Amplify Desmos Math

FLORIDA

Algebra 1

A1

Intervention and Extension Resources

Inside you'll find:

- Strategies for effective differentiation
- Mini-Lessons from prior grades
- Extensions

Amplify Desmos Math **FLORIDA**

Algebra 1

Intervention and Extension Resources

About Amplify

Amplify is dedicated to collaborating with educators to create learning experiences that are rigorous and riveting for all students. Amplify creates K–12 core and supplemental curriculum, assessment, and intervention programs for today's students.

A pioneer in K–12 education since 2000, Amplify is leading the way in next-generation curriculum and assessment. All of our programs provide teachers with powerful tools that help them understand and respond to the needs of every student.

Amplify gratefully acknowledges the work of distinguished program advisors from English Learners Success Forum (ELSF), who have been integral in the development of Amplify Desmos Math. ELSF is a 501(c)(3) nonprofit organization whose mission is to expand educational equity for multilingual learners by increasing the supply of high-quality instructional materials that center their cultural and linguistic assets.

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Intervention and Extension

Amplify Desmos Math Florida provides a comprehensive suite of intervention and extension resources designed to meet the needs of all students and are intended to be used outside of the core lesson.

Assess and Respond: After the Lesson

Each lesson has a formative assessment called a ***Show What You Know***. This assessment illustrates students' progress toward key concepts in the lesson and is accompanied by a table with suggestions in three categories: students who need **support**, students who would benefit from more practice to **strengthen** their understanding, and students who are interested in a **stretch** to deepen their understanding.

Support	Strengthen	Stretch
Provide targeted intervention for students.	Reinforce students' understanding of the concepts assessed.	Challenge students and extend their learning.

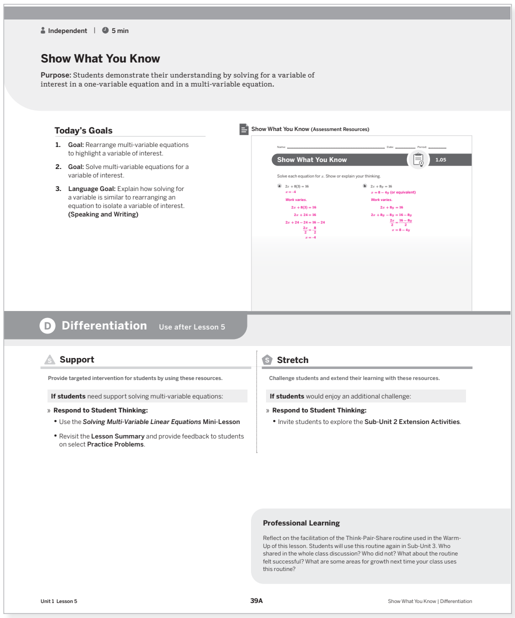
Differentiation Resources

The differentiation resources provide *beyond-the-lesson* support and challenge for all students.

Support, **Strengthen**, and **Stretch** print resources include:

- **Mini-Lessons:** Targeted intervention lessons to support students with a specific concept or skill.
- **Lesson Practices:** Practices to build and reinforce students' conceptual understanding, fluency, and application. Lesson Practices include fluency, test prep, and spiral review.
- **Extensions:** Problems aligned to the math of the sub-unit, designed for students who want to extend their thinking.

The differentiation table is available on the *Show What You Know* page of the Teacher Edition and in the *Differentiation Beyond the Lesson* tab of each lesson.


Independent | 5 min

Show What You Know

Purpose: Students demonstrate their understanding by solving for a variable of interest in a one-variable equation and in a multi-variable equation.

Today's Goals

1. **Goal:** Rearrange multi-variable equations to highlight a variable of interest.
2. **Goal:** Solve multi-variable equations for a variable of interest.
3. **Language Goal:** Explain how solving for a variable is similar to rearranging an equation to isolate a variable of interest. **(Speaking and Writing)**

Show What You Know (Assessment Resources)

Name: Date: Period:

Show What You Know 1.05

Solve each equation for x. Show or explain your thinking.

a. $2x + 8(3) = 16$
$x = -4$
Work varies.
$2x + 8(3) = 16$
$2x + 24 = 16$
$2x + 24 - 24 = 16 - 24$
$\frac{2x}{2} = \frac{-8}{2}$
$x = -4$

b. $2x + 8y = 16$
$x = 8 - 4y$ (or equivalent)
Work varies.
$2x + 8y = 16$
$2x + 8y - 8y = 16 - 8y$
$\frac{2x}{2} = \frac{16 - 8y}{2}$
$x = 8 - 4y$

Differentiation Use after Lesson 5

Support

Provide targeted intervention for students by using these resources.

If students need support solving multi-variable equations:

» **Respond to Student Thinking:**
- Use the ***Solving Multi-Variable Linear Equations*** **Mini-Lesson**
- Revisit the **Lesson Summary** and provide feedback to students on select **Practice Problems**.

Stretch

Challenge students and extend their learning with these resources.

If students would enjoy an additional challenge:

» **Respond to Student Thinking:**
- Invite students to explore the **Sub-Unit 2 Extension Activities**.

Professional Learning

Reflect on the facilitation of the Think-Pair-Share routine used in the Warm-Up of this lesson. Students will use this routine again in Sub-Unit 3. Who shared in the whole class discussion? Who did not? What about the routine felt successful? What are some areas for growth next time your class uses this routine?

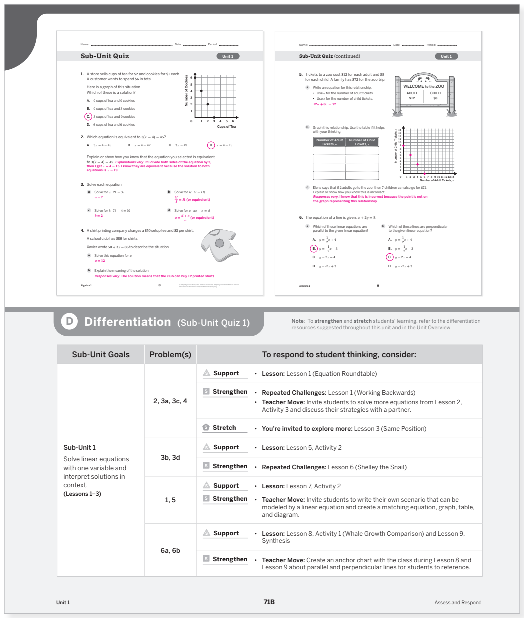
Unit 1 Lesson 5 39A Show What You Know | Differentiation

Differentiation Beyond the Lesson table from Algebra 1 Unit 1, Lesson 5.

Assess and Respond: After Unit Assessments

Embedded unit assessments offer key insights into students' understanding of the grade-level standards in the unit.

- Each unit includes an optional Pre-Unit Check, one or more Sub-Unit Quizzes, and an End-of-Unit Assessment.
- Each assessment is accompanied by an Assess and Respond Guide in the Teacher Edition, which includes responses to student thinking with resources that support, strengthen, and stretch learning.



Sub-Unit Quiz Differentiation table from Algebra 1 Unit 1, Sub-Unit 1.

About Mini-Lessons

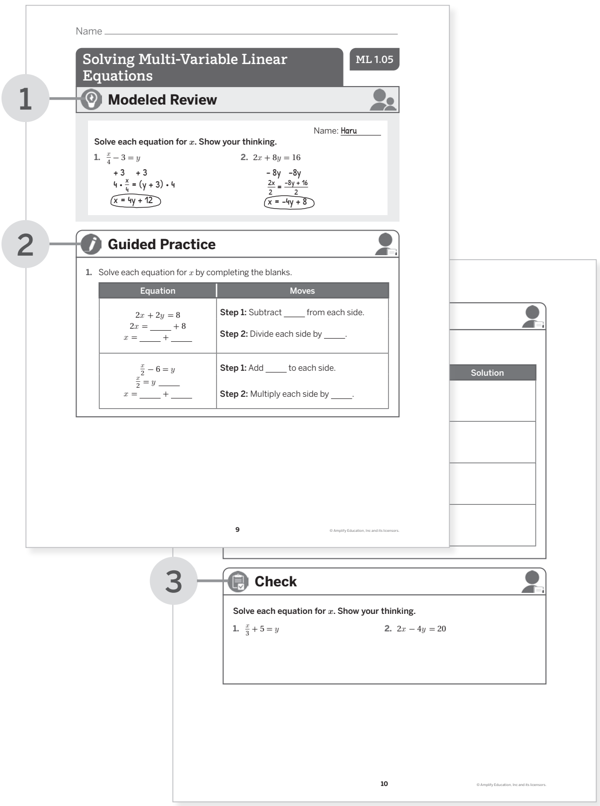
Amplify Desmos Math Florida Mini-Lessons are print activities aligned to the most critical content and skills in corresponding core lessons.

Mini-Lessons offer direct instruction and guided practice opportunities. These Mini-Lessons complement the problem-based approach, are ideal for small-group or whole-class instruction, as well as independent learning.

Student Page

Amplify Desmos Math Florida Mini-Lessons are designed based on extensive research around worked examples.[1]

The Student Page is organized to follow the flow of the Mini-Lesson: **Modeled Review**, **Guided Practice**, and **Check**.



Algebra 1 Unit 1, Lesson 5 Mini-Lesson

1 **Modeled Review:** A worked example designed to be discussed as a group. Students make sense of a concept or process by examining the modeled student thinking.

2 **Guided Practice:** A series of problems that fades away scaffolding as students progress. Teachers can approach the Guided Practice in various ways based on their expertise and understanding of their students' needs.

3 **Check:** An opportunity for students to show what they've learned. We recommend all students complete this independently.

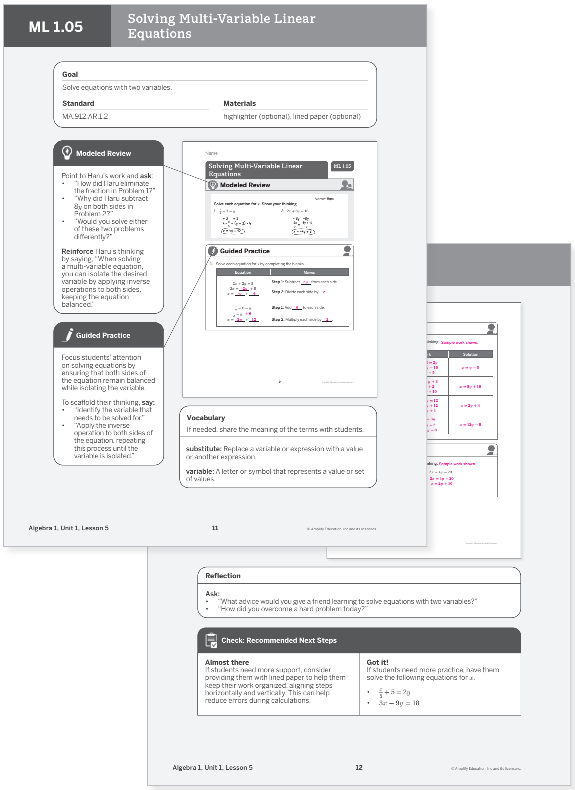
Considerations:

- Print or copy the Student Page in advance.
- Gather any needed materials. Any materials listed as optional on the Teacher Guide are not required to successfully implement the lesson.
- It may be helpful to have a whiteboard, but it's not required.

[1] Flores, R. and Inan, F. (2014). Examining the Impact of Adaptively Faded Worked Examples on Student Learning Outcomes. Journal of Interactive Learning Research, 467–485.

Teacher Guide

The Teacher Guide follows the same flow as the Student Page, with all the information needed to implement the Mini-Lesson.



Algebra 1 Unit 1, Lesson 5 Mini-Lesson

Every Teacher Guide includes:

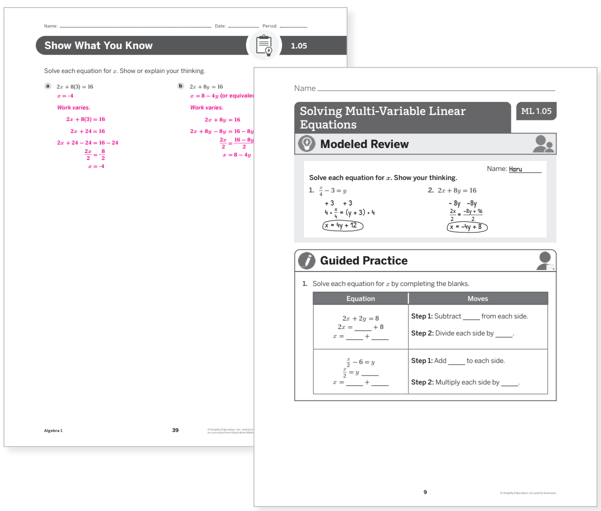
- The **lesson goal**, **materials**, and relevant **vocabulary terms**.
- Questions or statements to share with students in each part of the Mini-Lesson. **Reflection questions** are included as a way to close out the lesson.
- **Answer keys** with sample responses on the insets of the student pages.
- **Recommended next steps** for students needing more support or extra practice based on their performance on the Check problem(s).

Supporting All Learners

ELL This icon indicates suggestions for supporting **English Language Learners**.

A This icon appears at point-of-use and indicates suggestions for supporting the needs of all learners, based on the guidelines of **Universal Design for Learning (UDL)**.

Lesson and Mini-Lesson Alignment



Algebra 1 Unit 1, Lesson 5 SWYK and Mini-Lesson

Mini-Lessons are closely aligned to the core lesson they are connected to. The **Mini-Lesson Modeled Review** is built using the *Show What You Know* (SWYK) from the lesson. Teachers can use student thinking on the *Show What You Know* to identify students who might benefit from the extra support of a Mini-Lesson.

About Extensions

Amplify Desmos Math Florida Extensions are sets of problems aligned to the math of the sub-unit. They are useful for students interested in an additional challenge or for the whole class.

Extensions build on our **student-led**, **problem-based** approach because they provide more opportunities for students to engage in creative and rigorous problems that can be approached using different strategies.

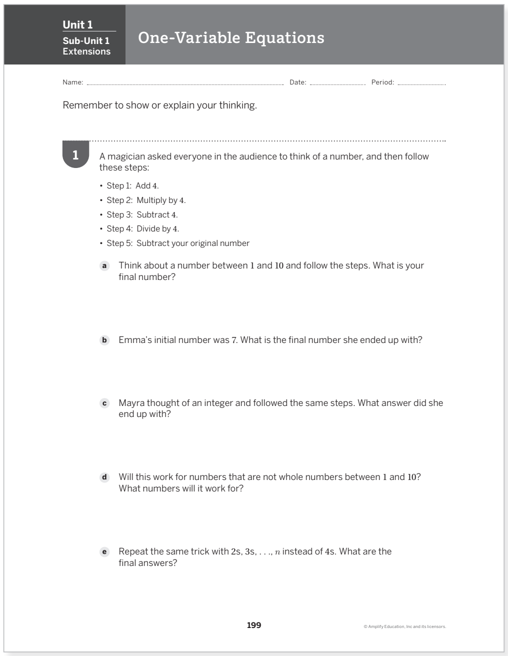
Student Page

Every sub-unit includes an Extensions problem set.

They are print-based, hands-on problems structured on the principle of student choice and designed to be student-led. The math is designed to be accessible to students at any time they are ready for more during the sub-unit.

Every sub-unit Extension includes:

- **Challenge:** Extensions focus on problem-solving and sharing thinking rather than answer-getting, with problems aligned to the math in the sub-unit.
- **Choice:** Extensions contain multiple open-ended problems, and students can start with what interests them.
- **Variety:** Some problems are designed with hands-on materials, others are discussion-based, and the rest require only a pencil and paper.


Unit 1
Sub-Unit 1
Extensions
One-Variable Equations

Name: ______ Date: ______ Period: ______

Remember to show or explain your thinking.

1 A magician asked everyone in the audience to think of a number, and then follow these steps:

- Step 1: Add 4.
- Step 2: Multiply by 4.
- Step 3: Subtract 4.
- Step 4: Divide by 4.
- Step 5: Subtract your original number

a. Think about a number between 1 and 10 and follow the steps. What is your final number?

b. Emma's initial number was 7. What is the final number she ended up with?

c. Mayra thought of an integer and followed the same steps. What answer did she end up with?

d. Will this work for numbers that are not whole numbers between 1 and 10? What numbers will it work for?

e. Repeat the same trick with 2s, 3s, . . ., n instead of 4s. What are the final answers?

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Algebra 1 Unit 1, Sub-Unit 1 Extension Student Page

Considerations:

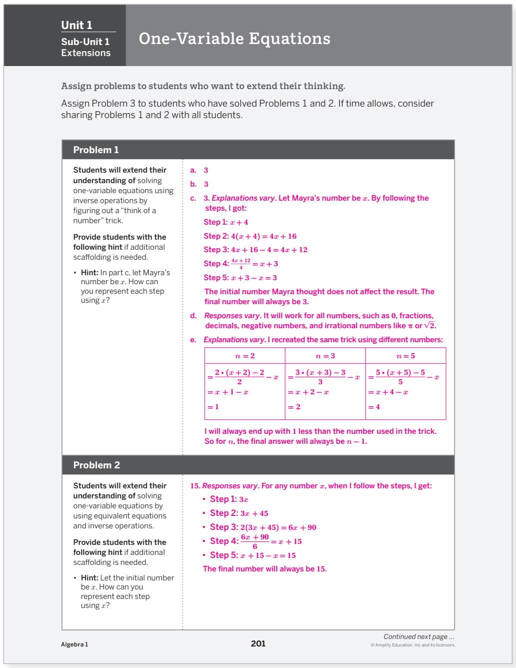
- Invite students to choose one problem to focus on at a time.
- Prepare enough Extensions at the start of the sub-unit for all students so you can be flexible with when different students work on them.
- Think about where you will want students to store their work on the Extensions for the duration of the sub-unit.

Teacher Guide

Extensions are designed to be a light lift for the teacher.

Every sub-unit Extension Teacher Guide includes:

- **Key background information** about the math in the problem.
- **Suggestions** for which problems to share with the whole class if time allows.
- **Hints** to share with students when needed.
- **Sample responses**.


Unit 1
Sub-Unit 1 Extensions
One-Variable Equations

Assign problems to students who want to extend their thinking.

Assign Problem 3 to students who have solved Problems 1 and 2. If time allows, consider sharing Problems 1 and 2 with all students.

Problem 1

Students will extend their understanding of solving one-variable equations using inverse operations by figuring out a "think of a number" trick.

Provide students with the following hint if additional scaffolding is needed.

- Hint: In part c, let Mayra's number be x. How can you represent each step using x?

a. 3

b. 3

c. 3. *Explanations vary.* Let Mayra's number be x. By following the steps, I got:

Step 1: $x + 4$

Step 2: $4(x + 4) = 4x + 16$

Step 3: $4x + 16 - 4 = 4x + 12$

Step 4: $\frac{4x+12}{4} = x + 3$

Step 5: $x + 3 - x = 3$

The initial number Mayra thought does not affect the result. The final number will always be 3.

d. *Responses vary.* It will work for all numbers, such as 0, fractions, decimals, negative numbers, and irrational numbers like π or $\sqrt{2}$.

e. *Explanations vary.* I recreated the same trick using different numbers:

$n = 2$	$n = 3$	$n = 5$
$= \frac{2 \cdot (x+2) - 2}{2} - x$	$= \frac{3 \cdot (x+3) - 3}{3} - x$	$= \frac{5 \cdot (x+5) - 5}{5} - x$
$= x + 1 - x$	$= x + 2 - x$	$= x + 4 - x$
$= 1$	$= 2$	$= 4$

I will always end up with 1 less than the number used in the trick. So for n, the final answer will always be $n - 1$.

Problem 2

Students will extend their understanding of solving one-variable equations by using equivalent equations and inverse operations.

Provide students with the following hint if additional scaffolding is needed.

- Hint: Let the initial number be x. How can you represent each step using x?

15. *Responses vary.* For any number x, when I follow the steps, I get:

- Step 1: $3x$
- Step 2: $3x + 45$
- Step 3: $2(3x + 45) = 6x + 90$
- Step 4: $\frac{6x+90}{6} = x + 15$
- Step 5: $x + 15 - x = 15$

The final number will always be 15.

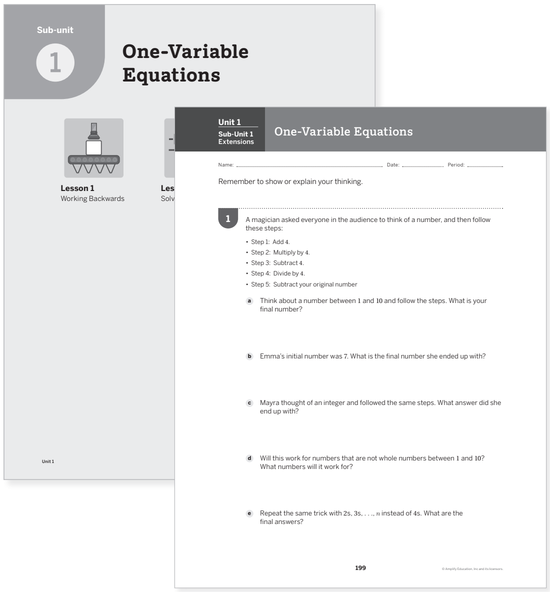
Algebra 1 201 *Continued next page ...*

Algebra 1 Unit 1, Sub-Unit 1 Extension Teacher Guide

Considerations:

- Look for opportunities to introduce an Extension problem with the whole class early in the sub-unit so all students can participate.
- Help students get started on the Extension task, then let them work independently or in pairs while you work with other groups of students who may benefit from more direct support from a teacher.

Lesson and Extension Alignment



Algebra 1 Unit 1 , Sub-Unit 1 Opener and Extension

Extensions are aligned to the math of the **sub-unit**, but they also go deeper. The problems in Extensions are designed to make connections between the math of the sub-unit and other concepts. In some cases, problems will involve content from prior grades or units.

Mini-Lessons

Unit 1

Mini-Lessons

Name ______________________________

Solving Linear Equations

ML 1.02

Modeled Review

Alina's work

$$x - 2x + 10 = \frac{2x - 28}{2}$$
$$-x + 10 = x - 14$$
$$10 = 2x - 14$$
$$24 = 2x$$
$$12 = x$$

Amir's work

$$x - 2x + 10 = \frac{2x - 28}{2}$$
$$2x - 4x + 20 = 2x - 28$$
$$-2x + 20 = 2x - 28$$
$$20 = 4x - 28$$
$$48 = 4x$$
$$12 = x$$

Guided Practice

1. Solve the equation by completing the blanks.

Equation	Moves
$-2x + 4 + 6x = 2(x - 6)$	**Step 1:** Combine like terms on the left side.
$\_\_\_\_ + 4 = 2(x - 6)$	**Step 2:** Distribute the 2 on the right side.
$\_\_\_\_ + 4 = \_\_\_\_\_\_$	**Step 3:** Subtract $2x$ from each side.
$\_\_\_\_ + 4 = \_\_\_\_$	**Step 4:** Subtract 4 from each side.
$\_\_\_\_ = \_\_\_\_$	**Step 5:** Divide by 2 on each side.
$x = \_\_\_\_$	

Guided Practice

Solve each equation. Show your thinking.

2. $2x - 6 = \frac{1}{2}(x - 2) + 4$

$4x - 12 =$ ________ $+ 8$

$4x - 12 =$ ________

$3x - 12 =$ ________

________ $=$ ________

________ $=$ ________

3. $2(3x - 1) + 4x = 5x + 8$

4. $x - 3x - 8 = \frac{3x + 12}{3}$

5. $-x + 2 + 5x = 2(x + 4)$

Check

Solve each equation. Show your thinking.

1. $5x - 3 = 3(x + 1) + 4$

2. $3x - 5x - 9 = \frac{2x + 18}{2}$

ML 1.02 Solving Linear Equations

Goal

Solve one-variable linear equations.

Standard

MA.912.AR.2.1

Materials

highlighter (optional)

Modeled Review

Point to the Modeled Review and **ask**:

- "How are Alina's and Amir's work similar? Different?"
- "In the second row, why did Amir write $2x - 28$ instead of $4x - 56$?"
- "How can you check that the solution is correct?"

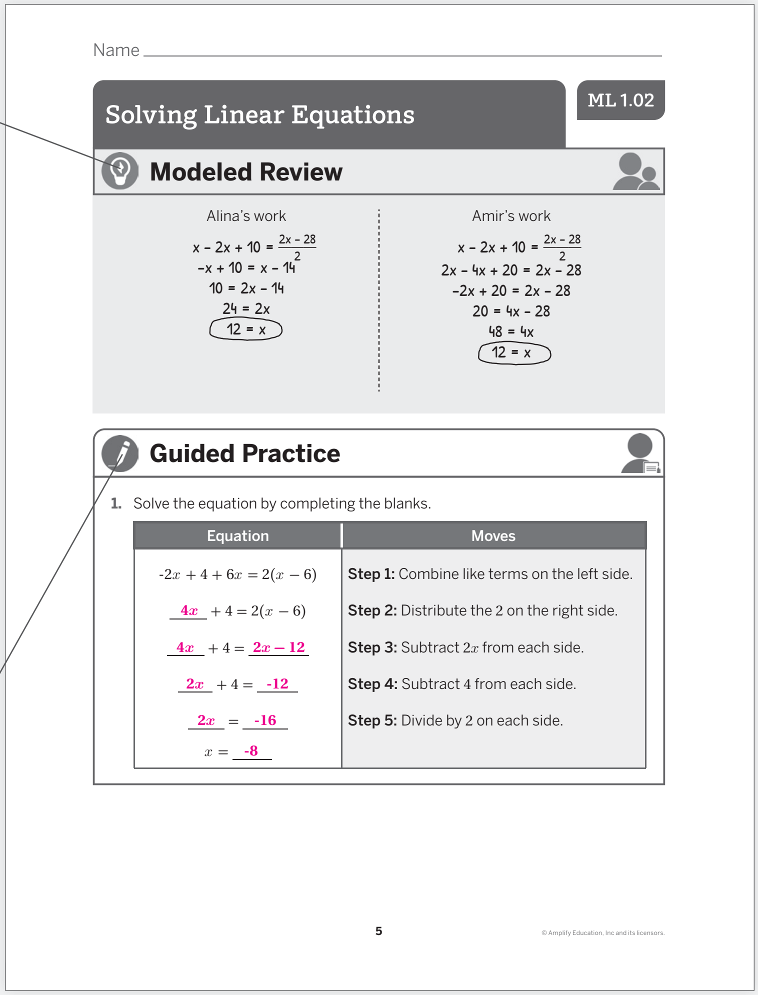
Reinforce the goal by saying, "When solving one-variable linear equations, you can use different methods to solve for the variable while maintaining equal balance throughout the calculation process."

Guided Practice

Focus students' attention on solving equations by ensuring that the equation remains balanced as they determine the value of the variable.

To scaffold their thinking, **say**:

- "Identify the variable that needs to be solved for."
- "If needed, simplify by distributing or combining like terms."
- "Apply the inverse operation to both sides of the equation, repeating this process until the variable is isolated."


Name ______

Solving Linear Equations ML 1.02

Modeled Review

Alina's work

$x - 2x + 10 = \frac{2x - 28}{2}$

$-x + 10 = x - 14$

$10 = 2x - 14$

$24 = 2x$

$12 = x$

Amir's work

$x - 2x + 10 = \frac{2x - 28}{2}$

$2x - 4x + 20 = 2x - 28$

$-2x + 20 = 2x - 28$

$20 = 4x - 28$

$48 = 4x$

$12 = x$

Guided Practice

1. Solve the equation by completing the blanks.

Equation	Moves
$-2x + 4 + 6x = 2(x - 6)$	**Step 1:** Combine like terms on the left side.
$\underline{4x} + 4 = 2(x - 6)$	**Step 2:** Distribute the 2 on the right side.
$\underline{4x} + 4 = \underline{2x - 12}$	**Step 3:** Subtract $2x$ from each side.
$\underline{2x} + 4 = \underline{-12}$	**Step 4:** Subtract 4 from each side.
$\underline{2x} = \underline{-16}$	**Step 5:** Divide by 2 on each side.
$x = \underline{-8}$	

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Vocabulary

If needed, share the meaning of the terms with students.

equivalent equations: Equations that have the exact same solution(s).

distributive property: Multiplying a number by the sum of two or more terms is equal to multiplying the number by each term separately before adding them together.

Solving Linear Equations

Guided Practice

(A) Guide student processing by providing questions students can ask themselves as they solve one-variable equations. For example:

- "How can you simplify the left side? The right side?"
- "Should you combine any like terms?"
- "Is the variable isolated?"

Note: Students can check the solution by substituting in the value for the variable and calculating both sides of the equation to determine whether it is true.

Key Takeaway:
Say, "One-variable equations can be solved using various strategies. Each step should maintain equal balance. This will ensure that every equation in the solving process has the same solution."

Guided Practice

Solve each equation. Show your thinking. Sample work shown.

2. $2x - 6 = \frac{1}{2}(x - 2) + 4$

$4x - 12 = \underline{x - 2} + 8$

$4x - 12 = \underline{x + 6}$

$3x - 12 = \underline{6}$

$\underline{3x} = \underline{18}$

$\underline{x} = \underline{6}$

3. $2(3x - 1) + 4x = 5x + 8$

$6x - 2 + 4x = 5x + 8$

$10x - 2 = 5x + 8$

$5x - 2 = 8$

$5x = 10$

$x = 2$

4. $x - 3x - 8 = \frac{3x + 12}{3}$

$-2x - 8 = \frac{3x + 12}{3}$

$-2x - 8 = x + 4$

$-8 = 3x + 4$

$-12 = 3x$

$-4 = x$

5. $-x + 2 + 5x = 2(x + 4)$

$4x + 2 = 2(x + 4)$

$4x + 2 = 2x + 8$

$2x + 2 = 8$

$2x = 6$

$x = 3$

Check

Solve each equation. Show your thinking. Sample work shown.

1. $5x - 3 = 3(x + 1) + 4$

$5x - 3 = 3x + 3 + 4$

$5x - 3 = 3x + 7$

$2x - 3 = 7$

$2x = 10$

$x = 5$

2. $3x - 5x - 9 = \frac{2x + 18}{2}$

$-2x - 9 = x + 9$

$-3x - 9 = 9$

$-3x = 18$

$x = -6$

6

Reflection

Ask:

- "Which strategy was the most helpful when determining the solution for each equation?"
- "How did you overcome a hard problem today?"

Check: Recommended Next Steps

Almost there
If students need more support, model highlighting all the coefficients. This can help students organize their work more effectively while they focus on isolating the variable.

Got it!
If students need more practice, ask them to solve $3x - 1 = \frac{1}{2}(x - 4) + 6$ and show their thinking.

Name ______________________________

Solving Multi-Variable Linear Equations

ML 1.05

Modeled Review

Name: Haru

Solve each equation for x. Show your thinking.

1. $\frac{x}{4} - 3 = y$

$+3 \quad +3$

$4 \cdot \frac{x}{4} = (y + 3) \cdot 4$

$x = 4y + 12$

2. $2x + 8y = 16$

$-8y \quad -8y$

$\frac{2x}{2} = \frac{-8y + 16}{2}$

$x = -4y + 8$

Guided Practice

1. Solve each equation for x by completing the blanks.

Equation	Moves
$2x + 2y = 8$ $2x =$ _____ $+ 8$ $x =$ _____ $+$ _____	**Step 1:** Subtract _____ from each side. **Step 2:** Divide each side by _____.
$\frac{x}{2} - 6 = y$ $\frac{x}{2} = y$ _____ $x =$ _____ $+$ _____	**Step 1:** Add _____ to each side. **Step 2:** Multiply each side by _____.

Guided Practice

2. Solve each equation for x. Show your thinking.

Equation	Work	Solution
$2x + 10 = 2y$		
$\frac{x}{5} + 1 = y + 3$		
$3x - 6y = 12$		
$\frac{x}{4} + 2 = 3y$		

Check

Solve each equation for x. Show your thinking.

1. $\frac{x}{3} + 5 = y$

2. $2x - 4y = 20$

ML 1.05 Solving Multi-Variable Linear Equations

Goal

Solve equations with two variables.

Standard

MA.912.AR.1.2

Materials

highlighter (optional), lined paper (optional)

Modeled Review

Point to Haru's work and **ask**:

- "How did Haru eliminate the fraction in Problem 1?"
- "Why did Haru subtract $8y$ on both sides in Problem 2?"
- "Would you solve either of these two problems differently?"

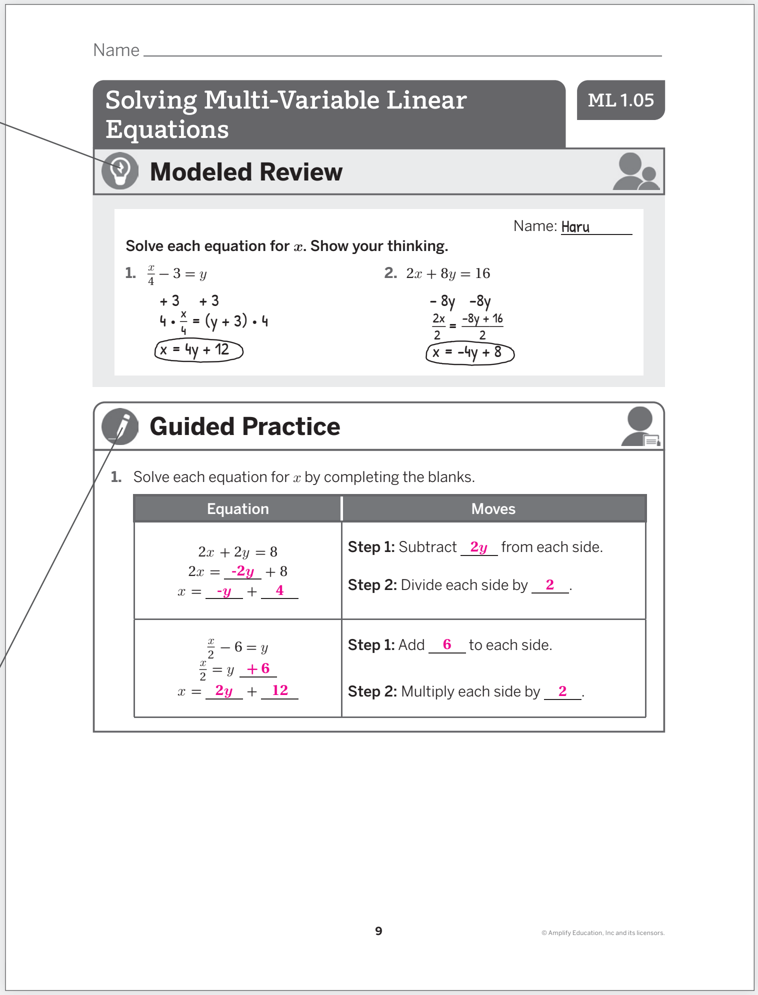
Reinforce Haru's thinking by saying, "When solving a multi-variable equation, you can isolate the desired variable by applying inverse operations to both sides, keeping the equation balanced."

Guided Practice

Focus students' attention on solving equations by ensuring that both sides of the equation remain balanced while isolating the variable.

To scaffold their thinking, **say:**

- "Identify the variable that needs to be solved for."
- "Apply the inverse operation to both sides of the equation, repeating this process until the variable is isolated."



Name ______

Solving Multi-Variable Linear Equations — ML 1.05

Modeled Review

Name: Haru

Solve each equation for x. Show your thinking.

1. $\frac{x}{4} - 3 = y$

$+3 \quad +3$

$4 \cdot \frac{x}{4} = (y+3) \cdot 4$

$x = 4y + 12$

2. $2x + 8y = 16$

$-8y \quad -8y$

$\frac{2x}{2} = \frac{-8y+16}{2}$

$x = -4y + 8$

Guided Practice

1. Solve each equation for x by completing the blanks.

Equation	Moves
$2x + 2y = 8$ $2x = \underline{-2y} + 8$ $x = \underline{-y} + \underline{4}$	**Step 1:** Subtract $\underline{2y}$ from each side. **Step 2:** Divide each side by $\underline{2}$.
$\frac{x}{2} - 6 = y$ $\frac{x}{2} = y \underline{+6}$ $x = \underline{2y} + \underline{12}$	**Step 1:** Add $\underline{6}$ to each side. **Step 2:** Multiply each side by $\underline{2}$.

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Vocabulary

If needed, share the meaning of the terms with students.

substitute: Replace a variable or expression with a value or another expression.

variable: A letter or symbol that represents a value or set of values.

Solving Multi-Variable Linear Equations

Guided Practice

A Consider providing students with a highlighter to color-code the variable x. This can help them better identify and isolate the variable they need to solve for.

Key Takeaway:
Say, "The strategies used for solving a one-variable equation, such as performing inverse operations on both sides to isolate the variable and maintain balance, can also be applied to solve for a specific variable in a multi-variable equation."

Guided Practice

2. Solve each equation for x. Show your thinking. Sample work shown.

Equation	Work	Solution
$2x + 10 = 2y$	$2x + 10 = 2y$ $2x = 2y - 10$ $x = y - 5$	$x = y - 5$
$\frac{x}{5} + 1 = y + 3$	$\frac{x}{5} + 1 = y + 3$ $\frac{x}{5} = y + 2$ $x = 5y + 10$	$x = 5y + 10$
$3x - 6y = 12$	$3x - 6y = 12$ $3x = 6y + 12$ $x = 2y + 4$	$x = 2y + 4$
$\frac{x}{4} + 2 = 3y$	$\frac{x}{4} + 2 = 3y$ $\frac{x}{4} = 3y - 2$ $x = 12y - 8$	$x = 12y - 8$

Check

Solve each equation for x. Show your thinking. Sample work shown.

1. $\frac{x}{3} + 5 = y$
   $\frac{x}{3} = y - 5$
   $x = 3y - 15$

2. $2x - 4y = 20$
   $2x = 4y + 20$
   $x = 2y + 10$

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Reflection

Ask:
- "What advice would you give a friend learning to solve equations with two variables?"
- "How did you overcome a hard problem today?"

Check: Recommended Next Steps

Almost there
If students need more support, consider providing them with lined paper to help them keep their work organized, aligning steps horizontally and vertically. This can help reduce errors during calculations.

Got it!
If students need more practice, have them solve the following equations for x.

- $\frac{x}{5} + 5 = 2y$
- $3x - 9y = 18$

Name ______________________________

Graphing Solutions of Two-Variable Linear Inequalities

ML 1.17

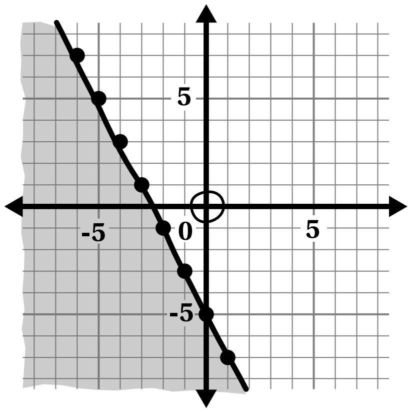
Modeled Review

Name: Jasmine

Graph the solutions of $2x + y \leq -5$.

2(0) + 0 ≤ -5
0 ≤ -5
false

(0,0) is not included in the solution, so I need to shade the other side.



Guided Practice

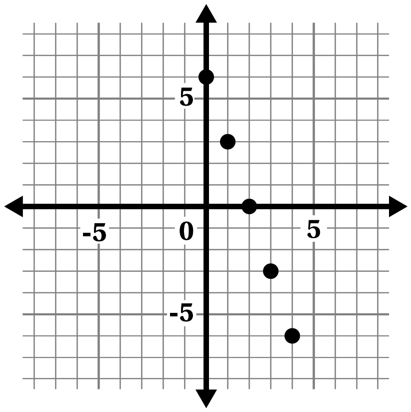
1. Complete the graph of $-3x - y > -6$.

$-3(0) -$ ______ > -6

$0 -$ ______ > -6

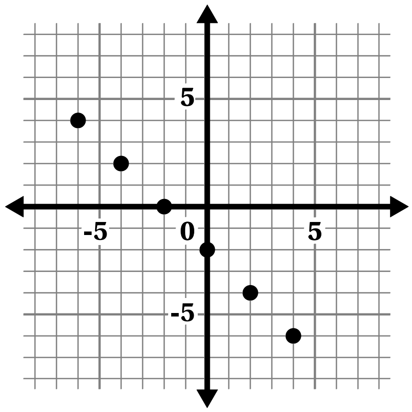
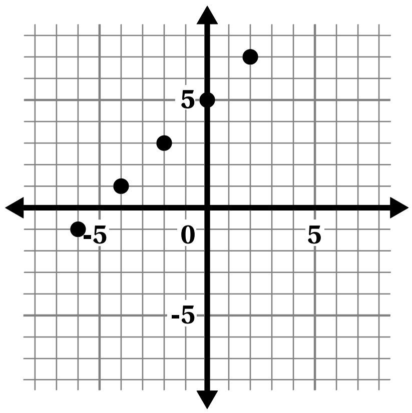
______ > -6

true or false



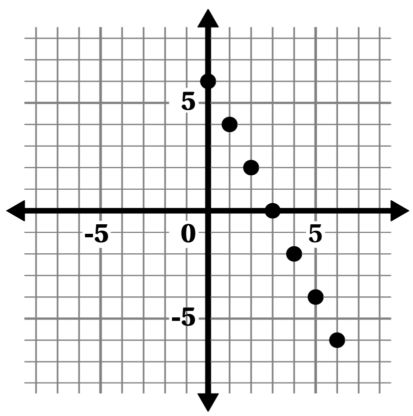
Guided Practice

2. Complete the graph for each two-variable linear inequality.

Inequality	Test point	Graph
$2x + 2y \geq -4$		
$x - y < -5$		

Check

Complete the graph of -$2x - y >$ -6.



Graphing Solutions of Two-Variable Linear Inequalities

Goal

Graph the solution set of two-variable linear inequalities on the coordinate plane.

Standard

MA.912.AR.2.8

Modeled Review

Point to Jasmine's work and **ask**:

- "Why did Jasmine use (0, 0) as a test point? When should (0, 0) not be used as a test point?"
- "Why did Jasmine not include (0, 0) when shading?"
- "How did Jasmine know the boundary line would be solid?"

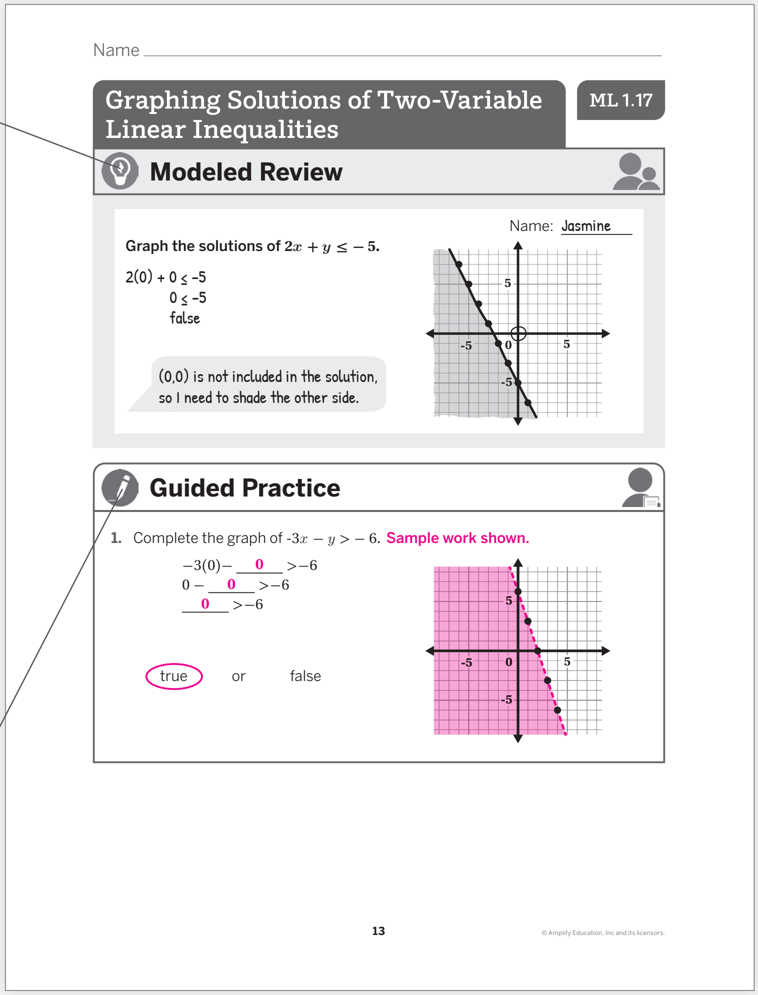
Reinforce the goal by saying, "When graphing solutions of two-variable inequalities, you can determine the solution region by substituting a test point, such as (0, 0), into the inequality to see if it is true."

Guided Practice

Focus students' attention on graphing the solution set of linear inequalities.

To scaffold their thinking, **say**:

- "First, choose a test point and substitute it into the inequality."
- "Then, determine if the result is true or false. If true, include the test point in the shaded region; if false, do not include it."
- "Last, draw a solid line if the inequality is ≤ or ≥, or a dashed line if the inequality is < or >."


Name

Graphing Solutions of Two-Variable Linear Inequalities

ML 1.17

Modeled Review

Name: Jasmine

Graph the solutions of $2x + y \le -5$.

$2(0) + 0 \le -5$

$0 \le -5$

false

(0,0) is not included in the solution, so I need to shade the other side.

Guided Practice

1. Complete the graph of $-3x - y > -6$. Sample work shown.

$-3(0) - \underline{0} > -6$

$0 - \underline{0} > -6$

$\underline{0} > -6$

true or false

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Vocabulary

If needed, share the meaning of the term with students.

boundary line: The line that separates the solution region of a linear inequality from non-solutions. A linear inequality (e.g., $y < 2x + 5$) has a boundary line that is represented symbolically by the corresponding equation (e.g., $y = 2x + 5$). A solid boundary line indicates that these points are included in the solution set (e.g., $y \le x$). A dashed boundary line indicates that they are not (e.g., $y < x$).

Graphing Solutions of Two-Variable Linear Inequalities

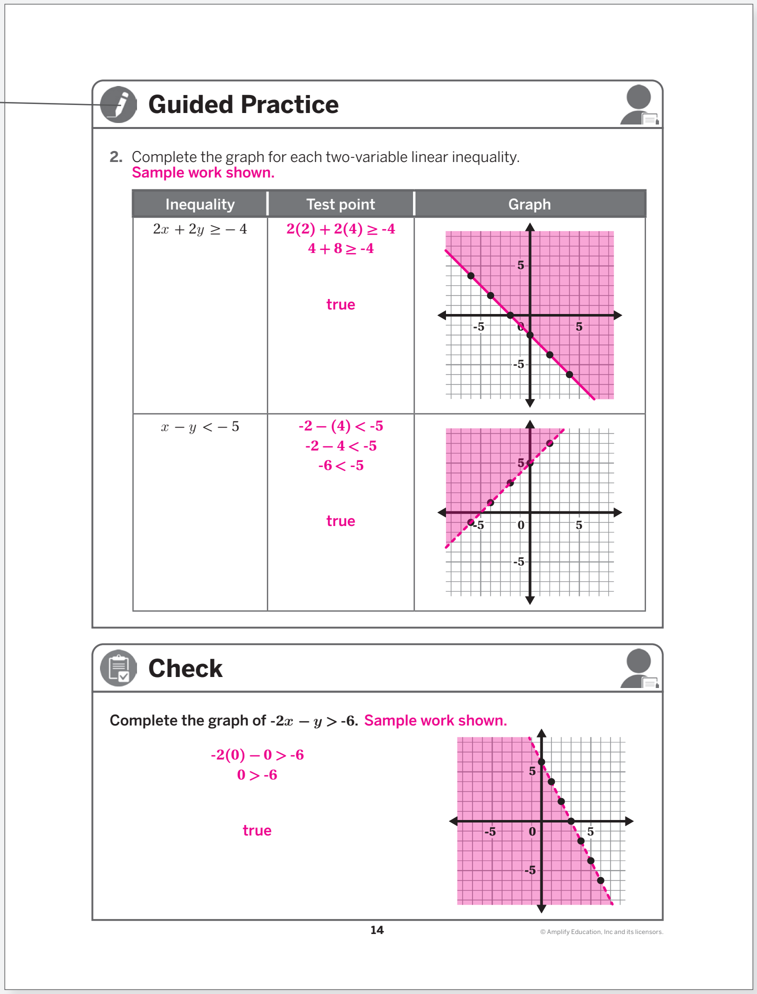
Guided Practice

A Guide student processing by providing questions they can ask themselves as they graph two-variable inequalities. For example:

- "What test point could you choose?"
- "Does the test point make the inequality true or false?"
- "Should the line be solid or dashed?"
- "How should you shade the graph?"

EL Use gestures to illustrate the meaning of the terms dashed and solid.

Key Takeaway:
Say, "The boundary line divides the plane into regions, and you can find the solution region by testing a point, such as (0, 0). The inequality symbol will indicate whether the boundary line is included."



Guided Practice

2. Complete the graph for each two-variable linear inequality.
Sample work shown.

Inequality	Test point	Graph
$2x + 2y \geq -4$	$2(2) + 2(4) \geq -4$ $4 + 8 \geq -4$ true	
$x - y < -5$	$-2 - (4) < -5$ $-2 - 4 < -5$ $-6 < -5$ true	

Check

Complete the graph of $-2x - y > -6$. Sample work shown.

$-2(0) - 0 > -6$
$0 > -6$
true

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Reflection

Ask:

- "Describe a strategy for graphing solutions of linear inequalities."
- "What makes sense? What is still confusing?"

Check: Recommended Next Steps

Almost there
If students need more support, consider demonstrating how to substitute a test point into the inequality. Then, determine if it is true or false by reading aloud and breaking down the meaning of each part of the inequality.

Got it!
If students need more practice, have them revisit Problem 2. Ask how the first graph in the table would change if the inequality were $2x + 2y < 4$.

Unit 2

Mini-Lessons

Name ____________________

Classifying Data

ML 2.02

Modeled Review

Name: Vin

Identify each data set as either univariate or bivariate data.

Data Set	Classification
Age and income of employees	Bivariate
Height of students in class	Univariate

I know that univariate data only contains one variable but bivariate data involves the relationship between two variables.

Guided Practice

1. A local ice cream shop recorded the number of ice cream cones sold each day for a month. On top of that, they also noted the daily temperature on each of those days to see if hotter days affected sales.

Which data collected is univariate? Which is bivariate?

Guided Practice

2. Identify each data set as either univariate or bivariate data.

Data Set	Classification
Number of steps taken in a day	Bivariate
Years of experience and salary	Univariate
Daily temperature in a city over a week's time	Bivariate
Age of cousins in a family	Bivariate

3. A coach records the free throw success rate for each player during a single practice. Later, the coach also compares each player's hours of practice per week with their success rate.

Which data collected is univariate? Which is bivariate?

Check

A group of students rated their favorite movie on a scale from 1 to 10. Then, they were also asked to record how many times they had seen the movie.

ML 2.02 Classifying Data

Goal

Identify whether a real world context represents univariate or bivariate data.

Standard

MA.912.DP.1.1

Materials

highlighter (optional)

Modeled Review

Point to Vin's work and **ask**:
- "What is a way to identify the difference between univariate and bivariate data?"
- "Is the data numerical or categorical? How can we tell?"

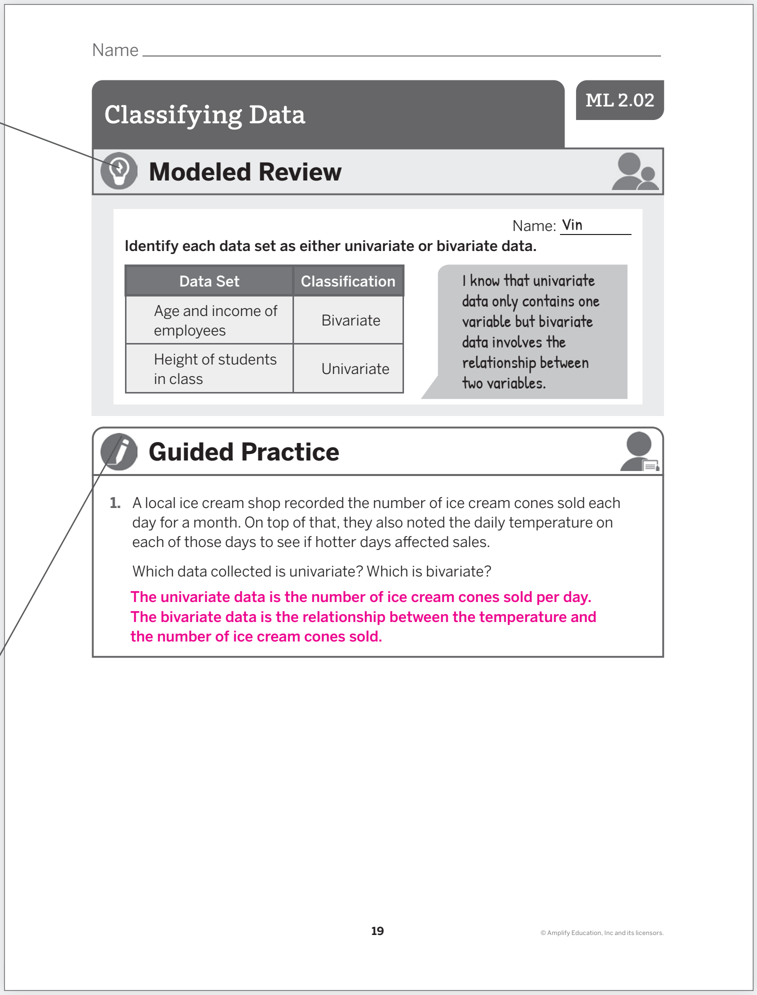
Reinforce Vin's thinking by saying "We can identify bivariate data by seeing if there is a relationship between two variables."

EL Model using a highlighter to identify the one (or two) variables identified in each problem.

Guided Practice

Focus students' attention on identifying whether a real world situation represents univariate or bivariate data.

Encourage students to identify what is changing in each context. Help guide their thoughts to recognize bivariate contexts as having two "things" that change in tandem.



Name ____________________

Classifying Data ML 2.02

Modeled Review

Name: Vin

Identify each data set as either univariate or bivariate data.

Data Set	Classification
Age and income of employees	Bivariate
Height of students in class	Univariate

I know that univariate data only contains one variable but bivariate data involves the relationship between two variables.

Guided Practice

1. A local ice cream shop recorded the number of ice cream cones sold each day for a month. On top of that, they also noted the daily temperature on each of those days to see if hotter days affected sales.

   Which data collected is univariate? Which is bivariate?

   The univariate data is the number of ice cream cones sold per day. The bivariate data is the relationship between the temperature and the number of ice cream cones sold.

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Vocabulary

If needed, share the meaning of the term with students.

bivariate data: Data that measures two characteristics of a population.

univariate data: Data that measures one characteristic of a population.

Guided Practice

A Co-construct a vocabulary guide/anchor chart with students who need additional support understanding the vocabulary terms involving classifying data sets.

Key Takeaway:
Say, "In order to differentiate between univariate and bivariate data, it is important to see if there is one characteristic or two characteristics being studied."

Guided Practice

2. Identify each data set as either univariate or bivariate data.

Data Set	Classification	
Number of steps taken in a day	**Univariate**	Bivariate
Years of experience and salary	Univariate	**Bivariate**
Daily temperature in a city over a week's time	**Univariate**	Bivariate
Age of cousins in a family	**Univariate**	Bivariate

3. A coach records the free throw success rate for each player during a single practice. Later, the coach also compares each player's hours of practice per week with their success rate.

Which data collected is univariate? Which is bivariate?

The univariate data is the number of ice cream cones sold per day. The bivariate data is the relationship between the temperature and the number of ice cream cones sold.

Check

A group of students rated their favorite movie on a scale from 1 to 10. Then, they were also asked to record how many times they had seen the movie.

The univariate data is the movie rating. The bivariate data is the relationship between the movie rating and the number of times it was watched.

20

Reflection

Ask:

- "How can you help a classmate understand the difference between univariate and bivariate data?"
- "What makes sense? What is still confusing?"

Check: Recommended Next Steps

Almost there

If students need more support, consider providing more examples of both types of data and modeling how you classify them as univariate or bivariate.

Got it!

If students need more practice, ask them to identify which part of the data is univariate and which part is bivariate in the context below:

A high school running club tracked the time it took each student to complete a 1-mile run. Later, they also recorded each student's age to see if younger students ran faster.

Name ______________________________

Estimating Population from a Sample Survey

ML 2.08

Modeled Review

Name: Mia

A city planner surveyed 40 homes and found that 28 recycle regularly. If 8400 homes used the recycling services last month, estimate how many homes are in the city.

I can set up a proportion comparing the part-to-whole results of the survey to the part-to-whole ratio of the households in the city.

$$\frac{28}{40} = \frac{8400}{x}$$

$$28x = 40 \cdot 8400$$

$$28x = 336{,}000$$

$$12{,}000 = x$$

It is estimated the city is composed of 12,000 homes.

Guided Practice

1. From a random sample of 72 college students, 54 said they drink coffee daily. Estimate how many students are on campus if about 6000 students drank coffee last month.

a. How many students of the sample drink coffee daily? ______________

b. How many total students were surveyed? ______________

c. How many total students drank coffee last month? ______________

d. Set up a proportion and solve.

e. About how many students are on campus? ______________

Guided Practice

2. At a local park, 15 out of 60 people surveyed were walking dogs. The park estimates it sees 225 people walking dogs on a busy Saturday. Estimate how many people visited the park that day. Show or explain your thinking.

3. A sample of 50 town residents showed that 8 of them have a gym membership. The gym has 320 members in total. Estimate the total population of the town. Show or explain your thinking.

4. In a sample of 60 people surveyed, 21 said they had visited the public library in the past month. The library's records show about 1,050 unique visitors last month. Estimate the population of the town. Show or explain your thinking.

Check

A survey of 20 students found that 6 are members of a school club. The school reports 180 total club members. Estimate the total student population. Show or explain your thinking.

ML 2.08 Estimating Population from a Sample Survey

Goal

Estimate a population total using data from a sample survey.

Standard

MA.912.DP.1.4

Modeled Review

Point to Mia's work and **ask**:
- "How did Mia know where to place the 8400 in the proportion?"
- "What did Mia do to solve the proportion for x?"

Reinforce Mia's thinking by saying "Determining the part-to-whole relationship of the sample and of the total population can help you set up a correct proportion."

EL Provide sentence frames to support students as they explain how they determined the part and whole relationships.
For example, "I know that _____ to _____ represents the part-to-whole ratio of the survey."

Guided Practice

Focus students' attention on finding the total number of students on campus.

Encourage students to identify what each number represents in the problem; Is it a part (or a whole) of the sample or of the total?

Name ______________________

Estimating Population from a Sample Survey ML 2.08

Modeled Review

Name: Mia

A city planner surveyed 40 homes and found that 28 recycle regularly. If 8400 homes used the recycling services last month, estimate how many homes are in the city.

I can set up a proportion comparing the part-to-whole results of the survey to the part-to-whole ratio of the households in the city.

$\frac{28}{40} = \frac{8400}{x}$

$28x = 40 \cdot 8400$

$28x = 336{,}000$

$12{,}000 = x$

It is estimated the city is composed of 12,000 homes.

Guided Practice

1. From a random sample of 72 college students, 54 said they drink coffee daily. Estimate how many students are on campus if about 6000 students drank coffee last month.

   a. How many students of the sample drink coffee daily? 54

   b. How many total students were surveyed? 72

   c. How many total students drank coffee last month? 6,000

   d. Set up a proportion and solve.

   Methods vary. $\frac{54}{72} = \frac{6000}{x}$

   $54x = 72(6000)$

   $x = 8000$

   e. About how many students are on campus? 8,000

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Vocabulary

If needed, share the meaning of the terms with students.

sample: subset of a larger population that is selected for the purpose of gathering data and making inferences about the larger group

survey: method of gathering information from a sample of a population.

ML 2.08 Estimating Population from a Sample Survey

Guided Practice

A Have the students create a table that can be used to organize the information in the problem.

Key Takeaway:
Say, "We can Estimate a population total using data from a sample survey using proportional reasoning."

Guided Practice

2. At a local park, 15 out of 60 people surveyed were walking dogs. The park estimates it sees 225 people walking dogs on a busy Saturday. Estimate how many people visited the park that day. Show or explain your thinking.

900 people. Methods vary.
$$\frac{15}{60} = \frac{225}{x}$$
$$15x = 60(225)$$
$$x = 900$$

3. A sample of 50 town residents showed that 8 of them have a gym membership. The gym has 320 members in total. Estimate the total population of the town. Show or explain your thinking.

2000 people. Methods vary.
$$\frac{8}{50} = \frac{320}{x}$$
$$8x = 50(320)$$
$$2000 = x$$

4. In a sample of 60 people surveyed, 21 said they had visited the public library in the past month. The library's records show about 1,050 unique visitors last month. Estimate the population of the town. Show or explain your thinking.

3675 people. Methods vary.
$$\frac{6}{21} = \frac{1050}{x}$$
$$6x = 21(1050)$$
$$3675 = x$$

Check

A survey of 20 students found that 6 are members of a school club. The school reports 180 total club members. Estimate the total student population. Show or explain your thinking.

3675 people. Methods vary.
$$\frac{6}{20} = \frac{180}{x}$$
$$6x = 20(180)$$
$$3675 = x$$

24

Reflection

Ask:
- "What are some other ways we can set up these problems?"
- "How do you know if your answers make sense?"

Check: Recommended Next Steps

Almost there
If students need more support, consider modeling your thinking for how you identify the part and whole parts of the sample and total representations in a problem.

Got it!
If students need more practice, ask them to solve the problem below with a partner and check their results.

In a sample of 60 people surveyed, 21 said they had visited the public library in the past month. The library's records show about 1,050 unique visitors last month. Estimate the population of the town.

Name ____________________

Calculating Values in Two-Way and Relative Frequency Tables

ML 2.12

Modeled Review

500 people were surveyed on their travel frequency and main mode of transportation: car or bicycle. The tables display the results.

two-way table

	Walk	Bike	Total
4th grade	300	50	350
5th grade	100	50	150
Total	400	100	500

relative frequency table

	Walk	Bike	Total
4th grade	$\frac{300}{350} = 86\%$	$\frac{50}{350} = 14\%$	100%
5th grade	$\frac{100}{150} = 67\%$	$\frac{50}{150} = 33\%$	100%

Guided Practice

For Problems 1–2, use the number bank to fill in the missing values in each two-way table and frequency table.

150	60%	40%	450	400	25%	40%	750

1. 1,600 students in 6th and 7th grade were asked about their favorite type of exercise: cardio or strength training. The tables display the results.

	Cardio	Strength	Total
6th grade	600		1,000
7th grade		450	600
Total	750	850	1,600

	Cardio	Strength	Total
6th grade	60%		100%
7th grade		75%	100%

2. 1,500 students in 11th and 12th grade were asked if they were a morning or night person. The tables display the results.

	Morning	Night	Total
11th grade		300	750
12th grade	300	450	750
Total	750		1,500

	Morning	Night	Total
11th grade		40%	100%
12th grade		60%	100%

Guided Practice

For Problems 3–4, 2,400 students in 9th and 10th grade were asked: Do you prefer to have lunch inside the cafeteria or outside in the courtyard?

3. Complete the two-way table.

	Inside	Outside	Total
9th grade	350		1,400
10th grade		700	
Total			2,400

4. Construct a relative frequency table showing the preferences for eating lunch inside or outside among 9th and 10th graders.

	Inside	Outside	Total
9th grade			100%
10th grade			100%

Check

For Problems 1–2, 3,200 students in 7th and 8th grade were asked: Do you prefer reading fiction or non-fiction books?

1. Complete the two-way table.

	Fiction	Non-Fiction	Total
7th grade		280	1,400
8th grade	1,080		
Total		1,000	3,200

2. Construct a relative frequency table showing the preferences for reading fiction or non-fiction among 7th and 8th graders.

	Inside	Outside	Total
7th grade			100%
8th grade			100%

ML 2.12

Calculating Values in Two-Way and Relative Frequency Tables

Goal

Identify whether a real world context represents univariate or bivariate data.

Standard

MA.912.DP.3.1

Materials

highlighter (optional)

Modeled Review

Point to the Modeled Review and **ask**:

- "How are two-way tables and relative frequency tables alike? Different?"
- "How is the information in the two-way table used to complete the relative frequency table?"
- "How can you use the row and column totals to check your work?"

Reinforce the goal by saying, "Categorical data can be displayed using two types of tables: two-way tables that present exact counts, and relative frequency tables that convert these counts into percentages."

Guided Practice

Focus students' attention on calculating the missing values in the two-way table and relative frequency table using the values given.

To scaffold their thinking, **say**:

- "How can you find the missing values in the two-way table?"
- "How can you use the row and column totals to check your work?"
- "What do you need to divide by to find the percentage?"

Name ______________________________

Calculating Values in Two-Way and Relative Frequency Tables

ML 2.12

Modeled Review

500 people were surveyed on their travel frequency and main mode of transportation: car or bicycle. The tables display the results.

two-way table

	Walk	Bike	Total
4th grade	300	50	350
5th grade	100	50	150
Total	400	100	500

relative frequency table

	Walk	Bike	Total
4th grade	$\frac{300}{350} = 86\%$	$\frac{50}{350} = 14\%$	100%
5th grade	$\frac{100}{150} = 67\%$	$\frac{50}{150} = 33\%$	100%

Guided Practice

For Problems 1–2, use the number bank to fill in the missing values in each two-way table and frequency table.

150	60%	40%	450	400	25%	40%	750

1. 1,600 students in 6th and 7th grade were asked about their favorite type of exercise: cardio or strength training. The tables display the results.

	Cardio	Strength	Total
6th grade	600	**400**	1,000
7th grade	**150**	450	600
Total	750	850	1,600

	Cardio	Strength	Total
6th grade	60%	**40%**	100%
7th grade	**25%**	75%	100%

2. 1,500 students in 11th and 12th grade were asked if they were a morning or night person. The tables display the results.

	Morning	Night	Total
11th grade	**450**	300	750
12th grade	300	450	750
Total	750	**750**	1,500

	Morning	Night	Total
11th grade	**60%**	40%	100%
12th grade	**40%**	60%	100%

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Vocabulary

If needed, share the meaning of the term with students.

relative frequency table: A type of two-way table used to compare data across two categorical variables. Relative frequency tables present the fraction or percent of the data that is in that category, instead of the actual number of data points.

ML 2.12 Calculating Values in Two-Way and Relative Frequency Tables

Guided Practice

A Guide student processing by providing questions they can ask themselves as they complete two-way and relative frequency tables. For example:

- “How can I check the two-way table?”
- “Which two values do I need to divide by?”
- “How can I convert the decimal into a percent?”

Note: Provide students with a calculator to efficiently compute the percentages for the relative frequency table.

Key Takeaway:
Say, “Categorical data can be displayed using two types of tables: two-way tables that present exact counts, and relative frequency tables that convert these counts into percentages.”

Guided Practice

For Problems 3–4, 2,400 students in 9th and 10th grade were asked: Do you prefer to have lunch inside the cafeteria or outside in the courtyard?

3. Complete the two-way table.

	Inside	Outside	Total
9th grade	350	**1,050**	1,400
10th grade	**300**	700	**1,000**
Total	**650**	**1,750**	2,400

4. Construct a relative frequency table showing the preferences for eating lunch inside or outside among 9th and 10th graders.

	Inside	Outside	Total
9th grade	**25%**	**75%**	100%
10th grade	**30%**	**70%**	100%

Check

For Problems 1–2, 3,200 students in 7th and 8th grade were asked: Do you prefer reading fiction or non-fiction books?

1. Complete the two-way table.

	Fiction	Non-Fiction	Total
7th grade	**1,120**	280	1,400
8th grade	1,080	**720**	**1,800**
Total	**2,200**	1,000	3,200

2. Construct a relative frequency table showing the preferences for reading fiction or non-fiction among 7th and 8th graders.

	Inside	Outside	Total
7th grade	**80%**	**20%**	100%
8th grade	**60%**	**40%**	100%

Reflection

Ask:

- “What strategies can be used to check the calculations in a two-way and relative frequency table?”
- “Reflect on your learning today. What were you most proud of?”

Check: Recommended Next Steps

Almost there
If students need more support, consider using a highlighter to annotate the counts and totals to make it easier to spot the missing values.

Got it!
If students need more practice, have them revisit Problems 3 and 4 and ask how the tables would change if the total number of students surveyed was 3,400 students (instead of 2,400 students).

Unit 3

Mini-Lessons

Name ____________________

Evaluating Function Notation

ML 3.02

Modeled Review

Function Notation
$f(x) = \underbrace{2x + 4}_{\text{output}}$ name of function (f of x) input
$f(3) = 2(3) + 4 = 10$ $f(\text{-}1) = 2(\text{-}1) + 4 = 2$

Guided Practice

1. Let $f(x)= 2x + 3$. Calculate the value of each expression. The first value is already completed.

Expression	Evaluate	Value
$f(0)$	$2(0) + 3$	3
$f(1)$	$2(1) + 3$	
$f(2)$	$2(\_\_\_\_) + 3$	
$f(3)$	$2(\_\_\_\_) + 3$	

2. Let $f(x) = 1.5x + 4$. Calculate the value of each expression.

Expression	Evaluate	Value
$f(2)$	$1.5(2) + 4$	
$f(4)$	$1.5(\_\_\_\_) + 4$	
$f(6)$	$1.5(\_\_\_\_) + 4$	
$f(8)$	$1.5(\_\_\_\_) + 4$	

Guided Practice

3. A baking class charges a flat fee of $\$10$ plus $\$4$ per participant. The function $c(p) = 4p + 10$ represents the total cost for p participants. Calculate the cost for 3 participants.

answer: ________

4. A pizza place charges $\$10$ per pizza and $\$1.50$ for each topping. The function $p(t) = 1.50t + 10$ represents the total cost for t toppings. Calculate the cost of a 4-topping pizza.

answer: ________

5. A parking lot charges $\$5$ per day. The function $p(d) = 5d$ represents the total cost for d days. Determine the total cost for 15 days.

answer: ________

6. A book club has an initial fee of $\$10$ and a monthly subscription cost of $\$11.50$. The function $c(m) = 11.50m + 10$ represents the total cost for m months. Determine the total cost after 5 months.

answer: ________

Check

A magazine subscription has an initial fee of $\$5$ and a monthly delivery fee of $\$6$. The function $s(m) = 5m + 6$ represents the total cost for m months. Determine the total cost after 6 months.

answer: ________

Goal

Evaluate functions written in function notation.

Standard

MA.912.F.1.2

Materials

coloring tools (optional)

Modeled Review

Point to the Modeled Review and **ask**:

- "How is function notation similar to an equation? Different?"
- "How do you identify each input?"
- "How do you use the input to calculate the output?"

Reinforce the goal by saying, "When evaluating a function written in function notation, substitute the input value into the equation given and calculate."

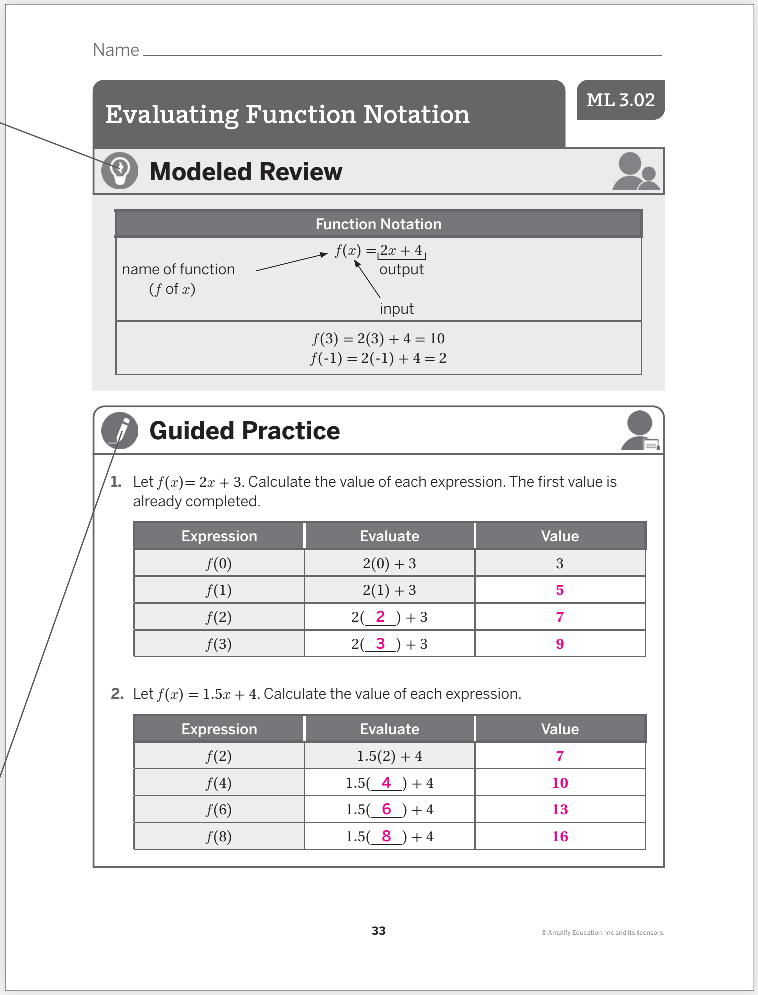
EL Use gestures to show input and output by miming the act of placing the input into a container (your cupped hands) and then extending your hands forward to reveal the calculated output.

Guided Practice

Focus students' attention on evaluating each function.

