Unit 2

# Introducing Proportional Relationships

Have you ever wondered what car can drive the fastest, what recipe contains the most sugar, or how many balloons it would take to float an object? In past lessons, you've made comparisons when determining the unit rate to score the best deal, or when creating scaled copies of figures. In this unit, you'll explore proportionality even further. And you'll discover how it can be useful when making comparisons about everyday situations!

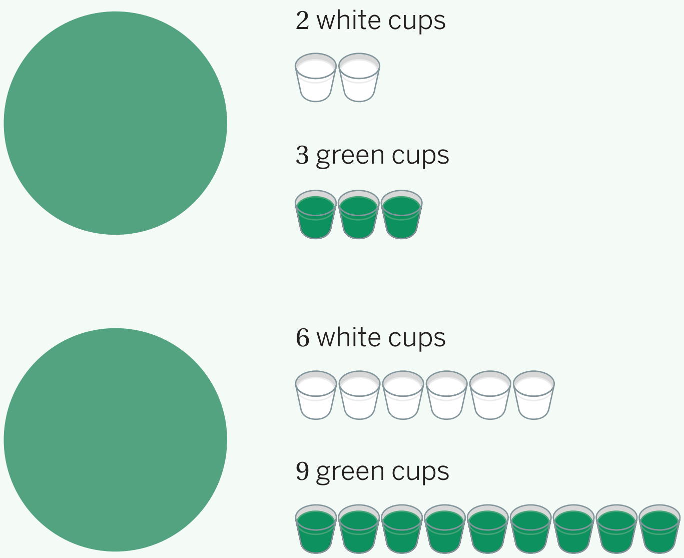
## Essential Questions

- What does it mean for two things to be proportionally related? How can you tell?
- What are the different ways you can represent proportional relationships? How are these representations related?

# Summary | Lesson 1

Ratios are said to be *equivalent ratios* when you can multiply the numbers in one ratio by the same factor to get the numbers in the other ratio.

For example, 2 cups of white paint mixed with 3 cups of green paint creates the same color as 6 cups of white paint mixed with 9 cups of green paint. You can multiply the number of white cups and green cups in the first ratio by 3 to get the second ratio.




When determining if two ratios are equivalent, it can be helpful to use a table of ratios, like this one:

| | White Paint (cups) | Green Paint (cups) | |
|---|---|---|---|
| | 2 | 3 | |
| $\times 3$ | 6 | 9 | $\times 3$ |

# Try This

Here is a recipe for pineapple soda:

For every 5 cups of soda water, mix in 2 cups of pineapple juice.

Complete the table to show three possible combinations of soda water and pineapple juice that will make pineapple soda.

| Soda Water (cups) | Pineapple Juice (cups) |
|---|---|
| | |
| | |
| | |

## Summary | Lesson 2

A **proportional relationship** is a set of *equivalent ratios*. The values for one quantity are each multiplied by the same number to get the values for the other quantity.

You can see this when moving between the columns of this table that shows the cost of varying amounts of soybeans. You can multiply the pounds of soybeans by 2 to get the cost.

When you multiply one quantity in a proportional relationship by a value, the other quantity will change by the same factor.

You can see this when moving between the rows of the table. When the pounds of soybeans is multiplied by $8$, the cost for them is multiplied by the same number.

| Soybeans (lb) | Cost ($) |
|---|---|
| $1$ | $2$ |
| $2$ | $4$ |
| $8$ | $16$ |
| $\frac{1}{2}$ | $1$ |
| $\frac{1}{4}$ | $0.50$ |

## Try This

Complete the tables so that one shows a proportional relationship and one does not.

**Proportional Relationship**

| $x$ | $y$ |
|---|---|
| $2$ | $8$ |
| $6$ | |
| | $4$ |

**Not a Proportional Relationship**

| $x$ | $y$ |
|---|---|
| $2$ | $8$ |
| $6$ | |
| | $4$ |

## Summary | Lesson 3

In a *proportional relationship*, you can multiply the values of one quantity by a **constant of proportionality** to get the values of the other quantity.

For example, 12 is a constant of proportionality in the relationship between feet and inches.

That means that you can multiply the number of feet by 12 to determine the number of inches.

| Feet | 1 | 2 | 6 |
|---|---|---|---|
| Inches | 12 | 24 | 72 |

## Try This

Here is a recipe for pineapple soda:

For every 5 cups of soda water, mix in 2 cups of pineapple juice.

