Unit 5

Lesson

8

Student Devices Recommended

This lesson uses digital student screens.
Student Notes are also available.



Clawdine the Crab

Equations of Circles

Let's make connections between the Pythagorean theorem and circles.

Focus and Coherence

Today's Goals

1. **Goal:** Derive an equation for any circle on the coordinate plane given a center and radius.
2. **Language Goal:** Explain how to determine the distance between two points on the coordinate plane. **(Writing, Speaking, and Listening)**

Students apply their knowledge of the Pythagorean theorem to write equations for circles given a center and a point on a circle. They engage with a delightful interaction to make sense of how changing the radius impacts its circle before computing the radius length. Students use repeated reasoning to make sense of the equation of any circle and then identify common misconceptions when writing equations that depict distance.

Prior Learning

In Unit 4, students revisited and proved the Pythagorean theorem. In Grade 8, students used the Pythagorean theorem to calculate distances on the coordinate plane. In Unit 1, students create constructions with and define a circle.

Future Learning

In Lessons 9–10, students identify the radius and center of circles for a variety of equations.

Rigor and Balance

- Students develop **conceptual understanding** of how to write the equation of a circle with a given center and point on the coordinate plane.
- Students develop **procedural fluency** as they use the Pythagorean theorem to solve for the distance of the radius of a circle.

Vocabulary

New Vocabulary

distance formula (optional)

Lesson at a Glance

~ 45 min



Why digital?

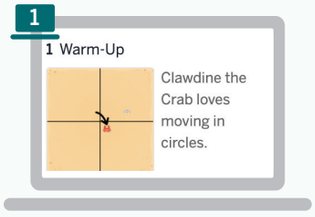
Students use digital interactions to make sense of writing equations of circles with a given center and point on the coordinate plane.

Warm-Up

Pairs Sharing a Device | 5 min

Pacing: Screen 1

Students make sense of the interaction by observing different circles created by the crab as they change the radius.



Activity 1

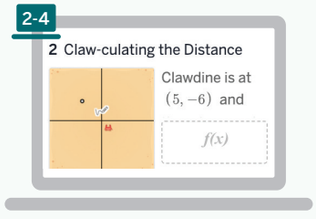
Pairs Sharing a Device | 10 min

Pacing: Screens 2–4

Students use the Pythagorean theorem to find the distance from Clawdine to a sand dollar on the coordinate plane before completing multiple challenges.

Key Takeaway (Screens 3–4): The Pythagorean theorem can be used to calculate the distance from the center of a circle to any point on a circle.

See the Activity 1 page for full Key Takeaway.

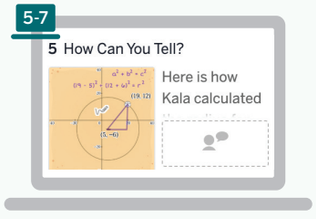


Activity 2

Pairs Sharing a Device | 10 min

Pacing: Screen 5, Screen 6, Screen 7

Students make connections between the distance on the coordinate plane and parts of a circle to write the equation of a circle. MLR2: Collect and Display, MLR3: Critique, Correct, Clarify

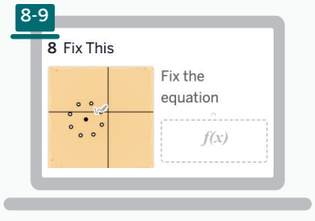


Activity 3 (Optional)

Pairs Sharing a Device | 10 min

Pacing: Screen 8, Screen 9

Students discuss common misconceptions when writing the equation of a circle and then practice writing equations for multiple circles.



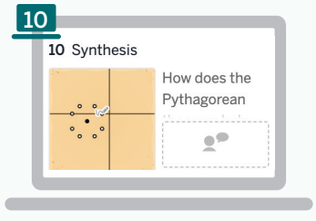
Synthesis

Whole Class | 5 min

Pacing: Screen 10

Students synthesize their understanding of using the Pythagorean theorem to write the equation of a circle. MLR2: Collect and Display

Lesson Takeaway: The Pythagorean theorem is a strategy to calculate distance from the center of a circle to any point on the circle. All of the points on a circle with center (h, k) and radius r can be represented with an equation in the form $(x - h)^2 + (y - k)^2 = r^2$.