To scaffold their thinking, **say**:

- "First, identify the function."
- "Then, substitute each input value from the table into the equation."
- "Last, calculate the output."



Name

Evaluating Function Notation

ML 3.02

Modeled Review

Function Notation

name of function (f of x) → $f(x) = 2x + 4$ output, input

$f(3) = 2(3) + 4 = 10$

$f(-1) = 2(-1) + 4 = 2$

Guided Practice

1. Let $f(x) = 2x + 3$. Calculate the value of each expression. The first value is already completed.

Expression	Evaluate	Value
$f(0)$	$2(0) + 3$	3
$f(1)$	$2(1) + 3$	5
$f(2)$	$2(\underline{2}) + 3$	7
$f(3)$	$2(\underline{3}) + 3$	9

2. Let $f(x) = 1.5x + 4$. Calculate the value of each expression.

Expression	Evaluate	Value
$f(2)$	$1.5(2) + 4$	7
$f(4)$	$1.5(\underline{4}) + 4$	10
$f(6)$	$1.5(\underline{6}) + 4$	13
$f(8)$	$1.5(\underline{8}) + 4$	16

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Vocabulary

If needed, share the meaning of the term with students.

function notation: A way of writing about the inputs and outputs of a function.

Guided Practice

A Guide student processing by providing questions students can ask themselves as they evaluate function notation. For example:

- "What function is given?"
- "What does it represent?"
- "What value do I need to calculate?"

Key Takeaway:
Say, "Evaluating functions in function notation involves substituting each given input into the equation to calculate the corresponding output."

Guided Practice

3. A baking class charges a flat fee of $10 plus $4 per participant. The function $c(p) = 4p + 10$ represents the total cost for p participants. Calculate the cost for 3 participants.

$c(3) = 4(3) + 10$
$c(3) = 22$

answer: $22

4. A pizza place charges $10 per pizza and $1.50 for each topping. The function $p(t) = 1.50t + 10$ represents the total cost for t toppings. Calculate the cost of a 4-topping pizza.

$p(4) = 1.50(4) + 10$
$p(4) = 16$

answer: $16

5. A parking lot charges $5 per day. The function $p(d) = 5d$ represents the total cost for d days. Determine the total cost for 15 days.

$p(15) = 5(15)$
$p(15) = 75$

answer: $75

6. A book club has an initial fee of $10 and a monthly subscription cost of $11.50. The function $c(m) = 11.50m + 10$ represents the total cost for m months. Determine the total cost after 5 months.

$c(5) = 11.50(5) + 10$
$c(5) = 67.50$

answer: $67.50

Check

A magazine subscription has an initial fee of $5 and a monthly delivery fee of $6. The function $s(m) = 5m + 6$ represents the total cost for m months. Determine the total cost after 6 months.

$s(6) = 5(6) + 6$
$s(6) = 36$

answer: $36

34

Reflection

Ask:

- "What strategy did you find most helpful when evaluating functions?"
- "How does what you learned today connect to your prior learning?"

Check: Recommended Next Steps

Almost there
If students need more support, consider using a coloring tool to help them annotate the function. Highlighting the input, x, and where it should be substituted into the equation to calculate the output.

Got it!
If students need more practice, have them solve the following problem:

A subscription to a streaming service has a one-time setup fee of $12 and a monthly charge of $8. The function $s(m) = 8m + 12$ represents the total cost for m months. Determine the total cost after 6 months.

Name ____________________

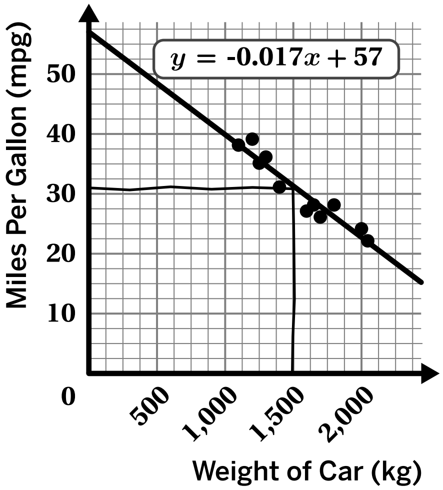
Interpreting Slope and y-intercept in Context

ML 3.04

Modeled Review

Name: Evan

For Problems 1–2, use the scatterplot to answer the problems.



1. What does the slope mean for the relationship between fuel economy (mpg) and weight of car (kg)?

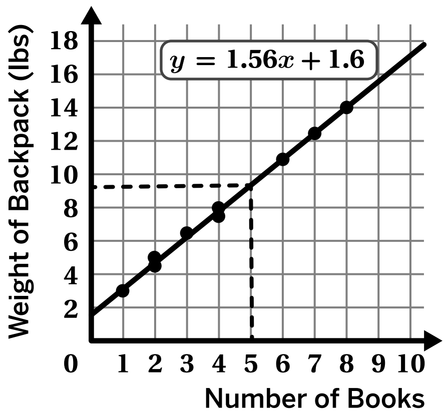
The slope means that as the mass of a car increases by 1 kg, the fuel economy tends to decrease by about 0.017 miles per gallon.

2. A car weighs $1{,}500$ kg. Predict the fuel economy (mpg) of the car.

31.5 mpg

Guided Practice

For Problems 1–2, use the scatterplot to answer the problems.



1. What does the slope mean for the relationship between the weight of the backpack (lbs) and the number of books?

A. As the number of books increases by 1.56, the weight increases by about 1 pound.

B. As the number of books increases by 1, the weight increases by about 1.56 pounds.

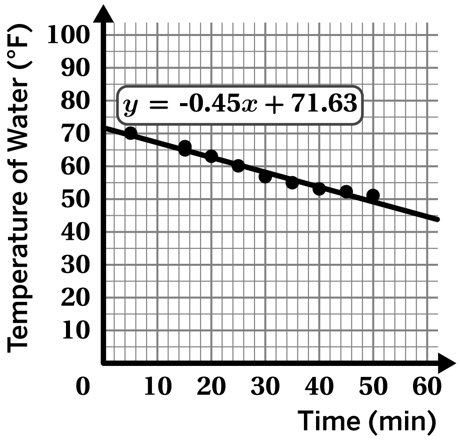
2. A backpack has 5 books. Predict the weight (lbs) of the backpack.

A. 10 lbs **B.** 9.4 lbs **C.** 8 lbs **D.** 10.5 lbs

Guided Practice

For Problems 3–5, use the scatter plot to answer the problems.

3. What does the slope mean for the relationship between the temperature of water (°F) and the time that has passed?



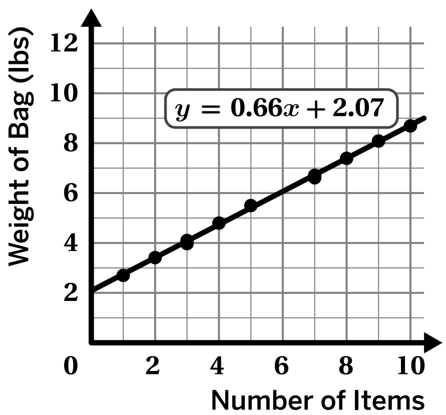
4. What does the y-intercept mean in this context?

5. 10 minutes have passed. Predict the temperature (°F) of the water.

Check

For Problems 1–2, use the graph to answer the problems.

1. What does the slope mean for the relationship between the weight of the bag (lbs) and the number of items?



2. 6 items need to be packed. Predict the weight (lbs) of the bag.

ML 3.04 Interpreting Slope and y-intercept in Context

Goal

Interpret the slope and y-intercept for a linear model in context.

Standard

MA.912.AR.2.2

Modeled Review

Point to Evan's work and **ask**:

- "How did Evan know that the fuel economy is decreasing with weight and not increasing?"
- "How did Evan estimate the fuel economy of a car that weighs 1,500kg?"

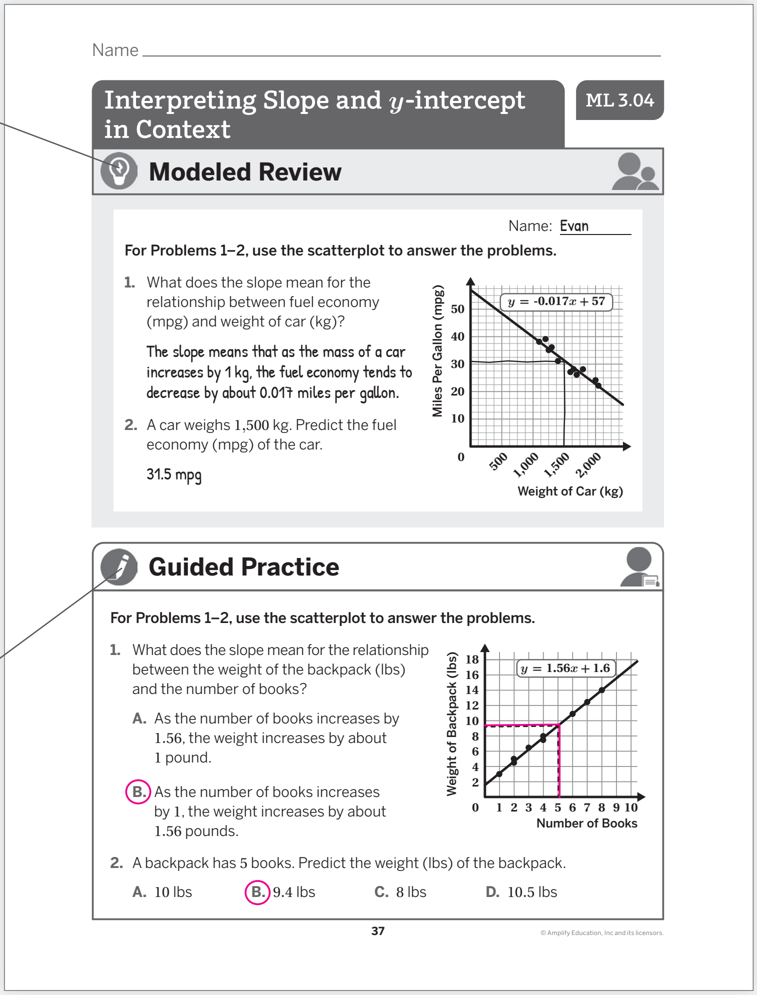
Reinforce Evan's thinking by saying, "When interpreting the slope and y-intercept in context, use the x-axis and y-axis labels to make sense of the data."

Guided Practice

Focus students' attention on using the attributes of the graph to answer the problems.

To scaffold their thinking, **ask**:

- "What are the labels for the x-axis and y-axis?"
- "What is the slope? Is it positive or negative?"
- "What value do we need to estimate? Is it an x-coordinate or y-coordinate?"



Name

Interpreting Slope and y-intercept in Context ML 3.04

Modeled Review

Name: Evan

For Problems 1–2, use the scatterplot to answer the problems.

1. What does the slope mean for the relationship between fuel economy (mpg) and weight of car (kg)?

   The slope means that as the mass of a car increases by 1 kg, the fuel economy tends to decrease by about 0.017 miles per gallon.

2. A car weighs 1,500 kg. Predict the fuel economy (mpg) of the car.

   31.5 mpg

Guided Practice

For Problems 1–2, use the scatterplot to answer the problems.

1. What does the slope mean for the relationship between the weight of the backpack (lbs) and the number of books?

   A. As the number of books increases by 1.56, the weight increases by about 1 pound.

   B. As the number of books increases by 1, the weight increases by about 1.56 pounds.

2. A backpack has 5 books. Predict the weight (lbs) of the backpack.

   A. 10 lbs B. 9.4 lbs C. 8 lbs D. 10.5 lbs

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Vocabulary

If needed, share the meaning of the terms with students.

line of fit: A line that shows, or models, the general direction or trend of a group of points in a data set. We can use linear models to make predictions about values related to a given data set.

slope: A number that describes the direction and steepness of a line. Slope represents the amount that y changes when x increases by 1.

y-intercept: A point where the graph of an equation or function crosses the y-axis or when $x = 0$.

ML 3.04 Interpreting Slope and y-intercept in Context

Guided Practice

A To support students in making predictions for a specific value, model annotating the graph. For example, in Problem 5, annotate a vertical line from 10 on the x-axis up to the line of fit. Then, draw a horizontal line from this point to determine its corresponding value on the y-axis.

Key Takeaway:
Say, "A line of fit with a strong correlation coefficient can be used to understand and predict the relationship between two variables."

Guided Practice

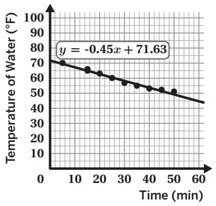
For Problems 3–5, use the scatter plot to answer the problems.
Sample responses shown.

3. What does the slope mean for the relationship between the temperature of water (°F) and the time that has passed?

   As the amount of time increases by 1 minute, the temperature of the water decreases by about 0.45°F.

4. What does the y-intercept mean in this context?

   The temperature of the water started at about 71.63°F.



5. 10 minutes have passed. Predict the temperature (°F) of the water.

   About 67.13 degrees (responses between 65 and 69 are considered correct).

Check

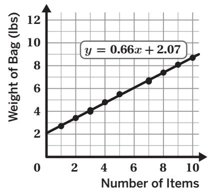
For Problems 1–2, use the graph to answer the problems.
Sample responses shown.

1. What does the slope mean for the relationship between the weight of the bag (lbs) and the number of items?

   As the number of items increases by 1, the weight increases by about 0.66 pounds.

2. 6 items need to be packed. Predict the weight (lbs) of the bag.

   About 6.03 pounds (responses between 5.8 and 6.2 are considered correct).



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Reflection

Ask:

- "How can a line help you make predictions about data?"
- "How was the lesson helpful to you today?"

Check: Recommended Next Steps

Almost there
If students need more support, consider expressing the slope as a fraction with a denominator of 1. This approach helps students better understand how to interpret the slope as a ratio of changes.

Got it!
If students need more practice, have them revisit the scatter plot in Problems 1–2. Ask them to predict the weight of the backpack with 9 books, and explain the meaning of the y-intercept in this context.

Name ___

Writing Domain and Range With Inequalities

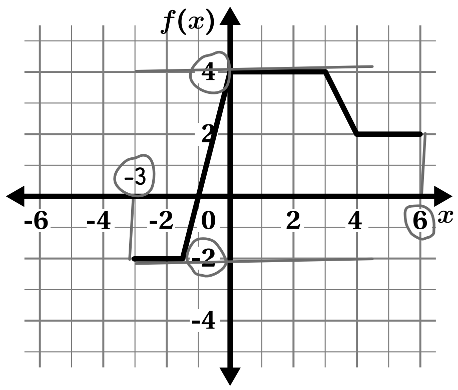
ML 3.10

Modeled Review

Name: Anushka

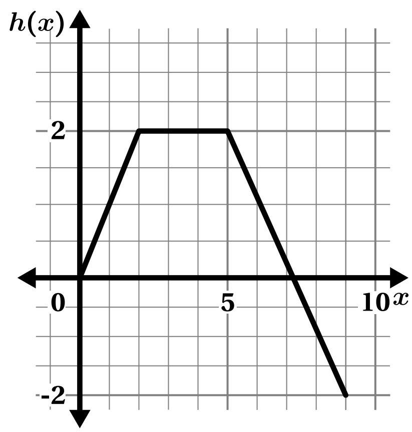
Select the domain and range that match the graph of $f(x)$.

- [x] $-3 \leq x \leq 6$
- [x] $-2 \leq f(x) \leq 4$
- [ ] $-2 \leq x \leq 4$
- [ ] $-3 \leq f(x) \leq 6$



To find the domain, I check from left to right across the x-axis. To find the range, I look up and down along the y-axis.

Guided Practice



1. According to the graph, which inequality describes the domain of $h(x)$?

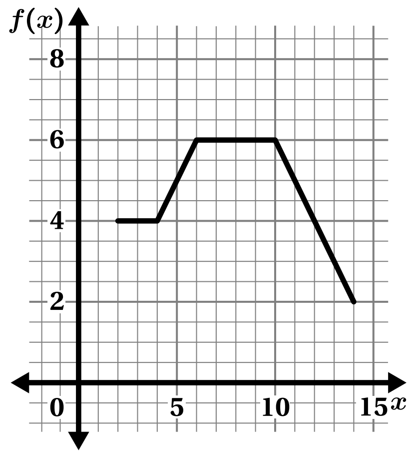
A. $0 \leq h(x) \leq 9$ **B.** $0 \leq x \leq 9$

2. According to the graph, which inequality describes the range of $h(x)$?

A. $-2 \leq x \leq 2$ **B.** $-2 \leq h(x) \leq 2$

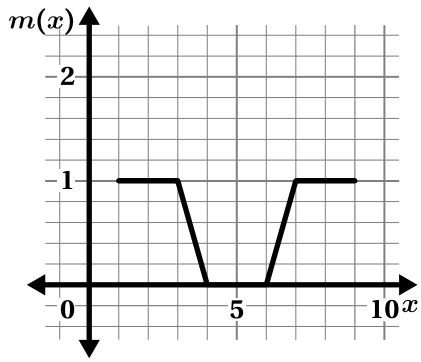
Guided Practice

3. Select the domain and range that match the graph of $f(x)$.



- ☐ $2 \leq x \leq 6$
- ☐ $2 \leq f(x) \leq 6$
- ☐ $2 \leq x \leq 14$
- ☐ $2 \leq f(x) \leq 14$

4. Select the domain and range that match the graph of $m(x)$.

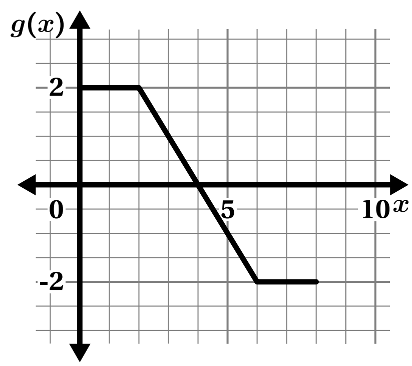


- ☐ $1 \leq x \leq 9$
- ☐ $1 \leq m(x) \leq 9$
- ☐ $0 \leq x \leq 1$
- ☐ $0 \leq m(x) \leq 1$

Check

Select the domain and range that match the graph of $g(x)$.

- ☐ $-2 \leq x \leq 2$
- ☐ $0 \leq g(x) \leq 8$
- ☐ $0 \leq x \leq 8$
- ☐ $-2 \leq g(x) \leq 2$



ML 3.10 Writing Domain and Range With Inequalities

Goal

Determine the domain and range of a function using inequalities.

Standard

MA.912.F.1.6

Materials

coloring tools (optional)

Modeled Review

Point to Anushka's work and **ask**:

- "Where on the graph did Anushka find the domain and range?"
- "Why are all the inequality signs less than or equal to?"
- "Do x-values represent the domain or range? Do y-values represent the domain or range?"
- "How can Anushka visually check her work?"

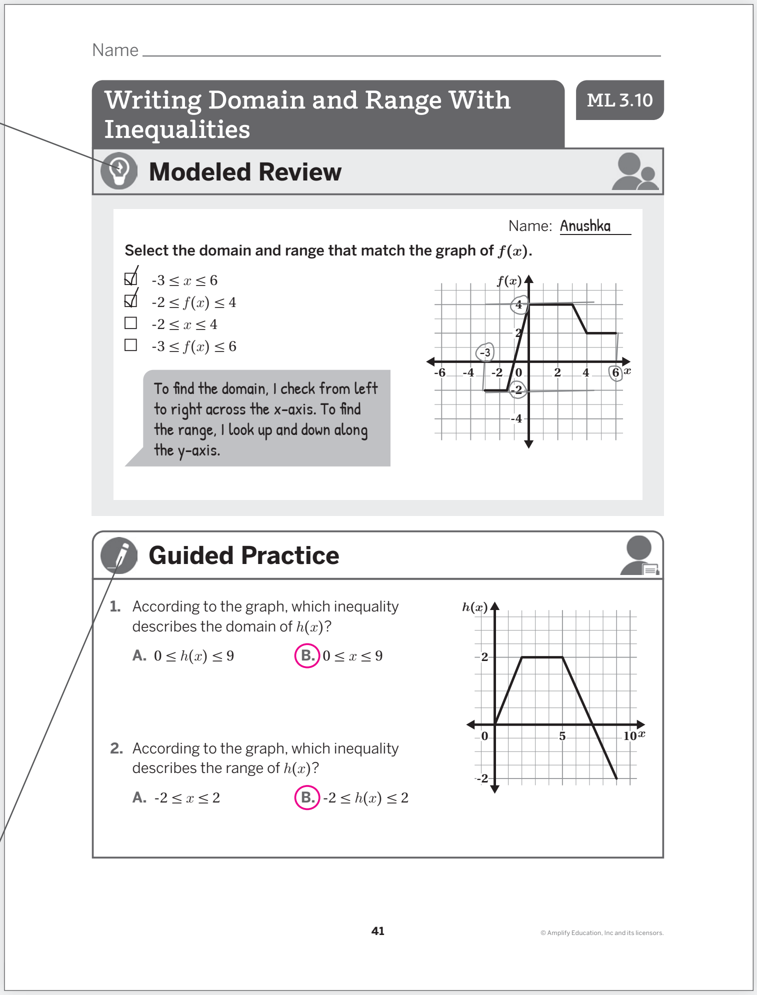
Reinforce the goal by saying, "You can use inequalities to describe a function's domain, which is all the possible x-values for which the function can be evaluated. You can also use inequalities to describe a function's range, which is all the possible y-values the function can have."

Guided Practice

Focus students' attention on determining the domain and range for the function.

To scaffold their thinking, **ask**:

- "To find the domain, what is the least x-value? The greatest x-value?"
- "To find the range, what is the least y-value? The greatest y-value?"
- "How can you check your work?"



Name

Writing Domain and Range With Inequalities

ML 3.10

Modeled Review

Name: Anushka

Select the domain and range that match the graph of $f(x)$.

- ☑ $-3 \le x \le 6$
- ☑ $-2 \le f(x) \le 4$
- ☐ $-2 \le x \le 4$
- ☐ $-3 \le f(x) \le 6$

To find the domain, I check from left to right across the x-axis. To find the range, I look up and down along the y-axis.

Guided Practice

1. According to the graph, which inequality describes the domain of $h(x)$?

   A. $0 \le h(x) \le 9$ B. $0 \le x \le 9$

2. According to the graph, which inequality describes the range of $h(x)$?

   A. $-2 \le x \le 2$ B. $-2 \le h(x) \le 2$

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Vocabulary

If needed, share the meaning of the terms with students.

domain: The set of all possible input values for a function or relation. The domain can be described in words or as an inequality.

range: The set of all possible output values for a function or relation. The range can be described in words or as an inequality.

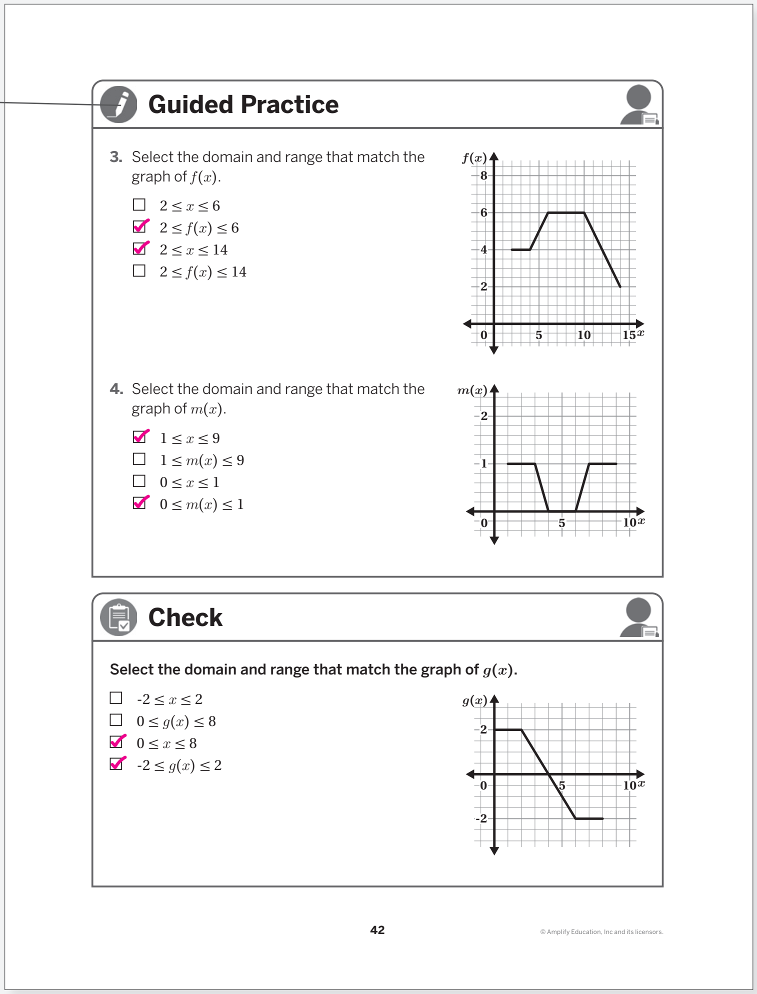
ML 3.10 Writing Domain and Range With Inequalities

Guided Practice

(A) Consider annotating the graph by using one color to mark the starting and ending points on the x-axis. Use another color to mark the endpoints on the y-axis.

(EL) Use gestures to represent the domain and range. For the domain, extend your hands horizontally to indicate the left-to-right span of the x-values. For the range, move your hands vertically to show the up-and-down span of the y-values.

Key Takeaway:
Say, "Inequalities can describe a function's domain, representing all possible x-values, and its range, representing all possible y-values."



Guided Practice

3. Select the domain and range that match the graph of $f(x)$.
- [ ] $2 \le x \le 6$
- [x] $2 \le f(x) \le 6$
- [x] $2 \le x \le 14$
- [ ] $2 \le f(x) \le 14$

4. Select the domain and range that match the graph of $m(x)$.
- [x] $1 \le x \le 9$
- [ ] $1 \le m(x) \le 9$
- [ ] $0 \le x \le 1$
- [x] $0 \le m(x) \le 1$

Check

Select the domain and range that match the graph of $g(x)$.
- [ ] $-2 \le x \le 2$
- [ ] $0 \le g(x) \le 8$
- [x] $0 \le x \le 8$
- [x] $-2 \le g(x) \le 2$

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Reflection

Ask:
- "How is the domain of a function different from its range?"
- "What is something you weren't sure about at the start of the lesson but understand better now?"

Check: Recommended Next Steps

Almost there	Got it!
If students need more support, review the definition for domain, which is the set of all possible inputs. Ask, "What are the inputs in this situation?" The range is the set of all possible outputs. Ask, "What are the outputs in this situation?"	If students need more practice, have them revisit the Check and ask why $-2 \le x \le 2$ and $0 \le g(x) \le 8$ are incorrect answers.

Name ______________________________

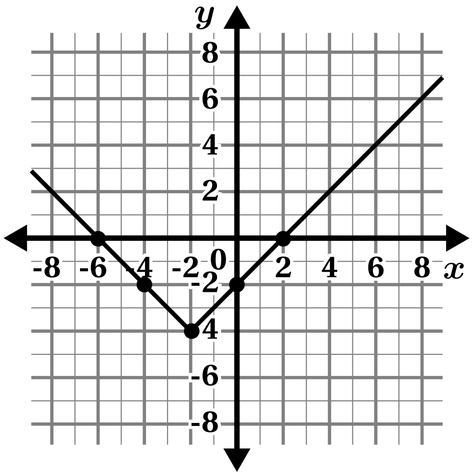
Absolute Value Functions

ML 3.12

Modeled Review

Name: Sandy

What is the equation of the absolute value function graphed below?

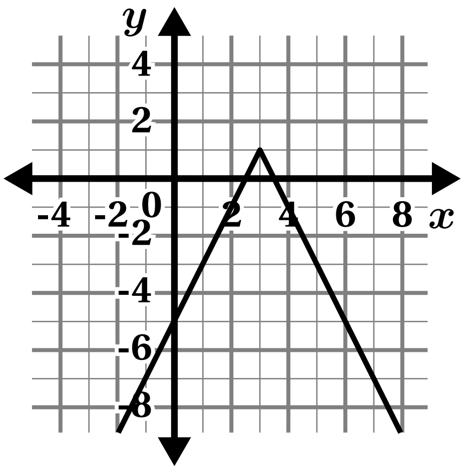


I know the equation of an absolute value function is f(x) = |x – h| +k with (h, k) representing the vertex of the graph.

Therefore, since the vertex of this graph is at (-2, -4), the equation of this graph is y = |x – (-2)| + (-4) or y = |x + 2| – 4.

Guided Practice

Which statement is true about the a-term of the equation of the function graphed below ?



A. $a=-2$

B. $a=-1$

C. $a=-\frac{1}{2}$

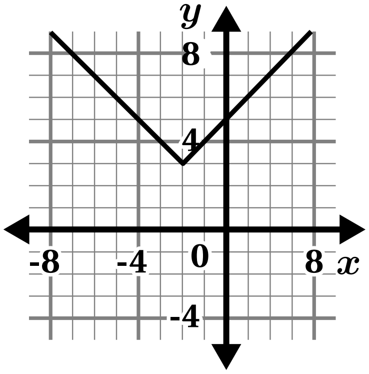
D. $a=1$

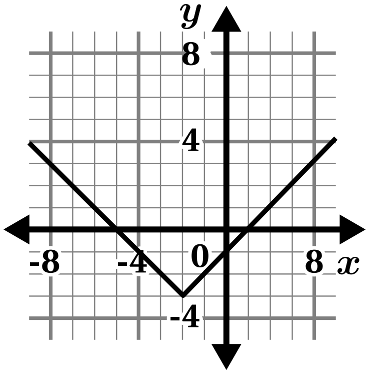
E. $a=2$

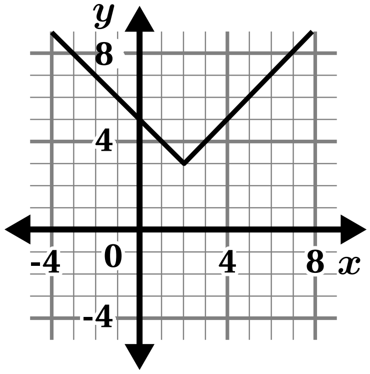
Guided Practice

2. Match each function with its graph.

A. $y = |x - 2| + 3$ B. $y = |x + 2| + 3$ C. $y = |x + 2| - 3$





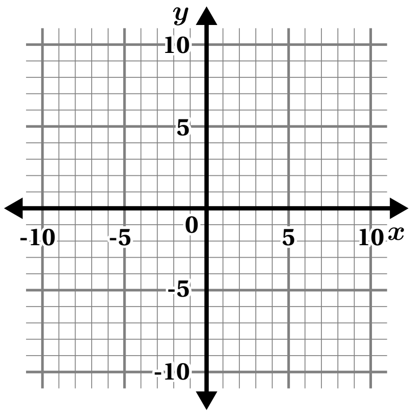


______ ______ ______

3. Graph the function below. A table is provided to help with your thinking.

$f(x) = |x + 4| - 2$

x	$f(x)$



Check

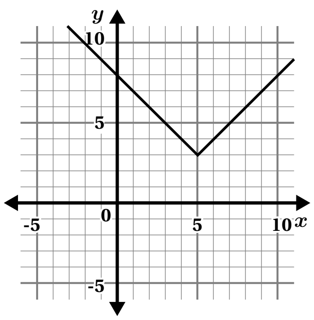
What is the equation of the function $g(x)$ graphed below?

A. $g(x) = |x + 5| + 3$

B. $g(x) = |x + 5| - 3$

C. $g(x) = |x - 5| - 3$

D. $g(x) = |x - 5| + 3$



ML 3.12 Absolute Value Functions

Goal

Graph absolute value functions and identify their key features.

Standard

MA.912.AR.4.3

Materials

Blank tables

Modeled Review

Point to Sandy's work and **ask**:

- "What part of the graph did Sandy identify first to help her write the equation of the function?"
- "Why does the function have $|x + 2|$ when the x-value of the vertex is -2?"

Reinforce Sandy's thinking by saying "if you know the vertex of an absolute value function, you can substitute the values of x for h and y for k into the equation $f(x) = |x - h| + k$."

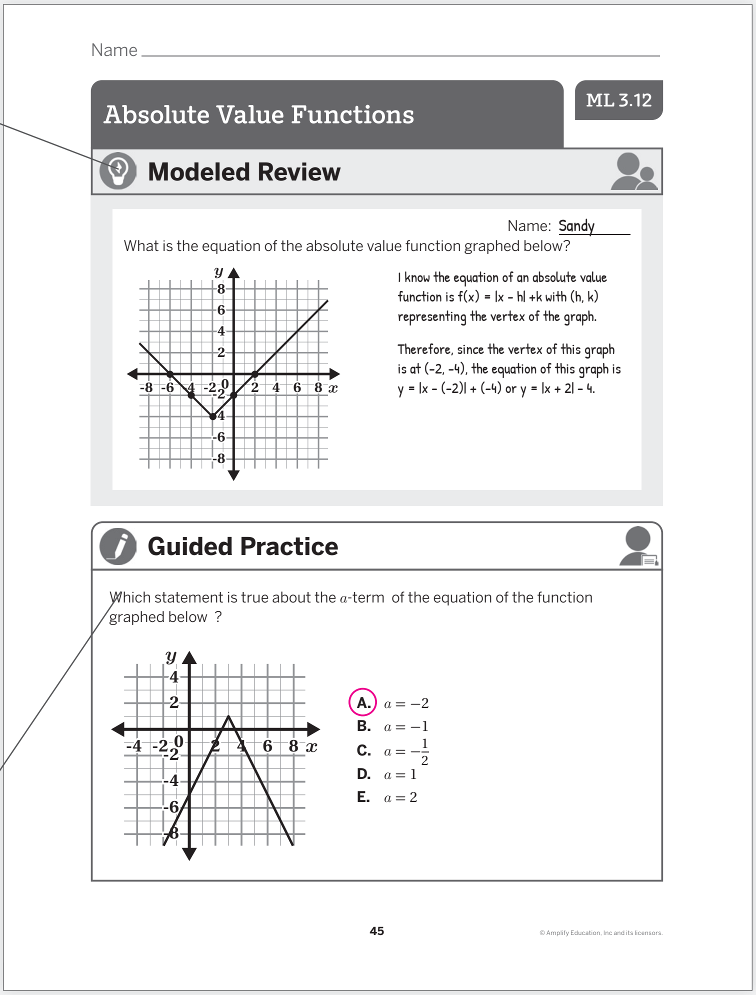
EL Display the graph without sharing the question and ask students to tell you what they know about the graph and/or the equation of the function.

Guided Practice

Focus students' attention on using direction and vertical stretch of the graph to identify the value of a.

To scaffold their thinking, **ask**:

- "What direction is the graph facing?"
- "What is the slope from the vertex to the next point on the graph, in either direction?"



Name

Absolute Value Functions

ML 3.12

Modeled Review

Name: Sandy

What is the equation of the absolute value function graphed below?

I know the equation of an absolute value function is f(x) = |x - h| +k with (h, k) representing the vertex of the graph.

Therefore, since the vertex of this graph is at (-2, -4), the equation of this graph is y = |x - (-2)| + (-4) or y = |x + 2| - 4.

Guided Practice

Which statement is true about the a-term of the equation of the function graphed below ?

A. $a = -2$
B. $a = -1$
C. $a = -\frac{1}{2}$
D. $a = 1$
E. $a = 2$

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Vocabulary

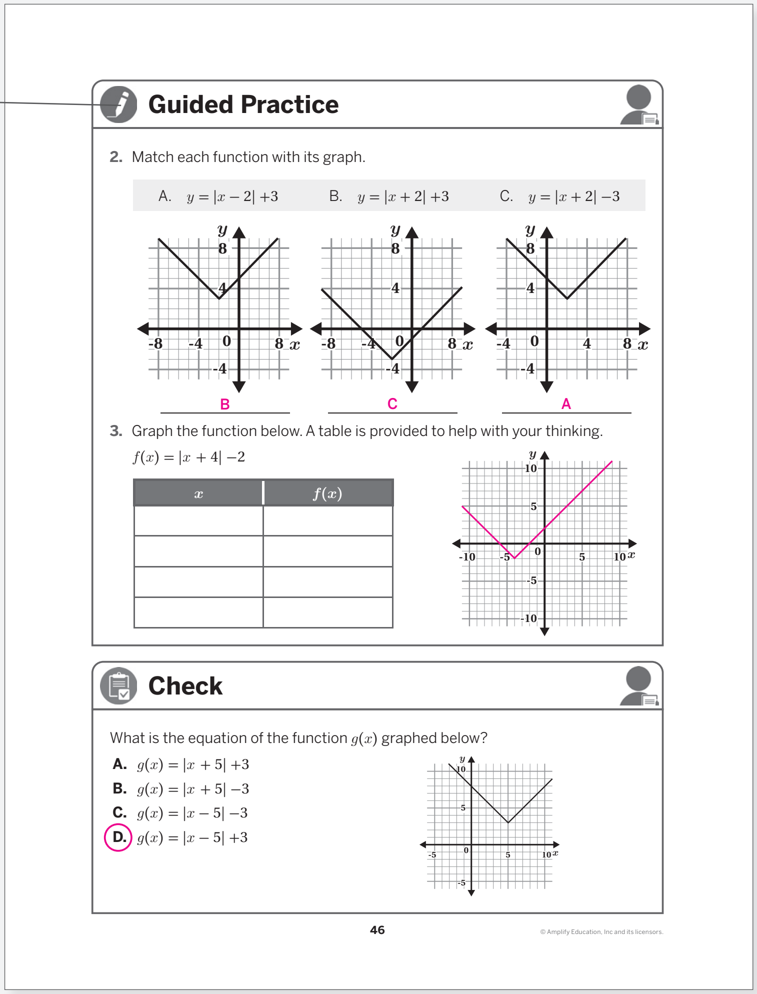
If needed, share the meaning of the term with students.

absolute value function: A function of the form $f(x) = a|x - h| + k$, where a represents the direction and vertical stretch of the function and (h, k) represents its vertex.

Guided Practice

A First, guide students to start by identifying the vertex of each graph. Second, have them identify the vertex of each equation. Finally, have them match the vertex with each other. Provide extra blank tables to use for graphing functions.

Key Takeaway:
Say, "Using key features of the function, such as its vertex and the direction/ slope, can help you graph the function."



Guided Practice

2. Match each function with its graph.

A. $y = |x - 2| + 3$ B. $y = |x + 2| + 3$ C. $y = |x + 2| - 3$

B C A

3. Graph the function below. A table is provided to help with your thinking.

$f(x) = |x + 4| - 2$

x	$f(x)$

Check

What is the equation of the function $g(x)$ graphed below?

A. $g(x) = |x + 5| + 3$
B. $g(x) = |x + 5| - 3$
C. $g(x) = |x - 5| - 3$
D. $g(x) = |x - 5| + 3$

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Reflection

Ask:

- "How can you determine the vertex of an absolute value function from its equation?"
- "How does what you learned today connect to your prior learning?"

Check: Recommended Next Steps

Almost there
If students need more support, consider modeling (or having a student) model how to identify the equation of an absolute value from its graph.

Got it!
If students need more practice, ask them to graph the function $y = -|x - 3| + 6$.

Unit 4

Mini-Lessons

Name ____________________

Solving Systems of Linear Equations by Substitution

ML 4.03

Modeled Review

Name: Ama

Determine the solution to this system of equations:

$y = 4x - 45$
$6x + 2y = 78$

$x =$ 12 $y =$ 3

$6x + 2y = 78$
$6x + 2(4x - 45) = 78$
$6x + 8x - 90 = 78$
$14x - 90 = 78$
$14x = 168$
$x = 12$

$y = 4x - 45$
$y = 4(12) - 45$
$y = 48 - 45$
$y = 3$

Guided Practice

1. Determine the solution to this system of equations by following each step.

$$2x + 3y = 9$$
$$y = x - 2$$

System of Equations	Moves
$2x + 3y = 9$ $\quad y = x - 2$	**Step 1:** Substitute the isolated expression, $x - 2$, in for y.
$2x + 3(\_\_\_\_) = 9$	**Step 2:** Distribute 3 on the left side.
$2x + \_\_\_\_ = 9$	**Step 3:** Combine like terms on the left side.
$\_\_\_\_ = 9$	**Step 4:** Add 6 on each side.
$\_\_\_\_ = \_\_\_\_$	**Step 5:** Divide by 5 on each side.
$x = \_\_\_\_$	
$y = x - 2$ $y = \_\_\_\_ - 2$ $y = \_\_\_\_$	**Step 6:** Substitute the value of x into the equation $y = x - 2$ and solve for y.

Guided Practice

For Problems 2–5, determine the solution to the system of equations.

2. $y = 2x - 2$
$y = x - 1$

$x =$ _____ $y =$ _____

3. $3x + 2y = 10$
$y = x - 5$

$x =$ _____ $y =$ _____

4. $y = 4x + 2$
$y = 3x - 6$

$x =$ _____ $y =$ _____

5. $2x - 3y = 16$
$y = 2x$

$x =$ _____ $y =$ _____

Check

Determine the solution to this system of equations.

$2x + 5y = 18$
$y = 2x - 6$

$x =$ _____ $y =$ _____

ML 4.03 Solving Systems of Linear Equations by Substitution

Goal

Use substitution to solve systems of equations.

Standard

MA.912.AR.9.1

Materials

highlighter (optional)

Modeled Review

Point to Ama's work and **ask:**

- "Why did Ama choose to substitute $4x - 45$ for y in the second equation?"
- "Why did Ama substitute 12 for x in the first equation? Could Ama have used the second equation instead?"
- "How can Ama check her work?"

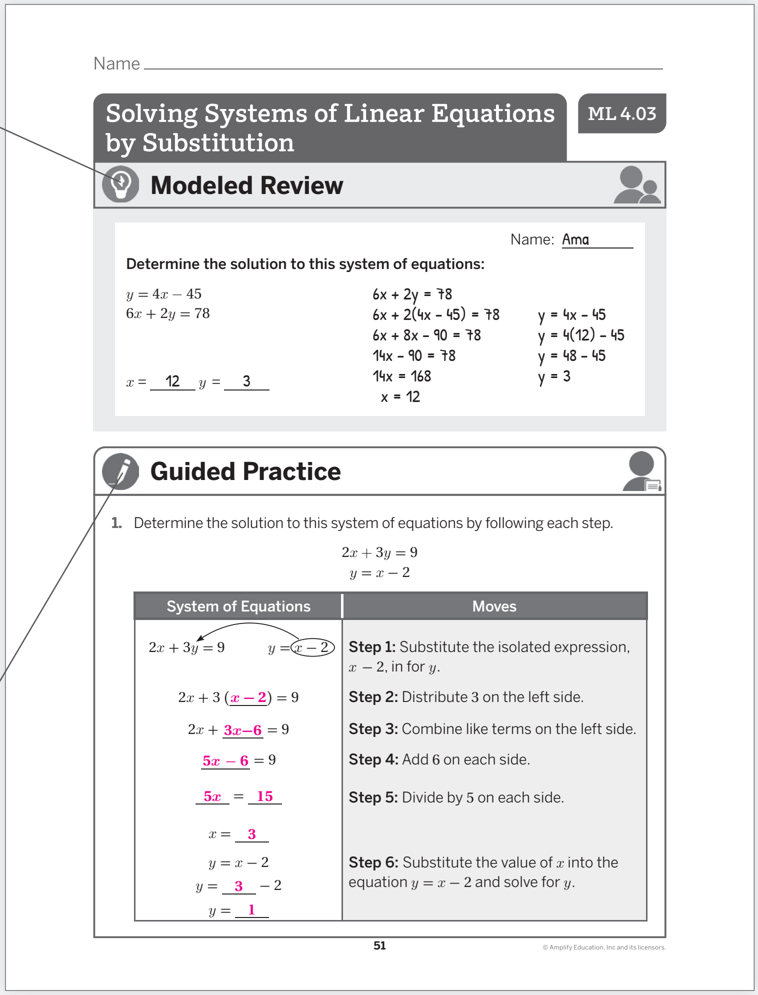
Reinforce Ama's thinking by saying, "Substitution is a method for solving systems of equations by solving for one variable at a time to determine the solution."

Guided Practice

Focus students' attention on solving systems of equations by using substitution.

To scaffold their thinking, **say**:

- "First, determine if a variable is isolated in an equation. Substitute this expression into the other equation to solve for the remaining variable."
- "Then, use this value to calculate the other variable."
- "Last, check your solution by substituting both values into the equations."



Name

Solving Systems of Linear Equations by Substitution — ML 4.03

Modeled Review

Name: Ama

Determine the solution to this system of equations:

$y = 4x - 45$
$6x + 2y = 78$

6x + 2y = 78
6x + 2(4x − 45) = 78
6x + 8x − 90 = 78
14x − 90 = 78
14x = 168
x = 12

y = 4x − 45
y = 4(12) − 45
y = 48 − 45
y = 3

$x =$ 12 $y =$ 3

Guided Practice

1. Determine the solution to this system of equations by following each step.

$$2x + 3y = 9$$
$$y = x - 2$$

System of Equations	Moves
$2x + 3y = 9$ $\quad y = x - 2$	**Step 1:** Substitute the isolated expression, $x - 2$, in for y.
$2x + 3(x - 2) = 9$	**Step 2:** Distribute 3 on the left side.
$2x + 3x - 6 = 9$	**Step 3:** Combine like terms on the left side.
$5x - 6 = 9$	**Step 4:** Add 6 on each side.
$5x = 15$	**Step 5:** Divide by 5 on each side.
$x = 3$	
$y = x - 2$ $y = 3 - 2$ $y = 1$	**Step 6:** Substitute the value of x into the equation $y = x - 2$ and solve for y.

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Vocabulary

If needed, share the meaning of the terms with students.

substitution: A method of solving systems of equations where a variable is replaced with an equivalent expression in order to produce a new equation with fewer variables.

distributive property: Multiplying a number by the sum of two or more terms is equal to multiplying the number by each term separately before adding them together.

Solving Systems of Linear Equations by Substitution

Guided Practice

A Guide student processing by providing questions they can ask themselves as they solve the system of equations.
For example:

- "Which equation has an isolated variable?"
- "What should you do if the same variable is isolated in both equations?"
- "What should you do after solving for one variable?"

Key Takeaway:
Say, "Substitution is a strategy for solving systems of equations by solving for one variable at a time. This strategy can be applied to any system of equations, but it's especially useful when at least one of the variables is already isolated."

Guided Practice

For Problems 2–5, determine the solution to the system of equations.
Sample work shown.

2. $y = 2x - 2$
$y = x - 1$

$2x - 2 = x - 1$
$x - 2 = -1$
$x = 1$
$y = x - 1$
$y = 1 - 1$
$y = 0$

$x =$ 1 $y =$ 0

3. $3x + 2y = 10$
$y = x - 5$

$3x + 2y = 10$
$3x + 2(x - 5) = 10$
$3x + 2x - 10 = 10$
$5x - 10 = 10$
$5x = 20$
$x = 4$

$y = x - 5$
$y = 4 - 5$
$y = -1$

$x =$ 4 $y =$ −1

4. $y = 4x + 2$
$y = 3x - 6$

$4x + 2 = 3x - 6$
$x + 2 = -6$
$x = -8$
$y = 3(-8) - 6$
$y = -30$

$x =$ −8 $y =$ −30

5. $2x - 3y = 16$
$y = 2x$

$2x - 3y = 16$
$2x - 3(2x) = 16$
$2x - 6x = 16$
$-4x = 16$
$x = -4$

$y = 2x$
$y = 2(-4)$
$y = -8$

$x =$ −4 $y =$ −8

Check

Determine the solution to this system of equations. Sample work shown.

$2x + 5y = 18$
$y = 2x - 6$

$2x + 5(2x - 6) = 18$
$2x + 10x - 30 = 18$
$12x - 30 = 18$
$12x = 48$
$x = 4$

$y = 2(4) - 6$
$y = 2$

$x =$ 4 $y =$ 2

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Reflection

Ask:

- "Where might you make mistakes when using substitution, and how can you check your solution?"
- "What is something you weren't sure about at the start of the lesson but understand now?"

Check: Recommended Next Steps

Almost there
If students need more support, model the substitution strategy for the problem in the Check. Highlight the fact that in this system of equations, y is already isolated so the expression $2x - 6$ can be substituted in for y in the first equation. Encourage students to annotate the equations to visualize replacing the variable with an equivalent expression.

Got it!
If students need more practice, ask them to solve the following system of equations.

$y = -2x + 6$

$y = 3x - 9$

Name ____________________

Graphing Solutions to Systems of Inequalities

ML 4.09

Modeled Review

Name: Caleb

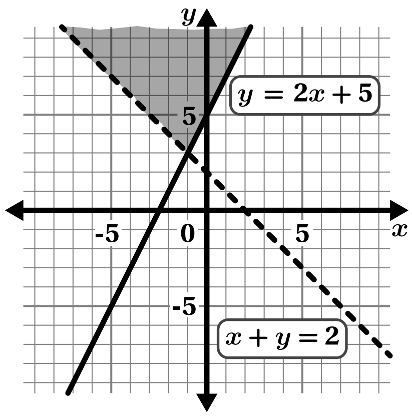
The graph shows the boundary lines and equations for this system of inequalities.

$$x + y > 2$$
$$y \geq 2x + 5$$

Shade the solution region for the system of inequalities.

x + y > 2	y ≥ 2x + 5
0 + 0 > 2	0 ≥ 2(0) + 5
0 > 2	0 > 5
false	false

The point (0, 0) makes both inequalities false. That means the solution for each inequality does not include (0, 0).

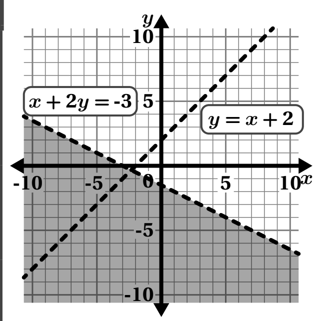


Guided Practice

1. Ella graphed the first inequality and the boundary line of the second inequality. Complete the graph of the second inequality.

$$x + 2y < -3$$
$$y < x + 2$$

Inequality	Moves
$y < x + 2$ __< __ + 2 __< __ **Circle one:** true false	**Step 1:** Choose a point that does not touch the dashed line and substitute its coordinates into the inequality. **Step 2:** Determine if the point makes a true or false statement. **Step 3:** Shade the region depending on whether the point should be included or not.

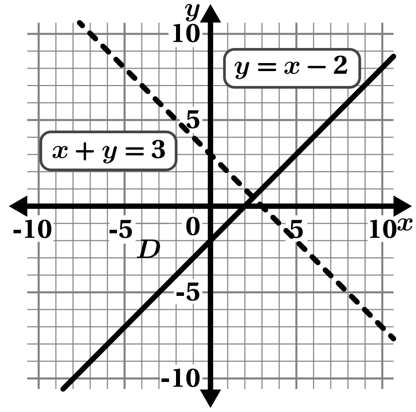


Guided Practice

This graph shows the boundary lines and equations for this system of inequalities.

$$x + y < 3$$
$$y \geq x - 2$$

2. Shade the solution region to the system of inequalities.



3. Is the point $(0, -5)$ a solution to the system? Circle one:

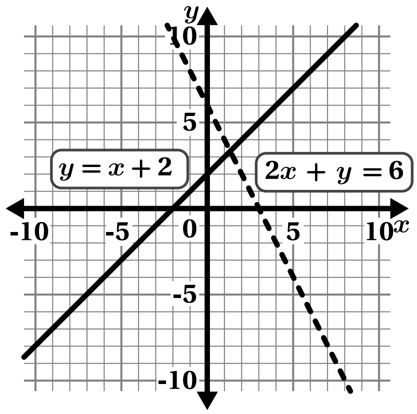
Yes **No**

Check

This graph shows the boundary lines and equations for this system of inequalities.

$$2x + y > 6$$
$$y \leq x + 2$$

1. Shade the solution region to the system of inequalities.



2. Is the point (6, 4) a solution to the system? Circle one:

Yes **No**

ML 4.09 Graphing Solutions to Systems of Inequalities

Goal

Graph solutions to systems of inequalities.

Standard

MA.912.AR.9.4

Materials

highlighter (optional)

Modeled Review

Point to Caleb's work and **ask:**

- "Why did Caleb use $(0, 0)$ as a test point? Could he have used another point?"
- "Why is $(0, 0)$ not shaded in the graph?"
- "Name a point that's a solution for the system."

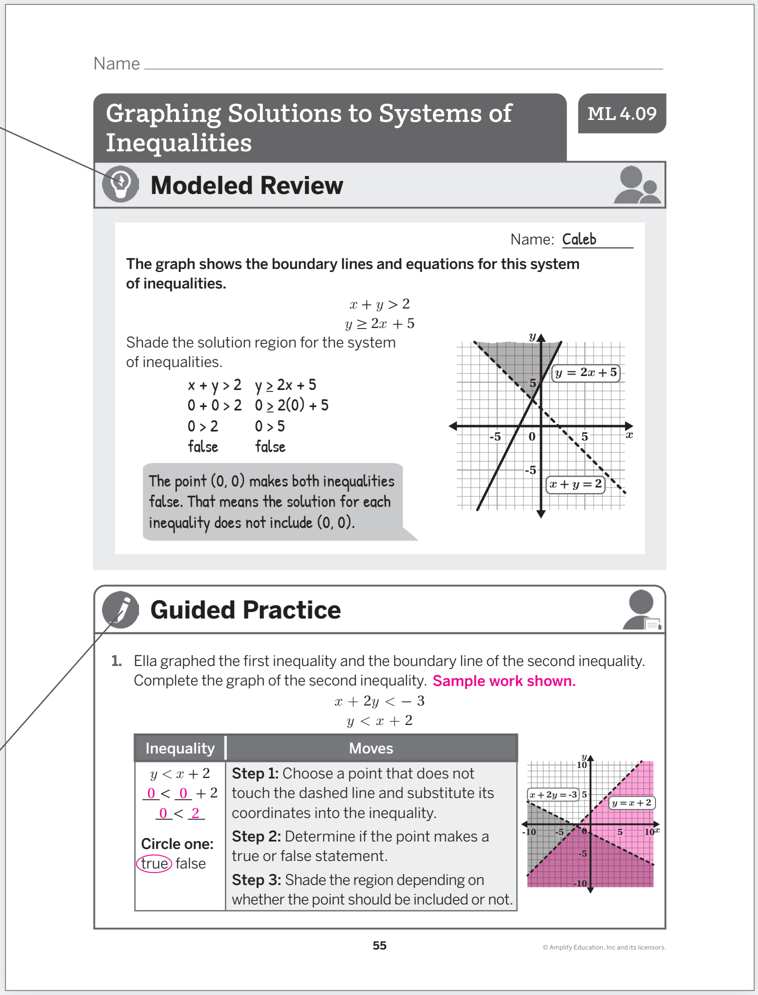
Reinforce Caleb's thinking by saying, "When graphing solutions to systems of inequalities, you can determine the solution region by substituting a test point, such as $(0, 0)$, into each inequality to see if it is true and shade the regions where the inequalities are true."

Guided Practice

Focus students' attention on using an ordered pair to complete the graph of the second inequality.

To scaffold their thinking, **ask**:

- "What point can you use?"
- "How can you determine if the inequality is true or false?"
- "How do you shade the graph to include $(0, 0)$?"



Name

Graphing Solutions to Systems of Inequalities — ML 4.09

Modeled Review

Name: Caleb

The graph shows the boundary lines and equations for this system of inequalities.

$x + y > 2$
$y \geq 2x + 5$

Shade the solution region for the system of inequalities.

$x + y > 2$	$y \geq 2x + 5$
$0 + 0 > 2$	$0 \geq 2(0) + 5$
$0 > 2$	$0 > 5$
false	false

The point (0, 0) makes both inequalities false. That means the solution for each inequality does not include (0, 0).

Guided Practice

1. Ella graphed the first inequality and the boundary line of the second inequality. Complete the graph of the second inequality. Sample work shown.

$x + 2y < -3$
$y < x + 2$

Inequality	Moves
$y < x + 2$ $0 < 0 + 2$ $0 < 2$ Circle one: true false	**Step 1:** Choose a point that does not touch the dashed line and substitute its coordinates into the inequality. **Step 2:** Determine if the point makes a true or false statement. **Step 3:** Shade the region depending on whether the point should be included or not.

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Vocabulary

If needed, share the meaning of the terms with students.

solution (to a system of inequalities): An ordered pair that makes each inequality in a system true. Every ordered pair that is a solution to a system is located in the solution region where the graphs overlap.

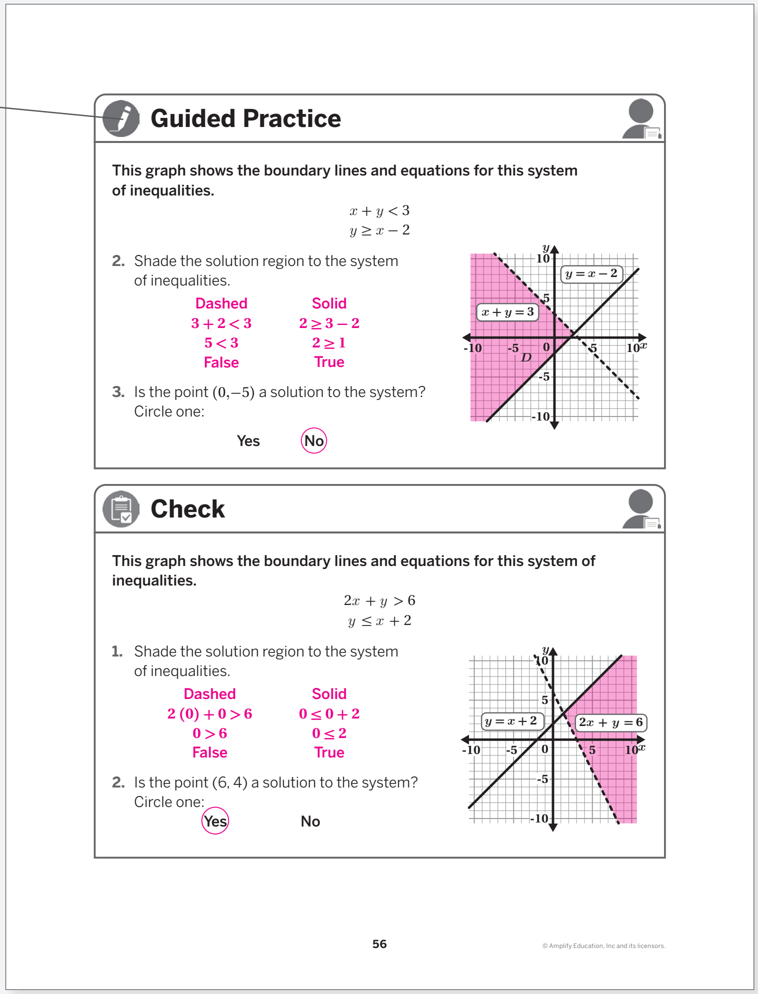
solution region: The set of all ordered pairs that make an inequality or each inequality in a system true.For a system of inequalities, the solution region is located where the graphs overlap.

Graphing Solutions to Systems of Inequalities

Guided Practice

A Guide student processing by providing questions they can ask themselves while graphing a system of inequalities.
For example:
- “What test point can you use?”
- “Does the test point satisfy each inequality?”
- “Which part of the graph should be shaded to represent each inequality?”
- “How do you identify the solution region?”

Key Takeaway:
Say, “To find the solution region for a system of inequalities, use a test point and shade the regions where each inequality is true. The overlap represents the solution.”



Guided Practice

This graph shows the boundary lines and equations for this system of inequalities.

$$x + y < 3$$
$$y \geq x - 2$$

2. Shade the solution region to the system of inequalities.

Dashed	Solid
$3 + 2 < 3$	$2 \geq 3 - 2$
$5 < 3$	$2 \geq 1$
False	True

3. Is the point $(0, -5)$ a solution to the system? Circle one:

Yes No

Check

This graph shows the boundary lines and equations for this system of inequalities.

$$2x + y > 6$$
$$y \leq x + 2$$

1. Shade the solution region to the system of inequalities.

Dashed	Solid
$2\,(0) + 0 > 6$	$0 \leq 0 + 2$
$0 > 6$	$0 \leq 2$
False	True

2. Is the point (6, 4) a solution to the system? Circle one:

Yes No

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Reflection

Ask:
- “What should you consider when determining the solution region for a system of inequalities?”
- “How does what you learned today connect to your prior learning?”

Check: Recommended Next Steps

Almost there
If students need more support, consider using two different colored highlighters to distinguish the shaded regions for each inequality. This visual aid helps students better identify where the shaded areas overlap, highlighting the solution region.

Got it!
If students need more practice, ask them to revisit the graph for Problems 2–3 and determine three points that are solutions to the system.

Unit 5

Mini-Lessons

Name ______________________________

Identifying Linear and Exponential Functions Using Tables

ML 5.01

Modeled Review

Linear	Exponential
Functions that change by a constant difference	Functions that change by a constant ratio

Example (Linear)

Term	Value
1	8
2	10
3	12
4	14

$+2$

Example (Exponential)

Term	Value
1	2
2	4
3	8
4	16

$\times 2$

Guided Practice

1. Match each table with its corresponding type of function.

Function

Term	Value
1	12
2	8
3	4

Term	Value
1	12
2	8
3	5

Term	Value
1	12
2	6
3	3

Type of Function

Linear

Exponential

Neither

Guided Practice

Determine if each function is linear, exponential, or neither. Circle the correct answer.

2.

Term	Value
1	2
2	6
3	18
4	54

Linear Exponential Neither

3.

Term	Value
1	5
2	10
3	15
4	30

Linear Exponential Neither

4.

Term	Value
1	5
2	2
3	-1
4	-4

Linear Exponential Neither

5.

Term	Value
1	4
2	6
3	9
4	13.5

Linear Exponential Neither

Check

Determine if each function is linear, exponential, or neither. Circle the correct answer.

1.

Term	Value
1	4
2	9
3	14
4	19

Linear Exponential Neither

2.

Term	Value
1	3
2	12
3	48
4	144

Linear Exponential Neither

ML 5.01 Identifying Linear and Exponential Functions Using Tables

Goal

Identify examples of linear and exponential functions from tables

Standard

F-LE.1.b

Modeled Review

Point to the Modeled Review and **ask**:

- "How are linear and exponential functions different?"
- "What patterns do you notice in linear functions? In exponential functions?"
- "Does every function have to be linear or exponential?"

Reinforce the goal by saying, Linear functions have a constant difference through addition or subtraction. Exponential functions have a constant ratio through multiplication or division."

Guided Practice

Focus students' attention on matching each table with the type of sequence.

To scaffold their thinking, **say**:

- "First, identify whether the function consistently adds, subtracts, multiplies, or divides from one value to the next."
- "If it's adding or subtracting, it's linear. If multiplying or dividing, it's exponential. If none of these apply, it's neither."

Name

Identifying Linear and Exponential Functions Using Tables

ML 5.01

Modeled Review

Linear

Functions that change by a constant difference

Example

Term	Value
1	8
2	10
3	12
4	14

+ 2

Exponential

Functions that change by a constant ratio

Example

Term	Value
1	2
2	4
3	8
4	16

× 2

Guided Practice

1. Match each table with its corresponding type of function.

Function

Term	Value
1	12
2	8
3	4

Term	Value
1	12
2	8
3	5

Term	Value
1	12
2	6
3	3

Type of Function

Linear

Exponential

Neither

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Vocabulary

If needed, share the meaning of the terms with students.

linear function: A function that changes by a constant difference.

exponential function: A function that changes by a constant ratio.

constant ratio: When the ratio between any two consecutive values in a pattern is the same, there is a constant ratio.

ML 5.01 Identifying Linear and Exponential Functions Using Tables

Guided Practice

A Guide student processing by providing questions they can reflect on when identifying the type of function. For example:

- "How does each term relate to the next?"
- "Should I add, subtract, multiply, or divide?"
- "Does the pattern consistently appear throughout the function?"

Key Takeaway:
Say, "Linear functions have a constant difference and create a linear graph. Exponential functions have a constant ratio and often create a curved graph. If a function is neither, it does not have a consistent pattern."

Guided Practice

Determine if each function is linear, exponential, or neither. Circle the correct answer.

2.

Term	Value
1	2
2	6
3	18
4	54

Linear Exponential Neither

3.

Term	Value
1	5
2	10
3	15
4	30

Linear Exponential Neither

4.

Term	Value
1	5
2	2
3	– 1
4	– 4

Linear Exponential Neither

5.

Term	Value
1	4
2	6
3	9
4	13.5

Linear Exponential Neither

Check

Determine if each function is linear, exponential, or neither. Circle the correct answer.

1.

Term	Value
1	4
2	9
3	14
4	19

Linear Exponential Neither

2.

Term	Value
1	3
2	12
3	48
4	144

Linear Exponential Neither

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Reflection

Ask:

- "What are some clues that a function might be linear? Exponential?"
- "What is something you weren't sure about at the start of the lesson but understand now?"

Check: Recommended Next Steps

Almost there
If students need more support, consider using operation symbols to indicate the jumps between terms on the table, highlighting the pattern.

Got it!
If students need more practice, have them determine if the following function is linear, exponential, or neither.

Term	Value
1	20
2	15
3	10
4	5

Name ______________________________

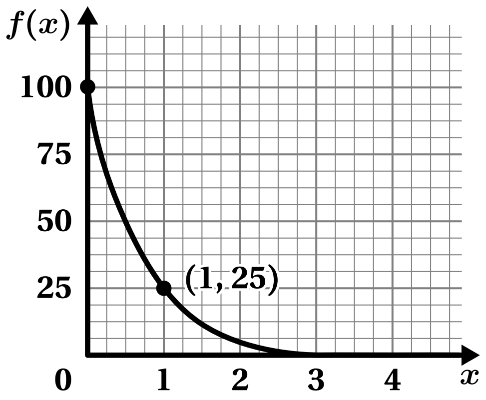
Writing Exponential Functions From Graphs

ML 5.02

Modeled Review

Name: Zion

Here is a graph of an exponential relationship. Write a function in the form $f(x) = a \cdot b^x$ that represents the graph.



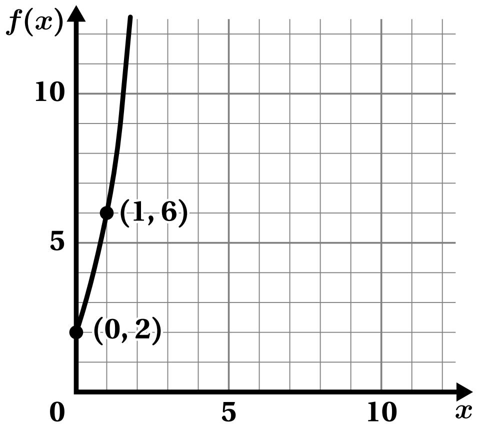
x	f(x)
0	100
1	25

$a = 100$ $b = \frac{25}{100} = \frac{1}{4}$

equation: $f(x) = 100 \cdot (\frac{1}{4})^x$

Guided Practice

1. Here is a graph of an exponential relationship. Fill in the table and write a function in the form $f(x) = a \cdot b^x$ that represents the graph.



x	$f(x)$
0	
1	

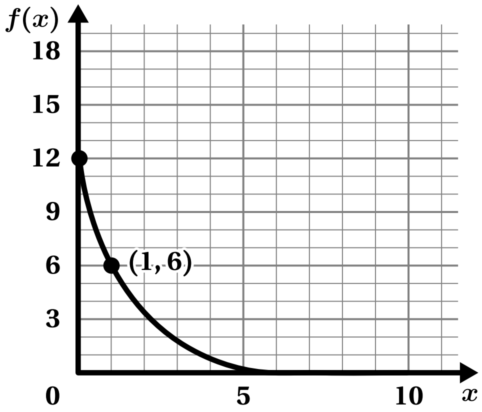
$a =$ ____ $b = \frac{\square}{\square} =$ ____

$f(x) =$ ______ $\cdot$ ______^{x}

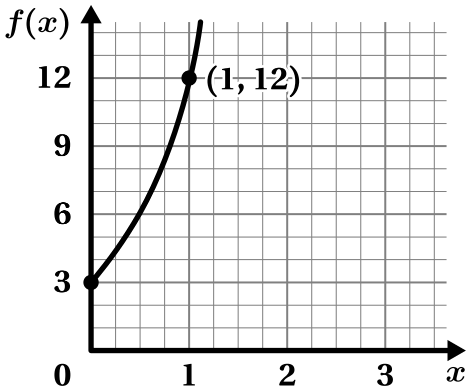
initial value   growth factor

Guided Practice

For Problems 2–3, write a function in the form $f(x) = a \cdot b^x$ that represents the graph.

2. 

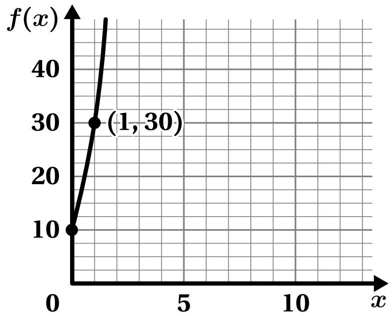
equation: ____________________

3. 

equation: ____________________

Check

Here is a graph of an exponential relationship. Write a function in the form $f(x) = a \cdot b^x$ that represents the graph.



equation: ____________________

ML 5.02 Writing Exponential Functions From Graphs

Goal

Use graphs to write equations to represent exponential functions.

Standard

MA.912.AR.5.4

Modeled Review

Point to Zion's work and **ask**:

- "Why did Zion create a table with the points $(0,100)$ and $(1,25)$?"
- "Where is the initial value located on the graph?"
- "Does the order Zion used to divide the y-values matter for calculating the growth factor?"

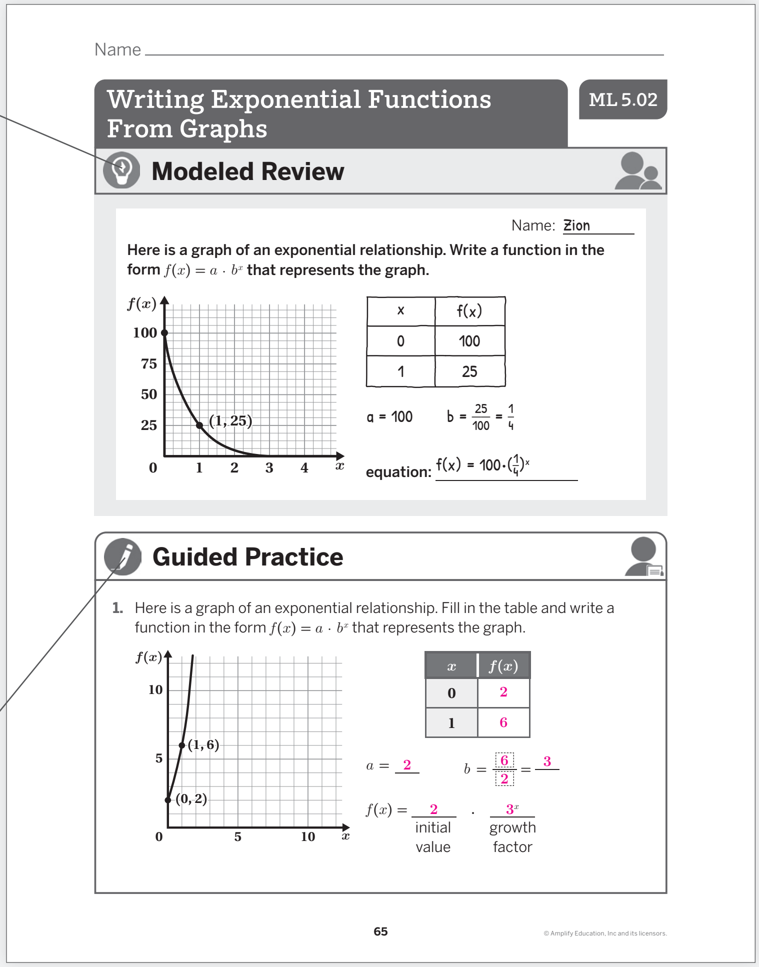
Reinforce Zion's thinking by saying, "To write an exponential function from a graph, you can use the coordinates of specific points on the graph. These can help you determine a, the initial value, and b, the growth factor."

Guided Practice

Focus students' attention on writing an exponential function for the graph.

To scaffold their thinking, **say**:

- "First, list the coordinates of given points from the graph in a table."
- "Then, find the initial value a, which is the y-intercept. Calculate the growth factor b by dividing the y-coordinates of points whose x-coordinates differ by 1."
- "Last, substitute the initial value and growth factor into the equation, $f(x) = a \cdot b^x$."



Name

Writing Exponential Functions From Graphs ML 5.02

Modeled Review

Name: Zion

Here is a graph of an exponential relationship. Write a function in the form $f(x) = a \cdot b^x$ that represents the graph.

x	f(x)
0	100
1	25

a = 100 b = $\frac{25}{100} = \frac{1}{4}$

equation: $f(x) = 100 \cdot (\frac{1}{4})^x$

Guided Practice

1. Here is a graph of an exponential relationship. Fill in the table and write a function in the form $f(x) = a \cdot b^x$ that represents the graph.

x	$f(x)$
0	2
1	6

$a =$ 2 $b = \frac{6}{2} =$ 3

$f(x) =$ 2 (initial value) $\cdot$ 3^x (growth factor)

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Vocabulary

If needed, share the meaning of the terms with students.

initial value: A point where the graph of an equation or function crosses the y-axis or when $x = 0$.

growth factor: The constant ratio (or common factor) that each term is multiplied by to generate an exponential pattern.

