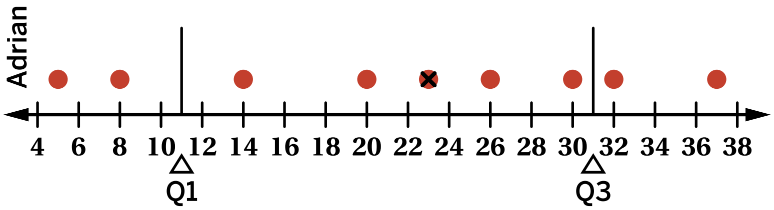
Pumpkin Weights (lb)



Reasoning About Quartiles (continued)

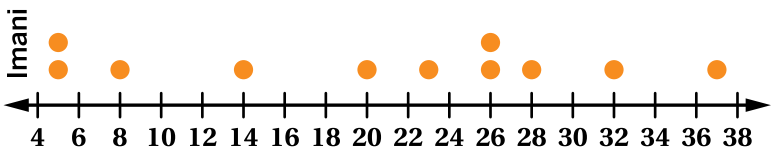
9. Statisticians agree that when you determine Q1 and Q3 for a data set that has an odd number of points, you do *not* include the median in the lower or upper half of the data.

Pumpkin Weights (lb)



This dot plot shows the weight of 11 pumpkins on Imani's farm.

Pumpkin Weights (lb)

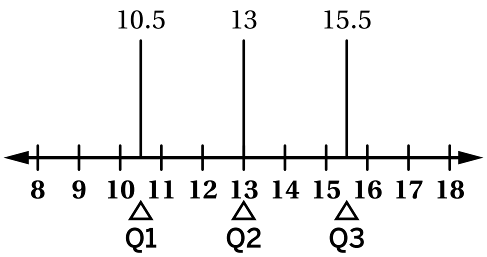


What is the value of Q3 for these pumpkin weights?

10. A store has 80 pumpkins for sale. Here are the values of the quartiles.

About how many of the 80 pumpkins would you expect to weigh *less* than 15.5 pounds?

Pumpkin Weights (lb)



Explain your thinking.

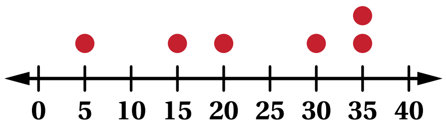
Activity 3

Name: ________ Date: ________ Period: ________

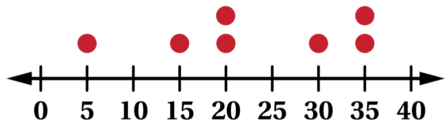
Repeated Challenges

11. Solve as many problems as you have time for. Sensemaking is more important than speed.

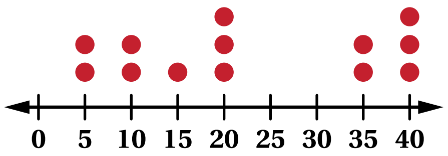
a The dot plot shows 6 data points. What is the value of Q1?



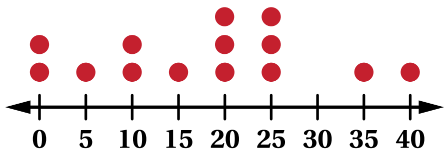
b The dot plot shows 7 data points. What is the value of Q2?



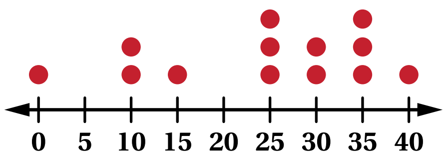
c The dot plot shows 13 data points. What is the value of Q1?



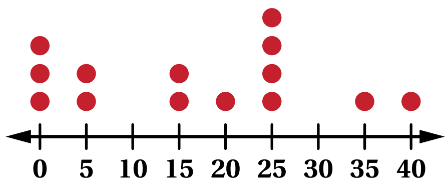
d The dot plot shows 14 data points. What is the value of Q3?



e The dot plot shows 13 data points. What is the value of Q2?



f The dot plot shows 14 data points. What is the value of Q3?

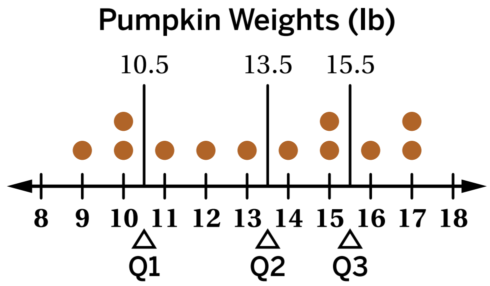


Synthesis

12. Discuss both questions with a partner. Then select *one* and write your response.

Use the example if it helps to show your thinking.

- How do quartiles relate to the middle half of a data set?
- How can you determine the value of the quartiles for a data set?

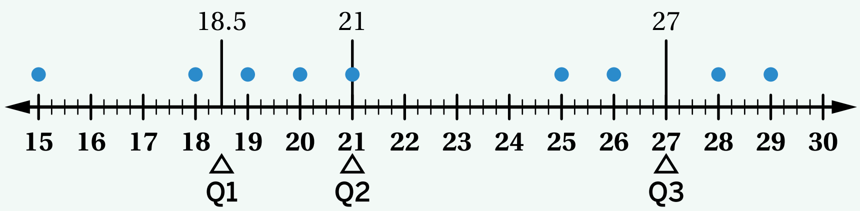


Lesson Practice 8.11

Lesson Summary

You can describe the middle half of a data set by dividing it into four equal sections called **quartiles**.

You can determine the value of the quartiles by splitting the entire data set in half and then splitting the halves again. The middle half is all the data points that are between Q1 and Q3. Representations such as dot plots are helpful for identifying quartiles to describe data sets.



The *first quartile (Q1)* is the median of the lower half of the data set.

The *second quartile (Q2)* is the median of the entire data set.

The *third quartile (Q3)* is the median of the upper half of the data set.

Lesson Practice
8.11

Name: .... Date: .... Period: ....

Problems 1–2: Here are the ages of 20 people at a family reunion, ordered from youngest to oldest.

3, 8, 9, 10, 11, 11, 12, 18, 18, 28, 30, 35, 37, 40, 53, 54, 58, 65, 70, 72

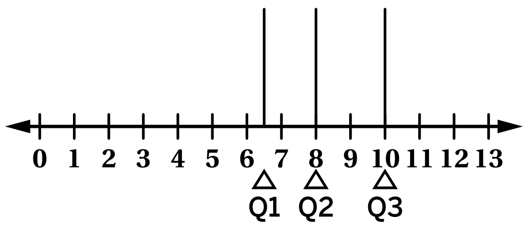
1. The value of Quartile 2 (Q2) is 29. What does that tell us about the people at the family reunion?

2. Determine the value of Q1 and Q3.

Quartile 1 (Q1): .... Quartile 3 (Q3): ....

Problems 3–5: Haru recorded how long his bus ride to school took for 16 days. Here are the values of the quartiles of his data.

Bus Ride Length (min)



3. About how many rides would you expect to be less than 6.5 minutes long?

4. About how many rides would you expect to be less than 10 minutes long?

5. About what percent of the rides would you expect to be between 6.5 minutes and 10 minutes long?

6. The heights, in inches, of Javier's classmates are 60, 68, 56, 60, 62, 58, 55, 67, 59, 61, 62, 64, 63, 63, 59, 62, 66, and 61. Determine the values of Q1, Q2 (median), and Q3.

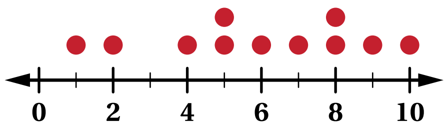
	Height (in.)
Q1	
Q2 (Median)	
Q3	

Lesson Practice
8.11

Name: .... Date: .... Period: ....

FAST Practice

7. Makayla and Axel both try to determine the quartiles for this dot plot.



Makayla says:

Q1: 4.5 Q2: 6 Q3: 8

Axel says:

Q1: 4 Q2: 6 Q3: 8

Who is correct? Explain your thinking.

A. Makayla **B.** Axel is correct. If the data set has an odd number of points, the median **A.** should **B.** should not be included when determining Q1 and Q2.

Spiral Review

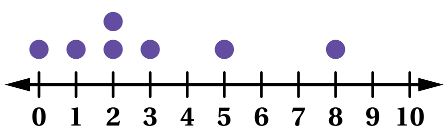
Problems 8–10: Calculate each percentage.

8. 25% of 40

9. 25% of 120

10. 25% of 90

11. Add 5 points to this dot plot without changing the median.

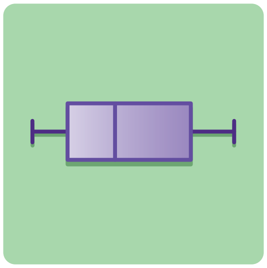


Unit 8
Lesson
12

Name: ________ Date: ________ Period: ________

MA.6.DP.1.3, MA.6.DP.1.5, MA.6.DP.1.6, MTR.1.1, MTR.4.1

Car, Plane, Bus, or Train?



Let's explore box plots to visualize data.

Warm-Up

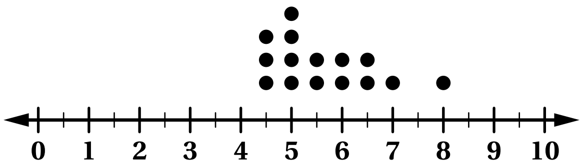
Jalen's family lives in St. Louis. They often visit relatives in Chicago.

1. **Discuss:**

- What forms of transportation do you think Jalen's family could take?
- Which would *you* take? Why?

2. Jalen recorded how long it took to drive from St. Louis to Chicago the last 15 times his family drove there.

Determine the values of Q1, Q2, and Q3. Label them on the dot plot.