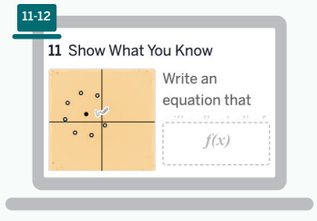


Show What You Know

Independent | 5 min

Pacing: Screens 11–12

Students demonstrate their understanding by writing the equation of a circle with a given center and point on the circle.



Prep Checklist

Assign the digital student screens. Student Notes are also available.

This lesson includes:

- Student Screens
- Lesson Practice
- Show What You Know (optional)

Student Notes sheet in Student Edition:

Consider providing the Student Notes to support class discussion and student thinking.



Pairs Sharing a Device | 5 min | Pacing: Screen 1

Warm-Up

Purpose: Students make sense of the interaction by observing different circles created by the crab as they change the radius.



Student Notes are also available

1 Launch

1

To connect to prior knowledge, consider asking, "What are some things you know about circles?"

Demonstrate how to drag Clawdine an arbitrary length and click 'Check'.

To encourage playfulness, invite students to collect as many sand dollars as they have time for, making note of strategies as they move Clawdine.

2 Connect

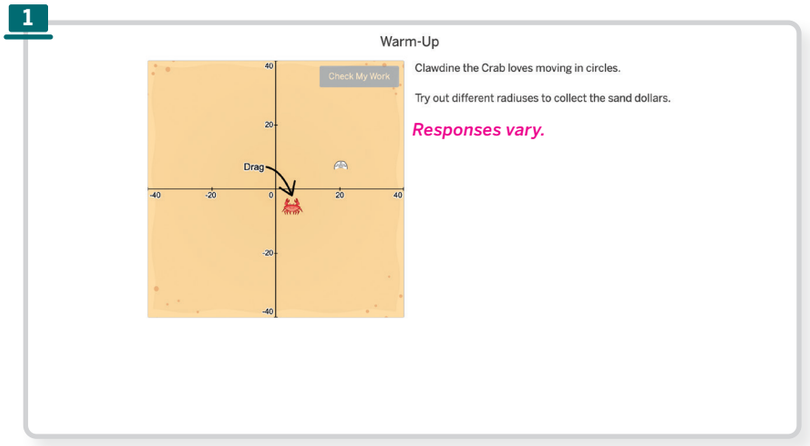
Invite students to share strategies that they found helpful or what they learned from the mistakes they made while moving Clawdine.

A **Accessibility: Conceptual Processing** Invite students to share how each problem is different from the previous one to help apply concepts to new situations.

Math Identity and Community Consider pausing to celebrate students who persisted through struggle. Invite them to share strategies for persistence that worked for them.

Note: If it does not come up naturally, consider asking, "What part of a circle does the distance between Clawdine and the sand dollar represent?"

1



Pairs Sharing a Device | 10 min | Pacing: Screens 2–4

Activity 1 Sandy Circles

Purpose: Students use the Pythagorean theorem to find the distance from Clawdine to the sand dollar on the coordinate plane before completing multiple challenges.



Student Notes are also available

1 Launch

2 **To support making connections**, consider asking, "How might knowing coordinates support us in determining the length of the radius?"

A **Accessibility: Visual-Spatial Processing** Invite students to use the sketch tool to support making connections between visual representations.

2 Monitor

To support students getting started, consider inviting students to estimate a radius and use the feedback to help them come up with a strategy.

3 **Capture** explanations to share during the Connect.

Consider asking pairs of students, "Which part of each coordinate would be helpful in calculating each leg?"

ML/EL **Multilingual/English Learners** Consider offering students the option to use a scribe or to verbally share their response instead of writing it. **(Writing and Speaking)**

4 **Listen for** strategies students use in discussing how to solve the challenges, such as sketching in the triangles or using the Pythagorean theorem.

D **Differentiation:** (Support) To support students as they work through the challenges, consider asking, "How could you use the structure of the coordinate plane to determine total distance, even without the squares drawn?"

A **Accessibility: Memory and Attention** Invite students who would benefit from having more time to think deeply to complete fewer challenges.

