Amplify Desmos Math
FLORIDA

Student Edition

A1

Volume 1

Amplify Desmos Math FLORIDA

Algebra 1

Volume 1: Units 1–4

Student Edition

About Amplify

Amplify is dedicated to collaborating with educators to create learning experiences that are rigorous and riveting for all students. Amplify creates K–12 core and supplemental curriculum, assessment, and intervention programs for today's students.

A pioneer in K–12 education since 2000, Amplify is leading the way in next-generation curriculum and assessment. All of our programs provide teachers with powerful tools that help them understand and respond to the needs of every student.

Amplify gratefully acknowledges the work of distinguished program advisors from English Learners Success Forum (ELSF), who have been integral in the development of Amplify Desmos Math. ELSF is a 501(c)(3) nonprofit organization whose mission is to expand educational equity for multilingual learners by increasing the supply of high-quality instructional materials that center their cultural and linguistic assets.

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Dear Student,

Welcome to Amplify Desmos Math Florida! We are excited to be partnering with you this year. You play an essential role in math class, so we wanted to reach out to introduce ourselves and tell you a bit about who we are.

Amplify Desmos Math Florida is a team of math educators on a mission to support you and your classmates in learning math. We hope each lesson inspires you to use your creativity, ask questions, and discover connections between math concepts and the world around us.

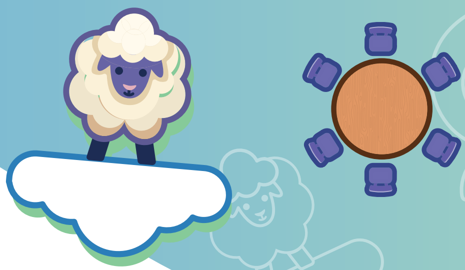
Here is what you can expect this year:

- Lessons that encourage you to ask questions, explore, settle disputes, create challenges for your classmates, and more!
- Activities that show what your ideas mean with plenty of chances for you to revise.
- Opportunities for you to engage with interesting and important ideas in mathematics.

We hope you enjoy exploring math this year as you work with friends to solve problems and learn about different concepts.

–The Amplify Desmos Math Florida Team

Unit 1 Linear Equations and Inequalities



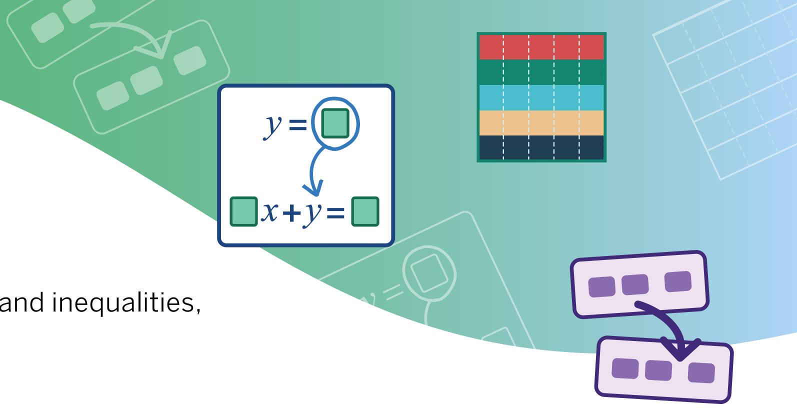
You will revisit strategies for solving one-variable equations and inequalities and extend your knowledge to make sense of multi-variable equations and two-variable inequalities.

Unit 2 Describing Data

You will analyze, describe, and compare one- and two-variable data sets.

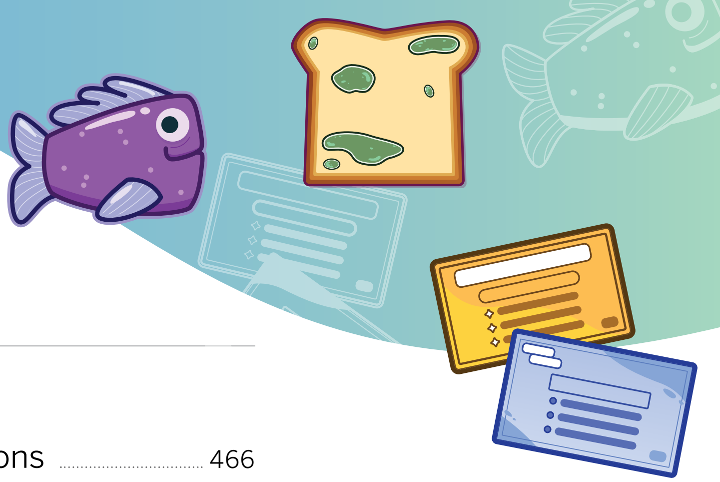
Unit 3 Describing Functions

You will model situations with functions and use function notation to describe key features of functions, compare different functions, and define functions.



Unit 4 Systems of Linear Equations and Inequalities

You will write and solve systems of linear equations and inequalities, interpreting their solutions in context.



Unit 5 Exponential Functions

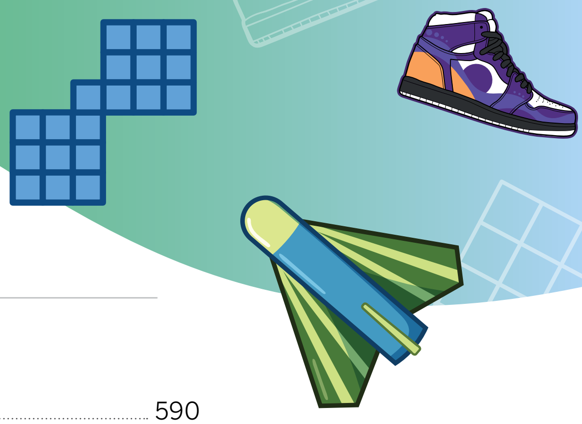
You will create and interpret exponential functions to model situations and data.

Sub-Unit 1 Comparing Linear and Exponential Functions 466

Sub-Unit 2 Exponential Growth and Decay 504

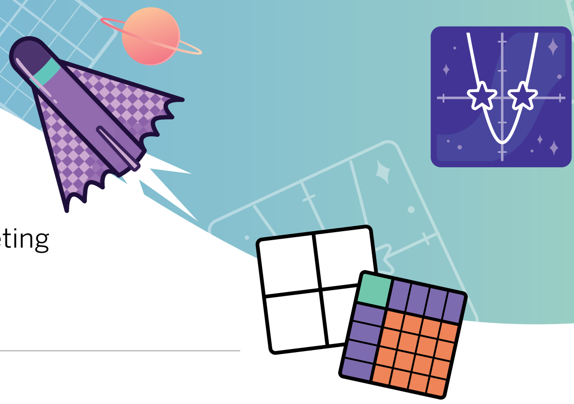
Sub-Unit 3 Modeling with Exponentials 522

Unit 6 Quadratic Functions



You will analyze graphs, tables, and equations in three forms to identify and interpret key features of quadratic functions.

Unit 7 Quadratic Equations



You will solve quadratic equations and systems of equations using reasoning, factoring, the zero-product property, completing the square, and the quadratic formula.

Florida B.E.S.T. Standards and Benchmarks for Algebra 1

Benchmark	B.E.S.T Mathematics Benchmark
Number Sense and Operations	
MA.912.NSO.1.1	Extend previous understanding of the Laws of Exponents to include rational exponents. Apply the Laws of Exponents to evaluate numerical expressions and generate equivalent numerical expressions involving rational exponents.
MA.912.NSO.1.2	Generate equivalent algebraic expressions using the properties of exponents.
MA.912.NSO.1.4	Apply previous understanding of operations with rational numbers to add, subtract, multiply and divide numerical radicals.
Algebraic Reasoning	
MA.912.AR.1.1	Identify and interpret parts of an equation or expression that represent a quantity in terms of a mathematical or real-world context, including viewing one or more of its parts as a single entity.
MA.912.AR.1.2	Rearrange equations or formulas to isolate a quantity of interest.
MA.912.AR.1.3	Add, subtract and multiply polynomial expressions with rational number coefficients.
MA.912.AR.1.4	Divide a polynomial expression by a monomial expression with rational number coefficients.
MA.912.AR.1.7	Rewrite a polynomial expression as a product of polynomials over the real number system.
MA.912.AR.2.1	Given a real-world context, write and solve one-variable multi-step linear equations.
MA.912.AR.2.2	Write a linear two-variable equation to represent the relationship between two quantities from a graph, a written description or a table of values within a mathematical or real-world context.
MA.912.AR.2.3	Write a linear two-variable equation for a line that is parallel or perpendicular to a given line and goes through a given point.
MA.912.AR.2.4	Given a table, equation or written description of a linear function, graph that function, and determine and interpret its key features.
MA.912.AR.2.5	Solve and graph mathematical and real-world problems that are modeled with linear functions. Interpret key features and determine constraints in terms of the context.
MA.912.AR.2.6	Given a mathematical or real-world context, write and solve one-variable linear inequalities, including compound inequalities. Represent solutions algebraically or graphically.
MA.912.AR.2.7	Write two-variable linear inequalities to represent relationships between quantities from a graph or a written description within a mathematical or real-world context.
MA.912.AR.2.8	Given a mathematical or real-world context, graph the solution set to a two-variable linear inequality.
MA.912.AR.3.1	Given a mathematical or real-world context, write and solve one-variable quadratic equations over the real number system.
MA.912.AR.3.4	Write a quadratic function to represent the relationship between two quantities from a graph, a written description or a table of values within a mathematical or real-world context.
MA.912.AR.3.5	Given the x-intercepts and another point on the graph of a quadratic function, write the equation for the function.
MA.912.AR.3.6	Given an expression or equation representing a quadratic function, determine the vertex and zeros and interpret them in terms of a real-world context.

Florida B.E.S.T. Standards and Benchmarks for Algebra 1

MA.912.AR.3.7	Given a table, equation or written description of a quadratic function, graph that function, and determine and interpret its key features.
MA.912.AR.3.8	Solve and graph mathematical and real-world problems that are modeled with quadratic functions. Interpret key features and determine constraints in terms of the context.
MA.912.AR.4.1	Given a mathematical or real-world context, write and solve one-variable absolute value equations.
MA.912.AR.4.3	Given a table, equation or written description of an absolute value function, graph that function and determine its key features.
MA.912.AR.5.3	Given a mathematical or real-world context, classify an exponential function as representing growth or decay.
MA.912.AR.5.4	Write an exponential function to represent a relationship between two quantities from a graph, a written description or a table of values within a mathematical or real-world context.
MA.912.AR.5.6	Given a table, equation or written description of an exponential function, graph that function and determine its key features.
MA.912.AR.9.1	Given a mathematical or real-world context, write and solve a system of two-variable linear equations algebraically or graphically.
MA.912.AR.9.4	Graph the solution set of a system of two-variable linear inequalities.
MA.912.AR.9.6	Given a real-world context, represent constraints as systems of linear equations or inequalities. Interpret solutions to problems as viable or non-viable options.
Functions	
MA.912.F.1.1	Given an equation or graph that defines a function, determine the function type. Given an input-output table, determine a function type that could represent it.
MA.912.F.1.2	Given a function represented in function notation, evaluate the function for an input in its domain. For a real-world context, interpret the output.
MA.912.F.1.3	Calculate and interpret the average rate of change of a real-world situation represented graphically, algebraically or in a table over a specified interval.
MA.912.F.1.5	Compare key features of linear functions each represented algebraically, graphically, in tables or written descriptions.
MA.912.F.1.6	Compare key features of linear and nonlinear functions each represented algebraically, graphically, in tables or written descriptions.
MA.912.F.1.8	Determine whether a linear, quadratic or exponential function best models a given real-world situation.
MA.912.F.2.1	Identify the effect on the graph or table of a given function after replacing $f(x)$ by $f(x)+k$, $kf(x)$, $f(kx)$ and $f(x+k)$ for specific values of k.
Financial Literacy	
MA.912.FL.3.2	Solve real-world problems involving simple, compound and continuously compounded interest.
MA.912.FL.3.4	Explain the relationship between simple interest and linear growth. Explain the relationship between compound interest and exponential growth and the relationship between continuously compounded interest and exponential growth.

Florida B.E.S.T. Standards and Benchmarks for Algebra 1

Data Analysis & Probability	
MA.912.DP.1.1	Given a set of data, select an appropriate method to represent the data, depending on whether it is numerical or categorical data and on whether it is univariate or bivariate.
MA.912.DP.1.2	Interpret data distributions represented in various ways. State whether the data is numerical or categorical, whether it is univariate or bivariate and interpret the different components and quantities in the display.
MA.912.DP.1.3	Explain the difference between correlation and causation in the contexts of both numerical and categorical data.
MA.912.DP.1.4	Estimate a population total, mean or percentage using data from a sample survey; develop a margin of error through the use of simulation.
MA.912.DP.2.4	Fit a linear function to bivariate numerical data that suggests a linear association and interpret the slope and y-intercept of the model. Use the model to solve real-world problems in terms of the context of the data.
MA.912.DP.2.6	Given a scatter plot with a line of fit and residuals, determine the strength and direction of the correlation. Interpret strength and direction within a real-world context.
MA.912.DP.3.1	Construct a two-way frequency table summarizing bivariate categorical data. Interpret joint and marginal frequencies and determine possible associations in terms of a real-world context.

Mathematical Thinking and Reasoning Standards

MA.K12.MTR.1.1	Actively participate in effortful learning both individually and collectively.
MA.K12.MTR.2.1	Demonstrate understanding by representing problems in multiple ways.
MA.K12.MTR.3.1	Complete tasks with mathematical fluency.
MA.K12.MTR.4.1	Engage in discussions that reflect on the mathematical thinking of self and others.
MA.K12.MTR.5.1	Use patterns and structure to help understand and connect mathematical concepts.
MA.K12.MTR.6.1	Assess the reasonableness of solutions.
MA.K12.MTR.7.1	Apply mathematics to real-world contexts.

English Language Arts B.E.S.T. Standards

ELA.K12.EE.1.1	Cite evidence to explain and justify reasoning.
ELA.K12.EE.2.1	Read and comprehend grade-level complex texts proficiently.
ELA.K12.EE.3.1	Make inferences to support comprehension.
ELA.K12.EE.4.1	Use appropriate collaborative techniques and active listening skills when engaging in discussions in a variety of situations.
ELA.K12.EE.5.1	Use the accepted rules governing a specific format to create quality work.
ELA.K12.EE.6.1	Use appropriate voice and tone when speaking or writing.

English Language Development Standards

ELD.K12.ELL.MA.1	English language learners communicate information, ideas and concepts necessary for academic success in the content area of Mathematics.

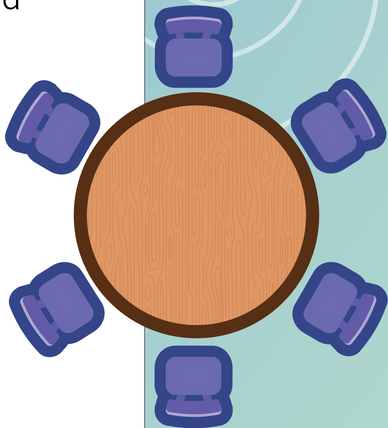
Unit 1

Linear Equations and Inequalities

In this unit, you will use what you know about solving one-variable equations and inequalities to make sense of multi-variable equations and two-variable inequalities.

Essential Questions

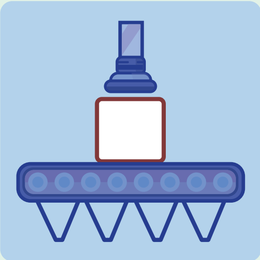
- What does it mean to be a solution to an equation or inequality?
- How are the different representations of equations or inequalities connected?
- How do you determine and graph the solution of an inequality?



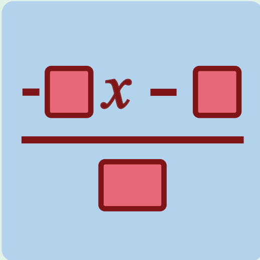
Sub-unit

1

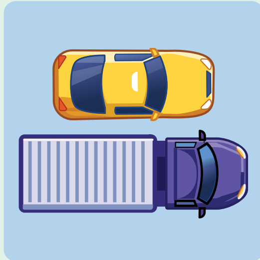
One-Variable Equations



Lesson 1
Working Backwards



Lesson 2
Solving Strategies



Lesson 3
Same Position

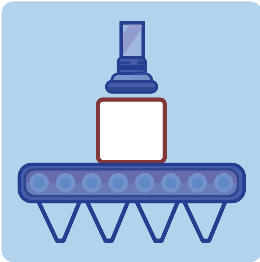
Unit 1
Lesson
1

Name: ______ Date: ______ Period: ______

MA.912.AR.2.1, MTR.1.1, MTR.2.1, MTR.5.1

Working Backwards

Let's solve equations by working backwards.

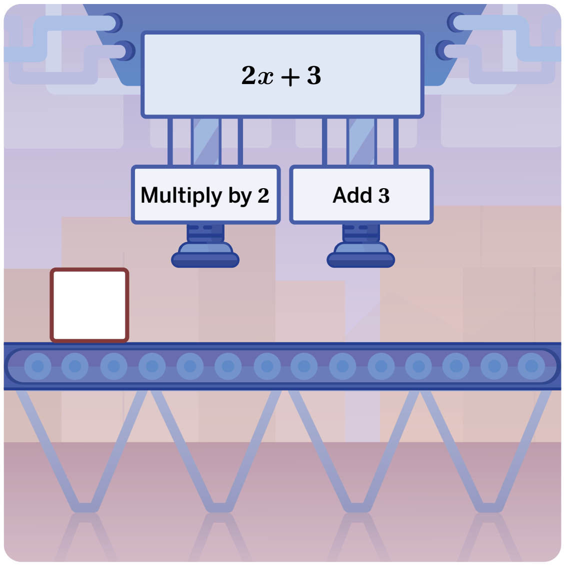


Warm-Up

1. Let's look at a number machine.

Complete the table for different values of x.

x	$2x$	$2x + 3$
5	10	13



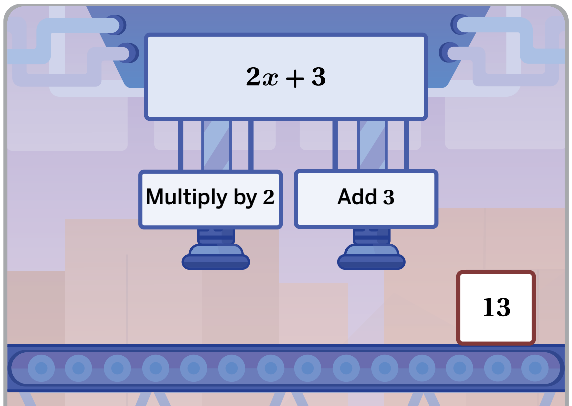
Name: .... Date: .... Period: ....

Number Machines

2. Alina put a number into this machine, and 13 came out.

What number did Alina put in?

Use the table if it helps with your thinking.



x	$2x$	$2x + 3$
		13

3. Alina's situation is represented by the equation $2x + 3 = 13$.

A **solution** to an equation is a value that makes the equation true.

Explain how you know that your answer from the previous problem is a solution to $2x + 3 = 13$.

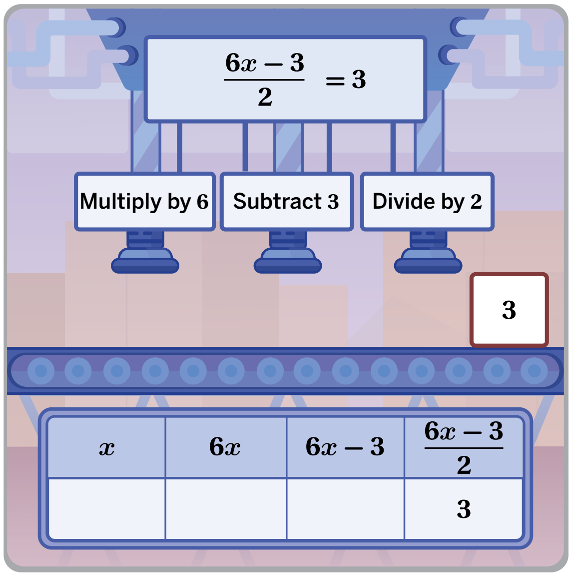
Name: ________ Date: ________ Period: ________

Number Machines (continued)

4. Here is a machine for the equation $\frac{6x-3}{2} = 3$.

Determine the solution to the equation.

Use the table if it helps with your thinking.



5. Let's look at Jin's and Nasir's strategies for determining what number has to go into the machine for 3 to come out.

Jin

÷6 +3 ·2

x	$6x$	$6x-3$	$\frac{6x-3}{2}$
1.5	9	6	3

Nasir

$$2 \cdot \frac{6x-3}{2} = 3 \cdot 2$$

$$6x - 3 = 6$$
$$+3 \quad +3$$

$$\frac{6x}{6} = \frac{9}{6}$$

$$x = 1.5$$

Discuss: How are their strategies alike and how are they different?

Activity 2

Name: .... Date: .... Period: ....

Solve It

6. Dakota says the solution to $2(3x - 9) = -6$ is $x = \frac{1}{2}$.

How can you convince Dakota that this is incorrect?

7. Solve $2(3x - 9) = -6$.

8. For each of the challenges:

- Decide with your partner who will complete Column A and who will complete Column B.
- Solve as many equations as you have time for.
- The solutions in each row should be the same. Compare your solutions, then discuss and resolve any differences.

Column A	Column B
$x + 3 = 7$	$x - 3 = 1$
$12 = 2x - 4$	$6 = 3x - 18$
$-4(x + 2) = 20$	$5(x + 15) = 40$
$\frac{2}{3}x - 2 = 5$	$2x - 6 = 15$
$-5x + 5 = -8x + 11$	$5x + 3 = 2x + 9$

Synthesis

What are some first steps you could take to determine the solution to $10 - 6 = -2(x + 4)$?

Lesson Practice A1.01

Lesson Summary

Solving an equation means taking steps to determine a **solution**. A solution to an equation is a value that makes the equation true. There are many ways to solve an equation, including working backwards, inverse operations, and moves that keep the equation balanced.

Here are two strategies for solving the equation $-4(x + 2) = 20$:

$x \xleftarrow{-2} x + 2 \xleftarrow{\div -4} -4(x + 2)$

x	$x + 2$	$-4(x + 2)$
-7	-5	20

$$\frac{-4(x + 2)}{-4} = \frac{20}{-4}$$

$$x + 2 = -5$$

$$-2 \quad -2$$

$$x = -7$$

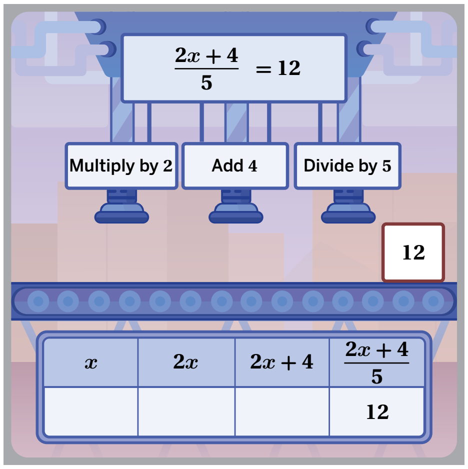
You can check that the value you determined is a solution to an equation by substituting the value back into the equation to see if it makes the equation true. The solution to the equation $-4(x + 2) = 20$ is $x = -7$ because $-4(-7 + 2) = 20$ is a true statement.

Lesson Practice
A1.01

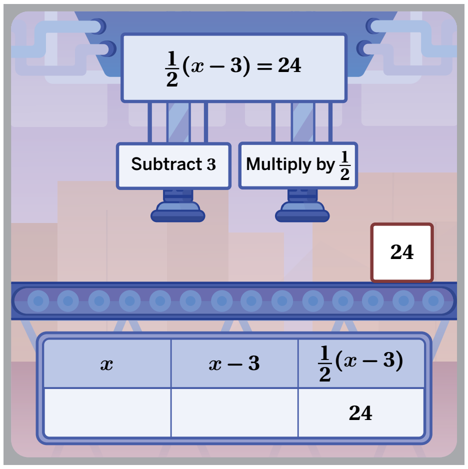
Name: .... Date: .... Period: ....

Problems 1–2: Solve each equation for x. Use the tables if they help with your thinking.

1. Here is a machine for the equation $\frac{2x+4}{5} = 12$.



2. Here is a machine for the equation $\frac{1}{2}(x-3) = 24$.



Problems 3–4: Solve each equation for x.

3. $5(3x-2) = \text{-}55$

4. $\frac{6x-2}{2} = x + 9$

Problems 5–6: Zwena made a mistake when solving $3(x-4) = 5x$ for x.

5. Show or explain her mistake.

6. What is the correct solution to this equation?

Zwena

Step 1: $3(x-4) = 5x$

Step 2: $3x - 12 = 5x$

Step 3: $8x - 12 = 0$

Step 4: $8x = 12$

Step 5: $x = 1.5$

Name: .... Date: .... Period: ....

Problems 7–10: Solve each equation for x.

7. $3x + 19 = 40$

8. $5 = x + 1 + 3x$

9. $3x + 5 = 4x + 1$

10. Determine the value of x that makes the given equation true: $2(5x + 6) = 17$.

Test Practice

11. $4x + 18 = 6x$

A. $x = 16$

B. $x = -9$

C. $x = -16$

D. $x = 9$

Spiral Review

12. Select *all* the equations where $x = 0.5$ is a solution.

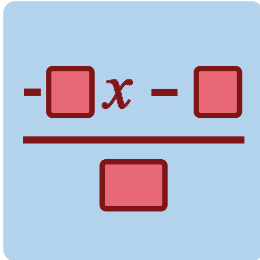
- ☐ **A.** $8 = 4x$
- ☐ **B.** $13x = 6.5$
- ☐ **C.** $3x + 8 = 9.5$
- ☐ **D.** $3 = 2x + 1$
- ☐ **E.** $5 - x = 5.5$

Name: ________ Date: ________ Period: ________

MA.912.AR.2.1, MTR.4.1, MTR.5.1

Solving Strategies

Let's explore strategies for solving equations.



Warm-Up

Equivalent equations have the exact same solution.

1. **Discuss:** Which of these equations are equivalent to each other? How do you know?

Equation A	Equation B
$-2(2x+3)=10$	$2x+3=-5$
Equation C	**Equation D**
$2x+3=12$	$-4x-6=10$

Activity 1

Name: ________ Date: ________ Period: ________

Equivalent or Not?

For each pair of equations, determine whether they are equivalent.

- If they're equivalent, explain how to get from one equation to the other.
- If they're not equivalent, explain how you know.

2. $5x = 24 + 2x$ Are the equations equivalent? ________

$3x = 24$ Explanation:

3. $-3(2x + 9) = 12$ Are the equations equivalent? ________

$-4 = 2x + 9$ Explanation:

4. $\frac{1}{2}x - 8 = 9$ Are the equations equivalent? ________

$x - 8 = 18$ Explanation:

5. **a** Write one equation that is equivalent to $12 = 5x - 30 + x$ and one equation that is not.

________ ________

b Trade papers with a classmate. Then circle the equation your classmate wrote that is equivalent to $12 = 5x - 30 + x$. Show or explain your thinking on your classmate's worksheet.

Activity
2

Name: ______ Date: ______ Period: ______

Step It Up

Problems 6–7: Here are Sadia's and Amir's steps for correctly solving the same equation.

Sadia

$$6 - 7x = \frac{-15x - 12}{3}$$
$$6 - 7x = -5x - 4$$
$$10 - 7x = -5x$$
$$10 = 2x$$
$$5 = x$$

Amir

$$6 - 7x = \frac{-15x - 12}{3}$$
$$18 - 21x = -15x - 12$$
$$18 - 6x = -12$$
$$-6x = -30$$
$$x = 5$$

6. **Discuss:** How did each student solve the equation?

7. How did Amir and Sadia take different first steps but have the same solution?

8. Caleb and Roberto also tried to solve the equation but made some errors. For each student's work:

a. **Discuss:** What is correct? What is incorrect?

b. Write a question to help the students see how they could revise their work.

Caleb

$$6 - 7x = \frac{-15x - 12}{3}$$
$$-1x = -5x - 4$$
$$4x = -4$$
$$x = -1$$

Roberto

$$6 - 7x = \frac{-15x - 12}{3}$$
$$6 - 7x = -5x - 4$$
$$2 - 7x = -5x$$
$$2 = 2x$$
$$1 = x$$

Activity 3

Name: ______ Date: ______ Period: ______

The Choice Is Yours

Equation A

$2(2a + 1.5) = 17 - 3a$

Equation B

$-\frac{1}{2}(b + 3) - 5 = -\frac{7}{2}$

Equation C

$\frac{-6 + 4c}{2} = 3(2c + 1)$

Equation D

$4d + 1 = -2(d - 5)$

Equation E

$\frac{x}{4} - 2 = 2x + 5$

Equation F

$9f + 3 - (f - 1) = 2(3f + 1)$

Equation G

$g - 4 = -\frac{8 + 4g}{8}$

Equation H

$h + 5h + 20 = h - 6 + h$

9. Examine these equations. Organize the equations into two or three groups based on patterns you notice.

Group A	Group B	Group C

10. **Discuss:** How did you group the equations?

11. Choose three equations to solve. (Choose at least one equation from each group.) Show your thinking.

You're invited to explore more.

12. Two of the equations in this activity are equivalent. Identify the two equivalent equations and explain your thinking.

Synthesis

13. **a** Write an equation you think is challenging to solve.

b What makes your equation challenging to solve?

c What are some strategies or tips for solving equations like this?

Lesson Practice A1.02

Lesson Summary

You can solve one-variable equations by creating **equivalent equations**. To create equivalent equations, use solving moves that keep the equation balanced, such as combining like terms or using inverse operations to move a variable from one side of the equation to the other.

Here is an example of a set of solving moves that keep an equation balanced:

$-3m + 5 + m = 2(6m + 3)$	This is the original equation.
$-2m + 5 = 12m + 6$	We combined like terms on the left and distributed on the right.
$5 = 14m + 6$	We added $2m$ to each side of the equation.
$-1 = 14m$	We subtracted 6 from each side of the equation.
$-\frac{1}{14} = m$	We divided each side of the equation by 14.

All of the equations created at each step of this solution process are equivalent equations because they have the same solution: $m = -\frac{1}{14}$.

Lesson Practice
A1.02

Name: ______________________ Date: __________ Period: __________

1. Which equation is equivalent to $6x + 9 = 12$?

A. $x + 9 = 6$

B. $2x + 3 = 4$

C. $3x + 9 = 6$

D. $6x + 12 = 9$

2. Write another equation that is equivalent to $6x + 9 = 12$.

3. Select *all* the equations that are equivalent to $\frac{-8x - 6}{2} = 15$.

☐ **A.** $4x + 3 = 15$

☐ **B.** $\frac{1}{2}(-4x - 3) = 15$

☐ **C.** $-4x - 3 = 15$

☐ **D.** $-8x - 6 = 30$

☐ **E.** $8x + 6 = 30$

Problems 4–6: Solve each equation.

4. $26 - 2x = 3(x + 2)$

5. $\frac{4x - 6}{2} = x - 8$

6. $\frac{1}{4}x - 5 = x - 14$

Problems 7–8: Polina made a mistake when solving $-3(x + 7) = 24$ for x.

Polina

Step 1: $-3(x + 7) = 24$

Step 2: $x + 7 = 27$

Step 3: $x = 20$

7. What is one thing that Polina did well?

8. What is one thing that she did incorrectly?

Name: .... Date: .... Period: ....

9. Which equation is equivalent to $0.05n + 0.1d = 3.65$?

A. $5n + d = 365$

B. $0.5n + d = 365$

C. $5n + 10d = 365$

D. $0.05d + 0.1n = 365$

10. Determine a value for x that makes this equation true: $9x - 4(x - 3) = 27 + 2x$. Show or explain your thinking.

Test Practice

11. Select *all* the moves that could be the first step to solving the equation $4(x + 3) = 8x - 4 + 12x$.

- ☐ **A.** Divide each side by 4.
- ☐ **B.** Take away $12x$ on the right side.
- ☐ **C.** Distribute 4 on the left side.
- ☐ **D.** Combine like terms on the right side.
- ☐ **E.** Add 4 to the right side.

Spiral Review

12. Renata scored 409 points in a video game. This was 223 more points than Sadia scored, s. Which equation represents the number of points that Sadia's scored??

A. $223 = 409 + s$ B. $s = 409 - 223$ C. $s = 409 + 223$ D. $223 - s = 409$

Name: .... Date: .... Period: ....

MA.912.AR.2.1, MTR.1.1, MTR.4.1, MTR.5.1

Same Position

Let's explore how many solutions are possible for a one-variable equation.

Warm-Up

1. Write a story about how the red truck and yellow taxi would catch up to the blue car, if they are all moving at constant speeds.

Activity 1

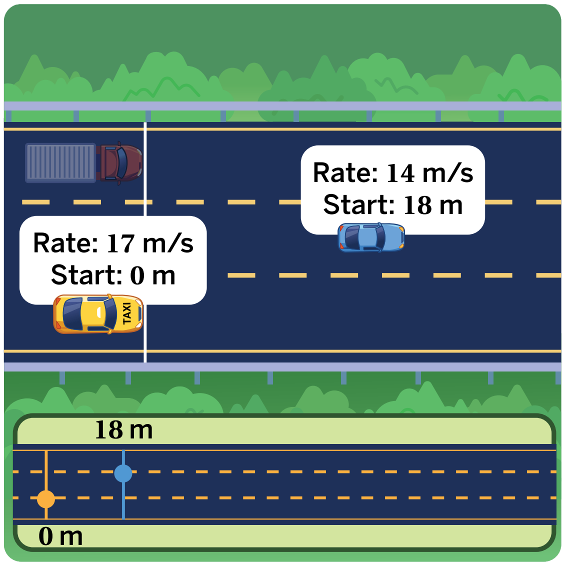
Name: ________ Date: ________ Period: ________

Same Position

2. The car and the taxi are moving at constant speeds.

Car Position Expression	Taxi Position Expression
$14t + 18$	$17t$

At what time, t, will the car and the taxi be in the same position?

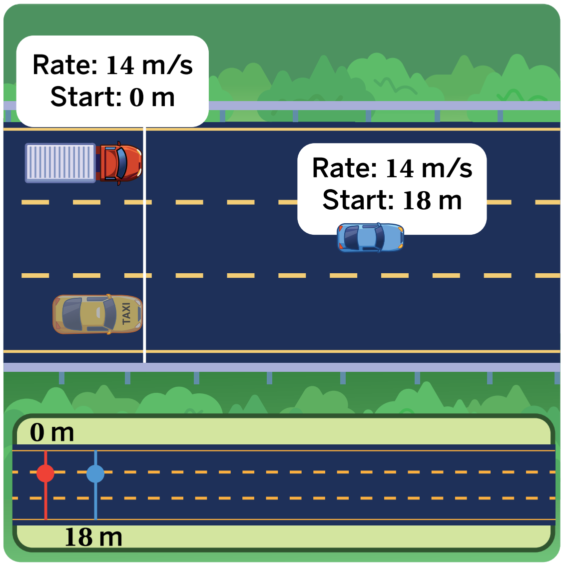


3. The truck and the car are moving at constant speeds.

Truck Position Expression	Car Position Expression
$14t$	$14t + 18$

a. Let's explore what happens at different times, t.

b. **Discuss:** What do you know about when the truck and the car will be in the same position?



4. Here is Antwon's work on the previous problem.

What does his work say about the time, t, when the truck and the car will be in the same position?

Antwon

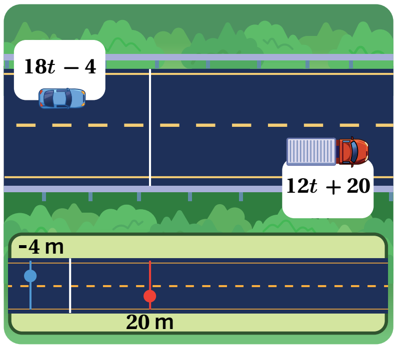
$$14t = 14t + 18$$
$$-14t \quad -14t$$
$$0 = 0 + 18$$

Name: .... Date: .... Period: ....

Same Position (continued)

5. The following equations represent when these vehicles will be in the same position.

a $18t - 4 = 12t + 20$

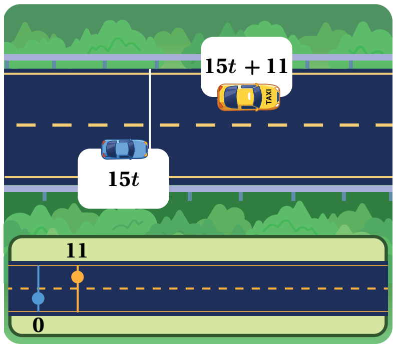


How often will they be in the same position? Circle one.

Once Never Always

If once, then after how many seconds? ....

b $15t + 11 = 15t$

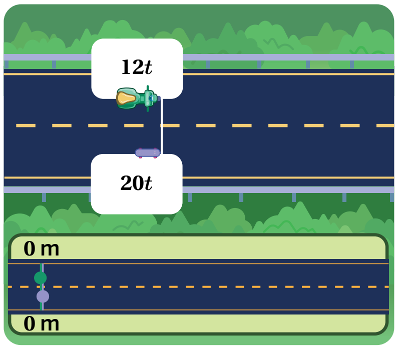


How often will they be in the same position? Circle one.

Once Never Always

If once, then after how many seconds? ....

c $12t = 20t$

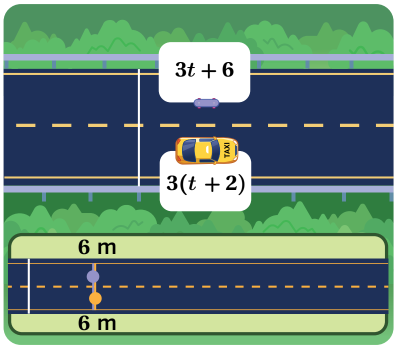


How often will they be in the same position? Circle one.

Once Never Always

If once, then after how many seconds? ....

d $3t + 6 = 3(t + 2)$



How often will they be in the same position? Circle one.

Once Never Always

If once, then after how many seconds? ....

Activity 2

Name: .... Date: .... Period: ....

Once, Never, Always

6. Here are Ella's and Nikhil's strategies for solving a challenge from the previous activity.

Ella	Nikhil
$12t = 20t$	$\frac{12t}{t} = \frac{20t}{t}$
$-12t \quad -12t$	$12 = 20$
$0 = 8t$	
$0 = t$	
they will meet when $0 = t$	they will never meet

Discuss: Is each strategy correct?

7. Each equation represents the time, t, when two vehicles will meet.

$12 - t = t - 12$	$t + 1 = t + 1$	$t = t + 2$
$2t + 6 = 2(t + 3)$	$2t = 8t$	$8(t + 1) = 8t - 8$

Sort the six equations based on how often the vehicles will be in the same position.

Once	Never	Always

Name: .... Date: .... Period: ....

Once, Never, Always (continued)

8. Darryl and Jasmine solved $t + 1 = t + 1$ and got $0 = 0$.

- Darryl says the vehicles will never be in the same position.
- Jasmine says the vehicles will always be in the same position.

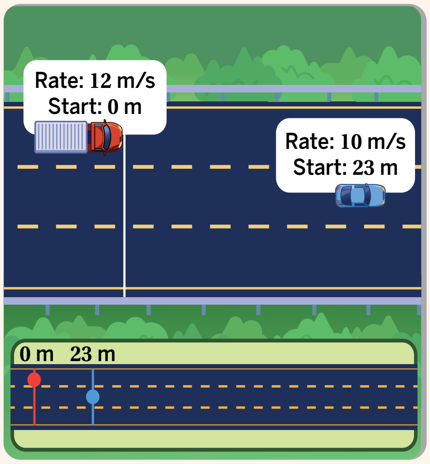
Who is correct? Explain your thinking.

You're invited to explore more.

9. The truck and the car are moving at constant speeds.

Write an expression, in terms of t, for the position of a taxi that makes both of these statements true:

- The taxi will never be in the same position as the truck.
- The taxi will be in the same position as the car when $t = 8$.



Vehicle	Position Expression
Truck	$12t$
Car	$10t + 23$
Taxi	

Synthesis

How can you tell whether an equation will have:

- No solution?
- Infinitely many solutions?

$t + 1 = t + 1$

$8(t + 1) = 8t - 8$

$2t = 8t$

$t = t + 2$

$12 - t = t - 12$

$2t + 6 = 2(t + 3)$

Lesson Practice A1.03

Lesson Summary

Not all one-variable linear equations have a single solution. Some linear equations have **infinitely many solutions**, and some have **no solution**.

In the process of solving, you will be able to see a difference between equations with one solution, no solution, or infinitely many solutions:

- In an equation with one solution, *a single value of* x will make the equation true.
- In an equation with no solution, *no value of* x will make the equation true.
- In an equation with infinitely many solutions, *any value of* x will make the equation true.

Here are examples of equations with one solution, no solution, and infinitely many solutions.

One Solution	No Solution	Infinitely Many Solutions
$3x + 4 = 2x + 10$	$2x + 4 = 2x + 10$	$2(x + 5) = 2x + 10$
$3x = 2x + 6$	$4 = 10$	$2x + 10 = 2x + 10$
$x = 6$	This is *never* true!	$10 = 10$
		This is *always* true!

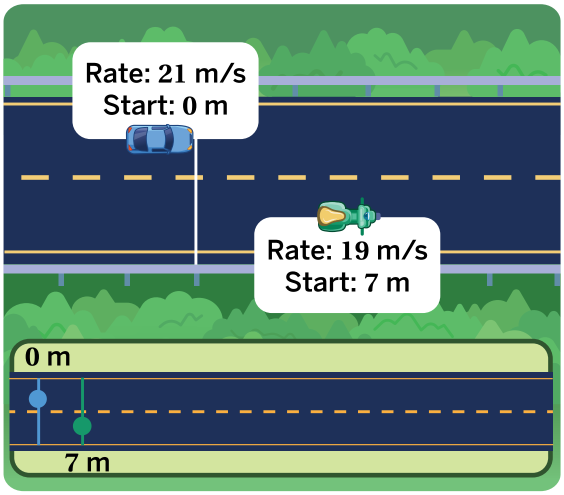
If the variable in an equation is eliminated during the solving process, that tells you that the equation has either no solution or infinitely many solutions. If the statement remaining is false, the equation has no solution. If the statement remaining is true, the equation has infinitely many solutions.

Lesson Practice
A1.03

Name: .... Date: .... Period: ....

1. The car and scooter are moving at constant speeds. The equation $21t = 19t + 7$ represents the time, t, when they will be in the same position.

When will the car and scooter be in the same position?



Problems 2–3: The equation $10t = 2.5t$ represents the time, t, when two vehicles will be in the same position.

2. When will these two vehicles be in the same position? Circle one.

Once Never Always

3. Explain how you know.

4. Here is Kiandra's work to solve $16x = 10x$. She says there is no solution.

Is this correct? Show or explain your thinking.

Kiandra

$$\frac{16x}{x} = \frac{10x}{x}$$

$$16 = 10$$

There is no solution.

Problems 5 and 6: Create two different equations that each have a solution of $x = 1$.

5. Fill in each blank using the digits 0 to 9 only once each.

$$\square x + \square = \square x + \square$$

$$\square x + \square = \square x + \square$$

6. Explain what you notice about your equations.

Lesson Practice
A1.03

Name: .... Date: .... Period: ....

Test Practice

7. Group the equations based on their number of solutions.

A. $5t = 3t$ **B.** $2t = 10 - 2t$ **C.** $15 - 3(t + 5) = -3t$

D. $4t + 7 = 4(t + 2)$ **E.** $6t + 2 = -3 + 6t$

One Solution	No Solution	Infinitely Many Solutions

Spiral Review

8. Select *all* the equations where $x = 2$ is a solution.

- ☐ **A.** $\frac{x}{4} = 8$
- ☐ **B.** $19 = 2(x + 6) + 3$
- ☐ **C.** $2x + 10 = 2x + 8$
- ☐ **D.** $5 - 3x = -1$
- ☐ **E.** $4 - x = x$

9. Select all the expressions that are equivalent to $2(x + 3)$.

- ☐ **A.** $(x + 3) \bullet 2$
- ☐ **B.** $2x + 5$
- ☐ **C.** $2x + 3 \bullet 2$
- ☐ **D.** $2x + 3$
- ☐ **E.** $2x + 6$

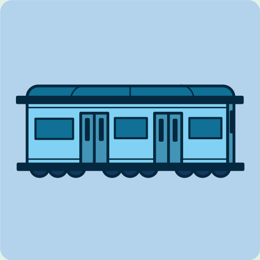
10. Select the expression that is equivalent to $6 - 2(x + 1)$.

- **A.** $4(x + 1)$
- **B.** $7 - 2x$
- **C.** $4 - 2x$
- **D.** $4x + 4$

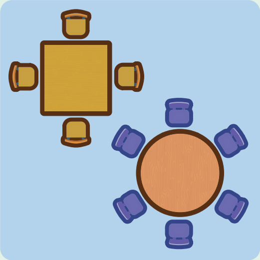
Sub-unit

2

Multi-Variable Equations



Lesson 4
Subway Seats



Lesson 5
Various Variables

Lesson 6
Shelley the Snail

Lesson 7
Five Representations

Lesson 8
Humpback Whale Growth Rates

Lesson 9
House Design

Name: ______ Date: ______ Period: ______

MA.912.AR.2.1, MA.912.AR.2.2, MTR.2.1, MTR.3.1, MTR.4.1

Subway Seats

Let's explore what different forms of linear equations reveal about a situation.

Warm-Up

1. Which one doesn't belong? Explain your thinking.

Equation A	Equation B	Equation C	Equation D
$x+y=5$	$x+y-5=0$	$x=5-y$	$5+x=y$

Activity
1

Name: .... Date: .... Period: ....

Crowded Subways

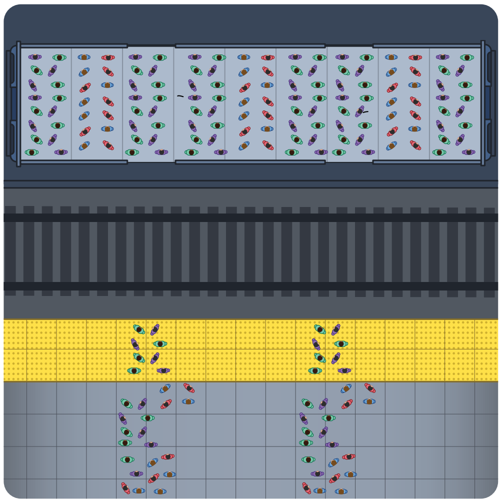
2. People in many large cities travel by subway.

To accommodate the growing number of people using the subway, the company has decided to redesign the cars, removing seats to allow for standing which will allow for more passengers. Why might people not like this new design?

3. Some subways removed all of the seats to allow more room for people to stand.

- A subway car has about 600 square feet.
- A standing passenger requires 2 square feet.

What is the *standing capacity* on this subway car with no seats?



4.
- A subway car has about 600 square feet.
- A seat requires 6 square feet.

What is the *seating capacity* on this subway car with no room to stand?

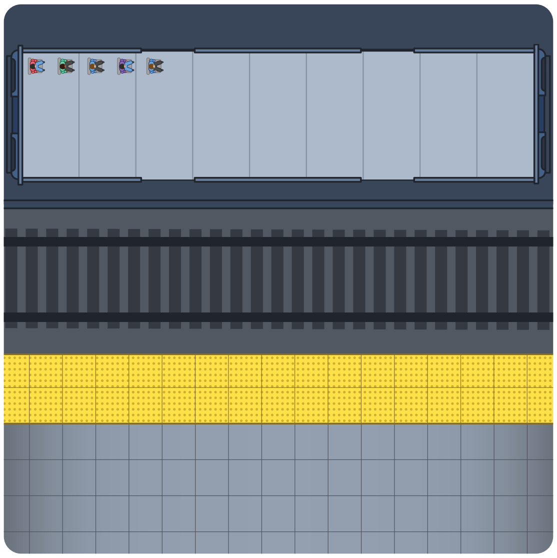
Name: .... Date: .... Period: ....

Crowded Subways (continued)

5. The capacity of this subway is modeled by $6t + 2d = 600$, where t is the seating capacity and d is the standing capacity.

For each number of seats, determine how many standing passengers can fit.

Seating Capacity, t	Standing Capacity, d
5	
10	
15	



6. Here is Tiam's strategy for determining the number of standing passengers that can fit when you know the number of seats.

- t is the seating capacity.
- d is the standing capacity.

What do 300 and -3 mean in this situation?

Tiam

$$6t + 2d = 600$$
$$-6t \qquad -6t$$
$$\frac{2d}{2} = \frac{600 - 6t}{2}$$
$$d = 300 - 3t$$

300:

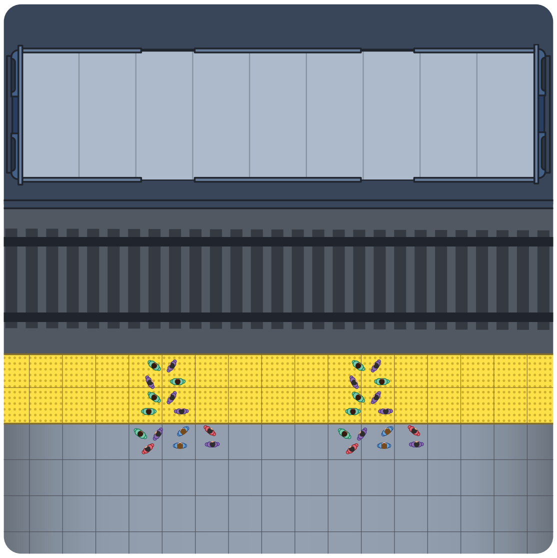
-3:

Activity 2

Name: ____________ Date: ________ Period: ________

Standing and Sitting

7. The capacity of this subway is modeled by $6t + 2d = 600$, where t is the seating capacity and d is the standing capacity.



For each number of standing passengers, determine the seating capacity.

Seating Capacity, t	Standing Capacity, d
	30
	150
	240

8. Solve for t so that the transit authority can calculate the seating capacity for any standing capacity.

9. Group the equations that represent the same relationship. One equation will have no match.

$x = \frac{50 - 3y}{2}$ $\quad x = \frac{50 - 2y}{3}$ $\quad y = \frac{50 - 3x}{2}$ $\quad y = \frac{50 - 2x}{2}$

$2x + 3y = 50$	$3x + 2y = 50$

Synthesis

Here are two equations that we considered in this lesson about subway capacity.

$$6t + 2d = 600$$
$$t = 100 - \frac{1}{3}d$$

- t is the seating capacity.
- d is the standing capacity.

Pick two numbers and explain what they mean in this situation.

6:

2:

600:

100:

$-\frac{1}{3}$:

Lesson Practice A1.04

Lesson Summary

Two-variable linear equations can be represented in different forms. Sometimes the different forms of an equation can reveal information that is useful for solving problems. Depending on what information you are looking for, you might choose to use one form or the other.

Here is an example of two equivalent equations. They are each represented in different forms, and they each reveal different information about seating and standing capacity. In each equation, t is the seating capacity and d is the standing capacity.

$$4t + 2d = 300$$

- Each seated passenger requires 4 square feet.
- Each standing passenger requires 2 square feet.
- The total area of the subway car is 300 square feet.

$$d = 150 - 2t$$

- When there are no seats ($t = 0$), 150 passengers can stand in the car.
- For every seat that is added, the standing capacity decreases by 2 square feet.

Name: .... Date: .... Period: ....

Problems 1–4: Adriana spent $24 on fruit punch and apple juice. Fruit punch costs $3 per bottle. Apple juice costs $2 per bottle.

1. How many bottles of *fruit punch* could Adriana buy if she didn't get any apple juice?

2. How many bottles of *apple juice* could Adriana buy if she did not get any fruit punch?

Adriana wrote the equation $3f + 2a = 24$ to represent the situation.

3. Use the equation to help you complete the table.

f	a
2	
6	

4. Which equation represents the same relationship?

A. $a = 8 - \frac{2}{3}f$ B. $a = 8 - \frac{3}{2}f$

C. $a = 12 - \frac{2}{3}f$ D. $a = 12 - \frac{3}{2}f$

Problems 5–6: Here is an equation: $2x + 4y = 80$.

5. Use the equation to help you complete the table.

x	y
6	
12	

6. Which equation represents the same relationship?

A. $y = 20 - 2x$ B. $y = 40 - 2x$

C. $y = 20 - \frac{1}{2}x$ D. $y = 40 - \frac{1}{2}x$

Test Practice

Problems 7–8: Nia is buying bananas and apples for her family. Bananas cost $0.50 each. Apples cost $1.00 each.

7. The equation $0.5b + 1a = 3.50$ represents the number of bananas and apples that Nia can buy for $3.50. Solve this equation for a.

8. Select *all* the combinations of bananas and apples that Nia could buy for exactly $3.50.

☐ A. 1 banana and 3 apples ☐ B. 5 bananas and 1 apple

☐ C. 1 banana and 2 apples ☐ D. 3 bananas and 2 apples

☐ E. 5 bananas and 2 apples

Name: .... Date: .... Period: ....

Spiral Review

9. Select *all* the expressions that are equivalent to $8 - 12 - (6 + 4)$.

- ☐ **A.** $(6 + 4) - 8 - 12$
- ☐ **B.** $8 - 6 - 12 + 4$
- ☐ **C.** $8 - 12 - 6 - 4$
- ☐ **D.** $8 - (6 + 4) - 12$
- ☐ **E.** $(8 - 12) - 6 + 4$

10. Which equation is equivalent to $\frac{1}{3}m + \frac{1}{2}n = 9$?

- **A.** $3m + 2n = 9$
- **B.** $2m + 3n = 54$
- **C.** $m + 3n = 27$
- **D.** $2m + n = 18$

11. Explain how you know that equation A and equation B are equivalent.

Equation A	**Equation B**
$48 - 5x = 13$	$5x = 35$

12. Create two equivalent equations by filling in the blanks using the digits 0 to 9 only once.

$$\square x + \square y = \square$$

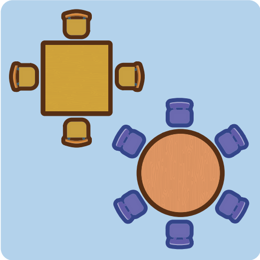
$$y = \square - \square x$$

Name: ______ Date: ______ Period: ______

MA.912.AR.1.2, MA.912.AR.2.2, MTR.3.1, MTR.4.1

Various Variables

Let's rearrange equations with multiple variables.



Warm-Up

1. Which equations are equivalent? How do you know?

Equation A	Equation B	Equation C	Equation D
$g = 3 - f$	$3 - g = f$	$3 = f - g$	$f + g = 3$

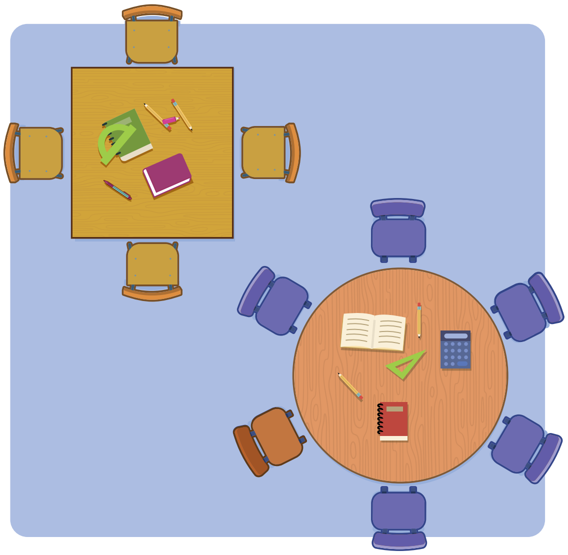
Name: .... Date: .... Period: ....

Sol's Strategies

Sol is planning a party.

He can use round tables and square tables.

- Round tables seat 6 people.
- Square tables seat 4 people.



2. Sol wrote $6r + 4s = 240$ to represent this situation. Explain what each number and variable in Sol's equation means.

3. How many square tables does Sol need if he has 16 round tables? Show or explain your thinking.

4. Sol solved the equation for s and got a new equation. How can he use this new equation?

Sol

$$\begin{array}{rcl} 6r + 4s & = & 240 \\ -6r & & -6r \\ \hline \frac{4s}{4} & = & \frac{240 - 6r}{4} \\ s & = & 60 - 1.5r \end{array}$$

Name: .... Date: .... Period: ....

Sol's Strategies (continued)

5. Use Sol's new equation, $s = 60 - 1.5r$, to decide how many square tables he needs if he uses 10 round tables. Show or explain your thinking.

6. How many round tables does Sol need if he will use 18 square tables? Show or explain your thinking.

7. Solve Sol's original equation $6r + 4s = 240$ for r. Show or explain your thinking.

Name: Date: Period:

Equations and Formulas

Solve each equation on the left. Show or explain your thinking. Then use the same strategy to solve each equation on the right.

8. Solve for t. $12 = 3t$ $D = rt$

9. Solve for b. $20 = b \cdot 5$ $A = bh$

10. Solve for x. $2x + 5 = 12$ $mx + b = y$

11. **Discuss:** How is solving the equations on the left like solving the equations on the right?

Name: ________ Date: ________ Period: ________

Equations and Formulas (continued)

Solve each equation on the left. Show or explain your thinking. Then use the same strategy to solve each equation on the right.

12. Solve for x. $-5x - 2 = 13$ $-kx - p = w$

13. Solve for a. $\frac{a}{5} - 18 = 2$ $\frac{a}{g} - r = c$

14. Solve for h. $289 = \pi(2)2 \cdot h$ $V = \pi r^2 \cdot h$

Synthesis

15. **a** How is solving an equation with many variables like solving an equation with one variable?

$2x + 5 = 12$

$mx + b = y$

b How is it different?

Lesson Practice A1.05

Lesson Summary

You can solve equations that contain multiple variables using some of the same strategies for solving equations with one variable. These strategies include working backwards, using inverse operations, and keeping the equation balanced.

When solving problems that model real-world situations, it can be helpful to rearrange an equation to highlight a variable of interest. Rearranging equations can also reveal different relationships between the variables and the quantities that they represent.

Here is how you could rearrange the equation $y = mx + b$ to solve for m or b.

Solving for m

$$y = mx + b$$
$$y - b = mx + b - b$$
$$\frac{y-b}{x} = \frac{mx}{x}$$
$$\frac{y-b}{x} = m$$

Solving for b

$$y = mx + b$$
$$y - mx = mx + b - mx$$
$$y - mx = b$$

Rearranging equations for a specific variable can help make some calculations easier. For example, if you know the values of r and s for $6r + 4s = 240$, solving the equation for s can make it simpler to test different values of r and s to see how they affect the value of s.

Lesson Practice
A1.05

Name: .... Date: .... Period: ....

Problems 1–2: Solve each equation for y.

1. $6(2.5) - 4y = 11$

2. $6x - 4y = 11$

Problems 3–4: Here is an equation: $2x - 4y - 31 = 123$.

3. Solve for x.

4. Solve for y.

5. Abdel has $12 to spend on beans and rice.

- Beans, b, cost $\$5$ per pound.
- Rice, r, costs $\$2$ per pound.

Abdel wrote $5b + 2r = 12$ to represent this relationship. Use the equation to complete the table.

Beans (lbs), b	Rice (lbs), r
2	
0.6	
b	

6. Solve for w.

$2\ell + 2w = P$

7. Solve for r.

$C = 2\pi r$

8. Here is an equation: $x + 3y = 6$.

Use the whole numbers 0 to 6 without repeating to create two pairs of x- and y-values that are solutions and one pair that is not.

	x	y
Solution		
Solution		
Not a Solution		

Name: ________ Date: ________ Period: ________

Test Practice

9. Solve for b.

$\frac{bh}{2} = A$

A. $b = \frac{2}{Ah}$

B. $b = \frac{2h}{A}$

C. $b = \frac{2A}{h}$

D. $b = \frac{Ah}{2}$

Spiral Review

10. Deven ran 27 miles last week, which was 3 times as far as Hailey ran. Select *all* the equations that represent the number of miles Hailey ran, h.

- ☐ A. $h = \frac{1}{3} \cdot 27$
- ☐ B. $\frac{1}{3} \cdot h = 27$
- ☐ C. $3 \cdot h = 27$
- ☐ D. $h = 3 \cdot 27$
- ☐ E. $h = 27 \div 3$

11. Which equation has the same solution as $2(x - 5) - 6 = 0$?

A. $2(3x + 8) = 0$

B. $\frac{1}{2}(x + 1) + 1 = 0$

C. $2x + 8 = 0$

D. $3(x - 4) - 12 = 0$

Name: .... Date: .... Period: ....

MA.912.AR.1.2, MA.912.AR.2.2, MA.912.AR.2.4, MTR.1.1, MTR.2.1

Shelley the Snail

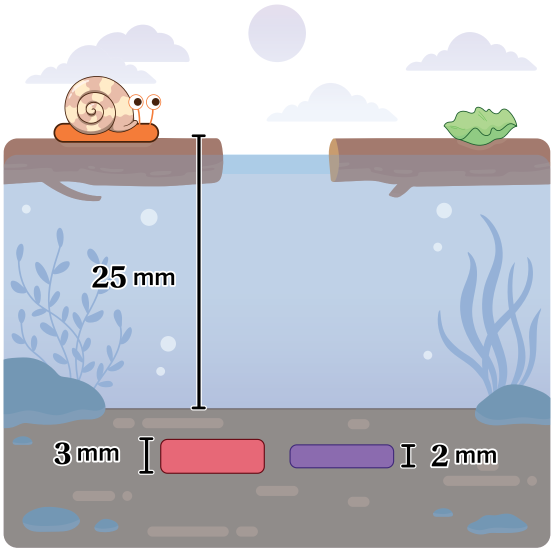
Let's connect graphs, tables, and equations to the situations they represent.

Warm-Up

1. Shelley the Snail loves eating lettuce leaves.

She needs to cross the gap to get to the lettuce.

Write different combinations of blocks that fill the gap.



Number of 3 mm Blocks	Number of 2 mm Blocks

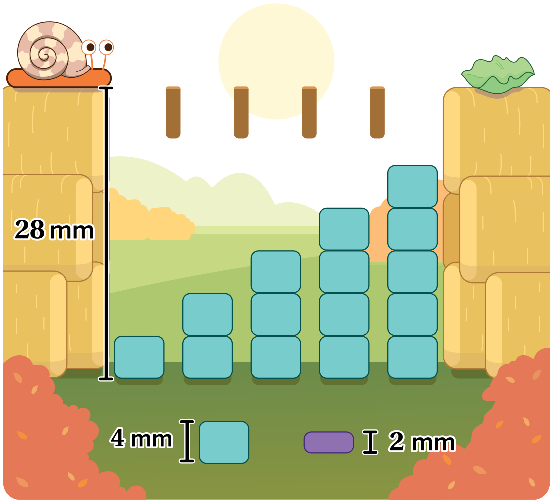
Activity 1

Name: ____ Date: ____ Period: ____

Mind the Gap

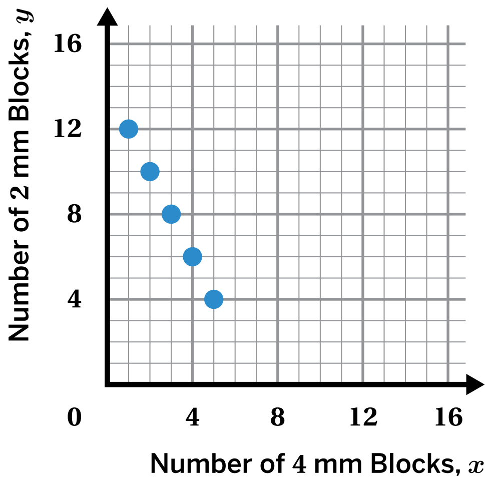
2. Complete the table to help Shelley get to the lettuce.

Number of 4 mm Blocks, x	Number of 2 mm Blocks, y
1	
2	
3	
4	
5	



3. Shelley has to cross a gap that is 28 mm deep using 4 mm and 2 mm blocks. This graph shows some of the possible combinations of blocks. Complete the table.

Number of 4 mm Blocks, x	Number of 2 mm Blocks, y
0	
6	
7	



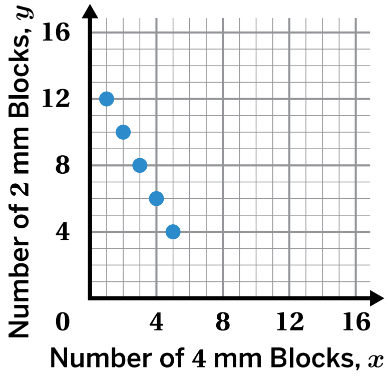
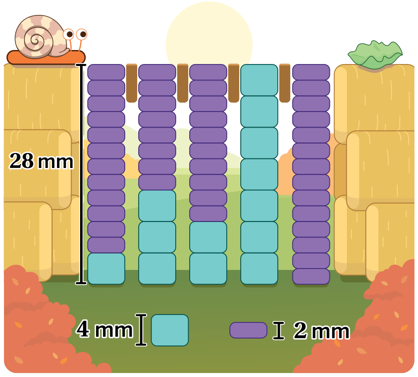
4. Here is Tyrone's work from the previous problem. Write an equation for the number of 2 mm blocks, y, needed for any number of 4 mm blocks, x.

Number of 4 mm Blocks, x	Number of 2 mm Blocks, y
0	14 – 2(0)
6	14 – 2(6)
7	14 – 2(7)
x	?

Name: .... Date: .... Period: ....

Mind the Gap (continued)

5. The x-**intercept** of the graph is $(7, 0)$. The y-**intercept** of the graph is $(0, 14)$.

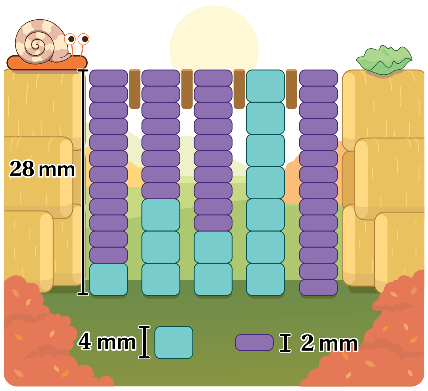
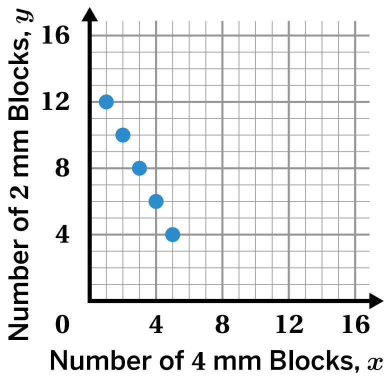
Graph	Situation	Equation
		$y = 14 - 2x$

a Select *one* representation

b Show or explain where you see the intercepts.

x-intercept: y-intercept:

6. Here is the same relationship represented in two different ways.

$4x + 2y = 28$	$y = 14 - 2x$
	

Discuss:

- How do you see the equation in each representation?
- Are these equations equivalent? Why or why not?

Activity 2

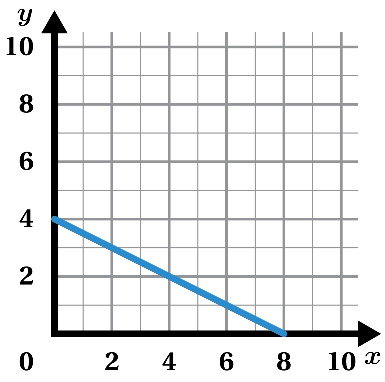
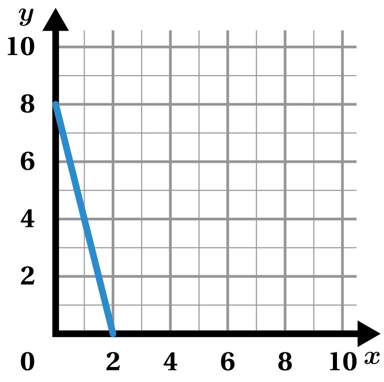
Name: ______________ Date: ________ Period: ________

Rearrange It

7. Solve the equation $4x + 2y = 28$ for y to show that it is equivalent to $y = 14 - 2x$. Show or explain your thinking.

8. Match each graph with two equations. Two equations will have no match.

$2x + 8y = 24$	$y = 8 - 2x$	$2x + 4y = 16$
$y = 4 - \frac{1}{2}x$	$8x + 2y = 16$	$y = 8 - 4x$

Graph		
Equations		

9. Rearrange each equation to solve for y.

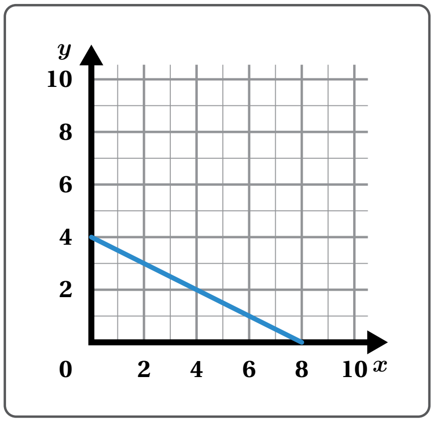
$6x + 2y = 34$ $y =$	$5x + 2y = 46$ $y =$
$2x + 4y = 26$ $y =$	$3x + 4y = 40$ $y =$

Synthesis

How would you convince a classmate that these three cards represent the same situation?

$2x + 4y = 16$

$y = 4 - \frac{1}{2}x$



Lesson Practice A1.06

Lesson Summary

Equations, tables, and graphs are all different ways to model a situation. The graph of a linear equation represents all the pairs of values that are solutions to the equation and make the equation true.

Linear equations can be written in different but equivalent forms. Rearranging equations into different but equivalent forms helps reveal new information, such as the x-**intercept** and y-**intercept**, which we can see in a graph, table, or description of a situation.

Let's say a lemonade stand sold lemonade for $3 per cup and cookies for $2 each. The stand made $12. ℓ represents the number of cups of lemonade sold and c represents the number of cookies sold. This situation can be represented in many different ways:

Equation

$3\ell + 2c = 12$

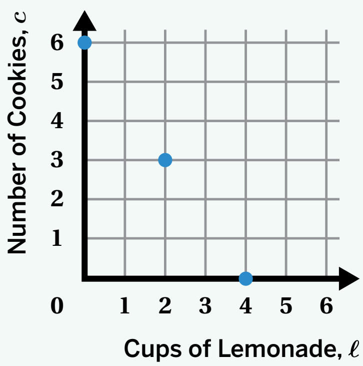
Equation Solved for c

$c = 6 - \frac{3}{2}\ell$

Table

ℓ	0	2	4
c	6	3	0

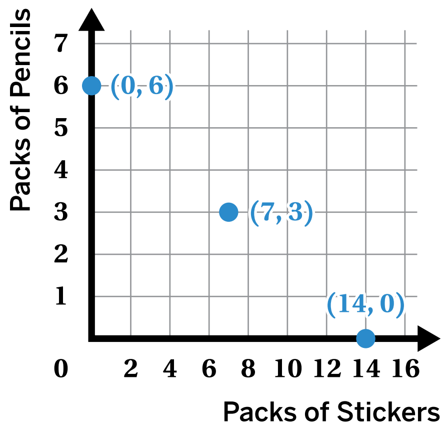
Graph



Lesson Practice
A1.06

Name: .... Date: .... Period: ....

Problems 1–2: Here is a graph about school supplies.



1. A teacher spent $21 on packs of stickers and packs of pencils for her class.

- Stickers cost $1.50 per pack.
- Pencils cost $3.50 per pack.

Show or explain how you know that this graph represents this situation.

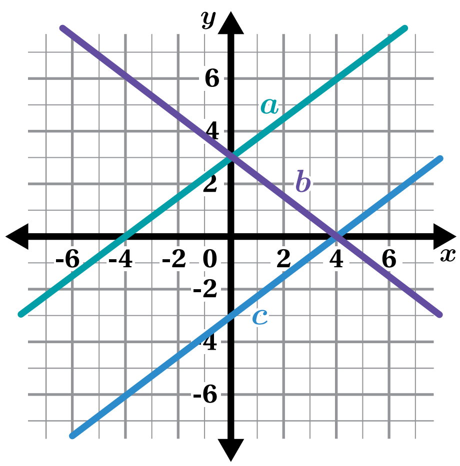
2. Circle a coordinate pair and explain what it means in this situation.

(0, 6) (7, 3) (14, 0)

3. Circle the line that represents $12 = 3x + 4y$.

Line a Line b Line c

Show or explain how you know.



4. Which equation is equivalent to $15x + 3y = 2$?

A. $y = \frac{2}{3} + 5x$ B. $y = \frac{2}{3} - 5x$ C. $y = 2 - 15x$ D. $y = 2 - 5x$

5. Match each equation with its equivalent equation.

a $4x + 6y = 20$ b $3x - 6y = 16$ c $2x - 3y = 10$ d $-3x + 6y = 16$

.... $y = \frac{8}{3} + \frac{1}{2}x$ .... $y = -\frac{10}{3} + \frac{2}{3}x$ .... $y = \frac{10}{3} - \frac{2}{3}x$ .... $y = -\frac{8}{3} + \frac{1}{2}x$

Name: .... Date: .... Period: ....

6. Makayla is trying to predict the price of her next taxi ride.

   She recorded her last three rides in the table.

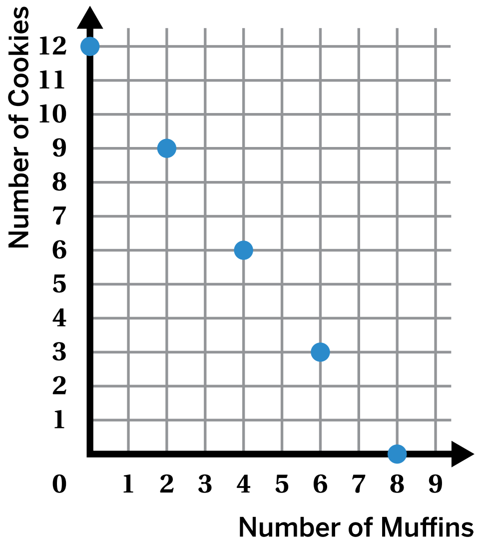
   How much can she expect to pay to travel 10 miles?

Distance (mi)	Price ($)
2	10
4	17
7	27.50

Test Practice

7. Jordan had a bake sale. Muffins were $3 each and cookies were $2 each. Jordan earned $24.

   Here is a graph of Jordan's situation.



   Select *all* the combinations of muffins and cookies Jordan could have sold.

   - ☐ A. 0 muffins and 8 cookies
   - ☐ B. 9 muffins and 2 cookies
   - ☐ C. 2 muffins and 9 cookies
   - ☐ D. 6 muffins and 4 cookies
   - ☐ E. 4 muffins and 6 cookies

Spiral Review

8. Select *all* the equations where $x = -2$ is a solution.

   - ☐ A. $4x = 4 + 2x$
   - ☐ B. $2(x + 5) = x + 8$
   - ☐ C. $3x - 5 = 1$
   - ☐ D. $19 = 2(x - 6) + 3$
   - ☐ E. $5 + 3x = -1$

9. Solve $-3x + 4y = 28$ for y.

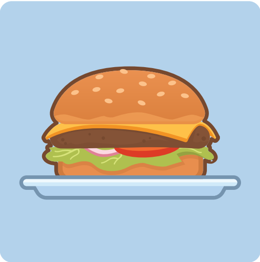
10. Solve $6x - 3y = 36$ for y.

Name: ________________ Date: ________ Period: ________

MA.912.AR.1.2, MA.912.AR.2.2, MA.912.AR.2.4, MTR.1.1

Five Representations

Let's explore different ways to represent linear relationships.



Warm-Up

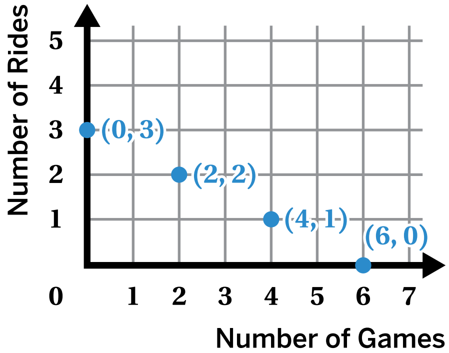
1. **Discuss:** Describe as many connections as you can between the representations.

Description

Caasi has \$12 to spend on games and rides at a fair.

Games cost \$2 each.
Rides cost \$4 each.

Graph



Equation in Standard Form

$$2x + 4y = 12$$

x represents the number of games.
y represents the number of rides.

Equation in Slope-Intercept Form

$$y = 3 - \frac{1}{2}x$$

x represents the number of games.
y represents the number of rides.

Activity 1

Name: .... Date: .... Period: ....

Match It

2. You will use a set of three cards: two graph cards and one equation card.

- Match each card to the correct description.
- Create the missing representation(s) in each column.

	Burgers	Bracelets
Description	A family bought 2 hamburgers and 4 salads at the fair. The total was $\$32$. Use x for the price of a hamburger. Use y for the price of a salad.	Vihaan sold 4 bracelets at the fair. To make the bracelets, he bought 8 packs of beads. Vihaan earned $\$32$ in total. Use x for the price of a bracelet. Use y for the cost of a pack of beads.
Graph		
Equation in Standard Form		
Equation in Slope-Intercept Form		

Activity 2

Name: .... Date: .... Period: ....

Make It

3. Situation 1: Complete all the representations for this situation.

Variables

- x represents the number of bus rides Kiana took last month.
- y represents the number of train rides Kiana took last month.

Description

- Each bus ride costs $\$2$.
- Each train ride costs $\$2.50$.
- Kiana spent $\$40$ riding the bus and the train last month.

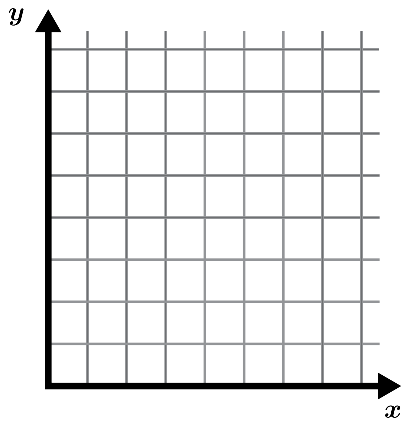
Equation in Standard Form

Equation in Slope-Intercept Form

Table

Number of Bus Rides, x	Number of Train Rides, y
0	
	12
10	
	4
20	0

Graph



Name: ________ Date: ________ Period: ________

Make It (continued)

4. Situation 2: Complete all the representations for this situation.

Variables

- x represents ________ .
- y represents ________.

Description

Equation in Standard Form

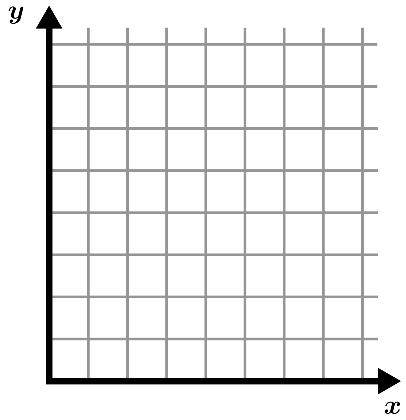
$$2x + y = 40$$

Equation in Slope-Intercept Form

Table

x	y
0	
5	30
	20
15	

Graph



Synthesis

5. Choose *one* representation on the right.

Explain how you could use your representation to create the other three.

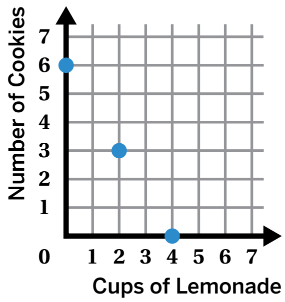
Equation in Standard Form

$3x + 2y = 12$

Equation in Slope-Intercept Form

$y = 6 - \frac{3}{2}x$

Graph



Description

A lemonade stand sold lemonade for \$3 per cup and cookies for \$2 each. The stand made \$12. x represents the number of cups of lemonade sold. y represents the number of cookies sold.

Lesson Practice A1.07

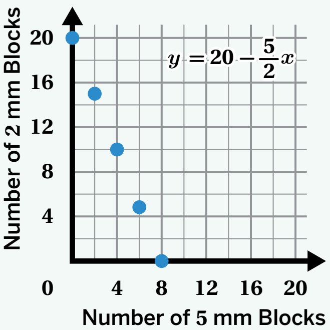
Lesson Summary

There are lots of ways to represent a situation that involves two variables. Each of these representations is connected and reveals information about how the quantities in the situation are related.

A description explains how the quantities are related verbally or in writing.
For example: A snail needs to cross a gap that is 40 mm tall using 5 mm and 2 mm blocks.

A graph reveals the slope and y-*intercept*.

This graph shows a slope of $-\frac{5}{2}$ and a y-intercept of 20.



A table reveals several combinations of 5 mm and 2 mm blocks that let the snail cross the gap.

x	0	2	4	6
y	20	15	10	5

The ordered pairs can be used to make a graph or find the rate of change (slope).

Different forms of an equation reveal unique information. In this situation, standard form reveals the relationship between combinations; slope-intercept form reveals how the blocks are changing in relationship with one another.

Standard Form

$5x + 2y = 40$

Slope-Intercept Form (solved for one variable)

$y = 20 - \frac{5}{2}x$

Name: .... Date: .... Period: ....

Problems 1–3: A school is putting on a play. Adult tickets cost \$8 each and student tickets cost \$4 each. The school collected \$320 total.

- a represents the number of adult tickets sold.
- s represents the number of student tickets sold.

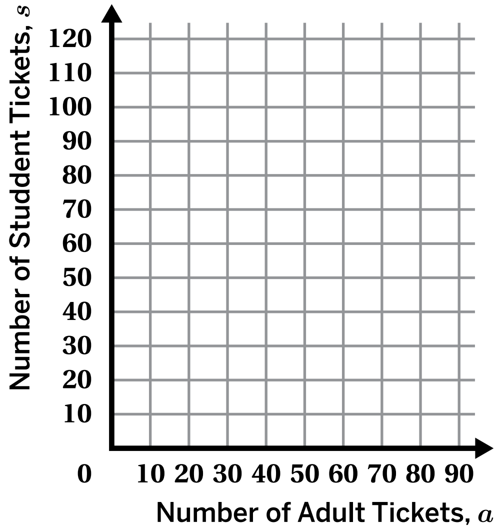
1. Which equation represents this relationship?

A. $a + s = 320$
B. $8a + 4s = 320$
C. $8a - 4s = 320$
D. $8s + 4a = 320$

2. Complete the table representing this relationship.

Number of Adult Tickets, a	Number of Student Tickets, s
0	
20	

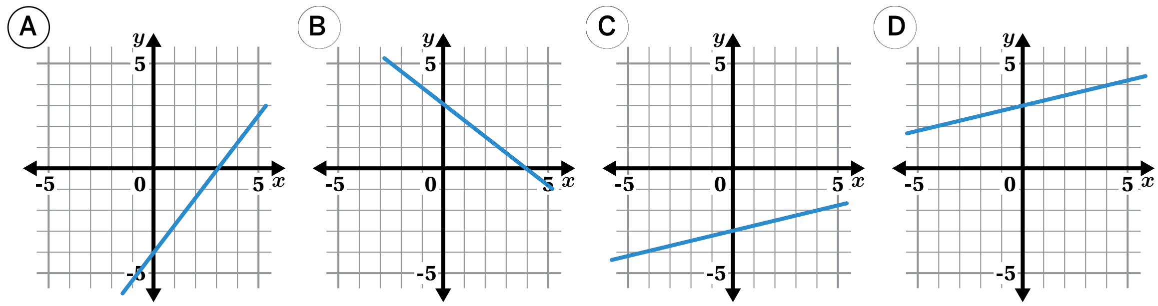
3. Make a graph showing this relationship.



Problems 4–5: Here is the equation $-x + 4y = 12$.

4. Explain how you can determine that $-x + 4y = 12$ is equivalent to $y = 3 + \frac{1}{4}x$.

5. Which graph matches the equations $-x + 4y = 12$ and $y = 3 + \frac{1}{4}x$?

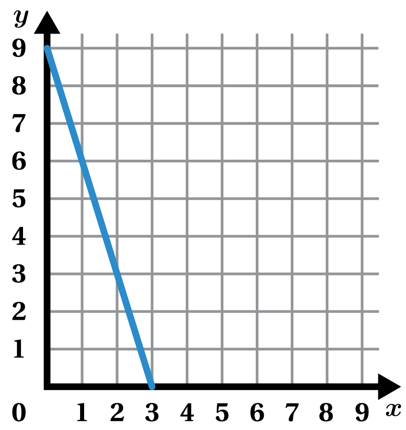


Test Practice

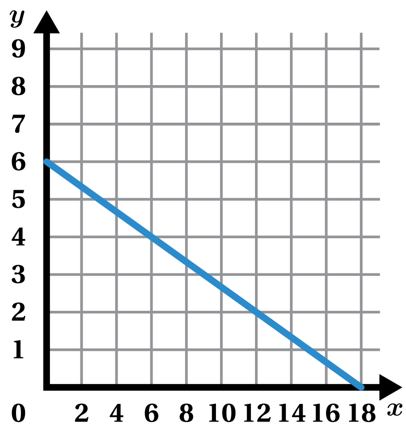
Problems 6–7: Match each graph to two equations. One equation will have no match.

Equation A	Equation B	Equation C	Equation D	Equation E
$y = 6 - \frac{1}{3}x$	$2y + 6x = 18$	$y = 9 - 3x$	$6y + 2x = 36$	$3y + 6x = 18$

6. Equation ______ Equation ______



7. Equation ______ Equation ______



Spiral Review

8. Write at least *three* different equations that are equivalent to $\frac{1}{2}(\text{-}10x + 2) = 6$.

Equation 1 ______ **Equation 2** ______ **Equation 3** ______

9. Select *all* the equations that have no solution.

- ☐ **A.** $2t + 1 = 1 + 2t$
- ☐ **B.** $2t + 1 = 2t$
- ☐ **C.** $2t + t = 2t + 1$
- ☐ **D.** $2t = 1$
- ☐ **E.** $2(t + 1) = 2t + 1$

10. Write an equation equivalent to $x = \frac{10 - 5y}{3}$ in the form $Ax + By = C$.

Name: Date: Period:

MA.912.AR.2.3, MTR.4.1, MTR.7.1

Whale Growth Rates

Let's write equations of parallel lines.

Warm-Up

The table provides information about two whales named Alpha and Beta.

Humpback Whale Age (years)	Alpha's Total Body Length (feet)	Beta's Total Body Length (feet)
0	3	1.9
0.5	5.5	4.4
1	8	6.9
3	18	16.9

1. Tell a story about these two whales. Include specific details about the whales' growth rates over time.

Activity 1

Name: ______ Date: ______ Period: ______

Whale Growth Comparison

A marine biologist has been tracking the growth of whales over time and found that their body lengths grow at approximately the same rate for the first 12 years of their lives.

2. Two juvenile whales, Gracie and Nala, have been tagged for tracking. Use the information provided in the table to determine the equation that models Nala's growth. Provide justification for your equation.

	Age (years)	Length (feet)	Equation Modeling Growth
Gracie	2	12.5	$y = 5x + 2.5$
Nala	3	16.2	

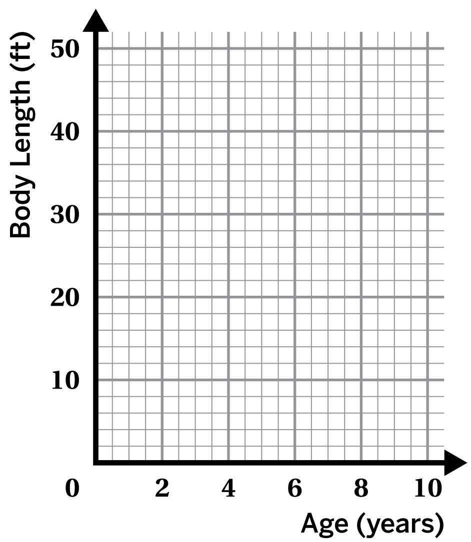
3. Graph the lines that model Gracie's and Nala's body lengths over time. Label Gracie's line G, and Nala's line N.

Coordinate plane here, Quadrant I only
x-axis goes 0 to 10, y-axis goes 0 to 50.

Grid marks along x-axis at every 0.5 units
x-axis labeled: **Age (years)**

with numbers on every whole unit of x-axis
y-axis labeled: **Body Length (ft)**

Gridlines at every whole unit with numbers on every 5 units of y-axis



Name: .... Date: .... Period: ....

Whale Growth Comparison (continued)

4. Use your equations and graphs for Gracie and Nala to compare the two whale growth equations and graphs.

a Which whale was larger at birth? At two years of age? At age 8?

b What is the same about the two whale's growths? What is different?

5. A third humpback whale, Omega, had a body length of 22.4 feet at 4 years of age. Since Omega grows at the same rate as other humpback whales, what is the equation, in slope-intercept form, that models Omega's growth? Explain your reasoning.

Name: Date: Period:

Humpback Whale Fin Length

Humpback whales also have side flipper fins that grow proportionally over the first 12 years of their lives. The growth of Delta's fin length is modeled by the equation, $y = \frac{3}{2}x + \frac{1}{8}$.

6. Pele is another humpback whale that was tagged at the age of 5 years with a fin length of 8.5 feet. Write an equation that models Pele's fin length for any age in years using the growth rate that was determined when analyzing Delta's growth.

7. Without graphing, compare the slopes and y-intercepts of Pele and Delta's growth models.

8. How would you describe the relationship of these two lines, if they were graphed?

Name: Date: Period:

Write a Story

9. Write a story that would define a line parallel to $y = \text{-}6x + 80$ and passes through the point $(7{,}26)$.

10. Show how to algebraically find the equation of the line, in slope intercept form, that is parallel to $y = \text{-}4$ and goes through the point $(8{,}\text{-}1)$. Describe a real-world scenario that these lines would model. Be sure to label all variables.

11. What is the equation of a line through $(3{,}9)$ and $(3{,}\text{-}9)$? What is the equation of the parallel line through $(\text{-}1{,}4)$? Describe a scenario that these two lines model.

Synthesis

12. When two lines are parallel, how are their slopes related?

13. If you know the slope of a line and you know the coordinates of a point on the line, how can you find the complete equation for the line?

14. If you know that Line A passes through the point (-2,10) and is parallel to the line $y = 4x + 6$, how do you find the slope-intercept form of the equation for Line A?

Lesson Practice A1.08

Lesson Summary

- The slope of a line describes the steepness of the line from left to right. When two lines have the same slope, they have the same steepness. So, they are parallel.
- If you know the slope of one line and you know that a second line is parallel to that line, then you know that the second line has the same slope as the first line.
- For example, if you know the equation for Line A is $y = \text{-}3x + 6$, and you know that line B is parallel to Line A, then you know that the slope of Line B is -3.
  - Once you know the slope of a line, you can write an equation for the line using the slope-intercept form of a line: $y = mx + b$, where m is the slope and b is the y-intercept. For example, if the slope of the line is 4, then the equation for the line is $y = \text{-}3x + b$. That is not the complete equation, but it is a good start.
  - If you know the slope and the location of one point on the line, then use the coordinates to find the y-intercept. Substitute the coordinates into the equation and solve for b.
  - For example, let's say that you know the equation of a line is $y = \text{-}3x + 6$ and you know that the point (-2,10) is on the line. Substitute $x = \text{-}2$ and $y = 10$ into $y = \text{-}3x + b$.

$$10 = \text{-}3(\text{-}2) + b$$

$$10 = 6 + b$$

$$4 = b$$

So the equation of the parallel line is $y = \text{-}3x + 4$.

Lesson Practice
A1.08

Name: .... Date: .... Period: ....

Problems 1–2: Determine which line is parallel to the given line.

1. Show how to algebraically find the slope intercept form for the equation of the line that is parallel to $3x + 3y = -18$ and goes through the point $(20,-13)$.

2. Which line is parallel to $3x - 4y = 24$?

A. $y = 3x + 4$

B. $y = 6 - \frac{4}{3}x$

C. $y = \frac{4}{3}x - 2$

D. $y = 2 + \frac{3}{4}x$

Problems 3–5: Find the equation of the line in slope-intercept form that meets the given conditions.

3. A line that passes through $(2,6)$ and is parallel to $y = 3x + 15$.

4. A line that passes through $(-10,-8)$ and is parallel to $x - 2y = 1$.

5. A line that passes through $(-4,0)$ and is parallel to the line that passes through $(2,12)$ and $(-1,3)$.

6. Road H passes through an intersection defined on a grid as $(7,9)$. Road H is parallel to road M. Road M passes through $(2,3)$ and $(9,3)$. What are the slopes and equations for these two roads?

Slope:	Equation for road	Equation for road
$m =$	H:	M: ....

Name: ____________ Date: ________ Period: ________

Problems 7–8: The populations of three small towns have been growing at consistent rates over the last 6 years. One of the towns in the table is growing at the same rate as Town C. Town C recorded a population of 532 in year 2 and a population of 556 in year 5. Use the information in the table provided.

	Town A	Town B
Year 1	1,215	893
Year 3	1,237	909
Year 6	1,259	925

7. What is the rate of change in population and the equation, in slope-intercept form, that represents the population growth, G, of Town C?

Rate of change: ____________ Equation: ____________

8. What is the equation that defines population, P, for any year, x, for the town with a population growth like Town C?

Town ____ is parallel to Town C and its population is modeled by the equation ____________

Test Practice

9. Which line passes through $(-6,5)$ and is parallel to $2x + y = 9$?

A. $y = -2x + 7$ B. $y = 2x - 7$ C. $y = 2x + 7$ D. $y = -2x - 7$

Spiral Review

10. Select *all* the equations that are equivalent to $\frac{2}{3}x - \frac{1}{6}y = -\frac{1}{9}$.

- ☐ **A.** $-6x + \frac{2}{3}y = 1$
- ☐ **B.** $3y = -12x - 2$
- ☐ **C.** $-3x + \frac{3}{4}y - \frac{1}{2} = 0$
- ☐ **D.** $6y - 24x = 4$
- ☐ **E.** $y = -4x - \frac{2}{3}$
- ☐ **F.** $y = 4x + \frac{2}{3}$

Name: ______ Date: ______ Period: ______

MA.912.AR.2.3, MTR.2.1, MTR.4.1, MTR.6.1, MTR.7.1

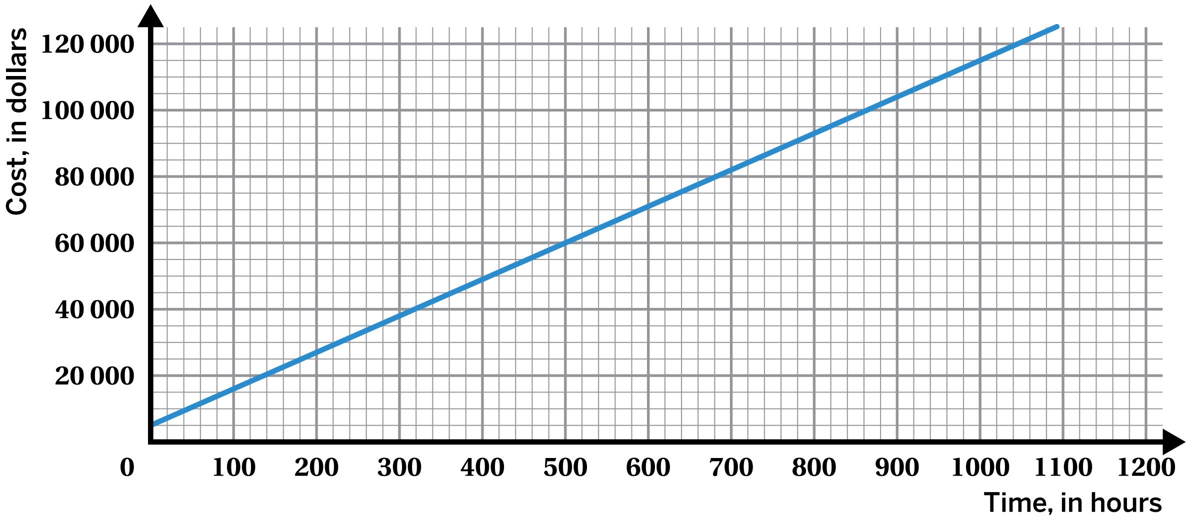
House Design

Let’s write equations of perpendicular lines.

Warm-Up

Mr. and Mrs. Walker are building a house. They plan to hire a contractor for the work. The graph below provides information about the cost to hire the contractor.

1. Tell a story using information from the graph. Include specific details to help the Walkers plan their budget.



Activity 1

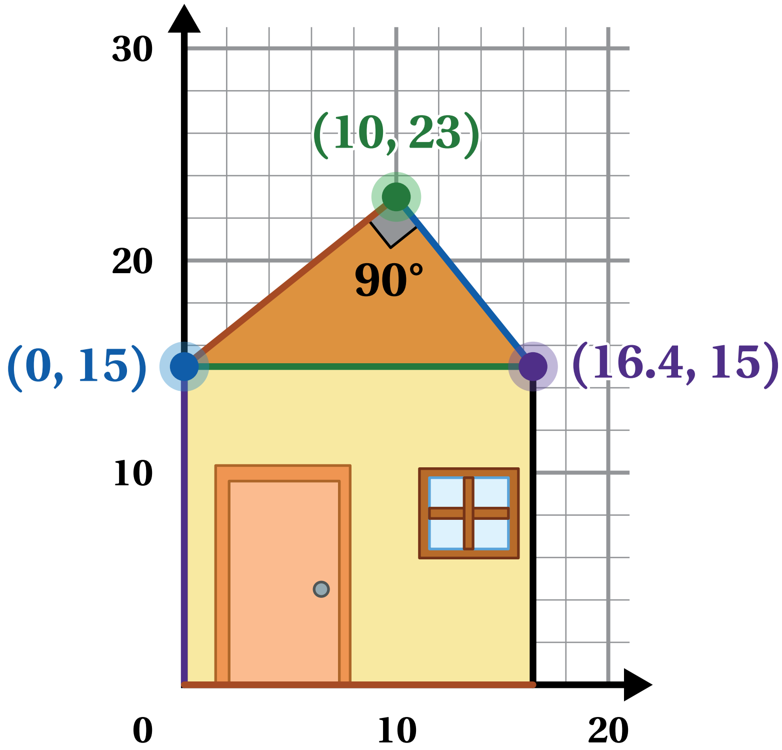
Name: ________ Date: ________ Period: ________

Exterior Design

First, the Walkers are focusing on designing the exterior of their house.

2. The front view of the house is drawn below. The left and right sides of the roof meet at a right angle at the peak.

a Write the equation of the line that represents the left side of the roof in slope-intercept form, $y = mx + b$.



b Write the equation of the line that represents the right side of the roof in slope-intercept form.

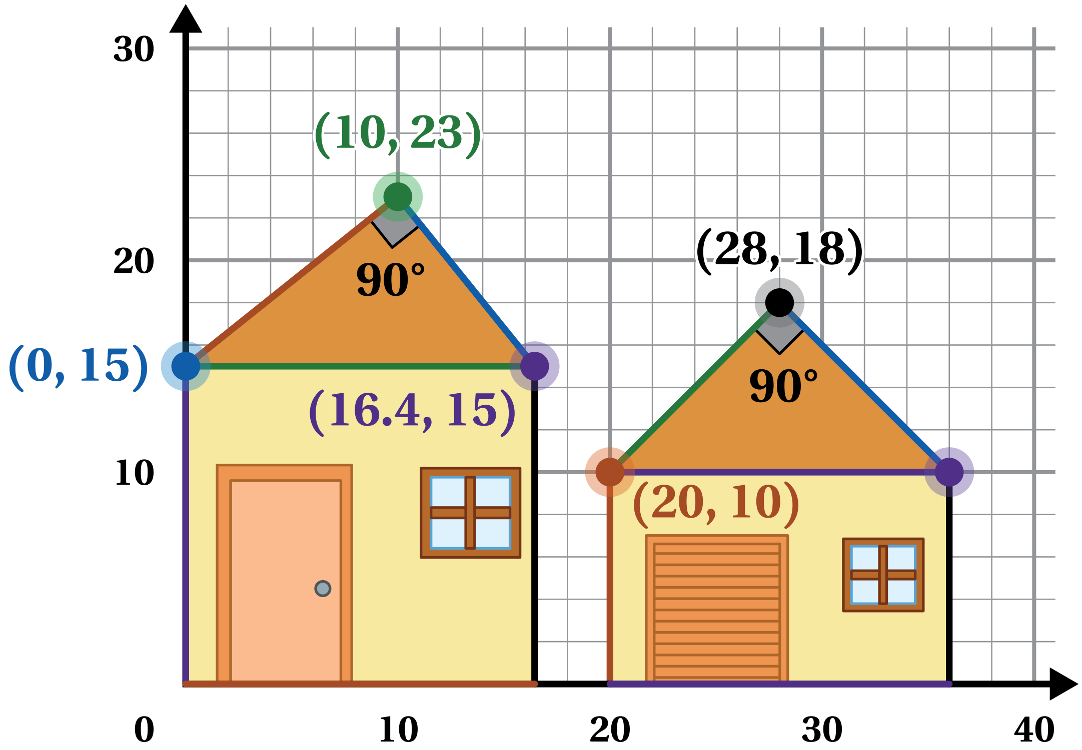
c What do you notice about the slopes of the two equations for the sides of the roof? What do you wonder?