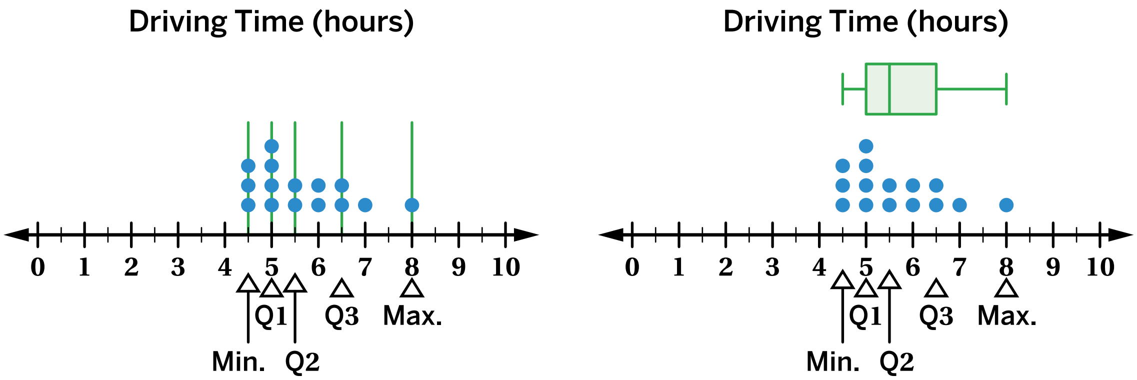


Name: .... Date: .... Period: ....

Car or Plane?

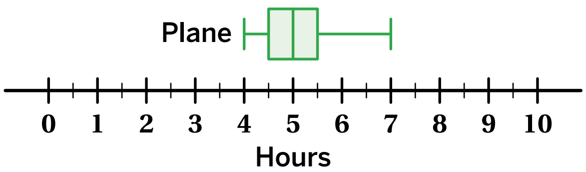
Jalen's family wonders: *What form of transportation should we use to travel between Chicago and St. Louis?*

3. One way to compare data sets is by using **box plots**. Let's look at how a box plot is drawn. Study the two drawings below.



Discuss: What do you notice? What do you wonder?

4. Jalen's family sometimes takes a plane to get to St. Louis. Here is a box plot that represents some of their travel times when they flew.



a Determine each statistic for the plane data.

- Minimum: ....
- Quartile 1: ....
- Median: ....
- Quartile 3: ....
- Maximum: ....

b What percent of the plane data was less than 5 hours long? Explain your thinking.

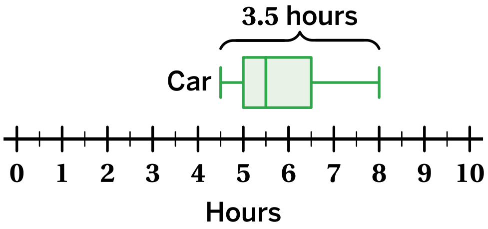
Activity 2

Name: .... Date: .... Period: ....

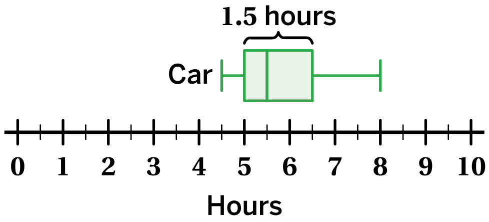
Measures of Spread

5. Jalen's family is also wondering: *How spread out are our travel times when we travel by car?*

Let's look at two ways to describe the spread of a box plot.



The range is the distance from the *minimum* to the *maximum*.



The **interquartile range (IQR)** is the distance from Q1 to Q3.

a Read the definitions of range and IQR.

How are they alike?

How are they different?

b Jalen says that one of these measures of spread describes the middle 50% of the data points.

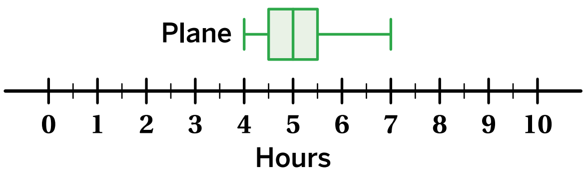
Which measure of spread do you think he is talking about? Circle one.

IQR Range Neither

Explain your thinking.

6. What is the range and IQR for the plane data?

Range: .... IQR: ....



7. Use the car data and the plane data to help you answer: *Which is more consistent, driving or flying?* Explain your thinking.

Activity 3

Name: ______________ Date: ______ Period: ______

Bus or Train?

8. Jalen's family has also traveled to Chicago by bus and by train.

You will use a set of cards for this activity.

- Match each dot plot, box plot, and statistic to a form of transportation. There will be two empty spaces in the table for you to complete.
- Draw a box plot on Card A and determine the values of the missing statistics to complete the table.

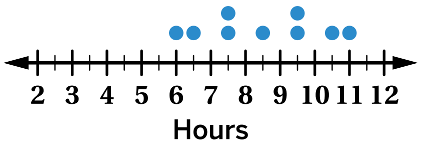
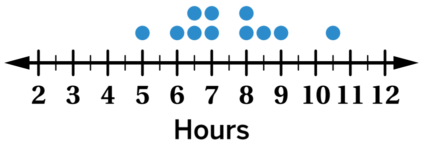
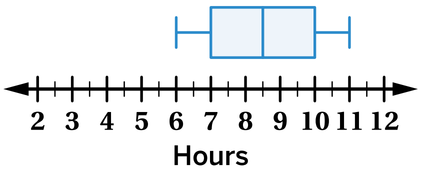
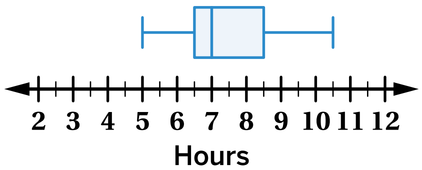
	Bus	Train
Travel Times (hours)	7.5, 10.5, 9.5, 9.5, 6 7, 6.5, 7.5, 11, 8.5	6.5, 8.5, 6, 10.5, 6.5 8, 7, 7, 9, 8
Dot Plot		
Box Plot		
Median		
IQR		
Range		

9. Which form of transportation (car, train, bus, or plane) would you recommend Jalen's family use to travel between Chicago and St. Louis? Use evidence to support your argument.

Name: .... Date: .... Period: ....

Bus or Train? (continued)

10. Jalen's sister did not travel by bus for one of the trips to Chicago and instead took the train. Let's look at her data.

	Bus	Train
Travel Times (hours)	7.5, 10.5, 9.5, 9.5, 6 6.5, 7.5, 11, 8.5	6.5, 8.5, 6, 10.5, 6.5, 5 8, 7, 7, 9, 8
Dot Plot		
Box Plot		
Median	8.5	7
IQR	3	2
Range	5	5.5

a What do you notice about the bus travel time removed and the train travel time added to the data sets?

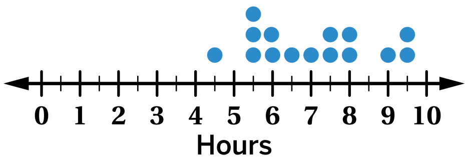
b How does the removal of the bus travel time affect the measures of center and spread?

c How does the addition of the train travel time affect the measures of center and spread?

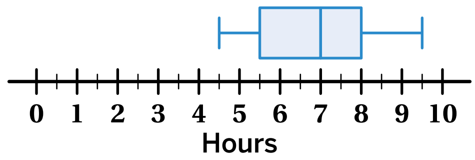
Synthesis

11. Here is a dot plot, a box plot, and several statistics for the car data.

Dot Plot



Box Plot



Statistics			
Median: 7 hours	Number of Data Points: 15	Range: 5 hours	IQR: 2.5 hours

Which statistics are more visible in the dot plot? In the box plot? Why do you think that is?

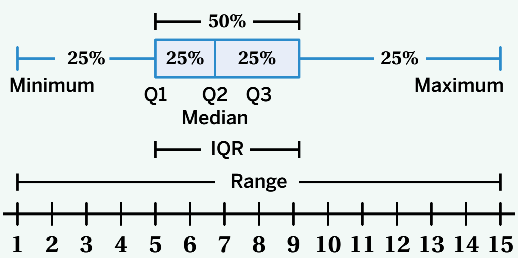
Lesson Practice 8.12

Lesson Summary

You can create a **box plot** to visualize and analyze a data set. While a box plot shows the same data as a dot plot, it gives us new information about the data. Rather than showing every data point, a box plot separates the data into quartiles.

We can use box plots to describe the spread of the data in two ways.

Min.	Q1	Median	Q3	Max.
1	5	6.5	9.25	15



- The range represents the difference between the *maximum* and *minimum* values of a data set. It describes the overall spread of the data.

  Range: $15 - 1 = 14$

- The **interquartile range (IQR)** represents the range of the middle 50% of the data (between Q3 and Q1). It describes how spread out the middle of the data is.

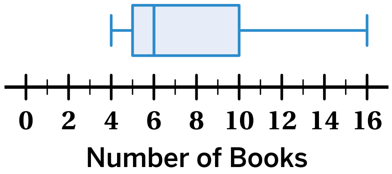
  IQR: $9.25 - 5 = 4.25$

Box plots do not show how many data points are in each set, or the values of any individual data points, except the minimum and maximum.

Lesson Practice
8.12

Name: .... Date: .... Period: ....

Problems 1–3: Each student in a class recorded how many books they read in a school year.



1. What is the greatest number of books that a student read this year?

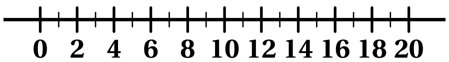
2. What is the median of this data?

3. How would removing a data value affect the range of this data?

Problems 4–5: Here are five statistics about a data set.

Minimum	Q1	Median	Q3	Maximum
4	6	9	13	19

4. Create a box plot that represents this data set.



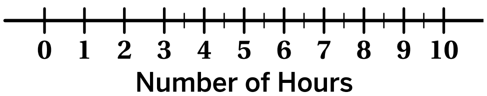
5. What is the IQR of this data set?

Problems 6–9: A group of seventh-grade students recorded the number of hours they spent doing homework in one week. Here is their data: 3, 4, 5, 5, 6, 7, 7, 9, 9, 10.

6. Determine each of the statistics.

Minimum	Q1	Median	Q3	Maximum

7. Create a box plot of this data.



8. Most seventh-graders do less than 5 hours of homework in a week. Circle one. True False

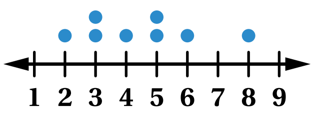
9. 25% of seventh-grade students spent between 9 and 10 hours doing homework in a week. Circle one. True False

Name: .... Date: .... Period: ....

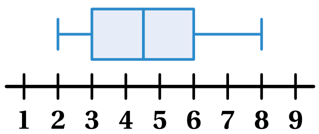
FAST Practice

10. Here are three representations of the same data set. One representation has a mistake.

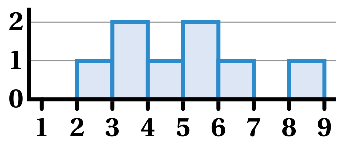
Dot Plot



Box Plot



Histogram



Select **ONE** correct answer in each box to complete the sentence to describe the mistake.