What is one constant of proportionality for this situation?

| Soda Water (cups) | Pineapple Juice (cups) |
|---|---|
| 5 | 2 |
| 10 | 4 |
| 20 | 8 |

# Summary | Lesson 4

*Proportional relationships* can be represented using the equation $y = kx$, where $k$ is the *constant of proportionality*.

For example, the table shows the proportional relationship between the number of pounds of soybeans and the cost at a certain store.

- The cost of the soybeans is proportional to the weight with a constant of proportionality of 2.
- If $c$ represents the cost and $w$ represents the weight, then you can represent the proportional relationship with the equation $c = 2w$.

| Weight (lb), $w$ | Cost ($), $c$ |
|---|---|
| $\frac{1}{2}$ | 1.00 |
| 1 | 2.00 |
| 2 | 4.00 |
| $w$ | $2w$ |

# Try This

Here is a recipe for pineapple soda:

For every 5 cups of soda water, mix in 2 cups of pineapple juice.

Write an equation that represents the relationship in the recipe. Use $s$ for cups of soda water and $p$ for cups of pineapple juice.

| Soda Water (cups), $s$ | Pineapple Juice (cups), $p$ |
|---|---|
| 5 | 2 |
| 10 | 4 |
| 20 | 8 |

## Summary | Lesson 5

When a vehicle is traveling at a constant speed, there is a *proportional relationship* between the time traveled and the distance traveled. This is true for any person, animal, or object traveling at a constant speed.

For example, imagine someone running 5 meters per second. The table shows the distance they travel over different periods of time. The table also shows their speed, in meters per second.

The last row in the table shows that we can multiply the amount of time, $t$, with the *constant of proportionality*, $5$, to determine the distance traveled, $d$.

The equation $d = 5t$ represents this relationship.

| Time (sec) | Distance Traveled (m) | Speed (m per sec) |
|---|---|---|
| $1$ | $5$ | $5$ |
| $2$ | $10$ | $5$ |
| $3$ | $15$ | $5$ |
| $7$ | $35$ | $5$ |
| $t$ | $5t$ | $5$ |

## Try This

A bakery uses the equation $f = 1.5h$ to decide how many tablespoons of honey, $h$, to add to $f$ cups of flour for bread.

**a** What does the $1.5$ mean in this situation?

**b** How many tablespoons of honey should be used in a loaf of bread with $18$ cups of flour?

## Summary | Lesson 6

When two quantities $x$ and $y$ are in a *proportional relationship*, you can represent the relationship in two ways:

- The equation $y = kx$, with $k$ as the *constant of proportionality*.
- The equation $x = \frac{1}{k}y$, with $\frac{1}{k}$ as the constant of proportionality.

Each equation highlights the relationship between the two quantities.

For example, if one pound of soybeans costs $\$2.00$, then:

- The cost, $c$, is proportional to the weight, $w$. The equation $c = 2w$ represents the situation because you can multiply the weight by $2$ to get the cost.
- The weight, $w$, is proportional to the cost $c$. The equation $w = \frac{1}{2}c$ represents the situation because you can multiply the cost by $\frac{1}{2}$ to get the weight.

The two constants of proportionality for a proportional relationship are reciprocals.

**Reciprocals** are two numbers with a product of $1$. For example, $\frac{1}{2}$ is the reciprocal of $2$.

## Try This

It took Jayden $6$ minutes to fill a bathtub with $24$ gallons of water from a faucet that was flowing at a steady rate.

| Time (min), $t$ | Water (gal), $w$ |
|---|---|
| $0$ | $0$ |
| $2$ | $8$ |
| $4$ | $16$ |
| $6$ | $24$ |

**a** What are the two constants of proportionality for this situation? How are they related?

**b** Write two equations that relate $w$ and $t$ in this situation.

The structure of an equation representing the relationship between two quantities can tell us whether that relationship is proportional. An equation in the form of $y = kx$ has a *constant of proportionality*, $k$, which means it represents a proportional relationship.

Equations like $y = 3x + 1$ and $y = x^2$ do not have a *constant of proportionality*, so they do *not* represent proportional relationships.

Rewriting an equation in another form can help make a proportional relationship easier to see. For example, $y = \frac{x}{3}$ and $y = \frac{1}{3}x$ both represent the same proportional relationship.

Tables can help you determine whether an equation can be rewritten in the form of $y = kx$.

## Try This

Select *all* the equations that represent a proportional relationship.