3 Connect

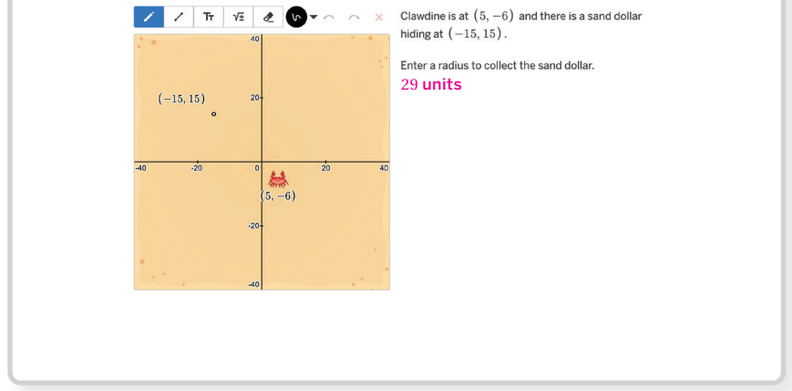
3-4 **Share** and discuss students' explanations to determine the length of the triangle legs.

To surface the Key Takeaway, consider asking, "How can you use coordinates and the Pythagorean theorem to determine the distance between these two points?"

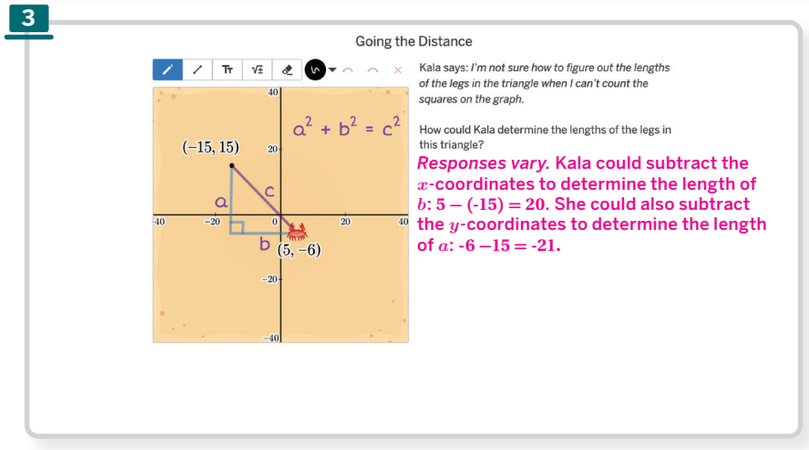
Note: Students do not need to know the term *distance formula* to engage in solving for the radius between two points, and this term will not be assessed. You may choose to introduce the term to your students if it helps with their thinking.

Students using digital

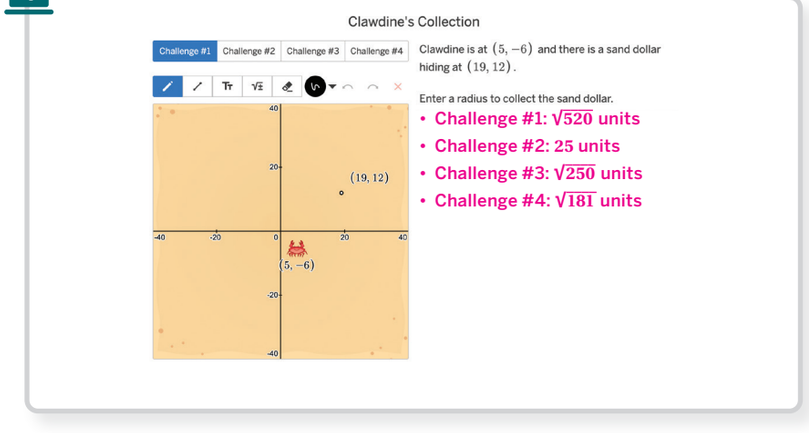
2



3



4



Key Takeaway: The Pythagorean theorem can be used to calculate the distance from the center of a circle to any point on a circle. The difference in the x-coordinates and the y-coordinates between these two points represents the legs of a right triangle. Then, apply the Pythagorean theorem to calculate the hypotenuse, which is the radius of the circle.

Pairs Sharing a Device | 10 min | Screen 5, Screens 6–7

Activity 2 Shelling out Circles

Purpose: Students make connections between the distance on the coordinate plane and parts of a circle to write the equation of a circle.



Student Notes are also available

1 Launch

5 **Invite students** to think independently before discussing their ideas with a partner.

2 Monitor

Capture and share student descriptions of how to calculate the radius for a generic point (x, y) on a circle.

MLR **MLR2: Collect and Display** Use this routine to collect students' ideas about writing a circle equation using the Pythagorean theorem during the class discussion. Display for students to reference and add to for the rest of the unit.