In the [(A.) dot plot (B.) box plot (C.) histogram], the mistake is that [(A.) Q3 should be 5.5 (B.) the maximum is 8 (C.) 6 should have two data values].

Spiral Review

11. Which of these has the largest value?

(A.) 25% of 80

(B.) 100% of 60

(C.) 75% of 100

(D.) 50% of 120

12. Nasir surveyed his classmates to determine how many miles they travel to school each day. This histogram shows the results. Select *all* the true statements.

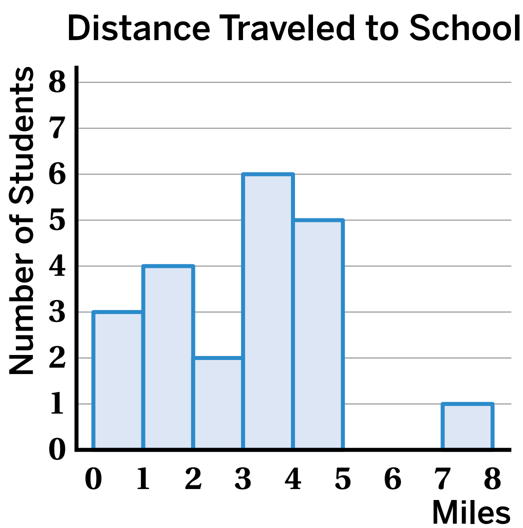
☐ **A.** Most of the data is between 4 and 7 miles.

☐ **B.** The median of the data values is between 3 and 4 miles.

☐ **C.** The interquartile range of the data values is greater than 5.

☐ **D.** No students travel between 2 and 3 miles.

☐ **E.** Most of the data is between 3 and 5 miles.



Career Connection

Models like the one shown in this picture represent large quantities of data. Once data is collected, it can be analyzed and used to make predictions. Data collection and analysis have a wide variety of applications in business, from how well a product is selling to how well machines are performing.


Vertigo 3d/Getty Images

Mathematicians and statisticians collect and analyze data. They use variables, expressions, and equations to study patterns in data over time. These studies help people make decisions about their finances as well as make businesses successful.

B.E.S.T. Mathematics Benchmark Connection

Scientists apply math extensively in their work. For example, they will often use measures like mean and median (MA.6.DP.1.2) to summarize large sets of data and explain the meaning for the real-world problem they are trying to solve. Data patterns are often described using simple and more complex equations (MA.6.AR.2.3) because equations can be useful to people who do not have the time or materials available to collect the data themselves.

Mathematical Thinking and Reasoning Connection

People involved in data collection and analysis use patterns and structure to help understand and connect mathematical concepts (MTR.5.1). They also look for their patterns to apply to different real-world situations (MTR.7.1).

Meet Alexander Aitken

In 1935, New Zealand mathematician Alexander Craig Aitken introduced the concept of analyzing data using special equations. Thanks to his achievements in algebra and statistics, we can collect and analyze data anywhere in the world, and use equations to make predictions for business, finance, weather, and many other applications.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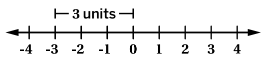
Glossary/Glosario

English

A

absolute value The distance from 0 to a number on a number line is its absolute value.

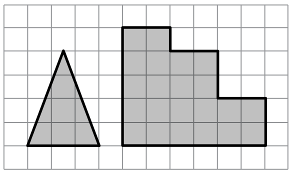


For example, the absolute value of -3 is 3 because -3 is 3 units away from 0. This is written as $|-3| = 3$.

additive inverses The additive inverse of a is $-a$. The sum of two additive inverses is 0.

For example, 5 and -5 are additive inverses because $5 + (-5) = 0$.

area Area measures the space inside a two-dimensional figure. It is expressed in square units.



The area of the triangle is 6 square units. The area of the other shape is 22 square units.

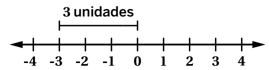
associative property The property says $a + (b + c) = (a + b) + c$ and $a \cdot (b \cdot c) = (a \cdot b) \cdot c$. This means that expressions with addition or multiplication have the same sum or product no matter how the numbers in the expression are grouped.

For example, $(2 + 1) + 3 = 2 + (1 + 3)$ and $(3 \cdot 4) \cdot 5 = 3 \cdot (4 \cdot 5)$.

Español

A

valor absoluto La distancia del 0 a un número en una recta numérica es su valor absoluto.

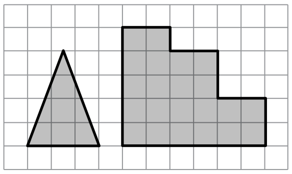


Por ejemplo, el valor absoluto de -3 es 3 porque -3 está a 3 unidades del 0. Esto se escribe $|-3| = 3$.

inversos aditivos El inverso aditivo de a es $-a$. La suma de dos inversos aditivos es 0.

Por ejemplo, 5 y -5 son inversos aditivos porque $5 + (-5) = 0$.

área El área mide el espacio dentro de una figura bidimensional. Se expresa en unidades cuadradas.



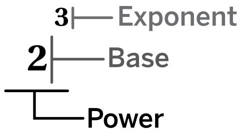
El área del triángulo mide 6 unidades cuadradas. El área de la otra figura mide 22 unidades cuadradas.

propiedad asociativa La propiedad indica que $a + (b + c) = (a + b) + c$ y $a \cdot (b \cdot c) = (a \cdot b) \cdot c$. Esto significa que las expresiones en las que se suma o se multiplica tienen la misma suma o el mismo producto independientemente de cómo se agrupen los números en la expresión.

Por ejemplo, $(2 + 1) + 3 = 2 + (1 + 3)$ y $(3 \cdot 4) \cdot 5 = 3 \cdot (4 \cdot 5)$.

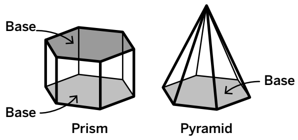
B

base (of a power) The number that is raised to an exponent. When determining the value of a power, the exponent tells you how many times the base should be multiplied.

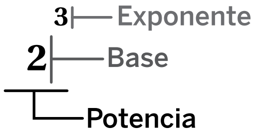


In the expression 2^3, 2 is the base.

base (of a pyramid or prism) The face that gives the solid its name. A prism has two identical bases that are parallel. A pyramid has one base.

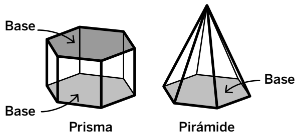


base (de una potencia) El número elevado a un exponente. Al determinar el valor de una potencia, el exponente indica cuántas veces debe multiplicarse la base.



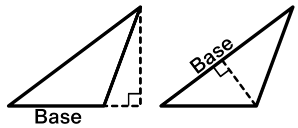
En la expresión 2^3, 2 es la base.

base (de una pirámide o un prisma) La cara que da el nombre al cuerpo geométrico. Un prisma tiene dos bases idénticas que son paralelas. Una pirámide tiene una base.



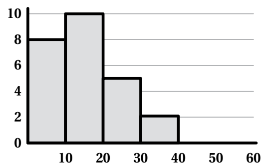
English

base (of a triangle) The base is one side of a triangle. We can choose any side to be the base.



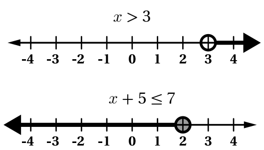
The base can also refer to the length of this side. The height of a shape is perpendicular to the base.

bin (of a histogram) The intervals used to group data values in a histogram.



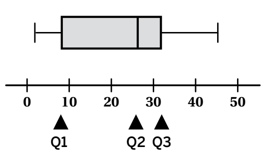
For example, this histogram shows bins of 10 units each.

boundary point The point on a number line or graph that separates solutions of an inequality from non-solutions. If the boundary point is a solution to the inequality (i.e., ≥ or ≤), it's represented with a closed circle on the graph. If it's not a solution (i.e., > or <), it's represented with an open circle on the graph.



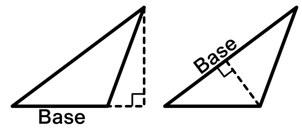
The boundary point of $x > 3$ is 3. The boundary point of $x \le 2$ is 2.

box plot A way to visualize numerical data sets. The data is divided into four sections using five values: the minimum, Q1, Q2 (or the median), Q3, and the maximum. A box is drawn between Q1 and Q3. The line inside the box represents the median.



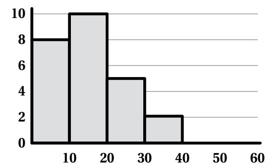
Español

base (de un triángulo) La base es un lado de un triángulo. Podemos elegir cualquier lado como base.



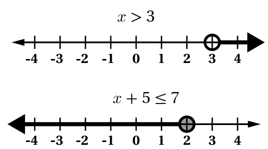
La base también puede referirse a la longitud de este lado. La altura de una figura es perpendicular a la base.

intervalo (de un histograma) Los intervalos que se usan para agrupar los valores de datos en un histograma.



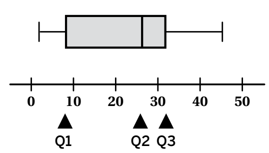
Por ejemplo, este histograma muestra intervalos de 10 unidades cada uno.

punto límite Punto en una recta numérica o una gráfica que separa las soluciones de una desigualdad de los valores que no son soluciones. Si el punto límite es una solución de la desigualdad (es decir, ≥ o ≤), se representa con un círculo cerrado en la gráfica. Si no es una solución (es decir, > o <), se representa con un círculo abierto en la gráfica.



El punto límite de $x > 3$ es 3. El punto límite de $x \le 2$ es 2.

diagrama de caja Una forma de visualizar conjuntos de datos numéricos. Los datos se dividen en cuatro secciones utilizando cinco valores: el mínimo, Q1, Q2 (o la mediana), Q3 y el máximo. Se dibuja una caja entre Q1 y Q3. La línea dentro de la caja representa la mediana.



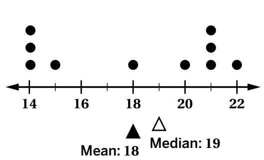
C

English

categorical data Data that can be sorted into categories instead of counted. Categorical data usually have values that are represented by words instead of numbers.

"What kind of pet do you have?" is a question that would result in categorical data.

center A single value that can be used to summarize the typical value in a data set.