- ☐ **A.** $K = C + 283$
- ☐ **B.** $m = \frac{1}{4}j$
- ☐ **C.** $V = s^3$
- ☐ **D.** $h = \frac{14}{2}$
- ☐ **E.** $c = 6.28r$

## Summary | Lesson 8

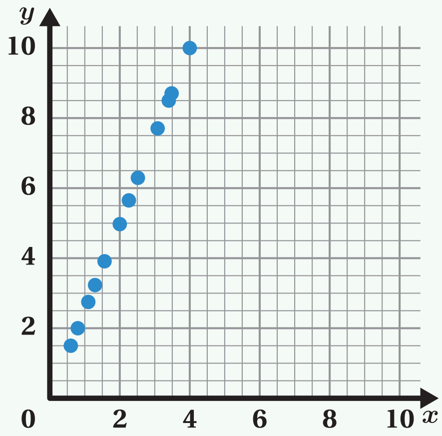
On a *coordinate plane*, if points all fall on a line that passes through $(0, 0)$, the relationship is proportional. The point $(0, 0)$ is known as the **origin**.

If it's unclear if the points form a line, you can test if the ratios of the coordinates are equivalent.

For example, the coordinates of two points on this line are $(2, 5)$ and $(4, 10)$.

$5 \div 2 = 2.5$ and $10 \div 4 = 2.5$

Since the ratio of the coordinates for both of these points is $2.5$, these points are part of a proportional relationship.

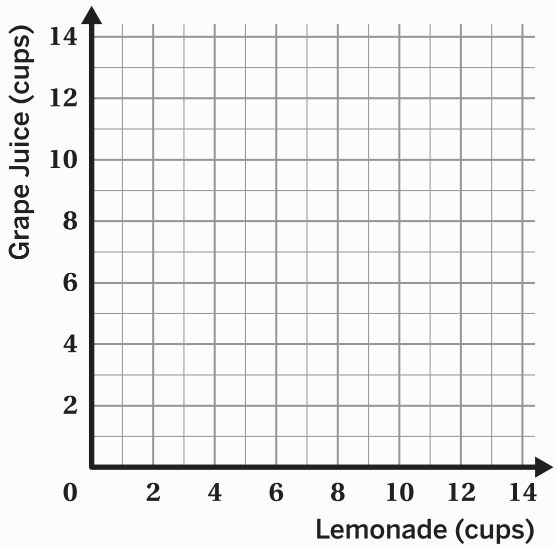



## Try This

Here is a recipe for Grape-Ade:

For every 6 cups of lemonade, mix in 3 cups of grape juice.

**a** Create a graph that represents the relationship between the amounts of lemonade and the amounts of grape juice in different-sized batches of Grape-Ade.




**b** Is this a proportional relationship? Explain your thinking.

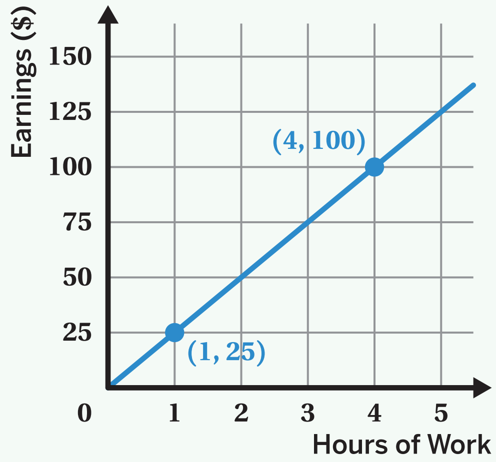
# Summary | Lesson 9

Each point on a graph of a proportional relationship tells a story using the quantities represented by $x$ and $y$. You can determine a constant of proportionality from a graph by using:

- The value of $y$ when $x$ is equal to $1$.
- The ratio of $\frac{y}{x}$ for any ordered pair.

For example, this graph shows a proportional relationship between hours worked, $x$, and money earned in dollars, $y$. One constant of proportionality is 25 because \$25 is earned for working 1 hour.

The ordered pair $(4, 100)$ shows that \$100 is earned for 4 hours of work, which is an equivalent ratio to earning \$25 per hour.