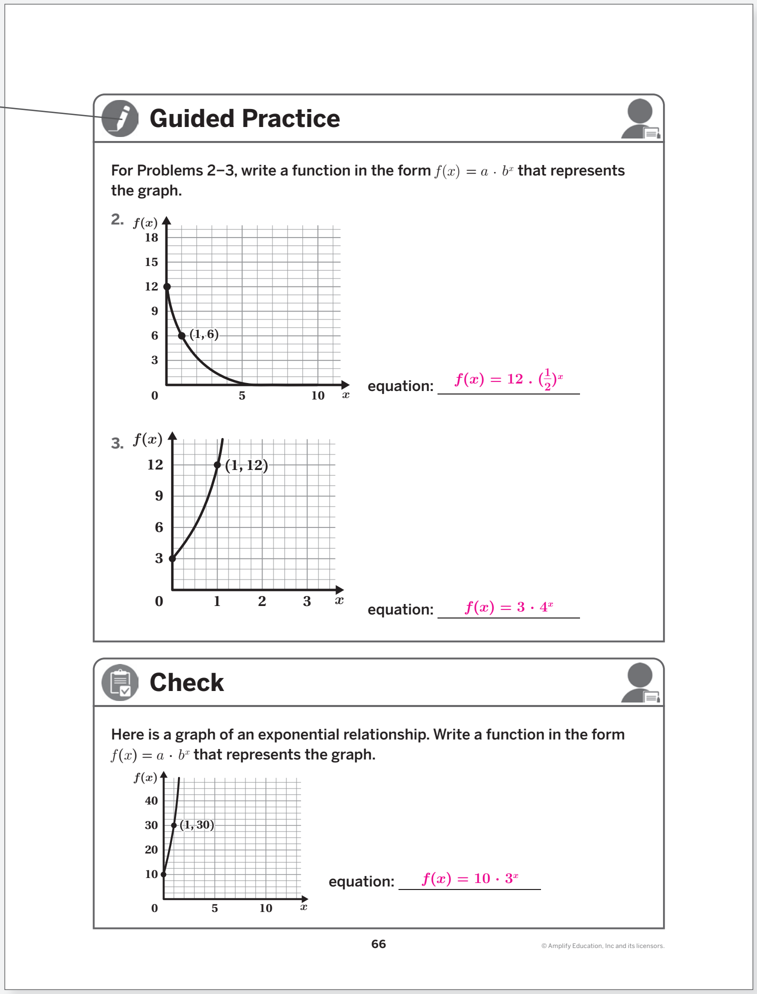
Writing Exponential Functions From Graphs

Guided Practice

A Guide student processing by providing questions they can ask themselves as they write exponential functions. For example:

- "Which point is the y-intercept?"
- "How can you calculate the growth factor?"
- "Does your equation match the graph? How do you know?"

Key Takeaway:
Say, "Exponential functions are written in the form $f(x) = a \cdot b^x$. To calculate the growth factor, divide the y-coordinate of a point by the y-coordinate of a point whose x-value is 1 less."



Guided Practice

For Problems 2–3, write a function in the form $f(x) = a \cdot b^x$ that represents the graph.

2. equation: $f(x) = 12 \cdot (\frac{1}{2})^x$

3. equation: $f(x) = 3 \cdot 4^x$

Check

Here is a graph of an exponential relationship. Write a function in the form $f(x) = a \cdot b^x$ that represents the graph.

equation: $f(x) = 10 \cdot 3^x$

66

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Reflection

Ask:

- "How can you determine the growth factor for an exponential function?"
- "What questions do you still have?"

Check: Recommended Next Steps

Almost there	Got it!
If students need more support, consider creating an anchor chart that displays the general form of an exponential function, $f(x) = a \cdot b^x$, where a is the y-intercept and b is the growth factor. Include an example like $f(x) = 2 \cdot 3^x$, showing how 2 is the initial value and 3 is the growth factor, along with a table of values and a simple graph for visual aid.	If students need more practice, revisit Problem 3 and ask them how the equation would change if the graph still passes through $(1,12)$, but now has an initial value at $(0,6)$.

Name ______________________________

Calculating Percent Change in Exponential Functions

ML 5.07

Modeled Review

Name: Victor

A town has a population of $200{,}000$ people. The population is decreasing by 3% each year.

1. Write a function in the form $f(x) = a \cdot b^x$ that represents the population of the town after x years.

   a = 200,000   b = 1 - 0.03
   b = 0.97

   It's decreasing, so I'll convert 3% to a decimal by dividing by 100 and subtract the value from 1.

   equation: $f(x) = 200{,}000(0.97)^x$

2. Use the function to determine how many people will be in the town after 10 years.

   $f(x) = 200{,}000(0.97)^{10}$
   $f(x) = 147{,}485$
   answer: About 147,485 people

Guided Practice

1. A forest initially has $1{,}500$ trees. The tree population is increasing by 5% each year due to ongoing tree-planting efforts. Write a function in the form $f(x) = a \cdot b^x$ that represents the number of trees after x years.

Step 1: Identify whether the scenario represents growth or decay.	**growth** **decay**
Step 2: Determine the initial value.	$a =$ ______
Step 3: Calculate the growth factor.	$b = (1$ ___ _____$)$ $b =$ _____
Step 4: Substitute the values into the formula and write the function.	

Guided Practice

A species of bird in a wildlife reserve is initially counted at $5{,}000$. The bird population is declining by 6% each year.

2. Write a function in the form $f(x) = a \cdot b^x$ that represents the population of the town after x years.

equation: ______________________________

3. Use the function to determine the number of birds in the wildlife reserve after 3 years.

answer: ______________

An ant colony initially containing $1{,}200$ ants is growing by 3% per day.

4. Write a function in the form $f(x) = a \cdot b^x$ that represents the population of the ant colony after x days.

equation: ______________________________

5. Use the function to determine the number of ants in the colony after 4 days.

answer: ________________

Check

A town has a population of $100{,}000$ people. The population is increasing by 2% each year.

1. Write a function in the form $f(x) = a \cdot b^x$ that represents the population of the town after x years.

equation: ____________________________________

2. Use the function to determine the number of people in the town after 10 years.

answer: ________________

ML 5.07 Calculating Percent Change in Exponential Functions

Goal

Calculate percent change in exponential functions.

Standard

MA.912.AR.5.4

Materials

graphing calculator (optional)

Modeled Review

Point to Victor's work and **ask**:

- "How did Victor know 200,000 was the initial value?"
- "Why did Victor subtract 0.03 from 1?"
- "How did Victor calculate the population, 147,485?"

Reinforce Victor's thinking by saying, "You can write an exponential function from a scenario by first determining if it's increasing or decreasing. If it is increasing, b is 1 plus the percent increase in decimal form; if it is decreasing, b is 1 minus the percent decrease."

Guided Practice

Focus students' attention on writing an exponential function involving exponential growth or decay.

To scaffold their thinking, **ask**:

- "How do you know if the scenario represents growth or decay?"
- "What is the initial amount?"
- "How can you calculate the growth factor?"
- "How can the initial amount and growth factor be used to write a function?"

Name

Calculating Percent Change in Exponential Functions

ML 5.07

Modeled Review

Name: Victor

A town has a population of 200,000 people. The population is decreasing by 3% each year.

1. Write a function in the form $f(x) = a \cdot b^x$ that represents the population of the town after x years.

   a = 200,000   b = 1 – 0.03
   b = 0.97

   It's decreasing, so I'll convert 3% to a decimal by dividing by 100 and subtract the value from 1.

   equation: $f(x) = 200{,}000(0.97)^x$

2. Use the function to determine how many people will be in the town after 10 years.

   $f(x) = 200{,}000(0.97)^{10}$
   $f(x) = 147{,}485$
   answer: About 147,485 people

Guided Practice

1. A forest initially has 1,500 trees. The tree population is increasing by 5% each year due to ongoing tree-planting efforts. Write a function in the form $f(x) = a \cdot b^x$ that represents the number of trees after x years.

Step 1: Identify whether the scenario represents growth or decay.	growth decay
Step 2: Determine the initial value.	$a = $ 1,500
Step 3: Calculate the growth factor.	$b = (1 + 0.05)$ $b = 1.05$
Step 4: Substitute the values into the formula and write the function.	$f(x) = 1{,}500\,(1.05)^x$ or $f(x) = 1{,}500\,(1 + 0.05)^x$

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Vocabulary

If needed, share the meaning of the terms with students.

initial value: A point where the graph of an equation or function crosses the y-axis or when $x = 0$.

percent decrease: Describes how much a quantity goes up, expressed as a percent of the starting amount.

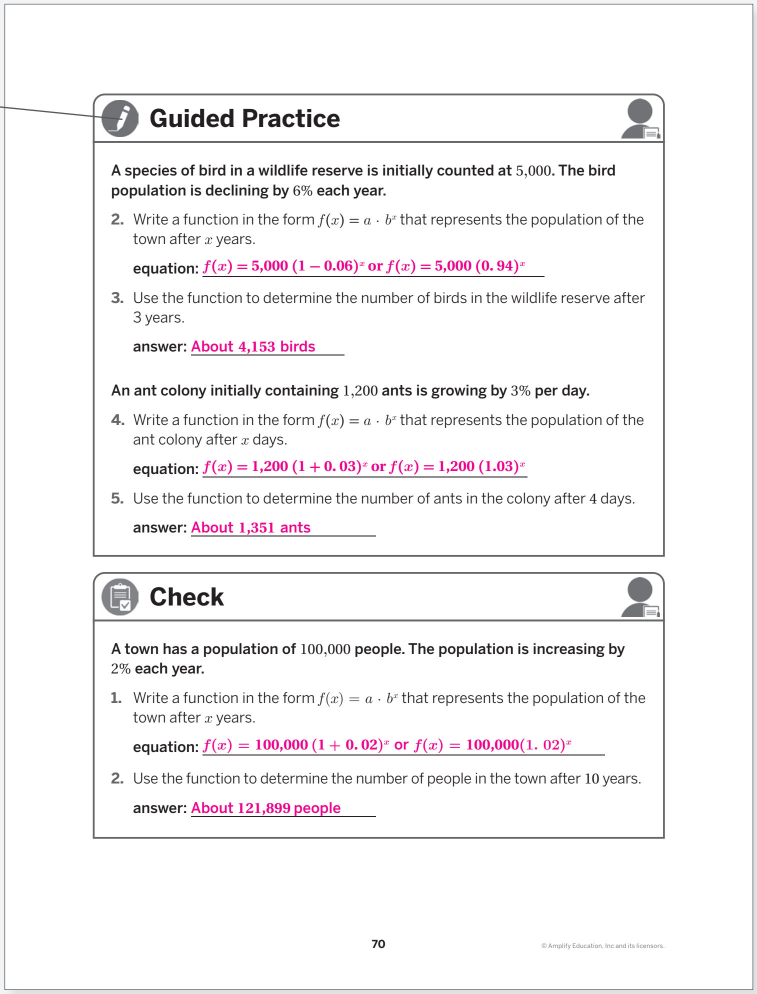
percent increase: Describes how much a quantity goes down, expressed as a percent of the starting amount.

Calculating Percent Change in Exponential Functions

Guided Practice

A Provide students with a graphing calculator to support them with performing calculations.

Key Takeaway:
Say, "Exponential functions in the form $f(x) = a \cdot b^x$ represent situations with a percent increase when b is greater than 1 and a percent decrease when b is less than 1."



Guided Practice

A species of bird in a wildlife reserve is initially counted at $5{,}000$. The bird population is declining by 6% each year.

2. Write a function in the form $f(x) = a \cdot b^x$ that represents the population of the town after x years.

   equation: $f(x) = 5{,}000\ (1 - 0.06)^x$ or $f(x) = 5{,}000\ (0.94)^x$

3. Use the function to determine the number of birds in the wildlife reserve after 3 years.

   answer: About $4{,}153$ birds

An ant colony initially containing $1{,}200$ ants is growing by 3% per day.

4. Write a function in the form $f(x) = a \cdot b^x$ that represents the population of the ant colony after x days.

   equation: $f(x) = 1{,}200\ (1 + 0.03)^x$ or $f(x) = 1{,}200\ (1.03)^x$

5. Use the function to determine the number of ants in the colony after 4 days.

   answer: About $1{,}351$ ants

Check

A town has a population of $100{,}000$ people. The population is increasing by 2% each year.

1. Write a function in the form $f(x) = a \cdot b^x$ that represents the population of the town after x years.

   equation: $f(x) = 100{,}000\ (1 + 0.02)^x$ or $f(x) = 100{,}000(1.02)^x$

2. Use the function to determine the number of people in the town after 10 years.

   answer: About $121{,}899$ people

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Reflection

Ask:

- "What is the difference between a growing exponential function and a decaying exponential function?"
- "What is something you weren't sure about at the start of the lesson but understand now?"

Check: Recommended Next Steps

Almost there
If students need more support, consider creating an anchor chart that shows $f(x) = a \cdot b^x$, where a is the initial amount, and b is 1 plus percent increase for growth and 1 minus percent decrease for decay.

Got it!
If students need more practice, ask them to solve the following problem.

A forest contains $15{,}000$ oak trees. A disease reduces the population by 8% each year.

1. Write a function in the form $f(x) = a \cdot b^x$ that represents the number of trees in the forest after x years.
2. Use the function to determine how many oak trees will be in the forest after 2 years.

Name ______________________________

Writing Equivalent Expressions Using Radicals and Rational Exponents

ML 5.08

Modeled Review

The symbol $\sqrt{\ }$ is called a **radical**. You can write equivalent expressions using radicals and **rational exponents**, which are exponents written as a fraction.

$$x^{\frac{1}{n}} = \sqrt[n]{x}.$$

In the following table, each row shows equivalent expressions.

Radical Expression	Exponential Expression
$\sqrt{4}$	$4^{\frac{1}{2}}$
$\sqrt[3]{64}$	$64^{\frac{1}{3}}$
$\sqrt[4]{81}$	$81^{\frac{1}{4}}$
$\sqrt[5]{32}$	$32^{\frac{1}{5}}$

Guided Practice

For Problems 1–2, choose the expression that is equivalent to the given radical or exponential form.

1. $\sqrt{10}$

A. $10^{\frac{1}{4}}$

B. $10^{\frac{1}{2}}$

2. $5^{\frac{1}{4}}$

A. $\sqrt[4]{5}$

B. $\sqrt{5}$

Guided Practice

3. Rewrite each exponential expression as a radical expression.

$4^{\frac{1}{2}}$	$6^{\frac{1}{3}}$	$11^{\frac{1}{4}}$

4. Rewrite each radical expression as an exponential expression.

$\sqrt{36}$	$\sqrt[3]{9}$	$\sqrt[4]{16}$

5. Rewrite each exponential expression as a radical expression.

$24^{\frac{1}{5}}$	$8^{\frac{1}{6}}$	$18^{\frac{1}{7}}$

6. Rewrite each radical expression as an exponential expression.

$\sqrt[6]{21}$	$\sqrt[7]{42}$	$\sqrt[8]{6}$

Check

1. Rewrite $17^{\frac{1}{4}}$ as a radical expression.

____

2. Rewrite $\sqrt[8]{11}$ as an exponential expression.

____

ML 5.08 Writing Equivalent Expressions Using Radicals and Rational Exponents

Goal

Use the properties of exponents to write equivalent expressions with radicals and rational exponents.

Standard

MA.912.NSO.1.1

Modeled Review

Point to the Modeled Review and **ask**:

- "What are the similarities and differences between radical and exponential expressions?"
- "How does the denominator of the exponent relate to the root in a radical expression?"
- "When you convert a radical into an exponent, how do you determine the base?"

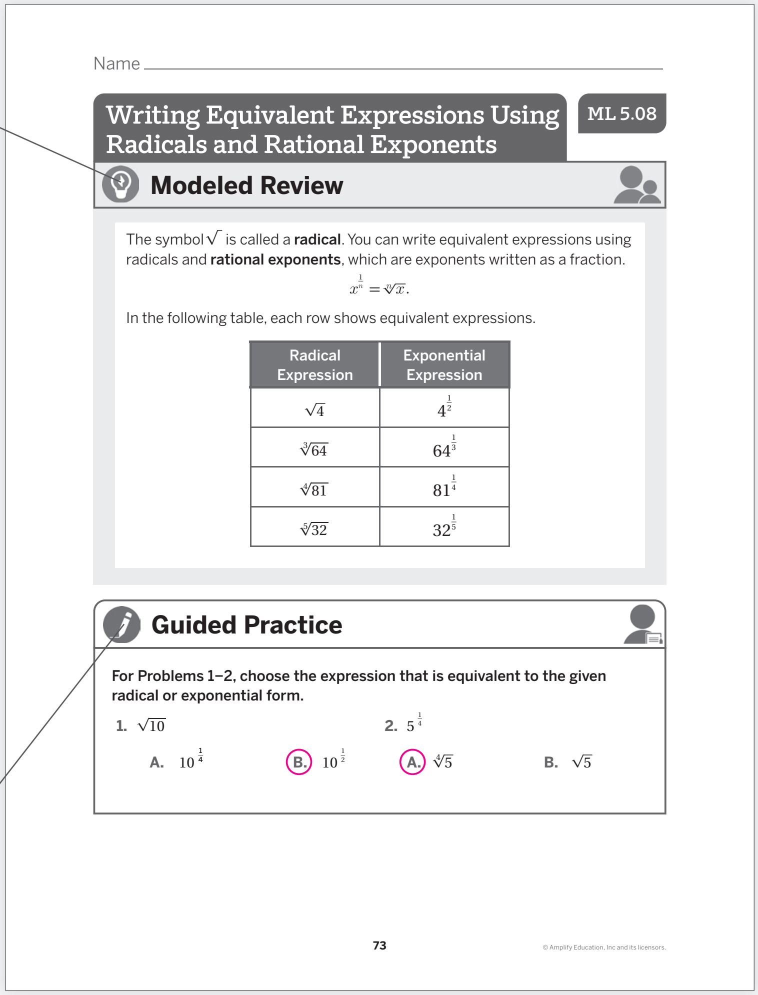
Reinforce the goal by saying "Raising a number or expression to the power of $\frac{1}{n}$ is equivalent to taking the nth root."

Guided Practice

Focus students' attention on choosing the equivalent expression for either the radical or exponential expression.

To scaffold their thinking, **ask**:

- "What root or exponent is given?"
- "Should it be expressed as a radical or with an exponent?"
- "How can you write the expression in the correct form?"



Name ______________________

Writing Equivalent Expressions Using Radicals and Rational Exponents ML 5.08

Modeled Review

The symbol $\sqrt{\ }$ is called a **radical**. You can write equivalent expressions using radicals and **rational exponents**, which are exponents written as a fraction.

$$x^{\frac{1}{n}} = \sqrt[n]{x}.$$

In the following table, each row shows equivalent expressions.

Radical Expression	Exponential Expression
$\sqrt{4}$	$4^{\frac{1}{2}}$
$\sqrt[3]{64}$	$64^{\frac{1}{3}}$
$\sqrt[4]{81}$	$81^{\frac{1}{4}}$
$\sqrt[5]{32}$	$32^{\frac{1}{5}}$

Guided Practice

For Problems 1–2, choose the expression that is equivalent to the given radical or exponential form.

1. $\sqrt{10}$

A. $10^{\frac{1}{4}}$ B. $10^{\frac{1}{2}}$

2. $5^{\frac{1}{4}}$

A. $\sqrt[4]{5}$ B. $\sqrt{5}$

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Vocabulary

If needed, share the meaning of the terms with students.

radical: A square root, cube root, fourth root, etc. The radical symbol is $\sqrt{\ }$.

rational exponent: An exponent that is written as a fraction.

base: A number or expression that is raised to an exponent. The exponent describes the number of times to multiply the base by itself.

ML 5.08

Writing Equivalent Expressions Using Radicals and Rational Exponents

Guided Practice

A Consider circling the root number or exponent to clarify how the radical or exponential expression is formed and understood.

Key Takeaway:
Say, "Raising a number or expression to the $\frac{1}{n}$ power is equivalent to taking its nth root. Written symbolically, this means that $x^{\frac{1}{n}} = \sqrt[n]{x}$."

Guided Practice

3. Rewrite each exponential expression as a radical expression.

$4^{\frac{1}{2}}$	$6^{\frac{1}{3}}$	$11^{\frac{1}{4}}$
$\sqrt{4}$	$\sqrt[3]{6}$	$\sqrt[4]{11}$

4. Rewrite each radical expression as an exponential expression.

$\sqrt{36}$	$\sqrt[3]{9}$	$\sqrt[4]{16}$
$36^{\frac{1}{2}}$	$9^{\frac{1}{3}}$	$16^{\frac{1}{4}}$

5. Rewrite each exponential expression as a radical expression.

$24^{\frac{1}{5}}$	$8^{\frac{1}{6}}$	$18^{\frac{1}{7}}$
$\sqrt[5]{24}$	$\sqrt[6]{8}$	$\sqrt[7]{18}$

6. Rewrite each radical expression as an exponential expression.

$\sqrt[6]{21}$	$\sqrt[7]{42}$	$\sqrt[8]{6}$
$21^{\frac{1}{6}}$	$42^{\frac{1}{7}}$	$6^{\frac{1}{8}}$

Check

1. Rewrite $17^{\frac{1}{4}}$ as a radical expression.

$\sqrt[4]{17}$

2. Rewrite $\sqrt[8]{11}$ as an exponential expression.

$11^{\frac{1}{8}}$

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Reflection

Ask:
- "How is writing a radical expression different from writing an exponential expression?"
- "What makes sense? What is still confusing?"

Check: Recommended Next Steps

Almost there
If students need more support, consider creating an anchor chart using the modeled review, and annotate both the root and exponent to emphasize their connection.

Got it!
If students need more practice, ask them to write the radical or exponential expressions of the following.

- $32^{\frac{1}{3}}$
- $\sqrt[3]{2}$
- 14^5

Name ______

Writing Exponential Functions Involving Compound Interest

ML 5.13

Modeled Review

Name: Tasia

A bank account with an initial balance of $\$500$ earns 4% monthly interest. Select the *three* expressions that represent the balance after 5 years of no payments or withdrawals.

- ☑ $500 \cdot (1.04)^{60}$
- ☐ $500 \cdot (1.04)^{5}$
- ☑ $500 \cdot (1.601)^{5}$
- ☑ $500 \cdot (1.04^{12})^{5}$

Since interest compounds monthly, I need to use exponent properties to account for the 12 months in a year.

Guided Practice

For Problems 1–5, Alina invests money into a college savings account. She writes the expression $200\,(1.02^{12})^4$ to help her calculate what the account balance will be in 4 years.

1. What does 200 represent in the expression?

A. The account balance after 4 years **B.** The initial amount invested

2. What does 1.02 represent in the expression?

A. The monthly growth factor **B.** The annual growth factor

3. What does 12 represent in the expression?

A. The interest rate **B.** The number of times interest is compounded per year

4. What does 4 represent in the expression?

A. The number of years for the investment **B.** The number of times interest is compounded per year

5. Which *two* expressions are equivalent to Alina's expression?

A. $200\,(1.02)^4$ **B.** $200(1.02)^{48}$ **C.** $200(1.268)^4$

Guided Practice

For Problems 6–8, select the *three* expressions that can be used to represent the balance after t years.

6. A bank account with a balance of $300 earns 5% monthly interest for 3 years with no payments or withdrawals.

- ☐ $300 \cdot (1.05)^3$
- ☐ $300 \cdot (1.05)^{36}$
- ☐ $300 \cdot (1.05^{12})^3$
- ☐ $300 \cdot (1.796)^3$

7. A bank account with a balance of $500 earns 4% monthly interest for 4 years of no payments or withdrawals.

- ☐ $500 \cdot (1.601)^4$
- ☐ $500 \cdot (1.04^{48})$
- ☐ $500 \cdot (1.04^{12})^4$
- ☐ $500 \cdot (1.04)^4$

8. A bank account with a balance of $400 earns 3% monthly interest for 5 years of no payments or withdrawals.

- ☐ $400 \cdot (1.03^{60})$
- ☐ $400 \cdot (1.03)^5$
- ☐ $400 \cdot (1.426)^5$
- ☐ $400 \cdot (1.03^{12})^5$

Check

A bank account with a balance of $600 earns 5% monthly interest. Select *three* of the expressions that can be used to represent the balance after 6 years of no payments or withdrawals.

- ☐ $600 \cdot (1.05^{12})^6$
- ☐ $600 \cdot (1.05^{72})$
- ☐ $600 \cdot (1.05)^6$
- ☐ $600 \cdot (1.796)^6$

Writing Exponential Functions Involving Compound Interest

Goal

Write exponential functions involving compound interest.

Standard

MA.912.FL.3.2, MA.912.NSO.1.2

Materials

graphing calculator (optional)

Modeled Review

Point to Tasia's work and **ask**:

- "How did Tasia determine the initial amount?"
- "What does 1.601 represent in the third expression? How did Tasia calculate it?"
- "Why is the exponent 12 used in the fourth expression?"
- "Why was the second expression not selected?"

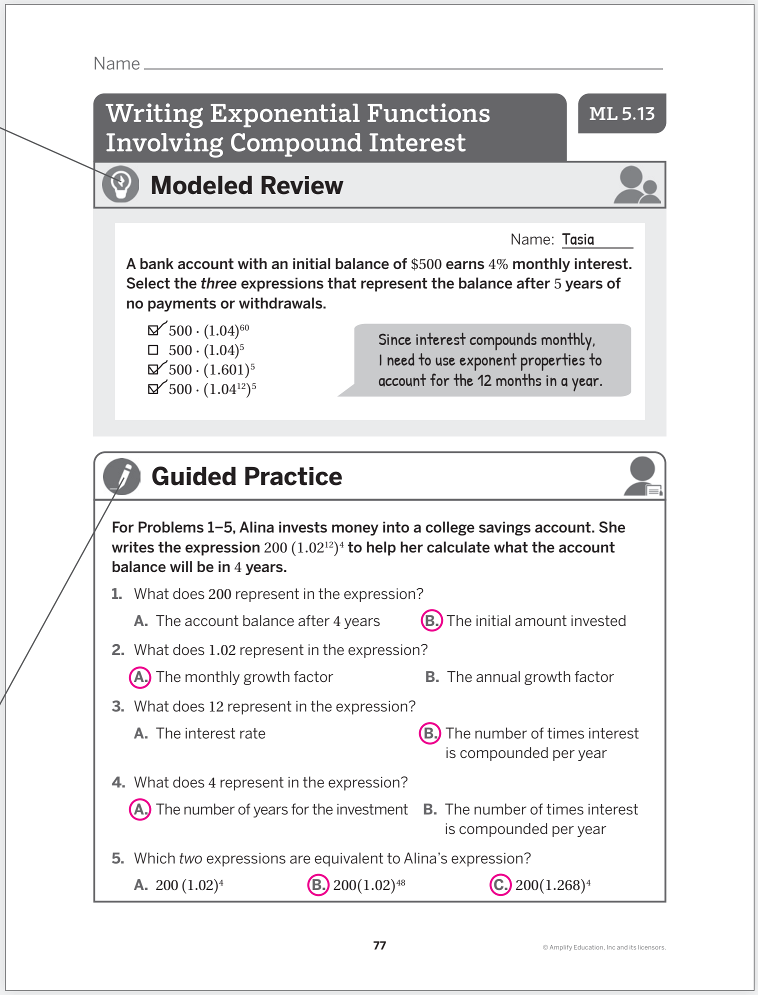
Reinforce the goal by saying, "You can write exponential functions to represent an account balance, using an initial investment and a monthly interest rate."

Guided Practice

Focus students' attention on understanding each component of an exponential function related to compound interest.

To scaffold their thinking, **ask**:

- "How do you identify the initial amount and the monthly interest rate?"
- "How can you determine the monthly growth factor? Annual growth factor?"
- "How can you determine which expressions are equivalent?"



Name

Writing Exponential Functions Involving Compound Interest — ML 5.13

Modeled Review

Name: Tasia

A bank account with an initial balance of $500 earns 4% monthly interest. Select the *three* expressions that represent the balance after 5 years of no payments or withdrawals.

- ☑ $500 \cdot (1.04)^{60}$
- ☐ $500 \cdot (1.04)^{5}$
- ☑ $500 \cdot (1.601)^{5}$
- ☑ $500 \cdot (1.04^{12})^{5}$

Since interest compounds monthly, I need to use exponent properties to account for the 12 months in a year.

Guided Practice

For Problems 1–5, Alina invests money into a college savings account. She writes the expression $200\,(1.02^{12})^4$ to help her calculate what the account balance will be in 4 years.

1. What does 200 represent in the expression?
   A. The account balance after 4 years
   B. The initial amount invested
2. What does 1.02 represent in the expression?
   A. The monthly growth factor
   B. The annual growth factor
3. What does 12 represent in the expression?
   A. The interest rate
   B. The number of times interest is compounded per year
4. What does 4 represent in the expression?
   A. The number of years for the investment
   B. The number of times interest is compounded per year
5. Which *two* expressions are equivalent to Alina's expression?
   A. $200\,(1.02)^4$
   B. $200(1.02)^{48}$
   C. $200(1.268)^4$

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Vocabulary

If needed, share the meaning of the term with students.

compound interest: Interest calculated based on the initial amount and the interest from previous periods. It is calculated at regular intervals (daily, monthly, annually, etc.). The balance in an account that earns compound interest can be modeled by an exponential function.

Writing Exponential Functions Involving Compound Interest

Guided Practice

A Consider showing students how to input each expression into a calculator and compare their outputs to check equivalence.

Key Takeaway:
Say, "When working with exponential functions, it's often useful to express them in different, yet equivalent, forms using the properties of exponents."

Guided Practice

For Problems 6–8, select the *three* expressions that can be used to represent the balance after t years.

6. A bank account with a balance of \$300 earns 5% monthly interest for 3 years with no payments or withdrawals.
   - [ ] $300 \cdot (1.05)^3$
   - [x] $300 \cdot (1.05)^{36}$
   - [x] $300 \cdot (1.05^{12})^3$
   - [x] $300 \cdot (1.796)^3$

7. A bank account with a balance of \$500 earns 4% monthly interest for 4 years of no payments or withdrawals.
   - [x] $500 \cdot (1.601)^4$
   - [x] $500 \cdot (1.04^{48})$
   - [x] $500 \cdot (1.04^{12})^4$
   - [ ] $500 \cdot (1.04)^4$

8. A bank account with a balance of \$400 earns 3% monthly interest for 5 years of no payments or withdrawals.
   - [x] $400 \cdot (1.03^{60})$
   - [ ] $400 \cdot (1.03)^5$
   - [x] $400 \cdot (1.426)^5$
   - [x] $400 \cdot (1.03^{12})^5$

Check

A bank account with a balance of \$600 earns 5% monthly interest. Select *three* of the expressions that can be used to represent the balance after 6 years of no payments or withdrawals.

- [x] $600 \cdot (1.05^{12})^6$
- [x] $600 \cdot (1.05^{72})$
- [ ] $600 \cdot (1.05)^6$
- [x] $600 \cdot (1.796)^6$

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Reflection

Ask:
- "How is determining the monthly growth factor different from determining the annual growth factor?"
- "Reflect on your learning today. What were you most proud of?"

Check: Recommended Next Steps

Almost there
If students need additional support, consider using Problems 1–5 as an anchor chart. Use the solutions from these problems to label the exponential functions on the chart.

Got it!
If students need more practice, ask them to write three equivalent expressions for the following situation.

A bank account with a balance of \$400 earns 5% monthly interest for 3 years of no payments or withdrawals.

Unit 6

Mini-Lessons

Name ___________________________

Describing Key Features of Parabolas

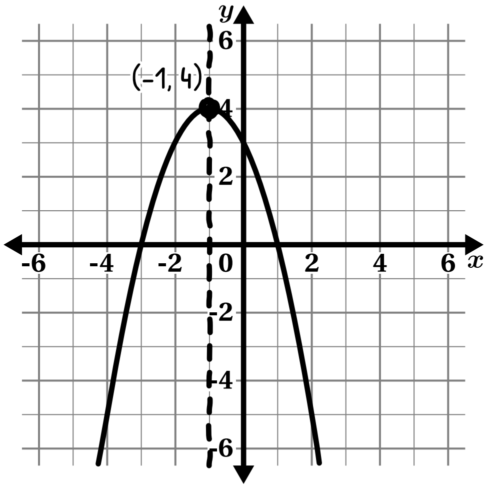
ML 6.06

Modeled Review

Name: Binta

Use the graph to determine each key feature.

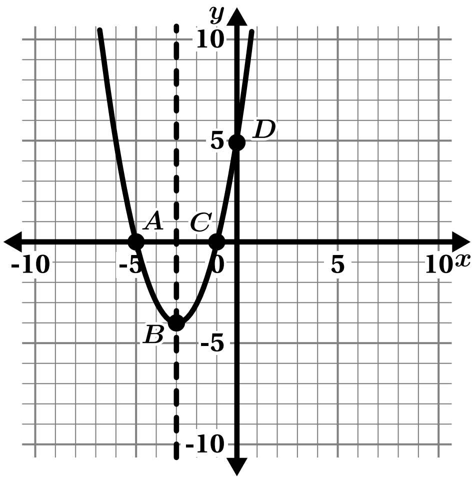
Key Features	
vertex	(−1, 4)
x-intercept(s)	(−3, 0) and (1, 0)
y-intercept	(0, 3)
axis of symmetry	x = − 1
Circle one: concave up	concave down



Guided Practice

1. The key features of this parabola are labeled A, B, C, D. Match each key feature with a term in the table.

Key Features	
vertex	
x-intercept(s)	
y-intercept	
axis of symmetry	$x = -3$
Circle one: concave up	concave down

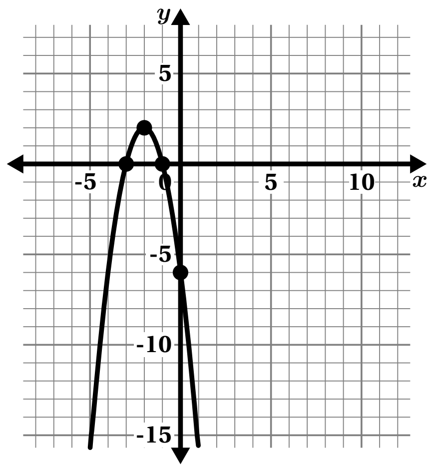


Guided Practice

For Problems 2–3, use the graph to determine each key feature.

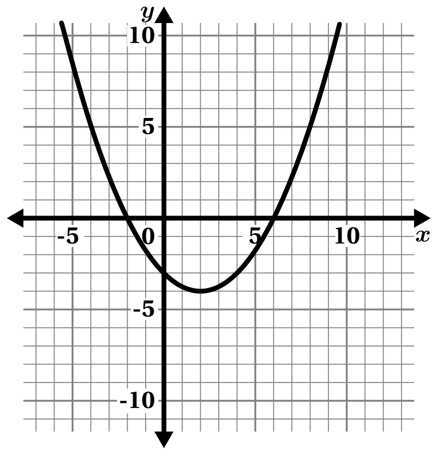
2.

Key Features	
vertex	
x-intercept(s)	
y-intercept	
axis of symmetry	
Circle one: concave up concave down	



3.

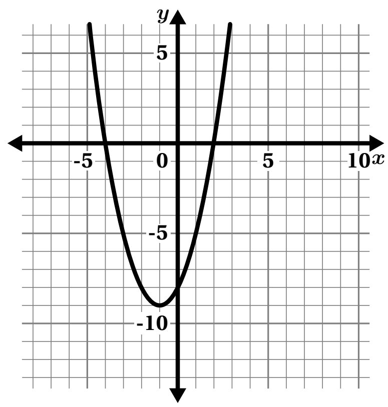
Key Features	
vertex	
x-intercept(s)	
y-intercept	
axis of symmetry	
Circle one: concave up concave down	



Check

Use the graph to determine each key feature.

Key Features	
vertex	
x-intercept(s)	
y-intercept	
axis of symmetry	
Circle one: concave up concave down	



Describing Key Features of Parabolas

Goal	**Standard**
Describe the key features of a parabola from its graph (vertex, intercepts, concavity, and axis of symmetry).	MA.912.AR.3.7

Modeled Review

Point to Binta's work and **ask**:

- "How did Binta identify the vertex of the parabola?"
- "How did Binta determine the x-and y-intercepts?"
- "How was the vertex used to find the axis of symmetry?"
- "Why did Binta say the parabola is concave down?"

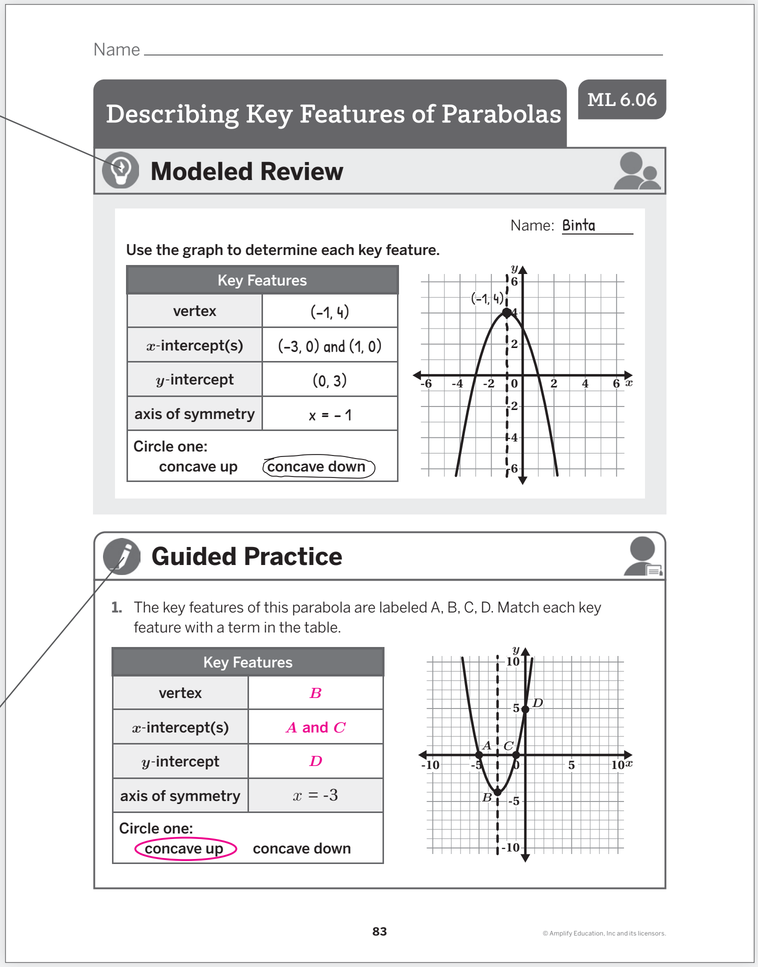
Reinforce Binta's thinking by saying, "You can describe parabolas by their key features, such as the vertex, intercepts, concavity and axis of symmetry.

Guided Practice

Focus students' attention on using the graph to determine the key features of the parabola.

To scaffold their thinking, **ask**:

- "What steps do you take to identify the vertex of a parabola?"
- "How can you locate the x-intercepts and y-intercept on a graph?"
- "How do you determine the axis of symmetry?"
- "Is this parabola concave up or concave down? How do you know?"



Name

Describing Key Features of Parabolas ML 6.06

Modeled Review

Name: Binta

Use the graph to determine each key feature.

Key Features	
vertex	(−1, 4)
x-intercept(s)	(−3, 0) and (1, 0)
y-intercept	(0, 3)
axis of symmetry	x = −1
Circle one: concave up	concave down

Guided Practice

1. The key features of this parabola are labeled A, B, C, D. Match each key feature with a term in the table.

Key Features	
vertex	B
x-intercept(s)	A and C
y-intercept	D
axis of symmetry	$x = -3$
Circle one: concave up	concave down

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Vocabulary

If needed, share the meaning of the term with students.

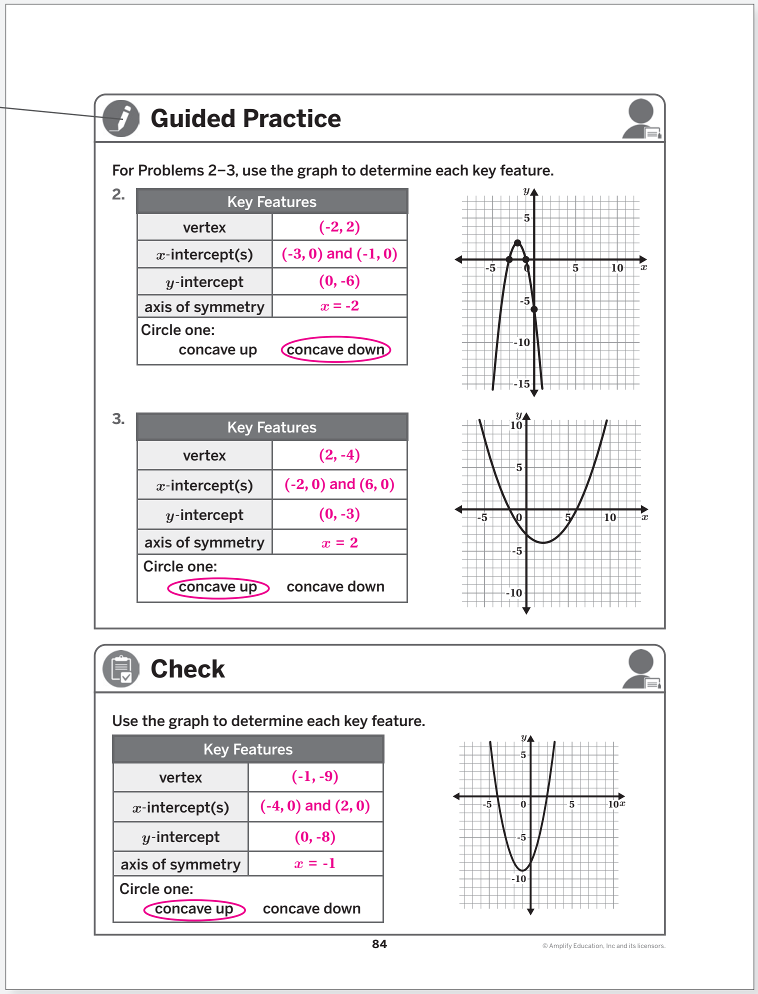
vertex: On the graph of a quadratic or absolute value function, the vertex is the maximum or minimum point. The vertex is also where the function changes from increasing to decreasing, or vice versa.

Guided Practice

A Consider modeling by drawing a point at the vertex, and marking the x-intercepts and y-intercept to help students accurately record the ordered pairs. Additionally, draw a vertical dashed line from the vertex to represent the axis of symmetry.

EL Use gestures to illustrate the concepts of concave up and concave down for parabolas. For example, use an upward-curving motion with your hand to represent concave up, and a downward-curving motion to represent concave down.

Key Takeaway:
Say, "Using key feature language like vertex, concave up, concave down, x-intercept, y-intercept, and line of symmetry can be helpful for describing parabolas."



Guided Practice

For Problems 2–3, use the graph to determine each key feature.

2.

Key Features	
vertex	(-2, 2)
x-intercept(s)	(-3, 0) and (-1, 0)
y-intercept	(0, -6)
axis of symmetry	x = -2
Circle one: concave up	concave down

3.

Key Features	
vertex	(2, -4)
x-intercept(s)	(-2, 0) and (6, 0)
y-intercept	(0, -3)
axis of symmetry	x = 2
Circle one: concave up	concave down

Check

Use the graph to determine each key feature.

Key Features	
vertex	(-1, -9)
x-intercept(s)	(-4, 0) and (2, 0)
y-intercept	(0, -8)
axis of symmetry	x = -1
Circle one: concave up	concave down

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Reflection

Ask:

- "How is the vertex related to a parabola's axis of symmetry?"
- "What advice would you give a friend for describing the key features of parabolas?"

Check: Recommended Next Steps

Almost there
If students need more support, consider displaying an anchor chart that illustrates a sketch of a parabola along with its key features, such as: vertex, x-intercept, y-intercept, concave up or down, and axis of symmetry.

Got it!
If students need more practice, ask them to revisit Problem 1 and write the coordinates for each key feature.

Name ______________________________

Graphing Quadratic Functions in Factored Form

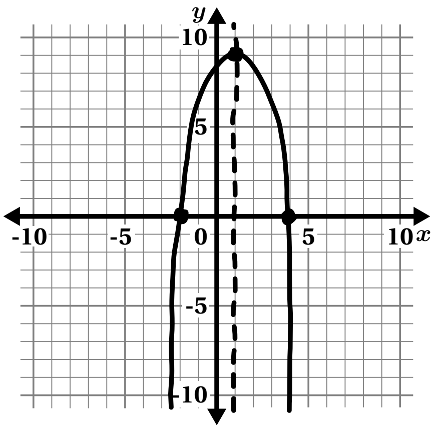
ML 6.10

Modeled Review

Name: Carlos

Consider the function $g(x) = (x + 2)\ (-x + 4)$. Determine the x-intercepts and the vertex of $g(x)$. Sketch the graph.

y-intercept	(−2, 0)
x-intercept	(4, 0)
vertex	(1, 9)



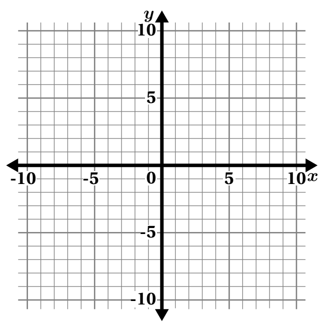
x + 2 = 0 −x + 4 = 0 g(1) = (1 + 2)(−1 + 4)
−2 + 2 = 0 −(4) + 4 = 0 g(1) = (3)(3)
(−2, 0) (4, 0) g(1) = 9

The line x = 1 is halfway between the x-intercepts, −2 and 4, making it the axis of symmetry.

Guided Practice

1. Determine the x-intercepts and the vertex of $h(x) = (-x + 3)(x + 1)$ to help graph the function. Show your thinking.

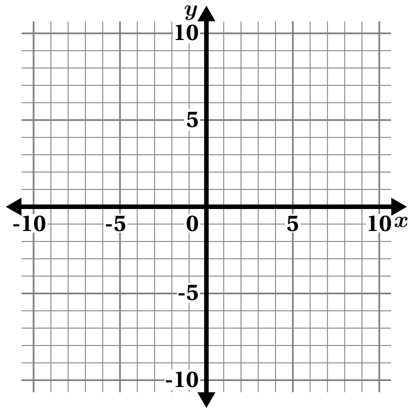
Step 1: Determine the x-intercepts.	$-x + 3 = 0$ $x + 1 = 0$ $-\_\_ + 3 = 0$ $\_\_ + 1 = 0$ $(\_\_\ 0)$ $(\_\_\ 0)$
Step 2: Determine the axis of symmetry.	$x = \_\_$
Step 3: Determine the vertex.	$h(\quad) = (-\_\_ + 3)\ (\_\_ + 1)$ $h(\_\_) = (\_\_)\ (\_\_)$ $h(\_\_) = \_\_$



Guided Practice

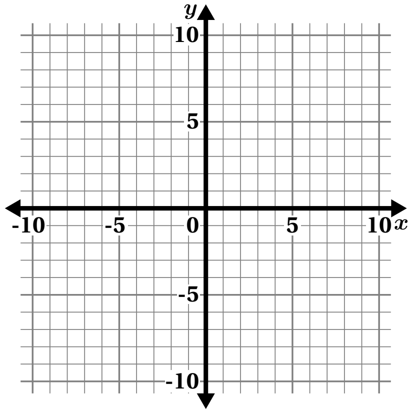
2. Consider the function $b(x) = (x + 4)\ (x + 6)$. Determine the x-intercepts and the vertex of $b(x)$. Sketch the graph.

x-intercept	
x-intercept	
vertex	



3. Consider the function $a(x) = (-x + 2)\ (x + 4)$. Determine the x-intercepts and the vertex of $a(x)$. Sketch the graph.

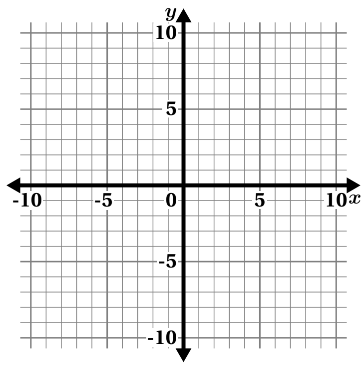
x-intercept	
x-intercept	
vertex	



Check

Consider the function $m(x) = (x + 1)(x - 5)$. Determine the x-intercepts and the vertex of $m(x)$. Sketch the graph.

x-intercept	
x-intercept	
vertex	



ML 6.10 Graphing Quadratic Functions in Factored Form

Goal	Standard
Graph quadratic functions written in factored form using their key features.	MA.912.AR.3.7

Modeled Review

Point to Carlos' work and **ask**:

- "Why did Carlos set each factor equal to 0?"
- "How did Carlos determine which values would make each factor equal to 0?"
- "What steps did Carlos take after finding the x-intercepts?"
- "How did Carlos use the information to graph the function?"

Reinforce Carlos' thinking by saying, "You can graph quadratic functions written in factored form by setting each factor equal to zero and solving for the value of x. Then, use the x-intercepts to calculate for the vertex."

Guided Practice

Focus students' attention on graphing the quadratic function.

To scaffold their thinking, **ask**:

- "What steps do you take to determine the x-intercepts?"
- "How do you use the
- x-intercepts to determine the vertex?"
- "How do you use the
- x-intercepts and vertex to determine the axis of symmetry?"

Name

Graphing Quadratic Functions in Factored Form

ML 6.10

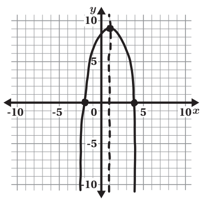
Modeled Review

Name: Carlos

Consider the function $g(x) = (x + 2)\ (-x + 4)$. Determine the x-intercepts and the vertex of $g(x)$. Sketch the graph.

y-intercept	(-2, 0)
x-intercept	(4, 0)
vertex	(1, 9)

x + 2 = 0   -x + 4 = 0   g(1) = (1 + 2)(-1 + 4)
-2 + 2 = 0   -(4) + 4 = 0   g(1) = (3)(3)
(-2, 0)   (4, 0)   g(1) = 9

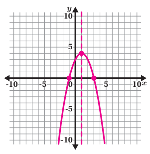


The line x = 1 is halfway between the x-intercepts, -2 and 4, making it the axis of symmetry.

Guided Practice

1. Determine the x-intercepts and the vertex of $h(x) = (-x + 3)(x + 1)$ to help graph the function. Show your thinking.

Step 1: Determine the x-intercepts.	$-x + 3 = 0$ $-\underline{3} + 3 = 0$ $(\underline{3}, 0)$	$x + 1 = 0$ $\underline{-1} + 1 = 0$ $(\underline{-1}, 0)$
Step 2: Determine the axis of symmetry.	$x = \underline{1}$	
Step 3: Determine the vertex.	$h(1) = (-\underline{1} + 3)\ (\underline{1} + 1)$ $h(\underline{1}) = (\underline{2})\ (\underline{2})$ $h(\underline{1}) = \underline{4}$	



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Vocabulary

If needed, share the meaning of the terms with students.

x-intercept: A point where the graph of an equation or function crosses the x-axis or when $y = 0$.

axis of symmetry: A line that divides a figure or the graph of a function into two halves. For every point (except the vertex), there is a corresponding point on the other side of the line that is the same distance from the line.

ML 6.10 Graphing Quadratic Functions in Factored Form

Guided Practice

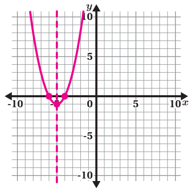
A Consider creating a graphic organizer to help identify key features of a quadratic function. This organizer should include sections for x-intercepts, the vertex, and the axis of symmetry.

Key Takeaway:
Say, "When a quadratic function is written in factored form, the x-intercepts are the values of x that makes a factor equal to zero. By plotting the x-intercepts and applying symmetry, you can identify the vertex."

Guided Practice

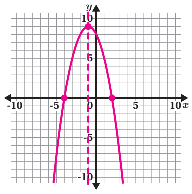
2. Consider the function $b(x) = (x + 4)(x + 6)$. Determine the x-intercepts and the vertex of $b(x)$. Sketch the graph.

x-intercept	(-6, 0)
x-intercept	(-4, 0)
vertex	(-5, -1)



3. Consider the function $a(x) = (-x + 2)(x + 4)$. Determine the x-intercepts and the vertex of $a(x)$. Sketch the graph.

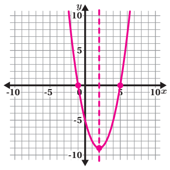
x-intercept	(-4, 0)
x-intercept	(2, 0)
vertex	(-1, 9)



Check

Consider the function $m(x) = (x + 1)(x - 5)$. Determine the x-intercepts and the vertex of $m(x)$. Sketch the graph.

x-intercept	(-1, 0)
x-intercept	(5, 0)
vertex	(2, -9)



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Reflection

Ask:
- "Describe how to identify the key features of parabola when it is given in factored form."
- "How did you overcome a hard problem today?"

Check: Recommended Next Steps

Almost there	Got it!
If students need more support, consider modeling how to find the axis of symmetry. Annotate the graph by marking the vertex and drawing a vertical dashed line through it.	If students need more practice, ask them to graph the function $c(x) = (x + 3)(x + 5)$.

Name ______________________________

Writing Quadratic Functions in Vertex Form

ML 6.13

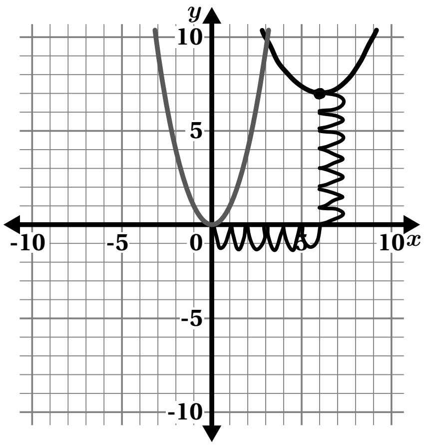
Modeled Review

Name: Deja

Write a quadratic function with a vertex at (6, 7), in the form $f(x) = (x - h)^2 + k$. Use the graph of the parent function if it is helpful.

(6,7)
h k
$f(x) = (x - 6)^2 + 7$

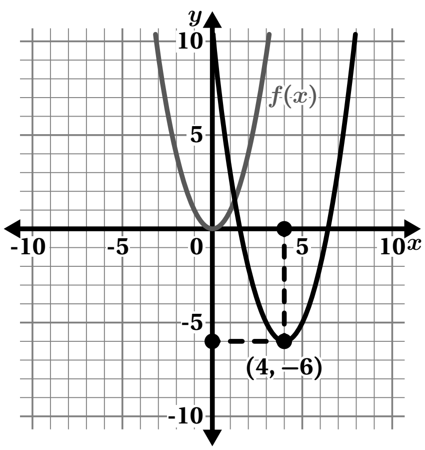
equation: $f(x) = (x - 6)^2 + 7$



Guided Practice

1. The graph shows two functions: the parent function $f(x) = x^2$, and another function with a vertex at (4, -6). Choose the correct equation for the function with a vertex at (4, -6).

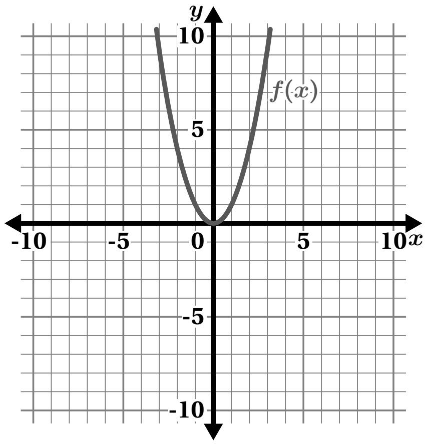
A. $f(x) = (x + 4)^2 - 6$ **B.** $f(x) = (x - 4)^2 - 6$



Guided Practice

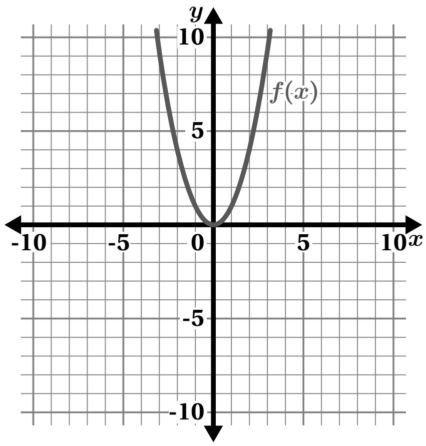
For Problems 2–3, write a quadratic function with the given vertex in the form $f(x) = (x - h)^2 + k$. Use the graph of the parent function if it is helpful.

2. $(5, 6)$



Equation: ____________________

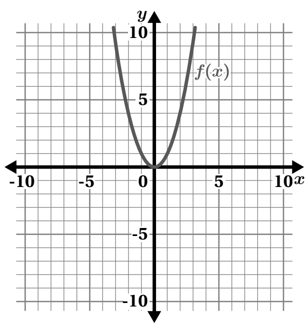
3. $(-3, 4)$



Equation: ____________________

Check

Write a quadratic function with a vertex at $(5, -2)$, in the form $f(x) = (x - h)^2 + k$. Use the graph of the parent function if it is helpful.



Equation: ____________________

ML 6.13 Writing Quadratic Functions in Vertex Form

Goal

Given the vertex, write quadratic functions in vertex form, $f(x) = (x - h)^2 + k$.

Standard

MA.912.AR.3.5

Modeled Review

Point to Deja's work and **ask**:

- "How did Deja use the graph to help with her thinking?"
- "What do the values h and k represent in the quadratic function?"
- "Why did Deja write the equation as $f(x) = (x-6)^2 + 7$ and not $f(x) = (x + 6)^2 + 7$?"

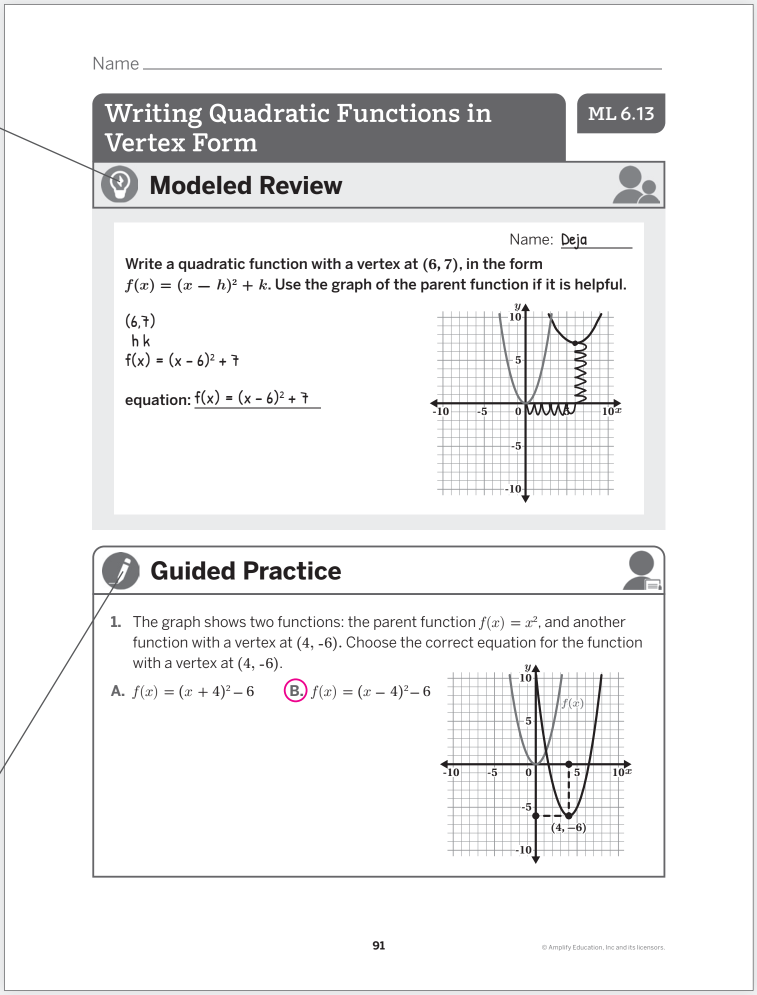
Reinforce Deja's thinking by saying, "You can write a parabola in vertex form, $f(x) = (x - h)^2 + k$. The parabola's vertex is at (h, k), indicating how it has been shifted from the origin horizontally by h units and vertically by k units."

Guided Practice

Focus students' attention on using the graph to determine the correct equation.

To scaffold their thinking, **say**:

- "First, identify the vertex."
- "Then, if necessary, use the graph to understand the horizontal and vertical translations."
- "Last, determine which equation includes the correct values of h and k."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Writing Quadratic Functions in Vertex Form

ML 6.13

Modeled Review

Name: Deja

Write a quadratic function with a vertex at $(6, 7)$, in the form $f(x) = (x - h)^2 + k$. Use the graph of the parent function if it is helpful.

(6,7)
h k
f(x) = (x - 6)² + 7

equation: f(x) = (x - 6)² + 7

Guided Practice

1. The graph shows two functions: the parent function $f(x) = x^2$, and another function with a vertex at $(4, -6)$. Choose the correct equation for the function with a vertex at $(4, -6)$.

A. $f(x) = (x + 4)^2 - 6$ B. $f(x) = (x - 4)^2 - 6$

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Vocabulary

If needed, share the meaning of the terms with students.

vertex form: One of three common forms of a quadratic equation. The equation of a quadratic function in vertex form looks like $f(x) = (x - h)^2 + k$.

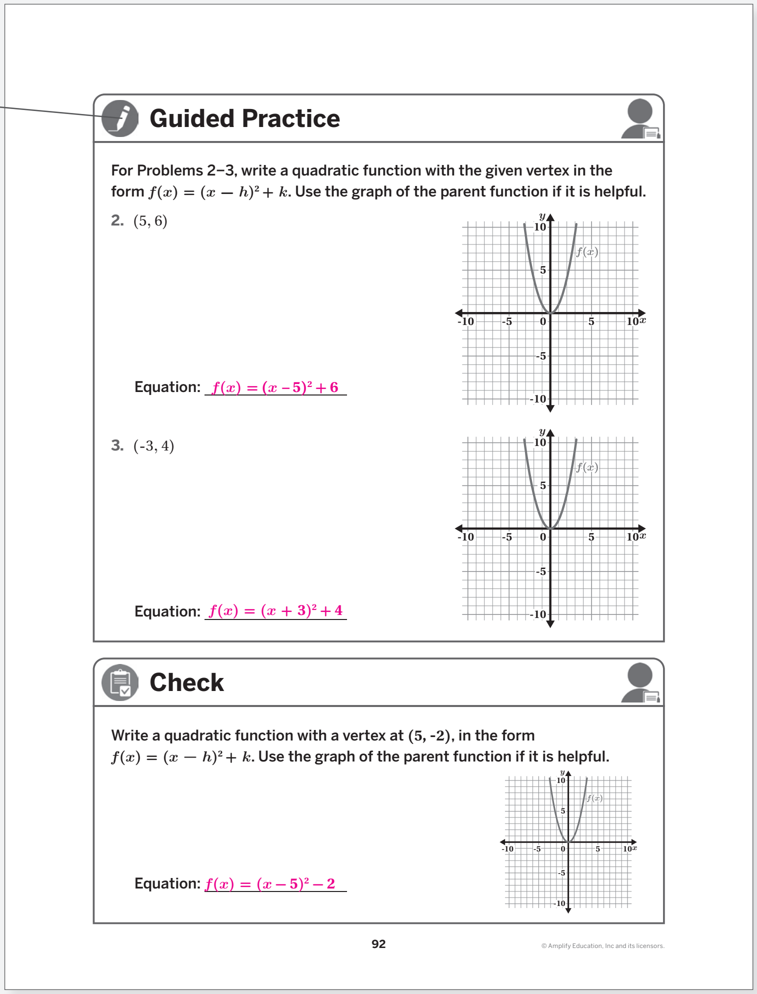
translation: A transformation that moves every point in a function a given distance in a given direction. A translation changes the location of a function, but does not change its shape.

ML 6.13 Writing Quadratic Functions in Vertex Form

Guided Practice

A Consider modeling how to substitute the values of h and k into the equation. This will help ensure students understand the correct use of the vertex formula, and, in particular, how to handle the positive and negative values involved.

Key Takeaway:
Say, "Writing a parabola in vertex form helps show how it has been translated horizontally and vertically from the origin, and reveals the coordinates of the vertex. A parabola with a vertex at (h, k) can be written in vertex form as $f(x) = (x - h)^2 + k$."



Guided Practice

For Problems 2–3, write a quadratic function with the given vertex in the form $f(x) = (x - h)^2 + k$. Use the graph of the parent function if it is helpful.

2. (5, 6)

Equation: $f(x) = (x - 5)^2 + 6$

3. (-3, 4)

Equation: $f(x) = (x + 3)^2 + 4$

Check

Write a quadratic function with a vertex at (5, -2), in the form $f(x) = (x - h)^2 + k$. Use the graph of the parent function if it is helpful.

Equation: $f(x) = (x - 5)^2 - 2$

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Reflection

Ask:

- "What steps can you take to write a quadratic function in vertex form?"
- "What strategy did someone else share today that was helpful?"

Check: Recommended Next Steps

Almost there	Got it!
If students need more support, consider creating an anchor chart that includes a graph of the parent function, a translated function in vertex form, and the vertex shown as an ordered pair on the graph.	If students need more practice, have them write quadratic functions in vertex form using the following vertices: • (8, 10) • (-2, 7) • (-5, -9)

Unit 7

Mini-Lessons

Name ____________________

Rewriting Factored-Form Expressions in Standard Form

ML 7.02

Modeled Review

Name: Luis

Multiply to rewrite $(x-4)(2x-10)$ in standard form.

	x	-4
$2x$	$2x \cdot x$ $2x^2$	$2x \cdot -4$ $-8x$
-10	$-10 \cdot x$ $-10x$	$-10 \cdot -4$ 40

$2x^2 - 8x - 10x + 40$

standard form: $2x^2 - 18x + 40$

Guided Practice

1. Multiply to rewrite $(3x-1)(x+2)$ in standard form by completing the diagram.

	x	2
$3x$	$3x \cdot x$ $3x^2$	$3x \cdot 2$
-1	$-1 \cdot x$	$-1 \cdot 2$

standard form: ____________________

Guided Practice

2. Multiply to rewrite $(x - 3)(4x - 2)$ in standard form.

x	-3

standard form: ________________

3. Multiply to rewrite $(5x + 1)(x - 4)$ in standard form.

standard form: ________________

Check

Multiply to rewrite $(2x - 4)(x + 5)$ in standard form.

standard form: ________________

ML 7.02 Rewriting Factored-Form Expressions in Standard Form

Goal	**Standard**
Use an area model to convert a quadratic expression from factored form to standard form.	MA.912.AR.1.3

Modeled Review

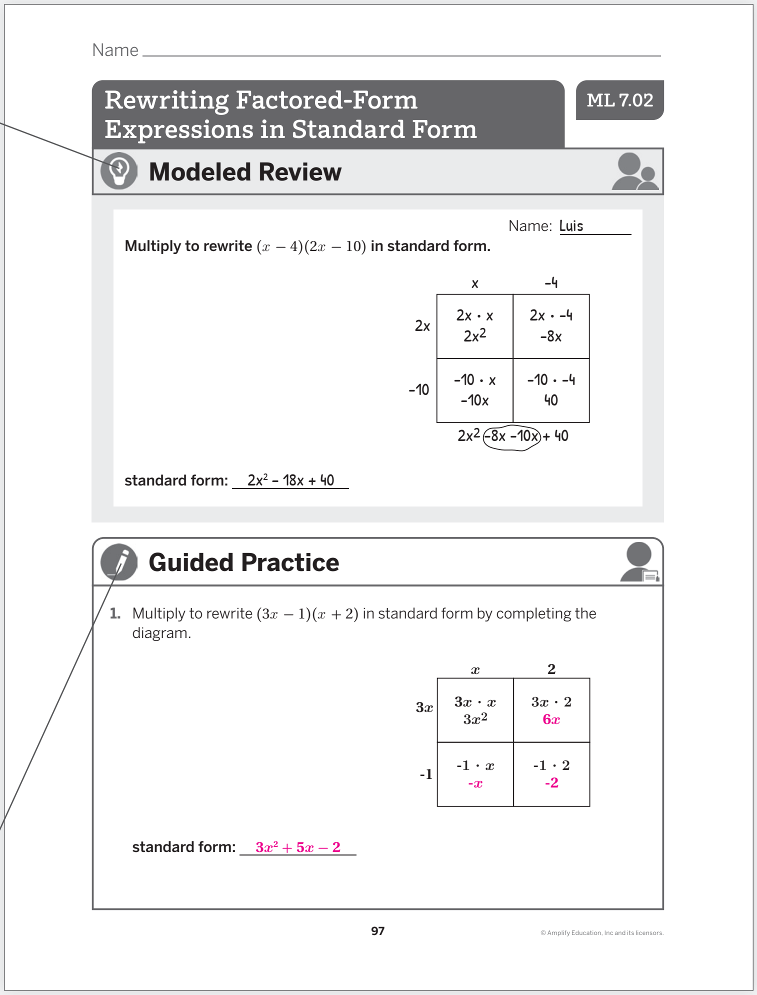
Point to Luis' work and **ask**:
- "Is the order in which Luis wrote each factor important? Would it change the answer?"
- "How did Luis calculate the product in each section?"
- "Why did Luis circle the terms $-8x$ and $-10x$?"
- "How did Luis know his expression was written in standard form?"

Reinforce Luis' thinking by saying, "You can rewrite a quadratic expression in standard form by using an area model and treating each factor as a length and width. Multiply these factors and combine like terms to write an expression in standard form."

Guided Practice

Focus students' attention on completing the diagram to rewrite the expression in standard form.

To scaffold their thinking, **say**:
- "First, determine the area of each section using multiplication."
- "Then, list all the terms."
- "Last, combine the like terms to rewrite the expression in standard form."


Name

Rewriting Factored-Form Expressions in Standard Form

ML 7.02

Modeled Review

Name: Luis

Multiply to rewrite $(x-4)(2x-10)$ in standard form.

	x	-4
2x	2x · x 2x²	2x · -4 -8x
-10	-10 · x -10x	-10 · -4 40

2x² -8x -10x + 40

standard form: 2x² – 18x + 40

Guided Practice

1. Multiply to rewrite $(3x-1)(x+2)$ in standard form by completing the diagram.

	x	2
$3x$	$3x \cdot x$ $3x^2$	$3x \cdot 2$ $6x$
-1	$-1 \cdot x$ $-x$	$-1 \cdot 2$ -2

standard form: $3x^2 + 5x - 2$

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Vocabulary

If needed, share the meaning of the terms with students.

factored form: One of three common forms of a quadratic equation. A quadratic equation in factored form looks like $f(x) = a(x - m)(x - n)$.

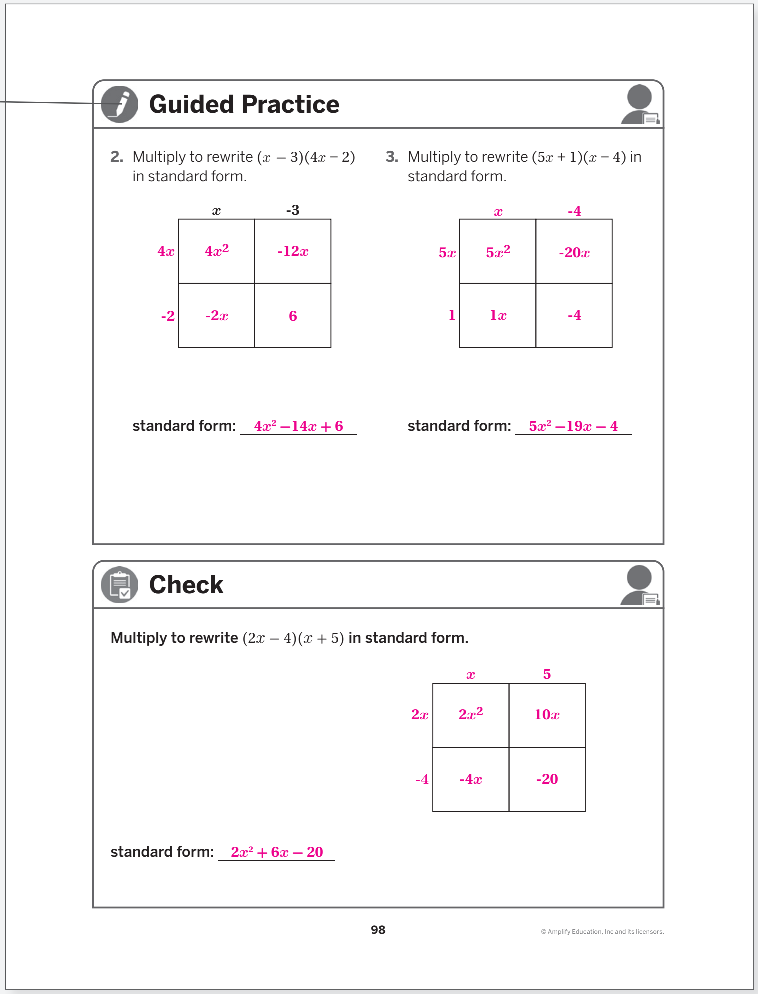
standard form (of a quadratic equation): One of three common forms of a quadratic equation. A quadratic equation in standard form looks like $f(x) = ax^2 + bx + c$.

Rewriting Factored-Form Expressions in Standard Form

Guided Practice

A Consider having students select a section of the diagram and identify its dimensions using their fingers to trace the corresponding column and row labels. Model this process by writing inside each section what is being multiplied.