Invite students using notes to record some important considerations from the class discussion.

6 **Encourage students** to share their reasoning with a partner and work together to reach an agreement about how to sort the circles and equations.

ML/EL **Multilingual/English Learners** Provide sentence frames to help students explain their strategy (e.g., "I matched _____ with _____ because..." or "These two cards don't match because...") **(Writing and Speaking)**

A **Accessibility: Executive Function** Invite students to focus on one card at a time to support students in carrying out multiple steps.

3 Connect

6 MLR **MLR3: Critique, Correct, Clarify** Use this routine to help students describe strengths, errors, and ambiguities in Kala's work. Then invite students to:

- Discuss what is *correct* about the sorting.
- Discuss what is *incorrect* or *unclear*.
- Write a question to help Kala see how her work can be improved, or create a revised response.

Invite students to describe the features of the circle represented on the equation card and explain how they know from the equation.

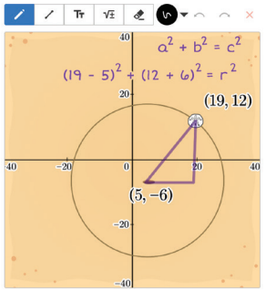
Invite students using notes to record some important considerations from the class discussion, including annotating the corrected equation.

Activity 2 continued >

Students using digital

5

How Can You Tell?



Here is how Kala calculated the radius for Clawdine's circle to get the sand dollar at $(19, 12)$.

a. **Discuss:** How does each part of Kala's equation relate to the circle?

b. How can you calculate the radius of a circle through any point (x, y) with center $(5, -6)$?

a. In Kala's equation, a is the horizontal distance and b is the vertical distance from the center to the sand dollar. c is the radius of the circle.

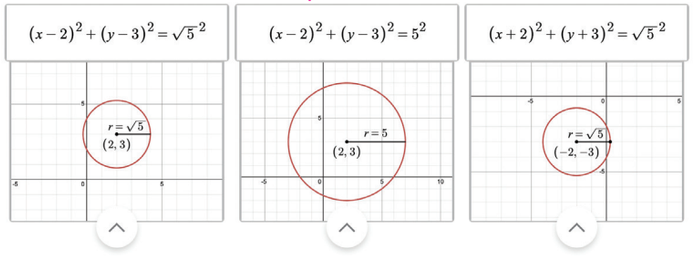
b. *Responses vary.* I can calculate the radius of any circle with center (5, -6) by subtracting the x-coordinates and subtracting the y-coordinates and then using those values for a and b in the Pythagorean theorem.

6

Card Sort

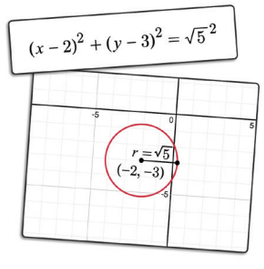
Match each equation with its graph.

Response shown.



7

Help Kala



Kala incorrectly matched these two cards.

What could you say to convince Kala that these cards don't match?

Responses vary. Since the center of this circle is at (-2, -3), you would need to subtract each number from x and y to determine the equation of a circle. The left side of the equation must be $(x-(-2))^2 + (y-(-3))^3$, which is equivalent to $(x+2)^2 + (y+3)^2$.

Pairs Sharing a Device | 10 min | Pacing: Screen 8, Screen 9

Activity 3 Circle Challenges (Optional)

Purpose: Students discuss common misconceptions when writing the equation of a circle and then practice writing equations for multiple circles.

Short on time? Consider omitting Activity 3, if students show a strong understanding of writing circle equations.



Student Notes are also available

1 Launch

8 **To support making connections**, consider asking, "What would be the center of the circle and length of the radius for the equation as written?"

Discuss students' strategies for fixing the equation.

9 **Invite students** to solve as many challenges as they have time for. Share that it's more important to make sense of the challenges than it is to work quickly.

2 Monitor

Listen for strategies students use in discussing how to write each equation, such as writing out the Pythagorean theorem or identifying parts of the circle equation.

D Differentiation

Look for students who:	Respond to Student Thinking
Need support getting started.	**Support** Consider asking, "How might a drawing help you? How might the equation for a circle help you?"
Swap the sign when putting the coordinates of the circle center into the equation.	**Support** Invite them to use the sketch tool to draw a right triangle and write a subtraction expression for the length of each leg.
Want to further explore writing equations of circles, given the center and a point on the circle.	**Stretch** Invite students to use a graphing calculator to make a design using only circles.