Name: .... Date: .... Period: ....

Exterior Design (continued)

3. The Walkers also want to build a garage next to their house. They add it to their drawing shown below.

a Given the labeled points, write the equation of the line that represents the left side of the garage roof.



b The left and right sides of the garage roof meet at the peak at a right angle. Write the equation of the line that represents the right side of the garage roof.

c Mr. Walker decides he wants to expand the width of the garage while keeping the slope of the roof the same. Write the equation of the line that would represent the right side of the roof if its peak was at $(30,20)$.

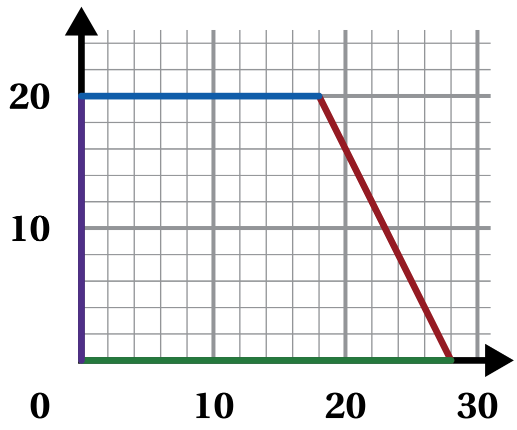
Activity 2

Name: ________ Date: ________ Period: ________

Living Room Layout

Mrs. Walker wants to design the living room to be functional and inviting.

4. An overhead view of the living room floor plan is drawn to the right.



a The television will be mounted on the wall represented by the line $y = -2x + 56$. Mrs. Walker wants to arrange two couches in an L shape that meet at $(12,2)$, one parallel to the television and one perpendicular to the television. Write the equation of the line that would represent the couch parallel to the television.

b Write the equation of the line that represents the couch perpendicular to the television.

Name: ______ Date: ______ Period: ______

Living Room Layout (continued)

c Mrs. Walker has a corner bookshelf that she wants to position perpendicular to the TV at point $(4,18)$. Write an equation of a line to represent the open face of the bookshelf.

d The door to the living room is on the line $x = 0$. Mrs. Walker wants to place an entry table perpendicular to the door at point $(4,0)$. Write an equation to represent the line that the entry table would lie on.

5. Mrs. Walker maps out where she wants to place a table. The four corners are located at A $(15,17)$, B $(20.1,10.2)$, C $(15,9)$, and D $(12,13)$.

a How many pairs of parallel sides does the table have?

b How many pairs of perpendicular sides does the table have?

c What is the shape of the table?

Synthesis

6. When two lines are perpendicular, how are their slopes related?

7. If you know the slope of a line and the coordinates of a point on the line, how can you find the equation of the line?

Lesson Practice A1.09

Lesson Summary

- The slope of a line describes the steepness of the line from left to right. When two lines have slopes that are opposite reciprocals of each other, they intersect at a 90-degree angle, making them perpendicular.
- If you know a line's slope, you know a line perpendicular to it will have the opposite reciprocal slope. For example, if you know the equation for Line G is $y = \frac{1}{3}x + 2$, and you know that Line H is perpendicular to Line G, then you know that the slope of Line H is $m = -\frac{3}{1} = -3$.
  - Once you know the slope of a line, you can write its equation using slope-intercept form: $y = mx + b$, where m is the slope and b is the y-intercept. So far, you know the equation for Line H is $y = -3x + b$.
  - If you know the slope and are given the location of one point on the line, you can substitute the coordinates of that point into the equation to find the y-intercept, b.
  - If Line H passes through (4,7), you can substitute 4 for x and 7 for y into $y = -3x + \text{b}$.

$$7 = -3(4) + b$$
$$7 = -12 + b$$
$$19 = b$$

So, the equation of Line H is $y = -3x + 19$.

Lesson Practice
A1.09

Name: .... Date: .... Period: ....

1. Without graphing, find the slope-intercept equation of the line perpendicular to $4x - 12y = 32$ and passing through (-6,19).

2. Which line is perpendicular to $y = 8x + \frac{1}{2}$?

A. $y = \frac{1}{2}x + 8$

B. $y = 8x - \frac{1}{2}$

C. $y = -\frac{1}{8}x - 2$

D. $y = -8x + 2$

Problems 3–4: Triangle JKL has vertices J (0,4), K (1,2), and L (4,6).

3. Determine whether triangle JKL is a right triangle. Explain.

4. Imagine you moved vertex J so that triangle JKL now formed a right angle at vertex K. Find the new equation for line JK.

Name: .... Date: .... Period: ....

5. Find the equation of Line M that passes through (-3,2) and (-3,7). Then, find the equation of Line N which is perpendicular to Line M and passes through (6,-1).

Equation for Line M: .... Equation for Line N ....

Test Practice

6. Consider line L_1 with the equation $5x + 2y = 14$. A second line, L_2, is perpendicular to L_1 and passes through the point (-5,3).

Select *all* of the true statements.

- ☐ **A.** The slope of L_1 is $-\frac{5}{2}$.
- ☐ **B.** The slope of L_2 is $\frac{2}{5}$.
- ☐ **C.** The y-intercept of L_1 is 14.
- ☐ **D.** The equation for L_2 is $y = \frac{2}{5}x + 5$.
- ☐ **E.** Lines L_1 and L_2 intersect at a 90-degree angle.

Spiral Review

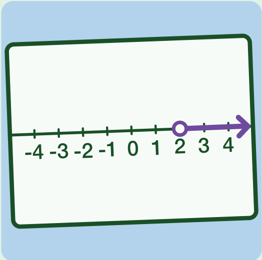
7. Select *all* the equations that are equivalent to $\frac{1}{8}x + \frac{3}{4}y = \frac{1}{12}$.

- ☐ **A.** $4 = 6x + 36y$
- ☐ **B.** $3y = \frac{1}{3} - \frac{1}{2}x$
- ☐ **C.** $\frac{4}{3}y - \frac{1}{12} = 8x$
- ☐ **D.** $6y = \frac{2}{3} - x$
- ☐ **E.** $x = 3y + \frac{2}{3}$
- ☐ **F.** $x + 6y - \frac{2}{3} = 0$

Sub-unit 3

One-Variable and Two-Variable Inequalities

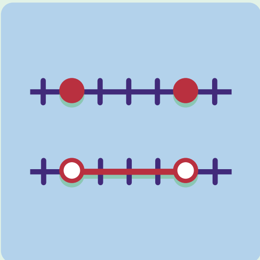
Lesson 10
Pizza Delivery



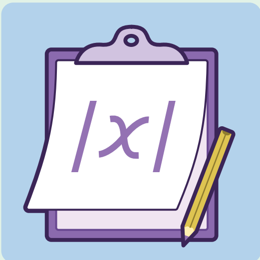
Lesson 11
Graphing Inequalities

Lesson 12
Solutions and Sheep

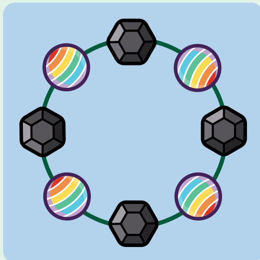
Lesson 13
Tick Tock



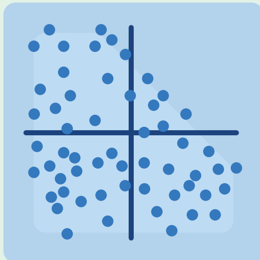
Lesson 14
Absolute Value Solutions



Lesson 15
Getting Absolute



Lesson 16
Bracelet Budgets



Lesson 17
All of the Solutions

Lesson 18
Concert Planning

Name: ________________ Date: ________ Period: ________

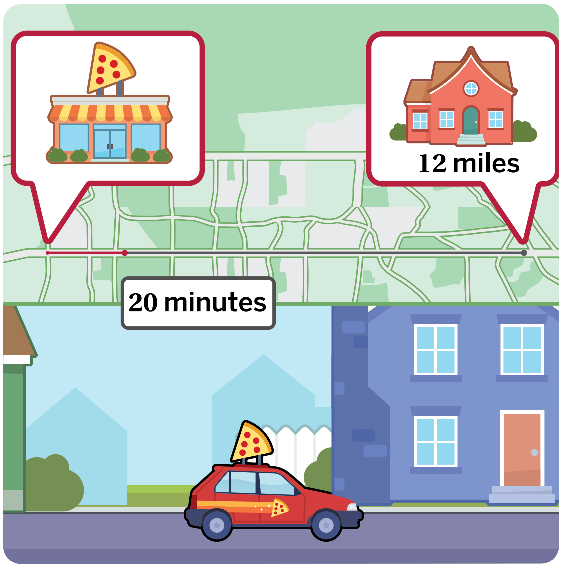
MA.912.AR.2.6, MTR.1.1, MTR.2.1, MTR.4.1

Pizza Delivery

Let's write inequalities to represent constraints.

Warm-Up

1. Write a story about what you see.



Activity
1

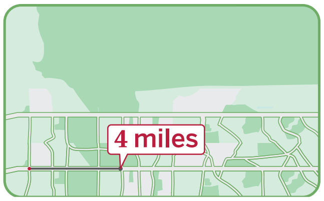
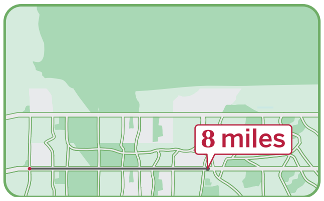
Name: ____________ Date: ________ Period: ________

Order Up!

2. Desmos Pizza is now offering delivery!

It takes 15 minutes to prepare an order and 3 minutes to drive each mile.

How long would it take to deliver each order?

Distance (mi)			
Time (min)			

3. Here is Mariana's work from the previous problem.

Write an expression for the number of minutes it would take to deliver a pizza x miles away.

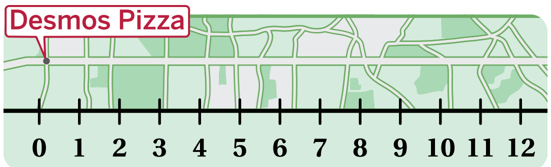
Distance (mi)	Time (min)
2	15 + 3(2)
4	15 + 3(4)
8	15 + 3(8)
x	?

4. Desmos Pizza wants to deliver orders in 30 minutes or less.

a Which inequality represents this situation?

A. $15 + 3x \leq 30$ B. $15 + 3x \geq 30$ C. $15 + 3x > 30$ D. $15 + 3x < 30$

b Show or describe all the distances they can deliver to in 30 minutes or less.



Activity 1

Name: .... Date: .... Period: ....

Order Up! (continued)

5. Here is how Mariana and Camila figured out all of the distances in which pizza could be delivered in 30 minutes or less.

Mariana

Miles, x	Minutes
1	18
2	21
3	24
4	27
5	30
6	33

$x \leq 5$

You have to Live 5 miles away or Less.

Camila

$$15 + 3x = 30$$
$$-15 \quad -15$$
$$3x = 15$$
$$x = 5$$

5 miles is the farthest you can live from Desmos Pizza.

Discuss: Why is each strategy helpful? How could each strategy be improved?

6. A large pizza is \$12 and toppings are \$2 each. Mariana can spend as much as \$20 on a pizza.

a If x is the number of toppings, which of these inequalities represents the situation?

A. $12 + 2x \geq 20$ **B.** $12x + 2 \geq 20$ **C.** $12 + 2 \leq 20x$ **D.** $12 + 2x \leq 20$

b Show or describe all the numbers of toppings, x, that Mariana can get for \$20 or less.

7. A **constraint** is a limitation on possible values in a model. Here are some examples of constraints:

- Desmos Pizza wants to deliver in 30 minutes or less.
- Mariana can spend as much as \$20 on a pizza.
- Desmos Pizza wants to sell more than 50 pizzas per day.

What is a constraint in your life?

Activity **2**

Name: ______________________ Date: __________ Period: __________

Trampoline World

8. Jamir is planning to host a party at Trampoline World.

Discuss: What constraints might Jamir think about when planning this party?

9. Hosting a party at Trampoline World costs a flat fee of $80, plus $30 per hour for the small room or $50 per hour for the large room.

Match each constraint to an inequality, where x represents the number of hours for the party. One inequality will have no match.

$30 + 80x \geq 140$	$80 + 30x \leq 140$	$80 + 50x \geq 140$	$80 + 80x \leq 140$

Mariana's party in the small room costs at most $140.	**The owner wants to earn at least $140 for a party in the large room.**	**Amoli can spend up to $140 for a party that uses both the big and small rooms.**

Name: Date: Period:

Trampoline World (continued)

10. Jamir can spend as much as $155 for a party in the large room.

Write an inequality to match this new constraint, where x represents the number of hours for the party.

11. Select *all* the possible numbers of hours, x, that could work with Jamir's constraints.

- ☐ **A.** 0.75 hours
- ☐ **B.** 1 hour
- ☐ **C.** 1.5 hours
- ☐ **D.** 2.25 hours
- ☐ **E.** 3 hours

Synthesis

12. What are some suggestions you have for writing a constraint as an inequality?

Lesson Practice A1.10

Lesson Summary

We can use *inequalities* to model situations with constraints. A **constraint** is a limitation on what values are possible in a model or situation.

Here is an example of a situation, the constraint, and the inequality that models them.

Situation	Constraint	Inequality
Tasia is ordering pizza for a party. Each plain pizza costs \$12 and there is a delivery fee of \$8.	Tasia can spend *up to* $\$140$.	$12p + 8 \leq 140$ Where p represents the number of pizzas.

When writing inequalities to model situations, you can use the symbols $>$, $\geq$, $<$, and $\leq$ to represent the nature of the constraint. Terms like *greater than, less than, at most, at least*, or *up to* can help you determine which inequality symbol to use.

Name: .... Date: .... Period: ....

1. For each constraint, write the letter of the matching inequality.

A. $x \leq 10$	**B.** $x > 10$	**C.** $x \geq 10$

.... x is less than or equal to 10.

.... x is at most 10.

.... x is greater than or equal to 10.

.... x is greater than 10.

.... x is at least 10.

2. Demetrius can spend as much as $\$50$ on shirts. Shirts, s, cost $\$16$ each at a nearby store. Which inequality represents this situation?

A. $16s \geq 50$ **B.** $16s \leq 50$ **C.** $50s \geq 16$ **D.** $50s \leq 16$

Explain your thinking.

3. List *three* values for x that would make $8 + 2x \leq 20$ true.

$x =$ $x =$ $x =$

Problems 4–6: Write an inequality for each constraint. Use t for time (in hours).

4. Trevon practices his clarinet at least 1 hour each day.

5. At some colleges, students must work 20 hours or less per week.

6. The American Academy of Pediatrics recommends teenagers play video games for no more than 2 hours each day.

Lesson Practice
A1.10

Name: ________________ Date: ________ Period: ________

Problems 7–8: Tell a story about each inequality. Specify the constraint and what the variable represents.

7. $x \geq 3$

8. $5 > y$

9. Fatima makes $\$9.25$ per hour, h, plus $\$150$ in commissions. She created an inequality that could be used to find the number of hours she needs to work to make at least $\$510$ on her next paycheck.

Her inequality is $9.25 + 150h \leq 510$. Find the mistakes in Fatima's inequality and correct it.

Test Practice

10. Marquis wants to work at least 20 hours a week to earn enough money to go to a concert. Which inequality represents x, the number of hours Marquis wants to work?

A. $x > 20$ **B.** $x < 20$ **C.** $x \leq 20$ **D.** $x \geq 20$

Spiral Review

Problems 11–13: Solve each equation.

11. $4x - 6 = 12 - 2x$

12. $\frac{1}{3}x - 8 = 12 - 3x$

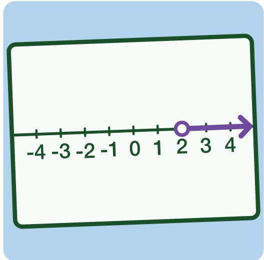
13. $2x + 7 - 3x = \frac{5}{2}$

Name: ______ Date: ______ Period: ______

MA.912.AR.2.6, MTR.1.1, MTR.2.1, MTR.4.1

Graphing Inequalities

Let's represent solutions to inequalities
on a number line.

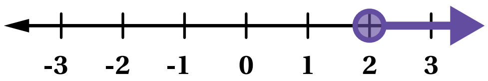


Warm-Up

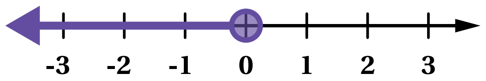
1. Here are four different inequalities.

Discuss: What do you notice?

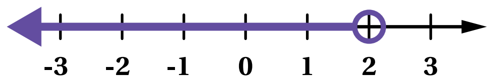
$x \geq 2$



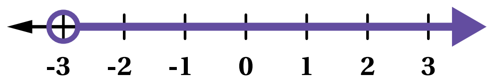
$x \leq 0$



$x < 2$

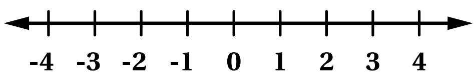


$x > -3$

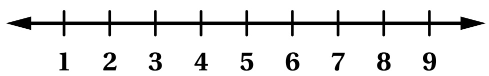


2. Make a graph of *all* the solutions to each inequality.

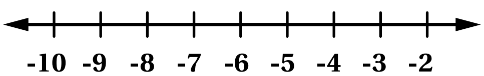
$x > -1.5$



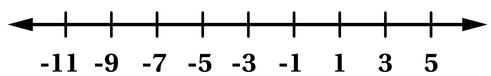
$x \leq 7$



$x < -5$



$x \leq -6$



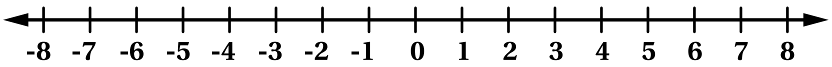
Activity 1

Name: ________ Date: ________ Period: ________

Show a Solution

3. **a** Plot a solution to this inequality.

$$2x - 8 > 0$$

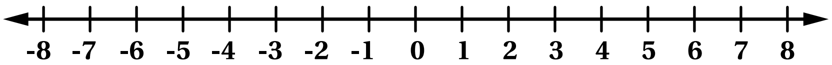


Share your response with your classmates.

b **Discuss:** Are any of the points incorrect? How do you know?

4. **a** Plot a solution to this inequality.

$$6x \leq 3x$$



b Explain how you know that your point is a solution (or why there is no solution).

5. Lan explains that $6x \leq 3x$ does not have a solution: *6 of something is always more than 3 of the same thing*.

Is Lan's statement correct? Circle one.

Yes No I'm not sure

Show or explain your thinking.

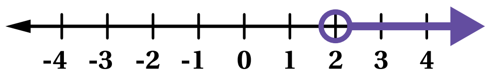
Activity
2

Name: ____________ Date: ________ Period: ________

Solution Sets

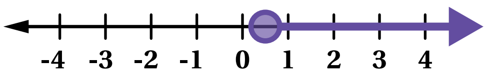
6. How can you check that this inequality and graph represent the solutions to $\frac{1}{4}x > \frac{1}{2}$?

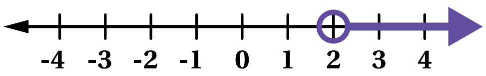
$x > 2$

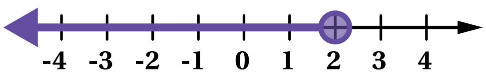


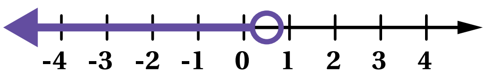
7. Match each inequality to a number line that represents its solutions.

$5x + 4 \geq 7x$	$3 - x < 1$	$2(x + 3) \geq 7$	$8x - 2 < 4x$









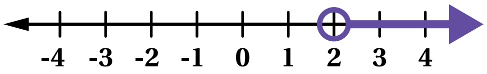
Activity 2

Name: ______ Date: ______ Period: ______

Solution Sets (continued)

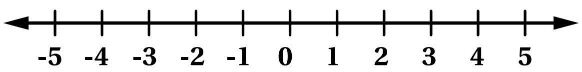
8. Lan matched this inequality and number line.

$$5x + 4 \geq 7x$$



How could you convince Lan that this inequality and number line don't match?

9. Create a graph of the solutions to the inequality $15 - x < 11$.

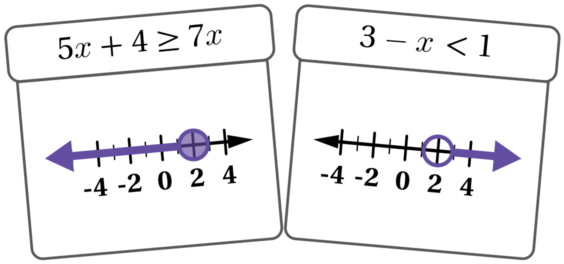


Explain your thinking.

Synthesis

10. Explain how you can determine whether a number line represents the solutions to an inequality.

Use these examples if they help with your thinking.



Lesson Practice A1.11

Lesson Summary

The *solution set* of an inequality contains all the values that make the inequality true. You can represent a solution set on a number line by marking the *boundary point* and then shading the region of values that make the inequality true. To identify the boundary point, you can solve the equation that corresponds to the inequality. Then you can test one or more values to determine whether the *solution region* is greater than or less than the boundary point.

Here is an example of how you can determine and represent the solution set for $2x - 4 \geq 8$:

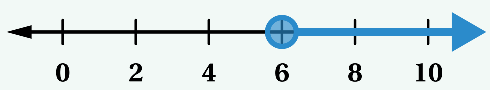
Determine the boundary point:

$$2x - 4 = 8$$
$$2x = 12$$
$$x = 6$$

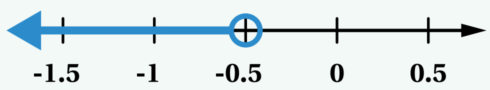
Determine the region:

$$2x - 4 \geq 8$$
$$2(0) - 4 \geq 8$$
$4 \geq 8$ ***False!***

- 6 is the boundary point and the ≥ symbol means it is included in the solution set.
- Since this statement is false when we substitute 0, we know 0 is not in the solution set.

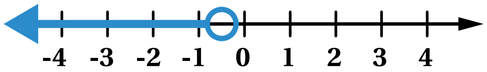


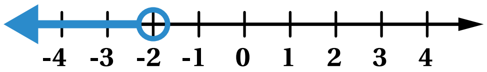
When a boundary point is not included in the solution set, this is represented with an open circle on the number line. Here is an example of the solution $-0.5 > x$ graphed on a number line.

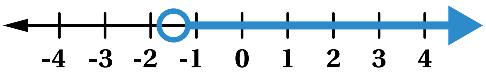


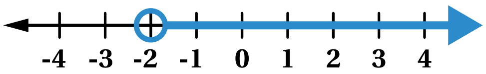
Name: .... Date: .... Period: ....

1. Which graph represents the solutions to $2x < -4$?

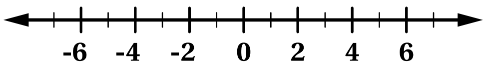
A.


B.


C.


D.


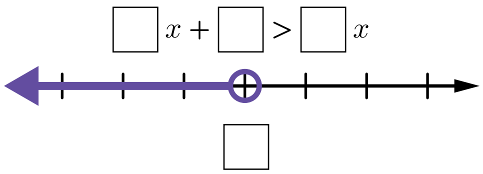
2. Create a graph of the solutions to $2x \leq 10$.



3. Leo is trying to solve $-15 + x < -14$. He knows the boundary point is at $x = 1$. How can he determine whether the solutions are $x < 1$ or $x > 1$?

4. Diego says that $x = 5$ is a solution to $-3x > 9$ because when you divide both sides by -3, you get $x > -3$. Is this correct? Explain your thinking.

5. Create an inequality and the graph of its solutions by filling in each blank using the numbers 0 to 9 only once.



6. Write an inequality where all the solutions are $x > 2$.

Lesson Practice
A1.11

Name: .... Date: .... Period: ....

Test Practice

7. What are the solutions to $-2(2 + c) + 4 < 6$?

A. $c > -3$

B. $c < -3$

C. $c > -2$

D. $c < -2$

Spiral Review

Problems 8–10: A community pool offers two different membership plans.

Plan A	Plan B
\$4 per visit	An initial \$12 fee, then \$2 per visit

Brielle wants to spend no more than \$48 at the community pool this month.

8. How many times could she visit the pool with Plan A?

9. How many times could she visit the pool with Plan B?

10. After how many visits will the cost of both plans be the same?

Unit 1
Lesson
12

Name: ________ Date: ________ Period: ________

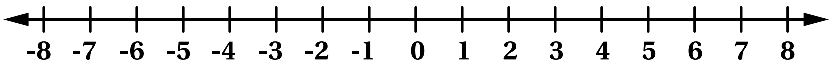
MA.912.AR.2.6, MTR.1.1, MTR.4.1

Solutions and Sheep

Let's make connections between solving one-variable equations and solving inequalities.

Warm-Up

1. **a** Plot *three* different solutions to the inequality $10 - 5x < 0$.



Share your response with your classmates.

b **Discuss:** Are any of the points incorrect? How do you know?

2. Let's look at all the correct solutions to $10 - 5x < 0$.

Kayleen and Leo are discussing how to write the solutions to this inequality.

Kayleen says the solution is $x < 2$.

Leo says the solution is $x > 2$.

Who is correct? Circle one.

Kayleen Leo Both Neither

Explain your thinking.

Activity
1

Name: ______ Date: ______ Period: ______

Feed the Sheep

3. Shira the Sheep loves eating grass. She does not like water.

Discuss: What do you notice? What do you wonder?

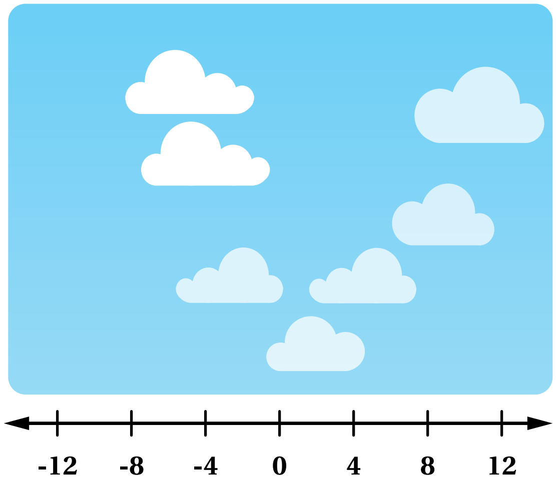
4. Here is an inequality: $\frac{x}{3} \geq 5$

Solve the inequality to help Shira eat all the grass.

Feed the Sheep (continued)

5. Here is a new inequality: $9 > 2x - 4$

Solve the inequality to help Shira eat all the grass.



6. Here is some of Kayleen's work from the previous problem.

$\boxed{9 > 2x - 4}$

$9 = 2x - 4$

$13 = 2x$

$6.5 = x$

Test

$x = 6$	$x = 7$
$9 > 2(6) - 4$	$9 > 2(7) - 4$
$9 > 8$	$9 > 10$
✓	False!

a **Discuss:** What do you notice and wonder about Kayleen's strategy?

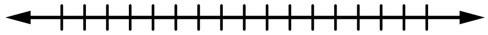
b Describe how Kayleen's work can help her decide which way to shade the solution set on the number line.

Activity
2

Name: ________ Date: ________ Period: ________

Solving for Sheep

7. Here is a new inequality: $8x - 4 < 10x + 2$.



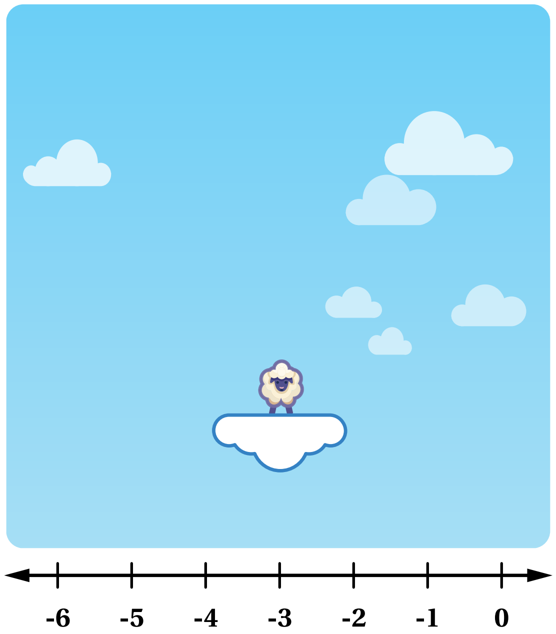
Solve the inequality to help Shira eat all the grass.

8. Leo was trying to solve the previous inequality: $8x - 4 < 10x + 2$.

He knew the sheep needed to land at -3, but didn't know if the grass was to the *right* or the *left*.

He wrote $8(0) - 4 < 10(0) + 2$.

How might that help Leo decide where the grass is?



Name: Date: Period:

Solving for Sheep (continued)

9. Here is a new inequality: $-2(x+2) \le -23$

Solve the inequality to help Shira eat all the grass.

10. For each of the challenges:

- Decide with your partner who will complete Column A and who will complete Column B.
- Solve as many inequalities as you have time for.
- The solutions in each row should be the same. Compare your solutions, then discuss and resolve any differences.

Column A	Column B
$2x + 10 > 20$	$\frac{x}{5} + 9 > 10$
$\frac{x}{5} + 10 \le 12$	$5x - 10 \le 40$
$-10 \ge 4x + 4$	$14 \ge 2x + 7$
$3 - 2x > 3$	$8 > 4x + 8$
$\frac{x+3}{5} < 2$	$\frac{x+8}{5} < 3$
$5x + 10 \ge 3x + 12$	$2x - 4 \ge x - 3$

Synthesis

Describe a strategy for solving any inequality.

Use the examples if they help with your thinking.

$10 - 5x < 0$

$9 > 2x - 4$

$8x - 4 < 10x + 2$

Lesson Practice A1.12

Lesson Summary

One strategy that you can use to solve any one-variable inequality is to:

- Solve the equation that corresponds with the inequality to determine the boundary point.
- Then test value(s) greater than or less than the boundary point to determine where the solution region is located.

Here is an example of how you could solve the inequality $10 > -3x - 2$.

Solve the corresponding equation:

$10 = -3x - 2$

$12 = -3x$

$-4 = x$

The boundary point is $x = -4$.

Test $x = -5$	Test $x = 0$
$10 > -3(-5) - 2$	$10 > -3(0) - 2$
$10 > 13$	$10 > -2$
False!	***True!***

When $x = -5$, the inequality is false, so the solutions are greater than -4.

The solution to an inequality does not always have the same inequality symbol as the original inequality. Since the value that is greater than -4 is true, the solution set to the inequality can be written as $-4 < x$ or $x > -4$.

Lesson Practice
A1.12

Name: .... Date: .... Period: ....

Problems 1–4: Solve each inequality.

1. $4x + 5 \geq 37$

2. $\text{-}3x + 4 \geq 12$

3. $\text{-}6 + \frac{x}{2} < 7$

4. $8x - 6 > 2x - 26$

5. Here is an inequality: $7x + 6 < 3x + 2$. Select all values that are solutions.

- ☐ **A.** $x = 1$
- ☐ **B.** $x = 0$
- ☐ **C.** $x = \text{-}1$
- ☐ **D.** $x = \text{-}2$
- ☐ **E.** $x = \text{-}8$

6. Write three values of n that make this inequality true: $\text{-}5(n + 1) > 10$.

7. Make the two inequalities equivalent by filling in each blank using the numbers 0 to 9 only once.

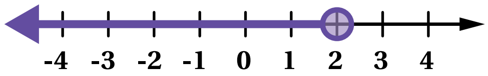
$$\square\, x + \square < \square\, x + \square$$

$$x > \square$$

Name: .... Date: .... Period: ....

Test Practice

8. Which inequality is represented by the graph?



A. $2x + 6 < 10$

B. $2x + 6 \geq 10$

C. $-2x + 6 \geq 10$

D. $-2x - 6 \geq -10$

Spiral Review

Problems 9–10: Rewrite each expression as a single power of 10.

9. $\frac{10^2 \cdot 10^5}{10^4}$

10. $(10^4)^2 \cdot (10^2)^2$

11. Imani is going shopping with a budget of \$125. Which inequality represents the amount of dollars, x, that Imani can spend while shopping?

A. $x \leq 125$

B. $x \geq 125$

C. $x > 125$

D. $x < 125$

Name: ________ Date: ________ Period: ________

MA.912.AR.2.6, MTR.1.1, MTR.2.1, MTR.4.1, MTR.6.1

Tick Tock

Let's solve one-variable linear compound inequalities.

Warm-Up

According to the American Academy of Sleep Medicine, teenagers aged $13–18$ should sleep $8–10$ hours per night. However, most teens only get about $6.5–7.5$ hours of sleep per night.

1. Do you sleep the same number of hours every night or do you have a range?

2. What is the maximum amount of hours you typically sleep?

3. What is the minimum amount of hours you typically sleep?

4. Shade the squares to show all the hours your classmates might sleep in one night.

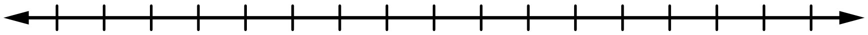
0	1	2	3	4	5	6	7	8	9	10	11	12

Activity 1

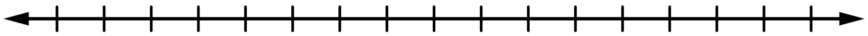
Name: ______________ Date: ________ Period: ________

Need More Sleep

5. Write an inequality and use a number line to plot less than the **maximum** hours (h) that you typically sleep.

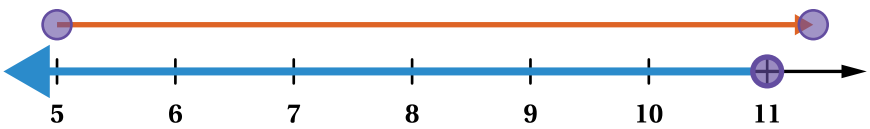


6. Write an inequality and use a number line to plot more than the **minimum** hours (h) you typically sleep.

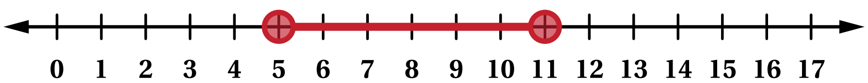


7. A teenager sleeps a minimum of 5 hours and a maximum of 11 hours per night. Look at the stacked inequality graph and the compound inequality graph.

Stacked Inequality



Compound Inequality



8. **Discuss:** Which graph of this inequality do you agree with and why ?

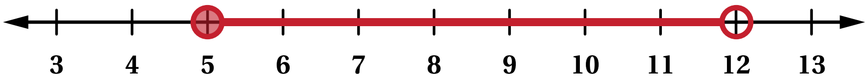
Activity 1

Name: ______ Date: ______ Period: ______

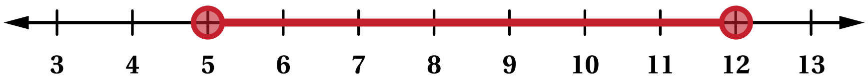
Need More Sleep (continued)

9. Describe the solution for the given graph.

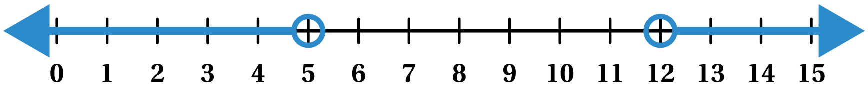
Graph A:



Graph B:



Graph C:



Complete the table to show the range of hours slept, the compound inequality, and the matching graph from Problem 9.

Range of Hours Slept	Compound Inequality	Graph
5 or more hours and less than 12 hours		
	$5 \leq h \leq 12$	
less than 5 hours or greater than 12 hours		

10. **a** Tell a story about sleep that represents this compound inequality graph.

b Naoki wrote the compound inequality $3 < s \leq 10$ to represent the graph. Do you agree or disagree with the inequality she wrote and why?

Activity 2

Name: ______ Date: ______ Period: ______

Solving Compound Inequalities

In a compound inequality, "**and**" and "**or**" have different meanings. "**And**" means **both** conditions must be true. "**Or**" means ***at least one*** condition is true.

Example: Let's start with a compound inequality from Activity 1: $5 \le h < 12$. If 2 is added to the variable h, you get $5 \le h + 2 < 12$. How could you solve for h? You can break this into two separate inequalities and use "and."

Solve for h if $5 \le h + 2$ **and** $h + 2 < 12$.

$5 \le h + 2$	$h + 2 < 12$
$3 \le h$ and $h \le 10$	

11. What do you notice and wonder about the example?

Example: Solve for x if $x + 2 < 1$ or $x - 3 > 2$

$x + 2 < 1$	$x - 3 > 2$
$x + 2 - 2 < 1 - 2$ $x <$ -1	$x - 3 + 3 \le 2 + 3$ $x \le 5$
$x <$ -1 or $x \le 5$	

12. What do you notice and wonder about the example?

Activity 2

Name: ________ Date: ________ Period: ________

Solving Inequalities (continued)

You can solve a single compound inequality by breaking it into two separate inequalities. The answer will be “and.”

13. Solve $-2 \leq 4c - 6 \leq 10$ by creating two inequalities.

$-2 \leq 4c - 6$	$4c - 6 \leq 10$

Another way to solve a compound inequality is by isolating the variable.

14. Solve the compound inequalities by isolating the variable.

$7 < y + 2 \leq 15$	$21 < 3t \leq 30$

15. **Discuss:** How did you solve the inequalities?

16. Imani’s solution for $19 < 3 - 4m \leq 35$ is $-4 < m \leq -8$. Is Imani’s solution correct? Circle one.

Yes No I’m not sure

Show or explain your thinking.

Activity 3

Name: ______ Date: ______ Period: ______

Pizza Party

The math club earned a pizza party with a budget between $50 and $100. They aren't sure how many students will attend. Let s equal the number of students who will attend the party.

17. The students put together four options based on local restaurants. Write and solve a compound inequality with one variable representing each option's cost based on the budget minimum and maximum. Show your work.

By the Slice	Whole Pizzas
• Pizza is $3 a slice. • Fountain drinks are $2 each. • 2 gallons of ice cream is $15.	• A whole pizza is $20 and feeds 5 students. • Cans of soda are $1 each. • 2 cakes is $20.
All You Can Eat Buffet	**Buffet Group Discount**
• The buffet includes drinks and desserts. • The buffet is $10 per person.	• The buffet includes drinks and desserts. • The buffet is $10 per person. • There is a $40 discount for over 14 people.

Name: ______ Date: ______ Period: ______

Pizza Party (continued)

18. **Discuss:** Which option could serve the most students within the budget?

19. **Discuss:** Can the buffet with a discount be used for this group's budget?

20. **Discuss:** If there are 15 students, which one is the best deal? Can they all be within the budget? Explain your reasoning.

Synthesis

21. What do the numbers on either side of a compound inequality represent?

22. How can you solve a compound inequality?

Lesson Practice A1.13

Lesson Summary

- A compound inequality represents a solution set with both a maximum and a minimum.
- There are two types of compound inequalities.
  - **"And" Inequalities:** These inequalities are written as a single statement, like $a < x \leq b$, or as two separate inequalities, like $x > 4$ and $x \leq 10$. The solution to an "and" inequality is the **intersection**, which satisfies both inequalities at *the same time*.
  - **"Or" Inequalities:** These inequalities have two separate conditions, like $2x \leq 2$ or $x > 7x$. The solution is the **union** of both conditions and satisfies **at least one part** of the inequality.
- You solve compound inequalities by two options:
  - Create two inequalities and solve them separately before combining the solution set.
  - Isolate the variable by performing the same operations on each part of the inequality.
- When plotting a compound inequality, you'll have either a solid or open circle on both ends of the line segment.
- Remember, reverse the direction of the inequality sign when multiplying or dividing by a **negative number** in an inequality.

Lesson Practice
A1.13

Name: ________ Date: ________ Period: ________

1. Choose the words to complete the sentence:

range	minimum	variable	maximum

A compound inequality represents the ________ and the ________ for the ________ of the solution set.

Problems 2 and 3: Select the compound variable that represents the scenario.

2. Part-time employees can work no fewer than 20 hours and no more than 30 hours per week

A. $20 \le h \le 30$ B. $20 < h < 30$ C. $20 < h \le 30$ D. $20 \le h < 30$

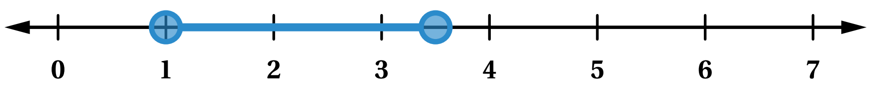
3. The temperature for the day will be more than 70 degrees but not more than 90 degrees Fahrenheit.

A. $70 \le t \le 90$ B. $70 < t < 90$ C. $70 < t \le 90$ D. $70 \le t < 90$

Problems 4–6: You are a part of a work-study program. The guidelines state you must work between 5 and 10 hours each week. You always work for 3 hours on Saturday. During the weekdays, you can work for 2 hours each day. How many days a week must you work to meet the program requirements?

4. Write and solve a compound inequality to determine the acceptable range of weekday (d) work shifts to meet the program requirements.

5. Plot the solution set on a line graph.



Lesson Practice
A1.13

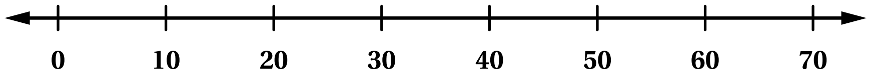
Name: .... Date: .... Period: ....

6. How do you interpret the maximum?

Problems 7 and 8: Your monthly budget includes spending between \$200 and \$400 on groceries and dining out. You know you will spend \$160 on groceries, and you want to budget to go out to eat once a week.

7. Let x represent the amount you spend dining out each week. Write and solve a compound inequality to determine the range of spending on dining out to stay within your monthly budget.

8. Plot the solution set on a line graph.



Test Practice

9. A recipe requires the oven temperature, measured in Farenheit, to be between 350 and 375 degrees, but it includes a 5-degree buffer for cooking effectiveness.

Which compound inequality represents this situation?

A. $350 \le c + 5 \le 375$ B. $350 < c - 5 < 375$ C. $350 \le c + 5 < 375$ D. $350 < c - 5 \le 375$

Spiral Review

Problems 10 and 11: Simplify the expression.

10. $\frac{x^{10}}{x^2}$

11. $(x^3 \cdot y^{-2})^2$

Unit 1
Lesson
14

Name: .... Date: .... Period: ....

MA.912.AR.4.1, MTR.2.1, MTR.5.1

Absolute Value Solutions

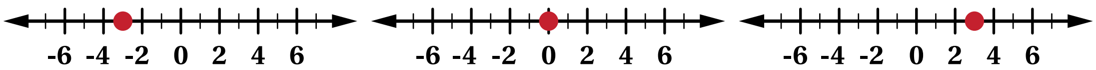
Let's analyze and solve absolute value equations.

Warm-Up

1. $|x|$ is pronounced "the **absolute value** of x."

a Here are three different values of $|x|$.

$|-3| = 3$ $\quad$ $|0| = 0$ $\quad$ $|3| = 3$



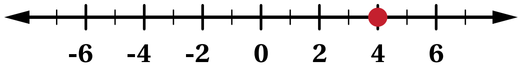
b How would you explain to someone else what $|x|$ means?

Activity 1

Name: ______ Date: ______ Period: ______

Showing Solutions

2. Here is one solution to $|x| = 4$.

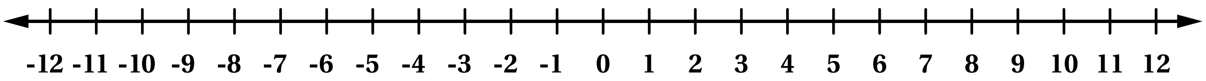


Show another solution to $|x| = 4$.

Explain your thinking.

3. **a** Determine the solutions for $|x| + 3 = 4$? You may use the number line, if needed.

Explain how you know the solutions are correct.



b Write an equivalent equation for $|x| + 3 = 4$. Justify your reasoning algebraically.

c What do you notice about the relationship between the solutions you found in part b? How are they related to each other on the number line?

4. **Discuss:**

- Based on problems 2 and 3, how many solutions should an equation in the form of $|x| = a$, where $a > 0$, have?
- How do the locations of the solutions relate to the position of 0?

Activity 2

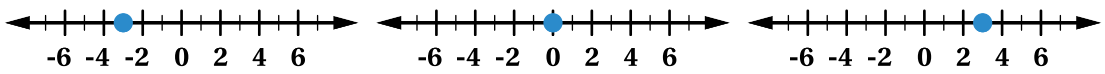
Name: .... Date: .... Period: ....

Solving Strategies

5. Polina says the value of $|x - 2|$ is the distance between any number and 2.

a Here are three different values of $|x - 2|$.

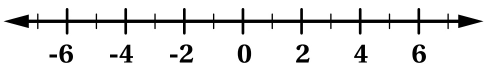
$|-3 - 2| = 5$ $\quad$ $|0 - 2| = 2$ $\quad$ $|3 - 2| = 1$



b **Discuss:** Do you agree with Polina?

6. **a** Here are three different values of $|x - 2| = 4$.

$|x - 2| = 4$



b Use substitution to show that your solutions from part a are correct.

7. Tay thinks that the solutions to absolute value equations can be found algebraically by rewriting the absolute value equation as two separate equations and then solving each one.

a Solve these equations using Tay's method to see if it works.

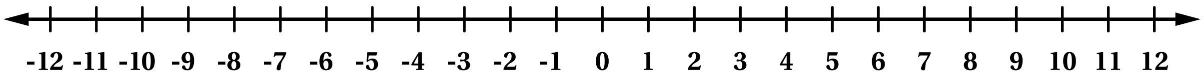
$(x - 2) = 4$ $\quad$ or $\quad$ $-(x - 2) = 4$

b Explain whether Tay's method of solving worked.

Name: ______________________ Date: __________ Period: __________

Solving Strategies (continued)

8. **a** Show or describe all the solutions for $|x + 3| = 5$.



b What do you notice about the relationship between the equation and the solutions you found on the number line?

9. Ramon and Tyler each solve $|x + 3| = 5$ algebraically.

Ramon says that two related equations should be written and solved to find the solutions. Tyler says that only one equation should be written and solved, obtaining two solutions by using the positive and negative values of the solution. Analyze the reasoning shown below.

Ramon's Solution Method	Tyler's Solution Method
$(x + 3) = 5$ or $(x + 3) = -5$ $x + 3 - 3 = 5 - 3$ or $x + 3 - 3 = -5 - 3$ $x = 2$ or $x = -8$ The solutions are $x = 2$ and $x = -8$.	$(x + 3) = 5$ $x + 3 - 3 = 5 - 3$ $x = 2$ The solutions are $x = 2$ and $x = -2$.
Ramon states that the two equations reflect the outcomes. "Either what is in the parentheses is equal to 5 or the opposite of what is in the parentheses is equal to 5."	Tyler states that the absolute value equation means there is a positive and negative solution, and the solutions are opposites.

a Whose method of solving is correct? Circle one.

Ramon Tyler Both Neither

b Explain your thinking.

Name: ______ Date: ______ Period: ______

Absolute Challenge

10. Once an absolute value equation has been written as two equations to represent each possible solution, continue to apply properties of equality to solve each one.

a Solve $|2x| = 8$ by writing two equations that represent each possible solution.

b How can you interpret your solutions in terms of distance?

11. Eva and Mai both solve the absolute value equation $|6 - 2x| = 8$ using different reasoning. Analyze the solution methods and answer the questions that follow.

Eva's Solution Method	Mai's Solution Method
$(6 - 2x) = 8$ or $-(6 - 2x) = 8$ $-6 \quad -6$ or $(6 - 2x) = -8$ $-2x = 2$ or $-6 \quad -6$ $x = -1$ or $-2x = -14$ $x = 7$	$(6 - 2x) = 8$ or $-(6 - 2x) = 8$ $-6 \quad -6$ or $-6 - 2x = 8$ $-2x = 2$ or $+6 \quad +6$ $x = -1$ or $-2x = 14$ $x = -7$

a Which student made an error? What was the error?

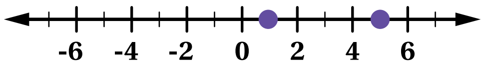
b Show how the method being used can work if the error is fixed.

12. Solve the absolute value equation $|-3x - 2| = 16$ by first creating two equations that represent the two possible solutions.

Synthesis

13. Consider strategies you could use to solve the absolute value equation $|x - 3| = 2$.

$| x - 3 | = 2$



a How does a number line help you understand the solutions to this equation?

b How can you solve this equation algebraically?

Lesson Practice A1.14

Lesson Summary

The **absolute value** of a number is its distance from 0 on a number line. Use the definition of absolute value to help with understanding how to solve absolute value equations.

For an absolute value equation, there can be two input values with the same output.

When solving an absolute value equation that contains values on the outside of the absolute value, first isolate the absolute value by using properties of equality to rewrite the equation.

- Example: $|x| - 5 = 3$

  Add 5 to each side:

  $|x| - 5 + 5 = 3 + 5$

  $|x| = 8$

- Rewrite the absolute value as two separate equations and solve.

  $(x) = 8$ or $-(x) = 8$

  $x = 8$ or $x = -8$

- Check your solutions by substituting them into the original absolute value equation.

  $|8| - 5 = 3$ or $|-8| - 5 = 3$

  $8 - 5 = 3$ or $8 - 5 = 3$

When solving an equation that contains an expression inside the absolute value, use the definition of absolute value to create two equations that each represent the solutions to the original equation. Then apply properties of equality to solve each equation.

- Example: $|2x + 1| = 3$ can be written as two equations:

  $(2x + 1) = 3$ or $-(2x + 1) = 3$

  $2x = 2$ $(2x + 1) = -3$

  so $x = 1$ or $2x - 5$ so $x = -2$

  $x = 1$ **or** $x = -2$

- Check your solutions by substituting them into the original absolute value equation.

  $|2(1) + 1| = 3$ or $|2(-2) + 1| = 3$

  $|3| = 3$ or $|-3| = 3$

Lesson Practice
A1.14

Name: .... Date: .... Period: ....

1. Match each absolute value equation to its meaning in words.

A. $|x| + 4 = 6$ .... The distance of four times x from 0 is 6.

B. $|4x| = 6$ .... The distance of x from 4 is 6.

C. $|x - 4| = 6$ .... Four times the distance of x from 0 is 6.

D. $4|x| = 6$ .... Four more than the distance of x from 0 is 6.

Problems 2–5: Solve the absolute value equations algebraically.

2. $|x - 5| = 8$

$x =$ .... or $x =$ ....

3. $4|x + 3| = 24$

$x =$ .... or $x =$ ....

4. $|1 - 2x| = 7$

$x =$ .... or $x =$ ....

5. $|\frac{1}{2}x| - 3 = 11$

$x =$ .... or $x =$ ....

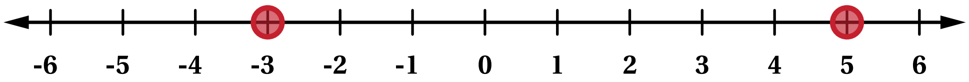
6. Which equations show the cases that you must consider when solving $|6x - 12| = 18$?

- ☐ A. $6x - 12 = 18$
- ☐ B. $6x + 12 = 18$
- ☐ C. $-6x - 12 = 18$
- ☐ D. $-6x + 12 = 18$

Lesson Practice
A1.14

Name: .... Date: .... Period: ....

7. The graph shows two solutions to an equation that contains an absolute value. Explain how you can find an equation with these solutions. Include your equation in your response.



8. Solve the absolute value equation: $8 - |\frac{2}{3}x + 3| = -13$.

Test Practice

9. Solve the absolute value equation: $2 + |3x - 7| = 6$. Choose all solutions.

A. $x = \frac{4}{3}$

B. $x = -\frac{11}{3}$

C. $x = \frac{11}{3}$

D. $x = -\frac{4}{3}$

E. $x = -3$

F. $x = 1$

G. $x = 3$

H. $x = -1$

Spiral Review

Problems 10–11: Use the expression: $-\frac{5}{6}x\left(\frac{3}{4}x - \frac{2}{5}\right)$.

10. What is the product of the expression?

11. What is the value of the expression when $x = \frac{4}{3}$?

Unit 1
Lesson
15

Name: ________ Date: ________ Period: ________

MA.912.AR.4.1; MTR.1.1, MTR.4.1, MTR.5.1, MTR.6.1, MTR.7.1

Getting Absolute

Let's create and solve absolute value equations.

Warm-Up

1. Dylan and Elena use different methods to solve $|x - 7| = 2$.

Dylan's work:		**Elena's work:**	
$|x - 7| = 2$		$|x - 7| = 2$	
$x - 7 = 2$	$x - 7 = -2$	$x - 7 = 2$	$-(x - 7) = 2$
$x = 9$	$x = 5$	$x = 9$	$x - 7 = -2$
			$x = 5$

a What do you notice? What do you wonder?

b The teacher says both Dylan's and Elena's work is correct; they just used different methods. Explain why both methods are correct.

c Whose method do you prefer and why?

Activity 1

Name: ________ Date: ________ Period: ________

How Many Solutions?

2. In order to determine how many solutions an absolute value equation has, it must first be simplified. Ethan and Kai each try to simplify $-3|x - 2| = -9$. Explain what each student did, who is correct, and why.

Ethan's work:	Kai's work:
$-3\|x - 2\| = -9$ $\|-3x + 6\| = -9$	$-3\|x - 2\| = -9$ $\|x - 2\| = 3$

3. The table below shows absolute value equations sorted by the number of solutions they have.

Two Solutions	One Solution	No Solution
$\|x\| = 2$	$\|x\| = 0$	$\|x\| = -2$
$\|5x - 9\| = 6$	$\|5x - 9\| - 6 = -6$	$\|5x - 9\| = -6$
$\|\frac{3}{4}x - 5\| = 19$	$\|\frac{3}{4}x - 5\| = 0$	$-2\|\frac{3}{4}x - 5\| = 38$

a **Discuss:** What do you notice about the equations in each column?

b Sort the following absolute value equations by writing them in the appropriate column of the table.

$|x| = -18$ $|x| = 68$ $|x - 12| = -4$ $|\frac{2}{3}x - 9| = -13$

$|2x + 5| = 7$ $|\frac{5}{6}x + 3| = 0$ $|7 - 9x| + 3 = 3$ $-4|x - 2| = -24$

Two Solutions	One Solution	No Solution

Activity 2

Name: ________ Date: ________ Period: ________

Analyze and Solve

4. Oscar and Neo both go to the same high school. Oscar lives 6 miles from the school. Neo lives 5 miles from Oscar.

a Arjun and Eliza each write absolute value equations to represent the location of Neo's house in relation to the high school.

Arjun's equation: $|x - 5| = 6$

Eliza's equation: $|x - 6| = 5$

Who is correct? Circle one and explain your thinking.

Arjun Eliza Both Neither

b How far could Neo live from his school?

5. Taylor is playing a trivia game against her friends. She gets 15 points for each correct answer and loses 15 points for each incorrect answer. Her current score is 45 points.

a Write an absolute value equation to model Taylor's score after the next question.

b What are the possible scores Taylor could have after the next question?

Name: .... Date: .... Period: ....

Analyze and Solve (continued)

6. A car dealership is having a contest where the prize is a new car. In order to win a chance at the prize, participants must first guess the number of keys in a jar within 3 of the actual number. Participants who guess within this range get to try a key in the ignition of the car.

a Suppose there are 437 keys in the jar. Write an absolute value equation that will reveal the highest and lowest guesses participants could make to get a chance at winning the car.

b What are the highest and lowest guesses that will qualify for a chance to win?

7. Lucy and Joel are on a scavenger hunt. Lucy is 4.5 miles east of the campsite and discovers a clue. Joel realizes that his distance to the clue is 3 miles more or less than twice his distance to the campsite.

a Write an absolute value equation to represent Joel's distance from the campsite. Explain the meaning of your equation in the context.

b What are Joel's possible locations relative to the campsite?

Activity 3

Name: ________ Date: ________ Period: ________

Road Trip

8. You and your friends are on a road trip. The last rest stop you passed, Rest Stop A, is 8 miles behind you. You know there is another rest stop on the same road, Rest Stop B, located four times as far from your current position as Rest Stop A, but you are not sure which direction it is in.

a Write an absolute value equation to represent the distance between your current position and Rest Stop B.

Explain the meaning of your equation in the context.

b How many solutions are there to your equation? How do you know?

c Where is Rest Stop B relative to your current position?

Activity 3

Name: ____________ Date: ______ Period: ______

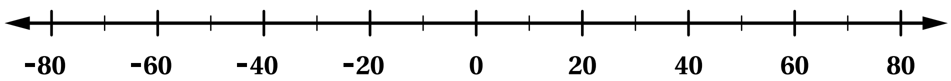
Road Trip (continued)

9. Your friend Jalen is driving. Jalen is always careful to go exactly the speed limit.

a The speed limit is 60 mph. Write an absolute value equation to represent Jalen’s speed.

b How many solutions are there to your equation? How do you know?

c Solve the absolute value equation, then graph the solution on the number line. Discuss how the algebra validates the graph.



10. Another passenger, Mariana, writes the equation $|x - 3| = -25$ to represent the gas mileage of the vehicle.

a How many solutions are there to Mariana’s equation? How do you know?

b Rewrite and explain Mariana’s equation so it makes sense in the context of gas mileage.

c What is the highest and lowest gas mileage the vehicle could have?

Synthesis

11. Describe strategies you could use to write an absolute value equation based on a real-world context.

12. How can you identify how many solutions an absolute value equation has?

Lesson Practice A1.15

Lesson Summary

- The absolute value of a number is always non-negative. It represents the distance of the number from zero on the number line.
- For any absolute value equation in the form $a|bx + c| + d = e$ where a,b,c,d, and e are all real numbers, rewrite as $|bx + c| = f$.
- If $f > 0$, there will be two solutions: one where the expression inside the absolute value equals the positive number and one where it equals the opposite.
  - Example: $|3x - 4| = 5$

    Write and solve two equations.

    $3x - 4 = 5$, $3x = 9$, $x = 3$

    $3x - 4 = -5$, $3x = -1$, $x = -\frac{1}{3}$

    Solutions: $x = 3$ or $x = -\frac{1}{3}$
- If $f = 0$, there will be only one solution because the expression inside the absolute value must be equal to zero.
  - Example: $|x - 7| = 0$

    Write and solve one equation.

    $x - 7 = 0$, $x = 7$

    Solution: $x = 7$
- If $f < 0$, there are no solutions because an absolute value cannot be negative.
  - Example: $|\frac{1}{3}x + 2| = -8$

  This equation has no solution because it is an absolute value set equal to a negative number.

Lesson Practice
A1.15

Name: .... Date: .... Period: ....

1. Circle the equations that show the cases that must be considered when solving $\frac{7}{2} = |3x - 6| + \frac{1}{2}$.

A. $3x - 6 = -3$ **B.** $-4 = 3x - 6$ **C.** $4 = 3x - 6$ **D.** $3x - 6 = 3$

2. The NBA™ regulates the air pressure of basketballs used during games. The highest acceptable air pressure is 8.5 pounds per square inch (psi), and the lowest acceptable air pressure is 7.5 psi. Write an absolute value equation to define the highest and lowest possible pressures, *p*, for NBA™ basketballs.

3. Alejandro and Zoe are making corrections to practice problems they missed. They each have one incorrect problem left, but neither can figure out why their answers are wrong. Describe and correct each of their errors.

Alejandro's incorrect problem:	Zoe's incorrect problem:
$\lvert 2x - 1\rvert = -9$	$\lvert 5x + 8\rvert = x$
$2x - 1 = -9$ $-(2x - 1) = -9$	$5x + 8 = x$ $-(5x + 8) = x$
$2x = -8$ $2x - 1 = 9$	$4x + 8 = 0$ $5x + 8 = -x$
$x = -4$ $2x = 10$	$4x = -8$ $6x + 8 = 0$
$x = 5$	$x = -2$ $6x = -8$
	$x = -\frac{4}{3}$
The solutions are $x = -4$ and $x = 5$.	The solutions are $x = -2$ and $x = -\frac{4}{3}$.

Name: .... Date: .... Period: ....

4. Without solving completely, sort each equation into one of the three categories.

$|x+13|-2=7$ $\quad 1+|x-8|=-2$ $\quad |\frac{3}{4}x-5|+4=4$

$|5x-2|+9=0$ $\quad -3+|\frac{1}{2}x-8|=\frac{1}{3}$ $\quad |x-2|-6=0$

No Solution	One Solution	Two Solutions

Test Practice

5. Consider the absolute value equation $7-|x+5|=3$. Select *all* of the true statements.

A. $x=1$ is a solution.

B.

C. The equation has no solutions.

D.

E.

F. The solutions are numbers that are 5 units away from -4.

Spiral Review

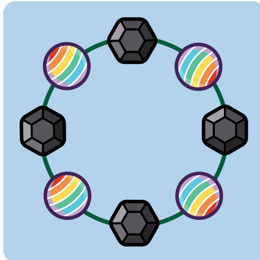
6. Solve the absolute value equation: $|2x-5|+8=22$. Choose *all* solutions.

☐ **A.** $x=-4.5$ ☐ **B.** $x=\frac{19}{2}$ ☐ **C.** $x=-9.5$ ☐ **D.** $x=-\frac{9}{2}$

☐ **E.** $x=-\frac{2}{19}$ ☐ **F.** $x=\frac{2}{9}$ ☐ **G.** $x=9.5$ ☐ **H.** $x=4.5$

Name: ____________ Date: ________ Period: ________

MA.912.AR.2.8, MTR.1.1, MTR.2.1



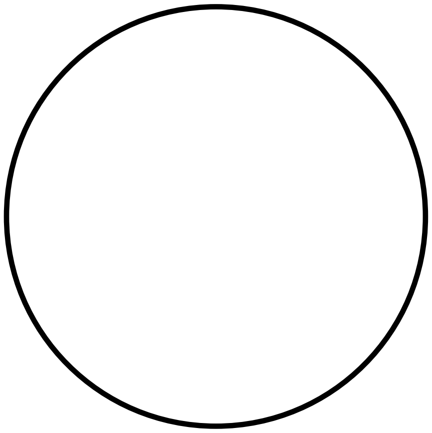
Bracelet Budgets

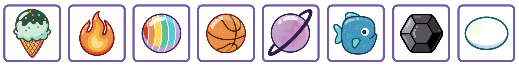
Let's explore solutions to two-variable inequalities graphically and symbolically.

Warm-Up

1. Draw or describe a bracelet. You can use any combination of the beads shown.

Tell us about your bracelet design.





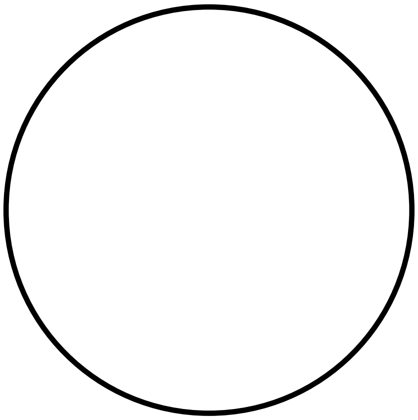
Name: .... Date: .... Period: ....

Modeling with Inequalities

2. Here are two types of beads:

- Black beads are $\$0.50$ each.
- Basketball beads are $\$1$ each.

Draw or describe a $5 bracelet.



 **$0.50 each**

 **$1.00 each**

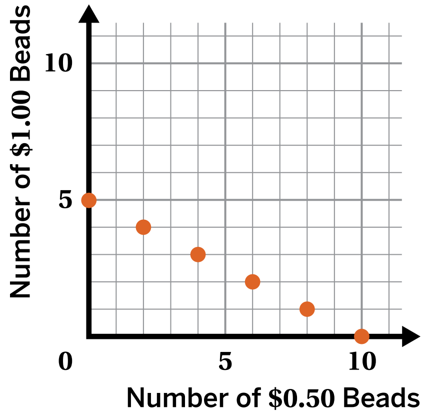
3. Each of these points represents a $5 bracelet.

- x is the number of $\$0.50$ beads.
- y is the number of $\$1.00$ beads.

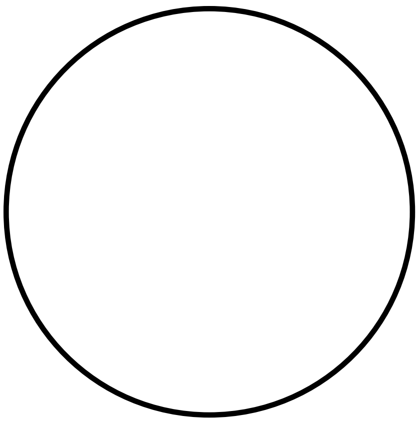
Which equation represents all the $5 bracelets? Circle one.

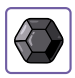
$x + y = 5$ $\quad$ $0.5x + y = 5$ $\quad$ $y = 0.5x + 5$

Explain your thinking.

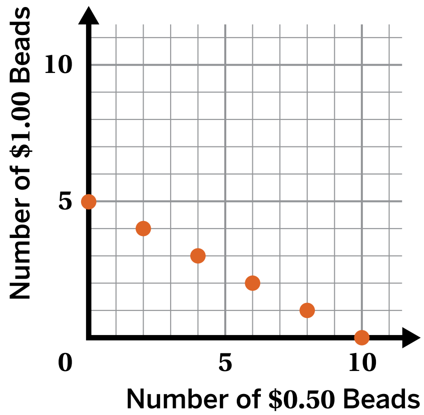


4. Binta can spend $5 or less on a bracelet. Graph some bracelets that Binta could buy. Draw or describe them if it helps with your thinking.



 **$0.50 each**

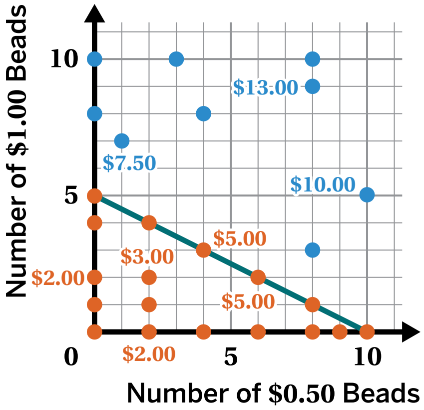
 **$1.00 each**



Name: Date: Period:

Modeling with Inequalities (continued)

5. Here are some bracelets that Binta could buy and some she could not buy for $5 or less.



What do you notice? What do you wonder?

I notice . . .

I wonder . . .

6. Binta can spend $5 or less on a bracelet.

- x is the number of $0.50 beads.
- y is the number of $1.00 beads.

Which statement describes all the bracelets that Binta can buy? Circle one.

$0.5x + y \leq 5$ $\quad$ $0.5x + y \geq 5$ $\quad$ $0.5x + y = 5$

Explain your thinking.

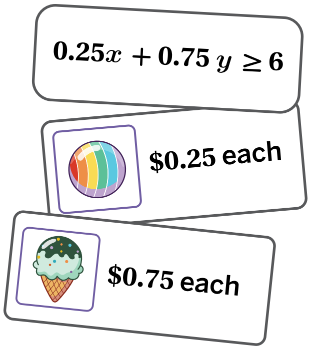
Name: .... Date: .... Period: ....

Solutions to Inequalities

7. Caleb loves Binta's bracelet and wants to make his own.

The inequality represents all the bracelets he can make.

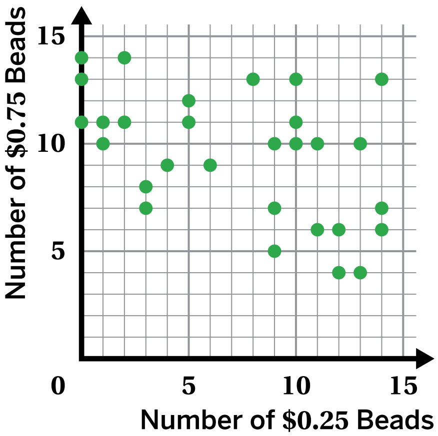
Discuss: What does each number and variable in the inequality represent about Caleb's bracelet?



8. Each of these points is a solution to $0.25x + 0.75y \geq 6$.

a Let's look at several points to see what this means.

b In your own words, what is a solution to an inequality with two variables?



9. Is $(2, 10)$ also a solution to $0.25x + 0.75y \geq 6$? Circle one.

Solution   Not a solution   I'm not sure

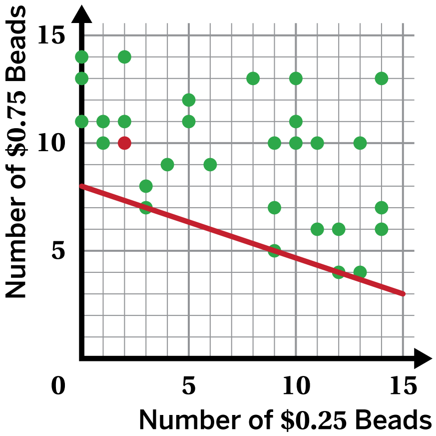
Explain your thinking.

Name: Date: Period:

Solutions to Inequalities (continued)

10. Here is how two students determined that $(2,10)$ is a solution to $0.25x + 0.75y \geq 6$.

Marco



Jada

$0.25x + 0.75y \geq 6$

$0.25(2) + 0.75(10) \geq 6$

$0.50 + 7.50 \geq 6$

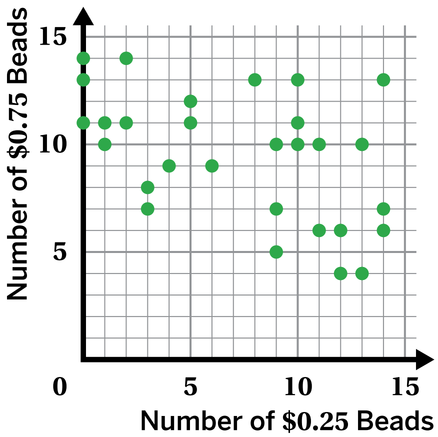
$8 \geq 6$

Explain each person's strategy to a partner.

11. The graph shows some solutions to $0.25x + 0.75y \geq 6$.

Select *all* of the other points that are also solutions.

- ☐ **A.** $(14, 8)$
- ☐ **B.** $(13, 1)$
- ☐ **C.** $(7, 5)$
- ☐ **D.** $(1, 13)$
- ☐ **E.** $(0, 8)$



Activity 3

Name: ______ Date: ______ Period: ______

All About Form

Problems 12–14: Use the given inequality to answer the questions.

12. $3x + 4y \leq 24$

a. Circle the form the inequality is in.

______ slope-intercept form point-slope form

b. Write the inequality in slope-intercept form.

13. $y - 3 \geq 2(x + 1)$

a. Circle the form the inequality is in.

standard form slope-intercept form ______

b. Write the inequality in standard form.

c. Is $(-2,5)$ a solution to the inequality?

14. $y > -\frac{1}{2}x + 5$

a. Circle the form the inequality is in.

standard form ______ point-slope form

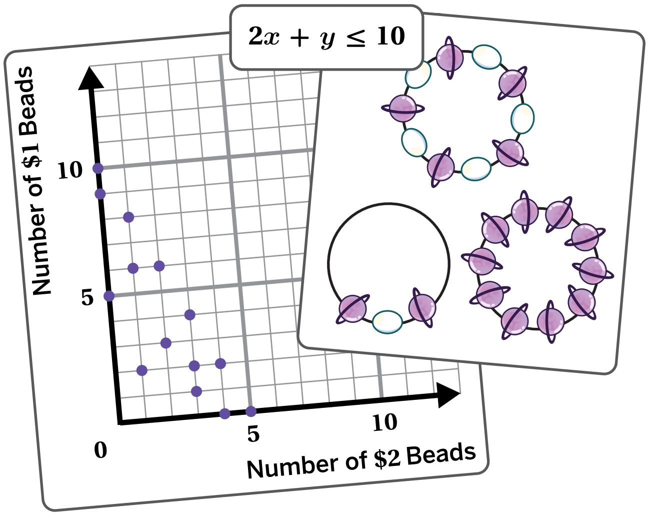
b. Write the inequality in point-slope form.

c. Identify two points that are in the solution region of the inequality.

Synthesis

15. How can you tell if a point is a solution to a two-variable inequality?

Use the example if it helps with your explanation.



Lesson Practice A1.16

Lesson Summary

Two-variable inequalities can be written in standard form, slope-intercept form, or point-slope form. Their solutions are all of the ordered pairs that make the inequality true. Graphs can help us visualize all of these solutions. To check if an ordered pair is a solution symbolically, you can substitute the x- and y-values into the inequality to see if it makes the inequality true.

Here is an example of how you can determine if an ordered pair is a solution to an inequality:

Marco is making bracelets. Each bracelet needs to cost no more than $\$10$. Planet beads cost $\$1$ and oval beads cost $\$2$. Marco wants to know if he can make a bracelet with 3 planet beads and 4 oval beads.

To check, Marco looked at the graph of this situation to see if the point $(4{,}3)$ was in the solution region, but he wasn't sure. Marco substituted 4 and 3 into the inequality $2x + y \leq 10$:

$$2(4) + (3) \leq 10$$

$$8 + (3) \leq 10$$

$$11 \leq 10 \quad \textit{False}!$$

That means Marco cannot make a bracelet with 3 planet beads and 4 oval beads while staying within his budget.

Name: .... Date: .... Period: ....

1. Write at least three coordinate pairs that are solutions to the inequality $x \le y$.

Problems 2–3: Tyler can spend up to $\$45$ on hats and socks. A hat costs $\$10$ and a pair of socks costs $\$2.50$.

- h is the number of hats.
- s is the number of pairs of socks.

2. Which inequality represents this situation?

A. $10h + 2.50s > 45$

B. $10h + 2.50s < 45$

C. $10h + 2.50s \ge 45$

D. $10h + 2.50s \le 45$

3. Explain how you know that $h = 2$ and $s = 1$ are solutions to this situation.

4. Marco is making a bracelet for his sister using beads that cost $\$0.50$ and $\$0.75$. He cannot spend more than $\$8.00$ on the bracelet.

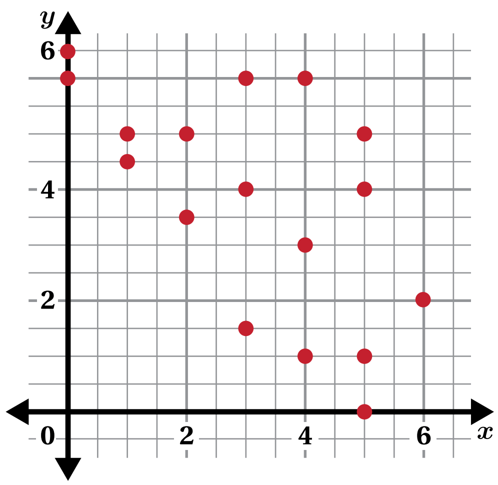
- x is the number of $\$0.50$ beads.
- y is the number of $\$0.75$ beads.

Marco says that the inequality $0.75y \ge 8 - 0.5x$ represents all the bracelets he can make. Do you agree with him? Explain your thinking.

5. Write an ordered pair for a point that is not a solution to $x + 3y \ge 24$.

6. This graph shows some solutions to $12 - 3x \le 2y$.

Write an ordered pair for another point that is a solution to this inequality.

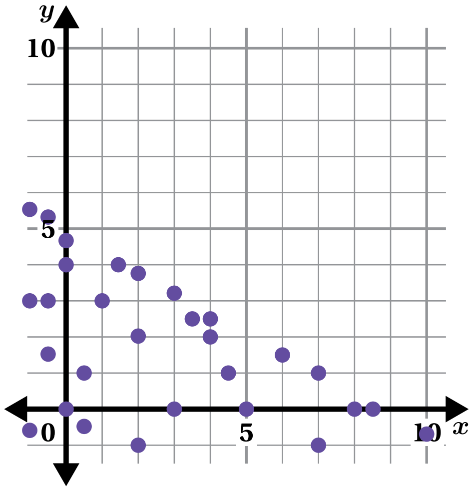


Lesson Practice
A1.16

Name: ______ Date: ______ Period: ______

Test Practice

7. This graph shows some solutions to $5x + 9y < 45$. Select *all* of the points that are also solutions.



- ☐ A. $(1, 1)$
- ☐ B. $(4, 0)$
- ☐ C. $(10, 4)$
- ☐ D. $(0, 10)$
- ☐ E. $(6, -1)$

Spiral Review

Problems 8–9: Write an inequality for each situation.

8. Duri will stay warm in her sleeping bag when the temperature is at least 30°F. Use t to represent temperatures at which Duri will stay warm.

9. Duri wants her backpack to weigh less than 45 pounds. Use w to represent the weight of the backpack.

Problems 10–11: Here is an equation: $6x + 2y = 36$.

10. For each value of x, determine the value of y.

x	y
2	
4	

11. Which equation represents the same relationship?

A. $y = 6 - 3x$

B. $y = 18 - 3x$

C. $y = 18 - \frac{1}{3}x$

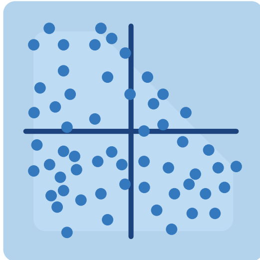
D. $y = 6 - \frac{1}{3}x$

Name: ______ Date: ______ Period: ______

MA.912.AR.2.8, MTR.2.1, MTR.3.1, MTR.4.1

All of the Solutions

Let's represent all of the solutions to two-variable inequalities graphically.



Warm-Up

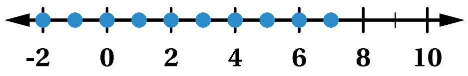
1. Two students created graphs of the solutions to $x \leq 7$.

   How are their graphs alike? How are they different?

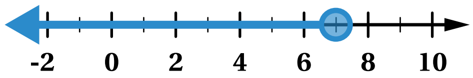
   Alike:

   Different:

Student A



Student B



Activity
1

Name: ______ Date: ______ Period: ______

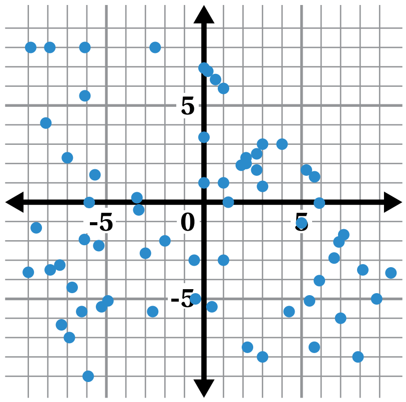
Some to All

2. Write three solutions to $7 - y \geq x$. Try thinking of x- and y-values that no one else will!

Solution 1 (x, y)	Solution 2 (x, y)	Solution 3 (x, y)

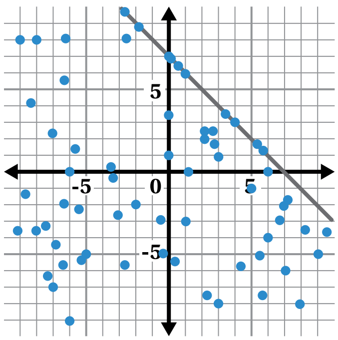
3. Here is a graph of some of the solutions to $7 - y \geq x$.

Sketch what you think the graph of all the solutions to $7 - y \geq x$ looks like.

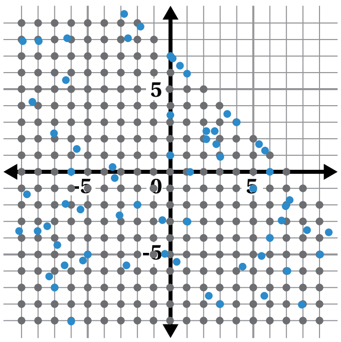


4. Here are Pablo's, Deven's, and Adhira's sketches of *all* the solutions to $7 - y \geq x$.

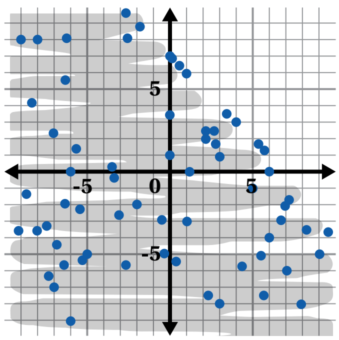
Pablo



Deven



Adhira



a Select *one* sketch.

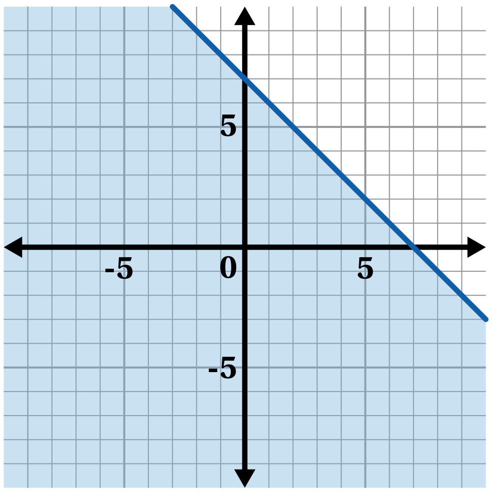
b **Discuss:** What do you like about this sketch? What would you change?

Name: .... Date: .... Period: ....

Shading the Solutions

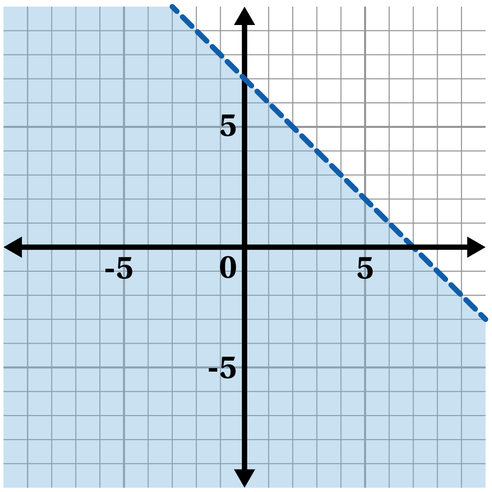
5. **a** Here is what a graph of all solutions looks like.

b How does this graph represent all the solutions to $7 - y \geq x$?

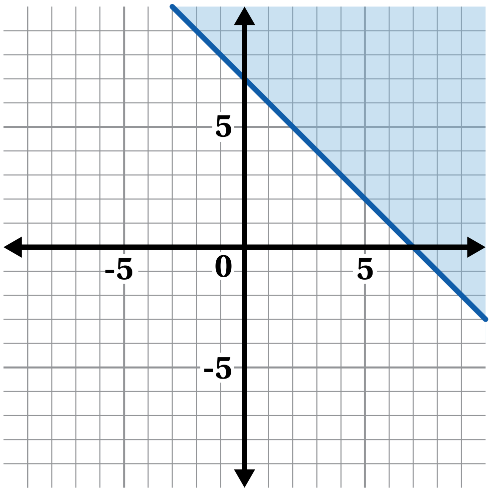


6. Here are the graphs of two other inequalities.

$x + y < 7$



$x + y \geq 7$



Discuss:

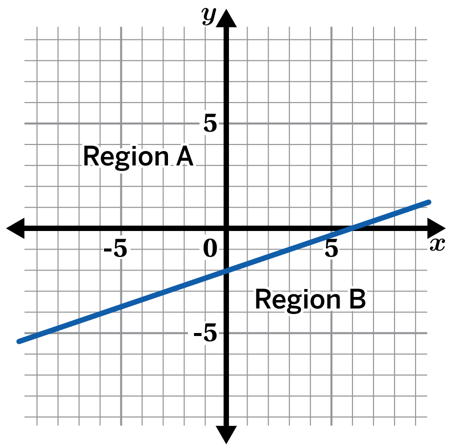
- How are the graphs of the two inequalities different?
- What does the **boundary line** represent?

Activity 3

Name: ______ Date: ______ Period: ______

Which Region?

7. Here is the graph of $x - 3y = 6$.



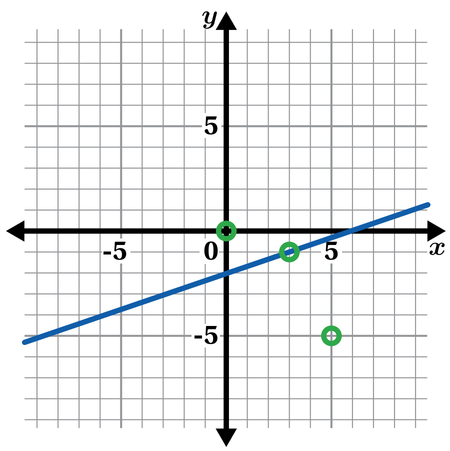
Where will the solutions to $x - 3y \geq 6$ be? Circle one.

Region A Region B I'm not sure

Explain your thinking.

8. Rebecca thought that checking points would help her decide how to graph the solutions to $x - 3y \geq 6$.

She chose the points $(0, 0)$, $(3, -1)$, and $(5, -5)$.

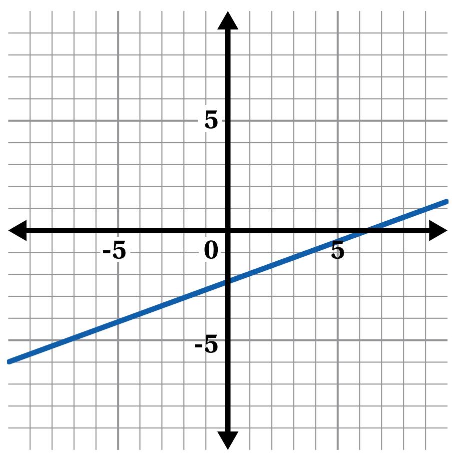


a **Discuss:** Why do you think Rebecca chose these points?

b Select *all* the points that are solutions to $x - 3y \geq 6$.

- ☐ **A.** $(0, 0)$
- ☐ **B.** $(3, -1)$
- ☐ **C.** $(5, -5)$

9. Graph *all* the solutions to $x - 3y \geq 6$.

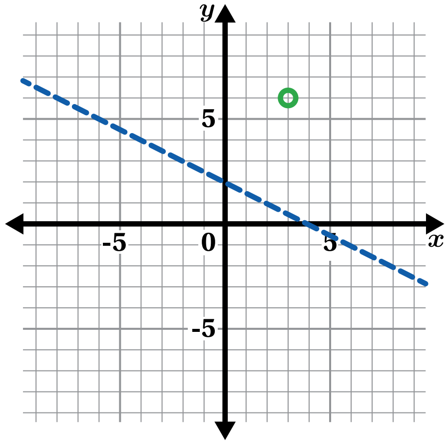


Activity 3

Name: ____________ Date: ________ Period: ________

Which Region? (continued)

10. Nathan is graphing the solutions to $y < -\frac{1}{2}x + 2$.



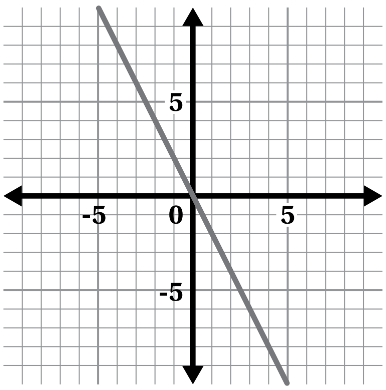
a **Discuss:** Why is his line dashed?

b Nathan determined that $(3, 6)$ is not a solution.

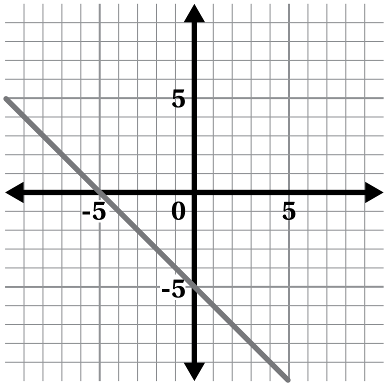
Does he have enough information to graph all the solutions?

11. Graph all the solutions to the following inequalities. The graph of each corresponding equation has been given to you.

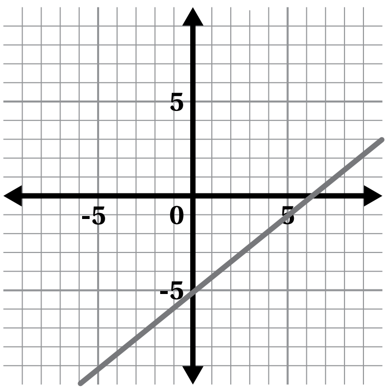
$2x + y \leq 0$



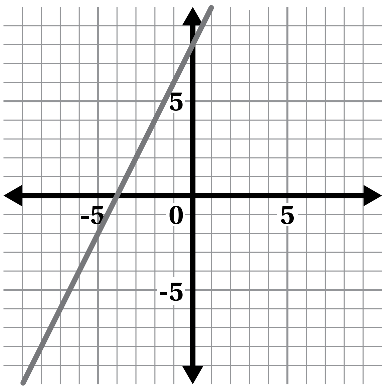
$y > -x - 5$



$y + 5 > \frac{5}{6}(x - 0)$



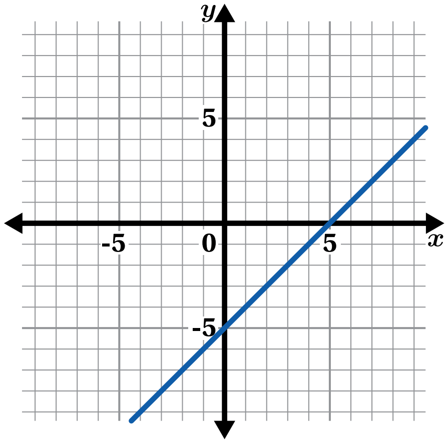
$-2x + y \geq 8$



Synthesis

12. Here is the graph of $y = x - 5$.

Describe a strategy for graphing all the solutions to $y < x - 5$.

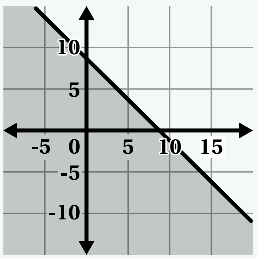


Lesson Practice A1.17

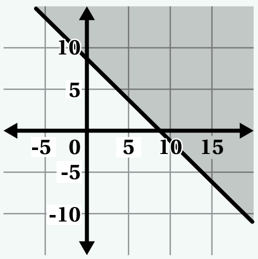
Lesson Summary

The solutions to a two-variable linear inequality can be represented on a graph as a half-plane. A **boundary line** separates the plane into the region that contains solutions and the region that does not. The shaded area represents all of the solutions, which are the values of (x, y) that make the inequality true.

$y \leq -x + 9$

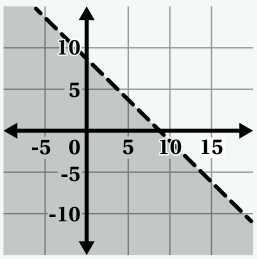


$y \geq -x + 9$

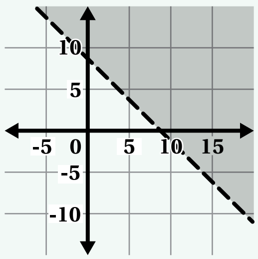


A solid line means that the points on the boundary line *are* included in the solutions. This is represented by the $\leq$ and $\geq$ symbols.

$y < -x + 9$



$y > -x + 9$



A dashed line means the points on the boundary line are *not* included in the solutions.

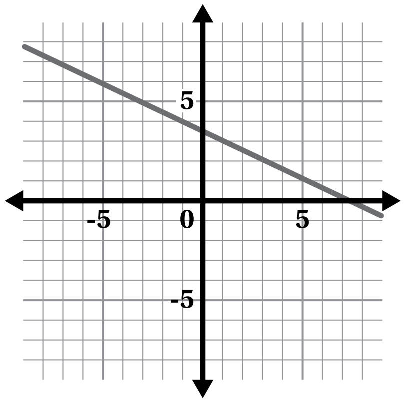
This is represented by the $<$ and $>$ symbols.

To determine which of the half-planes is the *solution region*, you can test points on either side of the *boundary line* to see whether they make the inequality true or false.

Lesson Practice
A1.17

Name: .... Date: .... Period: ....

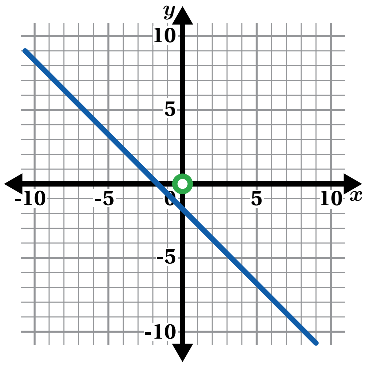
1. Graph all the solutions to $x + 2y < 7$.
Explain your thinking.



2. Here is an inequality: $y \le -x - 2$.

- Ada graphed the equation $y = -x - 2$.
- Ada noticed that $(0, 0)$ is not a solution to $y \le -x - 2$.

How can Ada use this information to graph the solutions to this inequality?



3. Brianna is creating a graph for the inequality $y + 2 < \frac{2}{5}(x - 0)$. She says that since the inequality has a greater-than symbol, she should shade the region above the line $y + 2 = \frac{2}{5}(x - 0)$. Is Brianna correct? Explain your thinking.

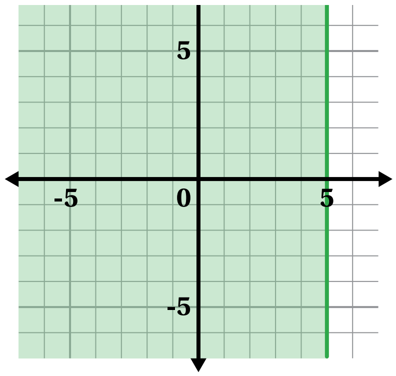
4. Which inequality is shown on the graph?

A. $y \le 5$

B. $y \ge 5$

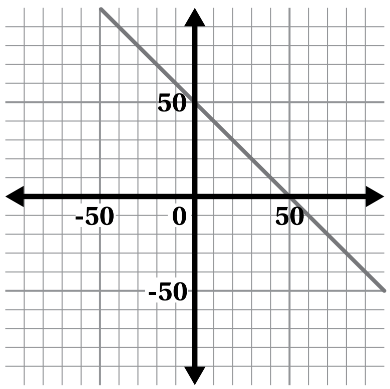
C. $x \ge 5$

D. $x \le 5$



Problems 5–6: A food truck only sells hot dogs and hamburgers. They want to sell 50 items or more each day.

- x represents the number of hot dogs sold.
- y represents the number of hamburgers sold.



5. Which inequality represents this situation?

A. $x + y > 50$

B. $x + y < 50$

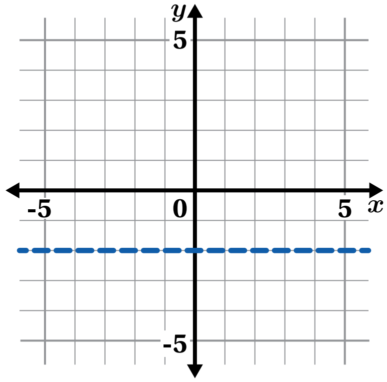
C. $x + y \ge 50$

D. $x + y \le 50$

Name: ______ Date: ______ Period: ______

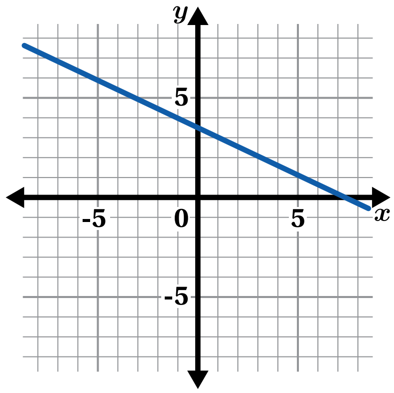
6. Complete the graph so that it represents all the solutions to the inequality for this situation.

7. Lucy started to graph the inequality $y > -2$ by graphing a dashed line at $y = -2$. How might Lucy decide where to shade?



Test Practice

8. Here is a graph of the equation $x + 2y = 7$.



Which of these points is a solution to the inequality $x + 2y < 7$?

A. $(0, 0)$

B. $(10, 0)$

C. $(7, 0)$

D. $(0, 7)$

Spiral Review

9. The equation of line a is $y = -2x - 1$. Select *all* the points that are on line a.

- ☐ A. $(-2, 3)$
- ☐ B. $(0, 4)$
- ☐ C. $(1, -3)$
- ☐ D. $(-1, -3)$
- ☐ E. $(2, 5)$

10. Lola can spend up to \$15 on pens and notebooks. A pen costs \$2 and a notebook costs \$1.50. Using p for the number of pens and n for the number of notebooks, write an inequality that represents this situation.

11. A golf ball weighs 1.6 ounces and an empty metal bucket weighs 12 ounces. Neel adds golf balls one at a time to the empty metal bucket. How many golf balls will be in the bucket when the total weight is 20 ounces?

Name: .... Date: .... Period: ....

MA.912.AR.2.7, MA.912.AR.2.8, MTR.1.1, MTR.5.1, MTR.6.1, MTR.7.1

Concert Planning

Let's represent constraints by graphing two-variable inequalities in context.

Warm-Up

Here is a situation with hidden information. Let's make sense of it together as a class.

The Funk-tions want to raise at least $............ for their band by holding multiple concerts at SoundZone. Each concert at SoundZone costs the band $............. They typically sell tickets for $............ each.

1. Read this information three times.
2. Choose a value for each blank that could make sense. Record your values in this table.

Amount to Raise ($)	Cost of SoundZone Venue ($)	Price of Each Ticket ($)

3. For the values you chose, will the band reach their goal if they hold 2 concerts and sell 100 total tickets? Explain your thinking.

Activity 1

Name: .... Date: .... Period: ....

SoundZone Concerts

The Funk-tions want to raise at least $2,000 by holding multiple concerts at SoundZone.

- Each concert at SoundZone costs the band $500.
- They typically sell tickets for $20 each.

4. Will the band reach their goal if they hold 2 concerts and sell 100 tickets total?

5. The band manager wrote the equation $-500c + 20t = 2000$. Explain what each part of the equation represents.

c:

t:

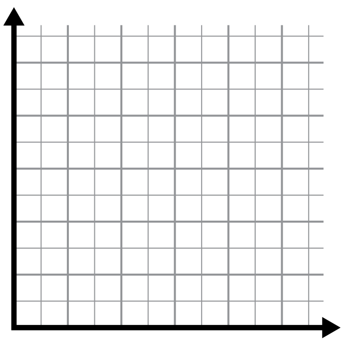
-500:

20:

2,000:

6. Graph the band manager's equation. Be sure to label and scale each axis. Complete the table of values for the band manager's equation if it helps with your thinking.

Number of Concerts, c	Number of Tickets Sold, t



7. Write an inequality that represents all the combinations of concerts and tickets that would raise at least $2,000.

8. Shade the region on the graph that represents all the solutions to the inequality you wrote. What does the solution mean in this situation?

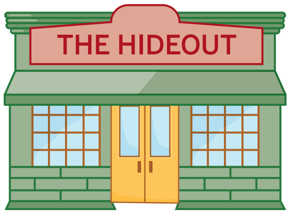
9. Are all the solutions to the inequality realistic in this context? Explain your thinking.

10. Write a question that the Funk-tions could answer using the graph.

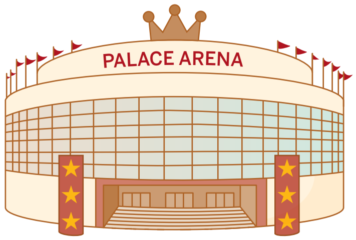
Name: .... Date: .... Period: ....

Which Venue?

The Funk-tions are considering two other venues for their concerts.



Cost per concert: $250
Ticket price: $10

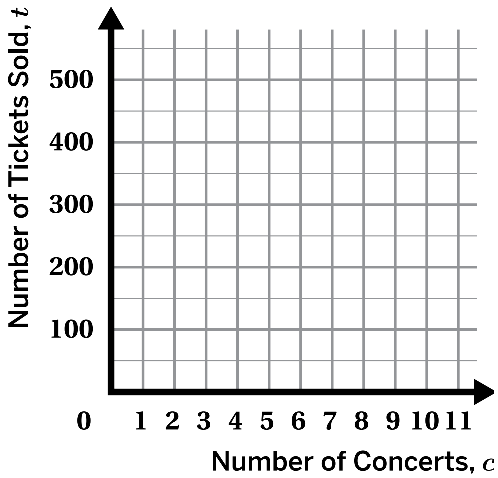


Cost per concert: $4,000
Ticket price: $40

In this activity, you will choose one venue to explore. Your partner will choose the other.

11. Write the name of the venue you chose. Then write an inequality that represents the number of concerts and tickets sold that would raise at least $2,000 at that venue.

12. Graph the solutions to your inequality.



13. Explain how you determined the boundary line and the shading.

14. **Discuss:** Consider your and your partner's graphs. How are they alike? How are they different?