Key Takeaway:
Say, "An area model is a helpful tool for rewriting a factored-form expression in standard form. Factored form can be seen along the length and width of the area model, while standard form can be seen as the sum of the individual areas inside the model."



Guided Practice

2. Multiply to rewrite $(x-3)(4x-2)$ in standard form.

	x	-3
$4x$	$4x^2$	$-12x$
-2	$-2x$	6

standard form: $4x^2 - 14x + 6$

3. Multiply to rewrite $(5x+1)(x-4)$ in standard form.

	x	-4
$5x$	$5x^2$	$-20x$
1	$1x$	-4

standard form: $5x^2 - 19x - 4$

Check

Multiply to rewrite $(2x-4)(x+5)$ in standard form.

	x	5
$2x$	$2x^2$	$10x$
-4	$-4x$	-20

standard form: $2x^2 + 6x - 20$

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Reflection

Ask:
- "Desribe how to rewrite a factored-form expression in standard form."
- "Reflect on your learning today. What were you most proud of?"

Check: Recommended Next Steps

Almost there
If students need more support, consider color coding the terms within each product to visually group like terms after multiplying factors.

Got it!
If students need more practice, have them multiply to rewrite each expression in standard form.
- $(x+3)(5x-7)$
- $(2x-1)(x+6)$
- $(x-2)(3x-9)$

Name ______________________________

Solving Quadratic Equations by Graphing

ML 7.11

Modeled Review

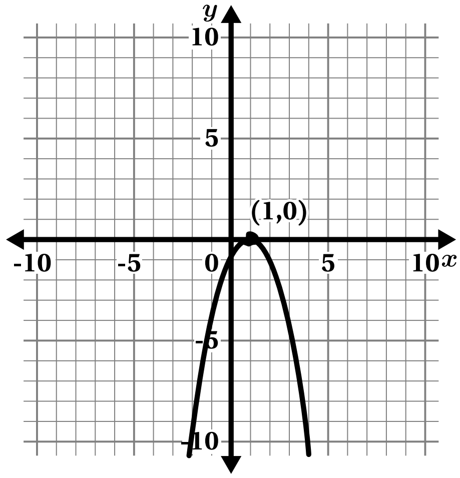
Name: Mai

Use the graph to solve $y = -x^2 + 2x - 1$. Record any solution(s) here:

$x =$ 1

$x =$ ________

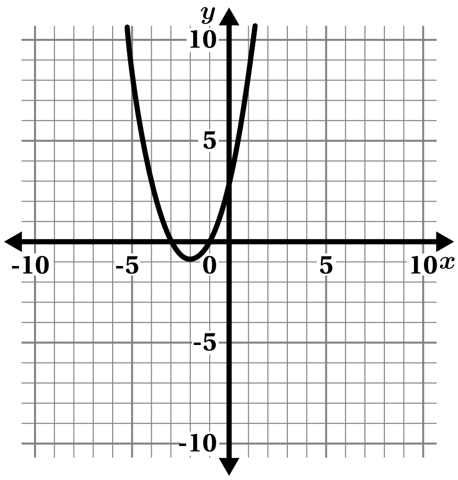
The solutions are the x-intercepts, where the graph crosses the x-axis. Since it only crosses once there is only one solution.



Guided Practice

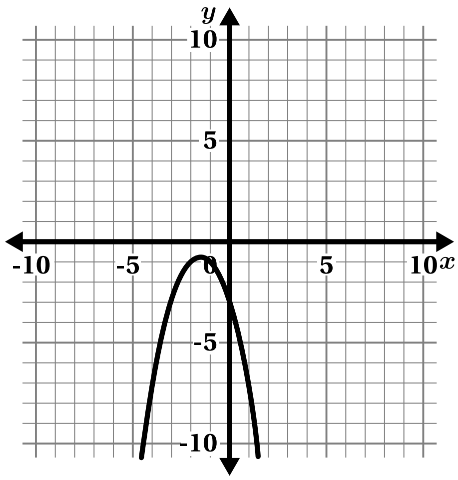
Determine the number of solutions for each graph.

1.



A. No solutions **B.** One solutions **C.** Two solutions

2.



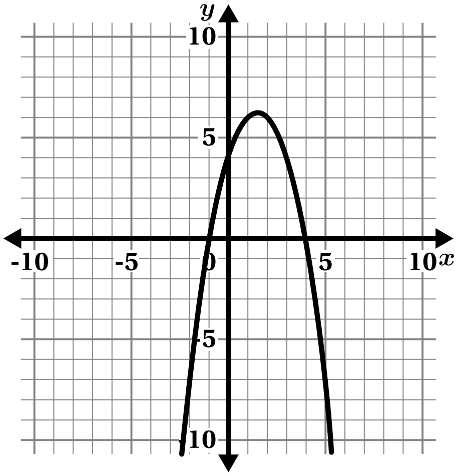
A. No solutions **B.** One solutions **C.** Two solutions

Guided Practice

3. Use the graph to solve $y = -x^2 + 3x + 4$.
Record any solution(s) here:

$x =$ ________

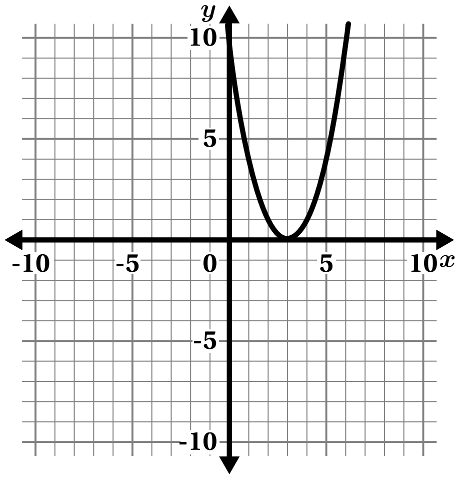
$x =$ ________



4. Use the graph to solve $y = -x^2 - 6x + 9$.
Record any solution(s) here:

$x =$ ________

$x =$ ________

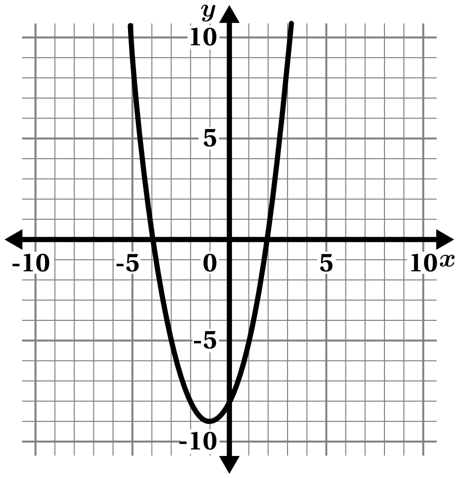


Check

Use the graph to solve $y = x^2 + 2x - 8$.
Record any solution(s) here:

$x =$ ________

$x =$ ________



ML 7.11 Solving Quadratic Equations by Graphing

Goal

Use graphs to solve quadratic equations with zero, one, or two solutions.

Standard

MA.912.AR.3.1

Materials

highlighter (optional)

Modeled Review

Point to Mai's work and **ask**:

- "How did Mai use the graph to solve the quadratic equation?"
- "Why did Mai only write down one solution?"
- "Can a quadratic equation ever have no solutions?"

Reinforce Mai's thinking by saying, "You can find the solutions to a quadratic equation by looking at its graph to see where it intersects the x-axis."

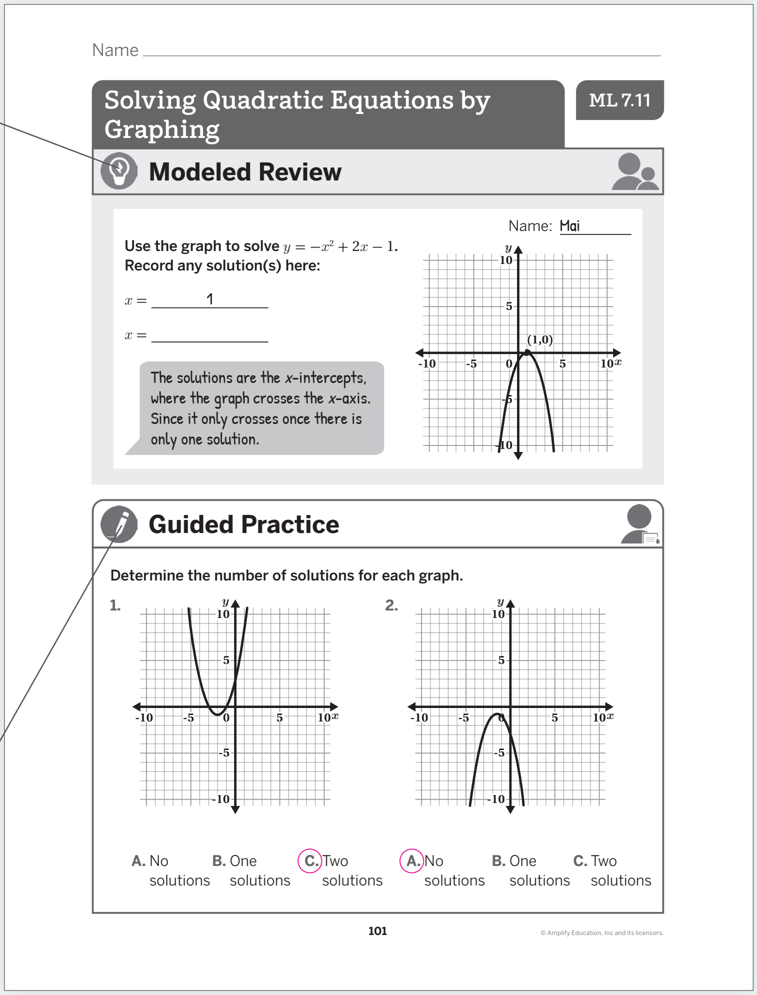
EL Emphasize that x-intercepts, solutions, and zeros are synonymous.

Guided Practice

Focus students' attention on determining the number of solutions using each graph.

To scaffold their thinking, **ask**:

- "First, identify where the graph intersects the x-axis."
- "Then, determine the number of x-intercepts."
- "If there are no x-intercepts, there are no solutions."



Name

Solutions Quadratic Equations by Graphing — ML 7.11

Modeled Review

Name: Mai

Use the graph to solve $y = -x^2 + 2x - 1$.
Record any solution(s) here:

$x =$ 1

$x =$

The solutions are the x-intercepts, where the graph crosses the x-axis. Since it only crosses once there is only one solution.

Guided Practice

Determine the number of solutions for each graph.

1\.

A. No solutions B. One solutions C. Two solutions

2\.

A. No solutions B. One solutions C. Two solutions

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Vocabulary

If needed, share the meaning of the terms with students.

point of intersection: A point where two lines or curves meet.

zeros: The x-values that make a function equal zero, or $f(x) = 0$.

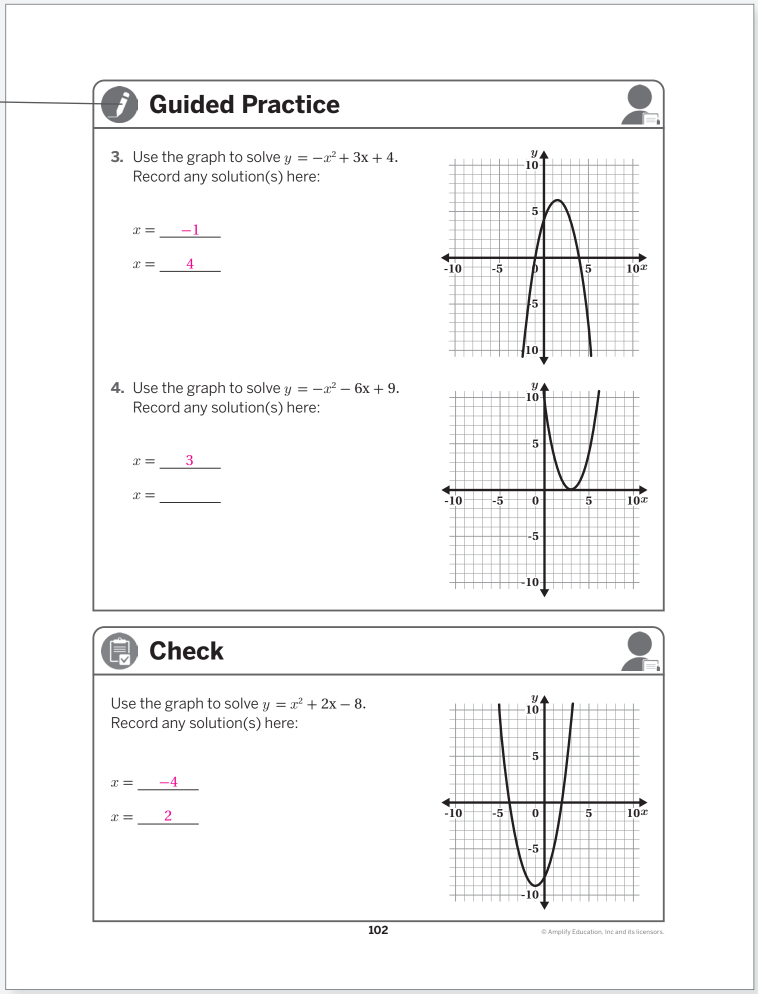
x-intercept: A point where the graph of an equation or function crosses the x-axis or when $y = 0$.

Solving Quadratic Equations by Graphing

Guided Practice

A Consider circling the intersection points on the x-axis and and writing the (x, y) coordinates to clearly highlight the solutions.

Key Takeaway:
Say, "Graphs can be used to determine the solutions to a quadratic equation by finding the x-intercepts."



Guided Practice

3. Use the graph to solve $y = -x^2 + 3x + 4$.
Record any solution(s) here:

$x =$ -1

$x =$ 4

4. Use the graph to solve $y = -x^2 - 6x + 9$.
Record any solution(s) here:

$x =$ 3

$x =$ ______

Check

Use the graph to solve $y = x^2 + 2x - 8$.
Record any solution(s) here:

$x =$ -4

$x =$ 2

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Reflection

Ask:
- "Describe how you can use a graph to determine the number of solutions to a quadratic equation."
- "What strategy was helpful today?"

Check: Recommended Next Steps

Almost there	**Got it!**
If students need more support, use a highlighter to mark the x-axis to clarify that solutions are found where the graph intersects it.	If students need more practice, ask them to revisit Problems 1–2, and determine the solution(s) for each graph.

Name ______________________________

Solving Quadratic Equations by Completing the Square

ML 7.14

Modeled Review

Name: Miko

Solve the equation $x^2 - 16x = -5$ by completing the square.

$x =$ $8 + \sqrt{59}$

$x =$ $8 - \sqrt{59}$

$x^2 - 16x = -5$
$x^2 - 16x + 64 = -5 + 64$
$(x - 8)^2 = 59$
$x - 8 = \pm\sqrt{59}$
$x = 8 \pm \sqrt{59}$

Guided Practice

1. Follow the steps to solve the quadratic equation $x^2 + 6x = 2$ by completing the square.

Steps	Completing the square
Step 1: Add the same number to both sides to create a perfect square.	$x^2 + 6x = 2$ $x^2 + 6x$ ____ $= 2$ ____
Step 2: Rewrite the perfect square $x^2 + 6x + 9$ in factored form.	$(x +$ ____ $)^2 =$ ____
Step 3: Take the square root and include both possibilities by writing ±.	______ = ______
Step 4: Solve for x.	$x =$ ________ $x =$ ________

Guided Practice

For Problems 2–5, solve each quadratic equation by completing the square.

2. $x^2 - 8x = 5$

3. $x^2 + 4x = 5$

$x =$ ________ $x =$ ________ $x =$ ________ $x =$ ________

4. $x^2 - 10x = -9$

5. $x^2 + 12x = 2$

$x =$ ________ $x =$ ________ $x =$ ________ $x =$ ________

Check

Solve the equation $x^2 + 14x = -13$ by completing the square.

$x =$ ________ $x =$ ________

ML 7.14 Solving Quadratic Equations by Completing the Square

Goal

Solve quadratic equations by completing the square.

Standard

MA.912.AR.3.1

Modeled Review

Point to Miko's work and **ask**:

- "Why did Miko add 64 to both sides?"
- "How did Miko factor the perfect square?"
- "Why is it important for Miko to use ± when taking the square root of a number?"

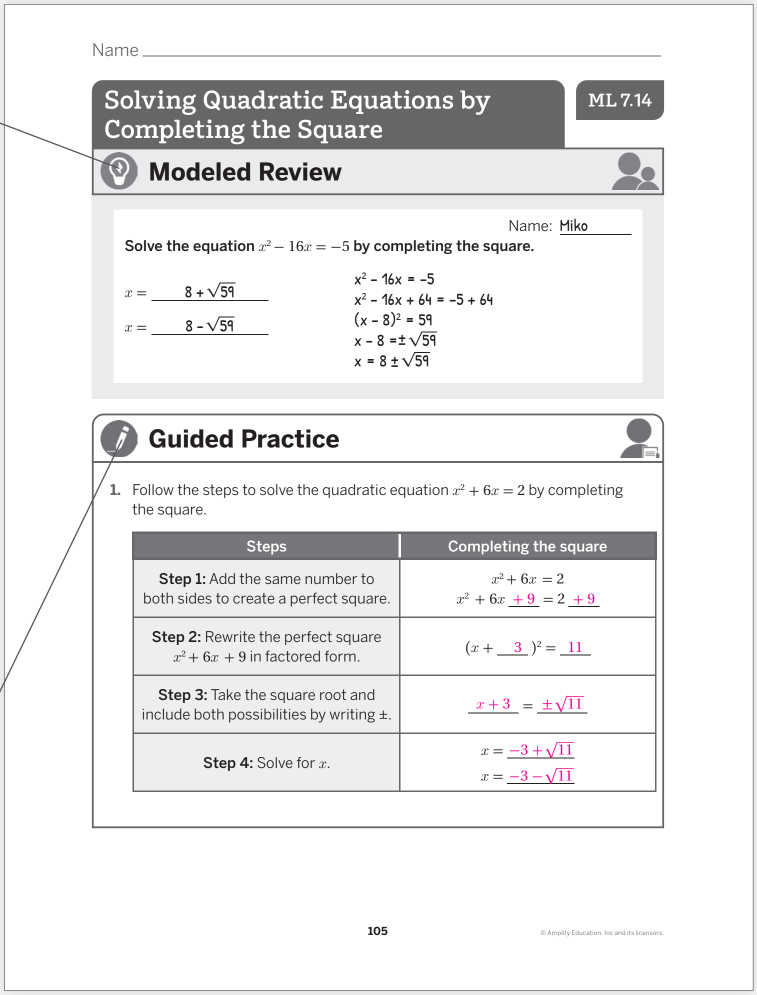
Reinforce Miko's thinking by saying "You can solve quadratic equations by completing the square, which involves adding a constant value to each expression to create a perfect square, and then taking the square root to solve for x."

Guided Practice

Focus students' attention on completing the square to solve the quadratic equation.

To scaffold their thinking, **ask**:

- "What constant should be added to make a perfect square?"
- "How do you rewrite the perfect square in factored form?"
- "How do you solve for x?"
- "Why do you need to include both solutions?"



Name ____________________

Solving Quadratic Equations by Completing the Square ML 7.14

Modeled Review

Name: Miko

Solve the equation $x^2 - 16x = -5$ by completing the square.

$x =$ $8 + \sqrt{59}$

$x =$ $8 - \sqrt{59}$

$x^2 - 16x = -5$
$x^2 - 16x + 64 = -5 + 64$
$(x - 8)^2 = 59$
$x - 8 = \pm\sqrt{59}$
$x = 8 \pm \sqrt{59}$

Guided Practice

1. Follow the steps to solve the quadratic equation $x^2 + 6x = 2$ by completing the square.

Steps	Completing the square
Step 1: Add the same number to both sides to create a perfect square.	$x^2 + 6x = 2$ $x^2 + 6x$ $\underline{+9}$ $= 2$ $\underline{+9}$
Step 2: Rewrite the perfect square $x^2 + 6x + 9$ in factored form.	$(x + \underline{3})^2 = \underline{11}$
Step 3: Take the square root and include both possibilities by writing ±.	$\underline{x+3} = \underline{\pm\sqrt{11}}$
Step 4: Solve for x.	$x = \underline{-3+\sqrt{11}}$ $x = \underline{-3-\sqrt{11}}$

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Vocabulary

If needed, share the meaning of the terms with students.

completing the square: The process of rewriting a quadratic expression or equation to include a perfect square.

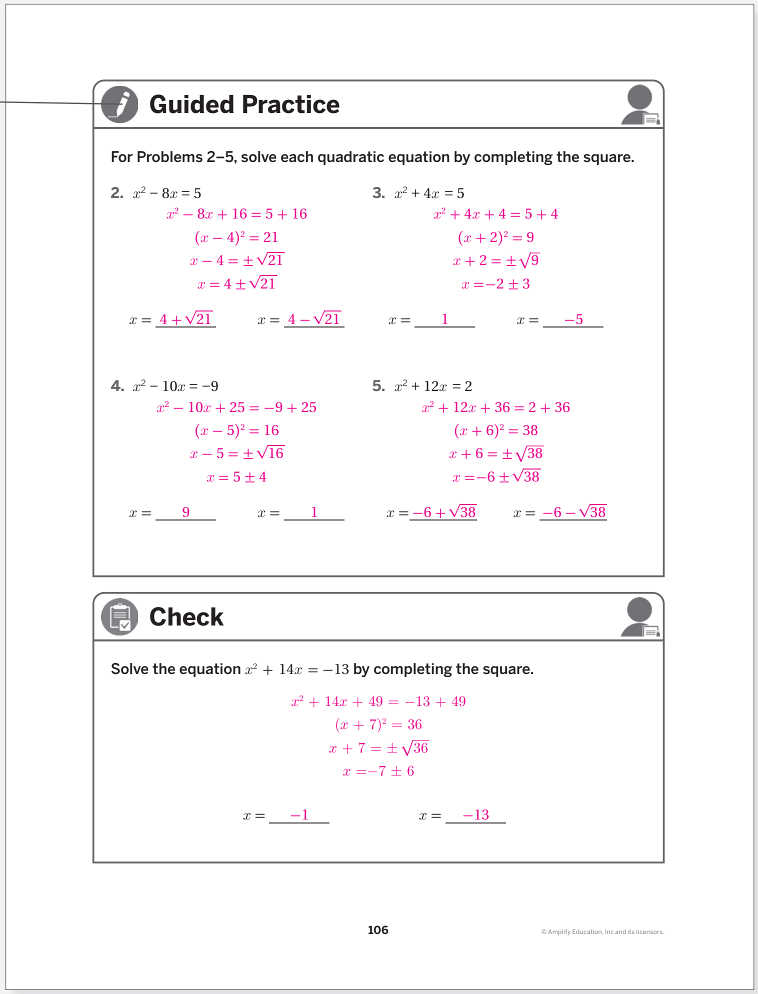
perfect square: An expression that can be written as something multiplied by itself.

Solving Quadratic Equations by Completing the Square

Guided Practice

A Guide students by providing a list of perfect squares to help them determine which constant value is needed to complete the square.

Key Takeaway:
Say, “Completing the square is the process of rewriting quadratic equations to include perfect squares, often by adding the same number to both sides. Once the perfect square is isolated, it is possible to take the square root of both sides to find the solutions to the quadratic equation.”



Guided Practice

For Problems 2–5, solve each quadratic equation by completing the square.

2. $x^2 - 8x = 5$

$x^2 - 8x + 16 = 5 + 16$
$(x - 4)^2 = 21$
$x - 4 = \pm\sqrt{21}$
$x = 4 \pm \sqrt{21}$

$x =$ $4 + \sqrt{21}$ $x =$ $4 - \sqrt{21}$

3. $x^2 + 4x = 5$

$x^2 + 4x + 4 = 5 + 4$
$(x + 2)^2 = 9$
$x + 2 = \pm\sqrt{9}$
$x = -2 \pm 3$

$x =$ 1 $x =$ -5

4. $x^2 - 10x = -9$

$x^2 - 10x + 25 = -9 + 25$
$(x - 5)^2 = 16$
$x - 5 = \pm\sqrt{16}$
$x = 5 \pm 4$

$x =$ 9 $x =$ 1

5. $x^2 + 12x = 2$

$x^2 + 12x + 36 = 2 + 36$
$(x + 6)^2 = 38$
$x + 6 = \pm\sqrt{38}$
$x = -6 \pm \sqrt{38}$

$x =$ $-6 + \sqrt{38}$ $x =$ $-6 - \sqrt{38}$

Check

Solve the equation $x^2 + 14x = -13$ by completing the square.

$x^2 + 14x + 49 = -13 + 49$
$(x + 7)^2 = 36$
$x + 7 = \pm\sqrt{36}$
$x = -7 \pm 6$

$x =$ -1 $x =$ -13

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Reflection

Ask:
- “How do you determine what constant to add to create a perfect square?”
- “What is something you weren’t sure about at the start of the lesson but understand now?”

Check: Recommended Next Steps

Almost there
If students need more support, consider reviewing how to maintain balance in an equation when solving for a variable, and clarify that taking the square root is the inverse operation of squaring.

Got it!
If students need more practice, have them solve the following quadratic equations by completing the square.
- $x^2 + 8x = -8$
- $x^2 - 4x = 10$
- $x^2 + 16x = 8$

Name ____________________

Solving Quadratic Equations Using the Quadratic Formula

ML 7.17

Modeled Review

Name: Raven

Use the quadratic formula to solve the equation $2x^2 + 5x - 12 = 0$.

Quadratic formula: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

a = 2 b = 5 c = -12

$$\frac{-(5) + \sqrt{5^2 - 4(2)(-12)}}{2(2)}$$

$$\frac{-5 \pm \sqrt{121}}{4} = \frac{-5 \pm 11}{4} \qquad \frac{-5 + 11}{4} = 1.5 \qquad \frac{-5 - 11}{4} = -4$$

$x =$ 1.5 $x =$ -4

Guided Practice

1. Use the quadratic formula to solve the equation $x^2 - 3x - 7 = 0$.

Quadratic formula: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Steps	Quadratic formula
Step 1: Identify the a-, b-, and c-values from the standard form quadratic equation.	$a =$ ___ $b = -3$ $c =$ ___
Step 2: Substitute each value into the quadratic formula.	$\frac{-(-3) \pm = \sqrt{(-3)^2 - 4(\ )(\ \ )}}{2(\ )}$
Step 3: Use order of operations to simplify the expression.	$x = \frac{3 \pm \sqrt{37}}{}$
Step 4: Write the solutions.	$x =$ ______ $x =$ ______

Guided Practice

For Problems 2–5, use the quadratic formula to solve each equation.

Quadratic formula: $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

2. $x^2 + 3x - 5 = 5$

$a =$ ____ $b =$ ____ $c =$ ____

$x =$ ________ $x =$ ________

3. $x^2 - 5x + 6 = 0$

$x =$ ________ $x =$ ________

4. $4x^2 - 6x + 2 = 0$

$x =$ ________ $x =$ ________

5. $3x^2 + 2x - 4 = 0$

$x =$ ________ $x =$ ________

Check

Use the quadratic formula to solve the equation $3x^2 - 10x + 6 = 0$

Quadratic formula: $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$x =$ ________ $x =$ ________

Solving Quadratic Equations Using the Quadratic Formula

Goal

Use the quadratic formula to solve quadratic equations.

Standard	Materials
MA.912.AR.3.1	graphing calculator (optional)

Modeled Review

Point to Raven's work and **ask**:

- "Why did Raven write a negative sign next to the first 5 in the formula?"
- "Is the square of a number always positive?"
- "What steps did Raven take to determine the solutions, 1.5 and –4?"

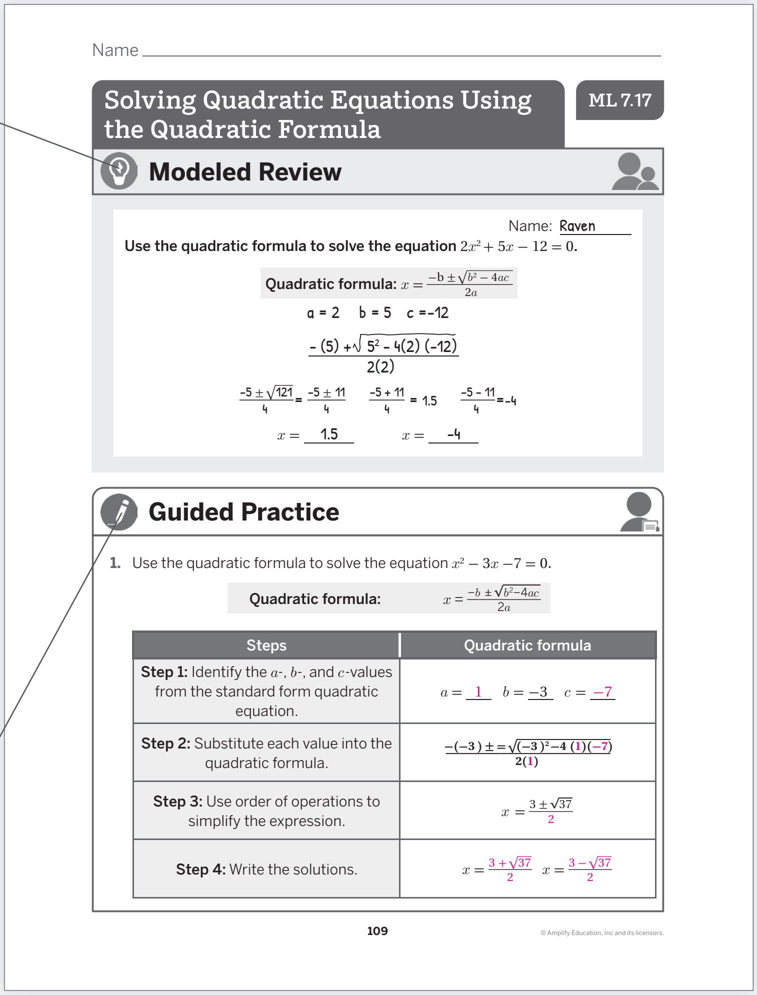
Reinforce Raven's thinking by saying, "You can solve quadratic equations by applying the quadratic formula and substituting the known values of a, b and c. Then, use the order of operations to calculate the solution(s)."

Guided Practice

Focus students' attention on using the quadratic formula to solve the equation.

To scaffold their thinking, **ask**:

- "How do you determine which terms are a, b, and c?"
- "How do you substitute the values into the formula?"
- "Which operations under the square root should you perform first?"
- "How can you check your solution(s)?"



Name

Solving Quadratic Equations Using the Quadratic Formula

ML 7.17

Modeled Review

Name: Raven

Use the quadratic formula to solve the equation $2x^2 + 5x - 12 = 0$.

Quadratic formula: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

a = 2 b = 5 c = –12

$\frac{-(5) \pm \sqrt{5^2 - 4(2)(-12)}}{2(2)}$

$\frac{-5 \pm \sqrt{121}}{4} = \frac{-5 \pm 11}{4}$ $\frac{-5 + 11}{4} = 1.5$ $\frac{-5 - 11}{4} = -4$

$x =$ 1.5 $x =$ –4

Guided Practice

1. Use the quadratic formula to solve the equation $x^2 - 3x - 7 = 0$.

Quadratic formula: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Steps	Quadratic formula
Step 1: Identify the a-, b-, and c-values from the standard form quadratic equation.	$a =$ 1 $b =$ –3 $c =$ –7
Step 2: Substitute each value into the quadratic formula.	$\frac{-(-3) \pm \sqrt{(-3)^2 - 4(1)(-7)}}{2(1)}$
Step 3: Use order of operations to simplify the expression.	$x = \frac{3 \pm \sqrt{37}}{2}$
Step 4: Write the solutions.	$x = \frac{3 + \sqrt{37}}{2}$ $x = \frac{3 - \sqrt{37}}{2}$

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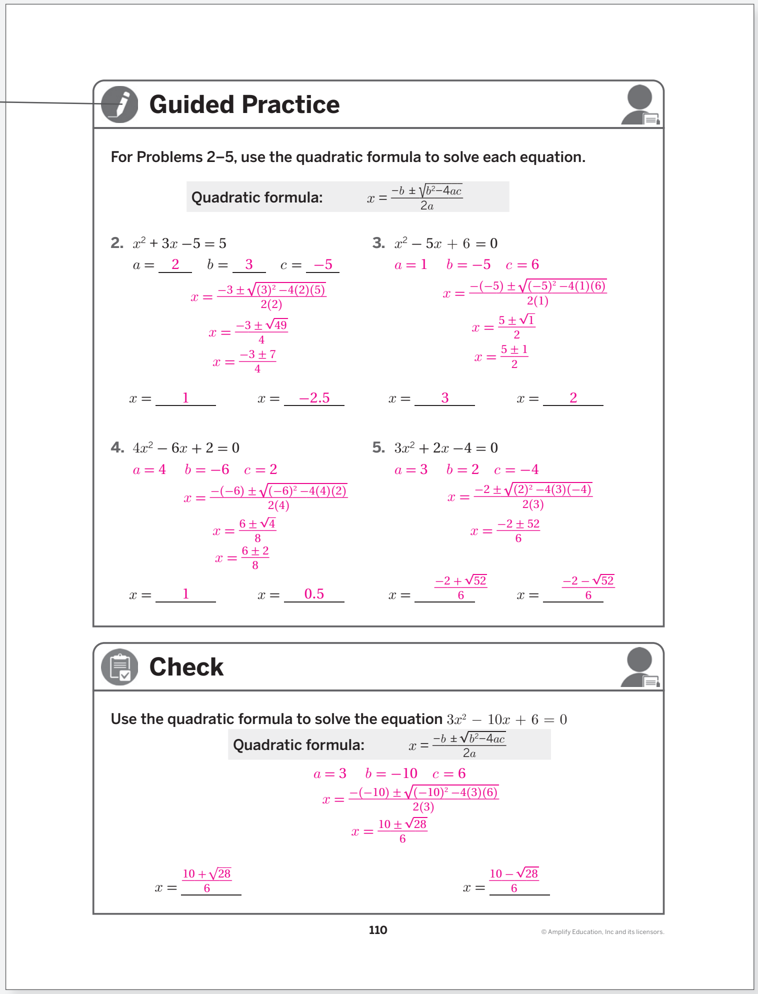
Solving Quadratic Equations Using the Quadratic Formula

Guided Practice

A Provide students with a graphing calculator to support them with calculations.

Key Takeaway:
Say, "The quadratic formula can be used to solve any quadratic equation. It is a strategy that is always useful, although sometimes it can involve more calculations than other strategies like factoring."



Guided Practice

For Problems 2–5, use the quadratic formula to solve each equation.

Quadratic formula: $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

2. $x^2 + 3x - 5 = 5$

$a =$ 2 $b =$ 3 $c =$ -5

$x = \frac{-3 \pm \sqrt{(3)^2 - 4(2)(5)}}{2(2)}$

$x = \frac{-3 \pm \sqrt{49}}{4}$

$x = \frac{-3 \pm 7}{4}$

$x =$ 1 $x =$ -2.5

3. $x^2 - 5x + 6 = 0$

$a = 1$ $b = -5$ $c = 6$

$x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(1)(6)}}{2(1)}$

$x = \frac{5 \pm \sqrt{1}}{2}$

$x = \frac{5 \pm 1}{2}$

$x =$ 3 $x =$ 2

4. $4x^2 - 6x + 2 = 0$

$a = 4$ $b = -6$ $c = 2$

$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(4)(2)}}{2(4)}$

$x = \frac{6 \pm \sqrt{4}}{8}$

$x = \frac{6 \pm 2}{8}$

$x =$ 1 $x =$ 0.5

5. $3x^2 + 2x - 4 = 0$

$a = 3$ $b = 2$ $c = -4$

$x = \frac{-2 \pm \sqrt{(2)^2 - 4(3)(-4)}}{2(3)}$

$x = \frac{-2 \pm 52}{6}$

$x = \frac{-2 + \sqrt{52}}{6}$ $x = \frac{-2 - \sqrt{52}}{6}$

Check

Use the quadratic formula to solve the equation $3x^2 - 10x + 6 = 0$

Quadratic formula: $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$a = 3$ $b = -10$ $c = 6$

$x = \frac{-(-10) \pm \sqrt{(-10)^2 - 4(3)(6)}}{2(3)}$

$x = \frac{10 \pm \sqrt{28}}{6}$

$x = \frac{10 + \sqrt{28}}{6}$ $x = \frac{10 - \sqrt{28}}{6}$

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Reflection

Ask:

- "What are some advantages of using the quadratic formula?"
- "What advice would you give a friend about how to use the quadratic formula to solve quadratic equations?"

Check: Recommended Next Steps

Almost there
If students need more support, consider reviewing foundational prerequisite skills such as square roots, negative numbers, and algebraic operations and rules.

Got it!
If students need more practice, have them solve the quadratic equations below using the quadratic formula.

- $4x^2 - 7x + 3 = 0$
- $2x^2 + 5x + 1 = 0$
- $2x^2 - 5x + 3 = 0$

Prerequisite Skills and Concepts

Mini-Lessons

Name ____________________

Determining Relationships Between Rules

ML 7.09

Modeled Review

Name: Jada

1. Use the rules to complete the table.

Rule 1: Start with 0 and keep adding 9.

Rule 2: Start with 0 and keep adding 3.

Rule 1	0	9 ~~×3 = 27~~	18	27	36	45
Rule 2	0	÷ 3 3	÷ 3 6	÷ 3 9	÷ 3 12	÷ 3 15

2. Describe the relationship between the numbers in Rule 1 and Rule 2.

Divide the number in Rule 1 by 3.

Guided Practice

1. Use the rules to complete each table.

Rule 1: Start with 0 and keep adding 5.

Rule 2: Start with 0 and keep adding 10.

Rule 1	0	5	10				
Rule 2							

2. **Discuss:** How could you describe the relationship between the numbers in Rule 1 and Rule 2?

Guided Practice

3. Use the rules to complete the table.

   Rule 1: Start with 0 and keep adding 8.

   Rule 2: Start with 0 and keep adding 2.

Rule 1						
Rule 2						

4. Describe the relationship between the numbers in Rule 1 and Rule 2.

____

____

Check

1. Use the rules to complete the table.

   Rule 1: Start with 0 and keep adding 6.

   Rule 2: Start with 0 and keep adding 12.

Rule 1						
Rule 2						

2. Describe the relationship between the numbers in Rule 1 and Rule 2.

____

____

ML 7.09 Determining Relationships Between Rules

Goal

Generate patterns from two given rules and relate their corresponding terms.

Standard

MA.5.AR.3.2

Modeled Review

Point to Jada's work and **ask**:

- "How did Jada use the rules to complete the table?"
- "What relationship did Jada notice between 9 and 3? The other numbers in Rule 1 and Rule 2?"
- "How did Jada describe that relationship?"

Reinforce the goal by saying, "You can use two rules to generate a pattern that has a relationship between corresponding terms. You can describe that relationship in different ways."

Guided Practice

Focus students' attention on using the rules to describe the relationship between the numbers.

To scaffold their thinking, **ask**:

- "How can you use the rules to find the next number?"
- "Look at the second number in Rule 1 and the second number in Rule 2. What do you notice? How could you describe that relationship?"
- "When the number in Rule 1 is 50, what will be the corresponding term in Rule 2? How do you know?"

Name

Determining Relationships Between Rules

ML 7.09

Modeled Review

Name: Jada

1. Use the rules to complete the table.

Rule 1: Start with 0 and keep adding 9.

Rule 2: Start with 0 and keep adding 3.

Rule 1	0	9 ~~×3 = 27~~	18	27	36	45
Rule 2	0	÷3 3	÷3 6	÷3 9	÷3 12	÷3 15

2. Describe the relationship between the numbers in Rule 1 and Rule 2.

Divide the number in Rule 1 by 3.

Guided Practice

1. Use the rules to complete each table.

Rule 1: Start with 0 and keep adding 5.

Rule 2: Start with 0 and keep adding 10.

Rule 1	0	5	10	15	20	25	30
Rule 2	0	10	20	30	40	50	60

2. **Discuss:** How could you describe the relationship between the numbers in Rule 1 and Rule 2? Oral activity: No writing expected.

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Vocabulary

If needed, share the meaning of the term with students.

pattern: A specific, replicable sequence of shapes or numbers; some patterns repeat and some patterns grow.

Determining Relationships Between Rules

Guided Practice

A Encourage students to annotate the tables to support their understanding of the relationship between each rule.

Key Takeaway:
Say, "Two rules can be used to generate a pattern that has a relationship between corresponding terms. The relationship between corresponding terms can be described in different ways, including using multiplication or division."

Guided Practice

3. Use the rules to complete the table.

Rule 1: Start with 0 and keep adding 8.

Rule 2: Start with 0 and keep adding 2.

Rule 1	0	8	16	24	32	40
Rule 2	0	2	4	6	8	10

4. Describe the relationship between the numbers in Rule 1 and Rule 2.
Sample response shown.
Divide the number in Rule 1 by 4.

Check

1. Use the rules to complete the table.

Rule 1: Start with 0 and keep adding 6.

Rule 2: Start with 0 and keep adding 12.

Rule 1	0	6	12	18	24	30
Rule 2	0	12	24	36	48	60

2. Describe the relationship between the numbers in Rule 1 and Rule 2.
Sample response shown.
Multiply the number in Rule 1 by 2.

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Reflection

Ask:
- "What is one way you can describe the relationship between two rules?"
- "How was the lesson helpful to you today?"

Check: Recommended Next Steps

Almost there
If students need more support, consider using a think-aloud to complete the table in Problem 3 and describe the relationship between the numbers. Then have students revisit the Check.

Got it!
If students need more practice, invite them to describe the relationship between the two rules in the Check in a different way.

Name ____________________

Writing Equivalent Expressions Using the Area Model

ML 6.08

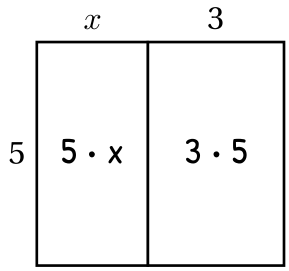
Modeled Review

Name: Jada

Write *two* equivalent expressions that represent the area of the rectangle.

Expression 1: $5(x + 3)$

Expression 2: $5x + 15$



5 times $x + 3$ or $5(x + 3)$

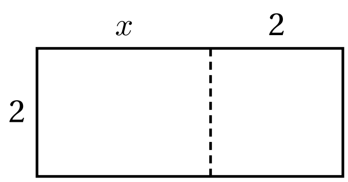
$5 \cdot x + 5 \cdot 3$ or $5 \cdot (x + 3)$

$5x + 15$

Guided Practice

Write *two* equivalent expressions using the area of a rectangle.

1.



$2 \cdot (x +$ ____$)$

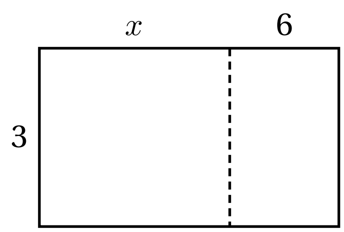
$2 \cdot x +$ ____

____ $+$ ____

Expression 1: ____________

Expression 2: ____________

2.



$3 \cdot ($____ $+$ ____$)$

____ $+$ ____

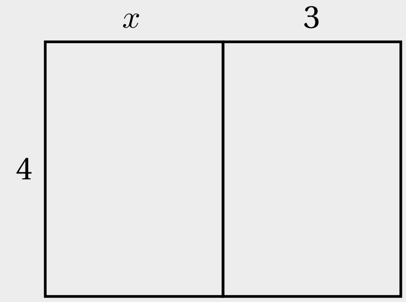
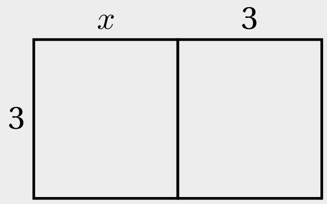
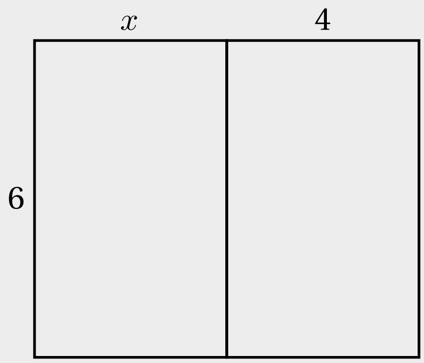
____ $+$ ____

Expression 1: ____________

Expression 2: ____________

Guided Practice

3. Write *two* equivalent expressions that represent the area of each rectangle.

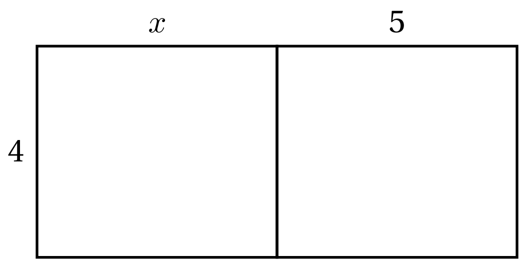
Area Model	Expression 1	Expression 2
	$4(x+3)$	
		
		

Check

Write *two* equivalent expressions that represent the area of the rectangle.

Expression 1: ____________

Expression 2: ____________



ML 6.08 Writing Equivalent Expressions Using the Area Model

Goal

Write equivalent expressions using the areas of rectangles.

Standard

MA.6.AR.1.4

Materials

algebra tiles (optional), highlighter (optional)

Modeled Review

Point to Jada's work and **ask**:
- "How did Jada use the area of the rectangle to write equivalent expressions?"
- "How are the two expressions alike? Different?"

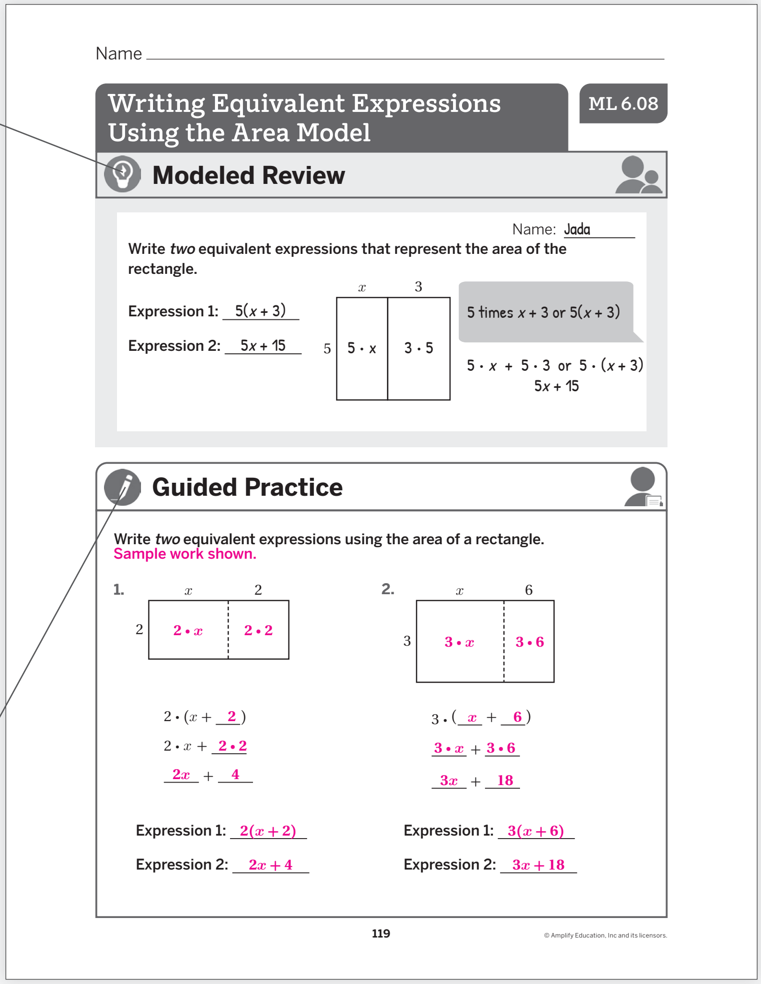
Reinforce Jada's thinking by saying, "The area of a rectangle can be used to create equivalent expressions."

ML/EL Provide sentence frames to help students explain their thinking (e.g., Expression 1 is equivalent to Expression 2 because ______.).

Guided Practice

Focus students' attention on using the area of a rectangle to write equivalent expressions.

To scaffold their thinking, **say**:
- "Determine the product of the length and width."
- "Determine the sum of the two smaller areas within the rectangle."


Name

Writing Equivalent Expressions Using the Area Model ML 6.08

Modeled Review

Name: Jada

Write *two* equivalent expressions that represent the area of the rectangle.

Expression 1: $5(x + 3)$

Expression 2: $5x + 15$

x 3

5   $5 \cdot x$ $3 \cdot 5$

5 times x + 3 or $5(x + 3)$

$5 \cdot x + 5 \cdot 3$ or $5 \cdot (x + 3)$

$5x + 15$

Guided Practice

Write *two* equivalent expressions using the area of a rectangle.

Sample work shown.

1. x 2

2   $2 \cdot x$ $2 \cdot 2$

$2 \cdot (x + \underline{2})$

$2 \cdot x + \underline{2 \cdot 2}$

$\underline{2x} + \underline{4}$

Expression 1: $2(x + 2)$

Expression 2: $2x + 4$

2. x 6

3   $3 \cdot x$ $3 \cdot 6$

$3 \cdot (\underline{x} + \underline{6})$

$\underline{3 \cdot x} + \underline{3 \cdot 6}$

$\underline{3x} + \underline{18}$

Expression 1: $3(x + 6)$

Expression 2: $3x + 18$

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Vocabulary

If needed, share the meaning of the terms with students.

equivalent expressions: Expressions that are equal for every value of a variable.

product: The value of two or more quantities when multiplied.

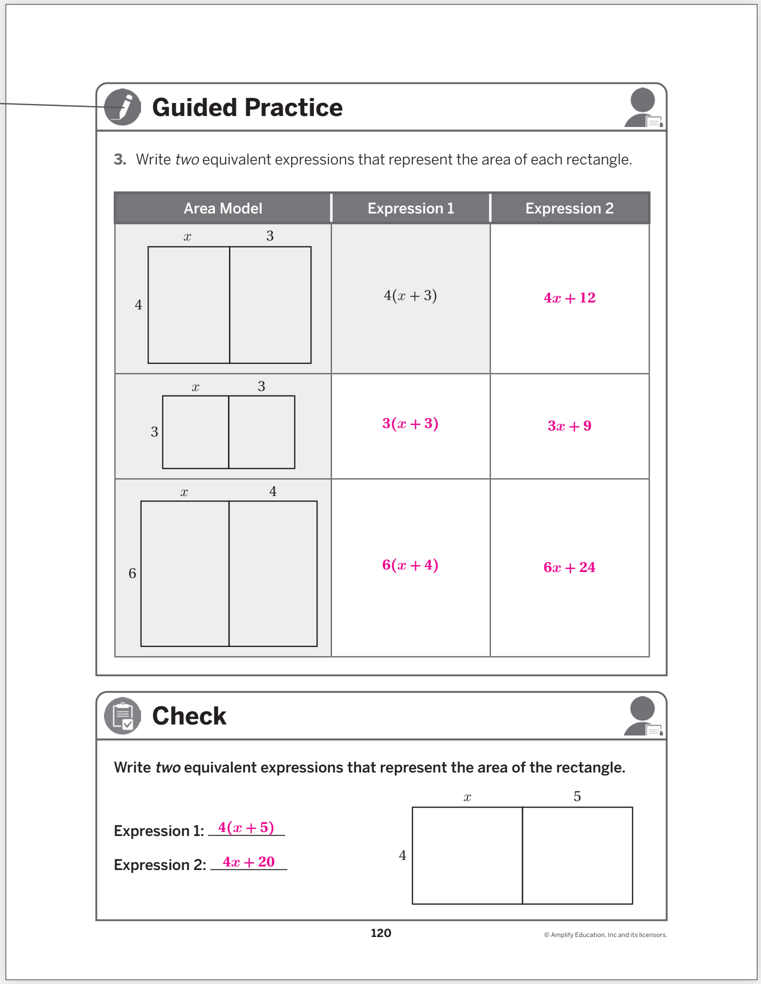
sum: The value of two or more quantities when added together.

Writing Equivalent Expressions Using the Area Model

Guided Practice

A Invite students to use algebra tiles to represent each expression.

Key Takeaway:
Say, "An area model is one way to generate equivalent expressions. The product of the length and width of a rectangle is one expression and the sum of the two smaller areas is another."



Guided Practice

3. Write *two* equivalent expressions that represent the area of each rectangle.

Area Model	Expression 1	Expression 2
x, 3; 4	$4(x+3)$	$4x+12$
x, 3; 3	$3(x+3)$	$3x+9$
x, 4; 6	$6(x+4)$	$6x+24$

Check

Write *two* equivalent expressions that represent the area of the rectangle.

Expression 1: $4(x+5)$

Expression 2: $4x+20$

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Reflection

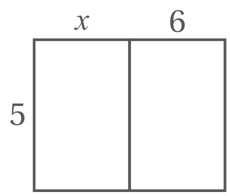
Ask:

- "How can areas of rectangles, also called area models, help you identify or create equivalent expressions?"
- "What questions do you still have?"

Check: Recommended Next Steps

Almost there
If students need more support, consider color coding the diagram in the Check to show the connection to the expressions. Highlight the length and width of the large rectangle in one color and the length and width of the two small rectangles in another color.

Got it!
If students need more practice, sketch the following area model and ask them to write two equivalent expressions that represent the area of the rectangle.



Name ______________________________

Evaluating Expressions With Exponents

ML 6.11

Modeled Review

Name: Kai

Determine the value of each expression.

1. $2 \cdot 3^2$ = 18

$2 \cdot (3 \cdot 3)$
$2 \cdot (9)$
18

I need to evaluate the part of the expression with the exponent first and then multiply.

2. $5 + (4 - 3)^2$ = 6

$5 + (1)^2$
$5 + (1 \cdot 1)$
$5 + 1$
6

I need to evaluate the grouped part of the expression first, then the exponent part. Then I can add.

Guided Practice

Determine the value of each expression.

1. 2^2

$2 \cdot$ ___

______

2. $2 + 3^2$

$2 + ($___ $\cdot$ ___$)$

___ $+$ ___

______

3. $(6 - 2)^2$

$($___$)^2$

$($______$)$

______

4. $1 + (3 - 2)^2$

$1 + ($___$)^2$

____

______

______

Guided Practice

5. Determine the value of each expression.

Expression	Value
$3^2 + 6$	
$(4 + 3)^2$	
$3 + (4 + 1)^2$	
$2 \cdot 4^2$	
$(7 - 1)^2 + 2$	
$6 + 7^2$	

Check

Determine the value of each expression.

1. $3 \cdot 5^2$ ______

2. $7 + (5 - 2)^2$ ______

ML 6.11 Evaluating Expressions With Exponents

Goal

Evaluate expressions with exponents using the order of operations.

Standard

MA.6.NSO.3.3

Materials

highlighter (optional)

Modeled Review

Point to Kai's work and **ask**:

- "How did Kai know which operation to evaluate first in each expression?"
- "Can you explain the order of operations Kai used to evaluate the expressions?"
- "What does the exponent 2 represent in each expression?"

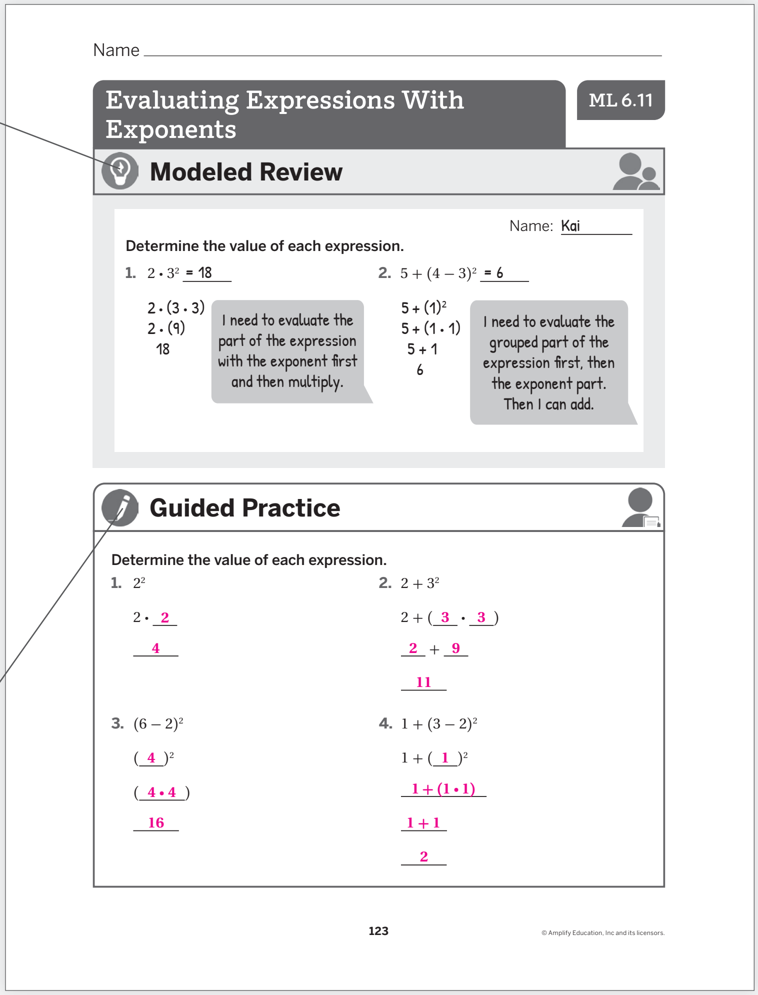
Reinforce Kai's thinking by saying, "The order of operations can be used to efficiently evaluate expressions with exponents."

Guided Practice

Focus students' attention on using the order of operations to evaluate each expression.

To scaffold their thinking, **say**:

- "First, evaluate brackets or parentheses."
- "Next, evaluate exponents."
- "Then, evaluate multiplication or division (left to right)."
- "Lastly, evaluate addition or subtraction (left to right)."



Name

Evaluating Expressions With Exponents ML 6.11

Modeled Review

Name: Kai

Determine the value of each expression.

1. $2 \cdot 3^2$ = 18

$2 \cdot (3 \cdot 3)$
$2 \cdot (9)$
18

I need to evaluate the part of the expression with the exponent first and then multiply.

2. $5 + (4 - 3)^2$ = 6

$5 + (1)^2$
$5 + (1 \cdot 1)$
$5 + 1$
6

I need to evaluate the grouped part of the expression first, then the exponent part. Then I can add.

Guided Practice

Determine the value of each expression.

1. 2^2

$2 \cdot \underline{2}$
$\underline{4}$

2. $2 + 3^2$

$2 + (\underline{3} \cdot \underline{3})$
$\underline{2} + \underline{9}$
$\underline{11}$

3. $(6 - 2)^2$

$(\underline{4})^2$
$(\underline{4 \cdot 4})$
$\underline{16}$

4. $1 + (3 - 2)^2$

$1 + (\underline{1})^2$
$\underline{1 + (1 \cdot 1)}$
$\underline{1 + 1}$
$\underline{2}$

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Vocabulary

If needed, share the meaning of the terms with students.

order of operations: A consistent order applied to an expression with multiple operations so that the expression is evaluated the same way by everyone.

exponent: A number used to describe repeated multiplication. Exponents are sometimes called powers.

Evaluating Expressions With Exponents

Guided Practice

A Use visual aids, such as anchor charts or diagrams, to illustrate the order of operations and guide students in understanding the correct sequence of steps to evaluate expressions with exponents.

ML/EL Demonstrate the step-by-step process of evaluating expressions with exponents, narrating your thinking aloud to show students how to apply the order of operations correctly.

Key Takeaway:
Say, "When evaluating expressions, evaluate exponents first, unless there are grouping symbols, like parentheses or a fraction bar. When there are grouping symbols, perform the operation(s) inside them first."

Guided Practice

5. Determine the value of each expression.

Expression	Value
$3^2 + 6$	15
$(4 + 3)^2$	49
$3 + (4 + 1)^2$	28
$2 \cdot 4^2$	32
$(7 - 1)^2 + 2$	38
$6 + 7^2$	55

Check

Determine the value of each expression.

1. $3 \cdot 5^2$ = 75

2. $7 + (5 - 2)^2$ = 16

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Reflection

Ask:

- "What are some things to remember when determining the value of expressions with exponents?"
- "How did you overcome a hard problem today?"

Check: Recommended Next Steps

Almost there
If students need more support, consider using a highlighter to color-code different operations and parts of the expression.

Got it!
If students need more practice, have them evaluate the following expressions:

- $9 \cdot 5^2$
- $20 - (5 - 1)^2$
- $36 - 2 \cdot 3^2$

Name ______________________________

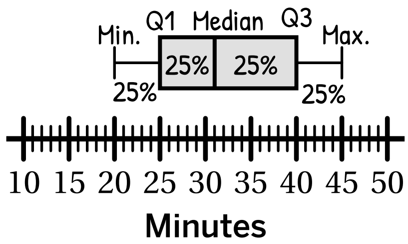
Interpreting Box Plots

ML 8.12

Modeled Review

Name: Santiago

Inola took the bus to school most days in January. She wrote down how many minutes her journey took each day and made this box plot.



1. Determine the median, IQR, and range for this data.

median: 30   **IQR:** 15   **range:** 25

40 – 25 = 15   45 – 20 = 25

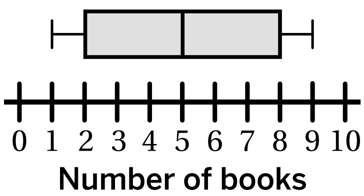
2. What percent of Inola's journey to school took 40 minutes or less?

A. 25%   **B.** 50%   **C.** 75%   **D.** 100%

Guided Practice

For Problems 1–2, refer to the box plots to identify the statistics of the data set.

1. Mia measured the number of books she read each day for a week in June.



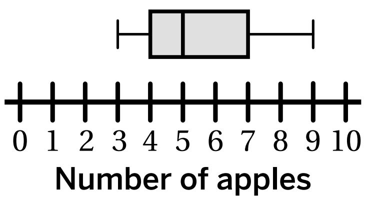
Min.	Q1	Median	Q3	Max.
1	2			

What percent of the data was more than **5** books?

A. 25%   **B.** 50%

C. 75%   **D.** 100%

2. Zoe tracked the number of apples she ate each day for a week in January.



Min.	Q1	Median	Q3	Max.

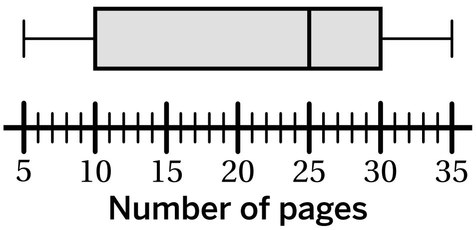
What percent of the data was between **4** and **5** apples?

A. 25%   **B.** 50%

C. 75%   **D.** 100%

Guided Practice

Eliza tracked the number of pages she read each day for a week. She wrote down the amounts and created this box plot.



3. Determine the median, IQR, and range for this data.

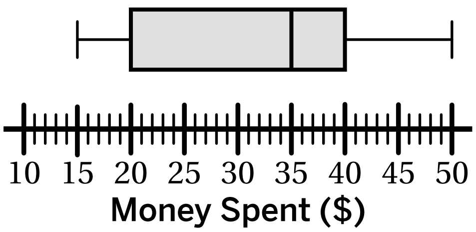
median: ______ IQR: ______ range: ______

4. On what percent of days did Eliza read less than 25 pages?

A. 25% B. 50% C. 75% D. 100%

Check

Evan tracked the amount of money he spent on clothes in January. He wrote down the amounts and created this box plot.



1. Determine the median, IQR, and range for this data.

median: ______ IQR: ______ range: ______

2. What percent of Evan's clothes cost at least $15?

A. 25% B. 50% C. 75% D. 100%

Goal

Determine and interpret the median, interquartile range (IQR), and range of a given data set in a box plot.

Standard

MA.6.DP.1.3

Materials

colored pencils (optional), highlighter (optional)

Modeled Review

Point to Santiago's work and **ask**:

- "How did Santiago determine the median? IQR?"
- "How did Santiago identify the minimum and maximum on the box plot to calculate the range?"
- "How did Santiago know that 75% of Inola's journeys took 40 minutes or less?"

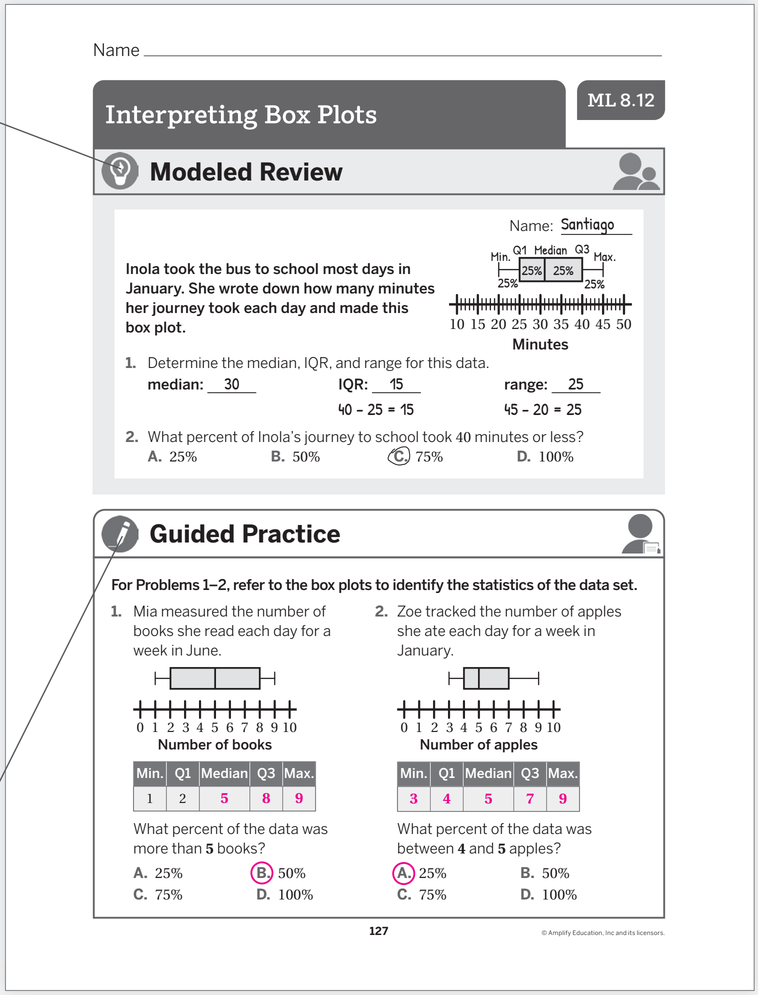
Reinforce Santiago's thinking by saying, "The median, IQR, and range can be determined by identifying and measuring the distances between key points on a box plot, with each section representing 25% of the data."

Guided Practice

Focus students' attention on identifying key points on each box plot to find the statistics for each data set.

To scaffold their thinking, **say**:

- "Identify the median of the box."
- "To calculate the IQR, determine the distance from Q1 to Q3."
- "To calculate the range, determine the distance from minimum to the maximum."
- "Each section of the box plot represents 25%."



Name

Interpreting Box Plots

ML 8.12

Modeled Review

Name: Santiago

Inola took the bus to school most days in January. She wrote down how many minutes her journey took each day and made this box plot.

1. Determine the median, IQR, and range for this data.
   median: 30 IQR: 15 range: 25
   40 − 25 = 15 45 − 20 = 25
2. What percent of Inola's journey to school took **40** minutes or less?
   A. 25% B. 50% C. 75% D. 100%

Guided Practice

For Problems 1–2, refer to the box plots to identify the statistics of the data set.

1. Mia measured the number of books she read each day for a week in June.

Min.	Q1	Median	Q3	Max.
1	2	5	8	9

What percent of the data was more than **5** books?

A. 25% B. 50%
C. 75% D. 100%

2. Zoe tracked the number of apples she ate each day for a week in January.

Min.	Q1	Median	Q3	Max.
3	4	5	7	9

What percent of the data was between **4** and **5** apples?

A. 25% B. 50%
C. 75% D. 100%

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Vocabulary

If needed, share the meaning of the terms with students.

box plot: A way to visualize quantitative data. The data is divided into four sections using five values: the minimum, Q1, Q2 (the median), Q3, and the maximum. A box is drawn between Q1 and Q3, and the line inside the box represents the median.

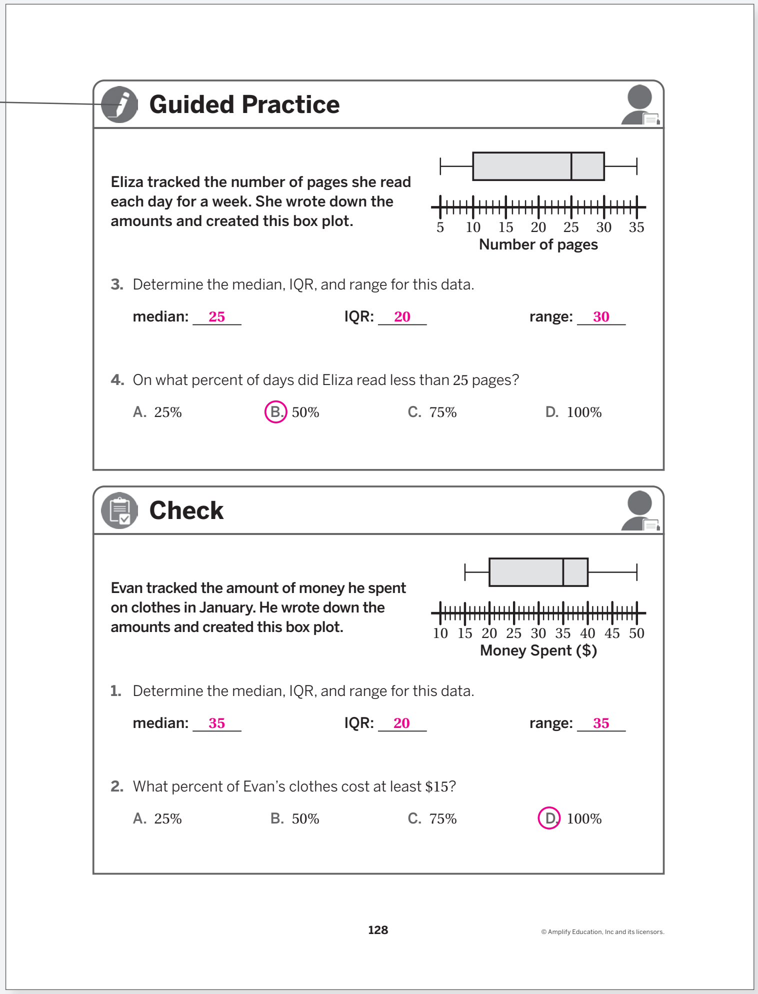
range: A measure of spread. It is the difference between the maximum and minimum values in a data set.

Guided Practice

A Invite students to use colored pencils or arrows to annotate the box plot and make connections between the median, IQR, and range.

ML/EL Consider providing key vocabulary in students' home language to scaffold their understanding of box plots.

Key Takeaway:
Say, "Identifying key points on a box plot and measuring the distances between them can help you determine the median, interquartile range (IQR), and range."



Guided Practice

Eliza tracked the number of pages she read each day for a week. She wrote down the amounts and created this box plot.

3. Determine the median, IQR, and range for this data.

median: 25 IQR: 20 range: 30

4. On what percent of days did Eliza read less than 25 pages?

A. 25% B. 50% C. 75% D. 100%

Check

Evan tracked the amount of money he spent on clothes in January. He wrote down the amounts and created this box plot.

1. Determine the median, IQR, and range for this data.

median: 35 IQR: 20 range: 35

2. What percent of Evan's clothes cost at least $15?

A. 25% B. 50% C. 75% D. 100%

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Reflection

Ask:
- "What is important to remember when determining the median, IQR, and range in a box plot?"
- "What questions do you still have?"

Check: Recommended Next Steps

Almost there
If students need more support, consider annotating or highlighting the percentage each section of the box plot represents. For example, highlight the area from the minimum to Q1 and write "25%" next to it. Repeat the same for Q1 to median, median to Q3, and Q3 to maximum by labeling each section as "25%."

Got it!
If students need more practice, ask them to revisit the box plot in Problems 3–4 and answer the following questions:

- What percent of days did Eliza read at least 5 pages?
- What percent of days did Eliza read between 10 and 35 pages?

Name ______________________________

Calculating Percentages

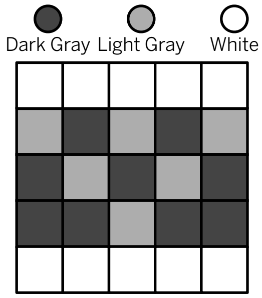
ML 4.03

Modeled Review

Name: Dylan

What percentage of the larger grid is white? Explain your thinking.

I counted 25 squares. Each square has a value of 4% because 100 divided by 25 is 4. There are 10 white squares. I can multiply 4 by 10 to get 40%.



$\frac{100}{25} = 4\%$

$4 \cdot 10 = 40$

40%

Guided Practice

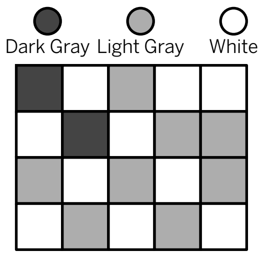
1. What percentage of the grid is light gray or white? Show your thinking.