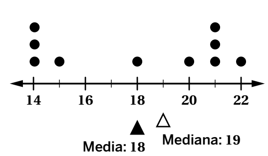
Mean and median are measures of center.

Español

datos categóricos Datos que pueden clasificarse en categorías en lugar de contarse. Los datos categóricos suelen tener valores que se representan mediante palabras en lugar de números.

"¿Qué tipo de mascota tienes?" es una pregunta que produciría datos categóricos.

centro Un solo valor que puede utilizarse para resumir el valor típico de un conjunto de datos.

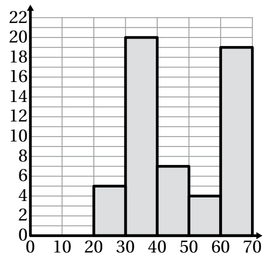


La media y la mediana son medidas de tendencia central, o sea, medidas de centro.

English

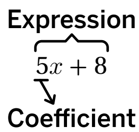
cluster A group of data values that are close together.

For example, this histogram shows a cluster between 30 and 40 and between 60 and 70.



coefficient A number that is multiplied by a variable. Usually, there is no symbol between the coefficient and the variable.

In the expression $5x + 8$, 5 is the coefficient of x.



common denominator The denominator is the bottom number in a fraction. Two fractions have a common denominator when their denominators are the same.

For example, $\frac{3}{4}$ and $\frac{5}{4}$ have a common denominator because they each split the whole into fourths.

common factor When two numbers have the same factor, we call that a common factor.

For example, the factors of 8 are 1, 2, 4, and 8 and the factors of 12 are 1, 2, 3, 4, 6, and 12.

Since 2 is a factor of 8 and also of 12, 2 is a common factor of 8 and 12.

common multiple When two numbers have the same multiple, we call that a common multiple.

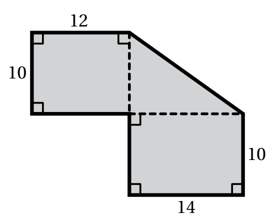
For example, 12 is a multiple of 2 and also of 3, so 12 is a common multiple of 2 and 3.

commutative property The property says $a + b = b + a$ and $a \cdot b = b \cdot a$. This means that expressions with addition or multiplication have the same sum or product no matter what order the numbers are in.

For example, $2 + 1 = 1 + 2$ or $3 \cdot 4 = 4 \cdot 3$.

composite figure A two- or three-dimensional figure that can be decomposed into smaller figures.

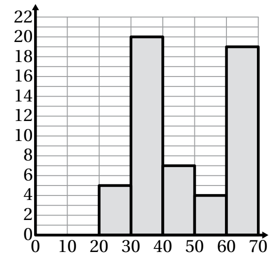
The composite figure can be decomposed into 2 rectangles and a triangle.



Español

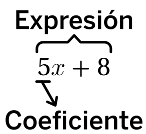
agrupación Un grupo de valores de datos que están próximos entre sí.

Por ejemplo, este histograma muestra una agrupación entre 30 y 40, y entre 60 y 70.



coeficiente Un número que se multiplica por una variable. Por lo general, no hay ningún símbolo entre el coeficiente y la variable.

En la expresión $5x + 8$, el coeficiente de x es 5.



denominador común El denominador es el número de abajo en una fracción. Dos fracciones tienen un denominador común cuando los denominadores son iguales.

Por ejemplo, $\frac{3}{4}$ y $\frac{5}{4}$ tienen un denominador común porque cada uno divide el todo en cuartos.

factor común Cuando dos números tienen el mismo factor, lo llamamos factor común.

Por ejemplo, los factores de 8 son 1, 2, 4 y 8 y los factores de 12 son 1, 2, 3, 4, 6 y 12.

Ya que 2 es un factor de 8 y también de 12, 2 es un factor común de 8 y 12.

múltiplo común Cuando dos números tienen el mismo múltiplo, lo llamamos múltiplo común.

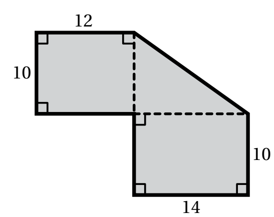
Por ejemplo, 12 es un múltiplo de 2 y también de 3, por lo tanto, 12 es un múltiplo común de 2 y 3.

propiedad conmutativa La propiedad indica que $a + b = b + a$ y $a \cdot b = b \cdot a$. Esto significa que las expresiones en las que se suma o se multiplica tienen la misma suma o el mismo producto, independientemente del orden en el que estén los números.

Por ejemplo, $2 + 1 = 1 + 2$ o $3 \cdot 4 = 4 \cdot 3$.

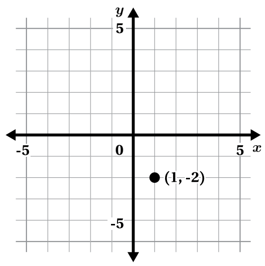
figura compuesta Figura bidimensional o tridimensional que puede descomponerse en figuras más pequeñas.

La figura compuesta se puede descomponer en 2 rectángulos y un triángulo.

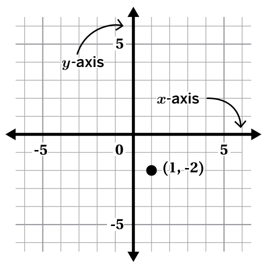


English

coordinate plane
The coordinate plane consists of two axes that intersect at 0: one horizontal (often called the x-axis) and one vertical (often called the y-axis).



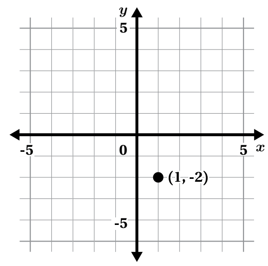
coordinates A pair of numbers that shows an exact position on the coordinate plane. The first number represents a position on the x-axis and is called the x-coordinate. The second number represents a position on the y-axis and is called the y-coordinate.



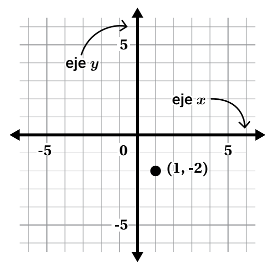
The coordinates of the point on the graph are $(1, \text{-}2)$.

Español

plano de coordenadas
El plano de coordenadas consta de dos ejes que se intersecan en 0: uno horizontal (a menudo llamado el eje x) y uno vertical (a menudo llamado el eje y).



coordenadas Un par de números que muestran una posición exacta en el plano de coordenadas. El primer número representa una posición en el eje x y se denomina coordenada x. El segundo número representa una posición en el eje y y se denomina coordenada y.



Las coordenadas del punto en la gráfica son $(1, \text{-}2)$.

D

decompose Decompose means "take apart." We use the word *decompose* to describe taking a figure apart to make more than one new shape.

distributive property The property that says $a(b+c) = ab + ac$. This means that multiplying a number by the sum of two or more terms is equal to multiplying the number by each term separately before adding the products together.

Area as a Product
$3(2x+5)$

For example, $3(2x+5) = 3 \cdot 2x + 3 \cdot 5 = 6x + 15$.

dividend The number in a division statement that is being divided.

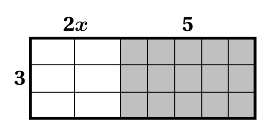
For example, in the equation $12 \div 3 = 4$, the dividend is 12.

divisor In a division statement, the divisor describes the number of equal-sized groups or the size of each group being created.

For example, in the equation $12 \div 3 = 4$, the divisor is 3.

descomponer Descomponer significa "desmontar." Usamos la palabra *descomponer* para describir que una figura se desmonta para formar más de una figura nueva.

propiedad distributiva
La propiedad que indica que $a(b+c) = ab + ac$. Significa que multiplicar un número por la suma de dos o más términos equivale a multiplicar el número por cada término individualmente y luego sumar los productos.



El área como un producto
$3(2x+5)$

Por ejemplo, $3(2x+5) = 3 \cdot 2x + 3 \cdot 5 = 6x + 15$.

dividendo El número que se está dividiendo en un enunciado de división.

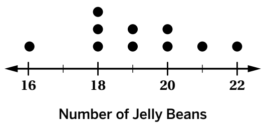
Por ejemplo, en la ecuación $12 \div 3 = 4$, el dividendo es 12.

divisor En un enunciado de división, el divisor describe la cantidad de grupos de igual tamaño o el tamaño de cada grupo que se produce.

Por ejemplo, esta ecuación $12 \div 3 = 4$, el divisor es 3.

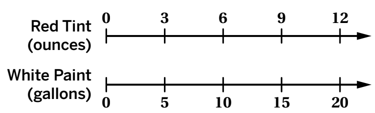
English

dot plot A way to visualize numerical data sets, where each data point is represented by a dot on a number line. Data points with the same value are stacked on top of each other. A dot plot is sometimes called a line plot.



For example, this dot plot shows that 3 students guessed that there were 18 jelly beans in a jar.

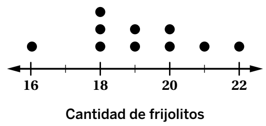
double number line A pair of parallel number lines showing equivalent ratios. The tick marks are labeled so that the marks that line up vertically are equivalent ratios.



This double number line shows a ratio of 3 ounces of red tint : 5 gallons of white paint.

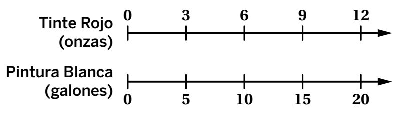
Español

diagrama de puntos Una forma de visualizar conjuntos de datos numéricos, en la que cada punto de datos se representa mediante un punto en una recta numérica. Los puntos de datos con el mismo valor se apilan unos sobre otros. Un diagrama de puntos a veces se conoce como gráfica de puntos.



Por ejemplo, este diagrama de puntos muestra que 3 estudiantes estimaron que había 18 frijolitos de jalea en un tarro.

recta numérica doble Un par de rectas numéricas paralelas que muestran razones equivalentes. Las marcas indicadoras se denominan de forma tal que las marcas alineadas verticalmente sean razones equivalentes.



Esta recta numérica doble muestra una razón de 3 onzas de tinte rojo : 5 galones de pintura blanca.

E

edge Each straight side of a polygon is called an edge. An edge is also a line segment where two faces of a 3-D figure meet.

This rectangle has four edges.

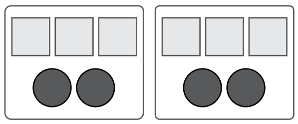
equation A mathematical statement made up of two expressions with an equal sign between them.