Name: ________ Date: ________ Period: ________

Which Venue? (continued)

15. For each situation, work with your partner to determine which concert venue you would recommend. Explain your thinking.

a 1 concert and 200 tickets total. The Hideout Palace Arena

b 5 concerts and 400 tickets total. The Hideout Palace Arena

c 2 concerts and 250 tickets total. The Hideout Palace Arena

16. Victor knows that $(2.5, 500)$ represents a solution to both the Hideout inequality and the Palace Arena inequality but says it is not realistic. Is Victor correct? How do you know?

Synthesis

17. A group of students is installing a garden at their school.

- A vegetable bed will cost $15 per square foot to install.
- A flower bed will cost $12 per square foot to install.
- Their budget for the project is $300.

Describe how to define variables and write an inequality that represents this context.

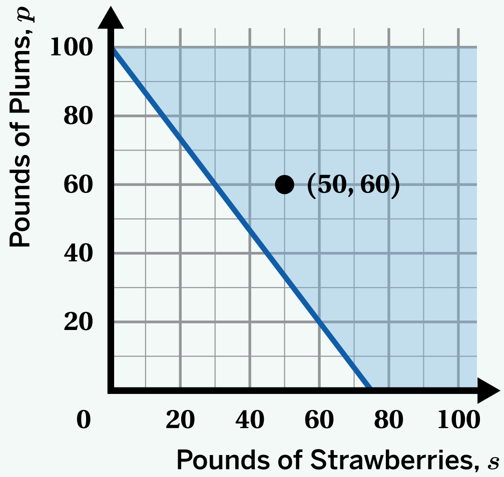
Lesson Practice A1.18

Lesson Summary

Looking at solutions to two-variable inequalities on a graph can help us make sense of different situations.

Here is an example: Angel makes $4 per pound of strawberries and $3 per pound of plums that she sells. The inequality $4s + 3p \geq 300$ represents the pounds of strawberries, s, and pounds of plums, p, that Angel needs to sell to meet her goal of making at least $300.

- To determine the solutions to the inequality, graph the corresponding equation
- $4s + 3p = 300$. Decide whether the points on the line will reach the goal by looking at the original inequality symbol.
- Then test a value, such as $(50, 60)$, to identify the solution region. $4(50) + 3(60) \geq 300$ ***True!***



Because the point $(50, 60)$ makes the inequality true, the half-plane that includes $(50, 60)$ is the solution region. So any combination of strawberries and plums in the shaded region, including those on the line, would meet Angel's goal.

But not all solutions to the inequality will make sense for the situation. For example, the point $(90, -20)$ makes the inequality true, but it doesn't make sense for Angel to sell -20 pounds of plums.

Lesson Practice
A1.18

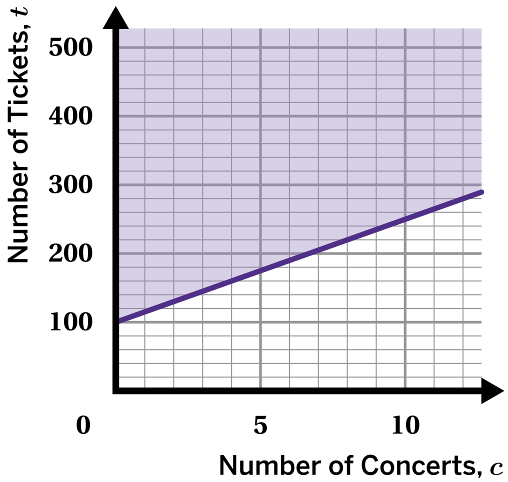
Name: .... Date: .... Period: ....

1. Mia is buying snacks for a movie night with her friends. Her budget is $60.

- A package of snack mix costs $5.00.
- A package of popcorn costs $2.50.

Write an inequality that represents all the snacks that Mia can buy for $60 or less. Use s for packages of snack mix and p for packages of popcorn.

Problems 2–3: The Funk-tions are looking into one more venue for their concerts. The Depot charges $300 per concert and tickets cost $20 each. The Funk-tions want to raise at least $2,000. The graph shows the solution set of $-300c + 20t \geq 2000$.



2. Write one solution to the inequality.

3. If the Funk-tions have 4 concerts and sell 200 tickets, will they reach their fundraising goal? Explain how you know.

Problems 4–6: Aditi's soccer team is selling bags of trail mix for $3 each and cups of lemonade for $2 each. To make a profit, they need to earn more than $120.

4. b represents the number of bags of trail mix and c represents the number of cups of lemonade. Which inequality describes all the ways Aditi's soccer team can make a profit?

A. $3b + 2c > 120$

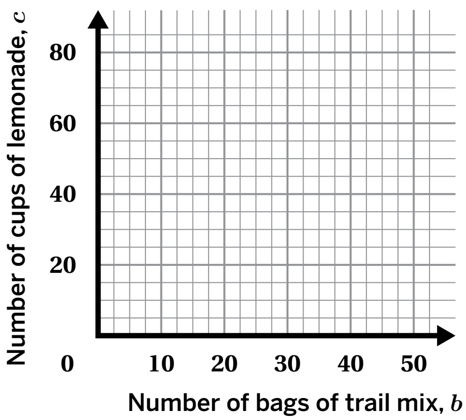
B. $c - 60 < -\frac{3}{2}b$

C. $c \geq -\frac{3}{2}b + 60$

D. $120 \geq 3b + 2c$

5. Graph all of the solutions to the inequality you chose.

6. Explain how you could check if the boundary is included or not included from the solution region.



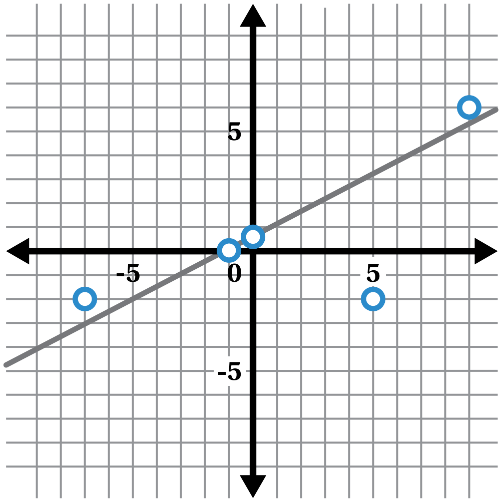
Lesson Practice
A1.18

Name: ______________________ Date: __________ Period: __________

7. Jasmine says that $(3, 10)$ is a solution to $y < \frac{1}{2}x + \frac{15}{4}$. Is she correct? Explain how you know.

Test Practice

Problems 8–9: Here is an inequality: $2y - x > 1$.



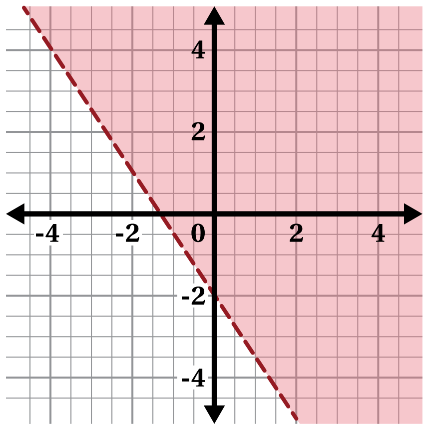
8. Graph *all* the solutions to $2y - x > 1$.

9. Select *all* the points that are solutions to this inequality.

- ☐ **A.** (-7, -2)
- ☐ **B.** (-1, 0)
- ☐ **C.** (0, 0.5)
- ☐ **D.** (5, -2)
- ☐ **E.** (9, 6)

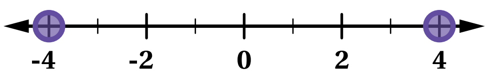
Spiral Review

10. The graph shows the solution set to an inequality. Select *all* the inequalities that produce this solution set.

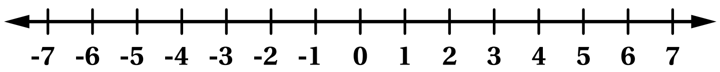


- ☐ **A.** $y < -1.5x - 2$
- ☐ **B.** $y > -1.5x - 2$
- ☐ **C.** $1.5x + y > -2$
- ☐ **D.** $y > 1.5x + 2$
- ☐ **E.** $y \leq 1.5x - 2$

11. Create an inequality using x that has the solution set shown on the graph.

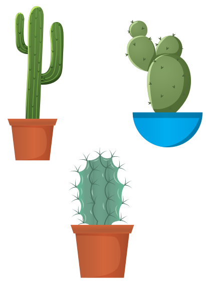


12. Graph all the solutions to the inequality $|x + 2| = 3$.



Career Connection

Did you know that there are over $1,000$ different types of cactus plants in the world? Most cacti are found in desert regions in states like Arizona, New Mexico, and Nevada. The prickly pear cactus grows in many U.S. states, including Florida, and produces prickly pear dragon fruit.



What do you know about cactus plants? How do you think they compare to other plants?

Botanists are among the many careers that involve plant life. They are people who are considered to be experts in the scientific study of plants. They use equations to determine measurements and to describe the characteristics of the plants they are studying.

B.E.S.T. Mathematics Benchmark Connection

Botanists and others involved in the study of plants use many math concepts in their work. For example, they apply inequality concepts and use number lines to track temperature and growth rates (MA.912.AR.2.6) so that they can compare different varieties of plants, even within the same plant family. This is especially important for cactus plants because they grow very slowly and because some species can live over 100 years. IT specialists will use diagrams and equations (MA.912.AR.1.2, MA.912.AR.2.4) as they use technology tools to monitor the growth of plants in a botanical garden.

Mathematical Thinking and Reasoning Connection

Botanists use thinking and reasoning skills like the ones you use for your math work! For example, they represent instructions in different forms, such as pictures, diagrams, or tables (MTR.2.1).

Meet Helia Bravo Hollis

Helia Bravo Hollis was a botanist who is known for her work in studying cactus plants. She developed her interest in plants when she was young, taking walks with her family in forests and along the river near her home. She traveled extensively for her work, collecting samples and taking photos of different cacti. Helia Bravo Hollis wrote books about her studies, and mentored many others who were interested in the science of plants, including her sister Margarita!

Photo Credit

Unit 2

Describing Data

In this unit, you will use dot plots, histograms, box plots, line graphs, and measures of center and spread to analyze and describe one-variable data sets. You will use two-way tables to make decisions about data and make predictions based on data. You will also use scatter plots and lines of best fit to describe two-variable data sets.

Essential Questions

- What tools can you use to help determine if there is an association in a set of data?
- How can you compare data using measures of center and spread?
- How can you use the line of best fit to describe the relationship between two variables?
- What is the difference between correlation and causation?

Sub-unit

1

Classifying Categorical Data

Lesson 1
Survey Says

Lesson 2
Data Dimensions

Name: ________________ Date: ________ Period: ________

MA.912.DP.1.1, MA.912.DP.1.2, MTR.4.1

Survey Says

Let's make sense of the kinds of data that can be collected and write questions to get to know each other better.

Warm-Up

1. Every year, the U.S. government sends the American Community Survey (ACS) to thousands of people.

The goal is to gather "vital information about our nation and its people."

What would you like to know about our nation and its people? List at least three questions.

Activity 1

Name: ______ Date: ______ Period: ______

Types of Data

2. Here are the initials of seven people who took the American Community Survey and their responses.

a Here are the responses to each question.

Anonymous People	What is your age?	How many people live with you, including you?	What is your main occupation?	How did you get to work or school last week?	How many miles away do you live from the closest grocery store?
M.A.	19	1	Cook	Bicycle	2
P.Y.	25	3	Construction worker	Car	10
Z.M.	53	2	Nurse	Walked	0.5
B.H.	62	5	I don't work	–	–
J.S.	44	7	Accountant	Worked from home	15
O.L.	12	3	Student	Car	–
A.B.	29	4	Carpenter	Bus	0.25

b What do you notice about the responses? What do you wonder?

I notice:

I wonder:

Name: .... Date: .... Period: ....

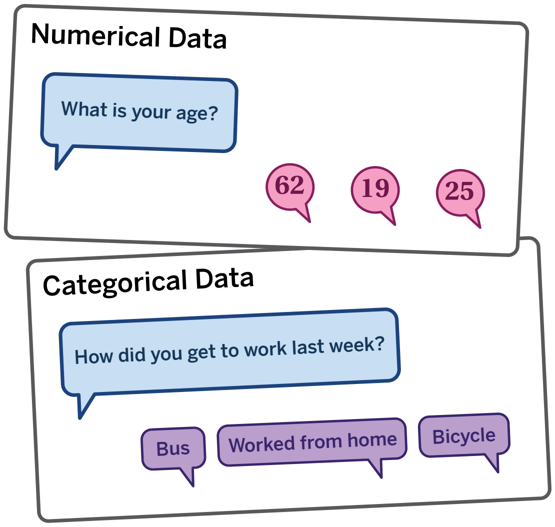
Types of Data (continued)

3. Asking different types of questions can produce different types of data.

What is your age? produces **numerical data** (also called *quantitative data*).

How did you get to work last week? produces **categorical data**.

Describe what you think these terms mean.



Numerical data:

Categorical data:

4. Which type of data would each of these questions produce?

Question	Categorical or Numerical?
Do you live within two miles of a grocery store?	
What type of fuel is used to heat your home?	
How long have you lived in your current home?	
How old is the youngest person in your family?	
How many minutes does it take you to get to work or school?	
What type of building do you live in?	

5. Convince someone about which type of data this question would produce: *Do you live within two miles of a grocery store?*

Activity 2

Name: ______ Date: ______ Period: ______

Crafting and Asking Questions

6. Jalen is making a survey that includes a question about people's television-watching habits.

He wrote three different ways to ask the question.

- Do you watch a lot of television?
- How many hours of TV did you watch last night?
- How much TV did you watch last month?

a **Discuss:** What are some advantages and disadvantages of each question?

b If you were making the survey, how would you write the question?

7. Think of something you want to learn about your classmates.

a Write a question for your classmates to answer. Will the data you collect be numerical or categorical?

Question:

Circle one: Categorical Numerical

b Collect responses to your question.

Initials	Response

Initials	Response

Name: .... Date: .... Period: ....

Crafting and Asking Questions (continued)

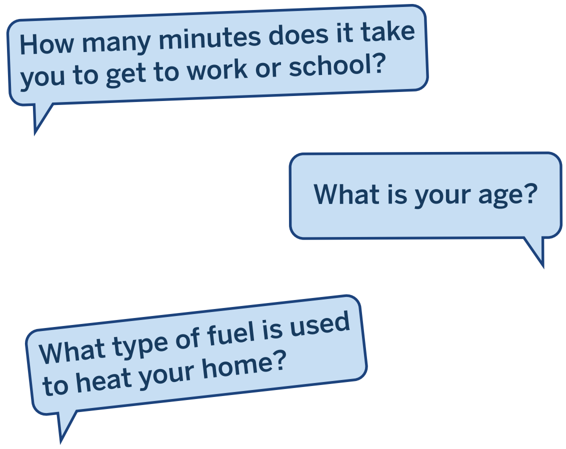
8. Look at your classmates' responses to your question.

a Summarize the data in some way.

b Now that you've seen the responses, how would you change your survey question?

Synthesis

9. How can you decide whether a question will produce numerical or categorical data?



Lesson Practice 2.01

Lesson Summary

Different types of questions lead to different types of data:

- **Numerical data** has values that are numbers, measurements, or quantities instead of words. It's sometimes called *quantitative data*.

  How many pets do you have? is a question that produces Numerical data.

- **Categorical data** has values that are categories, such as colors, words, or zip codes.

  What's your favorite animal? is a question that produces categorical data.

It's important to be specific when writing survey questions, so you can gather the exact type of data you need.

Lesson Practice
2.01

Name: ______ Date: ______ Period: ______

1. Which would *not* be a good survey question?

A. What grade are you in?
B. How many books did you read last year?
C. How many inches are in 1 foot?
D. How many pets do you have?

2. Determine which type of data these questions produce.

Question	Categorical or Numerical?
How old are you?	
Do you have any pets?	
How many siblings do you have?	

3. Callen claims that players on the school basketball teams are taller than players on the soccer teams. Write two survey questions that Callen could ask to investigate the claim.

1. ______

2. ______

4. Nikolai wants to know about the types of food his classmates prefer. Write a survey question that would give him *categorical* data about his classmates' food preferences.

Problems 5–6: Here are some responses to the question: *What is your birthday?*

January 7 March 18 December 23

5. Jaylin is not sure whether the data is categorical or numerical. Explain why this type of data is unclear.

6. What is another question that might generate data that is unclear?

Name: .... Date: .... Period: ....

7. Think about your community. What information would you like to know about the people in it? Write an example of one question that will produce numerical data and one question that will produce *categorical* data about your community.

Numerical	Categorical

Test Practice

8. Select *all* the questions that would produce numerical data.

- ☐ **A.** How many people live in your home?
- ☐ **B.** What is your favorite breakfast food?
- ☐ **C.** How did you travel to school this morning?
- ☐ **D.** How many minutes did it take you to get ready this morning?
- ☐ **E.** What is the last thing you ate or drank?

Spiral Review

9. Determine the value of x that makes the equation $5(3x - 2) = -55$ true.

Problems 10–11: Tasia is organizing two sets of marbles.

	Green Marbles	Marbles That Are Not Green
Set A	17	23
Set B	13	12

10. How many marbles are in each set?

Set A: ....
Set B: ....

11. Across both sets, how many marbles does Tasia have in total that are *not* green?

Unit 2
Lesson
2

Name: ______ Date: ______ Period: ______

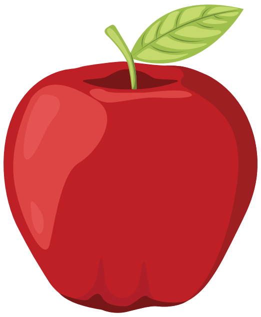
MA.912.DP.1.1, MA.912.DP.1.2, MTR.3.1, MTR.4.1

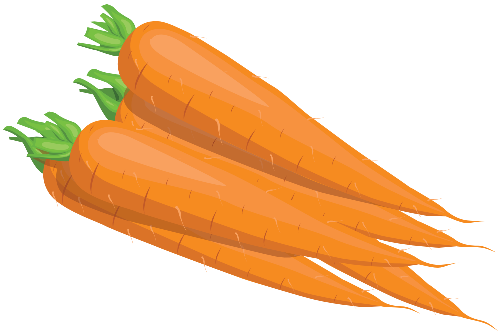
Data Dimensions

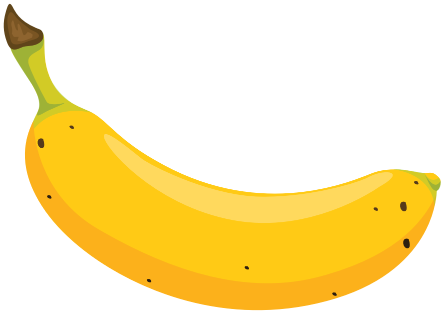
Let's look at different data sets and determine if they are univariate or bivariate.

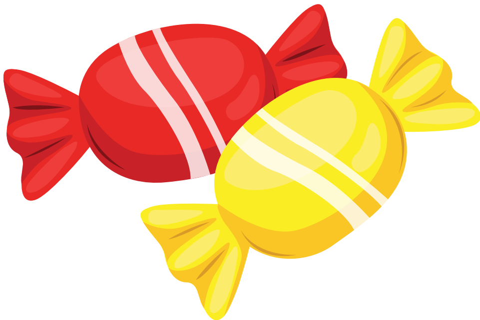
Warm-Up

1. Which one doesn't belong?









Activity
1

Name: .... Date: .... Period: ....

Identifying Data

You can classify data using the terms **univariate** or **bivariate**.

a Consider the data you collect on how long each student rode the bus to school today. This data set would be **univariate** because it focuses only on one variable: "minutes on the bus."

Student	Minutes on a Bus
A	15
B	17
C	23
D	30
E	32

Discuss: What do you wonder or notice about the data? What are some other examples of **univariate** data?

b Now, suppose you collect data that includes the number of minutes on the bus and the number of stops it made. This data set would be **bivariate** because it includes two variables: "minutes on the bus" and "stops made."

Student	Minutes on a Bus	Stops Made
A	15	4
B	17	5
C	23	3
D	30	7
E	32	6

Discuss: What do you wonder or notice about the data? What are some other examples of **bivariate** data?

Name: ______ Date: ______ Period: ______

Organizing Data

Alani and José are planning a field trip for their class and need to gather specific information to make sure the day goes smoothly. Help them organize their data by deciding if they need univariate or bivariate data to answer each question.

2. How many buses are needed?

3. How much money to budget for food?

4. What activities to plan for the day?

a **Think-Pair-Share.** What are the variables in each data set?

Question	Data Type	Variable(s)
Buses		
Cost		
Activity		

Synthesis

5. What is the key difference between univariate data and bivariate data?

a What is a strategy to remember what univariate means?

b What is a strategy to remember what bivariate means?

Lesson Practice 2.02

Lesson Summary

- When working with **univariate data**, you are dealing with only one variable. This type of data focuses on analyzing a single characteristic or measurement.
  - For example, if you collect the heights of students in your class or track the number of books you've read over the summer, you're working with univariate data because you're focusing on just one variable.
- On the other hand, **bivariate data** involves two variables, allowing you to explore the relationship between them. This can help you see how one variable affects or correlates with another.
  - For example, if you want to study how the number of hours you spend studying relates to your test scores or how exercise affects your resting heart rate, you're working with bivariate data.
  - Bivariate data allows you to compare and identify trends between the two variables you're examining.

Lesson Practice
2.02

Name: .... Date: .... Period: ....

Problems 1–4: Label each data set as either **univariate** or **bivariate** based on whether it involves one or two variables.

1. The number of siblings each student in a classroom has.
2. The relationship between distance biked and speed.
3. The time it takes different athletes to complete a marathon.
4. The relationship between the price of gasoline and the number of miles driven by car owners.
5. **Describe** two additional categories of univariate data.

Problem 6: Analyze the following situation.

6. Kinsley says, "Data involving multiple factors is always bivariate." Claude believes that Kinsley is correct because "bi" means more than one. What do you think?

Test Practice

7. A library requires that members borrow **at least 3 books** per month to maintain their membership. Let x represent the number of books a member borrows in a month. Write an inequality that represents this situation. Record your answer in the space provided.

Name: ________ Date: ________ Period: ________

Spiral Review

Problems 8–9: Marco can spend up to $\$55$ on pants and shirts. A pair of pants costs $\$20$ and a shirt costs $\$10$.

- p is the number of pants.
- s is the number of shirts.

8. Which inequality represents this situation?

A. $20p + 10s \le 55$

B. $20p + 10s \ge 55$

C. $20p + 10s < 55$

D. $10p + 20s \le 55$

9. Explain how you know that $p = 1$ and $s = 3$ solutions to this situation. make the inequality true.

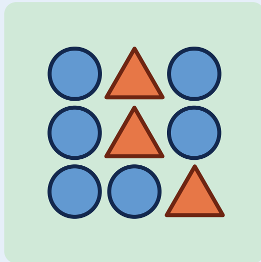
Sub-unit

2

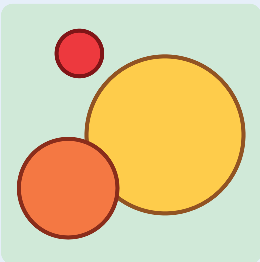
Summarizing One-Variable Data

Lesson 3
Data Driven

Lesson 4
Better Weather?

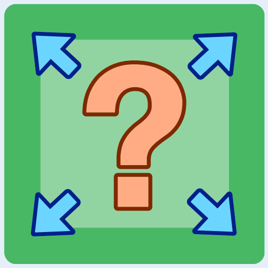


Lesson 5
Quick Pick



Lesson 6
Far Out

Lesson 7
Dynamic Decades



Lesson 8
How Big?

Lesson 9
How Many?

Lesson 10
Dribble, Draw, and Decide

Name: .... Date: .... Period: ....

MA.912.DP.1.1, MA.912.DP.1.2, MTR.4.1

Data Driven

Let's make sense of different ways to represent univariate numerical data and analyze the patterns and insights they reveal.

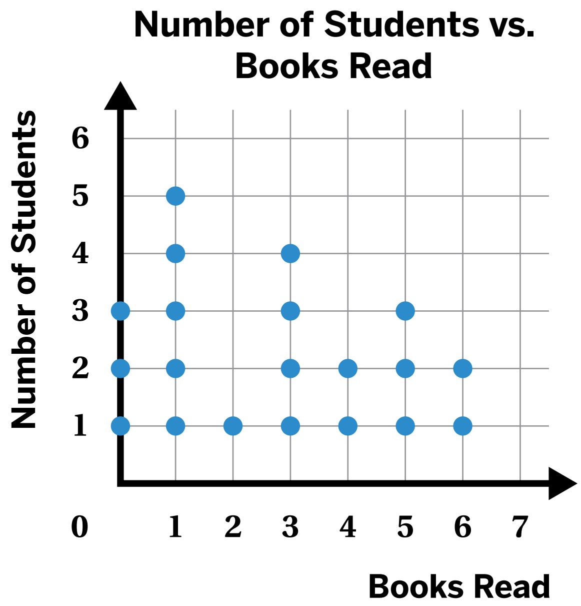
Warm-Up

1. Mr. Shae asked students in his class how many books they read over the summer break. He collected the data and created a line plot. Let's analyze this line plot together.

   a. What is the most common number of books read?

   b. What is the range of books read?

   c. What does this data tell you?



Activity 1

Name: .... Date: .... Period: ....

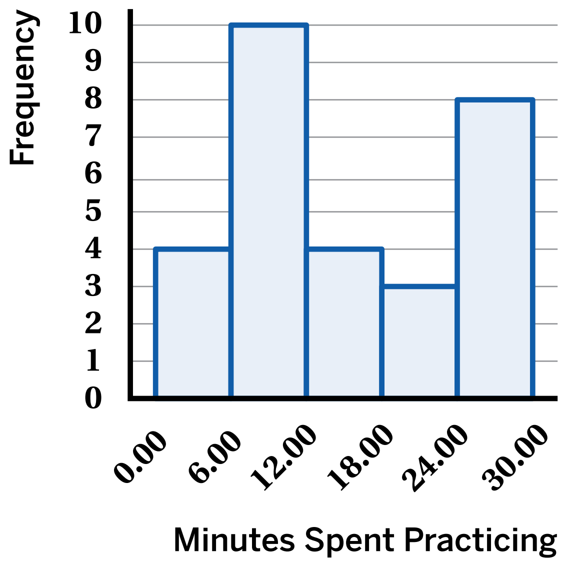
Four Different Displays

A group of students in music class recorded the number of minutes they spent practicing their musical instruments each day over one week. The students compiled their times (in minutes) in a list. They created a **histogram**, a **stem-and-leaf plot**, a **box plot**, and a **line plot** to represent the data.

Here is the data set:

Activity 1	
Minutes Spent Practicing	**Count**
0	1
0	2
5	1
5	2
6	1
6	2
6	3
9	1
10	1
10	2
10	3
11	1
11	2
11	3
15	1
15	2
15	3
15	4
19	1
19	2
20	1
24	1
24	2
24	3
24	4
25	1
25	2
25	3
25	4

Histogram of Minutes Spent Practicing



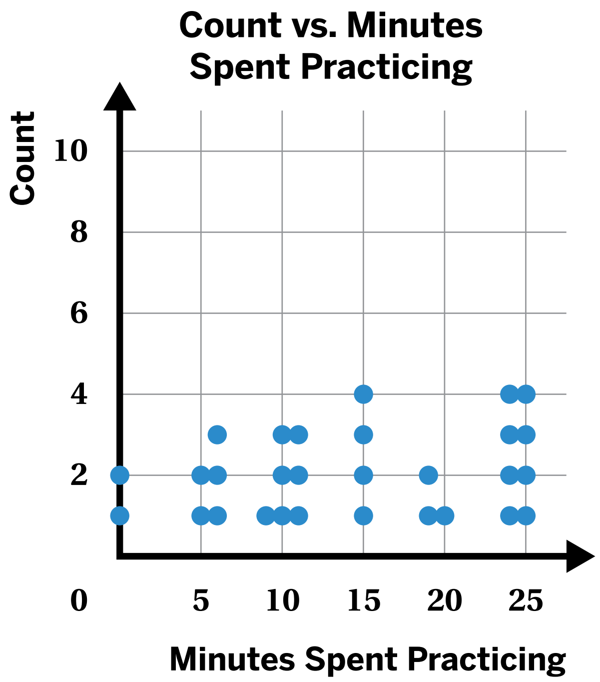
Stem	Leaf
0	0 0 5 5 6 6 6 9
1	0 0 0 1 1 1 5 5 5 5 9 9
2	0 4 4 4 4 5 5 5 5

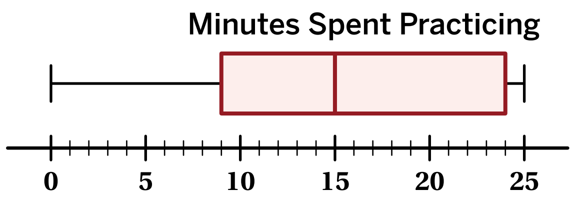
Key: 1 | 1 means 11 minutes

Activity 1

Name: Date: Period:

Four Different Displays (continued)





2. How many students practiced for less than 10 minutes? Which display did you use to find the answer?

3. Which displays make it easiest to see how many students practiced for exactly 15 minutes?

4. Which displays allow you to determine the minimum and maximum quickly?

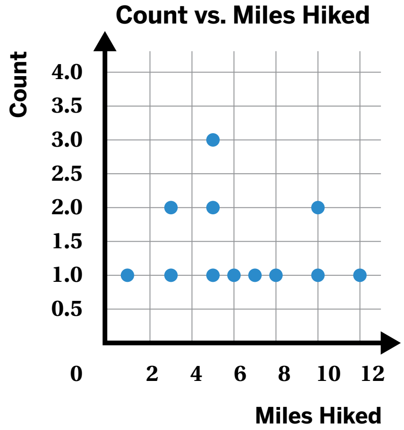
5. Which displays allow you to determine the most frequent number of minutes spent practicing?

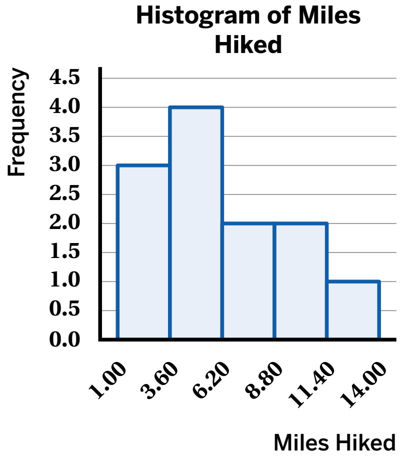
Name: ______ Date: ______ Period: ______

Comparing Data Displays

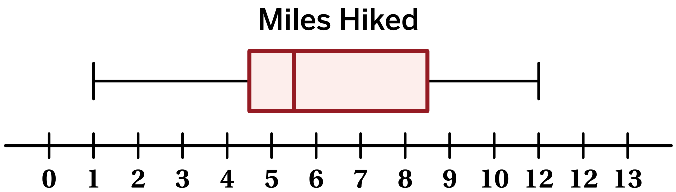
A group of hikers recorded the number of miles they hiked each weekend over a month. Their distances are listed below in miles. Use each data display to answer the following questions.

Data Set: $\{1, 3, 3, 5, 5, 5, 6, 7, 8, 10, 10, 12\}$





Stem	Leaf
0	1 3 3 5 5 5 6 7 8
1	0 0 2



6. Which data display makes it easiest to see how many hikes were less than 10 miles? Why?

7. Which data display makes it easiest to identify the range of the data? Why?

8. Which display makes it easiest to identify the mode of the data? Why?

9. Which display would you choose to compare how spread out the data is? Why?

10. Which displays do *not* show individual data values? How might that be a limitation?

Name: ______ Date: ______ Period: ______

Create Your Display

Problems 11–12: A wildlife team recorded the number of bird species observed in different parks over a month. The counts of bird species for each park are listed below:

Data Set: $\{12, 15, 15, 18, 18, 18, 20, 22, 22, 22, 22, 25, 28, 30\}$

Create a data display that you feel best represents this data set. Choose from a **histogram, stem-and-leaf plot, box plot, or line plot**.

11. Why did you choose this type of display?

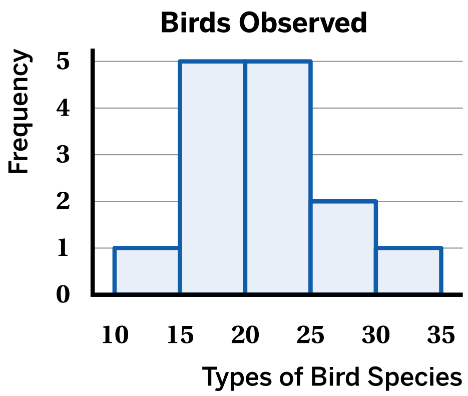
12. What questions could someone answer using your display?

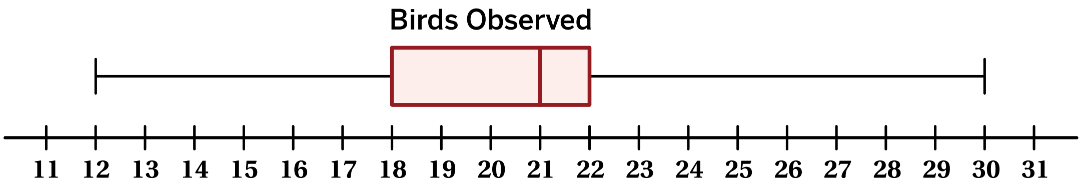
Activity 3

Name: Date: Period:

Create Your Display (continued)

Problems 13–17: Here are two displays that were made by other students. Use them to answer the following questions.





13. How does the histogram highlight the distribution of the bird species counts?

14. What does the box plot make clear about the spread of the data?

15. Which display cannot answer this question: "How many parks observed exactly 22 birds?"

16. Which display can answer this question: "What is the range of the number of bird species observed?"

17. Which display cannot answer this question: "How is the data distributed across intervals of 5 bird species?"

Synthesis

In this lesson, we examined four ways of visualizing data.

What are the key features of a line plot?

What are the key features of a histogram?

What are the key features of a box plot?

What are the key features of a stem-and-leaf-plot?

Lesson Practice 2.03

Lesson Summary

In this lesson, we explored four different types of data displays—**line plots**, **histograms**, **box plots**, and **stem-and-leaf plots**—and how they are used to represent **numerical univariate data**. Each display has unique strengths that make it suitable for specific purposes.

- **Line Plots:** These plots show individual data points along a number line, making it easy to identify modes, outliers, and exact values. They are ideal for small data sets where precise frequencies are important.
- **Histograms:** By grouping data into intervals (bins), histograms provide a clear picture of the distribution and spread of a larger data set. They are especially useful for identifying trends and the overall shape of the data but do not preserve exact values.
- **Box Plots:** Box plots summarize data using five-number summaries (minimum, Q1, median, Q3, and maximum). They highlight the spread, center, and any potential outliers, making them valuable for comparing multiple data sets.
- **Stem-and-Leaf Plots:** These plots organize data by place value, showing exact values while maintaining a visual representation of the distribution. They are particularly helpful for identifying individual data points and small-scale frequencies.

By understanding the strengths and limitations of each display, we can choose the most appropriate method to effectively represent numerical univariate data and answer specific questions about a data set.

Name: .... Date: .... Period: ....

Problems 1–4: Arjun and Lorenzo were given a data set about the average daily rainfall (in inches) in different cities during May. They were asked to create a data display using the data. Arjun created a histogram and Lorenzo created a box plot. Below is the data set:

Data: $0.8, 1.2, 1.3, 0.7, 1.9, 2.0, 0.6, 1.5, 1.4, 2.1, 1.8, 2.2, 0.9, 1.1, 1.7$

1. Why do you think Arjun chose to create a histogram?

2. What are some limitations of using a histogram to display this data?

3. Why do you think Lorenzo chose to create a box plot?

4. Which display would you use if you wanted to explain the spread of the data? Why?

Problems 5–7: A biology class is studying the average growth (in centimeters) of a certain plant species under different light conditions over 10 weeks. Below is the data set:

Data: $2.1, 2.3, 2.5, 3.0, 3.2, 3.5, 3.5, 3.8, 4.0, 4.2, 4.5, 4.5, 4.8, 5.0$

5. Create your own data display to represent this data. Choose from a **histogram, stem-and-leaf plot, box plot, or line plot**. Then, answer the following questions.

6. What are the strengths of your chosen display in representing the data?

7. What are the weaknesses or limitations of your chosen display for this data?

Lesson Practice
2.03

Name: .... Date: .... Period: ....

Test Practice

8. Which of the following data displays can be used to determine the exact values of a data set? Select all that apply.

- ☐ **A.** *Line Plot*
- ☐ **B.** Box and Whisker Plot
- ☐ **C.** Histogram
- ☐ **D.** Stem-and-Leaf Plot

Spiral Review

Problems 9–10: Vishant is investigating whether there is an association between wearing sneakers and participating on an athletic team. This table summarizes the data he collected from a survey of students.

	On Athletic Team	Not on Athletic Team	Total
Wear Sneakers	16%	17%	33%
Don't Wear Sneakers	32%	35%	67%
Total	48%	52%	100%

9. Interpret what 32% means in this situation.

10. Is there evidence of an association between wearing sneakers and participating on an athletic team?

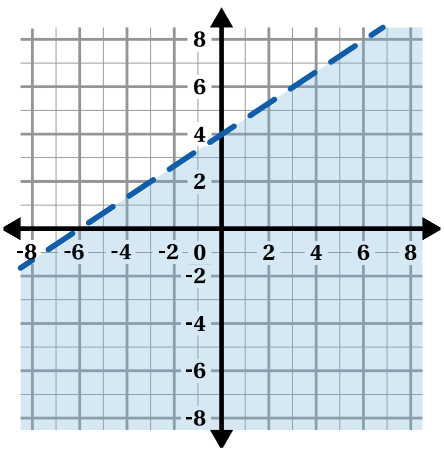
11. Which inequality is shown on the graph?

A. $y > -\frac{2}{3}x + 4$

B. $y < -\frac{2}{3}x + 4$

C. $y > \frac{2}{3}x + 4$

D. $y < \frac{2}{3}x + 4$



Name: ______ Date: ______ Period: ______

MA.912.DP.1.1, MA.912.DP.1.2, MTR.2.1, MTR.4.1, MTR.7.1

Better Weather?

Let’s analyze categorical univariate data.

Warm-Up

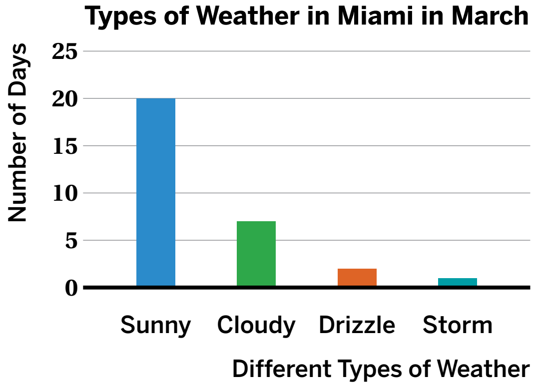
1. Mia lives in Seattle, Washington. Her best friend Bao lives in Miami, Florida. Mia and Bao are debating whose city has better weather. How could they compare the weather in Seattle and Miami?

Activity 1

Name: .... Date: .... Period: ....

Data Displays

Problems 2–5: Bao collects data about the weather daily for a month. The data is **categorical univariate data** because observations are not numerical.

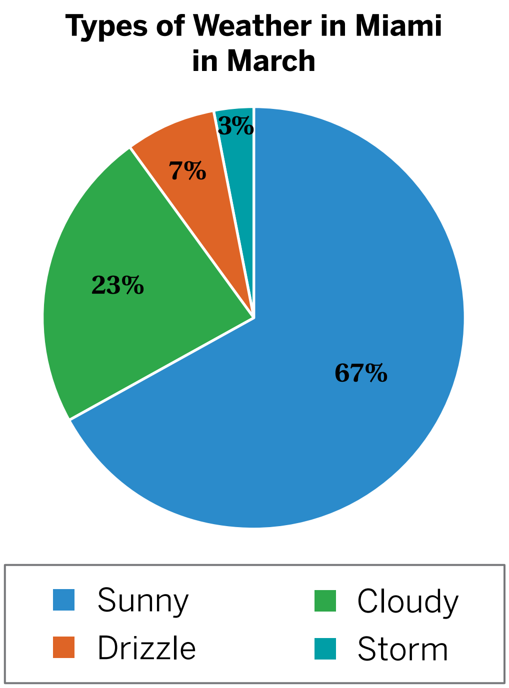


2. He shows the data on a bar chart. A **bar chart** shows bars with heights equal in length to the numbers in each category.

a What information can be interpreted from the graph?

b What can you learn from comparing the heights of the different bars?

3. Bao draws another data display of the same data using a circle graph. A **circle graph** has sectors that are proportional to the relative frequency of the data in each category. How does the information on the circle graph differ from that on the bar chart?



Name: ______ Date: ______ Period: ______

Data Displays (continued)

4. Bao also presents the data in a frequency table. A frequency table shows the number of observations in each category as a number.

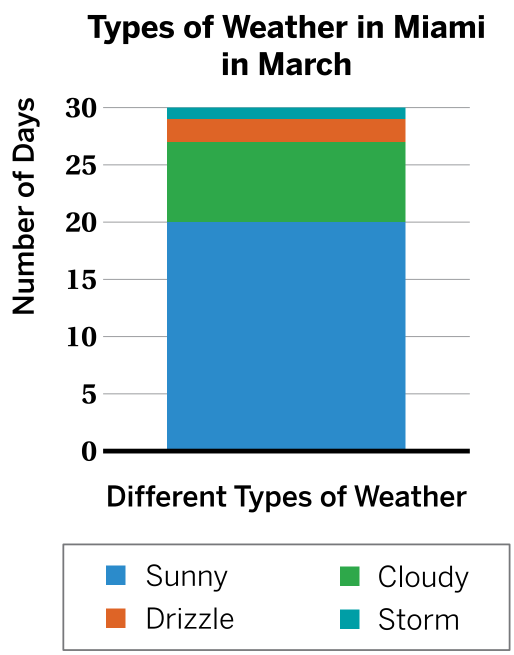
Types of Weather in Miami in March

Types of Weather	Number of Days
Sunny	20
Cloudy	7
Drizzle	2
Storm	1

a When would a frequency table be most useful for looking at data?

b When would a bar chart be more useful than a frequency table?

5. Lastly, Bao presents the data on a **segmented bar graph.** Here, the number of observations in each category is stacked on top of each other.



a How is the segmented bar graph similar to a bar chart?

b How is the segmented bar graph different from a bar chart?

c How is the segmented bar graph similar to a circle graph?

d How is the segmented bar graph different from a circle graph?

Activity 2

Name: ______ Date: ______ Period: ______

Comparing Data Displays

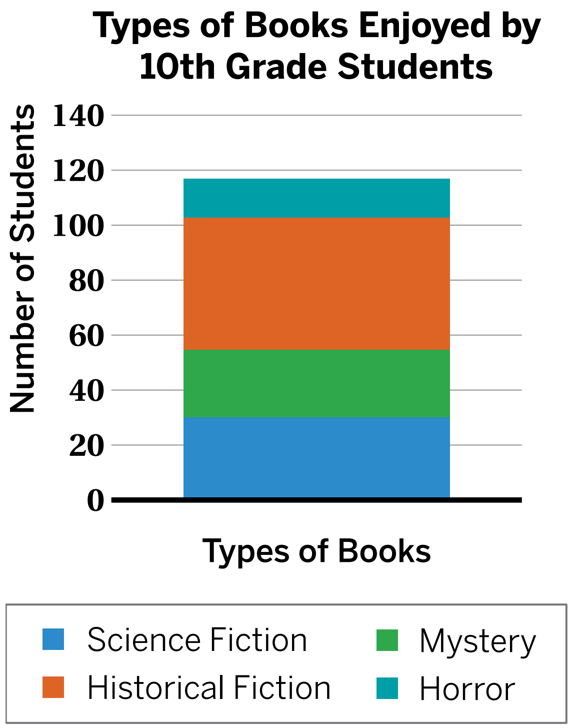
Problems 6–7: The ninth and tenth grade students collect data on what type of books their classmates enjoy most. The ninth grade students presented their data in a frequency table. The tenth grade students presented their data on a segmented bar graph.

Types of Books Enjoyed by 9th Grade Students

Types of Books	Number of Students
Science Fiction	82
Mystery	14
Historical Fiction	27
Horror	1

6. **a** Which data display is most useful to find out how many students participated in the survey?

b Which data display is most useful to find out how many students liked each type of book?



7. The school will use the data to decide what types of books to order for the library. Which data display is more useful for this purpose?

8. What type of data display would you have used to present this type of data?

Activity 3

Name: Date: Period:

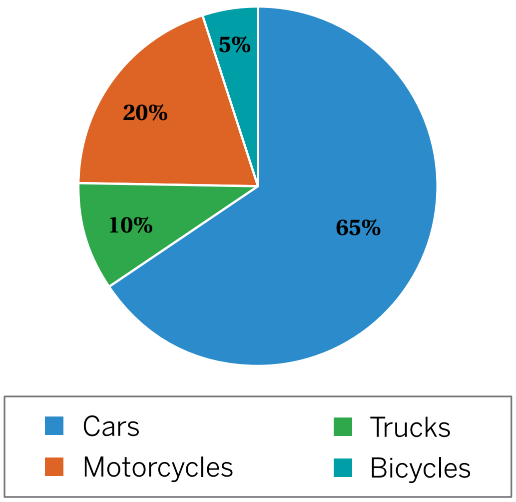
Creating Data Displays

Problems 9–11: A traffic engineer researches the vehicles that pass through an intersection for an hour. The engineer counts 26 cars, 4 trucks, 8 motorcycles, and 2 bicycles.

9. Construct a data display that best presents this data.

10. The engineer constructed the two data displays shown. What type of question can be answered by the frequency table but not the circle graph?

Types of Vehicles Passing Through an Intersection Over 1 Hour



Types of Vehicles Passing Through an Intersection Over 1 Hour

Types of Vehicles	Number of Vehicles
Car	26
Trucks	4
Motorcycles	8
Bicycles	2

11. How can the circle graph help the engineer analyze data?

Synthesis

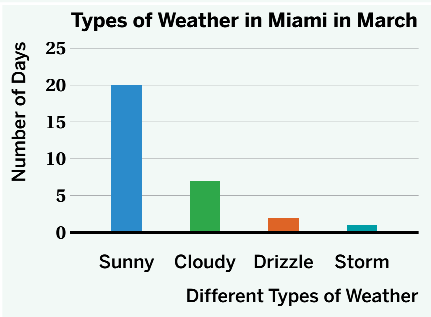
12. How are different types of data displays for categorical univariate data similar? How do they differ? Use the data displays if it helps your thinking.

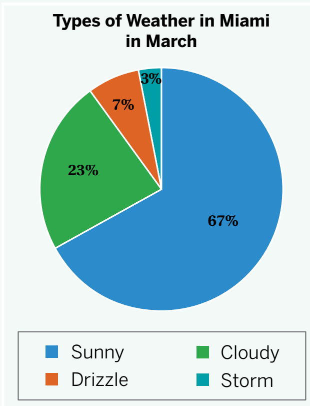
Lesson Practice 2.04

Lesson Summary

Categorical univariate data are observations that are collected in non-numerical categories. The data can be presented using different data displays.

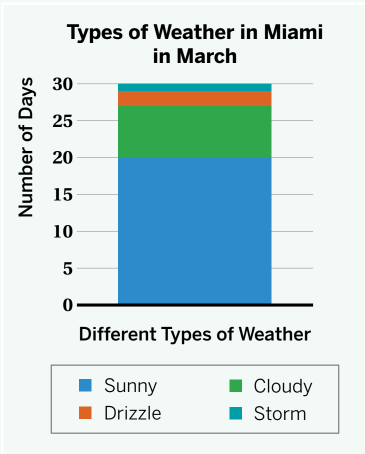
- A bar graph shows the number of observations in each category as the height of the bar.
- A circle graph shows the relative frequencies of each category proportional to the area of each sector.
- A frequency table shows the observations in each category numerically. It is often the easiest way to display categorical data.
- A segmented bar graph stacks the number of observations in each category on top of each other to show how each category compares both to each other and to the whole.





Types of Weather in Miami in March

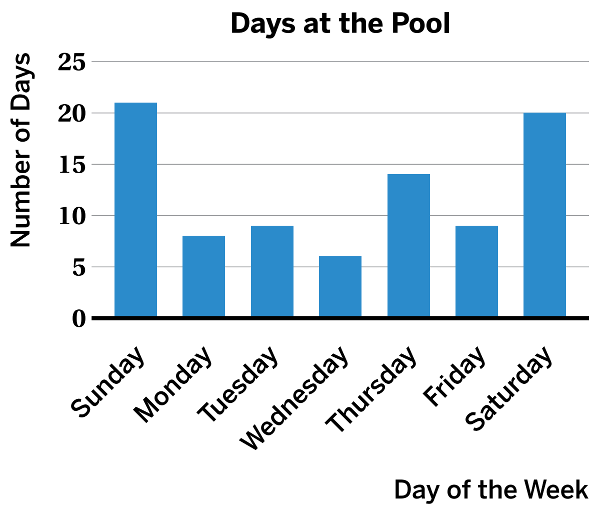
Types of Weather	Number of Days
Sunny	20
Cloudy	7
Drizzle	2
Storm	1



Name: .... Date: .... Period: ....

Problems 1–4: Ama records the number of times she has gone to the pool each day of the week this year in the bar chart.

1. Construct a frequency table with the data.



2. How can the frequency table be easier to use than the bar chart? How can the bar chart be easier to use than the frequency table?

3. How would a segmented bar graph appear different from what Ama has drawn?

4. Use Ama's data to construct a circle graph.

Name: ____________ Date: ________ Period: ________

5. Mateo receives survey data presented in a circle graph. What information does he need to construct a frequency table with the graph? Explain how he would use the information to construct the frequency table.

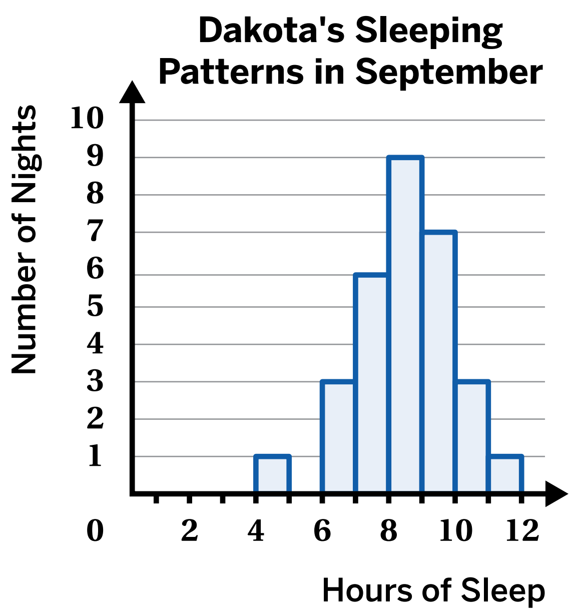
Test Practice

6. A survey collects data about students' favorite subjects in school. Which of these data displays can be used to present this data? Select all that apply.

- ☐ **A.** Histogram
- ☐ **B.** Bar chart
- ☐ **C.** Scatter plot
- ☐ **D.** Box plot
- ☐ **E.** Circle graph

Spiral Review

7. This histogram shows how many hours of sleep Dakota got each night in September. How many nights did Dakota sleep less than 7 hours?

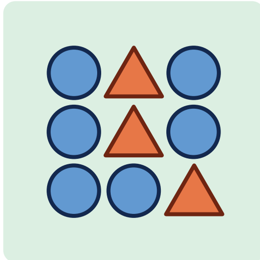


8. Calculate the mean of the data set:
1, 4, 5, 6, 7, 8, 9, 10, 11, 12, 15

Name: ____________ Date: ________ Period: ________

MA.912.DP.1.2, MTR.3.1, MTR.4.1

Quick Pick

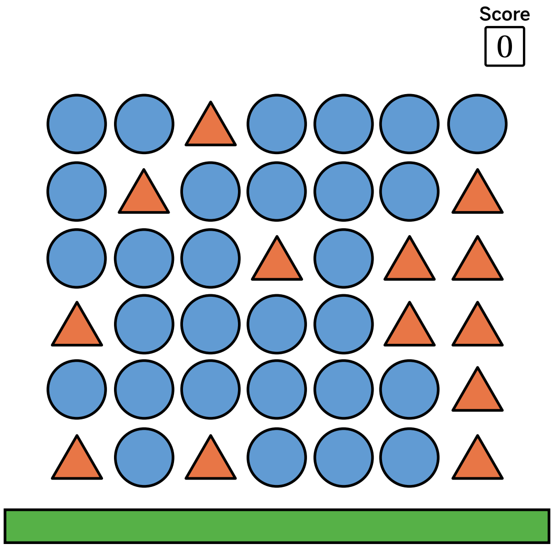


Let's explore how extreme values impact mean and median.

Warm-Up

1. This game is called *Quick Pick*. Here's how it works:

- You pick different shapes to get points.
- Triangles are 3 points. Circles are 2 points.
- A five-second timer starts after your first pick.
- You can only select one shape at a time by pointing to the shape with your pencil with your eyes closed.



Discuss:

- Play the game with your partner 10 times and record the points you earn each time.
- Will the data you collect be numerical or categorical? How do you know?
- Will the data you collect be univariate or bivariate? How do you know?

Pick Number	Partner A Points	Partner B Points
1		
2		
3		
4		
5		
6		
7		
8		
9		
10		

Activity 1

Name: Date: Period:

Measures of Center

2. Look at your scores and the scores of three other students from the warm-up activity.

Choose a student and describe their scores.

3. A **measure of center** is a single number that represents a central value in a data set.

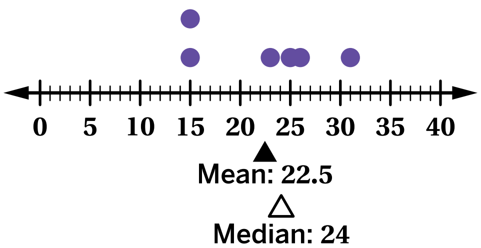
Mean and *median* are two measures of center.

Here are the mean and median for Oscar's scores.

What are some things you know about calculating the mean and median?

Oscar's Scores

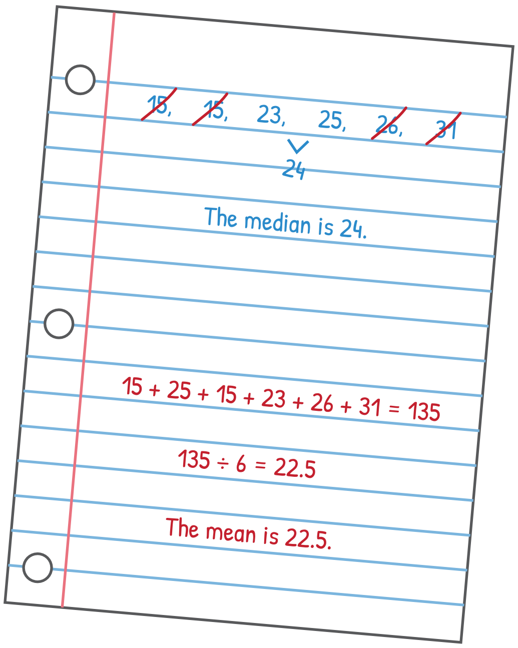
15	25	15	23	26	31



4. Here is the work Oscar did to calculate the median and the mean.

Discuss: How did Oscar calculate each measure of center?

Oscar



Name: .... Date: .... Period: ....

Shape and Center

5. Here are Dhruv's scores from the game. Determine the median and mean.

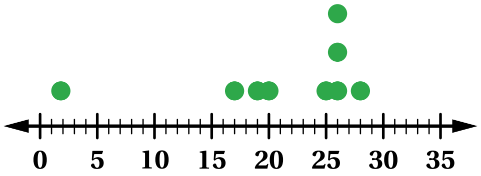
Use a calculator if it helps with your thinking.

Median: ....

Mean: ....

Dhruv's Scores

2	17	26	26	20	28	25	19	26



6. Analyze the mean and median for Dhruv's scores.

a **Discuss:** Why do you think the mean and median are far apart?

b Which measure of center would you use to represent Dhruv's typical score? Circle one.

Mean Median

Explain your thinking.

7. Here are Chloe's scores from the game.

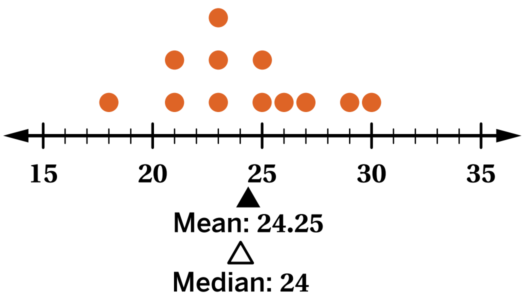
Imagine that her highest score was 300 instead of 30.

Which measure(s) of center would increase? Circle one.

Mean Median Both Neither

Chloe's Scores

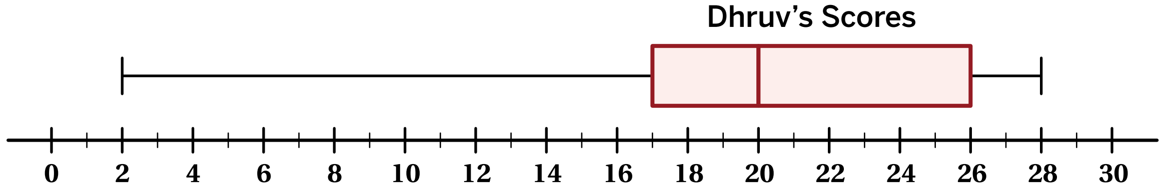
18	30	23	23	25	29	27	26	25	21	23	21

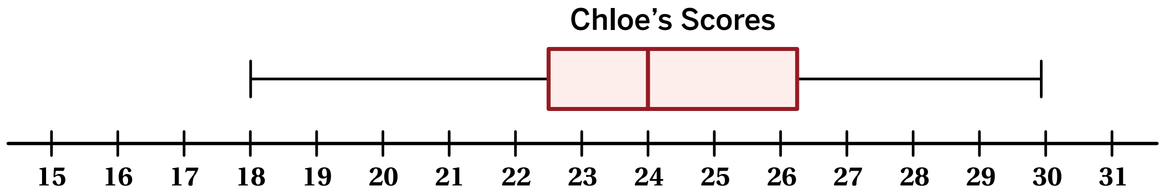


Name: ______ Date: ______ Period: ______

Shape and Center (continued)

8. Here are Dhruv's and Chloe's scores in the form of box and whisker plots. Compare these representations to the dot plots. Does one of these data plots tell you more about the scores than the other? Explain your reasoning.





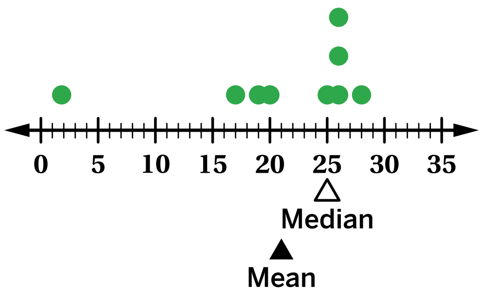
Synthesis

9. How are median and mean *alike*? How are they *different*?

Use the data set if it helps with your thinking.

Dhruv's Scores

2	17	26	26	20	28	25	19	26



Alike:

Different:

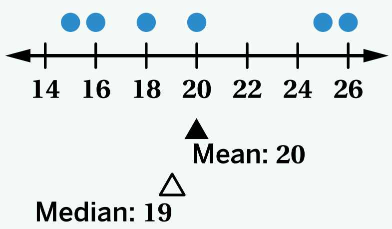
Lesson Practice 2.05

Lesson Summary

A **measure of center** is a single number that summarizes all of the values in a data set. *Mean* and *median* are measures of center that are used to describe a typical value of a data set.

- The mean is also called the average of a data set. To calculate the mean, you can add up all the data values, and divide by the number of data points.
- The median is the middle value of a data set when the values are in numerical order. If there are two values in the middle of the data set, then the median is the middle of those two values.

Here are the steps you can use to determine the mean of the data in this dot plot.



$15 + 16 + 18 + 20 + 25 + 26 = 120$

$120 \div 6 = 20$

The median is the value halfway between 18 and 20, so the median is 19.

When a data set includes extreme values that are much larger or smaller than most of the data, the value of the mean and median will be very different. Extreme values impact the mean more than they impact the median.

Lesson Practice
2.05

Name: ______ Date: ______ Period: ______

1. Determine the mean and median of each data set.

	Set A [27, 30, 33]	Set B [0, 100, 100, 100, 100]	Set C [3, 5, 7, 15]
Mean			
Median			

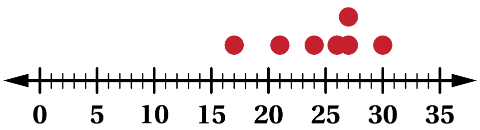
2. Seven people estimated how many marbles there were in a jar. Determine the mean and median of the estimates.

Mean: ______

Median: ______

Marble Jar Estimates

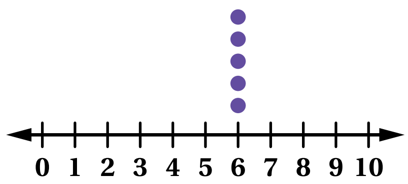
17	21	27	27	24	26	30



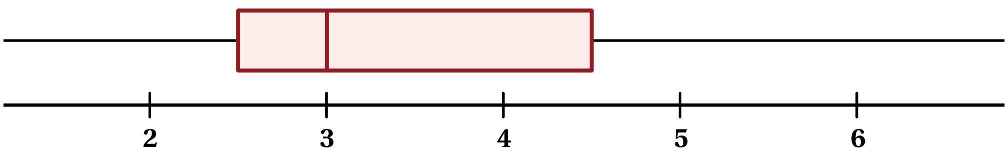
3. Here is a dot plot. If you added 4 and 9 to the data set, which statistic would change? Circle one.

Mean Median Both Neither

Explain your thinking.



4. Create a box and whisker plot that has a median of 3 and a range of 6.

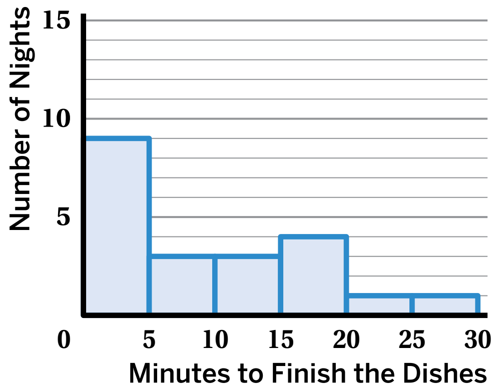


5. Find the median of the stem-and-leaf plot below.

Stem	Leaf
0	3 3 4 5 7 9
1	2 4 5
2	7 9

Name: .... Date: .... Period: ....

Problems 6–7: For the past few weeks, Anand kept track of how long it took him to do the dishes each night. This histogram shows the results organized in 5-minute bins.



6. Circle the bin that contains the median.

7. Explain how you know.

8. The following data set was given to Carl's class:
$12, 14, 17, 7, 15, 21, 19, 32, 22$

Carl concluded that the median of the data is 15. Explain the error in his reasoning?

Lesson Practice
2.05

Name: ______ Date: ______ Period: ______

Test Practice

9. A study looks at data to determine how the number of hours studied relates to test scores. The data in this study could be described as:

- **A.** bivariate and numerical
- **B.** bivariate and categorical
- **C.** univariate and numerical
- **D.** univariate and categorical

Spiral Review

10. Select all the expressions that are equivalent to $2(x + 3)$.

- ☐ **A.** $2x + 6$
- ☐ **B.** $2 \cdot x + 3$
- ☐ **C.** $x + 6$
- ☐ **D.** $(x + 3) \cdot 2$
- ☐ **E.** $2 \cdot x + 3 \cdot 2$

Problems 11–12: Evaluate each expression. Write your solution in scientific notation.

11. $(2 \cdot 10^4)(6 \cdot 10^5)$

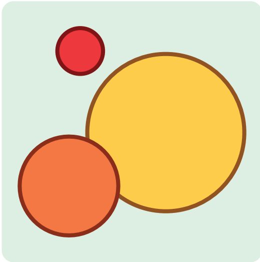
12. $\frac{3 \cdot 10^{-8}}{2 \cdot 10^{-3}}$

Name: .... Date: .... Period: ....

MA.912.DP.1.2, MTR.2.1, MTR.4.1, MTR.7.1

Far Out

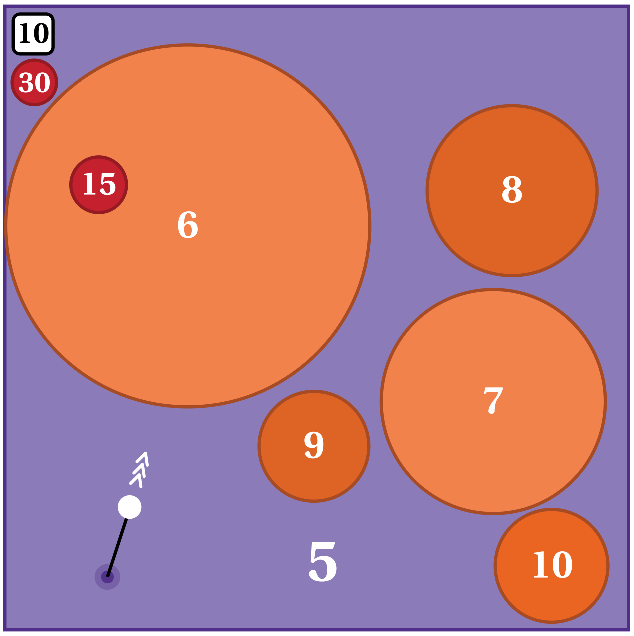
Let's determine whether a data point is an outlier and consider its effect on the mean and median.



Warm-Up

1. A game is played where a number cube is rolled on the game board. The number the cube lands on is the score for the turn. The purple area is all "5".

Play the game at least 5 times and record your scores.



Trial	Score

Activity 1

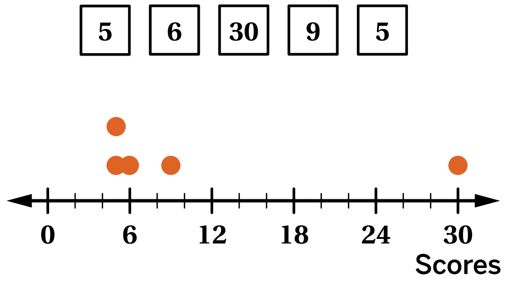
Name: ______ Date: ______ Period: ______

Outliers and Their Effects

2. Koharu played the game 5 times.

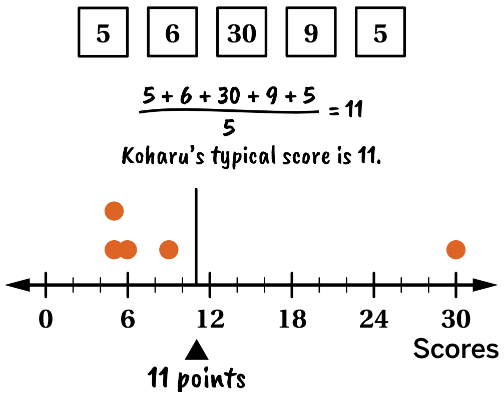
   Here are her scores.

   What do you think is her typical score?

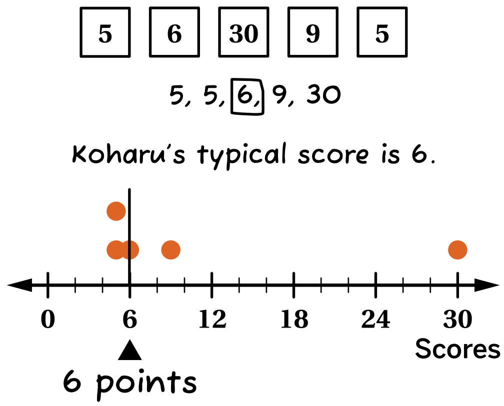


3. Here are two different strategies for determining Koharu's typical score.

Student 1



Student 2



Discuss: How are Student 1's and Student 2's strategies alike? How are they different?

Name: .... Date: .... Period: ....

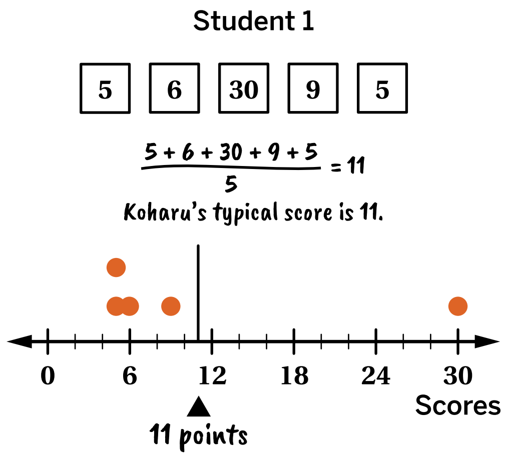
Outliers and Their Effects (continued)

4. Koharu says: *You shouldn't use the mean for the typical score because the* 30 *messes it up.*

Do you agree? Circle one.

Yes No I'm not sure

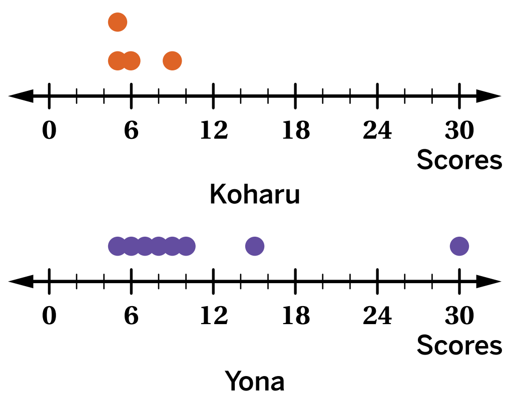
Explain your thinking.



5. In Koharu's data, 30 is an **outlier** because it's far from other values in the data set.

Outliers can have a big impact on the mean of a data set.

Circle the point(s) in Yona's data that you think are outliers.

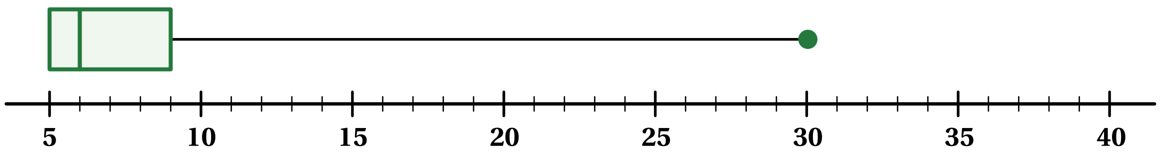


Activity 2

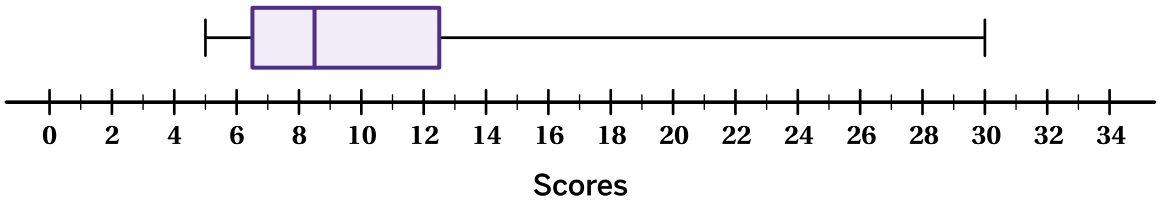
Name: Date: Period:

Outliers and Box Plots

6. Create a box and whisker with Kaharu's data.



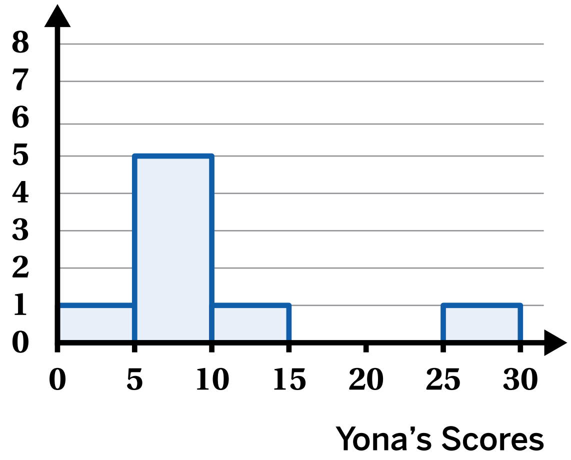
7. Create a box and whisker plot with Yona's data. $5, 6, 7, 8, 9, 10, 15, 30$



Name: .... Date: .... Period: ....

Outliers and Box Plots (continued)

8. Use a histogram to plot Yona's data.

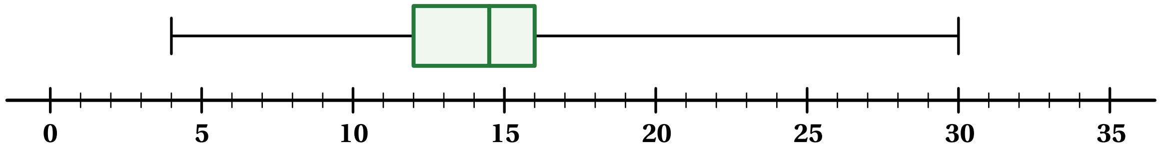


9. Use a stem-and-leaf plot to plot Kaharu's data.

Stem	Leaf

Synthesis

10. Describe how to use the data displays to determine which values are outliers.

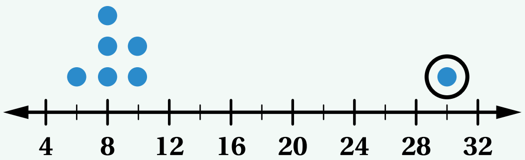


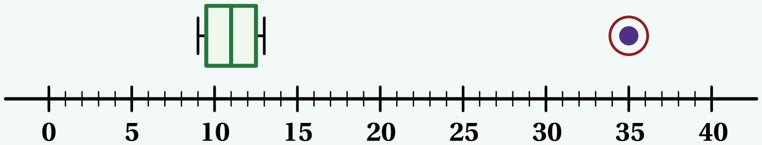
Lesson Practice 2.06

Lesson Summary

Outliers are data values that are far from the other values in the data set. In this data set, the circled data point is an outlier.

Stem	Leaf
0	1 3 4 5 7 7 9
1	0 1 4 8
2	5
3	
4	7 9
5	1 2 2 3 6 8
6	
7	
8	2





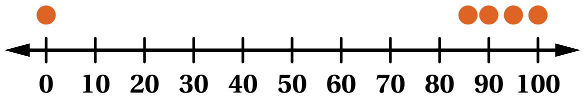
You can identify outliers using dot plots, box plots, and technology tools such as graphing calculators. You can also identify outliers using the *IQR*. Outliers are values further than 1.5 times the IQR below Q1 or above Q3.

When deciding which measure of center is appropriate to represent a data set, it's important to identify any outliers. Outliers have a big impact on the *mean*, but they don't impact the *median* very much.

Lesson Practice
2.06

Name: .... Date: .... Period: ....

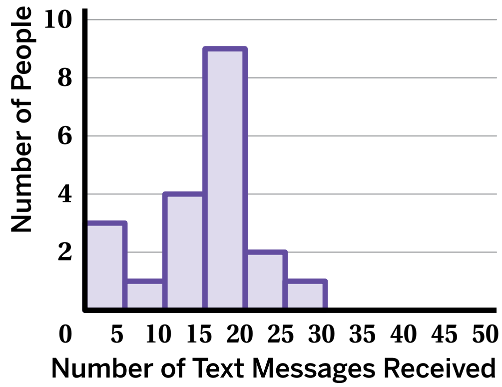
Problems 1–2: Ricardo got the following scores on five class assignments: 87, 90, 0, 95, and 100. His teacher lets each student decide whether the student's final score will be the mean or the median of those five scores.



1. Would using the mean or the median give Ricardo the higher final score?

2. Explain your thinking.

Problems 3–4: This histogram represents the number of text messages that 20 people received in one day.



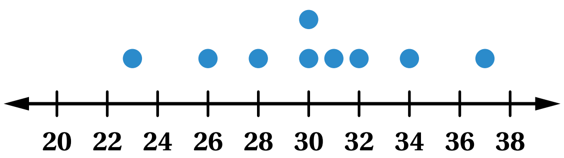
3. If you removed one person who received 3 text messages from the data set, would the median increase, decrease, or stay the same?

4. If you added a person who received 52 text messages to the data set, would the mean or median change more? Circle one.

Mean Median

Explain your thinking.

5. Here is a data set:
$26, 30, 31, 32, 28, 30, 34, 37, 23$.



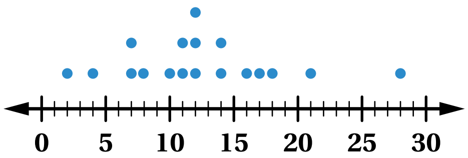
a Julian says that 37 is an outlier because it is the maximum value. What could you say to Julian to help him understand his mistake?

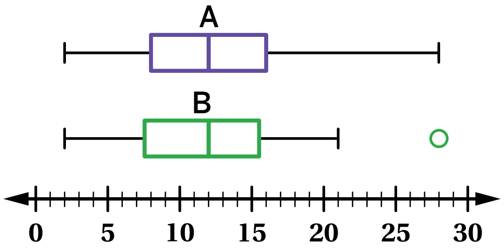
b Would this data be considered numerical or categorical? Explain.

c Would this data be considered univariate or bivariate? Explain.

Name: ________ Date: ________ Period: ________

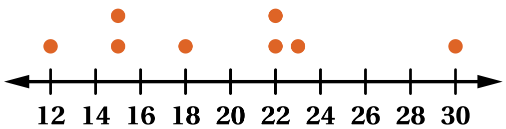
6. The dot plot represents the data in box plot A. Circle one point to remove from the dot plot so that it represents box plot B instead.





Test Practice

7. This dot plot represents the ages of different people in a bike shop. Which data point would be an outlier if it were added to the data set?



A. 44 B. 8 C. Both D. Neither

Spiral Review

Problems 8–10: Evaluate each expression. Write your answer as a fraction.

8. $\frac{2}{3} + \frac{1}{5}$

9. $\frac{2}{5} - \frac{3}{8}$

10. $\frac{6}{15} + \frac{8}{20}$

Name: ______ Date: ______ Period: ______

MA.912.DP.1.2, MTR.1.1, MTR.4.1, MTR.7.1

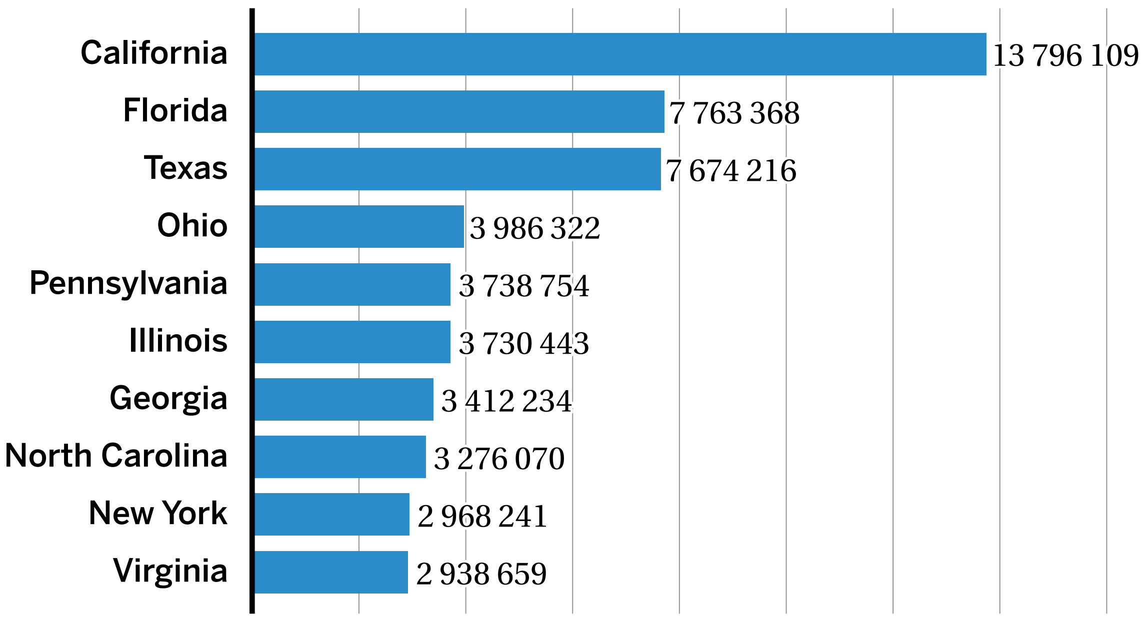
Dynamic Decades

Let’s use measures of center and spread to analyze aspects of how population changes over time.

Warm-Up

1. The chart below shows the number of cars registered for the ten states that had the most registered cars in the year 2022.

Number of Registered Automobiles



Discuss:

- What do you notice? What do you wonder?

Activity 1

Name: .... Date: .... Period: ....

Driving Away

2. Think about the car data from the Warm-up.

Write a question you could answer using that data. Then write one you could *not* answer.

Question You Could Answer	**Question You Could Not Answer**

Omari wondered: *How has the number of cars registered in the United States changed over time?*

3. Omari thinks that the number of cars registered was more consistent across states in 2012 than 2022. Which statistical measure could Omari use to help him back up his claim?

4. Omari researched and found the following data for registered vehicles in 50 states and Washington, D.C. in 2012 and 2022. The results are in millions rounded to the nearest hundredth.

Number of Registered Vehicles across the U.S. (in millions)

Year	Min	Max	Median	Mean	IQR	Standard Deviation	Total
2012	0.19	13.22	1.5	2.18	1.94	2.37	111.29
2022	0.13	13.8	1.44	1.96	1.82	2.35	99.95

5. Help Omari answer: *How has the number of registered vehicles changed over time?* Use statistics about center and spread to support your ideas.

6. **Discuss** Did the number of cars change in the ways you expected or were you surprised?

Name: Date: Period:

Shape of the Data

7. Look again at the data from Activity 1, but with the Q1 and Q3 values added. Create a box plot for each year.

Number of Registered Vehicles across the U.S. (in millions)

Year	Min	Max	Median	Mean	Q1	Q3	IQR	Standard Deviation	Total
2012	0.19	13.22	1.5	2.18	.57	2.51	1.94	2.37	111.29
2022	0.13	13.8	1.44	1.96	.51	2.33	1.82	2.35	99.95

8. What observations can you make comparing the two box plots?

9. For both 2012 and 2022, Alaska had the minimum number of registered vehicles. What are some factors that could have contributed to the state's decreases over that decade?

10. Which measure of center and spread would be best to use to compare the two years? What can you conclude when you make a comparison between the two? Use the data to support your conclusion.

Synthesis

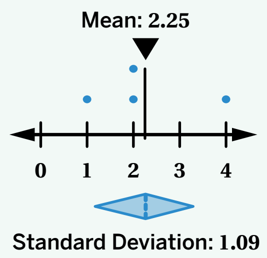
What statistics did you find most helpful for comparing data sets? Why?

Lesson Practice 2.07

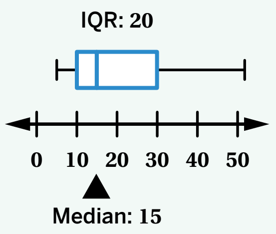
Lesson Summary

It can be helpful to use measures of center and measures of spread to compare data sets. You can choose which measure of center or spread to use based on the shape of the data:

- When data distributions are symmetric or bell-shaped, you can use the mean and standard deviation to compare.
- When data distributions are skewed or contain outliers, you can use the median and IQR because the mean and standard deviation are both affected by extreme values.



Comparing different data sets can reveal important information about change over time in different situations, like minimum wage or median rent. Comparing measures of center, like mean or median, can help you determine if the data has increased, decreased, or stayed the same. Comparing measures of spread, like standard deviation or IQR, can help you determine if the data has become more or less consistent over time.



Lesson Practice
2.07

Name: .... Date: .... Period: ....

Problems 1–3: Yona is interested in the lengths of online videos. She found this data about the lengths of thousands of videos, organized by category.

	Mean Length (min)	Standard Deviation (min)
Music	4.15	0.6
Gaming	6.38	2.1

1. Write a question you could answer using this data.

2. Write a question you could *not* answer using this data.

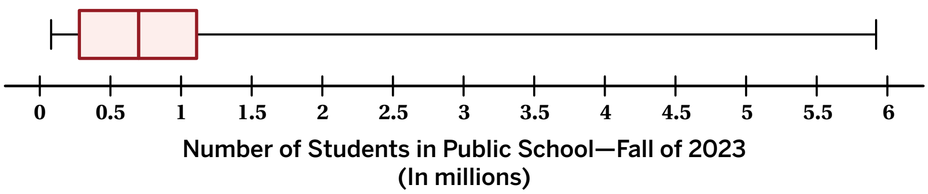
3. How do the lengths of the music videos and the gaming videos compare? Use statistics to support your answer.

4. The table shows some statistics about the points per game scored by the Chicago Bulls in 1978 and 2020.

	Median (pts)	Mean (pts)	Standard Deviation (pts)	IQR (pts)
1978	102	104.9	10.99	15
2020	110	110.70	11.36	17.5

How has the number of points per game changed from 1978 to 2020? Use statistics to support your answer.

Problems 5–6: The box plot represents the number of students in public school in all 50 states and Washington, D.C. in the fall of 2023.



5. Describe what you notice about the data using the center and spread.

Lesson Practice
2.07

Name: .... Date: .... Period: ....

Test Practice

6. Is the data numerical or categorical? Bivariate or Univariate?

A. The data is numerical and bivariate.
B. The data is numerical and univariate.
C. The data is categorical and bivariate.
D. The data is categorical and univariate.

Spiral Review

Problems 7–9: Use mental math to determine each value.

7. 50% of 120

8. 10% of 120

9. 90% of 120

Problems 10–11: How many solutions does each equation have? Circle one.

10. $6t + 2 = -3 + 6t$

A. One solution
B. No solution
C. Infinitely many solutions

11. $15 - 3t - 15 = -3t$

A. One solution
B. No solution
C. Infinitely many solutions

Problems 12–13: Here is a two-way table with data about people who live on two different islands and whether they prefer to wear a hat outside.

12. Complete the two-way table and relative frequency tables about the islands.

13. Is there an association between which island a person lives on and whether they prefer to wear a hat? Explain your thinking.

	Hat	No Hat	Total
Island A	2,394	6,128	
Island B	7,911		
Total		39,446	

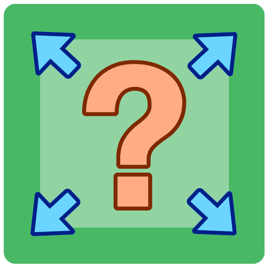
	Hat	No Hat	Total
Island A			100.0%
Island B			100.0%

Name: ________ Date: ________ Period: ________

MA.912.DP.1.4, MTR.4.1, MTR.5.1, MTR.7.1

How Big?

Let's estimate a population total using data from a sample survey.



Warm-Up

Solve each problem for x to the nearest tenth. Show your process to explain your thinking.

1. $\frac{4}{7} = \frac{x}{85}$

2. $\frac{x}{30} = \frac{2}{12}$

3. $\frac{5}{8} = \frac{16}{x}$

Name: ______________ Date: ______ Period: ______

How Big is the School?

4. The table below shows how a sample of students voted when asked about their favorite school subject in a survey.

	Math	Science	English	History
Number of Students	21	37	15	27

Out of the entire school, there were actually 432 students who chose math as their favorite subject. How many students are there in the entire school?

5. From the sample ______ students chose math as their favorite subject.

6. The survey asked ______ students about their favorite subject.

7. The ratio of students who chose math in the survey to the total number of students in the survey is ______.

8. Write a proportion to find the total number of students in the school.

Survey Results **Entire Population**

$$\frac{\square}{\square} = \frac{\square}{\square}$$

9. Use cross-multiplication to write an equation to solve for how many students are in the school.

10. There are approximately ______ students in the entire school.

Activity
2

Name: .... Date: .... Period: ....

Puzzler

A cookie maker surveyed some of the workers in the factory about their favorite cookie.

11. Use the clues to complete the table to solve the puzzle.

Clue 1: There were 150 people surveyed.

Clue 2: Twice as many people chose chocolate chip over sugar cookies.

Clue 3: In the factory, 90 total workers chose sugar cookies.

	Chocolate Chip	Peanut Butter	Sugar	Other
Number of People		48		27

12. What is the total number of people that work in the cookie factory? Use mathematics to explain your reasoning.

13. What percent of the sample chose “other” flavors of cookies?

A. 18%

B. 27%

C. 32%

D. 30%

14. Use the percent from #13 to estimate the total number of workers that you would expect to like “other” flavors.

Synthesis

15. What do the numbers in a survey represent?

16. How can you find an approximation of a total population given survey results?

17. How can you work to solve a proportion to find a total population?

The table shows how a sample of students voted when asked about their favorite school subject in a survey.

Subject	Number of Students
Math	21
Science	37
English	15
History	27

Lesson Practice 2.08

Lesson Summary

- A **survey** is a way to gather information from a **sample** of a **population**.
- You can use a survey to make predictions about an entire population.
- Calculating the proportion of people in a survey who choose a certain characteristic in a sample can help you estimate the number of people in the entire population that could choose that same characteristic.
    - First, set up a **proportion** where one ratio represents the sample and one ratio represents the entire population.

$$\frac{\text{(Part of sample)}}{\text{(Whole sample)}} = \frac{\text{(Part of population)}}{\text{(Whole population)}}$$

    - Use a **variable** for the missing quantity.
    - Use **cross-multiplication** to solve for the missing quantity.
    - Always check to make sure your answer makes sense in the problem.
        - For example, when talking about people, round to the nearest whole number (you cannot have “part” of a person).

Name: ________ Date: ________ Period: ________

1. Choose the words to fill in each blank.

Word Bank			
predictions	sample	population	survey

A ________ is given to find a ________ to be used to make ________ about a ________.

Problems 2–4:

2. A group of 40 ninth graders was surveyed about how many times a week they eat lunch from the cafeteria. Of these, 9 buy three times a week. If the actual number of ninth graders that buys three times a week is 62, about how many students are in the ninth grade?

3. A movie theater surveyed 35 people as they entered on a Friday. Of these, 19 said they were going to see a scary movie. That same day, 67 people actually saw a scary movie. About how many people came to the movie theater on that day?

4. A zoo surveyed 140 people on a Saturday about their favorite animal in the zoo. Of these, 84 said they were going to see giraffes during their visit. That day, 410 people actually visited the giraffes. About how many total people visited the zoo that day?

Name: ______ Date: ______ Period: ______

Problems 5–6: Solve each problem. Show all work.

5. A zoo surveyed one day's worth of visitors about their favorite animal. On that day, there were 215 visitors and 48 of them said that the giraffe is their favorite. If in a week, 730 visitors actually said the giraffe is their favorite, how many people visited the zoo that week?

Survey

Which animal is your favorite?

Giraffe	
Llama	
Two-toed sloth	

6. A teacher surveys one class about how many hours they sleep a night. Out of 24 students, 7 said that they get 8 hours of sleep a night. Out of four classes, 36 said they get 8 hours of sleep. What is the total number of students the teacher has in all four classes?

Test Practice

7. A group of 40 people were surveyed about their favorite restaurant in town. Of these, 15 chose The Variety Cafe. If 630 people in town actually like that restaurant, how many people are in the entire town? Which equation correctly describes this situation?

A. $\frac{15}{40} = \frac{630}{x}$

B. $\frac{40}{630} = \frac{15}{x}$

C. $\frac{40}{x} = \frac{630}{15}$

D. $\frac{15}{40} = \frac{x}{630}$

Spiral Review

Problems 8–10: Simplify using the laws of exponents.

8. $x^2 \cdot x^6$

9. $\frac{x^5 y^4}{x^2 y^9}$

10. $(x^6 y^3)^3$

Name: ______ Date: ______ Period: ______

MA.912.DP.1.4, MTR.1.1, MTR.4.1, MTR.7.1

How Many?

Let's use data to estimate mean and percentage.

Warm-Up

1. A city conducts a survey on the number of people living within a household. The city samples 250 addresses randomly.

Number of People	Number of Households	Percent
1	57	22.8%
2	95	38.0%
3	53	21.2%
4	27	10.8%
5	13	5.2%
6+	5	2.0%

What do you notice? What do you wonder?

I notice:

I wonder:

Activity 1

Name: ____________ Date: ________ Period: ________

Lots of Cars

A traffic engineer wants to know how many cars are in a particular town. The engineer conducts a survey of 40 residential addresses in the town to see how many cars are at each address.

Number of Cars				
0	2	1	2	2
3	0	0	4	1
2	2	1	1	3
4	0	1	3	2
6	1	3	1	1
0	1	2	2	1
3	2	2	1	0
0	1	2	4	1

2. **Discuss:** How should the survey be conducted to ensure that the results are representative of the population?

3. What is the **sample mean** of the survey?

4. How many cars would each address in town be expected to have present based on the data?

5. If the town has 52,050 addresses, how many cars would be expected in town based on the **population mean**?

Name: .... Date: .... Period: ....

Customer Satisfaction

An electric scooter company wants to know how satisfied consumers are with their products. They send a survey to 25 random customers and ask them to indicate if they are satisfied with the purchase.

Survey Results				
Yes	Yes	No	Yes	No
Yes	Yes	No	Yes	Yes
Yes	No	Yes	Yes	Yes
Yes	Yes	No	Yes	No
No	Yes	Yes	Yes	Yes

6. What percentage of the sample customers are satisfied with the purchase?

7. What percentage of the population would be expected to be satisfied with their purchase?

8. The company sold 219,325 electric scooters last year. How many satisfied customers would they expect to find in the population?

Synthesis

9. How can sampling surveys be useful for understanding the characteristics of a population?

Lesson Practice 2.09

Lesson Summary

- A representative sample can be used to draw conclusions about the entire population.
- The **sample mean** is the mean of the data collected in a sample.
  - Calculate the sample mean by finding the mean of all the values in the sample.
- The **population mean** is the mean of all the values in a population.
  - The population mean is approximately equal to the sample mean.
- The percentage of a sample that satisfies a given condition will be approximately equal to the percentage of the population that satisfies the same condition.
  - Calculate the percentage of a sample by counting the number of responses that satisfies a condition and dividing it by the total number of responses.
  - The sample percentage is approximately equal to the population percentage.

Name: ______________ Date: ________ Period: ________

Problems 1–2: A local transportation organization conducted a survey to understand how many miles drivers commute to work each day. The survey results are shown.

Miles Driven					
12.5	15.9	12.8	12.9	18.0	10.6
7.2	23.5	17.5	27.3	14.2	18.3
15.5	1.65	19.9	19.1	16.8	11.0
38.9	22.2	11.3	45.2	13.1	13.4
26.4	31.2	48.1	13.9	25.0	16.3

1. What is the average distance that drivers in the town commute each day?

2. What information would you need to know in order to calculate the total distance driven for commuting in this town?

Problems 3–4: The student government at a school conducts a survey for the theme of the fall dance.

Survey Results	
Fall Festival	12
Pumpkin Spice & Everything Nice	26
Starry Night	18
Time Traveler's Ball	4

3. What percentage of students at the school prefer to have a Fall Festival dance?

4. The school has 380 students. How many students would be expected to prefer a Starry Night theme?

Lesson Practice
2.09

Name: .... Date: .... Period: ....

Test Practice

5. In a survey of 75 students, 12 students indicated that their favorite class in school was math. What percentage of the student population is expected to prefer math over all other subjects?

A. 12%

B. 16%

C. 75%

D. 625%

Spiral Review

Problems 6–7: Select the equivalent number.

6. $4.05 \times 10^2 + 5.03 \times 10^3$

A. 9.08×10^2

B. 9.08×10^3

C. 5.435×10^2

D. 5.435×10^3

7. $(7.1 \times 10^2 + 2.0 \times 10^{-1}) \times (6.4 \times 10^1)$

A. 467.2

B. 45,452.8

C. 45,568

D. 58,240

Name: .... Date: .... Period: ....

MA.912.DP.1.4, MTR.1.1, MTR.4.1, MTR.7.1

Dribble, Draw, and Decide

Let's understand how to analyze the margin of error using real-world simulations.

Warm-Up

Imagine our class is trying to figure out the favorite color among all the 14-year-olds at our school. The table shows the results of the survey.

Number of Students	Color
31	Black
42	Blue
47	Orange
40	Purple
40	Red

1. What is the ratio of students that say their favorite color is orange?

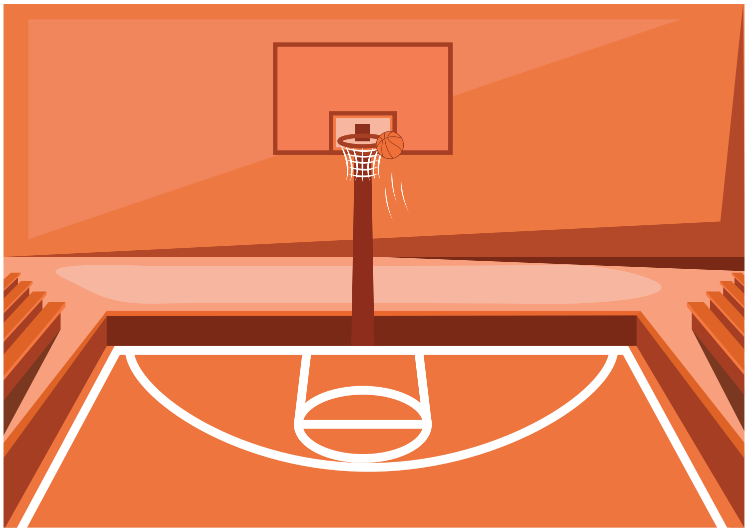
2. A random sample of 20 students is pulled from the survey and 7 students in the sample choose orange as their favorite color. What is the proportion of the random sample?

3. What do you notice about the random sample proportion compared to the population?

Name: .... Date: .... Period: ....

Swish or Miss?

4. A basketball player has taken 200 shots over several recent games. Out of these, the player made 120 shots. Makayla wants to use this sample to estimate a range for the player's overall shooting accuracy with a 95% confidence level with a margin of error of $\pm 6.78\%$.



a. Calculate the shooting percentage.

b. Makayla determines that the confidence interval of the player's shooting accuracy, with a 95% confidence level, is between 53.22% and 66.78%. Explain why you agree or disagree with Makayla's confidence interval.

c. Explain what the shooting accuracy confidence interval means.

5. The margin of error at the confidence level of 99% is $\pm\ 8.91\%$. The confidence interval of the player's shooting accuracy at this confidence level is between:

A. 51.09% and 68.91%

B. 90.09% and 107.91%

C. 41.09% and 48.91%

D. 41.09% and 68.91%

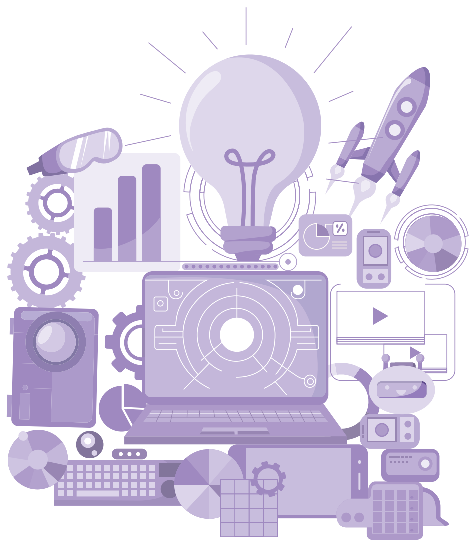
6. What do you notice about the margin of error at the different confidence levels? Explain.

Activity 2

Name: ______ Date: ______ Period: ______

What Do Customers Really Think?

A tech company launches a new product and wants to estimate the proportion of users who are satisfied with their purchase. They conduct a (“yes” or “no”) survey among their customers. They expect around 60% of customers to be satisfied. They want to be 95% confident that the sample results accurately represent their customers so they can use this data in their advertising.



Fill in the table with the proportions and confidence intervals, then analyze the results.

Sample Number	Sample Size	“Yes” Results	Proportion	Margin of Error	Lower confidence interval	Higher confidence interval
1	100	61		9.8%		
2	1250	627		2.8%		
3	2500	1228		2.0%		

7. Which sample provides the most reliable estimate of customer satisfaction and why?

8. Which sample most closely represents the company's expectation for customer satisfaction and why?

9. Do you think that the company's expectation that 60% of the customers like the new product is accurate? Explain your answer.