Light Gray:

$\frac{100}{20} =$ ______

White:

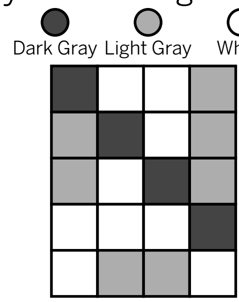
Color	Percentage
Dark Gray	10%
Light Gray	
White	



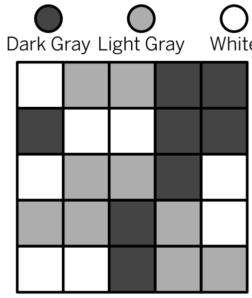
Guided Practice

Determine the percentages of the grids that are shaded each color.

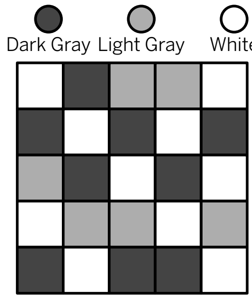
2. What percentage of the grid is dark gray? Show your thinking.



3. What percentage of the grid is white? Show your thinking.

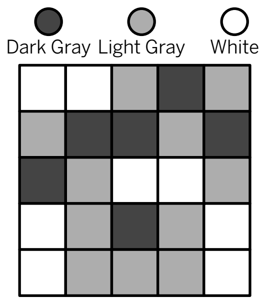


4. What percentage of the grid is light gray? Show your thinking.



Check

What percentage of the grid is white? Show your thinking.



Goal	Standard
Calculate the percentage of a quantity on a grid.	MA.7.AR.3.1

Modeled Review

Point to Dylan's work and **ask**:
- "What percentage represents 1 whole?"
- "How did Dylan use the grid to calculate the percentage of the white squares?"

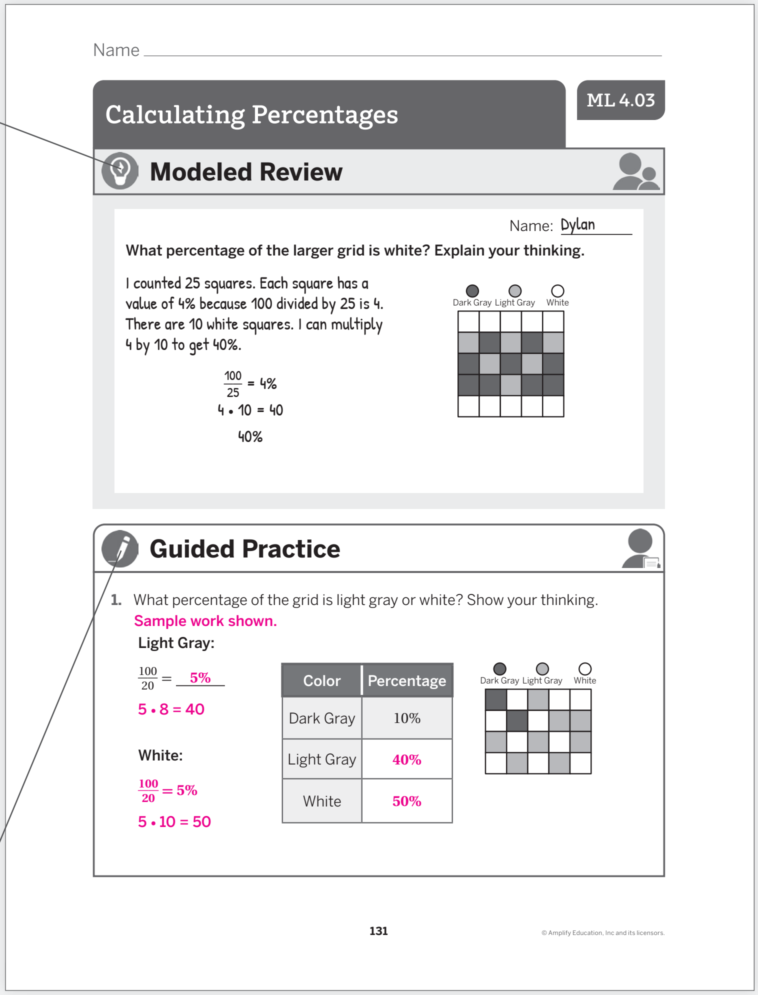
Reinforce Dylan's thinking by saying, "Finding the percentage each square represents can help you determine the total percentage of each color."

ML/EL Provide sentence frames to support students as they explain how they determined the percentage shaded for each colored section. For example, "I counted _______ sections in the larger square." or "There are _____ dark gray/light gray/white sections, so I _____."

Guided Practice

Focus students' attention on calculating the percentage of each color in the grid.

To scaffold their thinking, **ask**:
- "How many sections is the grid broken into?"
- "How many sections are there of each color?"
- "How do you calculate the percentage of each color?"


Name

Calculating Percentages ML 4.03

Modeled Review

Name: Dylan

What percentage of the larger grid is white? Explain your thinking.

I counted 25 squares. Each square has a value of 4% because 100 divided by 25 is 4. There are 10 white squares. I can multiply 4 by 10 to get 40%.

$\frac{100}{25} = 4\%$

$4 \cdot 10 = 40$

40%

Guided Practice

1. What percentage of the grid is light gray or white? Show your thinking.

Sample work shown.

Light Gray:

$\frac{100}{20} = 5\%$

$5 \cdot 8 = 40$

White:

$\frac{100}{20} = 5\%$

$5 \cdot 10 = 50$

Color	Percentage
Dark Gray	10%
Light Gray	40%
White	50%

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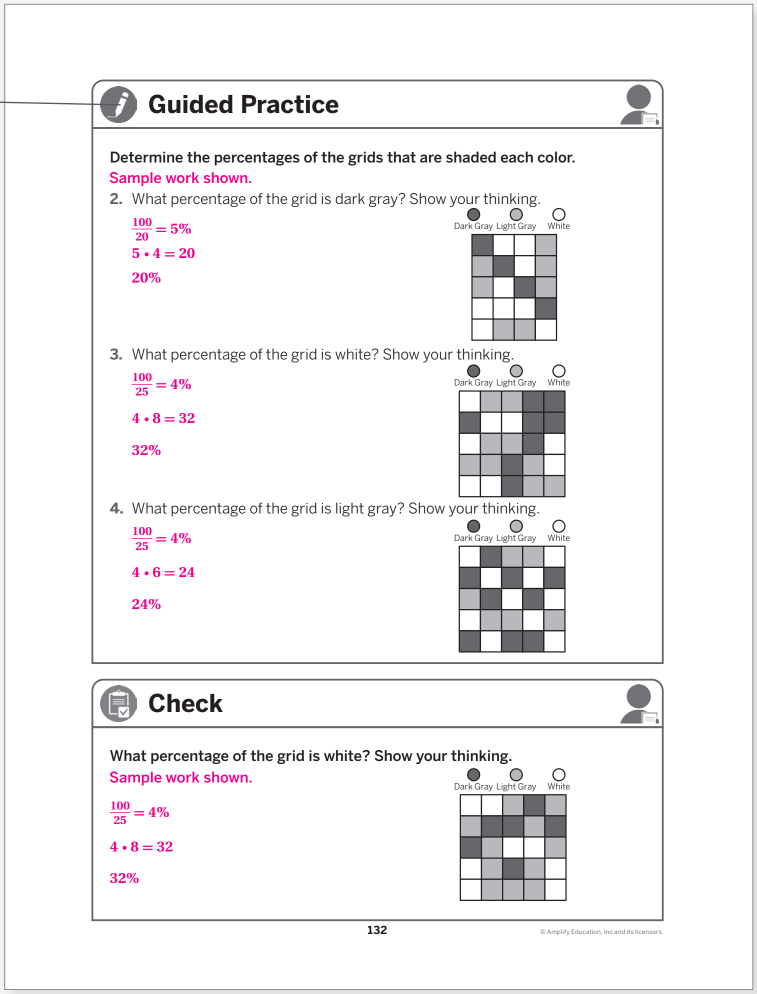
Guided Practice

A Chunk this task into smaller, more manageable parts by having students create a table that can be used to organize their data, including how many sections the grid is broken into and how many sections there are of each color.

Note: Students can check if they accurately determined the percentage for each color by ensuring all of the percentages add to 100%.

Key Takeaway:
Say, "Several strategies can be used to calculate the percent of a number such as multiplying the percentage by the whole, using benchmark fractions, or using a visual representation."



Guided Practice

Determine the percentages of the grids that are shaded each color.
Sample work shown.

2. What percentage of the grid is dark gray? Show your thinking.

$\frac{100}{20} = 5\%$

$5 \cdot 4 = 20$

20%

Dark Gray Light Gray White

3. What percentage of the grid is white? Show your thinking.

$\frac{100}{25} = 4\%$

$4 \cdot 8 = 32$

32%

Dark Gray Light Gray White

4. What percentage of the grid is light gray? Show your thinking.

$\frac{100}{25} = 4\%$

$4 \cdot 6 = 24$

24%

Dark Gray Light Gray White

Check

What percentage of the grid is white? Show your thinking.
Sample work shown.

$\frac{100}{25} = 4\%$

$4 \cdot 8 = 32$

32%

Dark Gray Light Gray White

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Reflection

Ask:

- "How is using a grid helpful in visualizing benchmark percentages?"
- "What questions do you still have?"

Check: Recommended Next Steps

Almost there	**Got it!**
If students need more support, consider using Grade 6 Mini-Lesson 3.12: *Calculating Unknown Percentages*.	If students need more practice, ask them to look at Problem 2 and determine the percentage of the remaining colors in the grid.

Name ____

Solving Equations With Positive and Negative Numbers

ML 6.06

Modeled Review

Name: Jack

Solve the equation. Show your thinking.

$-4(x - 2) = 12$

$$\begin{aligned} \underline{-4}(x-2) &= \underline{12} \\ -4 \quad\quad &\quad\ -4 \\ x - 2 &= -3 \\ +2 \quad &\ +2 \\ x &= -1 \end{aligned}$$

Check:

$$\begin{aligned} -4(x-2) &= 12 \\ -4(-1-2) &= 12 \\ -4(-3) &= 12 \\ 12 &= 12 \end{aligned}$$

Guided Practice

1. Solve the equations by completing the blanks in the equations and descriptions.

Equation	Moves
$-2x + 3 = 7$ $-2x =$ ____ $x =$ ____	**Step 1:** Subtract ____ from each side. **Step 2:** Divide each side by ____.
$2(x + 1) = 6$ $x + 1 =$ ____ $x =$ ____	**Step 1:** ______________ each side by ____. **Step 2:** ______________ from each side.

Guided Practice

For Problems 2–5, solve the equation. Show your thinking.

2. $-4x - 2 = 10$

3. $3(x - 2) = 9$

4. $-3x + 5 = -1$

5. $-4(x + 3) = 20$

Check

Solve each equation. Show your thinking.

1. $-4x - 3 = 13$

2. $-3(x - 4) = 27$

ML 6.06 Solving Equations With Positive and Negative Numbers

Goal

Solve linear equations that involve positive and negative numbers.

Standard

MA.7.AR.2.2

Materials

highlighter (optional)

Modeled Review

Point to Jack's work and **ask**:

- "Why did Jack use inverse operations to solve for the variable?"
- "Why did Jack do the same operation on both sides of the equal sign?"
- "How did Jack check his solution?"

Reinforce Jack's thinking by saying, "When solving an equation, the same operations should be applied to both sides at each step, so that the equation remains true."

ML/EL Invite students to check the solution by substituting in the value for the variable and evaluating to determine whether the equation is true.

Guided Practice

Focus students' attention on solving equations by ensuring that both sides of the equation remain balanced while determining the value of the variable.

To scaffold their thinking, **say**:

- "Identify the variable that needs to be solved for."
- "Apply the inverse operation to both sides of the equation, repeating this process until the variable is isolated."

Name ______

Solving Equations With Positive and Negative Numbers — ML 6.06

Modeled Review

Name: Jack

Solve the equation. Show your thinking.

$-4(x-2)=12$

$\underline{-4}(x-2) = \underline{12}$

$-4 \qquad -4$

$x - 2 = -3$

$+2 \qquad +2$

$x = -1$

Check:

$-4(x-2) = 12$

$-4(-1-2) = 12$

$-4(-3) = 12$

$12 = 12$

Guided Practice

1. Solve the equations by completing the blanks in the equations and descriptions.

Equation	Moves
$-2x + 3 = 7$ $-2x = \underline{4}$ $x = \underline{-2}$	**Step 1:** Subtract _3_ from each side. **Step 2:** Divide each side by _-2_.
$2(x+1) = 6$ $x + 1 = \underline{3}$ $x = \underline{2}$	**Step 1:** _Divide_ each side by _2_. **Step 2:** _Subtract 1_ from each side.

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ML 6.06 Solving Equations With Positive and Negative Numbers

Guided Practice

A Invite students to highlight the variable in each equation. Then have them make a step by step plan as to how they will isolate the variable. (For example, first I will divide. Next, I will add and finally, I will subtract.)

Key Takeaway:
Say, “The first step in solving equations may be different depending on the structure of the equation, but it is important that both sides of the equation remain balanced.”

Guided Practice

For Problems 2–5, solve the equation. Show your thinking.
Sample work shown.

2. $-4x - 2 = 10$

$$\begin{aligned} -4x - 2 &= 10 \\ +2 \quad &\quad +2 \\ \frac{-4x}{-4} &= \frac{12}{-4} \\ x &= -3 \end{aligned}$$

3. $3(x - 2) = 9$

$$\begin{aligned} \frac{3(x-2)}{3} &= \frac{9}{3} \\ x - 2 &= 3 \\ +2 \quad &\quad +2 \\ x &= 5 \end{aligned}$$

4. $-3x + 5 = -1$

$$\begin{aligned} -3x + 5 &= -1 \\ -5 \quad &\quad -5 \\ \frac{-3x}{-3} &= \frac{-6}{-3} \\ x &= 2 \end{aligned}$$

5. $-4(x + 3) = 20$

$$\begin{aligned} \frac{-4(x+3)}{-4} &= \frac{20}{-4} \\ x + 3 &= -5 \\ -3 \quad &\quad -3 \\ x &= -8 \end{aligned}$$

Check

Solve each equation. Show your thinking. Sample work shown.

1. $-4x - 3 = 13$

$$\begin{aligned} -4x - 3 &= 13 \\ +3 \quad &\quad +3 \\ \frac{-4x}{-4} &= \frac{16}{-4} \\ x &= -4 \end{aligned}$$

2. $-3(x - 4) = 27$

$$\begin{aligned} \frac{-3(x-4)}{-3} &= \frac{27}{-3} \\ x - 4 &= -9 \\ +4 \quad &\quad +4 \\ x &= -5 \end{aligned}$$

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Reflection

Ask:

- “What suggestions would you share with a friend about solving equations that involve positive and negative numbers?”
- “Why is it helpful to check your solutions?”

Check: Recommended Next Steps

Almost there
If students need more support, invite them to draw a line separating the two sides of the equation to support them in organizing their work on each side of the equation.

Got it!
If students need more practice, ask them to solve the following equations:

- $-3(x - 5) = 21$
- $-5x + 6 = 21$

Name ______________________________

Solving Linear Equations With Parentheses

ML 4.01

Modeled Review

Name: Kayla

Solve the equation.

$-3(2x - 4) = 18$

$$\begin{aligned} -3(2x - 4) &= 18 \\ -6x + 12 &= 18 \\ -12 \quad & \;\; -12 \\ -6x &= 6 \\ x &= -1 \end{aligned}$$

Name: Isaiah

Solve the equation.

$-3(2x - 4) = 18$

$$\begin{aligned} \frac{-3(2x - 4)}{-3} &= \frac{18}{-3} \\ 2x - 4 &= -6 \\ +4 &= +4 \\ 2x &= -2 \\ x &= -1 \end{aligned}$$

Guided Practice

Solve each equation. Complete the missing steps.

1.

$$\begin{aligned} 4x + 9 &= 5x + 7 \\ -4x \qquad & \;\; -4x \\ 9 &= \underline{\qquad} + 7 \\ -7 \qquad & \qquad -7 \\ \underline{\qquad} &= x \end{aligned}$$

solution: $x =$ ___

2.

$$\begin{aligned} x - 8 &= 10 - 5x \\ + \underline{\qquad} & \quad + \underline{\qquad} \\ 6x - 8 &= 10 \\ + \underline{\qquad} &+ \underline{\qquad} \\ 6x &= 18 \\ x &= \underline{\qquad} \end{aligned}$$

solution: $x =$ ___

Solve the equation $\frac{1}{2}(4x - 2) = 11$ using two different methods.

3.

$$\begin{aligned} \tfrac{1}{2}(4x - 2) &= 11 \\ \times 2 \qquad & \quad \times 2 \\ 4x - 2 &= \underline{\qquad} \\ \underline{\qquad} & \;\; \underline{\qquad} \\ 4x &= 24 \\ x &= \underline{\qquad} \end{aligned}$$

solution: $x =$ ___

4.

$$\begin{aligned} \tfrac{1}{2}(4x - 2) &= 11 \\ 2x - \underline{\qquad} &= 11 \\ +\;1 \quad &+ \underline{\qquad} \\ 2x &= \underline{\qquad} \\ x &= \underline{\qquad} \end{aligned}$$

solution: $x =$ ___

Guided Practice

Solve each equation.

5. $4(3x + 5) = \text{-}2(-8x + 6)$

$\_\_\_\,x + 20 = \_\_\_\,x - \_\_\_$

$20 = \_\_\_\,x - 12$

$32 = \_\_\_\,x$

$\_\_\_ = x$

solution: x = ___

6. $\text{-}2(2x + 3) = 3(x + 5)$

solution: x = ___

7. $\frac{1}{3}(x - 5) = 2(x - 1)$

solution: x = ___

8. $0.5(x - 4) = \text{-}3(2x + 5)$

solution: x = ___

Check

Solve the equation.

$$\text{-}2(5x - 3) = 3(\text{-}4x + 8)$$

solution: x = ___

ML 4.01 Solving Linear Equations With Parentheses

Goal

Solve single-variable equations with parentheses using multiple balanced steps, including the distributive property.

Standard

MA.7.AR.2.2

Materials

calculator (optional)

Modeled Review

Point to Kayla's and Isaiah's work and **ask**:

- "What steps did each student take to solve the equation? What was each student's first step?"
- "What is the same about each student's work? What is different?"
- "Whose strategy do you think is more efficient for solving this equation?"

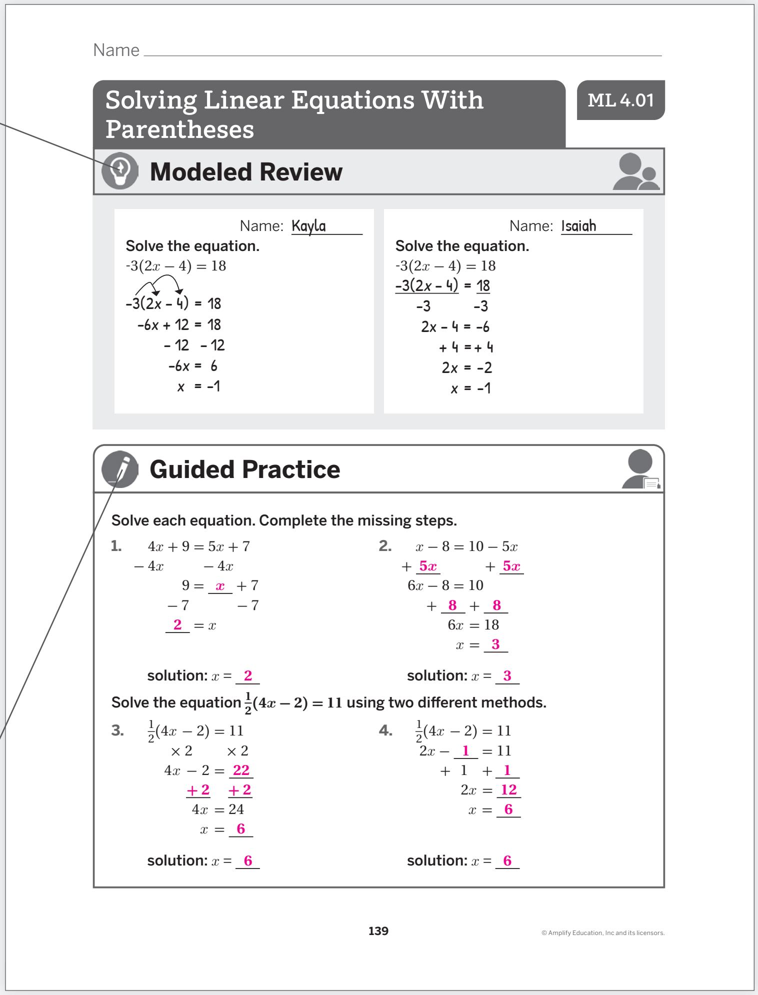
Reinforce the goal by saying, "Different strategies can be used to solve the same equation. When you have finished solving, you could substitute the value of *x* into the original equation to check that it is true."

Guided Practice

Focus students' attention on solving the linear equations.

To scaffold their thinking, **ask**:

- "What is the first step you could take to solve this equation?"
- "How could you check if your solution is correct?"


Name

Solving Linear Equations With Parentheses

ML 4.01

Modeled Review

Name: Kayla

Solve the equation.

$-3(2x - 4) = 18$

$-3(2x - 4) = 18$

$-6x + 12 = 18$

$-12 \quad -12$

$-6x = 6$

$x = -1$

Name: Isaiah

Solve the equation.

$-3(2x - 4) = 18$

$\frac{-3(2x - 4)}{-3} = \frac{18}{-3}$

$2x - 4 = -6$

$+4 = +4$

$2x = -2$

$x = -1$

Guided Practice

Solve each equation. Complete the missing steps.

1. $4x + 9 = 5x + 7$

$-4x \quad -4x$

$9 = \underline{x} + 7$

$-7 \quad -7$

$\underline{2} = x$

solution: $x = \underline{2}$

2. $x - 8 = 10 - 5x$

$+\underline{5x} \quad +\underline{5x}$

$6x - 8 = 10$

$+\underline{8} \quad +\underline{8}$

$6x = 18$

$x = \underline{3}$

solution: $x = \underline{3}$

Solve the equation $\frac{1}{2}(4x - 2) = 11$ using two different methods.

3. $\frac{1}{2}(4x - 2) = 11$

$\times 2 \quad \times 2$

$4x - 2 = \underline{22}$

$\underline{+2} \quad \underline{+2}$

$4x = 24$

$x = \underline{6}$

solution: $x = \underline{6}$

4. $\frac{1}{2}(4x - 2) = 11$

$2x - \underline{1} = 11$

$+1 \quad +\underline{1}$

$2x = \underline{12}$

$x = \underline{6}$

solution: $x = \underline{6}$

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Vocabulary

If needed, share the meaning of the term with students.

solution to an equation: The value or set of values that makes the equation true.

ML 4.01 Solving Linear Equations With Parentheses

Guided Practice

Ⓐ Invite students to use a calculator to support them with performing calculations.

Key Takeaway:
Say, "Different strategies can be used to solve the same equation. Using the structure of an equation can be helpful in determining effective steps to solving the equation."

Guided Practice

Solve each equation. Sample work shown for Problems 6–8.

5. $4(3x + 5) = -2(-8x + 6)$

$12x + 20 = 16x - 12$

$-12x \quad -12x$

$20 = 4x - 12$

$+12 \quad +12$

$32 = 4x$

$8 = x$

solution: $x = 8$

6. $-2(2x + 3) = 3(x + 5)$

$-4x - 6 = 3x + 15$

$+4x \quad +4x$

$-6 = 7x + 15$

$-15 \quad -15$

$-21 = 7x$

$-3 = x$

solution: $x = -3$

7. $\frac{1}{3}(x - 5) = 2(x - 1)$

$\times 3 \quad \times 3$

$x - 5 = 6(x - 1)$

$x - 5 = 6x - 6$

$-x \quad -x$

$-5 = 5x - 6$

$+6 = +6$

$1 = 5x$

$\frac{1}{5} = x$

solution: $x = \frac{1}{5}$

8. $0.5(x - 4) = -3(2x + 5)$

$\times 2 \quad \times 2$

$x - 4 = -6(2x + 5)$

$x - 4 = -12x - 30$

$+12x \quad +12x$

$13x - 4 = -30$

$+4 \quad +4$

$13x = -26$

$x = -2$

solution: $x = -2$

Check

Solve the equation. Sample work shown.

$-2(5x - 3) = 3(-4x + 8)$

$-10x + 6 = -12x + 24$

$+12x \quad +12x$

$2x + 6 = 24$

$-6 \quad -6$

$2x = 18$

$x = 9$

solution: $x = 9$

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Reflection

Ask:

- "Which strategy was the most helpful when determining the solution for each equation?"
- "After today's lesson, what made sense? What is still confusing?"

Check: Recommended Next Steps

Almost there
If students need more support, consider using Grade 7 Mini-Lesson 6.07: *Solving Equations With Positive and Negative Numbers*.

Got it!
If students need more practice, have them solve the following equations:

- $-5(x + 4) = -3(-2x - 8)$
- $\frac{1}{4}(8x - 4) = 3(-x + 1)$

Name ____________________

Solving Inequalities

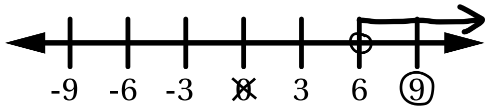
ML 6.15

Modeled Review

Name: Priya

Solve and graph the solution to $x - 1 > 5$.

$x > 6$

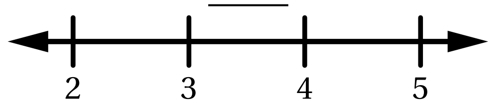


$x - 1 = 5$
$+1 \quad +1$
$x = 6$

$x < 6$	$x > 6$
$0 - 1 > 5$	$(9) - 1 > 5$
$-1 > 5$	$8 > 5$
False	True

Guided Practice

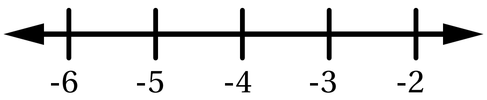
1. Graph the solution to the inequality $3x \geq 9$ by finding the boundary point and testing values on both sides of the boundary point.

Moves	Work
Move 1: Find the boundary point.	$\underline{3x} = \underline{9}$ $x = \underline{\quad}$
Move 2: Test the values on both sides of the point and determine whether the statement is true or false.	**Less than or equal to** $3(\_\_\_\_) \geq 9$ $\_\_\_\_ \geq 9$ ________ **Greater than or equal to** $3(\_\_\_\_) \geq 9$ $\_\_\_\_ \geq 9$ ________
Move 3: Graph the solution.	$x \_\_\_\_ 3$ 

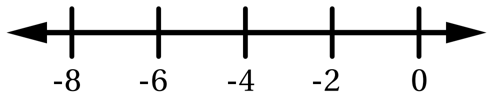
Guided Practice

Solve and graph the solution to each of the inequalities.

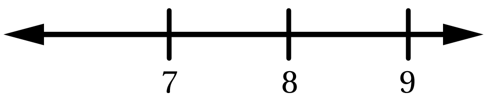
2. $2x \leq -8$ __________



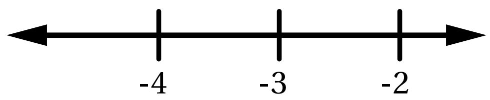
3. $x - 1 > -7$ __________



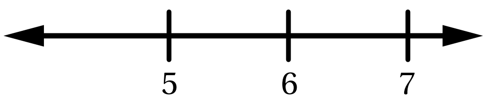
4. $x + 11 \geq 18$ __________



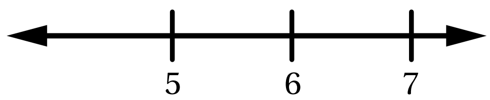
5. $-6x > 18$ __________



6. $x - 12 \geq -7$ __________



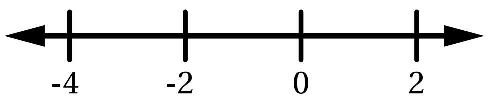
7. $-5x > -30$ __________



Check

Solve and graph the solution to $-4x < 8$**.**

____



Solving Inequalities

Goal	Standard
Solve linear equations that involve positive and negative numbers.	MA.7.AR.2.1

Modeled Review

Point to Avery's work and **ask**:

- "How did Avery determine the boundary point?"
- "Why did Avery substitute 0 and 9 into the inequality?"
- "How does testing numbers help to determine which inequality symbol to use?"

Reinforce Avery's thinking by saying, "After finding the boundary point, test values to determine which inequality symbol makes the solution statement true."

Guided Practice

Focus students' attention on finding the boundary point and testing values on both sides of the point.

To scaffold their thinking, **say**:

- "Determine the boundary point."
- "Choose numbers to the left and right of the boundary point and substitute them into the inequality."
- "Decide which number satisfies the inequality. Based on this, determine the correct inequality sign to use."

Name ______________________________

Solving Inequalities

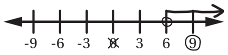
ML 6.15

Modeled Review

Name: Priya

Solve and graph the solution to $x - 1 > 5$.

x > 6

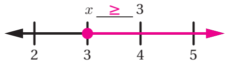


x - 1 = 5
+1 +1
x = 6

x < 6	x > 6
0 - 1 > 5	(9) - 1 > 5
-1 > 5	8 > 5
False	True

Guided Practice

1. Graph the solution to the inequality $3x \geq 9$ by finding the boundary point and testing values on both sides of the boundary point.

Moves	Work
Move 1: Find the boundary point.	$\frac{3x}{3} = \frac{9}{3}$ $x = 3$
Move 2: Test the values on both sides of the point and determine whether the statement is true or false.	**Less than or equal to:** $3(2) \geq 9$; $6 \geq 9$; False **Greater than or equal to:** $3(4) \geq 9$; $12 \geq 9$; True
Move 3: Graph the solution.	$x \geq 3$ 

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Vocabulary

If needed, share the meaning of the terms with students.

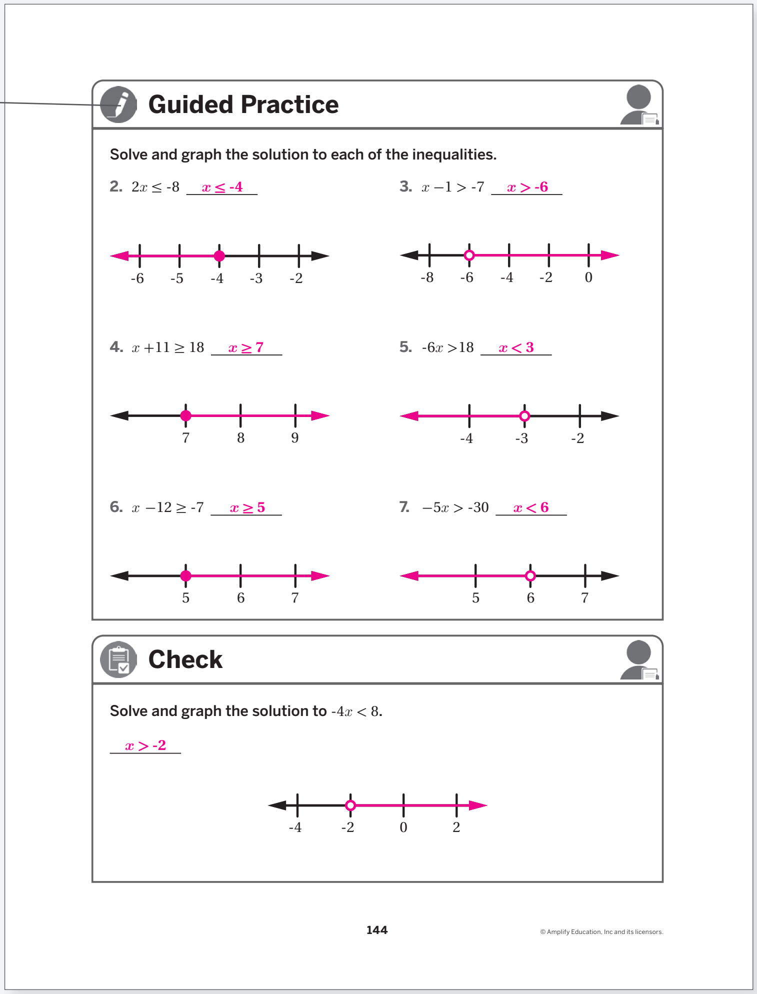
inequality: A comparison statement that uses the symbols < or >. Inequalities are used to represent the relationship between numbers, variables, or expressions that are not always equal.

solution to an inequality: All of the values of a variable that make that inequality true.

Guided Practice

(A) Emphasize that inequality signs that include ‘equal to’ should be represented with a shaded circle on the number line.

Key Takeaway:
Say, “Solving inequalities is similar to solving equations. The solutions to an inequality can be determined by first solving an equation, then testing whether the values on the left or right side of the boundary point make the inequality true.”



Guided Practice

Solve and graph the solution to each of the inequalities.

2. $2x \le -8$ $x \le -4$

3. $x - 1 > -7$ $x > -6$

4. $x + 11 \ge 18$ $x \ge 7$

5. $-6x > 18$ $x < 3$

6. $x - 12 \ge -7$ $x \ge 5$

7. $-5x > -30$ $x < 6$

Check

Solve and graph the solution to $-4x < 8$.

$x > -2$

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Reflection

Ask:

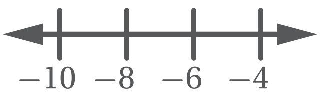
- “What steps can you take to graph the solutions to an inequality?”
- “How are solutions to an inequality different from an equation?”

Check: Recommended Next Steps

Almost there
If students need more support, consider having them revisit the problem in the Check. Then model graphing the solution by marking off the value on the number line that does not make statement true and circling the one that does. Invite them to use that information to determine how they should draw the arrow.

Got it!
If students need more practice, present students with the following problem and ask them to graph the solution:

$-7x \le 42$



Name ______________________________

Calculating Slope By Drawing Triangles on a Coordinate Plane

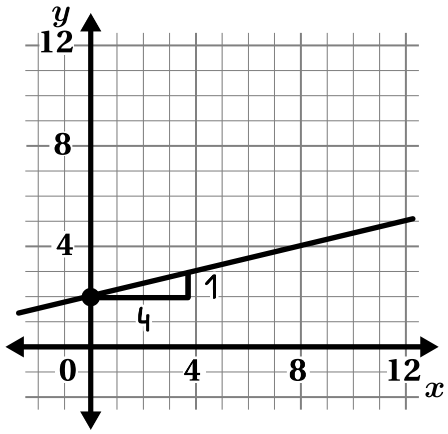
ML 2.10

Modeled Review

Name: Priya

Determine the slope of the line. Show your thinking.

The slope of the line is the ratio between the height of the triangle to its base.

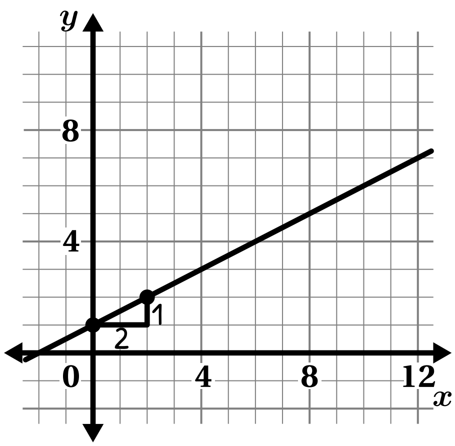


Slope is $\frac{1}{4}$.

Guided Practice

Determine the slope of the line using a slope triangle.

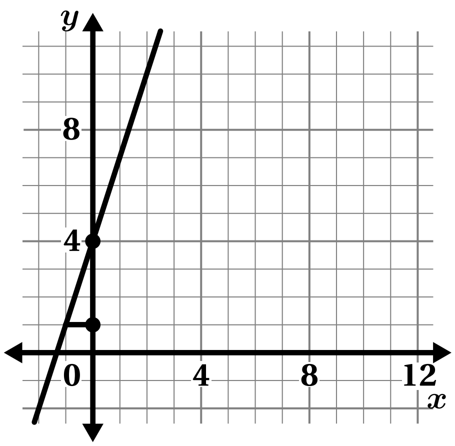
1.



slope = $\frac{\textit{height of slope triangle}}{\textit{base of slope triangle}} = \frac{\square}{\square}$

____

2.



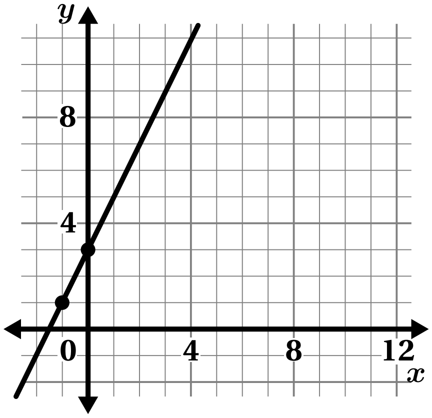
slope = $\frac{\textit{height of slope triangle}}{\textit{base of slope triangle}} = \frac{\square}{\square} = \square$

____

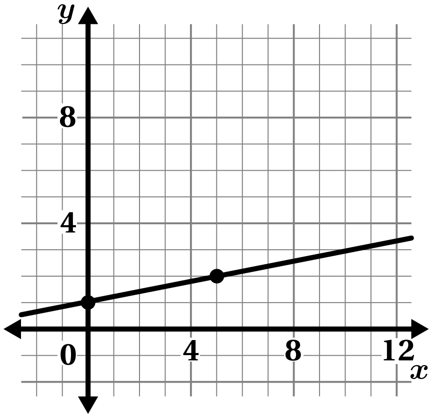
Guided Practice

Determine the slope of the line. Show your thinking.

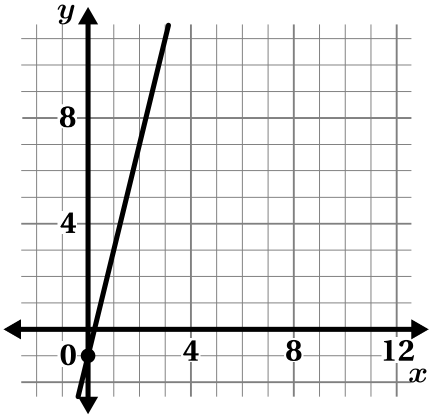
3.



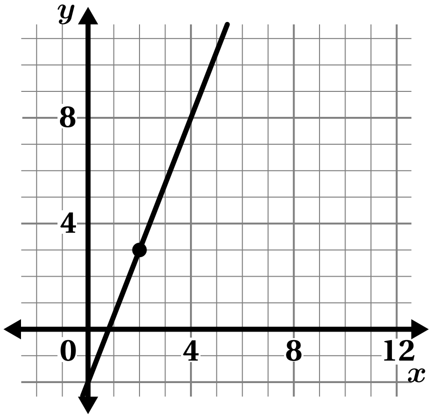
4.



5.

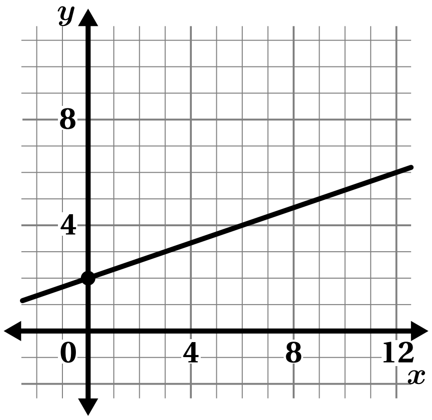


6.



Check

Determine the slope of the line. Show your thinking.



ML 2.10 Calculating Slope By Drawing Triangles on a Coordinate Plane

Goal

Calculate the slope of a line using triangles on a coordinate plane

Standard

MA.8.AR.3.2

Materials

straightedge (optional), highlighter (optional)

Modeled Review

Point to Priya's work and **ask**:
- "How did Priya use a triangle to find the slope of the line?"
- "Is there another similar triangle Priya could have drawn?"
- "How did Priya calculate the slope?"

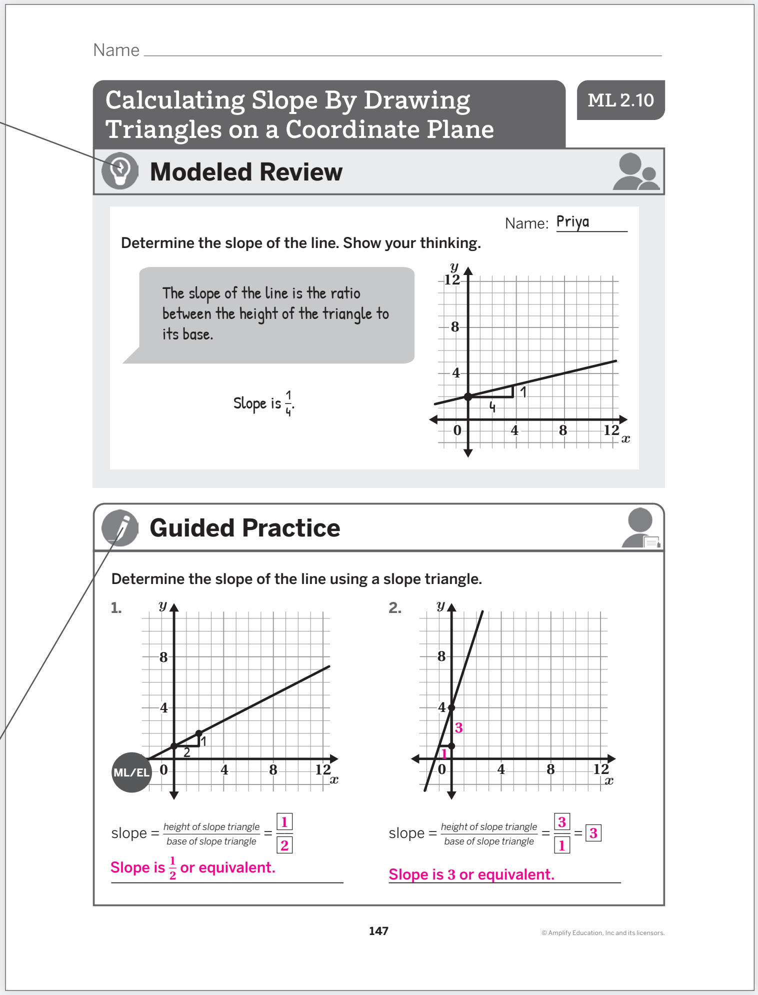
Reinforce the goal by saying, "The slope can be calculated by drawing similar right triangles, called slope triangles, between two points on the line."

ML/EL Model drawing a slope triangle on the coordinate plane.

Guided Practice

Focus students' attention on the line and points given.

To scaffold their thinking, **say**:
- "Identify the points given on the coordinate plane."
- "Create a slope triangle by connecting the two points."
- "Count the height and length of the base of the slope triangle."
- "Divide the height by the length of the base to find the slope."


Name

Calculating Slope By Drawing Triangles on a Coordinate Plane — ML 2.10

Modeled Review

Name: Priya

Determine the slope of the line. Show your thinking.

The slope of the line is the ratio between the height of the triangle to its base.

Slope is $\frac{1}{4}$.

Guided Practice

Determine the slope of the line using a slope triangle.

1. slope = $\frac{\text{height of slope triangle}}{\text{base of slope triangle}} = \frac{1}{2}$

Slope is $\frac{1}{2}$ or equivalent.

2. slope = $\frac{\text{height of slope triangle}}{\text{base of slope triangle}} = \frac{3}{1} = 3$

Slope is 3 or equivalent.

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Vocabulary

If needed, share the meaning of the terms with students.

slope: A number that describes the direction and steepness of a line.

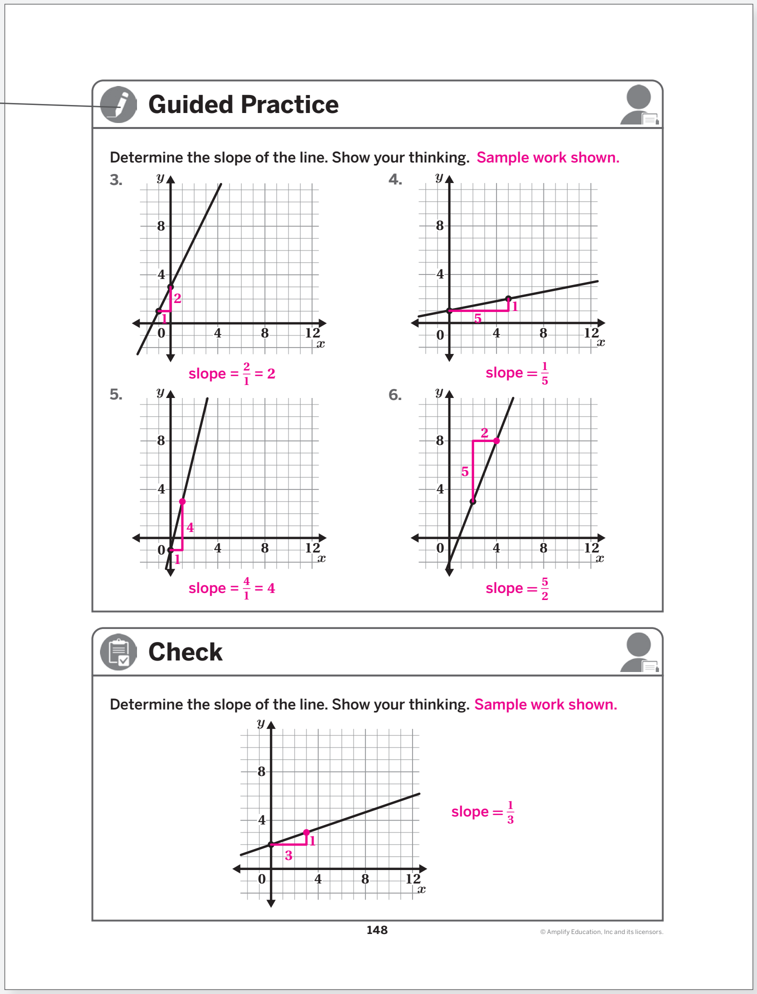
slope triangle: A slope triangle for a line is a triangle whose longest side lies on the line and whose other two sides are vertical and horizontal.

Calculating Slope By Drawing Triangles on a Coordinate Plane

Guided Practice

A As students interpret visual representations, provide straightedges that students can use to create slope triangles. Invite students to count the number of squares on the height and the base to determine slope.

Key Takeaway:
Say, "Constructing a slope triangle and calculating its height to the length of its base is a strategy for determining slope."



Guided Practice

Determine the slope of the line. Show your thinking. Sample work shown.

3. slope = $\frac{2}{1}$ = 2

4. slope = $\frac{1}{5}$

5. slope = $\frac{4}{1}$ = 4

6. slope = $\frac{5}{2}$

Check

Determine the slope of the line. Show your thinking. Sample work shown.

slope = $\frac{1}{3}$

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Reflection

Ask:

- "How can you use a slope triangle to find the slope of a line?"
- "How did you overcome a hard problem today?"

Check: Recommended Next Steps

Almost there
If students need more support, have them use two different colors to highlight the height and the base, and divide the height by the length of the base to determine the slope.

Got it!
If students need more practice, refer to Problem 2 and ask how the slope triangle and the slope would change if the coordinate (2, 10) was defined on the line.

Name ____________________

Interpreting Slope and Intercepts of Linear Relationships

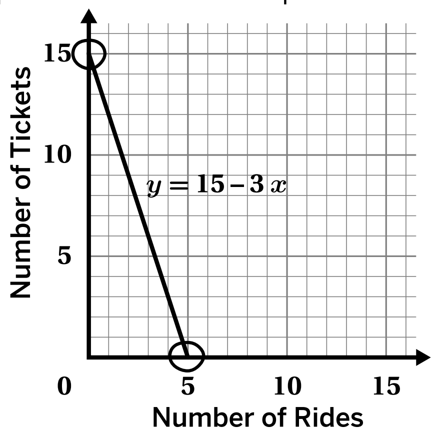
ML 3.05

Modeled Review

Name: Jack

The line $y = 15 - 3x$ represents Marc purchasing tickets at a carnival to go on rides.

1. Write the vertical intercept as a coordinate pair. What does it represent in this situation?
   (0,15); Marc started with 15 tickets.



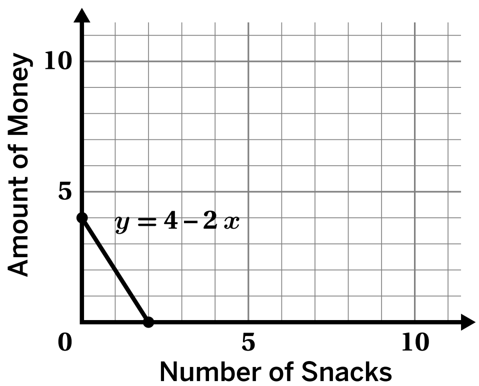
2. Write the horizontal intercept as a coordinate pair. What does it represent in this situation?
   (5, 0); Marc went on 5 rides and then ran out of tickets.

3. What is the slope and what does it represent?
   -3; Marc uses 3 tickets for each ride.

Guided Practice

The line $y = 4 - 2x$ represents Mia purchasing snacks at the theater.

1. Write the vertical intercept as a coordinate pair. What does it represent in this situation?
   (0,__);



2. Write the horizontal intercept as a coordinate pair. What does it represent in this situation?
   (0);

3. What is the slope, and what does it represent?
   ___; This means that Mia's ________ ____________ at a constant rate of ________________________________

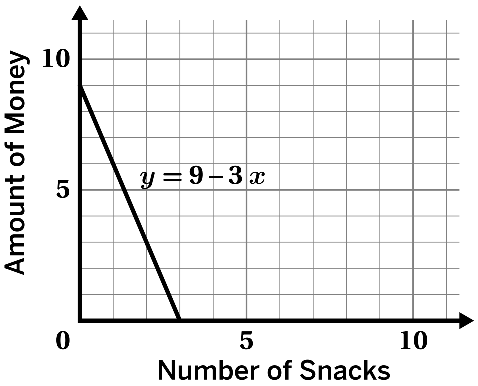
Guided Practice

The line $y = 9 - 3x$ represents Kai purchasing snacks at a soccer game.

4. Write the vertical intercept as a coordinate pair. What does it represent in this situation?

5. Write the horizontal intercept as a coordinate pair. What does it represent in this situation?

6. What is the slope, and what does it represent?



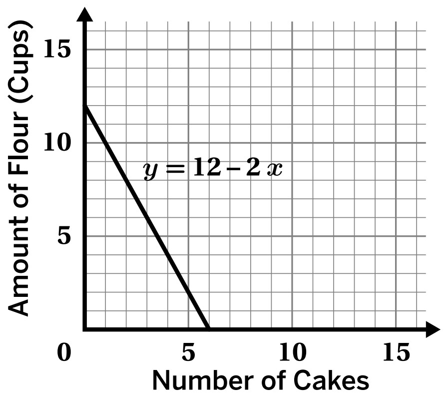
Check

The line $y = 12 - 2x$ represents Amy using flour to make cakes.

1. Write the vertical intercept as a coordinate pair. What does it represent in this situation?

2. Write the horizontal intercept as a coordinate pair. What does it represent in this situation?

3. What is the slope, and what does it represent?



Interpreting Slope and Intercepts of Linear Relationships

Goal

Interpreting representations of the slope, vertical intercept, and horizontal intercept between a graph, equation, and situation.

Standard

MA.8.AR.3.5

Materials

colored pencils or highlighter (optional)

Modeled Review

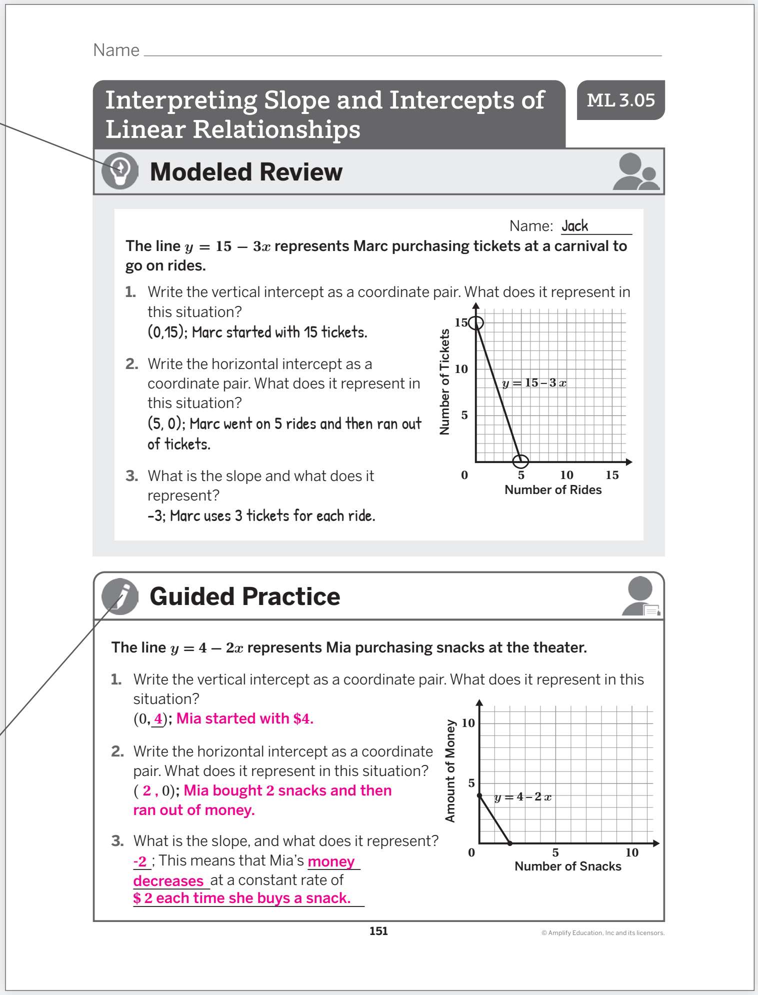
Point to Jack's work and **ask**:
- "How did Jack use the situation and graph given to identify and describe the vertical intercept and horizontal intercept?"
- "How did Jack use the labels of the graph to help describe the slope?"

Reinforce he goal by saying, "The slope and intercepts can be interpreted from a graph by looking at the labels of the x-axis and y-axis and features of the graph."

Guided Practice

Focus students' attention on the equation and axes on the graph given.

To scaffold their thinking, **ask**:
- "Where do you see the vertical intercept in the equation?"
- "How is this shown on the graph?"



Name

Interpreting Slope and Intercepts of Linear Relationships — ML 3.05

Modeled Review

Name: Jack

The line $y = 15 - 3x$ represents Marc purchasing tickets at a carnival to go on rides.

1. Write the vertical intercept as a coordinate pair. What does it represent in this situation?
   (0,15); Marc started with 15 tickets.
2. Write the horizontal intercept as a coordinate pair. What does it represent in this situation?
   (5, 0); Marc went on 5 rides and then ran out of tickets.
3. What is the slope and what does it represent?
   -3; Marc uses 3 tickets for each ride.

Guided Practice

The line $y = 4 - 2x$ represents Mia purchasing snacks at the theater.

1. Write the vertical intercept as a coordinate pair. What does it represent in this situation?
   (0, 4); Mia started with $4.
2. Write the horizontal intercept as a coordinate pair. What does it represent in this situation?
   (2 , 0); Mia bought 2 snacks and then ran out of money.
3. What is the slope, and what does it represent?
   -2 ; This means that Mia's money decreases at a constant rate of $ 2 each time she buys a snack.

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Vocabulary

If needed, share the meaning of the terms with students.

horizontal intercept: The horizontal intercept, sometimes called the x-intercept, is the point where the graph of a line crosses the horizontal axis or when $y = 0$.

vertical intercept: The vertical intercept, sometimes called the y-intercept, is the point where the graph of a line crosses the vertical axis or when $x = 0$.

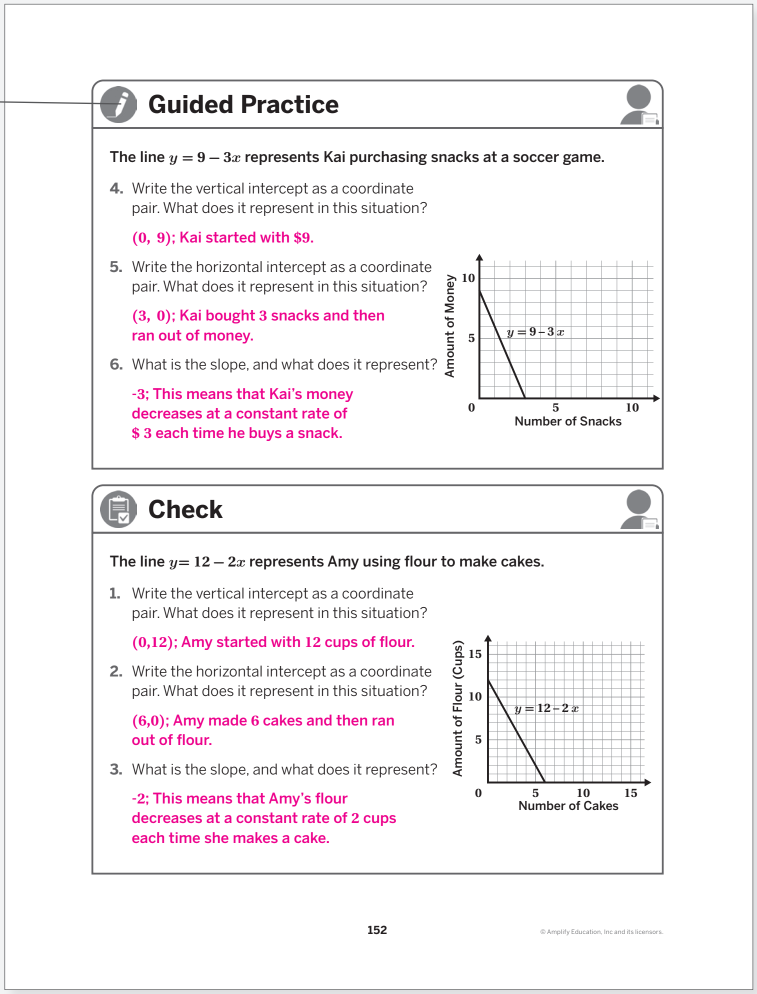
ML 3.05 Interpreting Slope and Intercepts of Linear Relationships

Guided Practice

A As students connect visual representations to linear equations, provide colored pencils or highlighters for them to color-code elements from the equation and their corresponding representations on the graph.

ML/EL Provide sentence frames for students to interpret the slope and intercepts, such as "The vertical intercept means ________.", "The horizontal intercept means ________.", or "The slope shows the rate at which________changes for every ________."

Key Takeaway:
Say, "The slope, horizontal intercept, and vertical intercept of a graph representing a linear relationship can be used to determine important information."



Guided Practice

The line $y = 9 - 3x$ represents Kai purchasing snacks at a soccer game.

4. Write the vertical intercept as a coordinate pair. What does it represent in this situation?

   $(0, 9)$; Kai started with \$9.

5. Write the horizontal intercept as a coordinate pair. What does it represent in this situation?

   $(3, 0)$; Kai bought 3 snacks and then ran out of money.

6. What is the slope, and what does it represent?

   -3; This means that Kai's money decreases at a constant rate of \$ 3 each time he buys a snack.

Check

The line $y = 12 - 2x$ represents Amy using flour to make cakes.

1. Write the vertical intercept as a coordinate pair. What does it represent in this situation?

   $(0,12)$; Amy started with 12 cups of flour.

2. Write the horizontal intercept as a coordinate pair. What does it represent in this situation?

   $(6,0)$; Amy made 6 cakes and then ran out of flour.

3. What is the slope, and what does it represent?

   -2; This means that Amy's flour decreases at a constant rate of 2 cups each time she makes a cake.

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Reflection

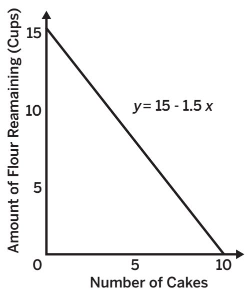
Ask:

- "How does a graph help identify the slope, vertical intercept, and horizontal intercept?"
- "What is something you weren't sure about at the start of the lesson but understand now?"

Check: Recommended Next Steps

Almost there
If students need more support, have them highlight the x-axis and y-axis to guide in identifying the vertical intercept and horizontal intercept. Then have them highlight the labels of both axes to help interpret the slope and intercepts.

Got it!
If students need more practice, sketch the problem and have them interpret the vertical intercept, horizontal intercept, and slope.



Name ______________________________

Calculating Slope Given Two Points

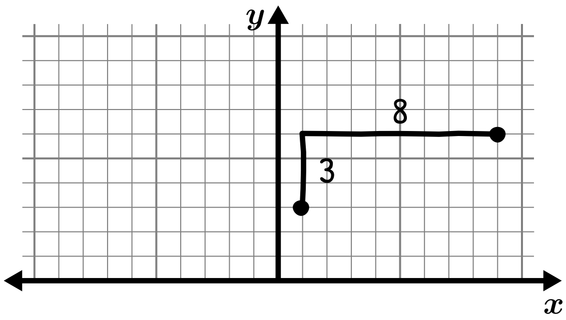
ML 3.09

Modeled Review

Name: Han

Calculate the slope of the line that passes through $(1, 3)$ and $(9, 6)$. Use the graph if it is helpful.

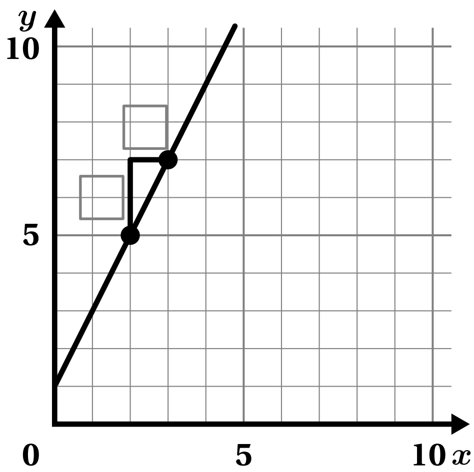
slope $= \frac{3}{8}$



Guided Practice

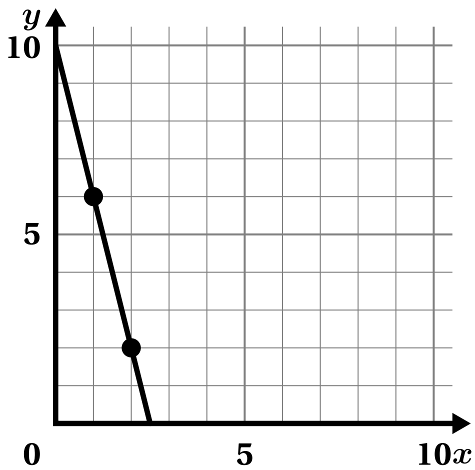
Calculate the slope of the line that passes through the given points.

1.



slope: ______________________

2.



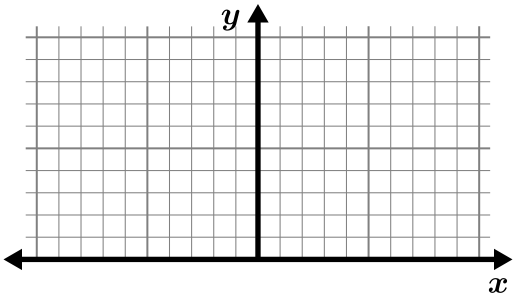
slope: ______________________

Guided Practice

Calculate the slope of the line that passes through the given points. Use the graph if it is helpful.

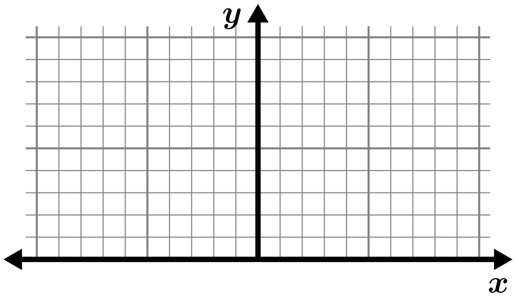
3. $(2, 4)$ and $(3, 1)$

slope: ______________________



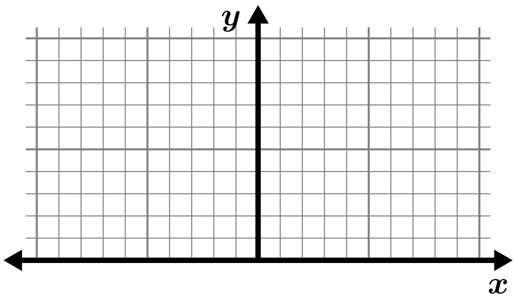
4. $(-1, 2)$ and $(4, 4)$

slope: ______________________



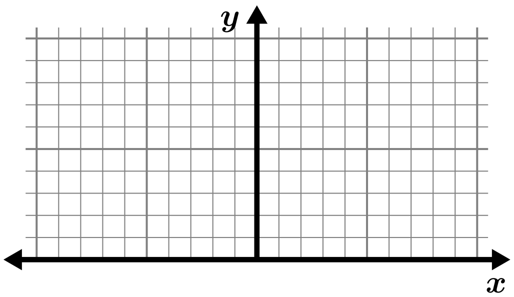
5. $(4, 2)$ and $(8, 5)$

slope: ______________________



6. $(6, 3)$ and $(9, 1)$

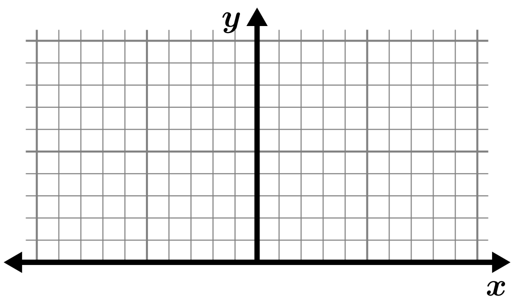
slope: ______________________



Check

Calculate the slope of the line that goes through $(2, 1)$ and $(4, 6)$. Use the graph if it is helpful.

slope: ______________________



Calculating Slope Given Two Points

Goal	Standard
Calculating the slope of a line through two given points.	MA.8.AR.3.2

Modeled Review

Point to Han's work and **ask**:

- "How did Han use a slope triangle to calculate the slope?"
- "How did Han know to label the side lengths as positive?"

Reinforce Han's thinking by saying, "Slope triangles can be used to calculate the slope of a line through two given points."

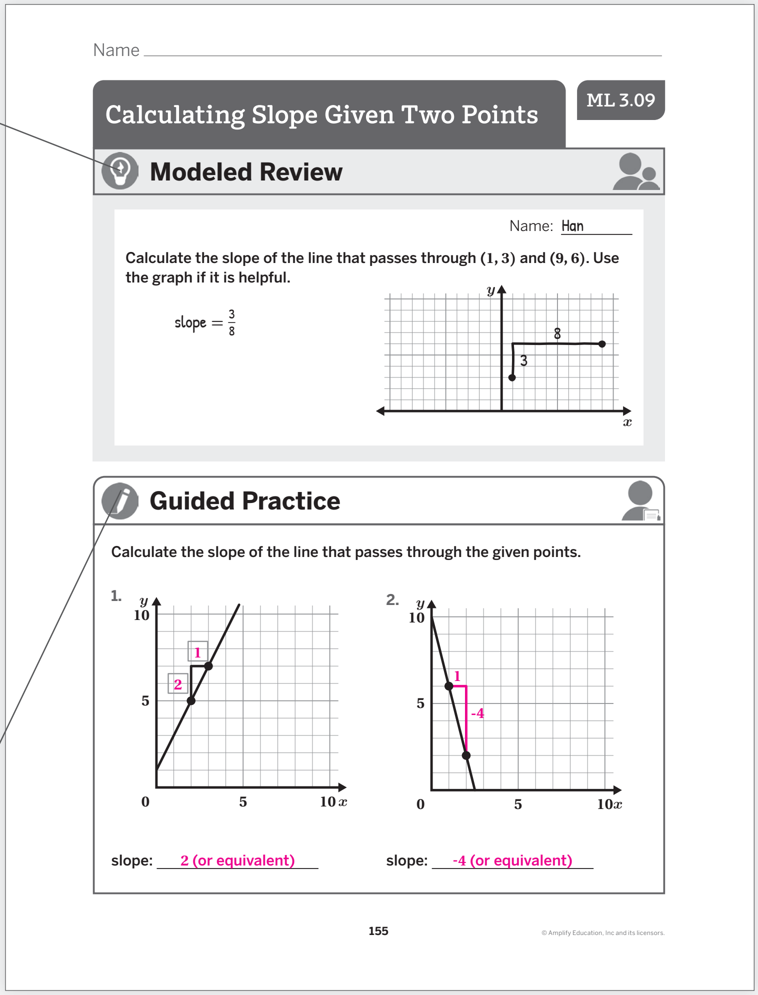
ML/EL Provide sentence frames to support students as they explain their strategies for calculating slope. For example, "I noticed that ______, so I ______." or "First, I ________ because ______."

Guided Practice

Focus students' attention on using slope triangles to calculate the slope.

To scaffold their thinking, **ask**:

- "Where are the points located on the coordinate plane?"
- "How do you create and label a slope triangle?"
- "How do you know if the side lengths are positive or negative?"



Name ______

Calculating Slope Given Two Points — ML 3.09

Modeled Review

Name: Han

Calculate the slope of the line that passes through $(1, 3)$ and $(9, 6)$. Use the graph if it is helpful.

slope $= \frac{3}{8}$

Guided Practice

Calculate the slope of the line that passes through the given points.

1. slope: 2 (or equivalent)

2. slope: -4 (or equivalent)

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Vocabulary

If needed, share the meaning of the terms with students.

slope: A number that describes the direction and steepness of a line.

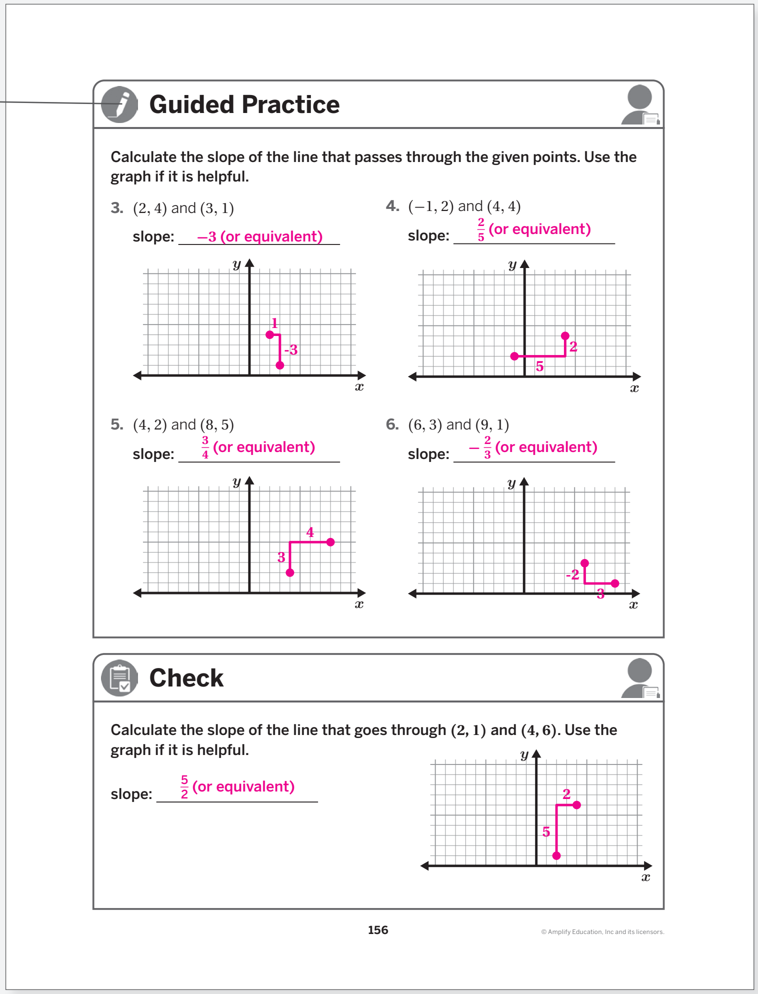
linear relationship: A relationship between two quantities is linear if there is a constant rate of change.

Guided Practice

(A) Chunk this task into smaller, more manageable parts by having students first number the scales then plot and label each point on the grid before drawing the slope triangle.

Note: Students may draw the slope triangle differently for each problem.

Key Takeaway:
Say, “The slope of a line can be calculated by dividing the difference in the values of *y* by the difference in the values of *x* using the coordinates of any two points on the line.”



Guided Practice

Calculate the slope of the line that passes through the given points. Use the graph if it is helpful.

3. $(2, 4)$ and $(3, 1)$
slope: -3 (or equivalent)

4. $(-1, 2)$ and $(4, 4)$
slope: $\frac{2}{5}$ (or equivalent)

5. $(4, 2)$ and $(8, 5)$
slope: $\frac{3}{4}$ (or equivalent)

6. $(6, 3)$ and $(9, 1)$
slope: $-\frac{2}{3}$ (or equivalent)

Check

Calculate the slope of the line that goes through $(2, 1)$ and $(4, 6)$. Use the graph if it is helpful.

slope: $\frac{5}{2}$ (or equivalent)

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Reflection

Ask:

- “How is using a slope triangle helpful in calculating the slope of a line?”
- “What is something you weren’t sure about at the start of the lesson but understand now?”

Check: Recommended Next Steps

Almost there If students need more support, consider asking them to draw an arrow to show if each length is positive or negative. Ask them to draw an arrow up or right to show a positive length and an arrow down or left to show a negative length. This will help them see which directions represent positive and negative values.	If students need more practice, invite them to look at Problem 3 and ask how the slope would change if the second point given was (5, 2).