Note: Use the dashboard's Teacher View to see how many problems each student has completed correctly.

A Accessibility: Conceptual Processing Invite students to revisit their work on the previous screen to help apply concepts to new situations.

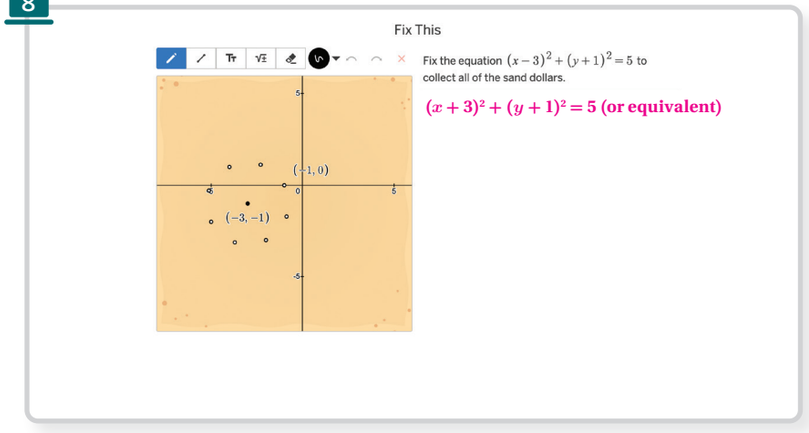
3 Connect

Invite students to share strategies that they found helpful or what they learned from the mistakes they made while solving these challenges.

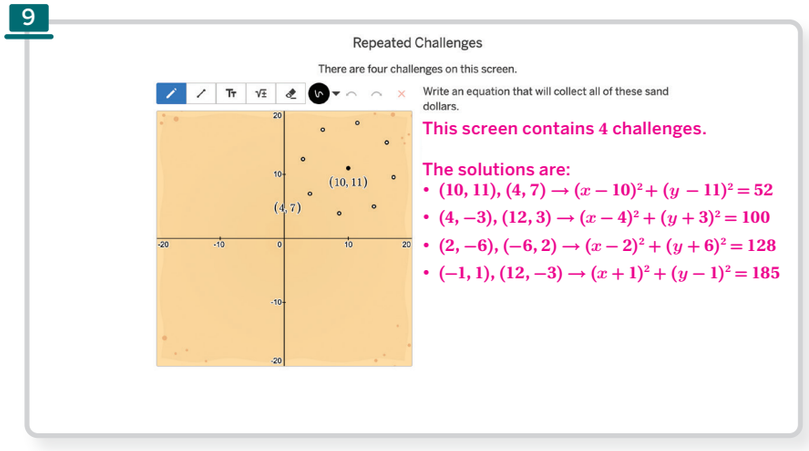
Math Identity and Community Celebrate students who keep an oversight of the process, while attending to the details of each problem. Consider naming this a strong mathematician move.

Students using digital

8



9



Whole Class | 5 min | Pacing: Screen 10

Synthesis



Student Notes are also available

10 **Invite students** to respond independently, and then share their thinking with a partner.

Capture and share a variety of ideas, such as:

- By making a right triangle with the distance between the points as the hypotenuse, the Pythagorean theorem can be a tool to calculate the radius.
- The x-coordinates and y-coordinates can be subtracted from one another to determine the length of each leg.
- The Pythagorean theorem can solve for the radius in the equation of a circle.

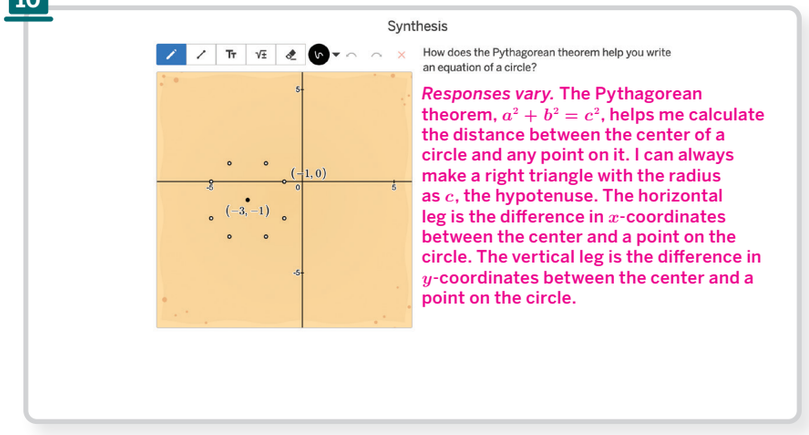
MLR **MLR2: Collect and Display** Use students' language to formalize the equation for a circle. Display any new language, examples, or diagrams for students to reference later.