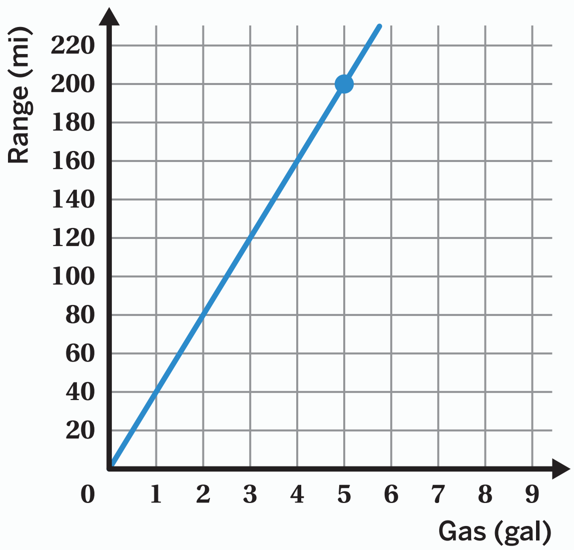



## Try This

The graph shows how far a car travels using any amount of gas.

**a** What is the constant of proportionality for the relationship between gallons of gas and miles?

**b** What does the point $(5, 200)$ mean in this situation?

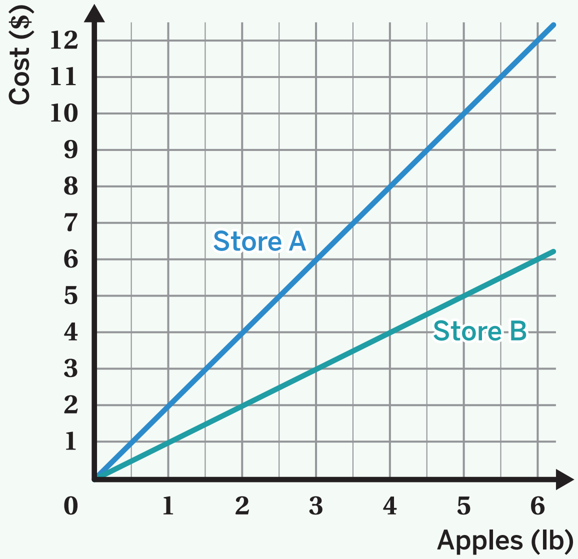


## Summary | Lesson 10

You can compare graphs of proportional relationships when they're on the same coordinate plane. The steeper the line, the greater the constant of proportionality.

For example, you can use this graph to compare the cost of apples at two different stores.

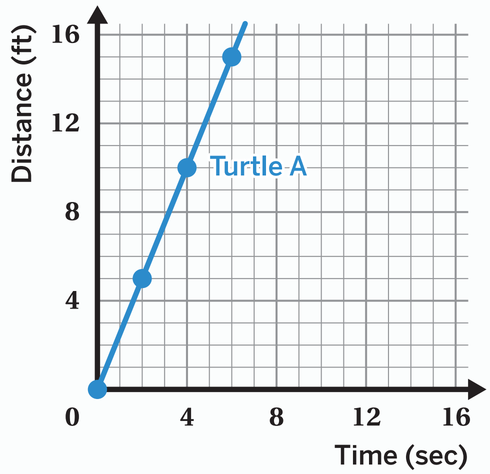
- The line representing Store A is steeper than the line representing Store B, so it has a greater constant of proportionality. This means Store A charges more per pound than Store B.
- Store A charges $2 for one pound ($k = 2$), while Store B charges $1 for one pound ($k = 1$). This is another way you can determine that the constant of proportionality at Store A is greater than that at Store B.




## Try This

Two turtles went for a walk. They each walked a distance, $d$, after $t$ seconds.

**a** The graph represents Turtle A's walk. Write an equation to represent this relationship.




**b** The equation $d = 2t$ represents Turtle B's walk. Sketch a line on the graph to represent Turtle B.

**c** Which turtle walked faster? Explain how you know.

## Summary | Lesson 11

You can determine the constant of proportionality, $k$, in different ways using different representations.

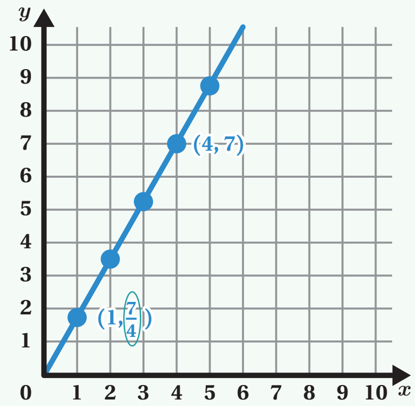
**Equation**

$y = \frac{7}{4}x$

**Table**

| $x$ | $y$ |
|---|---|
| 0 | 0 |
| 1 | $\frac{7}{4}$ |
| 4 | 7 |

**Graph**




In a table or graph, the constant of proportionality is the $y$-value that is paired with the $x$-value of 1. The constant of proportionality is also $k = \frac{y}{x}$, where $(x, y)$ is any ordered pair.