Synthesis

10. Evaluate how sample size affects the margin of error in estimating a population proportion.

11. Compare and contrast the margin of error results from Activity 1 (Swish or Miss?) and Activity 2 (Customer Satisfaction). How does the context (basketball shooting accuracy vs. customer satisfaction) affect how we interpret the margin of error, even when the sample sizes and confidence levels differ?

Lesson Practice 2.10

Lesson Summary

The margin of error shows how much to expect the sample results to vary from the actual value for the whole population. The margin of error can be used to calculate a range called a confidence interval, which shows how much higher or lower the real answer might be from the sample result.

The confidence interval is a range of values that is likely to contain the true population value, based on a sample.

Imagine the Florida Tourist Bureau surveys thousands of people to find out how many enjoy sunsets on a Florida beach. You are hired by their marketing department to determine an accurate range that they can use in their advertising campaigns that represents the results of the survey with a 95% confidence level.

You choose a random sample of 1000 people. 700 of them have answered yes to the survey.

Calculate the proportion of the sample: $\frac{700}{1000} = 70\%$

The margin of error at a 95% confidence level is $\pm 2.84\%$.

To calculate the confidence interval, you add and subtract margin of error to the proportion of the sample. $70\% \pm 2.84\%$.

$70 + 2.84 = 72.84 \qquad 70 - 2.84 = 67.16$

So, you can recommend the tourist bureau can advertise with 95% confidence that the true percentage of people who enjoy sunsets on a Florida beach is between 67% and 73%.

Lesson Practice
2.10

Name: ______ Date: ______ Period: ______

Problems 1–2: Determine the confidence interval of possible values.

1. A survey found that 60% of 200 participants prefer product A over product B. The survey reports a margin of error of $\pm 4\%$. What is the confidence interval of possible values for the true proportion of people who prefer product A? ______ to ______

2. At a local dog park, 45% of dog owners, when asked, supported the new policy to leave the park open an extra hour each day. With a margin of error of $\pm 2\%$, what is the confidence interval of possible values for the true proportion of supporters? ______ to ______

Problems 3–4: Determine the confidence interval or margin of error.

3. A company claims that 32% to 38% of their product is made from recycled materials. The actual amount of recycled materials is 35%.
What is the margin of error?

4. A national poll on sock preferences sampled 1000 people. The margin of error, for brand X, at a 95% confidence level was $\pm 3.08\%$. The confidence interval is between 51.92% and 58.08%. What proportion of people surveyed preferred brand X?

Problems 5–7: Explain your answer.

5. A survey finds that 48% of people prefer pens over pencils. The margin of error is $\pm 5\%$. If the sample size increases significantly, what is likely to happen to the margin of error?

6. After collecting location data from 250 visitors, a history center decides to advertise that 40% of their visitors come from out of state. The calculated margin of error for the claim is $\pm 6\%$ at a 90% confidence level. Explain how the margin of error would change if the confidence level increased to 95%, without recalculating.

7. A research company surveyed 1200 people to estimate the percentage of city residents who regularly bike on local trails. They found 68% ride regularly, with a margin of error of $\pm 2.5\%$ at a 95% confidence level. The city government plans to increase the budget for trail maintenance if at least 65% of residents bike regularly. Based on this survey, is it likely the true percentage meets this target? Explain your reasoning using the margin of error.

Name: .... Date: .... Period: ....

Test Practice

8. Select all three true statements about margin of error.

- ☐ **A.** The margin of error decreases when the sample size increases, making the estimate more precise.
- ☐ **B.** The margin of error increases as the sample size gets larger.
- ☐ **C.** A larger margin of error means the sample is more accurate.
- ☐ **D.** Larger sample sizes generally result in a smaller margin of error.
- ☐ **E.** The margin of error helps estimate how close the sample results are to the actual population value.

Spiral Review

Problems 9–11: Simplify each expression.

9. $(x^5)(x^3)$

10. $\frac{y^7}{y^4}$

11. $(3a^2b^4)^3$

Problem 12: Rewrite the expression with positive exponents.

12. $\frac{5x^{-3}}{y^2}$

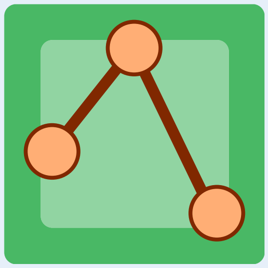
Sub-unit

3

Summarizing Two-Variable Data

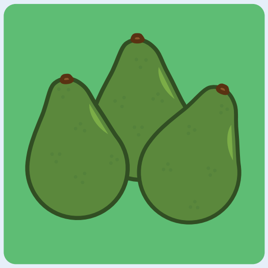
Lesson 11
Trains and Traffic

Lesson 12
Remodel Choices



Lesson 13
Connecting the Dots

Lesson 14
City Slopes



Lesson 15
Residual Fruit

Lesson 16
Penguin Populations

Lesson 17
Behind the Headlines

Unit 2
Lesson
11

Name: ______________ Date: ________ Period: ________

MA.912.DP.3.1, MTR.4.1, MTR.7.1

Trains and Traffic

Let's use categorical data to make an argument.

Warm-Up

1. **a** Order these sounds from *quietest* to *loudest*.

- Traffic noise inside a car
- Siren
- Sporting event
- Washing machine
- Leaf blower
- Rock concert

	Quietest
	Loudest

b How did you order the sounds? Explain your reasoning.

Activity 1

Name: ______ Date: ______ Period: ______

Traffic Delays

People in Metropolis are worried that having trains pass through their town might be associated with traffic delays. Trains have been running in the nearby city of Springtown for four years.

They compare traffic delays in their town to traffic delays in Springtown to see if there is an **association** between trains and traffic delays.

2. Here is a **two-way frequency table** that displays the frequencies of these two categorical variables.

a Complete the table.

	No Traffic Delays	Traffic Delays	Total
People in Metropolis	228,816		269,196
People in Springtown	34,929	9,852	
Total	263,745		313,977

b Write a statement that is true based on the data in the table.

c Make a claim about whether traffic delays are more common in Springtown or in Metropolis based on the data in the table.

d The frequencies representing each combination of the two variables are called **joint frequencies**. The frequencies representing the total number for each variable are called **marginal frequencies**. Which numbers in the table represent joint frequencies? Which represent marginal frequencies?

Name: ______ Date: ______ Period: ______

Traffic Delays (continued)

Rishi notices that the population of Metropolis is larger than the population of Springtown.

He says: *Comparing the total number of people that have experienced traffic delays in each city doesn't make sense.*

He uses the values in the table to calculate what percent of people in Metropolis that have not experienced traffic delays, and then makes a **relative frequency table** to show the percentage of the data that falls into each category.

	No Traffic Delays	Traffic Delays	Total
People in Metropolis	$\frac{228816}{269196} = 85\%$		100%
People in Springtown			

3. **a** **Discuss:** Explain how Rishi calculated the percent of people in Metropolis who have not experienced traffic delays.

b Complete the table.

c If you're concerned about traffic delays in your community, does it matter whether you live in Metropolis or Springtown? Explain your thinking.

Activity 2

Name: ______ Date: ______ Period: ______

Traffic Delays and Age

Tyani says: *I don't have enough information to decide whether there is an association between traffic delays and the city you live in because age might also be relevant to traffic delays.* She decides to explore data by age for both cities.

4. Complete the tables.

Metropolis Population

	No Traffic Delays	Traffic Delays	Total
18 or Younger	53,669		
Over 18		40,110	
Total	228,816	40,380	269,196

Springtown Population

	No Traffic Delays	Traffic Delays	Total
18 or Younger		179	8,956
Over 18	26,152		
Total			44,781

5. Construct a relative frequency table for Metropolis and Springtown by age groups. Round your answers to the nearest tenth of a percent.

Metropolis Population

	No Traffic Delays	Traffic Delays	Total
18 or Younger			100%
Over 18			100%

Springtown Population

	No Traffic Delays	Traffic Delays	Total
18 or Younger			100%
Over 18			100%

Name: .... Date: .... Period: ....

Traffic Delays and Age (continued)

6. Adhira says: *Out of all the people who are over the age of* 18 *in Metropolis, about* 19% *of them already have experienced traffic delays, so the trains aren't going to make a big difference.* Do you agree or disagree with Adhira? Explain your thinking.

7. Based on this data, do you think there is an association between the city that people live in and traffic delays? Explain your thinking.

8. Is the association strong enough to cause the people of Metropolis to act?

9. Traffic delays is one impact that trains might have on the population of Metropolis. What are some other impacts that trains might have on the population of Metropolis?

Synthesis

10. When might a two-way frequency table be useful? When might a relative frequency table be useful?

Lesson Practice 2.11

Lesson Summary

It can be helpful to represent categorical data in a **two-way frequency table**. Two-way tables show the distribution of a population according to two categorical variables, including joint frequencies and marginal frequencies.

A **relative frequency table** shows the same data as percents relative to the totals for one of the variables. You can use this representation to see if the data presents evidence that there is an **association** between the two variables.

In these tables, there is evidence of an association in the data between age and whether a student takes dance class. This association is suggested because the students under 13 are much more likely (≈58.9%) to take dance class than the students 13 or older (≈46.5%).

Arts Studio Students

	Takes Dance Class	Does Not Take Dance Class	Total
13 or Older	105	121	226
Under 13	89	62	151
Total	194	183	377

Arts Studio Students

	Takes Dance Class	Does Not Take Dance Class	Total
13 or Older	≈46.5%	≈53.5%	100.0%
Under 13	≈58.9%	≈41.1%	100.0%

Lesson Practice
2.11

Name: .... Date: .... Period: ....

1. A random sample of teenagers were asked about their preferences when organizing a birthday party for a friend. Use this information to complete the two-way frequency table:

- 122 people responded to the survey.
- 50 of the people who said they prefer surprise parties also prefer that guests bring individual gifts.
- 68 people prefer organizing one group gift.
- 56 people prefer not to have a surprise party.

	Surprise Party	No Surprise	Total
Group Gift			
Individual Gifts			
Total			

Problems 2–3: Students in the 7th, 8th, and 9th grade were asked whether they prefer to write in pen or pencil. 40 students prefer to write in pen and 60 students prefer to write in pencil.

2. Create values that could represent the number of students in the 7th, 8th, and 9th grade that responded to the survey.

	Pen	Pencil
7th Grade		
8th Grade		
9th Grade		
Total		

3. Write a statement about the data in your table.

4. This relative frequency table shows data on whether students listen to music while they do their math and Spanish homework. Based on this data, is there an association between the homework subject and whether students listen to music?

Circle one: Yes No

Explain your thinking.

	Music	No Music
Math Homework	61%	39%
Spanish Homework	28%	72%

Name: .... Date: .... Period: ....

5. 1,430 people are surveyed about whether they take daily vitamins and whether they eat breakfast. The table shows the results. Create a relative frequency table that shows the percentage of the entire group that is in each cell.

	Take Daily Vitamins	Don't Take Daily Vitamins
Eat Breakfast	384	476
Don't Eat Breakfast	268	302

	Take Daily Vitamins	Don't Take Daily Vitamins
Eat Breakfast		
Don't Eat Breakfast		

Problems 6–9: This two-way frequency table shows student responses to the question: *Do you prefer to have gym class in the morning or in the afternoon?*

	Morning	Afternoon	Total
6th Grade	15	8	23
8th Grade	18	21	39
10th Grade	12	26	38
Total	45	55	100

6. How many students participated in the survey? ....

7. How many 8th grade students prefer to have gym class in the morning? ....

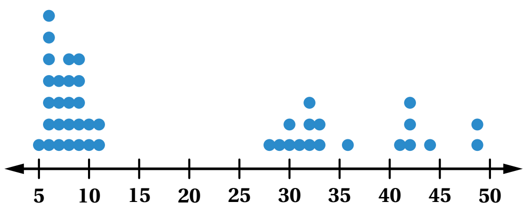
8. How many 10th grade students participated in the survey? ....

Test Practice

9. How many students prefer to have gym class in the afternoon? ....

Spiral Review

Problems 10–11: This dot plot represents the ages of people who watched a movie at a movie theater on a Saturday.



10. What kind of movie do you think this is? Explain your thinking.

11. How many people over 40 years old watched the movie on Saturday?

Unit 2
Lesson
12

Name: ______ Date: ______ Period: ______

MA.912.DP.3.1, MTR.2.1, MTR.4.1, MTR.7.1

Remodel Choices

Let's use data to make decisions and write arguments.

Warm-Up

Benjamin Banneker Secondary School is planning a renovation to remodel their outdoor space.

1. **Discuss:** If your school wanted to renovate their outdoor space, what are some options you would suggest?

2. The principal thinks it would serve the school best if the renovation covered making new basketball courts or a new picnic area. Which option would you choose to build for your school? Circle your choice.

   Basketball courts   Picnic area

   Explain your thinking.

Name: Date: Period:

Courtyard Questions

The principal surveyed the students to help the school decide whether to build basketball courts or a picnic area. The school has students in middle school (Grades 6–8) and high school (Grades 9–12).

Here is some of the data the principal collected.

3. Complete the two-way relative frequency table.

	Basketball Courts	Picnic Area	Total
Middle School	30%		37%
High School		41%	
Total	52%		

4. Does the school have more students in middle school or high school? Explain how you know.

5. Kadeem says: *The grade level with the largest number of students should get to decide what we do.*

Do you agree or disagree? Explain your thinking.

Name: Date: Period:

Courtyard Questions (continued)

Problems 6–9: Take a closer look at the *total relative frequency* of the survey results.

6. What percent of the total school population chose basketball courts?

7. What does the 41% mean in this situation?

8. Which choice was more popular for each group of students? Place a check mark in the correct column.

Group of Students	Basketball Courts	Picnic Area
Middle School		
High School		
All Students		

9. **Discuss:** Is there an association between grade level and whether the students want to build basketball courts or a picnic area? Use data from the table to support your claim.

Name: ________ Date: ________ Period: ________

Making Decisions

Sothy wanted to know how many people voted for each category. He knows that the school has 800 students total and that all students participated in the survey.

	Basketball Courts	Picnic Area	Total
Middle School	30%	7%	37%
High School	22%	41%	63%
Total	52%	48%	100%

10. Complete the table with the number of students in each category.

	Basketball Courts	Picnic Area	Total
Middle School			
High School			
Total			800

11. Write a true statement about the situation using each fraction.

a. $\frac{240}{416}$

b. $\frac{416}{800}$

c. $\frac{328}{800}$

12. Demetrius says: *About 58% of people who want to build basketball courts are middle school students.*

Nikolai says: *81% of middle school students want to build basketball courts.*

Who do you agree with? Circle one.

Demetrius Nikolai Both Neither

Activity 3

Name: ______ Date: ______ Period: ______

Closing Arguments

One middle schooler and one high schooler decided to talk to the principal about what the school should choose.

Kayleen (a middle schooler) said: *The school should build basketball courts because it's the most popular across the whole school.*

Mariam (a high schooler) said: *High school students have been here the longest, and we think a picnic area would benefit the entire school the most.*

13. Imagine you want to convince the principal which choice the school should make. Write an argument using data, reasoning, and your personal preference about what choice the school should make.

Synthesis

14. Describe a strategy for recognizing whether there is an association in a frequency table. Use the example if it helps with your thinking.

	Basketball Courts	Picnic Area	Total
Middle School	30%	7%	37%
High School	22%	41%	63%
Total	52%	48%	100%

Lesson Practice 2.12

Lesson Summary

Two-way tables can be helpful when making a decision. The distribution of data in a two-way frequency table can indicate whether there are associations in the data or other information that might support you in making a fair and informed decision.

For example, compare this relative frequency table and its corresponding two-way table. The tables show the data gathered by Tyani's principal about which field trip the students would prefer to take: going whale watching or going to an art museum.

The relative frequency table shows that Tyani's homeroom has a stronger preference for whale watching than the other homerooms, but the two-way table shows that there are more students overall who would prefer to go to the art museum.

Relative Frequency Table

	Whale Watching	Art Museum	Total
Tyani's Homeroom	≈61.5%	≈38.5%	100.0%
Other Homerooms	≈46.1%	≈53.9%	100.0%

Two-Way Frequency Table

	Whale Watching	Art Museum	Total
Tyani's Homeroom	16	10	26
Other Homerooms	88	103	191
Total	104	113	217

Lesson Practice
2.12

Name: .... Date: .... Period: ....

1. A group of student athletes set personal goals for a big race. After the race, they were asked whether they achieved their goal and whether they practiced daily.

This table shows the results. What does 35% represent?

	Daily Practice	No Daily Practice
Achieved Goal	35%	14%
Did Not Achieve Goal	11%	40%

2. A fashion magazine asked 250 people whether they wear a belt and a watch each day. Use the data in the relative frequency table to complete the frequency table.

Relative Frequency

	Watch	No Watch
Belt	44%	22%
No Belt	20%	14%

Frequency

	Watch	No Watch
Belt		55
No Belt		

Problems 3–4: A group of people were asked whether they like oranges and whether they like granola bars. This two-way frequency table shows the results.

	Like Oranges	Dislike Oranges	Total
Like Granola Bars	28	30	58
Dislike Granola Bars	23	38	61
Total	51	68	119

3. What percent of people who like oranges also like granola bars?

4. What percent of people who dislike oranges also dislike granola bars?

Lesson Practice
2.12

Name: .... Date: .... Period: ....

Problems 5–6: A random sample of people were asked about their preferences in home decoration and their interest in fashion.

5. Use this information to complete the two-way frequency table.

- 150 people completed the survey.
- 70% of the people do not enjoy fashion.
- 33 of the people who prefer neutral decorations also do not enjoy fashion.
- 20 of the people who enjoy fashion also prefer colorful decorations in their homes.

	Prefer Colorful Decorations	Prefer Neutral Decorations	Total
Enjoy Fashion			
Don't Enjoy Fashion			
Total			150

6. Create a total relative frequency table for this data.

	Prefer Colorful Decorations	Prefer Neutral Decorations	Total
Enjoy Fashion			
Don't Enjoy Fashion			
Total			100%

Test Practice

7. A group of people were asked whether they have any siblings and whether they have any pets.

What percent of people have no siblings and no pets?

	Have Siblings	Have No Siblings
Have Pets	20.5%	26%
Have No Pets	35.5%	?

Spiral Review

Problems 8–9: Which has a different solution? Circle your choice.

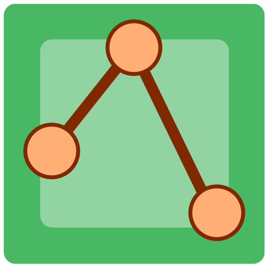
8. $4(x-3)+2=2(x+5)+10$ $\quad$ $2(3x-5)-8=4x+12$ $\quad$ $2(x+5)-3=3(x-1)+9$

9. $3(2y-5)+4 <= 2y+11$ $\quad$ $4(y-3)+6 \leq 2(y+2)+1$ $\quad$ $2(y+4)-5 \geq 3y+7$

Unit 2
Lesson
13

Name: ______ Date: ______ Period: ______

MA.912.DP.1.1, MA.912.DP.1.2, MTR.1.1, MTR.3.1, MTR.6.1



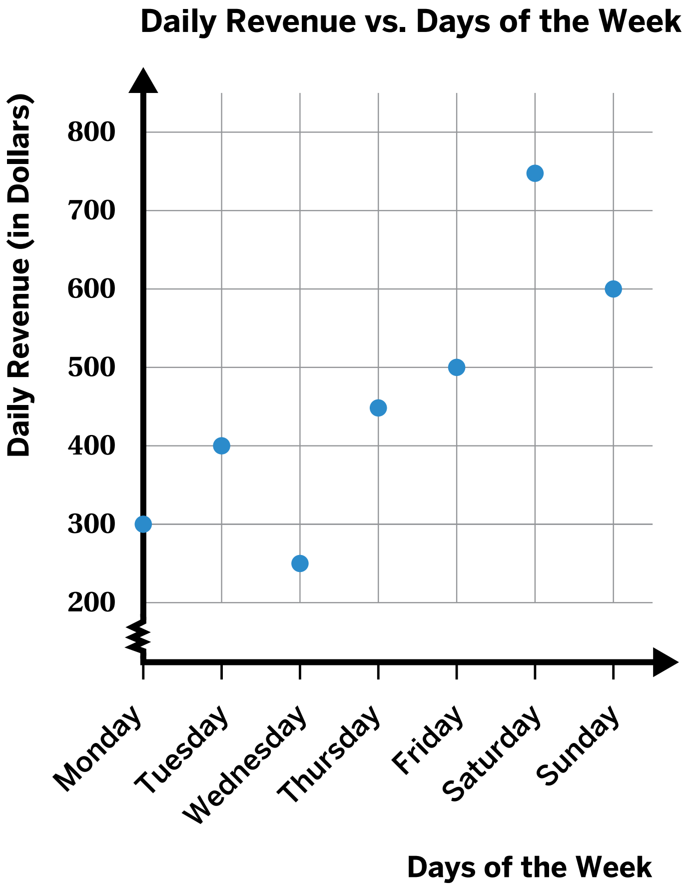
Connecting the Dots

Let's represent and analyze data sets in a line graph.

Warm-Up

1. A line graph can quickly provide information and visually analyze it. Draw a line to connect each dot in the order of the days of the week.

Discuss: What do you notice? What do you wonder?



Activity
1

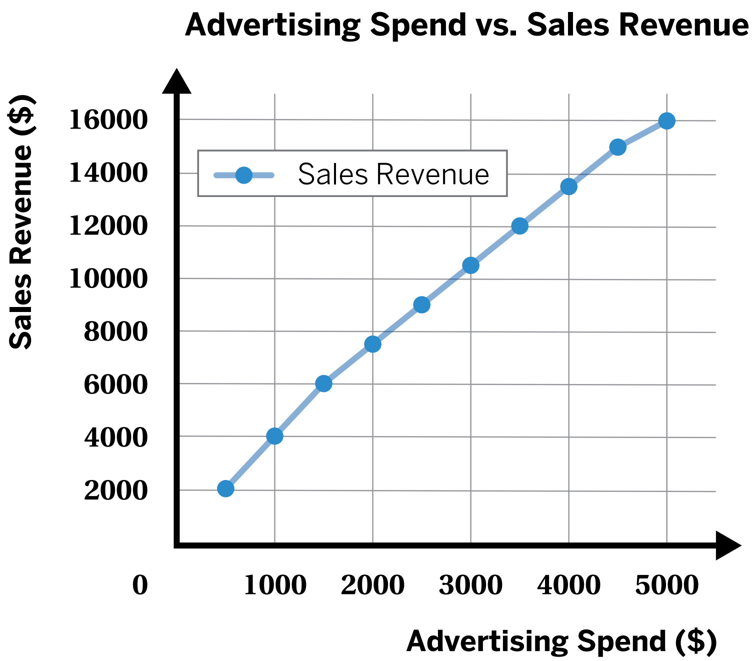
Name: ______ Date: ______ Period: ______

Advertising Goals

Line graphs can be a tool for identifying trends and analyzing relationships when plotting two continuous variables. They can help visualize upward or downward trends, cyclical patterns, or other fluctuations in the data and reveal a correlation, or relationship, between variables.

A small business owner wanted to grow her business, so she started investing in advertising. In January, she spent $500 and saw $2,000 in sales. Encouraged, she increased her monthly advertising spending: $1,000 for $4,000 in sales, $1,500 for $6,000, and so on. By the end of the year, she was spending $5,000 on advertising and earning $16,000 in sales. The consistent increase in advertising spending led to steady revenue growth, proving that strategic advertising was key to her business success.

Advertising Spend ($)	Sales Revenue ($)
500	2000
1000	4000
1500	6000
2000	7500
2500	9000
3000	10,500
3500	12,000
4000	13,500
4500	15,000
5000	16,000



Use the data in both the table and graph to answer questions.

2. What does the x-axis represent?

3. What does the y-axis represent?

4. What do the data points represent?

5. What does the line graph show you?

6. Describe what you see happening with the data set.

Activity 2

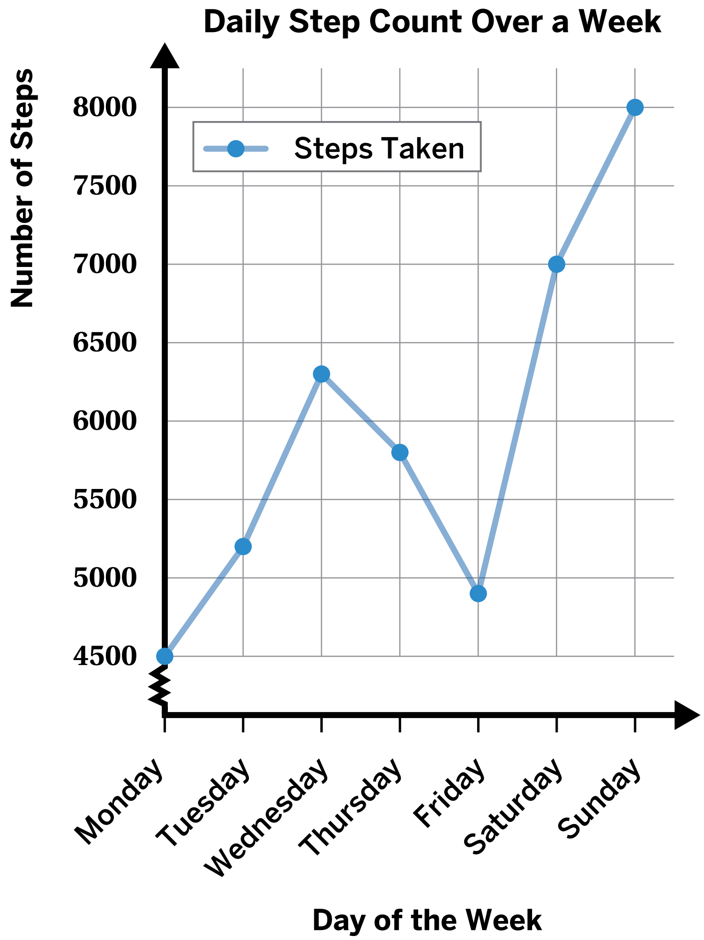
Name: ______ Date: ______ Period: ______

Making Steps

Line graphs can display numerical or categorical data and univariate or bivariate data.

A student tracked the number of steps that he took every day for one week.

Day of the Week	Number of Steps
Monday	4500
Tuesday	5200
Wednesday	6300
Thursday	5800
Friday	4900
Saturday	7000
Sunday	8000



7. What type of data are the days of the week?

8. What type of data are the steps taken?

9. Is this bivariate or univariate?

10. What trend do you observe in step counts between weekdays and weekends? Is there a noticeable difference?

Name: Date: Period:

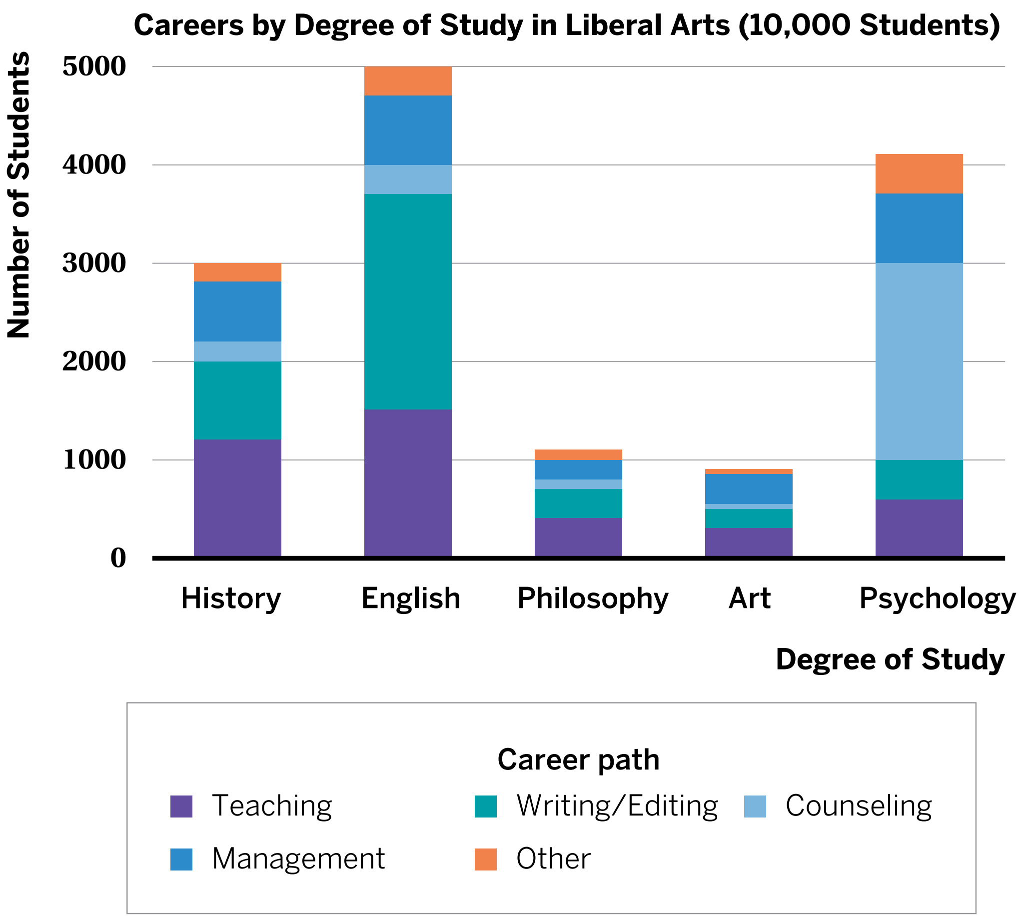
Making Steps (continued)

Problems 11–13. A college surveyed students about their degree of study and career path. Use this graph to answer these questions.

11. What type of data is represented in this segmented bar chart?

12. What does this representation help you to visualize regarding the relationship between the data?

13. Would this data be better represented as a line graph? Why or why not?



Activity 3

Name: ________ Date: ________ Period: ________

A Race in the Serengeti

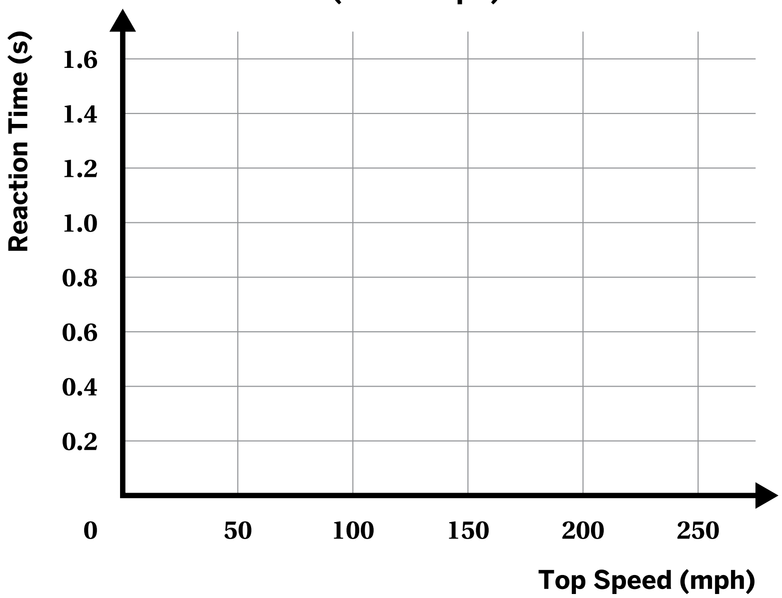
In the Serengeti, scientists studied how different animals use their speed and reflexes to survive. A cheetah sprinting after prey, a falcon diving through the sky, or a rabbit zigzagging to evade predators—each had its own unique combination of speed, in miles per hour, and reaction time, in seconds.

Animal	Top Speed (mph)	Reaction Time (s)
Cheetah	70	0.2
Falcon	240	0.1
Rabbit	35	0.3
Dog	42.5	0.25
Human	28	0.35
Elephant	25	0.4
Turtle	1	1.5
Horse	55	0.3
Dolphin	25	0.2
Kangaroo	44	0.35

14. In the grid below, create a line graph that best describes the data.

15. Write a question to share with your group that can be answered using your graph.

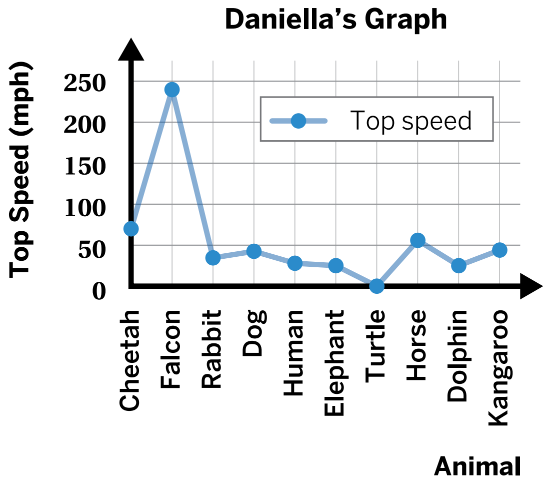
Reaction Time vs. Speed by Animal (Line Graph)

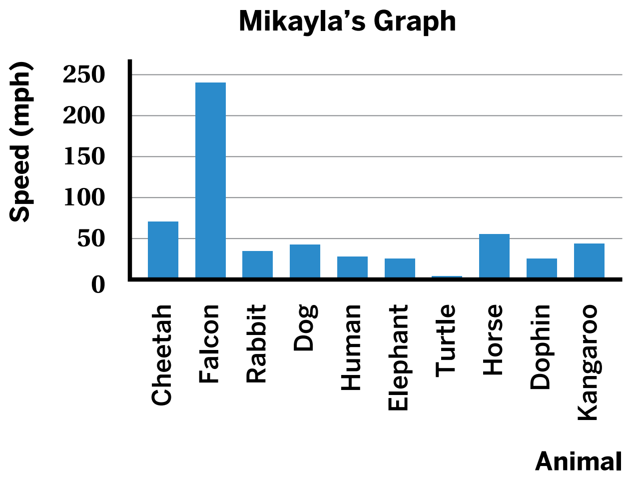


Name: ______________________ Date: __________ Period: __________

A Race in the Serengeti (continued)

For Problems 16–19, use the different graphs made by Daniella and Makayla representing some of the same data set.





16. What could be the title of each graph?

17. Which animal is the second fastest?

18. Which animals are outliers for speed?

19. Which graph is a better choice for this data set? Why or why not?

20. When is a line graph an appropriate choice to represent a data set?

21. Give an example for each type of data that a line graph could represent.

Data Type	Examples	Data Type	Examples
Continuous Data		Rate of Change	
Time Series Data		Sequential Data	

Synthesis

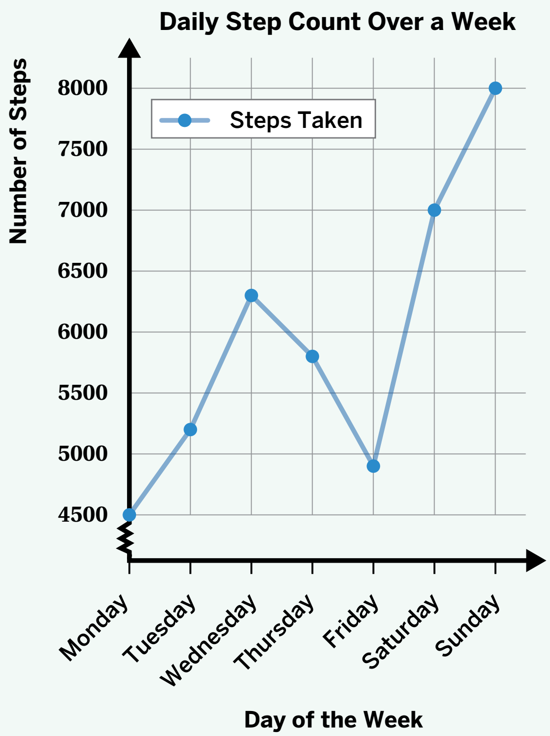
22. What types of data can be used on a line graph?

23. When is it best to use a line graph?

Lesson Practice 2.13

Lesson Summary

- Line graphs plot data points along the x-**axis** and y-**axis**.
- A **line graph** is good for showing trends or cyclical patterns in numerical bivariate data sets in which there is an independent variable, such as time, and a dependent variable, like the growth of a plant.
- Numerical bivariate data points are plotted and connected by lines to reveal **trends, patterns,** or **fluctuations**.
- **Strengths:** Ideal for tracking changes over time, identifying peaks and troughs, and comparing datasets.



Lesson Practice
2.13

Name: ________ Date: ________ Period: ________

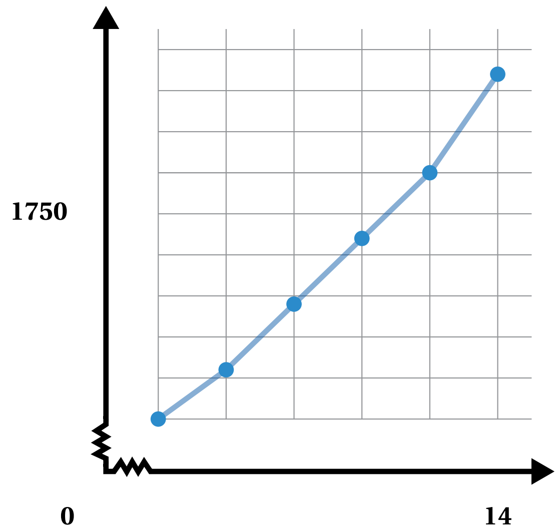
Problems 1–5: Use the scenario and graph provided to answer the questions.

A retail store tracks the data on the hours it stays open and the total daily sales. For example, when the store is open for 4 hours, it earns \$500 in daily sales, and when it is open for 12 hours, it earns \$2000. This helps the management assess whether extending store hours increases revenue proportionally.

Add the following elements to the line graph.

1. Give the graph a title.
2. Label the y-axis and intervals.
3. Label the x-axis and intervals.
4. Fill in the table with the data from the graph. Estimate appropriately for any numbers not on a grid line.

Time Store is Open (hrs) (x)	Daily Sales (\$) (y)
14	



5. What does the line graph of the data show you?

Problems 6–8: Describe each data set (Numerical or Categorical/univariate or bivariate). Then, decide if a line graph would be a good way to represent the information. Explain your reasoning.

6.

Type of Transportation	Number of Wheels
Car	4
Unicycle	1
Motorcycle	2
Semi Truck	18

7.

Distance Ran (Miles)	Calories Burned
1	100
2.5	250
3	300
5.5	550

8.

Time (Min)	Number of People
0–50	5
51–100	7
101–150	8
151–200	3

Lesson Practice
2.13

Name: ________ Date: ________ Period: ________

Test Practice

9. Select all the true statements about using line graphs to represent data sets.

- ☐ **A.** Line graphs are best for showing relationships between two numerical variables over time.
- ☐ **B.** Line graphs cannot display variability in data trends.
- ☐ **C.** Line graphs can help identify trends, peaks, and valleys in data.
- ☐ **D.** Line graphs are suitable for categorical data comparisons.
- ☐ **E.** The x-axis typically represents the independent variable, such as time or another numerical measurement.

Spiral Review

Fill in the table for each law of exponents and solve the sample problem.

Laws of Exponents	Rule	Sample
10. Product of Powers Rule	$a^m \bullet a^n =$	$j^2 \bullet j^3 =$
11. Quotient of Powers Rule	$\frac{a^m}{a^n} =$	$\frac{x^8}{x^2} =$
12. Power of a Power Rule	$(a^m)^n =$	$(y^3)^4 =$
13. Power of a Product Rule	$(ab)^m =$	$(xy)^5 =$
14. Power of a Quotient Rule	$\left(\frac{a}{b}\right)^m =$	$\left(\frac{k}{n}\right)^2 =$

Name: .... Date: .... Period: ....

MA.912.DP.2.4, MA.912.DP.2.6, MA.912.AR.2.5, MTR.1.1, MTR.4.1, MTR.6.1, MTR.7.1

City Slopes

Let's use a line of fit to describe the relationship between two variables and make predictions.

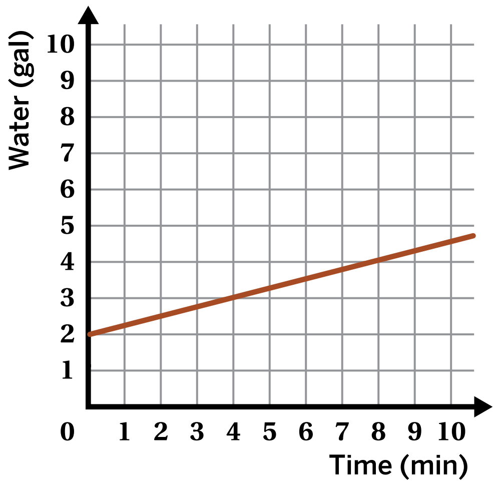
Warm-Up

1. An equation for this line is $y = \frac{1}{4}x + 2$.

Show or explain where you see $\frac{1}{4}$ and 2 in the graph.

$\frac{1}{4}$:

2:



Activity 1

Name: ________ Date: ________ Period: ________

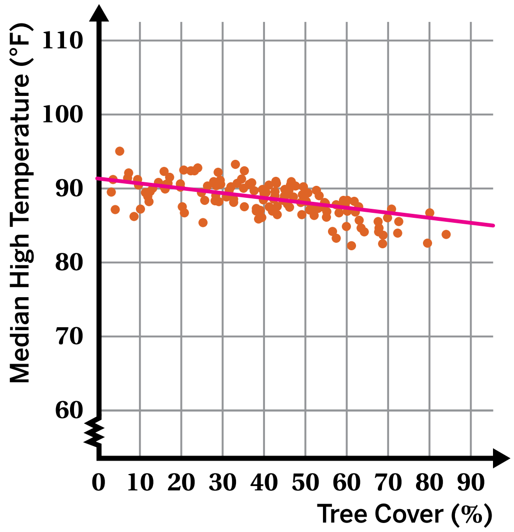
Lines of Fit

2. Here's the graph of temperature and tree cover for a city, City A.

Mathematicians use a *line of fit* to describe relationships and make predictions.

a **Discuss:** Why is a line a good fit for this data?

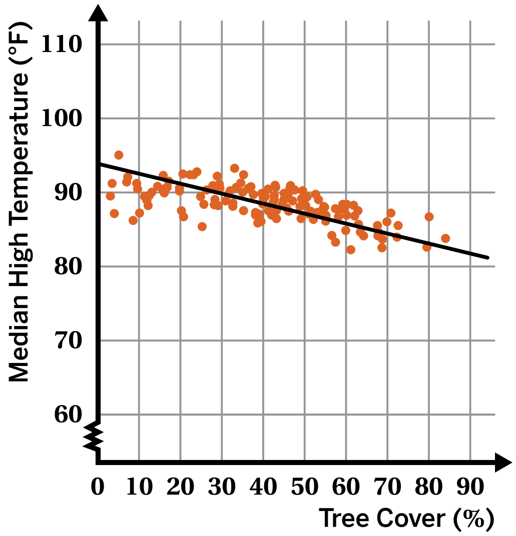
b Draw a line of fit for the data.



3. Here is the line of fit a student drew.

Jamal lives in City A, on a block that has 75% tree cover.

What might the median high temperature be on his block?

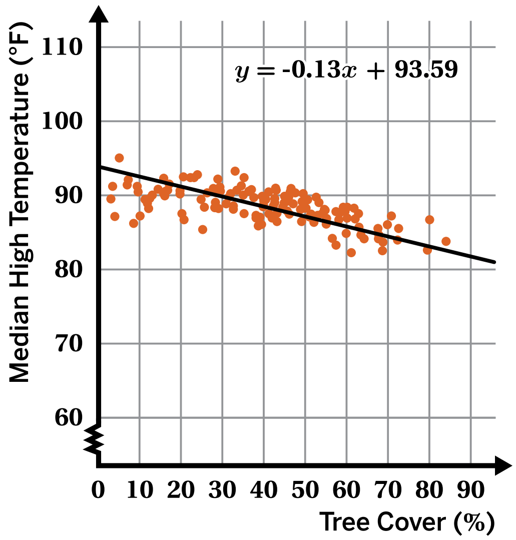


4. Here is an equation for this line of fit.

What does the *slope* mean for the relationship between temperature and tree cover? Circle one.

The slope means that when the tree cover increases by 1%, the temperature decreases by 0.13°F.

The slope means that when the tree cover increases by 1%, the temperature decreases by 93.59°F.



Name: ______ Date: ______ Period: ______

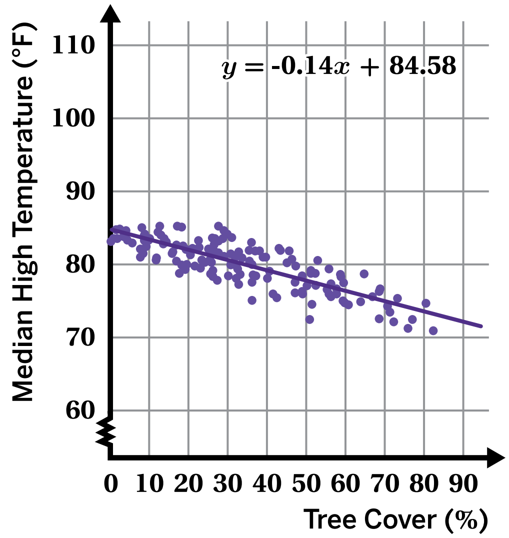
Interpreting in Context

5. Here is an equation that models this relationship for another that models this relationship for another city, City B.

What do the -0.14 and 84.58 mean about the relationship between temperature and tree cover?

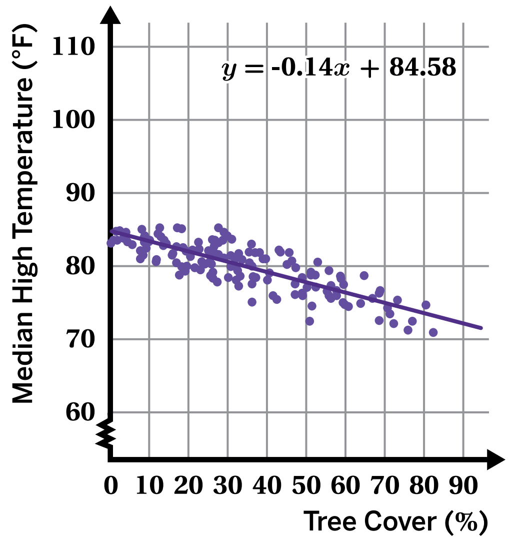
-0.14:

84.58:



6. A community in City B wants to build a park.

If the park has 80% tree cover, what might its median high temperature be?

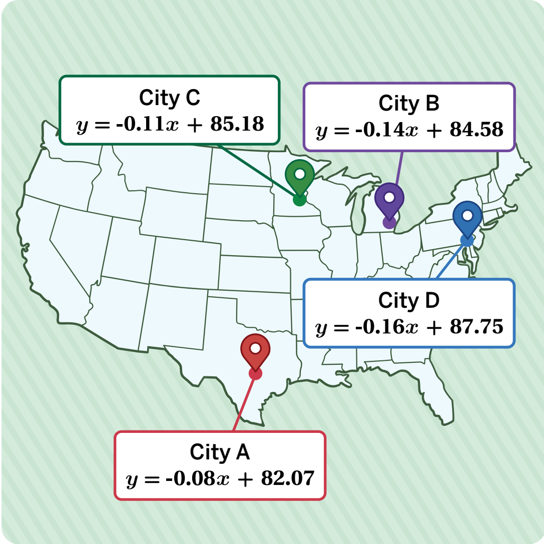


Name: ______ Date: ______ Period: ______

Interpreting in Context (continued)

7. a Order the cities according to where tree cover has the greatest impact on temperature.

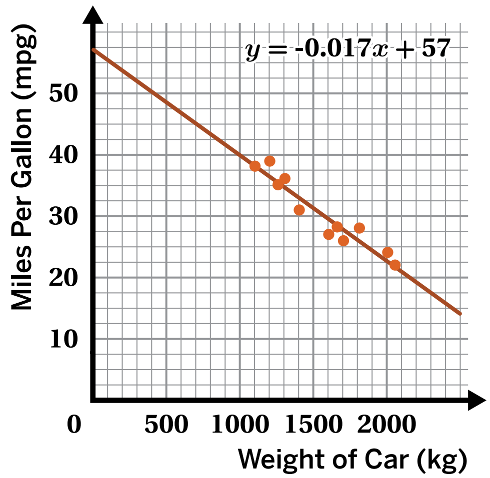
	Greatest
	Least



b **Discuss:** How did you order each city?

Synthesis

8. How can a line help us make predictions about data?



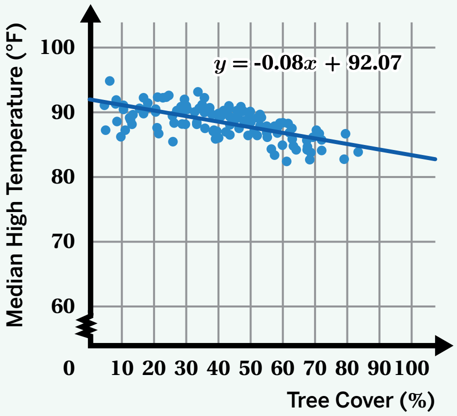
Lesson Practice 2.14

Lesson Summary

A *line of fit* can help you make predictions about specific values in a data set. Points along the line of fit represent the likely value of unknown data in the data set.

You can use the equation of the line of fit to better understand the data. The y-intercept of the line represents a potential initial value and the slope describes the rate that the variables change in relationship to one another.

For example, this scatter plot shows data on tree cover and temperature for 150 blocks in City A. The equation of the line of fit is $y = -0.08\text{x} + 92.07$.



The slope is -0.08. This means that when the tree cover increases by 1% in City A, the predicted temperature decreases by 0.08°F.

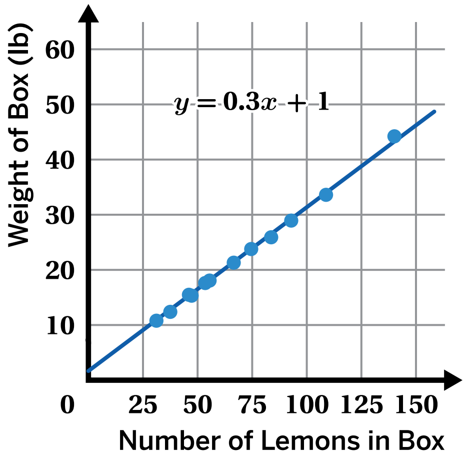
The y-intercept is 92.07. This means that if the tree cover in City A is 0%, the predicted temperature is 92.07°F.

You can use the line to predict that if a block in City A has 80% tree cover, the temperature will be about 85°F.

Lesson Practice
2.14

Name: ______ Date: ______ Period: ______

Problems 1–2: A store receives 12 boxes that contain many different numbers of lemons. They weigh each box and then count the lemons. Here is a scatter plot of that data.



1. A new box weighs 30 pounds. Approximately how many lemons are in the box?

2. The equation for the line that best fits this data is $y = 0.3x + 1$. What do the numbers 0.3 and 1 mean in this context?

Problems 3–4: The store collects data about the weight of other types of fruit boxes. The table shows equations for the line of best fit for each type of fruit box, where y represents the weight of a box and x represents the number of fruits in a box.

Fruit	Line of Best Fit
Orange	$y = 0.45x + 0.95$
Lemon	$y = 0.3x + 1$
Pomegranate	$y = 1.05x + 1.15$
Mango	$y = 0.85x + 1.25$

3. Which fruit is heavier? Circle your choice.

Lemons Pomegranates They are the same

Explain your thinking.

4. Order these fruits by their weight.

	Lightest
	Heaviest

5. Sora collects data about the number of bananas she buys at the store, x, and the total weight of the bananas in pounds, y. If Sora graphed her data, which value would be closest to the slope of the line of best fit?

A. -4

B. -0.4

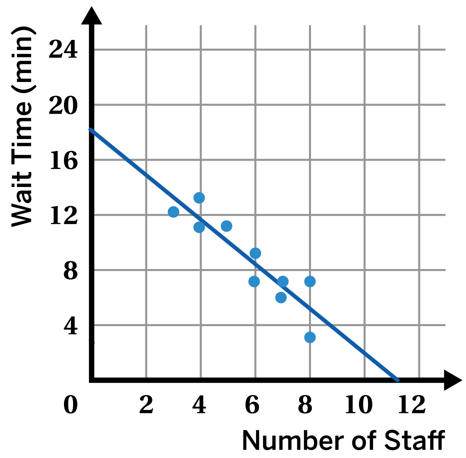
C. 0.4

D. 4

Lesson Practice
2.14

Name: .... Date: .... Period: ....

Problems 6–7: A restaurant gathered data about how long customers had to wait and how many staff members were working. The slope of the line of fit is -1.62.



6. What does the slope of -1.62 mean in this situation?

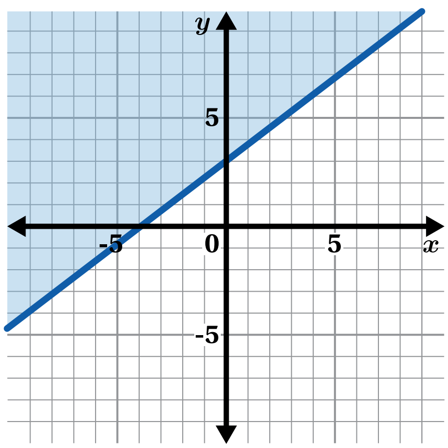
7. What equation might reasonably represent the line of fit? Explain your thinking.

Test Practice

8. Select *all* the coordinate pairs that are solutions to $y = -\frac{1}{2}x + 10$.

☐ **A.** $\left(0, \frac{2}{3}\right)$ ☐ **B.** $(-10, 15)$ ☐ **C.** $(4, 12)$ ☐ **D.** $(2, 9)$ ☐ **E.** $(5, 8)$

9. Here is the graph of the inequality $y \geq 0.75 + 3$. Select *all* the coordinate pairs that are solutions to this inequality.



☐ **A.** $(-9, -5)$
☐ **B.** $(-3, 2)$
☐ **C.** $(0, 3)$
☐ **D.** $(2, -3)$
☐ **E.** $(6, 8)$

Unit 2

Lesson

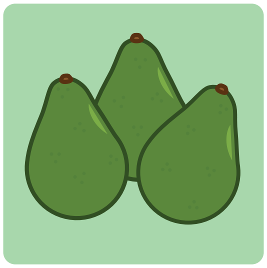
15

Name: ________ Date: ________ Period: ________

MA.912.DP.2.4, MA.912.DP.2.6, MA.912.AR.2.5, MTR.1.1, MTR.4.1, MTR.7.1

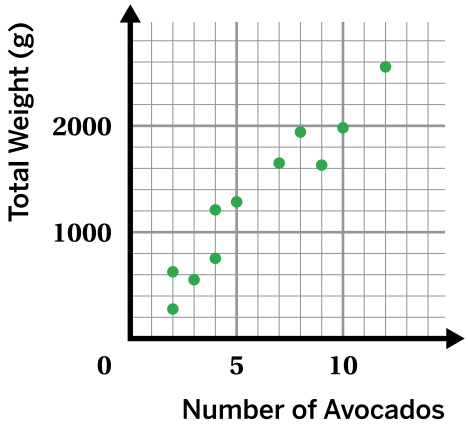
Residual Fruit

Let's use residual plots to determine how well a line fits data.



Warm-Up

1. Brianna has a business that ships different kinds of fruit.



Discuss: How much do you think 11 avocados will weigh?

Activity 1

Name: ______ Date: ______ Period: ______

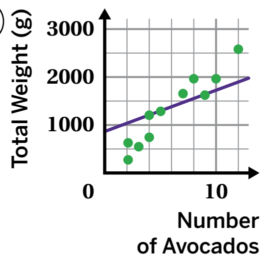
Predicting With Lines

2. Lines that fit the data well can help us make predictions.

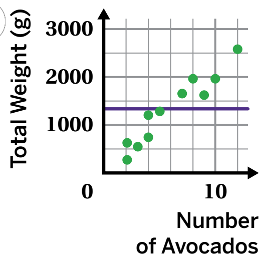
None of these lines fit the data well.

Circle a scatter plot. Explain why the line does not fit the data well.

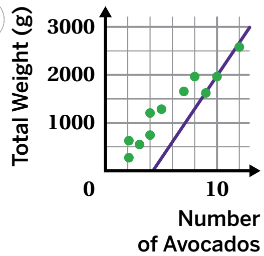
A



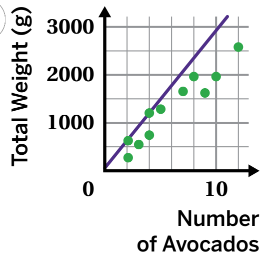
B



C

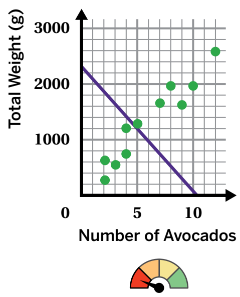


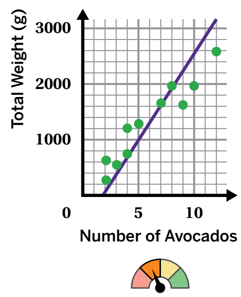
D

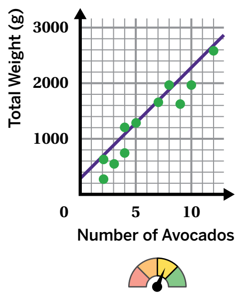


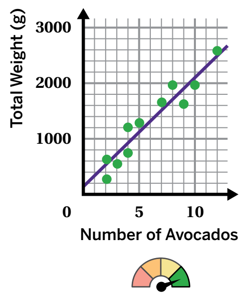
3. **a** Take a look at these scatter plots. The meters show how well each line fits the data.

b Explain to a classmate how to get a high score on the meter.









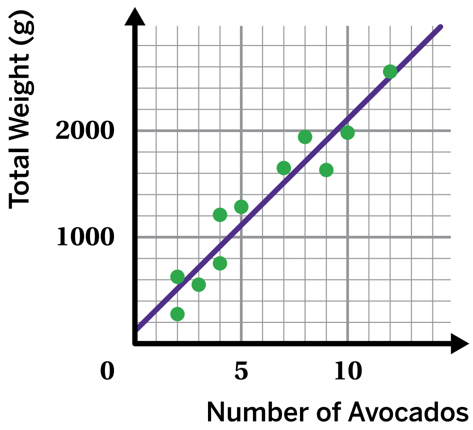
Activity 1

Name: .... Date: .... Period: ....

Predicting With Lines (continued)

4. Brianna has an order to ship 6 avocados.

How could you use this line to predict the weight of the order?



5. The line predicts that 6 avocados will weigh $1{,}316$ grams, but 6 avocados actually weigh $1{,}740$ grams.

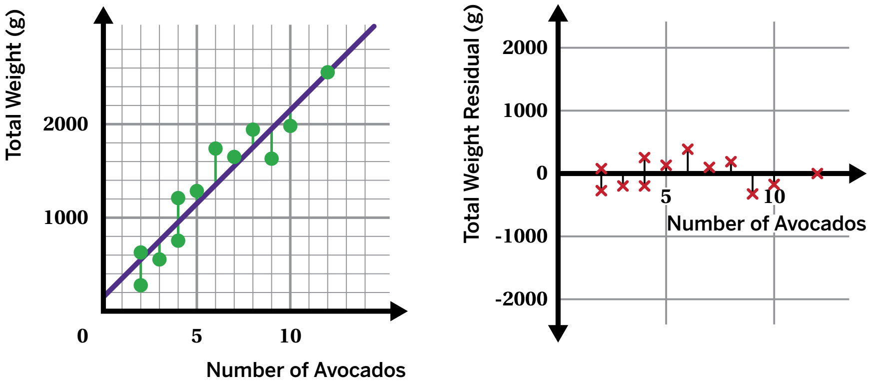
What is the difference, in grams, between the predicted weight and the actual weight?

Name: ______ Date: ______ Period: ______

Residual Plots

6. A **residual** is the difference between the predicted and measured weight.

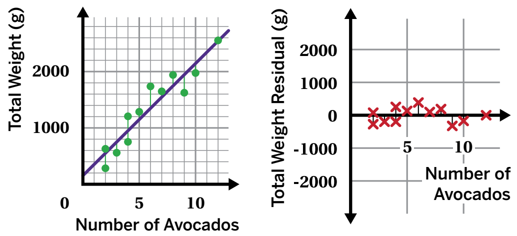
a Let's see how to plot residuals.



b **Discuss:** How could you make a residual plot?

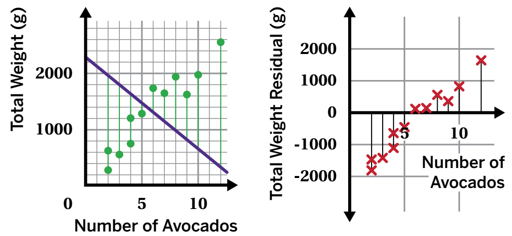
7. A **residual plot** shows how far each point is from the line of fit.

Fits the Data Well



Number of Avocados	Total Weight (g)	Residual
2	280	-237
2	630	113
3	556	-161
4	754	-163
4	1210	293
5	1286	170
7	1650	134
8	1944	288
9	1630	-285
10	1976	-139
12	2560	46

Doesn't Fit the Data Well



Number of Avocados	Total Weight (g)	Residual
2	280	-1680
2	630	-1330
3	556	-1244
4	754	-886
4	1210	-430
5	1286	-194
7	1650	490
8	1944	944
9	1630	790
10	1976	1296
12	2560	2200

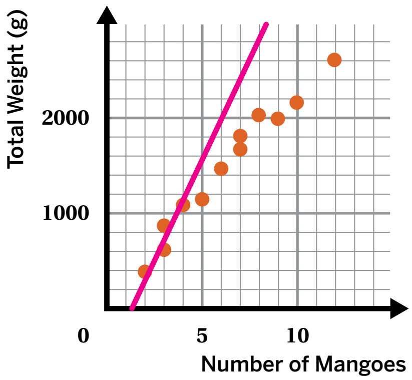
What does the residual plot look like when the line fits the data well? What about when it doesn't fit the data well?

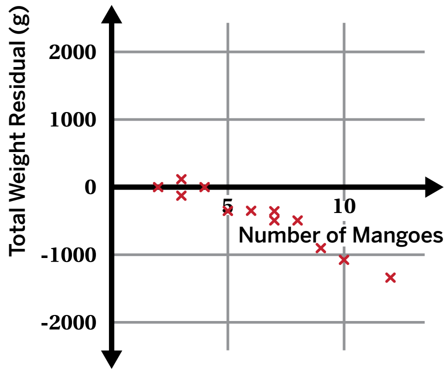
Name: .... Date: .... Period: ....

Residual Plots (continued)

8. Here is the residual plot for a line of fit Brianna created.

Sketch the line of fit you think Brianna made.

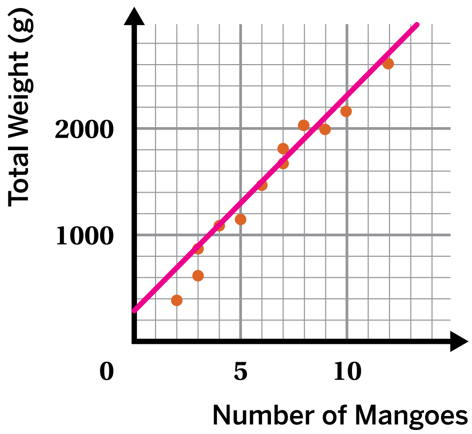




9. Let's look at Brianna's line.

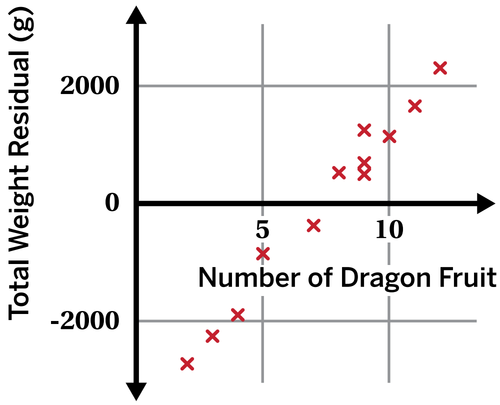
a Draw another line that is a better fit for the data.

b **Discuss:** How will the residual plot change once the line is a better fit for the data?



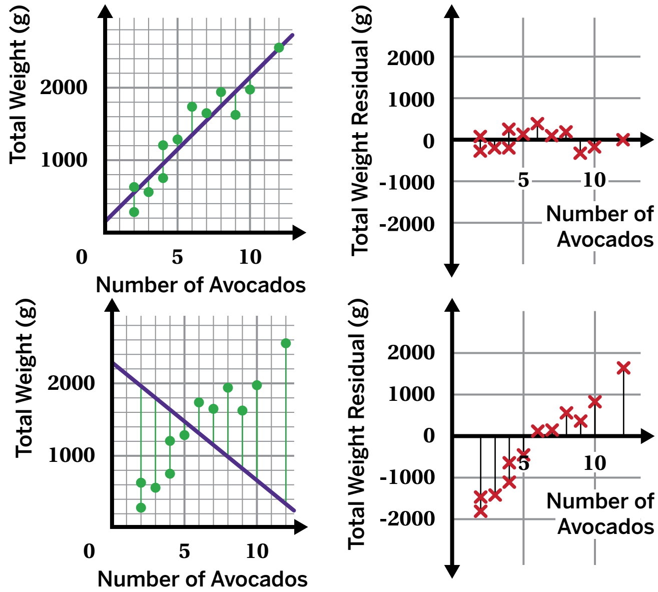
10. Aditi made a line of fit for this data showing the total weight of different numbers of dragon fruit.

Here is the graph of the residuals from Aditi's line. Based on the graph of the residuals, interpret the strength and direction of the correlation of the line of fit as it relates to number of dragon fruit and total weight in grams.



Synthesis

11. How can you use a residual plot to determine if a line fits the data well?

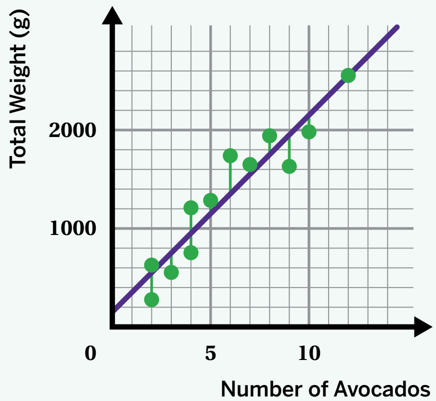


Lesson Practice 2.15

Lesson Summary

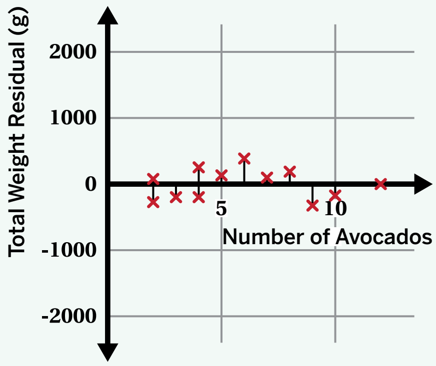
You can use residuals to determine how well a line fits a data set. A **residual** is the difference between the y-value for a point in a scatter plot and the value predicted by the line of best fit.

Here is a scatter plot with data on the number of avocados and their weights. The residuals are represented with lines connecting each point to the line of best fit.



You can also create a residual plot to analyze how well a line fits a data set. A **residual plot** is a scatter plot of residual values for a data set where the x-axis represents the value predicted by the line of best fit, and the y-value of each point represents the value of the residual.

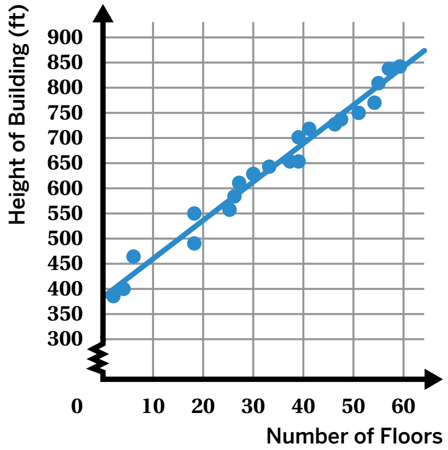
Here is the residual plot of the graph of avocado weights. The closer a point is to the x-axis, the closer that point is to the line of best fit.

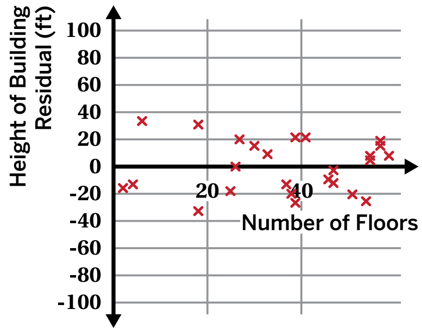


Lesson Practice
2.15

Name: .... Date: .... Period: ....

Problems 1–2: The scatter plot shows the heights of a group of buildings, the number of floors in each building, and a line that best fits the data. The residual plot is also shown.



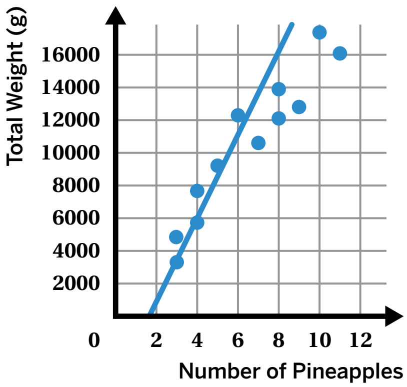


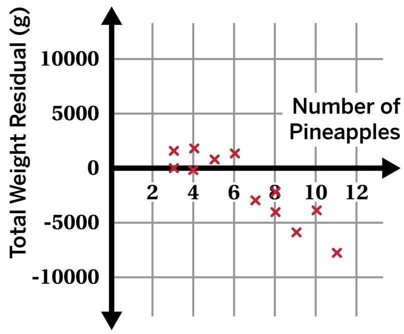
1. Predict the height of a building that has 50 floors.

2. How can you tell that the graphed line is a good fit for the data? Use the residual plot if it helps with your thinking.

3. Here is a scatter plot and its corresponding residual plot.

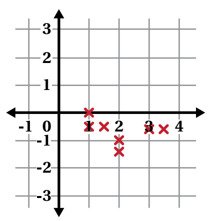
Draw a better line of fit on the scatter plot.



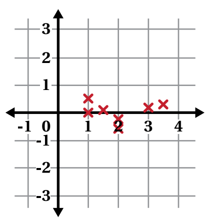


4. These residual plots are from the same set of data, but each one represents a different line of fit. Which residual plot shows the best line of fit?

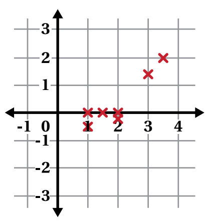
A.



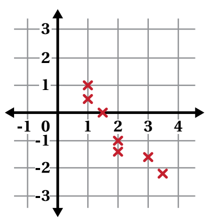
B.



C.



D.



Explain your thinking.

Problems 5–6: This scatter plot shows the number of traffic tickets and the cost of car insurance for 16 people.

5. Which equation could represent the line of best fit?

A. $y = -62x + 220$ **B.** $y = 62x + 220$ **C.** $y = -220x + 62$ **D.** $y = 220x + 62$

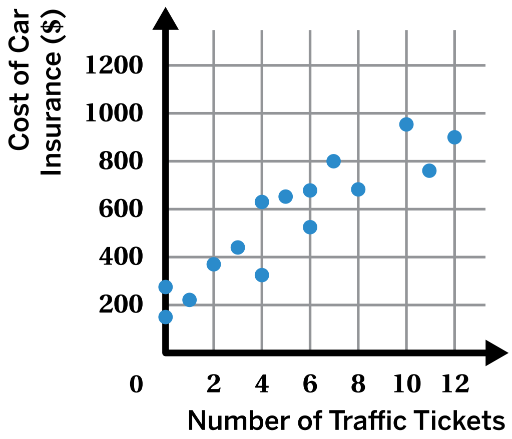
Explain your thinking.

Test Practice

6. Which of the following could be a reasonable prediction for the cost of car insurance for an individual with 15 traffic tickets?

A. \$750 **B.** \$900

C. \$1150 **D.** \$2000



Spiral Review

Problems 7–9: Solve each equation for x.

7. $y = 3x + 5$

8. $4y - 2x = 10$

9. $y = \frac{1}{6}x$

10. Are there any outliers in this data set? Use a calculator if it helps with your thinking.

$$[1, 1, 1, 1, 2, 2, 3, 3, 4, 4, 5, 5, 5, 6, 20]$$

Circle one. Yes No

Explain your thinking.

Unit 2
Lesson
16

Name: ________ Date: ________ Period: ________

MA.912.DP.2.4, MA.912.DP.2.6, MA.912.F.1.5, MA.912.AR.2.5, MA.912.AR.2.2, MTR.1.1, MTR.4.1, MTR.7.1

Penguin Populations

Let's generate and analyze lines of best fit to explore how penguin populations have changed over time.

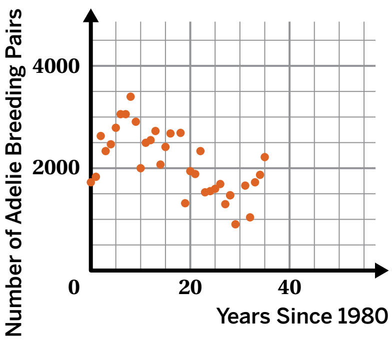
Warm-Up

1–2. Researchers are conducting a long-term study on the South Orkney Islands in Antarctica.

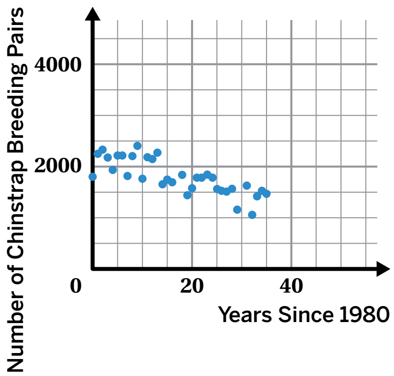
The goal of this study is to understand how the populations of three species of penguins have changed over the last 40 years.

Here is the study's data for each of the penguin species.

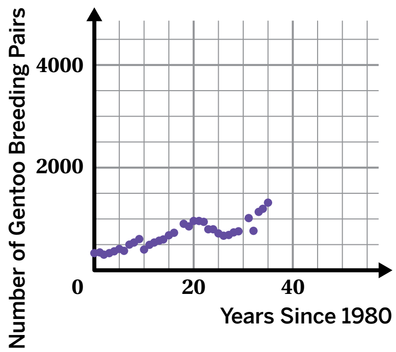
Adélie



Chinstrap



Gentoo



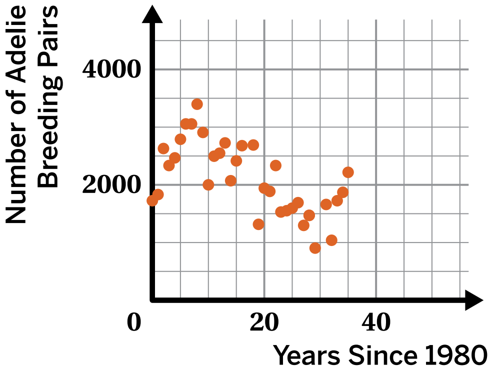
How is the population of each species changing over time? What might be affecting these changes?

Activity 1

Name: .... Date: .... Period: ....

Using the Line of Best Fit

3. Let's explore how the Adélie penguin population on the South Orkney Islands has changed over time.



a Draw a line that fits the data.

b What does the line tell you about the penguin population?

4. A graphing calculator or graphing tool can compute the **line of best fit**.

a Let's look at the line of best fit for the Adélie penguin data.

b **Discuss:** What does the residual plot tell you about the penguin population?

5. Let's look at the equation of the line of best fit for the Adélie penguin data.

a **Discuss:** What year would 2030 be on the x-axis?

b Use the line of best fit to predict how many breeding pairs of penguins there will be in 2030.

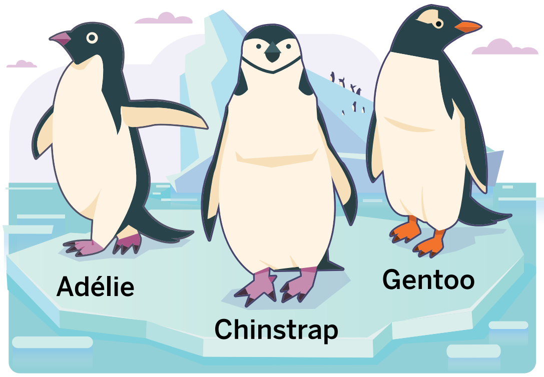
Activity 2

Name: ______ Date: ______ Period: ______

Generating a Line of Best Fit

6–7. Use the table of data showing the Adélie penguin population over time from the Activity Sheet.

Then use a graphing calculator or graphing tool to generate the line of best fit for this data.



Discuss: What does each part of $y_1 \sim mx_1 + b$ represent?

8. Use the data from the Activity Sheet to analyze the Chinstrap data.

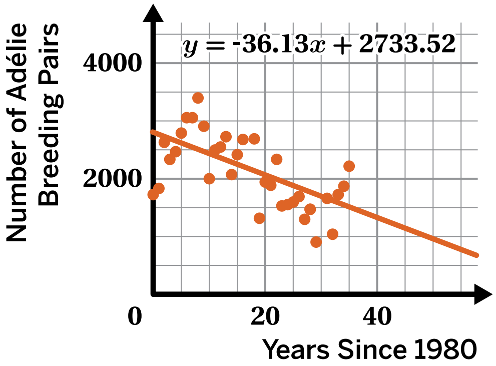
a Generate a line of best fit for the Chinstrap data using a graphing calculator or graphing tool.

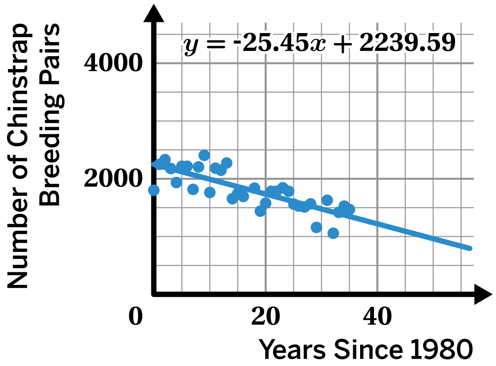
b Write an equation for the line of best fit, in the form $y = mx + b$.

Name: .... Date: .... Period: ....

Generating a Line of Best Fit (continued)

9. Compare the data of these two penguin populations.





Discuss: How are they alike? How are they different?

10. Recall that a linear relationship has a constant rate of change and thus, its graph is a straight line. A linear equation can be written in the form $y = mx + b$.

a. Are the equations shown on the graphs in Problem 9 linear? How do you know?

b. Compare these two equations and their graphs. What do you notice?

Activity 3

Name: Date: Period:

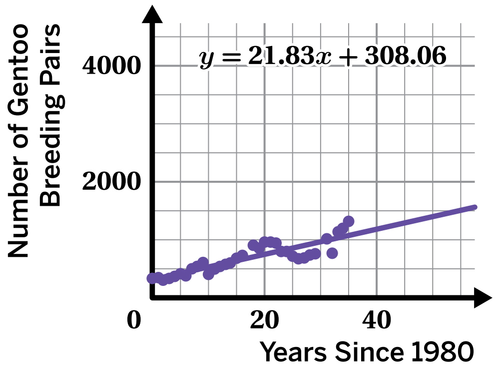
Making Predictions

11. Use the data from the Activity Sheet to analyze the Gentoo penguin population.

a Generate a line of best fit for the data using a graphing calculator or graphing tool. Write the equation for the best-fit line.

12. Here is the line of best fit for the Gentoo data.

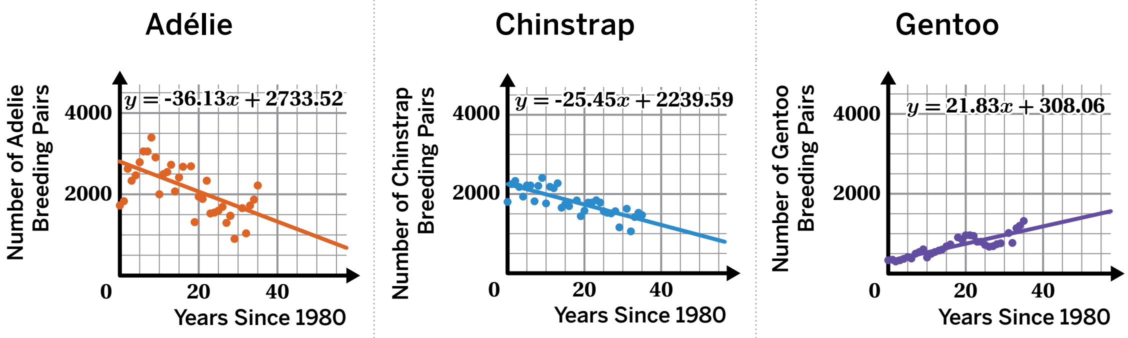
How many breeding pairs of penguins does the line of best fit predict there will be in 2030?



Name: .... Date: .... Period: ....

Making Predictions (continued)

13. What questions might researchers have after analyzing the data from the Adélie, Chinstrap, and Gentoo penguin populations?



14. Here's a quote about modeling that you may remember:

All models are wrong, but some are useful.

a Select a model we've explored today.

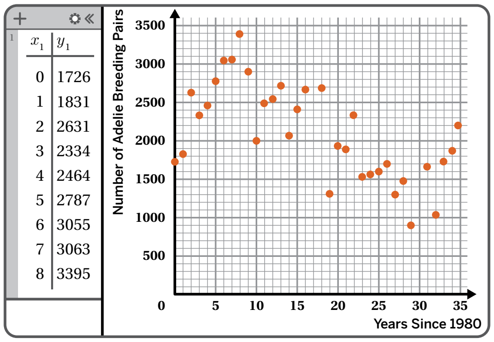
b Explain how that model is wrong and how it is useful.

The model is wrong because . . .

It is useful because . . .

Synthesis

15. Describe how to use a graphing tool to generate the line of best fit for data.



Lesson Practice 2.16

Lesson Summary

The **line of best fit** is the line on a scatter plot that best represents the trend created by the points in a data set.

Instead of sketching a line of fit, you can use a graphing calculator to precisely generate the equation of the line of best fit from a scatter plot. The equation of the line of best fit can help you interpret information about a situation, or allow you to substitute values into the equation to make predictions.

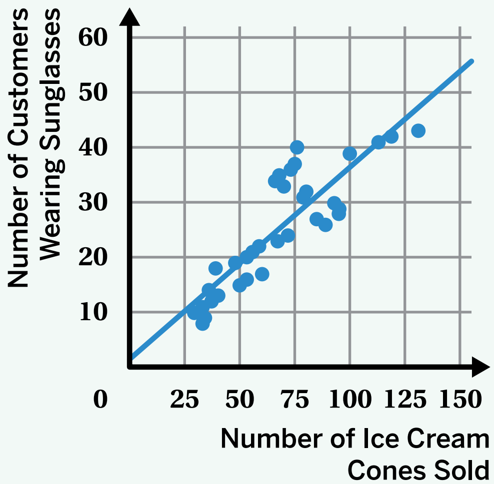
Here is an example of a line of best fit generated by a graphing calculator, and the information about the line that a graphing calculator will show you. Recall that m represents the slope of the line and b represents the initial value.

$y_1 = mx_1 + b$

PARAMETERS

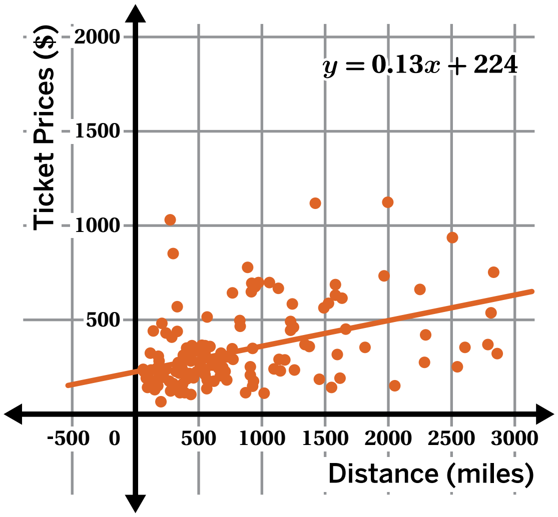
$m = 0.351312$

$b = 1.31984$



Name: .... Date: .... Period: ....

Problems 1–3: This scatter plot shows the distances of one way flights and their ticket prices.



1. The equation for the line of best fit is $y = 0.13x + 224$.

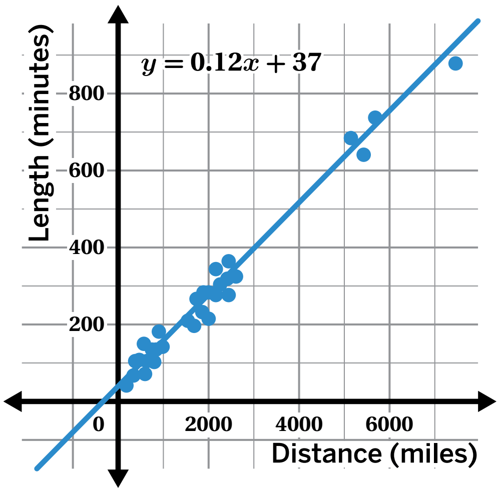
What does 0.13 mean in this situation?

2. The distance from Phoenix, Arizona, to Jacksonville, Florida, is $1{,}795$ miles.

Use the equation of line of best fit to predict the cost of a plane ticket from Phoenix to Jacksonville.

3. Do you think the prediction is accurate? Explain your thinking.

Problems 4–6: This scatter plot shows the distances of non-stop flights and their lengths in minutes.



4. The equation for the line of best fit is $y = 0.12x + 37$.

What does 0.12 mean in this situation?

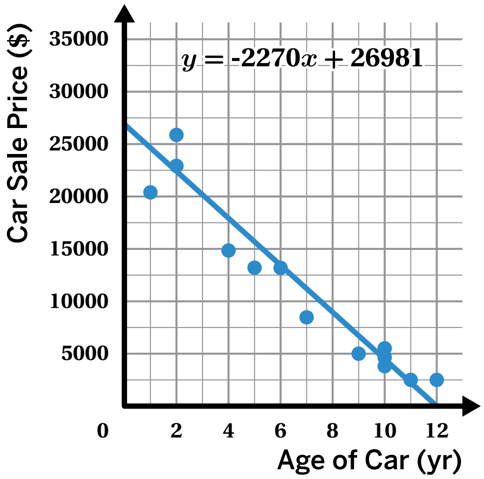
5. Use the equation of line of best fit to predict the length, in minutes, of a direct flight from Phoenix to Jacksonville ($1{,}795$ miles).

6. Do you think the prediction is accurate? Explain your thinking.

Lesson Practice
2.16

Name: .... Date: .... Period: ....

Problems 7–9: Kwasi wanted to know the relationship between the ages of cars and their values. He found data on the ages of several cars (in years) and their sale prices (in dollars).



7. Describe the relationship between the age of a car and its sale price.

8. Do you think one of the variables causes the other? Explain your thinking.

Test Practice

9. What else might affect this relationship? Select your answer.

- A. Option 1: How many owners have owned the car.
- B. Option 2: The mileage on the car and its maintenance.
- C. Option 3: The color of the car.
- D. Option 4: The car's brand logo placement.

Spiral Review

Problems 10–11: Here is a data set: $[1, 1, 2, 2, 3, 3, 7, 8, 9, 10, 11, 35]$

10. Complete the table. Use a graphing calculator if it helps with your thinking.

Min.	Q1	Median	Q3	Max.

11. Are there any outliers in this data set? Circle one. Yes No

Problems 12–14: Solve each equation for y.

12. $7 = 6x - y$

13. $3y + 15x = 24$

14. $4y - x = 44$

Name: ________________ Date: ________ Period: ________

MA.912.DP.2.4, MA.912.DP.2.6, MA.912.DP.1.3, MTR.1.1, MTR.4.1, MTR.7.1

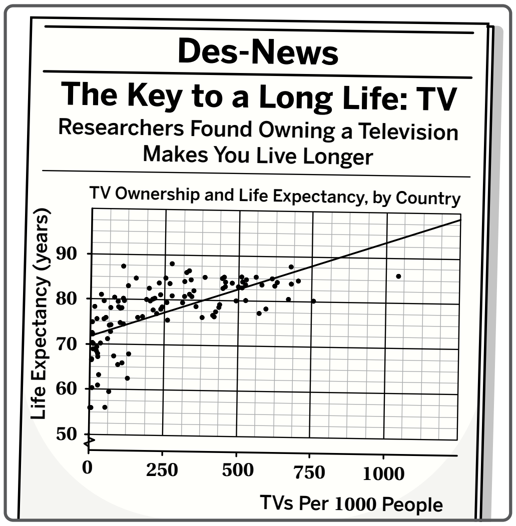
Behind the Headlines

Let's consider the differences between correlation and causation.

Warm-Up

1. Let's look at a headline.

Discuss: What do you notice? What do you wonder?

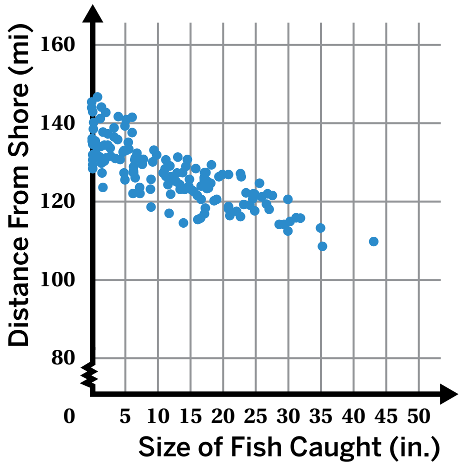


Activity 1

Name: Date: Period:

Correlation, Yes. But Causation?

Here is data from a tour company that sponsors an ocean sports fishing trip.

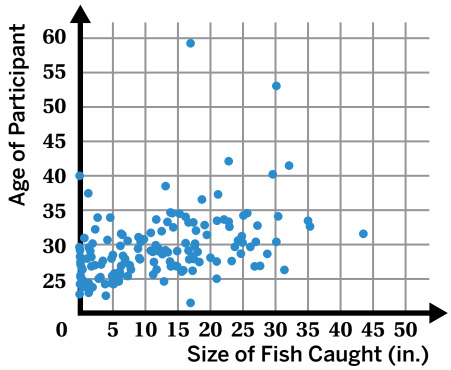


2. Describe the relationship between the size of the fish caught and the distance the boat is from shore.

3. Do you think one of the variables causes the other? If not, what else could be affecting the relationship? Explain your thinking.

4. Here is a headline: *Living Close to Shore Increases the Size of a Fish*. Does the headline suggest **causation**? Explain your thinking.

5. Describe the relationship between the age of the sports fisher and the size of the fish caught.



6. Do you think one of the variables *causes* the other? If not, what else could be affecting the relationship? Explain your thinking.

Activity 1

Name: .... Date: .... Period: ....

Correlation, Yes. But Causation? (continued)

7. Here is a headline: *Older Fishers Lure Larger Fish*. How could you rewrite the headline to suggest **correlation** but *not* causation?

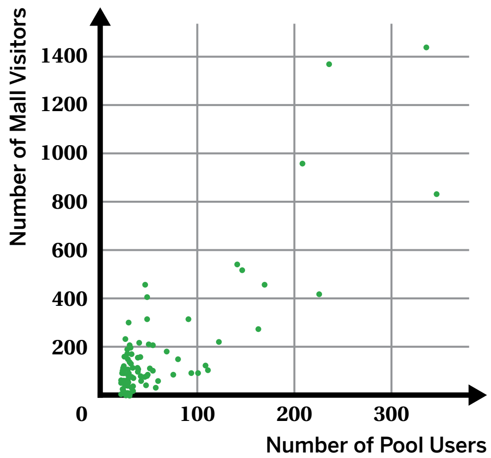
8. Here are two headlines for the same data.

   A: *Swimming Causes an Urge to Shop*

   B: *Mall Shopping Correlated with Pool Swimming*

   Which headline do you think is more accurate?

   Explain your thinking.



9. The two-way frequency table shows the number of families living in a certain town who have at least one pet and whether those families recycle. Here are two headlines.

   A: *Pet Ownership Associated with Recycling*

   B: *More Pets in Your Home Leads to Greater Recycling*

   Which headline do you think is more accurate? Explain your thinking.

	Have at least one pet	Do not have at least one pet	Total
Recycles	244	68	312
Does not recycle	49	139	188
Total	293	207	500

Activity 2

Name: ______ Date: ______ Period: ______

Card Sort

You will use a set of cards. Each card has a headline about the relationship between two variables.

10. Sort the cards based on the type of relationship the headline is suggesting.

Causation	Correlation but Not Causation

11. Select two cards. Read the article summaries for your headlines. Then answer:

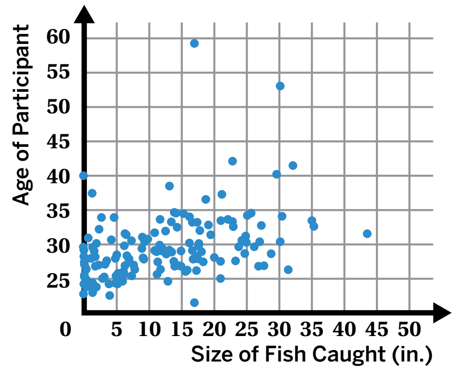
- Does the information in the summary support the headline? Explain your thinking.
- How could you rewrite the headline to make it more accurate?

Card ___	Card ___

Synthesis

12. How can you tell if a headline is suggesting *causation* or *correlation but not causation*?

Use the example if it helps with your thinking.



Lesson Practice 2.17

Lesson Summary

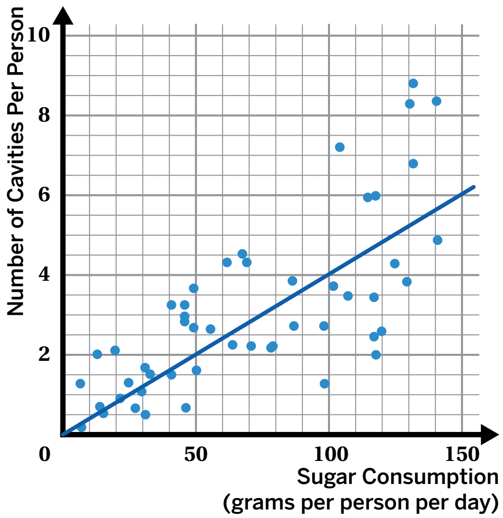
A **correlation**, sometimes called an association, is a relationship between two or more variables. One type of correlation is **causation**, which describes a relationship where a change in one variable causes a change in the other variable.

However, not every correlation is a *causal relationship*. For example, a third variable can cause the relationship between two variables to change. Correlation can even be caused by coincidence: if lots of variables are considered, then it's very likely that at least two of them will be somewhat correlated.

Media, such as article headlines, might present relationships that are correlated as causal. To be a critical consumer of information, check the sources behind claims and ask questions about what conclusions can be drawn from a data set.

Name: .... Date: .... Period: ....

Problems 1–2: This scatter plot shows data from different countries about average sugar consumption and tooth cavities.



1. Which statement best describes the data in the graph?

   A. The rate of tooth cavities has increased in many countries.

   B. The rate of sugar consumption has increased in many countries.

   C. People who eat more sugar are likely to have more cavities.

   D. People brush their teeth more frequently in some countries than others.

2. Describe the relationship between sugar consumption and teeth cavities.

3. A farmer notices that the apple trees in the orchard produce more fruit when more fertilizer is applied. Can one conclude that this positive correlation shows a causal relationship?

   A. Yes, because it is not a negative correlation.

   B. Yes, because fertilizers cause trees to grow larger.

   C. No, because fertilizers only work on flowers and grass.

   D. No, because it is possible that the trees produce more fruit due to another factor.

Problems 4–5: Circle the type of relationship each headline suggests.

4. Brushing Twice a Day Can Prevent Gum Disease

   Causation Correlation Only

5. Sitting Too Much Decreases Life Expectancy

   Causation Correlation Only

Lesson Practice
2.17

Name: .... Date: .... Period: ....

Test Practice

6. Which statements are true regarding correlation and causation?

- ☐ **A.** Correlation is when an increase in one variable causes another variable to increase.
- ☐ **B.** Correlation is when a change in one variable happens at the same time as another variable.
- ☐ **C.** Correlation is when a change in one variable causes a change in another variable.
- ☐ **D.** Causation is when an increase in one variable causes another variable to increase.
- ☐ **E.** Causation is when a change in one variable happens at the same time as another variable.
- ☐ **F.** Causation is when a change in one variable causes a change in another variable.

Spiral Review

7. Select all of the equations that are equivalent to $20 + 2m - 8 = m + 17$.

- ☐ **A.** $20 - 8 + 17 = -m$
- ☐ **B.** $2m + 12 = m + 17$
- ☐ **C.** $3m = 17 - 20 + 8$
- ☐ **D.** $m = 5$
- ☐ **E.** $2m = m + 5$

8. Write an equation to match the table.

x	0	1	2	3	4
y	5	15	45	135	405

Unit 3

Describing Functions

Functions can be represented using tables, graphs, equations, and words. In this unit, you will explore what makes a relationship a function and how functions can model situations and tell stories. You will use function notation to describe key features of functions, compare different functions, and define functions. Finally, you will explore new types of functions that can model situations that have different rules for different inputs.

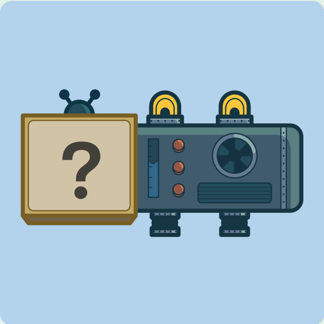
Essential Questions

- What are the characteristics of a function and how can a function tell a story?
- What are key features of functions and how can you describe them?
- How can you use function notation as a tool to communicate precisely?

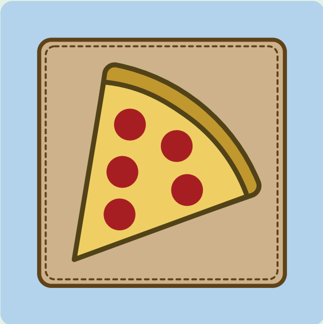
Sub-unit

1

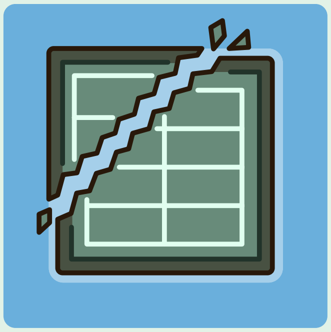
Function Notation



Lesson 1
Mystery Rule



Lesson 2
Pricing Pizzas



Lesson 3
Rule Breakers

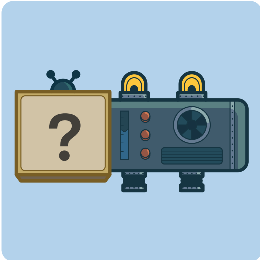
Lesson 4
Toy Factory

Name: ____________ Date: ________ Period: ________

MTR.3.1, MTR.4.1

Mystery Rule

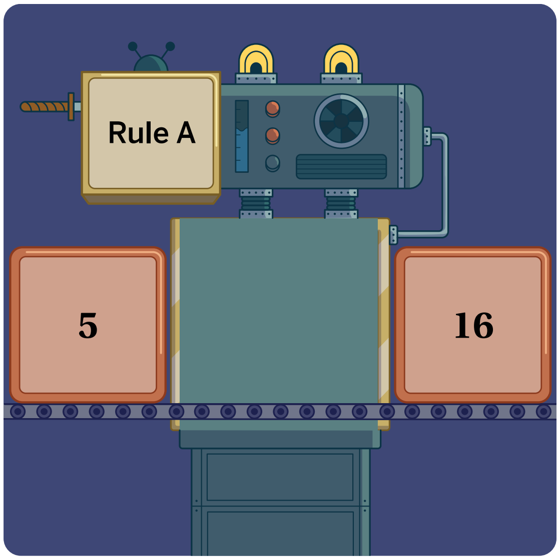
Let's consider whether or not rules are functions.



Warm-Up

1. This machine uses Rule A to turn *inputs* into *outputs*.

Let's test several inputs to see how Rule A works. Use the table below to see if you can determine how Rule A works.



Input	Output
5	16
6	19
0	1

2. Predict the output for 101. Explain your reasoning.

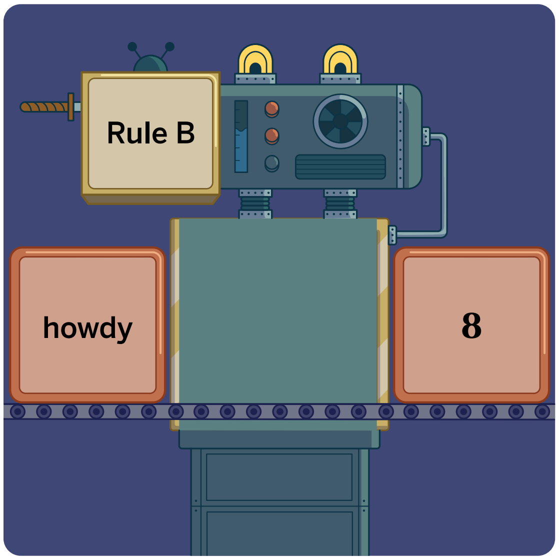
Activity 1

Name: ______ Date: ______ Period: ______

What Is a Function?