Math Identity and Community Invite students to share ideas they found helpful and celebrate those who shared them.

Note: If time allows, invite students to revise their previous thinking based on the discussion.

Students using digital

10

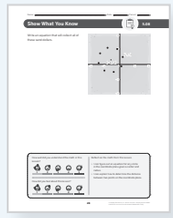


Lesson Takeaway: The Pythagorean theorem is a strategy to calculate distance from the center of a circle to any point on the circle. All of the points on a circle with center $(h,\ k)$ and radius r can be represented with an equation in the form $(x - h)^2 + (y - k)^2 = r^2$.

Independent | 5 min | Pacing: Screens 11–12

Show What You Know

Purpose: Students demonstrate their understanding by writing the equation of a circle with a given center and point on the circle.

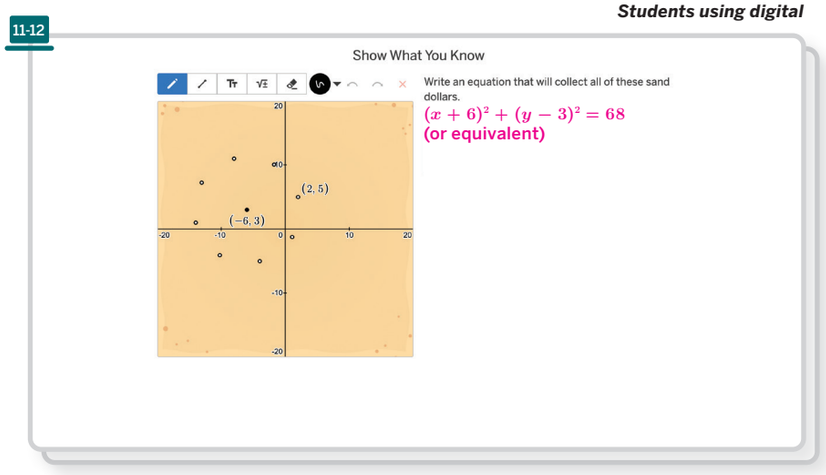


Students using print

11-12 Today's Goals

1. **Goal:** Derive an equation for any circle on the coordinate plane given a center and radius.
2. **Language Goal:** Explain how to determine the distance between two points on the coordinate plane. **(Writing, Speaking, and Listening)**

Students using digital



D Differentiation Beyond the Lesson

Here are options for responding to student work on the Show What You Know and throughout the lesson.

Support

Provide targeted intervention.

If student work shows **partial understanding** of using the Pythagorean theorem to write the equation of a circle, consider:

- Inviting students to read the **Summary** and highlight key components.
- Selecting a few related **practice problems** and giving individualized feedback.

Strengthen

Reinforce students' understanding.

If student work shows **conceptual understanding**, consider:

- Assigning the **Lesson Practice**.
- Inviting students to review the **Summary** and complete the **Try This** problem.

Stretch

Challenge students and extend their learning.

If students would enjoy an **additional challenge**, consider:

- Inviting them to write their own problem that is similar to the **Try This** problem.
- Encouraging them to complete the **DOK3 practice problem**.

Professional Learning

Reflect on how students' language for describing the Pythagorean theorem and circles developed throughout this lesson.

- How did your use of **MLR2: Collect and Display** support students in making sense of writing an equation for a circle using the Pythagorean theorem?

Lesson Practice

Independent

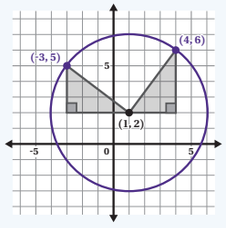
Students continue developing their conceptual understanding, fluency, and application of topics from this lesson and previous lessons/units (spiral review). Invite students to refer to the **Lesson Summary** to support them with this practice and their learning throughout the year.

Lesson 8
Practice

Students using digital

Lesson Summary

Here is a circle centered at $(1, 2)$. The points $(4, 6)$ and $(-3, 5)$ are on this circle.