In an equation in the form of $y = kx$, the number multiplying $x$ (called the *coefficient*) is the constant of proportionality.

## Try This

A bakery uses 7 scoops of chocolate for every 2 cups of milk to make chocolate milk.

**a** Create a table, an equation, and a graph of this proportional relationship.

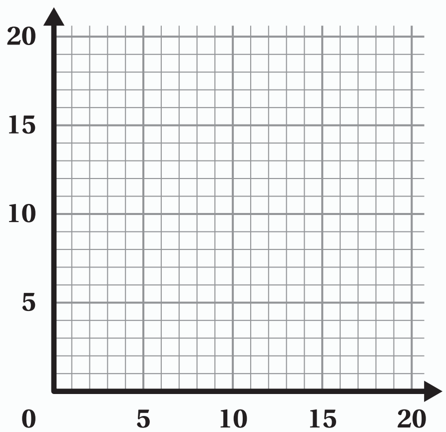
**Equation**

__________

**Table**

| | |
|---|---|
| | |
| | |
| | |
| | |

**Graph**




**b** Circle or show where you can see the constant of proportionality in each representation.

## Summary | Lesson 12

You can use proportional relationships to model real-world questions, like whether you use more water when taking a bath or a shower.

Choosing *how* to represent your model is important because it affects how the information will be interpreted by others.

By creating a model, you can turn a problem without a clear answer into information that someone can actually use. That's important work!

## Try This

Here is information about the fuel usage of two cars.

**a** Which car can go farther on 1 gallon of gas?

**b** Which car can go farther on a full tank of gas?

| Car A | Car B |
|---|---|
| 20-gallon tank | 12-gallon tank |
| 24.9 miles per gallon | 552 miles per tank |

## Summary | Lesson 13

Identifying whether a relationship is proportional is an important step when you're working on a mathematical problem.

If the relationship is proportional, there are multiple ways of representing it. The representation you choose can affect how well your audience understands the relationship and the information you want to communicate about it.

Having someone else review and critique your work can help to make it stronger. Sometimes there are perspectives that you can't see until someone shares them with you.

## Try This

Give an example of a real-world situation that can be modeled with a proportional relationship. Explain how you know the relationship is proportional.

## Lesson 1

*Responses vary.*

| Soda Water (cups) | Pineapple Juice (cups) |
|---|---|
| **10** | **4** |
| **15** | **6** |
| **20** | **8** |

## Lesson 2

Proportional Relationship

| $x$ | $y$ |
|---|---|
| 2 | 8 |
| 6 | **24** |
| **1** | 4 |

*Responses vary.*

Not a Proportional Relationship

| $x$ | $y$ |
|---|---|
| 2 | 8 |
| 6 | **16** |
| **4** | 4 |

## Lesson 3

Both $\frac{2}{5}$ and $\frac{5}{2}$ are constants of proportionality for this situation. $\frac{2}{5}$ is the number of cups of pineapple juice per cup of soda water. $\frac{5}{2}$ is the number of cups of soda water per cup of pineapple juice.

## Lesson 4

*Responses vary.* Some equations that represent this situation are $p = 0.4s$, $s = 2.5p$, and $2s = 5p$.

### Lesson 5

**a** *Responses vary*. $1.5$ means that there are $1.5$ cups of flour for every tablespoon of honey in the bread recipe.

**b** $12$ tablespoons of honey.

*Caregiver Note: One method is to substitute $18$ for $f$ in the equation $f = 1.5h$ to get the equation $18 = 1.5h$. From there, you can calculate $\frac{18}{1.5} = 12$ tablespoons of honey.*