For example, $6m + 5 = 17$ and $12 - 15 = -3$ are equations, but $2n$ and $x > 5$ are not equations.

equivalent expressions Expressions that are equal for every value of a variable.

$x + x + x$ is equivalent to $3x$ because they both describe three copies of an unknown number, x.

equivalent ratio Two ratios are equivalent if you can multiply each of the values in the first ratio by the same number to get the values in the second ratio.



$3 : 2$ is equivalent to $6 : 4$ because $3 \cdot 2 = 6$ and $2 \cdot 2 = 4$.

One lemonade recipe uses 3 cups of water and 2 lemons. Another uses 6 cups of water and 4 lemons. The second recipe will make twice as much lemonade but both recipes will taste the same.

lado, arista Cada borde recto de un polígono se llama arista o lado. Una arista también es un segmento de recta donde se unen dos caras de una figura tridimensional.

Este rectángulo tiene cuatro aristas.

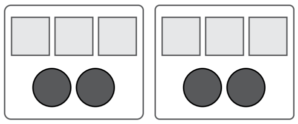
ecuación Un enunciado matemático formado por dos expresiones con un signo igual entre ellas.

Por ejemplo, $6m + 5 = 17$ y $12 - 15 = -3$ son ecuaciones, pero $2n$ y $x > 5$ no son ecuaciones.

expresiones equivalentes Expresiones que son iguales para cualquier valor de una variable.

$x + x + x$ equivale a $3x$ porque ambas describen tres copias de un número desconocido, x.

razón equivalente Dos razones son equivalentes si cada uno de los valores de la primera razón se puede multiplicar por el mismo número para obtener los valores de la segunda razón.



$3 : 2$ es equivalente a $6 : 4$ porque $3 \cdot 2 = 6$ y $2 \cdot 2 = 4$.

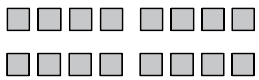
Una receta de limonada lleva 3 tazas de agua y 2 limones. Otra lleva 6 tazas de agua y 4 limones. La segunda receta rendirá el doble de limonada, pero ambas tendrán el mismo sabor.

English

evaluate To evaluate is to determine a single number that represents an expression's value.

For example, to evaluate $5x + 2$ when $x = 3$, we substitute 3 for x and then calculate $5(3) + 2 = 17$.

exponent A number used to describe repeated multiplication. Exponents are sometimes called powers.



$2^4 = 16$

For example, $2 \cdot 2 \cdot 2 \cdot 2 = 16$ can be represented by the equation $2^4 = 16$, where 4 is the exponent. We can read this equation as "2 to the power of 4 equals 16" or "2 to the fourth equals 16."

expression A set of numbers, variables, operations, and grouping symbols that represent a quantity.

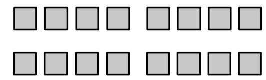
For example, $2n - 8$ and $21 + 37$ are expressions.

Español

evaluar Evaluar significa determinar el número individual que representa el valor de una expresión.

Por ejemplo, para evaluar $5x + 2$ cuando $x = 3$, sustituimos x por 3 y luego calculamos $5(3) + 2 = 17$.

exponente Un número que se usa para describir multiplicaciones repetidas. A los exponentes a veces se les conoce como potencias.



$2^4 = 16$

Por ejemplo, $2 \cdot 2 \cdot 2 \cdot 2 = 16$ puede representarse con la ecuación $2^4 = 16$, donde 4 es el exponente. Podemos leer esta ecuación como "2 a la potencia de 4 es igual a 16" o "2 a la cuarta potencia es igual a 16".

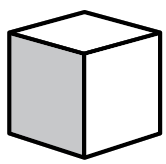
expresión Un conjunto de números, variables, operaciones y símbolos de agrupación que representan una cantidad.

Por ejemplo, $2n - 8$ y $21 + 37$ son expresiones.

F

face Each flat side of a polyhedron is called a face.

A cube has six faces and they are all squares.



factor (of a number) A whole number that divides evenly into the given number (with no remainder).

For example, 1, 2, 4, and 8 are all factors of the number 8.

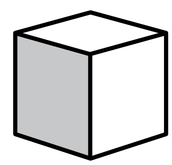
frequency table A table that shows the number of times each value or category occurs in a data set.

For example, this frequency table shows that 5 students selected red as their favorite color.

Favorite Color	Frequency
Red	5
Blue	3
Pink	4

cara Cada lado plano de un poliedro se llama cara.

Un cubo tiene seis caras y todas son cuadradas.



factor (de un número) Un número natural por el que se puede dividir en partes iguales el número dado (sin resto).

Por ejemplo, 1, 2, 4 y 8 son factores del número 8.

tabla de frecuencia Una tabla que muestra el número de veces que aparece cada valor o categoría en un conjunto de datos.

Por ejemplo, esta tabla de frecuencia muestra que 5 estudiantes seleccionaron el rojo como su color favorito.

Color favorito	Frecuencia
Rojo	5
Azul	3
Rosado	4

G

≥ (greater-than-or-equal-to) This symbol means "greater than or equal to."

For example, the inequality $x \geq 3$ says the value of x is either exactly 3 or any value greater than 3.

≥ (mayor que o igual a) Este símbolo significa "mayor que o igual a".

Por ejemplo, la desigualdad $x \geq 3$ indica que el valor de x es exactamente 3 o cualquier valor mayor que 3.

Glossary/Glosario

English	Español

greatest common factor (GCF) The largest number that is a common factor of two numbers.

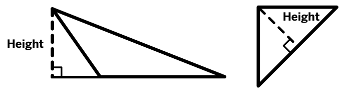
The common factors of 8 and 12 are 1, 2, and 4. The greatest common factor is 4.

máximo común divisor (MCD) El número mayor que es factor común de dos números.

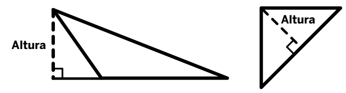
Los factores comunes de 8 y 12 son 1, 2 y 4. El máximo común divisor es 4.

H

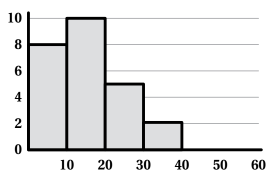
height (of a triangle) The shortest distance between a base and its opposite vertex. Sometimes the height falls outside the shape. The height is always perpendicular to the base.



altura (de un triángulo) La distancia más corta entre una base y su vértice opuesto. A veces, la altura cae fuera de la figura. La altura siempre es perpendicular a la base.

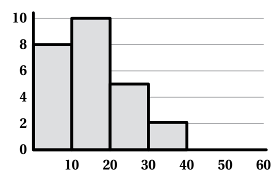


histogram A way to visualize numerical data where the data is grouped into bins represented by rectangles. The height of each rectangle shows how many values are in that bin.



For example, this histogram shows that there are 8 data values from 0 up to (but not including) 10.

histograma Una forma de visualizar datos numéricos en la que los datos se agrupan en intervalos que se representan con rectángulos. La altura de cada rectángulo muestra cuántos valores hay en ese intervalo.



Por ejemplo, este histograma muestra que hay 8 valores de datos del 0 al 10 (sin incluirlo).

I

inequality A comparison statement that uses the symbols $<$ or $>$. Inequalities are used to represent the relationship between numbers, variables, or expressions that are not always equal.

For example, the inequality $y > 30$ means that the value of the expression y is any number greater than 30.

desigualdad Un enunciado de comparación que utiliza los símbolos $<$ o $>$. Las desigualdades se usan para representar la relación entre números, variables o expresiones que no siempre son iguales.

Por ejemplo, la desigualdad $y > 30$ significa que el valor de la expresión y es cualquier número mayor que 30.

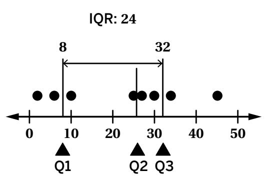
integers All whole numbers and their opposites.

For example, 35, -15, and 1 are integers. 0.3 and $\frac{1}{3}$ are not.

enteros Todos los números enteros y sus opuestos.

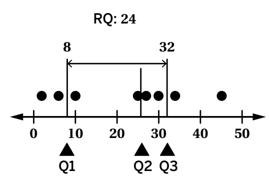
Por ejemplo, 35, -15 y 1 son números enteros. 0.3 y $\frac{1}{3}$ no lo son.

interquartile range (IQR) A measure of spread. The IQR is calculated as the distance from Q1 to Q3, or the width of the box in a box plot.



For example, the IQR of this data set is $32 - 8 = 24$.

rango intercuartílico (RQ) Una medida de dispersión. El RQ se calcula como la distancia de Q1 a Q3, o el ancho de la caja en un diagrama de caja.



Por ejemplo, el RQ de este conjunto de datos es $32 - 8 = 24$.

L

least common multiple (LCM) The smallest number that is a common multiple of two numbers.

The common multiples of 2 and 3 are 6, 12, 18, . . .
The least common multiple is 6.

mínimo común múltiplo (MCM) El número menor que es múltiplo común de dos números.

Los múltiplos comunes de 2 y 3 son 6, 12, 18. . .
El mínimo común múltiplo es 6.

English

≤ (less-than-or-equal-to) This symbol means “less than or equal to.”

For example, the inequality $x \leq 9$ says the value of x is either exactly 9 or any value less than 9.

like terms Terms with variables and exponents that are the same.

For example, $8x$ and $12x$ are like terms because both terms have a variable of x. $3x$ and $3x^2$ are not like terms because they have different exponents.

long division A way to divide numbers. When we use long division, we determine the quotient one digit at a time, from left to right.

For example, here is the long division for $106 \div 8$.

```
    13.25
8 ) 106.00
   - 8
     26
   - 24
      20
    - 16
       40
       40
        0
```

Español

≤ (menor que o iqual a) Este símbolo significa “menor que o iqual a”.

Por ejemplo, la desigualdad $x \leq 9$ indica que el valor de x es exactamente 9 o cualquier valor menor que 9.

términos semejantes Términos con variables y exponentes iguales.

Por ejemplo, $8x$ y $12x$ son términos semejantes porque ambos tienen una variable que incluye x. $3x$ y $3x^2$ no son términos semejantes porque tienen exponentes diferentes.

división larga Una forma de dividir números. Al usar la división larga, determinamos el cociente de izquierda a derecha, un dígito a la vez.

Por ejemplo, esta es la división larga de $106 \div 8$.