3. Rule B's inputs are words.

Let's test several inputs to see how Rule B works. Use the table below to see if you can determine how Rule B works.



Input	Output
howdy	8

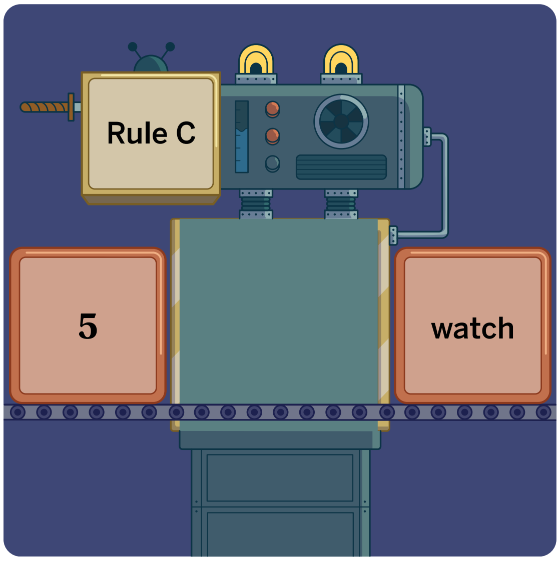
4. Predict the output for "give". Explain your reasoning.

Name: .... Date: .... Period: ....

What Is a Function? (continued)

5. Rule C's inputs are whole numbers from 1 to 15.

Let's test several inputs to see how Rule C works. Use the table below and the machine to the right to see if you can determine how Rule C works.



Input	Output
5	watch
9	vegetable

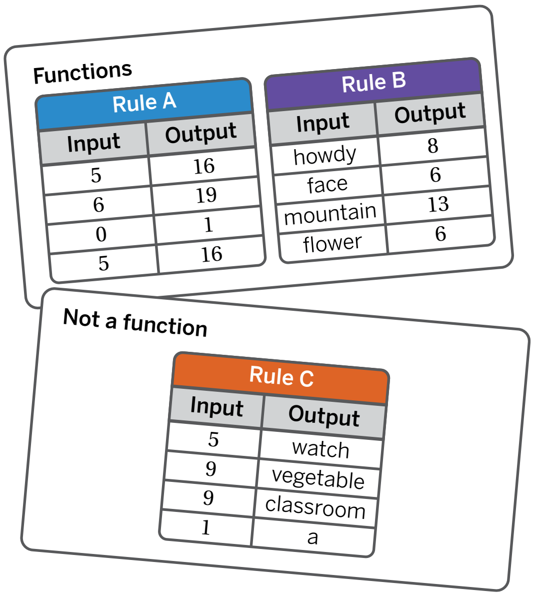
6. **a** Predict the output for 6.

b Compare your response with a partner's. How are your responses alike and different?

7. Rules A and B are examples of a **function**.

Rule C is *not* a function.

What do you think makes Rule C not a function?



Activity 2

Name: ______ Date: ______ Period: ______

Which Rules Are Functions?

8. Here are four rules.

Discuss: Is each rule a function? Why or why not?

Rule D takes temperatures in Fahrenheit and outputs temperatures in Celsius.

Input	Output
50	10
77	25
100	37.8
212	100

Rule E takes any integer and outputs one of its factors.

Input	Output
24	6
13	1
13	13
9	3

Rule F takes any number and rounds it to a whole number.

Input	Output
32.5	33
$\frac{4}{3}$	1
0.1	0
23	23

Rule G takes any word and shifts each letter one place in the alphabet.

Input	Output
bird	cjse
house	ipvtf
hello	ifmmp
world	xpsme

Name: .... Date: .... Period: ....

Which Rules Are Functions? (continued)

9. Rule E is *not* a function.

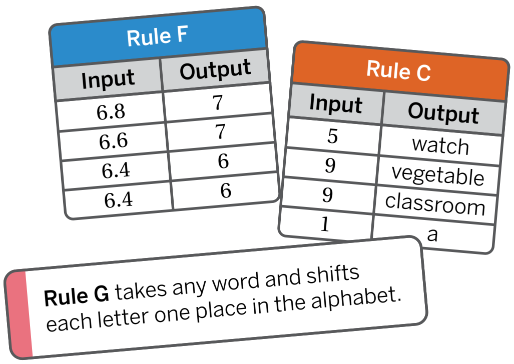
How can you tell by looking at the table?

Rule E takes any integer and outputs one of its factors.

Rule E	
Input	**Output**
1200	30
1200	10
20	10

Synthesis

10. How can you decide whether a rule is a function?



Lesson Practice 3.01

Lesson Summary

We can represent rules with verbal descriptions or as a table of *inputs* and *outputs*. All sets of inputs and outputs are called relations. A **function** is a special kind of relation that assigns exactly one output to each possible input.

You can determine whether a rule is a function by organizing the inputs and outputs into a table. If one input has multiple possible outputs, then the rule is not a function.

Here are two examples.

Rule A takes an integer and outputs an integer that is one less.

Input	Output
1	0
2	1
2	1
4	3

In this relationship, Rule A is a function because each input has exactly one output.

Rule B takes a number and outputs a random number that is greater.

Input	Output
0	2
0	10
-2	0
-1.6	-1.2

In this relationship, Rule B is *not* a function because each input has multiple outputs.

Lesson Practice
3.01

Name: .... Date: .... Period: ....

1. Rule A takes any word as an input and writes the word backwards as an output.

   Is Rule A a function? Explain your thinking.

Rule A

Input	hat	sock	racecar
Output	tah	kcos	racecar

Problems 2–3: Here is Rule B.

Rule B

Input	4	6	6	5
Output	blue	purple	yellow	white

2. Is Rule B a function? Explain your thinking.

3. Predict what the output could be when the input is 3.

Problems 4–5: Here is Rule C.

Rule C

Input	6	8	12	14
Output	4	6	10	?

4. Is Rule C a function? Explain your thinking.

5. Predict the missing output for Rule C.

6. This table shows the total number of days in each month of a given year.

	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
28		×										
29		×										
30				×		×			×		×	
31	×		×		×		×	×		×		×

Imagine a rule where the input is a month and the output is the number of days in that month. Does this rule represent a function? Explain your thinking.

7. A machine uses Rule D to turn inputs into outputs. The table shows two inputs and their outputs.

Hoang tried the input 4 again and the output was not 27. He claims that this is enough information to determine whether or not Rule D is a function.

Is Hoang correct? Explain how you know.

Rule D

Input	Output
3	21
4	27

Test Practice

8. Which output for an input of 4 could make Rule D a function?

- ☐ **A.** 7
- ☐ **B.** 21
- ☐ **C.** 28
- ☐ **D.** 40

Spiral Review

9. Complete the table using this rule: Add 2 to the input, then multiply by 3 to get the output.

Input	-5	0	4
Output	-9		

Problems 10–11: A school sells two types of tickets for a play: adult tickets and student tickets. Adult tickets are \$5 each and student tickets are \$2 each. The school collects \$400 total.

10. Write an equation where x represents the number of adult tickets sold and y represents the number of student tickets sold.

11. How many of each ticket type could the school sell to collect \$400?

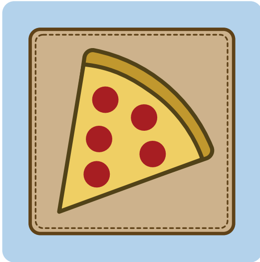
Adult tickets: .... Student tickets: ....

Name: ______________________ Date: __________ Period: __________

MA.912.F.1.2, MTR.2.1, MTR.3.1, MTR.6.1

Pricing Pizzas

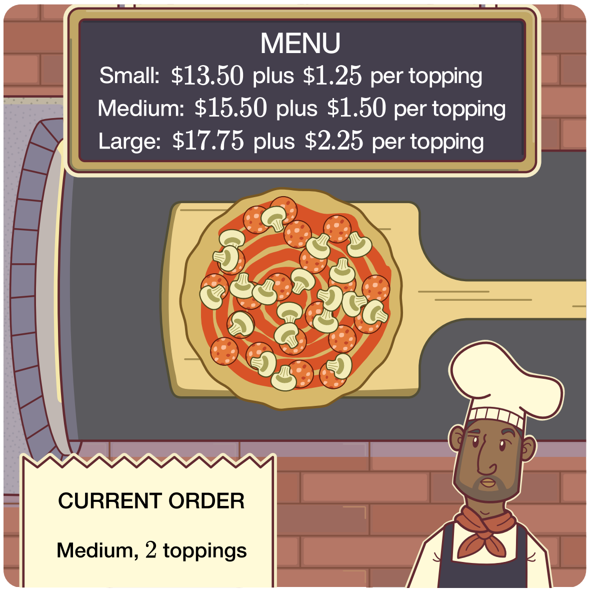
Let's learn what function notation is and interpret function notation statements in context.



Warm-Up

1. Desmos Pizza offers small, medium, and large pizzas.

Use the menu to determine the price of each pizza order.



Pizza Order	Price ($)
Medium, 2 toppings	
Large, 4 toppings	
Small, 3 toppings	
Large, 6 toppings	

Activity 1

Name: ______ Date: ______ Period: ______

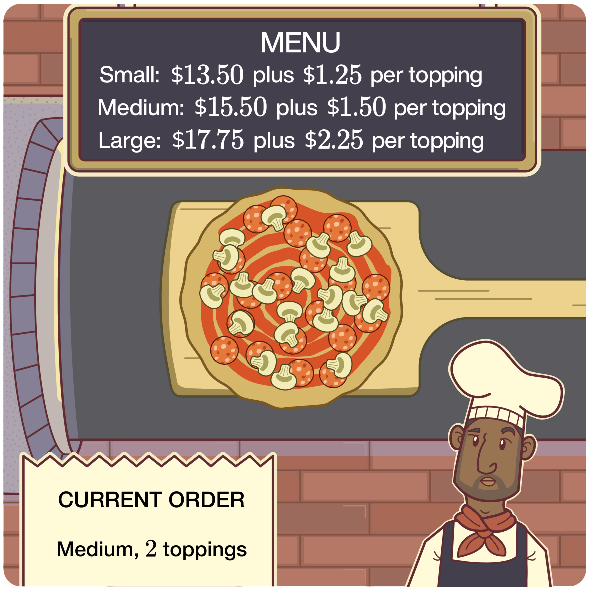
Pricing Pizzas

2. A worker at Desmos Pizza made a list of all the large pizza orders one night.

Discuss: How is this list like a function?

3. Desmos Pizza offers small, medium, and large pizzas. You can create a function using the price of the pizza as your constant and the cost per topping, t, as your variable. Use the menu to the right to complete the table using the same format as shown in the first output.

Pizza Order	Price ($)
Small, 3 toppings	$13.50 + 1.25t = 17.25$
Large, 2 toppings	
Medium, 1 topping	



Name: ________ Date: ________ Period: ________

Pricing Pizzas (continued)

4. $s(3)$ is an example of a statement in **function notation**.

We read $s(3)$ as "s of three."

a Say $s(3) = 17.25$ aloud to a classmate.

b Select *all* the ideas that $s(3)$ represents.

- ☐ **A.** The price of a small pizza with 3 toppings.
- ☐ **B.** The price of a small pizza multiplied by 3.
- ☐ **C.** The function s with an input of 3.
- ☐ **D.** The function s with an output of 3.
- ☐ **E.** The price of 3 small pizzas.

5. On the previous problem, Luca said:

s times 3 is 17.25, so a small pizza with 3 toppings will cost $5.75.

What would you say to help him understand his mistake?

Luca

$$\frac{s(3)}{3} = \frac{17.25}{3}$$

$$s = 5.75$$

Activity
2

Name: ______ Date: ______ Period: ______

Interpreting Function Notation

6. Match each function notation statement with its correct interpretation(s). Three cards will have no match.

Card A	**Card B**	**Card C**	**Card D**
The function s with the input 0.	A small pizza that costs $\$0$.	The price of a medium pizza with x toppings.	The price of 5 large pizzas.
Card E	**Card F**	**Card G**	
The output of l when the input is 5.	The price of a small pizza with 0 toppings.	The price of a medium pizza multiplied by x.	

$l(5)$	$s(0)$	$m(x)$

7. Emma and her friends are texting about their pizza order.

Emma writes: $m(7) < l(5)$.

What do you think this means?

MENU

Small: $\$13.50$ plus $\$1.25$ per topping

Medium: $\$15.50$ plus $\$1.50$ per topping

Large: $\$17.75$ plus $\$2.25$ per topping

8. Select *all* the true statements.

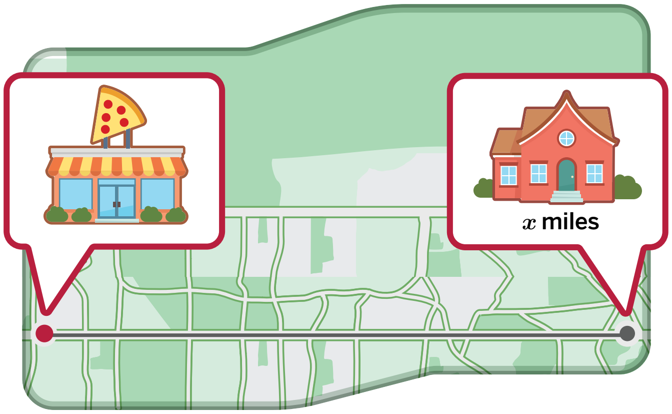
- ☐ **A.** $s(2) > s(1)$
- ☐ **B.** $m(4) < s(4)$
- ☐ **C.** $l(4) > s(3)$
- ☐ **D.** $l(1) = m(3)$
- ☐ **E.** $l(0) > m(2)$

Name: .... Date: .... Period: ....

Delivering Pizzas

9. Desmos Pizza uses the function $d(x)$ to estimate the number of minutes it takes to deliver pizza x miles.

What would $d(2) = 30$ mean in this situation?



10. $d(x)$ estimates the number of minutes it takes to deliver pizza x miles. Match each function notation card to a description. One card will have no match.

Card A

It takes longer to deliver 5 miles away than 1 mile away.

Card B

The number of minutes to make a delivery 5 miles away.

Card C

A delivery 1 mile away will take more than 5 minutes.

Card D

A delivery 5 mile away will take more than 1 minutes.

Card E

Delivering 5 pizzas takes longer than delivering 1 pizza.

$d(5) > 1$	$d(5)$	$d(5) > d(1)$	$d(1) > 5$

Synthesis

11. This lesson introduced function notation.

a Say the equation $m(5) = 23$ aloud to a classmate

b Describe what each part of the equation means.

Lesson Practice 3.02

Lesson Summary

Function notation is a way to write the inputs and outputs of a function. For example, $f(4) = 9$ is a statement written in function notation. It says that when the input of the function f is 4, the output is 9. In other words, when the value of the *independent variable* is 4, the value of the *dependent variable* is 9.

Here is another example. We can use a function to determine the price of a slice of pizza based on the number of toppings.

This table shows some input-output pairs for the function $s(t)$.

$s(2) = 2.75$ is a statement written in function notation.

- $s(2)$ can be read as "s of two."
- For this situation, the number of toppings is the *independent variable*, t, and the price of a slice of pizza is the *dependent variable*, $s(t)$.
- $s(2) = 2.75$ means the price of a slice of pizza with 2 toppings is $2.75.

Menu

Slice of Pizza $1.75 plus $0.50 per topping

Number of Toppings	Price ($)
0	1.75
1	2.25
2	2.75

Lesson Practice
3.02

Name: .... Date: .... Period: ....

Problems 1–2: The function $f(t)$ models the temperature, in degrees Celsius, t hours after midnight.

1. Select the equation that represents the statement: *At* 1 *AM, the temperature was 20° C.*

A. $f(100) = 20$ B. $f(20) = 100$ C. $f(1) = 20$ D. $f(20) = 1$

2. Use function notation to represent each statement.

The temperature at 2 AM. ....

The temperature was the same at 9 AM and at 11 AM. ....

The temperature was higher at 9 AM than at 2 AM. ....

t hours after midnight, the temperature was 24°C. ....

3. A restaurant sells three different salads. They use the functions $c(x)$, $g(x)$, and $p(x)$ to represent the cost of their caesar, garden, and pasta salads in dollars, with x additional ingredients added. Explain the meaning of each statement.

$g(0) = 10$	
$g(3) > c(1)$	
$p(2) < g(3)$	

Problems 4–6: Use the table to determine the missing values in the function statements.

x	$f(x)$
-5	17
-2	-5
5	23

4. $f($........$) = 23$

5. $f(-5) =$ ....

6. $f($........$) = -5$

Lesson Practice
3.02

Name: .... Date: .... Period: ....

Test Practice

7. Which orders can be made from this menu?

- A. Small with 3 toppings for \$15
- B. Medium with 3 toppings for \$22
- C. Large with 4 toppings for \$30
- D. Medium with 1 topping for \$17

MENU
Small: \$12 plus \$1 per topping
Medium: \$15 plus \$2 per topping
Large: \$18 plus \$3 per topping

Spiral Review

8. Here are Rules A and B.

Which rule is a function? Circle one.

Rule A   Rule B   Both   Neither

Rule A

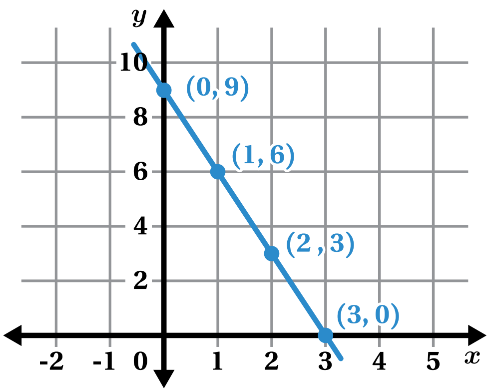
Input	Output
4	2
9	-3
9	3

Rule B

Input	Output
1	5
2	9
2	9

9. Solve for x: $3(2x - 4) + 5 = 2(x + 6) + x$

10. Here's a graph of a relationship and its table of values.



x	y
0	9
1	6
2	3
3	0

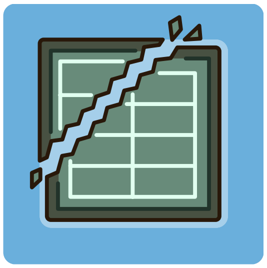
Write an equation to represent this relationship.

Unit 3
Lesson
3

Name: ______ Date: ______ Period: ______

MA.912.F.1.1, MA.912.F.1.5, MA.912.F.1.6, MTR.1.1, MTR.4.1, MTR.5.1

Rule Breakers



Let’s write function rules using set-builder notation.

Warm-Up

1. Each function group has something in common.

Discuss: What do you notice? What do you wonder?

Group A	Group B	Input (x)
$y = \sqrt{-x}$	$y = \sqrt{x}$	4
$\lvert 6 \rvert = 2x$	$\lvert 6 \rvert = -2x$	-3
$17 > -8x$	$17 > 8x$	-9
$y = \frac{5}{x}$	$y = \frac{x}{5}$	0

I notice:

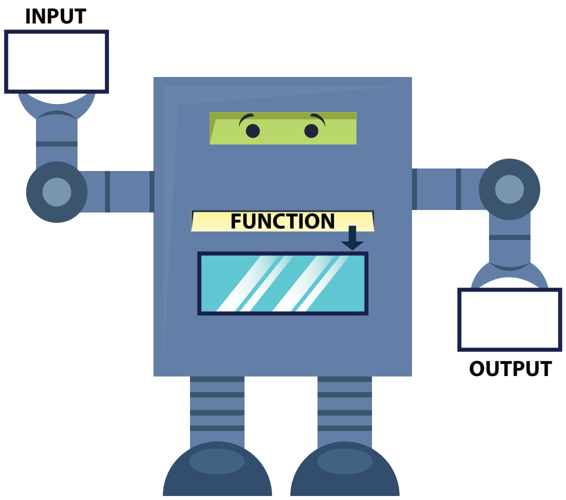
I wonder:

Activity 1

Name: .... Date: .... Period: ....

Broken Function Machine

Function notation describes the relationship between inputs and outputs of a function, describing the x- and y-values for which the function is defined.



2. With a partner, circle each function with inputs that would break the machine or make the function undefined.

Function	Input
☐ **A.** $\lvert y\rvert = 4 - x$	5
☐ **B.** $y = \frac{1}{x}$	0
☐ **C.** $y = x$	7
☐ **D.** $y = x - 2$	2
☐ **E.** $y = \sqrt{x - 5}$	1
☐ **F.** $y = 2x + 1$	8

3. **Discuss:** Choose one function from Problem 2 and explain how you determined the input value(s) that would "break" the function machine.

4. Recall that a linear function has a constant rate of change and can be written in the form $y = mx + b$. Which relationships above are linear functions? How do you know?

Name: ______ Date: ______ Period: ______

Notation, Notation, Notation

There are two types of notations to help describe the range and domain of a function: inequality and set-builder. Each uses mathematical symbols to express meaning.

Inequality		Set-Builder	
Expression with inequality		$\{x \mid \text{Conditions}\}$	
$<, >, \leq, \geq$	Inequality Symbols	$\mathbb{R}$	Real Numbers
$\neq, =$	Equality Symbols	$\mathbb{Z}$	Integers
		$\mathbb{N}$	Natural Numbers
$2 < x \leq 5$		$\{x \mid 2 < x \leq 5\}$	
x is greater than 2 and less than or equal to 5.		The set of all real numbers x such that x is greater than 2 and less than or equal to 5.	

5. **Notice and Wonder:** What do you notice is the same and different between the types of notations?

Activity 3

Name: ______ Date: ______ Period: ______

Function Notation

6. Complete the missing domains and ranges in set-builder and inequality notation. Remember that the domain is the possible values of x and the range is the possible values of y.

a

$\frac{y = 3}{(x - 2)}$	Domain	Range
Inequality		
Set-Builder		

b

$y = \sqrt{x - 5}$	Domain	Range
Inequality		
Set-Builder		

c

$y \geq x - 23$	Domain	Range
Inequality		
Set-Builder		

7. **Discuss:** Mario used set-builder notation to describe the domain and range for the function $|y| = (\text{-}2x)$. Did he use the correct notations to help describe the range and domain of the function? Why or why not?

Domain	$\lvert y\rvert = (\text{-}2x)$	Range
$x \leq 0$	Set-Builder	$y \geq 0$

Synthesis

8. **a** What does notation help you understand about a function?

b What symbols are included in set-builder notation?

Lesson Practice 3.03

Lesson Summary

- Function notation describes the relationship between inputs and outputs of a function, describing the values for which the function is defined.
- Two common reasons an algebraic expression can be undefined are **division by zero** and **taking the square root of a negative number**.
- Function notation is also needed for **absolute value functions** where the absolute value of a number cannot be negative.
- Another place where the domain or range might not be all real numbers is with **inequalities**.
- **Set-builder notation** uses the variable and its conditions separated by a | and within { }.
- **Inequality notation** uses inequality symbols (<,>,≥,≤) to describe input and output values for a function.

Lesson Practice
3.03

Name: .... Date: .... Period: ....

Problems 1–4: Describe the value of x, if any, that makes the function undefined and explain your answer.

1. $y = \sqrt{x}$

2. $y = \frac{1}{x}$

3. $y = 2x$

4. $y = \sqrt{x - 16}$

Problems 5–7: Write the domain and range for each function in set-builder notation.

5. $y = \lvert x \rvert$	Domain: Range:
6. $y = \sqrt{x + 10}$	Domain: Range:
7. $y = \frac{1}{x - 5}$	Domain: Range:

Problems 8–10: Write the domain and range for each function in inequality notation.

8. $y = \lvert -3x \rvert$	Domain: Range:
9. $y = \sqrt{x - 6}$	Domain: Range:
10. $y = \frac{1}{x + 12}$	Domain: Range:

Lesson Practice
3.03

Name: .... Date: .... Period: ....

Test Practice

11. Which of the following functions have a domain where x is all real numbers? Select all that apply.

- ☐ **A.** $y = 3x + 8$
- ☐ **B.** $y = \frac{1}{x+4}$
- ☐ **C.** $|y| = x^2 + 2$
- ☐ **D.** $y = \sqrt{x - 10}$
- ☐ **E.** $17 \geq 22 - x$

Spiral Review

12. Express $78{,}000{,}000$ in scientific notation.

13. Express 0.000045 in scientific notation.

14. Find the product and express the result in scientific notation: $(3.2 \cdot 10^4) \cdot (5.6 \cdot 10^2)$.

Unit 3
Lesson
4

Name: ________ Date: ________ Period: ________

MA.912.F.1.1, MA.912.F.1.2, MA.912.F.1.5, MA.912.F.1.6, MA.912.AR.2.2, MTR.4.1, MTR.7.1

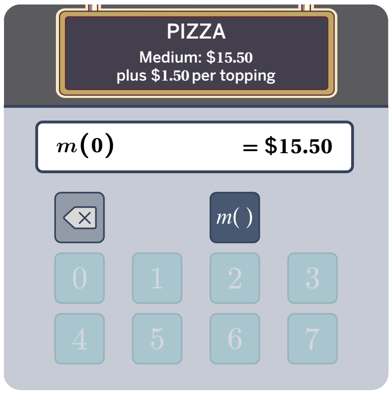
Toy Factory

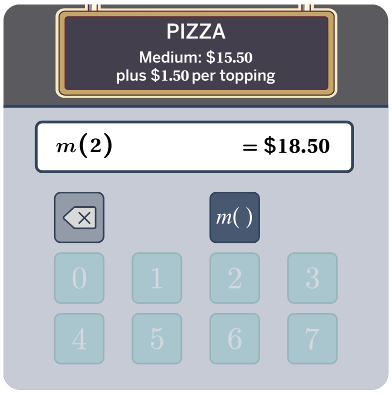
Let's explore functions represented as equations written in function notation.

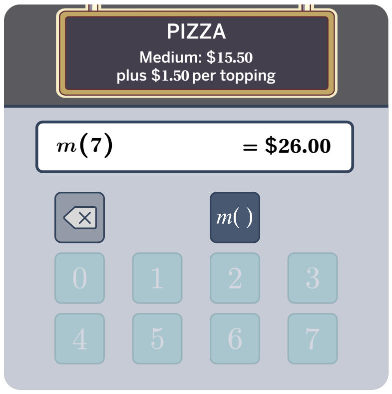
Warm-Up

1. The cash register uses $m(x)$ to determine the price of a medium pizza with x toppings.

a Here are three pizzas and their prices.







b Describe how the cash register calculates prices.

2. Which equation represents $m(x)$?

A. $m(x) = 15.5 + 1.5$

B. $m(x) = 15.5x + 1.5$

C. $m(x) = 15.5 + 1.5x$

Explain your thinking.

Name: .... Date: .... Period: ....

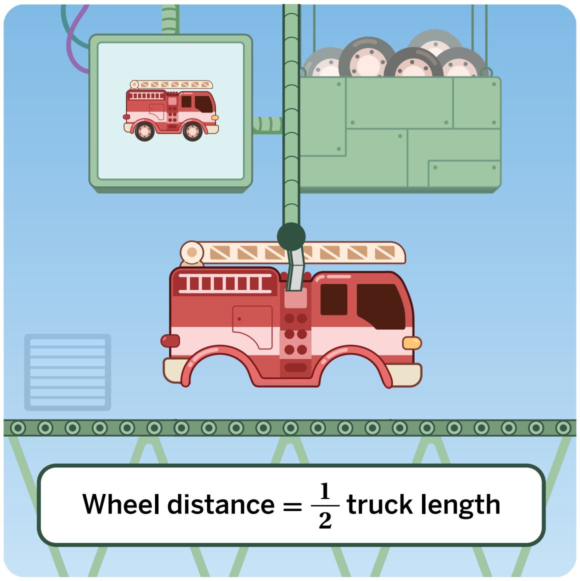
Exploring Equations of Functions

3. A toy factory makes fire trucks in a variety of sizes.

The distance between the truck's wheels is always half the length of the truck.

Complete the table to put wheels on the toy truck.

Truck Length (in.)	Wheel Distance (in.)
10	
6	
3	



4. Kanna wrote the function $d(x) = \frac{1}{2}x$ to determine the wheel distance for a truck length of x.

a. **Discuss:** What does $d(x) = \frac{1}{2}x$ mean?

b. What is the value of $d(7)$?

c. Is $d(x)$ a linear function? Explain your reasoning.

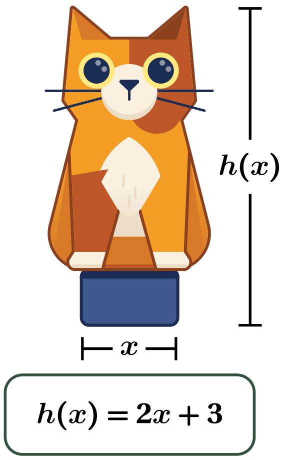
Name: .... Date: .... Period: ....

Exploring Equations of Functions (continued)

5. The factory also makes toy cats.

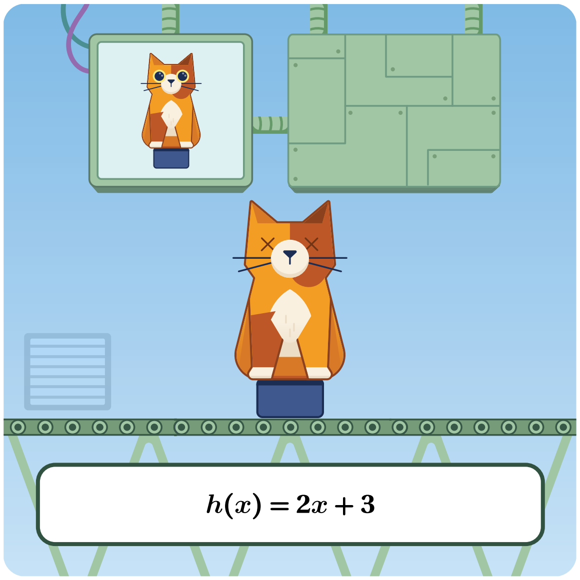
They use this diagram and function to determine where to place the cat's eyes. All units are in inches.

What does $h(x) = 2x + 3$ mean in this situation?



6. Calculate the value of each function notation expression.

Expression	Eye Height (in.)
$h(5)$	
$h(3)$	
$h(7.5)$	



Discuss: Why is it useful to write a function as an equation?

Activity 2

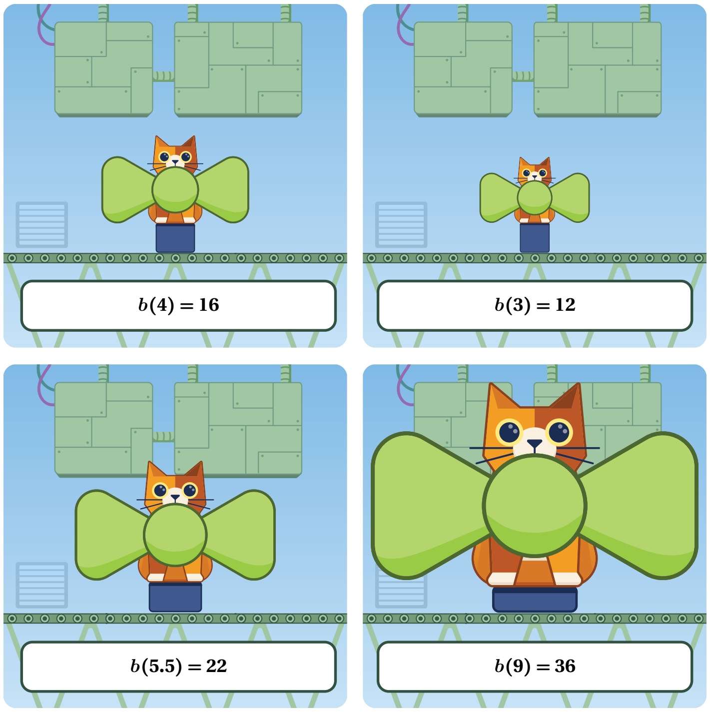
Name: ______ Date: ______ Period: ______

Writing Equations of Functions

7. Each toy cat needs a bow tie. The function $b(x)$ determines the width of the bow tie, where x is the width of the base of the cat toy. All units are in inches.

a The images to the right are the result of the function $b(x) = 4x$. What do you notice?

b What is an equation that will make the bow ties fit better? Compare your equation with a classmate's and share your thinking.



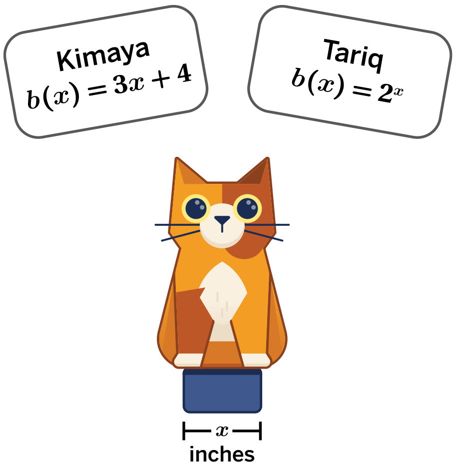
8. $b(x)$ determines the width of the bow tie, where x is the width of the base.

Kimaya says that her function will produce a wider bow tie than Tariq's function for any base width.

Is she correct? Circle one.

Yes No

Explain your thinking.



9. Refer to Problem 8. Is Kimaya's function linear? Is Tariq's function linear? Why or why not?

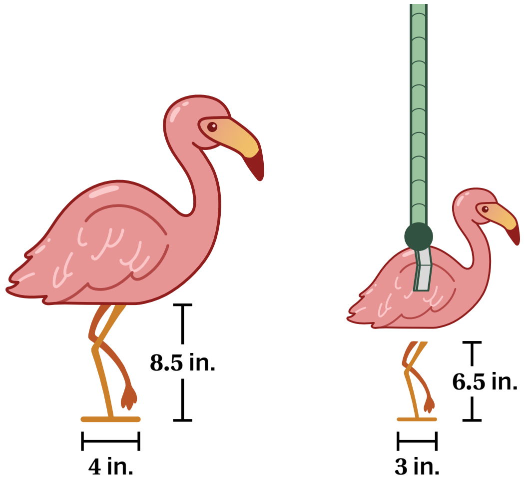
Name: ________ Date: ________ Period: ________

Writing Equations of Functions (continued)

10. Flamingo Frank is made by attaching a flamingo to legs at a specific height.

This height is determined by the width of the base.

Base Width (in.)	Height (in.)
4	8.5
3	6.5
1	2.5
5.5	11.5



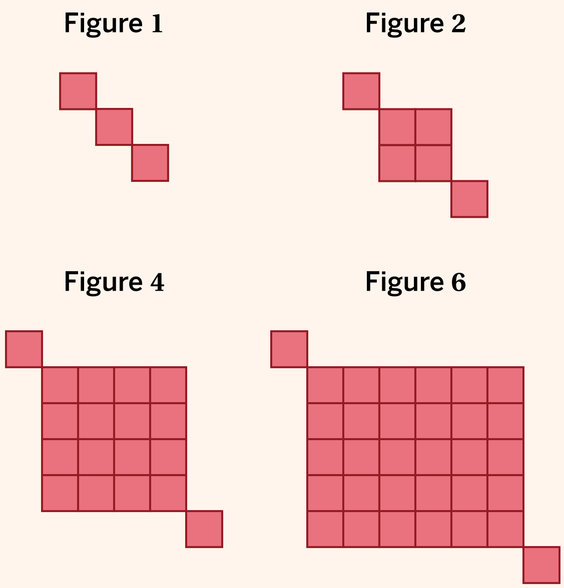
How can you determine the height for any base width?

11. This machine assembles Flamingo Frank by attaching it to legs at a specific height. That height, $f(x)$, depends on the width of the base, x. Write an equation for $f(x)$.

You're invited to explore more.

12. Here are four figures in a visual pattern. The number of tiles is a function of the figure number.

Figure, n	Number of Tiles, $t(n)$
1	3
2	6
4	18
6	38



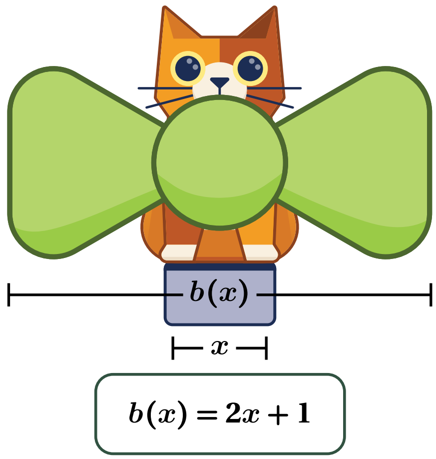
Write an equation for $t(n)$.

Synthesis

13. A toy factory uses this diagram and function to determine the width of the bow tie. All units are in inches.

What does $b(x) = 2x + 1$ mean in this situation?

What does $b(4) = 9$ mean?



Lesson Practice 3.04

Lesson Summary

You can represent function rules with equations, verbal descriptions, and tables.

For example, the function $s(t)$ describes the relationship between the cost of a slice of pizza and the number of toppings, t. Let's represent this function rule with an equation and a table.

Description

Menu

Slice of Pizza
$1.75 plus $0.50 per topping

Table

Number of Toppings	Price ($)
0	1.75
1	2.25
2	2.75

Equation

$s(t) = 1.75 + 0.50t$

You can use the equation to determine different values of the function.

Let's determine the value of $s(4)$:

$s(4) = 1.75 + 0.50(4)$

$s(4) = 3.75$

This means the price of a slice of pizza with 4 toppings is $\$3.75$.

Lesson Practice
3.04

Name: .... Date: .... Period: ....

1. Let $f(x) = 2x + 5$.

Calculate the value of each function notation expression.

The first value is already completed.

Expression	Value
$f(0)$	5
$f(4)$	
$f(6)$	
$f(\text{-}3)$	

Problems 2–3: A toy factory makes toy bunnies. Each toy bunny holds a carrot. A bunny's height, $h(x)$, is three times the length of the carrot, x.

2. Complete the table.

x	1	2	3	4	5	6
$h(x)$						

3. Write an equation for the function $h(x)$.

4. The functions $f(x)$ and $g(x)$ are defined by these equations:

- $f(x) = \text{-}15x + 80$
- $g(x) = 10x + 25$

Circle which is greater: $f(2)$ or $g(2)$. Explain your thinking.

Problems 5–6: The function $p(s)$ models the perimeter of a square of side length s. The perimeter is represented by the equation $p(s) = 4s$.

5. What is the value of $p(20)$?

6. What does your answer mean in this situation?

Problems 7–8: The function $w(t)$ models the weight of a pumpkin, in pounds, as a function of how many months, t, it has been growing. Explain the meaning of each statement.

7. $w(2) = 5$

8. $w(6) > w(4)$

Lesson Practice
3.04

Name: ______ Date: ______ Period: ______

Test Practice

9. Model rockets are created in various sizes. The height of a rocket in inches, $h(x)$, depends on the radius of the base of the rocket in inches, x.

Use the table to select an equation for $h(x)$ that outputs the height of the rocket with a base radius of x.

Radius (in.), x	Height (in.), $h(x)$
1	5
3	13
5	21
10	41

A. $h(x) = 4x - 1$

B. $h(x) = 3x + 2$

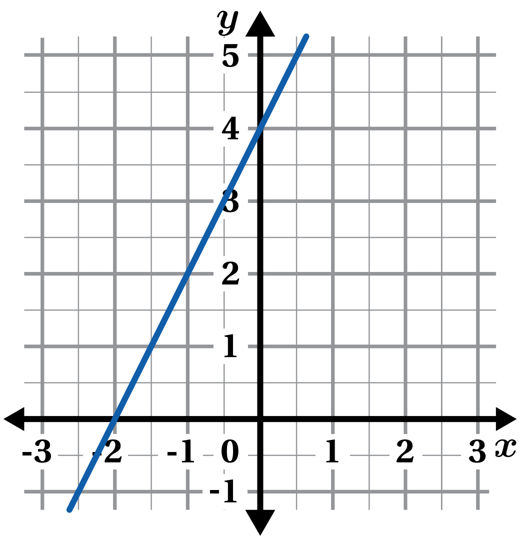
C. $h(x) = 4x + 1$

D. $h(x) = 5x - 3$

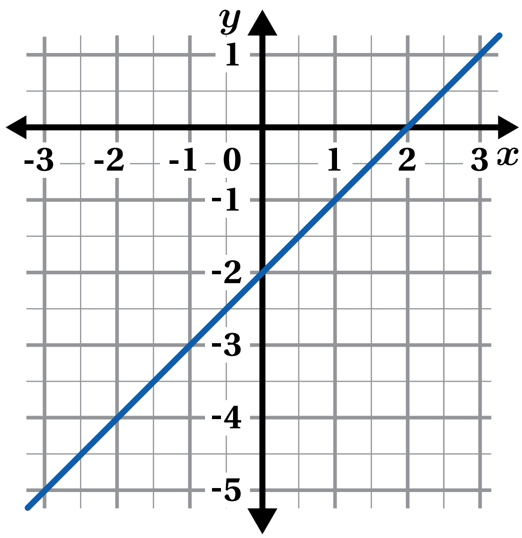
Spiral Review

Problems 10–11: Select all functions represented by the graph.

10.
- ☐ a. $y = 2x + 4$
- ☐ b. $2y = 4x - 8$
- ☐ c. $y = 2x - 4$
- ☐ d. $2y = 4x + 8$
- ☐ e. $y = -2x + 4$



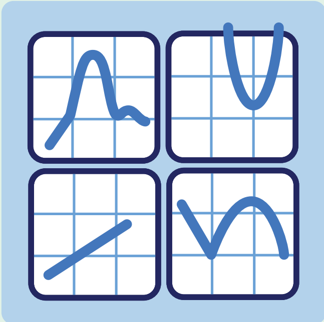
11.
- ☐ a. $y = x + 2$
- ☐ b. $y = x - 2$
- ☐ c. $2y = 2x + 4$
- ☐ d. $3y = 3x - 6$
- ☐ e. $y = 2 - x$



Sub-unit 2

Key Features of Functions

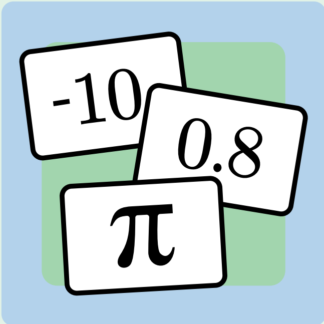
Lesson 5
Function Carnival



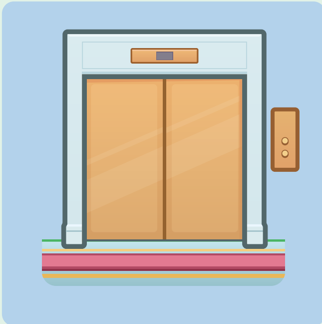
Lesson 6
Craft-a-Graph

Lesson 7
Plane, Train, and Automobile

Lesson 8
Space Race



Lesson 9
Ins and Outs



Lesson 10
Elevator Stories

Name: .... Date: .... Period: ....

MA.912.F.1.6, MA.912.F.1.2, MTR.2.1, MTR.4.1, MTR.6.1, MTR.7.1

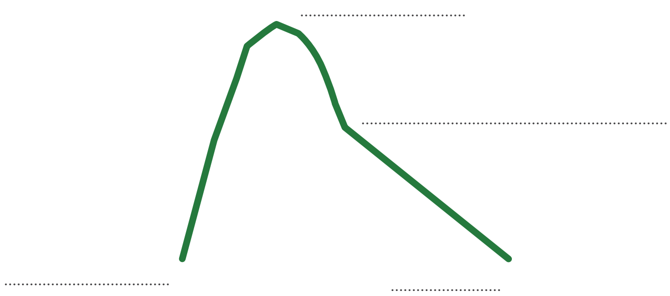
Function Carnival

Creating and Interpreting Graphs of Functions

Let's create and analyze graphs that represent stories.

Warm-Up

The curve represents a man's path from a cannon straight into the air. The man soars upward, reaches a peak, and then starts falling back to Earth. Midway down, he pops open the parachute, slowly descending until he lands safely.



1. Use the descriptions to help you label the stages on the curve.

- **Initial Height**—Launch from the cannon
- **Peak Height**—The highest point
- **Parachute Deployment**—Reduce the speed of the fall
- **Landing**—Reach the ground

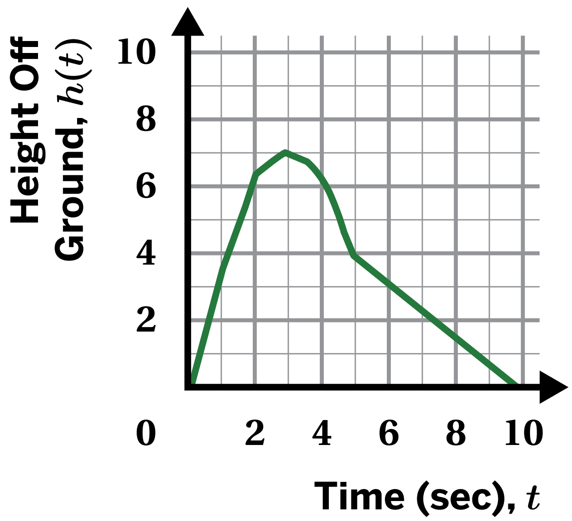
2. What do you notice and wonder about the overall motion and speed change?

Activity
1

Name: ______ Date: ______ Period: ______

Rocket Ride

The path of the curve of the man can also be plotted on a graph where $h(t)$ represents the man's height off the ground at time t.



3. Let's look at the precise graph of $h(t)$. Select *all* true statements.

- ☐ **A.** $h(3) > h(5)$
- ☐ **B.** $h(3) = h(5)$
- ☐ **C.** $h(0) < h(10)$
- ☐ **D.** $h(0) = h(10)$

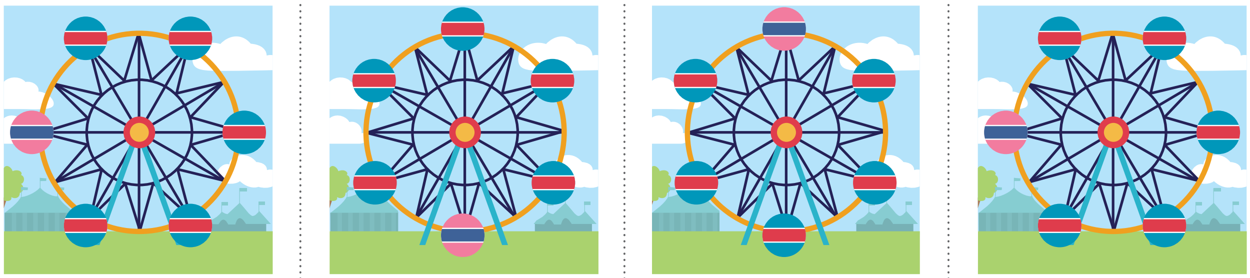
4. Explain what the true statements say about the height of the man.

5. Is $h(t)$ a linear function? How do you know?

Activity 2

Name: ____________ Date: ________ Period: ________

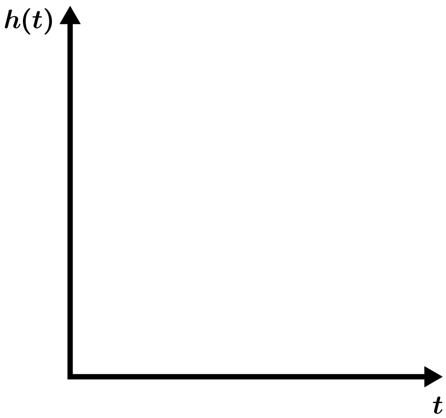
Ferris Wheel



Empty Car	Pick Up Rider	Maximum Height	Full Cycle

6. Observe the pink car on the Ferris wheel at the four different stages in the counterclockwise rotation cycle. Then describe what happens to the height of the pink car.

7. $h(t)$ represents the height of the Ferris wheel at time t. Sketch a graph of $h(t)$.



8. Is $h(t)$ a linear function or a nonlinear function? How do you know?

Activity
3

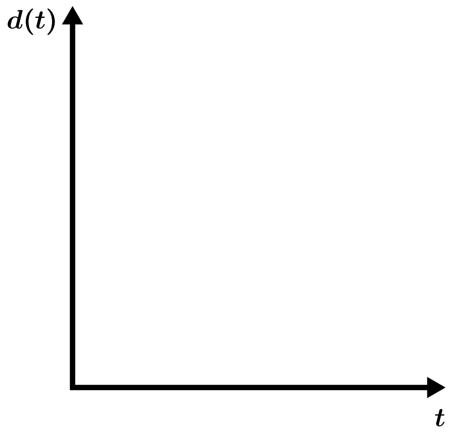
Name: .... Date: .... Period: ....

The Daring Dash

The carnival has a new attraction called “The Daring Dash”. Riders board the car, which starts at the entrance and speeds through a winding track toward the carnival's highest point: the Sky Tower.

- The car speeds steadily toward the Sky Tower at a constant speed, covering an equal distance every minute.
- When the car reaches the base of the Sky Tower, where the track levels off, the car continues at the same constant speed, moving in a straight line across the carnival grounds toward the food court.

9. $d(t)$ represents the distance traveled by the pink car at time t. Sketch a graph of $d(t)$.

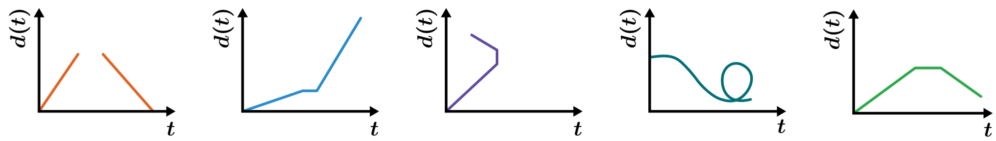


Name: .... Date: .... Period: ....

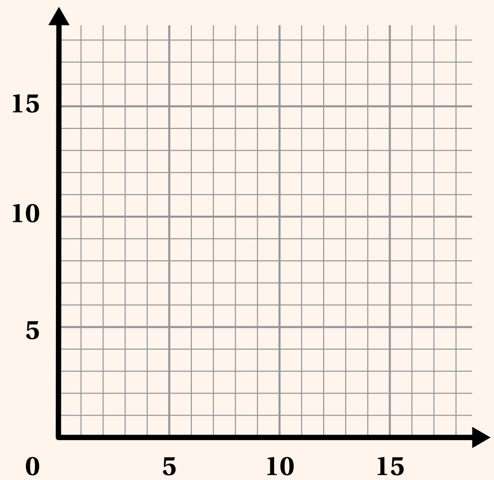
The Daring Dash (continued)

10. $d(t)$ represents a roller coaster's distance from the start at time t. Match each description to its graph. Two graphs will have no match.

a. The roller coaster moves slowly, stops for a moment then goes very fast.	**b.** The roller coaster moves forward, stops, and then backs up.	**c.** The roller coaster starts in the middle, backs up, and then splits into multiple roller coasters.

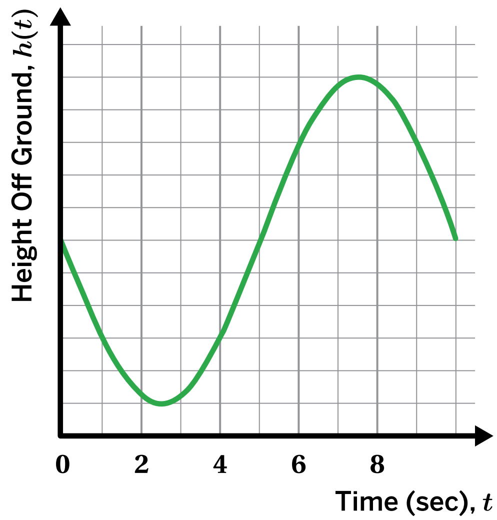


11. Draw your own graph and label the x-axis and y-axis. Then write a story about what it describes.



Synthesis

12. How can a graph help tell a story about a situation? Use this example of a Ferris wheel graph to support your explanation.



Lesson Practice 3.05

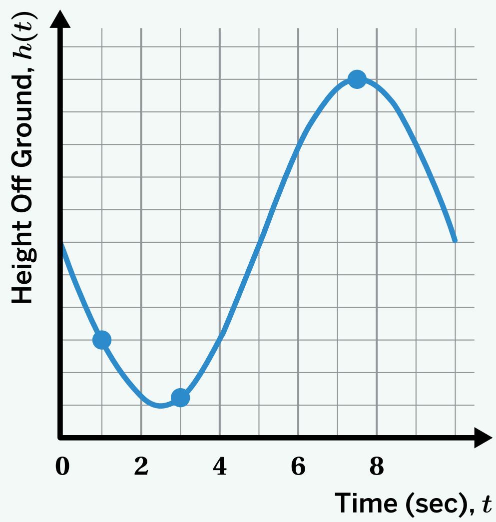
Lesson Summary

A graph can reveal in more detail what is happening during a situation. Here is an example.

The function $h(t)$ represents the height of the cart on the Ferris wheel at time t.

We can use the graph to describe many parts of the situation. For example:

- At around 7.5 seconds, the Ferris wheel cart is at its maximum height.
- $h(1)$ is greater than $h(3)$. This means the Ferris wheel cart was higher off the ground at 1 second than at 3 seconds.



While we can use the graph to describe many things, there are lots of things the graph cannot describe.

For example:

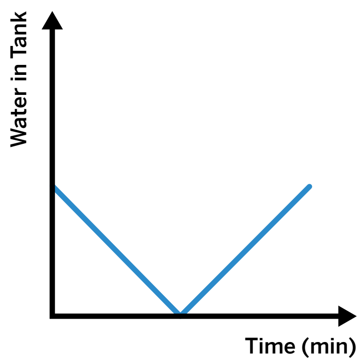
- How much fun the people are having
- How many people are riding the Ferris wheel

Lesson Practice
3.05

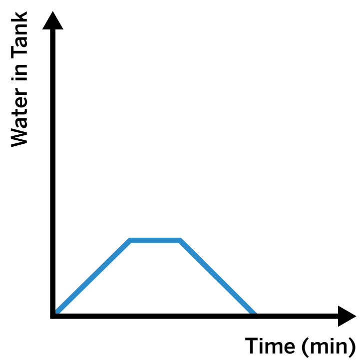
Name: ____________ Date: ________ Period: ________

1. An empty water tank is filled until it is half full. Two minutes later, it drains until it is empty again. Which graph could represent this situation? Circle your choice.

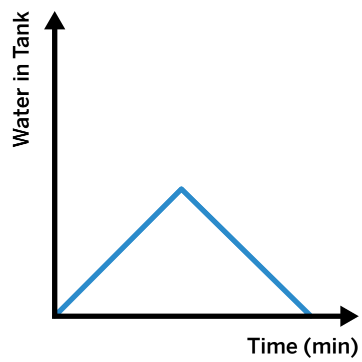
Graph A



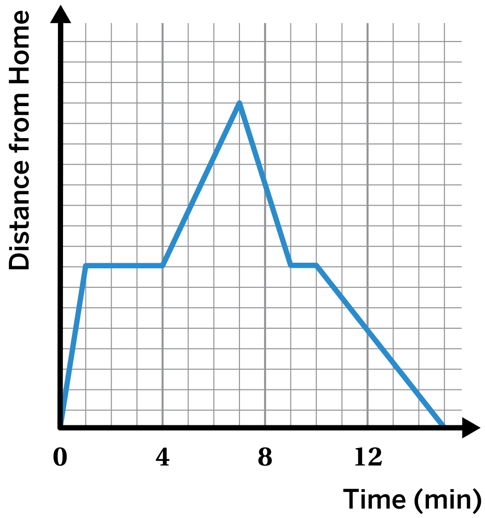
Graph B



Graph C



Problems 2–4: Prisha rode her bike around town. Her fitness tracker made a graph to represent the distance she was from her home at any given time during her ride.



2. How many minutes was Prisha's bike ride?

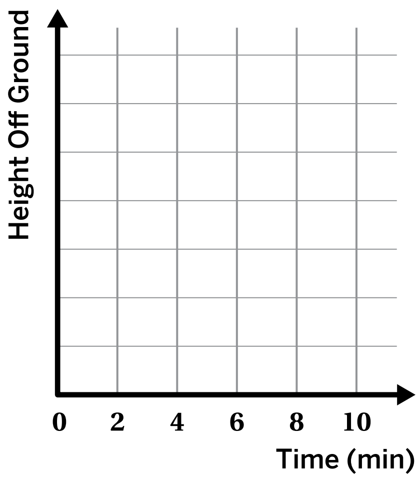
3. At what time was Prisha the farthest distance from her home?

4. How long did Prisha rest during her ride?

5. Here is some information about a hot air balloon ride.

- Ascends (goes up) quickly for 2 minutes.
- Ascends slowly for another minute until it reaches its maximum height.
- Maintains its maximum height for 3 minutes.
- Descends (goes down) for the next 4 minutes until it lands on the ground.

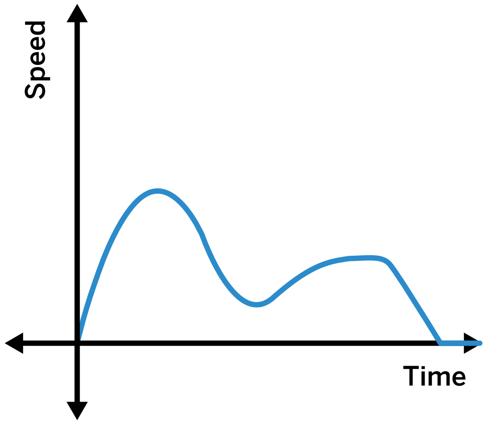
Make a graph that could represent the height of this hot air balloon over time.



Lesson Practice
3.05

Name: .... Date: .... Period: ....

Problems 6–7 Here is a graph of speed and time.



6. Which sport could this graph represent?

A. Fishing
B. Skydiving
C. 100-yard sprint
D. Golf
E. Soccer

Test Practice

7. Describe how you think that sport fits the graph.

Spiral Review

8. $p(t)$ represents the height of water in a bathtub, in inches, after t minutes. Match each sentence to its equation.

A. After 20 minutes, the bathtub is empty.
B. The bathtub starts with no water.
C. After 10 minutes, the height of the water is 4 inches.
D. The height of the water is 10 inches after 4 minutes.
E. The height of the water is w inches after t minutes.

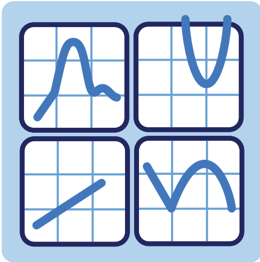
.... $p(10) = 4$
.... $p(t) = w$
.... $p(20) = 0$
.... $p(0) = 0$
.... $p(4) = 10$

Name: ________ Date: ________ Period: ________

MA.912.F.1.6, MTR.6.1

Craft-a-Graph

Let's describe and create graphs of functions using key features.



Warm-Up

1. Play a few rounds of Polygraph with your classmates!

You will use a Warm-Up Sheet with functions. In each round:

- You and your partner will take turns being the Picker and the Guesser.
- Picker: Select a function from the Warm-Up Sheet. Keep it a secret!
- Guesser: Ask the Picker yes-or-no questions, eliminating functions until you're ready to guess which function the Picker chose.

Record helpful questions from each round in the space below.

Activity 1

Name: ______ Date: ______ Period: ______

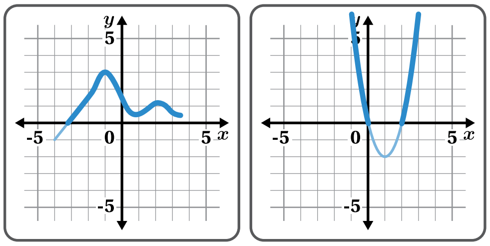
Describe It

2. Here are some functions from Polygraph, along with some terms that describe parts of their graphs.

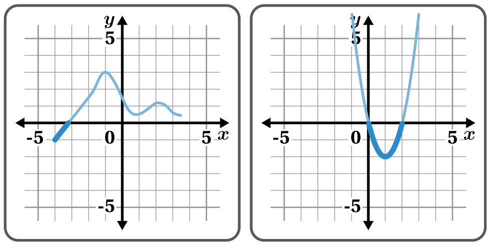
a Take a look at each term and where it appears on the graph.

b **Discuss:** What does each term mean?

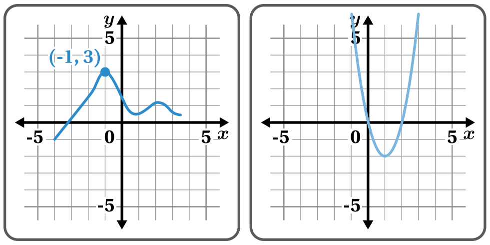
Positive



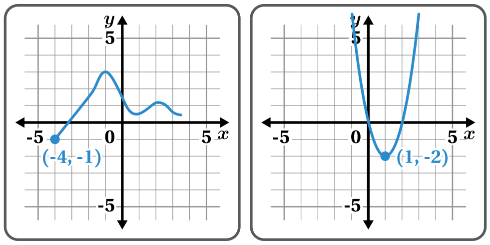
Negative



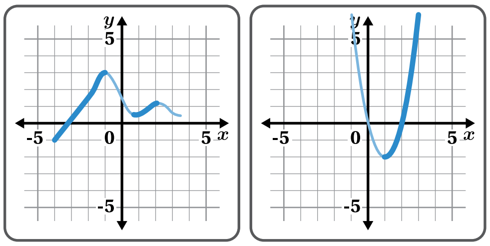
Maximum



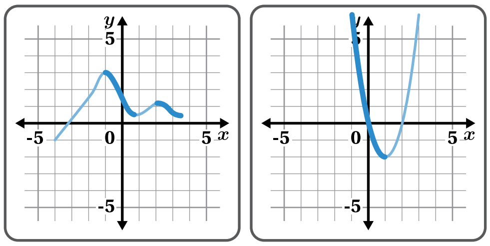
Minimum



Increasing



Decreasing

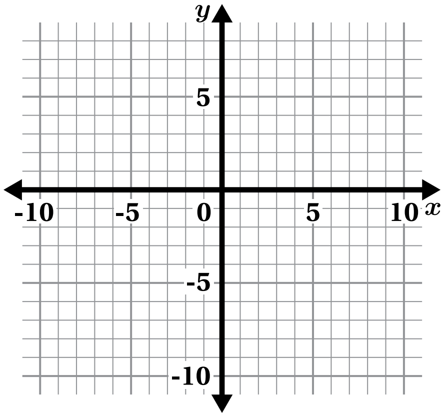


Name: Date: Period:

Build It

3. Now it's your turn to make a function!

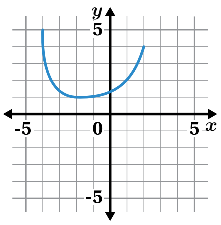
Describe your function using some of the terms from the previous problem.

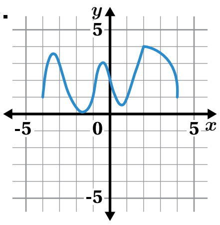


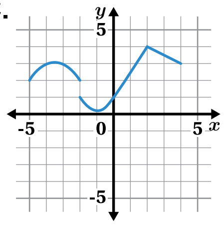
4. Latifah described her function this way:

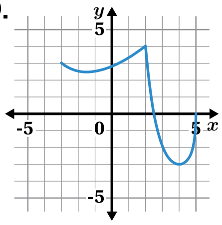
- My function is always positive.
- The maximum is at (2, 4).

Select *all* the functions that could be Latifah's.

☐ A. 

☐ B. 

☐ C. 

☐ D. 

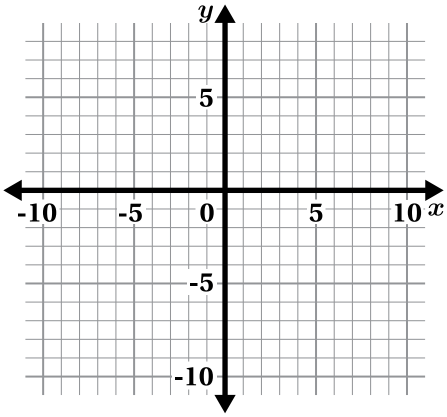
Name: .... Date: .... Period: ....

Build It (continued)

5. Haruto described his function this way:

- My function is increasing when $x > 0$.
- It has a minimum at $(-3, -4)$.

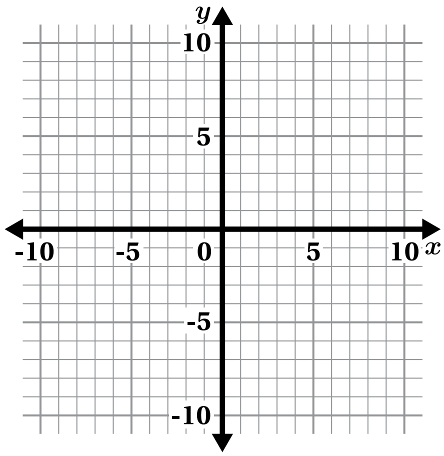
Create a function that could be Haruto's.



6. Andrea described her function this way:

- Positive when $x > -2$.
- Decreasing when $x > 1$.

Is it possible for a function to have both features? Explain your thinking. Use the graph if it helps with your thinking.



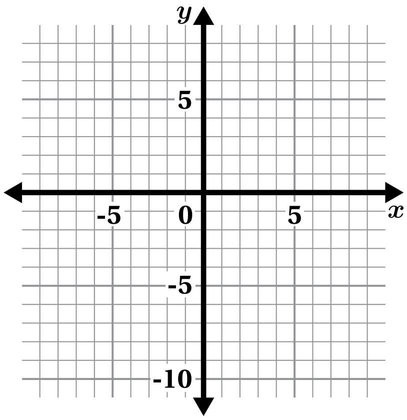
Activity 3

Name: Date: Period:

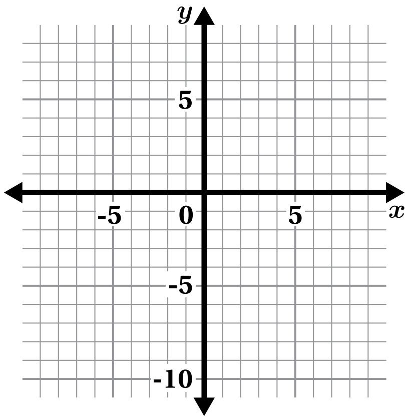
Repeated Challenges

7. Build a function that meets these criteria.

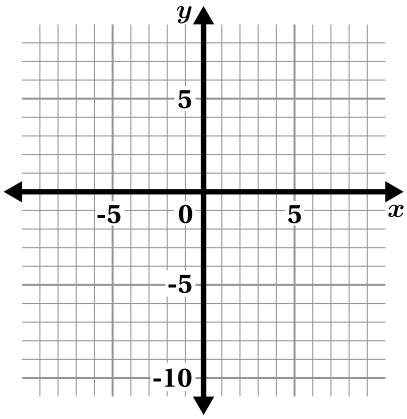
Positive when $x < 4$
Decreasing when $x >$ -1



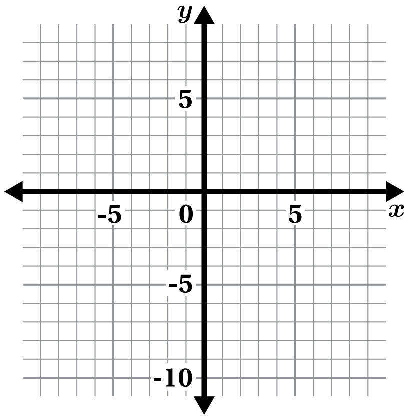
Increasing when $x > 1$
Maximum at (-3, 5)



Negative when $x < 2$
Decreasing when $x >$ -5



Positive when $x >$ -2
Minimum at (-3, -1)

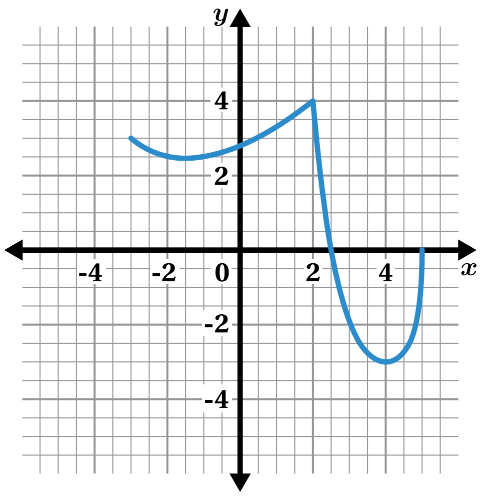


Synthesis

8. Here are some of the terms to describe functions that we learned about today.

Positive	Negative
Maximum	Minimum
Increasing	Decreasing

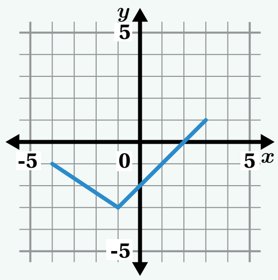
Select three terms. Write the meaning of each term you selected.



Lesson Practice 3.06

Lesson Summary

We can use the key features of a graph to help us describe a function or sketch a possible graph of a function. Here is an example. Let's analyze the graph of this function.



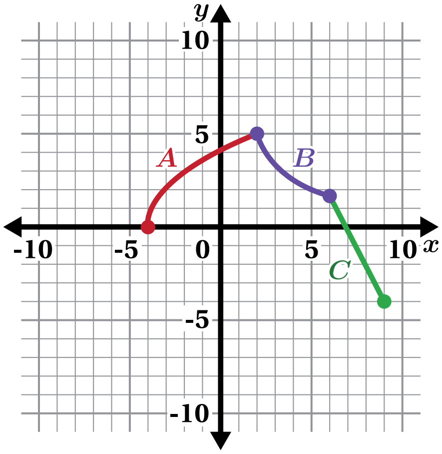
Minimum: The lowest point on a graph.	$(-1, -3)$
Maximum: The highest point on a graph.	$(3, 1)$
Positive: The x-values where the function has positive outputs; the graph is above the x-axis.	$x > 2$
Negative: The x-values where the function has negative outputs; the graph is below the x-axis.	$x < 2$
Increasing: The x-values where the graph is sloping upward as you read the graph from left to right. As the inputs increase, the outputs also increase.	$x > -1$
Decreasing: The x-values where the graph is sloping downward as you read the graph from left to right. As the inputs increase, the outputs decrease.	$x < -1$

Lesson Practice
3.06

Name: .... Date: .... Period: ....

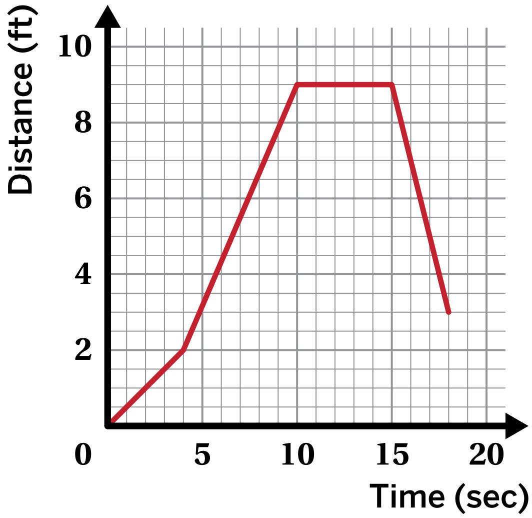
1. Select *all* the true statements about this graph.

- ☐ **A.** This graph is a function.
- ☐ **B.** Part A is decreasing.
- ☐ **C.** Part B is decreasing.
- ☐ **D.** The maximum is at $(2, 5)$.
- ☐ **E.** The minimum is at $(-4, 0)$.



2. Manuel is watching his little brother at the park. The graph represents the distance Manuel is from his brother as a function of time.

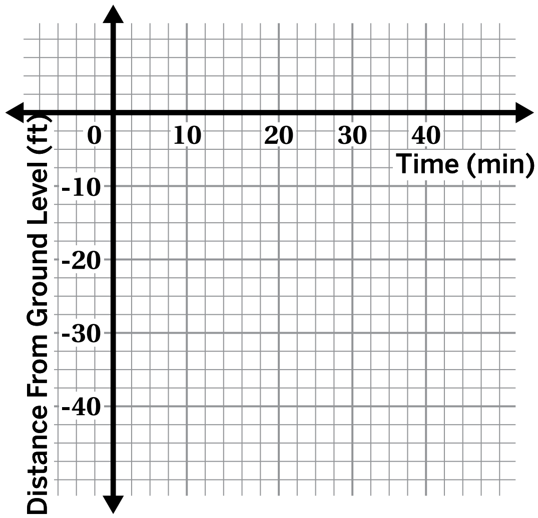
Describe Manuel's distance from his brother over time. Use terms that you learned in this lesson.



3. Ivory goes on a tour of a cave. The tour starts at ground level.

- The tour stays at ground level for 15 minutes.
- Then the tour descends (goes down) for 15 minutes to a depth of 20 feet below ground level.
- The tour stays at this level for 10 minutes.
- The tour spends the last 5 minutes ascending (going up) to ground level.

Sketch a graph describing Ivory's elevation as a function of time.



Lesson Practice
3.06

Name: ______________ Date: ________ Period: ________

Problems 4–6: Here is a table that lists Seattle's temperatures for one day.

Hours After Midnight	Temperature (°F)
1	32
2	33
3	35
4	36
5	35

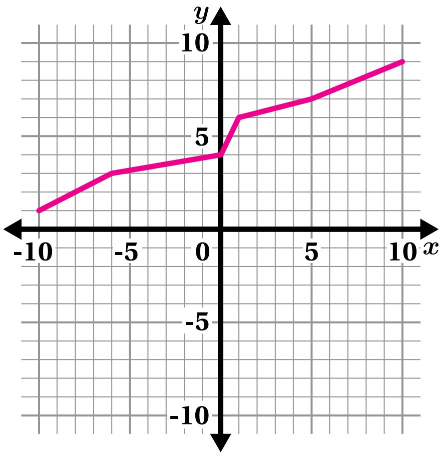
4. How many hours after midnight is the minimum temperature?

5. How many hours after midnight is the maximum temperature?

6. Between what hours is the temperature decreasing?

Test Practice

7. Which two statements are true about the graph of a function?



- ☐ **A.** The function is always positive.
- ☐ **B.** The function is always increasing.
- ☐ **C.** The function is always decreasing.
- ☐ **D.** The function goes through (5, 0).
- ☐ **E.** The function goes through (0,5).

Spiral Review

Problems 8–10: Determine each quotient.

8. $\frac{5}{6} \div \frac{2}{3}$

9. $\frac{8}{6} \div \frac{4}{3}$

10. $\frac{12}{20} \div \frac{18}{16}$

Name: ________ Date: ________ Period: ________

MA.912.F.1.3, MA.912.F.1.5, MTR.1.1, MTR.2.1, MTR.4.1

Plane, Train, and Automobile

Let's calculate the average rate of change over a specified interval.

Warm-Up

1. A wedding is happening in New York City! Many relatives are coming from out of town.

a Here are three wedding guests and how they traveled to the wedding.

Arjun	Troy	Mayra

b What do you notice? What do you wonder?

Activity 1

Name: Date: Period:

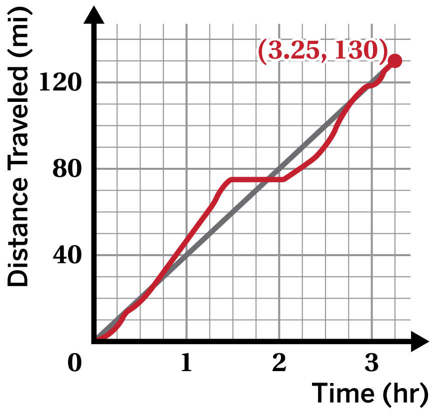
Arjun's Automobile Trip

2. Arjun's car trip was 130 miles and took 3.25 hours.

Imagine Arjun traveled at a constant speed the whole trip.

What would be Arjun's speed in miles per hour?

3. **a** Take a look at the map and the graph of Arjun's actual trip from Hartford, Connecticut to New York City.



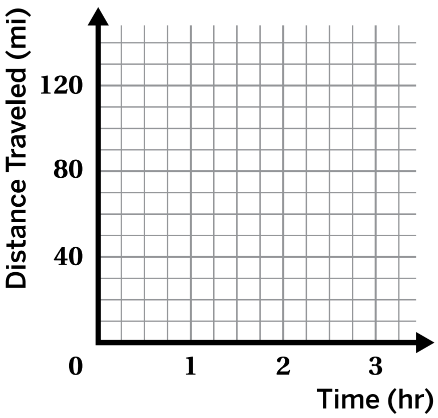
b Tell a story about Arjun's trip.

4. The **average rate of change** is the *slope* of the line that connects two points.

For Arjun's trip, the average rate of change was 40 miles per hour.

We can also look at average rate of change for an **interval**, such as 0 to 1.5 hours (highlighted in the graph).

How would you calculate the average rate of change for that interval?



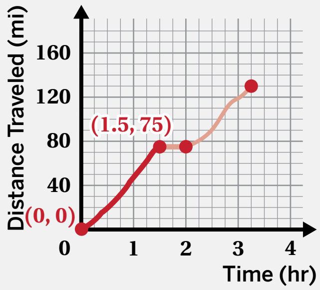
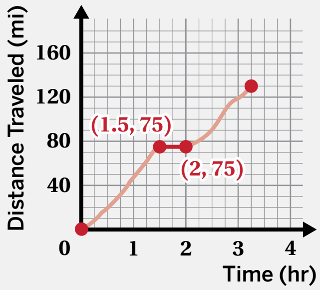
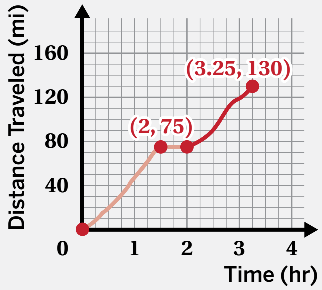
Activity 1

Name: Date: Period:

Arjun's Automobile Trip (continued)

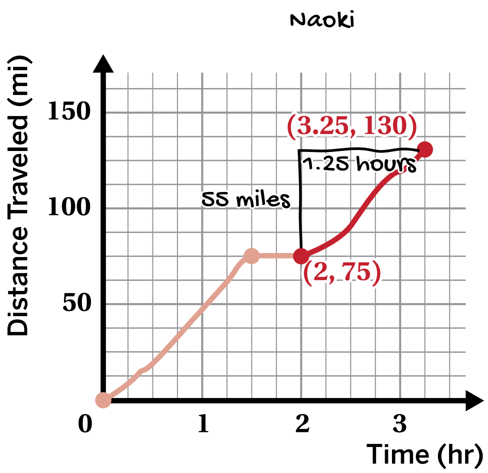
5. Let's examine some intervals of Arjun's trip.

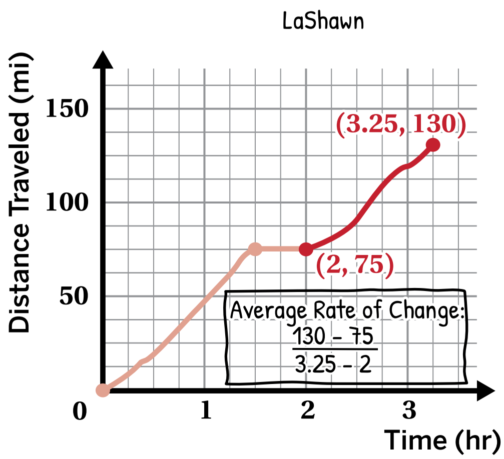
Determine the average rate of change of each interval.

Interval	0 to 1.5 hours	1.5 to 2 hours	2 to 3.25 hours
Graph			
Average Rate of Change (mph)			

6. Two students calculated the average rate of change between 2 and 3.25 hours.

a Take a look at Naoki's and LaShawn's work.





b How do their methods compare?

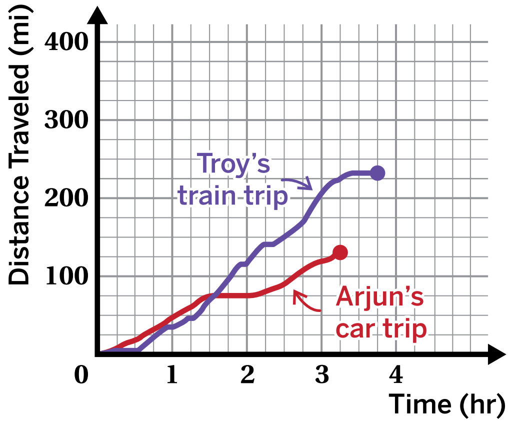
Activity 2

Name: ____ Date: ____ Period: ____

Troy's Train Trip

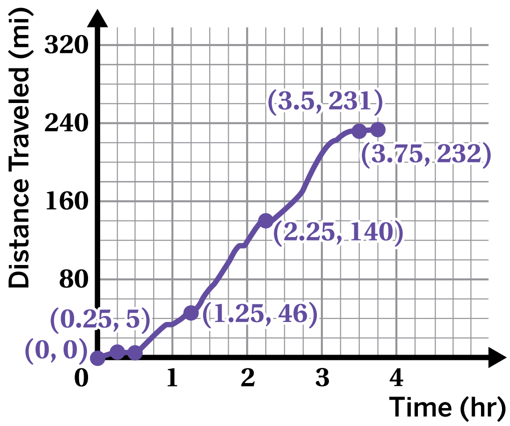
7. Here is the graph for Arjun's and Troy's trips.

What are some ways Troy's trip is different from Arjun's?



8. Let's examine some intervals of Troy's trip.

Choose two points on the graph. Then calculate the average rate of change for the interval you selected.



Can you find:

- Troy's average rate of change for the full trip?
- An interval where Troy moved fast? Slow?

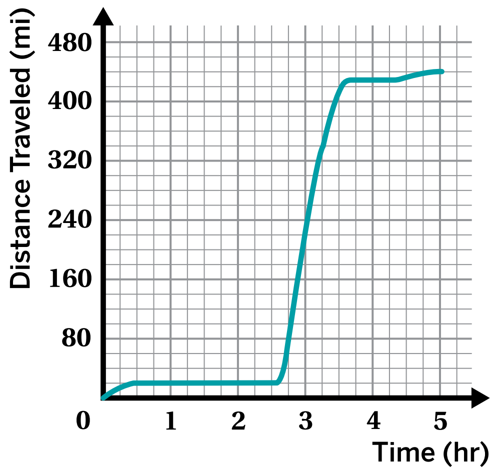
Interval	Average Rate of Change (mph)

Activity
3

Name: ________ Date: ________ Period: ________

Mayra's Flight

9. Mayra's plane trip from Pittsburgh was 440 miles and took 5 hours.

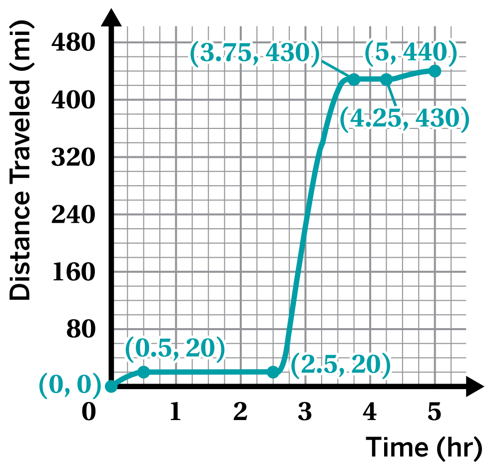


a **Discuss:** What questions do you have about Mayra's trip?

b Label three or more intervals of the graph with what you think was happening at that time.

10. Mayra said: *The flight felt fast, but the trip felt slow.*

Choose two points on the graph. Calculate the average rate of change for the interval you selected.



Can you find:

- Mayra's average rate of change for the full trip?
- Mayra's average rate of change *during the flight*?

Interval	Average Rate of Change (mph)

Name: ________ Date: ________ Period: ________

Mayra's Flight (continued)

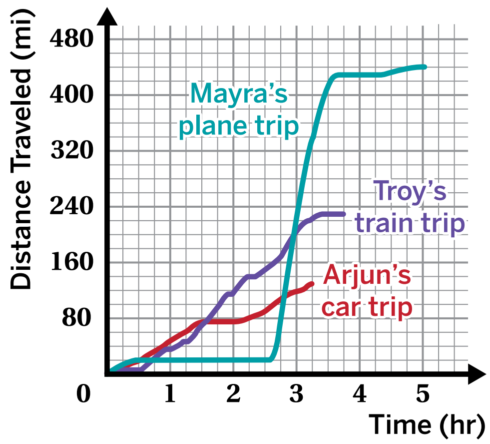
11. There are many reasons why someone may choose to travel by car, train, or plane.

Here are the graphs for all three people's trips.

Circle the trip you think is best.

Arjun (car)   Troy (train)   Mayra (plane)

Explain why you chose that trip.

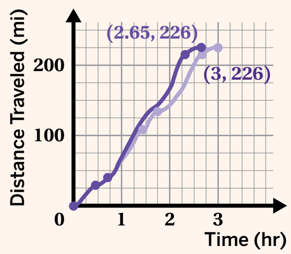


You're invited to explore more.

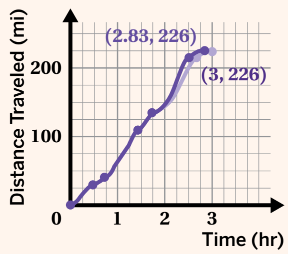
12. Public transit agencies often look at average speed (rate of change) when they look for ways to improve service.

a What is the train's average speed in each Proposal?

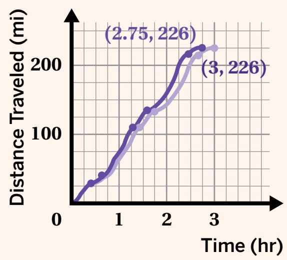
Proposal A: Skip stops along the way.



Proposal B: Upgrade one section of the tracks.



Proposal C: Buy trains with a higher top speed.



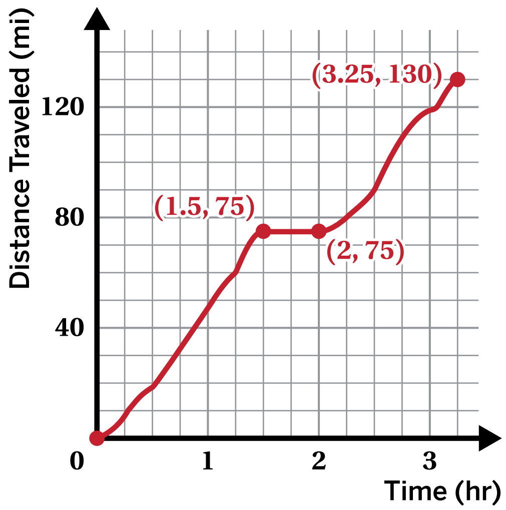
b **Discuss:**

- What is the train's average speed?
- What are some benefits and challenges for each proposal?

Synthesis

13. How can you calculate the average rate of change for an interval of a function?

Use this graph if it helps to show your thinking.

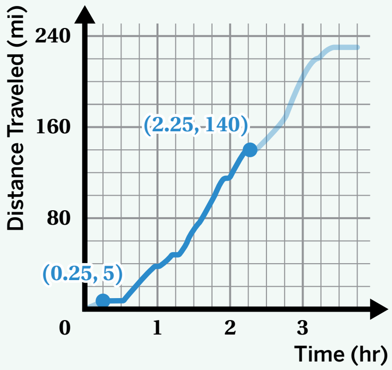


Lesson Practice 3.07

Lesson Summary

Functions can have different rates of change over different intervals. The **average rate of change** is equivalent to the slope of the line between two points.

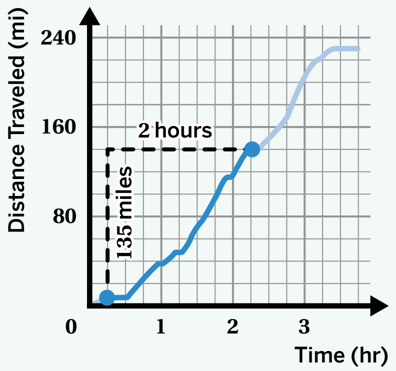
This graph represents Troy's car trip.



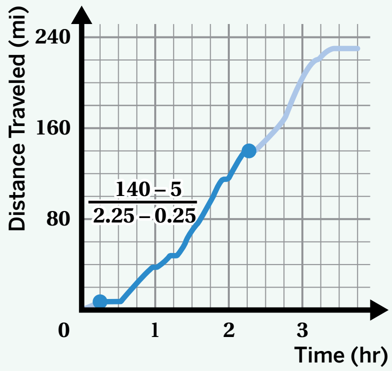
We can calculate the average rate of change over an **interval**, a specific length between two points, like the interval from 0.25 to 2.25 hours.

Here are two different strategies.

Strategy 1



Strategy 2



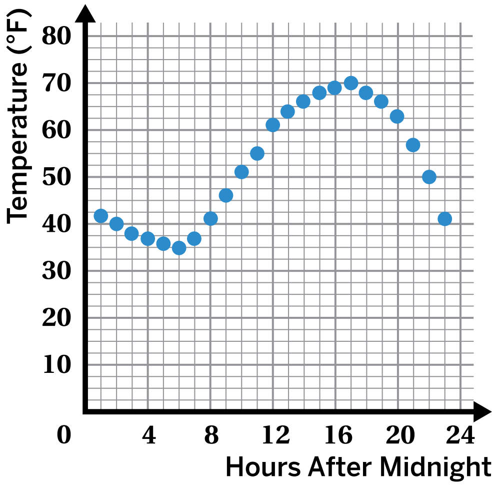
The average rate of change for the interval 0.25 to 2.25 hours is 67.5.

That means that Troy's average speed was 67.5 miles per hour in that interval.

Lesson Practice
3.07

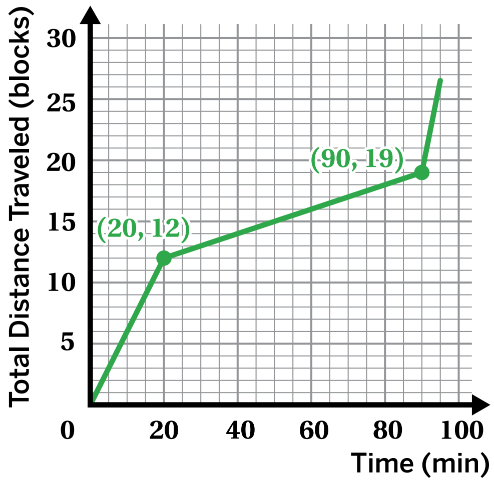
Name: .... Date: .... Period: ....

1. The temperature was recorded at several times in a 24-hour period. Function $t(n)$ gives the temperature in degrees Fahrenheit n hours after midnight. Use the graph to determine if the average rate of change for each interval is *positive* (+), *negative* (-), *or zero*. Place a check in the appropriate column.



	+	-	0
$n = 1$ to $n = 5$	◯	◯	◯
$n = 5$ to $n = 7$	◯	◯	◯
$n = 10$ to $n = 20$	◯	◯	◯

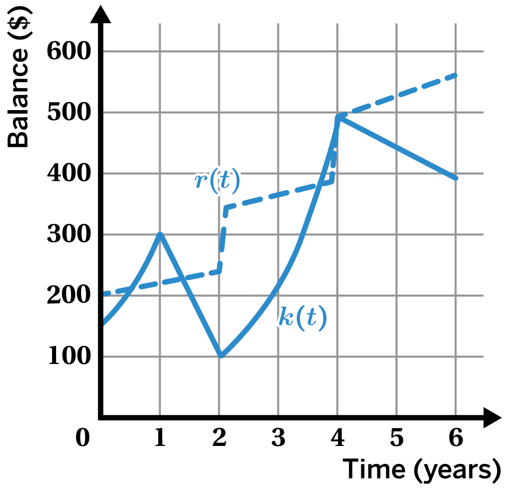
Problems 2–3: This graph shows the total distance in city blocks, $d(t)$, that Pilar walked as a function of time in minutes, t.



2. Determine the average rate of change between $t = 20$ and $t = 90$.

3. What do you think the average rate of change you calculated means in this situation?

4. r(t) and $k(t)$ model the savings account balances of Rafael and Katie after t years. Select *all* the statements that are true.



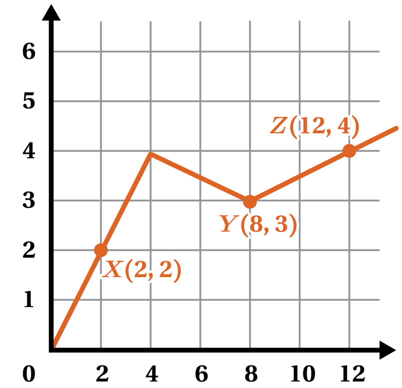
- ☐ **A.** Katie has a lower average rate of change in the last two years.
- ☐ **B.** Katie's balance is always less than Rafael's.
- ☐ **C.** $r(2) = 100$
- ☐ **D.** Rafael's balance is increasing from year 0 to year 6.
- ☐ **E.** Rafael has a higher average rate of change in the first four years.

Lesson Practice
3.07

Name: .... Date: .... Period: ....

5. Match each interval to its average rate of change.

Interval	Average Rate of Change
A. X to Y	.... $\frac{1}{5}$
B. Y to Z	.... $\frac{1}{4}$
C. X to Z	.... $\frac{1}{6}$



Test Practice

Problems 6–8: This table shows the population of a city from 1988 to 2016.

Year, t	Population, $p(t)$
1988	35,700
1992	42,700
1996	33,100
2000	33,700
2004	45,000
2008	48,400
2012	40,900
2016	43,000

6. Choose the average rate of change for $p(t)$ between 1992 and 2000.

A.

B. -9,000 people per year

C. 1,125 people per year

D. 9,000 people per year

7. State two values of t that create an interval with a *negative* rate of change.

8. State two values of t that create an interval with a *positive* rate of change.

Spiral Review

9. Jada is walking to school. The function $d(t)$ gives her distance from school, in meters, t minutes since she left home. Which equation represents this statement? *Jada is 600 meters from school after 5 minutes.*

A. $d(5) = 600$

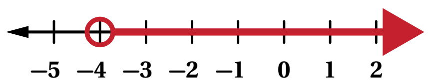
B. $d(600) = 5$

C. $t(5) = 600$

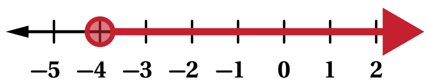
D. $t(600) = 5$

10. Select the graphed solution set of $-2x + 4 < 12$.

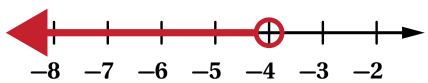
A.



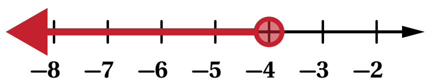
B.



C.



D.



Name: ______ Date: ______ Period: ______

MA.912.F.1.2, MA.912.F.1.3, MA.912.F.1.5, MA.912.F.1.6, MTR.4.1

Space Race

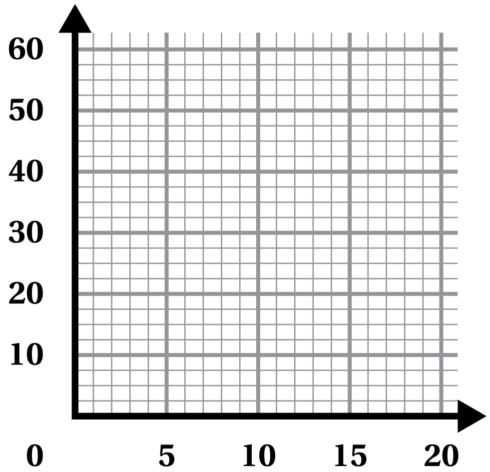
Let's make connections between function notation and key features of graphs.

Warm-Up

1. It's time to play a game called Space Race!

Your mission: Get your spaceship to the finish line as quickly as possible.

What do you notice about the game graph? What do you wonder?



Activity **1**

Name: ______ Date: ______ Period: ______

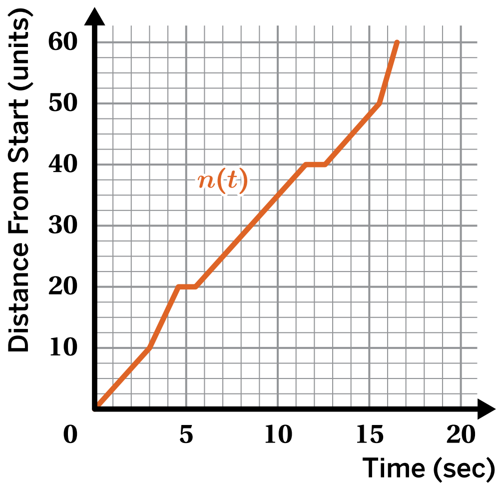
Comparing Graphs By Key Features

2. Nekeisha also played a round of Space Race.

$n(t)$ represents the distance of Nekeisha's spaceship after t seconds.

What is a value of t for which $n(t) = 10$?

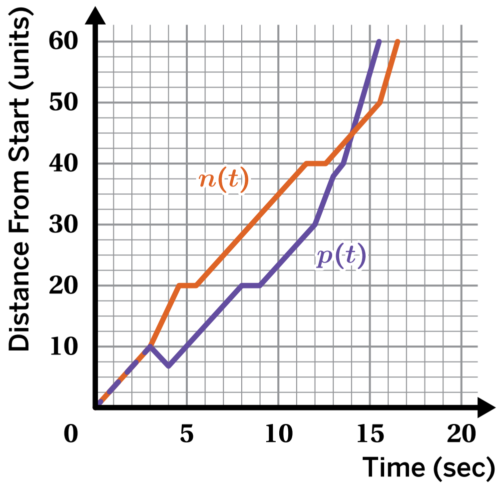
What does that tell you about the situation?



3. Nekeisha and Polina race their spaceships. The graph shows their results.

Discuss:

- What were some interesting moments during this race?
- Where do you see those moments on the graph?



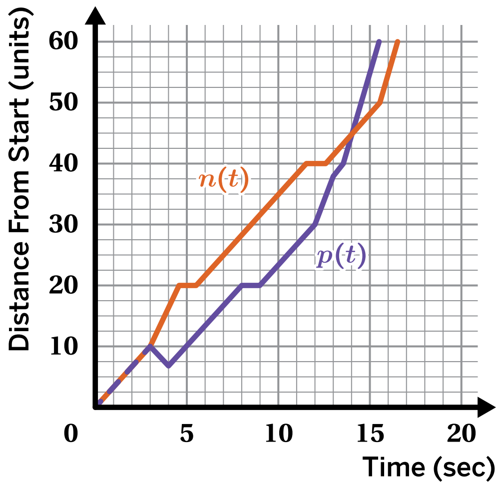
Name: Date: Period:

Comparing Graphs By Key Features (continued)

4. $n(t)$ and $p(t)$ represent the distances of Nekeisha's and Polina's spaceships after t seconds.

One student described part of the graph like this: *Polina's function was decreasing, and Nekeisha's function was increasing.*

Write a value of t that is in this interval of time.



Describe what was happening in the race at that moment.

5. Who had a greater average rate of change from 12 to 14 seconds?

A. Polina **B.** Nekeisha **C.** Their average rates of change were the same.

Explain your thinking.

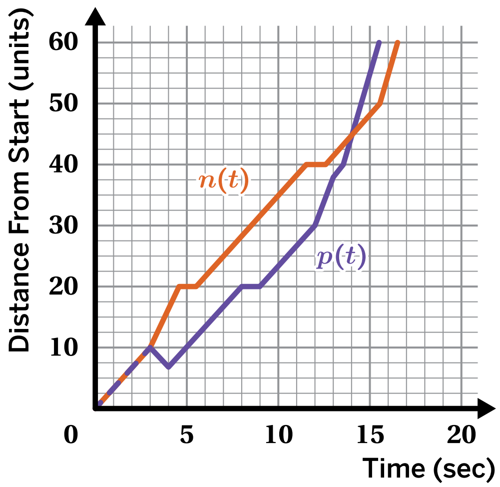
Activity 2

Name: Date: Period:

Comparing Using Function Notation

6. Nekeisha traveled farther than Polina after 11 seconds.

You can express that by writing $n(11) > p(11)$.



a **Discuss:** How can you use the graph to tell that $n(11) > p(11)$?

b Which statement could you use to show who traveled farther after 15 seconds?

A. $n(15) > p(15)$

B. $n(15) = p(15)$

C. $p(15) \geq n(15)$

D. $p(15) > n(15)$

7. We know that $n(11) > p(11)$.

What is a different value of t for which $n(t) > p(t)$?

Name: .... Date: .... Period: ....

Comparing Using Function Notation (continued)

8. What is a value of t for which $n(t) = p(t)$?

Describe what was happening in the race at that moment.