Name ____________________

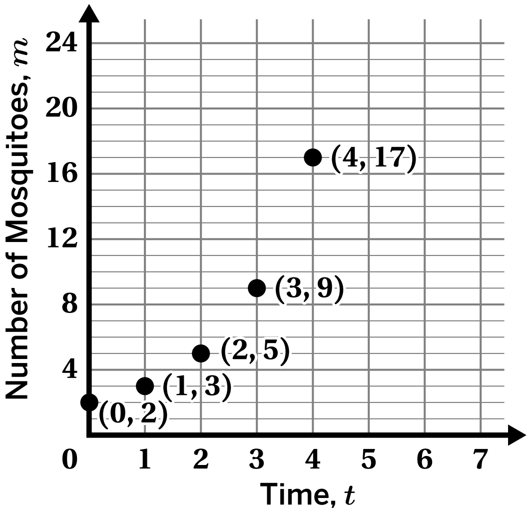
Interpreting Graphs

ML 5.04

Modeled Review

Name: Shawn

The graph represents the relationship between time, t, and the number of mosquitoes, m. Complete the table so it reflects the values in the graph.

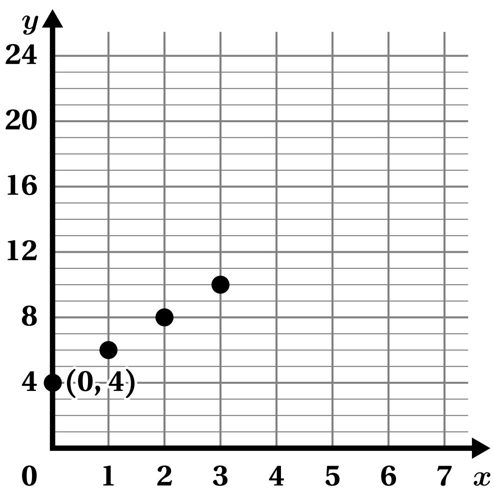


t	m
0	2
1	3
2	5
3	9
4	17

Time is the independent variable, so it represents the x-values. The number of mosquitos is the dependent variable, so it represents the y-values.

Guided Practice

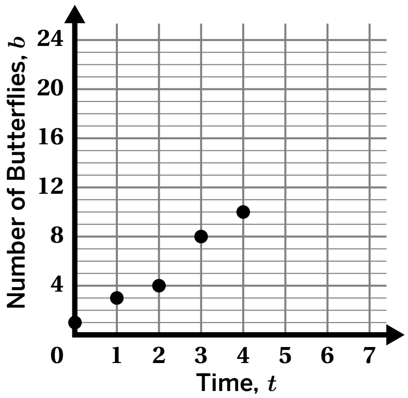
1. The graph represents the relationship between the independent variable, x, and the dependent variable, y. Complete the table so it reflects the values in the graph.



x	y
0	4

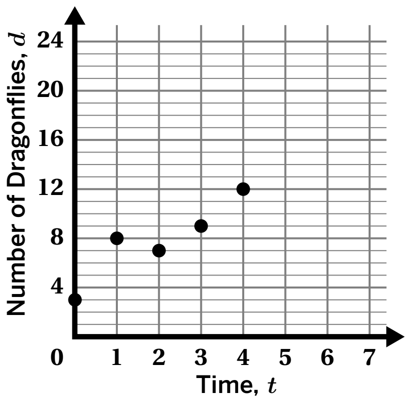
Guided Practice

2. This graph represents the relationship between time, t, and the number of butterflies, b. Complete the table so it reflects the values in the graph.



t	b

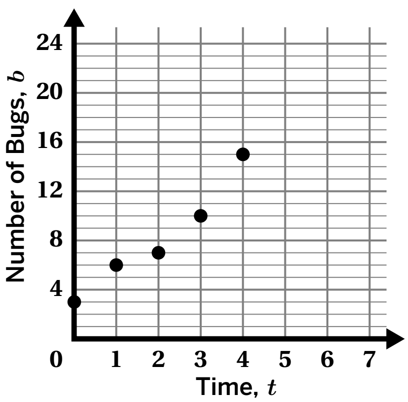
3. This graph represents the relationship between time, t, and the number of dragonflies, d. Complete the table so it reflects the values in the graph.



t	d

Check

This graph represents the relationship between time, t, and the number of bugs, b. Complete the table so it reflects the values in the graph.



t	b

ML 5.04 Interpreting Graphs

Goal

Interpret a graph to create a table that represents a relationship between two quantities.

Standard

MA.8.F.1.1

Modeled Review

Point to Shawn's work and **ask**:

- "How does Shawn know how to write the ordered pairs?"
- "Why is t the independent variable and m the dependent variable?"
- "How are the graph and table similar? How are they different?"

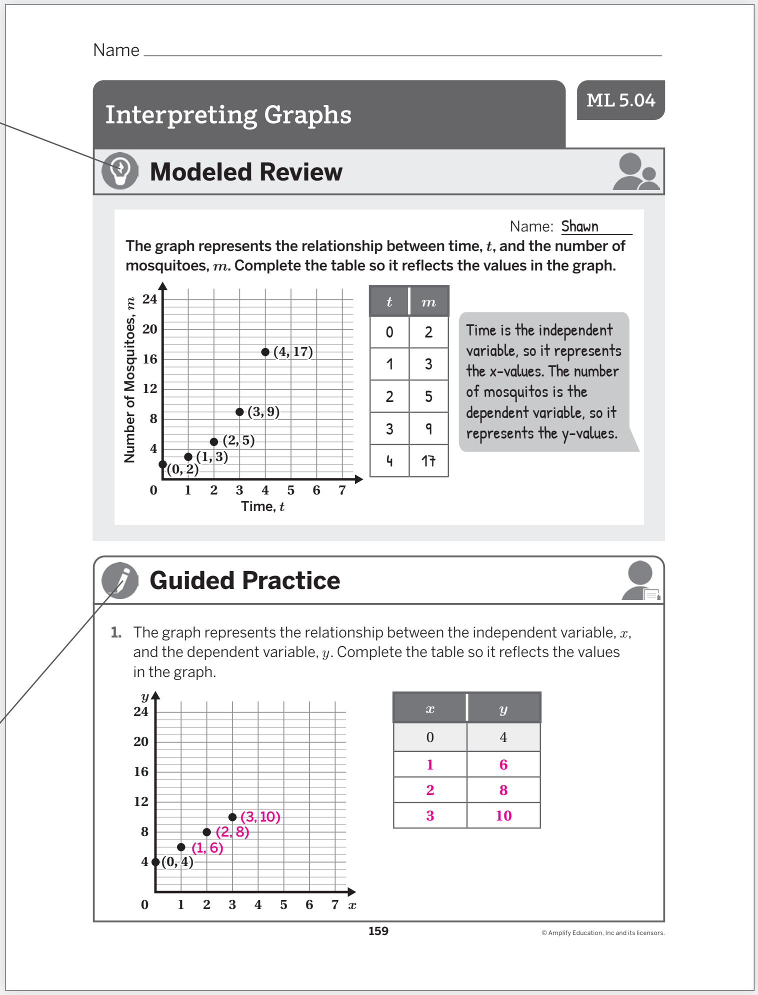
Reinforce Shawn's thinking by saying, "A graph can be used to create a table by focusing on the independent and dependent variables represented by each ordered pair."

Guided Practice

Focus students' attention on using the graph's ordered pairs to create a table that represents the data.

To scaffold their thinking, **say**:

- "Determine the independent and dependent variables."
- "Identify the ordered pairs on the graph."
- "Write the ordered pairs in the table by putting the x-values in the first column and y-values in the second column."



Name

Interpreting Graphs ML 5.04

Modeled Review

Name: Shawn

The graph represents the relationship between time, t, and the number of mosquitoes, m. Complete the table so it reflects the values in the graph.

t	m
0	2
1	3
2	5
3	9
4	17

Time is the independent variable, so it represents the x-values. The number of mosquitos is the dependent variable, so it represents the y-values.

Guided Practice

1. The graph represents the relationship between the independent variable, x, and the dependent variable, y. Complete the table so it reflects the values in the graph.

x	y
0	4
1	6
2	8
3	10

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Vocabulary

If needed, share the meaning of the terms with students.

dependent variable: The variable in a relationship that is the effect or result. The dependent variable is typically on the vertical axis of a graph and in the right-hand column of a table.

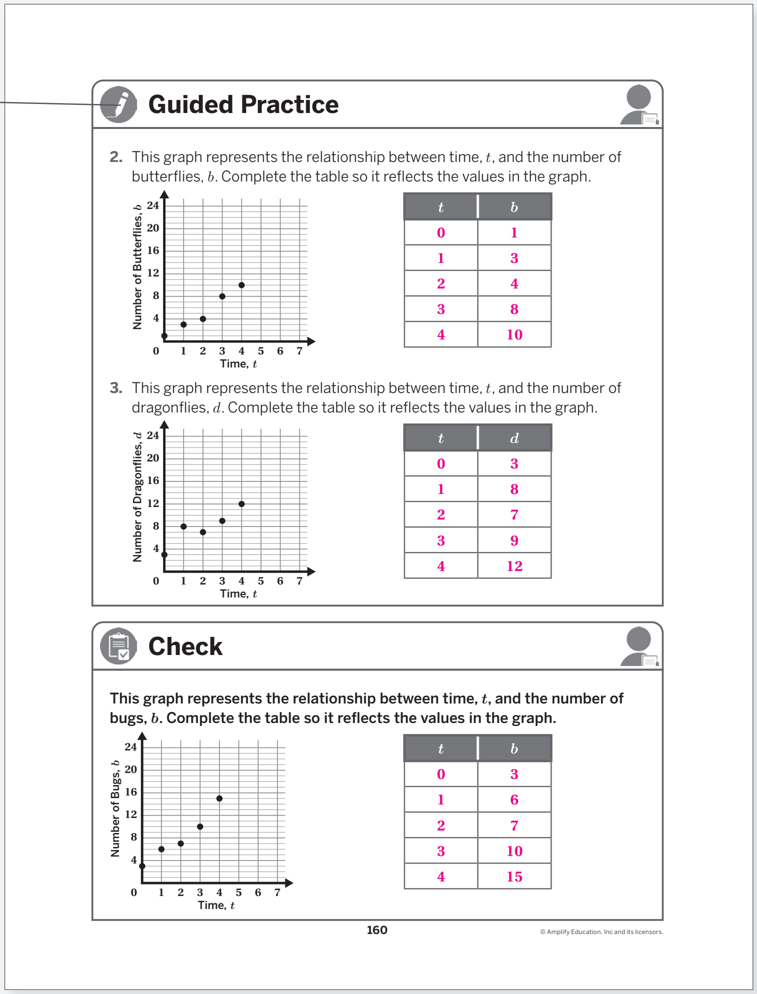
independent variable: The variable in a relationship that is the cause. The independent variable is typically on the horizontal axis of a graph and in the left-hand column of a table.

Guided Practice

A Break down the task by covering points to focus on one ordered pair at a time, and then transfer them to the table.

ML/EL To increase accessibility, offer bilingual support materials by including key vocabulary in the students' home language to scaffold their understanding of the task.

Key Takeaway:
Say, "Each point on the graph represents the relationship between an independent variable (x) and the dependent variable (y)."



Guided Practice

2. This graph represents the relationship between time, t, and the number of butterflies, b. Complete the table so it reflects the values in the graph.

t	b
0	1
1	3
2	4
3	8
4	10

3. This graph represents the relationship between time, t, and the number of dragonflies, d. Complete the table so it reflects the values in the graph.

t	d
0	3
1	8
2	7
3	9
4	12

Check

This graph represents the relationship between time, t, and the number of bugs, b. Complete the table so it reflects the values in the graph.

t	b
0	3
1	6
2	7
3	10
4	15

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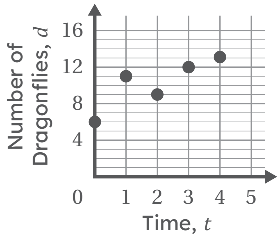
Reflection

Ask:
- "How can you tell that a table and graph show the same relationship?"
- "What is something you weren't sure about at the start of the lesson but understand now?"

Check: Recommended Next Steps

Almost there
If students need more support, consider using the graph in the Check to model writing the ordered pairs by drawing a vertical line from each ordered pair to the x-axis and a horizontal line from each ordered pair to the y-axis.

Got it!
If students need more practice, sketch the graph and ask them to create a table that represents the data.



Name ____________________

Solving Systems of Linear Equations by Graphing

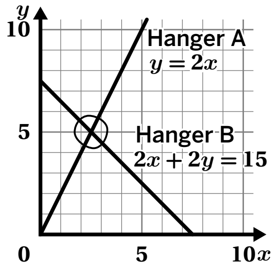
ML 4.08

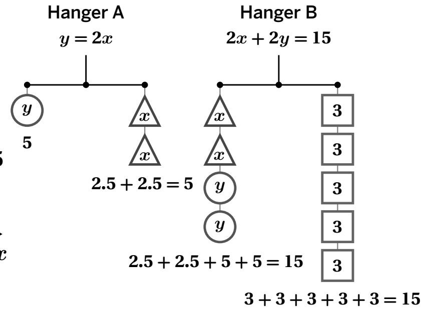
Modeled Review

Name: Clare

Use these hanger diagrams and the related graph to complete the problems.

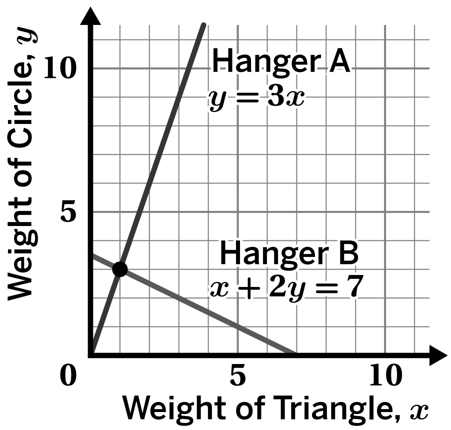
1. Determine the solution to the system of equations.
   (2.5, 5)
2. What does the solution tell you about the weight of the circle and the triangle to balance both hangers?
   Each triangle weighs 2.5 and each circle weighs 5.

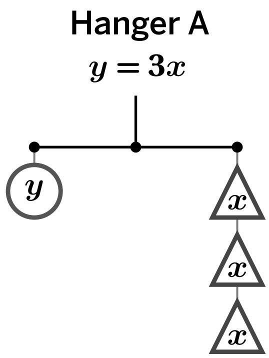


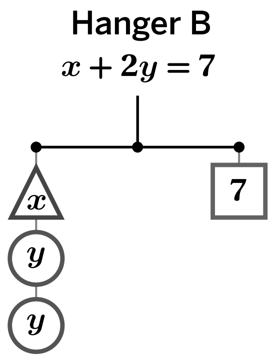


Guided Practice

Use these hanger diagrams and the related graph to complete the problems.







1. Determine the solution to the system of equations.

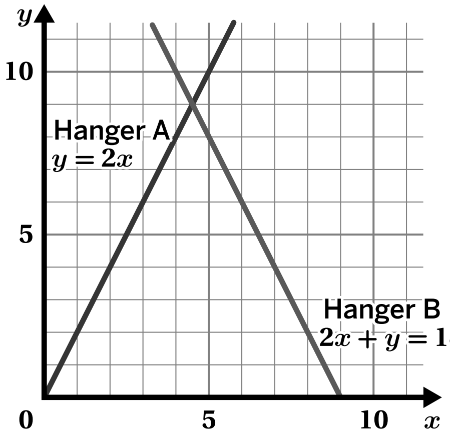
   ______

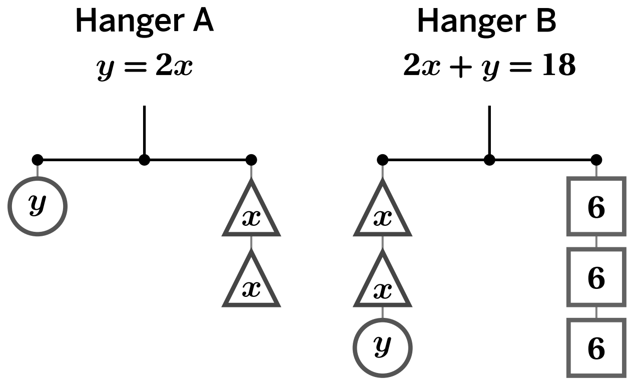
2. What does the solution tell you about the weight of the circle and the triangle to balance both hangers?

   Each triangle weighs 1 and each circle weighs ___.

Guided Practice

Use these hanger diagrams and the related graph to complete the problems.





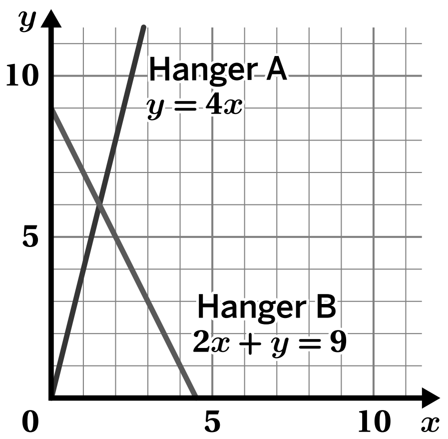
3. Determine the solution to the system of equations.

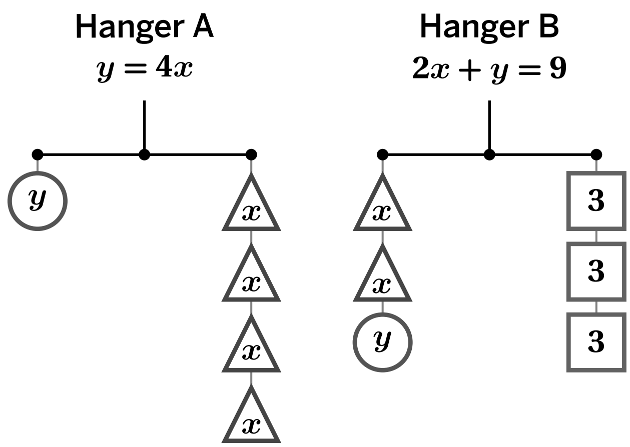
____

4. What does the solution tell you about the weight of the circle and the triangle to balance both hangers?

Check

Use these hanger diagrams and the related graph to complete the problems.





3. Determine the solution to the system of equations.

____

4. What does the solution tell you about the weight of the circle and the triangle to balance both hangers?

Solving Systems of Linear Equations by Graphing

Goal

Solve systems of equations by determining the values that satisfy both equations.

Standard

MA.8.AR.4.3

Modeled Review

Point to Clare's work and **ask**:

- "How did Clare identify the solution to the system of equations?"
- "What did Clare use from the graph to help determine how both hangers relate to the solution?"

Reinforce Clare's thinking by saying, "The solution to a system of equations is the point of intersection."

ML/EL To support students as they share their responses, provide the following sentence frames:

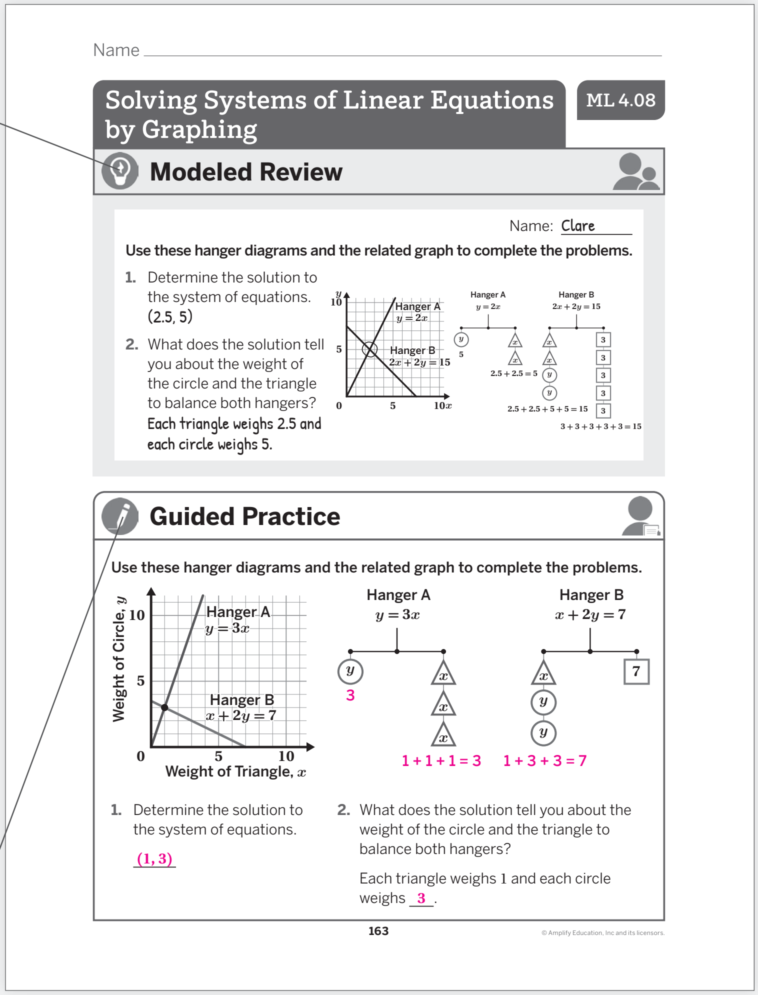
- "The point ___ balances just [Hanger A/B]."
- "When x is __ and y is __, both hangers are balanced."
- "The point __ will balance neither hanger."

Guided Practice

Focus students' attention on using the given graph to determine the solution to the system of equations.

To scaffold their thinking, **ask**:

- "Where is the point of intersection to the system of equations?"
- "Does this solution work to make both hangers true?"


Name

Solving Systems of Linear Equations by Graphing ML 4.08

Modeled Review

Name: Clare

Use these hanger diagrams and the related graph to complete the problems.

1. Determine the solution to the system of equations.
(2.5, 5)

2. What does the solution tell you about the weight of the circle and the triangle to balance both hangers?
Each triangle weighs 2.5 and each circle weighs 5.

Guided Practice

Use these hanger diagrams and the related graph to complete the problems.

1. Determine the solution to the system of equations.
(1, 3)

2. What does the solution tell you about the weight of the circle and the triangle to balance both hangers?
Each triangle weighs 1 and each circle weighs 3.

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Vocabulary

If needed, share the meaning of the terms with students.

system of equations: A system of equations is two or more equations that represent the constraints on a shared set of variables.

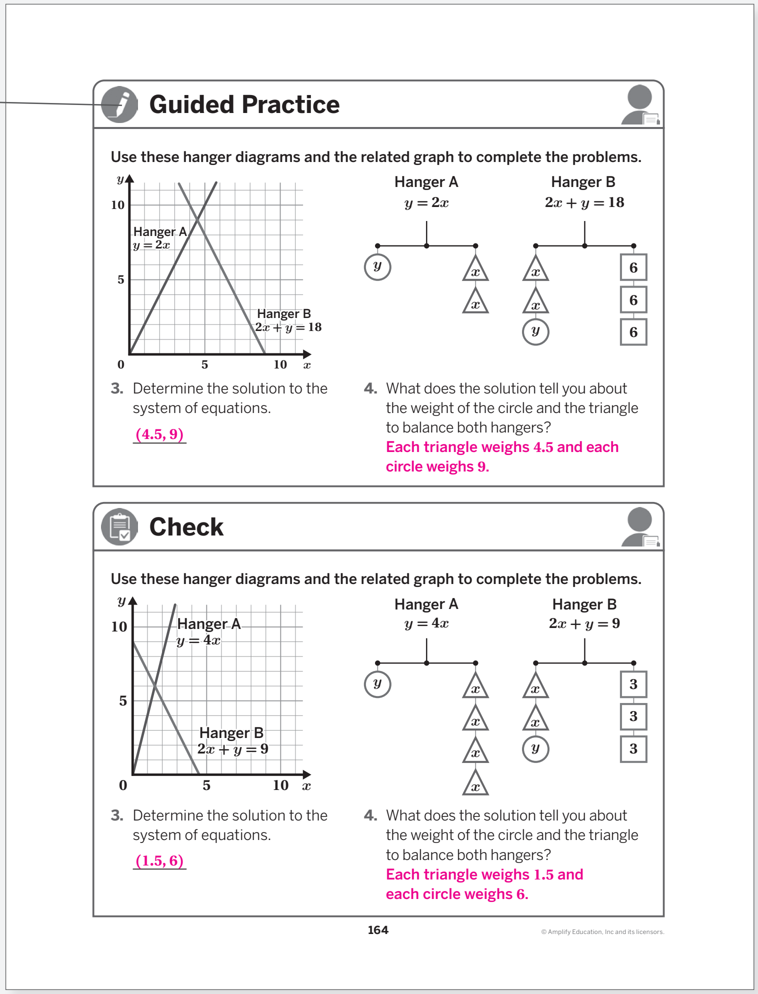
solution to a system of equations: A solution to a system of equations is a set of values that makes all equations in that system true.

Solving Systems of Linear Equations by Graphing

Guided Practice

A Chunk this task into smaller, more manageable parts by having students identify the point of intersection on the graph before looking at the hangers.

Key Takeaway:
Say, "A solution to a system of equations must satisfy all equations in the system."



Guided Practice

Use these hanger diagrams and the related graph to complete the problems.

3. Determine the solution to the system of equations.
   (4.5, 9)

4. What does the solution tell you about the weight of the circle and the triangle to balance both hangers?
   Each triangle weighs 4.5 and each circle weighs 9.

Check

Use these hanger diagrams and the related graph to complete the problems.

3. Determine the solution to the system of equations.
   (1.5, 6)

4. What does the solution tell you about the weight of the circle and the triangle to balance both hangers?
   Each triangle weighs 1.5 and each circle weighs 6.

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Reflection

Ask:

- "Where is the solution to a system of equations located on a graph?"
- "What is something you weren't sure about at the start of the lesson but understand now?"

Check: Recommended Next Steps

Almost there	**Got it!**
If students need more support, consider asking them to label both the *x*- and *y*-axes with all numbers to help them determine the correct solution.	If students need more practice, refer to Problem 3 and ask them to write an ordered pair that would not be a solution to the system of equations.

Name ______________________________

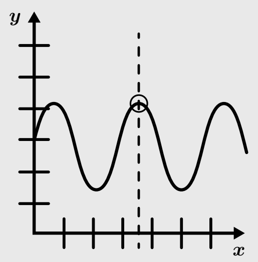
Justifying Whether a Graph Represents a Function

ML 5.03

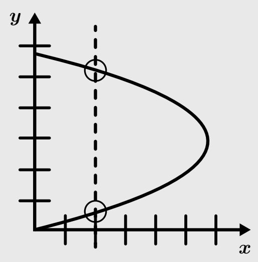
Modeled Review

Function: a rule that assigns exactly one output to each possible input.

Function



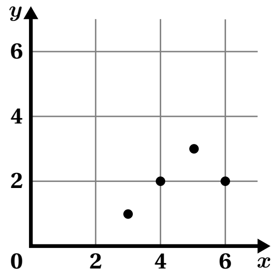
Not a function



Guided Practice

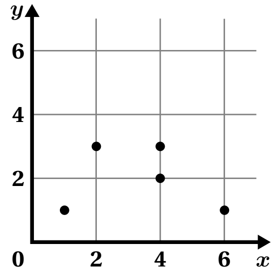
Determine whether the graph is a function. If the graph is not a function, draw a vertical line where the graph shows more than one output for the same input.

1.



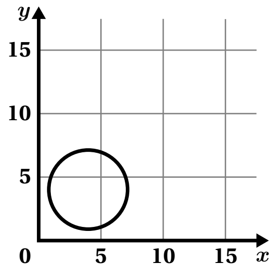
Function   Not a function

2.



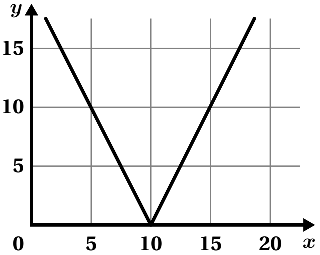
Function   Not a function

3.



Function   Not a function

4.

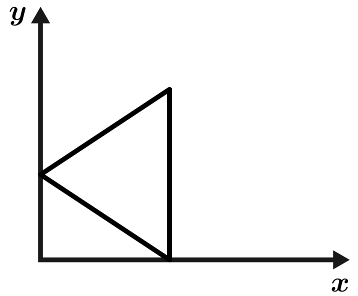


Function   Not a function

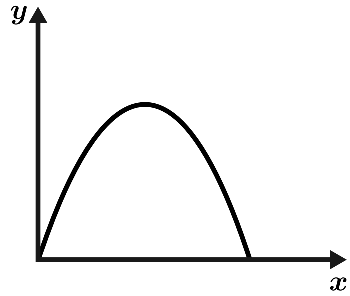
Guided Practice

Determine whether y is a function of x and write "function" or "not a function." If the graph is not a function, draw a vertical line where the graph shows more than one output for the same input.

5.



6.



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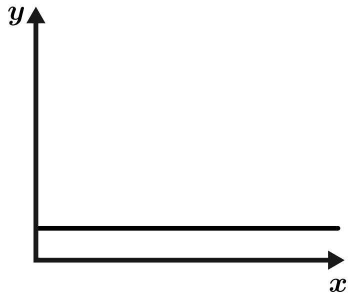

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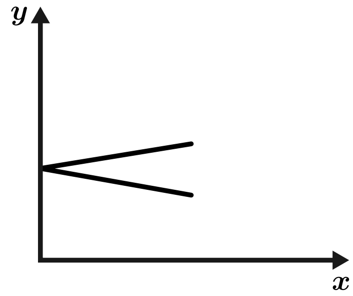
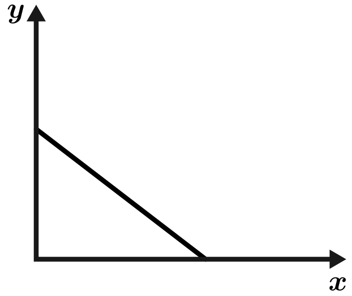


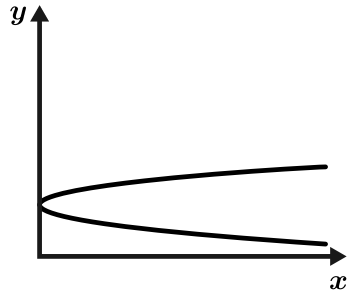
Check

Select the graph in which y is a function of x. On the graph that is not a function, draw a vertical line where the graph shows more than one output for the same input.

A.



B.



ML 5.03 Justifying Whether a Graph Represents a Function

Goal

Justify whether or not y is a function of x using a graph.

Standard

MA.8.F.1.1

Materials

coloring tools (optional)

Modeled Review

Point to the Modeled Review and **ask**:

- "In your own words, what is a function?"
- "What makes the graph on the left a function?"
- "What makes the graph on the right *not* a function?"

Reinforce the goal by saying, "You can determine whether a graph is a function or not by analyzing whether each input value has exactly one output."

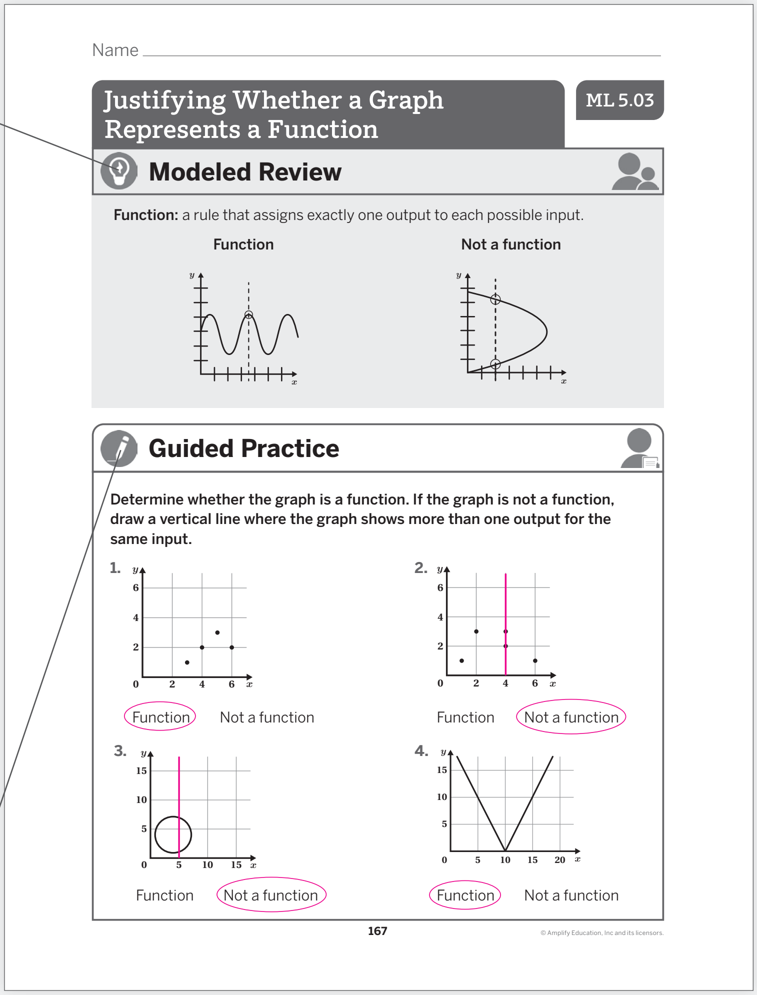
ML/EL Invite students to share what a new term reminds them of (e.g., "When I think of input, I think of someone giving advice or an opinion."). This may surface multiple meanings and support students as they connect to their prior knowledge.

Guided Practice

Focus students' attention on determining whether the graphs represent a function or not.

To scaffold their thinking, **ask**:

- "What is a function?"
- "What is not a function?"
- "How can you use a vertical line to help determine if the graph is a function or not?"



Name

Justifying Whether a Graph Represents a Function

ML 5.03

Modeled Review

Function: a rule that assigns exactly one output to each possible input.

Function | Not a function

Guided Practice

Determine whether the graph is a function. If the graph is not a function, draw a vertical line where the graph shows more than one output for the same input.

1. Function Not a function

2. Function Not a function

3. Function Not a function

4. Function Not a function

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Vocabulary

If needed, share the meaning of the term with students.

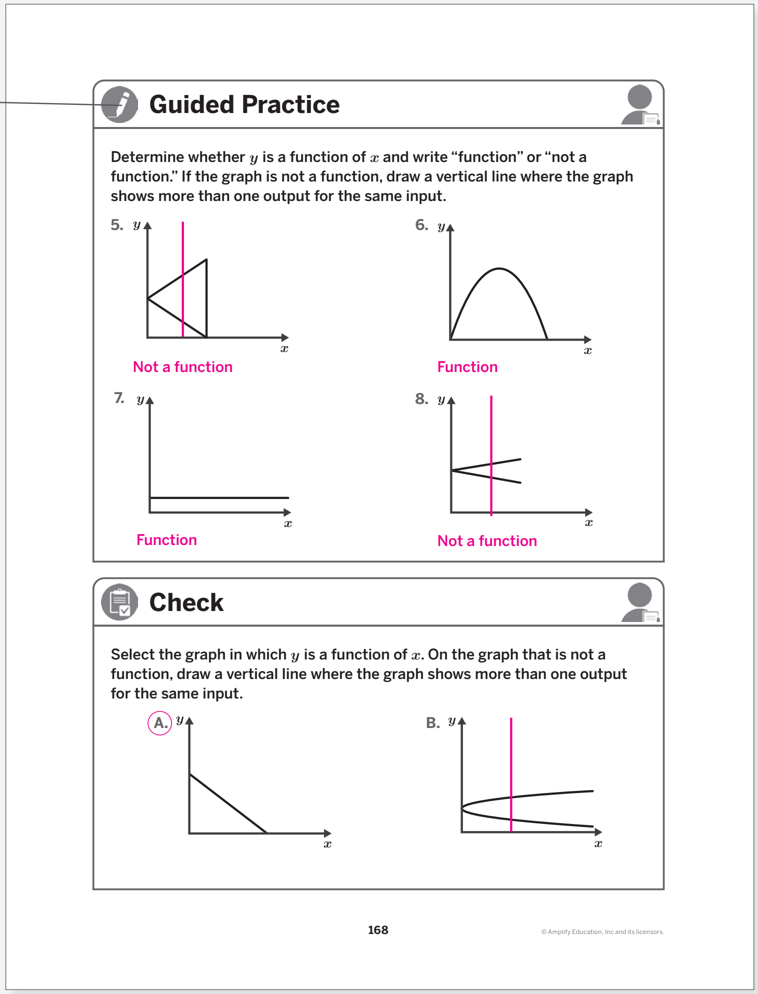
function: A function is a rule that assigns exactly one output to each possible input.

ML 5.03 Justifying Whether a Graph Represents a Function

Guided Practice

A Invite students to use coloring tools to determine if the graph is a function or not by having them denote the input in one color and the output in another color. Then have them use the colored graph to determine if each input has exactly one output.

Key Takeaway:
Say, “The graph of a function is a set of ordered pairs where each input has exactly one output.”



Guided Practice

Determine whether y is a function of x and write “function” or “not a function.” If the graph is not a function, draw a vertical line where the graph shows more than one output for the same input.

Check

Select the graph in which y is a function of x. On the graph that is not a function, draw a vertical line where the graph shows more than one output for the same input.

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Reflection

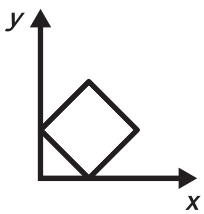
Ask:

- “How can you use a vertical line to determine if a graph is a function or not?”
- “What was easy for you? What questions do you still have?”

Check: Recommended Next Steps

Almost there
If students need more support, model representing the ordered pairs from a graph in a table or mapping diagram. Then check that each input has exactly one output.

Got it!
If students need more practice, sketch the following graph and have them determine if the graph is a function or not.



Name ____________________

Comparing Properties of Linear Functions

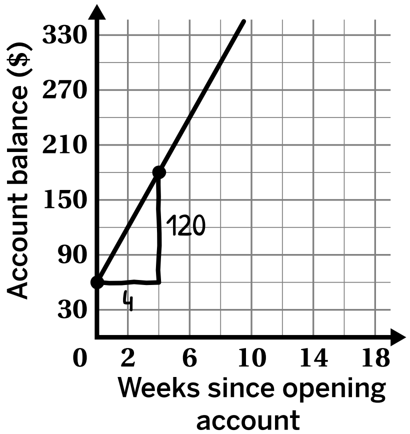
ML 5.07

Modeled Review

Name: Maya

The graph and table each show a different savings account with changes occurring at a constant rate.

Kai



Slope = $\frac{120}{4}$ = 30 y- intercept = 60

Jada

Number of weeks	Account balance ($)
+1 1	75 −15
2	60
3	45

Slope = $\frac{-15}{1}$ = −15

$y = mx + b$ (1,75)
$y = -15x + b$
$75 = -15(1) + b$
$75 = -15 + b$
$90 = b$

1. Whose balance changes at a faster rate? Explain your thinking.
   Kai; Kai's account is increasing by $30 every week, while Jada's account is decreasing by $15 every week.
2. Who started with the larger amount in their account? Explain your thinking.
   Jada; At 0 weeks she had $90 in her account, while Kai at 0 weeks had $60.

Guided Practice

1. Which representation has the fastest rate of change?

   A. $y = -3x + 1$ **B.** $y = 2x + 1$ **C.** $y = -\frac{1}{2}x + 1$

2. Which representation has the least y-intercept?

   A. The account balance, *a*, starts at $30 and increases by $5 per week.

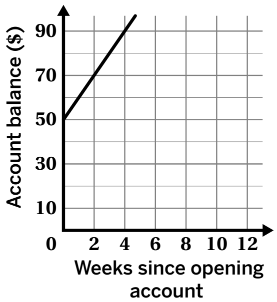
   B. The account balance, *a*, starts at $50 and increases by $8 per week.

   C. The account balance, *a*, starts at $20 and increases by $9 per week.

Guided Practice

3. The savings accounts of three customers are being compared. Circle the representation with the fastest rate of change. Explain your reasoning.

A.



B.

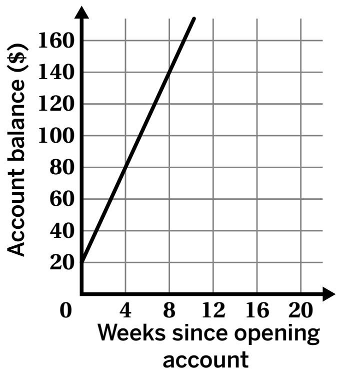
Number of weeks	Account balance ($)
1	90
2	70
3	50

C. The account balance, *a*, starts at $30 and increases $18 per week.

Check

The graph and table each show a different person's savings account.

Jack



Han

Number of weeks	Account balance ($)
1	100
2	80
3	60

1. Whose account balance changes at a faster rate? Explain your thinking.

2. Who started with the larger initial amount saved? Explain your thinking.

ML 5.07 Comparing Properties of Linear Functions

Goal	Standard
Compare y-intercepts and slopes of linear functions.	MA.8.AR.3.5

Modeled Review

Point to Maya's work and **ask**:

- "How did Maya use slope triangles to calculate the slope of the graph?"
- "How did Maya calculate the y-intercept from the table? The graph?"
- "How did Maya use the independent and dependent values to calculate the slope from the table?"

Note: Here, "faster rate" refers to the rate of change with the greatest magnitude and can be positive or negative.

Reinforce Maya's thinking by saying, "The slope and y-intercept can be used to compare linear functions in different representations."

Guided Practice

Focus students' attention on determining the slope and y-intercept to compare the linear functions.

To scaffold their thinking, **ask**:

- "How do you find the rate of change in a linear equation?"
- "How do you find the rate of change in a graph? The y-intercept?"
- "Which keywords in the description represent the slope and y-intercept?"

Name

Comparing Properties of Linear Functions

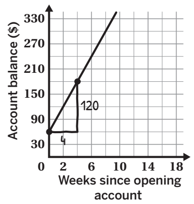
ML 5.07

Modeled Review

Name: Maya

The graph and table each show a different savings account with changes occurring at a constant rate.

Kai



Slope = $\frac{120}{4}$ = 30   y- intercept = 60

Jada

Number of weeks	Account balance ($)
+1 1	75 −15
2	60
3	45

Slope = $\frac{-15}{1}$ = −15

y = mx + b (1,75)
y = −15x + b
75 = −15(1) + b
75 = −15 + b
90 = b

1. Whose balance changes at a faster rate? Explain your thinking.
   Kai; Kai's account is increasing by $30 every week, while Jada's account is decreasing by $15 every week.
2. Who started with the larger amount in their account? Explain your thinking.
   Jada; At 0 weeks she had $90 in her account, while Kai at 0 weeks had $60.

Guided Practice

1. Which representation has the fastest rate of change?
   A. $y = -3x + 1$ B. $y = 2x + 1$ C. $y = -\frac{1}{2}x + 1$
2. Which representation has the least y-intercept?
   A. The account balance, a, starts at $30 and increases by $5 per week.
   B. The account balance, a, starts at $50 and increases by $8 per week.
   C. The account balance, a, starts at $20 and increases by $9 per week.

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Vocabulary

If needed, share the meaning of the term with students.

linear function: A function that can be defined by an equation in the form $y = mx + b$, where m represents the slope and b represents the vertical intercept. A vertical line is not a linear function because it has an input with different outputs.

ML 5.07 Comparing Properties of Linear Functions

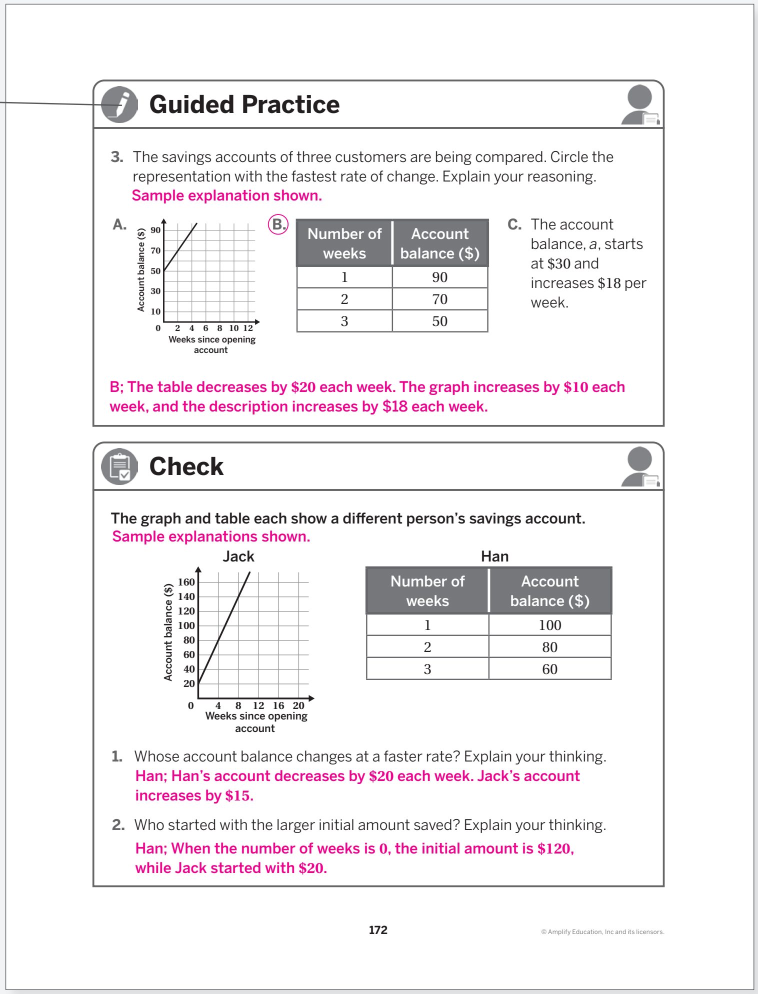
Guided Practice

A Chunk the problem by evaluating one representation at a time for students to determine the slope or y-intercept, based on what is being asked.

ML/EL Display the graph and table to students without revealing the questions that follow. Then ask students to calculate the slope and y-intercept of each.

Note: Remind students that slope and rate of change can be used interchangeably.

Key Takeaway:
Say, "Multiple linear functions represented in different ways can be compared by determining the slope and the y-intercept of the functions."



Guided Practice

3. The savings accounts of three customers are being compared. Circle the representation with the fastest rate of change. Explain your reasoning.
Sample explanation shown.

A.

B.

Number of weeks	Account balance ($)
1	90
2	70
3	50

C. The account balance, a, starts at \$30 and increases \$18 per week.

B; The table decreases by \$20 each week. The graph increases by \$10 each week, and the description increases by \$18 each week.

Check

The graph and table each show a different person's savings account.
Sample explanations shown.

Jack

Han

Number of weeks	Account balance ($)
1	100
2	80
3	60

1. Whose account balance changes at a faster rate? Explain your thinking.
Han; Han's account decreases by \$20 each week. Jack's account increases by \$15.

2. Who started with the larger initial amount saved? Explain your thinking.
Han; When the number of weeks is 0, the initial amount is \$120, while Jack started with \$20.

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Reflection

Ask:

- "How can you determine the slope and y-intercept from a graph, table, and description?"
- "How does what you learned today connect to your prior learning?"

Check: Recommended Next Steps

Almost there

If students need more support, consider modeling how to find the y-intercept from a table when it is not given by counting backwards using the pattern in the table to find what y equals when x is zero.

Got it!

If students need more practice, have them look at Problem 3 part C and ask if changing the description to say, "The account balance, a, starts at \$30 and increases \$25 per week." would increase its current rate of change.

Name ______________________________

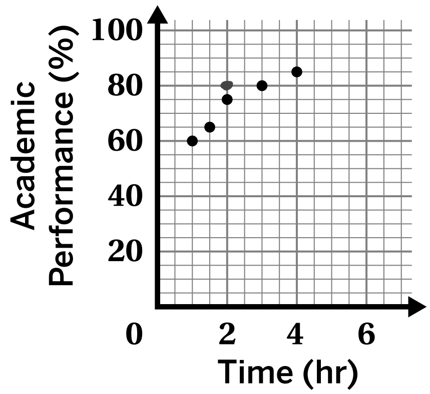
Interpreting Points on a Scatter Plot

ML 6.03

Modeled Review

Name: Diego

This scatter plot shows the academic performance and the number of hours students studied for a math test.

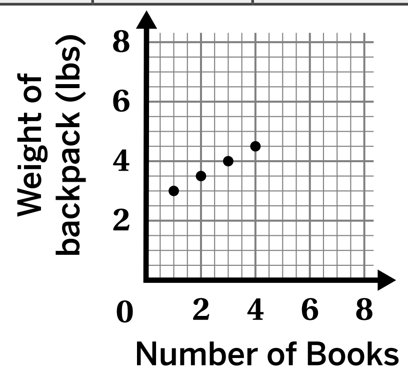


1. What is the highest academic performance a student received?
   85%
2. What is the academic performance of the student who studied the least?
   60%
3. Another student studied 2 hours and earned an academic performance of 80%. Plot a point on the graph that represents this student.

Guided Practice

The table and scatter plot show the number of books each student carried in their backpack and the corresponding weight of each backpack.

Student	Number of books	Backpack weight (lbs)
Isaiah	1	3
Kayla	3	4
Mia	2	3.5
Felipe	4	4.5



1. Circle the point on the scatter plot that represents the data for Mia.
2. What does the point $(3, 4)$ represent?
3. In the same study, the data showed that Arjun had a backpack weighing 5 pounds with 6 books. Add a point to the scatter plot to represent Arjun.

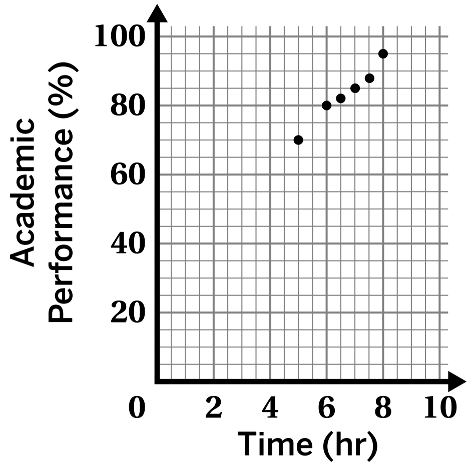
Guided Practice

This scatter plot illustrates the connection between the amount of sleep students get each night and their academic performance.

4. What is the highest academic performance a student received?

5. How many hours did the student who received the highest academic performance sleep?

6. The academic performance for a student who slept 5 hours is 50%. Plot a point on the graph that represents this student.



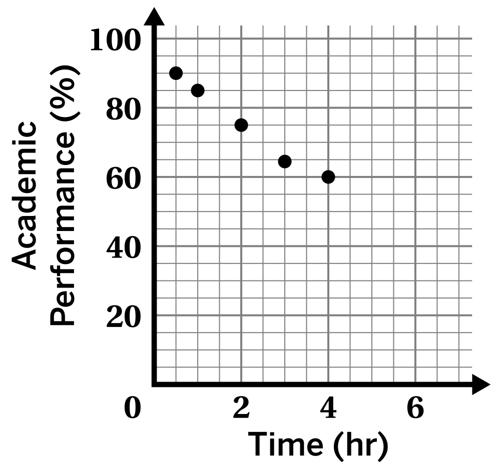
Check

This scatter plot shows the academic performance and the number of hours of screen time students have.

1. What is the highest academic performance observed in a student?

2. What is the lowest academic performance observed in a student?

3. The academic performance for a student who had 4 hours of screen time is 75%. Plot a point on the graph that represents this student.



ML 6.03 Interpreting Points on a Scatter Plot

Goal

Interpret points on a scatter plot in context.

Standard

MA.8.DP.1.1

Materials

highlighter (optional)

Modeled Review

Point to Diego's work and **ask**:

- "What specific information does each axis give you?"
- "What does a point represent?"
- "How did Diego find the academic performance of the student who studied the least?"

Reinforce the goal by saying, "The axis labels tell us how to interpret the coordinates of each point on the scatter plot."

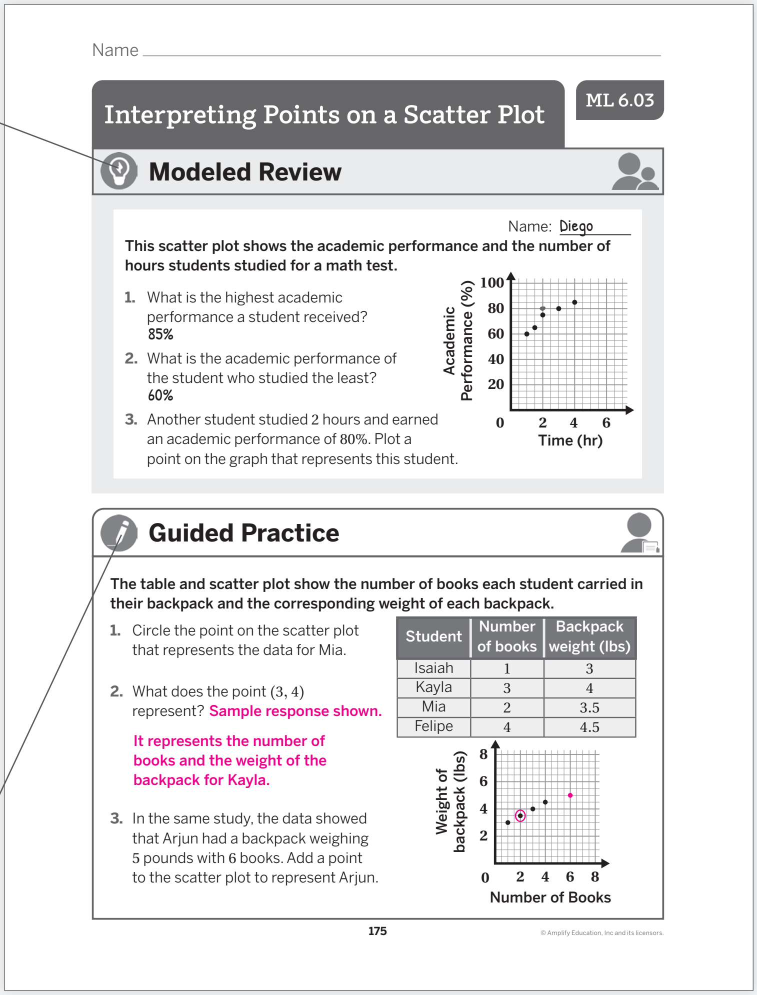
ML/EL Engage students in a discussion about everyday contexts where scatter plots might be useful, such as tracking hours studied vs. test scores or height vs. shoe size.

Guided Practice

Focus students' attention on using the table to interpret the scatter plot and answer the questions.

To scaffold their thinking, **ask**:

- "What information does the *x*-axis give you? Where do you see that in table?"
- "What information does the *y*-axis give you? Where do you see that in table?"
- "What does each point represent?"



Name

Interpreting Points on a Scatter Plot ML 6.03

Modeled Review

Name: Diego

This scatter plot shows the academic performance and the number of hours students studied for a math test.

1. What is the highest academic performance a student received?
   85%
2. What is the academic performance of the student who studied the least?
   60%
3. Another student studied 2 hours and earned an academic performance of 80%. Plot a point on the graph that represents this student.

Guided Practice

The table and scatter plot show the number of books each student carried in their backpack and the corresponding weight of each backpack.

1. Circle the point on the scatter plot that represents the data for Mia.
2. What does the point $(3, 4)$ represent? Sample response shown.
   It represents the number of books and the weight of the backpack for Kayla.
3. In the same study, the data showed that Arjun had a backpack weighing 5 pounds with 6 books. Add a point to the scatter plot to represent Arjun.

Student	Number of books	Backpack weight (lbs)
Isaiah	1	3
Kayla	3	4
Mia	2	3.5
Felipe	4	4.5

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Vocabulary

If needed, share the meaning of the term with students.

scatter plot: A set of disconnected data points plotted on a coordinate plane. Scatter plots allow us to investigate connections between two variables.

Guided Practice

(A) Invite students to highlight the axis labels to help them interpret the scatter plot.

Key Takeaway:
Say, "The values of a point and the axis labels can be used to interpret the coordinates of each point on a scatter plot."

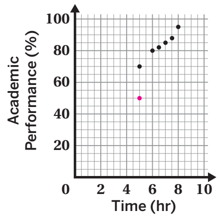
Guided Practice

This scatter plot illustrates the connection between the amount of sleep students get each night and their academic performance.

4. What is the highest academic performance a student received?
95%

5. How many hours did the student who received the highest academic performance sleep?
8

6. The academic performance for a student who slept 5 hours is 50%. Plot a point on the graph that represents this student.



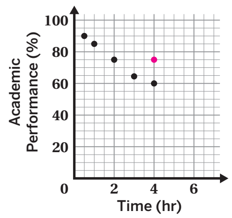
Check

This scatter plot shows the academic performance and the number of hours of screen time students have.

1. What is the highest academic performance observed in a student?
90%

2. What is the lowest academic performance observed in a student?
60%

3. The academic performance for a student who had 4 hours of screen time is 75%. Plot a point on the graph that represents this student.



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Reflection

Ask:
- "How do you know what a point represents on a scatter plot?"
- "What questions do you still have?"

Check: Recommended Next Steps

Almost there
If students need more support, consider having them revisit the scatter plot in the Check. Then have them represent the points as ordered pairs in a table.

Got it!
If students need more practice, have them revisit the scatter plot in the Check and ask the following questions:

- How many hours of screen time did the student who received the lowest academic performance have?
- What was the academic performance of the student who had 2 hours of screen time?

Name

Using Lines of Fit to Make Predictions

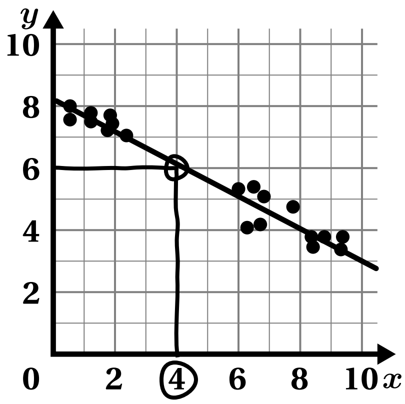
ML 6.09

Modeled Review

Name: Tristan

Use the line of fit to predict the y-value of a new data point whose x-value is 4.

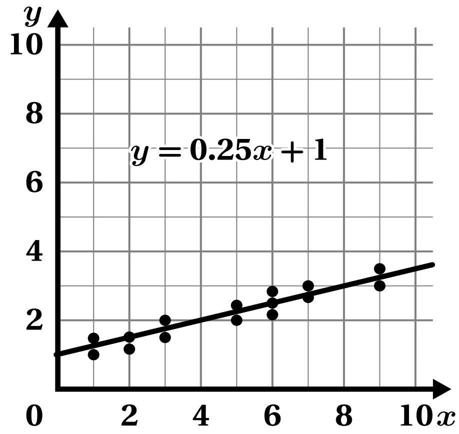
6



Guided Practice

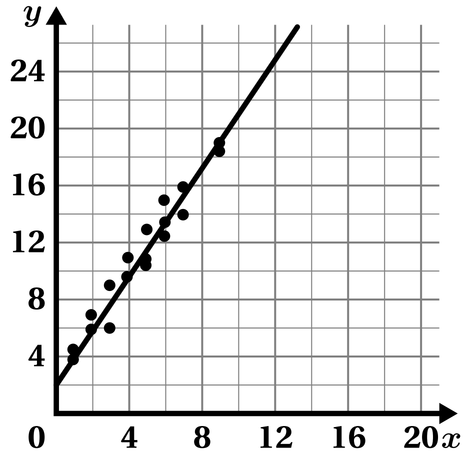
1. Use the line of fit to make predictions and complete the table.

x	y
4	
8	

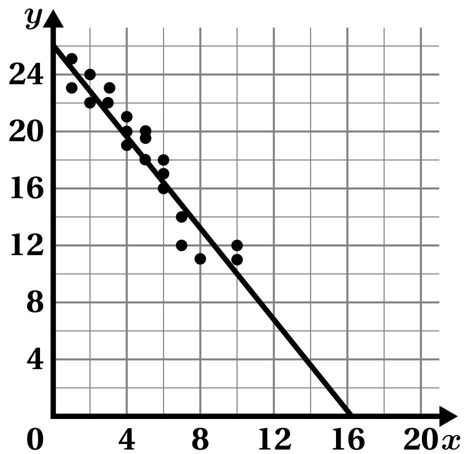


Guided Practice

2. Use the line of fit to predict the y-value of a new data point whose x-value is 8.

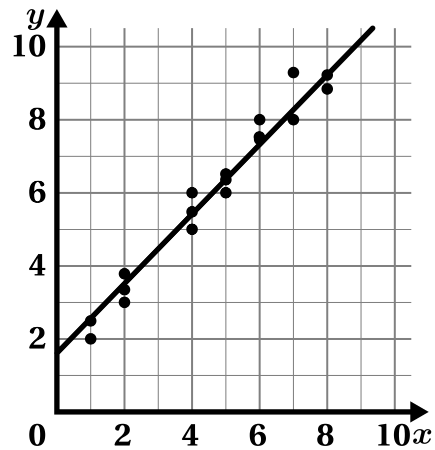


3. Use the line of fit to predict the y-value of a new data point whose x-value is 12.



Check

Use the line of fit to predict the y-value of a new data point whose x-value is 3.



ML 6.09 Using Lines of Fit to Make Predictions

Goal

Use the line of fit to make predictions about data points on a scatter plot.

Standard

MA.8.DP 1.3

Materials

highlighter (optional), ruler (optional)

Modeled Review

Point to Tristan's work and **ask**:

- "How did Tristan use the line of fit to make the prediction?"
- "How did Tristan use the given *x*-value of 4 to find the corresponding *y*-value?"

Reinforce Tristan's thinking by saying, "A line of fit can be used on scatter plots to make predictions for a given data set."

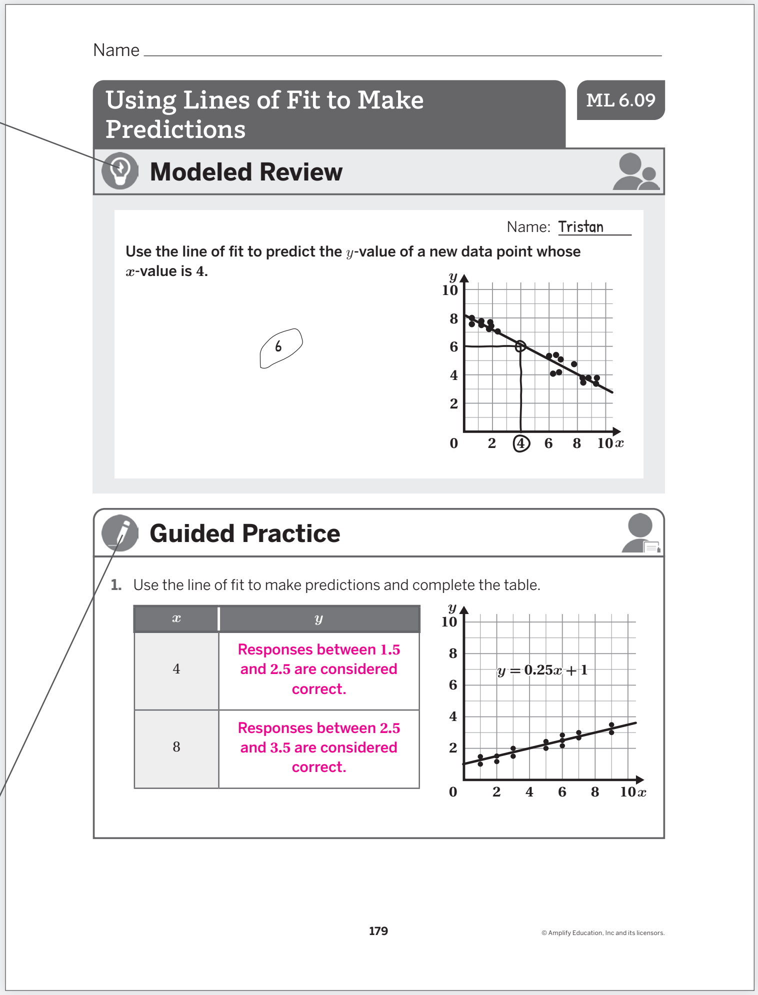
ML/EL Provide sentence frames to help students explain their strategy (e.g., "First, I see the *x*-value is ____, so I follow it vertically to meet the line of fit and predict the *y*-value is _____.").

Guided Practice

Focus students' attention on using the line of fit to make predictions.

Encourage students to annotate the graph as they find the corresponding *y*-value.

Note: Students can choose any number within the given range since it's an estimated value based on the line of fit.



Name

Using Lines of Fit to Make Predictions — ML 6.09

Modeled Review

Name: Tristan

Use the line of fit to predict the y-value of a new data point whose x-value is 4.

6

Guided Practice

1. Use the line of fit to make predictions and complete the table.

x	y
4	Responses between 1.5 and 2.5 are considered correct.
8	Responses between 2.5 and 3.5 are considered correct.

$y = 0.25x + 1$

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ML 6.09

Using Lines of Fit to Make Predictions

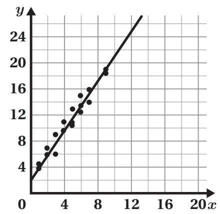
Guided Practice

A Model highlighting the provided *x*-value and drawing a vertical line to assist students in identifying the corresponding *y*-value.

Key Takeaway:
Say, "A line of fit can be useful for making predictions for data being represented in scatter plots."

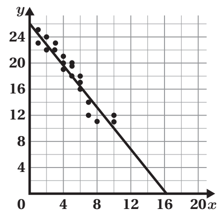
Guided Practice

2. Use the line of fit to predict the y-value of a new data point whose x-value is 8.



Responses between 16 and 18 are considered correct.

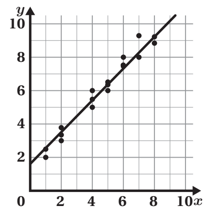
3. Use the line of fit to predict the y-value of a new data point whose x-value is 12.



Responses between 6 and 8 are considered correct.

Check

Use the line of fit to predict the y-value of a new data point whose x-value is 3.



Responses between 4 and 5 are considered correct.

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Reflection

Ask:

- "What does a line of fit tell you about the data?"
- "How was the lesson helpful to you today?"

Check: Recommended Next Steps

Almost there
If students need more support, model using a ruler to guide them in determining where to stop when making a prediction for a given *x*-value using the line of fit.

Got it!
If students need more practice, have them revisit the scatter plot in Problem 3. Then have them use the line of fit to predict the *y*-value for a data point whose *x*-value is 14.

Name ______________________________

Rules for Exponents

ML 7.03

Modeled Review

Name: Kira

Decide if the expressions in each pair are equivalent. Show your thinking.

Expression 1	Expression 2	Equivalent?
$(3^2 \cdot 3)^3$ $3^2 \cdot 3^2 \cdot 3^2 \cdot 3^3$ 3^9	$3^5 \cdot 3^{3}$ 3^8	Yes No
$\left(\frac{2}{3}\right)^{-3}$ $\left(\frac{3}{2}\right)^3$ $\frac{3^3}{2^3}$	$(3 \cdot 2^{-1})(3 \cdot 2^{-1})(3 \cdot 2^{-1})$ $\left(\frac{3}{2}\right) \cdot \left(\frac{3}{2}\right) \cdot \left(\frac{3}{2}\right)$ $\frac{3^3}{2^3}$	Yes No

Guided Practice

1. Match each expression to the correct equivalent form on the right.

Expression	**Expanded Form**
$\left(\frac{3}{5}\right)^{-2}$	$\frac{9}{25}$
$\left(\frac{5}{3}\right)^{-2}$	$-\frac{25}{9}$
$\left(-\frac{3^2}{5^2}\right)^{-1}$	$\frac{25}{9}$

Guided Practice

2. Decide if the expressions in each pair are equivalent. Show your thinking.

Expression 1	Expression 2	Equivalent?
$\dfrac{4^2 \cdot 4^3}{4^{-1} \cdot 4^4}$	$\left(\dfrac{4^0}{4}\right)^{-2}$	Yes No
$\left(\dfrac{5}{3}\right)^{-2}$	$\dfrac{3 \cdot 3}{5 \cdot 5}$	Yes No
$\dfrac{(2^{-1} \cdot 3)^3}{(2^{-3}3^2)^2}$	$\left(\dfrac{3}{2 \cdot 2 \cdot 2}\right)^{-1}$	Yes No

Check

Decide if the expressions in each pair are equivalent. Show your thinking.

Expression 1	Expression 2	Equivalent?
$\left(\left(\dfrac{5}{7}\right)^{-2}\right)^{-1}$	$(5^4 \cdot 7^{-5}) \cdot (5^{-2}7^{-3})$	Yes No
$(3^0 7^{-2})^{-2}$	$\dfrac{3^3 7^{-1}}{3^{-3} 7^{-5}}$	Yes No

ML 7.03 Rules for Exponents

Goal

I can apply the rules of exponents to rewrite expressions in equivalent forms.

Standard

MA.8.NSO.1.3

Materials

Whiteboards and dry erase markers (optional)

Modeled Review

Point to Kira's work and **ask**:

- "What strategy did Kira use to determine that $\left(\frac{2}{3}\right)^{-3}$ is equivalent to $(3 \cdot 2^{-1})(3 \cdot 2^{-1})(3 \cdot 2^{-1})$?"
- "How did Kira determine the expressions were equivalent without simplifying them completely?"

Reinforce Kira's thinking by saying, "Rewriting expressions using positive exponents is helpful when determining if they are equivalent."

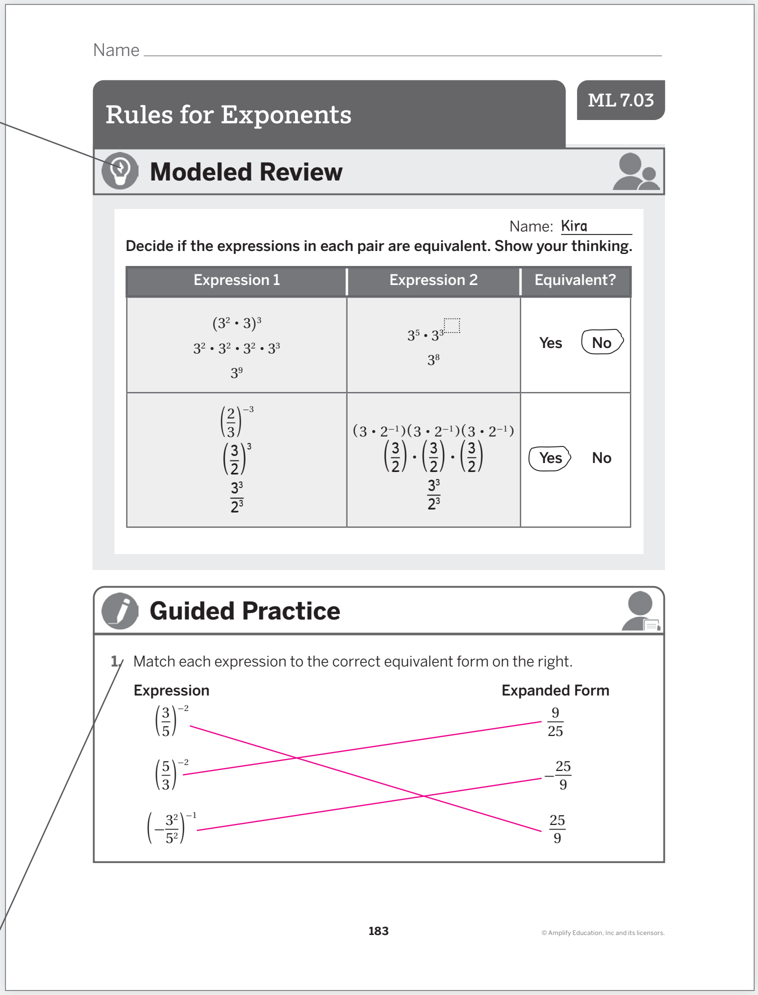
ML/EL Provide sentence frames to help students explain strategies for simplifying expressions with negative exponents. For example, "First, I need to rewrite expressions with negative exponents as their inverse with a positive exponent because _____."

Guided Practice

Focus students' attention on matching each expression to its equivalent expanded form.

To scaffold their thinking, **say**:

- "First, rewrite expressions using positive exponents."
- "Then, rewrite each expression in expanded form and rearrange the factors if needed."



Name

Rules for Exponents ML 7.03

Modeled Review

Name: Kira

Decide if the expressions in each pair are equivalent. Show your thinking.

Expression 1	Expression 2	Equivalent?
$(3^2 \cdot 3)^3$ $3^2 \cdot 3^2 \cdot 3^2 \cdot 3^3$ 3^9	$3^5 \cdot 3^3$ 3^8	Yes No
$\left(\frac{2}{3}\right)^{-3}$ $\left(\frac{3}{2}\right)^3$ $\frac{3^3}{2^3}$	$(3 \cdot 2^{-1})(3 \cdot 2^{-1})(3 \cdot 2^{-1})$ $\left(\frac{3}{2}\right) \cdot \left(\frac{3}{2}\right) \cdot \left(\frac{3}{2}\right)$ $\frac{3^3}{2^3}$	Yes No

Guided Practice

1. Match each expression to the correct equivalent form on the right.

Expression	Expanded Form
$\left(\frac{3}{5}\right)^{-2}$	$\frac{9}{25}$
$\left(\frac{5}{3}\right)^{-2}$	$-\frac{25}{9}$
$\left(-\frac{3^2}{5^2}\right)^{-1}$	$\frac{25}{9}$

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Guided Practice

A Chunk this task into smaller, more manageable parts by having students evaluate one expression at a time before determining if the expressions are equivalent. Allow students to work problems out on a whiteboard for extra room and ease of practice.

Key Takeaway:
Say, "Rewriting expressions using positive exponents is helpful when determining if they are equivalent."