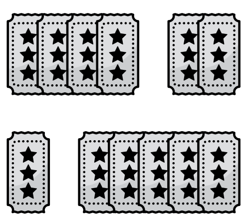
```
    13.25
8 ) 106.00
   - 8
     26
   - 24
      20
    - 16
       40
       40
        0
```

M

English

maximum The greatest value in a data set.

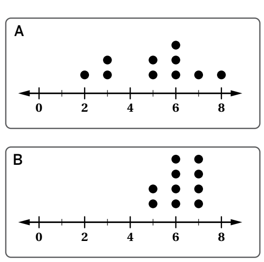
mean (average) A measure of center. If you equally distribute a set of items into different groups, the mean is the number of items in each group. It is also the balance point of a dot plot. To calculate the mean, you can add the values of all the data points, then divide by the number of data points.



The mean in this example is 3 tickets because that is the number each person would get if the tickets were distributed equally among four people.

measure of center A single number that summarizes all of the values in a data set. It is usually a typical value for a data set. Mean and median are measures of center.

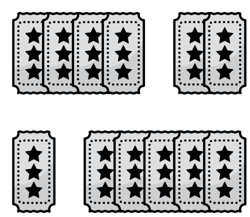
measure of spread A single number that describes the spread of a data set. Range and interquartile range are measures of spread.



Español

máximo El mayor valor en un conjunto de datos.

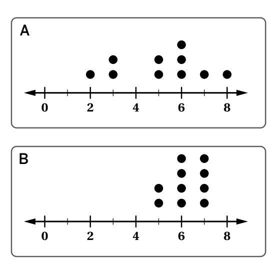
media (promedio) Una medida de tendencia central. Si se distribuye equitativamente un conjunto de elementos en diferentes grupos, la media es la cantidad de elementos en cada grupo. También es el punto de equilibrio de un diagrama de puntos. Para calcular la media, se pueden sumar los valores de todos los puntos de datos y luego dividir por la cantidad de puntos de datos.



La media en este ejemplo es 3 boletos porque es la cantidad que obtendría cada persona si los boletos se distribuyeran equitativamente entre cuatro personas.

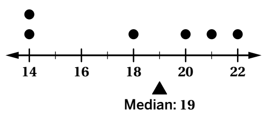
medida de tendencia central, medida de centro Un solo número que resume todos los valores de un conjunto de datos. Suele ser un valor típico de un conjunto de datos. La media y la mediana son medidas de tendencia central.

medida de dispersión Un solo número que describe la dispersión de un conjunto de datos. El rango y el rango intercuartílico son medidas de dispersión.



English

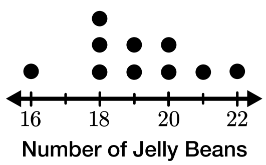
median A measure of center. It is the middle value of a data set when the values are in numerical order. When there is an even number of data points, the median is the average of the two middle values.



The median of this data set is 19 because it is the average of the two middle values: 18 and 20.

minimum The least value in a data set.

mode A measure of center. It is the value that occurs most often in a data set. There may be no mode, one mode, or more than one mode.



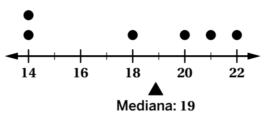
For example, the mode of this data set is 18 jelly beans because the value 18 occurs more times than any other data value.

multiple The result of multiplying a number by a whole number.

For example, 10, 15, and 20 are all multiples of the number 5.

Español

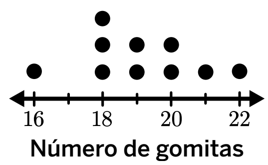
mediana Una medida de tendencia central. Es el valor del medio de un conjunto de datos cuando los valores se clasifican en orden numérico. Cuando hay un número par de puntos de datos, la mediana es el promedio de los dos números centrales.



La mediana de este conjunto de datos es 19 porque es el promedio de los dos valores centrales: 18 y 20.

mínimo El menor valor en un conjunto de datos.

moda Una medida de centro. Es el valor que aparece con mayor frecuencia en un conjunto de datos. Puede no haber ninguna moda, haber una moda o más de una moda.



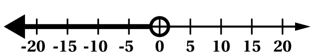
Por ejemplo, la moda de este conjunto de datos es 18 gomitas, porque el valor 18 aparece más veces que cualquier otro valor de datos.

múltiplo El resultado de multiplicar un número por un número natural.

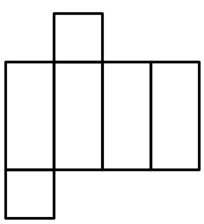
Por ejemplo, 10, 15 y 20 son múltiplos del número 5.

N

negative number A number that is less than 0. On a horizontal number line, negative numbers are to the left of 0.



net A two-dimensional representation of a three-dimensional shape. It can be folded to make a polyhedron.

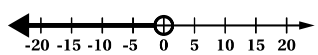


Here is a net for a rectangular prism.

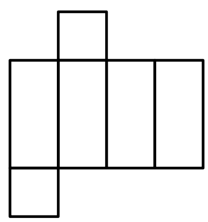
numerical data Data that includes values that are numbers and can be measured and meaningfully compared.

"How many pets do you have?" is a question that would result in numerical data.

número negativo Un número que es menor que 0. En una recta numérica horizontal, los números negativos están a la izquierda del 0.



red Una representación bidimensional de una figura tridimensional. Puede plegarse para formar un poliedro.



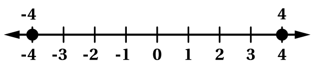
Esta es una red de un prisma rectangular.

datos numéricos Datos que incluyen valores numéricos que pueden medirse y compararse de forma significativa.

"¿Cuántas mascotas tienes?" es una pregunta que produciría datos numéricos.

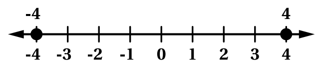
O

opposite Two numbers that are the same distance from 0 and on different sides of 0 on the number line are opposites.



For example, 4 and -4 are opposites.

opuesto Dos números son opuestos si están a la misma distancia del 0 y en diferentes lados del 0 en la recta numérica.



Por ejemplo, 4 y -4 son opuestos.

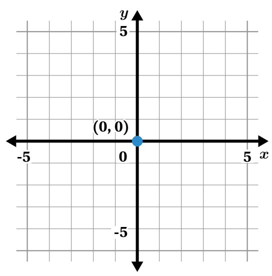
English

order of operations A consistent order applied to an expression with multiple operations so that the expression is evaluated the same way by everyone. The standard order of operations is parentheses/grouping symbols, exponents/roots, multiplication/division, and then addition/subtraction.

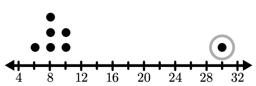
ordered pair Two values of x and y, written as (x, y), that represent a point on the coordinate plane.

For example, $(3, 5)$ represents the point where $x = 3$ and $y = 5$.

origin The point $(0, 0)$ on the coordinate plane. This is where the x-axis and the y-axis intersect.



outlier A data value that is far from the other values in a data set.



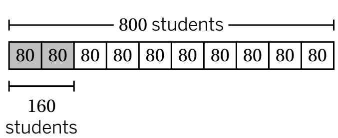
The circled point is an outlier.

P

per The word *per* means "for each."

For example, if the price is \$5 per ticket, that means that each ticket costs \$5. Buying 4 tickets would cost $5 \cdot 4 = \$20$.

percent (percentage) Percent means "for every 100." It is represented by the percent symbol, %. We use percentages to represent ratios and fractions.



For example, 20% means 20 for every 100. 20% of a number means $\frac{20}{100}$ or $\frac{1}{5}$ of that number. Let's say there are 800 students in a school. If 20% of them are on a field trip, that means 160 students because 20 students are on the trip for every 100 students total.

perpendicular Describes a line that crosses or meets another line at a 90° angle.

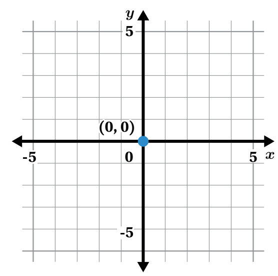
Español

orden de las operaciones Un orden coherente aplicado a una expresión con múltiples operaciones para que cualquiera pueda evaluar la expresión de la misma manera. El orden estándar de las operaciones es paréntesis/símbolos de agrupación, exponentes/raíces, multiplicación/división y, luego, suma/resta.

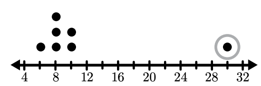
par ordenado Dos valores de x y y, escritos como (x, y), que representan un punto en el plano de coordenadas.

Por ejemplo, $(3, 5)$ representa el punto donde $x = 3$ y $y = 5$.

origen El punto $(0, 0)$ en el plano de coordenadas. El punto en el que se intersecan los ejes x y y.



valor atípico Un valor de datos que está lejos de los demás valores del conjunto de datos.

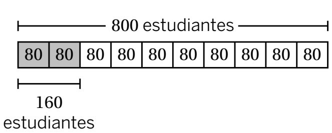


El punto encerrado en un círculo es un valor atípico.

por, por cada La palabra *por* puede significar "por cada".

Por ejemplo, si el precio es \$5 por boleto, esto significa que cada boleto cuesta \$5. Comprar 4 boletos costaría $5 \cdot 4 = \$20$.

por ciento (porcentaje) Por ciento significa "por cada 100". Se representa con el símbolo de porcentaje, %. Usamos porcentajes para representar razones y fracciones.

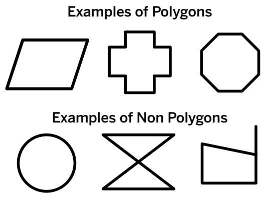


Por ejemplo, 20% significa 20 por cada 100. 20% de un número significa $\frac{20}{100}$ o $\frac{1}{5}$ de dicho número. Supongamos que hay 800 estudiantes en una escuela. Si el 20% de ellos está en una excursión, eso significa 160 estudiantes porque 20 están de viaje por cada 100 estudiantes.

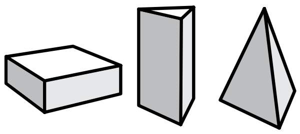
perpendicular Describe una línea que cruza o se une con otra línea formando un ángulo de 90°.

English

polygon A closed two-dimensional shape with straight sides that do not cross each other.

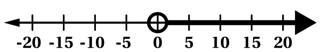


polyhedron A polyhedron is a closed three-dimensional shape with flat sides. The plural of polyhedron is polyhedra. Prisms and pyramids are types of polyhedra.



Here are some drawings of polyhedra.