### Lesson 6

**a** $4$ and $\frac{1}{4}$. They are reciprocals. If you multiply them, they make $1$.

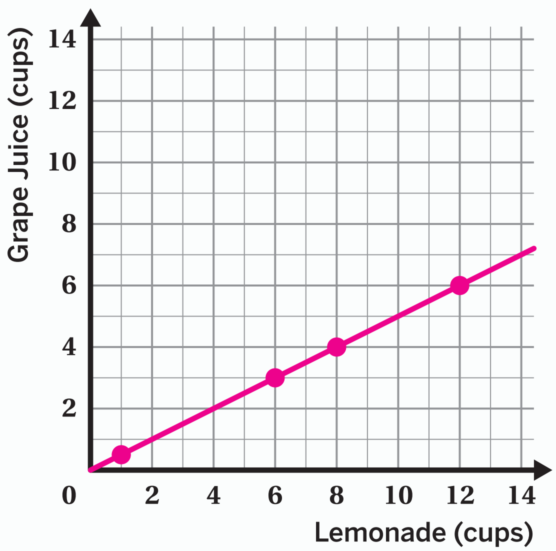
**b** $w = 4t$, $t = \frac{1}{4}w$

### Lesson 7

B. $m = \frac{1}{4}j$, E. $c = 6.28r$

### Lesson 8

**a**




**b** Yes, it's a proportional relationship. The points fall on a straight line through the origin, and there is a constant of proportionality of $2$ cups of lemonade per $1$ cup of grape juice, or $0.5$ cups of grape juice per $1$ cup of lemonade.

### Lesson 9

**a** $40$ miles per gallon. The point $(1, 40)$ represents this constant of proportionality.

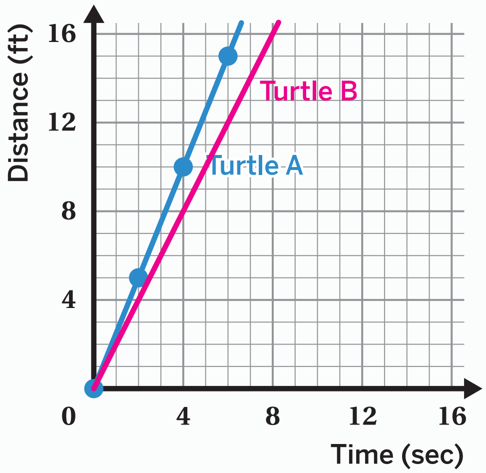
**b** *Responses vary*. The point $(5, 200)$ means that the car uses $5$ gallons of gas to travel $200$ miles.

## Lesson 10

**a** $d = 2.5t$ (or equivalent).

*Caregiver Note: Choose a point on the graph and divide the $y$-coordinate by the $x$-coordinate to find the constant of proportionality. For example, the point $(4, 10)$ is on the graph, so the constant of proportionality is $\frac{10}{4} = 2.5$.*

**b**




**c** Turtle A. The line is steeper and the constant of proportionality is 2.5 for Turtle A, which is greater than 2 for Turtle B.

## Lesson 11

**a**

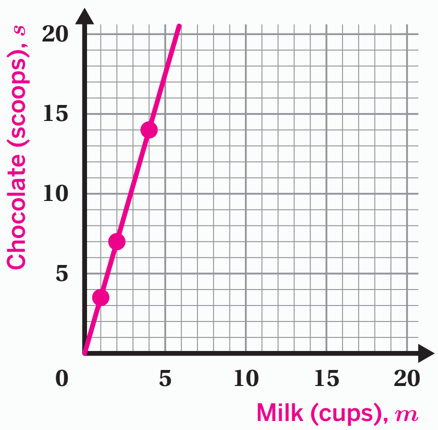
Equation

$s = 3.5m$

Table

| Milk (cups), $m$ | Chocolate (scoops), $s$ |
|---|---|
| 2 | 7 |
| 4 | 14 |
| 1 | 3.5 |

Graph




**b** *Responses vary*. In the table, the constant of proportionality is the scoops of chocolate for each cup of milk, 3.5. In the equation, it's the 3.5 that is multiplied by $m$ to calculate $s$. On the graph, the constant of proportionality is the $y$-coordinate of the point $(1, 3.5)$.

## Lesson 12

a. Car B can go farther on $1$ gallon of gas. Car A goes $24.9$ miles per gallon, while Car B goes $\frac{552}{12} = 46$ miles per gallon.

b. Car B can go farther on a full tank of gas. Car A can go $20 \cdot 24.9 = 498$ miles on a tank of gas, while Car B can go $552$ miles on a full tank of gas.

## Lesson 13

*Responses vary*. The amounts of each ingredient in a recipe are in a proportional relationship. If you double the recipe, you double all of the ingredients, and there is still the same ratio between any two of them.