Guided Practice

2. Decide if the expressions in each pair are equivalent. Show your thinking.

Expression 1	Expression 2	Equivalent?
$\frac{4^2 \cdot 4^3}{4^{-1} \cdot 4^4}$	$\left(\frac{4^0}{4}\right)^{-2}$	Yes No
$\left(\frac{5}{3}\right)^{-2}$	$\frac{3 \cdot 3}{5 \cdot 5}$	Yes No
$\frac{(2^{-1} \cdot 3)^3}{(2^{-3}3^2)^2}$	$\left(\frac{3}{2 \cdot 2 \cdot 2}\right)^{-1}$	Yes No

Check

Decide if the expressions in each pair are equivalent. Show your thinking.

Expression 1	Expression 2	Equivalent?
$\left(\left(\frac{5}{7}\right)^{-2}\right)^{-1}$	$(5^4 \cdot 7^{-5}) \cdot (5^{-2} 7^{-3})$	Yes No
$(3^0 7^{-2})^{-2}$	$\frac{3^3 7^{-1}}{3^{-3} 7^{-5}}$	Yes No

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Reflection

Ask:
- "What are some important things to remember when determining whether expressions with exponents are equivalent?"
- "What questions do you still have?"

Check: Recommended Next Steps

Almost there
If students need more support, consider using visual aids, such as anchor charts, to illustrate the order of operations and guide students through the correct sequence of steps to evaluate expressions with exponents.

Got it!
If students need more practice, ask them to decide if the following expressions are equivalent or not.

Expression 1	Expression 2
$\frac{3^6 \cdot 3^3 \cdot 3^{-2}}{3^{-1} \cdot 3^4}$	$\left(\frac{1}{3^2}\right)^{-2}$
$\frac{(4^2)^3}{(4^3)^{-1}}$	$4^2 \cdot 4^2 \cdot 4^2 \cdot 4^3$

Name ______________________________

Approximating Square Roots

ML 8.03

Modeled Review

Name: Evan

Approximate $\sqrt{18}$.

$\sqrt{18}$ is between 4.2 and 4.3

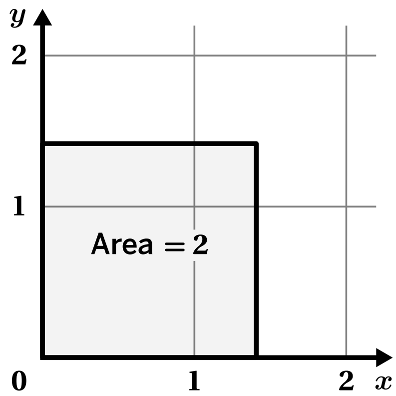
I multiplied 4.1, 4.2, and 4.3 by themselves to find their squares.

n	n^2
4.1	16.81
4.2	17.64
4.3	18.49

Guided Practice

Approximate the square root using the side lengths of the squares and the tables.

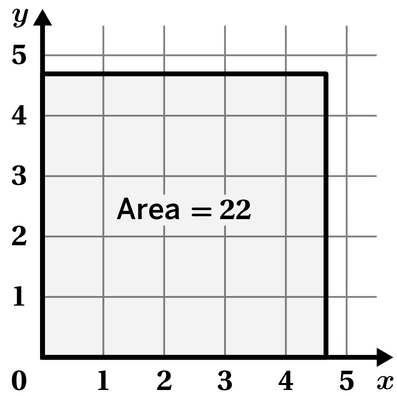
1. $\sqrt{2}$



n	n^2
1.4	1.96
1.5	

$\sqrt{2}$ is between ______ and ______

2. $\sqrt{22}$



n	n^2
4.6	

$\sqrt{22}$ is between ______ and ______

Guided Practice

For Problems 3–6, complete the table to approximate the square root.

3. $\sqrt{6}$ is between ______ and ______

n	n^2
2.3	5.29
2.4	

4. $\sqrt{10}$ is between ______ and ______

n	n^2
3.1	

5. $\sqrt{15}$ is between ______ and ______

n	n^2

6. $\sqrt{40}$ is between ______ and ______

n	n^2

Check

Approximate $\sqrt{28}$.

$\sqrt{28}$ is between ______ and ______

n	n^2

ML 8.03 Approximating Square Roots

Goal

Approximate square roots as decimals.

Standard

MA.8.NSO.1.1

Materials

calculator (optional), number line (optional)

Modeled Review

Point to Evan's work and **ask**:

- "Why is $\sqrt{18}$ not a perfect square root?"
- "How did Evan know to start with numbers slightly greater than 4?"
- "How did Evan calculate 4.1^2, 4.2^2, and 4.3^2?"

Reinforce Evan's thinking by saying, "Knowing perfect squares and calculating the nearest decimals helps you estimate square roots."

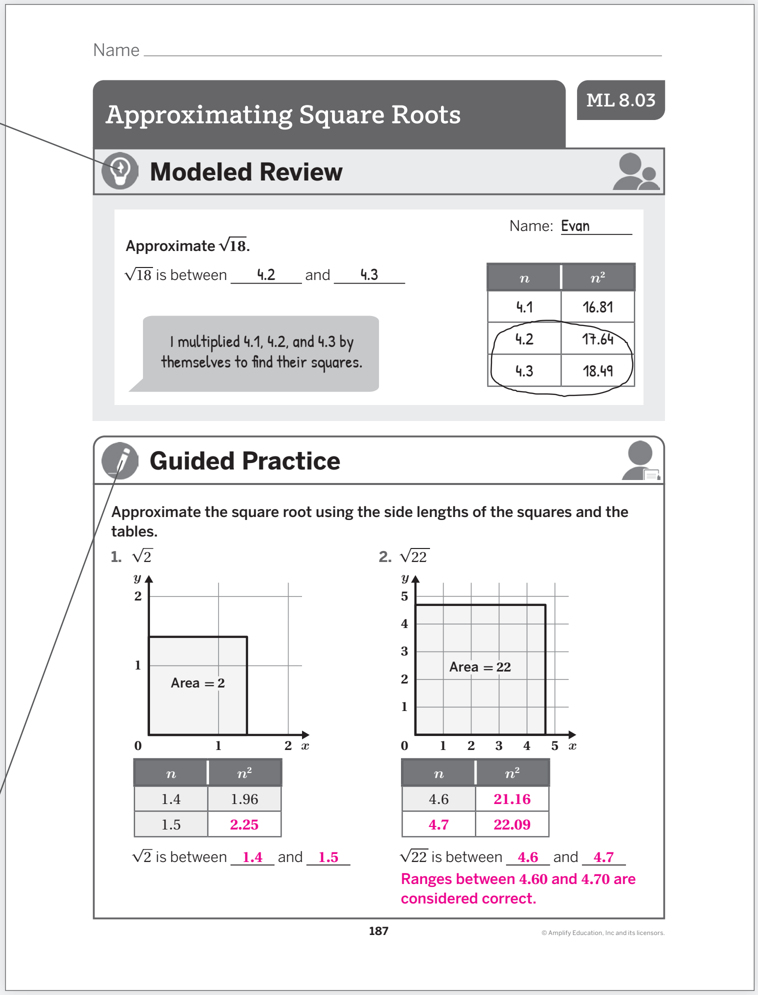
ML/EL Provide students with sentence frames to use when discussing how to approximate square roots. For example, "I need to find perfect squares near ______." or "I guess a number and square it to see if it's close to ______."

Guided Practice

Focus students' attention on estimating the square root.

To scaffold their thinking, **say**:

- "First, identify a perfect square closest to the number."
- "Next, select possible values for n and calculate n^2."
- "Last, find the range of n on either side of the square root."



Name ______

Approximating Square Roots ML 8.03

Modeled Review

Name: Evan

Approximate $\sqrt{18}$.

$\sqrt{18}$ is between 4.2 and 4.3

I multiplied 4.1, 4.2, and 4.3 by themselves to find their squares.

n	n^2
4.1	16.81
4.2	17.64
4.3	18.49

Guided Practice

Approximate the square root using the side lengths of the squares and the tables.

1. $\sqrt{2}$

Area = 2

n	n^2
1.4	1.96
1.5	2.25

$\sqrt{2}$ is between 1.4 and 1.5

2. $\sqrt{22}$

Area = 22

n	n^2
4.6	21.16
4.7	22.09

$\sqrt{22}$ is between 4.6 and 4.7

Ranges between 4.60 and 4.70 are considered correct.

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Vocabulary

If needed, share the meaning of the terms with students.

perfect square: A number that is the square of an integer. For example $\sqrt{16}$ is a perfect square because $4^2 = 16$.

square root: The square root of a positive number $\sqrt{p}$ is a positive solution to equations of the form $x^2 = p$. Write the square root of p as $\sqrt{p}$.

Guided Practice

A Provide students with a calculator to help them efficiently square decimal numbers.

Key Takeaway:
Say, "Calculating the nearest decimal approximations of square roots can help estimate the value of a square root."

Guided Practice

For Problems 3–6, complete the table to approximate the square root.

3. $\sqrt{6}$ is between 2.4 and 2.5

Ranges between 2.40 and 2.50 are considered correct.

n	n^2
2.3	5.29
2.4	5.76
2.5	6.25

4. $\sqrt{10}$ is between 3.1 and 3.2

Ranges between 3.10 and 3.20 are considered correct.

n	n^2
3.1	9.61
3.2	10.24
3.3	10.89

5. $\sqrt{15}$ is between 3.8 and 3.9

Ranges between 3.80 and 3.90 are considered correct.

n	n^2
3.7	13.69
3.8	14.44
3.9	15.21

6. $\sqrt{40}$ is between 6.3 and 6.4

Ranges between 6.30 and 6.40 are considered correct.

n	n^2
6.2	38.44
6.3	39.69
6.4	40.96

Check

Approximate $\sqrt{28}$.

$\sqrt{28}$ is between 5.2 and 5.3

Ranges between 5.20 and 5.30 are considered correct.

n	n^2
5.1	26.01
5.2	27.04
5.3	28.09

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Reflection

Ask:
- "Describe your favorite strategy to approximate a square root."
- "Reflect on your learning today. What were you most proud of?"

Check: Recommended Next Steps

Almost there
If students need more support, consider using a number line to visually estimate the position of the square root between two known whole numbers.

Got it!
If students need more practice, invite them to estimate each square root.

- $\sqrt{20}$
- $\sqrt{52}$
- $\sqrt{37}$

Name ______________________________

Identifying Rational and Irrational Numbers

ML 8.13

Modeled Review

Rational	Irrational
all positive and negative numbers that can be written as fractions, including whole numbers	a number that cannot be written as a fraction of two non-zero integers

Examples (Rational)

$\frac{3}{4}$	$2.5 = \frac{5}{2}$	$0.\overline{66}$	$\sqrt{16}$

Examples (Irrational)

π	$\sqrt{7}$	$\sqrt{3} = 1.732050...$

Guided Practice

1. Determine if the numbers in the bank are rational or irrational. Then add them to the correct column.

1.75	$\sqrt{3}$	9π	$\frac{1}{2}$
$-\frac{1}{4}$	$0.\overline{33}$	$\sqrt{10}$	$\sqrt{15} = 3.872983...$

Rational	Irrational
$\frac{1}{2}$	$\sqrt{15} = 3.872983...$

Guided Practice

2. Is the number rational or irrational? Add a check mark to the correct column.

Rational	Rational	Irrational
$\sqrt{36}$		
$0.\overline{72}$		
$1 + \sqrt{2} = 2.414213...$		
$-\frac{7}{8}$		
$\sqrt{11}$		
$\pi^2 = 9.869604...$		
8π		
$0.\overline{11}$		
4		

Check

Is the number rational or irrational? Add a check mark to the correct column.

Number	Rational	Irrational
$\sqrt{14}$		
5π		
$-\frac{11}{5}$		

ML 8.13 Identifying Rational and Irrational Numbers

Goal	Standard
Identify and categorize examples of rational and irrational numbers.	MA.8.NSO.1.1

Modeled Review

Point to the Modeled Review and **ask**:
- "How are rational and irrational numbers different?"
- "How can you tell if a square root is rational or irrational?"
- "What is the difference between repeating and terminating decimals?"

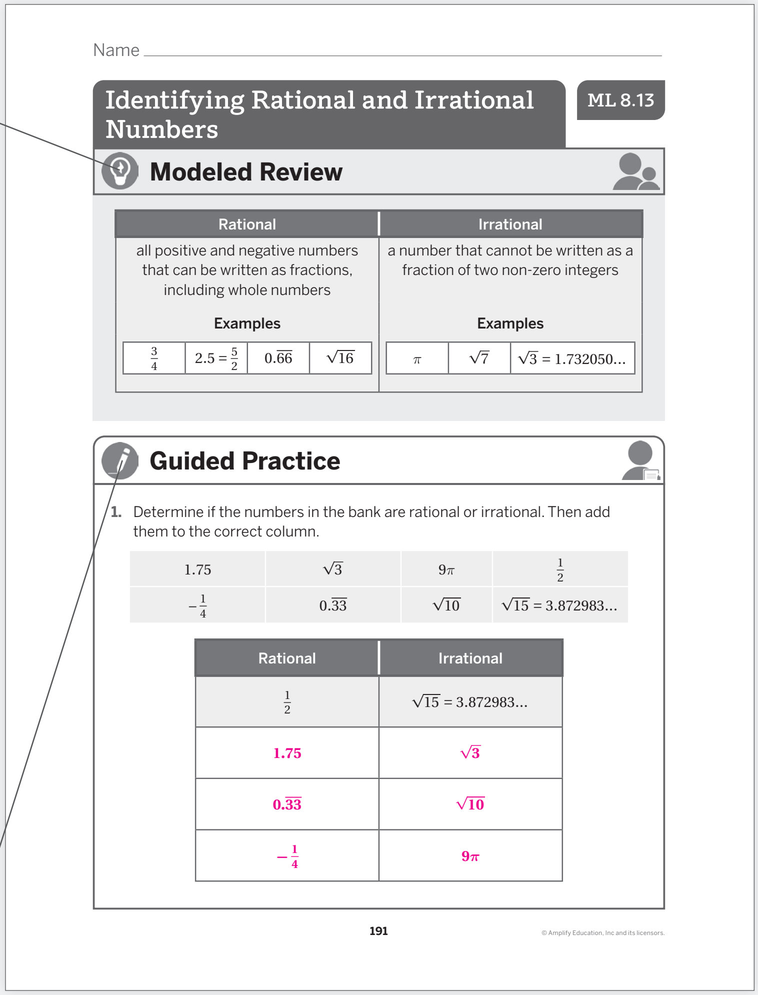
Reinforce the goal by saying, "An irrational number can't be written as a fraction of integers. It has a never-ending, non-repeating decimal."

ML/EL Provide sentence frames to help students explain their strategy. For example, "I noticed that ______ is an irrational number because _______." or "______ is a repeating decimal, so it is rational."

Guided Practice

Focus students' attention on determining whether each number is rational or irrational.

To scaffold their thinking, **say**:
- "Determine if the number is a fraction, repeating decimal, or terminating decimal."
- "If the number is none of these, it is irrational."



Name

Identifying Rational and Irrational Numbers ML 8.13

Modeled Review

Rational	Irrational
all positive and negative numbers that can be written as fractions, including whole numbers	a number that cannot be written as a fraction of two non-zero integers
Examples	Examples
$\frac{3}{4}$, $2.5 = \frac{5}{2}$, $0.\overline{66}$, $\sqrt{16}$	π, $\sqrt{7}$, $\sqrt{3} = 1.732050...$

Guided Practice

1. Determine if the numbers in the bank are rational or irrational. Then add them to the correct column.

1.75	$\sqrt{3}$	9π	$\frac{1}{2}$
$-\frac{1}{4}$	$0.\overline{33}$	$\sqrt{10}$	$\sqrt{15} = 3.872983...$

Rational	Irrational
$\frac{1}{2}$	$\sqrt{15} = 3.872983...$
1.75	$\sqrt{3}$
$0.\overline{33}$	$\sqrt{10}$
$-\frac{1}{4}$	9π

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Vocabulary

If needed, share the meaning of the terms with students.

rational: All positive and negative numbers that can be written as fractions, including whole numbers, are called rational numbers.

irrational: A number that cannot be written as a fraction of two non-zero integers.

ML 8.13 Identifying Rational and Irrational Numbers

Guided Practice

A Provide a checklist with the terms *fraction*, *repeating decimal*, and *terminating decimal*. Students can check the appropriate term as they go, helping them determine if the number is rational or irrational."

Key Takeaway:
Say, "An irrational number cannot be written as a fraction with integers. It goes on forever without repeating a pattern, unlike terminating or repeating decimals."

Guided Practice

2. Is the number rational or irrational? Add a check mark to the correct column.

Rational	Rational	Irrational
$\sqrt{36}$	✔	
$0.\overline{72}$	✔	
$1 + \sqrt{2} = 2.414213...$		✔
$-\frac{7}{8}$	✔	
$\sqrt{11}$		✔
$\pi^2 = 9.869604...$		✔
8π		✔
$0.\overline{11}$	✔	
4	✔	

Check

Is the number rational or irrational? Add a check mark to the correct column.

Number	Rational	Irrational
$\sqrt{14}$		✔
5π		✔
$-\frac{11}{5}$	✔	

192

Reflection

Ask:
- "What are some characteristics of irrational numbers? Rational numbers?"
- "How does what you learned today connect to your prior learning?"

Check: Recommended Next Steps

Almost there
If students need more support, consider providing examples of a terminating decimal, a repeating decimal, and a non-terminating decimal for reference.

Got it!
If students need more practice, ask them to determine if each of the following numbers are rational or irrational:
- $\frac{1+\sqrt{5}}{2} = 1.618033...$
- $0.\overline{77}$
- $\sqrt{49}$

Algebra 1

Extensions

Unit 1

Extensions

One-Variable Equations

Name: .... Date: .... Period: ....

Remember to show or explain your thinking.

1 A magician asked everyone in the audience to think of a number, and then follow these steps:

- Step 1: Add 4.
- Step 2: Multiply by 4.
- Step 3: Subtract 4.
- Step 4: Divide by 4.
- Step 5: Subtract your original number

a Think about a number between 1 and 10 and follow the steps. What is your final number?

b Emma's initial number was 7. What is the final number she ended up with?

c Mayra thought of an integer and followed the same steps. What answer did she end up with?

d Will this work for numbers that are not whole numbers between 1 and 10? What numbers will it work for?

e Repeat the same trick with 2s, 3s, . . ., n instead of 4s. What are the final answers?

One-Variable Equations (continued)

Name: .... Date: .... Period: ....

2

A magician asked everyone in the audience to think of a number, and then follow these steps:

- Step 1: Triple the number.
- Step 2: Add 45 to the result.
- Step 3: Double the result.
- Step 4: Divide the result by 6.
- Step 5: Subtract the number with the first number you started with.

Then she added, "When you are finished, you should all have the same number."

What is the number? How did the magician know it would be the same for everyone?

3

Design your own math magic trick that ends with the number 8. Your trick should have multiple steps.

One-Variable Equations

Assign problems to students who want to extend their thinking.

Assign Problem 3 to students who have solved Problems 1 and 2. If time allows, consider sharing Problems 1 and 2 with all students.

Problem 1

Students will extend their understanding of solving one-variable equations using inverse operations by figuring out a "think of a number" trick.

Provide students with the following hint if additional scaffolding is needed.

- **Hint:** In part c, let Mayra's number be x. How can you represent each step using x?

a. 3

b. 3

c. 3. *Explanations vary*. Let Mayra's number be x. By following the steps, I got:

Step 1: $x + 4$

Step 2: $4(x + 4) = 4x + 16$

Step 3: $4x + 16 - 4 = 4x + 12$

Step 4: $\frac{4x + 12}{4} = x + 3$

Step 5: $x + 3 - x = 3$

The initial number Mayra thought does not affect the result. The final number will always be 3.

d. *Responses vary*. It will work for all numbers, such as 0, fractions, decimals, negative numbers, and irrational numbers like π or $\sqrt{2}$.

e. *Explanations vary*. I recreated the same trick using different numbers:

$n = 2$	$n = 3$	$n = 5$
$= \frac{2 \cdot (x + 2) - 2}{2} - x$ $= x + 1 - x$ $= 1$	$= \frac{3 \cdot (x + 3) - 3}{3} - x$ $= x + 2 - x$ $= 2$	$= \frac{5 \cdot (x + 5) - 5}{5} - x$ $= x + 4 - x$ $= 4$

I will always end up with 1 less than the number used in the trick. So for n, the final answer will always be $n - 1$.

Problem 2

Students will extend their understanding of solving one-variable equations by using equivalent equations and inverse operations.

Provide students with the following hint if additional scaffolding is needed.

- **Hint:** Let the initial number be x. How can you represent each step using x?

15. *Responses vary*. For any number x, when I follow the steps, I get:

- Step 1: $3x$
- Step 2: $3x + 45$
- Step 3: $2(3x + 45) = 6x + 90$
- Step 4: $\frac{6x + 90}{6} = x + 15$
- Step 5: $x + 15 - x = 15$

The final number will always be 15.

Continued next page ...

One-Variable Equations (continued)

Problem 3

Students will extend their understanding of solving one-variable equations by designing their own math magic trick.

Provide students with the following hint if additional scaffolding is needed.

- **Hint:** How can you use Problem 1 to create a "think of a number" trick that ends with 8?

Responses vary.

Think of a number.

- Step 1: Add 9.
- Step 2: Multiply by 9.
- Step 3: Subtract 9.
- Step 4: Divide by 9.
- Step 5: Subtract your original number.

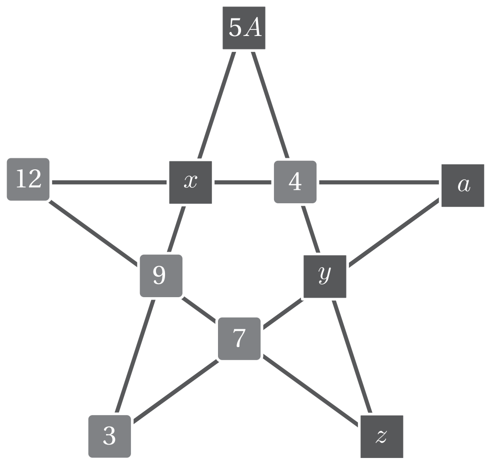
Multi-Variable Equations

Name: .... Date: .... Period: ....

Remember to show or explain your thinking.

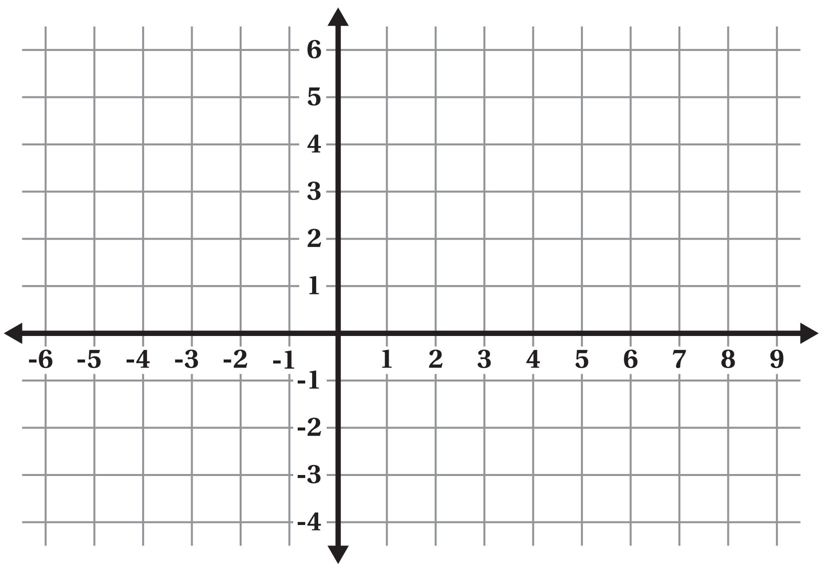
1 Here is a figure where the four values along each line have a sum of 30.

Determine the values of x, y, z, and a.



2

a A triangle is formed by the lines $y = \text{-}2$, $y = 2x + 8$, and $2x + 3y = 12$. What is the area of the triangle?



b John added a line that divides the triangle into two equal areas. What could be an equation of John's line?

Multi-Variable Equations (continued)

Name: .... Date: .... Period: ....

3

Let x and y be the first two terms of a Fibonacci-like sequence, where each term is the sum of the two preceding terms

a Write the next five terms. What do you notice?

b The fifth term of the sequence is 19 and the eighth term is 81. What are the first two terms, x and y, of this sequence?

4

A set of five weights can balance any object with an integer weight from 1 to 120 kilograms on a two-sided scale.

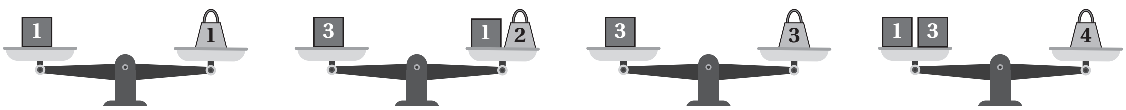
Here are examples of two of the five weights.

To weigh a 1 kg object, you can use the 1 kg weight.

To weigh a 2 kg object, you can use the 1 kg and 3 kg weights.

To weigh a 3 kg object, you can use the 3 kg weights.

To weigh a 4 kg object, you can again use the 1 kg and 3 kg weights.



a What one additional weight can you add to the set to weigh any object up to 13 kilograms?

b Complete the set of five weights so that it can balance any object with a weight up to 120 kilograms.

Multi-Variable Equations

Assign problems to students who want to extend their thinking.

Problems 1 and 2 can be solved in any order. Assign Problem 3 to students who have solved Problem 1. If time allows, consider sharing Problems 1, 2, and 3 with all students.

Problem 1

Students will extend their understanding of writing and solving multi-variable equations by solving a puzzle.

Provide students with the following hint if additional scaffolding is needed.

- **Hint:** Start with writing and simplifying a separate equation for each line in the star diagram.

$a = 1$, $x = 13$, $y = 19$ and $z = 2$.

Explanations vary. First I wrote five equations for all the lines in the star and simplified them;

- $12 + 9 + 7 + z = 30$, so $z = 2$
- $2 + 4 + y + 5a = 30$, $y + 5a = 24$
- $3 + 7 + y + a = 30$, $y + a = 20$
- $3 + 9 + x + 5a = 30$, $x + 5a = 18$
- $12 + 4 + x + a = 30$, $x + a = 14$

Next I solved $y + 5a = 24$ and $y + a = 20$ for y and set them equal to each other, resulting in $4a = 4$, so $a = 1$ and $y = 19$. Lastly I substituted $a = 1$ into $x + a = 14$ and determined the value of x as 13.

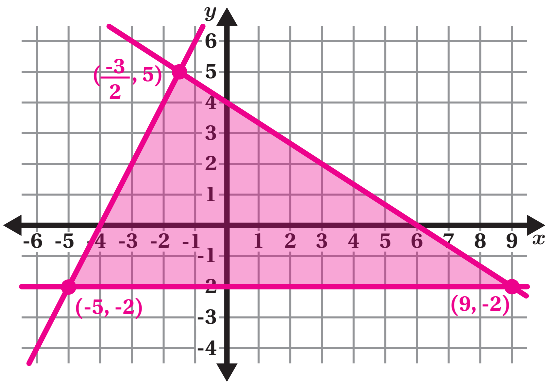
Problem 2

Students will extend their understanding of writing and solving multi-variable equation systems by graphing linear equations on the coordinate plane.

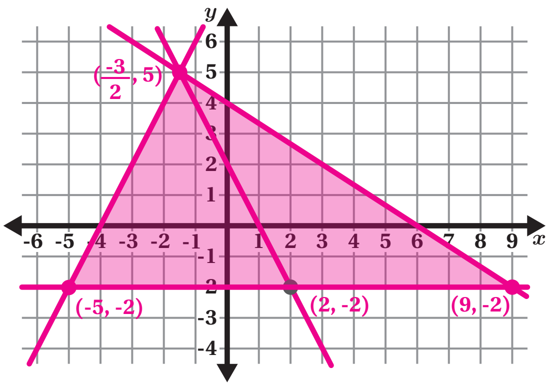
Provide students with the following hints if additional scaffolding is needed.

- **Hint 1:** In part a, what information do you need to determine the area? Can you determine the ordered pairs that represent each intersection point? How can that help you determine the area?
- **Hint 1:** In part b, draft some lines that divide the triangle into two equal areas. Which line's equation can you determine?

a. 49 square units



b. *Responses vary.* $y = -2x + 2$



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Problem 3

Students will extend their understanding of writing and solving multi-variable equation systems by representing each number of a Fibonacci-like sequence algebraically.

Provide students with the following hint if additional scaffolding is needed.

- **Hint:** In the Fibonacci sequence, every new term is the sum of the two previous terms. If the first term is x, and the second term is y, how can you write the third term?

a.

1	2	3	4	5	6	7
x	y	$x+y$	$x+2y$	$2x+3y$	$3x+5y$	$5x+8y$

Explanation vary. I notice that the coefficients of each term are the Fibonacci sequence: 1, 1, 2, 3, and so on.

b. $x = 2$ and $y = 5$. *Explanations vary*. If the fifth term of the sequence is 19, and the eighth term is 81, then $2x + 3y = 19$ and $8x + 13y = 81$. When I solve the system of equations, I determine $x = 2$ and $y = 5$. The sequence is 2, 5, 7, 12, 19, 31, 50, 81, 131 . . .

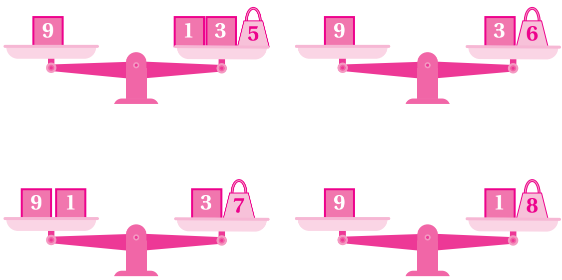
Problem 4

Students will extend their understanding of writing and solving multi-variable equations by solving a weight puzzle.

Provide students with the following hints if additional scaffolding is needed.

- **Hint 1:** What are the first two weights in the set of five? What do you predict for the next weight?
- **Hint 2:** For part b, what one weight can you add to your set to weigh anything up to 40 kilograms?

a. 9 kilograms. *Explanations vary*.



b. 1, 3, 9, 27, and 81 kilograms. *Explanations vary*. After I determined the first three weights were 1, 3, and 9, I tried 27. I realized that using only 1, 3, 9, and 27, I can balance all the weights from 1–40 kilograms. Continuing the pattern, the fifth weight must be 81. For example,

- $40 = 27 + 9 + 3 + 1$
- $70 + 9 + 3 = 81 + 1$
- $100 + 9 = 81 + 27 + 1$
- $119 + 1 = 81 + 27 + 9 + 3$

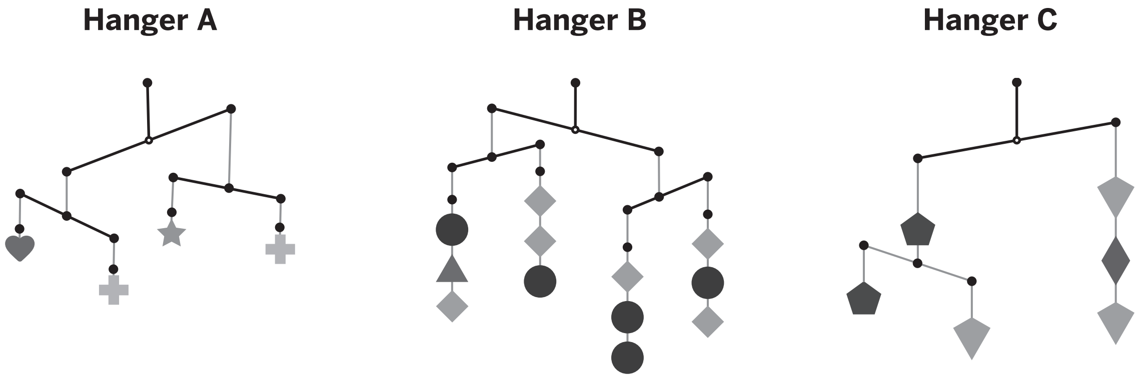
One-Variable and Two-Variable Inequalities

Name: .... Date: .... Period: ....

Student Choice Remember to show or explain your thinking.

1 Here are three imbalanced hangers.

For each hanger, write an inequality statement that orders the shapes from lightest to heaviest.



2 Create your own imbalanced hanger where there is only one way to order the shapes from lightest to heaviest.

One-Variable and Two-Variable Inequalities (continued)

Name: .... Date: .... Period: ....

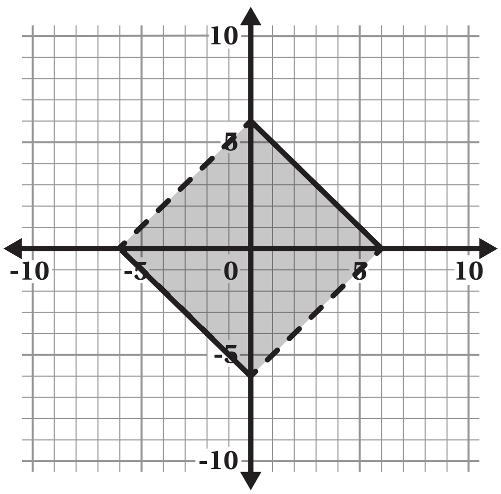
3 Determine a pair of values x and y for which each statement is true and false.

	Statement	True	False
a	If $x \geq y$, then $\|x\| \leq \|y\|$		
b	If $x \leq 0$, and $y > 0$, then $x \cdot y \geq 0$		
c	If $x > y$, then $x \cdot y \geq y^2$		
d	$\|x + y\| > x + y$		

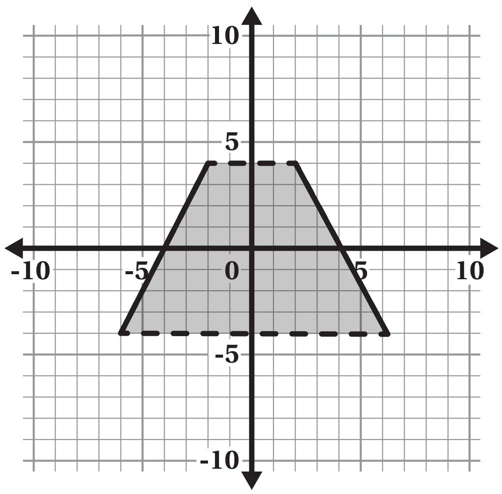
4 Anna draws polygons on a coordinate plane using inequalities.

Determine the set of inequalities she used to create each polygon.

a



b



One-Variable and Two-Variable Inequalities

Assign problems to students who want to extend their thinking.

Problems 1, 3, and 4 can be solved in any order. Assign Problem 2 to students who have solved Problem 1. If time allows, consider sharing Problems 1 and 4 with all students.

Problem 1

Students will extend their understanding of determining solutions to an inequality algebraically by finding the relationships in imbalanced hangers.

Provide students with the following hint if additional scaffolding is needed.

- **Hint:** How many different inequalities can you write for each hanger diagram? How can you connect these inequalities?

a. ★ < ♥ < ✚

Explanations vary. On the left side, I see ♥ < ✚ and on the right side I see ★ < ✚. When I look at the entire hanger, I see ★ + ✚ < ♥ + ✚ which means ★ < ♥.

b. ◆ < ▲ < ●

Explanations vary. On the left side I see ◆ < ▲, and on the right side I see ◆ < ●. When I look at the entire hanger, and eliminate the 3 ◆ and 2 ●, I see that ▲ < ●.

c. ⬧ < ⬟ < ▼

Explanations vary. The little hanger at the left reveals that ⬟ < ▼, and the entire diagram reveals that ⬧ + ▼ < ⬟ + ⬟, so ⬧ < ⬟.

Problem 2

Students will extend their understanding of determining solutions to an inequality algebraically by designing their own imbalanced hangers.

Provide students with the following hint if additional scaffolding is needed.

- **Hint:** Start by choosing how many different variables you would like to use and ordering them.

Responses vary.

Continued next page ...

One-Variable and Two-Variable Inequalities (continued)

Problem 3

Students will extend their understanding of determining solutions to an inequality algebraically by making sense of different inequality statements.

Provide students with the following hint if additional scaffolding is needed.

- **Hint:** Start by finding an example that makes each inequality true. What do you notice about the signs of these numbers? What happens when you change the sign of one variable? Both?

Responses vary.

	Statement	True	False
a.	If $x \geq y$, then $\lvert x \rvert \leq \lvert y \rvert$	$x = -3$ $y = -4$	$x = 4$ $y = 2$
b.	If $x \leq 0$, and $y > 0$, then $x \bullet y \geq 0$	$x = 0$ $y = 2$	$x = -1$ $y = 2$
c.	If $x > y$, then $x \bullet y \geq y^2$	$x = 4$ $y = 2$	$x = -3$ $y = -4$
d.	$\lvert x + y \rvert > x + y$	$x = -4$ $y = 1$	$x = 0$ $y = 0$

Problem 4

Students will extend their understanding of determining solutions to an inequality graphically by writing inequalities in one and two variables to represent constraints.

Provide students with the following hint if additional scaffolding is needed.

- **Hint:** Highlight and extend each side of the polygons to determine the equations for those lines before trying to write the inequalities.

Responses vary.

a. $x + y \leq 6$
$x + y \geq -6$
$x - y < -6$
$x - y > 6$

b. $y < 4$
$y > -4$
$y \leq 2x + 8$
$y \leq -2x + 8$

Unit 2

Extensions

Representing Categorical Data

Name: .... Date: .... Period: ....

Student Choice Start with any problem. Remember to show or explain your thinking.

1

On April 15, 1912, the Titanic struck an iceberg and rapidly sank with only 710 of the 2,204 passengers and crew surviving.

Data on the survival of first and second-class passengers are summarized in the following two-way table.

	Survived	Did not survive
First-class passengers	201	123
Second-class passengers	118	166

a Create a total relative frequency table to represent the data.

b Interpret the data. Is there evidence to support any associations in this data? Could you generalize these results? Be sure to include which data displays and statistical calculations you used when explaining your thinking.

c Data on the third-class passenger is added with 181 surviving and 528 not surviving. Create a relative frequency table including the new data and interpret your findings.

Representing Categorical Data

Assign problems to students who want to extend their thinking.

Problems 1 and 2 can be solved in any order. If time allows, consider sharing Problems 1 and 2 with all students.

Problem 1

Students will extend their understanding of relative frequency tables by interpreting data about the Titanic.

Provide students with the following hint if additional scaffolding is needed.

- **Hint:** What do you need to calculate first to determine the relative frequencies?

a. *Responses vary.*

	Survived	Did not survive
First-class passengers	~ 62%	~ 38%
Second-class passengers	~ 41.5%	~ 58.5%

Explanations vary. First I used the two-way table to determine the totals. Then I determined the relative frequencies using the total number of passengers in each class.

	Survived	Did not survive	Total
First-class passengers	201	123	324
Second-class passengers	118	166	284
Total	319	289	608

b. *Responses vary*.

- The rate of survival of the first-class passengers is almost 20% more than the second-class passengers.
- Only 37% of total survivors are second-class passengers.

c.

	Survived	Did not survive
First-class passengers	~ 62%	~ 38%
Second-class passengers	~ 41.5%	~ 58.5%
Third-class passengers	~ 25.5%	~ 74.5%

Responses vary. The survival rate decreases drastically with each class change.

Continued next page ...

Summarizing One-Variable Data

Name: .... Date: .... Period: ....

Remember to show or explain your thinking.

1

a If a fixed number n is added to all measurements in a data set, how does the mean of the new data set change?

b If all measurements in a data set are multiplied by a fixed number n, how does the mean of the new data set change?

c Are these properties of mean true for median? Give an example.

d If a fixed number n is added to all measurements in a data set, how do the deviations $(x - \overline{x})$ of the new data set change?

e If all measurements in a data set are multiplied by a fixed number n, how do the deviations $(x - \overline{x})$ of the new data set change?

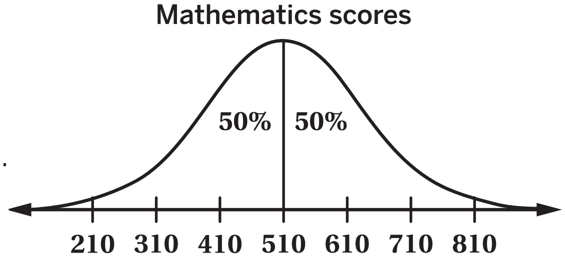
Summarizing One-Variable Data (continued)

Name: .... Date: .... Period: ....

2

When working with normally distributed data:

- The mean is equivalent to the median.
- 50% of the data lies to the left of the mean.
- 50% of the data lies to the right of the mean.
- 68% of the data are within 1 standard deviation of the mean.
- 95% of the data are within 2 standard deviations of the mean.
- 99.7% of the data are within 3 standard deviations of the mean.



The scores shown are for a standardized mathematics test in a particular year.

a Based on the given data, identify the mean and the standard deviation for the mathematics test.

b Based on the given bell curve, if 150 students are in a given sample, how many would you expect to score between 410 and 710?

Summarizing One-Variable Data

Assign problems to students who want to extend their thinking.

Assign Problem 2 to students who have solved Problem 1. If time allows, consider sharing Problems 1 and 2 with all students.

Problem 1

Students will extend their understanding of mean, median and deviations by exploring how they change when the data set changes.

Provide students with the following hints if additional scaffolding is needed.

- **Hint:** Verify your conclusions using several data sets such as $5, 9, 9, 8, 10, 7$.

a. original mean $+ n$

b. original mean $\cdot n$

c. Yes. *Explanations vary*.

- For example in the data set $4, 8, 8, 7, 9, 6$, the median is 7.5
- When $n = 2$ is added to each measurement of the data, the new set will be $6, 10, 10, 9, 11, 8$ and the new median will be 9.5, which is $7.5 + 2$.
- When each measurement of the data is multiplied by 2, the new set will be $8, 16, 16, 14, 18, 12$ and its median will be 15, which is $7.5 \cdot 2$.

d. *Explanations vary.* It will remain unchanged. Consequently variance, s^2, and standard deviation, s, will remain unchanged.

e. *Explanations vary*. $(x - \overline{x})$ will get multiplied by n too. Consequently variance, s^2, gets multiplied by n^2, and standard deviation, s, will get multiplied by $|n|$.

Problem 2

Students will extend their understanding of standard deviation by interpreting a normal distribution bell curve and applying the empirical rule.

Provide students with the following hint(s) if additional scaffolding is needed.

- **Hint 1:** What percent of data is between 1 standard deviation and the mean? What percent of data is between 2 standard deviation and the mean?

a. The mean is 510 and the standard deviation is 100.

b. About 122 students. *Responses vary*. 34% of the data is between 1 standard deviation, 410, from the mean. 47.5% of the data is between 2 standard deviations from the mean. If the sample has 150 students, and 81.5% are expected to score between 410 and 710, then $815 \cdot 150 = 122.25$, or about 122 students.

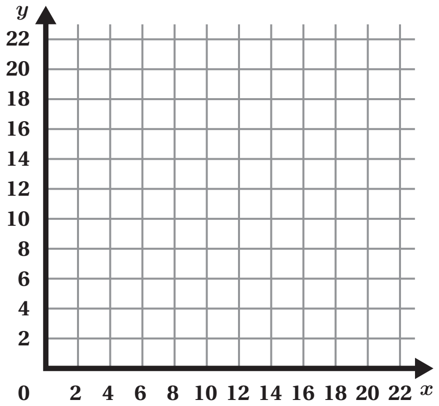
Summarizing Two-Variable Data

Name: .... Date: .... Period: ....

Remember to show or explain your thinking.

1 Here's a set of data.

a Plot a scatter plot diagram and add what you think is the line of best fit.



Days since Team Trees' campaign launch	Total donations (in millions of dollars)
1	1
2	4
4	6
6	10
7	11
8	12
10	14
12	14
14	16
19	20

b Another strategy for fitting a line to data is called the *three-median method*. Divide the data set into thirds. (If they do not divide evenly and there is one extra data value, put it in the middle group, If there are two extra values, put an extra value in each of the outer groups.)

- **Step 1:** Determine the medians of each group.

- **Step 2:** Draw an initial line to connect the medians of the lower and upper groups. Calculate its slope and y-intercept.

- **Step 3:** Move this line $\frac{1}{3}$ of the way towards the median of the central group by maintaining the slope of your initial line.

Summarizing Two-Variable Data (continued)

Name: .... Date: .... Period: ....

c Plot the line of the three-median method on the same scatter plot diagram in part a.

d Compare the lines you drew in part a and part c. Why do you think the *three-median method* works? Can there be examples where this method does not work well?

e Use the three-median regression line to predict the value in donations Team Trees can expect to receive in 30 days.

f Use the three-median regression line to predict how many days it will take for Team Trees to reach their goal of 25 million dollars.

Summarizing Two-Variable Data

Assign problems to students who want to extend their thinking.

If time allows, consider sharing Problem 1 with all students.

Problem 1

Students will extend their understanding of lines of best fit by using another strategy, three-median regressions, to determine a line of best fit.

Provide students with the following hint if additional scaffolding is needed.

- **Hint 1:** In part b, how many data values must there be in each group? What is the median of the lower three data points? What is the median of the central four data points? What is the median of the upper three data points?
- **Hint 2:** In part b, Step 3, Use the x-coordinate of the middle median to determine the y-value. Determine $\frac{1}{3}$ of the distance between the y-value and the y-coordinate of the middle median. Then add this value to the y-intercept of the equation in Step 2.

a. *Scatter plots vary*.

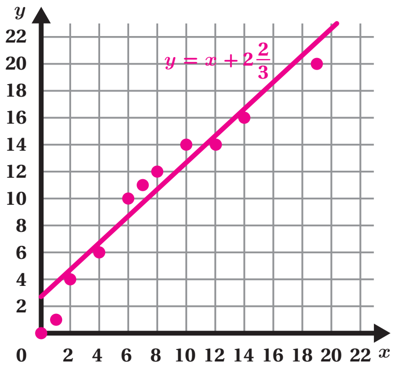
b. Step 1: $(2, 4)$, $(7.5, 11.5)$, $(14, 16)$

Step 2: The slope of the line passing through $(2, 4)$ and $(14, 16)$ is $\frac{16-4}{14-2} = \frac{12}{12} = 1$.

Since the slope is 1 and the line is passing through $(2, 4)$, substitute those values into $y = mx\ b$ to get $4 = 2 + b$, $b = 2$. The equation of the line is $y = x + 2$.

Step 3: Using the middle median, $y = 7.5 + 2 = 9.5$. The difference between the y-values is $11.5 - 9.5 = 2$. One third of this distance is $\frac{2}{3}$, so the line of best fit's equation is $y = x + 2\frac{2}{3}$

Days since campaign launch	Total donations (in millions of dollars)
1	1
2	4
4	6
6	10
7	11
8	12
10	14
12	14
14	16
19	20



c. *Responses vary*. The three-median method works because it finds the line of best fit by looking at the middle values in smaller groups of data, so it's not easily affected by extreme values.
Responses vary. The three-median method might not work well if the data is very skewed, has gaps, or isn't spread out evenly, or when the data doesn't have a strong linear pattern.

About $32.7 million. *Explanations vary*.
I substituted $x = 30$ into the three-median regression equation to determine the total donations in millions, $y = 30 + 2\frac{2}{3} \approx 32.7$.

d. About 23 days. *Explanations vary*.
I substituted $y = 25$ into the three-median regression equation to determine x, the number of days since the campaign launched, $25 = x + 2\frac{2}{3}$, $x \approx 22.3$.

Unit 3

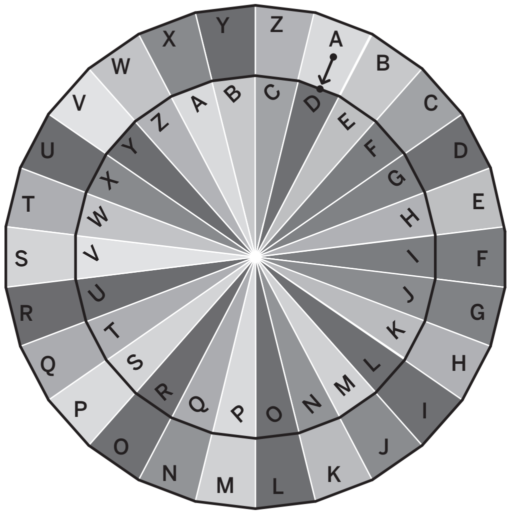
Extensions

Function Notation

Name: .... Date: .... Period: ....

Student Choice Complete Problem 1 before starting Problem 2. Remember to show or explain your thinking.

1 The *Caesar Cipher* is simply a type of substitution cipher, i.e., each letter of a given text is replaced by a letter with a fixed number of positions down the alphabet.



a The smaller disk is rotated to shift each letter 3 steps forward: $s(x) = x + 3$

Encode the word "Amplify" using the Caesar Cipher with the shift $s(x) = x + 3$.

b Encode a word you like and exchange with a classmate to decode.

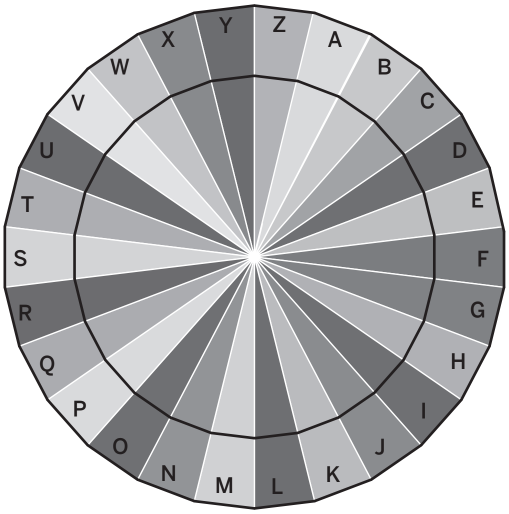
c Decode "Ohw'v ohduq wrjhwkhu" using $s(x) = x + 3$

d What other way can you write the same function, $s(x) = x + 3$, on the Caesar wheel?

Function Notation (continued)

Name: .... Date: .... Period: ....

2 Here is a blank cipher where the coding rule is not known.



a Decode the text.

X jhts id utta vjxain iwpi X hetci paa spn eapnxcv vpbth, lwxat X lph hjeedhts id qt sdxcv bpiwtbpixrh. iwtc, lwtc X sxhrdktgts hjggtpa cjbqtgh, X gtpaxots iwpi eapnxcv vpbth xh bpiwtbpixrh.Ydwc Rdclpn

b Write the rule of the cipher using function notation.

3 The mathematician Hedy Lamarr's patented "frequency hopping" allowed for secret codes to be passed over the radio. It was not possible for the code to be cracked without the key. A coded message was created by assigning each letter of the alphabet a number from 1 to 26 in chronological order, A is 1, B is 2, etc. Then it was multiplied by 4 and added with 3.

a Write a function that represents how the message was encoded.

b Decode the message $79\ 87\ 67\ 23\ 75$.

"Hedy Lamarr in 'The Heavenly Body'. Movie of MGM (1944)" by MGM Public Domain via Wikimedia Commons.

Unit 3

Sub-Unit 1 Extensions

Function Notation

Assign problems to students who want to extend their thinking.

Problems 1 and 3 can be solved in any order. Assign Problem 2 to students who have solved Problem 1. If time allows, consider sharing all with all students.

Problem 1

Students will extend their understanding of finding and interpreting functions by using Caesar Cipher to code a message.

Provide students with the following hint if additional scaffolding is needed.

- **Hint:** In Part c, the text is coded by shifting the letters 3 units right. How can you write the inverse function to decode the text?

a. DPSOLIB

b. *Responses vary.*

c. Let's learn together

d. *Responses vary.*

$s(x) = x - 23$

Problem 2

Students will extend their understanding of finding and interpreting functions by using Caesar Cipher to decode a message.

Provide students with the following hint if additional scaffolding is needed.

- **Hint:** How can the number of letters in words help you decode the text?

a. I used to feel guilty that I spent all day playing games, while I was supposed to be doing mathematics. Then, when I discovered surreal numbers, I realized that playing games is mathematics. John Conway

b. $s(x) = x - 11$ or $s(x) = x + 15$

Problem 3

Students will extend their understanding of finding and interpreting inverse functions by using inverse functions to decode a message.

Provide students with the following hint if additional scaffolding is needed.

- **Hint:** How can you write the function $c(n)$ if n represents the original, decoded number?

a. $c(n) = 4n + 3$, where c represents the coded number and n represents the original, decoded number.

b. SUPER. *Explanations vary.*

I reversed the encoding function by subtracting 3 from the given number and dividing the difference by 4, $\frac{c-3}{4}$.

Unit 3
Sub-Unit 2
Extensions

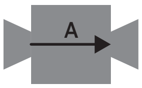
Key Features of Functions

Name: .... Date: .... Period: ....

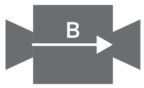
Student Choice Start with any problem. Remember to show or explain your thinking.

1

Here are two function machines with their input/output table.



Input	Output
2	5
3	7
4	9



Input	Output
5	11
7	17
9	23

a Fabiana wants to reverse the way that function machine A works. What is the rule she needs to write?

b Diego wants to reverse the way that function machine B works. What is the rule he needs to write?

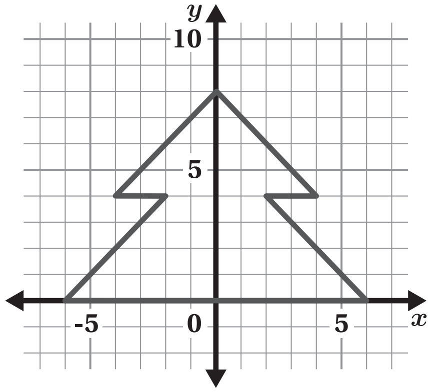
c Ada wants to combine two function machines by writing a single rule for a new machine. What is the new function Ada needs to use to get the same outputs of machine B using the inputs of machine A?

Key Features of Functions (continued)

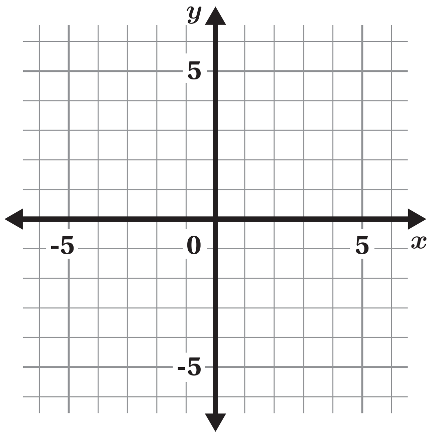
Name: .... Date: .... Period: ....

2 Here is the logo of a holiday store company designed in the Desmos Graphing Calculator using the graphs of several functions.

a Write the functions and their domains needed to create this logo.



b Design your own logo using graphs of functions and their domains.



Key Features of Functions

Assign problems to students who want to extend their thinking.

Problems 1 and 2 can be solved in any order. If time allows, consider sharing Problems 1 and 2 with all students.

Materials

- Desmos Graph Calculator (optional) **(Problem 2)**

Problem 1

Students will extend their understanding of functions and their key features by redesigning input-output tables to determine inverse and composite functions.

Provide students with the following hint if additional scaffolding is needed.

- **Hint:** Create the new input-output tables to help you determine the rules in each part.

a. $\frac{(x-1)}{2}$. *Explanations vary.*

- The new input output table Fabiana needs is

Input	Output
5	2
7	3
9	4

Then I determined the rule as $\frac{x-1}{2}$.

- Function A multiplies each input by 2, then adds 1 to the product. To reverse these operations. I need to subtract 1 and divide the difference by 2.

b. $\frac{x+4}{3}$. *Explanations vary.*

- The new input output table Diego needs is

Input	Output
11	5
17	7
23	9

Then I determined the rule as $\frac{x+4}{3}$.

- Function A multiplies each input by 3, then subtracts 4 to the product. To reverse these operations. I need to add 4 and divide the sum by 3.

c. $6x-1$

Explanations vary. $A(x)=2x+1$, and $B(x)=3x-4$. Ada needs a function machine with the input/output table

Input	Output
2	11
3	17
4	23

Then I determined the rule as $6x-1$.

Continued next page ...

Problem 2

Students will extend their understanding of functions and their key features by determining the functions and their domains for a given image and creating their own image.

Provide students with the following hint(s) if additional scaffolding is needed.

- **Hint 1:** In Part a, how many functions do you need to write to complete the image?
- **Hint 2:** In Part b, consider using Desmos Graphing Calculator to design your logo.

a. $y = x + 8 \; \{-4 \leq x \leq 0\}$

$y = -x + 8 \; \{0 \leq x \leq 4\}$

$y = 4 \; \{-4 \leq x \leq -2\}$

$y = 4 \; \{2 \leq x \leq 4\}$

$y = x + 6 \; \{-6 \leq x \leq -2\}$

$y = -x + 6 \; \{2 \leq x \leq 6\}$

$y = 0 \; \{-6 \leq x \leq 6\}$

b. *Graphs vary.*

Special Types of Functions

Name: .... Date: .... Period: ....

Student Choice Start with any problem. Remember to show or explain your thinking.

1

The Lucas Number sequence, like the Fibonacci sequence, is a sequence in which each term is the sum of the two terms that come before it.

a Take a look at the Lucas sequence below. Complete the recursive definition for it.
2, 1, 3, 4, 7, 11, 18, 29, 47, 76, 123, 199, 322, 521

$$L(n) = \begin{cases} 2 & n = 0 \\ \text{........} & n = 1 \\ \text{........} & \text{........} \end{cases}$$

b As in Fibonacci sequence, one can extend the Lucas numbers to negative integers such as..., -11, 7, -4, 3, -1, 2, 1, 3, 4, 7, 11,...

Write the function $L(\text{-}n)$ in terms of $L(n)$.

c The Lucas numbers are related to the Fibonacci numbers in so many ways. For example:

$$L(n) = F(n-1) + F(n+1)$$

n:	0,	1,	2,	3,	4,	5,	6,	7,	8,	...
F(n):	0,	1,	1,	2,	3,	5,	8,	13,	21,	...
L(n):	2,	1,	3,	4,	7,	11,	18,	29,	47,	...

Write at least two more relationships using the function notation.

2

a The *floor function* $\lfloor x \rfloor$ is defined to be the greatest integer less than or equal to the real number x.

For example:

$\lfloor 3 \rfloor = 3$

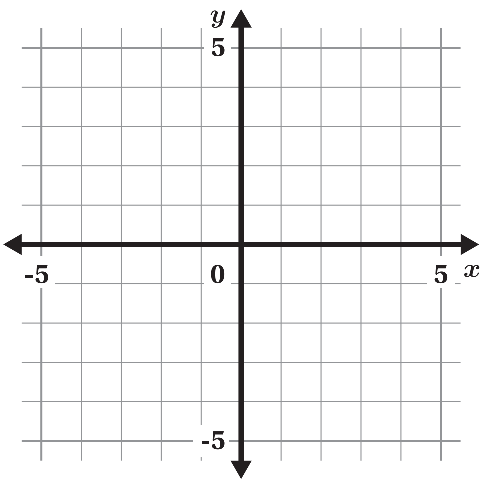
$\lfloor \text{-}3 \rfloor = \text{-}3$

$\lfloor 4.26 \rfloor = 4$

$\lfloor \text{-}4.26 \rfloor = \text{-}5$

$\lfloor \pi \rfloor = 3$

Plot the graph of the floor function.



Name: .... Date: .... Period: ....

b The *fractional part function* $\{x\}$ is defined to be the difference $x - \lfloor x \rfloor$.

For example:

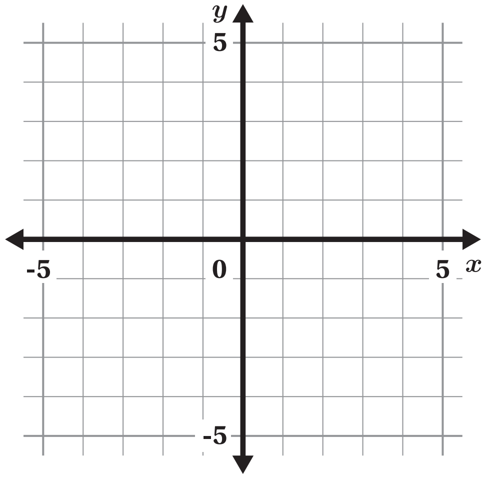
$\{3\} = 0$

$\{-3\} = 0$

$\{4.26\} = 4.26 - \lfloor 4.26 \rfloor = 4.26 - 4 = 0.26.$

$\{-4.26\} = -4.26 - \lfloor -4.26 \rfloor = -4.26 - (-5) = 0.74.$

Plot the graph of the fractional part function.



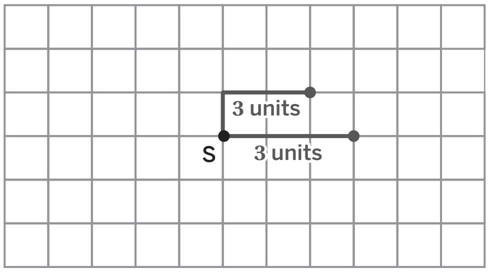
c What is the range of the fractional part function $\{x\}$?

d What is the sum of $\{x\}$ and $\{-x\}$?

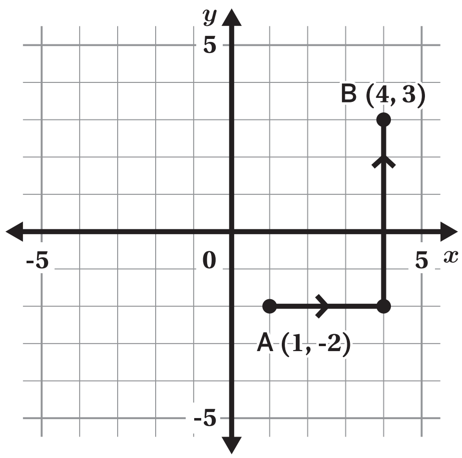
3

Taxicab geometry is a form of geometry in which you can only move along the lines of a grid. It gets its name from taxis since they can only drive along streets rather than moving as the bird flies.

a Determine all the points which are 3 units away to the given point S.



b What is the taxicab distance function between the points (x_1, y_1) and (x_2, y_2).



Special Types of Functions

Assign problems to students who want to extend their thinking.

Problems 1, 2 and 3 can be solved in any order. If time allows, consider sharing Problem 3 with all students.

Problem 1

Students will extend their understanding of piecewise-defined function by exploring the Lucas Numbers.

Provide students with the following hints if additional scaffolding is needed.

- **Hint 1:** In Part a, what is $L(1)$? How many pieces are needed to define the Lucas numbers function?
- **Hint 2:** In Part b, what is $L(\text{-}1)$? $L(\text{-}2)$? $L(\text{-}3)$? How are those numbers alike and different from the positive Lucas numbers?
- **Hint 3:** IIn Part c, how can we use the function $L(n) = F(n-1) + F(n+1)$ to create similar recursive formulas?

a. $L(n) = \begin{cases} 2 & n = 0 \\ 1 & n = 1 \\ L(n-1) + L(n-1) & n > 1 \end{cases}$

b. $L(\text{-}n) = (\text{-}1)^n\, L(n)$

c. *Responses vary.*

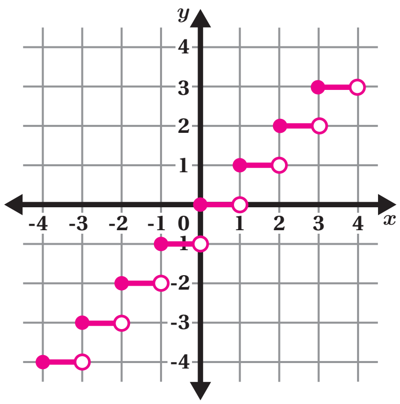
- $L(n) = 2 \bullet F(n+1) \text{ -} F(n)$
- $F(2n) = L(n) \bullet F(n)$

Problem 2

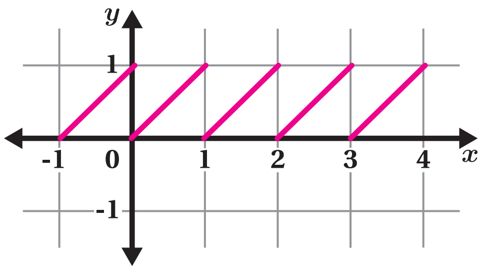
Students will extend their understanding of piecewise-defined function by exploring the floor and fractional part functions.

Continued next page ...

a.



b.



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Problem 2 (Continued)

Provide students with the following hint(s) if additional scaffolding is needed.

- **Hint 1:** In Part c, what is $\{x\}$ If x is an integer? What is the maximum value $\{x\}$ can get?
- **Hint 2:** In Part d, try several opposite numbers to determine the sum of $\{x\}$ and $\{-x\}$.

c. $0 \leq \{x\} < 1$

Explanations vary. If x is an integer, $\{x\}$ is 0, if it is a noninteger, $\{x\}$ is always between 0 and 1.

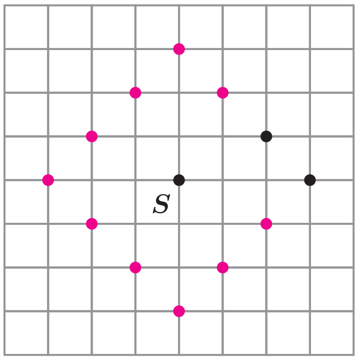
d. *Responses vary*. if x is an integer, $\{x\} + \{-x\}$ is 0, if x is not an integer, then the sum is always equal to 1.

Problem 3

Students will extend their understanding of absolute value function by exploring the distance function in Taxicab geometry.

Provide students with the following hints if additional scaffolding is needed.

- **Hint 1:** In Part a, is there a pattern all the points that are 3 units away from the given point follow?
- **Hint 2:** Try as many points as you can to determine the distance function in the Taxicab geometry.

a. 

Explanations vary. I noticed that in taxicab geometry the set of points equidistant to a certain point forms a square instead of a circle.

b. *Responses vary*. The taxicab distance function is the sum of the absolute differences of their coordinates.

$d = |x_1 - x_2| + |y_1 - y_2|$

For example, between $A(1, -2)$ and $B(4, 3)$, the taxicab distance is 8 units.

$d = |1 - 4| + |-2 - 3|$

$d = |-3| + |-5| = 8$ units

Unit 4

Extensions

Systems of Equations

Name: .... Date: .... Period: ....

Student Choice Start with any problem. Remember to show or explain your thinking.

1

Here is a 3×3 square and a 5×5 square. Except for the last row and column of each square, the sums of each column and row is given. Determine the value of each letter.

a

X	Z	Y	10
Y	X	X	14
Y	Z	Z	?
13	7	?	

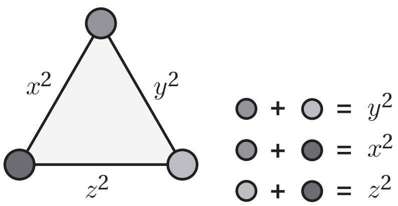
b

C	A	C	C	A	16
D	A	A	D	B	19
C	A	D	A	B	17
B	A	A	C	E	16
E	C	C	E	B	?
12	22	18	16	?	

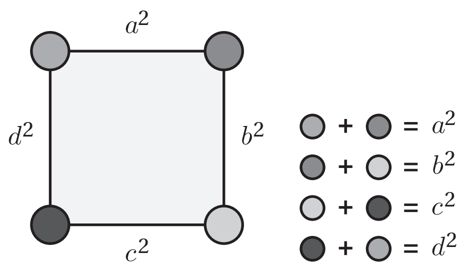
2

Determine the whole number value of each circle so that the sum of the two numbers along any given side is a square of another number.

a



b



Systems of Equations (continued)

Name: .... Date: .... Period: ....

3 A rectangle has a perimeter of 29 centimeters. If Trevon adds 3 centimeters to the width and subtracts 4 centimeters from the length, the area does not change. Determine the length and width of the rectangle.

4 The point $(8, -3)$ is the intersection of the graphs of $ax + by = 25$ and $3ax - 5by = 3$. Determine the values of a and b.

5 Prisha has 30 coins, consisting of nickels, dimes, and quarters, worth $\$4.60$. There are two more dimes than quarters. How many of each kind of coin does she have?

Unit 4
Sub-Unit 1 Extensions
Systems of Equations

Assign problems to students who want to extend their thinking.

Problems can be solved in any order. If time allows, consider sharing Problems 1, 3, and 4 with all students.

Problem 1

Students will extend their understanding of systems of equations by solving a puzzle with five variables.

Provide students with the following hint if additional scaffolding is needed.

- **Hint:** Start with writing the equations for each row and column. Which two equations can be used to determine the value of a variable?

a. $X = 5$, $Y = 4$, and $Z = 1$

The unknown sum of the last row is 6 and column is 10. *Explanations vary.*

b. $A = 5$, $B = 1$, $C = 2$, $D = 4$, and $E = 3$

The unknown sum of the last row and column are both 11. *Explanations vary.*

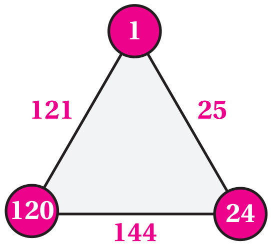
Problem 2

Students will extend their understanding of systems of equations by solving a puzzle using different strategies.

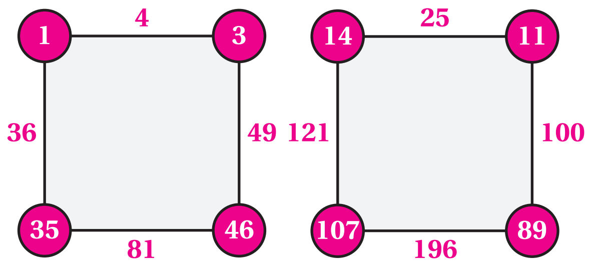
Provide students with the following hints if additional scaffolding is needed.

- **Hint 1:** In part a, consider choosing one of the circles as 1.
- **Hint 2:** In part b, consider choosing one of the circles as 1.
- **Hint 3:** In part b, consider choosing another circle as 3.

a. *Responses vary.*



b. *Responses vary.*



Problem 3

Students will extend their understanding of systems of equations by solving a puzzle using different strategies.

Provide students with the following hint if additional scaffolding is needed.

- **Hint:** Let a and b represent the length and width of the original rectangle. What are the new side lengths? What is the new area?

10 and 4.5. *Explanations vary.*
Let a and b represent the length and width of the original rectangle.
$2a + 2b = 29$
The area with the new side lengths is
$(a - 4)(b + 3) = ab + 3a - 4b - 12$.
This also equals the original area, ab, so
$ab + 3a - 4b - 12 = ab$.
Then I have
$3a - 4b = 12$.
Solving as a system of equations,
$2a + 2b = 29$ and $3a - 4b = 12$,
I determined a as 10 and b as 4.5.

Continued next page ...

Problem 4

Students will extend their understanding of systems of equations by solving a system with the given intersection point.

Provide students with the following hint if additional scaffolding is needed.

- **Hint:** How can the intersection point be used to determine the variables a and b?

$a = 2$ and $b = -3$. *Explanations vary*.

Because the point $(8, -3)$ is the intersection of the graphs, it means both equations are true when $x = 8$ and $y = -3$.

By using substitution, I have

$a \cdot 8 + b \cdot (-3) = 25$ and $3a \cdot 8 - 5b \cdot (-3) = 3$;

$8a - 3b = 25$

$24a + 15b = 3$

By solving this system, I determined a as 2 and b as -3.

Problem 5

Students will extend their understanding of systems of equations by solving a system with three variables.

Provide students with the following hint if additional scaffolding is needed.

- **Hint:** How can you use the values of each coin to write an equation which is equal to their total amount?

4 nickels, 14 dimes, and 12 quarters. *Explanations vary*.

By using the information given in the problem, I can write the following equations:

$n + d + q = 30$

$5n + 10d + 25q = 460$

$d = 2 + q$

I substituted d with $2 + q$ in each equation.

$n + 2 + q + q = 30$, so $n + 2q = 28$.

$5n + 10(2 + q) + 25q = 460$, so $5n + 35q = 440$.

By solving this system, I determined $n = 4$, $d = 14$, and $q = 12$.

Systems of Inequalities

Name: .... Date: .... Period: ....

Remember to show or explain your thinking.

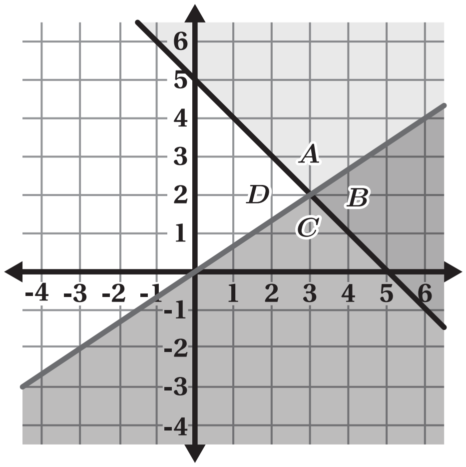
1 Determine the system of inequalities. . .

a whose solution is Region A.

b whose solution is Region B.

c whose solution is Region C.

d whose solution is Region D.



2 Here is a system of inequalities with missing inequality symbols.

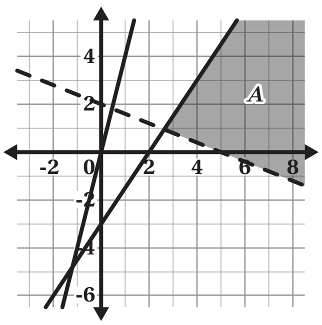
$$y \square 4x$$
$$2x + 5y \square 10$$
$$3x - 2y \square 6$$

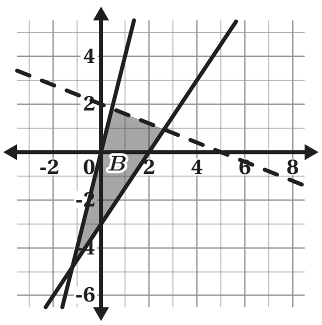
a Determine the symbols if there is no solution.