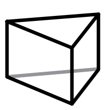
positive number
A number that is greater than 0. On a horizontal number line, positive numbers are to the right of 0.



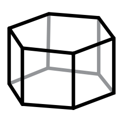
prime factorization The expression of a number as the product of prime factors.

The prime factorization of 28 is $2 \cdot 2 \cdot 7$.

prism A three-dimensional shape, or polyhedron, with two bases that are identical copies.

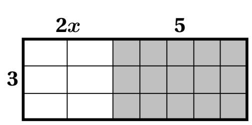


Triangular Prism



Hexagonal Prism

product The value of two or more quantities when multiplied.

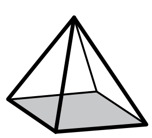


Area as a Product
$3(2x+5)$

For example, the area of this rectangle is the product of 3 and $2x + 5$ or $3(2x + 5)$.

profit The money left after paying for business expenses.

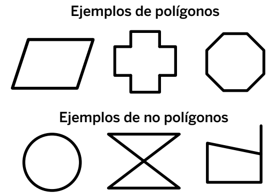
pyramid A three-dimensional shape, or polyhedron, that has one base. All of the other faces are triangles that meet at a single vertex.



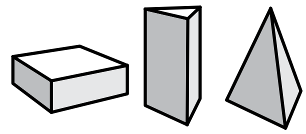
Pyramid

Español

polígono Una figura bidimensional cerrada con lados rectos que no se cruzan.

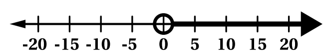


poliedro Un poliedro es una figura tridimensional cerrada con caras planas. En inglés, usamos las palabras polyhedron (singular) y polyhedra (plural). Los prismas y las pirámides son tipos de poliedros.



Estos son algunos dibujos de poliedros.

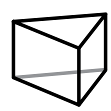
número positivo
Un número que es mayor que 0. En una recta numérica horizontal, los números positivos están a la derecha del 0.



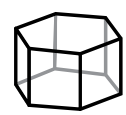
factorización prima La expresión de un número como producto de factores primos.

La factorización prima de 28 es $2 \cdot 2 \cdot 7$.

prisma Una figura tridimensional, o poliedro, con dos bases que son copias idénticas.

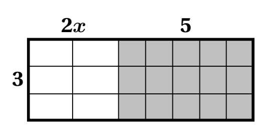


Prisma triangular



Prisma hexagonal

producto El valor de dos o más cantidades cuando se multiplican.

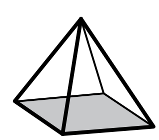


El área como un producto
$3(2x+5)$

Por ejemplo, el área de este rectángulo es el producto de 3 y $2x + 5$ o $3(2x + 5)$.

ganancia El dinero que sobra después de pagar los gastos del negocio.

pirámide Una figura tridimensional, o poliedro, que tiene solo una base. Todas las demás caras son triángulos que se unen en un único vértice.



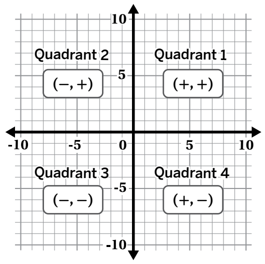
Pirámide

English

Q

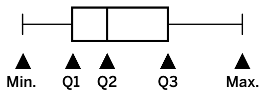
quadrant The coordinate plane is divided into 4 regions called quadrants.

The image shows each quadrant, along with the sign of the x- and y-values in that quadrant.



quantity The amount or the number of a thing.

quartile Quartiles divide an ordered data set into four equal sections.
Quartile 1 (Q1) is the median of the lower half of the data.
Quartile 2 (Q2) is the median.
Quartile 3 (Q3) is the median of the upper half of the data.



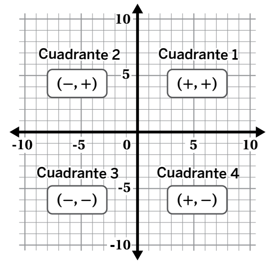
quotient The result of dividing two numbers is called the quotient.

For example, in the equation $12 \div 3 = 4$, the quotient is 4.

Español

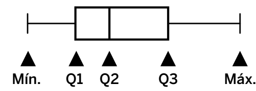
cuadrante El plano de coordenadas se divide en 4 regiones llamadas cuadrantes.

La imagen muestra cada cuadrante junto con el signo de los valores x y y del cuadrante correspondiente.



cantidad Cierta cantidad o número de algo.

cuartil Los cuartiles dividen un conjunto de datos ordenado en cuatro secciones iguales.
El cuartil 1 (Q1) es la mediana de la mitad inferior de los datos.
El cuartil 2 (Q2) es la mediana.
El cuartil 3 (Q3) es la mediana de la mitad superior de los datos.



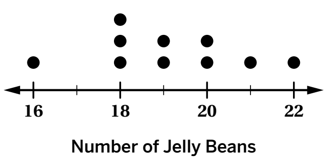
cociente Se denomina cociente al resultado de dividir dos números.

Por ejemplo, en la ecuación $12 \div 3 = 4$, el cociente es 4.

R

English

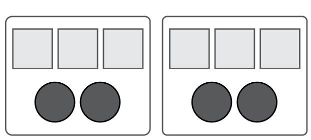
range A measure of spread. It is the difference between the maximum and minimum values in a data set.



For example, the range of this data set is 6 jelly beans because $22 - 16 = 6$.

rate A comparison, or ratio, that describes how two quantities change together.

ratio A ratio $a : b$ is a relationship between two quantities. For every a of the first, there are b of the second.

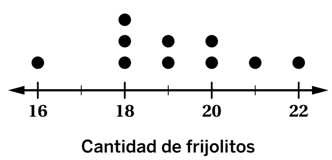


There are several ways to describe ratios.
- For every 3 squares, there are 2 circles.
- The ratio of squares to circles is 3 to 2.
- The ratio of squares to circles is 3 : 2.

If the ratio of apples to oranges in a fruit bowl is 2 : 3, then for every 2 apples, there are 3 oranges.

Español

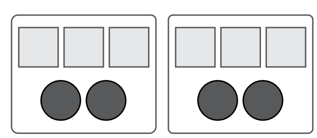
rango Una medida de dispersión. Es la diferencia entre los valores máximo y mínimo de un conjunto de datos.



Por ejemplo, el rango de este conjunto de datos es 6 frijolitos de jalea porque $22 - 16 = 6$.

tasa Una comparación, o razón, que describe cómo cambian juntas dos cantidades.

razón Una razón $a : b$ es una relación entre dos cantidades. Por cada a del primero, hay b del segundo.



Hay varias formas de describir razones.
- Por cada 3 cuadrados hay 2 círculos.
- La razón de cuadrados a círculos es de 3 a 2.
- La razón de cuadrados a círculos es 3 : 2.

Si la razón de manzanas a naranjas en un frutero es 2 : 3, entonces por cada 2 manzanas hay 3 naranjas.

English

rational number Any number that can be written as a fraction, including whole numbers and negative numbers.

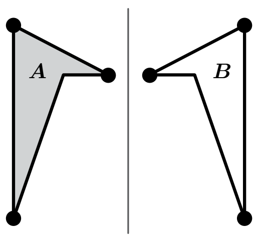
For example, -10, 2.5, $\frac{3}{7}$, and 82 are all rational numbers.

rearrange Change the position of something. We use the word *rearrange* to describe moving pieces of a figure to make a new shape.

reciprocal The reciprocal of a fraction $\frac{a}{b}$ is $\frac{b}{a}$. The product of two fractions that are reciprocals of one another is 1.

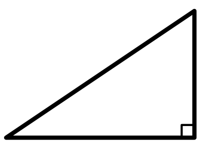
For example, $\frac{3}{2}$ and $\frac{2}{3}$ are reciprocals because $\frac{3}{2} \cdot \frac{2}{3} = 1$.

reflection A reflection moves each point on a figure across a line of reflection to a point on the opposite side of the line. The new point is the same distance from the line as it was in the original figure.



This diagram shows a reflection of A over a line that makes the mirror image B.

right triangle A triangle containing an interior right angle, or angle measuring exactly $90°$.



Español

número racional Cualquier número que pueda escribirse como una fracción, incluidos los números enteros y los números negativos.

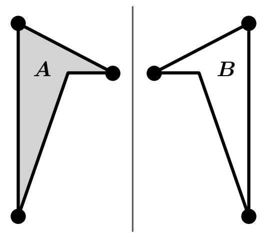
Por ejemplo, -10, 2.5, $\frac{3}{7}$ y 82 son números racionales.

reordenar Cambiar la posición de algo. Usamos la palabra *reordenar* para describir el movimiento de las partes de una figura para formar una nueva figura.

recíproco El recíproco de una fracción $\frac{a}{b}$ es $\frac{b}{a}$. El producto de dos fracciones que son recíprocas entre sí es 1.

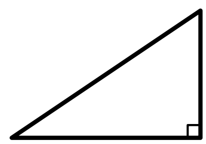
Por ejemplo, $\frac{3}{2}$ y $\frac{2}{3}$ son recíprocos porque $\frac{3}{2} \cdot \frac{2}{3} = 1$.

reflexión Una reflexión mueve cada punto de una figura sobre una línea de reflexión a un punto en el lado opuesto de la línea. El nuevo punto está a la misma distancia de la línea que estaba en la figura original.



Este diagrama muestra una reflexión de A sobre una línea que produce la imagen espejo B.

triángulo recto Un triángulo que contiene un ángulo recto interior o un ángulo que mide exactamente $90°$.



S

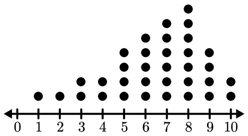
English

shape A description of the distribution (or pattern) of the data within a data set. Descriptions of the shape of a distribution can include: symmetry, skew, clusters, and outliers.

sign The sign of a number (other than 0) is either positive or negative.

For example, the sign of 4 or $+4$ is positive. The sign of -4 is negative.

skewed A data distribution where there are more values concentrated on one end of the data and few values on the other end.



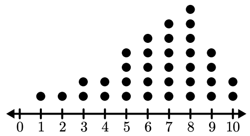
Español

figura Una descripción de la distribución (o patrón) de los datos en un conjunto de datos. Las descripciones de la forma de una distribución pueden incluir: simetría, asimetría, agrupaciones y valores atípicos.

signo El signo de un número (que no sea 0) es positivo o negativo.