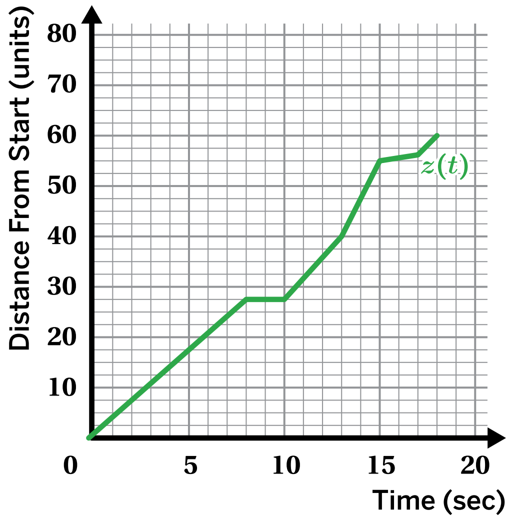
9. Valeria and Zion also raced their spaceships.

$v(t)$ and $z(t)$ represents the distances of Valeria's and Zion's spaceships after t seconds.

Here is some information about their races:

- $v(0) = z(0)$
- $v(4) < z(4)$
- $v(10) = 20$
- $v(13) = z(13)$
- $v(15) > z(15)$

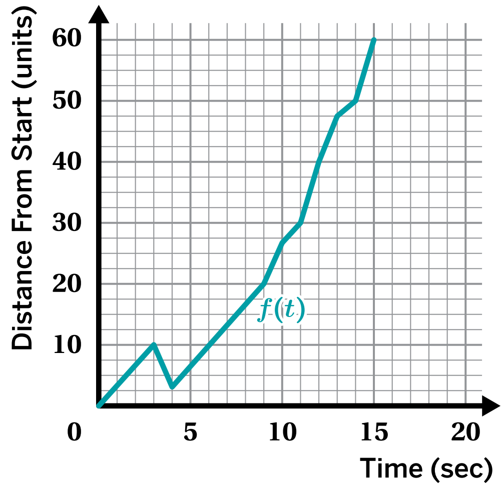
Make a graph that could represent Valeria's distance traveled after t seconds.



Synthesis

10. How can statements in function notation and terms like *maximum, increasing*, and *average rate of change* help us compare graphs of functions?

Use the example if it helps to show your thinking.



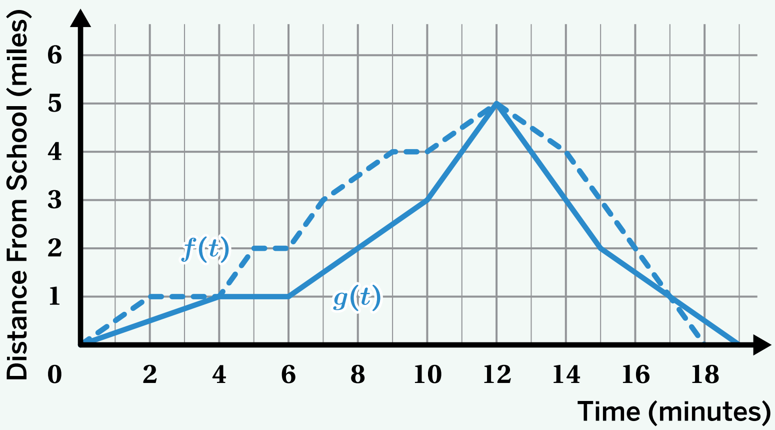
Lesson Practice 3.08

Lesson Summary

We can analyze functions by comparing the key features of different intervals on a graph, then using function notation to describe them.

For instance, here are some true statements about these two graphs:

- When $t = 4$, $f(t) = g(t)$.
- $f(8) > g(8)$
- $f(12) = g(12)$
- $f(15) > g(15)$
- $f(t)$ and $g(t)$ have the same maximum.
- $f(t)$ and $g(t)$ are both decreasing from 12 to 15 minutes.



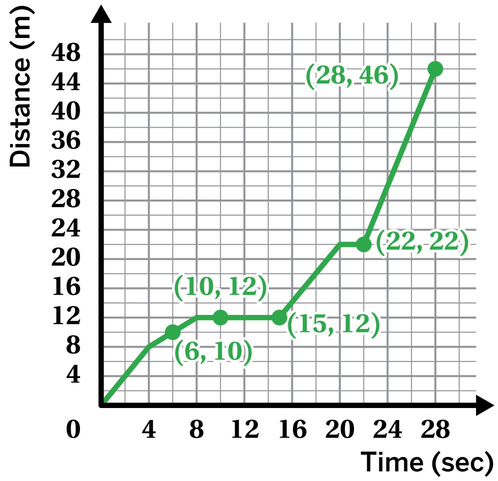
- $f(t)$ and $g(t)$ have the same average rate of change from 5 to 6 minutes.

Lesson Practice
3.08

Name: .... Date: .... Period: ....

Problems 1–2: Mai built a model race car for a school competition.

$m(t)$ represents the distance of Mai's car, in meters, after t seconds.



1. Use the graph to determine the missing value in each function statement.

$m(\text{.........}) = 10$

$m(10) = \text{.........}$

$m(22) = \text{.........}$

$m(\text{.........}) = 46$

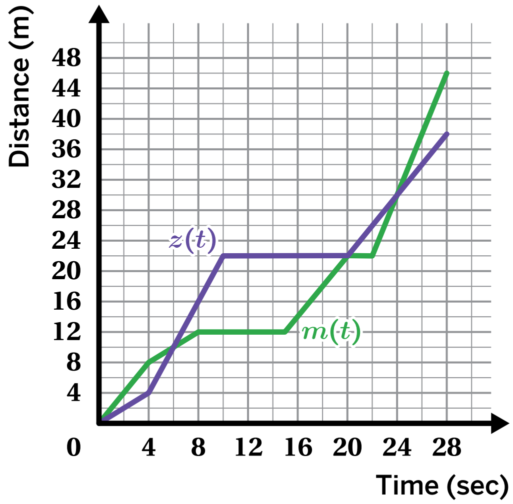
2. Over what interval did Mai's car travel the slowest?

A. 0 to 4 sec B. 4 to 8 sec C. 8 to 15 sec D. 15 to 20 sec

Problems 3–6: Zion also built a model race car for the school competition.

$z(t)$ represents the distance of Zion's car, in meters, after t seconds.

Did Zion or Mai have the greater average rate of change over the following intervals? Explain your thinking.



3. $t = 4$ to $t = 8$

4. $t = 10$ to $t = 20$

5. Name a time when Zion's and Mai's cars had traveled the same distance.

6. Select *all* the true statements.

- ☐ **A.** $m(t)$ has a greater maximum than $z(t)$.
- ☐ **B.** $z(t)$ and $m(t)$ have the same minimum at $(6, 10)$.
- ☐ **C.** $z(20) = m(20)$
- ☐ **D.** $m(15) > z(15)$
- ☐ **E.** $m(t)$ and $z(t)$ both increase from 22 to 28 sec.

Lesson Practice
3.08

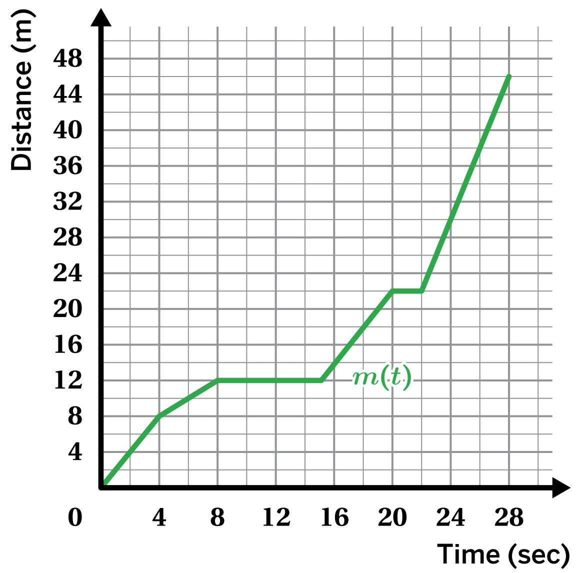
Name: .... Date: .... Period: ....

Test Practice

Problems 7–8: Parv built a race car to race against Mai.

7. Use this information to make a graph that could represent the distance of Parv's race car, $p(t)$, after t seconds:

- $p(8) < m(8)$
- $p(12) = m(12)$
- The average rate of change of $p(t)$ and $m(t)$ is the same from $t = 22$ to $t = 28$.
- $m(t)$ has a greater maximum than $p(t)$.



8. Which of the following equations is true?

A. $m(4) = 2$

B. $m(8) = 12$

C. $m(12) = 14$

D. $m(22) = 20$

Spiral Review

Problems 9–11: Nekeisha goes for a bike ride. $d(t)$ represents Nekeisha's distance from home, in miles, t minutes after she leaves.

Explain the meaning of each statement in context:

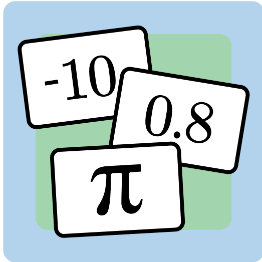
$d(0) = 0$	
$d(30) = d(60)$	
$d(90) = 0$	

Unit 3
Lesson
9

Name: ____________ Date: ________ Period: ________

MA.912.F.1.2, MA.912.F.1.5, MA.912.F.1.6, MTR.4.1

Ins and Outs



Let's explore the possible inputs and outputs of functions.

Warm-Up

1. Here are four sets of numbers.

Which set doesn't belong? Explain your thinking.

A 1, 2, 3, 4	B 0, 1, 2, 3, 4, . . .
C 5, 10, 15, 20, . . .	D -1, -2, -3, -4, . . .

Activity
1

Name: Date: Period:

Possible or Impossible?

You will need a set of cards for this activity. For each function, determine which numbers are possible inputs and which ones are impossible. Use the same set of cards for each function.

2. Brielle's Bike Rentals charges a $\$10$ rental fee plus an additional $\$2$ for every hour you rent a bike. $b(h) = 2h + 10$ represents the cost of a bike rental for h hours.

Possible inputs:

Impossible inputs:

What do the possible inputs have in common?

3. The price of a medium pizza is $\$10$ plus an additional $\$2$ for every topping (up to 4 toppings). $p(t) = 2t + 10$ represents the cost of a pizza with t toppings.

Possible inputs:

Impossible inputs:

What do the possible inputs have in common?

4. A student is graphing the function $f(x) = 2x + 10$.

Possible inputs:

Impossible inputs:

What do the possible inputs have in common?

Name: Date: Period:

Possible or Impossible? (continued)

5. The set of all possible inputs of a function is called the **domain**.

Match each domain description with $p(t)$, $b(h)$, or $f(x)$. Be prepared to share your thinking.

Function	Domain
	All Numbers
	All Numbers Greater Than 0
	Whole Numbers From 0 to 4

6. Each of the scenarios $p(t)$, $b(h)$, and $f(x)$ can be modeled by the same linear relationship.

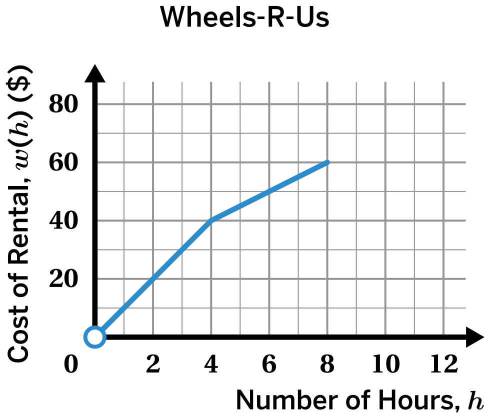
Discuss: Why are the domains of these three functions different from each other?

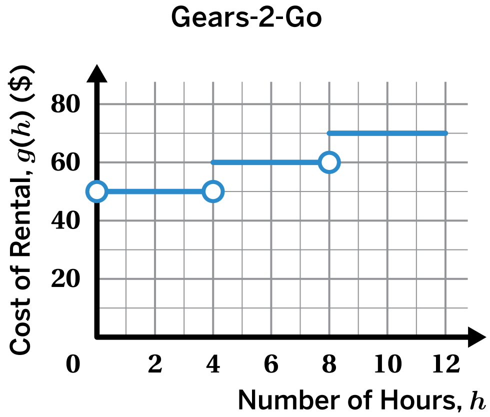
Activity 2

Name: .... Date: .... Period: ....

What About the Outputs?

Two bike rental companies decided to graph the cost of a bike as a function of time. The functions $w(h)$ and g(h) represent the cost of a bike rental for h hours.





7. Deja paid \$40 to rent a bike.

Discuss: Is this enough information for you to know which company she rented from? Why or why not?

8. Explain why Deja would never pay \$55 to rent her bike from Gears-2-Go.

9. The set of all possible outputs of a function is called the **range**.
How would you describe the range of each function?

$w(h)$:

$g(h)$:

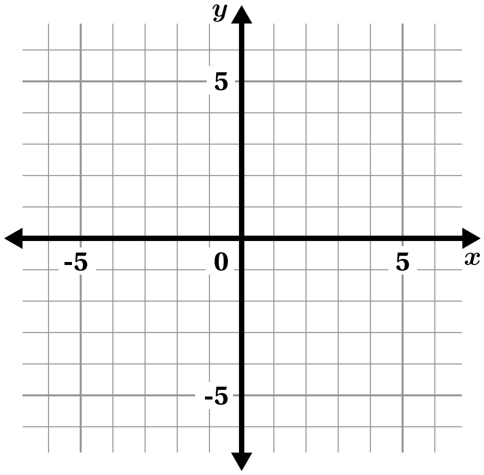
Name: Date: Period:

What About the Outputs? (continued)

Here is the domain and range of a function:

- Domain: All numbers from -3 to 4.
- Range: All numbers from 0 to 5.

10. Sketch a graph that matches the domain and range.



11. Compare your graph with a partner.

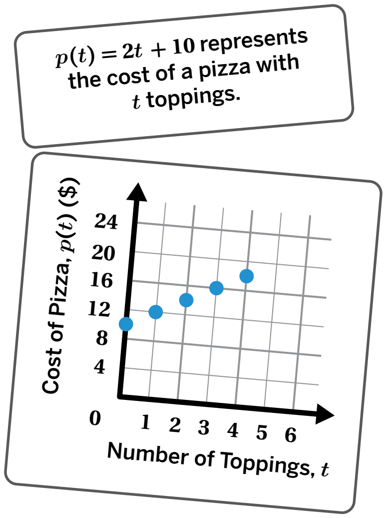
Discuss:

- What's the same about your graphs?
- What's different?

Synthesis

12. **a** How can you determine if a number is in the domain of a function?

b How can you determine if a number is in the range of a function?



Lesson Practice 3.09

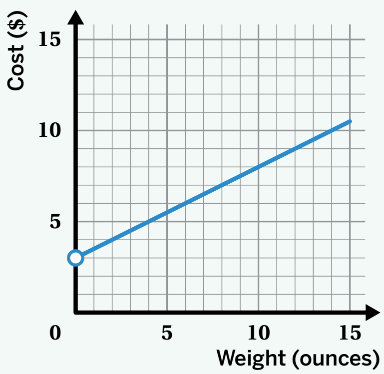
Lesson Summary

Domain and **range** are used to describe the inputs and outputs of a function. The domain is the set of all possible input values (or all possible values for the *independent variable*). The range is the set of all possible output values (or all possible values for the *dependent variable*).

Having context for what the function is describing helps to make sense of possible inputs and outputs. Some domains and ranges are discrete while others are *continuous*. *Discrete* means that the possible values aren't connected or *continuous*, like whole *numbers*. *Continuous* means that the possible values are all connected, like the values along the graph of a line.

You can find the domain by looking at the x-values as you read the graph from left to right. You can determine the range by looking at the y-values as you read the graph from bottom to top.

Here the function $f(w) = 3 + 0.5w$ represents the cost of a frozen yogurt that weighs w ounces.



To determine the domain, consider which inputs make sense for the situation:

- Someone could buy 0.5 and 2 ounces of frozen yogurt.
- Someone cannot buy 0 or -2 ounces of frozen yogurt.

The domain of this function is all numbers greater than 0.

To determine the range, consider which outputs make sense.

- Some possible outputs are $4, $6, and $6.25.
- Someone cannot pay a negative amount of dollars for frozen yogurt. It also doesn't make sense to have a cost of $3 since that would mean a customer purchased a yogurt weighing 0 ounces.

The range of this function is all the dollar amounts greater than 3 up to the nearest cent.

Lesson Practice
3.09

Name: ________ Date: ________ Period: ________

Problems 1–2: Aniyah is working at the mall wrapping presents during the holidays. She earns $2 for each present she wraps. The function $f(x) = 2x$ represents Aniyah's daily earnings for x number of presents wrapped.

1. Are the values 2.3, -10, and 0.5 possible inputs for this situation? Explain your thinking.

2. Why is $\$24.50$ an impossible daily earning amount for Aniyah?

Problems 3–6: A concert is being held at a local venue. Tickets are $25 per person. The concert venue has a maximum capacity of 500 people. The function $f(t) = 25t$ represents the amount of money raised for t tickets sold.

3. Select *all* the possible values in the domain of $f(t)$.

- ☐ **A.** -0.1
- ☐ **B.** 25
- ☐ **C.** 3.7
- ☐ **D.** 525
- ☐ **E.** 173

4. Describe the domain of $f(t)$.

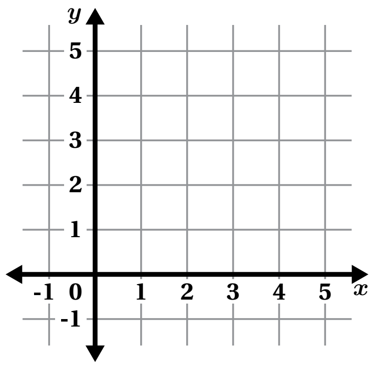
5. Select all the possible values in the range of $f(t)$.

- ☐ **A.** -25
- ☐ **B.** 0
- ☐ **C.** 25
- ☐ **D.** 860
- ☐ **E.** $12{,}500$

6. Describe the range of $f(t)$.

7. Sketch a graph of $f(x)$ that meets the following conditions:

- Domain: All numbers from 0 to 4.
- Range: All whole numbers from 0 to 3.



Lesson Practice
3.09

Name: .... Date: .... Period: ....

Test Practice

8. Choose the correct domain for the graph of the linear function: $p(t) = 10t + 1$.

A. All numbers

B. Whole numbers from 0 to 5

C. All numbers from 0 to 24

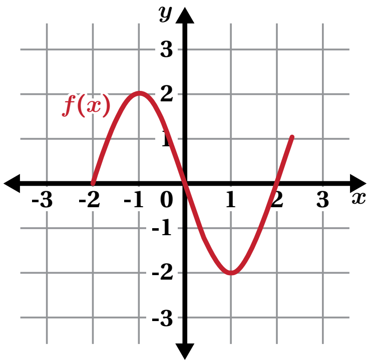
D. All numbers from 1 to 10

Spiral Review

9. Use the equations for $f(x)$ and $g(x)$ to determine the outputs for each input.

	$x = -5$	$x = 0$	$x = 7$
$f(x) = 2x + 5$			
$g(x) = -x + 2$			

10. Emmanuel is describing the graph of $f(x)$. Select *all* the true statements.

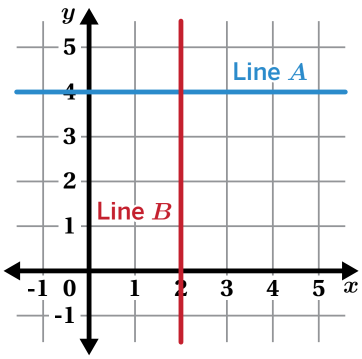


- ☐ **A.** The minimum of $f(x)$ is at $(1, -2)$.
- ☐ **B.** $f(0) = -2$
- ☐ **C.** The graph is decreasing for x, where $x \geq 1$.
- ☐ **D.** $f(0) > f(1)$
- ☐ **E.** The maximum of $f(x)$ is at $(-1, 2)$.

11. Write the equation for each line.

Line *A*: ....

Line *B*: ....



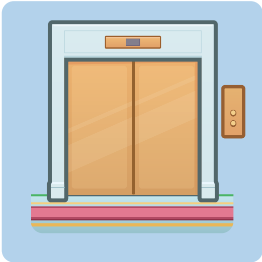
Unit 3
Lesson
10

Name: ______ Date: ______ Period: ______

MA.912.F.1.2, MA.912.F.1.5, MA.912.F.1.6, MTR.1.1, MTR.2.1, MTR4.1

Elevator Stories

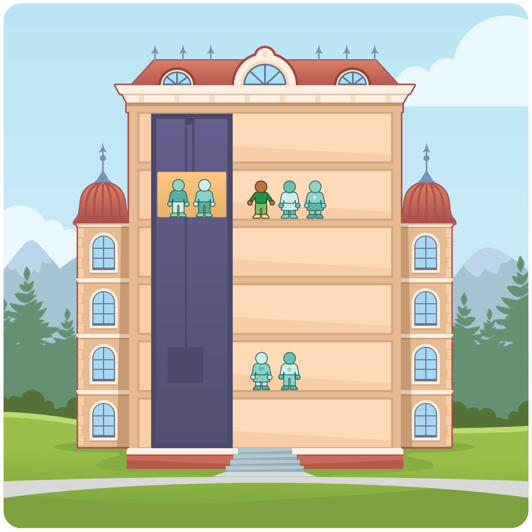
Let's use compound inequalities to describe the domain and range of functions from their graphs.

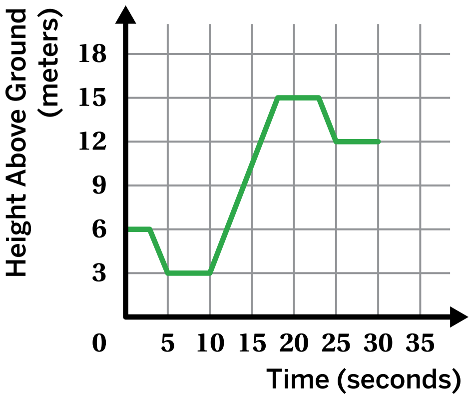


Warm-Up

1. Amari is taking an elevator ride in her apartment building. The graph below shows Amari's height above ground, in meters, over a period of 30 seconds.

Tell a story about what you see.





Activity 1

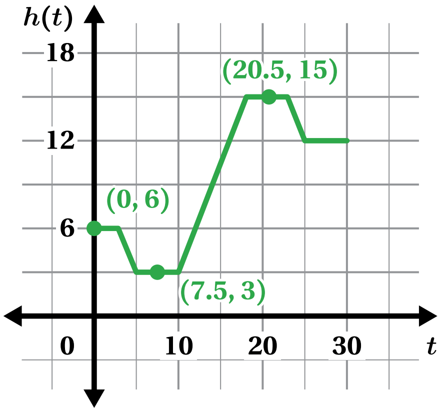
Name: .... Date: .... Period: ....

Describing the Domain

2. Here is a graph of $h(t)$, which represents the height of the elevator at a certain time, t, during Amari's ride.

Complete the input and output table for $h(t)$.

t	$h(t)$
5	
20	
30	



3. **a** Select *all* the numbers that are in the domain of $h(t)$.

- ☐ **A.** -5
- ☐ **B.** $\frac{1}{2}$
- ☐ **C.** 2
- ☐ **D.** 12
- ☐ **E.** 18
- ☐ **F.** 23.5
- ☐ **G.** 32
- ☐ **H.** 60

b Describe the domain of $h(t)$.

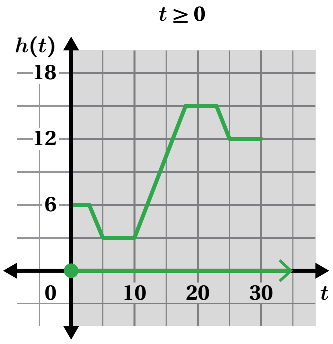
Activity 1

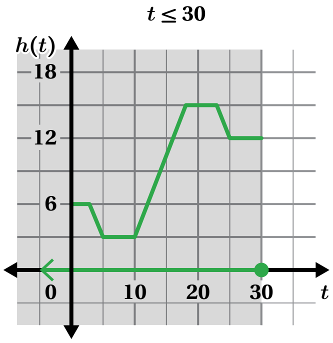
Name: ____________ Date: ________ Period: ________

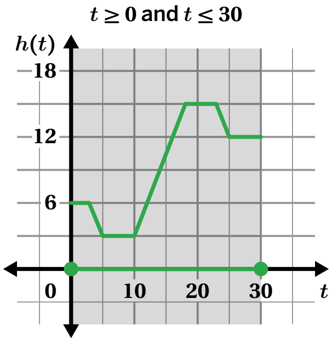
Describing the Domain (continued)

4. The domain of $h(t)$ is all the numbers from 0 to 30.

a Think about these inequalities and their relationships with the graph of $h(t)$.







b **Discuss:**

- Which inequality describes the domain of $h(t)$?
- Why don't the other inequalities describe the domain?

5. Each of these **compound inequalities** accurately describes the domain of $h(t)$.

$t \geq 0$ and $t \leq 30$
$0 \leq t$ and $t \leq 30$
$0 \leq t \leq 30$

a **Discuss:** How are they alike? How are they different?

b Explain how a compound inequality can help you describe the domain of $h(t)$.

Activity 2

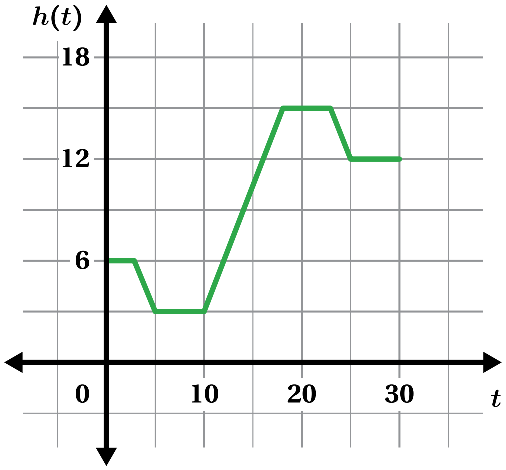
Name: .... Date: .... Period: ....

Distinguishing Domain and Range

6. **a** Write a compound inequality that describes the range of $h(t)$.

.... $\leq h(t) \leq$ ....

b Describe in words what the compound inequality says about the range.



7. Here is the graph of another elevator ride.

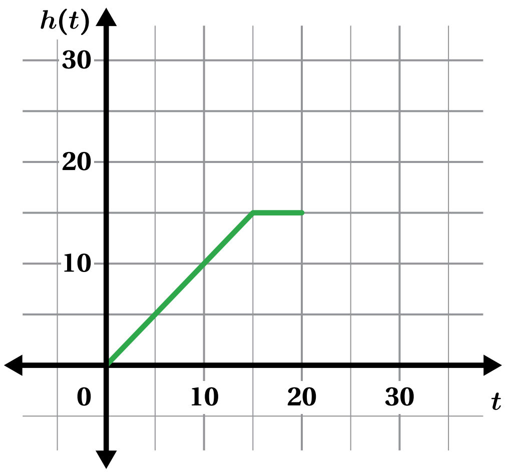
Two students described the range of this function.

- Ali says the range is $0 \leq t \leq 15$.
- Shanice says the range is $0 \leq h(t) \leq 15$.

Whose thinking is correct? Circle one.

Ali's Shanice's Both Neither

Explain your thinking.



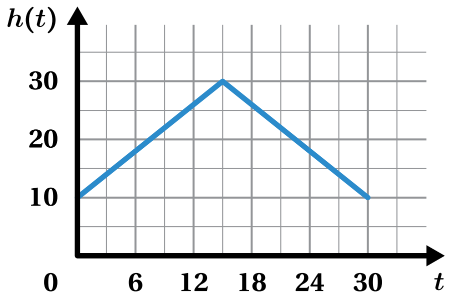
Activity 2

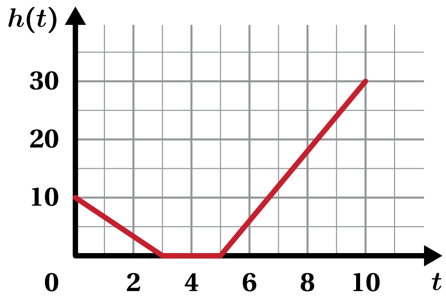
Name: ______ Date: ______ Period: ______

Distinguishing Domain and Range (continued)

8. Match each graph of a function with its domain and range. Two inequalities will have no match.

$0 \leq t \leq 10$	$0 \leq h(t) \leq 30$	$0 \leq t \leq 30$
$10 \leq h(t) \leq 30$	$0 \leq h(t) \leq 10$	$10 \leq t \leq 30$





9. The function $h(t)$ represents the height of the elevator at a certain time, t.

The range of $h(t)$ is $-30 \leq h(t) \leq 0$.

What could you say about this elevator ride?

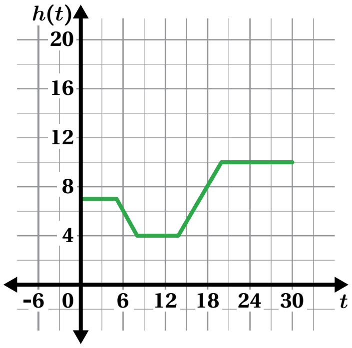
Activity
3

Name: ______ Date: ______ Period: ______

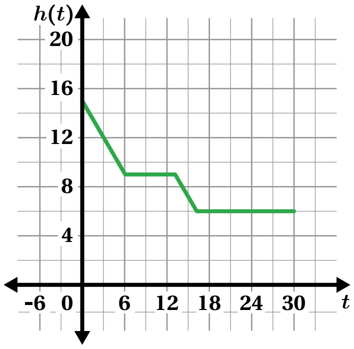
Writing Compound Inequalities

10. Complete the compound inequality to describe the range of $h(t)$.

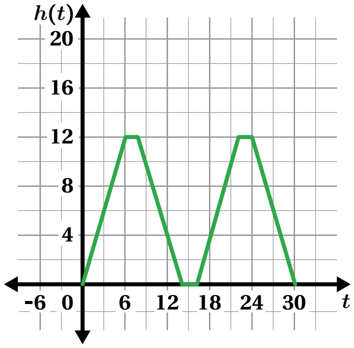
______ $\leq h(t) \leq$ ______



______ $\leq h(t) \leq$ ______



______ $\leq h(t) \leq$ ______



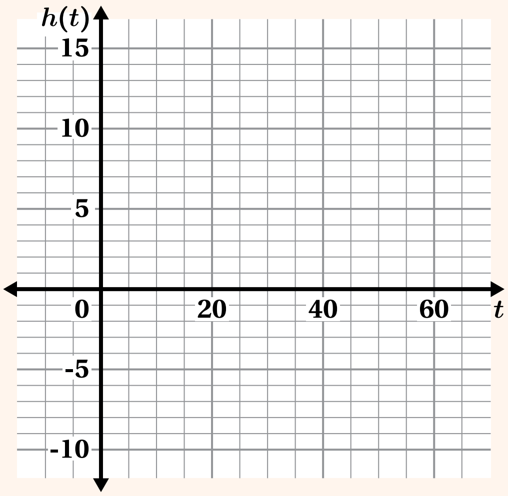
You're invited to explore more.

11. $h(t)$ represents the height of an elevator at a certain time, t, during an elevator ride.

- The domain is $0 \leq t \leq 60$.
- The range is $-6 \leq h(t) \leq 12$.

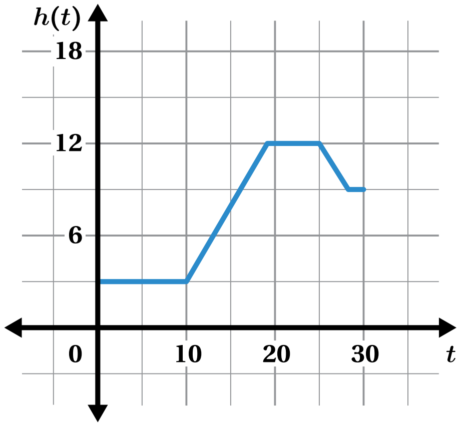
a Create a graph that matches the domain and range.

b Tell a story about your graph.



Synthesis

12. How can you determine the domain and range of a function from its graph? Draw on the graph if it helps with your thinking.

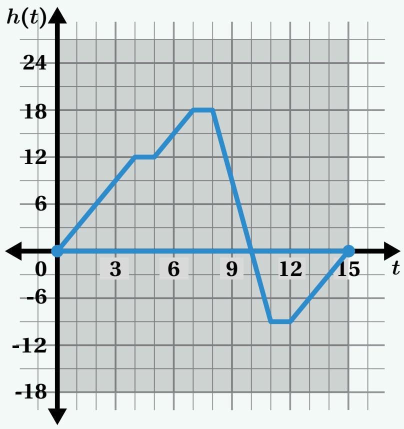


Lesson Practice 3.10

Lesson Summary

The *domain* and *range* of a function can each be described using a **compound inequality**, which is two or more inequalities joined together. You can write a compound inequality using symbols or using the words "and" or "or."

A graph can help you visualize the domain and range of a function, making it easier to describe them using compound inequalities.

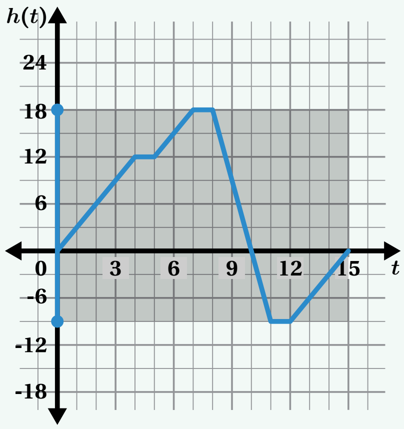


Domain: The domain describes the width of the function, or how far left and right the function goes. The domain is also the set of all inputs to the function, or all values of the independent variable. The domain of this function is all the t values from 0 to 15.

$$t \geq 0 \text{ and } t \leq 15$$

$$0 \leq t \text{ and } 15 \geq t$$

$$0 \leq t \leq 15$$



Range: The range describes the height of the function, or how far up and down the function goes. The range is also the set of all outputs of the function, or all values of the dependent variable. The range of this function is all the $h(t)$ values from -9 to 18.

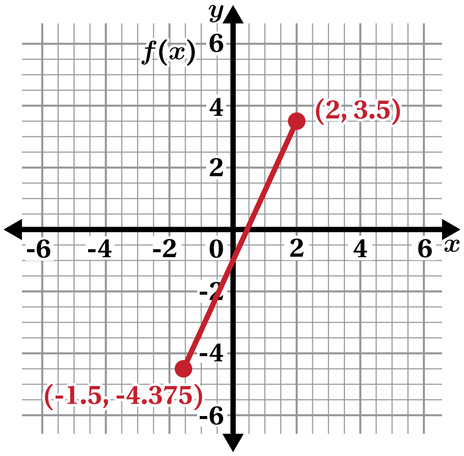
$$h(t) \geq -9 \text{ and } h(t) \leq 18$$

$$-9 \leq h(t) \text{ and } 18 \geq h(t)$$

$$-9 \leq h(t) \leq 18$$

Name: ____________ Date: ____________ Period: ____________

Problems 1–2: Valeria and Thiago disagree about the domain of $f(x)$.



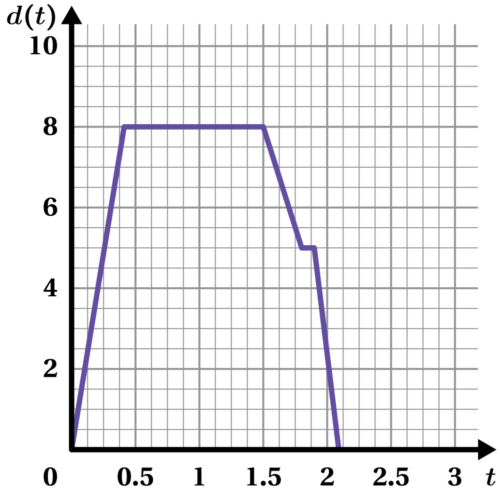
- Valeria says the domain is $-1.5 \leq x \leq 2$
- Thiago says the domain is $-4.375 \leq x \leq 3.5$

1. Whose answer is correct? Circle one.

Valeria's Thiago's

2. Explain why the other person's answer is incorrect.

Problems 3–5: Haru bikes to his friend's house. After a while, he heads home. On the way, he stops at the store to buy a bottle of water. $d(t)$ represents Haru's distance from his house, in kilometers, after t hours. This graph shows Haru's distance over time.



3. Which inequality describes the domain of $d(t)$?

A. $0 \leq d(t) \leq 2.1$ B. $0 \leq d(t) \leq 8$

C. $0 \leq t \leq 2.1$ D. $0 \leq t \leq 8$

4. Which inequality describes the range of $d(t)$?

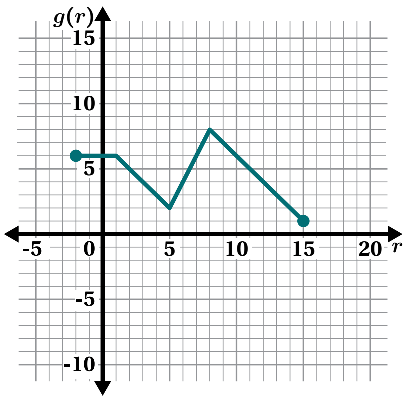
A. $0 \leq d(t) \leq 2.1$ B. $0 \leq d(t) \leq 8$

C. $0 \leq t \leq 2.1$ D. $0 \leq t \leq 8$

5. If Haru had not stopped at the store, would that change the domain or the range? Circle one.

Domain Range Both Neither

Problems 6–7: Here is the graph of $g(r)$.



6. Write a compound inequality to describe the domain.

7. Write a compound inequality to describe the range.

Name: .... Date: .... Period: ....

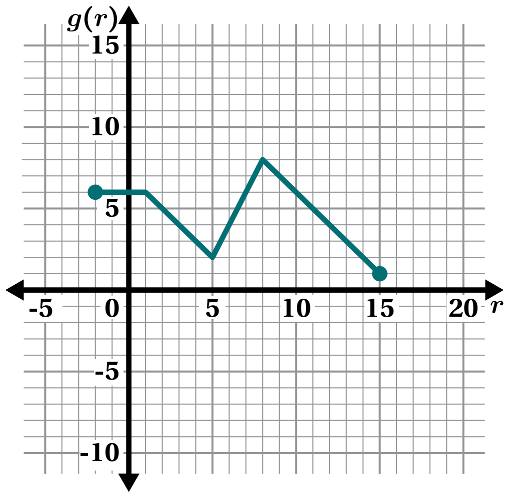
Test Practice

8. Which of the following compound inequalities correctly describes the domain of the function $f(x) = 2x + 5$ if the graph of $f(x)$ is only shown between $x = -3$ and $x = 4$?

- ☐ **A.** $-3 < x < 4$
- ☐ **B.** $-3 \leq x \leq 4$
- ☐ **C.** $-3 \leq x < 4$
- ☐ **D.** $-3 < x \leq 4$

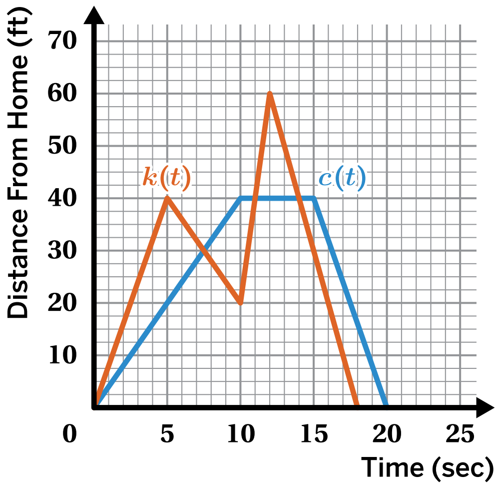
Spiral Review

9. Determine the average rate of change for each interval in the graph of $g(r)$.



Interval	Average Rate of Change
$r = -2$ to $r = 1$	
$r = -2$ to $r = 5$	
$r = -2$ to $r = 8$	

10. Functions $c(t)$ and $k(t)$ represent the distance of two cats from home after t seconds. Select *all* of the true statements.



- ☐ **A.** $k(5) > c(5)$
- ☐ **B.** $k(t)$ and $c(t)$ have the same domain and range.
- ☐ **C.** $k(t)$ keeps increasing from 0 to 13 seconds.
- ☐ **D.** $k(11) = c(11)$
- ☐ **E.** Both cats return home.

11. Tickets to the state fair cost \$10 each. The function $c(t) = 10t$ gives the total cost in dollars, $c(t)$, for the number of tickets purchased, t. Select *all* the values that are possible outputs for $c(t)$.

- ☐ **A.** 0
- ☐ **B.** 70
- ☐ **C.** 105
- ☐ **D.** 880
- ☐ **E.** 963

Sub-unit

3

Special Types of Functions

Lesson 11
What's Your Score?

Lesson 12
Absolute Value Machines

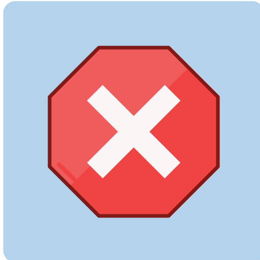
Unit 3
Lesson
11

Name: ______________________ Date: __________ Period: __________

MA.912.AR.4.3, MA.912.F.1.2, MA.912.F.1.6, MTR2.1, MTR.5.1

What's Your Score?

Let's practice restricting the domain and range of a graph.



Warm-Up

1. Which one doesn't belong?

A. $x = |\text{-}3|$

B. $x| = 3$

C. $x = |9| - |12||$

D. $|9 - 12| = x$

Explain your thinking.

Activity 1

Name: Date: Period:

Target Numbers

For a certain game, when you press “Stop” it stops the arrow at a location on a number line and you get a score.

2. Here are Adriana’s scores.

Adriana got a score of 4 on her next try.

What number do you think she stopped on? Why?

Number	Score
5	3
1	1
2	0
-4	6

3. Now there is a mystery target in this game!

a. Adriana played 2 rounds of the game with the mystery target. Those scores are in the table. She got a score of 7 on her next try. What number do you think she stopped on? Why?

b. **Discuss:** What do you think the target is? Why?

Number	Score
6	2
-6	10

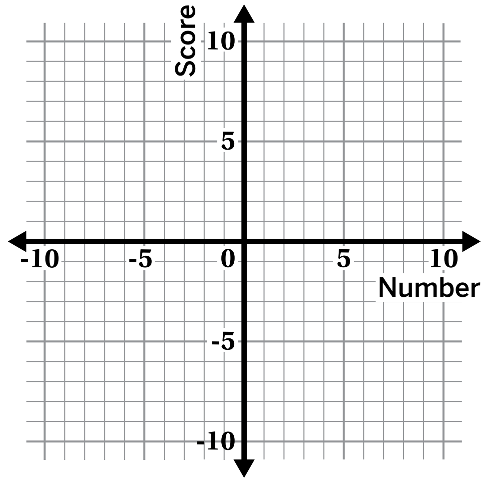
Activity 1

Name: Date: Period:

Target Numbers (continued)

4. a Plot the scores on the graph.

Number	Score
5	1
-1	5
3	1
0	4



Discuss: What do you think the target is? Why?

- What do you think the graph of all possible scores looks like?
- Where can you see the mystery target?

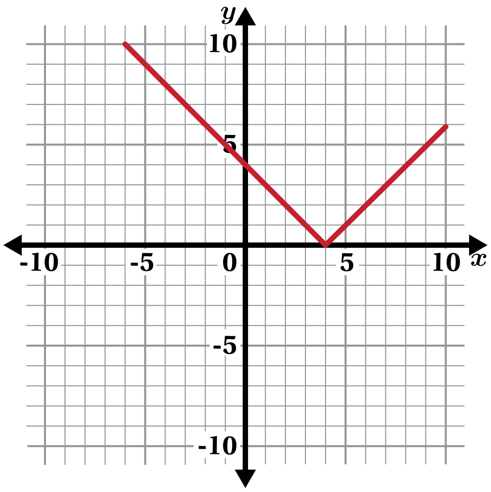
Problems 5–6: The function $f(x) = |x - 4|$ is an example of an **absolute value function**.

This particular function tells you how far away you are from a target value of 4.

5. What is the value of $f(-2)$?

6. Fill in the table for the function.

Number	Score



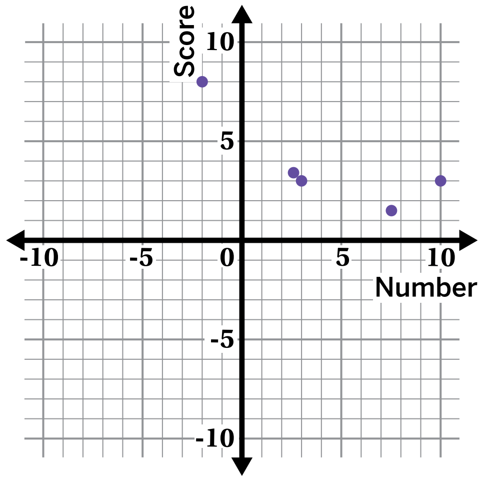
Activity
2

Name: ______ Date: ______ Period: ______

Absolute Value Functions

7. Here are some guesses and scores.

Which function gives the score for each guess in this game?



- A. $a(x) = |x| + 6$
- B. $b(x) = |x + 6|$
- C. $c(x) = |x| - 6$
- D. $d(x) = |x - 6|$

Explain your thinking.

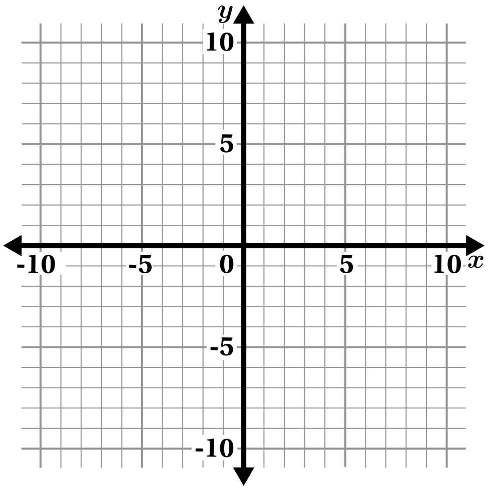
Name: ________ Date: ________ Period: ________

Absolute Value Functions (continued)

8. There is a new mystery number. The function $f(x) = |x + 3|$ gives the score for each guess, x.

Complete the table and plot the ordered pairs.

x	$f(x)$
5	
-1	
-5	
-3	
3	



9. Compare the function $f(x) = |x + 3|$ to the function $g(x) = x + 3$ by analyzing their equations, graphs, or a table of values. What do you notice?

Explore More

10. Here are some guesses and scores for a new mystery number. Can these be scores for the same mystery number? Circle one.

Yes No

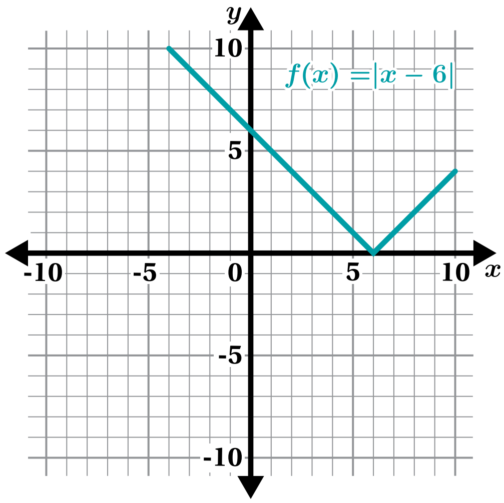
Explain your thinking.

Guess	Score
1	4
6	2

Synthesis

11. How is an absolute value function related to the distance from a number?

Use the graph and equation if they help with your thinking.

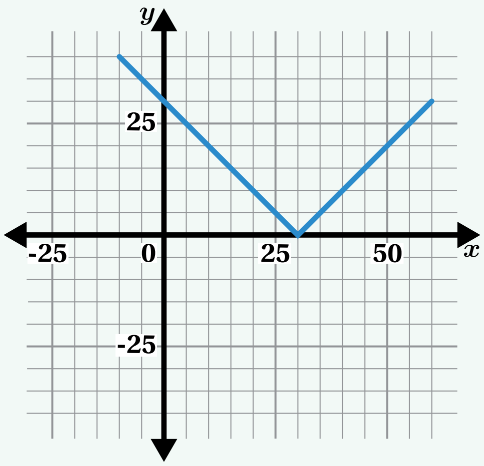


Lesson Practice 3.11

Lesson Summary

The output of an **absolute value function** is the distance of its input from a given value. The equation of an absolute value function is defined using absolute value symbols, and its graph forms the shape of a V. We can write absolute value functions in the form $f(x) = |x - h|$, where $f(x)$ gives the distance of any input, x, from h. Let's look at an example.

Mr. DeAndre asked his students to guess a mystery number and gave each student a score. Each person's score was how far away their guess was from his mystery number, 30.



Here is the graph of the function $f(x) = |x - 30|$, which gives the score for each guess, x.

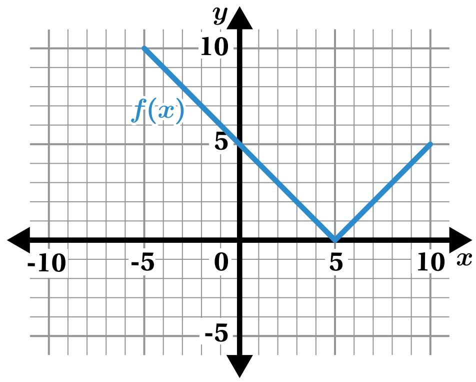
We can use the equation to determine the value of $f(25)$ and interpret its meaning.

$$\begin{aligned} f(25) &= |25 - 30| \\ &= |\text{-}5| \\ &= 5 \end{aligned}$$

This means a student who guessed 25 was 5 away from the mystery number.

Name: .... Date: .... Period: ....

Problems 1–3: Use the graph of $f(x)$ to determine each value.

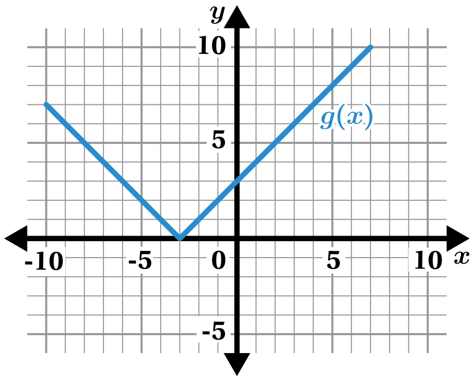


1. $f(0) =$ ....

2. $f(8) =$ ....

3. $f(5) =$ ....

4. Which equation represents the graph of $g(x)$? Circle your choice.



A. $g(x) = |x| - 3$

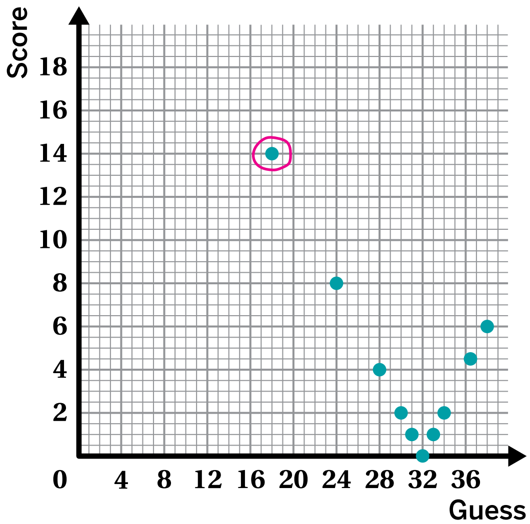
B. $g(x) = |x - 3|$

C. $g(x) = |x| + 3$

D. $g(x) = |x + 3|$

Problems 5–7: Ricardo's teacher challenged his class to guess how many marbles were in a jar. The graph shows each guess, x, and score, $m(x)$.

Each student was given a score equal to how far away their guess was from the actual number of marbles in the jar.



5. How many marbles are in the jar?

6. Circle the point that represents the furthest guess from the actual number of marbles.

Lesson Practice
3.11

Name: ________ Date: ________ Period: ________

Test Practice

7. Ricardo writes the equation $m(25) = 7$.

What does his equation mean?

A. Ricardo's equation shows that 25 is the correct number of marbles in the jar.

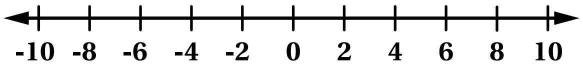
B. Ricardo's equation means a guess of 25 would give a score of 7 because it is 7 numbers away from the correct number of marbles in the jar.

C. Ricardo's equation means the correct number of marbles in the jar is 7.

D. Ricardo's equation shows that 25 and 7 are unrelated numbers.

Spiral Review

8. Select all of the true statements. Use the number line if it helps with your thinking.



☐ **A.** $-4 > -2$

☐ **B.** The distance from -2 to 2 is equal to the distance from 6 to 10.

☐ **C.** $-4 \geq -6$

☐ **D.** 8 is the only number 2 units away from 6.

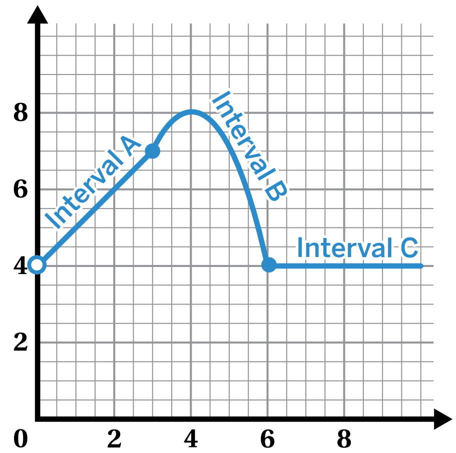
☐ **E.** $|3| = |-3|$

Problems 9–11: Match each domain to an interval of the piecewise function on the graph.

9. $3 \leq x \leq 6$ ________

10. $x \geq 6$ ________

11. $0 < x \leq 3$ ________



12. The California Department of Fish and Wildlife estimated there were 460,420 deer in the state in 2021. They estimated that the deer population in 2018 was 470,000.

Calculate the average rate of change during this time interval and explain what it tells us about the deer population.

Name: ______________ Date: ________ Period: ________

MA.912.AR.4.3, MA.912.F.1.2, MTR.4.1, MTR.5.1

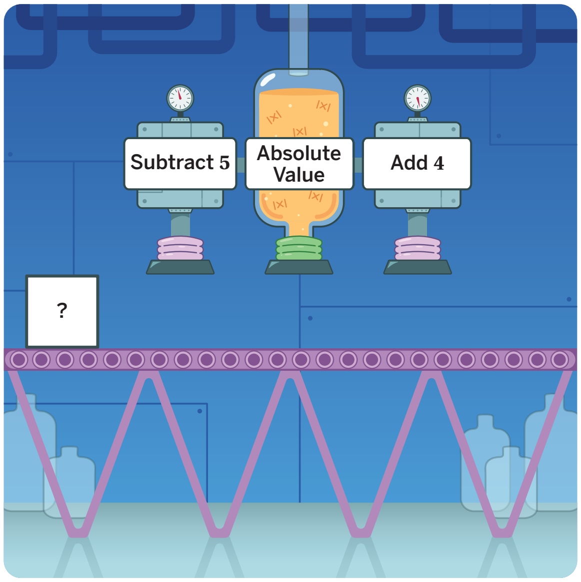
Absolute Value Machines

Let's graph absolute value functions.

Warm-Up

1. Here is a machine for $f(x) = |x - 5| + 4$.

What numbers will come out of the machine if we enter $-2, 1, 3, 8$?



x	$x - 5$	$\lvert x - 5 \rvert$	$\lvert x - 5 \rvert + 4$

Discuss: What do you notice and wonder?

Activity 1

Name: ________ Date: ________ Period: ________

Features of Absolute Value Functions

2. Kiri tried the numbers in the table.

She says: *The minimum value the machine can make is* 4.

Do you agree? Circle one.

Yes No Not enough information

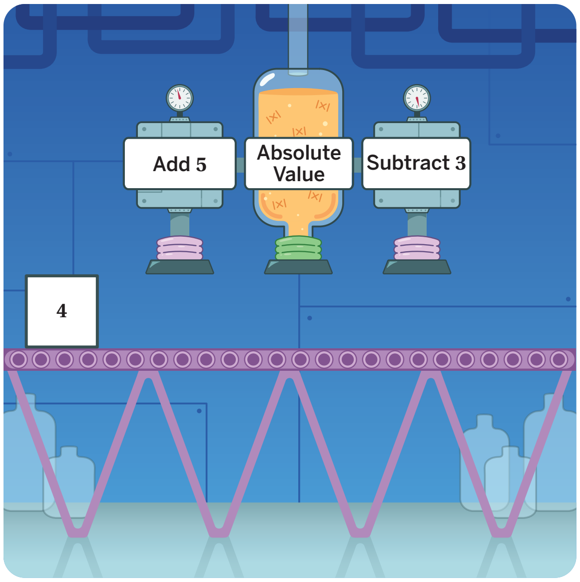
Explain your thinking.

x	$x-5$	$\lvert x-5\rvert$	$\lvert x+5\rvert+4$
-1	-6	6	10
2	-3	3	7
5	0	0	4
6	1	1	5

3. Here is a machine for $g(x) = \lvert x+5\rvert - 3$.

What number will come out of the machine if we enter 4, -1, and -6?

Use the table if it helps with your thinking.



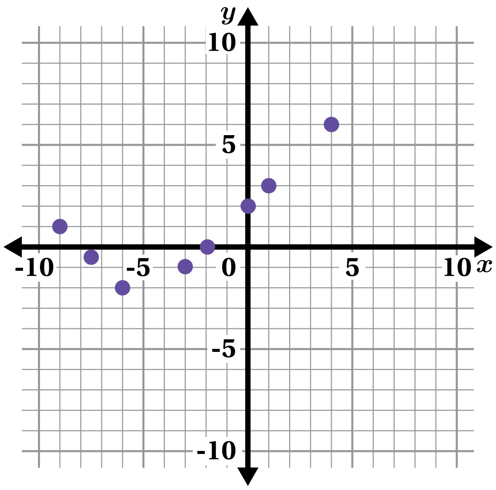
	x	$x+5$	$\lvert x+5\rvert$	$\lvert x+5\rvert-3$
a	4			
b	-1			
c	-6			

Activity 1

Name: ________ Date: ________ Period: ________

Features of Absolute Value Functions (continued)

4. Here are some points on the graph of $g(x) = |x + 5| - 3$.



a Draw a sketch that shows what all the points look like.

b Describe your sketch using some of these terms:

positive	maximum	increasing	domain
negative	minimum	decreasing	range
symmetry		intercepts, vertex, end behavior	

5. Here are descriptions of $g(x) = |x + 5| - 3$ from other students.

Select *all* the descriptions that are true.

- ☐ **A.** The domain is all numbers.
- ☐ **B.** The minimum is at (-6, -2).
- ☐ **C.** The range is $g(x) \geq -3$.
- ☐ **D.** $g(x)$ is increasing when $x >$ -6.
- ☐ **E.** The minimum is at (-5, -3).

Activity 2

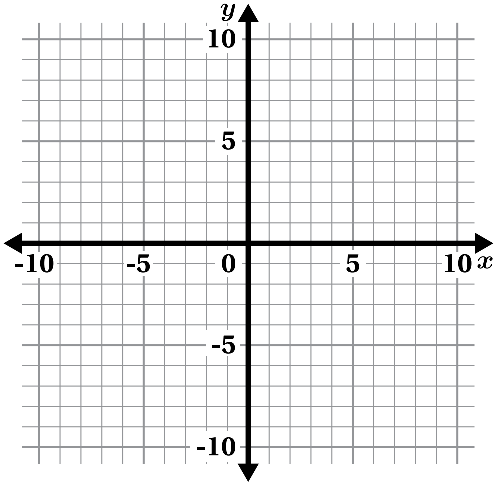
Name: ____ Date: ____ Period: ____

Graphing Absolute Value Functions

6. Here is a new function: $f(x) = |x - 6| + 2$.

Complete the table and then plot the graph of $f(x)$.

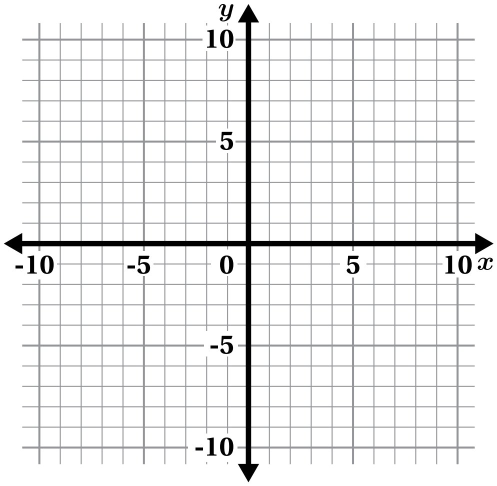
x	$f(x)$
9	
2	
0	
6	
-1	



7. Draw a graph of $j(x) = |x + 3| + 1$.

Use the table if it helps with your thinking.

x	$j(x)$



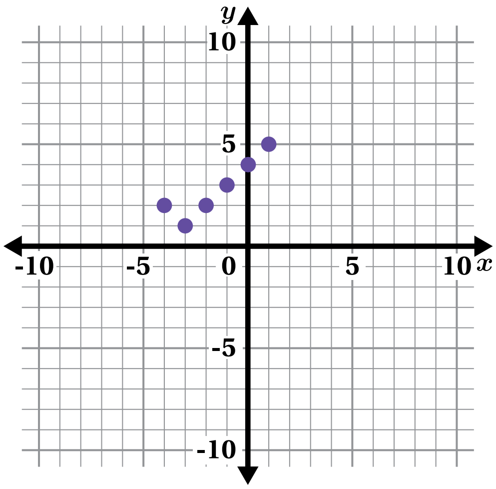
Name: .... Date: .... Period: ....

Graphing Absolute Value Functions (continued)

8. Here are the points that Tiana plotted for $j(x) = |x + 3| + 1$.

Tiana says: *I can use symmetry to plot more points on the graph.*

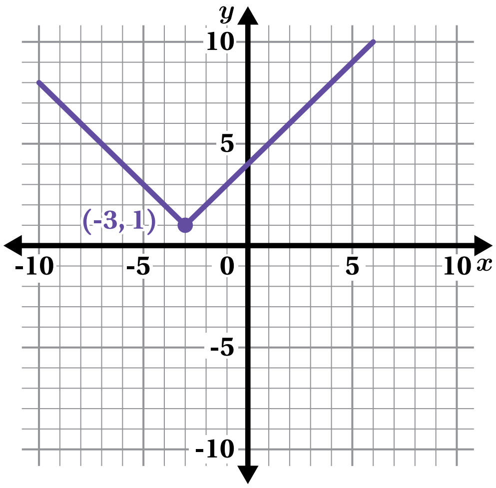
Show or describe what you think this means.



9. Here is the graph of $j(x) = |x + 3| + 1$.

The minimum value is shown.

How can you see the minimum value in the equation?

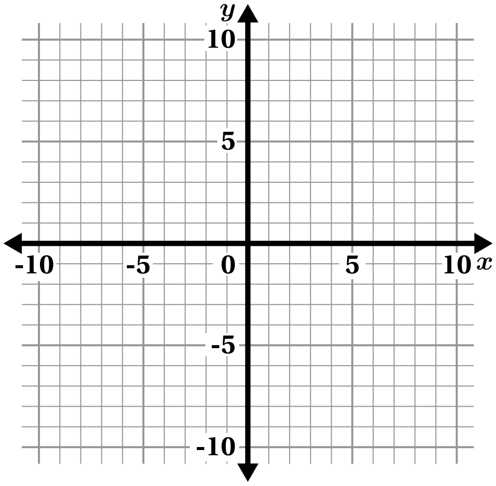


Name: ______________ Date: ______ Period: ______

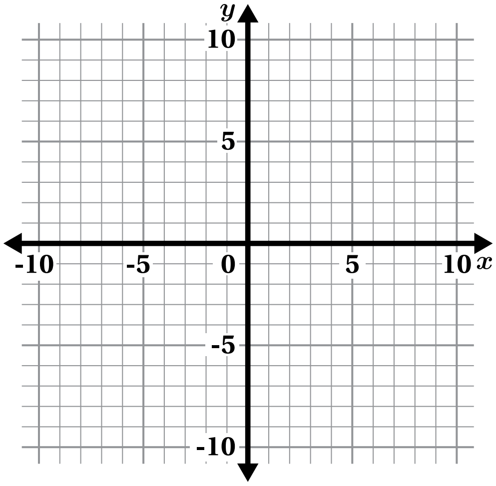
Repeated Challenges

10. Draw the graph of each function.

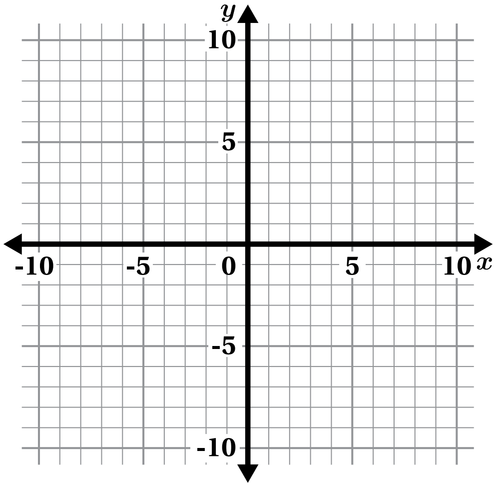
a $f(x) = |x - 4| + 3$



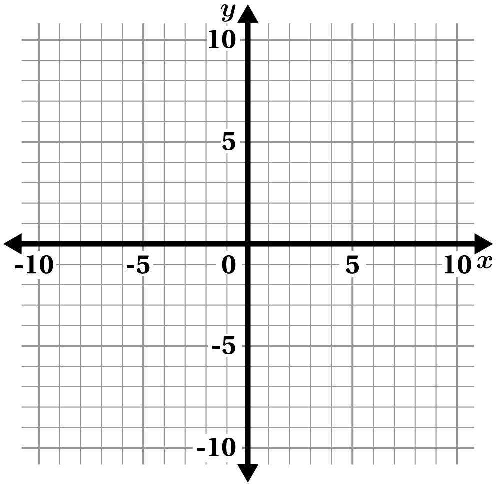
b $f(x) = |x + 2| + 1$



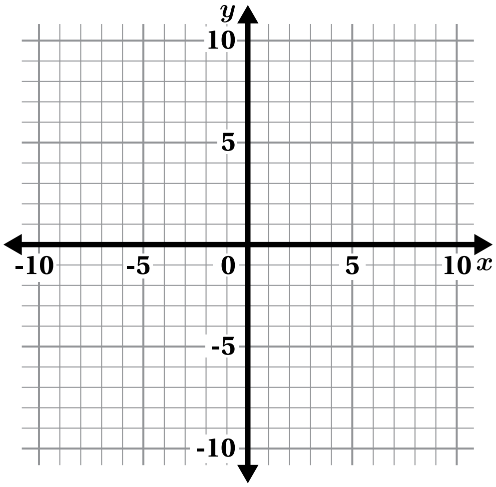
c $f(x) = |x - 1| + 7$



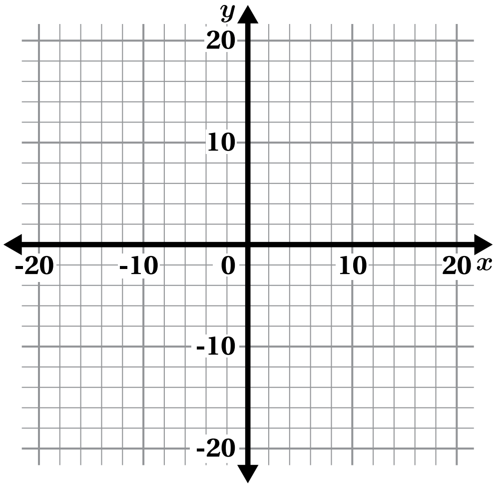
d $f(x) = |x - 5| - 4$



e $f(x) = -4|x + 4| + 3$



f $f(x) = 2|x + 6| - 1$



Synthesis

11. What can you know about the graph of an absolute value function by looking at its table or equation?

Use the example if it helps with your thinking.

$f(x) = |x - 4| + 3$

x	$f(x)$
-2	9
0	7
2	5
4	3
6	5

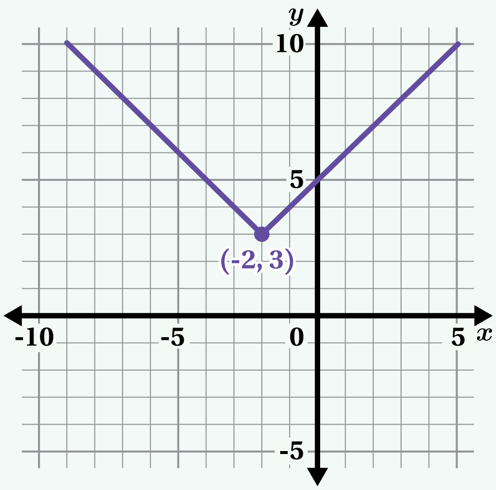
Lesson Practice 3.12

Lesson Summary

You can determine key features of the graph of an absolute value function by analyzing its table or equation, which are both helpful in sketching its graph.

Here are the graph and table for the absolute value function $f(x) = |x + 2| + 3$.

x	$f(x) = \lvert x + 2 \rvert + 3$
-4	5
-2	3
0	5
2	7



Domain: $\{x | \ x \in \mathbb{R}\}$

Range: $\{y | \ y \geq 3\}$

x-ntercepts: none

y-ntercept: $(0, 5)$

Vertex: (-2, 3)

Intervals of Increase: $x \geq$ -2

Intervals of Decrease: $x <$ -2

Symmetry: about the line $x =$ -2

End Behavior: ∞

Positive Intervals: -$\infty < x < \infty$

Negative Intervals: none

Lesson Practice
3.12

Name: ______ Date: ______ Period: ______

Problems 1–3: Write each expression as a single integer.

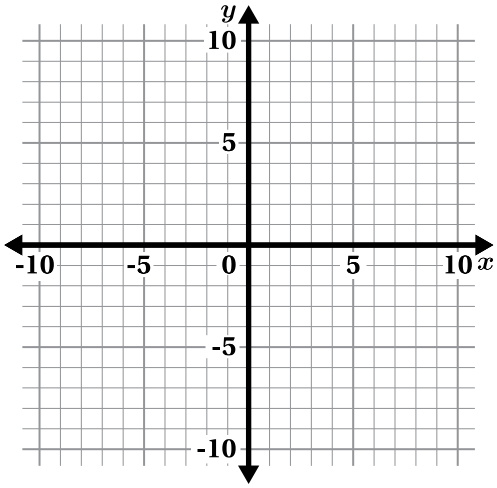
1. $|\text{-}4|$

2. $|7| - 2$

3. $|\text{-}8| + 1$

4. Graph $g(x) = |x + 1| + 4$. Use the table if it helps with your thinking.

x	$g(x)$
-3	
-1	
0	



Problems 5–9: The function $g(x) = 3|x - 2| - 4$ is represented by the graph.

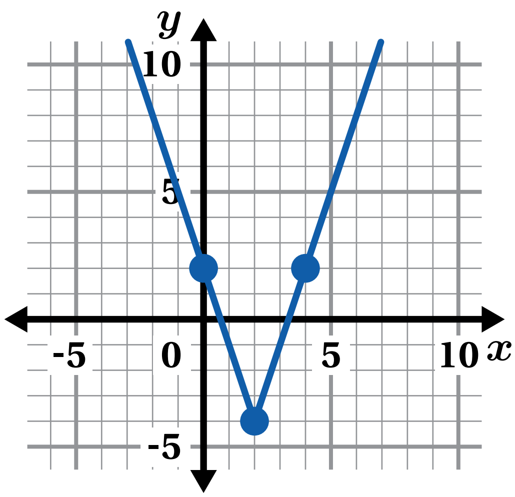
5. Identify the domain.

6. Identify the range.

7. Identify the minimum value/vertex.

8. Identify the increasing interval.

9. Identify the decreasing interval.



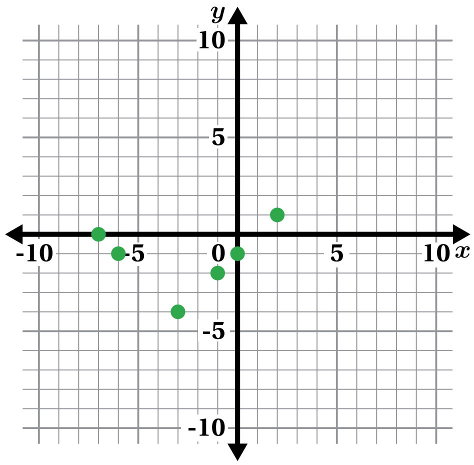
Lesson Practice
3.12

Name: ______ Date: ______ Period: ______

10. Here are some points on the graph of $h(x) = |x + 3| - 4$.

- Sketch a graph of $h(x)$.
- Describe the graph using some of these terms:

positive	maximum	increasing	domain
negative	minimum	decreasing	range
symmetry	intercepts, vertex, end behavior		



Test Practice

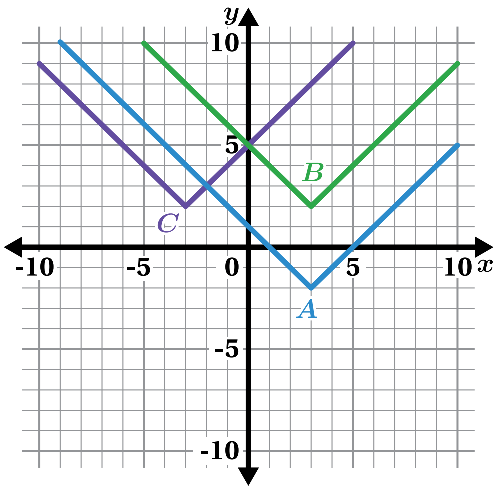
11. Choose the function that matches graph A.

A. $f(x) = |x + 3| + 2$

B. $g(x) = |x - 3| - 2$

C. $h(x) = |x - 3| + 2$

D. $k(x) = |x + 3| - 2$



Spiral Review

Problems 12–14: For each square root, write which two consecutive whole numbers the value is between.

12. $\sqrt{14}$ **13.** $\sqrt{60}$ **14.** $\sqrt{88}$

Career Connection

From Pole to Pole – the same or different?

Both the North and South Poles are very cold because they do not get as much direct sunlight as the rest of Earth. But the South Pole averages much colder temperatures! Why do you think that might be?

Climate scientists or *climatologists* study the long-term pattern of weather conditions for a particular area or areas. They use climate models to analyze large data sets which can be used to help make future predictions.

Mean Temperature

	North Pole	South Pole
Summer	32° F	-18° F
Winter	-40° F	-76° F

Source: NASA

B.E.S.T. Mathematics Benchmark Connection

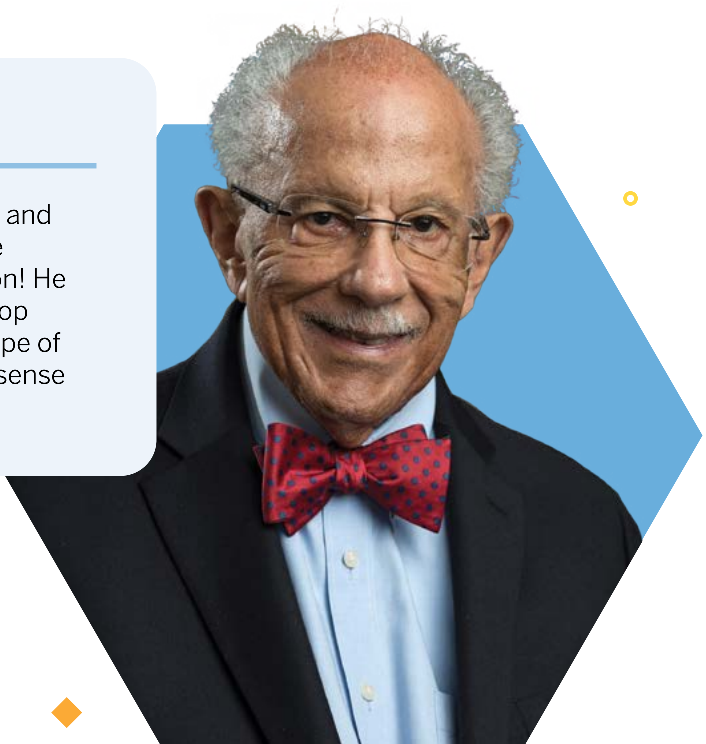
Many scientists rely on algebra concepts in their work. For example, they calculate and interpret the average rate of change (MA.912.F.1.3) when they analyze an equation, such as models that represent data collected over short and long periods of time. They may also compare key features of functions (MA.912.F.1.5) to study temperatures at different locations at the same time of year or at different times of year.

Mathematical Thinking and Reasoning Connection

Scientists who collect data to develop models use thinking and reasoning skills like the ones you use for your math work! For example, they discuss their work in multiple ways (MTR.2.1). When they study data collected for the same locations at different times, they learn to complete the tasks of organizing and displaying the data with mathematical fluency (MTR.3.1).

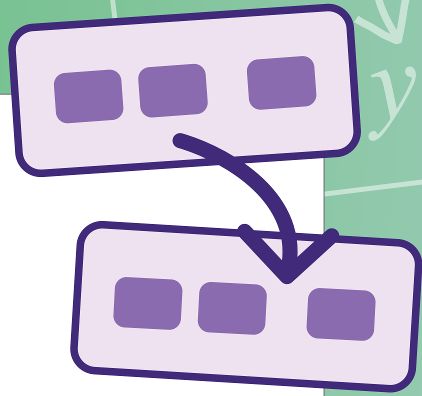
Meet Warren Washington

Warren M Washington developed his interest in weather and science while working weekends during his early college years operating a weather radar on a mountain in Oregon! He applied his interest in physics and meteorology to develop some of the first atmospheric computer models. This type of modeling combines mathematics and physics to make sense of the weather.



Unit 4

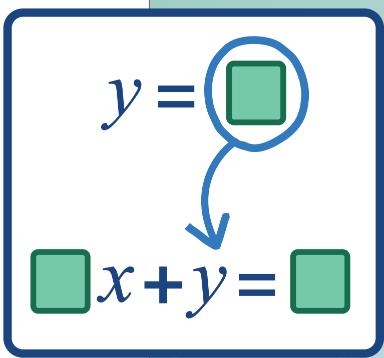
Systems of Linear Equations and Inequalities

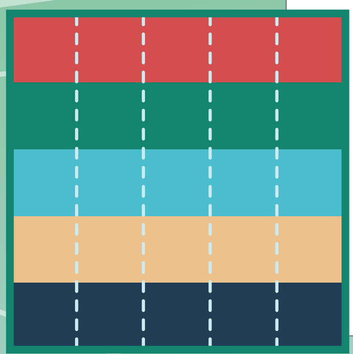


Have you ever been in a situation where you had to accomplish two goals at once? Systems of equations and inequalities are helpful for finding a set of values that meets both constraints at the same time. In this unit, you'll explore how to solve systems of equations and inequalities using different strategies. You will also analyze the structure of the equations in a system to strategize about which solving method you choose.

Essential Questions

- How can you solve systems of equations and inequalities symbolically and graphically?
- How can you use the structures of the equations, available tools, and knowledge of your personal mathematical preferences to select a solving method strategically?
- How can constraints be represented using systems of equations or inequalities?

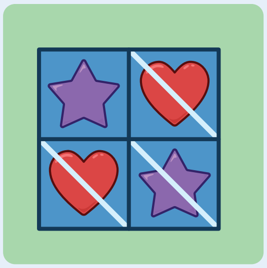




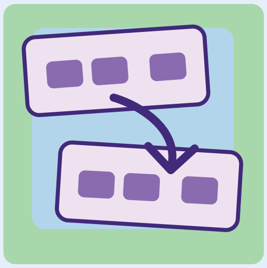
Sub-unit

1

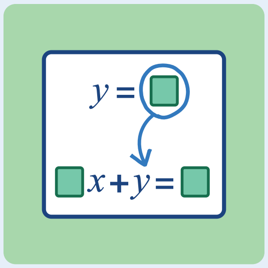
Systems of Equations



Lesson 1
Eliminating Shapes



Lesson 2
Process of Elimination



Lesson 3
Solution by Substitution

Lesson 4
Lizard Lines

Lesson 5
City Development

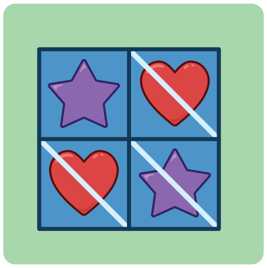
Lesson 6
Bus Systems

Unit 4
Lesson
1

Name: ________ Date: ________ Period: ________

MA.912.AR.9.1 MTR.2.1, MTR.3.1, MTR.4.1, MTR.5.1

Eliminating Shapes



Let's solve systems of equations by adding or subtracting the equations to eliminate a variable.

Warm-Up

Determine an expression that makes each equation true for any value of x and y.

1. $3x + \underline{\hspace{3cm}} = 0$

2. $3x - \underline{\hspace{3cm}} = 0$

3. $(3x + y) - (\underline{\hspace{3cm}}) = 0$

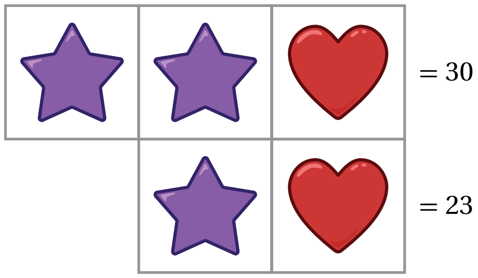
4. $(3x + y) + (\underline{\hspace{3cm}}) = 0$

Activity 1

Name: ______ Date: ______ Period: ______

Adding and Subtracting Equations

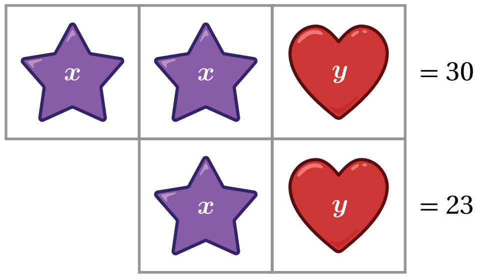
5. Here is a shape puzzle. The sum of each row is shown.



Determine the solution for this puzzle.

Shape	Value
Heart	
Star	

6. This shape puzzle could be written as a **system of equations**, where x is the value of each star and y is the value of each heart.



$$2x + y = 30$$

$$x + y = 23$$

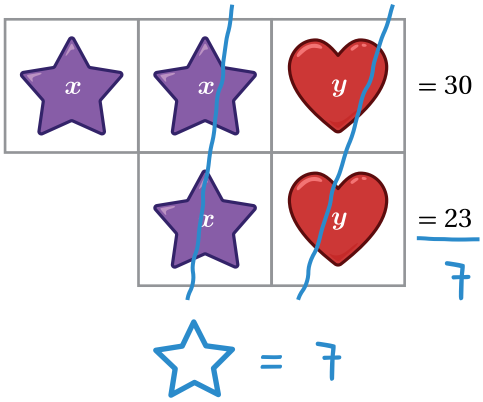
Explain how this system of equations is like the puzzle.

Name: .... Date: .... Period: ....

Adding and Subtracting Equations (continued)

7. Here is how Ebony and Nia each determined the value of a star.

Ebony



Nia

$$\begin{array}{r} 2x + y = 30 \\ -(x + y = 23) \\ \hline x + 0 = 7 \\ x = 7 \end{array}$$

Discuss how you see subtraction in each strategy.

Name: .... Date: .... Period: ....

Elimination

8. Here is a new system of equations.

$$x + 2y = 10$$
$$x + y = 7$$

Determine the values of x and y that make both equations true (the *solution to the system*).

Draw a puzzle if it helps with your thinking.

$x =$ ...., $y =$ ....

9. Ebony and Nia want to eliminate the y's in this system of equations.

$$-2x + y = 9$$
$$8x - y = 3$$

- Ebony says to *add* the equations.
- Nia says to *subtract* the equations.

Whose strategy will eliminate the y's ?

A. Ebony's B. Nia C. Both D. Neither

Name: .... Date: .... Period: ....

Elimination (continued)

10. Determine the **solution to the system of equations** from the previous problem:

$-2x + y = 9$

$8x - y = 3$

$x =$ ...., $y =$ ....

11. The strategy of adding or subtracting equations to eliminate a variable is called **elimination**.

Nia says elimination works because it's like adding or subtracting the same value from each side of an equation.

Explain what Nia is saying in your own words.

Nia

$$\begin{array}{r} 2x + y = 30 \\ -(x + y = 23) \\ \hline x + 0 = 7 \\ x = 7 \end{array}$$

12. Determine the solution to this system of equations:

$-7x - 5y = 15$

$7x + 3y = 12$

$x =$ ...., $y =$ ....

Name: .... Date: .... Period: ....

Elimination Repeated Challenges

13. Choose four of the systems of equations below and solve them using elimination.

A. $5x + 3y = 21$
$2x + 3y = 12$

B. $8x + 5y = 12$
$8x + 3y = 4$

C. $2x + 3y = 14$
$-2x + 7y = 6$

D. $9x + 3y = -3$
$4x - 3y = -23$

E. $2x + 3y = 4$
$2x + 7y = -12$

F. $y = 4x - 1$
$y = 6x - 7$

Synthesis

14. How can you determine whether to add or subtract equations in order to eliminate a variable?

$$\begin{aligned} 2x + y &= 30 \\ x + y &= 23 \end{aligned}$$

$$\begin{aligned} x + 2y &= 10 \\ x + y &= 7 \end{aligned}$$

$$\begin{aligned} -2x + y &= 9 \\ 8x - y &= 3 \end{aligned}$$

$$\begin{aligned} -7x - 5y &= 15 \\ 7x + 3y &= 12 \end{aligned}$$

Lesson Practice 4.01

Lesson Summary

A **system of equations** is two or more equations that represent the same constraints using the same variables. The **solution to a system of equations** is the (x, y) ordered pair(s) that makes every equation in the system true. There are many solutions to a linear equation in two variables, but there might be only one (or no) solution to a system of linear equations in two variables.

There are many strategies for determining the ordered pair that makes both equations in a system true. One strategy is called **elimination**, where you add or subtract the equations to produce a new equation with one variable. Let's look at some examples.

If the equations in the system share the same *coefficient* with opposite signs on the same variable, you can eliminate a variable by adding. You can solve this system by adding to eliminate the y-variable.

$$\begin{aligned} -2x + y &= 9 \\ +(8x - y &= 3) \\ \hline 6x + 0 &= 12 \\ x &= 2 \end{aligned}$$

$$\begin{aligned} -2(2) + y &= 9 \\ y &= 13 \end{aligned}$$

If the equations in the system share the same *coefficient* with the same signs on the same variable, you can eliminate a variable by subtracting. You can solve this system by subtracting to eliminate the y-variable.

$$\begin{aligned} x + 2y &= 30 \\ -(x + y &= 23) \\ \hline y &= 7 \end{aligned}$$

$$\begin{aligned} x + (7) &= 23 \\ x &= 16 \end{aligned}$$

Lesson Practice
4.01

Name: ________ Date: ________ Period: ________

Problems 1–2: Mateo made a mistake as he started to solve this system of equations.

1. Describe one thing Mateo did *correctly*.

$$2x + y = 19$$
$$-(x - y = 11)$$
$$x + 0 = 8$$
$$x = 8$$

2. Describe one thing Mateo did incorrectly.

Problems 3–4: Determine the solution for each system of equations.

3. $3x + 4y = 6$
$3x + 2y = 18$

$x =$ ________ $y =$ ________

4. $5x + 6y = 26$
$-5x + 2y = -18$

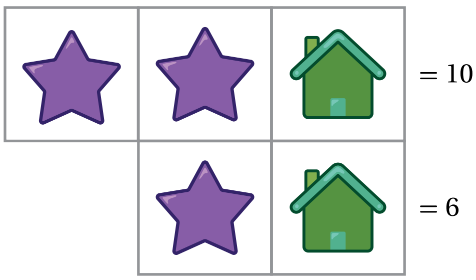
$x =$ ________ $y =$ ________

Test Practice

5. Solve this system of equations. Use the shape puzzle if it helps with your thinking.

$2x + y = 10$

$x + y = 6$



$x =$ ________ $y =$ ________

Name: ______________ Date: ________ Period: ________

Spiral Review

Problems 6–8: Write an equivalent expression by combining like terms.

6. $5a + 3b - 2a$

7. $3(c - 2) + 2c$

8. $5d - 2(7d + 3g)$

9. The function $f(t)$ models a hiker's elevation above or below sea level, in meters, t hours after noon. Circle the equation that represents this statement:
At 7 PM, the hiker was 3 meters below sea level.

A. $f(7) = 3$

B. $f(19) = \text{-}3$

C. $f(7) = \text{-}3$

D. $f(\text{-}3) = 7$

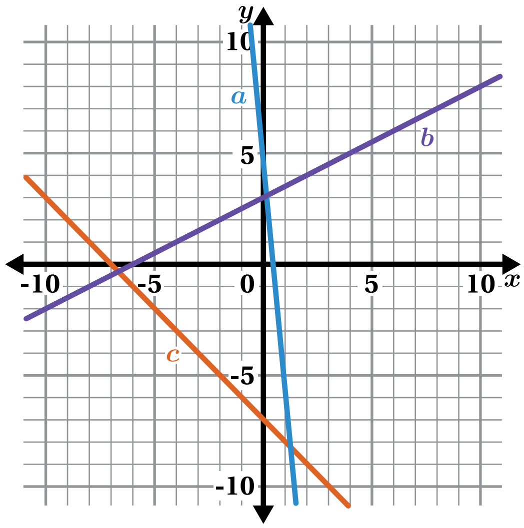
10. Which line represents $x - 2y = \text{-}6$?

A. Line a

B. Line b

C. Line c

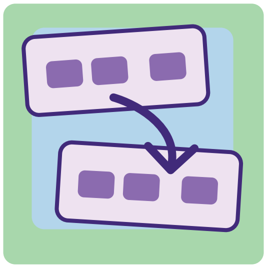
Explain your thinking.



Name: ________________ Date: ________ Period: ________

MA.912.AR.9.1,MTR.2.1, MTR.3.1, MTR.6.1, MTR.7.1

Process of Elimination



Let’s create equivalent equations to eliminate a variable.

Warm-Up

Here are some linear equations.

Equation A	Equation B	Equation C
$x + 2y = 11$	$4x + y = 2$	$5x + 10y = 55$
Equation D	**Equation E**	**Equation F**
$y = 2 - 4x$	$2x + \frac{1}{2}y = 1$	$x + 2y - 11 = 0$

1. Sort the equivalent linear equations into two groups. Record your groupings.

Group 1	Group 2

2. Choose one equation. Write a new *equivalent equation* that would belong in that group.

________________ *is equivalent to* ________________ *because . . .*

Activity
1

Name: ________ Date: ________ Period: ________

First Steps of Elimination

Caasi is solving this system of equations, but she got stuck.

Here's how Caasi started.

Caasi

$$\begin{array}{r} x + 2y = 11 \\ -(4x + y = 2) \\ \hline -3x + y = 9 \end{array}$$

3. **Discuss:**

- What was Caasi's first step?
- Why do you think she got stuck?

Diego is trying to solve this system of equations. Here's how Diego started.

Diego

$$\begin{array}{r} x + 2y = 11 \\ 4x + y = 2 \end{array} \longrightarrow \begin{array}{r} x + 2y = 11 \\ 8x + 2y = 4 \end{array}$$

$$\begin{array}{r} x + 2y = 11 \\ -(8x + 2y = 4) \\ \hline -7x = 7 \\ x = -1 \end{array}$$

4. What was Diego's first step?

5. Diego got stuck using his method after solving for $x = -1$. What do you think he should do next?

6. Ariel thinks that Diego can solve this system:

$$\begin{array}{r} x = -1 \\ x + 2y = 11 \end{array}$$

Discuss: Do you think this system will have the same solution as Diego's original system?

7. Finish Diego's work to solve the system.

$x =$ ________ and $y =$ ________

Activity
2

Name: .... Date: .... Period: ....

More Than One Way?

8. Caasi and Kwabena started solving this system in different ways.

$$\begin{aligned} 4x - y &= 5 \\ x + 2y &= 8 \end{aligned}$$

With a partner, solve the system both ways. Compare your solutions.

Caasi: *Multiply the first equation by* 2.

Kwabena: *Multiply the second equation by* -4.

9. **Discuss:**

- What is similar about Caasi's and Kwabena's methods for solving the linear system?
- What is different about their methods?

10. Constanza is trying to solve this system of equations.

$$\begin{aligned} -7x + 5y &= -19 \\ -3x + 3y &= -9 \end{aligned}$$

Review her process. What do you notice about the steps taken to solve the system?

$3(-7x + 5y) = 3(-19)$
$-5(-3x + 3y) = -5(-9)$

$-21x + 15y = -57$
$15x - 15y = 45$

$-6x = -12$
$x = 2$

$-3(2) + 3y = -9$
$-6 + 3y = -9$
$3y = -3$
$y = -1$

$(2,1)$

Activity 3

Name: ________ Date: ________ Period: ________

Prepare to Be Eliminated

You will use a set of cards for this activity.

11. Here are the instructions for each round.

Select a card from A–F.

- Discuss two possible first steps you could take to solve the system.
- Choose a different first step from your partner. Solve your system individually.
- Compare your solutions and support each other to make adjustments as needed.

Round 1, Card ________

Equation 1: ________

Equation 2: ________

Solution: $x =$ ________ and $y =$ ________

Round 2, Card ________

Equation 1: ________

Equation 2: ________

Solution: $x =$ ________ and $y =$ ________

Name: ________ Date: ________ Period: ________

Prepare to Be Eliminated (continued)

Round 3, Card ________

Equation 1: ________

Equation 2: ________

Solution: $x =$ ________ and $y =$ ________

Round 4, Card ________

Equation 1: ________

Equation 2: ________

Solution: $x =$ ________ and $y =$ ________

You're invited to explore more.

12. The solution to this system of equations is $x = 5$ and $y = 2$.

$$Ax - By = 24$$
$$Ax + By = 16$$

What are possible values for A and B?

$A =$ ________ and $B =$ ________

Synthesis

13. Describe how writing equivalent equations can help you solve systems of equations.

Use this system if it helps you explain your thinking.

$$x + 3y = 6$$
$$2x + y = 7$$

Lesson Practice 4.02

Lesson Summary

It can be helpful to write *equivalent equations* when using elimination to solve systems of equations. You can create equivalent equations by multiplying each term of the first or second equation by a number. Sometimes you may even need to multiply both equations before you can add or subtract them to eliminate a variable. Your goal is to end up with a system of equations where one variable has the same or *opposite* coefficients so you can add or subtract them to eliminate a variable.

Here is a system of equations:

$$9x - 4y = 2$$
$$3x + y = 10$$

You can multiply the second equation by -3 to eliminate the x-variables.

$$9x - 4y = 2$$
$$\text{-3}\,(3x + y = 10)$$

$$\begin{array}{rr} & 9x - 4y = 2 \\ + & \text{-9}x - 3y = \text{-30} \\ \hline & 0 - 7y = \text{-28} \\ & \boxed{y = 4} \end{array}$$

$$3x + (4) = 10$$
$$3x = 6$$
$$\boxed{x = 2}$$

Or you can multiply the second equation by 4 to eliminate the y-variables.

$$9x - 4y = 2$$
$$4\,(3x + y = 10)$$

$$\begin{array}{rr} & 9x - 4y = 2 \\ + & 12x + 4y = 40 \\ \hline & 21x + 0 = 42 \\ & \boxed{x = 2} \end{array}$$

$$9(2) - 4y = 2$$
$$18 - 4y = 2$$
$$\text{-4}y = \text{-16}$$
$$\boxed{y = 4}$$

Lesson Practice
4.02

Name: ____________ Date: ________ Period: ________

1. Select *all* expressions that are equivalent to $6x + 8y = 10$.

- ☐ **A.** $12x + 16y = 20$
- ☐ **B.** $2x + 6y = 10$
- ☐ **C.** $3x + 4y = 5$
- ☐ **D.** $1.5x + 2y = 2.5$
- ☐ **E.** $4x + 5y = 4$

2. Arnav and Omari are solving this system of equations. They disagree about what the first step should be to eliminate a variable.

$$4x + 2y = 62$$
$$-8x - y = 59$$

Arnav's strategy: Multiply $4x + 2y = 62$ by 2

Omari's strategy: Multiply $-8x - y = 59$ by 2

Whose strategy will eliminate a variable once the equations are added? Circle your choice.

A. Arnav's **B.** Omari's **C.** Both **D.** Neither

Explain your thinking.

Problems 3–4: Determine the solution to each system of equations.

3. $2x - 4y = 10$
$x + 5y = 40$

$x =$ ________ $y =$ ________

4. $5x + 2y = 20$
$2x - 3y = -11$

$x =$ ________ $y =$ ________

Lesson Practice
4.02

Name: ______________ Date: ________ Period: ________

Test Practice

5. Which ordered pair represents the solution to the system of linear equations?

$$9x + 8y = -5$$
$$4x + 10y = -28$$

A. (-3, -4) B. (-4, 3) C. (3, -4) D. (4, -3)

Spiral Review

Problems 6–8: Solve each equation.

6. $x - 2 = 5$

7. $3(x - 2) = 3 \cdot 5$

8. $8x - 16 = 40$

9. Solve the equation $3x - 9y = 72$ for y.

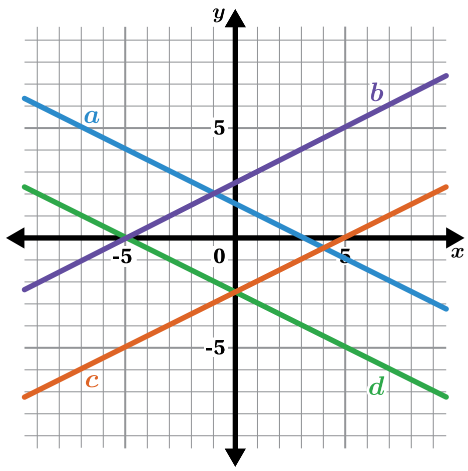
10. Which line represents $5x + 10y = 15$?

A. Line a

B. Line b

C. Line c

D. Line d

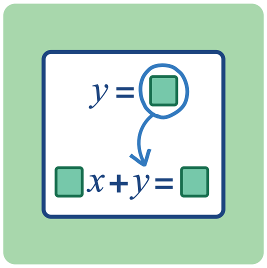


Unit 4
Lesson
3

Name: ______________ Date: ________ Period: ________

MA.912.AR.9.1, MTR.2.1, MTR.3.1, MTR.6.1, MTR.7.1

Solution by Substitution

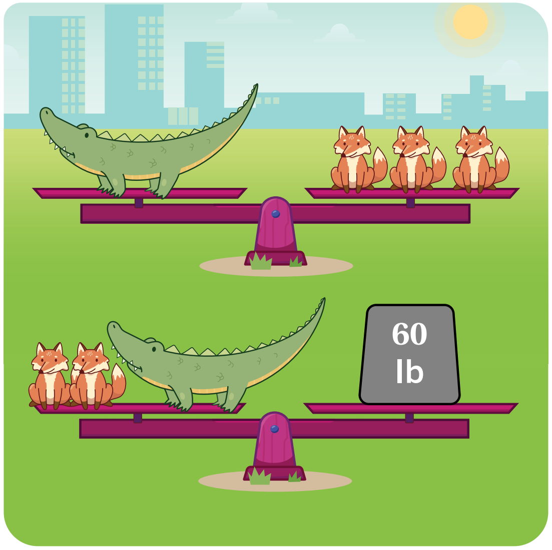


Let's use substitution to solve systems of equations.

Warm-Up

1. Here are two scales showing the weights of foxes and alligators.

What do you notice? What do you wonder?



2. Let's look at the scale. Use the information from the scales to find the weights of each animal.

Activity 1

Name: .... Date: .... Period: ....

Introducing Substitution

3. Riya determined the weight of a fox by writing a system of equations and doing these steps.

Then she needed to solve a new system of equations.

Show or explain what Riya's first step might be as she solves the new system.

Riya

$y = 3x \qquad 2x + y = 60$

$2x + (3x) = 60$

$5x = 60$

$x = 12$

New

$y = 2x + 3 \qquad 4x + y = 15$

4. Here is the system from the previous problem:

$y = 2x + 3$

$4x + y = 15$

Determine the solution.

$x =$ .... , $y =$ ....

5. Riya's strategy is called solving by **substitution**.

Substitution is when a variable is replaced with an expression that is equal to it.

Show or explain the first step to solving the new system of equations with substitution.

$y = 3x \qquad 2x + y = 60$

$2x + (3x) = 60$

$y = 2x + 3 \qquad 4x + y = 15$

$4x + (2x + 3) = 15$

New

$5x - 2y = 15 \qquad y = 3x - 12$

Activity
1

Name: ________ Date: ________ Period: ________

Introducing Substitution (continued)

6. Natalia made a mistake as she solved the system of equations from the previous problem.

What did Natalia do well? What should she fix?