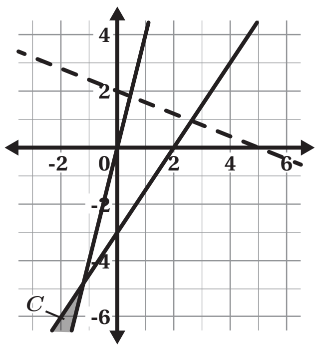
b Determine the symbols if the solution is Region A.

c Determine the symbols if the solution is Region B.

d Determine the symbols if the solution is Region C.





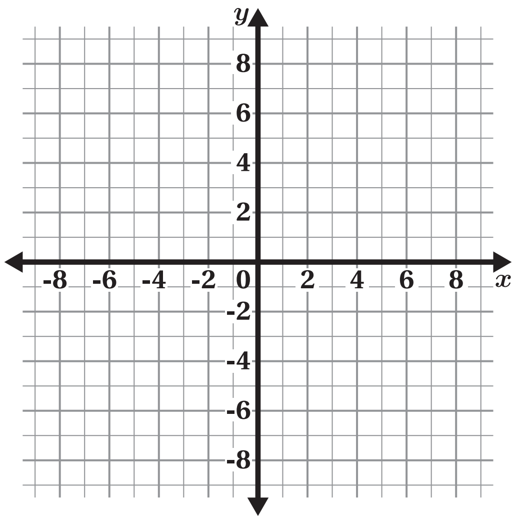


Systems of Inequalities (continued)

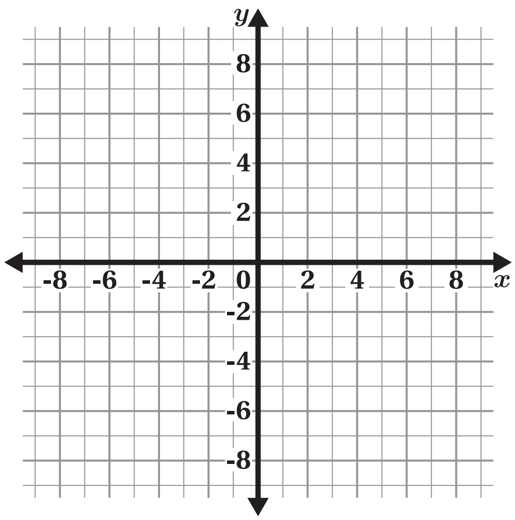
Name: .... Date: .... Period: ....

3

a Create your own system of inequalities whose solution is a single point. How do you know the solution is a single point?



b Create your own system of inequalities whose solution is a square. How do you know the solution is a square?



Systems of Inequalities

Assign problems to students who want to extend their thinking.

Problems 1 and 3 can be solved in any order. Assign Problem 2 to students who have solved Problem 1. If time allows, consider sharing Problems 1 and 2 with all students.

Problem 1

Students will extend their understanding of systems of inequalities by determining the inequalities that represent the given graphs.

Provide students with the following hint if additional scaffolding is needed.

- **Hint:** Start with determining the equations of the lines.

Students may write equivalent inequalities.

a. Region A:
$x + y \geq 5$
$3y \geq 2x$

b. Region B:
$x + y \geq 5$
$3y \leq 2x$

c. Region C:
$x + y \leq 5$
$3y \leq 2x$

d. Region D:
$x + y \leq 5$
$3y \geq 2x$

Problem 2

Students will extend their understanding of systems of inequalities by determining the inequality symbols to represent graphs.

Provide students with the following hint if additional scaffolding is needed.

- **Hint:** Consider picking a point from each region to substitute in the inequalities.

a. *Responses vary.*
$y \geq 4x$
$2x + 5y > 10$
$3x - 2y \geq 6$

b. $y \leq 4x$
$2x + 5y > 10$
$3x - 2y \geq 6$

c. $y \leq 4x$
$2x + 5y < 10$
$3x - 2y \leq 6$

d. $y \geq 4x$
$2x + 5y < 10$
$3x - 2y \geq 6$

Continued next page ...

Problem 3

Students will extend their understanding of writing systems of inequalities by creating different polygons on a coordinate plane.

Provide students with the following hints if additional scaffolding is needed.

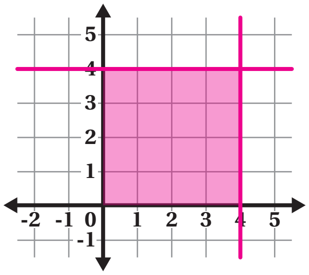
- **Hint 1:** In part a, think about the inequality symbols that can result with only one solution. How many inequalities are needed?
- **Hint 2:** In part b, think about the properties of a square such as opposite sides being parallel and congruent.

a. *Responses vary*. There is one solution, the point $(0, 0)$.

$$\begin{cases} x \geq 0 \\ x \leq 0 \\ y \geq 0 \\ y \leq 0 \end{cases}$$

b. *Responses vary*. The length of each side is equivalent to 4.

$$\begin{cases} x \geq 0 \\ x \leq 4 \\ y \geq 0 \\ y \leq 4 \end{cases}$$



Unit 5

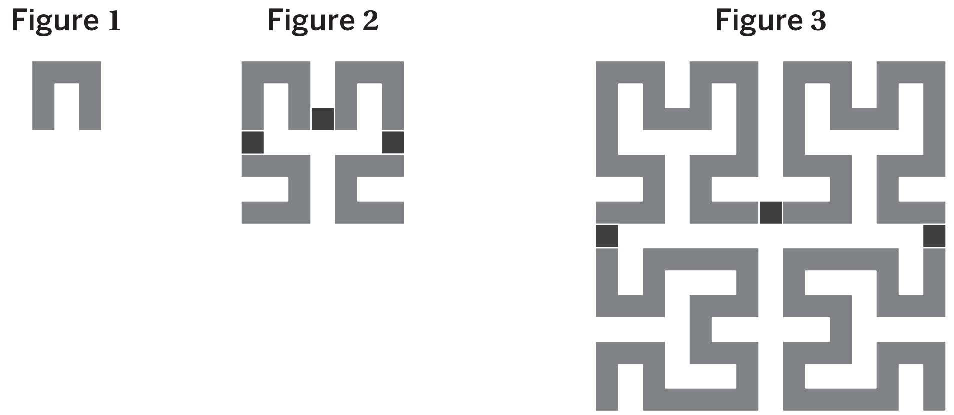
Extensions

Comparing Linear and Exponential Functions

Name: .... Date: .... Period: ....

Student Choice Start with any problem. Remember to show or explain your thinking.

1 Here is a pattern called the *Hilbert Curve*.

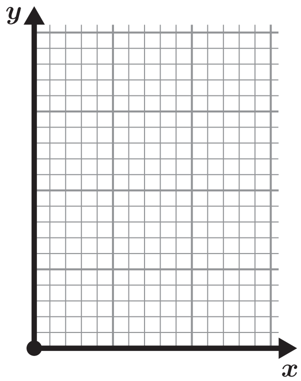


7 units long $\quad 4 \cdot 7 + 3$ units long $\quad$ .... units long

a Complete the table for the first five figures of the Hilbert Curve.

Figure	Length
1	7
2	31
3	
4	
5	

b Plot the points on a graph.



c Write a recursive and/or explicit expression to represent the growth.

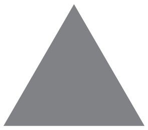
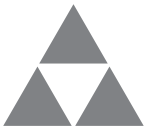
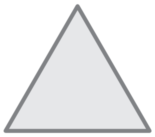
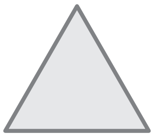
Comparing Linear and Exponential Functions (continued)

Name: .... Date: .... Period: ....

2

Here is a fractal called the *Sierpinski triangle*.

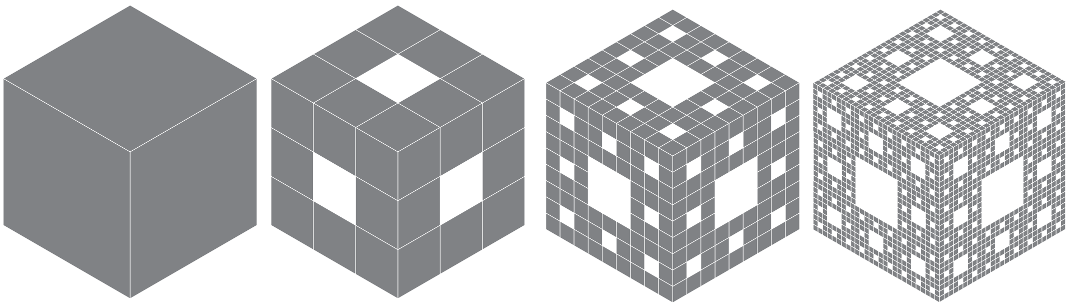
a Complete the table.

	Figure 0	Figure 1	Figure 2	Figure 3	Figure 4	Figure n
						
Number of shaded triangles:	1	3				
Shaded Area:	1	$\frac{3}{4}$	$\frac{9}{16}$			

b Which measure(s) of the Sierpinski triangle demonstrate exponential growth, and which one(s) show decay?

c What about the total perimeter of the Sierpinski triangle? What do you notice about the perimeter and the area of the Sierpinski triangle?

d Here is another fractal called the *Menger Sponge*. You start with a solid cube, and repeatedly drill smaller and smaller holes into its faces. Every new iteration of holes has $\frac{1}{3}$ of the width of the previous iteration of holes.



Which measure(s) of the Menger Sponge do you think will demonstrate exponential growth, and which one(s) will show decay?

Comparing Linear and Exponential Functions

Assign problems to students who want to extend their thinking.

Problems 1 and 2 can be solved in any order. If time allows, consider sharing both problems with all students.

Problem 1

Students will extend their understanding of exponential growth by investigating the space filling curves particularly the Hilbert curve.

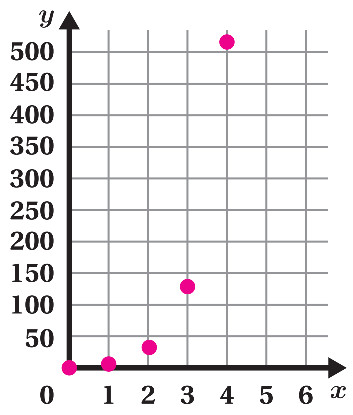
Provide students with the following hint if additional scaffolding is needed.

- **Hint:** The second figure has 4 times the original figure and 3 connecting pieces. What about the third figure? How can this rule help you write the recursive formula for growth?

a.

Figure	Length
1	7
2	$31 = (4 \bullet 7 + 3)$
3	$127 = (4 \bullet 31 + 3)$
4	$511 = (4 \bullet 127 + 3)$
5	$2047 = (4 \bullet 511 + 3)$

b.



c. Recursive formula for growth is

$a_{n+1} = 4 \bullet a_n + 3$

Explicit formula for growth is

$2^{2n+1} - 1$

Continued next page ...

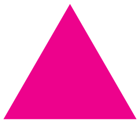
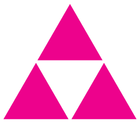
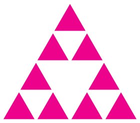
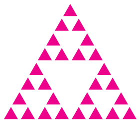
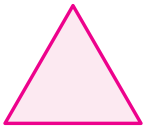
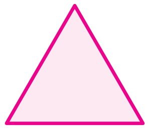
Problem 2

Students will extend their understanding of comparison properties by solving the *Sierpinski triangle*.

Provide students with the following hints if additional scaffolding is needed.

- **Hint 1:** How does the number of shaded triangles grow with each figure?
- **Hint 2:** How does the total area decay with each figure?

a.

	Figure 0	Figure 1	Figure 2	Figure 3	Figure 4	Figure n
						
Number of shaded triangles:	1	3	9	27	81	3^n
Shaded Area:	1	$\frac{3}{4}$	$\frac{9}{16}$	$\left(\frac{3}{4}\right)^3$	$\left(\frac{3}{4}\right)^4$	$\left(\frac{3}{4}\right)^n$

b. Exponential Growth: Number of triangles, and the total perimeter.
Exponential Decay: Remaining area and side length of the triangles.
Explanations vary. The number of triangles increases exponentially with each iteration. At each step, each existing triangle is divided into three smaller triangles. So, the total number of triangles for figure n is 3^n. This represents exponential growth because the number of triangles triples at each step. At each iteration, $\frac{1}{4}$ of the area is removed. The remaining area after n iterations is $\left(\frac{3}{4}\right)^n$. This shows exponential decay since the area is multiplied by $\frac{3}{4}$ at each iteration.
Also, the side length of each smaller triangle decreases by half with each iteration.

c. *Responses vary*. Considering the finer and finer details of the Sierpinski triangle's boundary, the perimeter theoretically grows infinitely because more and more small segments are added with every iteration. So, the "measured" perimeter increases without bound, demonstrating exponential growth. While the area of the Sierpinski triangle exponentially decays, its perimeter grows to infinity!

d. *Explanations vary*.
Exponential Growth: Total number of cubes, and surface area.
Exponential Decay: Volume of the remaining sponge, and side length of the smaller cubes.

Exponential Growth and Decay

Name: ______ Date: ______ Period: ______

Remember to show or explain your thinking.

1

A certain bacteria population doubles in size every 12 hours.

a By how much will it grow in 2 days?

b By how much will it grow in 10 days?

c Complete the *doubling-time growth formula* for a population of size N_0, that doubles every d years (where d is measured in years, not hours, days, or any other unit of time) to determine the number N of the population at a time t.

$$\boldsymbol{N} = \boxed{} \bullet \boxed{}^{\frac{t}{d}}$$

2

All radioactive elements decay at a very predictable rate — this is determined by their *half-life*. A half-life is the time required for something's quantity to decrease by half. For example, the half-life of radioactive radium is 1600 years. (That's why Marie Curie's notebooks are still stored in lead-lined boxes!)

a Complete the table to determine by how much radium will remain after each year.

Time in years	0	1600	3200	4800	5400	t
Amount left	N_0	$N_0 \bullet \frac{1}{2}$				

b Compare the *half-life decay* and *doubling-time growth* formulas. How are they similar? How are they different?

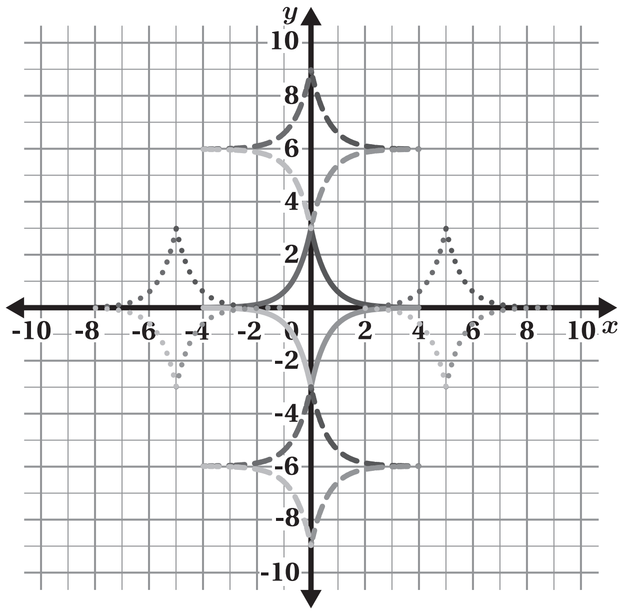
c Carbon-14 has a half-life of approximately 6,000 years. Scientists use C-14 in a process called *Carbon dating* to determine the ages of organic materials such as ancient fossils or mummies. If the amount N of C-14 is $\frac{1}{8}$ of the estimated amount N_0 that was in the sample when it was alive. What is the approximate age of the sample?

Name: ________ Date: ________ Period: ________

3 The central diamond-like shape made up of the exponential graphs:

- $y = 3 \cdot 5^{x}$ {$-4 < x < 4$} {$-3 < y < 3$}
- $y = (-3) \cdot 5^{-x}$ {$-4 < x < 4$} {$-3 < y < 3$}
- $y = 3 \cdot 5^{-x}$ {$-4 < x < 4$} {$-3 < y < 3$}
- $y = (-3) \cdot 5^{x}$ {$-4 < x < 4$} {$-3 < y < 3$}

The set of graphs is translated in four directions to create an image. Determine as many functions as you can to create the given image. Focus only on the functions rather than the range and domain restrictions.



Upper four graphs	Right four graphs	Lower four graphs	Left four graphs

Exponential Growth and Decay

Assign problems to students who want to extend their thinking.

Problems 1 and 3 can be solved in any order. Assign Problem 2 to students who have solved Problem 1. If time allows, consider sharing Problem 3 with all students.

Problem 1

Students will extend their understanding of writing exponential equations to model situations by exploring the doubling-time growth formula.

Provide students with the following hint if additional scaffolding is needed.

- **Hint:** Determine the percent of growth if you double the amount. How can you adapt it to your formula?

a. Let the initial size of the bacteria be n. Then its growth function can be written as $b(x) = n \cdot 2^x$.
In two days:
$b(x) = n \cdot 2^{\frac{48}{12}}$
$b(x) = n \cdot 2^4$
The size of the bacteria will be 16 times its initial size.

b. In 10 days, $b(x) = n \cdot 2^{\frac{240}{12}}$, its size will be around a million times its initial size.

c. At a time t, doubling–time growth formula is $N = N_0 \cdot 2^{\frac{t}{d}}$

Problem 2

Students will extend their understanding of writing exponential equations to model situations by exploring the half-life decay formula.

Provide students with the following hint if additional scaffolding is needed.

- **Hint:** Determine the percent of decay if you half the amount. How can you adapt it to your formula?

a.

Time in years	Amount left
0	N_0
1600	$N_0 \cdot \frac{1}{2}$
3200	$N_0 \cdot \left(\frac{1}{2}\right)^2$
4800	$N_0 \cdot \left(\frac{1}{2}\right)^3$
5400	$N_0 \cdot \left(\frac{1}{2}\right)^4$
t	$N_0 \cdot \left(\frac{1}{2}\right)^{t/d}$

b. *Explanations vary*. At a time t, doubling-time growth formula is: $N = N_0 \cdot 2^{\frac{t}{d}}$ and the half-life decay formula is $N = N_0 \cdot \left(\frac{1}{2}\right)^{\frac{t}{d}}$ They are similar because they are both exponential functions. Both equations include Initial size and the time calculations. The rate in the doubling-time formula is 2 whereas it is $\frac{1}{2}$ in the half-life formula.

c. 18,000 years. *Explanations vary*.
$\frac{N}{N_0} = \frac{1}{8} = \frac{1}{2^3} = \left(\frac{1}{2}\right)^{\frac{t}{6000}}$ then $3 = \frac{t}{6000}$.
Therefore t is 18,000 years.

Continued next page ...

Problem 3

Students will extend their understanding of graphing exponential functions by identifying the effects of vertical and horizontal translations on the graph of an exponential function.

Provide students with the following hint if additional scaffolding is needed.

- **Hint:** Consider using the Desmos Graphing Calculator to help to identify the unknown functions of graphs.

Responses vary.

Upper four graphs

a. $y = 3 \cdot 5^{-x} + 6 \, \{-4 < x < 4\} \, \{3 < y < 9\}$

b. $y = 3 \cdot 5^{x} + 6 \, \{-4 < x < 4\} \, \{3 < y < 9\}$

c. $y = -3 \cdot 5^{x} + 6 \, \{-4 < x < 4\} \, \{3 < y < 9\}$

d. $y = -3 \cdot 5^{-x} + 6 \, \{-4 < x < 4\} \, \{3 < y < 9\}$

Left four graphs

e. $y = 3 \cdot 5^{(-x-5)} \, \{-8 < x < -1\} \, \{-3 < y < 3\}$

f. $y = 3 \cdot 5^{(x+5)} \, \{-8 < x < -1\} \, \{-3 < y < 3\}$

g. $y = -3 \cdot 5^{(x+5)} \, \{-8 < x < -1\} \, \{-3 < y < 3\}$

h. $y = -3 \cdot 5^{(-x-5)} \, \{-8 < x < -1\} \, \{-3 < y < 3\}$

Lower four graphs

i. $y = 3 \cdot 5^{-x} - 6 \, \{-4 < x < 4\} \, \{-9 < y < -3\}$

j. $y = 3 \cdot 5^{x} - 6 \, \{-4 < x < 4\} \, \{-9 < y < -3\}$

k. $y = -3 \cdot 5^{x} - 6 \, \{-4 < x < 4\} \, \{-9 < y < -3\}$

l. $y = -3 \cdot 5^{-x} - 6 \, \{-4 < x < 4\} \, \{-9 < y < -3\}$

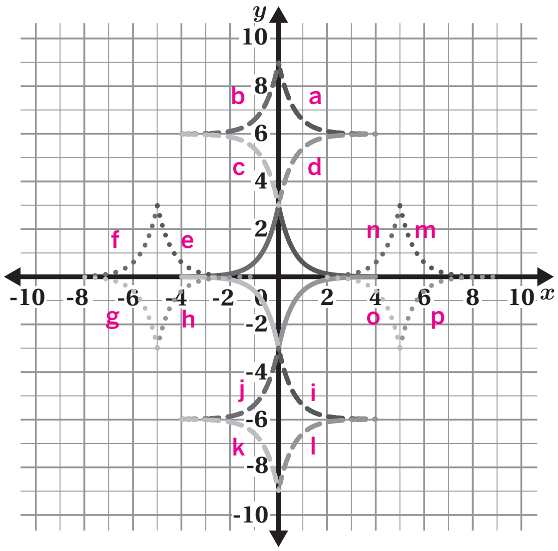
Right four graphs

m. $y = 3 \cdot 5^{(-x+5)} \, \{1 < x < 8\} \, \{-3 < y < 3\}$

n. $y = 3 \cdot 5^{(x-5)} \, \{1 < x < 8\} \, \{-3 < y < 3\}$

o. $y = -3 \cdot 5^{(x-5)} \, \{1 < x < 8\} \, \{-3 < y < 3\}$

p. $y = -3 \cdot 5^{(-x+5)} \, \{1 < x < 8\} \, \{-3 < y < 3\}$



Modeling With Exponentials

Name: .... Date: .... Period: ....

Student Choice Start with any problem. Remember to show or explain your thinking.

1

You are given $\$50{,}000$ that will double every year for the next 5 years.

a Write an equation that represents this situation where y represents the value of the investment after 5 years.

b Your equation has three numbers: starting value, growth factor, and the exponent. You are allowed to double one of these numbers. Which one do you think will result in the highest outcome?

2

Kiran wants to invest $\$1{,}000$ in an account with a 5% annual interest rate. He knows if the money is compounded semi-annually, he will earn more interest than if it is compounded annually. He wonders how much interest can be earned by compounding more and more often.

a Complete the table to investigate what happens to the end-of-year balance as the interest is compounded more and more frequently. Let n represent the number of times the interest is compounded in one year.

n	Year-end balance
1	
2	
3	
5	
10	
20	
50	

b What do you notice?

Modeling With Exponentials (continued)

Name: .... Date: .... Period: ....

3 Consider the following approximate data on the human population over time.

Year	1804	1927	1960	1974	1987	1999	2011	2024
World population in billions	1	2	3	4	5	6	7	8

a Generate an approximate model to represent the scenario.

b Use your model to determine when the human population will exceed 10 billion, 50 billion, and 100 billion.

c Do you think your model is accurate for these years?

Modeling With Exponentials

Assign problems to students who want to extend their thinking.

Problems 1, 2, and 3 can be solved in any order. If time allows, consider sharing Problems 1 and 3 with all students.

Problem 1

Students will extend their understanding of representing situations involving compound interest by writing and using exponential functions.

Provide students with the following hints if additional scaffolding is needed.

- **Hint 1:** In part a, what is the growth factor for the formula $y = 50{,}000 \cdot [?]^t$?
- **Hint 2:** In part b, double the starting value, growth factor, and time, and calculate the investment to determine which one leads to the highest outcome.

a. $y = 50{,}000 \cdot 2^t$ where the starting value is 50,000, and the growth factor is 2 (since the amount doubles every year). In 5 years,
$y = 50{,}000 \cdot 2^5$
$y = 1{,}600{,}000$

b. Growth factor or time. *Explanations vary*.
- *Doubling the starting value*: $y = 100{,}000 \cdot 2^5$, $y = 32,000,000$
- *Doubling the growth factor*: $y = 50{,}000 \cdot 4^5$, $y = 51,200,000$
- *Doubling the time*: $y = 50,000 \cdot 2^{10}$, $y = 51,200,000$

Problem 2

Students will extend their understanding of different compounding intervals by examining the results of continuous compounding.

Provide students with the following hint if additional scaffolding is needed.

- **Hint:** Use a calculator to determine if the interval numbers make a difference in the interest.

a.

n	Year-end balance
1	$1,050
2	$1,050.63
3	$1,050.84
5	$1,051.01
10	$1,051.14
20	$1,051.21
50	$1,051.24

b. *Responses vary*. I notice that the values seem to stay around $1,051 and do not get too much larger.

Continued next page ...

Modeling With Exponentials (continued)

Problem 3

Students will extend their understanding of modeling exponential behavior by generating a model to describe the growth of the human population over time.

Provide students with the following hint if additional scaffolding is needed.

- **Hint:** Consider plotting the points to help you write the function.

a. *Responses vary*.

$f(x) = 1 \cdot (1.01)^x$

where x is number of years since 1804. The y-intercept is $(0,1)$. Because the x-values do not increase at the same rate, I used trial and error to determine a possible growth factor.

b. *Responses vary*. In the years 2036, 2198, and 2267.

c. *Responses vary*. I do not think my model is accurate; In more recent years, the population is growing more quickly than in the 1800s and early 1900s. Also, I don't know if Earth can sustain 100 billion people!

Unit 6

Extensions

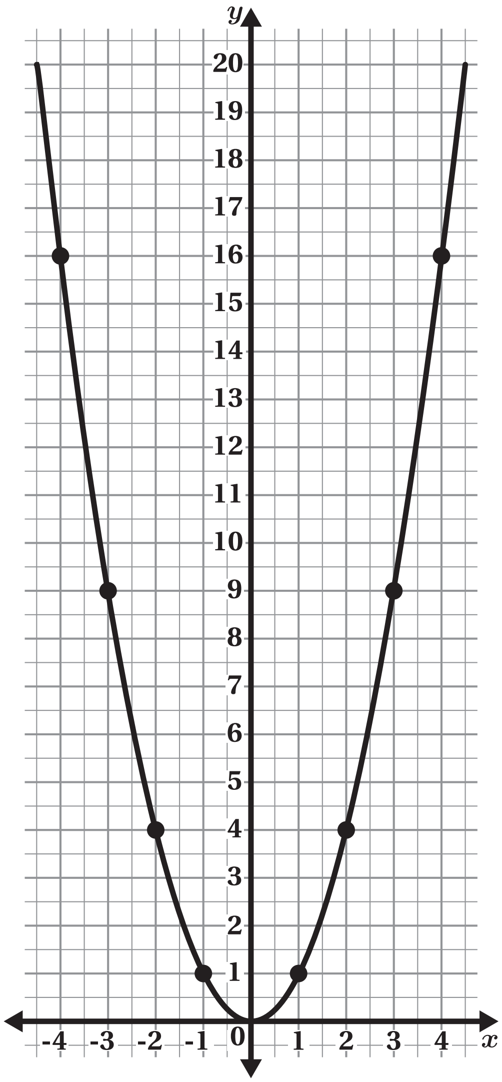
Introduction to Quadratic Functions

Name: .... Date: .... Period: ....

Remember to show or explain your thinking.

1 How many rectangles with whole-number side lengths have numerical perimeters equal to their numerical areas? What are the side lengths?

2 Here is the graph of $y = x^2$.



a Select any two points on the graph, one with a negative x-value and another with a positive x-value. Connect these two points with a line, then determine the y-intercept of the line.

b Repeat the steps in Problem 2a several times. What do you notice?

c Predict the y-intercept of the line connecting the points (-6, 36) and (8, 64)?

d Investigate the slope of each line. What do you notice?

Name: ______ Date: ______ Period: ______

3

Here are three quadratic number sequences given as tables. Determine the second difference for each. Is there a connection between the value of the second difference and the coefficient of the n^2 term?

a

Term number	Term
1	3
2	6
3	11
4	18
5	27
n	$n^2 + 2$

b

Term number	Term
1	6
2	8
3	14
4	24
5	38
n	$2n^2 - 4n + 8$

c

Term number	Term
1	2
2	5
3	14
4	29
5	50
n	$3n^2 - 6n + 5$

4

Here are new sequences. Determine the first, second, third, . . . differences if needed. What do you notice? How do you think the coefficient of the largest power of the rule is connected to the value of the difference?

a

Term number	Term
1	1
2	8
3	27
4	64
5	125
6	216
n	n^3

b

Term number	Term
1	3
2	17
3	55
4	129
5	251
6	433
n	$2n^3 + 1$

c

Term number	Term
1	0
2	14
3	78
4	252
5	620
6	1290
n	$n^4 - n$

Introduction to Quadratic Functions

Assign problems to students who want to extend their thinking.

Problems 1–3 can be solved in any order. Assign Problem 4 to students who have solved Problem 3. If time allows, consider sharing Problems 1–4 with all students.

Problem 1

Students will extend their understanding of areas of rectangles by examining a specific relationship between the area and perimeter of a rectangle.

Provide students with the following hint if additional scaffolding is needed.

- **Hint:** Can you model the scenario using an equation? How can this equation help you simplify the problem?

Two rectangles, a 4-by-4 square and a 6-by-3 rectangle. *Explanations vary*.
A rectangle with side lengths a and b has a perimeter of $2a + 2b$ and an area of ab. When they are equal numerically, $2a + 2b = ab$.

- Then I rearranged the equation as $a + b = \frac{ab}{2}$. Then using trial and error, I looked for two numbers whose sum is equal to the half of their product.
- Then I rearranged the equation as $ab - 2a - 2b = 0$, then added 4 to both sides to factorize the equation $ab - 2a - 2b + 4 = 4$ as $(a - 2)(b - 2) = 4$. Then I determined all a and b pairs that made the equation true to find the side lengths.

Problem 2

Students will extend their understanding of of the graphs of quadratic relationships by exploring the properties of $y = x^2$.

Provide students with the following hint if additional scaffolding is needed.

- **Hint:** Determine the product of x-coordinates of the points you have connected. How does the product relate to the y-intercept.

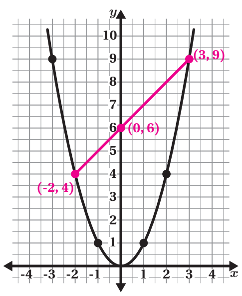
a. *Responses vary.* I have connected (-2, 4) and (3, 9). The line has a y-intercept as (0, 6).

b. I noticed that the value of the y-intercept is the absolute value of the product of the x-coordinates of the points.

c. 48 is the y-intercept of the line connecting the points (-6, 36) and (8, 64).

Points vary.

Point 1	Point 2	y-intercept
(-2, 4)	(1, 1)	(0, 2)
(-3, 9)	(3, 9)	(0, 9)
(-3, 9)	(4, 16)	(0, 12)



d. The slope of a line that connects any two points on the $y = x^2$ is the sum of the x-coordinates. *Explanations vary*.

Point 1	(-2, 4)	(-3, 9)	(-3, 9)	(-6, 36)	(-2, 4)
Point 2	(1, 1)	(3, 9)	(4, 16)	(8, 64)	(5, 25)
Slope	-1	0	1	14	3

Continued next page ...

Problem 3

Students will extend their understanding of quadratic relationships by using tables to connect the differences between terms with the equations.

Provide students with the following hint if additional scaffolding is needed.

- **Hint:** Use tables to mark the first and second differences between the consecutive terms.

The coefficient of the n^2 term is always half of the second difference.

a.

Term number	Term		
1	3		
2	6	+3	
3	11	+5	+2
4	18	+7	+2
5	27	+9	+2

b.

Term number	Term		
1	6		
2	8	+2	
3	14	+6	+4
4	24	+10	+4
5	38	+14	+4

c.

Term number	Term		
1	2		
2	5	+3	
3	14	+9	+6
4	29	+15	+6
5	50	+21	+6

Problem 4

Students will extend their understanding of quadratic relationships by using tables to connect the differences between terms with the equations.

Provide students with the following hints if additional scaffolding is needed.

- **Hint 1:** Use tables to mark the first and second differences between the consecutive terms.
- **Hint 2:** Is there a relationship between the degree of polynomial (the largest power) and the number of differences it takes to get to a constant difference.

a. The third difference is constant, it is 6.

b. The third difference is constant, it is 12.

c. The third difference is constant, it is 24.

a.

Term number	Term			
1	1			
2	8	+7		
3	27	+19	+12	
4	64	+37	+18	+6
5	125	+61	+24	+6
6	216	+91	+30	+6

b.

Term number	Term			
1	3			
2	17	+14		
3	55	+38	+24	
4	129	+74	+36	+12
5	251	+122	+48	+12
6	433	+182	+60	+12

c.

Term number	Term				
1	0				
2	14	+14			
3	78	+64	+50		
4	252	+174	+110	+60	
5	620	+368	+194	+84	+24
6	1290	+670	+302	+108	+24

I notice:
Responses vary.

- **The degree of polynomial (the largest power) and the number of differences to get to a constant difference are the same.**
- **In parts a and b, the coefficient of the n^3 term is $\frac{1}{6}$ of the third difference. In part c, the coefficient of the n^4 term is $\frac{1}{24}$ of the fourth difference.**
- **For a polynomial of degree k, the k-th difference is constant and is equal to $k!$ times the leading coefficient.**

Unit 6
Sub-Unit 2
Extensions

Standard Form and Factored Form

Name: .... Date: .... Period: ....

Remember to show or explain your thinking.

1 Han created a table to match the quadratic functions that are in *standard form* and *factored form*. Here is Han's table. He wants to determine the axis of symmetry of each parabola.

a Complete the table.

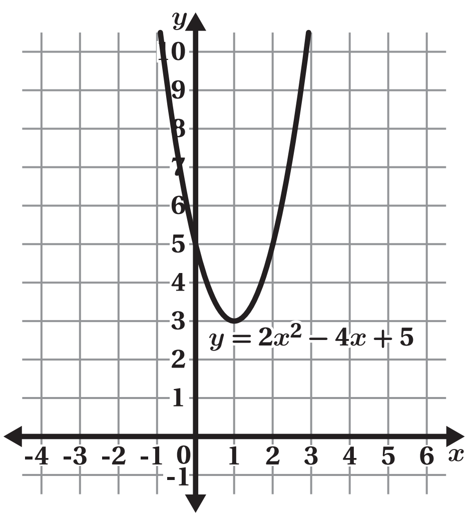
Standard form	Factored form	x-intercepts (m and n)	Axis of symmetry
$y = x^2 - 2x - 15$	$y = (x - 5)(x + 3)$		
$y = x^2 + 6x + 5$	$y = (x + 1)(x + 5)$		
$y = 2x^2 + 6x + 4$	$y = (2x + 4)(x + 1)$		
$y = 2x^2 + 6x - 36$	$y = (2x - 6)(x + 6)$		
$y = 3x^2 + 9x - 30$	$y = (x + 5)(3x - 6)$		

b Han thinks that there is a relationship between the axis of symmetry of each graph and the coefficients of x^2 and x in standard form. He determined the equation of the axis of symmetry as $x = \frac{-b}{a}$ for any quadratic function $f(x) = ax^2 + bx + c$. Is he correct? Explain why or why not.

c Here is a new quadratic function given in the standard form.

$y = 2x^2 - 4x + 5$

Use its graph to verify your conclusion in part b for the axis of symmetry.



Standard Form and Factored Form (continued)

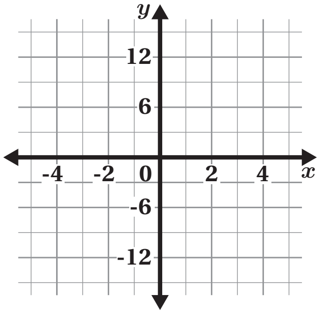
Name: .... Date: .... Period: ....

2

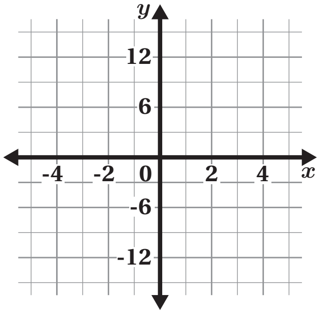
Here are three more functions.

a Graph each one on the given coordinate planes.

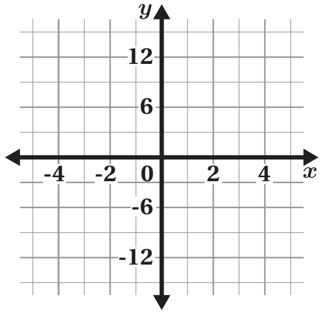
$y = (x - 3)^2$



$y = (x + 1)^2$



$y = (2x - 1)^2$

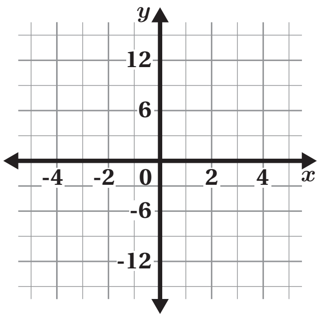


b What do you notice about the x- and y-intercepts, vertex, and axis of symmetry for each?

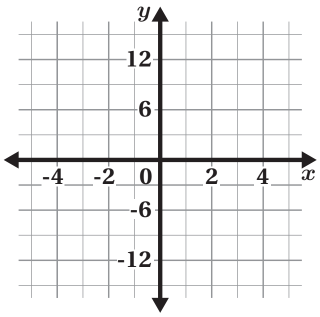
3

a Determine the x- and y-intercepts and sketch the graphs of the functions.

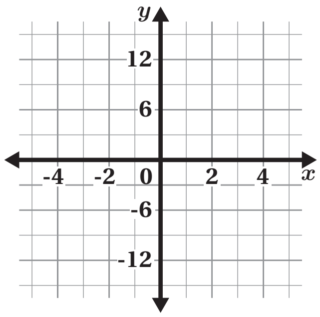
$(x + 2)(x - 1)$



$(x + 2)(x - 1)(x - 3)$



$(x + 2)(x - 1)(x - 3)\ (x + 1)$



b Use Desmos Graphing Calculator to graph the equations in part a. Which parts of your sketches match the actual graph?

Standard Form and Factored Form

Assign problems to students who want to extend their thinking.

Problems 1 and 2 can be solved in any order. Assign Problem 3 to students who have solved Problems 1 and 2. If time allows, consider sharing Problems 1–3 with all students.

Problem 1

Students will extend their understanding of key features and graphs of quadratic functions by exploring the relationship between the axis of symmetry and the quadratic equation in the standard form.

Provide students with the following hint if additional scaffolding is needed.

- **Hint:** Calculate Han's suggestion for each equation. How does Han's equation relate to the axis of symmetry?

Standard form	Factored form	x-intercepts	Axis of Symmetry	Han's equation $\left(\frac{-b}{a}\right)$
$x^2 - 2x - 15$	$(x-5)(x+3)$	5, -3	$x = 1$	$x = 2$
$x^2 + 6x + 5$	$(x+1)(x+5)$	-5, -1	$x = -3$	$x = -6$
$2x^2 + 6x + 4$	$(2x+4)(x+1)$	-2, -1	$x = -1.5$	$x = -3$
$2x^2 + 6x - 36$	$(2x-6)(x+6)$	3, -6	$x = -1.5$	$x = -3$
$3x^2 + 9x - 30$	$(x+5)(3x-6)$	-5, -2	$x = -3.5$	$x = 7$

a. Han is not correct. The equation for the axis of symmetry must be $\frac{-b}{2a}$.

Explanations vary. I determined the axis of symmetry for each equation and compared it with the Han's formula. The axis of symmetry was always half of Han's formula, so I have doubled the denominator to correct his equation.

b. $x = 1$. *Explanations vary*. I used the formula $\frac{-b}{2a}$ to determine the equation and then compared it with the graph. So I have concluded that the formula $\frac{-b}{2a}$ is correct.

Continued next page ...

Standard Form and Factored Form (continued)

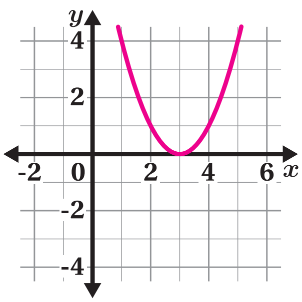
Problem 2

Students will extend their understanding of key features and graphs of quadratic functions by graphing perfect square functions.

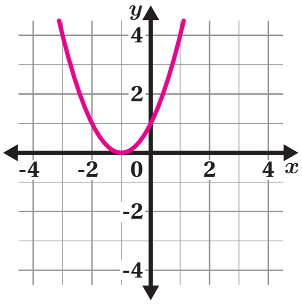
Provide students with the following hint if additional scaffolding is needed.

- **Hint:** You can rewrite $(x - 3)^2$ as $(x - 3)(x - 3)$. How can rearranging the equation help you graph the equation?

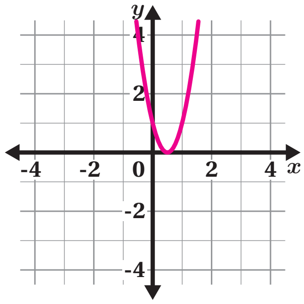
a. $y = (x - 3)^2$



$y = (x + 1)^2$



$y = (2x - 1)^2$



b. *Responses vary.* They all have a single x-intercept at the vertex, and the axis of symmetry passes through this point.

Problem 3

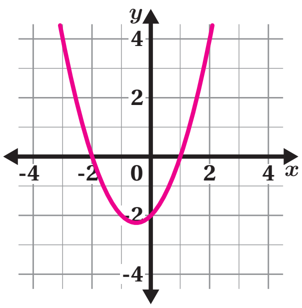
Students will extend their understanding of graphs of functions by predicting the graphs of third and fourth degree functions.

Provide students with the following hint if additional scaffolding is needed.

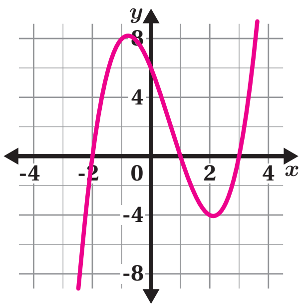
- **Hint:** How many x-intercepts does each graph have? What about the y-intercepts?

a. *Responses vary.*

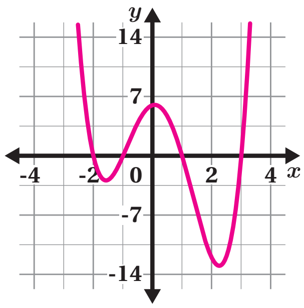
$(x + 2)(x - 1)$



$(x + 2)(x - 1)(x - 3)$



$(x + 2)(x - 1)(x - 3)(x + 1)$



In the first equation, there are two x-intercepts; -2 and 1. Its y-intercept is -2. In the second equation, there are three x-intercepts; -2, 1 and 3. Its y-intercept is 6. The last one has four x-intercepts; -2, 1, 3 and -1. Its y-intercept is 6 too.

b. *Responses vary.*

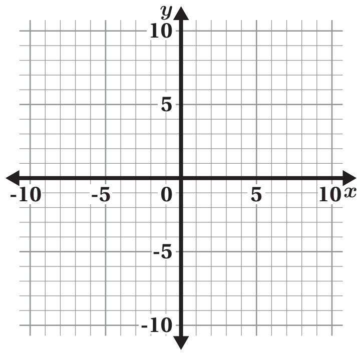
Vertex Form

Name: .... Date: .... Period: ....

Remember to show or explain your thinking.

1 Consider the equation $x = (y - 2)^2 + 1$.

a Determine the vertex and the line of symmetry.



b Graph the equation.

c Is y a function of x?

2

a Describe the graph of a parabola with the equation $y = x(b - x)$.

b Write expressions for the coordinates of its intercepts and vertex in terms of b.

c Do these expressions work for the negative values of b?

Vertex Form (continued)

Name: .... Date: .... Period: ....

3 The graph of $y = x^2$ has one x-intercept. Using the equation $y = a(x - h)^2 + k$, describe how to translate the graph of $y = x^2$ by changing the values of a, h, and k in order to create equations with the specified number of x-intercepts.

a No x-intercepts

b Only one x—intercept

c Two x-intercepts

Vertex Form

Assign problems to students who want to extend their thinking.

Problems 1 and 3 can be solved in any order. Assign Problem 2 to students who have solved Problem 1. If time allows, consider sharing Problems 1–3 with all students.

Problem 1

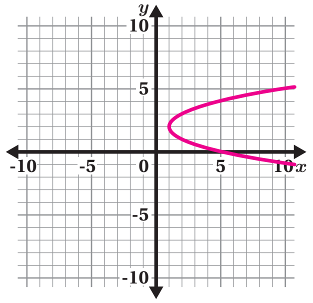
Students will extend their understanding of quadratic expressions in vertex form by examining a quadratic equation in terms of y.

Provide students with the following hint if additional scaffolding is needed.

- **Hint:** In part c, think about the conditions a relation must meet to qualify as a function.

a. $(1, 2)$, $y = 2$

b.



c. No, y is not a function of x, because there is more than one output, x-value, for each input, y-value.

Problem 2

Students will extend their understanding of quadratic expressions in vertex form by making generalizations.

Provide students with the following hint if additional scaffolding is needed.

- **Hint:** In part a, how does the coefficient of x^2 affect the parabola?

a. *Explanations vary*. Since the coefficient of x^2 is negative, the parabola opens downward. The graph of this parabola will be symmetrical about its vertex, and it will cross the x-axis at 0 and b.

b.

x-intercept	y-intercept	Vertex
$(0, 0)$ $(b, 0)$	$(0, 0)$	$\left(\frac{b}{2}, \frac{b^2}{4}\right)$

c. Yes, these expressions work for negative values of b.

Continued next page ...

Vertex Form (continued)

Problem 3

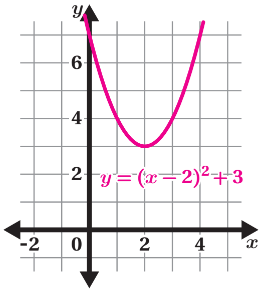
Students will extend their understanding of quadratic expressions in vertex form by making generalizations for translations on the coordinate plane.

Provide students with the following hint if additional scaffolding is needed.

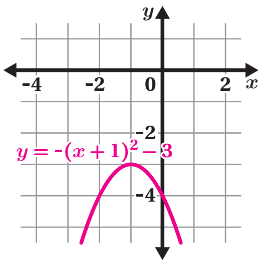
- **Hint:** In the vertex form of a parabola, $y = a(x - h)^2 + k$,
  - h represents a horizontal shift
  - k represents a vertical shift
  - a determines the "width" and direction (upward or downward) of the parabola

a. *Explanations vary*. For the parabola not to intersect with the x-axis, it must be entirely above or below the x-axis. This occurs if the vertex is above or below the x-axis and the parabola opens away from it.

- I can choose $a > 0$ and $k > 0$, then the parabola opens upward and is shifted up. For example $y = (x - 2)^2 + 3$ opens upward with a vertex at (2, 3) remaining entirely above the x-axis.

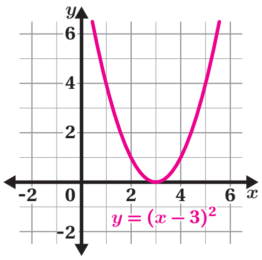


- I can choose $a < 0$ and $k < 0$, then the parabola opens downward and is shifted down. For example $y = -(x + 1)^2 - 3$ opens downward with a vertex at (-1, -3) remaining entirely below the x-axis.



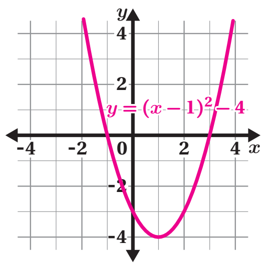
b. *Explanations vary*. For the parabola to intersect the x-axis at exactly one point, its vertex must lie on the x-axis.

- I can choose $k = 0$, and the vertex will be on the x-axis. For example, $y = (x - 3)^2$. The vertex is at $(3, 0)$, which is also the x-intercept.

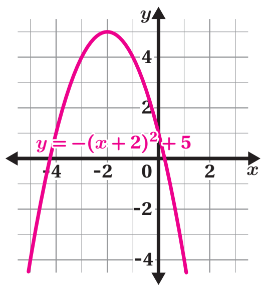


c. *Explanations vary*. For the parabola to intersect the x-axis at two points, its vertex must be above or below the x-axis, but open toward it .

- I can choose $a > 0$ and $k < 0$, then the parabola opens upward with a vertex below the x-axis. For example, for $y = (x - 1)^2 - 4$, the vertex is at $(1, -4)$, and the parabola opens upward, crossing the x-axis at two points.



- I can choose $a < 0$ and $k > 0$, then the parabola opens downward with a vertex above the x-axis. For example, for $y = -(x + 2)^2 + 5$, the vertex is at $(-2, 5)$, and the parabola opens downward, also crossing the x-axis at two points.



Unit 7

Extensions

Unit 7
Sub-Unit 1
Extensions

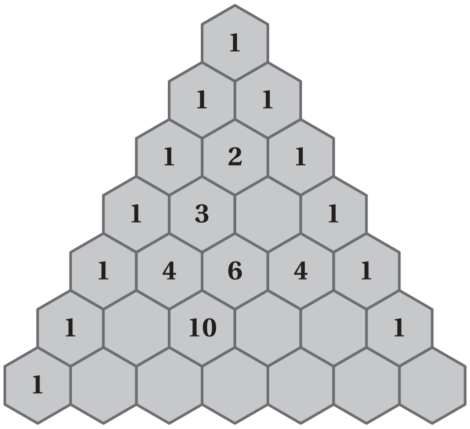
Multiplying and Factoring

Name: .... Date: .... Period: ....

Student Choice Start with any problem. Remember to show or explain your thinking.

1

Pascal's triangle is an arrangement of numbers in a triangular array. It starts with a single 1 at the top, and every following cell is the sum of the two cells directly above.



a Complete the missing cells of Pascal's triangle.

b Multiply the given expressions.

$(a + b)^2 = (a + b)\ (a + b)$

$(a + b)^3 = (a + b)(a + b)(a + b)$

$(a + b)^4 = (a + b)(a + b)(a + b)(a + b)$

c What do you notice? Can you connect the numbers in each row with the product expressions you wrote in part b?

d Amy is working on the standard form of $(a + b)^5$. Help her to complete the equation using Pascal's triangle.

$$[...]a^5 + [...]a^4b + [...]a^3b^2 + [...]a^2b^3 + [...]ab^4 + [...]b^5$$

e Can you predict the $(a + b)^6$ based on Amy's expansion of the fifth power and Pascal's triangle?

Multiplying and Factoring (continued)

Name: .... Date: .... Period: ....

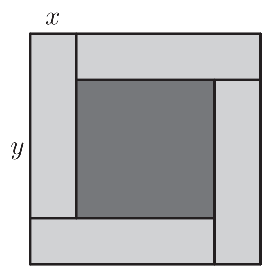
2

Here are commonly used identities to help writing the factored form and standard form of a given quadratic expression.

Square of the sum of two numbers $(a+b)^2$	Square of the difference of two numbers $(a-b)^2$	Product of the sum and difference of two numbers $(a+b)(a-b)$
$(a+b)(a+b) = a^2 + 2ab + b^2$	$(a-b)(a-b) = a^2 - 2ab + b^2$	$(a+b)(a-b) = a^2 - b^2$

These identities help solve many puzzle-like problems in mathematics. Here are some examples.

a Four identical rectangles with dimensions x and y enclosed the shaded region. Select the expression that represents the area of the shaded region.



A. $(x-y)^2$

B. $y^2 - x^2$

C. $x^2 - y^2$

D. $(x+y)^2$

b The perimeter of a rectangle is 46 cm and the length of one of its diagonals is 17 cm. Determine the area of the rectangle.

c How can you use the identities above to calculate the quotient of $\frac{105^2 - 2 \cdot 105 \cdot 64 + 64^2}{21^2 - 20^2}$ without a calculator?

d $9^x - 18 \cdot 3^x + 81 = 0$. Determine the value of x.

e Use $\frac{1}{a} - a = 7$ to determine the value of $\frac{1}{a} + a$.

Multiplying and Factoring

Assign problems to students who want to extend their thinking.

Problems 1 and 2 can be solved in any order. If time allows, consider sharing Problems 1 and 2 with all students.

Problem 1

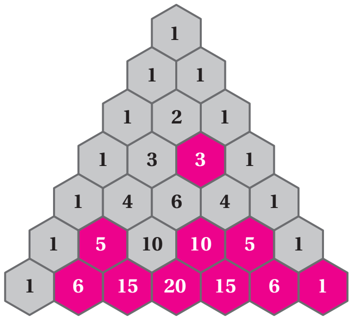
Students will extend their understanding of multiplying linear factors by describing connections between factored form, and standard form using Pascal's triangle.

Provide students with the following hints if additional scaffolding is needed.

- **Hint 1:** In part b, you can use the standard form of $(a + b)(a + b)$, which is $a^2 + 2ab + b^2$ and multiply it with $(a + b)$ to determine the next product. In other words

  $(a^2 + 2ab + b^2)(a + b) = (a + b)^3$
- **Hint 2: In part d, how many** factors are there in the factored form? Which row of the Pascal's triangle do you need to look at to determine the coefficients?
- **Hint 3:** In part e, what is the highest power of a? What can be the second term using Amy's way of organizing the equation?

a.



b. $(a + b)(a + b) = a^2 + 2ab + b^2$

$(a + b)(a + b)(a + b) = a^3 + 3a^2b + 3ab^2 + b^3$

$(a + b)(a + b)(a + b)(a + b) = a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$

c. *Responses vary.* I notice:

- Each row of Pascal's triangle corresponds to the coefficients of the product expressions.
- When you write the coefficients in the standard form of $(a + b)^n$, you need to look at the row that includes n after 1 in Pascal's triangle.
- The coefficients in the standard forms are symmetric.
- The exponents of a and b in each term always add up to the number of factors (or n) in the factored form.
- The total number of the terms in the standard form is one more than the number of factors in the factored form.

d. $1a^5 + 5a^4b + 10a^3b^2 + 10a^2b^3 + 5ab^4 + 1b^5$

e. *Predictions vary*.

$a^6 + 6a^5b + 15a^4b^2 + 20a^3b^3 + 15a^2b^4 + 6ab^5 + b^6$

Continued next page ...

Multiplying and Factoring (continued)

Problem 2

Students will extend their understanding of factoring and expanding using special identities by solving puzzle-like problems.

Provide students with the following hints if additional scaffolding is needed.

- **Hint 1:** In part a, what is the area of the outer square? How can you write in the standard form? What is the area of each square?
- **Hint 2:** In part b, start with writing the expressions for area and perimeter of the rectangle.
- **Hint 3:** In part c, how can the identity $(a-b)(a-b) = a^2 - 2ab + b^2$ help you organize the numerator of the expression?
- **Hint 4:** In part d, start with rearranging the given equation using $3^x = a$.
- **Hint 5:** In part e, how can squaring the equation $\frac{1}{a} - a = 7$ help you determine the value of $\frac{1}{a^2} + a^2$?

a. $(x-y)^2$. *Explanations vary*. The side length of the outer square is $x+y$. Therefore its area is $(x+y)^2$. Each rectangle has an area of xy, so the total area of rectangles is $4xy$. Then the inner square has an area of $(x+y)^2 - 4xy$. I rearranged the equation as $x^2 + 2xy + y^2 - 4xy = x^2 - 2xy + y^2 = (x-y)^2$.

b. 240 square centimeters. *Explanations vary*. If the perimeter of the rectangle, $2(a+b)$ is 46 cm, then $a+b = 23$. The diagonal of a rectangle can be determined using the Pythagorean theorem, $a^2 + b^2 = 17^2 = 289$. I squared the equation $a+b=23$ to write $(a+b)^2 = 23^2$, so $a^2 + b^2 + 2ab = 529$. Then I substituted the value of $a^2 + b^2$ to get $289 + 2ab = 529$. After simplifying, $ab = 240$, which is the area of the rectangle.

c. 41. *Explanations vary*. In the expression, $\frac{105^2 - 2 \cdot 105 \cdot 64 + 64^2}{21^2 - 20^2}$, the expression in the numerator can be rewritten, using the identity $(a-b)(a-b) = a^2 - 2ab + b^2$, as $(105-64)^2$ which is 41^2. The expression in the denominator can be rewritten, using the identity $(a+b)(a-b) = a^2 - b^2$, as $(21+20)(21-20) = 41 \cdot 1$. So I wrote the final products as $\frac{41 \cdot 41}{41 \cdot 1} = 41$.

d. $x = 2$. *Explanations vary*. $9^x - 12 \cdot 3^x + 27 = 0$. I rewrote this equation, using the identity $(a-b)(a-b) = a^2 - 2ab + b^2$, by using $3^x = a$. The new equation became $a^2 - 18a + 81$, which can be written in factored form as $(a-9)^2 = 0$. Therefore $a = 9$, so $3^x = 3^2$, and $x = 2$.

e. $\sqrt{53}$. *Explanations vary*. I squared the equation $\left(\frac{1}{a} - a\right)^2 = 7^2$ and wrote as $\frac{1}{a^2} + a^2 - 2a\frac{1}{a} = 49$. So $\frac{1}{a^2} + a^2 - 2 = 49$, then $\frac{1}{a^2} + a^2 = 51$. The unknown expression is $\frac{1}{a} + a$. I squared this expression and wrote $\left(\frac{1}{a} + a\right)^2 = \frac{1}{a^2} + a^2 + 2a\,\frac{1}{a} = \frac{1}{a^2} + a^2 + 2$. I know the value of $\frac{1}{a^2} + a^2$ is 51 so this expression will have a value of $51 + 2 = 53$. Now I know that $\left(\frac{1}{a} - a\right)^2 = 53$. If I determine the square root of both sides, $\sqrt{\left(\frac{1}{a} + a\right)^2} = \sqrt{53}$ so $\frac{1}{a} + a = \sqrt{53}$.

Unit 7

Sub-Unit 2
Extensions

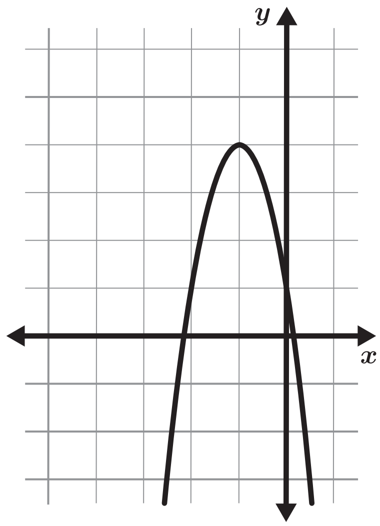
Solving Equations and Completing the Square

Name: .... Date: .... Period: ....

Student Choice Start with any problem. Remember to show or explain your thinking.

1

Here is the graph of the function $y = ax^2 + bx + c$.



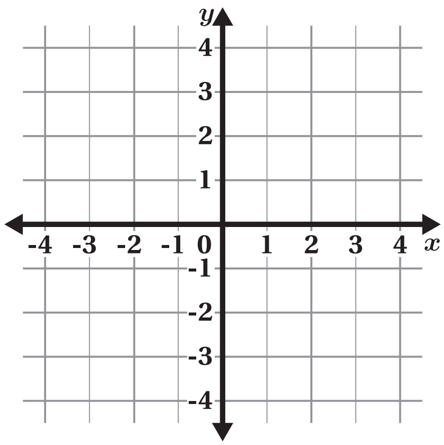
a Determine the signs of a, b, and c.

b Determine its axis of symmetry.

c Determine the coordinates of the vertex.

d Determine the equation of the graph.

e Draw a graph of the function $y = cx^2 + bx + a$.



2

$5x^2 + 20x + 25$ is not a perfect square, so you cannot rearrange into a single square. However, it can be arranged into the sum of squares. How can you rewrite this equation as the sum of two squares?

Unit 7
Sub-Unit 2
Extensions

Solving Equations and Completing the Square (continued)

Name: .... Date: .... Period: ....

3

Determine a value of n if $x^2 + 16x = n$ has

a no real solution

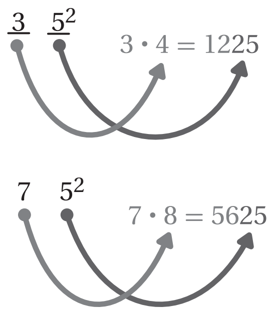
b one solution

c two solutions

4

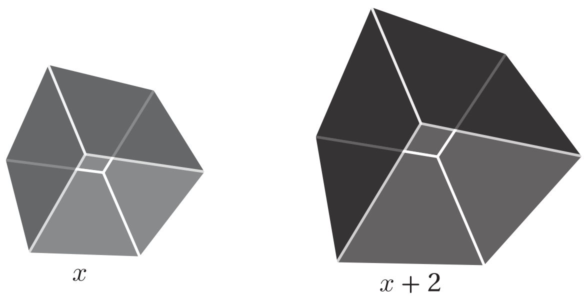
The square of a two-digit number $a5$ can easily be calculated using an algebraic trick of finding the product $a(a+1)$, then placing the result as digits in front of 25.

Can you explain why this trick works?



5

Each edge of a cube is 2 inches longer than each edge of another cube. If the difference between the volumes is 98 cubic inches, find the edge lengths of the cubes.



Solving Equations and Completing the Square

Assign problems to students who want to extend their thinking.

Problems 1–5 can be solved in any order. If time allows, consider sharing Problems 3–5 with all students.

Problem 1

Students will extend their understanding of solving quadratic equations by determining the values of coefficients.

Provide students with the following hint if additional scaffolding is needed.

- **Hint:** In part d, you can use the equation of the axis of symmetry, $x = \frac{-b}{2a}$.

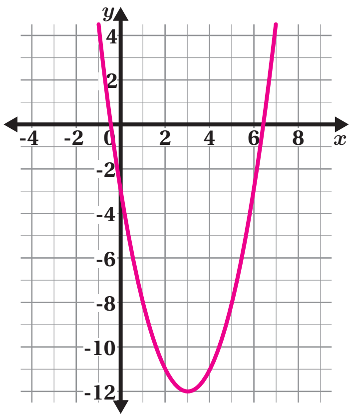
a. a is negative, b is negative, and c is positive. *Explanations vary*. Because the graph is facing downwards, I concluded $a < 0$. I see one of the x-intercepts is positive and the other one is negative, and the negative one is larger in magnitude, so b must be negative. I saw that the y-intercept is 1, so c is positive

b. $x = -1$

c. $(-1, 4)$

d. $y = -3x - 6 + 1$. *Explanations vary*. I used the vertex point to substitute for x. When $x = -1$, $a(-1)^2 + b(-1) + 1 = 4$, so $a - b = 3$. I know that axis of symmetry is $\frac{-b}{2a} = -1$, so $2a = b$. Solving both equations, I determined $a = -3$, and $b = -6$.

e.



Explanations vary. The new equation is $y = x^2 - 6x - 3$. I used the completing the square method, and rewrote the equation as $y = x^2 - 6x - 3 + 12 - 12$, then $y = (x - 3)^2 - 12$. So the y-intercept is $(0, -3)$, the vertex is $(3, -12)$, and the x-intercepts are $3 \pm \sqrt{12}$, which are approximately -0.5 and 6.5.

Problem 2

Students will extend their understanding of solving quadratic equations by writing a quadratic equation as the sum of two perfect square expressions

Provide students with the following hint if additional scaffolding is needed.

- **Hint:** What must be the value of a if you have a square expression as $ax^2 + 20x + 25$?

$x^2 + (2x + 5)^2$. *Explanations vary*. Because the problem says two square expressions, I tried to write $5x^2$ as the sum of two squares, $x^2 + 4x^2$. So now I have $x^2 + 4x^2 + 20x + 25$. Then I noticed $4x^2 + 20x + 25$ is the square of $(2x + 5)^2$, so I concluded that the solution is $x^2 + (2x + 5)^2$.

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Solving Equations and Completing the Square (continued)

Problem 3

Students will extend their understanding of solving quadratic equations by making generalizations about the number of solutions.

Provide students with the following hint if additional scaffolding is needed.

- **Hint:** Does the equation $x^2 = n$ where n is a negative number have a solution in real numbers? How can you use this fact to come up for the value of n to create an equation with no solution for $x^2 + 16x = n$?

Responses vary.

a. No real solution: $n <$ -64. For example, $x^2 + 16x + 80 = 0$

b. One solution: $n =$ -64 $x^2 + 16x + 64 = 0$

c. Two solutions: $n >$ -64. For example, $x^2 + 16x = 0$

Problem 4

Students will extend their understanding of solving quadratic equations by solving a calculation trick.

Provide students with the following hint if additional scaffolding is needed.

- **Hint:** Any two digit number as ab can be written as $10a + b$ using their place value.

Responses vary.

$a5$ can be written as $10a + 5$. When squaring $a5$, this expression becomes $(10a + 5)^2 = 100a^2 + 100a + 25$. The standard form can then be rewritten as $100a^2 + 100a + 25 = 100a(a + 1) + 25$. For example, we know that $35^2 = 1225$.
100 represents the place value of the first two numbers such as 1200 is 12 hundreds. So I multiply 3 and 4, which gives me 12 hundreds. When I place the 12 in front of the 25, I get 1225.

Problem 5

Students will extend their understanding of solving quadratic equations by attempting to solve a third degree equation.

Provide students with the following hints if additional scaffolding is needed.

- **Hint:** To multiply $(x + 2)$ three times with itself, first you can determine $(x + 2)(x + 2)$ and then multiply that expression with $(x + 2)$ again.

3 inches and 5 inches. *Responses vary*. The difference of the volumes of the cubes can be written as $(x + 2)^3 - x^3$ which is given as 98 cubic inches. So $(x + 2)^3 - x^3 = 98$.

$x^3 + 6x^2 + 12x + 8 - x^3 = 98$

$6x^2 + 12x - 90 = 0$

$6(x^2 + 2x - 15) = 0$

$6(x + 5)(x - 3) = 0$

So $x =$ -5 and $x = 3$. Because the length cannot be negative, I concluded that x is 3.

The first cube has 3 inch side lengths and has a volume of 27 cubic inches, and the second cube has 5 inch side lengths and has a volume of 125 cubic inches. The difference between 125 and 27 is 98 cubic inches.

The Quadratic Formula and More

Name: .... Date: .... Period: ....

Student Choice Start with any problem. Remember to show or explain your thinking.

1

In a chess tournament where each player competes against every other player, 36 games were played. How many people participated in this tournament?

2

Here is the Fibonacci sequence again: $1, 1, 2, 3, 5, 8, 13, 21, 34, 55, \ldots$.

Let a, b, c be the three consecutive numbers in this sequence. As one of the many properties of the Fibonacci sequence, it is known that the ratio between the consecutive terms converges to the same number, i.e., $\frac{a}{b} = \frac{b}{a}$ which can also be written as $\frac{a+b}{4} = \frac{b}{a}$ because $c = b + a$.

Eva uses the following steps to write a quadratic equation to determine the value of this ratio.

Eva's Work

$\frac{a+b}{b} = \frac{b}{a}$

$\frac{a}{b} + \frac{b}{b} = \frac{b}{a}$ substitute $x = \frac{a}{b}$

$x + 1 = \frac{1}{x}$

$x^2 + x - 1 = 0$

a Help Eva to solve $x^2 + x - 1 = 0$ to determine the ratio between the consecutive Fibonacci numbers.

b What is so special about the solutions

3

Solve each equation.

a $x^4 - 7x^2 + 12 = 0$

b $x^4 - x^2 - 30 = 0$

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4 Here is a table showing the standard form and factored form of the equations.

a Complete the table.

Standard form	Factored form	x-intercepts (m and n)
$x^2 + x - 6$	$(x - 2)(x + 3)$	2, -3
$x^2 - 2x - 3$		
	$(x - 6)(x + 6)$	
$2x^2 - 11x + 12$	$(2x - 3)(x - 4)$	
$3x^2 - 9x + 30$		

a Where do you see $m + n$ in standard form?

c Where do you see $m \cdot n$ in standard form?

d Determine two numbers whose sum is -7 and product is 12. How can the numbers you determined help you write the factored form of $x^2 + 7x + 12$?

e Suppose m and n are the solutions of the equation $x^2 - 7x + 5 = 0$. Determine the value of $m^2 + n^2$.

f Suppose m and n are the solutions of the equation $2x^2 + 15x + 16 = 0$.

Determine the value of $\frac{1}{m} + \frac{1}{n}$.

The Quadratic Formula and More

Assign problems to students who want to extend their thinking.

Problems 1–3 can be solved in any order. If time allows, consider sharing Problem 3 with all students.

Problem 1

Students will extend their understanding of solving quadratic equations by solving the chess tournament problem.

Provide students with the following hint if additional scaffolding is needed.

- **Hint:** If there are n players, each of these players will play with how many other players?

9 players. *Explanations vary*. In the tournament, each player will play against every other player. Therefore if there are n players, they will be playing with $n-1$ other players, so the total number of matches will be $\frac{n(n-1)}{2}$.

$$\frac{n(n-1)}{2} = 36$$

$$n^2 - n - 72 = 0$$

$$(n-9)(n+8) = 0$$

I discarded the negative value because the number of players cannot be negative.

Problem 2

Students will extend their understanding of solving quadratic equations by using the quadratic formula to determine the connection between the golden ratio and the Fibonacci sequence.

Provide students with the following hint if additional scaffolding is needed.

- **Hint:** You can use the quadratic formula to solve quadratic equations written in standard form. The quadratic formula states that the solutions to any quadratic equation $ax^2 + bx + c = 0$ are $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

a. **$x = \frac{-1 \pm \sqrt{5}}{2}$ (or equivalent). *Explanations vary*.**
I used the quadratic formula to solve the equation $x^2 + x - 1 = 0$ where $a = 1, b = 1$, and $c = -1$.
$x = \frac{-1 \pm \sqrt{1^2 - 4 \cdot 1 \cdot (-1)}}{2 \cdot 1} = \frac{-1 \pm \sqrt{1+4}}{2} = \frac{-1 \pm \sqrt{5}}{2}$. So the solutions are $x \approx 0.618$ and $x \approx -1.618$.

b. ***Responses vary*. The solutions to the equation $x^2 + x - 1 = 0$ are special because they involve the golden ratio! The golden ratio, often represented by the Greek letter ϕ, is a mathematical constant approximately equal to 1.618. The golden ratio appears throughout nature, art, and architecture, particularly in structures and patterns that are aesthetically pleasing. This connection makes equations like $x^2 + x - 1 = 0$ special, as they have deep ties to both mathematics and natural patterns.**

Continued next page ...

The Quadratic Formula and More (continued)

Problem 3

Students will extend their understanding of solving quadratic equations by solving 4th degree equations using substitution

Provide students with the following hint if additional scaffolding is needed.

- **Hint:** If you substitute $x^2 = n$, how can you rearrange the equations?

a. -2, 2, -$\sqrt{3}$, $\sqrt{3}$. *Explanations vary*. I substituted $x^2 = n$ and rewrote the equation as $n^2 - 7n + 12 = 0$. I factored as $(n-3)(n-4) = 0$, so $n = 3$ and $n = 4$. Then in the last step, $x^2 = 4$ and $x^2 = 3$. I have four solutions: -2, 2, -$\sqrt{3}$, and $\sqrt{3}$.

b. -$\sqrt{6}$ and $\sqrt{6}$. *Explanations vary*. I substituted $x^2 = n$ and rewrote the equation as $n^2 - n - 30 = 0$. I factored as $(n-6)(n+5) = 0$, so $n = 6$ and $n =$ -5. Then in the last step, $x^2 = 6$ and $x^2 =$ -5. I have two solutions: -$\sqrt{6}$ and $\sqrt{6}$ in real numbers.

Problem 4

Students will extend their understanding of solving quadratic equations by exploring Vieta's formula.

Provide students with the following hints if additional scaffolding is needed.

- **Hint 1:** Start with focusing on the equations with $a = 1$, what does $m + n$ equal? What about $m \cdot n$?
- **Hint 2:** Then focus on the equations with $a \neq 1$, how does the value of a effect $m + n$? What about $m \cdot n$?

a.

Standard form	Factored form	x-intercepts (m and n)
$x^2 + x - 6$	$(x-2)(x+3)$	2, -3
$x^2 - 2x - 3$	$(x-3)(x+1)$	3, -1
$x^2 - 36$	$(x-6)(x+6)$	6, -6
$2x^2 - 11x + 12$	$(2x-3)(x-4)$	1.5, 4
$3x^2 - 9x + 30$	$(3x+6)(x-5)$	-2, 5

b. $m + n = \frac{-b}{a}$ for a quadratic equation $ax^2 + bx + c = 0$.

c. $m \cdot n = \frac{c}{a}$ for a quadratic equation $ax^2 + bx + c = 0$.

d. -3 and -4. *Explanations vary*. When $a = 1$ then the opposite of the sum of the numbers is b, and the product of the numbers is c. Then $x^2 + 7x + 12 = (x+3)(x+4)$, so the solutions of the equation are -3 and -4.

e. 39. *Explanations vary*. I know that $m + n = 7$ and $m \cdot n = 5$. Then $m^2 + n^2$ can be written as $(m+n)^2 - 2mn$. So $m^2 + n^2 = 7^2 - 2 \cdot 5 = 39$.

f. -$\frac{15}{16}$. *Explanations vary*. $m + n =$ -$\frac{15}{16}$ and $m \cdot n = \frac{16}{2} = 8$. Therefore $\frac{1}{m} + \frac{1}{n} = \frac{m+n}{m \cdot n} =$ -$\frac{15}{16}$.

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