To calculate the radius of this circle, you can use the center, a point on the circle, $(4, 6)$ and the Pythagorean theorem:

$$(4 - 1)^2 + (6 - 2)^2 = r^2$$

Evaluate the left side of the equation, $25 = r^2$ so the radius of this circle is $\sqrt{25}$, or 5 units.

Then check this radius using another point on the circle, like $(-3, 5)$:

$$(-3 - 1)^2 + (5 - 2)^2 = r^2$$

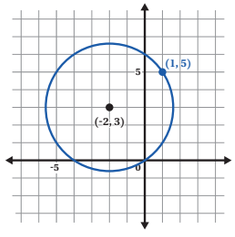
Evaluate the left side of the equation, $25 = r^2$, so the radius is $\sqrt{25}$, or 5 units.

You can use this equation to describe every point (x, y) on this circle: $(x - 1)^2 + (y - 2)^2 = 5^2$.

In general, a circle with center (h, k) and radius r can be written as $(x - h)^2 + (y - k)^2 = r^2$.

Try This

Here is a circle centered at $(-2, 3)$. The point $(1, 5)$ is on this circle.



a. Determine the length of the radius for this circle.
$\sqrt{13}$ (or equivalent)

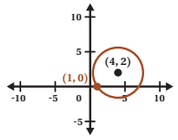
b. Write an equation for this circle.
$(x + 2)^2 + (y - 3)^2 = 13$ (or equivalent)

Lesson Practice
Geo.5.08

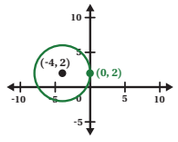
Name: _____ Date: _____ Period: _____

1. Determine the distance between (3, 5) and (-2, -3).
$\sqrt{89}$ (or equivalent)

2. What is the length of the radius of the circle?
$\sqrt{13}$ (or equivalent)

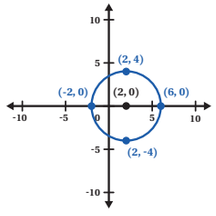


3. Which equation matches the graph?



A. $(x - 4)^2 + (y + 2)^2 = 4$
B. $(x - 4)^2 + (y + 2)^2 = 16$
C. $(x + 4)^2 + (y - 2)^2 = 4$
D. $(x + 4)^2 + (y - 2)^2 = 16$

Problems 4–5: Here is a circle with center (2, 0).



4. Is $(3, \sqrt{15})$ on the circle? Circle one.

(Yes) No

Show or explain your thinking. *Explanations vary.*
$(2 - 3)^2 + (0 - \sqrt{15})^2 = 4^2$
$1 + 15 = 16$
$16 = 16$

5. Is (5, 2) on the circle? Circle one.

Yes (No)

Show or explain your thinking. *Explanations vary.*
$(2 - 5)^2 + (0 - 2)^2 = 4^2$
$9 + 4 = 16$
$13 \neq 16$

Lesson Practice
Geo.5.08

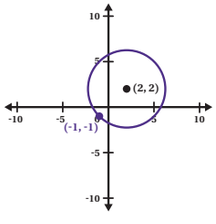
Name: _____ Date: _____ Period: _____

Problems 6–7: Create an equation of a circle with . . .

6. Center $(-2, 5)$ and radius $\sqrt{15}$.
$(x - \underline{-2})^2 + (y - \underline{5})^2 = \underline{15}$

7. Center $(1, -\sqrt{7})$ and radius $\sqrt{29}$.
$(x - \underline{1})^2 + (y - \underline{-\sqrt{7}})^2 = \underline{29}$

8. Write the coordinates of at least two more points that are on this circle.
Responses vary.
- $(5, -1)$
- $(2, 2 - \sqrt{18})$
- $(2 + \sqrt{18}, 2)$



Spiral Review

Problems 9–10: Fill in the blanks to make equivalent expressions.

9. $x^2 + 2x + 1 = (x + \underline{1})(x + \underline{1})$

10. $x^2 - x - 6 = (x + \underline{2})(x + \underline{-3})$

Reflection

1. Put a question mark next to a problem you were feeling stuck on.
2. Use this space to ask a question or share something you're proud of.

Lesson Practice Item Analysis

	Problem(s)	DOK
On-lesson		
	1	1
	2	2
Test Practice	3	2
	4–7	2
	8	3
Spiral Review		
Fluency	9–10	1

Need More Practice?

Additional Practice resources coming soon.