Natalia

$5x - 2y = 15 \qquad y = 3x - 12$

$5x - 2(3x - 12) = 15$

$5x - 6x - 24 = 15$

$-x = 39$

$x = -39$

7. Determine the solution to the previous problem.

$5x - 2y = 15$

$y = 3x - 12$

$x =$ ________ , $y =$ ________

Activity 2

Name: ________ Date: ________ Period: ________

Practicing Substitution

8. Here are three systems of equations.

Discuss:

- What would be your first step in solving each of these systems using substitution?
- Would you prefer to solve each system using substitution or elimination? Why?

A
$$-2x + y = 9$$
$$x = 14$$

B
$$x + 2y = 10$$
$$x + y = 7$$

C
$$y = 4x + 63$$
$$y = 7x + 15$$

9. Determine the solution to each system of equations.

$y = 7x + 12$

$y = -3x + 2$

$x =$ ________ , $y =$ ________

$2x + 2y = 8$

$x = 4 + 3y$

$x =$ ________ , $y =$ ________

$-2x + 4y = 9$

$y = x - 1$

$x =$ ________ , $y =$ ________

$3x - 2y = 14$

$x + 3y = 1$

$x =$ ________ , $y =$ ________

Synthesis

10. *Substitution* and *elimination* are two strategies for solving systems of equations.

How are these strategies alike? How are they different?

Use the examples if they help with your thinking.

A
$$-2x + y = 9$$
$$x = 14$$

B
$$x + 2y = 10$$
$$x + y = 7$$

C
$$y = 4x + 63$$
$$y = 7x + 15$$

Lesson Practice 4.03

Lesson Summary

One strategy you can use to solve a system of equations is **substitution**, where you replace a variable with an equivalent expression. Substitution is a useful strategy when one variable is already isolated in an equation.

Here are two examples of systems of equations where substitution may be a useful strategy.

In this system, both y-variables are already isolated. We can substitute the expression $-4x + 6$ in for y in the second equation.

$$y = -4x + 6$$
$$y = 3x - 15$$

$$y = -4x + 6 \qquad y = 3x - 15$$
$$-4x + 6 = 3x - 15$$
$$-7x = -21$$
$$\boxed{x = 3}$$
$$y = 3(3) - 15$$
$$\boxed{y = -6}$$

In this system of equations, y is already isolated, so we can substitute the expression $2x - 5$ in for y in the first equation.

$$-3x - 2y = 3$$
$$y = 2x - 5$$

$$y = 2x - 5 \qquad -3x - 2y = 3$$
$$-3x - 2(2x - 5) = 3$$
$$-3x - 4x + 10 = 3$$
$$-7x + 10 = 3$$
$$-7x = -7$$
$$\boxed{x = 1}$$
$$y = 2(1) - 5$$
$$\boxed{y = -3}$$

Lesson Practice
4.03

Name: ______________ Date: ________ Period: ________

Problems 1–2: Show or explain what your first step would be for solving each system of equations.

1. $4x - y = 20$
$x + y = 5$

2. $6x - 12y = 24$
$y = 2x - 1$

3. Determine the solution to this system of equations:

$7x - y = \text{-}3$

$y = x - 3$

Test Practice

Problems 4–5: Alma made a mistake as she started to solve this system of equations.

$y = \frac{1}{2}x - 1$

$4x - 2y = 11$

Alma

$$4x - 2\left(\frac{1}{2}x - 1\right) = 11$$
$$4x - x - 2 = 11$$
$$3x - 2 = 11$$
$$3x = 13$$
$$x = \frac{13}{3}$$

4. What is the error in Alma's work?

5. Determine the solution to this system of equations.

$x =$
$y =$

Lesson Practice
4.03

Name: ______________ Date: ________ Period: ________

Spiral Review

Problems 6–7: Kadeem made a mistake as he started to solve this system of equations.

$$5x - 4y = 6$$
$$5(x + y = 25)$$

Kadeem

$5x - 4y = 6$
$5x + 5y = 125$
$-1y = 131$
$y = -131$

6. Show or explain one thing Kadeem did correctly.

7. Show or explain Kadeem's mistake.

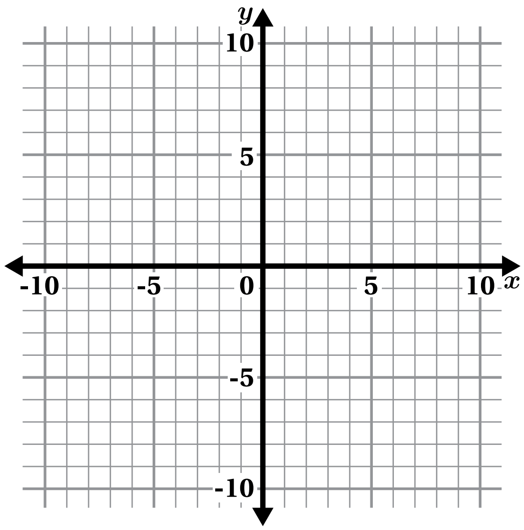
Problems 8–10: Solve each equation for the given variable.

8. Solve for k.
$2t + k = 6$

9. Solve for x.
$4x + 3y = 12$

10. Solve for y.
$4x + 3y = 12$

11. Graph the equation $4x - 6y = 24$.



Unit 4
Lesson
4

Name: ______ Date: ______ Period: ______

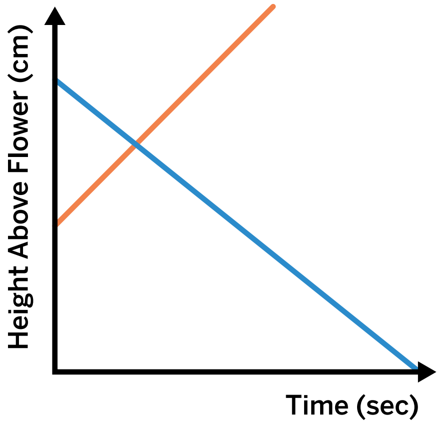
MA.912.AR.9.1, MTR.2.1, MTR.4.1, MTR.5.1

Lizard Lines

Let's explore systems of equations using graphs.

Warm-Up

1. Let's watch different pairs of lizards walk along a tree trunk together.



Discuss: What do you notice about the lizards and the graphs? What do you wonder?

Activity 1

Name: ____________ Date: ____________ Period: ____________

Making Connections

2. Here are equations for each lizard's height above the flower, y, as a function of time, x, in seconds:

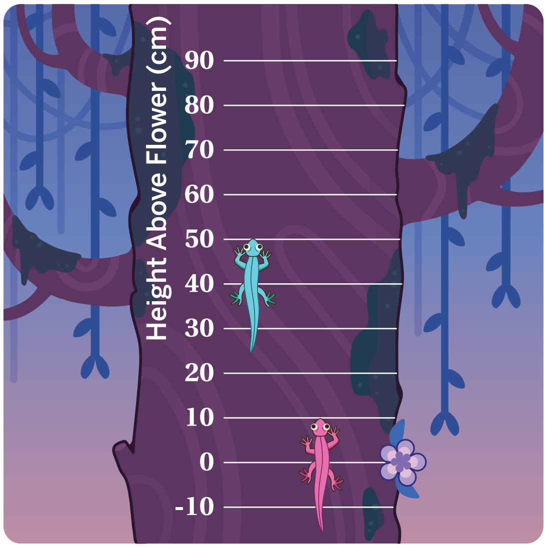
$$y = \text{-}5x + 50$$

$$y = 5x + 10$$

When and where will the lizards have the same position?

Time (sec), x: ____________

Height (cm), y: ____________



3. Here are Jin's and Nasir's strategies for figuring out when the lizards will be in the same position.

Jin

y = -5x + 50

y = 5x + 10

5x + 10 = -5x + 50

10x = 40

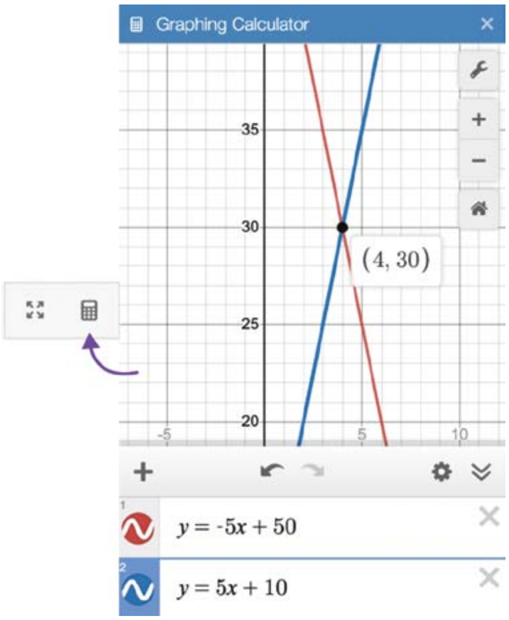
x = 4

y = 5x + 10

y = 5(4) + 10

y = 30

Nasir



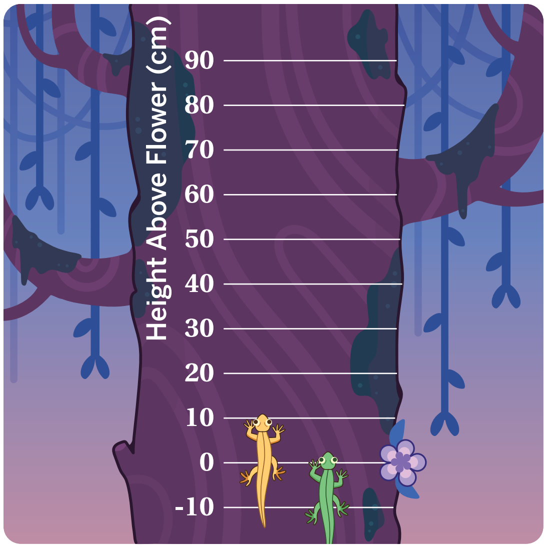
Discuss: Where do you see the solution in each strategy?

Activity 2

Name: ______ Date: ______ Period: ______

Will They Meet?

You will use a graphing calculator for this activity.



4. Here are equations for each lizard's height above the flower, y, as a function of time, x, in seconds:

$$y = -2x + 11$$

$$y = 4x + 2$$

When and where will the lizards have the same position?

Use a graphing calculator if it helps with your thinking.

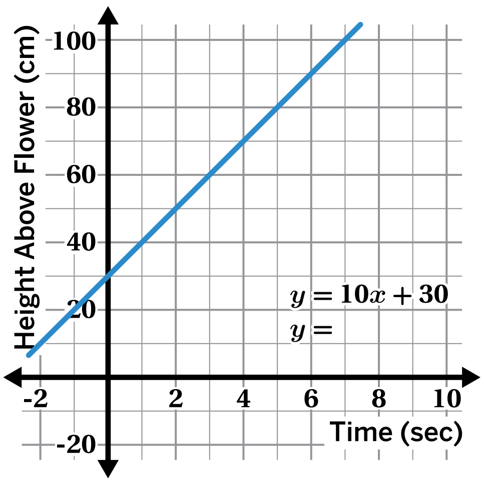
Time (sec), x: ______

Height (cm), y: ______

5. The blue line represents the graph of a blue lizard.

Create a line for a green lizard so that the lizards meet at exactly 5 seconds.

Try to make a line that none of your classmates will make.



6. Here are equations for two lizards' heights above the flower, y, as a function of time, x, in seconds.

$$y = 8x + 60$$

$$y = 8x + 35$$

Will these lizards meet? Explain your thinking.

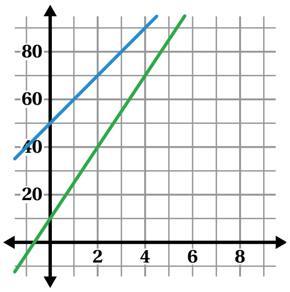
Activity 3

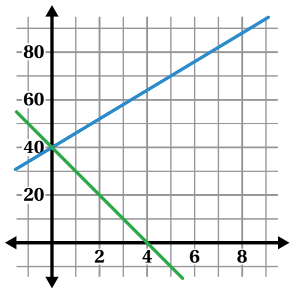
Name: ______ Date: ______ Period: ______

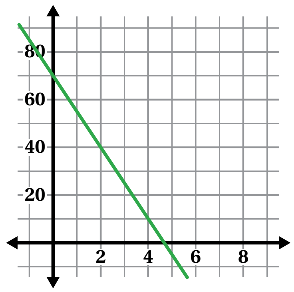
Graphing Systems

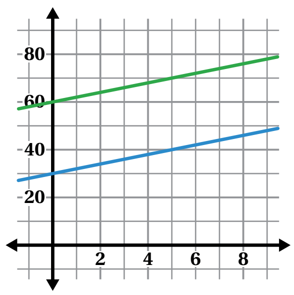
7. Here are some systems of equations.

$y = 10x + 50$ $y = 15x + 10$	$y = 6x + 40$ $y = -10x + 40$	$y = -15x + 70$ $y = -15x + 70$	$y = 2x + 60$ $y = 2x + 30$









Select each type of system that is possible to make.

- ☐ **A.** No solutions
- ☐ **B.** Exactly one solution
- ☐ **C.** Exactly two solutions
- ☐ **D.** Infinitely many solutions

8. For each system of equations, circle the number of solutions that it has. If there is one solution, what is the solution?

a $y = \frac{1}{2}x - 1$, $y = \frac{1}{2}x + 2$ — No solutions — One solution (______, ______) — Infinitely many solutions

b $y = x + 2$, $y = -3x - 2$ — No solutions — One solution (______, ______) — Infinitely many solutions

c $y = 2x + 6$, $y = 2(x + 3)$ — No solutions — One solution (______, ______) — Infinitely many solutions

d $y - 5x = -7$, $y = 5x$ — No solutions — One solution (______, ______) — Infinitely many solutions

e $y = 20x$, $20y = x$ — No solutions — One solution (______, ______) — Infinitely many solutions

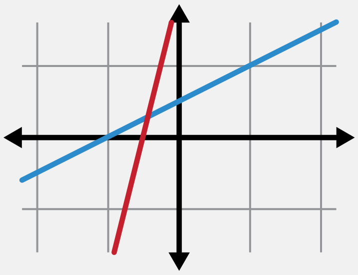
Name: ______ Date: ______ Period: ______

Graphing Systems (continued)

9. Group each system of equations based on the number of solutions it has.

A	B	C	D
$2x + 4y = 16$	$y = \frac{2}{3}x + 10$	$y = 2x + \frac{1}{4}$	$y = \frac{1}{2}x + 3$
$y = \frac{1}{2}x + 2$	$y = \frac{2}{3}x - 7$	$y = 4x + \frac{1}{4}$	$2y = x + 6$

E



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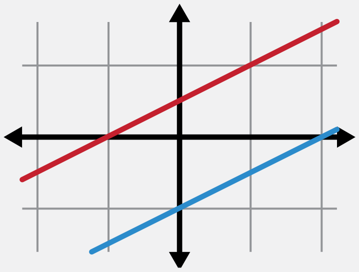


No Solutions	One Solution	Infinitely Many Solutions

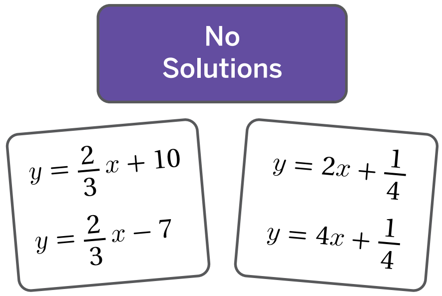
10. Jaleel and Irene are trying to decide when a system of equations has *no solutions*.

A system of equations has no solutions when . . .

Jaleel: . . . the *slopes* are the same.

Irene: . . . the y-*intercepts* are the same.

Whose claim is correct? Explain your thinking.

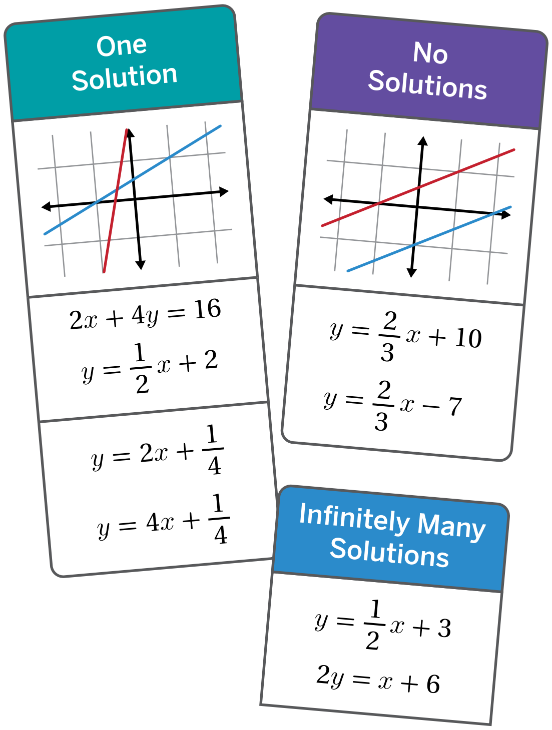


Synthesis

11. Select one and explain.

How can you tell if a system of equations has:

- ☐ **A.** No solutions?
- ☐ **B.** One solution?
- ☐ **C.** Infinitely many solutions?



Lesson Practice 4.04

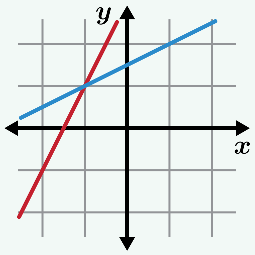
Lesson Summary

You can solve systems of equations using strategies like elimination, substitution, or graphing.

On a coordinate plane, you can see the solution of a system of equations at the point(s) where the two lines intersect. A system of linear equations can have:

One Solution

The lines intersect at (-2, 2).

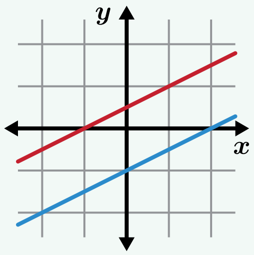


$$y = 2x + 6$$
$$y = \frac{1}{2}x + 3$$

The equations have different *slopes* and *y-intercepts*.

No Solutions

The lines are *parallel*.

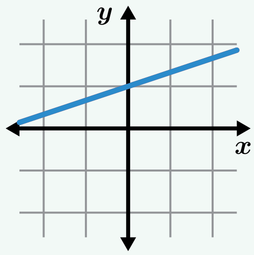


$$y = \frac{1}{2}x + 1$$
$$y = \frac{1}{2}x - 2$$

The equations have the same slope and different y-intercepts.

Infinitely Many Solutions

The lines are the same.



$$y = \frac{1}{3}x + 2$$
$$3y - x = 6$$

The equations are equivalent.

Lesson Practice
4.04

Name: .... Date: .... Period: ....

1. The point $(-2, 2)$ is on the line $y = x + 4$. Explain how you can determine if this point is the solution to this system of equations:

$y = x + 4$
$y = 2x - 1$

2. Solve this system of equations. Write the solution as a coordinate pair.

$y = -\frac{1}{2}x - 8$

$y = 3x + 6$

3. Match each system of equations to the number of solutions it has.

a $y = -2x + 1$
$2y = -4x + 2$

b $y = -2x + 1$
$y = -2x + 4$

c $y = -2x + 1$
$y = 2x + 1$

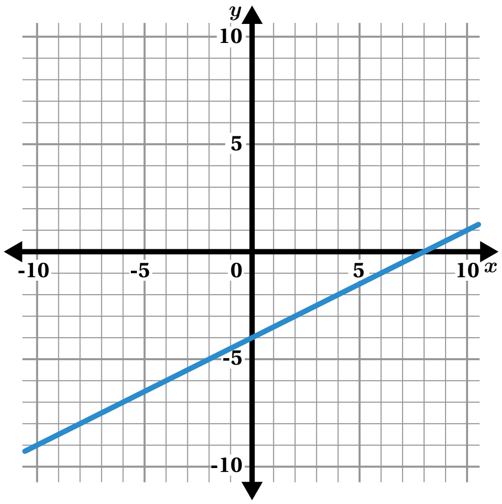
.... No solutions

.... One solution

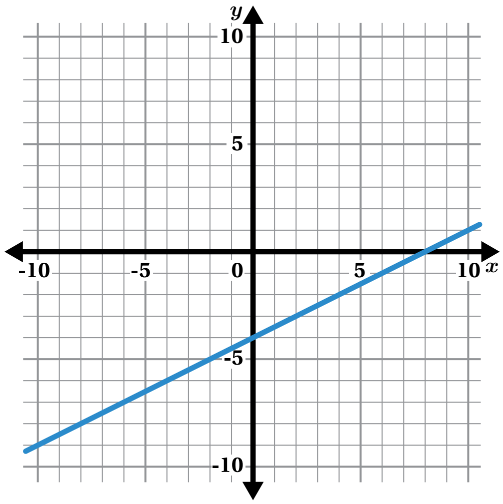
.... Infinitely many solutions

Problems 4–5: Here is a graph of $y = \frac{1}{2}x - 4$. Graph a second line to make this system of equations have:

4. No solutions.



5. One solution at $(2, -3)$.



6. Here is one equation in a system of equations: $y = 3x - 2$. Write a different second equation so that the system of equations has infinite solutions.

Name: ________ Date: ________ Period: ________

Test Practice

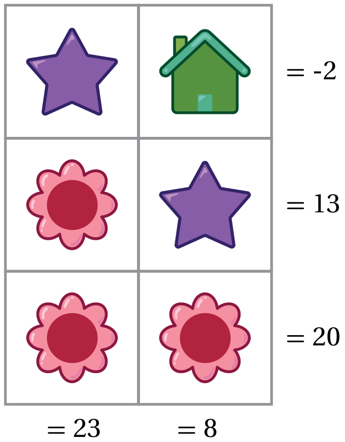
7. A system of equations has no solutions. Select *all* of the statements that must be true about the equations in this system.

- ☐ **A.** The equations have different slopes
- ☐ **B.** The equations have the same slope
- ☐ **C.** The equations have different y-intercepts
- ☐ **D.** The equations have the same y-intercepts

Spiral Review

8. Here is a shape puzzle. What is the value of each shape?

Shape	Value
Star	
House	
Flower	

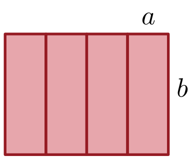


9. Solve this inequality: $3(x - 3) > 2x - 6$

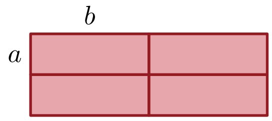
10. Here are two arrangements of identical rectangles. Determine the dimensions a and b.

$a =$ ________ units

$b =$ ________ units



Perimeter: 56 units



Perimeter: 64 units

Name: ________________ Date: ________ Period: ________

MA.912.AR.9.1, MA.912.AR.9.6, MTR.1.1, MTR.6.1

City Development

Let's solve systems of equations using multiple methods and interpret the solution in context.

Warm-Up

1. Which one doesn't belong? Routine

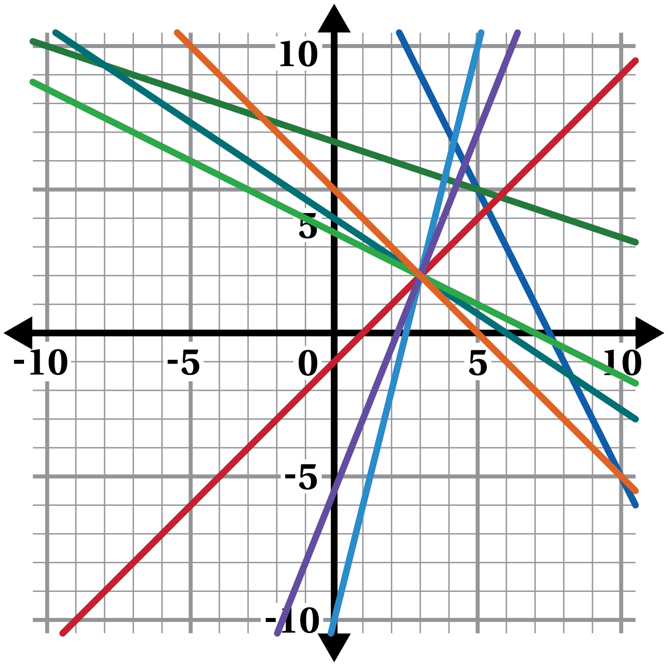
Identify the system that is different and why. Use the graph to support your thoughts.

$2x + 3y = 12$
$x - y = 1$

$2x + y = 15$
$x + 3y = 20$

$4x - y = 10$
$x + 2y = 7$

$5x - 2y = 11$
$x + y = 5$



2. Solve the system that has a different solution than the others.

Name: Date: Period:

Apartments and Green Space

Metropolis received 4 million dollars to develop 14 plots of unused land. They want to use all of the money and build on every plot.

Proposal 1: They can build an apartment building on two plots of land for $\$800,000$ and green space on one plot of land for $\$200,000$.

Here are two equations about this situation.

$$2a + g = 14$$
$$0.8a + 0.2g = 4$$

- a is the number of apartments.
- g is the number of green spaces for parks and playgrounds.

3. Show or explain how each part of these equations connect to the situation.

Name: .... Date: .... Period: ....

Apartments and Green Space (continued)

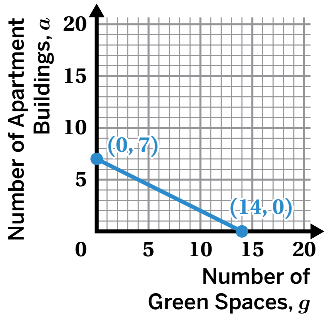
4. Here is incomplete work from three students who were solving the system. With your group, identify the strategy each student is using to solve the system and then complete the strategy.

Diamond

$g + 2a = 14$

a-intercept	g-intercept
$(0) + 2a = 14$	$g + 2(0) = 14$
$a = 7$	$g = 14$
$(0, 7)$	$(14, 0)$

$0.2g + 0.8a = 4$



Jamal

$-0.2 \cdot (g + 2a = 14)$

$0.2g + 0.8a = 4$

$-0.2g + -0.4a = -2.8$

$+ \quad 0.2g + 0.8a = 4$

Angela

$g + 2a = 14$

$-2a \quad -2a$

$g = 14 - 2a$

$0.2g + 0.8a = 4$

$0.2(14 - 2a) + 0.8a = 4$

$2.8 - 0.4a + 0.8a = 4$

5. In your group, compare your solutions and work together to revise any mistakes.

Discuss:

- What strategy did the student start with?
- How did you complete the strategy?

6. What does the solution represent about the Metropolis building project?

Activity 2

Name: Date: Period:

Houses and Green Space

Metropolis received $\$4$ million to develop 14 plots of unused land. They want to use all of the money and build on every plot.

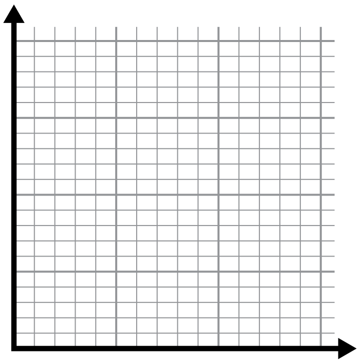
Proposal 2: They can build a house on a plot of land for $\$500{,}000$ and green space on a plot of land for $\$200{,}000$.

Here are two equations about this situation.

$$h + g = 14$$
$$0.5h + 0.2g = 4$$

- h is the number of houses.
- g is the number of green spaces.

7. Solve this system graphically and symbolically.

Graphically	Symbolically
	

8. What does your solution tell you about the city's plan to develop the land?

9. Which solving strategy did you prefer? Explain your thinking.

Activity 3

Name: ________ Date: ________ Period: ________

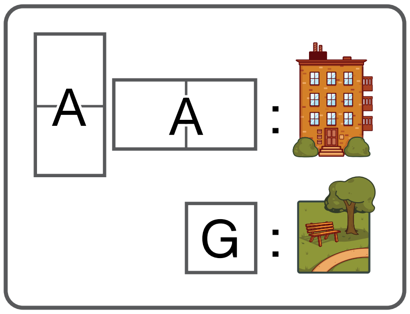
Comparing Proposals

Metropolis City Council is deciding between Proposal 1 and Proposal 2.

10. Use the solution to the linear system to design a map of a neighborhood that meets Proposal 1's *constraints*.

Proposal 1: Apartments and Green Space

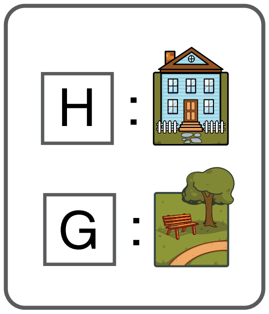
Mark the location of each apartment building, A, and each plot of green space, G.



11. Use the solution to the linear systems to design a map of a neighborhood that meets Proposal 2's constraints.

Proposal 2: Houses and Green Space

Mark the location of each house, H, and each plot of green space, G.



12. Compare your Metropolis proposals.

Discuss: What are some advantages and disadvantages of building houses versus apartments?

Synthesis

13. When would you choose to solve a system of equations graphically? Symbolically?

$$\begin{aligned} 2a + g &= 14 \\ 0.8a + 0.2g &= 4 \end{aligned}$$

Lesson Practice 4.05

Lesson Summary

Systems of equations can represent *constraints* in a situation. There are different ways to solve systems of equations to determine the values that satisfy these constraints.

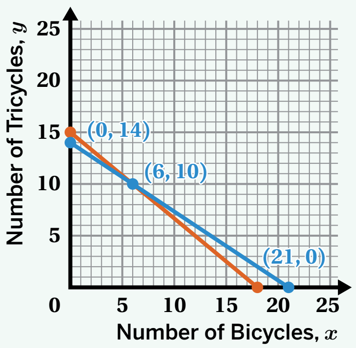
For example, a bike shop makes 2-wheel bicycles and 3-wheel tricycles. This week, they have 42 wheels and enough materials to make 16 bikes total.

Here is a system of equations about this situation:

- x is the number of bicycles
- y is the number of tricycles

$$\begin{aligned} x + y &= 16 \\ 2x + 3y &= 42 \end{aligned}$$

Graphing



The point of intersection $(6, 10)$ is the solution.

Substitution

$$\begin{aligned} x + y &= 16 \\ -y \quad & -y \\ x &= 16 - y \end{aligned}$$

$$\begin{aligned} 2x + 3y &= 42 \\ 2(16 - y) + 3y &= 42 \\ 32 - 2y + 3y &= 42 \\ \boxed{y = 10} & \end{aligned}$$

$$x = 16 - 10$$

$$\boxed{x = 6}$$

Elimination

$$\begin{aligned} -2 \cdot (x + y &= 16) \\ 2x + 3y &= 42 \end{aligned}$$

$$\begin{aligned} -2x - 2y &= -32 \\ + \quad 2x + 3y &= 42 \\ \hline 0 + y &= 10 \\ \boxed{y = 10} & \end{aligned}$$

$$x + (10) = 16$$

$$\boxed{x = 6}$$

In this situation, the solution $x = 6$ and $y = 10$ means that the bike shop will use all of their wheels and materials if they make 6 bicycles and 10 tricycles.

Lesson Practice
4.05

Name: .... Date: .... Period: ....

1. Match each system of equations to its solution.

a $y = 3x + 1$
$y = -3x + 25$

b $y = -2x$
$y = x - 12$

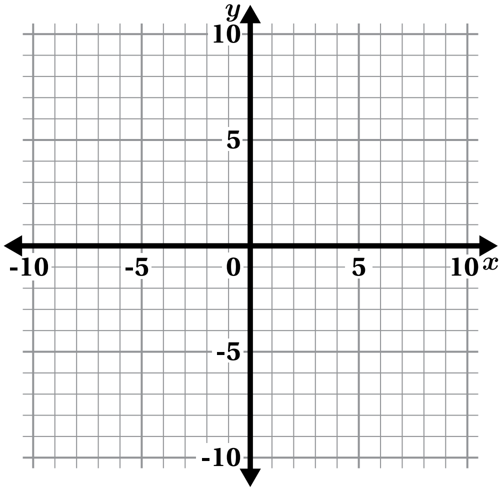
c $4x + 2y = 12$
$y = 3 + x$

.... $(1, 4)$ .... $(4, -8)$ .... $(4, 13)$

Problems 2–3: Here is a system of equations:

$$y = 8x - 6$$
$$8x + 2y = 12$$

2. Graph the system of equations.

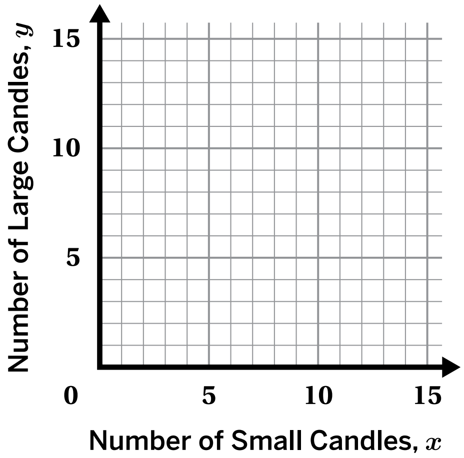


3. Determine the solution to the system of equations.

Problems 4–6: Dylan needs to use 40 ounces of wax to make 7 candles. A small candle requires 4 ounces of wax and a large candle requires 8 ounces of wax. Dylan wrote this system of equations:

$$x + y = 7$$
$$4x + 8y = 40$$

4. Graph the system of equations.



5. Determine the solution to the system.

6. What does the solution to the system tell you?

Lesson Practice
4.05

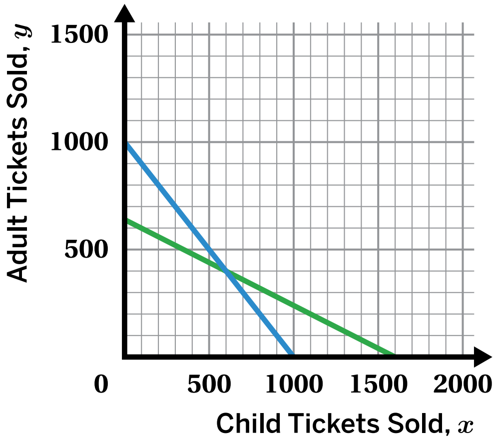
Name: .... Date: .... Period: ....

Test Practice

Problems 7–8: A child ticket to a state fair costs $10. An adult ticket costs $25. The fair sold 1,000 tickets and made $16,000 total. Here is a system of equations and a graph that represent the situation.

$$10x + 25y = 16000$$
$$x + y = 1000$$

7. Label each line with the equation it represents.



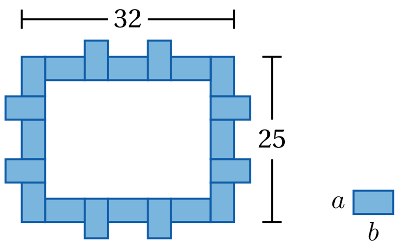
8. Determine the solution to the system.

Spiral Review

9. All the small rectangles are identical. Determine the dimensions, a and b, of one rectangle.

$a =$ ....

$b =$ ....



Problems 10–11: Using only the number 4 exactly four times and the four basic operations, write an expression to create each number.

10. 1

11. 15

Name: Date: Period:

MA.912.AR.9.1, MA.912.AR.9.6, MA.912.AR.2.2, MTR.2.1, MTR.4.1,MTR.5.1

Bus Systems

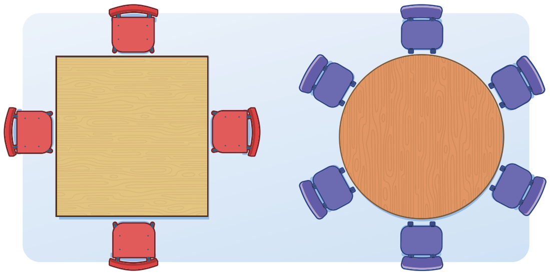
Let's model real-world situations using systems of equations.

Warm-Up

1. Sol is a party planner and uses square and round tables for each party. For each situation, write an equation to represent the number of tables Sol will use.

x: number of round tables

y: number of square tables



Situation 1: A party will have 240 people. Round tables seat 6 people and square tables seat 4 people.

Situation 2: There will be 3 round tables for every square table.

Situation 3: The number of square tables will be 10 more than double the number of round tables.

Situation 4: An event space will have 18 total tables.

Activity 1

Name: ______ Date: ______ Period: ______

Seats and Handholds

The Metropolis Transit Association (MTA) is deciding how many seats and handholds will be on their new buses. There are four configurations with different ratios of seats and handholds.

2. With your group, define the variables in this situation.

x represents . . . y represents . . .

3. Choose *two* configurations and write a system of equations for them.

	Configuration A	Configuration B	Configuration C	Configuration D
Ratio	4 handholds for every seat	1 handhold for every seat	2 handholds for every 5 seats	1 handhold for every 7 seats
Space	• A bus has 240 sq. ft of space to fill with seats and handholds. • Each seat uses 4 sq. ft of space and each handhold uses 2 sq. ft of space.			
System of Equations				

4. Solve each of the two systems of equations that you wrote using any strategy. Use the next page for your thinking.

5. With your group, come to an agreement on the solutions for each configuration.

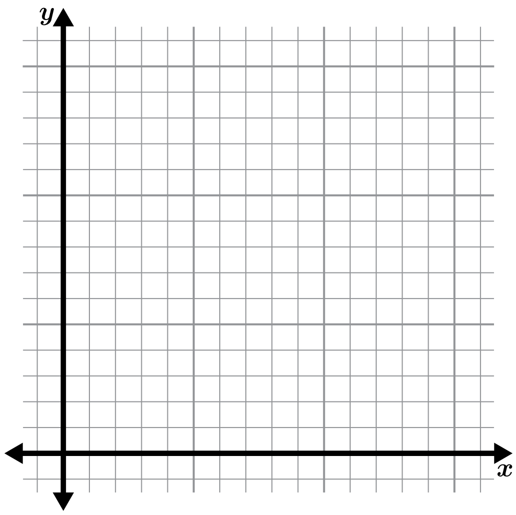
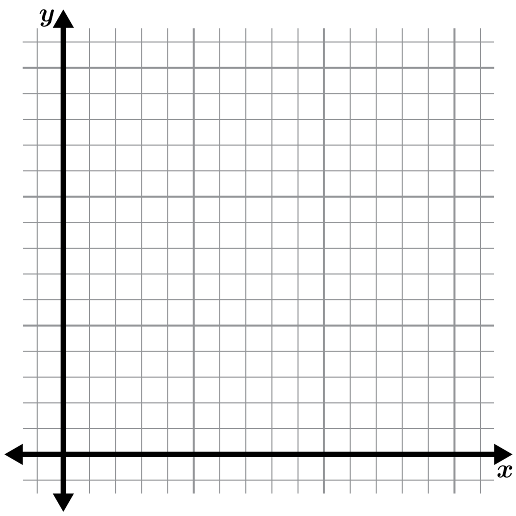
	Configuration A	Configuration B	Configuration C	Configuration D
Solution				

6. **Discuss:** What does each solution mean about the number of seats and handholds?

Name: ________ Date: ________ Period: ________

Seats and Handholds (continued)

Use this space to solve the systems of equations you wrote.

Configuration ________		
Configuration ________		

Activity
2

Name: Date: Period:

Making Recommendations

The MTA is considering using the new buses from Activity 1 for two existing bus lines.

Bus Line 1	Bus Line 2
• **Route:** Between the airport and parking garage. • **Average time per person:** 5 minutes • Buses often have a large number of people and their bags.	• **Route:** Along the main street of a neighborhood. • **Average time per person:** 15 minutes • Popular stops include: the grocery store, the laundromat, and the post office.

7. Which configuration would you recommend for Bus Line 1? Circle one.

Configuration A Configuration B Configuration C Configuration D

Explain your thinking.

8. Which configuration would you recommend for Bus Line 2? Circle one.

Configuration A Configuration B Configuration C Configuration D

Explain your thinking.

Synthesis

9. Describe how to write a system of equations to represent a situation.

Use the example if it helps with your explanation.

Bus Configuration A

- The bus has 240 sq. ft of space for seats and handholds.
- Each seat uses 4 sq. ft of space and each handhold uses 2 sq. ft of space.
- There are 4 handholds for every seat on the bus.

System of Equations

$$4x + 2y = 240$$

$$y = 4x$$

x: number of seats

y: number of handholds

Lesson Practice 4.06

Lesson Summary

You can write systems of equations to help represent constraints in real-world situations.

Here are some things to consider when you write a system of equations:

- Identify the constraints of the situation.
- Identify what each variable will represent.
- Determine whether you want to use *standard form* ($Ax + By = C$) or *slope-intercept form* ($y = mx + b$) to represent each constraint.

Let's look at an example. The architect of an apartment building has enough space to fit 50 apartments and 80 parking spaces. The city requires 2 parking spaces for every big apartment, and 1 parking space for every small apartment.

Constraints	Variables	System of Equations
• There is space to fit 50 apartments and 80 parking spaces. • Every big apartment has 2 parking spaces and every small apartment has 1 parking space.	x represents the number of big apartments. y represents the number of small apartments.	$x + y = 50$ $2x + y = 80$

Lesson Practice
4.06

Name: .... Date: .... Period: ....

1. Match the system of equations to the situation.

a $2x + 2y = 17$
$y = 5 + x$

b $x + y = 17$
$x - y = 5$

c $x + y = 17$
$x = 5$

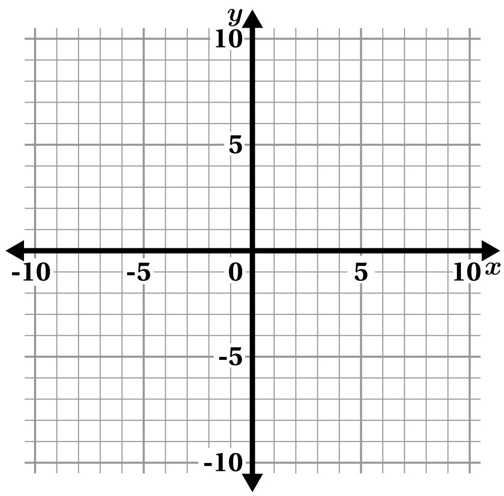
.... The sum of two numbers is 17. The difference between two numbers is 5.

.... A rectangle has a perimeter of 17 units. The length of the rectangle is 5 more units than the width.

.... The sum of two numbers is 17. One of the numbers is 5.

2. Solve this system of equations. Sketch a graph if it helps with your thinking.

$3x + 4y = 24$
$y = 2x - 5$



Problems 3–5: A school choir has a concert coming up. They are figuring out what prices to set for student tickets, s, and adult tickets, a.

- They estimate that 150 students and 50 adults will buy tickets.
- They want to make $1,000 in ticket sales.
- They would like the adult ticket price to be double the student ticket price.

3. Write a system of equations that represents this situation.

4. Solve the system.
$s =$ .... $a =$ ....

5. Explain the solution in context.

Name: .... Date: .... Period: ....

6. Determine values for A and B so that the system has infinitely many solutions.

$12x + Ay = 8$

$Ax + By = 4$

A: ....

B: ....

Test Practice

7. Determine the solution to the system of equations.

$y = 2x - 8$

$3x + y = 2$

$\boxed{}$

Spiral Review

Problems 8–10: For each equation, identify the slope of its graph.

8. $y = 5 + 2x$

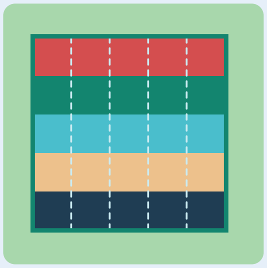
9. $2x + 4y = 4$

10. $y + 1 = 9x$

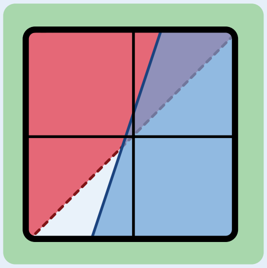
Sub-unit

2

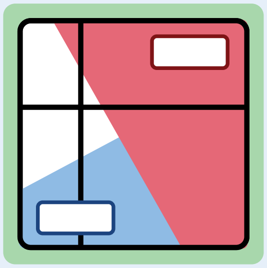
Systems of Inequalities



Lesson 7
Quilts



Lesson 8
Seeking Solutions



Lesson 9
Boundaries and Shading

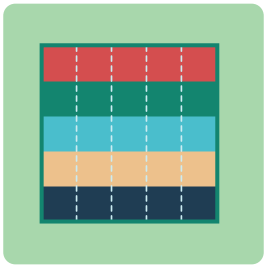
Lesson 10
Restaurant Meals

Name: ________ Date: ________ Period: ________

MA.912.AR.9.6, MTR.2.1, MTR.3.1, MTR.6.1, MTR.7.1

Quilts

Let's explore what solutions to systems of inequalities mean.



Warm-Up

1. Quilts are a warm bed covering made of padding enclosed between layers of fabric. The fabric is often a combination of patterned and solid prints. It is kept in place by lines of stitching. The stitching is usually applied in a decorative design.

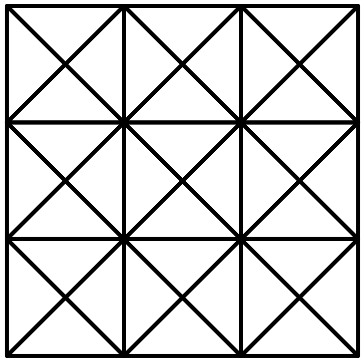
What are some decisions people might make when designing a quilt?

Name: .... Date: .... Period: ....

Sai’s Quilt

2. **a** Let’s make a quilt square together.

b **Discuss:** Why did you choose to design your quilt the way you did?



Solid Fabric

Patterned Fabric

3. Sai is making a quilt using solid and patterned fabric.

He needs *at least* 35 sq. ft of fabric to cover his bed.

Fabric Constraint

$$x + y \geq 35$$

What are some combinations of fabric Sai could use?

Solid Fabric (sq. ft), x	Patterned Fabric (sq. ft), y

4. Sai wants to spend no more than $\$30$ on fabric.

- Solid fabric costs $\$0.50$ per sq. ft.
- Patterned fabric costs $\$1$ per sq. ft.

Cost Constraint

$$0.50x + y \leq 30$$

What are some combinations of fabric Sai could use?

Solid Fabric (sq. ft), x	Patterned Fabric (sq. ft), y

Name: .... Date: .... Period: ....

Sai's Quilt (continued)

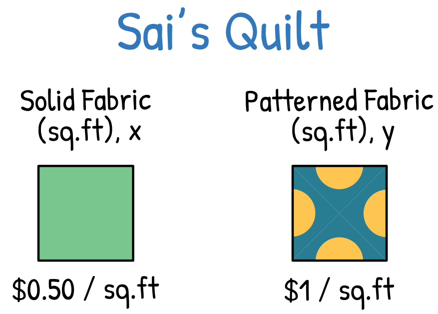
5. Sai wants his quilt to have at least 35 sq. ft of fabric and cost no more than $30. He wrote a **system of inequalities** to represent these two constraints.

$$x + y \geq 35$$

$$0.50x + y \leq 30$$

Sai designed a quilt using 10 sq. ft of solid fabric and 28 sq. ft of patterned fabric.

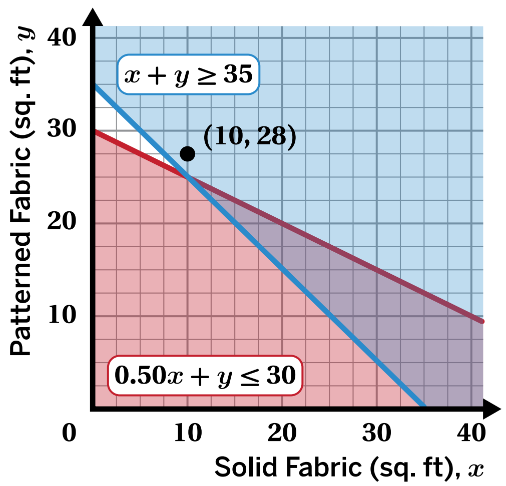
Does the design meet both constraints?



6. This graph represents Sai's system of inequalities.

The fabric used in his design is represented by the point $(10, 28)$.

How can the graph help Sai decide whether his design meets both constraints?



7. Let's test several fabric combinations.

a **Discuss:** How can you see which constraints a point meets by looking at the graph?

b Test these ordered pairs to see if they are a solution.

$(10, 10)$, $(20, 30)$, and $(30, 10)$

Name: ______ Date: ______ Period: ______

Evan’s Quilt

8. Evan is making a quilt using different fabrics.

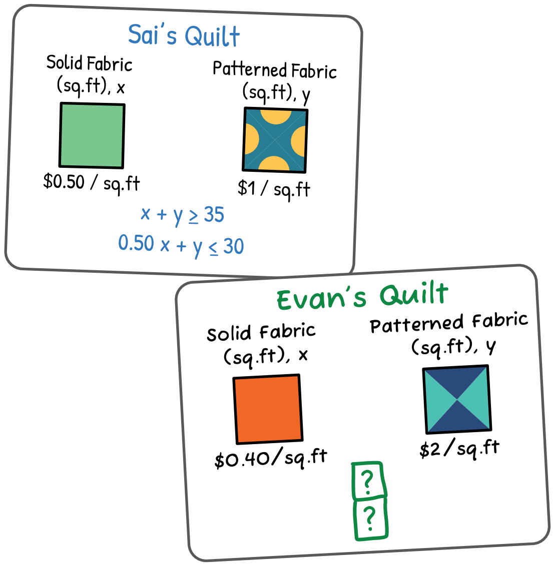
- Solid fabric costs $\$0.40$ per sq. ft.
- Patterned fabric costs $2 per sq. ft.

Evan wants his quilt to have *at least* 35 sq. ft of fabric and cost *no more than* $\$30$.

Write a system of inequalities to represent Evan’s quilt.

Fabric inequality: ______

Cost inequality: ______



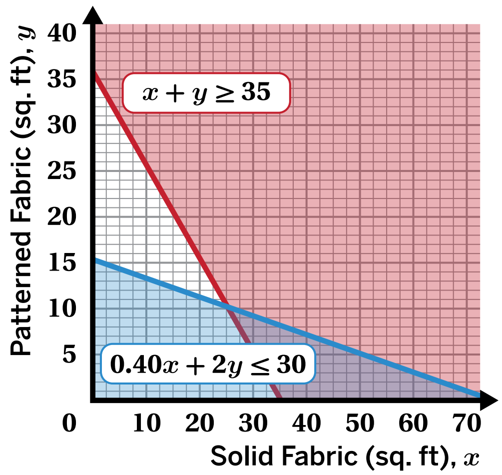
9. This graph represents Evan’s system of inequalities:

$$x + y \geq 35$$
$$0.40x + 2y \leq 30$$

Determine a combination of solid and patterned fabric that meets both constraints.

Solid Fabric (sq. ft), x: ______

Patterned Fabric (sq. ft), y: ______



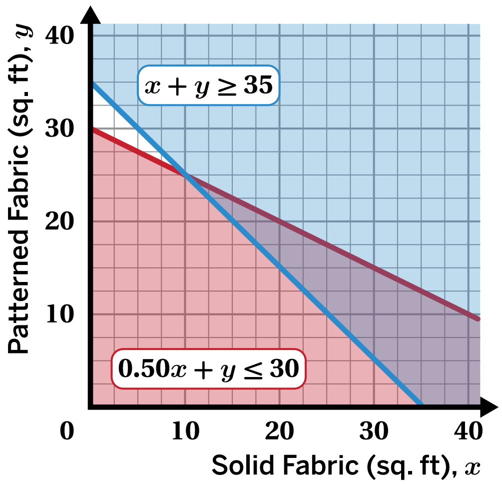
10. Titus says $(10, 10)$ *is* a solution to Evan’s system of inequalities. Alma says it *is not* a solution.

Whose thinking is correct? Explain your thinking.

Synthesis

11. How can you determine if a point is a solution to a system of inequalities?

Draw on the graph if it helps to show your thinking.



Lesson Practice 4.07

Lesson Summary

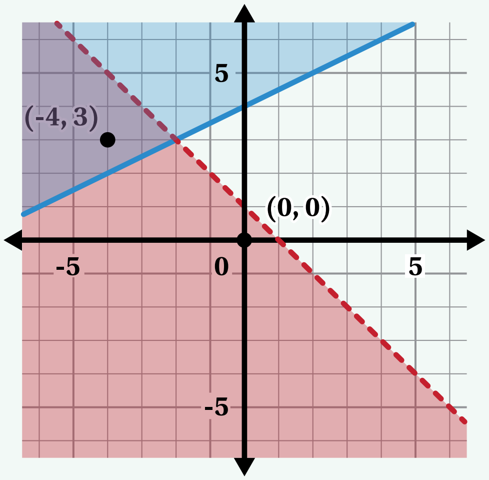
A **system of inequalities** is a system of two or more inequalities that represent the *constraints* on a shared set of variables.

You can use different strategies to determine if a point is a solution to a system of inequalities.

- If the point is in the shaded region for both inequalities, then it is a solution to the system.
- If the x- and y-values of the point are substituted into both inequalities and the inequalities are true, then the point is a solution to the system.

Here is a graph for this system of inequalities.

$$x + y < 1$$
$$y \geq \frac{1}{2}x + 4$$



You can see that the point $(-4, 3)$ is a solution because it is in the shaded region for both inequalities.

You can also substitute points into both inequalities to determine if they are solutions. $(0, 0)$ is not a solution and $(-4, 3)$ is a solution.

Name: .... Date: .... Period: ....

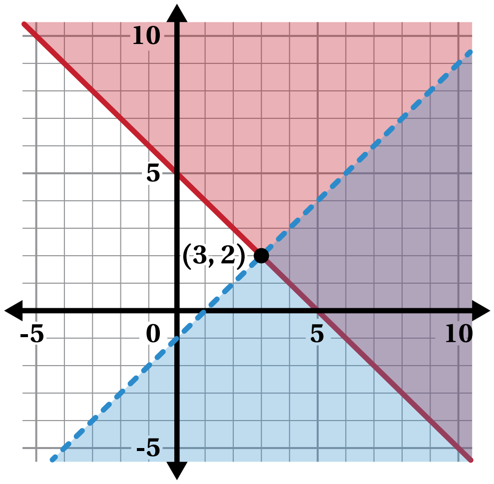
1. The graph shows this system of inequalities.

$$x + y \geq 5$$

$$x - y > 1$$

Is the point $(3, 2)$ a solution to the system?

Explain your thinking.



Problems 2–4: It costs Lukas $\$5.00$ to mail a package. Lukas has *postcard stamps*, p, that are worth $\$0.34$ each and *first-class stamps*, f, that are worth $\$0.49$ each.

2. Lukas wrote the inequality $0.34p + 0.49f \geq 5$. What does this inequality represent?

3. Lukas wrote another inequality: $p + f \leq 12$. What does this inequality represent?

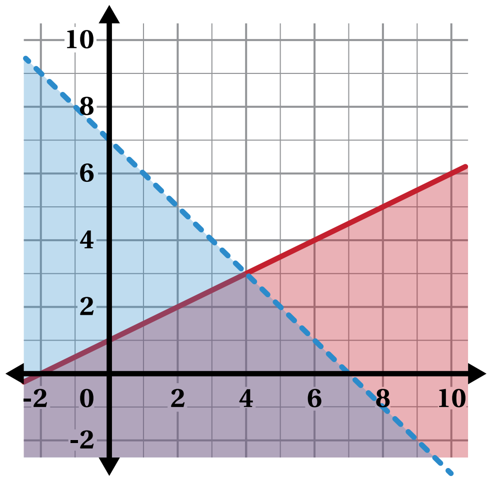
4. If Lukas uses 1 postcard stamp and 9 first-class stamps, will this satisfy both constraints? Explain your thinking.

5. Using the digits 0–9 without repeating, fill in each blank such that each statement is true:

- A is a solution to both inequalities
- B is a solution to only one inequality
- C is a solution to only the other inequality
- D is not a solution to either inequality

$A = (\square, \square)$ $B = (\square, \square)$

$C = (\square, \square)$ $D = (\square, \square)$



Lesson Practice
4.07

Name: .... Date: .... Period: ....

Test Practice

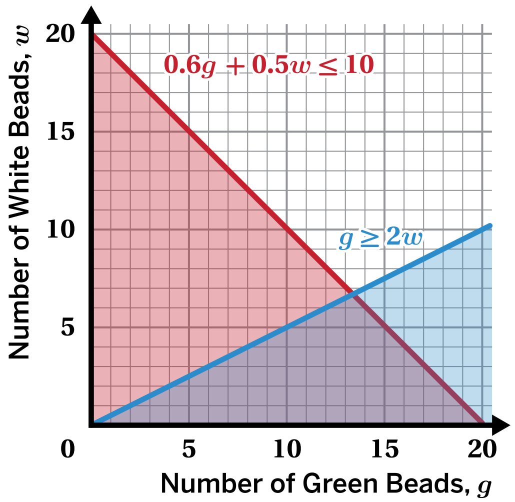
6. Arjun is making a bracelet.

He has $10 to spend on beads.

Green beads cost $0.60 each and white beads cost $0.50 each.

His bracelet design needs at least twice as many green beads as white beads.

The graph shows the system of inequalities that represents this situation.



What is a combination of green and white beads that meets both constraints?

.... green beads and .... white beads

Spiral Review

7. Solve this system of equations. Write your solution as a coordinate pair.

$$2x + y = 8$$
$$y = 2x + 4$$

8. Jayla is at the market with $14 to buy fruit. She decides to buy apples and grapes. Apples, a, cost $1.67 per pound and grapes, g, cost $1.87 per pound. Write an inequality to represent this situation.

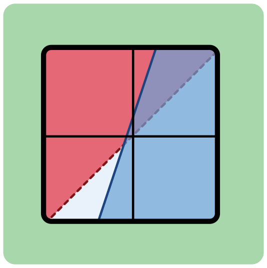
9. Esteban claims that $(5, 10)$ is *not* a solution to the inequality $y < 3x - 5$. Do you agree with Esteban's claim? Explain your thinking.

Name: ______________________ Date: __________ Period: __________

MA.912.AR.9.4, MTR.2.1, MTR.3.1, MTR.6.1, MTR.7.1

Seeking Solutions

Let’s explore strategies for determining the solution region for a system of inequalities.

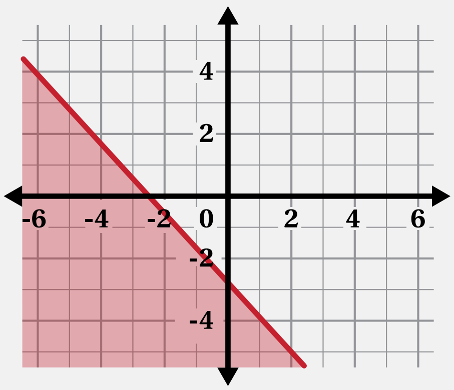


Warm-Up

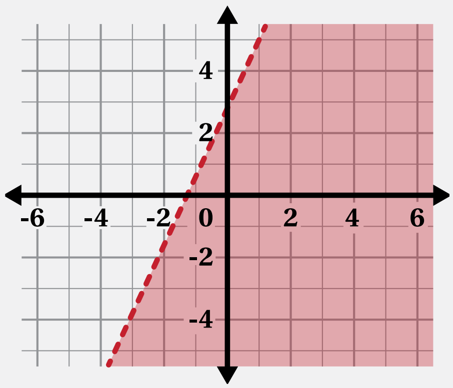
1. Match each inequality to its graph. There will be two graphs without a match.

$x + y \geq -3$	$y > 2x + 3$

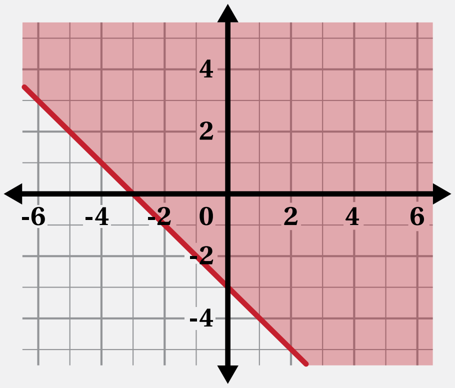
A.



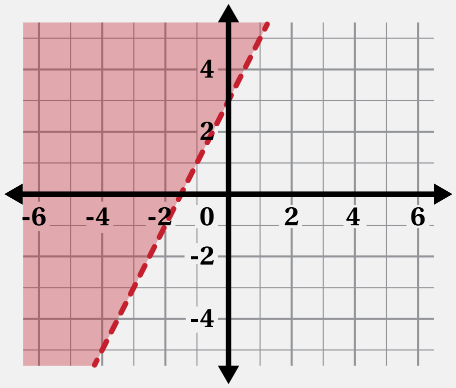
B.



C.



D.

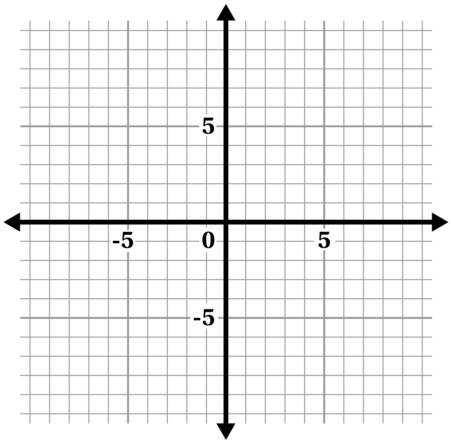


Name: ______ Date: ______ Period: ______

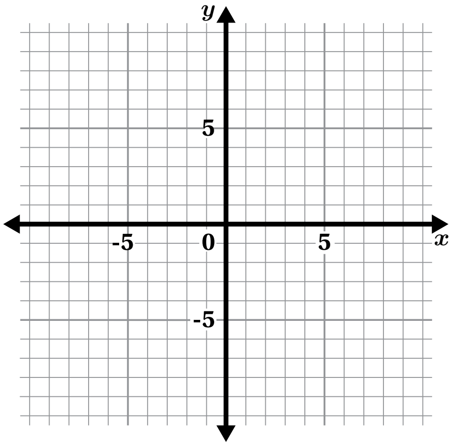
The Overlap

2. Graph the solution to each inequality.

$y > x$

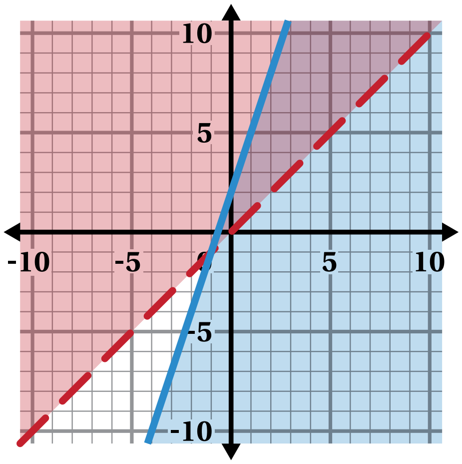


$-3x + y \leq 2$



$y > x$

$-3x + y < = 2$



3. Let's look at a graph that shows the system of inequalities from the previous question.

Discuss:

- How many regions do you see?
- In the system, what do these points represent? $(4, 2)$, $(-2, 2)$, $(2, 4)$, and $(-4, -6)$.
- What happens when the point is on the dashed line? On the solid line?

Activity 1

Name: .... Date: .... Period: ....

The Overlap (continued)

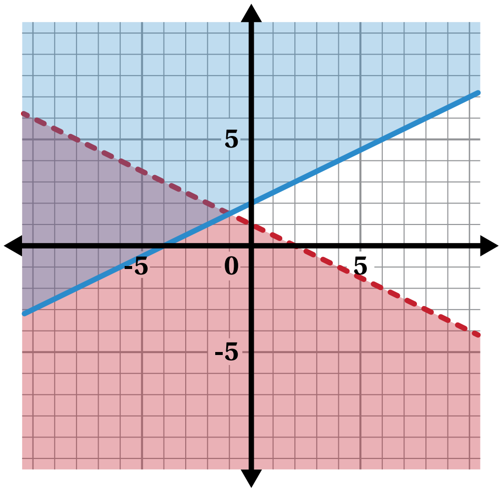
4. The **solutions to a system of inequalities** are all the points that make both inequalities true.

On a graph, the solutions are located in the same region.

Draw a point in the **solution region** of this system of inequalities:

$$\frac{1}{2}x + y < 1$$

$$-x + 2y \geq 4$$



Name: ______ Date: ______ Period: ______

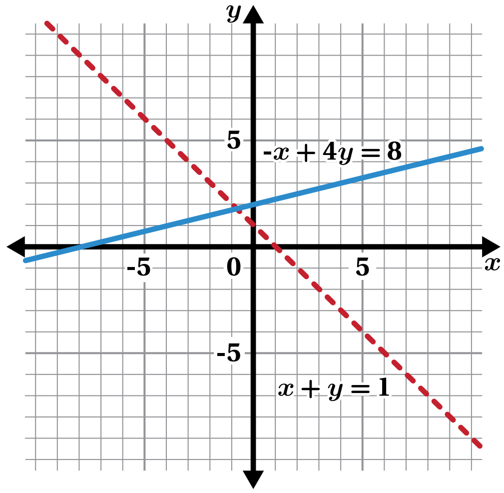
Where Is the Solution Region?

5. This graph shows the boundary lines and their equations for this system of inequalities:

$$x + y > 1$$

$$-x + 4y \leq 8$$

How can you determine where the solution region is?

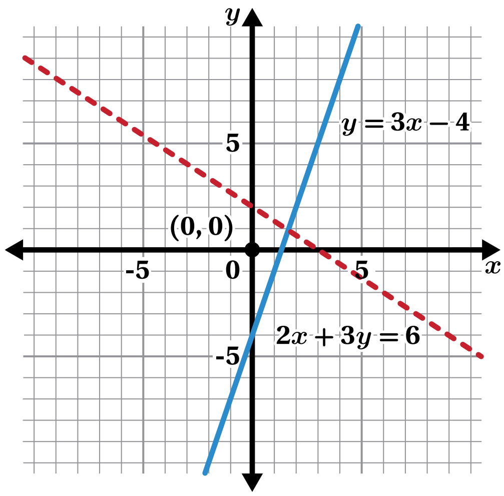


6. Plot a point on the solution region for the system of inequalities in the previous problem.

7. Terrance is trying to graph the solutions to this system of inequalities. First, he tests the point $(0, 0)$.

$2x + 3y > 6$
$y \geq 3x - 4$

Dashed Line	Solid Line
$2(0) + 3(0) > 6$	$0 \geq 3(0) - 4$
$0 + 0 > 6$	$0 \geq 0 - 4$
$0 > 6$	$0 \geq -4$
False!	True!



Discuss: What can Terrence do next to determine the solution region?

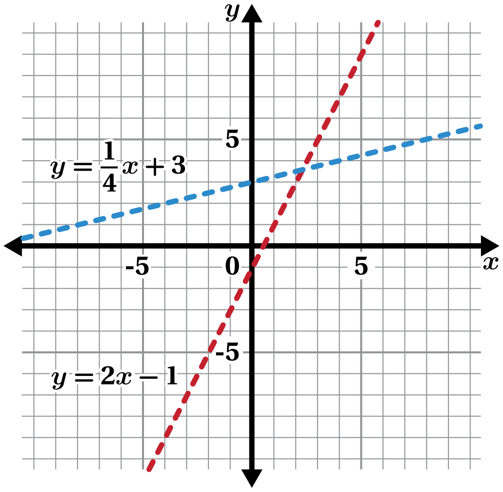
Name: .... Date: .... Period: ....

Solution Region Practice

8. Plot a point in the solution region for each system of inequalities.

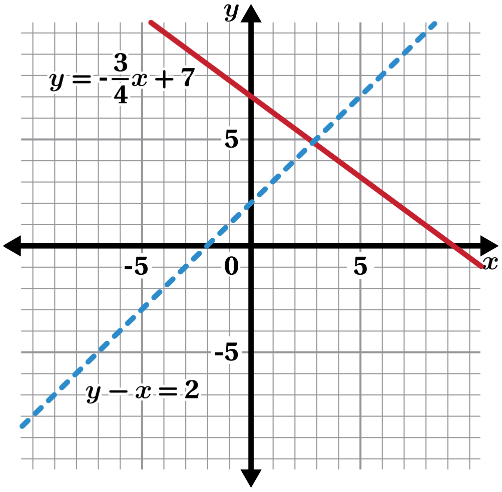
a $y > 2x - 1$

$y < \frac{1}{4}x + 3$



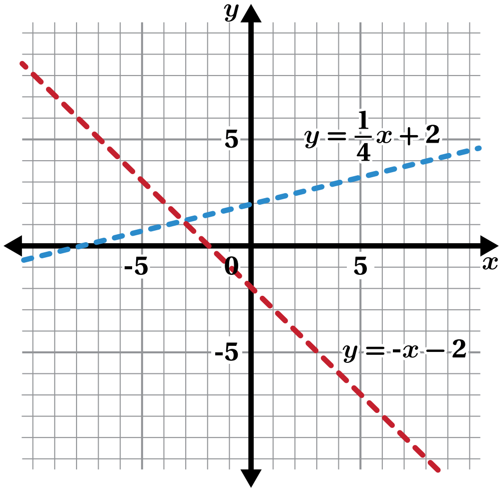
b $y \leq -\frac{3}{4}x + 7$

$y - x < 2$



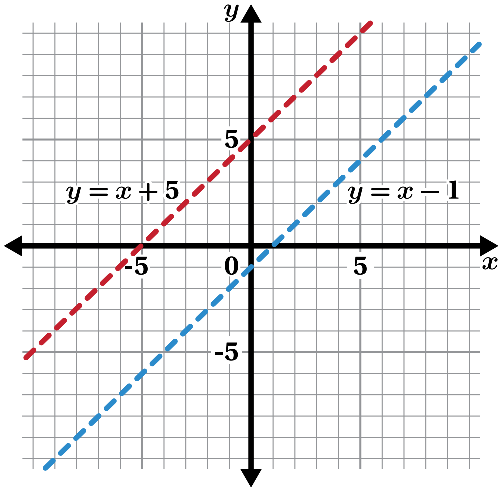
c $y < -x - 2$

$y < \frac{1}{4}x + 2$



d $y > x + 5$

$y > x - 1$



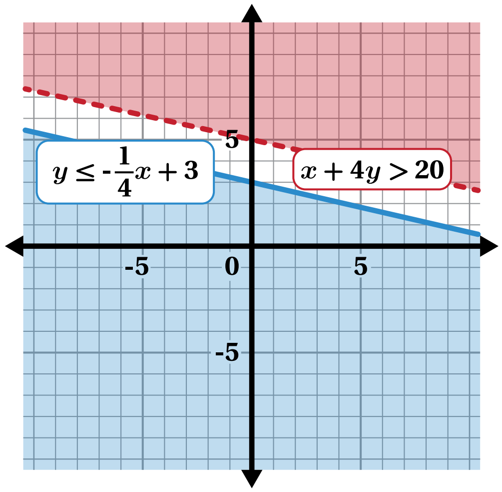
Activity 3

Name: .... Date: .... Period: ....

Solution Region Practice (continued)

9. This system of inequalities has no solutions.

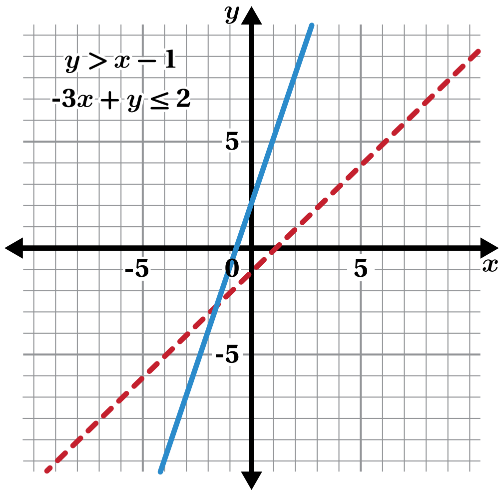
How would you convince a classmate that there are no solutions to this system?



Synthesis

10. What are some things you should keep in mind when determining the solution region of a system of inequalities?

Use the graph if it helps with your thinking.



Lesson Practice 4.08

Lesson Summary

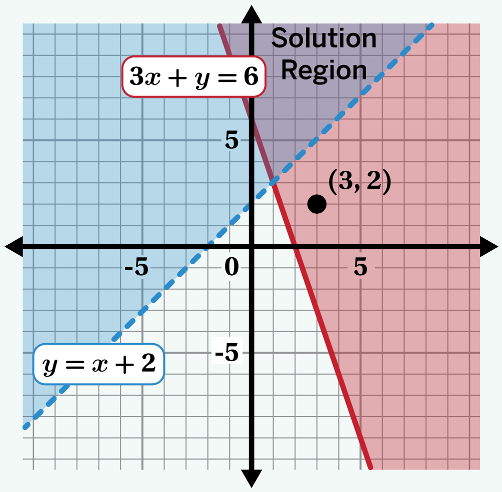
The **solutions to a system of inequalities** are all the points that make both inequalities true. The solutions can be seen in the region where the graphs overlap, called the **solution region**.

One strategy for determining the location of the solution region is to test a point. Choose a point that is not on either *boundary line*, substitute the x- and y-values into each inequality to see if it makes the statement true, and shade based on the results of the test.

Let's look an example for this system of inequalities:

$$3x + y \geq 6$$
$$y > x + 2$$

You can test the point (3, 2) to help determine the solution region.



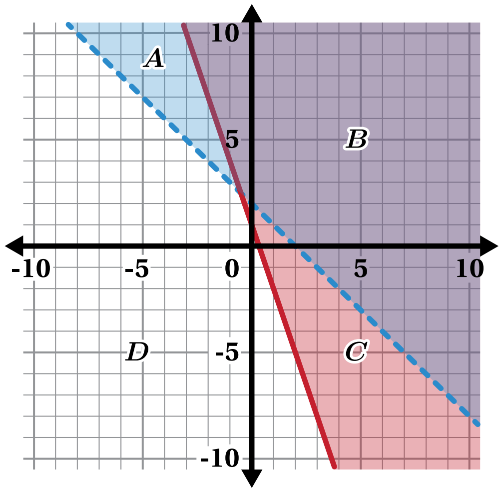
Solid Line	Dashed Line
$3x + y \geq 6$	$y > x + 2$
$3(3) + 2 \geq 6$	$0 > 3 + 2$
$11 \geq 6$	$0 > 5$
True ✓	False ✗
Shade the side of the solid line that includes $(3, 2)$	Shade the side of the dashed line that does not include $(3, 2)$

Lesson Practice
4.08

Name: .... Date: .... Period: ....

Problems 1–2: Here is the graph of a system of inequalities.

$$y > -x + 2$$
$$3x + y \geq 1$$



1. Which letter represents the solution region to the system of inequalities? Circle one:

A B C D

2. Is the point $(5, -4)$ a solution to the system? Circle one:

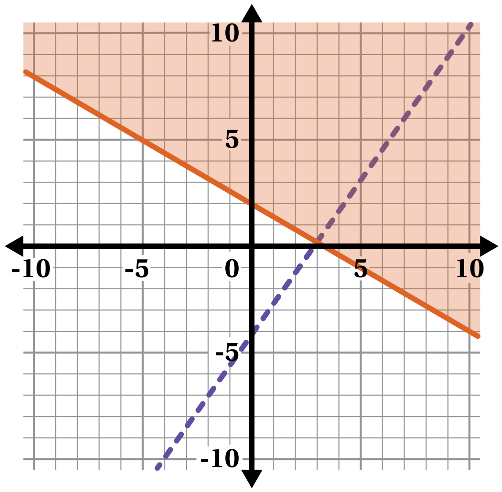
Yes No

Problems 3–4: Javier graphed the first inequality and the boundary line of the second inequality.

$$3x + 5y \geq 10$$
$$4x - 3y < 9$$

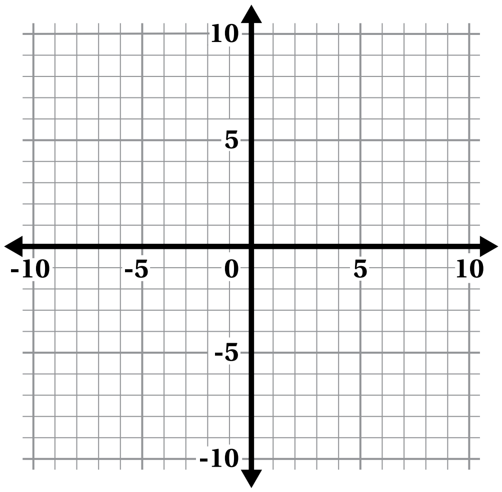
3. Complete the graph of the second inequality.

4. Explain how you knew where to shade the second inequality.



5. Make a graph of a system of inequalities that has no solutions.

6. Explain how you know it has no solutions.



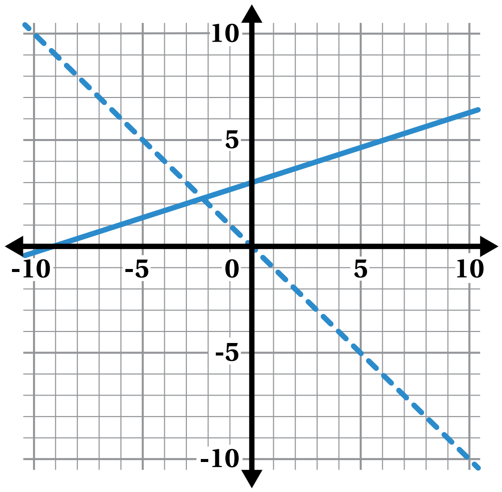
Lesson Practice
4.08

Name: .... Date: .... Period: ....

Problems 7–8: Nyanna graphed the boundary lines of this system of inequalities:

$$x + y > 0$$
$$-x + 3y \leq 9$$

7. Complete the graph of the system of inequalities.



8. Identify a coordinate pair that is in the solution region.

Test Practice

Problems 9–10: Fill in each blank with an inequality symbol such that:

9. The system has no solutions.

$x - y \ \boxed{}\ 0$

$x - y \ \boxed{}\ 0$

10. Only points with matching x- and y-coordinates are a solution.

$x - y \ \boxed{}\ 0$

$x - y \ \boxed{}\ 0$

Spiral Review

Problems 11–13: Determine the value of each expression.

11. $\frac{4}{6} \cdot 8$

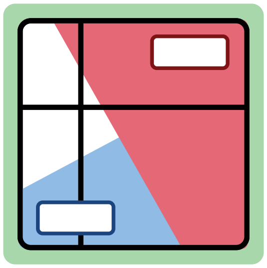
12. $\frac{3}{5} \cdot 2 \cdot \frac{10}{9}$

13. $\frac{3}{6} \cdot 3 \cdot \frac{6}{9} \cdot 2$

Name: ______ Date: ______ Period: ______

MA.912.AR.9.4, MTR.4.1, MTR.5.1, MTR.6.1

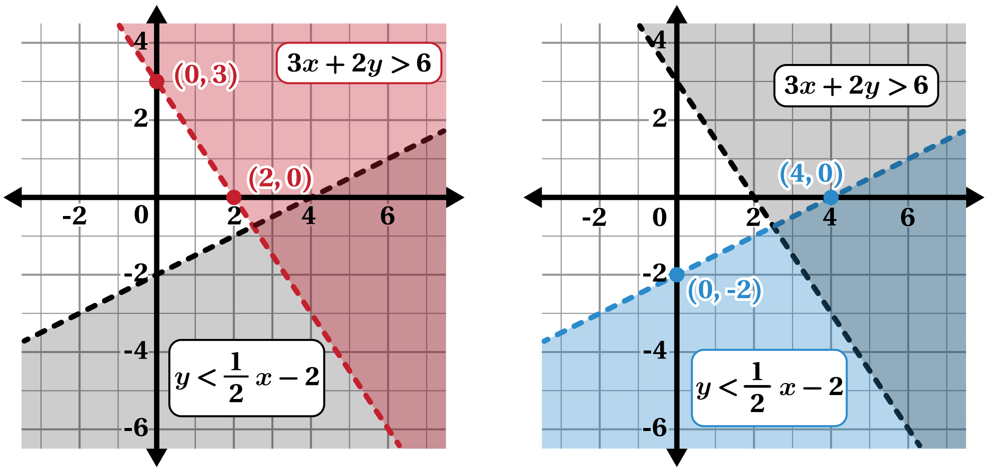
Boundaries and Shading



Let’s write and graph systems of inequalities.

Warm-Up

The graphs show a system of inequalities. The x- and y-*intercepts* are shown for the boundary lines of two inequalities. Choose one inequality to talk about with a partner.



1. **Discuss:** For the inequality you chose, how can you determine the intercepts of a boundary line without a graph?

2. **Discuss:** How can you use the intercepts of the boundary line to graph this inequality?

Name: .... Date: .... Period: ....

Rounds of Systems

You and your partner will use a set of cards for this activity.

3. Graph a system of inequalities for each round.

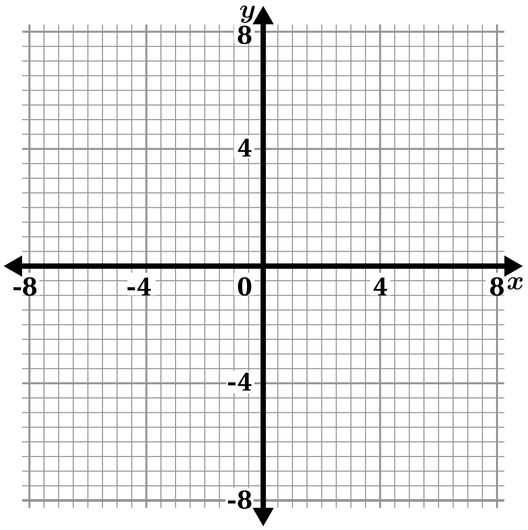
Instructions for Each Round

Step 1. Select a card. Graph the inequality on your card.
Step 2. Trade cards with your partner. This will be the second inequality in your system.
Step 3. Graph the second inequality on the same set of axes as the first.
Step 4. Determine one solution to the system. Explain or show how you know it is a solution.
Step 5. Highlight the solution region on the graph.

Round 1

My inequality: ....

My partner's inequality: ....

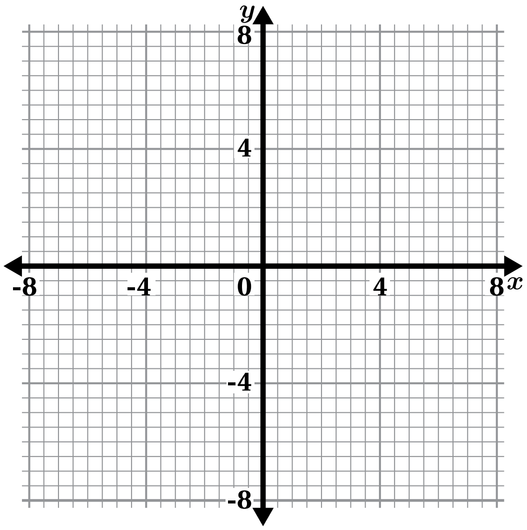


What is one solution to the system? Show or explain your thinking.

Round 2

My inequality: ....

My partner's inequality: ....



What is one solution to the system? Show or explain your thinking.

Activity 2

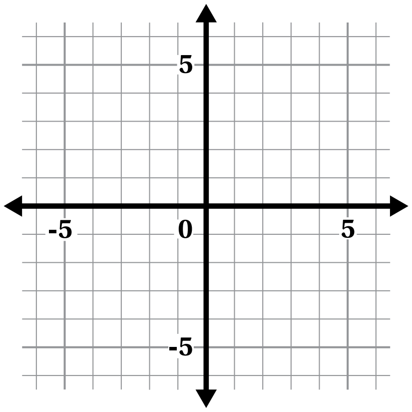
Name: ________ Date: ________ Period: ________

Graphing Systems

4. Select four of these six problems to complete with your partner. Use the graphing calculator to check your work.

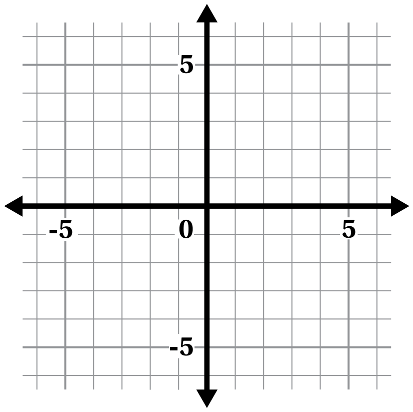
A. Graph the system of inequalities and mark the solution region.

$$2x - y < 4$$
$$y \geq \frac{1}{4}x + 3$$

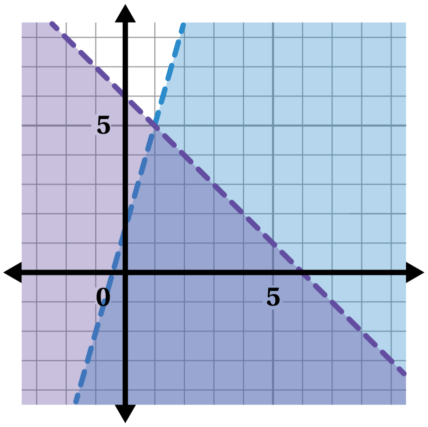


B. Graph the system of inequalities and mark the solution region.

$$3x - 6y \geq 12$$
$$y \leq 1$$



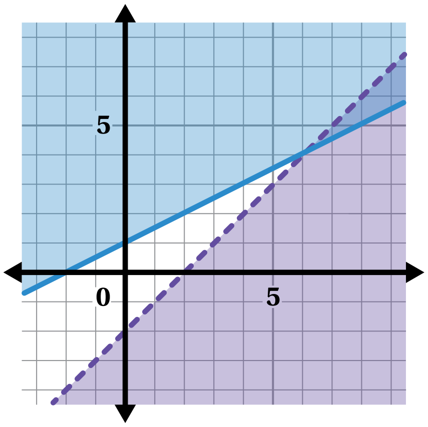
C. Write a system of inequalities that this graph could represent.



Inequality 1: ________

Inequality 2: ________

D. Write a system of inequalities that this graph could represent.



Inequality 1: ________

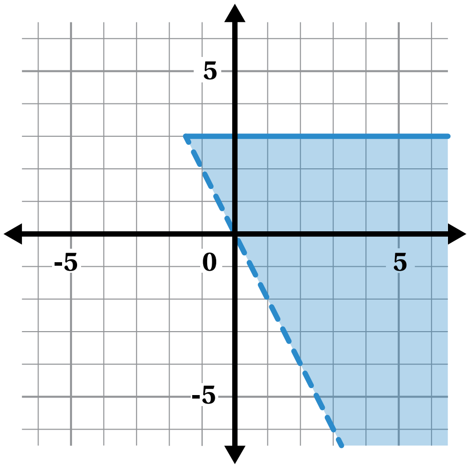
Inequality 2: ________

Name: .... Date: .... Period: ....

Graphing Systems (continued)

E. Here is the graph of a solution region.

Write a system of inequalities with this solution region.

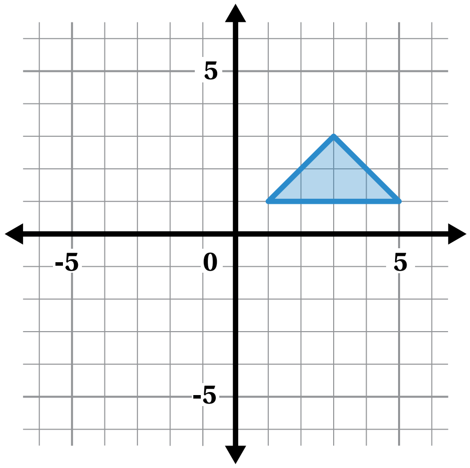


Inequality 1: ....

Inequality 2: ....

F. Here is the graph of a solution region.

Write a system of inequalities with this solution region.



Inequality 1: ....

Inequality 2: ....

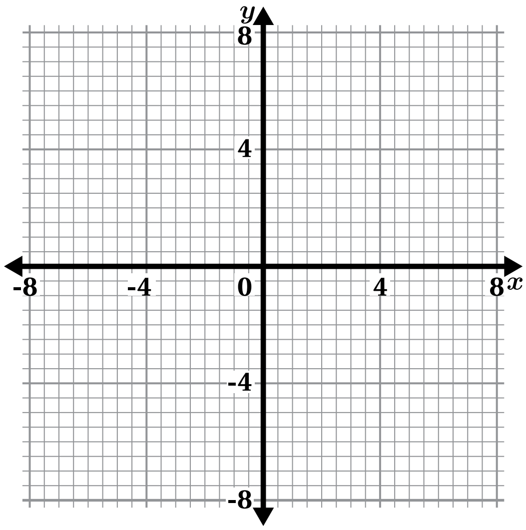
Inequality 3: ....

Synthesis

5. Describe a strategy for graphing the solution region for a system of inequalities.

Use this example if it helps with your thinking.

$$x + y < 3$$
$$y \geq 2x$$



Lesson Practice 4.09

Lesson Summary

You can graph a system of linear inequalities by graphing the boundary line of each inequality and testing a point to determine which side of the boundary lines to shade.

You can use different strategies to help you graph boundary lines of inequalities.

- A strategy to graph boundary lines written in *slope-intercept form* ($y = mx + b$) is to plot the y-intercept and use the slope to determine other points.
- A strategy to graph boundary lines written in *standard form* ($Ax + By = C$) is to plot and connect the x- and y-*intercepts*.

If an inequality uses a $\leq$ or $\geq$ symbol, then the boundary line is solid and included in the solution region. If an inequality uses a $<$ or $>$ symbol, then the boundary line is dashed and not part of the solution region.

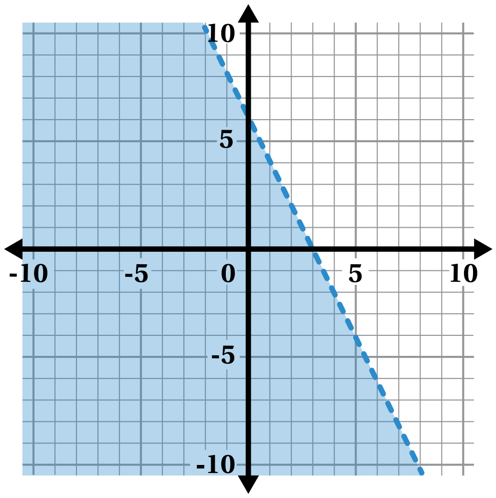
You can also write a system of linear inequalities from a graph. Use the boundary lines and test points to help you determine the inequality symbols. You can also use test points to check the accuracy of your system of inequalities.

Name: ________ Date: ________ Period: ________

Problems 1–3: The first inequality of this system of inequalities is graphed below.

$$5y < 30 - 10x$$
$$2x + 3y > 12$$

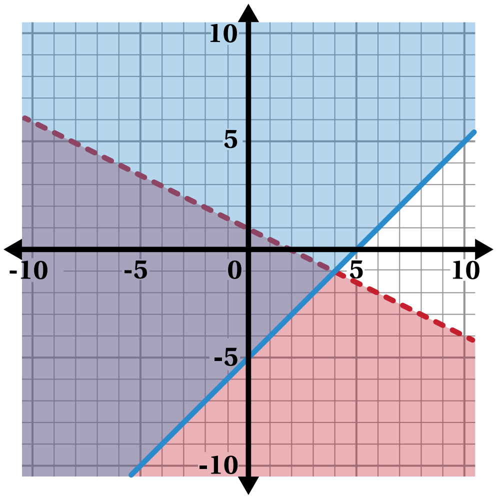
1. Graph the second inequality.



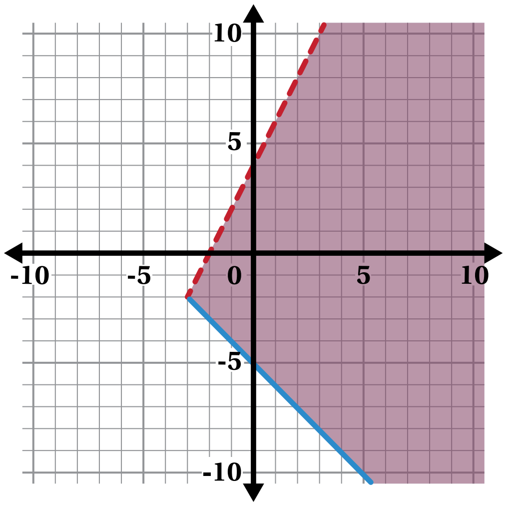
2. Write a point that is *not* a solution to this system.

3. Write a point that *is* a solution to this system.

4. Write a system of inequalities to represent this graph.



5. Here is the graph of a solution region. Write a system of inequalities that has this solution region.



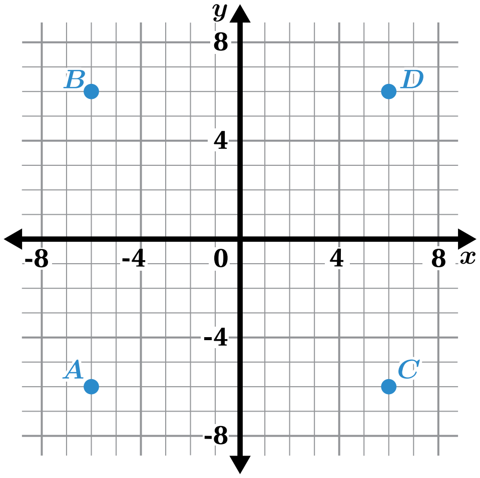
Lesson Practice
4.09

Name: ______ Date: ______ Period: ______

6. Using the digits 0–9 without repeating, fill in each blank such that all of the following statements are true.

- A is a solution to both inequalities.
- B is a solution to only one inequality.
- C is a solution to only the other inequality.
- D is not a solution to either inequality.

$$\square x + \square y < \square$$
$$\square x + \square y < \square$$

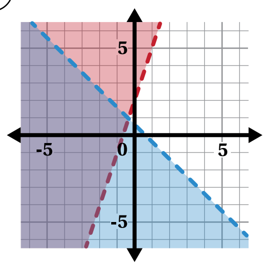


Test Practice

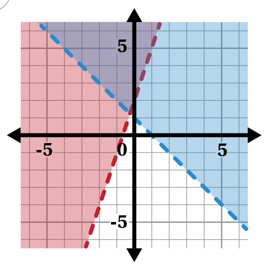
7. Which graph matches this system of inequalities?

$$-3x + y \geq 2$$
$$2x + 2y \geq 2$$

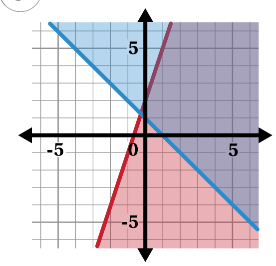
A.



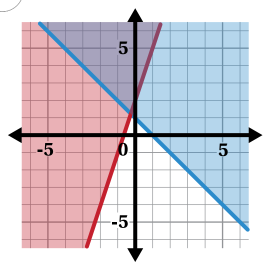
B.



C.



D.



Spiral Review

Problems 8–10: Consider the line $f(x) = 3x - 4$ and the point $(5, c)$. Write a value for c so that the point is:

8. On the line.

$c =$ ______

9. Above the line.

$c =$ ______

10. Below the line.

$c =$ ______

Unit 4
Lesson
10

Name: ____________ Date: ____________ Period: ____________

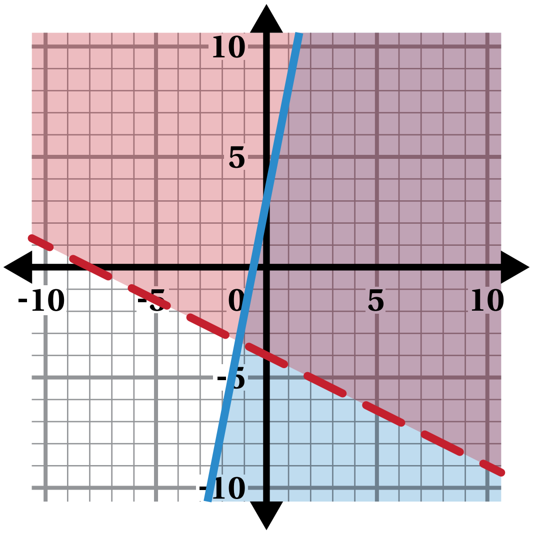
MA.912.AR.9.4, MA.912.AR.9.6, MTR.2.1, MTR.3.1, MTR.6.1, MTR.7.1

Restaurant Meals

Let's use systems of inequalities to model real-world situations.

Warm-Up

1. Aziza thinks that $(4\ ,2)$ is a solution to the system because it lays in the overlapping shaded region, do you agree? What is another strategy to see if $(4, 2)$ is a solution? Explain your thinking.



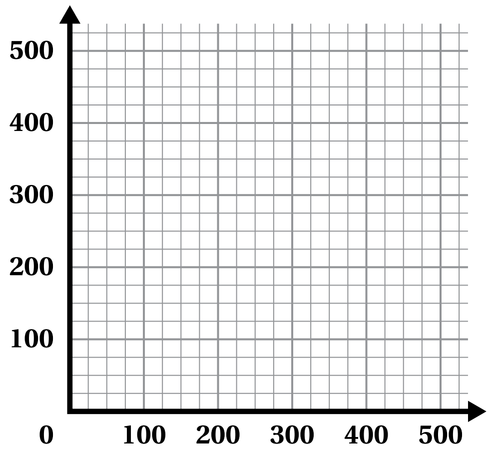
Name: Date: Period:

Meal Prep

Each day, Chef Malik plans to make at least 325 meals, some vegetarian and some with meat. Preparing a vegetarian meal costs $\$1.50$, while a non-vegetarian meal costs $\$2$. Chef Malik's total daily budget is $\$600$.

2. Define the variables in this situation.

- x represents . . .
- y represents . . .



3. Write a system of inequalities to represent these *constraints*.

4. Graph the system of inequalities you wrote. Mark the solution region. Be sure to label each axis.

5. Liam thinks that Chef Malik can make 400 vegetarian and 100 non-vegetarian meals each day and meet his constraints.

Is Liam's thinking correct? Show or explain your thinking.

6. How many vegetarian meals and non-vegetarian meals do you recommend Chef Malik plan to make? Explain your thinking.

Activity 2

Name: .... Date: .... Period: ....

Make It To-Go

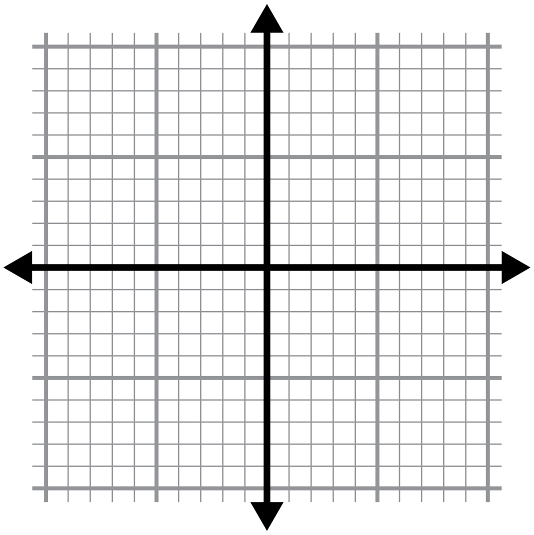
Chef Malik is considering packaging and delivering his meals in order to reach more customers. Each to-go vegetarian meal would cost $2.25 to make and each to-go non-vegetarian meal would cost $2.75.

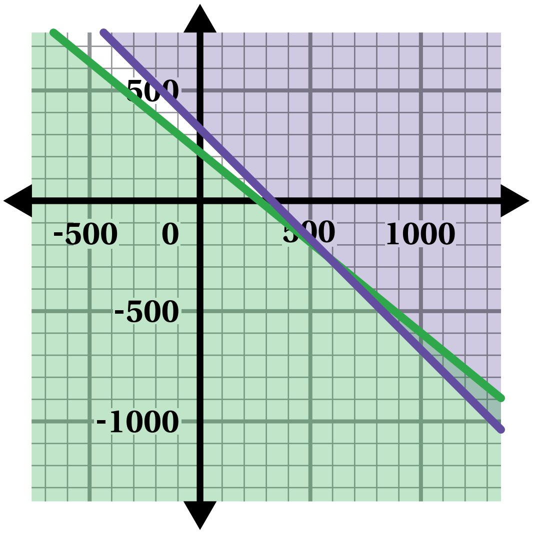
Chef Malik wants to serve at least 325 meals and spend no more than $600.

7. Write a system of inequalities to represent these constraints. Be sure to define your variables.

- x represents . . .
- y represents . . .

8. Graph the system you created.





9. What does the solution region for this system tell us about Chef Malik's plan?

10. Chef Malik was able to increase his budget to help with the added cost of the to-go meals.

Will that additional money allow him to meet the constraints for his meals? Explain your thinking.

Synthesis

11. How can systems of inequalities help solve real-world problems in your community?

Lesson Practice 4.10

Lesson Summary

Systems of inequalities can help you solve problems involving real-world *constraints*.

Here is an example about a juice shop.