Por ejemplo, el signo de 4 o $+4$ es positivo. El signo de -4 es negativo.

asimétrico Una distribución de datos en la que hay valores concentrados en un extremo de los datos y pocos valores en el otro extremo.



English

solution to an equation A value of a variable that makes the equation true. "Solving an equation" is any work you do to answer the question "Which values make the equation true?"

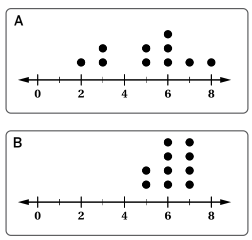
$$3x = 15$$
$$x = 15$$
$$3(5) = 15$$

For example, 5 is a solution to the equation $3x = 15$ because $3(5) = 15$ is true. 6 is not a solution to the equation $3x = 15$ because $3(6) = 15$ is not true.

solution to an inequality Any value of a variable that makes the inequality true.

For example, 5 is a solution to the inequality $x < 10$ because $5 < 10$. Some other solutions to $x < 10$ are 9.99, 0, and -4.

spread A description of how alike or different the values in a distribution are, often in relation to the center. The spread also describes the variability of a distribution.



For example, Dot Plot A has a larger spread than Dot Plot B.

statistic A single number that measures something about a data set.

Examples of statistics include: mean, median and IQR.

statistical question A question that requires more than one piece of data to answer.

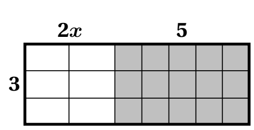
Here are some examples of statistical questions:
- What is the most popular band at your school?
- When do students in your class typically eat dinner?

substitute To replace a variable with a value or other expression.

$$4x = 4(5)$$
$$= 20$$

In this example, 5 is substituted for x in the expression $4x$.

sum The value of two or more quantities when added together.



Area as a Sum
$6x + 15$

For example, the area of this rectangle is the sum of $6x$ and 15, or $6x + 15$.

Español

solución de una ecuación Un valor de una variable que hace que la ecuación sea verdadera. "Resolver una ecuación" es cualquier trabajo que se hace para responder la pregunta: "¿Qué valores hacen que la ecuación sea verdadera?".

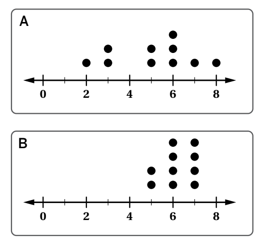
$$3x = 15$$
$$x = 15$$
$$3(5) = 15$$

Por ejemplo, 5 es una solución de la ecuación $3x = 15$ porque $3(5) = 15$ es verdadero. 6 no es una solución de la ecuación $3x = 15$ porque $3(6) = 15$ no es verdadero.

solución de una desigualdad Cualquier valor de una variable que hace que la desigualdad sea verdadera.

Por ejemplo, 5 es una solución de la desigualdad $x < 10$ porque $5 < 10$. Algunas otras soluciones de $x < 10$ son 9.99, 0 y -4.

dispersión Una descripción de las semejanzas o diferencias de los valores en una distribución, a menudo en relación con el centro. La dispersión también describe la variabilidad de una distribución.



Por ejemplo, el diagrama de puntos A tiene una dispersión mayor que el diagrama de puntos B.

dato estadístico, estadística Un número único que mide algo de un conjunto de datos.

Ejemplos de datos estadísticos: media, mediana y RQ.

pregunta estadística Una pregunta que requiere más de un dato para responder.

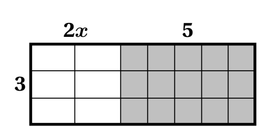
Estos son algunos ejemplos de preguntas estadísticas:
- ¿Cuál es la banda más popular en tu escuela?
- ¿Cuándo suelen cenar los estudiantes de tu clase?

sustituir Reemplazar una variable por un valor u otra expresión.

$$4x = 4(5)$$
$$= 20$$

En este ejemplo, el 5 sustituye a la x en la expresión $4x$.

suma El valor de dos o más cantidades que se suman.



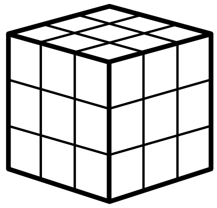
El área como una suma
$6x + 15$

Por ejemplo, el área de este rectángulo es la suma de $6x$ y 15, o $6x + 15$.

English

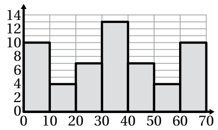
surface area The sum of the areas of a polyhedron's faces.

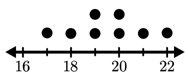
If the six faces of a cube each have an area of 9 square centimeters, then the surface area of the cube is 6 • 9, or 54 square centimeters.



symmetry (of a data distribution) A property of a data display where the left side of the distribution is a mirror image of the right side.

Here are some examples of data distributions that have symmetry.





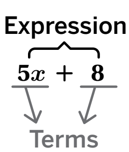
T

table A table organizes information into horizontal rows and vertical columns. The first row or column usually tells what the numbers represent.

Pet	Tail Length (in.)
Dog	22
Cat	12
Mouse	2

Here is a table showing the tail lengths of three different pets. This table has four rows and two columns.

term A part of an expression. A term can be a single number, a variable, or a number and variable multiplied together.



For example, the expression $5x + 8$ has two terms. The first term is $5x$ and the second term is 8.

U

unit price The price per unit of an item. You can count how many units by counting the items or the weight.

For example, if 4 avocados cost \$12, then the unit price is $\frac{12}{4} = \$3$ per avocado.

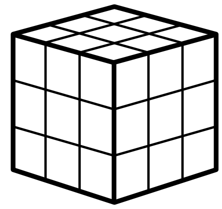
unit rate A rate that describes how one quantity changes when the other quantity changes by exactly 1 unit.

For example, if 12 people share 3 pizzas equally, then one unit rate is 4 people per pizza. Another unit rate in this situation is $\frac{1}{4}$ pizza per person.

Español

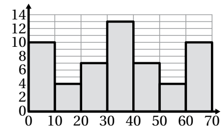
área de superficie La suma de las áreas de sus caras.

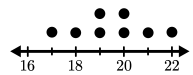
Si cada una de las seis caras de un cubo tiene un área de 9 centímetros cuadrados, el área de superficie del cubo mide 6 • 9, o 54 centímetros cuadrados.



simetría (de una distribución de datos) Una propiedad de una representación de datos donde el lado izquierdo de la distribución es una imagen reflejada del lado derecho.

Estos son algunos ejemplos de distribuciones de datos que tienen simetría.





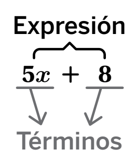
T

tabla Una tabla organiza la información en filas horizontales y columnas verticales. La primera fila o columna suele indicar lo que representan los números.

Mascota	Longitud de cola (pulg.)
Perro	22
Gato	12
Ratón	2

Esta es una tabla que muestra la longitud de la cola de tres mascotas diferentes. Esta tabla tiene cuatro filas y dos columnas.

término Una parte de una expresión. Un término puede ser un número individual, una variable, o una variable y un número multiplicados.



Por ejemplo, la expresión $5x + 8$ tiene dos términos. El primer término es $5x$ y el segundo término es 8.

U

precio unitario El precio por unidad de un artículo. Puedes contar la cantidad de unidades contando los artículos o el peso.

Por ejemplo, si 4 aguacates cuestan \$12, el precio unitario es $\frac{12}{4} = \$3$ por aguacate.

tasa unitaria Una tasa que describe cómo cambia una cantidad cuando la otra cantidad cambia en exactamente 1 unidad.

Por ejemplo, si 12 personas se reparten 3 pizzas en partes iguales, entonces una tasa unitaria es 4 personas por pizza. Otra tasa unitaria en esta situación es $\frac{1}{4}$ de pizza por persona.

English

Español

V

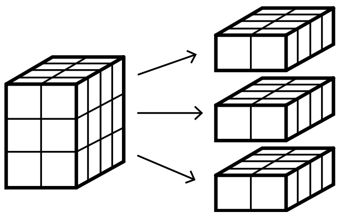
variable A letter or symbol that represents a value or set of values.

In the expression $10 - x$, the variable is x.

variable Una letra o un símbolo que representa un valor o un conjunto de valores.

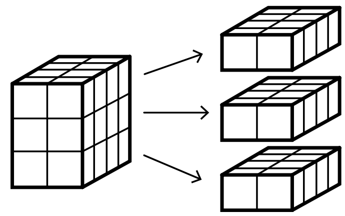
En la expresión $10 - x$, la variable es x.

volume The number of unit cubes needed to fill a three-dimensional shape without gaps or overlaps.



The volume of this rectangular prism is 24 cubic units because it is composed of 3 layers that are each 8 cubic units.

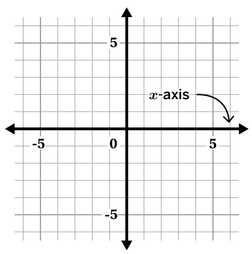
volumen La cantidad de cubos unitarios que se necesitan para llenar una figura tridimensional sin vacíos ni superposiciones.



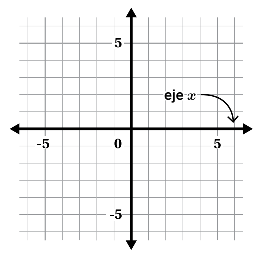
El volumen de este prisma rectangular es de 24 unidades cúbicas porque se compone de 3 capas de 8 unidades cúbicas cada una.

X

x-axis One of the perpendicular number lines that form the coordinate plane. The x-axis is the horizontal number line.



eje x Una de las rectas numéricas perpendiculares que forman el plano de coordenadas. El eje x es la recta numérica horizontal.

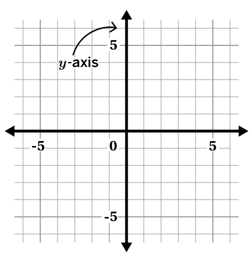


x-coordinate See *coordinates*.

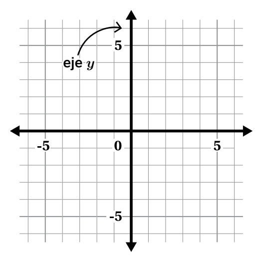
coordenada x Ver *coordenadas*.

Y

y-axis One of the perpendicular number lines that form the coordinate plane. The y-axis is the vertical number line.



eje y Una de las rectas numéricas perpendiculares que forman el plano de coordenadas. El eje y es la recta numérica vertical.



y-coordinate See *coordinates*.

coordenada y Ver *coordenadas*.

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