A juice shop has 144 ounces of orange juice to fit in jars of 12 and 16 ounces. They earn \$2.50 for each 12-ounce jar and \$4.50 for each 16-ounce jar. They need to earn more than \$33.50 from the juice.

You can write and graph a system of inequalities to represent these constraints.

Let x represent the number of 12-ounce jars and y can represent the number of 16-ounce jars.

$$12x + 16y \leq 144$$

$$2.50x + 4.50y > 33.50$$

You can graph each boundary line and use the test point (2, 7) to help us determine the solution region.

Solid Line

$$12x + 16y \leq 144$$
$$12(2) + 16(7) \leq 144$$
$$136 \leq 144$$

True ✓

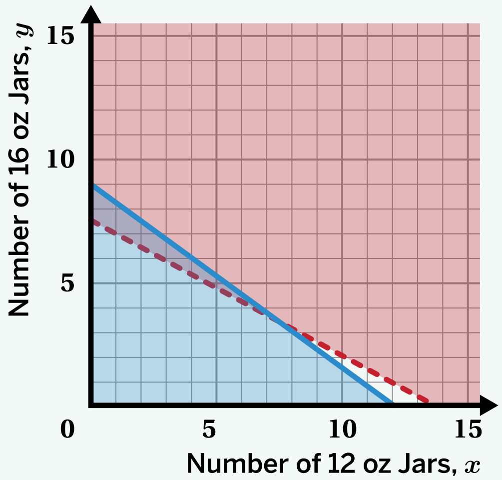
Shade the side of the solid line that includes (2, 7)

Dashed Line

$$2.50x + 4.50y > 33.50$$
$$2.50(2) + 4.50(7) > 33.50$$
$$36.50 > 33.50$$

True ✓

Shade the side of the dashed line that includes (2, 7)



You can use the graph to help you determine some possible combinations of 12- and 16-ounce jars of juice that meet the constraints. Such as:

- Zero 12 oz jars of juice and eight 16 oz jars of juice
- Zero 12 oz jars of juice and nine 16 oz jars of juice
- One 12 oz jars of juice and eight 16 oz jars of juice

Lesson Practice
4.10

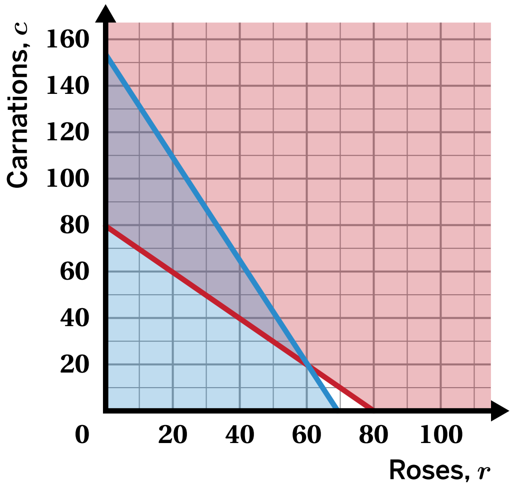
Name: .... Date: .... Period: ....

Problems 1–4: Victor has $\$100$ to spend on flowers for a school celebration. Roses, r, cost $\$1.45$ each and carnations, c, cost $\$0.65$ each. Victor wants to buy enough flowers so that all 80 people can take home at least one flower.

1. Select *all* inequalities that represent the constraints in this situation.

- ☐ **A.** $r + c \leq 80$
- ☐ **B.** $r + c \geq 80$
- ☐ **C.** $1.45r + 0.65c \leq 80$
- ☐ **D.** $1.45r + 0.65c \geq 100$
- ☐ **E.** $1.45r + 0.65c \leq 100$

2. Victor created a graph using inequalities. What is a combination of carnations and roses that meets the constraints?



3. What is a combination of carnations and roses that *does not* meet the constraints?

4. Victor wants to get *the most roses possible* while still meeting the constraints. How many roses and carnations should he get?

Explain your thinking.

Problems 5–6: Cho needs at least 3 pounds of fruit to make a fruit salad for a family gathering. She decides to buy blueberries and apples. She has $\$12$ to spend. Blueberries cost $\$4.85$ per pound and apples cost $\$1.31$ per pound.

- b represents pounds of blueberries.
- a represents pounds of apples.

5. Write a system of inequalities to represent Cho's constraints.

Test Practice

6. Can Cho buy 1.5 pounds of blueberries and 2.5 pounds of apples? Circle your choice.

Yes   No

Lesson Practice
4.10

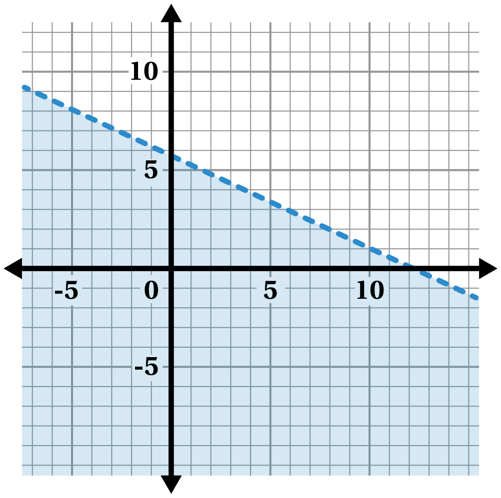
Name: .... Date: .... Period: ....

Spiral Review

7. The first inequality of this system of inequalities is graphed. Graph the second inequality.

$x + 2y < 12$

$y \leq x - 4$



Problems 8–9: Solve each system. Write the solution as a coordinate pair.

8. $2x - 3y = 14$
$2x + y = 6$

9. $y = 9x + 17$
$y = 6 - 2x$

Problems 10–12: Evaluate each expression.

10. 25% of 200

11. 12% of 200

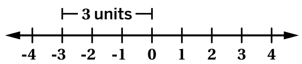
12. 8% of 200

Glossary/Glosario

English

A

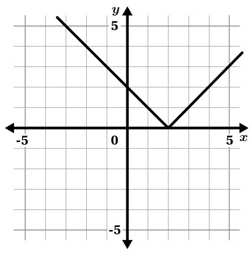
absolute value The absolute value of a number is its distance from 0 on the number line.



For example, the absolute value of -3 is 3 because -3 is 3 units away from 0. This is written as $|-3| = 3$.

$|4| = 4$ and $|-4| = 4$. They are both 4 units away from 0.

absolute value function A function that is defined using absolute value symbols. When written in the form $f(x) = |x - h|$, its output is the distance of its input from a given value, h.



For example, $f(x) = |x - 2|$ outputs the distance from 2 for every input value.

area The amount of space inside a two-dimensional shape. It is measured in square units, such as square inches or square centimeters.

arithmetic sequence A sequence that changes by a constant difference.

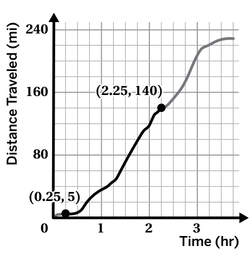
+6 +6 +6

5, 11, 17, 23, ...

In this arithmetic sequence, the first term is 5 and the constant difference is 6.

association If two variables are statistically related to one another, we say that there is an association between the two variables. A *correlation* is a type of association.

average rate of change A measure of how much a function changes, on average, over an interval. To calculate the average rate of change over an interval, find the slope between the point on the graph of the function where the interval begins and the point where it ends.

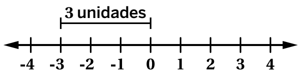


To calculate the average rate of change over the interval from 0.25 hours to 2.25 hours, calculate the change in y-values $(140 - 5)$ and divide by the change in x-values $(2.25 - 0.25)$. The average rate of change from 0.25 to 2.25 is 67.5, which means the average speed on that trip was 67.5 miles per hour in that interval.

Español

A

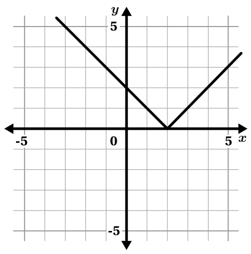
valor absoluto El valor absoluto de un número es su distancia al 0 en la recta numérica.



Por ejemplo, el valor absoluto de -3 es 3 porque -3 está a 3 unidades del 0. Esto se escribe $|-3| = 3$.

$|4| = 4$ y $|-4| = 4$. Ambos están a 4 unidades del 0.

función de valor absoluto Una función que se define utilizando signos de valor absoluto. Cuando se escribe en la forma $f(x) = |x - h|$, su salida es la distancia de su entrada a un valor determinado, h.



Por ejemplo, $f(x) = |x - 2|$ arroja la distancia de cada valor de entrada al 2.

área La cantidad de espacio dentro de una figura bidimensional. Se mide en unidades cuadradas, como pulgadas cuadradas o centímetros cuadrados.

secuencia aritmética Una secuencia que cambia con una diferencia constante.

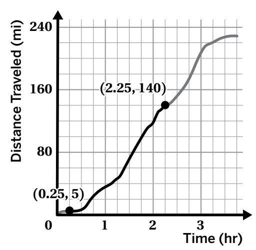
+6 +6 +6

5, 11, 17, 23, ...

En esta secuencia aritmética, el primer término es 5 y la diferencia constante es 6.

asociación Si dos variables están relacionadas estadísticamente, decimos que existe una asociación entre ambas variables. Una *correlación* es un tipo de asociación.

tasa de cambio promedio Una medida de cuánto cambia una función, en promedio, en un intervalo. Para calcular la tasa de cambio promedio a lo largo de un intervalo, se halla la pendiente entre el punto donde empieza el intervalo y el punto donde termina en la gráfica de la función.

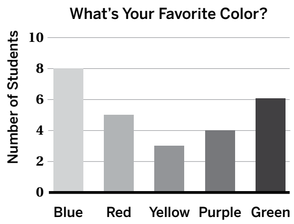


Para calcular la tasa de cambio promedio en el intervalo de 0.25 horas a 2.25 horas, se calcula el cambio en los valores de y $(140 - 5)$ y se divide por el cambio en los valores de x $(2.25 - 0.25)$. La tasa de cambio promedio de 0.25 a 2.25 es 67.5, lo que significa que la velocidad media en ese trayecto fue de 67.5 millas por hora en ese intervalo.

English

B

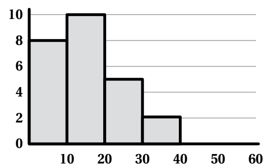
bar chart A visual display of categorical data values that uses bars to show frequencies for each category.



base A number or expression that is raised to an exponent. The exponent describes the number of times to multiply the base by itself.

For example, in the expression 2^3, 2 is the base and 3 is the exponent. In the expression $50(1.6)^{12x}$, 1.6 is the base and $12x$ is the exponent.

bin (of a histogram) The intervals used to group data values in a histogram are called *bins*. The distance of each interval is called the *bin width*.

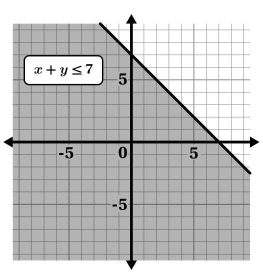


For example, this histogram shows 4 bins, each with a bin width of 10 units.

bivariate data Data that involves two variables. Each data point contains two pieces of information.

For example, a collection of students' heights and shoe sizes would be a bivariate data set.

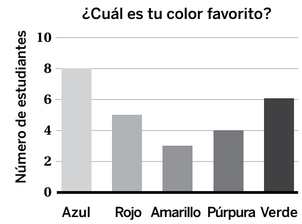
boundary line The line that separates the solution region of a linear inequality from non-solutions. A linear inequality (e.g., $y < 2x + 5$) has a boundary line that is represented symbolically by the corresponding equation (e.g., $y = 2x + 5$). A solid boundary line indicates that these points are included in the solution set (e.g., $y \leq x$). A dashed boundary line indicates that they are not (e.g., $y < x$).



The solution region to $x + y \leq 7$ has a boundary line at $x + y = 7$. The line is solid because the points on the line $x + y = 7$ are included in the solution region.

Español

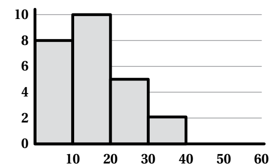
gráfico de barras Una representación visual de valores de datos categóricos que utiliza barras para mostrar frecuencias para cada categoría.



base Un número o una expresión que se eleva a un exponente. El exponente describe el número de veces que se multiplica la base por sí misma.

Por ejemplo, en la expresión 2^3, 2 es la base y 3 es el exponente. En la expresión $50(1.6)^{12x}$, 1.6 es la base y $12x$ es el exponente.

intervalo (de un histograma) Se denominan intervalos a los espacios que se usan para agrupar los valores de datos en un histograma. La distancia de cada intervalo se denomina *ancho del intervalo*.

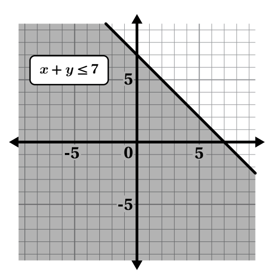


Por ejemplo, este histograma muestra 4 intervalos, cada uno con un ancho de intervalo de 10 unidades.

datos bivariados Datos en los que se incluyen dos variables. Cada punto de datos contiene dos informaciones.

Por ejemplo, una colección de estaturas de estudiantes y tamaños de zapatos sería un conjunto de datos bivariados.

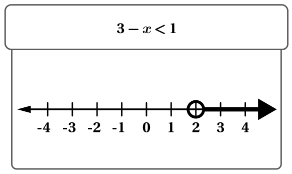
recta límite La línea que separa la región solución de una desigualdad lineal de todos los valores que no son soluciones. Una desigualdad lineal (p. ej., $y < 2x + 5$) tiene una recta límite representada simbólicamente por la ecuación correspondiente (p. ej., $y = 2x + 5$). Una recta límite continua indica que esos puntos están incluidos en el conjunto de soluciones (p. ej., $y \leq x$). Una recta límite discontinua indica que no lo están (p. ej., $y < x$).



La región solución de $x + y \leq 7$ tiene una recta límite en $x + y = 7$. La recta es continua porque los puntos en la recta $x + y = 7$ se encuentran dentro de la región solución.

English

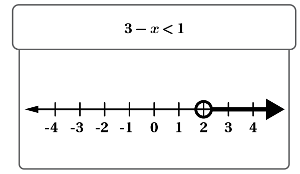
boundary point The value that separates the solution set of an inequality from non-solutions. A solid boundary point indicates that the point is included in the solution set. An empty boundary point indicates the point is not included in the solution set.



The solution set to $3 - x < 1$ has a boundary point at $x = 2$. The point at 2 is empty because 2 is not included in the solution set.

Español

punto límite El valor que separa el conjunto de soluciones de una desigualdad de todos los valores que no son soluciones. Un punto límite sólido (relleno) indica que el punto está incluido en el conjunto de soluciones. Un punto límite vacío indica que el punto no está incluido en el conjunto de soluciones.



El conjunto de soluciones de $3 - x < 1$ tiene un punto límite en $x = 2$. El punto en 2 no está relleno porque 2 no está incluido en el conjunto de soluciones.

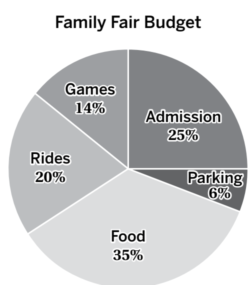
C

categorical data A type of data that is divided into groups or categories. Categorical data describes qualities or characteristics, such as colors, names, or zip codes, rather than numerical values.

For example, the breeds of 10 dogs are categorical data. Another example is the colors of 100 flowers.

causation Describes a relationship between two variables in which a change in one variable causes a change in the other variable. A special type of *correlation*.

circle graph A visual display of categorical data. The whole set of data is represented by a circle and its interior. The categories are represented by fractional parts of the circle. Also called a *pie chart*.



coefficient The number that multiplies a variable in an algebraic expression. If no number is shown, the coefficient is understood to be 1.

For example, in the expression $5x$, the coefficient is 5. In y, the coefficient is 1.

completing the square The process of rewriting a quadratic expression or equation to include a perfect square.

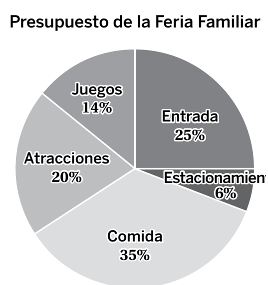
$$x^2 - 6x + 17$$
$$(x^2 - 6x + 9) - 9 + 17$$
$$(x - 3)^2 - 9 + 17$$
$$(x - 3)^2 + 8$$

datos categóricos Un tipo de datos que se dividen en grupos o categorías. Los datos categóricos describen cualidades o características, como colores, nombres o códigos postales, en vez de valores numéricos.

Por ejemplo, las razas de 10 perros diferentes son datos categóricos. Otro ejemplo son los colores de 100 flores diferentes.

causalidad Describe una relación entre dos variables en la que un cambio en una variable provoca un cambio en la otra variable. Un tipo especial de *correlación*.

gráfico circular Una representación visual de datos categóricos. Todo el conjunto de datos está representado por un círculo y su interior. Las categorías están representadas por partes fraccionarias del círculo. También llamado *gráfico de torta*.



coeficiente El número que multiplica una variable en una expresión algebraica. Si no se muestra ningún número, se entiende que el coeficiente es 1.

Por ejemplo, en la expresión $5x$, el coeficiente es 5. En y, el coeficiente es 1.

completación del cuadrado El proceso de reescribir una expresión o ecuación cuadrática para incluir un cuadrado perfecto.

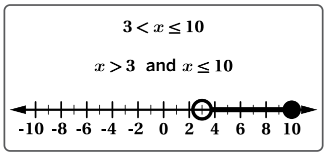
$$x^2 - 6x + 17$$
$$(x^2 - 6x + 9) - 9 + 17$$
$$(x - 3)^2 - 9 + 17$$
$$(x - 3)^2 + 8$$

English

compound To increase in value by applying an operation repeatedly. In mathematics, compounding can involve repeated multiplication or growth over time.

For example, when bacteria multiply by splitting in half every hour, the number of bacteria compounds, doubling each hour.

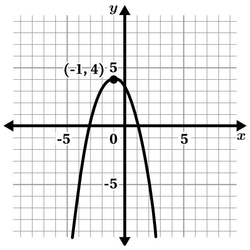
compound inequality Two or more inequalities joined together. A compound inequality can be written using symbols or the words "and" or "or."



The numbers greater than 3 and less than or equal to 10 can be written as: $x > 3$ and $x \leq 10$ or $3 < x \leq 10$.

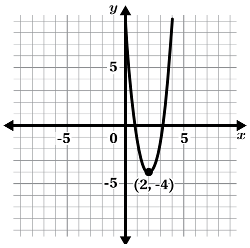
compound interest Interest calculated based on the initial amount and the interest from previous periods. It is calculated at regular intervals (daily, monthly, annually, etc.). The balance in an account that earns compound interest can be modeled by an exponential function.

concave down A parabola that opens downward is described as *concave down*. A function with a negative a-value will produce a concave down parabola.



This parabola is concave down. Two ways to write the equation are $f(x) = -1(x + 1)^2 + 4$ and $f(x) = -x^2 - 2x + 3$.

concave up A parabola that opens upward is described as *concave up*. A function with a positive a-value will produce a concave up parabola.



This parabola is concave up. Two ways to write the equation are $f(x) = 4(x - 2)^2 - 4$ and $f(x) = 4x^2 - 16x + 12$.

confidence interval A range of values used to estimate an unknown population value. It shows how likely it is that the true value is within the range.

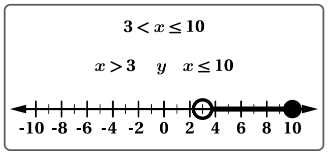
A survey estimates that 60% of students prefer online learning, with a confidence interval of 55% to 65%. This means the true percentage is likely between 55% and 65%.

Español

crecimiento compuesto Aumentar en valor aplicando una operación repetidamente. En matemáticas, el crecimiento compuesto puede implicar la multiplicación repetida o el crecimiento a lo largo del tiempo.

Por ejemplo, cuando las bacterias se multiplican dividiéndose por la mitad cada hora, el número de bacterias crece de forma compuesta y se duplica cada hora.

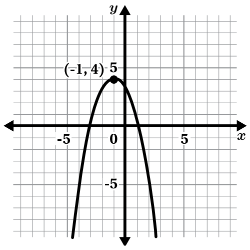
desigualdad compuesta Dos o más desigualdades juntas. Una desigualdad compuesta puede escribirse con signos o las palabras "y" u "o".



Los números mayores que 3 y menores o iguales que 10 pueden escribirse de la siguiente forma: $x > 3$ y $x \leq 10$ o $3 < x \leq 10$.

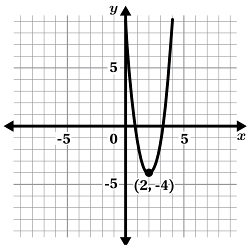
interés compuesto Interés que se calcula según una cantidad inicial y el interés de los períodos anteriores. Se calcula en intervalos regulares (diariamente, mensualmente, anualmente, etc.). El saldo de una cuenta que produce interés compuesto puede modelarse con una función exponencial.

cóncava hacia abajo Una parábola que se abre hacia abajo se describe como cóncava hacia abajo. Una función con un valor a negativo producirá una parábola cóncava hacia abajo.



Esta parábola es cóncava hacia abajo. Dos formas de escribir la ecuación son $f(x) = -1(x + 1)^2 + 4$ y $f(x) = -x^2 - 2x + 3$.

cóncava hacia arriba Una parábola que se abre hacia arriba se describe como cóncava hacia arriba. Una función con un valor a positivo producirá una parábola cóncava hacia arriba.



Esta parábola es cóncava hacia arriba. Dos formas de escribir la ecuación son $f(x) = 4(x - 2)^2 - 4$ y $f(x) = 4x^2 - 16x + 12$.

intervalo de confianza Un rango de valores utilizado para estimar un valor poblacional desconocido. Muestra la probabilidad de que el valor real esté dentro del rango.

Una encuesta estima que el 60 % de los estudiantes prefieren el aprendizaje en línea, con un intervalo de confianza del 55 % al 65 %. Esto significa que el porcentaje real probablemente esté entre el 55 % y el 65 %.

Glossary/Glosario

English

constant difference When the difference between every two consecutive values in a pattern is the same, there is a *constant difference*.

The pattern in the table has a constant difference of 2.

x	y	
0	5	+2
1	7	+2
2	9	+2
3	11	

constant ratio When the ratio between every two consecutive values in a pattern is the same, there is a *constant ratio*.

The pattern in the table has a constant ratio of 3.

x	y	
0	1	·3
1	3	·3
2	9	·3
3	27	

constraint A limitation on the possible values of variables in a model. Equations and inequalities are often used to represent constraints.

The constraint that "you must be 36 inches or taller to ride the Ferris wheel" can be represented by the inequality $h \geq 36$.

correlation A statistical relationship between two or more variables. Also called an *association*.

cube root The cube root of a number n (written as $\sqrt[3]{n}$) is the number that can be cubed to get n. The cube root is also the edge length of a cube with a volume of n.

For example, the cube root of 64 ($\sqrt[3]{64}$) is 4 because 4^3 is 64. 4 is also the edge length of a cube that has a volume of 64 cubic units.

Español

diferencia constante Cuando la diferencia entre dos valores consecutivos en un patrón permanece igual, hay una diferencia constante.

El patrón en la tabla tiene una diferencia constante de 2.

x	y	
0	5	+2
1	7	+2
2	9	+2
3	11	

razón constante Cuando la razón entre dos valores consecutivos en un patrón permanece igual, hay una razón constante.

El patrón en la tabla tiene una razón constante de 3.

x	y	
0	1	·3
1	3	·3
2	9	·3
3	27	

restricción Una limitación de los posibles valores de las variables en un modelo. Suelen usarse ecuaciones o desigualdades para representar restricciones.

La restricción "debes medir 36 pulgadas o más para subirte a la rueda de la fortuna" puede representarse con la desigualdad $h \geq 36$.

correlación Una relación estadística entre dos o más variables. También se denomina *asociación*.

raíz cúbica La raíz cúbica de un número n (se escribe $\sqrt[3]{n}$) es el número que puede elevarse al cubo para obtener n. La raíz cúbica también es la longitud de la arista de un cubo con un volumen de n.

Por ejemplo, la raíz cúbica de 64 ($\sqrt[3]{64}$) es 4 porque 4^3 es 64. 4 también es la longitud de arista de un cubo que tiene un volumen de 64 unidades cúbicas.

D

data A collection of information, usually in the form of numbers, words, or measurements, that can be analyzed.

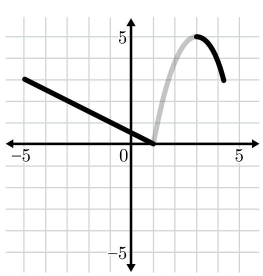
A list of students' test scores is an example of numerical data.

data sets A collection of values gathered for reference, analysis, or interpretation.

A list of test scores collected from a class is an example of data.

decreasing A function, or interval of a function, is decreasing if the y-values go down when the x-values go up.

The bolded parts of this function are decreasing.



datos Una recopilación de información, generalmente en forma de números, palabras o medidas, que se puede analizar.

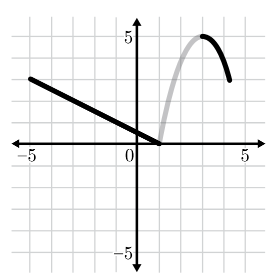
Una lista de las calificaciones de los exámenes de unos estudiantes es un ejemplo de datos numéricos.

conjuntos de datos Una recopilación de valores reunidos para referencia, análisis o interpretación.

Una lista de las calificaciones de exámenes recopiladas de una clase es un ejemplo de datos.

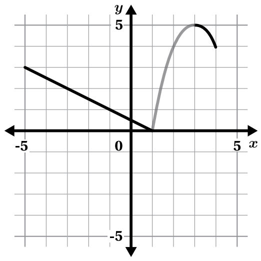
decreciente Una función, o un intervalo de una función, es decreciente si los valores de y disminuyen cuando los valores de x aumentan.

Las partes resaltadas de esta función son decrecientes.



English

decreasing (interval or function) A function is decreasing when its outputs decrease as its inputs increase. A function can be decreasing for its entire domain or over an interval.



For example, the function $f(x)$ is decreasing when $-5 < x < 1$ and when $3 < x < 4$.

dependent variable The value of a dependent variable is based on the value of another variable or set of variables. In a function, the value of the dependent variable represents the output. The dependent variable is typically on the vertical axis of a graph and in the right-hand column of a table.

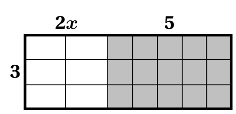
difference of squares An expression that can be written as one perfect square subtracted from another perfect square. It has the structure $r^2 - s^2$ in standard form and $(r - s)(r + s)$ in factored form.

These expressions are differences of squares: $x^2 - 25$, $9x^2 - 100$, and $16x^2 - 1$.

discrete A set of values is discrete if there is separation between the values. A graph can be described as discrete when it consists of unconnected points or intervals.

The set of numbers 1, 2, 3 is discrete, while the set of all values between 1 and 3 is not discrete.

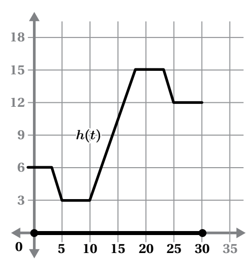
distributive property Multiplying a number by the sum of two or more terms is equal to multiplying the number by each term separately before adding them together.



Area as a Product
$3(2x + 5)$

For example, $3(2x + 5) = 3 \cdot 2x + 3 \cdot 5 = 6x + 15$.

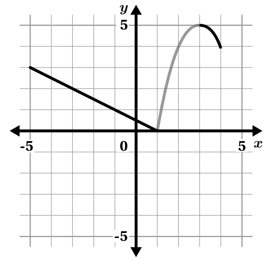
domain The set of all possible input values for a function or relation. The domain can be described in words or as an inequality.



The domain of this graph can be described as: All numbers from 0 to 30 or $0 \leq t \leq 30$.

Español

decreciente (intervalo o función) Una función es decreciente cuando sus valores de salida decrecen a medida que crecen sus valores de entrada. Una función puede ser decreciente en todo su dominio o en un intervalo.



Por ejemplo, la función $f(x)$ es decreciente cuando $-5 < x < 1$ y cuando $3 < x < 4$.

variable dependiente El valor de una variable dependiente se basa en el valor de otra variable o un conjunto de variables. En una función, el valor de la variable dependiente representa la salida.
La variable dependiente suele estar en el eje vertical de una gráfica y en la columna derecha de una tabla.

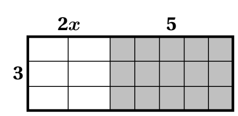
diferencia de cuadrados Una expresión que puede escribirse como un cuadrado perfecto que se resta a otro cuadrado perfecto. Tiene la estructura $r2 - s2$ en forma estándar y $(r - s)(r + s)$ en forma factorizada.

Estas expresiones son diferencias de cuadrados: $x^2 - 25$, $9x^2 - 100$ y $16x^2 - 1$.

discreto Un conjunto de valores es discreto si hay separación entre los valores. Una gráfica puede describirse como discreta cuando consta de puntos o intervalos que no se conectan.

El conjunto de números 1, 2, 3 es discreto, mientras que el conjunto de todos los valores entre 1 y 3 no es discreto.

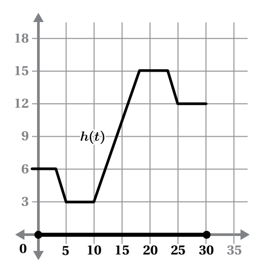
propiedad distributiva Multiplicar un número por la suma de dos o más términos equivale a multiplicar el número por cada término individualmente antes de sumarlos.



El área como un producto
$3(2x + 5)$

Ejemplo: $3(2x + 5) = 3 \cdot 2x + 3 \cdot 5 = 6x + 15$.

dominio El conjunto de todos los valores de entrada posibles de una función o relación. El dominio puede describirse con palabras o como una desigualdad.



El dominio de esta gráfica puede describirse de la siguiente manera: Todos los números del 0 al 30 o $0 \leq t \leq 30$.

Glossary/Glosario

English

E

elimination A method of solving systems of equations where you add or subtract the equations to produce a new equation with fewer variables.

$$\begin{array}{r} 9x + y = 2 \\ -(3x + y = 10) \\ \hline 6x + 0 = -8 \end{array}$$

In the example, subtraction is used to eliminate y and create an equation that can be solved for x.

equation Two expressions with an equal sign between them.

$-3(2x + 5) = 5 + x$ is an example of an equation.

equivalent equations Equations that have the same solution(s).

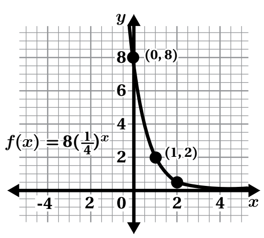
$3x + 4 = 10$ and $9x + 12 = 30$ are equivalent equations because if you multiply the first equation by 3, you create the second. The solution to each equation is $x = 2$.

equivalent systems Two or more systems of equations that have the same set of solutions.

exponent Exponents describe repeated multiplication. The exponent describes the number of times to multiply the base by itself.

For example, in the expression 2^3, 2 is the base and 3 is the exponent. In the expression $50(1.6)^{12x}$, 1.6 is the base and $12x$ is the exponent.

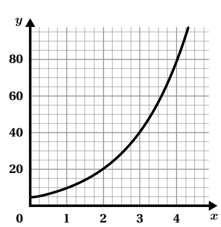
exponential decay An exponential relationship that decreases. An exponential decay relationship has a growth factor between 0 and 1.



For example, $f(x) = 8\left(\frac{1}{4}\right)^x$ is an example of exponential decay because the growth factor, $\frac{1}{4}$, is between 0 and 1.

exponential function A function that increases or decreases by a constant percent rate of change. A function that changes by equal factors over equal intervals.

x	$f(x)$
0	5
1	10
2	20
3	40



For example, the table and graph show the exponential function $f(x) = 5 \cdot (2)^x$, which has an initial value of 5 and has a growth factor of 2.

Español

eliminación Un método para resolver sistemas de ecuaciones en el que se suman o restan las ecuaciones para producir una nueva ecuación con menos variables.

$$\begin{array}{r} 9x + y = 2 \\ -(3x + y = 10) \\ \hline 6x + 0 = -8 \end{array}$$

En el ejemplo, se usa la resta para eliminar y y producir una ecuación que puede resolverse para determinar el valor de x.

ecuación Dos expresiones con un signo igual entre ambas.

$-3(2x + 5) = 5 + x$ es un ejemplo de una ecuación.

ecuaciones equivalentes Ecuaciones que tienen exactamente la misma o las mismas soluciones.

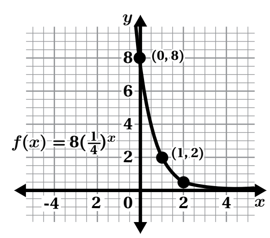
$3x + 4 = 10$ y $9x + 12 = 30$ son ecuaciones equivalentes porque si se multiplica la primera por 3, se forma la segunda. La solución de cada ecuación es $x = 2$.

sistemas equivalentes Dos o más sistemas de ecuaciones que tienen el mismo conjunto de soluciones.

exponente Los exponentes describen una multiplicación repetida. El exponente describe el número de veces que se multiplica la base por sí misma.

Por ejemplo, en la expresión 2^3, 2 es la base y 3 es el exponente. En la expresión $50(1.6)^{12x}$, 1.6 es la base y $12x$ es el exponente.

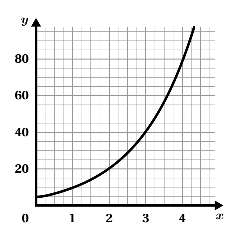
decaimiento exponencial Una relación exponencial que decrece. Una relación de decaimiento exponencial tiene un factor de crecimiento entre 0 y 1.



Por ejemplo, $f(x) = 8\left(\frac{1}{4}\right)^x$ es un ejemplo de decaimiento exponencial porque el factor de crecimiento, $\frac{1}{4}$, está entre 0 y 1.

función exponencial Una función que crece o decrece con una tasa porcentual de cambio constante. Una función que cambia por factores iguales en intervalos iguales.

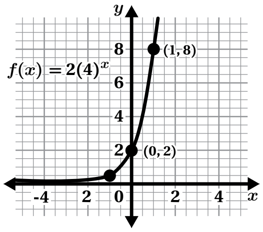
x	$f(x)$
0	5
1	10
2	20
3	40



Por ejemplo, la tabla y la gráfica muestran la función exponencial $f(x) = 5 \cdot (2)^x$, que tiene un valor inicial de 5 y un factor de crecimiento de 2.

English

exponential growth An exponential relationship that increases. An exponential growth relationship has a growth factor greater than 1.



For example, $f(x) = 2(4)^x$ is an example of exponential growth because the growth rate, 4, is greater than 1.

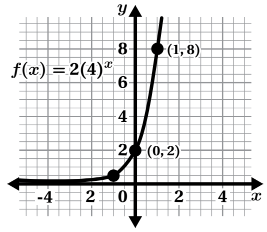
exponential relationship See *exponential function*.

expression A set of numbers, variables, operations, and grouping symbols that represent a quantity.

$5,\ 2x,\ 7 - 3^p$, and $4(2 - 5x^2) + 8$ are all examples of expressions.

Español

crecimiento exponencial Una relación exponencial que crece. Una relación con crecimiento exponencial tiene un factor de crecimiento mayor que 1.



Por ejemplo, $f(x) = 2(4)^x$ es un ejemplo de crecimiento exponencial porque la tasa de crecimiento, 4, es mayor que 1.

relación exponencial Ver función exponencial.

expresión Un conjunto de números, variables, operaciones y signos de agrupación que representan una cantidad.

$5,\ 2x,\ 7 - 3^p$ y $4(2 - 5x^2) + 8$ son ejemplos de expresiones.

F

factor (of a number or expression) A number or expression multiplied with other numbers or expressions to make a product.

For example, $1, 2, 4,$ and 8 are all factors of the number 8 because $1 \cdot 8 = 8$ and $2 \cdot 4 = 8$. Also, $(x + 3)$ and $(x - 5)$ are factors of $x^2 - 2x - 15$ because $(x + 3)(x - 5) = x^2 - 2x - 15$.

factored form One of three common forms of a quadratic equation. A quadratic equation in factored form looks like $f(x) = a(x - m)(x - n)$.

These equations are in factored form:
$g(x) = x(x + 10)$
$2(x - 1)(x + 3) = y$
$y = (5x + 2)(3x - 1)$

frequency table A table that shows the number of times each value or category occurs in a data set.

Favorite Color	Frequency
Red	5
Blue	3
Pink	4

For example, this frequency table shows that 5 students selected red as their favorite color.

function A rule, or relation, that assigns exactly one output to each possible input. Every function is a relation, but not every relation is a function. In a function, the value of the output variable depends on the value of the input variable.

factor (de un número o una expresión) Un número o una expresión que se multiplica por otros números o expresiones para dar como resultado un producto.

Por ejemplo, $1, 2, 4$ y 8 son factores del número 8 porque $1 \cdot 8 = 8$ y $2 \cdot 4 = 8$. Además, $(x + 3)$ y $(x - 5)$ son factores de $x^2 - 2x - 15$ porque $(x + 3)(x - 5) = x^2 - 2x - 15$.

forma factorizada Una de las tres formas comunes de una ecuación cuadrática. Una ecuación cuadrática en forma factorizada tiene el siguiente orden: $f(x) = a(x - m)(x - n)$.

Estas ecuaciones están en forma factorizada:
$g(x) = x(x + 10)$
$2(x - 1)(x + 3) = y$
$y = (5x + 2)(3x - 1)$

tabla de frecuencia Una tabla que muestra el número de veces que aparece cada valor o categoría en un conjunto de datos.

Color favorito	Frecuencia
Rojo	5
Azul	3
Rosado	4

Por ejemplo, esta tabla de frecuencia muestra que 5 estudiantes seleccionaron el rojo como su color favorito.

función Una regla, o relación, que asigna exactamente una salida a cada entrada posible. Toda función es una relación, pero no toda relación es una función. En una función, el valor de la variable de salida depende del valor de la variable de entrada.

English

function notation A notation used to represent the inputs and outputs of a function. For a function f, when x is an input (or domain value), the symbol $f(x)$ shows the corresponding output (or range value).

$$f(x) = 2x + 1$$
$$f(4) = 9$$

For example, $f(4) = 9$ is a statement written in function notation. It says that when the input of the function f is 4, the output is 9. In other words, when the value of the independent variable is 4, the value of the dependent variable is 9.

Español

notación de función Una notación utilizada para representar las entradas y las salidas de una función. Para una función (x), cuando x es una entrada (o valor de dominio), el símbolo $f(x)$ muestra la salida correspondiente (o valor de rango).

$$f(x) = 2x + 1$$
$$f(4) = 9$$

Por ejemplo, $f(4) = 9$ es una expresión escrita en notación de funciones. Indica que cuando la entrada de la función f es 4, la salida es 9. En otras palabras, cuando el valor de la variable independiente es 4, el valor de la variable dependiente es 9.

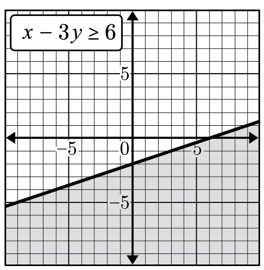
G

growth factor The constant ratio (or common factor) that each term is multiplied by to generate an exponential pattern.

factor de crecimiento La razón constante (o el factor común) que multiplica a cada término para generar un patrón exponencial.

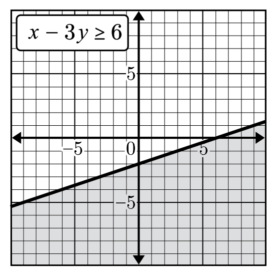
H

half-plane A region that defines the solution set of a single two-variable linear inequality. The boundary line of the inequality splits the coordinate plane into two equal half-planes.



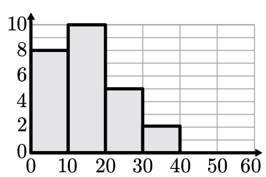
The graph shows the solution region to $x - 3y \geq 6$. The boundary line $x - 3y = 6$ divides the coordinate plane into two equal half-planes.

semiplano Una región que define el conjunto de soluciones de una desigualdad lineal con dos variables. La recta límite de la desigualdad divide el plano de coordenadas en dos semiplanos iguales.



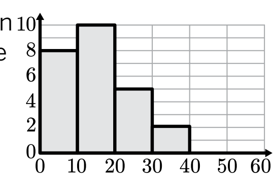
La gráfica muestra la región solución de $x - 3y \geq 6$. La recta límite $x - 3y = 6$ divide el plano de coordenadas en dos semiplanos iguales.

histogram A visual display of quantitative data that groups values into intervals, called *bins*. Each bin is represented by a rectangle, where the height shows the frequency or relative frequency of the data in that bin.



For example, this histogram shows that there are 8 values between 0 and 10.

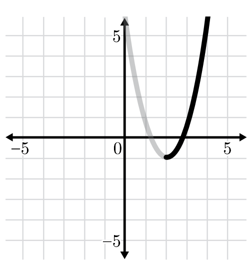
histograma Una representación visual de datos cuantitativos que agrupa los valores en intervalos, llamados *contenedores*. Cada contenedor está representado por un rectángulo, donde la altura muestra la frecuencia o la frecuencia relativa de los datos en ese contenedor.



Por ejemplo, este histograma muestra que hay 8 valores entre 0 y 10.

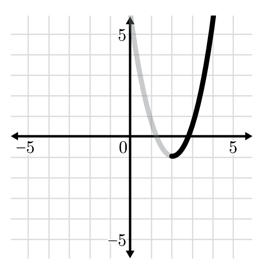
I

increasing (interval or function) A function is increasing when its outputs increase as its inputs increase. A function can be increasing for its entire domain or over an interval.



For example, the function $f(x)$, whose graph is shown, is increasing when $x > 2$.

creciente (intervalo o función) Una función es creciente cuando sus valores de salida crecen a medida que crecen sus valores de entrada. Una función puede ser creciente en todo su dominio o en un intervalo.



Por ejemplo, la función $f(x)$, cuya gráfica se muestra, es creciente cuando $x > 2$.

English

independent variable The value of an independent variable is not based on the value of any other variable. In a function, the value of the independent variable represents the input. The independent variable is typically on the horizontal axis of a graph and in the left-hand column of a table.

index In a radical $\sqrt[n]{x}$ the quantity n is called the *index*. (See *radical*.)

inequality A comparison statement that uses the symbols $<$, $>$, $\leq$, or $\geq$. Inequalities are used to represent the relationship between numbers, variables, or expressions that are not always equal.

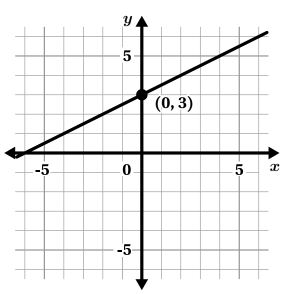
For example, the inequality $y + 2 \geq 30$ means that the value of the expression $y + 2$ will be greater than or equal to 30.

infinitely many solutions An equation has infinitely many solutions if it is true for any value of the variable. A system of equations has infinitely many solutions if the equations in the system are equivalent. In a system of equations with infinitely many solutions, every point on the graph is a solution to the system.

For example, the equation $3x + 6 = 3(x + 2)$ has infinitely many solutions because the equation is true for any value of x.

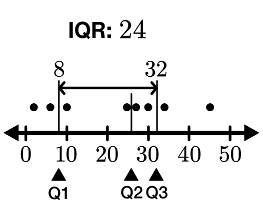
initial value A point where the graph of an equation or function crosses the y-axis or when $x = 0$.

The initial value of the graph $-2x + 4y = 12$ is $(0, 3)$, or just 3.



input In a function, any value that you substitute for x is called an input. The input is sometimes called the *independent variable*. The input typically appears on the horizontal axis of a graph and in the left-hand column of a table.

interquartile range (IQR) A measure of variation in a set of numerical data. The interquartile range is the distance between the first quartile (Q1) and the third quartile (Q3) of the data set.



For example, the IQR of this data set is $32 - 8 = 24$.

Español

variable independiente El valor de una variable independiente no depende del valor de ninguna otra variable. En una función, el valor de la variable independiente representa la entrada. La variable independiente suele estar en el eje horizontal de una gráfica y en la columna izquierda de una tabla.

índice En un radical $\sqrt[n]{x}$, la cantidad n se llama índice. (Ver *radical*).

desigualdad Un enunciado de comparación que utiliza los signos $<$, $>$, $\leq$ o $\geq$. Las desigualdades se usan para representar la relación entre números, variables o expresiones que no siempre son iguales.

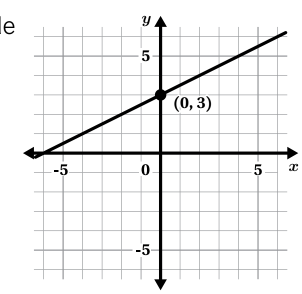
Por ejemplo, la desigualdad $y + 2 \geq 30$ significa que el valor de la expresión $y + 2$ será mayor o igual que 30.

infinitas soluciones Una ecuación tiene infinitas soluciones si es verdadera sea cual sea el valor de la variable. Un sistema de ecuaciones tiene infinitas soluciones si las ecuaciones del sistema son equivalentes. En un sistema de ecuaciones con infinitas soluciones, cada punto de la gráfica es una solución del sistema.

Por ejemplo, la ecuación $3x + 6 = 3(x + 2)$ tiene infinitas soluciones porque la ecuación es verdadera con cualquier valor de x.

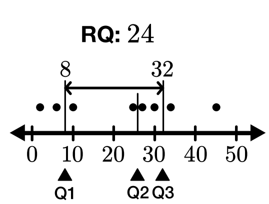
valor inicial Un punto donde la gráfica de una ecuación o función cruza el eje y, o cuando $x = 0$.

El valor inicial de la gráfica de $-2x + 4y = 12$ es $(0, 3)$, o simplemente 3.



entrada En una función, todo valor que sustituya a x se denomina entrada. La entrada a veces se denomina variable independiente.La entrada suele estar en el eje horizontal de una gráfica y en la columna izquierda de una tabla.

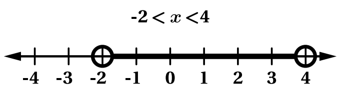
rango intercuartil (RIC) Una medida de variación en un conjunto de datos numéricos. El rango intercuartil es la distancia entre el primer cuartil (Q1) y el tercer cuartil (Q3) del conjunto de datos.



Por ejemplo, el RQ de este conjunto de datos es $32 - 8 = 24$.

English

interval A set of values between two points.



In words: All numbers between -2 and 4.
Inequality: $-2 < x < 4$.

interval notation A way to represent a range of values on a number line using brackets and parentheses.

$(2, 6]$ represents all values between 2 and 6, not including 2 but including 6. $[-3, 5)$ represents all values between -3 and 5, including -3 but not including 5.

inverse operations Two operations that undo each other are called *inverse operations*.

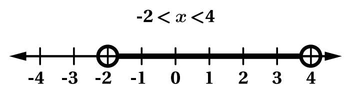
Adding and subtracting are inverse operations. Multiplying and dividing are inverse operations.

irrational number A number that cannot be written as a fraction of two integers, where the denominator is not zero.

2 is a rational number because it can be written as $\frac{2}{1}$, whereas $\sqrt{3}$ is irrational because it cannot be written as a fraction made up of two integers.

Español

intervalo Un conjunto de valores entre dos puntos.



Con palabras: Todos los números entre el -2 y el 4. Desigualdad: $-2 < x < 4$.

notación de intervalo Una forma de representar un rango de valores en una recta numérica usando corchetes y paréntesis.

$(2, 6]$ representa todos los valores entre 2 y 6, excluyendo 2 pero incluyendo 6. $[-3, 5)$ representa todos los valores entre -3 y 5, incluyendo -3 pero excluyendo 5.

operaciones inversas Dos operaciones que se anulan entre sí se llaman operaciones inversas.

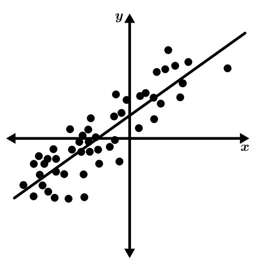
La suma y la resta son operaciones inversas. La multiplicación y la división son operaciones inversas.

número irracional Un número que no se puede escribir como una fracción de dos números enteros, donde el denominador es diferente de cero.

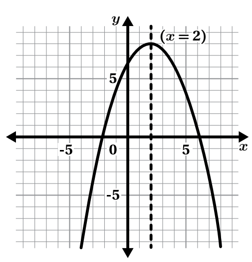
2 es un número racional porque se puede escribir como $\frac{2}{1}$, mientras que $\sqrt{3}$ es irracional porque no se puede escribir como una fracción de dos números enteros.

L

line of fit The line on a scatter plot that best represents the trend created by the points in a data set. There are many lines of fit for a single data set, but there is only one line of best fit.

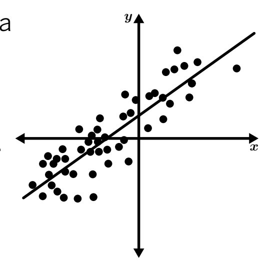


line of symmetry A line that divides a figure or the graph of a function into two halves. For every point (except the vertex), there is a corresponding point on the other side of the line that is the same distance from the line.

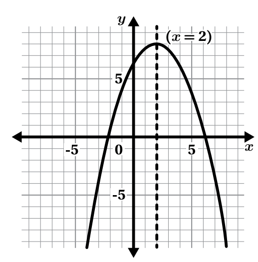


The equation of this line of symmetry is $x = 2$.

línea de (mejor) ajuste La línea de un diagrama de dispersión que mejor representa la tendencia que definen los puntos de un conjunto de datos. Existen muchas líneas de ajuste para un mismo conjunto de datos, pero solo hay una línea de mejor ajuste.



eje de simetría Una línea que divide una figura o la gráfica de una función en dos mitades. Cada punto (excepto el vértice) tiene un punto correspondiente en el otro lado de la línea, el cual está a la misma distancia de la línea.



La ecuación de este eje de simetría es $x = 2$.

English

linear function A function that increases or decreases by a constant rate of change, or shows a linear relationship. A function that changes by equal differences over equal intervals. The graph of a linear function is a line.

	x	f(x)	
	0	5	
+1	1	11	+6
+1	2	17	+6
+1	3	23	+6

For example, $f(x) = 5 + 6x$ represents a linear function that has an initial value of 5 and increases by a constant difference of 6.

linear relationship See *linear function*.

lurking variable An additional variable that has an effect on the other variables being analyzed. Lurking variables can lead us to make conclusions about data and relationships that are inaccurate.

M

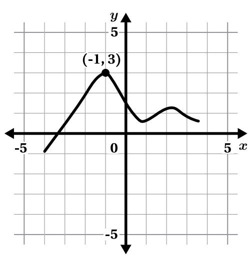
margin of error A measure that shows how much the results of a survey or study might differ from the true population value.

If a survey shows that 60% of people like a product with a margin of error of $\pm 3\%$, the true percentage is likely between 57% and 63%.

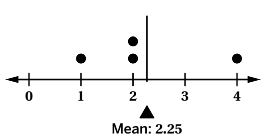
maximum The greatest value in a data set.

maximum (of a function) The highest point on the graph.

The maximum of this function is $(-1, 3)$.



mean A statistic used to describe the typical value of a data set. To calculate the mean, you can add up all the data values and divide by the number of data points. The mean is a measure of center and is also called the *average*.



In this data set, the mean is 2.25.

$1 + 2 + 2 + 4 = 9$

$\frac{9}{4} = 2.25$

Español

función lineal Función que crece o decrece con una tasa de cambio constante o muestra una relación lineal. Una función que cambia con diferencias iguales en intervalos iguales. La gráfica de una función lineal es una recta.

	x	f(x)	
	0	5	
+1	1	11	+6
+1	2	17	+6
+1	3	23	+6

Por ejemplo, $f(x) = 5 + 6x$ representa una función lineal que tiene un valor inicial de 5 y crece con una diferencia constante de 6.

relación lineal Ver *función lineal*.

variable de confusión Una variable adicional que tiene un efecto sobre las demás variables que se analizan. Las variables de confusión pueden producir conclusiones imprecisas en torno a los datos y las relaciones.

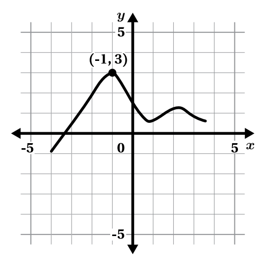
margen de error Una medida que muestra cuánto pueden diferir los resultados de una encuesta o un estudio del valor real de la población.

Si una encuesta muestra que al 60 % de las personas les gusta un producto con un margen de error de ± 3 %, el porcentaje real probablemente esté entre el 57 % y el 63 %.

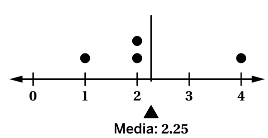
máximo El mayor valor en un conjunto de datos.

máximo (de una función) El punto más alto en una gráfica.

El máximo de esta función está en $(-1, 3)$.



media Una estadística que se usa para describir el valor típico de un conjunto de datos. Para calcular la media, se suman todos los valores y el resultado se divide por el número de puntos de datos. La media es una medida de tendencia central y también se denomina promedio.



En este conjunto de datos, la media es 2.25.

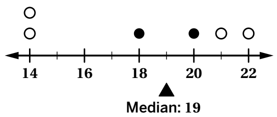
$1 + 2 + 2 + 4 = 9$

$\frac{9}{4} = 2.25$

English

measure of center A numerical value that describes the overall center or clustering of data in a set. The three most common measures of central tendency are the mean, median, and mode.

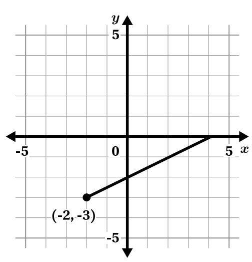
median A statistic used to describe the typical value of a data set. It is the middle value of a data set when the values are in numerical order. If there are two values in the middle of the data set, then the median is the mean of those two values. The median is a measure of center.



maximum The greatest value in a data set.

minimum (of a function) The lowest point on the graph.

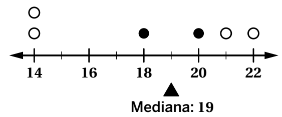
The minimum of this function is (-2, -3).



Español

medida de centro Un valor numérico que describe el centro general o la agrupación de datos en un conjunto. Las tres medidas de tendencia central más comunes son la media, la mediana y la moda.

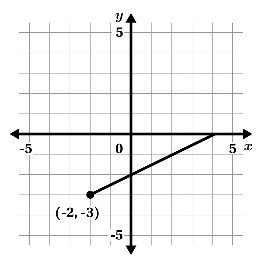
mediana Una estadística que se usa para describir el valor típico de un conjunto de datos. Es el valor del medio de un conjunto de datos cuando los valores se clasifican en orden numérico. Cuando hay dos valores en el medio del conjunto de datos, la mediana es la media de esos dos valores. La mediana es una medida de tendencia central.



máximo El mayor valor en un conjunto de datos.

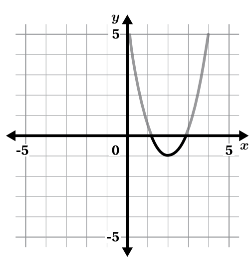
mínimo (de una función) El punto más bajo en una gráfica.

El mínimo de esta función está en (-2, -3).



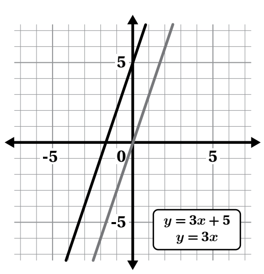
N

negative (interval or function) A function is negative when its outputs are negative and its graph is below the x-axis. A function can be negative for its entire domain or over an interval.



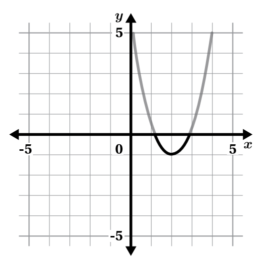
This function is negative when $1 < x < 3$.

no solution An equation has no solution if there is no value of the variable that will make the equation true. A system of equations has no solution if there is no set of values that makes all the equations in the system true. In a system of equations with no solution, there is no point that is on the graph of every equation in the system.



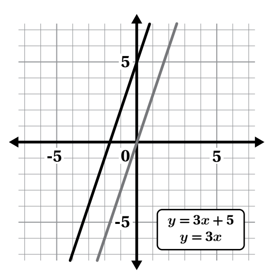
For example, the system of equations containing $y = 3x + 5$ and $y = 3x$ has no solution because the graphs are parallel and never intersect.

negativo (intervalo o función) Una función es negativa cuando sus salidas son negativas y su gráfica está por debajo del eje x. Una función puede ser negativa en todo su dominio o en un intervalo.



Esta función es negativa cuando $1 < x < 3$.

sin solución Una ecuación no tiene solución si no hay ningún valor de la variable que haga que la ecuación sea verdadera. Un sistema de ecuaciones no tiene solución si no hay ningún conjunto de valores que haga que todas las ecuaciones de ese sistema sean verdaderas. En un sistema de ecuaciones sin solución, no hay ningún punto que esté en la gráfica de cada una de las ecuaciones del sistema.



Por ejemplo, el sistema de ecuaciones que contiene $y = 3x + 5$ y $y = 3x$ no tiene solución porque las gráficas son paralelas y nunca se intersecan.

English

Español

non-linear relationship A pattern, table, scenario, graph, or equation that does not have a constant rate of change. A non-linear relationship is called non-linear because its graph is not a line.

Day	Teal Globs
0	1
1	2
2	5
3	10

The equation $y = x^2 + 1$ represents a non-linear relationship because its rate of change is not constant and its graph is not a line.

relación no lineal Patrón, tabla, contexto, gráfica o ecuación que no tiene una tasa de cambio constante. Una relación no lineal se denomina no lineal porque su gráfica no es una recta.

Día	Manchitas turquesa
0	1
1	2
2	5
3	10

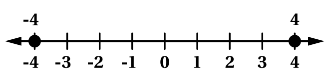
La ecuación $y = x^2 + 1$ representa una relación no lineal porque su tasa de cambio no es constante y su gráfica no es una recta.

numerical data See *quantitative data*.

datos numéricos Ver *datos cuantitativos*.

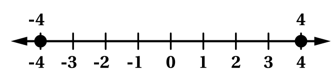
O

opposite Two numbers that are the same distance from 0 and on different sides of the number line. Two terms that are opposites are also referred to as zero pairs and additive inverses.



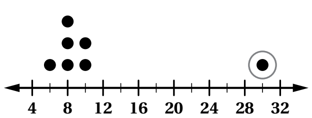
For example, 4 and -4 are opposites.

opuestos Dos números que están a la misma distancia del 0 y en diferentes lados de la recta numérica. Dos términos que son opuestos también se conocen como inversos aditivos.



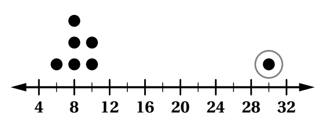
Por ejemplo, 4 y -4 son opuestos.

outlier A data value that is far from the other values in the data set. Values that are 1.5 times the IQR from either quartile 1 or quartile 3 are outliers.



The circled point is an outlier.

valor atípico Un valor de datos que está lejos de los demás valores del conjunto de datos. Son valores atípicos aquellos que están a 1.5 veces el RQ del cuartil 1 o del cuartil 3.



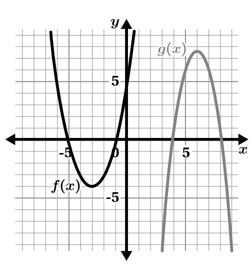
El punto encerrado en un círculo es un valor atípico.

output The value of a function after it has been evaluated for an input value of x is called the *output*. The output is sometimes called the *dependent variable*. The output typically appears on the vertical axis of a graph and in the right-hand column of a table.

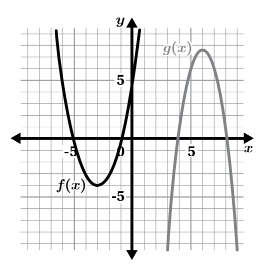
salida El valor de una función tras haber sido evaluada con un valor de entrada x se denomina salida. La salida a veces se denomina variable dependiente. La salida suele estar en el eje vertical de una gráfica y en la columna derecha de una tabla.

P

parabola The graph of a quadratic function, which is a U-shaped curve.

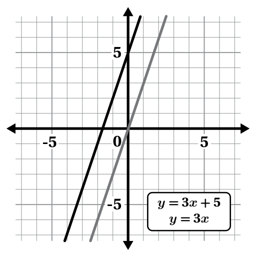


parábola La gráfica de una función cuadrática, que es una curva en forma de U.



English

parallel lines Lines that never cross or intersect. On a graph, two lines with the same slope and different y-intercepts are parallel.

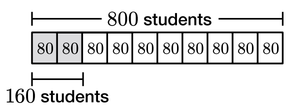


For example, the lines $y = 3x + 5$ and $y = 3x$ are parallel and never intersect.

percent decrease Describes how much a quantity goes down, expressed as a percent of the starting amount.

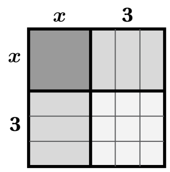
For example, the function $f(x) = 1000(.78)^x$ represents the value of a phone with an initial value of $1,000 with a percent decrease in value of 22% each year, x.

percent (percentage) Percent means for every 100. It is represented by the percent symbol: %. We use percentages to represent ratios and fractions.



For example, 20% means 20 : 100. 20% of a number means $\frac{20}{100}$ or $\frac{1}{5}$ of that number.
Let's say there are 800 students in a school. If 20% of them are on a field trip, that means 160 students because 20 students are on the trip for every 100 students total.

perfect square An expression that can be written as something multiplied by itself.



These expressions are perfect squares: 3^2, 9, $(x + 3)^2$, and $x^2 + 6x + 9$.

perimeter The sum of the lengths of all the sides of a polygon.

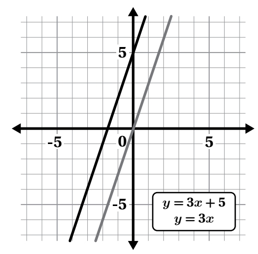
perpendicular Describes a line that crosses or meets another line at a 90° angle.

± (plus/minus symbol) A symbol used to represent both the positive and negative values of a number. It also can be used to represent two expressions.

± 9 represents -9 and +9.
2 ± 3 represents $2 - 3$ and $2 + 3$.

Español

líneas (o rectas) paralelas Líneas que nunca se cruzan o intersecan. En una gráfica, dos rectas con la misma pendiente e intersecciones diferentes con el eje y son paralelas.

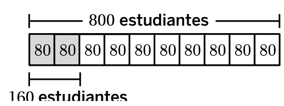


Por ejemplo, las rectas $y = 3x + 5$ y $y = 3x$ son paralelas y nunca se intersecan.

disminución porcentual Describe cuánto disminuye una cantidad y se expresa como un porcentaje de la cantidad inicial.

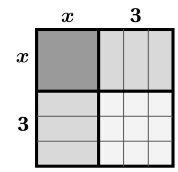
Por ejemplo, la función $f(x) = 1000(.78)^x$ representa el valor de un teléfono con un valor inicial de $1,000 y una disminución porcentual del 22% de su valor cada año, x.

por ciento (porcentaje) Por ciento significa por cada 100. Se representa con el símbolo de porcentaje: %. Usamos porcentajes para representar razones y fracciones.



Por ejemplo, 20% significa 20 : 100. 20% de un número significa $\frac{20}{100}$ o $\frac{1}{5}$ de dicho número.
Supongamos que hay 800 estudiantes en una escuela. Si el 20% de ellos están en una excursión, entonces eso es 160 estudiantes porque 20 están de viaje por cada 100 estudiantes en total.

cuadrado perfecto Una expresión que puede escribirse como algo multiplicado por sí mismo.



Estas expresiones son cuadrados perfectos: 3^2, 9, $(x + 3)^2$ y $x^2 + 6x + 9$.

perímetro La suma de las longitudes de todos los lados de un polígono.

perpendicular Describe una línea que cruza o se une con otra línea formando un ángulo de 90°.

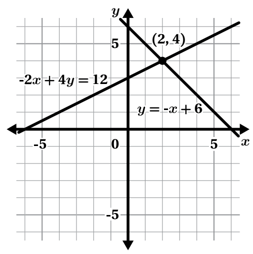
± (signo más menos) Un símbolo que se usa para representar los valores positivos y negativos de un número. También puede usarse para representar dos expresiones.

± 9 representa -9 and +9.
2 ± 3 representa $2 - 3$ y $2 + 3$.

English

point of intersection A point where two lines or curves meet.

For example, $(2, 4)$ is the point of intersection for the lines $-2x + 4y = 12$ and $y = -x + 6$.



polynomial The sum or difference of terms that include variables raised to non-negative integer powers, with coefficients that can be real or complex numbers.

$2x - 7$ and $3x^2 - x + 4$ are polynomial expressions.
$f(x) = 2x - 7$ and $g(x) = 3x^2 - x + 4$ are polynomial functions.

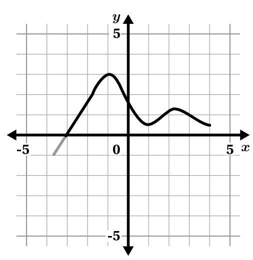
population A set of people or things that are being studied.

For example, if we want to study the heights of people on different sports teams, the population would be all the people on the teams.

population mean The average of all the values in a population. It is calculated by adding all the values and dividing by the total number of values.

positive (interval or function) A function is positive when its outputs are positive and its graph is above the x-axis. A function can be positive for its entire domain or over an interval.

This function is positive when $-3 < x < 4$.



principal The total amount (initial value) of money borrowed or invested, not including any interest.

proportional relationship A set of equivalent ratios. The values for one quantity are each multiplied by the same number to get the values for the other quantity.

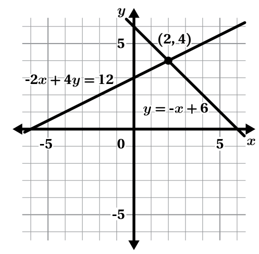
Carpet (sq. ft.)	Cost (dollars)
$10 \xrightarrow{\times 1.5}$	15.00
$20 \xrightarrow{\times 1.5}$	30.00
$50 \xrightarrow{\times 1.5}$	75.00

For example, every cost in the table is equal to 1.5 times the number of square feet of carpet.

Español

punto de intersección Un punto donde se cruzan dos rectas o curvas.

Por ejemplo, $(2, 4)$ es el punto de intersección de las rectas $y = -x + 6$ y $-2x + 4y = 12$.



polinomio La suma o la diferencia de términos que incluyen variables elevadas a potencias enteras no negativas, con coeficientes que pueden ser números reales o complejos.

$2x - 7$ y $3x^2 - x + 4$ son expresiones polinómicas.
$f(x) = 2x - 7$ y $g(x) = 3x^2 - x + 4$ son funciones polinómicas.

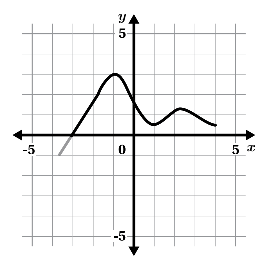
población Conjunto de personas o cosas que se estudian.

Por ejemplo, si queremos estudiar las alturas de las personas de diferentes equipos deportivos, la población sería todas las personas de los equipos.

media poblacional El promedio de todos los valores en una población. Se calcula sumando todos los valores y dividiendo por el número total de valores.

positivo (intervalo o función) Una función es positiva cuando sus salidas son positivas y su gráfica está por encima del eje x. Una función puede ser positiva en todo su dominio o en un intervalo.

Esta función es positiva cuando $-3 < x < 4$.



principal La cantidad total (valor inicial) de un préstamo o una inversión de dinero, sin incluir los intereses.

relación proporcional Un conjunto de razones equivalentes. Cada uno de los valores de una cantidad se multiplica por el mismo número para obtener los valores de la otra cantidad.

Alfombra (pies cuadrados)	Costo (dólares)
$10 \xrightarrow{\times 1.5}$	15.00
$20 \xrightarrow{\times 1.5}$	30.00
$50 \xrightarrow{\times 1.5}$	75.00

Por ejemplo, cada costo en la tabla es igual a 1.5 veces el número de pies cuadrados de alfombra.

Glossary/Glosario

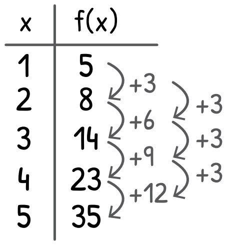
English

Q

quadratic formula A formula that can be used to determine the solutions of a quadratic equation $ax^2 + bx + c = 0$, where $a \neq 0$.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

quadratic function A function with output values that change by a constant second difference. Equations of quadratic functions have a squared term when written in standard form. The graph of a quadratic function is a parabola.

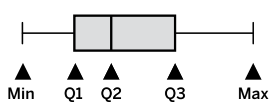


quadratic relationship See *quadratic function*.

quantitative data Data represented as numbers, quantities, or measurements that can be meaningfully compared is called *quantitative data*, or *numerical data*.

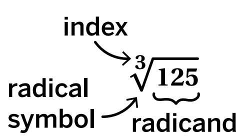
How many pets do you have? is a question that produces quantitative data. This is different from categorical data.

quartile Quartiles divide a data set into four sections. Quartile 1 is the median of the lower half of the data. Q2 is the median. Q3 is the median of the upper half of the data. Q4 is the maximum.



R

radical A square root, cube r oot, fourth root, etc. The radical symbol is $\sqrt{}$.



radicand The expression under a radical symbol. In the radical $\sqrt[n]{x}$ the quantity x is called the *radicand*. (See *radical*).

random Happening without a predictable pattern or order. In mathematics, a random event has outcomes that occur by chance.

Rolling a die is a random event because any number from 1 to 6 can appear with equal probability.

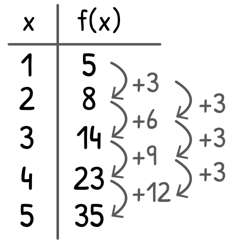
Español

Q

fórmula cuadrática Una fórmula que puede usarse para determinar las soluciones de una ecuación cuadrática $ax2 + bx + \mathrm{c} = 0$, donde $a \neq 0$.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

función cuadrática Una función con valores de salida que cambian de acuerdo con una segunda diferencia constante. Las ecuaciones de las funciones cuadráticas tienen un término elevado al cuadrado cuando se escriben en forma estándar. La gráfica de una función cuadrática es una parábola.

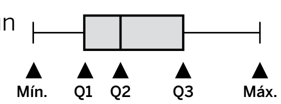


relación cuadrática Ver *función cuadrática*.

datos cuantitativos Los datos que se representan como números, cantidades o medidas y que pueden compararse de forma significativa se denominan datos cuantitativos o datos numéricos.

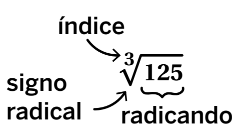
¿Cuántas mascotas tienes? es una pregunta que produce datos cuantitativos, que son diferentes de los datos categóricos.

cuartil Los cuartiles dividen un conjunto de datos en cuatro secciones. El cuartil 1 es la mediana de la mitad inferior de los datos. Q2 es la mediana. Q3 es la mediana de la mitad superior de los datos. Q4 es el máximo.



R

radical Una raíz cuadrada, r aíz cúbica, raíz cuarta, etc. El signo radical es $\sqrt{}$.



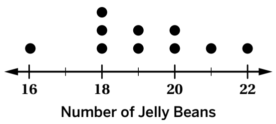
radicando La expresión bajo un signo radical. En un radical $\sqrt[n]{x}$, la cantidad x se llama radicando. (Ver *radical*).

aleatorio Algo que ocurre sin un patrón o un orden predecibles. En matemáticas, un suceso aleatorio tiene resultados que ocurren por casualidad.

Lanzar un dado es un suceso aleatorio porque cualquier número del 1 al 6 puede aparecer con la misma probabilidad.

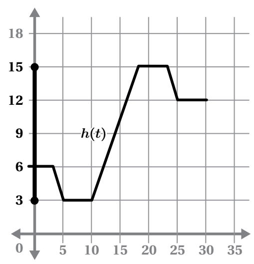
English

range The difference between the maximum and minimum values in a data set. Range is a measure of spread.



For example, the range of this data set is 6 jelly beans because $22 - 16 = 6$.

range (of a function) The set of all possible output values for a function or relation. The range can be described in words or as an inequality.



The range of this graph can be described as: All numbers from 3 to 15 or $3 \leq h(t) \leq 15$.

rate of change The ratio of the change in one quantity to the corresponding change in another quantity. In a linear relationship, the rate of change is the change in y divided by the change in x, which is also the slope of the graph.

rational exponent A real number that can be expressed as the ratio of two integers, where the denominator is not zero. Rational numbers can also appear as fractional exponents in mathematical expressions.

In the expression $5^{\frac{1}{2}}$, $\frac{1}{2}$ is a rational exponent.

rational number A number that can be written as a fraction of two integers, where the denominator is not zero.

$\frac{1}{3}$, $-\frac{7}{4}$, 0, 0.2, -5, and $\sqrt{9}$ are rational numbers.

reciprocal The reciprocal of a fraction $\frac{a}{b}$ is $\frac{b}{a}$. The product of two fractions that are reciprocals of one another is 1.

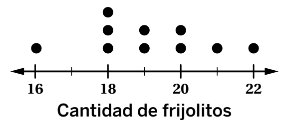
For example, $\frac{3}{2}$ and $\frac{2}{3}$ are reciprocals because $\left(\frac{3}{2}\right)\left(\frac{2}{3}\right) = 1$.

relation A way of creating input-output pairs. When a relation assigns exactly one output to every input, it is called a *function*.

relative frequency table A type of two-way table used to compare data across two categorical variables. Relative frequency tables present the fraction or percent of the data that is in that category, instead of the actual number of data points. You can use this representation to see if the data presents evidence that there is an association between the two variables.

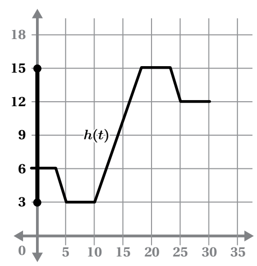
Español

rango La diferencia entre los valores máximo y mínimo de un conjunto de datos. El rango es una medida de dispersión.



Por ejemplo, el rango de este conjunto de datos es 6 frijolitos de jalea porque $22 - 16 = 6$.

fluctuación (de una función) El conjunto de todos los posibles valores de salida de una función o relación. La fluctuación puede describirse con palabras o como una desigualdad.



La fluctuación de esta gráfica puede describirse de la siguiente manera: Todos los números del 3 al 15 o $3 \leq h(t) \leq 15$.

tasa de cambio La razón entre el cambio en una cantidad y el cambio correspondiente en otra cantidad. En una relación lineal, la tasa de cambio es el cambio en (x) dividido por el cambio en x, que también es la pendiente del gráfico.

exponente racional Un número real que puede expresarse como la razón de dos números enteros, donde el denominador no es cero. Los números racionales también pueden aparecer como exponentes fraccionarios en expresiones matemáticas.

En la expresión $5^{\frac{1}{2}}$, $\frac{1}{2}$ es un exponente racional.

número racional Un número que se puede escribir como una fracción de dos números enteros, donde el denominador es diferente de cero.

$\frac{1}{3}$, $-\frac{7}{4}$, 0, 0.2, -5 y $\sqrt{9}$ son números racionales.

recíproco El recíproco de una fracción $\frac{a}{b}$ es $\frac{b}{a}$. El producto de dos fracciones que son recíprocas entre sí es 1.

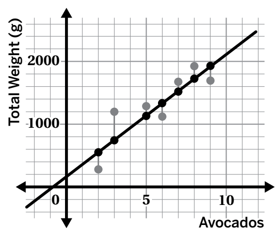
Por ejemplo, $\frac{3}{2}$ y $\frac{2}{3}$ son recíprocos porque $\left(\frac{3}{2}\right)\left(\frac{2}{3}\right) = 1$.

relación Una forma de establecer pares de entrada y salida. Cuando una relación asigna exactamente una salida a cada entrada, se denomina función.

tabla de frecuencia relativa Un tipo de tabla de doble entrada que se usa para comparar datos entre dos variables categóricas. Las tablas de frecuencia relativa presentan la fracción o el porcentaje de los datos que están en esa categoría, en lugar de la cantidad de puntos de datos. Esta representación puede emplearse para saber si los datos muestran que existe una asociación entre las dos variables.

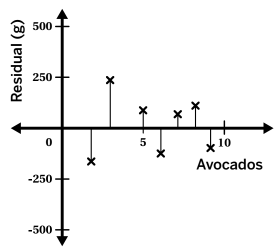
English

residual The difference between the y-value for a point in a scatter plot and the value predicted by the line of best fit.



The short lines connecting each point to the line of fit show the residual value for that point.

residual plot A scatter plot of residual values for a data set. The x-axis represents the value predicted by the line of best fit, and the y-value of each point represents the value of the residual.



The residual plot shows how the total weight of different numbers of avocados vary from their predicted values.

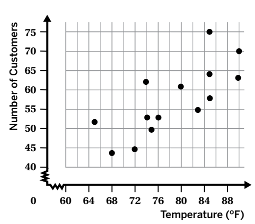
revenue The amount of money generated by selling a product or service.

S

sample A part of a population.

For example, a population could be all the seventh-grade students at one school. One sample of that population is all the seventh-grade students who are in band.

scatter plot A graph in the coordinate plane that shows the relationship between two variables by plotting individual data points. Scatter plots are used to observe patterns in bivariate numerical data.



second difference The differences between consecutive output values in the table of a function are called *first differences*. The differences between those values are called *second differences*. Quadratic functions have constant second differences.

x	f(x)
1	5
2	8
3	14
4	23
5	35

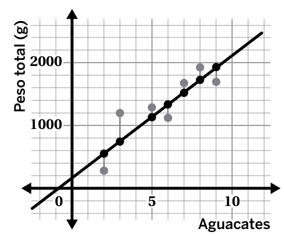
+3 +6 +9 +12

+3 +3 +3

In this example, the first differences are 3, 6, 9, and 12. The second differences are constant at 3, so $f(x)$ is a quadratic function.

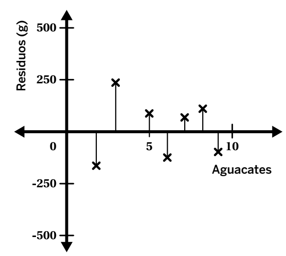
Español

residuo La diferencia entre el valor y de un punto en un diagrama de dispersión y el valor que predice la línea de mejor ajuste.



Las líneas cortas que conectan cada punto con la línea de ajuste muestran el valor del residuo del punto en cuestión.

diagrama de residuos Un diagrama de dispersión de los valores de los residuos de un conjunto de datos. El eje x representa el valor que predice la línea de mejor ajuste, y el valor y de cada punto representa el valor del residuo.



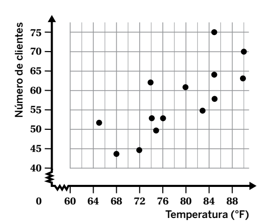
El diagrama de residuos muestra que el peso total de diferentes cantidades de aguacates varía con respecto a sus valores esperados.

ingresos La cantidad de dinero que genera la venta de un producto o servicio.

muestra Una parte de una población.

Por ejemplo, una población podría ser todos los estudiantes de séptimo grado de una escuela. Una muestra de esa población son todos los estudiantes de séptimo grado que están en la banda.

diagrama de dispersión Un gráfico en el plano de coordenadas que muestra la relación entre dos variables mediante el trazado de puntos de datos individuales. Los diagramas de dispersión se utilizan para observar patrones en datos numéricos bivariados.



segunda diferencia Las diferencias entre valores de salida consecutivos en la tabla de una función se llaman primeras diferencias. Las diferencias entre dichos valores se llaman segundas diferencias. Las funciones cuadráticas tienen segundas diferencias constantes.

x	f(x)
1	5
2	8
3	14
4	23
5	35

+3 +6 +9 +12

+3 +3 +3

En este ejemplo, las primeras diferencias son 3, 6, 9 y 12. Las segundas diferencias son constantes, de 3, por lo que $f(x)$ es una función cuadrática.

English

segmented bar graph A bar graph where each bar is divided into segments that represent different categories, showing how the total is split into parts.

set-builder notation A shorthand used to describe sets, often those with an infinite number of elements. It is written in the form $\{x : x > 5\}$, which is read as "the set of all x such that x is greater than 5." The colon (:) can also be replaced by a vertical line (|), as in $\{x \mid x > 5\}$, and is read the same way.

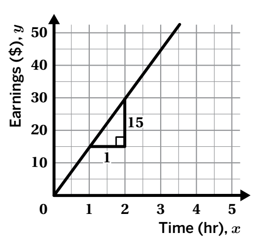
The set of all numbers greater than 8 can be written in set-builder notation as: $\{x \mid x>8\}$
This is read as "the set of all x such that x is greater than 8."

simple interest Interest that is calculated solely based on the initial amount. The balance in an account with simple interest is modeled by a linear function.

simulation An approximate imitation of a statistical experiment, often performed using a computer program to analyze the results of a large number of trials.

A simulation can be used to predict the likelihood of getting heads or tails by flipping a virtual coin many times.

slope (rate of change) The ratio of the change in the vertical direction (y-axis) to the change in the horizontal direction (x-axis).



In this graph, y increases by 15 dollars when x increases by 1 hour. The slope of the line is 15, and the rate of change is 15 dollars per hour.

slope-intercept form A way to write a linear equation that highlights the slope and the y-intercept of the line it represents. Slope-intercept form equations are written as $y = mx + b$, where m represents the slope, b represents the y-intercept of the line, and x and y are variables.

The equations $y = 2x + 4$ and $y = -5x - 10$ are in slope-intercept form. The equation $2x + 5y = 20$ is not in slope-intercept form.

Español

gráfico de barras segmentadas Un gráfico de barras donde cada barra está dividida en segmentos que representan diferentes categorías y muestran cómo el total se divide en partes.

notación de construcción de conjuntos Una abreviatura utilizada para describir conjuntos, a menudo aquellos con un número infinito de elementos. Se escribe en la forma $\{x : x > 5\}$, que se lee como "el conjunto de todas las x, de modo que x es mayor que 5". Los dos puntos (:) también se pueden reemplazar por una línea vertical (|), como en $\{x \mid x > 5\}$, y se leen de la misma manera.

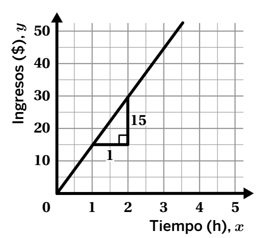
El conjunto de todos los números mayores que 8 se puede escribir en notación de construcción de conjuntos como: $\{x \mid x > 8\}$
Esto se lee como "el conjunto de todas las x, de modo que x es mayor que 8".

interés simple Interés que se calcula únicamente en función de la cantidad inicial. El saldo de una cuenta con interés simple se modela con una función lineal.

simulación Una imitación aproximada de un experimento estadístico, a menudo realizada mediante un programa informático para analizar los resultados de una gran cantidad de ensayos.

Se puede utilizar una simulación para predecir la probabilidad de obtener cara o cruz lanzando una moneda virtual muchas veces.

pendiente (tasa de cambio) La razón entre el cambio en la dirección vertical (eje y) y el cambio en la dirección horizontal (eje x).



En esta gráfica, y incrementa en 15 dólares cuando x incrementa en 1 hora. La pendiente de la recta es 15 y la tasa de cambio es de 15 dólares por hora.

forma pendiente-intersección, forma pendiente-ordenada al origen Una forma de escribir una ecuación lineal que destaca la pendiente y la intersección con el eje y (o la ordena al origen) de la recta que representa. Las ecuaciones en forma pendiente-intersección se escriben $y = mx + b$, donde m representa la pendiente, b representa la intersección con el eje y de la recta, y tanto x como y son variables.

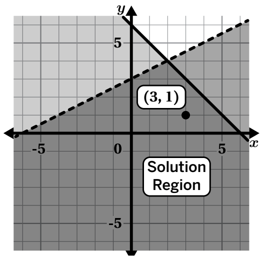
Las ecuaciones $y = 2x + 4$ y $y = -5x - 10$ están en la forma pendiente-intersección. La ecuación $2x + 5y = 20$ no está en la forma pendiente-intersección.

English

solution A value or set of values that makes an equation or inequality true.

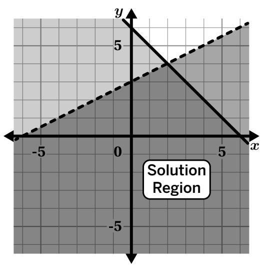
For example, $x = 2$ is a solution to the equation $3x + 4 = 10$.
$x > 2$ are the solutions to the inequality $3x + 4 > 10$.
The ordered pair $(1, 2)$ is a solution to the equation $3x + 4y = 11$.

solution (to a system of inequalities) An ordered pair that makes each inequality in a system true. Every ordered pair that is a solution to a system is located in the solution region where the graphs overlap.

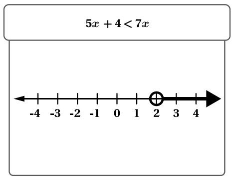


For system $y \leq -x + 6$ and $-2x + 4y < 12$, $(3,1)$ is a solution to this system of inequalities because it makes both inequalities true and falls in the region where the inequalities overlap.

solution region The set of all ordered pairs that make an inequality or each inequality in a system true. For a two-variable linear inequality, the solution region is a half-plane. For a system of inequalities, the solution region is located where the graphs overlap.



solution set The set of all values that makes an inequality true. To describe a solution set symbolically, we often use inequalities. To describe a solution set graphically, we often shade a portion of a number line or a region of the coordinate plane.



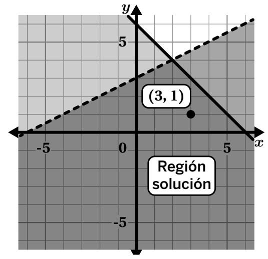
The solution set for the inequality $5x + 4 < 7x$ includes all of the values that are larger than 2.
2 is not included in the solution set.

Español

solución Un valor o conjunto de valores que hacen que una ecuación o una desigualdad sean verdaderas.

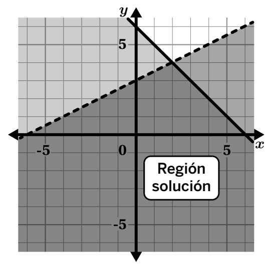
Por ejemplo, $x = 2$ es una solución de la ecuación $3x + 4 = 10$.
$x > 2$ son las soluciónes de la desigualdad $3x + 4 > 10$.
El par ordenado $(1, 2)$ es una solución de la ecuación $3x + 4y = 11$.

solución (de un sistema de desigualdades) Un par ordenado que hace que cada desigualdad de un sistema sea verdadera. Cada par ordenado que sea una solución de un sistema se encuentra en la región solución donde se superponen las gráficas.

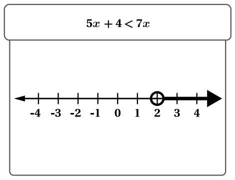


Por ejemplo, en el sistema de desigualdades $y \leq -x + 6$ y $-2x + 4y < 12$, la solución es $(3,1)$ porque hace que ambas desigualdades sean verdaderas y se ubica en la región donde las desigualdades se suporponen.

región solución El conjunto de todos los pares ordenados que hacen que una desigualdad, o cada desigualdad de un sistema, sean verdaderas. En una desigualdad lineal de dos variables, la región solución es un semiplano. En un sistema de desigualdades, la región solución se encuentra donde se superponen las gráficas.



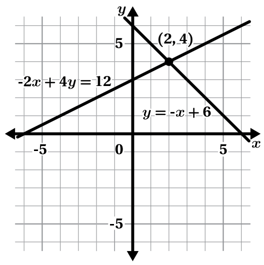
conjunto de soluciones El conjunto de todos los valores que hacen verdadera una desigualdad. Para describir con signos un conjunto de soluciones suelen emplearse desigualdades. Para describir con gráficas un conjunto de soluciones suele colorearse o sombrearse una parte de una recta numérica o una región del plano de coordenadas.



El conjunto de soluciones de la desigualdad $5x + 4 \geq 7x$ incluye todos los valores mayores que 2. El 2 no está incluido en el conjunto de soluciones.

English

solution to a system of equations
A solution to a system of equations is a set of values that makes all equations in that system true. When the equations are graphed, the solution to the system is the point of intersection.



For the system $y = -x + 6$ and $-2x + 4y = 12$, $(2, 4)$ is the solution to this system of equations and the point of intersection on the graph.

square root The square root of a number n (written as $\sqrt{n}$) is the positive number that can be squared to get n. The square root is also the side length of a square with an area of n.

The square root of 16 ($\sqrt{16}$) is 4 because 4^2 is 16. The $\sqrt{16}$ is also the side length of a square that has an area of 16.

standard form (of a linear equation) Linear equations that are written in the form $ax + by = c$, where a, b, and c are constants and x and y are variables.

The equations $2x + 5y = 20$ and $3x - 4y = -10$ are in standard form.
The equation $y = 2x + 4$ is not in standard form.

standard form (of a quadratic equation) One of three common forms of a quadratic equation. A quadratic equation in standard form looks like $f(x) = ax^2 + bx + c$.

These equations are in standard form:
$y = 2x^2 + 5x + 3$
$h(x) = x^2 + 3x$
$4x^2 - 7 = f(x)$

substitute Replace a variable or expression with a value or another expression.

$$4x = 4(5)$$

In this example, 5 is substituted for x in the expression $4x$.

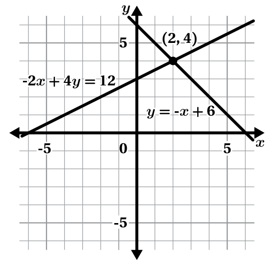
substitution A method of solving systems of equations where a variable is replaced with an equivalent expression in order to produce a new equation with fewer variables.

$y = \boxed{-4x + 6}$ $\quad y = 3x - 15$
$-4x + 6 = 3x - 15$
$-7x = -21$
$\boxed{x = 3}$
$y = 3(3) - 15$
$\boxed{y = -6}$

For example, we can substitute $-4x + 6$ in for y in $y = 3x - 15$ because they are equivalent.

Español

solución de un sistema de ecuaciones Una solución de un sistema de ecuaciones es un conjunto de valores que hace que todas las ecuaciones de ese sistema sean verdaderas. Al graficar las ecuaciones, la solución del sistema es el punto de intersección.



Por ejemplo, en el sistema de ecuaciones $y = -x + 6$ y $-2x + 4y = 12$, la solución y el punto de intersección de la gráfica es $(2, 4)$.

raíz cuadrada La raíz cuadrada de un número n (se escribe $\sqrt{n}$) es el número positivo que puede elevarse al cuadrado para obtener n. La raíz cuadrada también es la longitud de lado de un cuadrado con un área de n.

La raíz cuadrada de 16 ($\sqrt{16}$) es 4 porque 4^2 es 16. La $\sqrt{16}$ también es la longitud de lado de un cuadrado que tiene un área de 16.

forma estándar (de una ecuación lineal) Las ecuaciones lineales que se escriben en la forma $ax + by = c$, donde a, b y c son constantes, y tanto x como y son variables, se conocen como ecuaciones en forma estándar.

Las ecuaciones $2x + 5y = 20$ y $3x - 4y = -10$ están en forma estándar.
La ecuación $y = 2x + 4$ no está en forma estándar.

forma estándar (de una ecuación cuadrática) Una de las tres formas comunes de una ecuación cuadrática. Una ecuación cuadrática en forma estándar tiene el siguiente orden: $f(x) = ax^2 + bx + c$.

Estas ecuaciones están en forma estándar:
$y = 2x^2 + 5x + 3$
$h(x) = x^2 + 3x$
$4x^2 - 7 = f(x)$

sustituir Reemplazar una variable o expresión por un valor u otra expresión.

$$4x = 4(5)$$

En este ejemplo, el 5 sustituye a la x en la expresión $4x$.

sustitución Un método para resolver sistemas de ecuaciones donde una variable se reemplaza con una expresión equivalente para producir una nueva ecuación con menos variables.

$y = \boxed{-4x + 6}$ $\quad y = 3x - 15$
$-4x + 6 = 3x - 15$
$-7x = -21$
$\boxed{x = 3}$
$y = 3(3) - 15$
$\boxed{y = -6}$

Por ejemplo, podemos introducir $-4x + 6$ en lugar de y en $y = 3x - 15$ porque son equivalentes.

Glossary/Glosario

English

survey A method of collecting data by asking people to answer a given set of questions.

system of equations Two or more equations that represent the constraints on a shared set of variables form a system of equations.

These equations make a system:
$3b + c = -2$
$b - 5c = 12$

system of inequalities Two or more inequalities that represent the constraints on a shared set of variables form a system of inequalities.

These inequalities make a system:
$10m + 5n > -2$
$m - 5n \leq 12$

T

total relative frequency table A type of two-way table used to compare data across two categorical variables. Total relative frequency tables present the fraction or percentage of the data that is in that category, instead of the number of data points. You can use this representation to see if the data presents evidence that there is an association between the two variables.

	Meditated	Did Not Meditate	Total
Calm	46%	8%	54%
Agitated	24%	22%	46%
Total	70%	30%	100%

translation A transformation that moves every point in a function a given distance in a given direction. A translation changes the location of a function, but does not change its shape.

two-way table A way to compare two categorical variables. It shows one of the variables across the top and the other down one side. Each entry in the table is the frequency or relative frequency of the category shown by the column and row headings.

	Meditated	Did Not Meditate	Total
Calm	45	8	53
Agitated	23	21	44
Total	68	29	97

Español

encuesta Un método de recopilación de datos que consiste en pedir a personas que respondan una serie de preguntas.

sistema de ecuaciones Dos o más ecuaciones que representan las restricciones de un conjunto compartido de variables forman un sistema de ecuaciones.

Estas ecuaciones forman un sistema:
$3b + c = -2$
$b - 5c = 12$

sistema de desigualdades Dos o más desigualdades que representan las restricciones de un conjunto compartido de variables forman un sistema de desigualdades.

Estas desigualdades forman un sistema:
$10m + 5n > -2$
$m - 5n \leq 12$

tabla de frecuencia relativa total Un tipo de tabla de doble entrada que se usa para comparar datos entre dos variables categóricas. Las tablas de frecuencia relativa total presentan la fracción o el porcentaje de los datos que se encuentran en esa categoría, en lugar de la cantidad de puntos de datos. Esta representación puede emplearse para saber si los datos muestran que existe una asociación entre las dos variables.

	Meditaron	No meditaron	Total
Calmados	46%	8%	54%
Agitados	24%	22%	46%
Total	70%	30%	100%

traslación Una transformación que mueve cada punto de una función una determinada distancia en una determinada dirección. Una traslación cambia la ubicación de una función, pero no su forma.

tabla de doble entrada Una manera de comparar dos variables categóricas. Muestra una de las variables en la fila superior y la otra variable en la columna de un costado. Cada entrada de la tabla es la frecuencia o frecuencia relativa de la categoría que describen los títulos de la columna y la fila.

	Meditaron	No meditaron	Total
Calmados	45	8	53
Agitados	23	21	44
Total	68	29	97

English	Español

U

univariate data A data set that involves one variable. Each data point contains one piece of information.

A collection of students' heights is a univariate data set. Tables, dot plots, and bar graphs are useful for displaying univariate data.

datos univariados Un conjunto de datos que incluye una variable. Cada punto de datos contiene una información.

Una colección de las estaturas de estudiantes es un conjunto de datos univariado.
Las tablas, los diagramas de puntos y los diagramas de barras son útiles para mostrar datos univariados.

V

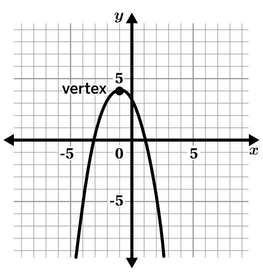
variable A letter or symbol that represents a value or set of values.

In the expression $10 - x$, the variable is x.

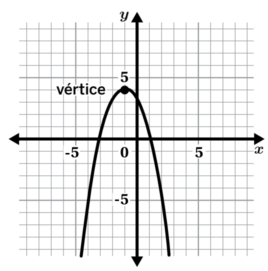
variable Una letra o un símbolo que representa un valor o un conjunto de valores.

En la expresión $10 - x$, la variable es x.

vertex On the graph of a quadratic or absolute value function, the vertex is the maximum or minimum point. The vertex is also where the function changes from increasing to decreasing, or vice versa.



vértice En la gráfica de una función cuadrática o una función de valor absoluto, el vértice es el punto máximo o mínimo. El vértice también es donde la función cambia de creciente a decreciente, o viceversa.



vertex form One of three common forms of a quadratic equation. A quadratic equation in vertex form looks like $f(x) = a(x - h)^2 + k$.

These equations are in vertex form:
$(x - 3)^2 + 10 = g(x)$
$y = 2(x + 8)^2 - 1$
$f(x) = -(x - 6)^2 + 15$

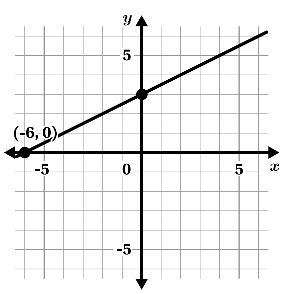
forma de vértice Una de las tres formas comunes de una ecuación cuadrática. Una ecuación cuadrática en forma de vértice tiene el siguiente orden: $f(x) = a(x - h)2 + k$.

Estas ecuaciones están en forma de vértice:
$(x - 3)^2 + 10 = g(x)$
$y = 2(x + 8)^2 - 1$
$f(x) = -(x - 6)^2 + 15$

X

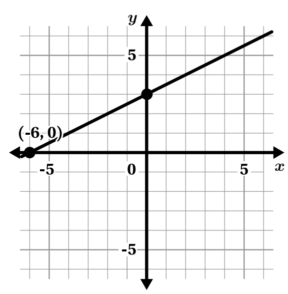
x-intercept A point where the graph of an equation or function crosses the x-axis or when $y = 0$.

The x-intercept of the graph of $-2x + 4y = 12$ is $(-6, 0)$, or just -6.



intersección con el eje x, abscisa al origen Un punto donde la gráfica de una ecuación o función cruza el eje x, o cuando $y = 0$.

La intersección con el eje x de la gráfica de $-2x + 4y = 12$ es $(-6, 0)$, o simplemente -6.



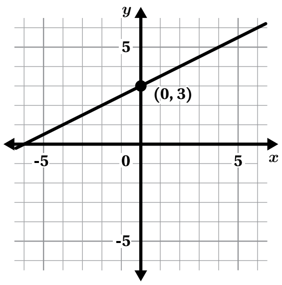
Glossary/Glosario

English

Y

y-intercept A point where the graph of an equation or function crosses the y-axis or when $x = 0$.

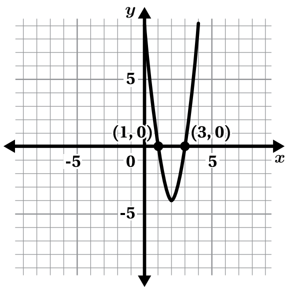
The y-intercept of the graph $-2x + 4y = 12$ is $(0, 3)$, or just 3.



Z

zero-product property A property which states that if the product of two or more factors is 0, then at least one of the factors is 0. This property can be used to help solve equations.

If $(2x - 3)(x + 1) = 0$, then either $2x - 3 = 0$ or $x + 1 = 0$.



zeros The x-values that make a function equal zero, or $f(x) = 0$.

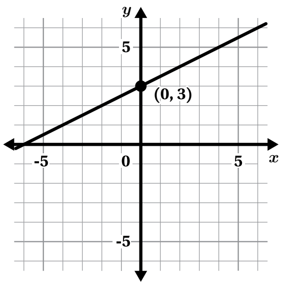
The zeros of $f(x) = 4(x - 1)(x - 3)$ are 1 and 3.

Español

Y

intersección con el eje y, ordenada al origen Un punto donde la gráfica de una ecuación o función cruza el eje y, o cuando $x = 0$.

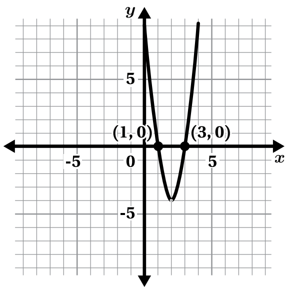
La intersección con el eje y de la gráfica de $-2x + 4y = 12$ es $(0, 3)$, o simplemente 3.



Z

propiedad del producto cero Una propiedad que establece que si el producto de dos o más factores es 0, entonces al menos uno de los factores es 0. Esta propiedad puede usarse como ayuda para resolver ecuaciones.

Si $(2x - 3)(x + 1) = 0$, entonces $2x - 3 = 0$ o $x + 1 = 0$.



ceros Los valores x que hacen que una función sea igual a cero, o $f(x) = 0$.

Los ceros de $f(x) = 4(x - 1)(x - 3)$ son 1 y 3.

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- **Unit 2: Describing Data**
- **Unit 3: Describing Functions**
- **Unit 4: Systems of Linear Equations and Inequalities**

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